Optimization of Retailers' Strategies in Price- and Carbon Emission- Sensitive Market

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Optimisation des Stratégies des Détaillants sur Un Marché Sensible Au Prix et Aux Émissions de Carbone

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I dedicate this thesis to my family
for their constant support and unconditional love
I love you all dearly.

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Acronyms and Abbreviation

Abbreviations	Explanation
ANR	Agence Nationale de la Recherche
CEA	Customers Environmental Awareness
GC	Green Credit
GDS	Greenness-Driven Switchovers
MILP	Mixed Integer Linear Programming
MS	Manufacturer Subsidy
MTO	Make-To-Order
MTS	Make-To-Stock
PDS	Price-Driven Switchovers
SC	Supply Chain
SS	Sales Subsidy

Résumé en Français

Chapitre 1: Introduction

La performance environnementale d'un produit mis sur le marché, souvent évaluée en fonction de la quantité d'émissions de carbone libérées pendant les phases de production et de transport, devient un critère d'achat important pour de nombreux clients (Palacios-Argüello et al., 2020; Hammami et al., 2018; Borin et al., 2013). Le rapport (AACE, 2015) sur l'analyse des émissions de carbone du produit révèle qu'environ 70% des émissions totales de carbone produites au cours du cycle de vie du produit sont liées au processus de production. Selon une enquête menée par Accenture en avril 2019 auprès de 6 000 consommateurs dans 11 pays d'Amérique du Nord, d'Europe et d'Asie, 72% des consommateurs ont déclaré qu'ils achetaient actuellement plus de produits respectueux de l'environnement qu'il y a cinq ans, et 81% qu'ils prévoyaient d'en acheter davantage encore au cours des cinq prochaines années. En outre, plus de la moitié des consommateurs paieraient plus cher pour des produits respectueux de l'environnement (Accenture, 2019). Les enquêtes de Carbon Trust indiquent qu'environ 20% des clients préfèrent acheter des produits verts même s'ils sont plus chers que les produits ordinaires (Hong and Guo, 2019).

La plupart des études dans le domaine de la gestion de la chaîne d'approvisionnement partent du principe que la demande du client est connue (demande exogène). Cependant, nous savons que la demande est fortement influencée par les décisions internes (demande endogène). Le facteur le plus connu qui affecte le comportement d'achat du client est le prix. Mais il existe différents types de facteurs qui influent la demande d'un produit ou d'un service. Huang et al. (2013) ont fourni une étude intéressante sur les facteurs influençant la demande. Ils les ont répartis en six catégories : prix, rabais, délai, espace, qualité et publicité. Il est bien connu dans la littérature sur la logistique commerciale que l'un des éléments les plus importants du service à la clientèle, outre le prix, est le délai de livraison. Pour comprendre la logique, les idées et la méthodologie de résolution de modèles avec des demandes endogènes, nous avons commencé notre travail en considérant une demande endogène sensible au prix et au délai de livraison. Nous considérons un système hybride de vente de produits substituables, qui consiste en un système de fabrication à la commande (MTO) et un système de fabrication en stock (MTS) (voir l'annexe Appendix 1 pour plus de détails). Cependant, ce doctorat se concentre sur une demande endogène sensible au prix et à la qualité environnementale du produit, que l'on appellera greeness dans cette thèse.

La conscience environnementale modifie les décisions des consommateurs en matière d'achat de biens. Ils sont prêts à payer plus cher pour avoir des produits plus écologiques (Brécard, 2014). Par conséquent, la plus ou moins grande sensibilité des clients à l'environnement (CEA pour Customers' Environmental Awareness) va influencer directement la stratégie de l'entreprise à produire des produits plus ou moins verts et donc à investir dans des technologies plus ou moins propres (Yalabik and Fairchild, 2011). Ce mouvement conduit aussi les fournisseurs à fournir des produits plus écologiques (Gu et al., 2015; Sheu and Chen, 2012; Ding et al., 2015; Cheng et al., 2017). Le consommateur est prêt à payer un prix plus élevé pour un produit vert, comme des voitures émettant moins de CO2 (Costa et al., 2019), mais le produit vert nécessite plus d'investissements dans la recherche et le développement (R&D), ce qui générera des coûts plus élevés (Krass et al., 2013).

De nombreux détaillants ont bien saisi les préférences des clients pour des produits respectueux de l'environnement et leur acceptation à les payer plus chers et, par conséquent, ont adapté leurs stratégies d'achat pour offrir des alternatives plus écologiques. Ainsi, comme le soulignent Ramanathan et al. (2012), ces détaillants ont demandé à leurs fournisseurs de réduire les émissions de carbone dans les phases de production et de transport. Benjaafar et al. (2012) ont souligné que divers détaillants commencent à mettre des étiquettes d'empreinte carbone sur leurs produits. Deux grands détaillants, Tesco au Royaume-Uni et Casino en France, se sont déjà lancés dans des efforts d'étiquetage de ce type. De nombreux produits font l'objet d'un bilan carbone. Citons par exemple les smoothies aux fruits, les chaussures et la bière... Walmart, le plus grand détaillant au monde, a établi en octobre 2016 un plan de réduction des émissions et a invité ses fournisseurs à s'engager à réduire les émissions de gaz à effet de serre résultant de leurs activités et de leurs chaînes de valeur. L'objectif est de travailler avec les fournisseurs pour réduire les émissions de carbone de 1 gigatonne dans la production et l'utilisation des aliments et des produits à l'échelle mondiale entre 2015 et 2030. Bestseller, l'un des principaux détaillants de mode au Danemark, donne à ses clients la possibilité d'acheter une large gamme de produits respectueux de l'environnement. Le détaillant s'est engagé à améliorer en permanence l'empreinte écologique de ses produits, de ses activités et de sa chaîne d'approvisionnement. Ainsi, pour gérer la demande et augmenter la rentabilité, les détaillants vont se faire concurrence non seulement sur le prix mais aussi sur la performance environnementale (la greenness) de leurs produits (qui est ici évaluée en termes d'émissions de carbone).

Notre revue de la littérature, présentée dans le chapitre 2, a permis d'identifier de nouvelles perspectives pour le problème de l'optimisation des opérations d'un détaillant dans un contexte de demande sensible au prix et à la greeness. En particulier,

1. La plupart des études sont réalisées sur la base du système MTO (Make-To-Order). Elles ne sont donc pas adaptées au contexte de la vente au détail et de la gestion des stocks dans ce domaine. En effet, dans de nombreux cas, les détaillants passent commande à leurs fournisseurs et constituent un stock pour servir les clients immédiatement.

2. Dans un environnement compétitif, la plupart des articles ont considéré le jeu de Stackelberg dans lequel un joueur agit (prend des décisions) en premier, appelé leader, et le second, appelé suiveur, prend les décisions sur la base des décisions prises par le leader. Peu d'études ont considéré qu'il existe une compétition dynamique entre les joueurs qui peuvent prendre des décisions en même temps, et

donc des décisions mutuellement influencées.

3. Dans la littérature, les chercheurs considèrent la demande linéaire sans aucune contrainte, bien que nos collègues (Rennes School of Business et Ecole des Mines de St Etienne dans le projet ANR) aient constaté que la demande n'est pas linéaire et de plus ne n'augmentera pas au-delà d'un certain niveau de demande avec comme seul levier l'amélioration de la greeness.

Sur la base des observations ci-dessus, et après une revue de la littérature dans le chapitre 2, nous avons étudié les problèmes suivants, résumés Figure 1.

- Nous développons tout d'abord dans le chapitre 3 un modèle de base : un seul détaillant offre un seul produit aux clients. La demande du produit est sensible au prix du produit et à son intensité carbone, dans un environnement stochastique pour la demande et la durée de livraison du fournisseur. Le détaillant conserve les produits en stock dans un entrepôt proche des clients pour les servir, et passe des commandes pour remplir le stock de produits auprès du fournisseur qui adopte une politique de fabrication MTS. Le stock est géré selon la politique standard (q,S) où q est le point de commande et S est la taille du lot de réapprovisionnement. Ce problème est considéré comme le modèle de référence pour les chapitres suivants.

- Différenciation des produits : un détaillant offre deux produits substituables à ses clients. L'objectif est de trouver les stratégies optimales, en termes de prix et de greeness, du détaillant, et ceci dans un environnement stochastique. La fonction de demande de chaque produit dépend non seulement de son prix et de son intensité carbone, mais aussi du prix et de l'intensité carbone de l'autre produit. Le problème est formulé dans un environnement stochastique. L'analyse est fournie dans le chapitre 4.

- Un jeu dynamique entre deux détaillants en concurrence est considéré dans un environnement sensible au prix et à la greeness, à nouveau dans un environnement stochastique, dans le chapitre 5. Nous considérons la situation dans laquelle un détaillant agit sur le marché et où un nouveau venu arrive. Le nouveau venu offre un produit substituable. La part de marché potentielle n'est pas nécessairement égale pour les deux détaillants.

- Le modèle de référence est considéré avec différentes fonctions de demande (plus complexes) dans le chapitre 6. Nous obtenons les résultats avec les différentes fonctions de demande, puis les comparons pour analyser si la fonction de demande linéaire classique est satisfaisante ou non. Le chapitre 6 fournit plus de détails et de discussion.

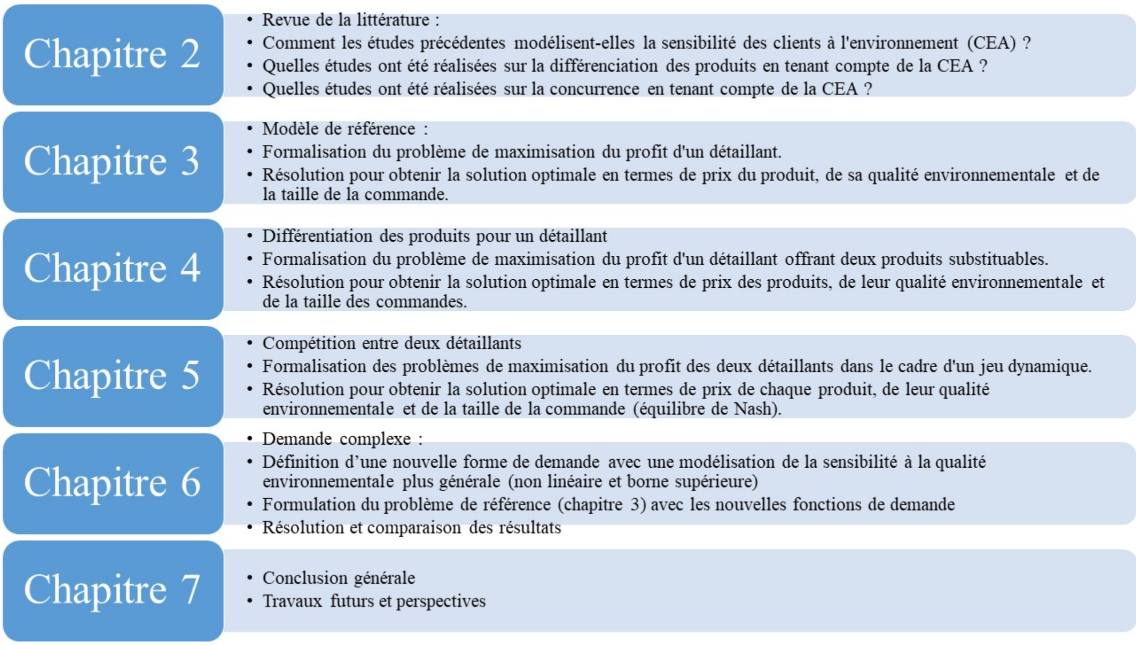

Figure 1: Cadre de la thèse

Chapitre 2: Revue de la Littérature

Notre étude est dans le champ des modèles analytiques de gestion des opérations dans une Supply Chain, avec des considérations environnementales et, en particulier, des études qui se concentrent sur la demande sensible au prix et à la greeness des produits. Nous avons distingué trois thèmes de la littérature sur la gestion d'une chaîne logistique verte. Dans un premier temps nous avons présenté les études qui considèrent la sensibilité des clients à la qualité environnementale (CEA pour Customers' Environmental Awareness), et donc le niveau des émissions comme variable de décision. Puis dans ce même contexte de marché dépendant du prix et de la qualité environnementale, nous avons présenté les études portant sur la différenciation des produits pour un acteur donné, et enfin les études concernant les stratégies dans un environnement concurrentiel entre acteurs.

Chapitre 3: Modèle de Référence

Ce chapitre étudie le problème de la maximisation des profits d'un détaillant comme modèle initial et de référence pour le reste de la thèse. Nous comparerons en effet les résultats obtenus dans les autres chapitres aux résultats de ce chapitre. Ce premier cas nous permet également de présenter les principales idées utilisées pour résoudre analytiquement les problèmes futurs plus complexes. Nous considérons un détaillant qui vend un produit, acheté à un fournisseur, sur un marché sensible aux prix et à la qualité environnementale du produit. Le détaillant décide du prix du produit, du niveau d'émission de carbone et de la taille du lot de commande des produits à son fournisseur afin de maximiser le profit total attendu. Les clients arrivent selon un processus de Poisson avec un taux d'arrivée moyen λ. La demande moyenne diminue linéairement en fonction du niveau des émissions de carbone et du prix. Les principales sources d'émission de carbone sont les activités de transport et de production. L'émission liée au transport par unité de produit, désignée par e, dépend de la distance parcourue par le produit depuis le site du fournisseur jusqu'à l'entrepôt du détaillant. Comme nous ne traitons pas de la sélection des fournisseurs, e n'est pas une variable de décision. Cependant, nous conservons e pour montrer l'effet des émissions de carbone dues au transport sur les décisions du détaillant. Sans perte de généralité, nous supposerons que les émissions liées au transport de l'entrepôt du détaillant vers les clients finaux peuvent être négligées, ce qui suppose implicitement que le détaillant est situé à proximité de la zone de demande. Pour les émissions liées à la production, nous considérons un contexte dans lequel le détaillant peut choisir le niveau d'émission de la production et demander au fournisseur de fabriquer le produit en conséquence. Ce contexte correspond aux exemples pratiques fournis en introduction où nous présentions des cas où de grands détaillants, tels que Walmart et Bestseller, demandent à leurs fournisseurs de fabriquer des produits plus écologiques. Nous appelons x_0 la quantité d'émissions de carbone en production par unité de produit fabriqué selon un processus de fabrication standard, et x la quantité d'émissions de production par unité de produit fabriqué selon le processus de fabrication effectivement mis en place. Offrir un produit plus vert (avec des émissions en production plus faibles) conduit à un coût de production et donc d'achat plus élevé pour le détaillant. Le coût d'achat unitaire d'un produit est donné par $c+b(x_0-x)^2$, où c est le coût unitaire du produit standard, et b est le facteur de coût de la réduction des émissions de production. Nous considérons une fonction de coût quadratique comme il est d'usage dans la littérature correspondante (par exemple, Liu et al., 2012, Ghosh and Shah, 2015).

Description du problème

Nous considérons donc un détaillant qui commande un produit à son fournisseur et le vend à des clients qui tiennent compte du prix et de l'intensité carbone du produit dans leur décision d'achat. La Figure 2 montre la chaîne d'approvisionnement considérée.

L'arrivée du client suit une distribution de Poisson de taux λ. Ce taux est une fonction linéaire du prix et des émissions. A représente le potentiel du marché. Nous appelons respectivement α_p et β_e la sensibilité du marché au prix et à l'intensité carbone. Le taux de demande est donné comme suit :

$$\lambda = A - \alpha_p p - \beta_e(x+e) \tag{1}$$

Cette fonction de demande traduit le fait que le détaillant peut attirer α_p clients supplémentaires avec une baisse de prix d'une unité et β_e clients supplémentaires avec une baisse de l'intensité carbone d'une unité.

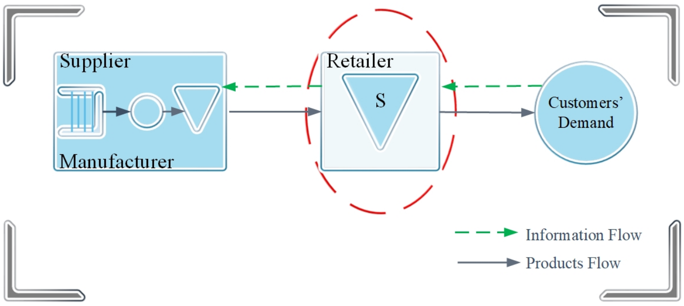

Figure 2: Benchmark SC

S est la taille de l'ordre de réapprovisionnement du détaillant. Le taux de service pour le réapprovisionnement du stock est réparti de manière exponentielle, avec un taux μ. Le délai de réapprovisionnement ne dépend pas de la taille de la commande puisque le produit est supposé être toujours disponible sur le site du fournisseur, ce qui est une hypothèse courante (Zhu, 2015). Ainsi, le délai de réapprovisionnement correspond essentiellement aux activités de préparation et de transport. La Figure 3 montre le modèle réseau de files d'attente.

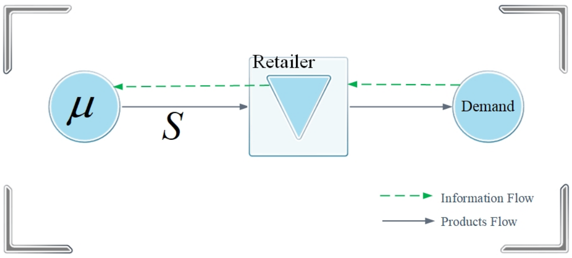

Figure 3: Modèle de référence du réseau de files d'attente

Le détaillant passe une commande pour remplir le stock auprès d'un fournisseur qui adopte une politique de fabrication MTS. Le stock est géré selon la politique standard (q, S) où q est le point de commande et S est la taille du lot de réapprovisionnement. Pour simplifier l'analyse, nous supposons que le détaillant passe une commande de réapprovisionnement lorsqu'il n'y a plus de produit dans le stock principal ($q = 0$). Pendant la période de réapprovisionnement, la demande est alors satisfaite par un stock de sécurité. Nous supposons que le stock de sécurité est suffisamment important pour satisfaire toutes les demandes dans la grande majorité des cas et nous ignorons les rares cas où la demande ne peut être satisfaite. Le dimensionnement et la gestion du stock de sécurité n'entrent pas dans le cadre de cette étude. La politique de réapprovisionnement du stock utilisée pour calculer la probabilité de rupture de stock est illustrée dans la Figure 4.

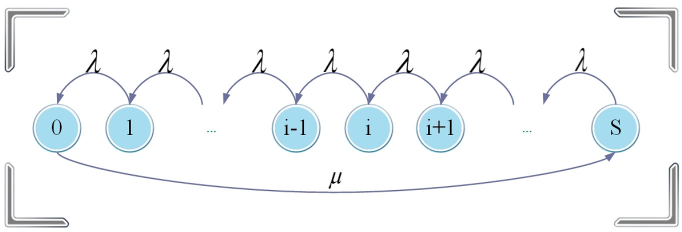

Figure 4: Politique de réapprovisionnement des stocks utilisée pour estimer la probabilité de rupture de stock

Nous appelons ψ la probabilité de rupture de stock principal du produit. Pour éviter des niveaux de stock irréalistes, nous imposons que la probabilité de satisfaire la demande à partir du stock principal, $(1-\psi)$, doit être supérieure à un niveau de service minimum prédéterminé. Nous appelons 1-r ce niveau de service minimum. Par conséquent, la contrainte de niveau de service pour le détaillant est donnée par $1-\psi \geq 1-r$, ce qui équivaut à $\psi \leq r$. Notez que ψ représente la probabilité d'avoir zéro article dans le stock principal lorsqu'un client arrive et représente donc aussi la probabilité de servir un client à partir du stock de sécurité.

Étant donné que l'arrivée des clients et le temps de service de réapprovisionnement des stocks suivent des distributions exponentielles le nombre de pièces dans le stock principal est un processus markovien. Nous pouvons calculer la probabilité de rupture de stock, dans laquelle il n'y a aucun article dans l'entrepôt, en utilisant les propriétés du processus markovien continu. Après des calculs classiques on obtient :

Ainsi, la contrainte de niveau de service pour le détaillant est donnée par $\dfrac{\lambda}{\lambda+\mu S} \leq r$. Nous pouvons également calculer le niveau de stock moyen $\bar{S}$ et on obtient

Le niveau moyen du stock est complexe, mais en considérant l'hypothèse $1 >> dfrac\lambda\mu S$ (qui est une hypothèse logique puisqu'elle traduit simplement que le temps de réapprovisionnement moyen $(frac1\mu)$ est très inférieur au temps moyen de consommation des S pièces du stock $(\frac{S}{\lambda})$, on peut utiliser l'approximation largement utilisée dans la littérature (par exemple, Cargal, 2003) consistant à considérer que la valeur moyenne du stock principal du détaillant est donnée par $fracS2$. Si $2h$ indique le coût unitaire des stocks, le coût d'inventaire pour le détaillant sera donc de hS.

Les paramètres et les variables de décision sont présentés ci-après.

Paramètres:

A	: Potentiel de marché,
c	: Coût fixe unitaire du produit,
μ	: Taux de remplissage moyen du produit,
$2h$	: Coût unitaire de détention des stocks pour le produit,
x_0	: Émission de carbone par le processus de production standard,
e	: L'émission de carbone par le transport du produit,
b	: Facteur de coût de la réduction des émissions de carbone,
r	: Stockage maximum garanti.

Variables de décision indépendantes:

x	: Émission de carbone par la production du produit,
p	: Prix du détaillant
S	: Taille de la commande.

Variables de décision dépendantes:

λ	: Taux de demande moyen pour le produit (donné par l'équation 1),
ψ	: La probabilité d'une rupture de stock.

Modèle mathématique et solution analytique

Dans ce modèle de référence, le détaillant est seul sur le marché. Le problème consiste à décider du prix p, du niveau d'émission de carbone x, et de la taille de la commande S pour maximiser son profit. Un seul produit est proposé aux clients dans ce cas; rappelons que le taux de demande est donné par l'équation 1. Définissons $A' = A - \beta_e e$, qui est une constante puisque e est connu. Le modèle (M_0) est donné ci-dessous.

$$\underset{x,p,S}{\text{Maximize}}\ \pi = \Big(p - \big(c - b(x_0 - x)^2\big)\Big)\lambda - hS \tag{2}$$

Sous réserve des

$$\psi = \frac{\lambda}{\lambda + S\mu} \leq r \tag{3}$$

$$\lambda = A' - \alpha_p p - \beta_e x \tag{4}$$

$$\lambda, p, S \geq 0, 0 \leq x \leq x_0 \tag{5}$$

L'objectif est de maximiser le profit total attendu donné dans l'équation 2. Ce bénéfice est égal au revenu (c'est-à-dire $p\lambda$) - le coût d'approvisionnement (c'est-à-dire $(c+b(x_0 - x)^2)\lambda$) - le coût d'inventaire (c'est-à-dire hS). La contrainte 3 correspond au taux de service par le stock principal souhaité. Elle garantit que la probabilité de rupture du stock principal ne dépasse pas un niveau prédéterminé de r ($1-r$ est le niveau de service minimum). Le taux de demande est donné dans l'équation 4. La contrainte 5 exprime que les variables de décision doivent être positives (et pour x inférieur à x_0, niveau d'émission pour un processus de fabrication standard).

Le problème est formulé dans un environnement stochastique. Nous avons utilisé une approche analytique pour résoudre de façon exacte le problème et obtenir les expressions des solutions optimales. Le niveau optimal d'émission de carbone diminue linéairement en fonction de β_e et augmente non linéairement en fonction de α_p et b. Le prix optimal est convexe en fonction de β_e ; il diminue jusqu'à une valeur seuil, puis il augmente. La solution optimale est donné ci-après:

$x^* = max\{0, x_0 - \dfrac{\beta_e}{2\alpha_p b}\}$,

$$p^* = \begin{cases} \dfrac{A - \beta_e e + \alpha_p\left(c + b{x_0}^2 + \dfrac{h(1-r)}{r\mu}\right)}{2\alpha_p} & \text{for } x^* = 0 \\ \dfrac{A - \beta_e(x_0+e) + \dfrac{3{\beta_e}^2}{4\alpha_p b} + \alpha_p\left(c + \dfrac{h(1-r)}{r\mu}\right)}{2\alpha_p} & \text{for } x^* \neq 0 \end{cases},$$

$$S^* = \begin{cases} \dfrac{(1-r)\left(A - \beta_e e - \alpha_p\left(c + b{x_0}^2 + \dfrac{h(1-r)}{r\mu}\right)\right)}{r\mu} & \text{for } x^* = 0 \\ \dfrac{(1-r)\left(A - \beta_e(x_0+e) + \dfrac{{\beta_e}^2}{4\alpha_p b} - \alpha_p\left(c + \dfrac{h(1-r)}{r\mu}\right)\right)}{r\mu} & \text{for } x^* \neq 0 \end{cases}, \text{ and}$$

$$\pi^* = \begin{cases} \dfrac{\left(A - \beta_e e - \alpha_p\left(c + b{x_0}^2 + \dfrac{h(1-r)}{r\mu}\right)\right)^2}{4\alpha_p} & \text{for } x^* = 0 \\ \dfrac{\left(A - \beta_e(x_0+e) + \dfrac{{\beta_e}^2}{4\alpha_p b} - \alpha_p\left(c + \dfrac{h(1-r)}{r\mu}\right)\right)^2}{4\alpha_p} & \text{for } x^* \neq 0 \end{cases}.$$

Chapitre 4: Différenciation des produits d'un détaillant dans le cadre d'une demande sensible au prix et à la qualité environnementale

Le chapitre précédent a abordé le problème de la maximisation des profits du détaillant, lorsque ce dernier offre un seul produit aux clients. Les résultats ont montré l'impact des facteurs de sensibilité aux émissions de carbone et au prix sur le profit optimal, et les variables de décision optimales (prix, niveau des émissions de carbone, taille des commandes). Dans ce chapitre, nous allons étudier les meilleures stratégies du détaillant lorsqu'il propose un deuxième produit substituable au produit initial, sur à nouveau un marché sensible au prix et à la qualité environnementale. Le nouveau produit est identique au premier produit en termes de performance, de fonction et d'utilisation, c'est-à-dire qu'il est substituable. Toutefois, ils peuvent différer en termes de prix et d'intensité carbone. Définissons le premier produit comme le produit "existant" et le second comme le "nouveau" produit. Les fournisseurs envoient les produits à l'entrepôt du détaillant, qui est proche des clients. Le détaillant conserve les produits pour servir les clients dès leur arrivée. La demande pour chaque produit dépend non seulement de son prix et de l'intensité de ses émissions de carbone, mais aussi du prix et de l'intensité des émissions de carbone de l'autre produit. Nous considérons un problème de maximisation du profit du détaillant, formulé dans un environnement stochastique. Les principales questions importantes de ce chapitre sont les suivantes:

- Le détaillant a-t-il intérêt à proposer un nouveau produit substituable ?
- Comment la différenciation des produits affecte-t-elle les stratégies optimales du détaillant ?
- Les différentes structures de marché affectent-elles les stratégies optimales du détaillant ?

La description du problème est présentée ci-après. Nous discutons des hypothèses du modèle et développons le cadre général. Nous considérons successivement différents ensembles de variables décisions et donc de modèles d'optimisation et les résolvons par des approches analytiques. Ensuite, nous en avons tirer des enseignements qualitatifs et quantitatifs.

Description du problème

Comme nous l'avons indiqué ci-dessus, nous allons étudier le problème d'un détaillant proposant deux produits substituables. La Figure 5 montre le détaillant (ellipse rouge) et les fournisseurs des produits. Le détaillant passe des commandes auprès de deux fournisseurs différents. Chaque fournisseur prépare un type de produit.

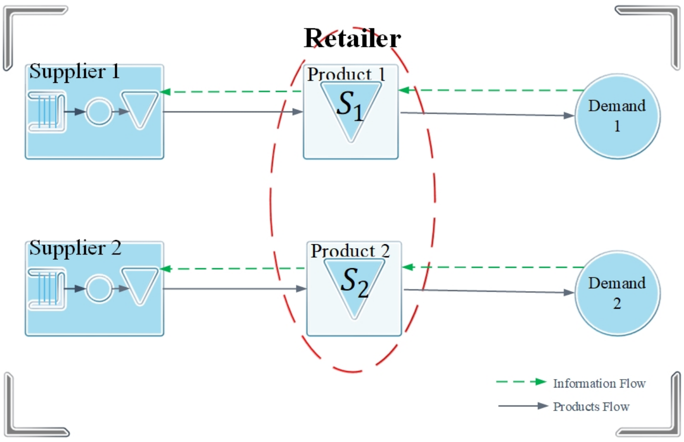

Figure 5: Différenciation des produits de la chaîne d'approvisionnement

Nous considérons une demande linéaire en fonction du prix et de l'intensité en carbone avec substitution. Les clients arrivent selon un processus de Poisson avec un taux d'arrivée moyen λ_i pour le produit i ($i = 1,2$; produit existant et nouveau, respectivement). La demande moyenne de chaque produit diminue avec son niveau des émissions de carbone et son prix, mais augmente avec le niveau des émissions de carbone et le prix de l'autre produit. Le marché potentiel est indiqué par 2A (nous supposons qu'il est divisé en parts égales pour chaque produit). Les paramètres α_p et β_e représentent la sensibilité des clients au prix et à l'intensité des émissions de carbone (CEA), respectivement. α_s et β_s représentent la sensibilité à respectivement la différence des prix et la différence entre les niveaux des émissions de carbone. Comme pour Liu et al. (2012) and Xiong et al. (2014), les fonctions de demande sont données comme suit.

$$\lambda_1 = A - \alpha_p p_1 + \alpha_s(p_2 - p_1) - \beta_e(x_1 + e_1) + \beta_s(x_2 + e_2 - x_1 - e_1)$$

$$\lambda_2 = A - \alpha_p p_2 + \alpha_s(p_1 - p_2) - \beta_e(x_2 + e_2) + \beta_s(x_1 + e_1 - x_2 - e_2)$$

moyenne totale est sensible aux prix et aux intensités d'émissions de carbone mais pas aux différences de prix et d'émissions de carbone entre produits. Pour un produit i, la diminution du prix d'une unité attire $\alpha_p + \alpha_s$ plus de clients et la diminution d'une unité d'intensité de carbone attire $\beta_e + \beta_s$ plus de clients. Il est à noter que seule une partie de ces clients représente des nouvelles demandes du marché (pour le produit i, plus précisément α_p nouveaux clients pour une baisse unitaire du prix et β_e nouveaux clients pour une baisse unitaire de l'intensité des émissions de carbone). Les autres clients attirés changent de produit (α_s changements de clients pour une baisse unitaire du prix et β_s changements de clients pour une baisse unitaire de l'intensité de carbone).

Les produits sont fournis par des fournisseurs différents. Nous considérons (sans perte de généralité) que le second fournisseur est plus proche du détaillant que le premier. Par conséquent, $e_1 \geq e_2$. Puisque e_1 et e_2 sont des paramètres fixes, nous pouvons supposer, sans perte de généralité, que $e_2 = 0$ et, donc, e_1 peut être interprété comme la différence d'émissions du transport. En outre, nous considérons que l'intensité carbone du produit existant, x_1, est connue (c'est-à-dire un paramètre fixe) dans notre étude. Soient $A_1 = A - \beta(x_1 + e_1)$, $A_2 = A + \beta_s(x_1 + e_1)$, $\beta = \beta_e + \beta_s$, $\alpha = \alpha_p + \alpha_s$. Notons que $\beta > \beta_e, \beta_s$ et $\alpha > \alpha_p, \alpha_s$. Après simplification, les demandes moyennes sont données comme suit.

$$\lambda_1 = A_1 - \alpha p_1 + \alpha_s p_2 + \beta_s x_2$$

$$\lambda_2 = A_2 - \alpha p_2 + \alpha_s p_1 - \beta x_2$$

La politique de gestion des stocks du détaillant est similaire à celle du chapitre précédent. Il passe une commande pour remplir les stocks de produits auprès de son fournisseur qui adopte une politique de fabrication MTS. Pour le produit i, l'inventaire est géré selon la politique standard (q_i, S_i) où q_i est le point de commande, et S_i est la taille du lot de réapprovisionnement. Le temps pour remplir le stock est distribué de manière exponentielle avec un taux moyen μ_i pour le produit i, qui ne dépend pas de la taille du lot. Rappelons que nous considérons le deuxième fournisseur comme étant plus proche du détaillant, donc, nous avons $\mu_2 \geq \mu_1$. Le cadre général de la problématique de ce chapitre est présenté sur la figure 6.

Nous avons formulé les problèmes dans différents contextes (en considérant différents ensembles de variables de décision) et les avons résolus par une approche analytique. Les solutions optimales sont fournies explicitement. Enfin, nous avons distingué différentes catégories de marché afin d'extraire des informations importantes de nos résultats. Ces

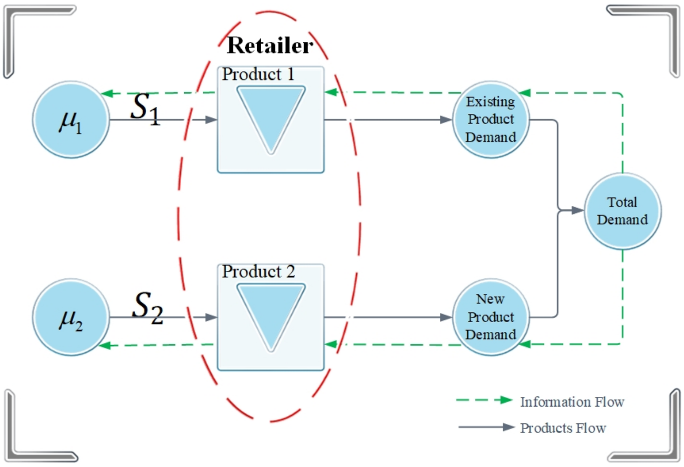

Figure 6: Le cadre général

marchés selon caractérisées selon les importances relatives de la sensibilité au prix ou aux émissions de carbone. Les résultats ont montré l'impact de ces caractéristiques du marché sur la stratégie du détaillant. Les résultats sont fournis dans différentes propositions pour couvrir tous les problèmes considérés dans cette étude. Un exemple numérique a également été présenté pour montrer la différence entre le profit du détaillant lorsqu'il offre un seul produit, et lorsqu'il offre deux produits substituables.

Chapitre 5: Optimisation des stratégies des détaillants dans un environnement de concurrence dynamique

Le chapitre précédent se situait dans un contexte de différenciation des produits pour un détaillant donné. Dans ce chapitre, nous considérons deux détaillants concurrents qui offrent deux produits substituables sur un marché sensible à la qualité environnementale et au prix des produits. Chaque détaillant a son propre fournisseur. La demande et les délais de réapprovisionnement des stocks des détaillants sont aléatoires. La demande moyenne pour chaque produit diminue en fonction du niveau de ses émissions de carbone et de son prix, mais augmente en fonction du niveau des émissions de carbone et du prix de l'autre produit. Les émissions liées au transport dépendent essentiellement de la localisation du fournisseur. Les émissions liées à la production se produisent sur les sites des fournisseurs. Elles peuvent être réduites, mais cela entraîne un coût de fabrication plus élevé pour le fournisseur (Conrad, 2005), ce qui implique un coût d'achat plus élevé pour le détaillant. Dans le cas général, chaque détaillant décide du prix, de l'intensité des émissions de carbone et de la taille du lot de commande du produit qu'il propose afin de maximiser le profit attendu tout en satisfaisant à une contrainte de niveau de service.

Dans ce chapitre nous abordons les principales questions de recherche suivantes :

- Comment la concurrence affecte-t-elle les stratégies optimales des détaillants?

- Les différentes structures de marché ont-elles une incidence sur les meilleures stratégies des détaillants?

Description du problème

Nous considérons deux détaillants qui vendent des produits substituables, distincts en termes de qualité environnementale (c'est-à-dire d'intensité des émissions de carbone) et de prix, sur un marché sensible à la greenness et au prix (voir la figure 6 ci-dessous). Les clients arrivent selon un processus de Poisson avec un taux d'arrivée λ_i pour le détaillant i. Les émissions liées au transport par unité de produit, désignées par e_i pour le détaillant i, dépendent de la distance parcourue par le produit entre le site du fournisseur et l'entrepôt du détaillant. Comme nous ne traitons pas de la sélection des fournisseurs, e_i n'est pas une variable de décision. Sans perte de généralité, nous supposons que les émissions dues au transport des entrepôts des détaillants aux clients finaux peuvent être négligées, ce qui signifie implicitement que les détaillants sont situés à proximité de la zone de demande. En ce qui concerne les émissions liées à la production, nous considérons un contexte dans lequel le détaillant peut choisir le niveau d'émission de la production et, par conséquent, demander au fournisseur de fabriquer le produit en conséquence. Nous avons fourni au chapitre 1 quelques exemples qui illustrent la manière dont les principaux détaillants, tels que Walmart, demandent à leurs fournisseurs de fabriquer des produits plus écologiques. x_0 est la quantité d'émissions en production par unité d'un produit fabriqué par un processus standard. Proposer un produit plus écologique (avec des émissions de production plus faibles) implique un coût d'achat plus élevé pour le détaillant, car le fournisseur encourt un coût de fabrication plus élevé. Nous notons x_i la quantité d'émissions de production par unité de produit P_i. L'intensité des émissions de carbone du produit P_i est donc donnée par $x_i + e_i$. Le coût d'achat unitaire de P_i est donné par $c + b(x_0 - x)^2$, où c_i est le coût unitaire du produit standard et b est le facteur de coût pour la réduction des émissions de la production. A nouveau nous considérons une fonction de coût quadratique comme il est d'usage dans la littérature correspondante (par exemple, Liu et al., 2012, Ghosh and Shah, 2015).

La taille de l'ordre de réapprovisionnement du détaillant i est dénotée par S_i. Le temps du réapprovisionnement du stock est réparti de manière exponentielle avec un taux μ_i pour le détaillant i. Le délai de réapprovisionnement ne dépend pas de la taille de la commande puisque les produits sont supposés être toujours disponibles sur le site du fournisseur, ce qui est une hypothèse courante (Zhu, 2015). La politique de gestion de stocks des détaillants est similaire à celle des chapitres précédents. La probabilité de satisfaire la demande à partir du stock standard, dénotée par $1 - \psi_i$ pour le détaillant i, doit être supérieure à $1 - r$, pour les deux détaillants. Ainsi, la contrainte de niveau de service pour le détaillant i est donnée par $(1 + \frac{\mu_i S_i}{\lambda_i})^{-1} \leq r$ (voir le chapitre 3, pour plus d'informations).

Comme chaque détaillant a son propre fournisseur, nous avons généralement une distance différente qui sépare chaque détaillant de son fournisseur. Sans perdre de vue la

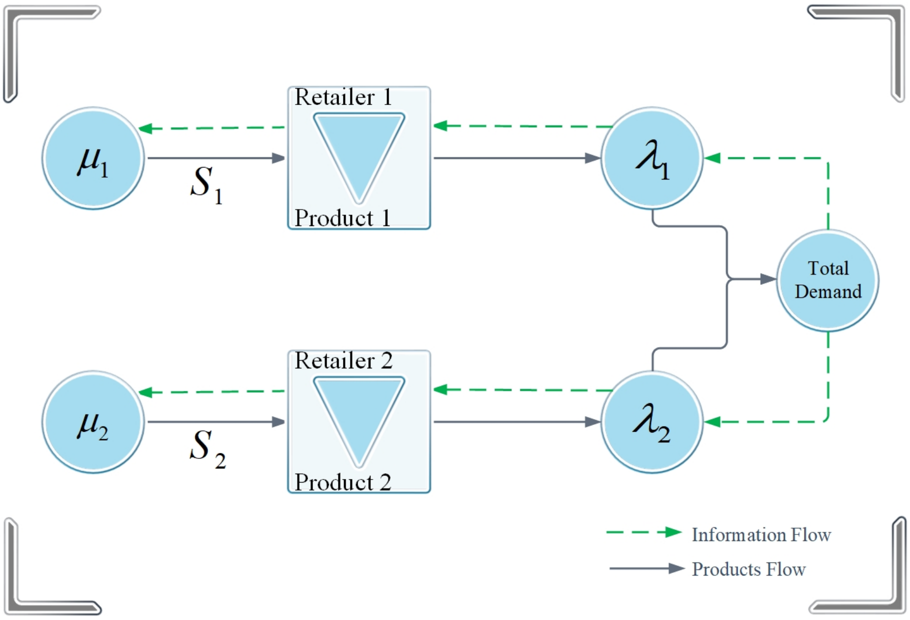

Figure 7: Competitive supply chains structure

généralité, nous laissons le détaillant 1 représenter le détaillant qui a le fournisseur le plus éloigné. Nous avons donc $e_1 \geq e_2$ et $\mu_1 \leq \mu_2$.

Comme chaque détaillant a son propre fournisseur, nous avons généralement une distance différente qui sépare chaque détaillant de son fournisseur. Sans perdre en généralité, nous considérons que le détaillant 1 a le fournisseur le plus éloigné. Nous avons donc $e_1 \geq e_2$ et $\mu_1 \leq \mu_2$.

Notre modèle de demande est linéaire avec substitution. Nous rappelons que λ_i fait référence au taux de la demande pour le détaillant i. Cette demande est très similaire à celle présentée au chapitre 4 (pour un produit sensible au prix et à la greeness de ce produit, mais aussi sensible à la différence de prix et de greeness entre les 2 produits).

Dans la pratique, les détaillants peuvent avoir des parts de marché différentes, par exemple lorsqu'un détaillant leader est établi et qu'un nouveau détaillant entre sur le marché. Dans ce cas, le premier a plus de parts de marché et attire donc plus de clients, même si le même produit est proposé par les deux détaillants. $\omega \in [0,1]$ représente la part de marché du détaillant 1 et, par conséquent, $(1-\omega)$ est la part de marché du détaillant 2, lorsque les deux détaillants offrent le même produit (c'est-à-dire avec le même prix p et la même intensité d'émission de carbone $x+e$). Les taux de demande sont donc donnés comme suit.

$$\lambda_1 = \omega A - \omega \alpha_p p_1 + \alpha_s (p_2 - p_1) - \omega \beta_e (x_1 + e_1) + \beta_s ((x_2 + e_2) - (x_1 + e_1)) \tag{6}$$

$$\lambda_2 = (1-\omega)A - (1-\omega)\alpha_p p_2 + \alpha_s (p_1 - p_2) - (1-\omega)\beta_e (x_2 + e_2) + \beta_s ((x_1 + e_1) - (x_2 + e_2)) \tag{7}$$

On a donc: $\lambda_1 + \lambda_2 = A - \omega\alpha_p p_1 - (1-\omega)\alpha_p p_2 - \omega\beta_e(x_1+e_1) - (1-\omega)\beta_e(x_2+e_2)$. Cela signifie que la demande moyenne totale est sensible aux prix et aux intensités des émissions de carbone.

Si le même produit est proposé par les deux détaillants, alors la demande moyenne est de $\omega(A - \alpha_p p - \beta_e(x+e))$ pour le détaillant 1 et de $(1-\omega)(A - \alpha_p p - \beta_e(x+e))$ pour le détaillant 2, et la demande moyenne totale ne dépend pas de ω. Le cas où les deux détaillants ont la même part de marché, qui est le cas typique étudié dans la littérature, correspond à $\omega = 0.5$. Dans notre modèle, la part de marché ω représente la part de marché du détaillant 1. Si $\omega > 0.5$, alors le détaillant 1 a plus de parts de marché que le détaillant 2, et vice versa. Avec la prise en compte de cette notion de part de marché, nous généralisons la demande linéaire avec substitution qui est généralement adoptée dans la littérature correspondante.

Pour mieux comprendre notre fonction de demande, notons que le détaillant 1 peut attirer $\omega\alpha_p + \alpha_s$ clients avec une diminution d'une unité de son prix (respectivement, $(1-\omega)\alpha_p + \alpha_s$ clients pour le détaillant 2) et $\omega\beta_e + \beta_s$ clients avec une diminution d'une unité de son intensité d'émissions de carbone (respectivement, $(1-\omega)\beta_e + \beta_s$ our le détaillant 2). Seule une partie de ces clients représente une nouvelle demande créée sur le marché (pour le détaillant 1, $\omega\alpha_p$ nouveaux clients pour une unité de baisse du prix et $\omega\beta_e$ nouveaux clients pour une unité de baisse de l'intensité des émissions de carbone), les autres clients attirés sont des clients qui changent de détaillant (α_s changements de clients pour une unité de baisse de prix et β_s changement de clients pour une unité de baisse de l'intensité des émissions de carbone).

Puisque $e_1 \geq e_2$ et que les deux sont des paramètres fixes, nous pouvons considérer, sans perte de généralité, que $e_2 = 0$ et, donc, e_1 peut être interprété comme la différence des émissions de transport. Pour simplifier la notation, nous notons $A_1 = \omega A - \theta_1 e_1$, $A_2 = (1-\omega)A + \beta_s e_1$, $\delta_1 = \omega\alpha_p + \alpha_s$, $\delta_2 = (1-\omega)\alpha_p + \alpha_s$, $\theta_1 = \omega\beta_e + \beta_s$ et $\theta_2 = (1-\omega)\beta_e + \beta_s$. Les taux de demande sont finalement donnés comme suit.

$$\lambda_1 = A_1 - \delta_1 p_1 + \alpha_s p_2 - \theta_1 x_1 + \beta_s x_2 \tag{8}$$

$$\lambda_2 = A_2 - \delta_2 p_2 + \alpha_s p_1 - \theta_2 x_2 + \beta_s x_1 \tag{9}$$

Sur la base du cadre décrit ci-dessus, nous étudions ensuite différents scénarios de concurrence et étudions leur impact sur le niveau de greeness et le prix des produits. Plus précisément nous considérons trois situations de concurrence, en partant du fait qu'il y a un détaillant établi et qu'un nouveau détaillant entre sur le marché. Plus précisément nous considérons que le détaillant établi réagit de 3 façons différentes : 1) le détaillant 1 ne réagit pas à l'apparition du nouveau détallant et donc au nouveau produit sur le marché ; 2) il réagit partiellement en ajustant son prix p_1 et sa taille de commande S_1, mais sans modifier le produit et donc sans demander à son fournisseur de modification du niveau de greeness, x_1; 3) il réagit en ajustant son prix p_1 et sa taille de commande S_1, mais aussi en demandant à son fournisseur de modifier le niveau de greeness, x_1.

Dans les scénarios de concurrence avec réaction (partielle ou complète), nous considérons un jeu non coopératif et déterminons la stratégie optimale de chaque détaillant par une recherche de l'équilibre de Nash.

Nous avons utilisé les résultats pour obtenir des considérations managériales, qui mettent en lumière l'impact de la concurrence sur les performances environnementales des produits. Par exemple nous avons constaté que lorsque le marché est davantage régi par la différence entre les qualités environnementales des produits (marché dit GDS pour Green Driven Switch), la concurrence conduit à proposer un produit plus vert. En revanche, lorsque le marché est davantage régi par la différenciation des prix (marché dit PDS pour Price Driven Switch), la concurrence conduit à proposer un produit moins vert. De plus, un détaillant qui acquiert un plus grand pouvoir de marché diminuera la greeness de son produit sur le marché du GDS et l'augmentera sur le marché du PDS.

Nous avons enfin étudié l'impact de la différence entre le niveau des émissions de carbone dans les transports sur la stratégie optimale des détaillants. Nous avons constaté que lorsque les deux détaillants ont la même part de marché, le détaillant qui s'appuie sur une chaîne d'approvisionnement plus courte offre un prix plus élevé si et seulement si la différence entre les émissions de carbone du transport est supérieure à une valeur seuil donnée.

Chapitre 6: Fonctions de demandes complexes

Jusqu'à présent, nous avons considéré une demande linéaire, sensible au prix et aux émissions de carbone. À notre connaissance, la très grande majorité des études dans la littérature utilisent cette forme linéaire, avec laquelle nous avons formulé et résolu différents modèles. Dans ce chapitre, nous questionnons la pertinence de cette hypothèse de linéarité en nous intéressant à des fonctions de demande plus générale (en particulier non linéaire) concernant l'influence de la greeness. Pour cela, nous étudions le problème de base introduit dans le chapitre 3 sous différentes fonctions de demande. Nous étudions donc la stratégie optimale d'un détaillant qui offre un produit à ses clients sur un marché sensible au prix et aux émissions de carbone. Le détaillant conserve un stock, S, pour servir les clients. Le détaillant vend les produits au prix de détail, p. Nous considérons x_0 comme le niveau d'émission de référence, c'est-à-dire pour un processus de fabrication standard. x_0 peut donc être vu comme l'amélioration maximale possible des émissions de carbone. Le coût d'une amélioration x est donné par bx^2, où b est le facteur de coût, et x est l'amélioration, c'est à dire la diminution du niveau des émissions. Rappelons que x dans les chapitres précédents représentait le niveau d'émission de carbone, alors que dans ce chapitre, pour faciliter le développement mathématique, x représente donc la diminution du niveau des émissions de carbone du produit. Le détaillant maximisera son profit en décidant du prix optimal, de l'amélioration des émissions de carbone et de la taille du lot de commande. La Figure 8 montre le problème décrit.

Comme nous l'avons dit précédemment, la fonction de demande linéaire est bien connue dans la littérature. Nos partenaires du projet ANR CONCLUDE à l'École des Mines de Saint-Étienne ont montré que la modélisation linéaire de la sensibilité de la demande aux émissions de carbone est insuffisante (Palacios-Argüello et al., 2020). Nous nous sommes donc intéressés à l'étude de nouvelles fonctions de demande pour voir en quoi les résultats diffèreraient de ceux obtenus avec des fonctions de demande linéaires tradi-

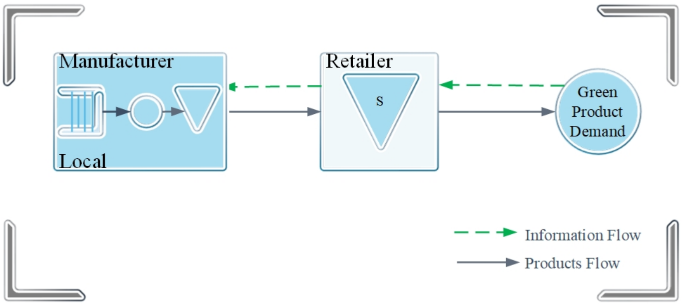

Figure 8: Supply chain

tionnelles, en procédant comme suit. Nous considérons que nous connaissons la demande du produit lorsque le détaillant offre le produit standard (c'est-à-dire sans amélioration). Nous supposons aussi connaître la forme de la courbe de la demande (par exemple, racine carrée de la diminution des émissions). Nous pouvons donc en déduire la courbe de pour x allant de 0 à 100% et en particulier le point à 100%. Nous pouvons en déduire l'équation de la demande si on la suppose linéaire en rejoignant les deux points extrêmes (0 et 100%) et on en déduit aussi le paramètre de sensibilité à x correspondant, $\beta_{e(L)}$. Par ailleurs dans la littérature, les chercheurs considèrent la demande linéaire sans aucune contrainte, c'est-à-dire considère que l'amélioration est linéaire de 0 à 100%. Mais nous savons qu'en réalité, et cela a été aussi montré par nos partenaires du projet ANR, la demande n'augmente pas au-delà d'une valeur maximale. Cette valeur est obtenue pour un x donné que nous appelons x_l. En d'autres termes, la demande augmente en fonction de x jusqu'à x_l, et après cela, la demande est constante même si on continuait à diminuer les émissions. Ainsi, nous considérons une contrainte sur la demande que nous appelons cap. Nous considérons que l'augmentation maximale obtenue grâce à l'amélioration des émissions de carbone est de ηA, pour un prix donné p. La Figure 9 montre alors les quatre fonctions de demande pour un prix donné que nous considérons : non linéaire, non linéaire avec cap, linéaire et linéaire avec cap.

Nous avons analysé le modèle de référence du chapitre 3, avec les trois nouvelles demandes : demande avec x agissant selon une racine nième, avec ou sans cap, et demande linéaire avec cap. Les nouveaux problèmes ont été résolus par une approche analytique. Nous avons alors comparé les résultats en terme de profit et de solution optimale pour les 3 demandes linéaire avec ou sans cap, et demande non linéaire (racine nième) sans cap avec la demande que l'on considère de référence : non linéaire (racine nième) et cap. Les exemples numériques, faits pour une demande non linéaire de type racine carrée, montrent que lorsque le cap de la demande est faible (jusqu'à 20% environ), le modèle de linéaire avec cap fournit la meilleure approximation de la demande de référence, tandis que le modèle non linéaire fournit la meilleure approximation pour un cap plus élevé. Ce résultat est intéressant car le cas d'une augmentation limitée à 20% est très significatif dans de nombreuses applications et la résolution des modèles avec une demande type linéaire avec cap est une simple extension des résolutions présentées dans les chapitres

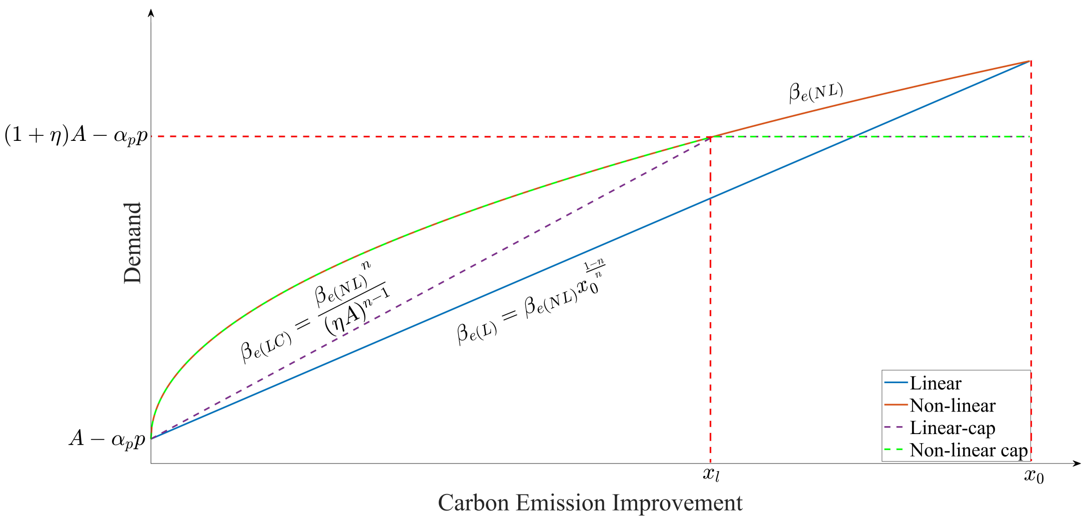

Figure 9: Demands functions for a given price

précédents.

Chapitre 7: Conclusion

Les études sur les problèmes considérant un détaillant dans le cadre d'une demande sensible au prix et à la qualité environnementale des produits révèlent plusieurs lacunes dans ce domaine. Premièrement, la plupart des études considèrent un système de fabrication à la commande alors qu'il existe de nombreux cas dans la pratique qui suivent un système de fabrication pour stock et nécessitent donc de considérer les questions de politique de gestion de stocks, de taille des commandes, etc. Deuxièmement, dans un cadre de compétition, la plupart des études ont considéré un jeu de Stackelberg, où le leader décide d'abord et le suiveur agit ensuite, alors que dans la pratique, les différents aceteurs (par exemple, les entreprises, les détaillants, etc.) changent leurs stratégies en fonction de celles des autres concurrents. Troisièmement, la fonction de demande considérée est généralement linéairement décroissante en fonction du prix et de la qualité environnementale. Cependant, la qualité environnementale d'un produit peut avoir un effet non linéaire sur la demande. De plus, souvent au-delà d'un niveau donné d'amélioration de la qualité environnementale n'attire pas plus de clients. Ainsi, dans cette thèse, nous avons proposé différents problèmes et formulations pour combler les lacunes existantes dans la littérature. Nous avons commençé notre travail par l'étude d'un modèle de référence simple, qui comprend un fournisseur et un détaillant qui offre un produit aux clients sur un marché dépendant du prix et du niveau de la qualité environnementale. Le temps de remplissage de l'entrepôt du détaillant suit une distribution exponentielle. L'arrivée des clients suit un processus de Poisson. Ainsi, l'évolution du stock est un processus Markovien. Le détaillant décide du prix du produit, du niveau d'émission de carbone et de la taille de la commande afin de maximiser son profit. Nous avons résolu le problème par une approche analytique. Ensuite, nous avons étendu le problème précédent en considérant un nouveau produit substituable. Dans ce cas, le problème inclut deux fournisseurs et un détaillant. Chaque fournisseur fournit au détaillant un produit qui permet au détaillant d'offrir deux produits substituables aux clients. La fonction de demande pour chaque produit est similaire au modèle de référence, sauf qu'il y a un effet de substitution entre produits. Cela signifie que la demande de chaque produit n'est pas seulement affectée par le prix et la qualité environnementale du produit, mais aussi par le prix et qualité environnementale de l'autre produit. Comme pour le problème précédent, ce problème est formulé dans un environnement stochastique et résolu, pour différents ensembles de variables de décision, par une approche analytique. Afin d'effectuer une analyse adéquate et de mettre en lumière les principaux éléments de gestion et de technique, nous distinguons le marché en fonction des paramètres de sensibilité. Le marché est catégorisé comme suit 1. Marché PDS, où les changements sont davantage régis par la différenciation des prix que par la différenciation des qualités environnementales. 2. Marché GDS, où les changements sont davantage régis par la différenciation de la qualité environnementale que par la différenciation des prix. 3. Marché neutre, où les disparités de prix et de qualité environnemtale ont la même importance en ce qui concerne les changements de fournisseur. Les résultats montrent que les stratégies optimales des détaillants sont fortement impactées par ces caractéristiques du marché (PDS, GDS ou neutre). Par ailleurs les exemples numériques montrent que le détaillant est bien sûr gagnant lorsqu'il offre un produit nouveau et substituable, et nous avons quantifié ces gains. Nous avons poursuivi nos travaux sur les produits substituables dans un contexte de concurrence dynamique. Deux détaillants concurrents proposent deux produits substituables (un produit par détaillant) sur un marché sensible aux prix et à la qualité environnementale. Chaque détaillant a son fournisseur. Nous considérons la même politique de gestion de stock que pour les problèmes précédents. La fonction de demande pour chaque détaillant (produit) est similaire au problème précédent, sauf que nous avons introduit et appliqué le pouvoir du détaillant (sa part de marché) dans la fonction de demande. La demande et les délais de réapprovisionnement des entrepôts des détaillants sont aléatoires et suivent une distribution exponentielle. Les détaillants décident de leurs stratégies en fonction des stratégies des autres détaillants. Les stratégies optimales de chaque détaillant sont déterminées à l'équilibre de Nash. Nous avons étudié le modèle général dans lequel chaque détaillant décide du prix du produit, de la qualité environnementale et de la taille de la commande. Mais dans la pratique, il existe de nombreuses situations où un détaillant existant opère déjà sur le marché, et un nouveau détaillant entre sur le marché et propose un produit substituable. Nous avons donc proposé deux scénarios complémentaires : 1. La concurrence sans réaction, dans laquelle le détaillant existant ne réagit pas aux décisions du nouveau détaillant. 2. La concurrence avec réaction partielle, dans laquelle le détaillant existant met à jour son prix et la taille de sa commande mais ne change pas l'intensité des émissions de carbone car cela nécessite de nouveaux accords avec le fournisseur. Grâce aux résultats obtenus, nous avons étudié l'effet de la concurrence sur la performance environnementale des produits (par rapport au modèle de référence) ainsi que l'impact des émissions de carbone du transport. Nous avons également étudié les effets du pouvoir des détaillants (part de marché) et des structures de marché (c.-à-d., PDS, GDS et marché neutre) sur la performance environnementale des produits. Les résultats ont montré que le détaillant qui a le plus de parts de marché diminuera la qualité environnementale de son produit sur le marché GDS et l'augmentera sur le marché PDS. Nous avons également constaté que lorsque les deux détaillants ont la même part de marché, le détaillant qui s'appuie sur une chaîne d'approvisionnement plus courte offre un prix plus élevé si et seulement si la différence des émissions de carbone liées au transport est supérieure à une valeur seuil donnée. Dans les chapitres 3 à 5, nous avons considéré une fonction de demande linéairement décroissante en fonction du prix et du niveau d'émission de carbone. Dans le chapitre 6, nous avons considéré une fonction de demande linéairement décroissante en fonction du prix, mais qui augmente de façon non linéaire en fonction de l'amélioration des émissions de carbone. Nous avons également considéré une contrainte qui prend en compte que la demande ne peut pas augmenter au-delà d'un certain niveau. Les modèles avec les nouvelles fonctions ont été formulés et résolus, sur le problème de référence, étudié au chapitre 3. Les exemples numériques montrent que lorsque la borne supérieure de la demande est relativement faible, le modèle linéaire avec borne fournit une très bonne approximation, ce qui est très intéressant car l'extension des modèles étudiés dans les chapitres 3 à 5 est relativement simple avec cette nouvelle forme de demande.

Travaux futurs et perspectives

Notre étude peut être étendue de différentes manières. Comme nous l'avons mentionné dans l'introduction, notre travail a commencé par un système hybride en considérant une demande sensible au délai et au prix. Comme première extension, il serait intéressant d'étudier deux modes d'approvisionnement en compétition en considérant une demande sensible au délai, à la qualité environnementale et au prix. Une autre extension de notre travail serait de reprendre les modèles étudiés dans les chapitres 4 et 5 en considérant des fonctions de demande introduites dans le chapitre 6. Le chapitre 6 introduit de nouvelles fonctions de demande et les compare à la demande linéaire. Ce travail peut être étendu en considérant d'autres formes de fonction de demande comme la fonction log-linéaire. Encore une fois, les problèmes de collaboration et de concurrence sont également intéressants à étudier avec d'autres fonctions de demande. Notre travail considère le CEA comme la seule source d'effet environnemental provenant des clients sur les stratégies des détaillants. Il serait également intéressant d'y ajouter la prise en compte de réglementations gouvernementales telles que la taxe carbone, le carbone cap, etc.

CHAPTER 1
Introduction

The environmental performance of a product, often assessed by the amount of carbon emissions released during the production and transportation phases, is becoming an essential purchase criterion for many customers (Palacios-Argüello et al., 2020; Hammami et al., 2018; Borin et al., 2013). Even it is highly dependent of products the relevant report on the product's carbon emission analysis reveals that approximately 70% of the total carbon emissions produced in the product's life cycle is related to the production process (AACE, 2015). According to a survey of 6,000 consumers, which is conducted by Accenture in April 2019, in 11 countries across North America, Europe, and Asia, 72% of consumers said they are currently buying more environmentally-friendly products than they were buying five years ago. Also, 81% said they expect to buy more over the next five years. Moreover, more than half of consumers would pay more for environmentally-friendly products (Accenture, 2019). Carbon Trust surveys indicate that approximately 20% of customers prefer to buy green products even if they are more expensive than regular products (Hong and Guo, 2019).

Most studies in supply chain management assume that customer's demand is known (exogenous demand). However, we know that demand is strongly impacted by internal decisions (endogenous demand). The most known factor that affects the customer's purchasing behavior is the price. However, there are different kinds of factors that affect the demand for a product or service. Huang et al. (2013) provided an extensive survey of papers where demand function depends on a factor. Those demand functions' models are in six categories: Price-Dependent, Rebate-Dependent, Lead Time-Dependent, Space-Dependent, Quality-Dependent, Advertising-Dependent Demand Model. It is well known in the business logistics literature that one of the most important customer service elements, in addition to price, is the delivery time. To understand the logic, the ideas, and solving methodology behind the endogenous demands, we started our work by considering an endogenous demand sensitive to price and lead time. We consider a hybrid system of selling substitutable products, which is consists of Make-To-Order (MTO) and Make-To-Stock (MTS) systems (see Appendix A for more details). However, this Ph.D. focuses on endogenous demand sensitive to the product's price and greenness.

Motivated by empirical studies that show evidence of the relation between demand and environmental performance, modeling-based research has recently revised some classical problems in the operations management and Supply Chain (SC) literature while assuming the demand to be dependent on the environmental performance (see, e.g., Zhang et al., 2015, Hovelaque and Bironneau, 2015, and Liu et al., 2012). Environmental awareness changes consumer's decisions on purchasing goods. European Commission pointed out that 75% of Europeans choose green products. They are willing to pay more price to have greener products (Brécard, 2014). Consequently, the Customers' Environmental Awareness (CEA) will directly influence the enterprise's strategy to produce green products and to invest and adopt cleaner technology (Yalabik and Fairchild, 2011). By recognizing these shifts in the marketplace, suppliers are considering to supply greener products (Gu et al., 2015; Sheu and Chen, 2012; Ding et al., 2015; Cheng et al., 2017). The consumer is willing to pay a premium price for a green product, such as lower CO2-emitting cars (Costa et al., 2019), but the green product needs more investment in research and development (R&D), which will generate more cost (Krass et al., 2013). Therefore, the product's price as an important element of customer's purchasing decisions has been considered by many studies (e.g., Conrad, 2005; Sengupta, 2012).

Many retailers have developed a thorough understanding of customers' preferences for environmentally-friendly products and their willingness to pay more. Consequently, they have adapted their procurement strategies to offer greener alternatives to their customers. Thus, as highlighted by Ramanathan et al. (2012), these retailers asked their suppliers to reduce carbon emissions in production and transportation phases. Benjaafar et al. (2012) highlighted that various retailers are starting to attach carbon footprint labels to their products and position these products as greener alternatives. Two leading retailers, Tesco in the UK and Casino in France, have already embarked on aggressive labeling efforts. Recently, different products are becoming the object of carbon footprinting. Examples include fruit smoothies, shoes, and beer. Walmart, the world's biggest retail company, has set an emissions-reduction plan in October 2016 and invited the suppliers to join Walmart in committing to reduce greenhouse gas emissions resulting from their operations and value chains. The objective is to work with the suppliers to reduce carbon emissions by 1 Gigatonne in the production and use of food and products globally between 2015 and 2030 (Walmart, 2017). Bestseller (Bestseller, 2018), one of the leading fashion retailers in Denmark, allows its customers to buy a wide range of environmentally-friendly products. The retailer (Bestseller) has committed to continuously improve the environmental footprint of its products, operations, and SC. Thus, to manage demand and increase profitability, retailers would have to compete not only on the price but also on their products' environmental performance (greenness) (which is here assessed in terms of carbon emissions).

Our review of the literature, presented in chapter 2, allowed to identify new perspectives for the problem of optimizing retail operations under a greenness- and price-sensitive demand context with endogenous demand. In particular,

1. Most studies are done based on MTO (Make-To-Order) system. As an exact term, they did not adapt to the retail context and inventory management in this area. However, in practice, we know that retailers make an order to their suppliers in many cases and make a stock to serve customers immediately. Therefore, retailers need to decide about order size and order point to optimize their expected profit.

2. In a competitive environment, most papers considered the Stackelberg game in which one player acts (makes decisions) first, called the leader. The second one follows and makes the decisions based on the leader's decisions, called the fol-

lower. Few studies considered a dynamic competition between players that they can make the decision simultaneously, and each player's decisions are affected by others' decisions.

3. In the literature, the researchers consider the linear demand without any constraint. However, our colleagues (Rennes School of Business and School of Mines of St Etienne in ANR project) found out that the demand is not linear and will not increase beyond a certain level of demand (Palacios-Argüello et al., 2020).

Based on the observations explained above, we propose the following extensions in this thesis:

- We first develop a basic model: a single retailer that offers one product to customers. The product's demand is sensitive to the product's price and carbon intensity. The retailer keeps the products as inventory in a warehouse near to customers to serve them. The retailer makes an order to fill the product's inventory from the supplier that adopts a MTS manufacturing policy. The inventory is managed according to the standard (q,S) policy where q is the reorder point, and S is the replenishment lot size. As demand and inventory replenishment time are random, we develop the mathematical model in a stochastic environment (which is the case for the all models in this thesis). This problem is considered the benchmark model for the remaining chapters. More discussion and details are provided in chapter 3.

- Product differentiation, where a retailer offers two substitutable products to customers, is considered. The goal is to find the retailer's optimal strategies. The demand function of each product depends on its price and carbon intensity and depends on other product's price and carbon intensity. The detailed analysis is provided in chapter 4.

- A dynamic game between two retailers in a competition is considered in a price- and greenness- environment under a stochastic environment. We consider the situation where a retailer acts in the market, and a newcomer arrives. Newcomer offers a substitutable product. The market potential's share is not necessarily equal for both retailers. The detailed discussion is presented in chapter 5.

- The benchmark model is considered with different (more complex) demands functions. We obtain and compare results thanks to different demand functions to analyze if the classical linear demand function is satisfying or not. More discussions and details are provided in chapter 6.

Chapter 2 gives an overview of the related literature. The framework of the thesis is presented in the following.

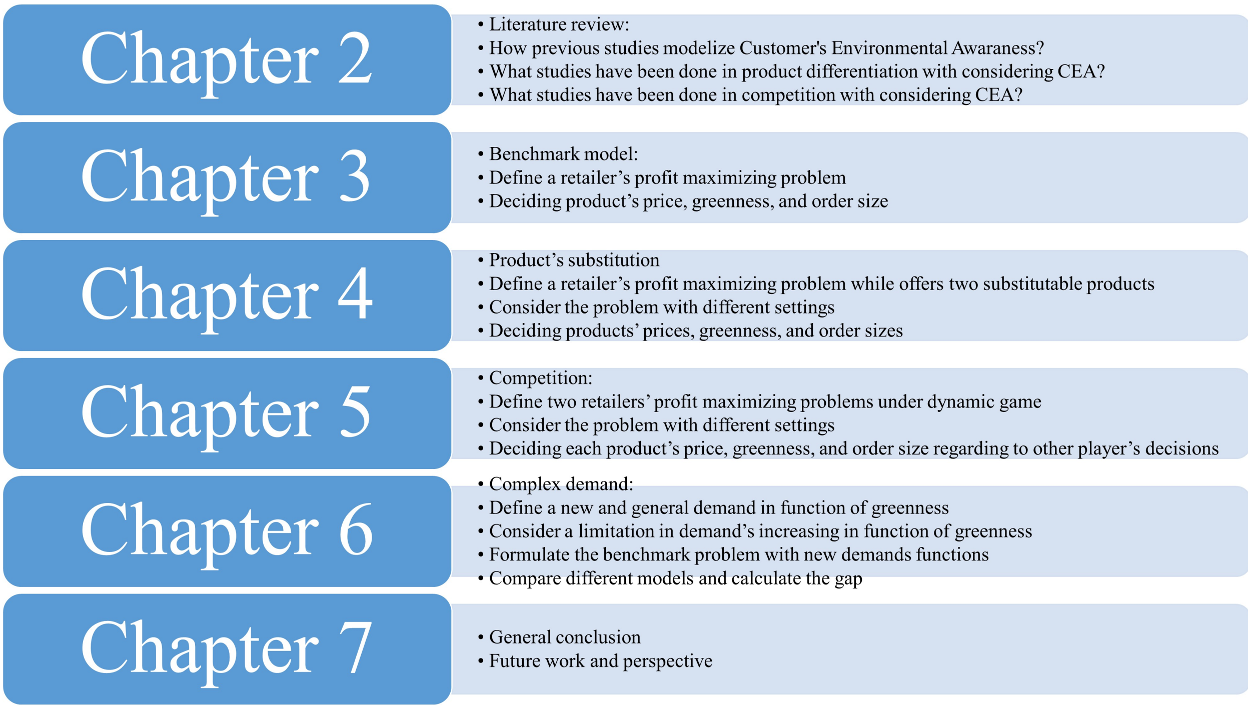

Figure 1.1: Framework

CHAPTER 2

Literature review

Our study relates to the analytical research in operations and SC management with environmental considerations and, particularly, those studies that focus on price- and greenness- sensitive demand in SCs. Our work involves three streams of related literature on green supply chain management. The first section presents the studies that consider CEA (Customer Environmental Awareness) and, thus, carbon emission as a decision variable in a price- and greenness-dependant market. The second section of the literature review presents the studies that mainly worked on product differentiation. The third section reviews the literature concerning the best players' (retailers') strategies under a competitive environment.

The Growing number of customers who consider the environmental-friendly aspect of a product during their purchase inspires suppliers (e.g., manufacturers) to slowly begin to realize that they should use more advanced green production systems and green energies as an alternative to the traditional production system. The new alternative (green production system) decreases carbon emissions released into the environment during production. The green products release less carbon emission than the standard products, which refers to the friendliness of products to humans and the environment (Dangelico and Pontrandolfo, 2010). Carbon emission is becoming an essential factor that affects consumer purchase decisions. However, green products are often costlier to produce than the standard products produced traditionally, which makes green products are offered at higher prices than the standard products (Conrad, 2005). The stronger the consumers' environmental awareness is, the more the consumers are willing to pay a higher price for these green products (Chitra, 2007). Suchlike awareness and willingness may be considerably different among companies and industries, changeable during the time, and be different between consumer groups based on demographics, knowledge, values, attitudes, and behavior (Laroche et al., 2001; Carlson, 2005). Moon et al. gathered a survey of customers in prior "West" and "East" Berlin after Germany united. The survey provides the customers' willingness to spend a higher price for green foods produced by an environmental-friendly process (Moon et al., 2002). The results are presented for different groups of customers distinguished by geographic region and age. In the end, Moon et al. (2002), reported that a higher willingness to pay for green products would result in an instant effect that the higher price (or higher margin profit) will persuade more manufacturers to go after using environmentally-friendly production.

2.1 Customers' Environmental Awareness

Ghosh and Shah (2012) investigated the impact of greening decisions when one manufacturer and one retailer cooperate (i.e., centralized SC) or act individually (i.e., decentralized SC). Under the decentralized policy, the manufacturer moves first and decides the greenness level and the wholesale price. Improving greenness requires an investment that is modeled as a quadratic cost in greenness improvement. The retailer reacts by deciding the final price. Under the cooperative policy, the retailer and manufacturer first bargain on the greenness level; the manufacturer then decides the wholesale price, and the retailer reacts by deciding the final price. The study showed how channel structures influence greenness levels, prices, and profits.

Ma et al. (2013) considered a two-stage supply chain composed of one manufacturer and one retailer to investigate the effect of different contracts on the supply chain's profit and green improvement of product. There is one product to offer. The demand is a linear function of the retailer's price, product's quality level, and retailer's marketing efforts. A deterministic model is developed to adopt the contracts and analyze them. They consider two scenarios, centralized and decentralized supply chain. In a decentralized setting, three different contracts are presented. In the first contract, the retailer has to pay a fixed payment, which leads to a lower wholesale price. In the second contract, the quality improvement cost is shared between the retailer and the manufacturer. In the third contract, the quality improvement cost and marketing efforts cost are shared between the retailer and the manufacturer. The results show that the retailer will exert the highest marketing effort in the second contract if the retailer shares' quality cost is considerably high.

Nouira et al. (2014) presented the impact of considering environmental aspects and producing green products to industries. A linear demand function, which is increasing in the product's greenness, is considered. Each product has two attributes of greenness: the first one is related to the released carbon emission (which depends on the manufacturing process, etc.). The second one is related to raw materials. Two strategies are considered. In the first strategy, the manufacturer offers a single product to both customers (ordinary and green). The second strategy offers a particular type of product to each customer. A Mixed Integer Linear Programming (MILP) model is presented in order to maximize the profit. The model is solved by using real case study data. Results show that the second strategy leads the manufacturer to higher profit.

The studied problem in Ghosh and Shah (2015) is similar to Ghosh and Shah (2012). A two-stage supply chain with a manufacturer and a retailer under sensitive demand to the retailer price and green improvement is considered. The manufacturer produces one product to offer. However, in this study, "Cost Sharing Contract" is introduced. According to this contract, the retailer takes a share of green improvement costs. The results show that the green improvement, wholesale and retailer's price under this contract are higher than the decentralized policy. Under this contract (cost-sharing contract), the manufacturer's and retailer's profits also are higher than with the decentralized policy. Further, they discuss bargaining and take into the cost-sharing contract. The result shows that bargaining on the cost-sharing parameter provides a greater green improvement and greater manufacturer's and retailer's profits.

Xu et al. (2016) considered a MTO closed-loop SC, composed of a manufacturer and a retailer, to investigate the effects of the CEA on the best decisions of the SC. In this SC, used products are not abandoned and returned to the life cycle by re-manufacturing. The product's demand increases linearly in the product's eco-friendly level and decreases the price's linear function. They formulated the problem in two ways to find out more about the supply chain system in economic, environmental, and social aspects: centralized and decentralized approaches. The manufacturer plays as the leader in the decentralized model. They also developed a revenue-sharing contract model to study more about collaboration (between manufacturer and retailer) in the decentralized model. The results showed that the centralized model provides better economic, environmental, and social performance than the decentralized model.

Li et al. (2016) investigated the pricing and greening strategies in a dual-channel SC. The manufacturer makes green products and sells them directly or through a retailer. The demand function is linearly decreasing in price and increasing in greenness level. The problem is formulated and solved in centralized and decentralized SCs. In the decentralized model, they propose coordination that provides the condition such that both the manufacturer and the retailer obtain win-win results. They also examine the effects of the greenness parameters (i.e., cost and sensitivity) on the manufacturer's and retailer's pricing decisions.

Liu and Yi (2017) brought the Big DATA (BDATA) concept into the green SC. The SC is composed of a manufacturer that produces green products and sells them through a retailer. The demand for products is sensitive to price and influenced by advertising. They presented four models: 1- M model in which the manufacturer decides the product's wholesale price and greenness level and then the retailer decides the product's retail price; 2- R Model in which the retailer decides the product's retail price and the input level of BDATA and then the manufacturer decides the product's wholesale price and greenness level; 3- N Model in which is similar to R model in terms of decision variables, but, there is no leader and follower, and both manufacturer and retailer decide at the same time; and 4- C Model that is the centralized setting, which the product's price is the only decision variable. They presented the close-form expression of optimal solutions and compared the profits of different models.

Hammami et al. (2018) studied the joint and alternative effect of the CEA and environmental regulations on the production policy, price, and greenness level of a product offered by a manufacturer. The authors considered linear and exponential demand functions that decrease in price and carbon intensity. Reducing the carbon intensity leads to increasing the production cost. The main results indicated that CEA is an efficient driver for better environmental performance, acting as a substitute for a carbon tax, unlike a carbon tax, leading to a lower price for customers. In the same research stream, many studies consider a SC that consists of one manufacturer and one retailer. These studies typically consider a linear demand function.

Hong and Guo (2019) considered a supply chain composed of one manufacturer and one retailer under different cooperation contracts (Price-only contract, Green-marketing cost-sharing contract, Full channel coordination). The system follows a MTO production with a sensitive demand to the price and product's greenness. The demand function is increasing in the product's greenness and decreasing in price. A stochastic model is presented in order to maximize the profit (manufacturer and retailer). Manufacturer's decisions (wholesale price and greenness of product) and retailer's decisions (selling price and exerted effort for green marketing) are obtained by following the Stackelberg game. The retailer decides after the manufacturer's decisions. In conclusion, they declared that cooperation is needed to make the supply chain's environmental efficiency better. Especially in a high CEA market, the cooperative contracts are worth it.

Liu et al. (2020) considered a SC that is composed of one manufacturer and one retailer. The manufacturer invests in green innovation, produces a product, and sells it to the retailer. The retailer offers the product to customers in a price- and greenness- sensitive market. They consider the Stackelberg game between two players. The manufacturer acts as the leader and decides wholesale price and innovation investment, while the retailer, as the follower, decides the retail price. They also consider that the retailer acts as the leader, and the manufacturer acts as the follower. They attempt to investigate the effects of cooperative mechanisms (i.e., revenue sharing and cost-sharing contracts) on innovation investment and pricing strategies under different scenarios. The results show that when the retailer acts as the leader, the revenue sharing contract provides lower profit for both players (manufacturer and retailer) and lower investment in green innovation. They suggest that the cost-sharing contract provides a higher profit.

Huang et al. (2020) focused on government and bank loan effects on the green SC. The SC consists of a manufacturer who produces green products and sells them through a retailer. The government helps the manufacturer to produce greener products by offering the green credit (GC) that provides subsidies directly to the bank, manufacturer subsidy (MS) that provides subsidies directly to the manufacturer, and sales subsidies (SS) that provides subsidies directly to the retailer. The total subsidies of different modes are equal. The product's demand is decreasing in its price and increasing in greenness. They considered four scenarios: no-subsidy, green credit, manufacturer subsidy, and sales subsidy. The closed-form solutions are presented for all scenarios. The results showed that GC setting has better performance in terms of returns considering a green degree, market demand, social welfare, and environmental benefit when the bank loan is infinite.

The studies that have been done and presented in this section provide a comprehensive knowledge of customer environmental awareness subject, and guide us in the choices of the hypotheses we will make from chapter 3.

2.2 Product Differentiation In Supply Chain Models

This section consists of two research areas: optimal pricing strategies of substitutable products, optimal pricing strategies under the green supply chains.

2.2.1 Optimal pricing strategies of substitutable products

The existence of substitutable products becomes an interesting area since the customers have more than one option in purchasing products. In pricing, customers switch between existing alternatives if they do not meet a preferred product (Shin et al., 2015). Therefore, the pricing strategies of substitutable products have become a popular research area.

Hsieh and Wu (2009) considered a SC, composed of two suppliers and one retailer, in a price-sensitive market. The suppliers produce two substitutable products and sell them through the common retailer. The problem consists of the retailer's pricing and ordering decisions in a stochastic environment. They considered different scenarios: revenue sharing, return policy, and combination of revenue sharing and return policy. The results reveal that in identical suppliers, the retailer's ordering and pricing decisions do not depend on demand or supply uncertainty. Also, they mentioned that whenever the retailer's cost per unit sold is smaller, he/she makes a higher order to suppliers.

Gürler and Yılmaz (2010) studied on a SC that includes one manufacturer and one retailer in a newsboy setting. The retailer makes an order for two substitutable products to the manufacturer. They assumed that each product's demand is independent of another. The customers switch between products (with a predetermined probability) when one of them is out of stock. The retailer has the right to return some or all of the unsold products to the manufacturer with some credit. The problem is formulated in a stochastic environment. They provided SC's total profit, manufacturer's profit, and retailer's profit.

Bish and Suwandechochai (2010) considered a SC that includes one supplier and one retailer. The supplier produces two substitutable products and sells them through a common retailer. The demand for each product is sensitive to its price (decreasing) and other product's price (increasing). They focused on the degree of products' substitution and the level of operational postponement. The problem is formulated in a stochastic environment that consists of deciding products' price and quantity. The results show the impacts of substitution and operational postponement on the optimal solution.

Zhao et al. (2014) considered a two-stage SC that consists of two manufacturers and one retailer to investigate the pricing strategies. The manufacturers sell the products through a common retailer. The products are substitutable. Each product's demand is sensitive to its and other product's prices. Different scenarios are considered to analyze the effects of the different SC's structures on the optimal pricing decisions. They are: 1- Centralized pricing model, 2- Manufacturer Stackelberg pricing model, 3- Retailer Stackelberg pricing model, and 4- Vertical Nash model. The closed-form expressions of all scenarios are provided by using an analytical approach. They derived some managerial insights.

Şen (2016) considered two suppliers that produce two substitutable products and sell them in a selling season. Each product's inventory is fixed; however, each product's demand is sensitive to the prices. At the beginning of the season, suppliers offer their products at a predetermined price and can change it once during the season. The goal is to know when is the best time for suppliers to reduce their products' prices to maximize the objective function (profit). The problem is formulated in a deterministic environment. The results showed that if a firm marks its price down after the season starts, it ensures that it runs out of inventory precisely when it ends.

Zhou et al. (2018) investigated the carbon tax policy's effect on a two-stage supply chain, which follows the MTO system. The model is deterministic, and the demand is a linear function of price and carbon emission's tax price. Demand is decreasing in price and carbon emission's tax price. The system is composed of three players. The government determines the carbon tax rate for the manufacturer. The manufacturer determines the wholesale price. The retailer determines the retail price. They declared that retail and wholesale prices are increasing in the carbon tax rate. The objective function is to maximize social welfare.

Luo et al. (2018) focused on customer values and different scenarios on the products' pricing strategy in a two-stage SC. They consider a manufacturer who produces two substitutable products and sells them through a retailer. The manufacturer and the retailer decide products' wholesale and retail prices, respectively. Each product's demand depends on the products' prices and customer acceptance. The goal is to maximize manufacture's and retailer's profit, and they formulate the problem in a deterministic environment. Different scenarios (i.e., manufacturer Stackelberg, retailer Stackelberg, and vertical Nash) are considered to investigate the effect of SC's structure on optimal solutions. They demonstrate that the vertical Nash structure creates a highly competitive environment between the manufacturer and the retailer, which retailer offers products at a lower price than other structures.

Ceryan (2019) investigated the pricing and replenishing strategies of two substitutable products: Regular and Seasonal. The regular products are refilled during horizon time, although the seasonal products are offer once. The products are different in the inventory management by the supplier. The supplier decides the regular product's price once (for all horizon times), while the seasonal product's price is determined at the beginning of the season. Each product's demand is sensitive to products' prices (i.e., decreases in its price and increases in other product's price). He formulated the problem through a multi-period stochastic dynamic model. In addition to the initial problem, he considered two sub-models: Pricing with partial replenishment and Replenishment with Partial Pricing. The results provide a simple and effective heuristic policy.

Our study is different from the presented studies in the way that we not only consider price as a decision variable in product differentiation problem but also consider the carbon intensity of products as a decision variable. More details of the difference between our study and the presented studies are presented in Table 2.1.

2.2.2 Optimal pricing and greenness strategies of substitutable products

CEA's effect on the increasing demand for green products is another research area related to the current work. With the rise of CEA, it is expected to increase the pleasure of consuming a product for which an environment-friendly substitute is available (Conrad, 2005). With the increase of customer knowledge about environmental issues, the market purchase behavior is changed; people prefer to buy green products instead of usual ones but do not pay too much money (Datta et al., 2011). Green advertising is one of the best-known methods to increase CEA and change the people living style to a green life (Haytko and Matulich, 2008; Abd Rahim et al., 2012). A reasonable pricing strategy is critical for green products to acquire market share.

Liu et al. (2012) studied product differentiation in a SC that is composed of two manufacturers and one retailer. The manufacturers sell the products through a common retailer in a price and greenness sensitive market. A Stackelberg game is considered, such as manufacturers decide the product's price and greenness level as the game leader and then retailer follows and decides the retail price. The problem is formulated in a deterministic environment. They also considered other scenarios that we discuss in the next sub-section.

Zhang et al. (2015) considered a stochastic model to investigate the impact of CEA on the order quantity and channel coordination in a problem that the model is built on the multi-product newsvendor model. Demand is sensitive to price and product's environmental quality. The system is composed of one manufacturer that produces two substitutable products and one retailer. It is assumed that the products have one sale season. Products, which remain at the end of the season are sold at a lower price (first and second scenarios) or returned to the manufacturer with a return contract. The goal is to maximize the profit function in both centralized and decentralized policies. They showed that in decentralized policy, given that the manufacturer's profit is convex in CEA, the retailer's profit is increasing in CEA. Thus, the retailer makes more profit when offered products are better (greener). Also, they declared that the return contract is a win-win situation, even though the retailer's profit is less than the manufacturer's profit. In the end, the CEA's impact on traditional products depends on the differentiation between the greenness levels of two products.

Basiri and Heydari (2017) considered a two-stage supply chain, composed of a manufacturer and a retailer. The supply chain offers two substitutable products, which are traditional and green. The demand function is sensitive to price, green quality, and also sales efforts. The manufacturer decides the product's green quality, while, retailer decides retail price and sales efforts. Three scenarios are considered and compared: 1- Decentralized model, where both parties try to maximize their profits; 2- Integrated model, where one decision-maker exists; and 3- Collaborative model, which is supposed to offer a win-win situation. All of the scenarios' models are deterministic. An analytical approach solves the model. The numerical example results show that the manufacturer's profit in the collaboration setting is higher than in two other situations. However, for the retailer, the integrated setting's profit is higher than the profits in other settings.

Qin et al. (2017) considered a nonlinear stochastic model. The demand function is sensitive to price and carbon emission reduction. They studied a MTO supply chain system composed of one manufacturer and one retailer. This study's main point is to combine various demand forecasting and supply chain strategies (e.g., price and carbon emission reduction). In all scenarios, which are Without Information Sharing, Full Information Sharing, and Retailer-only Forecasting, the manufacturer is the leader that determines wholesale price and carbon emission reduction level, and the retailer is the follower which determines selling price. The results show that both the manufacturer and the retailer in the high uncertainty market tend to choose the third forecasting scenario (retailer-only Forecasting). In a low uncertainty market, the manufacturer chooses the second scenario, while the retailer chooses the first scenario.

To highlight the contributions of our work, Table 2.1 presents a summary of the existing literature. As Table 2.1 shows, there are few studies in this area that consider the order size as a decision variable. However, these studies consider endogenous demand sensitive only to the product's price, and then, the product's price and order size are the decision variables. The other studies that consider carbon emission as a decision variable do not consider the product's order size as a decision variable. Therefore, there is a gap, such as considering a stochastic system with these three variables (product's price, carbon emission, and order size). Our work that has been done, in Chapter 4, closes this gap of literature.

Table 2.1: Products differentiation comparison table

Paper	Demand			Decision Variables				Model	
	Price	Carbon emission	Other	Price	Carbon emission	Order size	Other	Deterministic	Stochastic
Hsieh and Wu (2009)	•			•		•			•
Gürler and Yılmaz (2010)				•		•			•
Bish and Suwandechochai (2010)	•			•		•			•
Liu et al. (2012)	•	•		•	•			•	
Zhao et al. (2014)	•			•				•	
Zhang et al. (2015)	•			•		•			•
Şen (2016)	•			•				•	
Basiri and Heydari (2017)	•		•	•			•	•	
Qin et al. (2017)	•	•		•	•				•
Zhou et al. (2018)	•		•	•			•	•	
Luo et al. (2018)	•			•				•	
Ceryan (2019)	•				•		•		•
Our Study	•	•		•	•	•			•

2.3 Competition Environment

Our study relates to the research on analytical models for operations and SC management with environmental considerations and, particularly, those studies that focus on greenness-based competition in SCs. There are valuable works on retailers' competition. Most of these studies, however, do not consider environmental aspects. In this section, we review the extant research in these areas and highlight our work's contributions.

Cachon (2001) considered a two-stage supply chain composed of a supplier and N retailers to study competitive and cooperative situations. The retailers hold the inventory in order to serve the customers, which has a cost. Also, there is a backorder cost if the stock is empty and a customer arrives. The supplier also has the inventory and backorder (situation that retailers face to empty stock) costs. The objective function is to find the optimal reorder points in order to minimize the cost. They formulated the problem in a stochastic demand by considering exogenous demand for each retailer. They use the Nash equilibrium to find the best solution. They declared that competition does not necessarily lead to supply chain inefficiency. In other settings, competition leads to costs that are substantially higher than optimal.

Dai et al. (2005) considered competitive firms to investigate pricing strategies. They considered a profit-maximizing problem in which the demand function is sensitive to the product's price. Each firm has its capacity (of satisfying demands), and so, the satisfied demand is equal to the minimum of the demand and the capacity. They solved the model under two situations according to the deterministic demand or stochastic demand. They did a sensitivity analysis of the optimal prices concerning cost and capacity parameters. They also declared that when the retailer's capacity is big enough, the optimal solution is independent of the capacities; otherwise, the optimal solutions depend on them.

McGuire and Staelin (2008) considered a price competition between two SCs. They considered different scenarios and models. However, the demand in all scenarios depends only on products' prices. Each manufacturer distributes its products through a single exclusive retailer, either a franchised outlet or a factory store. The problem is formulated in a deterministic environment. The results show that manufacturers prefer to follow a centralized system when competition's degree is low. However, the manufacturers prefer to follow a decentralized system when competition's degree is low. They mentioned that the products' substitutions have any effect on the Nash equilibrium. However, later in this work (Chapter 5), we show that the products' substitutions have significant effects on optimal solutions.

Xiao and Yang (2008) investigated the competition between two supply chains, which each of them is composed of one supplier and one retailer under demand uncertainty. They offer two substitutable products. The demand function is sensitive to the product's price and service level. The supplier leads and determines the wholesale price in each supply chain, while the retailer follows and determines the retail price and service level. They focused on the effects of the demand uncertainties, the service investment efficiencies, and the wholesale prices. They found that increasing one retailer's service investment efficiency leads to a lower optimal retail price and the other retailer's service level.

Wu et al. (2009) investigated the optimal strategies of two competing SCs with considering uncertain demand. They assume that the demand is uncertain with a high value, h, with probability u, and a low value, l, with probability $1-u$. They considered demand functions depend on three factors: the primary uncertain demand of the state, its retail price, and other product's retail price. The problem is formulated in a stochastic environment. They considered different games between SCs: 1- Manufacturer Stackelberg; 2- Vertical Integration; and 3- Bargaining over the wholesale price. They considered a single period ' and infinitely periods ' competition to compare the models. In infinity periods, a discount value is considered for each period. The products that are not sold have no value. They presented that vertical integration provides the unique Nash Equilibrium over single period, while other games may provide Nash Equilibrium over infinitely many periods.

Wu et al. (2012) investigated the competition effects on a two-stage supply chain composed of a supplier and two retailers. The supplier produces a product, and two retailers offer the same product to customers. They formulated the problem in a deterministic model with endogenous demand, sensitive to price. Six different scenarios that are the combination of horizontal, vertical, and Bertrand games are considered. The results showed that when retailers offer the same product, the vertical game has more influence on the scenarios' performances than the horizontal game.

Hafezalkotob (2015) studied the effects of government regulations on competing SCs. The government decides tariffs on products first, and the SCs follow. The government's goal is to maximize its profit or minimize pollution, or both. Each SC is composed of one manufacturer and one retailer that offers two substitutable products (regular and green) to customers (each offers one kind of product). The demand for each product decreases in its price and government tariffs and increases in other product's price and government tariffs. He considered six scenarios based on government tendencies and SC structures. The results show that: 1- in centralized or decentralized SCs, the environmental impact increases when the government's goal is to maximize its profit. 2- whatever the government's goal is, centralized SCs have a better environmental impact than decentralized SCs.

Baron et al. (2016) considered a competition between two two-stage SCs. Each SC is composed of one manufacturer and one retailer. They offer two substitutable products in a price-sensitive market. The problem is formulated in a deterministic environment. They focused on the effect of bargaining power within the SCs over wholesale prices. They considered three types of games: 1- Manufacturer Stackelberg; 2- Vertical Integration; and 3- Nash bargaining over the wholesale price. They showed that the first two games are a special case of the third game. They found out that just competition degree affects the optimal bargaining power, and thus, other parameters in the competing SCs have any effect on it. Also, the results revealed that when the SC is not alone in the market, vertical integration does not coordinate the SC, while manufacturer Stackelberg coordinates when the manufacturer has all the bargaining power.

Qi et al. (2017) considered a two-stage MTO supply chain composed of a supplier and two retailers to investigate the optimal pricing strategies under carbon cap policy.

The supplier produces a product and is the primary source of releasing carbon emission. The retailers sell the product to the customers. The retailers offer the same product to customers and compete to maximize their profits. The products' demand is sensitive to the retail price, decreasing in its retail price and increasing in other product's price. The model is deterministic, and a Stackelberg game is used between the supplier and retailers in the decentralized scenario, where the supplier is the leader and retailers are the followers. In the coordination scenario, they considered two settings: centralized and transfer payment mechanism. They analyzed and compared the performance of scenarios. The results showed that the best pricing decision for supply chain members could be achieved when wholesale prices (for both retailers) are the same, and retail prices are different from a transfer payment mechanism.

Jamali and Rasti-Barzoki (2018) considered two two-stage SC that produces two substitutable products and offer them to customers at a price- and greenness- sensitive market. They considered that one product is traditional, which they do not change its greenness level, and another one is green, in which the greenness level is considered as a decision variable. The problem is formulated in a deterministic environment for two scenarios; Centralized and Decentralized. They solved the models by an analytical approach and provided the closed-form expressions of optimal solutions. The results showed that the centralized setting provides more profit for the SC and higher greenness for the green product.

Zhou et al. (2018), which is presented earlier in the previous sub-section, considered a competition between N retailers who offer substitutable products. As we mentioned, the SC includes government, manufacturer, and retailers. The problem consists of retailers' pricing decisions. In a scenario with multiple retailers, the government has to impose the carbon tax policy in order to decrease social welfare losses.

These studies by McGuire and Staelin (2008), Qi et al. (2017), and Zhou et al. (2018), however, do not consider a greenness-sensitive demand and do not investigate the greenness-based competition (in both studies, the greenness level is not a decision variable). Moreover, they assumed a deterministic setting which does not fit with most retail operations.

Giri et al. (2019) studied the effects of the government's policies on a two-stage SC that includes two manufacturers and one retailer. The manufacturers follow the MTO policy. The substitutable products, produced by manufacturers, are sold through a common retailer. The government's goal is to maximize its profit (not maximizing social welfare) by imposing carbon taxes, caps, and trade policies. They formulated the problem as a non-linear bi-level model such that the government is the leader of the Stackelberg game in a deterministic environment. The manufacturers horizontally follow Nash-equilibrium. Vertically, manufacturers are the leader in Stackelberg's leader-follower decisions and cooperating scenarios.

Sim et al. (2019) considered two two-stage SCs; each SC includes one manufacturer and one retailer. The products produced by manufacturers are sold through retailers. Each retailer offers only one product. The demand for each product depends on the products' quantity. The objective is to maximize social welfare. In their model, the manufacturers decide wholesale prices and abatement efforts concerning pollution emissions related to manufacturing processes, whereas the retailers compete in quantities instead of prices. Vertical integration generally results in higher social welfare but more polluting emissions than vertical competition, whereas horizontal integration leads to lower welfare and lower emissions than horizontal competition.

Heydari et al. (2019) considered a three-stage supply chain that consists of a manufacturer, a distributor, and a retailer. The products are offered to customers either directly by the distributor (e-retailing) or by the retailer. The demand for each channel (distributor or retailer) is sensitive to the products' price and green level. The problem is modeled in different structures; open triad, the closed triad, and transitional triad that guarantees the manufacturer's profit. The results show that the closed triad provides more profits than the open triad to the supply chain.

Xu et al. (2020) considered different game models based on model structures that consist of two manufacturers and two retailers. The products produced by the two manufacturers are substitutable. In the model, the government subsidizes consumers who buy low carbon products but imposes a carbon tax on the manufacturer producing high carbon products. They found out that horizontal integration decreases competition. As a result, it causes higher prices and a lower value of social welfare.

Karray et al. (2020) investigated the pricing strategies of two retailers to answer the following question. "Can ignoring cross-category effect be a smart choice?". The retailers compete in the market to maximize their profit by offering two products. They categorized products' relations such as cross-category pricing effects are positive (complementary), negative (substitutable), or null (independent). The demand function is sensitive to the prices. Precisely, the product's demand is decreasing in both products' prices for the complementary category, while it is decreasing in its price and increasing in other product's price for substitutable products. A deterministic mathematical model is developed for different scenarios: 1- when both retailers choose to disregard cross-category effects; 2- when both retailers account for cross-category effects; and 3- when one retailer disregards cross-category effects while the other does not. The results revealed that higher complementary effects could invert prices' sensitivity to substitution levels within a category. The results also revealed that, in particular, ignoring the cross-category effect leads to lower prices when the two categories are substitutable or highly complementary and to higher prices otherwise.

The study that comes closest to our work is Liu et al. (2012). This study investigated the impact of CEA and competition intensity on the decision-makers' profits in different SC structures (one manufacturer and one retailer, two manufacturers and one retailer, and two manufacturers and two retailers). The manufacturer decides the greenness level and the wholesale price. The retailer decides its selling price. A Stackelberg game is used to model the manufacturer's problems as a first-mover and the retailer as a follower. The demand for each product linearly decreases in its price and increases in its greenness level. It also increases in the other product's price and decreases in the other product's greenness level. The main findings suggested that, as CEA increases, retailers and manufacturers with superior eco-friendly operations will benefit. In contrast, the inferior eco-friendly firm's profitability will tend to increase if the production competition level is low and decrease if the production competition level is high. Our study differs from that of Liu et al. (2012) in many aspects. First, we focus on the retailer's problem and consider that the retailer undertakes all the decisions (greenness level and price of the product and inventory decisions) while Liu et al. (2012) focused on the manufacturer's problem and assumed that the retailer's role is to price the product. Second, we consider the retailers' inventory decisions in a context of stochastic demand and stochastic inventory replenishment time. Liu et al. (2012), however, ignored the inventory and replenishment aspects, which does not fit with the retail context. Moreover, although there is a random parameter in their demand function, they worked with an average demand equivalent to a deterministic setting. Stockouts are, thus, not considered in their models. Third, concerning managerial implications, our main objective is to investigate the impact of the greenness- and price-based competition on the greenness of products, while Liu et al. (2012) analyzed the profits of the different SC actors.

To highlight our work's contribution, Table 2.2 presents a summary of the existing literature and our work that has been done to close the gap of literature in Chapter 5.

Table 2.2: Competition comparison table

Paper	Demand			Decision Variables				Model		Product	
	Price	Carbon emission	Other	Price	Carbon emission	Order size	Other	Deterministic	Stochastic	One	Substitutable
Cachon (2001)						•	•		•	•	
Dai et al. (2005)	•			•				•	•		•
McGuire and Staelin (2008)	•			•				•			•
Xiao and Yang (2008)	•		•	•				•	•		•
Wu et al. (2009)	•			•		•			•		•
Liu et al. (2012)	•	•		•	•			•			•
Wu et al. (2012)	•			•				•		•	
Hafezalkotob (2015)	•		•	•			•	•			•
Baron et al. (2016)	•			•			•	•			•
Qi et al. (2017)	•			•				•		•	
Zhou et al. (2018)	•		•	•			•	•			•
Jamali and Rasti-Barzoki (2018)	•	•		•	•			•			•
Giri et al. (2019)	•	•		•	•		•	•			•
Sim et al. (2019)			•	•	•	•		•			•
Heydari et al. (2019)	•	•		•	•			•	•	•	
Xu et al. (2020)	•			•				•			•
Karray et al. (2020)	•			•				•			•
Our Study	•	•		•	•	•			•		•

The results of the studies that had been done bring so many helpful references and great ideas for further studies in the green supply chain, but rarely combine consumer environmental awareness with dynamic competition in a stochastic environment. Approaching the green production process is important to construct supply chains with environmental benefits and high economic efficiency when we speak of green products. So, we are focusing on optimizing the carbon emission production process. Therefore, our works will fill the lack of green process consideration in the stochastic environment under SCs (players) competition.

In chapter 3, we present the benchmark model that is our initial model. We study an optimal retailer strategy that offers a product in a price- and greenness- sensitive market. The problem is formulated in a stochastic environment. Chapter 4 extends the benchmark model. In this chapter, we will study product differentiation. The retailer offers two substitutable products and decides price, greenness, and order size for each of them. In chapter 5, we study product differentiation in a competition context. We consider two retailers that each offers one product. We discuss the dynamic competition between them and their optimal strategies. Chapter 6 presents new demands functions (a general demand in the function of greenness, specifically non-linear demand). Finally, in chapter 7, we conclude the works that have been done in this thesis and provide future work directions.

CHAPTER 3

Benchmark Model

This chapter studies a retailer's profit-maximizing problem as an initial and benchmark model for the rest of the thesis. We will compare the results of the remaining chapters to this chapter's results. This first case also allows us to present the main ideas that we will use to solve analytically future problems. We consider one retailer that sells one product in a price- and greenness- sensitive market. The retailer decides the product's price, carbon emission level, and order size in order to maximize the total expected profit. Customers arrive according to a Poisson process with mean arrival rate of λ. The mean demand for the product is linearly decreasing in its carbon intensity and price. The demand function will be introduced and discussed later in this chapter. The retailer has his/her supplier. The main sources of carbon emission are transportation and production activities. The transportation emission per unit of product, denoted by e, depends on the distance traveled by the product from the supplier's site to the retailer's warehouse. As we do not deal with supplier selection, e is not a decision variable. Without losing generality, we will assume that the retailer's warehouse's transportation emissions to end customers can be neglected, which implicitly assumes that the retailer is located close to the demand zone. However, we keep e to show the transportation carbon emission's effect on the retailer's decisions. As for production emissions, we consider a context where the retailer can choose the production emission level and ask the supplier to manufacture the product accordingly. This context fits with the practical examples provided in Section 1 to illustrate how leading retailers, such as Walmart and Bestseller, are asking their suppliers to produce greener products. We let x_0 denote the amount of production emissions per unit of a standard product produced by a standard process. Offering a greener product (with lower production emissions) implies a higher purchasing cost for the retailer as it incurs a higher manufacturing cost. We let x denote the amount of production emissions per unit of product. The carbon intensity is thus given by $x+e$. The unitary purchasing cost of a product is given by $c+b(x_0-x)^2$, where c is the unitary cost of the standard product, and b is the cost factor of the production's emission reduction. We consider a quadratic cost function as usual in the related literature (e.g., Liu et al., 2012, Ghosh and Shah, 2015).

3.1 Problem description

We consider a retailer who orders a product from his/her supplier and sells to customers who consider the product's price and carbon intensity in their purchase decision. Figure 3.1 shows the considered supply chain.

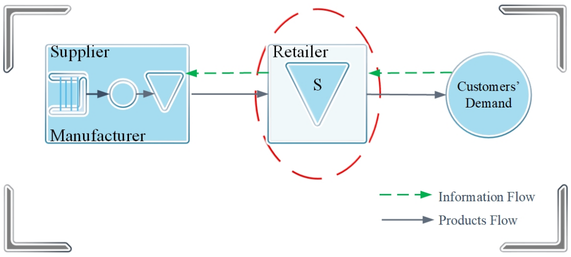

Figure 3.1: Benchmark SC

The customer's arrival follows a Poisson distribution with a rate of λ. Our demand model is linear in the function of price and greenness. A denotes the market potential. We let α_p and β_e respectively denote the market sensitivity to the price and the carbon intensity. The mean demand rate is given as follows.

$$\lambda = A - \alpha_p p - \beta_e (x + e) \tag{3.1}$$

To better understand our demand function, be aware that the retailer can attract α_p more customers with one unit decrease in price and β_e more customers with one unit decrease in carbon intensity.

S denotes the replenishment order size of the retailer. The service rate to refill the stock is exponentially distributed, with a rate of μ. The replenishment time does not depend on the order size since the product is assumed to be always available at the supplier's site, which is a common assumption (Zhu, 2015). Thus, the replenishment time corresponds basically to preparation and transportation activities. Figure 3.2 shows the Queueing network.

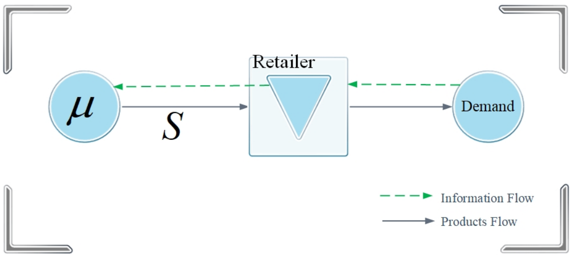

Figure 3.2: Benchmark queuing network

The retailer makes an order to fill inventory from a supplier that adopts a MTS manufacturing policy. The inventory is managed according to the standard (q,S) policy where q is the reorder point, and S is the replenishment lot size. To simplify the analysis, we assume that the retailer places a replenishment order when there is no stock (product) in the warehouse. During the replenishment time, demand is satisfied with safety stock. We assume that the safety stock is large enough to satisfy all demands in the vast majority of cases and ignore the rare cases in which demand cannot be satisfied. The sizing and management of the safety stock are out of the scope of this study.

We have the main stock that serves the customers in $(1-r)\%$ of the time. A safety stock serves the customer that faces an empty main stock. To simplify the analysis without losing the system's main trade-offs, we approximate the retailer's warehouse's stock-out probability with the stock-out probability obtained for $q=0$. This approximation is accurate when the value of q considered in practice is relatively small, which is the case in many warehouses with storage capacity constraints. When the value of q is relatively high, the impact of this approximation on the model's outcomes can be offset by increasing the minimum service level, $1-r$, to a higher value than that usually used in practice. The inventory refilling policy used to calculate the stock-out probability is illustrated in figure 3.3.

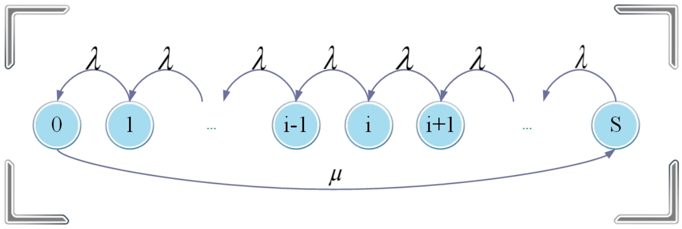

Figure 3.3: Inventory refilling policy used to approximate the stock-out probability

As demand and inventory replenishment time are stochastic, there is a probability that the main stock is stock-out. We let ψ denote the probability of stock-out of product. To avoid unrealistic inventory levels, we impose that the probability of satisfying demand from the main stock, denoted by $1-\psi$ for the retailer, must be greater than a predetermined minimum service level. We consider the minimum service level, $1-r$, for the retailer. Hence, the service level constraint for retailer is given by $1-\psi \geq 1-r$, which is equivalent to $\psi \leq r$. Notice that ψ represents the probability of having zero items in the main stock when a customer arrives. Thus, it also refers to the probability of serving this customer from the safety stock.

Since customers' arrival and inventory refilling service time follow the Exponential distribution with mean rates λ and μ, respectively, the number of part in main stock is a Markovian process. In this case, we calculate the probability of stock-out, in which there is no item in the warehouse, using continuous Markov chain process properties. Regarding Figure 3.3, let us define ψ_i, which stands for the probability of system in state i (i.e., having i item in stock). For each state, the classical balance equations give:

for state 0: $\quad \mu\psi_0 = \lambda\psi_1 \Leftrightarrow \psi_1 = \dfrac{\mu}{\lambda}\psi_0$

for state 1 to S-1: $\quad \lambda\psi_i = \lambda\psi_{i+1} \Leftrightarrow \psi_i = \psi_{i+1}$

for state S: $\quad \mu\psi_0 = \lambda\psi_S \Leftrightarrow \psi_S = \dfrac{\mu}{\lambda}\psi_0$

The summation of states' probability should be equal to 1. Therefore:

$$\sum_{i=0}^{S}\psi_i = 1 \Leftrightarrow \psi_0+\psi_1+\cdots+\psi_S = 1 \Leftrightarrow \psi_0+\frac{\mu}{\lambda}\psi_0+\frac{\mu}{\lambda}\psi_0+\cdots+\frac{\mu}{\lambda}\psi_0 = 1$$
$$\Leftrightarrow \psi_0+S\frac{\mu}{\lambda}\psi_0 = 1 \Leftrightarrow \psi_0(1+S\frac{\mu}{\lambda}) = 1 \Leftrightarrow \psi_0 = (1+S\frac{\mu}{\lambda})^{-1} = \frac{\lambda}{\lambda+\mu S}$$

Hence, the service level constraint for retailer is given by $\frac{\lambda}{\lambda+S\mu} \leq r$. Also, we are going to calculate the expected inventory level.

$$\overline{S} = \sum_{i=0}^{S} i\psi_i = \psi_1+2\psi_2+3\psi_3+\ldots+S\psi_S$$
$$\Leftrightarrow \overline{S} = \frac{\mu}{\lambda}\psi_0+2\Big(\frac{\mu}{\lambda}\psi_0\Big)+3\Big(\frac{\mu}{\lambda}\psi_0\Big)+\ldots+S\Big(\frac{\mu}{\lambda}\psi_0\Big) = \Big(\frac{S(S+1)}{2}\Big)\Big(\frac{\mu}{\lambda}\psi_0\Big)$$
$$\Leftrightarrow \overline{S} = \Big(\frac{S+1}{2}\Big)\Big(\frac{\mu}{\frac{\lambda}{S}+\mu}\Big) = \Big(\frac{S+1}{2}\Big)\Big(\frac{1}{1+\frac{\lambda}{\mu S}}\Big)$$

The expected inventory level is a complex equation in function of S. However, by considering a simple assumption such that $S \gg \frac{\lambda}{\mu}$ (which is a logical assumption; otherwise, the system often faces empty stock), a close approximation that is widely used in the inventory literature (e.g., Cargal, 2003) can be considered. Thus, the expected value of the average inventory level for the retailer can be given by $\frac{S}{2}$. We let $2h$ denote the unit inventory cost. Thus, the inventory cost for the retailer is hS.

The parameters and the decision variables are presented in the following.

Parameters:

A	: Market potential,
c	: Unit fixed cost of the product,
μ	: Mean refilling rate of the product,
$2h$	: Unit inventory holding cost for product,
x_0	: Standard production process carbon emission,
e	: Product's transportation carbon emission,
b	: Carbon emission reduction cost factor,
r	: Maximum guaranteed stock-out.

Independent Decision Variables:

x	: Production carbon emission of product,
p	: Retail price
S	: Order size.

Dependent Decision Variables:

λ	: Mean demand rate for product (given by Equation 3.1),
ψ	: The probability of stock out of product.

3.2 Mathematical model and analytical solution

In this benchmark model, the retailer is alone in the market. The problem consists of deciding price p, carbon emission level x, and order size S to maximize the retailer's expected profit. A single product is offered to customers in this case; let us recall that the mean demand rate is given by equation 3.1. Let us define $A' = A - \beta_e e$ since e is known. The model (M_0) is given below.

$$\underset{x,p,S}{\text{Maximize}}\ \pi = \Big(p - \big(c - b(x_0 - x)^2\big)\Big)\lambda - hS \tag{3.2}$$

Subject to

$$\psi = \frac{\lambda}{\lambda + S\mu} \leq r \tag{3.3}$$

$$\lambda = A' - \alpha_p p - \beta_e x \tag{3.4}$$

$$\lambda, p, S \geq 0, 0 \leq x \leq x_0 \tag{3.5}$$

The objective is to maximize the total expected profit given in equation 3.2. This profit is equal to the revenue (i.e., $p\lambda$) – the procurement cost (i.e., $(c + b(x_0 - x)^2)\lambda$) – the inventory cost (i.e., hS). Constraint 3.3 is the service level constraint. It ensures that stock-out probability does not exceed a predetermined level of r (i.e., $1 - r$ is the minimum service level). The mean demand rate is given in equation 3.4. Constraint 3.5 presents decision variables and demand positivity. To solve this model, we first transform it into a single-variable model based on the results of Lemma 3.1 and 3.2 given below.

Lemma 3.1. *For any given values of p and x, service level constraint 3.3 is binding and the optimal order size is* $S^*(p,x) = \dfrac{(1-r)(A' - \alpha_p p - \beta_e x)}{r\mu}$.

Proof. The first derivative of profit function, π, with respect to order size, S, is negative (i.e. $\dfrac{\partial \pi}{\partial S} < 0$). Considering that the second derivative of profit function with respect to S is zero (i.e. $\dfrac{\partial^2 \pi}{\partial S^2} = 0$), then, the smallest S is the optimal solution. Regarding to service level constraint (equation 3.3), we have $S \geq \dfrac{(1-r)(A' - \alpha_p p - \beta_e x)}{r\mu}$. Therefore the optimal order size is $S^*(p,x) = \dfrac{(1-r)(A' - \alpha_p p - \beta_e x)}{r\mu}$. ■

Thus, we replace S^* with its expression given in Lemma 3.1 and obtain the following equivalent formulation of the model (M_0) with two variables p and x.

$$\underset{x,p}{\text{Maximize}}\ \pi = \left(p - \left(c - b(x_0 - x)^2 + \frac{h(1-r)}{r\mu}\right)\right)\lambda \tag{3.6}$$

Subject to

$$\lambda = A' - \alpha_p p - \beta_e x$$
$$\lambda, p \geq 0,\ x \leq x_0$$

In the following Lemma, we determine the optimal price for any given carbon emission of x.

Lemma 3.2. *For any given value of x, the optimal price is*
$$p^*(x) = \frac{\alpha_p b x^2 - (2\alpha_p b x_0 + \beta_e)x + A' + \alpha_p(c + b x_0^2 + \frac{h(1-r)}{r\mu})}{2\alpha_p}.$$

Proof. The second derivative of π with respect to p, which is presented in the following, is negative and, thus, the profit function is concave in p. Therefore, the root of the first derivative maximizes the objective function. Let us call this value p^{*max}.

$$\frac{\partial \pi}{\partial p} = -2\alpha_p p + \alpha_p b x^2 - (2\alpha_p b x_0 + \beta_e)x + A' + \alpha_p(c + b x_0^2 + \frac{h(1-r)}{r\mu})$$

$$\frac{\partial^2 \pi}{\partial p^2} = -2\alpha_p < 0$$

$$\frac{\partial \pi}{\partial p} = 0 \leftrightarrow p^{*max} = \frac{\alpha_p b x^2 - (2\alpha_p b x_0 + \beta_e)x + A' + \alpha_p(c + b x_0^2 + \frac{h(1-r)}{r\mu})}{2\alpha_p}$$

If $p^{*max} > 0$ and $\lambda(p^{*max}) \geq 0$, then it can be considered as the optimal price. The condition that p^{*max} is positive is presented in the following.

$$p^{*max} = \frac{\alpha_p b x^2 - (2\alpha_p b x_0 + \beta_e)x + A' + \alpha_p(c + b x_0^2 + \frac{h(1-r)}{r\mu})}{2\alpha_p} > 0$$

$$\leftrightarrow \alpha_p b x^2 - (2\alpha_p b x_0 + \beta_e)x + A' + \alpha_p(c + b x_0^2 + \frac{h(1-r)}{r\mu}) > 0$$

The discriminant of the above equation is equal to $\Delta_1 = -4\alpha_p b\Big(A' - \beta_e x_0 - \frac{\beta_e^2}{4\alpha_p b} + \alpha_p\big(c + \frac{h(1-r)}{r\mu}\big)\Big)$. Let us notice that we consider assumptions such that the market potential is sufficiently high to satisfy forthcoming conditions (in this thesis), which $A' = A - \beta_e e > \beta_e x_0 + \frac{\beta_e^2}{4\alpha_p b} - \alpha_p\big(c + \frac{h(1-r)}{r\mu}\big)$ is one of them. Thus, the Δ_1 is negative and there is no root. Because $\alpha_p b$ (coefficient of x^2) is positive and Δ_1 is negative, therefore the equation is always positive. Therefore the p^{*max} is always positive. The next condition that need to be satisfied is $\lambda(p^{*max}) \geq 0$.

$$\lambda \geq 0 \leftrightarrow \frac{-\alpha_p b x^2 + (2\alpha_p b x_0 - \beta_e)x + A' - \alpha_p(c + bx_0^2 + \frac{h(1-r)}{r\mu})}{2\alpha_p} \geq 0$$

$$\leftrightarrow -\alpha_p b x^2 + (2\alpha_p b x_0 - \beta_e)x + A' - \alpha_p(c + bx_0^2 + \frac{h(1-r)}{r\mu}) \geq 0$$

The discriminant of the above equation is equal to $\Delta_2 = 4\alpha_p b\Big(A' - \beta_e x_0 - \frac{\beta_e^2}{4\alpha_p b} + \alpha_p\big(c + \frac{h(1-r)}{r\mu}\big)\Big)$. $\Delta_2 = -\Delta_1$. Earlier we assume that $A' > \beta_e x_0 + \frac{\beta_e^2}{4\alpha_p b} - \alpha_p(c + \frac{h(1-r)}{r\mu})$, therefore, the Δ_2 is positive and the above equation has two roots (called R_1 and R_2). The demand is positive between these two roots and negative outside them.

$$R_1 = \frac{-(2\alpha_p b x_0 - \beta_e) - \sqrt{\Delta_2}}{-2\alpha_p b} = x_0 + \frac{-\beta_e + \sqrt{\Delta_2}}{2\alpha_p b}$$

$$R_2 = \frac{-(2\alpha_p b x_0 - \beta_e) + \sqrt{\Delta_2}}{-2\alpha_p b} = x_0 - \frac{\beta_e + \sqrt{\Delta_2}}{2\alpha_p b}$$

We have $R_1 > x_0$ and $R_2 < 0$ (see Appendix B for more details). Therefore, demand is positive in the feasible region ($[0, x_0]$). Considering the condition that are mentioned before $p^{*max} > 0$ and $\lambda(p^{*max}) \geq 0$, thus, we can consider p^{*max} as optimal price (p^*). ■

We can now replace p^* with its expression given in Lemma 3.2 and obtain the following equivalent formulation of the model (M_0) with only one variable x.

$$\underset{0 \leq x \leq x_0}{\text{Maximize}}\ \pi = \frac{\Big(-\alpha_p b x^2 + (2\alpha_p b x_0 - \beta_e)x + A' - \alpha_p\big(c + bx_0^2 + \frac{h(1-r)}{r\mu}\big)\Big)^2}{4\alpha_p} \tag{3.7}$$

Subject to

$$\frac{-\alpha_p b x^2 + (2\alpha_p b x_0 - \beta_e)x + A' - \alpha_p\big(c + bx_0^2 + \frac{h(1-r)}{r\mu}\big)}{2} \geq 0 \tag{3.8}$$

We finally solve this single-variable model and derive the optimal solution in Proposition 3.1.

Proposition 3.1. *The optimal solution of benchmark model (M_0) is the following.*
$x^* = max\{0, x_0 - \frac{\beta_e}{2\alpha_p b}\}$,

$$p^* = \begin{cases} \dfrac{A-\beta_e e+\alpha_p\left(c+bx_0{}^2+\dfrac{h(1-r)}{r\mu}\right)}{2\alpha_p} & for\ x^*=0 \\ \dfrac{A-\beta_e(x_0+e)+\dfrac{3\beta_e{}^2}{4\alpha_p b}+\alpha_p\left(c+\dfrac{h(1-r)}{r\mu}\right)}{2\alpha_p} & for\ x^*\neq 0 \end{cases},$$

$$S^* = \begin{cases} \dfrac{(1-r)\left(A-\beta_e e-\alpha_p\left(c+bx_0{}^2+\dfrac{h(1-r)}{r\mu}\right)\right)}{r\mu} & for\ x^*=0 \\ \dfrac{(1-r)\left(A-\beta_e(x_0+e)+\dfrac{\beta_e{}^2}{4\alpha_p b}-\alpha_p\left(c+\dfrac{h(1-r)}{r\mu}\right)\right)}{r\mu} & for\ x^*\neq 0 \end{cases}, \text{ and}$$

$$\pi^* = \begin{cases} \dfrac{\left(A-\beta_e e-\alpha_p\left(c+bx_0{}^2+\dfrac{h(1-r)}{r\mu}\right)\right)^2}{4\alpha_p} & for\ x^*=0 \\ \dfrac{\left(A-\beta_e(x_0+e)+\dfrac{\beta_e{}^2}{4\alpha_p b}-\alpha_p\left(c+\dfrac{h(1-r)}{r\mu}\right)\right)^2}{4\alpha_p} & for\ x^*\neq 0 \end{cases}.$$

Proof. Considering equation 3.8, the objective function (equation 3.7) can be presented as $\frac{\lambda^2}{\alpha_p}$. Since $\lambda \geq 0$, then maximizing λ is equivalent to maximizing $\frac{\lambda^2}{\alpha_p}$ (profit function). The second derivative of demand with respect to x, which is presented in the following, is negative and the root of the first derivative, called x^{*max}, maximizes the demand (as well as objective function).

$$\frac{\partial\lambda}{\partial x} = \frac{-2\alpha_p bx + (2\alpha_p bx_0 - \beta_e)}{2}$$

$$\frac{\partial^2\lambda}{\partial x^2} = -\alpha_p b < 0$$

$$\frac{\partial\lambda}{\partial x} = 0 \leftrightarrow x^{*max} = x_0 - \frac{\beta_e}{2\alpha_p b}$$

It is clear that $x^{*max} < x_0$. While the x^{*max} is in the feasible region, the optimal solution is equal to x^{*max} ($x^* = x^{*max}$), otherwise, if $x^{*max} < 0$ then the optimal solution is equal to zero. So $x^* = max\{0, x_0 - \frac{\beta_e}{2\alpha_p b}\}$.

We obtain p^* by substituting x^* into 3.2. Then, we can obtain S^* by substituting x^* and p^* into Lemma 3.1. ■

We deduce that the optimal carbon emission linearly decreases in CEA (i.e., β_e). The optimal price is convex functions in CEA. It first decreases until a threshold value of CEA ($\frac{\partial p^*}{\partial \beta_e} = 0 \Leftrightarrow \beta_e = \frac{2}{3}\alpha_p b(x_0 + e)$), and then, increases. The calculations to obtain this threshold value are given in the following.

$$\frac{\partial p^*}{\partial \beta_e} = \begin{cases} \frac{-e}{2\alpha_p} & \text{for } x^* = 0 \\ \frac{-(x_0+e) + \frac{6\beta_e}{4\alpha_p b}}{2\alpha_p} & \text{for } x^* \neq 0 \end{cases}$$

$$\frac{\partial^2 p^*}{\partial {\beta_e}^2} = \begin{cases} 0 & \text{for } x^* = 0 \\ \frac{3}{4{\alpha_p}^2 b} & \text{for } x^* \neq 0 \end{cases}$$

In the case of $x^* = 0$, the optimal price is linearly decreasing in CEA ($\frac{\partial p^*}{\partial \beta_e} < 0$ and $\frac{\partial^2 p^*}{\partial {\beta_e}^2} = 0$). However in the case of $x^* \neq 0$, the optimal price is convex ($\frac{\partial^2 p^*}{\partial {\beta_e}^2} > 0$), and it is decreasing until $\beta_e = \frac{2}{3}\alpha_p b(x_0 + e)$ and increasing after that. In addition, we deduce that the optimal price and order size are linearly decreasing in transportation carbon emission (i.e., e). It is an intuitive result. Increasing e means that retailer offers dirtier product (i.e., higher carbon intensity). Increasing one unit of e leads to increasing carbon intensity (dirtier product) that makes decreasing $\frac{\beta_e}{2\alpha_p}$ unit of price and $\frac{\beta_e(1-r)}{r\mu}$ of order size.

Proposition 3.2. *The optimal profit is convex with respect to β_e and increasing β_e leads profit to zero.*

Proof. The first and second derivatives of π^* with respect to β_e are presented in the following.

$$\frac{\partial \pi^*}{\partial \beta_e} = \frac{-(x_0 - \frac{\beta_e}{2\alpha_p b})\left(A - \beta_e(x_0+e) + \frac{{\beta_e}^2}{4\alpha_p b} - \alpha_p\left(c + \frac{h(1-r)}{r\mu}\right)\right)}{2\alpha_p}$$

The first derivative is negative that means the optimal profit is decreasing.

$$\frac{\partial^2 \pi^*}{\partial {\beta_e}^2} = \frac{(x_0 - \frac{\beta_e}{2\alpha_p b})^2 + \frac{1}{2\alpha_p b}\left(A - \beta_e(x_0+e) + \frac{{\beta_e}^2}{4\alpha_p b} - \alpha_p\left(c + \frac{h(1-r)}{r\mu}\right)\right)}{2\alpha_p} > 0$$

The second derivative is positive when $A > \beta_e(x_0+e) - \frac{{\beta_e}^2}{4\alpha_p b} + \alpha_p\left(c + \frac{h(1-r)}{r\mu}\right)$. We recall that this condition is already considered in Lemma 3.2. Therefore, optimal profit has a decreasing convex shape in β_e. ■

As we mentioned earlier, the optimal carbon emission is linearly decreasing in β_e and leads to zero. Thus, the optimal demand also goes to $\lambda = A - \alpha_p p$. However, decreasing optimal carbon emission means increasing total production cost and consequently increasing the optimal price (after a threshold value that we mentioned earlier). Finally, increasing β_e is decreasing product's demand and the margin profit that leads to decreasing profit. As an illustration, figure 3.4 shows the behavior of optimal solutions in β_e. To illustrate what it is explained, we consider the following numerical example: $A = 1000$, $\alpha_p = 8$, $c = 20$, $x_0 = 100$, $\mu = 30$, $b = 0.01$, $h = 5$, $r = 0.05$. We vary β_e from 2 to 10.

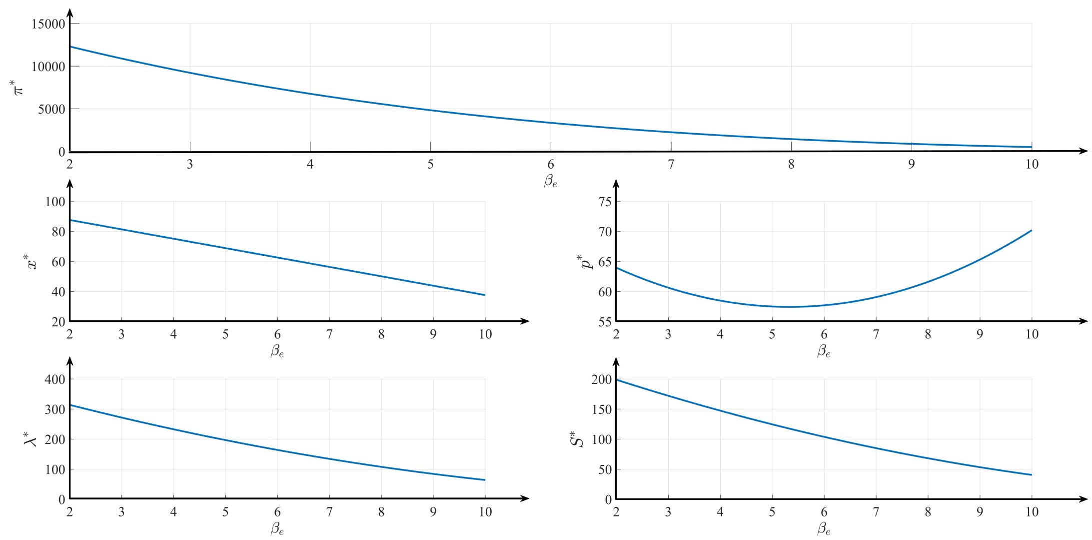

Figure 3.4: Optimal solutions' behavior in function of β_e

3.3 Conclusion

This chapter contains a retailer's profit maximization problem in a price- and greenness-sensitive market. The product's demand is a random variable that follows a Poisson distribution with a mean rate of λ. The mean rate is decreasing in the product's price and carbon intensity level. The retailer keeps the product to serve the customers immediately. The refilling time also is a random variable that follows Exponential distribution with a mean rate of μ. The problem is formulated in a stochastic environment. We use an analytical approach to solve the problem and obtain the optimal solutions' expressions. The optimal carbon emission level is linearly decreasing in β_e and non-linearly increasing in α_p and b. However, the optimal price is convex with respect to β_e, and it is decreasing until a threshold value, and after that, it is increasing.

CHAPTER 4

Product differentiation in retailing under price- and greenness-sensitive demand

The previous chapter discussed the retailer's profit maximization problem, while the retailer offers one product to customers. The results showed the impacts of carbon emission and price sensitivity factors on optimal profit, price, carbon emission level, and order size. In this chapter, we are going to investigate the retailer's best strategies when he/she offers a new and substitutable product in a price- and greenness- sensitive market, in addition to the existing product. The new product is the same as the first product in terms of performance, function, and usage, i.e., substitutable. However, they can differ from pricing and carbon intensity. Let us define the first product as the "Existing" product and the second one as the "New" product. The suppliers send products to the retailer's warehouse, which is near to customers. The retailer keeps products to serve the customers as soon as one arrives. The demand for each product not only depends on its price and carbon emission intensity but also depends on the other product's price and carbon emission intensity. We consider a retailer-maximizing profit problem that is formulated in a stochastic environment. The main important questions of this chapter are:

- How much the retailer benefit from offering a new substitutable product?
- How product differentiation affects the retailer's best strategies?
- Do different market structures affect the retailer's best strategies?

The problem description is presented in the following. We discuss the model's assumptions and develop the general model framework. We consider different settings (sets of decisions) based on the general model and solve them by an analytical approach. After that, we derive analytical and numerical insights. Finally, the conclusion of this chapter is presented.

4.1 Problem description

As we briefly discussed at the beginning of this chapter, we are going to study the problem that a retailer offers two substitutable products. The retailer is already offering a product (called "Existing" product) to customers at a price- and carbon emission- sensitive market and plans to offer a new product (called "New" product). The new product is substitutable to the existing product (i.e., similar in terms of usage, performance, function, etc.). However, they can be different from greenness and price points of view. The products' demands not only depend on their price and carbon emission but also depend on other product's price and carbon emission. Figure 4.1 shows the retailer (red ellipse) and the products' suppliers. The retailer makes orders from two different suppliers. Each supplier prepares one kind of product.

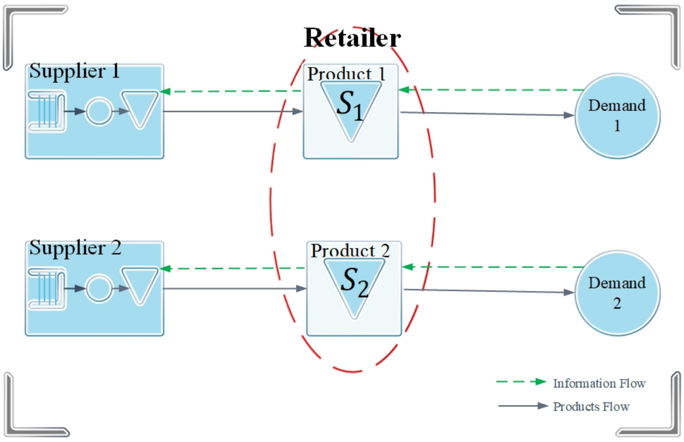

Figure 4.1: Collaboration supply chain

We consider a linear demand in the function of price and carbon intensity with substitution. Customers arrive according to a Poisson process with mean arrival rate λ_i for product i ($i = 1,2$; existing and new product, respectively). A unit of demand corresponds to one customer (order). Each product's mean demand decreases in its carbon intensity and price and increases in other products' carbon intensity and price. The market potential is denoted by $2A$ (which we assume it is equally divided for each product). Parameters α_p and β_e represent the customers' sensitivity to the price and the carbon emission intensity, respectively. Note that β_e represents the CEA. We let α_s and β_s represent the switchover's sensitivity toward price difference and carbon intensity difference, respectively. Similar to Liu et al. (2012) and Xiong et al. (2014), the demand functions given as follows.

$$\lambda_1 = A - \alpha_p p_1 + \alpha_s(p_2 - p_1) - \beta_e(x_1 + e_1) + \beta_s(x_2 + e_2 - x_1 - e_1) \tag{4.1}$$

$$\lambda_2 = A - \alpha_p p_2 + \alpha_s(p_1 - p_2) - \beta_e(x_2 + e_2) + \beta_s(x_1 + e_1 - x_2 - e_2) \tag{4.2}$$

Obviously, $\lambda_1 + \lambda_2 = 2A - \alpha_p(p_1 + p_2) - \beta_e(x_1 + e_1 + x_2 + e_2)$. This implies that total mean demand is sensitive to prices and carbon emission intensities; the switchovers do not affect the total demand.

Decreasing one unit of product i's price attracts $\alpha_p + \alpha_s$ more customers, and decreasing one unit of carbon intensity attracts $\beta_e + \beta_s$ more customers. It is noted that only a part of these customers represent the new created demand in the market (for product i, α_p new customers for one unit decrease in price and β_e new customers for one unit decrease in carbon emission intensity). The other attracted customers are switching from the other product (α_s switching customers for a unit decrease in price and β_s switching customers for a unit decrease in carbon intensity).

The products are supplied by different suppliers. We consider (without loss of generality) that second supplier is closer to the retailer than first supplier. Therefore, $e_1 \geq e_2$. Since e_1 and e_2 are fixed parameters, we can assume, without loss of generality, that $e_2 = 0$ and, thus e_1 can be interpreted as the difference in transportation emissions. In addition, we consider that the existed product's carbon intensity, x_1, is known (i.e.; fixed parameter) in our study. We let $A_1 = A - \beta(x_1 + e_1)$, $A_2 = A + \beta_s(x_1 + e_1)$, $\beta = \beta_e + \beta_s$, $\alpha = \alpha_p + \alpha_s$. Note that $\beta > \beta_e, \beta_s$ and $\alpha > \alpha_p, \alpha_s$. After simplifications, the mean demands are given as follows.

$$\lambda_1 = A_1 - \alpha p_1 + \alpha_s p_2 + \beta_s x_2 \tag{4.3}$$

$$\lambda_2 = A_2 - \alpha p_2 + \alpha_s p_1 - \beta x_2 \tag{4.4}$$

The retailer's inventories policy is similar to the previous chapter. He/She makes an order to fill products' inventory from their supplier that adopts an MTS manufacturing policy. For product i, the inventory is managed according to the standard (q_i, S_i) policy where q_i is the reorder point, and S_i is the replenishment lot size. The service rate to refill the buffer is exponentially distributed with a mean rate of μ_i for product i, which does not depend on the lot size. Thus, the replenishment time corresponds basically to preparation and transportation activities. Recall that we consider the second supplier closer to the retailer. Thus, we have $\mu_2 \geq \mu_1$. The general framework of this chapter's problem is presented in Figure 4.2.

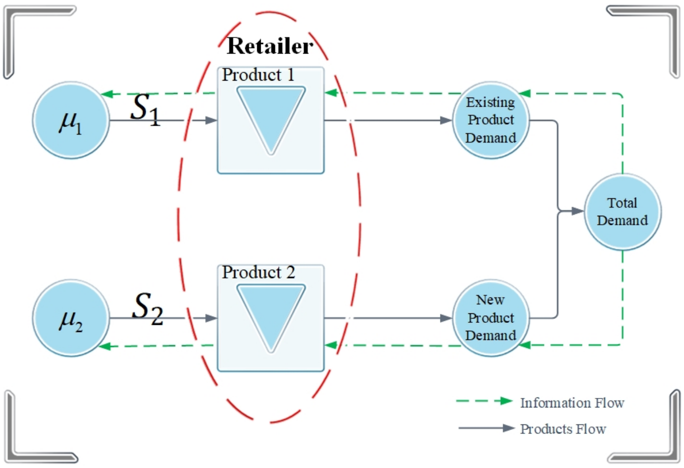

Figure 4.2: General framework

As demands and inventories' replenishment time are stochastic, there is a probability of stock-out for each product. We let ψ_i denote the probability of stock-out of product i. The probability of serving customers from the stock (i.e., $1 - \psi_i$) must be higher than a predetermined minimum service level. We consider the same service level, $1-r$, for both products. This assumption is motivated by the fact that we consider the same activity sector and the same product. To simplify the analysis without losing the system's main trade-offs, we approximate the retailer's warehouse's stock-out probability with the stock-out probability obtained for $q_i = 0$ (see Chapter 3 for more details). The inventory refilling policy used to calculate the stock-out probability is illustrated in Figure 4.3.

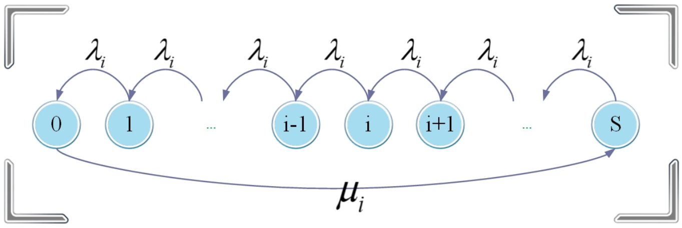

Figure 4.3: Inventory refilling policy used to approximate the stock-out probability

Like the previous chapter, we consider the inventory cost for product i as hS_i, where $2h$ is the unit inventory cost. Since customers' arrival and inventory refilling time follow the exponential distribution, the system follows the Markov chain process. Using continuous Markov chain process properties and the same approach and calculations as previous chapter, we obtain $\psi_i = \left(1 + \frac{\mu_i S_i}{\lambda_i}\right)^{-1}$. The parameters and decision variables that we consider in this chapter are presented in the following.

Parameters:

A	: Each product's market potential,
c_1, c_2	: Unit fixed cost of existing and new products, respectively,
μ_1, μ_2	: Mean refilling rate of existing and new products, respectively,
$2h$	: Unit inventory holding cost for products,
x_0	: Standard production process carbon emission,
x_1	: Existing product's production process carbon emission,
e_1, e_2	: Existing and new products transportation carbon emission of, respectively,
b	: Carbon emission reduction cost factor,
r	: Maximum guaranteed stock-out.

Independent Decision Variables:

x_2	: New product's production process carbon emission,
p_1, p_2	: Retail price of existing and new products, respectively,
S_1, S_2	: Order size of existing and new products, respectively.

Dependent Decision Variables:

λ_1, λ_2	: Mean demand rate for existing and new products (given by Equations 4.3 and 4.4), respectively,
ψ_1, ψ_2	: The probability of stock out of existing and new products, respectively.

We study different scenarios to investigate their impact on the greenness level and price of both products.

1. Retailer offers a new and substitutable product to the market and decides both products' prices, p_1 and p_2, and order sizes, S_1 and S_2. This scenario is the most common behavior of retailers in the real world.

2. The retailer offers a new and substitutable product to the market and decides the new product's carbon emission intensity, x_2, price, p_2, and both products' order sizes, S_1 and S_2. In this scenario, the retailer does not change the existed product price and carbon emission intensity but has the power of asking the second supplier (manufacturer) to provide him/her the product with x_2 carbon emission intensity.

3. Retailer offers a new and substitutable product to the market and decides new product's carbon emission intensity, x_2, both products' prices, p_1 and p_2, and order sizes, S_1 and S_2. This scenario is the general model of the retailer's discussed problem.

We compare the results of the above-described scenarios to investigate the impact of product differentiation.

4.2 Mathematical models and analytical solutions

In this technical section, we solve the problem under different settings (decision variables) and provide the closed-form expressions of the optimal solutions. Then, we analyze these optimal solutions to derive managerial insights.

4.2.1 Prices and stocks optimization

The retailer offers a new product to customers and decides the prices, p_1 and p_2, and the order sizes, S_1 and S_2, of products in order to maximize his/her expected profit. The retailer keeps the existing and new products' carbon intensity (i.e. $x_1 + e_1$ and x_2, respectively). Here, the mean demand for existing (new) product, namely λ_1 (λ_2), depends not only on its price, p_1 (p_2), but also on the new (existing) product's price, p_2 (p_1). However, in this case x_1 and x_2 are known. Therefore, λ_1 is here given by $A'_1 - \alpha p_1 + \alpha_s p_2$ and λ_2 is here given by $A'_2 - \alpha p_2 + \alpha_s p_1$. As x_1 and x_2 are fixed, we let $A'_1 = A - \beta(x_1 + e_1) + \beta_s x_2$ and $A'_2 = A - \beta x_2 + \beta_s(x_1 + e_1)$.

In prices and stocks optimization, the retailer's profit optimization problem is denoted by (M_1). We use the same methodology used for (M_0) to solve the model (M_1). However, the calculation steps are different since we do not have the same demand.

$$\underset{p_1,p_2,S_1,S_2}{\text{Maximize}}\ \pi = (p_1 - c_1)\lambda_1 + (p_2 - c_2)\lambda_2 - hS_1 - hS_2 \tag{4.5}$$

Subject to

$$\psi_1 = \frac{\lambda_1}{\lambda_1 + S_1\mu_1} \leq r \tag{4.6}$$

$$\psi_2 = \frac{\lambda_2}{\lambda_2 + S_2\mu_2} \leq r \tag{4.7}$$

$$\lambda_1 = A'_1 - \alpha p_1 + \alpha_s p_2 \tag{4.8}$$

$$\lambda_2 = A'_2 - \alpha p_2 + \alpha_s p_1 \tag{4.9}$$

$$p_1, p_2, S_1, S_2, \lambda_1, \lambda_2 \geq 0$$

The retailer's objective function (π) is to maximize his total profit (i.e., net profit of selling products – total inventory costs). Since the model is stochastic, the expected value of the stock is equal to $\sum_{i=0}^{S} i\psi_{ij}$ (where ψ_{ij} represents the probability product $j; j = 1,2$, represent the existing and the new product, respectively, of having $i; i = 1,\ldots,S$ items in stock when a random customer arrives). The expected value is too complicated. Therefore we consider a close approximation. The inventory cost is proportional to S, and we consider it as hS_j. Two service level constraints ensure that the probability of facing product j's customer(s) to empty stock is less than or equal r. Other constraints are related to demands, prices, and order sizes positivity. To solve this model, we first transform it into a single-variable model based on the results of Lemma 4.1 and 4.2 given below.

Lemma 4.1. *For any given values of p_1 and p_2, service levels constraints 4.6 and 4.7 are binding and the optimal stocks are* $S_1^* = \dfrac{(1-r)(A'_1 - \alpha p_1 + \alpha_s p_2)}{r\mu_1}$ *and* $S_2^* = \dfrac{(1-r)(A'_2 - \alpha p_2 + \alpha_s p_1)}{r\mu_2}$.

Proof. Proof. Since the objective function is linearly decreasing in S_1, the smallest possible S_1 is the optimal stock. According to the service level constraint, $S_1 \geq \dfrac{1-r}{r\mu_1}\lambda_1$, therefore, the optimal order size of the existing product is $S_1^* = \dfrac{(1-r)(A'_1 - \alpha p_1 + \alpha_s p_2)}{r\mu_1}$, which implies that service level constraint is binding. In the same way, the optimal value of the new product's order size is equal to $S_2^* = \dfrac{(1-r)(A'_2 - \alpha p_2 + \alpha_s p_1)}{r\mu_2}$. ∎

Since we assume that $\lambda_1, \lambda_2 \geq 0$, thus, $S_1^*, S_2^* \geq 0$. Thanks to Lemma 4.1, we can substitute S_1^* and S_2^* by their expressions, $\dfrac{(1-r)(A'_1 - \alpha p_1 + \alpha_s p_2)}{r\mu_1}$ and $\dfrac{(1-r)(A'_2 - \alpha p_2 + \alpha_s p_1)}{r\mu_2}$

respectively, and obtain the following equivalent model with only two variables, p_1 and p_2.

$$\underset{p_1,p_2}{\text{Maximize}}\ \pi = \left(p_1 - \left(c_1 + \frac{h(1-r)}{r\mu_1}\right)\right)\lambda_1 + \left(p_2 - \left(c_2 + \frac{h(1-r)}{r\mu_2}\right)\right)\lambda_2 \tag{4.10}$$

Subject to

$$\lambda_1 = A_1' - \alpha p_1 + \alpha_s p_2 \tag{4.11}$$

$$\lambda_2 = A_2' - \alpha p_2 + \alpha_s p_1 \tag{4.12}$$

$$\lambda_1, \lambda_2, p_1, p_2 \geq 0$$

Lemma 4.2. *For any given value of p_1, new product's optimal price is*
$$p_2^* = \frac{2\alpha_s p_1 + A_2' + \alpha\left(c_2 + \frac{h(1-r)}{r\mu_2}\right) - \alpha_s\left(c_1 + \frac{h(1-r)}{r\mu_1}\right)}{2\alpha}.$$

Proof. The second derivative of the objective function with respect to p_2, which is presented in the following, is negative and demonstrates that the objective function is concave with respect to p_2. Therefore the root of the first derivative (called p_2^{*max}) maximizes the objective function.

$$\frac{\partial\pi}{\partial p_2} = -2\alpha p_2 + 2\alpha_s p_1 + A_2' + \alpha\left(c_2 + \frac{h(1-r)}{r\mu_2}\right) - \alpha_s\left(c_1 + \frac{h(1-r)}{r\mu_1}\right)$$

$$\frac{\partial^2\pi}{\partial {p_2}^2} = -2\alpha < 0$$

$$\frac{\partial\pi}{\partial p_2} = 0 \leftrightarrow p_2^{*max} = \frac{2\alpha_s p_1 + A_2' + \alpha\left(c_2 + \frac{h(1-r)}{r\mu_2}\right) - \alpha_s\left(c_1 + \frac{h(1-r)}{r\mu_1}\right)}{2\alpha}$$

If ${p_2}^{*max} \geq 0$, then it can be considered as the optimal price. The condition that ${p_2}^{*max}$ is positive is presented in the following.

$$p_2^{*max} = \frac{2\alpha_s p_1 + A_2' + \alpha\left(c_2 + \frac{h(1-r)}{r\mu_2}\right) - \alpha_s\left(c_1 + \frac{h(1-r)}{r\mu_1}\right)}{2\alpha} \geq 0$$

$$2\alpha_s p_1 + A_2' + \alpha\left(c_2 + \frac{h(1-r)}{r\mu_2}\right) - \alpha_s\left(c_1 + \frac{h(1-r)}{r\mu_1}\right) \geq 0$$

We know that $p_1 \geq 0$. Then, ${p_2}^{*max} \geq 0$ while $A_2' \geq -\alpha\left(c_2 + \frac{h(1-r)}{r\mu_2}\right) + \alpha_s\left(c_1 + \frac{h(1-r)}{r\mu_1}\right)$, which means that market potential is sufficiently high. The positivity of ${p_2}^{*max}$

is a necessary condition but not sufficient. We have to also verify positivity of λ_1 and λ_2. By substituting ${p_2}^{*max}$ into λ_1, λ_2 we get following equivalent equations.

$$\lambda_1 = \frac{-2(\alpha^2 - \alpha_s^2)p_1 + 2\alpha A'_1 + \alpha_s A'_2 + \alpha\alpha_s\Big(c_2 + \dfrac{h(1-r)}{r\mu_2}\Big) - \alpha_s^2\Big(c_1 + \dfrac{h(1-r)}{r\mu_1}\Big)}{2\alpha} \geq 0$$

$$\lambda_2 = \frac{A'_2 - \alpha\Big(c_2 + \dfrac{h(1-r)}{r\mu_2}\Big) + \alpha_s\Big(c_1 + \dfrac{h(1-r)}{r\mu_1}\Big)}{2} \geq 0$$

The existing product's demand, λ_1, is positive while $p_1 \leq \frac{1}{2(\alpha^2 - \alpha_s^2)}\Bigg(2\alpha A'_1 + \alpha_s A'_2 + \alpha\alpha_s\Big(c_2 + \frac{h(1-r)}{r\mu_2}\Big) - \alpha_s^2\Big(c_1 + \frac{h(1-r)}{r\mu_1}\Big)\Bigg)$. The new product's demand, λ_2, is positive while we consider that $A'_2 \geq \alpha\Big(c_2 + \frac{h(1-r)}{r\mu_2}\Big) - \alpha_s\Big(c_1 + \frac{h(1-r)}{r\mu_1}\Big)$. Again, these conditions are satisfying when we consider sufficiently high market potential. Under consideration of above conditions, ${p_2}^{*max}$ is considered as optimal solution. ■

Finally, we substitute p_2^* with its expression, given in Lemma 4.2, and obtain the following equivalent single-variable model.

$$\text{Maximize}_{p_1 \geq 0}\ \pi = \Big(p_1 - \big(c_1 + \frac{h(1-r)}{r\mu_1}\big)\Big)$$

$$\Big(\frac{-2(\alpha^2 - \alpha_s^2)p_1 + 2\alpha A'_1 + \alpha_s A'_2 + \alpha\alpha_s\big(c_2 + \frac{h(1-r)}{r\mu_2}\big) - \alpha_s^2\big(c_1 + \frac{h(1-r)}{r\mu_1}\big)}{2\alpha}\Big)$$
$$+ \Big(\frac{2\alpha_s p_1 + A'_2 - \alpha\big(c_2 + \frac{h(1-r)}{r\mu_2}\big) - \alpha_s\big(c_1 + \frac{h(1-r)}{r\mu_1}\big)}{2\alpha}\Big)$$
$$\Big(\frac{A'_2 - \alpha\big(c_2 + \frac{h(1-r)}{r\mu_2}\big) + \alpha_s\big(c_1 + \frac{h(1-r)}{r\mu_1}\big)}{2}\Big) \tag{4.13}$$

Subject to

$$p_1 \leq \frac{2\alpha A'_1 + \alpha_s A'_2 + \alpha\alpha_s\Big(c_2 + \dfrac{h(1-r)}{r\mu_2}\Big) - \alpha_s^2\Big(c_1 + \dfrac{h(1-r)}{r\mu_1}\Big)}{2(\alpha^2 - \alpha_s^2)} \tag{4.14}$$

Proposition 4.1. *The optimal solutions for prices and stocks optimization model (M_1) are properly presented in the following.*
$p_1^* = \frac{\alpha}{2(\alpha^2-\alpha_s^2)}\Big(\frac{\alpha+\alpha_s}{\alpha}A - \frac{\alpha\beta-\alpha_s\beta_s}{\alpha}(x_1+e_1)+\frac{\alpha\beta_s-\beta\alpha_s}{\alpha}x_2\Big)+\frac{1}{2}\big(c_1+\frac{h(1-r)}{r\mu_1}\big)$,
$p_2^* = \frac{\alpha_s}{2(\alpha^2-\alpha_s^2)}\Big(\frac{\alpha+\alpha_s}{\alpha_s}A - \frac{\alpha\beta_s-\beta\alpha_s}{\alpha_s}(x_1+e_1)+\frac{\alpha\beta-\alpha_s\beta_s}{\alpha_s}x_2\Big)+\frac{1}{2}\big(c_2+\frac{h(1-r)}{r\mu_2}\big)$,
$S_1^* = \frac{1-r}{2r\mu_1}\Big(A-\beta(x_1+e_1)+\beta_s x_2-\alpha(c_1+\frac{h(1-r)}{r\mu_1})+\alpha_s(c_2+\frac{h(1-r)}{r\mu_2})\Big)$, *and*
$S_2^* = \frac{1-r}{2r\mu_2}\Big(A-\beta x_2+\beta_s(x_1+e_1)-\alpha(c_2+\frac{h(1-r)}{r\mu_2})+\alpha_s(c_1+\frac{h(1-r)}{r\mu_1})\Big)$.

Proof. The second derivative, which is presented below, is negative with respect to p_1. Therefore, the root of the first derivative when it is equal to zero maximizes the objective function.

$$\frac{\partial\pi}{\partial p_1} = -\frac{2(\alpha^2-\alpha_s^2)}{\alpha}p_1+\Big(\frac{\alpha+\alpha_s}{\alpha}A-\frac{\alpha\beta-\alpha_s\beta_s}{\alpha}(x_1+e_1)+\frac{\alpha\beta_s-\beta\alpha_s}{\alpha}x_2\Big)+ \frac{\alpha^2-\alpha_s^2}{\alpha}\big(c_1+\frac{h(1-r)}{r\mu_1}\big)$$

$$\frac{\partial^2\pi}{\partial {p_1}^2} = -\frac{2(\alpha^2-\alpha_s^2)}{\alpha} < 0$$

$$\frac{\partial\pi}{\partial p_1} = 0 \leftrightarrow p_1^{*max} = \frac{\alpha}{2(\alpha^2-\alpha_s^2)}\Big(\frac{\alpha+\alpha_s}{\alpha}A-\frac{\alpha\beta-\alpha_s\beta_s}{\alpha}(x_1+e_1)+\frac{\alpha\beta_s-\beta\alpha_s}{\alpha}x_2\Big)+ \frac{1}{2}\big(c_1+\frac{h(1-r)}{r\mu_1}\big)$$

While $A \geq \frac{\alpha\beta-\alpha_s\beta_s}{\alpha+\alpha_s}(x_1+e_1)-\frac{\alpha\beta_s-\beta\alpha_s}{\alpha+\alpha_s}x_2$, p_1^{*max} is positive. We also need to make sure that p_1^{*max} satisfies constraint 4.14. Thus, we have:

$$p_1^{*max} \leq \frac{2\alpha A_1'+\alpha_s A_2'+\alpha\alpha_s\Big(c_2+\frac{h(1-r)}{r\mu_2}\Big)-\alpha_s^2\Big(c_1+\frac{h(1-r)}{r\mu_1}\Big)}{2(\alpha^2-\alpha_s^2)}$$
$$\Leftrightarrow \frac{\alpha}{2(\alpha^2-\alpha_s^2)}\Big(\frac{\alpha+\alpha_s}{\alpha}A-\frac{\alpha\beta-\alpha_s\beta_s}{\alpha}(x_1+e_1)+\frac{\alpha\beta_s-\beta\alpha_s}{\alpha}x_2\Big)+\frac{1}{2}\big(c_1+\frac{h(1-r)}{r\mu_1}\big) \leq \frac{1}{2(\alpha^2-\alpha_s^2)}\Bigg(2\alpha A_1'+\alpha_s A_2'+\alpha\alpha_s\Big(c_2+\frac{h(1-r)}{r\mu_2}\Big)-\alpha_s^2\Big(c_1+\frac{h(1-r)}{r\mu_1}\Big)\Bigg)$$
$$A \geq \beta(x_1+e_1)-\alpha_s\Big(c_2+\frac{h(1-r)}{r\mu_2}\Big)+\alpha\Big(c_1+\frac{h(1-r)}{r\mu_1}\Big)$$

Under above condition, $p_1^{*max} = p_1^*$. By substituting p_1^* into lemma 4.2 we get new product's optimal price, which is $p_2^* = \frac{\alpha_s}{2(\alpha^2-\alpha_s^2)}$

$\Big(\frac{\alpha+\alpha_s}{\alpha_s}A-\frac{\alpha\beta_s-\beta\alpha_s}{\alpha_s}(x_1+e_1)+\frac{\alpha\beta-\alpha_s\beta_s}{\alpha_s}x_2\Big)+\frac{1}{2}\big(c_2+\frac{h(1-r)}{r\mu_2}\big)$. Then by substituting p_1^* and p_2^* into lemma 4.1 we get S_1^* and S_2^*, which are $S_1^*=\frac{1-r}{2r\mu_1}\Big(A-\beta(x_1+e_1)+\beta_s x_2-\alpha(c_1+\frac{h(1-r)}{r\mu_1})+\alpha_s(c_2+\frac{h(1-r)}{r\mu_2})\Big)$ and $S_2^*=\frac{1-r}{2r\mu_2}\Big(A-\beta x_2+\beta_s(x_1+e_1)-\alpha(c_2+\frac{h(1-r)}{r\mu_2})+\alpha_s(c_1+\frac{h(1-r)}{r\mu_1})\Big)$, respectively. ■

The prices and stocks optimization problem is solved by the analytical approach, and the closed-form expressions of optimal solutions are presented in Proposition 4.1. The optimal prices and order sizes have a linear relation with the inventory cost factor. Increasing one unit h leads to increase $\frac{1-r}{2r\mu_1}$, $\frac{1-r}{2r\mu_2}$, $\frac{\alpha_s(1-r)^2}{2r^2\mu_1\mu_2}$ unit of optimal existing and new product price, respectively. However, increasing one unit h leads existing and new products' order sizes to increase $\frac{\alpha_s(1-r)^2}{2r^2\mu_1\mu_2}$. We will use optimal solutions that are presented by Proposition 4.1 to pull managerial insights out of the shadow.

4.2.2 New product optimization (M_2)

Like previous case, the retailer offers a new product to customers. But in this case, the retailer keeps the existing product's price and carbon intensity (i.e., p_1 and x_1). Hence, we define m_1 as profit of selling one unit of existed product. This problem consists in deciding new product's price p_2, carbon emission intensity x_2, and both products' order sizes S_1 and S_2. Because in this case x_1 and p_1 are known, the demands functions (equation 4.1 and 4.2) can be simplified as follows. λ_1 is here given by $A_1''+\alpha_s p_2+\beta_s x_2$ and λ_2 is here given by $A_2''-\alpha p_2-\beta x_2$. We let $A_1''=A-\alpha p_1-\beta(x_1+e_1)$ and $A_2''=A+\alpha_s p_1+\beta_s(x_1+e_1)$.

In new product optimization, the retailer's profit optimization problem is denoted by (M_2). We use the same methodology used for (M_1) to solve model (M_2). However, the calculation steps are different since we do not have the same decision variables.

$$\underset{x_2,p_2,S_1,S_2}{\text{Maximize}}\ \pi = m_1\lambda_1+\Big(p_2-\big(c_2+b(x_0-x_2)^2\big)\Big)\lambda_2-hS_1-hS_2 \tag{4.15}$$

Subject to

$$\psi_1=\frac{\lambda_1}{\lambda_1+S_1\mu_1}\le r \tag{4.16}$$

$$\psi_2=\frac{\lambda_2}{\lambda_2+S_2\mu_2}\le r \tag{4.17}$$

$$\lambda_1=A_1''+\alpha_s p_2+\beta_s x_2 \tag{4.18}$$

$$\lambda_2=A_2''-\alpha p_2-\beta x_2 \tag{4.19}$$

$$p_2,S_1,S_2,\lambda_1,\lambda_2\ge 0, 0\le x_2\le x_0$$

The mathematical model is similar to the prices and stocks optimization problem. Thus the explanation will be skipped. The rest of this sub-section aims to present theoretical proofs of optimal solutions.

Lemma 4.3. *For any given values of x_2 and p_2, service levels constraints 4.6 and 4.7 are binding at optimality and the optimal stocks are $S_1^* = \frac{(1-r)(A_1'' + \alpha_s p_2 + \beta_s x_2)}{r\mu_1}$ and $S_2^* = \frac{(1-r)(A_2'' - \alpha p_2 - \beta x_2)}{r\mu_2}$.*

The proof is similar to Lemma 4.1. Since we assume that $\lambda_1, \lambda_2 \geq 0$, so, $S_1^*, S_2^* \geq 0$. Thanks to Lemma 4.3, we can substitute S_1^* and S_2^* by their expressions, $\frac{(1-r)(A_1'' + \alpha_s p_2 + \beta_s x_2)}{r\mu_1}$ and $\frac{(1-r)(A_2'' - \alpha p_2 - \beta x_2)}{r\mu_2}$, respectively, and obtain the following equivalent model with only two variables, p_1 and p_2.

$$\underset{x_2, p_2}{\text{Maximize}}\ \pi = \Big(m_1 - \frac{h(1-r)}{r\mu_1}\Big)\lambda_1 + \Big(p_2 - \big(c_2 + b(x_0 - x_2)^2 + \frac{h(1-r)}{r\mu_2}\big)\Big)\lambda_2 \tag{4.20}$$

Subject to

$$\lambda_1 = A_1' - \alpha p_1 + \alpha_s p_2 \tag{4.21}$$

$$\lambda_2 = A_2' - \alpha p_2 + \alpha_s p_1 \tag{4.22}$$

$$\lambda_1, \lambda_2, p_2 \geq 0, 0 \leq x_2 \leq x_0$$

Lemma 4.4. *For any given value of x_2, optimal price is*

$$p_2^* = \frac{\alpha b x_2^2 - (\beta + 2\alpha b x_0)x_2 + A_2'' + \alpha_s\big(m_1 - \frac{h(1-r)}{r\mu_1}\big) + \alpha\big(c_2 + b x_0^2 + \frac{h(1-r)}{r\mu_2}\big)}{2\alpha}.$$

The proof approach is similar to Lemma 4.2. Finally, we substitute p_2^* with its expression given in Lemma 4.4 and obtain the following equivalent single-variable model.

$$\underset{0\leq x_2\leq x_0}{\text{Maximize}}\ \pi = \Big(m_1 - \frac{h(1-r)}{r\mu_1}\Big)\Bigg(A_1'' + \frac{\alpha\alpha_s b x_2^2 - (\alpha_s\beta + 2\alpha\alpha_s b x_0 - 2\alpha\beta_s)x_2 + \alpha_s^2 m_1 + \alpha_s A_2'' + \alpha\alpha_s\big(c_2 + bx_0^2 + \frac{h(1-r)}{r\mu_2}\big)}{2\alpha}\Bigg)$$
$$+\left(\frac{\Big(-\alpha b x_2^2 - (\beta - 2\alpha b x_0)x_2 + A_2'' - \alpha\big(c_2 + bx_0^2 + \frac{h(1-r)}{r\mu_2}\big)\Big)^2 - \Big(\alpha_s\big(m_1 - \frac{h(1-r)}{r\mu_1}\big)\Big)^2}{4\alpha}\right) \tag{4.23}$$

Proposition 4.2. *In the case of new product's optimization, if the root of following cubic equation is in the feasible region, it is the new product's optimal carbon emission.*

$$\alpha b^2 x_2^3 + \frac{3}{2}b(\beta - 2\alpha b x_0)x_2^2 + \Big(-bA_2'' + 3\alpha b^2 x_0^2 - 2b\beta x_0 + \alpha b\big(c_2 + \frac{h(1-r)}{r\mu_2}\big) + \alpha_s b\big(m_1 - \frac{h(1-r)}{r\mu_1}\big) + \frac{\beta^2}{2\alpha}\Big)x_2 + \Bigg(\Big(m_1 - \frac{h(1-r)}{r\mu_1}\Big)\Big(\beta_s - \frac{\alpha_p(\beta + 2\alpha b x_0)}{2\alpha}\Big) - \frac{1}{2\alpha}\Big((\beta - 2\alpha b x_0)\Big(A_2'' - \alpha\big(c_2 + bx_0^2 + \frac{h(1-r)}{r\mu_2}\big)\Big)\Big)\Bigg) = 0$$

Otherwise, in the case that there is no root in the feasible region, either 0 *or* x_0 *(based on the profit in these two points) is the optimal solution.*

Proof. The first and the second derivatives of objective function with respect to x_2 are presented in the following.

$$\frac{\partial\pi}{\partial x_2} = \alpha b^2 x_2^3 + \frac{3}{2}b(\beta - 2\alpha b x_0)x_2^2 + \Big(-bA_2'' + 3\alpha b^2 x_0^2 - 2b\beta x_0 + \alpha b\big(c_2 + \frac{h(1-r)}{r\mu_2}\big) + \alpha_s b\big(m_1 - \frac{h(1-r)}{r\mu_1}\big) + \frac{\beta^2}{2\alpha}\Big)x_2 + \Bigg(\Big(m_1 - \frac{h(1-r)}{r\mu_1}\Big)\Big(\beta_s - \frac{\alpha_p(\beta + 2\alpha b x_0)}{2\alpha}\Big) - \frac{1}{2\alpha}\Big((\beta - 2\alpha b x_0)\Big(A_2'' - \alpha\big(c_2 + bx_0^2 + \frac{h(1-r)}{r\mu_2}\big)\Big)\Big)\Bigg)$$

$$\frac{\partial^2\pi}{\partial {x_2}^2} = 3\alpha b^2 x_2^2 + 3b(\beta - 2\alpha b x_0)x_2 + \Big(-bA_2'' + 3\alpha b^2 x_0^2 - 2b\beta x_0 + \alpha b\big(c_2 + \frac{h(1-r)}{r\mu_2}\big) + \alpha_s b\big(m_1 - \frac{h(1-r)}{r\mu_1}\big) + \frac{\beta^2}{2\alpha}\Big)$$

The second derivative is a quadratic function of x_2 that is negative between to roots and positive outside of them (because ${x_2}^2$ coefficient is positive), only if the discriminant is positive; otherwise, the second derivative is always positive (i.e., convex). In the case that discriminant is positive, the objective function is concave between two roots and convex outside of them. Let's define Δ_3 the discriminant, $\Delta_3 = \Big(3b(\beta - 2\alpha b x_0)\Big)^2 - 12\alpha b^2 \Bigg(- bA_2'' + 3\alpha b^2 x_0^2 - 2b\beta x_0 + \alpha b\Big(c_2 + \frac{h(1-r)}{r\mu_2}\Big) + \alpha_s b\Big(m_1 - \frac{h(1-r)}{r\mu_1}\Big) + \frac{\beta^2}{2\alpha}\Bigg)$. As said above, if $\Delta_3 < 0$, then the second derivative is always positive. It means that the profit function is always convex. However, if $\Delta_3 > 0$, the second derivative is negative between the two roots and positive out of this range (two roots). Thus, the profit function is concave between two roots and convex out of them.

The condition that declares the Δ_3's different situations are presented in the following.

$$\Delta_3 = 12\alpha b^3 \Bigg(A_2'' - \beta x_0 - \Big(c_2 + \frac{h(1-r)}{r\mu_2}\Big) - \alpha_s\Big(m_1 - \frac{h(1-r)}{r\mu_1}\Big) + \frac{\beta^2}{4\alpha b}\Bigg)$$

Thus, while $A_2'' \geq \beta x_0 + \Big(c_2 + \frac{h(1-r)}{r\mu_2}\Big) + \alpha_s\Big(m_1 - \frac{h(1-r)}{r\mu_1}\Big) - \frac{\beta^2}{4\alpha b}$, the $\Delta_3 \geq 0$ and, thus, there are two roots (of the second derivative) that are:

$$R_3 = x_0 + \frac{-3\beta b + \sqrt{\Delta_3}}{6\alpha b^2}$$

$$R_4 = x_0 - \frac{3\beta b + \sqrt{\Delta_3}}{6\alpha b^2}$$

The objective function is concave between two roots, and out of them is convex. In addition, the first root is greater than x_0, and the second root is negative (see Appendix C for more details). According to the feasible region of x_2 ($[0, x_0]$), it can be said that the objective function is concave in the feasible region. Therefore, there is one optimal solution, at most, in the feasible region. Therefore, the optimal value of x_2 is the root of the first derivative ($\frac{\partial \pi}{\partial x_2} = 0$), only if there is a root in the feasible region. Suppose there is no root in the feasible region. In that case, the profit of the start point of the feasible region, 0, and the profit of the last point of the feasible region, x_0, will be compared, and the point that has higher profit is the optimal solution. ∎

4.2.3 General products differentiation model (M_3)

This case is the general scenario of retailer's maximizing profit problem. Like previous case, the retailer offers a new product to customers. In addition to prices and order sizes, the retailer also makes the decision of new product's carbon intensity, x_2. The existing product's carbon emission, x_1, remains without any changes. Equations 4.3 and 4.4 represent each product's demand. Therefore, λ_1 is here given by $A_1 - \alpha p_1 + \alpha_s p_2 + \beta_s x_2$ and λ_2 is here given by $A_2 - \alpha p_2 + \alpha_s p_1 - \beta x_2$, which $A_1 = A - \beta(x_1 + e_1)$ and $A_2 =$

$A+\beta_s(x_1+e_1)$. The mathematical model of discussed problem is presented in the following.

$$\underset{x_2,p_1,p_2,S_1,S_2}{\text{Maximize}}\ \pi = (p_1-c_1)\lambda_1+\Big(p_2-\big(c_2+b(x_0-x_2)^2\big)\Big)\lambda_2-hS_1-hS_2 \tag{4.24}$$

Subject to

$$\psi_1=\frac{\lambda_1}{\lambda_1+S_1\mu_1}\leq r \tag{4.25}$$

$$\psi_2=\frac{\lambda_2}{\lambda_2+S_2\mu_2}\leq r \tag{4.26}$$

$$\lambda_1=A_1-\alpha p_1+\alpha_s p_2+\beta_s x_2 \tag{4.27}$$

$$\lambda_2=A_2-\alpha p_2+\alpha_s p_1-\beta x_2 \tag{4.28}$$

$$p_1,p_2,S_1,S_2,\lambda_1,\lambda_2\geq 0, 0\leq x_2\leq x_0$$

Lemma 4.5. *For any given values of x_2, p_1, and p_2 service level constraints 4.25 and 4.26 are binding at optimality and the optimal stocks are* $S_1^*=\dfrac{(1-r)(A_1-\alpha p_1+\alpha_s p_2+\beta_s x_2)}{r\mu_1}$ *and* $S_2^*=\dfrac{(1-r)(A_2-\alpha p_2+\alpha_s p_1-\beta x_2)}{r\mu_2}$.

Proof is similar to that has been already given for Lemma 4.1. Since we assume that $\lambda_1,\lambda_2\geq 0$ and thanks to Lemma 4.5, $S_1^*,S_2^*\geq 0$ and we subsitute them by their expressions. By substituting S_1^* and S_2^*, the model can be rewritten as presented in the following:

$$\underset{x_2,p_1,p_2}{\text{Maximize}}\ \pi = \Big(p_1-\big(c_1+\frac{h(1-r)}{r\mu_1}\big)\Big)\lambda_1+\Big(p_2-\big(c_2+b(x_0-x_2)^2+\frac{h(1-r)}{r\mu_2}\big)\Big)\lambda_2 \tag{4.29}$$

Subject to

$$\lambda_1=A_1-\alpha p_1+\alpha_s p_2+\beta_s x_2$$

$$\lambda_2=A_2-\alpha p_2+\alpha_s p_1-\beta x_2$$

$$p_1,p_2,\lambda_1,\lambda_2\geq 0, 0\leq x_2\leq x_0$$

Lemma 4.6. *For any given value of x_2 and p_1, new product's optimal price is*

$$p_2^*=\frac{\alpha b x_2^2-(\beta+2\alpha b x_0)x_2+A_2+2\alpha_s p_1-\alpha_s\Big(c_1+\dfrac{h(1-r)}{r\mu_1}\Big)+\alpha\Big(c_2+bx_0^2+\dfrac{h(1-r)}{r\mu_2}\Big)}{2\alpha}.$$

The proof approach is similar to Lemma 4.2. Based on Lemma 4.6, we substitute p_2^* by its expressions and obtain the following equivalent model with only two variables p_1 and x_2.

$$\underset{x_2,p_1}{\text{Maximize}}\ \pi = \Big(p_1 - \big(c_1 + \frac{h(1-r)}{r\mu_1}\big)\Big)\lambda_1 +$$
$$\left(\frac{-\alpha b x_2^2 + (-\beta + 2\alpha b x_0)x_2 + A_2 + 2\alpha_s p_1 - \alpha_s\big(c_1 + \frac{h(1-r)}{r\mu_1}\big) - \alpha\big(c_2 + b x_0^2 + \frac{h(1-r)}{r\mu_2}\big)}{2\alpha}\right)$$
$$\left(\frac{-\alpha b x_2^2 + (-\beta + 2\alpha b x_0)x_2 + A_2 + \alpha_s\big(c_1 + \frac{h(1-r)}{r\mu_1}\big) - \alpha\big(c_2 + b x_0^2 + \frac{h(1-r)}{r\mu_2}\big)}{2\alpha}\right) \tag{4.30}$$

Subject to
$\lambda_1 = A_1 - \alpha p_1 + \alpha_s p_2 + \beta_s x_2$
$\lambda_2 = A_2 - \alpha p_2 + \alpha_s p_1 - \beta x_2$
$p_1, \lambda_1, \lambda_2 \geq 0, 0 \leq x_2 \leq x_0$

Lemma 4.7. *For any given value of x_2, existed product's optimal price is*
$$p_1^* = \frac{\alpha}{2(\alpha^2 - \alpha_s^2)}\left(A_1 + \frac{\alpha_s}{\alpha}A_2 + \frac{\alpha\beta_s - \beta\alpha_s}{\alpha}x_2\right) + \frac{1}{2}\big(c_1 + \frac{h(1-r)}{r\mu_1}\big).$$

The proof is similar to Proposition 4.1. Finally, we substitute p_1^* with its expression given in Lemma 4.7 and obtain the following equivalent single-variable model.

$$\underset{0\leq x_2\leq x_0}{\text{Maximize}}\ \pi = \left(\frac{\alpha}{2(\alpha^2 - \alpha_s^2)}\Big(A_1 + \frac{\alpha_s}{\alpha}A_2 + \frac{\alpha\beta_s - \beta\alpha_s}{\alpha}x_2\Big) - \frac{1}{2}\big(c_1 + \frac{h(1-r)}{r\mu_1}\big)\right)$$
$$\Bigg(A_1 - \alpha\Big(\frac{\alpha}{2(\alpha^2 - \alpha_s^2)}\big(A_1 + \frac{\alpha_s}{\alpha}A_2 + \frac{\alpha\beta_s - \beta\alpha_s}{\alpha}x_2\big) - \frac{1}{2}\big(c_1 + \frac{h(1-r)}{r\mu_1}\big)\Big) +$$
$$\alpha_s\Big(\frac{\alpha b x_2^2 - (\beta + 2\alpha b x_0)x_2 + A_2 + 2\alpha_s p_1 - \alpha_s\big(c_1 + \frac{h(1-r)}{r\mu_1}\big) + \alpha\big(c_2 + b x_0^2 + \frac{h(1-r)}{r\mu_2}\big)}{2\alpha}\Big) + \beta_s x_2\Bigg)$$
$$+\left(\frac{-\alpha b x_2^2 + (-\beta + 2\alpha b x_0)x_2 + A_2 + 2\alpha_s p_1 - \alpha_s\big(c_1 + \frac{h(1-r)}{r\mu_1}\big) - \alpha\big(c_2 + b x_0^2 + \frac{h(1-r)}{r\mu_2}\big)}{2\alpha}\right) \tag{4.31}$$

$$\left(\frac{-\alpha b x_2^2+(-\beta+2\alpha b x_0)x_2+A_2+\alpha_s\big(c_1+\dfrac{h(1-r)}{r\mu_1}\big)-\alpha\big(c_2+bx_0^2+\dfrac{h(1-r)}{r\mu_2}\big)}{2\alpha}\right) \tag{4.32}$$

Subject to
$\lambda_1 = A_1 - \alpha p_1 + \alpha_s p_2 + \beta_s x_2$
$\lambda_2 = A_2 - \alpha p_2 + \alpha_s p_1 - \beta x_2$
$\lambda_1, \lambda_2 \geq 0$

Proposition 4.3. *In the case of general products differentiation, if the root of following cubic equation is in the feasible region, is the new product's optimal carbon emission.*

$$\alpha b^2 {x_2}^3 - 3\alpha b^2(x_0-\frac{\beta}{2\alpha b}){x_2}^2+\Bigg(\frac{\alpha\beta_s-\beta\alpha_s}{2(\alpha^2-{\alpha_s}^2)}\Big(\frac{4\alpha\beta_s-3\beta\alpha_s-2\alpha\alpha_s b x_0}{2\alpha}+b\alpha_s(x_0-\frac{\beta}{2\alpha b})\Big)-b\Big(A_2+\alpha_s\big(c_1+\frac{h(1-r)}{1\mu_1}\big)-\alpha\big(c_2+b{x_0}^2+\frac{h(1-r)}{1\mu_2}\big)\Big)+2\alpha b^2(x_0-\frac{\beta}{2\alpha b})^2\Bigg)x_2$$
$$+\Bigg(\frac{\alpha\beta_s+\beta\alpha_s}{2\alpha}\Bigg(\frac{1}{2}(A_1-\frac{\alpha_s}{\alpha}A_2)-\alpha\big(c_1+\frac{h(1-r)}{1\mu_1}\big)+\frac{\alpha_s}{2\alpha}\Big(A_2+\alpha\big(c_2+b{x_0}^2+\frac{h(1-r)}{1\mu_2}\big)\Big)\Bigg)+\Big(\frac{\alpha\beta_s+\beta\alpha_s}{2\alpha}-\alpha_s b(x_0+\frac{\beta}{2\alpha b})\Big)\Big(\frac{\alpha}{2(\alpha^2-{\alpha_s}^2)}(A_1+\frac{\alpha_s}{\alpha}A_2)-\frac{1}{2}\big(c_1+\frac{h(1-r)}{r\mu_1}\big)\Big)+$$
$$\frac{1}{4\alpha}\Big(2\alpha b(x_0-\frac{\beta}{2\alpha b})+2\alpha_s\frac{\alpha\beta_s-\beta\alpha_s}{2(\alpha^2-{\alpha_s}^2)}\Big)\Big(A_2+\alpha_s\big(c_1+\frac{h(1-r)}{r\mu_1}\big)-\alpha\big(c_2+b{x_0}^2+\frac{h(1-r)}{1\mu_2}\big)\Big)+\alpha b(x_0-\frac{\beta}{2\alpha b})\Big(\frac{A_2-\alpha\big(c_2+b{x_0}^2+\frac{h(1-r)}{1\mu_2}\big)\big)+\alpha b(x_0-\frac{\beta}{2\alpha b})}{2\alpha}+$$
$$\frac{\alpha_s}{2(\alpha^2-{\alpha_s}^2)}\big(A_1+\frac{\alpha_s}{\alpha}A_2\big)\Big)\Bigg)=0$$

Otherwise, in the case of having no root in the feasible region, either 0 *or* x_0 *(based on the profit in these two points) is the optimal solution.*

Proof is similar to Proposition 4.2.

In the following section, we are going to use the optimal values that we obtain to pull important managerial insights out from the shadow.

4.3 Analysis and managerial insights

In this section, we present important insights of problems that are presented and solved earlier. First, we present a comprehensive numerical result to answer the first question of this study; how much the retailer benefit from offering a new substitutable product? We consider sets of parameters to do the numerical example. The parameters are presented in Table 4.1. The result is an average of 160K different combinations.

Table 4.1: Parameters

Parameter	Value	Parameter	Value
A	1200:100:2000	x_0	100
α_p	4:1:8	e_1	0:0.2x_0:0.4x_0
α_s	3:1:$\alpha_p - 1$	c_1	10
β_e	3:1:5	c_2	c_1:0.2c_1:1.4c_1
β_s	1:1:$\beta_e - 1$	h	1:1:3
μ_1	1.5μ_2:μ_2:3.5μ_2	b	0.005:0.005:0.015
μ_2	30	r	0.05

Observation 4.1. *On our large numerical example, the product differentiation provides 12.7% more profit on average than one product for retailer.*

How much the retailer benefit from offering a new substitutable product? To answer the research question, we consider over 160k combinations of parameters' value that are presented in Table 4.1. The gap between the optimal profits when the retailer offers one product (π_{M_0}) and when the retailer offers two substitutable products (π_{M_2}) is calculated as $\frac{(\pi_{M_2} - \pi_{M_0}) * 100}{\pi_{M_2}}$. To be more precise, we confirm that in especial case when the retailer offers two identical products, total demand is the same as when he/she offers one product and so the profit is similar to the case we offer only one product (chapter 3). Table 4.2 shows the details concerning this gap. According to the results in table 4.2, the retailer gains 12.7 percent more profit on average when offers two substitutable product than offers one product. A more in-depth look into the results reveals that product differentiation, at the very least, provides the same profit as when retailer offers one product, which means that the retailer does not interest in offering a new substitutable product. It is intuitive, because the retailer offers two identical products in the worst case. The numerical results confirm that. The minimum value of the gap between the optimal profits is almost zero. However, the maximum value of the gap between the optimal profits is 46.93%. So from a managerial perspective, when the gap between profits is sufficiently high enough, it is interesting to offer a new substitutable product. To completing the analysis, a T-student test is performed on the results. With regard to the confidence level 95%, the mean gap lies in [12.66 12.74], the confidence interval. In simple words, with a probability of 95%, the retailer gains between 12.66 and 12.74 percent more profit when offers a new product to customers.

Table 4.2: Gap between profits

	Mean	Standard Deviation	95% Confidence Interval
π_{M_2} V.s. π_{M_0}	12.7%	9.04%	[12.66 12.74]%

Comparing the optimal carbon emission level in benchmark and product differentiation models to highlight the substitutable effect is presented in the following results. The comparison is made in different market structures in order to answer the research questions.

4.3.1 Market Structures

To understand the following results, we distinguish the market based on price and greenness sensitivity factors (similar to Boyaci and Ray, 2003 and Boyaci and Ray, 2006).

In the case of $\frac{\alpha_s}{\alpha_p} < \frac{\beta_s}{\beta_e}$, a unit decrease in carbon intensity leads to a higher proportion of cannibalized customers (with respect to "new" customers) than the proportion resulting from a unit decrease in price. Thus, the switchovers are more governed by greenness differentiation than price differentiation, which will henceforth be referred to as a greenness-driven switchovers (GDS) market. To show that, let us first proof that GDS condition (i.e., $\frac{\alpha_s}{\alpha_p} < \frac{\beta_s}{\beta_e}$) is also equivalent to $\frac{\alpha_s}{\alpha_p + \alpha_s} < \frac{\beta_s}{\beta_e + \beta_s}$.

$$\frac{\alpha_s}{\alpha_p} < \frac{\beta_s}{\beta_e} \Leftrightarrow \alpha_s\beta_e < \alpha_p\beta_s \Leftrightarrow \alpha_s\beta_e + \alpha_s\beta_s < \alpha_p\beta_s + \alpha_s\beta_s \Leftrightarrow \alpha_s(\beta_e + \beta_s) < \beta_s(\alpha_p + \alpha_s)$$
$$\Leftrightarrow \frac{\alpha_s}{\alpha_p + \alpha_s} < \frac{\beta_s}{\beta_e + \beta_s}$$

Recall Equations 4.3 and 4.4 that are demand functions. If we decrease one unit of product 1's price, its demand increases of α units with $\alpha = \alpha_p + \alpha_s$, which α_p are the new customers and α_s are the customers who switch. Therefore, the fraction of customers who switch to the total customers is $\frac{\alpha_s}{\alpha_p + \alpha_s}$. In the same way for carbon emission, the fraction of customers who switch to the total customers (when we decrease one unit of carbon emission) is $\frac{\beta_s}{\beta_e + \beta_s}$. The fraction of "cannibalized" demand (with respect to the total demand generated) is higher for a carbon emission reduction rather than a price reduction. Therefore, in the GDS market, the switchover effect is stronger for carbon emission.

With the same analysis, we deduce in the case of $\frac{\alpha_s}{\alpha_p} > \frac{\beta_s}{\beta_e}$ that the switchovers are more governed by price differentiation, which will henceforth be referred to as a price-driven switchovers (PDS) market.

Finally, when $\frac{\alpha_s}{\alpha_p} = \frac{\beta_s}{\beta_e}$, price and greenness disparities have the same importance with respect to switchovers, which is therefore referred to as a neutral market. The above market segregation builds an analysis framework that helps to understand the following findings.

Most of our results depend on the market type (PDS or GDS). We recall that a PDS market characterizes the case where switchovers are more governed by price differentiation than greenness differentiation, which does not necessarily mean that customers are less sensitive to greenness but may refer to situations where it is not possible for the customers to compare the greenness levels of products. For instance, for many products, there is no green labeling or the labeling does not allow customers to make reliable comparisons, such as when firms just highlight that the product is made from sustainable materials (e.g., Matt & Nat handbags and wallets, some products offered by H&M and Zara) or with an environmentally-friendly process (e.g., Lobodis coffee). In these cases, the customers can compare the products only in terms of price, and switchovers will then be more governed by price differentiation (PDS market) even in the presence of environmentally-conscious customers. However, in a market characterized by a high customers' environmental awareness (e.g., agribusiness) and the possibility for customers to compare the greenness levels of products, switchovers can be more governed by greenness differentiation (GDS market). This motivated many agribusiness companies to focus on offering greener products while simplifying the greenness comparison for consumers. For instance, the British firm Innocent indicates the carbon footprint of some fruit smoothies on the packaging (Hammami et al., 2018). A GDS market may also correspond to products for which the greenness level implies a lower use cost (e.g., energy saving). The example of the environmentally-friendly bulb of the Philips-Carrefour SC in Europe, provided by Hong et al. (2019), is a good one. For this product, greenness is associated with energy labeling, which is a color-coded performance scale from G to A+++. The customers use this scale to compare green lamps with the conventional ones, and this comparison significantly influences their purchasing decisions (Hong et al., 2019).

4.3.2 Insights

Proposition 4.4. *In prices and stocks optimization problem (M_1), the greater the new product's carbon emission is, the lower the new product's price will be, and (i) the lower the existing product's price will be in the PDS market, (ii) the higher the existing product's price will be in the GDS market, and (iii) same price in the Neutral market (does not change).*

Proof. Thank to proposition 4.1, the optimal prices are presented in the following.

$$p_1^* = \frac{\alpha}{2(\alpha^2-\alpha_s^2)}\Big(\frac{\alpha+\alpha_s}{\alpha}A - \frac{\alpha\beta-\alpha_s\beta_s}{\alpha}(x_1+e_1) + \frac{\alpha\beta_s-\beta\alpha_s}{\alpha}x_2\Big) + \frac{1}{2}\big(c_1 + \frac{h(1-r)}{r\mu_1}\big)$$

$$p_2^* = \frac{\alpha_s}{2(\alpha^2-\alpha_s^2)}\Big(\frac{\alpha+\alpha_s}{\alpha_s}A - \frac{\alpha\beta_s-\beta\alpha_s}{\alpha_s}(x_1+e_1)+\frac{\alpha\beta-\alpha_s\beta_s}{\alpha_s}x_2\Big)+\frac{1}{2}\big(c_2+\frac{h(1-r)}{r\mu_2}\big)$$

The first derivatives of prices (p_1^* and p_2^*, respectievly) with respect to the new product's carbon emission, x_2, are presented in the following.

$$\frac{\partial p_1^*}{\partial x_2} = \frac{\alpha\beta_s-\beta\alpha_s}{2(\alpha^2-\alpha_s^2)}$$
$$\frac{\partial p_2^*}{\partial x_2} = -\frac{\alpha\beta-\beta_s\alpha_s}{2(\alpha^2-\alpha_s^2)}$$

Let us remember that $\alpha=\alpha_p+\alpha_s$ and $\beta=\beta_e+\beta_s$. Therefore, $\alpha\beta_s-\beta\alpha_s=(\alpha_p+\alpha_s)\beta_s-(\beta_e+\beta_s)\alpha_s=\alpha_p\beta_s-\beta_e\alpha_s$. The existing product's price behavior is strongly related to the behavior of the customers (or market). In case that $\alpha\beta_s>\beta\alpha_s$ (equivalently, $\alpha_p\beta_s>\beta_e\alpha_s$) the existing product's price is increasing in x_2, while in case that $\alpha\beta_s<\beta\alpha_s$ (equivalently, $\alpha_p\beta_s<\beta_e\alpha_s$) the existing product's price is decreasing in x_2. In neutral market when $\alpha\beta_s=\beta\alpha_s$ (equivalently, $\alpha_p\beta_s=\beta_e\alpha_s$) the existing product's price is independent from x_2. However, the new product's price is always decreasing in x_2. ∎

Intuitively, increasing the new product's carbon intensity (offer dirtier product) makes the retailer decrease its price. However, our findings show that the behavior of the existing product's price depends on market characteristics. In the neutral market, increasing the new product's carbon intensity has no effect on the existing product's price, while other markets either decrease or even increase its price. In the PDS market, the price switchover is stronger than the greenness switchover. Decreasing one unit of price attracts customers at a rate of $\alpha_p+\alpha_s$ for the existing product, which, α_p are "new" attracted customers and α_s are cannibalized from the new product. In this situation, the retailer gains more profit by decreasing the existing product's price. In the GDS market, the greenness switchover is stronger than the price. Unlike the previous case, the retailer is going to have more customers because of the cannibalization effect; therefore, the retailer increases the greener product's (new product's) price. In the neutral market, price and greenness are equally paid attention; the retailer's best strategy is to keep the existing product's price.

Proposition 4.5. *In prices and stocks optimization problem (M_1), increasing the gap between products' transportation carbon emission, e_1, leads to lower existing product's price and (i) lower the new product's price will be in PDS market and (ii) higher the new product's price will be in GDS market (iii) new product's price in the neutral market is independent of e_1.*

Proof. The first derivatives of optimal prices (p_1^* and p_2^*, respectively), which are presented in proposition 4.1, with respect to the gap of the transportation carbon emission, e_1, are presented in the following.

$$\frac{\partial p_1^*}{\partial e_1} = -\frac{\alpha\beta-\beta_s\alpha_s}{2(\alpha^2-\alpha_s^2)}$$

$$\frac{\partial p_2^*}{\partial e_1} = \frac{\alpha\beta_s - \beta\alpha_s}{2(\alpha^2 - \alpha_s^2)}$$

As we can see, the existing product's price is always decreasing in e_1 ($\frac{\partial p_1^*}{\partial e_1} < 0$). However, the new product's price behavior is strongly related to the behavior of the customers (or market). In case that $\alpha\beta_s > \beta\alpha_s$ (equivalently, $\alpha_p\beta_s > \beta_e\alpha_s$) the new product's price is increasing in e_1, while in case that $\alpha\beta_s < \beta\alpha_s$ (equivalently, $\alpha_p\beta_s < \beta_e\alpha_s$) the new product's price is decreasing in e_1. In neutral market when $\alpha\beta_s = \beta\alpha_s$ (equivalently, $\alpha_p\beta_s = \beta_e\alpha_s$) the new product's price is independent from e_1. ■

The first part of proposition 4.3 is intuitive. Increasing e_1 means that the existing product's carbon intensity (i.e. $x_1 + e_1$) increases. Therefore it is logical that the price decreases as dirtier as the existing product is to attract new customers. On the other hand, the new product's price behavior has a similar explanation as to the existing product's price in Proposition 4.3. As a result, we can say that in a product differentiation model, offering a dirtier product leads to a lower retail price for it. In contrast, the other product's retail price depends highly on the market. It can be higher in the GDS market or lower in the PDS market, or constant in the neutral market.

Proposition 4.6. *In prices and stocks optimization problem (M_1), increasing the gap between products' transportation carbon emission, e_1, decreases the existing product's order size and increases the new product's order size.*

Proof. Thank to Proposition 4.1, the optimal order sizes are presented in the following.

$$S_1^* = \frac{1-r}{2r\mu_1}\Big(A - \beta(x_1+e_1) + \beta_s x_2 - \alpha(c_1 + \frac{h(1-r)}{r\mu_1}) + \alpha_s(c_2 + \frac{h(1-r)}{r\mu_2})\Big)$$
$$S_2^* = \frac{1-r}{2r\mu_2}\Big(A - \beta x_2 + \beta_s(x_1+e_1) - \alpha(c_2 + \frac{h(1-r)}{r\mu_2}) + \alpha_s(c_1 + \frac{h(1-r)}{r\mu_1})\Big)$$

The first derivatives of stocks (S_1^* and S_2^*, respectievly) with respect to the gap of the transportation carbon emission, e_1, are presented in the following.

$$\frac{\partial S_1^*}{\partial e_1} = -\frac{\beta(1-r)}{2r\mu_1}$$
$$\frac{\partial S_2^*}{\partial e_1} = \frac{\beta_s(1-r)}{2r\mu_2}$$

As you can see, the existing product's stock is always decreasing in e_1. However, the new product's stock is increasing in e_1. ■

The result of Proposition 4.4 is quite intuitive. As we explained in the previous proposition, the retailer offers a dirtier product (existing product) to customers when e_1 increases. Despite that, the carbon intensity is not a decision variable in this model, but the demand is still sensitive. Thanks to Lemma 4.1, we know that the order sizes of products have a positive and direct relation with demands, which means the order size increases when the demand increases. Thus, the retailer makes a higher order when there are more customers to respect the service level. Thus, as a result, there will be fewer customers for existing products and more customers for new products. Consequently, the existing product's order size decreases, and the new product's order size increases. Figure 4.4 illustrates what we discuss above.

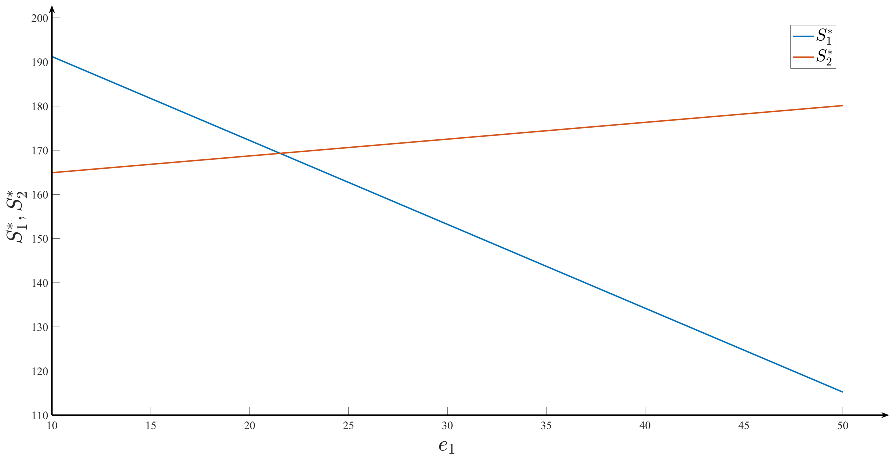

Figure 4.4: Impact of e_1 on optimal order sizes

Proposition 4.7. *In prices and stocks optimization problem (M_1), the optimal profit is decreasing convex functions in transportation carbon emission, e_1.*

Proof. The first and second derivatives of profit function are presented in the following.

$$\frac{\partial \pi^*}{\partial e_1} = \frac{\partial\left(p_1^* - (c1 + \frac{h(1-r)}{r\mu 1})\right)}{\partial e_1}(A_1' - \alpha p_1^* + \alpha_s p_2^*) + \frac{\partial(A_1' - \alpha p_1^* + \alpha_s p_2^*)}{\partial e_1}\left(p_1^* - (c1 + \frac{h(1-r)}{r\mu 1})\right)$$
$$+ \frac{\partial\left(p_2^* - (c2 + \frac{h(1-r)}{r\mu 2})\right)}{\partial e_1}(A_2' - \alpha p_2^* + \alpha_s p_1^*) + \frac{\partial(A_2' - \alpha p_2^* + \alpha_s p_1^*)}{\partial e_1}\left(p_2^* - (c2 + \frac{h(1-r)}{r\mu 2})\right)$$
$$= -\frac{\alpha\beta - \alpha_s\beta_s}{2(\alpha^2 - \alpha_s^2)}\left(\frac{A - \beta(x_1 + e_1) + \beta_s x_2 - \alpha\left(c_1 + \frac{h(1-r)}{r\mu_1}\right) + \alpha_s\left(c_2 + \frac{h(1-r)}{r\mu_2}\right)}{2}\right)$$
$$-\left(\frac{\beta}{2}\right)\left(\frac{\alpha}{2(\alpha^2 - \alpha_s^2)}\left(\frac{\alpha + \alpha_s}{\alpha}A - \frac{\alpha\beta - \alpha_s\beta_s}{\alpha}(x_1 + e_1) + \frac{\alpha\beta_s - \beta\alpha_s}{\alpha}x_2\right) - \frac{1}{2}\left(c_1 + \frac{h(1-r)}{r\mu_1}\right)\right)$$
$$+\frac{\alpha\beta_s - \beta\alpha_s}{2(\alpha^2 - \alpha_s^2)}\left(\frac{A - \beta x_2 + \beta_s(x_1 + e_1) - \alpha\left(c_2 + \frac{h(1-r)}{r\mu_2}\right) + \alpha_s\left(c_1 + \frac{h(1-r)}{r\mu_1}\right)}{2}\right)$$

$$-\Big(\frac{\beta_s}{2}\Big)\left(\frac{\alpha_s}{2(\alpha^2-\alpha_s^2)}\Big(\frac{\alpha+\alpha_s}{\alpha_s}A+\frac{\alpha\beta_s-\beta\alpha_s}{\alpha_s}(x_1+e_1)-\frac{\alpha\beta-\alpha_s\beta_s}{\alpha_s}x_2\Big)-\frac{1}{2}\Big(c_2+\frac{h(1-r)}{r\mu_2}\Big)\right)$$

$$\frac{\partial^2\pi^*}{\partial {e_1}^2}=\frac{\alpha\beta-\alpha_s\beta_s}{\alpha^2-\alpha_s^2}\Big(\frac{\beta}{2}\Big)+\frac{\alpha\beta_s-\beta\alpha_s}{\alpha^2-\alpha_s^2}\Big(\frac{\beta_s}{2}\Big)=\frac{\alpha\beta^2+\alpha\beta_s^2-2\alpha_s\beta_s\beta}{2(\alpha^2-\alpha_s^2)}$$
$$=\frac{\alpha_p\beta^2+\alpha_s(\beta_e+\beta_s)^2+(\alpha_p+\alpha_s)\beta_s^2-2\alpha_s\beta_s(\beta_e+\beta_s)}{2(\alpha^2-\alpha_s^2)}=\frac{\alpha_p(\beta^2+\beta_s^2)+\alpha_s\beta_e^2}{2(\alpha^2-\alpha_s^2)}$$

It is proven that the second derivative of profit with respect to e_1 is positive and, then, the profit is convex in e_1.

$$\frac{\partial\pi^*}{\partial e_1}=-\frac{\alpha_p\beta_e+2\alpha_s\beta_e}{2(\alpha^2-\alpha_s^2)}A+\frac{\alpha_p(\beta^2+\beta_s^2)+\alpha_s\beta_e^2}{2(\alpha^2-\alpha_s^2)}(x_1+e_1)-\frac{\alpha_s\beta_e^2+2\alpha_p\beta_s^2+2\alpha_p\beta_e\beta_s}{2(\alpha^2-\alpha_s^2)}x_2$$
$$+\Big(\frac{\beta}{2}\Big)\Big(c_1+\frac{h(1-r)}{r\mu_1}\Big)-\Big(\frac{\beta_s}{2}\Big)\Big(c_2+\frac{h(1-r)}{r\mu_2}\Big)$$

The first derivative of optimal profit with respect to e_1 is negative considering that A is sufficiently high to satisfy following condition:

$$A\geq\frac{2(\alpha^2-\alpha_s^2)}{\alpha_p\beta_e+2\alpha_s\beta_e}\left(\frac{\alpha_p(\beta^2+\beta_s^2)+\alpha_s\beta_e^2}{2(\alpha^2-\alpha_s^2)}(x_1+e_1)-\frac{\alpha_s\beta_e^2+2\alpha_p\beta_s^2+2\alpha_p\beta_e\beta_s}{2(\alpha^2-\alpha_s^2)}x_2\right.$$
$$\left.+\Big(\frac{\beta}{2}\Big)\Big(c_1+\frac{h(1-r)}{r\mu_1}\Big)-\Big(\frac{\beta_s}{2}\Big)\Big(c_2+\frac{h(1-r)}{r\mu_2}\Big)\right)$$

■

This result is intuitive. The market is sensitive to greenness, therefore, it is intuitive that the retailer loses profits when he/she offers a dirtier product. Increasing transportation carbon emission results in losing customers for the retailer at rate β_e, because total demand is $\lambda=\lambda_1+\lambda_2=2A-\alpha_p(p_1+p_2)-\beta_e(x_1+x_2+e_1)$. This proposition demonstrates the importance of transportation carbon emission, which depends on the distance between the retailer and the supplier. The retailer's best strategy would be choosing the closer supplier if we assume that the other factors like production cost are the same.

Proposition 4.8. *Increasing market potential to infinite leads to (i) increasing the new product's carbon intensity that approaches to x_1 (optimal decision in benchmark model) in the PDS market and (ii) decreasing the new product's carbon intensity that approaches to x_1 in GDS market. The new product's carbon intensity is constant in the neutral market.*

Proof. The roots of cubic equation, which is presented in proposition 4.7, can be found using Cardano formula:

$$r_1=\sqrt[3]{R+\sqrt{Q^3+R^2}}+\sqrt[3]{R-\sqrt{Q^3+R^2}}+T$$

$$r_2 = -\frac{\sqrt[3]{R+\sqrt{Q^3+R^2}}+\sqrt[3]{R-\sqrt{Q^3+R^2}}}{2}+T+\frac{i\sqrt{3}}{2}(\sqrt[3]{R+\sqrt{Q^3+R^2}}-\sqrt[3]{R-\sqrt{Q^3+R^2}})$$

$$r_3 = -\frac{\sqrt[3]{R+\sqrt{Q^3+R^2}}+\sqrt[3]{R-\sqrt{Q^3+R^2}}}{2}+T-\frac{i\sqrt{3}}{2}(\sqrt[3]{R+\sqrt{Q^3+R^2}}-\sqrt[3]{R-\sqrt{Q^3+R^2}})$$

where

$$T = x_0 - \frac{\beta}{2\alpha b}$$

$$Q = \frac{A_2' - \beta x_1 - \alpha\big(c_2 + \frac{h(1-r)}{r\mu_2}\big) - \alpha_s\big(m_1 - \frac{h(1-r)}{r\mu_1}\big) + \frac{\beta^2}{4\alpha b}}{-3\alpha b}$$

$$R = \frac{\big(m_1 - \frac{h(1-r)}{r\mu_1}\big)(\alpha\beta_s - \beta\alpha_s)}{-2\alpha^2 b^2}$$

Without loss of generality, let us consider $r_1 = \sqrt[3]{R+\sqrt{Q^3+R^2}}+\sqrt[3]{R-\sqrt{Q^3+R^2}}+T$ as the optimal solution. We can rewrite r_1 as $\sqrt[3]{Q(\frac{R}{Q}+\sqrt{Q+\frac{R}{Q}^2})}+\sqrt[3]{Q(\frac{R}{Q}-\sqrt{Q+\frac{R}{Q}^2})}+T$ (we can do the same thing for r_2 and r_3). Increasing A results in:

$$\lim_{A\to\infty}\frac{R}{Q} = \lim_{A\to\infty}\frac{\dfrac{\big(m_1 - \frac{h(1-r)}{r\mu_1}\big)(\alpha\beta_s - \beta\alpha_s)}{-2\alpha^2 b^2}}{\dfrac{A_2' - \beta x_1 - \alpha\big(c_2 + \frac{h(1-r)}{r\mu_2}\big) - \alpha_s\big(m_1 - \frac{h(1-r)}{r\mu_1}\big) + \frac{\beta^2}{4\alpha b}}{-3\alpha b}} = 0$$

As much as A increases, $\frac{R}{Q}$ goes to zero. In GDS market $\lim_{A\to\infty} r_1 = T+\varepsilon$, which means that the optimal carbon emission converge towards T (decreasing). On the other hand, in PDS market $\lim_{A\to\infty} r_1 = T-\varepsilon$, which means that optimal carbon emission converge towards T (increasing). ■

In a huge market that each product has an infinite demand, the difference between demands based on their price and carbon intensity is negligible. In other words, the differentiation between products has a small effect on the retailer's optimal profit. It means that even the retailer offers just one product instead of two substitutable products, he/she does not lose a considerable amount of profit. Consequently, the product differentiation problem becomes the benchmark problem where he/she offers one product. As a result, it is logical that we say as market potential increases, the more product differentiation model becomes to benchmark, which means products become more similar to price and greenness point of view. Figure 4.5 illustrates the Proposition 4.8.

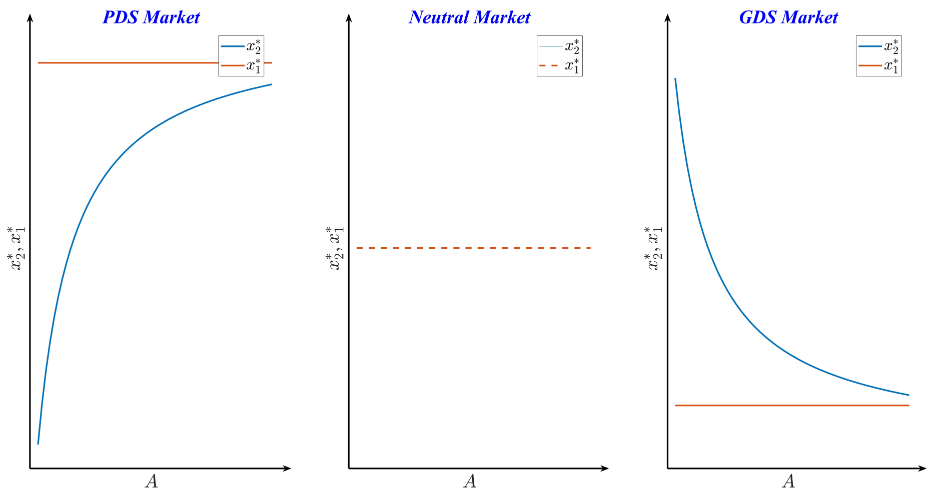

Figure 4.5: Impact of market potential on optimal carbon emission level

Proposition 4.9. *The new products that are offered in neutral market have, (i) higher price than new products in GDS market and (ii) lower price than new products in PDS market.*

The first derivative of optimal price of new product (p_2) is negative with respect to its carbon intensity (x_2), consequently, p_2 has a negative relation with x_2. Thanks to proposition 4.8, we know that the new product is offered with higher carbon intensity in the GDS market (Let us call it x_2^{*GDS}) than the neutral market (Let us call it x_2^{*N}). In the same way, it is offered with higher carbon intensity in the neutral market than PDS (Let us call it x_2^{*PDS}). Briefly, $x_2^{*GDS} > x_2^{*N} > x_2^{*PDS}$. Intuitively, dirtier product (i.e., higher carbon intensity) has a lower price. Therefore, the retailer offers a new product with a higher price in the PDS market than the neutral market and a higher price in the neutral market than the GDS market.

4.4 Conclusion

In this chapter, a product differentiation problem is considered such that a retailer offers two substitutable products to customers in a price and carbon emission sensitive market. The retailer follows the MTS policy, keeps the products, and serves them as soon as a customer arrives. Since the products are substitutable, the products' demand not only depends on their own price and carbon emission but also depends on other product's price and carbon emission. As a benchmark, we consider the retailer when it offers one product to customers. We formulate the problems under different settings (considering different sets of variables) and solve them through an analytical approach. The optimal solutions are provided by closed-form expressions. Finally, we distinguished different markets' categories to extract important insights into our results. The results showed the impact of market characteristics and substitution on the retailer's strategy. The insights are provided in different propositions to cover all the problems considered in this study. A numerical example is also done to show the difference between the retailer's profit when it offers one product, and when it offers two substitutable products. As future work, we would like to consider a more general demand function (e.g., non-linear concerning greenness) and compare the product differentiation scenario results.

CHAPTER 5

Optimization of Retailers' Strategies Under Dynamic Competition Environment

The previous chapter discussed about products differentiation context. In this chapter, we consider two competing retailers that offer two substitutable products in a greenness- and price-sensitive market. Each retailer has its own supplier. Demands and the replenishment lead times of retailers' stocks are random. The mean demand for each product decreases in its carbon emission intensity and price, and increases in other product's carbon emission intensity and price. Like previous chapters, the emissions of production and transportation activities are considered as the carbon emission intensity. The transportation emissions depend basically on the location of the supplier. The production emissions occur at suppliers' sites. They can be reduced, but this leads to a higher manufacturing cost for the supplier (Conrad, 2005), which implies a higher purchasing cost for the retailer. In the general case, each retailer decides the price, the carbon emission intensity, and the order size of the product that he/she is offering to maximize the expected profit while satisfying a service level constraint. This chapter closes the gap by studying the dynamic competition from a carbon emission perspective. We address these main research questions in this chapter:

- How greenness-driven competition affects the retailers' best strategies?
- Do different market structures affect retailers' best strategies?

5.1 Problem description

We consider two retailers that sell substitutable products, differentiated in terms of greenness level (i.e., carbon emission intensity) and price, in a greenness- and price-sensitive market (see Figure 5.1 below). Customers arrive according to a Poisson process with mean arrival rate λ_i for the retailer i. The mean demand of each product is decreasing in its carbon emission intensity and price and increasing in other product's carbon emission intensity and price. The demand function will be introduced and discussed later in this section. Each retailer has its own supplier. The product sold by retailer i is denoted by P_i. The main sources of carbon emissions are the transportation and production activities. The transportation emissions per unit of product, denoted by e_i for retailer i, depend on the distance traveled by the product from the supplier's site to the retailer's warehouse.

As we do not deal with supplier selection, e_i is not a decision variable. Without loss of generality, we assume that the transportation emissions from the retailers' warehouses to end customers can be neglected, which implicitly means that the retailers are located close to the demand zone. As for production emissions, we consider a context where the retailer can choose the production emission level and, thus, ask for the supplier to manufacture the product accordingly. We provided chapter 1 some examples that illustrate how leading retailers, such as Walmart, are asking their suppliers to produce greener products. We let x_0 denote the amount of production emissions per unit of a standard product. Offering a greener product (with lower production emissions) implies a higher purchasing cost for the retailer as it incurs a higher manufacturing cost. We let x_i denote the amount of production emissions per unit of product P_i. The carbon emission intensity of P_i is thus given by $e_i + x_i$. The unitary purchasing cost of P_i is given by $c_i + b(x_0 - x_i)^2$, where c_i is the unitary cost of the standard product and b is the cost factor for production emissions reduction. We consider a quadratic cost function as usual in the related literature (e.g., Liu et al., 2012, Ghosh and Shah, 2015).

The replenishment order size of retailer i is denoted by S_i. The service rate to refill the stock is exponentially distributed with mean rate μ_i for retailer i. The replenishment time does not depend on the order size since the products are assumed to be always available at the supplier's site, which is a common assumption (Zhu, 2015). Thus, the replenishment time corresponds basically to preparation and transportation activities. The retailers' inventory policy is similar to the previous chapter and, thus, we are going to skip explanation to avoid redundancy. As a reminder, the probability of satisfying demand from the standard stock, denoted by $1 - \psi_i$ for retailer i, must be greater than $1 - r$, for both retailers. Hence, the service level constraint for retailer i is given by $(1 + \frac{\mu_i S_i}{\lambda_i})^{-1} \leq r$ (See Chapter 3 for more information).

Since each retailer has its own supplier, we typically have a different distance separating each retailer from its supplier. Without loss of generality, we let retailer 1 represents the retailer that has the farthest supplier. Therefore, we have $e_1 \geq e_2$ and $\mu_1 \leq \mu_2$.

Our demand model is linear with substitution. We recall that λ_i refers to the mean demand rate for retailer i. The market potential is denoted by A. We let α_p and β_e respectively denote the market sensitivity to the price and the carbon emission intensity. As for switchovers, we respectively denote by α_s and β_s, the sensitivity of switchover toward price difference and carbon emission intensity difference.

In practice, the retailers may have different market powers such as the case where there is an established leading retailer and a new retailer entering the market. In this case, the leading retailer has more market power and, thus, attracts more customers even when the same product is offered by both retailers. We let $\omega \in [0,1]$ denote the market share of retailer 1 and, thus, $(1 - \omega)$ is the market share of retailer 2, when both retailers offer the same product (i.e., with the same price p and carbon emission intensity $x + e$). The mean demand rates are given as follows.

$$\lambda_1 = \omega A - \omega \alpha_p p_1 + \alpha_s (p_2 - p_1) - \omega \beta_e (x_1 + e_1) + \beta_s ((x_2 + e_2) - (x_1 + e_1)) \tag{5.1}$$

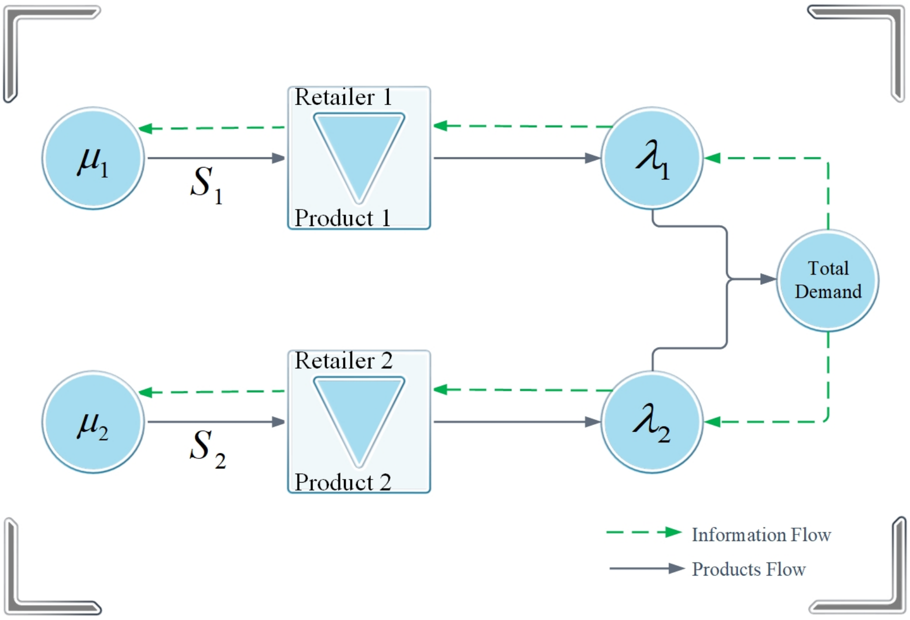

Figure 5.1: Competitive supply chains structure

$$\lambda_2 = (1-\omega)A-(1-\omega)\alpha_p p_2+\alpha_s(p_1-p_2)-(1-\omega)\beta_e(x_2+e_2)+\beta_s((x_1+e_1)-(x_2+e_2)) \tag{5.2}$$

Notice that $\lambda_1+\lambda_2 = A-\omega\alpha_p p_1-(1-\omega)\alpha_p p_2-\omega\beta_e(x_1+e_1)-(1-\omega)\beta_e(x_2+e_2)$. This means that the total mean demand is sensitive to prices and carbon emission intensities.

If the same product is offered by both retailers, then the mean demand is $\omega\big(A-\alpha_p p-\beta_e(x+e)\big)$ for retailer 1 and $(1-\omega)\big(A-\alpha_p p-\beta_e(x+e)\big)$ for retailer 2, and the total mean demand does not depend on ω. The case where both retailers have the same market power, which is the typical case studied in the literature, corresponds to $\omega = 0.5$. In our model, the market share ω represents the market power of retailer 1. If $\omega > 0.5$, then retailer 1 has more market power than retailer 2, and vice versa. With the consideration of $\omega \in [0,1]$, we generalize the linear demand with substitution that is usually adopted in the related literature.

To better understand our demand function, notice that retailer 1 can attract $\omega\alpha_p+\alpha_s$ more customers with one unit decrease in price (respectively, $(1-\omega)\alpha_p+\alpha_s$ more customers for retailer 2) and $\omega\beta_e+\beta_s$ more customers with one unit decrease in carbon emission intensity (respectively, $(1-\omega)\beta_e+\beta_s$ for retailer 2). Only a part of these customers represent a new created demand in the market (for retailer 1, $\omega\alpha_p$ new customers for one unit decrease in price and $\omega\beta_e$ new customers for one unit decrease in carbon emission intensity), and the other attracted customers are switching from the other retailer (α_s switching customers for a unit decrease in price and β_s switching customers for a unit decrease in carbon emission intensity).

Since $e_1 \geq e_2$ and both of them are fixed parameters, we can consider, without loss of generality, that $e_2 = 0$ and, thus, e_1 can be interpreted as the difference in transportation emissions. To simplify notation, we let $A_1 = \omega A - \theta_1 e_1$, $A_2 = (1-\omega)A + \beta_s e_1$, $\delta_1 = \omega\alpha_p + \alpha_s$, $\delta_2 = (1-\omega)\alpha_p + \alpha_s$, $\theta_1 = \omega\beta_e + \beta_s$ and $\theta_2 = (1-\omega)\beta_e + \beta_s$. The mean demand rates are finally given as follows.

$$\lambda_1 = A_1 - \delta_1 p_1 + \alpha_s p_2 - \theta_1 x_1 + \beta_s x_2 \tag{5.3}$$

$$\lambda_2 = A_2 - \delta_2 p_2 + \alpha_s p_1 - \theta_2 x_2 + \beta_s x_1 \tag{5.4}$$

Based on the framework describe above, we then study different competition scenarios and investigate their impact on the greenness level and price of the products. In the competition with full reaction case, we consider that each retailer decides the greenness level of its product (i.e., x_i), the price (p_i) and the order size (S_i) to maximize its expected profit under the service level constraint while considering other retailer's decisions. We also consider two other particular settings that can be particularly relevant when there is an established retailer and a new retailer that enters the market. In fact, we consider that retailer 2 enters the market and decides x_2, p_2 and S_2 whereas, in the first particular setting, retailer 1 does not react and, in the second particular setting, retailer 1 reacts by adjusting its price p_1 and order size S_1, but without changing the greenness level, x_1, as this was agreed upon with its supplier. In the scenarios of competition with reaction (i.e., the general case and the second particular case), we consider a non-cooperative game and determine the optimal strategy of each retailer at the Nash equilibrium. In the following section, we formulate the different models and present the analytical solving approaches and the closed-from expressions of optimal solutions.

5.2 Mathematical models and analytical solutions

In this technical section, we solve the problem under different settings (decision variables) and provide the closed-form expressions of the optimal solutions. Then, we analyze these optimal solutions to derive managerial insights.

5.2.1 Competition with full reaction (M_4)

In this sub-section, we are looking forward to finding out that what first retailer's optimal strategies will be if he/she changes the product's carbon emission level also (i.e. considering x_1 as variable) with considering newcomer's decisions are and how newcomer reacts to the other retailer's new strategies. This chain of reaction between these two retailers, old one and newcomer, continues until there are no new strategies that improve both retailers' profit. The newcomer decides product's carbon emission level (x_2), price (p_2) and order size (S_2). Meanwhile, the existed retailer decides product's carbon emission level (x_1), price (p_1) and order size (S_1). We formulate the mathematical model of the problem as:

Retailer 1's model

$$\underset{x_1,p_1,S_1}{\text{Max}}\ \pi_1 = \Big(p_1 - \big(c_1 + b(x_0 - x_1)^2\big)\Big)\lambda_1 - hS_1$$

Subject to

$$\psi_1 = \frac{\lambda_1}{\lambda_1 + S_1\mu_1} \leq r$$

$$\lambda_1 = A_1 - \delta_1 p_1 + \alpha_s p_2 - \theta_1 x_1 + \beta_s x_2$$

$$p_1, S_1, \lambda_1 \geq 0, 0 \leq x_1 \leq x_0$$

Retailer 2's model

$$\underset{x_2,p_2,S_2}{\text{Max}}\ \pi_2 = \Big(p_2 - \big(c_2 + b(x_0 - x_2)^2\big)\Big)\lambda_2 - hS_2$$

Subject to

$$\psi_2 = \frac{\lambda_2}{\lambda_2 + S_2\mu_2} \leq r$$

$$\lambda_2 = A_2 - \delta_2 p_2 + \alpha_s p_1 - \theta_2 x_2 + \beta_s x_1$$

$$p_2, S_2, \lambda_2 \geq 0, 0 \leq x_2 \leq x_0$$

Each retailer objective is to maximize its profit function under other retailer's decisions. The profit function is the net profit of selling product, which is total revenue of selling product minus production and inventory costs. Each retailer has a service level constraint that ensures probability of having no stock is less than a predetermined amount. It is recalled that $A_1 = \omega A - \theta_1 e_1$, $A_2 = (1-\omega)A + \beta_s e_1$, $\delta_1 = \omega\alpha_p + \alpha_s$, $\delta_2 = (1-\omega)\alpha_p + \alpha_s$, $\theta_1 = \omega\beta_e + \beta_s$ and $\theta_2 = (1-\omega)\beta_e + \beta_s$. It is also recalled that $\omega \in [0,1]$ represents the market power of retailer 1 and, thus, $1-\omega$ is the market power for retailer 2.

To find the Nash equilibrium strategies, we first consider that retailer 2's decisions are known and determine retailer 1's best response, which means that we need to solve retailer 1's model. Then we consider that retailer 1's decisions are known and solve retailer 2's model to obtain retailer 2's best response. We finally determine the Nash equilibrium of the game.

We now determine the best strategy of retailer 1 given retailer 2's decisions. Thus, we need to solve model for a given values of x_2, p_2, and S_2. To solve this model, we first transform it into a single-variable model based on the results of Lemma 5.1 and 5.2 given below.

Lemma 5.1. *For a given strategy of retailer 2 (i.e., given x_2,p_2, and S_2), the retailer 1's optimal order size as a function of x_1 and p_1 is*
$$S_1^* = \frac{(1-r)(A_1 - \delta_1 p_1 + \alpha_s p_2 - \theta_1 x_1 + \beta_s x_2)}{r\mu_1}$$

Proof. Since the objective function is linearly decreasing in S_1, the smallest possible S_1 is the optimal stock. According to the service level constraint, $S_1 \geq \dfrac{(1-r)\lambda_1}{r\mu_1}$, therefore, the optimal value is $S_1{}^* = \dfrac{(1-r)}{r\mu_1}(A_1 - \delta_1 p_1 + \alpha_s p_2 - \theta_1 x_1 + \beta_s x_2)$, which implies that service level constraint is binding. ■

We, thus, replace S_1^* with its expression given in Lemma 5.1 and obtain the following equivalent formulation of model (M_4) with two variables p_1 and x_1.

$$\underset{p_1,x_1}{\text{Maximize}}\ \pi = \left(p_1 - \left(c_1 - b(x_0 - x_1)^2 + \frac{h(1-r)}{r\mu_1} \right) \right) \lambda_1 \tag{5.5}$$

Subject to

$$\lambda_1 = A_1 - \delta_1 p_1 + \alpha_s p_2 - \theta_1 x_1 + \beta_s x_2$$
$$\lambda_1, p_1 \geq 0, 0 \leq x_1 \leq x_0$$

In the following Lemma, we determine the optimal price.

Lemma 5.2. *For a given strategy of retailer 2 (i.e., given x_2,p_2, and S_2), the retailer 1's optimal price as a function of x_1 is*

$$p_1^* = \frac{\delta_1 b{x_1}^2 - (\theta_1 + 2\delta_1 b x_0)x_1 + A_1 + \alpha_s p_2 + \beta_s x_2 + \delta_1 (c_1 + b{x_0}^2 + \frac{h(1-r)}{r\mu_1})}{2\delta_1}$$

Proof. The second derivative of objective function with respect to p_1, which is presented in the following, is negative and demonstrates that objective function is concave in p_1. Therefore the root of first derivative (called ${p_1}^{*max}$) maximize the objective function.

$$\frac{\partial \pi}{\partial p_1} = -2\delta_1 p_1 + \delta_1 b{x_1}^2 - (\theta_1 + 2\delta_1 b x_0)x_1 + A_1 + \alpha_s p_2 + \beta_s x_2 + \delta_1 \left(c_1 + b{x_0}^2 + \frac{h(1-r)}{r\mu_1} \right)$$
$$\frac{\partial^2 \pi}{\partial {p_1}^2} = -2\delta_1$$
$$\frac{\partial \pi}{\partial p_1} = 0 \Leftrightarrow {p_1}^{*max} = \frac{\delta_1 b{x_1}^2 - (\theta_1 + 2\delta_1 b x_0)x_1 + A_1 + \alpha_s p_2 + \beta_s x_2 + \delta_1 \left(c_1 + b{x_0}^2 + \frac{h(1-r)}{r\mu_1} \right)}{2\delta_1}$$

If ${p_1}^{*max} > 0$, then it can be considered as the optimal price. The condition that ${p_1}^{*max}$ is positive, is presented in the following.

$${p_1}^{*max} = \frac{\delta_1 b{x_1}^2 - (\theta_1 + 2\delta_1 b x_0)x_1 + A_1 + \alpha_s p_2 + \beta_s x_2 + \delta_1 \left(c_1 + b{x_0}^2 + \frac{h(1-r)}{r\mu_1} \right)}{2\delta_1} \geq 0$$
$$\Leftrightarrow \delta_1 b{x_1}^2 - (\theta_1 + 2\delta_1 b x_0)x_1 + A_1 + \alpha_s p_2 + \beta_s x_2 + \delta_1 \left(c_1 + b{x_0}^2 + \frac{h(1-r)}{r\mu_1} \right) \geq 0$$

The discriminant of above equation is $\Delta_4 = -4\delta_1 b(A_1 + \alpha_s p_2 + \beta_s x_2 + \delta_1 \left(c_1 + \frac{h(1-r)}{r\mu_1} \right) - \theta_1 x_0 - \frac{\theta_1^2}{4\delta_1 b})$. We know that $x_2, p_2 \geq 0$, then, Δ_4 is negative as long as we consider a constraint such that $A_1 \geq \theta_1 x_0 + \frac{\theta_1^2}{4\delta_1 b} - \delta_1\left(c_1 + \frac{h(1-r)}{r\mu_1}\right)$. Because $\delta_1 b$ (coefficient of ${x_1}^2$) is positive and $\Delta_4 < 0$, then, ${p_1}^{*max}$ is always positive.

The last constraint is related to the demand positivity. The ${p_1}^{*max}$ is the optimal solution if $\lambda_1({p_1}^{*max}) \geq 0$. The condition that $\lambda_1 \geq 0$ is presented in the following.

$$\lambda_1 = \frac{-\delta_1 b{x_1}^2 + (-\theta_1 + 2\delta_1 bx_0)x_1 + A_1 + \alpha_s p_2 + \beta_s x_2 - \delta_1\left(c_1 + b{x_0}^2 + \frac{h(1-r)}{r\mu_1}\right)}{2} \geq 0$$

$$\Leftrightarrow -\delta_1 b{x_1}^2 + (-\theta_1 + 2\delta_1 bx_0)x_1 + A_1 + \alpha_s p_2 + \beta_s x_2 - \delta_1\left(c_1 + b{x_0}^2 + \frac{h(1-r)}{r\mu_1}\right) \geq 0$$

The discriminant of the above equation is equal to $\Delta_5 = 4\delta_1 b\left(A_1 + \alpha_s p_2 + \beta_s x_2 - \delta_1\left(c_1 + \frac{h(1-r)}{r\mu_1}\right) - \theta_1 x_0 + \frac{\theta_1^2}{4\delta_1 b}\right)$. Like earlier, We know that $x_2, p_2 \geq 0$ and Δ_5 is positive as long as we assume a constraint such that $A_1 \geq \theta_1 x_0 + \delta_1\left(c_1 + \frac{h(1-r)}{r\mu_1}\right) - \frac{\theta_1^2}{4\delta_1 b}$. Therefore, the above equation has two roots (called R_5 and R_6), which is positive between these two roots and negative outside them. Therefore, the demand is positive between these roots.

$$R_5 = \frac{-(-\theta_1 + 2\delta_1 bx_0) - \sqrt{\Delta_5}}{-2\delta_1 b} = x_0 + \frac{-\theta_1 + \sqrt{\Delta_5}}{2\delta_1 b},$$
$$R_6 = \frac{-(-\theta_1 + 2\delta_1 bx_0) + \sqrt{\Delta_5}}{-2\delta_1 b} = x_0 - \frac{\theta_1 + \sqrt{\Delta_5}}{2\delta_1 b}.$$

We have $R_5 > x_0$ and $R_6 < 0$ (more details in Appendix D). Therefore, demand is positive in the feasible region ($[0, x_0]$). As a result ${p_1}^{*max} = {p_1}^*$. ■

Thanks to the result of the previous Lemma, we formulate model with only one variable x_1 as follows.

$$\underset{0 \leq x_1 \leq x_0}{\text{Maximize}}\ \pi =$$
$$\frac{\left(-\delta_1 b{x_1}^2 + (-\theta_1 + 2\delta_1 bx_0)x_1 + A_1 + \alpha_s p_2 + \beta_s x_2 - \delta_1\left(c_1 + b{x_0}^2 + \frac{h(1-r)}{r\mu_1}\right)\right)^2}{4\delta_1} \tag{5.6}$$

Subject to

$$\lambda_1 = \frac{-\delta_1 b{x_1}^2 + (-\theta_1 + 2\delta_1 bx_0)x_1 + A_1 + \alpha_s p_2 + \beta_s x_2 - \delta_1\left(c_1 + b{x_0}^2 + \frac{h(1-r)}{r\mu_1}\right)}{2} \tag{5.7}$$

Proposition 5.1. *The optimal retailer 1's carbon emission is* $x_1^* = x_0 - \dfrac{\theta_1}{2\delta_1 b}$.

Proof. The objective function can be presented as $\dfrac{{\lambda_1}^2}{\delta_1}$. Since $\lambda_1 \geq 0$ (see Lemma 5.2), maximizing λ_1 is equivalent to maximizing $\dfrac{{\lambda_1}^2}{\delta_1}$ (objective function). The second derivative of demand with respect to the x_1, which is presented in the following, is negative and the root of the first derivative is the ${x_1}^{*max}$ that maximize the demand (equivalently objective function).

$$\frac{\partial \lambda_1}{\partial x_1} = \frac{-2\delta_1 b x_1 + (2\delta_1 b x_0 - \theta_1)}{2}$$

$$\frac{\partial^2 \lambda_1}{\partial {x_1}^2} = -\delta_1 b$$

$$\frac{\partial \lambda_1}{\partial x_1} = 0 \Leftrightarrow {x_1}^{*max} = x_0 - \frac{\theta_1}{2\delta_1 b}$$

It is obvious that ${x_1}^{*max} < x_0$. While the ${x_1}^{*max}$ is in the feasible region, it is the optimal solution, otherwise, is equal to zero. Thus ${x_1}^* = max\{0, x_0 - \dfrac{\theta_1}{2\delta_1 b}\}$ ■

We now assume that retailer 1's decisions (i.e., x_1, p_1 and S_1) are known and solve model for retailer 2. It is noted that models are symmetric. Therefore, we use the same approach to solve retailer 2's model.

Lemma 5.3. *For a given strategy of retailer 1 (i.e., given* x_1, p_1*, and* S_1*), the retailer 2's optimal order size as a function of* x_2 *and* p_2 *is*
$S_2^* = \dfrac{(1-r)(A_2 - \delta_2 p_2 + \alpha_s p_1 - \theta_2 x_2 + \beta_s x_1)}{r\mu_2}$

The proof is similar to Lemma 5.1. We, thus, replace S_2^* with its expression given in Lemma 5.3 and obtain the following equivalent model with two variables p_2 and x_2.

$$\underset{p_2, x_2}{\text{Maximize}}\ \pi = \left(p_2 - \left(c_2 - b(x_0 - x_2)^2 + \frac{h(1-r)}{r\mu_2} \right) \right) \lambda_2 \tag{5.8}$$

Subject to

$$\lambda_2 = A_2 - \delta_2 p_2 + \alpha_s p_1 - \theta_2 x_2 + \beta_s x_1$$
$$\lambda_2, p_2 \geq 0, 0 \leq x_2 \leq x_0$$

In the following Lemma, we determine the optimal price.

Lemma 5.4. *For a given strategy of retailer 1 (i.e., given x_1,p_1, and S_1), the retailer 2's optimal price as a function of x_2 is*

$$p_2^* = \frac{\delta_2 b {x_2}^2 - (\theta_2 + 2\delta_2 b x_0)x_2 + A_2 + \alpha_s p_1 + \beta_s x_1 + \delta_2\Big(c_2 + b{x_0}^2 + \dfrac{h(1-r)}{r\mu_2}\Big)}{2\delta_2}$$

The proof is similar to Lemma 5.2. Thanks to the result of the previous Lemma, we formulate model with only one variable x_2 as follows.

$$\underset{0\leq x_2 \leq x_0}{\text{Maximize}}\ \pi =$$

$$\frac{\left(-\delta_2 b {x_2}^2 + (-\theta_2 + 2\delta_2 b x_0)x_2 + A_2 + \alpha_s p_1 + \beta_s x_1 - \delta_2\left(c_2 + b{x_0}^2 + \dfrac{h(1-r)}{r\mu_2}\right)\right)^2}{4\delta_2} \tag{5.9}$$

Subject to

$$\lambda_1 = \frac{-\delta_2 b {x_2}^2 + (-\theta_2 + 2\delta_2 b x_0)x_2 + A_2 + \alpha_s p_1 + \beta_s x_1 - \delta_2\Big(c_2 + b{x_0}^2 + \dfrac{h(1-r)}{r\mu_2}\Big)}{2} \tag{5.10}$$

Proposition 5.2. *The optimal retailer 2's carbon emission is* $x_2^* = x_0 - \dfrac{\theta_2}{2\delta_2 b}$.

The proof is similar to Proposition 5.1.

Based on the analysis presented above, we can now derive the best response of each retailer to other retailer's decisions. Thanks to Proposition 5.1 and 5.2, we substitute x_1 and x_2 by their expression $x_0 - \dfrac{\theta_1}{2\delta_1 b}$ and $x_0 - \dfrac{\theta_2}{2\delta_2 b}$, respectively. Therefore, the optimal expression of p_1 given in Lemma 5.2 becomes:

$$p_1^*(p_2) = \frac{\alpha_s}{2\delta_1}p_2 + \frac{A_1 + (\beta_s - \theta_1)x_0 + \dfrac{3{\theta_1}^2}{4\delta_1 b} - \dfrac{\beta_s\theta_2}{2\delta_2 b} + \delta_1\Big(c_1 + \dfrac{h(1-r)}{r\mu_1}\Big)}{2\delta_1} \tag{5.11}$$

Similarly, the optimal expression of p_2 given in Lemma 5.4 becomes:

$$p_2^*(p_1) = \frac{\alpha_s}{2\delta_2}p_1 + \frac{A_2 + (\beta_s - \theta_2)x_0 + \dfrac{3{\theta_2}^2}{4\delta_2 b} - \dfrac{\beta_s\theta_1}{2\delta_1 b} + \delta_2\Big(c_2 + \dfrac{h(1-r)}{r\mu_2}\Big)}{2\delta_2} \tag{5.12}$$

The response of each player is, thus, linear in other player's decision. Consequently, the intersection of the two best response curves is the Nash equilibrium (Osborne et al., 2004), as illustrated in Figure 5.2. Based on the above analysis, we finally derive in the following

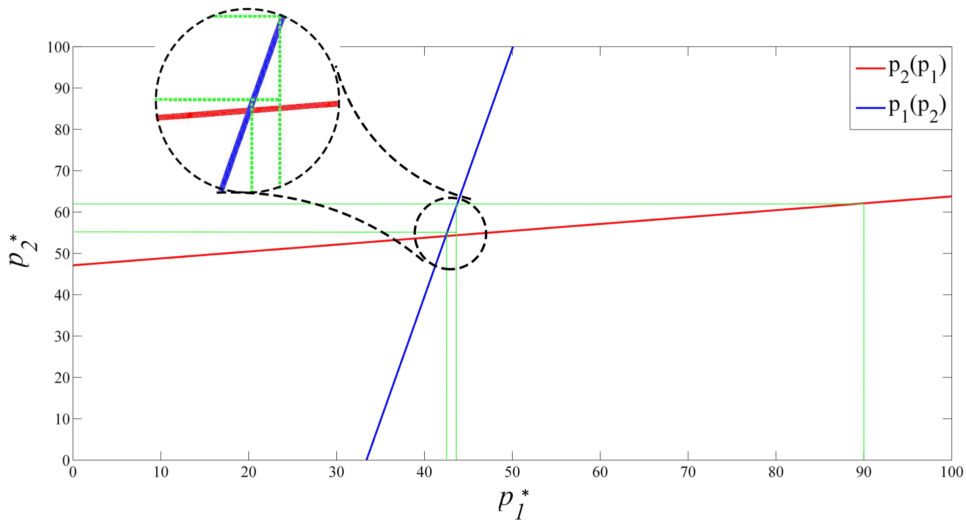

Figure 5.2: Illustration of the Nash equilibrium

Proposition the optimal strategy of each retailer at the Nash equilibrium.

Proposition 5.3. *The optimal strategy of each retailer at the Nash equilibrium is the following.*

For retailer 1: $x_1^* = x_0 - \dfrac{\theta_1}{2\delta_1 b}$, $p_1^* = \dfrac{2\delta_2(2\delta_2 z_1 + \alpha_s z_2)}{4\delta_1\delta_2 - \alpha_s{}^2}$, *and*

$$S_1^* = \frac{1-r}{r\mu_1}\Big(A_1 + \frac{2\delta_1(-2\delta_1\delta_2 + \alpha_s{}^2)z_1 + 2\alpha_s\delta_1\delta_2 z_2}{4\delta_1\delta_2 - \alpha_s{}^2} - \omega\beta_e\big(x_0 - \frac{\theta_1}{2\delta_1 b}\big) + \frac{\beta_s}{2b}\big(\frac{\theta_1}{\delta_1} - \frac{\theta_2}{\delta_2}\big)\Big).$$

For retailer 2: $x_2^* = x_0 - \dfrac{\theta_2}{2\delta_2 b}$, $p_2^* = \dfrac{2\delta_1(2\delta_2 z_2 + \alpha_s z_1)}{4\delta_1\delta_2 - \alpha_s{}^2}$, *and*

$$S_2^* = \frac{1-r}{r\mu_2}\Big(A_2 + \frac{2\delta_2(-2\delta_1\delta_2 + \alpha_s{}^2)z_2 + 2\alpha_s\delta_1\delta_2 z_1}{4\delta_1\delta_2 - \alpha_s{}^2} - (1-\omega)\beta_e\big(x_0 - \frac{\theta_2}{2\delta_2 b}\big) + \frac{\beta_s}{2b}\big(\frac{\theta_2}{\delta_2} - \frac{\theta_1}{\delta_1}\big)\Big).$$

Where $z_1 = \dfrac{A_1 + (\beta_s - \theta_1)x_0 + \dfrac{3\theta_1{}^2}{4\delta_1 b} - \dfrac{\beta_s\theta_2}{2\delta_2 b} + \delta_1\Big(c_1 + \dfrac{h(1-r)}{r\mu_1}\Big)}{2\delta_1}$ *and*

$$z_2 = \frac{A_2 + (\beta_s - \theta_2)x_0 + \dfrac{3\theta_2{}^2}{4\delta_2 b} - \dfrac{\beta_s\theta_1}{2\delta_1 b} + \delta_2\Big(c_2 + \dfrac{h(1-r)}{r\mu_2}\Big)}{2\delta_2}.$$

Proof. We know that the optimal prices p_1^* and p_2^* are obtained at the intersection of the two best response curves. If we take the optimal price p_1^*, we can determine its associated p_2^* as a function of p_1^* by using equation 5.12. The optimal price of retailer 1's product that is associated with this p_2^* is, on the one hand, obtained by equation 5.11 and, on the other hand, equal to p_1^* since we are at the intersection point.

Consequently, it comes that $p_1 = \frac{\alpha_s}{2\delta_1}\Bigg(\frac{\alpha_s}{2\delta_2}p_1 + \frac{1}{2\delta_2}\Bigg(A_2 + (\beta_s - \theta_2)x_0 + \frac{3\theta_2{}^2}{4\delta_2 b} - \frac{\beta_s\theta_1}{2\delta_1 b} + \delta_2\Big(c_2 + \frac{h(1-r)}{r\mu_2}\Big)\Bigg)\Bigg) + \frac{1}{2\delta_1}\Bigg(A_1 + (\beta_s - \theta_1)x_0 + \frac{3\theta_1{}^2}{4\delta_1 b} - \frac{\beta_s\theta_2}{2\delta_2 b} + \delta_1\Big(c_1 + \frac{h(1-r)}{r\mu_1}\Big)\Bigg).$
We then deduce by standard calculus that $p_1^* = \frac{2\delta_2(2\delta_2 z_1 + \alpha_s z_2)}{4\delta_1\delta_2 - \alpha_s{}^2}$ and, consequently, $p_2^* = \frac{2\delta_1(2\delta_2 z_2 + \alpha_s z_1)}{4\delta_1\delta_2 - \alpha_s{}^2}$. The optimal expressions of order sizes are given in Lemma 5.1 and 5.3. The optimal order sizes are deduced from the formula $S_1^* = \frac{1-r}{r\mu_1}\Bigg(A_1 + \frac{1}{4\delta_1\delta_2 - \alpha_s{}^2}\Big(2\delta_1(-2\delta_1\delta_2 + \alpha_s{}^2)z_1 + 2\alpha_s\delta_1\delta_2 z_2\Big) - \omega\beta_e\Big(x_0 - \frac{\theta_1}{2\delta_1 b}\Big) + \frac{\beta_s}{2b}\Big(\frac{\theta_1}{\delta_1} - \frac{\theta_2}{\delta_2}\Big)\Bigg)$ and $S_2^* = \frac{1-r}{r\mu_2}\Bigg(A_2 + \frac{1}{4\delta_1\delta_2 - \alpha_s{}^2}\Big(2\delta_2(-2\delta_1\delta_2 + \alpha_s{}^2)z_2 + 2\alpha_s\delta_1\delta_2 z_1\Big) - (1-\omega)\beta_e\Big(x_0 - \frac{\theta_2}{2\delta_2 b}\Big) + \frac{\beta_s}{2b}\Big(\frac{\theta_2}{\delta_2} - \frac{\theta_1}{\delta_1}\Big)\Bigg)$ by replacing $p_1, x_1, p_2,$ and x_2 with their optimal expressions.

■

Until now, we have solved the competition with full reaction problem where both retailers undertake price, carbon emission, and inventory decisions. However in practice, there are many situations where an existing retailer (retailer 1) is already operating in the market, and a new retailer (retailer 2) enters the market and offers a substitutable product. In this case, two situations are relevant to study:

- Competition without reaction. The existing retailer does not react to the new retailer's decisions because, for instance, it has a much higher market power.
- Competition with partial reaction. The existing retailer just updates its price and order size but does not change the carbon emission intensity as this requires new deals with the supplier.

With respect to modeling and solving approaches, the case of competition without reaction is similar to the benchmark model and the case of competition with partial reaction is a particular case of our general competition model. We present in what follows the optimal solutions in these two particular cases, but do not provide the details of modeling and solving approaches to avoid redundancy.

5.2.2 Competition without reaction (M_5)

In this case, retailer 1 is already selling its product in the market and retailer 2 enters the market and offers a substitutable product. Retailer 2 decides its product's greenness level, price, and order size (x_2, p_2, and S_2, respectively) while retailer 1 changes nothing (S_1, p_1, and x_1 are known). Hence, the problem consists in finding the optimal decisions for retailer 2 given that retailer 1 already operates in the market. Here, the mean demand for retailer 2's product, namely λ_2, depends not only on its carbon emission intensity and price (x_2 and p_2) but also on the carbon emission intensity and price of retailer 1's product (see equations 5.3 and 5.4). Therefore, λ_2 is here given by $\omega A + \alpha_s p_1 + \beta_s(x_1 + e_1) - \delta_2 p_2 - \theta_2 x_2$. It is recalled that ω represents the market share of retailer 1 if both retailers offer products with the same characteristics. In the case of competition without reaction, x_1, p_1, and S_1 are fixed. We let $A'_2 = \omega A + \alpha_s p_1 + \beta_s(x_1 + e_1)$. Thus, the expression of λ_2 can be simplified as follows.

$$\lambda_2 = A'_2 - \delta_2 p_2 - \theta_2 x_2 \tag{5.13}$$

In the case of competition without reaction, the formulation of retailer 2's model, denoted by (M_5), is similar to the benchmark model, although, the effectual market potential, the price sensitivity parameter, and the greenness sensitivity parameter are different. Consequently, we use the same methodology used for (M_0) to solve model (M_5). The calculation steps are similar, although, the demands are not the same (in aspect of parameters). The optimal solutions are provided in Proposition 5.4.

Mathematical model:

$$\underset{x_2, p_2, S_2}{\text{Maximize}}\ \pi = (p_2 - (c_2 - b(x_0 - x_2)^2))\lambda_2 - hS_2 \tag{5.14}$$

Subject to

$$\psi_0 = \frac{\lambda_2}{\lambda_2 + S_2\mu_2} \leq r \tag{5.15}$$

$$\lambda_2 = A'_2 - \delta_2 p_2 - \theta_2 x_2 \tag{5.16}$$

$$\lambda_2, p_2, S_2 \geq 0, 0 \leq x_2 \leq x_0$$

Thanks to following Lemma 5.5 and 5.6, the model transforms into a single-variable model. The optimal solution is provided in Proposition 5.4.

Lemma 5.5. *For a given values of p_2 and x_2, the optimal stock is* $S_2^* = \dfrac{(1-r)(A_2' - \delta_2 p_2 - \theta_2 x_2)}{r\mu_2}$

The proof is similar to Lemma 5.3. Lemma 5.5 provides the optimal amount of order size that the retailer 2 needs to order in function of the product's price and carbon emission level. The service level (equation 5.13) is binding. Hence, we deduce that service level constraint will be relaxed after substituting S_2^* by its expression into the model.
Since we assume that $\lambda_2 \geq 0$, S_2^* is positive too and can be substituted. By substituting S_2^* in equation 5.12, the model can be rewritten as presented in the following:

$$\underset{x_2,p_2,S_2}{\text{Maximize}}\ \pi = \left(p_2 - \left(c_2 - b(x_0 - x_2)^2 - \frac{h(1-r)}{r\mu_2}\right)\right)\lambda_2 \tag{5.17}$$

Subject to

$$\lambda_2 = A_2' - \delta_2 p_2 - \theta_2 x_2$$
$$\lambda_2, p_2 \geq 0, 0 \leq x_2 \leq x_0$$

Lemma 5.6. *For a given value of x_2, optimal price is*

$$p_2^* = \frac{\delta_2 b {x_2}^2 - (2\delta_2 b x_0 + \theta_2)x_2 + A_2' + \delta_2\left(c_2 + b{x_0}^2 + \dfrac{h(1-r)}{r\mu_2}\right)}{2\delta_2}$$

The proof methodology is similar to Lemma 5.4. Lemma 5.6 provides the optimal price of retailer 2's product in function of carbon emission level. The optimal price is decreasing in carbon emission level ($\dfrac{dp_2^*}{dx_2} < 0$) that makes sense as we expected. As much as the carbon emission level of product decreases, retail price non-linearly increases. By substituting p_2^* by its expression, the model transforms into single-variable model as presented in the following:

$$\underset{0\leq x_2\leq x_0}{\text{Maximize}}\ \pi = \frac{\left(-\delta_2 b{x_2}^2 + (2\delta_2 b x_0 - \theta_2)x_2 + A_2' - \delta_2\left(c_2 + b{x_0}^2 + \dfrac{h(1-r)}{r\mu_2}\right)\right)^2}{4\delta_2} \tag{5.18}$$

Subject to

$$\lambda_2 = \frac{-\delta_2 b{x_2}^2 + (2\delta_2 b x_0 - \theta_2)x_2 + A_2' - \delta_2\left(c_2 + b{x_0}^2 + \dfrac{h(1-r)}{r\mu_2}\right)}{2}$$

Proposition 5.4. *The optimum amount of carbon emission, price and order size are equal to* $x_2{}^* = x_0 - \frac{\theta_2}{2\delta_2 b}$, $p_2{}^* = \frac{A_2' - \theta_2 x_0 + \frac{3\theta_2{}^2}{4\delta_2 b} + \delta_2(c_2 + \frac{h(1-r)}{r\mu_2})}{2\delta_2}$, *and* $S_2{}^* = \frac{(1-r)\left(A_2' - \theta_2 x_0 + \frac{3\theta_2{}^2}{4\delta_2 b} - \delta_2\big(c_2 + \frac{h(1-r)}{r\mu_2}\big)\right)}{2r\mu_2}$, *respectively.*

The proof methodology is similar to Proposition 5.2.

The optimal carbon emission level is affected by competition. The competition's effect and comparison are presented in the next section.

Like the optimal results of monopoly scenario, the optimal carbon emission level linearly decreases as customers' carbon emission sensitivity, θ_2, increases. While optimal price and stock level non-linearly decrease until $\theta_2 < \frac{2}{3}\delta_2 b x_0$ and after that increase. When customers take more greenness level of products, when purchasing, into account, the retailer offers a greener product that leads to a higher price. In addition, optimal carbon emission level non-linearly decreases as carbon emission reduction cost factor, b, increases, while, optimal price and stock level non-linearly increase.

5.2.3 Competition with partial reaction (M_6)

In this section, retailer 2 offers a new substitutable product along with retailer 1's product to customers. Retailer 2's decisions are the carbon emission level x_2, the price p_2, and the order size S_2. Under the competition of retailer 2, retailer 1 can also update its retail price p_1 and order size S_1 strategy. Retailer 1 decides to keep his/her product's carbon emission level x_1. We formulate the problem as a non-cooperative game where there is no dominant retailer. Each retailer makes its decisions to maximize its expected profit, taking into account other retailer's decisions. We aim to determine the Nash equilibrium of this game, i.e., the set of optimal decisions such as no retailer (player, from a game theory perspective) can benefit by changing its own decisions while the other player keeps its decisions unchanged.

Since x_1 is known, we let $A_1'' = \omega A - \theta_1(e_1 + x_1)$ and $A_2'' = (1-\omega)A + \beta_s(e_1 + x_1)$ to simplify the notation. Thus, the mean demand for each retailer is given as follows.

$$\lambda_1 = A_1'' - \delta_1 p_1 + \alpha_s p_2 + \beta_s x_2 \tag{5.19}$$

$$\lambda_2 = A_2'' - \delta_2 p_2 + \alpha_s p_1 - \theta_2 x_2 \tag{5.20}$$

The mathematical model for each retailer is given in the following.

Retailer 1's model

$$\max_{p_1,S_1} \pi_1 = (p_1 - c_1)\lambda_1 - hS_1$$

Subject to

$$\psi_1 = \frac{\lambda_1}{\lambda_1 + S_1\mu_1} \le r$$

$$\lambda_1 = A_1'' - \delta_1 p_1 + \alpha_s p_2 + \beta_s x_2$$

$$p_1, S_1, \lambda_1 \ge 0$$

Retailer 2's model

$$\max_{x_2,p_2,S_2} \pi_2 = (p_2 - (c_2 + b(x_0 - x_2)^2))\lambda_2 - hS_2$$

Subject to

$$\psi_2 = \frac{\lambda_2}{\lambda_2 + S_2\mu_2} \le r$$

$$\lambda_2 = A_2'' - \delta_2 p_2 + \alpha_s p_1 - \theta_2 x_2$$

$$p_2, S_2, \lambda_2 \ge 0, 0 \le x_2 \le x_0$$

We use same approach and steps since the problem is similar to general model M_4. To find the Nash equilibrium strategies, we first consider that retailer 2's decisions are known and solve retailer 1's problem. Second, we consider that retailer 1's decisions are known and solve retailer 2's problem. We finally deduce the Nash equilibrium.

Lemma 5.7. *For a given strategy of retailer 2 (i.e., given x_2, p_2, and S_2), service constraint is binding and, thus, the optimal order size is* $S_1^* = \dfrac{(1-r)(A_1'' - \delta_1 p_1 + \alpha_s p_2 + \beta_s x_2)}{r\mu_1}$.

The proof is similar to Lemma 5.1. We substitute S_1^* by its expression and transform model into a single-variable model in p_1 as the unique variable.

$$\underset{p_1}{\text{Maximize}}\ \pi_1 = \Big(p_1 - \big(c_1 + \frac{h(1-r)}{r\mu_1}\big)\Big)\lambda_1 \tag{5.21}$$

Subject to

$$\lambda_1 = A_1'' - \delta_1 p_1 + \alpha_s p_2 + \beta_s x_2 \ge 0 \tag{5.22}$$

$$p_1 \ge 0$$

The retailer 1's optimal response to retailer 2's strategy is provided in the following.

Lemma 5.8. *For a given strategy of retailer 2 (i.e., given x_2, p_2, and S_2), the optimal retailer 1's price is* $p_1^* = \dfrac{A_1'' + \alpha_s p_2 + \beta_s x_2 + \delta_1\Big(c_1 + \dfrac{h(1-r)}{r\mu_1}\Big)}{2\delta_1}$.

The proof is similar to Lemma 5.2. Increasing one unit of second retailer product's price increases $\dfrac{\alpha_s}{2\delta_1}$ unit of first retailer product's price.

So far, the first retailer's problem is solved. We now assume that retailer 1's decisions (i.e., p_1 and S_1) are known and solve second retailer's model.

Lemma 5.9. *For a given values of p_1, p_2, and x_2, service constraint is binding at optimality and, thus, the optimal order size is* $S_2^* = \dfrac{(1-r)(A_2'' - \delta_2 p_2 + \alpha_s p_1 - \theta_2 x_2)}{r\mu_2}$.

The proof is similar to Lemma 5.3. We substitute S_2^* by its expression and obtain an equivalent formulation of model $(M_{2.2})$ with only two variables p_2 and x_2. The resulting model is given below.

$$\underset{x_2,p_2}{\text{Maximize}}\ \pi_2 = \left(p_2 - \left(c_2 + b(x_0 - x_2)^2 + \frac{h(1-r)}{r\mu_2}\right)\right)\lambda_2 \tag{5.23}$$

Subject to

$$\lambda_2 = A_2'' - \delta_2 p_2 + \alpha_s p_1 - \theta_2 x_2 \tag{5.24}$$

$$p_2, \lambda_2 \geq 0, 0 \leq x_2 \leq x_0$$

The following Lemma provides the retailer 2's optimal price.

Lemma 5.10. *For a given values of p_1 and x_2, the optimal price of retailer 2's product is* $p_2^* = \dfrac{\delta_2 b {x_2}^2 - (2\delta_2 b x_0 + \theta_2)x_2 + A_2'' + \alpha_s p_1 + \delta_2\left(c_2 + b{x_0}^2 + \frac{h(1-r)}{r\mu_2}\right)}{2\delta_2}$.

The proof is similar to Lemma 5.4. The second retailer's product price is increasing in first retailer's product price and decreasing in its carbon emission. Increasing one unit of first retailer product's price increases $\dfrac{\alpha_s}{2\delta_2}$ unit of second retailer product's price.

Thanks to the result of the previous Lemma, the mathematical model transforms into a single-variable model as function of x_2, which is presented in the following.

$$\underset{0\leq x_2\leq x_0}{\text{Maximize}}\ \pi_2 = \frac{\left(-\delta_2 b{x_2}^2 + (2\delta_2 b x_0 - \theta_2)x_2 + A_2'' + \alpha_s p_1 - \delta_2\left(c_2 + b{x_0}^2 + \frac{h(1-r)}{r\mu_2}\right)\right)^2}{4\delta_2} \tag{5.25}$$

Subject to

$$\lambda_2 = \frac{-\delta_2 b{x_2}^2 + (2\delta_2 b x_0 - \theta_2)x_2 + A_2'' + \alpha_s p_1 - \delta_2\left(c_2 + b{x_0}^2 + \frac{h(1-r)}{r\mu_2}\right)}{2}$$

Proposition 5.5. *In case of competition with partial reaction, the optimal carbon emission level of retailer 2's product is* $x_2^* = x_0 - \dfrac{\theta_2}{2\delta_2 b}$. *Consequently by substituting* x_2^*, *the optimal prices of retailer 1 and retailer 2 transform into*
$p_1^* = \dfrac{A_1'' - \dfrac{\beta_s\theta_2}{2\delta_2 b} + \beta_s x_0 + \alpha_s p_2 + \delta_1\big(c_1 + \dfrac{h(1-r)}{r\mu_1}\big)}{2\delta_1}$ *and*
$p_2^* = \dfrac{A_2'' + \dfrac{3{\theta_2}^2}{4\delta_2 b} - \theta_2 x_0 + \alpha_s p_1 + \delta_2\big(c_2 + \dfrac{h(1-r)}{r\mu_2}\big)}{2\delta_2}$ *for a given strategy of* p_1 *and* p_2, *respectively.*

The proof is similar to Proposition 5.2.

The optimal carbon emission level is decreasing in θ_2. It means that either more sensitive and concern customers to greenness (i.e., higher β_e and β_s) or higher power and market share (i.e., higher $1-\omega$) push second retailer to offer products with lower production's carbon emission.

Similar to general model M_4, the Nash equilibrium is the intersection of the retailers' optimal price response, the point that neither retailer 1 nor retailer 2 can not gain higher expected profit under other opponent's strategies. Thus, we derive in the following Proposition the optimal strategy of each retailer at the equilibrium.

Proposition 5.6. *The optimum prices of retailer 1 and retailer 2 are*
$p_1^* = \dfrac{2\delta_2(2\delta_2 z_1' + \alpha_s z_2')}{4\delta_1\delta_2 - {\alpha_s}^2}$ *and* $p_2^* = \dfrac{2\delta_1(2\delta_2 z_2' + \alpha_s z_1')}{4\delta_1\delta_2 - {\alpha_s}^2}$, *respectively,*
where $z_1' = \dfrac{\omega A - \theta_1(e_1 + x_1) - \dfrac{\beta_s\theta_2}{2\delta_2 b} + \beta_s x_0 + \delta_1\big(c_1 + \dfrac{h(1-r)}{r\mu_1}\big)}{2\delta_1}$ *and*
$z_2' = \dfrac{(1-\omega)A + \beta_s(e_1 + x_1) + \dfrac{3{\theta_2}^2}{4\delta_2 b} - \theta_2 x_0 + \delta_2\big(c_2 + \dfrac{h(1-r)}{r\mu_2}\big)}{2\delta_2}$.

The proof is similar to Proposition 5.3.

In this technical section, different competition scenarios have been considered, formulated, and solved. The next section aims to bring different types of important managerial insights to the light.

5.3 Analysis and managerial insights

In this section, we drive important insights of problems that are presented and solved earlier. We investigate in this section the effect of competition on products' environmental performance as well as the impact of transportation carbon emissions. First, we present the comparison of the optimal carbon emission level in benchmark and competitive scenarios to highlight the competition's effect. The comparison is done in different market structures (that are presented in Chapter 4), in order to answer the research questions. After that, we focus on the impact of competition on the second retailer's retail price. Finally, we drive some insights of important parameters' effect on benchmark and competitive scenarios.

5.3.1 Impact of competition on products' environmental performance

To understand the impact of competition on greenness, we compare the greenness level of retailer 2's product in competition scenarios to that obtained in the benchmark situation. We recall that the greenness level is measured in terms of carbon emission intensity and that the carbon emission intensity for retailer 2 is given by x_2. We focus on retailer 2's strategy because it is the retailer that undertakes all decisions (i.e., greenness, price and order size) in all studied situations, whereas retailer 1 does not decide the greenness level in the case of competition with partial reaction, and does not undertake any decision in the case of competition without reaction.

It is first noted that in all competition scenarios (regardless of whether and how retailer 1 reacts), retailer 2 offers a product with carbon emission $x_2^* = x_0 - \dfrac{\theta_2}{2\delta_2 b} = x_0 - \dfrac{(1-\omega)\beta_e + \beta_s}{2\big((1-\omega)\alpha_p + \alpha_s\big)b}$. It is interesting to figure out that the carbon emission reduction (i.e., $x_0 - x_2^*$) depends only on market characteristics and emission reduction cost, but does not depend on other retailer's strategy. The price and inventory decisions, however, depend on whether the other retailer reacts or not and the nature of decisions it makes. Hence, the retailer chooses its greenness strategy based on market characteristics and reacts to other retailer's decisions only by adjusting pricing and inventory policies.

We now compare the carbon emission obtained in the benchmark situation for retailer 2 (i.e.; $x_2^* = x_0 - \dfrac{\beta_e}{2\alpha_p b}$) to that resulting from the different competitions scenarios (i.e.; $x_2^* = x_0 - \dfrac{(1-\omega)\beta_e + \beta_s}{2\big((1-\omega)\alpha_p + \alpha_s\big)b}$). We obtain the following main result presented in Proposition 5.7.

Proposition 5.7. *When the switchovers are more governed by greenness differentiation (i.e., $\frac{\alpha_s}{\alpha_p} < \frac{\beta_s}{\beta_e}$), the competition leads to offering a greener product.*
When the switchovers are more governed by price differentiation (i.e., $\frac{\alpha_s}{\alpha_p} > \frac{\beta_s}{\beta_e}$), the competition leads to offering a dirtier product.
In a neutral market (i.e., $\frac{\alpha_s}{\alpha_p} = \frac{\beta_s}{\beta_e}$), the competition does not affect the greenness of the product. The retailer offers the product with the same greenness level in both monopoly and competition situations.

Proof. Let us define x_2^{*B} and x_2^{*C} as optimal carbon emissions in benchmark and competition scenarios, respectively. The carbon emissions' difference in benchmark and competition scenario is $\Delta_x = x_2^{*B} - x_2^{*C}$.

$$x_2^{*B} - x_2^{*C} = \Big(x_0 - \frac{\beta_e}{2\alpha_p b}\Big) - \Big(x_0 - \frac{(1-\omega)\beta_e + \beta_s}{2\big((1-\omega)\alpha_p + \alpha_s\big)b}\Big) = \frac{(1-\omega)\beta_e + \beta_s}{2\big((1-\omega)\alpha_p + \alpha_s\big)b} - \frac{\beta_e}{2\alpha_p b}$$
$$\Leftrightarrow = \frac{\alpha_p\beta_s - \beta_e\alpha_s}{2\alpha_p\big((1-\omega)\alpha_p + \alpha_s\big)b}$$
$$\Rightarrow \Delta_x = \begin{cases} > 0 & \text{, if } \alpha_p\beta_s > \beta_e\alpha_s \\ = 0 & \text{, if } \alpha_p\beta_s = \beta_e\alpha_s \\ < 0 & \text{, if } \alpha_p\beta_s < \beta_e\alpha_s \end{cases}$$

As we can see, the sign of Δ_x depends on market's structures. ■

Compared to the benchmark situation, one may expect that greenness- and price-based competition will lead to enhance the product's greenness. However, our results show that this holds only for some specific market conditions while, for other types of markets, competition either has no effect or even deteriorates the product's greenness. As we will see afterwards, the market characteristics influence many of our results, so they deserve deeper investigation.

We first recall that a unit decrease in carbon emission intensity of retailer 2's product (P2) generates customers at a rate of $(1-\omega)\beta_e + \beta_s$ for P2, out of which $(1-\omega)\beta_e$ are "new" attracted customers and β_s are "cannibalized" customers, substituting P2 for P1. Similarly, a unit price decrease generates customers at a rate of $(1-\omega)\alpha_p + \alpha_s$ for P2, out of which $(1-\omega)\alpha_p$ are "new" attracted customers and α_s are cannibalized from P1.

As a quick reminder, we briefly mention present the market's structures. $\frac{\alpha_s}{\alpha_p} < \frac{\beta_s}{\beta_e}$ implies GDS (governed by greenness differentiation) market, $\frac{\alpha_s}{\alpha_p} > \frac{\beta_s}{\beta_e}$ implies PDS (governed by price differentiation) market, and $\frac{\alpha_s}{\alpha_p} = \frac{\beta_s}{\beta_e}$ implies neutral market.

Back to the findings of Proposition 5.7. The results indicate that whether switching of the customers is influenced more by the disparity between the prices or carbon emissions, governs how the competition influences the optimal greenness strategy and changes it with comparison to the monopoly case. The main difference between the competition case and the monopoly case is the structure of demand. In competition, the retailer's demand is sensitive not only to its own price and carbon emission level but also to the price and carbon emission differentiations with the other retailer as this determines the number of customers switching from one retailer to another. When the switchovers are more governed by greenness differentiation (i.e., in GDS market), there is a fierce competition on greenness and the retailer should therefore capitalize more on greenness performance, which leads to offering a greener product with comparison to the product offered in the monopoly case. In PDS market, in which the switchovers are more governed by price differentiation, the retailer should offer a more competitive price which requires to reducing the cost and, consequently, reducing the greenness performance. This leads to offering a dirtier product. Finally, in the neutral market, the switchovers are equally governed by price and carbon emission disparities. The impact of competition on the optimal carbon emission level is neutralized in this case, and the retailer offers the product with the same greenness level of the monopoly case. The results of Proposition 5.7 are summarized in the following table.

Table 5.1: Impact of competition on the environmental performance

Market structure	$\frac{\alpha_s}{\alpha_p} < \frac{\beta_s}{\beta_e}$	$\frac{\alpha_s}{\alpha_p} = \frac{\beta_s}{\beta_e}$	$\frac{\alpha_s}{\alpha_p} > \frac{\beta_s}{\beta_e}$
Impact on the environmental performance	Competition improves the environmental performance	Competition does not impact the environmental performance	Competition deteriorates the environmental performance

The above discussion explains how competition impacts the greenness level with comparison to the monopoly case. In what follows, we focus on the impact of the market power on the greenness strategy of the retailer. The main result is given in Proposition 5.8.

Proposition 5.8. *In competition with full reaction, a retailer that gains more market power will decrease its product's greenness in GDS market and increase it in PDS market.*

Proof. For retailer 2, we have $x_2^* = x_0 - \frac{\theta_2}{2\delta_2 b} = x_0 - \frac{(1-\omega)\beta_e + \beta_s}{2\big((1-\omega)\alpha_p + \alpha_s\big)b}$. Retailer 2 has more market power when ω decreases (i.e.; $1-\omega$ increases). We have $\frac{\partial x_2^*}{\partial \omega} = \frac{\beta_e\alpha_s - \alpha_p\beta_s}{2b\big((1-\omega)\alpha_p + \alpha_s\big)^2}$. Therefore, $\frac{\partial x_2^*}{\partial \omega} < 0$ in GDS market (i.e., when $\frac{\alpha_s}{\alpha_p} < \frac{\beta_s}{\beta_e}$) and

$\dfrac{\partial x_2^*}{\partial \omega} > 0$ in PDS market. Hence, an increase in market power for retailer 2 leads to an increase in x_2^* in GDS market and a decrease in x_2^* in PDS market, as stated in this Proposition.

As for retailer 1, we have $x_1^* = x_0 - \dfrac{\theta_1}{2\delta_1 b} = x_0 - \dfrac{\omega\beta_e + \beta_s}{2(\omega\alpha_p + \alpha_s)b}$. Retailer 1 has more market power when ω increases. We have $\dfrac{\partial x_1^*}{\partial \omega} > 0$ in GDS market and $\dfrac{\partial x_1^*}{\partial \omega} < 0$ in PDS market.

■

Let us recall that ω represents the market power of retailer 1 and, thus $1-\omega$ is the market power of retailer 2. One may expect that a retailer that sees an increase in its market power (i.e., for retailer 1, ω increases and, for retailer 2, ω decreases) will increase its price and, thus, will have more margin that justifies offering a greener product. However, our results indicate again that the retailer can have two different reactions according to market characteristics. A deeper analysis of the retailer's strategy under each market structure helps to shed light on the main trade-offs resulting from an increase in ω, as we explain in the following. In GDS market, a retailer that loses market power needs to offer a greener product in order to attract more switching customers since switchovers are here governed by greenness. Offering a greener product increases the unit cost but the priority is here given to maintaining a profitable amount of demand. If we make this same analysis in the opposite way (increasing market power), we deduce that this retailer should offer a dirtier product when its market power increases under GDS market, as stated in Proposition 6. In PDS market, a retailer with decreasing market power needs to offer a cheaper product in order to attract more switching customers since switchovers are here governed by price. Offering a cheaper product requires to decreasing the unit cost which means reducing the greenness level. This analysis also means that this retailer should offer a greener product when its power increases under PDS market, wich explains the result of Proposition 5.8.

The results of Proposition 5.8 are illustrated in Figure 5.3. We see that an increase in the market power of retailer 1 (i.e., increase in ω) leads an increase in x_1^* (i.e., decrease in greenness) in GDS market and a decrease in x_1^* (i.e., increase in greenness) in PDS market. For retailer 2, an increase in the market power (i.e., decrease in ω) leads an increase in x_2^* (i.e., decrease in greenness) in GDS market and a decrease in x_2^* (i.e., increase in greenness) in PDS market.

We now compare the optimal carbon emissions x_1^* and x_2^* to determine which retailer buys the greener product (i.e., asks its supplier to provide the greener product). Notice that $x_2^* > x_1^*$ means that retailer 1 buys a greener product than the product bought by retailer 2. However, this does not necessarily mean that retailer 1 offers a greener product to its customers because we must add the transportation carbon emissions e_1 for P1 while we have $e_2 = 0$ for P2. We consider here the general competition model, in which both retailers undertake greenness decisions. The result is given in Proposition 5.9.

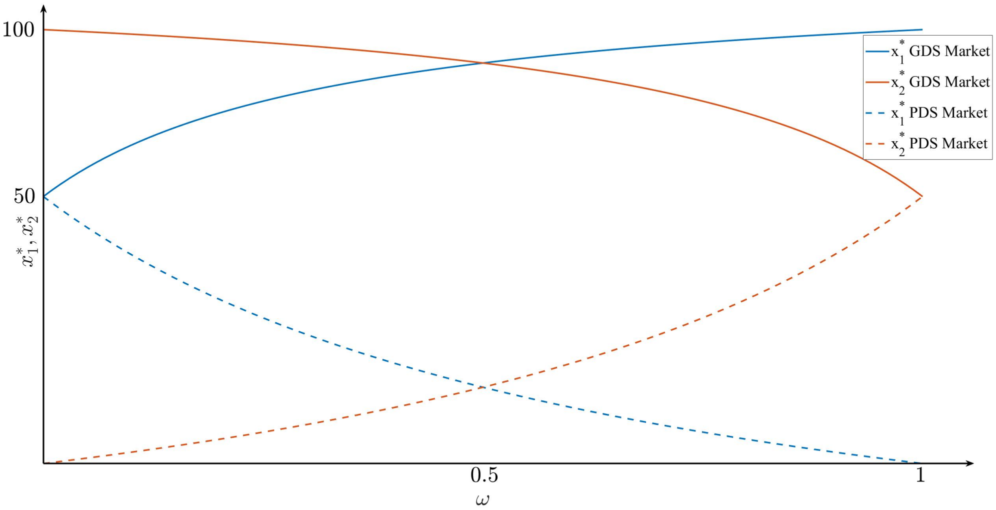

Figure 5.3: Optimal carbon emissions in function of market power

Proposition 5.9. *The retailer that has a smaller market power buys a greener product (than the product bought by the other retailer) under GDS market, and a dirtier product (than the product bought by the other retailer) under PDS market.*

Proof. We have $x_2^* - x_1^* = \left(x_0 - \frac{\theta_2}{2\delta_2 b}\right) - \left(x_0 - \frac{\theta_1}{2\delta_1 b}\right) = \frac{1}{2b}\left(\frac{(1-2\omega)(\alpha_p\beta_s - \beta_e\alpha_s)}{(\omega\alpha_p + \alpha_s)\big((1-\omega)\alpha_p + \alpha_s\big)}\right)$.
We know that $(\omega\alpha_p + \alpha_s)\big((1-\omega)\alpha_p + \alpha_s\big) > 0$. Hence, the sign of $x_2^* - x_1^*$ is given by the sign of $(1-2\omega)(\alpha_p\beta_s - \beta_e\alpha_s)$. Notice that $\omega > 0.5$ (respectively, $\omega < 0.5$) means that retailer 1 has more power (respectively, less power). Thus, for $\omega < 0.5$, we have $x_2^* > x_1^*$ if $\frac{\alpha_s}{\alpha_p} > \frac{\beta_s}{\beta_e}$ (PDS market) and $x_2^* < x_1^*$ if $\frac{\alpha_s}{\alpha_p} < \frac{\beta_s}{\beta_e}$ (GDS market). For $\omega > 0.5$, we have $x_2^* > x_1^*$ if $\frac{\alpha_s}{\alpha_p} < \frac{\beta_s}{\beta_e}$ (GDS market) and $x_2^* < x_1^*$ if $\frac{\alpha_s}{\alpha_p} > \frac{\beta_s}{\beta_e}$ (PDS market). This demonstrates the results of this Proposition. ■

We provide a qualitative explanation of Proposition 5.9 in the following. If a given retailer (let us say retailer 2) has a smaller market power, then it needs to increase its market share. Thus, in GDS market, retailer 2 buys a greener product than the product bought by retailer 1 in order to convince more customers to substitute P2 for P1 since switchovers are, in this case, more governed by greenness. In PDS market, however, retailer 2 adopts an aggressive pricing strategy to attract more switching customers. Consequently, retailer 2 needs to decrease the cost and, thus, to reduce its greenness level which results in buying a dirtier product than the product bought by retailer 1. The result of Proposition 5.9 also means that the retailer that has a larger market power (let us say retailer 1) buys a dirtier product in GDS market and a greener product in PDS market. In the case where retailer 2 enters a market in which retailer 1 is already established, we can imagine that retailer 2 will start with a smaller market power. In this case, retailer 2 will buy a greener product than retailer 1's product under GDS market and a dirtier product under PDS market. This means that the new retailer will not always buy a greener product than the product bought by the existing retailer.

Table 5.2: Impact of market share on the environmental performance

Market structure	$\frac{\alpha_s}{\alpha_p} < \frac{\beta_s}{\beta_e}$	$\frac{\alpha_s}{\alpha_p} = \frac{\beta_s}{\beta_e}$	$\frac{\alpha_s}{\alpha_p} > \frac{\beta_s}{\beta_e}$
$\omega < 0.5$	$x_1^* > x_2^*$	$x_1^* = x_2^*$	$x_1^* < x_2^*$
$\omega = 0.5$	$x_1^* = x_2^*$	$x_1^* = x_2^*$	$x_1^* = x_2^*$
$\omega > 0.5$	$x_1^* < x_2^*$	$x_1^* = x_2^*$	$x_1^* > x_2^*$

5.3.2 Impact of transportation carbon emissions disparity

We now focus on the impact of transportation carbon emissions disparity on the optimal strategy of retailers. We recall that we considered, without loss of generality, that retailer 1 has the farthest supplier and, thus, generates more transportation carbon emissions per unit of product than retailer 2. Since we considered, without loss of generality, that $e_2 = 0$, the transporation emissions per unit of retailer 1's product, namely e_1, also refers here to the transportation carbon emissions disparity. Moreover, given that retailer 1 relies on a longer SC (i.e., a more distant supplier), we have $\mu_1 \leq \mu_2$. In this section, we assume that the retailer that relies on a longer SC has a smaller base cost. Therefore, we have $c_1 \leq c_2$. This is the typical situation when, for instance, one retailer relies on a low-cost abroad supplier whereas the other retailer relies on a local but more expensive supplier. We consider here the general competition model. We have seen that the transportation emissions do not affect the optimal greenness levels of the products purchased from the suppliers (i.e., x_1^* and x_2^*). However, they impact the pricing decisions. Hence, we focus on pricing and present the first result in Proposition 5.10.

Proposition 5.10. *When both retailers have the same market power, the retailer that relies on a shorter SC offers a higher price if and only if the difference in transportation carbon emissions (i.e., e_1) is higher than a threshold value*

$$e_0 = \frac{(\alpha_p + 2\alpha_s)\left(\frac{h(1-r)}{r}\left(\frac{1}{\mu_1} - \frac{1}{\mu_2}\right) - (c_2 - c_1)\right)}{\beta_e + 4\beta_s}.$$

Proof. Recall Proposition 5.3. We have $p_2^* > p_1^* \Leftrightarrow \frac{2\delta_1(2\delta_2 z_2 + \alpha_s z_1)}{4\delta_1\delta_2 - {\alpha_s}^2} > \frac{2\delta_2(2\delta_2 z_1 + \alpha_s z_2)}{4\delta_1\delta_2 - {\alpha_s}^2}$

$\Leftrightarrow \frac{z_2}{z_1} > \frac{2\delta_1\delta_2 - \delta_1\alpha_s}{2\delta_1\delta_2 - \delta_2\alpha_s}$. When both retailers have the same market power, we have $\omega = 0.5$ and $\delta_1 = \delta_2$. Therefore, the above condition can be simplified as $\frac{z_2}{z_1} > 1$, which is equivalent to $A_2 - A_1 + \delta_2\left(c_2 - c_1 + \frac{h(1-r)}{r}\Big(\frac{1}{\mu_2} - \frac{1}{\mu_1}\Big)\right) > 0$. We know that $A_2 - A_1 = (\omega\beta + 2\beta_s)e_1$. Hence, the above inequality is equivalent to $(0.5\beta_e + 2\beta_s)e_1 + \delta_2\left(c_2 - c_1 + \frac{h(1-r)}{r}\Big(\frac{1}{\mu_2} - \frac{1}{\mu_1}\Big)\right) > 0$, which is equivalent to

$$e_1 > e_0 = \frac{(\alpha_p + 2\alpha_s)\Big(\frac{h(1-r)}{r}(\frac{1}{\mu_1} - \frac{1}{\mu_2}) - (c_2 - c_1)\Big)}{\beta_e + 4\beta_s}. \quad \blacksquare$$

One may expect that the retailer that has a shorter SC and bears a higher base cost (here, retailer 2) will always offer a higher price than the other retailer. This is true only when $e_0 \le 0$, which is verified when $\frac{h(1-r)}{r}(\frac{1}{\mu_1} - \frac{1}{\mu_2}) \le (c_2 - c_1)$. In other words, the retailer that has a shorter SC will always offer a higher price when the difference in cost is too high (i.e., greater than $\frac{h(1-r)}{r}(\frac{1}{\mu_1} - \frac{1}{\mu_2})$). However, when the difference in cost is not that big (i.e., $c_2 - c_1 \le \frac{h(1-r)}{r}(\frac{1}{\mu_1} - \frac{1}{\mu_2})$), e_0 is positive and, thus, we may have $e_1 \le e_0$. In this case, retailer 2 may offer a smaller price than retailer 1. Indeed, if the disparity in transportation carbon emissions is not that big (i.e., $e_1 < e_0$), then retailer 2 offers a smaller price despite its higher base cost. The qualitative explanation is the following. A small disparity in transportation carbon emissions implies that retailer 2 should do more effort to offer a product that has an attractive carbon emission intensity level that enables to attract a profitable amount of greenness-sensitive customers. However, this increases the cost and reduces the net margin. Hence, it becomes more profitable for retailer 2 to capitalize on price-sensitive customers and, thus, offer a lower price than retailer 1's price.

We finally investigate the impact of the transportation carbon emissions disparity on the price differentiation between products. The result is given in Proposition 5.11.

Proposition 5.11. *As long as the transportation carbon emissions gap (i.e., e_1) is smaller than e_0, the higher the transportation emission gap, the smaller the price differentiation between the products.*
When the transportation carbon emission gap is greater than e_0, the higher the transportation emission gap, the higher the price differentiation between the products.

Proof is actually presented in the previous Proposition. It can be verified that p_1^* is decreasing in e_1, whereas p_2^* is increasing in e_1. When $e_1 < e_0$, P2 is offered at a lower price than P1 (see Proposition 5.10). A higher disparity in transportation emissions (i.e., an increase in e_1) will increase p_2^* and decrease p_1^*, which closes the price gap. However, when $e_1 > e_0$, P2 is offered at a higher price than P1 (Proposition 5.10), which implies that a higher disparity in transportation emissions will increase the price gap. This is illustrated in Figure 5.4.

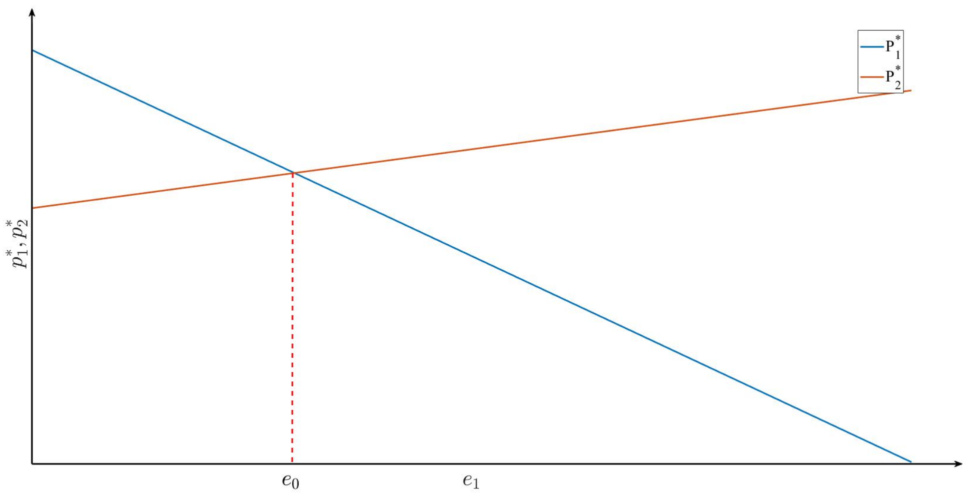

Figure 5.4: Optimal prices in function of disparity in transportation emissions

5.3.3 Impact of refilling time on order size decisions

We have obtained complex expressions of the optimal order size for each retailer. Theses expressions show that the inventory policy is impacted by all the input parameters of the problem (market characteristics, cost factors, supply characteristics, etc.). It is known in the inventory literature that the order size of a given retailer highly depends on its inventory refilling rate. In our context, we observe that the order size also depends on the refilling rate of the other retailer, as highlighted in the following proposition.

Proposition 5.12. *An increase in the inventory refilling rate of one retailer not only leads to reducing the optimal order size of this retailer but also to reducing the optimal order size of the other retailer.*

In Figure 5.5, we increase retailer 1's refilling rate (μ_1) and represent the optimal order size of each retailer. This provides an illustration of Proposition 5.12. Note that we obtain the same behavior when we increase μ_2.

An increase in the inventory refilling rate of a given retailer implies a shorter expected lead time between the moment when the retailer places the order and when it receives the products from the supplier. This means that the retailer can still satisfy the service level constraint (expressed in terms of maximum stock out probability level) while holding a lower stock. It is therefore expected that this retailer will reduce its order size as this allows to reducing the inventory cost. However, the inventory refilling rate of the other retailer does not change and, thus, it is not intuitive that this second retailer will also reduce its order size (even though this order size reduction is relatively small). This reaction is due to the price competition. Indeed, with an increase in its inventory refilling rate, the first retailer will decrease its price as its inventory cost goes down. This leads the other retailer to react and decrease its price (to limit the switchovers) but this price reduction is smaller than that of the first retailer. Hence, the second retailer will necessarily lose demand. Thus, it will need less inventory level which implies a smaller order size.

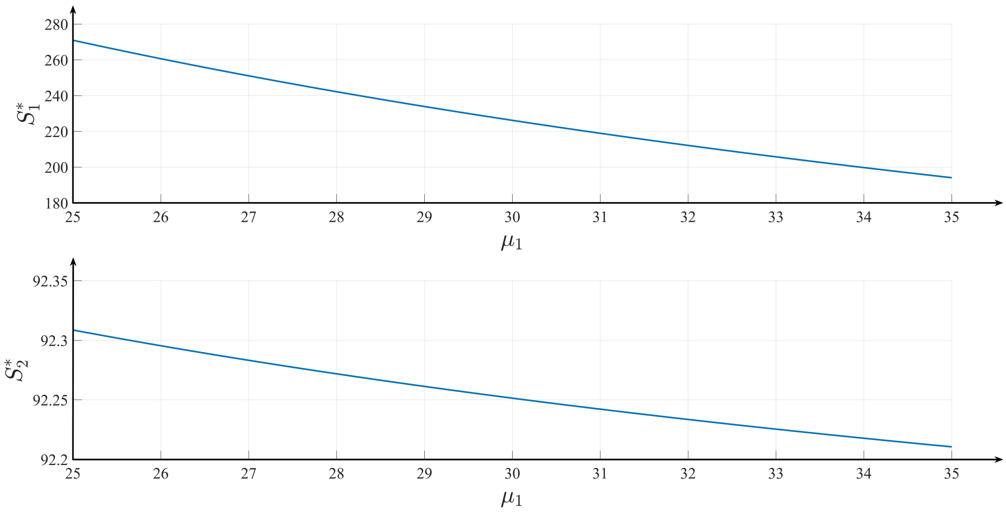

Figure 5.5: Effect of μ_1 on retailers' optimal order sizes

5.3.4 Impact of competition on retailers' optimal profits

In this section, we aim to compare the optimal retailers' profits under the different competition scenarios. Given the complexity of the expressions of optimal profits, we base our analysis on numerical experiments. We conducted experiments to assess how the optimal profit of each retailer varies in function of the market power under the different competition scenarios. To obtain robust results, we considered different values of models parameters under the different market types (PDS, GDS, and neutral market). We tested 10 instances for each market type. In all cases, we obtained the same behavior for each retailer. This behavior is illustrated in Figures 5.6 and 5.7 for the existing retailer and the new retailer, respectively.

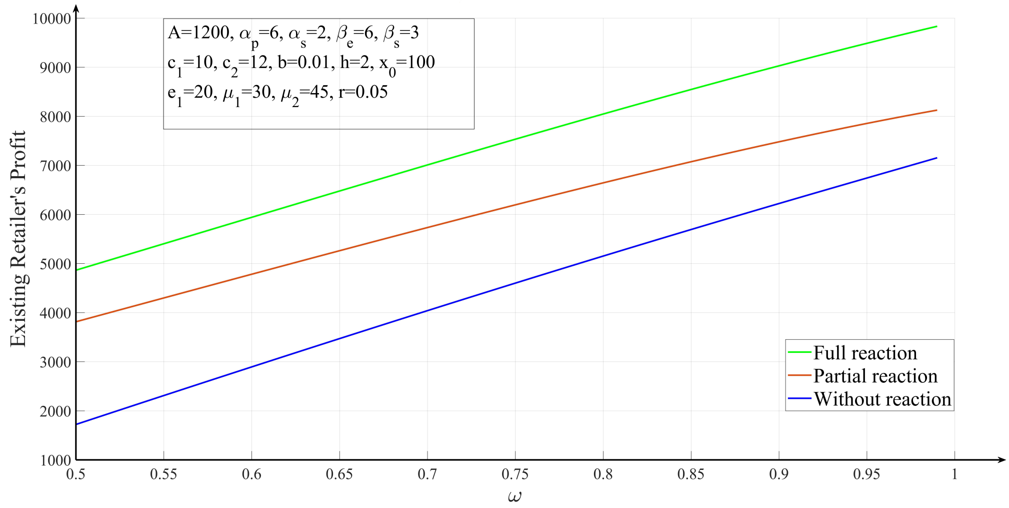

Figure 5.6: Existing retailer's profit under the different competition scenarios

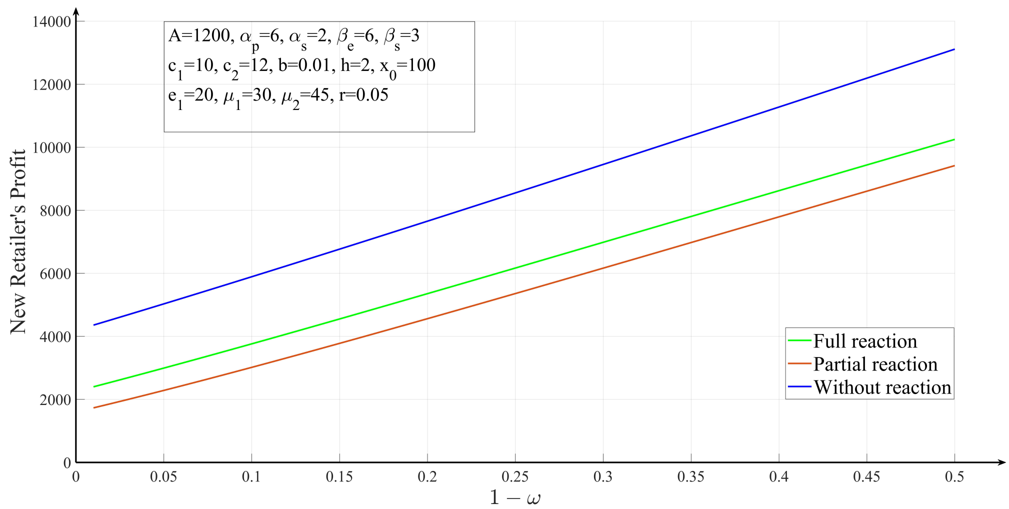

Figure 5.7: New retailer's profit under the different competition scenarios

When a new retailer enters the market, the existing retailer obtains the highest profit if it adopts a full reaction by adjusting its pricing, greenness and inventory policies (see Figure 5.6). We also observe that the partial reaction scenario is preferred to the scenario without reaction. All these observations are expected. As for the new retailer entering the market, we see in Figure 5.7 that this retailer makes the highest profit when the existing retailer does not react, which is also expected.

Less intuitive is that a full reaction of the existing retailer can be more beneficial for the new retailer than a partial reaction (see Figure 5.7). This result deserves a deeper investigation. Under partial reaction, the existing retailer reacts by adjusting its price and order size. Thus, as the existing retailer does not change its product's greenness level, this retailer is obliged to adopt an aggressive pricing strategy to avoid a big loss of demand. This leads the new retailer to react by reducing its price, and finally leads to a significant price decrease for the new retailer at the equilibrium. This has a significant impact on new retailer's profit since its market potential is relatively low. However, under a full reaction scenario, the price reduction performed by the existing retailer is not that high, which incurs a smaller price reduction for the new retailer and leads to a relatively higher profit.

5.4 Conclusion

While considering a setting with two retailers that offer two substitutable products, this study investigated the effect of greenness- and price-based competition on the environmental performance of products. In the general case, each retailer decides the greenness of its product (carbon emission intensity), the price and the order size to maximize its expected profit under a service level constraint. We derived analytically the best response of each retailer to other retailer's decisions at the Nash equilibrium. We used the results to derive managerial insights that explain how competition affect the products' environmental performances. Some of our results are not intuitive.

We found that when switchovers are more governed by greenness differentiation (GDS market), the competition leads to offering a greener product. However, when switchovers are more governed by price differentiation (PDS market), the competition leads to offering a dirtier product. Moreover, a retailer that gains more market power will decrease its product's greenness in GDS market and increase it in PDS market. Our results also indicated that the retailer that has a smaller market power will buy a greener product (than the product bought by the other retailer) when switchovers are more governed by price differentiation, and will buy a dirtier product when switchovers are more governed by price differentiation.

We finally investigated the impact of transportation carbon emissions disparity on the optimal strategy of retailers. We found that when both retailers have the same market power, the retailer that relies on a shorter supply chain offers a higher price if and only if the difference in transportation carbon emissions is higher than a given threshold value.

CHAPTER 6

Complex Demands Functions

So far, we consider a linear demand that is sensitive to price and carbon emission. To the best of our knowledge, most studies in the literature used the same linear demand. We formulate different models and solved them. In this chapter, we are interested in considering a more general (especially non-linear) demand function concerning greenness. The objective of this chapter is to investigate the effects and differences of non-linear demand. For that, we would like to study the retailer's profit optimization problem under different demands functions for the benchmark model that was introduced in Chapter 3. We study the optimal strategies of a retailer who offers a product to customers at a price- and carbon emission- sensitive market. The retailer keeps a stock, S, to serve customers. The inventory level decreases one unit as soon as a customer arrives. The retailer sells products at the retail price, p. We consider x_0 as the product's carbon emission reference. Therefore, x_0 is the maximum Carbon Emission Improvement (CEI) possible. CEI's cost is given by bx^2, where b is the CEI's cost factor, and x is the CEI. Let us remind that x in previous chapters presents the carbon emission level. In this chapter for easier mathematical development, x represents the product's carbon emission improvement (not the level of carbon emission). The retailer will maximize its profit by deciding the optimal price, carbon emission improvement, and order size. Figure 6.1 shows the described problem.

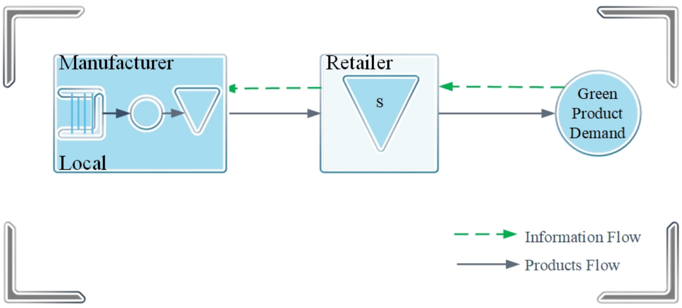

Figure 6.1: Supply chain

As we already said above, the linear demand function is well known in the literature. Our partners in project ANR CONCLuDE at École des Mines de Saint-Étienne found out that the linear sensitivity of demand to carbon emissions is insufficient (Palacios-Argüello et al., 2020). Therefore, we are interested in considering a new demand function (precisely, non-linear) to see how results differ from traditional linear demand functions. We consider that we know the product's demand when the retailer offers the standard product (i.e., without improvement). We also know the demand's shape of the curve (e.g., square root). Therefore, we can deduce the last point (when we improve 100%). Hence, we can deduce the linear demand from the two extreme points and calculate the corresponding greenness sensitivity parameter, $\beta_{e(L)}$.

In the literature, the researchers consider the linear demand without any constraint, but we know that in reality, the demand does not increase beyond a maximum value. This value is obtained for a given x that we call x_l. In other words, the demand increases in CEI until x_l, and after that, the demand is constant even when CEI increases. Thus, we consider a constraint on the demand that we call cap. We consider that the maximum increasing obtained thanks to carbon emissions improvement is ηA, for a given price p. Figure 6.2 shows four demands functions for a given price that we consider: non-linear, non-linear cap, linear, and linear cap.

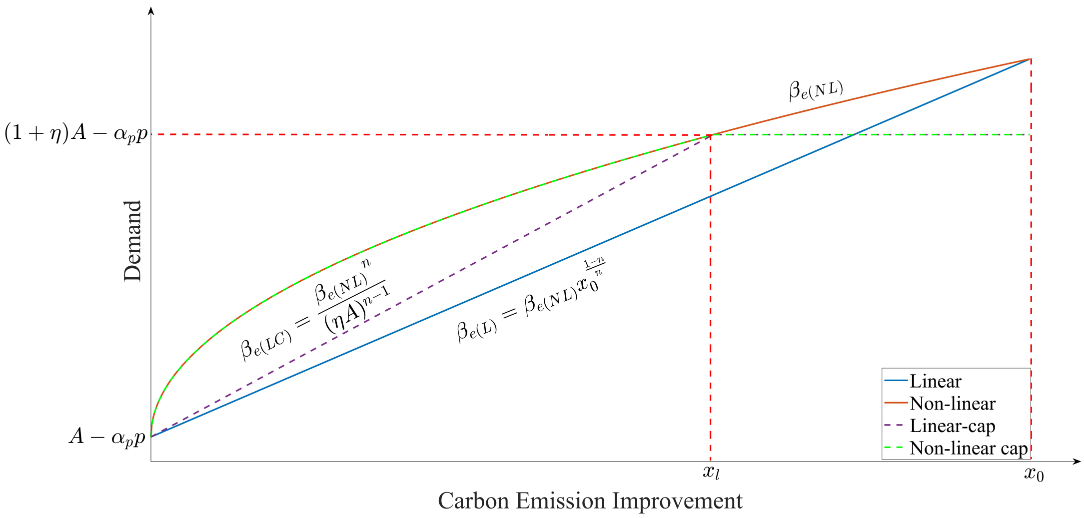

Figure 6.2: Demands functions for a given price

We are going to analyze the benchmark model (M_0) with three new demands: nth root demand (M_7), Capacitated nth root demand (M_8), and capacitated linear demand (M_9).

6.1 nth root demand (M_7)

In this problem, we consider a general demand function. However, when $n = 2$, the demand function is the square root that we presented earlier (Figure 6.2). The demand function is linearly decreasing in retail price and non-linearly (for $n \geq 2$), increasing in carbon emission improvement. The demand function is presented in the following.

$$\lambda = A - \alpha_p p + \beta_{e(NL)} \sqrt[n]{x} \tag{6.1}$$

Where A is market potential, α_p and $\beta_{e(NL)}$ are price's- and CEI's- sensitivity parameters, respectively. p and x represent the product's retail price and carbon emission improvement, respectively, which are decision variables. Note that x_0 indicates the carbon emission reference (thus, maximum carbon emission improvement is equal to x_0; $x \leq x_0$). The last decision variable of our model is order size, S, as we mentioned earlier.

The customer's arrival follows Poisson distributions with a rate of λ. The retailer's warehouse's refilling rate follows an exponential distribution with a mean rate of μ. The mathematical model of the stochastic problem is presented in the following:

$$\underset{x,p,S}{\text{Maximize}}\ \pi = (p - (c + bx^2))\lambda - hS \tag{6.2}$$

Subject to

$$\psi_0 = \frac{\lambda}{\lambda + S\mu} \leq r \tag{6.3}$$

$$\lambda = A - \alpha_p p + \beta_{e(NL)} \sqrt[n]{x} \tag{6.4}$$

$$\lambda, p, S \geq 0, 0 \leq x \leq x_0 \tag{6.5}$$

The mathematical model and formulations are similar to the benchmark model that is presented in Chapter 3. In order to avoid redundancy, we skip the model explanation. The following of this section aims to present analytical proofs to obtain optimal solutions.

Lemma 6.1. *For any given values of p and x, service level constraint (6.3) is binding and the optimal amount of order size is* $S^* = \frac{(1-r)}{r\mu}(A - \alpha_p p + \beta_{e(NL)} \sqrt[n]{x})$.

Proof. Since the objective function is linearly decreasing in S ($\frac{\partial \pi_M}{\partial S} < 0$ and $\frac{\partial^2 \pi_M}{\partial S^2} = 0$), the smallest possible S is the optimal stock. According to the constraint 6.3, $S \geq \frac{(1-r)\lambda}{r\mu}$, therefore, the optimal value is $S^* = \frac{(1-r)}{r\mu}(A - \alpha_p p + \beta_{e(NL)} \sqrt[n]{x})$, which implies that service level constraint (Equation 6.3) is binding. ■

Since we assume that $\lambda \geq 0$, then, S^* is positive and can be substituted by its expression. The equivalent model with two variables, p and x, is presented in the following.

$$\begin{aligned}
\underset{x,p}{\text{Maximize}}\ \pi =&(p-(c+bx^2+\frac{h(1-r)}{r\mu}))\lambda \qquad (6.6)\\
\text{Subject to}&\\
&\lambda = A-\alpha_p p+\beta_{e(NL)}\sqrt[n]{x}\\
&\lambda, p \geq 0, 0 \leq x \leq x_0
\end{aligned}$$

Lemma 6.2. *For any given value of x, the optimal price is*
$p^* = \dfrac{\alpha_p bx^2+\beta_{e(NL)}\sqrt[n]{x}+A+\alpha_p\Big(c+\dfrac{h(1-r)}{r\mu}\Big)}{2\alpha_p}.$

Proof. The first and second derivatives of π with respect to p are presented in the following. The second derivative is negative that means objective function is concave in p. Therefore, the root of first derivative (called p^{max}) maximizes the objective function.

$$\frac{\partial \pi}{\partial p} = -2\alpha_p p+\alpha_p bx^2+\beta_{e(NL)}\sqrt[n]{x}+A+\alpha_p\Big(c+\frac{h(1-r)}{r\mu}\Big)$$

$$\frac{\partial^2 \pi}{\partial p^2} = -2\alpha_p < 0$$

$$\frac{\partial \pi}{\partial p} = 0 \Leftrightarrow p^{max} = \frac{\alpha_p bx^2+\beta_{e(NL)}\sqrt[n]{x}+A+\alpha_p\Big(c+\frac{h(1-r)}{r\mu}\Big)}{2\alpha_p}$$

To consider p^{max} as an optimal solution, we need to verify $p^{max} \geq 0$ and $\lambda(p^{max}) \geq 0$. Since $x > 0$, this is obvious that $p^{max} \geq 0$. Then we need to verify that $\lambda(p^{max})$ is greater than zero. To show demand positivity, we substitute p^{max} in the demand function, which is presented in the following:

$$\lambda(p^{max}) \geq 0 \Leftrightarrow \frac{-\alpha_p bx^2+\beta_{e(NL)}\sqrt[n]{x}+A-\alpha_p\Big(c+\frac{h(1-r)}{r\mu}\Big)}{2} \geq 0$$

$$\Leftrightarrow -\alpha_p bx^2+\beta_{e(NL)}\sqrt[n]{x}+A-\alpha_p\Big(c+\frac{h(1-r)}{r\mu}\Big) \geq 0$$

The first and second derivatives of $\lambda(p^{max})$ with respect to x are presented in the following.

$$\frac{\partial \lambda(p^{max})}{\partial x} = \frac{1}{2}\Big(-2\alpha_p bx+\frac{\beta_{e(NL)}}{n}x^{\frac{n-1}{2}}\Big)$$

$$\frac{\partial^2 \lambda(p^{max})}{\partial x^2} = \frac{1}{2}\Big(-2\alpha_p b - \frac{\beta_{e(NL)}(n-1)}{n^2} x^{\frac{2n-1}{2}}\Big) < 0$$

The second derivative is negative, which means the demand is concave in x. The demand function is increasing from zero ($\lim_{x\to+\varepsilon} \frac{\partial \lambda}{\partial x} > 0$) to the root of the first derivative. If the first root is higher than x_0, it means that the demand function is positive in the feasible region. Otherwise, we have to make sure that the demand is positive when $x = x_0$. At first, we are going to consider that $\lambda(p^{max})$ is positive, when $x = 0$, that lead us to $A \geq \alpha_p\Big(c + \frac{h(1-r)}{r\mu}\Big)$. Figure 6.3 shows the possible shapes of the demand.

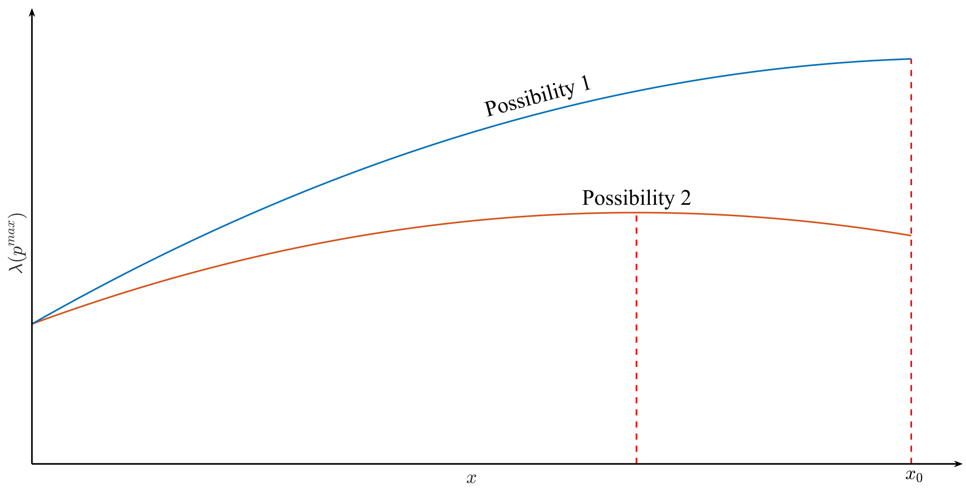

Figure 6.3: Possible demands functions

As it has been described, there are two possibilities. The first possibility is that the root of the first derivative is higher than the upper bound, which is x_0. In this case, $\lambda(p^{max})$ is increasing in the feasible region ($[0, x_0]$). Therefore if start point, $x = 0$, is positive (we presented the condition earlier), then, the demand is always positive.

In the second possibility, the root of first derivative is in the feasible region, and the demand is decreasing after this root. Thus, also, we would need to make sure that demand is positive when $x = x_0$, which the condition is $A \geq \alpha_p b {x_0}^2 - \beta_{e(NL)} \sqrt[n]{x_0} + \alpha_p\Big(c + \frac{h(1-r)}{r\mu}\Big)$. However, it is not necessary because we show in the following (Proposition 6.1) that the root of the first derivative is also the point that maximizes the profit function. Therefore we do not interest in the region after the first derivative's root. As summary, while $A \geq max\Big\{\alpha_p\Big(c + \frac{h(1-r)}{r\mu}\Big), \alpha_p b {x_0}^2 - \beta_{e(NL)} \sqrt[n]{x_0} + \alpha_p\Big(c + \frac{h(1-r)}{r\mu}\Big)\Big\}$ the demand function is positive in the feasible region. Therefore we can consider p^{max} as the optimal solution (i.e., $p^* = p^{max}$). ■

By substituting p^* in equation 6.6, the equivalent model with only one variable, x is presented in the following:

$$\underset{0\le x\le x_0}{\text{Maximize}}\ \pi_M = \frac{\left(-\alpha_p b x^2 + \beta_{e(NL)}\sqrt[n]{x} + A - \alpha_p\left(c + \dfrac{h(1-r)}{r\mu}\right)\right)^2}{4\alpha_p} \tag{6.7}$$

Subject to

$$\lambda = \frac{-\alpha_p b x^2 + \beta_{e(NL)}\sqrt[n]{x} + A - \alpha_p\left(c + \dfrac{h(1-r)}{r\mu}\right)}{2} \ge 0 \tag{6.8}$$

Proposition 6.1. *The optimal solutions for nth root demand model (M_7) are properly presented in the following.*

$$x^* = min\left\{\left(\frac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\dfrac{n}{2n-1}}, x_0\right\},$$

$$p^* = \begin{cases} \dfrac{\dfrac{(2n+1)\beta_{e(NL)}}{2n}\left(\dfrac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\dfrac{1}{2n-1}} + A + \alpha_p\left(c + \dfrac{h(1-r)}{r\mu}\right)}{2\alpha_p} & \textit{for } x^* \ne x_0\textit{, and} \\[2ex] \dfrac{\alpha_p b {x_0}^2 + \beta_{e(NL)}\sqrt[n]{x_0} + A + \alpha_p\left(c + \dfrac{h(1-r)}{r\mu}\right)}{2\alpha_p} & \textit{for } x^* = x_0 \end{cases}$$

$$S^* = \begin{cases} \dfrac{(1-r)\left(\dfrac{(2n-1)\beta_{e(NL)}}{2n}\left(\dfrac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\dfrac{1}{2n-1}} + A - \alpha_p\left(c + \dfrac{h(1-r)}{r\mu}\right)\right)}{2r\mu} & \textit{for } x^* \ne x_0 \\[2ex] \dfrac{(1-r)\left(-\alpha_p b {x_0}^2 + \beta_{e(NL)}\sqrt[n]{x_0} + A - \alpha_p\left(c + \dfrac{h(1-r)}{r\mu}\right)\right)}{2r\mu} & \textit{for } x^* = x_0 \end{cases}.$$

Proof. From Equation 6.8, the objective function (Equation 6.7) is equivalent to $\dfrac{\lambda^2}{\alpha_p}$. Since $\lambda \ge 0$, then maximizing λ is equivalent to maximizing $\dfrac{\lambda^2}{\alpha_p}$. We showed earlier that the demand function is concave in x. Therefore, the root of the first derivative (called x^{*max}) maximizes the demand (equivalently, objective function).

$$\frac{\partial \lambda}{\partial x} = \frac{1}{2}\Big(-2\alpha_p bx + \frac{\beta_{e(NL)}}{n} x^{-\frac{n-1}{n}}\Big) = 0 \Leftrightarrow -2\alpha_p bx + \frac{\beta_{e(NL)}}{n} x^{-\frac{n-1}{n}} = 0$$

$$\Leftrightarrow 2\alpha_p bx = \frac{\beta_{e(NL)}}{n} x^{-\frac{n-1}{n}} \Leftrightarrow x.x^{\frac{n-1}{n}} = \frac{\beta_{e(NL)}}{2\alpha_p bn} \Leftrightarrow x^{*max} = \left(\frac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\frac{n}{2n-1}}$$

If x^{*max} is in the feasible region, then, it is the optimal solution. We know that $0 \leq x \leq x_0$. Therefore, $x^* = min\left\{\left(\frac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\frac{n}{2n-1}}, x_0\right\}$. By substituting $x^* \neq x_0$ into $p^*(x)$, the optimal price is $p^*(x^* \neq x_0) = \dfrac{\frac{(2n+1)\beta_{e(NL)}}{2n}\left(\frac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\frac{1}{2n-1}} + A + \alpha_p\Big(c + \frac{h(1-r)}{r\mu}\Big)}{2\alpha_p}$. In the same way, $S^*(x^* \neq x_0) = \dfrac{(1-r)\left(\frac{(2n-1)\beta_{e(NL)}}{2n}\left(\frac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\frac{1}{2n-1}} + A - \alpha_p\Big(c + \frac{h(1-r)}{r\mu}\Big)\right)}{2r\mu}$.

■

6.2 Capacitated nth root demand (M_8)

In previous section, we have studied the problem with a non-linear demand function (precisely, nth root). In this section, we consider the same problem with considering one more constraint on demand function. As we mentioned in the beginning of this chapter, our colleagues found out that demand can not go beyond a certain value. Thus, we are introducing a new constraint that ensures demand function cannot go beyond a predetermined level. Therefore as already said previously, we define a new parameter, η such that $0 \leq \eta \leq 1$. We consider that carbon emission improvement can at maximum attract ηA more customers. Thus for a given p, the demand non-linearly (for $n \geq 2$) increases in CEI up to $(1+\eta)A - \alpha_p p$, and after that, it is constant. Since improving carbon emission level after that point just increases the cost, it is logical that the optimal solution can not be after that. Therefore, we can introduce the constraint such that carbon emission improvement has a new upper bound, x_l, obtained from this inequality: $\beta_{e(NL)}\sqrt[n]{x} \leq \eta A$. Figure 6.4 shows the capacitated demand function under given price.

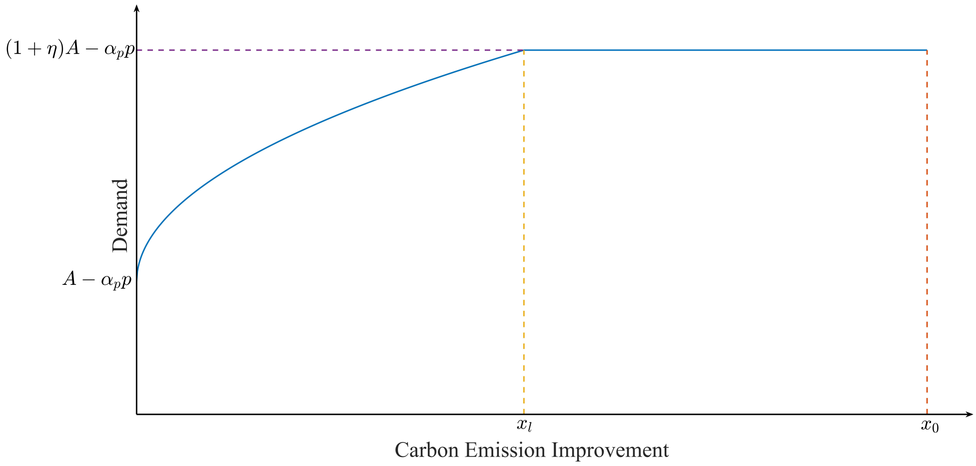

Figure 6.4: Capacitated nth root demand for a given price

The mathematical model of described problem is presented in the following:

$$\underset{x,p,S}{\text{Maximize}}\ \pi = (p-(c+bx^2))\lambda - hS \tag{6.9}$$

Subject to

$$\psi_0 = \frac{\lambda}{\lambda+S\mu} \leq r \tag{6.10}$$

$$\beta_{e(NL)}\sqrt[n]{x} \leq \eta A \tag{6.11}$$

$$\lambda = A-\alpha_p p+\beta_{e(NL)}\sqrt[n]{x} \tag{6.12}$$

$$\lambda, p, S \geq 0, 0 \leq x \leq x_0 \tag{6.13}$$

The mathematical model is similar to the previous section. Therefore we skip their explanations. The optimal solutions are presented in the following.

Lemma 6.3. *For any given value of x, the optimal order size and price are* $S^* = \frac{(1-r)}{r\mu}(A-\alpha_p p+\beta_{e(NL)}\sqrt[n]{x})$ *and* $p^* = \frac{\alpha_p bx^2+\beta_{e(NL)}\sqrt[n]{x}+A+\alpha_p\left(c+\frac{h(1-r)}{r\mu}\right)}{2\alpha_p}$, *respectively.*

The proof is similar to Lemma 6.1 and 6.2. By substituting optimal order size and price by their expressions, the equivalent model with only one variable, x is presented in the following:

$$\underset{0\leq x\leq x_0}{\text{Maximize}}\ \pi_M = \frac{\left(-\alpha_p bx^2+\beta_{e(NL)}\sqrt[n]{x}+A-\alpha_p\left(c+\dfrac{h(1-r)}{r\mu}\right)\right)^2}{4\alpha_p} \tag{6.14}$$

Subject to

$$x \leq \left(\frac{\eta A}{\beta_{e(NL)}}\right)^n \tag{6.15}$$

$$\lambda = \frac{-\alpha_p bx^2+\beta_{e(NL)}\sqrt[n]{x}+A-\alpha_p\left(c+\dfrac{h(1-r)}{r\mu}\right)}{2} \geq 0 \tag{6.16}$$

Proposition 6.2. *The optimal carbon emission improvement for capacitated nth root demand model (M_8) is* $x^* = min\left\{\left(\dfrac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\dfrac{n}{2n-1}}, \left(\dfrac{\eta A}{\beta_{e(NL)}}\right)^n, x_0\right\}$.

The proof is similar to the previous model except there is an extra constraint on x that concerns maximum customers that can be attract when product's carbon emission level improves.

6.3 Capacitated linear demand (M_9)

In this section, we study the benchmark problem with capacitated traditional linear demand function. Like previous section, we consider that the retailer cannot keep getting more costumers than ηA. So, after some point (x_l), the demand is constant as carbon emission improvement is increasing. The figure 6.5 shows the demand function. The demand function is linearly increasing in carbon emission improvement until x_l and is constant after that. The mathematical model of the stochastic problem is presented in the following:

$$\underset{x,p,S}{\text{Maximize}}\ \pi = (p-(c+bx^2))\lambda - hS \tag{6.17}$$

Subject to

$$\psi_0 = \frac{\lambda}{\lambda+S\mu} \leq r \tag{6.18}$$

$$\beta_{e(LC)}x \leq \eta A \tag{6.19}$$

$$\lambda = A-\alpha_p p+\beta_{e(LC)}x \tag{6.20}$$

$$\lambda, p, S \geq 0, 0 \leq x \leq x_0 \tag{6.21}$$

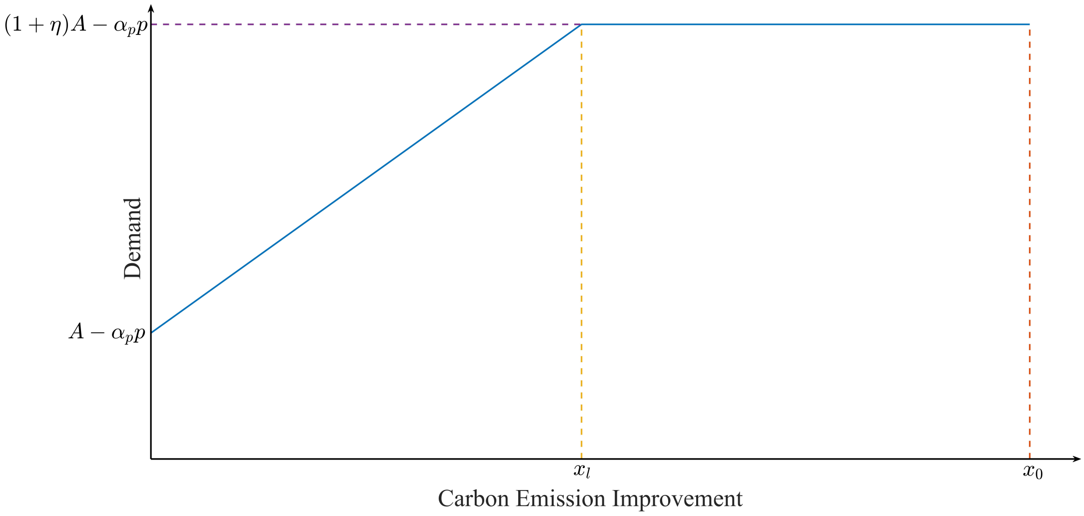

Figure 6.5: Capacitated linear demand for a given price

We skip model explanation since it is similar to benchmark model. The optimal solution is presented in the following.

Proposition 6.3. *The optimal carbon emission improvement for capacitated linear demand model* (M_9) *is* $x^* = min\left\{\frac{\beta_{e(LC)}}{2\alpha_p b}, \frac{\eta A}{\beta_{e(LC)}}, x_0\right\}$.

The proof is similar to the benchmark model that is presented in chapter 3 (see Proposition 3.1 for more details) except there is an extra constraint on x (Equation 6.19) that concerns maximum customers that can be attract when product's carbon emission level improves.

6.4 Analysis and managerial insights

In this section, we provide some important insights. At first, we start to provide insights concerning to the optimal solutions that we obtained. Then, we compare the optimal profits of models with different demands functions.

Table 6.1 presents the parameters that are used to compare the gap between square root cap demand ($n = 2$), as the reference, and other demands' (square root, linear, and linear cap) profits. To present a comprehensive result, we vary the parameters to consider a large number of combinations.

Table 6.1: Parameters

Parameter	Value	Parameter	Value
A	1200:100:2000	c	10:5:20
α_p	4:1:8	h	1:1:3
β_e	3:1:6	b	0.005:0.005:0.015
μ	30:10:50	r	0.05
x_0	100	η	0.1:0.3

Observation 6.1. *The linear cap model is the closest approximation to the square root cap for small η, while increasing η favors the square root demand model.*

We consider a variation of parameters, which are presented in Table 6.1, that leads to 7.2K different combinations. The gap between the optimal profit of the square root cap model (π_{M_8}) and other models ($\pi_{M_{0,7,9}}$) is calculated as $\frac{(\pi_{M_8} - \pi_{M_{0,7,9}}) * 100}{\pi_{M_8}}$.The results are presented in Table 6.2.

Table 6.2: Gap between profits

Profit	Linear (M_0)	Linear Cap (M_9)	Square Root (M_7)
$\eta = 0.02$	-11.87%	0.00%	-25.36%
$\eta = 0.04$	-7.41%	0.00%	-20.35%
$\eta = 0.06$	-3.39%	0.00%	-15.84%
$\eta = 0.08$	0.003%	0.003%	-12.00%
$\eta = 0.10$	2.8%	0.21%	-8.88%
$\eta = 0.12$	4.97%	0.61%	-6.44%
$\eta = 0.14$	6.63%	1.24%	-4.56%
$\eta = 0.16$	7.93%	1.71%	-3.35%
$\eta = 0.18$	8.96%	2.28%	-2.41%
$\eta = 0.19$	9.39%	2.60%	-2.03%
$\eta = 0.20$	9.73%	3.15%	-1.64%
$\eta = 0.30$	12.18%	6.49%	-0.28%

As Table 6.2 shows, the linear cap demand is the closest approximation to the square root cap when η is less than 0.18, while, the square root demand provides better approximation to the square root cap when $\eta \geq 0.19$.

Table 6.3: Gap between optimal carbon emissions improvements

x^*	Linear (M_0)	Linear Cap (M_9)	Square Root (M_7)
$\eta = 0.02$	-117.11%	0.00%	-91.20%
$\eta = 0.04$	-28.52%	0.00%	-22.05%
$\eta = 0.06$	-12.12%	0.00%	-9.24%
$\eta = 0.08$	-6.39%	-0.01%	-4.77%
$\eta = 0.10$	-3.76%	-0.04%	-2.72%
$\eta = 0.12$	-2.36%	-0.07%	-1.65%
$\eta = 0.14$	-1.55%	-0.10%	-1.03%
$\eta = 0.16$	-1.11%	-0.12%	-0.69%
$\eta = 0.18$	-0.83%	-0.14%	-0.46%
$\eta = 0.19$	-0.72%	-0.15%	-0.38%
$\eta = 0.20$	-0.61%	-0.16%	-0.30%
$\eta = 0.30$	-0.31%	-0.25%	-0.05%

As Table 6.3 shows, the product obtained with a linear cap demand has the closest optimal carbon emission improvement approximation with respect to the product obtained with a square root cap when η is less than 0.20, while, the product obtained with a square root demand provides better approximation (closer carbon emission improvement) to the product obtained with a square root cap when $\eta \geq 0.20$.

Table 6.4: Gap between optimal prices

p^*	Linear (M_0)	Linear Cap (M_9)	Square Root (M_7)
$\eta = 0.02$	-17.87%	0.00%	-18.65%
$\eta = 0.04$	-15.70%	0.00%	-16.47%
$\eta = 0.06$	-13.52%	0.00%	-14.27%
$\eta = 0.08$	-11.31%	-0.08%	-12.04%
$\eta = 0.10$	-9.19%	-0.27%	-9.91%
$\eta = 0.12$	-7.28%	-0.47%	-7.99%
$\eta = 0.14$	-5.59%	-0.55%	-6.30%
$\eta = 0.16$	-4.50%	-0.71%	-5.13%
$\eta = 0.18$	-3.56%	-0.92%	-4.08%
$\eta = 0.19$	-3.15%	-1.02%	-3.61%
$\eta = 0.20$	-2.60%	-1.16%	-3.08%
$\eta = 0.30$	-1.25%	-2.40%	-0.83%

As Table 6.4 shows, the product obtained with a linear cap demand has the closest optimal price approximation with respect to the product obtained with a square root cap when η is less than 0.20, while, the product obtained with a square root demand provides better approximation (closer price) to the product obtained with a square root cap when $\eta \geq 0.20$.

Table 6.5: Gap between optimal order sizes

S^*	Linear (M_0)	Linear Cap (M_9)	Square Root (M_7)
$\eta = 0.02$	-5.61%	0.00%	-11.82%
$\eta = 0.04$	-3.48%	0.00%	-9.56%
$\eta = 0.06$	-1.53%	0.00%	-7.49%
$\eta = 0.08$	0.15%	0.01%	-5.71%
$\eta = 0.10$	1.53%	0.11%	-4.25%
$\eta = 0.12$	2.61%	0.31%	-3.10%
$\eta = 0.14$	3.45%	0.63%	-2.20%
$\eta = 0.16$	4.12%	0.87%	-1.62%
$\eta = 0.18$	4.65%	1.16%	-1.17%
$\eta = 0.19$	4.87%	1.33%	-0.99%
$\eta = 0.20$	5.04%	1.61%	-0.80%
$\eta = 0.30$	6.33%	3.34%	-0.14%

As Table 6.5 shows, the product obtained with a linear cap demand has the closest optimal order size approximation with respect to the product obtained with a square root cap when η is less than 0.19, while, the product obtained with a square root demand provides better approximation (closer order size) to the product obtained with a square root cap when $\eta \geq 0.19$.

The results show that the linear demand that is well known in the literature is never the best one. However, the linear cap is a good approximation when η is small. The bright thing about linear cap is that we can easily adapt it to previous chapters (Chapter 4 and 5).

Proposition 6.4. *In non-linear models (M_7 and M_8), increasing n leads to zero carbon emission improvements and in this case* $p^* = \dfrac{A + \alpha_p(c + \dfrac{h(1-r)}{r\mu})}{2\alpha_p}$.

Proof. Thanks to Propositions 6.1 and 6.2, we know that the optimal carbon emission improvements of non-linear and non-linear cap models are $min\left\{\left(\dfrac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\dfrac{n}{2n-1}}, x_0\right\}$ and $min\left\{\left(\dfrac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\dfrac{n}{2n-1}}, \left(\dfrac{\eta A}{\beta_{e(NL)}}\right)^n, x_0\right\}$, respectively. It is clear that n has no effect on x_0. However, the other elements, $\left(\dfrac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\dfrac{n}{2n-1}}$ and $\left(\dfrac{\eta A}{\beta_{e(NL)}}\right)^n$, depend on n. In the following, we show values of these elements when n increases and goes to infinity.

$$\lim_{n\to\infty}\left(\frac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\frac{n}{2n-1}} = 0$$

$$\lim_{n\to\infty}\left(\frac{\eta A}{\beta_{e(NL)}}\right)^{n} = \begin{cases} 0 & \text{for } \dfrac{\eta A}{\beta_{e(NL)}} < 1 \\ 1 & \text{for } \dfrac{\eta A}{\beta_{e(NL)}} = 1 \\ \infty & \text{for } \dfrac{\eta A}{\beta_{e(NL)}} > 1 \end{cases}$$

The upper bound's value in non-linear cap, $\left(\frac{\eta A}{\beta_{e(NL)}}\right)^{n}$, when n goes to infinity has different values that depends on different situations. However, the other element's value, $\left(\frac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\frac{n}{2n-1}}$, goes to zero. Therefore, increasing n in non-linear model leads to decrease x^* to zero. It is true for non-linear cap also (there is at least one element that goes to zero when n increases to infinity). The optimal price when optimal CEI is equal to zero is $p^* = \dfrac{A+\alpha_p(c+\frac{h(1-r)}{r\mu})}{2\alpha_p}$. ■

The n is a introduction of customers' desire for carbon emission improvement. Increasing n means the product's carbon emission improvement becomes less and less important to them. In other words, improving the carbon emission level attracts fewer and fewer new customers. So, improving one unit's carbon emission becomes less interesting from economical point of view. As a result, retailer's (or manufacturer's) motivation to offer greener products becomes fewer and fewer. Therefore for a given η, increasing n leads to lower $\left(\frac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\frac{n}{2n-1}}$, which is optimal solution without considering improvement constraint, and after some point, the constraint becomes non-binding at optimality.

Proposition 6.5. *In non-linear cap model (M_8), increasing η from zero to $\dfrac{\left(\frac{\beta_{e(NL)}^{2n}}{2\alpha_p bn}\right)^{\frac{1}{2n-1}}}{A}$ not only leads to increasing optimal carbon emission improvement, but also leads to increasing profit.*

Proof. Thanks to proposition 6.2, we know that the optimal solution is $x^* = min\left\{\left(\frac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\frac{n}{2n-1}}\right.$

$, \left(\dfrac{\eta A}{\beta_{e(NL)}}\right)^n, x_0 \Bigg\}. \left(\dfrac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\dfrac{n}{2n-1}}$ is the value that maximizes the profit function without considering any constraints. However, we know that Equation 6.15 does not allow to have the optimal CEI higher than $\left(\dfrac{\eta A}{\beta_{e(NL)}}\right)^n$, which ensures demand does not go beyond a certain level.

$$\left(\frac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\dfrac{n}{2n-1}} \geq \left(\frac{\eta A}{\beta_{e(NL)}}\right)^n \Leftrightarrow \frac{\eta A}{\beta_{e(NL)}} \leq \left(\frac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\dfrac{1}{2n-1}}$$

$$\Leftrightarrow \eta \leq \frac{\left(\dfrac{\beta_{e(NL)}^{\;2n}}{2\alpha_p bn}\right)^{\dfrac{1}{2n-1}}}{A}$$

As we presented, when $0 \leq \eta \leq \dfrac{\left(\dfrac{\beta_{e(NL)}^{\;2n}}{2\alpha_p bn}\right)^{\frac{1}{2n-1}}}{A}$, the optimal solution is $\left(\dfrac{\eta A}{\beta_{e(NL)}}\right)^n$, which means Equation 6.15 is binding, and so, increasing η leads to increase optimal carbon emission's improvement. As we mentioned earlier, $\left(\dfrac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\dfrac{n}{2n-1}}$ is the value that maximizes the profit function when Equation 6.15 is non-binding in optimal solution. Recall that optimal profit is concave with respect to x and $\left(\dfrac{\beta_{e(NL)}}{2\alpha_p bn}\right)^{\dfrac{n}{2n-1}}$ is the value that maximize the profit function. Therefore, as long as η increases (between $0 \leq \eta \leq \dfrac{\left(\dfrac{\beta_{e(NL)}^{\;2n}}{2\alpha_p bn}\right)^{\frac{1}{2n-1}}}{A}$, which means x^* increases) the optimal profit increases.

When $\eta > \dfrac{\left(\dfrac{\beta_{e(NL)}^{\;2n}}{2\alpha_p bn}\right)^{\frac{1}{2n-1}}}{A}$, the optimal solution is independent of η, which means Equation 6.15 is not binding, and η's changes do not affect the optimal solution (carbon emission improvement and profit). ■

Proposition 6.6. *In case that* $\eta \leq \dfrac{\left(\dfrac{\beta_{e(NL)}{}^{2n}}{2\alpha_p bn}\right)^{\dfrac{1}{2n-1}}}{A}$*, the carbon emission improvement constraint is binding at optimality, and the optimal product's carbon emission improvement and price of the capacitated model is less than the non-capacitated model.*

Proof. Thanks to Proposition 6.5, we know that when $\eta \leq \dfrac{\left(\dfrac{\beta_{e(NL)}{}^{2n}}{2\alpha_p bn}\right)^{\dfrac{1}{2n-1}}}{A}$ Equation 6.15 is binding and as a result, the optimal carbon emission's improvement in binding situation (called x^{*B}) is less than the optimal carbon emission's improvement in the non-linear model (M_7), which called x^{*NB} ($x^{*B} < x^{*NB}$). Also, the optimal price in the function of x, which has presented in Lemma 6.2 and 6.3, is strictly increasing in x. Indeed:

$$\frac{dp}{dx} = 2\alpha_p bx + \frac{\beta_{e(NL)}}{n} x^{\dfrac{1-n}{n}} > 0$$

Therefore $x^{*B} < x^{*NB} \Rightarrow p(x^{*B}) < p(x^{*NB})$. ■

Proposition 6.7. *In case such that optimal carbon emissions are less than* x_0*, the product obtained with a linear demand has a higher price than the product obtained with a square root demand if* $\dfrac{\beta_{e(L)}}{\alpha_p bx_0} > \sqrt{\dfrac{125}{108}}$*, otherwise, product obtained with a linear demand has equal or lower price than the product obtained with a square root demand.*

Proof. The optimal CEIs are equal to $\dfrac{\beta_{e(L)}}{2\alpha_p b}$ and $(\dfrac{\beta_{e(NL)}}{4\alpha_p b})^{\dfrac{2}{3}}$ for linear and square root demand (considering that both of them are lower than x_0), respectively. The optimal prices are presented in the following (p^1 and p^2 indicate the linear and square root demands' prices, respectively).

$$p^1 = \frac{\alpha_p b x^2 + \beta_{e(L)} x + A + \alpha_p\big(c + \dfrac{h(1-r)}{r\mu}\big)}{2\alpha_p}$$

$$= \frac{\alpha_p b\Big(\dfrac{\beta_{e(L)}}{2\alpha_p b}\Big)^2 + \beta_{e(L)}\Big(\dfrac{\beta_{e(L)}}{2\alpha_p b}\Big) + A + \alpha_p\big(c + \dfrac{h(1-r)}{r\mu}\big)}{2\alpha_p} = \frac{\dfrac{3\beta^2_{e(L)}}{4\alpha_p b} + A + \alpha_p\big(c + \dfrac{h(1-r)}{r\mu}\big)}{2\alpha_p}$$

$$p^2 = \frac{\alpha_p b x^2 + \beta_{e(NL)}\sqrt{x} + A + \alpha_p\big(c + \dfrac{h(1-r)}{r\mu}\big)}{2\alpha_p}$$

$$= \frac{\alpha_p b\left(\Big(\dfrac{\beta_{e(L)}}{2\alpha_p b}\Big)^{\frac{2}{3}}\right)^2 + \beta_{e(NL)}\sqrt[3]{\Big(\dfrac{\beta_{e(L)}}{2\alpha_p b}\Big)^2} + A + \alpha_p\big(c + \dfrac{h(1-r)}{r\mu}\big)}{2\alpha_p}$$

$$= \frac{5\sqrt[3]{\dfrac{\beta^4_{e(NL)}}{4^4\alpha_p b}} + A + \alpha_p\big(c + \dfrac{h(1-r)}{r\mu}\big)}{2\alpha_p}$$

The condition that $p^1 > p^2$ is provided in the following.

$$p^1 > p^2 \Leftrightarrow \frac{\dfrac{3\beta^2_{e(L)}}{4\alpha_p b} + A + \alpha_p\big(c + \dfrac{h(1-r)}{r\mu}\big)}{2\alpha_p} > \frac{5\sqrt[3]{\dfrac{\beta^4_{e(NL)}}{4^4\alpha_p b}} + A + \alpha_p\big(c + \dfrac{h(1-r)}{r\mu}\big)}{2\alpha_p}$$

$$\Leftrightarrow \frac{3\beta^2_{e(L)}}{4\alpha_p b} > 5\sqrt[3]{\frac{\beta^4_{e(NL)}}{4^4\alpha_p b}} \Leftrightarrow \frac{27\beta^6_{e(L)}}{64\alpha_p^3 b^3} > \frac{125\beta^4_{e(NL)}}{256\alpha_p b} \Leftrightarrow \frac{\beta^6_{e(L)}}{\alpha_p^2 b^2 \beta^4_{e(NL)}} > \frac{125}{108}$$

We know $\beta_{e(NL)} = \beta_{e(L)}\sqrt{x_0}$, therefore, $\dfrac{\beta^6_{e(L)}}{\alpha_p^2 b^2 \beta^4_{e(NL)}} = \dfrac{\beta^2_{e(L)}}{\alpha_p^2 b^2 {x_0}^2}$. Thus, while $\dfrac{\beta_{e(NL)}}{\alpha_p b x_0} > \sqrt{\dfrac{125}{108}}$ the product with linear demand function has higher price than the product with a square root demand function. ■

6.5 Conclusion

In this chapter, we consider more general and complex demand functions. The demand functions are linearly decreasing in price, while, non-linearly increasing in carbon emission improvement. We consider a similar problem as benchmark of chapter 3 and we formulated under new demand function. The new problem is solved by an analytical approach. We consider furthermore a constraint that allows demand to increase to a certain level and we call it cap. The problems with new demands functions (i.e. linear cap and non-linear cap) are formulated and solved.

The numerical examples show that when demand's cap is small, the linear cap model provides the better approximation than others, whereas, the square root model provides the better approximation for the bigger cap.

The work that has been done in this chapter can extend in several ways. First, the new demand function can be used in product differentiation and competition models, which are presented in previous chapters. It should be interesting to see the effect of cap on optimal strategies of retailers, specially on greenness, along with retailers' power (i.e. market share). Another extension can be the government regulations. The government regulations such as carbon tax, carbon cap, and etc. have their own effects on greenness decisions. However, it is interesting to see the effects of demand's cap alongside the government regulations' effect.

CHAPTER 7

General conclusion

7.1 Conclusion

The studies on retailing problems under the consideration of a sensitive demand to the price and greenness reveal several gaps in this area. First, most of the studies considered a Make-To-Order system while there are many cases in practice that follows Make-To-Stock system and deal with problems such as inventory policy, order size and etc. Second, most studies considered a Stackelberg game, where the leader decides first and the follower acts later, while in practice, there are many players (e.g., companies, retailers, etc.) that changes their strategies based on other competitors' strategies. Third, the considered demand function is generally linearly decreasing in price and greenness. However, greenness may have a non-linear effect on demand. Also, improving greenness level does not always attract more customers, beyond a given level.

Thus, in this study, we propose different problems and formulation to close the existed gaps in the literature. We start our work by a simple benchmark model, which includes one supplier and one retailer who offer one product to customers in a price- and greenness-dependent market. The retailer's warehouse refilling time follows Exponential distribution. The customers' arrival follows Poisson process. Thus, the system follows Markov chain process. The retailer decides product's price, carbon emission level, and order size to maximize his/her profit. We solve the problem by an analytical approach.

Then, we extended previous problem by considering a new and substitutable product. In this case, the problem includes two suppliers and one retailer. Each supplier provides retailer with one kind of products that makes retailer to offer two substitutable products to customers. The demand function for each product is similar to the benchmark model except there are switchovers' effect in the demand function. It means that each product's demand not only is affected by its product's price and greenness, but also, affected by other product's price and greenness. Like previous problem, this problem is formulated in a stochastic environment under different settings (considering different sets of decision variables) and solved by an analytical approach. To do a proper analyze and bring most important managerial and technical insights to the light, we distinguish the market according to sensitivity parameters. The market is categorized as: 1. PDS market, the switchovers are more governed by price differentiation rather than greenness differentiation 2.GDS market, the switchovers are more governed by greenness differentiation rather than price differentiation 3. Neutral market, price and greenness disparities have the same importance with respect to switchovers. The results show that retailer's optimal strategies depend on the market characteristics (i.e. PDS, GDS, and Neutral markets).

The numerical examples show that retailer benefits when it offers a new and substitutable product.

We continued our work on substitutable products in dynamic competition context. Two competing retailers offer two substitutable products (one product per retailer) in a grenneess- and price-sensitive market. Each retailer has its supplier. We consider the similar inventory policy as previous problems. The demand function for each retailer (product) is similar to previous problem, except that we introduced and applied retailer's power (market share) into the demand function. Demand and the replenishment lead times of retailers' warehouse are random and follow Exponential distribution. The retailers (players) decide their strategies based on other retailer's (player's) strategies. It is a non-dominated and non-cooperative game. Optimal strategies of each retailer are determined at the Nash equilibrium. We start with a general model which in each retailer decides product's price, greenness level, and order size. In practice there are many situations where an existing retailer is already operating in the market, and a new retailer enters the market and offers a substitutable product. Therefore we proposed two different scenarios: 1. Competition without reaction, in which the existing retailer does not react to the new retailer's decisions 2. Competition with partial reaction, in which the existing retailer just updates its price and order size but does not change the carbon emission intensity as this requires new deals with the supplier. Both problems are solved by analytical approaches. Then, we investigated the effect of competition on products' environmental performance (compared to benchmark model) as well as the impact of transportation carbon emissions. We also investigated the effects of the retailers' power (market share) and the market structures (i.e., PDS, GDS, and Neutral market) on products' environmental performance (compare retailers' greenness level). The results showed that the retailer that gains more market power will decrease its product's greenness in GDS market and increase it in PDS market. We also found that when both retailers have the same market power, the retailer that relies on a shorter supply chain offers a higher price if and only if the difference in transportation carbon emissions is higher than a given threshold value.

So far, we considered a demand function that is linearly decreasing in price and carbon emission level. Finally, we consider the demand function that is linearly decreasing in price, while, non-linearly increasing in carbon emission improvement. We consider a similar problem as benchmark and formulated under new demand function. The new problem is solved by an analytical approach. Also, we considered a constraint that allows demand to increase only to a certain level (cap). The new demands functions (i.e. linear cap and non-linear cap) are formulated and solved. Finally, we highlight some technical insights to enrich this chapter. The results showed that increasing demand's cap not only leads to increasing carbon emission improvement, but also leads to increasing profit. The numerical examples show that when demand's cap is small, the linear cap model provides a better approximation than others, whereas, the square root model provides the better approximation for the bigger cap.

7.2 Future works and perspectives

Our study can be extended in different ways. As we mentioned in introduction, our work started by a hybrid system with considering a Lead time- and price- sensitive demand. As the first extension, it would be interesting to investigate two competitive supply chains systems while considering Lead time-, greenness-, and price sensitive demand. Thus in this case, there are two competitive retailers in the market, each of them offers one product. The demand of each product is decreasing in its price, carbon intensity, and lead time and increasing in other product's price, carbon intensity, and lead time. This work can be followed under different scenarios such as: Centralized supply chain and Decentralized supply chain (which two sub-scenarios; supplier as the leader, and retailer as the leader).

Another extension of our work would be to consider the works that have been done in chapters 4 and 5 with linear cap and non-linear demands functions. Thus, it is interesting to consider product differentiation and competition problems with new demands that are introduced in chapter 6.

Chapter 6 introduced new demands functions and compared them with linear demand. This work can be extended with considering different shapes of demand function like the log-linear. Different demands functions prepare the opportunities to develop our understanding of customers' behavior. Again both collaboration and competition problems are also interesting to investigate with other demands function.

Considering the government regulations such as carbon tax, carbon cap, and etc. would also be interesting. Our work considers the CEA as the only source of environmental effect that comes from customers on retailer's strategies. However, the government regulations can be considered as other sources (in addition to CEA) that affect the retailer's strategies (especially carbon intensity). This extension can be applied even with new demands functions that are introduced in chapter 6.

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APPENDIX A

Time- and price-based product differentiation in a hybrid MTO/MTS system with stockout-triggered customer switching

A.1 Introduction

Many online retailers rely on a delivery mix that includes traditional delivery from stock (DFS) and drop-shipping to satisfy demand. With drop-shipping, a retailer simply forwards customer orders to manufacturers or wholesalers who fulfill those orders directly for a predetermined price to be paid by the retailer (Khouja and Stylianou 2009). Drop-shipping has significant advantages for the retailer. There are savings in holding cost, but drop-shipping typically incurs longer delivery times (DTs), which may deter time-sensitive customers (Khouja 2001, Khouja and Stylianou 2009, Rabinovich et al. 2008). DFS implies a shorter and more reliable DT, but incurs a holding cost that may increase with uncertainty. A mix of DFS and drop-shipping has proven to be an efficient distribution strategy, particularly when there is high uncertainty in demand and/or replenishment lead time. In such cases, drop-shipping provides retailers with the option to reserve stock internally for high-priority orders while fulfilling regular orders through drop-shipping (Ayanso et al. 2006).

With a mix of DFS and drop-shipping, a product is offered with two varieties that have different DTs and can generate different margins (Ayanso et al. 2006). Rabinovich et al. (2008) studied the handbag and luggage segment of online retailing (e-Bags.com) and provided empirical evidence suggesting that an internet retailer can command higher margins from customers when it is willing and able to promise a shorter DT. While shopping on Cdiscount.com (a French leading online retailer) over the last Christmas Holidays, one of the authors has been offered two options to buy a product. In the first option, the product is immediately delivered from Cdiscount store in France. In the second option, the product is delivered directly from the supplier in China. This second option is less expensive for the customer but leads to a much longer and less reliable DT. Two real examples are presented below.

Many other online retailers use this hybrid distribution mode to sell substitutable products, as highlighted by Li et al. (2019), Tian et al. (2018) and Hagiu and Wright (2015).

Table A.1: Real examples of products delivered with a mix of DFS and drop-shipping

		
Option 1. Delivery from Cdiscount store	Delivery: 21-22 November Price: 179.99€	Delivery: 14-15 November Price: 10.13€
Option 2. Delivery from Chinese supplier	Delivery: 05-19 December Price: 169.99€	Delivery: 13-17 December Price: 7.99€

Examples include, but are not limited to, some globally recognized retailers such as Amazon, Home Depot, and NewEagle. For instance, Amazon offers substitutable goods that can be either drop-shipped from upstream suppliers or shipped directly from Amazon fulfillment centers (Ayanso et al. 2006, Rabinovich et al. 2008). Figure 2 shows a generic supply chain (SC) for a retailer that relies on the hybrid distribution mode presented in the above examples, where regular product refers to the drop-shipped product, and express product refers to the product delivered from stock. In compliance with the above examples, the express product has a shorter DT but a higher price than the regular product.

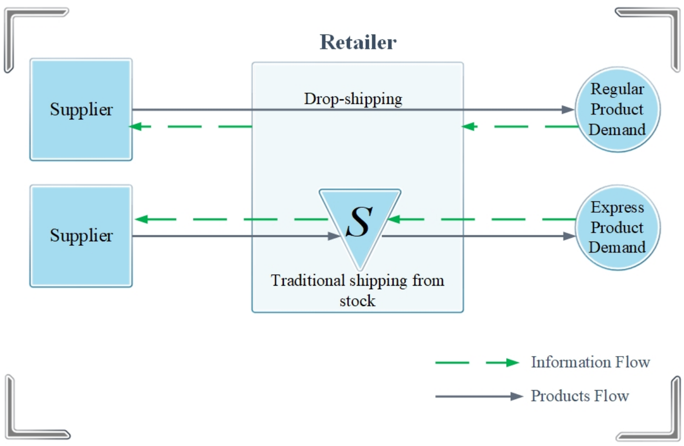

Figure A.1: Illustration of the studied hybrid system with a mix of DFS and drop-shipping

The customers observe the pair of quoted DTs and prices provided by the retailer for each product and make their purchase decisions. In case of stockout, some customers initially interested in the express product may switch to the regular product, whereas others may leave the system (lost sales). Many authors have provided practical examples to illustrate how customers can be served with drop-shipping in case of stockout (e.g., Khouja and Stylianou 2009, Ayanso et al. 2006). Stockout-based substitution (S-b-S) impacts the demand of both express and regular products. It is, therefore, important to consider S-b-S and study its effect on DTs, price, and inventory decisions. Overall, the rate of S-b-S increases when the retailer is the exclusive seller of the product and decreases if the product can be found elsewhere by customers.

This study develops a modeling framework that fits the practical situations described above in order to investigate the DT, price, and inventory decisions faced by a retailer that uses a mix of DFS and drop-shipping to serve a time- and price-sensitive market with two substitutable products that differ in terms of the guaranteed DT and price considering S-b-S. In its general form, the problem is deciding how to differentiate the products in terms of DT and price, and determining the stock level in retailer's warehouse while satisfying the service level constraint for each product. We examine this problem when the demand process is stochastic, inventory replenishment lead time is stochastic, and drop-shipping lead time is also stochastic. The products are substitutable, so the mean demand rate for each product decreases in price and quoted DT and increases in price and quoted DT for the competing product. In addition, as we consider S-b-S, the effective mean demand rate of each product depends also on the stockout probability, which leads to a non-linear demand function that is sensitive to DT, price, and stock level. Finally, we consider a time- and price-sensitive market and, consequently, the total initial demand (for both products) is not constant but depends on the quoted prices and DTs.

There are several trade-offs in the model. The express product can be sold with a higher margin but requires holding inventory, which implies an additional cost. If the express product is profitable enough, it may be interesting to hold more stock than the minimum level imposed by the service constraint as this reduces the stockout probability and potentially could satisfy more express customers. To increase the initial demand for the express product, the retailer can either lower the price or quote a longer DT for the regular product. However, an increase in the express demand may require holding more inventory to satisfy the service level constraint. S-b-S is expected to have a significant impact on such trade-offs. Additionally, with the presence of S-b-S, it may be interesting to hold less stock (to reduce the inventory cost) although this implies a loss of a certain amount of express demand since this loss will be partially transformed into additional demand for the regular product. However, this may lead to a longer waiting time in the drop-shipping channel and, consequently, a higher risk of violating the service level constraint. To provide insights into these complex trade-offs, we study different variants of the problem: (i) The retailer decides the DT differentiation where the stock and prices are fixed. (ii) The retailer decides the stock level and the DT differentiation where the prices are fixed. (iii) The retailer decides the DT and price differentiations as well as the stock level (general model). In the first setting, we determine the closed-form expression of the optimal solution. We show that it is not always optimal to adopt an extreme time differentiation strategy and that an intermediate strategy can be more valuable but should be adjusted as a function of the stock level. In the second setting, we determine the optimal solution when S-b-S is ignored and provide a near-optimal solution in the presence of S-b-S. In the case without S-b-S, there exists a threshold price below which it is optimal to offer both products with minimum time differentiation and above which only the express product should be offered. Considering S-b-S, the profit function profile changes. We show that quoting the DT without considering the effect of S-b-S leads to considerable loss for the retailer. Finally, using the analytical results obtained in the first two settings, and based on an extensive numerical study, we transform the general model into a quasi-equivalent single-variable model. We show that a higher rate of S-b-S should normally lead to greater time differentiation and more stock. However, this would not impact the price differentiation. As we shall show in the next section, this is the first paper that investigates time- and price-based differentiation along with inventory decisions for a retailer that relies on a hybrid distribution system (with a mix of DFS and drop-shipping) to satisfy the demand of a time- and price-sensitive market subject to S-b-S behavior. Section 2 gives an overview of the literature. In Section 3, we develop the general model framework. In Sections 4, 5 and 6 we solve different variants of the model and derive analytical and numerical insights. Finally, in Section 7, we conclude and discuss the practical implications of our findings.

A.2 Literature Review

This study relates to two streams of research: models on product differentiation in time-sensitive markets and models on the design and/or management of hybrid distribution modes. In this section, we review the extant research in those areas and discuss the contributions of our work. Studies on time-based product differentiation are relatively scarce in the literature (Albana et al. 2018). Most studies focus on the problem of an MTO manufacturer that differentiates products by allocating a different production capacity to each product variety. Boyaci and Ray (2003) considered a firm selling two substitutable products to satisfy a linear price- and DT-sensitive demand. Demand is served from two separate facilities; one facility per type of product, and each facility is modeled as an M/M/1 queue. In its general form, the problem is choosing the price of each product and the DT of the express product where there is a cost associated with each capacity. The numerical illustrations show that the firm should reduce the time differentiation when the capacity cost for the regular product is ignored. When capacity costs are considered, the optimal strategy should be to guarantee a shorter DT for the express product. Boyaci and Ray (2006) extended the previous work by considering delivery reliability (i.e., the minimum service level) as a decision variable. The authors found that as long as the proportion of time-sensitive customers is approximately equal to the proportion of price-sensitive customers, the firm would typically offer the express product with a lower guarantee than the regular product. Zhao et al. (2012) compared the uniform quotation mode, when a firm offers a single DT and price, to the differentiated quotation mode, when a firm offers a menu of DT and prices. As with the previous works, the production system is modeled as an M/M/1 queue, the demand is linear in price and DT, and there is a dedicated capacity for each product. The results indicated that when DT-sensitive customers value a product no more than price-sensitive customers, a firm should not use the differentiated quotation mode. Our research differs from the cited studies in three key aspects. First, we focus on a different and more complex SC configuration since we consider a retailer that relies on a hybrid distribution channel with a mix of DFS and drop-shipping. Second, we study product differentiation along with inventory decisions while the above studies consider MTO manufacturing systems without inventory considerations, which does not fit with the retailing context. Third, in our work, switchovers are governed not only by price and DT disparities but also by stockout, which enables us to investigate the impact of S-b-S on the main trade-offs of the system and derive new managerial insights. Our study also contributes to the research literature on hybrid distribution systems. In this stream of research, there are valuable studies on dual-channel structure and procurement strategies. For instance, Hagiu and Wright (2015) investigated the decision of an intermediary to function as a marketplace or as a reseller. This decision is driven by the level of demand-enhancing marketing activity, which is controlled by suppliers in the pure marketplace mode and the intermediary in the pure reseller mode. Which mode is preferred depends on whether independent suppliers or the intermediary have more important information relevant to the optimal tailoring of marketing activities for each specific product. Tian et al. (2018) considered an SC composed of two competing suppliers. The suppliers sell two substitutable products through a common online retailer that acts as a Stackelberg leader and decides whether to function as a pure reseller, a pure marketplace, or with a hybrid mode. The main findings indicated that the retailer's preferred mode depends on the level of order-fulfillment costs and on the level of competition among suppliers. Li et al. (2019) considered a similar framework to investigate the online retailer's choice between pure batch ordering (traditional shipping) and hybrid shipping where drop-shipping is used with one supplier and batch ordering is used with the other. The authors found that the retailer prefers simultaneous structure (when it approaches both suppliers simultaneously for a price quotation) over sequential structure in the traditional mode while, in the hybrid mode, the sequential structure is preferred. The cited works differ from our study in many ways. In particular, they do not consider time-based product differentiation and do not deal with operations related to inventory and lead times. More closely related to our study, another stream of research on hybrid distribution channels focuses on inventory control with the consideration of the drop-shipping option. Ayanso et al. (2006) considered an internet retailer relying on an (r, q) inventory system and facing two customer classes with respectively short DT needs satisfied via air-shipment and regular needs satisfied via ground-shipment. The authors developed a Monte Carlo simulation model to identify the stock level below which low-priority orders are to be drop-shipped directly from the supplier rather than fulfilled from in-house stock, which is, in this case, exclusively reserved for high-priority orders. The authors demonstrated that differentiating customer orders in terms of their priority and reserving inventory for high-priority orders can provide retailers with greater profit opportunities. Our study builds on this finding and investigates further challenges, such as how to differentiate customer orders in terms of DT and price and how to adjust the stock level accordingly. Khouja and Stylianou (2009) revisited the (r, q) inventory model by allowing a retailer to use the drop-shipping option in case of a shortage during replenishment lead time. Numerical experiments showed that the drop-shipping option is more valuable when the replenishment lead time is long, the ordering cost relative to the holding cost is small, and the back order cost relative to the holding cost is also small. These two papers (Ayanso et al. 2006 and Khouja and Stylianou 2009) focused only on inventory decisions while respectively relying on simulation and numerical experiments and adopted a framework in which the demand, DT, and price of each product are exogenous. Our research adopts an analytical approach to study product differentiation with endogenous DT, price, inventory, and demand. Finally, an interesting study on DT quotation and pricing in dual-channel SCs has been proposed by Hua et al. (2010). The authors considered the problem of a manufacturer that sells a product through a retailer or directly to customers. The decision variables are the retail price, the direct channel price, and the quoted DT for the direct channel. Quoting a shorter DT requires a higher delivery cost. Similar to our model, demand is sensitive to price and DT. The authors studied both a centralized and decentralized setting (where the manufacturer is the leader, and the retailer reacts by choosing its selling price) and showed that DT strongly influences the manufacturer and the retailer's pricing strategies and profits. Our study differs from that of Hua et al. (2010)'s in three ways. First, we focus on the problem of a retailer that relies on a hybrid channel and, consequently, we also consider inventory decisions. Second, we consider a stochastic setting where demand, stock replenishment DT, and drop-shipping DT are random while Hua et al. (2010) assumed a deterministic context. Third, in our model, customer switching depends on the differentiation in terms of price and DT and on the inventory level (stockout). In summary, there are valuable studies on time-based differentiation for manufacturers, particularly in the MTO context where differentiation typically results from allocating different production capacity to each product variety (e.g., Zhao et al. 2012, Boyaci and Ray 2003, 2006). There is also a growing body of literature on dual-channel distribution with a focus on situations where the manufacturer (and not the retailer) introduces the direct channel (e.g., Hua et al. 2010), the retailer rethinks its inventory policy with drop-shipping (e.g., Ayanso et al. 2006, Khouja and Stylianou 2009), and the retailer is concerned with the choice of distribution channel structure (e.g., Hagiu and Wright 2015, Tian et al. 2018). However, the problem studied in this paper has not been thoroughly explored in the extant literature. Our findings can provide useful insights into product differentiation and inventory decisions for retailers relying on hybrid distribution with DFS and drop-shipping to serve substitutable products to time-and price-sensitive customers with S-b-S behavior.

A.3 Modeling framework

We consider a profit-maximizing retailer that uses a hybrid distribution with a mix of DFS and drop-shipping and, thus, offers two substitutable products differentiated in terms of DT and price to satisfy random time- and price-sensitive demand. With drop-shipping, the retailer forwards the customer order to the supplier (wholesaler or manufacturer). The order is then processed by the supplier and delivered to the customer directly. In this case, the product is sold as a regular product. With traditional DFS, the product is immediately delivered to the customer from the retailer's stock (if stock is available) and is sold as an express product. This SC structure has been illustrated in Figure 1.

Our initial demand model (i.e., without incorporating S-b-S) is linear with substitution. Customers arrive according to a Poisson process with mean arrival rates λ_1 and λ_2 for regular and express products, respectively. One unit of demand corresponds to one customer (order). The mean demand of each product is decreasing in its DT and price and increasing in the other product's DT and price. The regular product is offered at a standard price p_1 and a guaranteed DT l_1. The express product is offered by the retailer at a higher price p_2 and a shorter DT l_2. The mean demand rates are given in equations (1) and (2) below. Notice that the total potential market size is $2A$. Parameters α_p and β_l are price-sensitivity and time-sensitivity, respectively, and α_2 and β_2 represent the sensitivity of switchover toward price difference and DT difference, respectively.

$$\lambda_1 = A - \alpha_p p_1 + \alpha_2(p_2 - p_1) - \beta_l l_1 + \beta_2(l_2 - l_1) \tag{A.1}$$

$$\lambda_2 = A - \alpha_p p_2 + \alpha_2(p_1 - p_2) - \beta_l 2_1 + \beta_2(l_1 - l_2) \tag{A.2}$$

Thus, if the DT l_1 increases by one unit, $(\beta_2 + \beta_l)$ units of demand (customers) will be lost from the drop-shipping channel of which β_2 units will transfer to the DFS channel, and β_l units of the demand will be lost from the two channels. Note that the total initial demand, given by $\lambda_1 + \lambda_2 = 2A - \alpha_p(p_1 + p_2) - \beta_l(l_1 + l_2)$, decreases in price charged and DT quoted.

For the regular product, the supplier processes each single order forwarded by the retailer and ships it to the customer without passing through the retailer's stock. The drop-shipping channel is then modeled as an MTO M/M/1 queue where the service time is exponentially distributed with mean rate μ_1. The M/M/1 queue has been widely used in DT quotation literature since the pioneering paper of Palaka et al. (1998). The retailer targets a minimum service level r_1 for the regular product. This guarantees that the probability that a customer will be served within the quoted DT l_1 is greater than r_1 (i.e., $Pro(w \leq l_1) \geq r_1$ where $w$is the expected waiting time for a customer), which prevents an unreliable DT quote.

As the express product is immediately delivered from stock, our entire analysis, without loss of generality, considers that l_2 is normalized to be equal to zero. Thus, the DT of regular product represents also the DT difference between express and regular products. DT l_1 is a decision variable. It is, of course, affected by the drop-shipper service time (capacity), but the retailer controls l_1 with pricing and inventory decisions as these decisions impact the demand rate in the drop-shipping channel. Hua et al. (2010) argued that the decision on the DT for the direct channel is crucial for dual-channel management.

Each replenishment of retailer's inventory refills the stock to its target level S. The service time to refill the inventory is exponentially distributed with mean rate μ_2. The service time does not depend on the replenishment lot size since the products are assumed to be always available at the supplier's site, which is a common assumption (Zhu, 2015). Thus, replenishment time corresponds to preparation and transportation activities. We let ψ_0 denote the probability of stockout in the retailer's warehouse. The probability of serving customers (i.e., $1 - \psi_0$) must be greater than a predetermined minimum service level r_2. In case of stockout, the customer can either switch to the regular product (i.e., accept a longer DT l_1 but with a lower price p_1) or leave the system (lost sale). We let $\phi \in [0,1]$ denote the S-b-S rate; it represents the percentage of customers initially interested in the express product but accepting the regular product in case of stockout. In particular, $\phi = 1$ means that all customers switch to the regular product, whereas $\phi = 0$ means that all customers leave the system. Note that a smaller ϕ can represent a more competitive market; the S-b-S typically increases when the retailer is the exclusive seller of the product and decreases if the product can be found elsewhere by customers.

We let $\overline{\lambda_1}$ and $\overline{\lambda_2}$, respectively, denote the effective mean demand rate of the regular and express products. $\overline{\lambda_1}$ is the sum of customers initially interested in the regular product (i.e., λ_1) and customers interested in the express product but deciding to switch to the regular product in case of stockout (i.e., $\phi\psi_0\lambda_2$). $\overline{\lambda_2}$ is the difference between the customers initially interested in the express product (i.e., λ_2) and the customers that are likely to find empty stock (i.e., $\psi_0\lambda_2$). This system is illustrated in Figure 3. To obtain

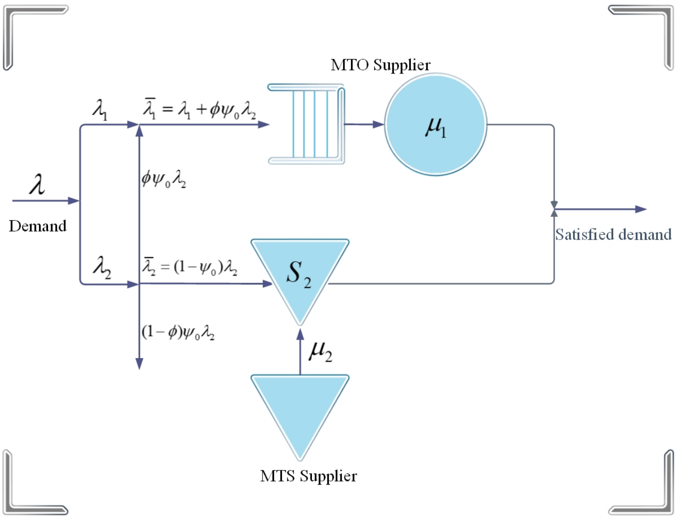

Figure A.2: A hybrid delivery system with stock-out based substitution

a tractable model while capturing the essential features and the key trade-offs, we make two approximations that we will discuss. The first approximation concerns the service constraint for the regular product. When S-b-S is considered (i.e., for $\phi > 0$), $\overline{\lambda_1}$ is no longer exponentially distributed. In this case, we conducted extensive experiments to check whether it is still acceptable to use the service constraint formula of the M/M/1 queue (which assumes that demand is exponentially distributed). Thus, we simulated our hybrid system with Arena and ran it for the equivalent of 500,000 hours. We found that the probability of serving customers on time, obtained with simulation of the real system, is equal to 0.25495, whereas this probability is equal to 0.25727 if $\overline{\lambda_1}$ is assumed to be exponentially distributed. Therefore, we deduced that the service constraint formula of the M/M/1 queue provides an acceptable approximation. Note that this constraint is exact when $\phi = 0$.

The second approximation relates to the stockout probability which is given by a very complex formula in the case of a general inventory refilling policy. To approximate the stockout probability in our system, we use the stockout probability obtained in the case where the buffer is refilled to stock level S when it is empty. This approximation is accurate when the reorder point considered in practice is relatively small, which is the case in many warehouses that have storage capacity constraints or in the presence of a second delivery channel as we consider in our model. When the reorder point is relatively high, the impact of this approximation on the model's outcomes can be offset by increasing the minimum service level r_2 to a higher value than that typically used in practice. Based on this assumption, we obtain $\psi_0 = (1+\frac{\mu_2 S}{\lambda_2})^{-1}$.

Thus, the effective mean demand rates are given by the following equations where $\alpha_1 = \alpha_p + \alpha_2$ and $\beta_1 = \beta_l + \beta_2$ (l_2 is normalized to be equal to zero without loss of generality).

$$\overline{\lambda_1} = \lambda_1 + \phi\psi_0\lambda_2 = \lambda_1 + \phi\frac{{\lambda_2}^2}{\lambda_2+\mu_2 S} = A - \alpha_1 p_1 + \alpha_2 p_2 - \beta_1 l_1 + \phi\frac{(A-\alpha_1 p_2 + \alpha_2 p_1 + \beta_2 l_1)^2}{A - \alpha_1 p_2 + \alpha_2 p_1 + \beta_2 l_1 + \mu_2 S} \quad \text{(A.3)}$$

$$\overline{\lambda_2} = (1-\psi_0)\lambda_2 = \frac{\mu_2 S\lambda_2}{\lambda_2+\mu_2 S} = \frac{\mu_2 S(A-\alpha_1 p_2+\alpha_2 p_1+\beta_2 l_1)}{A-\alpha_1 p_2+\alpha_2 p_1+\beta_2 l_1+\mu_2 S} \quad \text{(A.4)}$$

In the traditional shipping mode, retailers purchase products from suppliers for a wholesale price and then determine retail prices for consumers. We let c_2 denote the purchasing cost per unit of the product (shipping cost included), which incurs the retailer's unit margin $m_2 = p_2 - c_2$ (this does not account for the inventory holding cost). In contrast, in the drop-shipping mode, suppliers determine retail prices and share revenue with the retailer. Consequently, the retailer loses the flexibility of setting market prices (Khouja and Stylianou 2009, Tian et al. 2018). Hence, as in practice, we consider that the regular product has a given fixed price and generates a fixed unit margin $m_1 = p_1 - c_1$ for the retailer.

Hence, in its general form, the problem is determining (i) the DT differentiation by quoting the DT l_1, (ii) the price differentiation by setting the price p_2, and (iii) the stock level S with the objective of maximizing the total expected profit under service level constraints for both regular and express products. The general model is given below.

General Model ($M_{l_1,p_2,S}$)

$$\underset{l_1,p_2,S}{\text{Maximize}}\ \pi = (p_1-c_1)\overline{\lambda_1} + (p_2-c_2)\overline{\lambda_2} - hS \quad \text{(A.5)}$$

Subject to

$$1 - e^{-(\mu_1-\overline{\lambda_1})l_1} \geq r_1 \quad \text{(A.6)}$$

$$\frac{\mu_2 S}{\lambda_2+\mu_2 S} \geq r_2 \quad \text{(A.7)}$$

$$S, \lambda_1, \lambda_2 \geq 0, l_1 > 0, p_2 > p_1, \overline{\lambda_1} < \mu_1$$

Objective function (5) represents the total expected profit to be maximized (i.e., net revenue – inventory cost). Note that $\overline{\lambda_1}$ and $\overline{\lambda_2}$ are, respectively, given by equations (3) and

(4). The average inventory level is given by $\frac{S}{2}$ and the unit inventory cost is denoted by 2h, which incurs the total average inventory cost hS. Constraint (6) expresses the service level constraint for the regular product, i.e., $Pr(w \leq l_1) \geq r_1$. Using the M/M/1 queue properties (as discussed earlier), we obtain $Pr(w \leq l_1) = 1 - e^{-(\mu_1 - \overline{\lambda_1})l_1}$ (see Hammami et al. 2020). The service level constraint for the express product, i.e., $1 - \psi_0 \geq r_2$, is given by constraint (7). We recall that $1 - \psi_0 = \frac{\mu_2 S}{\lambda_2 + \mu_2 S}$. The other constraints specify the variable domains. Note that the stability condition of the M/M/1 queue (i.e., $\overline{\lambda_1} < \mu_1$) is automatically satisfied when constraint (6) is satisfied, so we ignore the stability condition for the remainder of the paper.

For convenience, we let l denote the DT of the regular product (instead of l_1) and p denote the price of the express product (instead of p_2) in the rest of this manuscript. In the following sections, we solve and analyze different variants of this model. In Section 4, we study the case where l is a decision variable while S and p are fixed. In Section 5, we study the case where l and S are decision variables while p is fixed. The general case, in which l, S, and p are decision variables, is studied in Section 6.

A.4 Delivery time quotation model with fixed stock and price

In this section, we determine the optimal DT l when S and p are fixed. To simplify the notation, we let $\lambda_1 = a_{1,p} - \beta_1 l$ and $\beta_2 = a_{2,p} + \beta_2 l$, where $a_{1,p} = A - \alpha_1 p_1 + \alpha_2 p$ and $a_{2,p} = A + \alpha_2 p_1 - \alpha_1 p$. In addition, we let $z = ln(\frac{1}{1-r_1})$. The resulting model, denoted by (M_l), is given below.

$$\text{Maximize } \pi(l) = m_1\Big(a_{1,p} - \beta_1 l + \phi\frac{(a_{2,p}+\beta_2 l)^2}{a_{2,p}+\beta_2 l+\mu_2 S}\Big) + \frac{m_2\mu_2 S}{a_{2,p}+\beta_2 l+\mu_2 S}\big(a_{2,p}+\beta_2 l\big) - hS \tag{A.8}$$

Subject to

$$\mu_1 - a_{1,p} + \beta_1 l - \phi\frac{(a_{2,p}+\beta_2 l)^2}{a_{2,p}+\beta_2 l+\mu_2 S} - \frac{z}{l} \geq 0 \tag{A.9}$$

$$\frac{\mu_2 S}{a_{2,p}+\beta_2 l+\mu_2 S} \geq r_2 \tag{A.10}$$

$$max\{0, \frac{-a_{2,p}}{\beta_2}\} < l \leq \frac{a_{1,p}}{\beta_1}$$

Note that constraint (10) imposes lower and upper bounds on l to guarantee positive values for λ_1 and λ_2 (also note that $\frac{-a_{2,p}}{\beta_2}$ is not necessarily negative). As highlighted by many authors (e.g., Ayanso et al. 2006, Rabinovich et al. 2008), the express product generates a higher margin than the regular product. Hence, we focus here on the case of $m_2 \geq m_1$. Since $\phi \leq 1$, we also have $m_2 \geq \phi m_1$. We first solve model (M_l) and then discuss the insights.

A.4.1 Model solving

Notice that constraint (8) is equivalent to $f_{\phi(l)} \geq 0$ where $f_{\phi(l)} = \beta_2(\beta_1 - \phi\beta_2)l^3 + (\beta_1(a_{2,p}+\mu_2 S) - \beta_2(a_{1,p}-\mu_1+2\phi a_{2,p}))l^2 - ((a_{1,p}-\mu_1)(a_{2,p}+\mu_2 S)+\beta_2 z+\phi {a_{2,p}}^2)l - z(a_{2,p}+\mu_2 S)$. To solve model (M_l), we first provide a simpler formulation of constraint (8). All proofs are given in the appendix.

Lemma A.1. *For $\phi = 0$, the service constraint for the regular product (constraint (8)) is equivalent to $l \geq l_0$, where $l_0 = \dfrac{a_{1,p}-\mu_1+\sqrt{(a_{1,p}-\mu_1)^2+4\beta_1 z}}{2\beta_1}$.*
For $\phi > 0$, constraint (8) is equivalent to $l \geq l_\phi$, where l_ϕ is the unique root in $[l_0, \dfrac{a_{1,p}}{\beta_1}]$ of the cubic equation: $f_\phi(l) = 0$.

Lemma 1 provides a new lower bound on l. Furthermore, constraint (9) is equivalent to $l \leq \dfrac{(1-r_2)\mu_2 S - r_2 a_{2,p}}{r_2\beta_2}$. Hence, we deduce that the feasible domain for model (M_l) is $[l_{min}, l_{max}]$, where $l_{min} = \max\{l_\phi, \dfrac{-a_{2,p}}{\beta_2}\}$ and $l_{max} = \min\{\dfrac{a_{1,p}}{\beta_1}, \dfrac{(1-r_2)\mu_2 S - r_2 a_{2,p}}{r_2\beta_2}\}$. The optimal solution of model (M_l) is given below.

Lemma A.2. *The optimal DT of the regular product is:*

$$l^* = \begin{cases} l_{th} & \text{if } l_{min} \leq l_{th} \leq l_{max} \\ l_{min} & \text{if } l_{th} < l_{min} \\ l_{max} & l_{th} > l_{max} \end{cases}$$

where $l_{th} = \dfrac{-a_{2,p}+\mu_2 S(\sqrt{\dfrac{\beta_2(m_2-\phi m_1)}{m_1(\beta_1-\phi\beta_2)}}-1)}{\beta_2}$, $l_{min} = max\{l_\phi, \dfrac{-a_{2,p}}{\beta_2}\}$ and $l_{max} = min\{\dfrac{a_{1,p}}{\beta_1}, \dfrac{(1-r_2)\mu_2 S - r_2 a_{2,p}}{r_2\beta_2}\}$. For $\phi = 0$ (i.e., in the case without S-b-S), $l_\phi = l_0 = \dfrac{a_{1,p}-\mu_1+\sqrt{(a_{1,p}-\mu_1)^2+4\beta_1 z}}{2\beta_1}$.
For $\phi > 0$, l_ϕ is the unique solution in $[l_0, \dfrac{a_{1,p}}{\beta_1}]$ of the cubic equation $f_\phi(l) = 0$ (see Lemma 1).

A.4.2 Model analysis and insights

The cases of $l^* = \frac{-a_{2,p}}{\beta_2}$ or $\frac{a_{1,p}}{\beta_1}$ correspond to extreme situations where only one product is offered to customers. These cases are not relevant to our study, so we exclude them from the analysis. Based on the result of Lemma 2, we deduce a set of managerial implications in the following series of propositions. In Proposition 1, we analyze the optimal time differentiation strategy.

Proposition A.1. *If* $\frac{m_2}{m_1} \leq \frac{\beta_1}{\beta_2}$*, then the retailer should adopt a minimum time differentiation strategy (i.e., offering the regular product with the shortest feasible DT).*
If $\frac{\beta_1}{\beta_2} < \frac{m_2}{m_1} \leq \frac{1}{{r_2}^2}(\frac{\beta_1}{\beta_2} - \phi(1-{r_2}^2))$*, then the optimal strategy depends on the stock level S. The retailer should adopt minimum time differentiation when S is smaller than a given threshold value and should increase the time differentiation (without reaching the maximum).*
If $\frac{m_2}{m_1} > \frac{1}{{r_2}^2}(\frac{\beta_1}{\beta_2} - \phi(1-{r_2}^2))$*, then the optimal strategy is maximum time differentiation (i.e., offering the regular product with the longest feasible DT).*

The managerial guidelines provided in Proposition 1 are graphically illustrated in Figure 4.

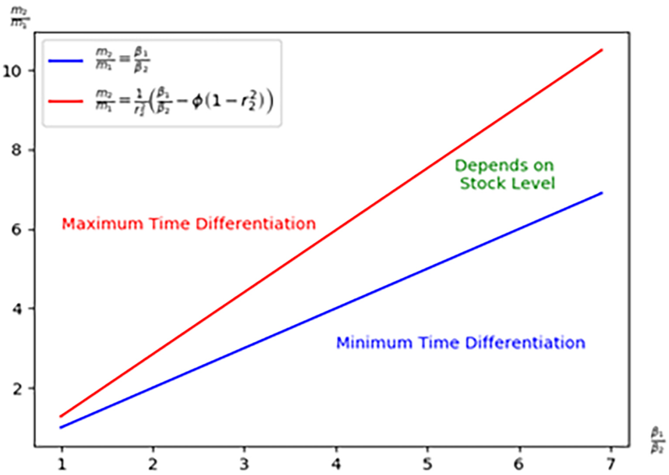

Figure A.3: Time differentiation strategy when the characteristics of the express product are fixed

Note that $\frac{m_2}{m_1}$ represents the price differentiation between the express product and the regular product. Furthermore, $\frac{\beta_1}{\beta_2}$ represents the ratio of customers lost from the drop-shipping channel to customers transferring to the DFS channel due to a unitary increase in the DT of the regular product. A higher ratio $\frac{\beta_1}{\beta_2}$ means a higher number of lost customers relative to the number of customers switching to the express product when the retailer quotes a longer DT. This characterizes, for instance, a more competitive external market or more time-sensitive customers but who are not sensitive enough to switch to the express product and pay a higher price.

When $\frac{m_2}{m_1} \leq \frac{\beta_1}{\beta_2}$, quoting a longer DT results in loss of revenue caused by a decrease in regular demand that is greater than the additional revenue generated from the increase in the express demand. In this case, the retailer has an interest in quoting the shortest feasible DT given by $l^* = l_{min} = l_\phi$.

Then, when $\frac{\beta_1}{\beta_2} < \frac{m_2}{m_1} \leq \frac{1}{{r_2}^2}(\frac{\beta_1}{\beta_2} - \phi(1 - {r_2}^2))$, the trade-off between the regular demand and the express demand is affected by the stock level. Compared to the first case, the express product becomes more profitable. However, when S is relatively small, it is not in the retailer's interest to favor the express demand since the service level constraint for the express product cannot be satisfied. Thus, it is still optimal to adopt minimum time differentiation. When the stock level increases, it is possible to capitalize more on the express demand, which leads to greater time differentiation (i.e., $l^* = l_{th}$). Nevertheless, the price differentiation is here not large enough to justify maximizing the express demand, so there is no need for maximum time differentiation. This situation is interesting as it shows that the optimal strategy is neither minimum nor maximum time differentiation but an intermediate strategy where $l^* = l_{th}$, could be preferred.

Finally, when $\frac{m_2}{m_1} > \frac{1}{{r_2}^2}(\frac{\beta_1}{\beta_2} - \phi(1 - {r_2}^2))$, the express product becomes profitable. Thus, the loss of revenue for the regular product resulting from increasing the DT is offset by the gain in revenue from the express product. It is then optimal to maximize the express demand (the only limitation is the service constraint), which requires maximum time differentiation (i.e., $l^* = l_{max}$).

In summary, the optimal strategy is minimum time differentiation when the price differentiation relative to the ratio $\frac{\beta_1}{\beta_2}$ is small, and the optimal strategy is maximum time differentiation when the price differentiation relative to the ratio $\frac{\beta_1}{\beta_2}$ is large. For intermediate levels of price differentiation, neither minimum nor maximum time differentiation is preferred, but the optimal strategy depends on the express product stock levels. We now investigate in Proposition 2 the effect of stock level on time differentiation.

Proposition A.2. *A higher stock level S leads to the retailer offering the regular product with a shorter DT when the retailer adopts a minimum time differentiation strategy and with a longer DT in all other cases.*

We expect that holding more inventory leads to quoting a longer DT for the regular product to favor the express demand since more of this demand can be satisfied. However, in case of minimum time differentiation, we demonstrate that the optimal DT decreases in S (see the proof of Proposition 2 in the appendix). The qualitative explanation is the following. An increase in S reduces the number of switching customers in case of stockout, which implies lower regular demand and, consequently, leads to a non-binding service constraint for the regular product. Since we are analyzing the case where the shortest feasible DT is offered to customers, the model reacts by quoting a shorter DT until it reaches the binding situation once again. We now study in Proposition 3 the effect of S-b-S.

Proposition A.3. *An increase in the S-b-S rate ϕ leads to the retailer offering the regular product with a longer DT as long as the retailer does not adopt a maximum time differentiation strategy. In case of maximum time differentiation, the optimal DT does not depend on ϕ.*

Recall that a greater ϕ implies a higher percentage of express demand transformed into regular demand in the case of stockout. To understand the effect of S-b-S, three cases should be distinguished. In the case of minimum time differentiation, the service constraint for the regular product is binding. Consequently, when ϕ increases, the system must quote a longer DT to decrease the regular demand and satisfy the service constraint.

In the case of intermediate time differentiation (i.e., when $l^* = l_{th}$), none of the service constraints are binding (neither for the regular product nor for the express product). This situation is less intuitive since the optimal strategy is composed of a trade-off between both types of demand and not by maximizing one of the demand types. In this case, the model reacts to an increase in ϕ by quoting a longer DT (as in the first case) as this enables offsetting the increase in regular demand and returning to a more profitable trade-off.

Finally, in case of maximum time differentiation, the express product is much more profitable than the regular product, but the system cannot increase the express demand any more since the service constraint for the express product is already binding. Thus, an increase in ϕ does not lead to any reaction. The analysis of this first model variant revealed the significant impact of S on the optimal strategy and, consequently, the importance of optimizing S.

A.5 Delivery time quotation and stock level determination model with fixed price

We now study a new variant of the model where l and S are decision variables, but p is fixed. Recall that $\lambda_1 = a_{1,p} - \beta_1 l$ and $\beta_2 = a_{2,p} + \beta_2 l$. The resulting model, denoted by $(M_{l,S})$, is given below.

$$\underset{l,S}{\text{Maximize}}\ \pi = m_1\Big(a_{1,p} - \beta_1 l + \phi\frac{(a_{2,p}+\beta_2 l)^2}{a_{2,p}+\beta_2 l+\mu_2 S}\Big) + \frac{m_2\mu_2 S}{a_{2,p}+\beta_2 l+\mu_2 S}\big(a_{2,p}+\beta_2 l\big) - hS \tag{A.11}$$

Subject to

$$\mu_1 - a_{1,p} + \beta_1 l - \phi\frac{(a_{2,p}+\beta_2 l)^2}{a_{2,p}+\beta_2 l+\mu_2 S} - \frac{z}{l} \geq 0 \tag{A.12}$$

$$\frac{\mu_2 S}{a_{2,p}+\beta_2 l+\mu_2 S} \geq r_2 \tag{A.13}$$

$$l > 0, S, \lambda_1, \lambda_2 \geq 0$$

Note that $\pi(l,S)$ is concave in S (for a given l) and reaches its maximum in $S_{th}(l) = \frac{\lambda_2}{\mu_2}\Big(\sqrt{\frac{\mu_2(m_2-\phi m_1)}{h}} - 1\Big)$. To obtain a relevant problem, we must have $\frac{\mu_2(m_2-\phi m_1)}{h} \geq 1$. Service constraints (11) and (12) are, respectively, equivalent to $S \geq S_1(l) = \frac{\lambda_2}{\mu_2}(\frac{\phi\lambda_2}{\mu_1-\lambda_1-\frac{z}{l}} - 1)$ and $S \geq S_2(l) = \frac{\lambda_2}{\mu_2}\frac{r_2}{1-r_2}$. Hence, for a given feasible l, the optimal stock level is $S^*(l) = max\{S_{th}(l), S_1(l), S_2(l)\}$. To make more progress, we now need to distinguish two cases: $\phi = 0$ and $\phi > 0$. We first solve analytically the model for $\phi = 0$ and derive managerial insights. Then, we analyze the case of $\phi > 0$ and derive insights based on numerical experiments.

A.5.1 Model ($M_{l,S}$) without stockout-based substitution ($\phi = 0$): Analytical resolution and insights

To solve model ($M_{l,S}$) for $\phi = 0$, we first provide a simpler equivalent formulation in Lemma A.3.

Lemma A.3. *For $\phi = 0$, model ($M_{l,S}$) is equivalent to the following single-variable model.*

$$Max\pi(l) = \begin{cases} m_1(a_{1,p} - \beta_1 l) + r_2(m_2 - \frac{h}{(1-r_2)\mu_2})(a_{2,p} + \beta_2 l) & \text{if } m_2 \leq \frac{h}{(1-r_2)^2\mu_2} \\ m_1(a_{1,p} - \beta_1 l) + (\sqrt{m_2} - \sqrt{\frac{h}{\mu_2}})(a_{2,p} + \beta_2 l) & m_2 > \frac{h}{(1-r_2)^2\mu_2} \end{cases}$$

$$\text{subject to } \max\{l_0, \frac{-a2,p}{\beta_2}\} \leq l \leq \frac{a_{1,p}}{\beta_1}, \text{ where } l_0 = \frac{a_{1,p} - \mu_1 + \sqrt{(a1,p-\mu_1)^2 + 4\beta_1 z}}{2\beta_1}.$$

Note that $\pi(l)$ is linear in l. This result is not intuitive; it implies that the optimal DT is either the shortest feasible DT (i.e., $\max\{l_0, \frac{-a2,p}{\beta_2}\}$) or the longest DT (i.e., $\frac{a_{1,p}}{\beta_1}$). When it is optimal to quote the shortest DT, we assume that $l_0 \geq \frac{-a2,p}{\beta_2}$ since, otherwise, the retailer offers only the regular product, which is an extreme case that we do not study. We provide the optimal solution in Lemma 4.

Lemma A.4. *For $\phi = 0$, the optimal solution of model ($M_{l,S}$) is given as follows.*

case of $m_2 \leq \frac{h}{(1-r_2)^2\mu_2}$

- *if* $m_2 \leq \frac{m_1\beta_1}{r_2\beta_2} + \frac{h}{(1-r_2)\mu_2}$, *then* $\begin{cases} l^* = l_0 = \frac{a_{1,p} - \mu_1 + \sqrt{(a_{1,p}-\mu_1)^2 + 4\beta_1 z}}{2\beta_1} \\ S^* = \left(\frac{r_2}{1-r_2}\right)\frac{2\beta_1 a_{2,p} + \beta_2\left(a_{1,p} - \mu_1 + \sqrt{(a_{1,p}-\mu_1)^2 + 4\beta_1 z}\right)}{2\beta_1\mu_2} \end{cases}$

- *if* $m_2 > \frac{m_1\beta_1}{r_2\beta_2} + \frac{h}{(1-r_2)\mu_2}$, *then* $\begin{cases} l^* = \frac{a_{1,p}}{\beta_1} \\ S^* = \left(\frac{r_2}{1-r_2}\right)\frac{\beta_1 a_{2,p} + \beta_2 a_{1,p}}{\beta_1\mu_2} \end{cases}$

case of $m_2 > \frac{h}{(1-r_2)^2\mu_2}$

- *if* $m_2 \leq \left(\sqrt{\frac{m_1\beta_1}{\beta_2}} + \sqrt{\frac{h}{\mu_2}}\right)^2$, *then* $\begin{cases} l^* = l_0 = \dfrac{a_{1,p} - \mu_1 + \sqrt{(a_{1,p} - \mu_1)^2 + 4\beta_1 z}}{2\beta_1} \\ S^* = \left(\sqrt{\dfrac{m_2\mu_2}{h}} - 1\right)\dfrac{2\beta_1 a_{2,p} + \beta_2\big(a_{1,p} - \mu_1 + \sqrt{(a_{1,p} - \mu_1)^2}}{2\beta_1\mu_2} \end{cases}$

- *if* $m_2 > \left(\sqrt{\frac{m_1\beta_1}{\beta_2}} + \sqrt{\frac{h}{\mu_2}}\right)^2$, *then* $\begin{cases} l^* = \dfrac{a_{1,p}}{\beta_1} \\ S^* = \left(\sqrt{\dfrac{m_2\mu_2}{h}} - 1\right)\dfrac{\beta_1 a_{2,p} + \beta_2 a_{1,p}}{\beta_1\mu_2} \end{cases}$

Hence, according to Lemma 4, if $m_2 \leq \frac{h}{(1-r_2)^2\mu_2}$, then the minimum time differentiation is adopted when we also have $m_2 \leq \frac{m_1\beta_1}{r_2\beta_2} + \frac{h}{(1-r_2)\mu_2}$, and only the express product is offered when $m_2 > \frac{m_1\beta_1}{r_2\beta_2} + \frac{h}{(1-r_2)\mu_2}$. Furthermore, if $m_2 > \frac{h}{(1-r_2)^2\mu_2}$, then the minimum time differentiation is adopted when we also have $m_2 \leq \left(\sqrt{\frac{m_1\beta_1}{\beta_2}} + \sqrt{\frac{h}{\mu_2}}\right)^2$, and only the express product is offered when $m_2 > \left(\sqrt{\frac{m_1\beta_1}{\beta_2}} + \sqrt{\frac{h}{\mu_2}}\right)^2$. This is recapitulated in Table 1.

Table A.2: Optimal retailer's strategy under the setting of model $(M_{l,S})$ with $\phi = 0$

$m_2 \leq \frac{h}{(1-r_2)^2\mu_2}$		$m_2 > \frac{h}{(1-r_2)^2\mu_2}$	
$m_2 \leq \frac{m_1\beta_1}{r_2\beta_2} + \frac{h}{(1-r_2)\mu_2}$	$m_2 > \frac{m_1\beta_1}{r_2\beta_2} + \frac{h}{(1-r_2)\mu_2}$	$m_2 \leq \left(\sqrt{\frac{m_1\beta_1}{\beta_2}} + \sqrt{\frac{h}{\mu_2}}\right)^2$	$m_2 > \left(\sqrt{\frac{m_1\beta_1}{\beta_2}} + \sqrt{\frac{h}{\mu_2}}\right)^2$
Minimum time differentiation	Only the express product is offered (DFS channel)	Minimum time differentiation	Only the express product is offered (DFS channel)

Based on the result of Lemma 4, we derive insights into the optimal strategy of the retailer in the following two propositions. In Proposition 4, we investigate the effect of prices on time differentiation.

Proposition A.4. *There exists a threshold value defined by* $\frac{\beta_1 m_1}{r_2\beta_2} + \frac{h}{(1-r_2)\mu_2} + c_2$, *if* $\frac{\beta_1 m_1}{\beta_2} \leq \frac{h}{\mu_2}\left(\frac{r_2}{1-r_2}\right)^2$ *and by* $\left(\sqrt{\frac{\beta_1 m_1}{\beta_2}} + \sqrt{\frac{h}{\mu_2}}\right)^2$, *if* $\frac{\beta_1 m_1}{\beta_2} > \frac{h}{\mu_2}\left(\frac{r_2}{1-r_2}\right)^2$, *such that:*
If the express product's price is smaller than this threshold value, then it is optimal to offer both products with a minimum time differentiation; otherwise, only the express product should be offered (i.e., the retailer must rely only on the DFS channel).

Observing Table 1, we expect that an increase in m_2 would have a non-monotonous effect on the time differentiation strategy. However, whatever the values of the parameters, we demonstrate that the intermediate intervals (second and third columns of Table 1) cannot be feasible simultaneously (see the proof of Proposition 4 in the appendix). Therefore, when m_2 increases, the model reacts only by moving from minimum time differentiation to giving up the drop-shipping channel. Thus, unlike model (M_l) (in which the stock level was fixed), it is never optimal here to have an intermediate time differentiation strategy. The retailer should either offer both products with minimum time differentiation or only offer the express product. In Proposition 5, we investigate the case where both products are offered to customers.

Proposition A.5. *If both products are offered to customers, then the higher the express product's price, the longer the DT of the regular product, and the higher the stock level.*

From Proposition 4 we deduce that if both products are offered to customers, then the optimal strategy is minimum time differentiation. In this case, Proposition 5 states, as expected, that the optimal DT and the optimal stock are increasing in the express product's price as this favors express demand when the express product becomes more profitable.

Finally, note that the results discussed in this section are relative to the case without S-b-S and that the consideration of S-b-S will likely impact such trade-offs. This is the focus of the next section.

A.5.2 Model ($M_{l,S}$) with stockout-based substitution ($\phi > 0$): Analysis and numerical insights

When $\phi > 0$, it is not possible to solve model ($M_{l,S}$) analytically. Hence, we first provide a simpler formulation of the model and then rely on this formulation to conduct our analysis.

Lemma A.5. *For $\phi > 0$, Model ($M_{l,S}$) is equivalent to the following single-variable model:*

$$Max\pi(l) = \begin{cases} \pi_1(l) = \Big(m_1 - \frac{m_2}{\phi}\Big)\mu_1 + m_2 a_{2,p} + \frac{m_2 a_{1,p}}{\phi} + \frac{h a_{2,p}}{\mu_2} - \Big(m_1 - \frac{m_2}{\phi}\Big)\frac{z}{l} + \\ \Big(m_2\beta_2 - \frac{m_2\beta_1}{\phi} + \frac{h\beta_2}{\mu_2}\Big)l - \big(\frac{h\phi}{\mu_2}\big)\frac{(a_{2,p}+\beta_2 l)^2}{\mu_1 - a_{1,p} + \beta_1 l - \frac{z}{l}} if\, l_0 \le l \le l_\theta \\ \pi_2(l) = m_1(a_{1,p} - \beta_1 l) + (m_2 + \frac{h}{\mu_2}(1-\theta) - \frac{m_2 - \phi m_1}{\theta})(a_{2,p} + \beta_2 l) if\, l_\theta \le l \le \frac{a_{1,p}}{\beta_1} \end{cases}$$

where $\theta = max\{\sqrt{\frac{\mu_2(m_2 - \phi m_1)}{h}}, \frac{1}{1-r_2}\}$ *and*

$$l_\theta = \frac{\big(\theta(a_{1,p} - \mu_1) + \phi a_{2,p}\big) + \sqrt{\big(\theta(a_{1,p} - \mu_1) + \phi a_{2,p}\big)^2 + 4\theta(\theta\beta_1 - \phi\beta_2)z}}{2(\theta\beta_1 - \phi\beta_2)}.$$

Observing Lemma 5, note that when $\phi = 0$, we have $l_\theta = l_0$, implying that $\pi_1(l)$ is no longer defined and that $\pi(l) = \pi_2(l)$ over $\big[l_0, \frac{a_{1,p}}{\beta_1}\big]$. Thus, we obtain the same linear profit function given in the previous section (Lemma 3). When $\phi > 0$, we have two different behaviors of profit in function of l represented by the two functions $\pi_1(l)$ and $\pi_2(l)$ defined, respectively, over $[l_0, l_\theta]$, and $\big[l_\theta, \frac{a_{1,p}}{\beta_1}\big]$. $\pi_1(l)$ is not linear while $\pi_2(l)$ is linear. Therefore, the optimal DT is either the DT that maximizes $\pi_1(l)$ over $[l_0, l_\theta]$, or l_θ or $\frac{a_{1,p}}{\beta_1}$. Given the complexity of $\pi_1(l)$, it is not possible to obtain a closed-form expression of the optimal solution. Hence, the remainder of our analysis will be based on numerical experiments.

To conduct our experiments, we consider three market structures. The first market is characterized by $\frac{\beta_2}{\beta_l} > \frac{\alpha_2}{\alpha_1}$, i.e., the switchovers are more governed by time difference than price difference, which is referred to as a STD market. The second market is characterized by $\frac{\beta_2}{\beta_l} < \frac{\alpha_2}{\alpha_1}$, i.e., the switchovers are more governed by price difference than time difference, which is referred to as a SPD market. When $\frac{\beta_2}{\beta_l} = \frac{\alpha_2}{\alpha_1}$, the market is neutral (neither STD nor SPD). We consider the following parameters for each market structure. For the neutral market, $\alpha_p = 50$, $\alpha_2 = 10$, $\beta_l = 50$, and $\beta_2 = 10$. For the STD market, $\alpha_p = 50$, $\alpha_2 = 10$, $\beta_l = 25$, and $\beta_2 = 25$. For the SPD market, $\alpha_p = 25$, $\alpha_2 = 25$, $\beta_l = 50$, and $\beta_2 = 10$. For each type of market, we generated 168,000 instances by varying the values of the other parameters, as shown in Table 2.

For the neutral market, we obtained 56,705 feasible instances. In all of them, we have $\Big(-\beta_1 m_1 + \beta_2\big(m_2 + \frac{h}{\mu_2}(1-\theta) - \frac{m_2 - \phi m_1}{\theta}\big)\Big) < 0$, implying that the linear function $\pi_2(l)$

Table A.3: Test cases

Parameter	Value
A	From 800 to 1,200 with a step of 100
μ_1	Fixed to 150
μ_2	From 200 to 400 with a step of 50
c_1	Fixed to 3
c_2	$c_1+0.5$, c_1+1, $c_1+1.5$, and c_1+2
h	From 0.5 to 2 with a step of 0.5
p_1	From 6 to 12 with a step of 1
p_2	From m_1+c_2 to 18 with a step of 1
ϕ	From 0.2 to 1 with a step of 0.2
$r_1=r_2$	Fixed to 0.98

is decreasing in l. For the SPD market, we obtained 32,031 feasible instances, and $\pi_2(l)$ is also decreasing in all of them. Finally, for the STD market, $\pi_2(l)$ is decreasing in 24,904 instances and increasing in 39,875 instances. The analysis of the results leads to the following observations. We first highlight in observations 1 and 2 the implications in terms of DT quotation and then turn to the impact of stock-out in observation 3.

Observation A.1. *If* $\left(-\beta_1 m_1+\beta_2\left(m_2+\frac{h}{\mu_2}(1-\theta)-\frac{m_2-\phi m_1}{\theta}\right)\right)>0$ *(i.e., if* $\pi_2(l)$ *is increasing in l), then it is more profitable to offer only the express product (i.e.,* $l^*=\frac{a_{1,p}}{\beta_1}$*) and to use the drop-shipping channel only in case of stock-out as a backup solution.*

This result has been observed in all instances that have an increasing $\phi_2(l)$ (39,875 instances in total). To illustrate, we consider the example of Figure 5 ($A=1000$, $p_1=10$, $c_1=3$, $\mu_1=150$, $p_2=18$, $c_2=4.5$, $\mu_2=400$, $h=1$, $r_1=r_2=0.98$, and a STD market with $\phi=0.80$). We observe that the profit is increasing in l and, consequently, the optimal solution is offering the longest DT for the regular product. This means that the initial regular demand is equal to zero and that only the express product is to be offered. In this case, the drop-shipping channel is used only as a backup solution to serve the customers that choose to switch to the regular product in case of stock-out. The whole system can thus be viewed as a traditional shipping system with a combination of lost sales (customers who leave in case of stock-out) and back orders (customers who switch to the regular product in case of stock-out).

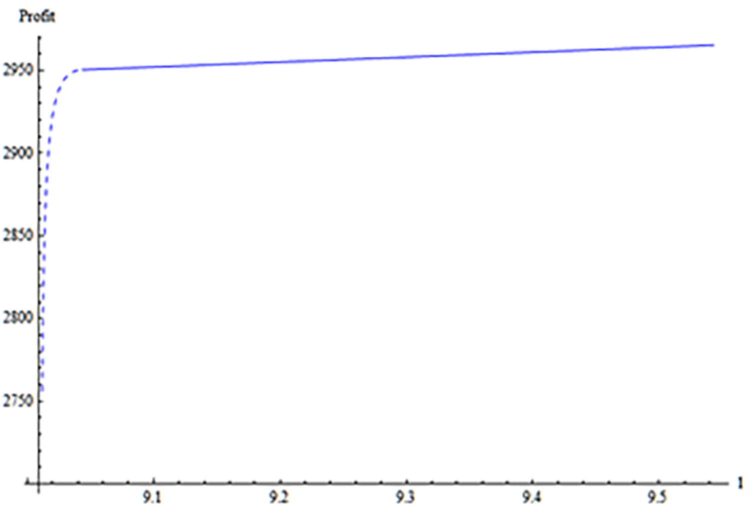

Figure A.4: Profit as a function of DT for increasing $\pi_2(l)$

Observation A.2. *If* $\Big(-\beta_1 m_1 + \beta_2\big(m_2 + \frac{h}{\mu_2}(1-\theta) - \frac{m_2 - \phi m_1}{\theta})\Big) < 0$ *(i.e., if* $\pi_2(l)$ *is decreasing in l) then it is optimal to offer both products. In this case, quoting the DT* l_θ *is a near-optimal solution.*

For each generated feasible instance that has a decreasing $\pi_2(l)$ (113,640 instances in total), we calculated numerically the optimal DT and deduced the profit gap resulting from quoting l_θ (which is the intersection point between $\pi_1(l)$ and $\pi_2(l)$) instead of the real optimal DT. The gap is calculated as follows: $\frac{Profit(optimalDT) - Profit(l_\theta)}{Profit(optimalDT}) \times 100$. For all types of markets, we obtained a minimal mean gap (0.007% for the neutral market, 0.0002% for the STD market, and 0.012% for the SPD market). This shows that l_θ is a near-optimal DT. To illustrate, we consider the following example: $A = 1000$, $p_1 = 10$, $c_1 = 3$, $\mu_1 = 150$, $p_2 = 14$, $c_2 = 4.5$, $\mu_2 = 400$, $h = 1$, $r_1 = r_2 = 0.98$, with a neutral market structure and different values of ϕ. The profiles of the profit functions are shown in Figure 6 (note that $\pi_1(l)$ and $\pi_2(l)$ are, respectively, represented by dashed and continuous lines over $[l_0, l_\theta]$ and $\big[l_\theta, \frac{a_{1,p}}{\beta_1}\big]$, and that l_θ is the intersection point). For $\phi = 0.2$, $\pi_1(l)$ is increasing over $[l_0, l_\theta]$ and, consequently, $l^* = l_\theta$ (since $\pi_2(l)$ is decreasing). For the other values of ϕ, $\pi_1(l)$ first increases and then slightly decreases before l_θ. We observe that the curve is flat between the optimal DT and l_θ, which means that l_θ is near-optimal.

Observation A.3. *S-b-S leads to higher time differentiation (i.e., offering the regular product with a longer DT), and ignoring S-b-S (by quoting the optimal DT obtained for $\phi = 0$, namely $l_0\theta$) leads to considerable loss.*

Since $l_\theta > l_0$, it is clear that the consideration of S-b-S leads to a retailer quoting a longer DT. In addition, similar to the results obtained from model (M_l), we observe in Figure 6 that the higher the value of ϕ, the longer the optimal DT for the regular product. Note that $\pi_1(l)$ increases rapidly at the neighborhood of l_0 (for all values of ϕ). This means that ignoring S-b-S, and, consequently, quoting the optimal DT obtained for $\phi = 0$ (i.e., l_0) can lead to substantial loss. We quantified this loss by calculating the gap $\frac{Profit(l_\theta) - Profit(l_0)}{Profit(l_\theta)} \times 100$. We found a mean gap of 39.36% for the neutral market, 57.18% for the SPD market, and 45.28% for the STD market. This shows the motivation to consider the effect of S-b-S, as we do in our models.

A.6 General model and effect of S-b-S

In this section, we study the general model ($M_{l,S,p}$) where l, S, and p are decision variables and the S-b-S is considered. Given the complexity of this model, we first provide a simpler quasi-equivalent formulation using the results of the previous section. Then, we rely on the new formulation to conduct experiments and derive managerial implications.

To simplify the analysis, we focus on the more general case where both regular and express products are offered to customers. Therefore, we consider the values of p such that $\Big(-\beta_1 m_1 + \beta_2\big(p - c_2 + \frac{h}{\mu_2}(1-\theta) - \frac{p - c_2 - \phi m_1}{\theta}\big)\Big) < 0$ (see Observations 1 and 2). Under this condition, and for a given price p, we have shown in the previous section that the model can be written in the function of the single variable l and that $l = l_\theta$ is a near-optimal solution (for $\phi = 0$, $l_\theta = l_0$ is the exact optimal solution). Hence, for a given price p, we replace l with l_θ. Then, we show by standard (but long) calculus, that model ($M_{l,S,p}$) can be written in the function of the single variable p. We denote this new model by (M) and provide it below. Note that we add the constraint $p \geq p_{min} = max\{m_1 + c_2, \frac{h}{\mu_2} + \phi m_1 + c_2\}$ to guarantee model consistency as explained in the previous sections.

Before proceeding further, we must verify that model (M) provides a good approximation of the original model ($M_{l,S,p}$). We generate instances according to Table 2 (for $\phi = 0$ and $\phi > 0$) and calculate the gap between the optimal profit obtained with model (M) and that of the original model ($M_{l,S,p}$). To solve model ($M_(l,S,p)$, we used the algorithm SLSQP (sequential least squares programming) of the SciPy module. This is a recommended procedure for nonlinear problems with nonlinear constraints. For our modified model (M), it can be easily solved to optimality with any computational software. The gap between both models is calculated as $\frac{(\pi_{originalmodel} - \pi_{modifiedmodel}) \times 100}{\pi_{originalmodel}}$. Table 3 shows the results.

Table A.4: Comparison between the original model and the modified model

	$\phi = 0$				$\phi > 0$			
	Test cases: 2800				Test cases: 14000			
Market	Nb. Relevant Instances	Mean Gap	Standard Deviation	Confidence Interval	Nb. Relevant Instances	Mean Gap	Standard Deviation	Confidence Interval
Neutral	2449	0.0002	0.0003	(0.00022, 0.00024)	11955	0.0087	0.0294	(0.0082, 0.0092)
SPD	2798	0.00014	0.0001	(0.00013, 0.00015)	13917	0.0102	0.0494	(0.0094, 0.0110)
STD	2154	0.95	3.0392	(0.8216, 1.0783)	10491	0.8560	2.7181	(0.8036, 0.9076)

The results confirm that the modified model (M) is quasi-equivalent to the original model ($M_{l,S,p}$), particularly for the neutral and SPD markets. Thus, the remainder of our analysis will be based on model (M). Our main objective is to investigate the effect of S-b-S. We consider the basic example: $A = 1000$, $p_1 = 8$, $c_1 = 3$, $\mu_1 = 150$, $c_2 = 4.5$, $\mu_2 = 400$, $h = 1$, and $r_1 = r_2 = 0.98$. We vary the value of ϕ and report the results in Tables 4, 5, and 6 for the neutral, the SPD, and the STD markets, respectively.

Table A.5: Effect of stock-out based substitution for the neutral market

ϕ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
p^*	12.10	12.10	12.10	12.10	12.10	12.10	12.10	12.10	12.10	12.10	12.10
l^*	8.19	8.20	8.22	8.23	8.25	8.26	8.28	8.29	8.31	8.32	8.34
S^*	59	57	55	54	54	54	54	54	54	54	54

Table A.6: Effect of stock-out based substitution for the SPD market

ϕ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
p^*	16.25	16.25	16.25	16.25	16.25	16.25	16.25	16.25	16.25	16.25	16.25
l^*	14.28	14.29	14.30	14.32	14.33	14.35	14.37	14.38	14.40	14.42	14.45
S^*	90	88	86	84	82	80	77	75	73	70	68

The analysis of the numerical results leads to the following main finding.

Table A.7: Effect of stock-out based substitution for the STD market

ϕ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
p^*	13.80	13.80	13.80	13.80	13.80	13.80	13.80	13.80	13.80	13.80	13.80
l^*	10.18	10.19	10.20	10.22	10.24	10.27	10.29	10.31	10.33	10.35	10.37
S^*	76	74	72	70	68	65	63	63	63	63	63

Observation A.4. *In the general model, a higher rate of S-b-S would normally lead to greater time differentiation and to holding less stock. However, it would not impact the price differentiation.*

When ϕ increases, different trade-offs between price, DT, and stock govern the system. Nevertheless, we observe for all types of markets that an increase in the S-b-S rate does not affect the optimal price p^*. This result is not intuitive and implies that the price quoted for the express product should be the same whatever the extent of S-b-S. In particular, the optimal price is unchanged with or without S-b-S. To confirm this result, we extend our experiments to test, for each type of market, all the instances that are generated according to Table 2. For each instance, we calculate the difference between its optimal price (for $\phi > 0$) and the optimal price obtained for $\phi = 0$. The results, provided below in Table 7, confirm that the optimal price is not significantly sensitive to the S-b-S rate. The

Table A.8: Effect of stock-out based substitution for the SPD market

Market	Mean price difference	Standard deviation	Confidence interval
Neutral	0.0046	0.0066	(0.0044, 0.0049)
SPD	0.0080	0.0083	(0.0077, 0.0083)
STD	0.0769	0.2326	(0.0683, 0.0856)

numerical results presented in Tables 4, 5, and 6 also show that an increase in the S-b-S rate should lead to greater time differentiation and the holding of less stock. We illustrate in Figure 7 how the optimal stock varies as a function of ϕ. When ϕ increases, it is preferable to capitalize on the express demand since the loss of regular demand can be offset by the additional number of customers switching from the express product to the regular product in case of stockout. Thus, as it is not profitable to reduce the price of the express product (we have just shown that this price does not change), the system reacts by quoting a longer DT for the regular product. In addition, when ϕ increases, the stockout becomes less penalizing for the retailer as there is a smaller proportion of customers leaving the system. Thus, the system favors reducing the inventory cost over satisfying more express demand. This explains why the stock decreases when ϕ increases. Nevertheless, beyond a threshold value, it is no longer possible to reduce the stock since the system must satisfy the service constraint. Therefore, the stock remains constant, as we observe in Figure 7.

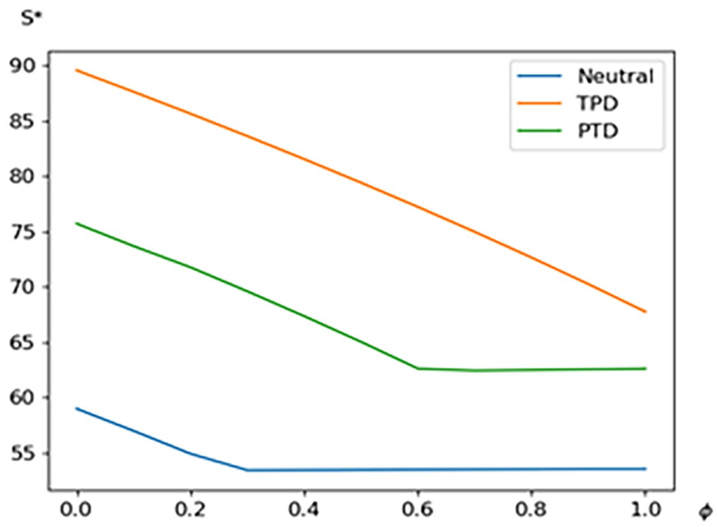

Figure A.5: Impact of the stock-out based substitution on the stock level

A.7 Conclusion and practical implications of our findings

We studied the problem of a retailer serving a time- and price-sensitive market with two substitutable products that differ in the guaranteed DT and price. The express product is delivered from stock, whereas the regular product is delivered directly from the supplier, which requires operating a hybrid distribution system with a mix of DFS and drop-shipping. In case of stockout, some customers initially interested in the express product may switch to the regular product, which is referred to as S-b-S behavior.

When the characteristics of the express product (price and stock) are fixed and the DT of the regular product is a decision variable, we provided the closed-form expression of the optimal solution. We found that is not always optimal to adopt an extreme strategy (maximum or minimum time differentiation), and that an intermediate strategy could be preferred. An increase in the stock level leads to a retailer offering the regular product with a shorter DT when the retailer adopts a minimum time differentiation strategy and with a longer DT in all other cases. When the DT and the stock level are decision variables but the price is fixed, we distinguished two situations depending on whether S-b-S is considered. In cases without S-b-S, we solved the model to optimality and found that there exists a threshold value for the express product's price below which it is optimal to offer both products with minimum time differentiation and above which only the express product should be offered. The consideration of S-b-S renders the model much more complex. We characterized the situations where it is optimal to offer both products. In this case, we provided a near-optimal solution and showed that ignoring S-b-S by quoting the optimal DT obtained without S-b-S leads to considerable loss. Finally, we studied the general model where DT, stock, and price are decision variables. We used the previous results to develop a quasi-equivalent formulation of the general model with the price as a unique variable. Based on numerical studies, we found that a higher rate of S-b-S would normally lead to greater time differentiation and less stock. However, it would not impact the price differentiation.

Practitioners might benefit from our findings in many ways. First, our study shows that similar to manufacturers, retailers can also use time-based product differentiation and segment pricing to enhance performance. In the retailing context, time-based differentiation can result from relying on a delivery mix including DFS and drop-shipping. Second, our results offer insights for retailers into how to differentiate products in terms of DT and price. Third, retailers can rely on our findings to rethink the inventory level for a given product when the drop-shipping option is also available. Finally, our study demonstrates that retailers must account for the effect of S-b-S, particularly in low competitive environments that are typically characterized by high S-b-S rate. Ignoring S-b-S leads to retailer undertaking operational decisions that are far from optimality, which implies a considerable loss.

This is the first study to investigate the time and price differentiation problem from the perspective of a hybrid distribution system and to include the effect of S-b-S. Our work can be extended to consider inventory capacity constraints in the retailer's warehouse as this may prevent the model from excessively increasing the time differentiation to favor the express demand. A future work can also investigate a more complex SC where a customer order may be delivered from the stock or from one of several potential suppliers. The retailer can quote a specific DT per supplier and, consequently, offer a menu of DTs to customers or quote a unique DT from all suppliers, which raises the issue of allocating orders to suppliers.

Appendix B
Demand function's positivity in Lemma 3.2

According to Lemma 3.2, there are two roots (R_1 and R_2) that demand is positive between them and negative outside of them.

$$R_1 = \frac{-(2\alpha_p b x_0 - \beta_e) - \sqrt{\Delta_2}}{-2\alpha_p b} = x_0 + \frac{-\beta_e + \sqrt{\Delta_2}}{2\alpha_p b}$$

$$R_2 = \frac{-(2\alpha_p b x_0 - \beta_e) + \sqrt{\Delta_2}}{-2\alpha_p b} = x_0 - \frac{\beta_e + \sqrt{\Delta_2}}{2\alpha_p b}$$

where $\Delta_2 = 4\alpha_p b\Big(A - \beta_e x_0 - \dfrac{\beta_e^2}{4\alpha_p b} + \alpha_p\big(c + \dfrac{h(1-r)}{r\mu}\big)\Big)$.

For the first root, we will prove that $x_0 + \dfrac{-\beta_e + \sqrt{\Delta_2}}{2\alpha_p b} > x_0$; for that let us prove:

$$-\beta_e + \sqrt{\Delta_2} > 0 \Leftrightarrow \sqrt{\Delta_2} > \beta_e \Leftrightarrow \Delta_2 > \beta_e^2$$

$$\Leftrightarrow \Delta_2 = 4\alpha_p b\Big(A - \beta_e x_0 - \frac{\beta_e^2}{4\alpha_p b} + \alpha_p\big(c + \frac{h(1-r)}{r\mu}\big)\Big) > \beta_e^2$$

$$\Leftrightarrow 4\alpha_p b\Big(A - \beta_e x_0 + \alpha_p\big(c + \frac{h(1-r)}{r\mu}\big)\Big) > 0$$

While we assume that $A > \beta_e x_0 - \alpha_p\big(c + \dfrac{h(1-r)}{r\mu}\big)$, the above condition ($R_1 > x_0$) is true.

The same approach will be used for the second root. We will prove that $R_2 < 0$ and we have:

$$x_0 - \frac{\beta_e + \sqrt{\Delta_2}}{2\alpha_p b} < 0 \Leftrightarrow x_0 < \frac{\beta_e + \sqrt{\Delta_2}}{2\alpha_p b} \Leftrightarrow \sqrt{\Delta_2} > 2\alpha_p b x_0 - \beta_e$$

$$\Leftrightarrow 4\alpha_p b\Big(A - \beta_e x_0 - \frac{\beta_e^2}{4\alpha_p b} + \alpha_p\big(c + \frac{h(1-r)}{r\mu}\big)\Big) > (2\alpha_p b x_0 - \beta_e)^2$$

$$\Leftrightarrow 4\alpha_p b\Big(A - \alpha_p\beta_e x_0^2 - \alpha_p\big(c + \frac{h(1-r)}{r\mu}\big)\Big) > 0$$

While $A > \alpha_p\beta_e x_0^2 + \alpha_p\big(c + \dfrac{h(1-r)}{r\mu}\big)$, the above conclusion is true. Since we assume that A is sufficiently high, it considers that the condition is authenticated. Thus, we have $R_2 < 0$.

We conclude that under a condition such that $A > max\left\{\beta_e x_0 - \alpha_p\left(c + \frac{h(1-r)}{r\mu}\right), \alpha_p \beta_e x_0^2 + \alpha_p\left(c + \frac{h(1-r)}{r\mu}\right)\right\}$, we have $R_1 > x_0$ and $R_2 < 0$, and thus, demand function is positive in feasible region ($[0, x_0]$).

APPENDIX C

Objective function concavity in Proposition 4.6

According to Proposition 4.6, there are two roots (R_3 and R_4) that objective function is concave between them and convex outside of them.

$$R_3 = x_0 + \frac{-3\beta b + \sqrt{\Delta_3}}{6\alpha b^2}$$

$$R_4 = x_0 - \frac{3\beta b + \sqrt{\Delta_3}}{6\alpha b^2}$$

where $\Delta_3 = 12\alpha b^3 \left(A_2'' - \beta x_0 - \left(c_2 + \frac{h(1-r)}{r\mu_2} \right) - \alpha_s \left(m_1 - \frac{h(1-r)}{r\mu_1} \right) + \frac{\beta^2}{4\alpha b} \right)$.
For the first root, we will prove that $x_0 + \frac{-3\beta b + \sqrt{\Delta_3}}{6\alpha b^2} > x_0$; for that let us prove:

$$-3\beta b + \sqrt{\Delta_3} > 0 \Leftrightarrow \sqrt{\Delta_3} > 3\beta b \Leftrightarrow \Delta_3 > 9\beta^2 b^2$$
$$\Leftrightarrow \Delta_3 = 12\alpha b^3 \left(A_2'' - \beta x_0 - \left(c_2 + \frac{h(1-r)}{r\mu_2} \right) - \alpha_s \left(m_1 - \frac{h(1-r)}{r\mu_1} \right) + \frac{\beta^2}{4\alpha b} \right) > 9\beta^2 b^2$$
$$\Leftrightarrow 12\alpha b^3 \left(A_2'' - \beta x_0 - \left(c_2 + \frac{h(1-r)}{r\mu_2} \right) - \alpha_s \left(m_1 - \frac{h(1-r)}{r\mu_1} \right) - \frac{\beta^2}{2\alpha b} \right) > 0$$

While we assume that $A_2'' > \beta x_0 + \left(c_2 + \frac{h(1-r)}{r\mu_2} \right) + \alpha_s \left(m_1 - \frac{h(1-r)}{r\mu_1} \right) + \frac{\beta^2}{2\alpha b}$, the above condition ($R_3 > x_0$) is true.
The same approach will be used for the second root. We will prove that $R_4 < 0$ and we have:

$$x_0 - \frac{3\beta b + \sqrt{\Delta_3}}{6\alpha b^2} < 0 \Leftrightarrow x_0 < \frac{3\beta b + \sqrt{\Delta_3}}{6\alpha b^2} \Leftrightarrow \sqrt{\Delta_3} > 6\alpha b^2 x_0 - 3\beta b$$
$$\Leftrightarrow 12\alpha b^3 \left(A_2'' - \beta x_0 - \left(c_2 + \frac{h(1-r)}{r\mu_2} \right) - \alpha_s \left(m_1 - \frac{h(1-r)}{r\mu_1} \right) + \frac{\beta^2}{4\alpha b} \right) > (6\alpha b^2 x_0 - 3\beta b)^2$$
$$\Leftrightarrow 12\alpha b^3 \left(A_2'' - 3\alpha b {x_0}^2 + 2\beta x_0 - \left(c_2 + \frac{h(1-r)}{r\mu_2} \right) - \alpha_s \left(m_1 - \frac{h(1-r)}{r\mu_1} \right) - \frac{\beta^2}{2\alpha b} \right) > 0$$

While $A_2'' > 3\alpha b{x_0}^2 - 2\beta x_0 + \left(c_2 + \frac{h(1-r)}{r\mu_2}\right) + \alpha_s\left(m_1 - \frac{h(1-r)}{r\mu_1}\right) + \frac{\beta^2}{2\alpha b}$, the above conclusion is true. Since we assume that A is sufficiently high, it considers that the condition is authenticated. Thus, we have $R_4 < 0$.

We conclude that under a condition such that $A_2'' > max\left\{\beta x_0 + \left(c_2 + \frac{h(1-r)}{r\mu_2}\right) + \alpha_s\left(m_1 - \frac{h(1-r)}{r\mu_1}\right) + \frac{\beta^2}{2\alpha b}, 3\alpha b{x_0}^2 - 2\beta x_0 + \left(c_2 + \frac{h(1-r)}{r\mu_2}\right) + \alpha_s\left(m_1 - \frac{h(1-r)}{r\mu_1}\right) + \frac{\beta^2}{2\alpha b}\right\}$, we have $R_3 > x_0$ and $R_4 < 0$, and thus, objective function is concave in feasible region ($[0, x_0]$).

APPENDIX D

Demand function's Positivity in Lemma 5.2

According to Lemma 5.2, there are two roots (R_5 and R_6) that the demand function is positive between these two roots and negative out of them.

$$R_5 = x_0 + \frac{-\theta_1 + \sqrt{\Delta_5}}{2\delta_1 b},$$
$$R_6 = x_0 - \frac{\theta_1 + \sqrt{\Delta_5}}{2\delta_1 b},$$

where $\Delta_5 = 4\delta_1 b\left(A_1 + \alpha_s p_2 + \beta_s x_2 - \delta_1\left(c_1 + \frac{h(1-r)}{r\mu_1}\right) - \theta_1 x_0 + \frac{\theta_1^2}{4\delta_1 b}\right)$.

For the first root, we will prove that $x_0 + \frac{-\theta_1 + \sqrt{\Delta_5}}{2\delta_1 b} > x_0$; for that let us prove:

$$-\theta_1 + \sqrt{\Delta_5} > 0 \Leftrightarrow \sqrt{\Delta_5} > \theta_1 \Leftrightarrow \Delta_5 > \theta_1^2$$
$$\Leftrightarrow \Delta_5 = 4\delta_1 b\left(A_1 + \alpha_s p_2 + \beta_s x_2 - \delta_1\left(c_1 + \frac{h(1-r)}{r\mu_1}\right) - \theta_1 x_0 + \frac{\theta_1^2}{4\delta_1 b}\right) > \theta_1^2$$
$$\Leftrightarrow 4\delta_1 b\left(A_1 + \alpha_s p_2 + \beta_s x_2 - \delta_1\left(c_1 + \frac{h(1-r)}{r\mu_1}\right) - \theta_1 x_0\right) > 0$$

While we assume that $A_1 > \delta_1\left(c_1 + \frac{h(1-r)}{r\mu_1}\right) + \theta_1 x_0$, the above condition ($R_5 > x_0$) is true.

The same approach will be used for the second root. We will prove that $R_6 < 0$ and we have:

$$x_0 - \frac{\theta_1 + \sqrt{\Delta_5}}{2\delta_1 b} < 0 \Leftrightarrow x_0 < \frac{\theta_1 + \sqrt{\Delta_5}}{2\delta_1 b} \Leftrightarrow \sqrt{\Delta_5} > 2\delta_1 b x_0 - \theta_1$$
$$\Leftrightarrow 4\delta_1 b\left(A_1 + \alpha_s p_2 + \beta_s x_2 - \delta_1\left(c_1 + \frac{h(1-r)}{r\mu_1}\right) - \theta_1 x_0 + \frac{\theta_1^2}{4\delta_1 b}\right) > (2\delta_1 b x_0 - \theta_1)^2$$
$$\Leftrightarrow 4\delta_1 b\left(A_1 + \alpha_s p_2 + \beta_s x_2 - \delta_1\left(c_1 + \frac{h(1-r)}{r\mu_1}\right) - \delta_1 b {x_0}^2\right) > 0$$

While $A_1 > \delta_1\left(c_1 + \frac{h(1-r)}{r\mu_1}\right) + \delta_1 b{x_0}^2$, the above conclusion is true. Since we assume that A is sufficiently high, it considers that the condition is authenticated. Thus, we have $R_6 < 0$.

We conclude that under a condition such that $A_1 > max\left\{\delta_1\left(c_1 + \frac{h(1-r)}{r\mu_1}\right) + \theta_1 x_0, \delta_1\left(c_1 + \frac{h(1-r)}{r\mu_1}\right) + \delta_1 b{x_0}^2\right\}$, we have $R_5 > x_0$ and $R_6 < 0$, and thus, demand function is positive in feasible region ($[0, x_0]$).

Optimization of Retailers' Strategies in Price- and Carbon Emission- Sensitive Market

Abstract – This work studies the retailer's profit maximization problem and investigates his/her optimal strategies in a price- and greenness- sensitive market. This work starts with a benchmark model where a retailer offers one kind of product to customers. The products are produced by a supplier and sent to the retailer. The retailer keeps the products in a warehouse near to the customers to serve them as soon as one arrives. The demand for the products is random and follows Poisson process. The customers' arrival mean rate is sensitive to retail price and carbon emissions level of the product. The refilling time of the retailer's warehouse is also random and follows Exponential distribution. The problem consists in deciding product's price, carbon emission level, and order size. We solve the problem by an analytical approach and provide the closed-form expressions of the optimal solutions.
In the next step, the benchmark model is extended in the way that retailer offers two products to customers. The products, which are supplied by different suppliers, are the same in terms of performance, function, and etc., thus, they are substitutable. The demand function in this case is also affected by the substitution's effect. In another words, the demand for each product not only depends on its price and carbon emission level, but also, depends on the other product's price and carbon emission level. Like benchmark model, the retailer's profit maximization problem is formulated in an stochastic environment under different settings (decision variables) and are solved by an analytical approach. According to the results, the market is distinguished in three categories: 1- Greenness-Driven Switchovers (GDS) market, 2- Price-Driven Switchovers (PDS) market, and 3- Neutral market. These different types of market make it possible to structure and analyze the obtained results.
A dynamic competition between two retailers, which each of them has its supplier, is considered. Retailers offer two substitutable products that each of them just offers one kind of product. There are two symmetric mathematical model that consist in deciding products' prices, carbon emission levels, and order sizes. Each retailer's decision affects the other retailer's decision. The general problem (price, greenness, and order size for each retailer as decision variables) is solved by an analytical approach and determined the Nash equilibrium. However, in practice, there are many situations where an existing retailer is already operating in the market, and a new retailer enters the market and offers a substitutable product. In this case, two situations are relevant to study are considered and solved: 1- Competition without reaction and 2- Competition with partial reaction. The close-form expressions of the optimal solutions are presented for all scenarios.
This work ends its studies by introducing a non-linear demand function. In the literature, all studies consider a linear demand function (to the best of our knowledge). However, our partners in project ANR CONCLuDE found out that the linear function is not sufficient enough. Thus, a new non-linear demand function is considered with respect to carbon emission improvement. Furthermore, our partners' studies also reveal that product's demand can not go beyond a given level as carbon emission improvement increases. More precisely, they express that improving greenness leads to increase the demand to a certain amount of market potential and after that it is constant. The benchmark model is re-formulated with different demand functions (non-linear, non-linear cap, and linear cap) and solved by an analytical approach. Then, close-form expressions of optimal solutions are presented. A numerical example is conducted to compare profits with different demand functions. The non-linear cap is considered as reference and compare to other, the results reveal that when maximum attracted costumers possible is low (below than 20%) the linear cap model performs better than others, but beyond that, the non-linear model performs better.

Keywords: supply chain management, retailers, endogenous demand, greenness, pricing, queuing theory, game theory

Optimisation des Stratégies des Détaillants sur Un Marché Sensible Au Prix et Aux Émissions de Carbone

Résumé – Ce travail étudie le problème de maximisation des profits d'un détaillant et examine ses stratégies optimales dans un marché sensible aux prix et à l'environnement (la greenness de produit). Ce travail commence par un modèle de référence où un détaillant propose un type de produit aux clients. Les produits sont fabriqués par un fournisseur et envoyés au détaillant. Le détaillant garde les produits dans un entrepôt à proximité des clients pour les servir dès leur arrivée. La demande pour les produits est aléatoire et suit un processus de Poisson. Le taux moyen d'arrivée des clients est sensible au prix de détail et au niveau des émissions de carbone du produit. Le temps de réapprovisionnement de l'entrepôt du détaillant est également aléatoire et suit la distribution exponentielle. Le problème consiste à décider du prix du produit, du niveau d'émission de carbone et de la taille de la commande. Nous résolvons le problème par une approche analytique et fournissons les expressions explicites des solutions optimales.
Dans l'étape suivante, le modèle de référence est étendu à un situation où le détaillant propose deux produits aux clients. Les produits, qui sont fournis par différents fournisseurs, sont les mêmes en termes de performances, de fonctions, etc., ils sont donc substituables. La fonction de demande dans ce cas est également affectée par l'effet de substitution. En d'autres termes, la demande pour chaque produit ne dépend pas seulement de son prix et de son niveau d'émission de carbone, mais aussi du prix de l'autre produit et de son niveau d'émission de carbone. Comme le modèle de référence, le problème de maximisation des profits du détaillant est formulé dans un environnement stochastique sous différents paramètres (variables de décision) et est résolu par une approche analytique. Les marchés se distinguent en trois catégories: 1- Greenness-Driven Switchovers (GDS) market, 2- Price-Driven Switchovers (PDS) market, et 3- Neutral market. Ces différentes types de marché permettent de structurer et analyser les résultats obtenu.
Une concurrence dynamique entre deux détaillants, dont chacun a son fournisseur, est ensuite envisagée. Les détaillants proposent deux produits substituables (chaque détaillant propose un type de produit). Le problème consiste à optimiser deux modèles mathématiques symétriques qui consistent à décider des prix des produits, des niveaux d'émission de carbone et des tailles de commande. La décision de chaque détaillant affecte la décision de l'autre détaillant. Le problème général (prix, greenness et taille de la commande pour chaque détaillant en tant que variables de décision) est résolu par une approche analytique en déterminant l'équilibre de Nash. Cependant, dans la pratique, il existe de nombreuses situations où un détaillant existant opère déjà sur le marché et un nouveau détaillant entre sur le marché et propose un produit substituable. Dans ce cas, deux situations pertinentes à étudier sont envisagées et résolues: 1- Compétition sans réaction et 2- Compétition avec réaction partielle. Les expressions des solutions optimales sont présentées pour tous les scénarios.
Ce travail se termine en introduisant une fonction de demande non linéaire. Dans la littérature, toutes les études considèrent une fonction de demande linéaire (au meilleur de nos connaissances). Cependant, nos partenaires du projet ANR CONCLuDE ont montré que la fonction linéaire n'est pas suffisante. Ainsi, une nouvelle fonction de demande non linéaire est considérée pour l'amélioration des émissions de carbone. De plus les études de nos partenaires révèlent également que la demande de produits ne peut pas dépasser un niveau donné à mesure que l'amélioration des émissions de carbone augmente. Plus précisément, l'amélioration de la greenness conduit à augmenter la demande jusqu'à un certain potentiel de marché et ensuite elle est constante. Le modèle de référence est reformulé avec trois différentes fonctions de demande (non linéaire bornée, non linéaire et linéaire bornée) et résolu par une approche analytique. Ensuite, des expressions des solutions optimales sont présentées. Un exemple numérique est effectué pour comparer les bénéfices avec différentes fonctions de demande. La demande non linéaire bornée est considérée comme une référence et est comparée aux autres, les résultats révèlent que lorsque la borne est relativement faible (moins de 20% pouvant être obtenue par l'amélioration des émissions), le modèle linéaire borné fonctionne mieux que les autres mais au delà, le modèle non linéaire fonctionne mieux.

Mots clés: gestion de la chaîne d'approvisionnement, détaillants, demande endogène, la greenness, tarification, théorie des files d'attente, théorie des jeux