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Multiple Criteria Spatial Risk Rating

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Contents

1 Résumé

1.1 Introduction

Aider à décider dans la plupart des cas nécessite des informations liées aux préférences des décideurs. Cependant, dans de nombreux problèmes d'aide à la décision, le client (l'entité nécessitant une aide à la décision) n'est pas un décideur, ou du moins, pas le seul décideur. Par exemple, le client peut être une entreprise chargée de fournir une expertise à un décideur final. De même, il est possible que plusieurs parties prenantes soient prises en considération. C'est le cas dans de nombreux problèmes liés à la prise de décision dans le secteur publique, et notamment dans l'évaluation des risques environnementaux. Par exemple, les citoyens n'interviennent pas directement dans le processus de prise de décision, mais ils seront touchés par le résultat de la décision. Par conséquent, il est important qu'une aide à la décision soit “ convaincante ” (cohérence et possibilité de justification du résultat), afin qu'elle puisse être rapportée au décideur final ou justifiée aux différentes parties prenantes.

Nos recherches s'appuient sur un vrai cas d'étude, concernant l'évaluation des risques environnementaux, plus précisément en cas d'accident nucléaire en milieu marin. Le terme “risque environnemental” couvre un large éventail de phénomènes liés à l'interaction entre les activités humaines et les espaces environnementaux. Le risque est généralement défini comme une association du hasard à la vulnérabilité. Par conséquent, d'une part, la prise en compte du hasard dans le processus d'aide à la décision, nécessite des outils de modélisation de l'incertitude. D'autre part, la prise en compte de la vulnérabilité nécessite l'évaluation de l'impact d'un accident nucléaire sur les différents enjeux caractérisant les différentes zones étudiées. Ainsi, l'évaluation de la vulnérabilité passe par une analyse spatiale et des outils d'aide à la décision multicritère afin d'évaluer l'impact d'un accident nucléaire sur les enjeux impliqués dans la zone étudiée. Dans notre étude de cas, le client est l'IRSN (Institut de Radioprotection et de Sûreté Nucléaire `https://www.irsn.fr/FR/Pages/Home.aspx`), une agence gouvernementale française chargée de fournir une expertise aux pouvoirs publics pour les problèmes liés au secteur nucléaire. Par conséquent, notre client n'est pas le décideur final.

Plusieurs problèmes d'aide à la décision sont caractérisés par la présence de plusieurs dimensions, des critères, sous lesquelles un problème peut être décrit, et donc une aide à la décision peut être construite. De tels problèmes sont mentionnés dans la littérature associée par des problèmes d'analyse de décision multicritères. Dans ce travail, nous sommes concernés par un type particulier de problèmes multicritères, dans lesquels nous partitionnons un ensemble d' **objets étudiés**, également appelés dans la littérature associée alternatives ou actions, en classes d'équivalence ordonnées et prédéfinies, appelées catégories. Ce type de problèmes est appelé “problèmes de notation” ou de classification ordinale (rating problem statement) [25]. Notre objectif dans ce travail est de fournir une évaluation complète et “convaincante” de ces problèmes. Par complet, nous entendons toute procédure de classification ordinale qui peut évaluer de manière cohérente l'ensemble des objets étudiés.

Ce chapitre est organisé comme suit. La seconde section est consacrée à présenter et décrire le cas d'étude motivant notre travail et les hypothèses de simplification. Dans la troisième section, nous présentons le projet de recherche.

1.2 Cas d'étude

L'idée de développer une méthode de classification ordinale convaincante, vient de l'étude de cas concernant l'évaluation des risques nucléaires post-accidentels, en particulier un cas d'évaluation des risques environnementaux. Les problèmes de risques environnementaux impliquent différents enjeux et différents types de parties prenantes, tels que les citoyens, les entreprises, les ONG (organisations non gouvernementales), les autorités publiques...etc. Dans le cadre de ce projet, notre est l'IRSN [1], une agence gouvernementale française en charge de l'expertise et de recherche en radioprotection et sûreté des installations nucléaires. Dans cette section, nous présenterons le problème de l'IRSN et l'objectif de l'aide à la décision demandée. Ensuite, nous présenterons les caractéristiques des problèmes et les hypothèses faite dans le cadre de ce travail.

1.2.1 Le cas de l'IRSN

Notre cas d'étude concerne l'évaluation du risque associé à un accident nucléaire majeur en milieu marin. Il est étudié à travers la simulation de plusieurs rejets de sous-marins nucléaire dans la baie de Toulon, où se trouve l'une des plus importantes bases de la marine française. En cas d'accident nucléaire, le préfet en exercice (l'autorité régionale) et les autres parties prenantes ont besoin d'informations synthétiques pour appuyer les décisions, telles que l'interdiction de certaines activités économiques, la mise en place d'une nouvelle politique de gestion de l'eau dans chaque zone concernée ou l'interdiction d'accès à des zones spécifiques. L'IRSN est en charge d'un projet de recherche visant à améliorer les modèles de prévision de la dispersion des substances

[1] Plus d'informations sur http://www.irsn.fr

radioactives et à évaluer leur impact sur l'environnement[2]. Avant cette étude, l'évaluation de l'impact d'un accident sur le milieu marin se basait sur la concentration du contaminant dans chaque unité géographique: Une zone fortement impactée est une zone à haut niveau de contamination. En se basant sur la concentration de certains radionucleides dans l'eau et dans les organismes marins, de nombreuses décisions ont été prises sur la base de certaines normes, par ex. les niveaux maximaux de radioactivité admissibles. Sur la base de ces normes, les décisions prises sont des mesures d'interdiction, telles que l'interdiction de certaines activités économiques. Cependant, la population peut réagir à une information sur la présence de faibles niveaux de concentration dans une zone.

Malgré la littérature existante visant à comprendre les processus régissant le devenir des radionucléides dans l'environnement, [5, 6, 7], nous notons que la concentration d'un radionucleide donné est une information nécessaire mais pas suffisante pour prendre des décisions éclairées. Prenons l'exemple de deux zones géographiques: la première est caractérisée par un niveau de concentration moyen et des enjeux économiques et environnementaux très importants tandis que la seconde est fortement contaminée mais ne présente aucune pertinence économique ou environnementale. De toute évidence, les parties prenantes impliquées seront différemment sensibles aux impacts dans les deux zones géographiques.

Plusieurs articles ont été proposés pour aborder le problème de l'évaluation du risque nucléaire, voir [9, 86, 99, 103, 107]. La plupart des travaux antérieurs reposent sur des analyses de recherche opérationnelle et très peu d'articles traitaient de l'évaluation du risque post-accidentel d'un point de vue analyse de décision à multicritères, voir [8, 77]. Notre rôle dans ce projet consiste à développer des outils d'aide à la décision afin de:

1. évaluer les impacts de différents scénarios de pollution marine acciden-

[2] https://www.irsn.fr/FR/Larecherche/Organisation/Programmes/Amorad/Pages/projet-Amorad.aspx#.XJeYzi17RQI

telle sur les différents actifs impliqués dans la baie de Toulon;

2. synthétiser ces impacts en structurant les indicateurs afin de fournir aux acteurs intéressés, une connaissance commune permettant de prendre des décisions éclairées.

1.2.2 Caractéristiques du Problème

La baie de Toulon est entourée de cinq communes: Toulon, La Seyne-Sur-Mer, Pradet, Saint-Mandrier et La Garde. La baie est également caractérisée par des activités maritimes telles que la pêche professionnelle, l'aquaculture et des activités touristiques telles que la natation et la plongée. De plus, c'est une zone naturelle importante présentant des intérêts faunistiques et floristiques en raison de la présence de l'herbier de Posidonie, qui est très importante pour la vie marine. Ainsi, notre étude de cas se caractérise par plusieurs acteurs représentant les pouvoirs publics présent dans les différents territoires, les citoyens et les chefs d'entreprises. De plus, l'étude de cas requiert différentes compétences en science de la décision afin de développer un modèle d'aide à la décision, principalement:

- L'analyse multicritère afin de synthétiser l'impact d'une concentration donnée sur les actifs impliqués dans la zone.
- Modélisation de l'incertitude afin de synthétiser les impacts liés aux différents scénarios d'accident. Les scénarios doivent être indépendants et représenter différentes configurations possibles de la contamination.
- Analyse spatiale pour évaluer l'impact d'une contamination sur la globalité de l'espace étudié. Les caractéristiques spatiales pourraient inclure la possibilité d'interaction entre des zones géographiques contiguës ainsi que la possibilité de la présence de plusieurs décompositions spatiales.

Le problème de l'IRSN est un problème de classification ordinale (notation du risque) [25]. il consiste à partitionner un ensemble d'objets étudiés en classes d'équivalence ordonnées prédéfinies, appelées catégories. Dans notre cas, les objets étudiés sont des unités géographiques. La notation de l'impact d'un scénario donné sur une unité géographique doit prendre en compte la manière dont les différents enjeux sont impactés à la fois au niveau de l'unité géographique et au niveau des unités voisines. Ainsi, les caractéristiques spatiales, les critères multiples et les sources d'incertitudes doivent être synthétisés.

1.2.3 Formulation du problème

L'IRSN intervient dans le nucléaire en tant qu'expert et analyste pour l'ASN [3] et les pouvoirs publics. Il les fournit par des jugements d'experts, des informations synthétisées et des arguments solides soutenant et justifiant une action à entreprendre. L'IRSN n'a donc pas la légitimité pour prendre des décisions. Dans une telle perspective, l'aide à la décision requise dans cette étude de cas, vise à fournir des outils complémentaires de gestion post-accidentelle, permettant d'évaluer les impacts environnementaux et économiques, en tenant compte des jugements d'experts. Ces impacts devraient être structurés sur différents indicateurs et synthétisés dans un seul indicateur représentant l'impact global. Le résultat de l'aide à la décision doit être justifié afin de convaincre le décideur final.

Les objets étudiés dans ce travail sont les unités géographiques de la zone étudiée. Ces unités sont évaluées selon plusieurs critères représentant les impacts sur les enjeux économiques et environnementaux caractérisant la baie. Ces impacts peuvent dépendre, d'une part, des paramètres d'accident, de scénarios d'accident, tels que la position de l'accident, l'intensité de l'accident, les courants marins et, d'autre part, de la façon dont les zones voisines sont affectées. Les impacts évalués sont représentés par des indices de risque. Par conséquent, nous avons affaire à un problème de notation du risque spatiale à

[3] Autorité de Sûreté Nucléaire (`https://www.asn.fr`)

critères multiples.

Cette étude de cas consiste à développer une aide à la décision pour:

- la modélisation des impacts des scénarios de pollution marine accidentelle;
- synthétiser les impacts sur les actifs considérés pour chaque zone;
- synthétiser les scénarios envisagés pour chaque zone;
- synthétiser les caractéristiques spatiales.

Le problème est complexe pour des raisons techniques et scientifiques. Les difficultés techniques du problème résident dans:

1. la façon dont la baie doit être décomposée et le nombre d'unités géographiques à considérer;
2. les scénarios d'accident à considérer parmi l'infinité de scénarios possibles;
3. la manière de prendre en compte les jugements et les croyances des experts dans notre analyse;
4. la façon dont nous pouvons recueillir ou estimer les données relatives à certaines activités telles que la natation, la plongée...etc.

Les difficultés scientifiques du problème peuvent être décomposées en:

- difficultés spatiales: En supposant que les zones géographiques sont homogènes et indépendantes, ce sont des hypothèses très fortes. Cependant, ils sont nécessaires pour évaluer les impacts.
- difficultés à critères multiples: Dans l'évaluation des risques, les impacts sont représentés par des indices (notes). Afin d'agréger les impacts d'un accident sur les différents enjeux, nous avons besoin d'une

méthode multicritère basée sur la règle de la majorité telle que les variantes d'ELECTRE-TRI ou sur des méthodes utilisant des règles de décision telles que DRSA. De telles méthodes nécessitent de connaître les préférences du décideur (par exemple à travers un ensemble d'apprentissage) et de faire l'hypothèse de l'indépendance et de l'homogénéité des zones géographiques.

- difficultés de la modélisation de l'incertitude: il est difficile d'évaluer la probabilité qu'un scénario de pollution marine donné se produise en cas d'accident. Cependant, ces informations sont très importantes au cas où nous souhaiterions évaluer l'impact attendu en tenant compte des différents scénarios envisagés.

- difficultés liées à l'ordre d'agrégation: il existe six chemins d'agrégation possibles:

  - Multicritère → scénarios → notation des cartes;
  - Multicritère → notation des cartes → scénarios;
  - Notation des cartes → Multicritère → scénarios;
  - Notation des cartes → scénarios → Multicritère;
  - Scénarios → Notation des cartes → Multicritère;
  - Scénarios → Multicritère → Notation des cartes;

  La difficulté réside ici dans la possibilité que le résultat dépende du chemin choisi.

1.2.4 Hypothèses

Plusieurs problèmes et questions ouvertes sont liés à notre cas d'étude. Afin de résoudre le problème, nous avons fait les hypothèses suivantes:

- Nous ne considérerons pas la santé publique comme un attribut. Cela est dû à la politique publique adoptée en France: dès qu'il y a un accident, basé sur la qualité de l'eau, certains plans d'urgence sont activés comme l'interdiction de certaines activités comme la baignade ou l'évacuation des personnes vivant dans une zone.

- Nous ne considérerons pas les problèmes sociaux, tels que la pauvreté, comme des attributs. La prise en compte de tels attributs sera redondante, car nous considérerons l'impact sur les différentes activités économiques qui auront à leur tour un impact social.

- Nous supposerons que les unités géographiques sont homogènes. L'utilisation de zones homogènes est particulièrement utile pour intégrer et utiliser des méthodes de surclassement dans les SIG, comme l'ont souligné plusieurs auteurs, dont [24, 61]. La raison en est que les méthodes de surclassement peuvent rencontrer des difficultés car elles ont de sérieuses limitations de calcul en ce qui concerne le nombre d'alternatives de décision, comme l'a remarqué Marinoni dans [73]. Outre les avantages mentionnés précédemment, le fait de supposer que les unités géographiques sont homogènes facilite l'ajustement des données collectées sur la décomposition spatiale.

- Nous supposerons que les unités géographiques contiguës sont indépendantes dans l'évaluation des impacts sur les enjeux caractérisant les unités géographiques. Après avoir évalué les impacts, nous pourrions considérer l'interaction entre les unités voisines si nécessaire, soit pendant l'agrégation multicritère (pour synthétiser les impacts sur les différents actifs dans chaque unité géographique) ou pendant l'agrégation spatiale (pour évaluer les cartes) comme dans [78]. Dans le cas où des unités géographiques ayant les mêmes impacts sont regroupées, il n'est pas nécessaire de faire cette hypothèse.

- À des fins de simulation, nous substituons les préférences du décideur aux préférences de l'expert.

1.3 Un processus d'aide à une décision convaincante

Plusieurs méthodes d'aide à la décision fonctionnent comme des boîtes noires où il est difficile de comprendre et d'appuyer une recommandation ou une aide à la décision. Dans de nombreux problèmes, un résultat non justifiable pourrait être acceptable. Cependant, dans le contexte de la prise de décision publique, telle que l'évaluation des risques, une aide à la décision devrait être convaincante et justifiable.

1.3.1 Caractéristiques des problèmes d'aide à la décision

Aider à décider ou à appuyer une décision, dans la plupart des cas, nécessite trois éléments principaux:

1. au moins deux entités doivent être impliquées dans les processus d'aide à la décision: la première entité est une personne ou un organisme demandant une aide à la décision. Dans ce travail, nous ferons référence à cette entité par **client**. La deuxième entité est une personne ou un organisme à qui le client demande conseil. Dans notre contexte, ces conseils interviennent lors de la formulation et de la résolution du problème d'aide à la décision. Nous ferons référence tout au long de ce manuscrit à cette entité par **analyste de décision**;

2. un contexte (caractéristiques du problème) qui comprend: une connaissance de l'objectif de l'aide à la décision; les parties prenantes impliquées ou concernées par le résultat de la décision; la légitimité des parties

prenantes; les caractéristiques des objets étudiés; et les caractéristiques sous lesquelles les objets étudiés sont évalués;

3. différents types d'informations liées au contexte ou aux parties prenantes: aider à la décision nécessite une connaissance, d'une part, des informations techniques liées au contexte, telles que les différents types d'incertitudes entourant l'environnement du problème, ou les interactions possibles entre les objets étudiés; et d'autre part des préférences de certaines parties prenantes et leur légitimité pour influencer la décision finale.

Ainsi, un modèle d'aide à la décision est construit en fonction des caractéristiques du problème et des préférences des décideurs. Il consiste à agréger les différentes informations de ces deux paramètres afin de dériver une recommandation. Le choix des méthodes et des outils d'agrégation dépend du type d'aide à la décision demandée et de l'analyse du problème sur la base de critère(s) unique ou multiples; un unique ou multiple décideur(s), un unique ou multiples scénario(s).

1.3.2 Problèmes pratiques

Dans plusieurs problèmes, aider à décider peut être difficile. C'est le cas lorsque:

1. certaines parties prenantes ne sont pas directement impliquées dans le processus de décision et devraient être prises en compte dans l'analyse, comme les citoyens dans les problèmes de prise de décision de politique publique;

2. plusieurs parties prenantes sont impliquées dans le processus de prise de décision avec différents degrés d'influence;

3. le client n'est pas un décideur.

En ce qui concerne le point 3., le lecteur doit noter que toute méthode d'aide à la décision nécessite les préférences des décideurs afin d'élaborer une solution. Il existe différentes techniques pour obtenir ces préférences au cas ou notre client est l'unique décideur: soit par le biais d'un processus interactif [50, 58], soit en apprenant des décisions passées [56, 72]. Cependant, dans le cas où le client n'est pas un décideur ou n'est pas le seul décideur, l'apprentissage de préférences ainsi que la validation du modèle d'aide à la décision deviennent plus complexes. Dans le cas où les préférences d'un expert sont utilisées pour remplacer les préférences des décideurs, une solution convaincante (stable sous critique) devrait être dérivée afin d'être communiquée aux différentes parties prenantes.

1.3.3 Projet de recherche

Afin de résoudre les problèmes mentionnés ci-dessus, nous nous sommes concentrés sur le développement d'une méthode d'aide à la décision multicritère convaincante pour les problèmes dans laquelle certains critères sont évalués sur une échelle ordinale. Le choix de travailler sur des problèmes muticritères de classification ordinale (notation) est dû au problème de l'étude de cas. Ce dernier sera présenté dans la section suivante.

Dans le contexte de l'agrégation multicritère, où certains critères sont évalués sur des échelles ordinales, nous utilisons souvent des méthodes basées sur la règle d'agrégation majoritaire. De telles méthodes peuvent conduire à différents problèmes d'intransitivité [15] liés au théorème d'impossibilité d'Arrow [65]. Cela rend l'apprentissage des préférences des décideurs, lorsque le client n'est pas un des décideurs, à partir de décisions historiques insignifiant, car nous ne pouvons pas valider le modèle d'aide à la décision et nous pouvons avoir des cycles de préférences. Ainsi, nous appelons une classification ordinale ou une notation "convaincante":

- une classification monotone: **pas de meilleur objet assigné à une**

catégorie pire. Un tel résultat n'est pas atteint pour les méthodes utilisant la règle majoritaire, car des cycles de préférences traversant différentes catégories peuvent se produire;

- une évaluation efficace: une évaluation complète où tous les objets sont évalués dans un temps polynomial, sans être forcés d'être affectés à la même catégorie.

- une classification justifiable: la capacité de l'analyste à justifier la classification.

L'avantage d'utiliser une méthode de notation multicritère convaincante réside dans la flexibilité du résultat à communiqués et justifiés aux différentes parties prenantes, et ceci quelle que soit leurs légitimités et leurs qualités d'être client ou non. En outre, tirer des enseignements des décisions passées est logique en raison de la caractéristiques susmentionnées d'une classification ordinale "convaincante".

Nous devons mentionner que malgré l'importance des sujets de recherche suivants, ces derniers sont hors de la portée de ce travail:

- La méthodologie de prise de décision dans le contexte des problèmes de la pollution environnementale. Nous nous concentrerons sur les processus d'aide à la décision et en particulier sur la mise à la disposition de notre client une **description** (notation) convaincante des impacts induits par un accident nucléaire sans s'aventurer sur le processus de prise de décision. Des références liées aux processus de prise de décision peuvent être trouvées dans [4, 95, 97].

- La prise de décision par un groupe, plus de références liées à ce problème dans notre contexte peuvent être trouvées dans [12, 13, 41, 63, 74].

- Protocoles d'élicitation de préférences [50, 56, 58, 72].

1.4 Conclusion

Il n'est pas rare dans les problèmes environnementaux de faire face à des situations où le client a soit besoin d'avoir des arguments justifiant une aide à la décision soit n'a pas un pouvoir décisif. Notre objectif dans cette thèse est de résoudre ces problèmes dans le cas particulier des problèmes de classification ordinales (problèmes de notation) en analyse multicritère, où certains critères sont évalués sur des échelles ordinales (problèmes dans lesquels on utilise des approches de surclassement). Ainsi, l'objectif de cette recherche est de développer une méthode de multicritère de classification ordinale fournissant un résultat "convainquant". Un tel objectif consiste à résoudre plusieurs problèmes théoriques en relation avec le paradoxe de Condorcet, la question de l'incomparabilité, la séparation entre profils limites et centraux dans les approches existantes et l'interdiction des comparaisons entre objets.

Ce sujet de recherche est motivé par un cas d'étude réel, où notre client (l'IRSN) demande des outils d'aide à la décision permettant d'évaluer les risques. L'évaluation du risque consiste à noter les unités géographiques en fonction de leurs impacts sur les différents enjeux présent dans chaque unité. Les objets étudiés, dans notre travail, sont les unités géographiques. Les enjeux sont considérés comme des critères permettant d'évaluer les unités géographiques. Par conséquent, nous avons affaire à un problème de classification multicritère spatiale. En plus, plusieurs scénarios d'accident doivent être envisagés avec des probabilités différentes. Ainsi, nous avons affaire à un problème de notation du risque multicritère spatiale.

2 Introduction

Environmental risk problems, are in general characterized by the presence of different dimensions from which the problem might be apprehended. This thesis is motivated by an interesting case study related to environmental risk assessment. The case study problem consists in assessing the impact of a nuclear accident in the marine environment. The assessment of the marine pollution is widely studied in the literature. Most of these studies are focused on simulating the physical dispersion process of pollutants. Such information is not sufficient to take informed decisions. This is because different impacts might be assessed based on the intensity of the accident, the sea currents and the vulnerability of the assets characterizing the geographic area. As far as we are concerned, we aim at studying the way risk associated to the marine pollution can be assessed in order to provide the interested stakeholders by a common knowledge allowing to take informed decisions.

The problem we are dealing with is characterized by spatial characteristics, different assets characterizing the spatial area, incomplete knowledge about the possible stakeholders, and a high number of possible accident scenarios. Our focus in this work will be on:

- assessing the impact of a radioactive substance concentration on the assets in each geographic unit (small zones);
- aggregating the different impacts, for each accident scenario, on assets

in order to rate the geographic units vulnerability;

- aggregating the different accident scenarios in order to assess the risk of a nuclear accident in the studied area.

To do so, decision science propose numerous tools allowing to deal with this type of problems. A first solution of the case study problem was proposed where different decision analysis techniques were used such as lotteries comparison, and MCDA, Multiple Criteria Decision Analysis, tools. Different theoretical problems raised from this case study, such as:

1. should we aggregate scenarios before criteria, or should we proceed in the reverse order?

2. should we consider the interaction between neighboring geographic units?

3. In case our client is not the final decision maker, how should we provide a "convincing"[1] solution?

In this work, we decided to develop the third point. For the first point the aggregation order is very important and represent an interesting research question. However, in my opinion, the question depends on the client needs. A way to tackle this problem is to define with a set of examples where the client is able to rate the risk, and to chose the aggregation order with the highest accuracy. Considering interaction between neighboring units is also an interesting question, which was studied in the literature and solved using Choquet integrals and by considering geographic units as criteria. Also, in problems of the type we are dealing with, it is very frequent to have geographic units with the same impact. On the one hand, this can be due to the physical continuity. On the other hand it is very likely that the assets involved in neighboring units have the same importance.

[1] A solution that can be reported and justified by our client to his client

The method used to solve the MCDA problem associated to the case study is the rating method ELECTRE-Tri, because the criteria are assessed on ordinal scales. ELECTRE-Tri, as most of the majority principle[2] based methods, is subject to Condorcet Paradox[3]. The idea behind the ELECTRE TRI method is to not compare the studied objects to each other, but to rate them based only on their relative position with typical profiles characterizing the ratings. However, on the one hand, a decision analyst has no right to forbid his client from questioning the relative position of two rated studied objects, and on the other hand, it is not acceptable that an object x worse than y is rated better than y. Even if we assume that the parameters used to rate the studied objects might be different from the ones used to compare objects, our literature review revealed that there is no rating procedure aiding to rate some objects better than others which are worse when directly compared. For these reasons, we decided to undergo the construction of a new MCDA method. The aim of the developed rating method is to provide a rating that we are completely able to justify: no better object is assigned to a worse category. This method provided interesting results to the case study, and very interesting theoretical properties that will be detailed later in this manuscript.

The manuscript is organized, in five chapters, as follow:

1. Chapter 1. Problem setting: In this chapter, we present the general context related to both the practical and the theoretical problems characterizing this thesis.

2. Chapter 2. State of art: In this chapter, we present the main theoretical tools in decision science aiming to deal with multicriteria decision support systems, risk analysis and spatial decision support.

3. Chapter 3. Case study: In this chapter, we model the client's problem. This consists on modeling impacts induced by nuclear accident scenarios

[2]Making an action corresponding to a sufficient majority of criteria
[3]Cycles of preferences

on the involved assets in the studied area. Then we solve the multiple criteria problem. Several conclusions are drown, mainly concerning incoherences over the rating.

4. Chapter 4. Dynamic-R and its variants: In this chapter, we present the new MCDA rating method. The proposed method aims at providing a justifiable rating, which is also consistent.

5. Chapter 5. Experimental study. The first part of this chapter, is dedicated to present algorithms related to the main concepts introduced in Dynamic-R. We then apply the developed method to an imaginary example with randomly generated data, where we experiment different intuitions that drove the development of the method. We end the chapter by an application on the case study where the resulting rating is justified.

3 Problem Setting

3.1 Introduction

Aiding to decide or supporting a decision in most of the cases requires information related to decision-makers preferences. However, in many decision aiding problems, the client (the entity requiring a decision aiding) is not a decision maker, or at least, not the only decision maker. For instance, the client can be a company in charge of providing expertise to a final decision maker. Likewise, it is possible that several stakeholders have to be taken into consideration. It is the case in many problems related to public decision making, and particularly in environmental risk assessment. For instance, citizens do not intervene directly in the decision making process, however, they will be impacted by the outcome of the decision; hence, it is important for a decision aiding to be "convincing" (stable under criticism), so that it can be reported to the final decision maker or justified to the different stakeholders.

Our research is driven by a real case study, concerning environmental risk assessment, specifically, a case of a nuclear accident in a marine environment. The term "environmental risk" covers a wide range of phenomena related to the interaction between human activities and environmental areas. Risk is generally defined as an association of the hazard to the vulnerability. Hence, taking into account the hazard in the decision aiding process requires uncertainty modeling tools while taking into account the vulnerability requires assessing the impact of a given nuclear accident on different assets present in different geographic areas. Thus, assessing the vulnerability involves spatial analysis and MCDA tools in order to evaluate the impact of a nuclear accident over the assets involved in the studied area. In our case study, the client is the IRSN (Institut de Radioprotection et de Sûreté Nucléaire `https://www.irsn.fr/FR/Pages/Home.aspx`), a French government agency in charge of providing expertise to public authorities for problems in relation with the nuclear sector. Hence, our client is not the final decision maker.

Several decision aiding problems are characterized by the presence of several dimensions, named criteria, under which a problem can be described, and hence a decision aiding can be built. Such problems are referred to in the associate literature by Multiple Criteria Decision Analysis (MCDA) problems. In this work, we are concerned by a particular type of MCDA problems, in which we partition a set of **studied objects**, also called in the associate literature alternatives or actions, into predefined ordered equivalence classes, called categories. Such problems are called rating problem statements [25]. Our aim in this work is to provide a complete and "convincing" rating for such problems. By complete, we mean any rating procedure which can rate consistently the whole set of objects given. The term convincing will be discussed in section 3.3.

This chapter is organized as follow. The second section is dedicated to present and describe the case study motivating our work and the hypothesis made for simplification. In the third section, we present the research project.

3.2 Case Study

The idea of developing a convincing MCDA rating method, comes from a case study concerning post-accident nuclear risk assessment, specifically a case of environmental risk assessment. Environmental risk problems involve different assets, and different types of stakeholders, such as citizens, companies, NGO (Non-Governmental Organizations), public authorities, to name but a few. Our client in this work is the IRSN[1], a French government agency of expertise and research in radiation protection and safety of nuclear installations. In this section, we will present the IRSN's problem and the objective of the decision aiding required. Then, we will present the characteristics of the problems and the hypothesis made in this work.

3.2.1 IRSN's Problem

Our case study concerns the management of a major nuclear accident in a marine environment and is studied through the simulation of releases from a nuclear submarine at the bay of Toulon, where one of the most important bases of the French Navy is located. In case of a nuclear accident, the incumbent Prefect (the regional authority), and other stakeholders, need synthetic information to support decisions, such as banning certain economic activities, setting a new water management policy at each relevant zone or impeding the access to specific areas. The IRSN is in charge of a research project aiming at improving models predicting the radioactive substances dispersion and assessing their impact on the environment[2]. The impact assessment of an accident on the marine environment, before this work, was based on the concentration of different isotopes in each geographic unit: A highly impacted area is an area with a high contamination level. Based on the concentration in water and in the marine organisms, many decisions were undertaken based on norms, e.g. the Maximum Eligible Levels. Based on these norms, the undertaken decisions are prohibition measures, such as banning some economic activities. However, the population may react to an information about the presence of a low concentration levels in an area.

[1] More information can be found at http://www.irsn.fr

[2] `https://www.irsn.fr/FR/Larecherche/Organisation/Programmes/Amorad/Pages/projet-Amorad.aspx#.XJeYzi17RQI`

Despite the existing literature aiming at understanding the processes governing the fate of radionuclides in the environment, [5, 6, 7], we note that the concentration of a given isotope is a necessary but not sufficient information for making informed decisions. Let us consider the example of two geographic zones: the first one is characterised by an average concentration level and very important economic and environmental assets while the second one is highly contaminated but does not present any economic or environmental relevance. Clearly, the involved stakeholders will be differently sensitive to the impacts in both geographic plots.

Several papers have been proposed to address the problem of nuclear risk assessment, see [9, 86, 99, 103, 107]. Most of previous works rest upon operational research analyses and very few papers addressed the post-accident risk assessment from a multiple criteria decision analysis point of view, see [8, 77]. Our role in this project consists on developing decision aiding tools in order to:

1. assess the impacts of different accidental marine pollution scenarios on the different assets involved in the Bay of Toulon;
2. synthesize these impacts by structuring indicators in order to provide the interested stakeholders, by a common knowledge allowing to take informed decisions.

3.2.2 Problem's Characteristics

The bay of Toulon is surrounded by five municipalities: Toulon, La Seyne-Sur-Mer, Pradet, Saint-Mandrier, and La Garde. The bay is also characterized by maritime activities such as professional fishers, fish-framing, and tourism activities such as swimming and diving. Moreover, it is an important natural area of ecological faunistic and floristic interest due to the presence of seagrass Posidonia, which is very important for sea-life. Hence, our case study is characterized by several stakeholders representing the public authority in the different territories, the citizens and the heads of the companies. In addition, the case study requires different competencies in decision science in order to develop a decision aiding model, mainly:

- MCDA in order to synthesize the impact of a given concentration on the assets involved on the area.
- Uncertainty modeling in order to synthesize the impacts related to different accident scenarios. The scenarios should be independent and represent different possible configurations of contamination.
- Spatial analysis in order to rate the impact of a concentration over a map. The spatial characteristics might include the possibility of interaction between contiguous geographic zones as well as the possibility of the presence of several spatial decompositions.

The IRSN's problem is a rating problem [25]. It consists on partitioning a set of studied objects into predefined ordered equivalence classes, called categories. In our case, the studied objects are geographic units. Rating the impact of a given scenario on a geographic unit has to take into account the way the different assets are impacted in both the geographic unit and in neighboring units. Thus, spatial characteristics, multiple criteria, and uncertainties have to be synthesized in order to provide the IRSN by a common knowledge.

3.2.3 Problem Formulation

The IRSN acts in the nuclear sector as an expert and analyst for the ASN[3] and the public authorities. It provides them by expert judgments, synthesized information and strong arguments supporting and justifying an action to take. Thus, IRSN does not have the legitimacy to take decisions. Under such a perspective, the decision aiding required in this case study, aims at providing supplementary post-accident management tools allowing to evaluate

[3]Autorité de Sûreté Nucléaire (`https://www.asn.fr`)

environment and economic impacts, taking into account expert's judgments. These impacts should be structured on different indicators, and synthesized in a single indicator representing the global impact. The outcome of the decision aiding should be justified in order to convince the final decision maker.

The studied objects in this work are the geographic units in the studied area. These units are assessed under several criteria representing the impacts on economic and environment assets characterizing the Bay. These impacts might depend, on the one hand, upon accident parameters, named accident scenarios, such as the accident position, the intensity of the accident, the sea currents, and on the other hand, upon the way neighboring zones are impacted. The assessed impacts are represented by risk rates. Hence, we are dealing with a spatial multiple criteria rating risk problem.
This case study consists on developing decision aiding for:

- modeling the impacts of accidental marine pollution scenarios;
- synthesizing the impacts over the considered assets for each zone;
- synthesizing the considered scenarios for each zone;
- synthesizing the spatial characteristics.

The problem is complex for technical and scientific reasons. The technical difficulties of the problem lie on

1. the way the Bay should be decomposed and on the number of geographic units;
2. the accident scenarios that should be considered, since in reality, there exist infinite possible scenarios;
3. the way to take into account experts judgments and beliefs in our analysis;

4. the way we can collect or estimate data related to some activities such as swimming, diving, to name but a few.

The scientific difficulties of the problem can be decomposed on:

- spatial difficulties: Assuming that the geographic zones are homogeneous and that they are independent, are very strong assumptions. However, they are necessary in order to assess the impacts.

- multiple criteria difficulties: In risk assessment, the impacts are represented by rates. In order to aggregate the impacts of an accident on different assets, we need an MCDA method based on either the majority rule such as the variants of ELECTRE-TRI or methods using decision rules such as DRSA. Such methods require knowing the decision maker's preferences, for instance through a learning set, and the independence and the homogeneity of the geographic zones.

- uncertainty modeling difficulties: It is difficult to assess the likelihood of a given marine pollution scenario occurring in case of an accident. However, such information is very important in case we want to assess the expected impact taking into account the different considered scenarios.

- order of aggregation difficulties: there are six possible aggregation paths:

  - Multiple criteria → scenarios → maps rating;
  - Multiple criteria → maps rating → scenarios;
  - Maps rating → multiple criteria → scenarios;
  - Maps rating → scenarios → multiple criteria;
  - Scenarios → maps rating → multiple criteria;
  - Scenarios → multiple criteria → maps rating;

  The difficulty here lies on the possibility that the result depends on the chosen path.

3.2.4 Hypothesis

Several problems and open questions are related to our case study. In order to solve the problem, we made the following hypothesis:

- We will not consider the public health as an attribute. This is due to the public policy adopted in France: as soon as there is an accident, based on water quality, some emergency plans are activated such as forbidding some activities such as swimming or evacuating people living in some area.

- We will not consider as attributes social issues such as poverty. Considering such attributes will be redundant, since we will consider the impact upon different economic activities which in their turn will have a social impact.

- We will assume that the geographic units are homogeneous. The use of homogeneous zones is particularly useful for integrating and using outranking methods in GIS, as underlined by several authors, including [24, 61]. The reason is that outranking methods may run into difficulties since they have serious computational limitations with respect to the number of decision alternatives, as remarked by Marinoni in [73]. Assuming that the geographic units are homogeneous facilitates fitting the collected data on the decomposition of the spatial area.

- We will assume that the contiguous geographic units are independent in the assessment of the impacts on the assets present in the geographic units. After assessing the impacts, we might consider the interaction between neighboring units if it is needed either during the MCDA aggregation (to synthesise the impacts on the different assets in each geographic unit) or during the spatial aggregation (to rate the maps) such as in [78]. In case geographic units having the same impacts are grouped, there is no need to revise such hypothesis.

- For simulation purposes, we substitute the decision maker's preferences by the expert's preferences.

3.3 Convincing Decision Aiding Process

Several decision aiding methods work like black-boxes where it is difficult to understand and support a recommendation or a decision aiding. In many problems, a non-justifiable result could be acceptable. However, the decision aiding in the context of public decision making, such as risk assessment, should be convincing and justifiable.

3.3.1 Characteristics of Decision Aiding Problems

Aiding to decide or supporting a decision in most of the cases requires three main components:

1. at least two entities must be involved in the decision aiding processes: the first entity is a person or an organism requesting a decision aiding. In this work, we will refer to this entity by **client**. The second entity is a person or an organism to whom the client demands an advice. In our setting this advice occurs under formulating and solving the decision aiding problem. We will refer all along this manuscript to this entity by **decision analyst**;

2. a context (problem's characteristics) which includes: a knowledge about the objective of the decision aiding; the stakeholders involved or concerned by the outcome of the decision; the legitimacy of the stakeholders; characteristics of the studied objects; and the features under which the studied objects are assessed;

3. different types of information either related to the context or to the stakeholders: aiding to decide requires knowing on the one hand tech-

nical information related to the context, such as the different types of uncertainties surrounding the environment of the problem, or the possible interactions between the studied objects; and on the other hand some stakeholder's preferences and their legitimacy to influence the final decision.

Thus, a decision aiding model is built based on the problem's characteristics and the decision makers preferences. It consists on aggregating the different information from these two parameters in order to derive a recommendation. The choice of the aggregation methods and tools depends on the type of the decision aiding required and on whether the problem should be analysed based on single or multiple criteria; single or multiple decision makers, single or multiple scenarios.

3.3.2 Practical Problems

In several problems, aiding to decide can be hard. It is the case when:

1. some stakeholders are not directly involved in the decision process and should be taken into account in the analysis such as citizens in public policy decision making problems;

2. multiple stakeholders are involved in the decision making process with different degrees of influence;

3. the client is not a decision maker.

With respect to point 3. the reader should note that any decision aiding method requires preferences in order to elaborate a solution. There exist different techniques to elicit preferences in case the client is a decision maker: either through an interactive process [50, 58], or through learning from past decisions [56, 72]. However, in case the client is not a decision maker or is not the only decision maker, learning preferences as well as validating the decision aiding model becomes more complex. In case an expert's preferences are used in order to substitute the decision makers preferences, a convincing (stable under criticism) solution should be derived in order to be communicated to the different stakeholders.

3.3.3 Research Project

In order to tackle the above mentioned problems, we focused on developing a convincing multiple criteria decision aiding method for problems in which some criteria are assessed on ordinal scales. The choice of working on multiple criteria rating problems is due to the case study problem. This last will be presented in the next section.

In the context of MCDA, where some criteria are assessed on ordinal scales, we often use methods based on the majority principle. Such methods may lead to different problems [15] related to Condoret paradox (as a special case of the Arrow's impossibility theorem [65]). This makes learning the decision makers preferences, when the client is not a decision maker, from historical decisions meaningless, as we cannot validate the decision aiding model and we might have cycles of preferences. Thus, we refer to a "convincing" rating as:

- a monotonic rating: **no better object assigned to a worse category**. Such result is not fulfilled for methods using the majority rule, since cycles of preferences crossing different categories might occur.
- an efficient rating: a complete rating where all objects are rated in a polynomial time, without being forced to be assigned to the same category.
- a justifiable rating: the ability of the decision analyst to justify the rating.

The advantage of using a convincing MCDA rating method, lies on the flexibility of the result to be communicated and justified to the different stakeholders,

regardless of their legitimacy and their quality of being clients or not. Moreover, learning from past decisions makes sense because of the above mentioned characteristics of a “convincing” rating.
We have to mention that the following research topics are out of the scope of this work:

- Decision making methodology in environmental pollution. We will focus on decision aiding processes and particularly on providing a client by a convincing description (rating) of the impacts induced by a nuclear accident without making any recommendation. More references can be found in [4, 95, 97].

- Group decision making. More references related to this problem in our context can be found in [12, 13, 41, 63, 74].

- Protocols for preferences elicitation [50, 56, 58, 72].

3.4 Conclusion

It is not seldom in environmental problems to face situations where the client has no decisive power, or s.he requires a justifiable decision aiding. Solving these issues is our goal in this thesis, in the particular case of MCDA rating problems where some criteria are assessed on ordinal scales (problems to which outranking methods fit). Thus, the objective of this research is to develop an MCDA rating method providing a “convinicing” rating. Such aim consists on solving several theoretical problems in relation with the Condorcet Paradox, the incomparability issue, the separation between limiting and central profiles in the existing approaches, the prohibition of comparisons among objects.

This research topic is motivated by a real case study, where our client (The IRSN) asks for decision aiding tools allowing risk assessment. Assessing risk comes to rating the geographic units based on their impacts regarding the different assets in each unit. The studied objects, in our work, are the geographic units. The assets are considered as criteria allowing to evaluate the geographic units. Likewise, several scenarios of accident should be considered with different likelihoods. Thus, we are dealing with a multiple criteria spatial risk rating problem.

4 State of art

4.1 Introduction

For centuries voices have been raised advising us to take time to reflect, to calculate, to anticipate before reaching a decision and acting on it. (B. Roy[92]).

In the previous chapter, we presented different problems associated to my research topic and to the case study. We also mentioned different hypothesis that will be considered as our starting point, and different topics that will be out of the scope of this work but that might be seen as perspectives. Our research project involves different theoretical and technical problems. The theoretical problems, in our concern, consist on developing a "convincing" MCDA rating method for problems requiring the use of the majority rule. The term "convincing", in our context, refers to a complete and consistent rating such that **"no better object assigned to a lower category"**. Such results are achieved, through a new method that will be presented later in this manuscript. The different preference modeling and multicriteria aggregation techniques, used in the developed method, will be presented in this chapter. The technical problems are related to the case study. As mentioned in the previous chapter, our client is not a decision maker and the decision maker is not clearly identified, leading to several questions about the way we might take into account the decision maker's preferences over the different impacted assets. The case study, at the origin of this work, is in a crossroad between MCDA modeling, risk assessment, spatial analysis, leading to a complex situation. The methods and the recent developments that will be required to address these technical problems will be also presented in this chapter.

The chapter is organized as follows: the first sections we will be dedicated into introducing the main decision aiding concepts and the preference modeling tools. Then, we will present different decision aiding approaches for problems involving multiple criteria (MCDA problems). We end by a litterature review about spatial decision support systems and risk analysis.

4.2 Main Definitions

Before any further developments, I will introduce the definitions of *"decision aiding process"*, *"client"*, *"decision maker"*, and *"decision analyst"*.

Definition 1. *(Decision aiding process)*

According to Tsoukiàs [100], a decision aiding process is an interactive process between an entity having a decision problem, and an entity in charge of building a consensual representation of the problem.

Definition 2. *(Client)*

Client is the intervening entity in the decision process, on whose account the decision aiding is exercised. Such entity might be a person or an organism.

Definition 3. *(Decision maker)*

A decision maker is the entity having the power and the responsibility of taking a decision.

Definition 4. *(Decision Analyst)*

Decision analyst is the entity in charge of analysing the problem in order to support the client in the decision problem. For simplicity we will use the "analyst" instead of "decision analyst".

To synthesize the above mentioned definitions, decision aiding can be seen as the activity of the analyst: providing answers to the client's problem based on, explicit but not necessarily formalized, models. The client is not necessarily a decision maker, he can be also an analyst for another entity. For example, in case of a nuclear accident, the prefect might ask the IRSN for expertise. This last might ask a decision analyst to provide a formalism, allowing the aggregation of the way different assets are impacted in each zone.

4.3 Decision Aiding Process

In this section, I will start by a joke that my PhD supervisor[1], told me when I chose to work on multiple criteria decision aiding science.

A tailor (T) conceived a bent suit to one of his clients (C).

- *C: Sir the suit is bent.*
- *T: No sir, you have just to bend your shoulders to fit on it.*

When people saw the client C leaving the T shop, they said:" what a wonderful tailor, he succeeded on conceiving an adjusted suit to a bent person."

This story illustrates the possibility to solve correctly a wrong problem when we force a decision to fit on a predefined model. The origin of the problem might come from the nature of the decision aiding activity. In fact, decision aiding is not limited to well structured problems, and can constitute an interesting subject of scientific investigation. To provide the client by elements allowing him to take an informed decision, the analyst and the client are committed in an interactive process aiming at defining the client's problem, its characteristics and the objective of the requested decision aiding (see Bouyssou et al in [19]).

[1] Eric Chojnacki

Identifying and defining decision aiding problems depend on four main features:

- the identification of the context for which the decision aiding is required. This axis concerns the possible outcomes and consequences of decisions. For instance, in environmental risk problems, the decision aiding required in situations where human lives are involved is different from the decision aiding required in situations where only the economic stakes characterize the decision problem.

- the identification of the client's level of implication in the decision process. Is the client the final decision maker? Does the client has the legitimacy to take decisions or is he an analyst in his turn? In case the client has the legitimacy to take decisions, the analyst needs to know the level of the client's intervention.

- the identification of the quality of information and knowledge that the client has about the problem. Such axis can be seen as a part of an exogenous source of uncertainty. This might be related to either the non ability of the client to express clearly his problem or to some uncertain characteristics of the decision problem, due to:

  - uncertainty about the actual or the future "state of the world". To illustrate, we can imagine a prefect wanting to develop an emergency plan in case of a nuclear accident. However, the day, the position, the sea currents, the intensity, and the probability of the accident to take place in his region is unknown;

  - due to the complexity of the problem, it might be difficult for the client to describe clearly the decision problem.

- the problem statement [25]. Depending on the client's needs we have to identify the type of decision aiding corresponding to his problem. In environmental risk several types of decision aiding might be required. For instance, the decision aiding required to choose a good public policy corresponds to a ranking problem. However, in case the client needs to identify the vulnerability of the territories, the decision aiding needed might be a rating.

The conduction of decision aiding processes has not been frequently studied in the literature. One of the main contributors are Roy in [92] and Tsoukiàs in [100]. According to Tsoukiàs [100], the conduction of a decision aiding process consists on an interactive process between the analyst and the client following four cognitive steps:

- **Representation of the problem situation:** To establish a representation of the problem, the analyst should start by listing the actors involved in the decision aiding process. This step allows the client to situate himself in the decision process and the analyst to structure the problem by answering the following questions:

  - the objectives of the decision aiding;
  - the reason for which the client considers this situation as a problem;
  - the identification of the situation before the formalization;
  - the client's social position, his legitimacy, and the level of his intervention in the decision aiding process;
  - the characteristics of the problem situation and the parameters that should be included in the analysis according to the client.

  Formally, the representation of the problem situation is a triplet:

  $$\mathscr{P} =< \mathscr{A}, \mathscr{O}, \mathscr{S} >$$

  where:

  - $\mathscr{A}$ is the set of stakeholders involved in the decision process;
  - $\mathscr{O}$ is the set of objects (or stakes) involved the decision process;
  - $\mathscr{S}$ is the set of resources.

- **Problem formulation:** It may exist different formulations for the same representation of the problem situation. The objective is to model the rationality and structure the problem. To transform the client's problem into a formal problem, the analyst needs to find answers to the following questions:

  - What are all possible objects of our investigation?
  - Which attributes should we consider to evaluate the problem?
  - What kind of problems should we solve?

  Formally, a problem formulation is expressed by a triplet:

$$\Gamma =< \mathbb{A}, V, \Pi >$$

  where:

  - $\mathbb{A}$ refers to the set of all potential studied objects related to the problem situation. Such set might be modified;
  - V represents the set of points of views describing and evaluating the set of potential actions;
  - Π refers to the problem statement. It can be seen as the type of the decision aiding requested by the client.

  Remark 1. *The decision problem can be analyzed according to several points of views, representing the axes under which we observe, analyze, describe, evaluate and compare the elements of* $\mathbb{A}$.

  According to Colorni and Tsoukias [25], there exist four types of problem statements: rating, ranking, assigning, and clustering. The four types *of problem statements consist on partitioning the studied objects into equivalence classes depending on whether or not the equivalence classes are predefined or ordered. Such formalism will be detailed in Section 4.5.1.*

- **The evaluation model:** The aim of this step, is to apply decision analysis tools, by the analyst, in order to synthesize the generated information and to build a model allowing the decision maker to take informed decisions.

  Formally, the evaluation model can be represented by the n-uplet:

  $$\mathscr{M} = < OS, \{D, \mathscr{E}\}, \mathscr{F}, \mathscr{U}, \mathscr{R} >$$

  where:

  - OS is the final set of the studied objects;
  - D is a non-empty set of attributes describing elements of OS, it is the set of functions mapping each element of OS to a co-domain, called a scale;
  - $\mathscr{E}$ is the set of co-domains associated to D;
  - $\mathscr{F}$ is the set of criteria evaluating the elements of OS. A criterion is a dimension to which we can associate a preference model. This set should fulfill some few conditions:
    * Separability of criteria: The capability of each criterion, taken separately, to discriminate the alternatives.
    * Consistent family of criteria: The exact strict necessary set of criteria allowing to evaluate OS.
  - $\mathscr{U}$ is a set of uncertainties and or imprecision of information. We can distinguish two types of uncertainties:

    * Exogenous: It represents different possible scenarios and states of nature which affect the performances of elements in OS. It includes the imprecision or the missing information during the interaction process between the decision analyst and the client.
    * Endogenous: It represents uncertainties coming from parameters used in the decision aiding models or the missing information during the aggregation process.
  - $\mathscr{R}$ represents a set of operators to synthesize the information expressed over OS through the different dimensions D. The choice of the method depends on the characteristics of the problem statement and the client's preferences and should satisfy two criteria:
    * Theoretical meaningfulness: The aggregation operator should be conform to properties characterizing the problem.
    * Operational meaningfulness: The result of the method should correspond to the client's needs.

- **Final recommendation:** The result of the evaluation model may be seen as a first solution to the client's problem. It is possible that this solution does not satisfy the client or it is not consistent or less robust in case we consider multiple scenarios. Thus a final recommendation should be valid in case it is:
  - not sensitive to non-significant variation of parameters;
  - robust regarding the possible scenarios;
  - Legitimate regarding the client's position. Is our client legitimate to take such decision or not?

The legitimacy and the validity, was integrated in the evaluation model [75] in order to study biases in the decision making and decision aiding processes.

In my opinion, as defined in [100], the third dimension of the problem situation (the set of resources $\mathscr{S}$) should be revised. A decision aiding problem, in Corlorni and Tsoukias [25], is described by three dimensions: the dimension of opinions (the stakeholders in $\mathscr{A}$); the dimension of values (the attributes in $\mathscr{O}$); and the dimension of likelihoods instead of $\mathscr{S}$. I would substitute such dimension by the problem's environment, which might include likelihoods, resources, and spatial characteristics or any variable characterizing the context of the client's problem. For instance, in case of environmental risk problems, it is possible to have one or multiple involved stakeholders, one or multiple involved attributes, and one or multiple scenarios of consequences or causes related to the same problem.

4.4 Preference Modeling

Facing the same decision problem, different clients might have different behaviors with respect to their preference. Hence, modeling the client's preferences is one of the main steps in the majority of decision aiding processes. In this section, we present different properties and characterizations of binary relations allowing to compare objects of OS.

4.4.1 Perspectives of Preference Modeling

Bell in [10] states that preference modeling in decision science can be studied from three perspectives:

- Normative perspective: Models in normative approaches are based on the economical rationality, the rational behavior, where models are based on some coherence axioms independently from the client.

- Descriptive perspective: Models in these approaches are based on observations, where the objective is to describe a phenomena.

- Prescriptive perspective: In this approach we do not assume neither the pre-existence of any rational hypothesis or axiom nor the existence of a learning data or similar information. The decision analyst has to collect these information to build a preference model that is able to lead to an adequate recommendation. The given recommendation should be validated by the client.

Many environmental risk problems can be apprehended by one of these three perspectives. Our case study corresponds to descriptive perspective as we aim to rate the environmental risk, based on several attributes, in order to describe the consequences induced by a marine pollution.

4.4.2 Binary relations

Definition 5. *(binary relation)*

The developed rating method is based on the manipulation of binary relations. A binary relation R, defined between a set X and a set Y, is a subset of the Cartesian product $(X \times Y) \cup (Y \times X)$, i.e. a set of pairs (x, y) or (y, x), with $x \in X$ and $y \in Y$.

Obviously, we admit the case where $X = Y$, the binary relation being a subset of X^2.

Remark 2. *By convention, we will denote xRy any couple $(x, y) \in R$. In the opposite case, $(x, y) \notin R$, will be referred to by $\neg(xRy)$.*
The sets X or Y might be sets of sets.

Definition 6. *(Classical operations)*

Binary relations are sets, thus we can apply on them the classical operations of set theory. For given $x \in X$, $y \in Y$ we consider the following definitions:

- *Inclusion:* $R_1 \subseteq R_2 \implies [xR_1y \Rightarrow xR_2y]$;

- *Union:* $x(R_1 \cup R_2)y \iff xR_1y$ *or* xR_2y;
- *Intersection:* $x(R_1 \cap R_2)y \iff xR_1y$ *and* xR_2y;
- *Inverse relation* (R^-): $xR^-y \iff yRx$;
- *Complement relation* (R^c): $xR^cy \iff \neg(xRy)$;
- *Dual relation* (R^d): $xR^dy \iff \neg(yRx)$;
- *Symmetric part of a relation R* (I_R): $xI_Ry \iff xRy$ *and* yRx;
- *Asymmetric part of a relation R* (P_R): $xP_Ry \iff xRy$ *and* $\neg(yRx)$;
- *Product* ($R_1.R_2$): $xR_1.R_2y \iff \exists z \in OS : xR_1z$ *and* zR_2y.

Each binary relation has some characteristics and properties. Based on the previous definition, we shall mention:

Definition 7. *For all* $x \in X$, $y \in Y$, *a binary relation* R *on* OS *is said to be*

- *Reflexive if* xRx;
- *Irreflexive if* $\neg(xRx)$;
- *Symmetric if* $xRy \implies yRx$;
- *Antisymmetric if* xRy *and* $yRx \implies x = y$;
- *Asymmetric if* $xRy \implies \neg(yRx)$;
- *Weakly complete if* $x \neq y \implies xRy$ *or* yRx;
- *Complete if* xRy *or* yRx;
- *Transitive if* xRy *and* $yRz \implies xRz$;

4.4.3 Preference structures

In decision aiding processes, the evaluation model is based on establishing preference structures over the studied objects or dimensions characterizing the problem situation.

A client aiming to compare two objects x and y in OS, might be either indifferent or prefers x over y, in this case a suitable semantic formulation might be "x is at least as good as y" or may prefer strictly y over x. A third possible answer might be "I do not know" either because of a missing data, imprecision over the values of x or y, or because of some ambiguity around the client's preferences.

A preference structure can be seen as a binary relation S over the set of studied objects OS where: xSy **iff** the response of the answer "x is at least as good as y" is positive. Bouyssou et al in [16] defined a preference structure in OS as *"a binary reflexive relation S"* on OS.

Let us consider the preference structure S over the set OS such that: for a given x, y in OS, one of the following situations may occur:

- xSy and ySx. This situation can be interpreted as "x is at least as good as y" and "y is at least as good as x". Hence, a corresponding formulation might be "x is indifferent to y", noted xI_Sy.

- xSy and $\neg(ySx)$. In this case "x is at least as good as y" but this last is not as good as x. Thus, "x is strictly preferred to y", noted xP_Sy.

- $\neg(xSy)$ and ySx. As for the last case "y is strictly preferred to x", and it is noted as yP_Sx or xP_S^-y.

- $\neg(xSy)$ and $\neg(ySx)$. In this situation, the preference structure over the set $\{x,y\}$ does not allow the comparability. We say that "x is incomparable to y. Such relation is noted J_S.

Properties 1. *(Characteristics of I_S, J_S and P_S)*

- *I_S is reflexive, symmetric and transitive;*
- *J_S is irreflexive, symmetric;*
- *P_S is irreflexive, asymmetric and transitive;*
- *P_S, I_S and J_S are mutually exclusive: $P_S \cap I_S = I_S \cap J_S = J_S \cap P_S = \emptyset$;*
- *P_S, P_S^-, I_S and J_S are exhaustive: $P_S \cup I_S \cup J_S \cup P_S^- = OS^2$.*

In case we consider S being a complete relation we obtain a preference structure, noted in literature as $< P_S, I_S >$ structures. In case S also satisfies the transitivity property, we have a representation theorem (see Roberts [89]) such that $\exists u : OS \to \mathbb{R}$:

$$\forall x, y \in OS : \begin{cases} xP_Sy \iff u(x) > u(y) \\ xI_Sy \iff u(x) = u(y) \end{cases}$$

Such structures might be obtained by defining utilities using conjoint measurement tools introduced by Fred Roberts [89]. In this case, S is called a weak order.

Definition 8. *(Weak order or partial order)*

Let S be a binary relation associated to the preference structure $< P_S, I_S >$. The following propositions are equivalent:

1. *S is a weak order;*
2. *S is a reflexive, complete and transitive;*
3. $\begin{cases} I_S \text{ is transitive} \\ P_S \text{ is transitive} \\ P_S \cup I_S \text{ is reflexive and complete} \end{cases}$;

Weak order structure is also called total pre-order or complete pre-order.

Theorem 1. *(numerical representation of weak order)*

A preference structure S over a finite set OS is a weak order structure iff there exist a mapping $u : OS \rightarrow \mathbb{R}$, such that: $\forall x, y \in OS$

$$xSy \iff u(x) \geq u(y)$$

In case incomparability is not empty (S being not complete), we can still have a preference structure when S is transitive, called partial pre-order.

Definition 9. *(Partial pre-order)*

Let $S = P_S \cup I_S \cup J_S$ be a binary relation over OS associated to the structure $< P_S, I_S, J_S >$, the following propositions are equivalent:

1. *S is a partial preorder;*
2. *S is a reflexive, and transitive;*
3. $$\begin{cases} P_S \text{ is irreflexive, asymmetric and transitive} \\ I_S \text{ is reflexive, symmetric and transitive} \\ J_S \text{ is irreflexive, and symmetric} \\ P_S.I_S \cup I_S.P_S \subset P_S \end{cases}.$$

Theorem 2. *(numerical representation of partial pre-order)*

A preference structure S over finite set OS is a partial pre-order structure iff there exist a mapping $u : OS \rightarrow \mathbb{R}$, such that: $\forall x, y \in OS$

$$xSy \implies u(x) \geq u(y)$$

Remark 3. *The numerical representation and the partial pre-order are not homomorphic.*

4.5 Multiple criteria decision aiding

Multiple criteria decision aiding can be viewed as the field of decision aiding where multiple dimensions need to be explicitly considered. This branch of decision science allows to take into account different perspectives from which the decision aiding can be apprehended.

Unlike mono-criterion approaches, multiple criteria decision aiding gives up to the notion of optimum and the uniqueness of the solution. This is in order to find a good compromised solution with respect to the multiple and conflicting facets of real world decision problems. As claimed by Roy and Bouyssou [93], this approach is justified by the following considerations:

- the decision can be seen as a compromised solution between several conflicting objectives.
- the conflicting nature of criteria contains meaningful information serving as a basis of justification, transformation and expression of preferences;
- the different criteria allow to manage, in each axis of information, uncertainties and imprecision related to the decision problem.

4.5.1 Main Multiple Criteria Problem Statements

After defining the problem situation, and before any further developments concerning the establishment of an evaluation model, we need to formulate a decision aiding problem. Formally a decision aiding problem consists in partitioning the set of studied objects. For instance, a university may need to rank the candidates in order to select and recruit new students. This procedure can be seen as a partition of students in two categories: Admitted applications and rejected ones.

According to Bouyssou et al [19], the concept of deciding is associated to the choice problem. This last is a particular case of the ranking problem in the formalism of Tsoukias [100]. The formalism proposed by Tsoukias consists on four problem statements:

- the problem statement of clustering: consisting in partitioning the set of alternatives into unordered not pre-defined equivalence classes (clusters). Clustering problems are very frequent when the client aims to behave differently on each group of elements having close characteristics.
- the problem statement of assignment: consisting on partitioning the set of alternatives into unordered predefined equivalence classes. This kind of problems can be encountered, for instance, when we aim to assign a person to the most appropriate coalition.
- the problem statement of rating: consisting on partitioning the set of alternatives into ordered predefined equivalence classes). Such kind of problems is very frequent in rating the vulnerability of a spatial area, the impact of a nuclear accident or in rating a financial product, to name but a few.
- the problem statement of ranking: consisting on partitioning the set of alternatives into ordered not predefined equivalence classes. Very frequent to establish an order structure among a set of elements such as ranking students.

The choice of the problem statement depends on the characteristics of the client's problem. In most of the cases, the client understands the differences between problem statements. In case it is not easy for the client to express what type of outcome he needs, an interaction process is required between the decision analyst and the client aiming at presenting different possible types of outcomes. Generally, an inappropriate problem statement will immediately generate information the client will realize being useless. The problem statement, in this case, will be defined through feedbacks.

4.6 Formulation and the main multiple criteria rating methods

An MCDA problem is characterized by a set of criteria $\mathscr{F} = \{g_1, ..., g_m\}$, with $g_j : OS \rightarrow \mathbb{R}$, $\forall j \in \{1, ..., m\}$. In MCDA, the set of objects OS should verify the preference independence condition [64, 70, 104]. The preference independence condition means that the comparison of two objects is independent from their common evaluations: Let $G = \{1, ..., n\}$ be the set of indices of criteria, for x, y, z, $v \in OS$

$$(x_K, z_{-K})S(y_K, z_{-K}) \iff (x_K, v_{-K})S(y_K, v_{-K})$$

where K denote a subset of G, and (x_K, y_{-K}) denotes the performance vector associated to an object e in OS such that $e_i = x_i \forall i \in K$ and $e_i = y_i \forall i \in G \backslash K$.

Rating problems consist on partitioning a set of studied objects into predefined ordered categories. The categories are identified by rates. Let's call $\mathscr{C} = \{\mathscr{C}_1, ..., \mathscr{C}_q\}$, $q \geq 2$, the set of the considered categories, with $\mathscr{C}_i$ refers to the equivalence class of objects rated i. Without loss of generality, we assume that categories are ranked as follow: $C_h \gg C_{h+1}$ $\forall h \in \{1, ..., q-1\}$ where $\gg$ refers to a total order. Thus, $\mathscr{C}_1$ is the best category.

Several methods have been developed to solve different real world MCDA rating problems [20, 53, 76, 96, 105]. Rating problems are also referred to as sorting problems in the associate literature. MCDA rating problems are solved either based on the majority rule, called *outranking* methods, see [2, 3, 44, 72, 93, 101, 106]; or using utility functions, see [21, 30, 31, 57, 69, 59, 111], or using decision rules (such as DRSA or Decision Tree Classifier), see [29, 54, 55, 56, 94]. Most of supervised learning techniques might also be used for rating purposes.

4.6.1 Utility based methods

Utility functions are widely used in decision science, particularly in MCDA problems. Considering the set A on which a weak order holds: $\succcurlyeq_i \subseteq A^2$. For each such a weak order we get: $\exists u_i : A \longrightarrow \mathbb{R}$. Under appropriate conditions [64] for which the value functions u_i allow to measure differences of preferences, an additive numerical representation can be used, of the type:

$$U : \left| \begin{array}{lcl} A & \longrightarrow & \mathbb{R} \\ x & \longmapsto & \sum_{i=1}^{m} u_i(x) \end{array} \right.$$

where $u_j(x)$ refers to the marginal utility function of criterion g_j. This last transforms the scale of criterion g_j into utility terms. Marginal functions should be monotone (either increasing or decreasing functions). The shape of the marginal utility function depends on the client's preferences (judgment policy). Its form can be calibrated through either a direct protocol or indirect protocol as in UTA method[2] [60].. A detailed presentation of the assessment procedure of the marginal utility function with illustrations can be found in [18].

The transformation of criteria functions into a global utility using marginal utilities offers two major advantages:

1. It allows the model to take into account the non-linearity of the client's behavior;

2. It presents a flexibility when criteria are expressed on a qualitative scale.

Such methods are suitable in multiple criteria problems where trade-offs among criteria are possible and meaningful. The global utility of an object x constitutes its overall performance measure when all criteria are considered. Hence, it can also be used for rating purposes by using thresholds defining

[2] UTilités Additives the French translation of additive utility

lower bounds of each category. This can be formulated as:

$$\begin{aligned}
&U(x) \geq u_1 && \Longrightarrow\ x \in \mathscr{C}_1 \\
&u_1 > U(x) \geq u_2 && \Longrightarrow\ x \in \mathscr{C}_2 \\
&\dots\dots\dots\dots\dots\dots && \\
&u_{q-1} > U(x) && \Longrightarrow\ x \in \mathscr{C}_q
\end{aligned}$$

Such approach is used in the variant of UTA method, named UTADis method [36, 59, 111]. UTADis is based on an aggregation disaggregation procedure: disaggregation of examples of rated objects in order to calibrate the utility function, so the no rated objects can be assigned to the corresponding categories.

An other utility based method, very close to UTADis and named M.H.Dis, is proposed by Zopounidis and Doumpos [112]. It consists on a hierarchical rating of the objects by assessing $2(q-1)$ additive utility functions instead of a single one in UTADis. The first two utility functions that are assessed are $U_1(g)$ describing the objects of category $\mathscr{C}_1$ and $U_{\sim 1}(g)$ describing the objects of categories $\mathscr{C}_2, ..., \mathscr{C}_q$. When objects that should be assigned to $\mathscr{C}_1$ are distinguished from the others, we proceed on discriminating objects in $\mathscr{C}_2$ from objects in $\mathscr{C}_3, ..., \mathscr{C}_q$ by assessing $U_2(g)$ and $U_{\sim 2}(g)$. We then reiterate this procedure until we discriminate objects in $\mathscr{C}_{q-1}$ from objects in $\mathscr{C}_q$ by assessing $U_{q-1}(g)$ and $U_{\sim q-1}(g) = U_q(g)$.

4.6.2 Outranking based methods

The name of Outranking methods comes from Outranking binary relations. Outranking binary relations are built based on two concepts:

- the majority rule: sufficient majority of criteria supporting the statement. Such concept is called the concordance principle.

- the respect of the minority rule: no significant minority of criteria rejecting the statement. Such concept is named the discordance principle.

Thus the outranking relation can be defined as follow:

Definition 10. *(Outranking relation)*

Outranking relation S_λ is a binary relation defined on OS^2. x outranks y can be interpreted as "x is at least as good as y". S_λ can be formulated as:

$$xS_\lambda y \iff w(\{j \in \mathcal{F} : x \succeq_j y\}) \geq \lambda \wedge \neg(\mathcal{V}(\{j \in \mathcal{F} : y \succeq_j x\}) \geq v) \quad (4.1)$$

where $\mathcal{V}$ the importance of the discordant criteria to reject a preference relation, defined as a capacity $\mathcal{V} : 2^{\mathcal{F}} \to [0,1]$, v the veto threshold, w is the importance of coalitions of criteria defined as a capacity $w : 2^{\mathcal{F}} \to [0,1]$, and λ is a majority threshold.

Three binary relations results from outranking relation. These binary relations are used to conclude different degrees of preferences:

- the strict preference P_λ:

$$xP_\lambda y \iff xS_\lambda y \wedge \neg(yS_\lambda x) \quad (4.2)$$

- the indifference I_λ:

$$xI_\lambda y \iff xS_\lambda y \wedge yS_\lambda x \quad (4.3)$$

- the incomparability J_λ:

$$xJ_\lambda y \iff \neg(xS_\lambda y \vee yS_\lambda x) \quad (4.4)$$

The ELECTRE-TRI family [48, 49, 93] is an outranking based procedure used for rating purposes. It is based on pairwise comparisons, using outranking relations, between objects and reference profiles characterizing the categories. Most of the developed outranking based rating methods are variants of ELECTRE TRI. One of the major differences among the ELECTRE

TRI variants is the way categories are characterized. The original version of ELECTRE TRI, renamed ELECTRE TRI-B, is introduced in the thesis of Yu [93, 106]. Categories, in ELECTRE TRI-B, are bounded by single reference profiles. These reference profiles are called limiting profiles and they are related by the dominance relation. The method is characterized by two different exploitation procedures:

- Pessimistic (pseudo-conjunctive) procedure. It consists on the pairwise comparison between each object and limiting profils starting from the worst to the best category. We stop this procedure when an object outranks a limiting profile.

- Optimistic (pseudo-disjunctive) procedure. The comparison between objects and limiting profiles starts from the best to the worst category. The procedure stops when a limiting profile outranks the object we aim to rate.

An other variant of ELECTRE TRI method, named ELECTRE TRI-C [2], was developed by Almeida-Dias. In ELECTRE TRI-C, categories are characterized by central profiles. Unlike ELECTRE TRI-B, the method uses simultaneously an ascending and descending exploitation procedures.

These two variants of ELECTRE TRI were studied by Bouyssou and Marchant in [17]. The object of this study was to verify whether both methods may lead to the same rating with suitable changes over parameters. The study is motivated by the intuition that we might find between each successive two limiting profiles, a central profile, for which ELECTRE TRI-C lead to the same result obtained by ELECTRE TRI-B and vice-versa. The answer to this intuition is no: they are two different methods, as proved in [17].

Characterizing a category by a single profile is a hard task. During the last decade, different variants of ELECTRE TRI family were developed to characterize categories by a set of profiles instead of a single one, mainly:

- ELECTRE TRI-nB [44] in which each category is characterized by a set of limiting profiles;
- ELECTRE TRI-nC [3] in which each category is characterized by a set of central profiles.

Unlike machine learning techniques dedicated for rating problems, MCDA methods require a knowledge about the client's preferences. The direct elicitation of the client's preferences [50] is a difficult task. In fact, it is more complicated than an answer to the question: "what are the decisive coalitions of criteria". A decisive coalition of criteria, in the context of ELECTRE TRI family methods, depends on the criteria importance which also depend on other parameters such as the number of categories and the definition of profiles used for discrimination. Hence, several ordinal regression and machine learning techniques were developed in order to infer ELECTRE TRI parameters from assignment examples. Most of these method avoid eliciting the veto threshold. Learning all ELECTRE TRI parameters requires a non-linear and non-convex model [32, 34, 33, 35, 80, 81, 98, 110], making resolving large size real world problems computationally impossible [35]. During the last decade, very interesting works were published concerning the elicitation of the whole parameters used in Outranking approaches based on evolutionary optimisation [26, 27, 46, 47]. Learning these parameters is out of scope for this work.

The ELECTRE TRI family is characterized by the following properties, called structural requirements:

- Uniqueness: Each object is assigned to a unique category;
- Independence: The assignment of an object does not depend on the assignment of the other objects;
- Conformity:
  - if an object x outranks a limiting profile characterizing a given rating k and strictly less preferred than a limiting profile characterizing

a better rating j, then x will be rated between j and k;

  - each limiting profile characterizing a category j, should be rated j;

- Monotonicity: If an object x dominates an object y, and if y is rated k, then x should be at least rated k;

- Homogeneity: If two objects compare the same way to the limiting profiles, they must be assigned to the same category;

- Stability: When applying either the merging or the splitting operation, the objects previously assigned to the non-modified categories will remain in the same categories. After merging two consecutive categories the objects belonging previously to those categories will be assigned to the new one. After splitting a category, objects belonging to the old category will be assigned to one of new categories.

Despite these interesting properties, outranking relations do not have any remarkable ordering properties [15]. For this reason, most of outranking methods concerned by rating purposes do not take into consideration the way objects compare to each other, making the result potentially "non-convincing" for the client or in the context of automatic decision making. This will be detailed and illustrated in the chapter dedicated to the presentation of the developed MCDA rating method.

For this reason, several rating methods have been developed aiming at rating a set of objects, taking into account the way objects compare to each other, with respect to a consistency rule. For example, C. Rocha and L.C. Dias in [90] developed the PASA (Progressive Assisted Sorting Algorithm) algorithm, respecting the following consistency principle: an object cannot be assigned to a category in case it is outranked by any example (reference profile) assigned to a worse category. This principle seems very close to our work since we also characterize the categories by a set of reference profiles and we have a consistency rule. However, this method presents also many disadvantages such as:

- the order of the selected objects for rating might bias the ratings of the next selected objects;
- in case of an imprecise rating, either the decision maker is needed or the rating is postponed;
- forcing the consistency might lead to bad quality of rating: objects involved in cycles are placed in the same category (the worse category among the ones to which objects can be assigned).

The THESEUS method [45] is another rating method, aiming at providing a rating, minimizing inconsistencies with respect to a learning set (reference profiles in our case). This method is based on an original approach, transforming a rating problem into a ranking problem. Such transformation consists on associating to each non rated object x, new alternatives x_k: "assign x to the category k". The generated alternatives x_k are assessed under the following criteria: inconsistencies with respect to the strict preference, the weak preference, and the indifference. Hence, the problem of rating x, comes to a ranking problem associated to selecting the best x_k, minimizing the inconsistencies. We note the following weaknesses of THESEUS method:

- The provided rating minimizes inconsistencies. However, it does not always provide a convincing rating;
- The dependency on the learning set: both small and very big learning sets may lead to a poor rating either because of incomparabilities or the high number of inconsistencies.

4.6.3 Decision rule based methods

Using utility based or outranking based methods, in order to solve MCDA rating problems, requires a strong assumption that a relation or a function is able to represent the complexity of the client's preferences. Methods based on decision rules are function-free. They are represented by symbolic forms of the type "if ... then ..." decision rules or decision trees. Perhaps the most widely used MCDA rating methods based on the majority rule is DRSA (Dominance-based Rough-Set Approach).

DRSA [29, 54, 55, 56, 94] is based on rough set theory [84, 85]. The main novelty in DRSA is replacing the indescernibility relation between objects, used in the rough set theory, by the dominance relation. This is because, MCDA rating problems we are dealing with, are ordinal classification problems and objects are assessed, under each criterion, on ordinal scales. The steps in DRSA can be described as follow:

- Defining lower and upper approximation sets of the objects "at least" and "at most" belonging to each category using the dominance relation upon a learning set. The lower approximation is the set of objects that we are sure they belong to a class. The upper approximation consists on the set of objects that possibly can belong to a class. The lower approximation is included in the upper approximation set. These approximations represent the client's preference model;

- Assessing the boundary set which can be seen as a set of confusion. The boundary is the difference between the upper and the lower approximations for each class;

- Deriving two types of decision rules:

  - certain rules: they are derived from lower approximations.

  - approximate rules: they are derived from boundaries.

  Each "if ... then ..." decision rule is composed by:

  - a condition part. It consists on a partial profile upon a subset of criteria.

  - a decision part. It consists on a suggestion of the possible ratings of an object. The possible rating are described by “at least” and “at most” ratings.

- Validation based on the quality of the approximation. The quality of the approximation is based on the cardinality of the lower approximation.

There are many advantages of using methods based on decision rules. For instance, the most widely mentioned qualities are their nature of being easy for interpretation, and their flexibility in case of inconsistent learning set.

4.7 Risk in the context of decision science

Studying risk comes to adopting an approach oriented by two questions:

- How to identify unexpected potential damages?

- What measures to take in order to be protected against them?

Answering the first question might be called risk assessment and to the second question risk management. Our concern in this work is related to risk assessment. But how can we define risk?

4.7.1 Evolution of risk’s definition in time

From a decision science point of view, risk was first defined by the economist Frank Knight [68] in 1921 from an economic perspective. He proposed to name risk, a measurable negative event under an objective probability. The definition consists on the context in which a rational person has to choose the best from different alternatives, with future consequences. To choose the best alternative, we should be able to assess their expected values. The use of objective probability in risk assessment was invalidated as it differs from a person to an other depending on the individual perception. In 1944, Von Neumann and Oscar Morgenstern (VN-M) [102], were interested to the concept of lotteries, in order to take into account the human subjectivity regarding risk, with a single decision criterion consisting on maximizing an expected utility. Maximizing the expected utility as a decision criterion was questioned by Allais [1] in his Allais Paradox criticizing the foundations of VN-M axioms. In case it is not possible to estimate future values of the alternatives we are dealing with an uncertain context. With the industrial and technological advances, after 1960, the definition of risk evolved to cover, in addition to the hazard, the intensity of a bad event, as stated by Kamper [62].

In the literature we find both “natural risks” and “industrial risks”. Fortunately both concepts share the same definition with different terminologies. Natural risk is defined as a combination of the hazard and the vulnerability of the exposed stakes. Industrial risk is defined as a combination of the occurrence probability and the severity of its consequences. In our work, we will use the risk under the following definition:

$$Risk = Vulnerability \times Hazard$$

4.7.2 Uncertainty modelling

Uncertainty modelling tools are very important in risk assessment. According to Klir and Smith [67] an operational methodology to handle uncertainty must:

- propose a mathematical representation;
- develop a calculation process to propagate uncertainty;
- measure uncertainty explicitly;
- propose a practical methodology for implementing the concepts.

Several theoretical frameworks are developed to represent uncertain and imprecise information, frequently characterizing decision under risk. Probability theory is perhaps the most widely used. For long time, the approach used in the context of risk assessment to manipulate and characterize uncertainty, are mainly part of the probabilistic framework with an evolution from a frequentist to a Bayesian approach [40, 79].

A probability space is defined by a triple $(\Omega, \mathscr{A}, P)$, with Ω the set of all possible outcomes, $\mathscr{A}$ a set of events where each event is a set containing zero or more outcomes, and P an application, named probability defined as $P : \mathscr{A} \to [0, 1]$. P is a probability measure if it verifies the following three axioms, called Kolmogorov axioms:

- For any event A in $\mathscr{A}$: $P(\overline{A}) = 1 - P(A)$;
- The probability of the certain event Ω is 1: $P(\Omega) = 1$;
- The probability is σ-additive: the probability of any countable sequence of disjoint sets $E_1, E_2, \ldots$ satisfies: $P(\cup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} P(E_i)$.

Two approaches are distinguished by practitioners of probability theory: Frequentist and Bayesian approaches [51]. The first approach is based on the frequency of the achievements of an event. The second approach is based on updating a probability prior given by the client (probably the decision maker).

Even if it is widely used in order to take decisions in risk context, probability theory is not always adapted to represent a subjective view of information, and thus its use can be criticized [14] in the context of environment risk assessment. This is because, of two issues characterizing many environmental risk problems:

- the hazard assessment is based on the expert judgment, which is imperfect and subjective;
- the frequentist approach is adapted to non-reproducible events [82], which is the case of nuclear risk assessment, as a particular case of environmental risk.

For these reasons, De Finetti introduced the concept of subjective probability [28], by quantifying the qualitative and subjective evaluation of uncertainty of the type an event A is more (respectively less or equally) likely than an other event B. Limits of probability theory are listed by Dubois and Prade [38]. Hence, in many risk assessment studies, probability theory was substituted by possibility measures [37, 108, 109], in order to deal with the ambiguity as a form of uncertainty. Other approaches were also developed to model uncertainty, such as the mathematical theory of the evidence, called also Dempster-Shafer (DST) theory. The different models of uncertainty used in risk are out for the scope of this work.

4.7.3 Vulnerability assessment based on Expert judgment

In risk assessment, the vulnerability is defined as the intensity of impact in case an uncertain bad event occurs. Assessing the vulnerability is a very hard task as it is subjective and differs from a person to another. For this reason, in environmental risk problems such as nuclear accidents, experts judgment is used in order to estimate the way the different stakes involved in the studied area might be impacted.

The elicitation process of an expert judgment might be subject to preference modeling tools. One way of learning the expert judgments is through an interactive process where the analyst provide the expert with different examples of events and extract decision rules by using, for example, rough set theory. This way of proceeding will not be developed in this section as the concept of Rough Set was introduced in section 4.6.3. Another way, consists on assessing a utility function under the hypothesis that this last takes into account its "evaluation[3]" model. Calibrating such utility might be done using

[3] I used here the word evaluation as I think that the word preference in risk assessment area is not adapted

the probability equivalent method [43]. The method is based on assessing the median of an uncertain event through lottery comparison. The analyst asks the expert to choose between two lotteries A and B where:

- Lottery A, represents throwing a fair coin in which the expert wins 100 € if he obtains Head and 0 € is he obtains "Tail". This serves as reference.

- Lottery B, represents the calibrated event and gives the expert 100 € if the intensity of a bad event $I(E)$ exceeds a given q (the calibrating value) and 0 € otherwise.

We ask the expert whether he prefers A to B ($A \succeq B$). In such case, we have: $100 \times \frac{1}{2} + 0 \times \frac{1}{2} \geq 100 \times P(I(E) \geq q) + 0 \times (1 - P(I(E) \geq q))$ (respectively $\frac{1}{2} \leq P(I(E) \geq q)$ in case the expert prefers B over A); In order to adjust q to approximate the median, an iterative procedure to converge to the median, consists on bounding it from below and above. Initially, the bounding interval is $[0, 1]$ and we iteratively split it depending on the responses of the expert. Specifically, we use $q = \frac{Y+X}{2}$, for $[X, Y]$ and adjust X and Y according to expert responses, with $X = 0, Y = 1$ initially. The way of proceeding is as follows:

- Step 0: Y=1, X=0;

- Step 1: Update B;

- Step 2: We ask the expert to choose either the lottery A or B:

  – if the player prefers lottery A to lottery B: Y=Y and X=$\frac{Y+X}{2}$

  – if the player prefers lottery B to lottery A: Y=$\frac{Y+X}{2}$ and X=X

  and go to step 1.

For a large number of iterations this will converge to the median.

4.8 Spatial decision support systems

The spatial dimension is very important in the context of environmental risk problems. This dimension taken independently, demonstrated a real interest in addressing real-world problem. Also, due to the evolution of computational capabilities, the ability of analyzing geographic information has been improved and evolved to represent today an interesting part of decision making processes characterized by spatial characteristics. For instance, in some problems related to environmental risk assessment, such as the marine pollution problems, maps represent a good decision aiding tool describing a studied phenomena such as the concentration evolution of a marine pollutant in real time.

Many well known decision problems require spatial information, I shall mention the traveling salesman problem (TSP) [11], environmental management, land use planning, to name but a few. The spatial dimension in environment risk problems involve generally:

- several interveners in space with different powers, responsibilities and levels of legitimacy.
- conflicting objectives stemming directly from the multidimensional nature of spatial problems. For example, the impact of a nuclear accident will be viewed differently from the environmental, the economical and the political perspectives.
- Criteria expressed on different scales. Problems related to marine pollution are necessarily interdisciplinary, requiring the consideration of several criteria of a quantitative and qualitative natures with different importances.

Such kind of problems represent a high level of complexity that require the use of decision aiding tools. Spatial decision support systems (SDSS) represent a class of tools used to deal with some of these complicated spatial decision problems.

Spatial Decision Making/Aiding

Spatial characteristics represent a rich source of information to be taken into account and analyzed during the decision aiding and making processes. A decision aiding model is characterized by two main components: the objects of study and dimensions under which the objects are evaluated. In spatial decision making / aiding, the studied objects, generally, have a spatial nature. For instance, in the TSP problem, the studied object are the alternatives of different possible paths; in marine pollution the studied objects might be the marine areas having the same level of pollution. In geographic information systems, we shall distinguish four types of spatial representations of information:

- points spatial objects: This type of objects is used to represent discrete, and 0-dimensional spatial information. An example of this type of spatial data, would be the tracking points, commonly used in order to compute the concentration of a pollutant.

- arc or line spatial objects: This type of data is commonly used to represent 1-dimensional spatial information such as information about roads, rivers, to name but a few. This type of data are commonly used in logistic decision problems such as TSP problems, multimodal chains problems.

- polygons spatial objects: This type of objects is used to represent 2-dimensional spatial information displayed on an area. An example of this type of spatial data would be the color gradation scheme. For instance, in marine pollution problems, this type of data is commonly used to assess risk levels on geographic areas.

- raster spatial objects: This type of objects is used to represent information evaluated on surfaces. We shall distinguish two types of raster data continuous and discrete raster data. An example of continuous raster data, would be the physical dispersion of the radioactive substances in the marine environment due to a nuclear accident. Discrete data, can be viewed in the same example through the contamination of the marine organisms due to a nuclear accident, since this evaluation depends on the density of fish in the marine area.

The three first types of objects are called, in geographic information systems, vector data.

Remark 4. *In our case, we will use points spatial information through tracking points, in order to generate information about the intensity of pollution due to a given accident in the marine environment. We will also use polygon spatial objects in the decomposition of the studied area.*

In case we are dealing with the same type of spatial data, we can reduce the decision aiding space into three dimensions: dimension of scenarios, dimension of stakeholders, and dimension of criteria. In such case, the type of spatial information can be modeled either as:

- studied objects: The different spatial information might be assessed as studied objects under different criteria, involving different stakeholders in the presence of different possible scenarios. This situation is more common when we aim at aggregating different decision maps. For instance, in risk environmental problems associated to marine pollution, the same intensity of a pollutant in a polygon may induce a different impact on each asset present in the spatial area. In such case, the problem becomes a simple Multicriteria problem and the operation can be seen as an overlay operation. Chakhar et al [23, 24] worked on the integration of geographic information systems in multicriteria decision aiding.

- criteria: In some decision problems we aim to compare decision maps and thus we have to aggregate the spatial information. In such cases, the objects of study are the decision maps evaluated by several geographic units having the same spatial representation of information. In such cases, the geographic units play the role of criteria under which the decision maps are evaluated. Metchebon et al [78], worked on maps comparaison problem, where maps are evaluated by criteria (the geographic units) in the two cases when criteria may interact, and when they do not.

Kemp in [66], distinguish four types of spatial decisions:

- Site selection: This category of spatial decision problems is characterized in general by points spatial objects (sites) evaluated under multiple criteria. For example, based on maps displaying the marine pollution under different assets (criteria), an investor may decide to chose among different locations of restaurants. Each location is characterized by different levels of vulnerability obtained taking into account different assets.

- Location allocation: This category of spatial decision problems concerns cases where we aim at choosing a location optimizing an allocation. This category of spatial decision problems is very common in logistic decision problems.

- Land use selection: This category concerns the optimal use of a geographic area. The spatial decision problems in this category consist on choosing an activity for the studied land. Thus, the studied objects in such case, are alternatives representing the suitable activities for the studied land based on its characteristics.

- Land use allocation: This category consists on finding a suitable decomposition of the studied land for different purposes. By its nature, spatial decision problems, in this category, are most of the time characterized by polygons spatial objects (parcels in the studied land). The objective of the decision aiding is to solve for each parcel a land use selection problem.

All these categories might be involved in public policy decision making problems.

4.9 Conclusion and discussion

Decision aiding is a science opened to different domains. The aim of a decision aiding methodology is to propose different tools allowing to provide a client with "informed decisions", depending on the circumstances surrounding the decision problem.

In this chapter, we started by presenting the decision aiding methodology, its concepts, and perspectives. The decision aiding methodology is a general framework that provides the decision analyst with very interesting tools and concepts which are common in all decision aiding types of problems, we shall mention:

- The co-construction procedure of the decision aiding problem, between the client and the analyst;
- the methodology for modeling the client's preferences.

The decision aiding problem, we are studying, is characterized by multiple criteria. In environmental risk problems, this dimension is frequently present through the presence of different assets that should be taken into account in the decision problem and thus in the decision aiding problem as well. In addition to the different dimension under which the problem might be analysed, our problem is characterized by the presence of uncertainty related to the assessment of risk. Another dimension characterizing our problem is the spatial dimension. This last can be apprehended in several ways: Spatial items can be either considered as criteria or as studied objects depending on whether we aim at evaluating, or comparing, maps or parcels. To do this we discussed spatial decision aiding analysis from a multiple criteria and rating problem statements perspectives. Different theoretical aspects are detailed in this section in order to address these issues.

Most of the concepts defined in this chapter are used whether in the case study (Chapter 4), or in the developed rating method (Chapter 5).

5 Case Study

This chapter was published in the Socio-Economic Planning Sciences journal [87]

5.1 Introduction

This work is part of a larger project aimed at developing theoretical and practical tools aiding to synthesize multiple criteria spatial risks in case of multiple nuclear release scenarios. A literature review with relevant papers on the integration of multiple criteria decision analysis tools in spatial decision problems until 2006 can be viewed at `http://publish.uwo.ca/~jmalczew/list.htm`. Despite the existing literature aiming to understand the processes governing the fate of radionuclides in the environment, [5], [6], [7], we note that the concentration of a given isotope is a necessary but not sufficient information for making informed decisions. Let us consider the example of two geographic zones: the first one is characterised by an average concentration level and very important economic and environmental assets while the second one is highly contaminated but does not present any economic or environmental relevance. Clearly, the involved stakeholders will be differently sensitive to the impacts in both geographic plots.

Our case study deals with simulated releases from a nuclear submarine at the bay of Toulon, where one of the most important bases of the French Navy is located. In case of a nuclear accident, the incumbent Prefect (the regional authority) needs synthetic information to support decisions, such as banning certain economic activities, setting a new water management policy at each relevant zone or impeding the access to specific areas. The IRSN[1] is in charge of a project aimed at improving models predicting dispersion and assessing the impact of radionuclides in the environment, see https://www.irsn.fr/FR/Larecherche/Organisation/Progra- mmes/Amorad/Pages/projet-Amorad.aspx#.XJeYzi17RQI. Several papers have been proposed to address the problem of nuclear risk assessment, see [9] [86] [99] [103] [107]. Most of previous works rest upon operational research analyses and very few papers addressed the post-accident risk assessment from a multiple criteria decision analysis point of view, see [8] [77]. In order to provide supplementary post-accident management tools allowing to evaluate environment and economic impacts, we have developed an approach in which data associated to assets involved in the bay are paired with maps displaying the concentration level of a given isotope generating criteria maps. Each map describes the impact of a release concentration for a given criterion. We then use a multiple criteria aggregation procedure generating impact maps taking into account all assets. The final step consists of aggregating uncertain information over release scenarios (release positions, sea conditions,...) through an outranking approach. Our case study serves as a template that can be extended to other release events and geographical areas.

The originality of our work stands on the way we structured and modeled a practical issue, starting from the raw question "How can we evaluate the impact of a nuclear accident, similar to that of Fukushima, in the marine area?" The practical case was offered by the bay of Toulon, due to the presence

[1] Institut de Radioprotection et de Sureté Nucléaire is a French center of expertise and research in radioprotection and safety of nuclear installations. More information can be found at `http://www.irsn.fr`.

of nuclear submarines in its port, characterised by the presence of multiple assets and two levels of spatial decomposition. In this chapter, we propose the models used to assess the impact of a nuclear release on each asset involved, in case we are interested in identifying the most impacted assets or areas with respect to each asset, as well as to evaluate the global impact taking into account all considered assets.

The chapter is organised as follows. Section 2 describes the case study including different decompositions of the area of interest and the associated data. In Section 3, we introduce the main theoretical concepts used in this work. We present in Section 4 the construction procedure of the criteria functions characterizing and evaluating the Bay. In Section 5, we show the results of the multiple criteria aggregation and the aggregation of release scenarios. We end up with a discussion. Several appendices provide additional details about this work and its results.

5.2 Case study

The area of interest is the Bay of Toulon (in what follows, we will use the Bay to refer to it), where a major basis of the French naval force is located, including nuclear submarines, besides being a densely inhabited area with important economic activities. Thus, there is a possibility of major negative impacts in case an accidental nuclear release takes place. Two features are identified in this study:

- Multiple impacts over different assets characterising the Bay.
- Uncertainties relative to accident parameters, to be modeled through scenarios.

In a radioactive release several isotopes may be present such as cesium-137, cesium-134, silver-110 or iodine-131. In our case, we will focus on cesium-137 characterised by a half-life of 30.17 years. However, the developed methodology does not depend on the considered radionuclide.

In our problem context, our objective is to set a decision aiding model based on consequences induced by an accident. The available information includes:

- scientific facts and results: The dispersion model of radionuclides in the marine environment;
- geographic features: Each geographic zone has special characteristics such as the income associated with tourism or fishing;
- norms: Including the maximum allowable levels of concentration for fishing or forbidding an activity.

5.2.1 Assets data

A decomposition of the Bay was carried out within the "Bay contract" by the "Syndicat Intercommunal de l'Aire Toulonnaise" (SIAT, 1998 and 2002). This decomposition was based on the following criteria:

- A physical criterion, relying mainly on the geomorphology and local hydrodynamics of water bodies.
- A biological criterion, taking into account the presence of particular ecosystems.
- A socioeconomic criterion, based upon the presence of certain special activities such as ports and military activities.

In what follows, we adopt the above mentioned division, with seven homogeneous zones illustrated in Figure 5.1:

1. The north of the small bay, characterised by maritime and military activities. It includes a military port, freight, passenger transport, boaters and professional fishers.

2. The bay of Lazaret, characterised by aquaculture and tourism activities.

3. From the beaches Mourillon, Saint-Mandrier, until Cape Brown. The entrance to the small harbour is also characterised by military activity, a port, boaters and maritime transport. Its particularity lies in the fact that it represents a natural area of ecological faunistic and floristic interest, due to the presence of seagrass Posidonia.

4. From zone 3 to "Commune le Pradet". This part is characterised by an important fishing activity, tourism activities and a high presence of seagrass Posidonia.

5. From Cap Sicié to Saint-Elme, characterised by several seaside activities. There is mainly swimming, boating, diving and professional fishing activities. This area is characterised by ecological richness, particularly a high presence of seagrass Posidonia. Moreover, there are three protected zones at "Anse des Sablettes", the "Islands of the Two Brothers" and Cape Sicié.

6. From Marégau Point to Cape Cepet. This area is mainly dedicated to military activities. There is also tourism activities and seagrass Posidonia. This last is an important asset for sea life.

7. The rest of the bay with no land boundary is mainly characterised by professional fishing.

In order to evaluate the consequences of accidents, we distinguished two types of attributes: economic and environmental.

Remark 5. *Although public health is an important variable, we will consider it out of scope for this chapter. This is mainly due to the public policy adopted by France: as soon as there is an accident, depending on water quality, some emergency plans are activated such as forbidding some activities such as swimming or evacuating people living in some area. These decisions might*

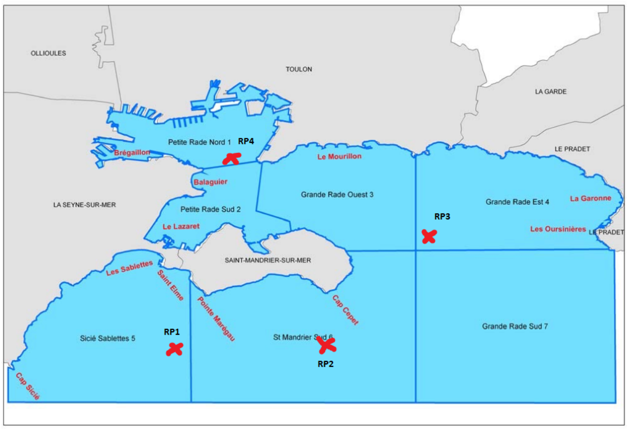


Figure 5.1 – Decomposition of the bay into seven homogeneous zones.

have economical consequences such as an impact on tourism fish farming and fishing.

Economic attributes

Two types of activities are present in the Bay:

- commercial activities linked with water quality: fishing, water sports, diving, professional fishing and aquaculture;
- Non-commercial activities such as swimming and leisure fishing.

Non-commercial activities seem to be not directly linked with economic assets. However, they have a strong influence over the touristic attractiveness of each zone which might induce an economic impact.

As far as the economic axis is concerned, we shall evaluate the impact of a released cesium-137 concentration based on three attributes:

- Professional fishing (F), based on an estimation of the annual economic impact of the fish caught. The data comes from the "Système d'Informations Halieutiques" (SIH-2007). Table 5.1 provides the annual turnover associated with professional fishing at each of the zones.

Zones	1	2	3	4	5	6	7
Annual turnover (k€)	300	300	965	965	934	1286	1000

Table 5.1 – Annual turnover of professional fishing in 2007.

- Fish farming (FF), supports raising fish and shellfish. The area characterised by this activity is zone 2, representing an important economic asset for Toulon. The main characteristic is that the fish are more impacted by water quality as they cannot swim outside the breeding areas. The turnover generated by this activity in 2007 was 2129 (k€).

- Tourist attractiveness (T), refers mainly to swimming, diving and water sports. The economic value of swimming is assessed based on the income of restaurants located at sea shore and accommodations at each municipality. Thus, the value associated with the commune of Toulon will be assigned to zones 1 and 3; that of Seyne-Sur-Mer to zone 2; Pradet to zone 4; and, finally the commune of Saint-Mandrier to zone 6 (zone 7 has no coastline). Data associated with this attribute come from INSEE-Sirene 2007 for the catering sector, Chambre de Commerce et d'Industrie CCI-PACA 2007 for water sports and boaters and BVA-Ifremer 2007 for non professional fishing. Table 5.2 summarises the turnover associated with touristic activities in the Bay.

Zones	1	2	3	4	5	6	7
Annual turnover (k€)	34 839.5	29 593	20 828.5	13 591.5	23 113	24 483	1 131

Table 5.2 – Annual turnover of Tourism in 2007.

Remark 6. *The social attributes, such as poverty, might be considered as a part of economic attributes. In fact, Fishing attribute is assessed based on the annual turnover associated to fish caught, the Touristic attractiveness is assessed based on accomodations, restaurants and many other economical activities. Hence, an impact on these assets will lead to an impact on the* *social issues. The social issues will not be considered in this work, because the dimensions already considered will not be independent regarding the social impact.*

Environmental axis

As far as the environmental axis is concerned, we shall focus our attention on the presence of seagrass Posidonia. This is one of the most important ecosystems in Mediterranean coastal zones, playing the same importance as forests in terrestrial areas: It is essential for the preservation of the balance of sea-life, [22], [52], as it:

1. Influences coastal water quality, through significant oxygen production and sediment trapping.

2. Is at the base of many trophic networks, for the production of plant and animal biomass.

3. Plays a fundamental role in the hydrodynamic protection of the coastline and beaches.

4. Fixes sediments and reduces the turbidity of the water, preventing their resuspension during storms.

Data on the mapping of seagrass Posidonia are rare, mostly very old, and its evolutionary dynamics are poorly known. Nevertheless, we have qualitative information on its presence at each geographic zone. Table 5.3 summarises its presence in the Bay.

Zones	1	2	3	4	5	6	7
Degree of the presence of seagrass Posidonia	Absent	Absent	Average	High	High	High	Absent

Table 5.3 – Presence of seagrass Posidonia in the Bay (2002).

Cesium concentration might be included as a relevant environmental indicator representing water quality. However, we will not consider it independently, since we use it to assess criteria and we are interested in its impact on assets characterising the bay.

5.2.2 Generating concentration data

Many studies have been conducted to model the physical dispersion process of radioactive substances in the marine environment, e.g. [5], [6], [7], [39], [42], [71]. These have led to the development of simulation tools, such as STERNE [2], which we have used in our case study. The input parameters required by this tool are the type of sea currents, the release position and the quantity initially released.

Sources of uncertainty

Since accidental nuclear releases are related with the routes undertaken by the submarines, there will be two main sources of uncertainty in our case study [3]:

- The sea conditions (wind, currents, ...), at the time of the release, identified by a parameter β. In the case of Toulon, they are dominated by wind [71], and their probabilities can be estimated using a meteorologic database.

- The position $RP = (x_{RP}, y_{RP})$ where the release takes place, being, respectively, the latitude and longitude. The choice of the release positions is made based on an expert, in marine radioprotection, point of view, in order to have different release scenarios (low similarity).

[2] Simulation du Transport et du transfert d'Eléments Radioactifs dans l'environNEment marin, translated as Simulation of radionuclide transport and transfer in marine environments

[3] The amount initially released can be also considered as a source of uncertainty. However, in this work we shall fix it to $10^{15} Bq$, i.e. a very important release.

We modeled uncertainty about the accident parameters through representative scenarios. We shall consider three sea conditions with their associated probabilities, as described in Table 5.4: and four initial release positions with

Scenario	Prevailing wind	Probability
β_1	Mistral	q_1
β_2	East	q_2
β_3	Steady	q_3

Table 5.4 – Discretisation of sea conditions.

their associated probabilities, as specified in Table 5.5, displayed by red crosses in Figure 5.1.

Position in the map	Scenarios	Probability
Zone 5	RP_1= (43.053 , 5.89)	r_1
Zone 6	RP_2= (43.053 , 5.96)	r_2
Zone 4	RP_3= (43.079 , 5.975)	r_3
Zone 1	RP_4= (43.103 , 5.918)	r_4

Table 5.5 – Discretisation of initial release positions.

The corresponding probabilities will be assessed in Section 5.5.2, where we shall synthesise the twelve scenarios.

Assessing cesium concentration

The approach proposed here is driven by the contaminant concentration at each plot of the bay. This, in turn, will be driven by the amount initially released as well as the release position RP and sea conditions β. Based on a hydrodynamic model [39] sketched in Appendix A, we may estimate the concentration of the radioactive substance in water (respectively in a marine organism) at any point $z = (x, y)$ in the map, which we designate $c_w(z, RP, \beta)$ (respectively $c_o(z, RP, \beta)$).

STERNE offers the possibility of using tracking points to simulate the concentration evolution of a given isotope. We discretised the bay into several geographic units, represented by their tracking points in the center. The previous decomposition of the Bay in 7 homogeneous zones was too rough to be applied for the estimation of the contaminant concentration, as it may lead to missing significant concentrations. We instead defined 97 geographical units adjusted to the map of the Bay. Each geographic unit is defined by two representative points: a tracking point at the bottom of the sea and another one at 1m depth. The reasons for choosing two depth levels are related with the nature of the chosen attributes. In what follows, we shall use “geographic zone” to refer to the first decomposition in 7 zones and “geographic units” to the second decomposition, in 97 units. Figure 5.2 displays 10 evolution curves of cesium concentration at 1m depth at the 10 most contaminated zones based on the maximum concentration attained.

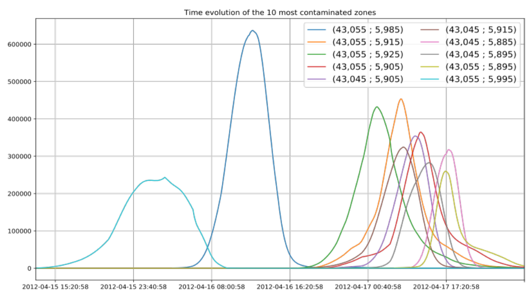


Figure 5.2 – 10 highest evolutions of cesium concentration in water over time.

We summarize the concentration evolution curves through their mean and maximum values. Figures 5.3 and 5.4 display, respectively, the maximum and mean values corresponding to tracking points at 1m depth for the release scenario $(mistral; RP_2)$.

Empty cells in both figures correspond to land space. In all simulations we face a factor of 10 between the average and maximum values. We aggregate both values and move from a cardinal to an ordinal scale by assigning each zone to a corresponding concentration level. This can be achieved in several ways depending on the eventual compensation between both values. In our work we considered a geometric mean between them as their is a scale factor

	5,86_5,87	5,87_5,88	5,88_5,89	5,89_5,9	5,9_5,91	5,91_5,92	5,92_5,93	5,93_5,94	5,94_5,95	5,95_5,96	5,96_5,97	5,97_5,98	5,98_5,99	5,99_6	6_6,01	6,01_6,02
43,11_43,12			3,09E-10	1,09E-09	3,53E-08	4,87E-07	1,44E-04									
43,1_43,11			2,09E-09	1,09E-08	2,63E-06	7,85E-05	1,00E+00	3,24E+01	4,52E+01	6,54E+01	7,77E+01	5,84E+01	9,02E+01	1,55E+03	2,89E+03	
43,09_43,1						3,23E-03	4,62E+00	3,35E+01	6,63E+01	4,34E+01	5,29E+01	2,24E+02	1,02E+03	1,80E+03	7,72E+03	1,51E+04
43,08_43,09					2,66E-03	6,13E-02	1,14E+00	3,46E+01	2,50E+01	8,36E+01	6,50E+02	4,14E+03	8,56E+03	3,58E+04	7,31E+04	8,42E+04
43,07_43,08			1,45E+03	1,25E+03						2,39E+04	1,07E+04	2,95E+04	1,54E+05	2,55E+05	2,45E+05	1,70E+05
43,06_43,07		3,42E+03	2,00E+03	5,21E+03	2,82E+04	6,96E+04	8,04E+04	3,96E+05	9,64E+05	6,84E+04	1,10E+05	1,55E+05	3,07E+05	4,11E+05	4,05E+05	3,37E+05
43,05_43,06	2,75E+03	8,86E+02	1,71E+03	4,38E+03	1,20E+04	2,09E+04	1,30E+05	3,62E+05	3,30E+05	1,87E+05	1,73E+05	2,59E+05	1,09E+05	1,85E+05	3,31E+05	1,85E+05
43,04_43,05	2,01E+03	1,64E+03	4,06E+03	3,53E+03	2,06E+04	2,48E+04	4,51E+04	9,23E+04	2,56E+05	2,61E+05	7,11E+04	4,92E+04	5,74E+04	1,39E+05	1,08E+05	1,61E+07

Figure 5.3 – Maximum concentration, 1m depth, at the 97 geographic units for $(mistral; RP_2)$.

	5,86_5,87	5,87_5,88	5,88_5,89	5,89_5,9	5,9_5,91	5,91_5,92	5,92_5,93	5,93_5,94	5,94_5,95	5,95_5,96	5,96_5,97	5,97_5,98	5,98_5,99	5,99_6	6_6,01	6,01_6,02
43,11_43,12			1,27E-12	3,75E-11	1,35E-09	2,08E-08	4,21E-06									
43,1_43,11			3,24E-11	4,24E-10	1,82E-07	6,21E-06	3,05E-02	3,00E+00	5,59E+00	8,53E+00	9,65E+00	7,28E+00	1,31E+01	1,31E+02	3,19E+02	
43,09_43,1						2,35E-04	1,67E-01	5,13E+00	1,18E+01	7,32E+00	9,17E+00	1,98E+01	7,05E+01	1,92E+02	4,07E+02	1,15E+03
43,08_43,09					1,47E-04	1,88E-03	5,47E-02	5,37E+00	4,46E+00	6,85E+00	5,45E+01	2,81E+02	8,48E+02	1,99E+03	3,42E+03	5,99E+03
43,07_43,08			2,10E+02	1,91E+02						1,22E+03	8,22E+02	2,41E+03	1,25E+04	2,77E+04	2,55E+04	1,90E+04
43,06_43,07		4,56E+02	2,79E+02	6,90E+02	3,80E+03	7,25E+03	4,50E+03	1,98E+04	4,37E+04	5,73E+03	9,49E+03	1,72E+04	3,85E+04	4,05E+04	3,79E+04	2,74E+04
43,05_43,06	4,32E+02	1,42E+02	2,52E+02	5,93E+02	1,76E+03	2,58E+03	6,92E+03	2,11E+04	1,56E+04	1,35E+04	1,64E+04	2,50E+04	1,08E+04	1,95E+04	2,70E+04	1,36E+04
43,04_43,05	3,76E+02	3,69E+02	5,58E+02	7,99E+02	1,85E+03	2,95E+03	4,19E+03	5,82E+03	1,15E+04	1,31E+04	4,72E+03	3,45E+03	3,16E+03	8,44E+03	9,61E+03	1,57E+05

Figure 5.4 – Average concentration, 1m depth, at the 97 geographic units for $(mistral; RP_2)$.

between both values. Figure 5.5 illustrates their aggregation considering the same level of importance for both evaluations at each zone.

Csw	5,86_5,87	5,87_5,88	5,88_5,89	5,89_5,9	5,90_5,91	5,91_5,92	5,92_5,93	5,93_5,94	5,94_5,95	5,95_5,96	5,95_5,97	5,97_5,98	5,98_5,99	5,99_6	6_6,01	6,01_6,02
43,11_43,12			3,44E-10	1,13E-09	3,67E-08	5,08E-07	1,48E-04									
43,1_43,11			2,79E-09	1,13E-08	2,82E-06	8,47E-05	1,03E+00	3,54E+01	5,08E+01	7,39E+01	8,73E+01	6,57E+01	1,03E+02	1,68E+03	3,21E+03	
43,09_43,1						3,47E-03	4,78E+00	3,87E+01	7,82E+01	5,08E+01	6,21E+01	2,44E+02	1,09E+03	1,99E+03	8,13E+03	1,62E+04
43,08_43,09					2,81E-03	6,32E-02	1,20E+00	4,00E+01	2,94E+01	9,04E+01	7,05E+02	4,42E+03	9,40E+03	3,78E+04	7,65E+04	9,01E+04
43,07_43,08			1,66E+04	1,45E+04						2,51E+04	1,15E+04	3,19E+04	1,67E+05	2,83E+05	2,70E+05	1,89E+05
43,06_43,07		3,87E+03	2,28E+03	5,90E+03	3,20E+04	7,69E+04	8,49E+04	4,16E+05	1,01E+06	7,41E+04	1,20E+05	1,72E+05	3,45E+05	4,51E+05	4,43E+05	3,64E+05
43,05_43,06	3,18E+03	1,03E+03	1,97E+03	4,98E+03	1,38E+04	2,35E+04	1,37E+05	3,83E+05	3,45E+05	2,01E+05	1,90E+05	2,84E+05	1,20E+05	2,04E+05	3,58E+05	1,99E+05
43,04_43,05	2,39E+03	2,01E+03	4,62E+03	4,33E+03	2,24E+04	2,78E+04	4,93E+04	9,81E+04	2,67E+05	2,74E+05	7,59E+04	5,26E+04	6,05E+04	1,47E+05	1,18E+05	1,63E+07

Figure 5.5 – Contamination level for 97 geographic units for $(mistral; RP_2)$.

A colour coding will reflect the contamination level at each geographic unit. We consider 5 levels from less to more contaminated. The cutting levels are fixed based on expert judgment. Level 1 is displayed in blue, 2 in green, 3 in yellow, 4 in orange and 5 in red. We shall use this grading colour in the rest of the chapter. As a first way to display the information, we could present the map $(z, c_w(z, RP, \beta))$, which provides, for each geographic unit z, the estimated contamination level, in an ordinal scale, given specific initial conditions. Figure 5.6 displays the contamination level induced by the release scenario $(mistral; RP_2)$ using the previous colour code.

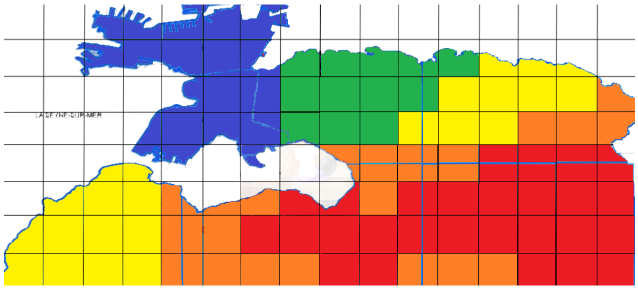


Figure 5.6 – Map displaying the contamination level corresponding to $(mistral; RP_2)$.

5.3 Multiple criteria decision analysis

In Section 5.2.1, we described the Bay of Toulon as a rich area where several assets are involved and can be impacted in case of a nuclear release. Our first objective is to define functions, which we shall call criteria, allowing us to assess the impact on each asset at each geographic unit. Each function evaluates a geographic unit from a single perspective. In consequence, we shall associate with each criterion a map evaluating the impact on the corresponding asset [4]. We shall consider the four criteria expressed on an ordinal scale, see section 5.4, all of which need to be taken into account in an appropriate multiple criteria formulation.

The field of multiple criteria decision analysis (MCDA) offers a set of operational tools and methodologies to incorporate the decision maker's preferences as well as any information allowing the decision analyst to evaluate a set of actions described by multiple attributes. In real-world cases, several problem statements can be considered referring to the way in which decision aiding is envisaged, see [100]:

[4] Considering each criterion function separately, we can either identify the most impacted geographic units or compute the expected impact.

- clustering (partition the set of alternatives into unordered not pre-defined equivalence classes; the clusters).
- assignment (partition the set of alternatives into unordered pre-defined equivalence classes).
- rating (partition the set of alternatives into ordered pre-defined equivalence classes).
- ranking (partition the set of alternatives into ordered not pre-defined equivalence classes).

Modeling MCDA problem requires representing preferences either measuring their values, as in the case of multi-attribute value theory, or directly using binary relations, as in the case of social choice theory and outranking based methods, see [18].

In our case, we aim at assigning each geographic unit to the corresponding impact level. We consider five predefined and ordered impact categories $C_1, ..., C_5$, ranked from not risky to very risky $C_h \gg C_{h+1} \forall h \in \{1, ..., 4\}$ where $\gg$ refers to a complete order on the set of categories. Hence, the type of decision aid required here is a rating problem statement.

Two main methods corresponding to two different approaches deal with rating problems: UTADis and ELECTRE-TRI. The UTADis method was first presented in [31], being a variant of the well-known UTA method [60]. UTADis consists in defining a marginal utility function over criteria, taking respectively the value 0 and 1 for the least and most preferred values of each criterion, and evaluating each action with an additive utility function. Such methods are suitable in multiple criteria problems where trade-offs among criteria are possible and meaningful. Alternatively, the ELECTRE-TRI method is an outranking based procedure first introduced in [106]. This method uses a majority rule, while respecting a minority using a veto rule, to compare the actions to the profiles characterizing categories; ELECTRE-TRI method is detailed in Appendix B. The MCDA procedure used in this work is based on ELECTRE-TRI, as trade-offs among the criteria were not interpretable.

5.4 Construction of criteria

The multiple criteria problem at hand adopts a rating formulation in which we consider the four criteria reflected in Table 5.6, with scales referring to the raw impact of a nuclear accident at each geographic unit. All criteria considered to evaluate the Bay are based on water quality through the concentration of cesium in water. Hence, the criteria will measure the impact of a given concentration on the assets involved at each geographic zone.

	Criteria	scale
1	Fishing	impact level
2	Fish Farming	impact level
3	Seagrass Posidonia	Impact level
4	Tourism	impact level

Table 5.6 – Criteria and scales.

We start by presenting the typology of impact functions, allowing to associate with each concentration level an impact on an asset. For example, given a concentration level, the impact function will assess the proportion of tourists giving up visiting a geographic unit, the proportion of fishes not allowed to be commercialised or the impact on seagrass Posidonia. In the second part of this section, we construct the criteria functions, taking into account the impact function and the data associated with the assets. For instance, the tourism criterion is evaluated based on the income in a geographic zone, when there is no accident, multiplied by the proportion of tourists giving up visiting the such unit given a concentration level (impact function).

5.4.1 Typology of impact functions

We aim now at evaluating the impact of a given level of contamination on each asset. The considered impact functions are based on two hypotheses:

- independent geographic units. As units are small, we do not consider mutual influences between neighbouring units. Thus, the impact on a geographic unit will only depend on its concentration level.
- The impact function does not depend on geographic units, as it depends on the characteristics of the assets.

Three types of impact functions will be considered. Choosing among them will depend on the characteristics of attributes and the decision maker's preferences:

- Heaviside function: We consider that a given asset is impacted from a certain level of concentration. This function is used in evaluating the impact on seagrass Posidonia.
- Linear function: no impact is considered before a first threshold is met while an important impact is assumed after the second one. Between both thresholds, the impact is linear. This type of function can be chosen when the population response is linearly proportional to pollution levels.
- Cumulative function: It is more suitable for modeling social phenomena for which the number of people influencing the evaluation of areas is important. We will use this function to assess the impact on tourism and fishing.

The cumulative impact function requires calibration reflecting the impact of different levels of concentration on a given asset. For example, qualitatively, the higher the concentration, the less tourists will visit the corresponding polluted area. This function can be derived through a weighted sum of linear

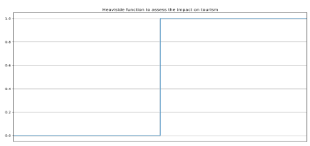


Figure 5.7 – Heaviside impact function.

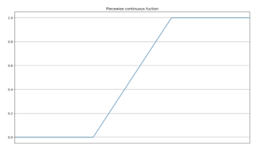


Figure 5.8 – Linear impact function.

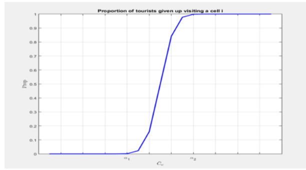


Figure 5.9 – Cumulative impact function.

functions, of type 2, representing each the impact assessment by a pool of experts, assessing a “tolerance threshold” and a “reaction threshold”. Alternatively, we can calibrate the median for each contamination level as we do here. This approach is inspired by the probability equivalent method for assessing utilities [43]. Let us call the cumulative impact function $prop_i(c_k)$, where i refers to a geographic unit i and c_k is the level of contamination in the marine organism $k = o$ or in seawater $k = w$. Our objective is to find for a few concentrations $c_{k_1}, ..., c_{k_5}$[5], the corresponding $prop_i(c_{k_1}), ..., prop_i(c_{k_5})$, through expert judgment, and then adjust a curve. Note that $prop_i(c_k)$ will essentially be uncertain and we shall focus on assessing its median using lottery comparison.

In what follows we apply this approach to the attribute Tourism and thus $k = w$. For this we compare two lotteries:

- Lottery A, represents throwing a fair coin in which the expert wins 100 € if he obtains ”Head” and 0 € is he obtains ”Tail”. This serves as reference.
- Lottery B, represents the calibrated event and gives the expert 100 € if $prop \geq q$ and 0 € otherwise, where $prop = prop_i(c_w)$ is the proportion of tourists giving up visiting a geographic unit in case c_w is high enough and q is the calibrating value.

We ask the expert whether he prefers A to B ($A \succeq B$). In such case, we have: $100 \times \frac{1}{2} + 0 \times \frac{1}{2} \geq 100 \times P(prop \geq q) + 0 \times (1 - P(prop \geq q))$ (respectively

[5] 5 represents the number of contamination levels introduced in Section 5.2.2

$\frac{1}{2} \leq P(prop \geq q)$ in case the expert prefers B over A); we need to adjust q to approximate the median. For this, we can design an iterative procedure to converge to it, bounding it from below and above. Initially, the bounding interval is $[0, 1]$ and we iteratively split it depending on the responses of the expert. Specifically, we use $q = \frac{Y+X}{2}$, for $[X, Y]$ and adjust X and Y according to expert responses, with $X = 0, Y = 1$ initially. For a large number of iterations this will converge to the median. Figure 5.10 displays the calibration for a few concentration levels using the above procedure. The same approach remains valid for the fishing attribute, for which $k = o$.

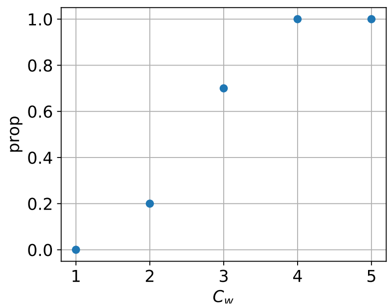


Figure 5.10 – Calibration of proportion of tourists giving up visiting a cell.

5.4.2 Tourism

We construct first the criterion function for tourism (T), referring to the level of economic loss related with the tourism sector. This last is assessed as $prop_i(c_w(z_i, s), T)Inc_i(T)$ where $prop_i(c_w(z_i, s), T)$ represents the proportion of tourists refraining from visiting the geographic unit i under the incumbent release scenario and $Inc_i(T)$ represents the income associated with the geographic unit i. The function $prop_i(c_w(z_i, s), T)$ has been assessed in section 5.4.1, Figure 5.10.

In order to evaluate the economic importance of each geographic unit, an issue with the spatial decomposition arises:

- Data associated with tourism revenues are available just for the seven geographic zones. We partitioned the annual turnover proportionally between all geographic units constituting each of the seven geographic zones.

- Some geographic units are shared between several homogeneous zones. The solution adopted is to evaluate the geographic units by considering the turnover proportionally to the surface occupied by geographic zones at the geographic unit. This entails the use of the same decomposition as for cesium concentration simulations.

Thus, the estimated annual turnover at each geographic unit is

$$Inc_i(T) = \sum_{j \in Z} \frac{S_{ij}}{\sum_{i \in U} S_{ij}} Tur_j(T),$$

where U and Z represent, respectively, the set of geographic units (decomposition of the Bay adopted to forecast cesium concentration) and the set of geographic zones (decomposition made to describe the attributes); T refers to the asset Tourism; S_{ij} the maritime surface (land excluded) belonging both to the geographic unit i of U and the zone j of Z; $Tur_j(T)$ the turnover associated with geographic zone j.

We denote by $g_T(i,s)$, the function of the tourism criterion rating the geographical unit i, given a scenario s. Such function would be [6]:

$$g_T(i,s) = \begin{cases} 1, \text{ if } prop_i(c_w(z_i,s),T)Inc_i(T) \times 97 < 10^4 \\ 2, \text{ if } 10^4 \leq prop_i(c_w(z_i,s),T)Inc_i(T) \times 97 < 10^6 \\ 3, \text{ if } 10^6 \leq prop_i(c_w(z_i,s),T)Inc_i(T) \times 97 < 10^7 \\ 4, \text{ if } 10^7 \leq prop_i(c_w(z_i,s),T)Inc_i(T) \times 97 < 10^8 \\ 5, \text{ if } 10^8 \leq prop_i(c_w(z_i,s),T)Inc_i(T) \times 97 \end{cases}$$

[6] 97 in the criterion function refers to the number of geographic units

where $10^4, 10^6, 10^7, 10^8$ represent the economic losses delimiting each impact category. The cutting thresholds used in the different criteria are assessed based on expert judgment. Figure 5.11 shows the assessment of the tourism criterion for the mistral-type marine currents and release point RP_2.

Tourism	5,86_5,87	5,87_5,88	5,88_5,89	5,89_5,9	5,90_5,91	5,91_5,92	5,92_5,93	5,93_5,94	5,94_5,95	5,95_5,96	5,95_5,97	5,97_5,98	5,98_5,99	5,99_6	6_6,01	6,01_6,02
43,11_43,12			1	1	1	1	1									
43,1_43,11			1	1	1	1	1	3	3	3	3	3	4	4	4	
43,09_43,1						1	1	3	3	3	3	4	4	4	4	5
43,08_43,09					1	1	1	3	3	3	3	4	4	5	5	5
43,07_43,08			4	4						5	5	5	5	5	5	5
43,06_43,07		4	4	4	5	5	5	5	5	5	5	5	5	5	5	5
43,05_43,06	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5
43,04_43,05	4	4	4	4	5	5	5	5	5	5	5	5	5	5	5	5

Figure 5.11 – evaluation of the tourism criterion under the scenario $(Mistral, RP_2)$

We can display the above results through maps [7], see Table 5.7.

RP_1 RP_2 RP_3 RP_4

β_1 β_2 β_3

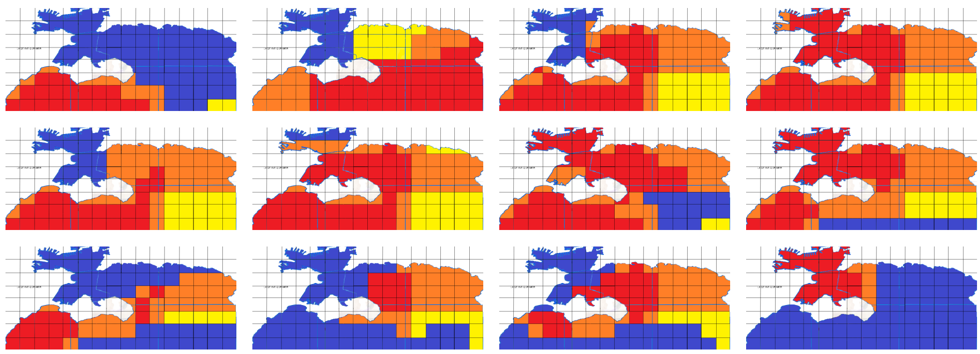


Table 5.7 – Tourism criterion maps for the twelve scenarios

Some relevant information can be assessed in this way. For example we can identify areas which are most at risk from the perspective of tourism. (e.g. the red ones)

The economic loss in the bay associated with scenario $s = (\beta_h, RP_k)$ can be obtained through spatial aggregation, without considering interactions be-

[7]We used the same colour coding as in Figure 5.6

tween neighbouring geographic units based on:

$$\sum_i prop_i(c_w(z_i, \beta_h, RP_k), T)Inc_i(T).$$

The expected economic loss in the whole area, through aggregating uncertainties over initial conditions, $s = (\beta_h, RP_k)$, would be:

$$\varphi_{TA} = \sum_i Inc_i(T) \sum_{h=1}^{3} \sum_{k=1}^{4} prop_i(c_w(z_i, (\beta_h, RP_k), T)q_h r_k,$$

which we denote $\varphi_{TA} = \sum Inc_i(T)prop_i(c_w, T)$.

The expected income in tourism sector when there is no accident would be:

$$\varphi_T = \sum_i Inc_i(T),$$

Then, the expected income on the whole area after an accidental release would be:

$$\varphi_T - \varphi_{TA}.$$

We could also use relative losses. For example, for the income from tourism, it would be:

$$\frac{\varphi_{TA}}{\varphi_T}.$$

All these indices, derived from the process of the construction of the tourism criterion may help the decision maker assessing the impact of an eventual accident over the tourism sector.

5.4.3 Fishing

We now assess the fishing criterion function, focusing on the economic loss on the fishing sector at each geographic unit. Such loss is evaluated by coupling the proportion of fish not authorised for sale and the economic income before the accident in a geographic unit. Thus, the economic loss would be $prop_i(c_o(z_i, s), P_e)Inc_i(P_e)$, where $prop_i(c_o(z_i, s), P_e)$ represents the impact function associated with the fishing sector, $c_o(z_i, s)$ denotes the contamination level in fish and $Inc_i(P_e)$ represents the income from the fishing sector at the geographic unit i. It should be mentioned that, for this criterion, we will consider tracking points both at 1m depth and at the bottom of the sea. This is justified by the presence of fish at all sea levels in this region.

The impact function $prop_i(c_o(z_i, s), P_e)$, is characterised by two thresholds:

- The first one reflects the level at which responsible authorities begin to control the cesium concentration in fish before selling.

- The second one represents the level at which authorities prohibit consumption of fish caught at a given geographic unit. We shall consider the second threshold to be $500Bq/kg$ equal to the maximum allowable level of contamination for authorising fish consumption.

Between both thresholds, the impact is considered non-linear. The calibration process in section 5.4.1 is applicable. The only modification would be to use $prop = prop_i(c_o)$ in lottery B, reflecting the proportion of fish not allowed for sale given the level of cesium concentration c_o in fish.

In order to evaluate the economic importance of a geographic unit, $Inc_i(P_e)$, we use the same solution for the two spatial decompositions as for tourism. Thus, the annual turnover at each geographic unit is defined as:

$$Inc_i(P_e) = \sum_{j \in Z} \frac{S_{ij}}{\sum_{i \in U} S_{ij}} Tur_j(P_e),$$

where $Tur_j(P_e)$ refers to the turnover of fishing associated with geographic zone j.

We denote by $g_{P_e}(i, s)$, the fishing criterion rating the geographic unit i

under scenario s. Such function would be

$$g_{P_e}(i,s) = \begin{cases} 1, \text{ if } prop_i(c_o(z_i,s),P_e)Inc_i(P_e) \times 97 < 10^3 \\ 2, \text{ if } 10^3 \leq prop_i(c_o(z_i,s),P_e)Inc_i(P_e) \times 97 < 10^5 \\ 3, \text{ if } 10^5 \leq prop_i(c_o(z_i,s),P_e)Inc_i(P_e) \times 97 < 10^6 \\ 4, \text{ if } 10^6 \leq prop_i(c_o(z_i,s),P_e)Inc_i(P_e) \times 97 < 5.10^6 \\ 5, \text{ if } 5.10^6 \leq prop_i(c_o(z_i,s),P_e)Inc_i(P_e) \times 97 \end{cases}$$

Table 5.8 shows the assessment of the fishing criterion maps for the twelve scenarios. As for tourism, we can derive the economic loss in the bay, the expected loss, the relative loss and the expected income in relation with the fishing asset.

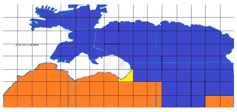
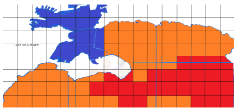
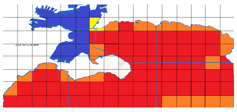
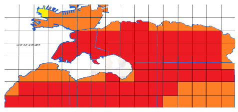
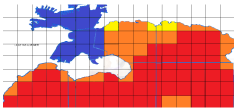
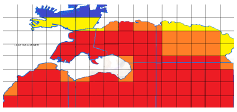
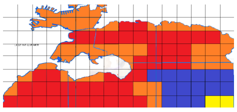
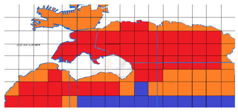
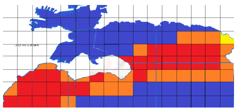
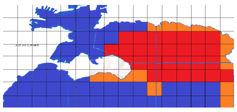
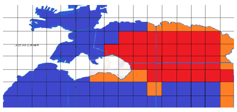
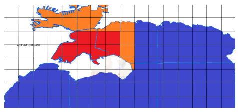
	RP_1	RP_2	RP_3	RP_4
β_1				
β_2				
β_3				

Table 5.8 – Fishing criterion maps for the twelve scenarios

5.4.4 Fish Farming

For this criterion, as for fishing, we use the concentration level of cesium in organisms (fish and shellfish farming). Because of the special characteristics of the fish farming activity, the impact on this sector will not be assessed at a geographic unit but at the whole geographic zone 2:

- Unlike the fishing indicator, where fish can swim through many geographic units, fish in aqua-farms cannot leave geographic zone 2 and, thus, they are just impacted by the water quality of this zone.

- The economic relevance of all geographic units in zone 2 is the same.

To assess this criterion, we consider $\overline{c_o(s)} = max_i(c_o(z_i, s))$, where z_i is a geographic unit in zone 2. The economic income associated with the fish farming sector will not be considered on the criterion evaluation, as it is the same in all geographic units of zone 2. However, this last will represent a relevant information to assess the criterion's importance during the multicriteria aggregation procedure.

We denote by $g_{F_f}(s)$, the fish farming criterion evaluating the geographic zone 2 under scenario s. Such function can be interpreted as a rate representing the impact on the fish farming sector.

$$g_{F_f}(s) = \begin{cases} 1, \text{ if } \overline{c_o(s)} < 100 \\ 2, \text{ if } 100 \leq \overline{c_o(s)} < 200 \\ 3, \text{ if } 200 \leq \overline{c_o(s)} < 300 \\ 4, \text{ if } 300 \leq \overline{c_o(s)} < 400 \\ 5, \text{ if } 500 \leq \overline{c_o(s)} \end{cases}$$

$500Bq/kg$ and $100Bq/kg$ are respectively the maximum allowable level to consume fishes from Fukushima before and after the accident. Table 5.9 shows the assessment of the fish-farming criterion maps for the twelve scenarios.

5.4.5 Seagrass "Posidonia Oceanica"

We assess now a criterion function in relation with the impact of a radioactive release scenario on seagrass for each geographic unit. Unlike the previous ones, the seagrass Posidonia criterion rates the impact of a concentration level at a geographic unit level. Such impact represents a coupling between

RP_1 RP_2 RP_3 RP_4

β_1 β_2 β_3

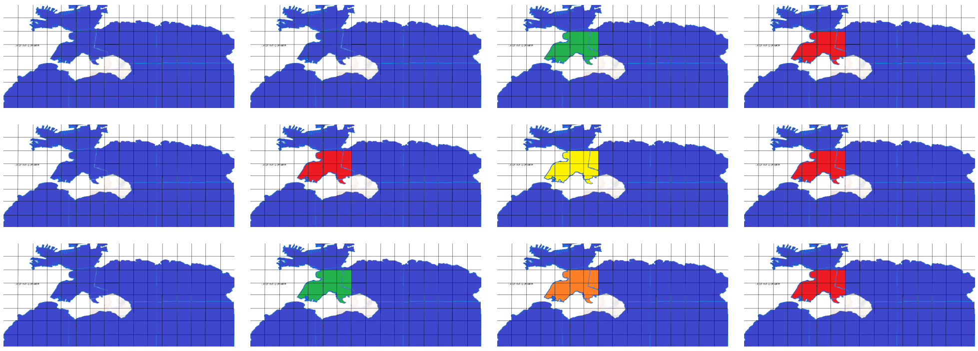


Table 5.9 – Fish-Farming criterion maps for the twelve scenarios

contamination levels, through the corresponding impact function, and scores associated with the presence of seagrass Posidonia at each geographic unit. To assess the corresponding impact function, we consider a Heaviside function, Figure 5.7, defined through

$$Imp_i(c_w) = \begin{cases} 0, \text{ if } c_w \in \{1, 2\} \\ 1, \text{ if } c_w \in \{3, 4, 5\} \end{cases}$$

where c_w refers to the level of cesium concentration in seawater. Degrees of the presence of seagrass Posidonia are described on an ordinal scale in Table 5.3. We denote by $LHp(j)$ the score associated with the degree of presence of seagrass Posidonia in geographic zone j, with the following scores:

- 0: Absence;
- 1: Weak presence;
- 2: Average presence;
- 3: Strong presence.

At this level, we need to solve the problem of both spatial decompositions in our problem. This asset is characterised by the lack of information about the exact distribution of seagrass Posidonia in the geographic units. Thus, we shall assume that its presence is uniform in all of them. This generates the following cases:

- For each geographic unit entirely included in a geographic zone, we consider that it has the same degree of presence of seagrass Posidonia as for the geographic zone;

- For geographic units shared between several geographic zones, we consider a weighted sum of the different degrees of presence of the seagrass in geographic zones. Weights in this work represent the relative surface at each geographic unit belonging to a given geographic zone.

The function describing these two cases would be

$$Sc(i) = \sum_{j \in Z} \frac{S_{ij}}{S_i} LHp(j)$$

where $Sc(i)$ represents the score associated with the presence of Posidonia at zone i, S_{ij} the surface (land excluded) of the geographic zone j and geographic unit i and S_i the surface of geographic unit i. We denote by $RSc(i)$, the rounded value of $Sc(i)$. We denote by $g_{Sp}(i, s)$, the seagrass Posidonia criterion rating the geographic unit i, under scenario s

$$g_{Sp}(i, (\beta_k, z_R P_j)) = Imp_i(z_i, c_w((\beta_k, RP_j), Sp))RSc(i) + 1.$$

Table 5.10 shows the assessment of the seagrass Posidonia criterion for the twelve scenarios.

Again, we could compute various aggregated indices.

RP_1 RP_2 RP_3 RP_4

β_1

β_2

β_3

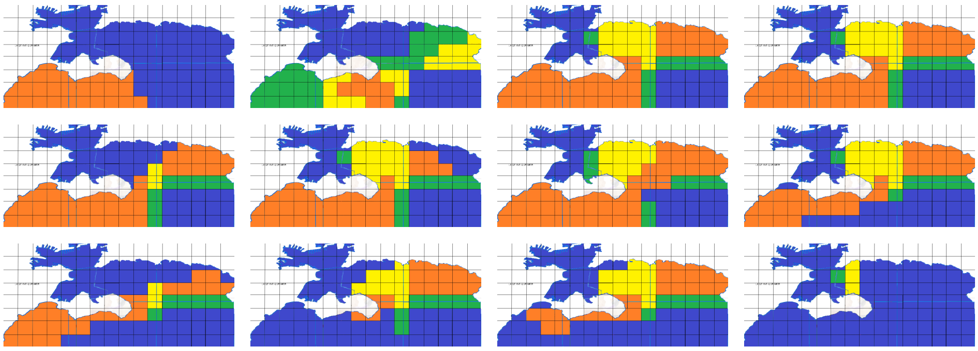


Table 5.10 – Seagrass-Posidonia criterion maps for the twelve scenarios

5.5 Multiple impacts

We finally consider the case with multiple criteria. Recall that for each scenario (β_i, RP_j), which occurs with probability $p_{ij} = q_i \times r_j$, we obtain four criterion maps:

- Fishing: $g_{P_e}(\beta_i, RP_j)$
- Fish farming: $g_{F_f}(\beta_i, RP_j)$
- Seagrass Posidonia: $g_{S_p}(\beta_i, RP_j)$
- Tourism: $g_T(\beta_i, RP_j)$

The aim of this section is to aggregate effects due to:

- multiple criteria.
- uncertainty.

In the first part of this section, we shall solve the multiple criteria problem. In the second part, we aggregate uncertainties by considering scenarios as criteria evaluating the geographic units in the aggregated maps with respect to their corresponding importance (probabilities).

5.5.1 ELECTRE-TRI for multiple criteria aggregation

The problem at hand is a rating one. To solve it we use the ELECTRE TRI method. The first step consists of rating each geographic unit X for each scenario (β_i, RP_j):

$$X(\beta_i, RP_j) = (g_{P_e}(\beta_i, RP_j), g_{F_f}(\beta_i, RP_j), g_{S_p}(\beta_i, RP_j), g_T(\beta_i, RP_j)).$$

We consider the following notation:

- the set of criteria $\mathscr{F}$, with criteria $\mathscr{F}_j$ characterised by an importance (weight) w_j.
- the set $\mathscr{C}$ of predefined impact categories. Each category $\mathscr{C}_k$ is characterised by a lower bound, called limiting profile, which we denote $r^k = (r_j^k)_{j\in\mathscr{F}}$.

The idea is, then, to compare the performance of each geographic unit with the limiting profiles to assign it to the corresponding category. Figure 5.12 illustrates the issue where the axes represent the criteria and we aim to assign x, a geographic unit, to one of the five predefined categories by comparing it with the limiting profiles.

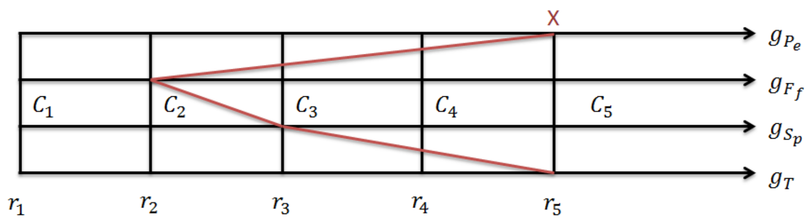


Figure 5.12 – Illustration of the multiple criteria problem

Assessing criteria weights

The literature reports several methods for assessing ELECTRE-TRI parameters from assignment examples, [34], [80], [81], [98], [110]. We use a simplified version of the optimisation model in [81], by assuming that we are able to assess, with the aid of the decision maker, the limiting profiles. In this work, we substituted the decision maker's opinion by technical experts from the IRSN. In general, the Prefect might be a decision maker in case of a nuclear accident. We denote by $A = \{A_1, \dots, A_5\}$ the learning set where the assignments are previously known, with $A_k = \{a_{ki}; a_{ki} \in \mathscr{C}_k\}$. The learning set consists on assignment examples: examples of performance vectors, for which the rating is previously known.

Under the previous assumption, and based on the majority rule, an alternative $a_k \in A_k$ from the learning set is assigned to category $\mathscr{C}_k$ if there is a weighted-majority of criteria in favour of "a_k is at least as good as the limiting profile r^k" and there is no weighted-majority in favour of "a_k is at least as good as the limiting profile r^{k+1}". This can be written as

$$\sum_{j \in \mathscr{F}, g_j(a_{ki}) \geq r_j^k} w_j \geq c,$$

and

$$\sum_{j \in \mathscr{F}, g_j(a_{ki}) \geq r_j^{k+1}} w_j < c,$$

where c is the concordance threshold. Such inequalities are equivalent to the following equalities, introducing the slack variables x_{ki} and y_{ki}:

$$\sum_{j \in \mathscr{F}, g_j(a_{ki}) \geq r_j^k} w_j - x_k = c$$

and

$$\sum_{j \in \mathscr{F}, g_j(a_{ki}) \geq r_j^{k+1}} w_j + y_k = c.$$

If the slack variables x_{ki} and y_{ki} are positive, the assignment made by the decision maker corresponds to the assignment done through the pessimistic procedure of ELECTRE TRI: the lower the minimum of these values, the less adapted is the model. In case one of these slack variables is negative, the concordance principle is not sufficient to justify the assignment and we need to assess the veto threshold. Thus, we need to maximise the minimum of both slack variables to take into account the worst assignment from the decision maker through

$$\max \min_{\substack{a_{ki} \in A_k \\ A_k \in A}} (x_{ki}, y_{ki})$$

and we also need to maximise the ability of the model to assign alternatives correctly through

$$\max \sum_{\substack{a_{ki} \in A_k \\ A_k \in A}} (x_{ki} + y_{ki}).$$

We then consider the following decision variables:

- Weight vector. $w_j, \forall j \in \mathscr{F}$
- Concordance threshold c
- Slack variables $x_{ki}, y_{ki}, \forall a_{ki} \in A_k, \forall k$

and the following objective function to be maximised

$$\text{maximise} \left(\min_{\substack{a_{ki} \in A_k \\ A_k \in A}} (x_{ki}, y_{ki}) + \epsilon \sum_{\substack{a_{ki} \in A_k \\ A_k \in A}} (x_{ki} + y_{ki}) \right) \tag{5.1}$$

Problem (5.1) is equivalent to

$$\begin{aligned} \text{maximise} \quad & \delta + \epsilon \sum_{\substack{a_{ki} \in A_k \\ A_k \in A}} (x_{ki} + y_{ki}) \\ \text{s.t} \quad & \delta \leq x_{ki}, \forall a_{ki} \in A_k, \forall A_k \in A \\ & \delta \leq y_{ki}, \forall a_{ki} \in A_k, \forall A_k \in A. \end{aligned}$$

In order to assess criteria weights, we add to the previous model the following constraints:

- Two constraints related with the slack variables $\forall a_{ki} \in A_k, \forall A_k \in A$, $\sum_{\substack{j\in\mathscr{F} \\ g_j(a_{ki})\geq r_j^k}} w_j - x_{ki} = c$ and $\sum_{\substack{j\in\mathscr{F} \\ g_j(a_{ki})\geq r_j^{k+1}}} w_j + y_{ki} = c$.
- The majority constraint related to the concordance principle $c > 0.5$.
- We assume that all criteria are relevant, $w_j < c, \forall j \in \mathscr{F}$.
- The strict positivity and normalisation of weights: we respectively have $\forall j \in \mathscr{F}, w_j > 0$ and $\sum_j w_j = 1$.

We finally use the following model:

$$\begin{aligned}
\text{maximise} \quad & \delta + \epsilon \sum_{\substack{a_{ki}\in A_k \\ A_k\in A}} (x_{ki} + y_{ki}) \\
\text{s.t} \quad & \delta \leq x_{ki}, \forall a_{ki} \in A_k, \forall A_k \in A, \\
& \delta \leq y_{ki}, \forall a_{ki} \in A_k, \forall A_k \in A, \\
& \sum_{\substack{j\in\mathscr{F} \\ g_j(a_{ki})\geq r_j^k}} w_j - x_{ki} = c, \forall a_{ki} \in A_k, \forall A_k \in A, \\
& \sum_{\substack{j\in\mathscr{F} \\ g_j(a_{ki})\geq r_j^{k+1}}} w_j + y_{ki} = c, \forall a_{ki} \in A_k, \forall A_k \in A, \\
& \sum_j w_j = 1, \\
& w_j < c, \forall j \in \mathscr{F}, \\
& w_j > 0, \forall j \in \mathscr{F}, \\
& 0.5 < c < 1.
\end{aligned} \tag{5.2}$$

Example: We consider the following learning sets:

$$A_2 = \{(1,3,2,1),(1,2,3,1),(3,1,1,1)\}$$

$$A_3 = \{(2,4,3,3),(4,2,3,2),(3,3,2,4)\}$$

$$A_4 = \{(2,4,4,5),(4,4,4,3),(5,5,3,3)\}$$

$$A_5 = \{(5,4,4,5),(5,4,5,3),(3,3,5,5)\}$$

The limiting profile of a category $\mathscr{C}_k$ is the vector (k,k,k,k). Therefore, we will not consider a learning set associated with category $\mathscr{C}_1$ (no impact), since it does not provide us with any relevant information. A profile in A_1 will always dominate $(1,1,1,1)$; based on the majority principle it will always be outranked by $(2,2,2,2)$, otherwise it will not be assigned to $\mathscr{C}_1$. Hence, both x_1 and y_1 are positive.

The tourism and fishing sectors are more sensitive than that of fish farming since they are present in most of the geographic units. Hence, we consider two additional constraints, $w_1 \geq w_2$ and $w_4 \geq w_2$.

The solution of the model (5.2) is:

- weights: $w_1 = 0.33$; $w_2 = 0.1$; $w_3 = 0.23$; $w_4 = 0.34$;
- concordance threshold: $c = 0.54$;
- the slack variables:
  - slacks associated with $A_2 = \{(1,3,2,1),(1,2,3,1),(3,1,1,1)\}$ are $(x_{21} = -0.17, y_{21} = 0.41)$, $(x_{22} = -0.07, y_{22} = 0.17)$, $(x_{23} = -0.17, y_{23} = 0.17)$;
  - slacks associated with $A_3 = \{(2,4,3,3),(4,2,3,2),(3,3,2,4)\}$ are $(x_{31} = 0.15, y_{31} = 0.41)$, $(x_{32} = 0.35, y_{32} = 0.17)$, $(x_{33} = 0.25, y_{33} = 0.17)$;
  - slacks associated with $A_4 = \{(2,4,4,5),(4,4,4,3),(5,5,3,3)\}$ are $(x_{41} = 0,15, y_{41} = 0.17)$, $(x_{42} = 0.15, y_{42} = 0.51)$, $(x_{43} = -0.7, y_{43} = 0.07)$;

– slacks associated with $A_5 = \{(5,4,4,5),(5,4,5,3),(3,3,5,5)\}$ are $x_{51} = 0.15$, $x_{52} = 0.05$, $x_{53} = 0.05$.

Negative slack variables can be justified due to the non consideration of the veto threshold in our linear model. For example, vector $(1,3,2,1)$, using the majority principle, should be assigned to category $\mathscr{C}_1$, since $w_1 + w_4 > c$. However, $(1,3,2,1)$ is assigned to 2 because of its performance (a rate 3) under the fish farming criterion, and thus we cannot consider that there is no considerable impact. A similar remark is valid for $(1,2,3,1)$, $(3,1,1,1)$ and $(5,5,3,3)$. Based on an observation over assignment examples with negative slack variables, a threshold value equal to 2 is the minimum value justifying the assignments.

Assignment zones to the predefined categories

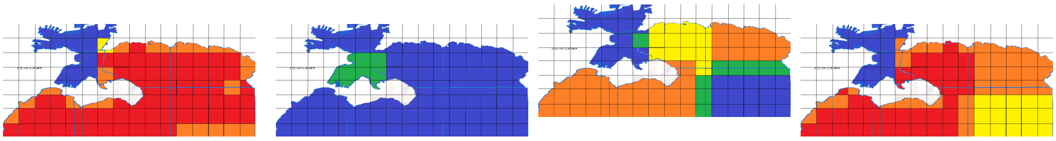
We show now the results of the multiple criteria aggregation procedure using ELECTRE-TRI. The parameters we use are derived from the example in Section 5.5.1:

- criteria weights: $w_1 = 0.33$; $w_2 = 0.1$; $w_3 = 0.23$; $w_4 = 0.34$;
- concordance threshold: $c = 0.54$;
- veto threshold $v = 2$;

In what follows, we display the criteria-maps associated with the scenarios $(Mistral, RP_3)$, and the corresponding aggregated map.

Scenario $(Mistral, RP_3)$

These maps display the criteria for fishing, fish farming, seagrass Posidonia and tourism, respectively. They are assessed based on the level of cesium concentration, from 1 to 5, where level 1 refers to low concentration and level 5 to a high concentration, and the vulnerability of each geographic unit from a given asset point of view. For example, zones 1 and 2 are not very impacted



Fishing Fish Farming Seagrass Posidonia Tourism

Figure 5.13 – Criteria maps for the $(mistral, RP_3)$

because of a low level of concentration; however, zone 3 is characterised by a level 5 of cesium concentration, crossed with important tourist and fishing activities, an average presence of seagrass Posidonia and no activity of fish farming. Thus, the outcome of the multiple criteria aggregation mostly associate a rate 1 to geographic units in zones 1 and 2 and a rate 5 in zone 3 (recall that $w_1 + w_4 = 0.67 > 0.54$ and there is no discordance). The result of the aggregation is displayed in Figure 5.14.

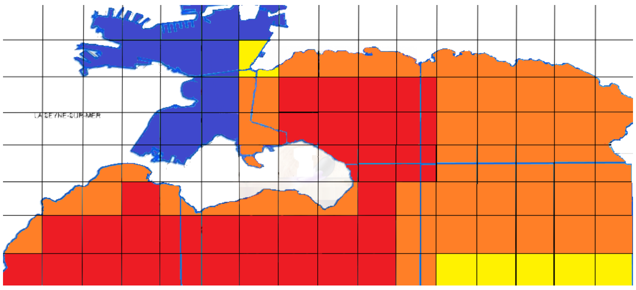


Figure 5.14 – The aggregated map for $(mistral, RP_3)$ scenario

5.5.2 Uncertainty aggregation

The aim of this section is to model uncertainties represented through different accident scenarios. We need to establish a global rate for geographic units. We can aggregate the impact induced by different scenarios either before aggregating criteria or after the aggregation. The most common technique synthesising uncertainties is to compute expected values. We used the expected impact in section 5.4, before the multiple criteria aggregation procedure, in order to evaluate the sensitivity of each geographic unit from a single criterion point of view. In this section, we deal with the case in which we want to synthesize uncertainties related with the accident scenarios after the multiple criteria aggregation, section 5.5.1.

Computing the expected impact at each geographic unit allows for compensation between rates with respect to the probabilities over scenarios. In our context, such compensation is not desirable since the performance of geographic units under each scenario is modeled through rates. The aggregation procedure proposed in this section is based on the concordance and discordance principles, reflected in ELECTRE TRI, and can be solved as a multiple criteria rating problem, by considering scenarios as criteria, probabilities as weights and geographic units as alternatives to be evaluated.

Probabilities and ELECTRE TRI parameters

In this section, we assess uncertainties over the initial conditions and the ELECTRE TRI parameters to rate the geographic units. In section 5.2.2, we defined three sea conditions, corresponding to different types of wind. In what follows, we associate to the types of wind the following probabilities [39]: For mistral $q_1 = 0.4$, for east wind $q_2 = 0.4$ and for steady wind $q_3 = 0.2$.

To assess probabilities over the four release positions, we assume that the closer we are to the naval base, the greater the probability of a release. Such hypothesis can be transcribed through the following inequalities $r_i > r_j$ where $i > j$, with $r_i > 0$, $\sum_{i=1}^{4} r_i = 1$. One possible assessment would be $r_1 = 0.5, r_2 = 0.25, r_3 = 0.15$ and $r_4 = 0.1$, which we use in our initial analysis.

In what follows, we shall assume such values. A sensitivity analysis with respect to them, based on intervals, would be necessary, but we shall not include it in this chapter. Observe now that this "multiple criteria decision making problem" is characterised by:

- The criteria evaluating the geographic units: the release scenarios;

- The weights of criteria: the probabilities $p_{ij} = q_i \times r_j$;
- Under each scenario, impacts on geographic units are rated from 1 to 5. We shall consider the same scale for the aggregated rate;
- The veto threshold: $v = 2$.

Results of the aggregation

We represent now the results of the aggregation over the 12 scenarios using ELECTRE TRI and the parameters in section 5.5.2.

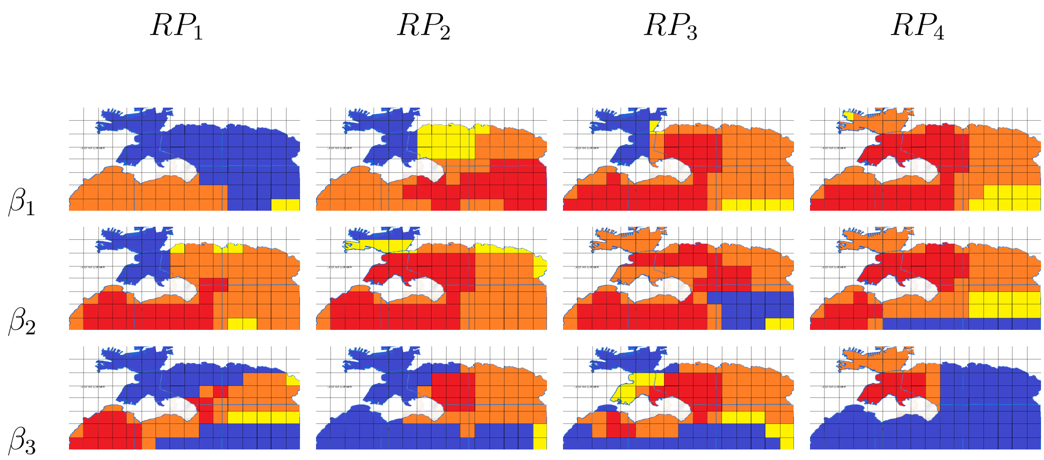


Table 5.11 – The aggregated maps for the twelve scenarios

From Table 5.11, we notice that for the release position RP_1, the geographic zone 5 is highly impacted. This is justified by the simulated release position in Zone 5, and the high importance of economic environmental assets in this area. This remark is still valid for Zones 3 and 6 for RP_2 and 3 and 4 for RP_3. depending on the direction of wind, other zones might be highly impacted. For instance, considering the scenarios characterised by a Steady wind, the impacted zones are those close to the release position. We also note from Table 5.11 that the most impacting scenarios are those corresponding to East wind. The main reason is the high dispersion of radionuclides in the majority of geographic zones due to the sea currents, which impact many assets.

Zone 1 where the simulated RP_4 took place is highly impacted, rated 4, but less impacted than other neighbouring zones, even if the contamination level is the highest. This is due to several reasons, such as the non presence of seagrass Posidonia and fish farming activity, representing a total weight $w_2 + w_3 = 0.33$, the low income from fishing activity compared to the other geographic zones.

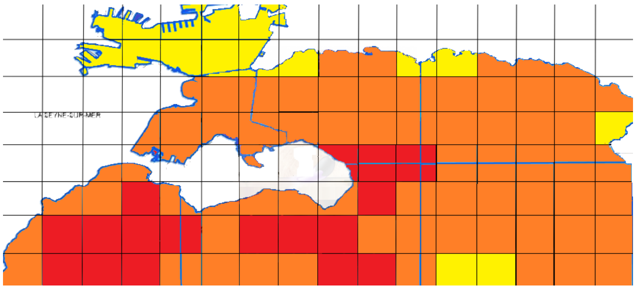


Figure 5.15 – The aggregated map for the 12 release scenarios

Aggregating the release scenarios, we note from Figure 5.15 that geographic zone 1 seems the less impacted. The reason of such level of impact in zone 1 is justified by the low presence or absence of the majority of assets and the low level of concentration at several release scenarios. The other geographic zones are either rated 4 or 5, since scenarios corresponding to East wind, occurring with a probability of 0.4, impact highly the majority of the Bay and scenarios RP_2, RP_3 and RP_4 in the case of Mistral type of wind, occurring with a total probability of 0.2, impact highly zones 3, 4, 5, 6 and 7. Figure 5.15 is oriented to illustrate the post-accidental risk in the Bay in case of a nuclear accident. This post-accidental risk map takes into consideration the vulnerability of the geographic units regarding the importance of the assets and the different scenarios. The resulting map might be not very informative for the decision maker, since the majority of geographic unit are either risky or very risky. However, the used procedure still interesting to be applied in wider areas.

5.6 Discussion

The chapter presents an approach to assess the impact of a nuclear accident on the Bay of Toulon. This work can be extended to handle the case of different release accidents that are spatially and temporally very close by only updating the concentrations of radioactive substences in each geographic unit. The study remains valid also if the accidents are not spatially close. However in case they are not temporally close, the results will be biased by the short terms impacts before the second release.

For the typical reasons related to a real world application we were induced to make a number of simplification hypotheses which we discuss here. Clearly these also indicate relevant research directions to explore.

5.6.1 The use of homogeneous zones

The assumption of homogeneous zones is very strong. However, it is an important assumption since on one hand we use a different decomposition to simulate the dispersion of the contaminant in the Bay. This decomposition, into 97 geographic, is dictated by the numerical model associated to the dispersion process of the contaminant, see section 5.2.2. On the other hand, data describing the assets involved in the Bay, characterize each of the seven zones. This lead to two possible solutions: The first one consists on considering the geographic units (the 97) as homogeneous and assess the impacts on micro zones based on the contamination levels. The second solution consists on aggregating the contamination levels for the 97 geographic units in order to rate the contamination level in each of the 7 geographic zones, then apply the data associated to each asset. Both approaches lead to the same result in case we assume that the geographic zones are homogeneous.

The use of homogeneous zones is also particulary useful for integrating and using outranking methods in GIS, as underlined by several authors, including Chakhar and Mousseau [24] and Joerin et al. [61]. This is because outranking methods may face difficulties since they have serious computational limitations with respect to the number of decision alternatives, as remarked by Marinoni [73].

5.6.2 Analysis of the multiple criteria aggregation procedures

The multiple criteria aggregation procedure used in this chapter is based upon the concordance and non discordance principles. The obtained results are coherent. Nevertheless, the use of ELECTRE TRI method might lead to inconsistent results. For instance, let us consider the impact vector $(5, 1, 1, 3)$ characterizing the geographic units in zone 7 in case of scenario $(Mistral, RP_3)$. Because of the discordance principle $(5, 1, 1, 3)$ will be rated 4. Let us consider a fictitious geographic unit characterized by $(4, 3, 3, 3)$, using the same parameters, this last will be rated 3. However, $(5, 1, 1, 3)$ is strictly preferred to $(4, 3, 3, 3)$, due to a veto of 2. Other inconsistencies, might come from the Condorcet Paradox due to the concordance principle.

5.6.3 Evaluating a map

A relevant question for the decision maker can be, how can we rate a geographic area? The answer to this question is not simple. The rate of a geographic space depends upon:

- the characteristics of the problem, e.g. we may have interactions between geographic units (or not) [78];
- the aggregation path, e.g. one possible path is aggregating the multiple criteria problem, then synthesising uncertainties before rating the global

map. Changing this order may lead to a different result.

In this work, the interaction effects between geographic units is not taken into account because, in all simulations, geographic units belonging to the same category of impact are grouped together.

5.7 Conclusion

We have presented an approach to assess spatial risks, in cases characterised by the presence of several assets, spatial characteristics and uncertainties over the accident parameters (mainly the release position and sea currents). The developed approach is illustrated through an application of nuclear releases in the marine environment. The methodology aims to assess the impact of a nuclear accident at a geographic space (in our case the Bay of Toulon) as part of a post-accident analysis. In order to evaluate the impact of a nuclear release on a geographic space, several methods were used for decision aiding purposes. The procedure developed consists of representing uncertainties through accident scenarios, structuring impact indices for each asset and under each scenario, and synthesising these indices using a multiple criteria aggregation procedure, describing the general impact over the studied area. We then aggregated uncertainties to evaluate the vulnerability of the studied area regarding the accident scenarios. At a next step, we shall establish a robustness analysis and study the possible recommendations to one or several decision markers, depending on their risk aversion.

6 Dynamic-R

A particular case of Dynamic-R was published in the proceeding of ICDSST conference [88]. This chapter is submitted to the European Journal of Operational Research.

6.1 Introduction

In this chapter, we propose a new MCDA (Multiple Criteria Decision Analysis) method aiming at providing a "convincing" rating to a set of objects, here after named A, such as geographic units, financial products, clients in an insurance company, to name but a few, evaluated by ordinal information under at least one dimension. A rating problem statement [25] consists in partitioning A into predefined and ordered equivalence classes, called categories, identified by ratings. Since we are dealing with objects evaluated under several dimensions, called criteria, we will consider rating problem statements in the context of MCDA. By "convincing", we refer to the following claims:

Claim 1. *No better object is assigned to a worse category when objects are compared.*

Claim 2. *We provide a complete rating.*

Several MCDA methods have been developed to deal with rating problems. These methods can be partitioned into three categories:

1. methods based on the majority principle, called *outranking* methods, see [2] [3] [44] [72] [101] [106];

2. methods based on the assessment of utility functions, see [21] [30] [31] [57] [69];

3. methods based on rough sets, see [29] [56] [55] [54].

In this work, we are interested in the same type of problems for which Outranking methods fit. Outranking methods, in the context of rating problems, are based on preference relations established between the set A and reference profiles without considering comparisons among objects. Because of this feature, Outranking methods may lead to non-convincing ratings, because of cycles of preferences or because of incomparabilities. This is because Outranking relations do not have any remarkable ordering properties, see [15]. Consider the following example:

Example 1. *(Non convincing rating due to the Condorcet Paradox)*

Let us consider a rating problem characterized by three necessary and sufficient criteria, i.e. the three are exhaustive and none of them is a dictator. This comes to considering any coalition of two criteria as a decisive coalition. We consider that each criterion evaluates the set A on an ordinal scale: $\{B, A, A^+\}$. In this problem we aim at assigning two objects $x = (A^+, A, B)$ and $y = (A, B, A^+)$ into two predefined ordered categories $\mathscr{C}_1$ (rate 1) and $\mathscr{C}_2$ (rate 2) such that $\mathscr{C}_1$ is the best. The two categories are separated by a lower bound of $\mathscr{C}_1$: $p = (B, A^+, A)$. Using the majority rule to rate x and y, we obtain: $y \succ p$ and $p \succ x$, where $\succ$ refers to the strict preference relation. Thus, y will be rated 1 while x will be rated 2. The decision maker might be not convinced by the result: indeed x is strictly better than y (assuming the same majority rule).

The originality of this work consists in handling this type of inconsistencies, through a new "dynamic" and "convincing" MCDA rating method, named

"Dynamic-R", for problems characterized by ordinal information under at least one criterion. A "convincing" rating is based on clear positive and negative reasons, respectively supporting and opposing a rating, and solves any potential contradiction. The dynamic aspect of the method is related to the rating procedure associated to the method: the rated objects are added to the profiles characterizing the categories and are used in the next time step when new objects are considered for rating. Hence, each time step, a new set A is considered for rating, the positive and negative reasons will be updated in order to take into account preferential information coming from these just-rated objects. In order to obtain a "convincing" rating, we address the following features:

- We allow comparison among elements in the set A;
- We allow both limiting and typical profiles;
- We separate positive and negative reasons for and against a rating;
- We provide a monotonic and complete rating, as a result of our rating procedure.

The chapter is organized as follows. Section 2, introduces notations used all along the chapter. In section 3, we introduce the developed method and its properties. In section 4, we introduce the basic concepts within Dynamic-R. In section 5, we present the theoretical foundations of the method. In section 6, we present an analysis of the performance quality of Dynamic-R. We then present a variant of Dynamic-R, named Dynamic-R 2.0 ans we end by a conclusion and discussion.

6.2 Notations and concepts

All along this document we will use the following notation:

- A set of time steps $T = \{1, 2, 3, ...\}$. Generally we will use $t \in T$ to refer to a time step in the process. For simplicity we will use the term step to refer to a time step in which a new set of objects is considered for rating.

- At each step $t \in T$, a set of studied objects $A^t = \{x, y, z, w, ...\}$ is considered for rating. The set A^t can be either known previously, or elicited during an interactive process between a decision analyst and a client. This set is traditionally called in the literature associated to decision sciences, alternatives or actions [18].

- In this work we will use the mathematical notation "$[\![\ ;\ ]\!]$", to refer to the integer interval.

- A set of predefined ordered categories $\mathscr{C} = \{\mathscr{C}_1, ..., \mathscr{C}_q\}$, $q \geq 2$, where $\mathscr{C}_k$ refers to a category where all objects are rated k. Without loss of generality, we assume that,$\forall k \in [\![1\ ;\ q-1]\!]$: $\mathscr{C}_k$ is better than $\mathscr{C}_{k+1}$. Hence, $\mathscr{C}_1$ is the best category.

- Reference profiles, at a step $t \in T$: $Z^t = \{Z_1^t, ..., Z_q^t\}$, where $Z_h^t = \{z_{h,k}, k = 1, ..., i_{h,t}\}$, $i_{h,t} \geq 1$, represents the set of reference profiles characterizing the category $\mathscr{C}_h$, at the step t. The initial set of reference profiles Z^0 is used as a learning set to generate the preferential information. At the end of each step $t \in T$, objects in A^t will be assigned to the sets of reference profiles associated to the corresponding categories. We will use also the notation: $\forall j, k \in [\![1\ ,\ q]\!], j < q$: $Z_{j,k}^t$ to refer to $\bigcup\limits_{i \in [\![j\ ,\ k]\!]} Z_i^t$.

- A set of minimal requirements $\mathscr{B} = \{b_1, ..., b_q\}$, characterizing categories where performances of the profile $b_k = (\underline{b}_{j,k})_{j \in \mathscr{F}}$ characterizing $\mathscr{C}_k$, are the minimal performances in order to be admissible in $\mathscr{C}_k$. These minimum requirements are characterized by the following condition: We assume that $\forall j \in \mathscr{F}, \forall k \in [\![1\ ;\ q-1]\!] : b_k \succsim_j b_{k+1}$. The profile b_k should not be confused with a limiting profile since it does not necessarily belong to $\mathscr{C}_k$.

- The set of all objects $\mathscr{A}^t = \cup_k Z_k^t \cup A^t \cup \mathscr{B}$ considered at the step t of the rating aggregation procedure.
- A Family of criteria $\mathscr{F} = \{1, ..., m\}$ with $m \geq 3$ under which objects are evaluated. We associate to each criterion $j \in \mathscr{F}$ a weak order $\succsim_j$ upon $\mathscr{A}^t$.
- Importance of coalitions of criteria, w. It is a capacity defined as: $w : 2^{\mathscr{F}} \to [0, 1]$. By definition of capacity we have $w(\mathscr{F}) = 1$, $w(\emptyset) = 0$, and for all $A, B \in 2^{\mathscr{F}}$ such that $A \subseteq B$, $w(A) \leq w(B)$. To simplify notations, we will use w_j to refer to $w(\{j\})$.
- Importance of the discordant criteria to reject a preference relation, $\mathscr{V}$. It is a capacity defined as: $\mathscr{V} : 2^{\mathscr{F}} \to [0, 1]$. By definition of capacity we have $\mathscr{V}(\mathscr{F}) = 1$ (all criteria reject a given preference), $\mathscr{V}(\emptyset) = 0$, and for all $A, B \in 2^{\mathscr{F}}$ such that $A \subseteq B$, $\mathscr{V}(A) \leq \mathscr{V}(B)$.
- Parameters: λ the majority, considered sufficient, enabling a coalition to be decisive, called concordance threshold; v the veto threshold.
- The set of objects having negative reasons to be rated k or better, based on the comparison with reference profiles: $\forall k \in [\![1\ ;\ q-1]\!] : U^-_{r,k}$. The notation $U^-_{r,0}$ will be used to refer to the set of objects not having negative reasons, against being rated 1, based on the comparison with reference profiles: $U^-_{r,0} = A^t \cap \neg(U^-_{r,1})$
- The set of objects having positive reasons to be rated k or worse, based on the comparison with reference profiles: $U^+_{r,k}$.
- The set of objects for which negative reasons are enriched, due to the comparison with the other objects in A^t, to a worse category k: $U^-_{er,k}$.

- The set of objects for which negative reasons are withdrawn, due to the comparison with the other objects in A^t, to a better category k: $U^-_{rr,k}$.
- The set of objects for which positive reasons are enriched, due to the comparison with the other objects in A^t, to a better category k: $U^+_{ur,k}$.
- The set of objects for which the worst possible rating is k (without taking into account the way objects compare to each other): L^t_k.
- The set of objects for which the best possible rating is k (without taking into account the way objects compare to each other): H^t_k.
- The set of objects for which the worst possible rating is k, with respect to reference profiles and objects in A^t: $L^t_{u,k}$.
- The set of objects for which the best possible rating is k, with respect to reference profiles and objects in A^t: $H^t_{u,k}$.
- The set of objects in $H^t_{u,h} \cap L^t_{u,l}$ (for $h \leq k$ and $l \geq k$) rated k, based on a distance from reference profiles: $U^+_{2r,k}$.

6.3 General overview of Dynamic-R

In this section, we outline the developed method, named Dynamic-R. At first, we present the general architecture of the existing multiple criteria rating methods based on the majority rule and their extensions with a consistency checking. Then we provide a description of the Dynamic-R and we present the problems to which it fits. Finally we will present and discuss the sets of positive and negative reasons used in this work and we address the rating procedure.

6.3.1 Outranking methodology for rating problems

The existing rating procedures based upon the use of Outranking relations use a majority principle applied on positive reasons, this being bounded by a minority principle (usually a veto condition) which can invalidate the aggregation of the positive reasons. Positive reasons are typically obtained comparing objects either to limiting profiles (a vector or a set of vectors) separating categories, or to typical profiles (a vector or a set of vectors) characterising the categories. In the first case we make use of asymmetric comparisons (intuitively an object x is rated k if it is better than the profile separating category k from category $k+1$), while in the second case we make use of symmetric comparisons (intuitively an object x is rated into category k if it is similar to a typical profile of such category). In both approaches objects are never compared to each other.

Several rating methods have been developed aiming at rating a set of objects with respect to a consistency rule. For example, C. Rocha and L.C. Dias in [90] developed the PASA (Progressive Assisted Sorting Algorithm) method, respecting the following consistency principle: an object cannot be assigned to a category in case it is outranked by any example (reference profile) assigned to a worse category. This principle seems very close to our work since we also characterize the categories by a set of reference profiles and we have a consistency rule. However, this method presents also many disadvantages such as:

- the order of the selected objects for rating might bias the ratings of the next selected objects;
- in case of an imprecise rating, either the decision maker is needed or the rating is postponed;
- forcing the consistency might lead to bad quality of rating: objects involved in cycles are placed in the same category (the worse category

among the ones to which objects can be assigned).

The THESEUS method [45] is another rating method, aiming at providing a rating minimizing inconsistencies with respect to a learning set (reference profiles in our case). This method is based on an original approach, transforming a rating problem into a ranking problem. Such transformation consists on associating to each non rated object x, new alternatives x_k: "assign x to the category k". The generated alternatives x_k are assessed under the following criteria: inconsistencies with respect to the strict preference, the weak preference, and the indifference. Hence, the problem of rating x, comes to a ranking problem associated to selecting the best x_k, minimizing the inconsistencies. We address the following weaknesses of THESEUS method:

- The provided rating minimizes inconsistencies. However, it does not prevent an inconsistent rating;
- The dependency on the learning set: both small and very big learning sets may lead to a poor rating either because of incomparabilities or the high number of inconsistencies.

It is true that some machine learning methods based on decision rules such as DRSA, provide a rating respecting the convincing claims. However, these methods require a large learning set. In many decision aiding problems, all what we can have are few assignment examples given by the decision maker (the client).
The next section will be dedicated to present Dynamic-R and its advantages.

6.3.2 Description of Dynamic-R and problems to which it fits

Dynamic-R introduces three new ideas:

1. it does not make any distinction between limiting and typical profiles since both of them might be available and provide positive or negative reasons about the rating of a given object x;

2. it explicitly introduces the concept of minimal requirements, a disjunctive constraint among the criteria, providing strong evidence that an object CANNOT be rated to a certain category (because it fails to satisfy a requirement on any of the criteria), without the vector of minimal requirements being a profile of any category;

3. it accumulates reference profiles since objects, that are rated at step t, are used as profiles both at step $t+1$and as consistency checking within step t, thus allowing comparisons among objects.

Dynamic-R is a MCDA rating method extending the use of the concordance/discordance principles through the use of generalised positive and negative reasons for which a given object can belong to a given category. The main inputs required by the method are: the set of partitions of reference profiles characterizing the categories Z^t, and the set of minimum requirements $\mathscr{B}$. At the basic level, the developed rating procedure, at each time step $t \in T$, is based on the assessment of: on the one hand, subsets of objects $U^+_{r,k} \subseteq A^t$, $k \in [\![1\ ,\ q]\!]$, having reasons supporting their rating at most k (k or worse). Such set is based on the presence of a sufficient majority of criteria, not disqualified by a veto, in favor of an object in A^t, compared to a reference profile characterizing the category k.

Example 2. *(Example of the set of positive reasons)*

A new student in a school, might have positive reasons to be in the category of excellent students, if he is better, according to a majority of criteria, than a former excellent student. Having positive reasons to be in the category of excellent students implies having positive reasons to be in any worse category, such as the one of good or even bad students.

On the other hand, subsets of objects $U^{-}_{r,k} \subseteq A^t$, $k \in [\![1\ ,\ q-1]\!]$, having reasons opposing their rating at least k (opposing a rating to k or better). Such negative reasons might come either from the incompatibility with category k due to the violation of the minimum requirements, or the dominance or the strict preference (depending on the way negative reasons are defined) in favor of a reference profile characterizing a worse category. The concept of minimum requirements consists on profiles representing the minimum acceptable performances, under each criterion, regardless the global performance, in order to be admissible in a category, as illustrated in the following example:

Example 3. *(Example of negative reasons due to incompatibility)*

Regardless the global mark, a student cannot be considered a good student if he performs worse than 7/20 in any of the lectures. Minimum requirements should not be confused with limiting profiles: in the previous example $(7/20, ..., 7/20)$ is the minimum requirement associated to the category of good students, however, a student performing 7/20 in all the lectures "$(7/20, ..., 7/20)$" is not a good student.

Remark 7. *In case the set of minimum requirements $\mathscr{B}$ is not empty, and the number of objects to be rated and reference profiles is important, it is better to not consider the strict preference relation in negative reasons, for two reasons:*

1. *The negative discrimination power due to vetoes, with respect to limiting profiles, might be substituted by the minimum requirements in the case where reference profiles are not necessarily limiting profiles. This substitution provide many advantages as the assessment of minimum requirements is directly related to the categories while their might exist a very high number of limiting profiles and thus an object discriminated by a limiting profile might be not discriminated by another.*

2. *Negative reasons based on strict preference might influence badly the quality of the obtained rating, due to the non-transitivity: discriminating*

the assignment of an object to a category due to a strict preference in favor of a reference profile might be criticized since we might have cycles.

Hence, the use of the strict preference in negative reasons will be limited to the cases where $\mathscr{B} = \emptyset$ *and the number of objects to be assigned is low. Here after, negative reasons will be treated in two cases, whether strict preference is considered or not.*

When the decision maker or the quality of the rating problem require taking into account the way objects compare to each other, new positive and negative reasons might appear, and some reference profiles might need to be updated. Considering the way objects compare to each other may lead to either enriching negative reasons, in case strict preference is used in the assessment of negative reasons, or enriching positive reasons, or withdrawing negative reasons.

The rating process associated to Dynamic-R, at a step t, can be structured as follow:

1. For each object $x \in A^t$, we compute for each category k, the sets of objects having respectively positive and negative reasons to be rated k: $U^+_{r,k}$ and $U^-_{r,k}$.

2. We revise the positive and negative reasons for each object, and the reference profile based on the way they compare to each other. The possible updates lead to

   (a) a set of objects $U^-_{er,k}$ not having initially negative reasons opposing rating k(not in $U^-_{r,k}$), but for which their negative reasons were enriched to oppose rating k.

   (b) a set of objects $U^+_{ur,k}$ not having initially positive reasons supporting rating k(not in $U^+_{r,k}$), but for which their positive reasons were enriched to support rating k.

(c) a set of objects $U^{-}_{rr,k}$ for which negative reasons opposing rating worse than k are withdrawn to oppose a rating k.

We then compute the updated reference profiles Z^t_u and the updated set of objects to be rated A^t_u.

3. We compute $H^t_{u,k}$ and $L^t_{u,k}$, $\forall k \in [\![1\ ;\ q]\!]$. All objects in $H^t_{u,k} \cap L^t_{u,k}$ will be assigned to Z^{t+1}_k. We distinguish two cases:

   (a) Objects belonging to any among the sets $H^t_{u,1} \cap L^t_{u,1}, ..., H^t_{u,q} \cap L^t_{u,q}$. In other terms, objects having the same maximum and minimum rating. These objects are rated k.

   (b) Objects have different minimum and maximum rating $(A^t_u \backslash \cup_k (H^t_{u,k} \cap L^t_{u,k}))$ we can consider them as interval rated. In such a case, we compute a distance between objects and reference profiles characterizing the possible categories and we choose the "nearest" one. This is done through the use of $U^{+}_{2r,k}$. The distance is computed first over objects in $H^t_{u,1}$, then $H^t_{u,2}$,..., and we end by objects in $H^t_{u,q}$. Each time an object is rated based on the distance, we assign it to the corresponding set in Z^{t+1}_u and we update positive reasons for objects in worse categories. This procedure is repeated until all objects are rated.

Example 4. Imagine the situation of two students x and y, such that, one the one hand, x might be either exceptional, or excellent, or good student. On the other hand, y might be either excellent or good student. In case there is a sufficient majority of criteria in favor of y, with respect to x, and x is close to former exceptional students, x will provide y by positive reasons to be assigned to the category of exceptional students. However, since the best possible rating for y is excellent student, then y will be rated as excellent student without computing his distance with former students in each category (thanks to x). We will note $U^{+}_{2r,k}$ the set of objects close to a category k, for which they have neither positive reasons or valid negative reasons.

Remark 8. *In case the strict preference relation is not considered in the assessment of negative reasons, these will not be enriched:* $\forall k \in [\![1\ ,\ q-1]\!] : U^{-}_{er,k} = \emptyset$. *This is due to the transitivity of both the dominance relation and the non violation of minimum requirements (more details will be provided in section 6.5.5).*

The order of the assessment of the updated sets of positive and negative reasons is important. The following example illustrates the case.

Example 5. let's consider three new students x, y and s, such that: x, y and s have positive and no negative reasons to be considered as a good student, an excellent student and an average student respectively. Let's assume that according to a sufficient majority of criteria, the student x is at least as good as y. Hence, based on this information, positive reasons will be enriched in order to support rating x as an excellent student. However, in case the student s is strictly preferred (better) to y, and there are negative reasons against being considered as an excellent student, the student y cannot be considered anymore as excellent, this corresponds to the enrichment of negative reasons. As a result of enriching negative reasons of y, the enrichment of positive reasons of x is no more valid. For this reason, $\forall k \in [\![1\ ,\ q-1]\!] : U^{-}_{er,k}$ should be computed before $\forall k \in [\![1\ ,\ q]\!] : U^{+}_{ur,k}$. Furthermore, withdrawing negative reasons takes into account the enriched negative reasons, and can be generated by the enriched positive reasons. Let's consider that the student s is considered as average because his performance is strictly worst, according to a sufficient majority of criteria, than the performance of a former average student z (a reference profile in the category of average students). In this case, the rating of z has no negative reasons against being in the category of excellent students, and his positive reasons were enriched based on his comparison with the new students. Thus, the rating of z will be improved leading to withdrawing negative reasons against s.

For all $k \in [\![1\ ,\ q]\!]$, the sets H_k^t and L_k^t might give an idea about the quality of the rating, by drawing a distribution of the precision of the rating. We can also provide the decision maker by statistics such as the median and the mode of the rated objects among the categories, or the percentage of objects rated at this level: In other terms the cardinality of $H_k^t \cap L_k^t$ for all $k \in [\![1\ ,\ q]\!]$, might be a good indicator for the quality of the rating.

The aggregation of all these reasons leads to the assessment of subsets of objects H_k^t and L_k^t, $k \in [\![1\ ,\ q]\!]$, for which respectively the best and the worst possible rating is the same: k. For $k \in [\![1\ ,\ q]\!]$ objects in $H_k^t \cap L_k^t$ will be rated k. For the remaining objects, for which the best and the worst possible ratings are different (objects in $A^t \setminus (\cup_{k \in [\![1\ ,\ q]\!]} H_k^t \cap L_k^t)$), we compute how close are objects from sets of reference profiles through a distance between objects and reference profiles characterizing each category between the best and the worst possible rating. Again, this operation takes into account the relative preference among the remaining objects: each assigned object based on this distance will possibly generate new positive reasons. This is illustrated in the following example

Example 6. Imagine the situation of two students x and y, such that x might be either exceptional, or excellent, or good student, and y might be either excellent or good student. In case there exist a sufficient majority of criteria in favor of y, with respect to x, and x is close to former exceptional students, x will provide y by positive reasons to be assigned to the category of exceptional students. However, since the best possible rating for y is excellent student, then y will be rated as excellent student without computing his distance with former students in each category (thanks to x). We will note $U_{2r,k}^+$ the set of objects close to a category k, for which they have neither positive reasons or valid negative reasons.

The reader should note that Dynamic-R is a whole rating process, rather a simple rating procedure. Under such a perspective the "convincing" property of Dynamic-R refers to the outcome of the whole process. The flowchart of Dynamic-R, is displayed in Figure 6.1, representing the main operations in the rating procedure.

6.4 Basic concepts within Dynamic-R

Dynamic-R is a method based on defining and aggregating positive and negative reasons respectively supporting and opposing a rating. These reasons are based on some concepts used in different MCDA methods, and the new concept of minimum requirements. These concepts will be used at the basic level. In this section, we will present the way to define the set of minimum requirements and the basic tools used in order to assess positive and negative reasons.

6.4.1 Methodology for assessing the minimum requirements

The minimum requirements represent the minimum performance, that can be taken by an object, under each criterion and regardless on its performance on the other criteria, in order to be admissible in a category. Hence, the profiles in the set of minimum requirements $\mathscr{B}$ have to be dominated by an object x to be admissible in a category: Let's consider $b_k = (\underline{b}_{j,k})_{j\in\mathscr{F}}$, characterizing the category $\mathscr{C}_k$, an object $x = (x_j)_{j\in\mathscr{F}}$ cannot be rated k if $\exists j \in \mathscr{F} : \underline{b}_{j,k} \succ_j x_j$. In this section, we will present a methodology to assess the minimum requirements characterizing each category.

Let's name x^* the ideal object: an object consisting on the best possible performance under each criterion $x^* = (x^*_j)_{j\in\mathscr{F}}$, where x^*_j is the best possible performance under the scale of the criterion j (the best if j needs to be maximized, and the lowest if j needs to be minimized). In this section, we will use the following notation: $x = (x_j, x_{-j})$ where x_j is the performance of x under the criterion j, and x_{-j} the performance of x under the criteria $\mathscr{F} \setminus \{j\}$.

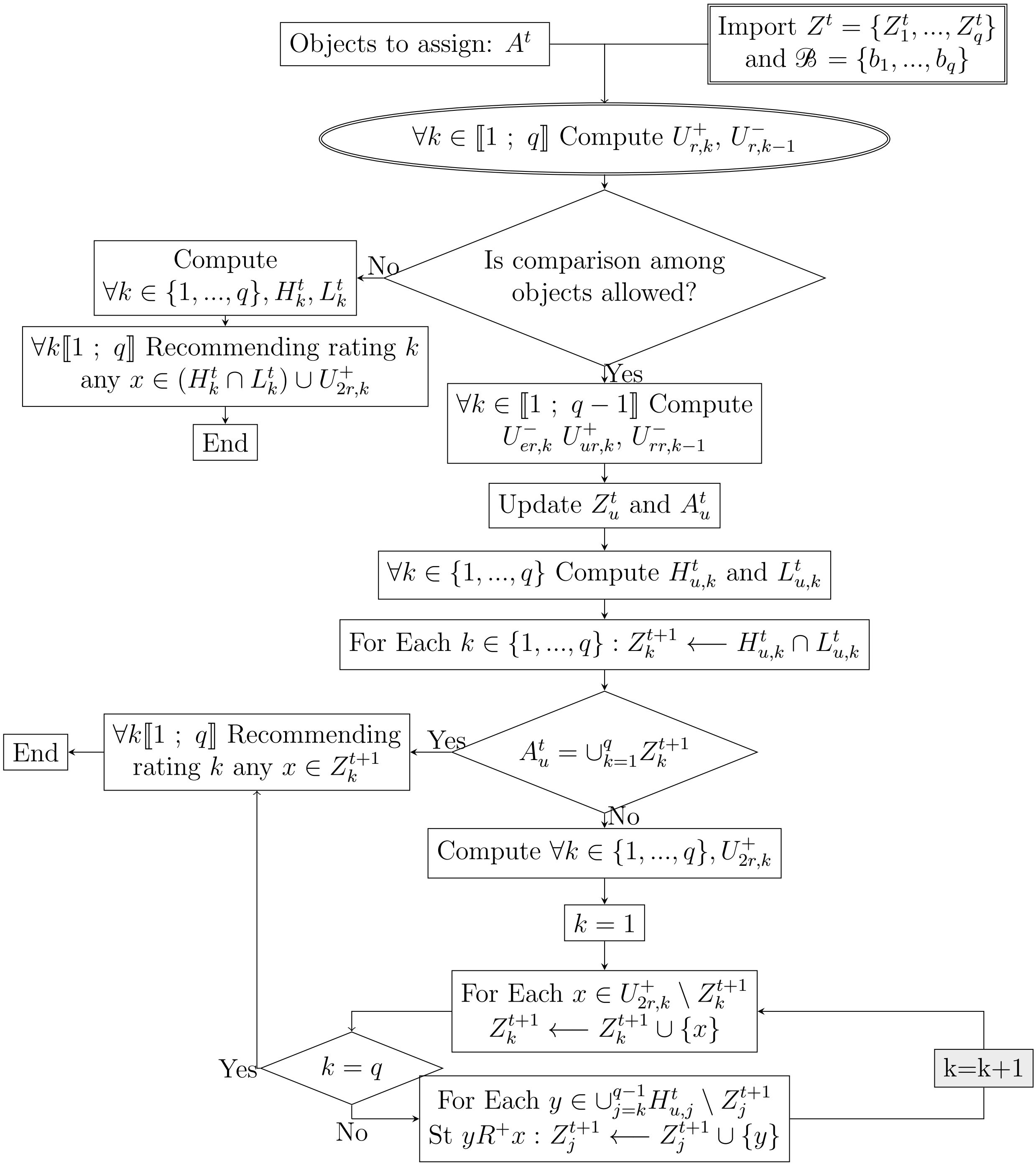


Figure 6.1 – Dynamic-R rating algorithm.

In case the number of categories and criteria is not very important, the developed procedure consists on asking the decision maker the following questions: "What is the worst performance that can be taken by x_j to rate the vector (x_j, x^*_{-j}), k (respectively the highest in case the criterion j needs to be minimized)?". Asking this question m (m being the cardinal of $\mathscr{F}$) times leads to determining the minimum requirement to be rated k: $b_k = (x_1, ..., x_m)$. This procedure requires $(k-1) \times m$ questions, to assess the minimum requirements of all the categories (we assume that the worst category does not require a minimum requirement by its nature).

Example 7. Imagine a headhunter aiming at performing an ordinal classification of candidates (graduated students) for a client based on the following criteria:

1. The global mark: to be maximized, assessed on a cardinal scale $[0, 20]$, representing the general mark of the degree;

2. Assiduity: to be maximized, assessed on an ordinal scale $\{1, ..., 10\}$, 1: refers to a not serious student and 10: refers to a very serious student;

3. The physical aptitude: to be maximized, assessed on an ordinal scale $\{1, ..., 5\}$, such that 1: not able to move, 2: bad health, 3: average health, 4: good health, 5: very high aptitude.

4. The requested annual salary: to be minimized, assessed on a cardinal scale $[40k, 70k]$ euros.

The headhunter aims at partitioning the candidates into three categories: Good opportunities for the client; opportunities that need to be discussed with the client; bad candidates for the client.

In this example the ideal candidate is characterized by the following performance vector $x^* = (20, 10, 5, 40k)$. In order to assess the minimum requirements, the headhunter might ask the client about the minimum acceptable mark (*mark*), assiduity level (*assid*), and physical aptitude (*apt*), and the maximum possible annual wage ($wage$), such that candidates performing $(mark, 10, 5, 40k)$, $(20, assid, 5, 40k)$, $(20, 10, apt, 40k)$, $(20, 10, 5, wage)$ could be considered respectively as at least good opportunities, and at least opportunities that need to be discussed.

The result might be $(12, 7, 3, 50k)$ for the minimum requirement for being a good candidate. This means that a candidate having less than 12 for Mark could never be considered good, the same reasoning applies for candidates with less than 7 for Assiduity, less than 3 for Physical aptitude, and more than 50k for wage.

6.4.2 Basic definitions

In this section, we will present the basic definitions used in order to assess positive and negative reasons for a given rating. Some of these definitions are commonly used in Outranking methods.

Definition 11. *(Positive reason for an Outranking)*

Positive reasons for Outranking relations are binary relations R^+ defined on $(\mathscr{A}^t)^2$ representing the capacity of a sufficient coalition of criteria, to influence the relative preference between two objects. This can be expressed as:

$$xR^+y \iff w(\{j \in \mathscr{F} : x \succeq_j y\}) \geq \lambda \tag{6.1}$$

where λ is the majority threshold

Remark 9. *Recall that $\mathscr{A}^t = \cup_k Z_k^t \cup A^t \cup \mathscr{B}$.*

Remark 10. *In case the measure associated with the decisive coalition of criteria is additive, the previous formulation would be:*

$$xR^+y \iff \sum_{j \in \mathscr{F} : x \succeq_j y} w_j \geq \lambda \tag{6.2}$$

Definition 12. *(Negative Reason against an Outranking)*

Negative reason against an Outranking R^- is a binary relation defined on $(\mathscr{A}^t)^2$ displaying the capacity of a subset of criteria to reject a possible Outranking in case its importance is greater than a veto v. This can be formulated by:

$$xR^-y \iff \mathscr{V}(\{j \in \mathscr{F} : y \succeq_j x\}) \geq v \tag{6.3}$$

Remark 11. *A negative reason in many Outranking methods [83, 91, 101], called discordance principle, is defined as the minimal difference v_j under each criterion $g_j \in \mathscr{F}$ not allowed to be compensated.*

Definition 13. *(Outranking relation)*

Outranking relation S_λ is a binary relation defined on $(\mathscr{A}^t)^2$. x outranks y can be interpreted as "x is at least as good as y". S_λ can be formulated as:

$$xS_\lambda y \iff xR^+y \wedge \neg(xR^-y) \tag{6.4}$$

Definition 14. *(Basic binary relations)*

Based on the Outranking relation, three possible binary relations might be defined: for $x, y \in \mathscr{A}^t$

- Strict Preference (P_λ): $xP_\lambda y \iff xS_\lambda y \wedge \neg(yS_\lambda x)$
- Indifference (I_λ): $xI_\lambda y \iff xS_\lambda y \wedge yS_\lambda x$
- Incomparability (J_λ): $xJ_\lambda y$ iff non of the previous binary relations hold.

Definition 15. *(Weak dominance relation)*

Weak dominance relation D is a binary relation defined on $(\mathscr{A}^t)^2$. For $x, y \in \mathscr{A}^t$, we say that xDy if x is at least as good as y under each criterion and strictly better than y under at least one criterion. This can be formulated by:

$$xDy \iff \exists i \in \mathscr{F}, \forall j \in \mathscr{F} : x \succeq_j y \wedge x \succ_i y \tag{6.5}$$

Remark 12. $xDy \implies xS_\lambda y$.

In this chapter, many definitions involve binary relations between objects and the sets of reference profiles. We propose the following two definitions:

Definition 16. *(Binary relations used in positive and negative reasons)*

Consider the set A and a set of sets B. A binary relation $\mathscr{R} \subseteq A \times B$, such that $\forall (x, Y) \in A \times B : x\mathscr{R}Y$ should be read as "there are negative reasons opposing x to belong to Y", or "there are positive reasons for x belonging to Y".

Definition 17. *(Preference between $2^{A_u^t}$ and Z_u^t)*

Consider the power set 2^A and a set of sets B. A binary relation $\mathscr{R} \subseteq (2^A \times B) \cup (B \times 2^A)$, such that $\forall (X, Y) \in (2^A \times B) \cup (B \times 2^A) : X\mathscr{R}Y$ should be read as "The class X is at least as good as the class Y".

Remark 13. *In this work, we will consider only singletons in $2^{A_u^t}$.*

In assignment problems, the case where categories are not necessarily ordered, the assignment is based on a similarity index. This last can be seen as a distance between an object we aim to assign and a set of objects characterizing a class. We will adapt this idea to the context where the objects are described by ordinal information under at least one dimension.

Definition 18. *(Distance between an object and a set of characteristic profiles)*

Let Z_k^t be a set of reference profiles characterizing the category k at the step t. We define the distance of an object $x \in A^t$ from the set Z_k^t as:

$$dist(x, Z_k^t) = \min \left(\min_{z \in Z_k^t} |c(x,z) - c(z,x)|; \frac{1}{|Z_k^t|} |\sum_{z \in Z_k^t} c(x,z) - c(z,x)| \right) \tag{6.6}$$

where $c(x, y) = w(\{j \in \mathscr{F} : x \succeq_j y\})$.

This distance represents the relative position of an object with respect to a set of reference profiles. It computes the minimum between two values:

- on the one hand, the way the object compares to the closest reference profile;
- on the other hand, the way the object compares to all the reference profiles

The first component of the distance, $\min_{z \in Z_k^t} |c(x, z) - c(z, x)|$, represents the minimum of distances between "x" and each profile in Z_k^t. Intuitively, it can be seen as an answer to the question "is there any profile in Z_k^t close to x?". The second component of the distance, $\frac{1}{|Z_k^t|} |\sum_{z \in Z_k^t} c(x, z) - c(z, x)|$, represents the net flow evaluation: The difference between the total importance of criteria in favor of x compared to the profiles in Z_k^t and the total importance of criteria in favor of the reference profiles in Z_k^t compared to x. This last can be seen as an evaluation of the distance with the center of reference profiles in Z_k^t. Figure 6.2 illustrates the defined distance.

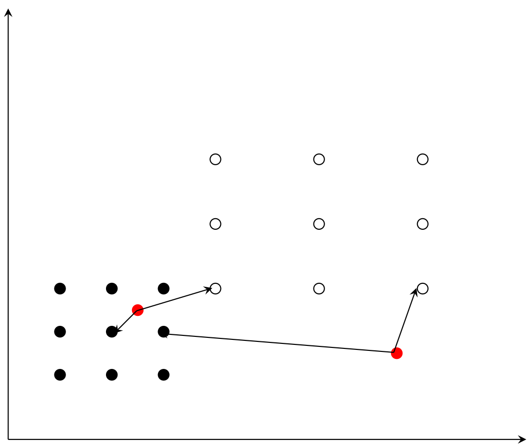


Figure 6.2 – Illustration of distances between objects and the majority of reference profiles or the closest profile

In case the set of minimum requirements is not empty, we define an incompatibility binary relation between categories and objects.

Definition 19. *(Incompatibility binary relation)*

Incompatibility binary relation $Incomp_{lower}$ defined on $A^t \times Z^t$, represents the illegibility of an object to characterize a given category with respect to some minimum requirements. For $x \in A^t$, $Z_k^t \in Z^t$:

$$xIncomp_{lower}Z_k^t \iff \exists b_k \in \mathscr{B} : \neg(xDb_k) \tag{6.7}$$

This means that, at a given step $t \in T$, if for an object $x \in A^t, \exists j \in \mathscr{F}$: $x \prec_j \underline{b}_k$, the assignment of x to the category $\mathscr{C}_k$ should be "questioned", thus, x cannot be rated k. The incompatibility binary relation and the discordance index represent close concepts related to the respect of minority principle. It consists on the existence of strong reasons to not approve a preference relation, between two objects, even in the presence of a sufficient majority of concordant criteria. However, these two concepts are different: the discordance index characterizes the Outranking between two objects while the incompatibility binary relation characterizes the illegibility of an object to belong to a category.

6.5 Characteristic properties of Dynamic-R

In this section, we present the theoretical results about the developed method. We will detail all the concepts presented in section 6.3, and we will provide formal definitions, as well as the properties of the introduced concepts.

6.5.1 Negative reasons against a rating

Negative reasons represent information or premises against a rating. In our approach, negative reasons represent, on the one hand, the "negative consistency" of a rating, due to the strict preferences or the dominance between objects and reference profiles at a given step. On the other hand, the non-compatibility of an object with a category. In this section, we will define negative reasons in two different ways, depending on whether they are based on the strict preference relation or the incompatibility.

The "negative consistency" should be considered as a situation where an object being potentially rated k is either weakly dominated or strictly preferred by a reference profile of rate $k+1$ which is worse. Incompatibility should be understood as the situation where an object being potentially rated k fails to meet one of the minimal requirements of category $\mathscr{C}_k$.

In order to assess the negative reasons against a rating, the assignment of objects to a given category will depend on the relative position of the non assigned objects with reference profiles using the weak dominance and either the strict preference relations or the absence of the incompatibility of objects with the categories.

Definition 20. *($U^-_{r,k}$, For $k \in [\![1\ ;\ q-1]\!]$)*

The set of objects having negative reasons against being assigned to a given category k, $U^-_{r,k}$ can be formulated as

$$U^-_{r,k} = \{x \in A^t \cup Z^t_{1,q}, xR^-_r Z^t_k\}, \forall k \in [\![1\ ;\ q-1]\!] \tag{6.8}$$

where R^-_r is a binary relation defined on $(A^t \cup Z^t_{1,q}) \times Z^t$. $xR^-_r Z^t_k$ should be read as: "there are negative reasons against rating x, k": For $x \in A^t \cup Z^t_{1,q}$, $Z^t_k \in Z^t$:

- Case using the incompatibility and the strict preference relation:

$$xR^-_r Z^t_k \iff \exists h \in [\![k+1\ ;\ q]\!], \exists z \in Z^t_h : zP_\lambda x \vee xIncomp_{lower} Z^t_k. \tag{6.9}$$

- Case using the incompatibility and the dominance:

$$xR^-_r Z^t_k \iff \exists h \in [\![k+1\ ;\ q]\!], \exists z \in Z^t_h : zDx \vee xIncomp_{lower} Z^t_k. \tag{6.10}$$

Remark 14. Objects $\underline{b}_1, ..., \underline{b}_q$ do not necessarily belong respectively to the categories $\mathscr{C}_1, ..., \mathscr{C}_q$.

If Definition 8 holds then:

Proposition 1. *(Monotonicity of negative reasons)*

1. If there exist negative reasons against assigning an object to a given category then there exist negative reasons against assigning it to any better category:

$$\forall x \in A^t \cup Z^t_{1,q}, \forall Z^t_h \in Z^t : \; xR^-_r Z^t_h \implies \forall k \in [\![1 \; ; \; h]\!] : xR^-_r Z^t_k; \quad (6.11)$$

2. If there are no negative reasons to assign an object to a given category then there are no negative reasons to assign it to any worse category:

$$\forall x \in A^t \cup Z^t_{1,q}, \forall Z^t_h \in Z^t : \; \neg(xR^-_r Z^t_h) \implies \forall k \in [\![h \; ; \; q]\!] : \neg(xR^-_r Z^t_k). \quad (6.12)$$

Proof. (properties of negative reason)

$\forall x \in A^t \cup Z^t_{1,q}, \forall Z^t_h \in Z^t$

1. Let us assume that $xR^-_r Z^t_h$. Then we have either:

$$xR^-_r Z^t_h \iff xIncomp_{lower} Z^t_h \vee (\exists r > h, \exists w \in Z^t_r : wP_\lambda x)$$

or

$$xR^-_r Z^t_h \iff xIncomp_{lower} Z^t_h \vee (\exists r > h, \exists w \in Z^t_r : wDx)$$

By definition: $xIncomp_{lower} Z^t_h \iff \neg(xDb_h)$

Since $\forall k \in \{1, ..., h\}$: $\forall j \in \mathscr{F}$, $\underline{b}_{j,k} \geq \underline{b}_{j,h}$: $xIncomp_{lower} Z^t_h \implies \neg(xDb_k)$ Thus $\forall k \in \{1, ..., h\}$:

$$xIncomp_{lower} Z^t_h \implies xIncomp_{lower} Z^t_k \quad (6.13)$$

also since $k \leq h$, we have:

$$\exists r > h, \exists w \in Z_r^t : wP_\lambda x \implies \exists r > k, \exists w \in Z_r^t : wP_\lambda x \tag{6.14}$$

and

$$\exists r > h, \exists w \in Z_r^t : wDx \implies \exists r > k, \exists w \in Z_r^t : wDx \tag{6.15}$$

Hence, from 6.13, 6.14, and 6.15, we have $xR_r^- Z_k^t$. (The same proof is valid in both cases of Definition 35)

2. Suppose that $\neg(xR_r^- Z_h^t)$ and that $\exists k \in \{h, ..., q\} : xR_r^- Z_k^t$.

   Based on (1.), $\forall h \leq k : xR_r^- Z_h^t$. Absurd.

■

Corollary 1. The monotonicity of negative reasons can also be formulated as:

$$\forall k \in [\![2 \ ; \ q-1]\!] : U_{r,k}^- \subseteq U_{r,k-1}^- \tag{6.16}$$

Proof. Direct consequence of Proposition **??**. ■

Negative reasons prevent a rating that can be criticized. To confirm a rating, we need to verify the existence of reasons supporting an assignment to categories for which no negative reasons are involved. In the next subsection, we will define and discuss the forms of the reasons supporting a rating, called positive reasons.

6.5.2 Positive reasons supporting a rating

Positive reasons represent information or premises supporting a rating. These reasons are built with respect to the "positive consistency" of the rating, that could be understood as the situation where an object can be rated k because it is at least as good as at least one reference profile belonging to $\mathscr{C}_k$.

Definition 21. *($U^+_{r,k}$, For $k \in [\![1\ ;\ q]\!]$)*

The set of objects having positive reasons supporting the assignment to a given category k, named $U^+_{r,k}$, can be formulated as:

$$\forall k \in [\![1\ ;\ q]\!] : U^+_{r,k} = \{x \in A^t \cup Z^t_{1,q}, xR^+_{1r}Z^t_k\} \tag{6.17}$$

where R^+_{1r} is a binary relation defined on $(A^t \cup Z^t_{1,q}) \times Z^t$ representing the possibility to be at least as good as reference profiles characterizing a category. R^+_{1r} can be formulated as: For $x \in A^t \cup Z^t_{1,q}, Z^t_k \in Z^t$:

$$xR^+_{1r}Z^t_k \iff \exists h \leq k, \exists z \in Z^t_h : xS_\lambda z. \tag{6.18}$$

Proposition 2. *(Monotonicity of positive reasons)*

1. If there exist positive reasons supporting the assignment of an object to a given category then there exist positive reasons supporting its assignment to any worse category:

$$\forall x \in A^t \cup Z^t_{1,q}, \forall Z^t_h \in Z^t :\ xR^+_{1r}Z^t_h \implies \forall k \in [\![h\ ;\ q]\!] : xR^+_{1r}Z^t_k; \tag{6.19}$$

2. If there are no positive reasons to assign an object to a given category then there are no positive reasons to assign it to any better category:

$$\forall x \in A^t \cup Z^t_{1,q}, \forall Z^t_h \in Z^t :\ \neg(xR^+_{1r}Z^t_h) \implies \forall k \in [\![1\ ;\ h]\!] : \neg(xR^+_{1r}Z^t_k). \tag{6.20}$$

Proof. Obvious, by construction of R^+_{1r} in Definition 36. ∎

Corollary 2. The monotonicity of positive reasons can also be formulated as:

$$\forall k \in [\![2\ ;\ q]\!] : U^+_{r,k-1} \subseteq U^+_{r,k} \tag{6.21}$$

Proof. Direct consequence of Proposition 2. ∎

6.5.3 Characteristics of reference profiles and assessment of object's priority

The aim of this chapter is to provide a "convincing" rating. Hence, at all steps in T of the process, the set of reference profiles should respect the following "convincing" condition:

Definition 22. *"Convincing" property*

$$\forall z \in Z_k^t, \nexists y \in Z_h^t (h > k) : y S_\lambda z \wedge y \notin U_{r,k}^- \tag{6.22}$$

A second version of the Definition 34 is:

Proposition 3. The "Convincing" property can be equivalently formulated as:

$$\forall y \in Z_k^t, \nexists z \in Z_h^t (k > h) : y S_\lambda z \wedge y \notin U_{r,h}^- \tag{6.23}$$

Proof. Condition 1 is equivalent to $\forall y \in Z_k^t, \nexists z \in Z_h^t (k > h), y R^+ z \wedge y \notin U_{r,h}^-$

$$\begin{aligned}
\forall z \in Z_k^t, \nexists y \in Z_h^t (h > k) : y S_\lambda z \wedge y \notin U_{r,k}^- &\iff \forall z \in Z_k^t, \nexists y \in Z_h^t (h > k) : y S_\lambda z \wedge \neg (y R_r^- Z_k^t) \\
&\iff \forall z \in Z_k^t, \forall y \in Z_h^t (h > k) : \neg (y S_\lambda z) \vee y R_r^- Z_k^t) \\
&\iff \forall y \in Z_h^t, \nexists z \in Z_k^t (h > k) : y S_\lambda z \wedge \neg (y R_r^- Z_k^t)
\end{aligned}$$

∎

In the next subsection, we will present the aggregation procedure of positive and negative reasons without considering the way objects compare to each other (without a consistency checking).

6.5.4 Aggregating of $U_{r,l}^+, U_{r,h}^-$, without consistency checking

Let's assume that the decision maker only needs a rating without any consistency checking. We need to aggregate the sets $U_{r,l}^+$ and $U_{r,h}^-$, for all $l \in [\![1 \; ; \; q]\!]$ $h \in [\![1 \; ; \; q-1]\!]$.

Rating an object comes to its assignment to the best possible category, for which there are no negative reasons. Thus, the aggregation is made in a hierarchical way: We first verify the absence of negative reasons, then the existence of positive ones. Under this principle, we will assess two partitions of A^t: H_h^t, for all $h \in [\![1\ ;\ q]\!]$, representing the objects for which the best possible rating is h; and L_l^t, for all $l \in [\![1\ ;\ q]\!]$, representing the objects for which the worst possible rating is l. These assessments are based only on the way objects compare to reference profiles.

Definition 23. *(H_h^t and L_l^t, for $h, l \in [\![1\ ;\ q]\!]$)*

For a given $t \in T$, the partitions of A^t, H_h^t and L_l^t, for which the best and the worst possible ratings are respectively $h, l \in [\![1\ ;\ q]\!]$, can be formulated as:

$$H_h^t = U_{r,h-1}^- \setminus U_{r,h}^- \tag{6.24}$$

$$L_l^t = U_{r,l}^+ \setminus \left(U_{r,l}^- \cup (U_{r,l-1}^+ \setminus U_{r,l-1}^-)\right) \tag{6.25}$$

Proposition 4. *(properties of H_h^t and L_l^t)*

For a given $t \in T$, the sets $H_1^t, ..., H_q^t$ and $L_1^t, ..., L_q^t$, are two partitions of A^t.

Proof. For a given $t \in T$, let's prove that

1. $H_1^t, ..., H_q^t$ is a partition of A^t:

   For all $h, k \in [\![1\ ;\ q]\!]$, we have $H_h^t \cap H_k^t = \emptyset$, $(h < k)$, since:

   Using the Definition 23, we have $H_h^t \cap H_k^t = (U_{r,h-1}^- \setminus U_{r,h}^-) \cap (U_{r,k-1}^- \setminus U_{r,k}^-)$.

   Due to the monotonicity of negative reasons (see Corollary 1), $U_{r,k-1}^- \subseteq U_{r,h}^-$. Hence:

$$H_h^t \cap H_k^t = \emptyset \tag{6.26}$$

It is also easy to check that

$$\cup_h H_h^t = A^t \tag{6.27}$$

since $U_{r,0}^- = A^t \setminus U_{r,1}^-$ and $U_{r,q}^- = \emptyset$. Hence: $\cup_{h=1}^q H_h^t = A^t \setminus U_{r,q}^- = A^t$. '

2. $L_1^t, ..., L_q^t$ is a partition of A^t:

   For all $l, k \in [\![1\ ;\ q]\!]$, we have $L_l^t \cap L_k^t = \emptyset$, $(k < l)$, since:

$$\begin{aligned} L_l^t \cap L_k^t &= \left[U_{r,l}^+ \setminus \left(U_{r,l}^- \cup (U_{r,l-1}^+ \setminus U_{r,l-1}^-)\right)\right] \cap \left[U_{r,k}^+ \setminus \left(U_{r,k}^- \cup (U_{r,k-1}^+ \setminus U_{r,k-1}^-)\right)\right] \\ &= \left((U_{r,l}^+ \setminus U_{r,l}^-) \setminus (U_{r,l-1}^+ \setminus U_{r,l-1}^-)\right) \cap \left((U_{r,k}^+ \setminus U_{r,k}^-) \setminus (U_{r,k-1}^+ \setminus U_{r,k-1}^-)\right) \end{aligned}$$

   Based on Corollary 1: $U_{r,l-1}^- \subseteq U_{r,k}^-$. And based on Corollary 2: $U_{r,k}^+ \subseteq U_{r,l-1}^+$.

   Hence: $U_{r,k}^+ \setminus U_{r,k}^- \subseteq U_{r,l-1}^+ \setminus U_{r,l-1}^-$.

   Thus:

$$L_l^t \cap L_k^t = \emptyset \tag{6.28}$$

   It is easy to check that

$$\cup_l L_l^t = A^t \tag{6.29}$$

   Since:

$$\begin{aligned} \cup_l L_l^t &= \cup_l\, (U_{r,l}^+ \setminus U_{r,l}^-) \setminus (U_{r,l-1}^+ \setminus U_{r,l-1}^-) \\ &= (U_{r,q}^+ \setminus U_{r,q}^-) \setminus (U_{r,0}^+ \setminus U_{r,0}^-) \end{aligned}$$

   $U_{r,0}^+ \setminus U_{r,0}^- = \emptyset$, $U_{r,q}^+ = A^t$ and $U_{r,q}^- = \emptyset$. Thus, $\cup_l L_l^t = A^t$.

■

Having computed the two series of sets H_h^t and L_l^t we can identify the objects for which the best possible rating and the worst possible rating is the same $(H_k^t \cap L_k^t,\ \forall k \in [\![1\ ;\ q]\!])$. It is clear that after performing this step there will exist objects for which the best possible rating and the worst possible rating do not coincide. We have two options here:

- either present an "interval rating" (x is rated between l and h);
- or try to reduce this imprecision by computing the "distance" of x with respect to all such possible categories, as defined in Definition 18, and choosing the rate $k = \arg\min_j dist(x, Z_j^t)$.

Such a way of rating is close to the method developed by C. Rocha and L.C. Dias [90] based on comparing objects with assignment examples, under the principle that an object assigned to a category should not be outranked by any example (reference profile) assigned to a worse category. In our work, negative reasons consists partially on either strict preference or the dominance rather than an Outranking. This is due to the separation between positive and negative reasons: being indifferent in Outranking approaches is based on ordinal comparison under each criterion and the indifference between two objects does not take into account the differences of performances under each criterion. Hence, the following situation might happens: $xI_\lambda y$, $xI_\lambda z$ and yDz. This is illustrated in the following example:

Example 8. *Consider an MCDA problem characterized by four criteria* $\{1, 2, 3, 4\}$*, such that the coalition of each two criteria is a decisive coalition. We consider three objects* $x = (16, 16, 8, 8)$*,* $y = (15, 15, 10, 10)$*, and* $z = (10, 10, 10, 10)$*. In such case,* $xI_\lambda y$*,* $xI_\lambda z$*, but* yDz*. Let's note* $D_{i,j}$ *a strict dominance under the subset of criteria criteria* i, j*. Such result can be obtained by any* $xD_{1,2}y$*,* $yD_{3,4}x$*, and* z *having the same performances of* y *under criteria* $\{3, 4\}$ *and* $yD_{1,2}z$*. Hence,* x *will be indifferent at the same time to* y *and* z*, however,* y *dominates* z*.*

In our work, y and z might be reference profiles characterizing two different categories, Z_l^t for a worse category and Z_h^t for a better one (h and l are not necessarily consecutive). Defining negative reasons based on "Not being outranked by any reference profile characterizing a worse category", as it is the case in [90], will lead to assigning x and y to at most to the same category characterized by z. However, the way we defined positive and negative reasons will lead to having positive reasons for x from y and no negative reasons generated to x by z.

In the next section, the negative and positive reasons will be updated to take into account the way objects compare to each other. This operation is important in order to enrich the sets of reference profiles in the next section, with respect to the "convincing" condition.

6.5.5 Updating positive and negative reasons

For $t \in T$, the assessment of $U^-_{r,h}$ and $U^+_{r,l}$, for $l \in [\![1\ ;\ q]\!]$, $h \in [\![1\ ;\ q-1]\!]$ is based on the sets of reference profiles. These last were updated in the previous step. In this section, we will discuss and analyse two major features: the way objects in A^t might modify the sets of reference profiles (for example by enriching their positive reasons); and the way objects in A^t might change positive and negative reasons supporting or against a given assignment. Analysing preferential information originated by A^t leads to three possible treatments: enriching negative reasons (only when strict preference is considered in the assessment of negative reasons), enriching positive reasons and withdrawing negative reasons.

Definition 24. *($U^-_{er,k}$, For $k \in [\![1\ ;\ q-1]\!]$)*

For a given $k \in [\![1\ ;\ q-1]\!]$, the set of objects, $U^-_{er,k}$, for which negative reasons were enriched to prevent a rating $k \in [\![1\ ;\ q-1]\!]$, can be formulated as

$$U^-_{er,k} = \{x \in Z^t_{1,q} \cup A^t : (xR^-_{er}Z^t_k) \wedge \neg(xR^-_{er}Z^t_{k+1})\} \tag{6.30}$$

where R^-_{er} is binary relation representing enriched negative reasons against a rating. R^-_{er} can be formulated as:

For $x \in U^-_{r,h} \setminus U^-_{r,h+1}$:

$$xR^-_{er}Z^t_k \iff \exists y \in \cup^q_{j=k}U^-_{er,j} \cup U^-_{r,k} : yP_\lambda x \wedge \neg(yIncomp_{lower}Z^t_{h+1}). \quad (6.31)$$

Definition 24, represents the assessment of the sets of objects or reference profiles for which the negative reasons were enriched to prevent a rating to a worse category. The enrichment of negative reasons at a step $t \in T$, based on new information not available in previous steps comes from either A^t or reference profiles for which negative reasons were enriched. Enriching negative reasons associated to an object (or reference profile) x having negative reasons against being rated h or better ($x \in U^-_{r,h} \setminus U^-_{r,h+1}$), at a step $t \in T$, consists on the existence of y, either a reference profile for which its negative reasons were enriched or an object in A^t, having negative reasons to be assigned to a category strictly worst than $\mathscr{C}_h$ without being incompatible with this last (otherwise the rating of x is not compromised), which is either strictly preferred to or dominating x. The condition of enriching negative reasons is the use of the strict preference in the assessment of $U^-_{r,1}, ..., U^-_{r,q-1}$ The enrichment of negative reasons for x due to a dominance in favor of y occurs only in case negative reasons of y were enriched too. Hence, because of the transitivity of the dominance relation and the monotonicity of the incompatibility, negative reasons cannot be enriched in case negative reasons are assessed based only on these two binary relations. The algorithm assessing $U^-_{er,k}$, for $k \in [\![1 \; ; \; q-1]\!]$, will be presented in section the next chapter, Algorithm 2.

The following proposition presents a characteristic of the binary relation used in the assessments of $U^-_{er,1}, ..., U^-_{er,q-1}$.

Proposition 5. *(properties of R^-_{er})*

For $t \in T$, for $x \in U^-_{r,h} \setminus U^-_{r,h+1}$, if there are enriched negative reasons opposing x being rated k then there are enriched negative reasons opposing any rating between k and h. This can be formulated as:

$$xR_{er}^{-}Z_k^t \implies \forall j \in [\![h \ ; \ k]\!] : xR_{er}^{-}Z_j^t \tag{6.32}$$

Proof. Obvious since, $\forall j \in [\![h \ ; \ k]\!]$ we have $\cup_{h=k}^{q} U_{er,h}^{-} \cup U_{r,k}^{-} \subseteq \cup_{h=j}^{q} U_{er,h}^{-} \cup U_{r,j}^{-}$. ($\cup_{h=k}^{q} U_{er,h}^{-} \subseteq \cup_{h=j}^{q} U_{er,h}^{-}$ and $z \in U_{r,k}^{-} \implies z \in U_{r,j}^{-}$, see equation 2) ■

Remark 15. Under the hypothesis that at a given step $t \in T$, reference profiles are "convincing", the enrichment of positive reasons or the withdrawn of negative reasons cannot lead to the enrichment of negative reasons related to any object or reference profile. This is justified by the fact that improving the assignment of an object to a given category $\mathscr{C}_k$ cannot influence negatively the assignment of any object in a better category (see proposition 10). For this reason, enriching negative reasons will be processed first.

Definition 25. *($U_{ur,k}^{+}$, For $k \in [\![1 \ ; \ q-1]\!]$)*

For a given $k \in [\![1 \ ; \ q-1]\!]$, the set of objects, $U_{ur,k}^{+}$, for which positive reasons were enriched to support a rating k, can be formulated as:

$$U_{ur,k}^{+} = \{x \in Z_{1,q}^t \cup A^t : xR_{ur}^{+}Z_k^t \wedge \neg(xR_{ur}^{+}Z_{k-1}^t)\} \tag{6.33}$$

where R_{ur}^{+} is a binary relation representing enriched positive reasons supporting a rating. R_{ur}^{+} can be formulated as:

For $x \in U_{r,l}^{+} \setminus U_{r,l-1}^{+}$, $k < l$:

$$xR_{ur}^{+}Z_k^t \iff \exists y \in (\cup_{j=1}^{k} U_{ur,j}^{+} \cup U_{r,k}^{+}) \setminus (\cup_{j=k}^{q-1} U_{er,j}^{-} \cup U_{r,k}^{-}) : xS_{\lambda}y \tag{6.34}$$

Definition 37, represents the assessment of the sets of objects for which positive reasons were enriched to support the assignment to a better category. Enriching positive reasons for a given x is mainly due to the presence of $y \in (\cup_{j=1}^{k} U_{ur,j}^{+} \cup U_{r,k}^{+}) \setminus (\cup_{j=k}^{q-1} U_{er,j}^{-} \cup U_{r,k}^{-})$, having positive and no negative reasons to be assigned to a category better than x, such that xR^+y. Hence, y will provide x by new positive reasons that will potentially improve its possible rating.

The following proposition presents a characteristic of the binary relation used in the assessments of $U^+_{ur,1}, ..., U^+_{ur,q}$.

Proposition 6. *(properties of R^+_{ur})*

For $t \in T$, for $x \in U^+_{r,l} \setminus U^+_{r,l-1}$, for a given k better than l ($k < l$), if there are enriched positive reasons supporting x being rated k then there are enriched positive reasons supporting any rating between l and k. This can be formulated as:

$$xR^+_{ur}Z^t_k \implies \forall j \in [\![k\ ;\ l-1]\!] : xR^+_{ur}Z^t_j \tag{6.35}$$

Proof. Obvious since $\forall j \in [\![k\ ;\ t-1]\!]$ we have

$(\cup_{h=1}^{k}U^+_{ur,h} \cup U^+_{r,k}) \setminus (\cup_{h=k}^{q-1}U^-_{er,h} \cup U^-_{r,k}) \subseteq (\cup_{h=1}^{j}U^+_{ur,h} \cup U^+_{r,j}) \setminus (\cup_{h=j}^{q-1}U^-_{er,h} \cup U^-_{r,j})$,

since:

$(\cup_{h=1}^{k}U^+_{ur,h} \cup U^+_{r,k}) \subseteq (\cup_{h=1}^{j}U^+_{ur,h} \cup U^+_{r,j})$ and $(\cup_{h=j}^{q-1}U^-_{er,h} \cup U^-_{r,j}) \subseteq (\cup_{h=k}^{q-1}U^-_{er,h} \cup U^-_{r,k})$. ■

Definition 26. *($U^-_{rr,k}$, For $k \in [\![1\ ;\ q-1]\!]$)*

For a given $k \in [\![1\ ;\ q-1]\!]$, the set of objects, $U^-_{rr,k}$, for which negative reasons are withdrawn to prevent a better rating k, can be formulated as:

$$U^-_{rr,j} = \{x \in Z^t_{1,q} \cup A^t : xR^-_{rr}Z^t_j \wedge \neg(xR^-_{rr}Z^t_{j+1})\} \tag{6.36}$$

where R^-_{rr} is binary relation representing withdrawn negative reasons against a rating. R^-_{rr} can be formulated as:

For $x \in U^-_{er,h} \cup (U^-_{r,h} \setminus U^-_{r,h+1})$, $k < h$:

- case of negative reasons with strict preference

$$xR^{-}_{rr}Z^{t}_{k} \iff \begin{cases} \forall z \in (U^{-}_{r,h} \cup U^{-}_{er,h}) \setminus \cup^{h-1}_{j=1}U^{-}_{rr,j} & : \neg(zP_{\lambda}x \vee zDx) \\ \text{And} & \\ \exists y \in \left[\cup^{h}_{j=1}U^{+}_{ur,j} \cup U^{+}_{r,h}\right] \cap U^{-}_{h,k} & : yP_{\lambda}x \vee xIncomp_{lower}Z^{t}_{k} \end{cases} \tag{6.37}$$

- case of negative reasons without strict preference

$$xR^{-}_{rr}Z^{t}_{k} \iff \begin{cases} \forall z \in (U^{-}_{r,h}) \setminus \cup^{h-1}_{j=1}U^{-}_{rr,j} & : \neg(zDx) \\ \text{And} & \\ \exists y \in \left[\cup^{h}_{j=1}U^{+}_{ur,j} \cup U^{+}_{r,h}\right] \cap U^{-}_{h,k} & : yDx \vee xIncomp_{lower}Z^{t}_{k} \end{cases} \tag{6.38}$$

with $U^{-}_{h,k} = \cup^{h-1}_{j=k}(U^{-}_{er,j} \cup U^{-}_{rr,j}) \cup U^{-}_{r,k} \setminus \left(\cup^{q-1}_{j=h}U^{-}_{er,j} \cup (\cup^{k-1}_{j=1}U^{-}_{rr,j}) \cup U^{-}_{r,h}\right)$ representing objects with valide negative reasons against ratings between $h-1$ and k.

Definition 38, represents the assessment of the sets of objects for which negative reasons were withdrawn to prevent a rating to a better category. The binary relation R^{-}_{rr} associated to these sets, and characterizing the operation of withdrawing negative reasons for a given x from a worse rating l, to a better rating h, are defined by two conditions:

1. Eligibility for withdrawing negative reasons for an object x: The existence of another object or reference profile y, having valid negative reasons against being rated $l-1$, and either strictly preferred to x or dominating x (in case strict preference is not considered), will invalidate the ability of x to improve its position (x will still have valid negative reasons against being rated l).

2. New negative reasons against a rating k: the improvement of the rating of x will be at most limited by the improvement of the object or refer-

ence profile, let's name it y, at the origin of x's negative reasons. The limitation might also come from an other element strictly preferred or dominating x, limiting its improvement to at most $k+1$ (since the withdrawn negative reasons will oppose being rated k). It is also possible that the withdrawn of x's negative reasons will not be limited by any object or reference profile, but by its own performance not dominating the minimum requirement b_k.

The algorithm assessing $U^-_{rr,h}$, for all $j \in [\![1 \ ; \ q-1]\!]$, can be found in the next chapter, Algorithm 4.

Proposition 7. *(properties of R^-_{rr})*

For $t \in T$, $\forall x \in U^-_{er,h} \cup (U^-_{r,h} \setminus U^-_{r,h+1})$, if there are withdrawn negative reasons opposing x being rated k then there are withdrawn negative reasons opposing any rating better than k. This can be formulated as:

$$xR^-_{rr}Z^t_k \implies \forall j \in [\![1 \ ; \ k]\!] : xR^-_{rr}Z^t_j \tag{6.39}$$

Proof. Suppose that for a given object $x \in U^-_{er,h} \cup (U^-_{r,h} \setminus U^-_{r,h+1})$ there exists a $k \in [\![1 \ ; \ h-1]\!]$ such that $xR^-_{rr}Z^t_k$:
$\forall j \in [\![1 \ ; \ k]\!]$, we have: $U^-_{h,k} \subseteq U^-_{h,j}$ since $U^-_{r,k} \subseteq U^-_{r,j}$; $\cup_{l=k}^{h-1}(U^-_{er,l} \cup U^-_{rr,l}) \subseteq \cup_{l=j}^{h-1}(U^-_{er,l} \cup U^-_{rr,l})$; and $\cup_{l=1}^{j-1}U^-_{rr,l} \subseteq \cup_{l=1}^{k-1}U^-_{rr,l}$.
Hence $\exists y \in \left[\cup_{l=1}^{h}U^+_{ur,l} \cup U^+_{r,h}\right] \cap U^-_{h,k} \subseteq \left[\cup_{l=1}^{h}U^+_{ur,l} \cup U^+_{r,h}\right] \cap U^-_{h,j}$ such that $yP_\lambda x \vee yDx \vee xIncomp_{lower}Z^t_k$
Thus, $xR^-_{rr}Z^t_j$. ∎

Remark 16. *The updates of negative and positive reasons leads to a change of some reference profiles, either to a better or to a worse category. In case the strict preference relation is not used in the assessment of negative reasons, positions of reference profiles cannot change to a worse category. The updated* *sets of reference profiles can be formulated as:*

$$Z^t_{uk} = Z^t_k \backslash \Big[\Big(\cup_{j=1}^{k-1} U^+_{ur,j} \cap (\cup_{j=1}^{k-2} U^-_{rr,j}) \Big) \cup \Big(\cup_{j=1}^{k-1} U^+_{ur,j} \setminus U^-_{r,k-1} \Big) \cup \Big(\cup_{j=k}^{q-1} U^-_{er,j} \setminus (\cup_{j=1}^{k-1} U^-_{rr,j}) \Big) \Big] \tag{6.40}$$

Since the sets of reference profiles are updated, we have more objects to rate. Thus, we note A^t_u the new set of objects that need to be rated at the the current step $t \in T$. A^t_u can be formulated as:

$$A^t_u = A^t \cup \Big(Z^t_{1,q} \setminus (\cup_{j=1}^{q-1} Z^t_{uj}) \Big) \tag{6.41}$$

6.5.6 Recommendation

The assessments of the $U^+_{r,k}$, $U^+_{ur,k}$, $U^-_{r,k}$, $U^-_{er,k}$, and $U^-_{rr,k}$, for all k, needs to be used to have a "convincing" rating. For this aim, A^t_u will be partitioned into $H^t_{u,1}, ..., H^t_{u,q}$, and $L^t_{u,1}, ..., L^t_{u,q}$. These partitions will be defined using a binary relation between A^t_u and Z^t_{uk}, for all k, as follows:

Definition 27. *($H^t_{u,h}$ and $L^t_{u,l}$, for $h, l \in [\![1\ ; q]\!]$)*

For a given $t \in T$, the partitions of A^t_u, $H^t_{u,h}$ and $L^t_{u,l}$, for which the best and the worse possible ratings are respectively $h, l \in [\![1\ ; q]\!]$, can be formulated as:

$$\begin{cases} H^t_{u,h} = \{x \in A^t_u, Z^t_{u,h} \succcurlyeq^t \{x\} \wedge \neg(Z^t_{u,h+1} \succcurlyeq^t \{x\})\} & h \neq q \\ H^t_{u,q} = \{x \in A^t_u, Z^t_{u,q} \succcurlyeq^t \{x\}\} \end{cases} \tag{6.42}$$

$$\begin{cases} L^t_{u,l} = \{x \in A^t_u, \{x\} \succcurlyeq^t Z^t_{u,l} \wedge \neg(\{x\} \succcurlyeq^t Z^t_{u,l-1})\} & l \neq 1 \\ L^t_{u,1} = \{x \in A^t_u, \{x\} \succcurlyeq^t Z^t_{u,1}\} \end{cases} \tag{6.43}$$

Where $\succcurlyeq^t$ is a weak order built on $(2^{A^t_u} \times Z^t_u) \cup (Z^t_u \times 2^{A^t_u})$, representing the preference between a subset of A^t_u and sets in Z^t_u. $\forall t \in T, \succcurlyeq^t$, defined as follows :

1. On $Z^t_u \times 2^{A^t_u}$: $\forall x \in A^t_u, \exists k \in [\![1\ ;\ q]\!]$ such that:

$$Z^t_{u,k} \succsim^t \{x\} \iff \begin{cases} x \in \left(\cup_{j=k-1}^{q-1} U^-_{er,j} \cup U^-_{r,k-1}\right) \setminus \cup_{j=1}^{k-2} U^-_{rr,j} & k > 2; \\ x \in \cup_{j=k-1}^{q-1} U^-_{er,j} \cup U^-_{r,k-1} & k \leq 2; \end{cases} \tag{6.44}$$

2. On $2^{A^t_u} \times Z^t_u$: $\forall x \in A^t_u, \exists k \in [\![1\ ;\ q]\!]$ such that:

$$\{x\} \succsim^t Z^t_{u,k} \iff \begin{cases} \neg(Z^t_{u,k+1} \succsim^t \{x\}) \wedge (x \in \cup_{j=1}^{k} U^+_{ur,j} \cup U^+_{r,k}) & k \neq q \\ x \in \cup_{j=1}^{q} U^+_{ur,j} \cup U^+_{r,q} \end{cases} \tag{6.45}$$

Definition 27, represents the assessment of the two partitions of A^t_u: $H^t_{u,h}$ and $L^t_{u,l}$ for all $h, l \in [\![1\ ; q]\!]$. A set $H^t_{u,h}$ contains objects for which the best possible rating is h. In other terms the best category for which x has no valid negative reasons is h.

Valid negative reasons preventing being rated $h-1$ or better are formulated as:

$$\left(\cup_{j=h-1}^{q-1} U^-_{er,j} \cup U^-_{r,h-1}\right) \setminus \cup_{j=1}^{h-2} U^-_{rr,j}$$

. To detail this formula, negative reasons against a rating $h-1$ or better contains:

- the negative reasons against being rated $h-1$ or better unless they were withdrawn to a better category $U^-_{r,h-1} \setminus \cup_{j=1}^{h-2} U^-_{rr,j}$.
- the enriched negative reasons to a worse category than $h-1$ they were withdrawn to a better category $\cup_{j=h-1}^{q-1} U^-_{er,j} \setminus \cup_{j=1}^{h-2} U^-_{rr,j}$

$L^t_{u,l}$ contains objects for which the worst possible rating is l. The worst possible rating for an object x is the best category for which x has no valid negative reasons and valid positive reasons for being rated l. The absence of valid negative reasons are represented by $\neg(Z^t_{u,k+1} \succsim^t \{x\})$. Valid positive reasons are presented by $\cup_{j=1}^{k} U^{+}_{ur,j} \cup U^{+}_{r,k}$. Algorithmic details about the assessment of the sets $H^t_{u,h}$ and $L^t_{u,l}$ can be found in the next chapter, Algorithm 5.

The binary relation $\succsim^t$, used in the assessment of $H^t_{u,h}$ and $L^t_{u,l}$ for all $h, l \in [\![1\ ; q]\!]$, is characterized by the following proposition:

Proposition 8. *(properties of $\succsim^t$)*

For a given $t \in T$, $x \in A^t_u$, $Z^t_{u,k} \in Z^t_u$, we have the following properties:

If a set of reference profiles characterizing a rating k is at least as good as x than any set of reference profiles characterizing a better rating is at least as good as x. This can be formulated as:

$$Z^t_{u,k} \succsim^t \{x\} \implies \forall s \in [\![1\ ;\ k]\!], Z^t_{u,s} \succsim^t \{x\} \tag{6.46}$$

If a set of reference profiles characterizing a rating k is at most as good as x than any set of reference profiles characterizing a worse rating is at most as good as x. This can be formulated as:

$$\{x\} \succsim^t Z^t_{u,k} \implies \forall s \in [\![k\ ;\ q]\!], \{x\} \succsim^t Z^t_{u,s} \tag{6.47}$$

Proof. For a given step $t \in T$, and $x \in A^t_u$,

1. Let us assume that $\exists Z^t_{u,k} \in Z^t_u$ such that $Z^t_{u,k} \succsim^t \{x\}$. We aim to prove that $\forall s \in [\![1\ ;\ k]\!], Z^t_{u,s} \succsim^t \{x\}$.

   Since for $s \in [\![1\ ;\ k]\!]$, we have: $\cup_{j=k-1}^{q-1} U^{-}_{er,j} \subseteq \cup_{j=s-1}^{q-1} U^{-}_{er,j}$; $U^{-}_{r,k-1} \subseteq U^{-}_{r,s-1}$; $\cup_{j=1}^{s-1} U^{-}_{rr,j} \subseteq \cup_{j=1}^{k-1} U^{-}_{rr,j}$, then $x \in (\cup_{j=s-1}^{q-1} U^{-}_{er,j} \cup U^{-}_{r,s-1}) \setminus (\cup_{j=1}^{s-1} U^{-}_{rr,j})$. Hence, $Z^t_{u,s} \succsim^t \{x\}$.

2. Let us assume that $\exists Z^t_{u,k} \in Z^t_u$ such that $\{x\} \succsim^t Z^t_{u,k}$. We aim to prove that $\forall s \in [\![k\ ;\ q]\!], \{x\} \succsim^t Z^t_s$.

   Since for $s \in [\![k\ ;\ q]\!]$, we have: $\neg(Z^t_{u,k+1} \succsim^t \{x\}) \implies \neg(Z^t_{u,s+1} \succsim^t$

$\{x\}$) (justified by 6.46); Also $\cup_{j=1}^{k} U_{ur,j}^{+} \subseteq \cup_{j=1}^{s} U_{ur,j}^{+}$; $U_{r,k}^{+} \subseteq U_{r,s}^{+}$ (see proposition 2 "1.").

Hence, $\{x\} \succcurlyeq^t Z_{u,k}^t \implies \neg(Z_{u,s+1}^t \succcurlyeq^t \{x\}) \wedge (x \in \cup_{j=1}^{s} U_{ur,j}^{+} \cup U_{r,s}^{+})$. Thus, $\{x\} \succcurlyeq^t Z_{u,s}^t$.

■

Proposition 9. For a given $t \in T$, the sets $H_{u,1}^t, ..., H_{u,q}^t$ and $L_{u,1}^t, ..., L_{u,q}^t$, are two partitions of A_u^t.

Proof. At a given step $t \in T$:

1. Let's prove that $H_{u,1}^t, ..., H_{u,q}^t$ is a partition of A_u^t.

   Since, based on Proposition 8, for all $h, j \in [\![1\ ; q]\!]$, $h < j$, $Z_{u,j}^t \succcurlyeq^t \{x\} \implies Z_{u,h+1}^t \succcurlyeq^t \{x\}$, we have:

$$H_{u,h}^t \cap H_{u,j}^t = \emptyset \tag{6.48}$$

   For all $x \in A_u^t$, we have $Z_{u,1}^t \succcurlyeq^t \{x\}$ (even in case x has no valid negative reasons $x \in U_{r,0}^{-} = A^t \setminus U_{r,1}^{-}$) Hence:

$$\begin{aligned} x \in A_u^t &\implies Z_{u,1}^t \succcurlyeq^t \{x\} \\ &\implies \Big(Z_{u,1}^t \succcurlyeq^t \{x\} \wedge \neg(Z_{u,2}^t \succcurlyeq^t \{x\})\Big) \vee ... \vee \\ &\Big(Z_{u,q-1}^t \succcurlyeq^t \{x\} \wedge \neg(Z_{u,q}^t \succcurlyeq^t \{x\})\Big) \vee Z_{u,q}^t \succcurlyeq^t \{x\} \\ &\implies \cup_{j=1}^{q} H_{u,j}^t \end{aligned}$$

   Also since for all $h \in [\![1\ ;\ q]\!]$: $H_{u,h}^t \subseteq A_u^t$, we have:

$$\cup_{h=1}^{q} H_{u,h}^t = A_u^t \tag{6.49}$$

   From 6.48 and 6.49, $H_{u,1}^t, ..., H_{u,q}^t$ is a partition of A_u^t.

2. Let's prove that $L^t_{u,1}, ..., L^t_{u,q}$ is a partition of A^t_u.

   Based on Proposition 8, for all $l, j \in [\![1\ ; q]\!]$, $j < l$, $\{x\} \not\succsim^t Z^t_{u,j} \implies \{x\} \not\succsim^t Z^t_{u,l-1}$, we have:

$$L^t_{u,l} \cap L^t_{u,j} = \emptyset \tag{6.50}$$

   For all $x \in A^t_u$, we have $\{x\} \succsim^t Z^t_{u,q}$ (since the way positive and negative reasons are assessed, we will always have valid positive reasons and no valid negative reasons to be in the worst category). Hence:

$$\begin{aligned} x \in A^t_u &\implies \{x\} \succsim^t Z^t_{u,q} \\ &\implies \Big(\{x\} \succsim^t Z^t_{u,q} \wedge \neg(\{x\} \succsim^t Z^t_{u,q-1})\Big) \vee ... \vee \\ \Big(\{x\} &\succsim^t Z^t_{u,2} \wedge \neg(\{x\} \succsim^t Z^t_{u,1})\Big) \vee \{x\} \succsim^t Z^t_{u,1} \\ &\implies \cup^q_{j=1} L^t_{u,j} \end{aligned}$$

   Also since for all $l \in [\![1\ ;\ q]\!]$: $L^t_{u,l} \subseteq A^t_u$, we have:

$$\cup^q_{l=1}\, L^t_{u,l} = A^t_u \tag{6.51}$$

   From 6.50 and 6.51, $L^t_{u,1}, ..., L^t_{u,q}$ is a partition of A^t_u.

■

A first rating might be established based on the latest developments. This rating concerns objects for which the best and worst possible rating lead to the same category: objects in $H^t_{u,k} \cap L^t_{u,k}$, for all k. However, the rating of objects is not always precise: objects in $A^t_u \setminus (\cup^q_{k=1} H^t_{u,k} \cap L^t_{u,k})$. Such objects require additional information in order to be rated. This information can be seen as additional positive reasons supporting a rating to one of the categories located between the best and the worst possible categories. For this aim, we define a symmetric binary relation based in the distance function *dist*, see definition 18. This function represents a similarity measure evaluating how close is an object from an updated set of reference profiles Z^t_u.

Definition 28. *($U^+_{2r,k}$, for $k \in [\![1\ ;\ q]\!]$)*

$U^+_{2r,k}$, for $k \in [\![1\ ;\ q]\!]$, refers to the set of objects for which the rating is not precise and the closest updated reference profiles are the ones rated k. for $k \in [\![1\ ;\ q]\!]$, $U^+_{2r,k}$ can be formulated as:

$$U^+_{2r,k} = \{x \in \Big((\cup_{j=1}^{k} H^t_{u,j}) \cap (\cup_{j=k}^{q} L^t_{u,j})\Big) \setminus (H^t_{u,k} \cap L^t_{u,k}); xR^+_{2r}Z^t_{u,k}\} \tag{6.52}$$

where R^+_{2r} is a binary relation defined on $A^t_u \times Z^t_u$, that can be interpreted for $(x, Z^t_{u,k})$ as "x is as good as reference profiles characterizing $\mathscr{C}_k$". For $x \in A^t_u$, R^+_{2r} can be formulated as:

$$xR^+_{2r}Z^t_{u,k} \implies Z^t_{u,k} = \arg \min_{Z \in K_x \subseteq Z^t_u} dist(x, Z) \tag{6.53}$$

where $K_x = \{Z^t_{u,k} \in Z^t_u; x \in \Big((\cup_{j=1}^{k} H^t_{u,j}) \cap (\cup_{j=k}^{q} L^t_{u,j})\Big) \setminus (H^t_{u,k} \cap L^t_{u,k})\}$.

K_x consists on sets of reference profiles characterizing categories for which the rating of the object x is not precise based on $\succsim^t$.

The use of the second level of positive reasons may lead to the violation of the convincing "condition". For this aim, Algorithm 1, starts by rating objects for which the rating is precise, then the ones for which the rating requires using the distance. The assignment of objects for which the rating is not precise is computed from the best to the worst category. This direction of rating is because each object x rated k based on the second level of positive reasons lead to enriching positive reasons of other objects in worse categories: an object $y \in H^t_{u,s}$ with $k < s$ (worse than k), such that yR^+x will be rated s and thus assigned to Z^{t+1}_s. Also, the objects for which the best possible rating, $H^t_{u,j}$ with $j \leq k$ (obviously their worst possible rating l is worst then k: $k < l$, otherwise they would be previously rated) will be assigned to Z^{t+1}_s. x will be then removed from the considered objects (will be added to the set Z_{2r} see Algorithm 1) and we will move to the next object having the best second level of positive reasons.

Algorithm 1: Rating Algorithm

Input: $\forall s \in [\![1 \;;\; q]\!] : H^t_{u,s}, U^+_{2r,s}, Z^t_{u,s}, L^t_{u,s}$;
Output: $\forall s \in [\![1 \;;\; q]\!] : Z^{t+1}_s$;

Function `Rating algorithm(`$\forall s \in [\![1 \;;\; q]\!] : H^t_{u,s}, U^+_{2r,s}, Z^t_{u,s}, L^t_{u,s}$`):`
  $Z_{2r} \longleftarrow \emptyset$;
  $Z^{t+1}_s \longleftarrow \emptyset$;
  for *s=1 to q* **do**
    $Z^{t+1}_s \longleftarrow Z^t_{u,s} \cup (L^t_{u,s} \cap H^t_{u,s})$;
    $Z_{2r} \longleftarrow Z^{t+1}_s$;
  end
  for *s=1 to q-1* **do**
    foreach $x \in (U^+_{2r,s} \cup Z^{t+1}_s) \setminus Z_{2r}$ **do**
      $Z^{t+1}_s \longleftarrow Z^{t+1}_s \cup \{x\}$;
      $Z_{2r} \longleftarrow Z_{2r} \cup \{x\}$;
      for *j=s+1 to q* **do**
        foreach $y \in H^t_j \setminus Z^{t+1}_j$ *st:* yR^+x **do**
          $Z^{t+1}_j \longleftarrow Z^{t+1}_j \cup \{y\}$;
        end
      end
      for *j=1 to s* **do**
        foreach $y \in H^t_j \setminus Z^{t+1}_s$ *st:* yR^+x **do**
          $Z^{t+1}_s \longleftarrow Z^{t+1}_s \cup \{y\}$;
        end
      end
    end
  end
  return $\forall s \in [\![1 \;;\; q]\!] : Z^{t+1}_s$;
End Function

This procedure provide a complete rating and respecting the convincing condition as it will be announced in the next section.

6.6 Performance quality of Dynamic-R

In this section, we will show that the obtained rating is convincing in case the initial set of reference profiles Z^0 is convincing. We will also provide statistics about the precision of the rating before using the symmetric relation *dist*.

6.6.1 The respect of the convincing condition

Obtaining a "convincing" rating is garanteed by the following theorem:

Theorem 1. For $t \in T$, if Z^t respects the convincing condition then Z^{t+1} respects the convincing condition.

Proof. For $t \in T$, for $k \in [\![1\ ;\ q]\!]$, let x be a reference profile in Z_k^{t+1}.

Let us consider that there exists a reference profile $y \in Z_s^{t+1}$ characterizing a category worse than k ($k < s$) such that such that yR^+x and $y \notin U_{r,k}^-$ at beginning of the step $t+1$.
We have: $x \in Z_k^{t+1} \implies \exists h \in [\![1\ ;\ k]\!], l \in [\![k\ ;\ q]\!] : x \in H_{u,h}^t \cap L_{u,l}^t$
We distinguish two cases:

Case 1 $h = l$**:** In such case, since yR^+x, x would provide positive reasons to y supporting its rating k. Hence:

$$y \in U_{r,k}^+ \cup (\cup_{j=1}^k U_{ur,j}^+) \tag{6.54}$$

Based on Definition 27, we have:

$$\neg(yR_r^- Z_k^{t+1}) \implies \exists j \geq k : y \in H_{u,j}^t \tag{6.55}$$

Thus from 6.54 and 6.55, we have $\exists j \leq k : y \in L_{u,j}^t$ (better than k). Absurde since y was assigned to a worst category $s > k$.

Case 2: $h < l$. In such case, since x was assigned by Algorithm 1 to a category better than the one to which y was assigned, then x will provide positive reasons to y (because yR^+x) to be assigned to the best possible (for which it has no valid negative reasons) category worse than k. Since $y \notin U_{r,k}^-$, at the beginning of the step $t+1$, then y had no valid negative reasons preventing being rated k at the end of the step t. Hence, it would be assigned by the algorithm to at least Z_k^{t+1}. Absurde since y was assigned to a set of reference profiles Z_s^{t+1} characterizing a worse category. ■

Theorem 1 guarantees that the obtained rating is convincing at each step. A direct deduction of this theorem is the following:

Corollary 3. *If Z^0 is convincing, then for all $t \in T$, Z^t is convincing.*

Proof. Obvious: direct conclusion of Theorem 1. ■

At the end of each step, the obtained rating is complete. This is formulated in the following theorem.

Theorem 2. *For $t \in T$, the resulting rating of Dynamic-R is complete: $Z_{1,q}^{t+1} = Z_{1,q}^t \cup A^t$.*

Proof. By construction we have $Z_{1,q}^{t+1} \subseteq Z_{1,q}^t \cup A^t$. By construction we have $Z_{1,q}^{t+1} \subseteq Z_{1,q}^t \cup A^t$. Let's consider $z \in Z_k^t$. In case neither positive nor negative reasons were updated, z will be in $Z_{u,k}^t$ and thus in Z_k^{t+1}. Otherwise z will be in A_u^t. By construction we have $A^t \subseteq A_u^t$. Using Proposition 9: $H_{u,1}^t, ..., H_{u,q}^t$ and $L_{u,1}^t, ..., L_{u,q}^t$, are two partitions of A_u^t. Also R_{2r}^+ is computed for all objects in $A_u^t \setminus \cup_k((H_{u,k}^t \cap L_{u,k}^t))$. Hence $\cup_k(H_{u,k}^t \cup L_{u,k}^t)) = \cup_k(U_{2r,k}^+ \cup (H_{u,k}^t \cap L_{u,k}^t))$. ■

These results are very interesting, and make the method adapted to the context of automatic decision making. In fact, in automatic decision making the result should be convincing, justifiable and all objects should be rated.

6.6.2 Statistics about the precision of the rating

In order to derive statistics about the precision of Dynamic-R before using R_{2r}^+, we will define a fitness index for each object. The fitness index represents the precision of a given rating associated to an object. For instance, the best fitness index corresponds to the case where the best and the worst possible rating for any object refers to the same category, while the worst fitness index corresponds to the case where for all objects the best possible rating is 1 and the worst is q.

Definition 29. *(fitness index)*

The fitness index, for $t \in T$, is a function $f_t : A_u^t \to [1/q, 1]$ assessing the precision level of rating associated to objects based on best and the worst possible rating. "f_t" is defined as follow:

$$\forall t \in T, \forall x \in A_u^t, f_t(x) = \frac{q + h_t(x) - l_t(x)}{q} \tag{6.56}$$

where $h_t, l_t : A_u^t \to [\![1 \ ; \ q]\!]$, and $h_t \leq l_t$, being respectively the best and the worst ratings that can be taken by an object at a step $t \in T$.

Remark 17. $\forall l, h \in [\![1 \ ; \ q]\!], x \in L_l^t \cap H_h^t \implies (h_t(x) = h) \wedge (l_t(x) = l)$

We might define equivalence classes of objects having the same fitness. These equivalence classes can be defined as follow:

Definition 30. *(The class of objects with an equivalent priority)*

The class of objects with equivalent priority B_j^t, at the step t, is an equivalence class where all objects have the same fitness value. Such equivalence class can be defined as follow: $\forall j \in [\![0 \ ; \ q-1]\!]$,

$$B_j^t = \{x \in A_u^t; f_t(x) = \frac{q-j}{q}\} \tag{6.57}$$

Remark 18. B_j^t represents the set of objects for which the imprecision is $\frac{j}{q}$: for an object $x \in B_j^t$, $j = l_t(x) - h_t(x)$.

The set of equivalence classes can be used to describe the quality of the rating based on the previously defined positive and negative reasons. Based on the cardinality of B_j^t, $H_{u,j}^t$ and $L_{u,j}^t$ for all j, we can draw a distribution function related to the precision and the diversity of the rating (based on $H_{u,j}^t$ and $L_{u,j}^t$ for all j) before computing a symmetric binary relation. The mode, the median and the mean can be provided to the decision maker.

These distributions can be also indicators about the quality of the reference profiles and the objects to be rated: In case the number of objects rated with a high precision is important, and the cardinalities of $H^t_{u,1} \cap L^t_{u,1}, ..., H^t_{u,q} \cap L^t_{u,q}$ converge to a discrete uniform distribution, this means that the set of reference profiles and the objects to be rated are very rich.

6.7 Discussion

Automatic decision making requires some properties such as the decisiveness and the consistency. Dynamic-R is a method developed for automatic decision making purposes, particularly rating problems. In the context of rating, the decisiveness property is represented by the completeness and the consistency by the convincing property. Also, the resulting rating can be justified by a set of positive and negative.

6.7.1 Discussion about negative reasons

When strict preference relation is used in the assessment of negative reasons, Dynamic-R presents some weaknesses. Imagine a reference profile characterizing a worse category having the best possible performances with respect to all criteria except one for which it has a performance violating the minimum requirements. Such reference profile might provide false negative reasons for many objects. The same situation might hold during the enrichment of negative reasons. This lead to the possibility of having a non efficient rating. Also, Forcing the consistency while using strict preference relation in the assessment of negative reasons, may lead to assigning all objects to the worse category. Although different tools are present in the developed method to break cycles, such as not considering non-compatible objects as origins of enriching negative reasons. For instance, let's consider a graph $G = (V, E)$ of negative reasons where vertices display objects and reference profiles while edges represent the negative reasons among the vertices. In case the graph is constituted of non disjointed cycles, then all objects will belong to the same category because of the Condorcet paradox.

One solution might consist on deleting the strict preference relation from negative reasons. In case there exist criteria for which categories are characterized by the minimum requirements, the negative discrimination can be based on the minimum requirements and the dominance relation. In such case, deleting the strict preference relation will lead to an efficient rating since the use of minimum requirement or the dominance allows the transitivity of the order. In case categories are not characterized by minimum requirements, deleting the strict preference relation from negative reasons may lead to a luck of discrimination (which is the objective of using negative reasons), since the set of negative reasons characterizing categories $\forall k \in [\![1 \; ; \; q-1]\!] : U^-_{r,k}, U^-_{er,k}, U^-_{rr,k}$ might be empty due to the absence of dominance among objects $D = \emptyset$. This is due to the possibility of having no dominance among objects.

An interesting possible solution consists on optimizing the majority threshold λ. On one hand, setting $\lambda = 1$ leads to a weak dominance (unless objects are identical or indifference thresholds are involved). In such case, the obtained rating is efficient: convincing and reference profiles will not be updated. On the other hand, the more λ is far from $\lambda \leq 1 - min\{w(A); \forall A \subseteq \mathscr{F}\}$ the more we might obtain cycles of strict preferences. Hence, a research question might consist on developing a model determining the majority threshold that should be considered in the rating procedure.

6.7.2 Discussion about the second level of positive reasons

Definition 28 represents the assessment of the objects having a second level of positive reasons. This last represents additional reasons supporting a rating. These reasons can be interpreted as the capability of an object to describe a category based on how close it is from the sets of reference profiles. The second level of positive reasons might also provide additional positive reasons for other objects: each object assigned based on the second level of positive reasons may enrich positive reasons for objects in worse categories and more precisely the ones for which the assignment is not precise. Hence, it is possible to not assess the second level of positive reasons for all objects for which the assignment is not precise. However, the rating algorithm should be aware of respecting the convincing condition since using a symmetric binary relation does not respect the monotonicity of the assignment. For instance, consider two objects x and y such that $xR^{+}_{2,r}Z^{t}_{u,j}$ and $yR^{+}_{2,r}Z^{t}_{u,j+1}$; we have no guarantee that $\neg(yP_{\lambda}x)$. Hence, the order of assessing the second level of positive reasons is very important.

Remark 19. *(Some advantages of considering the second level of positive reasons)*

- The only case where $dist(x, Z^{t}_{u,k}) = 1$ is when x dominates all the reference profiles in Z^{t}_{k} or the inverse. Such case cannot occur since it means that either we have negative reasons or the first level of positive reasons, which is not possible since the distance is evaluated upon a set of categories $K_x \subseteq Z^{t}_{u}$ for which we have neither positive nor negative reasons.

- The distance catches some natural interesting situations such as, the case in which there exists $z \in Z^{t}_{u,k}$ such that $c(x, z) = c(z, x)$, this means that criteria in favor of x and the ones in favor of z have the same importance. This implies that $|c(x, z) - c(z, x)| = 0$, thus, $dist(x, Z^{t}_{u,k}) = 0$, However the first level of positive reasons can ignore this possibility in case the considered level of majority is $c(x, z) < \lambda$, x and z will be incomparable.

- It might be possible to have $k \in [\![1\ ;\ q-1]\!]$ for which $Z^{t}_{u,k}, Z^{t}_{u,k+1} \in K_x \subseteq Z^{t}_{u}$ such that $xR^{+}_{2r}Z^{t}_{u,k}$ and $xR^{+}_{2r}Z^{t}_{u,k+1}$. If such a situation occurs,

x might be assigned to either k or $k+1$ taking into account the opinion of the decision maker.

6.8 Solution to problems mentioned in section 6.7.1

Dynamic-R 2.0 is an extension of Dynamic-R introducing the following characteristics, in addition to the ones proposed in Dynamic-R:

1. Outranking relations are extended to sets comparisons and used to support and to oppose a rating.

2. effect of cycles of preference crossing different categories is minimized.

3. positive and negative reasons are kept separated, and we only consider at the last step how to aggregate them;

4. negative reasons use both the strict preference (it will be defined in the next subsection) between sets of reference profiles and objects; and the minimal requirements;

5. positive reasons use the Outranking of objects over a set of reference profile.

6.8.1 Additional basic materials

The rating process associated to Dynamic-R 2.0 is similar to the one of dynamic-R, with few modifications. Positive and negative reasons characterizing Dynamic-R 2.0 are based on the following materials:

- The respect of the minimum requirements characterizing the categories;

- The presence of a sufficient majority supporting a set of reference profiles[1] “to be at least as good as an object” and the absence of any sufficient majority of criteria supporting this object “To be at least as good as this set of reference profiles”;

- The Presence of a sufficient majority of criteria supporting an object “To be at least as good as a set of reference profiles” characterizing a category.

- The presence of similarities between an object and a set of reference profiles in case of interval rating.

In this section, we will detail the materials that are not previously mentioned in Dynamic-R.

In what follows, we will use binary relations between objects and the sets of reference profiles. For this aim, we propose the following three definitions:

Definition 31. *(Outranking relations between objects and reference profiles)*

Consider the set A and a set of sets B. A binary relation $\mathcal{O} \subseteq A \times B \cup B \times A$, such that $\forall (x, Y) \in A \times B : x\mathcal{O}Y$ should be read as “x is at least as good as Y”, and $\forall (Y, x) \in B \times A : Y\mathcal{O}x$ should be read as “Y is at least as good as x”.

Remark 20. *In this work, we will consider only singletons in $2^{A_u^t}$.*

Definition 32. *(Extended Outranking relation)*

Extended Outranking O_λ is a binary relation defined on $A^t \times (Z^t \cup 2^{A^t}) \cup (Z^t \cup 2^{A^t}) \times A^t$. O_λ can be defined as follows:

1. on $A^t \times (Z^t \cup 2^{A^t})$ the extended Outranking relation is defined as

$$xO_\lambda Y \iff \exists y \in Y : xS_\lambda y \tag{6.58}$$

[1]Note that we are dealing with the whole sets of reference profiles instead of the elements in these sets.

2. on $(Z^t \cup 2^{A^t}) \times A^t$ the extended Outranking relation is defined as

$$YO_\lambda x \iff \exists y \in Y : yS_\lambda x \tag{6.59}$$

The extended Outranking relation leads to the following preference structures:

Definition 33. *Based on the extended Outranking relation, we define four relations on $A^t \times (Z^t \cup 2^{A^t}) \cup (Z^t \cup 2^{A^t}) \times A^t$:*

- *the strict preference P_λ on $A^t \times (Z^t \cup 2^{A^t})$:*

$$xP_\lambda Y \iff xO_\lambda Y \wedge \neg(YO_\lambda x) \tag{6.60}$$

- *the strict preference P_λ on $(Z^t \cup 2^{A^t}) \times A^t$:*

$$YP_\lambda x \iff \neg(xO_\lambda Y) \wedge YO_\lambda x \tag{6.61}$$

- *the indifference I_λ:*

$$xI_\lambda Y \iff xO_\lambda Y \wedge YO_\lambda x \tag{6.62}$$

- *the incomparability J_λ:*

$$xJ_\lambda Y \iff \neg(xO_\lambda Y \vee YO_\lambda x) \tag{6.63}$$

This new definition of the Outranking relation over sets is natural: a new student is better (in terms of strict preference) than the category of good students in a school if he is at least as good as all former good students in the school and non of them is at least as good as him. He has the same level (here we are dealing with the case of indifference) of former good students if he outranks at least one former good student and at least one former good student outranks the new one. The case of incomparability is when the new student is incomparable with all former good students.

Remark 21. *This definition of strict preference relation is weaker than the original one, but more informative: By definition*

$$xP_\lambda Y \iff (\exists y \in Y : xS_\lambda y) \wedge (\forall y \in Y : \neg(yS_\lambda x))$$

and

$$YP_\lambda x \iff (\exists y \in Y : yS_\lambda x) \wedge (\forall y \in Y : \neg(xS_\lambda y))$$

Using the extended Outranking relation lower the chances to have a "bad" quality of rating, where all objects are assigned to the same category.

6.8.2 Definitions of positive and negative reasons in Dynamic-R 2.0

This section is dedicated to present the hypothesis made in this work as well as the assessments of positive and negative reasons used in Dynamic-R 2.0.

6.8.3 Hypothesis of Dynamic-R 2.0

In order to avoid the situation where $xO_\lambda Z_k^t \wedge Z_{k+h}^t P_\lambda x$, with $h > 0$, we make the following hypothesis, called the separability Hypothesis:

Hypothesis. *We assume that at the time step $t = 0$, each reference profile characterizing a category k, should dominates a reference profile in the category $k+1$. This can be formulated as: $\forall k \in [\![1\ ;\ q-1]\!]$*

$$\forall x \in Z_k^0, \exists y \in Z_{k+1}^0 : xDy \tag{6.64}$$

This hypothesis is important for the rating to be convincing.

Within this new extension, we propose the following new definition of the "convincing" property. At all steps in T of the process, the set of reference profiles must respect the following basic "convincing" property:

Definition 34. *"Convincing" property can be formulated, in the context of Dynamic-R 2.0 as:*

$$\forall Z_k^t \in Z^t, \nexists y \in Z_h^t(h > k) : yO_\lambda Z_k^t \wedge y \notin U_{r,k}^- \tag{6.65}$$

Negative and positive reasons

Negative reasons against rating k, for an object x, represent the situation where either at least one reference profile characterizing a worse category h is strictly preferred to x and x does not outrank any of the other reference profiles characterizing h or x is incompatible with k.

Definition 35. *($U_{r,k}^-$, For $k \in [\![1\ ;\ q-1]\!]$)*

The set of objects having negative reasons against being assigned to a given category k, $U_{r,k}^-$ can be formulated as

$$U_{r,k}^- = \{x \in A^t \cup Z_{1,q}^t, xR_r^- Z_k^t\}, \forall k \in [\![1\ ;\ q-1]\!] \tag{6.66}$$

where R_r^- is a binary relation representing negative reasons against a rating. R_r^- can be defined on $(A^t \cup Z_{1,q}^t) \times Z^t$ as: For $x \in A^t \cup Z_{1,q}^t, Z_k^t \in Z^t$,

$$xR_r^- Z_k^t \iff \exists h \in [\![k+1\ ;\ q]\!] : Z_h^t P_\lambda x \vee xIncomp_{lower} Z_k^t. \tag{6.67}$$

Positive reasons supporting rating k, for an object x, represent the situation where x is at least as good as (outrank) a set of reference profiles characterizing a better category.

Definition 36. *($U_{r,k}^+$, For $k \in [\![1\ ;\ q]\!]$)*

The set of objects having positive reasons supporting the assignment to a given category k, named $U^+_{r,k}$, can be formulated as:

$$\forall k \in [\![1\ ;\ q]\!] : U^+_{r,k} = \{x \in A^t \cup Z^t_{1,q}, xR^+_{1r}Z^t_k\} \tag{6.68}$$

where R^+_{1r} is a binary relation defined on $(A^t \cup Z^t_{1,q}) \times Z^t$ representing the possibility to be at least as good as reference profiles characterizing a category. R^+_{1r} can be formulated as: For $x \in A^t \cup Z^t_{1,q}, Z^t_k \in Z^t$,

$$xR^+_{1r}Z^t_k \iff \exists h \in [\![1\ ;\ k]\!] : xO_\lambda Z^t_h. \tag{6.69}$$

Proposition 10. *(The monotonicity of positive and negative reasons)*

$$\forall k \in [\![2\ ;\ q]\!] : U^+_{r,k-1} \subseteq U^+_{r,k} \text{ (Monotonicity of positive reasons.)} \tag{6.70}$$

$$\forall k \in [\![2\ ;\ q-1]\!] : U^-_{r,k} \subseteq U^-_{r,k-1} \text{ (Monotonicity of negative reasons.)} \tag{6.71}$$

Proof. The monotonicity of the set of positive reasons is by definition.
The monotonicity of the set of negative reasons is due to the transitivity of the dominance with respect to the minimum requirements characterizing different categories and also since:

$$\exists h \in [\![k+1\ ;\ q]\!] : Z^t_h P_\lambda x \implies \exists h \in [\![k\ ;\ q]\!] : Z^t_h P_\lambda x$$

■

Reasons based on objects comparisons

Since the last rated objects at each time step are added to the sets of reference profiles used in the next step, positive and negative reasons need to take into consideration the way objects compare to each other. This leads to the possible enriching of positive reasons, or to the withdrawing negative ones.

Remark 22. *In Dynamic-R 2.0, negative reasons are not enriched. This is by construction of the extended Outranking relation: Even if an object in a worse category is strictly preferred to an object in a better category, the fact that the second object outranks another in same category of the first one, will disqualify the strict preference and will make it an indifference.*

Definition 37. *($U^+_{ur,k}$, For $k \in [\![1\ ;\ q-1]\!]$)*

For a given $k \in [\![1\ ;\ q-1]\!]$, the set of objects, $U^+_{ur,k}$, for which positive reasons were enriched to support a rating k, can be formulated as:

$$U^+_{ur,k} = \{x \in Z^t_{1,q} \cup A^t : xR^+_{ur}Z^t_k \wedge \neg(xR^+_{ur}Z^t_{k-1})\} \tag{6.72}$$

where R^+_{ur} is a binary relation representing enriched positive reasons supporting a rating. R^+_{ur} can be formulated as: For $x \in U^+_{r,l} \setminus U^+_{r,l-1}$, $k < l$:

$$xR^+_{ur}Z^t_k \iff xO_\lambda(\cup^k_{j=1}U^+_{ur,j} \cup U^+_{r,k}) \tag{6.73}$$

Enriching positive reasons for a given x to be classified in a better category k, lies on the existence of an other object y outranked by x, having positive reasons to be assigned to k. Hence, at the end of a step t, if y is assigned to a category k, we will have $xO_\lambda Z^{t+1}_k$.

Definition 38. *($U^-_{rr,k}$, For $k \in [\![1\ ;\ q-1]\!]$)*

For a given $k \in [\![1\ ;\ q-1]\!]$, the set of objects, $U^-_{rr,k}$, for which negative reasons were withdrawn in order to prevent a better rating k, can be formulated as:

$$U^-_{rr,k} = \{x \in Z^t_{1,q} \cup A^t : xR^-_{rr}Z^t_k \wedge \neg(xR^-_{rr}Z^t_{k+1})\} \tag{6.74}$$

where R^-_{rr} is binary relation representing withdrawn negative reasons against a rating. R^-_{rr} can be formulated as: For $x \in U^-_{r,h} \setminus U^-_{r,h+1}$, $k < h$:

$$xR^-_{rr}Z^t_k \iff \begin{cases} \neg(U^-_{r,h} \setminus \cup_{j=1}^{h-1} U^-_{rr,j} P_\lambda x \vee x Incomp_{lower} Z^t_h) \\ \text{And} \\ \Big[(\cup_{j=1}^{h} U^+_{ur,j} \cup U^+_{r,h}) \cap N^-_{h,k} P_\lambda x\Big] \vee x Incomp_{lower} Z^t_k \end{cases} \tag{6.75}$$

with $N^-_{h,k} = \cup_{j=k}^{h-1} U^-_{rr,j} \cup U^-_{r,k} \setminus \left(\cup_{j=1}^{k-1} U^-_{rr,j} \cup U^-_{r,h}\right)$ representing objects with valide negative reasons against ratings between $h-1$ and k.

The rest of the procedure remains same as defined in Dynamic-R.

6.9 Conclusion

This chapter presents a new MCDA method, aiming at providing a “convincing” rating to decision aiding problems for which the Outranking approaches are useful and without considering the IIA axiom. This method is based on a dynamic rating of objects by aggregating positive and negative reasons for and against a rating. The basic idea is to learn from an evolving set of reference profiles characterizing each category. At each time step of the process, reference profiles are constituted by reference profiles and objects rated at the previous step: the rated objects are assigned to the sets of reference profiles corresponding to the categories they characterize.

The developed rating method has interesting properties such as the convincing property (Definition 34). Also the violation of the IIA axiom allow as to reinforce the justification of our rating by the main of positive and negative reasons. Many perspectives might be associated to the developed method, such as: the importance of criteria might change during the process; the possibility to assess positive and negative reasons on coalitions of objects such as the case of an insurance company aiming at rating a package of clients or products that might interact; to name but a few. A specific mention should be given to the development of an argumentation framework for explaining/justifying/defending a rating thanks to the explicit representation of the positive and negative reasons on which such rating has been established.

7 Experimental results

7.1 Introduction

This chapter is dedicated to present major algorithmic and experimental results related to Dynamic-R. The experimental study is based on an implementation, on python, of all the operations used in the developed method and applied to randomly generated data. The aim of this study is to verify the behavior of the method with a large amount of data and a small learning set. Also, we aim at verifying an intuition consisting on abandoning the strict preference relation in the assessment of negative reasons. This intuition will be explained and discussed based on some illustrative examples. After studying the outcome of the method over randomly generated data, the method will be applied to our case study and compared to the results obtained in the chapter "Case Study". The results have been discussed with the IRSN expert with whom we are working. By the end we will present different perspectives of the current work. In this chapter we will not present experiments related to Dynamic-R 2.0. However, we will refer to this variant all along this chapter in the discussions.

This chapter is organized as follow. In the next section, we will present algorithmic aspects related to the tools used in Dynamic-R. In section 3, we will present an experiment upon randomly generated data with a discussion illustrating and supporting different intuitions we had in work. Section 4 is dedicated to present IRSN's case study using Dynamic-R. We end by a conclusion and perspectives.

7.2 Algorithms used in Dynamic-R

Dynamic-R is a method based on defining and aggregating positive and negative reasons respectively supporting and against a rating. These reasons are based on the dominance, the majority rule, and the new concept of minimum requirements. This section is dedicated to present and discuss algorithms implementing the different tools used in the Dynamic-R rating process. In the discussions related to the algorithms, we will mention the modifications that should be done in order to make algorithms operational for Dynamic-R 2.0. The assessment of elementary positive and negative reasons, the ones where we compare objects with profiles, will not be presented in this section as its a direct application of the dominance and the majority rule.

7.2.1 The sets of objects for which negative reasons are enriched

Taking into account the way objects compare in order to each other might lead to new negative reasons in case strict preference relation is used to compare objects: if an object x is "worst" than an object y ($yP_\lambda x$), x should not be assigned to a better category than the one for which y was assigned to. However, such situation might occur due to cycles of preferences. This is because negative reasons opposing the assignment of an object to a given category are not based only on strict preference. An object y might be forbidden from being assigned to a better category not because of its performance, a strict preference in favor of a reference profile, but because of the incompatibility: an almost good object according to a majority of criteria with a bad performance. Thus, stating that x is better or worst than y is not obvious. For this aim, only objects for which negative reasons are not due to minimum requirements can be used to enrich negative reasons for other objects. Algorithm 2, represents a way to enrich negative reasons, in the context of the original version of Dynamic-R.

Algorithm 2: Enriching Negative reasons

Input: $\forall l \in [\![0\ ;\ q-1]\!] : U^-_{r,l}$;
Output: $\forall l \in [\![1\ ;\ q-1]\!] : U^-_{er,l}$;
Function `Enriching Negative Reasons(`$\forall l \in [\![0\ ;\ q-1]\!] : U^-_{r,l}$`):`
    $\forall l \in [\![1\ ;\ q-1]\!] : U^-_{er,l} \longleftarrow \emptyset$;
    for *k=q-1 to 1* **do**
        for *l=k-1 to 0* **do**
            $Y \longleftarrow U^-_{er,k} \cup (U^-_{r,k} \setminus U^-_{r,k+1})$; $NU^-_{er,l} \longleftarrow \emptyset$;
            foreach $x \in (U^-_{r,l}) \setminus (U^-_{er,k} \cup NU^-_{er,l})$ **do**
                if $\exists y \in Y$ *such that* $(yP_\lambda x) \wedge \neg(yIncomp_{lower} Z^t_l)$ **then**
                    $U^-_{er,k} \longleftarrow U^-_{er,k} \cup \{x\}$;
                else
                    $NU^-_{er,l} \longleftarrow NU^-_{er,l} \cup \{x\}$;
                end
            end
        end
        return $U^-_{er,k}$;
    end
End Function

Algorithm 2 consists on defining sets of objects for which negative reasons were enriched. This procedure starts from the worst to the best category, since each object for which negative reasons were enriched can be used to enrich negative reasons for other objects in better categories. Enriching negative reasons consists on verifying whether any object having negative reasons (or enriched negative reasons) preventing a given rating, is strictly preferred or dominates an other object that can potentially be assigned to a better category.

Remark 23. The strict preference might be substituted by the dominance or the extended Outranking relation defined in the previous chapter. In such case, negative reasons should not be enriched.

7.2.2 The sets of objects for which positive reasons are enriched

Due to the non-transitivity of the Outranking relations, positive reasons might be enriched when a "worst" object x outranked by y has positive reasons and no negative reasons to be assigned to a better category. The object y will have new positive reasons supporting its assignment to the same category of x. Algorithm 3 presents the algorithmic aspects related to the assessment of the set of objects for which positive reasons are enriched in Dynamic-R.

Algorithm 3: Updating Positive reasons

Input: $\forall h \in [\![1\ ;\ q]\!] : U^-_{er,h}; U^-_{r,h}; U^+_{r,h}$;
Output: $\forall h \in [\![1\ ;\ q]\!] : U^+_{ur,h}$;
Function `Updating Positive Reasons(`$\forall h \in [\![1\ ;\ q]\!] : U^-_{er,h}; U^-_{r,h}; U^+_{r,h}$`)`**:**
    $\forall h \in [\![1\ ;\ q]\!] : U^+_{ur,h} \longleftarrow \emptyset$;
    for *k=1 to q* **do**
        for *h=k+1 to q* **do**
            $Y \longleftarrow \Big((U^+_{ur,k} \cup U^+_{r,k} \setminus U^+_{r,k-1}) \setminus (\cup_{j=k}^{q-1} U^-_{er,j} \cup U^-_{r,k}) \Big)$;
            $NU^+_{ur,h} \longleftarrow \emptyset$;
            foreach $x \in \Big(U^+_{r,h} \setminus U^+_{r,h-1} \Big) \setminus \Big(U^+_{ur,k} \cup NU^+_{ur,h} \Big)$ **do**
                if $\exists y \in Y$ *such that* $xS_\lambda y$ **then**
                    $U^+_{ur,k} \longleftarrow U^+_{ur,k} \cup \{x\}$;
                else
                    $NU^+_{ur,h} \longleftarrow NU^+_{ur,h} \cup \{x\}$;
                end
            end
        end
        return $U^+_{ur,k}$;
    end
End Function

The assessment of the sets of objects for which positive reasons are enriched is computed from the best to the worst category. This is due to the possibility of an object, for which its positive reasons were enriched, to enrich positive reasons for other objects.

Enriching positive reasons consists on verifying the existence an object y "better", according to a sufficient majority of criteria, than any object having positive and no valid negative reasons against being assigned to a given category k: an object x in $\Big((U^+_{ur,k} \cup U^+_{r,k} \setminus U^+_{r,k-1}) \setminus (\cup_{j=k}^{q-1} U^-_{er,j} \cup U^-_{r,k})\Big)$. Negative reasons are used in order to avoid cases where all positive reasons for objects get enriched which may occur when we consider a high number of objects. This will be illustrated later in the next section. In such a case, positive reasons of y will be improved to support its rating k. Hence, y will be assigned to $U^+_{ur,k}$, and can be used in order to enrich positive reasons of other objects in worse categories.

For Dynamic-R 2.0, the assessment procedure of $U^+_{ur,j}$ for a given j, should take into account the extended Outranking relation, used in the assessment of positive reasons. To achieve this aim, we have to delete negative reasons from the formula in line 5, since negative reasons are not enriched in the case of Dynamic-R 2.0, and substitute the Outranking relation S_λ by the extended Outranking relation $xO_\lambda Y$ in line 7.

7.2.3 The sets of objects for which negative reasons are withdrawn

Due to the possibility of enriching positive reasons, the improvement of the positions of some objects lead to withdrawing the negative reasons they caused to other objects. This leads to improving the positions of the objects for which negative reasons were withdrawn and withdrawing negative reasons they caused to other objects. Algorithm 4, is dedicated to withdraw negative reasons for Dynamic-R.

The assessment $U^-_{rr,h}$, $h \in [\![1 \ ; \ q]\!]$, the set of objects for which negative reasons were withdrawn, is computed for Dynamic-R, in Algorithm 4, from

Algorithm 4: Withdrawing Negative reasons

Input: $\forall h \in [\![1 \ ; \ q]\!] : U_{er,h}^-, U_{ur,h}^+, U_{r,h}^-, U_{r,h}^+$;
Output: $\forall h \in [\![1 \ ; \ q]\!] : U_{rr,h}^-$;
Function `Releasing Negative Reasons(`$\forall h \in [\![1 \ ; \ q]\!] : U_{er,h}^-, U_{ur,h}^+, U_{r,h}^-, U_{r,h}^+$`):`
  $\forall h \in [\![1 \ ; \ q]\!] : U_{rr,h}^- \longleftarrow \emptyset$;
  for *k=q-2 to 0* **do**
    for *h=k+1 to q-1* **do**
      $U_k^- \longleftarrow (U_{er,k}^- \cup U_{rr,k}^- \cup U_{r,k}^-) \setminus (\cup_{j=k+1}^{q-1} U_{er,j}^- \cup (\cup_{j=1}^{k-1} U_{rr,j}^-) \cup U_{r,k+1}^-)$;
      $Y_k \longleftarrow \left[\cup_{j=1}^{k+1} U_{ur,j}^+ \cup U_{r,k+1}^+\right] \cap U_k^-$; $NU_{rr,h}^- \longleftarrow \emptyset$;
      foreach $x \in \left(U_{er,h}^- \cup (U_{r,h}^- \setminus U_{r,h+1}^-)\right) \setminus \left(U_{rr,k}^- \cup NU_{rr,h}^-\right)$ *such that* $\forall z \in Y_h : \neg(zP_\lambda x \vee zDx)$ **do**
        if $\exists y \in Y_k : yP_\lambda x \vee yDx \vee xIncomp_{lower}Z_k^t$ **then**
          $U_{rr,k}^- \longleftarrow U_{rr,k}^- \cup \{x\}$;
        else
          $NU_{rr,h}^- \longleftarrow NU_{rr,h}^+ \cup \{x\}$;
        end
      end
    end
    return $U_{rr,k}^-$;
  end
End Function

the worst to the best category. We start from the worst category since each object for which negative reasons were withdrawn up to a better category $\mathscr{C}_k$, can be used to provide negative reasons stopping the improvement (the withdrawing of negative reasons) for other objects. Withdrawing negative reasons for an object x, in the algorithm, consists on being eligible for withdrawing its negative reasons (see explanation of Definition 38 in the previous chapter), and checking the presence of new negative reasons to a better category. In case, no new negative reasons stop the improvement of x, this last will be assigned to $U_{rr,0}^-$ (objects not having negative reasons). A modification should be done in this algorithm by deleting the strict preference relation when this last is not considered in the assessment of negative reasons of Dynamic-R.

The same principle still valid for Dynamic-R 2.0, with the following modifications: the sets of enriched negative reasons should be deleted from the algorithm in the input line and lines 1, 5 and 7; the condition in line 7 should be substituted by $\neg(U_h^- P_\lambda x \vee x Incomp_{lower} Z_h^t)$ and the line 8 should be substituted by $Y_k P_\lambda x \vee x Incomp_{lower} Z_k^t$.

7.2.4 The partitions of A_u^t into $H_{u,h}^t$ and $L_{u,l}^t$, for all $h, l \in [\![1\ ;\ q]\!]$

In the above, we assessed different equivalence classes: For all h: $U_{r,h}^-$, $U_{er,h}^-$, $U_{rr,h}^-$, $U_{r,h}^+$, $U_{ur,h}^+$. The binary relations, used in the assessment of $U_{er,h}^-$, $U_{rr,h}^-$, and $U_{ur,h}^+$, for all h, might lead to modifying the sets of reference profiles, and increasing the number of objects to be rated. Thus, we defined a new set of objects A_u^t and a new set of reference profiles Z_u^t. The aggregation of all positive and negative reasons leads to a best and worst possible ratings for each object. Based on these two values for each object, we defined an equivalence classes of objects $H_{u,h}^t$ (respectively $L_{u,l}^t$) having the same best possible rating h (respectively the worst possible rating l). In Algorithm 5, we present the way we compute two partitions of the set of A_u^t, $H_{u,h}^t$ and $L_{u,l}^t$, for all $h, l \in [\![1\ ;\ q]\!]$.

Objects in $H_{u,h}^t$ are characterized by a best possible rating h. The assessment of the best possible rating is based on the presence of valid negative reasons in the better categories. For this reasons, the partitioning A_u^t, into $H_{u,1}^t, ..., H_{u,q}^t$ is performed by Algorithm 5, from the worst to the best category: Valid negative reasons leads to the assessment of $\not\succcurlyeq^t$, on $Z_u^t \times 2^{A_u^t}$ between the sets of reference profiles in Z_u^t and objects; such relation is monotonic based on the Proposition 8 ($Z_{u,h}^t \not\succcurlyeq^t \{x\} \implies \forall s \leq h : Z_{u,s}^t \not\succcurlyeq^t \{x\}$). Assessing $H_{u,h}^t$ comes to verifying the existence of objects having valid negative reasons preventing their rating $h-1$, which were not assigned to $H_{u,h+1}^t$ (not having valid negative reasons against being rated h).

Objects in $L_{u,l}^t$ are characterized by a worst possible rating l. The worst

Algorithm 5: Assessment of $H^t_{u,h}$ and $L^t_{u,l}$, for all $h, l \in [\![1\ ;\ q]\!]$

Input: $\forall s \in [\![1\ ;\ q]\!] : U^-_{er,s}, U^-_{r,s}, U^-_{rr,s}, U^+_{ur,s}, U^+_{r,s}, U^-_{r,0}$;
Output: $\forall s \in [\![1\ ;\ q]\!] : L^t_{u,s}, H^t_{u,s}$;
Function `Best & Worse Possible Assignments(`$\forall s \in [\![1\ ;\ q]\!] : U^-_{er,s}, U^-_{r,s}, U^-_{rr,s}, U^+_{ur,s}, U^+_{r,s}, U^-_{r,0}$`):`
  $\forall s \in [\![1\ ;\ q]\!] : L^t_{u,s} \longleftarrow \emptyset$;
  $\forall s \in [\![1\ ;\ q]\!] : H^t_{u,s} \longleftarrow \emptyset$;
  for *k=q to 2* **do**
    $H^t_{u,k} \longleftarrow \Big(U^-_{er,k-1} \cup U^-_{rr,k-1} \cup (U^-_{r,k-1} \setminus U^-_{r,k})\Big) \setminus \Big((\cup_{j=1}^{k-2} U^-_{rr,j}) \cup (\cup_{j=k+1}^{q} H^t_{u,j})\Big)$;
  end
  $H^t_{u,1} \longleftarrow (A^t \cup Z^t_{1,q}) \setminus (\cup_{j=2}^{q} H^t_{u,j})$
  $L^t_{u,1} \longleftarrow \Big(U^+_{ur,1} \cup U^+_{r,1}\Big) \setminus \Big(\cup_{j=2}^{q} H^t_{u,j}\Big)$
  for *k=2 to q-1* **do**
    $L^t_{u,k} \longleftarrow \Big((\cup_{j=1}^{k} U^+_{ur,j}) \cup U^+_{r,k}\Big) \setminus \Big((\cup_{j=k+1}^{q} H^t_{u,j}) \cup ((\cup_{j=1}^{k-1} L^t_{u,j}))\Big)$;
  end
  $L^t_{u,q} \longleftarrow (A^t \cup Z^t_{1,q}) \setminus (\cup_{j=1}^{q-1} L^t_{u,j})$
  return $\forall s \in [\![1\ ;\ q]\!] : L^t_{u,s}, H^t_{u,s}$;
End Function

possible rating l corresponds to the best possible category for which there exists positive reasons and the best possible rating, regarding the absence of valid negative reasons, is at least l. The partitioning A^t_u, into $L^t_{u,1}, ..., L^t_{u,q}$ is due to the monotonicity of $\succsim^t$, on $2^{A^t_u} \times Z^t_u$, see Proposition 8. Hence, in order to assess the worst possible rating l, we start from the best to the worst category, since we delete at each iteration all objects for which the worst possible category is assessed (objects in $\cup_{j=1}^{k-1} L^t_{u,j}$), otherwise we will have $L^t_{u,k} \subseteq L^t_{u,k+1}$.
Algorithm 5 still valid for Dynamic-R 2.0 as we can consider that $\forall h : U^-_{er,h} = \emptyset$.

7.2.5 Assessment of Z^{t+1}

Partitioning the set of objects to be rated into equivalence classes $H^t_{u,h}$ $\forall h$, and $L^t_{u,l}$ $\forall l$, leads to rating k any object in $H^t_{u,k} \cap L^t_{u,k}$. However, $H^t_{u,1} \cap$

$L^t_{u,1}, ..., H^t_{u,q} \cap L^t_{u,q}$ is not a partition of A^t_u. Hence, some objects require more information to be rated. For this purpose, we use the second level of positive reasons representing a distance between objects for which the assignment is not precise and the reference profiles. However, the use of the second level of positive reasons may lead to a violation of the the convincing "condition". For this aim, Algorithm 1, starts by rating objects for which the rating is precise, then the ones for which the rating requires using the distance: we refer to the set objects having a second level of positive reasons to be rated k, $U^+_{2r,k}$. More details, as well as a deep analysis, about the rating algorithm can be found in the previous chapter.

The algorithm remains valid for Dynamic-R 2.0. This procedure provides a complete rating, while respecting the convincing condition.

7.3 Experiments with randomly generated data

The aim of this section is to apply Dynamic-R on randomly generated data in order to analyse the rating procedure. For this aim we use an imaginary MCDA rating problem, in which we generated randomly 200 performance vectors. In this section, we will focus on Dynamic-R and we will not experiment on its variant Dynamic-R 2.0.

7.3.1 Description of the fictitious problem

We consider the imaginary MCDA rating problem, presented in Example 7, in which a headhunter is in charge of finding new candidates to one of his clients. The headhunter does not know exactly the number of employees required by the client. Thus, he decided to run a first analysis in which he has to classify the candidates to three categories: "good opportunities for the client", "probably good opportunities for the client", and "bad opportunities". For simplicity we will rename these profiles respectively "good", "average" and "bad". The candidates are evaluated based on their graduation mark, the assiduity, the physical aptitude, and the requested salary. In this example criteria are assessed on both ordinal and cardinal scales. Details about the used scales are presented in the Example 7.

The veto can be used to disqualify an Outranking relation. In case categories are characterized by limiting profiles, veto can be seen as a negative discrimination power against a rating. This is because, a difference of performance between an object and a limiting profile greater than a veto might be interpreted as a violation of the minimum requirements. **Such interpretation is not correct**: on the one hand, a category might be characterized by a high number of limiting profiles: an object discriminated by a limiting profile due to a veto might be not discriminated by another limiting profile. On the other hand, a minimum requirement characterizing a category is unique since it is the worst possible assignment under each criterion to be admissible in a category regardless of the performance under the other criteria.

In this example, substituting the notion of veto by the minimum requirements is justified by the problem's nature:

- Defining the performance vectors for limiting profiles is a hard task for the client: What is the performance vector separating the "good" candidates and the "possibly good" ones?

- It is easy to assess minimum requirements as we ask about the worst acceptable performance to be in a category from a single criterion point of view, even with the best possible performances on the other criteria.

Performance vectors for the minimum requirements are displayed in Table 7.1:

Vetoes considered in the assessment of the strict preference relation are displayed in Table 7.2.

The sets of reference profiles, respecting the minimum requirements, characterizing categories good candidates (rate 1), average candidates (rated 2),

	Marks	Assiduity	Physical Aptitude	Requested salary
b0	12.0	7.0	3.0	55.0
b1	8.0	5.0	2.0	60.0
b2	20.0	10.0	5.0	70.0

Table 7.1 – Minimum requirements

	Marks	Assiduity	Physical Aptitude	Requested salary
v	7.0	4.0	2.0	8.0

Table 7.2 – Veto thresholds vector

and bad candidates (rated 3) are respectively displayed in Tables 7.3, 7.4, and 7.5.

	Marks	Assiduity	Physical Aptitude	Requested salary
z1,0	20.0	10.0	5.0	55.0
z1,1	20.0	10.0	3.0	40.0
z1,2	20.0	7.0	5.0	40.0
z1,3	12.0	10.0	5.0	40.0
z1,4	16.0	8.0	4.0	45.0
z1,5	14.0	9.0	4.0	40.0
z1,6	14.0	10.0	4.0	45.0

Table 7.3 – Reference profiles for “good” candidates

	Marks	Assiduity	Physical Aptitude	Requested salary
z2,0	20.0	10.0	5.0	60.0
z2,1	20.0	10.0	2.0	40.0
z2,2	20.0	5.0	5.0	40.0
z2,3	8.0	10.0	5.0	40.0
z2,4	12.0	8.0	3.0	50.0
z2,5	16.0	7.0	3.0	45.0
z2,6	14.0	7.0	4.0	45.0

Table 7.4 – Reference profiles for “average” candidates

Applying Dynamic-R to this set of reference profile might lead to improving the ratings of $z2, 5$ and $z2, 6$. This is because, on the one hand, no object in the

	Marks	Assiduity	Physical Aptitude	Requested salary
z3,0	8.0	5.0	2.0	60.0
z3,1	6.0	10.0	5.0	40.0
z3,2	10.0	8.0	4.0	55.0
z3,3	12.0	8.0	3.0	55.0

Table 7.5 – Reference profiles for "bad" candidates

set of reference profiles characterizing the ratings "good" or "average" is strictly preferred to these objects and they do not violate the minimum requirements. On the other hand, $z2,5$ has positive reasons to be "good" coming from $z1,6$ and $z2,5$ might enrich positive reasons for $z2,6$. Although, we will assume that the following set of reference profiles is correct, and at worst positions of reference profiles might be updated during the rating process.

A short version of the performance vectors for the 200 considered candidates is displayed in Table 7.6.

	Mark	Assiduity	Physical Aptitude	Requested salary
A0	11.111125	7.0	4.0	47.669977
A1	13.098216	7.0	2.0	51.011259
A2	13.842586	8.0	4.0	51.914856
...	...	...	...	...
A198	12.669920	9.0	2.0	41.083305
A199	17.972142	2.0	3.0	52.315349

Table 7.6 – Performances of candidates

7.3.2 Assessment of negative reasons

Negative reasons represent the negative discrimination opposing a rating. Reasons for rejecting a rating differ from an object to another. They might be due to the dominance, or the strict preference, or to a violation of the minimum requirements. In this section, we will detail the different origins of negative reasons preventing a rating and we will discuss the possibility of substituting the strict preference by the dominance from the definition of negative reasons.

In our work, we had the intuition to disgard strict preference from the definition of negative reasons, without losing on the discrimination effect of negative reasons. This intuition is justified by:

- In the rating process of Dynamic-R, objects are compared to each other in order to provide a rating. Thus, even with a small set of reference profiles, the effect of discrimination power due to the dominance grows with the number of objects. Unfortunately, in the current state, we are not able to provide a numerical approximation.

- being preferred to a limiting profile can be seen as the existence of a majority of criteria in favor of the objects and the non-violation of the veto. This is represented in our case by the positive reasons and the non-violation of the minimum requirements which are more interesting in the context of rating than vetoes.

Hence, intuitively, even without considering the strict preference in the definition of the negative reasons, we think that we will have a better discrimination. One of the objectives of this section is to verify our intuition by this experiment. However, this experiment does not represent a proof that the quality of rating improves when the strict preference relation is not considered.

To proceed, we will run separately negative reasons due to the strict preference, the violation of minimum requirements and the dominance. Then, we will compare and analyse the different negative reasons.

Negative reasons opposing a rating "good"

Negative reasons due to the strict preference, consist on the existence of a reference profile characterizing a lower category that is strictly preferred to the object we aim to rate. Such strict preference might be due to a high performance of a reference profile according to a sufficient majority of criteria and at least a performance violating the minimum requirements.

Based only on strict preference, 198 objects have negative reasons opposing a rating "good". The only two objects without negative reasons caused by strict preference are displayed in Table 7.7:

	Marks	Assiduity	Physical Aptitude	Requested salary
A57	19.279540	9.0	4.0	48.198923
A132	19.443986	9.0	4.0	44.264673

Table 7.7 – Objects without negative reasons due to strict preference opposing a rating "good"

Negative reasons due to a violation of the minimum requirements, consist on not dominating the minimum requirement characterizing a given category. 171 objects have negative reasons originated by the violation of the minimum requirements. The 29 Objects with no negative reasons due to violation of minimum requirements for a rating "good" are displayed in Table 7.8.

	Marks	Assiduity	Physical Aptitude	Requested salary
A2	13.842586	8.0	4.0	51.914856
A12	17.257793	8.0	3.0	52.198923
A20	12.883421	8.0	3.0	53.707417
A25	12.534812	7.0	4.0	41.521228
A28	18.648225	8.0	4.0	53.028754
A30	15.514905	7.0	3.0	42.716819
A32	13.990524	8.0	4.0	49.995556
A33	19.352406	7.0	3.0	46.049301
A34	12.408294	7.0	4.0	44.266146
A37	18.588024	7.0	3.0	46.076652
A41	17.697286	9.0	3.0	50.372820
A42	14.829143	7.0	3.0	45.372046
A45	14.195109	8.0	3.0	52.243190
A46	18.972247	9.0	3.0	48.991131
A53	16.358145	9.0	3.0	53.317693
A55	17.663634	8.0	3.0	45.418242
A56	14.351084	8.0	4.0	43.550296
A57	19.279540	9.0	4.0	48.198923
A63	15.386028	9.0	3.0	51.054856
A66	13.572791	8.0	3.0	48.018649
A69	15.577211	8.0	3.0	48.479280

	Marks	Assiduity	Physical Aptitude	Requested salary
A75	19.646005	8.0	4.0	46.935005
A91	16.190796	7.0	3.0	44.306487
A97	17.013776	7.0	3.0	48.200286
A99	19.497170	8.0	3.0	53.624683
A112	19.124406	8.0	3.0	52.749381
A132	19.443986	9.0	4.0	44.264673
A187	13.697872	7.0	4.0	46.339493
A196	15.410242	7.0	3.0	53.058647

Table 7.8 – objects not violating the minimum requirements of the category "good"

The last type of negative reasons considered in Dynamic-R are the ones caused by the weak dominance. They consist on the existence of a reference profile characterizing a lower category that weakly dominates objects we aim to rate. 152 objects have negative reasons due to the weak dominance. The remaining Objects with no negative reasons caused by the weak dominance, and opposing being rated "good", are displayed in Table 7.9.

	Marks	Assiduity	Physical Aptitude	Requested salary
A2	13.842586	8.0	4.0	51.914856
A3	18.364864	6.0	4.0	42.999055
A9	12.725630	8.0	3.0	58.516870
A12	17.257793	8.0	3.0	52.198923
A17	14.662998	6.0	3.0	41.365675
A20	12.883421	8.0	3.0	53.707417
A23	15.385430	6.0	4.0	49.461605
A24	9.305283	7.0	3.0	44.872529
A25	12.534812	7.0	4.0	41.521228
A26	9.035486	8.0	4.0	49.287263
A28	18.648225	8.0	4.0	53.028754
A29	16.411948	8.0	4.0	55.915578
A30	15.514905	7.0	3.0	42.716819
A32	13.990524	8.0	4.0	49.995556
A33	19.352406	7.0	3.0	46.049301
A34	12.408294	7.0	4.0	44.266146
A35	18.216540	6.0	4.0	57.468397
A37	18.588024	7.0	3.0	46.076652

	Marks	Assiduity	Physical Aptitude	Requested salary
A41	17.697286	9.0	3.0	50.372820
A45	14.195109	8.0	3.0	52.243190
A46	18.972247	9.0	3.0	48.991131
A53	16.358145	9.0	3.0	53.317693
A55	17.663634	8.0	3.0	45.418242
A56	14.351084	8.0	4.0	43.550296
A57	19.279540	9.0	4.0	48.198923
A62	13.397716	9.0	4.0	55.341421
A63	15.386028	9.0	3.0	51.054856
A64	19.100698	8.0	4.0	56.318051
A66	13.572791	8.0	3.0	48.018649
A67	19.519486	6.0	4.0	52.862740
A69	15.577211	8.0	3.0	48.479280
A72	16.888600	6.0	4.0	51.549270
A75	19.646005	8.0	4.0	46.935005
A78	19.377280	8.0	3.0	58.437695
A91	16.190796	7.0	3.0	44.306487
A92	16.112011	6.0	3.0	54.554546
A94	11.829614	6.0	4.0	41.692489
A97	17.013776	7.0	3.0	48.200286
A98	11.617134	8.0	4.0	45.547574
A99	19.497170	8.0	3.0	53.624683
A112	19.124406	8.0	3.0	52.749381
A128	15.478409	6.0	4.0	57.505178
A130	11.536346	6.0	4.0	41.345661
A132	19.443986	9.0	4.0	44.264673
A148	19.243159	8.0	4.0	56.373017
A152	18.791540	7.0	4.0	57.981986
A178	8.577377	8.0	4.0	40.031666
A189	16.743959	6.0	4.0	59.098639

Table 7.9 – Objects without negative reasons due to weak dominance opposing a rating “good”

Discussion: In our Example, 174 objects are discriminated based on both weak dominance and the minimum requirements (which are also based on the dominance relation). All objects discriminated by these two origins of negative reasons are also discriminated by the main of strict preference. In my opinion, the 24 additional objects that are discriminated by the use of strict preference either should not be disgarded from being in the category of "good" or should not be discriminated by the main of negative reasons but by the lack of positive reasons. $A75$ is an example of object that should not have negative reasons against being in the category "good". However, $A75$ has negative reasons originated by the strict preference but neither by the weak dominance nor by a violation of the minimum requirements. Such candidate is clearly "good": s.he graduated by 19.646005/20, with an assiduity of 8/10, a physical aptitude of 4/5 and requesting a salary close to the minimum possible wage given his degree 46.935005k/40k euros. The only reason why he was not selected as a good candidate based on strict preference is because $z2, 2$ is strictly preferred to $A75$. This might be seen as an inconsistency, but let's analyse the numbers: $z2, 2 = (20, 5, 5, 40)$ has a performance violating the minimum requirement under the assiduity (5/10) and the best possible values on all the remaining criteria. Also the difference of performances between $A75$ and $z2, 2$ for the "assiduity" is not vetoed.

The example shows that the discrimination power associated to the strict preference can be substituted by the dominance with the minimum requirements.

Negative reasons opposing a rating "average"

113 objects have negative reasons due to strict preference opposing being rated "average". Table 7.11, displays objects not having negative reasons opposing being rated "average".

	Marks	Assiduity	Physical Aptitude	Requested salary
A2	13.842586	8.0	4.0	51.914856
A3	18.364864	6.0	4.0	42.999055
A6	19.279089	8.0	2.0	48.976294
A7	18.679046	5.0	4.0	48.153211
A12	17.257793	8.0	3.0	52.198923
A15	19.388006	7.0	2.0	54.722837
A16	14.898717	5.0	4.0	48.524141
A17	14.662998	6.0	3.0	41.365675

	Marks	Assiduity	Physical Aptitude	Requested salary
A18	18.884273	5.0	2.0	51.314288
A19	15.595875	7.0	2.0	40.340243
A21	17.145249	5.0	3.0	55.947240
A23	15.385430	6.0	4.0	49.461605
A27	19.658051	6.0	2.0	48.666054
A28	18.648225	8.0	4.0	53.028754
A29	16.411948	8.0	4.0	55.915578
A30	15.514905	7.0	3.0	42.716819
A32	13.990524	8.0	4.0	49.995556
A33	19.352406	7.0	3.0	46.049301
A35	18.216540	6.0	4.0	57.468397
A36	14.544670	5.0	4.0	51.953968
A37	18.588024	7.0	3.0	46.076652
A40	16.639287	5.0	2.0	42.838971
A41	17.697286	9.0	3.0	50.372820
A42	14.829143	7.0	3.0	45.372046
A43	19.933678	6.0	2.0	41.511887
A44	17.914896	7.0	2.0	47.273567
A45	14.195109	8.0	3.0	52.243190
A46	18.972247	9.0	3.0	48.991131
A47	16.638501	5.0	4.0	59.636934
A49	14.844106	5.0	4.0	56.853771
A50	14.867992	5.0	4.0	53.489655
A51	18.384079	5.0	3.0	41.270831
A53	16.358145	9.0	3.0	53.317693
A55	17.663634	8.0	3.0	45.418242
A56	14.351084	8.0	4.0	43.550296
A57	19.279540	9.0	4.0	48.198923
A59	19.733880	8.0	2.0	55.360254
A62	13.397716	9.0	4.0	55.341421
A63	15.386028	9.0	3.0	51.054856
A64	19.100698	8.0	4.0	56.318051
A65	19.703168	5.0	4.0	57.371329
A66	13.572791	8.0	3.0	48.018649
A67	19.519486	6.0	4.0	52.862740
A69	15.577211	8.0	3.0	48.479280
A70	19.401251	7.0	2.0	43.303077
A71	19.357219	6.0	2.0	55.918664
A72	16.888600	6.0	4.0	51.549270
A73	16.856523	5.0	3.0	41.064763
A74	19.409748	5.0	4.0	42.865066
A75	19.646005	8.0	4.0	46.935005
A76	17.563898	5.0	4.0	49.412348
A78	19.377280	8.0	3.0	58.437695
A79	19.568255	5.0	3.0	48.236885

	Marks	Assiduity	Physical Aptitude	Requested salary
A81	19.654610	5.0	3.0	43.035904
A83	13.358012	5.0	3.0	54.264068
A87	16.417978	5.0	3.0	54.392683
A90	14.939107	5.0	3.0	47.496803
A91	16.190796	7.0	3.0	44.306487
A92	16.112011	6.0	3.0	54.554546
A93	14.890635	7.0	2.0	43.965448
A97	17.013776	7.0	3.0	48.200286
A99	19.497170	8.0	3.0	53.624683
A103	19.710520	2.0	3.0	67.531649
A109	17.380778	0.0	2.0	41.372638
A111	15.261858	6.0	3.0	51.282795
A112	19.124406	8.0	3.0	52.749381
A113	18.261422	2.0	4.0	44.795162
A118	13.630164	2.0	4.0	46.218542
A123	15.765601	8.0	2.0	42.155756
A128	15.478409	6.0	4.0	57.505178
A131	13.524325	1.0	4.0	43.497326
A132	19.443986	9.0	4.0	44.264673
A134	15.309736	9.0	2.0	43.467431
A137	16.338662	6.0	2.0	40.140900
A139	19.288261	4.0	2.0	53.003902
A145	15.514119	5.0	4.0	40.435674
A148	19.243159	8.0	4.0	56.373017
A152	18.791540	7.0	4.0	57.981986
A167	19.935674	9.0	1.0	42.402738
A169	19.340006	5.0	2.0	44.206725
A174	15.630494	4.0	3.0	43.171332
A177	16.711575	2.0	2.0	43.938207
A182	18.670565	2.0	2.0	45.661877
A186	14.683137	9.0	2.0	45.702999
A187	13.697872	7.0	4.0	46.339493
A189	16.743959	6.0	4.0	59.098639
A196	15.410242	7.0	3.0	53.058647

Table 7.10 – Objects without negative reasons due to strict preference opposing a rating “average”

83 objects have negative reasons due to weak dominance or the violation of minimum requirements opposing rating “average”. Table 7.11 display these 83 objects.

	Marks	Assiduity	Physical Aptitude	Requested salary
A100	16.6038	3	3	54.6917
A101	5.31712	9	1	53.542
A102	11.9036	7	1	55.2678
A103	19.7105	2	3	67.5316
A104	18.8051	9	3	63.4427
A105	11.0703	3	4	40.5095
A106	4.93345	1	2	49.6318
A107	14.5037	9	1	63.0533
A108	5.6388	0	4	42.7027
A109	17.3808	0	2	41.3726
A110	11.5913	2	2	49.6413
A113	18.2614	2	4	44.7952
A114	2.40252	3	2	45.1533
A115	9.95909	2	2	46.9593
A116	1.22885	4	3	63.1747
A117	1.92479	4	1	66.2391
A118	13.6302	2	4	46.2185
A119	15.8479	2	4	67.8106
A120	15.1111	2	1	51.8437
A122	8.3909	9	3	61.3552
A124	7.37971	6	1	49.262
A125	8.5096	1	1	53.3314
A126	5.21392	2	3	63.35
A127	14.624	7	3	68.0546
A129	10.7231	4	1	63.9068
A131	13.5243	1	4	43.4973
A133	11.2249	1	3	65.4873
A135	18.924	9	1	55.115
A138	4.89853	9	2	65.2376
A139	19.2883	4	2	53.0039
A140	12.7571	2	1	48.9126
A141	14.2266	9	2	61.3856
A142	8.72121	1	4	62.6875
A143	11.7908	6	3	62.4381
A144	15.1177	4	3	60.7131
A147	18.1297	3	2	68.9811
A149	17.4856	3	3	62.083
A153	3.19001	7	3	48.3835
A154	1.79196	4	2	44.0953
A155	10.8758	1	1	40.595
A156	0.408811	9	3	55.7939
A157	11.4466	4	3	48.9215

	Marks	Assiduity	Physical Aptitude	Requested salary
A158	4.57156	1	1	53.7284
A159	1.3592	6	2	50.3091
A160	5.51947	3	2	46.0756
A161	4.29826	1	1	65.1674
A162	0.569976	4	1	47.0869
A163	15.1463	1	3	48.4092
A164	1.2057	3	3	58.6773
A165	3.5855	5	1	42.6042
A166	17.4672	4	3	68.9009
A167	19.9357	9	1	42.4027
A168	0.914148	0	3	62.3448
A170	10.5039	3	3	66.2385
A171	9.31393	9	1	51.168
A172	6.79833	5	1	64.6122
A173	2.34387	4	3	51.0549
A174	15.6305	4	3	43.1713
A175	8.17421	2	4	48.2159
A176	3.4417	9	4	52.7731
A177	16.7116	2	2	43.9382
A179	15.6706	1	4	52.8902
A180	16.143	4	1	67.0467
A181	15.4555	1	3	50.9754
A182	18.6706	2	2	45.6619
A183	6.35637	2	3	53.1716
A184	4.378	3	4	61.6003
A185	6.0726	8	3	54.7623
A188	12.0391	2	1	51.866
A190	7.20602	9	2	61.5979
A191	12.8087	1	4	57.1615
A192	10.2778	3	1	53.6661
A193	13.2178	9	4	65.2007
A194	16.7448	4	2	68.8281
A195	17.3982	8	1	59.1267
A197	10.3334	4	2	40.3378
A199	17.9721	2	3	52.3153
A22	9.97894	6	3	58.7015
A60	9.1689	7	3	58.3634
A61	9.84351	8	4	58.938
A86	9.4464	5	3	57.5322
A96	8.30755	6	2	57.692
A151	11.6845	5	2	57.2315

Table 7.11 – Objects having negative reasons due to weak dominance or minimum requirements opposing a rating “average”.

10 of the objects that have negative reasons because of inadmissible performances under some criteria were missed by the strict preference. This prove that the strict preference does not have a discrimination power stronger than the weak dominance and minimum requirements. For instance, the object $A103 = (19.7105, 2, 3, 67.5316)$ cannot be discriminated by the strict preference, because of the veto. However, it can be discriminated the violation of the minimum requirements, with respect to reference profiles characterizing the category “bad”.

7.3.3 Assessment of positive reasons

Positive reasons supporting a rating represent the existence of a sufficient majority of criteria supporting assigning an object to a given category. This is done by outranking a reference profile characterizing a given category. We use Outranking relation to support a rating instead of the concordance in order to avoid having a sufficient majority of criteria supporting the rating with bad performances. 26 objects have positive reasons supporting being rated “good”, as displayed in Table 7.12.

	Marks	Assiduity	Physical Aptitude	Requested salary
A3	18.3649	6	4	42.9991
A7	18.679	5	4	48.1532
A12	17.2578	8	3	52.1989
A25	12.5348	7	4	41.5212
A30	15.5149	7	3	42.7168
A34	12.4083	7	4	44.2661
A41	17.6973	9	3	50.3728
A46	18.9722	9	3	48.9911
A51	18.3841	5	3	41.2708
A55	17.6636	8	3	45.4182
A56	14.3511	8	4	43.5503
A57	19.2795	9	4	48.1989
A67	19.5195	6	4	52.8627
A72	16.8886	6	4	51.5493
A73	16.8565	5	3	41.0648

	Marks	Assiduity	Physical Aptitude	Requested salary
A74	19.4097	5	4	42.8651
A75	19.646	8	4	46.935
A76	17.5639	5	4	49.4123
A81	19.6546	5	3	43.0359
A91	16.1908	7	3	44.3065
A94	11.8296	6	4	41.6925
A112	19.1244	8	3	52.7494
A130	11.5363	6	4	41.3457
A132	19.444	9	4	44.2647
A145	15.5141	5	4	40.4357
A178	8.57738	8	4	40.0317

Table 7.12 – Objects having positive reasons supporting a rating “good”.

Table 7.13 displays the 94 objects having positive reasons supporting rating “average”.

	Marks	Assiduity	Physical Aptitude	Requested salary
A0	11.1111	7	4	47.67
A2	13.8426	8	4	51.9149
A3	18.3649	6	4	42.9991
A4	11.9387	5	4	48.2789
A6	19.2791	8	2	48.9763
A7	18.679	5	4	48.1532
A8	10.8619	7	4	52.2906
A10	12.9453	6	3	49.214
A12	17.2578	8	3	52.1989
A13	11.669	9	2	45.5956
A16	14.8987	5	4	48.5241
A17	14.663	6	3	41.3657
A19	15.5959	7	2	40.3402
A20	12.8834	8	3	53.7074
A21	17.1452	5	3	55.9472
A23	15.3854	6	4	49.4616
A24	9.30528	7	3	44.8725
A25	12.5348	7	4	41.5212
A26	9.03549	8	4	49.2873
A27	19.6581	6	2	48.6661
A28	18.6482	8	4	53.0288

	Marks	Assiduity	Physical Aptitude	Requested salary
A29	16.4119	8	4	55.9156
A30	15.5149	7	3	42.7168
A32	13.9905	8	4	49.9956
A33	19.3524	7	3	46.0493
A34	12.4083	7	4	44.2661
A35	18.2165	6	4	57.4684
A36	14.5447	5	4	51.954
A37	18.588	7	3	46.0767
A39	9.79947	5	4	45.5512
A40	16.6393	5	2	42.839
A41	17.6973	9	3	50.3728
A42	14.8291	7	3	45.372
A43	19.9337	6	2	41.5119
A44	17.9149	7	2	47.2736
A45	14.1951	8	3	52.2432
A46	18.9722	9	3	48.9911
A49	14.8441	5	4	56.8538
A50	14.868	5	4	53.4897
A51	18.3841	5	3	41.2708
A53	16.3581	9	3	53.3177
A55	17.6636	8	3	45.4182
A56	14.3511	8	4	43.5503
A57	19.2795	9	4	48.1989
A59	19.7339	8	2	55.3603
A62	13.3977	9	4	55.3414
A63	15.386	9	3	51.0549
A64	19.1007	8	4	56.3181
A65	19.7032	5	4	57.3713
A66	13.5728	8	3	48.0186
A67	19.5195	6	4	52.8627
A69	15.5772	8	3	48.4793
A70	19.4013	7	2	43.3031
A72	16.8886	6	4	51.5493
A73	16.8565	5	3	41.0648
A74	19.4097	5	4	42.8651
A75	19.646	8	4	46.935
A76	17.5639	5	4	49.4123
A77	11.8709	5	3	46.3185
A79	19.5683	5	3	48.2369
A81	19.6546	5	3	43.0359
A82	15.9519	6	2	47.2095
A83	13.358	5	3	54.2641

	Marks	Assiduity	Physical Aptitude	Requested salary
A85	8.52956	6	4	48.8138
A87	16.418	5	3	54.3927
A88	12.1351	6	3	52.1519
A90	14.9391	5	3	47.4968
A91	16.1908	7	3	44.3065
A92	16.112	6	3	54.5545
A93	14.8906	7	2	43.9654
A94	11.8296	6	4	41.6925
A95	12.7187	6	3	45.2457
A97	17.0138	7	3	48.2003
A98	11.6171	8	4	45.5476
A99	19.4972	8	3	53.6247
A111	15.2619	6	3	51.2828
A112	19.1244	8	3	52.7494
A123	15.7656	8	2	42.1558
A128	15.4784	6	4	57.5052
A130	11.5363	6	4	41.3457
A132	19.444	9	4	44.2647
A134	15.3097	9	2	43.4674
A137	16.3387	6	2	40.1409
A145	15.5141	5	4	40.4357
A146	9.02831	5	3	48.7988
A148	19.2432	8	4	56.373
A152	18.7915	7	4	57.982
A169	19.34	5	2	44.2067
A174	15.6305	4	3	43.1713
A178	8.57738	8	4	40.0317
A186	14.6831	9	2	45.703
A187	13.6979	7	4	46.3395
A196	15.4102	7	3	53.0586
A198	12.6699	9	2	41.0833

Table 7.13 – Objects having positive reasons supporting a rating “Average”.

7.3.4 Assessment of updated reasons and the derived rating

In this section, we will consider separately the both cases where strict preference relation is used and the one where it is not.

Case where strict preference is used in the assessment of negative reasons

Enriching negative reasons

When strict preference relation is used in the assessment of negative reasons, it is possible that some objects get additional negative reasons opposing a worse rating. No object has enriched negative reasons against being rated "good", while 64 objects got their negative reasons enriched to prevent being rated "average". Table 7.14 display the list of objects for which negative reasons were enriched to oppose rating "average".

	Marks	Assiduity	Physical Aptitude	Requested salary
A19	15.5959	7	2	40.3402
A21	17.1452	5	3	55.9472
A40	16.6393	5	2	42.839
A44	17.9149	7	2	47.2736
A47	16.6385	5	4	59.6369
A49	14.8441	5	4	56.8538
A83	13.358	5	3	54.2641
A87	16.418	5	3	54.3927
A93	14.8906	7	2	43.9654
A111	15.2619	6	3	51.2828
A137	16.3387	6	2	40.1409
A29	16.4119	8	4	55.9156
A92	16.112	6	3	54.5545
A128	15.4784	6	4	57.5052
A189	16.744	6	4	59.0986
z2,4	12	8	3	50
A6	19.2791	8	2	48.9763
A15	19.388	7	2	54.7228
A18	18.8843	5	2	51.3143
A27	19.6581	6	2	48.6661
A59	19.7339	8	2	55.3603
A71	19.3572	6	2	55.9187
A90	14.9391	5	3	47.4968
A196	15.4102	7	3	53.0586
A17	14.663	6	3	41.3657

	Marks	Assiduity	Physical Aptitude	Requested salary
A45	14.1951	8	3	52.2432
A63	15.386	9	3	51.0549
A69	15.5772	8	3	48.4793
A36	14.5447	5	4	51.954
A50	14.868	5	4	53.4897
A79	19.5683	5	3	48.2369
A78	19.3773	8	3	58.4377
A65	19.7032	5	4	57.3713
A76	17.5639	5	4	49.4123
A169	19.34	5	2	44.2067
A23	15.3854	6	4	49.4616
A35	18.2165	6	4	57.4684
A62	13.3977	9	4	55.3414
A67	19.5195	6	4	52.8627
A72	16.8886	6	4	51.5493
A2	13.8426	8	4	51.9149
A12	17.2578	8	3	52.1989
A32	13.9905	8	4	49.9956
A41	17.6973	9	3	50.3728
A53	16.3581	9	3	53.3177
A99	19.4972	8	3	53.6247
A112	19.1244	8	3	52.7494
A16	14.8987	5	4	48.5241
A152	18.7915	7	4	57.982
A64	19.1007	8	4	56.3181
A148	19.2432	8	4	56.373
A123	15.7656	8	2	42.1558
A43	19.9337	6	2	41.5119
A134	15.3097	9	2	43.4674
A186	14.6831	9	2	45.703
A7	18.679	5	4	48.1532
A70	19.4013	7	2	43.3031
A81	19.6546	5	3	43.0359
A42	14.8291	7	3	45.372
A30	15.5149	7	3	42.7168
A66	13.5728	8	3	48.0186
A97	17.0138	7	3	48.2003
A28	18.6482	8	4	53.0288
A37	18.588	7	3	46.0767

Table 7.14 – Objects with enriched negative opposing rating “Average”.

Let's analyse the objects for which negative reasons were enriched to oppose rating "average". $A19$ had negative reasons opposing being rated "good" which were enriched to oppose being rated "average". These negative reasons are caused by $A0$ ($A0P_\lambda A19$) which has negative reasons, not due to the incompatibility, opposing being rated "average". This last is in the origin of new negative reasons for many other objects such as $A21$, $A40$, to name but a few. This is because of its high performances under the three last criteria. An interesting case is displayed through this example is the update of negative reasons for objects assigned in previous time steps (reference profiles). The object $A19$ for which negative reasons were enriched provided new negative reasons to the reference profile $z2,4$ because $A19P_\lambda z2,4$.

Updating positive reasons

When we take into account the way objects compare to each other in the rating process, some objects get new positive reasons supporting a better rating. This occur when an object assigned to a better category is outranked by an object assigned to a worse category.

Table 7.15 displays the 97 objects for which positive reasons were enriched to support being rated "good".

	Marks	Assiduity	Physical Aptitude	Requested salary
z2,6	14.000000	7.0	4.0	45.000000
z2,2	20.000000	5.0	5.0	40.000000
z2,3	8.000000	10.0	5.0	40.000000
z2,5	16.000000	7.0	3.0	45.000000
z2,1	20.000000	10.0	2.0	40.000000
z2,4	12.000000	8.0	3.0	50.000000
z3,1	6.000000	10.0	5.0	40.000000
z3,2	10.000000	8.0	4.0	55.000000
z3,3	12.000000	8.0	3.0	55.000000
z2,0	20.000000	10.0	5.0	60.000000
A0	11.111125	7.0	4.0	47.669977
A1	13.098216	7.0	2.0	51.011259
A2	13.842586	8.0	4.0	51.914856

	Marks	Assiduity	Physical Aptitude	Requested salary
A4	11.938737	5.0	4.0	48.278879
A5	9.334663	7.0	3.0	53.449001
A6	19.279089	8.0	2.0	48.976294
A8	10.861877	7.0	4.0	52.290606
A9	12.725630	8.0	3.0	58.516870
A10	12.945317	6.0	3.0	49.213972
A13	11.669020	9.0	2.0	45.595611
A14	10.678502	6.0	4.0	56.541756
A15	19.388006	7.0	2.0	54.722837
A16	14.898717	5.0	4.0	48.524141
A17	14.662998	6.0	3.0	41.365675
A18	18.884273	5.0	2.0	51.314288
A19	15.595875	7.0	2.0	40.340243
A20	12.883421	8.0	3.0	53.707417
A21	17.145249	5.0	3.0	55.947240
A23	15.385430	6.0	4.0	49.461605
A24	9.305283	7.0	3.0	44.872529
A26	9.035486	8.0	4.0	49.287263
A27	19.658051	6.0	2.0	48.666054
A28	18.648225	8.0	4.0	53.028754
A29	16.411948	8.0	4.0	55.915578
A31	16.266510	8.0	2.0	58.435891
A32	13.990524	8.0	4.0	49.995556
A33	19.352406	7.0	3.0	46.049301
A35	18.216540	6.0	4.0	57.468397
A36	14.544670	5.0	4.0	51.953968
A37	18.588024	7.0	3.0	46.076652
A39	9.799472	5.0	4.0	45.551153
A40	16.639287	5.0	2.0	42.838971
A42	14.829143	7.0	3.0	45.372046
A43	19.933678	6.0	2.0	41.511887
A44	17.914896	7.0	2.0	47.273567
A45	14.195109	8.0	3.0	52.243190
A47	16.638501	5.0	4.0	59.636934
A49	14.844106	5.0	4.0	56.853771
A50	14.867992	5.0	4.0	53.489655
A53	16.358145	9.0	3.0	53.317693
A54	10.918926	5.0	4.0	54.581566
A58	8.313407	8.0	3.0	50.176427
A59	19.733880	8.0	2.0	55.360254
A61	9.843509	8.0	4.0	58.937990
A62	13.397716	9.0	4.0	55.341421
A63	15.386028	9.0	3.0	51.054856

	Marks	Assiduity	Physical Aptitude	Requested salary
A64	19.100698	8.0	4.0	56.318051
A65	19.703168	5.0	4.0	57.371329
A66	13.572791	8.0	3.0	48.018649
A68	10.227295	6.0	4.0	59.206283
A69	15.577211	8.0	3.0	48.479280
A70	19.401251	7.0	2.0	43.303077
A77	11.870872	5.0	3.0	46.318524
A78	19.377280	8.0	3.0	58.437695
A79	19.568255	5.0	3.0	48.236885
A82	15.951900	6.0	2.0	47.209508
A83	13.358012	5.0	3.0	54.264068
A85	8.529562	6.0	4.0	48.813826
A87	16.417978	5.0	3.0	54.392683
A88	12.135127	6.0	3.0	52.151867
A90	14.939107	5.0	3.0	47.496803
A92	16.112011	6.0	3.0	54.554546
A93	14.890635	7.0	2.0	43.965448
A95	12.718736	6.0	3.0	45.245691
A97	17.013776	7.0	3.0	48.200286
A98	11.617134	8.0	4.0	45.547574
A99	19.497170	8.0	3.0	53.624683
A111	15.261858	6.0	3.0	51.282795
A122	8.390905	9.0	3.0	61.355203
A123	15.765601	8.0	2.0	42.155756
A128	15.478409	6.0	4.0	57.505178
A134	15.309736	9.0	2.0	43.467431
A136	8.747799	7.0	3.0	52.833705
A137	16.338662	6.0	2.0	40.140900
A141	14.226598	9.0	2.0	61.385614
A146	9.028313	5.0	3.0	48.798806
A148	19.243159	8.0	4.0	56.373017
A152	18.791540	7.0	4.0	57.981986
A169	19.340006	5.0	2.0	44.206725
A174	15.630494	4.0	3.0	43.171332
A176	3.441702	9.0	4.0	52.773058
A185	6.072598	8.0	3.0	54.762286
A186	14.683137	9.0	2.0	45.702999
A187	13.697872	7.0	4.0	46.339493
A189	16.743959	6.0	4.0	59.098639
A196	15.410242	7.0	3.0	53.058647
A198	12.669920	9.0	2.0	41.083305

Table 7.15 – Objects with enriched positive reasons supporting rating “good”.

Only two objects had new negative reasons supporting being rated "average" as displayed in Table 7.16. This is because the number of objects having positive reasons supporting being rated average is very high.

	Marks	Assiduity	Physical Aptitude	Requested salary
A105	11.070267	3.0	4.0	40.509455
A113	18.261422	2.0	4.0	44.795162

Table 7.16 – Objects with enriched supporting rating "average".

A legitimate question could be: **Why in our rating method, only objects with no valid negative reasons against being assigned to a given category can be used to enrich other object's positive reasons?** Here are two answers:

1. Objects with valid negative reasons will not be in the origin of inconsistency regarding objects in better categories;

2. We risk to enrich positive reasons to all objects by not considering negative reasons for the objects in the origin of enriching positive reasons.

Let's play with the numbers and see what do we get when we delete negative reasons from objects in the origin of enriching positive reasons. Surprise, 183 objects and reference profiles had enriched positive reasons supporting their rating "good". Taking into account the 26 objects having positive reasons supporting rating "good", we can state that all objects and reference profiles have now positive reasons to be "good", even $A172 = (6.798330, 5.0, 1.0, 64.612236)$ which is clearly bad. This makes positive reasons useless. Let's see what happened: $A12 = (17.2578, 8, 3, 52.1989)$ enriched positive reasons for the following objects:

	Marks	Assiduity	Physical Aptitude	Requested salary
A0	11.111125	7.0	4.0	47.669977
A2	13.842586	8.0	4.0	51.914856
A6	19.279089	8.0	2.0	48.976294

	Marks	Assiduity	Physical Aptitude	Requested salary
A10	12.945317	6.0	3.0	49.213972
A13	11.669020	9.0	2.0	45.595611
A16	14.898717	5.0	4.0	48.524141
A18	18.884273	5.0	2.0	51.314288
A23	15.385430	6.0	4.0	49.461605
A27	19.658051	6.0	2.0	48.666054
A29	16.411948	8.0	4.0	55.915578
A32	13.990524	8.0	4.0	49.995556
A35	18.216540	6.0	4.0	57.468397
A37	18.588024	7.0	3.0	46.076652
A42	14.829143	7.0	3.0	45.372046
A44	17.914896	7.0	2.0	47.273567
A53	16.358145	9.0	3.0	53.317693
A59	19.733880	8.0	2.0	55.360254
A62	13.397716	9.0	4.0	55.341421
A64	19.100698	8.0	4.0	56.318051
A66	13.572791	8.0	3.0	48.018649
A69	15.577211	8.0	3.0	48.479280
A77	11.870872	5.0	3.0	46.318524
A79	19.568255	5.0	3.0	48.236885
A88	12.135127	6.0	3.0	52.151867
A90	14.939107	5.0	3.0	47.496803
A95	12.718736	6.0	3.0	45.245691
A97	17.013776	7.0	3.0	48.200286
A99	19.497170	8.0	3.0	53.624683
A111	15.261858	6.0	3.0	51.282795
A123	15.765601	8.0	2.0	42.155756
A134	15.309736	9.0	2.0	43.467431
A148	19.243159	8.0	4.0	56.373017
A152	18.791540	7.0	4.0	57.981986
A169	19.340006	5.0	2.0	44.206725
A186	14.683137	9.0	2.0	45.702999
A198	12.669920	9.0	2.0	41.083305
z2,0	20.000000	10.0	5.0	60.000000
z2,2	20.000000	5.0	5.0	40.000000
z2,4	12.000000	8.0	3.0	50.000000
z2,6	14.000000	7.0	4.0	45.000000

Table 7.17 – Objects with enriched positive reasons supporting generated by $A12$.

Then $A25 = (12.5348, 7, 4, 41.5212)$, which also have positive reasons supporting its rating "good", enriched positive reasons for the following objects:

	Marks	Assiduity	Physical Aptitude	Requested salary
A17	14.662998	6.0	3.0	41.365675
A26	9.035486	8.0	4.0	49.287263
A33	19.352406	7.0	3.0	46.049301
A98	11.617134	8.0	4.0	45.547574
A187	13.697872	7.0	4.0	46.339493
z3,1	6.000000	10.0	5.0	40.000000
z2,3	8.000000	10.0	5.0	40.000000

Table 7.18 – Objects with enriched positive reasons supporting generated by $A25$.

Then $A30 = (15.5149, 7, 3, 42.7168)$, enriched positive reasons for the following objects:

	Marks	Assiduity	Physical Aptitude	Requested salary
A19	15.595875	7.0	2.0	40.340243
A43	19.933678	6.0	2.0	41.511887
A70	19.401251	7.0	2.0	43.303077
A137	16.338662	6.0	2.0	40.140900
A174	15.630494	4.0	3.0	43.171332
z2,1	20.000000	10.0	2.0	40.000000

Table 7.19 – Objects with enriched positive reasons supporting generated by $A30$.

Then $A34 = (12.4083, 7, 4, 44.2661)$ enriched positive reasons for the following objects:

	Marks	Assiduity	Physical Aptitude	Requested salary
A36	14.544670	5.0	4.0	51.953968
A45	14.195109	8.0	3.0	52.243190
A63	15.386028	9.0	3.0	51.054856
z2,5	16.000000	7.0	3.0	45.000000

Table 7.20 – Objects with enriched positive reasons supporting generated by $A34$.

Objects, $A41 = (17.6973, 9, 3, 50.3728)$, $A91 = (16.1908, 7, 3, 44.3065)$, and $A112 = (19.1244, 8, 3, 52.7494)$ enriched positive reasons for the following objects:

	Marks	Assiduity	Physical Aptitude	Requested salary
A28	18.648225	8.0	4.0	53.028754
A8	10.861877	7.0	4.0	52.290606
A40	16.639287	5.0	2.0	42.838971
A93	14.890635	7.0	2.0	43.965448
A65	19.703168	5.0	4.0	57.371329
A78	19.377280	8.0	3.0	58.437695

Table 7.21 – Objects with enriched positive reasons supporting generated by $A34$.

Then the objects for which positive reasons were enriched to the best category start enriching positive reasons for the remaining objects. $A0$ which is an object for which positive reasons were enriched but which violates the minimum requirements of the category "good", enriched positive reasons for the following objects, which don't seem suitable to support the category "good":

	Marks	Assiduity	Physical Aptitude	Requested salary
A4	11.938737	5.0	4.0	48.278879
A20	12.883421	8.0	3.0	53.707417
A24	9.305283	7.0	3.0	44.872529
A39	9.799472	5.0	4.0	45.551153
A50	14.867992	5.0	4.0	53.489655
A196	15.410242	7.0	3.0	53.058647
z3,2	10.000000	8.0	4.0	55.000000

Table 7.22 – Objects with enriched positive reasons supporting generated by $A34$.

The first wave of candidates for which positive reasons were enriched to a rating "good", enriched on their turn the following objects (in addition to the ones enriched by $A0$):

	Marks	Assiduity	Physical Aptitude	Requested salary
A47	16.638501	5.0	4.0	59.636934
A49	14.844106	5.0	4.0	56.853771
A58	8.313407	8.0	3.0	50.176427
A85	8.529562	6.0	4.0	48.813826
A128	15.478409	6.0	4.0	57.505178
A189	16.743959	6.0	4.0	59.098639
A15	19.388006	7.0	2.0	54.722837
A71	19.357219	6.0	2.0	55.918664
A82	15.951900	6.0	2.0	47.209508
A167	19.935674	9.0	1.0	42.402738
A1	13.098216	7.0	2.0	51.011259
A14	10.678502	6.0	4.0	56.541756
A38	8.434261	6.0	2.0	42.450930
A52	9.786406	6.0	2.0	47.182113
A83	13.358012	5.0	3.0	54.264068
A87	16.417978	5.0	3.0	54.392683
A92	16.112011	6.0	3.0	54.554546
A100	16.603848	3.0	3.0	54.691704
A105	11.070267	3.0	4.0	40.509455
A136	8.747799	7.0	3.0	52.833705
A146	9.028313	5.0	3.0	48.798806
A150	18.506636	6.0	2.0	55.436744
A157	11.446642	4.0	3.0	48.921470
A185	6.072598	8.0	3.0	54.762286
z3,3	12.000000	8.0	3.0	55.000000
A89	11.130205	7.0	2.0	45.023631
A21	17.145249	5.0	3.0	55.947240
A113	18.261422	2.0	4.0	44.795162
A118	13.630164	2.0	4.0	46.218542
A175	8.174207	2.0	4.0	48.215922
A9	12.725630	8.0	3.0	58.516870
A31	16.266510	8.0	2.0	58.435891
A135	18.923966	9.0	1.0	55.114995
A139	19.288261	4.0	2.0	53.003902
A177	16.711575	2.0	2.0	43.938207
A182	18.670565	2.0	2.0	45.661877
A54	10.918926	5.0	4.0	54.581566
A104	18.805051	9.0	3.0	63.442651
A193	13.217798	9.0	4.0	65.200695
A141	14.226598	9.0	2.0	61.385614
A5	9.334663	7.0	3.0	53.449001
A84	9.835730	5.0	2.0	44.240655
A199	17.972142	2.0	3.0	52.315349
A48	11.273867	6.0	2.0	50.980548

	Marks	Assiduity	Physical Aptitude	Requested salary
A61	9.843509	8.0	4.0	58.937990
A60	9.168904	7.0	3.0	58.363376
A176	3.441702	9.0	4.0	52.773058
A197	10.333376	4.0	2.0	40.337819
A68	10.227295	6.0	4.0	59.206283

Table 7.23 – Objects with enriched positive reasons due to the first wave of objects with enriched positive reasons supporting “good”

For the second wave we will consider two examples: $A52 = (9.78641, 6, 2, 47.1821)$ and $A136 = (8.7478, 7, 3, 52.8337)$ enriching positive reasons for the following objects:

	Marks	Assiduity	Physical Aptitude	Requested salary
A11	10.812270	5.0	2.0	53.458575
A160	5.519469	3.0	2.0	46.075599
A173	2.343866	4.0	3.0	51.054877

Table 7.24 – Objects with enriched positive reasons by $A52$ and $A136$

The head hunter will be fired if he says to his client that $A160$ and $A173$ have positive reasons to be rated “good”.

We will not develop further. The main idea is that enriching positive reasons by only objects with no valid negative reasons preventing their rating to a given category is important for convincing reasons and also to keep the interest of having positive reasons.

In our example, non of the objects has withdrawn negative reasons.

Rating

The head hunter went back to his client by the following results: When strict preference is considered in the assessment of negative reasons, two objects are rated “good” as displayed in Table 7.25.

	Marks	Assiduity	Physical Aptitude	Requested salary
A57	19.2795	9	4	48.1989
A132	19.444	9	4	44.2647

Table 7.25 – Objects rated “good”

Candidates rated “average” are displayed in Table 7.26.

	Marks	Assiduity	Physical Aptitude	Requested salary
A51	18.3841	5	3	41.2708
A73	16.8565	5	3	41.0648
A74	19.4097	5	4	42.8651
A145	15.5141	5	4	40.4357
A187	13.6979	7	4	46.3395
A3	18.3649	6	4	42.9991
A33	19.3524	7	3	46.0493
A46	18.9722	9	3	48.9911
A55	17.6636	8	3	45.4182
A56	14.3511	8	4	43.5503
A75	19.646	8	4	46.935
A91	16.1908	7	3	44.3065

Table 7.26 – Objects rated “average”

All the remaining objects were rated “bad”.

The headhunter, curious about the significance of his results, decided to run further analysis without considering the strict preference relation in the assessment of negative reasons. We will follow his analysis in the next section.

Case of not using strict preference

29 of the candidates and three reference profiles are rated “good” as displayed in Table 7.27. We captured more interesting profiles which were discriminated by the main of strict preference in the previous procedure.

	Marks	Assiduity	Physical Aptitude	Requested salary
z2,5	16	7	3	45
z2,6	14	7	4	45
z2,4	12	8	3	50
A2	13.8426	8	4	51.9149
A12	17.2578	8	3	52.1989
A20	12.8834	8	3	53.7074
A25	12.5348	7	4	41.5212
A28	18.6482	8	4	53.0288
A30	15.5149	7	3	42.7168
A32	13.9905	8	4	49.9956
A33	19.3524	7	3	46.0493
A34	12.4083	7	4	44.2661
A37	18.588	7	3	46.0767
A41	17.6973	9	3	50.3728
A45	14.1951	8	3	52.2432
A46	18.9722	9	3	48.9911
A53	16.3581	9	3	53.3177
A55	17.6636	8	3	45.4182
A56	14.3511	8	4	43.5503
A57	19.2795	9	4	48.1989
A63	15.386	9	3	51.0549
A66	13.5728	8	3	48.0186
A69	15.5772	8	3	48.4793
A75	19.646	8	4	46.935
A91	16.1908	7	3	44.3065
A97	17.0138	7	3	48.2003
A99	19.4972	8	3	53.6247
A112	19.1244	8	3	52.7494
A132	19.444	9	4	44.2647
A42	14.8291	7	3	45.372
A187	13.6979	7	4	46.3395
A196	15.4102	7	3	53.0586

Table 7.27 – Objects rated “good”

91 candidates were rated average and further analysis should be conducted. The candidates rated average are dispayed in Table 7.28.

	Marks	Assiduity	Physical Aptitude	Requested salary
z3,2	10.0	8.0	4.0	55.0
z3,3	12.0	8.0	3.0	55.0
A0	11.1111	7	4	47.67
A1	13.0982	7	2	51.0113
A2	18.3649	6	4	42.9991
A4	11.9387	5	4	48.2789
A5	9.33466	7	3	53.449
A6	19.2791	8	2	48.9763
A7	18.679	5	4	48.1532
A8	10.8619	7	4	52.2906
A9	12.7256	8	3	58.5169
A10	12.9453	6	3	49.214
A11	10.8123	5	2	53.4586
A13	11.669	9	2	45.5956
A14	10.6785	6	4	56.5418
A15	19.388	7	2	54.7228
A16	14.8987	5	4	48.5241
A17	14.663	6	3	41.3657
A18	18.8843	5	2	51.3143
A19	15.5959	7	2	40.3402
A21	17.1452	5	3	55.9472
A23	15.3854	6	4	49.4616
A24	9.30528	7	3	44.8725
A26	9.03549	8	4	49.2873
A27	19.6581	6	2	48.6661
A29	16.4119	8	4	55.9156
A31	16.2665	8	2	58.4359
A35	18.2165	6	4	57.4684
A36	14.5447	5	4	51.954
A38	8.43426	6	2	42.4509
A39	9.79947	5	4	45.5512
A40	16.6393	5	2	42.839
A43	19.9337	6	2	41.5119
A44	17.9149	7	2	47.2736
A47	16.6385	5	4	59.6369
A48	11.2739	6	2	50.9805
A49	14.8441	5	4	56.8538
A50	14.868	5	4	53.4897
A51	18.3841	5	3	41.2708
A52	9.78641	6	2	47.1821
A54	10.9189	5	4	54.5816
A58	8.31341	8	3	50.1764
A59	19.7339	8	2	55.3603

	Marks	Assiduity	Physical Aptitude	Requested salary
A62	13.3977	9	4	55.3414
A64	19.1007	8	4	56.3181
A65	19.7032	5	4	57.3713
A67	19.5195	6	4	52.8627
A68	10.2273	6	4	59.2063
A70	19.4013	7	2	43.3031
A71	19.3572	6	2	55.9187
A72	16.8886	6	4	51.5493
A73	16.8565	5	3	41.0648
A74	19.4097	5	4	42.8651
A76	17.5639	5	4	49.4123
A77	11.8709	5	3	46.3185
A78	19.3773	8	3	58.4377
A79	19.5683	5	3	48.2369
A80	10.5324	5	2	50.2714
A81	19.6546	5	3	43.0359
A82	15.9519	6	2	47.2095
A83	13.358	5	3	54.2641
A84	9.83573	5	2	44.2407
A85	8.52956	6	4	48.8138
A87	16.418	5	3	54.3927
A88	12.1351	6	3	52.1519
A89	11.1302	7	2	45.0236
A90	14.9391	5	3	47.4968
A92	16.112	6	3	54.5545
A93	14.8906	7	2	43.9654
A94	11.8296	6	4	41.6925
A95	12.7187	6	3	45.2457
A98	11.6171	8	4	45.5476
A111	15.2619	6	3	51.2828
A121	18.4012	5	2	59.9193
A123	15.7656	8	2	42.1558
A128	15.4784	6	4	57.5052
A130	11.5363	6	4	41.3457
A134	15.3097	9	2	43.4674
A136	8.7478	7	3	52.8337
A137	16.3387	6	2	40.1409
A145	15.5141	5	4	40.4357
A146	9.02831	5	3	48.7988
A148	19.2432	8	4	56.373
A150	18.5066	6	2	55.4367
A152	18.7915	7	4	57.982
A169	19.34	5	2	44.2067

	Marks	Assiduity	Physical Aptitude	Requested salary
A178	8.57738	8	4	40.0317
A186	14.6831	9	2	45.703
A189	16.744	6	4	59.0986
A198	12.6699	9	2	41.0833

Table 7.28 – Objects rated "average"

The remaining candidates are considered "bad".

By increasing the majority threshold and reducing the vetoes, we might have more interesting results as the cardinality of the sets of positive reasons will decrease and in case where strict preference is used in the assessment of positive reasons, the cardinality of the set of negative reasons will increase.

7.4 Application to the IRSN case study

In this section, we applied the method Dynamic-R to the IRSN's case study. To remind, we aim at rating the impact of a nuclear accident upon each geographic unit based on their impact in each asset characterizing the studied area. The considered assets are respectively Fishing (F), Fish Farming (FF), Seagrass-Posidonia (SP) and Tourism (T). The rating corresponds to five levels: Safe (coded by a rate 1, low risk (coded by a rate 2), average risk (coded by a rate 3), high risk (coded by a rate 4), and extreme risk (coded by a rate 5).

7.4.1 Dynamic-R parameters

To apply Dynamic-R, we defined an exhaustive set of non redundant zones over the twelve scenarios. We used as a set of reference profiles the learning set used in the assessment of ELECTRE-TRI parameters and the limiting profiles. We then added reference profiles characterizing category "Safe" as displayed in Table 7.29.

	F	FF	SP	T
z1,0	1	1	1	2
z1,1	1	2	1	1
z1,2	2	1	1	1

Table 7.29 – Reference profiles for the category Safe

We also used the same preference parameters.

For the minimum requirements, it seems reasonable to consider the performance vector $(k+v-1, k+v-1, k+v-1, k+v-1)$ to characterize a rating k. Hence, the zone $(1, 1, 1, 3)$ cannot be considered safe but with a low risk, as the risk of impact in tourism is average. The set of minimum requirements are displayed in Table 7.30.

	F	FF	SP	T
safe	2	2	2	2
LowRisk	3	3	3	3
AverageRisk	4	4	4	4
HighRisk	5	5	5	5
ExtremeRisk	5	5	5	5

Table 7.30 – Minimum requirements

The non redundant set of zones we aim to rate are displayed in Table 7.31.

	F	FF	SP	T
Zone0	1	1	1	1
Zone1	4	1	4	4
Zone2	4	1	4	5
Zone3	3	1	4	4
Zone4	4	1	1	4
Zone5	4	1	1	3
Zone6	4	1	2	4
Zone7	4	1	3	5
Zone8	4	1	2	5
Zone9	5	1	3	5

	F	FF	SP	T
Zone10	5	1	4	5
Zone11	5	1	1	5
Zone12	4	1	1	5
Zone13	3	1	1	4
Zone14	4	1	3	4
Zone15	5	1	3	4
Zone16	1	2	1	1
Zone17	5	2	2	4
Zone18	5	1	4	4
Zone19	4	2	1	5
Zone20	5	1	2	4
Zone21	5	1	1	3
Zone22	5	5	1	5
Zone23	5	5	2	5
Zone24	1	4	1	1
Zone25	4	4	1	5
Zone26	5	4	1	5
Zone27	5	2	3	1
Zone28	1	1	1	3
Zone29	4	1	1	1
Zone30	5	5	3	5
Zone31	5	3	1	5
Zone32	5	3	2	5
Zone33	4	3	1	4
Zone34	4	3	2	5
Zone35	5	1	1	4
Zone36	3	1	1	3
Zone37	1	1	1	4

Table 7.31 – Zones we aim to rate

7.5 Results

By applying Dynamic-R we obtained the following rating: The two zones displayed in Table 7.32 are safe: They respect the minimum requirements, they are not dominated by any non-updated reference profile, they have positive reasons coming from $z1, 1$.

	F	FF	SP	T
Zone0	1	1	1	1
Zone16	1	2	1	1

Table 7.32 – Safe Zones

Only one object is rated as low risk as displayed in Table 7.33. Zone28 is a safe zone according to a majority of criteria. However, it does not have neither positive nor enriched positive reasons (because of the veto), and it has negative reasons due to the violation of the minimum requirements. Zone28, has positive reasons coming from the vector $(2, 2, 2, 2)$ characterizing the rating 2.

	F	FF	SP	T
Zone28	1	1	1	3

Table 7.33 – Low risk Zone

For zones with average risk, all zones violate the minimum requirements characterizing the category Low risk. Non of them violate the minimum requirements of being rated Average Risk and non of the objects rated High risk is in the origin of negative reason opposing their rating average. All zones have positive reasons to be rated average except zone3. However, this last had enriched positive reasons originated by zone6. The result of zones rated average risk is displayed in Table 7.34.

	F	FF	SP	T
Zone3	3	1	4	4
Zone4	4	1	1	4
Zone5	4	1	1	3
Zone6	4	1	2	4
Zone13	3	1	1	4
Zone14	4	1	3	4
Zone24	1	4	1	1
Zone29	4	1	1	1

	F	FF	SP	T
Zone33	4	3	1	4
Zone37	1	1	1	4

Table 7.34 – Average Risk Zones

All the remaining zones are rated high risk. It is easy to justify the rating of every single zone displayed in Table 7.35. All zones, except zone Zone1, violates the minimum requirements of being rated Average Risk. Zone1 was discriminated because both the lack of positive reasons or the enriched positive reasons.

	F	FF	SP	T
Zone2	4	1	4	5
Zone9	5	1	3	5
Zone18	5	1	4	4
Zone32	5	3	2	5
Zone34	4	3	2	5
Zone1	4	1	4	4
Zone7	4	1	3	5
Zone15	5	1	3	4
Zone8	4	1	2	5
Zone11	5	1	1	5
Zone12	4	1	1	5
Zone17	5	2	2	4
Zone19	4	2	1	5
Zone20	5	1	2	4
Zone21	5	1	1	3
Zone25	4	4	1	5
Zone26	5	4	1	5
Zone27	5	2	3	1
Zone31	5	3	1	5
Zone35	5	1	1	4

Table 7.35 – High Risk Zones

7.6 Conclusion

In this chapter we presented a compilation of experiments we run in relation with Dynamic-R. In this experimental study, we simulated an imaginary MCDA rating problem, with randomly generated data. Based on this example we discussed an intuition we had during the development of Dynamic-R consisting on abandoning the strict preference relation in the assessment of negative reasons. Such intuition might provide very important theoretical results. We illustrated this intuition by an example where strict preference might be biased by a profile with high performances but characterizing a worse category because it does not dominate the minimum requirements. We also experimented the case in which negative reasons are not considered for objects in the origin of enriching positive reasons. The method was also applied to the case study. The results were justifiable with a clear separation of positive and negative reasons.

8 General Conclusion

This thesis is motivated by a real case study concerning marine pollution problems. Marine pollution problems might, at least in our case, lead to potential impacts over several assets characterizing the studied area. Thus assessing the impact of a marine pollution in a geographic area, comes to, on the one hand, studying and assessing the way such pollution impacts the different involved assets, and on the other hand, aggregating these impacts.

Our client's problem, consists on proposing a methodology allowing the assessment of a nuclear accident impact, taking place in the marine area, on the Bay of Toulon. Their problem can be seen as post-accident risk assessment. As decision analysts, a first solution was proposed in which different decision aiding tools were used. This solution consists on the following steps:

1. Defining the different possible accident scenarios and running simulations of the accidents;
2. Defining the different assets involved in the Bay of Toulon, and their importance;
3. Assessing the impact of a given concentration on each asset involved in the Bay of Toulon;
4. For each accident scenario, rating the impact on each asset, with respect to its importance in the geographic unit;

5. For each accident scenario, aggregating the different ratings over all the assets in order to assess the overall impact of the nuclear accident in the geographic unit.

Nevertheless, our client is not a decision maker and the question about the nature of the decision maker is still ambiguous. Hence, as decision scientists researchers, we start thinking about answers to the following research question: "Is our methodology convincing in order to be reported to our client's client? Trying to answer this question lead to the following questions:

1. Can we rely on expert judgment in case of a nuclear accident, and the way we elicited the subjectivity of expert judgment is it convincing?

2. Our problem is characterized by different types of aggregations, mainly, the spatial aggregation of the impacts in different zones, the multicriteria aggregation to assess the overall impact over the assets, the aggregation of scenarios. The order of these aggregations might influence the resulting rating. On which order should we proceed?

3. The aggregation of criteria, spatial impacts and scenarios might be done using a method based on the majority rule, as criteria are assessed on ordinal scales and we do not have a rich learning set to use decision rule based methods. The majority rule based methods, such as the well known ELECTRE TRI method and its variants, is subject to a lack of ordering properties. This might lead to a non-convincing rating. Hence, how can we develop a convincing rating method, using the majority rule, and where no better object is assigned to a lower category?

For many reasons, mainly the limited time of the thesis, we decided to work on the third question.

To answer the third method, we start by defining the properties characterizing, what we name a "convincing" rating method. A "convincing" rating method should provide a complete rating (all objects should be rated), a justifiable rating (we should be able to support a rating), a consistent rating (according to a sufficient majority of criteria, while comparing two objects, a better object should be assigned at least to the same category of the worse one), a good quality of rating (the rating should represent as much as possible the reality). Forcing the "convincing" property may lead to all objects assigned to the same category, because of cycles of preferences, which is a bad rating. So how can we conceive such method without withdrawing the majority rule? The original idea behind the developed method is to use different types of negative reasons in order to disqualify the resulting rating generated by the majority rule.

It is not a secret that research is full of surprises, in addition to the fact that the developed method is able to verify the "convincing" properties, this last seems very promising to be used in automatic decision making context. For this reason, I propose the following two perspectives in relation with automatic decision making:

1. Developing a mechanism allowing to detect changes in the environment (for example, a client rated good yesterday might be not good anymore), without withdrawing all the set of reference profiles.

2. The use of positive and negative reasons, always within the framework of automatic decision making, and in the context of muti-agent systems. The idea is to have different machines, each of them is based on the history of decisions taken by an expert. So we will have d convincing rating result at each time step (with d number of decider machines). These machines are equipped with a communication protocol, the communication generates new positive and negative reasons. Each rating results from the use of our method Dynamic-R, by each machine, will be involved in a deliberation process in order to converge to a recommendation. One way of proceeding, consists on introducing a mediator agent

who, at each time step, recommends a compromised rating based on the d ratings proposed by the deciders machines. An other way, consists on aggregating the d different ratings at the time step 0, in the mediator agent and then apply the method based on the mediator agent's reference profiles. Are there any communication protocol between the machines making the ratings proposed by both approaches converges at a given time step.

9 Appendices

Appendix A. The hydrodynamic model

1.1. The advection diffusion equation

To assess the radiological impact of an accidental release in seawater, the IRSN has developed a hydrodynamic model tool called STERNE (*Simulation du Transport et du transfert d'Eléments Radioactifs dans l'environNEment marin*, translated as *Simulation of radionuclide transport and transfer in marine environments*) to simulate the dispersion of radionuclides in the marine area. This tool is based on the tracer advection diffusion equation estimating the dispersion of radionuclides:

$$\frac{\partial C}{\partial t} - \frac{\partial}{\partial x}\left[D_x\frac{\partial c}{\partial x} - u_x c\right] - \frac{\partial}{\partial y}\left[D_y\frac{\partial c}{\partial y} - u_y c\right] - \frac{\partial}{\partial z}\left[D_z\frac{\partial c}{\partial z} - u_z c\right] = F(c,t)$$

where C is the radionuclide concentration; u, the advection current; and, finally, D is the turbulent diffusion tensor. The model is illustrated with Figure 9.1:

Since it is difficult to solve this equation analytically, the most common procedure consists of discretising time, the choice of time step depending on the mesh size and maximum sea current velocity for the area considered and sigma-coordinates and calculates this concentration at each grid point and

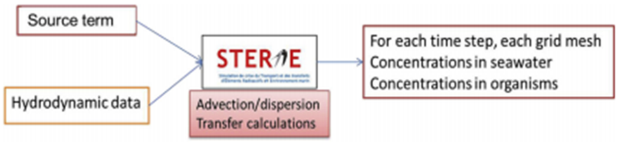


Figure 9.1 – Schematic diagram of STERNE implementation principle

time step. This model takes into account the half life of each radionuclide considered.

1.2. Input data

For each time step and mesh, the hydrodynamic data required as input to dispersion calculations includes

- The cumulative water fluxes in x, y and z directions; free surface elevation and diffusion coefficients (set to calculate the exact quantity of water passing through the grid meshes at each instant and should satisfy the continuity equation)
- The free surface elevation and diffusion coefficients.

Hydrodynamic models are generated based on hindcasts and forecasts of meteorological and tidal forcing. Source terms are characterised by:

- known quantities of radionuclide releases.
- known localisations (Release point coordinates).
- instants of releases.

Appendix B. Brief introduction of ELECTRE TRI

ELECTRE TRI is a rating method, aiming to assign elements of a set A to one of predefined ordered categories $C_1, ..., C_p$. Such categories are ranked from the worst to the best: $C_{h+1} \gg C_h \; \forall h \in \{1, ..., p-1\}$ where $\gg$ refers to a complete order on the set of categories, [106]. This method uses a majority rule while respecting a minority using a veto rule, in order to compare elements of a set A (representing actions) to the profiles characterising categories. Let us denote $r_1, ..., r_p$ the limiting profiles characterising the p categories, r_k refers to the upper limit of category C_k and the lower limit of category C_{k+1}, $k = 1, 2, ..., p$ and R the set of the associate indices. Let F denote the set of the indices of the criteria $g_1, g_2, ..., g_m$. Without loss of generality, we make the assumption that preferences increase with the value on each criterion. ELECTRE TRI is based on an outranking relation S. Roughly speaking, an outranking relation can be interpreted as, "at least as good as". In a first step, we aim at constructing an outranking relation S characterising how actions compare to each limiting profile. Thus, we use S to assign each action to a specific category. The procedure can be described as follows:

- Partial concordance index $c_j(a, r_h) \in [0, 1], \forall j \in F h \in R$: IT represents a weight of the proposition *a is at least as good as a certain* r_h from the criterion j point of view. The formulation of partial concordance index is:

  $$c_j(a, r_h) = \begin{cases} 1, \text{ if } g_j(r_h) - g_j(a) \leq 0 \\ 0, \text{ if } g_j(r_h) - g_j(a) > 0 \end{cases}$$

  This index takes 1 to denote a full approval of the proposition "a is at least as good as r_h" from the criterion j point of view.

- Global concordance index $c(a, r_h) \in [0, 1], \forall h \in R$: represents the majority rule, i.e. the global weight of all criteria approving the proposition "a is at least as good as r_h".

  $$c(a, r_h) = \frac{\sum_{j \in F} w_j c_j(a, r_h)}{\sum_{j \in F} w_j}$$

  where $w_j, j \in F$ refers to the weight associated to the criterion j.

- Discordance index $d_j(a, r_h) \in [0, 1], \forall j \in F\ h \in R$: represent the respect of minority rule, i.e. when the difference between a certain r_h and a for a given criterion j is greater than a threshold, called veto threshold, the outranking relation between a and r_h is vetoed.

$$d_j(a, r_h) = \begin{cases} 1, \text{ if } g_j(r_h) - v_j(r_h) \geq g_j(a) \\ 0, \text{otherwise} \end{cases}$$

  where $v_j(r_h), j \in F, h \in R$ refers to the veto threshold associated with the criterion j.

- Credibility index or the outranking relation $\sigma(a, r_h)$ aggregating the concordance and the discordance.

In the ELECTRE TRI method, the assignment of a depends on the values of $\sigma(a, r_h)$, $\sigma(r_h, a)$ and a cutting threshold λ. When $\sigma(a, r_h) \geq \lambda$, a outranks r_h, denoted aSr_h. Four possible situation may occur:

- $\sigma(a, r_h) \geq \lambda$, $\sigma(r_h, a) \geq \lambda \implies aIr_h$, i.e. a is indifferent to r_h
- $\sigma(a, r_h) < \lambda$, $\sigma(r_h, a) < \lambda \implies aRr_h$, i.e. a is incomparable to r_h
- $\sigma(a, r_h) \geq \lambda$, $\sigma(r_h, a) < \lambda \implies aPr_h$, i.e. a is preferred to r_h
- $\sigma(a, r_h) < \lambda$, $\sigma(r_h, a) \geq \lambda \implies r_hPa$, i.e. r_h is preferred to a

The assignment is done using two procedures:

- Pessimistic (conjunctive) procedure. It consists on the pairwise comparaison between each action a and the limiting profil r_h starting from $h = p$ to $h = 0$. We stop this procedure when aSr_h, and potentially a will be assigned to C_{h+1}.
- Optimistic (disjunctive) procedure. We compare each action a and the limiting profil r_h starting from $h = 1$ to $h = p$. We stop this procedure when r_hSa, and potentially a will be assigned to C_h.

The imperfection of knowledge about evaluations of criteria can be taken into account when defining the thresholds of the aggregation model. However, it is not easy for the decision maker to provide precise and complete information about weights and thresholds. Numerous technics were proposed in the literature to elicit these parameters, [34], [80], [81], [98], [110].

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RÉSUMÉ

La thèse est motivée par une étude de cas intéressante liée à l'évaluation du risque nucléaire. Le cas d'étude consiste à évaluer l'impact d'un accident nucléaire survenu dans le milieu marin. Ce problème comporte des caractéristiques spatiales, différents enjeux économiques et environnementaux, des connaissances incomplètes sur les potentiels acteurs et un nombre élevé de scénarios d'accident possibles. Le cas d'étude a été résolu en utilisant différentes techniques d'analyse décisionnelle telles que la comparaison des loteries et les outils MCDA (Multiple Criteria Decision Analysis). Une nouvelle méthode de classification ordinale, nommée Dynamic-R, est née de cette thèse, visant à fournir une notation complète et convaincante. La méthode développée a fourni des résultats intéressants au cas d'étude et des propriétés théoriques très intéressantes qui sont présenté dans les chapitres 6 et 7 de ce manuscrit.

MOTS CLÉS

Analyse de décision multicritère, Classification ordinale, Problématiques de notation, Théorie de décision algorithmique, Pollution marine, Evaluation du risque environnementale, Notation du risque

ABSTRACT

The thesis is motivated by an interesting case study related to environmental risk assessment. The case study problem consists on assessing the impact of a nuclear accident taking place in the marine environment. This problem is characterized by spatial characteristics, different assets characterizing the spatial area, incomplete knowledge about the possible stakeholders, and a high number of possible accident scenarios. A first solution of the case study problem was proposed where different decision analysis techniques were used such as lotteries comparison, and MCDA (Multiple Criteria Decision Analysis) tools. A new MCDA rating method, named Dynamic-R, was born from this thesis, aiming at providing a complete and convincing rating. The developed method provided interesting results to the case study, and very interesting theoretical properties that will be presented in chapters 6 and 7 of this manuscript.

KEYWORDS

Multicriteria Decision Analysis, Ordinal classification, Rating problem statements, Decision supporting systems, Algorithmic Decision Theory, Marine pollution, Environmental risk assessment, Risk rating.