Marchés pair-à-pair de l'électricité dans les réseaux électriques

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Peer-to-peer electricity markets in power systems

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Preface

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[1] F. Moret, T. Baroche, E. Sorin, and P. Pinson, "Negotiation Algorithms for Peer-to-Peer Electricity Markets: Computational Properties," in 2018 Power Systems Computation Conference (PSCC). IEEE, 6 2018, pp. 1–7.

[2] T. Baroche, P. Pinson, R. Le Goff Latimier, and H. Ben Ahmed, "Exogenous Cost Allocation in Peer-to-Peer Electricity Markets," IEEE Transactions on Power Systems, vol. 34, no. 4, pp. 2553–2564, 7 2019.

[3] T. Sousa, T. Soares, P. Pinson, F. Moret, T. Baroche, and E. Sorin, "Peer-to-peer and community-based markets: A comprehensive review," Renewable and Sustainable Energy Reviews, vol. 104, pp. 367–378, 4 2019.

[4] T. Baroche, F. Moret, and P. Pinson, "Prosumer Markets: A Unified Formulation," 2019 IEEE Milan PowerTech, pp. 1–6, 6 2019.

[5] R. L. G. Latimier, T. Baroche, and H. Ben Ahmed, "Mitigation of Communication Costs in Peer-to-peer Electricity Markets," in 2019 IEEE Milan PowerTech. IEEE, 6 2019, pp.1–6.

[6] P. Pinson, F. Moret, T. Baroche, and A. Papakonstantinou, "Negotiation Approaches for Sharing Systems," in Analytics for the Sharing Economy: Mathematics, Engineering and Business Perspectives. Springer International Publishing, 2020, pp. 151–171.

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Mots clés: Marché pair-à-pair, Flux optimal de puissance, Répartition de puissance, Marché de l'énergie, Marché des capacités, ADMM

Résumé: Le déploiement de ressources énergétiques distribuées, combiné à une gestion plus pro-active de la demande et à l'intégration de systèmes de gestion d'énergie, fait entrer l'exploitation des systèmes électriques et des marchés de l'électricité dans un nouveau paradigme. En partie liés à leur structure décentralisée, les marchés dits pair-à-pair ont gagné un intérêt considérable. Les marchés pair-à-pair reposent sur des négociations bilatérales entre les agents pour faire correspondre l'offre et la demande. De plus, ils peuvent cartographier l'ensemble des échanges possibles, ce qui permet de repenser ces interactions avec le réseau.

Ces travaux de thèse traitent de trois défis majeurs dont la résolution est essentielle avant d'envisager le passage à des applications réelles : *(i)* le passage à l'échelle pour gérer un nombre croissant d'acteurs et de ressources distribués, *(ii)* le respect des contraintes du réseau électrique, et *(iii)* la résilience du marché à la présence d'agents stochastiques.

Une analyse de complexité a permis de montrer que le passage à l'échelle des marchés pair-à-pair et le mécanisme de résolution peut être renforcé par trois améliorations réduisant les complexités algorithmiques et structurelles. Pour le respect des contraintes réseau, le manuscrit propose d'introduire des redevances qui seraient liées à l'utilisation du réseau électrique. Deux approches sont considérées pour déterminer ces redevances réseau. La première, exogène, exige que le gestionnaire de réseau les fournisse a priori avant le début des négociations. Dans la seconde, le gestionnaire de réseau actualise les redevances réseau de manière endogène à chaque itération pour mieux tenir compte de l'état actuel du réseau. Enfin, les prévisions de production et de consommation des agents stochastiques sont mieux prises en compte par la création d'un marché pair-à-pair de l'énergie et des capacités de réserve, pour corriger un éventuel déséquilibre de puissance due à des erreurs de prévision.

Title: Peer-to-peer electricity markets in power systems

Keywords: Peer-to-peer markets, Optimal power flow, Economic dispatch, Energy market, Capacity Market, ADMM

Abstract: The deployment of distributed energy resources, combined with a more proactive demand side management and energy management systems, is inducing a new paradigm in power system operation and electricity markets. Within a consumer-centric market framework, peer-to-peer approaches have gained substantial interest. Peer-to-peer markets rely on multi-bilateral negotiation among all agents to match supply and demand. These markets can yield a complete mapping of exchanges onto the grid, hence allowing to rethink market–grid interactions.

This thesis treats three main challenges which needs to be overcome before considering real world implementations: *(i)* scalability to host a growing number of distributed users and resources, *(ii)* compati-bility with grid constraints, and *(iii)* resilience to stochastic power injections.

After a complexity analysis, scalability of peer-to-peer markets and the proposed negotiation mechanism to solve them is enhanced by three improvements reducing algorithmic and structural complexities. Feasibility of the peer-to-peer electricity market is eventually obtained with the use of network charges. Two approaches are proposed to handle these network charges. The first, exogenous, requires the system operator to provide them a priori before negotiations start. In the second, the system operator updates network charges endogenously at each iteration to better account for the current grid status. Finally, power forecasts of stochastic agents are taken in a more comprehensive way by the developpement of peer-to-peer market on both energy and capacities, used to restore power balance in case of misdipatch due to forecast errors.

French résumé

Contexte et défis actuels

Le concept des systèmes d'alimentation électrique a débuté en 1881 avec la première fourniture publique d'électricité au monde qui a illuminé les rues locales de la ville de Godalming dans le Surrey [7]. Utilisant l'eau, la centrale de Godalming était donc aussi la première centrale hydroélectrique de Grande-Bretagne. Des copies et des extensions de cet exemple ne tardèrent pas à fleurir au fil des ans jusqu'à devenir aujourd'hui le coeur de la vie moderne et des facteurs clés du développement technologique et économique. Pour témoigner de l'hégémonie de l'électricité dans nos systèmes économiques modernes, il est possible de citer l'exemple d'une panne d'électricité subit aux États-Unis en 2003. Cette panne massive aurait causé des pertes économiques estimées à 6 milliards de dollars par le ministère de l'énergie états-unien. Or, pour des raisons historiques de coûts d'investissement, de gestion et de rentabilité, la production d'énergie électrique c'est concentrée autour de grandes centrales thermiques au charbon, gaz ou nucléaire et d'immenses barrages hydroélectriques. Et c'est précisément pour des raisons de fiabilité que les réseaux électriques ont, pour la plupart, été développés suivant une architecture hiérarchique mieux adaptée à la production massive et localisée de l'énergie. Ainsi, un système électrique est composé d'un réseau de transport pour acheminer de grandes quantités d'énergie électrique entre les unités de production, de plusieurs centaines de MVA, et des gros consommateurs de plusieurs MVA tels que des usines. Considérés comme une charge connectée au réseau de transport, les réseaux de distribution dirigent ensuite l'énergie à des charges plus petites allant de quelques kVA à plusieurs centaines de kVA, telles que des maisons, des bâtiments et l'éclairage public. Les réseaux de transport adoptent une topologie maillée pour améliorer la fiabilité de l'approvisionnement en énergie. Cette architecture permet d'adapter les flux d'énergie en cas de défaillance, que ce soit d'une ligne ou d'une centrale électrique. Comme les réseaux de distribution fournissent des installations moins critiques, une architecture radiale est utilisée pour réduire les coûts d'installation. En conséquence, l'exploitation des réseaux électriques est divisée en zones. Les réseaux de transport sont contrôlés par des gestionnaires de réseaux de transport (GRT), typiquement un par pays en Europe. Et les réseaux de distribution peuvent être contrôlés indépendamment les uns des autres par différents gestionnaires de réseaux de distribution (GRD).

Très pratique et adapté à cette façon hiérarchique de produire et de consommer l'électricité, la répartition économique de la production d'électricité dans les systèmes électriques est organisée sous la forme de structures centralisées dans un marché de gros. En revanche, les petits consommateurs doivent d'abord s'abonner à un fournisseur d'électricité. Il est à noter qu'en fonction des régions et de la législation, ils peuvent avoir le choix entre un ou plusieurs fournisseurs. Ensuite, avec les gros producteurs et consommateurs, les fournisseurs d'électricité peuvent à leur tour participer au marché de gros centralisé sur une plate-forme. Toutefois, l'augmentation des quantités d'électricité produite à partir de sources d'énergie renouvelables, avec par exemple des champs éoliens ou des toits solaires utilisant des onduleurs, une demande plus souple et un engagement plus important des consommateurs ont entraîné une réduction de la prévisibilité de l'offre et de la demande. Parallèlement à la libéralisation des marchés de l'électricité, ayant entraîné des flux transfrontaliers plus importants et un fonctionnement plus volatil en raison des échanges à court terme, le système est exposé à des écarts d'injections et de flux de puissance de plus en plus importants avec la planification initiale. Mais ce changement majeur de produire l'énergie électrique de manière plus diffuses sur les territoires bouleversent les méthodes d'administration utilisées par les gestionnaires de réseau.

En effet, l'électricité est un vecteur énergétique transportée par un système électrique et doit donc obéir à ses règles physiques. Le principe fondamental auquel sont confrontés les gestionnaires de réseau est qu'il doit y avoir à tout moment l'équilibre des puissances entre la production et la consommation. Les réseaux électriques n'ont pas de capacité de stockage interne durable comme peuvent en avoir les réseaux de gaz avec l'augmentation de la pression ou comme les réseaux d'eau pouvant inclure des réservoirs tampons. Par conséquent, les gestionnaires de réseaux sont de plus en plus souvent amenés à interférer avec les décisions du marché de l'électricité et d'effectuer des actions coûteuses de ré-allocation de puissance pour maintenir l'équilibre des puissances. Pour assurer la qualité d'approvisionnement, les gestionnaires de réseaux disposent également de mesures de sécurité. Ils peuvent par exemple être plus restrictifs que les limites réelles du réseau pour conserver des marges de sécurité. Historiquement, l'énergie électrique est produite avec d'énormes machines électriques offrant une certaine inertie mécanique, les gestionnaires de réseaux les utilisent notamment comme un tampon à très court terme leur donnant le temps de prendre des mesures pour rétablir l'équilibre des puissances. Ce tampon à court terme est observé à travers la vitesse de variation de la fréquence des courants et tensions alternatifs, par exemple 50 Hz avec une marge de ±0,2 Hz en Europe. Même dans un vaste réseau électrique aussi interconnecté qu'en Europe, le présence de se tampon de court terme n'est pas suffisant pour palier aux éventuelles variations rapides de production et de consommation. Pour cela, des marchés de capacités ont été mis en afin de compenser un déséquilibre prolongé. Ces marchés de capacités permettent donc aux producteurs de vendre une part de leur puissance installée aux gestionnaires de réseaux de transport qu'ils pourront ensuite préempter et utiliser pour rétablir l'équilibre entre la production et la consommation.

Objectifs et contribution de la thèse

Cependant les ressources énergétiques distribuées, conjointement avec les techniques d'information et de communication et la gestion des systèmes énergétiques des bâtiments et maisons, nous font repenser notre approche de l'exploitation des systèmes électriques. En particulier, en descendant aux niveaux inférieurs du réseau, de nouveaux types d'agents apparaissent, les "consommacteurs", ayant la capacité de produire et de consommer (et très probablement de stocker dans un avenir proche). Bien que des efforts considérables soient déployés pour faire évoluer l'exploitation du réseau électrique, les marchés de l'électricité n'ont pas encore suivi le même processus d'adaptation à ce nouveau contexte, avec ses défis et ses opportunités. La piste principale d'évolution serait l'adaptation des marchés de l'électricité d'un système centralisé sur les gros producteur vers un système centré sur les consommacteurs [8,9]. Il est notamment probable que les futures organisations de marche comprennent une composante d'échange pair-à-pair ou communautaire [10]. Un marché dit pair-à-pair repose sur de multiples échanges bilatéraux qui relient directement deux consommacteurs coopérants. Le recours à de multiples échanges bilatéraux pourrait présenter un certain nombre d'avantages, par exemple grâce à la différenciation de produit et à sa nature centrée sur le consommacteur, ce qui permettrait de créer une multitude de nouveaux modèles commerciaux.

La nécessité du recours aux marchés pair-à-pair provient non seulement du passage à une large échelle des ressources énergétiques distribuées et des consommacteurs mais aussi de l'intérêt que ceux-ci portent pour les énergies renouvelables et leur envie de plus s'impliquer dans les choix énergétiques avenirs. Ce souhait d'implication pourra par exemple se faire par le biais de la différentiation de produit. La différenciation de produit est ici vue comme le fait que les acteurs du marché peuvent exprimer des préférences sur le type et la qualité de l'énergie qu'ils vont s'échanger. Ces préférences peuvent concerner une production locale d'énergie, une énergie à faible émission de CO_2, etc. Lorsqu'ils ont des préférences et des intérêts communs, un groupe de consommacteurs peut souhaiter se regrouper et former une communauté comme dans [11]. Une telle communauté peut être considérée comme un marché de gros entre les membres de la communauté, organisé autour d'un gestionnaire de communauté à but non lucratif. Le gestionnaire de la communauté gérerait alors le marché de gros local et servirait d'interface avec le reste du monde extérieur (soit en marché de gros, soit en pair-à-pair) pour fournir ou vendre l'excédent d'énergie. Les communautés locales peuvent également être représentées par des échanges bilatéraux, chaque membre commerçant uniquement avec son gestionnaire de communauté. Partant de ce constat, on peut noter qu'une collectivité n'est pas spécifiquement liée à une zone géographique ou électrique. Les communautés peuvent donc être composées de membres reliés à différentes zones du réseau électrique. De plus, une communauté de consommacteurs peut également faire partie d'une communauté plus large. Les centrales électriques et les agrégateurs virtuels pourraient également être représentés dans ce cadre comme des communautés dont les "actifs" seraient membres. Les échanges bilatéraux multiples sont donc une façon plus générale de représenter les marchés. C'est pour cette raison, que ces travaux de thèse se concentrent sur les échanges bilatéraux multiples à la base des marchés pair-à-pair.

Plusieurs défis s'opposent l'introduction de ces futurs marchés de consommacteurs. Tout d'abord, l'implication de tous les consommacteurs, même les plus petits, pose la question de savoir si le mécanisme de négociation est évolutif et capable de traiter un nombre aussi important de transactions bilatérales. Si ce mécanisme devait être appliqué en Europe, le marché qui en résulterait concernerait plusieurs dizaines de millions de consommateurs. Plusieurs améliorations, allant de l'augmentation du taux de convergence algorithmique à la structure des transactions bilatérales, sont proposées ici pour résoudre ce problème. Il est toutefois possible d'améliorer continuellement cet algorithme au fur et à mesure du déploiement des marchés de consommacteurs. Au-delà de ce défi technique, les deux autres défis sont conceptuels et mettent à l'épreuve une mise en œuvre réelle de tels marchés. En effet, il peut y avoir des divergences entre la négociation du marché et la répartition réalisable sur le réseau électrique en raison des contraintes liées à son exploitation. Pourtant, ces réseaux sont gérés de manière centralisé par les gestionnaires de réseau. L'incohérence entre la nature décentralisée des marchés de consommacteurs et le fonctionnement centralisé des réseaux électriques doit donc être traitée avant d'envisager sa réelle mise en place. Ainsi, le second objectif de ces travaux de thèse fut de concilier ces deux points de vue en proposant un marché pair-à-pair dont les échanges bilatéraux satisfont les contraintes du système électrique. La méthode proposée pour déterminer les échanges bilatéraux doit également être adaptée aux consommacteurs stochastiques et non contrôlables tels que les parcs éoliens et les centrales solaires qui sont sujets à des erreurs de prévisions météorologiques. Or, les marchés de capacités sont traditionnellement traités de manière centralisée pour bénéficier au mieux du foisonnement d'un nombre important de consommacteurs stochastiques. Ainsi, le troisième et dernier défi traité dans cette thèse est le développement d'un marché de capacités décentralisé compatible avec le marché pair-à-pair de l'énergie. À cette fin, le marché des capacités proposé utilise également des échanges bilatéraux multiples sur les capacités afin de préserver une prise de décision décentralisée. De cette façon, le marché pair-à-pair de l'énergie et des capacités qui en résulte ne se contente pas de distribuer de l'énergie électrique mais, en même temps, constitue des réserves de capacité. Ces capacités seront alors disponibles pour compenser d'éventuelles erreurs d'acheminement dues aux incertitudes de la production d'électricité.

Description des travaux

L'objectif de ces travaux de thèse est donc de proposer un marché pair-à-pair adapté aux réseaux électriques. Le marché pair-à-pair de l'électricité proposé doit être *(i)* améliorable pour faire face au défi du passage à l'échelle, *(ii)* compatible avec les contraintes des réseaux électriques, et *(iii)* résilient face aux consommacteurs stochastiques. Avant de répondre à ces objectifs, le chapitre 2 présente les formulations usuellement utilisés pour modéliser les marchés électriques centralisés. Le chapitre présente ensuite une formulation généralisée des problèmes de coordination et propose un algorithme de résolution décentralisée associé. La résolution des marchés de l'électricité se fait essentiellement sur plusieurs pas de temps. Cela permet notamment aux différents acteurs du marché de minimiser leur coûts d'exploitation et de maximiser leur profits. Pour ce faire, ils modulent leur production au cours de la journée pour tirer le meilleur parti de la courbe de consommation, si contrôlables comme les centrales thermiques, ou de leur productible estimé, si non-contrôlables comme les producteurs éoliens ou solaires. Bien que les développement de cette thèse sont compatibles avec plusieurs pas de temps, l'accent est mis sur des solutions à un seul pas de temps afin de clarifier les propos et de mieux expliciter leurs bénéfices.

Les chapitres suivants visent alors à répondre aux interrogations et à proposer des solutions pour surmonter les trois défis majeurs énoncés précédemment. Ces chapitres s'organisent comme suit. Le chapitre 3 fait une analyse de convergence de l'algorithme de résolution associée au problème de coordination présenté au chapitre 2. À l'issue de cette analyse, le chapitre 3 présente plusieurs contributions qui soit améliorent intrinsèquement l'algorithme, soit modifient la structure du marché pair-à-pair pour en diminuer la complexité. Ensuite, deux approches visant à prendre en compte les contraintes du système électrique dans le marché pair-à-pair sont exposées dans le chapitre 4. Après avoir rappelé le problème en question, le chapitre développe les deux techniques envisagées, à savoir une technique exogène et l'autre endogène. Le chapitre 5 discute des moyens de gérer l'incertitude à laquelle sont confrontés les consommacteurs stochastiques tels que les parcs éoliens et les centrales solaires. Pour ce faire le chapitre 5 fait appel à un marché pair-à-pair des capacités résolue conjointement au marché pair-à-pair de l'énergie introduit au chapitre 2. Le chapitre compare ainsi différentes manières de répartir les incertitudes entre les consommacteurs stochastiques.

Les chapitres 3 à 5 sont résumés ci-dessous suivant le même ordre.

Amélioration des performance pour un passage à l'échelle des marchés pair-à-pair

Dans la description du problème plus général de coordination, le chapitre 2 propose l'utilisation d'un algorithme itératif de résolution basé sur l'*alternating direction method of multipliers* (ADMM). Le premier objectif du chapitre 3 fut donc de vérifier ce choix en le comparant à un second basé sur l'algorithme de consensus et d'innovation relaxé (*relaxed consensus and innovation* en anglais) lui aussi itératif. À l'occasion de la validation de l'utilisation de l'ADMM, une analyse de complexité est réalisée pour déceler les points faible des marchés pair-à-pair. Cette analyse a notamment permis fournir des orientations d'améliorations algorithmiques et structurelles. Ainsi, le chapitre propose dans un second temps d'utiliser des communications asynchrones entre les consommacteurs plutôt que les communications synchrones usuellement utilisées. En effet, dans des applications réelles l'hypothèse d'itérations synchrones implique que la durée entre chaque itération est dictée par le consommacteur le plus lent. Des retards de calcul peuvent aussi apparaître en cas de matériels peu performants ou lorsque les sous-problèmes locaux d'optimisation sont compliqués à résoudre. Des retards de communication causés par de faibles bandes passantes ou un trafic internet important peuvent aussi se présenter. La probabilité non négligeable d'avoir des retards importants justifie l'analyse des effets liés à l'utilisation d'itérations asynchrones. Cette étude a ainsi révéler la résilience de l'algorithme basé sur l'ADMM aux retards de communication. Cette constatation positive est essentielle pour les futurs marchés pair-à-pair car ceux-ci augmenteront le nombre d'échanges entre les différents acteurs du marché et, par conséquent, ces retards en raison d'un trafic internet. L'autre algorithme testé étant incapable de gérer de tels retards, l'analyse a définitivement confirmé l'utilisation de l'ADMM pour le reste du manuscrit.

Une seconde voie d'amélioration a ensuite été proposé par le chapitre afin de réduire le nombre de total de messages nécessaires pour parvenir à un consensus de négociation entre tous les consommacteurs. Il est à noter que, selon l'analyse de complexité en début de chapitre, la réduction du nombre de messages échanger pour parvenir à un consensus permet de réduire la complexité algorithmique d'un marché pair-à-pair. En outre, les informations sont potentiellement coûteuses lorsqu'elles doivent être échangées rapidement en très grand nombre. Cela implique un risque de surcharge des infrastructures existantes et la nécessité de développer des protocoles et des canaux spécifiques [12]. Le chapitre présente et test des critères d'arrêt alternatif permettant aux consommacteurs de décider le moment auquel ils souhaitent conclure les négociations avec leurs partenaires plutôt que d'avoir à continuer jusqu'au consensus de tous les échanges bilatéraux. Appliquée sur les marchés pair-à-pair, cette amélioration algorithmique épuise potentiellement le nombre de messages échangés requis pour obtenir les échanges bilatéraux optimaux. Grâce à ces critères d'arrêt alternatif des négociations, il est possible de réduire considérablement le nombre de messages échangés tout en préservant le même respect du bilan de puissance global et, donc la qualité énergétique.

Un avantage supplémentaire du marché pair-à-pair est qu'il permet également de pouvoir adopter la structure de la plupart des autres marchés de consommacteurs. Par exemple, un marché pair-à-pair peut prendre la forme d'un marché communautaire simplement en regroupant les consommacteurs autour de gestionnaires de communauté. Ces communautés pourraient ensuite rester isoler ou être interconnectées entre elles par des échanges bilatéraux via les gestionnaires de communauté. En somme, cela correspond simplement à modifier le graphe de communication entre consommacteurs. La définition intelligente de ce graphe de communication est donc identifiée comme une troisième voie d'améliorer du temps de convergence et de réduction du nombre de messages des marchés pair-à-pair. Il est à noter que la modification du graphe de communication permet de faire varier la complexité structurelle des marchés pair-à-pair. Lors d'une analyse sur la densité du graphe de communication par méthode de Monte-Carlo, le chapitre 3 a montré qu'il est possible de trouver un compromis entre optimalité du marché pair-à-pair, rapidité de convergence et réduction du nombre de messages échangés. Ainsi, il est possible de tirer parti de la flexibilité structurelle des marchés pair-à-pair.

Marchés pair-à-pair de l'énergie électrique respectant les contraintes réseaux

La nature décentralisée et indépendante des marchés pair-à-pair peut naturellement conduire à des injections de puissance ne respectant pas les contraintes du réseau électrique. Ainsi, cela produirait des écarts entre les injections de puissance issues d'un marché pair-à-pair et celles effectivement acceptables pour le gestionnaire réseau, soumis aux contraintes physiques du réseau. Parallèlement, s'il semble normal de socialiser les coûts liés au réseau dans la forme actuelle des marchés de gros et de détail, l'utilisation d'échanges bilatéraux ferait repenser la manière dont ces coûts sont attribués. L'objectif du chapitre 4 est donc de décrire comment les coûts liés au réseau peuvent d'être attribués dans un marché pair-à-pair. Pour ce faire, le chapitre décrit d'abord dans quelles mesures le problème classique des flux de puissances optimal peut être adapté pour inclure les échanges bilatéraux multiples dans sa formulation. Il est notamment montré que les contraintes de réseau peuvent être condensées dans une fonction barrière qui restreindrait le marché pair-à-pair à un certain domaine fixé. Deux approches ont ensuite été proposées pour traiter cette fonction barrière contenant les contraintes du réseau électrique. La première approche propose de remplacer la fonction barrière évaluant par une fonction d'allocation des coûts réseau jouant le rôle de redevances pour l'utilisation du réseau. Conçues individuellement pour chaque échange bilatéral, les redevances d'utilisation réseau doivent être considérées comme un outil permettant au gestionnaire réseau de facturer le consommacteur d'une manière qui reflète l'impact qu'ont ces échanges bilatéraux sur le système qu'il gère. Le montant perçu par le gestionnaire réseau grâce aux redevances peut alors ensuite, par exemple, servir à financer l'investissement de nouvelles lignes, à la maintenance, aux taxes ou aux sur-coûts de congestion du réseau. Le gestionnaire réseau fournirait les tarifs de ces redevances réseau à l'avance afin que les consommacteurs puissent adapter leurs stratégies de production ou de consommation. Par conséquent, les redevances doivent être estimées au préalable de manière exogène par le gestionnaire réseau. Établies de manière exogène et a priori, ces redevances réseau ne donneraient donc aucune garantie quant au respect des contraintes réseau.

Pourtant, le non respect des contraintes réseau peut mettre en péril tout le système électrique, d'autant plus au sein de réseaux électriques faibles ou sous-dimensionnés. Les redevances réseau exogènes, fournissant uniquement une incitation économique, ne sont toujours suffisante. Une participation plus forte du gestionnaire réseau au mécanisme de négociation des échanges bilatéraux serait donc nécessaire. C'est pour cette raison que la seconde approche proposée dans le chapitre 4 traite directement la fonction barrière des contraintes réseau en même temps que les négociations sur les échanges bilatéraux. Dans ce cadre, les consommacteurs ne négocieraient alors pas seulement entre eux mais chercheraient également le consensus avec les injections de puissance certifiées comme réalisables par le gestionnaire réseau. Pour les certifier, le gestionnaire réseau chercherait les flux de puissance optimaux ayant les injections de puissance les plus proches possibles de celles souhaitées par les consommacteurs. Du point de vue des consommacteurs, ce consensus avec le gestionnaire réseau prendrait aussi la forme de redevances réseau mais qui seraient cette fois liées à leurs injections de puissance sur le réseau. Issues d'un flux de puissance optimal et étant constamment vérifiées par le gestionnaire réseau, ces redevances réseau englobent les contraintes du réseau électrique de manière endogène. L'avantage de ces redevances réseau endogènes est qu'elles conduisent les consommacteurs à des injections de puissance identiques à celles d'un flux de puissance optimal centralisé classique. En effet, les échanges bilatéraux sont uniquement vus comme des variables de relaxation et n'altèrent donc pas les coûts finaux des consommacteurs. Les redevances réseau endogène présentent également l'avantage de demander moins d'efforts financiers des consommacteurs que sur les redevances réseau exogènes pour obtenir des injections de puissance réalisables. En revanche, comme cela était attendu, les redevances réseau endogènes ralentissent la vitesse de convergence des négociations car elles nécessitent la cherche du flux de puissance optimal à chaque itération.

En conséquence, les deux approches proposées pour tenir compte des contraintes réseau dans les marchés pair-à-pair de l'électricité sont de bonnes candidates, mais elles doivent toutefois être améliorées avant d'être mises en place dans des applications réelles. Convergeant rapidement, les redevances réseau exogènes ne proposent cependant pas toujours des solutions réalisables sur le réseau électrique et ce en particulier lorsqu'elles ne sont pas conçues de manière adéquate. En revanche, les redevances réseau endogènes compensent cette lacune en incluant le gestionnaire réseau dans la boucle de décision, permettant ainsi de garantir le respect des contraintes réseau. En revanche, cette garantie est obtenue au prix d'un processus de décision plus lent que l'approche exogène. Plusieurs pistes d'amélioration sont envisageables. Premièrement, les redevances réseau exogènes peuvent être renforcées en passant à une allocation nodale. Cette répartition nodale des coûts pourrait même être étayée par l'utilisation de l'apprentissage machine automatique, des chaînes de Markov ou d'outils liés aux séries temporelles pour anticiper les meilleures redevances réseau en chaque noeud. Pour améliorer les redevances réseau endogènes, il est possible d'envisager des communications asynchrones entre les consommacteurs et le gestionnaire réseau. Pour être plus conforme au concept décentralisé des marchés pair-à-pair, la recherches des plus proches injections de puissance pourraient être effectuées sur la base de résolutions distribuées ou décentralisées du flux de puissance optimal. Enfin, en guise de compromis entre les redevances réseau exogènes et endogènes, l'utilisation de l'approche exogène nodale peut être envisagée dans un premier temps. Puis, après quelques itérations, de les actualiser de manière endogène pour les adapter au niveau actuel de l'utilisation du réseau.

Marché pair-à-pair de l'énergie électrique et des capacités

L'inconvénient de ces nouveaux marchés de consommacteurs est qu'ils considèrent souvent l'intégration de consommacteurs déterministes. Hors, la majorité des nouveaux moyens de production d'énergie électrique sont issues de sources renouvelable et intermittentes tels que les champs éoliens et solaires. Pour prendre les incertitudes de production en compte, un marché classique et centralisé de l'électricité considère deux types distincts. Tout d'abord, le marché de l'énergie électrique cherche à déterminer le plan de production des différents consommacteurs participants. Puis, un marché des capacités vient supplanter le marché de l'énergie en attribuant un plan de provision des puissances restantes qui pourront être mobilisée en cas de déséquilibre si l'un des consommacteurs stochastique et non-contrôlables faillis à ces engagement d'injection de puissance. Ces deux marchés, d'énergie et de capacité, sont résolus de manière indépendante et, généralement, séquentielle. Cependant, les considérer de manière indépendante amène à des résultats sous-optimaux. Ainsi, l'actuel marché centralisé de l'électricité conduit inévitablement à mauvaise utilisation des ressources primaires [13]. Pour obtenir la solution optimal, les incertitudes doivent être considérées conjointement avec les prévisions de production et de consommation dans une formulation stochastique du marché. Les marchés stochastiques de l'énergie sont résolus à l'aide d'algorithmes centralisés, ou bien en se basant sur des résolutions par scénarios coûteux en échange d'information [14]. Les marchés stochastiques de l'énergie ne sont donc pas adaptés aux marchés de consommacteurs, décentralisés de part leur nature. Néanmoins, l'emploi de contraintes probabilistes semble être une bonne alternative aux approches purement stochastiques, [15]. Dès lors, le chapitre 5 de ce manuscrit investigue la formulation et la résolution d'un marché pair-à-pair de l'énergie et des capacités à l'aide de contraintes probabilistes.

L'approvisionnement des capacités est aussi historiquement géré de manière centralisée afin de bénéficier du foisonnement entre les prévisions de production et de consommation des consommacteurs stochastiques. Il était dès lors cohérent de faire appel à une unique contrainte probabiliste globale pour l'ensemble des incertitudes présentes dans le marché. Cependant, cette contrainte probabiliste global fait appel à des variables agrégeant de nombreux consommacteurs ce qui fait obstacle à la décentralisation de l'affectation des capacités de réserve. En particulier, la contrainte probabiliste global permet de décider avec quel niveau de confiance global, ou pourcentage de chance, la quantité de capacités globalement réservée sera suffisante pour palier aux erreurs de prévision et assurer l'équilibre des puissances. Pour surmonter cette difficulté technique et conceptuel. Le chapitre 5 propose de séparer la contrainte probabiliste global en de multiples contraintes probabilistes localisées au niveau de chaque consommacteur stochastique. Ceci correspond en réalité à un changement de paradigme car à l'issue de cette séparation, chaque consommacteur stochastique se voit maintenant responsable de constituer suffisamment de capacité de réserve pour couvrir ces propres erreurs de prévision. La quantité de capacité qu'un consommacteur stochastique devra apporté sera déterminée par le niveau de confiance local de sa contrainte probabiliste. Hors, les consommacteurs stochastiques ne possèdent pas nécessairement les installations suffisantes pour approvisionner suffisamment de capacité de réserve. Dès lors, le chapitre propose également de permettre à ceux-ci d'acheter des capacités supplémentaires au près des consommacteurs contrôlable présent sur le marché via des échanges bilatéraux multiples. Les consommacteurs n'échangeraient donc pas uniquement de l'énergie de manière multi-bilatéral mais aussi des capacités de réserve. Le marché pair-à-pair ainsi obtenu permettrait la détermination conjointe et simultanée des quantités d'énergie échanger entre les consommacteurs et des capacités que les consommacteurs contrôlables doivent mettre à disposition des consommacteurs stochastiques.

Se pose maintenant la question de la manière avec laquelle les taux de couverture locaux des incertitudes, c'est à dire les niveaux de confiance locaux des consommacteurs stochastiques, seront déterminés. Pour illustrer cette difficulté, le chapitre 5 propose deux approches principales pour allouer les niveaux de confiance locaux. La première approche, plus simple, consiste à les définir proportionnellement au niveau de confiance global. La seconde, plus complète, consiste à allouer une proportion de la quantité total de capacités nécessaire aux consommacteurs stochastiques. Le chapitre a testé les performances de ces deux approches au travers de quatre configuration, la première suivant trois types de coefficient proportionnel et la seconde avec un seul d'entre eux. Deux de ces quatre configurations ont montré leur aptitude à fournir des capacités de réserve globales suffisantes avec un faible écart d'optimalité par rapport à l'approche centralisée classique.

Mais elles se sont toutefois montrées insuffisances lorsque les consommacteurs stochastiques prévoyaient de faibles productions. Incapable de déterminer si ces lacunes étaient inhérentes à la nature des approches proposées ou si elles provenaient de la faible taille du cas d'étude, la création d'un cas d'étude d'ampleur est nécessaire à l'avenir. Les deux configurations s'étaient constamment montrées insuffisantes pourraient être améliorées avec l'augmentation du coefficient multiplicateur. Cette augmentation pourrait être choisie judicieusement sur la base des données historiques observées par le gestionnaire du réseau électrique. Un autre travail futur intéressant serait l'extension de l'actuel marché pair-à-pair de l'énergie et des capacités à plusieurs pas de temps. Cela permettrait entre autre d'étudier la possibilité pour un élément de stockage de tirer parti des sources d'énergie renouvelables lorsqu'elles sont excédentaires. En effet, un élément de stockage pourrait absorber la surproduction ou la sous-consommation des consommacteurs stochastiques pour la revendre ensuite soit sous forme d'énergie, soit comme capacité de réserve. Cette étude pourrait ensuite être étendue pour savoir s'il serait plus intéressant que cette élément de stockage soit localisé au niveau de chaque consommacteur stochastiques ou plutôt mutualisé au marché dans son ensemble.

Conclusions

Pour conclure, l'objectif de cette thèse était d'étudier la possibilité d'utiliser le concept de marché pair-à-pair dans les marchés de l'électricité. Il est important de noter que le passage des marchés classiques de l'électricité qui sont gérés de manière centralisé vers des marchés pair-à-pair de l'électricité, décentralisés par nature, correspond à un vrai changement de paradigme. Il était donc essentiel de vérifier les points clés qu'un marché de l'électricité doit remplir. Ce manuscrit s'attache tout particulièrement aux trois points clés suivants: *(i)* le passage à l'échelle pour inclure un nombre important de consommacteurs, *(ii)* vérifier la compatibilité des puissances injectés avec les contraintes du réseau électrique, et *(iii)* être résilient à la présence de consommacteurs stochastiques. Après avoir rappelé la formulation classique des marchés électriques centralisés, le manuscrit a tout d'abord décrit comment ceux-ci pouvaient être modifiés afin d'intégrer les échanges bilatéraux multiples entre consommacteurs qui sont à la base des marchés pair-à-pair. Également, un effort de généralisation théorique a été mené pour montrer que les travaux de cette thèse peuvent être étendus et appliqués à de nombreux autres domaines. Ainsi, la formulation obtenue permet de résoudre des problèmes de coordination généralisée. Comme le nom l'indique, les problèmes de coordination consistent en un ensemble d'acteurs ayant des variables locales d'optimisation interdépendantes et qui, par conséquent, doivent se coordonner pour s'entendre sur les valeurs de celles-ci. Le manuscrit propose notamment de résoudre ce type de problème avec un algorithme décentralisé inspiré de l'ADMM. C'est sur cette base que le manuscrit développe ensuite les solutions pour surmonter les difficultés à surmonter pour atteindre les trois points clés énoncés.

En effet, à l'occasion d'une analyse de complexité, le processus de négociation basé sur l'ADMM à prouvé son intérêt pour les marchés pair-à-pair et, plus largement, les marchés de consommacteurs. Pour répondre au premier défis du passage à l'échelle, le manuscrit propose deux améliorations algorithmiques, permettant de réduire le temps de convergence des négociations, et une amélioration structurelle, permettant de réduire la complexité du problème. Ensuite, pour assurer la faisabilité des échanges bilatéraux et des injections de puissance qu'ils induisent sur le réseau électrique, le manuscrit propose l'introduction de redevances réseau illustrant les coûts de gestion qu'ils engendrent pour le gestionnaire du réseau. Deux approches de calcul de ces redevances réseau sont proposées. La première, exogène, les déterminent a priori et les transmets aux consommacteurs avant le début des négociations. La seconde, endogène, requière au gestionnaire du réseau de les mettre à jour au fur et à mesure du processus de négociation. Enfin, une meilleure inclusion des consommacteurs est étudié notamment pour mieux en compte leur prévision de production et les incertitudes autour de celle-ci. Pour cela, le manuscrit élabore un marché pair-à-pair de l'électrique dans lequel les consommacteurs échangerait non-seulement de manière bilatéral en énergie mais aussi sur les capacités de réserves à constituer pour palier aux éventuelles erreurs de prévision. Dans ce marché pair-à-pair de l'énergie et des capacités chaque consommacteur stochastique serait dorénavant seul responsable de la constitution d'une quantité suffisante de capacité capable de compenser ces propres erreurs de prévision. Ce manuscrit montre ainsi la possibilité théorique d'utiliser un marché pair-à-pair de l'électrique dans des applications réelles.

Nomenclature

Acronyms

ADMM *Alternating direction method of multipliers*

CED Community-based economic dispatch

HPC High-performance computing machine

MBED Multi-bilateral economic dispatch

PCMBED Power consensus multi-bilateral economic dispatch

RCI *Relaxed consensus and innovation*

Time symbols

$\cdot^t$ Denotes time step t (when absent the symbol is taken constant in time)

ΔT Time increment between two time steps

T Time horizon

Energy markets

$\cdot_m$ Denotes prosumer m

$\cdot_n$ Denotes prosumer n

$\cdot_{nm}$ Denotes a bilateral trade from prosumer n to prosumer m

$\Delta p_n^{\max}$ Prosumer n's upper ramp bound

$\Delta p_n^{\min}$ Prosumer n's lower ramp bound

γ_{nm} Prosumer n's preference when trading with prosumer m

Λ_n Prosumer n's energy trade prices

λ_{nm} Prosumer n's energy trading price with prosumer m

$\mathbf{P}$ Matrix of bilateral power trades

μ_n Prosumer n's perceived price

Ω Set of prosumers in the market

ω_n Prosumer n's list of trading partners

$\tilde{\zeta}_\cdot$ Regularization function of the peer-to-peer market

c_n Prosumer n's energy cost function

E_n Prosumer n's stored energy

$E_n^{\max}$ Prosumer n's upper stored energy bound

$E_n^{\min}$ Prosumer n's lower stored energy bound

p_n Prosumer n's power set-point

$p_n^{\max}$ Prosumer n's upper power bound

$p_n^{\min}$ Prosumer n's lower power bound

p_n^{sto} Prosumer n's power injected in the storage

p_n^{tot} Prosumer n's total amount of power

p_{nm} Prosumer n's bilateral power trade with prosumer m

Reserve markets

$\alpha_n^{\cdot}$ Uncertain prosumer n's uncertainty allocation factor

$\cdot_{\mathrm{nc}}$ Denotes non-controllable, uncertain prosumers

δ Global confidence level

δ_n Uncertain prosumer n's local confidence level

λ_{nm}^{+} Prosumer n's upward reserve trading price with prosumer m

λ_{nm}^{-} Prosumer n's downward reserve trading price with prosumer m

$\mathbf{R}^{+}$ Matrix of bilateral upward reserve trades

$\mathbf{R}^{-}$ Matrix of bilateral downward reserve trades

Ω^{n} Prosumer n's set of the node on which it is connected (singleton)

Ω_{nc} Set of non-controllable, uncertain prosumers in the market

$\mathbb{P}$ Probability operator

σ_n Uncertain prosumer n's power forecast standard deviation

σ_{nc} Uncertain prosumers' global power forecast standard deviation

$\tilde{p}_n$ Uncertain prosumer n's random variable

$\tilde{p}_{\mathrm{nc}}$ Uncertain prosumers' global random variable

c_n^{+} Prosumer n's upward reserve cost function

c_n^{-} Prosumer n's downward reserve cost function

F_n Prosumer n's cumulative distribution function

f_n Prosumer n's probability distribution function

F_{nc} Uncertain prosumers' global cumulative distribution function

f_{nc} Uncertain prosumers' global probability distribution function

p_n^{μ} Uncertain prosumer n's expected power forecast

p_{nc} Uncertain prosumers' global contracted power

p_{nc}^{μ} Uncertain prosumers' global expected power forecast

r^+ Global upward reserve

r^- Global downward reserve

$r_n^{+,\min}$ Prosumer n's maximal upward reserve

r_n^+ Prosumer n's upward reserve

$r_n^{-,\max}$ Prosumer n's maximal downward reserve

r_n^- Prosumer n's downward reserve

r_{nm}^+ Prosumer n's bilateral upward reserve trade with prosumer m

r_{nm}^- Prosumer n's bilateral downward reserve trade with prosumer m

Optimal power flows

$\boldsymbol{B}$ Power system's susceptance matrix

$\boldsymbol{G}$ Power system's conductance matrix

$\cdot_i$ Denotes node $i \in \mathcal{N}$

$\cdot_j$ Denotes node $j \in \mathcal{N}$

$\cdot_{ij}$ Denotes a line from node i to node j with $(i,j) \in \mathcal{L}$

ℓ^{B} Power transfer distribution factor

$\ell_{ij}^{\max}$ Line's capacity limit between node i and node j

η_n^p Prosumer n's active power endogenous network charge

η_n^q Prosumer n's reactive power endogenous network charge

γ Cost allocation function

$\gamma_n^{\cdot}$ Prosumer n's total amount of money paid via network charges

$\gamma_{\mathrm{SO}}^{\cdot}$ System operator's total amount of money collected through network charges

$\gamma_{nm}^{\cdot}$ Prosumer n's exogenous network charge when trading with prosumer m

$\mathbf{Z}^{\mathrm{th}}$ Thevenin electrical distance matrix between nodes

$\mathcal{L}$ Set of lines in the power system

$\mathcal{N}$ Set of nodes in the power system

$\mathcal{N}_i$ Set of prosumers connected to node i

θ_i Node i's voltage angle

$\theta_i^{\max}$ Node i's upper voltage angle bound

$\theta_i^{\min}$ Node i's lower voltage angle bound

$\underline{\boldsymbol{Y}}$ Power system's complex admittance matrix

$\underline{\boldsymbol{Z}}$ Power system's complex impedance matrix

$\underline{S}$ Vector of complex/apparent power injections on the power system

$\underline{s}_{ij}$	Complex line power flow from node i to node j
$\underline{V}$	Vector of node's complex voltages
d_{nm}^{PT}	Power transfer electrical distance between prosumers n and m
d_{nm}^{th}	Thevenin electrical distance between prosumers n and m
N_{nm}^{zone}	Number of crossed zones for the trade between prosumers n and m
P^{SO}	Power system's feasible active power injection plan
Q^{SO}	Power system's feasible reactive power injection plan
q_n	Prosumer n's reactive power injection
$q_n^{\max}$	Prosumer n's upper reactive power bound
$q_n^{\min}$	Prosumer n's lower reactive power bound
$u^{\cdot}$	Network charges' unit fee
v_i	Node i's voltage magnitude
$v_i^{\max}$	Node i's upper voltage magnitude bound
$v_i^{\min}$	Node i's lower voltage magnitude bound

Generalized coordination problem

$\boldsymbol{E}$	Global exchange matrix between agents
$\boldsymbol{E}^{(n)}$	Global exchange matrix arranged to agent n's point of view
$\boldsymbol{E}_{\cdot}^{(n)}$	Sub block matrix of $\boldsymbol{E}^{(n)}$
$\cdot_m$	Denotes agent m
$\cdot_n$	Denotes agent n
$\cdot_u$	Denotes element u of a local variable
$\cdot_v$	Denotes element v of a local variable
Λ_n	Agent n's dual variable of variable exchanges
λ_{nm}	Dual variable exchange between agent n and agent m
$\mathcal{G}$	Function mapping indexes of local variables X_n to indexes of global variable X
$\mathcal{H}$	Function routing exchanged information to an agent from its partners
$\mathcal{X}_n$	Agent n's feasibility set
$\overline{X}_n$	Agent n's average between X_n's exchanged variables and X_n'
ϕ	Number of agents
ϕ_n	Size of agent n's local variable
φ_n	Agent n's local objective function
e	Total number of exchanges

X Global variable collection of local variables X_n

X_n Agent n's local variable

X'_n Agent n's copy of its partners shared variables

Main operators

$(\cdot)_{\cdot,h}$ Denotes the h-th column vector of a matrix

$(\cdot)_{g,\cdot}$ Denotes the g-th row vector of a matrix

$(\cdot)_{g,h}$ Denotes the g-th row element on the h-th column of a matrix

$(\cdot)_g$ Denotes the g-th element of a vector

$\cdot\angle\cdot$ Denotes a complex number given in its magnitude and angular form

$\cdot^*$ Denotes the complex conjugate

$\frac{\partial\cdot}{\partial\cdot}$ Partial derivative

$|\cdot|$ Denotes the cardinal of a set and the element-wise absolute value of a matrix, vector or scalar

$\|\cdot\|_2$ Denotes the Euclidean norm of a matrix or a vector

$\mathrm{diag}(\cdot)$ Returns a square matrix with the provided vector on the main diagonal

$\tilde{\cdot}$ Extended value of a function/Random variable

Negotiation mechanisms

$\cdot^k$ Denotes the k-th iteration of negotiation

$\epsilon^{\mathrm{d,tol}}$ Dual global feasibility tolerance

$\epsilon^{\mathrm{p,tol}}$ Primal global feasibility tolerance

ϵ_n^{d} Prosumer/Agent n's dual local residual

ϵ_n^{p} Prosumer/Agent n's primal local residual

π_n^{ρ} Coordination problem's disagreement function of agent n

ρ Penalty factor (>0)

σ_ρ Peer-to-peer market's disagreement function

L_ρ Augmented Lagrangian of a problem

$L_{n,\rho}$ Prosumer n's piece of the augmented Lagrangian of a problem

Introduction 1

1.1 Background and motivation

The concept of power systems started in 1881 with the world's first public electricity supply illuminating local streets of the Surrey town of Godalming [7]. Driven by water, Godalming Power Station was thus also the first hydroelectric power station in Britain. Copies and extensions of this living example soon flourished over the years. Being produced by bigger and bigger facilities such as coal and hydro power plants, power systems were mostly developed in a hierarchical architecture. First, a power system is composed of a transmission network to transmit large amounts of electrical energy between production units and large consumers such as factories and primary substation, injection points toward distribution networks. Seen as a large load connected to the transmission network, distribution networks distribute the energy to smaller loads such as residential, small stores, small and medium-sized enterprises, and public lights. As illustrated in Figure 1.1, transmission networks adopt a meshed topology to improve reliability of the energy supply. This architecture allows to adapt energy flows in case of a fault, either of a line or a power plant. Since distribution networks provide less critical facilities, a radial architecture is used to lower installation costs. In consequence, operation of power systems is divided in areas. Transmission networks are controlled by transmission system operators (TSOs), one or more than one per country in European's transmission grid for example. And, distribution networks can be controlled independently from each others by different distribution system operators (DSOs).

Flourishing since the end of the 1880's, electric power systems became the backbone of modern life and key enablers of technological and economical development. Interruptions of their

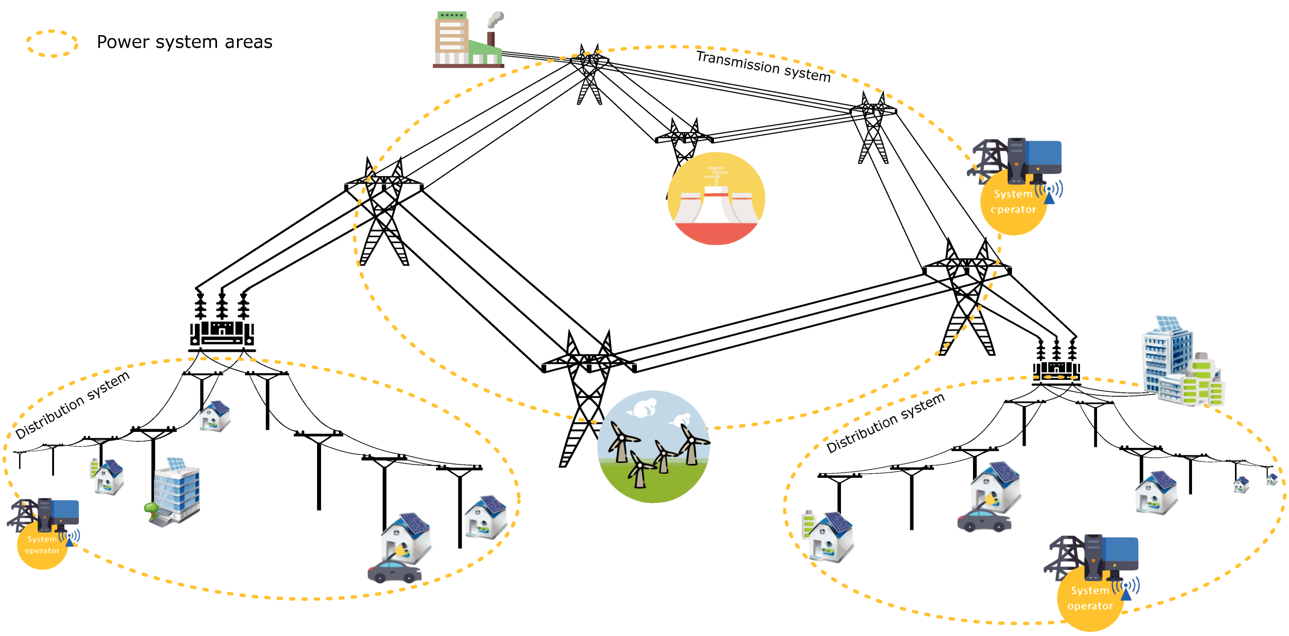


Figure 1.1: Physical assets connected on a power system

services such as blackouts are thus particularly onerous and costly, incurring physical damages as well as long-term economic losses and social effects. Rapidly becoming vital for the development of the economy, reliability of the power system has been an important issue. Extreme weather events can lead to long lasting and severe power outages. For example, North America suffered from a blackout in 2003 which caused economic losses estimated at $6 billion by the US Department of Energy [16]. While such extreme events are rare, partial interruptions may happen due to factors such as poor system planning and operation, aging infrastructure, occasional faults, system overloading or insufficient generation [17]. Despite usually being less severe, their associated losses accumulate to several billions of dollars annually in the US alone and are projected to increase under the current planning and operational paradigm [18]. As synthesized in [19], power systems reliability are becoming more and more pressing both for transmission and distribution networks, many power quality assessment and enhancement tools have been developed in the 1980's and the 1990's. Yet, over the past two decades, power systems have undergone significant changes, unlocking new opportunities but also introducing additional challenges. Thus, novel techniques constantly emerge to improve power systems' reliability [20–23].

Very convenient and adapted to this top-down hierarchical way to produce and consume electricity, the economic dispatch of power production in power systems is organized in the form of centralized pool market structures such as the one illustrated in Figure 1.2. First, small consumers have to subscribe to an electricity provider. Note that depending on regions and legislation they might be able to choose between one or several providers. Then, along with producers and large consumers, electricity providers can in turn participate to a pool market centralized on a platform. However increasing amounts of renewable electricity production from e.g., inverter-based wind and solar generators, more flexible demand and higher consumer engagement have led to a reduced predictability of both supply and demand. Along with the liberalization of electricity markets, which has led to larger cross-border flows and more

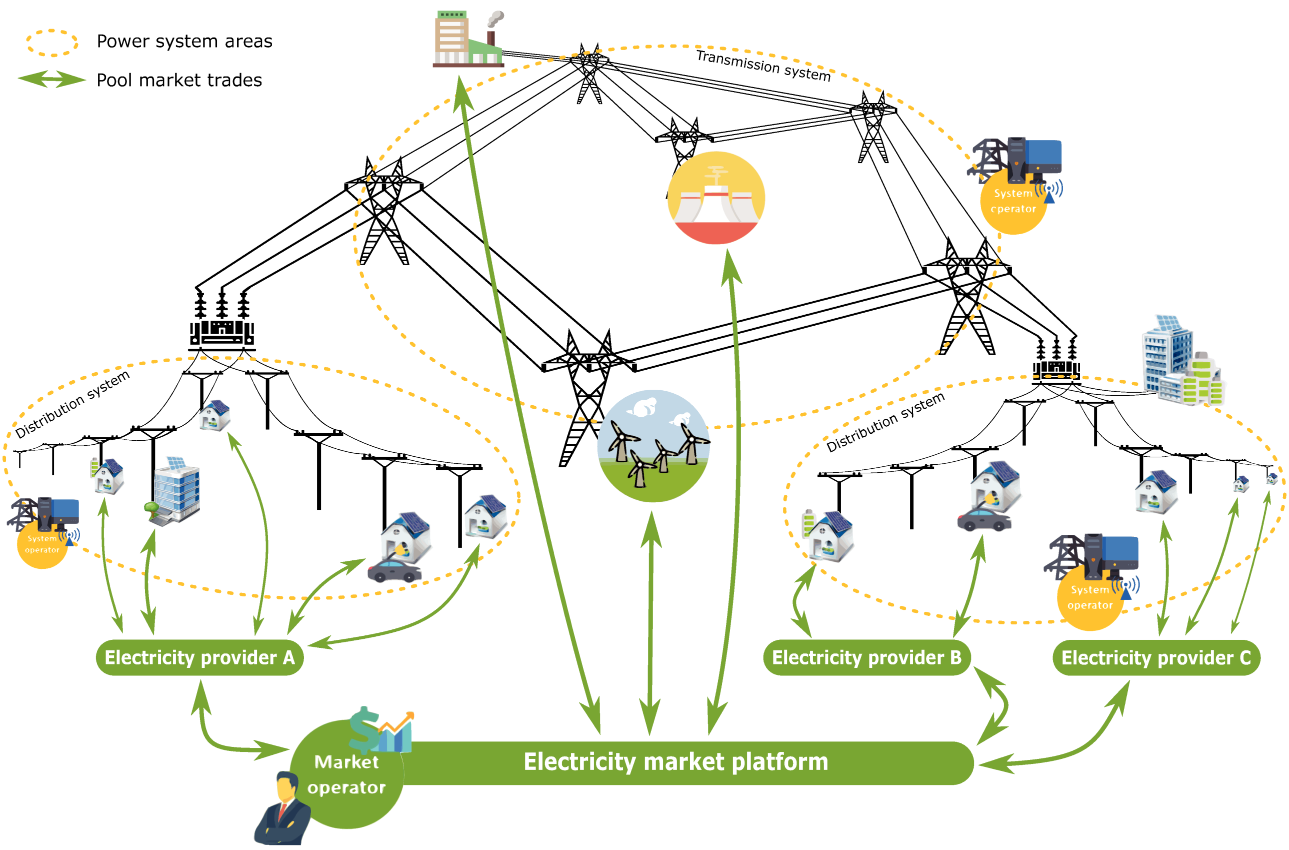


Figure 1.2: Centralized pool market structure

volatile operation due to short-term trading, the system is exposed to larger deviations of power injections and flows from their planned schedules.

Yet, electricity is a physical commodity carried by the power system and has thus to obey its physical rules. The fundamental rule faced by system operators is that there must be a power balance between production and consumption at all times. Indeed, power systems do not have an internal lasting storage capacity as gas networks can have by increasing pressure or as water networks which can add tanks. As a result, transmission system operators are more and more often required to limit market outcomes and to perform expensive re-dispatching actions to maintain power balance. To insure a certain quality of supply, system operators also have security measures such as being slightly more restrictive than the actual system capabilities to offer security margins. Historically being produced with large electrical machines offering some mechanical inertia, system operators also uses them as a very short term buffer giving them time to take actions. The state of this short term buffer is observed through the variation of the alternative currents and voltages' frequency, for example around 50 Hz with a margin of ± 0.2 Hz in Europe. More recent advances in power electronics have also allowed for the deployment of high voltage direct current (HVDC) lines between countries. The power flow of HVDC lines being controllable, they provide additional operational margins to system operators by adjusting their power flow, which would change power flows on other lines. Finally, larger energy storage can be found in pumped-storage hydroelectricity (PSH), or pumped hydroelectric energy storage (PHES). PSH allow to smooth the power demand by pumping water into a higher reservoir at low prices and releasing it into the lower reservoir to produce electricity at high prices. However, they are considered as simple consumers or producers from the power system's point of view as they are not naturally part of the grid. That is why system operators required another market on reserves of capacity which they can rapidly be controlled to serve as a viable buffer if the state of electrical machine's short buffer signals a power unbalance.

1.2 Research objectives and contributions

But distributed energy resources (DERs), jointly with information and communication technologies and energy system management for residential homes and buildings, are making us

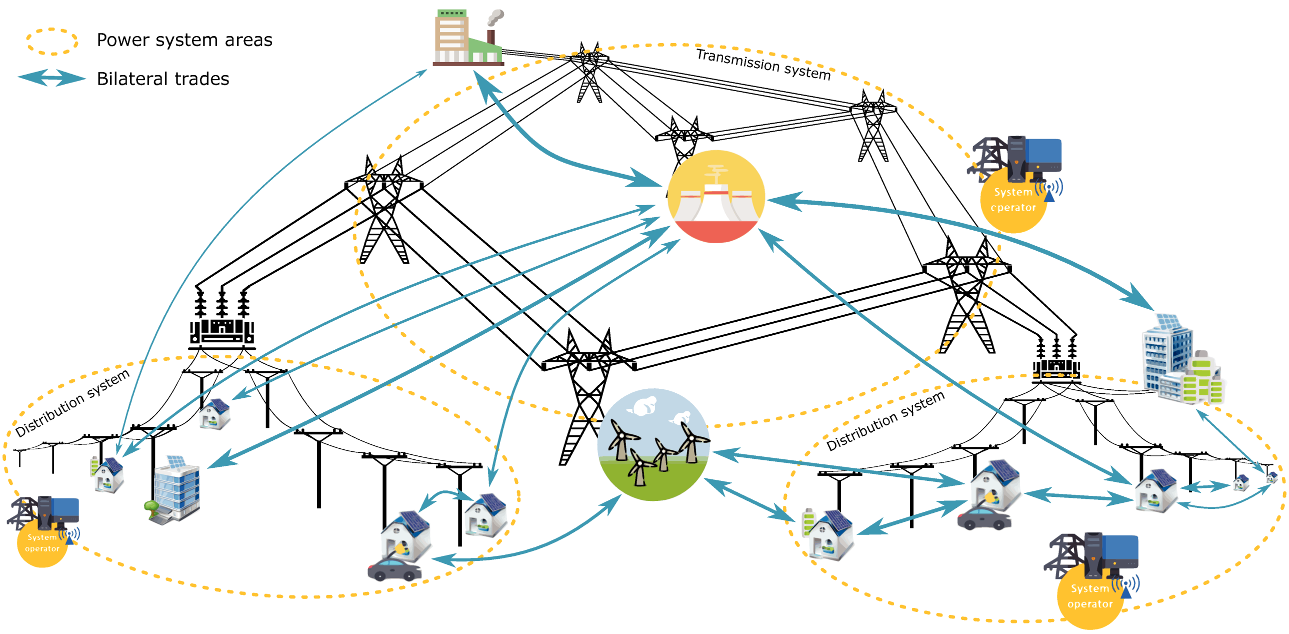


Figure 1.3: Decentralized market structure based on multiple bilateral trades

rethink our approach to power system operation. Especially, going down to lower levels of the network, new type of agents are appearing, the so called prosumers, with the ability to produce and consume (and most likely store in a very near future). While substantial efforts are made to have power system operation evolve in view of that new context, electricity markets have not gone yet through the same process of accommodating this new context with its challenges and opportunities. Electricity markets are expected to go from producer-centric to consumer-centric [8, 9], while they will most likely include a peer-to-peer and community-based component [10]. As illustrated in Figure 1.3, a peer-to-peer market relies on multiple bilateral trades which directly links two cooperating prosumers. Employing multiple bilateral trades could yield a number of advantages, for instance thanks to product preference and its consumer-centric nature, allowing for a wealth of new business models.

Product preference is to be understood here as the fact that market players can express preferences on the type and quality of the energy they will exchange. Such preferences could be for local energy generation, for energy with limited CO_2 emissions, etc. When having common preferences and interests, a group of prosumers may wish to gather themselves and form a community as in [11]. Such community can be seen as a local pool market between community members organized around a non-profit community manager. The community manager would then operate the local pool market and serve as an interface with the rest of the outside world (either pool or peer-to-peer based) to provide or sell energy excess. As displayed in Figure 1.4, local communities may also be represented with bilateral trades, each member solely trading with their manager. Arising from this observation, one may note that a community is not specifically linked to a geographical or electrical area. Communities can thus be composed of members connected to different power system areas. On top of that, a community of prosumers can also be part of a larger community. This framework could also represent virtual power plants and aggregators as communities which assets' would be members of. Multiple bilateral trades are them a more general way to represent market frameworks. For this reason, this thesis focuses on multiple bilateral trades based peer-to-peer markets.

However, several challenges arise along with the introduction of these future prosumer markets and there new business opportunities. First, involving all prosumers, even the smaller ones as suggested in Figure 1.3 brings the question of whether the clearing mechanism is scalable and able to handle such large number of bilateral trades. If this were to be applied in Europe the resulting market would concern dozens of millions of prosumers. Several improvements

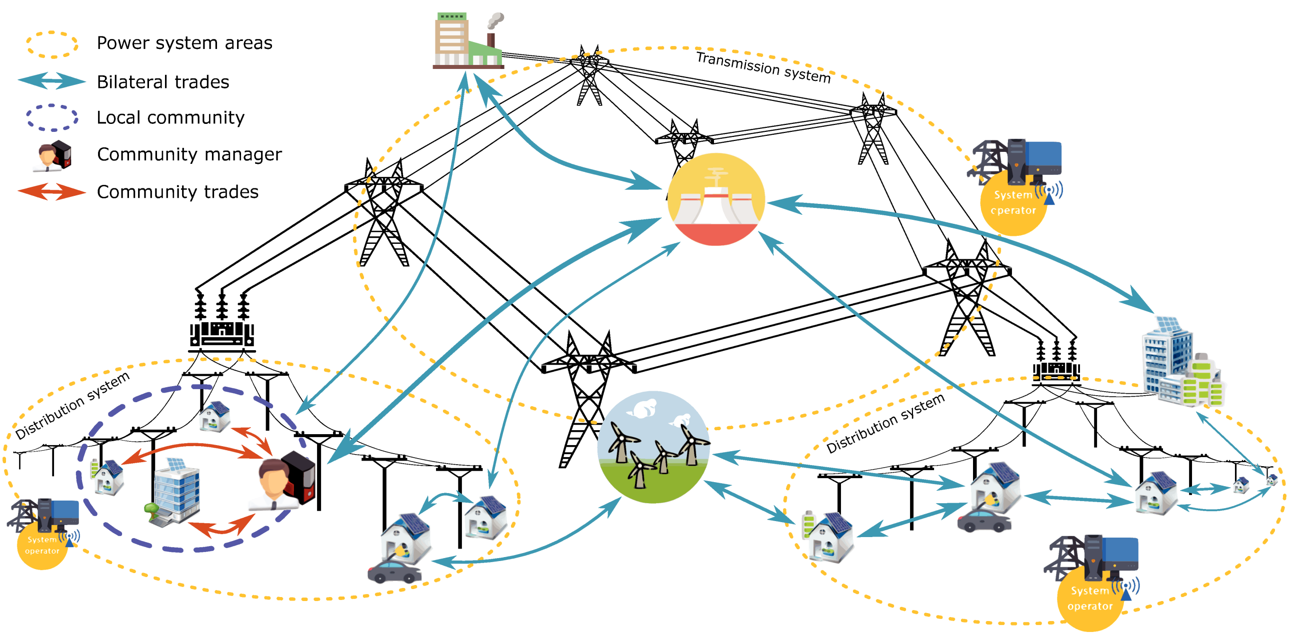


Figure 1.4: Decentralized market structure enabling local communities

ranging from algorithmic convergence rate increase to bilateral trades structure are proposed here to overcome this issue. But this can continuously be improved along with the deployment of prosumer markets. Beyond this technical challenge, the other two are more conceptual challenges testing the real implementation of prosumer markets. Indeed, there may be discrepancies between the market clearing and the actually feasible dispatch due to the grid-related and operational constraints. Yet, they are managed centrally by system operators. The inconsistency between decentralized prosumer markets and centralized power system operation must be treated before considering implementing them in the real world. Thus the second objective of this work is to reconcile these two point of views by proposing a peer-to-peer market which bilateral trades satisfy power system constraints. Moreover, the proposed method to determine bilateral trades also has to be adapted to stochastic, non-controllable prosumers such as wind farms and solar power plants which are subject to weather forecast errors. Yet, reserve market are traditionally handled in a central manner to benefit from large the number of non-controllable prosumers and the statistical compensation or correlation of their production forecasts. So the third, and last, challenge treated in this thesis is the development of a decentralized reserve market compatible with the peer-to-peer energy market. For this purpose, the proposed reserve market also uses multiple bilateral trades on reserves to preserve the decentralized decision making. This way, the resulting peer-to-peer energy and reserve market not only dispatches power but also procure reserves at the same time. These reserves will then available to compensate eventual mis-dispatches coming from power production uncertainties.

1.3 Thesis outline

The goal of this thesis is to propose a peer-to-peer electricity market suited for power systems. In consequence, the proposed peer-to-peer electricity market must be *(i)* scalable, *(ii)* compatible with grid constraints, and *(iii)* resilient to stochastic prosumers. To this end, Chapter 2 introduces the classical models used to solve the problems which can be encountered in Figures 1.1 and 1.2. After exposing the solutions considered to reach the decentralized decision making of Figures 1.3 and 1.4, the chapter further outlines a generalized formulation for coordination problems and proposes an associated decentralized solving algorithm. Even though compatible to multiple time steps, focus is eventually put on single time step solutions to add clarity to the proposed solutions. Chapter 3 makes a convergence analysis of this solving algorithm and presents several improvements, which either intrinsically boosts the algorithm or specifically targets the structure of the peer-to-peer market. Then, approaches to account for power system's constraints are exposed in Chapter 4. After recalling the problem at hand, the chapter develops the two possible techniques envisaged in this thesis, namely an exogenous and an endogenous one. Chapter 5 discusses ways to handle the uncertainty faced by stochastic prosumers such as wind farms and solar power plants. With the help of an additional decentralized reserve capacity market, in parallel of the decentralized energy market introduced in Chapter 2, Chapter 5 compares the different ways to allocate uncertainties. Finally, Chapter 6 concludes and discusses possible directions for future works. It can be noted that Chapters 3 to 5 are self-consistent and can thus be read independently from each other or from the manuscript.

This manuscript presents the contributions of this work either in already published papers or in working papers written during this Ph.D. project.

Decentralized coordination problems in electricity markets 2

This chapter provides an overview of the bases of the work made during this thesis. It introduces the problem formulations which are linked to electricity markets and gives bases for the issues tackled by this thesis. By rewriting these problems, an alternative formulation inserting the use of multiple direct bilateral trades presents the way decision making is decentralized in this thesis. This chapter then defines a more general framework, namely coordination problems, as well as a decentralized solving algorithm associated to it which are used in the rest of this thesis. Finally, the chapter makes a parallel between optimization and game theory to solving these generalized coordination problems.

Contents

2.1 Models and problems in electricity markets

In lights of past, present and future situations of the electrical system described in Chapter 1, it is important to define the outline of this chapter. For this purpose, Subsection 2.1.1 shows the classical problem formulations, usually centralized, used in the context of power systems and electricity markets. Then, Subsection 2.1.2 describes the basis on which this thesis builds upon to modify these classical problem formulations into problems which can be solved in a decentralized manner.

2.1.1 Classical problem formulation

As introduced in Figure 1.2 the electrical system is composed by a set Ω of prosumers. In this thesis prosumers are supposed rational as in [24], i.e. always objectively taking the most beneficial decisions, and non-strategic, i.e. not anticipating actions and reactions of other prosumers.

Centralized energy market

At a given time step t, the goal of each prosumer n is to minimize its cost c_n^t which is a function of power set-point p_n^t as in (2.1a). For this purpose, the prosumer can adapt p_n^t within flexibility a range defined by a lower $p_n^{t,\min}$ and an upper $p_n^{t,\max}$ bound, as in (2.1c). In the case of a fully flexible prosumer, such a generator, these bounds are constant in time and equal to the installed capacities, so $p_n^{\min}$ and $p_n^{\max}$. For a fixed prosumer, such as a non-controllable load, these bounds may depend on the time but set as both equal to the fixed set-point (so $p_n^{t,\min} = p_n^{t,\max}$). However, in the case of a new semi-flexible prosumer which has both a base load and controllable devices, such as a building and an energy manager system (EMS), both bounds can be different and time dependent. By convention the power set-point p_n^t is taken positive if prosumer n produces electricity, and negative when it consumes. As expressed in (2.1b), the power balance over the set Ω of prosumers is what couples prosumers' individual problem. Thus, the classical centralized energy market of the market operator in Figure 1.2 reads

Centralized energy market – Single time step

$$\min_{(p_n^t)_{n\in\Omega}} \quad \sum\nolimits_{n\in\Omega} c_n^t\left(p_n^t\right) \tag{2.1a}$$
$$\text{s.t.} \quad \sum\nolimits_{n\in\Omega} p_n^t = 0 \tag{2.1b}$$
$$p_n^{t,\min} \leqslant p_n^t \leqslant p_n^{t,\max} \qquad n \in \Omega \tag{2.1c}$$

which can independently be solved at each time step $t = 1 \ldots T$, with T the time horizon. Note that this problem can also be called economic dispatch or pool market in the literature.

However, from prosumers point of view market time steps are not completely independent and, thus, needs time coupling constraints. For example, prosumers such as flexible generators may have ramping constraint. In that case the difference between power set-points of two consecutive time steps p_n^t and $p_n^{t+\Delta T}$ would be limited within a lower $\Delta p_n^{t,\min}$ and an upper $\Delta p_n^{t,\max}$ ramp bound, as in (2.2d). These bounds may be infinite if the prosumer does not have any ramping constraints. Time coupling constraints also appear for prosumers with energy storage capabilities. As expressed in, the amount of energy charged in an ideal storage is proportional to the charging power $p_n^{t,\text{sto}}$ and the amount of time ΔT between two time steps. By convention, the power injected in the storage $p_n^{t,\text{sto}}$ is taken positive when charging, and negative when discharging. The amount of energy stored is limited by a lower $E_n^{\min}$ and an upper $E_n^{\max}$ stored energy bound, which is constant in time. If a prosumer does not have storage, then its energy bounds are both equal to zero. As expressed in (2.2e), the total amount of power $p_n^{t,\text{tot}}$ that a prosumer can provide is then the sum of both the power set-point p_n^t and the charging power $p_n^{t,\text{sto}}$. The storage model could be completed with some imperfections such as in [25–28] to account for charge/discharge efficiencies, self-discharge and aging. In light of this, it would be more optimal to directly account for all market time steps rather than solving each of them sequentially as suggested previously. Thus, the centralized energy market on multiple time steps reads

Centralized energy market – Multiple time steps

$$\min_{\left(p_n^t, p_n^{t,\text{sto}}, p_n^{t,\text{tot}}, E_n^t\right)_{n\in\Omega, t=1\ldots T}} \quad \sum\nolimits_{t=1}^{T} \sum\nolimits_{n\in\Omega} c_n^t\left(p_n^t\right) \tag{2.2a}$$

$$\text{s.t.} \quad \sum\nolimits_{n\in\Omega} p_n^{t,\text{tot}} = 0 \qquad t = 1\ldots T \tag{2.2b}$$

$$p_n^{t,\min} \leqslant p_n^t \leqslant p_n^{t,\max} \qquad n\in\Omega,\ t = 1\ldots T \tag{2.2c}$$

$$\Delta p_n^{t,\min} \leqslant p_n^t - p_n^{t-\Delta T} \leqslant \Delta p_n^{t,\max} \qquad n\in\Omega,\ t = 1\ldots T \tag{2.2d}$$

$$p_n^{t,\text{tot}} = p_n^t + p_n^{t,\text{sto}} \qquad n\in\Omega,\ t = 1\ldots T \tag{2.2e}$$

$$E_n^t = E_n^{t-\Delta T} + p_n^{t,\text{sto}} \cdot \Delta T \qquad n\in\Omega,\ t = 1\ldots T \tag{2.2f}$$

$$E_n^{\min} \leqslant E_n^t \leqslant E_n^{\max} \qquad n\in\Omega,\ t = 1\ldots T \tag{2.2g}$$

with given initial power set-points $(p_n^0)_{n\in\Omega}$ and stored energies $(E_n^0)_{n\in\Omega}$. Note that powers, set-points p_n^t, storage $p_n^{t,\text{sto}}$ and total $p_n^{t,\text{tot}}$, are all supposed constant during the length of the time step, so in $[t, t+\Delta T[$.

Centralized energy and reserve market

With the deployment of distributed energy resources, especially renewable energy sources, there is a need to better consider prosumers with stochastic behaviours in the market [13]. Even though classical problems also encounter stochastic non-controllable loads such as houses, these were not an important issue as uncertainties were aggregated and considered globally at the transmission level. Aggregating lower electrical grid levels notably allowed to improve forecast performances, since it is easier to forecast power consumption of a large district of houses rather an an individual one for example. For robustness, lines in distribution networks have been oversized, allowing to neglect congestion issues. Thus, the distribution system operator main focus resides in voltage limits. The recent change on the generation side, with the development of renewable generators, did not fundamentally changed system operators management as they represented a small part of the production. But, nowadays, renewable generators may represent an important part of the energy production such as in Denmark for example. Therefore it rapidly became vital to reinforce the accounting of both energy and reserves. Instead of clearing the market for each possible scenario, stochastic electricity markets usually split the problem in two. The first market aims to dispatch energy, such as showed previously in (2.1) and (2.2), while the second evaluates the amount of power capacity to reserve in case of dispatch errors [29]. Note that this second market handling errors can sometimes also be called capacity reserve or capacity market in the literature. Dispatch errors can either be handled in a robust way, e.g. with a scenario based approach [14, 30], or in a probabilistic way, e.g. with chance constraints [15,29]. Due to tractability issues [31], this thesis favors the use of chance constraints over the scenario based approach.

The central energy and reserve market for a given time step t can be expressed as follows

Centralized energy and reserve market – Single time step

$$\min_{\substack{(p_n^t, r_n^{-t}, r_n^{+t})_{n\in\Omega} \\ p_{\text{nc}}^t, r^{-t}, r^{+t}}} \quad \sum_{n\in\Omega} c_n^t(p_n^t) + c_n^{+t}(r_n^{+t}) + c_n^{-t}(r_n^{-t}) \tag{2.3a}$$

$$\text{s.t.} \quad \sum_{n\in\Omega} p_n^t = 0 \tag{2.3b}$$

$$p_n^{t,\min} + r_n^{-t} \leqslant p_n^t \leqslant p_n^{t,\max} - r_n^{+t} \qquad n \in \Omega \tag{2.3c}$$

$$0 \leqslant r_n^{+t} \leqslant r_n^{+t,\max} \qquad n \in \Omega \tag{2.3d}$$

$$0 \leqslant r_n^{-t} \leqslant r_n^{-t,\max} \qquad n \in \Omega \tag{2.3e}$$

$$r^{+t} = \sum_{n\in\Omega} r_n^{+t} \tag{2.3f}$$

$$r^{-t} = \sum_{n\in\Omega} r_n^{-t} \tag{2.3g}$$

$$p_{\text{nc}}^t = \sum_{n\in\Omega_{\text{nc}}} p_n^t \tag{2.3h}$$

$$\mathbb{P}_{f_{\text{nc}}^t}\left(-r^{-t} \leqslant p_{\text{nc}}^t - \tilde{p}_{\text{nc}}^t \leqslant r^{+t}\right) \geqslant \delta \tag{2.3i}$$

where the index $\cdot_{\text{nc}}$ relates to non-controllable prosumers as a group. Note that this problem assumes an hourly reserve time frame to simplify the formulation.

In addition to the centralized energy market (2.1), problem (2.3) includes a centralized reserve market to overcome uncertainties of non-controllable prosumers, such as wind farms for example. Gathered in $\Omega_{\text{nc}} \subset \Omega$, each uncertain prosumer n's power production or consumption is a random variable noted $\tilde{p}_n^t$ defined by probability distribution function f_n^t. In consequence, uncertain prosumer n may undergo a deviation $\Delta_n^t = p_n^t - \tilde{p}_n^t$ from the original energy unit-commitment p_n^t. A reserve market procures production and consumption margins to overcome the overall deviation of uncertain prosumers from their total energy unit-commitment p_{nc}^t, given by (2.3h). Random variable $\tilde{p}_{\text{nc}}^t$ denotes the overall uncertain power production and follows probability distribution function f_{nc}^t which is a copula of local probability distributions $(f_n^t)_{n\in\Omega_{\text{nc}}}$. Each prosumers n can either provide an upward reserve r_n^{+t}, i.e. a generation reserve, or a downward reserve r_n^{-t}, i.e. a demand reserve. The total upward r^{+t} and downward r^{-t} reserves available in the market, given by (2.3f)–(2.3g), must cover uncertainties up to a global confidence level δ as in (2.3i). The global confidence level can also be seen as an indication of market's aversion towards risk, so of its robustness.

Of course, the engagement of reserves may induce additional costs to the ones providing it. Hence, in (2.3a), prosumers also aim at minimizing cost functions c_n^{+t} and c_n^{-t} which are respectively linked to upward and downward reserves. As in (2.3d)–(2.3e), upward and downward reserves prosumers can provide might be limited by technical constraints, respectively noted $r_n^{+t,\max}$ and $r_n^{-t,\max}$, such as ramping limits for example. Moreover, a prosumer proposing reserves takes the responsibility of actually being able to provide them. So, prosumers' energy commitment and reserve procurement must be within their flexibility range. In other words, the feasible flexibility range accessible for energy unit-commitment is tightened by the promised amount of reserves, as in (2.3c). Being centralized both on energy and reserves it would be coherent to solve problem (2.3) using chance-constrained program algorithms such as the ones in [32–34].

The presence of variable, non-controllable, uncertain prosumers pleads even more for the use of storage units. For example, a fully (resp. half) charged storage unit would always be able to procure upward reserves (resp. upward and downward reserves) corresponding at its installed capacity. In the case where the uncertain prosumers are renewable sources which underestimates their power production most of the time, the storage unit would be able to extract the excess of energy whether than renewables shedding it. In consequence it would be more interesting to clear centralized energy and reserve markets on multiple time steps such as follows

Centralized energy and reserve market – Multiple time steps

$$\min_{\substack{\left(p_n^t, r_n^{-t}, r_n^{+t}\right)_{n\in\Omega} \\ \left(p_n^{t,\mathrm{sto}}, p_n^{t,\mathrm{tot}}, E_n^t\right)_{n\in\Omega} \\ p_{\mathrm{nc}}^t, r^{-t}, r^{+t};\ \forall t=1\dots T}} \sum_{t=1}^{T} \sum_{n\in\Omega} c_n^t(p_n^t) + c_n^{+t}(r_n^{+t}) + c_n^{-t}(r_n^{-t}) \tag{2.4a}$$

$$\text{s.t.} \quad \sum_{n\in\Omega} p_n^t = 0 \qquad t = 1\dots T \tag{2.4b}$$

$$p_n^{t,\min} + r_n^{-t} \leqslant p_n^t \leqslant p_n^{t,\max} - r_n^{+t} \qquad n\in\Omega,\ t = 1\dots T \tag{2.4c}$$

$$0 \leqslant r_n^{+t} \leqslant r_n^{+t,\max} \qquad n\in\Omega,\ t = 1\dots T \tag{2.4d}$$

$$0 \leqslant r_n^{-t} \leqslant r_n^{-t,\max} \qquad n\in\Omega,\ t = 1\dots T \tag{2.4e}$$

$$\Delta p_n^{t,\min} \leqslant p_n^t - p_n^{t-\Delta T} \leqslant \Delta p_n^{t,\max} \qquad n\in\Omega,\ t = 1\dots T \tag{2.4f}$$

$$p_n^{t,\mathrm{tot}} = p_n^t + p_n^{t,\mathrm{sto}} \qquad n\in\Omega,\ t = 1\dots T \tag{2.4g}$$

$$E_n^t = E_n^{t-\Delta T} + p_n^{t,\mathrm{sto}} \cdot \Delta T \qquad n\in\Omega,\ t = 1\dots T \tag{2.4h}$$

$$E_n^{\min} \leqslant E_n^t \leqslant E_n^{\max} \qquad n\in\Omega,\ t = 1\dots T \tag{2.4i}$$

$$r^{+t} = \sum_{n\in\Omega} r_n^{+t} \qquad t = 1\dots T \tag{2.4j}$$

$$r^{-t} = \sum_{n\in\Omega} r_n^{-t} \qquad t = 1\dots T \tag{2.4k}$$

$$p_{\mathrm{nc}}^t = \sum_{n\in\Omega_{\mathrm{nc}}} p_n^t \qquad t = 1\dots T \tag{2.4l}$$

$$\mathbb{P}_{f_{\mathrm{nc}}^t}\left(-r^{-t} \leqslant p_{\mathrm{nc}}^t - \tilde{p}_{\mathrm{nc}}^t \leqslant r^{+t}\right) \geqslant \delta \qquad t = 1\dots T \tag{2.4m}$$

where optimization variables are cleared over all time steps the once rather then sequentially as suggested for (2.3).

Centralized optimal power flow

But electricity markets are not isolated or except of physical constraints as a speculative market is. Electricity markets trade on a commodity which needs a physical support to actually be exchanged, namely the power system (also called electrical network or electrical grid). As explained in [35], the centralized energy market of (2.1) is transformed into an optimization called centralized optimal power to consider power flows and limits of the power system.

The electrical grid is a large-scale network, which connects electricity prosumers, and consists of a set of network nodes $\mathcal{N}$ and network lines $\mathcal{L}$. Each line is associated with a tuple (i, j) defining its sending and receiving nodes. Being sensitive to orientation, note that lines set $\mathcal{L}$ contains the two tuples (i, j) and (j, i) of each line. The AC power flow equations model the complex, steady-state network flows of a power system and determine the non-linear relationship between complex voltages $\underline{V}^t$ and complex/apparent power injections $\underline{S}^t$, both which are defined for all nodes of $\mathcal{N}$,

$$\underline{S}^t = \mathrm{diag}(\underline{V}^t)(\underline{\mathbf{Y}}\underline{V}^t)^* \tag{2.5}$$

where $\cdot^*$ gives the conjugate of a complex number or the element-wise conjugate of a vector or matrix. Function $\mathrm{diag}(\cdot)$ returns a square diagonal matrix with the elements of the provided vector on the main diagonal. $\underline{\mathbf{Y}}$ represents the complex $(|\mathcal{N}| \times |\mathcal{N}|)$ network admittance matrix [36], with $|\cdot|$ the cardinal of a set. In presence of multiple prosumers connected to the same node the complex power injection at a node would be the sum of these prosumers' injection, then power flow equations (2.5) can be written

$$\sum_{n \in \mathcal{N}_i} p_n^t + \mathrm{j}\, q_n^t = \sum_{(i,j) \in \mathcal{L}} \underline{s}_{ij}^t \qquad i \in \mathcal{N} \tag{2.6}$$

with j the imaginary number, $(\cdot)_g$ the g-th element of a vector. Set $\mathcal{N}_i$ lists the prosumers connected to node i. Variable q_n^t represents the imaginary/reactive power injected by prosumer n and p_n^t its active power injection. The complex power flowing through a line from node i to node j, denoted by $\underline{s}_{ij}^t$, can be expressed in two forms. First, the matrix form simply develops (2.5) for each line such that

$$\underline{s}_{ij}^t = (\underline{V}^t)_i \, (\underline{\mathbf{Y}})_{i,j}^* \, (\underline{V}^t)_j^* \qquad (i,j) \in \mathcal{L},\, i \in \mathcal{N} \tag{2.7}$$

where $(\cdot)_{g,h}$ denotes the g-th row element on the h-th column of a matrix. being complex values each element could be expressed in their magnitude and angular form. So, the second form classically used in power systems writes line power flows as

$$\underline{s}_{ij}^t = \left(v_i^t \angle \theta_i^t\right) (\underline{\mathbf{Y}})_{i,j}^* \left(v_j^t \angle -\theta_j^t\right) \qquad (i,j) \in \mathcal{L},\, i \in \mathcal{N} \tag{2.8}$$

where operator $\cdot\angle\cdot$ denotes a complex number given in its magnitude and angular form. Hence, node i's complex voltage can be written $(\underline{V}^t)_i = v_i^t \angle \theta_i^t$ with v_i^t and θ_i^t respectively voltage magnitude and angle. Note that it is also very common to split (2.6) in real and imaginary parts when using (2.8) for line flows. The resulting polar AC power flow equations are then expressed with

$$\sum_{n \in \mathcal{N}_i} p_n^t = v_i^t \sum_{j \in \mathcal{N}} v_j^t \big((\mathbf{G})_{i,j} \cos(\theta_i^t - \theta_j^t) + (\mathbf{B})_{i,j} \sin(\theta_i^t - \theta_j^t) \big) \qquad i \in \mathcal{N} \tag{2.9a}$$

$$\sum_{n \in \mathcal{N}_i} q_n^t = v_i^t \sum_{j \in \mathcal{N}} v_j^t \big((\mathbf{G})_{i,j} \sin(\theta_i^t - \theta_j^t) - (\mathbf{B})_{i,j} \cos(\theta_i^t - \theta_j^t) \big) \qquad i \in \mathcal{N} \tag{2.9b}$$

where $\mathbf{G}$ and $\mathbf{B}$ are real and imaginary parts of $\underline{\mathbf{Y}}$.

Thus, the problem satisfying electrical grid constraints reads

AC optimal power flow

$$\min_{\substack{(p_n^t, q_n^t)_{n \in \Omega} \\ (v_i^t \angle \theta_i^t)_{i \in \mathcal{N}},\, (\underline{s}_{ij}^t)_{(i,j) \in \mathcal{L}}}} \sum_{n \in \Omega} c_n^t \left(p_n^t\right) \tag{2.10a}$$

$$\text{s.t. } p_n^{t,\min} \leqslant p_n^t \leqslant p_n^{t,\max} \qquad n \in \Omega \tag{2.10b}$$

$$q_n^{t,\min} \leqslant q_n^t \leqslant q_n^{t,\max} \qquad n \in \Omega \tag{2.10c}$$

$$v_i^{\min} \leqslant v_i^t \leqslant v_i^{\max} \qquad i \in \mathcal{N} \tag{2.10d}$$

$$\theta_i^{\min} \leqslant \theta_i^t \leqslant \theta_i^{\max} \qquad i \in \mathcal{N} \tag{2.10e}$$

$$|\underline{s}_{ij}^t| \leqslant \ell_{ij}^{\max} \qquad (i,j) \in \mathcal{L} \tag{2.10f}$$

$$\underline{s}_{ij}^t = \left(v_i^t \angle \theta_i^t\right) (\underline{\mathbf{Y}})_{i,j}^* \left(v_j^t \angle -\theta_j^t\right) \qquad (i,j) \in \mathcal{L} \tag{2.10g}$$

$$\sum_{n \in \mathcal{N}_i} p_n^t + \mathrm{j}\, q_n^t = \sum_{(i,j) \in \mathcal{L}} \underline{s}_{ij}^t \qquad i \in \mathcal{N} \tag{2.10h}$$

for a given time t. The AC optimal power flow includes lower and upper limits on voltage magnitudes (2.10d), voltage angles (2.10e) and apparent power line flows (2.10f) to AC power flows, noted $\underline{s}_{ij}^t$ between node i and j as in (2.10g), and nodal power balances (2.10h). Note that there is a reference bus in voltage angle. Generally taken null, voltage angle limits of the reference bus are then both set to zero, so $\theta_{\text{ref}}^{\min} = \theta_{\text{ref}}^{\max} = 0$. Similarly to active power injections, prosumers' reactive power injection can vary within a range defined by a lower $q_n^{t,\min}$ and an upper $q_n^{t,\max}$ bound. To add time coupling prosumers such as in market (2.2), one would only need to replace power balance (2.2b) by power flows (2.10h) and reactive power and network limits (2.10c)–(2.10g) for each time step. Note that in (2.10h), the power injected by prosumer n would be $p_n^{t,\text{tot}}$ instead of p_n^t in presence of storage variables.

The optimal power flow based on the full AC power flow equations as presented in (2.10) is named AC-OPF in [37, 38]. The non-linearity of (2.10g) and the quadratic aspect of apparent power flow limits (2.10f) renders the AC-OPF non-linear and non-convex [39]. Potentially many local minima, saddle points and very flat regions still challenge existing solving algorithms. In particular, AC optimal power flows are, in general, NP-hard [40, 41]. Significant advances in non-convex optimization have been achieved. Many local search techniques exist for efficiently computed feasible solutions, such as interior point methods or sequential quadratic programming [42]. Nevertheless, none of them is guaranteed to converge for non-convex problems. Even if they do, the solution quality is determined by the initialization point chosen and there is no proof that the obtained solution is the global optimum. Thus, computational robustness remains the biggest challenge for both AC power flow and optimal power flow algorithms which often fail to succeed [43–46]. Note that the failure rate increases with the size of the electrical networks. Several convex approximations of the AC power flow equations are widely used for the optimal power flow today. Convex approximations help to increase the reliability of solving algorithms and foster widespread application of optimization tools which improve decision-making. These approximations leverage the benefits of convex programming to provide globally optimal solutions of the approximated problems they solve as well as computational robustness and efficiency. Three main convex approximations can be found in the literature. First, the fully linearized DC approximation neglects reactive power, voltage magnitudes and losses, leading to DC-OPF [47]. The second leverages the quadratic voltage dependency of the AC power flow equations, leading to second order cone optimal power flow (SOC-OPF) [48–50]. The third is a more general convexification technique based on semi-definite programming, leading to SDP-OPF [51]. It can be noted that the two last relaxations and approximations are more accurate than the first one. All these approaches lead to synthesis work such as [35] which lists them in greater detail and also associates a dedicated solving algorithm to each of them. Finally, to insure reliability of the obtained optimal operation points, models generally include additional security margins within their line capacities and voltage limits. For example, [23] studies security margins and uncertainty constraints in optimal power flows.

2.1.2 Decentralized formulation based on bilateral trades

The goal of this thesis is to proposed a decentralized way to clear these three problems historically centralized. For this purpose, this thesis focused on the use of multiple bilateral trades, also called peer-to-peer trades, as presented in Figure 1.3.

Decentralized energy market

Considering multiple bilateral trades in an energy market calls for a split of power set-points p_n^t, in the manner of [52], into a set of multiple bilateral trades p_{nm}^t such that

$$p_n^t = \sum\nolimits_{m \in \omega_n^t} p_{nm}^t \qquad n \in \Omega,\ t = 1 \dots T \tag{2.11}$$

where ω_n^t lists prosumer n's trading partners. Every possible bilateral power trades within the community can be condensed in a matrix $\mathbf{P}^t$ of the form

$$\mathbf{P}^t = \begin{pmatrix} p_{11}^t & \cdots & p_{1|\Omega|}^t \\ \vdots & \ddots & \vdots \\ p_{|\Omega|1}^t & \cdots & p_{|\Omega||\Omega|}^t \end{pmatrix} \tag{2.12}$$

where $|\cdot|$ denotes the cardinal of a set. Bilateral trade p^t_{nm} is necessarily equal to zero if prosumer m is not in prosumer n's trading partnership set ω^t_n. To insure that bilateral trades are reciprocal, i.e. agreed upon by both parties, they must be balanced in amount. So, each bilateral trade must verify that $p^t_{nm} = -p^t_{mn}$. As outlined in (2.13b), this power balance can be condensed by imposing $\mathbf{P}^t$ to be skew-symmetric, which also imposes that a prosumer can not trade with itself, i.e. $p^t_{nn} = 0$. By convention, a bilateral trade is taken positive, i.e. $p^t_{nm} > 0$, when prosumer n sells power to prosumer m, and negative when buying. In consequence, power set-points and total amount of power traded by prosumers must be equal for each prosumer, as in (2.13c), to guarantee its own balance. In other words, it guarantees that a prosumer sells, reps. buys, as must power than it produces, resp. consumes.

The main advantage of this approach is that it provides more procurement flexibility to prosumers. In other words, it is now possible for prosumers to select which peers supply its consumption or which peers it want to supply. This can either be done in a rigid way, through the definition of partnership sets ω^t_n, or a more adaptable way by favoring certain peers. To this end, as in (2.13a), a prosumer n can assign an additional undesirability charge γ^t_{nm}, reflecting its aversion to trade with prosumer m, in its minimization objective. A positive undesirability charge $\gamma^t_{nm} > 0$ is to be seen as a penalty while a negative undesirability charge $\gamma^t_{nm} < 0$ can be assimilated to a subsidy. Note that an undesirability charge equal to zero corresponds to a neutral point of view. Thus, the decentralized energy market at a time t based on multi-bilateral trades reads

$$\min_{\left(p^t_n, (p^t_{nm})_{m\in\omega^t_n}\right)_{n\in\Omega}} \quad \sum\nolimits_{n\in\Omega} \left[c^t_n\left(p^t_n\right) + \sum\nolimits_{m\in\omega^t_n} \gamma^t_{nm} p^t_{nm} \right] \tag{2.13a}$$

$$\text{s.t.} \quad \mathbf{P}^t = -{\mathbf{P}^t}^{\mathsf{T}} \tag{2.13b}$$

$$p^t_n = \sum\nolimits_{m\in\omega^t_n} p^t_{nm} \qquad n\in\Omega \tag{2.13c}$$

$$p^{t,\min}_n \leqslant p^t_n \leqslant p^{t,\max}_n \qquad n\in\Omega \tag{2.13d}$$

where $\cdot^{\mathsf{T}}$ denotes the transpose operator. Since, as mentioned in Subsection 2.1.1, all powers are supposed constant during the length of the time step $[t, t+\Delta T[$ it is still possible to talk about an energy market even though trades are made on power.

The extension of this multi-bilateral trades approach to the multiple time step energy market (2.4) is straight forward at it solely consist in verifying bilateral trades' reciprocity (2.13b) for each time step and to consider prosumers' the total amount of power $p^{t,\mathrm{tot}}_n$ as the total traded amount, in (2.13c), instead of their power-points. The final decentralized version of multiple time step energy market (2.2) can be written

$$\min_{\left(p^t_n, p^{t,\mathrm{sto}}_n, p^{t,\mathrm{tot}}_n, (p^t_{nm})_{m\in\omega^t_n}, E^t_n\right)_{n\in\Omega, t=1\dots T}} \quad \sum\nolimits_{t=1}^{T} \sum\nolimits_{n\in\Omega} \left[c^t_n\left(p^t_n\right) + \sum\nolimits_{m\in\omega^t_n} \gamma^t_{nm} p^t_{nm} \right] \tag{2.14a}$$

$$\text{s.t.} \quad \mathbf{P}^t = -{\mathbf{P}^t}^{\mathsf{T}} \qquad t = 1\dots T \tag{2.14b}$$

$$p^{t,\mathrm{tot}}_n = \sum\nolimits_{m\in\omega^t_n} p^t_{nm} \qquad n\in\Omega,\ t=1\dots T \tag{2.14c}$$

$$p^{t,\min}_n \leqslant p^t_n \leqslant p^{t,\max}_n \qquad n\in\Omega,\ t=1\dots T \tag{2.14d}$$

$$\Delta p^{t,\min}_n \leqslant p^t_n - p^{t-\Delta T}_n \leqslant \Delta p^{t,\max}_n \qquad n\in\Omega,\ t=1\dots T \tag{2.14e}$$

$$p^{t,\mathrm{tot}}_n = p^t_n + p^{t,\mathrm{sto}}_n \qquad n\in\Omega,\ t=1\dots T \tag{2.14f}$$

$$E^t_n = E^{t-\Delta T}_n + p^{t,\mathrm{sto}}_n \cdot \Delta T \qquad n\in\Omega,\ t=1\dots T \tag{2.14g}$$

$$E^{\min}_n \leqslant E^t_n \leqslant E^{\max}_n \qquad n\in\Omega,\ t=1\dots T \tag{2.14h}$$

Note that trade partnership sets are taken as time dependent ω^t_n to provide a more general formulation, allowing prosumers to adapt with whom they trade with in time.

Decentralized energy and reserve market

Even though bilateral power trades as described above were still used in the energy and reserve market of (2.3) and (2.4), the resulting market would solely be decentralized for the energy part. Indeed, it is important to notice that control variables $\{p_{\mathrm{nc}}^t, r^{-t}, r^{+t}\}$ respectively defines the global non-controllable present in the market and the global amount of upward and downward reserves in (2.4j)–(2.4l). Yet, the three of them are used in a chance constraints which can solely be handled centrally. Thus even with bilateral trades on energy, the energy and reserve market as currently formulated can be distributed but can not be solved in a decentralized manner. As it will be detailed later in Chapter 5, this thesis proposes to reformulate the problem in such way that it can be decentralized. For this, global chance constraint (2.4m) is split into local chance constraints such that each uncertain prosumer is responsible for the provision of its own reserve coverage. Then, Chapter 5 also adopts the strategy of using multi-bilateral trades on reserves to allow uncertain prosumers to acquire reserves from peers.

Decentralized energy market account for grid constraints

Contrary to the classical energy market (2.1), in the AC optimal power flow of (2.10), the balance of active power is not solely regulated through a simple null sum. Thus two main obstacles to the use of multi-bilateral trades arise. First, allowing bilateral trades implies the presence of trade reciprocity constraint (2.13b) in addition to nodal power balance equations (2.10h). This would lead to a problem with two contradictory active power balances as the AC power flow model leads to power losses and, hence, is not compatible with bilateral trades which are balanced. Secondly, being a complex set of equations by nature, computation of electrical constraints (2.10f)–(2.10h) can not be distributed among prosumers as they require power injections of all prosumers, in particular for nodal power balances (2.10h). As it will be detailed in Chapter 4, this thesis proposes two methods, one exogenous the other endogenous, to handle power flow equations in a decentralized energy market based on bilateral trades. It is important to note here that the goal of this thesis is solely to introduce a way for the peer-to-peer market to interact with the system operator such that market outcomes are feasible on the power system. In other words, the goal is not to propose a decentralized way to solve the optimal power flow problem.

2.2 Generalized decentralized coordination problem

Remark. *This section presents both a generalized coordination problem formulation and the associated decentralized solving algorithm. Self consistent, the following chapters will always present the studied problems and expose dedicated solving algorithms. Thus the theoretical developments proposed in Sections 2.2 and 2.3 are not required for the understanding of the rest of the thesis. Their purpose is to provide a theoretical generalization larger than the sole context of peer-to-peer markets. In consequence, notations used in them are distinct.*

The objective of this section is to extend the use of bilateral exchange variables to solve more general coordination problems, which encompass the different problems presented in Section 2.1. A coordination problem is to be seen here as a problem revolving around a set of agents with their own objectives and constraints which are trying to agree on a certain number of variables. Note that the proposed formulation is indeed not only a collaboration problem but a coordination since it is equivalent to a pure Nash equilibrium problem, arguments detailed in Section 2.3. Depending on the field of application other names could be used to denote agents. For example in game theory they would be called players, or actors. Prosumers and system operators as presented in the previous section correspond to different type of agents. First, Subsection 2.2.1 presents the formulation of this generalized coordination problem. Based on consensus *alternating direction method of multipliers* (ADMM), Subsection 2.2.2 develops a decentralized negotiation algorithm compatible with the formulation. Finally, line 11 and Subsection 2.2.3 show some extensions and practical examples in which the generalized coordination problem and the associated negotiation algorithm could be used.

2.2.1 Generalized coordination problem formulation

Being an association of multiple local problems linked by common coupling objectives and/or constraints, coordination problems are most likely to be used in distributed or decentralized frameworks. The general formulation of coordination problems considered in this thesis reads

$$\min_{X=\left(X_1^\intercal,\dots,X_\phi^\intercal\right)^\intercal} \quad \sum\nolimits_{n=1}^{\phi} \varphi_n(X_n) \tag{2.15a}$$

$$\text{s.t.} \quad \mathbf{E}X = 0 \tag{2.15b}$$

$$X_n \in \mathcal{X}_n \qquad n = 1 \dots \phi \tag{2.15c}$$

with column vectors $X_n \in \mathbb{R}^{\phi_n}$, sets $\mathcal{X}_n \subset \mathbb{R}^{\phi_n}$ and functions $\varphi_n : \mathbb{R}^{\phi_n} \to \mathbb{R}$ for any $n = 1 \dots \phi$, and matrix $\mathbf{E} \in \mathbb{R}^{e \times (\phi_1 + \dots + \phi_\phi)}$. Note that $\phi > 1$ denotes the total number of agents in the problem which aim to minimize their local cost φ_n by optimizing their local variable X_n within a feasibility set $\mathcal{X}_n$. In addition, agents may want to reach consensus or reciprocity[1] on variables related or common with some peers. All e consensus and reciprocity constraints are condensed in (2.15b) where $\mathbf{E}$ is the global exchange matrix between agents, which could also be called communication matrix, and vector $X \in \mathbb{R}^{\phi_1 + \dots + \phi_\phi}$ gathers all local vectors. Note that $e = 0$ would mean that agents were isolated.

As introduced in Subsection 2.1.2, these consensus and reciprocity constraints can be expressed as direct bilateral exchanges between coordinating peers. In other words, saying that agent n has to reach consensus on its u-th variable element with agent m's v-th variable element can be translated by the following constraint

$$(X_n)_u - (X_m)_v = 0 \tag{2.16}$$

where $(\,\cdot\,)_u$ denotes the u-th element of a vector. If this consensus is the g-th bilateral exchange constraint then the g-th row of global exchange matrix $\mathbf{E}$ is composed of zeroes except for elements

$$(\mathbf{E})_{g,\mathcal{G}(n,u)} = 1 \quad \text{and} \quad (\mathbf{E})_{g,\mathcal{G}(m,v)} = -1 \tag{2.17}$$

pointing the two targeted scalar variables, where $(\,\cdot\,)_{g,h}$ denotes the g-th row element on the h-th column of a matrix. Index mapping function $\mathcal{G}$ given by

$$\mathcal{G} : (n,u) \mapsto u + \sum\nolimits_{h=1}^{n-1} \phi_h \tag{2.18}$$

passes from the local index u of agent n's local variable X_n to its global index in X. By convention $\sum_{h=1}^{0} \phi_h = 0$ as the set on which the sum occurs is empty. In the case of a reciprocity between agents' variable elements constraint (2.16) becomes

$$(X_n)_u + (X_m)_v = 0 \tag{2.19}$$

in which case (2.17) becomes

$$(\mathbf{E})_{g,\mathcal{G}(n,u)} = 1 \quad \text{and} \quad (\mathbf{E})_{g,\mathcal{G}(m,v)} = 1. \tag{2.20}$$

One could note that in either case global exchange matrix $\mathbf{E}$ verifies the following properties: each matrix element $(\mathbf{E})_{g,h} \in \{-1,0,1\}$, each row $g = 1 \dots e$ of $\mathbf{E}$ is such that $\sum |(\mathbf{E})_{g,\cdot}| = 2$, so $\sum |\mathbf{E}| = 2\,e$, and $\sum (\mathbf{E})_{g,\cdot} \in \{0,2\}$, with $|\cdot|$ the element-wise absolute value operator.

[1] In analogy with physical systems, the consensus between states of two systems is an equality of potentials, so equal values (e.g. the temperature of two built-in pieces or the voltage potential of two free wires connected on the same source), while the reciprocity is an equality of flows, so opposite numbers (e.g. the heat transferred between two bodies or the electric current flowing between two connected devices).

2.2.2 Decentralized negotiation algorithm

The proposed decentralized negotiation algorithm is an iterative process based on the consensus version of ADMM. First synthesized in [53], the ADMM is a distributed algorithm. Through a decomposition inspired from [2], the problems tackled here can be solved in a decentralized manner[2]. Since [53] in 2011, many improvements have been proposed and adjoined to the original ADMM algorithm. As it is not the focus of this section, a straightforward adaptation of the consensus ADMM is used here without any convergence rate improvements. Several convergence rate improvements will be proposed in Chapter 3.

Reformulation of the problem

As proposed in [2], constraints (2.16) and (2.19) can be reformulated. For this, each agent n would have a local copy ${X'}_n$ of the exchanged variables from the point of view of its partners. This allows to pass from a classically distributed consensus ADMM to a decentralized form. Agent n's local copy ${X'}_n$ can be obtained by

$$ {X'}_n = \mathcal{H}(n, X) \tag{2.21} $$

where routing function $\mathcal{H}$ extracts peers' variables of X which are exchanged bilaterally with agent n. Put in a matrix form, suppose that rows of global matrix exchange $\mathbf{E}$ are sorted to obtain its per block form with respect to agent n's point of view, noted $\mathbf{E}^{(n)}$. This arranged matrix reads

$$ \mathbf{E}^{(n)} = \begin{array}{c} \begin{array}{ccc} X_1 \cdots X_{n-1} & X_n & X_{n+1} \cdots X_\phi \end{array} \\ \begin{pmatrix} \mathbf{E}_1^{(n)} & \mathbf{0} & \mathbf{E}_3^{(n)} \\ \mathbf{E}_4^{(n)} & \mathbf{E}_5^{(n)} & \mathbf{E}_6^{(n)} \\ \mathbf{E}_7^{(n)} & \mathbf{0} & \mathbf{E}_9^{(n)} \end{pmatrix} \end{array} \tag{2.22} $$

where $\mathbf{E}_\cdot^{(n)}$ are sub block matrices and $\mathbf{0}$ null matrices of adequate size. Then, routing function $\mathcal{H}$ can be written as

$$ \mathcal{H} : (n, X) \mapsto {X'}_n = \begin{array}{c} \begin{array}{ccc} X_1 \cdots X_{n-1} & X_n & X_{n+1} \cdots X_\phi \end{array} \\ \begin{pmatrix} \left|\mathbf{E}_4^{(n)}\right| & \mathbf{0} & \left|\mathbf{E}_6^{(n)}\right| \end{pmatrix} \end{array} X \tag{2.23} $$

which is linear. It is important to note here that the local copy vector ${X'}_n$ of peers' exchanged variables adopts the same sign convention as agent n's peers, so not the one of agent n. Routing function $\mathcal{H}$ actually plays the role of a communication network which carries the information between agents. It can be noted that ${X'}_n \in \mathbb{R}^{e_n}$ where $e_n = \sum |\mathbf{E}_{\cdot,\mathcal{G}(n,1\dots\phi_n)}|$ and matrices $\mathbf{E}_4^{(n)}$, $\mathbf{E}_5^{(n)}$ and $\mathbf{E}_6^{(n)}$ all have e_n rows.

Coupling consensus and reciprocity constraints (2.15b) can be split and replicated at agents' level. Thus consensus and reciprocity constraints undergone by an agent n would be expressed as

$$ \mathbf{E}_n \begin{bmatrix} X_n \\ {X'}_n \end{bmatrix} = 0 \tag{2.24} $$

[2]It is important to note that this thesis uses the terminology of optimization theory, not of computer systems. In consequence, the term distributed algorithm relates to an algorithm which possesses a central computation unit which supervises and helps local units to converge towards a solution. The term decentralized algorithm relates to an algorithm which does not have any supervisory entity but which still relies on communications between local units. In other words, contrary to computer systems, a decentralized algorithm here does not consider agents to be also isolated communication wise. Hence, a distributed algorithm adopts a star-like communication structure between computation units, while a decentralized algorithms has a meshed or fully meshed communication structure between computation units.

where local exchange matrix $\mathbf{E}_n$ is given by

$$\mathbf{E}_n = \begin{array}{c} \\ \left(\right. \end{array} \begin{array}{cc} X_n & {X'}_n \\ \mathbf{E}_5^{(n)} & \left[\mathbf{E}_4^{(n)} \mathbf{E}_6^{(n)}\right]_\emptyset \end{array} \begin{array}{c} \\ \left. \right) \end{array} \tag{2.25}$$

where $[\,\cdot\,]_\emptyset$ deletes empty columns of a matrix. Note that $\left[\mathbf{E}_4^{(n)} \mathbf{E}_6^{(n)}\right]_\emptyset$ returns a square matrix of size e_n which rows could be rearranged to obtain a diagonal matrix composed of ones or minus ones, so it also verifies $\sum \left| \left[\mathbf{E}_4^{(n)} \mathbf{E}_6^{(n)}\right]_\emptyset \right| = e_n$. It can be noted that

$$\overline{X}_n = \frac{1}{2} \mathbf{E}_n \begin{bmatrix} X_n \\ -{X'}_n \end{bmatrix} \in \mathbb{R}^{e_n} \tag{2.26}$$

is the mean value between agent n's local variables exchanged and the one from its exchanging peers and follows agent n's point of view of the sign convention in reciprocity exchanges. Thus, $\overline{X}_n - \mathbf{E}_n \begin{bmatrix} X_n \\ 0 \end{bmatrix}$ represents agent n's half distant to proceed in order to reach consensus and reciprocity with all its peers. This difference for all $n = 1 \dots \phi$ is also a way to verify if global consensus and reciprocity constraints (2.15b) are satisfied when they equal zero.

Based on these notations problem (2.15) is now reformulated as follows

$$\min_{\substack{X = \left(X_1^\intercal, \dots, X_\phi^\intercal\right)^\intercal \\ W = \left(W_1^\intercal, \dots, W_\phi^\intercal\right)^\intercal}} \quad \sum\nolimits_{n=1}^{\phi} \varphi_n(X_n) \tag{2.27a}$$

$$\text{s.t.} \quad \frac{1}{2} \mathbf{E}_n \begin{bmatrix} W_n \\ -\mathcal{H}(n, W) \end{bmatrix} = \mathbf{E}_n \begin{bmatrix} X_n \\ 0 \end{bmatrix} \qquad n = 1 \dots \phi \tag{2.27b}$$

$$X_n \in \mathcal{X}_n \qquad n = 1 \dots \phi \tag{2.27c}$$

where $W = \left(W_1^\intercal, \dots, W_\phi^\intercal\right)^\intercal$ is a global slack variable which copies global variable X through (2.27b) to insure that X satisfies consensus and reciprocity constraints (2.15b). Thus, W_n is a copy of agent n's local optimization variable X_n.

Decomposition of the problem

The augmented Lagrangian of problem (2.27) can be written

$$L^\rho(X, W, \Lambda) = \sum\nolimits_{n=1}^{\phi} L_n^\rho(X_n, W, \Lambda_n) \tag{2.28}$$

where $\rho > 0$ is the penalty factor, $\Lambda = \left(\Lambda_1^\intercal, \dots, \Lambda_\phi^\intercal\right)^\intercal$ is the collection of dual variables $\Lambda_n \in \mathbb{R}^{e_n}$ of local consensus and reciprocity constraints (2.27b) and L_n^ρ gathers Lagrangian terms involving agent n. Local parts of the augmented Lagrangian are such that

$$L_n^\rho(X_n, W, \Lambda_n) = \tilde{\varphi}_n(X_n) + \pi_n^\rho \left(X_n, \frac{1}{2} \mathbf{E}_n \begin{bmatrix} W_n \\ -\mathcal{H}(n, W) \end{bmatrix}, \Lambda_n \right) \tag{2.29}$$

where function $\tilde{\varphi}_n$ is the extended-value of φ_n, is the sense of [54], on the domain defined by $\mathcal{X}_n$. The disagreement of agent n with its exchanging partners is expressed through cost function π_n^ρ given by

$$\pi_n^\rho : (X_n, Z_n, \Lambda_n) \mapsto \Lambda_n^\intercal \left(Z_n - \mathbf{E}_n \begin{bmatrix} X_n \\ 0 \end{bmatrix} \right) + \frac{\rho}{2} \left\| Z_n - \mathbf{E}_n \begin{bmatrix} X_n \\ 0 \end{bmatrix} \right\|_2^2 \tag{2.30}$$

where $\rho > 0$ is a penalty factor scaling the weight given to the disagreement with respect to the agent's cost function φ_n. Operator $\|\cdot\|_2$ denotes the Euclidean norm of a vector or a matrix. So, for a matrix $\mathbf{Z}$, $\|\mathbf{Z}\|_2^2 = \sum(\mathbf{Z})^2_{:,:}$.

Thus, ADMM of (2.27) consists in iteration steps

$$X^{k+1} = \underset{X}{\text{argmin}}\ L^\rho(X, W^k, \Lambda^k) \tag{2.31a}$$

$$W^{k+1} = \underset{W}{\text{argmin}}\ L^\rho(X^{k+1}, W, \Lambda^k) \tag{2.31b}$$

$$\Lambda_n^{k+1} = \Lambda_n^k + \rho\left(\frac{1}{2}\mathbf{E}_n\begin{bmatrix} W_n^{k+1} \\ -\mathcal{H}(n, W^{k+1})\end{bmatrix} - \mathbf{E}_n\begin{bmatrix} X_n^{k+1} \\ 0\end{bmatrix}\right) \qquad n = 1\dots\phi. \tag{2.31c}$$

Since it applies to independent local Lagrangian parts $L_n^\rho(X_n, W^k, \Lambda_n^k)$, (2.31a) can be split among agents. Each agent n would then concurrently compute

$$X_n^{k+1} = \underset{X_n}{\text{argmin}}\ L_n^\rho(X_n, W^k, \Lambda_n^k). \tag{2.32}$$

As for the consensus ADMM of [53], update (2.31b) can analytically be obtained with

$$W_n^{k+1} = \frac{1}{2}\mathbf{E}_n\begin{bmatrix} X_n^{k+1} \\ -\mathcal{H}(n, X^{k+1})\end{bmatrix} - \frac{1}{2\rho}\mathbf{E}_n\begin{bmatrix} \Lambda_n^k \\ -\mathcal{H}(n, \Lambda^k)\end{bmatrix} \qquad n = 1\dots\phi \tag{2.33}$$

since there are no constraints and that agents' extended-value functions are evaluated on fixed variables X_n^{k+1}. Substituting (2.33) in (2.31c) implies that dual variables $(\Lambda_n^{k+1})_u$ and $(\Lambda_m^{k+1})_v$ on both sides of an exchange between agent n and m are equal of opposite sign for a consensus exchange, and are equal of the same sign for a reciprocity exchange. Thus, $\mathbf{E}_n\begin{bmatrix} \Lambda_n^{k+1} \\ -\mathcal{H}(n, \Lambda^{k+1})\end{bmatrix} = 0$ after the first iteration and (2.33) can be simplified in

$$W_n^{k+1} = \frac{1}{2}\mathbf{E}_n\begin{bmatrix} X_n^{k+1} \\ -\mathcal{H}(n, X^{k+1})\end{bmatrix} \qquad n = 1\dots\phi \tag{2.34}$$

In consequence, there is no need of an additional central entity to compute (2.31b) since its simplified form (2.34) can directly be substituted in (2.32) and (2.31c) which can both be computed locally by agents.

Final algorithm

The final decentralized negotiation algorithm solving (2.27) and, hence, coordination problem (2.15) reads

$$X_n^{k+1} = \underset{X_n}{\text{argmin}}\ \varphi_n(X_n) + \pi_n^\rho\left(x_n, \overline{X}_n^k, \Lambda_n^k\right) \quad \text{s.t.}\ X_n \in \mathcal{X}_n \qquad n = 1\dots\phi \tag{2.35a}$$

$$X'^{k+1}_n = \mathcal{H}(n, X^{k+1}) \qquad n = 1\dots\phi \tag{2.35b}$$

$$\overline{X}_n^{k+1} = \frac{1}{2}\mathbf{E}_n\begin{bmatrix} X_n^{k+1} \\ -X'^{k+1}_n\end{bmatrix} \qquad n = 1\dots\phi \tag{2.35c}$$

$$\Lambda_n^{k+1} = \Lambda_n^k + \rho\left(\overline{X}_n^{k+1} - \mathbf{E}_n\begin{bmatrix} X_n^{k+1} \\ 0\end{bmatrix}\right) \qquad n = 1\dots\phi \tag{2.35d}$$

where augmented Lagrangian terms of consensus and reciprocity constraints (2.24) are aggregated in local projection functions π_n^ρ which illustrates agent n's over costs relative to the lack of consensus and reciprocity with its exchanging peers. Agent n's disagreement cost function π_n^ρ is given by

$$\pi_n^\rho : (X_n, \overline{X}_n, \Lambda_n) \mapsto \Lambda_n^\top \left(\overline{X}_n - \mathbf{E}_n \begin{bmatrix} X_n \\ 0 \end{bmatrix} \right) + \frac{\rho}{2} \left\| \overline{X}_n - \mathbf{E}_n \begin{bmatrix} X_n \\ 0 \end{bmatrix} \right\|_2^2 \tag{2.36}$$

where $\rho > 0$ is a penalty factor scaling the weight given to the disagreement with respect to agents cost functions $(\varphi_n)_n$. Thus disagreement cost function π_n^ρ economically encourages agent n to reach consensus and reciprocity with its exchanging peers. Local exchange matrix $\mathbf{E}_n$ is extracted from global exchange matrix $\mathbf{E}$ to reflect how agent n's local optimization variable X_n is related to the variables X'_n copied from its exchanging peers. Note that ρ is common to all agents and as, hence, to be agreed upon in advance before launching the iterative process. Operands $\cdot^\top$ and $\|\cdot\|_2$ respectively denote the transposition and the Euclidean norm of a vector or matrix. Lagrangian multipliers $\Lambda_n \in \mathbb{R}^{e_n}$ associated to agent n's consensus and reciprocity constraints (2.24) can be seen as agent n's prices of coordination with the exchanging peers.

One can notice that agents can compute each iteration steps (2.35) independently. However an agent can not execute them all successively independently from others as (2.35b) requires to wait for all agents in order to gather their local variable update X_n^{k+1} in X^{k+1}. This negotiation algorithm guarantees at each iteration step that local variables are feasible at agents' level, i.e. (2.15c) are always satisfied, while consensus and reciprocity exchanges (2.15b) are only verified at convergence up to a primal $\epsilon^{\mathrm{p,tol}}$ and dual $\epsilon^{\mathrm{d,tol}}$ feasibility tolerance. Global stopping criteria associated to (2.35) read

$$\left\| (\epsilon_1^{\mathrm{p},k+1}, \dots, \epsilon_\phi^{\mathrm{p},k+1}) \right\|_2 \leqslant \epsilon^{\mathrm{p,tol}} \qquad \text{and} \qquad \left\| (\epsilon_1^{\mathrm{d},k+1}, \dots, \epsilon_\phi^{\mathrm{d},k+1}) \right\|_2 \leqslant \epsilon^{\mathrm{d,tol}} \tag{2.37}$$

for primal and dual local residuals respectively given by

$$\epsilon_n^{\mathrm{p},k+1} = \frac{1}{2} \left\| \overline{X}_n^{k+1} - \mathbf{E}_n \begin{bmatrix} X_n^{k+1} \\ 0 \end{bmatrix} \right\|_2 \qquad \text{and} \qquad \epsilon_n^{\mathrm{d},k+1} = \frac{1}{2} \left\| \overline{X}_n^{k+1} - \overline{X}_n^{k} \right\|_2 \tag{2.38}$$

which can be estimated locally by agents.

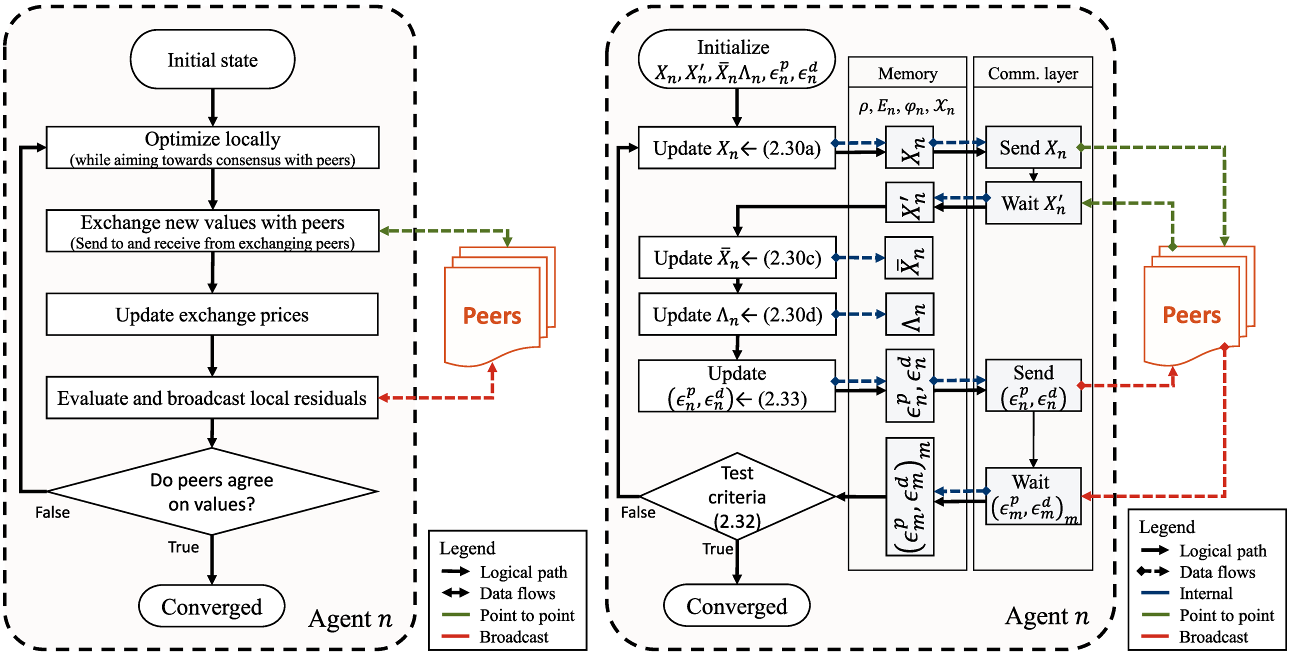
The overall negotiation algorithm, illustrated in Figure 2.1, occurs as follows. First, each local agents independently updates their local variables based on (2.35a) in a parallel manner. Once updated, each agent specifically sends the updated values to its exchanging peers and waits to receive back their counter values. Then, concurrently, each agent sequentially evaluates the new average $\overline{X}_n$ between its local values and the ones of its peers with (2.35c), updates exchange prices Λ_n with (2.35d) and estimates local primal r_n and dual s_n residuals of the current iteration with (2.38). Finally, agents share their local residuals in a broadcast manner so that they can all test stopping criteria (2.37). This process repeats until convergence of the algorithm. Convergence of the negotiation algorithm is ensured under the assumption of convexity of the problem. So, convexity of local functions φ_n and local sets $\mathcal{X}_n$ for each agent $n = 1 \dots \phi$ is then a sufficient condition to guarantee convergence of the negotiation algorithm towards the optimal value.

2.2.3 Practical examples

Decentralized energy market – Single time step

Writing the decentralized energy market (2.13) at time t into the form of (2.15) is rather straightforward. For this, prosumer n's local optimization variable X_n^t can gather its power set-point, p_n^t, and all possible bilateral trades which it could make, $(\mathbf{P})_{n,\cdot}$, as follows $X_n^t = \left(p_n^t, p_{n1}^t, \dots, p_{n|\Omega|}^t\right)$, so $\phi_n = |\Omega| + 1$. Then, time t's prosumers objective function φ_n^t would read

$$\varphi_n^t\left(X_n^t\right) = c_n^t\left(\left(X_n^t\right)_1\right) + \sum\nolimits_{m \in \omega_n^t} \gamma_{nm}^t \left(X_n^t\right)_{m+1} \tag{2.39}$$



(a) Conceptual version

(b) Symbolic version

Figure 2.1: Decentralized solving algorithm

and optimization sets $\mathcal{X}_n^t$ given by

$$\mathcal{X}_n^t = \left\{ X_n \in \mathbb{R}^{|\Omega|+1} \middle| \left(X_n^t\right)_1 = \sum\nolimits_{m \in \omega_n^t} \left(X_n^t\right)_{m+1},\; p_n^{t,\min} \leqslant \left(X_n^t\right)_1 \leqslant p_n^{t,\max} \right\} \tag{2.40}$$

which are convex. Finally, one needs to list the bilateral trades mutually accepted by prosumers, so if $m \in \omega_n$ and $n \in \omega_m$. To this end, it is possible to use Algorithm 1, where $\mathbf{0}_{g \times h}$ returns a matrix of g rows and h columns full of zeros, to build time t's global exchange matrix $\mathbf{E}^t$.

Algorithm 1: Algorithm to build (2.13)'s global exchange matrix $\mathbf{E}^t$

Data: $(\omega_n)_{n \in \Omega}$

Result: Global exchange matrix $\mathbf{E}^t$ corresponding to reciprocity constraints (2.13b)

Initialize $\mathbf{E}^t = \mathbf{0}_{0 \times |\Omega|(|\Omega|+1)}$;
for $n = 1 \ldots |\Omega| - 1$ **do**
  for $m \in \omega_n$ **do**
    if $n \in \omega_m$ **then**
      `/* Create new empty line */`
      $E_{\text{new}} = \mathbf{0}_{0 \times |\Omega|(|\Omega|+1)}$;
      `/* Link bilateral trades` p_{nm}^t `and` p_{mn}^t `*/`
      $(E_{\text{new}})_{\mathcal{G}(n,m+1)} = 1$;
      $(E_{\text{new}})_{\mathcal{G}(m,n+1)} = 1$;
      `/* Concatenate to global exchange matrix` $\mathbf{E}$ `*/`
      $\mathbf{E}^t = \begin{pmatrix} \mathbf{E}^t \\ E_{\text{new}} \end{pmatrix}$;
    end
  end
end

Decentralized energy market – Multiple time steps

As it can be observed in Subsection 2.1.1, the presence of storage implies the presence of a charge/discharge power $p_n^{t,\mathrm{sto}}$, a total amount of power traded $p_n^{t,\mathrm{tot}}$ and a stored amount of energy E_n^t at each time step t for each prosumer n. Then, the optimization variable of a prosumer n at time t becomes $X_n^t = \left(p_n^t, p_{n1}^t, \dots, p_{n|\Omega|}^t, p_n^{t,\mathrm{sto}}, p_n^{t,\mathrm{tot}}, E_n^t\right)$, so $\phi_n = |\Omega| + 4$. Time t's optimization set of time independent constraints $\mathcal{X}_n^t$ is now such that

$$X_n^t \in \mathcal{X}_n^t \Leftrightarrow \begin{cases} p_n^{t,\mathrm{tot}} = \sum_{m \in \omega_n^t} p_{nm}^t \\ p_n^{t,\min} \leqslant p_n^t \leqslant p_n^{t,\max} \\ p_n^{t,\mathrm{tot}} = p_n^t + p_n^{t,\mathrm{sto}} \\ E_n^{\min} \leqslant E_n^t \leqslant E_n^{\max} \end{cases} \tag{2.41}$$

which allows to define prosumer n's overall optimization set $\mathcal{X}_n$ as follows

$$\left(X_n^1, \dots, X_n^T\right) \in \mathcal{X}_n \Leftrightarrow \begin{cases} X_n^t \in \mathcal{X}_n^t & t = 1 \dots T \\ \Delta p_n^{t,\min} \leqslant p_n^t - p_n^{t-1} \leqslant \Delta p_n^{t,\max} & t = 1 \dots T \\ E_n^t = E_n^{t-1} + p_n^{t,\mathrm{sto}} \cdot \Delta T & t = 1 \dots T \end{cases} \tag{2.42}$$

To obtain decentralized energy market (2.14)'s objective function separated per prosumer, sums on time horizon T and prosumer set Ω must be swapped. Prosumer n's final objective function would then read

$$\varphi_n\left(X_n^1, \dots, X_n^T\right) = \sum_{t=1}^{T} \varphi_n^t\left(X_n^t\right) \tag{2.43}$$

with φ_n^t as defined above in (2.39). It can be recalled that an optimization point of view is taken here, so all market time steps are solved at the same time.

Then, if global optimization variable X is defined as follows

$$X = \left(X_1^1, \dots, X_\phi^1, \dots, X_1^T, \dots, X_\phi^T\right) \tag{2.44}$$

with $\phi = |\Omega|$, it is possible to write multiple time step decentralized energy market (2.14) such as

$$\min_{X = \left(X_1^1, \dots, X_\phi^1, \dots, X_1^T, \dots, X_\phi^T\right)} \quad \sum_{n=1}^{\phi} \varphi_n\left(X_n^1, \dots, X_n^T\right) \tag{2.45a}$$

$$\text{s.t.} \quad \mathbf{E}X = 0 \tag{2.45b}$$

$$\left(X_n^1, \dots, X_n^T\right) \in \mathcal{X}_n \qquad n = 1 \dots \phi \tag{2.45c}$$

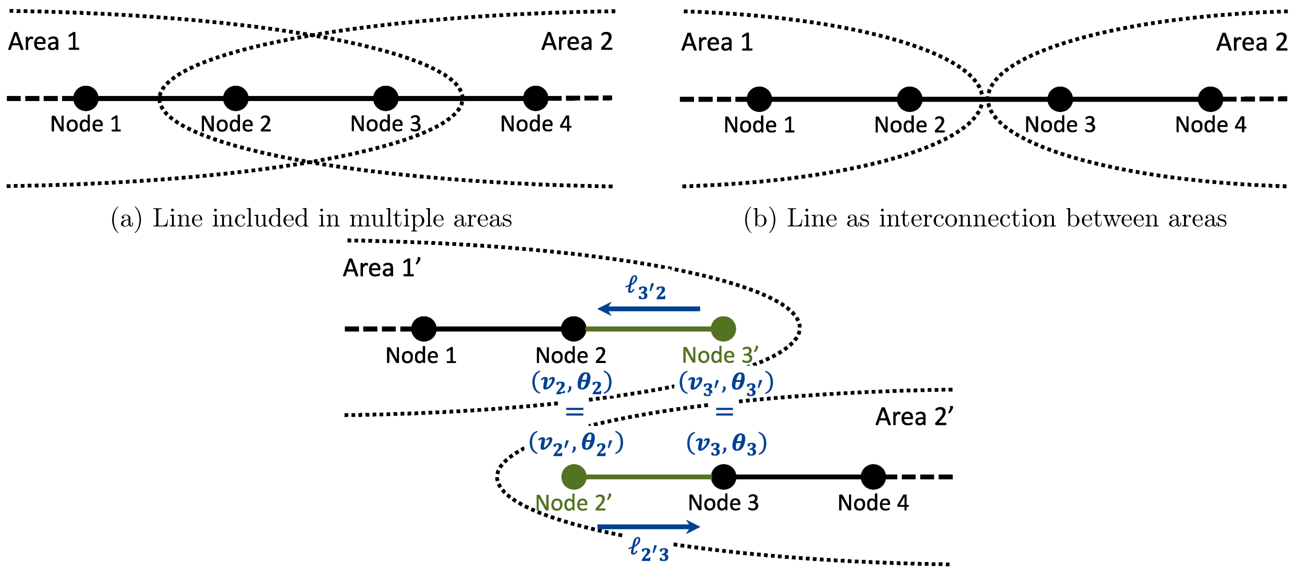
with a global exchange matrix $\mathbf{E} = \mathrm{diag}\left(\mathbf{E}^1, \dots, \mathbf{E}^T\right)$, where $\mathrm{diag}(\,\cdot\,)$ applied to matrices returns a diagonal per block matrix. In consequence, decentralized energy market (2.14) on multiple time steps can be defined in a form similar to its single time step formulation (2.13). Hence, for the sake of simplicity, in the rest of the thesis after this chapter all problems will be simplified to their single time step form and time step exponents $\cdot^t$ disappear.

Optimal power flow with multiple system operators

As pointed by [55], power systems are large interconnected systems with a high degree of complexity, so the control of such systems is a challenging task. For the determination of optimal settings for the controllable devices, optimal power flow (2.1.1) is a suitable method. But centralized optimal power flow taking the entire grid into account is often not feasible. Reasons are the size of the resulting optimization problem but also the concurrent control of the system by several independent entities. To facilitate the application of optimal control to large-scale systems, the overall problem can be decomposed into sub-problems which are solved in a coordinated way. This also complies with the fact that the task of controlling a system might be shared by several entities within each is in charge for a dedicated part of the system. In such cases, coordination is needed because the settings chosen by one entity will possibly influence the state of the entire system and thus, the choice of settings of the other entities. For power systems, which generally include hundreds or thousands of lines and buses, the sub-problems are very often associated with distinct areas. Traditionally, especially in Europe, the areas correspond to countries and the control entities are the transmission system operators. The overall electrical network of a country is also usually divided between distinct entities such as the national or federal transmission system operator and several regional distribution system operators.

As it can be testified in [56–60], coordination of system operators is still an open research field, in particular between transmission and distribution system operators. Figure 2.2 illustrates the two ways to model an interconnection between two system operators or areas. Put simply, a line interconnecting two areas can either be included in both areas, as in Figure 2.2a, or none, as in Figure 2.2b. These two interconnection models can be mixed in a way that is compatible with the proposed generalized coordination problem. For this, each area only needs to be extended to the other end of the interconnecting line and to end it with a copy of the node of the other area, as shown in Figure 2.2c. The sole purpose of this nodal copy is to allow the area to compute the power flowing between the two areas. Thus nodal copies are not balanced in power and do not follow classical nodal power balance (2.10h).

Suppose that the set Ξ lists the different system operators or areas of the problem. Then, similarly to the classical optimal power flow described in Subsection 2.1.1, sets Ω^ξ, $\mathcal{N}^\xi$ and $\mathcal{L}^\xi$ can respectively list prosumers, nodes and line tuples of an area $\xi \in \Xi$. Note that interconnection lines with area ξ are also accounted in $\mathcal{L}^\xi$ and are not discriminated from others and equally follow line flow equations (2.10g). Even though they are listed in $\mathcal{N}^\xi$, nodal copies of the area can be gathered in $\mathcal{N}^{\xi'} \subset \mathcal{N}^\xi$ and behave differently form the other nodes $\mathcal{N}^{\xi^*} = \mathcal{N}^\xi \setminus \mathcal{N}^{\xi'}$ which follow the classical nodal power balance (2.10h). Indeed, voltage magnitude and angle of nodal copies would be linked through consensus exchange constraints to there associated copied node, so that $(v_{2'}, \theta_{2'}) = (v_2, \theta_2)$ and $(v_{3'}, \theta_{3'}) = (v_3, \theta_3)$ in Figure 2.2c. Finally, copied nodes i and node copies j couples can be listed in form of tuples (i, j) in $\mathcal{N}^c$. Note that $\mathcal{N}^{\xi'}$ and $\mathcal{N}^{\xi^*}$ are complementary sets, so $\mathcal{N}^{\xi^*} \cap \mathcal{N}^{\xi'} = \emptyset$ and $\mathcal{N}^\xi = \mathcal{N}^{\xi^*} \oplus \mathcal{N}^{\xi'}$.



(a) Line included in multiple areas

(b) Line as interconnection between areas

(c) Model used for the coordination

Figure 2.2: Types of interconnection between system operators or areas

For a given time step t, the overall multi-area problem can be written

$$\min_{X=(X_\xi)_{\xi\in\Xi}} \quad \sum\nolimits_{\xi\in\Xi} c_\xi^t\left(X_\xi^t\right) \tag{2.46a}$$
$$\text{s.t.} \quad \mathbf{E}^t X^t = 0 \tag{2.46b}$$
$$X_\xi^t \in \mathcal{X}_\xi^t \qquad \xi\in\Xi \tag{2.46c}$$

where area objective functions read

$$c_\xi^t\left(X_\xi^t\right) = \sum\nolimits_{n\in\Omega^\xi} c_n^t\left(p_n^t\right) \tag{2.47}$$

and area electrical network constraints read

$$X_\xi^t \in \mathcal{X}_\xi^t \Leftrightarrow \begin{cases} p_n^{t,\min} \leqslant p_n^t \leqslant p_n^{t,\max} & n\in\Omega^\xi \\ q_n^{t,\min} \leqslant q_n^t \leqslant q_n^{t,\max} & n\in\Omega^\xi \\ v_i^{\xi,\min} \leqslant v_i^{\xi,t} \leqslant v_i^{\xi,\max} & i\in\mathcal{N}^\xi \\ \theta_i^{\xi,\min} \leqslant \theta_i^{\xi,t} \leqslant \theta_i^{\xi,\max} & i\in\mathcal{N}^\xi \\ |\underline{\ell}_{ij}^t| \leqslant S_{ij}^{\xi,\max} & (i,j)\in\mathcal{L}^\xi \\ \underline{\ell}_{ij}^t = \left(v_i^t\angle\theta_i^t\right)\left(\underline{\mathbf{Y}}\right)_{i,j}^*\left(v_j^t\angle-\theta_j^t\right) & (i,j)\in\mathcal{L}^\xi \\ \sum_{n\in\mathcal{N}_i^\xi} p_n^t + \mathsf{j}\, q_n^t = \sum_{j\in\mathcal{N}} \underline{\ell}_{ij}^t & i\in\mathcal{N}^{\xi *} \end{cases} \tag{2.48}$$

for an optimization variable $X_\xi^t = \left((p_n^t, q_n^t)_{n\in\Omega^\xi}, (v_i^t\angle\theta_i^t)_{i\in\mathcal{N}^\xi}, \left(\underline{\ell}_{ij}^t\right)_{(i,j)\in\mathcal{L}^\xi}\right)$. Global exchange matrix $\mathbf{E}^t$ of (2.46) would then be defined such that

$$v_i^t - v_j^t = 0 \qquad (i,j)\in\mathcal{N}^c \tag{2.49a}$$
$$\theta_i^t - \theta_j^t = 0 \qquad (i,j)\in\mathcal{N}^c \tag{2.49b}$$

are satisfied.

Multi-block ADMM

Multi-block ADMM is an important extension of the classical ADMM of [53] which is largely used in big data optimizations. Multi-block optimization problems such as in [61–63] takes the form of

$$\min_{(X_1,\dots,X_\phi)} \quad \sum\nolimits_{n=1}^{\phi} \varphi_n(X_n) \tag{2.50a}$$
$$\text{s.t.} \quad \sum\nolimits_{n=1}^{\phi} \mathbf{A}_n X_n = B \tag{2.50b}$$
$$X_n \in \mathcal{X}_n \qquad n = 1\dots\phi \tag{2.50c}$$

where $\mathbf{A}_n \in \mathbb{R}^{a\times\phi_n}$ are given matrices and $B\in\mathbb{R}^a$ is a given vector.

The two main approaches, namely Jacobian and Gauss-Seidel multi-block ADMMs, are historically used to solve them. But they both suffer from two main drawbacks. The first issue is that they both require the presence of a central entity which updates the Lagrangian multipliers of coupling constraint (2.50b) or, at least, which as to gather all the information needed to test the stopping criterion. This negative effect is even more present in the case of the Jacobian multi-block ADMM since local optimization variable updates are made sequentially, while the Gauss-Seidel multi-block ADMM makes them in a concurrent manner. The second, and non-negligible, drawback is that each algorithm requires the set of coupling matrices matrices $\{\mathbf{A}_1,\dots,\mathbf{A}_\phi\}$ to satisfy certain conditions in order to insure convergence towards the optimal solution. For instance, [61] shows (for $\phi = 3$, so for $\phi \geqslant 3$ by extension) that convergence of the Gauss-Seidel multi-block ADMM is ensured if there exist two integers n and m such that any two matrices of the different sets $\{\mathbf{A}_n, \dots, \mathbf{A}_{n+m}\}$ and $\{\mathbf{A}_{n+m+1}, \dots, \mathbf{A}_\phi, \mathbf{A}_1, \dots, \mathbf{A}_{n-1}\}$ are orthogonal. This condition allows to reorder a subset of agents in the proof of convergence by propagation from the original case of $\phi = 2$ (verified since convergence of [53]'s ADMM is proved) to the $\phi + 1$ case. And, according to [64], convergence of the Jacobian multi-block ADMM is ensured if matrices $\mathbf{A}_n$ are mutually near-orthogonal and have full column-rank. This condition is sufficient as well, but is more restrictive than for the Gauss-Seidel approach since near-orthogonality applies to all matrix couples $\mathbf{A}_n$ and $\mathbf{A}_{m \neq n}$ rather than between two subset partitions of coupling matrices.

However, by reformulating multi-block problem (2.50) into

$$\min_{X=\left(X_1^\intercal, \dots, X_\phi^\intercal, X_{\phi+1}^\intercal\right)^\intercal} \quad \sum\nolimits_{n=1}^{\phi+1} \varphi_n(X_n) \tag{2.51a}$$

$$\text{s.t.} \quad \mathbf{E}X = 0 \tag{2.51b}$$

$$X_n \in \mathcal{X}_n \qquad n = 1 \dots \phi + 1 \tag{2.51c}$$

with agent $\phi + 1$'s optimization variable $X_{\phi+1} \in \mathbb{R}^{\phi_1 + \dots + \phi_\phi}$, objective function $\varphi_{\phi+1}$ null (so $\varphi_{\phi+1} = 0$) and local optimization set

$$\mathcal{X}_{\phi+1} = \left\{ X_{\phi+1} \in \mathbb{R}^{\phi_1 + \dots + \phi_\phi} \middle| \sum\nolimits_{n=1}^{\phi} \mathbf{A}_n (X_{\phi+1})_{\mathcal{G}(n,1\dots\phi_n)} = B \right\} \tag{2.52}$$

where $\mathcal{G}_n = \mathcal{G}(n, 1 \dots \phi_n)$ points to agent n's copies with mapping function $\mathcal{G}$ defined in (2.18). The global exchange matrix $\mathbf{E}$ used to match (2.51) to (2.50) reads

$$\mathbf{E} = \begin{array}{c} \begin{array}{cccccccc} X_1 & X_2 & \cdots & X_m & (X_{\phi+1})_{\mathcal{G}_1} & (X_{\phi+1})_{\mathcal{G}_2} & \cdots & (X_{\phi+1})_{\mathcal{G}_\phi} \end{array} \\ \left(\begin{array}{cccccccc} 1 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & -1 \end{array} \right) \end{array}. \tag{2.53}$$

It is then possible to use the negotiation algorithm of Subsection 2.2.2 to solve multi-block problem (2.50). This method still requires an additional entity to handle multi-block constraint (2.50b) in the form of a projection of vector $X_{\phi+1}$, which copies $\left(X_1^\intercal, \dots, X_\phi^\intercal\right)^\intercal$, on $\mathcal{X}_{\phi+1}$. However, contrary to Jacobian and Gauss-Seidel multi-block ADMMs, there is no condition on coupling matrices $\{\mathbf{A}_1, \dots, \mathbf{A}_\phi\}$ to obtain convergence of the algorithm. Indeed, now, convexity of local functions φ_n and local sets $\mathcal{X}_n$ for each agent $n = 1 \dots \phi$ is the only sufficient condition to guarantee convergence of the negotiation algorithm towards the optimal value.

Note that this approach can be extended to any coupling constraints of the form

$$\mathcal{A}_{\text{eq}}(X_1, \dots, X_\phi) = 0 \tag{2.54a}$$

$$\mathcal{A}_{\text{ineq}}(X_1, \dots, X_\phi) \leqslant 0 \tag{2.54b}$$

where $\mathcal{A}_{\text{eq}}$ and $\mathcal{A}_{\text{ineq}}$ are any functions, which must be convex to insure convergence of the algorithm towards the optimal solution. Moreover, even though it may not be directly useful here, this approach can be extended to the presence of multiple coupling constraints (2.50b) simply by introducing multiple additional agents, null objective functions $\varphi_{\phi+m}$, local optimization set $\mathcal{X}_{\phi+m}$, optimization variable $X_{\phi+m}$ and to append the different global exchange matrices $\mathbf{E}_m$ such as defined above, for as much m as multi-block coupling constraints.

2.3 Equivalent game theory problem: A pure Nash game

This section makes a parallel between the optimization form of generalized coordination formulation (2.15) and the equilibrium approach that would be taken in a game theoretical context. This will especially explain why the presented generalized problem of in Section 2.2 can indeed be considered as a coordination problem and not only a collaboration problem. With an equivalent in game theory, coordination problems such as peer-to-peer markets would not have to suppose that agents are non-strategic. Indeed, the game theoretical properties would induce that it is more beneficial economically for agents to behave truthfully. First Subsection 2.3.1 explains the equivalent equilibrium form that is considered here. Then, Subsection 2.3.2 exposes eventual conditions and implications of such equivalence.

2.3.1 Equivalent game theory problem

Writing generalized coordination problem (2.15) as a game theory problem consists in splitting such that each agent, named player in such case, solely focus on their own local optimization problem. Player n's optimization problem would then read

$$
\begin{aligned}
\min_{X_n} \;\; & \varphi_n(X_n) + \Lambda_n^\mathsf{T} \mathbf{E}_n \begin{bmatrix} X_n \\ 0 \end{bmatrix} && \text{(2.55a)}\\
\text{s.t.} \;\; & X_n \in \mathcal{X}_n && \text{(2.55b)}
\end{aligned}
$$

where local exchange matrix $\mathbf{E}_n$ allows to link local optimization variables of X_n, named actions or decisions in game theory, with peers' exchanged actions, grouped in $X_n^{'}$, through local exchange constraints, as expressed in (2.24) or (2.27b) in presence of slack copy actions W_n. Dual variables Λ_n are the Lagrangian multipliers associated to these constraints and, in a costs optimization context, can be seen as exchange prices. Thus, local objective (2.55a) illustrates the fact that player n as to make a trade off between its own local costs, represented by function φ_n, and the cost of exchanging with outside peers, represented by the inner product term.

Since players can not act on dual variables Λ_n but only on their local actions X_n, there is a need for a bilateral exchange operator. The bilateral exchange operator would aim at matching exchanges by selecting dual variables Λ_n such that mismatches are minimized. Thus, bilateral exchange operator optimization simply consists in

$$
\begin{aligned}
\min_{\Lambda_1,\dots,\Lambda_\phi} \;\; & -\sum\nolimits_{n=1}^{\phi} \Lambda_n^\mathsf{T} \mathbf{E}_n \begin{bmatrix} X_n \\ X_n^{'} \end{bmatrix} && && \text{(2.56a)}\\
\text{s.t.} \;\; & \Lambda_n \in \mathbb{R}^{e_n} && n = 1 \dots \phi && \text{(2.56b)}
\end{aligned}
$$

where e_n is local exchange matrix $\mathbf{E}_n$'s number of rows. Remember that exchanged actions $X_n^{'}$ can be extracted from global actions' list $X = \left(X_1^\mathsf{T}, \dots, X_\phi^\mathsf{T}\right)^\mathsf{T}$ through mapping function $\mathcal{H}$, with $X'_n = \mathcal{H}(n, X)$, which specifically points elements of X which are exchanged with a player n. It can be noted that this problem can not only be solved on a per player basis but also per exchanges (i.e. for each line of $\mathbf{E}_n$) as X_n and $X_n^{'}$ are fixed. In consequence, this computation could be assign to each player on exchanges involving it. This allocated computations would then replace Lagrangian multiplier update (2.35d). However, to truly entrust players, these allocated computations would have to be carried by a sealed, certified software or hardware.

2.3.2 Conditions of equivalence and implications

The Lagrangian of game problem (2.55)–(2.56) reads

$$
L(X, \Lambda) = \sum\nolimits_{n=1}^{\phi} L_n(X_n, \Lambda_n) \qquad \text{(2.57)}
$$

where $\Lambda = \left(\Lambda_1^\top, \ldots, \Lambda_\phi^\top\right)^\top$ is the collection of all dual variables and L_n is the Lagrangian of player n. Player n's Lagrangian is such that

$$L_n(X_n, \Lambda_n) = \tilde{\varphi}_n(X_n) + \Lambda_n^\top \mathbf{E}_n \begin{bmatrix} X_n \\ 0 \end{bmatrix} \tag{2.58}$$

where $\tilde{\varphi}_n$ is the extended-value function of φ_n, in the sense of [54], on the domain defined by $\mathcal{X}_n$. An optimal decision, i.e. an equilibrium, of game theory problem (2.55)–(2.56) must satisfy its Karush - Kuhn – Tucker (KKT) conditions. One can note that KKT conditions of (2.58), reading

$$(X_n)_u : \qquad \frac{\partial \tilde{\varphi}_n}{\partial \left(X_n\right)_u} + \Lambda_n^\top \left(\mathbf{E}_n\right)_{\cdot,u} = 0 \qquad\qquad u = 1, \ldots \phi_n \tag{2.59a}$$

$$(\Lambda_n)_u : \qquad -\left(\mathbf{E}_n\right)_{u,\cdot} \begin{bmatrix} X_n \\ X_n' \end{bmatrix} = 0 \qquad\qquad u = 1, \ldots e_n,\ n = 1 \ldots \phi \tag{2.59b}$$

are identical to the ones of the optimization form of generalized coordination problem (2.15). There are also identical to the ones of the augmented Lagrangian in (2.28)–(2.29) of its modified form with the additional slack global variable W.

As seen in (2.55), it appears that no variables of a player materializes in the problem of another, so game theory problem (2.55)–(2.56) is a pure Nash game, or pure equilibrium problem. Moreover, KKT conditions (2.59) show that the Jacobian of game theory problem (2.55)–(2.56) is symmetric. Thus, according to the principle of symmetry as described in [65], (2.55)–(2.56) is a game theory problem equivalent to coordination problem (2.15). Note that the equivalence does not require any additional conditions than already mentioned, i.e. convexity. Similarly to optimization problems, a Nash game may have a unique or multiple solutions. Such as for convexity of an optimization problem, the easiest way is to check its Jacobian matrix. If it is positive definite, then the game is strongly monotone, and thereby it has a unique solution. Yet, KKT conditions (2.59) are met if and only if extended-value functions $\tilde{\varphi}_n$ are strongly convex, so if objective functions φ_n and optimization sets $\mathcal{X}_n$ are strongly convex.

Being equivalent to the game theory problem exposed in Subsection 2.3.1, agents of the generalized coordination problem in Subsection 2.2.1 have the incentive to be truthful, i.e. to act based on its "true" preferences and not to deviate from them. However, this is obtained by the presence of a bilateral exchange operator as a trusted third party. This trusted party could for example take the form of a centralized trading platform. As mentioned above, the computation carried by this operator can be entrusted at agents' level in the form of a sealed and certified software or hardware. Note that new cryptography technologies such as blockchain could be investigated in the future. Even though solutions of the coordination problem could be obtained with a decentralized algorithm, incentive compatibility can solely be reached by a distributed framework. So in absence of a trusted third party, rationality and non-anticipativity hypotheses must hold for the decentralized algorithm proposed in Subsection 2.2.2 to lead towards the same equilibrium as game theory problem (2.55)–(2.56).

2.4 Synthesis

This chapter first described the context and defined the perimeter in which this thesis applies. The complexity of electricity markets and the power systems on which they operate present many challenges which are still strong open research fields. Power systems are complex on several levels. First, the steady state AC model of the power system is non-linear, due to complex power flows, and has quadratic inequality constraints, due to line flow limits. The resulting optimal power flow problem is then strongly non-convex and NP-hard. Even with simplified models of the network, the scale of real power systems with its dozens of thousands of nodes such as, for example, in Europe. To overcome this structural complexity, control and operation of electrical networks has been assigned to multiple entities dividing the network not only in geographical areas, such as countries or regions, but also in power levels, leading to transmission or distribution system operators. This cut provided smaller, more manageable networks but lead to coordination issues between system operators. Another structural complexity has been increasing in the past years with the constant growth of distributed energy resources, such as renewables, and of pro-active consumers, the so called prosumers. The goal of this thesis is to study how new market frameworks, peer-to-peer markets in particular, can be adapted to the specific constraints of electricity markets.

In a second step, electricity markets and power systems have been assimilated to the more general framework which are coordination problems. A generalized formulation of coordination problems have been proposed in this chapter. In this formulation, agents are supposed to possess its own set of objective and constraints. Then, links between the variables of these agents are seen as bilateral exchanges of information. To respectively correspond to potential or flux information, bilateral exchanges can either be modeled as a consensus of the two variables, i.e. equal in value and sign, or as a reciprocity, i.e. equal in value but with opposite signs. Associated to this, the chapter developed an algorithm which can solve the generalized coordination problem in a decentralized manner. Based on consensus ADMM, the negotiation algorithm handles the problem in an iterative way such that agents concurrently aim at minimizing not only their own objective function but also additional terms compelling them to reach consensus and reciprocity with others. As presented in a practical example, this coordination framework allows to solve large electrical networks with multiple areas or system operators by coordinating them. Moreover, applying bilateral exchanges into bilateral power trades, also called peer-to-peer trades, allow to deal with energy markets in a decentralized way.

However, several issues are still at hand. Indeed, even though the approach proposed by the generalized coordination problem allows to decompose the problem into simpler, less complex ones, this may have been done at the price of an increasing algorithmic complexity with the multiplication of coordinating variables. After a convergence analysis, Chapter 3 proposes different techniques to improve the convergence rate of the negotiation algorithm. On top of that, the practical examples presented in this chapter distinguished the issue of multiple area optimal power flow and the one of decentralized energy markets. This chapter also showed that a combination of both, to obtain a decentralized energy market aware of the power system's limits, is not straightforward. Chapter 4 details the two approaches proposed in this thesis to reach such awareness. Finally, presence of stochastic behaviours such as renewable sources requires not only an energy market but also a reserve capacity market. When handled in a probabilistic way, the global chance constraint which determines the minimum amount of reserves is strongly centralized. Thus the reserve capacity market can not directly be treated in a decentralized way. Chapter 5 presents the approach proposed by this thesis to overcome this obstacle.

3 Computational properties and improvements

After a complexity analysis of the general problem and the associated solving algorithm, this chapter proposes multiple solutions to reduce the problem complexity or to improve the convergence rate of the algorithm. Overall these improvements aim to improve peer-to-peer markets and, thus in a larger way, of decentralized coordination problems applicability in real world implementations.

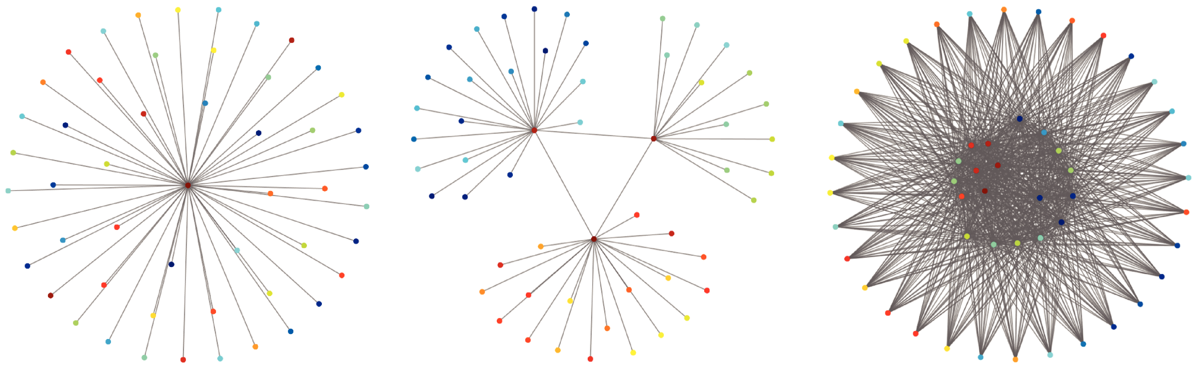
Contents

3.1 Introduction

Resource allocation in electricity markets is traditionally solved with a centralized clearing mechanism, where agents participate in a centralized market. Due to the energy transition [66], power systems are currently undergoing an important change of nature challenging the efficiency of the centralized market organization. Increasing deployment of distributed energy resources, prospects for increasing demand response and distributed storage (residential, electric vehicles, etc.), as well as the rapid progress of sensing and control systems based on information and communication technologies (ICT), enable a profound rethinking of electricity markets. Firstly this growth applies to high-capacity renewable power plants and large storage units. Such facilities are significant enough to be integrated into the operators port-folio and have a significant individual impact on the grid. But beyond these large installations, each consumer is in a position to potentially become a player on the electricity market, via domestic storage or roof-mounted photovoltaic panels. The introduction of these small-sized agents with both the ability to generate and consume energy, the so-called prosumers, pleads for a shift of the electricity markets to a more consumer-centric framework. As of current practice, small-sized prosumers are managed at retail level, since existing mechanisms, such as real time markets for distributed energy sources (DERs) and demand response [67], require thresholds on agents' size and often a strict dichotomy between consumers and producers. Extending these existing mechanisms to small-sized prosumers is not an option, since the amount of communications and data required can quickly become too large to be handled efficiently by a central agent. In order to improve the robustness and performance of electricity networks, it is necessary to involve these distributed actors as part of the management of the network [10]. Nevertheless, this is extremely difficult in the paradigm of centralized electricity markets, as such mechanisms cannot directly connect millions of players.

The aforementioned reasons justify the need for adapting electricity market designs to more decentralized organizations. Decentralized electricity markets were first introduced by Wu and Varaiya as coordinated multilateral transactions [68], now better known as peer-to-peer trades when solely involving two parties. In this framework, each market participant directly negotiates with a set of trading partners with the objective of minimizing their energy procurement costs. In view of large scale applications, regulation and other economic arguments – such as licensing and certification, data and employment regulation – are fundamental but still open topics [69]. Depending on the overall objectives and potential regulation, alternative organizations may be considered. An attempt of categorizing some of the possible organization layouts of decentralized electricity markets is proposed in [10], where additionally to a peer-to-peer market, the authors identify two other market organizations. In the first one, prosumers are connected to microgrids which can either be isolated or interconnected; while in the second one, prosumers are organized in groups, namely energy communities, in which resources, not necessarily geographically located close to each other, are managed in small local centralized markets. Other recent literature proposes peer-to-peer energy-trading markets either to incentivize prosumers to form virtual power plants [70] or for microgrid management [71]. Each organization has been investigated independently and through different market mechanisms. On one hand, peer-to-peer energy trading is proposed in the form of matching contracts [70], consensus-based optimization [72], microgrid management [73] and control systems [74]. On the other hand, community-based mechanisms are designed as control strategies [75], coalition games [76] and distributed optimization [11].

As suggested in Chapter 2, decentralized electricity markets can be conceived as consensus-based decentralized optimization of prosumers' energy procurement on a communication graph. As displayed in Figure 3.1, nodes of the graph represent market participants while edges are placed to connect two agents who can trade with each other. From this point of view, one can interpret a centralized market as a radial decentralized market (a). The market is cleared in a decentralized fashion, where agents do not disclose their assets' information but negotiate with a central agent, i.e. market maker or operator, to minimize their energy costs. In the same



(a) Centralized market (b) Interconnected communities (c) Peer-to-peer

Figure 3.1: Decentralized peer-to-peer electricity market layouts

way, energy communities (b) can be seen as smaller centralized markets, where market makers, or community managers as defined in [11], operate as interfaces with the outside world. Communities can operate in isolated mode, mimicking stand-alone microgrid operations, or connected with other market agents. At the extreme, peer-to-peer layouts (c) can be seen as singleton communities connected to any, or a subset, of market participants. Ultimately, peer-to-peer markets are a generalized form of consumer-centric markets defined by its communication matrix interconnecting agents. The final work of this chapter is to study the effects of the communication matrix, the way agents are organized, on market's optimality. It will notably allow to question whether the design of market structures can be used as improvement speed of consumer-centric market settlements without penalizing their efficiency.

Consumer-centric markets have substantial advantages such as product differentiation, consumer involvement and (potentially) low transaction costs. However, if interactions and negotiation mechanisms are not adequately designed, market outcomes may be clearly sub-optimal if compared to centralized market structures. Existing works in distributed optimization and coordination of actors on power networks, e.g. [47, 77, 78], support the proposal of decentralized and distributed algorithms to clear peer-to-peer and community-based markets. However, for applications of peer-to-peer markets to fully reach their potential, it can be argued that computation and communication complexity issues must be resolved as they represent one of the main threats to robust system operation. In practice, this may originate from a large number of agents involved in transactions, delays in the iterative exchange of information or simply the number of iterations needed by these iterative algorithms to converge to acceptable solutions. Consequently, this chapter starts with an extensive computational analysis of some already proposed distributed and decentralized consumer-centric market structures. Eventually, this analysis allows to draw conclusions on applicability of these approaches to real-world deployment, as well as providing directions for algorithmic and structural improvements.

In lights of this, the chapter then proposes and analyzes several manners to improve applicability of consumer-centric markets. When dealing with actual applications, the assumption of synchronous iterations implies that the time of each iteration is dictated by the slowest agent. Computation delays appear in case of non performing hardware or when the optimization subproblems are complicated to solve, while communication delays are caused by bandwidth limits or internet traffic. The non-negligible likelihood of having significant delays justifies the analysis on how the investigated algorithms behave in case of asynchronous iterations. Moreover, faster prosumers will be able to pursue their local computation and start another iteration and, therefore, may improve the overall convergence rate. In consequence, this chapter studies the resilience of the negotiation mechanisms subject to asynchronous communications and looks at whether the convergence rate is affected by it.

Employing a peer-to-peer market framework could yield a number of advantages, for instance thanks to product preference, allowing for a wealth of new business models. Nevertheless, consumer-centric and, in particular, peer-to-peer markets require a very large amount of information to be exchanged, much greater than that required for a centralized market [79]. In a real-time context, such exchanges run the risk of not having enough time to succeed if the deadline is reached before the end of the negotiation process. In addition, information is potentially expensive when it has to be exchanged rapidly in very large volumes. This implies a risk of overloading existing infrastructures and the necessity to develop specific protocols and channels [12]. These information costs – that are inherent to these markets – lead to the question of their value within the clearing and the potential trade-offs if they are to be reduced. Thus, the following stage of this chapter focuses on testing alternative stopping criterion allowing agents to decide when to conclude negotiations with their partners rather than having to pursue until every trades have reached agreement. Applied on peer-to-peer markets, this algorithmic improvement potentially depletes the number of exchanged messages required to obtain the optimal bilateral trades.

In consequence, the chapter proceeds as follows. First, Section 3.2 makes a complexity analysis of two consumer-centric markets, namely community-based and peer-to-peer. These two market frameworks will be associated to different solving algorithms to evaluate whether the use of consensus *alternating direction method of multipliers* (ADMM) in the previous chapter was a relevant choice as suggested in the literature. The goal being to compare them without being influenced on particularities of a given test case, Section 3.2 also introduces randomly generated setups to avoid specific configurations. Based on the same setups, Section 3.3 tests the resilience of the two consumer-centric market and their associated solving algorithms to computation and communication delays. This section also tests the benefits of using asynchronous communications in such conditions. Choosing the ADMM solving algorithm which showed more reliable in the two first sections, Section 3.4 builds upon this and proposes alternative stopping criteria and analyzes whether they allow to reduce the number of messages required to solve peer-to-peer markets without endangering market's power balance or optimality. Finally, Section 3.5 performs a sparsity analysis on peer-to-peer markets' communication graph before Section 3.6 concludes on the computation properties and proposed improvements of consumer-centric and, especially, peer-to-peer markets.

3.2 Complexity analysis

The emphasis is placed on two alternative paradigms represented by a community-based market and by a full peer-to-peer framework. In a Community-based Economic Dispatch (CED), all prosumers communicate with a supervisory node that coordinates the process to optimality in a distributed manner [11]. In case of a Multi-Bilateral Economic Dispatch (MBED), fully decentralized peer-to-peer trades among all participants are obtained without needing third-party supervision [52]. Following [80], a third market structure, called Power Consensus Multi-Bilateral Economic Dispatch (PCMBED), considers a full peer-to-peer negotiation process handled in a distributed fashion by means of a virtual supervisory node.

Consequently, a computational analysis of these consumer-centric market structures is conducted in this section using multiple-core simulations. First Subsection 3.2.1 describes formulations and algorithms of the evaluated consumer-centric markets. After a description in Subsection 3.2.2 on how test cases are generated, a convergence analysis is carried out in Subsection 3.2.3 to assess the trade-off between convergence speed and accuracy. Scaling properties are investigated in Subsection 3.2.4 as a function of the number of prosumers involved, considering both computational and communication burden. Finally, Subsection 3.2.5 gathers conclusions of this complexity analysis.

3.2.1 Evaluated market organisations

To make the consumer-centric market mechanisms comparable, for a given set of prosumers, all three market structures are based on total cost minimization, where each prosumer is either a consumer or a producer. All proposed structures aim at solving the economic dispatch problem of a local community that is assumed to be autonomous (no interaction with the system operator or grid services provided). In the case of a community-based market, a single price system is considered where prosumers, supposed rational and non-strategic, do not express individual preferences. In contrast in a peer-to-peer setup, the power balance on each trade yields differentiated electricity prices.

Community-based market

Since focusing on the computational properties only, we adopt a simplified version of the distributed CED, proposed in [11]. Community Ω's objective is to minimize the sum of the costs c_n of its prosumers $n \in \Omega$. The problem can be formulated as

$$\min_{(p_n)_{n\in\Omega}} \quad \sum\nolimits_{n\in\Omega} c_n(p_n) \tag{3.1a}$$
$$\text{s.t.} \quad \sum\nolimits_{n\in\Omega} p_n = 0 \tag{3.1b}$$
$$p_n^{\min} \leq p_n \leq p_n^{\max} \qquad n \in \Omega \tag{3.1c}$$

where the power set-point p_n of prosumer n (negative when consuming) range within a lower $p_n^{\min}$ and an upper $p_n^{\max}$ boundary, and (3.1b) grants Ω's power balance. Since the objective function and power boundaries are separable among prosumers, the market is cleared by means of a distributed optimization algorithm, i.e. [53]'s ADMM. A supervisory virtual prosumer, the so-called community manager, coordinates the negotiation process as in an optimal exchange problem. The solving iterative procedure is summarized by

$$p_n^{k+1} = \operatorname*{argmin}_{p_n} \left(f_n(p_n) + y^k \Delta^k + \frac{\rho}{2} \left\| \Delta^k - p_n^k + p_n \right\|_2^2 \right) \qquad n \in \Omega \tag{3.2a}$$
$$\Delta^{k+1} = \sum\nolimits_{n=1}^{n} p_n^{k+1} \tag{3.2b}$$
$$y^{k+1} = y^k + \rho\, \Delta^{k+1} \tag{3.2c}$$

where Δ^k represents the power balance residual constraint and y^k the electricity price at iteration k, both being computed by the central prosumer. Note that $\rho > 0$ denotes ADMM's penalty factor. The penalty factor used in this section and the next is $\rho = 1$.

Decentralized peer-to-peer based market

The formulation of the decentralized peer-to-peer market extends (3.1), as the power set-points p_n of each prosumer n are defined as the sum of the power p_{nm} bilaterally traded with a set of trading partners $m \in \omega_n$. The MBED problem reads as

$$\min_{(p_n,(p_{nm})_{m\in\omega_n})_{n\in\Omega}} \quad \sum\nolimits_{n\in\Omega} f_n(p_n) + \sum\nolimits_{m\in\omega_n} \gamma_{nm} p_{nm} \tag{3.3a}$$
$$\text{s.t.} \quad p_{nm} + p_{mn} = 0 \qquad n \in \Omega,\ m \in \omega_j \tag{3.3b}$$
$$p_n = \sum\nolimits_{m\in\omega_n} p_{nm} \qquad n \in \Omega \tag{3.3c}$$
$$p_n^{\min} \leq p_n \leq p_n^{\max} \qquad n \in \Omega \tag{3.3d}$$
$$p_{nm} \geq 0 \qquad n \in \Omega_p,\ m \in \omega_j \tag{3.3e}$$
$$p_{nm} \leq 0 \qquad n \in \Omega_c,\ m \in \omega_j \tag{3.3f}$$

where prosumers apply specific preferences on their trades with the use of product differentiation coefficient γ_{nm}, as in [52]. However, it may be noted that the problem resulting of (3.3a)–(3.3d) would be convex but not strictly convex. This comes from the fact that multiple combinations of bilateral trades $(p_{nm})_{m\in\omega_n}$ may be found to obtain the same power set-point p_n. In consequence, a strict convexification technique has to be found. The sign constraints on power trades (3.3e)-(3.3f) serve this purpose as they force producers Ω_p and consumers Ω_c respectively to only sell and buy energy. The downside of this is that it may shift the resulting optimal solution if it involves arbitrage of one prosumer. The trading reciprocity constraint (3.3b) imposes reciprocity of bilateral trades and allows for product differentiation, reflected by the price of each and every trade.

A *relaxed consensus and innovation* (RCI) method can be used to solve the optimization problem under the assumption that the cost functions (c_n) have a bijective gradient of inverse $(c_n^{'-1})$. Even if *consensus and innovation* (C+I) methods are slower to converge than direct methods, they present lighter computation and a higher algorithmic flexibility. The iterative process, for a producer, reads

$$y_{nm}^{k+1} = y_{nm}^k - \beta^k\left(y_{nm}^k - y_{mn}^k\right) - \alpha^k\left(p_{nm}^k + p_{mn}^k\right) \tag{3.4a}$$
$$\overline{\mu_n}^{k+1} = \left|\overline{\mu_n}^k + \eta^k\left(p_n^k - p_n^{\max}\right)\right|^+ \tag{3.4b}$$
$$\underline{\mu_n}^{k+1} = \left|\underline{\mu_n}^k + \eta^k\left(p_n^{\min} - p_n^k\right)\right|^+ \tag{3.4c}$$
$$g_{nm}^k = \frac{|p_{nm}| + \delta^k}{\sum_{l\in\omega_n}\left(|p_{nl}| + \delta^k\right)} \tag{3.4d}$$
$$p_{nm}^{k+1} = \left|p_{nm}^k + g_{nm}^k\left(c_n^{'-1}\left(y_{nm}^{k+1} - \gamma_{nm} - \overline{\mu_n}^{k+1} + \underline{\mu_n}^{k+1}\right) - p_n^k\right)\right|^+ \tag{3.4e}$$

where tuning parameters $\{\alpha^k, \beta^k, \eta^k, \delta^k\}$ are persistent sequences. Variables y_{nm}^k, $\underline{\mu_n}^k$ and $\overline{\mu_n}^k$ are the dual variables of trading reciprocity constraints and power boundary constraints, respectively. And variables g_{nm}^k are asymptotically proportional factors. Operator $|\ .\ |^+$ is the positive part operator (to be replaced by the negative part for consumers). The RCI implementation defines a fully decentralized negotiation process, where all calculations are made locally by each prosumer. The persistent sequences used in this section and the next are taken as defined in [52] and read

$$\alpha^k = \frac{0.01}{k^{0.01}} \qquad \beta^k = \frac{0.1}{k^{0.1}} \qquad \eta^k = 0.005 \qquad \delta^k = 1. \tag{3.5}$$

Distributed peer-to-peer based market

A distributed implementation of (3.3) is formulated through the PCMBED, proposed in [80]. In this formulation, prosumers focus on reaching consensus on their local trades p_{nm} by means of a global variable z_{nm}. The RCI method (3.4) is adjusted as (here for a producer)

$$z_{nm}^{k+1} = \left(p_{nm}^k - p_{mn}^k\right)/2 \tag{3.6a}$$
$$y_{nm}^{k+1} = y_{nm}^k - \beta^k\left(y_{nm}^k - y_{mn}^k\right) - \alpha^k\left(p_{nm}^k - z_{nm}^{k+1}\right) \tag{3.6b}$$
$$\overline{\mu_n}^{k+1} = \left|\overline{\mu_n}^k + \eta^k\left(z_n^{k+1} - p_n^{\max}\right)\right|^+ \tag{3.6c}$$
$$\underline{\mu_n}^{k+1} = \left|\underline{\mu_n}^k + \eta^k\left(p_n^{\min} - z_n^{k+1}\right)\right|^+ \tag{3.6d}$$
$$g_{nm}^k = \frac{|p_{nm}| + \delta^k}{\sum_{l\in\omega_n}\left(|p_{nl}| + \delta^k\right)} \tag{3.6e}$$
$$p_{nm}^{k+1} = \left|z_{nm}^{k+1} + g_{nm}^k\left(c_n^{'-1}\left(y_{nm}^{k+1} - \gamma_{nm} - \overline{\mu_n}^{k+1} + \underline{\mu_n}^{k+1}\right) - z_n^{k+1}\right)\right|^+ \tag{3.6f}$$

The z and y updates (3.6a)-(3.6b) are operated by the central prosumer while the others are computed locally by each prosumer. Implementing this PCMBED approach allows analyzing the benefits of a distributed implementation of a peer-to-peer market compared to a decentralized framework, as well as the impact of the intermediary z-update on the RCI process.

3.2.2 Test cases of reference

This subsection describes the simulation setups and computing framework to showcase the algorithms' properties.

Test case generation

To avoid possible dependencies on contingent combination of assets, simulations are performed using a sample of ten randomly generated setups. The samples are drawn from uniform distribution and in such way that extreme cases are avoided. In particular, the presence of flat utility curves and preponderant prosumers are eluded. At first given the fixed number of prosumers, the number of producers and consumers are sampled randomly such that there is at least a third of each type. In addition, variations of prices and power set points range are controlled in order to have a resilient tuning. The total consumption and production are sampled randomly within a range that is proportional to the number of prosumers and split randomly into the individual capacity of each prosumer. Following a common assumption in literature, the utility curves are assumed quadratic while built according to a price range of flexibility that is sampled randomly. The product differentiation in the peer-to-peer structures is here expressed as a preference for local consumption, with trading costs that are proportional to the euclidean distance between two prosumers calculated from their randomly generated positions on a two dimensional map.

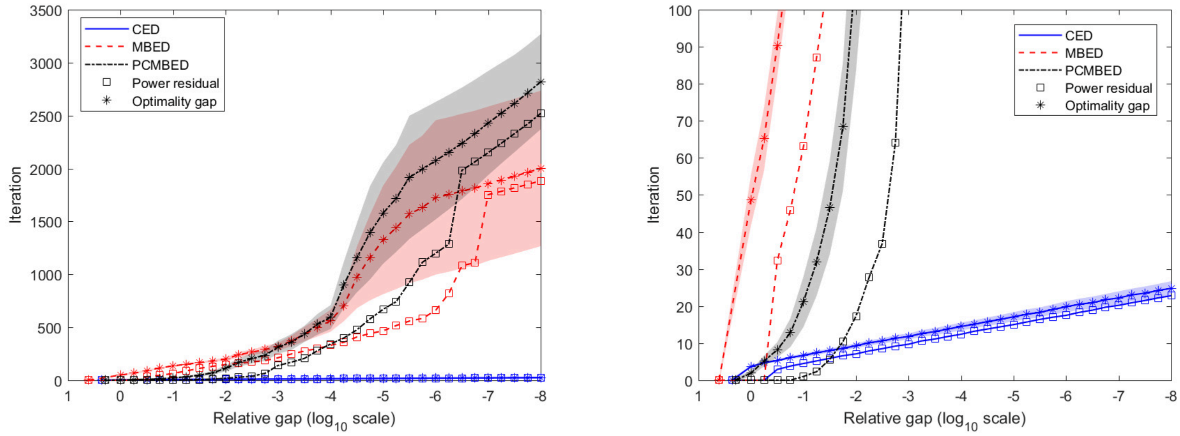
Computing infrastructure

Simulations are executed on a High-Performance Computing machine located at DTU of 2500 cores in total. For this work only 912 cores are accessible, divided in 38 nodes and connected by 10GB Ethernet cables. Each node is equipped with two Intel Xeon Processor 2650v4 (12 core, 2.20GHz) and 256 GB RAM and 480 GB-SSD disk. To study a more realistic application, we assign each prosumer of the community to a core of the HPC and we design a communication system through a message passing interface.

Using a parallel structure for the simulations allows to better describe prosumers' actual computational efforts as well as their interactions. By designing a message passing interface, communication architectures are modeled to match the three different negotiation processes [81]. For instance, in the CED and PCMBED a master core takes care of the collective computations and communications (master-to-prosumer and prosumer-to-master). On the other hand, for the MBED approach all calculations are done by the market participants and the communications are point-to-point. In section Subsection 3.2.3 and Subsection 3.2.4, synchronous communications are implemented through the message passing interface with blocking functions.

3.2.3 Convergence analysis

First, emphasis is placed on convergence properties. The fact that peer-to-peer markets reflect individual preferences, contrary to community-based structure, impacts negotiation mechanisms as well as computational efficiency. The convergence of each algorithm is benchmarked against centralized implementation formulated as the optimization problems in (3.1) and (3.3). Convergence for groups of 25 prosumers is considered to verify whether and how the investigated algorithms achieve optimality. Simulations' results are only expressed in terms of itera-



(a) Focus on MBED and PCMBED evolution range (b) Focus on CED evolution range

Figure 3.2: Number of iterations required to reach different levels of accuracy for the proposed algorithms

tions since synchronous communications are used for these simulations. The results discussed in this section will not depend on the hardware employed.

The average number of iterations required for the different algorithms to reach a given optimality gap is depicted in Figure 3.2 as well as the power residual used as a description of the feasibility of the solution. Additionally, the surface represents the mean absolute error at each optimality gap. Given the low complexity of the test case, a straightforward implementation of the ADMM is used for the solution of the CED. Even with this approach, the CED reaches small optimality gaps considerably faster than the peer-to-peer market frameworks. The use of the z-update (3.6a) speeds up the initial convergence and decreases the power residual for the PCMBED compared to the MBED. However, the PCMBED appears to be less efficient to reach low optimality gaps. For all algorithms, the optimality gap and of the power residual show similar patterns. This justifies for the rest of the section, that the optimality gap can be used to describe both the convergence of the algorithm and the feasibility of the solution found.

While the CED shows on average a linear convergence rate with a mean relative error of maximum 16%, both algorithms for peer-to-peer negotiation display a change in the convergence rate when the optimality gap is below 10^{-4}. This behaviour is mainly caused by few simulations for which the algorithms are much slower to reach small optimality gaps. Indeed, for the MBED the maximum detected mean relative error for optimality gaps above 10^{-3} is below 25%, while it increases to around 80% for optimality gaps below 10^{-4}. The PCMBED shows a similar behaviour with two simulations that push up the average number of iterations. However, the mean relative error is consistently between 30% and 60% which implies a more constant dependency of the convergence speed on the setup. The convergence patterns of the peer-to-peer markets show that the tuning parameters are not able to cope with all setups.

3.2.4 Scaling analysis

Intuitively, prosumer-centric markets such as those described here will be challenged by increasingly large numbers of participants. In this subsection, the ability of the proposed approaches to scale up their negotiation mechanisms is analyzed by investigating the time complexity of the implemented algorithms [82]. For this reason, each market framework is simulated on different community sizes – from 25 to 300 prosumers. Overall, the expected theoretical scaling trends can be well correlated with the simulations within the given range. By assigning each prosumer to a parallel thread, the maximum number of prosumers is limited but communication processes are introduced in the performance assessment. Time complexity T^a of an algorithm $a \in \{\text{CED, MBED, PCMBED}\}$ can be split as

$$T^a(N_\Omega) = t^a_{alg}(N_\Omega) t^a_{str}(N_\Omega). \tag{3.7}$$

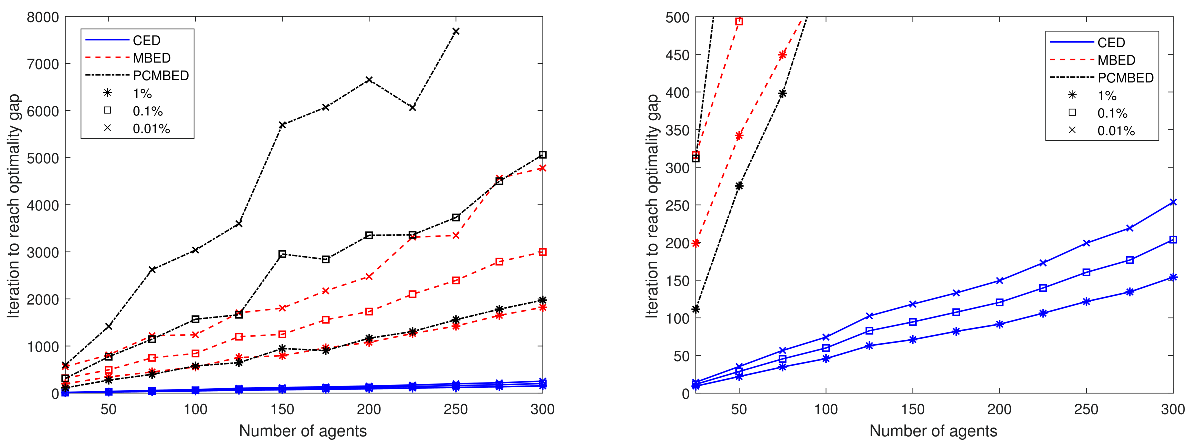
respectively depending on algorithmic times structural complexities. First, algorithmic complexity t^a_{alg} expresses the number of iterations required for the algorithm to converge. Thus, this term depends only on the algorithm implemented (in this case ADMM or RCI) but not on the structure of the implementation (distributed or decentralized). Second, structural complexity t^a_{str} expresses the average time required to compute an iteration. It can be noted that both algorithmic and structural complexities vary with the number of prosumers N_Ω, i.e. the size of the test case. For the investigated market frameworks, expressions of algorithmic and structural complexities are proposed here as function of the number of prosumers (expressed by means of operator $\sim$) and verified empirically.

Algorithmic complexity

As it is difficult to implement a comparable stopping criterion for different algorithms, the number of iterations for which each algorithm is above a certain optimality gap while increasing the number of prosumers is investigated and reported in Figure 3.3. While the CED and the MBED have a low spread between the different optimality gaps, the PCMBED is found more unstable when it comes to higher accuracy of the solution. Even if the benefit of a faster power consensus seems to fade for the PCMBED as the size of the setup increases, a linear algorithmic complexity can be extrapolated for all algorithms $t^a_{alg} \sim O(N_\Omega)$ (R^2 above 0.95).

Structural complexity

The algorithms' structural complexity is assessed through the average time to complete an iteration in a synchronous handling of the communication. As this analysis depends mostly on the structure of the implementation, the results of the PCMBED can also be transferred to a decentralized implementation as the MBED (i.e. without the z-update). The results, displayed in Figure 3.4, report a linear trend for all algorithms (R^2 between 0.97 and 0.995). However, in order to transcend from the hardware employed for these simulations and to provide a more general interpretation, a theoretical analysis of the structural complexity is carried out for each algorithm.



(a) Focus on MBED and PCMBED evolution range

(b) Focus on CED evolution range

Figure 3.3: Evolution of the number of iterations required to reach optimality gaps of 10^{-2}, 10^{-3} and 10^{-4} over the number of prosumers.

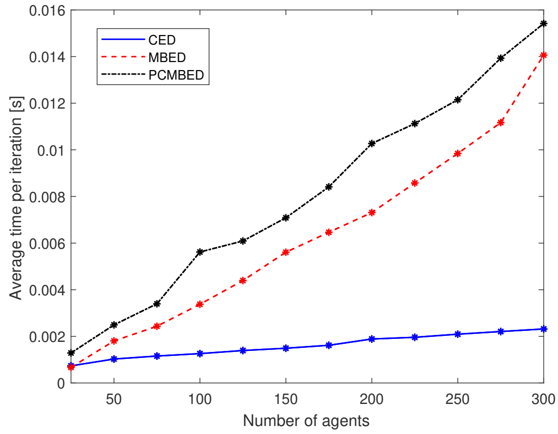


Figure 3.4: Impact of scale on the average time per iteration

The structural complexity is split into computation time and communication time. Under the assumption that the time to communicate a message of size S can be expressed through a linear function $h^a_{com}(S)$, the structural complexity becomes

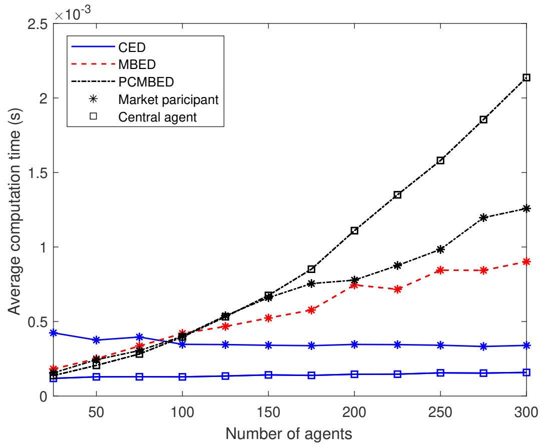
$$t^a_{str} = \delta^a_{com}(N_\Omega)h^a_{com}(S^a(N_\Omega)) + \delta^a_{comp}(N_\Omega)\Delta t_{comp} \tag{3.8}$$

where δ^a_{com} and δ^a_{comp} are respectively the number of communications and computations needed for algorithm a and Δt_{comp} is the time it takes to complete one operation. In the distributed structures, different complexity can apply for the central prosumers and for the market participants. However, as synchronous communication are used, the maximum of the two defines the general complexity. From the structure of the algorithms their complexity can be extrapolated, as reported in Table 3.1. It is important to notice that for peer-to-peer algorithms, all prosumers complexity (not the central prosumer in the case of the PCMBED) depend on the number of trading partners $N_{\omega_n} = \mid \omega_n \mid$ and not on the actual size N_Ω of the setup. Even if in these setups the trading partners are comparable to the total number of prosumers $N_{\omega_n} \sim N_\Omega$. Note that one could reduce the algorithmic complexity by limiting the number of trading partners per prosumer. Section 3.5 develops on this and points that cutting the number of partnerships may degrade optimality when not done wisely.

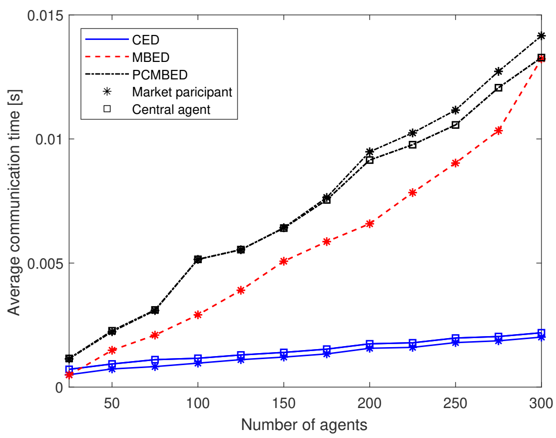
A difference appears between the expected structural complexity of the PCMBED (quadratic) and the empirical results (linear). However, when looking separately at the results for computation and communication time, respectively the average time that each participant takes to compute the local optimization, in Figure 3.5a, and to transmit its messages, in Figure 3.5b, the expected results are verified. The computation time of the central prosumer in the PCMBED

Table 3.1: Structural complexity of the different algorithms.

Model	CED		MBED	PCMBED	
Prosumer type	Central	Other	Any	Central	Other
$\delta^a_{com}(N_\Omega)$	O(N$_\Omega$)	O(1)	O(N_{ω_n})	O(N$_\Omega$)	O(1)
$S^a(N_\Omega)$	O(1)	O(1)	O(1)	O(N_{ω_n})	O(N_{ω_n})
$\delta^a_{comp}(N_\Omega)$	O(N$_\Omega$)	O(1)	O(N_{ω_n})	O($N_\Omega N_{\omega_n}$)	O(N_{ω_n})
t^a_{str}	O(N_Ω)		O(N_{ω_n})	O($N_\Omega N_{\omega_n}$)	



(a)



(b)

Figure 3.5: Evolution of average computation (a) and communication (b) time over the number of prosumers.

shows the expected quadratic increase, but as the values are smaller compared to the communication ones (even in the specific feature of the HPC implementation) the quadratic trend is not perceived in the total complexity. Only some small differences are noted: for instance, the synchronous communications give the exact same trend for all prosumers (both central and non) in the CED and the PCMBED. The communication time is only slightly dependent on the size of the data transmitted ($h^a_{com}(S) \sim O(1)$) thanks to the communication hardware used for the HPC. However, this might not be the case in practice, where different communication infrastructures can lead to more variable time to transmit messages of different sizes.

These results show the importance of the characteristics used for the implemented architecture. An efficient handling of the central prosumer in the distributed cases reduces the structural complexity, as it does for the CED and PCMBED in the exposed simulations. As for the communication framework, the distributed structure for instance could benefit from an efficient handling of large communications, while decentralized algorithms require more sparse and reliable communication framework to operate efficiently.

3.2.5 Conclusions

With the new usages of electricity, the classical centralized pool market is bound to be replaced by more consumer-centric market structures which can be named prosumer markets. Before reaching real world implementations, these prosumer markets have to overcome many challenges. Scalability can be pointed as the main issue due to the constant increase of distributed energy resources, local storage units, energy management systems, etc. This section assessed computational properties of three prosumer market configurations. The first considered a community-based framework solved in a distributed manner with ADMM. The two last considered a peer-to-peer market structure either solved in a decentralized or a distributed manner based on RCI. Their computation and communication complexities have been analyzed theoretically and verified via simulations by means of parallel programming.

As expected, the community-based approach was found faster and more scalable than the two peer-to-peer configurations. Yet, peer-to-peer markets are the only framework allowing for product differentiation. So improving them may be of interest such as to open for new business opportunities. Notably, the complexity analysis of this section identified sparsity of the communication matrix, defining prosumers' trading partners, as a way to improve scalability. This point is further developed later in Section 3.5. Moreover, this section highlighted that ADMM seemed more adequate to clear prosumer markets than the RCI. Before finally discrediting the

RCI in favor of ADMM, there is still a need to assess their robustness to computation and communication delays. Indeed, the complexity analysis showed that the messages exchanged over the communication infrastructure grows with the number of prosumers. Added with a diversity of computation units among prosumers, communication and computation delays are therefore very likely to occur. In consequence, the prosumer market clearings must be resilient to them. This aspect is studied in Section 3.3. Another way to improve scalability of prosumer markets would be to explore on better stopping criteria which are currently global. As a matter of fact, the relevance of considering them at a global level is questionable, in particular for peer-to-peer markets. To this end, Section 3.4 studies the use of several alternative stopping criteria in the case of a peer-to-peer market.

3.3 Resilience to asynchronous communications

As presented in Subsection 3.2.4, both computation and communication complexity impact on the average time per iteration. When dealing with actual applications, the assumption of synchronous communications implies that the time of each iteration is dictated by the slowest prosumer. Computation delays appear in case of non performing hardware or when sub-problems' optimization are hard to solve, while communication delays are caused by bandwidth limits or internet traffic. The non-negligible likelihood of having significant delays justifies the need of an analysis on how the algorithms investigated in Section 3.2 behave in case of asynchronous communications. First, Subsection 3.3.1 describes the methodology used for the analysis. While Subsection 3.3.2 presents simulation results in a second step, Subsection 3.3.3 concludes on the resilience to asynchronous communications.

3.3.1 Methodology

For the sake of simplicity, both computation and communication delays are modeled in order to control their disparities. Carried on the same test cases and HPC machine as described in Subsection 3.2.2, computation heterogeneity is accounted for by assigning different computation time to each prosumer. Computation time of the central prosumer is fixed to $\tau_C = 0.01$ seconds and computation time of each other prosumer is sampled with the following uniform distribution $\tau_i = \tau_C + U(-\frac{\epsilon}{2}, -\frac{\epsilon}{2})$. Using the uniform distribution models large diversity of local problem complexities and computation unit performances. The amplitude of $\epsilon \in [0, \tau_C]$ varies to investigate the resilience of the algorithms to an increasing diversity. The modelling of different hardware computing power is assumed by forcing the computation time of each prosumer with a sleep command. Note that can also model different complexity of the prosumers' routine. For this reason, for each simulation the sampled computation time is kept fixed, representing systematic delays in the negotiation mechanism.

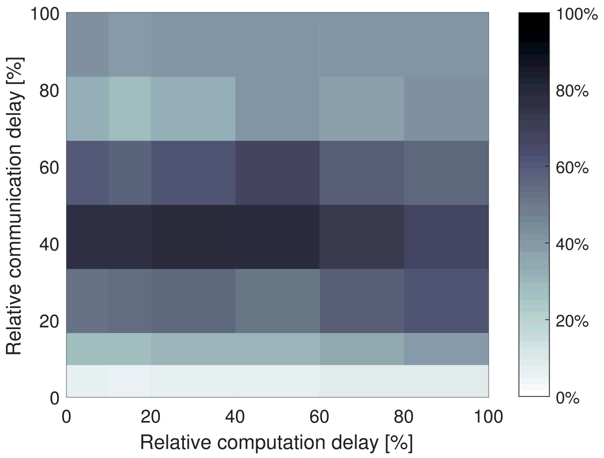
Communication delays are modeled as random variables $\tilde{X}$, following an exponential distribution $\lambda e^{-\lambda x}$, as proposed in [83]. Since accounting for internet traffic and bandwidth limitations, a new delay is sampled for each communication instance. By employing non blocking communication instances in the message passing interface, the prosumers can proceed with their optimization routine even if their communication is not finalized. To investigate the robustness towards different sizes of communication delays (simulating weaker and stronger networks), the expected value of the exponential distribution varies $\mathbb{E}[\tilde{X}] = \frac{1}{\lambda} \in [0, \tau_C]$.

In case of distributed or decentralized systems affected by computation or communication delays, each prosumer can receive multiple information (e.g. of price and power set point) at each iteration. In order to manage these multiple updates, three different strategies are commonly implemented in literature. A first attempt considers only the most recent information received. As not only the time stamp are communicated but also the number of iteration of the sending prosumer, it is possible to identify the last updated variables. However, this strategy does not exploit all the available information. In order to take into account all updates, each prosumer can average all the information received at each iteration. Finally, a compromise between these two approaches by implementing an exponential weighed average over the information received is also investigated. In that case, the most recent values have a larger impact, but all the information is taken into consideration.

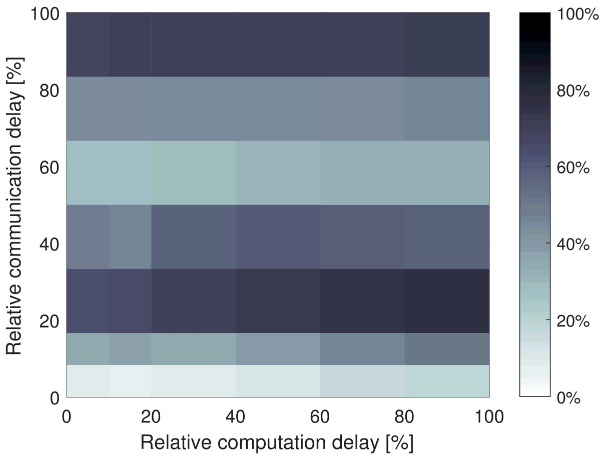
3.3.2 Simulation results

When simulating how the investigated algorithms respond to asynchronous updates, the peer-to-peer approaches are found to be unstable. Both communication and computation delays lead to oscillations of the negotiation process, especially when the bilateral trades have to be finely settled. Further investigation on efficient consensus algorithms in perspective with existing literature (e.g. [84]) are required to increase resilience of the RCI algorithm towards asynchronous behaviours. On the other hand, the CED is found very resilient to both computation and communication delays. The distributed structure of ADMM together with a lower number of variables allow the negotiation process to converge to optimality also when exposed to a highly asynchronous functioning.

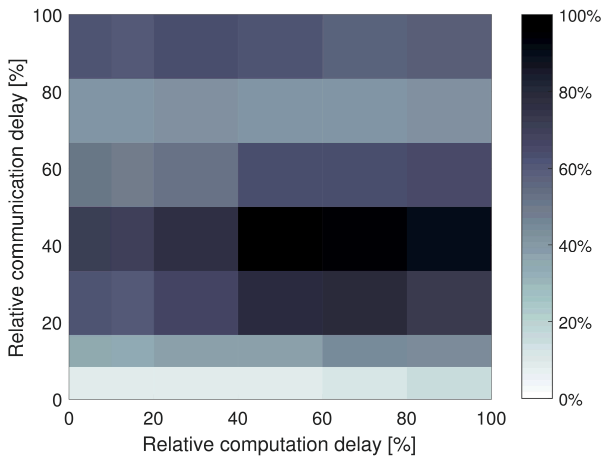
The performance of the CED are reported in Figure 3.6, presented as the relative time increase to reach convergence due to random computation and communication delays of amplitude given



(a) Last received information



(b) Mean of received information



(c) Exponential average of received information

Figure 3.6: Relative time increases of the three tested strategies to reach a 0.01% optimality gap for different levels of computation and communication delays.

in the axis. They show that the negotiation mechanisms are generally robust towards asynchronous information updates. The time to reach 0.01% of optimality gap is at maximum doubled if compared to a synchronous system. On one hand the heterogeneity of computation time impacts the algorithmic performance linearly and with small increases, on the other hand communication delays have a more complicated influence. Depending on the strategy used to handle multiple information, the results show that in some cases higher expected values of communication delays speed up convergence. This behaviour addresses a well-known issue of ADMM exchange algorithm for non-orthogonal multi-block problems [85]. In case of communication delays, the impact of unstable equilibrium is smoothed as prosumers change their optimal set-points at different rates.

Over all simulations, the strategy that leads to the lowest relative time increase (average 41.1% and standard deviation of 19.3%) is to consider only the most recent information. Using the average of the information received only slows down the process (average time increase of 55.6% and standard deviation of 19.5%). Employing an exponential weighted average on the multiple updates leads to a behaviour in between the two other strategies (average time increase of 50.6% and standard deviation of 19.9%). However, in case of less smooth convergence, this strategy can allow for a good trade-off between filtering the noise of oscillating phenomena and speed of convergence. Further work on an adaptive tuning of the exponential weights employed would be needed to achieve robust performances in case of more complicated negotiation mechanisms.

3.3.3 Conclusion

Distributed or decentralized negotiation algorithms of future consumer-centric markets will increase the number of exchanges. As a consequence, the traffic over communication network infrastructures would intensify which may in turn produce more communication delays. As exposed in Section 3.2, the increasing number of prosumers participating in such markets would not only challenge communication networks, but also prosumers' local computation units. Prosumers would deal with local optimization problems of variable complexities which would, thus, induce computation delays. This section conducted a first order analysis on the resilience of the proposed negotiation algorithms when facing communication and computation delays with asynchronous communications. It has been shown that the RCI algorithm is much less robust to such delays than the consensus ADMM based solving algorithm. This confirms the use of the ADMM based negotiation algorithm proposed in Chapter 2 to solve generalized coordination problems such as peer-to-peer electricity markets. Note that before its final publication in [2], [80] took Sections 3.2 and 3.3's conclusions, published in [1], into account and changes from RCI to consensus ADMM for its negotiation algorithm. From this point forward only the ADMM based algorithm will remain in the rest of the manuscript.

More than testifying to ADMM's resilience to communication and computation delays, the analysis of this section identified that asynchronous communications could also speed up the convergence in some cases. This observation opens the opportunity to study the conditions and improvement which can be brought to the decentralized peer-to-peer negotiation mechanism presented in Chapter 2 and largely use in the rest of the manuscript. Even though solely performed on a simplified communication network model for now, the use of more realistic delay characteristics or models would notably help show whether asynchronous communications will benefit the development of consumer-centric markets such as peer-to-peer markets.

3.4 Alternative stopping criteria

Consumer-centric markets require much more intensive information exchanges then classical centralized markets [79]. Yet Section 3.2 showed that would come at the disadvantage of involving larger communication delays for the peer-to-peer market structure in particular. In a real-time context, such exchanges would run the risk of not having enough time to succeed if the deadline is reached before the end of the negotiation process. Moreover, information is potentially expensive when it has to be exchanged rapidly in very large volumes. This implies a risk of overloading existing infrastructures and the necessity to develop specific protocols and channels [12]. These information costs – that are inherent to consumer-centric markets – lead to the question of their value within the clearing and the potential trade-offs if they are to be reduced. In the peer-to-peer market structure prosumers exchange information with each trading partners which puts more pressure on the communication infrastructure than the community-based structure. Moreover, as pointed in Section 3.1, the peer-to-peer market structure is able to capture the form of most consumer-centric market layouts solely by adapting its communication matrix. For both these reasons, this section as well as the rest of the manuscript focuses on the peer-to-peer market structure.

Within a peer-to-peer energy market, information exchanges make it possible to solve a constrained optimization problem. The objective is then to achieve maximum social welfare for the people involved while respecting the operating constraints of the electricity grid. On one hand, the costs associated with the exchange of information compete with the satisfaction of the various market prosumers. It is up to each prosumer to decide if he can have a profit from looking for a better trade. On the other hand, a compromise with the network's operating constraints appears. Indeed, exchanging too little information means that the constraints of the optimization problem may not be precisely respected. When the decided dispatch is enforced, this will imply non-compliance with grid codes, voltage values, overloads [86] or imbalances that will have to be compensated. This leads to increased stress on the network, a deterioration in energy quality and additional operating costs. In the subsequent stages of this section, only the power equilibrium constraint will be considered. What is the evolution of the imbalance of a peer-to-peer market according to the number of messages exchanged? How does the cost of communications evolve when trying to reduce the balancing cost for the system operator? Is it possible to propose alternative implementations reducing this number of messages?

To answer these questions the rest of the section is organized as follows. First, 3.4.1 recalls the formulation of the studied peer-to-peer market problem and describes the associated decentralized negotiation mechanism, based on consensus ADMM as recommended by the conclusions of Sections 3.2 and 3.3. Note that both are at the base of the rest of the manuscript and be largely reused. In a second step 3.4.2 presents the different stopping criteria that are proposed here in the objective of reducing the number of messages exchanged to reach consensus. Subsection 3.4.3 describes the case studies, similar to the ones of Subsection 3.2.2, on which the Monte Carlo simulations of the peer-to-peer market are performed. The evolution of the imbalance according to the different stopping criteria will then be presented. Finally, Subsection 3.4.4 gathers conclusions and perspectives for future work on the matter of mitigation of communication costs.

3.4.1 Standard peer-to-peer market design

A peer-to-peer market is based on a community of prosumers with flexible consumption or production. The scope here being centred on exchange mechanisms, a deterministic version of the market clearing is addressed. A single market time unit is considered. However, as depicted in Chapter 2, it may readily be extended to multiple time units with temporally binding constraints. As it is classically done, prosumers are supposed rational in the sense of [24], i.e. always objectively taking the most beneficial decisions, and non-strategic, i.e. not anticipating actions and reactions of other prosumers. In a first step, the peer-to-peer market design is recalled. The associated negotiation mechanism based on [53]'s consensus ADMM.

Problem formulation

The goal of any market-clearing, formed by a set Ω of N_Ω participants, is to match demand and supply while minimizing the total cost. The total cost sums all individual cost functions as in (3.9a). To minimize its cost function c_n, prosumer n is able to optimize its traded volume p_n within a flexibility range defined by a lower $p_n^{\min}$ and an upper $p_n^{\max}$ bound, as expressed in (3.9d). Note that a traded amount p_n is taken positive if prosumer n is selling electricity, and negative when buying.

However a peer-to-peer market is intrinsically based on multi-bilateral trades. This fundamental mechanism calls for a split of these net powers into a set of multiple bilateral trades p_{nm} in the manner of [72]. Every possible bilateral power trades within the peer-to-peer community can be gathered in matrix $\mathbf{P}$. Elements p_{nm} of this matrix which are not imposed to zero reflect prosumers m belonging to the trading partnership set ω_n of prosumer n. The total traded volume of a prosumer n is then obtained by $p_n = \sum_{m\in\omega_n} p_{nm}$ as in (3.9c). For a bilateral contract to be valid both partners need to agree on both a quantity and a price. Trade reciprocity on quantities is enforced by (3.9b), ensuring that $p_{nm} = -p_{mn}$. Price consensus is implicitly reached through the negotiation mechanism as detailed below. One can note that (3.9b) implies that $\mathbf{P}$ is skew-symmetric, so $p_{nn} = 0$.

The final peer-to-peer market problem can be formulated as

Standard peer-to-peer electricity market

$$
\begin{aligned}
\min_{\mathbf{P}, p_{n\in\Omega}} \quad & \sum\nolimits_{n\in\Omega} c_n(p_n) && && (3.9a)\\
\text{s.t.} \quad & \mathbf{P} = -\mathbf{P}^{\mathsf{T}} && [\boldsymbol{\Lambda}] && (3.9b)\\
& p_n = \sum\nolimits_{m\in\omega_n} p_{nm} && [\mu_n] \;\; n\in\Omega && (3.9c)\\
& p_n^{\min} \leqslant p_n \leqslant p_n^{\max} && [\nu_n^{\min}, \nu_n^{\max}] \;\; n\in\Omega. && (3.9d)
\end{aligned}
$$

Note that dual variable matrix $\boldsymbol{\Lambda} = (\lambda_{nm})$ gathers all trading prices while μ_n represents prosumer n's perceived price. Such as introduced in Subsection 3.2.1, this formulation allows for an additional specific cost on each bilateral trade to express preferences as in [72] or to allocate grid-related costs as in [2] and Chapter 4.

Negotiation mechanism

As developed in [2], peer-to-peer market (3.9) can be solved in a decentralized manner based on the consensus ADMM of [53], which seems appropriated according to [1,87]. The decentralized negotiation mechanism associated to (3.9) reads at each iteration k

$$
\begin{aligned}
P_n^{k+1} = \underset{P_n}{\text{argmin}} \quad & c_n(p_n) + \sum\nolimits_{m\in\omega_n} \sigma^\rho\left(p_{nm}, \tfrac{p_{nm}^k - p_{mn}^k}{2}, \lambda_{nm}^k\right) && (3.10a)\\
\text{s.t.} \quad & p_n = \sum\nolimits_{m\in\omega_n} p_{nm}\\
& p_n^{\min} \leqslant p_n \leqslant p_n^{\max}\\
\lambda_{nm}^{k+1} = \lambda_{nm}^k - \rho & \left(p_{nm}^{k+1} + p_{mn}^{k+1}\right)/2 && (3.10b)
\end{aligned}
$$

where penalty factor $\rho > 0$ and $P_n = (p_{nm})_{m\in\omega_n}$ groups trade proposals of prosumer n. Augmented Lagrangian terms gathered in

$$
\sigma^\rho : (x, y, z) \in \mathbb{R}^3 \mapsto z\,(y - x) + \frac{\rho}{2}(y - z)^2 \tag{3.11}
$$

represents an prosumer's overcosts due to the lack of consensus with its partners. It aims at economically encouraging an prosumer to reach power consensus with its partners. According to [53], convergence of negotiation mechanism (3.10) is ensured as long as cost functions c_n are closed, proper and convex. Note that in (3.10b) prices are updated with a symmetric corrective term. Hence, prices are identical on both ends of a trade when $\lambda^0_{nm} = \lambda^0_{mn}$. The global stopping criteria associated to (3.10) are such as

$$\sum\nolimits_{n\in\Omega} \epsilon_n^{\mathrm{p},k+1} \leqslant {\epsilon^{\mathrm{p,tol}}}^2 \quad \text{and} \quad \sum\nolimits_{n\in\Omega} \epsilon_n^{\mathrm{d},k+1} \leqslant {\epsilon^{\mathrm{d,tol}}}^2 \tag{3.12}$$

with, respectively, primal and dual local residuals

$$\epsilon_n^{\mathrm{p},k+1} = \sum\nolimits_{m\in\omega_n} \left(p_{nm}^{k+1} + p_{mn}^{k+1}\right)^2 \tag{3.13a}$$

$$\epsilon_n^{\mathrm{d},k+1} = \sum\nolimits_{m\in\omega_n} \left(p_{nm}^{k+1} - p_{nm}^{k}\right)^2 . \tag{3.13b}$$

Parameters $\epsilon^{\mathrm{p,tol}}$ and $\epsilon^{\mathrm{d,tol}}$ denotes primal and dual global feasibility tolerances, respectively.

Overall, the negotiation mechanism, described in Algorithm 2, occurs in the following steps. At first prosumers solve their local optimization (3.10a). Then, they send the new trade proposals $\left(p_{nm}^{k+1}\right)_{m\in\omega_n}$ to their respective partners. Once all counter proposals $\left(p_{mn}^{k+1}\right)_{m\in\omega_n}$ are received, prosumers can update trading prices $\left(\lambda_{nm}^{k+1}\right)_{m\in\omega_n}$ with (3.10b) and local residuals $\left(\epsilon_n^{\mathrm{p}}, \epsilon_n^{\mathrm{d}}\right)^{k+1}$ with (3.13). Finally, they broadcast local residuals to all such that they can test global stopping criterion (3.12)[1]. This process is repeated until convergence.

Algorithm 2: Standard peer-to-peer negotiation mechanism

Data: $\rho, \epsilon^{\mathrm{p,tol}}, \epsilon^{\mathrm{d,tol}}$
for $n \in \Omega$ **do in parallel**
  Data: $c_n, p_n^{\min}, p_n^{\max}, \omega_n$
  Initialize: $p_{nm} = p_{mn} = \lambda_{nm} = 0,\ \forall m \in \omega_n$;
  do
    $P_n \longleftarrow$ (3.10a); /* Update local proposals */
    foreach $m \in \omega_n$ **do**
      Send p_{nm} to and receive p_{mn} from prosumer m;
      $\lambda_{nm} \longleftarrow$ (3.10b); /* Update trading price */
    end
    $\left(\epsilon_n^{\mathrm{p}}, \epsilon_n^{\mathrm{d}}\right) \longleftarrow$ (3.13); /* Update local residuals */
    Broadcast $\left(\epsilon_n^{\mathrm{p}}, \epsilon_n^{\mathrm{d}}\right)$ and receive $\left(\epsilon_m^{\mathrm{p}}, \epsilon_m^{\mathrm{d}}\right)_{m\in\Omega\setminus\{n\}}$;
  while (3.12) *not True*;
end

3.4.2 Proposed alternative stopping criteria

The resolution of an electricity market in a peer-to-peer approach achieves a global optimum in a fully decentralized manner while respecting the physical constraints that are inherent in an electricity network [2] – the power balance will be the only constraint considered here. In this sense, this is therefore a relevant alternative to centralized or community electricity markets. Indeed, the latter require the participation of a coordinating prosumer even when they are resolved in a decentralized manner. However, peer-to-peer resolution inherently presents computational difficulties that were presented in [1] and Section 3.2. In particular the present section focuses on the very important number of messages that have to be exchanged between

[1] Besides costly communication, this could rise a privacy concern which is beyond the scope of this study.

peers. This issue is made even more critical as the number of messages $N_{message}$ increases with the square of the number of participants N_Ω:

$$N_{message} = O\left(N_\Omega^2\right) \tag{3.14}$$

whereas a classical pool-based market only requires the number of message to increase linearly with the number of prosumers. This characteristic is likely to be blocking for a real and operational deployment of peer-to-peer markets.

In order to reduce the number of messages exchanged when resolving a peer-to-peer market, this section implements and compares several alternative stop criteria regarding their communication needs.

- stopping criteria per prosumer: each prosumer can unilaterally decide to stop trying to improve his exchanges with his peers. The iteration when this decision occurs is denoted k_n. Then the powers that prosumer n will exchange with prosumers $m \in \omega_n$ no longer evolve and remain frozen at $(p_{nm})^{k_n}_{m\in\omega_n}$. The other prosumers continue their negotiations independently until each one of them meets their individual stopping criteria. This individual stopping criteria is based on the share of prosumer n into the global primal and dual residues r_n and s_n. A prosumer shall stop trading as soon as the primal and dual residues which concern his trades fall below a tolerance:

$$\epsilon_n^{\mathrm{p},k+1} \leqslant \epsilon_{\mathrm{pros}}^{\mathrm{p,tol}^2} \quad \text{and} \quad \epsilon_n^{\mathrm{d},k+1} \leqslant \epsilon_{\mathrm{pros}}^{\mathrm{d,tol}^2} \tag{3.15}$$

- stopping criteria per trade: each prosumer can unilaterally decide to stop trying to improve a particular trade with a peer while carrying on the negotiation with his other peers. The iteration when this decision occurs is denoted k_{nm}. Then the power prosumer n will exchange with prosumer m no longer evolves and remains frozen at $p_{nm}^{k_{nm}}$. The prosumer will continue to negotiate with his other peers until each one of his trades meets their individual stopping criteria. The global algorithm is stopped when all trades of all prosumers are finished. This stopping criteria per trade only considers the share of trade p_{nm} into the global primal and dual residues r_n and s_n. A prosumer shall stop trading as soon as the primal and dual residues which concern his trades fall below a tolerance:

$$\left(p_{nm}^{k+1} + p_{mn}^{k+1}\right)^2 \leqslant \epsilon_{\mathrm{trade}}^{\mathrm{p,tol}\ 2} \tag{3.16a}$$
$$\left(p_{nm}^{k+1} - p_{nm}^{k}\right)^2 \leqslant \epsilon_{\mathrm{trade}}^{\mathrm{d,tol}\ 2} \tag{3.16b}$$

For each one of these two criteria, thresholds $\epsilon_{\mathrm{pros}}^{\mathrm{p}}$, $\epsilon_{\mathrm{pros}}^{\mathrm{d}}$, $\epsilon_{\mathrm{trade}}^{\mathrm{p}}$ and $\epsilon_{\mathrm{trade}}^{\mathrm{d}}$ can either be defined in an absolute or a relative manner. In the latter case, they can be fixed as a fraction of the prosumer's nominal power:

$$\epsilon_{\mathrm{pros,trade}}^{\mathrm{p,d,tol}} = a \cdot \max\left(|p_n^{\min}|, |p_n^{\max}|\right) \tag{3.17}$$

It should be observed that these alternative stopping criteria no longer provide evidence of the convergence of the peer-to-peer negotiation algorithm towards a global optimum. Although the objective functions are unchanged as well as the decomposition scheme of the global problem, no demonstration of the convergence of a peer-to-peer market when some exchanges are frozen before the end of the global algorithm has been found in the literature so far. In addition, exchanges between peers are assumed to be synchronous in this case: each exchange of the k^{th} iteration takes place before all peers move on to the $k+1^{th}$ iteration. Although this assumption seems unrealistic in view of Section 3.3 and the potential large deployment of peer-to-peer markets. Indeed, more realistic communication models, such as of [88], may be required as power system performances may be affected by them in the case of asynchronous communications [89].

3.4.3 Simulation results

To obtain a comprehensive analysis, this subsection conducts Monte Carlo simulations for each of the studied market configurations. For this purpose the subsection first describes the test cases on which the study is based on. After observing performances of standard peer-to-peer negotiation mechanism, the subsection pursue analyzing performances of the alternative stopping criteria, their influence on the convergence rate and how they are impacted by the size of the peer-to-peer community.

Test cases description and reference performances

The previous resolution and the different stopping criteria are evaluated here on peer-to-peer market configurations. Based on Section 3.2's test case generator, prosumers' characteristics are drawn from a uniform distribution to avoid extreme cases such as low flexibility slope and market power. For this purpose the generator ensures that consumers and producers each presents at least a third of the total number of market players. Before being split between prosumers, total consumption and production capacities are randomly sampled within a range proportional to the market size. Moreover, prosumers follow a quadratic cost function, as commonly done in the literature, for which prices and power set points are sampled within a given range. Since the outcome of the market clearing depends on prosumers characteristics, the convergence speed of the negotiation mechanism is evaluated on 1000 generated cases to perform a Monte Carlo analysis. To give a first overview of alternative criteria's impact on performances, the configurations tested for a market size of 25 prosumers before being extended up to 200 to study scale's influence on performances.

In order to define a reference situation, these thousand resolutions are first executed without any stopping criteria but a maximum number of iterations. The first result of this study is thus the shape of the trade-off between an acceptable power balance and the number of messages that have to be exchanged to achieve it. Indeed if there is still any imbalance in the market – i.e. $\sum \mathbf{P} \neq 0$ – the grid operator will have to compensate it by adjusting the production of another power plant. The plunge of this imbalance along iterations of the peer-to-peer market clearing is then a tool to decide when power balance compensation costs become lower than communication costs. The left panel of Figure 3.7 illustrates the progress of this imbalance.

Center and right panels of the same figure display, respectively, the simultaneous evolution of per prosumer and per trade primal residuals as defined in Subsection 3.4.2. These two figures highlight the benefits of defining the proposed stopping criteria. Indeed, a significant proportion of prosumers converge quickly towards a solution that seems satisfactory to them. Their contribution to the overall residue therefore quickly becomes insignificant, i.e. about 2

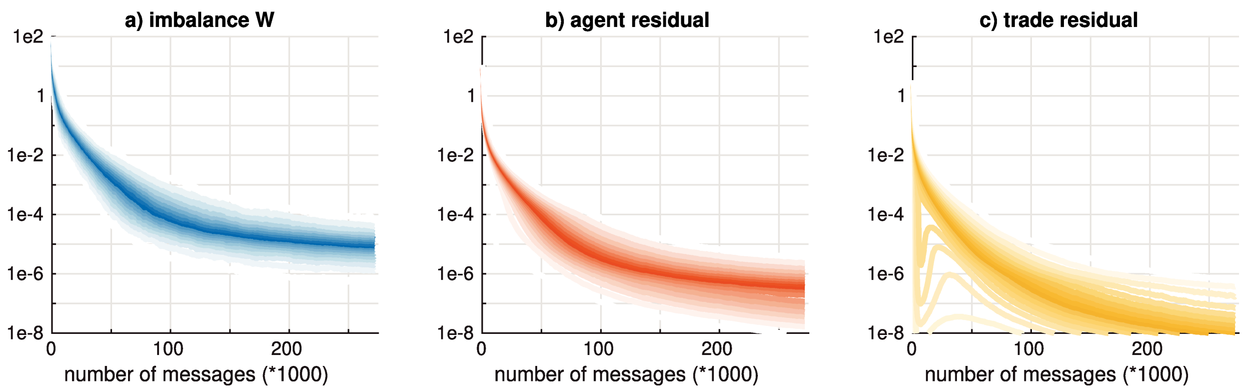


Figure 3.7: Quantiles (5%) of the power imbalance (left), and per prosumer (center) and per trade (right) residuals through messages in a 25 prosumer peer-to-peer market. The total traded volume is 1370 W in this test case.

orders of magnitude lower than the average of the prosumers. This phenomenon is even more pronounced when considering trades independently. Many converge towards their final value after a few iterations and therefore have no interest in being taken into account later. Thus communication would not need to be followed up once the prosumers or the trades reach a tolerance threshold. It seems a promising way to reduce the number of messages exchanged to reach market outcomes without deteriorating quality of the solutions.

Performances of alternative stopping criteria

As outlined above, this study considers the case of the use of peer-to-peer negotiation for an electricity market. The discussion therefore focuses on respecting the equilibrium constraint of the injected and subtracted powers $\sum P = 0$. Any violation of this constraint would entail additional costs to restore the balance of the network, by urgently using flexible means of production or consumption. Figure 3.8 represents the evolution of this coupling constraint depending on the number of messages exchanged for the different considered stopping criteria. The convergence speed of the reference solution is recalled in blue – average, 1^{st} and 3^{rd} quartiles, and 1^{st} and 9^{th} deciles.

The first panel in this figure refers to the stopping criterion where each prosumer stops when its residues fall below an absolutely fixed limit, as in (3.13). Each point on the figure represents one of the 1000 simulated situations. The threshold $\epsilon_{\text{pros}}^{\text{p,tol}}$ and $\epsilon_{\text{pros}}^{\text{d,tol}}$ takes values from 10^{-5} to $10^{0.5}$ to map a large part of the convergence. It should be noted that the speed of convergence under this strategy is almost similar to the baseline situation. It appears that very few, if any, prosumers can achieve convergence on all their trades in an earlier stage. They therefore all remain active for a very long time in the negotiation process. This is actually consistent with panels a) and b) of Figure 3.7. Indeed, the residues per prosumer decrease at the same rate as the global imbalance and all prosumers share the residuals equally.

The second panel of Figure 3.8 is dedicated to the stopping criterion on each trade, as in (3.16), with an absolute tolerance. A significant acceleration can then be noticed. Indeed, the coupling constraint decreases up to 10 times faster with this stopping criterion strategy. Due to the definition of $\epsilon_{\text{trade}}^{\text{p,tol}}$ and $\epsilon_{\text{trade}}^{\text{d,tol}}$ in absolute terms, the quantiles are grouped together, which reflects a low dispersion in convergence rates between the market simulations. The choice of such a shutdown criterion therefore seems highly appropriate to achieve market clearing that respects the network's equilibrium constraint while minimizing communication costs and delays.

The third panel represents the results for the stopping criterion per prosumer, defined as a proportion of its nominal power. Similarly to when the stopping criterion was defined in an

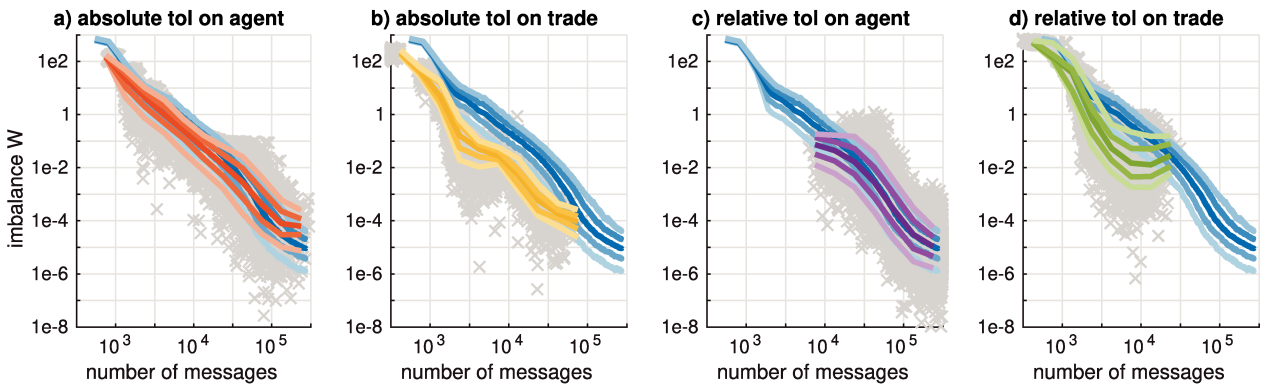


Figure 3.8: Convergence speed on the coupling constraint depending on the number of messages with different stopping criterions – average, 1^{st} and 3^{rd} quartile, 1^{st} and 9^{th} deciles. The reference behaviour is recalled in blue.

absolute way, this strategy gives results very close to the reference situation.

Finally, the fourth panel concerns the stopping criterion per trade, defined as a fraction of the nominal power of the prosumer. This strategy may have significant accelerations compared to the reference situation although these do not concern all values. It therefore seems generally less effective than when tolerance thresholds are defined in an absolute way.

Continuous curves observed in Figure 3.8 over wide ranges of accuracy have been obtained by setting various values of tolerance thresholds $\epsilon^{\text{p,d,tol}}_{\text{pros,trade}}$. For each value of tolerance threshold, the mean imbalance is represented in Figure 3.9 along with the different strategies. Hence, this figure represents what imbalance can be expected for a given couple. Once more, this figure shows the consistency between the global imbalance and the per prosumer absolute tolerance by a factor of 10, as observed in Figure 3.7. Besides, the per trade absolute strategy oscillates closely around the same correlation instead of below it as in the reference case. The difference is actually due to the fact that trades are taken as constant in the alternative strategy while they are always improved in the reference case. In other words, stopping negotiations on several trades have an impact on the remaining ones, which was to be expected. Even though the per prosumer relative approach appears interesting in terms of precision, its inefficiency in terms of messages as observed here rather puts it aside. Lastly, the use of a relative per trade method seems detrimental overall for the imbalance.

Final iteration distribution

The reduction in the number of messages globally exchanged is achieved by stopping many trades as soon as their primary and dual residues fall below a threshold. Figure 3.10 represents the histogram of the final iterations for each trade k_{nm} with a tolerance per trade $\epsilon^{\text{p,tol}}_{\text{trade}} = \epsilon^{\text{d,tol}}_{\text{trade}} = 10^{-1.5}$. The overwhelming majority of trades are therefore frozen from the very first iterations. After 15 iterations, 99.99% of the trades are concluded. For these trades, the observed prices are similar to those obtained in the reference solution to within 10^{-6}. However, the chosen threshold value allows to draw attention to the fact that the remaining fraction was no longer able to converge towards a solution that respected the tolerance. These trades therefore continued to iterate until they reached the maximum number of iterations set in this resolution at 1000. This observation stems in all likelihood from the absence of a guarantee of global convergence when introducing such stopping criteria. Such a behavior cannot be highlighted for lower ϵ_{trade}.

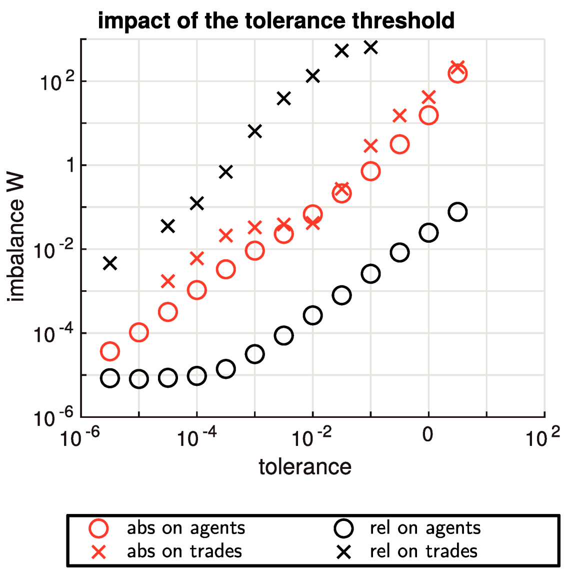


Figure 3.9: Impact of tolerance threshold on global imbalance

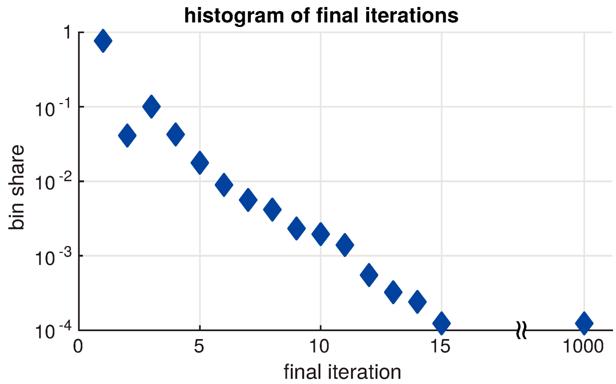


Figure 3.10: Histogram of final iterations for a peer-to-peer market of 25 prosumers and a tolerance per trade $\epsilon^{\text{p,tol}}_{\text{trade}} = \epsilon^{\text{d,tol}}_{\text{trade}} = 10^{-1.5}$

Impact of the number of prosumers

In order to estimate the impact of the number of prosumers on the stopping criteria previously introduced, the configuration identified as particularly interesting is reproduced for larger markets. This configuration is a stopping criterion per trade and a tolerance on primary and dual residues of 10^{-3}. Table 3.2 summarizes the variations for markets with between 25 and 100 prosumers. First, the imbalance of the injected powers is indicated in mean value and standard deviation. The number of messages exchanged using the stopping criterion per trade is designated $N^{stop}_{message}$. This number is compared to $N^{ref}_{message}$, the number of messages that must be exchanged to achieve the same accuracy with a global stopping criterion. The increase in the number of messages follows the square of the number of prosumers in both cases. Counter intuitively, the reduction factor of the number of exchanged messages therefore remains globally constant regardless of the number of peers in the market.

Table 3.2: Impact of the number of peers in a market on the stopping criterion.

N_{peers}	50	100	125	150	175	200
imbalance $\cdot 10^{-3}$	160	460	610	765	905	1050
$\sigma_{imbalance} \cdot 10^{-3}$	60	200	300	410	540	660
$N^{stop}_{message} \cdot 10^3$	11.5	49.5	100	140	177	225
$N^{ref}_{message} \cdot 10^3$	100	510	830	1220	1700	2290
reduction factor	10.4	9.7	10.3	8.7	9.6	10.1

3.4.4 Conclusion

Peer-to-peer markets, and more largely consumer-centric markets, require to exchange a number of messages that increases with the square of the number of participants. This property would entail significant communication costs in the case of their deployment in electricity markets where the constraint of balance between injected and consumed power is critical. In this section, alternative stopping criteria have been proposed in order to reduce the number of messages while preserving the same respect for the equilibrium constraint.

The most efficient of these criteria proposes to establish each trade independently as soon as its primal and dual residue falls below a tolerance. This results in a tenfold reduction in communication needs. The impact of the selected tolerance levels on the respect of the equilibrium constraint has been investigated, as well as the number of iterations allowing trades to be frozen. Finally, the scaling up of this stopping criterion was examined on markets with up to 200 prosumers.

However, the stopping criterion was considered here as an exogenous constraint, for example set by regulation to maximize the system's efficiency. In an individual approach, where each prosumer would bear the communication costs for each message he sends, the problem could then be formulated as a trade-off between the communication cost and the expected improvement of a trade. Such a resolution would require anticipating prosumers' decisions and, therefore, assuming that prosumers would behave strategically.

3.5 Sparsity analysis of the communication matrix

One of the pointed advantage of the peer-to-peer market structure is that it encompasses most consumer-centric market. This is made possible thanks to the flexibility enabled by the communication matrix. An important point is that the layout of the communication matrix does not affect the optimality of the economic dispatch in a peer-to-peer market as simple as the one of Section 3.4. Notably, Subsection 3.5.1 shows that the same peer-to-peer negotiation mechanism allows to reach the same optimal economic dispatch for different market layouts. In a second step, Subsection 3.5.2 demonstrates that this observation does not hold as soon as the individual cost terms are added, such as product differentiation coefficients γ_{nm} in MBED's objective function (3.3a). Indeed, these additional specific cost terms change the nature of the market and, hence, stir the market equilibrium as it would no longer be comparable to the classical centralized market. In spite of that Subsection 3.5.2 also shows that peer-to-peer market's flexibility may also allow for a compromise between the sparsity of its communication matrix and the convergence speed, so the number of exchanged messages.

3.5.1 Changes without optimality alterations

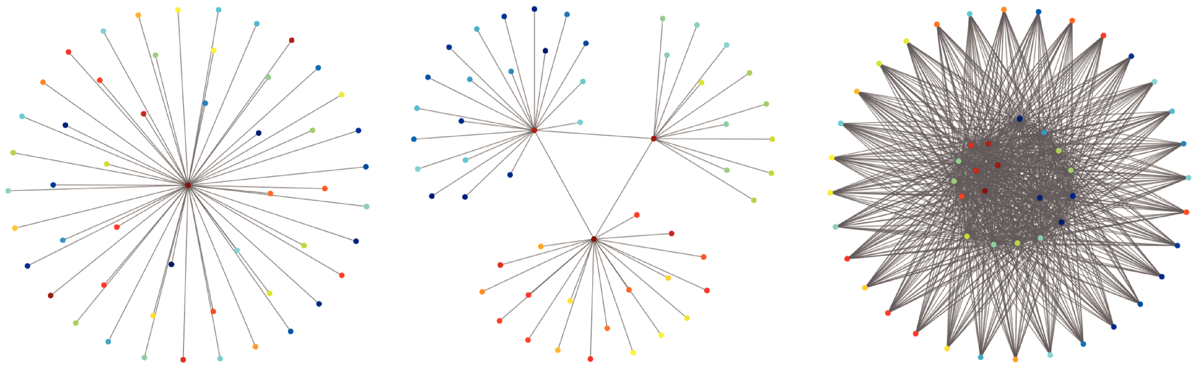
After a brief description of the test cases, this subsection analyzes the influence of different organization layouts on market equilibrium.

An attempt of categorizing some of the possible organization layouts of decentralized electricity markets is proposed in [10], where additionally to a peer-to-peer market, the authors identify two other market organizations. In the first one, prosumers are connected to microgrids which can either be isolated or interconnected; while in the second one, prosumers are organized in groups, namely energy communities, in which resources, not necessarily geographically located close to each other, are managed in small centralized markets.

Test case description

In an attempt to categorize prosumer market organizations, [10] identified two structures in addition to peer-to-peer markets. In the first one, prosumers are connected to microgrids which can either be isolated or interconnected. In the second one, prosumers are organized in groups, namely energy communities, in which resources, not necessarily geographically located close to each other, are managed in small centralized markets. As highlighted in Section 3.1, these two additional structures can be grouped in one family which could be named community-based. It is here proposed to compare both peer-to-peer and community-based market structures to the classical centralized organization.

Yet, two remarks could be made. First, one could note that the centralized organization could be solved with Subsection 3.4.1's peer-to-peer market model and the associated negotiation mechanism. As illustrated in Figure 3.11a, the prosumers would be grouped around an additional prosumer playing the role of the market operator. This market operator would be a non-profit entity without production or consumption capabilities. In other words, the market operator's cost function power bounds would be all set to zero. Thus, the market operator would simply gather prosumers power injections, in the form of single bilateral trades, and verify the overall power balance. Second, community-based layouts can also be modeled by



(a) Centralized market (b) Interconnected communities (c) Peer-to-peer

Figure 3.11: Decentralized peer-to-peer electricity market layouts

the peer-to-peer market structure. As illustrated in Figure 3.11b, the prosumers would be gathered in clusters, called communities, around additional prosumers playing the role of community managers. Such as the central market operator, the community managers would also be non-profit entities. Their role would thus consist in gathering community members' power injections and to insure power balance of the community. Contrary to the centralized market configuration, community managers would be able to import to or export power from the community and would, therefore, act as an interface with the outside world. Too reflect [10]'s different community-based layouts, the community managers could either stay isolated or be interconnected with each other through multiple bilateral trades. The centralized market could then be seen as a particular case of community-based market with one isolated community.

In the rest of this section, all market layouts of Figure 3.11 are modeled and cleared as peer-to-peer markets. The difference between these layouts consist in the communication matrix, representing the graph layout. To solely evaluate the impact of the communication graph, the three peer-to-peer market layouts will account for the exact same prosumers at the exception of the non-profit market operator and community managers which do not impact the optimal value. Thus, the sole variations between the tested configurations will only be the communication matrix and whether individual costs terms bilateral trades. The size of the test case in the present study is arbitrarily fixed at 50 prosumers. Prosumers' characteristics are generated in the same manner as described earlier in Subsection 3.2.2. The prosumers are then split into three communities, one lacking generation, another with extra non-dispatchable power and the last being balanced. The presence of two imbalanced communities insures that energy exchange among them will be necessary, making it easier to analyze simulation results.

Without per trade individual cost terms

First a market setup without any individual cost terms is considered. In consequence all γ_{nm} in (3.3) are equal to zero, thus leaving the simulated formulation as in (3.9). Yet, deriving

Table 3.3: Simulation results without per trade individual cost terms (with $\epsilon^{\mathrm{p,tol}} = \epsilon^{\mathrm{d,tol}} = 10^{-4}$)

	Centralized	Peer-to-peer	Community-based
Social Welfare ($)	125	125	125
No. of iterations	82	37	134
Penalty factor ρ	5.10^{-4}	0.01	5.10^{-4}
Avg. trading price (c$/kW)	15.22	15.22	15.22
Cons./prod. power (kW)	1440	1440	1440
Total traded power (kW)	2880	1440	2880

(3.9) Lagrangian with respect to power set-points p_n and bilateral trades p_{nm} provides the two following Karush–Kuhn–Tucker stationarity conditions

$$\mu_n = \frac{\partial \tilde{c}_n}{\partial p_n} \qquad n \in \Omega \tag{3.18a}$$

$$\mu_n = \lambda_{nm} \qquad n \in \Omega,\ m \in \omega_n \tag{3.18b}$$

where $\tilde{c}_n$ denotes cost function c_n's extended-value function, in the sense of [54], defined on power boundary range (3.9d). In this situation trading prices λ_{nm} are uniform and equal to the centralized market price λ^{Pool}, as (3.18a) would be replaced by $\lambda^{\text{Pool}} = \frac{\partial \tilde{c}_n}{\partial p_n}$ for all $n \in \Omega$. The communication graph is therefore expected to have no effect on the global social welfare. This fact is confirmed by the simulations as reported in Table 3.3. Centralized, peer-to-peer and community-based organizations reach the same social welfare optimum for a price of 15.22 c$/kW. The three structures reach the same level of total consumption and production of 1.44 MW confirming that, overall, prosumers obtain the same set-points. It can also be observed that the total power exchanged, i.e. $\sum |\mathbf{P}|/2$, is doubled for centralized and community-based structures due to the presence of a central entity, inducing a double counting of trades. This translates the presence of managers whose trades are also encompassed in the sum. The same remark can be done for the centralized organization for the centralized market structure, so with single community.

3.5.2 Changes with optimality losses

Now, increasing per trade individual cost terms are introduced to analyze their influence on both market outcomes and negotiations' convergence speed. Afterwards, a Monte Carlo analysis outlines the influence of communication matrix sparsity on the same performance indicators.

With per trade individual cost terms

Whenever per trade individual cost terms are not null, different market outcomes occur depending on the market layout. For the sake of this study, per trade individual cost terms are solely considered outside of communities, hence only on inter-community exchanges in Figure 3.11b and for all trades in Figure 3.11c. Consequently, the centralized structure is not affected by these cost terms as it behaves as a single community. As expected, in Table 3.4a

Table 3.4: Simulation results for 1 c$/kW per trade individual cost term (with $\epsilon^{\text{p,tol}} = \epsilon^{\text{d,tol}} = 10^{-4}$)

	Centralized	Peer-to-peer	Community-based
Social Welfare ($)	125	102	105
No. of iterations	82	80	195
Penalty factor ρ	5.10^{-4}	0.01	5.10^{-4}
Avg. trading price (c$/kW)	15.22	15.58	15.28
Cons./prod. power (kW)	1440	1152	1192
Tot. exchanged power (kW)	2880	1152	2880

(a) Overview

	Balance of trade	Interior price
Community 1	-908 kW	16.50 c$/kW
Community 2	590 kW	14.50 c$/kW
Community 3	319 kW	14.50 c$/kW

(b) Focus on communities' power balance

the social welfare of the peer-to-peer approach is negatively impacted by the use of a 1 c$/kW trade-based transaction cost. The effect is largest on prices and, hence, power set-points. It can be noted that, since the transaction costs are uniform, all participants are equally affected in the peer-to-peer structure. The community-based simulation also shows a decrease of social welfare.

In presence of per trade individual cost terms, such as in (3.3), Karush–Kuhn–Tucker stationarity conditions (3.18) would become

$$\mu_n = \frac{\partial \tilde{c}_n}{\partial p_n} \qquad n \in \Omega \tag{3.19a}$$

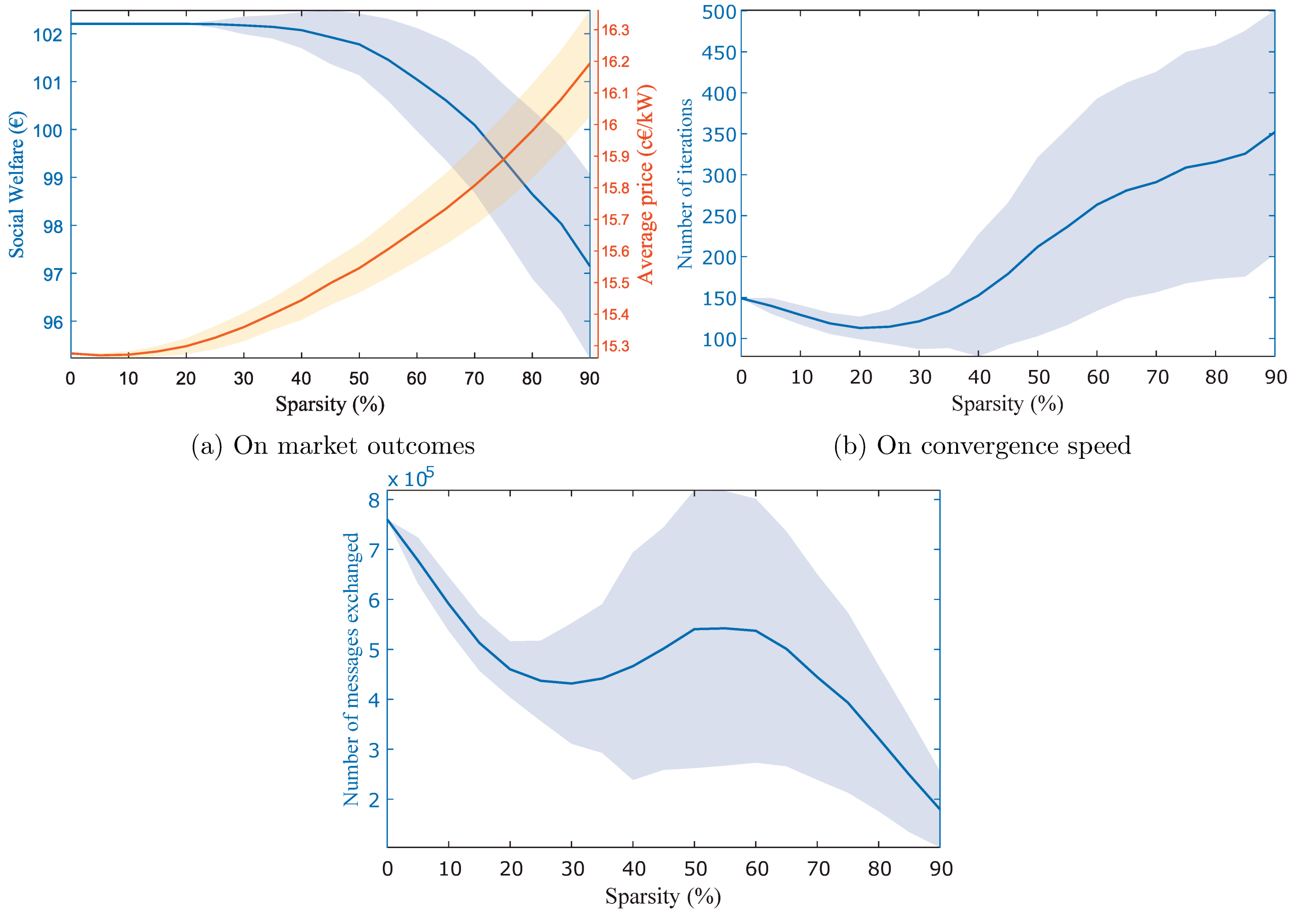
$$\mu_n = \lambda_{nm} - \gamma_{nm} \qquad n \in \Omega,\ m \in \omega_n \tag{3.19b}$$

with γ_{nm} the individual cost term on the bilateral trade from prosumer n to prosumer m. It can be observed in (3.19) that a difference of price appears at the community manager level between the inside, where $\gamma_{nm} = 0$, and the outside of a community, where $\gamma_{nm} \neq 0$. Table 3.4b shows that communities with a positive balance of trade perceive a lower price within the community, while communities with a negative balance are penalized with a higher interior price as they need to import power. By increasing the value of trade-based transaction costs, one can observe that these differences follows the same trend. In fact, the average trading price grows linearly in both peer-to-peer and community-based layouts. The social welfare is less impacted in the community-based layout than in the peer-to-peer one, as already pictured in Table 3.4. In addition, convergence speed of the negotiation mechanism appeared to linearly increase with transaction costs' intensity for both peer-to-peer and community-based approaches. However, in the community-based case the slope is rather flat compared to the peer-to-peer. A broader study should be conducted to evaluate in more comprehensively the influence of transaction costs on the negotiation mechanism.

Random sparsity

To evaluate the influence of communication structures on market outcomes, a Monte Carlo analysis is carried out on the peer-to-peer structure for different levels of sparsity of the communication graph. Starting from a fully connected peer-to-peer market, i.e. each prosumer is connected to all others, the communication graph is progressively altered by randomly deleting links. The Monte Carlo analysis, over 1000 cases for each 5% step of sparsity, allows to describe how peer-to-peer market outcomes evolve as communication links get sparser. Obtained in the presence of a unitary trade-based transaction cost, Figure 3.12a outlines means (lines) and standard deviations (shadows) of social welfare and average trade prices. The sparser communications are, the more likely it is for market outcomes to be affected and with a larger variety. This correlates with the increased possibility of prosumers to be unsatisfied, e.g. when a consumer is solely partnered to other consumers.

As it is harder for prosumers to match their requirements, negotiations' convergence speed can be significantly slowed down, as shown in Figure 3.12b. But even though increasing the number of iterations, Figure 3.12c shows that the increase of sparsity would have a benefit in the reduction of the overall number of messages to exchange to reach consensus and, thus, to reduce traffic on the communication network. Comparing Figures 3.12a and 3.12b, it is possible to notice that there exist situations where the trade-off between convergence speed and the amount of messages can be found without a too great impact on the social welfare. Hence, the development of methods to retrieve communication layouts that optimize this trade-off becomes fundamental in order to enhance the feasibility in real world implementations of decentralized electricity markets. Notably, it can be seen that good trade-offs can be found between 40% and 55% sparsity. In future works, one could use matching algorithms or machine learning techniques to find beneficial trade-offs.



(a) On market outcomes (b) On convergence speed

(c) On the number of exchanged messages

Figure 3.12: Effects of sparsity

3.5.3 Conclusion

With the deployment of distributed energy sources and home management systems, the role of prosumers in power systems will soon be fundamental. In the literature a variety of market structures adapted to particular situations are listed. The advantage of the peer-to-peer market structure is that it can be adapted to most of these consumer-centric frameworks. This section analyzed simulation results of two specific layouts: peer-to-peer and a community-based structures. It is outlined that the communication structure does not influence market outcomes in a standard situation without per trade individual cost terms. However, whenever these individual cost terms are not null such as in presence of preferences, the community-based structure seems better suited in terms of optimality while the peer-to-peer structure converged faster. But the convergence speed of the peer-to-peer structure appeared much more sensitive to the intensity of these individual cost terms than communities-based one. A Monte Carlo analysis revealed that sparsity of the peer-to-peer communication matrix influences market outcomes and convergence speed in a non-trivial way. Therefore, the development of methods exploiting sparsity to improve convergence speed while limiting the optimality gap is a fundamental future work. For example, matching algorithms or machine learning may be used to find beneficial trade-offs. This would notably improve the feasibility of prosumer markets in real world implementations as it also has the potential to reduce the number of messages to exchange to reach consensus. Naturally, a better understanding of sensitivities to the individual cost terms and prosumers' flexibility would also allow to improve the robustness of the negotiation mechanism.

3.6 Synthesis

The goal of this chapter was to challenge the suggestion made in Section 2.2 to solve peer-to-peer electricity markets based on consensus ADMM. To obtain these results the chapter first conducted a complexity analysis on two consumer-centric organization, namely peer-to-peer and community-based markets, each with a different associated algorithm, respectively *relaxed consensus and innovation* and ADMM. Testing the two different algorithms allowed to show that the ADMM based version was more efficient for consumer-centric market clearing. By means of a message passing interface these algorithms have been deployed on HPC which enabled investigation of their computation and communication complexities of the two market organizations. As expected, the community-based distributed approach was found faster and more scalable. However, peer-to-peer markets are the only framework allowing for product differentiation such as through preferences. This first analysis has then been deepened by studying the resilience of these algorithms to computation and communication delays. This study was essential mainly for communication delays as future consumer-centric markets will increase the number of exchanges and, thus, increase those delays due to a more intense traffic on communication network infrastructures. The other one being unable to handle such delays, this analysis finally confirmed the use of ADMM based algorithms for consumer-centric markets.

Validating the choice of algorithm made in Chapter 2 for generalized decentralized coordination problems which comprehend consumer-centric markets, the chapter then proposed improvements to make it more scalable and, hence, more practical for real world implementations. Being a challenge in particular for peer-to-peer markets, the first improvement aimed at reducing the number of messages required to reach consensus of peer-to-peer market clearing. By testing alternative, maybe more meaningful, stopping criteria, the chapter showed it was possible to greatly reduce the number of exchanged messages while preserving the same respect of the overall power balance and thus the same energy quality. Solely testing this on the peer-to-peer market structure does not mean the same can not be observed for the community-based market structure. Actually, peer-to-peer markets are shown to be able to take the form of most consumer-centric market structures. For example, a peer-to-peer market could take the form of a community-based structure simply by grouping prosumers in clusters and add non-profit entities to operate them. This change of the communication graph would then result in communities gathered around community managers which could be interconnected or stay isolated. In fact, the design of this communication graph has been identified as another possible way to improve peer-to-peer market's negotiation algorithm. Notably, modifying sparsity of the communication matrix allowed to reduce the number of exchanged messages as it reduced the problem's structural complexity.

This chapter also identified several further works to improve further peer-to-peer markets scalability for real world implementations. Firstly, the alternative stopping criteria approach could be enhanced by accounting for more strategic prosumers. However, considering strategic prosumers would also arise the issue of privacy. Indeed, strategic attitudes may push prosumers to reverse engineer their partners' characteristics, thus violating their wish of privacy. Secondly, using matching algorithms or machine learning techniques may help exploit sparsity of the communication graph to find beneficial trade-offs between low optimality gaps, convergence rate and the number of messages exchanged to reach consensus. Finally, prosumer markets may also be able to take advantage from asynchronous communications by using more comprehensive communication network models.

Decentralized coordination problems in shared infrastructures 4

Decentralized coordination problems such as the peer-to-peer markets may require a physical infrastructure to actually convey and operate the agreed bilateral trades. This chapter focuses on the case of electricity peer-to-peer markets and, hence, their interaction with the system operator of the electrical network on which they are connected. Two methods are highlighted in this chapter to include the system operator in the loop of bilateral trades' negotiation. First, the system operator could provide network charges a priori to stir agents towards a solution more acceptable for the infrastructure. Second infrastructures with critical constraints, the system operator would have to take a more active role by regularly updating network charges directly during the negotiation process.

Contents

4.1 Introduction

As introduced in Chapter 1, recent changes in the way electrical energy is generated and consumed make us rethink our approach to power systems operation and, in particular, how electricity markets are organized. Electricity markets are expected to go from producer-centric to consumer-centric [8, 9], while they will most likely include a peer-to-peer and community-based component [10]. It may be noted, as illustrated in Chapter 1, that community-based markets can also be seen exclusively with the multiple bilateral trades of the peer-to-peer market. In consequence, this chapter and the rest of the manuscript solely focuses on the peer-to-peer market structure. The decentralized and independent nature of peer-to-peer markets may naturally lead to power injections violating grid constraints. Thus, there would be a discrepancy between their market outcomes and the dispatch a system operator would obtain when accounting for grid-related and operational constraints. In parallel, while it appears normal to socialize grid-related costs in the current wholesale-retail market structure, a future with bilateral trades and preferences may allow to rethink the way we attribute such costs. Our objective here is hence to describe a consumer-centric market allowing to allocate grid-related costs in an exogenous manner. Grid related costs may refer to network investment cost as well as operating costs such as maintenance, power losses, etc.

The various attribution mechanisms are to impact trades and subsequent network usage. The first approach to coordinated multi-lateral[1] electricity trades was already proposed more than 20 years ago [68]. The original aim was to allow for the separation of economics and reliability of system operation, as is the case for the current European pool-based electricity markets. The framework developed in [68] involved an iterative process where all prosumers first propose their trades followed by the system operator estimating whether the requested trades respect operational constraints. This framework was enhanced in [90] with a game-theoretical analysis of the obtained solutions. In both cases, the authors pointed at the fact that charges for network usage were not considered.

A second approach may consist in relying on optimal power flow models, allowing to consider network constraints in an endogenous manner (see e.g. [36]). While those are traditionally solved in a centralized fashion, many decomposition techniques were proposed to solve them in a distributed manner. Based on approximate Newton directions, [55] proposed a decentralized method to solve optimal power flow control for power systems with overlapping areas. [91] followed by [77] respectively proposed distributed state estimation and multi-agent coordination in micro-grids based on consensus and innovation approach. Concurrently [92] used the alternating direction method of multipliers (ADMM), developed by [53], to solve optimal power flow in a distributed manner. [93] did the same with another consensus-based mechanism and applied it to energy management of cooperative micro-grids with peer-to-peer energy sharing in [94]. A comparison of different distributed and decentralized algorithms was finally made in [47]. More recently, works like that of [73] proposed to account for network limits in the presence of distributed renewable resources using decentralized consensus on a blockchain. Even though those operational problems are increasingly considered in decentralized manner, these do not comprise a market construct nor they account for how grid-related usage costs would be attributed.

In network-constrained economic dispatch problems, e.g. [95], nodal prices classically encompasses both energy generation and congestion-related costs. In such case, grid-related costs can not be dissociated from the final energy price. In contrast, the bilateral contracts considered here solely include the energy generation cost. They will be supplemented by network charges recovering the grid-related costs. This chapter proposes two ways to manage such network charges. The system operator can first estimate network charges exogenously, based on past data for example, so that prosumers know them *a priori* and can anticipate their actions.

[1] Note that bilateral trades are a specific case of multi-lateral trades involving only two prosumers. A multi-lateral trade can theoretically involve a very high number of prosumers.

Contrary to a classical economic dispatch, this transparency on network charges enables agents to anticipate on what it will cost them to trade on the network. In this exogenous approach, network charges can not only allow to recover all network costs but also other costs e.g. operational, taxes and policy-related costs. However network constraints would not be enforced directly but rather accommodated through these network charges. The resulting peer-to-peer market formulation comprises a simple tool, transparent to market participants, for system and market operators to limit potential detrimental effects that might be induced by peer-to-peer markets on power networks. Second, at the image of classical optimal power flows, network charges could also be estimated endogenously, so at the same time as prosumers proceed their trades negotiation. This method is particularly adapted to more critical or weak power systems for which network constraints must absolutely be guaranteed.

This chapter is structured as follows. Firstly, the peer-to-peer electricity market comprising network constraints is recalled in Section 4.2. To compare both exogenous and endogenous performances, the section also presents a standard test case specifically developed for joint peer-to-peer markets and optimal power flow, so for grid-aware consumer-centric markets. Exogenous network charges are then developed in Section 4.3 while Section 4.4 focuses on endogenous network charges. Note that in both cases the specific market design and, in the exogenous case, cost allocations are first described. Then simulation results of each approach is exposed. Finally, Section 4.5 syntheses the findings of this chapter.

4.2 Peer-to-peer electricity market with network constraints

A peer-to-peer market is based on a community of prosumers with flexible consumption or production. As it is classically done in the literature, prosumers are supposed rational as in [24], i.e. always objectively taking the most beneficial decisions, and non-strategic, i.e. not anticipating actions and reactions of other prosumers. In this chapter, emphasis is eventually placed on a deterministic clearing mechanism for a single market time unit. It may readily be extended to multiple time units with temporally binding constraints, such as explained in Chapter 2. First, the electricity peer-to-peer market formulation with grid constraints is exposed in Subsection 4.2.1 to recall the context of this chapter. Then, Subsection 4.2.2 describes the standard test case on which exogenous and endogenous network charges allocation methods are tested.

4.2.1 Problem Formulation

This chapter aims at proposing an alternative way to treat the following endogenous peer-to-peer electricity market

$$\min_{\mathbf{P}, p_{n\in\Omega}, \theta_{i\in\mathcal{N}}} \sum_{n\in\Omega} c_n\left(p_n\right) \tag{4.1a}$$

$$\text{s.t.} \quad \mathbf{P} = -\mathbf{P}^{\mathsf{T}} \tag{4.1b}$$

$$p_n = \sum\nolimits_{m\in\omega_n} p_{nm} \qquad n\in\Omega \tag{4.1c}$$

$$p_n^{\min} \leqslant p_n \leqslant p_n^{\max} \qquad n\in\Omega \tag{4.1d}$$

$$s_{ij} = \left|\left(\underline{\mathbf{Y}}\right)_{i,j}\left(\theta_j-\theta_i\right)\right| \leqslant \ell_{ij}^{\max} \qquad (i,j)\in\mathcal{L} \tag{4.1e}$$

$$\sum\nolimits_{n\in\mathcal{N}_i} p_n = \sum\nolimits_{(i,j)\in\mathcal{L}} s_{ij} \qquad i\in\mathcal{N} \tag{4.1f}$$

which straightly includes transmission network constraints, as in [95], in the context of a peer-to-peer market. In transmission networks, the admittance of electrical lines $\mathcal{L}$, noted $\left(\underline{\mathbf{Y}}\right)_{i,j}$ for the line connecting node i and j, are classically assumed to be driven by their inductance in presence of pure sinusoidal voltage and current. This assumption leads to real power flows $\underline{s}_{ij}$ proportional to the difference of voltage angles, noted θ_i at node i, between the two ends of the line as in (4.1e). To avoid any damage to transmission lines their flows are bounded by thermal limits $\ell_{ij}^{\max}$ related to the heat they can dissipate. Moreover, a power balance must be kept (4.1f) at each nodes $\mathcal{N}$ of the grid between line flows and power injections of prosumers connected to it, so in $\mathcal{N}_i$ at node i.

The goal of a peer-to-peer community Ω is to minimize the total cost which sums all individual cost functions as in (4.1a). To minimize its cost function c_n, prosumer n is able to optimize its volume traded p_n within a flexibility range defined by a lower $p_n^{\min}$ and an upper $p_n^{\max}$ bound, as expressed in (4.1d). Traded amount p_n is taken positive if prosumer n is selling electricity, and negative when buying. Considering multi-bilateral trades calls for a split of net powers, in the manner of [52], into a set of multiple bilateral trades p_{nm}. Every possible bilateral power trades within the community can be condensed in a matrix $\mathbf{P}$ such that

$$\mathbf{P} = \begin{pmatrix} p_{11} & \cdots & p_{1|\Omega|} \\ \vdots & \ddots & \vdots \\ p_{|\Omega|1} & \cdots & p_{|\Omega||\Omega|} \end{pmatrix} \tag{4.2}$$

where p_{nm} is necessarily equal to zero if prosumer m is not in prosumer n's trading partnership set ω_n. Net powers are then obtained by $p_n = \sum_{m\in\omega_n} p_{nm}$ as in (4.1c). Note that the definition of partnership set ω_n only inform on prosumer n's possible trades but does not enforce participation. As outlined in (4.1b), $\mathbf{P}$ is skew-symmetric to insure power balance of each trade, so $p_{nn} = 0$. This allows to potentially individualize prices per trade.

It is essential to notice that contrary to [95] dual variables of nodal power balances (4.1f), noted η_i, do not include the energy generation price but only prices derived from network operation. Note that congestion rights originate from (4.1e)'s dual variable. In this case, energy generation prices are given by the dual variables for trade reciprocity (4.1b), denoted $\mathbf{\Lambda} = (\lambda_{nm})$.

Directly coupling the peer-to-peer market to grid constraints as in (4.1) implies an intense involvement of the system operator at each step of the solving algorithm. To level this an exogenous approach of the network limitations could be used. Network constraints (4.1e)–(4.1f) can be condensed in a regularization function $\tilde{\zeta}_{\mathrm{DC}}$, equal to 0 if they are respected and $+\infty$ if they are violated. It can be noted that in this case $\tilde{\zeta}_{\mathrm{DC}}$ depends on the real power injections $P = (p_n)_{n\in\Omega}$. Then, problem (4.1) can be written as the following endogenous peer-to-peer electricity market regularized with DC power flows which reads

DC regularized peer-to-peer electricity market

$$\min_{P=(p_n)_{n\in\Omega},\mathbf{P}} \sum_{n\in\Omega} c_n(p_n) + \tilde{\zeta}_{\mathrm{DC}}(P) \tag{4.3a}$$

$$\text{s.t. } \mathbf{P} = -\mathbf{P}^{\mathsf{T}} \tag{4.3b}$$

$$p_n = \sum\nolimits_{m\in\omega_n} p_{nm} \qquad n \in \Omega \tag{4.3c}$$

$$p_n^{\min} \leqslant p_n \leqslant p_n^{\max} \qquad n \in \Omega. \tag{4.3d}$$

Note that 'regularized peer-to-peer market' denotes the generalized form of (4.3) with any regularization function $\tilde{\zeta}$.

4.2.2 Improved standard test case

To evaluate market responses flexible prosumers need to be defined. In addition, grid characteristics are needed to test feasibility of power commitments. Some test cases exist for peer-to-peer markets, such as in [77], but they do not provide power system's characteristics. Many standards exist for optimal power flow problems on transmission and distribution networks. However, they do not involve flexible loads similar to flexible generation. Following the notion of organized prosumer group model, proposed in [10], the focus can be put on transmission systems such as the IEEE 39-bus test system.

Hence, there is a need for a novel test case adapted to study peer-to-peer markets accounting for network constraints. The IEEE 39-bus test system is adapted to include flexible loads. Generators will keep their power boundaries. A wide flexibility range is given to consumers, with bounds from 10% to 150% of IEEE test system's fixed loads. The test case uses quadratic cost functions written as follows

$$c_n(p_n) = \frac{1}{2} a_n p_n^2 + b_n p_n. \tag{4.4}$$

Parameters a_n and b_n, inspired from [77], are summarized in Table 4.1. The final power system in Figure 4.1 could be referred as the *peer-to-peer New England* test case. The network is divided in four administrative zones close to the following states: Vermont, New Hampshire, Massachusetts and Maine (zone 1 to 4 in the same order).

4.3 Exogenous network charges

Even though (4.3) enables the system operator to recover congestion related costs, it does not guarantee the recovery of the costs of maintenance, modernization of power lines, taxes, and policies. This section proposes to replace the regularization function $\tilde{\zeta}_{\mathrm{DC}}$ with exogenous terms. First, Subsection 4.3.1 describes how these exogenous terms alter the formulation of Section 4.2 and, then, how the system operator can use them to allocate grid-related costs in Subsection 4.3.2. Simulation results are presented and discussed in Subsection 4.3.3 using the test case described in Subsection 4.2.2. Finally Subsection 4.3.4 concludes on the exogenous approach to provide network charges and gathers perspectives for further developments.

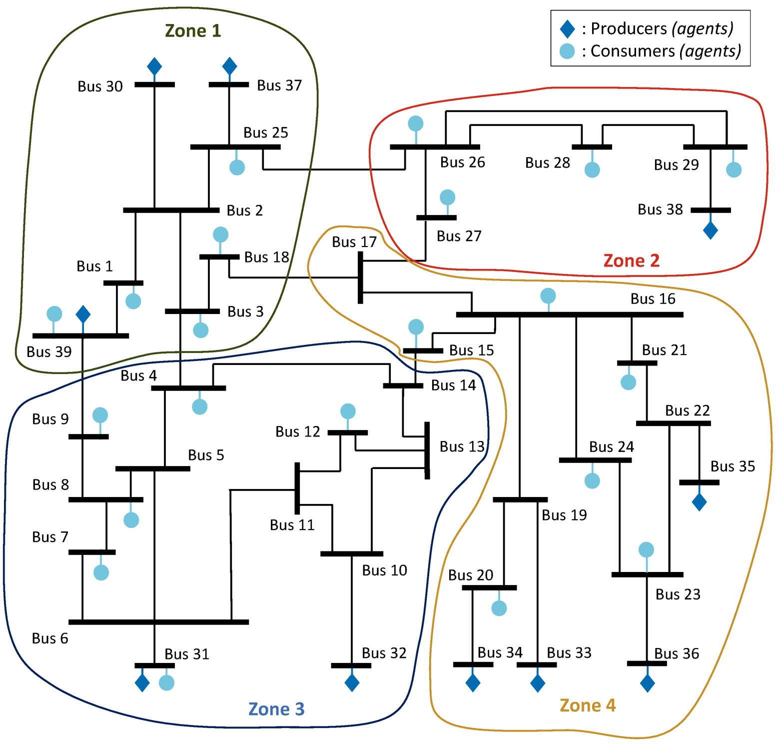


Figure 4.1: *Peer-to-peer New England* test case for joint peer-to-peer market and optimal power flow

Table 4.1: Prosumers' characteristics of *peer-to-peer New England* test case (with $p_n > 0$ when producing and < 0 when consuming)

Prosumer	Bus	a_n (€/MW^2)	b_n (€/MW)	$p_n^{\min}$ (MW)	$p_n^{\max}$ (MW)	$q_n^{\min}$ (MVar)	$q_n^{\max}$ (MVar)
1	1	0.067	64	−146.4	−9.76	−44.2	−44.2
2	3	0.047	79	−483	−32.2	−2.4	−2.4
3	4	0.047	71	−750	−50	−184	−184
4	7	0.053	62	−350.7	−23.38	−84	−84
5	8	0.082	65	−783	−52.2	−176.6	−176.6
6	9	0.052	83	−9.8	−0.65	66.6	66.6
7	12	0.087	63	−12.8	−0.853	−88	−88
8	15	0.057	81	−480	−32	−153	−153
9	16	0.050	73	−493.5	−32.9	−32.3	−32.3
10	18	0.052	69	−237	−15.8	−30	−30
11	20	0.071	62	−1020	−68	−103	−103
12	21	0.064	79	−411	−27.4	−115	−115
13	23	0.057	60	−371.3	−24.75	−84.6	−84.6
14	24	0.082	80	−462.9	−30.86	92.2	92.2
15	25	0.069	78	−336	−22.4	−47.2	−47.2
16	26	0.069	70	−208.5	−13.9	−17	−17
17	27	0.086	62	−421.5	−28.1	−75.5	−75.5
18	28	0.054	70	−309	−20.6	−27.6	−27.6
19	29	0.078	66	−425.3	−28.35	−26.9	−26.9
20	31	0.081	70	−13.8	−0.92	−4.6	−4.6
21	39	0.059	71	−1656	−110.4	−250	−250
22	30	0.089	18	0	1040	140	400
23	31	0.067	21	0	646	−100	300
24	32	0.055	37	0	725	150	300
25	33	0.082	25	0	652	0	250
26	34	0.088	17	0	508	0	167
27	35	0.076	38	0	687	−100	300
28	36	0.084	28	0	580	0	240
29	37	0.077	36	0	564	0	250
30	38	0.051	38	0	865	−150	300
31	39	0.087	19	0	1100	−100	300

4.3.1 Market design

The exogenous terms would aim not only at allocating congestion-related costs but also costs of maintenance and modernization of power lines, taxes, and policies such as e.g. renewable support schemes. Preference prices as introduced in [52], recalled in Subsection 2.1.2, seems a good candidate for this purpose. Thus, regularization function $\tilde{\zeta}_{\mathrm{DC}}$ evaluating network constraints can be replaced by the cost allocation function defined as

$$\gamma(\mathbf{P}) = \sum\nolimits_{n\in\Omega} \left[\gamma_n^0 + \sum\nolimits_{m\in\omega_n} \gamma_{nm} p_{nm}\right] \tag{4.5}$$

where parameter γ_{nm} is the network charge associated to power trade p_{nm} for the given time step. Constant terms γ_n^0, which do not affect the minimization outcome, allow to reflect costs that are independent of the power traded, such as power line investment and maintenance. Network charges γ_{nm}, detailed in Subsection 4.3.2, would then account for congestion-related costs and taxes. Function γ is separable among participants, and will be integrated in their objective function as it will be further discussed in Subsection 4.3.2. Note that γ also represents the amount of money collected by the system operator from community Ω for its use of the power system.

Exogenous peer-to-peer electricity market reformulation

In peer-to-peer electricity market (4.3) regularized with cost allocation function (4.5), reciprocity constraint (4.3b) is the only barrier to fully distribute the problem. To overcome this an additional slack variable $\mathbf{W}$ can be considered. Variable $\mathbf{W}$, which can contribute to reach consensus, aims at being the image of all possible trades $\mathbf{P}$. For this, reciprocity constraint (4.3b) is replaced by power consensus constraint (4.6b) leading to the deterministic, single time step, exogenous peer-to-peer electricity market

Exogenous peer-to-peer electricity market

$$\min_{P=(p_n)_{n\in\Omega},\mathbf{P},\mathbf{W}} \sum_{n\in\Omega} \left[c_n(p_n) + \gamma_n^0 + \sum_{m\in\omega_n} \gamma_{nm} p_{nm} \right] \tag{4.6a}$$

$$\text{s.t.} \quad \left(\mathbf{W} - \mathbf{W}^\intercal\right)/2 = \mathbf{P} \tag{4.6b}$$

$$p_n = \sum\nolimits_{m\in\omega_n} p_{nm} \qquad n \in \Omega \tag{4.6c}$$

$$p_n^{\min} \leqslant p_n \leqslant p_n^{\max} \qquad n \in \Omega \tag{4.6d}$$

$$p_{nm} \geqslant 0 \qquad n \in \Omega_g \tag{4.6e}$$

$$p_{nm} \leqslant 0 \qquad n \in \Omega_c \tag{4.6f}$$

$$p_n^{\min} \leqslant p_{nm} \leqslant p_n^{\max} \qquad n \in \Omega_p. \tag{4.6g}$$

In other words, prosumer n keeps the possibility to opt-out of the market, i.e. with outcome $(p_{nm})_m = 0$, if its power boundaries allow it.

It can be noted that problem (4.6a)–(4.6d) is convex, but not strictly convex. Indeed if parameters γ_{nm} were to be uniform overall trades, there would be a flat minima as the objective function would only be sensible to power set-points p_n which could be obtained by several combinations of prosumer n's bilateral trades $P_n = (p_{nm})_{m\in\omega_n}$. In this situation prosumers can buy a large amount of energy at a low price from one prosumer to sell it back at a higher price to another. This possibility of arbitraging can be proscribed by limiting the possible amounts traded. Generators, for which $p_n^{\min} \geqslant 0$ and grouped in Ω_g, are forbidden to buy power in (4.6e). On the other hand consumers, for which $p_n^{\max} \leqslant 0$ and grouped in Ω_c, are forbidden to sell power in (4.6f). However, prosumers, gathered in Ω_p, must still be able to either buy or sell power since they are such that $p_n^{\min} < 0 < p_n^{\max}$. Power trades of prosumers are bounded by their power boundaries as in (4.6g).

The augmented Lagrangian of (4.6) is such that

$$L_\rho(P, \mathbf{P}, \mathbf{W}, \mathbf{\Lambda}, M) = \sum\nolimits_{n\in\Omega} L_{n,\rho}(p_n, P_n, \mathbf{W}, \Lambda_n, \mu_n) \tag{4.7a}$$

with $\mathbf{W} = (w_{nm})_{n,m}$, $\mathbf{\Lambda} = (\lambda_{nm})_{n,m}$ and $M = (\mu_n)_{n\in\Omega}$. Local augmented Lagrangians read

$$\begin{aligned} L_{n,\rho}(p_n, P_n, \mathbf{W}, \Lambda_n, \mu_n) = \tilde{c}_n(p_n) + \mu_n \left(\sum\nolimits_{m\in\omega_n} p_{nm} - p_n\right) \\ + \gamma_n^0 + \sum_{m\in\omega_n} \Big[\gamma_{nm} p_{nm} + \lambda_{nm} \left((w_{nm} - w_{mn})/2 - p_{nm}\right) \\ + (\rho/2) \left((w_{nm} - w_{mn})/2 - p_{nm}\right)^2 \Big] \end{aligned} \tag{4.7b}$$

where $\rho > 0$ is the penalty factor, $P_n = (p_{nm})_{m\in\omega_n}$ and $\Lambda_n = (\lambda_{nm})_{m\in\omega_n}$. Function $\tilde{c}_n$ is the extended-value of c_n, in the sense of [53], with a domain defined by (4.6c)–(4.6g). Karush—Kuhn—Tucker (KKT) stationarity conditions of (4.7) allows to obtain equalities

$$\mu_n = \begin{cases} \frac{\partial \tilde{c}_n}{\partial p_n} & \\ \lambda_{nm} - \gamma_{nm} & m \in \omega_n \end{cases}, n \in \Omega \tag{4.8}$$

Note that the perceived price μ_n, which is the dual variable of constraint (4.6c), links prosumer n's energy cost $\frac{\partial \tilde{c}_n}{\partial p_n}$ to trading prices $(\lambda_{nm})_{m\in\omega_n}$ and network charges $(\gamma_{nm})_{m\in\omega_n}$.

The KKT conditions (4.8) of the exogenous peer-to-peer electricity market have to be compared to the ones obtained for endogenous problem (4.1). Yet, KKT optimality conditions of (4.1) can be written

$$\mu_n = \begin{cases} \frac{\partial \tilde{c}_n}{\partial p_n} \\ \lambda_{nm} - \eta_{i\in\Omega^n} \quad m \in \omega_n \end{cases}, n \in \Omega \tag{4.9}$$

where $\eta_{i\in\Omega^n}$ denotes the dual variable of nodal balance constraint (4.1f) associated to the node i on which prosumer n is connected which is stored in singleton set Ω^n. If one had complete prior knowledge of the market, they could solve the endogenous peer-to-peer economic dispatch (4.1) and deduce the optimal nodal energy prices, namely $\gamma_{nm} = \eta_{i\in\Omega^n}$. However, in doing so they would only be able to recover their costs of congestion but neither taxes nor other operation costs as proposed in this paper.

In a centralized energy market as exposed in Subsection 2.1.1, consensus constraint (4.6b) is replaced by power balance constraint $\sum_{n\in\Omega} p_n = 0$ and constraints (4.6c)–(4.6g) are non-existent. Hence, KKT stationarity conditions of the pool market give

$$\frac{\partial \tilde{c}_n}{\partial p_n} + \lambda^{\mathrm{PM}} = 0 \tag{4.10}$$

where dual variable λ^{PM} of the power balance constraint represents the pool market energy price. As both (4.6) and the pool market would be made of the same prosumers, one could readily notice that (4.6) without network charges, so with all $\gamma_{nm} = 0$, leads to a uniform trading price equal to the pool market price λ^{PM}. The system operator does not intervene in solving (4.6) as it only provides network charges γ_{nm} *a priori*. Hence, network charges are provided in a transparent manner before negotiations such that prosumers can anticipate on the over costs brought by the use of the power system.

Specific decentralized exogenous peer-to-peer electricity market algorithm

As developed in [2]'s appendix, a decentralized procedure based on the consensus ADMM of [53] can be used to solve (4.6). This decentralized method solves global problem (4.6) and, hence, leads to a competitive equilibrium which efficiency strongly depends on the chosen network charges. According to [87] ADMM seems well adapted for negotiation mechanisms in smart grids. Several extensions and convergence rate improvements have been proposed in [96–99]. Given the focus of this chapter is not on scalability a straightforward implementation of consensus ADMM is used.

The final decentralized exogenous negotiation mechanism reads

$$\begin{aligned} (p_n, P_n)^{k+1} = \underset{p_n, P_n}{\operatorname{argmin}} \quad & c_n(p_n) + \gamma_n^0 + \textstyle\sum_{m\in\omega_n} \left[\gamma_{nm} p_{nm} + \sigma^\rho\left(p_{nm}, \frac{p_{nm}^k - p_{mn}^k}{2}, \lambda_{nm}^k\right)\right] && \text{(4.11a)} \\ \text{s.t.} \quad & p_n = \textstyle\sum_{m\in\omega_n} p_{nm} \\ & p_n^{\min} \leqslant p_n \leqslant p_n^{\max} \\ & p_{nm} \geqslant 0 \qquad\qquad\quad \text{if } n \in \Omega_g \\ & p_{nm} \leqslant 0 \qquad\qquad\quad \text{if } n \in \Omega_c \\ & p_n^{\min} \leqslant p_{nm} \leqslant p_n^{\max} \quad \text{if } n \in \Omega_p \\ \lambda_{nm}^{k+1} = \lambda_{nm}^k - \rho & \left(p_{nm}^{k+1} + p_{mn}^{k+1}\right)/2 && \text{(4.11b)} \end{aligned}$$

where disagreement cost function σ^ρ is given by

$$\sigma^\rho : (x, y, z) \in \mathbb{R}^3 \mapsto z\,(y - x) + \frac{\rho}{2}(y - z)^2 \tag{4.12}$$

and the penalty factor $\rho > 0$. Element λ_{nm} of matrix $\mathbf{\Lambda}$ corresponds to generation price of electricity for traded volume p_{nm}. Possible trades of prosumer n can be grouped in variable $P_n = (p_{nm})_{m\in\omega_n}$. According to [53], supposing cost functions c_n to be closed, proper, and convex is a sufficient condition to ensure convergence of (4.11). This formulation allows to have primal feasibility of constraints (4.6c)–(4.6g) at each iteration step. However, primal feasibility of trades reciprocity (4.6b) is only verified at the limit after convergence. Note that additional terms of the augmented Lagrangian, represented by disagreement cost function σ^ρ in (4.11a), aim at encouraging, economically, a prosumer n to reach power consensus with its partners. Global stopping criteria associated to (4.11) are such as

$$\sum\nolimits_{n\in\Omega} \epsilon_n^{\mathrm{p},k+1} \leqslant \epsilon^{\mathrm{p,tol}^2} \quad \text{and} \quad \sum\nolimits_{n\in\Omega} \epsilon_n^{\mathrm{d},k+1} \leqslant \epsilon^{\mathrm{d,tol}^2} \tag{4.13}$$

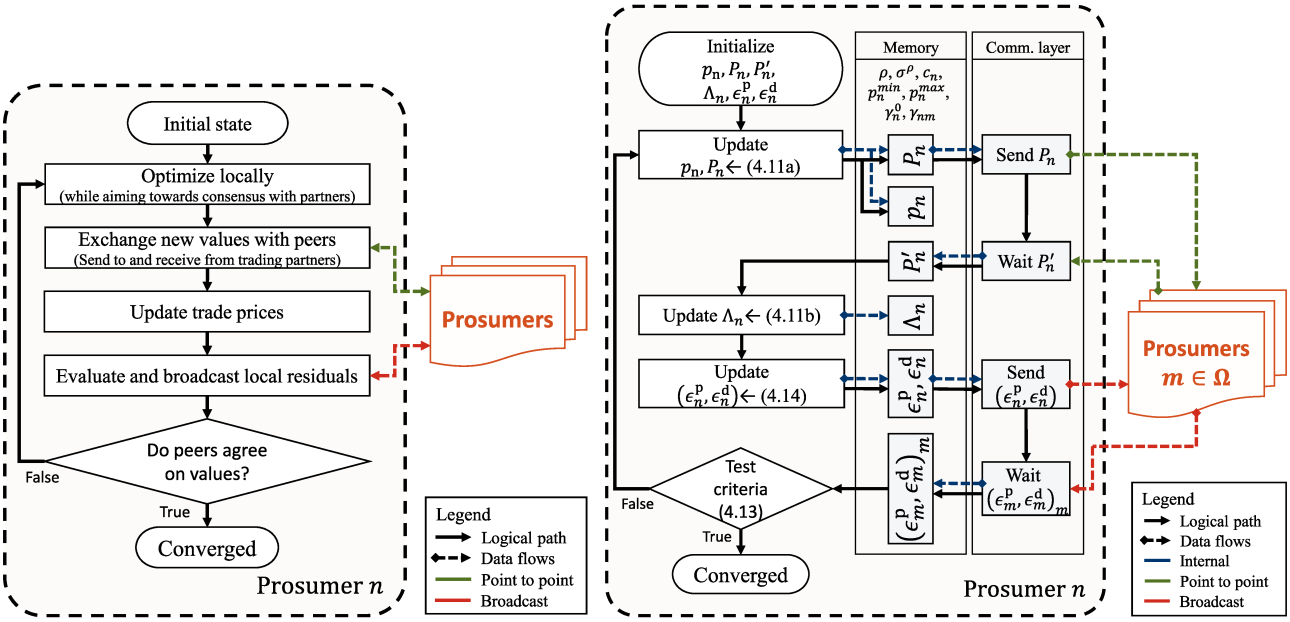
with, respectively, primal and dual local residuals

$$\epsilon_n^{\mathrm{p},k+1} = \frac{1}{4}\sum\nolimits_{m\in\omega_n} \left(p_{nm}^{k+1} + p_{mn}^{k+1}\right)^2 \tag{4.14a}$$

$$\epsilon_n^{\mathrm{d},k+1} = \sum\nolimits_{m\in\omega_n} \left(p_{nm}^{k+1} - p_{nm}^{k}\right)^2 \tag{4.14b}$$

where $\epsilon^{\mathrm{p,tol}}$ and $\epsilon^{\mathrm{d,tol}}$ denotes primal and dual global feasibility tolerances, respectively.

As illustrated in Figure 4.2, the overall exogenous negotiation mechanism occurs as follows. Each prosumer n first solves its own local optimization (4.11a) to update power set-point p_n^{k+1} and look for new bilateral trade proposals $P_n^{k+1} = (p_{nm}^{k+1})_{m\in\omega_n}$. Then, prosumer n individually sends trade proposals p_{nm}^{k+1} to each chosen partner $m \in \omega_n$. After receiving all counter proposals $P'_n = (p_{mn}^{k+1})_{m\in\omega}$, prosumer n can update the corresponding trade prices $\Lambda_n^{k+1} = (\lambda_{nm}^{k+1})_{m\in\omega}$ with (4.11b), and local residuals $\left(\epsilon_n^{\mathrm{p}}, \epsilon_n^{\mathrm{d}}\right)^{k+1}$ with (4.14). Finally, each prosumer n broadcasts its local residuals to all and, when all local residuals $\left(\epsilon_m^{\mathrm{p}}, \epsilon_m^{\mathrm{d}}\right)_{m\in\Omega\setminus\{n\}}^{k+1}$ are received, tests global stopping criteria (4.13). This process is repeated until convergence. Being decentralized the negotiation mechanism is not supported by a central entity. However, as most information exchanges, a communication standard needs to be defined by an institutional organization to associate a communication protocol between peers participating in the market. Note that



(a) Conceptual version (b) Symbolic version

Figure 4.2: Decentralized exogenous negotiation mechanism for peer-to-peer electricity markets

problem (4.6) and this specific algorithm are directly in line with the generalized coordination problem proposed in Section 2.2 since it corresponds to the one time step practical example of Subsection 2.2.3.

Privacy issues

This type of decentralized negotiation mechanism is believed to require solely local characteristics. However, a deeper analysis based on inverse problem theory [100] should be conducted to verify that exchanges of power proposals does not jeopardize this privacy. As illustrated in [101–103], privacy issues go beyond than the sole topic of multi-bilateral trades but also affect smart grids in general. Interestingly, the proposed negotiation mechanism limits the amount of transmitted information as prosumers only send their trade proposal to their direct partners and their local residuals. In this context it would be interesting to use a secured mechanism, as does [73] for prices updates. In addition, game theory studies on bounded rationality, as in [104, 105], still hold but might need some adaptations. As initiated in Section 2.3, a further game theoretical analysis may also determine the impacts of strategic behaviors on the exogenous peer-to-peer market.

4.3.2 Exogenous operation cost allocation

When the goal is to obtain a peer-to-peer market with allocation of grid-related costs it is possible to use network charges as in (4.5). Contrary to preference prices chosen by prosumers, parameters γ_{nm} are provided by the system operator *a priori* when used as network charges. As mentioned in Section 4.1, remember that, in this section, a cost allocation policy refers to the way costs are divided between peer-to-peer market participants. The proposed cost allocation policies will define how network charges are estimated. They will eventually be pondered by a coefficient named unit fee to allow a level of slackness for the system operator to reach cost recovery. Another objective of network charges may be to reduce congestion risks. In other words, it should allow the system operator to incite prosumers to behave in a beneficial way for the power system. This property is important because the outcome of (4.6) does not necessarily satisfy network constraints (4.1e)–(4.1f), as shown later in Subsection 4.3.3.

Finally, when an incident occurs on the electric network security dispositions are automated. However, the market as defined initially is not intrinsically considering this deteriorated mode. Partnership sets ω_n could be dynamically adapted in case of congestion. But this would require to duplicate the number of signals sent by the system operator to prosumers. In addition, prosumers would have to manage different routines. A simpler way to influence prosumers is to apply new unit fees. This way, network charges offer an indirect mechanism to handle deteriorated modes. Thus, network charges can push prosumers to shift from their usual partners to others unaffected by the malfunctions while keeping the same routine. At the limit, this operating mode of cost allocation policies enables market islanding. This corresponds to a security market procedure with the least grid stress while waiting for repairment.

After explicitly expressing the amount of money collected by the system operator, three cost allocation policies are proposed.

Total Fees

The money paid (resp. received) by prosumer n for buying from (resp. selling to) prosumer m is given by the perceived price $\mu_n = \lambda_{nm} - \gamma_{nm}$, as shown in (4.8). Network charges represent exogenous costs. Thus, when prosumer n consumes its γ_{nm} are negative which leads to perceived prices μ_n higher than trading prices λ_{nm}. When generating perceived prices are lower than trading prices since parameters γ_{nm} would be positive. The total money paid or received by a prosumer $n \in \Omega$, for real power bilateral trades per time step, is expressed by

$$\lambda_n^{\text{Exo}}\left(P_n\right) = \sum\nolimits_{m\in\omega_n} \lambda_{nm} p_{nm} - \underbrace{\left(\gamma_n^0 + \sum\nolimits_{m\in\omega_n} \gamma_{nm} p_{nm}\right)}_{=\gamma_n^{\text{Exo}}(P_n)} \tag{4.15}$$

where $\gamma_n^{\text{Exo}}\left(P_n\right)$ is the part reserved to the system operator.

From the system operator's point of view, the total amount of money collected through network charges is simply given by $\gamma_{\text{SO}}^{\text{Exo}}\left(\mathbf{P}\right) = \sum_{n\in\Omega} \gamma_n^{\text{Exo}}\left(P_n\right)$. As mentioned previously, the focus is put only on real power trades fees and not reactive power injection's. This money can be used to cover operation expenses – such as maintenance, power losses, power injection compensations, etc. – as well as investment cost when considering multiple time steps.

Unique Cost Allocation Policy

The way to allocate costs is to share them equally between community members. At the image of Paris' one-way trip public transportation ticket, no discrimination is made between trades. Because of the universality of this policy, prosumers in recurrently congested areas might not be spurred to behave in a responsible manner. Misbehavior of a few prosumers may penalize the rest of the community. If network charges are such that both ends of a trade are equally responsible, they can be written as

$$\gamma_{nm}^{\text{uniq}} = \pm \frac{u^{\text{uniq}}}{2}, \quad \forall (n,m) \in \Omega \times \omega_n, \tag{4.16}$$

where the sign of $\gamma_{nm}^{\text{uniq}}$ is such that $\gamma_{nm}^{\text{uniq}} p_{nm} \geqslant 0$, so $\geqslant 0$ for producers and $\leqslant 0$ for consumers. Unique unit fee u^{uniq} is expressed in €/MWh in the case of an hourly time step.

Electrical Distance Cost Allocation Policy

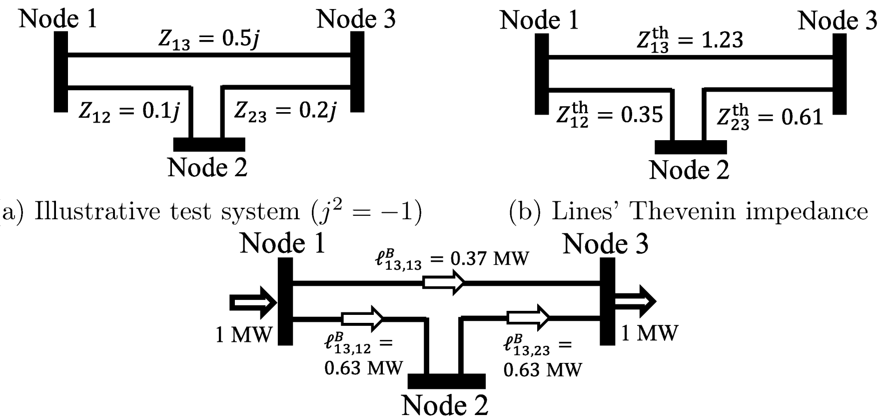
To be more precise in how costs are allocated it is possible to use network charges proportional to the electrical distance between prosumers. As for cab travels, this cost allocation policy would incite prosumers to trade with their closest electrical partners. Such policy would reflect that long electric distances are costlier to operate due to power losses for example. However, power losses can not directly be considered as they are quadratic, which can not be superposed. When both ends of a trade equally share responsibility and the previous sign convention is followed, network charges become

$$\gamma_{nm}^{\text{dist}} = \pm \frac{u^{\text{dist}} \mathrm{d}_{nm}}{2}, \quad \forall (n,m) \in \Omega \times \omega_n, \tag{4.17}$$

where d_{nm} is the electrical distance between prosumer n and prosumer m. Distance unit fee u^{dist} is expressed in €/MWh/distance unit if the same time step is used.

The definition of an electrical distance is a crucial issue for this cost allocation policy. [106] recommends two electrical distances, initially developed to allow a better vulnerability assessment through topological visualization of an electrical structure. It is possible to either consider

1. the *Thevenin Impedance Distance*, where each line is weighed by the norm of its Thevenin impedance after which a shortest path algorithm is performed to obtain the Thevenin electrical distance between two distant nodes, or
2. the *Power Transfer Distance*, where the absolute value of *Power Transfer Distribution Factors* induced by a unitary trade are summed.



(a) Illustrative test system ($j^2 = -1$)

(b) Lines' Thevenin impedance

(c) Power transfer distribution factor example

Figure 4.3: Illustrative test system for electrical distance estimations (normalized impedances)

Thevenin electrical distance $\left(\mathbf{Z}^{\text{th}}\right)_{i,l}$ between two connected nodes is evaluated by

$$\left(\mathbf{Z}^{\text{th}}\right)_{i,l} = \left|(\underline{\mathbf{Z}})_{i,i} + (\underline{\mathbf{Z}})_{l,l} - (\underline{\mathbf{Z}})_{i,l} - (\underline{\mathbf{Z}})_{l,i}\right| \tag{4.18}$$

where $(\underline{\mathbf{Z}})_{i,l} = 1/\left(\underline{\mathbf{Y}}\right)_{i,l}$ is an element of bus and branch impedance matrix. If applied to the small example of Figure 4.3a, Thevenin impedances are as in Figure 4.3b. Though multiple roads are possible for the power to flow between two nodes. For example, a power injection at node 1 and a withdrawal at node 3 only one Thevenin impedance distance value must prevail. It is proposed to use the shortest path as a metric to reflect the path on which the power exchange as the highest effect. For example, shortest path algorithm of [107] can be used. In the case of Figure 4.3b, Thevenin impedance distance between node 1 and node 3 is $d_{13}^{\text{th}} = \min\left(\left(\mathbf{Z}^{\text{th}}\right)_{1,2} + \left(\mathbf{Z}^{\text{th}}\right)_{2,3}, \left(\mathbf{Z}^{\text{th}}\right)_{1,3}\right) = 0.96$.

On the other hand, the power transfer distance between distant nodes is evaluated by

$$d_{il}^{\text{PT}} = \sum\nolimits_{(r,o)\in\mathcal{L}} \left|\ell^B_{il,ro}\right| \tag{4.19}$$

where $\ell^B_{il,ro}$ is the power transfer distribution factor, in the sense of [108], of the branch from node r to node o for an injection at node i and a withdrawal at node l. Power transfer distribution factors of Figure 4.3a for injection at node 1 and withdrawal at node 3 are given in Figure 4.3c. The resulting power transfer distance is $d_{13}^{\text{PT}} = 0.63 + 0.63 + 0.37 = 1.63$.

So the two approaches do not lead towards identical distance estimations. The Thevenin impedance distance, considering the shortest path, is more adapted to radial networks such as distribution grids. While the power transfer distance, considering a DC power flow approximation of the entire network, is better suited for meshed networks such as transmission grids. In consequence, the choice of electrical distance type has a strong impact on the efficiency of the electrical distance cost allocation policy.

Uniform Zonal Cost Allocation Policy

While the unique cost allocation policy does not differentiate prosumers, the electrical distance one might individualize too much grid tariffs. At the image of Danish public transportation system, using a zonal cost allocation policy seems a good compromise between both. In this situation the electrical network would be divided in several zones associated to distinct zonal unit fees. Each zone could be managed by a different system operator. A zone with a high price would incite outside prosumers not to trade with prosumers within, and push within prosumers towards self-consumption. In this sense, the zonal cost allocation policy allows to economically isolate an area. However, its efficiency strongly depends on zones' design.

A possible way to obtain the network charges is to sum zonal network fees of zones crossed by each trade. As mentioned previously, the electrical path is not unique which could lead to multiple lists of crossed zones. To select only one of them, the shortest electrical path criterion, as defined above with the Thevenin electrical distance, can be taken. Then, chosen crossed zones would reflect the most stressed one by a trade. For illustrative simplicity, in this section the mechanism is simplified by considering a uniform zonal unit fee. This way, the problem of how zonal unit fees are designed between zones is limited. When costs are equally shared on both ends of a trade and the sign convention is conserved, uniform zonal network charges become

$$\gamma_{nm}^{\text{zone}} = \pm \frac{u^{\text{zone}} \mathrm{N}_{nm}^{\text{zone}}}{2}, \quad \forall (n,m) \in \Omega \times \omega_n, \tag{4.20}$$

where $\mathrm{N}_{nm}^{\text{zone}}$ corresponds to the number of crossed zone for trade p_{nm}. Zonal unit fee u^{zone} is expressed in €/MWh.

Both electrical distance and zonal cost allocation policies depend on grid characteristics, supposed time independent. Their unit fees can be adapted to grid's status (e.g. between day and night). As any exogenous approach the proposed allocation policies may not ensure efficiency of the peer-to-peer market, as pointed in [95] in the case of transmission rights. Even though local marginal prices seem effective, they may be largely rejected for their opacity by peer-to-peer market participants anxious for transparency. To define the unit fees, as well as zones, the system operator can periodically, e.g. yearly, update unit fees based on the revenue adequacy and the congestion occurrence rate of the last period. This type of historical data analysis is a common practice. For example the French transmission operator, RTE, publishes[2] its transmission tariffs (or TURPE for *Tarif d'Utilisation du Réseau Public d'Electricité*) based on this type of analysis. More details on this method can be found in the "Study on tariff design for distribution systems"[3] prepared for the European Commission. Alternatively, zones can follow administrative delimitation such as states. Note that other allocation policies have recently been proposed such as in [109].

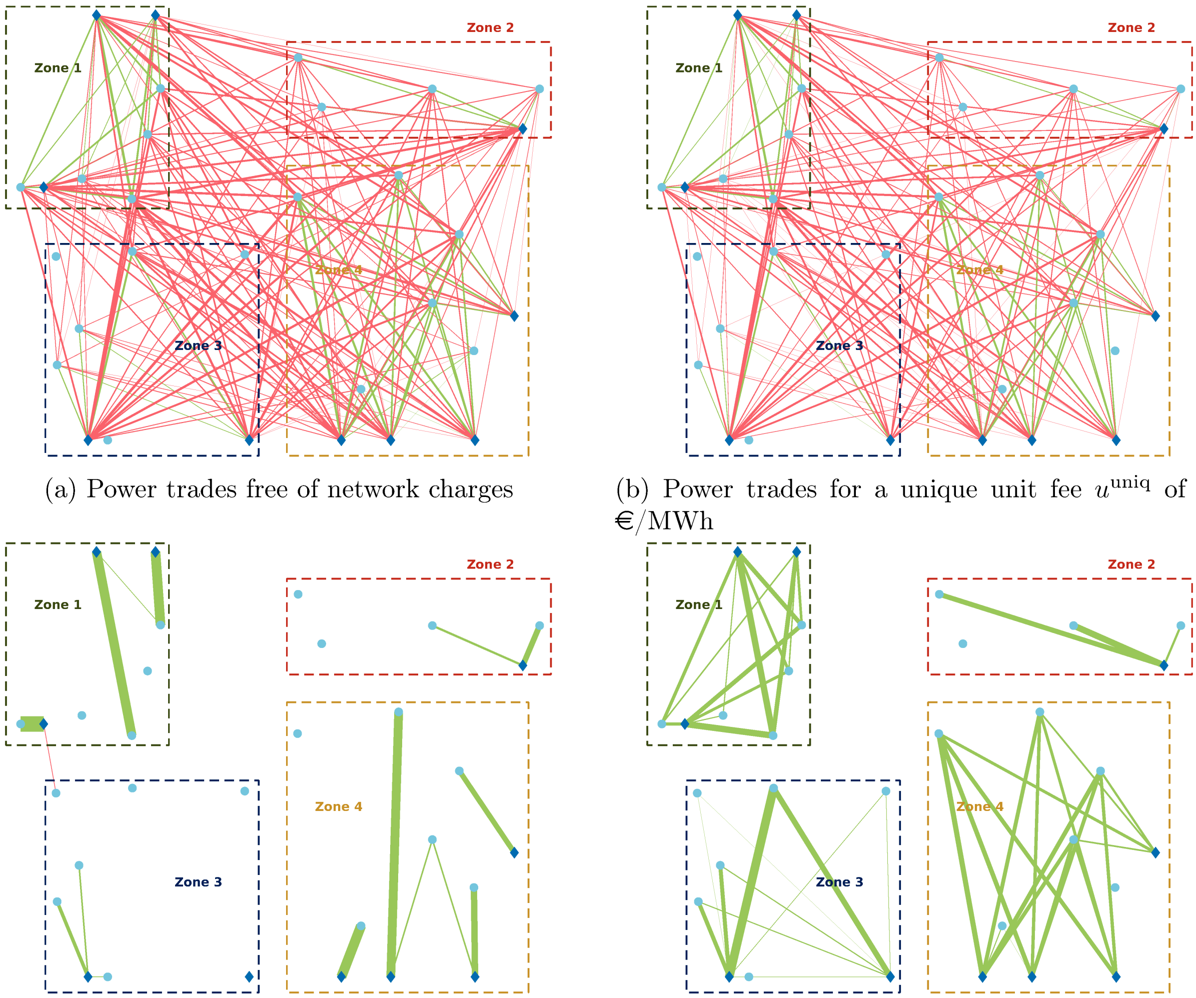
4.3.3 Simulation results

Not affecting market outcomes, constant terms γ_n^0 are set to zero. However, cost allocation policies will exert differently on the trade. For example, let consider the trade between node 16's consumer (middle left) and node 39's producer (middle right) of Figure 4.1. Since the test case is based on a meshed network the power transfer distance is used. Note that in such case unit fee u^{dist} is expressed in €/MWh. The power transfer electrical distance between node 16 and 39 is 7.3, without dimension. While Thevenin impedance distance's path, passing by nodes {16, 17, 18, 3, 2, 1, 39}, crosses two zones. This gives a ratio 7.3 between the electrical distance and uniform network charges, and a ratio of 2 in the case of zonal network charges.

Free Market

Free market refers to the peer-to-peer market without network charges. In the *peer-to-peer New England* test case, the free market leads to an electricity price of 57.2 €/MWh which is uniform as exposed in Subsection 4.3.1. Iterative process (4.11) converges in 9.5 seconds in MATLAB to primal and dual residuals below 10^{-4} when $\rho = 1$. Independently from the power system, it is important to study interactions between prosumers. Looking at how trades are distributed

[2] https://clients.rte-france.com/lang/an/clients_producteurs/services_clients/tarif.jsp
[3] https://ec.europa.eu/energy/sites/ener/files/documents/20150313%20Tariff%20report%20fina_revREF-E.PDF



(a) Power trades free of network charges

(b) Power trades for a unique unit fee u^{uniq} of 10 €/MWh

(c) Power trades for a distance unit fee u^{dist} of 10 €/MWh

(d) Power trades for a uniform zonal unit fee u^{zone} of 10 €/MWh

Figure 4.4: Cost allocation policies' influence on trades (red lines: inter-zone exchanges, green lines: intra-zone exchanges)

between prosumers and exchanges between zones seem also relevant. Moreover, grid usage can be studied in a second step to point the presence of potential congestion.

To visualize the bilateral trades it is possible to look into graph theory visualizations such as interaction diagrams. However, their interpretation in the case of multiple bilateral trades might be complex. A more intuitive visualization is to show the trades on power system's map Figure 4.1. For clarity reasons lines and buses will not be represented. To emphasize the difference between intra- and inter-zone exchanges, they will respectively be represented by green and red lines. To discriminate between main and small trades lines thickness will be proportional to the power traded. Finally, only trades over 10^{-2} MW are represented in Figure 4.4.

Figure 4.4a shows free market power exchanges between participants. Prosumers almost equally trade with all their partners. This results in many inter-zone exchanges which implies no correlation between market and power system. In other words, prosumers do not favor local trades. As a consequence a high amount of power flows between zones. The global absolute exchanges between zones is above 2 GW. However, a DC power flow analysis shows only one congested line located between node 16 and node 19, used up to 130% of its capacity. Hence the risk of congestion does not necessarily originate from exchanges between zones. Of course these quantitative, numerical results strongly depend on prosumers' characteristics.

Impact of network charges on prosumer interactions

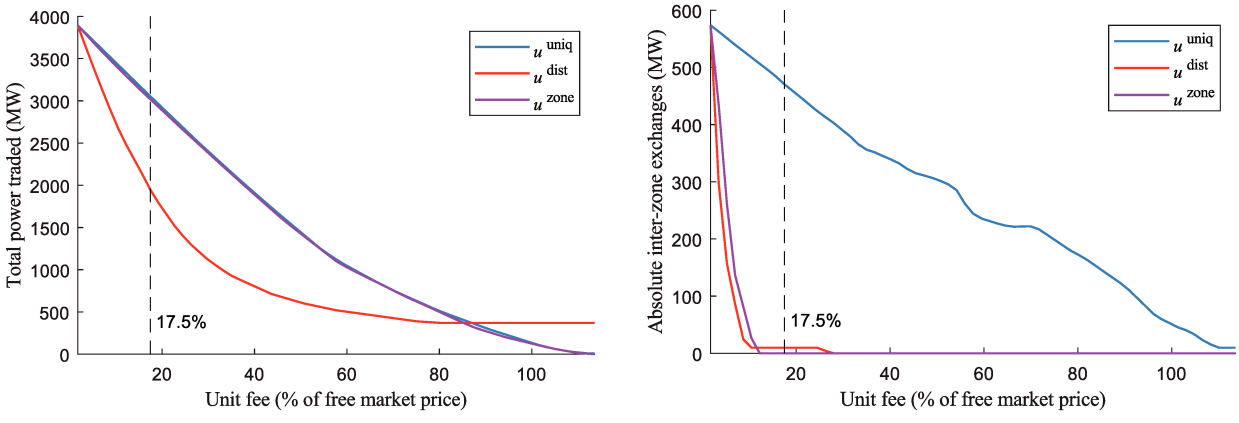
Free market results above manifest that no link exists between market operations and grid's infrastructure. Understandably, feasibility of market outcomes is not guaranteed. Even though commitments were applicable, the system operator would not be able to allocate operation costs without using network charges. The proposed cost allocation policies have a direct impact on relationships between community members and the increase of unit fees might incite prosumers to trade less or differently.

By definition the unique cost allocation policy does not discriminate trades. So, as shown in Figure 4.4b, strong interactions between zones persist even with a unique unit fee of 10 €/MWh, which is equivalent to 17.5% of the free market electricity price. This policy does not encouraged prosumers to reach a dispatch coherent with the power system. However, the level of trades and the number of relevant trades suffered from a small decrease. The possibility to impact the grid power flow thanks to a market fee is a general property.

On the other hand, with the electrical distance cost allocation policy, and the same value of unit fee, the number of relevant trades has plummeted, Figure 4.4c. Strongest prosumers appear to privilege single partnerships with each other while others are left with many small, negligible trades, so below the threshold. This reflects that market participants are restrained to a lower level of flexibility than previously, analysis deepen below. Power interactions now appear to be grouped by area making it more consistent with the network. Note that this assertion is not strict as two inter-zone trades remain relevant.

Finally, with the zonal cost allocation policy similar interactions can be reached with the same value of unit fee, Figure 4.4d. The pressure does not reach the same level of constraint on market's flexibility since more power is traded than in Figure 4.4c. In this case prosumers seem restrained to a small number of partnerships, instead of only one. Note that here no inter-zone trade continue to exist.

The sensitivities of power exchanges to unit fee variations, overall on the market and between zones, are presented in Figure 4.5. Inspection of Figure 4.5a reveals that the network charges actually decreases overall the traded volumes on the market which, as it will be detailed below, lowers the stress on the transmission network. Figure 4.5a also shows that prosumers chose to opt-out of the market when unique and zonal network charges are too high. The electrical distance policy does not steer trades between partners connected to the same electrical node. Hence, prosumers 21 and 31, both connected to bus 39, continue to trade with each other even in presence of high distance unit fees. As expected, Figure 4.5b shows that both electrical distance and zonal unit fees allow to annihilate all power exchanges between zones. This



(a) Total amount of power traded on the market (b) Total amount of power exchanged between zones

Figure 4.5: Sensitivities of power exchanges to unit fees (17.5%≡10€/MWh)

perfectly illustrate that, contrary to the unique cost allocation, electrical distance and zonal cost allocations enable to isolate zones economically.

Effects on Power Network Usage

Despite these results, market aggregation by zone does not reflect how the electrical grid is used. Cost allocation policies provides a tool for the system operator to affect the economic dispatch. Unit fees enable to tweak the expected outcomes of the peer-to-peer market in such way that line capacities might not be violated. They also offer to the system operator the chance to look for cost recovery.

Looking at power flows induced by power commitments is more relevant to evaluate the efficiency of cost allocation policies. The market does not follow any physical limitations unlike the electrical grid. Thus, the difference between a feasible and a none feasible market equilibrium lies in power network's feasibility set. In Figure 4.6 DC power flows are represented for various unique unit fees by steps of 1 €/MWh. For the given market community, in average, lines are used way below their capacity. As expected when the unit fee increases, lines usage decreases. However, one line risks congestion but a sufficient increase of unit fee allows its flow rate to fall below its capacity. More generally, the absolute disparity of line rates drops until market's flexibility is lost.

To compare the three cost allocation policies only the average (continuous) and the maximum (dash-dotted) line rates are plotted in Figure 4.7 (upper part). It can be seen that electrical distance cost allocation policy behaves differently from the other two. As an indication, a green zone corresponds to the possible range of average line rates. The lower and upper bounds are respectively defined as the average line rate for the global minimum and maximum consumption.

Both maximum and average line rates decrease linearly with for unique and zonal network charges. Their behavior is linear as they are defined uniformly or uniformly over zones. As noted above the most stressed line is within a zone. In consequence, the zonal cost allocation policy is not able to remedy the congestion in a better way than the unique cost allocation policy as zones are not defined properly. The electrical distance cost allocation policy generates a greater impact on the market which is translated in a faster decrease not only on the average but also on the maximum line rate. This allows to obtain feasibility of market commitments with a lower over-cost for market participants. However, the money collected by the system operator might not be sufficient in this case to reach cost recovery.

The total amount of money collected by the system operator given by

$$\gamma_{\mathrm{SO}}^{\mathrm{Exo}}\left(\mathbf{P}\right) = \sum\nolimits_{m\in\omega_n}\left(\gamma_n^0 + \sum\nolimits_{m\in\omega_n}\gamma_{nm}p_{nm}\right) \tag{4.21}$$

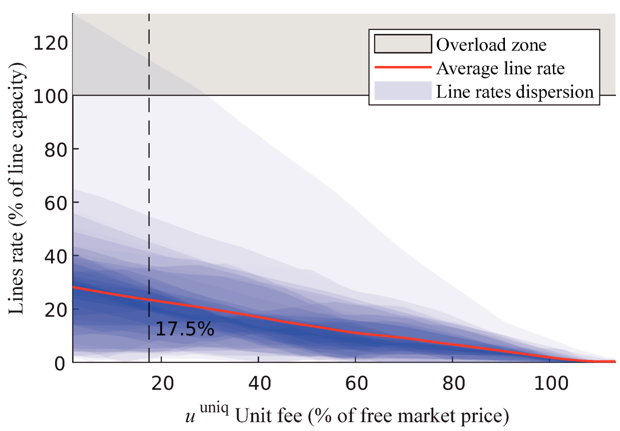


Figure 4.6: Line rates of power trades with unique network charges (17.5%≡10€/MWh)

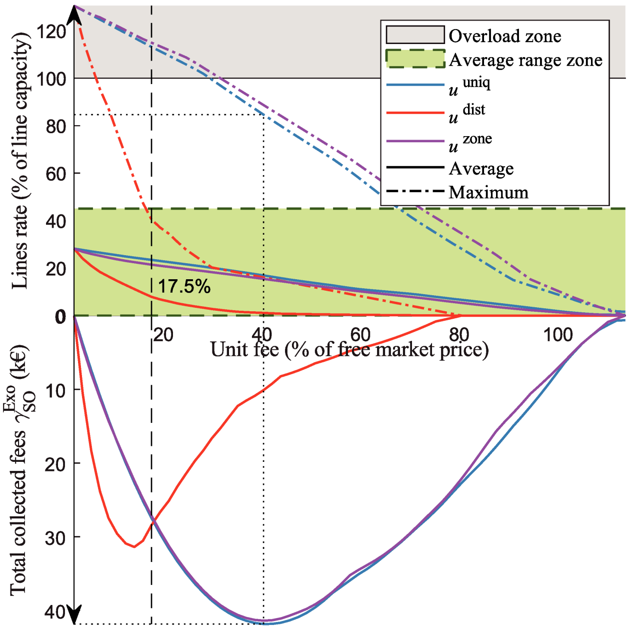


Figure 4.7: Effects of unit fees on line rates (upper part) and total money collected by the system operator (lower part) (17.5%≡10€/MWh)

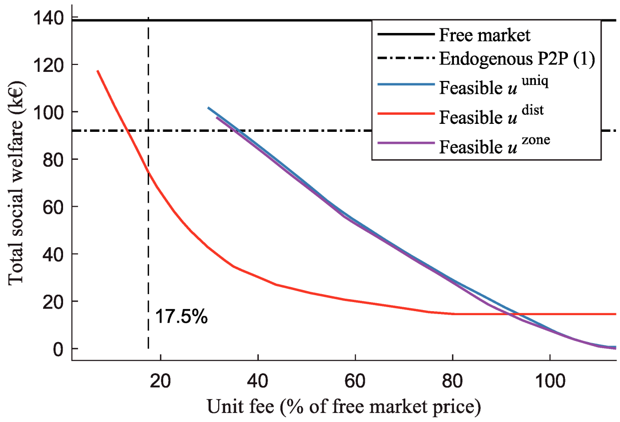


Figure 4.8: Exogenous peer-to-peer market efficiency (17.5%≡10€/MWh)

is illustrated by the lower part of Figure 4.7. Note that $\gamma_n^0 = 0$ in the current test case. From these curves it is possible to identify when the market is too constrained, so when network charges are too high. In this configuration, prosumers choose to opt-out which leads to an absence of collected money. As a consequence it can be observed that a maximum exists, dashed lines for the unique cost allocation policy. Then, the system operator can deduce if operation costs can be recovered for a given cost allocation policy. Thus, Figure 4.7 offers a graphic tool to the system operator. For a given cost allocation policy this helps choosing the unit fee to provide to market participants simply by following the proposed guide. For a given maximum acceptable line rate the corresponding unit fee and amount of money collected can be deduced. Or, for a given amount of money to collect the corresponding unit fee and maximum line rate can be deduced. One can notice that distance-based network charges seems advisable when they are low compared to the operational cost while zonal, or even unique, network charges seems more suited for network charges larger than the operational cost. A more advance study would be required to determine whether this observation is related to the test case or intrinsic to network charges' design.

The proposed network charges affect bilateral trades in a way that may introduce sub-optimalities. As shown in Figure 4.8, considering network charges, when feasible, deteriorates clearing's optimality compared to the free market without network limitations. Distance-based network charges rapidly degrades the social welfare since low unit fees suffice to impact market outcomes, as observed in Figures 4.5b and 4.7. Remember that the proposed network charges encompass more than just congestion-related costs. Hence, exogenous peer-to-peer market (4.6) can neither be compared to the endogenous peer-to-peer economic dispatch (4.1) nor to [95]. If network charges were including only congestion-related costs, one with complete prior knowledge of the market could choose them optimally as suggested by the KKT optimality conditions given in Subsection 4.3.1. Since no distributed solution approach exists, (4.1) is handled with a centralized interior-point solver. In comparison to the classical economic dispatch, the proposed method based on network charges brings transparency and is simple to implement in peer-to-peer markets. However, this may be done at the cost of technical and economical drawbacks as respectively pointed in Figures 4.7 and 4.8. This result reinforces the interest of developing a distributed approach solving (4.1) similarly to [90], which would require more involvement from the system operator.

4.3.4 Conclusions on exogenous network charges

Peer-to-peer markets are considered as a likely evolution of the power systems driven by distributed energy resources and ICT development. In this section a peer-to-peer electricity market including network charges has been considered. Network charges, provided *a priori*, have been used as incentives to account for grid-related costs in a simple and transparent way. This mechanism incites market participants to respect power system's limits, rather than enforcing them. Tested for three incentive frameworks, on a novel test case based on the IEEE 39-bus test system, it has been shown the ability of this mechanism to limit the stress put on the physical grid by the market. Network charges also allow the system operator to collect money from market participants for their use of the grid in the aim of reaching cost recovery. On the down side, the approach may lead to inefficient or unfeasible solutions when network charges are not chosen wisely.

This exogenous approach is a candidate for a future implementation of peer-to-peer markets with low involvement of the system operator. In addition, the development of network charges adapted to distribution networks, so considering reactive powers, would provide a more generic exogenous peer-to-peer market. As any consumer-centric system, it is essential to study the privacy and the security of market participants as well as the stability of the proposed design. In particular the presence of prosumers who could have the ability to be self sufficient represent a risk of snowballing effect. For each prosumer opting out, remaining prosumers would suffer from higher charges due to a redistribution between less participants. Finally, the resilience of the system to non-rational or strategic prosumers must be examined before a real world implementation.

4.4 Endogenous network charges

The above Section 4.3 showed how network charges could be simply encompassed in a decentralized peer-to-peer electricity market. The approach not only allowed the system operator to collect congestion related costs but also side costs to cover, for example, maintenance, modernization of power lines, taxes and policies. The low computation burden put on the system operator and on the market clearing was another advantage of the exogenous approach. However, the low participation of the system operator in the negotiation process came at a great price since it did not insure the respect of network constraints. Yet, network constraints violation may put the all power system at risk, in particular for weak and undersized (or at least not oversized) electrical grids. In such configuration, economical incentives of the exogenous approach are not sufficient and network constraints must actually be enforced by the system operator. Thus, another way to define network charges may be needed.

For this purpose, the system operator must take a more important part in the negotiation mechanism and determine network charges on the fly, in other words in an endogenous way. This section develops on how network charges can be determined endogenously by directly including the system operator's inputs in the negotiation process. First, Subsection 4.4.1 details the proposed market design to obtain endogenous network charges. Then Subsection 4.4.2 discusses simulation results of these endogenous network charges and compares their benefits to the exogenous approach presented above. Finally, Subsection 4.4.3 concludes on the endogenous approach to determine network charges and gathers perspectives for further developments.

4.4.1 Endogenous market design

Extending the application of the endogenous approach to weak and undersized power systems calls for considering a more precised power flow model than the DC one. The market design is described here for the full AC model and, more generally, for any network model including both active and reactive powers. The endogenous market design can then be straightly applied to not only transmission networks but also distribution networks or a mix of it as long as there is only one system operator.

Endogenous peer-to-peer electricity market with reactive power and power losses

The peer-to-peer economic dispatch considered here is a direct extension of (4.3) to AC network constraints (2.10d)–(2.10h) condensed in regularization function $\tilde{\zeta}_{\text{AC}}$, equal to 0 if they are respected and $+\infty$ if they are violated. In a direct implementation this regularization would be represented by a barrier function. It can be noted that $\tilde{\zeta}_{\text{AC}}$ would not only depend on real power injections $P = (p_n)_{n\in\Omega}$ but also on reactive power injections $Q = (q_n)_{n\in\Omega}$. Of course, prosumers would also need to acknowledge their reactive injections q_n. Prosumer n's cost function can then be written $c_n(p_n, q_n)$ to account for both active and reactive generation costs. Note that this cost function would allow to implement inverter's P-Q curve and account for their apparent power, which would be more relevant. Finally, prosumer n's reactive injection would also be ranged within a lower q_n^{min} and an upper q_n^{min} bound as expressed in the classical central optimal power flow problem (2.10).

It is essential to notice that there may be a discrepancy between real power injections of the peer-to-peer market, where supply equals demand, and power system's power balance, where the supply must also compensate for line losses. To alleviate this issue the system operator may participate in the peer-to-peer market to provide for its power losses. The system operator would then be considered as an additional agent in the peer-to-peer market buying power to compensate for active losses. By convention the peer-to-peer community Ω does not only include the prosumers but also the additional market participant, called loss provider, providing for system operator's power losses. The loss provider is identified by index $\cdot_{\text{Loss}}$. To distinguish this new peer-to-peer community $\Omega^* = \Omega \cup \{\text{Loss}\}$ from community Ω of Section 4.3, solely including prosumers, constituents of Ω^* are called market peers. Since system operators does not classically aim at losses minimization, the objective of the loss provider is the null function and, having to compensate power losses, its only local constraint is to match its active power set-point p_{Loss} equal to the power system's active power losses. Active power losses can simply be estimated by the system operator through the sum of active powers injected in the power system, as in (4.22f). Thus, loss provider's active power bounds are infinite and its reactive power bounds equal zero. Note that the system operator's constraint (4.22c) is redundant with the set of nodal power balances.

Hence, the new peer-to-peer electricity market regularized with AC power flows reads

AC regularized peer-to-peer electricity market

$$\min_{(p_n)_{n\in\Omega^*},(q_n)_{n\in\Omega^*},\mathbf{P},\mathbf{W}} \sum_{n\in\Omega^*} c_n(p_n,q_n)+\tilde{\zeta}_{\mathrm{AC}}(P,Q) \tag{4.22a}$$
$$\text{s.t.}\quad \left(\mathbf{W}-\mathbf{W}^\intercal\right)/2=\mathbf{P} \tag{4.22b}$$
$$p_n=\sum\nolimits_{m\in\omega_n} p_{nm} \qquad n\in\Omega^* \tag{4.22c}$$
$$p_n^{\min}\leqslant p_n\leqslant p_n^{\max} \qquad n\in\Omega^* \tag{4.22d}$$
$$q_n^{\min}\leqslant q_n\leqslant q_n^{\max} \qquad n\in\Omega^* \tag{4.22e}$$
$$p_{\mathrm{Loss}}=-\sum\nolimits_{n\in\Omega} p_n \tag{4.22f}$$

which adds the estimation of active power losses to the regularized peer-to-peer electricity market of (4.3). This problem is still compatible with the preferences of [52], for this one solely has to consider them in an extended form of prosumer cost function such that $c_n(p_n,q_n,P_n)$. Node voltages and line flows are not considered as optimization variables as they are side variables internal to $\tilde{\zeta}_{\mathrm{AC}}$. Moreover, it may be noted that the AC model could even be extended to add high-voltage DC power lines in the power system, for example such as expressed in [110]. Being internal to the regularized function, the change of model is transparent to prosumers.

Straightforward distributed endogenous peer-to-peer market algorithm

As developed in [2]'s appendix, bilateral trades can be negotiated in a decentralized manner based on the consensus ADMM. However, the network constraints, common to all, can not be directly decoupled from prosumers in a decentralized manner but must be handled centrally by the system operator. At the image of [53]'s consensus ADMM with regularization, the system operator does not directly verify network constraints on prosumers power injections $P=(p_n)_{n\in\Omega}$ and $Q=(q_n)_{n\in\Omega}$ but on a copy of them, respectively noted $P^{\mathrm{SO}}=(p_n^{\mathrm{SO}})_{n\in\Omega}$ and $Q^{\mathrm{SO}}=(q_n^{\mathrm{SO}})_{n\in\Omega}$. Consensus between power injections and their respective copy would then be insured by (4.23b)–(4.23c). Being a sum of power injections over prosumers, losses estimation takes [53]'s sharing ADMM form. Thus, losses estimation should be carried by the loss provider on $(p_n^{\mathrm{Loss}})_{n\in\Omega}$ copies of system operator's active power injections $P^{\mathrm{SO}}=(p_n^{\mathrm{SO}})_{n\in\Omega}$, transforming (4.22f) into (4.23i). Thus, (4.23b)–(4.23c) are common to prosumers but not to the loss provider.

The straightforward application of [53]'s consensus ADMM with regularization on (4.3) leads to the following problem reformulation

$$\min_{\substack{P_*=(p_n)_{n\in\Omega^*},Q_*=(q_n)_{n\in\Omega^*},\mathbf{P},\mathbf{W},(p_n^{\mathrm{Loss}})_{n\in\Omega},\\ P^{\mathrm{SO}}=(p_n^{\mathrm{SO}})_{n\in\Omega},Q^{\mathrm{SO}}=(q_n^{\mathrm{SO}})_{n\in\Omega}}} \sum_{n\in\Omega^*} c_n(p_n,q_n)+\tilde{\zeta}_{\mathrm{AC}}(P^{\mathrm{SO}},Q^{\mathrm{SO}}) \tag{4.23a}$$
$$\text{s.t.}\quad p_n^{\mathrm{SO}}=p_n \qquad n\in\Omega \tag{4.23b}$$
$$q_n^{\mathrm{SO}}=q_n \qquad n\in\Omega^* \tag{4.23c}$$
$$p_n^{\mathrm{SO}}=p_n^{\mathrm{Loss}} \qquad n\in\Omega \tag{4.23d}$$
$$\left(\mathbf{W}-\mathbf{W}^\intercal\right)/2=\mathbf{P} \tag{4.23e}$$
$$p_n=\sum\nolimits_{m\in\omega_n} p_{nm} \qquad n\in\Omega^* \tag{4.23f}$$
$$p_n^{\min}\leqslant p_n\leqslant p_n^{\max} \qquad n\in\Omega^* \tag{4.23g}$$
$$q_n^{\min}\leqslant q_n\leqslant q_n^{\max} \qquad n\in\Omega^* \tag{4.23h}$$
$$p_{\mathrm{Loss}}=-\sum\nolimits_{n\in\Omega} p_n^{\mathrm{Loss}} \tag{4.23i}$$

where the system operator solely handles optimization variables $P^{\mathrm{SO}} = (p_n^{\mathrm{SO}})_{n\in\Omega}$ and $Q^{\mathrm{SO}} = (q_n^{\mathrm{SO}})_{n\in\Omega}$, while the loss provider handles variables p_{Loss}, q_{Loss}, $P_{\mathrm{Loss}} = (p_{\mathrm{Loss}\,m})_{m\in\omega_{\mathrm{Loss}}}$ and $(p_n^{\mathrm{Loss}})_{n\in\Omega}$.

After simplifications such as in [2]'s appendix, for reciprocity constraint (4.23e), and in [53], for consensus constraints (4.23b)–(4.23c) and sharing constraints (4.23d) and (4.23i), the ADMM of (4.23) gives the following distributed endogenous negotiation mechanism

$$
\begin{aligned}
(p_n, q_n, P_n)^{k+1} = \quad & \underset{\substack{p_n, q_n,\\ P_n=(p_{nm})_{m\in\omega_n}}}{\operatorname{argmin}} \quad c_n(p_n,q_n) + \textstyle\sum_{m\in\omega_n} \sigma^\rho\left(p_{nm}, \frac{p_{nm}^k - p_{mn}^k}{2}, \lambda_{nm}^k\right) \\
& \qquad\qquad +\sigma^\rho\left(p_n, p_n^{\mathrm{SO},k}, \eta_n^{p,k}\right) + \sigma^\rho\left(q_n, q_n^{\mathrm{SO},k}, \eta_n^{q,k}\right) \\
& \qquad \text{s.t.} \quad p_n = \textstyle\sum_{m\in\omega_n} p_{nm} \\
& \qquad\qquad p_n^{\min} \leqslant p_n \leqslant p_n^{\max} \\
& \qquad\qquad q_n^{\min} \leqslant q_n \leqslant q_n^{\max}
\end{aligned} \tag{4.24a}
$$

$$
\begin{aligned}
(P^{\mathrm{SO}}, Q^{\mathrm{SO}})^{k+1} = \quad & \underset{\substack{P^{\mathrm{SO}}=(p_n^{\mathrm{SO}})_{n\in\Omega},\\ Q^{\mathrm{SO}}=(q_n^{\mathrm{SO}})_{n\in\Omega}}}{\operatorname{argmin}} \quad \tilde{\zeta}_{\mathrm{AC}}(P^{\mathrm{SO}}, Q^{\mathrm{SO}}) + \textstyle\sum_{n\in\Omega}\Big[\sigma^\rho\left(p_n^{k+1}, p_n^{\mathrm{SO}}, \eta_n^{p,k}\right) \\
& \qquad\qquad +\sigma^\rho\left(q_n^{k+1}, q_n^{\mathrm{SO}}, \eta_n^{q,k}\right) \\
& \qquad\qquad +\sigma^\rho\left(p_n^{\mathrm{SO},k} - \tfrac{1}{|\Omega|}p_{\mathrm{Loss}}^{k+1}, p_n^{\mathrm{SO}} - \tfrac{1}{|\Omega|}p_{\mathrm{Loss}}^{\mathrm{SO},k}, \tfrac{1}{|\Omega|}\eta_{\mathrm{Loss}}^{p,k}\right)\Big]
\end{aligned} \tag{4.24b}
$$

$$
p_{\mathrm{Loss}}^{\mathrm{SO},k+1} = -\sum_{n\in\Omega} p_n^{\mathrm{SO},k+1}, \quad q_{\mathrm{Loss}}^{\mathrm{SO},k+1} = q_{\mathrm{Loss}}^{k+1} \tag{4.24c}
$$

$$
\lambda_{nm}^{k+1} = \lambda_{nm}^k - \rho\left(p_{nm}^{k+1} + p_{mn}^{k+1}\right)/2 \tag{4.24d}
$$

$$
\eta_n^{p,k+1} = \eta_n^{p,k} + \rho\left(p_n^{\mathrm{SO},k+1} - p_n^{k+1}\right) \tag{4.24e}
$$

$$
\eta_n^{q,k+1} = \eta_n^{q,k} + \rho\left(q_n^{\mathrm{SO},k+1} - q_n^{k+1}\right) \tag{4.24f}
$$

where disagreement cost function σ^ρ is given by

$$
\sigma^\rho : (x,y,z) \in \mathbb{R}^3 \mapsto z\,(y-x) + \frac{\rho}{2}(y-x)^2 \tag{4.25}
$$

and the penalty factor $\rho > 0$. Lagrangian multipliers η_n^p and η_n^q are respectively the dual variables of constraints (4.23b) and (4.23c) for $n \in \Omega$, and of (4.23d) and (4.23c) for $n =$ Loss. As in Subsection 4.3.1's exogenous negotiation mechanism, element λ_{nm} of matrix $\mathbf{\Lambda}$ corresponds to generation price of electricity for traded volume p_{nm}. Possible trades of market peer n can be grouped in variable $P_n = (p_{nm})_{m\in\omega_n}$. According to [53], supposing cost functions $(c_n)_{n\in\Omega}$ and $\tilde{\zeta}_{\mathrm{AC}}$ to be closed, proper, and convex is a sufficient condition to ensure convergence of (4.24). Of course, this is not true for $\tilde{\zeta}_{\mathrm{AC}}$ modeling lines with the AC power flow model, which equations were recalled in Subsection 2.1.1. The grid would then need to be modeled with a convex relaxation of the AC power flow model such as with second order cone programming [48, 49], condensed in extended-value function $\tilde{\zeta}_{\mathrm{SOCP}}$, or semi-definite programming [51], condensed in $\tilde{\zeta}_{\mathrm{SDP}}$.

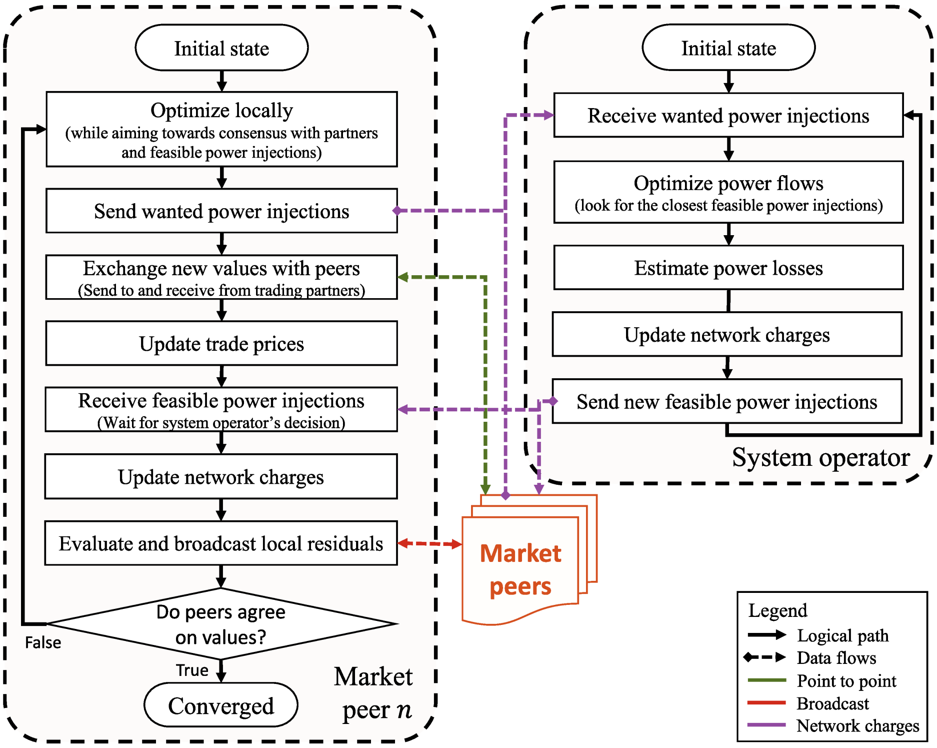
This negotiation mechanism allows to have primal feasibility of constraints (4.23f)–(4.23i) at each iteration step. However, primal feasibility of trades reciprocity (4.23e) and of power injections consensus (4.23b)–(4.23c) are only verified at the limit after convergence. Note that the terms encompassed in disagreement function σ^ρ aim at economically encouraging prosumers to reach consensus with their trading partners and the system operator. Global stopping criteria associated to (4.24) are such as

$$
\sum_{n\in\Omega^*} \epsilon_n^{\mathrm{p},k+1} \leqslant {\epsilon^{\mathrm{p,tol}}}^2 \quad \text{and} \quad \sum_{n\in\Omega^*} \epsilon_n^{\mathrm{d},k+1} \leqslant {\epsilon^{\mathrm{d,tol}}}^2 \tag{4.26}
$$

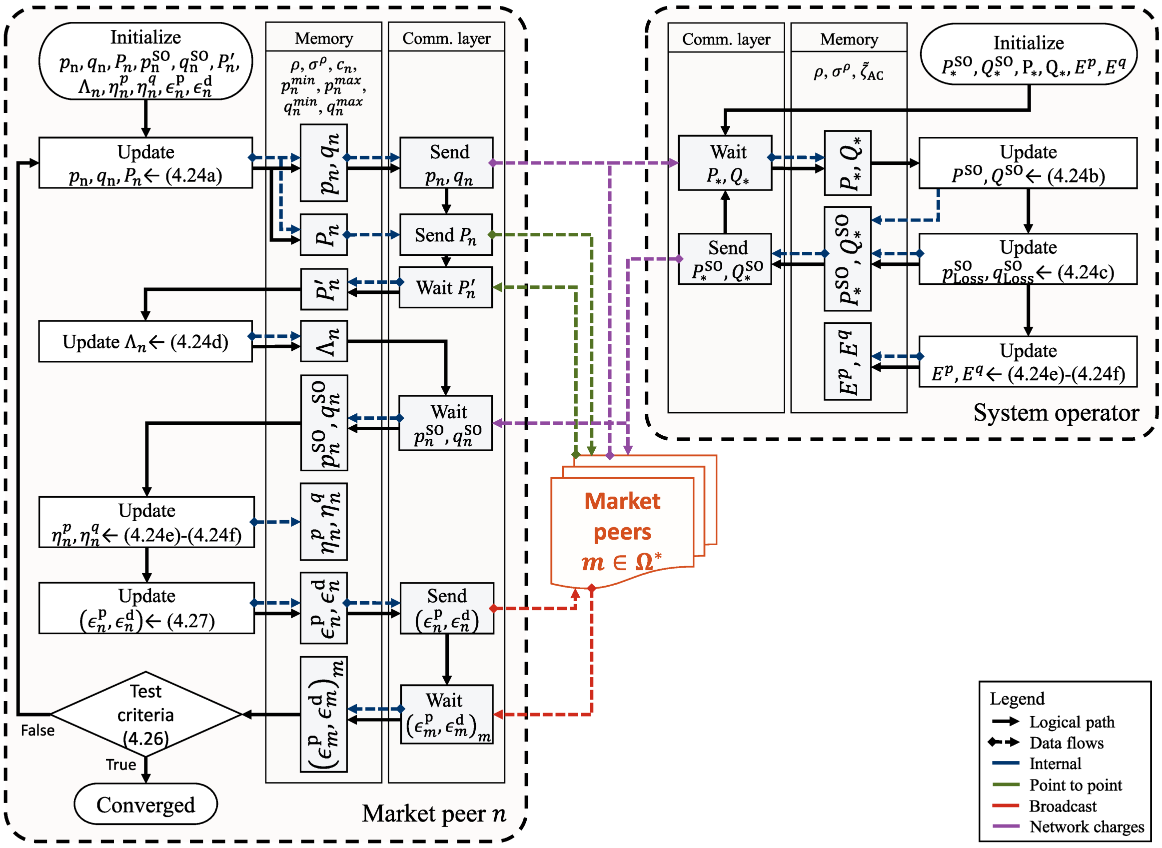
with, respectively, primal and dual local residuals

$$
\epsilon_n^{\mathrm{p},k+1} = \left(p_n^{\mathrm{SO},k+1} - p_n^{k+1}\right)^2 + \left(q_n^{\mathrm{SO},k+1} - q_n^{k+1}\right)^2 + \frac{1}{4}\sum_{m\in\omega_n}\left(p_{nm}^{k+1} + p_{mn}^{k+1}\right)^2 \tag{4.27a}
$$

$$
\epsilon_n^{\mathrm{d},k+1} = \left(p_n^{k+1} - p_n^k\right)^2 + \left(q_n^{k+1} - q_n^k\right)^2 + \sum_{m\in\omega_n}\left(p_{nm}^{k+1} - p_{nm}^k\right)^2 \tag{4.27b}
$$



(a) Conceptual version



(b) Symbolic version

Figure 4.9: Distributed endogenous negotiation mechanism for peer-to-peer electricity markets

where $\epsilon^{\mathrm{p,tol}}$ and $\epsilon^{\mathrm{d,tol}}$ denotes primal and dual global feasibility tolerances, respectively. Note that local residuals (4.27) of the decentralized exogenous peer-to-peer market algorithm have been extended into (4.27) to also account for active and reactive power injections consensus residuals.

As illustrated in Figure 4.9, the overall distributed endogenous negotiation mechanism occurs as follows. Each market peer n first solves its own local optimization (4.24a) to update active power set-point p_n^{k+1}, reactive power set-point q_n^{k+1} and look for better bilateral trade proposals $P_n^{k+1} = (p_{nm}^{k+1})_{m\in\omega_n}$. Then, market peer n shares its power set-points with the system operator and individually send trade proposals p_{nm}^{k+1} to each partner $m \in \omega_n$. When all power injections $P_*^{k+1} = (p_n^{k+1})_{n\in\Omega^*}$ and $Q_*^{k+1} = (q_n^{k+1})_{n\in\Omega^*}$ are gathered, the system operator looks for the closest feasible injection plan $P^{\mathrm{SO},k+1} = (p_n^{\mathrm{SO},k+1})_{n\in\Omega}$ and $Q^{\mathrm{SO},k+1} = (q_n^{\mathrm{SO},k+1})_{n\in\Omega}$, which satisfies the power system's constraints such as in (4.24b). The system operator then completes $P_*^{\mathrm{SO},k+1} = (p_n^{\mathrm{SO},k+1})_{n\in\Omega^*}$ and $Q_*^{\mathrm{SO},k+1} = (q_n^{\mathrm{SO},k+1})_{n\in\Omega^*}$ by estimating the resulting power losses $p_{\mathrm{Loss}}^{\mathrm{SO},k+1}$ and $q_{\mathrm{Loss}}^{\mathrm{SO},k+1}$ with (4.24c) and send overall power set-points back to market peers. To know what prosumers owes for their network usage, the system operator also keeps track of network charges $E^{p,k+1} = \left(\eta_n^{p,k+1}\right)_{n\in\Omega}$ and $E^{q,k+1} = \left(\eta_n^{q,k+1}\right)_{n\in\Omega}$ by updating them at each iteration with (4.24e)–(4.24f). While waiting for the system operator's feedback, market peers can update their trading prices $\Lambda_n^{k+1} = (\lambda_{nm}^{k+1})_{m\in\omega}$ with (4.24d) once it received all counter proposals $P_n' = (p_{mn}^{k+1})_{m\in\omega}$. When finally receiving feasible power injections $p_n^{\mathrm{SO},k+1}$ and $q_n^{\mathrm{SO},k+1}$, each market peer n can update the corresponding network charges $\eta_n^{p,k+1}$ and $\eta_n^{q,k+1}$ with (4.24e)–(4.24f). Market peers are now able to deduce their local residuals $\left(\epsilon_n^{\mathrm{p}}, \epsilon_n^{\mathrm{d}}\right)^{k+1}$ with (4.27). Finally, each market peer n broadcasts its local residuals to all and, when all local residuals $\left(\epsilon_m^{\mathrm{p}}, \epsilon_m^{\mathrm{d}}\right)_{m\in\Omega^*\setminus\{n\}}^{k+1}$ are received, tests global stopping criteria (4.26). This process is repeated until convergence.

Modified decentralized endogenous peer-to-peer market algorithm

The straightforward application of ADMM, in its consensus with regularization form, presents the main disadvantage to sequence bilateral market peer's local optimization (4.24a) and, then, system operator's power flow feasibility search (4.24b). It is proposed here to adapt the formulation of (4.23) such that (4.22) can be solved in a decentralized manner. This way system operator's computation can be done in parallel to the one of market peers (or at least of prosumers if it also handles loss providers computation). This potentially allows to gain computation time. For this purpose system operator's extended-value objective function $\tilde{\zeta}_{\mathrm{AC}}$ should be treated as any other market peer. Its goal would not be to reach reciprocity on bilateral trades such as between market peers, but to reach consensus with all market peers' power set-points. To do so, as done in [53]'s consensus ADMM, global slack variables are added to the problem. Equality constraints (4.23b)–(4.23d) are thus replaced by

$$
\begin{aligned}
p_n^{\mathrm{SO}} &= p_n^{\mathrm{c}} && n \in \Omega && (4.28a)\\
p_n^{\mathrm{c}} &= p_n && n \in \Omega && (4.28b)\\
q_n^{\mathrm{SO}} &= q_n^{\mathrm{c}} && n \in \Omega^* && (4.28c)\\
q_n^{\mathrm{c}} &= q_n && n \in \Omega^* && (4.28d)\\
p_n^{\mathrm{SO}} &= p_n^{\mathrm{cLoss}} && n \in \Omega && (4.28e)\\
p_n^{\mathrm{cLoss}} &= p_n^{\mathrm{Loss}} && n \in \Omega && (4.28f)
\end{aligned}
$$

where $\cdot^c$ denotes a copy variable. Variable $P^{\mathrm{c}} = (p_n^{\mathrm{c}})_{n\in\Omega}$ copies $P = (p_n)_{n\in\Omega}$ and $P^{\mathrm{SO}} = (p_n^{\mathrm{SO}})_{n\in\Omega}$ while variable $Q^{\mathrm{c}} = (q_n^{\mathrm{c}})_{n\in\Omega}$ copies $Q = (q_n)_{n\in\Omega}$ and $Q^{\mathrm{SO}} = (q_n^{\mathrm{SO}})_{n\in\Omega}$. And, variable $P^{\mathrm{cLoss}} = (p_n^{\mathrm{cLoss}})_{n\in\Omega}$ copies $P^{\mathrm{Loss}} = (p_n^{\mathrm{Loss}})_{n\in\Omega}$ and $P^{\mathrm{SO}} = (p_n^{\mathrm{SO}})_{n\in\Omega}$. Note that consensus constraints (4.23b)–(4.23d) are respectively equivalent to (4.28a) + (4.28b), (4.28c) + (4.28d) and (4.28e) + (4.28f).

After simplifications similar to the ones made for bilateral trades, the final decentralized endogenous negotiation mechanism, solving (4.23) with (4.28) instead of (4.23b)–(4.23d), reads

$$\begin{aligned}
(p_n, q_n, P_n)^{k+1} = &\underset{p_n,q_n,P_n=(p_{nm})_{m\in\omega_n}}{\operatorname{argmin}} \quad c_n(p_n,q_n) + \sum\nolimits_{m\in\omega_n} \sigma^\rho\left(p_{nm}, \tfrac{p_{nm}^k - p_{mn}^k}{2}, \lambda_{nm}^k\right) \\
&\qquad + \sigma^\rho\left(p_n, \tfrac{p_n^{\mathrm{SO},k}+p_n^k}{2}, \eta_n^{p,k}\right) + \sigma^\rho\left(q_n, \tfrac{q_n^{\mathrm{SO},k}+q_n^k}{2}, \eta_n^{q,k}\right) \\
&\quad \text{s.t.} \quad p_n = \sum\nolimits_{m\in\omega_n} p_{nm} \\
&\qquad\quad p_n^{\min} \leqslant p_n \leqslant p_n^{\max} \\
&\qquad\quad q_n^{\min} \leqslant q_n \leqslant q_n^{\max}
\end{aligned} \tag{4.29a}$$

$$\begin{aligned}
(P^{\mathrm{SO}}, Q^{\mathrm{SO}})^{k+1} = &\underset{\substack{P^{\mathrm{SO}}=(p_n^{\mathrm{SO}})_{n\in\Omega},\\ Q^{\mathrm{SO}}=(q_n^{\mathrm{SO}})_{n\in\Omega}}}{\operatorname{argmin}} \quad \tilde{\zeta}_{\mathrm{AC}}(P^{\mathrm{SO}}, Q^{\mathrm{SO}}) + \sum\nolimits_{n\in\Omega}\Bigg[\sigma^\rho\left(\tfrac{p_n^{\mathrm{SO},k}+p_n^k}{2}, p_n^{\mathrm{SO}}, \eta_n^{p,k}\right) \\
&\qquad + \sigma^\rho\left(\tfrac{q_n^{\mathrm{SO},k}+q_n^k}{2}, q_n^{\mathrm{SO}}, \eta_n^{q,k}\right) \\
&\qquad + \sigma^\rho\left(p_n^{\mathrm{SO},k} - \tfrac{1}{|\Omega|}\tfrac{p_{\mathrm{Loss}}^{\mathrm{SO},k}+p_{\mathrm{Loss}}^k}{2}, p_n^{\mathrm{SO}} - \tfrac{1}{|\Omega|}p_{\mathrm{Loss}}^{\mathrm{SO},k}, \tfrac{1}{|\Omega|}\eta_{\mathrm{Loss}}^{p,k}\right)\Bigg]
\end{aligned} \tag{4.29b}$$

$$p_{\mathrm{Loss}}^{\mathrm{SO},k+1} = -\sum\nolimits_{n\in\Omega} p_n^{\mathrm{SO},k+1}, \quad q_{\mathrm{Loss}}^{\mathrm{SO},k+1} = q_{\mathrm{Loss}}^k \tag{4.29c}$$

$$\lambda_{nm}^{k+1} = \lambda_{nm}^k - \rho\left(p_{nm}^{k+1} + p_{mn}^{k+1}\right)/2 \tag{4.29d}$$

$$\eta_n^{p,k+1} = \eta_n^{p,k} + \rho\left(p_n^{\mathrm{SO},k+1} - p_n^{k+1}\right)/2 \tag{4.29e}$$

$$\eta_n^{q,k+1} = \eta_n^{q,k} + \rho\left(q_n^{\mathrm{SO},k+1} - q_n^{k+1}\right)/2 \tag{4.29f}$$

with $\rho > 0$. Even though they are written on different lines, due to the difference of formulation, iteration steps (4.29a) and (4.29b)–(4.29c) are actually computed at the same time. This can be testified by the fact that, contrary to (4.24b)–(4.24c), system operator's steps (4.29b)–(4.29c) solely consider previous iteration step information. This time Lagrangian multiplier η_n^p for $n \in \Omega$ is the dual variable of both (4.28a) and (4.28b), since KKT stationarity condition of (4.28) instead of (4.23b)–(4.23d) on variable p_n^c shows that they are equal. Similarly η_n^q for any $n \in \Omega^*$ is the dual variable of (4.28c) or (4.28d). In the same way and as in its distributed version, Lagrangian multiplier η_{Loss}^p is the dual variable of both (4.28e) and (4.28f). Neither the fact that Lagrangian multipliers η_n^p and η_n^q do not refer to the same consensus constraints nor that they are computed differently in (4.24) and (4.29) alters their value after convergence. Indeed, if one looked at the KKT stationarity conditions of (4.23) and of (4.23) with (4.28) instead of (4.23b)–(4.23d), they would obtain the same equations when deriving on variables p_n, q_n, p_n^{SO} and q_n^{SO}.

Such as with (4.24), this negotiation mechanism allows to have primal feasibility of constraints (4.23f)–(4.23i) at each iteration step. However, primal feasibility of trades reciprocity (4.23e) and of power injections consensus (4.28) are only verified at the limit after convergence. Global stopping criteria associated to (4.29) are such as

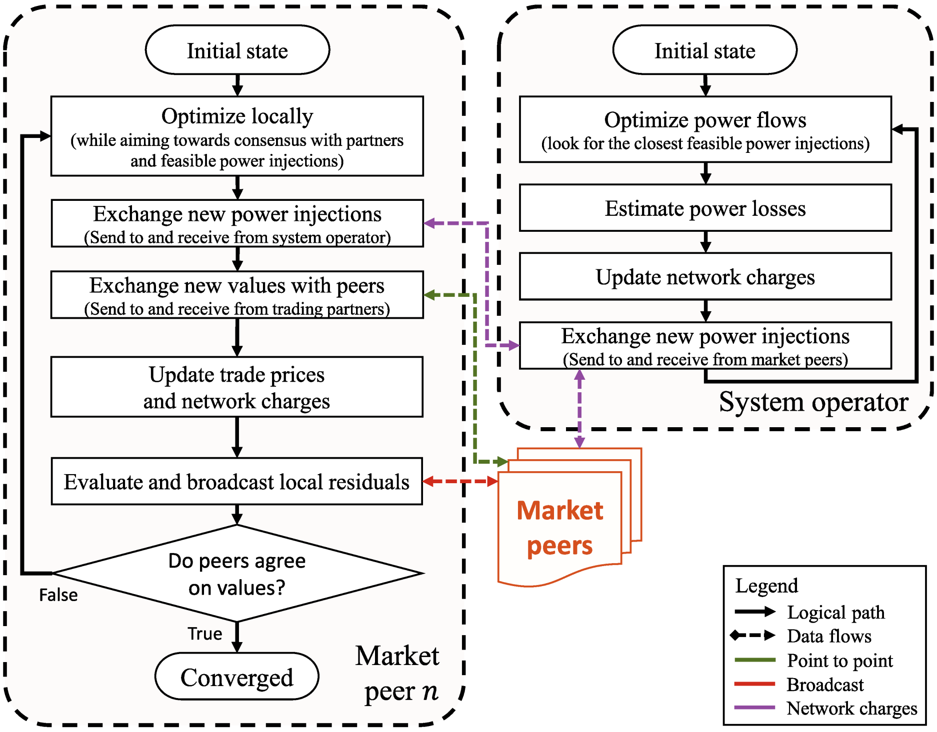
$$\sum\nolimits_{n\in\Omega^*} \epsilon_n^{\mathrm{p},k+1} \leqslant {\epsilon^{\mathrm{p,tol}}}^2 \quad \text{and} \quad \sum\nolimits_{n\in\Omega^*} \epsilon_n^{\mathrm{d},k+1} \leqslant {\epsilon^{\mathrm{d,tol}}}^2 \tag{4.30}$$

with, respectively, primal and dual local residuals

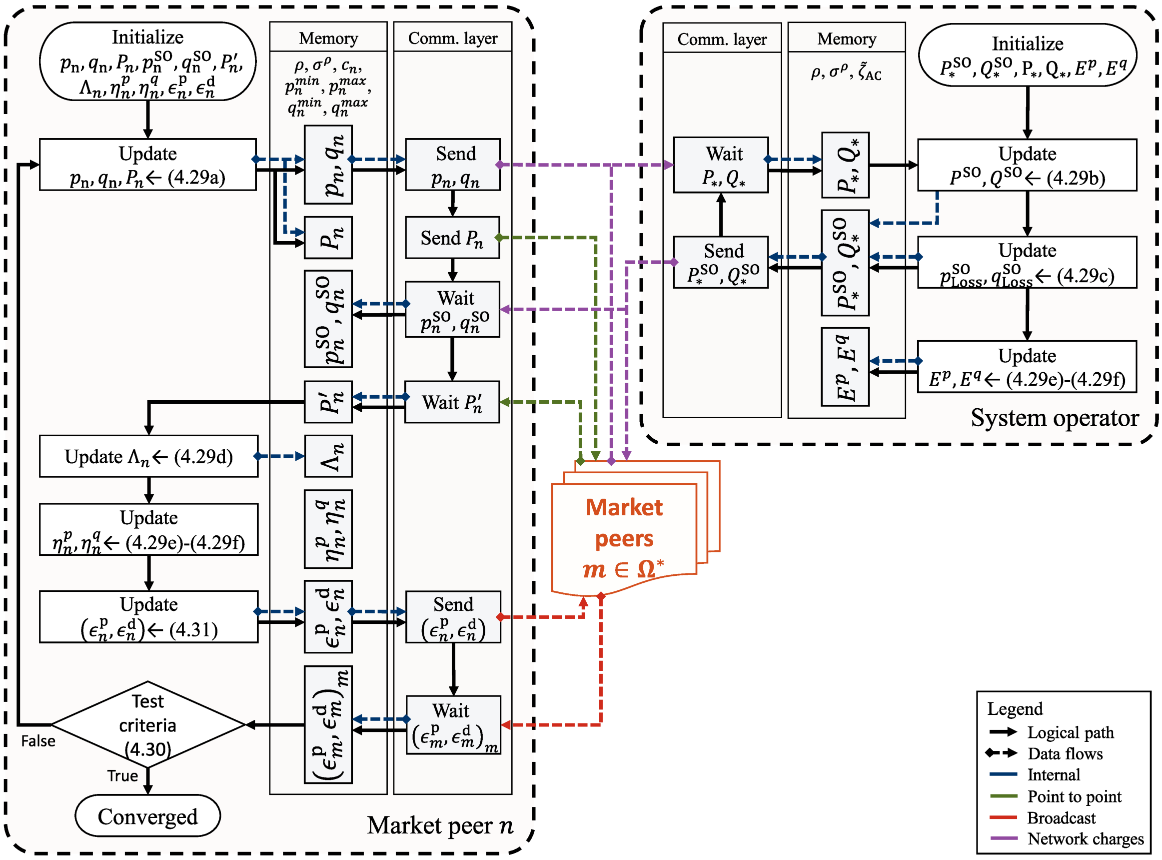
$$\epsilon_n^{\mathrm{p},k+1} = \frac{1}{4}\sum\nolimits_{m\in\omega_n}\left(p_{nm}^{k+1} + p_{mn}^{k+1}\right)^2 + \frac{1}{4}\left(p_n^{\mathrm{SO},k+1} - p_n^{k+1}\right)^2 + \frac{1}{4}\left(q_n^{\mathrm{SO},k+1} - q_n^{k+1}\right)^2 \tag{4.31a}$$

$$\epsilon_n^{\mathrm{d},k+1} = \sum\nolimits_{m\in\omega_n}\left(p_{nm}^{k+1} - p_{nm}^k\right)^2 + \left(p_n^{k+1} - p_n^k\right)^2 + \left(q_n^{k+1} - q_n^k\right)^2 \tag{4.31b}$$

where $\epsilon^{\mathrm{p,tol}}$ and $\epsilon^{\mathrm{d,tol}}$ denotes primal and dual global feasibility tolerances, respectively.



(a) Conceptual version



(b) Symbolic version

Figure 4.10: Decentralized endogenous negotiation mechanism for peer-to-peer electricity markets

As illustrated in Figure 4.10, the overall decentralized endogenous negotiation mechanism occurs as follows. Each market peer n first solves its own local optimization (4.29a) to update active power set-point p_n^{k+1}, reactive power set-point q_n^{k+1} and look for new bilateral trade proposals $P_n^{k+1} = (p_{nm}^{k+1})_{m\in\omega_n}$. Then, market peer n can share its power set-points with the system operator and individually send trade proposals p_{nm}^{k+1} to each partner $m \in \omega_n$. During that time, based on all previous injections $P_*^k = (p_n^k)_{n\in\Omega^*}$ and $Q_*^k = (q_n^k)_{n\in\Omega^*}$, the system operator can first look for the closest feasible injection plan $P^{\mathrm{SO},k+1} = (p_n^{\mathrm{SO},k+1})_{n\in\Omega}$ and $Q^{\mathrm{SO},k+1} = (q_n^{\mathrm{SO},k+1})_{n\in\Omega}$, which satisfies the power system's constraints such as in (4.29b) and estimate resulting power losses $p_{\mathrm{Loss}}^{\mathrm{SO},k+1}$ and $q_{\mathrm{Loss}}^{\mathrm{SO},k+1}$ with (4.29c). The system operator then shares feasible injections, $P_*^{\mathrm{SO},k+1} = (p_n^{\mathrm{SO},k+1})_{n\in\Omega^*}$ and $Q_*^{\mathrm{SO},k+1} = (q_n^{\mathrm{SO},k+1})_{n\in\Omega^*}$, and collects market peers' new asked injections $P^{k+1} = (p_n^{k+1})_{n\in\Omega^*}$ and $Q^{k+1} = (q_n^{k+1})_{n\in\Omega^*}$. To know what prosumers owes for their network usage, the system operator also keeps track of network charges $E^{p,k+1} = \left(\eta_n^{p,k+1}\right)_{n\in\Omega}$ and $E^{q,k+1} = \left(\eta_n^{q,k+1}\right)_{n\in\Omega}$ by updating them at each iteration with (4.24e)–(4.24f). Market peers can now update their trading prices $\Lambda_n^{k+1} = (\lambda_{nm}^{k+1})_{m\in\omega}$ with (4.29d) based on the counter proposals received from their partners, $P_n^{'} = (p_{mn}^{k+1})_{m\in\omega}$, and updates their network charges $\eta_n^{p,k+1}$ and $\eta_n^{q,k+1}$ with (4.29e)–(4.29f). Market peers are now able to deduce their local residuals $\left(\epsilon_n^{\mathrm{p}}, \epsilon_n^{\mathrm{d}}\right)^{k+1}$ with (4.31). Finally, each market peer n broadcasts its local residuals to all and, when all local residuals $\left(\epsilon_m^{\mathrm{p}}, \epsilon_m^{\mathrm{d}}\right)_{m\in\Omega^*\setminus\{n\}}^{k+1}$ are received, tests global stopping criteria (4.30). This process is repeated until convergence.

Several remarks can be made on this decentralized negotiation mechanism. First, the system operator's role solely consist in providing the new feasible power injections based on the previous ones sent by market peers. Moreover, optimal power flow (4.29b) may be complex and require some time to compute compared to market peers step (4.29a), particularly for a large power system. It is then possible to gain some computation time by simply starting the next feasibility search (of step $k+2$) as soon as the system operator received wished power injections $P_*^{k+1} = (p_n^{k+1})_{n\in\Omega^*}$ and $Q_*^{k+1} = (q_n^{k+1})_{n\in\Omega^*}$. To improve this even more, market peers may send their power set-points first so that the system operator can start the computation as soon as possible. This may be particularly beneficial since, as explained in [1] and recalled in Section 3.2, the communication time required to exchange many trade proposals may be longer than the computation time. Being decentralized, this negotiation mechanism is not supported by a central entity, at least for bilateral trades. The system operator is to be seen here as a support to market peers providing useful information to deduce their network charges and guiding them so they can attain acceptable trades. The system operator computation could even be handled asynchronously from market peers trade negotiations, and would then be seen as a small shared memory between market peers. Problem (4.23) with (4.28) instead of (4.23b)–(4.23i) and negotiation mechanism (4.29) are obviously in line with the generalized coordination problem proposed in Section 2.2 as it is similar to the concept multi-block ADMM in practical example line 11.

System operator implementation

Even though separated, the loss provider and the system operator can be carried by the same computation unit. This separation has actually been made to obtain a loss provider compatible with the generic model of prosumers since steps (4.24a) and (4.24d)–(4.24f) are valid for all market peers $n \in \Omega^*$. Compatibility of the loss provider iteration step with the one of prosumers came at the expense of using the fact that cost function c_{Loss} and reactive power set-point q_{Loss} are both null. Thus, if one wanted to consider a local objective such as losses minimization, they would have to multiply cost function c_{Loss} by $|\Omega|$ in order to preserve homogeneity of (4.24a)'s objective. Currently, the loss provider's reactive power set-point q_{Loss} is equal to zero, so the system operator's copy $q_{\mathrm{Loss}}^{\mathrm{SO}}$ will simply follow. But one could of course extend the concept with a peer-to-peer market on reactive power and, thus, reactive power losses.

It is important to note that the system operator's feasibility searches (4.24b) and (4.29b) take the form of a classical optimal power flow. In the straightforward distributed endogenous case, system operator step (4.24b) could be written such as (2.10) in Subsection 2.1.1 where prosumers' cost functions would read

$$c_n^{\mathrm{SO},k} : p_n^{\mathrm{SO}}, q_n^{\mathrm{SO}} \mapsto \sigma^\rho \left(p_n^{k+1}, p_n^{\mathrm{SO}}, \eta_n^{p,k}\right) + \sigma^\rho \left(q_n^{k+1}, q_n^{\mathrm{SO}}, \eta_n^{q,k}\right) + \sigma^\rho \left(p_n^{\mathrm{SO},k} - \frac{1}{|\Omega|} p_{\mathrm{Loss}}^{k+1}, p_n^{\mathrm{SO}} - \frac{1}{|\Omega|} p_{\mathrm{Loss}}^{\mathrm{SO},k}, \frac{1}{|\Omega|} \eta_{\mathrm{Loss}}^{p,k}\right) \tag{4.32}$$

and would have to be updated at each iteration step before (4.24b). In the decentralized endogenous case, system operator step (4.29b) prosumers' cost functions would read

$$c_n^{\mathrm{SO},k} : p_n^{\mathrm{SO}}, q_n^{\mathrm{SO}} \mapsto \sigma^\rho \left(\frac{p_n^{\mathrm{SO},k} + p_n^k}{2}, p_n^{\mathrm{SO}}, \eta_n^{p,k}\right) + \sigma^\rho \left(\frac{q_n^{\mathrm{SO},k} + q_n^k}{2}, q_n^{\mathrm{SO}}, \eta_n^{q,k}\right) + \sigma^\rho \left(p_n^{\mathrm{SO},k} - \frac{1}{|\Omega|} \frac{p_{\mathrm{Loss}}^{\mathrm{SO},k} + p_{\mathrm{Loss}}^k}{2}, p_n^{\mathrm{SO}} - \frac{1}{|\Omega|} p_{\mathrm{Loss}}^{\mathrm{SO},k}, \frac{1}{|\Omega|} \eta_{\mathrm{Loss}}^{p,k}\right) \tag{4.33}$$

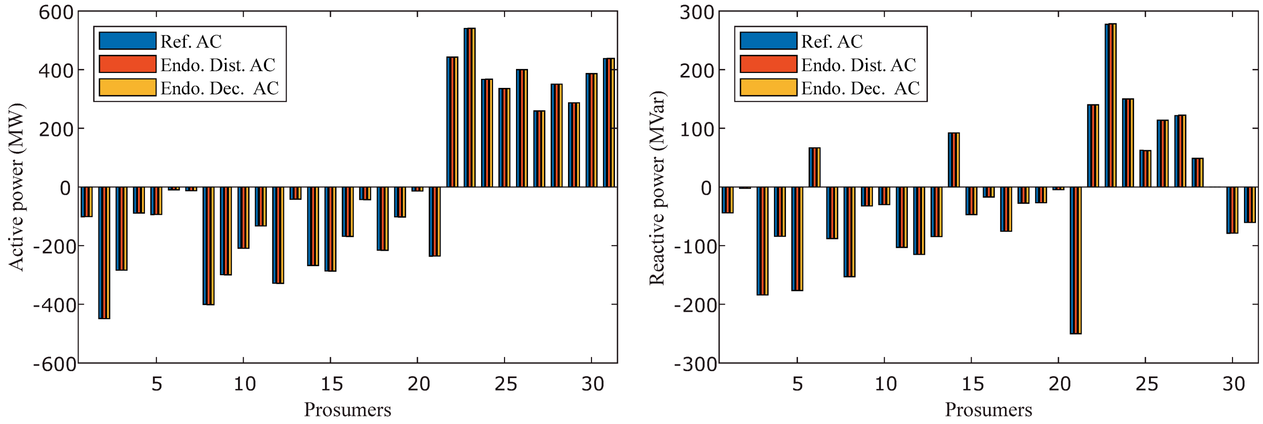
and would have to be updated at each iteration step before (4.29b). One could observe that these two formulations only differs in the way they take prosumers outputs into account. The distributed form considers updated variables p_n^{k+1}, p_n^{k+1} and p_{Loss}^{k+1}. While the decentralized form averages prosumers' past iteration variables p_n^k, q_n^k and p_{Loss}^k with system operator's past iteration variables $p_n^{\mathrm{SO},k}$, $q_n^{\mathrm{SO},k}$ and $p_{\mathrm{Loss}}^{\mathrm{SO},k}$. When using classical optimal power flow tools it is possible to preserve active and reactive power bound limits (2.10b)–(2.10c) to guide the process. The system operator would not have access to prosumers' boundaries of the current time step but it can, however, use installation capacities stipulated in network usage contracts.

4.4.2 Simulation results

Such for the exogenous peer-to-peer electricity market in Subsection 4.3.3, the endogenous peer-to-peer electricity market presented here is tested on the test case described in Subsection 4.2.2. Primal and dual tolerances used for the stopping criteria are both set to 10^{-3}, so $\epsilon^{\mathrm{p,tol}} = \epsilon^{\mathrm{d,tol}} = 10^{-3}$, and the penalty factor to one, so $\rho = 1$. First, it is important to verify the optimality, in other words the exactness, of the solutions reached by the endogenous peer-to-peer electricity market both in its distributed and decentralized versions. Especially, the solutions reached must have exactly the same power injections as the classical optimal power flow as expressed in (2.10). Used as reference in the literature [111, 112], it can be noted that simulations use MATPOWER both to provide the classical optimal power flow reference and for feasibility searches (4.24b) and (4.29b) of the system operator. Once their accuracy is verified, endogenous peer-to-peer electricity market's solutions are analyzed and compared to the exogenous approach as previously developed in Section 4.3. Finally, this subsection compares computation performances of both endogenous versions to the exogenous one.

Analysis on endogenous negotiation algorithms' optimality

Only being a relaxed version of classical optimal power flow (2.10), the presence of bilateral trades is not supposed to alter the equilibrium reached on power injections. It can be noted that convergence of ADMM towards the global optimal value is only guaranteed for a convex problem. Of course, being based on gradient descent, ADMM would also converged towards an optimal solution but which could be only local. However, even in the strongly non-convex AC line flow modelling, both proposed endogenous negotiation mechanisms converges towards the optimal solution as the reference MATPOWER centralized optimal power flow. Indeed, Figure 4.11 shows that the endogenous negotiation mechanisms leads to the same active and



(a) Active power injections (b) Reactive power injections

Figure 4.11: Power injections comparison of both AC endogenous negotiation mechanisms

reactive power injections plan as the reference AC optimal power flow. As it can be seen in Figure 4.12, the relative error of power injections made by the endogenous negotiation mechanism with the reference algorithm is below 1%. It can also be pointed that the average active power error is less then 0.1%. As consumers have fixed reactive power injections in this test case, the relative error on reactive power can only come from producers thus leading to an even lower average. It could thus be extrapolated that the endogenous negotiation mechanisms is optimal and do not alter the optimal power flow search. Of course, the mechanisms would have to be tested on more test cases in the future to confirm this assessment.

Another question now is whether both distributed and decentralized endogenous negotiation mechanisms lead to the same optimal solution. Indeed, being equivalent problem formulations, they should reach the same solution not only on power injections but also on power trades. The minor changes required to change from the distributed to the decentralized endogenous negotiation mechanism also pleads for them leading to identical solutions. Even though they may slightly differ from the reference optimal power flow, Figure 4.12 also shows that both endogenous negotiation mechanisms lead to the same exact injections. Indeed, maximum and average relative power injection differences between distributed and decentralized versions are respectively of 7.1 10^{-6} and 1.7 10^{-6} for active powers and of 30 10^{-6} and 3.0 10^{-6} for reactive powers. These differences are much lower than the tolerances used in the stopping criteria, it can be concluded that both versions lead to identical power injections. Note that Figure 4.13 allows to draw the same conclusion for the DC line flow model, which is convex.

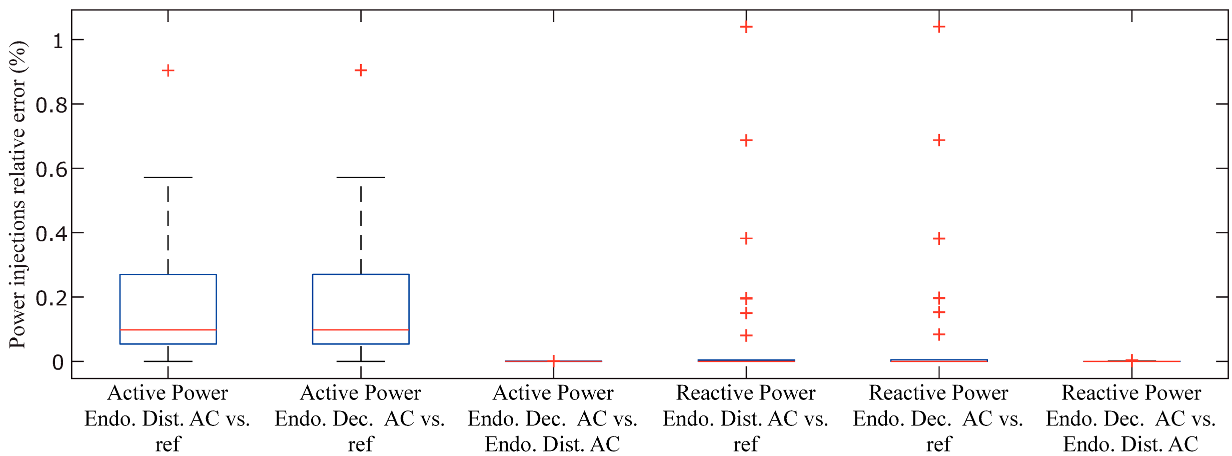
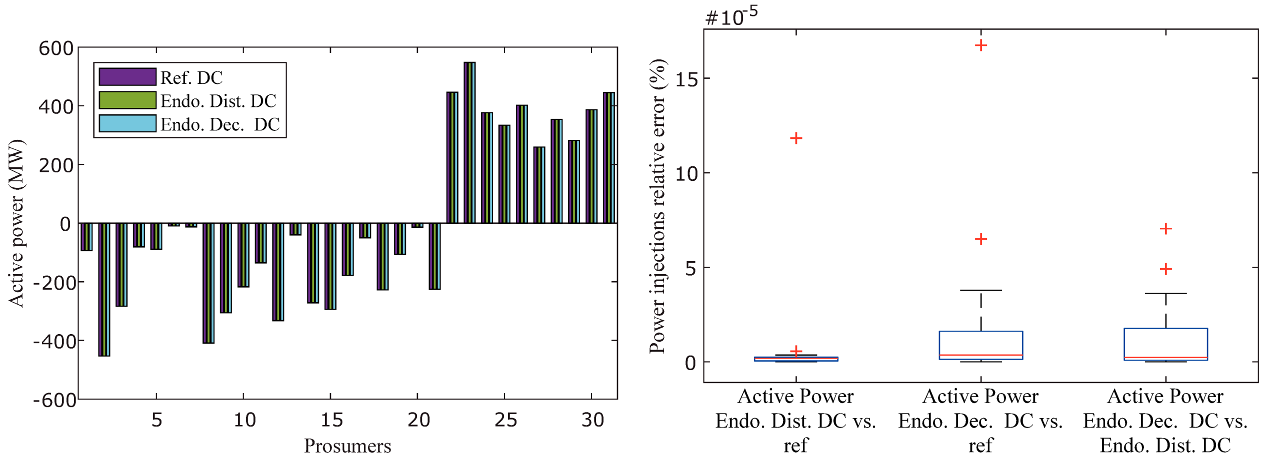


Figure 4.12: Relative power injection errors of AC endogenous negotiation mechanisms



(a) Power injections comparison

(b) Relative power injection errors

Figure 4.13: Power injections of DC endogenous negotiation mechanisms

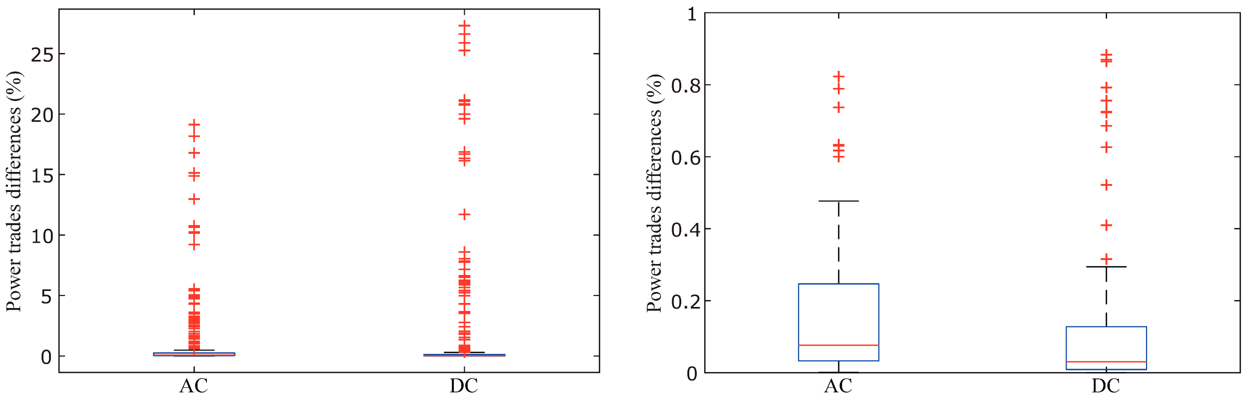


Figure 4.14: Power trade differences between distributed and decentralized versions both for AC and DC line flow modellings (full scale on the left, zoom on the right)

Moreover, Figure 4.14 shows the relative differences of power trades between the decentralized and distributed versions of the endogenous negotiation mechanism. The root mean square power trade difference of AC and DC line flow modellings are respectively of 7.7 10^{-4} and 3.0 10^{-4}. However, one could rightly object that some bilateral trades differ up to 19.1% and 27.3% for AC and DC modellings, respectively. An explanation of these important differences can be found in the non-strongly convex nature of bilateral trades and the tight range of nodal prices on active power. Indeed, prosumers cost functions apply to their active power set-point p_n which actually is a sum of bilateral trades $(p_{nm})_{m\in\omega_n}$. Thus, in absence of any cost differentiation between its trades an agent can obtain its active power set-point p_n with multiple combinations of bilateral trades $(p_{nm})_{m\in\omega_n}$. Hence, the peer-to-peer market without network charges is convex but not strongly convex by nature. To stir bilateral trades in a predefined direction the defined network charges would need to create price differences between agents. In the endogenous network charges case, the price differentiation can solely come from distinguished nodal prices of active power. Yet, Figure 4.15 shows that, even though presenting some diversity, observed nodal prices are close to each other. More pronounced with DC line flows this allows more liberty in bilateral trades combination to reach the same power set-point, as testifies Figure 4.13 and Figure 4.14. In consequence, even though probable, the current test case does not allow to confirm with certainty that decentralized and distributed versions lead exactly to the same optimal solution of power trades. To do so, one would need to test the endogenous negotiation mechanisms on a more congested test case for example. Another possible method would be to add convexification terms or preferences on bilateral trades. However, this method may shift the optimal solution of the initial problem.

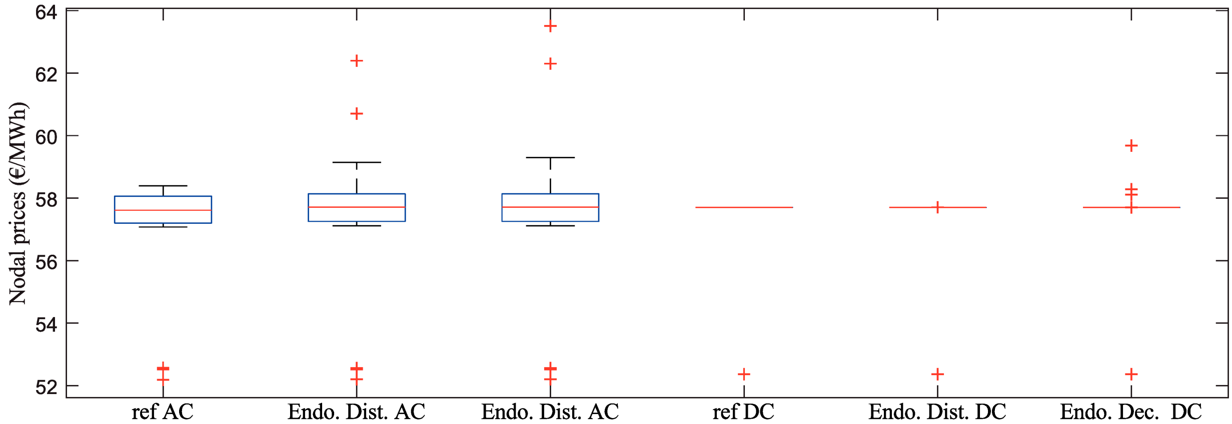
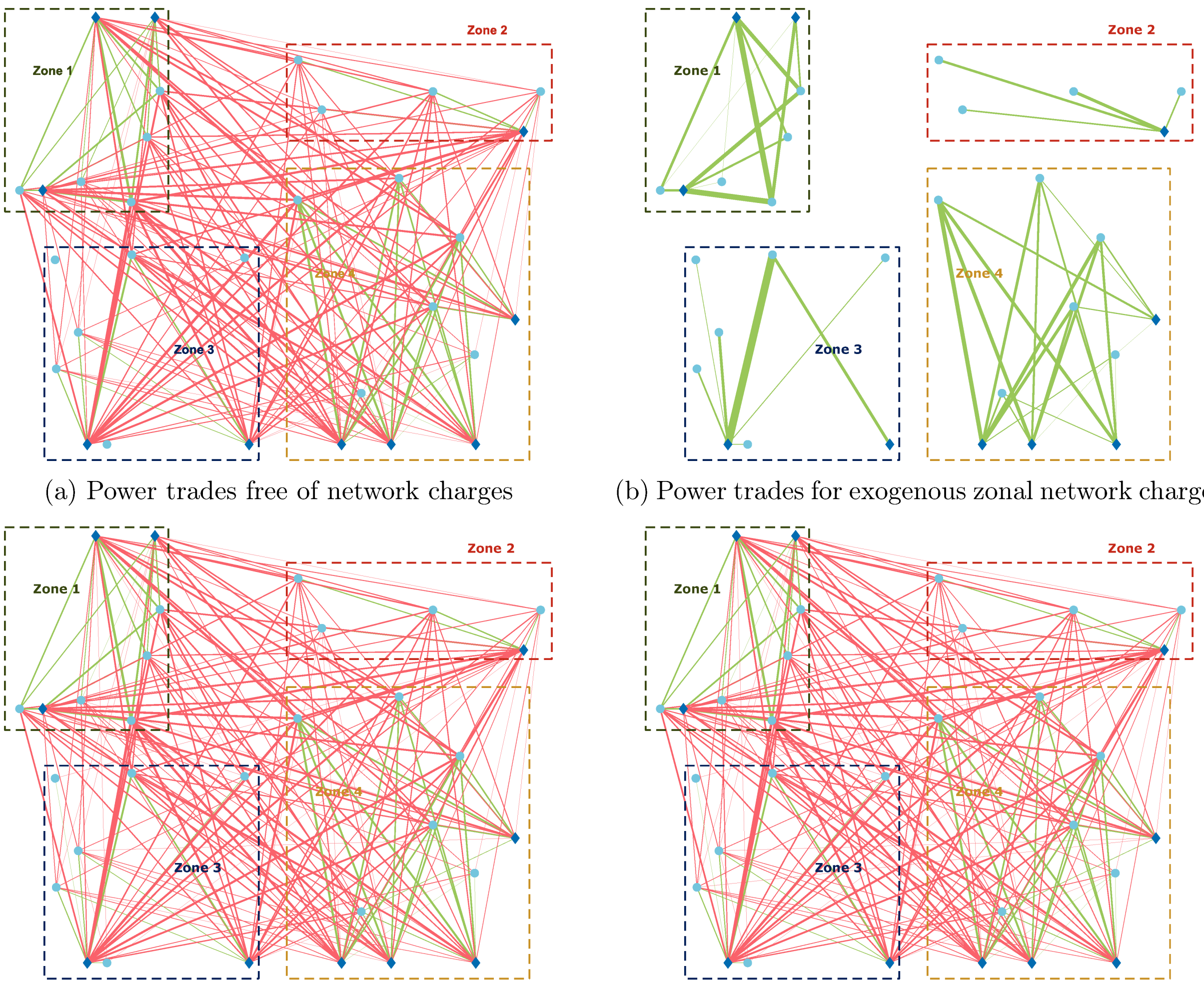


Figure 4.15: Active power nodal prices

Analysis on endogenous peer-to-peer electricity market solutions

Optimality of endogenous network charges exposed, it is now possible to compare the endogenous peer-to-peer market clearing they provide with exogenous peer-to-peer electricity markets. First, it should be recalled that involving the system operator in a more comprehensive manner, the endogenous network charges guarantee the respect of electrical network constraints while exogenous network charges do not since provided *a priori*, as stated in Section 4.3. A



(a) Power trades free of network charges

(b) Power trades for exogenous zonal network charges

(c) Power trades for DC endogenous network charges

(d) Power trades for AC endogenous network charges

Figure 4.16: Influence on trades (red lines: inter-zone exchanges, green lines: intra-zone exchanges)

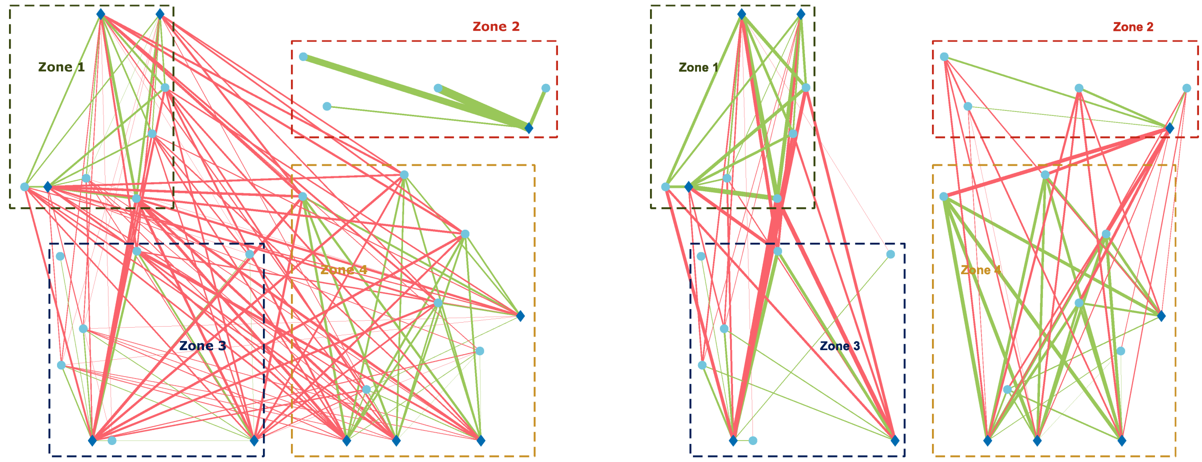
first way to compare the obtained bilateral trades would be to show them on test case's map such as in Figure 4.4. Using the exact same normalization for trades thickness and the same color chart, Figure 4.16 plots bilateral trade maps of the following simulations. To serve as reference and to facilitate comparisons, Figure 4.16a duplicates Figure 4.4a of the peer-to-peer electricity market in absence of any network consideration, so without network charges. Figure 4.16b presents trade results of the exogenous peer-to-peer electricity market based on the unique zonal cost allocation policy, as introduced in Subsection 4.3.2, for a unit fee u^{zone} of 20 €/MWh per crossed zone. This unit fees has been chosen based on Figure 4.7 such that it is the least costly and does not induce congestion. Note that a unit fee u^{zone} of 20 €/MWh corresponds to 35% of the free market price on the x-axis of Figure 4.7. Then, Figures 4.16c and 4.16d respectively plot bilateral trades of DC and AC endogenous peer-to-peer electricity markets.

One can note on Figure 4.16 that endogenous network charges have much less impact on bilateral trades than exogenous policies. Even though individualizing at the level of each node and hence each prosumer, endogenous charges are much more meticulously defined and adapted to the current power injection plan. Indeed, even in its more detailed form that is the electrical distance cost allocation policy, exogenous network charges still has one tuning parameter, namely the unit fee, which is common for the all network. The zonal cost allocation policy with different unit fee per zone would be more refined but still lack precision for trades happening within it. The optimal approach for exogenous network charges would then be the transformation of the zonal cost allocation policy into a nodal one by considering each node as a zone possessing its own individual price. This approach would then consist in forecasting nodal prices, so anticipating what would endogenous network charges be.

The global influence on bilateral trades can also be testified in Table 4.2. This table shows that endogenous network charges apply much less pressure on the peer-to-peer market to reach a feasible power dispatch. Indeed, DC and AC endogenous network charges respectively impose a 1.6% and 2.2% power trades reduction to reach feasible power injections while the exogenous approach requires a 25.6% reduction in its electrical distance form and more than 44% for the two others. One could remark that exogenous network charges at least allows to isolate zones from each others from an economical point of view, in particular the zonal cost allocation policy. However it may be remarked that, even if showing many inter-zone exchanges, the system operator could also perform this by altering its power system layout. To isolate zone 2 for example, the system operator would solely need to suppress the two lines linking it to zone 1 and 4 from the list of available lines. In such case, the feasibility search made at each iteration of the endogenous negotiation mechanism could even be decomposed as two independent power systems, each with their system operator, but without directly changing prosumers partnerships.

Table 4.2: Influence of network charges approaches on total traded amounts

	Unit fee (€/MWh)	Between peers (MW (%))	Between zones (MW)
Exogenous	$u^{\text{uniq}} =$ 20	2156 (55.4)	35
	$u^{\text{dist}} =$ 5	2901 (74.5)	24
	$u^{\text{zone}} =$ 20	2137 (54.9)	0
Endogenous	DC	3832 (98.4)	526
	AC	3808 (97.8)	480
No network charges		3894 (100)	574

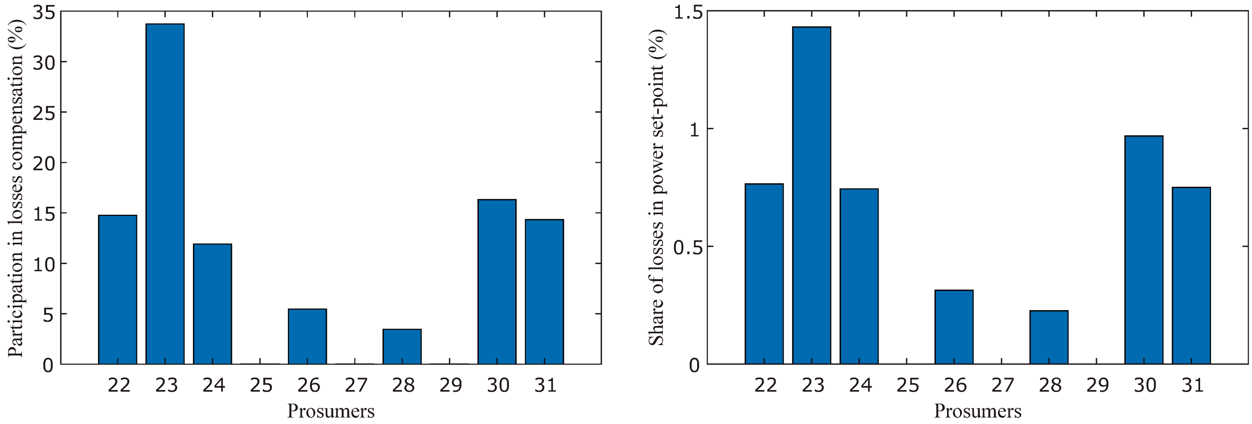


(a) When zone 2 is isolated from the rest (b) When zones 2 and 4 are separated from the rest

Figure 4.17: Power trades for AC endogenous network charges

For example, the *peer-to-peer New England* test case described in Subsection 4.2.2 can be altered solely by suppressing the two lines of zone 2 with zones 1 and 4. In that case, overall power balances would be affected such that an additional power balances between offer and demand would now occur in zone 2. Prosumers would reach a new consensus leading to the power bilateral trades illustrated in Figure 4.17a. It can be noted that in presence of losses, such as for the exposed AC line flow model used here, there must be one loss provider per isolated power system. For example, the price of losses observed in zone 2 is 18.75 €/MWh which is much higher than 4.23 €/MWh for the group of zones {1,3,4}. Indeed zone 2 has only one producer with a higher generation than it's competitors. Unable to get lower prices from producers outside, consumers of zone 2 thus have to cope with it by accepting higher prices and lower their consumption. The endogenous approach being very modular, the *peer-to-peer New England* test case could be modified a second time by now considering that zones {1,3} and {2,4} constitutes two separate electrical systems. Bilateral trades resulting of this configuration are represented in Figure 4.17b. Both set of zones being balanced in production costs, loss prices are now more leveled with 6.67 €/MWH for zones {1,3} and 6.92 €/MWh for zones {2,4}. These two tested configurations demonstrate the fact that the communication matrix layout between prosumers is not correlated to the power system's infrastructure. Thus prosumers in an endogenous peer-to-peer electricity market are free to choose whoever they want as trading partners. Prosumers connected at distribution level could, for example, directly trade with prosumers of the transmission level rather than passing through a distribution aggregator such as it is the case today.

Back to the initial *peer-to-peer New England* test case, an additional peer called loss provider participates in the endogenous peer-to-peer market when the line flow model includes losses. The loss provider buys power on the market to compensate the line losses observed on the power system. For example, in the AC endogenous peer-to-peer market there are 22.94 MW of line losses that must be compensated and, hence, bought by the loss provider on the market. Note that to avoid issues of arbitrage, trading partnerships are such that consumers can solely buy from producers while producers can solely sell to consumers. Thus, in the current example only prosumers from 22 to 31, which are producers, can sell power to the loss provider. Figure 4.18a shows that not all producers participate in the compensation of line losses as trading with the loss provider is not always interesting for them. Indeed, the cost of power losses is only of 3.78 €/MW, thus revenues generated for trading with the loss provider are low. As it can be observed in Figure 4.18b, when a producer sells power to the loss provider its quantity is always marginal compared to the amount of power sold to consumers.



(a) Participation share of prosumers

(b) Weight put on prosumers

Figure 4.18: Losses compensation in AC endogenous peer-to-peer markets

Analysis on computation performances

Endogenous network charges do not only need to respect grid constraints, they also need to offer appropriate computation performances to be applied in real world implementations. Indeed, at the image of the European power system, electrical grids are large scale networks composed of thousands or hundreds of thousands of nodes and connect hundreds of thousands, millions or dozens of millions of prosumers. In consequence, the proposed market frameworks must be able to support this type of scale. Even though leading to identical market equilibria, peer-to-peer markets are less efficient computationally than pool markets when they are fully connected. Chapter 3 presented several methods which could improve peer-to-peer markets performances. The scaling issue is even more pronounced in the domain of power system operation as optimal power flows can be very hard non-convex problems to solve [40, 41]. Of course, several improvements can be found in the literature such as in [39]. Thus it is important to verify that the proposed network charges do not slower further these processes as they combine both. As pure performances is not the main focus, but rather their comparison, computation performances gathered in Table 4.3 were run on a simple Intel Core i5-7200U 2.5GHz.

Table 4.3: Performances of negotiation mechanisms for the different network charges

		Number of iterations	Total time (s)	Average time per iteration (ms)	
				Peers	System operator
No network charges		141	1.60	7.05	–
Exo. Zonal		330	3.39	6.63	–
DC	MATPOWER	–	0.016	–	16.16
	Endo. Dist.	258	4.90	5.39	11.52
	Endo. Dec.	443	4.70	5.22	10.56
AC	MATPOWER	–	0.123	–	122.5
	Endo. Dist.	433	32.31	5.20	67.63
	Endo. Dec.	605	40.61	5.17	67.13

First, it can be noticed that the decentralized negotiation mechanism in presence of exogenous of network charges is slower than in absence of them. Only the example of zonal allocation policy is presented here, but the tendency is the same with the other allocation policies of Section 4.3. This speed decrease actually does not come from the fact that peers must solve hard problems at each iteration, on the contrary since the average computation time per iteration is lower, but from a higher number of iterations to settle the last trades. However, one can note that endogenous negotiation mechanisms are slower then these two previous cases. Even though simplifying peers' local computation, the time required for the system operator to compute its feasibility search dominates peers' computation time. For example in the DC line flow modelling case, the computation time required by the system operator at each iteration is of the same order as for peers (around the double). In consequence, the higher number of iterations needed for the decentralized endogenous approach is still faster than the distributed approach as each iteration takes as much time as the slowest between peers and the system operator instead of both the slowest peer plus the system operator. However, the benefit of the decentralized approach is blurred in the AC line flow modelling case as system operator's feasibility search is much slower than peers' computation. An interesting point is that the average time per iteration to compute the feasibility search is about half the time the classical optimal power flow. This mainly comes from the warm starts as this step is only slower for the first iteration.

But, it is too soon to conclude that endogenous network charges are too slow compared to exogenous charges, when scaling up, to reach real world implementation. Indeed, it is important to note that performances results presented here were obtained without any of the algorithm improvements proposed in Chapter 3 which could modify these conclusions. Notably, the use of asynchronous communications in the endogenous negotiation mechanism would largely help reduce the computation time as many bilateral trades iteration could be carried during a single feasibility search step. The system operator would then only be queried a handful of times before reaching consensus. Of course the mathematical proof of convergence would not hold any longer, such as it did not with the non-convex AC line flow model. But emulations in Section 3.3 and the literature, e.g. [83], showed the efficiency of asynchronous communications. Note that in an asynchronous setup, frontiers between distributed and decentralized approaches of network charges would be blurred and solely consist in whether peers consider system operator's corrective injections or its average with what they proposed, and vice-versa for the system operator. As prosumers' power set-point mainly evolves at the beginning of the negotiation process, it could also be considered to query the system operator at more specific moments such as at the first step and when power set-points almost stabilizes. In addition, it may not be coherent to use a centralized feasibility search to coordinate with a fully decentralized market decision making framework. Thus, combining the endogenous negotiation mechanism with a distributed or decentralized optimal power flow algorithm such as [113,114], with synchronized or asynchronized iteration steps, could help improve its convergence rate. Thus, there is a need in further works to study these possible improvements of the endogenous negotiation mechanism before ruling it out and solely envisage exogenous network charges.

4.4.3 Conclusions on endogenous network charges

To partially conclude, this section exposed another peer-to-peer electricity market design in which network charges would be defined in an endogenous manner. Contrary to the exogenous approach of Section 4.3, these endogenous network charges would directly encompass power system's constraints and estimated simultaneously with bilateral power trades. To solve this, the decentralized exogenous negotiation mechanism would be adjoined with an optimal power flow to search for the feasible power injection plan closest to the one requested by prosumers. Handled by the system operator, this closest feasibility search would then allow to replace exogenous network charges by endogenous ones updated at each iteration. In the case of a model with active power losses, an additional market participant would be required to buy power from prosumers to compensate for line losses. Tested in the novel case based on the IEEE

39-bus test system, endogenous network charges demonstrated to be efficient in guaranteeing power system constraints. It has also been confirmed that optimal power injections reached by a classical optimal power flow approach were not altered by the presence of bilateral trades, thus making endogenous network charges efficient from an economic perspective too.

However, the optimality allowed by endogenous network charges came at the price of a slower negotiation mechanism process. Because it implies a strong involvement of the system operator, obtaining endogenous network charges requires to conduct an optimal power flow at each negotiation iteration. Guarantee on the non-violation of power flow constraints being essential in most power systems, particularly in distribution and weak systems, future work on improving convergence speed of endogenous network charges is necessary. After choosing the most adequate line flow model, several leads of improvement can be listed. First, the use of asynchronous communications, at least for the interaction between peers and the system operator, would allow peers to carry bilateral power trade negotiation while waiting for the system operator's feedback. This would reduce the number of times the closest feasibility search has to be conducted and, overall, the convergence time. Independently from the type of communication, decentralized trade negotiations could be combined to a distributed or decentralized optimal power flow algorithm for rather than a fully centralized one. It would then be possible to synchronize iteration counters of both processes, thus concurrently searching for bilateral trades and feasible injections rather than fully computing the closest feasibility search.

4.5 Synthesis

The aim of this chapter was to study the way peer-to-peer electricity markets could interact and account for electrical network constraints. For this purpose, the chapter first described how the classical optimal power flow problem would be adapted to include multi-bilateral trading to its formulation. Even though another approach is favored in Chapter 5, note that this formulation could account for uncertainty by using a scenario-based stochastic optimization framework. This description notably showed that network constraints could be condensed in an extended-value function which would actually regularized peer-to-peer market's problem. Two approaches to treat network constraints' regularization function have then been proposed. The first approach proposed to replace regularization function evaluating network constraints by a cost allocation function introducing network charges. Individually designed for each bilateral trade, network charges are to be seen as a tool to allow the system operator to charge prosumer in a way which reflects how their bilateral trades impact the power system. The amount of money collected by the system operator through these charges could for example serve to finance power line investment and maintenance of the power system or congested-related costs and taxes. To allow prosumers to adapt their trading strategies, the system operator would provide these network charges *a priori* and, hence, would have to be evaluated before hand in an exogenous manner. In consequence, these exogenous network charges would not give any guarantee to the system operator that the electrical network constraints would be all respected.

Yet, network constraints violation may put all the power system at risk, in particular for weak and undersized electrical grids. Thus, network charges in the form of exogenous economical incentives would not suffice and need to be reinforced with a stronger involvement of the system operator in bilateral trades' negotiation mechanism. For this hand, rather than replacing it, the second proposed approach directly treats network constraints' regularization function at the same time as bilateral trade's negotiations. Prosumers would then not only negotiate with each other on bilateral trades but also aim at reaching consensus with the system operator on feasible power injections. The system operator would then conduct an optimal power flow on its power system with the objective of finding closest feasible injections from prosumers request. From prosumers perspective this consensus would take the form of network charges on their power set-points rather than on each bilateral trade. Being determined through an optimal power flow, these network charges would thus endogenously encompass electrical network constraints. The advantage of these endogenous network charges is that they lead prosumers to the same power injections as in optimal power flow. Bilateral trades are relaxation variables so they do not alter prosumers' objective functions. Endogenous network charges also presented the advantage of putting less pressure on bilateral trades than exogenous ones to obtain feasible, congestion free power injections. However, as expected, endogenous network charges came at the detriment of convergence speed. Indeed, requiring an optimal power flow at each iteration, endogenous network charges in their current framework strongly slows the negotiation mechanism.

In consequence, the two proposed approaches to account for network constraints in peer-to-peer electricity markets are good candidates but they both need further improvements before getting in future real world implementations. Fast to reach convergence, exogenous network charges however proved to have insufficient or unfeasible solutions when they are not designed wisely. In opposition, endogenous network charges made up for this deficiency by hardly including network constraints and, hence, guaranteeing their respect but lacked in rapidity. Several tracks of improvements can be foreseen. First, exogenous network charges could be enhanced by passing to a nodal based cost allocation policy. This nodal cost allocation could even be further strengthened with the use of machine learning, Markov switching or time series tool to anticipate the best nodal network charges. To improve endogenous network charges, it would be possible to consider asynchronous communications between market peers and the system operator. Moreover, to be more in line with the decentralized concept of peer-to-peer, the closest feasibility searches could be carried based on distributed or decentralized resolutions of the optimal power flow problem. Finally, as a trade off between exogenous and endogenous network charges, one could imagine first using the nodal exogenous approach for the first negotiation iterations and, then, update them endogenously once or twice to adapt the nodal network charges to the current situation.

Decentralized stochastic coordination problems 5

This chapter focuses on coordination problems in presence of stochastic behaviours. As a coordination problem, stochastic behaviours may be considered overall through a coupling constraint to take advantage of their spread or to overcome their correlation. This chapter proposes to split this common constraint into individual stochastic constraints. Different different methods to allocate the uncertainty among entities are presented. They notably allow to adapt the way risks are shared between them depending, for example, on the way they have on the overall uncertainty. Finally, performance analyses are conducted to evaluate these uncertainty allocation methods.

Contents

5.1 Introduction

Resource allocation in electricity markets is traditionally handled by centralized platforms gathering offers and demands of market participants to form the well known pool market. The centralized clearing mechanism as proven to be efficient and scaling well over the years. However, the efficiency of this organization is being challenged by the introduction of novel actors and behaviors. Distributed energy resources jointly with information and communications technology and energy management systems, either for residential homes or buildings, scatter the electrical power production and consumption throughout the electrical network. Going down to lower levels of the network, the appearance of prosumers with both the ability to produce and consume (and most likely store) electricity is emblematic of the transformations occurring in the domain. While power system operators are making substantial efforts to strengthen electrical networks to these new types of use, electricity markets have not gone yet through the same process of accommodating to the new context at hand. As of current practice, small-sized prosumers are managed at retail level in existing mechanisms such as real time market and demand response [67]. This particularly comes from regulatory thresholds put on prosumers' size and, often, a strict dichotomy between pure consumers and pure producers. One can understand that these exiting mechanisms are not adapted to small-sized prosumers as it would exponentially increase the amount of communications and data which must be centralized by the market operator.

As mentioned in previous chapters, in response to this issue, electricity markets are expected to go from producer-centric to more decentralized approaches [10] which can be qualified as prosumer-centric. Already 20 years ago, Wu and Varaiya proposed the first decentralized electricity market based on coordinated multilateral transactions [95]. Originally developed to separate economics from power systems' reliability, it has recently been enhanced with a game-theoretical properties analysis of the obtained solution in [90]. When solely involving two parties this approach can be assimilated to peer-to-peer trades in which each market participant directly negotiates with a set of trading partners such as in [2]. Prosumers' objective is then to minimize their energy procurement costs by favoring the most profitable partnerships. Alternatively, [10] categorized two other possible layouts of decentralized or distributed market structures. In the first one, prosumers are connected in microgrids, either interconnected or not, while prosumers are grouped in energy communities which are not necessarily connected on the same local microgrid in the second. Although other architectures have been proposed in recent literature [72–74, 115], [4] showed they all can be represented as particular cased of the peer-to-peer market structure.

However, these novel market structures often suppose deterministic prosumers and rarely considers their eventual uncertainties, which are inherent for some of them such as renewable sources. The classical centralized market framework considers two distinct markets, one trading energy and another bidding on reserves. Depending on one another, these two markets are usually dealt in a sequential manner. But this sequential approach of energy and reserve markets necessarily leads to sub-optimal solutions. Thus, the current centralized energy and reserve market inevitably leads to a sub-optimal use of primal resources [13]. To obtain the best solution, uncertainties must be considered jointly with power production and consumption forecasts in a stochastic market formulation. Even though leading to the optimal solution, a fully stochastic energy market is not adapted for future decentralized prosumer markets. Indeed, stochastic energy markets are solved with centralized algorithms such as two stage stochastic programs. An other way to solve them would be to use scenario based approaches, but these are not tractable communication wise [14]. Nevertheless, chance constraint based optimization seems a good alternative to scenario based approaches, [15]. One could rather formulate the stochastic energy market into the form of a chance constraint problem. The obtained energy and reserve market would thus propose both energy and reserve market products but would allocate them jointly, within a single market problem resolution. In this configuration, reserved capacities would be used to restrain the energy market such that prosumers are able to provide a sufficient amount of power capacity. The reserved capacities would then be used to compensate misdispatches if an error of consumption or production forecast occurred.

Of course, reserving the total amount of capacity in a robust way, i.e. allowing to overcome any worst case, would be costly and sub-optimal. For this reason, an energy and reserve market formulated as a chance constraint problem would provide the possibility to choose the confidence level with which one wants the market to allocate reserves. The confidence level is to be seen here as an indicator of how risk averse the electricity market is. So, a confidence level of 100% implies that the market is fully risk-averse and such must provide reserves for the worst case. On the contrary, a confidence level of 0% would mean that the market is willing to take all the risks, thus no capacity would have to be reserved and the energy side of the market can simply considers the expected forecasts. Note that one could use the notion of risk level, so opposite to the notion of confidence level mentioned above. For example [116] named it loss-of-load-expectation and [117] calls it risk threshold. For example, as in [116], the reserve procurement is classically done globally using a joint chance constraint. Such joint chance constraint requires global variables which aggregate uncertain prosumers' actions. Yet, an issue arises when going towards a decentralized framework such as peer-to-peer markets. Indeed, peer-to-peer markets call for a split of the global chance constraint into multiple individual chance constraints at uncertain prosumers' level.

This chapter proposes a peer-to-peer market with bilateral contracts both on energy and reserves. To stay coherent with the decentralized aspect of prosumer markets, reserves are considered at a local level by uncertain prosumers through individual chance constraints. Solved in a decentralized manner, this formulation leads to a fully decentralized market coupling both energy and reserve market products such that uncertain prosumers must constitute sufficient reserves to compensate their own unreliability. If technically possible, uncertain prosumers can constitute reserves locally with their own asset, such as load-shedding or renewable power spillage for example. If insufficient, uncertain prosumers can also buy additional reserves through bilateral contracts with other partners such as reliable, controllable prosumers. In other words, the proposed formulation leads to a combined peer-to-peer energy and reserve market which can be solved by a decentralized negotiation mechanism similar to the one proposed in previous chapters. Yet, splitting chance constraints may lead to sub-optimalities due to too restrictive or too slack individual chance constraints.

Individual chance constraints were the first chance constraints formulated in 1958 by [118]. But reaching a specific global level of confidence without optimality gaps was too complex. Thus, they were replaced by joint chance constraints later in 1965 by [119]. Since, the literature has mainly been focused on joint chance constraints due to their simplicity of design and their satisfying optimality performances. Chance-constrained programs solving this type of problem have been developed soon after, such as [33] in 1972 and [120] in 1974. Also called integrated chance constraints, improvements have been made through the years to solve joint chance constraint problems. Notably, [121] made efforts to obtain a reduced form of the solving algorithm and [34] used non-linear problem solvers. Moreover, theoretical extensions have been made in [122–124] on coupled random variables, approximations of non-convex chance constraints and two-sided linear chance constraints. Conditions on convexity of chance-constrained problems can be found in [124–126]. Joint chance constraints have been used for many applications such as resource allocation in [127, 128], optimal power flow in [129–131] and demand response in [132]. One of the main application of chance constraints is robust optimization and, in particular, distributionally robust optimization which leads to many theoretical studies. Contrary to classical chance constraints with known probability distributions, distributionally robust optimization rather focus on ambiguity sets of probability distributions as exposed in [133, 134]. Ambiguity sets are generally based on historical data and aim at estimating the worst possible probability distribution on which to optimize in a robust manner. Distributionally robust chance constraint can be approximated in various ways. For example, Bonferroni, Fréchet, Bernstein or worst-case conditional value at risk approximations can be used as in [122,123,135,136], respectively.

Applications of distributionally robust optimization in power systems can for example be found in [137–140].

However, the study on the conditions of equivalence between joint and individual chance constraints seemed almost non-existent in the explored literature. Some works by S. Ponda can be found on this topic. For example, [117, 141] proposed a formal approach to allocate risks in a distributed chance-constrained task allocation. Yet, their work was restricted to one-sided inequality chance constraints while an energy and reserve market requires two-sided inequality chance constraints. This chapter proposes to make the necessary extensions to handle a two-sided inequality chance constraint problem that is the peer-to-peer energy and reserve market. The allocation methods to separate the joint, global chance constraint into split, individual ones are proposed and tested in this chapter. The first method suggests to directly define local confidence levels as a proportion of the global one. In this case risks are allocated in a relative manner as they are allocated with respect to the overall probability area which an uncertain prosumer must cover. In the manner of [117], the second method rather allocates amounts of reserves that uncertain prosumers need to cover. In this case the amount of reserves that each uncertain prosumers must retain is a proportion of the global amount of reserved capacities required to reach the global confidence level. Thus, risks are allocated in an absolute manner as they are allocated with respect to quantities.

This chapter is structured as follows. After recalling the formulation of a centralized, joint chance-constrained, energy and reserve market, Section 5.2 describes the novel peer-to-peer energy and reserve market. A decentralized negotiation mechanism solving it is also exposed. Section 5.3 presents the different uncertainty allocation policies which can be used to define the local confidence levels observed by uncertain prosumers. Both relative and absolute methods are presented there. Proposed uncertainty allocation policies are then compared to the centralized form in Section 5.4 through simulations on a yearly test case, based on [3]'s modified IEEE 14-bus system. Finally, Section 5.5 concludes on uncertainty allocation policies and gathers perspectives for further work.

5.2 Decentralized energy and reserve market

A peer-to-peer market is based on a community of prosumers with flexible consumption or production. As it is classically done in the literature, prosumers are supposed rational as in [24], i.e. always objectively taking the most beneficial decisions, and non-strategic, i.e. not anticipating actions and reactions of other prosumers. After recalling the formulation of a centralized energy and reserve market in Subsection 5.2.1, bilateral trades are introduced in Subsection 5.2.2 to obtain a peer-to-peer energy and reserve market. A decentralized negotiation mechanism solving this problem is then described in Subsection 5.2.3. The emphasis being placed on allocating reserves in a decentralized way, no temporally binding constraint is considered and time steps are supposed independent. Thus, each time unit, e.g. hourly, will be treated as an isolated test case configuration. As explained in Chapter 2, note that it may readily be extended to multiple time units with temporally binding constraints.

5.2.1 Centralized energy and reserve market

Classically, a pool market which combines both energy and reserves can be expressed as the following chance-constrained problem

Centralized energy and reserve market

$$\min_{\substack{(p_n, r_n^+, r_n^-)_{n\in\Omega} \\ p_{\mathrm{nc}}, r^+, r^-}} \quad \sum\nolimits_{n\in\Omega} c_n(p_n) + c_n^+(r_n^+) + c_n^-(r_n^-) \tag{5.1a}$$

$$\text{s.t.} \quad p_n^{\min} + r_n^- \leqslant p_n \leqslant p_n^{\max} - r_n^+ \qquad n \in \Omega \tag{5.1b}$$

$$0 \leqslant r_n^+ \leqslant r_n^{+,\max} \qquad n \in \Omega \tag{5.1c}$$

$$0 \leqslant r_n^- \leqslant r_n^{-,\max} \qquad n \in \Omega \tag{5.1d}$$

$$\sum\nolimits_{n\in\Omega} p_n = 0 \tag{5.1e}$$

$$r^+ = \sum\nolimits_{n\in\Omega} r_n^+ \tag{5.1f}$$

$$r^- = \sum\nolimits_{n\in\Omega} r_n^- \tag{5.1g}$$

$$p_{\mathrm{nc}} = \sum\nolimits_{n\in\Omega_{\mathrm{nc}}} p_n \tag{5.1h}$$

$$\mathbb{P}_{f_{\mathrm{nc}}}\left(-r^- \leqslant p_{\mathrm{nc}} - \tilde{p}_{\mathrm{nc}} \leqslant r^+\right) \geqslant \delta \tag{5.1i}$$

where control variables $\{p_{\mathrm{nc}}, r^+, r^-\}$ defined in (5.1f)–(5.1h) are associated to the global level and, hence, handled by a central operator. Note that the index $\cdot_{\mathrm{nc}}$ relates to non-controllable prosumers as a group. In a deterministic clearing market, the goal would be to minimize the total cost of a group Ω of prosumers participating in the market, which would sum individual energy cost functions c_n as in (5.1a). To do so, each prosumer n can define its power set-point p_n within a feasible flexibility range bounded by lower $p_n^{\min}$ and upper $p_n^{\max}$ bounds. Power set-point p_n is taken positive if prosumer n is net producer and negative if net consumer. The global power balance of the market is ensured by constraint (5.1e).

As the energy market is associated to a reserve market, power set-points p_n are to be seen as prosumers' unit-commitment which must be satisfied. Any deviation from it would constitute a breach of commitment and call for the use of an additional capacity to compensate it. This additional capacity would have to be reserved before hand in a centralized reserve market. Reserves aim to overcome uncertainties of non-controllable prosumers, such as wind farms for example. Gathered in $\Omega_{\mathrm{nc}} \subset \Omega$, each uncertain prosumer n's power production or consumption forecast is a random variable noted $\tilde{p}_n$ defined by probability distribution function f_n. For example, if uncertain prosumer n was a wind farm, random variable $\tilde{p}_n$ would denote the amount of power that may be produced with a probability $\mathbb{P}_{f_n}(\tilde{p}_n)$. Thus, an uncertain prosumer n may undergo a deviation $\Delta_n = p_n - \tilde{p}_n$ from its original unit-commitment p_n. A reserve market procures reserves to overcome the global deviation of all uncertain prosumers. Their total unit-commitment p_{nc} is given by (5.1h). Random variable $\tilde{p}_{\mathrm{nc}}$ denotes the global uncertain power forecast following probability distribution function f_{nc}, which is a copula of local probability distributions $(f_n)_{n\in\Omega_{\mathrm{nc}}}$. Each prosumer n can either provide an upward reserve, i.e. a generation reserve, or a downward reserve, i.e. a demand reserve. Upward and downward reserve set-points are thus self-constituted and respectively noted r_n^+ and r_n^-. Global upward r^+ and downward r^- reserves available on the market are defined in (5.1f)–(5.1g) and must cover generation uncertainties up to a global confidence level δ as in (5.1i). The global confidence level can also be seen as an indication of market's aversion towards risk, so of its robustness.

Of course, the engagement of reserves may induce additional costs to the ones providing them. Hence, in (5.1a), prosumers also aim at minimizing cost functions c_n^+ and c_n^- which are respectively linked to the cost of providing upward and downward reserves. As in (5.1c)–(5.1d), upward and downward reserves provided by a prosumer may be limited by technical constraints, such as ramping limits for example. These reserve limits are respectively noted $r_n^{+,\max}$ and $r_n^{-,\max}$. Moreover, a prosumer proposing an amount of reserved capacity takes the responsibility of actually being able to provide it. So, prosumers' power set-point and reserve set-points must be within their flexibility range. In other words, the feasible flexibility range accessible to power set-point p_n is tightened by the promised amount of reserves, as in (5.1b).

5.2.2 Peer-to-peer energy and reserve market

As pointed earlier, the current change in the power system, with new ways to produce and to consume electricity, calls for a shift from centralized producer-centric to decentralized prosumer-centric market structures. Focusing on the so called peer-to-peer market architecture, this thesis proposes the use of multiple bilateral trades, which can be seen as small unit-commitments between trading partners. Considering multi-bilateral trades requires a split of net powers, in the manner of [52], into a set of multiple bilateral trades p_{nm}. Every possible bilateral power trades within the community can be condensed in a matrix $\mathbf{P}$ such that

$$\mathbf{P} = \begin{pmatrix} p_{11} & \cdots & p_{1|\Omega|} \\ \vdots & \ddots & \vdots \\ p_{|\Omega|1} & \cdots & p_{|\Omega||\Omega|} \end{pmatrix} \tag{5.2}$$

where p_{nm} is necessarily equal to zero if prosumer m is not in prosumer n's trading partnership set ω_n. Net powers are then obtained by $p_n = \sum_{m\in\omega_n} p_{nm}$ as in (5.5e). As outlined in (5.5b), $\mathbf{P}$ is skew-symmetric to insure power balance of each trade, so $p_{nn} = 0$. This allows to potentially individualize prices per trade.

As mentioned in the introduction, aiming for a decentralized reserve market implies the split of joint chance constraint (5.1i) into individual chance constraints

$$\mathbb{P}_{f_n}\left(-r_n^- \leqslant p_n - \tilde{p}_n \leqslant r_n^+\right) \geqslant \delta_n \tag{5.3}$$

for each uncertain prosumer $n \in \Omega_{\text{nc}}$. This way each uncertain prosumer takes the responsibility of covering its own variability up to local confidence level δ_n. The allocation of the global risk responsibility, illustrated by δ, into local confidence levels $(\delta_n)_{n\in\Omega_{\text{nc}}}$ is discussed in Section 5.3. But it can be noted that the eventual correlation of local probability functions $(f_n)_{n\in\Omega_{\text{nc}}}$ would have to be taken into account in the definition of local confidence levels $(\delta_n)_{n\in\Omega_{\text{nc}}}$. Despite that, uncertain agents may not be able to cover all their variability on their own with load-shedding or power spillage. In consequence, a way of exchanging reserves should be added to the market. At the image of power bilateral trades this chapter proposes the use of reserve bilateral trades. This would then allow flexible and controllable agents to sell their reserve to uncertain agents and, hence, have the possibility to monetize their full available capacity. Every possible upward and downward reserve bilateral trades within the community can reciprocally be condensed in matrices $\mathbf{R}^+$ and $\mathbf{R}^-$ such that

$$\mathbf{R}^+ = \begin{pmatrix} r_{11}^+ & \cdots & r_{1|\Omega|}^+ \\ \vdots & \ddots & \vdots \\ r_{|\Omega|1}^+ & \cdots & r_{|\Omega||\Omega|}^+ \end{pmatrix}, \qquad \mathbf{R}^- = \begin{pmatrix} r_{11}^- & \cdots & r_{1|\Omega|}^- \\ \vdots & \ddots & \vdots \\ r_{|\Omega|1}^- & \cdots & r_{|\Omega||\Omega|}^- \end{pmatrix} \tag{5.4}$$

where r_{nm}^+ and r_{nm}^- are necessarily equal to zero if prosumer m is not in prosumer n's trading partnership set ω_n. As outlined in (5.5c)–(5.5d), $\mathbf{R}^+$ and $\mathbf{R}^-$ are skew-symmetric to insure reciprocity of each trade. Note that reserve tradings could be extended to specific partnership sets, ω_n^+ and ω_n^- reciprocally for upward and downward reserves. As the goal is to introduce the proposed techniques, all trading partnership sets are supposed identical to clarify the message.

However, it is important to note that the sign convention used for reserve trades differs from the one of power trades. Indeed, a prosumer n buying upward reserve capacity from prosumer m needs to be credited of this amount, so $r_{nm}^+ \geqslant 0$, while this amount must be debited if it is sold, so $r^+_{nm} \leqslant 0$. These observations are also true for downward reserve trades. Total amount of upward $r^{+,\text{tot}}_n$ and downward $r^{-,\text{tot}}_n$ reserves available to prosumer n combine reserve set-points r^+_n and r^-_n, which are self procured, and the sum of reserves traded with partners. Thus, for example, prosumer n's total upward reserve is given by $r^{+,\text{tot}}_n = r^+_n + \sum_{m\in\omega_n} r^+_{nm}$, as in (5.5f)–(5.5g). With reserve trades sign convention, one could notice that if a controllable prosumer n were to sell all its self procured upward reserve r^+_n, its total upward reserve left and available would then equal zero. To ensure that a prosumer can not sell more reserve than it can actually provide, total reserves available locally are forced to be positive, i.e. $0 \leqslant r^{+,\text{tot}}_n$ and $0 \leqslant r^{-,\text{tot}}_n$ as in (5.5k).

Gathering all these changes, the proposed peer-to-peer energy and reserve market finally reads

Peer-to-peer energy and reserve market

$$\min_{\substack{(p_n, r^+_n, r^-_n, r^{+,\text{tot}}_n, r^{-,\text{tot}}_n)_{n\in\Omega} \\ \mathbf{P}, \mathbf{R}^+, \mathbf{R}^-}} \quad \sum_{n\in\Omega} c_n(p_n) + c^+_n(r^+_n) + c^-_n(r^-_n) \tag{5.5a}$$

$$\text{s.t.} \quad \mathbf{P} = -\mathbf{P}^\mathsf{T} \tag{5.5b}$$

$$\mathbf{R}^+ = -\mathbf{R}^{+,\mathsf{T}} \tag{5.5c}$$

$$\mathbf{R}^- = -\mathbf{R}^{-,\mathsf{T}} \tag{5.5d}$$

$$p_n = \sum_{m\in\omega_n} p_{nm} \qquad n \in \Omega \tag{5.5e}$$

$$r^{+,\text{tot}}_n = r^+_n + \sum_{m\in\omega_n} r^+_{nm} \qquad n \in \Omega \tag{5.5f}$$

$$r^{-,\text{tot}}_n = r^-_n + \sum_{m\in\omega_n} r^-_{nm} \qquad n \in \Omega \tag{5.5g}$$

$$p^{\min}_n + r^-_n \leqslant p_n \leqslant p^{\max}_n - r^+_n \qquad n \in \Omega \tag{5.5h}$$

$$0 \leqslant r^+_n \leqslant r^{+,\max}_n \qquad n \in \Omega \tag{5.5i}$$

$$0 \leqslant r^-_n \leqslant r^{-,\max}_n \qquad n \in \Omega \tag{5.5j}$$

$$0 \leqslant r^{+,\text{tot}}_n,\ r^{-,\text{tot}}_n \qquad n \in \Omega \tag{5.5k}$$

$$\mathbb{P}_{f_n}\left(-r^{-,\text{tot}}_n \leqslant p_n - \tilde{p}_n \leqslant r^{+,\text{tot}}_n\right) \geqslant \delta_n \qquad n \in \Omega_{\text{nc}} \tag{5.5l}$$

where all control variables are handled locally by prosumers. Prosumer n's control variables are power set-point p_n, reserve set-points r^+_n and r^-_n, total available reserves $r^{+,\text{tot}}_n$ and $r^{-,\text{tot}}_n$, bilateral power trades $P_n = (p_{nm})_{m\in\omega_n}$ and reserve bilateral trades $R^+_n = (r^+_{nm})_{m\in\omega_n}$ and $R^-_n = (r^-_{nm})_{m\in\omega_n}$. Note that reserve bilateral trades help prosumers to adjust their total available reserves. Thus, prosumer n's uncertainty coverage does not solely considers self procured reserves as in (5.3) but for the total amount of reserves at its disposable as in (5.5l). It can also be pointed that reserve set-points r^+_n and r^-_n are still the ones used to tighten power set-point p_n flexibility range in (5.5h). Indeed, while total available reserves $r^{+,\text{tot}}_n$ and $r^{-,\text{tot}}_n$ helps compensate forecast errors, this power would not actually flow towards or outwards of a prosumer. If activated, traded reserves R^+_n and R^-_n would be engaged by trading partners to balance the global power balance. Thus, total available reserves $r^{+,\text{tot}}_n$ and $r^{-,\text{tot}}_n$ are rather to be seen as contractual reserves.

5.2.3 Specific decentralized peer-to-peer energy and reserve market algorithm

As developed in [2]'s appendix, a decentralized procedure based on the consensus ADMM of [53] can be used to solve (5.5). This decentralized method solves global problem (5.5) and, hence, leads to a competitive equilibrium which efficiency strongly depends on the chosen network charges. According to [87] ADMM seems well adapted for negotiation mechanisms in smart grids. Several extensions and convergence rate improvements have been proposed in [96–99]. Given the focus of this chapter is not on scalability a straightforward implementation of consensus ADMM is used. It can be noted that convergence, so existence, and uniqueness of a solution are guaranteed as long as the problem is convex. This condition is verified if: *(i)* all cost functions $(c_n, c_n^+, c_n^-)_{n\in\Omega}$ are convex, and *(ii)* chance constraints (5.5l) are convex. Conditions on convexity of chance constraints can be found in the literature such as in [124–126].

The final decentralized energy and reserve negotiation mechanism reads

$$
\begin{aligned}
X_n^{k+1} = \underset{X_n}{\text{argmin}} \quad & c_n(p_n) + \sum\nolimits_{m\in\omega_n} \sigma^\rho\left(p_{nm}, \tfrac{p_{nm}^k - p_{mn}^k}{2}, \lambda_{nm}^k\right) && (5.6a)\\
& + c_n^+(r_n^+) + \sum\nolimits_{m\in\omega_n} \sigma^\rho\left(r_{nm}^+, \tfrac{r_{nm}^{+,k} - r_{mn}^{+,k}}{2}, \lambda_{nm}^{+,k}\right) \\
& + c_n^-(r_n^-) + \sum\nolimits_{m\in\omega_n} \sigma^\rho\left(r_{nm}^-, \tfrac{r_{nm}^{-,k} - r_{mn}^{-,k}}{2}, \lambda_{nm}^{-,k}\right) \\
\text{s.t.} \quad & p_n = \sum\nolimits_{m\in\omega_n} p_{nm} \\
& r_n^{+,\text{tot}} = r_n^+ + \sum\nolimits_{m\in\omega_n} r_{nm}^+ \\
& r_n^{-,\text{tot}} = r_n^- + \sum\nolimits_{m\in\omega_n} r_{nm}^- \\
& p_n^{\min} + r_n^- \leqslant p_n \leqslant p_n^{\max} - r_n^+ \\
& 0 \leqslant r_n^+ \leqslant r_n^{+,\max} \\
& 0 \leqslant r_n^- \leqslant r_n^{-,\max} \\
& 0 \leqslant r_n^{+,\text{tot}},\ r_n^{-,\text{tot}} \\
& \mathbb{P}_{f_n}\left(-r_n^{-,\text{tot}} \leqslant p_n - \tilde{p}_n \leqslant r_n^{+,\text{tot}}\right) \geqslant \delta_n \quad \text{if } n \in \Omega_{\text{nc}} \\
\lambda_{nm}^{k+1} = \lambda_{nm}^k - \rho & \left(p_{nm}^{k+1} + p_{mn}^{k+1}\right)/2 && (5.6b)\\
\lambda_{nm}^{+,k+1} = \lambda_{nm}^{+,k} - \rho & \left(r_{nm}^{+,k+1} + r_{mn}^{+,k+1}\right)/2 && (5.6c)\\
\lambda_{nm}^{-,k+1} = \lambda_{nm}^{-,k} - \rho & \left(r_{nm}^{-,k+1} + r_{mn}^{-,k+1}\right)/2 && (5.6d)
\end{aligned}
$$

where $X_n = \left\{p_n, r_n^{+,\text{tot}}, r_n^+, r_n^{-,\text{tot}}, r_n^-, \hat{X}_n\right\}$ gathers prosumer n's optimization variables, sub-variable $\hat{X}_n = \{P_n, R_n^+, R_n^-\}$ gathers all its bilateral trades, disagreement cost function σ^ρ is such as

$$\sigma^\rho : (x, y, z) \in \mathbb{R}^3 \mapsto z\,(y - x) + \frac{\rho}{2}(y - x)^2 \tag{5.7}$$

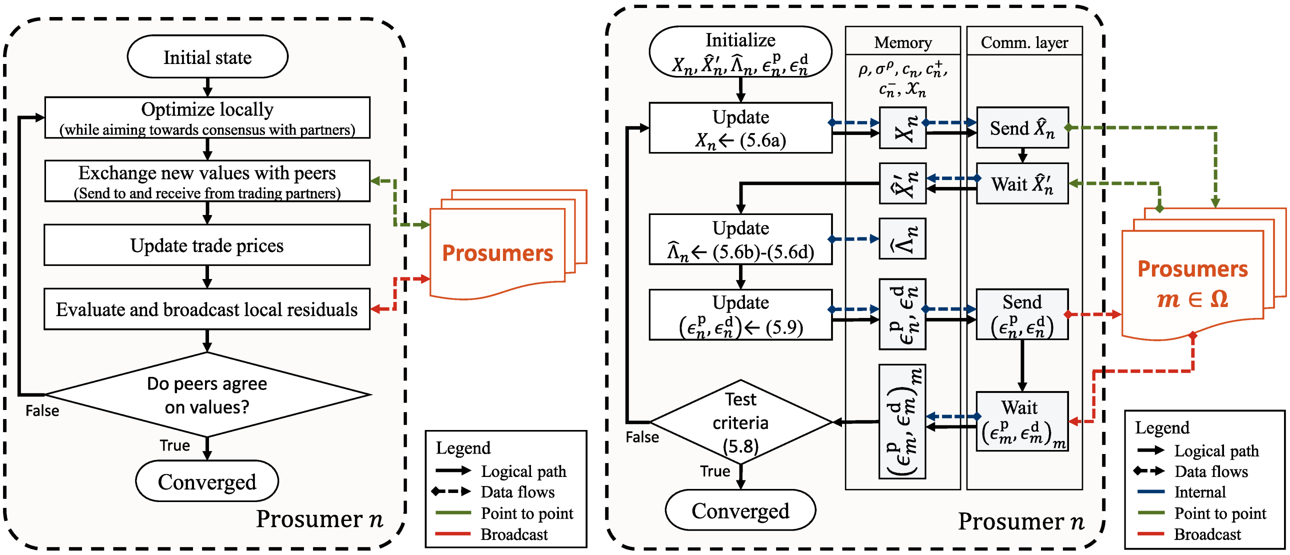
and penalty factor $\rho > 0$. Element λ_{nm} of matrix $\mathbf{\Lambda}$ corresponds to generation price of electricity for power trade p_{nm}. Similarly, λ_{nm}^+ and λ_{nm}^- are elements of matrices $\mathbf{\Lambda}^+$ and $\mathbf{\Lambda}^-$ and can be assimilated as procurement prices of upward and downward reserves. Note that $\mathbf{\Lambda}$, $\mathbf{\Lambda}^+$ and $\mathbf{\Lambda}^-$ are respectively the dual variables of (5.5b)–(5.5d). This formulation allows to have primal feasibility of constraints (5.5e)–(5.5l) at each iteration step. However, primal feasibility of trades reciprocity (5.5b)–(5.5d) is only verified at the limit after convergence. Note that additional terms of the augmented Lagrangian, represented by disagreement cost function σ^ρ in (5.6a), aim at encouraging, economically, a prosumer n to reach power consensus with its partners. Global stopping criteria associated to (5.6) are such as

$$\sum\nolimits_{n\in\Omega} \epsilon_n^{\text{p},k+1} \leqslant {\epsilon^{\text{p,tol}}}^2 \quad \text{and} \quad \sum\nolimits_{n\in\Omega} \epsilon_n^{\text{d},k+1} \leqslant {\epsilon^{\text{d,tol}}}^2 \tag{5.8}$$

with, respectively, primal and dual local residuals

$$
\begin{aligned}
\epsilon_n^{\text{p},k+1} &= \frac{1}{4}\sum\nolimits_{m\in\omega_n}\left[\left(p_{nm}^{k+1} + p_{mn}^{k+1}\right)^2 + \left(r_{nm}^{+,k+1} + r_{mn}^{+,k+1}\right)^2 + \left(r_{nm}^{-,k+1} + r_{mn}^{-,k+1}\right)^2\right] && (5.9a)\\
\epsilon_n^{\text{d},k+1} &= \sum\nolimits_{m\in\omega_n}\left[\left(p_{nm}^{k+1} - p_{nm}^{k}\right)^2 + \left(r_{nm}^{+,k+1} - r_{nm}^{+,k}\right)^2 + \left(r_{nm}^{-,k+1} - r_{nm}^{-,k}\right)^2\right] && (5.9b)
\end{aligned}
$$

where $\epsilon^{\text{p,tol}}$ and $\epsilon^{\text{d,tol}}$ denotes primal and dual global feasibility tolerances, respectively.



(a) Conceptual version (b) Symbolic version

Figure 5.1: Decentralized negotiation mechanism for peer-to-peer energy and reserve markets

As illustrated in Figure 5.1, the global energy and reserve negotiation mechanism occurs as follows. Each prosumer n first solves its own local optimization (5.6a) to update set-points p_n, r_n^+ and r_n^- while looking for new trade proposals $P_n^{k+1} = (p_{nm}^{k+1})_{m\in\omega_n}$, $R_n^{+,k+1} = (r_{nm}^{+,k+1})_{m\in\omega_n}$ and $R_n^{-,k+1} = (r_{nm}^{-,k+1})_{m\in\omega_n}$. Note that local constraints (5.5e)–(5.5l) are condensed in $\mathcal{X}_n$ which thus encompass memory of bounds $p_n^{\min}$, $p_n^{\max}$, $r_n^{+,\max}$ and $r_n^{-,\max}$, but also of f_n and δ_n for an uncertain prosumer $n \in \Omega_{\mathrm{nc}}$. The update of these local optimization variables also helps the prosumer keep track of its total available reserves $r_n^{+,\mathrm{tot}}$ and $r_n^{-,\mathrm{tot}}$. Then, prosumer n individually sends trade proposals p_{nm}^{k+1}. $r_{nm}^{+,k+1}$ and $r_{nm}^{-,k+1}$ to each partner $m \in \omega_n$. After receiving all counter proposals $\hat{X}_n^{\prime,k+1} = \left(p_{mn}^{k+1}, r_{mn}^{+,k+1}, r_{mn}^{-,k+1}\right)_{m\in\omega}$, prosumer n can update the corresponding trade prices $\hat{\Lambda}_n^{k+1} = \left(\lambda_{nm}^{k+1}, \lambda_{nm}^{+,k+1}, \lambda_{nm}^{-,k+1}\right)_{m\in\omega}$ with (5.6b)–(5.6d), and local residuals $\left(\epsilon_n^{\mathrm{p}}, \epsilon_n^{\mathrm{d}}\right)^{k+1}$ with (5.9). Finally, each prosumer n broadcasts its local residuals to all and, when all local residuals $\left(\epsilon_m^{\mathrm{p}}, \epsilon_m^{\mathrm{d}}\right)^{k+1}_{m\in\Omega\setminus\{n\}}$ are received, tests global stopping criteria (5.8). This process is repeated until convergence. Being decentralized the negotiation mechanism is not supported by a central entity. However, as most information exchanges, a communication standard needs to be defined by an institutional organization to associate a communication protocol between peers participating in the market.

It can be noted that problem (5.5) and this specific algorithm are in line with the generalized coordination problem proposed in Section 2.2. Moreover, peer-to-peer energy and reserve market (5.5) can be assimilated to exogenous peer-to-peer electricity market (4.6). Indeed, reciprocity constraints (5.5b)–(5.5d) could be condensed in a single reciprocity constraint $\hat{\mathbf{P}} = -\hat{\mathbf{P}}^{\mathsf{T}}$ where $\hat{\mathbf{P}}$ encompasses all bilateral trades. Whether they are power or reserve trades, bilateral trades matrix $\hat{\mathbf{P}} \in \mathbb{R}^{3|\Omega|\times 3|\Omega|}$ could be written such that

$$\hat{\mathbf{P}} = \begin{pmatrix} \mathbf{P} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}^+ & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R}^- \end{pmatrix} \tag{5.10}$$

with $\mathbf{0}$ zero matrices of the adequate size. The decomposition proposed in [2] would then directly apply to $\hat{\mathbf{P}}$ and lead to the same negotiation mechanism as (5.6).

5.3 Uncertainty allocation methods

While exposing peer-to-peer energy and reserve market (5.5), Section 5.2 voluntarily overlooked the way local confidence levels $(\delta_n)_{n\in\Omega_{\text{nc}}}$ are defined. Indeed, scattering the decision making should not deteriorate the quality of the solution and, in particular, the robustness provided by the global amount of reserves constituted. Local confidence levels should be such that local chance constraints (5.5l) provide the same global guarantee as with global confidence level δ in chance constraint (5.1i). This section exposes two approaches to allocate the uncertainty. The first way to allocate uncertainty would be to directly distribute portions of global confidence level. In other words, it allocates probability levels that each uncertain prosumer has to cover. Hence, it will be named relative uncertainty allocation. The second approach rather consists in sharing the amount of reserves necessary to reach the global confidence level. Uncertain prosumers would receive shares of it and would have to cover for a local amount of reserves to gather, which in turn could be translated in a local confidence level. This approach can be qualified as absolute uncertainty allocation since it distributes reserve amounts rather than their equivalent probabilities. It can be pointed that the two presented approaches can be employed independently to the correlation or not of uncertain prosumers. Naturally, these correlations would impact their performances.

5.3.1 Relative uncertainty allocation

An intuitive way to share risks between uncertain prosumers would be to simply distribute portions of global confidence level δ to relative local confidence levels $(\delta_n^{\text{rel}})_{n\in\Omega_{\text{nc}}}$. Local confidence levels would then be defined as follows

$$\delta_n^{\text{rel}} = \alpha_n\,\delta \tag{5.11}$$

with α_n is the uncertainty allocation factor adjusting responsibility shares among prosumers. For example, one could want to share risks proportionally to the part they take in the global uncertainty in average or in deviation. Corresponding uncertainty allocation factors would respectively take the forms

$$\alpha_n^{\mu} = p_n^{\mu}/p_{\text{nc}}^{\mu} \tag{5.12a}$$

$$\alpha_n^{\sigma} = \sigma_n/\sigma_{\text{nc}} \tag{5.12b}$$

with $p_{\text{nc}}^{\mu} = \mathbb{E}[\tilde{p}_{\text{nc}}]$ and $p_n^{\mu} = \mathbb{E}[\tilde{p}_n]$ where $\mathbb{E}[\,\cdot\,]$ denotes the expectation operator. It can be noted that p_{nc}^{μ} and p_n^{μ} respectively reflect global and local expected power outputs of uncertain prosumers. Similarly, σ_{nc} and σ_n are global and local standard deviations of global f_{nc} and local f_n power forecasts. Even though these two uncertainty allocation factors verify that $\sum_{n\in\Omega_{\text{nc}}}\delta_n \geqslant \delta$, there is no guarantee that they lead to a more robust solution than the centralized approach. Indeed, the Boole-Bonferroni inequality, classically used in distributionally robust optimization, is sufficient only in the case of worst case approximations. That is why one may be tempted to use a unitary uncertainty allocation factor such as

$$\alpha_n^{1} = 1 \tag{5.13}$$

which necessarily leads to a more robust solution than the centralized approach. However this increased robustness also induces a less optimal solution, i.e. a costlier reserve dispatch.

The relative uncertainty allocation distributes portions of the probability area δ, covered by global chance constraint (5.1i), to local probability areas δ_n^{rel} covered by local chance constraints (5.5l). As illustrated in Figure 5.2 these probability areas (in green) would be allocated regardless of probability distribution functions. The simplicity of this approach is a strong advantage for real world implementation as no additional knowledge on global and local power forecasts is required. However, similarly to exogenous network charges in Chapter 4, this apparent simplicity comes at the expense of a careful design of uncertainty allocation factors α_n. Indeed, there must be enough reserves to overcome misdispatches and, hence, guarantee a global power balance to the system operator. The challenge is even bigger as prosumers have to know uncertainty allocation factors before starting negotiation, so they have to be defined *a priori* by the market or system operator. The presented uncertainty allocation factors α_n could be extended to account for prosumers' local characteristics in a more comprehensive manner. For example, one could imagine using machine learning, Markov switching or other data analysis tools on their historical data to identify specific operational regimes. Another way would be to have a more comprehensive knowledge of prosumers' probability distribution forecasts, but this would be in opposition with the wish of privacy associated to peer-to-peer markets.

5.3.2 Absolute uncertainty allocation

The goal being to allocate amounts of reserve requirements, it may be more efficient to do it literally rather than doing it indirectly through risk indicators such as confidence levels. Thus, at the image of [117], the absolute uncertainty allocation distributes portions of the minimal reserves required, to obtain a global confidence of δ, to minimal local reserve requirements. As represented in Figure 5.2, this corresponds to allocating the spread of power which allows to cover a probability area of at least δ.

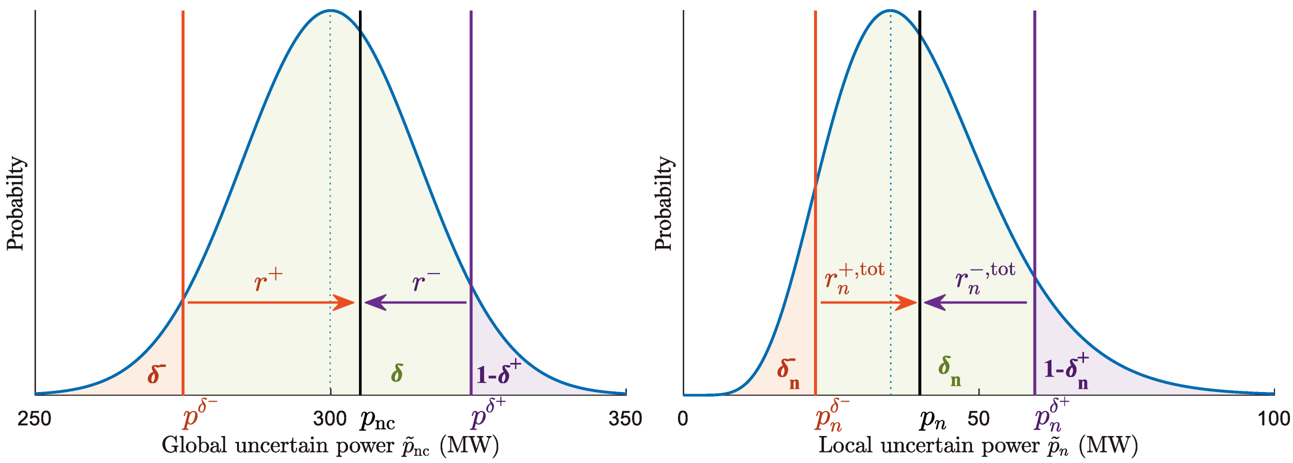
Let suppose that chance constraints (5.1i) and (5.5l) are respectively simplified into

$$\mathbb{P}_{f_{\mathrm{nc}}}\left(p^{\delta^-} \leqslant \tilde{p}_{\mathrm{nc}} \leqslant p^{\delta^+}\right) \geqslant \delta \quad \text{and} \quad \mathbb{P}_{f_n}\left(p_n^{\delta^-} \leqslant \tilde{p}_n \leqslant p_n^{\delta^+}\right) \geqslant \delta_n. \tag{5.14}$$

In a centralized context, p^{δ^+} and p^{δ^-} are maximal and minimal guaranteed power injections of uncertain prosumers. In a decentralized point of view, so with individual responsibilities, $p_n^{\delta^+}$ and $p_n^{\delta^-}$ are the maximal and minimal power injection an uncertain prosumer must be able to cover through the use of reserves. For example, if a wind farm were to overestimate the actual produced power, it would have to compensate contracted power p_n with additional power sells. For this, production capacity of another producer could be replaced, or a consumption capacity increase could be bought. Yet, this corresponds to a downward reserve, which is inline with global and local maximal guaranteed powers expressions

$$p^{\delta^+} = p_{\mathrm{nc}} + r^- \qquad \text{and} \qquad p_n^{\delta^+} = p_n + r_n^{-,\mathrm{tot}}. \tag{5.15}$$

Similarly, if that same wind farm were to underestimate its actual produced power, it would have to buy additional power capacity from another producer or replace a consumption capacity.



(a) Global probability distribution function f_{nc} (b) Local probability distribution function f_n

Figure 5.2: Uncertainty allocations illustrated on probability distribution functions (blue curves)

This corresponds to an upward reserve, which is inline with global and local maximal guaranteed powers expressions

$$p^{\delta^-} = p_{\text{nc}} - r^+ \qquad \text{and} \qquad p_n^{\delta^-} = p_n - r_n^{+,\text{tot}}. \tag{5.16}$$

The absolute uncertainty allocation consist in distributing the total amount of reserve required into local requirements on local available reserves. This can be written

$$\left(r_n^{+,\text{tot}} + r_n^{-,\text{tot}}\right) = \alpha_n \left(r^+ + r^-\right) \qquad \Leftrightarrow \qquad \left(p_n^{\delta^+} - p_n^{\delta^-}\right) = \alpha_n \left(p^{\delta^+} - p^{\delta^-}\right) \tag{5.17}$$

where both equations are equivalent with regard to expressions of p^{δ^+}, p^{δ^-}, $p_n^{\delta^+}$ and $p_n^{\delta^-}$. However, allocating the full reserve range such as (5.17) would not allow to deduce absolute local confidence levels $(\delta_n^{\text{abs}})_{n\in\Omega_{\text{nc}}}$. Cumulative distribution functions are not linear, so it is not possible to deduce local confidence levels of two-sided inequality chance constraints as it was in [117] for one-sided chance constraints.

In consequence, there is a need to split the coverage bands in two. Spread on each side of the expected power, one corresponds to the downward reserves to procure in the event of a power higher than expected. The second would represent the upward reserves required in the event of a power lower than expected. So, this chapter proposes to extend [117] to two-sided inequality probability constraints instead of one-sided ones. The distribution of reserves coverage on both sides of the expected value is illustrated in Figure 5.2 with the use of two different colors for downward (purple) and upward (orange) reserves. Absolute allocation (5.17) is now replaced by

$$\left(p_n^{\delta^+} - p_n^{\mu}\right) = \alpha_n \left(p^{\delta^+} - p_{\text{nf}}^{\mu}\right) \tag{5.18a}$$

$$\left(p_n^{\mu} - p_n^{\delta^-}\right) = \alpha_n \left(p_{\text{nf}}^{\mu} - p^{\delta^-}\right) \tag{5.18b}$$

with p^{δ^+}, $p_n^{\delta^+}$, p^{δ^-} and $p_n^{\delta^-}$ as defined in (5.15)–(5.16). Uncertainty allocation factors $(\alpha_n)_{n\in\Omega_{\text{nc}}}$ adjust responsibility shares among prosumers. To simplify the message the same uncertainty allocation factor has been chosen for both sides. But one could use two uncertainty allocation factors α_n^+ and α_n^- to respectively differentiate between downward and upward reserve allocations. Using two different uncertainty allocation factors may particularly be interesting in the case of non-symmetrical probability distributions or when there is a change of symmetry between global and local probability distribution functions as represented in Figure 5.2.

A two-sided inequality probability verifies that $\mathbb{P}_f(a \leqslant \tilde{X} < b) = F(b) - F(a)$, with $a < b \in \mathbb{R}$ and where f and F are respectively probability and cumulative distribution functions of random variable $\tilde{X}$. Hence, inequalities (5.14) can also be written as

$$\underbrace{F_{\text{nc}}\left(p^{\delta^+}\right)}_{\delta^+} - \underbrace{F_{\text{nc}}\left(p^{\delta^-}\right)}_{\delta^-} \geqslant \delta \qquad \text{and} \qquad \underbrace{F_n\left(p_n^{\delta^+}\right)}_{\delta_n^+} - \underbrace{F_n\left(p_n^{\delta^-}\right)}_{\delta_n^-} \geqslant \delta_n \tag{5.19}$$

where F_{nc} and F_n are respectively the cumulative distribution functions associated to probability distribution functions f_{nc} and f_n. Combining (5.18) with (5.19) returns

$$\delta_n^+ = F_n\Big(p_n^{\mu} + \alpha_n\big(F_{\text{nc}}^{-1}\left(\delta^+\right) - p_{\text{nf}}^{\mu}\big)\Big) \tag{5.20a}$$

$$\delta_n^- = F_n\Big(p_n^{\mu} - \alpha_n\big(p_{\text{nf}}^{\mu} - F_{\text{nc}}^{-1}\left(\delta^-\right)\big)\Big) \tag{5.20b}$$

where F_{nc}^{-1} is the inverse function of F_{nc}. Note that global probability distribution F_{nc} may include correlations between uncertain prosumers if present. The absolute uncertainty allocation finally gives

$$\delta_n^{\text{abs}} = F_n\Big(p_n^\mu + \alpha_n\big(F_{\text{nc}}^{-1}\left(\delta^+\right) - p_{\text{nf}}^\mu\big)\Big) - F_n\Big(p_n^\mu + \alpha_n\big(F_{\text{nc}}^{-1}\left(\delta^-\right) - p_{\text{nf}}^\mu\big)\Big) \tag{5.21}$$

as cumulative distribution functions are odd. Recall that $p_{\text{nc}}^\mu = \mathbb{E}[\tilde{p}_{\text{nc}}]$ and $p_n^\mu = \mathbb{E}[\tilde{p}_n]$ are global and local expected power outputs of uncertain prosumers, with $\mathbb{E}[\,\cdot\,]$ the expectation operator. The question now resides in the definition of δ^- and δ^+. Front-tail and end-tail global probabilities are linked to the global confidence level by

$$\underbrace{1-\delta}_{\text{Global probability error}} = \underbrace{\delta^-}_{\text{Front-tail probability}} + \underbrace{1-\delta^+}_{\text{End-tail probability}} \tag{5.22}$$

since the global probability error was split between both tails in (5.19). Thus, the definition of δ^+ and δ^- in (5.21) allows to emphasize on either tails, if one is more problematic for example. A barycenter of mass β can be defined to represent the importance of the front-tail in the global probability error. Front-tail and end-tail probabilities would then be given by

$$\delta^- = \beta\left(1-\delta\right) \qquad \text{and} \qquad 1-\delta^+ = \left(1-\beta\right)\left(1-\delta\right) \tag{5.23}$$

where $\beta \in [0,1]$. Since we are currently considering symmetrical probability distributions we can rightly assume both tails to be equally important. In such case, the barycenter is taken as follows $\beta = 1/2$ in the rest of the chapter.

As it is more aware of global and local forecast errors, the absolute uncertainty allocation approach may offer a lower optimality gap than the relative approach. Note that no hypothesis were required to obtain (5.21) and are, thus, also valid when uncertain prosumers are correlated. Yet, computing the global probability distribution function f_{nc} of correlated uncertain prosumers requires to conduct convolutions which would jeopardize their privacy. However, considering independent uncertain prosumers would help preserve their privacy. Indeed, if uncertain prosumers are supposed independent and are either in great numbers or follow Gaussian curves, the global probability distribution function would be a Gaussian with an expected value $p_{\text{nc}}^\mu = \sum_{n\in\Omega_{\text{nc}}} p_n^\mu$ and a standard deviation $\sigma_{\text{nc}}^2 = \sum_{n\in\Omega_{\text{nc}}} \sigma_n^2$. In such case, f_{nc} can be estimated without uncertain prosumers revealing too much information on themselves. Moreover, uncertainty allocation factors α_n^σ based on standard deviations as given in (5.12b) would be good candidates.

5.4 Simulation results

The objective of this section is to evaluate performances of the proposed uncertainty allocations by analyzing simulation results. After describing the case on which they have been tested on in Subsection 5.4.1, the section investigates several points. Firstly, Subsection 5.4.2 verifies the ability of the proposed uncertainty allocations to globally cover forecast errors by providing enough reserves. Secondly, Subsection 5.4.3 looks into the optimality gaps they might induce by comparing them to the centralized energy and reserve market recalled in Subsection 5.2.1. The same subsection also investigates the way prosumers are impacted by them. Note that the four decentralized uncertainty allocations tested in the following simulation results are reported in Table 5.1.

5.4.1 Test case description

To evaluate the proposed peer-to-peer energy and reserve market, flexible prosumers as well as uncertain prosumers need to be present within a single case study. Some test cases exist for peer-to-peer energy markets, such as in [2,3], but they do not provide uncertain prosumers' forecasts,

Table 5.1: Tested decentralized uncertainty allocations

Name (Acronym used in figures)	Allocation method ($\delta_n^{\cdot}$)	Allocation factor ($\alpha_n^{\cdot}$)
Relative expectation uncertainty allocation (Rel. α^μ)	Relative as in (5.11) (δ_n^{rel})	Expected power ratios as in (5.12a) (α_n^μ)
Relative standard deviation uncertainty allocation (Rel. α^σ)	Relative as in (5.11) (δ_n^{rel})	Standard deviation ratios as in (5.12b) (α_n^σ)
Relative unitary uncertainty allocation (Rel. α^1)	Relative as in (5.11) (δ_n^{rel})	Unitary ratios as in (5.13) (α_n^1)
Absolute standard deviation uncertainty allocation (Abs. α^σ)	Absolute as in (5.21) (δ_n^{abs})	Standard deviation ratios as in (5.12b) (α_n^σ)

actual realizations or forecast errors. Separately, there exist some open-source databases which provide such information for wind and solar productions, such as [142] and [143] respectively. But they do not describe characteristics of the market in which they are connected.

Hence, there is a need for a novel test case adapted to study peer-to-peer markets both on energy and reserves. The focus here is put on combining [3]'s test case with [142, 143]'s data sets. Illustrated in Figure 5.3, [3] is based on the IEEE 14-bus test system [144] and has cost functions c_n such that

$$c_n(p_n) = \frac{1}{2}a_n p_n^2 + b_n p_n \tag{5.24}$$

where a_n and b_n parameters of the 20 prosumers are given in [145]. Note that the connection to the main grid is represented by a prosumer with a quadratic term equal to zero and a linear term equal to pool market prices. Prosumers' flexibility as well as pool market prices present in [145] are based on Australian data sets [146, 147] and are normalized with installed capacities.

Data sets [142, 143] have been used to provide forecast errors to [145]'s renewable producers. Power production forecasts $\tilde{p}_n$ are given by

$$\tilde{p}_n = \overline{p}_n + \Delta_n \tag{5.25}$$

where realized power $\overline{p}_n$ comes from [145] and forecast error Δ_n is deduced from [142, 143], after being normalized with installed capacities. It can be noted that data sets [142, 143] aggregate productions of wind and solar on a geographical area over several years, while [145] concerns multiple assets over a single year. Thus, each uncertain prosumer represents a single year of [142, 143]. Uncertain prosumers can thus be considered independent. The rest of the chapter assumes that power forecast errors follow a normal distribution. Standard deviations σ_n are constant and deduced from forecast errors Δ_n observed over the year. In consequence, global probability distribution function f_{nc} follows a Gaussian curve centered on power forecast $\tilde{p}_{\text{nc}} = \sum_{n\in\Omega_{\text{nc}}} \tilde{p}_n$ with a standard deviation $\sigma_{\text{nc}} = \|(\sigma_n)_{n\in\Omega_{\text{nc}}}\|_2$.

To synthesize, the final test case is composed of: *(i)* 11 flexible consumers, *(ii)* 3 flexible producers, *(iii)* 5 independent uncertain renewable producers, 3 wind and 2 solar farms, and *(iv)* a prosumer representing the connection to the grid. All prosumers have quadratic energy cost functions c_n. Both quadratic and linear terms are null for renewables, while the linear term of the prosumers representing the grid is equal to the pool market price. Renewable producers are assumed independent and have normal probability distributions with a constant standard deviation. Global probability distribution f_{nc}, encompassing all uncertain prosumers, have a standard deviation $\sigma_{\text{nc}} = 5.20$ MW for a total installed capacity of 180 MW. Note that the average global power forecast error is solely of 6 kW. The test case includes hourly time steps over a year. Finally, upward and downward reserve cost functions (resp. c_n^+ and c_n^-) are supposed purely quadratic and equal to c_n's quadratic terms. Renewable prosumers can not provide upward reserves, $r_{n\in\Omega_{\text{nc}}}^{+,\max} = 0$, since they can not guarantee their provision. But they can always procure downward reserves simply by shading their production. Since shading

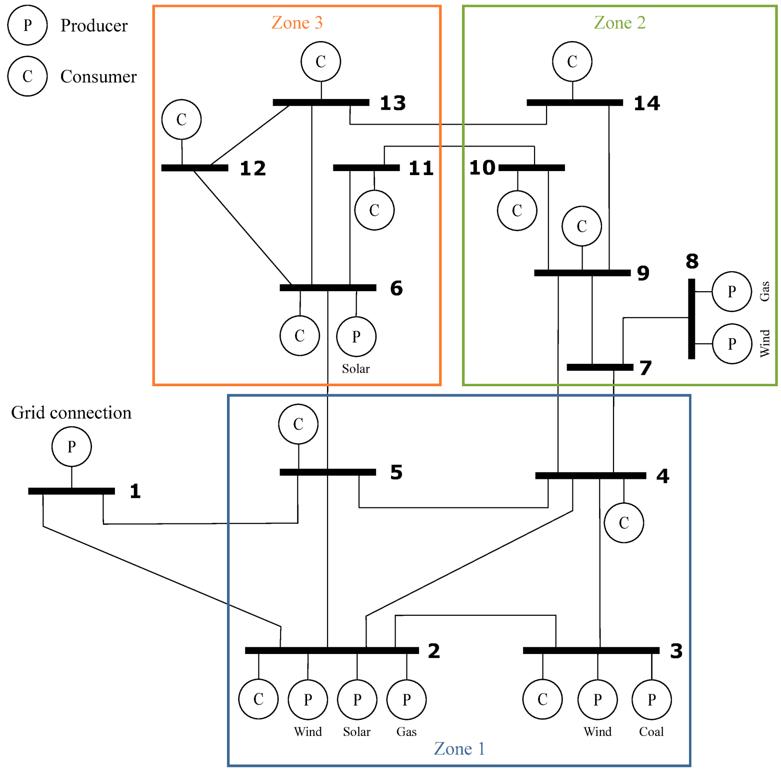
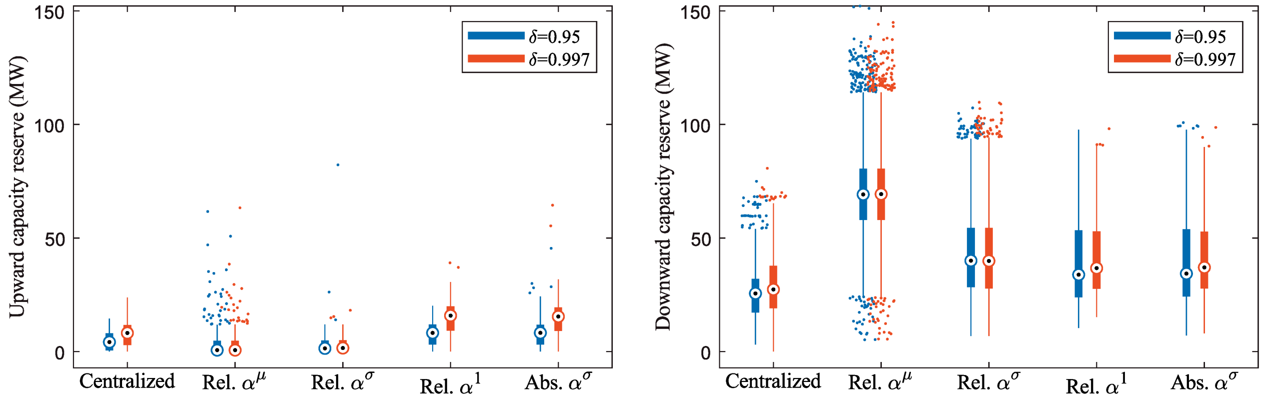


Figure 5.3: Modified IEEE 14-bus test system

does not induce any additional cost to renewable sources, downward reserve cost function of non-controllable prosumers are taken equal to zero, $c^{-}_{n\in\Omega_{\text{nc}}} = 0$.

5.4.2 Reliability of decentralized uncertainty allocations

When dealing with reserve markets the first and most crucial step is to verify whether enough amounts of reserves are procured to cover eventual lack of commitment and, thus, limit the risks of power unbalance. It is important to note that the tools proposed here only bear for non respect of commitments such as power forecast errors and do not consider technical breakdowns. To verify this, one could first look at the global amount of reserves made in the overall market. Figure 5.4 compares global amounts of global reserves constituted for the different approaches to allocate reserves. The analysis must be differentiated between upward and downward reserves. Indeed, on one hand, all non-controllable prosumers of the current test case are renewable sources which can freely provide downward reserves with no additional costs simply by shading their production. Thus, it is not surprising that downward reserves are provided abundantly by the peer-to-peer energy and reserve markets, as showed in Figure 5.4b. This observation is valid for any uncertainty allocation since largely providing downward reserves only helps to improve the coverage of power forecast errors. On the other hand, always being costly, the different energy and reserve markets, both centralized and peer-to-peer, are much more frugal in the amount of upward reserves. For example, it can be observed in Figure 5.4a that relative expectation and standard deviation uncertainty allocations with expectation α^{μ} or standard deviation α^{σ} allocation factors procure less upward reserves than with the centralized energy and reserve market. On the contrary, as it is more robust, the unitary allocation factor allows the relative uncertainty allocation factor to procure more upward reserves then the centralized approach. The tested absolute uncertainty allocation gave a similar result to the relative unitary uncertainty allocation. For the same reasons, it can be noted that when aiming for a higher confidence level, e.g. 99.7% instead of 95%, increases the amount of upward reserves but not necessarily for downward reserves.



(a) Upward reserves (b) Downward reserves

Figure 5.4: Global amount of reserves on the market level

A more relevant way at analyzing market outcomes would be to deduce the global confidence level actually reached. If noted δ^*, the reached global confidence level can be estimated by

$$\delta^* = \mathbb{P}_{f_{\text{nc}}}\left(-r^{-*} \leqslant p_{\text{nc}}^* - \tilde{p}_{\text{nc}} \leqslant r^{+*}\right) \tag{5.26}$$

where the $\cdot^*$ superscript denotes market outcomes obtained by simulations. Note that variables r^+, r^- and p_{nc} are respectively obtained by (5.1f)–(5.1h), even for the peer-to-peer energy and reserve markets *a posteriori*. The global confidence levels reached by the different market configurations are gathered in Figure 5.5. The first important observations is that the relative uncertainty allocation both associated to expectation and standard deviation allocation factors, by far, do not provide sufficient risk coverage. But these insufficient performances do not necessarily imply that the two approaches can not improve. As a matter of fact, one may choose to multiply the uncertainty allocation factors by a coefficient which may be adjusted with regard of the test case and historical data. In spite of that, relative unitary and absolute standard deviation uncertainty allocations present more satisfying performances. Indeed, the median value of the global confidence levels they reached, over the yearly data set, are respectively of 98.2% and 98.3%, and of 94.4% and 93.7% for the 25th percentile for a targeted 95% global confidence level. However, Figure 5.5 shows that their coverage is still lacking at times, detailed later in Subsection 5.4.3. This result is somewhat unexpected as, from a statistical point of view, the relative unitary uncertainty allocation should always be more restrictive than the centralized approach. This observation is also valid when aiming for a global confidence level of 99.7%.

Before excluding relative unitary and absolute standard deviation uncertainty allocations, it would be interesting to look at the specific conditions in which they do not perform as well

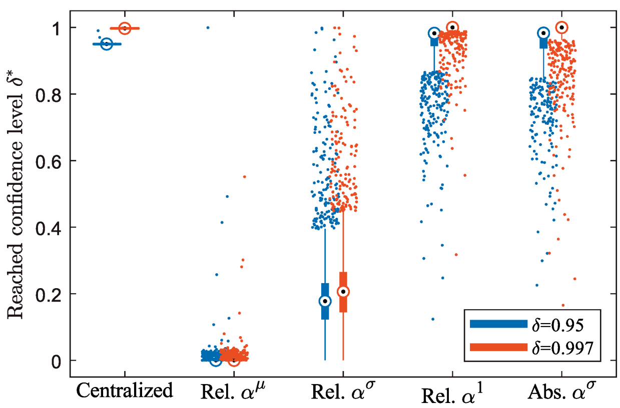
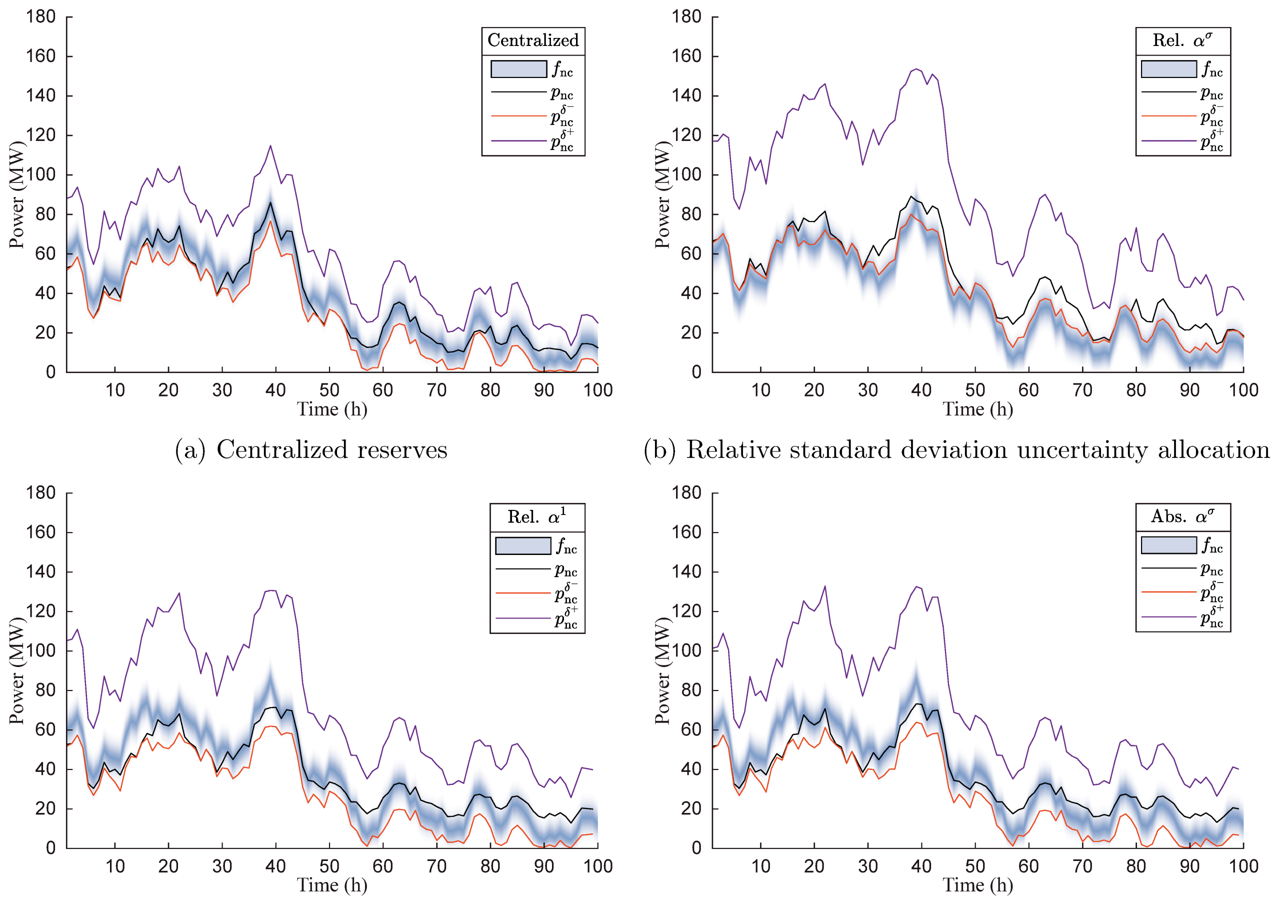


Figure 5.5: Confidence levels actually reached

anticipated. To have a first insight on this, one could look at the evolution of reserves r^+ and r^- as well as the total uncertain power p_{nc}. This would help to see how probability distribution f_{nc} of uncertain power forecast $\tilde{p}_{\text{nc}}$ is framed by them. Figure 5.6 illustrates this for the different uncertainty allocation approaches, centralized and decentralized, for the first 100 independent time steps. Note that relative expectation uncertainty allocation is not displayed here as it is way too ineffective and would not add to the comprehension of other uncertainty allocation approaches. For example Figure 5.6b shows that, overall, uncertain agents have a tendency to over sell power without contracting sufficient upward reserves. In view of the minimal covered power production, the lack of upward reserves appears highly probable for almost all time steps. This allows to understand that the relative standard deviation uncertainty allocation requires local confidence levels which are too low to obtain the desired global confidence level. However for relative unitary and absolute standard deviation uncertainty allocations in Figures 5.6c and 5.6d, uncertain prosumers gathered enough upward reserves to follow the under side of the probability distribution curve.

Though two remarks can be made on them. Firstly, it can be seen that they do not follow the distribution curve as tightly as the centralized approach which, as detailed later, may induce higher costs and, hence, optimality gaps. Secondly, when taking a better look at Figures 5.6c and 5.6d, one can see that the orange lines are crossing over the blue probability distributions in places such as between times 57 and 59, 72 and 75, or 90 and 95. Thus, at this time intervals, uncertain prosumers did not contracted enough upward reserves to cover their uncertain power forecast. Yet, these three pointed intervals happen at times when global expected power forecasts are low. It could then be conjectured that these two decentralized uncertainty allocation become inefficient only in cases of low uncertain power. Of course this speculation



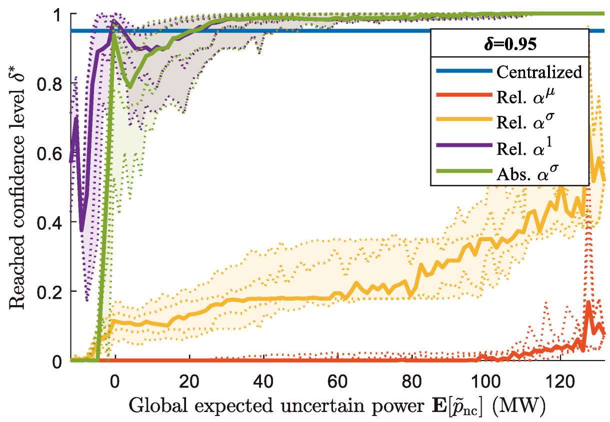
(a) Centralized reserves

(b) Relative standard deviation uncertainty allocation

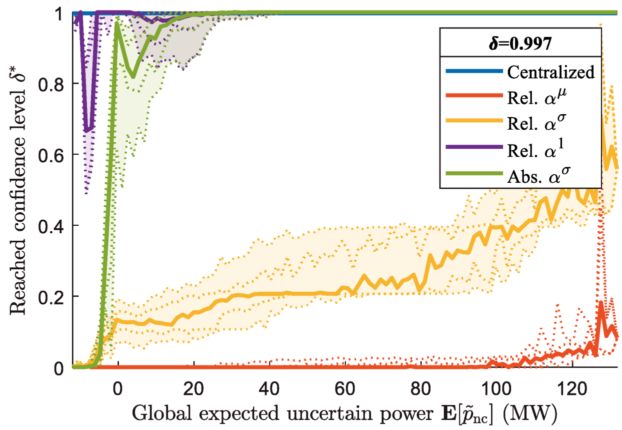
(c) Relative unitary uncertainty allocation

(d) Absolute standard deviation uncertainty allocation

Figure 5.6: Global coverage of reserves for a global confidence level aimed of 95%



(a) Global confidence level aimed of 95%



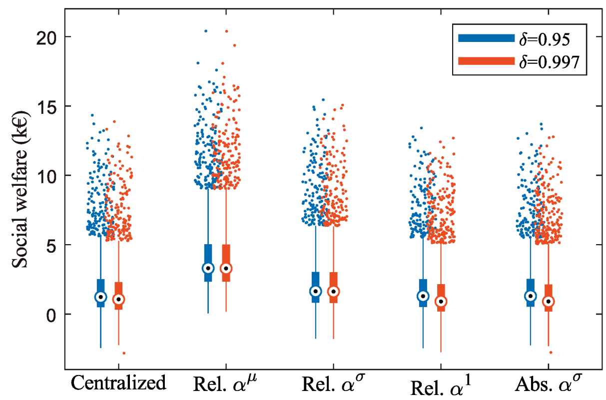
(b) Global confidence level aimed of 99.7%

Figure 5.7: Influence of the expected power forecast on uncertainty allocation performances

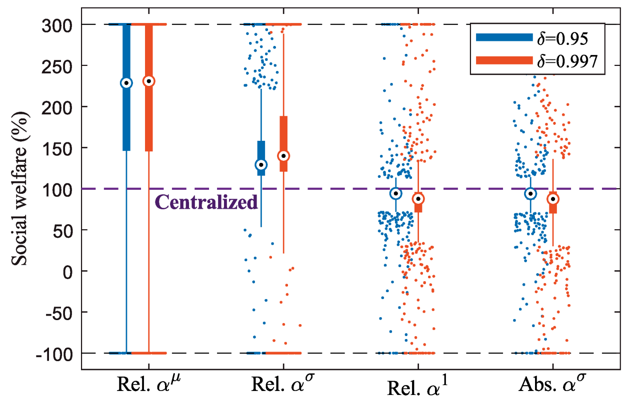
needs to be verified over the all data set. For this, Figure 5.7 plots the global confidence levels actually reached in function of the global expected uncertain power. Figure 5.7 shows that performances of decentralized approaches are all influence by the amount of uncertain power that is forecasted while the centralized approach has reliable performances. In particular, relative unitary and absolute standard deviation uncertainty allocations perform well and even above requirements at higher power forecasts. But their performances deteriorates as the forecasted amount of uncertain power diminishes, until it passes under the requested threshold of 95% such as in Figure 5.7a for example. Thus, Figure 5.7 confirms the conjecture that performances of these two decentralized uncertainty allocations are influenced by the global amount of uncertain power forecasted. However the small size of the current test case does not allow to say whether this deduction is inherent to the proposed approaches or solely that there are not enough uncertain prosumers to fully take advantage at the eventual compensations between probability distributions. In consequence, there is a need to create larger test cases not only including controllable but also many stochastic prosumers to test future consumer-centric markets accounting for both energy and reserves.

5.4.3 Optimality gap and impact on prosumers

Once uncertainty coverage performances have been checked, another important assessment is the optimality gap between the tested decentralized approaches and the centralized approach taken as reference. The analysis of these eventual differences provides information on whether their use is economically reasonable. Real world implementations make a trade off between per-



(a) Social welfare amounts

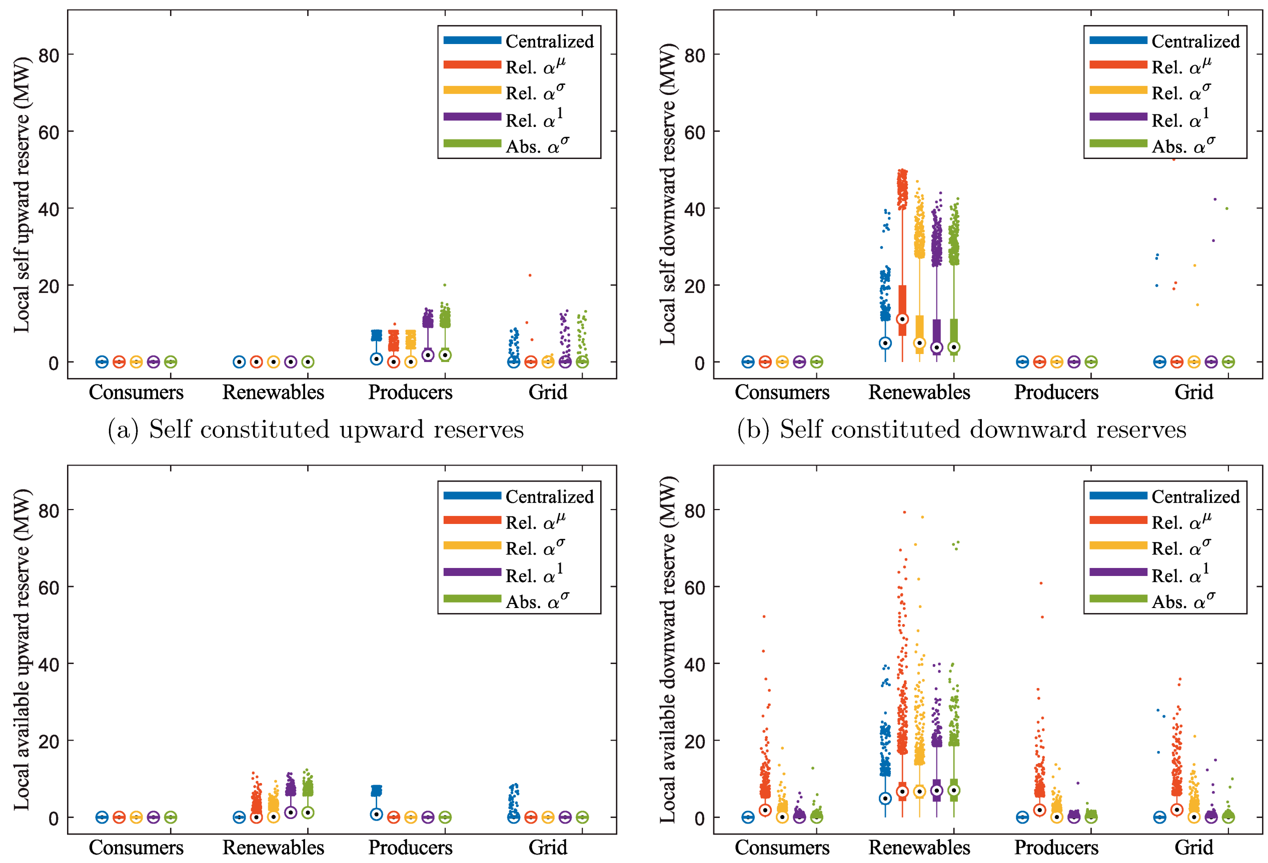


(b) Social welfare gains in percentage

Figure 5.8: Impact on the global social welfare

formances and costs. Hence, a very effective uncertainty allocation would not be implemented if the performances were obtained at the expense of cost efficiency. The choice would actually be made out of a compromise between costs and uncertainty coverage. The fastest way to observe how proposed approaches economically compare to the centralized approach is to show the global social welfare they induced. Global social welfares can be deduced by taking opposites of global objectives coming from centralized (5.1) and peer-to-peer (5.5) energy and reserve market outcomes. These global social welfares are gathered in Figure 5.8a while Figure 5.8b highlights ratios of decentralized approaches compared to the centralized one. Note that ratios in Figure 5.8b are made individually for each time step. Figure 5.8 shows that relative expectation and standard deviation uncertainty allocations have higher global social welfares than the centralized approach. But this economical benefit does not compensate for their important lack of uncertainty coverage. Relative unitary and absolute standard deviation uncertainty allocations are slightly less optimal than the centralized approach. This is explained by the fact that they often were more restrictive and, thus, required more reserves than strictly necessary to reach the targeted global confidence level, exposed in the previous subsection. Though it may be noted that their optimality gap to the centralized approach is small. For example, for an aimed global confidence level of 95%, relative unitary and absolute standard deviation uncertainty allocations respectively have a median optimality gap of 5.8% and 6.1%. However, their over cautiousness is costlier for an higher global confidence level target such as 99.7% since upward reserves become more expenses, thus leading to median optimality gaps of 12.3% and 12.6%. Note that the lower coverage observed in Subsection 5.4.2 at low expected global uncertain power induces higher social welfare at those times.

Even though slightly less effective than the centralized way to allocate reserves, the main advantage of decentralized uncertainty allocations is the transfer of responsibility from every



(a) Self constituted upward reserves

(b) Self constituted downward reserves

(c) Total available upward reserves

(d) Total available downward reserves

Figure 5.9: Local reserves

prosumers of the market to the prosumers creating the issue in the first place, so to uncertain prosumers. This transfer of responsibility can be seen in Figure 5.9 illustrating self constituted reserves and what is actually available at the end after prosumers made bilateral trades on reserves. As expected, renewable prosumers (here the sole uncertain prosumers) are the ones procuring downward reserves abundantly with their ability to shade production. It can be seen that controllable agents buys unnecessary downward reserves, but this is an artefact as it only come from the free of charge particularity of renewables shading. Unable to make their own, renewable prosumers are however obliged to buy upward reserves from the controllable prosumers participating in the market. For this specific test case, one could observe that renewable producers mainly get there available upward reserve form the producers and, marginally, from the connection to the main grid. Thus the peer-to-peer community appears to be rather autonomous on reserves in this test case. Note that this autonomy of reserves could even be enforced by taking out the grid prosumer from the reserve side of the peer-to-peer energy and reserve market, which would induce specific trading partnership sets for each market product.

5.5 Synthesis

To conclude on decentralized stochastic coordination problems, this chapter proposed a new decentralized way to handle reserve allocations by the mean of a peer-to-peer energy and reserve market. This decentralized approach notably calls for shift from the classical share of responsibility for all members of the market community to solely the responsibility of members at the origin of the issue, namely uncertain prosumers such as renewable sources. The transfer of responsibility was allowed by the introduction of new market products, namely upward and downward reserve bilateral trades, allowing uncertain prosumers to buy reserves from other, more reliable prosumers if they are unable to provide enough on their own. But the split of the global responsibility materialized by a joint chance-constraint into local individual ones rose the question of how the global level of confidence to reach should be allocated among uncertain prosumers into local level of confidences. Two main approaches to allocate the coverage of uncertainty have been proposed in the chapter. The first, simpler, approach consist in a direct proportional share of the global confidence level while the second, more comprehensive, rather shared portions of the global amounts of reserves required to reach the targeted global level of confidence.

Tested on an adapted form of the IEEE 14-bus test system, two of the four proposed uncertainty allocations proved to be promising. These two approaches have shown their ability to provide sufficient global reserves only with a low optimality gap compared to the centralized approach. But they had insufficient performances at low expected power forecasts of renewable sources. Unable to determine if these lower performances was inherent to their nature or whether coming from the small size of the test case, this calls for the creation of a much larger test case both involving controllable and stochastic prosumers in sufficient numbers to verify their performances in a future work. Even though showing insufficient performances, the two other approaches could be improved by the presence of a multiplicative coefficient, which could be called unit uncertainty allocation. At the image of Chapter 4's unit fees $u^{\cdot}$ for exogenous network charges, this additional parameter could be chosen wisely based on historical data by the system operator. Another interesting future work would be the extension from the current single time-step peer-to-peer energy and reserve market to a multiple time-step version. Doing so would for example allow to study the possibility for a storage unit to take advantage of renewable sources. Indeed, a storage unit could absorb their non-predicted over production, rather than shedding it, and to sell it back either as energy or upward reserve at latter times. This study could then be extended to the question of whether this storage should be localized at each uncertain prosumer or as a bigger independent actor.

6 Conclusions and perspectives

6.1 Summary and conclusions

The objective of this thesis was to study the feasibility of the concept of peer-to-peer markets to electricity markets. An important point was that considering a peer-to-peer electricity market does not only entail into a change of paradigm from a centralized to a decentralized structure of electricity market, but also requires an adaptation of the peer-to-peer market framework to the specific constraints existing in power systems. After a reminder of the classical problems at hand, the thesis first described how classically centralized electricity markets would be adapted to allow for bilateral trades between prosumers at the basis of peer-to-peer markets. While it appeared relatively straightforward in simple energy market, the thesis pointed strong conceptual difficulties when considering the electrical network's constraints or the procurement of reserves which are centralized by nature. An effort of theoretical generalization has been conducted to exhibit that the works of this thesis can be applied to many other fields. Notably, the proposed generalized coordination problem was able to ally the decentralized nature of peer-to-peer markets with the centralized nature of power systems. Indicated by the name, coordination problems consist in a set of problems within which numerous entities are interdependent and, thus, need to coordinate and agree on certain linking variables. Coordination problems are a large family of optimization problems as it not only includes most if not all type of markets, of goods in particular, but also problems in which systems much coexist with each other, such as energy systems (electricity, heat and gas systems) or a fleet of drones or vehicles, or even large systems integrating multiple complex subsystems, such as the power grid itself or a house with several intelligent appliances or a factory with robots. Based on *alternating direction method of multipliers* (ADMM), the thesis proposed a negotiation algorithm which orchestrates the resolution of such coordination problems in a decentralized way. Exposing how it could be applied on different examples, the decentralized algorithm showed its applicability and usefulness for peer-to-peer electricity markets.

Indeed, at the occasion of a complexity analysis, the thesis was able to show the interest of using the ADMM in comparison to another type of algorithm sometimes used in the literature, namely *consensus and innovation*. This confirmed comments made in other literature that ADMM based algorithm may be appropriated for consumer-centric and, hence, peer-to-peer markets. The complexity analysis also revealed that the structural complexity of the peer-to-peer market organization induced more scalability issues than other consumer-centric markets as the community-based one. Notably, even though also impacted by the size of the market, the analysis showed that the computational burden placed on prosumers during the negotiation mechanism was not as pregnant as the communication burden put on the communication infrastructure. Yet, the important traffic that would be induced by such algorithm would inevitably induce and increase communication delays. Thankfully the ADMM based negotiation mechanism proved rather resilient to such conditions, in particular when taking advantage of asynchronous communications so that fastest and closest prosumers could pursue the negotiation process without having to constantly wait for the slowest and farthest prosumer of the market. In the view of the important implications peer-to-peer electricity market would have on the communication infrastructure, two improvements have been identified with the common aim of reducing the number of messages that must be exchanged between the market participants to reach final consensus. The first focused on improving the negotiation mechanism itself. Alternative stopping criteria have been tested to allow prosumers to unilaterally conclude negotiations on bilateral trades which already reached consensus before the end of the negotiation process. So,by giving more meaning to stopping criteria, i.e. at the level of bilateral trades rather than overall, it has been possible to reduce the number of exchanged messages necessary to reach consensus without negatively impacting the overall power balance of the market. The second improvement focused on reducing the structural complexity of the peer-to-peer market framework. One of the main advantage of the peer-to-peer market is that it can change its structural complexity simply by changing the way prosumers are interconnected with one another. This notably allows the peer-to-peer market framework to assume most consumer-centric market organizations solely by adapting its interaction graph accordingly. For example, the peer-to-peer market can assume the pool and community-based market configurations respectively by gathering prosumers around one or multiple non-profit entities named market operator and community managers. This adaptability could be used to design interaction graphs reducing the structural complexity of the problem and, thus, reduce the number of information exchanged to reach consensus. In consequence, the chosen negotiation mechanism demonstrated its ability for further improvements to overcome the scalability issues of real world implementations.

Other conceptual dilemmas such as grid constraints and stochastic behaviors were still at hand before obtaining an actual peer-to-peer electricity market. First, contrary to speculative markets, electricity markets as well as most commodity markets have strong constraints coming from their physical nature requiring a physical infrastructure to carry them between market participants. Even though already hard to solve, electricity network models are still less complex than other commodity markets. Indeed, other energy carriers such as heat and gas infrastructures are not only non-convex but would also account for internal storage, named line packs in gas networks for example. This would have required to directly consider multiple time step problems rather than simpler single time step models first. In the case of good shipments, such as common goods transported in containers or cargoes, optimization variables would have been discrete by nature which would have inherently brought more complexity to the problem. Moreover, good markets usually enclose long horizon time coupling constraints as their transportation takes time. Having continuous variables and able to be model on independent time steps, electricity markets are a good starting applicative example to show how the centralized nature of physical infrastructures can be overcome to coordinate with decentralized peer-to-peer markets. This thesis showed that all constraints and objectives of the electrical network could be gathered in one unique entity, such as the system operator, with which prosumers would coordinate at the same time as they negotiate bilateral trades with each other. Two class of approaches have been identified for prosumers to coordinate with this system operator. The first approach proposes to account for power system constraints in an exogenous way. In this setup the system operator would provide network charges to prosumers before the start of the negotiations so that they are aware of the costs they will have to face for using the power system and act accordingly during the negotiations. Thus prosumers would simply account for additional costs they would have to pay, on a per trade basis, for using the grid. The main advantage of this approach is that the exogenous peer-to-peer electricity market unfolds as any standard peer-to-peer market as it would simply adapt the prosumers model to the electrical network on which they are connected to. However, being exogenous and, hence, given *a priori*, these exogenous network charges can not guarantee respect of power system constraints in every circumstances. For this, network charges would have to be estimated endogenously at the same time as prosumers have knowledge of their power injections on the grid. And that is exactly what the second approach proposes to account for power system constraints. In the resulting endogenous peer-to-peer electricity markets, prosumers send updates of their injected or withdrawn power to the system operator at each iteration step of bilateral trade negotiations. The system operator is then able to to re-estimate the network charges and to check feasibility of the inquired power injections. Now insuring the feasibility of the peer-to-peer market outcomes, this approach as the inconvenient of requiring a strong presence of the system operator in the negotiation process and, hence, a certain amount of computation power.

The second conceptual dilemma challenging feasibility of peer-to-peer electricity markets for real world applications is the issue of stochastic behaviors. Even though they originate locally at prosumers level, uncertain power productions have a global impact on the market as they can combine one another and aggravate their potential harmful consequences. Thus, procuring a sufficient amount of reserves to these probable dispatch errors is essential. Historically, these reserves have been managed centrally by the system operator as resulting power unbalances would occur on the power system its manages. Yet, one could say that uncertain prosumers such as renewable sources should be the only ones with the responsibility to provide reserves covering their own forecast errors. But uncertain prosumers, renewable sources in particular, would not be capable to provide sufficient reserves on their own as this would require them to have important local storage units which would render them economically non-viable. Uncertain prosumers would then need to contract reserves from other prosumers which are controllable. This thesis proposed these contracts of reserves to be made through bilateral contracts such as already made on energy. A peer-to-peer energy and reserve electricity market thus emerges and allows prosumers to provide and exchange not only electrical energy between them but also a way for uncertain prosumers to procure sufficient amounts of reserves. Controllable prosumers would then still be able to fully benefit from their installed power capacity by monetizing it on both energy and reserves market products. However simply saying that uncertain prosumers must procure as much coverage as it was aimed overall in a centralized approach may lead to sub-optimalities. Indeed this would result in a more robust solution which may bring reserves overcosts as it would not take advantage of positive statistical correlations between uncertain prosumers and solely suppose the negative ones. To tackle this the thesis proposed two approaches to allocate the overall coverage required at uncertain prosumers' level. One approach, named relative uncertainty allocation, proposes that uncertain prosumers directly considers a proportion of the global level of confidence that must be reached. While the other, named absolute uncertainty allocation, rather shares proportions of the total amount of reserves that are required to reach this global level of confidence. This second approach would lead to lower optimality gaps as it accounts for uncertain power probability distributions in a more comprehensive way. This second, absolute approach showed to have similar performances to the robust approach, where uncertain prosumers have to locally reach the same level of confidence as it would be asked globally, but at smaller costs, so lower optimality gaps. In spite of these promising performances, this absolute uncertainty allocation displayed weaknesses when the global amount of uncertain power forecasted is low. But the goal only being to show that it was possible to allocate uncertainty, the simulations have been conducted on a small sized test case which did not allow to deeply analyze the results and, hence, understand whether this weakness was inherent to the proposed technique or linked to the lack of disparity between prosumers.

In consequence, the developments and results presented in this thesis demonstrated that it was possible to adapt the simple standard peer-to-peer market framework to the specific technical needs of electricity markets. More specifically, this thesis proposed approaches to tackle the issue of power system constraints and stochastic behaviors. The solutions developed in this thesis can not only apply to peer-to-peer electricity markets but also to the large family of coordination problems. It can finally be noted that these developments have been tested on modified IEEE test systems as no general test cases has been standardize this type of consumer-centric markets, including both network constraints and power production forecasts.

6.2 Perspectives for future work

The work in this thesis has opened up a number of different directions for future research, some of which are discussed below.

Even though proving theoretical feasibility of peer-to-peer electricity markets, the most fundamental issue raised by this thesis revolves around the practicability of the proposed solutions in real world implementations. The question of scalability is particularly important to solve as power systems can connect hundreds of thousand, if not multiple millions of electrical prosumers. Thus, the possibility to use a peer-to-peer electricity market ubiquitously on a large power system highly depends on the ability of its decentralized negotiation mechanism to tackle large numbers of prosumers. This calls for algorithmic and structural improvements of the peer-to-peer negotiation mechanism. This thesis notably showed the potential gain of using alternative stopping criteria which are more meaningful. It has been pointed that these criteria could be even further perfected by allowing for more strategic behaviors during the negotiation process as the prosumers would be in better capacity to estimate available gains with their partners and, therefore, anticipate the final agreement. The number of iterations necessary for a bilateral trade to be settled would then decrease. In a similar way, one could inspire the design of prosumers' partnerships from matching algorithms. The structural complexity, so the density of prosumers interaction graph, would then sink as the number of bilateral negotiations initiated would be cut down. Another strong direction of communication reduction would be to take advantage of the time continuous nature of most actors in the power system. Indeed, all convergence speed results presented in this thesis have been obtained based on optimization variable initial points set to zero. A deep analysis on the use of warm start strategies, even as simple as persistence, may plummet the number of iterations and exchanged messages to reach consensus. Of course, all these technical improvements would not completely suppress the exchange of information between prosumers. A better understanding of communication delays, with more realistic communication network models for example, may allow to test further the resilience of the negotiation mechanism when using asynchronous communications. This analysis could in turn open new strategies of improvement in which, for example, communication delays would not be a liability but an advantage.

Naturally, this necessity of technical improvements to allow for large scale implementations does not replace the need to refine the proposed methods to account for electricity markets' specificities. Efficient computation wise, the exogenous peer-to-peer electricity market showed vulnerabilities in the way the network charges should be defined to offer sufficient guarantees of electrical network constraints' respect. The use of historical data would for example help the system operator to more accurately estimate the future power flows and thus provide network charges more adapted to the current situation of the power system. But before this more operational requirements, even the design of the power system's zones can be questioned. These same historical data analyses may also be able to answer to this question. In fact the use of machine learning, Markov switching or time series techniques may also help improve computation performances of the endogenous peer-to-peer electricity market as it would allow the system operator to start the negotiation process with nodal network charges closer to their final values. This would boost the endogenous negotiation process which main inconvenient was a much slower convergence rate than its exogenous counterpart. In a way the use of historical data analysis calls for an hybridization of the two approaches where the exogenous method would be called less frequently, e.g. every several market time units, and update exogenous network charges for the next following time steps. This would make a compromise between the need to guarantee power system's constraints and the necessity of negotiation's swiftness. Moreover, upgrading the endogenous peer-to-peer electricity market by the use of a decentralized resolution of power flow equations may both improve the overall convergence rate and foster more localized communications. In fact, these two characteristics not only lessen the amount of traffic to carry but also reduce the amount of messages which have to transit across the whole communication infrastructure. The decentralized resolution of power flows would equally facilitate the resolution of large power systems where multiple system operators must coordinate, not only because of interconnected transmission networks but also between transmission and distribution system operators.

Whereas respect of power flow constraints is the foundation, peer-to-peer electricity markets must also satisfy other points for power systems' stability and, therefore, their feasibility in real world applications. Notably, the ability of peer-to-peer electricity markets to guarantee the power balance at all times as well depends on their capacity to manage stochastic behaviors and to compensate for unfulfilled commitments at the origin of misdispatches. Although promising, the uncertainty allocations proposed in view of obtaining a decentralized peer-to-peer energy and reserve market showed some gaps. Before refining them, there is a need for a deeper analysis, e.g. on much larger cases, to identify whether the difficulties exposed in this thesis are inherent to the proposed approaches or linked to the specifics of the small case on which they were tested. Testing the proposed uncertainty allocations on larger and more realistic cases may also be the occasion to see if they can be extended to highly correlated probability distributions in a straightforward manner or if they need to be completed. Concurrently to these studies, one could investigate the benefits of including storage units in such framework. Especially, this would be the occasion to extend the current peer-to-peer energy and reserve market formulation to multiple time steps and to implement strategies of storage management. It would then be possible to analyze on the best economical compromise between large common storage units and small localized ones. Moreover, the question of power system constraints and reserves have been treated separately, but the final peer-to-peer electricity markets would have to combine both issues. Naturally endogenous peer-to-peer market and peer-to-peer energy and reserve market formulations are compatible as they are. Yet, the resulting endogenous peer-to-peer energy and reserve market would solely guarantee power flow feasibility of energy trades but not of reserves if they ever need to be engaged. Thus, there would be a need to extend power system constraint formulations so that they encompass the eventual participation of reserves to power flows.

Furthermore, the presence of storage units rises the question of how the presented peer-to-peer market formulations could be extended to multiple time steps. As a matter of fact a simple multiple time steps formulation has been proposed at the beginning of the manuscript but only in a deterministic configuration where the social welfare was optimized over all time units at the same time. Hence, the simple formulation results in a multiple time units dispatch decided before the first time step, so at the image of the current day-ahead pool market. Yet, one could operate on a gliding time window to constantly update its bilateral trades such as allowed by the current intraday market. It may be interesting to study a peer-to-peer electricity market on a gliding trading window or, even more, to not only negotiate on quantities/fluxes but also length of the time frame during which the quantity/flux is valid. In such case each bilateral trade would have its own time window. Of course such framework would have to account for past trades that are still active. The same negotiation mechanism would thus start before each market time unit in which prosumers would mostly be looking for recourse trades to refine their commitments or occasionally negotiate a new long term window trade. In consequence this framework would be the first step towards a continuous peer-to-peer electricity market where prosumers can asynchronously initiate a new negotiation whenever they desire. Aside from this, it may be interesting to apply the proposed generalized coordination problem to a coupled multi-energy market formulation such as a peer-to-peer electricity, heat and gas market. A coupled multi-energy market like this would greatly improve the interactions between energy vectors and may allow to consolidate each of them by using strengths of the others and, more simply, enable prosumers at interfaces to optimize their production or consumption process. On a more general level, the advances developed in this thesis for the particular case of electricity markets may straightforwardly be applied to the generalized coordination problem. Nevertheless, other theoretical developments may be necessary. For now, the conditions guaranteeing convergence and optimality of the proposed decentralized solving algorithm are solely satisfied for the small family of convex coordination problems. Yet the good performances of the non-convex AC endogenous negotiation mechanism questions the possibility of extending these conditions to other families of non-convex coordination problems, e.g. quadratically constrained quadratic problems such as the AC endogenous peer-to-peer electricity market. In addition, extending the conditions of convergence and optimality to the family of integer problems would largely extend the range of applications. For example, this would allow to apply the proposed negotiation algorithm to non-fluid commodity markets, i.e. markets which goods are transported discretely such as in containers or in cargoes.

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UNIVERSITÉ DE RENNES 1

Titre : Marchés pair-à-pair de l'électricité dans les réseaux électriques

Mots clés : Marché pair-à-pair, Flux optimal de puissance, Répartition de puissance, Marché de l'énergie, Marché des capacités, ADMM

Résumé : Le déploiement de ressources énergétiques distribuées, combiné à une gestion plus pro-active de la demande et à l'intégration de systèmes de gestion d'énergie, fait entrer l'exploitation des systèmes électriques et des marchés de l'électricité dans un nouveau paradigme. En partie liés à leur structure décentralisée, les marchés dits pair-à-pair ont gagné un intérêt considérable. Les marchés pair-à-pair reposent sur des négociations bilatérales entre les agents pour faire correspondre l'offre et la demande. De plus, ils peuvent cartographier l'ensemble des échanges possibles, ce qui permet de repenser ces interactions avec le réseau.

Ces travaux de thèse traitent de trois défis majeurs dont la résolution est essentielle avant d'envisager le passage à des applications réelles : *(i)* le passage à l'échelle pour gérer un nombre croissant d'acteurs et de ressources distribués, *(ii)* le respect des contraintes du réseau électrique, et *(iii)* la résilience du marché à la présence d'agents stochastiques.

Une analyse de complexité a permis de montrer que le passage à l'échelle des marchés pair-à-pair et le mécanisme de résolution peut être renforcé par trois améliorations réduisant les complexités algorithmiques et structurelles. Pour le respect des contraintes réseau, le manuscrit propose d'introduire des redevances qui seraient liées à l'utilisation du réseau électrique. Deux approches sont considérées pour déterminer ces redevances réseau. La première, exogène, exige que le gestionnaire de réseau les fournisse a priori avant le début des négociations. Dans la seconde, le gestionnaire de réseau actualise les redevances réseau de manière endogène à chaque itération pour mieux tenir compte de l'état actuel du réseau. Enfin, les prévisions de production et de consommation des agents stochastiques sont mieux prises en compte par la création d'un marché pair-à-pair de l'énergie et des capacités de réserve, pour corriger un éventuel déséquilibre de puissance due à des erreurs de prévision.

Title: Peer-to-peer electricity markets in power systems

Keywords: Peer-to-peer markets, Optimal power flow, Economic dispatch, Energy market, Capacity Market, ADMM

Abstract: The deployment of distributed energy resources, combined with a more proactive demand side management and energy management systems, is inducing a new paradigm in power system operation and electricity markets. Within a consumer-centric market framework, peer-to-peer approaches have gained substantial interest. Peer-to-peer markets rely on multi-bilateral negotiation among all agents to match supply and demand. These markets can yield a complete mapping of exchanges onto the grid, hence allowing to rethink market–grid interactions.

This thesis treats three main challenges which needs to be overcome before considering real world implementations: *(i)* scalability to host a growing number of distributed users and resources, *(ii)* compatibility with grid constraints, and *(iii)* resilience to stochastic power injections.

After a complexity analysis, scalability of peer-to-peer markets and the proposed negotiation mechanism to solve them is enhanced by three improvements reducing algorithmic and structural complexities. Feasibility of the peer-to-peer electricity market is eventually obtained with the use of network charges. Two approaches are proposed to handle these network charges. The first, exogenous, requires the system operator to provide them *a priori* before negotiations start. In the second, the system operator updates network charges endogenously at each iteration to better account for the current grid status. Finally, power forecasts of stochastic agents are taken in a more comprehensive way by the developpement of peer-to-peer market on both energy and capacities, used to restore power balance in case of misdipatch due to forecast errors.