Are polytopic control design methods suitable for the next robotic control challenges?

Alexandre Kruszewski

Habilitation à Diriger des Recherches

Génie informatique, automatique et traitement du signal
(Section N°61 du Conseil National des Universités)

Alexandre KRUSZEWSKI

Maître de conférences à Centrale Lille
Docteur en Automatique

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Soutenue le 12/12/2017 devant le jury suivant:

Président	Colot	Olivier	PR (61)	Université de Lille
Garant	Richard	Jean-Pierre	PR (61)	Centrale Lille
Rapporteur	Sala Piqueras	Antonio	Full Pr.	Universitat Polytecnica de Valencia
Rapporteur	Mounier	Hugues	PR (61)	Université Paris-Sud 11
Rapporteur	Poignet	Philippe	PR (61)	Université de Montpellier
Membre	Brogliato	Bernard	DR INRIA	INRIA Grenoble Rhône-Alpes
Membre	Duriez	Christian	DR INRIA	INRIA Lille Nord-Europe
Membre	Guerra	Thierry-Marie	PR (61)	UVHC

Centre de Recherche en Informatique, Signal et Automatique de Lille (CRIStAL)

Institut National de Recherche en Informatique et en Automatique (INRIA)

Centrale Lille,
Cité Scientifique, CS20048,
59651 Villeneuve-d'Ascq France.

Summary

I. Introduction and Curriculum Vitae

This manuscript has been written with the aim of passing my Habilitation à Diriger des Recherches (HDR) in the field of control theory. The HDR is a French diploma used to recognize the work of researchers and it allows them to get more autonomy. For example, the HDR is required to be the main supervisor of a PhD student or to apply to a position of Professeur des Universités. I must admit that it is far from being the manuscript I dreamed about. I would have preferred to write something closer to a book with a high tutorial value and enough wisdom to help the readers to choose and use the presented control techniques. Unfortunately, writing such a manuscript is very time consuming and I would not have been able to finish it in a reasonable time (the teaching part of my current position is also very time consuming). This manuscript is not an overview nor a tutorial but it provides the reader with the necessary information to help him in judging the quality of my research. It relates the evolution of my topics, how the different tracks are articulated and the positioning with the respect of the state of the art.

This manuscript is composed with four chapters:

- The first one is dedicated to my curriculum vitae. The information provided in this section is focussed on the research aspect of my productions. There, one finds my complete bibliography, the PhD thesis I co-supervised, my collaborations and my scientific responsibilities.
- The second one deals with my work on the control design by means of polytopic models. The beginning of this section gives an example of use of this technique and tries to depict all sources of conservatism created during the process of getting tractable conditions, *i.e.* all choices that could result in the loss of potential solutions to the control problem. The second part of this section summarizes a selection of results that I developed with the PhD student I co-supervised which tries to reduce these sources of conservatism.
- The third section is application-oriented. It describes the results obtained during two of the PhD supervision I have done in the field of Networked Control System. The first PhD thesis presented deals with the simplest NCS system setup and tries to improve its performances by adapting the control gain according to the Quality Of Service (QoS) of the network. The second PhD thesis is the application of the previously developed techniques on the problem of bilateral teleoperation systems.
- The final section depicts my future research directions which are heavily influenced by a new application topic: the control of deformable robots.

Curriculum Vitae

KRUSZEWSKI Alexandre
Nationality: French
Date of birth: October 10th, 1981
Place of birth: Dechy (59) France

Position: Maître de conférences (Associate professor) at Centrale Lille since 2007. Member of the DEFROST team (CRIStAL - INRIA).

Laboratories:

CRIStAL[1] UMR 8022,
https://www.cristal.univ-lille.fr/

INRIA[2] Lille – Nord Europe
https://www.inria.fr/centre/lille

Professional address:

Ecole Centrale de Lille
Cité Scientifique, CS20048,
59651 Villeneuve-d'Ascq France.
http://www.ec-lille.fr/fr/index.html

Tel. +33 3 20 67 60 13
e-mail: alexandre.kruszewski@centralelille.fr

Education

2006 PhD thesis
Title: « Lois de commande pour une classe de modèles non linéaires sous la forme Takagi-Sugeno : mise sous forme LMI »
Mention Très Honorable avec Félicitations du Jury:

M. Dambrine	Pr Univ. de Valenciennes	Jury member
G. Garcia	Pr Univ. Toulouse	Reviewer
T.-M. Guerra	Pr Univ. de Valenciennes	Supervisor
N. Manamanni	Pr Univ. de Reims Champagne-Ardenne	Jury member
D. Maquin	Pr INPL Nancy	Reviewer
J.-P. Richard	Pr Ecole Centrale de Lille	President
A. Sala-Piqueras	Pr Univ. Polytecnica Valencia, Spain	Jury member

[1] Centre de Recherche en Informatique, Signal et Automatique de Lille, Villeneuve d'Ascq
[2] Institut National de Recherche en Informatique et en Automatique

2004 Master degree: AISIH (Automatique et Informatique des Systèmes Industriels et Humains). Mention Bien, (Major)

2003 Titre d'ingénieur-Maître. Mention Bien (Major)

2002 Licence degree: GEII (Génie électrique et informatique industrielle)

1999 Scientific baccalaureate

Doctoral co-supervision

Defenced co-supervised PhD: 4
Actual PhD co-supervision: 2

2016 – 2019: Thieffry Maxime,
Modélisation et contrôle de robots déformables à grande vitesse, Co-supervisors: Guerra 20% Duriez 20% Kruszewski 60%

2015 – 2018: Zhang Zhongkai,
New methods of visual servoing for soft-robots,
Co-supervising: Duriez 30%, Dequit 40%, Kruszewski 30%

2011 – 2014: Maalej Sonia,
Commande Robuste des Systèmes à Paramètres Variables,
Co-supervisors: Belkoura 30% Kruszewski 70%

2012 – 2015: Marquez Borbon Raymundo,
Nouveaux schémas de commande et d'observation basés sur les modèles de T-S,
Co-supervisors: Guerra 40% Bernal 20% Kruszewski 40%

2009 – 2012: Zhang Bo,
Commande à retour d'effort à travers des réseaux non dédiés : stabilisation et performance sous retards asymétriques et variables,
Co-supervisors: Richard 50% Kruszewski 50%

2006 – 2009: Jiang Wenjuan,
A contribution to control and observation of networked control systems,
Co-supervisors: Richard 50% Toguyeni 20% Kruszewski 30%

Collaborations and responsibilities

Collaborations:

- ITS, Mexique: M. Bernal (1 PhD co-supervising)
- Université de Valenciennes: T.M. Guerra (2 PhD co-supervising)
- Université de Tel'Aviv, Israël: E. Fridman, (Publications on Network Control System)
- Université de Reims: K. Guelton, N. Manamanni, (Publications on polytopic models stabilization)
- ENSAM of Lille (L2EP): starting consulting activities on power grid control problems.

Implication in the scientific communities:

- International Program Committee member for:
    - IFAC ICONS 2011, 2013, 2015 and 2016

    - ETFA 2011
- Projects:
    - Member of the European project ‘SYSIASS 6-20' (Autonomous and Intelligent Healthcare System) [INTERREG IV A 2 Mers Seas Zeen 2007-2013]
    - Member of the ANR (Agence Nationale de la Recherche) project ROCC-SYS, 2014-2018
    - Leader of 2 BQR projects (Bonus Qualité Recherche) of Centrale Lille. 10k€ each.
- Expertise:
    - Reviewer of an IDEX (Initiative D’EXellence) project for the Univ. Strasbourg, 2017
    - In charge of 4 industrial contracts with the Société Industrielle de Chauffage of the Atlantic group, since 2012
    - Selection committee member of Univ. Valenciennes, 2008-2010
    - Recurrent reviewer for Automatica, IEEE TAC, IEEE TFS, Fuzzy sets and systems, NAHS, …
- Others:
    - In charge of some test bench of the SyNeR team (CRIStAL), 2007-2015
    - Laboratory council member of CRIStAL, since 2015

Teachings:

- In charge of the coordination of the automatic control teachings at Centrale Lille since 2015
- In charge of 7 modules in automatic control and robotics (linear control, mobile robot control, robust approaches, LMI, Lyapunov, …) for 3 different engineer formations
- Active participant to the last teaching reform of Centrale Lille
- Teaching charge of ≈240h/year

Other information:

- Change of research team (within CRIStAL) from SyNeR (more theoretical topics) to DEFROST team (pluridisciplinary team with application to deformable robotics) in 2015.

Personal publications

https://scholar.google.fr/citations?user=4eyt5ewAAAAJ&hl=fr&oi=ao

H-index google Scholar total = 14

H-index google Scholar 2012-2016 = 12

Journal papers

[1] R. Márquez, T. M. Guerra, M. Bernal, and A. Kruszewski, “Asymptotically necessary and sufficient conditions for Takagi--Sugeno models using generalized non-quadratic parameter-dependent controller design,” *Fuzzy Sets Syst.*, vol. 306, pp. 48–62, 2017.

[2] S. Maalej, A. Kruszewski, and L. Belkoura, “Robust Control for Continuous LPV System with Restricted-Model-Based Control,” *Circuits, Syst. Signal Process.*, vol. 36, no. 6, pp. 2499–25520, 2017.

[3] S. Maalej, A. Kruszewski, and L. Belkoura, “Stabilization of Takagi–Sugeno models with non-measured premises: Input-to-state stability approach,” *Fuzzy Sets Syst.*, 2017.

[4] R. Márquez, T. M. Guerra, M. Bernal, and A. Kruszewski, “A non-quadratic Lyapunov functional for H8 control of nonlinear systems via Takagi-Sugeno models,” *J. Franklin Inst.*, vol. 353, no. 4, pp. 781–796, 2016.

[5] Y.-J. Chen, M. Tanaka, K. Inoue, H. Ohtake, K. Tanaka, T. M. Guerra, A. Kruszewski, and H. O. Wang, “A nonmonotonically decreasing relaxation approach of Lyapunov functions to guaranteed cost control for discrete fuzzy systems,” *IET Control Theory Appl.*, vol. 8, no. 16, pp. 1716–1722, 2014.

[6] B. Zhang, A. Kruszewski, and J.-P. Richard, “A novel control design for delayed teleoperation based on delay-scheduled Lyapunov--Krasovskii functionals,” *Int. J. Control*, vol. 87, no. 8, pp. 1694–1706, 2014.

[7] R. Delpoux, L. Hetel, and A. Kruszewski, “Parameter-Dependent Relay Control: Application to PMSM,” *Trans. Control Syst. Technol.*, vol. 23, no. 4, pp. 1628–1637, 2014.

[8] R. Marquez, T. M. Guerra, A. Kruszewski, M. Bernal, and others, “Improvements on Non-Quadratic Stabilization of Takagi-Sugeno Models Via Line-Integral Lyapunov Functions,” in *Intelligent Control and Automation Science*, 2013, vol. 3, no. 1, pp. 473–478.

[9] K. Guelton, N. Manamanni, A. Kruszewski, T. M. Guerra, and B. Mansouri, “Commande sous-optimale par retour de sortie pour le suivi de trajectoires des mod{è}les TS incertains,” *J. Eur. des Syst. Autom.*, vol. 46, no. 4, p. 397, 2012.

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[11] A. Kruszewski, R. Bourdais, and W. Perruquetti, “Converging algorithm for a class of BMI applied on state-dependent stabilization of switched systems,” *Nonlinear Anal. Hybrid Syst.*, vol. 5, no. 4, pp. 647–654, 2011.

[12] L. Hetel, A. Kruszewski, W. Perruquetti, and J. P. Richard, “Discrete and intersample analysis of systems with aperiodic sampling,” *Autom. Control. IEEE Trans.*, vol. 56, no. 7, pp. 1696–1701, 2011.

[13] T. M. Guerra, A. Kruszewski, and M. Bernal, “Control law proposition for the stabilization of discrete Takagi--Sugeno models,” *Fuzzy Syst. IEEE Trans.*, vol. 17, no. 3, pp. 724–731, 2009.

[14] T. M. Guerra, A. Kruszewski, and J. Lauber, “Discrete Tagaki--Sugeno models for control: Where are we?,” *Annu. Rev. Control*, vol. 33, no. 1, pp. 37–47, 2009.

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International conference papers

[1] Z. Zhang, J. Dequidt, A. Kruszewski, F. Largilliere, and C. Duriez, “Kinematic modeling and observer based control of soft robot using real-time Finite Element Method,” in *Intelligent Robots and Systems (IROS), 2016 IEEE/RSJ International Conference on*, 2016, pp. 5509–5514.

[2] R. Márquez, T. M. Guerra, M. Bernal, and A. Kruszewski, “Nested control laws for h8 disturbance rejection based on continuous-time takagi-sugeno models,” in *IFAC-PapersOnLine*, 2015, vol. 28, no. 10, pp. 282–287.

[3] R. Delpoux, L. Hetel, and A. Kruszewski, “Permanent magnet synchronous motor control via parameter dependent relay control,” in *2014 American Control Conference*, 2014, pp. 5230–5235.

[4] S. Maalej, A. Kruszewski, R. Delpoux, and L. Belkoura, “Derivative based control for LPV system with unknown parameters: An application on a Permanent Magnet Synchronous Motors,” in *Systems, Signals & Devices (SSD), 2014 11th International Multi-Conference on*, 2014, pp. 1–6.

[5] R. Márquez, T. M. Guerra, A. Kruszewski, and M. Bernal, “Non-quadratic stabilization of second order continuous Takagi-Sugeno descriptor systems via line-integral Lyapunov function,” in *2014 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, 2014, pp. 2451–2456.

[6] R. Márquez, T. M. Guerra, A. Kruszewski, M. Bernal, and others, “Decoupled nested LMI conditions for Takagi-Sugeno observer design,” in *World Congress*, 2014, vol. 47, no. 1, pp. 7994–7999.

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[9] B. Zhang, A. Kruszewski, J. P. Richard, and others, “H_ {\ infty} Control Design for Novel Teleoperation System Scheme: A Discrete Approach,” in *10th IFAC Workshop on Time Delay Systems*, 2012.

[10] B. Zhang, A. Kruszewski, J. P. Richard, and others, “H_ {\ infty} Robust Control Design for Teleoperation Systems,” in *7th IFAC Symposium on Robust Control Design*, 2012.

[11] B. Zhang, A. Kruszewski, J. P. Richard, and others, “H_ {\ infty} Control of Delayed Teleoperation Systems under Polytopic-Type Uncertainties,” in *20th Mediterranean Conference on Control and Automation*, 2012.

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[23] M. Bernal, T. M. Guerra, and A. Kruszewski, “A membership-function-dependent H∞ controller design for Takagi-Sugeno models,” in *Fuzzy Systems, 2008. FUZZ-IEEE 2008.(IEEE World Congress on Computational Intelligence). IEEE International Conference on*, 2008, pp. 1139–1145.

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[25] M. Bernal, T. M. Guerra, and A. Kruszewski, “A membership-function-dependent stability analysis of Takagi--Sugeno models,” *Proc. IFAC World Congr.*, pp. 5611–5616, 2008.

[26] W.-J. Jiang, A. Kruszewski, J.-P. Richard, and A. Toguyeni, “A Gain Scheduling Strategy for the Control and Estimation of a Remote Robot via Internet,” in *Proc. Chinese Control Conference ({CCC}’08)*, 2008, pp. 793–799.

[27] A. Kruszewski, T. M. Guerra, and M. Bernal, “K-samples variation analysis for discrete-time Takagi-Sugeno models stabilization: Complexity reduction,” in *Advanced Fuzzy and Neural Control*, 2007, vol. 3, no. 1, pp. 37–42.

[28] B. Mansouri, A. Kruszewski, K. Guelton, and N. Manamanni, “Sub-optimal output tracking control design for uncertain TS models,” in *Advanced Fuzzy and Neural Control*, 2007, vol. 3, no. 1, pp. 91–96.

[29] T. M. Guerran, M. Bernal, A. Kruszewski, M. Afroun, and L. Vermeiren, “Descripteurs sous forme T-S : relaxation des résultats en stabilisation,” in *LFA’2007*, 2007.

[30] A. Kruszewski, A. Sala, T. M. Guerra, and C. Arino, “Sufficient and asymptotic necessary conditions for the stabilization of Takagi-Sugeno model,” in *Advanced Fuzzy and Neural Control*, 2007, vol. 3, no. 1, pp. 55–60.

[31] R. Wang, T. M. Guerra, A. Kruszewski, and J. Pan, “Guaranteed cost control for uncertain discrete delay TS fuzzy system,” in *Advanced Fuzzy and Neural Control*, 2007, vol. 3, no. 1, pp. 85–90.

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[33] T. M. Guerra, M. Bernal, A. Kruszewski, and M. Afroun, “A way to improve results for the stabilization of continuous-time fuzzy descriptor models,” in *Decision and Control, 2007 46th IEEE Conference on*, 2007, pp. 5960–5964.

[34] W. Renming, T. M. Guerra, A. Kruszewski, And P. A. N. Juntao, “Guaranteed cost control for uncertain discrete delay TS fuzzy system,” *IFAC Proc. Vol.*, vol. 40, no. 21, pp. 85–90, 2007.

[35] A. Kruszewski and T. M. Guerra, “Stabilization of a class of nonlinear model with periodic parameters in the Takagi-Sugeno form,” in *Periodic Control Systems*, 2007, vol. 3, no. 1, pp. 132–137.

[36] A. Kruszewski, R. Wang, and T. M. Guerra, “New approaches for stabilization of a class of nonlinear discrete time-delay models,” in *Time Delay Systems*, 2006, vol. 6, no. 1, pp. 308–313.

[37] A. Kruszewski and T. M. Guerra, “Conditions of output stabilization for uncertain discrete TS fuzzy models,” *16th IFAC Trienn. World Congr. Prague, Czech Repub.*, 2005.

[38] A. Kruszewski, T. M. Guerra, and A. Kruszewski, “New Approaches for the Stabilization of Discrete Takagi-Sugeno Fuzzy Models,” *IEEE Conf. Decis. Control*, vol. 44, no. 4, p. 3255, 2005.

[39] T. M. Guerra and A. Kruszewski, “Non-quadratic stabilization conditions for uncertain discrete nonlinear models in the Takagi-Sugeno’s form,” *Work. IFAC AFNC*, vol. 4, 2004.

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Book Chapters:

[1] A. Kruszewski, B. Zhang, and J.-P. Richard, “Control Design for Teleoperation over Unreliable Networks: A Predictor-Based Approach,” in in Delay Systems, Springer International Publishing, 2014, pp. 87–100.

[2] W. Jiang, A. Kruszewski, J. P. Richard, A. Toguyeni, and others, “Networked control and observation for Master-Slave systems,” in in Delay Differential Equations: Recent Advances and New Directions, Springer, 2009, pp. 31–54.

II. Polytopic model control design

Introduction – A control design workflow for polytopic models and its issues

This chapter is dedicated to the presentation of my research results as well as those developed under my supervision by PhD students (Márquez 2015; Maalej 2014). The results presented are dedicated improving the general control design methodology for a class of nonlinear systems. They are focused on the use of polytopic representations of nonlinear models to design a control law. This section focusses on TS (Takagi-Sugeno) (Takagi and Sugeno 1985) models but almost every result also holds for polytopic models in general like the Linear Parameter Varying ones (Briat 2014). The only difference between TS models and other polytopic ones is the background (TS models initially were fuzzy models and most of the results are published in the fuzzy community) and some implicit choices like whether or not to consider the dependency between the state and the scheduling parameters or the rate of variation of the parameter. My references will focus on the TS-related publications.

The TS control framework was successfully used to solve real control and estimation challenges for various application such as bioinspired robotics (Chang, Liou, and Chen 2011), heat exchanger state estimation (Delmotte et al. 2013), wastewater treatment plant parameter estimation (Bezzaoucha et al. 2013), motor cycle lateral dynamic state estimation (Dabladji et al. 2016), spark ignition engine control (Khiar et al. 2007),… I think that TS framework is a valuable tool for the following reasons:

- It allows the engineers to tackle nonlinear control/estimation design by relying on powerful tools coming from the association of Lyapunov theory and demi-definite problem numerical solver in particular Linear Matrix Inequalities solvers (Boyd et al. 1994).
- It removes most of the necessary manipulations of complex expressions and most manual researches of solutions that can be found in nonlinear classical framework. Of course, jumping quickly into numerical analysis has some side effects and one will see that the ease of use is paid by accepting to lose some solutions and mathematical beauty.

I recommend reading the following overviews concerning TS models (T M Guerra, Kruszewski, and Lauber 2009; Thierry M. Guerra, Sala, and Tanaka 2015; Lendek et al. 2011; Sala, Guerra, and Babuška 2005).

To illustrate my research, the rest of this section provides one classical model-based workflow used to find a controller for a nonlinear system using their polytopic representations. At each step, one will highlight my results, the strength and caveats. The last sections of this chapter detail some selected topics.

Polytopic-based control design workflow

This subsection presents one of the possible workflows to design a stabilizing controller. For sake of clarity, it will focus only on guaranteeing the stability property of the closed-loop. These results can be extended to performance guarantee by modifying some steps.

Assume that a nonlinear model is available for the control design in the following form:

$$\dot{x}(t) = a_0 x + \sum_{j=1}^{p} a_j \zeta_j\left(z(x)\right)x + b_0 u + \sum_{j=1}^{p} b_j \zeta_j\left(z(x)\right)u \tag{2.1}$$

$$y(t) = c_0 x + \sum_{j=1}^{p} c_j \zeta_j\left(z(x)\right)x + d_0 u + \sum_{j=1}^{p} d_j \zeta_j\left(z(x)\right)u \tag{2.2}$$

where $x(t) \in \mathbb{R}^{n_x}$ represents the system state vector, $u(t) \in \mathbb{R}^{n_u}$ the input vector, $y(t) \in \mathbb{R}^{n_y}$ the measured output vector. a_i,b_i,c_i and d_i are matrices with appropriate dimensions. $\zeta_j(\cdot)$, $j \in \{1, \dots, p\}$ and $z(\cdot)$ are sufficiently smooth nonlinear scalar functions bounded on a compact set of the state space denoted $\mathcal{C}_x$. Note that the class of models can be extended to the case where z is a function of the state, the input, some parameters or any external signal as long as the functions $\zeta_j\left(z(\cdot)\right)$ are bounded on the sets of interest (generally around an equilibrium point). A quite similar model can be defined in the discrete time domain by replacing $\dot{x}(t)$ by $x(t+1)$ in (2.1).

Notations:

As this chapter deals with polytopic models, one will encounter convex sums expressions. The following shorthand notations will be adopted for sufficiently smooth scalar functions h_i satisfying the convex sum property ($\sum_i h_i(z) = 1$, $h_i \geq 0$) and matrices Υ_i:

Description	Notation
Single convex sum	$\Upsilon_h = \sum_{i=1}^{r} h_i(z)\Upsilon_i$
Double convex sum	$\Upsilon_{hh} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\Upsilon_{ij}$
Double convex sum	$\Upsilon_{h\hat{h}} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(\hat{z})\Upsilon_{ij}$
“ q ” nested convex sum	$\Upsilon_{\underbrace{hh\cdots h}_{\bar{h}}} = \sum_{i_1=1}^{r}\sum_{i_2=1}^{r}\cdots\sum_{i_q=1}^{r} h_{i_1}(z)h_{i_2}(z)\cdots h_{i_q}(z)\Upsilon_{i_1 i_2 \cdots i_q}$
Inverse of a convex sum	$\Upsilon_h^{-1} = \left(\sum_{i=1}^{r} h_i\left(z(t)\right)\Upsilon_i\right)^{-1}$
Time-derivative of a convex sum	$\dot{\Upsilon}_h = \frac{d}{dt}\left(\sum_{i=1}^{r} h_i(z(t))\Upsilon_i\right)$

Table 2.1. Notation for convex sums

In the following, one will try to express the conditions as Linear Matrix Inequalities constraints. The following notations will be used to simplify their reading:

- $(*)$ stands for the smallest expression induced by symmetry.
- ">0" and "<0" means respectively positive definite and negative definite when applied to matrix expressions.

Polytopic reformulation

The sector nonlinear approach (Boyd et al. 1994; Tanaka and Wang 2001) is a useful method allowing for an exact polytopic representation of (2.1)-(2.2) over a compact set of the state space $\mathcal{C}_x$. One just needs to:

1. Construct the Weighting Functions (WFs) for each nonlinear function $\zeta_j(\cdot)$ in the following form:
$$\omega_0^j(z(\cdot)) = \frac{\overline{\zeta_j} - \zeta_j(z(\cdot))}{\overline{\zeta_j} - \underline{\zeta_j}}, \quad \omega_1^j(z(\cdot)) = 1 - \omega_0^j(z(\cdot)), \ j \in \{1, 2, \cdots, p\} \tag{2.3}$$
where $\overline{\zeta_j}$ and $\underline{\zeta_j}$ are respectively the maximum and the minimum of $\zeta_j(\cdot)$ for $x \in \mathcal{C}_x$.
2. Set the membership functions (MFs) as follows:
$$h_i(\cdot) = h_{1+i_1+i_2\times 2+\cdots+i_p\times 2^{p-1}}(\cdot) = \prod_{j=1}^{p} \omega_{i_j}^j(z_j), \ i \in \{1, 2, \cdots, 2^p\}, \ i_j \in \{0, 1\}.$$
3. Obtain the matrices at the polytope vertices:
$$A_i = A(z(\cdot))\big|_{h_i(\cdot)=1}, \qquad B_i = B(z(\cdot))\big|_{h_i(\cdot)=1}, \qquad C_i = C(z(\cdot))\big|_{h_i(\cdot)=1},$$
$$D_i = D(z(\cdot))\big|_{h_i(\cdot)=1}, \ i \in \{1, 2, \cdots, r\} \text{ with } r = 2^p \in \mathbb{N}.$$

Then one gets the following polytopic representation:

$$\dot{x} = \sum_{i=1}^{r} h_i(z)(A_i x + B_i u) = A_h x + B_h u \tag{2.4}$$

$$y = \sum_{i=1}^{r} h_i(z)(C_i x + D_i u) = C_h x + D_h u. \tag{2.5}$$

where $\sum_{i=1}^{r} h_i(\cdot) = 1$ and $h_i(\cdot) \geq 0$ and z a function of the state and the input.

Example:

Let consider the following nonlinear model:

$$\dot{x}_1 = \cos(x_1 x_2)\, x_1 + x_2 + u$$
$$\dot{x}_2 = \sin(x_1 x_2)\, x_1 + x_2$$

One can identify two nonlinear functions: $\zeta_1(\cdot) = \cos(x_1 x_2)$ and $\zeta_2(\cdot) = \sin(x_1 x_2)$. Other choices are also valid like $\zeta_1(\cdot) = \cos(x_1 x_2) + \sin(x_1 x_2)$ and $\zeta_2(\cdot) = \sin(x_1 x_2)$. Multiple choices are also available for $z(\cdot)$. One can choose for example $z(x) = x_1 x_2$ or,$z(x) = x$, of $z(x) = \begin{pmatrix} \cos(x_1 x_2) \\ \sin(x_1 x_2) \end{pmatrix}$ and so on. For the next step, one chooses $z(x) = x_1 x_2$ and $\zeta_1(\cdot) = \cos(x_1 x_2)$, $\zeta_2(\cdot) = \sin(x_1 x_2)$. The results of the sector nonlinear steps are:

1. $\omega_0^1(z(\cdot)) = \frac{1-\cos(x_1 x_2)}{2}, \omega_0^2(z(\cdot)) = \frac{1-\sin(x_1 x_2)}{2}, \omega_1^j = 1 - \omega_0^j,$
2. $h_1 = \omega_0^1 \omega_0^2, h_2 = \omega_1^1 \omega_0^2, h_3 = \omega_0^1 \omega_1^2, and\ h_1 = \omega_1^1 \omega_1^2$
3. $h_1 = 1 \Leftrightarrow \omega_0^1 = \omega_0^2 \Leftrightarrow \{\cos(x_1 x_2) \leftarrow 1, \sin(x_1 x_2) \leftarrow 1\}$ where $\leftarrow$ means 'subtitute'. The matrices A_i and B_i given by:

$$\begin{pmatrix} \cos(x_1 x_2)\, x_1 + x_2 + u \\ \sin(x_1 x_2)\, x_1 + x_2 \end{pmatrix}\Bigg|_{\substack{\zeta_1 = \underline{\zeta}_2 \\ \zeta_2 = \underline{\zeta}_2}} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}_{A_1} x + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B_1} u$$

$$\begin{pmatrix} \cos(x_1 x_2)\, x_1 + x_2 + u \\ \sin(x_1 x_2)\, x_1 + x_2 \end{pmatrix}\Bigg|_{\substack{\zeta_1 = \bar{\zeta}_2 \\ \zeta_2 = \underline{\zeta}_2}} = \underbrace{\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}}_{A_2} x + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B_2} u$$

…

The reader may refer to (Márquez 2015; A Kruszewski 2006; Tanaka and Wang 2001) for more details.

It is important to note that, because this transformation is not unique, it may impact the control design result and conservatism[3]. The only known result trying to optimize this choice is reported in (Robles et al. 2016).

Choose a control structure

Once the model has been redefined in a polytopic form one has to choose the right control structure. Being exhaustive would not help at all. One will focus on the classical tools for control: state feedback and state observer when necessary.

It can be proven that for quadratic stabilization in the continuous case, it is necessary and sufficient to consider a PDC control law (parallel distributed compensation) to stabilize a TS system. If one does not care about reference tracking, these control laws can be reduced to the following expression:

$$u(t) = -F_h x(t). \tag{2.6}$$

Despite this property, some results managed to get less conservative results using more complex control laws (Márquez et al. 2013; Márquez et al. 2015a; Márquez et al. 2015b; Márquez et al. 2017). These results are the topic of another subsection of this chapter and explains this strange result.

When it comes to the output feedback problem, the natural way is to consider the use of state observer which use the same structure as the model:

$$\dot{\hat{x}} = \sum_{i=1}^{r} h_i(\hat{z}) \left(A_i \hat{x} + B_i u + K_i (y - \hat{y}) \right) = A_{\hat{h}} \hat{x} + B_{\hat{h}} u + K_{\hat{h}} (y - \hat{y}) \tag{2.7}$$

$$\hat{y} = \sum_{i=1}^{r} h_i(\hat{z}) \left(C_i \hat{x} + D_i u \right) = C_{\hat{h}} x + D_{\hat{h}} u\,. \tag{2.8}$$

[3] one loses potential stabilizing solutions (or performances)

This observer structure allows to reuse a state feedback control law $u(t) = -F_{\hat{h}}\hat{x}(t)$. From here, one must study two cases: the case where the arguments of the nonlinear functions are measured ($\hat{z} = z$) and the case they are not ($\hat{z} \neq z$). The first case is quite nice since the separation principle applies and both problems (observation and control) can be studied separately (Tanaka and Wang 2001). The second one is still an open problem and often reduces the global stability property to local stability property. Some results about this topic and alternative possibilities are discussed later in this manuscript (Maalej, Kruszewski, and Belkoura 2017b; Thierry Marie Guerra et al. 2017; Maalej 2014; Márquez 2015).

Choose a Lyapunov function

For this class of model, it is convenient to use the direct Lyapunov method to deal with stability. An important choice is the formulation of the Lyapunov function (LF). Many papers consider the problem of finding a quadratic Lyapunov function, *i.e.*:

$$Find\ P \in \mathbb{R}^{n\times n}\ s.t.$$

$$V(x) = x^T P x > 0\ \forall x \neq 0$$

$\frac{d}{dt}V\big(x(t)\big) < 0\ \forall x(t) \neq 0$ along the trajectories of the system (2.7)

This choice is not suitable for all polytopic model and is quite conservative[4]. Many attempts were made to leave this class of LF and tried to use nonquadratic function (NQLF).

One should distinguish the continuous case from the discrete case ((2.4) with $x(t+1)$ instead of $\dot{x}(t)$ and $V\big(x(t+1)\big) - V\big(x(t)\big) < 0$ instead of $\dot{V} < 0$). The discrete case is the easiest one. By considering nonquadratic LF like $V(x) = x^T P_h x$ or non-monotically decreasing LF $V\big(x(t+k)\big) - V\big(x(t)\big) < 0$ with $k \geq 2$ leads to good relaxation of the results. Based on the later results one also find the Asymptotically Necessary and Sufficient (ANS) class of Lyapunov function (Hetel et al. 2011) and ANS LMI conditions (A Kruszewski and Guerra 2007b; A Kruszewski, Guerra, and Kruszewski 2005; A Kruszewski and Guerra 2005; Chen et al. 2014; T M Guerra and Kruszewski 2004; T M Guerra, Kruszewski, and Bernal 2009; A Kruszewski, Guerra, and Bernal 2007; T M Guerra, Kruszewski, and Lauber 2009; T M Guerra and Kruszewski 2005; A Kruszewski and Guerra 2007a; Alexandre Kruszewski, Guerra, and WANG 2008; A Kruszewski, Wang, and Guerra 2008) for this class of polytopic model (when ignoring the shape of the functions h_i). (Hetel et al. 2011) proposed a link between this approach and an equivalent LF with $k = 1$. Despite the ANS property, there is a lot a room for improvement to reduce the complexity of the condition and to make these results numerically tractable.

In the continuous-time case, the main problem comes from the introduction of the time derivative of the nonlinear parts of the LF while trying to ensure $\dot{V} < 0$. For example, if one chooses the candidate:

$$V(x) = x^T P_h x = x \sum_{i=1}^{r} h_i\big(z(\cdot)\big) P_i\, x$$

The variation of the LF along $x(t)$ becomes:

$$\dot{V}(x) = x^T P_h \dot{x} + \dot{x}^T P_h x + x^T \dot{P}_h x = x^T P_h \dot{x} + \dot{x}^T P_h x + x^T \sum_{i=1}^{r} \frac{d}{dt} h_i(z(\cdot)) P_i\, x$$

[4] one loses potential solution stabilizing solutions (or reduce performances)

In this expression, the two first terms are easily handled as in the quadratic case. The last term is not signed and needs to be bounded. The use of these bounds reduces the stability proof to a local region of the state space. Two kinds of assumption are used in this case:

- Consider $|\dot{h}_i| < \phi_i$ (Tanaka and Wang 2001; Mozelli, Palhares, and Avellar 2009) which is problematic in control design due to the link between the bounds ϕ_i and the control law, *i.e.*:
  $$\dot{h}_i = \frac{d}{dt} h\left(z\big(x(t)\big)\right) = \frac{\partial h(z(x))}{\partial x}\frac{dx}{dt} = \frac{\partial h\big(z(x)\big)}{\partial x}(Ax + Bu)$$
  In that case, the invariant set depends on the control law and can only be computed a posteriori. It is difficult in this case to optimize the invariant set of the closed loop.
- Consider a bound on the partial derivatives which helps in mastering the invariant set (Pan et al. 2012; T.-M. Guerra and Bernal 2009; Sala et al. 2010) by writing something like:
  $P_{\dot{h}} = \sum_{i=1}^{r}\sum_{k=1}^{p} h_i \frac{\partial \omega_0^k}{\partial z_k}\left(P_{g_1(i,k)} - P_{g_2(i,k)}\right)\dot{z}_k$ … $\left|\frac{\partial \omega_0^k}{\partial z_k} x_l\right| \le \lambda_{kl}$ where ω_i are the constitutives elements of the h_i.

Remark: Knowing that the use of NQLF extends the possibilities in stabilization, it is not sure that all the control law designed with such LF are still robust as we may be considering a limit case. That is why I think it is preferable to introduce some robustness guarantee when nonquadratic LF are used (in the form of artificial parameter uncertainty or a state disturbance for example).

LMI formulation

Once a LF has been selected, it is convenient to reformulate the stability/stabilization conditions as a set of Linear Matrix Inequalities (LMI). LMI formulation is the backbone of the polytopic approach as it provide an effective way to numerically solve the conditions (global convergence, feasibility check and polynomial time computation) (Boyd et al. 1994; Gahinet et al., n.d.).

For example, in continuous time (2.4) with a PDC control law (2.6) if there exist matrices $P = P^T$ such that:

$$V(x) = x^T P x > 0 \ \forall x \neq 0$$

$$\dot{V}\big(x(t)\big) = x^T P \dot{x} + \dot{x}^T P x = x^T P (A_h - B_h F_h) x + (*) < 0 \ \forall x \neq 0$$

then the closed loop is stable. This stability criterion is equivalent to the following matrix inequality conditions:

$$\begin{cases} P > 0 \\ P(A_h - B_h F_h) + (*) < 0 \end{cases} \quad (2.9)$$

Where "> 0" and "< 0" means respectively positive definite and negative definite. These conditions are not linear in the decision variables (the entries of P and the F_i enclosed in F_h)

Linearization of the stabilization problem:

Multiple properties are available which help to get LMI conditions. They can be classified in three groups:

- *Necessary and sufficient properties:*
  In this group we can find useful lemma and matrix transformations which do not introduce conservatism. The first transformations one should know are: the bijective changes of variables (like $Y = FX$ where X is an invertible matrix) and the congruence (left multiplication with a full rank matrix and right multiplication with its transpose). Other useful lemmas exist and helps in getting a LMI problem formulation: the Schur complement lemma which helps

removing some quadratic terms, the Finsler lemma and the S-procedure to check a condition under state restrictions. These manipulations can be found in (Boyd et al. 1994)

- *Bounding methods:*
  This group consists in all other tools that should be used when no other options are available. They consist in approximating (find a guaranteed bound) non-convex term with linear ones. For example, the matrix square completion $X^TY + Y^TX \leq X^TX + Y^TY$ and all its variations, Jensen inequality to deal with integral terms (Gu, Chen, and Kharitonov 2003), Finsler lemma with restrictions on slack matrices, and so on…
- *LMI sequence algorithms:*
  It consists in a loop of LMI problems of solve in which one solves the optimization problem for different set of decision variable. For example: if one wants to find a solution to:
  $$\begin{cases} \text{find } P, F \text{ s.t.:} \\ PA + PBF + (*) < 0 \\ P = P^T > 0 \end{cases}$$
  Then one can try:
  - Initialize $P_0 = I$ and $k = 0$
  - Do:
    - $k = k + 1$
    - Solve $\begin{cases} \text{find } F_k, \text{maximize } t \text{ s.t.:} \\ P_{k-1}A + P_{k-1}BF_k + (*) < -t \\ P_{k-1} = P_{k-1}^T > t \end{cases}$
    - Solve $\begin{cases} \text{find } P_k, \text{maximize } t \text{ s.t.:} \\ P_kA + P_kBF_k + (*) < -t \\ P_k = P_k^T > t \end{cases}$
  - Until $\begin{cases} P_kA + P_kBF_k + (*) < 0 \\ P_k = P_k^T > 0 \end{cases}$

These linearization techniques are presented in order of preference:

- the first one is exact and no solution are lost,
- the last one is the worst as it depends on the initialization of the algorithm, the choice of the decision variable sets (when multiple solutions are available) as well as the optimization criteria that is used.

If one wants to apply such techniques on the classical stabilization problem one should apply the congruence with P^{-1} on (2.9):

$$\begin{cases} P^{-1} > 0 \\ A_hP^{-1} - B_hF_hP^{-1} + (*) < 0 \end{cases} \tag{2.10}$$

Using bijective transformations $X = P^{-1}$ and $Y_i = F_iP^{-1}$ the stabilization problem (2.9) becomes:

$$\begin{cases} find\ X = X^T\ and\ F_i\ s.t. \\ X > 0 \\ A_hX - B_hY_h + (*) < 0 \end{cases} \tag{2.11}$$

Or by exposing the convex sums:

$$\begin{cases} find\ X = X^T\ and\ F_i\ s.t. \\ \forall h_i(\cdot) \in [0\ 1],\ \sum_{i=1}^{r} h_i(\cdot) = 1 \\ X > 0 \\ \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\cdot)h_j(\cdot)\left(A_iX - B_iY_j + (*)\right) < 0 \end{cases} \tag{2.12}$$

The optimization problem (2.12) is linear according to the decision variable but cannot be solve yet. It represents an infinite number of LMI constraint due to the dependence on the functions h_i.

Convex embedding

Convex embedding corresponds to the techniques transforming an infinite set of LMI into a finite one, more suitable for numerical solving. Most of the time it consists in transforming a multiple sum problem like (2.11):

$$\begin{cases} X > 0 \\ A_h X - B_h Y_h + (*) < 0 \end{cases} \Leftrightarrow \begin{cases} X > 0 \\ \sum_i \sum_j h_i(z) h_j(z) \left(A_i X - B_i Y_j + (*) \right) < 0 \end{cases} \tag{2.13}$$

Unfortunately, all results available to deal with this transformation are conservative. The roughest result is to simply consider that each term of the sum is negative definite:

$$A_i X - B_i Y_j + (*) < 0 \ \forall i, j$$

This is problematic because one can prove that these inequalities have solutions only if a linear state feedback is available (each the gain F_j stabilizes all the linear models (A_i, B_i)).

Several techniques are available in the literature (Kim and Lee 2000; Tuan et al. 2001; Sala and Ariño 2007; A Kruszewski et al. 2007; A Kruszewski et al. 2009) introducing more or less complexity to the numerical problem (number of decision variable and combined size of the problem). One should note the results of (Ario and Sala 2008; A Kruszewski et al. 2007; A Kruszewski et al. 2009) provide Asymptotically Necessary and Sufficient (ANCS) conditions. They allow to choose the accuracy/complexity ratio of the convex embedding with a proof of convergence to the infinite size problem (2.12). Unfortunately, the computational complexity is exponential and the ratio parameter must stay quite low to be numerically tractable. The results of (Tuan et al. 2001) are also interesting and I think they are the most efficient result for the moment, and should be tried first.

The conditions of (Tuan et al. 2001) are the following:

$$\begin{cases} \Upsilon_{ii} < 0, \ \forall i \in \{1, 2, \cdots, r\} \\ \dfrac{2}{r-1} \Upsilon_{ii} + \Upsilon_{ij} + \Upsilon_{ji} < 0, \ \forall (i, j) \in \{1, 2, \cdots, r\}^2, \ i \neq j. \end{cases} \Rightarrow \begin{cases} \displaystyle\sum_{i=1}^{r} \sum_{i=1}^{r} h_i h_j \Upsilon_{ij} < 0 \\ \forall h_i \in [0, 1] \ \displaystyle\sum_{i=1}^{r} h_i = 1 \end{cases}$$

Applying this lemma with $\Upsilon_{ij} = A_i X - B_i F_j$ makes (2.12) a LMI problem with a finite set of constraints. One can see here that the conditions lose some knowledge about the functions h_i as the conditions ensure (2.12) for any functions h_i with satisfying the convex sum property.

Only few results exploit the shape of the weighting function or optimize the choice of the h_i during the sector nonlinear approach which potentially reduces a lot the conservatism (Bernal, Guerra, and Kruszewski 2009; Robles et al. 2016; Bernal, Guerra, and Kruszewski 2008a; Bernal, Guerra, and Kruszewski 2008b). Most of the available results only rely on the convex sum property of the weighting functions h_i which is quite problematic because all the knowledge of the nonlinear function is lost and impossible situations may be considered unnecessarily (like $\cos(x_1 x_2) = 1$ and $\sin(x_1 x_2) = 1$ at the same time).

Get a solution

Solvers are available for this kind of convex optimization problem (Sturm 1999; Gahinet et al., n.d.; Löfberg 2004). When there are no numerical problems, they provide the global optimal solution if any and can state the unfeasibility of the conditions. Getting a solution is easy if the problem is not too large (size of the LMI problem + number of decision variable) or if not ill-conditioned.

When it fails

If the solver fails in finding a consistent solution (feasible solution + realistic control gains) because of the complexity or the nonlinear nature of the problem, some backup options are available:

- Rewrite the problem by splitting it in small pieces, study them separately and use for example Input-to-state gains (Sontag 2008) to glue them back together. This approach is conservative because it relies on the ISS small gain theorem but helps in exploring new solutions (for example see (Maalej, Kruszewski, and Belkoura 2017b)).
- Try to approximate the model to find control gains then check the stability on the former one. This helps in reducing the number of decision variables. However, the sequential implementation (find the gains, then find the LF) introduces conservatism.
- Try to apply redesign techniques, for example: design a linear state feedback with pole placement techniques and then convert back the solution to the former problem (Delpoux, Hetel, and Kruszewski 2014).
- Avoid completely the numerical resolution of the LMI problem. For example use a more generic control strategy (Maalej 2014).

This workflow proposition was restricted to stabilization. If one wants to deal with performances, one must change the model definition and the starting condition ($\dot{V} < 0$). Problems may appear in the linearization step where the introduction of performances may certainly introduce nonlinear terms. Due to the conservative nature of this workflow, the performances can only be guaranteed above the specified level. The real performances being always better than what was predicted/required (for some case, the difference is huge).

The next two sections provide some details about the works I supervised. These results comes from two PhD thesis (Maalej 2014; Márquez 2015). The first section focuses on output feedback solutions in the case where the premise vector $z(\cdot)$ is not measured. It proposes three solutions based on different control strategies (used of TS observer, linear observer or derivative estimator). The second one deals with the conservatism reduction of state-feedback stabilization conditions and present the results obtained with different Lyapunov functions.

My works on polytopic approaches are not limited to this perimeter and also includes results on tracking, performances, discrete time, descriptor, … (see the bibliography section).

Output feedback (Maalej 2014; Márquez 2015)

This section provides additional details about the problem of observer-based output feedback for TS models. It focusses on the hard case where the premises variables are not available.

Let us consider the following polytopic representation of the system:

$$\dot{x} = \sum_{i=1}^{r} h_i(z)\left(A_i x + B_i u\right) = A_h x + B_h u \tag{2.14}$$

$$y = \sum_{i=1}^{r} h_i(z)\left(C_i x + D_i u\right) = C_h x + D_h u\,. \tag{2.15}$$

and the standard observer for this class of polytopic models (Tanaka and Wang 2001):

$$\dot{\hat{x}} = \sum_{i=1}^{r} h_i(\hat{z})\left(A_i\hat{x} + B_i u + K_i(y-\hat{y})\right) = A_{\hat{h}}\hat{x} + B_{\hat{h}}u + K_{\hat{h}}(y-\hat{y}) \quad (2.16)$$

$$\hat{y} = \sum_{i=1}^{r} h_i(\hat{z})(C_i\hat{x} + D_i u) = C_{\hat{h}}x + D_{\hat{h}}u\,. \quad (2.17)$$

The observation error system writes:

$$\dot{e} = A_h x - A_{\hat{h}}\hat{x} + B_h u - B_{\hat{h}}u - K_{\hat{h}}\left(C_h x - C_{\hat{h}}\hat{x}\right)$$

or

$$\dot{e} = \left(A_{\hat{h}} - K_{\hat{h}}C\right)e + \left(A_h - A_{\hat{h}}\right)x + \left(B_h - B_{\hat{h}}\right)u + K_{\hat{h}}\left(C_h - C_{\hat{h}}\right)x \quad (2.18)$$

Separation principle applies when $\hat{z} = z$ as one can write the following error model:

$$\dot{e} = \left(A_h - K_h C_h\right)e \quad (2.19)$$

The evolution of (2.19) is completely independent from the one of (2.14). The closed loop (2.14) with a PDC $u = -F_{\hat{h}}\hat{x} = -F_h x + F_h e$ becomes:

$$\dot{x} = A_h x - B_h F_h x + B_h F_h e \quad (2.20)$$

Then by the input to state properties (Sontag 2008) one can state that if (2.20) is stable for $e = 0$ and if e is L_2 bounded then (2.20) is stable. It proves that if the solutions of the error model (2.19) converge and if $\dot{x} = A_h x - B_h F_h$ is stable then closed loop is be stable.

The case where $z = \hat{z}$ is reductive since it means that not only the output has to be measured but also the argument of the membership function. For the case $\hat{z} \neq z$, the theory is more complicated and further investigation must be done: one cannot separate the two stability problems and it leads to non-convex optimization problems when one is searching for the LF matrices P_e and P_x , the control gains F_i and observation gains K_i.

Some results from the literature propose solutions to this problem. The first one (T M Guerra et al. 2006) proposes a really rough manner to deal with the problem and is very conservative. This conservativeness comes from the fact that no connections are made between the functions h_i and the functions $\hat{h}_i$. To overcome this limitation, one need to consider additional assumptions like the knowledge of a Lipchitz constant of the functions h_i (D. Ichalal et al. 2010; Dalil Ichalal et al. 2010; P Bergsten and Palm 2000).

The next subsections present some results of thesis I co-supervised. The first one is related to the thesis (Márquez 2015) and provides new conditions for convergence of state observer in the case of non-measured premises. The second and the third ones are related to another thesis I co-supervised (Maalej 2014) in which one proposes to consider the problem of output feedback as an interconnection of two sub-systems. Then by using input-to-state properties one can derive conditions which does not depends on extra assumption of the membership functions. The last result proposed is to change the output feedback structure but the results only guarantee the stability and performances for a specific class of system. The resulting controller requires the knowledge of only three model parameters to be tuned.

Using the differential mean value theorem (Thierry Marie Guerra et al. 2017)

The paper (Thierry Marie Guerra et al. 2017) provides a result that was developed during the thesis of (Márquez 2015) that I have co-supervised. It presents a solution for observer design in the case of non-measured premises observers. This solution is based on the differential mean value theorem which helps in characterising the evolution of $h_i(z(t)) - h_i(\hat{z}(t))$. The resulting conditions ensure the observation error convergence as well as $L_2 \to L_2$ gain performances.

To illustrate this result, let us consider the following observer:

$$\begin{aligned}
\dot{\hat{x}} &= \sum_{i=1}^{r} h_i(\hat{z})\left(A_i\hat{x} + B_i u + K_i(y-\hat{y})\right) = A_{\hat{h}}\hat{x} + B_{\hat{h}}u + K_{\hat{h}}(y-\hat{y}) \\
\hat{y} &= \sum_{i=1}^{r} h_i(\hat{z}) C_i\hat{x} = C_{\hat{h}}\hat{x}.
\end{aligned}$$

To reduce the conservatism, one considers the separation $\hat{h} = \alpha\hat{\beta}$ between what is measured $\alpha(z(\cdot)) \triangleq \alpha$ in the $\hat{h}$ function and what is not $\beta(\hat{z}(\cdot)) \triangleq \hat{\beta}$. This decomposition is performed during the polytopic reformulation of the model by applying the sector nonlinear approach twice: on the measured part, then on the non-measured part of the nonlinearities.

With this decomposition, the nonmatched terms in the observation error equations (2.18) become:

$$\begin{cases}
A_h - A_{\hat{h}} = A_{\alpha\beta} - A_{\alpha\hat{\beta}} = \sum_{i=1}^{r_\alpha}\sum_{j=1}^{r_\beta} \alpha_i(z_\alpha)\left(\beta_j(z_\beta) - \beta_j(\hat{z}_\beta)\right)A_{ij}, \\
B_h - B_{\hat{h}} = B_{\alpha\beta} - B_{\alpha\hat{\beta}} = \sum_{i=1}^{r_\alpha}\sum_{j=1}^{r_\beta} \alpha_i(z_\alpha)\left(\beta_j(z_\beta) - \beta_j(\hat{z}_\beta)\right)B_{ij}, \\
C_h - C_{\hat{h}} = C_{\alpha\beta} - C_{\alpha\hat{\beta}} = \sum_{i=1}^{r_\alpha}\sum_{j=1}^{r_\beta} \alpha_i(z_\alpha)\left(\beta_j(z_\beta) - \beta_j(\hat{z}_\beta)\right)C_{ij}.
\end{cases} \tag{2.21}$$

Then, after applying the differential mean value theorem, one writes:

$$\begin{cases}
A_{\alpha\beta} - A_{\alpha\hat{\beta}} = \sum_{i=1}^{r_\alpha}\sum_{j=1}^{r_\beta} \alpha_i(z_\alpha)\nabla\beta_j(c)e_z A_{ij} = \sum_{j=1}^{r_\beta}\nabla\beta_j(c)e_z A_{\alpha j}, \\
B_{\alpha\beta} - B_{\alpha\hat{\beta}} = \sum_{j=1}^{r_\beta}\nabla\beta_j(c)e_z B_{\alpha j}, \\
C_{\alpha\beta} - C_{\alpha\hat{\beta}} = \sum_{j=1}^{r_\beta}\nabla\beta_j(c)e_z C_{\alpha j}
\end{cases} \tag{2.22}$$

with $\nabla\beta_j(c) = \left.\dfrac{\partial\beta_j(z)}{\partial z}\right|_{z=c}$ and $e_z = z_\beta - \hat{z}_\beta$.

For sake of conciseness, the technical developments towards the LMI formulation are omitted. The important fact is that one assume that the partial derivative of the non-measured membership functions $\frac{\partial\beta_j(z)}{\partial z}$ are bounded as well as the state $x(t)$ and the control input $u(t)$, *i.e:*. $\left\|\frac{\partial\beta_j(z)}{\partial z}\right\| \le \sigma_j$, $\|x\| \le \lambda_x$ and $\|u\| \le \lambda_u$

In order to illustrate the efficiency of this approach, let us consider the following nonlinear plant:

$$\dot{x} = \begin{bmatrix} \frac{a}{4}(x_1 + x_2) & b-3 \\ \frac{3}{4}(x_1 + x_2) & x_1 + x_2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = x_1$$

with the model validity set $\mathcal{C}_x = \{x : |x_1 + x_2| \leq 4\}$. Fig 2.1 shows the feasibility set of the LMI conditions if the space (a, b) of (Thierry Marie Guerra et al. 2017) compared to the Lipchitz based of (Dalil Ichalal et al. 2007) and (Pontus Bergsten, Palm, and Driankov 2001). The new conditions are able to find more solutions than older ones.

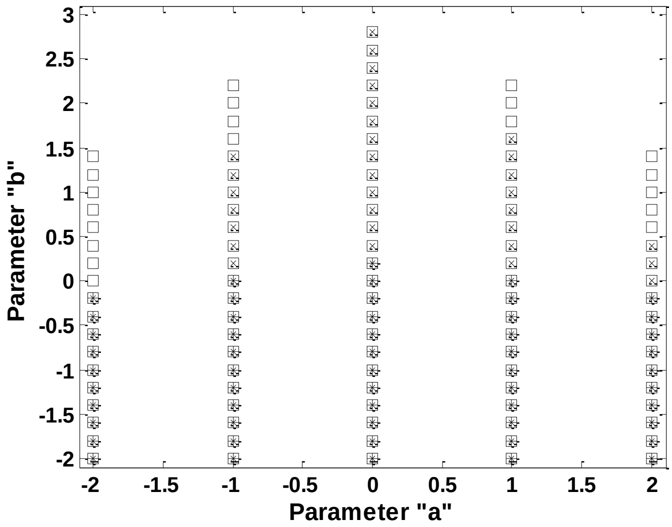

Fig 2.1 Feasibility region: "□" for conditions is (Thierry Marie Guerra et al. 2017), "×" for conditions in (Dalil Ichalal et al. 2007), and "+" for conditions in (Pontus Bergsten, Palm, and Driankov 2001)

An extension with $L_2 \to L_2$ attenuation (noise w to estimation error e) is also provided in of (Thierry Marie Guerra et al. 2017) and the numerical results are compared with (Dalil Ichalal et al. 2011) which is also based on the mean value theorem. The comparison is performed on the following plant:

$$\dot{x} = \begin{bmatrix} -1.5 - 0.5\xi_1(x) & 1.5a - 1.5\xi_2(x) \\ 2 & -4 + \xi_2(x) \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0.1 + 0.1\xi_2(x) \\ -\frac{1}{1+10\phi} - 0.1\xi_2(x) \end{bmatrix} w,$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x,$$

Fig 2.2 shows the guaranteed $L_2 \to L_2$ attenuation obtained for different values of the parameter ϕ.

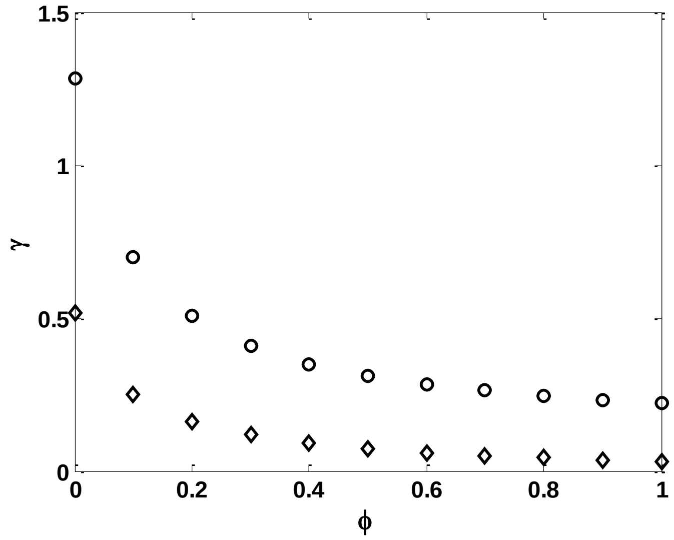

Fig 2.2. $L_2 \to L_2$ *attenuation values: "*o*" for Th. 3 in (Dalil Ichalal et al. 2011) and "*$\lozenge$*" for (Thierry Marie Guerra et al. 2017)*

All numerical examples tested in this work lead to the same conclusion about the feasibility sets inclusions: (Thierry Marie Guerra et al. 2017) showed more relaxed results. However, one is not able to state if this property is general.

Using a Linear observer (Maalej, Kruszewski, and Belkoura 2017b)

Another way to deal with non-measured premises is to avoid using them in the observer. Doing so leads to consider a linear observer for a nonlinear plant and it may increase the conservatism. The advantages of this method are:

1) no assumption about the boundedness of the state and the control is needed, *i.e.* the conditions obtained guarantee the global stability instead of local stability as in the previous results
2) the only assumption about h_i are their convex sum property and their smoothness
3) it allows the search of the best (A, B) matrices for the observer.

An observer design proposition is available in a PhD thesis that I co-supervised (Maalej 2014). It considers the following state observer:

$$\dot{\hat{x}} = A_e\hat{x} + B_e u + K\left(y - \hat{y}\right) \tag{2.23}$$

As in the classical problem, trying to find a triplet (LF, observer gains, state feedback gains) at the same time is not a LMI problem. The control design method proposed in (Maalej, Kruszewski, and Belkoura 2017b) is based on the analysis of the input-to-state properties (ISS) (Sontag 2008) of an interconnection of the two following subsystems:

1) The ideal closed loop (state feedback involving the plant state) disturbed by the estimation error:

$$\dot{x} = A_h x - B_h F x + B_h F e \tag{2.24}$$

2) The ideal observer disturbed by the non-measured premises:

$$\dot{e} = \left(A_e - KC - \Delta B_h F\right)e + \left(\Delta A_h + \Delta B_h F\right)x \tag{2.25}$$

where $\Delta A_h = A_h - A_e$ and $\Delta B_h = B_h - B_e$.

Because (2.23) it does not share the matrices with the system, one has to compute the matrices (A_e, B_e, K) such that one minimizes the input-to-state gain between x and e for the subsystem (2.25) It ensures that $\hat{x}$ is a good enough estimation of x to be used in the control law.

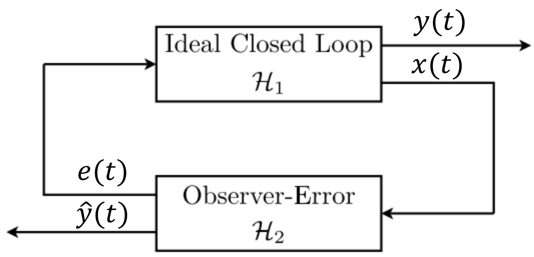

Fig. 2.3 interconnection between the ideal closed loop subsystem (2.24) and the observation error subsystem (2.25)

The design method involves the computation of the input-to-state gains (ISG) functions $\Gamma_x(s)$ and $\Gamma_e(s)$ of the two subsystems. For the subsystem 1, it consists in finding four scalar functions $\alpha_i(s)$ and a Lyapunov function $V_x(x)$ such that:

$$\alpha_1(\|x\|) \le V_x(x) \le \alpha_2(\|x\|)$$

$$\frac{d}{dt}V_x\big(x(t)\big) < \alpha_3(\|x\|) + \alpha_4(\|x\|) \text{ along the trajectories of (2.24)}$$

The ISG is given by

$$\Gamma_x(s) = \alpha_1^{-1} \circ \alpha_2 \circ \alpha_3^{-1} \circ \alpha_4^{-1}$$

In the quadratic LF case $V_x(x) = x^T P_x x$, if one chooses $\alpha_1(s) = \alpha_2(s) = s^2$, $\alpha_3(s) = 0.5s^2$, $\alpha_4(s) = 0.5\gamma_x^2 s^2$, $\|x\| = \sqrt{x^T P_x x}$ and $\|e\| = \sqrt{x^T P_e x}$ with P_e the LF matrix of the subsystem 2, one gets the following ISG function: $\Gamma_x(s) = \gamma_x s$

In the same way, one can obtain the ISG of the subsystem 2. One gets:

$$\beta_1(\|e\|) \le V_e(e) \le \beta_2(\|e\|)$$

$$\frac{d}{dt}V_e\big(e(t)\big) < \beta_3(\|e\|) + \beta_4(\|e\|)$$

The ISG is given by

$$\Gamma_e(s) = \beta_1^{-1} \circ \beta_2 \circ \beta_3^{-1} \circ \beta_4^{-1}$$

with $V_e(e) = e^T P_e e$, $\beta_1(s) = \beta_2(s) = s^2$, $\beta_3(s) = 0.5s^2$, $\beta_4(s) = 0.5\gamma_e^2 s^2$, $\|x\| = \sqrt{x^T P_x x}$ and $\|e\| = \sqrt{x^T P_e x}$, the ISG function is $\Gamma_e(s) = \gamma_e s$

If the composition of both gains functions is smaller than the identity function then the interconnection is input-to-state stable (Sontag 2008). Since our gain functions $\Gamma_x(s)$ and $\Gamma_e(s)$ are linear for the selected α_i and β_i, the condition of ISS stability of the interconnection reduces to $\gamma_x \gamma_e < 1$. This condition can be formulated in a matrix inequality problem. However, trying to find P_e, P_x as well as the controller and observer gains is not linear in the decision variables. This is due to the expression of the norms used which are be shared by both problems.

The control design consists in solving a sequence of LMI problems trying to minimize the products of ISG. Despite the sequential nature of the design, this method showed up better results than those presented in (T M Guerra et al. 2006; D. Ichalal et al. 2010; P Bergsten and Palm 2000) on very specific examples. It means that this approach should be tried when other LMI conditions fail, *i.e.* when (T M Guerra et al. 2006; D. Ichalal et al. 2010; P Bergsten and Palm 2000; Thierry Marie Guerra et al. 2017). The proposed result is very conservative but simple improvement can be performed like taking into account the measured membership functions, user better Lyapunov functions...

Using a differentiator and active rejection control (Maalej, Kruszewski, and Belkoura 2017a)

The result presented in this section is an alternative to the use of state observer. It is the core work of (Maalej 2014), a PhD thesis I co-supervised. By focusing on output derivative estimation and considering control law like active rejection control (Gao 2006; Han 2009) or model free control (Fliess and Join 2013; Choi et al. 2009; Menhour et al. 2017) one can remove the need of the estimation of the membership function value $h_i(\cdot)$. The publications (Maalej, Kruszewski, and Belkoura 2017a; Maalej, Kruszewski, and Belkoura 2017b; Maalej et al. 2014; Maalej 2014) formally prove that this technique can stabilize a complete class of model (nonlinear "all poles" SISO models), *i.e.:* for any set of parameters. The resulting control design method requires only few easy-to-get information about the system and the control goal. The controller and its gain are obtained directly from this information without the need of numerically solve any LMI problem.

To illustrate this result, let us consider the following nonlinear SISO model for the plant:

$$y^{(n)}(t) = f\left(y^{(n-1)}(t), \dots, y(t), w(t)\right) + \alpha u(t) \tag{2.26}$$

where $y(t) \in \mathbb{R}$ is the measured output, $u(t) \in \mathbb{R}$ the control input, $w(t) \in \mathbb{R}^{n_w}$ a bounded external input, f is a function modelling the plant dynamics, $n \in \mathbb{N}$ the order of the system and $\alpha \in \mathbb{R}$ the input gain.

According to (Fliess and Join 2013), one can chose the following controller structure:

$$u(t) = \frac{1}{\hat{\alpha}}\left(\hat{f}(t) - R\big(\hat{y}^{(n-1)}(t), \dots, \hat{y}^{(1)}, y(t) - r(t)\big)\right) \tag{2.27}$$

where $\hat{\alpha}$ is an approximation of α, $\hat{f}(t)$ is the estimation of the value of $f(\cdot)$ at time t, $\hat{y}^{(i)}$ the estimation of i^{th} time derivative of $y(t)$, $r(t)$ the reference and $R(\cdot)$ a polynomial such that $y^{(n)}(t) = R\big(y^{(n-1)}(t), \dots, y^{(1)}, y(t) - r(t)\big)$ represents the required dynamics. The term $\hat{y}^{(i)}$ are computed using differentiators and the value $\hat{f}(t)$ is calculated by considering the following equation:

$$\hat{f}(t) = \hat{y}^{(n)}(t) - \hat{\alpha}u(t)$$

For practical reason, a slightly delayed or filtered value of u is preferred in order to avoid algebraic loops:

$$\hat{f}(t) = \hat{y}^{(n)}(t) - \hat{\alpha}\hat{u}(t)$$

The key point is to provide a good enough estimation of the successive derivatives of the output signal $\hat{y}^{(i)}$. In (Fliess and Join 2013) the authors considered algebraic estimators (Mboup, Join, and Fliess 2009) which provide an estimation of the output derivatives based on their successive integrals.

However, to the best of our knowledge the closed loop stability in this case is not proved and no constructive tuning methodology is available. Furthermore, from our experience:

- These algorithms are tricky to tune mainly because one has to choose an algebraic annihilator, the integration window length and the integral discretization technique and sampling time. During this thesis and to the best of our knowledge no analysis of these choices was available.
- The real-time implementation of these estimators requires a large computational power compared to other solutions.
- Modelling this implementation as a dynamical system involves large scale discrete-time models with a model order greater than 100 which is not suitable for the stability analysis.

In the following, we consider linear differentiating filters instead of the algebraic ones. These filters are easier to implement and allow the stability analysis.

Because of the reasons aforementioned, (Maalej 2014; Maalej et al. 2014; Maalej, Kruszewski, and Belkoura 2017b; Maalej, Kruszewski, and Belkoura 2017a) considered linear differentiating filters instead of the algebraic ones as it simplifies both the implementation and the stability analysis. These works are dedicated to the stability analysis and they provide constructive methods to tune the controller parameters. The output derivative estimator and the control filter are expressed as continuous time filters:

$$\frac{\hat{y}^{(i)}}{y}(s) = \frac{s^i}{(\tau s+1)^i}, i = 1 \dots n \tag{2.28}$$

$$\frac{\hat{u}}{u} = \frac{1}{\tau s+1} \tag{2.29}$$

In order to prove stability properties, the work of (Maalej 2014) focuses on the use of Lyapunov functions in the state space. The plant model (2.26) is considered with the following class of function:

$$f(\cdot) = a_{n-1}\big(z(\cdot)\big)y^{(n-1)}(t) + \cdots + a_0\big(z(\cdot)\big)y(t) + w(t) \tag{2.30}$$

where, $a_i(\cdot)$ are bounded functions on the compact set of interest and $z(\cdot)$ represents a function of the output and external inputs. Then, the model (2.26) can be reformulated in a polytopic form:

$$\begin{cases} \dot{x} = A_h x + Bu + Dw \\ \quad\; y = Cx \end{cases} \tag{2.31}$$

where A_h is the convex embedding of $A(t) = \begin{bmatrix} 0 & I_{(n-1)\times(n-1)} \\ -a_0(\cdot) & [-a_1(\cdot) \quad \cdots \quad a_{n-1}(\cdot)] \end{bmatrix}$,

$C = [1 \quad 0_{1\times(n-1)}]$, $B = \begin{bmatrix} 0_{(n-1)\times 1} \\ \alpha \end{bmatrix}$, $D = \begin{bmatrix} 0_{(n-1)\times 1} \\ 1 \end{bmatrix}$

The output derivative estimator and the control law can be represented by:

$$\begin{cases} \dot{x}_e(t) = A_e x_e(t) + B_{ey} y(t) + B_{eu} u(t) \\ \hat{Y}(t) = C_{eyx} x_e(t) + C_{eyy} y(t) \\ \hat{u}(t) = C_{eu} x_e(t) \\ \hat{f}(t) = C_{fx} x_e(t) + C_{fy} y(t) \end{cases} \tag{2.32}$$

with: $$A_e = \begin{bmatrix} \mathcal{A} & 0_{n\times 1} \\ 0_{1\times n} & 0 \end{bmatrix}, B_{ey} = \begin{bmatrix} \frac{1}{\tau} \\ \vdots \\ \frac{1}{\tau^n} \\ 0 \end{bmatrix}, B_{eu} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \frac{1}{\tau} \end{bmatrix},$$

$$C_{eyx} = \begin{bmatrix} [1 & 0_{1\times n}] \\ [\mathcal{A} & 0_{n\times 1}] \end{bmatrix}, C_{eyy} = \begin{bmatrix} 0 \\ \frac{1}{\tau} \\ \vdots \\ \frac{1}{\tau^n} \end{bmatrix},$$

$$C_{eu} = [0_{1\times n} \quad 1], C_{fx} = \begin{bmatrix} \frac{-1}{\tau^n} & \cdots & \frac{-1}{\tau} & -\hat{\alpha} \end{bmatrix},$$

$$C_{fy} = \frac{1}{\tau^n} \text{ and } \mathcal{A} = \begin{bmatrix} \frac{-1}{\tau} & & 0 \\ \vdots & \ddots & \\ \frac{-1}{\tau^n} & \cdots & \frac{-1}{\tau} \end{bmatrix}$$

where $x_e \in \mathbb{R}^{n+1}$ is the estimator state, $\hat{Y} = [\hat{y}, \hat{y}^{(1)}, \dots, \hat{y}^{(n-1)}]$ is a vector with the successive estimations of the output derivatives.

The control law can be written:

$$u(t) = \frac{1}{\hat{\alpha}}\left(k_0 r(t) - K\hat{Y}(t) - \hat{f}(t)\right) \tag{2.33}$$

where K is a vector representing the coefficients k_i of the polynomial of the requirements:

$$R\left(\hat{y}^{(n-1)}(t), \dots, \hat{y}^{(1)}(t), y(t) - r(t)\right) = -k_0\left(y(t) - r(t)\right) - \sum_{i=1}^{r} k_i\, \hat{y}^{(i)}(t)$$

The closed loop composed with equations (2.31) and (2.32) can then be written in a polytopic form. This reformulation allows the stability analysis of the closed loop using the classical Lyapunov tools.

Remarks:

1) In the polytopic model framework, the basic stability conditions are given in the form of a LMI involving only the bounds of the functions $a_i(\cdot)$. So, if one can design a controller with these conditions for a given nonlinear model, then any nonlinear model sharing the same bounds will be also stabilized by the designed controller.
2) It is always possible to decrease the bounds of functions $a_i(\cdot)$ by considering a change of time scale $t \to \gamma^{-1} t$.

The previous remark means that if one can find a solution for a model such that the bounds $\bar{a}_i$ and $\underline{a}_i$ of the functions $a_i(\cdot)$ satisfy $\underline{a}_i < 0 < \bar{a}_i$, then for any model (2.31) of the same order there exist a stabilizing controller in the form of (2.32). To get this controller one just needs to apply the same change of time scale as the one used to make the bounds equivalent on the controller equations (basically it decreases the value of τ). The paper (Maalej, Kruszewski, and Belkoura 2017a) provides the details about this controller design techniques as well as a the solution in the case of models of order 1,2 and 3. It also prove through a robustness analysis with respect to the estimation error of the input gain $\hat{\alpha}/\alpha$ that the closed loop remains stable even is the input gain error is greater than 300%.

Remark: Despite the fact that considering a small enough τ stabilizes any model of this class up to the order 3, in practice it is not recommended to consider too low values of this parameter. The first reason is that the output derivative filter will increase its bandwidth and amplify measurement noises. The second reason is that a model as a specific order because we decided to neglect any dynamics faster than a chosen threshold and designing a control law with a too fast dynamic may "awake' these neglected behaviours and create instability.

I think that it may be a good alternative to the PID in the cases where it would be tuned empirically. The tuning involves only 3 parameters $(n, \tau, \hat{\alpha})$ which are easy to choose for a given system. It also shows a better performance robustness with respect to uncertainties and disturbances since, in the case of fast enough estimation, the plant dynamic and the disturbances (the function $f(\cdot)$) are completely replaced by the reference model. The experimental results provided in (Maalej 2014) on a permanent magnet stepper motor confirm its efficiency on a real case both in control quality and tuning time.

Other properties still need to be proved or analyzed like the disturbance rejection, its natural anti-windup property, robustness with respect to the zeros of the system or to the neglected dynamics. Also, an extension to the multi-input multi-output case is lacking.

Conservatism reduction Thesis of Raymundo Márquez (Márquez 2015)

This section provides details about some of the result obtained in the PhD thesis (Márquez 2015) that I co-supervised. The first subsection proposes a way to reduce the conservatism even in the quadratic case by choosing a new type of control law which seems counter-intuitive. The last results focus on the choice of the LF and provides a solution to escape from the quadratic case while keeping the global stability properties. It considers the control design for the following state-space model:

$$\dot{x} = \sum_i h_i\big(z(\cdot)\big)\,(A_i x + B_i u) = A_h x + B_h u \tag{2.34}$$

Playing with the control law (Márquez et al. 2017)

This subsection presents an interesting link between the choice of the control structure and the convex embedding step of the workflow presented in the first section. For TS models, it is easy to show that if the model is quadratically stabilizable then a stabilizing PDC controller exist, *i.e.* considering a control law $u(t) = F_h x(t)$ is necessary and sufficient for the quadratic stabilization of (2.34). The conditions of existence of a quadratically stabilizing controller in the form $u(t) = F(z)x(t)$ are:

$$\begin{cases} X > 0 \\ A_h X - B_h F(z) X + (*) < 0 \end{cases} \tag{2.35}$$

By using the elimination lemma applied on $Y(z) = F(z)X$ (Scherer and Weiland 2000; Boyd et al. 1994), (2.35) is equivalent to:

$$\begin{cases} X > 0 \\ A_h X + X A_h^T - 2\sigma B_h B_h^T < 0 \\ \sigma > 0 \end{cases} \tag{2.36}$$

By considering the controller $u(t) = F(z)x(t) = F_h x(t)$ with $F_h = \sigma B_h^T P$ in (2.35) one gets also the conditions (2.36) which proves that this controller is necessary and sufficient for quadratic stabilization.

Nevertheless, in (Márquez et al. 2015b; Márquez et al. 2013; Márquez et al. 2017; Márquez 2015) one managed to find examples in which a nested type controller $u(t) = F_{h\ldots h} H^{-1} x(t)$ can be designed to quadratically stabilize the plants while a PDC cannot. One was convinced that the improvement was due to the numerical relaxation made by adding some extra decision variables that improved the numerical conditioning of the LMI problems but the results were too efficient to be that case. In (Márquez 2015; Márquez et al. 2017), it is proved that the introduction of extra freedom in the control law and the use of some matrix inequality lemmas (Peaucelle et al. 2000) act as if one was using a better convex embedding technic. It proves that the solution set of LMI obtained with a nested control law and a basic embedding technic always includes the one of the LMI obtained with a PDC and one of the best embedding technic based on the Polya's theorem (Sala and Ariño 2007), for the same complexity level. So, as in (Sala and Ariño 2007), the theorems in (Márquez et al. 2017) are asymptotically necessary and sufficient for quadratic stabilization as the number of extra parameters in the control law increases. On all examples tested, the conditions of (Márquez et al. 2017) require a fewer number of decision variables and a reduced size of LMI than in (Sala and Ariño 2007).

This fact is important as choices made in early step may reduce the conservatism of another step. For example, one may choose a completely new Lyapunov function which brings nothing from the Lyapunov stability point of view (because it is inappropriate for the class of model) but still improve the conditions.

Discrete like integral Lyapunov functional (Márquez et al. 2016)

Considering a nonquadratic Lyapunov Function (LF) is easier in the discrete-time case than in the continuous and one can consider LF in the form:

$$V\big(x(t), z(t-1)\big) = x^T(t) \sum_{i=1}^{r} \big(h_i\big(z(t-1)\big)P_i\big)^{-1} x(t) = x^T(t) P_{h^-}^{-1} x(t)$$

The variation of this LF along the trajectories is given by:

$$\begin{aligned} \Delta V &= V\big(x(t+1), z(t)\big) - V\big(x(t), z(t-1)\big) \\ &= x(t+1)^T P_h^{-1} x(t+1) - x(t)^T P_{h^-}^{-1} x(t) \end{aligned}$$

Then ensuring $\Delta V < 0$ is generally done by ignoring completely the link between $h_i\big(z(t)\big)$ and $h_i\big(z(t-1)\big)$ as well as the shape of the h_i functions. Generally, only the convex sum property is exploited. Despite this loss of information, the LMI conditions obtained with this LF are ones of the most efficient found in the literature (T.-M. Guerra et al. 2012) and provide a good compromise between the conservatism and the numerical complexity.

The continuous case is much harder to deal with compared to the discrete case. Mainly because of introduction of the time derivative of the Lyapunov matrix:

$$V(x) = x^T P_h x$$

$$\dot{V}(x) = x^T P_h \dot{x} + \dot{x}^T P_h x + x^T \dot{P}_h x = x^T P_h \dot{x} + \dot{x}^T P_h x + x^T \sum_{i=1}^{r} \frac{d}{dt} h_i(z(\cdot)) P_i \, x$$

The term $x^T \sum_{i=1}^{r} \frac{d}{dt} h_i(z(\cdot)) P_i \, x$ has no straightforward properties to exploit and the only solution to get LMI conditions seems to bound it. This has two major drawbacks:

1) bounding this term often done by assuming that the state and the control input are bounded, thus limiting the analysis to local stability
2) the fact that term $x^T \sum_{i=1}^{r} \frac{d}{dt} h_i(z(\cdot)) P_i \, x$ may be negative for specific values of x and z is not well exploited. The only property that can be easily used is $\sum_{i=1}^{r} \frac{d}{dt} h_i(z(\cdot)) = 0$.

The idea proposed in (Márquez et al. 2016) is to mimic the discrete case result and build a Lyapunov functional making the history of the functions h_i appear in $\dot{V}$ instead of their derivatives. The considered Lyapunov functional is the following:

$$V(x) = x^T P_s^{-1} x = x^T \left(\sum_{i=1}^{r} s_i(z(t)) P_i \right)^{-1} x \text{ , } P_i = P_i^T > 0 \tag{2.37}$$

$$s_i(z(t)) = \frac{1}{\alpha} \int_{t-\alpha}^{t} h_i(z(\tau)) d\tau \geq 0 \text{ , } \alpha > 0 . \tag{2.38}$$

The integral in (2.38) is well defined as the Membership Functions (MF) $h_i(\cdot)$ are smooth and bounded. The function (2.37) is a valid LF candidate for any positive α as it is a sum of positive radially bounded functions and $V(x) = 0 \Rightarrow x = 0$.

The functions $s_i(\cdot)$, $i \in \{1, 2, \ldots, r\}$ in (2.38), keep also the convex sum property which is a useful property to use:

$$\sum_{i=1}^{r} s_i(z(t)) = \frac{1}{\alpha} \int_{t-\alpha}^{t} \left(\sum_{i=1}^{r} h_i(z(\tau)) \right) d\tau = 1. \tag{2.39}$$

Finally, the time-derivatives of the functions s_i are:

$$\dot{s}_i(z) = \frac{1}{\alpha} \left(h_i(z(t)) - h_i(z(t-\alpha)) \right), \tag{2.40}$$

with $x(t) = \phi(t)$, $t \in [-\alpha, 0]$, $\phi \in \ell([-\alpha, 0], \mathbb{R})$ being the initial function and $\ell([-\alpha, 0], \mathbb{R})$ the Banach space of real continuous functions on the interval $[-\alpha, 0]$ with $|\phi|_\alpha = \max_{t \in [-\alpha, 0]} |\phi(\theta)|$ (Gu, Chen, and Kharitonov 2003). The equation (2.40) is the key to make the conditions look like the discrete case. Therefore:

$$\frac{dP_s}{dt} = \frac{1}{\alpha}(P_h - P_{h^-}), \text{ with } h_i^- = h_i(z(t-\alpha)) \tag{2.41}$$

Considering the evolution of $\dot{V}$ along the trajectories of the closed loop composed with (2.34) and the control law $u = F_{hh^-s} P_s^{-1} x$, leads to the following condition of stability:

$$A_h P_s - B_h F_{hh^-s} + \left(A_h P_s - B_h F_{hh^-s} \right)^T - \frac{1}{\alpha}\left(P_h - P_{h^-} \right) < 0 \tag{2.42}$$

The main strengths of this condition are:

- It is checking the *global* stabilization by opposition to other NQ results (Jaadari et al. 2012)
- No need of additional assumptions by opposition to (Márquez et al. 2014; Rhee and Won 2006)
- Its solution set always includes the one of the classical condition using the quadratic LF. It is easy to prove by considering $P_i = P \; \forall i$, $F_{ijk} = F_i$ since the condition becomes:

$$\underset{A_h P}{A_h P_s} - \underbrace{B_h F_{hh^-s}}_{B_h F_h} + (*) \underbrace{- \frac{1}{\alpha}\left(P_h - P_{h^-} \right)}_{0} < 0$$

Numerical examples provided in (Márquez et al. 2016 shows that this condition and its extensions outperform existing results like (Jaadari et al. 2012).

The (2.42) condition induces the use of a quite complex control law which constitutes a notable drawback. The non-condensed expression of the control law:

$$u = -\frac{1}{\alpha}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r} h_i\left(z(t)\right) h_j\left(z(t-\alpha)\right)\int_{t-\alpha}^{t} h_k\left(z(\tau)\right) F_{ijk}\, d\tau$$

which is not realistic for real-time implementation. Further works are needed to try to reduce this complexity to a suitable level. It is also important to determine if the improvement is due to the choice of the LF, the control law or if it is an indirect improvement of another step of the method. In the latter case, one may be able to redesign a simpler control law for the same performances.

Line integral Lyapunov functions (Márquez et al. 2014; Marquez et al. 2013)

Another way to consider non-quadratic Lyapunov functions without the need of bounding the derivatives of the parameters is proposed in (Rhee and Won 2006). The main result is based on the use of path independent Lyapunov function (see details in (Khalil 2001)) and considers a line-integral Lyapunov functions in the form:

$$V(x) = 2\int_{\Gamma(0,x)} \mathfrak{F}(\psi)\, d\psi \tag{2.43}$$

where is a function $\mathfrak{F}(x) = \left[\mathfrak{F}_1(x), \ldots, \mathfrak{F}_{n_x}(x)\right]^T$ is such that $V(x)$ does not depend on the path of integration $\Gamma(0,x)$ (thus does not need to be chosen). This property can be checked by verifying $\frac{\partial \mathfrak{F}_i(x)}{\partial x_j} = \frac{\partial \mathfrak{F}_j(x)}{\partial x_i}$. A valid candidate LF can be obtained by choosing:

$$\mathfrak{F}(x) = P(x)x = \left(\sum_{i=1}^{r}\underbrace{\prod_{j=1}^{n_x}\omega_{i_j}^{j}\left(x_j\right)}_{h_i(x)}\left(\underbrace{\begin{bmatrix} 0 & p_{12} & \cdots & p_{1n_x} \\ p_{12} & 0 & \cdots & p_{2n_x} \\ \vdots & \vdots & \ddots & \vdots \\ p_{1n_x} & p_{2n_x} & \cdots & 0 \end{bmatrix}}_{\bar{P}} + \underbrace{\begin{bmatrix} d_{11}^{\alpha_{i1}} & 0 & \cdots & 0 \\ 0 & d_{22}^{\alpha_{i2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d_{n_x n_x}^{\alpha_{in_x}} \end{bmatrix}}_{\mathfrak{D}_i}\right)\right) x$$

such that $\bar{P} + \mathfrak{D}_i = (\bar{P} + \mathfrak{D}_i)^T > 0$.

Remark: The functions ω_j^0 and ω_j^1 are the weighing function (WF) that constitute the membership functions (MF) h_i. The fact that $\omega_{i_j}^{j}$ must be function of x_j to obtain a valid Lyapunov function restrict the class of TS model that can be studied. One can find more details about this limitation in (Rhee and Won 2006).

The use of this type of LF showed up good results in stability but up to now no LMI formulation is available in the control design case (Rhee and Won 2006). To illustrate this, one need to write the derivative of the LF along the trajectories of the system (2.34):

$$\dot{V}(x) = x^T\left(P(x)\left(A_h - B_h F_h\right) + \left(A_h - B_h F_h\right)^T P(x)\right)x$$

which is negative for all $x \neq 0$ iif:

$$P(x)(A_h - B_h F_h) + (A_h - B_h F_h)^T P(x) < 0$$

To linearize this inequality, one should apply the congruence property with $P(x)^{-1}$, then use bijective change of variables ($X = P^{-1}$, $Y = FX$ in the quadratic case):

$$A_h P(x)^{-1} - B_h F_h P(x)^{-1} + (*) < 0$$

The control term $B_h F_h P(x)^{-1}$ can be linearized by changing the control law. Calculating $\dot{V}$ with the control law $u = -Y_h P(x)x$ leads to:

$$A_h P(x)^{-1} - B_h F_h + (*) < 0 \tag{2.44}$$

The term $A_h P(x)^{-1}$ is nonlinear in the decision variables (the entries of the matrices $\bar{P}$ and $\mathfrak{D}_i$). Appling a change of variable like $X(x) = P(x)^{-1}$ do not linearize the problem because the specific structure of the matrices $\bar{P}$ and $\mathfrak{D}_i$ involved in $P(x)$. It implies constraints on the entries of the matrix $X(x)$ that are not linear. Moreover, a mistake I saw during the review of related paper was to consider a line-integral LF with $\mathfrak{F}(x) = P(x)^{-1}x$ which is wrong. With such a function $\mathfrak{F}(x)$, the LF loses its path independency property.

To the best of our knowledge, the control design problem can be written with LMI condition without adding extra conservatism in the second order case. In this case, one can explicitly write the inverse of the matrix $P(x)$:

$$P(x)^{-1} = \begin{bmatrix} d_{11}^{\alpha_{i1}} & q \\ q & d_{22}^{\alpha_{i2}} \end{bmatrix}^{-1} = \frac{1}{d_{11}^{\alpha_{i1}} d_{22}^{\alpha_{i2}} - q^2} \begin{bmatrix} d_{22}^{\alpha_{i2}} & -q \\ -q & d_{11}^{\alpha_{i1}} \end{bmatrix} = \frac{X_h}{|P(x)|}$$

With this property, (2.44) is equivalent to:

$$\frac{1}{|P(x)|}\left(A_h X_h - B_h F_h + (*)\right) < 0$$

because $P(x) > 0$ it can be written:

$$A_h X_h - B_h F_h + (*) < 0$$

This latter condition can be transformed into a finite set of LMI. The details of these results are available (Márquez et al. 2014; Marquez et al. 2013) as well as a numerical example showing how it outperforms the LMI sequence algorithm proposed in (Rhee and Won 2006).

These results are interesting, efficient in term of conservatism and original but the two limitations (the fact that $\omega_{i_j}^{i}$ must be a function of x_j and second order) need to be addressed.

Conclusions

This chapter presented some of my work related to the improvement of Takagi-Sugeno model based control design in term of conservatism reduction. The introduction depicted the classical workflow used in this framework and highlighted the positioning of some of my works.

The second section presented some techniques related to a key problem which is how to design an output controller when the premises variables are not measured. This problem is very important and corresponds to many situations. The propositions illustrated in this section considered two approaches, one avoiding the use of the expression of the membership functions and the other considering the use of additional knowledge about these functions. For both the approaches the advantage and the drawbacks were presented. This problematic is far from solved as either one can prove only the local stability, or a lot of conservatism is added. To improve these results, one should be to consider more accurate assumption on the difference between $h_i(z)$ and its estimation $h_i(\hat{z})$ by using the shape of the MF. Another track would be to consider and array of observer/controller pairs with different properties (ratio performances/domain of attraction). In this case, the less accurate observer/controller pair would have a global stability but with a large error of estimation like the linear observer that was presented in this section. This first state estimation could be used as an estimation of the value of $h_i(z)$ by a more restrictive observer which provides a better state estimation and so on… The last track presented in this section is based on to the use of differentiator and active dynamic cancellation which do not requires the full knowledge of a model of the plant. It is more subject to discussion as it does not provide enough guarantee (for the moment) to be convincing. Important features are missing like dealing with MIMO systems or zero dynamics and further analysis are required to complete the robustness analysis. The structure of this controller can still be used in a model-based context (Maalej et al. 2014) which helps in designing an output feedback which does not requires the measurement of the membership functions $h_i(z)$.

The last part presented a persistent topic in the TS or LPV framework: How to get the most 'efficient' stabilization conditions? However, the term 'efficient' lacks a good definition. Numerous works pretend to outperform others but when one looks at the additional complexity required to achieve, sometimes, a very small conservatism improvement, the word 'outperform' seems to lose its meaning. There are problems in the way the works are compared: I saw many paper comparing their new fancy LF or new control law with others without considering the same convex embedding techniques which makes the comparison 'unfair' and not rigorous. I think a good clean-up among these results is necessary if one wants the engineers and researchers choosing the right conditions for a given control problem these comparisons have to be done correctly and summarized somewhere. One should think of measurable indexes of 'efficiency' for stabilization conditions, some kind of numerical benchmark. In my mind, the efficiency is the ratio between the computational burden required to solve the problem and the conservatism. There are already indexes for the computational complexity but they do not always quantify the computational burden and ignore completely the complexity of the implementation of the control law. On some problems, adding extra variables and thus increasing the numerical complexity of the problem, reduces the number of iteration required by numerical solvers to find a solution with the same precision. On other problems, considering an extremely complex control law helps the numerical solver in finding quickly a solution but the implementation of that control law in real-time is unrealistic. One also lacks a way to quantify the conservatism to estimate the 'efficiency' and beside proposing unified numerical benchmarks, I don't see what else could be done. Finally, most of these results focus on adding more and more decision variables/constraints to reduce the conservatism. For the moment, no one cares about the scalability of these conditions. I am convinced that most of them may not be numerically tractable on moderate size problem (order greater than 4 with more than more than 2 nonlinearities) which is for me the class of model that these tools could be really useful. One last aspect of efficiency will be discussed in the conclusion chapter which is the ease-of-use of the control design methods. An aspect that seems to be neglected by researchers.

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III. Networked control system

This chapter describes my own application-oriented researches as well as the ones which have been developed under my co-supervision until 2012. They were the subject of two PhD thesis (W. Jiang 2009; Bo Zhang 2012) and concern Networked Control System (NCS). The first PhD focuses on how to design a networked control law that works over unreliable networks (with non-neglectable delay jitter and packet dropout like the Internet or Wi-Fi). This control law adapts its gains according to the Quality of Service (QoS) provided by the network in order to maximize the guaranteed closed-loop performances. The results of these works were illustrated with an academical test bench consisting in a robot controlled by a distant computer (40km away) over the internet. They were published in a journal paper, one book chapter and 5 international conferences. The second PhD is the follow-up of the first one and focuses on bilateral teleoperation over the internet case. This topic is a specific case of NCS where one needs to design two controllers, one at each side of the network, that ‘synchronize’ two robotic devices. These works were published in a journal, one book chapter and 6 international congresses.

The two first sections of this chapter provide some details about these works. The bibliography of these sections may be a bit outdated as I have put these topics in standby since 2012. I will mainly compare these works to state of the art of this period. The last section mentions some side applications that cannot be detailed in this manuscript.

Remote control through the Internet

Networked Control System (NCS) problems consist in the analysis and the design of closed loop involving communication phenomena between the entities (actuators, sensors, calculator). The reader may refer to (X. Zhang, Han, and Yu 2015) for an overview of recent advances in the field and to (Richard and Divoux 2007; Mounier et al. 2003) for complementary topics such as the control or the estimation of communication quality. The figure 3.1 depicts a typical setup of networked control systems.

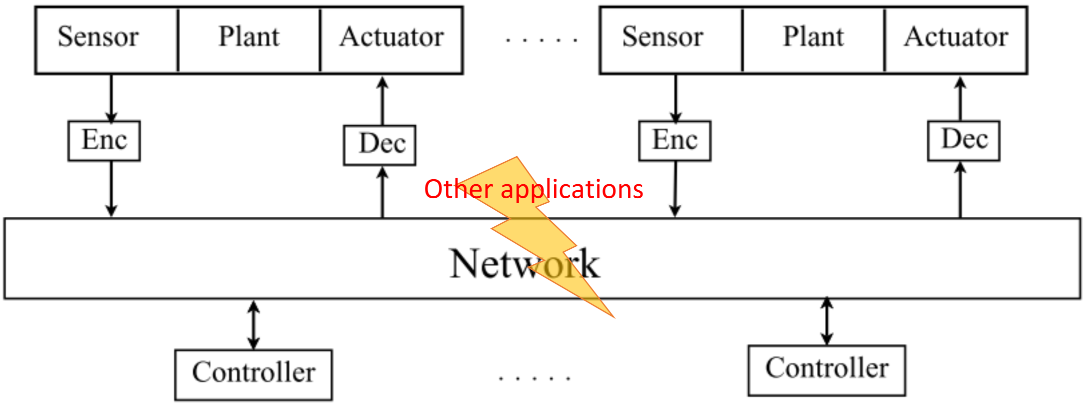

Fig. 3.1 General NCS architecture.

The trend of NCS appeared some decades ago and exploded when the actuator and the sensor became "smart" and where able to communicate through communication networks. When the network has a few nodes, a high bandwidth (compared to the plant) and a low load, then its effects on the dynamic may be assimilated to a zero-order sampler with a neglectable or small delay. Using scheduling like a static time window attribution to each entity solves the problem as the closed loop falls into the classical discrete framework with periodic sampling. In the case of heavily loaded, unreliable or non-scheduled networks, these kinds of solutions do not seem suitable. This is the case with the Internet: it is unreliable because of the number of devices, nodes, users, traffic involved. However, the use of such networks for controlling plants is still appealing has it may reduce the cost of an application or makes it more flexible. The classical example is the medical application where one surgeon wants to control a surgical robot elsewhere on the planet, in this case the Internet could be the medium of communication as its infrastructures are already in place. One can also consider robot cooperation where wireless network would be a nice solution because of the ubiquity of Wi-Fi access points or the actual 4G coverage. The main questions are: 'How to design a controller which still guarantee the correct behaviour of the controlled plant?' and 'How to minimize the impact of this closed loop on the load of the involved networks?'. My work only focuses on the first problematic. The reader may refer to event trigger techniques in (Hetel et al. 2017; X. Zhang, Han, and Yu 2015) for the second one.

From the theoretical point of view, networked control system problems are a particular case of sampled data systems in which one considers sampling and delay effect on the closed loop. These effects may come from networked communication medias, task scheduling, event triggers control law, quantized signals and so on… A good overview of the topic is available in (Hetel et al. 2017) which provides different modelling and analysis tools and an insight of the link between them.

NCS model and assumptions

This section focuses on the main results obtained in the thesis (W. Jiang 2009). It considers the simplest case of a networked control design problem over an unreliable network, the quality of service (QoS) of which is highly volatile. We have chosen to illustrate it in the Internet case because the communication media is shared with various other applications (Zitoune et al. 2009; Mounier et al. 2003). This configuration is depicted in Fig 3.2.

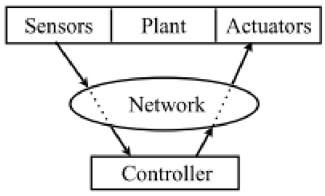

Fig 3.2 Control of a single plant over a network.

The assumptions made in these works are:

- The plant is linear.
- The network-induced delay is the result of three effects (Fig. 3.3):
    - the communication delay, *i.e.* the time taken by a data packet to go through the network (random bounded delay),
    - the zero-order-hold sampling effect due to the discrete nature of the computation and the communication (a time varying delay with a unitary derivative and a reset at the sampling instants),
    - the packet losses due to bad routing or buffer overflows which multiplies the sampling period by a natural number
- If the network-induced delay is larger than a specified value, the packet is considered as lost.
- If the packets arrive in disorder with respect to the sending time, only the most up-to-date is considered for the control task, others are considered lost.
- If too many packets are lost or if the communication delay becomes too large, the networked control application is stopped safely (the plant goes into a safe local controlled mode or proceed an emergency stop).

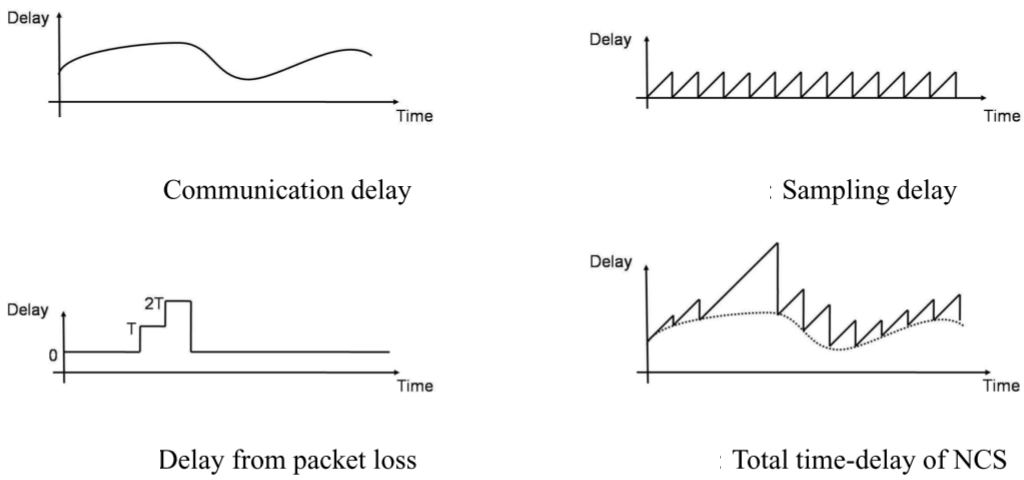

Fig 3.3 Illustration of the 3 network-induced delay sources.

The thesis (W. Jiang 2009) also discusses the choice of the protocol (UPD rather than TCP) and proposes some implementation methods based on multi-tasking techniques.

A way to deal with the control design of these setups is to consider the buffering of the received packets. Buffering consists in putting in memory the packets received and wait for specified date before using them. The date of application of the packed may be contained in the packed itself or may be calculated from date of sending of the packet. In those cases, a time stamp is sent in the packet[5]. The most common buffering strategy consists in making the perceived delay constant at the maximum network delay admissible. Buffering mechanics has two positive effects and a negative one:

+ It softens the delay jitter because only the sampling effect and packet dropout effects remain
+ It makes the packet use date more predictable by the sender.
- The extra-delay, artificially added, may reduce the performance of the NCS.

Following this strategy transforms the problem into a pure periodic sampling control design problem with constant communication delay disturbed by the packet dropout phenomena. The problem can then be studied in continuous time or in discrete case (Hetel et al. 2011; Donkers et al. 2011). This way of dealing with networked delay is efficient in term of the design effort but the performances are heavily impacted when one considers unreliable networks: the upper delay maybe large compared to mean value of the delay thus the closed loop loses precious reaction time most of the time. Moreover, implementing a buffer requires memory on both side of the network, which may be contradictory if the unreliable networks was chosen to reduce the cost of the application.

NCS control design

The goal of the works (W. Jiang 2009; Kruszewski et al. 2012) is to provide a design procedure allowing the stabilization of the closed loop under performance constraints while trying to reduce the length of the buffers. To do so, a QoS-dependent control law is designed such that it adapts the closed-loop performances according the state of the network. It reduces the buffer length and adapts the control gains according to the delay (Kruszewski et al. 2012). Another result, (W. Jiang et al. 2009) provides a procedure to get rid of the buffer mechanics but this design technique is more conservative than buffer-based ones (because it involves non-convex problem solving). Stability conditions were derived using Lyapunov-Krasovskii Functional (LKF) (Krasovskii 1963) and the resulting conditions were expressed as Linear Matrix Inequalities (LMI) (Boyd et al. 1994). The chosen control structure is composed with a state feedback controller and a Luenberger observer which estimate the current state of the remote plant from the sequence of input sent and the delayed output of the plant (see Fig. 3.4). In this sense, this observer is also a predictor.

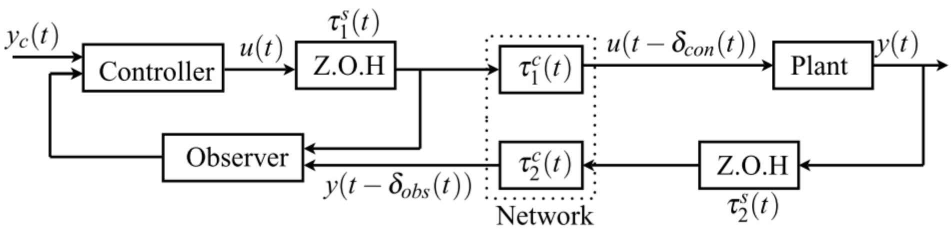

Fig 3.4 Observer-based networked control system

Putting this problem into equations gives:

$$\dot{x}(t) = Ax(t) + Bu\big(t - \delta_{con}(t)\big) \tag{1.1}$$

$$x(t_0+\theta) = \phi(\theta), \dot{x}(t_0+\theta) = \dot{\phi}(\theta)\ \forall\theta \in [-h_2, 0]$$

[5] Assuming that the clock of the sender and the receiver are synchronized

$$u(t) = Gr(t) - F\hat{x}(t) \tag{1.2}$$

$$\dot{\hat{x}} = A\hat{x}(t) + Bu\left(t - \hat{\delta}_{con}(t)\right) + K\left(y\left(t - \delta_{obs}(t)\right) - \hat{y}\left(t - \hat{\delta}_{obs}(t)\right)\right) \tag{1.3}$$

$$\hat{x}(t_0 + \theta) = \hat{\phi}(\theta), \dot{x}(t_0 + \theta) = \dot{\hat{\phi}}(\theta)\ \forall \theta \in [-h_2, 0]$$

where: $x \in \mathbb{R}^n, \hat{x} \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, $\hat{y} \in \mathbb{R}^p$ are respectively the plant instantaneous state, the estimated instantaneous plant state (observer state), the control vector, the measure vector and the estimated output vector. ϕ and $\hat{\phi}$ are the initial condition functions for the plant and the observer. $\delta_{con}(t) \in [h_1, h_2]$ and $\delta_{obs}(t) \in [h_1, h_2]$ are the total delays of the control channel and the measurement channel respectively, *i.e.* $\delta_.(t) = \tau_.^c(t) + \tau_.^s(t)$ where τ^c is the communication delay including packets dropouts delay and τ^s is the induced sampling delay. $\hat{\delta}_{con}(t)$ and $\hat{\delta}_{obs}(t)$ are the estimated delays. $\delta_{con}(t)$ and $\delta_{obs}(t)$ satisfies the properties: $\frac{d}{dt}\delta_{con}(t) \leq 1$ and $\frac{d}{dt}\delta_{obs}(t) \leq 1$.

Important remark: By considering a time stamping mechanic one can assume that a measure of the plant-to-controller delay is available $\hat{\delta}_{obs}(t) = \delta_{obs}(t)$: the date of acquisition of the measure is sent through the network with the measurement data. Then by simply computing the difference between the current time and the time stamp one gets the value of the delay. For the control channel, if no packet dropout occurs, the control signal delay can be predicted at the controller side as the controller sends the control value together its ideal application date. This way one can assume that $\hat{\delta}_{con}(t) = \delta_{con}(t)$ during the design phase of the controller, the robustness with respect to packet dropout being analyzed afterward. Under the that $\hat{\delta}_{con}(t) = \delta_{con}(t)$ assumption, the separation principle between the state feedback problem and the observation problem holds. It is easy to see since (1.1)-(1.3) becomes:

$$\dot{x}(t) = Ax(t) - BFx\big(t - \tau_{con}(t)\big) + BFe\big(t - \tau_{con}(t)\big) \tag{1.4}$$

$$\dot{e}(t) = Ae(t) - KCe\big(t - \tau_{obs}(t)\big) \tag{1.5}$$

with $e(t) = x(t) - \hat{x}(t)$ the instantaneous observation error state vector.

For the design of F and K in (1.4) and (1.5) a lot of LMI-based results can be applied. One must remind that $\frac{d}{dt}\delta_{con}(t) \leq 1$ and $\frac{d}{dt}\delta_{obs}(t) \leq 1$ when selecting a theorem since only a selection of the LKF-based result can include the case $\frac{d}{dt}\delta_{obs/con}(t) = 1$. The results in (W. Jiang 2009) are mainly based on the LFK developments of (He et al. 2007; Emilia Fridman, Seuret, and Richard 2004; E Fridman 2006). The LFK considered is:

$$V(t, x_t, \dot{x}_t) = x^T(t)Px(t) + \int_{t-h_1}^{t} x^T(s)Sx(s)ds + h_1 \int_{-h_1}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta$$
$$+ \int_{t-h_2}^{t} x^T(s)S_a x(s)ds + (h_2 - h_1)\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{x}^T(s)R_a\dot{x}(s)dsd\theta$$

where $P > 0$ and $R, R_a, S, S_a \geq 0$

The LMI conditions and proof are omitted in this document for the sake of conciseness. The reader may refer to (W. Jiang 2009) for the theorems and technical details and to (Hetel et al. 2017) for a more didactic point of view and a deeper insight in what is going on. Presently, more efficient LKF and stability results are available and can be adapted to the NCS problem (Park, Ko, and Jeong 2011).

QoS-dependent control design

In order to increase the performances and the responsiveness of the closed loop, one can consider QoS dependent control gains so that $u(t) = F(QoS(t))\hat{x}(t)$ and buffers lengths. The chosen performance index is expressed in α-stability terms (S.-I. Niculescu et al. 1998): $\|x(t, t_0, \phi)\| \leq F\|\phi\|e^{-\alpha(t-t_0)}$. These performances are ensured by proceeding with a change of state variable $x_\alpha(t) = e^{\alpha t}x(t)$ and studying the stability of $x_\alpha(t)$ as in (Seuret 2006). The QoS is measured by using the time stamps and thus is only available after the controller side received the feedback from the plant, *i.e.* the control law is given by:

$$u(t) = F\left(\delta_{con}\big(t - \delta_{obs}(t)\big)\right)\hat{x}(t)$$

On consider the following switching gain law:

$$F(\cdot) = \begin{cases} F_1 \quad if \quad h_1 \leq \delta_{con}\big(t - \delta_{obs}(t)\big) < h_2 & : small\ delay\ mode \\ F_2 \quad if \quad h_2 \leq \delta_{con}\big(t - \delta_{obs}(t)\big) \leq h_3 & : large\ delay\ mode \end{cases}$$

where $h_1 < h_2 < h_3$. The buffers lengths are adapted to match the upper bound of the current delay interval.

The closed loop becomes a time-dependent switching system (Liberzon 2003). Two stabilizing design strategies arise depending on the technical development one might choose:

- The first strategy is to consider a common LKF for every mode of the closed loop but with different performances. One gets more performances for small delays than in the non QoS dependent case. Moreover, this is a safe choice as the stability is guaranteed even if the wrong mode is selected, *i.e.:* the small delay mode gains still stabilize the system when the delays are large (W.-J. Jiang et al. 2008; Sun, Wang, and Xie 2006; Lee and Dullerud 2006; Lee and Dullerud 2007).
- The second strategy consists in using a different LKF (V_1 and V_2) for each mode (Hespanha and Morse 1999; Hirche, Chen, and Buss 2008). In this case, the conditions are significantly relaxed and allow for better performances. The main drawback with this approach is the creation of potentially unstable unwanted modes: During the transition between the small-delay mode to the large-delay mode, because of the delay in the switching signal, the high-performance gains will be active when the delay is large. Moreover, a bad sequence of switches may lead to instability, *i.e.* even is the created modes are stable, there may exist a destabilizing sequence of modes. Fortunately, these unwanted modes are only temporary and last only the time needed to detect the correct mode (the duration of the delay). To avoid these two unstable behaviours, two techniques are available and are both based on enforcing the NCS to be in the large-delay mode:
    - One can compute the authorized dwell time which is the minimal time the controller has to wait in the large-delay mode before switching back to the small-delay mode. This approach is quite conservative and the obtained dwell-times are often overestimated and larger than the time-response of the system.
    - One can monitor the value of the LKFs and switch back to the small-delay mode only when these functions are establishing a decreasing sequence (see Fig 3.5) This approach does not introduce extra conservatism. It provides higher performances than the former approach at the expense of the computational cost of the control law: LKF involves integral terms that need memory and computation power to be evaluated.

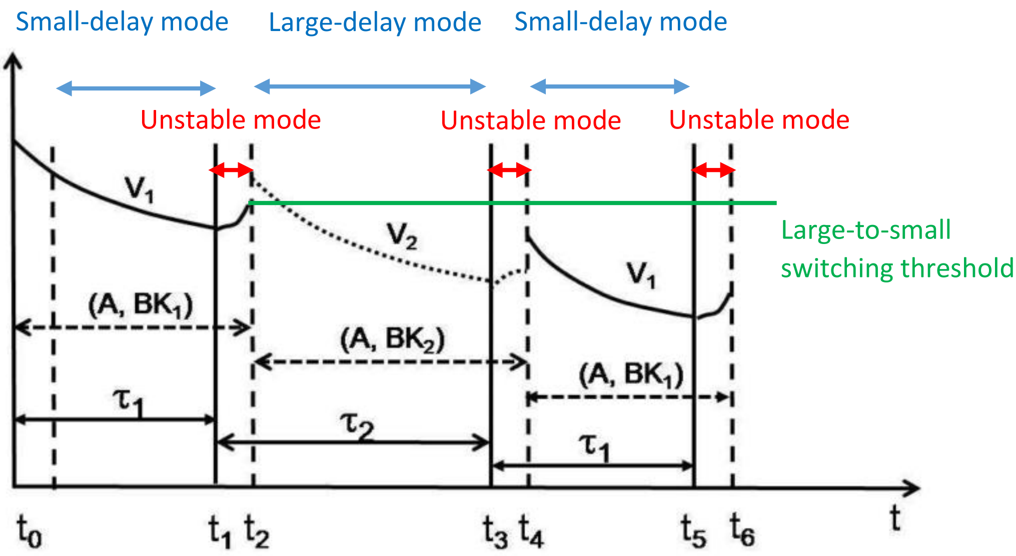

Fig 3.5 Evolution of the LKF values during a switching sequence

These approaches have been published in (Kruszewski et al. 2012). One can note that similar results exist with a constant delay assumption (Yan and Özbay 2008; Chen, Hirche, and Buss 2006; Hirche, Chen, and Buss 2008) which is may be used if the sampling time is very small compared to the network delay and if a very large buffer is used.

Experimental results

All these results were applied to the academic test bench depicted below:

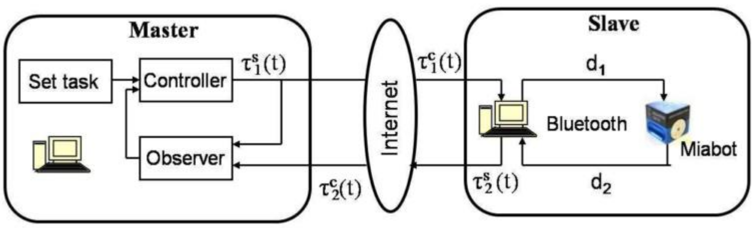

Fig 3.6 Evolution of the LKF values during a switching sequence

The test bench includes a computer playing the role of the controller. It communicates with a second computer through Internet (the two computers where 40km away from each other). This second computer plays the role of a special network node which communicates with the plant (a mobile robot) via Bluetooth. An example of the experimental results is depicted on Figs. 3.7-3.9. It corresponds to the application of the QoS-dependent strategy with buffering. The Fig 3.7 shows the evolution of the estimated state of the plant (longitudinal speed and position of the robot) and the output of the system. Fig 3.8 corresponds to control signal obtained with each control gain and the control signal resulting of the switching strategy. The control delay and the switching signal are depicted Fig 3.9. The last figure (Fig 3.10) shows the response of the system when constant gains are considered. The result obtained with QoS dependent strategy clearly outperforms the non-switching one.

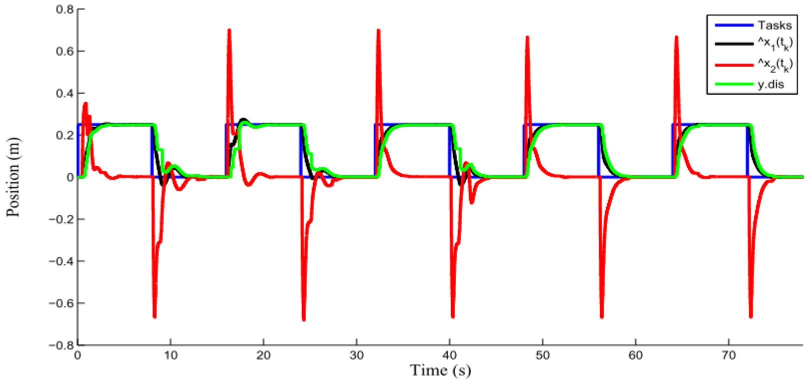

Fig 3.7 Time-response of robot and the remote estimator when the switching strategy is considered

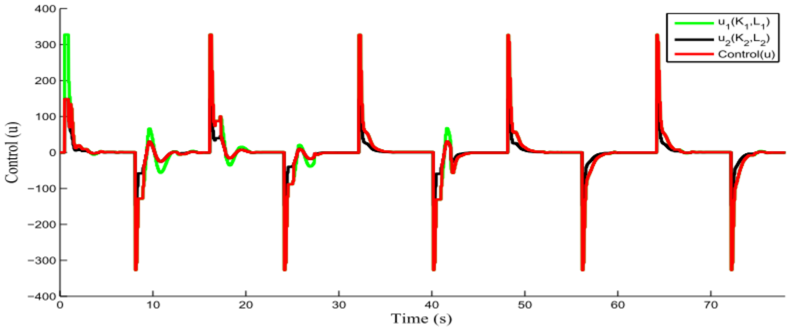

Fig 3.8 The control value of each modes and the resulting value

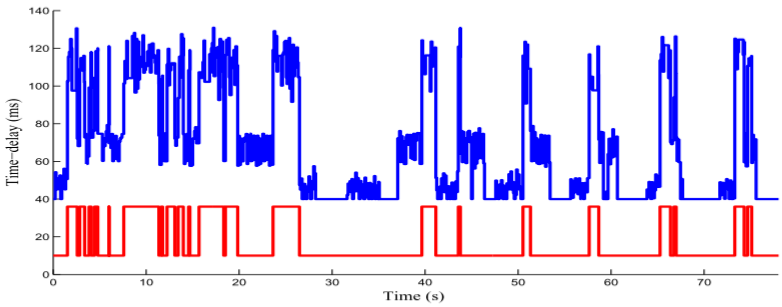

Fig 3.9 The delay and the switching signal

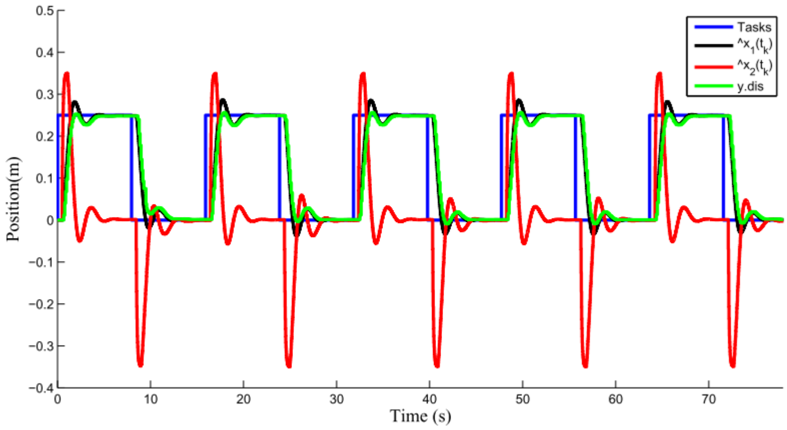

Fig 3.10 The time response of the robot and the estimator in the non-switching case

The experiments showed some interesting problem about clock synchronization. Buffer-based strategies rely on clock synchronization between all elements of the network to compute the delays and the application time of the packets. In the real setup, this synchronization is complex and does not hold for a long period of time. This is due to the disparity between clock rates and a periodic clock resynchronization is needed. Our experiments used NTP (Network Time Protocol) to perform this synchronization.

Conclusion

This section showed how one can control a plant over an unreliable network and take advantage of high QoS when available. These results were interesting from a theoretical point of view as well as from a feasibility point of view but we do not felt that the test bench was realistic enough: why should we implement the controller 40km away from the plant while this later could embed all the computation necessary to its control? Ultimately, handling the communication protocol and the buffers is costlier than computing the control itself for this test bench.

There were still some work to do on this topic: one can try to reduce the conservatism, make the no-buffer strategy gains computation LMI, find better control structures, reduce the consumed bandwidth or deal with the clock synchronization problem.

Bilateral teleoperation through unreliable network

In order to test the credibility of the previously presented NCS design methods, one starts to study a more realistic networked controlled system: a bilateral teleoperation setup (Fig 3.11). It consists in controlling two robots (the master and the slave) separated physically and communicating remotely. The goal is to make them “track” each other. What one calls the master robot is generally a human-manipulated haptic interface, whose goal is to provide the posture to be tracked by the slave robot. It transmits the forces sensed by the slave robot to the human operator. The slave manipulates the environment by tracking the master’s posture. The human operator should sense the efforts applied in the slave as if it was manipulating it directly. Teleoperation is a serious candidate because of the potential applications on the chirurgical field (a surgeon at one side of the earth, operating its robot doing surgery on a patient at the other side). But not only: one can think of the use of a Wi-Fi network to teleoperate a mobile manipulator which are used to move fragile, heavy or dangerous materials through a building.

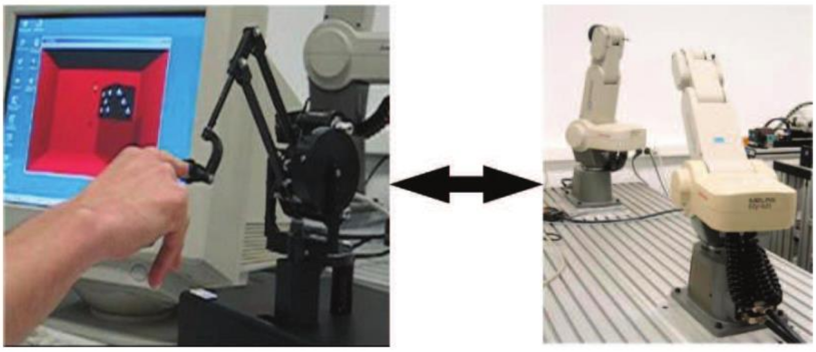

Fig 3.11 Bilateral teleoperation test-bench, CRIStAL

From the control point of view, the teleoperation problem can be seen as a NCS system with two controllers to be designed (Fig. 3.12). The particularity of theses controller is that they can act only their local plant. They share information over the network which are typically the state of their local plant x_m and x_s (vectors composed with generalized position and speed) and the sensed external forces coming from the human operator F_h for the master and from the environment F_e for the slave. Additionally, a visual feedback coming from the slave is needed for the human operator. All this setup must satisfy three goals which are:

- Stability: without interaction with the human operator and environment, the states must converge.
- Synchronization: without interactions, the states must converge to each other (with a scaling ratio) and with interactions, they must be as close as possible.
- Transparency/Telepresence: the control has to provide the human operator with the experience or the impression of acting at the slave's place.

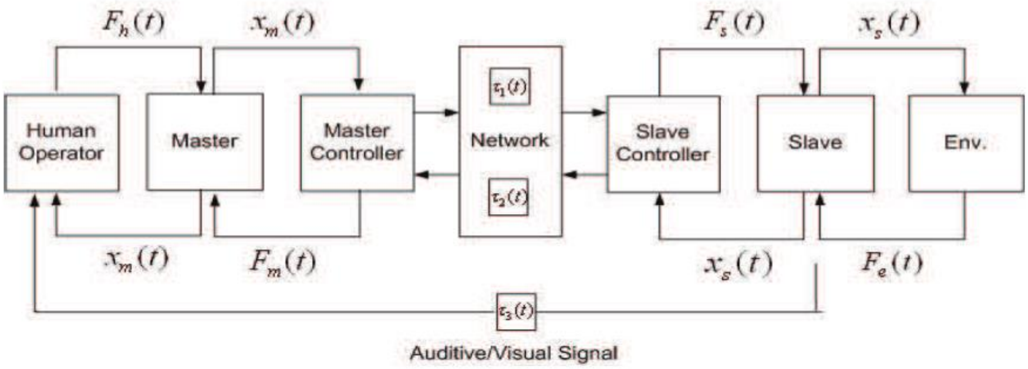

Fig 3.12 Bilateral seen as multiple closed-loops

Since we were not able to find mathematical criteria for these three goals in the literature, we decided to focus on the use H_∞ performance indexes for the posture tracking, as it is suitable when fast varying unknown external inputs are considered. Because we are not experts on the haptic or teleoperation field we were not able to judge if the teleoperation experience was good or not from the transparency point of view. Still, from the control point of view, the problem was interesting enough even if one focus on the two first goals.

I started to co-supervise a thesis (Bo Zhang 2012) on this subject in 2009. At that time, passivity-based techniques seemed to be the trend in the teleoperation field. The main limitation with passivity approaches, as they are used in (Matiakis, Hirche, and Buss 2005; Nuño, Basañez, and Ortega 2011; Ye, Pan, and Gupta 2009; Ye et al. 2011), is that they do not use explicitly the properties of the of network delay. They consider the network as a disturbance which may increase the overall energy of the closed loop. If global energy increases too much then the passivity controller lowers the control signal to reduce it. From our point of view, this design was efficient in the following sense: whatever happens to the closed loop, the controller always stabilizes it, even in the case of badly-modelled or ignored phenomenon. The drawback of this technique is the distortion applied to the input signals which lowers the tracking performances on the slave side and distorts the haptic rendering on the master side.

Other techniques were also available during this period but were not suitable for Internet-based bilateral teleoperation for the following reasons:

- Frequential methods like in (Tian, Yashiro, and Ohnishi 2011; Delgado and Barreiro 2009) need a constant delay (or the implementation of a buffer). Some other results in this field are limited to closed-loop analysis and are not suitable for control design (S. Niculescu, Taoutaou, and Lozano 2003; Taoutaou, Niculescu, and Gu 2004).

- Predictive control (Casavola, Mosca, and Papini 2006; Iqbal and Roth 2006; Sheng and Spong 2004) requires a prediction of the delay which is not suitable for unreliable networks.
- Sliding mode proposed in (Ye, Pan, and Gupta 2009) cannot handle the situation where the slave is in contact with a rigid environment.
- Adaptive methods like in (Leeraphan, Maneewarn, and Laowattana 2002) require constant delays. (Niemeyer and Slotine 1991; Hsu, Costa, and Lizarralde 2007; Nuño, Basañez, and Ortega 2011) also provide delay-independent results that may introduce conservatism.

LKF-based control design of bilateral teleoperation

The thesis (Bo Zhang 2012) proposes robust LKF-based conditions which have some advantages over aforementioned techniques. It can handle time-varying delays, considers the bounds of the delays, does not require each element of the closed loop to be passive, thus allows more energetic closed loop that can reach higher performances. It is also suitable for control design and can handle wall contacts scenarios. The downsides of this approach are the necessity of a state model of the master and the slave, the need of some assumptions on the external disturbances (bounded interaction forces) and knowledge of the bounds of the delays. For instance, with the passivity approaches, one just needs a guarantee of passivity for each element (the control design is energy based instead of model based).

The model used in (Bo Zhang 2012) is relatively simple. It only needs an uncertain linear second-order model per degree of freedom at both side of the network. The main reason is that one considers a linearizing control change of variable (computed torque techniques) which allows the decoupling and the removing/attenuating of the nonlinear effect. The matching errors are considered as parametric uncertainties and are handled via robust control approaches. Since all DoF are decoupled with these techniques, one assumes that each DoF can be controlled separately. The model of the master and the slave robot are:

$$\underbrace{\begin{pmatrix}\ddot{\theta}_m(t)\\ \dot{\theta}_m(t)\end{pmatrix}}_{\dot{x}_m} = \underbrace{\begin{bmatrix}\lambda_m & 0\\ 1 & 0\end{bmatrix}}_{A_m}\underbrace{\begin{pmatrix}\dot{\theta}_m(t)\\ \theta_m(t)\end{pmatrix}}_{x_m} + B_m\big(F_m(t)+F_h(t)\big)$$

$$\underbrace{\begin{pmatrix}\ddot{\theta}_s(t)\\ \dot{\theta}_s(t)\end{pmatrix}}_{\dot{x}_s} = \begin{bmatrix}\lambda_s & 0\\ 1 & 0\end{bmatrix}\underbrace{\begin{pmatrix}\dot{\theta}_s(t)\\ \theta_s(t)\end{pmatrix}}_{x_s} + B_s\big(F_s(t)+F_e(t)\big)$$

where θ_m and θ_s are the generalized position of the considered DoF of the master and the slave robot, respectively. x_m and x_s are the associated state vectors. λ_m and λ_s correspond to the nonzero eigenvalue related to the viscous frictions coefficient. $F_m(t)$ and $F_s(t)$ are the forces applied by the actuators on the master and the slave robot, respectively: They correspond to the control inputs of the problem. $F_h(t)$ is the unknown force applied by the human operator on the master robot and $F_e(t)$ is the force applied on the slave robot resulting of the interaction of the slave on the environment. One assumes that $F_e(t)$ and $F_h(t)$ belong to L_2. The control goal is to make θ_m and θ_s track each other.

The control problem to address is the following: Find the two control laws, one for the master controller C_1 computing the control signal $F_m(t)$ and one for the slave controller C_2 computing the control signal $F_s(t)$ (see Fig 3.12), such that:

- they stabilize the teleoperation system:

$$\begin{pmatrix} \ddot{\theta}_m \\ \ddot{\theta}_s \\ \dot{\theta}_m - \dot{\theta}_s \end{pmatrix} = \underbrace{\begin{bmatrix} \lambda_m & 0 & 0 \\ 0 & \lambda_s & 0 \\ 1 & -1 & 0 \end{bmatrix}}_{A_{ms}} \underbrace{\begin{pmatrix} \dot{\theta}_m \\ \dot{\theta}_s \\ \theta_m - \theta_s \end{pmatrix}}_{x_{ms}} + \underbrace{\begin{bmatrix} B_m & 0 \\ 0 & B_s \\ 0 & 0 \end{bmatrix}}_{B_{ms}} \begin{pmatrix} F_m \\ F_s \end{pmatrix} + \underbrace{\begin{bmatrix} B_m & 0 \\ 0 & B_s \\ 0 & 0 \end{bmatrix}}_{B_{ms}} \underbrace{\begin{pmatrix} F_h \\ F_e \end{pmatrix}}_{w}.$$

- The closed-loop meets the $L_2 \to L_2$ performance requirement:

$$\frac{\|\theta_m - \theta_s\|_2}{\|w\|_2} < \gamma.$$

Two control structures where proposed in (Bo Zhang 2012; B Zhang, Kruszewski, and Richard 2011; Bo Zhang, Kruszewski, and Richard 2014):

- **The Bilateral State Feedback Control Scheme:**
  This proposition is depicted in Fig. 3.13. It considers two actions for the controllers: a local state feedback which only modifies the apparent viscous friction and a delayed state feedback used for the tracking. For the master side, the control law is $C_1: F_m(t) = -K_0^m \dot{\theta}_m(t) - K_1^m x_{ms}(t - \tau_2(t))$. The vector $x_{ms}(t - \tau_2(t))$ is obtained using a time stamped data packet received form the slave and the local state x_m which is delayed with the measured delay on the network $\hat{\tau}_2 \approx \tau_2$. The same structure applied on the slave side leads to $C_2: F_s(t) = -K_0^s \dot{\theta}_s(t) - K_1^s x_{ms}(t - \tau_1(t))$.

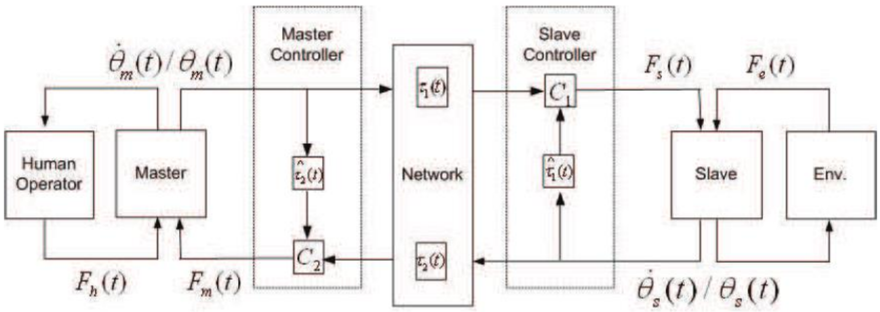

Fig. 3.13 Bilateral state feedback control scheme

The control design is based on the following closed-loop equations:

$$\dot{x}_{ms}(t) = (A_{ms} - B_{ms}K_0)x_{ms}(t) - \begin{bmatrix} B_m \\ 0 \\ 0 \end{bmatrix} K_1^m x_{ms}(t - \tau_1(t)) - \begin{bmatrix} 0 \\ B_s \\ 0 \end{bmatrix} K_1^s x_{ms}(t - \tau_1(t)) + B_{ms}w(t)$$

where $K_0 = \begin{bmatrix} K_0^m & 0 & 0 \\ 0 & K_0^s & 0 \end{bmatrix}$ is given a priori and the pair of matrices (K_1^m, K_1^s) have to be designed. The design procedure proposed in (Bo Zhang, Kruszewski, and Richard 2014; Bo Zhang 2012) is based on LKF-based LMI optimization problem under $L_2 \to L_2$ gain constraints. The $L_2 \to L_2$ gain γ chosen represents the energy transfert from the external input vector $w = (F_h \quad F_e)^T$ to the tracking error $z = \theta_m - \theta_s$. This way, one can fulfill the two first requirements of the bilateral teleoperation problem (stability and synchronization). The third goal (transparency) is not directly addressed which is the main drawback of this method, one just hope that a good synchronization will implicitly create a sufficiently convincing haptic feedback.

- **Force-Reflecting Proxy Control Scheme:**
  This proposition aims at improving the haptic feedback. It consists in applying the sensed force

F_e to the master robot with the smallest distortion possible. To do so, one chooses $F_m(t) = F_e\big(t - \tau_2(t)\big) - K_0^m\dot{\theta}(t)$. On the slave side, one keeps the same structure as in the latter scheme: $F_s(t) = -K_0^s\dot{\theta}_s(t) - K_1^s x_{ms}\big(t - \tau_1(t)\big)$. The problem with this structure was the behavior of the loop which, due to the delays, creates a limit cycle when the slave interacts with the environment. To reduce this effect and improve both the tracking performances and the haptic rendering we proposed the use of a master's state prediction on the slave side (also called proxy or emulator in other communities (Cheong and Niculescu 2008; Li and Constantinescu 2009)). This prediction is used by the slave controller in the following setup:

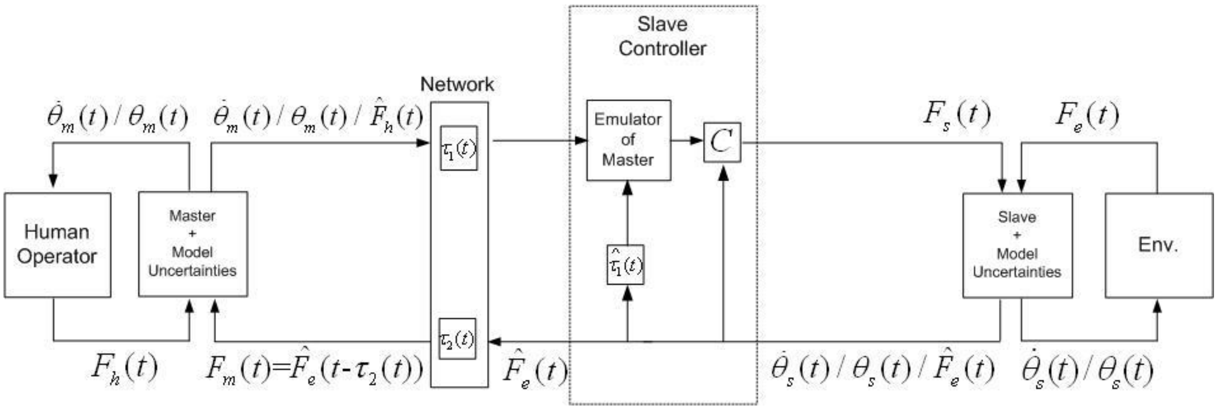

Fig. 3.14 Force-reflecting proxy control scheme.

The predictor equations are based on the master equations:

$$\dot{x}_p(t) = \underbrace{(A_m - B_m K_0)x_p(t) + B_m\big(F_e(t-\tau_1) + F_h(t-\tau_1)\big)}_{Master'\ Model} \quad \underbrace{-B_m F_p(t)}_{Synchronization\ term}$$

$$F_p(t) = L\begin{pmatrix} \dot{\theta}_p\big(t-\tau_1(t)\big) \\ \dot{\theta}_m\big(t-\tau_1(t)\big) \\ \theta_p\big(t-\tau_1(t)\big) - \theta_m\big(t-\tau_1(t)\big) \end{pmatrix}$$

where a virtual force $F_p(t)$ is added to make x_p converging to x_m.

The control design is the performed by following these steps:

- Choose the local gains K_0 preserving the stability of the plants.
- Design L such that the predictor state tracks correctly the master state, *i.e.* minimize the of $L_2 \rightarrow L_2$ gain $\frac{\|\theta_p(t)-\theta_m(t)\|_2}{\|w_{mp}\|_2}$, where w_{mp} is the external unknown input:
$$w_{mp} = \begin{pmatrix} F_e(t-\tau_1) + F_h(t-\tau_1) \\ F_m(t) + F_h(t) \end{pmatrix}$$
- Design the slave control law $F_s(t) = -K\big(\dot{\theta}_s(t) \quad \dot{\theta}_p(t) \quad \theta_s(t) - \theta_p(t)\big)^T$ by minimizing the gain $\frac{\|\theta_s(t)-\theta_p(t)\|_2}{\|w_{sp}\|_2}$, where w_{sp} is the external unknown input:
$$w_{sp} = \begin{pmatrix} F_e(t) \\ F_e\big(t-\tau_1(t)\big) + F_h(t-\tau_1) - F_p(t) \end{pmatrix}$$

The design of L and K are performed with LKF-based LMI conditions under $L_2 \rightarrow L_2$ gain constraints.

Other approaches where also studies like made considering discrete models (B Zhang et al. 2012a) as well as a robustness analysis (B Zhang et al. 2012b) of these setups.

Simulation: comparison

In order to compare the tracking performances of the different results, two cases have been considered:

- **Abrupt Tracking motion:**
  It consists in simulating the teleoperation system in closed loop with no environmental force $F_e = 0$ and F_h a step going from $10N$ to $0N$ at $t = 10s$.
- **Wall contact motion:**
  It consists in simulating the teleoperation system in closed loop with $F_h = 10N$ and $F_e(t)$ simulating a wall at the position $\theta_s = 1$ with a high stiffness contact model, *i.e.:*

$$F_e = \begin{cases} 0 \ if \ \theta_s < 1 \\ 30000\ (1 - \theta_s) \end{cases}$$

The delay used in the simulation is depicted in Fig 3.15 which is a recorded internet delay (France-China), so to compare the various solutions in a same situation.

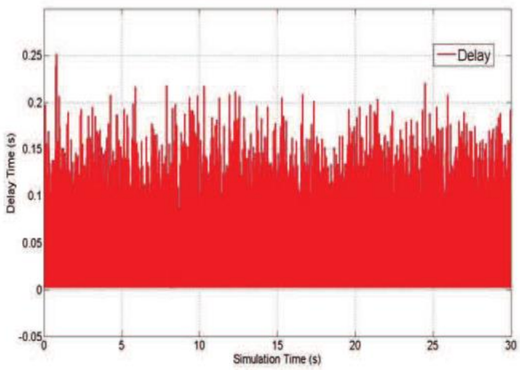

Fig. 3.15 The recorded delay used during the experiments.

These results are depicted in Figs. 3.16 and 3.17 for different controllers: the bilateral state feedback, the force-reflecting control scheme and passivity-based results of (Ye, Pan, and Gupta 2009; Hua and Liu 2010). The passivity-based control shows a drift in the position synchronization due to its lack of position-tracking criterion (lower figures). One can also notice the high responsiveness of the position tracking transient of the force-reflecting control scheme with proxy (upper-right figure).

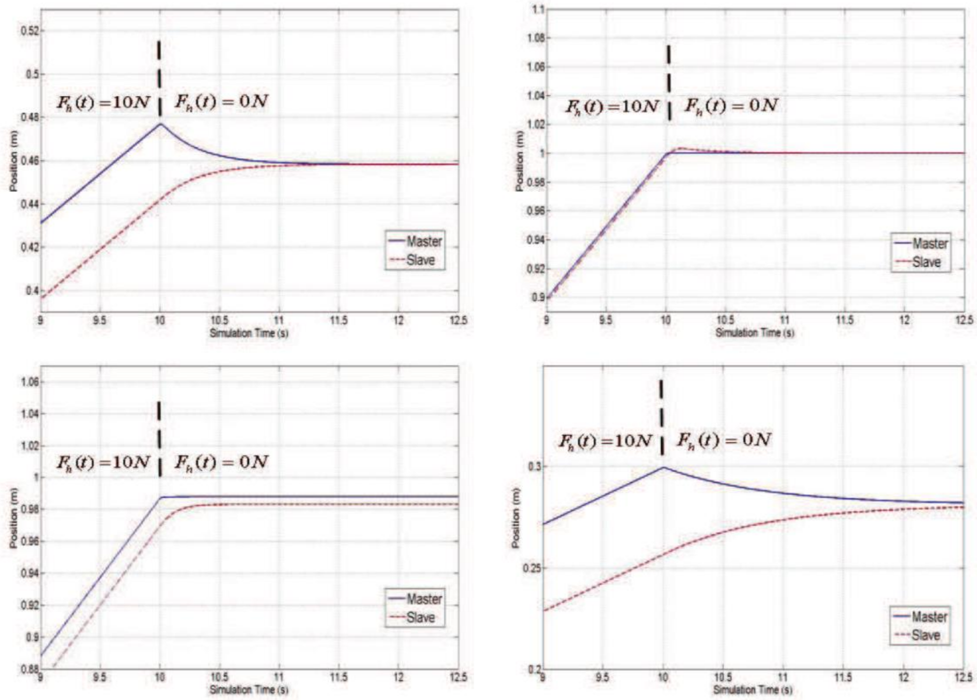

Fig.3.16 Position response in the abrupt tracking motion scenario (Upper left: Bilateral state feedback control scheme; upper right : force-reflecting proxy control scheme; lower left: (Ye, Pan, and Gupta 2009); lower right (Hua and Liu 2010))

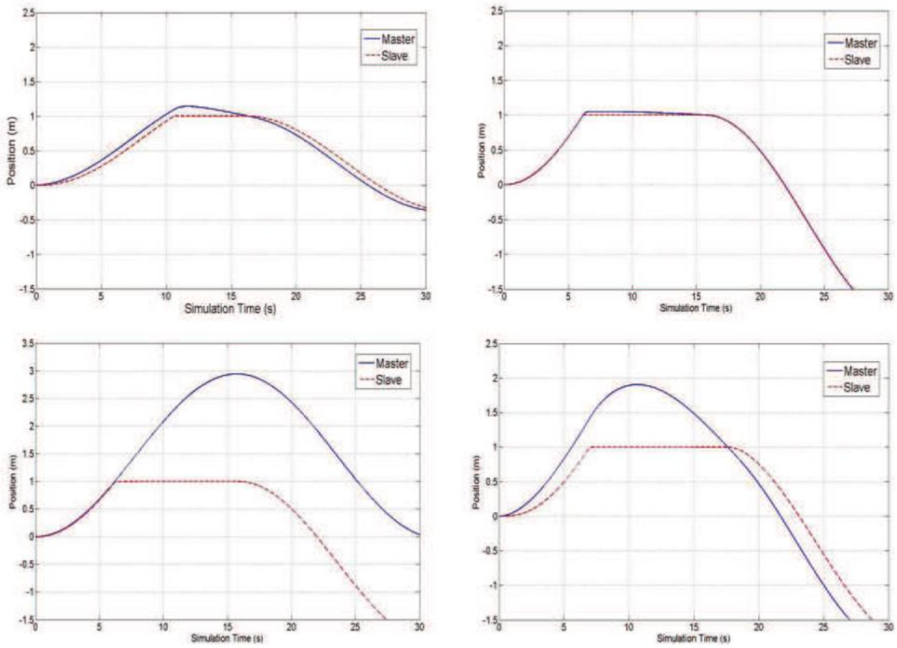

Fig. 3.17 Position response in the wall contact scenario (Upper left: Bilateral state feedback control scheme; upper right : force-reflecting proxy control scheme; lower left: (Ye, Pan, and Gupta 2009); lower right (Hua and Liu 2010))

The discrete case and the continuous methods were also compared in the case of the force reflecting proxy control scheme (see Fig. 3.18). Note that the discrete approaches showed better performance indexes resulting in a better position tracking.

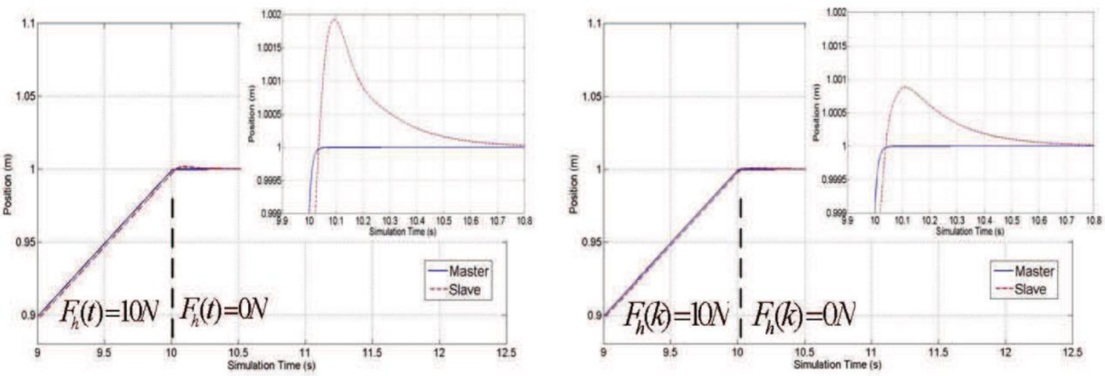

Fig. 3.18 Position response in the abrupt tracking motion scenario (left: continuous case: right: discrete case)

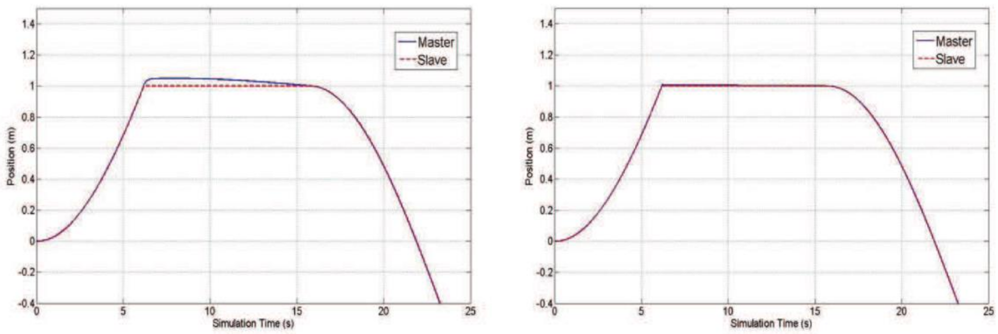

Fig. 3.19 Position response in the wall contact scenario (left: continuous case: right: discrete case)

Other simulation results are available in (Bo Zhang 2012) and related papers but are not provided here for sake of conciseness.

Experimental results

This subsection presents the result obtained on the CRIStAL teleoperation test bench which was founded by the FEDER, the regional council and government, as well as Centrale Lille BQR. The design and the realization of this experimental setup was made by the Centrale Lille support engineering team (thanks to Patrick Gallais, Jacques Lasue, Gilles Marguerite, Hilaire Rossi and Bernard Szukala) and myself. The definitive version of this setup is composed with a Sensable PHANToM 3dof haptic interface and a computer for the master side, a Mitsubishi Move Master robot controlled by a National Instrument Compact Rio embedded system for the slave side. The communication is made via the Ethernet network of Centrale Lille, the nodes of which can be disturbed by generating traffic. An additional computer monitors and records the states, control signals and the delays. The forces are generated by controlling the current of the electrical actuators, the environment force is sensed by a strain gauge (filtered) and the human force applied on the haptic device is estimated using an unknown-input Luenberger's observer.

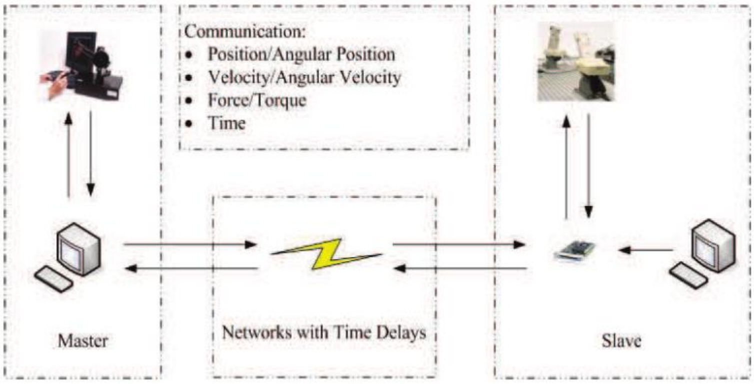

Fig. 3.20 the experimental setup

To increase the repeatability and facilitate the comparisons of the results, the experiments in this section are limited to only one axis of the robots (but the platform allows for 3 axis motion). Fig. 3.21 depicts the results in the free motion results when the master is actuated by a Human operator for the force reflecting scheme. Fig 3.22-3.24 shows the result in wall contact motion case with an emulated constant Human force (constant current injection into the motor) so to make the experiment repeatable. The wall is a steel tube placed at about $\theta_s = -0.2rad$. Three control schemes are tested: the bilateral state feedback, the force reflecting without the emulator and the force-reflecting proxy control scheme. The delay was varying in the $[0.01s\ 0.3s]$ range. The experiments comfort the observation made in simulation and the latter control scheme achieves a better tracking performance than the others.

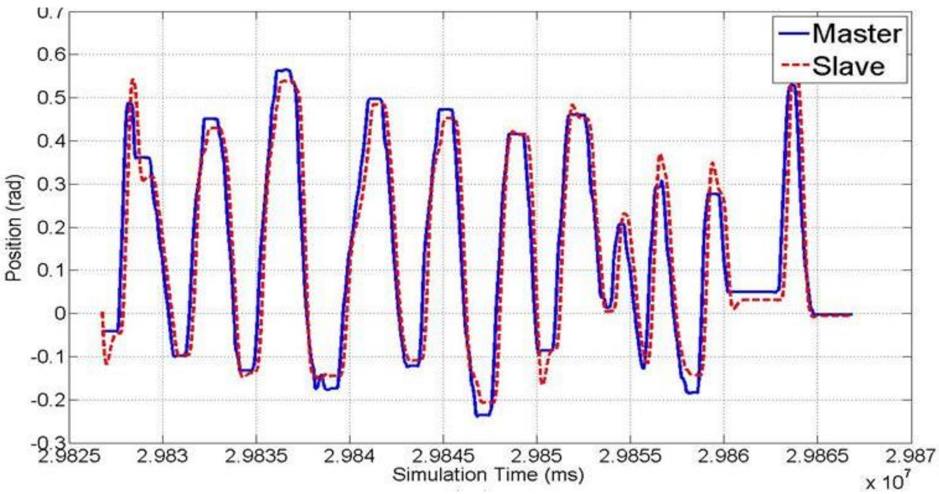

Fig. 3.21 Free motion experimental result with the force reflecting scheme.

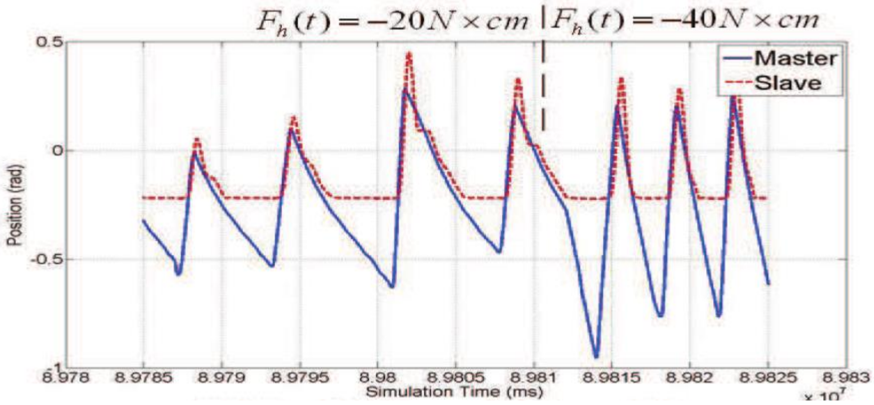

Fig. 3.22 Wall contact experimental result for the bilateral force feedback.

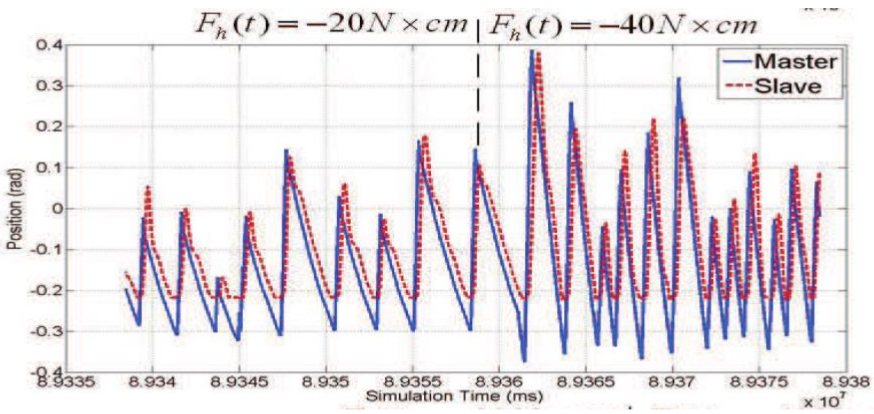

Fig. 3.23 Wall contact experimental result for the force reflecting scheme without the predictor.

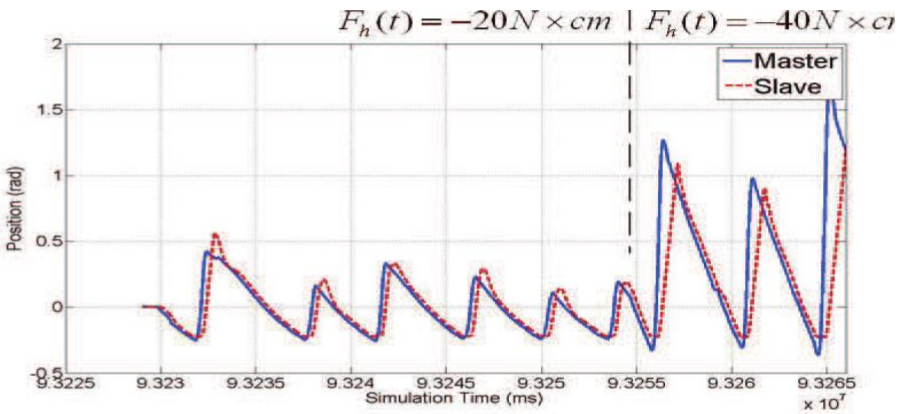

Fig. 3.24 Wall contact experimental result for the force reflecting scheme with the predictor.

Videos of the experiments can be found on the CO2 group YouTube channel:

https://www.youtube.com/watch?v=aVo6j9Rva40

Conclusion

These results have shown the feasibility of designing controllers for bilateral teleoperation over Internet using LKF-based techniques. Beside the straightforward technical improvement (better LKF, QoS dependent gains, …), what would have been the next steps of this work if we had found an industrial partner are the following:

- Consider a less reductive model for the Human operator: It turns out that humans are not simple norm-bounded unknown inputs as they react to the behaviour of the master behaviour. It impacts the stability of the setup which could help or disturb the control loop depending on the quality of the haptic rendering and its delay as well as the quality of the visual feedback.
- Consider a contact model in the control design. On the experiment, we were faced to bouncing effect which may be dampened with the correct control law parameters. Studying the contact stability seems necessary.
- Reduce the impact of bad synchronization of the clocks. As in the NCS section, we were faced to clock synchronization problem that disturbed the experimental setup: clocks were desynchronized by 1s every minute whereas the sampling rate was 0.01s!
- Find a mathematical model defining what is a good haptic rendering. Mimicking the environment force on the Human operator may be not necessary to make the illusion (it may be even worse because of sensor noise and disturbances).

Other application results

Around these two PhD thesis co-supervision, I have some other applicated results that are not related to NCS. They are mainly related to validate some theoretical aspects on a test-bench:

- A result about derivative-based control (same as in chapter 2 section 2) of a permanent magnet synchronous stepper motor (Maalej et al. 2014) in which one was able to tune a performant controller without considering the model of the plant (4^{th} order nonlinear ODE) expect for the sign of the input gain. The coding and the tuning of this controller took less than time we needed to tune a PID under the same specifications (less than 1 hour).
- A result about relay control of a permanent magnet synchronous stepper motor (Delpoux, Hetel, and Kruszewski 2014b; Delpoux, Hetel, and Kruszewski 2014a) in which we control directly the commutation of the power source instead of considering pulse-width modulation (PWM) techniques. The results showed better disturbance responsiveness. An original aspect of these results relies on the fact that the control design is performed considering a simple pole placement technique in a state feedback case, then a relay control is deduced from these gains and the Lyapunov matrices.

I was also in charge of 4 industrial contracts with the “Société Industrielle de Chauffage” of the Atlantic group about temperature control of buildings. I am not allowed to give much details about this work.

Currently, our group is in contact with a power electronics team of the L2EP (Lille) in which we try to design a control law for low storage capacity power sources connected to a power grid. The control objectives are under physical constraint and must provide a safe behaviour in case of brutal grid disconnection or shortcuts.

In 2015, I choose to move to another team of our research group CO2 (https://www.cristal.univ-lille.fr/?rubrique28&id=12) in 2015. This team is interdisciplinary and tackle the problem of simulation, design and control of soft robots (robot with a deformable structure) which will constitute my new application playground.

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IV. Perspectives

Here it is! The last chapter where I will give all my next secret research topics for the coming years. Before unrevealing anything, I would like to detail a new topic that I discovered when the two CNRS laboratories LAGIS and the LIFL labs merged to become the CRIStAL. During the merging process, a new team named DEFROST was created by some LIFL researchers who wanted to invest themselves in an emerging field: soft robotics. The main goal of DEFROST is to develop tools allowing to design, simulate and control deformable robots. They asked me whether I was interested to look at the control part of their project. This new field was the opportunity to get a better feeling of what is really required in term of control design tools on the application side, and to play with innovative and fun applications. Besides trying to find pragmatic solution to these new control problems, it was also the opportunity to see how my favourite theoretical tools will perform in this field.

The first section describes what is a soft robot, defines some of the new control problems linked to this fields and preliminary results obtained during the last two years.

The second and the third sections provide an insight of my future work in the application and theoretical sides and how they converge to the following problematics:

- How to reduce the gap between tools available and tools needed?
- How to design useful and appealing tools for engineers?

The last section is the final conclusion and a partial answer to the question “Are polytopic control design methods suitable for the next robotic control challenges?”.

Soft robotics control challenges

What is a soft robot?

According to (Majidi 2014; Trivedi et al. 2008; Penning et al. 2011), soft robots or deformable robot are the next revolution in robotics. These robots are mainly inspired by the nature and are made of different materials organized in a complex deformable structure. The main advantages of soft robotics are the reduction of the manufacturing cost, the robustness, the efficiency and the security. Because these robots are soft, they can safely interact with their environment and Human beings. These interactions are no more avoided, contrarily to most rigid robotics applications, and moreover become necessary so to reach the robots full potential: it can increase the task space and the configuration space significantly. It opens to new applications possibilities. For example, in surgery (Penning et al. 2011), endoscopic and catheter soft robot can safely use organics tissues of the patient as a support to reach new positions whereas a rigid robot would be limited by its joints and may damage these tissues. Medical applications are not the only field in which these robots can be useful, for example one can think about cobotics (Human-Robot cooperation), fragile object handling, constrained environment exploration and entertainment. However, these robotic devices are challenging to control (Lamnabhi-Lagarrigue et al. 2017; Trivedi et al. 2008) mainly because of the complexity of continuum mechanic equations needed to model their structure as well as the number and the nature of their actuators and sensors.

Among the challenges present in the soft robotics fields, I am interested in their real-time control with and without environment interaction. Later, I may study other fields like trajectory generation and tracking, structural properties, distributed actuators and sensors, smart meso-materials design that facilitate the control implementation, environment properties estimation, and so on.

Control challenges and preliminary solutions

The main problem when dealing with soft robots control comes from the complexity of their models. Today, there are two possible model-based approaches considering continuum mechanics: on one hand, one can exploit the Partial Differential Equations (PDE) directly. On the other hand, on can spatially discretise the PDE using the Finite Element Method (FEM). The difficulty using the first approach lies in the description of complex-shaped robots and the nature of these equations. On the other side, FEM models are ODE (ordinary differential equations) but with a large state vector (more than 5000 components) which complexify the analysis and the control design.

PDE model-based control design were investigated in (Marchese et al. 2014) where the robot is assimilated to multiple-rod segments with a variable curvature (Renda et al. 2014; Marchese, Tedrake, and Rus 2015). Then, a PID controller is used in conjunction with a model inversion in order to control the robot. This strategy is limited to soft robots which are similar to rigid ones in their structure, since one as to identify pseudo-joints to use this modelling strategy.

There are also some alternatives to model-based control approaches: In (Braganza et al. 2007; Li et al. 2012), machine learning approaches are considered (based on neural networks) avoiding the model complexity problem. In (Li et al. 2012), a neural network is trained directly on a real robot: 3000 steps of data under a sampling frequency of 1Hz where required so to train the neural network and make an octopus mobile robot move in the right direction. The application used a limited set of possible actuations (6 control vector values allowed) in order to reduce the learning phase. The numerical burden associated to this control design rises quickly with the number of inputs and outputs (I/O) of the robot, as well as its complexity.

The DEFROST team focuses on FEM-based modelling, simulation and, initially, open-loop control. So, I naturally started to deal with FEM-based control design. This approach as the advantage to be generic as any robot shape can be represented in a systematic manner. The team is also a main contributor to the development of a real-time simulation environment based on FEM models named Sofa (Allard et al. 2007). This software can simulate a soft robot, its control algorithm and its environment (with contacts). It is compatible with existing tools generating FEM mesh from Computer Aided Design software, which smoothen the workflow of designing a robot. The underlying ODE model matrices can also be extracted from this software

Since my integration in DEFROST, I am pursuing two goals related to soft robotics control. *The first goal* is to provide a systematic way of designing control algorithms so to help roboticists in testing the design of their robots. To be useful, these algorithms must provide a control law in reasonable time (for interactivity purpose), allowing the roboticist to iterate their design trials quickly enough. In this case, the control laws provided do not need to be the 'best' ones from the performances point of view, but must at least:

- ensure stability;
- be robust enough with respect to uncertainties and external forces;
- guarantee the steady-state precision (set-point tracking and constant disturbance rejection);
- be suitable for checking the feasibility of the design both in simulation and real experiments.

The second goal is to develop control design methods which optimize the performances of the soft robots. These control laws would be the ones that are implemented in the final design versions of the robot. The design of these controllers has less constraints on its computation time, however it will certainly need to handle more complex phenomena. The rest of this section provide preliminary results for both goals.

Control based on quasi-static models

The first goal (design control algorithms that help testing the robot design) can be reached by designing the control relying on the Inverse Kinematic Model (IKM) of the robot (Fig. 4.2). I have collaborated to this approach, which has been published in (Zhang et al. 2016; Morales-Biez 2017). It is assumed that for each admissible input vector:

- The robot has a unique stable equilibrium point
- There is no disturbance acting on the system.

Under quasi-static assumptions, *i.e.* low speed and the plant state is close enough to the equilibrium point, a model of the variation of the output $\delta y(k+1) = y(k+1) - y(k)$ at the equilibrium can be written:

$$\delta y(k+1) = J\big(x(k)\big)\delta u(k) \tag{4.1}$$

where J is the Jacobian matrix representing the variation of the input vector u according to the variation of the output vector y (*i.e.* linking the actuator space to the effector space). This J matrix is a function of the state x of the robot which can be estimated numerically by synchronizing a simulation of the robot such that converges to the robot measured output. This estimation is used to inverse the kinematic relation and simplify the model with the following change of variable:

$$\delta u(k) = \hat{J}\big(\hat{x}(k)\big)^{-1} v(k) \tag{4.2}$$

Fig. 4.1 depicts open loop experimental results showing the precision of this method on the FESTO CBHA soft robot (Fig. 4.2). The results obtained are acceptable (11% or error) for an open loop considering the fact that the constitutive law of the robot materials is badly known (this robot has significant unmodeled plastic behaviour).

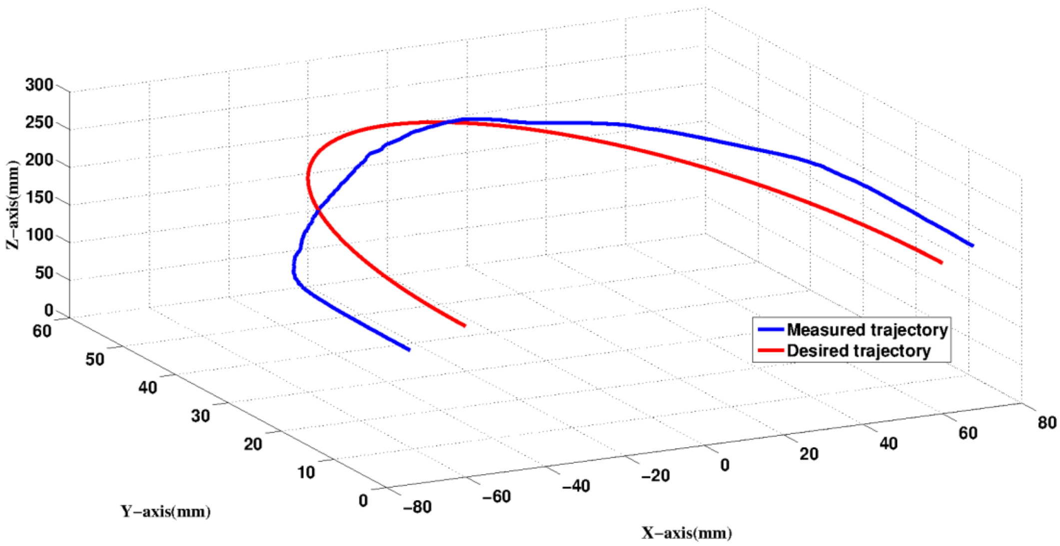

FIG. 4.1 TRACKING ERROR OF THE CBHA ROBOT DURING AN OPEN LOOP INVERSE KINEMATIC EXPERIMENT

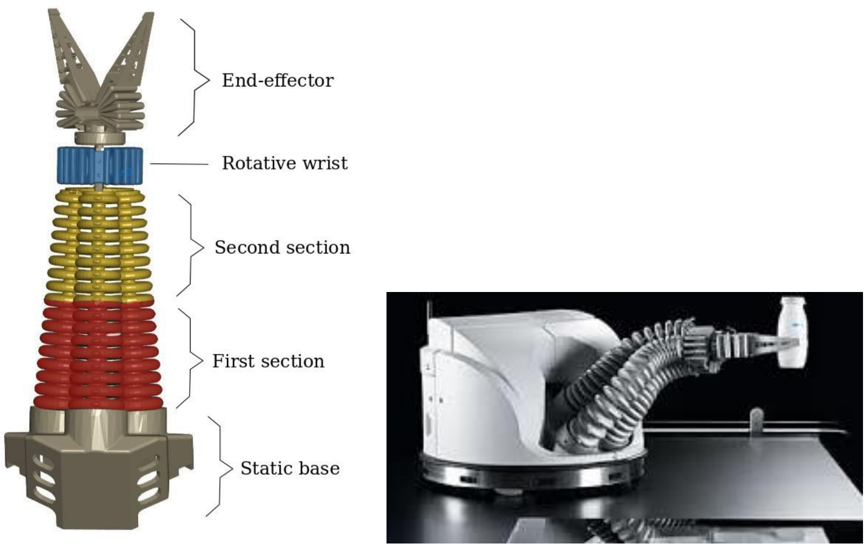

Fig. 4.2 The CBHA robot from the Festo company. Each section is actuated by an inflatable structure. String potentiometers are attached along the structure and measure sections lengths. Only the two first sections (red and yellow) are used during the experiment.

Using the change of variable (4.2) on the model (4.1) gives:

$$\delta y(k+1) = J(x(k))\hat{J}(\hat{x}(k))^{-1}v(k) \tag{4.3}$$

$$y(k+1) = y(k) + J(x(k))\hat{J}(\hat{x}(k))^{-1}v(k) \tag{4.4}$$

If the Jacobian estimation is accurate enough, then $y(k+1) \approx y(k) + v(k)$. A control loop is then designed so to stabilize this model and remove the steady state error using a Proportional-Integrator corrector (Fig. 4.3). The tuning method and the parameter this PI are available in (Morales-Biez 2017).

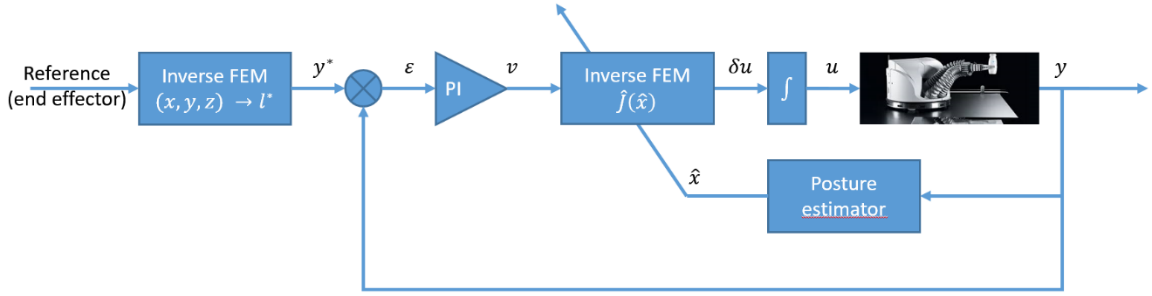

Fig. 4.3 The IKM-based control structure with a PI corrector.

Both experimental and simulation results are depicted in Fig. 4.4. The implementation is realized under the Sofa framework using the Python language with a sampling period of $0.1s$. One can see that, despite a badly known model, this setup is able to control the end effector position of the robot and nullify the steady state error (or, at least, to put them below the sensor resolution).

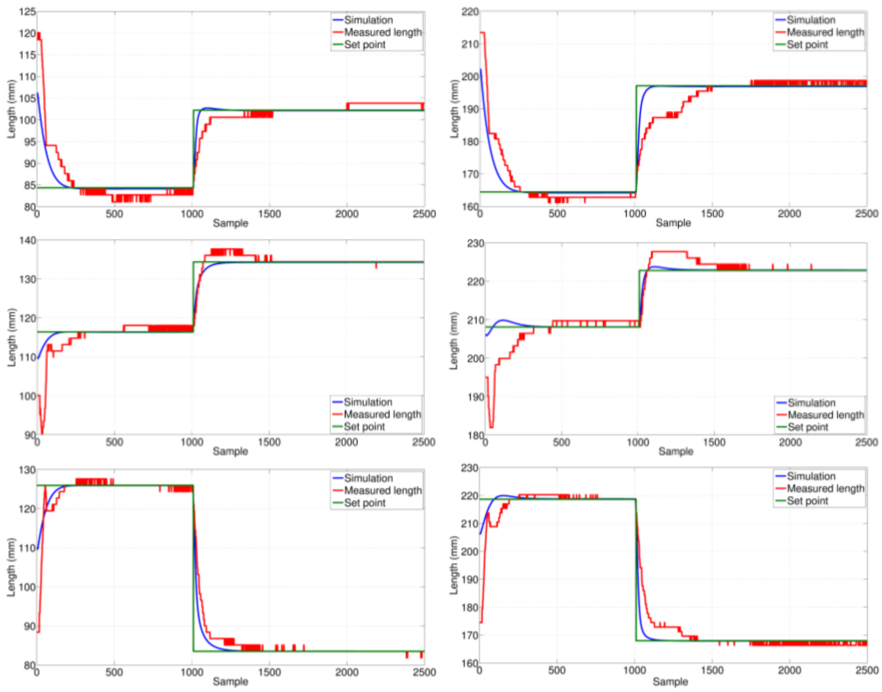

Fig. 4.4 CBHA tracking response (real robot and simulatio) using the IKM-based control structure

A second experiment is performed in order to test the disturbance rejection properties of the control loop (Fig. 4.5). A weight is attached at the effector around $= 1000\ samples$. Despite an erroneous Jacobian estimation coming from the no-disturbance assumption, the control is still able to reject the steady-state error.

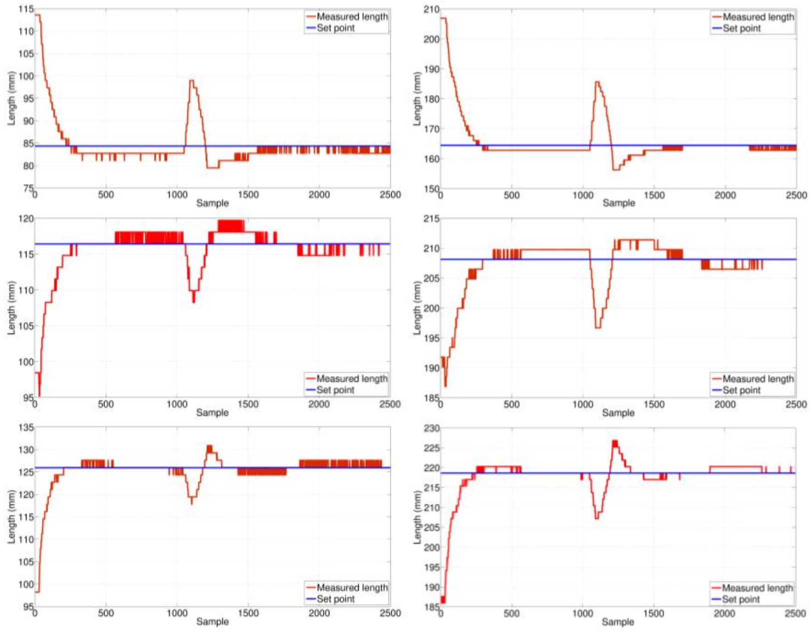

Fig. 4.5 CBHA regulation response (real robot) using the IKM-based control structure. A constant external force (disturbance) is added at time $k \approx 1000\ samples$.

To investigate the robustness of this control law with respect to the IKM error, one considers the following model:

$$y(k+1) = y(k) + v(k) + w(k) \tag{4.5}$$

$$w(k) = \left(J\big(x(k)\big)\hat{J}\big(\hat{x}(k)\big)^{-1} - I\right) v(k) \tag{4.6}$$

Because we assume that there exists a unique equilibrium for each constant control input, $J\big(x(k)\big)$ should never be singular, so there exists a scalar γ such that $\|w(k)\| < \gamma$. Then robust tools can be used in order to evaluate the maximum γ guaranteeing the stability when considering a PI controller. Concerning the PI controller used in the experiment, one can guarantee the stability for $\gamma < 0.98$ using those LMI robustness conditions. Note that when we reduce the knowledge about $w(k)$ to its only bound $\|w(k)\| < \gamma$, it can be proved that the LMI fails as soon as $\gamma \geq 1$. It intuitively corresponds to consider the case where $J\big(x(k)\big)\hat{J}\big(\hat{x}(k)\big)^{-1} < 0$, *i.e.* the Jacobian estimation suggests the opposite of the right control direction. In the perspective of (Morales-Biez 2017), a control design version of these robust control LMI conditions is provided.

The IKM error effect have been analysed, but the approach now lacks a rigorous analysis of the convergence of Jacobian estimator which is necessary for understanding its limits (and provide a guaranteed value of γ). What is promising in this approach is that successful experimental results have been obtained with this structure and this estimator. Moreover, this approach being numerical and generic, it can be performed automatically with few Human interventions and with a tuning computation done in few milliseconds, which clearly fits with my first goal. More investigations are needed so to understand the limits of this approach and remove some of the limiting assumptions like the open loop stability, the unicity of the equilibrium, the low speed framework… A last important point to be investigated is the reduction of the online computation burden of the controller: the Jacobian estimation computation is intensive and limits the control actualization rate.

Control based on dynamic models

To reach the second goal (designing a high-performance controller for a given robot), it seems mandatory to leave quasi-static case and consider the dynamic model of the robot. The complexity of the real-time implementation has also to be considered, as the performances and the robustness of the closed loop will now depend on the quality of the sampling rate. The computation time of the control algorithm must be guaranteed and low enough.

The work presented here concerns the control of the large-scale dynamical model under linearity assumption. Without going into details, one can write around the linearization point for any robot FEM model:

$$\begin{pmatrix} \ddot{q} \\ \dot{q} \end{pmatrix} = \underbrace{\begin{bmatrix} -M^{-1}D & -M^{-1}K \\ I & 0 \end{bmatrix}}_{A} \underbrace{\begin{pmatrix} \dot{q} \\ q \end{pmatrix}}_{x} + \underbrace{\begin{bmatrix} M^{-1}H^T \\ 0 \end{bmatrix}}_{B} u \tag{4.7}$$

where $q \in \mathbb{R}^n$, $n = 3 \times number\ of\ nodes$. Here, x is the vector composed with the positions q and the velocities $\dot{q}$ of the nodes, u is the actuation force. The matrix M is the mass matrix, K is the compliance matrix, D is the damping matrix and H is the control forces direction matrix. M, D and K have notable properties: they are square, sparse and positive definite.

In order to illustrate this, Fig. 4.6 shows a deformable pneumatic actuator, the red part of which can be inflated) (Mosadegh et al. 2014). The FEM model requires 1214 nodes to simulate this simple robot correctly. It leads to 7284 state variables.

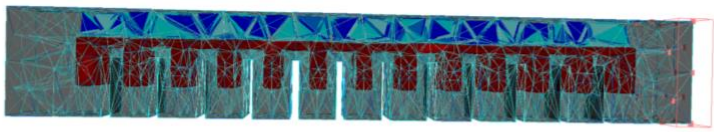

Fig. 4.6 FEM model of the pneumatic soft robot 'PneuNet' modelled with 1214 nodes.

This example shows that even in the linear case, the problem is complex as the dimension of the state obtained by FEM is larger than what most classical control design tools can handle. It is even impossible to check the stability without doubt, because of the potential numerical errors. Multiple tracks are currently investigated by a PhD student under my co-supervision and some preliminary results are available in (Thieffry et al. 2017).

The first track tries to get rid of the numerical computation of a Lyapunov function. It relies on mechanical energy formulation and the specific properties of the matrices involved in the model. If one assumes the open loop stability, then a Lyapunov function exists. This Lyapunov function can be obtained by computing the total mechanical energy of the robot which decreases when no inputs are applied to the system. The expression of this energy is quite direct form the model expression (4.7):

$$V(q, \dot{q}) = \frac{1}{2} \begin{pmatrix} \dot{q} \\ q \end{pmatrix}^T \underbrace{\begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix}}_{P} \begin{pmatrix} \dot{q} \\ q \end{pmatrix} \tag{4.8}$$

The control strategy is designed in such a way that (4.8) decreases faster in closed loop than in open loop by either choosing a control law like $u = -\sigma B^T P x$ or using numerical gradient techniques. This approach is also called damping control. Fig. 4.8 shows a simulation result obtained when this control law is applied to an academic example (Fig. 4.7) composed with a 2D deformable structure on which one can apply 2 forces at specific locations. The chosen output is a point at the top of the robot.

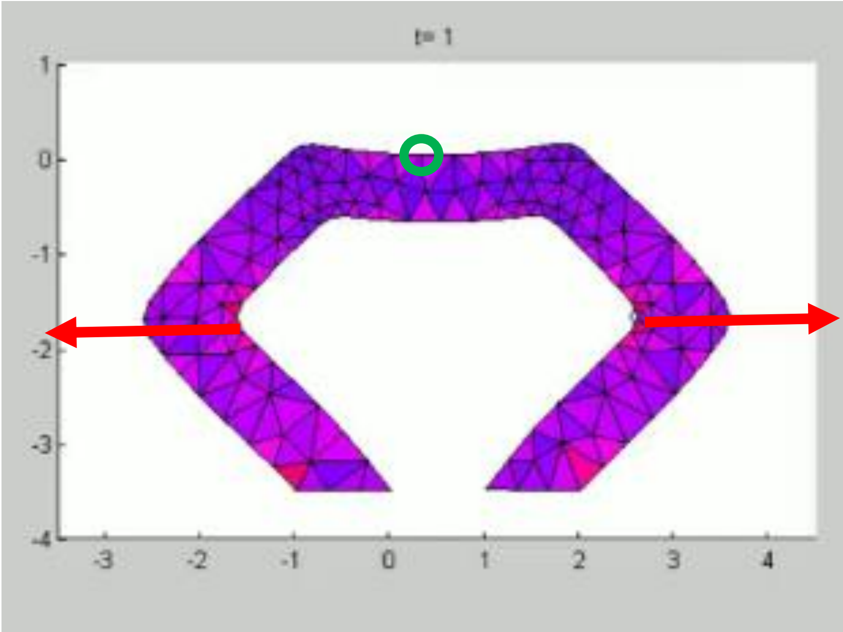

Fig. 4.7 A 2D robot simulation. The red arrows are the control forces and the green circle is the output position.

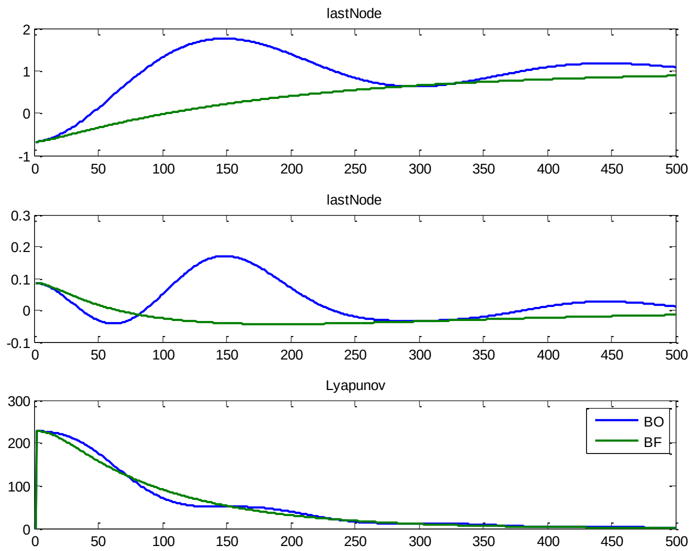

Fig. 4.8 The non-zero initial condition response of the output of the 2D robot in open loop (blue) and in closed loop (green)

The damping control approach helps in reducing the oscillations of the output response but cannot affect the settling time. A major flaw of this technique is that it requires an online estimation of the full state (≈600 state variables in this example). It also requires the open-loop stability of the considered equilibrium.

The second track explores the use of model reduction techniques and classical control design tools. Preliminary results obtained in the PneuNet case are encouraging, as the use of Proper Orthogonal

Decomposition (POD) (Volkwein 2005) allowed a model reduction from order 7284 to 4. An output feedback (state observer with a state feedback) has been applied on this example, the design of which was obtained from the reduced order model. Fig. 4.9 shows the non-zero initial condition response of the PneuNet model in open loop and in closed loop.

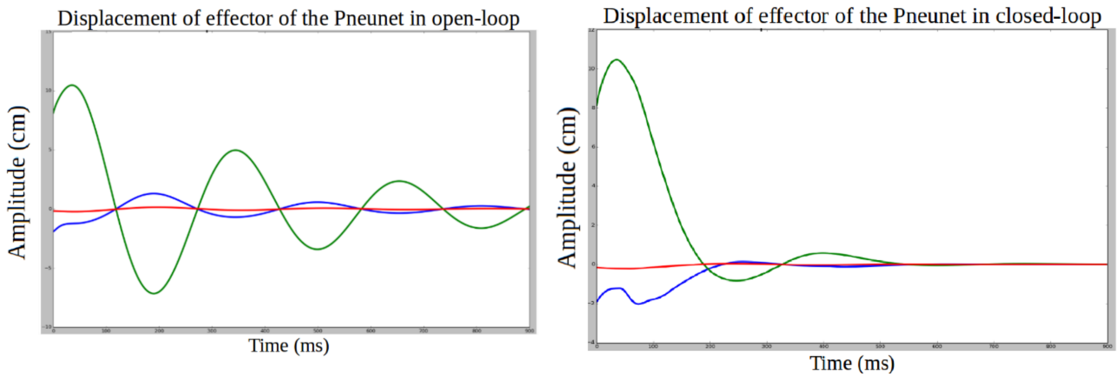

Fig. 4.9 time responses of the tip position of the Pneunet model in open loop (left), and in closed loop (right) to non-zero initial conditions.

The oscillations are damped and the time response is shorter in closed loop. POD may not be the most efficient approach in our case as it requires the generation of snapshots which are be obtained from intensive simulations. The reduced models obtained by POD are also sensitive to the excitation signal chosen when the snapshots were generated. Other techniques are available which are model-based and rely only on the model matrices (Poussot-Vassal and Vuillemin 2012; Antoulas 2010; Gugercin 2008), but are limited to the linear case. However, for the moment, no reduction technique can give guarantees for the success of the control design, *i.e.* one cannot be sure that the control law designed from the reduced order model will perform well on the large-scale model and even less on the real robot.

Application perspectives: Identify relevant control problems and provide efficient design tools.

The soft robot application field is a gold mine for control problems. This chapter only named a few and I intend to continue to explore them. I think that the first short term goal is to design a deformable robot which have a 'purpose' and is not only an academic benchmark. This way, one will be able to state what problem are really relevant and should be addressed with the top priority.

In parallel with this demonstrator preparation, I will investigate the extension the Inverse Kinematic Model approach to the dynamic case, because I think that this is a simple way to control this type of robot without too much design effort. Moreover, in this framework, it seem easy and natural to take into account the constraints on the input vector like saturations, rate limitation and so on…

Another track I will pursue is a bit more general and consists in investigating how one can design a control law robust with respect to the model reduction, *i.e.* extract some information about the model reduction error and use them in the control design. This latter topic is the subject of a new collaboration with the ONERA Laboratory and Universities of Poitier, Strasbourg and Limoges (through a second trial of ANR project proposal). This feature is an important one as, for the moment, the choice of an order of reduction is empirical and identifying the causes of the control loop failure is not easy: Is it because of the order reduction? Is there a suitable control law for this level of reduction? Is it because of unmodeled features? Still, the goal is to provide the roboticist with automated tools that can design a suitable control law to test their designs.

Beside these investigations, some other interesting problems should be studied:

- Shall we consider the nonlinear behaviour of these robots and how?
- How to design a control law that can handle contacts between the environment and the robot?
- How to estimate the properties of the environment?
- Which sensor and actuators are the most suitable to control such robots?
- … There are so many topics that I cannot be exhaustive ...

I also would like to pursue the fresh cooperation with the L2P (power source interaction with a power grid) and the Atlantic company in order to keep an eye on which theoretical tools are really useful to solve practically engineering problems.

Theory side perspectives: Efficiency, formal computation and Toolboxes.

From my experience in teaching, but also in PhD supervising, intern supervising and industrial contracts, I feel that the polytopic techniques presented in the chapter 2 are not the first reflex of engineers despite their similitudes with the linear framework. I am not sure that engineers are aware of their existence and the possibilities they offer. I think these techniques lack appeal because:

- They do not seem to be taught in graduate courses. Teachers often focus on the way they learned automatic control and thus teach techniques in chronological order.
- It requires to manipulate the nonlinear model which may be frightening at first (look at the face of a student while you are speaking about nonlinear techniques in general…).
- It requires to go in depth into the mathematics of the model (choose the membership functions, get the bounds required by the conditions one wants to use….). A lot of choices must be decided (membership function, Lyapunov function, control law, region of validity, performances indices…), making each numerical resolution failure frustrating and time consuming. It is not easy to identify if it failed because of one of these choices, because of a mistake in the implementation or simply because the problem has no solution.
- Performances indices in the Lyapunov framework are not the same as what is taught at school and are related to formulation of the plant model (choice of state and output variables). However, I think that Lyapunov framework performances indices are closer to the plant physics (energy, physical variable bounds) than the traditional one (poles and frequency domain).
- No toolbox that covers the essential possibilities offered by the LMI formulation is available. This enforces engineers to recode the linear matrix inequality problems in their solver which is tedious and a non-neglectable source of error, inefficiency and numerical problems.
- It is not sure to find the paper which matches exactly with the problem to solve. Often, the conditions have to be rewritten according to the model features (delay, uncertainties, …) which is possible only if the engineer has a full awareness of what is going on under the hood. I am convinced that engineers prefer to know what things do instead of the technical details of how things are done.

On another side, from the rewriting of the nonlinear model to the last step in which one gets the finite set of LMI, one has to follow a given set of rules. Modifying a given set of condition in order to take into account additional model feature (ex: an uncertainty, a delay) is often as simple as reapplying the same rules again. As shown in the second chapter, there are strong interaction between each step of the method and the choices available at each step lead to a large number of possible LMI conditions sets. Because following these steps is tedious and time consuming, one tries to guess what would be the best results in term of conservatism and focus on that track. However, the conservatism introduction of these choices may be mitigated if one was able to test systematically all known possibilities or at least many of them by automating their generation. One would not limit ourselves to test a single LMI condition and try a complete range of them for a given problem. It is even more relevant to automate this part of the design process as the time needed to solve numerically the problem is neglectable compared to the rest of the process. Finally, it may also change the way researchers try to improve these techniques as the proof of strict inclusion of a given approach wouldn't be mandatory anymore.

This LMI condition automatic generation, combined with the new trends in multi-physics modelling software and languages like Modelica[6], Maplesim[7] or Simscape[8] (non-exhaustive list) may become a powerful framework. These new software frameworks help the engineers to describes complex from the physics point of view and not from the mathematical point of view. They allow the simulation of complex model and to iteratively complexify them: adding a new physical phenomenon in your model does not require to completely rewrite the mathematical expression as it would be the case in the 'Simulink-like' solutions. These softwares also allow for the extraction of the linearized LTI model, the parameter-scheduled LPV models or the exact nonlinear differential equations.

That to say that a solid and turn-key toolbox/software (maybe based on symbolic manipulation) seems mandatory to convince engineers and to enlarge the basis of users of these techniques. If one looks to the success of gain-scheduling techniques (LPV assuming slow varying parameters, frequency based techniques, H_∞…), I really believe that it is mainly due to the existence of Matlab toolboxes (and other softwares) removing all the burden of manual programming, *i.e.* from a simple block diagram simulation, the toolbox extracts the LPV model and designs a control law trying that satisfies the requirement. The entire workflow has to be focused around what is a solution of a specific problem instead of focusing on how to find a solution.

Finally, despite these evolutions, when looking back to the robotics application side, these techniques might not be suitable to design a control law in the case a 6 DoF deformable robot for the following reasons: assuming that model-reduction techniques allow for a drastically reduced order, modelling the potential oscillations of a 6 DoF deformable robot leads to at least 12 state variables. If one wants to remove the steady state error, 6 extra states coming from the error integrators have to be added. In the most simplified case, the model finally has at least 18 state variables. Considering the size of this control problem, the LMI solver will likely either return inconsistent results due to numerical error, or report failure due to lack of memory, or take too long time. What I would like to investigate is a way to enlarge the possibilities of these LMI-based techniques to fill a bit the gap between what the theoretical tools can provide and what is really needed for this specific application. I will explore how to make polytopic tools scale with the complexity of these control problems instead of trying to reduce the conservatism at all cost. In other words, the next track of my theoretical research will be fed with new problems coming from soft robotics.

Conclusion

In this manuscript, I hope I have shown that my research activities in control design are mainly gravitating around the use of the Lyapunov's direct method and Linear Matrix Inequalities. The theoretical side of my research is dealing with the conservatism reduction of these techniques when they are based on polytopic models. I co-supervised two PhD thesis on the subject. In the framework of these control design tools, I have been able to identify three current drawbacks, which are:

[6] https://www.modelica.org/
[7] https://www.maplesoft.com/products/maplesim/index.aspx?L=E
[8] https://www.mathworks.com/products/simscape.html

1) The newest improvements coming after these works tend to be numerically less and less efficient, *i.e.* they are more and more complex for a small improvement (in term of size of solution sets). The results tends be less and less scalable.
2) They are not enough appealing toolboxes that remove the burden of redeveloping and coding the LMI conditions for a specific control setup.
3) Engineers do not have enough information about the existence of these efficient techniques.

The application aspect of my research started with the Networked Control System topic (2 PhD co-supervised) and moved to soft-robotics (2 PhD ongoing). This latter application is a fresh source of concrete control problem which helps me in focussing on the essential needs of engineers as well researchers coming from various fields different from control science (*i.e.* real-time computing and simulation, mechanics of soft materials, computer science, robotics…)..

The preliminary analysis of the soft-robot control topic has showed that the answer to the question: "Are polytopic control design methods suitable for the next robotic control challenges?" is "Yes... and no".

- They are suitable in the sense that they can deal with a large variety of model features like nonlinearities, delays, switches…
- But today, even the simplest stability conditions are not numerically efficient enough to deal with problems with such a high dimension (drawback n°1). Moreover, I don't think one can convince the roboticist to use them because of the drawbacks n°2 and n°3.

Today, I am really interested in filling the gap between the real needs of this field and the existing theoretical tools. This will start by addressing the challenge corresponding to drawback n°1 and try to improve the LMI based techniques in handling higher dimension problems, trying to mix them with model reduction techniques while preserving the stability and the performances on the high dimensional problem. I also want to contribute in spreading the use of control design techniques based on polytopic models, LMI and Lyapunov's methods. I believe that this combination is a good trade-off between ease-of-use and quality of the solutions but that they would be far more efficient (in term of development time) if the unnecessarily and repetitive tasks could be automated (drawback n°2). Also, I think that this spreading can start only if the next generation of engineers knows the existence of these control design techniques and during the next year I will try to incorporate them as early as possible in the engineer education I am invested in (drawback n°3).

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