Towards Rapid and Precise Parallel Kinematic Machines
Samah Aref Shayya

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Vers des Robots et Machines Parallèles Rapides et Précis
(Towards Rapid and Precise Parallel Kinematic Machines)
Foreword

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Samah Aref SHAYYA
Abstract

Parallel manipulators (PMs) have been there for more than half a century and they have been subject of intensive research. In comparison with their serial counterparts, PMs consist of several kinematic chains that connect the fixed base to the moving platform. The interest in such architectures is due to the several advantages they offer, among which we mention: high rigidity and payload-to-weight ratio, elevated dynamical capabilities due to reduced moving masses (especially when the actuators are at or near the base), better precision, higher proper frequencies, etc. Nevertheless, despite of the aforementioned merits, their exploitation as machine tools is still timid and limited, in which they most often do not exceed the research and prototyping stages at university laboratories and machine tool manufacturers. The main drawbacks that hinder the widespread of parallel kinematic machines (PKMs) are the following: limited operational workspace and tilting capacity, presence of singular configurations, design complexities, calibration difficulties, collision-related problems, sophistication of control (especially in the case of actuation redundancy), etc. Besides, though PMs have met a great success in pick-and-place applications, thanks to their rapidity (acceleration capacity), still their precision is less than what has been initially anticipated. On the other hand, extremely precise PMs exist, but unfortunately with poor dynamic performance. Starting from the aforementioned problematicas, the current thesis focuses on obtaining PKMs with a good compromise between rapidity and precision. We begin by providing a survey of the available literature regarding PKMs and the major advancements in this field, while emphasizing the shortcomings on the level of design as well as performance. Moreover, an overview on the state of the art regarding performance evaluation is presented, and the inadequacies of classical measures, when dealing with redundancy and heterogeneity predicaments, are highlighted. In fact, if finding the proper architectures is one of the prominent issues hindering PKMs’ widespread, the performance evaluation and the criteria upon which these PKMs are dimensionally synthesized are of an equal importance. Therefore, novel performance indices are proposed to assess precision, kinetostatic, and dynamic capabilities of general manipulators, while overcoming the aforementioned dilemmas. Subsequently, several novel architectures with 3T-2R and 3T-1R degrees of freedom (T and R signify translational and rotational degrees of freedom), namely MachLin5, ARROW V1, and ARROW V2 with its mutated versions ARROW V2 M1/M2, are presented. Furthermore, the dimensional synthesis of the executed PKM, namely ARROW V2 M2, is discussed with its preliminary performances and possible future enhancements, particularly regarding precision amelioration.

Key words:
Parallel kinematic machines (PKMs), rapidity, precision, large operational workspace, large tilting capacity, actuation redundancy, kinematic redundancy, performance indices, 3T-2R and 3T-1R PKMs, MachLin5 PKM, ARROW V1 PKM, ARROW V2 PKM, ARROW V2 M1/M2 PKMs
Résumé

Les machines parallèles (MPs) existent depuis plus d'un demi-siècle et ils ont fait l'objet d'études intensives. Par opposition avec leurs homologues de structure série, ces mécanismes sont constitués de plusieurs chaînes cinématiques qui relient la base fixe à la plateforme mobile. L'intérêt de ces architectures s'explique par les nombreux avantages qu'elles offrent, parmi lesquels: une rigidité élevée, un rapport important charge/poids global, des capacités dynamiques élevées en raison des masses en mouvement réduites (en particulier lorsque les actionneurs sont sur ou près de la base), une meilleure précision, des fréquences propres plus élevées, etc. Néanmoins, leur exploitation comme machines-outils reste timide et limitée, et le plus souvent elles ne dépassent pas le stade d'étude et de prototype de laboratoires universitaires ou de fabricants de machines-outils. Les principaux inconvénients qui entravent la généralisation des MPs dans l'industrie sont les suivants: un espace de travail limité, des débattements angulaires réduits, la présence de configurations singulières, la complexité de conception, les difficultés d'étalonnage, les problèmes causés par les collisions, la complexité du contrôle/commande (en particulier dans le cas de redondance à actionnement), etc. De plus, si les MPs ont rencontré un grand succès dans les applications de pick-and-place grâce à leur rapidité (capacité d'accélération), leur précision reste inférieure à ce qui a été prévu initialement. Par ailleurs, on trouve également des MPs de très précision, mais malheureusement avec de faibles performances dynamiques. En partant du constat précédant, cette thèse se concentre sur l'obtention de MPs avec un bon compromis entre rapidité et précision. Nous commençons par donner un aperçu de la bibliographie disponible concernant MPs et les avancées majeures dans ce domaine, tout en soulignant les limites de performance des MPs, ainsi que les limites des outils de conception classique. En outre, nous insistons sur les outils d'évaluation des performances, et montrons leurs limites dès qu'il s'agit de traiter le cas de la redondance ou l'hétérogénéité des degrés de liberté (ddl). En effet, si la synthèse architecturale est un point dur de la conception de MPs, la synthèse dimensionnelle reposant sur des indices de performances réellement significatifs l’est également. Par conséquent, de nouveaux indices de performance sont proposés pour évaluer la précision, les capacités cinéstato-spatiales et dynamiques des manipulateurs de manière générale qui apportent des solutions aux difficultés évoquées ci-dessus. Par la suite, plusieurs nouvelles architectures 3T2R et 3T-1R (T: signifie ddl en translation et R signifie un ddl de rotation) sont présentées, à savoir MachLin5, ARROW V1, et ARROW V2 et ses versions dérivées ARROW V2 M1 et M2. En outre, la synthèse dimensionnelle d’ARROW V2 M2 est réalisée, et les performances de la machine sont évaluées. Finalement, des améliorations futures concernant la précision sont proposées au regard de premiers résultats obtenus sur le prototype.

Mots clés:
Machines parallèles (MPs), rapidité, précision, grand espace de travail opérationnel, grande capacité d'inclinaison, redondance à actionnement, redondance cinématique, indices de performance, 3T-2R et 3T-1R MPs, MachLin5 MP, ARROW V1 MP, ARROW V2 MP, ARROW V2 M1/M2 MPs
# Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Introduction</td>
<td>xvi</td>
</tr>
<tr>
<td>Context</td>
<td>xvi</td>
</tr>
<tr>
<td>Motivations</td>
<td>xvii</td>
</tr>
<tr>
<td>Contributions</td>
<td>xviii</td>
</tr>
<tr>
<td>Notations</td>
<td>xix</td>
</tr>
<tr>
<td>Chapter 1: State of the Art</td>
<td></td>
</tr>
<tr>
<td>1.1- Generalities and Definitions</td>
<td>2</td>
</tr>
<tr>
<td>1.1.1- A Brief History on the First Parallel Robots and Parallel Kinematic Machine Tools</td>
<td>2</td>
</tr>
<tr>
<td>1.1.2- Serial Robots</td>
<td>4</td>
</tr>
<tr>
<td>1.1.3- Parallel Robots</td>
<td>5</td>
</tr>
<tr>
<td>1.2- State of the Art of PMs: Between Theoretical Synthesis and Industrial Implementation</td>
<td>10</td>
</tr>
<tr>
<td>1.2.1- Type Synthesis of Parallel Mechanisms</td>
<td>10</td>
</tr>
<tr>
<td>1.2.2- Some Parallel Robots and Parallel Kinematic Machine Tools</td>
<td>17</td>
</tr>
<tr>
<td>1.3- Precision-Related Performances of Some Prototypes</td>
<td>37</td>
</tr>
<tr>
<td>1.4- Performance Criteria and Parallel Kinematic Machine Tools</td>
<td>39</td>
</tr>
<tr>
<td>1.5- Thesis Outline</td>
<td>44</td>
</tr>
<tr>
<td>Chapter 2: Performance Evaluation of General Manipulators</td>
<td></td>
</tr>
<tr>
<td>2.1- Introduction</td>
<td>47</td>
</tr>
<tr>
<td>2.1.1- Kinematic Redundancy (KR)</td>
<td>47</td>
</tr>
<tr>
<td>2.1.2- Actuation Redundancy (AR)</td>
<td>48</td>
</tr>
<tr>
<td>2.1.3- Mixed-Redundancy Manipulators (MRMs)</td>
<td>49</td>
</tr>
<tr>
<td>2.2- General Rigid Manipulators</td>
<td>50</td>
</tr>
<tr>
<td>2.2.1- Generalities on the Kinetostatics of Redundant and Non-Redundant Manipulators</td>
<td>50</td>
</tr>
<tr>
<td>2.2.2- Precision-Related Performance Measures</td>
<td>52</td>
</tr>
<tr>
<td>2.2.3- Kinetostatic Performance Evaluation</td>
<td>58</td>
</tr>
<tr>
<td>2.2.4- Dynamic Performance Evaluation: A Novel Approach Based on Multi-Assessment of Isotropic Dynamic Capabilities</td>
<td>69</td>
</tr>
<tr>
<td>2.3- Cable-Driven Parallel Robots (CDPRs)</td>
<td>74</td>
</tr>
</tbody>
</table>
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

3.1- Introduction ................................................................................................................. 92

3.2- MachLin5: A 5-DoF (3T-2R) Parallel Mechanism with Articulated Platform ............... 93
   3.2.1- Description .............................................................................................................. 93
   3.2.2- Inverse Geometric Model (IGM) ........................................................................... 96
   3.2.3- Direct Geometric Model (DGM) ........................................................................... 97
   3.2.4- Kinematic Model .................................................................................................... 98
   3.2.5- Singularity Analysis .............................................................................................. 100
   3.2.6- Workspace and Kinetostatic Performance ............................................................. 102
   3.2.7- Synopsis ................................................................................................................ 105

3.3- ARROW V1: A Redundantly Actuated Four-DoF (3T-1R) Parallel Manipulator .......... 106
   3.3.1- Description .............................................................................................................. 106
   3.3.2- Inverse Geometric Model (IGM) ........................................................................... 108
   3.3.3- Direct Geometric Model (DGM) ........................................................................... 108
   3.3.4- Kinematic Model .................................................................................................... 110
   3.3.5- Singularity Analysis .............................................................................................. 111
   3.3.6- Simplified Dynamic Model (SDM) ........................................................................ 113
   3.3.7- Dimensional Synthesis Based on Dual Criteria: Precision and Dynamics ............ 115
   3.3.8- Synopsis ................................................................................................................ 123

3.4- ARROW V2: From Theoretical Concept to Prototype Execution ............................... 124
   3.4.1- ARROW V2: An Enhanced Version of ARROW V1 ............................................. 124
   3.4.2- Manufacturability Study of ARROW V2 and Its Mutated Versions ....................... 131
   3.4.3- ARROW V2 M2: The Implemented PKM ............................................................... 133

3.5- Conclusion ..................................................................................................................... 154
Chapter 4: ARROW PKM: Preliminary Performances, Geometric Sensitivity, and Possible Error Compensation ................................................................. 155

4.1- Introduction .................................................................................................................. 156
4.2- ARROW V2 M2 Prototype: The Basic Control Strategy and Treatment of Redundancy 156
4.3- Preliminary Precision Evaluation of ARROW V2 M2 PKM ........................................ 158
4.4- Geometric Calibration and Methodology of Error Compensation .......................... 161
  4.4.1- Geometric Sensitivity of ARROW V2 M2 ............................................................. 162
  4.4.2- Compensation of Geometric Errors of ARROW PKM ........................................ 169
  4.4.3- General Remarks on Geometric Calibration ..................................................... 172
4.5- Compensation of Elastic Deformation ..................................................................... 172
4.6- Conclusion ................................................................................................................... 174

General Conclusions and Perspectives ............................................................................. 175

  Possible Advancements Regarding the Suggested Performance Measures ................. 175
  Possible Improvements Regarding ARROW Machine .................................................. 176

Bibliography ....................................................................................................................... 178

Appendices .......................................................................................................................... 193

Appendix A: Precision-Related Performances of Some Rapid Industrial Robots and Prototypes at LIRMM ..................................................................... 193
  A.1- Generalities and Definitions .................................................................................... 193
  A.2- Briefing on the Procedure and Main Results ......................................................... 196

Appendix B: Derivation of Lower Bounds for the Dynamic Specific Isotropic Values in the Case of Rigid Manipulators ......................................................... 207
  B.1- The Derivation Procedure ....................................................................................... 207
  B.2- Optimization Using Lagrange Multipliers ............................................................. 217

B.3-Case Study: DUAL V ................................................................................................. 220

Appendix C: The Establishment of the Operational Wrench Zonotope of Cable-Driven Parallel Robots (CDPRS) in Statics ................................................. 224

Appendix D: Dynamic Performance Evaluation of CDPRs ............................................ 227
  D.1-Formulation of the Dynamic Model for CDPRs ...................................................... 227
  D.2- Case Study: A Fully-Constrained CDPR .............................................................. 228

Appendix E: Photos of ARROW Machine ....................................................................... 232

Publications ......................................................................................................................... 234
List of Figures

Fig. 1-1: Original Gough platform and shortly before its transfer into the British National Museum of Science and Industry in 2000 (Dunlop Tyres). ................................................................. 2
Fig. 1-2: FlexPicker (ABB Robotics) industrial robot: photo and graph diagram.......................... 3
Fig. 1-3: Variax parallel kinematics machine tool (Giddings & Lewis).......................................... 3
Fig. 1-4: Robot IRB 7600-150 (ABB Robotics): photo and graph diagram................................. 5
Fig. 1-5: Hexamove system (OHE Hagenbuch AG): photo and graph diagram........................... 5
Fig. 1-6: Singularities: series singularity (or RI singularity) (left), parallel singularity (or RO singularity) (middle), and internal singularity (or RPM singularity) (right). ....................... 10
Fig. 1-7: Two fully isotopic translational PMs: the left is one of the T3 family (it is more precisely maximally regular), whereas on the right is a type II of T4 family (CARRICATO & PARENTI-CASTELLI, 2002). ............................................................ 12
Fig. 1-8: The isotropic 3-CRR translational PM (KONG & GOSSELIN, 2002). ...................... 12
Fig. 1-9: Schematics of the Pantopteron (fully isotropic translational PM) (BRIOT & BONEV, 2009) .................................................................................................................. 13
Fig. 1-10: Pantopteron with four-dof (3T-1R) uncoupled PM (BRIOT & BONEV, 2009). ........ 13
Fig. 1-11: Models of two fully isotropic mechanisms for Schoenflies motion (CARRICATO, 2005) ....................................................................................................................... 14
Fig. 1-12: Example of basic kinematics structure of PM with decoupled Schönflies motions: Isoglide4-T3R1-A5 schematic and its constructed prototype at the French Institute of Advanced Mechanics (GOGU, 2007). ................................................................. 14
Fig. 1-13: Uncoupled (PMs) with Schönflies motion: Isoglide4-T3R1-B5 (GOGU, 2007). Note that this (PM) is similar to Isoglide4-T3R1-A5 in Fig. 1-12, except that 2c ≡1D. ................. 15
Fig. 1-14: Example of kinematic structure of fully isotropic PM with Schönflies motions: Isoglide4-T3R1-C5-2 (GOGU, 2007). .................................................................................. 15
Fig. 1-15: From left to right and top to bottom: Isoglide5-T3R2-A1 (S-PPPRR-type) (decoupled), Isoglide5-T3R2-B1 (uncoupled), Isoglide5-T3R2-C1 (maximally regular), and Isoglide5-T3R2-C3 (maximally regular) (GOGU, 2009)...................................................................................... 16
Fig. 1-16: HEXA Robot: prototype and graph diagram.............................................................. 18
Fig. 1-17: HexaM Machine Tool (Toyoda): CAD drawing, graph diagram and photo............. 18
Fig. 1-18: Hexaglide: schematic (from (HONEGGER, et al., 1997)) and graph diagram......... 19
Fig. 1-19: Dynamil PKM (ISW): photo and graph diagram. ............................................................ 19
Fig. 1-20: Lambda Kinematics machine tool: CAD drawing and graph diagram. ..................... 20
Fig. 1-21: Fanuc M-3iA robot (http://www.fanucrobotics.fr/fr/countries/frfr/news/m3ia). ........ 20
Fig. 1-22: Hexapteron: schematic drawing, the under-construction mechanical design, and the close-up view of the mobile platform at an extreme orientation (SEWARD & BONEV, 2014). 21
Fig. 1-23: Seyanka (Tekniker): photo and graph diagram. ....................................................... 22
Fig. 1-24: P 800 (Metrom): photo and graph diagram. ............................................................. 22
Fig. 1-25: Tricept 845 (Neos Robotics AB): photo and graph diagram................................. 23
Fig. 1-26: Exechon (Exechon AB): photo and graph diagram. ............................................... 23
Fig. 1-27: Sprint Z3 (DS Technology): photo of the wrist and graph diagram......................... 24
Fig. 1-28: Hermes (Fatronik, now Tecnalia): photo and graph diagram. ............................... 24
Fig. 1-29: Dumbo (IFW): photo and graph diagram. ............................................................... 25
Fig. 1-30: Orthoglide five-axis version (IRCCyN): schematic, graph diagram, simplified CAD drawing, and prototype photo. ...................................................................................... 25
Fig. 1-31: VERNE machine (Fatronik, now Tecnalia): a photo and a schematic depicting the top view of the parallel module along its direction (from (KANAAN, et al., 2009)).................. 26
Fig. 1-32: Hita-STT (parallel module): CAD drawing and graph diagram. ............................. 27
Fig. 1-33: A five-dof (3T-2R) PKM: global schematic, fifth limb illustration, and machine photo (GAO, et al., 2006). ........................................................................................................... 27
Fig. 1-34: The five-dof Gantry-Tau PKM (ABB Robotics): schematic and photo (from (TYAPIN & HOVLAND, 2013)) ......................................................................................................................... 28
Fig. 1-35: ROBOTEX machine (LIRMM): CAD drawing and photo....................................... 28
Fig. 1-36: Eureka (KRUT, et al., 2003): CAD design and graph diagram. ........................... 29
Fig. 1-37: Five-dof (3T-2R) manipulator (ANCUTA, 2008): schematic and graph diagram. .... 29
Fig. 1-38: Manta (ROLLAND, 1999): schematic and graph diagram ..................................... 30
Fig. 1-39: Kanuk (ROLLAND, 1999): schematic and graph diagram ................................. 30
Fig. 1-40: H4 robot (COMPANY, 2000): photo and graph diagram ....................................... 32
Fig. 1-41: H4 articulated platform: basic platform (left) and modified platform including amplification gear assembly (right) (image from (KRUT, 2003)). ......................................................... 32
Fig. 1-42: H4 modified version (KRUT, 2003): photo and graph diagram ........................... 32
Fig. 1-43: I4L prototype (KRUT, 2003): photo, platform close-up view, CAD drawing, and graph diagram................................................................................................................................. 33
Fig. 1-44: Par4 (NABAT, 2007): CAD drawing, graph diagram and the two platforms with gear and cable-pulley amplification mechanisms. ................................................................. 33

Fig. 1-45: Industrialized version of Par4 (Adept Quattro). .................................................. 34

Fig. 1-46: Héli4 (NABAT, 2007): CAD drawing, graph diagram, prototype photo, and close-up view of the platform................................................................. 34

Fig. 1-47: Veloce (Penta Robotics): photo and platforms (on the left corresponds to the case of three dofs (3T), and on the right corresponds to the case of four dofs (3T-1R)). ............ 35

Fig. 1-48: λ-Quadriglide-V1 (ANCUTA, 2008): schematic, graph diagram and prototype photo. ................................................................. 35

Fig. 1-49: The SMG of McGill University (ANGELES, et al., 2006): CAD drawing and clarifying schematic ................................................................. 36

Fig. 1-50: Four-dof (3T-1R) parallel manipulator (KIM, et al., 2009): photo of exemplary device, clarifying schematic, and graph diagram. ................................................................. 37

Fig. 2-1: Speed-R-Man (from http://www.onera.fr/fr/dc/sd/robots-paralleles?page=2). ....... 48

Fig. 2-2: A simple example on mixed redundant manipulators: two series prismatic joints in parallel with another prismatic joint (one dof and three actuators) ...................... 49

Fig. 2-3: DUAL V: prototype, CAD drawing and simplified schematic diagram showing the principal geometric parameters. ................................................................. 57

Fig. 2-4: DUAL V: translational and orientation precision amplification factors TPAF (left) and OPAF (right), for the case of zero rotation. ................................................................. 57

Fig. 2-5: Geometrical interpretation of specific isotropic values in the case of two-dof (1T-1R) robot. ........................................................................................................... 65

Fig. 2-6: Geometrical interpretation of specific isotropic values in the case of three-dof (2T-1R) robot ........................................................................................................... 66

Fig. 2-7: DUAL V kinetostatic performance evaluation: the specific isotropic values (a) and the satisfactory regions (b). ........................................................................................................... 68

Fig. 2-8: A partially constrained CDPR with three dofs (2T-1R) in the vertical plane: x, z and Θ (rotation about y-axis) motion. ........................................................................................................... 76

Fig. 2-9: A fully constrained CDPR with horizontal planar motion (2T-1R) (grey configuration corresponds to zero rotation). ........................................................................................................... 81

Fig. 2-10: Ratios of specific isotropic values to corresponding requirements for the CDPR in Fig. 2-9. ........................................................................................................... 82

Fig. 2-11: Satisfactory regions in terms of kinetostatic performances based on specific isotropic values for the CDPR in Fig. 2-9. ........................................................................................................... 83

Fig. 2-12: Actuator limits illustration. ........................................................................................................... 88
Fig. 3-1: MachLin5: simplified CAD drawing ................................................................. 94
Fig. 3-2: MachLin5: the articulated platform ............................................................... 94
Fig. 3-3: MachLin5: graph diagram ................................................................................. 94
Fig. 3-4: MachLin5 - $FVI_h$ performance: case of zero rotation (left) and case of rotation (i.e. $|\theta_z| \leq 90^\circ$ and $|\theta_y| \leq 45^\circ$) (right) ........................................................................................................ 103
Fig. 3-5: MachLin5 - $v_{iuo}$ performance: case of zero rotation (left) and full-range rotation (right). .................................................................................................................. 104
Fig. 3-6: MachLin5 - $f_{iuo}$ performance: case of zero rotation (left) and full-range rotation (right). .................................................................................................................. 104
Fig. 3-7: ARROW V1 PKM: CAD drawing and graph diagram. ........................................ 107
Fig. 3-8: Frontal view of ARROW V1 PKM and close-up side view of its platform. ........ 107
Fig. 3-9: Pareto diagram of ARROW V1 and the chosen geometric parameters .......... 118
Fig. 3-10: $TPAF$ of ARROW V1: case of $\theta_z = 0^\circ$ (left) and case of $|\theta_y| \leq 45^\circ$ (right) ................................................................. 120
Fig. 3-11: $ILA$ of ARROW V1: case of $\theta_z = 0^\circ$ (left) and case of $|\theta_y| \leq 45^\circ$ (right) ................................................................. 120
Fig. 3-12: $v_{iuo}$ of ARROW V1: case of $\theta_z = 0^\circ$ (left) and case of $|\theta_y| \leq 45^\circ$ (right) .................................................................................................................. 121
Fig. 3-13: $f_{iuo}$ of ARROW V1: case of $\theta_z = 0^\circ$ (left) and case of $|\theta_y| \leq 45^\circ$ (right) ................................................................. 121
Fig. 3-14: $PLA$ of ARROW V1: case of $\theta_z = 0^\circ$ (left) and case of $|\theta_y| \leq 45^\circ$ (right) ................................................................. 121
Fig. 3-15: Spatial workspace of ARROW VI in the case $\theta_z = 0^\circ$ and for stroke lengths: $SL_z = 1.49$ m (actuators 1, 2, 5 and 6) and $SL_p = 1.53$ m (actuators 3 and 4) .................................................................................. 122
Fig. 3-16: Spatial workspace of ARROW VI in the case $|\theta_y| \leq 45^\circ$ and for stroke lengths: $SL_z = 1.49$ m (actuators 1, 2, 5 and 6) and $SL_p = 1.53$ m (actuators 3 and 4) .................................................................................. 123
Fig. 3-17: ARROW V2: CAD views and notations .......................................................... 125
Fig. 3-18: The platform of ARROW V2 and its corresponding notations ...................... 126
Fig. 3-19: The five-dof (3T-2R) machine tool: ARROW V2 with turntable ................. 126
Fig. 3-20: Graph diagram of ARROW V2 M1/M2, the mutated versions of ARROW V2 .... 132
Fig. 3-21: Visualization of the offsets between the revolute joints in ARROW V2 M1/M2 versions (top view): ARROW V2 M1 corresponds to having $r_p = 0$ and $r \neq 0$, whereas ARROW V2 M2 corresponds to the situation where $r_p \neq 0$ and $r \neq 0$ .............................................................................. 132
Fig. 3-22: ARROW V2 M2: 3D CAD view................................................................... 132
Fig. 3-23: ARROW V2 M2: projective CAD views with notations ..................................... 133
Fig. 3-24: ARROW V2 M2: schematic of a kinematic chain and its different notations. .... 133
Fig. 3-25: The complex chains (III) and (IV) with notations................................. 139
Fig. 3-26: Singularity analysis: case $e_x^T m_o = 0$ .............................................. 143
Fig. 3-27: Singularity analysis: case $e_z^T (L_4 \times u_4) = 0$ ................................. 143
Fig. 3-28: Pareto diagram of ARROW V2 M2 with the two potential candidates encircled in red. ...................................................................................................................... 149
Fig. 3-29: ARROW V2 M2 PKM - $ILA$ capacity: case $\theta_z = 0^\circ$ (left) and case $\theta_z \in [-45^\circ;+45^\circ]$ (right). ...................................................................................................................... 150
Fig. 3-30: ARROW V2 M2 - $TPAF$ performance: case $\theta_z = 0^\circ$ (left) and case $\theta_z \in [-45^\circ;+45^\circ]$ (right). ...................................................................................................................... 151
Fig. 3-31: ARROW V2 M2 PKM - $v_{iso}^p$ capacity: case $\theta_z = 0^\circ$ (left) and case $\theta_z \in [-45^\circ;+45^\circ]$ (right). ...................................................................................................................... 151
Fig. 3-32: ARROW V2 M2 PKM - $f_{iso}^p$ capacity: case $\theta_z = 0^\circ$ (left) and case $\theta_z \in [-45^\circ;+45^\circ]$ (right). ...................................................................................................................... 152
Fig. 3-33: ARROW V2 M2 PKM - $PLA$ capacity: case $\theta_z = 0^\circ$ (left) and case $\theta_z \in [-45^\circ;+45^\circ]$ (right). ...................................................................................................................... 152
Fig. 3-34: ARROW V2 M2: illustrations of the spatial workspaces in the case of $\theta_z = 0^\circ$ (top) and $\theta_z \in [-45^\circ;+45^\circ]$ (bottom). ...................................................................................................................... 153
Fig. 3-35: ARROW V2 M2 PKM with turntable (rendered CAD drawing)................. 153
Fig. 4-1: ARROW V2 M2 PKM prototype: basic control model. .............................. 157
Fig. 4-2: The tested trajectory profile..................................................................... 159
Fig. 4-3: Actuated-joint tracking errors for the trajectory profile in Fig. 4-2. .......... 160
Fig. 4-4: Estimated tracking errors in the operational space. ............................. 160
Fig. 4-5: ARROW PKM geometric calibration: the main relations. ....................... 161
Fig. 4-6: Calibration parameters of the simple PSRR chain.................................. 163
Fig. 4-7: Calibration parameters of the complex P(SRR)$_2$ chain ....................... 163
Fig. 4-8: Mean absolute geometric sensitivity of ARROW V2 M2 prototype ........ 166
Fig. 4-9: Estimated standard deviation of absolute geometric sensitivity of ARROW V2 M2 prototype .......................................................... 167
Fig. 4-10: Schematic of the turntable of ARROW machine with notations............ 169
Fig. 4-11: Illustration of the impact of the rotation $e_i^T e_{PM}^{rot} \approx \theta_z$ of the turntable. 170
Fig. Ap-1: Static accuracy and repeatability illustration. .......................................................... 193

Fig. Ap-2: Illustration of multi-directional variation of accuracy ($V_{AP}$ in the figure) [ISO 9283; 1998 (F)]. ................................................................................................................................. 195

Fig. Ap-3: Illustration of trajectory positional accuracy ($AT_{P}$) and trajectory positional repeatability ($RT_{P}$) [ISO 9283; 1998 (F)]. ................................................................................................................................. 196

Fig. Ap-4: Metris K600 CMM measuring device ...................................................................... 197

Fig. Ap-5: Experimental setup: an illustrative schematic (left) and a close-up view on the LEDs assembly on the platform (right). ......................................................................................... 197

Fig. Ap-6: The largest inscribed cube or rectangular parallelepiped within the accessible workspace, the diagonal plane, and the test points ($P_1$...$P_2$). .......................................................... 198

Fig. Ap-7: Illustration of the possible cycle schemes. ................................................................. 198

Fig. Ap-8: Multi-directional variation of accuracy: poses ($P_1$, $P_2$, and $P_3$) and the three directions of approach for the pose $P_i$ ($A_iP_i$, $B_iP_i$, and $C_iP_i$, with $i=1,2,4$). ............................. 199

Fig. Ap-9: Adept Quattro platforms: articulated (four dofs 3T-1R on the left) and rigid (three dofs 3T on the right) (http://www.adept.com/products/robots/parallel/quattro/s650h/downloads). ........................................................................................................ 200

Fig. Ap-10: Par2 (on the left) and DUAL V (on the right) (LIRMM). ........................................ 204

Fig. Ap-11: DUAL V: schematic showing principal geometric parameters and center-of-mass positions of the different parts. ............................................................................................................. 220

Fig. Ap-12: DUAL V dynamic analysis: ratios of specific isotropic values’ lower bounds relative to their corresponding requirements in the case of zero rotation. ........................................... 222

Fig. Ap-13: DUAL V: the regions with specific isotropic values’ lower bounds greater or equal to their corresponding requirements in the case of zero rotation. ........................................... 222

Fig. Ap-14: Ratios of specific isotropic values’ lower bounds relative to corresponding requirements for the CDPR presented in Fig. 2-9. ................................................................. 230

Fig. Ap-15: Satisfactory regions based on lower bounds of specific isotropic dynamic values for the CDPR presented in Fig. 2-9. ......................................................................................................... 230

Fig. Ap-16: ARROW V2 M2 prototype. .................................................................................... 232

Fig. Ap-17: ARROW V2 M2 prototype: close-up image of the platform. .................................. 232

Fig. Ap-18: ARROW machine CAD drawing: close view on the turntable showing the springs used to counteract the gravity effect. ......................................................................................... 233
List of Tables

Table 2-1: DUAL V actuators’ characteristics and the desired kinetostatic requirements.* .... 67
Table 2-2: The significance of the general terms used in relations (2.53) and (2.54). ............. 73
Table 2-3: The speed and static load requirements for the CDPR in Fig. 2-9* ....................... 82
Table 2-4: The significance of the general terms used in relations (2.89) and (2.90). ........... 87
Table 3-1: Geometric parameters of MachLin5.* ..................................................................... 103
Table 3-2: Platform and actuator characteristics. ................................................................. 117
Table 3-3: ARROW V1: inertia parameters. ........................................................................... 117
Table 3-4: Inertia parameters of ARROW V2 M2 (with some safety margins). ...................... 148
Table 3-5: The fixed geometric parameters and limits of ARROW V2 M2 ......................... 148
Table Ap-1: Static positional accuracy and repeatability of Adept Quattro (case of articulated platform). ........................................................................................................... 200
Table Ap-2: Static positional accuracy and repeatability of Adept Quattro (case of rigid platform). .................................................................................................................. 200
Table Ap-3: Static orientation accuracy of Adept Quattro (case of articulated platform). ...... 200
Table Ap-4: Static orientation repeatability of Adept Quattro (case of articulated platform). .......................................................................................................................... 201
Table Ap-5: Static orientation accuracy of Adept Quattro (case of rigid platform) ............. 201
Table Ap-6: Static orientation repeatability of Adept Quattro (case of rigid platform). ....... 201
Table Ap-7: Multi-directional variation of positional accuracy of Adept Quattro. ............... 202
Table Ap-8: Static positional accuracy, repeatability, and multi-directional variation of accuracy of Veloce prototype. ................................................................................... 202
Table Ap-9: Static orientation accuracy of Veloce prototype. ............................................. 203
Table Ap-10: Static orientation repeatability of Veloce prototype ....................................... 203
Table Ap-11: Static positional accuracy and repeatability of Par2 ...................................... 204
Table Ap-12: Static accuracy and repeatability of DUAL V. ............................................... 204
Table Ap-13: Positional path accuracies and repeatabilities of Quattro and Par 2 (case of linear trajectory P2P4) ...................................................................................... 205
Table Ap-14: The inertia parameters of DUAL V ................................................................. 221

xiv
Table Ap-15: Actuators’ capacities and required dynamic performances for DUAL V............. 221

Table Ap-16: Actuators’ characteristics, inertia parameters, and required dynamic performances for the CDPR in Fig. 2-9.* ............................................................... 229
In these few pages, we provide a general overview on the thesis context within the inclusive ARROW project, clarifying the motivations and major contributions.

The ARROW project is a French project financed by the French National Research Agency (ANR) under the number ANR 2011 BS3 006 01. Its main objectives can be summarized as the design of Accurate and Rapid Robots with large Operational Workspace, from which the acronym “ARROW” has been derived. The project embraces three partners:

1. IRCCyN (Institut de Recherche en Communication et Cybernétique de Nantes);
2. LIRMM (Laboratoire d'Informatique, de Robotique et de Microélectronique de Montpellier);
3. And TECNALIA France.

The current PhD thesis falls within the Industrial Conventions for Research Training CIFRE (Conventions Industrielles de Formation par la Recherche in French), and it has been financed by TECNALIA France, and in a part by the ANR.

Context

The recent decades have witnessed an increased industrial interest in parallel manipulators (PMs). Undeniably, this increase has been due to the great success of the Delta robot introduced by (CLAVEL, 1991), and which opened a new era of lightweight robots (or so-called Delta-like robots). Unfortunately, despite of that, the major implementation of PMs in industry has been limited to pick-and-place applications, with rather few and shy exploitations in other industrial operations, such as machining processes.

In fact, the rapidity and highly decreased cycle times achievable by PMs, allowed them to be highly competitive compared with serial counterparts in the field of pick-and-place operations, especially where the precision demand is not that high.

However, in applications demanding both rapidity and elevated precision, the available PMs fall short of supplying these requirements. In fact, we have been able to design ultra-rapid robots with mediocre or poor precision on one hand, and high-precision robots but with almost quasi-static performance on the other hand. As exemplification on the former robots, we mention: Adept Quattro (PIERROT, et al., 2008) (the fastest pick-and-place industrial robot with up to 15 g acceleration¹), Par2 (BARADAT, et al., 2008) (with acceleration that reaches 43 g).

¹“g”: corresponds to gravitational acceleration (1 g = 10 m/s²).
and the exceptional R4 (CORBEL, et al., 2010) with its incredible 100 g acceleration capability (CHEMORI, et al., 2013). As for high-precision robots, we mention the hexapods of PI company that are designed for micro-positioning purposes (refer to http://www.physikinstrumente.com/en/products/prdetail.php?sortnr=1000770).

Based on this argument, the ARROW project has been initiated in the year 2011, with the aim of achieving rapid and precise parallel kinematic machines (PKMs). For this ultimate goal, two different scenarios have been proposed:

1. The first scenario has considered the development of robots with large acceleration capacity, small cycle time, and elevated precision only at the end-points of a given trajectory. The intended application of such robots is the assembly of electronic components and the alike. This scenario has been approached by IRCCyN.
2. The second scenario has been concerned with the development of robots with high acceleration capability and elevated precision following any trajectory within a desired workspace. Such robots can be implemented in industrial applications as laser or water-jet cutting, welding, rapid prototyping, etc. This scenario has been dealt with by LIRMM and Tecnalia.

This latter objective constitutes the main subject of the current PhD thesis that details the major achievements and contributions in this aspect.

**Motivations**

As it has been mentioned above, we have targeted the establishment of a PKM with high dynamic capability and precision. The numerical set goals have been achieving up to 20 g regarding linear acceleration and less than 20 µm absolute accuracy. As for the PKM’s operational degrees of freedom (dofs), they are supposed to be five dofs (3T-2R)\(^2\), as they are sufficient for most industrial applications.

However, although acceleration and precision are the main targets, they are not the sole ones. Actually, the PKM must be characterized by large singularity-free workspace and tilting capacity. This is not to mention the design simplicity that is essential, not just for having simple models, but also to facilitate the manufacturing of this PKM.

Based on all the above, several mechanisms have been synthesized and studied after the establishment of suitable performance measures, which constitute one of the major contributions of this research in addition to the PKM itself. These contributions will be detailed in the upcoming section.

---

\(^2\) “T” and “R” stand for translational and rotational dofs respectively.
Contributions

In the scope of the work to accomplish the preset objectives, the following contributions have been made:

1. Establishment of original performance measures: These can be used for the assessment and dimensional synthesis of a general robot based on precision, kinetostatic, and/or dynamic performances. They overcome two major issues as compared with the classical available ones, namely redundancy (whether of actuation or kinematic type) and heterogeneity relative to operational dofs or actuator types (i.e. having rotational and prismatic actuators at the same time). The generality of the approach embraces: serial, parallel, and hybrid robots. Furthermore, the approach is applicable on not only rigid manipulators, but also cable-driven ones.

2. Synthesis of several novel parallel architectures: Among these, we mention the MachLin5, ARROW V1, and ARROW V2 with its mutated versions, ARROW V2 M1/M2. MachLin5 is a five-dof (3T-2R) manipulator; whereas the rest are four-dof (3T-1R) redundantly actuated PKMs.

Finally, in what follows, we describe the general outline of the dissertation:

1. The first chapter will provide some generalities and basic definitions regarding PMs. Then, an exposition of the state of art regarding PMs and implemented PKMs will be presented, emphasizing the merits and demerits of some particular designs. After that, the issue of performance evaluation and the corresponding available literature will be discussed while highlighting the major encountered limitations, especially when dealing with redundant or heterogeneous-dof robots.

2. The second chapter will be dedicated to the presentation of the newly established approach for the performance assessment and optimization of general manipulators, relative to precision, kinetostatics, and/or dynamics. Besides, two case studies will be provided to demonstrate the methodology. The first analysis will be done on DUAL V (WIJK, et al., 2013), a redundantly actuated rigid robot with planar motion (three dofs, 2T-1R). As for the second study, it will be carried on a fully constrained cable-driven parallel robot (CDPR), with planar motion (three dofs, 2T-1R) and four active cables.

3. The third chapter will be devoted for the presentation of several novel mechanisms. This presentation will include geometric models, Jacobians, and singularity analysis. Also, the manufacturing procedure and the necessarily modifications done on the chosen robot for execution, will be discussed. Moreover, the dimensional synthesis of the implemented PKM and another architecture will be provided.

4. The fourth chapter will emphasize some points regarding the control of the ARROW PKM and the possible error compensations that can be made in the future.

5. Ultimately, the dissertation ends with general conclusions and perspectives regarding possible future research directions.
Notations

In this thesis, the equations are numbered in order of appearance and depending on the chapter number. Moreover, the following notations are adopted:

- Scalar variables are italicized and numbers are written in a regular font, for example: $i$, $j$, $1$, $2$, etc.
- Vectors and matrices are italicized in bold, for example: $v$, $M$, etc.
- $1_{n\times n}$ denotes the $n\times n$ identity matrix.
- $0_{m\times n}$ denotes the $m\times n$ zero matrix.
- $M^{-1}$ and $M^T$ correspond to the inverse and transpose of matrix $M$, respectively.
- $M^*$ denotes the pseudo-inverse of matrix $M$.
- $\text{diag}(\lambda_i)$ designates the $n\times n$ diagonal matrix whose diagonal terms are $\lambda_i$, with $i = 1...n$.
- $\text{eigs}(\text{Matrix})$ denotes the list of eigenvalues of the square matrix $\text{Matrix}$.
- $\text{sing}(\text{Matrix})$ denotes the list of singular values of the matrix $\text{Matrix}$.
- $\text{val}$ and $\text{val}$ correspond to lower and upper bounds of the term $\text{val}$.
- $\dot{f}$ and $\dot{q}$ represent the time derivative of the function, $f$, and the vector, $q$, respectively. Similarly, $\ddot{f}$ and $\ddot{q}$ represent the second time derivatives of the aforementioned terms.
- $e_x$, $e_y$, and $e_z$ are the unit vectors along the $x$, $y$, and $z$ axes of the base frame.
- $\text{Rot}_x(\theta_x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_x) & -\sin(\theta_x) \\ 0 & \sin(\theta_x) & \cos(\theta_x) \end{bmatrix}$: is the rotation matrix in the case of a rotation of $\theta_x$ about the $x$-axis.
- $\text{Rot}_y(\theta_y) = \begin{bmatrix} \cos(\theta_y) & 0 & \sin(\theta_y) \\ 0 & 1 & 0 \\ -\sin(\theta_y) & 0 & \cos(\theta_y) \end{bmatrix}$: is the rotation matrix in the case of a rotation of $\theta_y$ about the $y$-axis.
- $\text{Rot}_z(\theta_z) = \begin{bmatrix} \cos(\theta_z) & -\sin(\theta_z) & 0 \\ \sin(\theta_z) & \cos(\theta_z) & 0 \\ 0 & 0 & 1 \end{bmatrix}$: is the rotation matrix in the case of a rotation of $\theta_z$ about the $z$-axis.
Rot (u, θ) = \[
\begin{bmatrix}
    u_x^2 (1-c_θ) + c_θ & u_x u_y (1-c_θ) - u_z s_θ & u_x u_z (1-c_θ) + u_y s_θ \\
    u_y u_x (1-c_θ) + u_z s_θ & u_y^2 (1-c_θ) + c_θ & u_y u_z (1-c_θ) - u_x s_θ \\
    u_z u_x (1-c_θ) - u_y s_θ & u_z u_y (1-c_θ) + u_x s_θ & u_z^2 (1-c_θ) + c_θ
\end{bmatrix}
\]

is the rotation matrix in the case of θ rotation about an axis of direction \( u = (u_x, u_y, u_z) \).

\[\hat{a} = \begin{bmatrix}
    0 & -a_z & a_y \\
    a_z & 0 & -a_x \\
    -a_y & a_x & 0
\end{bmatrix} \]

meaning \( a \times b = \hat{a} b \) where \( b = (b_x, b_y, b_z) \).

Also, graph diagrams are used to depict mechanism topologies. For this purpose, the following notations are adopted:

- \( R, U, S, C, H, \) and \( P \): stand for revolute, universal, spherical, cylindrical, helical, and prismatic joints, respectively.
- \( X \) and \( \bar{X} \): represent passive and actuated joints, respectively.
- \( \bar{X} \): means that the joint \( X \) is equipped with a position sensor.

Finally, several acronyms are frequently used in the report. These are supplied here to serve as a quick reference for the reader:

- \( AR, KR, \) and \( TR \): stand for actuation, kinematic, and task redundancies, respectively.
- \( NRM, RAM, KRM, \) and \( MRM \): stand for non-redundant, redundantly actuated, kinematically redundant, and mixed-redundancy manipulators or machines, respectively.
- \( IBAR \) and \( BAR \): stand for in-branch and branch actuation redundancies, respectively.
- \( IGM \) and \( DGM \): stand for inverse and direct geometric models.
- \( IKM \) and \( DKM \): stand for inverse and direct kinematic models.
- \( DM (SDM) \): corresponds to dynamic model (respectively simplified dynamic model).
- \( IDM \) and \( DDM \): stand for inverse and direct dynamic models.
- \( DWS \): means desired workspace.
- \( TPAF \) and \( OPAF \): correspond to translational and orientation amplification factors, respectively.
- \( WTPAF_{DWS} \): is the worst value of TPAF over DWS, i.e. the maximum value.
- \( ILA \) and \( PLA \): are respectively the isotropic and peak linear accelerations starting from rest, and in the absence of any external non-gravitational wrench.
- \( WILA_{DWS} \): is the worst value of ILA over DWS, i.e. the minimum value.
Chapter 1: State of the Art

In this chapter:

Available literature is rich with parallel manipulators (PMs), and some have found their way into industrial applications. However, the number of parallel kinematic machines (PKMs) that has been implemented so far is still very low. This poor exploitation is due to several reasons. In this chapter, an overview on the state of art of PMs and available industrial PKMs will be exposed, highlighting the problematics that hinder their widespread. In addition, the available literature on performance evaluation of manipulators will be discussed, emphasizing the major limitations encountered in the case of robots with heterogeneous degrees of freedom (dofs) and/or with redundancy (whether of actuation or kinematic type). This latter problematic is crucial as the synthesis of both, the architecture and its geometrical dimensions, are supposed to be based on solid criteria well interpretable and that fit machine tool basic requirements. Starting from the aforementioned points, the thesis problematics will be clarified by the end of the chapter and the approach to overcome them will be outlined.
1.1- Generalities and Definitions

1.1.1- A Brief History on the First Parallel Robots and Parallel Kinematic Machine Tools

In most articles, it has been reported that the first PMs were the tyre-testing machine of (GOUGH & WHITEHALL, 1962) (see Fig. 1-1) and the flight simulator of (STEWART, 1965). However, according to (BONEV, 2003), the first PM perhaps dates back to 1931 (GWINNETT, 1931) and it has been proposed as an amusement device; but it is not known whether the aforementioned architecture has been built or not. Nevertheless, it is undeniable that Gough-Stewart platforms have played an essential role in popularizing PMs and inspiring new ones.

Actually, parallel robots at their earlier beginnings, were of full mobility\(^1\) and mainly based on the Gough-Stewart architectures. As for the trend towards lower mobility\(^2\) parallel mechanisms, it can be traced back, according to (ANGELES, 2004), to the work of (HUNT, 1983) and after which planar, spherical, and later spatial mechanisms have been synthesized.

Moreover, another remarkable milestone in the world of parallel mechanisms is perhaps the Delta robot introduced by (CLAVEL, 1991) (see Fig. 1-2). This robot not only has been vastly industrialized, but also it inspired many new ones with similar features.

According to (KRUT, 2003), one can speak of two generations of parallel mechanisms: the first being described by Gough-Stewart platforms and based upon architectures, while the second being embodied by Delta-like or lightweight structures. In fact, thanks to the features

---

\(^{1}\) With six dofs

\(^{2}\) With the number of dofs less than six
Chapter 1: State of the Art

present in the latter manipulators, such as fixed actuators at the base and lightweight components and parallelograms, exceptional performances with up to 10 m/s in speed and 15 g in acceleration can be reached, even more!

This was a very short briefing regarding parallel robots. Concerning machine tools, the first one based on parallel kinematics, as contrasted to the conventional or serial counterparts, was the Variax of Giddings & Lewis (see Fig. 1-3). It has been presented publically in 1994 within the International Machine Tool Show held at Chicago. It was based on the Gough-Stewart idea and intended for milling applications.

Afterwards, many industries started researching and developing parallel kinematic machine

![Image of FlexPicker](image1)

**Fig. 1-2:** FlexPicker (ABB Robotics) industrial robot: photo and graph diagram.

![Image of Variax](image2)

**Fig. 1-3:** Variax parallel kinematics machine tool (Giddings & Lewis).

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3 A video regarding this machine is available at: [https://www.youtube.com/watch?v=7TowJZQj-qY](https://www.youtube.com/watch?v=7TowJZQj-qY).
tools (PKMs) in parallel with university laboratories. Among these PKMs, we mention: H0H600 (Ingersoll), 6X Hexapod (Mikromat), G500 and G1000 (Geodetics), Cosmo Center PM-600 (Okuma), Tornado 2000 (Hexel), HEXACT (developed by INA and IFW), Hexapode 300 (CMW), Triaglide (Mikron), Quickstep (Krause & Mauser), UraneSX (Renault Automation/Comau), Georg V of IFW (University of Hanover), Eclipse (Seoul National University), etc. In (COMPANY, 2000) and (WECK & STAIMER, 2002), a more elaborate information is presented regarding the history and state of the art of machine tools in general, and PKMs in particular. Later in this chapter, we will expose only a sample of these machines that exhibit some interesting features.

However, despite the increased interest in PKMs, their spread in industrial and machining applications is rather shy, and still conventional or serial kinematic machines (SKMs) are the highly dominant, with PKMs not exceeding research stages. In fact, comparing the number of theoretically synthesized architectures to that of the implemented ones, clearly shows the huge gap between theory and industrial needs. While the higher motion coupling and insufficiency of the processors’ capabilities were a major hinder for PKMs spread before 1994, the present status, where large advancement in electronics yielded highly performant controllers, shows that other constraints have come into the light presenting new challenges and obstacles to overcome. This will become clear by the end of the chapter.

Nevertheless, before going any further in the world of parallel mechanisms, it is indispensable to provide some definitions to clarify their notion and particularities as compared to serial ones, emphasizing the merits and drawbacks of each. These are discussed in the following sections.

1.1.2- Serial Robots

The most industrial robots built until now are serial manipulators (SMs). An SM is an open chain formed by a series of links interconnected one to another by an actuated joint. An example of such manipulators is shown in Fig. 1-4.

While SMs are characterized by large workspaces and being rather simple to deal with regarding control, they suffer from the following drawbacks:

- High moving masses which limit their dynamic capability;
- Poor rigidity as a result of the series configuration and which leads to cumulative errors regarding the end-effector pose;
- Wear of the power and sensor connections (cables, flexible tubes) and which might lead to hazardous consequences;
1.1.3- Parallel Robots

A parallel manipulator (PM) is defined, according to (MERLET, 2006), as a closed-loop kinematic chain mechanism whose end-effector is linked to the base by several independent kinematic chains. Examples on such manipulators are those in Fig. 1-1, Fig. 1-2, Fig. 1-3, and Fig. 1-5.

Such mechanisms have their interesting features that made them suitable candidates to overcome the limitations of SMs. Unfortunately, they are not without their own demerits. In the upcoming subsections, we expose these virtues and hindrances.

A- Advantages

In general, PMs are characterized by the following advantages:
• High payload-to-weight ratio;
• High stiffness due to the parallel structure;
• High dynamic capabilities due to small moving masses, especially when the actuators are placed at or near the base;
• Improved accuracy due to the parallel structure and in which unlike SMs, the end-effector pose errors are non-cumulative;
• Higher proper frequency due to elevated rigidity and therefore, lessened repeatability errors due to the uncontrolled structural oscillations;
• Moreover, the possibility of placing the actuators at the base allows for the following additional benefits:
  ➢ Higher flexibility regarding the choice of motors and/or gearboxes, as their masses do not highly influence the eventual inertia of the robot (particularly the moving inertia);
  ➢ Reduction of the problems arising from cable connections between the motors, sensors, and controller.
  ➢ Enhanced cooling of the actuators resulting in reduced errors due to thermal expansions;
  ➢ Easier isolation of the motors from the possible detrimental environmental conditions that might be present in the workspace (e.g. applications that might require heavy water rinsing).

It is worth emphasizing that the above-mentioned features might not necessarily be present in all PMs. In fact, the première designs experienced problems of precision and rigidity (KRUT, 2003).

B- Inconveniences

Despite the interesting features described above, PMs suffer usually from the following inconveniences:

• Reduced workspace and tilting capacity: This results from the parallel structure itself, as the end-effector workspace is the intersection of the regions permissible by the individual kinematic chains. Another cause of this reduction is the possibility of inter-collisions. Additionally, we mention the usually rather complex shapes of the feasible workspaces, especially when internal singularities (i.e. inside the geometrically accessible workspace) exist.
• Singularities: Unlike SMs, PMs present, in addition to serial-type singularities, other singularity types with more subtleties regarding their identification and classification. These singularities are critical and can lead to uncontrollable motion of the platform (end-effector) or the deterioration of the mechanical system. The avoidance of such singularities constitutes an essential challenge in the design of a PM. This will be discussed in the subsequent part.
• The difficulty of having closed-form solution for the direct geometric model (DGM): This is nowadays less severe as numerical solutions can be implemented without influencing computation time. This is due to the advanced performance of modern controllers.
• Calibration difficulties whether relative to geometry, elasticity, and/or dynamics: This is due to the high coupling between the different chains and the large number of parameters involved as compared with SMs.
• High motion coupling and control complexity: Currently, with the advancement in electronics and development of highly performant processing units, these have become less critical.

C- Singularities of PMs

Singularities of PMs and their study constitute one of the prominent fields of research in robotics. In fact, the first attempt to set a general framework regarding singularity analysis of PMs can be traced back to (GOSSELIN & ANGELES, 1990). It has been based on the differentiation of the constraint equations, \( \mathbf{F}(\mathbf{q}, \mathbf{x}) = \mathbf{0} \), relating the joint positions, \( \mathbf{q} \), to the end-effector pose, \( \mathbf{x} \). This leads to the following relation between the joint velocities, \( \dot{\mathbf{q}} \), and the operational twist\(^4\), \( \mathbf{t}_{\text{red}} \) (the reduced \( n \)-dimensional twist for an \( n \)-dof robot):

\[
J_q \dot{\mathbf{q}} = J_x \mathbf{t}_{\text{red}}
\]  
(1.1)

According to (GOSSELIN & ANGELES, 1990), three types of singularities exist depending on the rank deficiency of \( J_q \) and/or \( J_x \). These are described as follows:

a) Type-1 singularity (aka\(^5\) inverse kinematic or serial singularity): It occurs when \( J_q \) is rank deficient. In such a case, the actuators can move while the platform is fixed.

b) Type-2 singularity (aka direct kinematic or parallel singularity): It corresponds to the case where \( J_x \) is rank deficient; meaning that the platform can move even when the actuators are fixed (at least one of the dofs is uncontrollable). We mention that type-2 singularity can be also termed as force singularity according to (MULLER, 2013), since certain operational wrenches cannot be supported by the manipulator in such a situation.

c) Type-3 singularity: It is the case where both matrices, \( J_q \) and \( J_x \), are simultaneously singular. The two phenomena described in types 1 and 2 can be noticed.

However, the above study does not embrace all possible singularities. An example on that is the well-known 3-UPU translational PM of Seoul National University (SNU). The mechanism was suggested by (TSAI, 1996) as a three-dof translational robot with actuated P joints. But the developed PM revealed peculiar motion for the case of equal limbs and fixed actuators. This was despite the absence of parallel singularities. The phenomenon was investigated by several researchers ((BONEV & ZLATANOV, 2001), (GREGORIO & PARENTI-CASTELLI, 2002), (JOSHI & TSAI, 2002), (WOLF, et al., 2002)) and became a popular example to demonstrate the insufficiency of considering only input-output kinematic relation in singularity analysis. Its singularity is what (BONEV & ZLATANOV, 2001) referred to as a constraint singularity.

\(^4\)In the general case, we have \( \mathbf{t}_{\text{red}} = (\mathbf{v}^T \ \omega^T)^T \) for a six-dof robot, with \( \mathbf{v} \) and \( \omega \) being the linear and angular velocities, respectively. The twist \( \mathbf{t}_{\text{red}} \) of a translational three-dof robot is \( \mathbf{t}_{\text{red}} = \mathbf{v} = (v_x \ v_y \ v_z)^T \).

\(^5\)"aka": also known as
A more generalized study was proposed by ([ZLATANOV, et al., 1995], [ZLATANOV, et al., 1998]). Unlike (GOSSELIN & ANGELES, 1990), the study of (ZLATANOV, et al., 1995) includes the passive joint velocities in the play. Accordingly, six types of singularities have been defined based on the solvability of the instantaneous forward and inverse kinematic problems: (IFKP) and (IIKP), respectively. Actually, the IFKP starts from given actuated joint velocities to establish both, the end-effector twist and the passive joint velocities. The failure of deriving a definite solution for any of the latter two quantities corresponds to a singularity. Similarly, starting from the end-effector twist, the IIKP seeks the establishment of definite actuated and passive joint velocities. Having any of the latter quantities indefinite implies a singularity as well. Therefore, a non-singular configuration of the mechanism is a one that does not present either type of singularities. Based on the above, the six singularity types are: redundant input (RI), redundant output (RO), redundant passive motion (RPM), impossible input (II), impossible output (IO), and increased instantaneous mobility (IIM). Moreover, n -order singularity types have been investigated by (WOHLHART, 1999) and (LIU, et al., 2003); however, they are rather difficult to put into practical use. In addition, another taxonomy of singularities has been suggested recently by (CONCONI & CARRICATO, 2009). For more information on singularities and their theory, the reader may refer to (MERLET, 2006) and (DONELAN, 2007).

In this section, we present the recommended singularity study following the general approach of (ZLATANOV, et al., 1998), but considering the full end-effector twist\(^6\),

\[
t = \begin{pmatrix} v^T & \omega^T \end{pmatrix}^T
\]

(instead of the full-cycle\(^7\) or n -dimensional one in the general case of an n -dof robot). Denote by \(\dot{\psi}\) and \(\ddot{\psi}\) the passive and actuated joint velocities, respectively. Also, consider the decomposition of \(t\) into: \(t_{\text{red}}\) and \(t_{\text{comp}}\). The term \(t_{\text{red}}\) denotes the n -dimensional twist for a mechanism whose full-cycle mobility is n (HUNT, 1978). As for \(t_{\text{comp}}\), it is the complementary twist\(^8\) of \(t_{\text{red}}\) or in other words, the constrained part of \(t\).

Then, the differential equation relating \(\dot{\psi}, t, \) and \(\ddot{\psi}\) is of the form:

\[
L_{\psi} \ddot{\psi} + L_{\dot{t}} \dot{\psi} + L_t t = 0 \tag{1.2}
\]

Relation (1.2) can also be written simply as below:

\[
L \begin{pmatrix} \dot{\psi}^T & \psi^T & t^T \end{pmatrix}^T = 0 \tag{1.3}
\]

---

\(^6\) \(t = \begin{pmatrix} v^T & \omega^T \end{pmatrix}^T\), where \(v\) and \(\omega\) are respectively the linear and angular velocities in the three-dimensional space.

\(^7\) Full-cycle mobility is the minimum instantaneous mobility of the platform for all the possible configurations (HUNT, 1978).

\(^8\) For instance, the complementary twist of a three-dof translational robot is \(t_{\text{comp}} = \omega = \begin{pmatrix} \omega_x & \omega_y & \omega_z \end{pmatrix}^T\), i.e. the angular velocity. If at the considered configuration there is no instantaneous increase of mobility, we should have \(t_{\text{comp}} = \omega = 0\) for the aforementioned robot.
Therefore, we categorize three singularity types, which are:

a) Redundant input singularity (RI): In this case, there exists $\dot{q} \neq 0$ for which $t = 0$. This is identical to type-1 singularity of (GOSSELIN & ANGELES, 1990) and indicates that the IIKP is not solvable.

b) Redundant output singularity (RO): In this case, for $\dot{q} = 0$, a non-zero twist, $t \neq 0$, is possible. In fact, RO embraces the type-2 singularity of (GOSSELIN & ANGELES, 1990), but the converse is not necessarily true. In particular, RO corresponds to type-2 only if $t_{\text{red}} \neq 0$. However, if $t_{\text{comp}} \neq 0$ and $t_{\text{red}} = 0$, it is an RO singularity that cannot be detected utilizing the classical input-output kinematic relation. Such a case is referred to as constraint singularity ((BONEV & ZLATANOV, 2001) and (ZLATANOV, et al., 2001)). Moreover, it is possible to distinguish a particular type of constraint singularities, namely the architectural ones. These occur when the motion of the travelling platform is finite and in this case, the robot is coined as self-motion robot (e.g. (KARGER, 2003)).

c) Redundant passive motion singularity (RPM): Despite having the actuators and the platform fixed (i.e. $\dot{q} = 0$ and $t = 0$), the passive joint velocities are not necessarily zero ($\dot{\psi} \neq 0$). Such singularity has been referred to as an actuator singularity in the work of (HAN, et al., 2002). An easy example would be having an arm between two passive spherical (S) joints. Then, although the extremities are fixed, the arm can rotate freely about the axis joining the centers of these two S-joints. Theoretically speaking, such a singularity would not produce a motion of the end-effector, but practically it must be rather circumvented due to the unavoidable imperfections in the joints (MERLET, 2006).

With the above, a non-exhaustive but sufficient overview on the PMs’ singularities has been given. Also, the general approach based on (ZLATANOV, et al., 1998) has been clarified. The essence to be kept in mind is what follows. To assure the absence of singularities of any type at a certain configuration, it is sufficient to show the following three points:

a) $t_{\text{comp}} = 0$: meaning that the undesired motion of the platform is always constrained and therefore, the end-effector twist is always the $n$-dimensional one associated with the full-cycle mobility, $n$;

b) For each $t_{\text{red}}$, there exist unique definite $\dot{q}$ and $\dot{\psi}$;

c) And finally for each $\dot{q}$, there exist unique definite $t_{\text{red}}$ and $\dot{\psi}$, with $t_{\text{comp}} = 0$ necessarily.

It is worth mentioning that different tools can be used to verify the aforementioned ideas, whether linear algebra that we are going to use later in the analysis of our mechanisms, or Grassmann geometry (e.g. (MERLET, 2006), (BEN-HORIN & SHOHAM, 2009 ), (MBAREK, et al., 2007)).
Furthermore, it is important to emphasize that in most of the studies of singularities done at LIRMM, the nomenclature as adopted by (KRUT, 2003) is usually used. This nomenclature categorizes singularities into: series singularities or under-mobilities, parallel singularities or over-mobilities, and the rest being called internal singularities. Note that the “internal” adjective used in this context refers to the internal structure of the mechanism and its internal mobilities, and not in the sense of having the singularity inside the accessible workspace.

The different three types of singularities are illustrated in Fig. 1-6. In the left drawing, the second slider can perform an infinitesimal motion while the TCP\(^{9}\) is fixed, which indicates series singularity. In the middle diagram, the TCP can do an infinitesimal motion along the \(x\)-axis while the actuators are fixed indicating a parallel-type singularity. In the right hand side of the same figure, we can fix both, the actuator and the TCP, but still the point \(B\) can do an infinitesimal motion along \(x\) (indicating an infinitesimal rotation of the passive revolute at \(B\)); this is an internal/RPM singularity.

1.2- State of the Art of PMs: Between Theoretical Synthesis and Industrial Implementation

1.2.1- Type Synthesis of Parallel Mechanisms

Before exploring the available PMs, it is important to talk first on their type synthesis. In fact, the existing approaches can be classified into three main classes ((MERLET, 2006), (SICILLIANO & KHATIB, 2008)):

a) Approaches based on graph theory (e.g. (EARL & ROONEY, 1983)): For a given number of dofs, these methods assume a set of finite number of possible kinematic pairs, and then enumerate all the possible architectures that can be obtained by the different combinations of these pairs. Such methods utilize classical mobility formulae, e.g.

\[^{9}\text{Tool Center Point}\]
Chapter 1: State of the Art

Chebychev–Grübler–Kutzbach formula, to establish a relation between the structural parameters (number of joints, joint constraints, number of links) and the end-effector dofs. Nonetheless, as these formulae do not account for the geometrical constraints, they might not correctly predict the actual number of dofs (refer to (GOGU, 2005a)). Therefore, synthesis based strictly on graph theory can yield limited results and has been largely outdated.

b) Approaches based on group theory (e.g. (HERVE, 1995), (SPARACINO & HERVE, 1993), (ANGELES, 2004), (LI, et al., 2004)): These methods make use of the fact that rigid body displacements can be described by the structure of a group, referred to as displacement group. Within it, we have different subgroups, such as the spatial translations and the Schönflies (3T-1R) motion. However, not all body displacements form a subgroup, such as the 3T-2R motion. Therefore, 3T-2R mechanisms, for instance, cannot be synthesized using this method. The methodology of synthesis, based on this approach, is done in the following manner:

i. First, the subgroup $S$ that describes the desired end-effector motion is determined.

ii. Then, all the possible subgroups whose intersection is $S$ are established.

iii. Eventually, all the motion generators of these subgroups are considered. These will constitute the kinematic chains of the PM.

c) Approaches based on screw theory (e.g.: (FRISOLI, et al., 2000), (KONG & GOSSELIN, 2001), (KONG & GOSSELIN, 2004a), (FANG & TSAI, 2002), (HUANG & LI, 2002), (CARRICATO, 2005)): In these approaches, the wrench system $S$ that is reciprocal to the desired end-effector twist $T$ is determined in a first step. Then, the wrenches of the kinematic chains of the PM whose union spans $S$ are enumerated. These latter wrenches will be used to determine all the possible structures of the kinematic chains constituting the PM. Nevertheless, as these wrenches and twists are instantaneous, it is mandatory to verify that the platform mobility is the full-cycle mobility and not only the instantaneous one (for more details, refer to (KONG & GOSSELIN, 2007)).

We may add here a fourth approach, which is that based on the theory of linear transformations adopted primarily in the synthesis of the mechanisms described by ((GOGU, 2004a), (GOGU, 2007), (GOGU, 2009)) and several books for the same author. In these works, the new mobility and spatiality formulae, which have been established in (GOGU, 2005b), are used to overcome the limitations of the classical mobility formulations.

Based on the above methods, thousands of mechanisms are and can be synthesized, and it is rather impossible to keep track of even the fewer interesting ones emphasized in literature. In this aspect, it is sufficient, for instance, to look on the gathered lists of articles and patents on the webpages\(^{10}\) of Dr. BONEV and Dr. MERLET to know the massiveness of these contributions,

reflecting both the interest and hopes held on such parallel structured architectures; also, they sadly highlight the gap between theory and industry.

Among the synthesized mechanisms, it is worth stopping at some intriguing architectures based on more recent trend towards maximally regular\(^{11}\), fully isotropic\(^{12}\), uncoupled\(^{13}\), or decoupled\(^{14}\) motions. Such mechanisms, especially the first three types, are interesting in the sense of their simplicity regarding control and energy saving (GOGU, 2007). This is due to that in the case of a unidirectional motion along one axis, only one motor is working while the others are locked. Nevertheless, such mechanisms are not fully virtuous and have their drawbacks, which we are going to emphasize at the end of the section. First, let us present some samples of such architectures, mainly with 3T, 3T-1R, and 3T-2R motions.

**Fig. 1-7:** Two fully isotopic translational PMs: the left is one of the T\(_3\) family (it is more precisely maximally regular), whereas on the right is a type II of T\(_4\) family (CARRICATO & PARENTI-CASTELLI, 2002).

**Fig. 1-8:** The isotropic 3-CRR translational PM (KONG & GOSSELIN, 2002).

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\(^{11}\) Meaning with identity Jacobian matrix throughout the workspace.

\(^{12}\) Meaning with diagonal Jacobian matrix such that the diagonal entries are the same throughout the workspace.

\(^{13}\) Meaning with diagonal Jacobian matrix such that the diagonal entries are not necessarily equal throughout the workspace.

\(^{14}\) In this case, the Jacobian is a triangular matrix throughout the workspace.
Regarding translational mechanisms, (CARRICATO & PARENTI-CASTELLI, 2002) presented a topological synthesis for fully isotropic symmetric PMs with 3T motion. Their approach has been based on constraint and direct singularity investigation, in the purpose of their elimination. Two examples of such PMs are depicted in Fig. 1-7. Among other works, we mention, for instance, the fully isotropic version of the 3-CRR of (KONG & GOSSLIN, 2002) (Fig. 1-8), the fully isotropic 3-PRRR manipulator of (KIM & TSAI, 2002), the fully decoupled (more precisely fully isotropic) Pantopteron of (BRIOT & BONEV, 2009), and the synthesized fully isotropic translational PMs of (GOGU, 2004c). In Fig. 1-9, an illustration of the Pantopteron is given. This manipulator has been synthesized for pick-and-place applications and is characterized by the use of pantograph linkages to amplify the actuators’ displacements. Also, a four-dof (3T-1R) version with uncoupled motions has been presented in (BRIOT & BONEV, 2009) as well and is depicted in Fig. 1-10.
Chapter 1: State of the Art

Fig. 1-11: Models of two fully isotropic mechanisms for Schoenflies motion (CARRICATO, 2005).

Fig. 1-12: Example of basic kinematics structure of PM with decoupled Schönflies motions: Isoglide4-T3R1-A5 schematic and its constructed prototype at the French Institute of Advanced Mechanics (GOGU, 2007).

As for fully isotropic parallel manipulators with Schönflies motion, they have been proposed in ((GOGU, 2004b), (GOGU, 2005c), (CARRICATO, 2005), (GOGU, 2007)) for the first time. Also, other uncoupled and decoupled PMs with 3T-1R motions have been suggested in (GOGU, 2007). In Fig. 1-11, two examples of the PMs suggested by (CARRICATO, 2005) are depicted, while in Fig. 1-12, Fig. 1-13, and Fig. 1-14, examples on decoupled, uncoupled and fully isotropic PMs among those synthesized in (GOGU, 2007) are given.
This was regarding 3T and 3T-1R PMs. In what concerns five-dof (3T-2R) PMs, the synthesized architectures are rather few in comparison with others (e.g. (FANG & TSAI, 2002), (GAO, et al., 2002), (HUANG & LI, 2002)) and are mostly coupled. In fact, the synthesis of the first maximally regular non-redundant or redundantly actuated PMs of 3T-2R motion can be tracked back to the recent work of ((GOGU, 2006a), (GOGU, 2006b), (GOGU, 2006c), and (GOGU, 2009)). In (GOGU, 2009), some decoupled and uncoupled five-dof (3T-2R) PMs, in addition to the maximally regular ones, have been synthesized as well. In Fig. 1-15, several examples of such PMs are presented.
Chapter 1: State of the Art

As we have given an overview on particular types of synthesized mechanisms, with decoupled and uncoupled motions, it is important to make some points in this aspect. In fact, despite the interesting features that uncoupled mechanisms provide, they suffer from the following limitations:

- Most of these mechanisms are characterized by large number of passive joints and links. This is discouraged from industrial perspective. The reasons behind that are the added complexity, increased masses, reduced stiffness, and reduced precision, especially when there are joint clearances.
- Most of these designs have limited workspaces due in one part to chain complexities and possible collisions, and in another part to the use of prismatic joints with non-parallel axes and/or use of rotational actuators.

In brief, this section has provided a summary on the type synthesis of PMs and highlighted some recent trends in this domain. In fact, while the innovative approaches of synthesis of the
last decades have led to thousands of architectures, yet those that have been implemented and adopted for industrial applications are extremely few and countable. This clearly highlights the huge gap between theory and application.

In the next section, we will present some of the parallel robots and parallel kinematic machine tools that have been so far implemented. In addition, we will talk about some contributions that have been made in this scope.

1.2.2- Some Parallel Robots and Parallel Kinematic Machine Tools

In this part, we are going to expose several industrial machine tools based on parallel kinematics, as well as some other implemented prototypes or industrial parallel robots. The exposition is done based on the number of dofs. The list presented is a non-exhaustive one. Here, we focus on four-dof (3T-1R), five-dof (3T-2R), and six-dof (3T-3R) designs only.

In fact, most industrial applications do not require more than five dofs of 3T-2R nature. In others, fewer might be even sufficient. However, these five dofs can be achieved via different means:

- Six-dof (3T-3R) design: In this case, one of the rotational dofs, particularly that whose axis is parallel to that of the spindle, is disregarded. The result is task redundancy, which helps in enlarging the workspace. Such machines have been firstly designed, yet not all made proper use of the aforementioned redundancy.
- Five-dof (3T-2R) structure: In this case, the design is intended for the real need of five axis machining or other industrial applications.

Note that within each of the above two categories, hybrid structure, whether in the form of series-parallel or right-hand left-hand paradigm (branched structure), can be used. Our presentation starts with six dofs down to four dofs, which constitute the scope of the current thesis.

A- Six DoFs (3T-3R)

In addition, to the Gough-Stewart (aka hexapod) based designs (e.g. Fig. 1-1, Fig. 1-3, Fig. 1-5), some others with slight or major modifications have been implemented in recent decades. Here, we are going to focus only on those that have some interesting features.

Hexa, HexaM, Hexaglide, Dynamil, and Lambda Kinematics

In fact, after the introduction of Delta robot and the great success it achieved, many researchers investigated the possibility of extending this idea to the synthesis of other types of PMs. In ((PIERROT, 1991), (PIERROT, et al., 1991a), (PIERROT, et al., 1991b)), the design of a six dof (3T-3R) robot, assimilating the features of Delta regarding rapidity and lightness, has been investigated and led to Hexa robot, which is depicted in Fig. 1-16. Later, the aforementioned robot has been industrialized by Toyoda Machine Works Ltd. under the name of HexaM
Chapter 1: State of the Art

(PIERROT & SHIBUKAWA, 1999), and it was intended for milling applications. The graph diagram and CAD drawing of HexaM are shown in Fig. 1-17. Compared with Hexa, the rotary actuators have been replaced by prismatic ones to avoid the high bending on the actuated arms. Thanks to having the actuators at the base and the light structure, HexaM is very similar in performance to Delta robot. It is worth mentioning here that HexaM, as compared with classical hexapods, has the placement of the prismatic and first universal joints interchanged in each kinematic chain (we have 6-PUS instead of 6-UPS).

Another interesting six-dof PKM is Hexaglide (WIEGAND, et al., 1996), a concept developed at the Institute of Machine Tools of the Swiss Federal Institute of Technology Zurich, for milling.
applications as well. The machine is similar to HexaM. The difference is in having all the linear actuators along one direction and hence, allowing independent motion along this axis. The schematic of the PKM is illustrated in Fig. 1-18.

Similar machines to the aforementioned ones exist. These have same graph diagram, but different geometric configurations (e.g.: Paralix PKM of IFW at Stuttgart and Dynamil PKM of ISW at Aachen, which is depicted in Fig. 1-19).

The Lambda Kinematics PKM, introduced in the Mach21\(^{15}\) project, is a slight variant of the aforementioned concepts. Instead of connecting the arms directly from the base into the platform, the arms are considered in pairs. In each pair, one of the arms is connected into the other that is then articulated to the platform. This limits the risk of internal collisions (see Fig. 1-20).

Unfortunately, still such machines suffer from workspace limitations and/or reduced tilting capacity, as it is the case with the classical hexapods.

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\(^{15}\) Mach21 (Multipurpose and cross-sectorial modernization of manufacturing processes through parallel kinematics) is a European research program that involved 12 partners: Fatronik (currently Tecnalia), WZL, INA, IWU, Karl Mayer, LIRMM, Anayak, Gamesa, ISW, Comau, Arjal, and Lernsttat. It began in January 2000 and ended in January 2003.
Fanuc M-3iA Robot

A different six-dof variant from the above and explicitly implementing the Delta structure exists. It is the Fanuc industrial robot with up to six dofs (3T dofs and up to 3R dofs). Actually, it is a Delta robot equipped with a wrist and it has been developed as high-speed and assembly robot (see Fig. 1-21). The actuators of the wrist are placed on the parallelograms, next to the actuated arms, as to have them as closer to the base as possible and hence, reducing their impact regarding dynamics. It is characterized by a cycle time of 0.3 s for 0.1 kg load and a pick-and-place path of $25 \text{ mm} \times 200 \text{ mm} \times 25 \text{ mm}$. Note that this robot is a complex parallel structure and not a hybrid one. This is because the actuation of the wrist dofs is not done in place, but by means of transmission chains from the actuators on the parallelograms.

Hexapteron

Finally, we end the panorama on six-dof 3T-3R category by highlighting an interesting recently introduced design, the Hexapteron of (SEWARD & BONEV, 2014). It is characterized by a simple direct geometric model and can be used for machining or rapid prototyping applications. The prototype for 3D printing application is under construction. First simulations show a tilting capacity of $45^\circ$ about any direction. The manipulator is depicted in Fig. 1-22.
Such a machine might suffer from inter-collisions and therefore, a considerable attention for parts design must be paid. Furthermore, the different directions of actuators and the limits imposed by singularities highly constrict the workspace.

**B- Five DoFs (3T-2R)**

**Seyanka**

Seyanka is a five-axis milling machine designed by Tekniker, as a prototype to demonstrate the suitability of PKMs for high-speed milling applications (HERRERO, et al., 2000). The machine was publically presented in the Machine Tool Show of Bilbao in 2000 (BIEMH 2000). In this design, the usually unused rotation of the platform (i.e. that about the same axis of the spindle) has been removed, thanks to the passive chain (see Fig. 1-23). Also, compared to other machines with similar structure, the configuration of the actuators in the initial position is done with 90° symmetry instead of 120°. This is more suitable for the intended application, since it allows the working volume to be almost prismatic-shaped. According to (HERRERO, et al., 2000).
In 2000, the machine is characterized by a speed of 60 m/min and an acceleration of 10 m/s² (i.e. 1 g).

**P 800**

In Fig. 1-24, we depict the P 800 (Metrom) machine. It obeys the right-hand left-hand paradigm, where a five-axis (3T-2R) parallel module is incorporated with a one-dof (1R) turntable. This machine is characterized by elevated tilting capacity, but the workspace is highly irregular, mainly due to arm inter-collisions. Actually, this was the reason behind equipping the machine with a turntable. The mechanism is kinematically redundant and perhaps, the only one so far that intentionally makes use of such redundancy to enhance the angular motion capability.

**Tricept and Exechon**

Among the successful five-axis machines based on parallel kinematics is the Tricept (Neos Robotics AB), which is depicted in Fig. 1-25. It is a typical hybrid machine with parallel structure followed by a series wrist, which therefore, allows large rotational motion. The machine, like the Seyanka, utilizes a passive kinematic chain (the U-P chain) to constrain the platform of the parallel structure. Among the characteristics of the machine, we mention the following: ±50 μm accuracy, ±10 μm repeatability, 90 m/min maximum feed rate, and 2 g maximum
acceleration\textsuperscript{16}. The applications of this PKM range from light machining (e.g. cutting aluminum, plastic, wood, composites) to applications where the required path accuracy and/or high process force cannot be handled by conventional robots (e.g. friction welding) (DONG, 2002).

Nevertheless, Tricept is not stiff enough for some high-precision manufacturing tasks, such as aircraft assembly (NEUMANN, 2006). To overcome this limitation, another machine tool, called Exechon (see Fig. 1-26), has been developed by Exechon AB. It is also formed of a parallel module followed by a series wrist. Exechon has shown better performance as compared to Tricept (JIN, et al., 2012).

![Fig. 1-25: Tricept 845 (Neos Robotics AB): photo and graph diagram.](image1)

![Fig. 1-26: Exechon (Exechon AB): photo and graph diagram.](image2)

**Sprint Z3 and Hermes**

Another hybrid PKM, the Sprint Z3 (DS Technology), is depicted in Fig. 1-27. The PKM has been developed for aeronautical industrial applications. It consists of series–structured carrier providing the \textit{xy}-motion. On this carrier, a parallel module is fixed. The latter provides the wrist motions (two rotations) and the translational motion along the \textit{z}-axis (the direction of the prismatic actuators). The design is quite simple, and it has met a real commercial success. Concerning its capabilities, it can achieve 50 m/min speed, 1g acceleration along each axis,

\textsuperscript{16}See: \url{http://www.pkm-news.de/deu/triceptmc845.html}.
±40° orientation (A/B)\(^{17}\), 80°/s maximum rotational speed (A/B), and 685°/s\(^2\) rotational acceleration (A/B). It is worth mentioning here that the Z3 head belongs to the group of so-called 3-[PP]-S\(^{18}\) parallel mechanisms, defined as those whose three spherical joints move in vertical planes intersecting at a common line (LIU & BONEV, 2008). Such architectures are referred to as zero-torsion\(^{19}\) mechanisms, and they are characterized by simple kinematic models (see BONEV, 2002)).

Inspired by Sprint Z3, another similar PKM has emerged, which is Hermes (Fatronik, now Tecnalia). The PKM and its graph diagrams are shown in Fig. 1-28. As it can be clearly noticed, the only difference is using (U-S)\(_2\) instead of R-S chains.

**Dumbo**

It is another hybrid machine (one dof carrier + parallel module + series wrist) developed at IFW of University of Hannover (Germany). It is presented in Fig. 1-29. Such a machine though has the advantages of large workspace and tilting capacity, it falls in the trap of a serial structure and the pyramidal effect regarding moving masses, thus influencing dynamics.

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\(^{17}\) “A” and “B” are the designation of rotations about x and y axes respectively (nomenclature adopted in machine tools)

\(^{18}\) [PP] denotes any combination of joints that allows two-dof planar motion (adapted from BONEV, 2008)).

\(^{19}\) Tilt-and-Torsion (T&T) is an orientation representation, like Euler angles, but that allows better interpretation and simpler presentation. For details, refer to BONEV, 2002, pp. 75-97.)
Orthoglide Five-Axis Version

An interesting five-dof (3T-2R) prototype, namely the Orthoglide five-axis version\(^\text{20}\) (CHABLAT & WENGER, 2003), has been implemented at IRCCyN. The design combines the

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Orthoglide three-axis version (Wenger & Chablat, 2000), (Chablat & Wenger, 2003) and the two-dof (2R) Agile Eye (Gosselin, et al., 1996), with the former being a translational parallel mechanism and the latter a parallel spherical wrist. The Orthoglide represents a complex parallel architecture, and not a hybrid one (similar to the Fanuc robot in Fig. 1-21). This becomes clear by inspecting its graph diagram. In fact, if the actuation of the wrist were done in place, we would have a hybrid structure. Here, it is not the case. Instead, two transmission chains are used to transfer the motion from the rotary motors at the base to the wrist (see Fig. 1-30).

**VERNE**

VERNE is a hybrid branched-structure machine developed by Fatronik (now Tecnalia). It consists of three-dof parallel module and two-dof (2R) turntable (see Fig. 1-31). For the first glance, the parallel module seems to be a Delta structure, but a closer look reveals that this is partially true. In fact, unlike Delta where all the three arms are parallelograms, VERNE is characterized by having one arm being a trapezium and the others being classical parallelograms. Also, the trapezium individual rods are shorter than the other ones. This asymmetric structure of VERNE results in a coupled rotation of the tool about one of the axes with its translational motion (Kanaan, et al., 2009).

**Hita-STT**

The Hita-STT hybrid machine introduced by (Clavel, 2002) falls also within the category of left-hand right-hand structures. It is composed of four-dof (3T-1R) PM (see Fig. 1-32) and a one-dof (1R) turntable. An interesting feature of this PM is having parallel sliders and thus, allowing independent motion along their direction. This PM is capable of $\pm 60^\circ$ rotation and its stiffness is ensured in any position. Despite of that, it is not able to handle large parts due to the vertical axis of the rotary table (Ancuta, 2008). Another disadvantage is the complicated mechanism in the architecture that provides rotation. Possible applications can be light finishing operations, such as polishing or deburring.
Chapter 1: State of the Art

Among other interesting five-dof (3T-2R) machines, we mention that of (GAO, et al., 2006) (see Fig. 1-33). It is composed of five limbs: four of them are of PSU type and the remaining fifth limb is a complex one denoted by PU*U. In all the limbs, the P-joint is actuated. The fifth limb can be described precisely as P-(U-U)₃-U, where (U-U)₃ corresponds to U* between the slider and the U joint. This fifth chain is the constraining one, which allows having the five dofs (3T-2R).

Fig. 1-32: Hita-STT (parallel module): CAD drawing and graph diagram.

Fig. 1-33: A five-dof (3T-2R) PKM: global schematic, fifth limb illustration, and machine photo (GAO, et al., 2006).

Other Interesting Machines
We terminate the section with the Reconfigurable Gantry-Tau PKM (ABB Robotics) and the ROBOTEX/Spider4 machine tool recently developed at LIRMM. The actuators in the former ([Fig. 1-34]) are: three fixed parallel electrically driven linear ball screws for positioning and two piston type electrically driven ball screws for tool orientation. The main reason for the industrial interest in this structure is that high machining performance can be obtained without needing the very heavy, expensive and difficult to install serial gantry robots used today. Moreover, its accessible workspace is larger than that based on serial kinematics (for more information, refer to [HUNT, 2007]). In fact, having the three positioning linear actuators along one direction, allows for independent motion along this direction, only limited by the available stroke. As for ROBOTEX/Spider4, it consists of three-dof (3T) redundantly actuated parallel structure and two-dof (2R) series wrist. The former structure is inherited from R4 robot (CORBEL, et al., 2010) that has been also developed at LIRMM and that achieved incredible acceleration of 100 g (CHEMORI, et al., 2013). The actuation redundancy helps in eliminating singularities as well as
homogenizing and elevating dynamic performance over the workspace. In numbers, it is expected to achieve an acceleration of about $4-5 \, \text{g}$, but still real performance is to be evaluated. The machine is depicted in Fig. 1-35. As it can be noticed, it is equipped with measuring struts to better measure the pose of the robot and hence, promote its precision characteristics. Finally, we mention that at LIRMM, two other designs that incorporate five dofs (3T-2R) in one non-hybrid structure and exploit both, actuation redundancy and parallel linear actuators at the base, have been suggested by (KRUT, et al., 2003) and (ANCUTA, 2008)(see Fig. 1-36 and Fig. 1-37). The former is interesting regarding its workspace, performance homogeneity, and large tilting capacity, but the main limitation is in the use of gear or tendon based mechanism that may influence accuracy. Regarding the second, the collision related aspect and its impact on tilting capacity could be a drawback, though it has the advantage of large spatial workspace and performance homogeneity. A more recent design by (SHAYYA, et al., 2014a) has been also proposed. It utilizes an articulated platform and parallel linear actuators, but this time with only five motors. This will be discussed later in Chapter 3.

![Fig. 1-36: Eureka (KRUT, et al., 2003): CAD design and graph diagram.](image1)

![Fig. 1-37: Five-dof (3T-2R) manipulator (ANCUTA, 2008): schematic and graph diagram.](image2)
C- Four DoFs (3T-1R)

Perhaps, it is classical to start with the well-known and industrialized Delta robot with RUPU chain of (CLAVEL, 1991) (see **Fig. 1-2**). As early mentioned, thanks to the fixed actuators, lightweight components and use of parallelograms with simple technology, such robot is characterized by an exceptional performance (10 m/s speed and 15 g for 1 kg charge\(^2\)). As for positional repeatability, it is about 0.1 mm\(^2\). Moreover, the robot is characterized by large workspace, especially if linear Delta is considered (i.e. using linear actuators instead of rotary ones for the parallelograms). Nonetheless, this workspace remains restricted by the RUPU chain and particularly the available stroke for its corresponding prismatic joint. Also, the rotational stiffness can be impaired due to the use of universal joints; this is not to mention the vibration issues. These latter shortcomings have motivated some researchers in the quest for even better designs.

In the work of (ROLLAND, 1999), two robots, called Manta and Kanuk, have been introduced for industrial handling purposes. These are depicted in **Fig. 1-38** and **Fig. 1-39**, respectively. In both designs, the two undesired rotational motions are constrained, thanks to the pair of parallelograms that does not permit except one rotational motion (specifically that about the common axis). Their interesting features are the parallel linear actuators that help enlarge the

![Fig. 1-38: Manta (ROLLAND, 1999): schematic and graph diagram.](image)

![Fig. 1-39: Kanuk (ROLLAND, 1999): schematic and graph diagram.](image)

\(^2\) See for example: [http://www05.abb.com/global/scot/scot241.nsf/veritydisplay/c1b594e2b0a6f035c1257403005371b2/$file/Fiche%20IR8%20360%20FR%20rev1.pdf](http://www05.abb.com/global/scot/scot241.nsf/veritydisplay/c1b594e2b0a6f035c1257403005371b2/$file/Fiche%20IR8%20360%20FR%20rev1.pdf) .
workspace along their direction on one hand, and having the sliders in the same plane on the other hand, which contributes to design simplicity. Moreover, the first robot, which has been implemented, has its rotation provided by a rotary actuator placed on the linear one and transmitted by a UU chain. Hence, its rotational capacity is unlimited and does not restrict the workspace, as compared with what we had in the case of Delta. Yet, it suffers from the possible impact on stiffness, in addition to the vibration issue. As for the Kanuk, the structure is fully parallel and replaces the PRUU chain by two PSS chains. This robot may be better than Manta for two reasons: the first is getting rid of the UU chain and the second is reducing the number of slider guides yielding an even simpler design, with fewer parameters. The latter point is particularly interesting, especially when dealing with calibration-related issues. Nevertheless, the Kanuk, as compared with Manta, is characterized by a limited rotational capacity.

In the same scope and in the attempt to eliminate the use of RUPU chain in Delta robot, several works have been done at LIRMM embodied in the doctoral theses of (COMPANY, 2000), (KRUT, 2003), (NABAT, 2007), and (ANCUTA, 2008). In fact, among the interesting contributions of (COMPANY, 2000), is the idea of articulated platforms and that led to the H4 family of symmetric and asymmetric robots. Later then, H4 concept evolved within the work of (KRUT, 2003) to the I4 version and afterwards, the concept of Par4 emerged with the work of (NABAT, 2007). This latter version has been industrialized under the name of Adept Quattro and is the world’s fastest-pick-and-place industrial robot. In (ANCUTA, 2008), an extension of those works has been done, and architectures with Schönflies motion and rigid platform have been proposed. In what follows, we emphasize some points regarding the aforementioned works.

Concerning the H4 robot, it consists of four parallelogram arms connected to three-part articulated platform. The H4 prototype is depicted in Fig. 1-40. The rotary actuators can be replaced with prismatic ones, and the platform can be substituted by another one including a gear sub-mechanism to amplify the rotational capability (see Fig. 1-41). Furthermore, in the latter case, a redundant sensor to measure the rotational angle can be added; this version is depicted in Fig. 1-42. Nonetheless, H4 has the following limitations:

- The conditioning of the Jacobian matrix may vary considerably with tool orientation and hence, implying a change in machine behavior (KRUT, et al., 2003).
- Depending on the practical design, internal collisions might occur.
- Also, as discussed in (PIERROT & COMPANY, 1999), the relative positions of the four “spatial parallelograms” must be properly selected to avoid singularities (particularly internal ones). Specifically, placing them at 90° from each other is not recommended, though it would be interesting to have a symmetrical design with respect to the vertical axis for practicality reasons.
Starting from the aforementioned issues, the I4 family has been suggested by (KRUT, et al., 2003). The basic idea behind I4, as compared with H4, is the replacement of pivot joints by prismatic ones and gears by rack-and-pinion assembly. In Fig. 1-43, I4L (“L” meaning with linear actuators) is shown with its graph diagram. However, for high-speed applications, the use of prismatic joints in the mobile platform is less recommended due to short-life service. Based on this, Par4 has been developed later by (NABAT, et al., 2005) utilizing only revolute joints in the
articulated platform. Also, two amplification systems for rotation have been proposed: the first utilizes a gear assembly and the other exploits pulleys and cables. The generic CAD drawing and graph diagrams are depicted in Fig. 1-44, and the industrialized version is shown in Fig. 1-45. Another robot, also proposed by (NABAT, 2007), is the Héli4. The idea of this latter robot is to achieve the four dofs (3T-1R) by a compact simple platform. This has been accomplished by means of two-part platform joined by screw-nut system. The Héli4 prototype and its graph
diagram are depicted in Fig. 1-46. The robot has been later industrialized by Penta Robotics under the name of Veloce\textsuperscript{22} (see Fig. 1-47). The robot has rotational capacity of up to 180°. Its cycle time is 0.32 s, for a 0.2 kg load and a pick-and-place path described by

\textsuperscript{22} See: http://pentarobotics.com/products/. Notice also that the slight difference between Veloce and Héli4 is the presence of prismatic guides in the former connecting the two parts of the platform.
25 mm×305 mm×25 mm. Positional repeatability of Veloce is about 0.1 mm (according to the technical specifications provided by Penta Robotics\textsuperscript{22}). We end the overview on the LIRMM robots by presenting the λ-Quadriglide-V1 of (ANCUTA, 2008). In (ANCUTA, 2008), the demerits of articulated platforms, such as reduced stiffness and accuracy, were the motive behind seeking a rigid-platform-based manipulator while maintaining large workspace and design
simplicity. The result of this work was the λ-Quadriglide-V1 consisting of parallel linear actuators and crank-like platform (see Fig. 1-48). In this design, two parallelogram arms are used to constrain undesired rotations with the other chains being of simple type. An interesting point in this mechanism is the pairing of arms, which is done in λ-shaped scheme, rendering it more feasible from manufacturability point of view. In this case, the use of spherical joints based on ball-socket and spring technology is eliminated together with their undesirable effects. The angular motion limits are in the vicinity of ±45°.

We end the panorama of four-dof (3T-1R) robots by the Schönflies Motion Generator (SMG) (ANGELES, et al., 2006) developed at McGill University, and the robot introduced by (KIM, et al., 2009). The SMG (see Fig. 1-49) consists of two R||T|R chains with the || indicating a parallelogram linkage playing the role of a kinematic pair, termed ||-joint. In each chain, the revolute and the first || (one revolute of ||) are active. The robot has been designed with the target of achieving a pick-and-place cycle within at most 500 ms for the path described by 25 mm×300 mm×25 mm, with a concomitant 180° turn. However, such design may suffer from poor rigidity and vibration. As for the robot of (KIM, et al., 2009) (see Fig. 1-50), it utilizes similar but non-identical kinematic chains. Each is composed of a simple actuated arm in series with a parallelogram one. The parallelogram is connected to the platform with a revolute joint parallel to the axis of rotation of the platform, as depicted in Fig. 1-50. However, having identical chains results in an architectural singularity, which occurs at the center of the workspace. Thus, to get rid of such singularity, an asymmetric design must be considered (similar to what we have in H4). Consequently, this would result in a highly inhomogeneous performance.

Finally, we mention that a more recent four-dof (3T-1R) PKM with actuation redundancy has been introduced and studied by (SHAYYA, et al., 2013a), (SHAYYA, et al., 2013b), (SHAYYA, et al., 2014c) and (SHAYYA, et al., 2014b)). This PKM and its different modifications will be discussed later in Chapter 3.
With this, we end the overview on PKMs. In the next part, the performance evaluation regarding the precision of LIRMM rapid prototypes will be discussed.

1.3- Precision-Related Performances of Some Prototypes

In order to have insights on the precision-related performances of the most rapid parallel manipulators, a precision evaluation of several prototypes and industrial robots has been done at LIRMM. This assessment considered accuracy\(^{23}\) and repeatability\(^{24}\), not only for static poses but also in dynamics, i.e. as the robot follows a pre-specified trajectory. Moreover, multi-

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\(^{23}\) It is the distance between the mean of the attained poses and the commanded one. The closer it is to zero, the better.

\(^{24}\) It is the radius of the smallest sphere centered at the mean pose and circumscribing all the attained ones.
directional variation of accuracy\textsuperscript{25}, in the case of statics, has been investigated as well. The norm adopted for all the aforementioned tests is [ISO 9283; 1998 (F)].

The study considered the following robots: Quattro, Par2, DUAL V, and Veloce (the prototype). However, regarding dynamic performance (path following), not all robots were possible to evaluate due to technical issues and constraints.

In this section, we are not interested in detailing the results but rather synthesizing the main points, and emphasizing the major aspects and conclusions. For more details and clarifications, refer to Appendix A.

Briefly, in the light of the aforementioned static and dynamic precision evaluation, we have concluded that the positional accuracies and repeatabilities are in the order of millimeters and tenths of millimeters. As for orientation errors, the accuracies and repeatabilities are mainly degraded for the controlled rotation. Regarding the constrained orientations, their accuracies and repeatabilities are usually very small and due to assembly errors. Furthermore, the static repeatability results of Par2 and DUAL V, being between 10\,\mu m and 25\,\mu m, are quite interesting\textsuperscript{26}, though their accuracies do not differ much from the other robots. In particular, the good overall performance of DUAL V has been of great impact and provided many insights. It accentuated that actuation redundancy is not problematic, especially when well treated on the control level. Based on the results of Quattro with rigid platform, such a conclusion is not possible to make. In addition, the use of revolute joints, in comparison with the joint technology used for parallelogram spherical joints in Quattro and Veloce, has been an essential factor in promoting DUAL V precision\textsuperscript{27}, and particularly its repeatability. We add to this, the use of direct actuation that eliminates backlash problems, which are usually inherent with the use of gear trains. Furthermore, the results of Veloce and Quattro (in its articulated version) emphasize the limitations of articulated platforms.

Based on the above, it becomes obvious that though the aforementioned robots are characterized by high rapidity, their precision is not sufficient for high-speed and high-precision applications. Thus, reaching a good compromise between rapidity and precision, the target of the ARROW project, should be planned along three directions:

1. The first direction is synthesizing the ideal mechanical structure and design of the PKM, while considering large workspace requirements and design simplicity.
2. The second direction is the proper choice of the optimization criteria for the dimensional synthesis of the geometric parameters of the PKM. This is essential and presents some

\textsuperscript{25}It is the largest distance between the means of attained poses along different directions.

\textsuperscript{26}Note that the repeatability of the measurement device is about 10\,\mu m. Note also that accuracy results, especially for DUAL V, are not very reliable due to several technical issues (refer to Appendix A).

\textsuperscript{27}Precision, as referred to in several places in literature, is more related to repeatability than accuracy. Here, we will use the words “precision” and “precise” to refer globally to all terms, such as accuracy, repeatability and resolution (operational resolution), which we will discuss later in Chapter 2.
Chapter 1: State of the Art

barriers to overcome. We expose this issue, highlighting the state of the art in this matter, in the upcoming section.

3. The final point is the proper control strategy, in which different advanced control laws are to be investigated. This point is not within the scope of the current thesis.

1.4- Performance Criteria and Parallel Kinematic Machine Tools

Thus far, we have exposed a non-exhaustive overview on parallel mechanisms and some of their precision evaluation. The main conflict that can be concluded regarding their poor exploitation in industry is the lack of architectures that have large singularity-free workspace, large tilting capacity, high stiffness, and design simplicity. Besides, so far we cannot find a PM that is both rapid and acceptably precise for handling high-speed and high-precision applications, such as laser cutting, welding, etc. Nevertheless, if finding the proper architectures is one of the prominent issues hindering PKMs’ wide-spread, the performance evaluation and the criteria upon which these PKMs are dimensionally synthesized are of an equal importance. As (WECK & STAIMER, 2002) mentioned, “A poor topology but optimally designed may perform better than a mechanism with appropriate topology, but poor design.” Moreover, if classically established performance measures have been more or less suitable for the synthesis of non-redundant robots with homogeneous dofs and same nature of actuators, still they are most often inadequate and debatable for robots characterized by redundancy or heterogeneous dofs. Actually, performance evaluation, despite of its importance and the rich research done in this scope, remains an open issue.

In this section, we discuss a non-extensive list of the available measures and their suitability for design and dimensional synthesis.

In general, these measures deal with singularity, precision, kinetostatics, dynamics, stiffness, natural frequencies, etc. In this thesis, we will be focusing on the first four domains, in which our contribution mostly falls.

Among the precision and kinetostatic measures, we mention for exemplification: the condition number $\kappa$ (SALISBURY & CRAIG, 1982), the manipulability index $\mu$ (YOSHIKAWA, 1985b), the global conditioning index GCI (GOSSELIN & ANGELES, 1991), the motion/force transmissibility indices ((XIE, et al., 2011), (CHEN & ANGELES, 2007), (WANG, et al., 2010)), the minimum singular value (STOUGHTON & KOKKINIS, 1987), the use of semi-axes lengths of largest inscribed ellipsoid and pseudo-condition numbers ((KRUT, et al., 2004b) (KRUT, et al., 2002), (KRUT, et al., 2004a)), etc. It is worth mentioning that the aforementioned indices hold also some singularity significance, particularly concerning singularities related to the Jacobian matrix. Even some of them (e.g. condition number, manipulability, etc.) have been established primarily with the target of measuring the quality of the pose and its farness from singularity,
all while embracing some physical significance. Nevertheless, as other types of singularities\textsuperscript{28} are not detectable by these measures, we prefer listing them under the aforementioned category, at least in what concerns PMs.

In fact, most of these works ([SALISBURY & CRAIG, 1982], [YOSHIKAWA, 1985b], [GOSELLEN & ANGELES, 1991], [STOUGHTON & KOKKINIS, 1987], [KRUT, et al., 2002], [KRUT, et al., 2004a], [KRUT, et al., 2004b]) have been based on quantifying the isotropy of kinetostatic performance (variation of force, velocity and accuracy\textsuperscript{29} capabilities with direction), but from two different perspectives that only become distinguishable upon studying redundant robots. For example, in ([SALISBURY & CRAIG, 1982], [YOSHIKAWA, 1985b], [GOSELLEN & ANGELES, 1991], [STOUGHTON & KOKKINIS, 1987]), the indices provide a measure of the variation of transmission values with direction, while in ([KRUT, et al., 2002], [KRUT, et al., 2004a], [KRUT, et al., 2004b]), the target has been quantifying the maximal output performance with direction, i.e. an isotropy measure of output performance based on zonotope approximation by largest inscribed ellipsoid (mathematically referred to as John’s Ellipsoid\textsuperscript{30} (BALL, 1992)). In non-redundant robots, however, both concepts coexist naturally\textsuperscript{31}. So while singular values and based upon indices serve well regarding singularity, precision, and kinetostatic performances in non-redundant robots, their kinetostatic significance, as described in ([KRUT, et al., 2002], [KRUT, et al., 2004a], [KRUT, et al., 2004b]), is lost in redundant ones. This is because the minimum and maximum output performances are not necessarily along the directions of the minimum and maximum transmission values. Not far from that, (MERLET, 2006) has revisited the concept of condition number and manipulability in the case of PMs, in which some important remarks regarding accuracy and the norms to be used have been pointed out. More precisely, the assumption that joint errors are described by unit-radius sphere has been criticized being non-realistic and therefore, its substitution by a unit-half-side cube has been favored. In addition, a comparison and a discussion on the suitability of the condition number and manipulability indices have been made. In ([KLEIN & BLAHO, 1987], a survey of the various local kinematic dexterity measures is provided and definitions for positional, orientation, and spatial isotropies are given in ([KLEIN & MIKLOS, 1991]. In ([PARK & KIM, 1998], the choice of the Riemannian metric to establish the manipulability of closed kinematic chains, including those with redundancy, has been discussed. To end the scrutiny on the above-cited works, we emphasize that motion/force transmissibility indices as compared with others, always serve as

\textsuperscript{28} Such as constraint singularities in parallel manipulators.

\textsuperscript{29} Note that accuracy and velocity are similarly treated as they are both related by the Jacobian matrix. In fact, they are antagonist. Improving velocity leads to deteriorating accuracy, as it means error transmission value is increased.

\textsuperscript{30} It is important to mention that John’s Ellipsoid is unique for any convex compact region. This uniqueness is important feature added to the methodology of using the largest inscribed ellipsoid semi-axis lengths and the pseudo-condition numbers, as they are unique as well.

\textsuperscript{31} Because the largest inscribed ellipsoid in the operational zonotope (whether velocity or force) is the image of the unit-radius sphere in the joint zonotope.
a measure to qualify good or bad transmission, but not indicative regarding the extreme output performances.

In a recent work (GUAY, et al., 2013), an index called “minimum degree of constraint satisfaction,” has been introduced and applied on cable-driven parallel robots (CDPRs) to assess the capability of producing a specified set of wrenches. It can be applied on rigid robots equally well. It considers the minimum distance between the specified set to the zonotope boundary. However, to deal with the case of heterogeneous-dof robots, an arbitrary homogenization has been adopted and hence, falling as most other indices in the debate that accompanies such procedure.

Actually, in addition to redundancy that has been particularly treated for example by the works of (KRUT, et al., 2004b) (KRUT, et al., 2002), (KRUT, et al., 2004a) and (XIE, et al., 2011), the heterogeneity problem (case of robots with heterogeneous dofs or actuators) has constituted another dilemma that attracted numerous researchers as well. For instance, among the homogenization techniques, we mention the use of some sort of weighting/normalizing matrix or characteristic length, as in ((IMA & ANGELES, 1991), (ANGELES, 1992), (STOCCO, et al., 1998)). In other works, such as ((GOSSELIN, 1992), (KIM & RYU, 2003), (KIM & RYU, 2004), (POND & CARRETERO, 2006)), the homogenization of the Jacobian has been achieved by considering the velocities (or forces) of two or three points situated on the end-effector, after relating them to the actuated joint velocities (or torques). Such methods are characterized by arbitrariness and lack of direct interpretable kinetostatic significance. Particularly, such homogenization would not affect the singularity assessment, though numerical value varies with the arbitrary homogenization. This is since if we are using, for instance, singular values to assess singularity, then theoretically and regardless of the homogenization technique\(^{32}\), the singular configurations remain invariant and occur for zero or infinite singular value. As for kinetostatics, the indication of the homogenized vector is not easy to interpret or relate to the end-effector twist (or wrench). In more recent works ((MANSOURI & OUALI, 2009), (MANSOURI & OUALI, 2011)), the issue has been addressed based on the apparent power concept to achieve homogeneous formulation of the problem. The methodology introduces some weighting factors \(K_{1}\) and \(K_{2}\), the ratio of which is important. Then, a minimization of some quantity is carried out to get a unique value of \(\beta = K_{1}/K_{2}\), upon which the final power manipulability depends. However, the relation of the established measures to the operational twist and wrench is rather vague. This is not to mention the associated complexity.

In fact, the arbitrariness involved in dealing with heterogeneity conflict is inevitable consequence of the absence of a natural metric for the geometry of \(SE(3)\) as stated by (PARK & KIM, 1998), although in the work of (PARK, 1995) some insight on homogenization technique has been provided. Actually, (PARK, 1995) has discussed left and right invariant distance metrics, and proved the non-existence of a bi-invariant one. Furthermore, a discussion on the

\(^{32}\) As long as the homogenized matrix and the initial one are equivalent regarding their rank.
invariance issue of manipulability indices can be found in (STAFFETTI, et al., 2002). Therefore, an intuitive approach to get rid of such predicament would be to study translational and rotational performances separately. This has been suggested by (YOSHIKAWA, 1991) through the translational and rotational manipulabilities, defined in the weak and strong senses. In a more recent work, Direction-Selective Indices (DSIs) have been proposed by (BOSCHETTI, et al., 2011). In this approach, the manipulabilities of the individual translational dofs are studied independently. Such method can be used equally with rotational and heterogeneous-dof robots. However, the individual study of translational and rotational performances resembles the redundancy situation; therefore, classical indices become less significant in this sense. In another work, the Force-Velocity Isotropy Index and the treatment of heterogeneous-dof robots by proper separation of translation and rotation have been discussed by (SHAYYA, et al., 2014d). This will be detailed later in Chapter 2. In the scope of precision, a similar approach can be considered. For example, the kinematic-sensitivity indices that separate orientation and position errors have been suggested by (CARDOU, et al., 2010). In this work, for bounded p-norm of joint errors, the maximal p-norm of orientation and positional errors are considered as to overcome the heterogeneity issue. This means if we consider, say infinity norm for joint errors, we end with same norm regarding orientation and positional ones. Therefore, the orientation (or positional) errors are geometrically interpreted as rectangular parallelepiped. Likewise, in the case of using Euclidean norm on joint errors, we end with an ellipsoid representing orientation (or positional) errors. In either cases (Euclidean norm or infinity norm), only the realistic aspect of one of the two errors, either joint or operational errors, is considered properly. More precisely, the infinity norm is the reasonable to use for joint errors, being generally independent. However, it is not consistent for those of the operational space. As the translation (similarly rotation) is a coupled motion, the Euclidean norm is the more consistent to use, as the spatial accuracy is the more interesting and not the axial one. This idea is what we are going to present and discuss in Chapter 2. An additional point to make is that while the proper separation of translation and rotation for precision evaluation is always justifiable, it is not the case regarding kinetostatics for only one exception. This is the case where the robot performs the two motions (or supports force and moment) in separate phases. Apart from that, such decomposition is not indicative. That is why a relevant approach to co-assess simultaneous performance in translation and rotation has been investigated in this thesis (see Chapter 2).

While most of the works were concentrated on investigating input-output performance, (BRIOT, et al., 2013) suggested the study of reactional forces in the passive joints. This is indispensable in the design of any manipulator. Nevertheless, it remains an intermediate step that influences dynamics. Knowing the magnitudes of these forces, proper sizing of the joints and components to sustain stiffness and prevent failure can be done. The impact would be

33 The independency of joint errors is true only for non-redundant robots. When the robot is redundant actuated, some dependency relations exist and therefore, its accuracy analysis must account for this particularity.
embodied then in the inertia whose effect can be inferred via dynamic measures. Therefore, considering these forces by themselves as an objective is less favored.

Moreover, in (VOGLEWEDE & EBERT-UPHOF, 2005), the authors investigated different indices in the attempt to identify the most suitable ones that can detect singularities (the classical ones), while providing both: a sort of distance measure to singularity on one hand, and interesting relevant physical significance on the other. They concluded that while power and input torque measures are not sufficient to detect all singularities, the natural frequency does and more importantly merges stiffness and mass inertia together.

Stiffness, whether the control-related one or that of the physical structure itself, is undeniably essential and especially for machine tools. The same applies for natural frequency. However, such criteria can be fulfilled at the design stage of the parts through both: their proper dimensioning (for structural stiffness and natural frequency), and tuning of the controller (for control-related stiffness). Based on this, such indices are not that critical to consider as main objectives.

It remains finally to discuss the literature in what concerns dynamics. Among the available indices, we exemplify: the dynamic manipulability (YOSHIKAWA, 1985a), the dynamic conditioning index ((MA & ANGELES, 1990), (MA & ANGELES, 1993)), the generalized inertia ellipsoid as means of evaluation of the capability of changing end-effector velocity in different directions for the given kinetic energy ((ASADA, 1983), (ASADA, 1984)), the maximum singular value of the generalized inertia matrix or its row vector matrix (LI, et al., 2005), (HUANG, et al., 2005)), the acceleration radius (GRAETTINGER & KROGH, 1988), the acceleration hyper-parallelepiped (KATIB & BURDICK, 1987), the Dynamic Load Carrying Capacity (DLCC) ( (WANG & RAVANI, 1988a), (WANG & RAVANI, 1988b)), the Acceleration Set Theory (KIM & DESA, 1993), the motion isotropy hypersurface (BOWLING & KATIB, 1998), the actuation efficiency measure (BOWLING & KATIB, 2000), the actuator-selection criteria based on dynamic performance discussed by (BOWLING & KATIB, 2003), the maximum-required-torque-based indices of (ZHAO & GAO, 2009), etc.

Unfortunately, each of these measures, in addition to its features, has its own drawbacks. For example, in ((YOSHIKAWA, 1985a), (MA & ANGELES, 1990), (MA & ANGELES, 1993), (ASADA, 1983), (ASADA, 1984), (LI, et al., 2005), (HUANG, et al., 2005)), the velocity and gravity terms are not considered. This makes them less suited for high-speed manipulators or heavy ones used for applications with large load requirements. Also, the arbitrary homogenization needed in ((YOSHIKAWA, 1985a), (MA & ANGELES, 1990), (MA & ANGELES, 1993), (ASADA, 1983), (ASADA, 1984), (LI, et al., 2005), (HUANG, et al., 2005), (GRAETTINGER & KROGH, 1988), (KATIB & BURDICK, 1987)) to deal with heterogeneous-dof robots renders these measures less significant. The Acceleration Set Theory (KIM & DESA, 1993) does not address the heterogeneity problem. As for DLCC ( (WANG & RAVANI, 1988a), (WANG & RAVANI, 1988b)), it is not influenced by heterogeneity conflict, but it is trajectory-based measure. It considers performance required to move in one direction along a particular path and therefore, it is more suitable for pick-and-place applications than machine tools. As for (ZHAO & GAO, 2009), an
implicit homogenization, in the case of heterogeneous-dof robots, is considered via the assumption of $\|\chi\| = 1$ (where $\chi$ is six-dimensional velocity or acceleration vector). Furthermore, the computation of the maximum torques is done assuming sole translation along or rotation about one of the coordinate axes with unitary value (in velocity or acceleration), as implied by the use of infinite-matrix norm. Hence, the suggested indices of (ZHAO & GAO, 2009) suffer from the same limitations and arguments accompanying arbitrary homogenization.

In the work of (BOWLING & KHATIB, 1998), the approach of overcoming arbitrary homogenization by decomposing each of the acceleration and twist into their composing translational and rotational parts is remarkable. However, the isotropy hypersurface itself is not that easy to use. For this purpose, the same authors introduced the actuation efficiency measure and discussed actuator-selection criteria in (BOWLING & KHATIB, 2000) and (BOWLING & KHATIB, 2003), respectively.

The actuation efficiency in (BOWLING & KHATIB, 2000), as its name indicates, is more suitable to assess actuation but cannot give an insight on the isotropic accelerations themselves. This is not to mention the complexity, as volume integration is required for the calculation of the aforementioned measure. The approach discussed in (BOWLING & KHATIB, 2003) allows the user to inspect whether the pre-specified performance at a configuration is attainable or not (depending on having $\alpha$, or $\alpha_y$, superior or inferior to one). An additional feature is including external forces and moments in the play, which is quite interesting. Among the cited works, (BOWLING & KHATIB, 2003) is the closer to our proposed methodology in Chapter 2. However, unlike (BOWLING & KHATIB, 2003), our indices tell not only whether certain dynamic performances are applicable simultaneously in all directions, but also the extreme that can be reached in each aspect while fulfilling the others. Thus, more interesting options and features are available, permitting better design optimization. Besides, the approach is not restricted to rigid manipulators, and its extension to deal with CDPRs is formulated as well.

With this, we end the overview on performance evaluation and the available literature. It remains to state the general outline of the thesis, the subject of the next section.

### 1.5- Thesis Outline

Having exposed the state of art of PMs and the literature concerning performance measures, the following chapters detail the different steps taken towards the achievement of the preset goals. The rest of the thesis is planned as follows:

- **Chapter 2** details the establishment of adequate precision, kinetostatic, and dynamic measures that consider machine tool requirements. In addition, their formulation to treat CDPRs is provided as well.
- Afterwards, **Chapter 3** presents several interesting novel architectures that have been synthesized during the course of the thesis. Additionally, the dimensional syntheses
Chapter 1: State of the Art

concerning two of them are provided, among which is that of the final implemented ARROW PKM.

- Chapter 4 exposes the geometric sensitivity of the eventual PKM and suggests some possible methodologies of compensation for geometric and elastic errors. This would be a second step to go for further amelioration of precision performance.
- Ultimately, a separate part concludes the major accomplishments and discusses the possible future research directions.
In this chapter:

This chapter introduces a relevant novel methodology to tackle the performance evaluation of general manipulators and machine tools, regardless of being based on serial, parallel or hybrid kinematics. The presented measures fall within precision, kinetostatic, and dynamic performance fields. They overcome two main quandaries that have been highlighted in the previous chapter, namely: heterogeneity problem (heterogeneous dofs or actuators) and redundancy. Regarding the latter issue, only manipulators with either pure actuation or kinematic redundancy are discussed. Therefore, robots that exhibit both types of redundancy are beyond the scope of the current chapter. However, the methodology is not restricted to rigid manipulators, but extensible to cable-driven ones equally well. The mathematical formulation for both cases are elaborated and supported by case studies.
2.1- Introduction

Before introducing our approach, we need to talk first of some generalities regarding manipulator kinetostatic relations. While such relations are straightforward for non-redundant manipulators, few particularities are worth emphasizing in what concerns redundancy. Moreover, it is equally important to discuss the latter’s major benefits and types. This is the subject of the current section.

While actuation redundancy (AR) is only applicable to PMs, kinematic redundancy (KR) is applicable to both serial and parallel structured mechanisms (SMs and PMs). A common point between the two is having at least one extra actuator as compared with the end-effector dofs. Thus, if we assume \( m \) and \( n \) the respective numbers of actuators and operational dofs\(^1\), then in the case of redundancy, we have \( m > n \). We emphasize here that redundancy and its definition can be slightly misleading as highlighted in (CONKUR & BUCKINGHAM, 1997). In particular, kinematic and task redundancies might be messed up. In what follows, we attempt to make all these terms as clear as possible.

2.1.1- Kinematic Redundancy (KR)

According to (CONKUR & BUCKINGHAM, 1997), a kinematically redundant manipulator (KRM) is a one whose joint-space dimension is greater than that of the end-effector. The joint space meant here is the consistent one, with consistency being related to the kinematic constraints. While in SMs the consistent joint space is always of dimension \( m \), it is not the case with PMs (case of AR for instance).

This means that in the case of a KRM, for each end-effector pose, \( x \), there corresponds infinite possible values of actuated-joint configurations, \( q \). The same can be translated regarding kinematics by considering the end-effector twist, \( \tau \), and the actuated-joint velocities, \( \dot{q} \).

This type of redundancy helps reducing or even eliminating serial-type singularities, enlarging the workspace, and elevating speed capacity. Yet, it adds additional moving masses that impairs dynamics. Moreover, while \( \dot{q} \) are freely chosen, the actuators torques, \( \tau \), are subject to \( (m-n) \) dependency relations. This indicates that load capability cannot directly be optimized in this sense. Nevertheless, the possibility of having different joint configurations for the same pose can be used as an alternative to change the torque distributions in the actuators.

An example of PMs with KR is the three-dof (3T) robot, called Speed-R-Man (depicted in Fig. 2-1), of (REBOULET, et al., 1992). The robot is characterized by an important working volume and speed capacity. It utilizes six prismatic actuators and therefore, the aforementioned

\(^1\) Meaning that of the end-effector.
advantages come with the drawbacks embodied by the additional costs (three extra actuators) and the added complexity at the control level.

To end this part, we highlight another type of redundancy that might be confused with KR, and which is task redundancy (TR). If only \( r \) dofs (with \( r < n \)) of the end-effector are used for particular application, the redundancy is referred to as TR. This redundancy enhances the task accomplishment and improves accessibility. However, it has nothing to do with the mechanical structure itself or its intrinsic properties, but with the intended task. Such redundancy has been already highlighted in the previous chapter in the use of six-dof PKMs for five-axis machining. In the scope of this report, we will not be dealing with it.

### 2.1.2- Actuation Redundancy (AR)

A redundantly actuated manipulator (RAM) is one that has greater number of actuators than its end-effector dofs (i.e. \( m > n \)), and in which there exists infinite possible values of actuated joint torques, \( \tau \), to counteract an external wrench, \( w \), on the end-effector (FIRMANI, et al., 2008).

More precisely, in the case of an AR, we have \( m > n \) with \((m−n)\) dependency relations imposed on \( q \). These latter conditions are usually referred to as synchronization relations. Therefore, commanding an RAM to move without respecting the aforementioned constraints would lead to system damage.

Furthermore, two main subtypes of AR can be distinguished, namely: in-branch actuation redundancy (IBAR) and branch actuation redundancy (BAR) (FIRMANI, et al., 2008). In the former (i.e. IBAR), one or more passive joints in the initially non-redundant PM are actuated. While this preserves the initial workspace, it adds more moving masses and as a result, can be less desirable. As for the latter case (i.e. BAR), additional chains with actuated joints are added.

Fig. 2-1: Speed-R-Man (from http://www.onera.fr/fr/dcsd/robots-paralleles?page=2).
to the structure. Though their added influence regarding moving masses may be less severe (not necessarily always), it restricts more the workspace.

Many examples on AR exist and we already mentioned several of them in the first chapter (see for instance Fig. 1-35, Fig. 1-36, and Fig. 1-37).

The major benefits of AR are reducing or eliminating parallel-type singularities, homogenizing performances, and improving wrench capabilities as well as dynamics. Moreover, it can be utilized to reduce the influence of passive joint clearances and backlashes, thanks to the possibility of applying pre-stress on the structure (MULLER, 2005).

2.1.3- Mixed-Redundancy Manipulators (MRMs)

An MRM is a one that exhibits both kinematic and actuation redundancies. To most of our knowledge, MRMs are not being researched as they combine the demerits of both types of redundancy rather than their virtues. But to make their idea clear, perhaps an example can be enough for this purpose. In Fig. 2-2, a simple example, yet sufficient to demonstrate the complexity, is shown.

The kinematic relation for the mechanism is expressed by:

$$\dot{q}_1 + \dot{q}_2 = \dot{q}_3 = \ddot{x} \tag{2.1}$$

Thus, infinite solutions are possible for the inverse kinematics. These are obtained by choosing any value for $$\dot{q}_1$$ (or $$\dot{q}_2$$) and adapting the other $$\dot{q}_2$$ (or $$\dot{q}_1$$) to satisfy (2.1). This explains the KR in the exemplified mechanism. However, not every combination of joint velocities is permissible. In particular, choosing $$\dot{q}_3 \neq \dot{q}_1 + \dot{q}_2$$ leads to system damage and implies the presence of AR as well. More clearly, we can explain AR by considering the force relationships. If we have a force $$f$$ acting on the platform, then the forces of the actuators should satisfy the following:

$$\begin{cases}
\tau_1 = \tau_2 \\
\tau_2 + \tau_3 = f
\end{cases} \tag{2.2}$$

Fig. 2-2: A simple example on mixed redundant manipulators: two series prismatic joints in parallel with another prismatic joint (one dof and three actuators).
From (2.2), it is clear that infinite possible distributions of forces among the actuators exist to counteract the same external force. These distributions consist of arbitrary choosing $\tau_1 = \tau_2$ or $\tau_3$, while satisfying the 2nd relation of (2.2). Hence, at the control level, we must deal with two problems, specifically the optimal choice of joint velocities, and that of actuation forces. Thus with this, we have clarified the concept of mixed redundancy and its inherent intricacy.

It is worth mentioning that another type of redundancy called metrological or measurement redundancy exists. This type of redundancy is characterized by having extra number of sensors than needed. For instance, if we have five sensors in a four-dof robot, then the robot is said to be metrologically redundant. Despite that such redundancy can help improve pose calculation (via simplifying direct geometric model), the way the measurements are exploited in control constitutes a non-trivial issue. A final word in this aspect is that in the case of RAMs, where all actuated joints are equipped with feedback sensors, we have also measurement redundancy.

These were the major points to emphasize regarding redundancy. The rest of the chapter is outlined as follows. First, rigid manipulators are considered and their precision, kinetostatic, and dynamic performances are addressed. Afterwards, cable-driven parallel robots (CDPRs) are considered and the extension of the proposed kinetostatic and dynamic measures is done. Alongside, case studies are provided to make better sense of the concepts and their implementations. Also, few remarks are made, and recommendations concerning design optimization are discussed. The chapter is eventually concluded by highlighting some essential points.

2.2- General Rigid Manipulators

2.2.1- Generalities on the Kinetostatics of Redundant and Non-Redundant Manipulators

Consider any rigid manipulator with the following system of constraint equations describing its kinematic structure:

\[
\begin{aligned}
F(q, x) &= 0, \\
\dim(q) &= m, \ \dim(x) = n, \\
\dim(F(q, x)) &= n_c, \ n \leq n_c \leq m
\end{aligned}
\] (2.3).

The differentiation of (2.3) with respect to time yields the following well-known kinematic relation:

\[
\begin{aligned}
J_q \dot{q} &= J_x \dot{t}, \\
\dim(J_q) &= n_c \times m, \ \dim(J_x) = n_c \times n
\end{aligned}
\] (2.4).

In the case of non-redundant manipulator (NRM), both $J_q$ and $J_x$ are square matrices (i.e. $n_c = n = m$). Nonetheless, in the situation of redundancy (KR or AR), we have $m > n$ and at least
one of the aforementioned matrices is rectangular. In specific, if $J_q$ is only rectangular, then pure KR exists. On the other hand, having $J_x$ solely non-square corresponds to pure AR. Having both matrices rectangular (i.e. $n < n_c < m$) results in mixed redundancy (KR and AR), and in which we are not concerned.

In what follows, we give the generally known kinetostatic relations for NRM, KRM, and RAM. Starting with NRMs, we always have (at regular non-singular pose$^2$):

$$\dot{q} = J_m^{-1} t \quad \text{or} \quad t = J \dot{q}, \quad \text{with:}$$

$$J_m = J_q^{-1} J_x \quad \text{and} \quad J = J_x J_q$$

and$^3$

$$w = J_m^T \tau \quad \text{or} \quad \tau = J^T w$$

(2.6).

The terms $J_m$ and $J$ are respectively the inverse and direct/forward Jacobian matrices. While both, $J_m$ and $J$, are naturally available$^4$ in NRMs, this is not the case with KRMs and RAMs where only one of the two is available—the other requiring pseudo-inversion operation based on some assumptions.

In the case of a KRM, we naturally have $J$ as the forward kinematic problem is unique, unlike the inverse one. Therefore, only the forms of the kinetostatic relations ((2.5) and (2.6)) that are based on $J$ are directly applicable. As for the inverse kinematic and forward static problems, they are expressed as follows:

$$\dot{q} = J^* t + \left[ \text{null}(J) \right] \lambda, \quad \text{with} \quad \lambda \in \mathbb{R}^{m-n} \quad \text{(arbitrary)}$$

(2.7)

and

$$w = J^T \tau, \quad \text{with} \quad \tau^T \left[ \text{null}(J) \right] = 0_{(m-n)}$$

(2.8).

The term $\left[ \text{null}(J) \right]$ corresponds to a null space basis vector of $J$. Note that as we are always considering regular non-singular pose, $\text{null}(J)$ is of exactly $(m-n)$ dimension.

On the other hand, in the case of RAMs, only the forms of the kinetostatic relations ((2.5) and (2.6)) that are based on $J_m$ are directly applicable, in contrast with what we had for KRMs. Additionally, the forward kinematic and inverse static solutions are given respectively by:

---

$^2$ Which is the always-assumed case throughout the chapter.

$^3$ Note that under the form of (2.6), we are describing how the actuators torques are transmitted into the operational space and vice-versa. In this sense, $w$ is the opposite of the real applied wrench. This form is considered here for convenience. However, later in cable-driven robots, the equilibrium form will be used instead, i.e.: $w + J_m^T \tau = 0$ (platform equilibrium) or $\tau + J^T w = 0$ (actuators equilibrium).

$^4$ Meaning they are computable with no assumptions, by means of matrix inversion operations (no pseudo-inverse procedure).
Chapter 2: Performance Evaluation of General Manipulators

\[ t = J_m^* \dot{q}, \text{ with } \dot{q}^T \left[ \text{null} \left( J_m^T \right) \right] = 0 \text{ in } (m-n) \]  
(2.9)

and

\[ \tau = J_m^{*T} w + \left[ \text{null} \left( J_m^T \right) \right] \lambda, \text{ with } \lambda \in \mathbb{R}^{m-n} \text{ (arbitrary)} \]  
(2.10).

The relations (2.7) and (2.10) show that when dealing with KR and AR, one should get rid of the arbitrariness of \( \lambda \). One way to do that is considering the solutions of \( \dot{q} \) and \( \tau \) that do not have any component along the null spaces of \( J \) and \( J_m^T \) respectively; this means setting \( \lambda = 0 \). These are mathematically referred to as minimum norm solutions. Note that choosing \( \lambda \neq 0 \) results in antagonist forces that can deform the robot structure; these forces are referred to as internal preloads and can be used to eliminate the influence of clearances and to control stiffness (MULLER, 2006).

The particular solutions with \( \lambda = 0 \) will be assumed in the kinetostatic analysis, and they can be practically achieved by control means (refer to §4.2 in Chapter 4 for more information on the methodology). Therefore, for rigid manipulators, regardless of being redundant or not, both forms of the relations ((2.5) and (2.6)) are applicable with the following assumption:

\[
\begin{align*}
J_m &= J^*, \text{ if KRM} \\
J &= J_m^*, \text{ if RAM}
\end{align*}
\]  
(2.11).

Nonetheless, in what concerns precision evaluation, the particularities of redundancy must be considered, as we are going to see in the following part. As for dynamics, a similar approach to deal with redundancy is adopted as well. This will be made clear later in place. It remains just to emphasize that we will consider the following decompositions of \( J_m \) and \( J \):

\[
\begin{align*}
J_m &= \begin{bmatrix} J_{mp} & J_{mo} \end{bmatrix} \\
J &= \begin{bmatrix} J_p \\ J_o \end{bmatrix}
\end{align*}
\]  
(2.12).

The subscripts “p” and “o” in (2.12) are used to denote the respective translational/positional and orientation/rotational parts of \( J_m \) and \( J \).

2.2.2- Precision-Related Performance Measures

When dealing with machine tool design, among the most important aspects to consider is precision. This precision embraces several terms, such as accuracy, repeatability, and operational resolution. The first two have been previously discussed and explained in Chapter 1, with further information being provided in Appendix A. It remains to shed the lights on the operational resolution of a robot or machine.

In fact, one is more familiar with measurement resolution of some measuring device, such as a scale. For instance, consider a ruler with 1 mm graduations. Then, we cannot measure any length inferior to this resolution. In fact, every measurement of a length \( L \) using the
aforementioned device is done with roughly $\pm 1 \text{ mm}$ error and thus, the resolution stands also for the measurement uncertainty (CORBEL, 2008).

If we consider the same notion applied to an actuator, the latter’s resolution is the smallest step that it can perform. According to (CORBEL, 2008), the resolution of a motor can be seen in several ways. The most common is to consider it equal to that of the associated encoder. However and according to the same reference, this value adopted for resolution is not necessarily equal to the physical one of the motor, but it may be lower or higher. (CORBEL, 2008) supports this idea by considering the results of the work of (BRETHE & LEFEBVRE, 2007), in which the granular space structure of some industrial robots has been discussed. Our opinion in this matter is quite different. From our point of view, the actuator resolution is not only the smallest step that can be done, but also detected. The necessity of detection makes resolution a characteristic of the actuator-encoder system rather than the actuator itself. As a result, it should be a multiple of the encoder resolution.

Regarding the robot, its operational resolution is the smallest detectable step that the end-effector can do in the operational space. In conventional machines where each axis of motion is activated by a sole actuator, we may speak of axial resolution, i.e. the smallest step that can be done along each Cartesian axis. Nonetheless, in machines based on parallel kinematics, though resolutions along particular directions can be defined, they are less indicative. This is due to the coupled nature of the end-effector motion. Consequently, it is preferable to speak of operational translational and orientation resolutions instead.

In the following, we define theoretical resolutions as well as accuracies and repeatabilities for all robots (NRMs, KMRs, and RAMs). But what we will be interested in later, is replacing those three measures by a single one, though this is possible only for the particular situation of having the same type of actuators.

**A- Theoretical Accuracy, Repeatability, and Resolution**

In this section, we assume that the actuators’ resolutions, accuracies, and repeatabilities are all sufficiently small to have the following assumption valid:

$$\delta q = J_m \delta x \text{ or } \delta x = J \delta q \quad (2.13).$$

Relation (2.13) means that for very small errors (which is usually the case for machine tools), the relation in the vicinity of the considered pose is linear. In this case, the actuated-joint displacement zonotope is transformed into an operational one. However, attention must be paid regarding RAMs and the synchronization imposed on their joint-displacements.

To calculate the theoretical operational resolutions at non-singular pose for given actuators’ resolutions, denoted by $R_q (\forall i = 1...m)$, we define the following joint zonotope $Z_q$:

---

$^5$ It is a rectangular parallelepiped in the case of NRM and KMR only. In the case of RAMs and due to synchronization constraints, it is part of a rectangular parallelepiped.
Chapter 2: Performance Evaluation of General Manipulators

\[ \mathcal{Z}_q = \begin{cases} \delta q \in \mathbb{R}^m; \ |\delta q_i| \leq Res_{qi}, \ \forall i = 1...m, \\ \text{and} \ \left[ \text{null} \left( J^T_m \right) \right]^T \delta q = 0 \end{cases}, \ \text{if RAM} \]

or:

\[ \{\delta q \in \mathbb{R}^m; \ |\delta q_i| \leq Res_{qi}, \ \forall i = 1...m\}, \ \text{if NRM or KRM} \]

(2.14).

Then, the theoretical translational (orientation) resolution is the smallest translational (orientation) step detectable in the operational space. In other words, it is the radius of the smallest sphere that circumscribes the translational (orientation) zonotope, the image of \( \mathcal{Z}_q \) under the mapping described by \( J_p \) (correspondingly \( J_o \)). Denoting translational and orientation resolutions by \( Res_p \) and \( Res_o \) respectively, we get:

\[ \begin{cases} Res_p = \max_{\delta q \in \mathcal{Z}_q} (J_p \delta q) \\ Res_o = \max_{\delta q \in \mathcal{Z}_q} (J_o \delta q) \end{cases} \]

(2.15).

Based on the above, \( Res_p \) (\( Res_o \)) is the maximal Euclidean distance from the origin of the translational (orientation) zonotope to the corresponding vertices (the image of those of \( \mathcal{Z}_q \)).

Denoting the \( N \) vertices of \( \mathcal{Z}_q \) by \( V_i \) (\( \forall i = 1...N \)), we can rewrite (2.15) as:

\[ \begin{cases} Res_p = \max_{i=1...N} (J_p V_i) \\ Res_o = \max_{i=1...N} (J_o V_i) \end{cases} \]

(2.16).

A similar approach can be used to estimate the theoretical translational or orientation accuracies and repeatabilities. This is done by replacing \( Res_{qi} \) by \( E_{qi} \) (i-th joint accuracy) to get the translational (orientation) accuracy \( E_p \) (respectively \( E_o \)). On the other hand, replacing \( Res_{qi} \) by \( Rep_{qi} \) (i-th joint repeatability), we get the estimated translational and orientation repeatabilities, denoted by \( Rep_p \) and \( Rep_o \) respectively. It is worth mentioning here that regarding accuracy and repeatability, another statistical approach exists. This is done by assuming a certain statistical distribution for joint errors with a specific mean and standard deviation, then establishing the mean and standard deviation of the operational error based on (2.13) (e.g. (BRETHE & LEFEBVRE, 2007)). However, it can be rather complex, especially that in the case of RAMs, the joint errors are dependent. Also in this scope, we mention the work of (MERLET & DANÉY, 2005), in which interval analysis has been used in the design of a robot with a specified accuracy over a given workspace. This has been done while taking into account not only the actuated-joint errors, but also the tolerances on the geometric parameters. Despite that theoretically such approach is the most relevant, putting it into practice is undeniably difficult and presents computational complexities. This is not to mention being time-expensive. Besides, in the design of machine tools, the geometric errors are supposedly small, and they are later compensated by means of calibration.

54
Chapter 2: Performance Evaluation of General Manipulators

So far, we have presented the methodology in the general case, yet our focus will be on the particular situation where all actuators are of same nature, meaning either linear or rotational.

In such a case, we set \( Res_q = \max_{x \in DWS} \left( \max_{i=1...m} \left( Res_{q_i} \right) \right) \), being the worst resolution among the actuators and over the whole desired workspace, \( DWS \). The following step is to replace the above definition of \( Z_q \) by:

\[
Z_q = \begin{cases} 
\{ \delta q \in \mathbb{R}^n; |\delta q| \leq R_q, \forall i = 1...m, \} & \text{if } \text{RAM} \\
\{ \delta q \in \mathbb{R}^n; |\delta q| \leq R_q, \forall i = 1...m \} & \text{if } \text{NRM or KRM}
\end{cases}
\]

(2.17).

Then, we can calculate the operational resolutions, accuracies, and repeatabilities\(^6\) via the same equation (2.16), and where \( E_q = \max_{x \in DWS} \left( \max_{i=1...m} \left( E_{q_i} \right) \right) \) and \( Rep_q = \max_{x \in DWS} \left( \max_{i=1...m} \left( Rep_{q_i} \right) \right) \).

Hence, we define what we may call precision amplification factors in translation and orientation, denoted by \( TPAF \) and \( OPAF \) correspondingly:

\[
\begin{align*}
TPAF &= \frac{Res_p}{Res_q} = \frac{E_p}{E_q} = \frac{Rep_p}{Rep_q} \\
OPAF &= \frac{Res_{\omega}}{Res_q} = \frac{E_\omega}{E_q} = \frac{Rep_\omega}{Rep_q}
\end{align*}
\]

(2.18).

Note that the calculation of \( TPAF \) and \( OPAF \) does not necessitate the knowledge of \( Res_q, E_q, \) or \( Rep_q \). This is very interesting, especially at the design level where we might have these terms still unknown. Optimizing any design in the sense of minimizing \( \max_{x \in DWS} \left( TPAF \right) \) and \( \max_{x \in DWS} \left( OPAF \right) \) results in a precise robot. This is what we are going to employ later in the dimensional synthesis of ARROW PKM in Chapter 3.

**B- Precision and Peak Operational Speeds**

Perhaps, it is worth emphasizing the relation between peak operational speeds on one side, and precision measures on the other side, though this relation can be intuitively noticed.

To demonstrate the case, the simplest way is to consider a robot with identical actuators of maximal speed being \( \dot{q}_{\text{max}} \). Then, one can easily show that the peak linear \( v_{\text{peak}} \) and angular \( \omega_{\text{peak}} \) speeds are related to precision measures as follows:

---

\(^6\) Regarding accuracy and repeatability, replace \( Res_q \) by \( E_q \) and \( Rep_q \) respectively.
\[
\begin{align*}
\frac{v_{\text{peak}}}{q_{\text{max}}} &= TPAF \\
\frac{\omega_{\text{peak}}}{q_{\text{max}}} &= OPAF
\end{align*}
\]

, all actuators being identical \hspace{1cm} (2.19).

Hence, improving precision (i.e. reducing \(TPAF\) and \(OPAF\)) is accompanied by decreasing peak speed capacities. While the relation is straightforward in the case of identical actuators, it becomes less direct in the general situation, being more or less coupled. For machine tools, the peak speeds are not the interesting ones to maximize, but rather the isotropic ones (those that can be attained in all directions). The antagonism between the latter speeds and precision factors is more intriguing and to be compromised in the optimization of a robotic design, in which rapidity and precision are among the sought goals.

As the main points in what concerns precision have been clarified, it remains to present a case study, the subject of the upcoming section.

**C- Case Study: DUAL V**

In Fig. 2-3, DUAL V is depicted with simplified schematic and CAD drawings. It is a redundantly actuated PM with three dofs (2T-1R), and four identical actuators. Its complete description with its models can be found in (WIJK, et al., 2013).

Its inverse Jacobian matrix \(J_m\) is given by:

\[
J_m = J_q^{-1} J_x
\]

with:

\[
J_q = \text{diag}(B_iC_i^T(e_i \times A_i B_i)), \quad \text{dim}(J_q) = 4 \times 4
\]  \hspace{1cm} (2.21)

and

\[
J_x = \begin{bmatrix}
B_1C_1^T e_x & B_1C_1^T e_y & -B_1C_1^T (PC_1 \times e_z) \\
\vdots & \vdots & \vdots \\
B_4C_4^T e_x & B_4C_4^T e_y & -B_4C_4^T (PC_4 \times e_z)
\end{bmatrix}, \quad \text{dim}(J_x) = 4 \times 3
\]  \hspace{1cm} (2.22)

The \(TPAF\) and \(OPAF\) for DUAL V have been calculated for the case of \(\theta = 0^\circ\). The results are depicted in Fig. 2-4. The actuators are of type ETEL RTMB0140-100. According to the data sheet, the accuracy and repeatability are about 25 arcsec and 5 arcsec, respectively. This means that the expected worst translational accuracy and repeatability are about 54 \(\mu\)m and 11 \(\mu\)m respectively (considering \(\max(TPAF) \approx 0.45\)). However, these values do not account for control errors and only serve as rough estimate. In Table Ap-12 of Appendix A.2, we can only consider repeatability, as accuracy is not that reliable due to the technical difficulties explained therein. Aside from pose \(P_i\), the results are somehow in close agreement. We should keep in mind the existence of other factors, such as the measurement device repeatability,
experimentation imperfections, etc. Also, we should know that the datasheet values are given for certain environmental conditions that can slightly differ from those at experimentation time.

By this case study, we end up the discussion on precision assessment measures. It remains to discuss kinetostatic measures regarding twists and static wrenches, before moving into dynamics.
2.2.3- Kinetostatic Performance Evaluation

In the design of machine tools, the most important points to investigate are the twist and static wrench capabilities. But as these are direction-dependent, possibly a relevant approach would be only to consider the isotropic capacities; i.e. the maximal values that can be achieved by the manipulator irrespective of direction. This part concentrates on these aspects. We start by considering a robot with homogeneous dofs and end with the most generic case of robots with heterogeneous dofs.

A- Kinetostatic Measures: Case of Robots with Homogeneous DoFs

Let us consider a manipulator with homogeneous dofs, say a translational one. Then, depending on the intended task, a certain minimal value of speed and static force are usually required to be achieved in all directions. So why do not we consider isotropic speed, \( v_{iso} \), and force, \( f_{iso} \), as kinetostatic measures?!

In fact, the first step we have done to deal with homogeneous-dof robots was considering the aforementioned values in one composite measure, the Force-Velocity Isotropy index (\( FVI \)) and that includes also the minimal requirements in speed and static force, denoted by \( v_{req} \) and \( f_{req} \) respectively. It is given as follows:

\[
FVI = \min \left( \frac{v_{iso}}{v_{req}}, \frac{f_{iso}}{f_{req}} \right)
\]

As for \( v_{iso} (f_{iso}) \), it is nothing other than the maximum value that can be achieved regardless of velocity (respectively force) direction. Geometrically, \( v_{iso} (f_{iso}) \) is the radius of the largest sphere inscribed in the operational velocity (respectively force) zonotope. Mathematically, \( v_{iso} \) and \( f_{iso} \) are calculated as follows\(^\text{7}\):

\[
v_{iso} = \min_{i=1...m} \left( \frac{\hat{q}_i^{\text{max}}}{\|J_{mr}\|} \right)
\]

and

\[
f_{iso} = \min_{i=1...m} \left( \frac{\tau_i^{\text{max}}}{\|J_{c}\|} \right)
\]

with \( J_{mr} \) and \( J_{c} \) being the i-th row and column vectors of \( J_m \) and \( J \) respectively, and where \( \hat{q}_i^{\text{max}} \) and \( \tau_i^{\text{max}} \) are the corresponding maximal speed and torque capacities of the i-th actuator (i.e. \( |\hat{q}_i| \leq \hat{q}_i^{\text{max}} \) and \( |\tau_i| \leq \tau_i^{\text{max}} \)). Note that regarding redundant manipulators, the calculation is based on the assumption (2.11). It can be easily shown that if the general solution with

\(^\text{7}\) The derivation is very simple and trivial. However, one can find it in **2.2.3.C** as a special case.
arbitrary $\lambda$ is considered ((2.7) and (2.10)) in the case of KRM$s$ and RAM$s$, then the isotropic speed will be larger than or equal to that of (2.24) for a KRM, and the isotropic static force will be greater than or equal to that of (2.25) for an RAM. This is since the minimum norm solution set (i.e. the case in which $\lambda = 0$) is a subset of the general one. Thus, with (2.24) and (2.25) the guaranteed minimum isotropic values are computed. Yet, later and after dealing with CDPRs, one will become capable of computing the isotropic values for the general solution with arbitrary $\lambda$, although we do not recommend it for rigid manipulators.

Finally, it remains to highlight that when $v_{iso} \rightarrow 0$ (or $f_{iso} \rightarrow 0$), we are close to series-type singularity (correspondingly parallel-type singularity). Thus, the higher $FVI$ is, the better the robot is regarding isotropic kinetostatic capabilities, and the farther the pose is from classical singularities. In particular, when $FVI \geq 1$, it means that all the requirements have been fulfilled. In this sense, $v_{req}$ and $f_{req}$ serve as numerical tolerance to indicate singularity, in the same way one puts a tolerance on singular values. Nevertheless, unlike what we had in the case of singular values and especially with redundant robots, $v_{req}$ and $f_{req}$ are clear and physically significant.

Despite the ease of their computation, these isotropic values necessitate arbitrary homogenization for heterogeneous-dof robots, and as the debate of Chapter 1 concluded, the result is lack of direct physical significance.

Therefore, to overcome this issue, one suggestion is to consider translation and rotation separately. This is justifiable as long as translation and rotation occur in two separate phases (e.g. pick-and-place applications\(^8\)). This is the subject of the upcoming section.

B- Kinetostatic Performance Evaluation by Separation of Translation and Rotation

Let us consider now a heterogeneous-dof robot regardless of being redundant or not. Then, to use the above-defined $FVI$ index, a prior homogenization is needed. However, unlike classical indices, such as those based on singular values, the homogenization required is only regarding the operational dofs; meaning no particular treatment is demanded in the case of having actuators of different nature.

Nevertheless, the $FVI$ index based on homogenization, called $FVI_h$, maintains only the significance regarding classical series and parallel type singularities. Regarding kinetostatics, the physical interpretation is no more concrete and is rather difficult to interpret. That is why studying translational and rotational performances separately, but in proper manner, is favored.

For instance, one can define pure isotropic linear speed, $v_{iso}^p$, for the case where angular velocity is assumed null (i.e. $\omega = 0$). Also, we may define the pure static isotropic force, $f_{iso}^p$,\(^8\) Where pure rotation occurs at the end-points and pure translation in between.
that can be supported by the manipulator in the case where no external moment is acting on the end-effector (i.e. \( m = 0 \)). Likewise, we define pure isotropic angular speed, \( \omega_{iso}^p \), and pure isotropic static moment, \( m_{iso}^p \). The aforementioned terms can be easily derived to get as a result:

\[
\begin{align*}
\mathbf{v}_{iso}^p &= \min_{i=1...n} \left( \frac{q_i^{max}}{\| \mathbf{j}_{mpr} \|} \right) \\
\mathbf{\omega}_{iso}^p &= \min_{i=1...n} \left( \frac{q_i^{max}}{\| \mathbf{j}_{mor} \|} \right) \\
\mathbf{f}_{iso}^p &= \min_{i=1...n} \left( \frac{q_i^{max}}{\| \mathbf{j}_{pc} \|} \right)
\end{align*}
\]

and

\[
\begin{align*}
\mathbf{m}_{iso}^p &= \min_{i=1...n} \left( \frac{q_i^{max}}{\| \mathbf{j}_{oc} \|} \right)
\end{align*}
\]

The terms \( \mathbf{j}_{mpr} \) (\( \mathbf{j}_{mor} \)) and \( \mathbf{j}_{pc} \) (\( \mathbf{j}_{oc} \)) correspond to the i-th row and column vectors of \( \mathbf{J}_{mp} \) (\( \mathbf{J}_{mo} \)) and \( \mathbf{J}_p \) (\( \mathbf{J}_o \)) respectively.

Thus, based on the requirements, one can define different FVI indices. For example, we can define translational and orientation indices, denoted by \( FVI_p \) and \( FVI_o \) respectively. These are expressed as follows:

\[
\begin{align*}
FVI_p &= \min \left( \frac{\mathbf{v}_{iso}^p}{\mathbf{v}_{req}}, \frac{\mathbf{f}_{iso}^p}{\mathbf{f}_{req}} \right) \\
FVI_o &= \min \left( \frac{\mathbf{\omega}_{iso}^p}{\mathbf{\omega}_{req}}, \frac{\mathbf{m}_{iso}^p}{\mathbf{m}_{req}} \right)
\end{align*}
\]

The terms \( \mathbf{v}_{req} \) and \( \mathbf{\omega}_{req} \) are the minimum required pure isotropic linear and angular speeds. Similarly, \( \mathbf{f}_{req} \) and \( \mathbf{m}_{req} \) are the minimal required pure static isotropic force and moment to be supported by the robot.

Note that other combinations of pure isotropic values can be used depending on what the user needs. However, indices built on the aforementioned values do not necessarily exhibit a singularity significance, but solely a kinetostatic one. To be more clear, having one of the
isotropic values zero, say $v_{iso}^p = 0$, implies a series-type singularity. Nonetheless, having $v_{iso}^p \neq 0$ is not sufficient to prove the non-existence of series-type singularity. Similar note can be made regarding $f_{iso}^p$ and parallel-type singularity.

The exploitation of pure isotropic values is justifiable only when translation and rotation motions are done separately and/or when only pure force or moment is being counteracted at a time. As for the other cases where simultaneous motion is to be done or a mixed-type wrench is to be supported, another alternative must be investigated. The subsequent section is dedicated for discussing this second alternative, which is founded on the simultaneous isotropic kinetostatic performance evaluation.

C- Kinetostatic Isotropic Performance Co-Assessment: A Novel Approach that Overwhelms Heterogeneity Predicament and Fits Adequately Machine Tools

The kinetostatic performance measures that we are to introduce here, not only respond directly to the requirements to be satisfied when designing a machine tool, but also overcome the dilemma embodied in both: heterogeneity and redundancy. Actually, a machine tool is designed to perform a coupled heterogeneous motion (simultaneous translation and rotation) on one hand, and to support an external mixed-type wrench (force and moment) on the other hand. Thus, based on the type of application or machining task, one can set minimum requirements for isotropic linear and angular speeds, static force, and static moment that must be achievable by the manipulator simultaneously and regardless of direction. Denote these in respective order by: $v_{req}$, $\omega_{req}$, $f_{req}$ and $m_{req}$.

Starting from the aforementioned requirements and to assess simultaneous mixed kinetostatic performances in translation and rotation, a novel concept has been introduced. It is the notion of specific isotropic values. These values will be discussed in the ensuing subsection.

Description and Mathematical Formulation

In what follows, we define isotropic values and derive their formulae. For instance, the specific isotropic linear speed, $v_{iso}^{sp}$, is the maximal linear speed attainable in all directions while allowing the angular velocity, $\omega$, to reach $\omega_{req}$ in all directions. Analogous definitions can be made for specific isotropic angular speed, $\omega_{iso}^{sp}$, specific isotropic static force, $f_{iso}^{sp}$, and specific isotropic static moment, $m_{iso}^{sp}$. These terms not only hold a physical kinetostatic interpretation, but also a singularity one as it will become clear by the end of the section.

---

9 In this case, the origin of the operational twist zonotope belongs to its boundary, i.e. the twist zonotope is collapsed, and which corresponds to series-type singularity.

10 We mean by mixed-type wrench the one with non-zero force and non-zero moment.
To calculate $v^p_{iso}$, we consider the condition $|\dot{q}_i| \leq \dot{q}^\text{max}_i$, $\forall i = 1...m$ and which can be rewritten as:

$$-\dot{q}^\text{max}_i \leq \dot{q}_i = \mathbf{j}^T_{mpr} \mathbf{v} + \mathbf{j}^T_{mor} \mathbf{\omega} \leq \dot{q}^\text{max}_i, \quad \forall i = 1...m$$  \hspace{1cm} (2.32).

Let us consider the right-hand side of the double inequality (2.32). It can be written as:

$$\mathbf{j}^T_{mpr} \mathbf{v} \leq \dot{q}^\text{max}_i - \mathbf{j}^T_{mor} \mathbf{\omega}, \quad \forall i = 1...m$$  \hspace{1cm} (2.33).

Then, to calculate $v^p_{iso}$, we need to consider the worst case of (2.33) in terms of $\mathbf{v}$ and $\mathbf{\omega}$, such that $\|\mathbf{\omega}\| \leq \omega_{eq}$. This corresponds to having $\dot{q}^\text{max}_i - \mathbf{j}^T_{mor} \mathbf{\omega}$ minimal and $\mathbf{j}^T_{mpr} \mathbf{v}$ maximal and which obviously occurs for:

\[
\begin{cases}
\mathbf{\omega} = \omega_{eq} \frac{\mathbf{j}_{mor}}{\|\mathbf{j}_{mor}\|}, & \text{if } \|\mathbf{j}_{mor}\| \neq 0 \\
\text{indifferent of } \mathbf{\omega}, & \text{if } \|\mathbf{j}_{mor}\| = 0
\end{cases}
\]  \hspace{1cm} (2.34)

and

\[
\begin{cases}
\mathbf{v} = \mathbf{v} \frac{\mathbf{j}_{mpr}}{\|\mathbf{j}_{mpr}\|}, & \text{with } \mathbf{v} = \|\mathbf{v}\| \geq 0, \text{ if } \|\mathbf{j}_{mpr}\| \neq 0 \\
\text{indifferent of } \mathbf{v}, & \text{if } \|\mathbf{j}_{mpr}\| = 0
\end{cases}
\]  \hspace{1cm} (2.35).

Hence, substituting (2.34) and (2.35) in (2.33) yields:

$$\|\mathbf{j}_{mpr}\| \mathbf{v} \leq \dot{q}^\text{max}_i - \mathbf{j}^T_{mor} \mathbf{\omega}_{eq}, \quad \forall i = 1...m$$  \hspace{1cm} (2.36).

As $\|\mathbf{j}_{mpr}\| \mathbf{v} \geq 0$, it is mandatory to have $\dot{q}^\text{max}_i - \mathbf{j}^T_{mor} \mathbf{\omega}_{eq} \geq 0$. Otherwise, it means that the robot cannot initially fulfill the rotational motion requirement in the absence of translation. In this latter case, $v^p_{iso}$ does not really exist and it is set to zero by convention. Aside from that, we get:

\[
\begin{cases}
\mathbf{v} \leq A^+_i, & \text{with:} \\
\begin{cases}
0, & \text{if } a^+_i < 0 \\
a^+_i = \frac{\mathbf{j}_{mpr}}{\|\mathbf{j}_{mpr}\|}, & \text{if } \|\mathbf{j}_{mpr}\| \neq 0 \text{ and } a^+_i \geq 0, \quad \forall i = 1...m \\
\infty, & \text{otherwise}
\end{cases}
\end{cases}
\]  \hspace{1cm} (2.37).

Proceeding in the same manner with left-hand inequality of (2.32), we obtain:

$$\mathbf{v} \leq A^-_i, \quad \text{with } A^-_i = A^+_i$$  \hspace{1cm} (2.38),

or simply we can say (based on (2.37) and (2.38)), we have:
As $v_{iso}^{sp}$ is the largest attainable isotropic linear speed while allowing angular speed to reach $\omega_{req}$ in all directions, $v_{iso}^{sp}$ is then:

$$v_{iso}^{sp} = \min_{i=1...m} (A_i), \text{ with:}$$

$$A_i = \begin{cases} 
0, & \text{if } a_i < 0 \\
\frac{a_i}{\|j_{mpr}\|}, & \text{if } \|j_{mpr}\| \neq 0 \text{ and } a_i \geq 0, \forall i = 1...m \\
\infty, & \text{otherwise}
\end{cases} \quad (2.39).$$

$$a_i = d_i^{max} - \|j_{mor}\| \omega_{req}, \forall i = 1...m$$

Following the same lead, the mathematical expressions of the other specific isotropic terms are as follows:

$$\omega_{iso}^{sp} = \min_{i=1...m} (B_i), \text{ with:}$$

$$B_i = \begin{cases} 
0, & \text{if } b_i < 0 \\
\frac{b_i}{\|j_{mor}\|}, & \text{if } \|j_{mor}\| \neq 0 \text{ and } b_i \geq 0, \forall i = 1...m \\
\infty, & \text{otherwise}
\end{cases} \quad (2.41),$$

$$b_i = d_i^{max} - \|j_{mor}\| \omega_{req}, \forall i = 1...m$$

$$f_{iso}^{sp} = \min_{i=1...m} (C_i), \text{ with:}$$

$$C_i = \begin{cases} 
0, & \text{if } c_i < 0 \\
c_i \frac{1}{\|j_{pc}\|}, & \text{if } \|j_{pc}\| \neq 0 \text{ and } c_i \geq 0, \forall i = 1...m \\
\infty, & \text{otherwise}
\end{cases} \quad (2.42)$$

$$c_i = d_i^{max} - \|j_{pc}\| m_{req}, \forall i = 1...m$$

and
Chapter 2: Performance Evaluation of General Manipulators

\[ m_{iso}^{\tau} = \min_{i=1..m} \left( D_i \right), \text{ with:} \]

\[
D_i = \begin{cases} 
0, & \text{if } d_i < 0 \\
\frac{d_i}{\| j_{oc} \|}, & \text{if } \| j_{oc} \| \neq 0 \text{ and } d_i \geq 0, \ \forall i = 1..m \\
\infty, & \text{otherwise}
\end{cases}
\]

(2.43).

To better understand the above values and have some insight on their geometrical interpretation, illustrations for the cases of two-dof (1T-1R) and three-dof (2T-1R) robots are provided in Fig. 2-5 and Fig. 2-6, respectively.

In these figures, the operational twist zonotopes as well as the geometrical interpretation of \( v_{iso}^{\tau} \) and \( \omega_{iso}^{\tau} \) are illustrated. Note that the interpretation of \( v_{iso}^{\tau} \) and \( \omega_{iso}^{\tau} \) that appears as rectangular regions in the case of (1T-1R) must be carefully understood, as it can be misleading. That is why, the second example of (2T-1R) robot has been provided in the hope of eliminating any ambiguity. Perhaps, an even better example would be to consider the case of at least (2T-2R) manipulator, but unfortunately, such a case is graphically impossible to represent or comprehend. Nonetheless, with Fig. 2-6, a deeper insight on the true nature of the regions described by:

\[
\left\{ \begin{array}{l}
\Omega = \left\{ t \in \mathbb{R}^{\eta}; \| \omega \| \leq \omega_{req} \text{ and } \| v \| \leq v_{req} \right\} \\
\mathcal{R}_1 = \left\{ t \in \mathbb{R}^{\eta}; \| \omega \| \leq \omega_{req} \text{ and } \| v \| \leq v_{iso}^{\tau} \right\} \\
\mathcal{R}_2 = \left\{ t \in \mathbb{R}^{\eta}; \| v \| \leq v_{req} \text{ and } \| \omega \| \leq \omega_{iso}^{\tau} \right\}
\end{array} \right\}
\]

(2.44)
can be achieved.

The Interesting Features of Specific Isotropic Values

It remains to discuss some interesting features of specific isotropic values. First, we emphasize that it is enough to consider one specific isotropic speed (\( v_{iso}^{\tau} \) or \( \omega_{iso}^{\tau} \)) to verify that both translational and rotational speed requirements are fulfilled simultaneously. This is done by checking if its value is greater or equal to the corresponding requirement. Likewise, it is sufficient to know one of the specific isotropic static loads (i.e. \( f_{iso}^{\tau} \) or \( m_{iso}^{\tau} \)) to verify whether the mixed external wrench capacity requirements are satisfied or not. In fact, we can easily prove that specific isotropic values satisfy the following properties\(^{11}\):

1. \( v_{iso}^{\tau} \geq v_{req} \iff \omega_{iso}^{\tau} \geq \omega_{req} \).

\(^{11}\)The proof is as follows. For instance, let us consider the first property. As \( v_{iso}^{\tau} \geq v_{req} \), then we can guarantee that we can simultaneously and isotropically achieve at least \( \omega_{req} \) and \( v_{req} \) (see definition of \( v_{iso}^{\tau} \)). Then, based on the definition of \( \omega_{iso}^{\tau} \), this latter is necessarily greater or equal to \( \omega_{req} \); otherwise, we have contradiction. For better understanding, the reader may refer to Fig. 2-5.
Chapter 2: Performance Evaluation of General Manipulators

2. \( f_{iso}^{sp} \geq f_{req} \iff m_{iso}^{sp} \geq m_{req} \).

3. \( v_{iso}^{sp} \leq v_{iso}^{p} , \omega_{iso}^{sp} \leq \omega_{iso}^{p} , f_{iso}^{sp} \leq f_{iso}^{p} \), and \( m_{iso}^{sp} \leq m_{iso}^{p} \) with the terms having “p” superscript and “iso” subscript indicating the same meaning and definition as introduced in §2.2.3.B. It is clear that \( v_{iso}^{sp} , \omega_{iso}^{sp} , f_{iso}^{sp} \), and \( m_{iso}^{sp} \) are special cases of the specific isotropic values. They

Fig. 2-5: Geometrical interpretation of specific isotropic values in the case of two-dof (1T-1R) robot.

Fig. 2-6: Geometrical interpretation of specific isotropic values in the case of three-dof (2T-1R) robot.
correspond to the case where \( \omega_{req} = 0 \), \( v_{req} = 0 \), \( m_{req} = 0 \), and \( f_{req} = 0 \), respectively. This can be directly noticed from equations (2.40) through (2.43).

4. \( v_{iso}^{sp} \geq v_{req} \) (or \( \omega_{iso}^{sp} \geq \omega_{req} \)) implies that all speed requirements are fulfilled and the pose is sufficiently far from series-type singularity. Similarly, \( f_{iso}^{sp} \geq f_{req} \) (or \( m_{iso}^{sp} \geq m_{req} \)) indicates satisfaction of mixed wrench requirements and that the pose is adequately far from classical parallel-type singularity.

It is worth emphasizing that \( v_{req} \) and \( \omega_{req} \) serve as some sort of numerical tolerances for series-type singularity. To clarify this further, consider the set \( \mathcal{Q} \). Then, to assure that the pose is sufficiently distant from series-type singularity, the aforementioned region should be included in the operational-twist zonotope. The region \( \mathcal{Q} \) is the grey rectangle and the black cylinder in Fig. 2-5 and Fig. 2-6, respectively. Similar argument can be considered regarding static wrench capacities and parallel-type singularity.

Ultimately, to be able to co-assess simultaneous mixed motion, mixed static wrench capacities, and assure farness from classical series and parallel type singularities, we need to build an index using at least one specific isotropic speed (\( v_{iso}^{sp} \) or \( \omega_{iso}^{sp} \))\(^{12}\) and one specific isotropic static load (\( f_{iso}^{sp} \) or \( m_{iso}^{sp} \))\(^{13}\). In fact, the choice depends on what the roboticist needs or favors. For example, if he/she is interested in obtaining the utmost of robot capability in linear speed and static moment capacity, then it is recommended to use \( v_{iso}^{sp} \) and \( m_{iso}^{sp} \). In this case, optimizing for \( v_{iso}^{sp} \) and \( m_{iso}^{sp} \), by maximizing the composite index \( CI = \min \left( \frac{v_{iso}^{sp}}{v_{req}}, \frac{m_{iso}^{sp}}{m_{req}} \right) \) for instance, will elevate \( v_{iso}^{sp} \) and \( m_{iso}^{sp} \) without caring for maximizing \( \omega_{iso}^{sp} \) and \( f_{iso}^{sp} \). But surely, these latter two cannot drop below \( \omega_{req} \) and \( f_{req} \), respectively. This is due to the mutuality described by the first and second properties above. Of course, if one would like to elevate all, then he/she should combine them all in a composite index, and this does not lead to any computational expenses, as all are very simple to get. Note also that to globally assess or optimize a robot over a desired workspace, \( DWS \), it is sufficient to consider the worst values of the aforementioned local measures over \( DWS \).

Hence, it is obvious that with these novel measures, we can assess and optimize heterogeneous-dof manipulators with confidence regarding classical singularity and kinetostatic aspects. Using this approach, one knows well what he/she is optimizing or assessing a robot for, as compared to the available techniques based on arbitrary homogenization.

**Case Study: DUAL V**

Now, as the mathematical formulation of the new kinetostatic measures has been made, we proceed by performing a case study on DUAL V to concretize the concepts. The actuator

\(^{12}\) To assess the twist capacity and the farness from series-type singularity.

\(^{13}\) To assess the wrench capability and the farness from parallel-type singularity.
Table 2-1: DUAL V actuators’ characteristics and the desired kinetostatic requirements.*

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{q}^{\text{max}}$</td>
<td>Actuator’s Maximum Speed</td>
<td>$550 \text{ rpm} = 57 \text{ rad/s}$</td>
</tr>
<tr>
<td>$\tau^{\text{max}}$</td>
<td>Actuator’s Maximum Torque</td>
<td>$127 \text{ N m}$</td>
</tr>
<tr>
<td>$v_{req}$</td>
<td>Required Linear Speed</td>
<td>$0.2 \dot{q}^{\text{max}}L = 3.2 \text{ m/s}$</td>
</tr>
<tr>
<td>$\omega_{req}$</td>
<td>Required Angular Speed</td>
<td>$0.2 \dot{q}^{\text{max}} = 110 \text{ rpm} = 11.5 \text{ rad/s}$</td>
</tr>
<tr>
<td>$f_{req}$</td>
<td>Required Static Force Capacity</td>
<td>$0.2 \tau^{\text{max}}/L = 90.6 \text{ N}$</td>
</tr>
<tr>
<td>$m_{req}$</td>
<td>Required Static Moment Capacity</td>
<td>$0.2 \tau^{\text{max}} = 25.4 \text{ N m}$</td>
</tr>
</tbody>
</table>

*All actuators are identical of type ETEL RTMB-0140-100.

characteristics and the desired minimum requirements in translation and rotation are given in Table 2-1.

The specific isotropic values and the satisfactory regions are presented in Fig. 2-7, in terms of $\dot{q}^{\text{max}}$ and $\tau^{\text{max}}$. In this analysis, the rotation is fixed to $\theta = 0^\circ$ to allow visualization. Note that the regions where $v_{iso}^{\text{req}} \geq v_{req}$ and $\omega_{iso}^{\text{req}} \geq \omega_{req}$ are the same. Similar observation can be mentioned regarding the regions where $f_{iso}^{\text{req}} \geq f_{req}$ and $m_{iso}^{\text{req}} \geq m_{req}$. These concurrences are expected as discussed previously (first and second properties).

At the end of the kinetostatic evaluation section, we mention that the introduced measures can be sufficient as sole performance criteria only in the case where the manipulator under study works in quasi-static conditions. An example would be a machining application where we have two phases: a transient phase and a machining one. In the transient phase, the end-effector is supposed to move fast, usually at constant speed, from the current position to nearby the piece. Afterwards, the machining stage starts in which the tool moves at low speeds but counteracts considerable external wrenches due to material removal. Thus, considering only kinetostatic measures in such situations is valid. Nevertheless, in the case where the application involves considerable acceleration, speed, and external wrenches at the same time, a dynamic assessment is inevitable to evaluate the expected capabilities of the manipulator. This will be dealt with in the next part.
Fig. 2-7: DUAL V kinetostatic performance evaluation: the specific isotropic values (a) and the satisfactory regions (b).
2.2.4- Dynamic Performance Evaluation: A Novel Approach Based on Multi-Assessment of Isotropic Dynamic Capabilities

When the intended task of the manipulator or machine tool cannot be assumed quasi-static, it is then mandatory to put the dynamics of the whole system into the play. Considering the dynamic model (DM), an estimation of the isotropic simultaneous dynamic performances can be established. This establishment follows a similar methodology to that adopted for kinetostatics.

The complexity of the method is rather that of obtaining the DM itself. In fact, once the DM in the required form is obtained, the dynamic measures can be computed or estimated effortlessly via analytical formulae. Only in the case of heterogeneous-dof manipulators, where the effects of linear and angular velocities are coupled in the DM, the computation of the exact values necessitates a numerical optimization. Nevertheless, this numerical optimization can be done relatively at ease or can be replaced by analytical lower bounds, as a safe approximation. Although this latter point can be considered as a drawback, it is not that severe considering the various benefits the method provides. This will become clear by the end of the section.

In what follows, we will establish the analytical lower bounds of the dynamic specific isotropic values in the most general case of heterogeneous-dof robots. These lower bounds become the exact values in the situation of homogeneous dofs or uncoupled velocity effects.

A- Generalities and Assumptions

As a starting point, we assume that the DM of the robot has been obtained as a function of the operational variables and actuated-joint torques only. In this aspect, several methods exist and are classified into four main categories (ZHAN & GAO, 2009): Newton-Euler method (e.g., (KHALIL & GUEGAN, 2002), (HARIB & SRINIVASAN, 2003)), Lagrangian method (e.g., (PANG & SHAHINPOOR, 1994), (LEE & LEE, 2003)), Kane’s method (e.g. (BEN-HORIN, et al., 1998), (LIU, et al., 2000)), and virtual work principle based method (e.g. (WANG & GOSSELIN, 1998), (SOKOLOV & XIROUCHAKIS, 2007)). We further mention that in the case of redundant manipulators, a pseudo-inversion would be needed to acquire the DM in the aforementioned form. This can be done by assuming the solution with zero components along the null space of the considered matrix.

Anyway, we should have the DM as follows:

\[
M(x) \dot{\rho} + C(x,t) \dot{t} + \tau_g(x) + D(x) w_e = \tau \tag{2.45}
\]

The terms \( \rho = (a^T \; a^T)^T \), \( \tau_g(x) \), and \( w_e = (f_e^T \; m_e^T)^T \) correspond orderly to the operational accelerations (\( a = \dot{v} \) and \( a = \dot{\omega} \)), the torques due to gravitational forces, and the non-gravitational external wrench\(^{14} \) acting on the end-effector (external force, \( f_e \), and external

\(^{14}\) This is the wrench due to machining, for instance.
moment, \( m_e \). \( M(x) \) is the generalized \( m \times n \) inertia matrix, whereas \( c(x,t) = C(x,t)t \) represents the generalized Coriolis and centrifugal effects, with \( C(x,t) \) being of \( m \times n \) dimension. As for \( D(x) \), it is an \( m \times n \) matrix as well. The rest of symbols hold exactly the same meanings as before. Notice that in (2.45), friction is not considered. This has been done to simplify the analysis, as we are interested in evaluating the preliminary expected performances. Yet, the effect of friction can be included if it can be written under a similar form as any of the aforementioned terms.

Note that \( c(x,t) \) can be written under the following form:

\[
c(x,t) = (t^T H_1 t \cdots t^T H_m t)^T = h^r(x,t) + h^\omega(x,t) + h^{\omega^o}(x,t) \tag{2.46},
\]

with:

\[
h^r(x,v) = (v^T H_1^r v \cdots v^T H_m^r v)^T \tag{2.47},
\]

\[
h^\omega(x,\omega) = (\omega^T H_1^\omega \omega \cdots \omega^T H_m^\omega \omega)^T \tag{2.48}
\]

and

\[
h^{\omega^o}(x,t) = (2v^T H_1^{\omega^o} \omega \cdots 2v^T H_m^{\omega^o} \omega)^T \tag{2.49}.
\]

The terms \( H_1, H_i^r, H_i^\omega, \) and \( H_i^{\omega^o} \) are pose-dependent\(^{15} \) matrices of respective dimensions: \( n \times n, \dim(v) \times \dim(v), \dim(\omega) \times \dim(\omega), \) and \( \dim(v) \times \dim(\omega). \)

As a result, (2.45) can be written as:

\[
K \xi + h^r + h^\omega + h^{\omega^o} + \tau_e = \tau \tag{2.50},
\]

where \( K = [M(x) \ D(x)] \) and \( \xi = (\rho^T \ w_e^T)^T. \) This is the favorable form for our analysis. Henceforth, we are going to drop the \( (x) \) and \( (x,t) \) notations in \( K, h^r, \) etc. for readability purpose.

Concerning the constraints on actuators' capabilities, we only account for the following speed and torque limits:

\[
\begin{align*}
|\dot{q}_i| & \leq \dot{q}_i^{\text{max}} \quad \forall i = 1...m \\
|\tau_i| & \leq \tau_i^{\text{max}}
\end{align*}
\tag{2.51}
\]

Based on these hypotheses, the subsequent section describes briefly the approach to analyze the dynamic performance.

\(^{15}\) Function of pose, \( x, \) only.
B- Brief Description and Mathematical Formulation

Suppose that based on the intended application, the machine tool must be able to achieve simultaneously certain minimal isotropic requirements in: \( a, \alpha, v, \omega, f_e \), and \( m_e \). Denote them in corresponding order by: \( d\text{req}_a, d\text{req}_\alpha, d\text{req}_v, d\text{req}_\omega, d\text{req}_f \), and \( d\text{req}_m \), all of which being positive scalars.

Then, we may define the specific dynamic isotropic values as means of dynamic assessment. For instance, the specific isotropic linear acceleration, \( d\text{iso}_{\text{isp}}^a \), is defined as the maximal value of linear acceleration attainable by the robot regardless of direction, while allowing \( a, v, \omega, f_e \), and \( m_e \) to reach their corresponding requirements in all directions. Similarly, we define the specific isotropic angular acceleration, \( d\text{iso}_{\text{isp}}^\alpha \), the specific isotropic linear speed, \( d\text{iso}_{\text{isp}}^v \), the specific isotropic angular speed, \( d\text{iso}_{\text{isp}}^\omega \), the specific isotropic force, \( d\text{iso}_{\text{isp}}^f \), and the specific isotropic moment, \( d\text{iso}_{\text{isp}}^m \).

As mentioned earlier, in the general case of heterogeneous-dof robots, we can consider some particular lower bounds for the specific isotropic values. These are denoted by: \( d\text{isp}_{\text{isp}}^a, d\text{isp}_{\text{isp}}^\alpha, d\text{isp}_{\text{isp}}^v, d\text{isp}_{\text{isp}}^\omega, d\text{isp}_{\text{isp}}^f, \) and \( d\text{isp}_{\text{isp}}^m \). The computation of all these terms is done starting from definition and considering the following system of relations:

\[
\begin{align*}
K \dddot{\zeta} + \dot{h} + h' + h'' + \tau_g &= \tau \\
\dot{q} &= J_m t \\
|\dot{q}_i| &\leq \dot{q}_{i\text{max}}, \forall i = 1...m \\
|\tau_i| &\leq \tau_{i\text{max}}, \forall i = 1...m
\end{align*}
\]  

(2.52).

Based on (2.52) and following the procedure detailed in Appendix B.1 and Appendix B.2, we get the following compact forms for the different specific isotropic values (note that \( \text{val} = a, \alpha, f_e, m_e \) and \( \text{vel} = v, \omega \)):

---

16 The superscript "d" is used to indicate the case of dynamics as distinguished from kinetostatics, particularly for: linear and angular speeds, external force, and external moment.
Chapter 2: Performance Evaluation of General Manipulators

\[ d \text{val}_{iso}^p \geq d \alpha_{iso}^p = \min_{i=1..m} \left( K_C, \text{val} E_i^-, \text{val} E_i^+ \right), \]

with:

\[ K_C = \begin{cases} 
0, & \text{if } k_{iso}^p \leq d \omega_{req}, \text{or } k_{iso}^p < d \omega_{req} \\
\infty, & \text{otherwise} 
\end{cases} \]

\[ \text{val} E_i^- = \begin{cases} 
\text{val} e_i^- \| k_{i}^\text{val} \|, & \text{if } \text{val} e_i^- \leq 0 \text{ and } \| k_{i}^\text{val} \| \neq 0, \\
\infty, & \text{otherwise} 
\end{cases} \]

\[ \text{val} e_i^- = -\tau_{i}^\text{max} + \sum_{\text{vel} = \text{vel}} -\zeta_{\text{min}, \text{vel}} \left( d \omega_{\text{req}} \right)^2 - \zeta_{\text{max}, \text{vel}} \left( d \omega_{\text{req}} \right)^2 + 2 \sigma_{\text{max}, \text{vel}} d \omega_{\text{req}} d \omega_{\text{req}} - \tau_{g_i}, \]

\[ \text{val} e_i^+ = +\tau_{i}^\text{max} - \sum_{\text{vel} = \text{vel}} -\zeta_{\text{max}, \text{vel}} \left( d \omega_{\text{req}} \right)^2 - \zeta_{\text{min}, \text{vel}} \left( d \omega_{\text{req}} \right)^2 - 2 \sigma_{\text{max}, \text{vel}} d \omega_{\text{req}} d \omega_{\text{req}} - \tau_{g_i}, \]

\[ \forall i = 1..m \]

and

\[ d \text{vel}_{iso}^p \geq d \text{vel} E_{i \text{vel}}^p = \min_{i=1..m} \left( k_{i \text{vel}}^p, \text{vel} E_i^-, \text{vel} E_i^+ \right), \]

with:

\[ \text{vel}_{i \text{vel}}^- = \begin{cases} 
\text{vel} e_i^- \text{val} \text{vel} \text{vel} \text{vel}, & \text{if } \text{vel} e_i^- \leq 0, \text{val}_i \neq 0, \text{and } \zeta_{\text{vel}, \text{vel}} = 0 \\
\text{vel} e_i^- \text{vel} \text{vel} \text{vel} \text{vel}, & \text{if } \text{vel} e_i^- \leq 0, \zeta_{\text{vel}, \text{vel}} \neq 0, \text{and } \text{vel} e_i^- \geq 0 \\
\infty, & \text{otherwise} 
\end{cases} \]

\[ \text{vel} e_i^- = -\tau_{i}^\text{vel} + \sum_{\text{vel} = \text{vel}} -\zeta_{\text{vel}, \text{vel}} \left( d \omega_{\text{vel}} \right)^2 - \zeta_{\text{vel}, \text{vel}} \left( d \omega_{\text{vel}} \right)^2 + 2 \sigma_{\text{vel}, \text{vel}} d \omega_{\text{vel}} d \omega_{\text{vel}} - \tau_{g_i}, \]

\[ \text{vel} e_i^+ = +\tau_{i}^\text{vel} - \sum_{\text{vel} = \text{vel}} -\zeta_{\text{vel}, \text{vel}} \left( d \omega_{\text{vel}} \right)^2 - \zeta_{\text{vel}, \text{vel}} \left( d \omega_{\text{vel}} \right)^2 - 2 \sigma_{\text{vel}, \text{vel}} d \omega_{\text{vel}} d \omega_{\text{vel}} - \tau_{g_i}, \]

\[ \forall i = 1..m \]

(2.54)
Table 2-2: The significance of the general terms used in relations (2.53) and (2.54).

<table>
<thead>
<tr>
<th>Definition of the terms appearing in relation (2.53)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( val )</td>
</tr>
<tr>
<td>( a )</td>
</tr>
<tr>
<td>( \alpha )</td>
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<tr>
<td>( f_{r} )</td>
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<tr>
<td>( m_{r} )</td>
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<table>
<thead>
<tr>
<th>Definition of the terms in relation (2.54)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( vel )</td>
</tr>
<tr>
<td>( v )</td>
</tr>
<tr>
<td>( \omega )</td>
</tr>
</tbody>
</table>

with:

\[
\begin{align*}
  k_{v_{iso}}^{isp} &= \min_{i=1\ldots m} \left( A_i \right), \text{ such that:} \\
  A_i &= \begin{cases} 
    0, & \text{if } a_i < 0 \\
    \frac{a_i}{\| j_{mpr} \|} & \text{if } \| j_{mpr} \| \neq 0 \text{ and } a_i \geq 0 \\
    \infty, & \text{otherwise} 
  \end{cases}, \\
  a_i &= q_i^{\max} \ - \| j_{mor} \| d \omega_{req} \\
  \forall i = 1\ldots m
\end{align*}
\]

\[
\begin{align*}
  k_{\omega_{iso}}^{isp} &= \min_{i=1\ldots m} \left( B_i \right), \text{ such that:} \\
  B_i &= \begin{cases} 
    0, & \text{if } b_i < 0 \\
    \frac{b_i}{\| j_{mor} \|} & \text{if } \| j_{mor} \| \neq 0 \text{ and } b_i \geq 0 \\
    \infty, & \text{otherwise} 
  \end{cases}, \\
  b_i &= q_i^{\max} \ - \| j_{mpr} \| d \omega_{req} \\
  \forall i = 1\ldots m
\end{align*}
\]
Chapter 2: Performance Evaluation of General Manipulators

and

\[
\begin{align*}
\xi_{\min}^v &= \min \left( \text{eigs} \left( H^v_i \right), 0 \right), \quad \xi_{\max}^v &= \max \left( \text{eigs} \left( H^v_i \right), 0 \right), \\
\xi_{\min}^\omega &= \min \left( \text{eigs} \left( H^\omega_i \right), 0 \right), \quad \xi_{\max}^\omega &= \max \left( \text{eigs} \left( H^\omega_i \right), 0 \right), \quad \forall i = 1...m \\
\text{and } \sigma_{\max}^{\omega} &= \max \left( \text{sing} \left( H^{\omega i} \right) \right)
\end{align*}
\]

As for the significance of the terms \( \text{val} \), \( \text{sum} \text{val} \), \( \text{vel} \), \( \text{val}_1 \), \( \text{val}_2 \), and \( \text{val}_3 \) in (2.53) and (2.54), refer to Table 2-2. Also, for explicit formulae of specific isotropic values, they are given in Appendix B.1.

Hence, we have established some particular lower bounds for the specific isotropic values in dynamics. These lower bounds become the exact values in the following cases:

1. Homogeneous-dof manipulators;
2. Heterogeneous-dof manipulators with \( h^m = \theta \) (uncoupled effect of \( v \) and \( \omega \));
3. And heterogeneous-dof manipulators with \( d v_{\text{req}} = 0 \) or \( d \omega_{\text{req}} = 0 \).

We emphasize that knowing one of the specific isotropic values in dynamics is sufficient to test for the fulfillment of all dynamic requirements. The same applies regarding the computed lower bounds. For instance, we have:

\[
\begin{align*}
\frac{d a_i^{\text{iso}}}{d a_{\text{req}}} &\geq \frac{d a_{\text{req}}}{d a_{\text{req}}} \\
\frac{d f_i^{\text{iso}}}{d f_{\text{req}}} &\geq \frac{d f_{\text{req}}}{d f_{\text{req}}} \\
\frac{d m_i^{\text{iso}}}{d m_{\text{req}}} &\geq \frac{d m_{\text{req}}}{d m_{\text{req}}}
\end{align*}
\]

Regarding design optimization or assessment, the roboticist can choose one or more of the specific isotropic values, depending on his/her interest. Finally, a case study on DUAL V is presented in Appendix B.3.

2.3- Cable-Driven Parallel Robots (CDPRs)

2.3.1- Few Words on CDPRs

CDPRs have been extensively investigated in the last decades, due to several merits as compared to classical rigid PMs. Perhaps, the most intriguing of these features are large workspace volume with respect to global machine size, design simplicity, low cost, high payload to weight ratio, flexibility, and ease of reconfiguration. Also, having low moving inertia\(^{18}\) allows high dynamical performances to be achievable (e.g. the FALCON robot with a peak linear speed

\[17\] Recall that \( \text{eigs} \left( \text{Mat} \right) \) and \( \text{sing} \left( \text{Mat} \right) \) denote the lists of eigenvalues and singular values of the matrix \( \text{Mat} \), respectively.

\[18\] Due to having only cables connecting the platform with the actuators instead of rigid limbs.
of 13 m/s and peak linear acceleration of 43 g (KAWAMURA, et al., 1995)). Besides, most often the wires can be considered massless and inextensible, which simplifies their modeling.

Yet, CDPRs are not without their own limitations. In particular, when cables become lengthy, their mass cannot be any more ignored. Also, when high tension forces exist, the hypothesis of inextensibility is no more justifiable. All these complicate the modeling of CDPRs, as sagging and deformations are to be necessarily considered. Furthermore, as cables can only pull and not push, the feasibility of a pose is not only dependant on length admissibility of the wires and collision avoidance, but also on the possibility of having static equilibrium. Actually, the problem of tension distribution in CDPRs is a non-trivial issue and one of the key research topics in this field. This is not to mention the challenges that CDPRs present on the control level. Additionally, CDPRs are usually characterized by poor precision and stiffness as compared to rigid robots. The major sources of precision errors are the incomplete modeling, deformations, and sagging of wires. These errors become higher with the increase of payload. For more information regarding CDPRs, their state-of-art, applications, and advancements, the reader may refer to (LAMAURY, 2013), (TANG, 2014), the articles cited here and therein.

This was a brief overview on CDPRs. It remains to talk a little about their classification. Generally, we may distinguish three groups of CDPRs, namely the partially constrained, the fully constrained, and the underactuated ones.

In the case of partially constrained CDPRs (e.g. (ALBUS, et al., 1993)), their functioning depends on an external action such as gravity, which helps put all cables under tension. In fully constrained CDPRs (e.g. (LAFOURCADE & LLIBRE, 2002), (KAWAMURA, et al., 2000)), additional active cables are added to fully constrain the moving platform. As for underactuated ones, these are characterized by having fewer active cables than the number of dofs of the end effector, such as the Winch-Bot presented in (CUNNINGHAM & H. H. ASADA, 2009). These latter CDPRs are beyond our scope. In fact, we only consider in the current report CDPRs having $m$ active cables and $n$ controllable dofs such that $m \geq n$.

In what follows, we only deal with the kinetostatic and dynamic analysis of CDPRs following the same paradigm used for rigid manipulators, but after doing the necessarily mathematical reformulations. As for precision, though the methodology used for rigid robots can be adopted here, it is unreasonable as control errors are not the primary source of pose inaccuracy. In fact, regarding this latter issue, the other sources of error, such as elasticity and sagging, are more worth to focus on.

2.3.2- Generalities on the Kinetostatics of CDPRs

Consider a CDPR with $m$ active cables and $n$ controllable dofs for the end-effector, such that $m \geq n$. Then, the kinetostatic relations at a feasible pose and that are always available for CDPRs are:
Chapter 2: Performance Evaluation of General Manipulators

\[ \dot{q} = J_m \, t \]  
\[ w + J_m^T \, \tau = 0 \]

with all symbols carrying the same meaning as before.

As for the direct Jacobian, \( J \), it is usually not available except in the case where we have \( m = n \), as in the case of the partially constrained CDPR depicted in Fig. 2-8. Such CDPR can perform a planar motion (2T-1R) in the vertical plane and have the additional two equations at a feasible pose:

\[ t = J \, \dot{q} \]  
\[ \tau + J^T \, w = 0 \]

with \( J = J_m^{-1} \).

Other CDPRs with \( m > n \) have their forward kinematic and inverse static models given by:

\[ t = J_m^* \, \dot{q}, \text{ with } \dot{q}^T \left[ \text{null} \left( J_m^T \right) \right] = 0 \in \mathbb{R}^{(m-n)} \]  
\[ \tau = -J_m^{**T} \, w + \left[ \text{null} \left( J_m^* \right) \right] \lambda, \lambda \in \mathbb{R}^{m-n} \text{ (quasi-arbitrary)} \]

This case resembles redundantly actuated rigid manipulators. Yet, it is not quite the same. As the cables can only pull, \( \lambda \) is not fully but quasi arbitrary. This is due to the fact that \( \lambda \) must be chosen in such a way to have positive cable tensions, \( f_c = \left(f_{c_1} \cdots f_{c_n}\right)^T \). Thus, choosing \( \lambda = 0 \) as in rigid robots is not justifiable anymore.

In the case of statics, the positiveness condition of cable tensions:
Chapter 2: Performance Evaluation of General Manipulators

\[ 0 \leq f_{c_i}^{\text{min}} \leq f_{c_i} \leq f_{c_i}^{\text{max}}, \ \forall i = 1\ldots m \]  
\[ (2.65), \]
can be written in terms of actuators’ torques as below:

\[ 0 \leq \tau_{i}^{\text{min}} \leq \tau_{i} \leq \tau_{i}^{\text{max}}, \ \forall i = 1\ldots m \]  
\[ (2.66). \]

This is since in statics, the cable tensions and actuators’ torques are related by \( \Psi \), a diagonal matrix of positive entries\(^{19} \), i.e.:

\[ \tau = \Psi \ f_c \]  
\[ (2.67). \]

As for the bounds \( \tau_{i}^{\text{min}} \) and \( \tau_{i}^{\text{max}} \) in (2.66), they depend on \( f_{c_i}^{\text{min}} \), \( f_{c_i}^{\text{max}} \), and the maximal torque capacity of the \( i \)-th actuator for \( i = 1\ldots m \).

Thus, with this basic knowledge, we can proceed with the formulation of the kinetostatic analysis in terms of the operational twist and wrench.

### 2.3.3- Kinetostatic Performance Evaluation of CDPRs: An Extension of the Approach Applied on Rigid Manipulators

In this section, we seek to extend the notion of kinetostatic isotropic values defined for rigid manipulators into the realm of CDPRs. Regarding specific isotropic speeds, \( v_{\text{iso}}^{\text{sp}} \) and \( \omega_{\text{iso}}^{\text{sp}} \), they can be computed exactly as in the case of rigid manipulators, since they depend on (2.59) and the maximal speeds of the actuators, \( \dot{q}_i^{\text{max}} \) (with \( i = 1\ldots m \)). This is valid provided the pose is feasible. The formulae for these speeds are found in (2.40) and (2.41).

Therefore, it only remains to establish the mathematical formulation of specific isotropic force and moment. For this purpose, we consider the two generic cases. The first case (A) is having CDPRs with \( m = n \), whereas the second one (B) is having CDPRs with \( m > n \).

For both cases, we need first to decompose the external wrench, \( \mathbf{w} \), into two parts: the gravitational \( \mathbf{w}_g = (f_g^T \ m_g^T)^T \) and non-gravitational \( \mathbf{w}_e = (f_e^T \ m_e^T)^T \) external wrenches. In the case of suspended partially constrained CDPRs, the presence of \( \mathbf{w}_g \) is indispensable for the functioning of the robot, whereas it is not the case for fully constrained ones. Thus, the study is now transformed from \( \mathbf{w} \)-space into the \( \mathbf{w}_e \)-space.

In all what follows, we assume that the demanded simultaneous minimum external force and moment requirements are: \( f_{\text{req}}^{\text{sp}} > 0 \) and \( m_{\text{req}}^{\text{sp}} > 0 \), respectively. So, how to calculate the specific isotropic force \( f_{\text{iso}}^{\text{sp}} \) and moment \( m_{\text{iso}}^{\text{sp}} \)?

---

\(^{19}\) Note that to have \( \Psi \) with positive diagonal terms, we should choose the positive sense of actuator torque as to generate tension in the corresponding cable.
A- Specific Isotropic Force and Moment for CDPRs with Square Inverse Jacobian Matrices

In this situation, the calculation is straightforward and almost the same as what we have done for rigid robots, except that we need to account for $w_g$ and the modified torque limits described in (2.66).

Considering the static model in (2.62) and replacing $w$ by $w_g + w_e$, we get:

$$\tau = -J^T w_g + \tau_g$$  \hspace{1cm} (2.68),

with $\tau_g = -J^T w_g$. Then, the inequalities of (2.66) can be reformulated as follows:

$$\tau_{i\min} \leq -j_{pc}^T f_e - j_{ac}^T m_e + \tau_{i\max} \leq \tau_{i\max}, \quad \forall i = 1...m$$  \hspace{1cm} (2.69),

where $\tau_{i\min}$ is the i-th component of $\tau_g$. Then, to calculate $f_{esio}^{sp}$ when the static moment can reach $m_{ereq}$ in all directions, we need to rewrite (2.69) isolating the term containing $f_e$ as follows:

$$\tau_{i\min} + j_{ac}^T m_e - \tau_{i\min} \leq -j_{pc}^T f_e \leq \tau_{i\max} + j_{ac}^T m_e - \tau_{i\max}, \quad \forall i = 1...m$$  \hspace{1cm} (2.70).

Hence, considering each side of the double inequality of (2.70) in the worst case, we get certain upper bounds on $f_e = \|f_e\|$ over the region described by $m_e = \|m_e\| \leq m_{ereq}$. As this step is analogous to the already detailed one in the case of rigid manipulators, we directly provide the result, which is:

$$f_{esio}^{sp} = \min_{i=1...(m=n)} \left( E_i^+, E_i^- \right), \quad \text{with:}$$

$$E_i^+ = \begin{cases} 0, & \text{if } e_i^+ < 0 \\ \frac{e_i^+}{\|j_{pc}\|}, & \text{if } \|j_{pc}\| \neq 0 \text{ and } e_i^+ \geq 0, \\ \infty, & \text{otherwise} \end{cases}, \quad E_i^- = \begin{cases} 0, & \text{if } e_i^- < 0 \\ \frac{e_i^-}{\|j_{pc}\|}, & \text{if } \|j_{pc}\| \neq 0 \text{ and } e_i^- \geq 0, \\ \infty, & \text{otherwise} \end{cases}$$

$$e_i^+ = \tau_{i\max} - \|j_{ac}\| m_{ereq} - \tau_{i\min} - \|j_{ac}\| m_{ereq} + \tau_{i\min}, \quad e_i^- = -\tau_{i\max} - \|j_{ac}\| m_{ereq} + \tau_{i\min}$$

$$\forall i = 1... (m = n)$$

Similarly, we get the value of $m_{esio}^{sp}$ to be:
Chapter 2: Performance Evaluation of General Manipulators

\[
m_{\text{ciso}}^{\text{vp}} = \min_{i = 1, \ldots, (m-n)} \left( H_i^+, H_i^- \right), \quad \text{with:}
\]

\[
H_i^+ = \begin{cases} 
0, & \text{if } h_i^+ < 0 \\
\frac{h_i^+}{\| j_{\text{oc}} \|}, & \text{if } \| j_{\text{oc}} \| \neq 0 \text{ and } h_i^+ \geq 0, \quad H_i^- = \begin{cases} 
0, & \text{if } h_i^- < 0 \\
\frac{h_i^-}{\| j_{\text{oc}} \|}, & \text{if } \| j_{\text{oc}} \| \neq 0 \text{ and } h_i^- \geq 0
\end{cases}
\end{cases}, \quad \text{otherwise}
\]

\[
h_i^+ = \tau_i^{\text{max}} - \| J_{\text{pc}} \| f_{\text{ereq}} - \tau_{g_i}, \quad h_i^- = -\tau_i^{\text{min}} - \| J_{\text{pc}} \| f_{\text{ereq}} + \tau_{g_i},
\]

\[
\forall i = 1 \ldots (m-n)
\]

Note that in (2.71) and (2.72), the terms \( j_{\text{pc}} \) and \( j_{\text{oc}} \) hold the same significance as before\(^\text{20}\).

As it can be noticed, only slight dissimilarities exist in comparison with the formulae of (2.42) and (2.43). Now, it is time to deal with the more generic case, which is having \( m > n \).

B- Specific Isotropic Force and Moment for CDPRs with Non-Square Inverse Jacobian Matrices

For this case, we need to consider (2.59) after substituting \( w = w_g + w_e \) to have:

\[
w = w_e + w_g = -J_m^T \tau
\]

(2.73).

Then, to be able to calculate \( f_{\text{ciso}}^{\text{vp}} \) and \( m_{\text{ciso}}^{\text{vp}} \), it is compulsory to get the operational wrench zonotope boundaries based on (2.66) and (2.73). For this purpose, we implement the quite efficient and quick method suggested by (BOUCHARD, et al., 2008). This procedure is described in Appendix C. It yields the following system of inequalities describing the operational wrench zonotope:

\[
\eta_k^T w \leq \eta_k^T w_{0k}, \quad \forall k = 1 \ldots 2n_p
\]

(2.74),

with \( \eta_k \) being the unit outward normal of the \( k \)-th hyperplane, \( w_{0k} \) a point on this plane, and \( 2n_p \) the total number of boundary planes.

The system of inequalities (2.74) can be rewritten under the following form:

\[
\eta_{\text{pc}}^T f_e + \eta_{\text{oc}}^T m_e + \eta_k^T w_g \leq \eta_k^T w_{0k}, \quad \forall k = 1 \ldots 2n_p
\]

(2.75),

with \( \eta_k \) being expressed as \( \eta_k = (\eta_{\text{pc}}^T \quad \eta_{\text{oc}}^T)^T \) for \( k = 1 \ldots 2n_p \). Based on (2.75), the terms \( f_{\text{ciso}}^{\text{vp}} \) and \( m_{\text{ciso}}^{\text{vp}} \) are derived in a similar fashion as before to get the following formulae:

\[\text{Note that in (2.71) and (2.72), the terms } j_{\text{pc}} \text{ and } j_{\text{oc}} \text{ hold the same significance as before.}\]

\[\text{As it can be noticed, only slight dissimilarities exist in comparison with the formulae of (2.42) and (2.43). Now, it is time to deal with the more generic case, which is having } m > n.\]

\[\text{B- Specific Isotropic Force and Moment for CDPRs with Non-Square Inverse Jacobian Matrices}\]

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(2.73).

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\[\eta_k^T w \leq \eta_k^T w_{0k}, \quad \forall k = 1 \ldots 2n_p\]

(2.74),

\[\text{with } \eta_k \text{ being the unit outward normal of the } k \text{-th hyperplane, } w_{0k} \text{ a point on this plane, and } 2n_p \text{ the total number of boundary planes.}\]

\[\text{The system of inequalities (2.74) can be rewritten under the following form:}\]

\[\eta_{\text{pc}}^T f_e + \eta_{\text{oc}}^T m_e + \eta_k^T w_g \leq \eta_k^T w_{0k}, \quad \forall k = 1 \ldots 2n_p\]

(2.75),

\[\text{with } \eta_k \text{ being expressed as } \eta_k = (\eta_{\text{pc}}^T \quad \eta_{\text{oc}}^T)^T \text{ for } k = 1 \ldots 2n_p. \text{ Based on (2.75), the terms } f_{\text{ciso}}^{\text{vp}} \text{ and } m_{\text{ciso}}^{\text{vp}} \text{ are derived in a similar fashion as before to get the following formulae:}\]

\[\text{Note that in (2.71) and (2.72), the terms } j_{\text{pc}} \text{ and } j_{\text{oc}} \text{ hold the same significance as before.}\]

\[\text{As it can be noticed, only slight dissimilarities exist in comparison with the formulae of (2.42) and (2.43). Now, it is time to deal with the more generic case, which is having } m > n.\]

\[\text{B- Specific Isotropic Force and Moment for CDPRs with Non-Square Inverse Jacobian Matrices}\]

\[\text{For this case, we need to consider (2.59) after substituting } w = w_g + w_e \text{ to have:}\]

\[w = w_e + w_g = -J_m^T \tau\]

(2.73).

\[\text{Then, to be able to calculate } f_{\text{ciso}}^{\text{vp}} \text{ and } m_{\text{ciso}}^{\text{vp}}, \text{ it is compulsory to get the operational wrench zonotope boundaries based on (2.66) and (2.73). For this purpose, we implement the quite efficient and quick method suggested by (BOUCHARD, et al., 2008). This procedure is described in Appendix C. It yields the following system of inequalities describing the operational wrench zonotope:}\]

\[\eta_k^T w \leq \eta_k^T w_{0k}, \quad \forall k = 1 \ldots 2n_p\]

(2.74),

\[\text{with } \eta_k \text{ being the unit outward normal of the } k \text{-th hyperplane, } w_{0k} \text{ a point on this plane, and } 2n_p \text{ the total number of boundary planes.}\]

\[\text{The system of inequalities (2.74) can be rewritten under the following form:}\]

\[\eta_{\text{pc}}^T f_e + \eta_{\text{oc}}^T m_e + \eta_k^T w_g \leq \eta_k^T w_{0k}, \quad \forall k = 1 \ldots 2n_p\]

(2.75),

\[\text{with } \eta_k \text{ being expressed as } \eta_k = (\eta_{\text{pc}}^T \quad \eta_{\text{oc}}^T)^T \text{ for } k = 1 \ldots 2n_p. \text{ Based on (2.75), the terms } f_{\text{ciso}}^{\text{vp}} \text{ and } m_{\text{ciso}}^{\text{vp}} \text{ are derived in a similar fashion as before to get the following formulae:}\]

\[\text{Note that in (2.71) and (2.72), the terms } j_{\text{pc}} \text{ and } j_{\text{oc}} \text{ hold the same significance as before.}\]

\[\text{As it can be noticed, only slight dissimilarities exist in comparison with the formulae of (2.42) and (2.43). Now, it is time to deal with the more generic case, which is having } m > n.\]
Chapter 2: Performance Evaluation of General Manipulators

\[ f_{\text{el}}^{\text{iso}} = \min_{i=1\ldots n_p} (\varphi_i), \text{ with:} \]

\[ \varphi_i = \begin{cases} 
0, & \text{if } \varphi < 0 \\
\varphi_i, & \text{if } \|\eta_p\| = 0 \text{ and } \varphi_i \geq 0, \forall i = 1\ldots n_p \\
\infty, & \text{otherwise}
\end{cases} \quad (2.76) \]

\[ \varphi_i = \eta_i^T w_{gi} - \|\eta_i\| m_{\text{req}} - \eta_i^T w_g, \forall i = 1\ldots n_p \]

and

\[ m_{\text{el}}^{\text{iso}} = \min_{i=1\ldots n_p} (\psi_i), \text{ with:} \]

\[ \psi_i = \begin{cases} 
0, & \text{if } \psi < 0 \\
\psi_i, & \text{if } \|\eta_o\| = 0 \text{ and } \psi_i \geq 0, \forall i = 1\ldots n_p \\
\infty, & \text{otherwise}
\end{cases} \quad (2.77) \]

\[ \psi_i = \eta_i^T w_{oi} - \|\eta_i\| f_{\text{req}} - \eta_i^T w_g, \forall i = 1\ldots n_p \]

With this, we terminate the mathematical formulation of the kinetostatic performance measures in the case of CDPRs. We emphasize that these measures admit the same properties discussed in §2.2.3.C. As a summary, this extension of specific isotropic values, though have necessitated some additional steps, still it is rather simple and computationally efficient. We also highlight that although we have treated case (A) separately, still it can be approached as in case (B) without any problem. This means that one can use the approach in (B) to evaluate all CDPRs having \( m \geq n \), with relatively no noticeable computational impact. In the following section, we provide a case study on a fully constrained CDPR to further clarify the methodology.

C- Case Study: A Fully Constrained CDPR

In Fig. 2-9, we depict the fully constrained CDPR that will be analyzed in this part. It can perform a planar motion (2T-1R) in the horizontal plane and consists of four active cables connected to the mobile platform. Straight-line models of the tensioned cables are assumed, while neglecting the deformation due to tension (so in this case, it is similar to a rigid robot except that the cables must be under tension).

Here, the gravitational wrench is not included in the study, as it has no effect (i.e. \( w_g = (f_{gs} f_{gy} m_{gx})^T = 0 \)). The cables are wound on reels of equal radii of value \( r_c = 0.1 \text{ m} \), and the actuated-joint variables are defined as follows:

\[ q_i = \frac{L_i^\text{ref} - \|A_i B_i\|}{r_c}, \forall i = 1\ldots 4 \quad (2.78) \]
Chapter 2: Performance Evaluation of General Manipulators

with \( L_i = 0.71 \text{ m} \) for \( i = 1 \ldots 4 \) (i.e. positive rotation of the actuator winds the cable), \( x_{A_{1,2,3}} = -x_{A_{2,3}} = 0.5 \text{ m} \), \( y_{A_{1,2,3}} = -y_{A_{1,2,3}} = 0.5 \text{ m} \), and \( B_1 B_2 B_3 B_4 \) a square of side \( \|B_1 B_2\| = 0.05 \text{ m} \) and center \( P \equiv TCP \) (see Fig. 2-9).

The inverse Jacobian matrix is given by:

\[
J_m = J_q^T J_x
\]  
(2.79),

with\(^{21}\):

\[
J_q = -\text{diag} (r_1, \ldots, r_4), \quad \dim (J_q) = 4 \times 4
\]  
(2.80),

\[
J_x = \begin{bmatrix}
n_1^T & -n_1^T (PB \times e_z) \\
\vdots & \vdots \\
n_4^T & -n_4^T (PB \times e_z)
\end{bmatrix}, \quad \dim (J_x) = 4 \times 3
\]  
(2.81),

and \( n_i \) the unit vector along \( A_i B_i \) for \( i = 1 \ldots 4 \).

\(^{21}\) To derive \( J_q \), \( J_x \), and \( J_m \), we assume that when \( \dot{q}_i > 0 \), the cable is being wound on the pulley and thus, the velocity of point \( A_i \) is in the opposite sense to \( n_i \) for \( i = 1 \ldots 4 \). This means \( n_i^T v_{A_i} = -r_i \dot{q_i} = n_i^T v_{B_i} = n_i^T \left( v - \left( PB \times e_z \right) \dot{\theta} \right) \) for \( i = 1 \ldots 4 \) and as a result, relations (2.80) and (2.81) follow.
For this study, we have considered the requirements regarding speeds and static force and moment as given in Table 2-3. The plots depicting the specific isotropic values are shown in Fig. 2-10, as ratios relative to their corresponding requirements for the ease of comprehension. Notice that the study has been done as a function of \( x \) and \( y \), while maintaining \( \theta = 0^\circ \). Moreover, the satisfactory regions regarding kinetostatic requirements are shown in Fig. 2-11.

### Table 2-3: The speed and static load requirements for the CDPR in Fig. 2-9*

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Significance</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{q}^{\text{max}} )</td>
<td>Maximum Actuator’s Speed</td>
<td>550 rpm = 57 rad/s</td>
</tr>
<tr>
<td>( \tau^{\text{max}} )</td>
<td>Maximum Actuator’s Torque</td>
<td>127 N m</td>
</tr>
<tr>
<td>( \tau^{\text{min}} )</td>
<td>Positive Minimum Allowed Torque</td>
<td>0.1 ( \tau^{\text{max}} )</td>
</tr>
<tr>
<td>( v_{\text{req}} )</td>
<td>Required Linear Speed</td>
<td>0.65 ( \dot{q}^{\text{max}} ) ( r_e )</td>
</tr>
<tr>
<td>( \omega_{\text{req}} )</td>
<td>Required Angular Speed</td>
<td>0.45 ( \dot{q}^{\text{max}} )</td>
</tr>
<tr>
<td>( f_{\text{req}} )</td>
<td>Required Static Force Capacity</td>
<td>0.5 ( \tau^{\text{max}} / r_e )</td>
</tr>
<tr>
<td>( m_{\text{req}} )</td>
<td>Required Static Moment Capacity</td>
<td>0.5 ( \tau^{\text{max}} )</td>
</tr>
</tbody>
</table>

*All actuators are identical. The positive sense of the actuator torque is that of the positive sense of rotation that leads to shortening of the cable.

Fig. 2-10: Ratios of specific isotropic values to corresponding requirements for the CDPR in Fig. 2-9.
Chapter 2: Performance Evaluation of General Manipulators

It is worth emphasizing that the variation of performances of the CDPR along $x$ and $y$ directions, for $\theta = 0^\circ$, are not the same (see Fig. 2-10 and Fig. 2-11). This is due to the asymmetry in the cable connections at the platform level (see the grey configuration in Fig. 2-9). Also, it can be noticed clearly from Fig. 2-11 that the region in which the CDPR fulfills both kinetostatic requirements in terms of twist and external wrench coincides with the wrench satisfactory one (i.e. the region in the second graph of Fig. 2-11).

With this case study, we end the kinetostatic performance assessment of CDPRs. This evaluation is sufficient as long as the intended application falls within the quasi-static field. Aside from that, it is not enough and a dynamic analysis is necessary. This is the objective of the upcoming section.

2.3.4- Dynamic Performance Analysis of CDPRs: An Extension of the Approach Applied on Rigid Manipulators

In this section, we provide the reformulation of the dynamic measures introduced in §2.2.4 in order to deal with CDPRs. For this purpose, the DM should be written under the following compact form\(^{22}\) (notice that friction is neglected here as in §2.2.4):

\(^{22}\) This is applicable to all CDPRs with $m \geq n$. 

![Satisfactory Region Regarding Speeds](image)

![Satisfactory Region Regarding Static Loads](image)

**Fig. 2-11: Satisfactory regions in terms of kinetostatic performances based on specific isotropic values for the CDPR in Fig. 2-9.**
Chapter 2: Performance Evaluation of General Manipulators

\[
K \zeta + h' + h'' + h''' - w_g = \Phi \gamma, \quad \text{with:}
\]
\[
K = \begin{bmatrix}
M' & -I_a J_m \\
I_a J_m & 0
\end{bmatrix}, \quad \zeta = \begin{bmatrix}
\rho & w_c \\
w_g & w_{geq}
\end{bmatrix}, \quad \Phi = \begin{bmatrix}
J_m T & 0 \\
0 & -\Psi
\end{bmatrix}, \quad \gamma = \begin{bmatrix}
f_c
\end{bmatrix}
\]
\[
h = h' + h'' + h''' = \begin{bmatrix}
C'
I_a J_m
\end{bmatrix} t = \begin{bmatrix}
t^T H_1 t \\
\vdots \\
t^T H_{n+m} t
\end{bmatrix},
\]
\[
H_i = H_i(x) = \begin{bmatrix}
H_i^T(x) & H_i''(x) \\
(H_i''(x))^T & H_i''(x)
\end{bmatrix} \forall i = 1 \ldots (n+m)
\]
\[
h' = h'(x,v) = \begin{bmatrix}
v^T H_1' v \\
\vdots \\
v^T H_{n+m} v
\end{bmatrix}^T
\]
\[
h'' = h''(x,\omega) = \begin{bmatrix}
\omega^T H_1'' \omega \\
\vdots \\
\omega^T H_{n+m}'' \omega
\end{bmatrix}^T
\]
\[
h''' = h'''(x,t) = \begin{bmatrix}
v^T H_1''' \omega \\
\vdots \\
v^T H_{n+m}''' \omega
\end{bmatrix}^T
\]

In (2.82), \(M' = M'(x), C' = C'(x,t), w_{geq} = w_{geq}(x),\) and \(w_g = w_{geq}(x)\) correspond to the generalized inertia matrix, generalized Coriolis and centrifugal effects matrix, the total gravitational wrench as seen from the platform, and the external non-gravitational wrench acting on the platform, respectively. On the other hand, \(I_a = I_a(x)\) and \(w_{ga} = w_{ga}(x)\) represent the \(m \times m\) diagonal inertia matrix of the actuators and the corresponding gravitational wrenches acting on them due to their proper weight. The dimension of \(w_{ga}\) is \(m \times 1\). Regarding \(\Psi = \Psi(x)\), it is the \(m \times m\) diagonal matrix that relates \(\tau, \dot{q},\) and \(f_c\). As for the other symbols, they hold the same meanings as defined earlier\(^{23}\). For more details on the establishment of (2.82), refer to Appendix D.1. It is worth emphasizing that (2.82) is similar to (2.50) except for the term \(\Phi \gamma\), where \(\Phi\) is an \((n+m) \times 2m\) matrix. This difference necessitates a slight modification that resembles what have been done in the case of kinetostatics (§2.3.3.B).

Regarding the imposed constraints, we have:
\[
|\dot{q}_i| \leq \dot{q}_i^{\max}, \quad \forall i = 1 \ldots m
\]
(2.83),
and

\(^{23}\) Meaning: \(\rho = (a^T a^T)^T = (v^T \omega^T)^T\) (operational linear and angular accelerations), and \(t = (v^T \omega^T)^T\) (operational twist).
Chapter 2: Performance Evaluation of General Manipulators

\[
\begin{align*}
\gamma_i^{\text{min}} &\leq \gamma_i \leq \gamma_i^{\text{max}}, \quad \forall i = 1\ldots 2m \\
\text{with:} \quad f_i^{\text{min}} &= \begin{cases} 
-\tau_i^{\text{max}}, & \text{if } i \leq m \\
f_i^{\text{cmin}}, & \text{otherwise}
\end{cases}, \quad \gamma_i^{\text{max}} = \begin{cases} 
\tau_i^{\text{max}}, & \text{if } i \leq m \\
f_i^{\text{cmax}}, & \text{otherwise}
\end{cases} \quad (2.84).
\end{align*}
\]

To be able to acquire the specific isotropic values, or more precisely their lower bounds in the most generic case, we need to determine the zonotope described by:

\[
f_d = \Phi \gamma
\]

and for the limits provided in (2.84). This can be easily done following the same method suggested by (BOUCHARD, et al., 2008) and which is summarized in Appendix C. As a result, we get the \( f_d \)-zonotope expressed as:

\[
\mu_i^T f_d \leq \mu_i^T f_{d0i}, \quad \forall i = 1\ldots n_d
\]

with \( \mu_i \) and \( f_{d0i} \) being the unit outward normal and a point belonging to the \( i \)-th hyperplane. As for \( n_d \), it is the total number of hyperplanes that constitute the aforementioned zonotope.

Substituting \( f_d = K \xi + h^r + h^{\text{mo}} + h^\omega - w_g \) in (2.86), we get:

\[
\begin{align*}
\forall i = 1\ldots n_d : \\
\left( \beta_i^a \right)^T a + \left( \beta_i^w \right)^T w + \left( \beta_i^{\text{cmax}} \right)^T f_c + \left( \beta_i^{\text{cmin}} \right)^T f_m + v^T \Pi_i^a \omega + v^2 \Pi_i^{\text{mo}} \omega &\leq c_i - \beta_i^v \\
&\text{with:} \\
\beta_i = (\mu_i^T K)^T, \quad \beta_i^v = \mu_i^T h^r + v^T \Pi_i^a \omega, \quad \beta_i^{\text{cmax}} = \mu_i^T h^{\text{mo}}, \quad \beta_i^{\text{cmin}} = \omega^T \Pi_i^{\text{mo}} \omega \\
\beta_i^{\omega} = \mu_i^T h^{\text{mo}} = 2 v^T \Pi_i^{\text{mo}} \omega, \quad \beta_i^v = -\mu_i^T w_g, \quad c_i = \mu_i^T f_{d0i} \\
\Pi_i^a = \sum_{j=1\ldots(n+m)} \mu_{ij} H_j^a, \quad \Pi_i^{\omega} = \sum_{j=1\ldots(n+m)} \mu_{ij} H_j^{\omega}, \quad \Pi_i^{\text{mo}} = \sum_{j=1\ldots(n+m)} \mu_{ij} H_j^{\text{mo}}, \quad \forall i = 1\ldots n_d \quad (2.88).
\end{align*}
\]

Based on the relations (2.59), (2.83), and (2.87), it is possible to assess the dynamic capabilities. The derivation is similar to what has been described in Appendix B.1.

In brief, suppose that the dynamic simultaneous requirements are given by \( d \alpha_{\text{req}}, d \omega_{\text{req}}, d f_{\text{ereq}}, d m_{\text{ereq}} \), with all symbols baring the same significance as before. Then, we get the following compact formulation of the specific isotropic values, or their lower bounds in the most generic case (note that \( val = a, \alpha, f, m \) and \( vel = v, \omega \)):
Chapter 2: Performance Evaluation of General Manipulators

\[
\begin{align*}
  d \text{val}^\text{val}_{i,o} \geq d \text{val}^\text{val}_{i,o} &= \min_{i=1...n_d}(K_C, \text{val} E_i), \text{ with } (\forall i = 1...n_d): \\
  K_C &= \begin{cases} 
  0, & \text{if } k \text{vel}^\text{val}_{i,o} < d v_{req} (\text{or } k \text{omega}^\text{val}_{i,o} < d \omega_{req}) \\
  \infty, & \text{otherwise}
  \end{cases} \\
  \text{val} E_i &= \begin{cases} 
  0, & \text{if } \text{val} e_i < 0 \\
  \|\text{beta}\_i\|, & \text{if } \text{val} e_i \geq 0 \text{ and } \|\text{beta}\_i\| \neq 0 (2.89), \\
  \infty, & \text{otherwise}
  \end{cases}
\end{align*}
\]

\[
\begin{align*}
  \text{val} e_i &= c_i - \beta_i - \text{sum}_{\text{val}} - \zeta_{\text{max}} (d v_{req})^2 - \zeta_{\text{max}} (d \omega_{req})^2 - 2 \alpha_{\text{max}} d v_{req} d \omega_{req}
\end{align*}
\]

and

\[
\begin{align*}
  d \text{vel}^\text{vel}_{i,o} \geq d \text{vel}^\text{vel}_{i,o} &= \min_{i=1...n_d}(k \text{vel}^\text{vel}_{i,o}, \text{vel} E_i), \text{ with } (\forall i = 1...n_d): \\
  \text{vel} E_i &= \begin{cases} 
  \text{vel} e_i, & \text{if } \text{vel} e_i \geq 0, \text{ val} \_i \neq 0, \text{ and } \zeta_{\text{max}} = 0 \\
  \text{vel} e_i, & \text{if } \text{vel} e_i \geq 0, \text{ and } \zeta_{\text{max}} \neq 0, \text{ and } \text{vel} e_i \geq 0 \\
  \infty, & \text{if } \text{vel} e_i \geq 0, \text{ val} \_i = 0, \text{ and } \zeta_{\text{max}} = 0 \\
  0, & \text{otherwise}
  \end{cases}
\end{align*}
\]

\[
\begin{align*}
  \text{vel} e_i &= c_i - \beta_i - \|\text{beta}\_i\| \frac{d \alpha_{\text{req}}}{\|\text{beta}\_i\|} - \frac{d \omega_{\text{req}}}{\|\text{beta}\_i\|} \frac{f_{\text{req}}}{\|\text{beta}\_i\|} - \frac{d m_{\text{req}}}{\|\text{beta}\_i\|} - \text{val} \_2
\end{align*}
\]

with:

\[
\begin{align*}
  \zeta_{\text{max}} &= \max \left( \text{eigs} \left( \Pi_i^\text{eig} \right), 0 \right) \\
  \zeta_{\text{max}} &= \max \left( \text{eigs} \left( \Pi_i^\text{omega} \right), 0 \right), \forall i = 1...n_d \\
  \sigma_{\text{max}} &= \max \left( \text{sing} \left( \Pi_i^\text{omega} \right) \right)
\end{align*}
\]

As for \( k \text{vel}^\text{val}_{i,o} \) and \( k \text{omega}^\text{val}_{i,o} \), they have the same expressions as given in (2.55) and (2.56), respectively. Regarding the significance of the terms \text{val}, \text{sum}_{\text{val}}, \text{vel}, \text{val} \_1, \text{ and } \text{val} \_2, \text{ refer to Table 2-4.}

Hence, the dynamic measures have been established for CDPRs. Despite the fact that a new reformulation has been required to deal with their particularities, these dynamic measures are still possible to compute or estimate analytically. Finally, to demonstrate the methodology, a case study on a fully constrained CDPR is provided in Appendix D.
Chapter 2: Performance Evaluation of General Manipulators

2.4- Few Remarks

In all the kinetostatic and dynamic measures based on the notion of specific isotropic values, we have these latter values always positive. This positiveness is imposed by definition. More precisely, a specific isotropic value is the maximum attainable norm of a vectorial quantity in all directions, while allowing other vectorial quantities to reach their minimum required norms irrespective of their directions. Nevertheless, for the purpose of optimization, we can ignore the test on positiveness. For instance, $v_{\text{vel}}^{\text{sp}}$ in kinetostatics can be computed as follows:

$$v_{\text{vel}}^{\text{sp}} = \min_{i=1..m} \left( A_i \right), \quad \text{with :}$$

$$A_i = \begin{cases} a_i, & \text{if} \quad \left\| j_{\text{mpr}} \right\| \neq 0 \\ -\infty, & \text{if} \quad \left\| j_{\text{mpr}} \right\| = 0 \text{ and } a_i < 0, \ \forall i = 1..m \\ +\infty, & \text{otherwise} \end{cases}$$

$$(2.92),$$

regardless of having $a_i$ positive or negative. With the form given in (2.92), the optimization problem defined by:

<table>
<thead>
<tr>
<th>Definition of the terms appearing in relation (2.89)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>val</strong></td>
</tr>
<tr>
<td>$a$</td>
</tr>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>$f_e$</td>
</tr>
<tr>
<td>$m_e$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Definition of the terms in relation (2.90)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>vel</strong></td>
</tr>
<tr>
<td>$v$</td>
</tr>
<tr>
<td>$\omega$</td>
</tr>
</tbody>
</table>

Table 2-4: The significance of the general terms used in relations (2.89) and (2.90).
Chapter 2: Performance Evaluation of General Manipulators

![Actuator Limits Diagram](image)

**Fig. 2-12: Actuator limits illustration.**

\[
\begin{cases}
\max_{\xi_x} \left( \min_{x \in DWS} \left( \nu_{iso}^{sp} (x, \xi_g) \right) \right), \\
\nu_{iso}^{sp} (x, \xi_g) \quad \text{at pose } x \text{ and geometric parameters } \xi_g
\end{cases}
\]

is easier to do in comparison with initial definition given in (2.40).

Additionally, in all the presented dynamic evaluations, we have considered the actuators’ speeds and torques to be independent. This is implied by using:

\[
\begin{cases}
|\dot{q}_i| \leq \dot{q}_i^{\max}, \\
|\tau_i| \leq \tau_i^{\max}, \quad \forall i = 1...m
\end{cases}
\]

(2.94).

Such region corresponds to “zone 1” in Fig. 2-12. However, the more general case is to have the actuator limits bounded by several segments, like the case of considering the union of zones 1 and 2 for instance (see Fig. 2-12). This latter case can be generalized as follows (for \(m\) actuators):

\[
\delta_{ij}^T \left( \dot{q}_i, \tau_i \right) \leq \delta_{ij}^T \varsigma_{ij}, \quad \forall i = 1...m, \quad \forall j = 1...n_{ab}
\]

(2.95),

with \(\delta_{ij}\) representing the unit-outward normal of the j-th limiting segment, \(\varsigma_{ij}\) being a point on this segment, and \(n_{ab}\) corresponding to the number of limiting segments for the i-th actuator.
These general limits in (2.95) can be easily accounted for in the dynamic study. Yet, we do not recommend it, especially if the region where the speed and torque are independent constitutes a large proportion of the global available one.

Another worthy point to mention is concerning the acceleration limits imposed on the actuators. So far, we presumed there are no restrictions in this matter. Nonetheless, it might not always be the case. Actually, if limits exist, they can be directly utilized or after some simplification. In particular, if the limits are constants or pose-dependent, then their incorporation is a straightforward problem. It is done through considering $\ddot{q} = J_m \rho + J_m t$, and the inequalities $|\ddot{q}_i| \leq \ddot{q}_{i}^{\text{max}}$ for $i = 1...m$. Aside from that, it might be more complex and/or time demanding.

2.5- General Guidelines and Recommendations for the Design Optimization of Machine Tools

Thus far, we have presented several performance indices related to precision, kinetostatics, and dynamics. But how to optimize a given design based on one or more of the aforementioned criteria?

In fact, while the approach is straightforward for single criterion optimization, it is more debatable in the case of multi-objective one. In this latter case, several methodologies exist in literature. These can be classified into two main categories (UNAL, et al., 2008): scalarization and Pareto methods.

Scalarization addresses the multi-criteria optimization problem in an indirect manner, through altering it into a single or a series of single objective problems. As instantiation on this, we mention the hierarchical optimization approach (HAYWARD, et al., 1994), the sequential optimization technique ((ALICI & SHIRINZADEH, 2004), (RISOLI, et al., 1999)), the probabilistic weighting strategy (MCGHEE, et al., 1994), the use of composite index (LEE, et al., 2001), etc. Nonetheless, such methods possess the inherent detriment of requiring preferences or weights to be determined apriori; i.e. before the results of the optimization process are actually known. Since assigning proper weights or prioritizing different criteria is a problem-dependent and a non-trivial task, these techniques fall short of providing a general framework to the design process.

Therefore, we do not recommend such approaches. Instead, we suggest the Pareto-based methods. These incorporate all performance criteria within the optimization process and address them simultaneously to find a set of non-dominated solutions in the objective space (UNAL, et al., 2008). Once the hypersurface resolving the design trade-offs is obtained, an appropriate solution can then be selected. Such techniques are transparent, highly efficient, and do not require high expertise. As examples on this approach, we mention the works of ((KREFFT & HESSELBACH, 2005), (COURTEILLE, et al., 2009), (UR-REHMAN, et al., 2010)), and the recent dimensional synthesis we have done on ARROW V1 (SHAYYA, et al., 2014b), which will
be discussed later in Chapter 3. Finally, we point out that in the case where the number of objectives is greater than three, there are some tools that can help visualize the solution sets, such as the use of parallel coordinates for instance (INSELBERG & DIMSDALE, 1990). Nevertheless, it is always favored to keep the number of the criteria as low as possible.

2.6- Conclusion

In this chapter, the performance evaluation of general manipulators has been addressed from non-classical perspective. It has been founded on newly suggested precision, kinetostatic, and dynamic measures that overcome two main predicaments: redundancy and heterogeneity.

Among the prominent features of the approach, we recall the following: applicability to different types of robots (whether of rigid or cable-driven nature), embracing physical significance, computational simplicity, and direct relation with the needs of roboticist, engineer, or end-user.

Furthermore, some refinements concerning these measures have been emphasized, and general recommendations on the design optimization have been made.

In the next chapter, we present some synthesized parallel architectures, the dimensional synthesis of two of them, and the manufactured ARROW prototype.
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

In this chapter: In this chapter, few novel architectures, among the synthesized PMs, are discussed. These include mechanisms with five and four dofs of respective 3T-2R and 3T-1R nature. Their exposition embraces: geometric and kinematic models, singularity analysis, and workspace evaluation. Moreover, the dynamic models of two of them are detailed, and their dimensional syntheses are presented. In particular, only one of the suggested architectures is considered for implementation. The practical execution procedure and its required modifications are detailed, and the eventual ARROW PKM is exposed.
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

3.1- Introduction

In the quest for rapid and precise PKMs, several essential points must be considered, starting from the architecture itself, passing through the manufacturing procedure, and ending with the control approach. These three points are not completely separable and one can influence the other, especially the first two. This will become clear when we discuss the prototyped PKM.

Concerning the architecture, certain requirements are to be met. These are derived from the targets of the ARROW project that can be summarized as follows: large singularity-free workspace, high tilting capacity, performance homogeneity over the workspace, design simplicity, and rigidity.

As for manufacturing procedure, the execution of each joint, especially the complex ones (e.g. universal and spherical joints), must be carefully thought of. This consideration should assure maintaining high stiffness and minimizing clearances, as they are indispensable for having a precise machine. Furthermore, the quality control of the manufacturing as well as the assembly of the different components is necessary. It not only aids in having a machine with good initial performances, but also permits future enhancements or refinements, via geometric calibration for instance.

Regarding control, it is indisputably of great prominence. In fact, having a good architecture but with poor control will eventually lead to a poor-performance PKM. As a result, control can be the weakest or the strongest point in any machine tool. Nevertheless, this point is beyond the scope of the current thesis.

For our intended PKM, it should be capable of performing any machining task, although the main target applications have been the contactless ones, such as laser cutting and the alike. These latter applications are among the potential candidates that usually demand or benefit from rapidity, in addition to precision. Nevertheless, all these applications do not require more than five dofs of 3T-2R nature. These dofs can be achieved by various means, for instance: one parallel structure (complex or fully parallel), left-hand right-hand paradigm (i.e. branched structure), or series-parallel hybrid design.

In our case, we have focused mainly on implementing left-hand right-hand paradigm, in which a four-dof (3T-1R) PM is equipped with a one-dof (1R) turntable. However, this does not mean we did not consider five-dof (3T-2R) solutions. In fact, we have synthesized some five-dof mechanisms, among which is MachLin5 described in (SHAYYA, et al., 2014a).

In this chapter, we do not intend detailing all the synthesized mechanisms, but rather discussing the most prominent ones for the sake of compactness.

Finally, the current chapter is organized as follows. First, the five-dof (3T-2R) mechanism, namely MachLin5, is presented. Then, the study of two simple four-dof (3T-1R) PKMs, called ARROW V1 and ARROW V2, follows and some of the mutated versions of ARROW V2 are described. In particular, the mutated version ARROW V2 M2 is ultimately implemented after
some detailed analysis and verifications. Eventually, the chapter is concluded by re-emphasizing the substantial aspects.

3.2- MachLin5: A 5-DoF (3T-2R) Parallel Mechanism with Articulated Platform

Having a PKM with all its desired dofs embedded in one parallel structure has its rewards, as compared with the generally referred to hybrid machines, those implementing series-parallel design or left-hand right-hand paradigm (branched structures). In fact, series-parallel architectures (e.g. Tricept in Fig. 1-25, Exechon in Fig. 1-26, Sprint Z3 in Fig. 1-27, Hermes in Fig. 1-28, Dumbo in Fig. 1-29, etc.) are not a recommendable choice, as they often possess increased moving masses due to having additional actuators on the platform. Thus, not only their dynamic performance is impaired, but also their global precision and stiffness.

On the other hand, branched-structure mechanisms (e.g. P 800 in Fig. 1-24, VERNE in Fig. 1-31, HITA-STT whose parallel module is shown in Fig. 1-32, etc.) are more favored than series parallel designs since they overcome the latter’s aforementioned downsides. But depending on the design itself, it might be less flexible. This is due to the possibility of introducing challenges regarding control and motion planning. This is not to mention the calibration-related issues.

Based on the above, we have been motivated to investigate PMs with five dofs (3T-2R) in a first step. Knowing the inconveniences of similar PMs reported in literature (see Chapter 1), we attempted to synthesize new ones that overcome the former limitations. The result has been few novel architectures, among which we are going to present the most intriguing one, called MachLin5. A simplified CAD of this PM and a close-up view of its articulated platform are given in Fig. 3-1 and Fig. 3-2, respectively. The graph diagram is provided in Fig. 3-3.

3.2.1- Description

The PM consists of five linear actuators along the same direction (x-axis). Thus, the motion along x is independent of the other dofs and only restricted by the available stroke length for the actuators. Regarding the platform, it is an articulated one and consists of two parts, which are coupled by means of translation-to-rotation transformation system.
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

Fig. 3-1: MachLin5: simplified CAD drawing.

Fig. 3-2: MachLin5: the articulated platform.

Fig. 3-3: MachLin5: graph diagram.
The functioning of the mechanism is straightforward. Chains (III) through (V) cooperate to position the segment \( B_4B_5 \), which remains always vertical thanks to the constraining parallelograms. Then, chains (I) and (II), together, control the two rotational dofs: \( \theta_z \) about the \( z \)-axis of the base frame, and \( \theta_y \) about the \( y \)-axis of the tool frame \( (M_f) \). This is done thanks to the rack-pinion mechanism shown in Fig. 3-2, which by fixing the distance from \( B_1 \equiv B_2 \) to \( B_4B_5 \) and rotating it, controls the first rotation \( \theta_z \). As for \( \theta_y \), it is controlled by moving the rack, which then turns the pinion an angle proportional to the latter’s displacement. Hence, the TCP position and the tool orientation are controlled.

It is worth mentioning that we have used a rack-pinion assembly here to simplify the analysis. In general, it can be replaced by any rigid linear-to-rotational motion transformation system. In such a case, the transformation device should not admit any singularities within the required range of rotation for \( \theta_y \), which is \( \pm 45^\circ \).

Thus, we have a PM characterized by a large workspace and a tilting capacity that can reach \( \pm 90^\circ \) for \( \theta_z \), and \( \pm 45^\circ \) for \( \theta_y \). These rotational capacities are usually sufficient to perform five face machining and similar tasks.

This has been a general description of the MachLin5 functioning. In what follows, we emphasize the following essential technical remarks:

1. The S (spherical) joints can be achieved by using three concurrent revolute joints. This helps in overcoming the limited rotational capacity in the case of commercial counterparts.
2. The three S-joints at \( B_5 \) can be vertically spaced instead of having them coincident, which is difficult to implement. Thus, we can consider a common S-joint for the lower rods of the parallelograms, and another one for the fifth kinematic chain.

It then remains to give the general geometric designations used in the modeling process. Some of these symbols, particularly \( a, b, c, r_p \), and \( t_p \), are depicted in Fig. 3-2. The rest are defined hereafter:

- \( L_i \): the length of the \( i \)-th chain for \( i=1...5 \), with \( L_1 = L_2 \) and \( L_3 = L_4 \);
- \( A_i = (x_i, y_i, z_i)^T = (q_i, y_i, z_i)^T \), \( \forall i=1...5 \) with: \( -y_1 = y_2 = L_{y_1}, \; -y_3 = y_4 = L_{y_2}, \; y_5 = 0, \; z_1 = z_2 = L_{z_1}, \; z_3 = z_4 = 0, \; z_5 = -L_{z_2} \) and where \( A_3 \) and \( A_4 \) correspond to the coordinates of points \( A_3 \) and \( A_4 \) situated on the imaginary mid-axes of the parallelograms (III) and (IV), respectively;
- \( B_i = (x_{bi}, y_{bi}, z_{bi})^T \), \( \forall i=1...5 \) and where \( B_3 \) and \( B_4 \) correspond to the coordinates of the points \( B_3 \) and \( B_4 \) situated on the imaginary mid-axes of the parallelograms (III) and (IV), respectively;
\[ P = (x \ y \ z)^T, \ \theta = (\theta_z \ \theta_y)^T, \ \text{and} \ x = (P^T \ \theta^T)^T : \text{the TCP position, the orientation} \]
\[ e_h = \begin{pmatrix} \cos(\theta_z) & -\sin(\theta_z) & 0 \end{pmatrix}^T : \text{the unit vector along the } x \text{-axis of the frame } (M_i); \]
\[ e_f = \begin{pmatrix} -\sin(\theta_z) & \cos(\theta_z) & 0 \end{pmatrix}^T : \text{the unit vector along the common } y \text{-axis of the frames } (M_i) \text{ and } (M_f); \]
\[ \Omega_i = [e_z \ 0_{3x1}] \ \text{and} \ \Omega = [e_z \ e_f]; \]
\[ R_z = \text{Rot}_z(\theta_z), \ R_y = \text{Rot}_y(\theta_y), \ \text{and} \ R = R_z R_y, \ \text{with} \ R \ \text{representing the rotation} \]
\[ N = (x_N \ y_N \ z_N)^T \ \text{and} \ N^m = (x_N^m \ y_N^m \ z_N^m)^T : \text{the coordinates of some point } N \ \text{in} \]
\[ v = \dot{P} : \text{the linear velocity of } P; \]
\[ \omega_i = \Omega_i \dot{\theta} \ \text{and} \ \omega = \Omega \dot{\theta}; \ \text{the angular velocities of the first and second parts of the} \]
\[ \text{Finally, to simplify the analysis, we assume that the points } B_3 \equiv B_4 \ \text{and} \ B_5 \ \text{are confounded} \]
\[ \text{Having clarified the principle of functioning of the current PM and the different symbols that} \]
\[ \text{3.2.2 - Inverse Geometric Model (IGM)} \]
\[ C = P + R C^{M_f} \]
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

and

\[ B_3 \equiv B_4 \equiv B_5 = C + R_z B_3^{M_i} \]  \hspace{1cm} (3.4),

where \( B_3^{M_i} \equiv B_4^{M_i} \equiv B_5^{M_i} \) are all known.

Then, it remains to get the \( x \)-component of the points \( A_i \) (\( \forall i = 1...5 \)). This can be done utilizing the following relation:

\[ A_i B_i^2 = L_i^2, \hspace{0.5cm} \forall i = 1...5 \]  \hspace{1cm} (3.5),

which after development yields:

\[ q_i = x_i = x_{bi} \pm \sqrt{L_i^2 - (y_{bi} - y_i)^2 - (z_{bi} - z_i)^2}, \hspace{0.5cm} \forall i = 1...5 \]  \hspace{1cm} (3.6).

Only one solution of (3.6) is acceptable depending on the assembly mode. In our case, we consider the assembly mode described by the following condition:

\[ q_i = x_i \leq x_{bi}, \hspace{0.5cm} \forall i = 1...5 \]  \hspace{1cm} (3.7).

Consequently, the solution is:

\[ q_i = x_i = x_{bi} - \sqrt{L_i^2 - (y_{bi} - y_i)^2 - (z_{bi} - z_i)^2}, \hspace{0.5cm} \forall i = 1...5 \]  \hspace{1cm} (3.8),

and hence, the IGM of the mechanism has been derived.

3.2.3- Direct Geometric Model (DGM)

While the IGM is always computable for PMs, DGM is most often difficult to establish. Fortunately, here we can derive the DGM analytically with ease.

Knowing the joint positions, \( q \), the coordinates \( A_j \) (\( \forall j = 1...5 \)) are available. The first step is then to compute \( B_j \) (\( \forall j = 1...5 \)). For this purpose, we need to consider the following system of equations:

\[
\begin{align*}
A_3 B_3^2 &= L_3^2 \\
A_4 B_4^2 &= L_4^2, \hspace{0.5cm} \text{with: } B_3 \equiv B_4 \equiv B_5 \\
A_5 B_5^2 &= L_5^2
\end{align*}
\]  \hspace{1cm} (3.9).

The system (3.9) can be translated geometrically as the intersection of the three spheres centered at \( A_3, A_4, \) and \( A_5 \), and with respective radii \( L_3, L_4, \) and \( L_5 \). This is a classical mathematical problem, which can be easily solved. Therefore, we omit the details. In general, we get two possible solutions, say: \( B_3^{11} \) and \( B_3^{12} \).

As \( B_3 \) is known, then the \( z \)-component of point \( B_i \equiv B_2 \) can be determined by:

\[ z_{bi} \equiv z_{b2} = z_{b3} + (c - b) \]  \hspace{1cm} (3.10).
Hence, two solutions exist in correspondence to $B_3^{11}$ and $B_3^{12}$. Let us denote them by: $z_3^{11}$ and $z_3^{12}$. Then, obtaining the remaining components of $B_i \equiv B_2$, namely $x_{bi} \equiv x_{b2}$ and $y_{bi} \equiv y_{b2}$, is done by considering the following system of equations:

$$\begin{align*}
A_i B_i^1 &= L_1^2, \\
A_i B_i^2 &= L_2^2,
\end{align*}$$

(3.11).

The solution of (3.11) is geometrically described as the intersection of two circles in the horizontal plane defined by $z = z_{b1}$. For each solution of $z_{b1}$, we obtain two possible solutions for the coordinates $B_i \equiv B_2$. For $z_{b1} = z_{b1}^{11}$, we get $B_i^{111}$ and $B_i^{112}$. Similarly for $z_{b1} = z_{b1}^{12}$, we have $B_i^{121}$ and $B_i^{122}$.

Hence, there are four possible solutions described by the set $S = \{S_1, S_2, S_3, S_4\}$, with:

$S_1 = \{B_3^{11}, B_i^{111}\}$, $S_2 = \{B_3^{12}, B_i^{112}\}$, $S_3 = \{B_3^{21}, B_i^{21}\}$, and $S_4 = \{B_3^{22}, B_i^{22}\}$. Among these solutions, only one satisfies the assembly mode condition described by (3.7). Thus, we obtain $B_i \equiv B_2$ and $B_i \equiv B_3$. It remains to compute $P$ and $\theta$.

Defining $\eta = (\eta_x, \eta_y, 0)^T = B_3B_i - (B_i B_i^T e_z) e_z$, the value of $\theta_z$ is obtained by:

$$\theta_z = \text{atan2}(\eta_x, \eta_y) \in [-\pi; \pi]$$

(3.12).

Knowing $R_z = \text{Rot}_z(\theta_z)$, we get:

$$C = B_3 + R_{\theta_z} B_i C^M,$$

(3.13)

and

$$D = C + r_p e_z$$

(3.14).

Consequently, $\theta_y$ and $P$ are computed by means of the following relations:

$$\theta_y = \frac{(DB_i - (c - r_p) e_z) e_h}{r_p}$$

(3.15)

and

$$P = (x \ y \ z \ \theta_z \ \theta_y)^T = C - R C^M_T$$

(3.16).

Hence, the pose $x = (x \ y \ z \ \theta_z \ \theta_y)^T$ is obtained, and the DGM is analytically established.

### 3.2.4- Kinematic Model

This section is dedicated for the derivation of the kinematic model of the current robot, i.e. the relation between $q$ and $\dot{x}$. This can be easily done by following the classical approach.
The first step is to obtain the velocities of all points $B_i \ (\forall i = 1...5)$ in terms of $\dot{x}$. These are given below as:

$$
\begin{align*}
\forall i = 1, 2 : \\
v_{B_i} &= v_D + \omega_i \times DB_i = v + \omega \times PD + \omega_i \times DB_i \\
&= v - (PD \Omega \dot{\Omega} - DB_i \Omega_i \dot{\Omega}_i) \left[1_{3 \times 3} - \left(PD \Omega + DB_i \Omega_i\right)\right] \dot{x} \\
\end{align*}
$$

\hspace{1cm} (3.17)

and

$$
\begin{align*}
\forall i = 3, 4, 5 : \\
v_{B_i} &= v_C + \omega_i \times CB_i = v + \omega \times PC + \omega_i \times CB_i \\
&= \left[1_{3 \times 3} - \left(PC \Omega + CB_i \Omega_i\right)\right] \dot{x} \\
\end{align*}
$$

\hspace{1cm} (3.18)

Then, as $A_i B_i$ is a rigid body, the projections of the velocities $v_A$ of point $A_i$ and $v_{B_i}$ of point $B_i$ along $A_i B_i$ are equal, for all $i = 1...5$. This can be written as:

$$
A_i B_i^T v_{A_i} = A_i B_i^T v_{B_i}, \ \forall i = 1...5
$$

(3.19),

with $v_{A_i} = \dot{q}_i e_x \ (\forall i = 1...5)$.

The development of (3.19) after substituting (3.17) and (3.18) gives the following relation:

$$
J_q \dot{q} = J_x \dot{x}
$$

(3.20),

with:

$$
\begin{align*}
J_q &= \text{diag}(n_1^T e_x, ..., n_5^T e_x) \\
J_x &= \begin{bmatrix}
n_1^T & -n_1^T (PD \Omega + DB_i \Omega_i) \\
n_2^T & -n_2^T (PD \Omega + DB_i \Omega_i) \\
n_3^T & -n_3^T (PC \Omega + CB_i \Omega_i) \\
n_4^T & -n_4^T (PC \Omega + CB_i \Omega_i) \\
n_5^T & -n_5^T (PC \Omega + CB_i \Omega_i)
\end{bmatrix}
\end{align*}
$$

\hspace{1cm} (3.21)

Based on the relation (3.20), we have the following inverse and direct kinematic models, denoted by IKM and DKM respectively:

IKM: $\dot{q} = J_m \dot{x}$, with $J_m = J_q^{-1} J_x$

(3.22)

and

DKM: $\dot{x} = J \dot{q}$, with: $J = J_m^{-1} = J_x^{-1} J_q$

(3.23)

The relations (3.22) and (3.23) are valid provided that $J_q$ and $J_x$ are non-singular matrices.
Having established the kinematic models, we can proceed by investigating the singularity status of MachLin5 machine.

### 3.2.5- Singularity Analysis

In the case of PMs, there are various singularities to investigate and usually it is not sufficient to consider the input-output kinematic relation, as previously discussed in Chapter 1. Fortunately, this mechanism and due to its particular features, it is possible to carry its singularity study in two simple independent steps.

The first step is characterized by the study of constraint singularities, whereas the second is concerned with the classical ones, i.e. those based on the kinematic relation expressed by (3.20).

**A- Constraint Singularities**

In §3.2.1, we have discussed the principle of functioning of the mechanism. Particularly, we have explained that its functionality is held on the premise that the pair of parallelograms fulfills its intended role; i.e. as long as the chains (III) and (IV) constrain the two undesired rotations of the first part of the platform. Hence, a constraint singularity exists when this latter condition is not satisfied.

Actually, this singularity occurs only in the case where the two parallelograms are collinear. Due to the assembly mode condition (3.7), it dictates having $A_b B_1 // A_1 B_4 // e_y$. Consequently, we should have:

$$3 L_2 + 2 y = + \quad (3.24).$$

This singularity can then be coined as an architectural one, since it exists for a particular set of geometric parameters (those satisfying (3.24)).

Nevertheless, this singularity is prevented by construction, as we have $L_3 + L_4 > 2 L_{y2}$. In fact, having $2 L_{y2} = L_3 + L_4$ renders the robot useless, as the $y$ and $z$ components of $B_3 \equiv B_4$ remain fixed.

In brief, the PM does not admit any constraint singularity provided that $L_3 + L_4 > 2 L_{y2}$, which is satisfied by construction. Therefore, there is no possibility for an instantaneous increase of mobility.

**B- Series-Type Singularities**

These occur in the case where $J_q$ (in (3.20)) is rank deficient, i.e. $\det(J_q) = 0$. As $J_q$ is a diagonal matrix, then this singularity occurs if:

$$\det(J_q) = 0 \iff \exists i_o \in \{1, \ldots, 5\}; \quad n_{i_o}^T e_x = 0 \iff n_{i_o} \perp e_x \quad (3.25).$$
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

Based on (3.25), series-type singularity occurs when one or more of the arms are completely stretched in the $yz$ plane. This occurs, therefore, on the boundary of the geometrically accessible workspace and forms no problem.

### C- Parallel-Type Singularities

These occur when $J_x$ (in (3.20)) is rank deficient, i.e. $\det (J_x) = 0$. To derive the singular configurations, we proceed by performing some linear operations on $J_x$.

Adding $(n_1^T (PC \times e_z) \cdots n_5^T (PC \times e_z))^T$ (linear combination of the first three columns of $J_x$) and $(n_1^T (PC \times e_f) \cdots n_5^T (PC \times e_f))^T$ to the fourth and fifth columns of $J_x$ respectively, we obtain the matrix $T_1$ of same rank as $J_x$. $T_1$ is given by:

$$
T_1 = \begin{bmatrix}
n_1^T & -n_5^T (CD \Omega + DB \Omega) \\
n_2^T & -n_5^T (CD \Omega + DB \Omega) \\
n_3^T & -n_5^T (CB_3 \Omega) \\
n_4^T & -n_5^T (CB_4 \Omega) \\
n_5^T & -n_5^T (CB_5 \Omega)
\end{bmatrix}
$$

Again, adding $(an_1^T e_f \cdots an_5^T e_f)^T$ to the fourth column of $T_1$ simplifies the latter into $T_2$ that admits the same rank as $T_1$ and $J_x$. It is given by:

$$
T_2 = \begin{bmatrix}
n_1^T (a + r_p \theta_s) n_1^T e_f & r_p n_1^T e_h \\
n_2^T (a + r_p \theta_s) n_2^T e_f & r_p n_2^T e_h \\
n_3^T & 0 \\
n_4^T & 0 \\
n_5^T & 0
\end{bmatrix}
$$

The study of rank deficiency of $J_x$ can then be partitioned into investigating the singularity of two simpler matrices: $TS_1$ and $TS_2$. These are defined hereafter:

$$
TS_1 = \begin{bmatrix} n_3 & n_4 & n_5 \end{bmatrix}^T, \quad \dim(TS_1) = 3 \times 3
$$

and

$$
TS_2 = \begin{bmatrix}
(a + r_p \theta_s) n_1^T e_f & r_p n_1^T e_h \\
(a + r_p \theta_s) n_2^T e_f & r_p n_2^T e_h
\end{bmatrix}, \quad \dim(TS_2) = 2 \times 2
$$

$J_x$ is rank deficient when $TS_1$ or $TS_2$ is singular. Starting with $TS_1$, its singularity mandates having the three vectors $n_3, n_4,$ and $n_5$ coplanar. Due to the assembly condition (3.7), this
cannot happen unless the aforementioned vectors are in the $yz$ plane and hence, being confounded with series-type singularity as well. Therefore, it is only possible to occur on the boundary of the geometrically accessible workspace and does not form a problem.

As for $TS_2$, developing its determinant yields:

$$
det(TS_2) = \left( a + r_p \theta_z \right) \left( n_{1x} n_{2y} - n_{1y} n_{2x} \right) \left( e_{f_x} e_{h_z} - e_{f_z} e_{h_x} \right)
$$

$$
= -\left( a + r_p \theta_z \right) \left( n_{1x} n_{2y} - n_{1y} n_{2x} \right)
$$

(3.30),

where $vect_x$ and $vect_y$ designate the $x$ and $y$ components of vector $vect$, respectively. Since $a$ and $r_p$ can be chosen in such a way to have $|a + r_p \theta_z| > 0$ for $\theta_z \in [-\pi/4; +\pi/4]$, then it remains to investigate the expression $expr = \left( n_{1x} n_{2y} - n_{1y} n_{2x} \right)$. The term $expr$ is null when the projections of $n_1$ and $n_2$ in the $xy$ plane are collinear. But due to condition (3.7), this is not possible unless $n_1$ and $n_2$ are in the $yz$ plane, which implies coincidence with series-type singularity. Therefore, such singularity is only possible on the boundary of the workspace and consequently non-problematic.

In conclusion, we have seen that parallel singularities can exist, but confounded with series type ones, which are restricted to the boundary of the geometrically accessible workspace.

D- Results’ Briefing

Based on the above analyses, MachLin5 is free of singularities of all types over the accessible region excluding its boundary. This highly prominent feature makes the current mechanism stand out, as compared with those available in literature.

3.2.6- Workspace and Kinetostatic Performance

As the essential steps of modeling and singularity investigation have been done, the current section presents the workspace analysis for some values of the geometric parameters. These values are provided in Table 3-1 and do not represent an optimal solution with respect to any criterion.

Since the $x$-motion is independent of the remaining dofs, only the workspace region in terms of $y$ and $z$ will be investigated, for $\theta_z = \theta_y = 0^\circ$ on one hand, and for $|\theta_z| \leq 90^\circ$ and $|\theta_y| \leq 45^\circ$ on the other.

As for the performance indices, we consider the following measures:
**Chapter 3: The Novel Synthesized Architectures and ARROW PKM**

Table 3-1: Geometric parameters of MachLin5.*

<table>
<thead>
<tr>
<th>Geometric Parameter</th>
<th>Value$^{(1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_i$, $i = 1...5$</td>
<td>1</td>
</tr>
<tr>
<td>$L_{z1}$</td>
<td>0.3</td>
</tr>
<tr>
<td>$a$</td>
<td>0.1</td>
</tr>
<tr>
<td>$c$</td>
<td>0.125</td>
</tr>
<tr>
<td>$t_p$</td>
<td>0.075</td>
</tr>
<tr>
<td>$t_l$</td>
<td>0.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Geometric Parameter</th>
<th>Value$^{(1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{z1} = L_{z2}$</td>
<td>0.5</td>
</tr>
<tr>
<td>$b$</td>
<td>0.0875</td>
</tr>
<tr>
<td>$r_p$</td>
<td>0.025</td>
</tr>
<tr>
<td>$t_c$</td>
<td>0.0175</td>
</tr>
</tbody>
</table>

$^{(1)}$All values are expressed in meters (m).

*The parameters $t_c$ and $t_l$ correspond to the minimum offsets from the sliders’ planes in case of zero rotation (i.e. $\theta_z = \theta_y = 0^\circ$) and full range of rotations (i.e. $|\theta_z| \leq 90^\circ$ and $|\theta_y| \leq 45^\circ$), respectively.

Fig. 3-4: MachLin5 - $FVI_h$ performance: case of zero rotation (left) and case of rotation (i.e. $|\theta_z| \leq 90^\circ$ and $|\theta_y| \leq 45^\circ$) (right).

- $FVI_h$: We compute $FVI_h$ at each pose, after homogenizing $J_m$ and $J$ using the matrix $W = \text{diag} (1, 1, 1, a, t_p)$. The aim is just to have some insight on singularity performance only. We recall that no kinetostatic significance is being sought from this study. We have set $v_{\text{req}}^h = \dot{q}_{\text{max}}$ and $f_{\text{req}}^h = \tau_{\text{max}}$, where $v_{\text{req}}^h$ and $f_{\text{req}}^h$ are the required minimal homogenized linear speed and static force.
Here, we are more interested in quantifying pure translational motion and pure static force capacities, as we usually have the requirements regarding rotational motion and static moment less demanding. That is why we limit the kinetostatic study to $p_{\text{iso}}^v$ and $p_{\text{iso}}^f$. For this study, we have set the following requirements:

$$v_{\text{req}}^p = \dot{q}_{\max}$$ and $$f_{\text{req}}^p = \tau_{\max}$$

The analysis based on $FVI_h$ is summarized in Fig. 3-4. As for $v_{\text{iso}}^p$ and $f_{\text{iso}}^p$, the results of their analyses are plotted in Fig. 3-5 and Fig. 3-6, respectively. In all these figures, we clarify the following points:
• The black-dotted box represents the $yz$ projection of the robot’s physical limits (i.e. vertical walls, floor, and ceiling).
• The vertical solid red lines represent the limits that should not be exceeded by the TCP to avoid collision with the sliders’ planes.
• Finally, in the graphs with rotation, the plotted values are obtained as follows. For each $(y, z)$ position, several values of $\theta_z$ and $\theta_y$ are assumed and the value of the indices $FVI_h$, $v_{iso}^p$, and $f_{iso}^p$ are calculated. Then, the worst value (minimal one) is associated with the corresponding $(y, z)$ position. The angles tested are: $\theta_z = \pm (90^\circ, 60^\circ, 45^\circ, 30^\circ, 0^\circ)$ and $\theta_y = \pm (45^\circ, 30^\circ, 0^\circ)$.

Analyzing the results of Fig. 3-4, we can clearly notice the performance degradation (low value of $FVI_h$) near the boundary. We can also notice that while $FVI_h$ smoothly varies within the geometrically accessible regions, it steeply goes down just in the very narrow vicinity of the boundary. This is due to the presence of series or parallel-series type singularities, as discussed in §3.2.5. On the other hand, we can notice, based on Fig. 3-5, an interesting isotropic linear speed capacity that surpasses 50% of $\dot{v}_{\text{max}}$ over a large proportion of the workspace. Regarding $\frac{f_{iso}^p}{\tau_{\text{max}}}$, though not as high as what we had in the case of $\frac{v_{iso}^p}{\dot{q}_{\text{max}}}$, still it is rather fairly well surpassing 18% in vast proportion of the workspace. Here, the reader must keep in mind that the geometrical parameters are not optimized and the values being discussed are of isotropic nature.

3.2.7- Synopsis

In this part (§3.2), we have studied a novel five-dof (3T-2R) PM, in which we have derived all of its geometric and kinematic models, investigated its singularities, and analyzed its workspace. The results have revealed that this mechanism has the following interesting features:

1. The design is quite simple and easy to model. In particular, it admits an analytical DGM, which can be helpful regarding control. Also, several options are available that can simplify it further, particularly in what concerns the spherical joints and their execution.
2. It admits vast workspace with interesting tilting capacity, and independent $x$-motion. Also, the design is characterized by the absence of singularities of all types within the geometrically accessible region excluding its boundary.
3. Its singularity and kinetostatic performances over the workspace are interesting. They are subject for further improvement upon the optimization of its geometric parameters.
4. Having all the arms under tension/compression forces means reduced deformation and consequently, better accuracy.
5. The design, being non-redundant, does not compel any special treatment on the control level.
6. The possibility of replacing the rack-pinion assembly by a more rigid linear-to-rotation transformation mechanism helps improving the structural rigidity. This also benefits precision.

Nonetheless, having five sliders with independent guides situated in two planes and along a line is not favored. This is because it will necessitate more severe quality control, such as assuring the surface parallelism and orthogonality on one hand, and the parallelism of the linear guides on the other. Moreover, having an articulated platform is another weak point in the current mechanism. In fact, it has been a discouraging factor, especially after our observation regarding the precision of Veloce prototype and Quattro, with their articulated platform versions.

Therefore, as it seemed rather difficult to have a five-dof architecture embedding all the yearned for characteristics, among which is platform rigidity, we have diverted our attention towards four-dof PMs. Among these PMs, we focus in what follows on two main designs, called ARROW V1 and ARROW V2. In particular, ARROW V2 can be considered as an advancement of ARROW V1. This will become clear as we go through.

3.3- ARROW V1: A Redundantly Actuated Four-DoF (3T-1R) Parallel Manipulator

This section is devoted for discussing the four-dof (3T-1R) PKM, namely ARROW V1 (an acronym for Accurate and Rapid Robot with large Operational Workspace Version 1), which has been the subject of ((SHAYYA, et al., 2013a), (SHAYYA, et al., 2013b), (SHAYYA, et al., 2014b) and (SHAYYA, et al., 2014c)). The CAD of the PKM and its graph diagram are provided in Fig. 3-7. Also, the frontal and close-up side views of the PKM and its platform are depicted in Fig. 3-8.

In the upcoming subsection, a brief description of ARROW V1 and its principle of functioning are provided.

3.3.1- Description

ARROW V1 is a redundantly actuated PKM consisting of six linear actuators for four dofs of 3T-1R type. The actuators are aligned along the same direction (x-axis) and partitioned into two sets. Each set consists of three actuators, equidistant one from another, and laying in the same vertical plane. The PKM is structurally symmetric, which is highly favorable from industrial and manufacturing perspectives. Besides, the parallelism of its actuators contributes to the independent motion along the x-axis and therefore, having a vast workspace. Moreover, it is capable of performing $\pm 90^\circ$ about the z-axis, which constitutes one of the significant merits of the design.
The principle of functioning of the mechanism is quite simple. The pair of parallelograms (i.e. chains (III) and (IV)) constrains all rotations except that about the \( z \)-axis of the base frame. Then, all the chains cooperate to position the TCP and control the tool orientation. But for the mechanism to function properly, it is mandatory to have \( L_z \neq a \) (see Fig. 3-8). In practice, \( L_z \) is chosen to be larger than \( a \). This condition and its importance will be clarified later.
Finally and before presenting the modeling of ARROW V1, we mention that the symbols that we are going to use are clarified in Fig. 3-8. Also, notice that $L_i$ designates the $i$-th arm length for all $i = 1...6$, such that $L_1 = L_2 = L_3 = L_6 = L_p$, and $L_3 = L_4 = L_p$.

### 3.3.2- Inverse Geometric Model (IGM)

As the design is simple and its modeling is similar to that of MachLin5, we are going to omit redundant details. Given the end-effector pose $x = (x, y, z, \theta)^T$, then the coordinates $P = (x, y, z)^T$ of $P$ (the TCP) and the rotation matrix $R = \text{Rot}_z(\theta)$ of the platform are all known. Thus, the coordinates $B_i$ of points $B_i$ $(\forall i = 1...6)$ are computable via:

$$B_i = (x_{bi}, y_{bi}, z_{bi})^T = P + R B_i^{\text{plat}}, \forall i = 1...6$$ \hspace{1cm} (3.31),

where $B_i^{\text{plat}}$ corresponds to the known coordinates of $B_i$ in the frame connected to the moving platform. Then, substituting (3.31) in the following relation:

$$A_i B_i = L_i^2, \forall i = 1...6, \text{ with:}$$

$$A_i = (x_i, y_i, z_i)^T = (q_i, y_i, -z_i - y_i)^T, y_i = (-1)^i L_y$$

$$z_i = \begin{cases} L_z, & \text{if } i = 1, 2, 5, 6 \\ 0, & \text{if } i = 3, 4 \\ -L_z, & \text{if } i = 5, 6 \end{cases}$$ \hspace{1cm} (3.32)

and considering the assembly mode condition described by:

$$q_i = x_i \leq x_{bi}, \forall i = 1...6$$ \hspace{1cm} (3.33),

we get the solution of the IGM as below:

$$q_i = x_i = x_{bi} - \sqrt{L_i^2 - (z_i - z_{bi})^2}, \forall i = 1...6$$

with:

$$L_i = \begin{cases} L_i, & \text{if } i = 1, 2, 5, 6 \\ L_p, & \text{otherwise} \end{cases}$$ \hspace{1cm} (3.34).

### 3.3.3- Direct Geometric Model (DGM)

Unlike most PKMs, ARROW V1 admits an analytical DGM. In fact, due to actuation redundancy there exists no unique way to establish it. In what follows, we describe one of the possible methods.

Assuming that the joint positions vector $q$ is known, then the coordinates $A_i$ are available as well, for all $i = 1...6$.

Let us consider the following system of equations:
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

\begin{align}
A_1B_1^2 &= L_1^2 \\
A_2B_2^2 &= L_2^2 \\
A_3B_3^2 &= L_3^2
\end{align}  \tag{3.35}.

But as \( B_1 \equiv B_2 \) and \( B_1B_3 = -2a e_z \), the system \eqref{3.35} is transformable into the following form:

\begin{align}
A_1B_1^2 &= L_1^2 \\
A_2B_2^2 &= L_2^2, \text{ with: } A_3' = A_3 + 2a e_z \\
A_3B_3^2 &= L_3^2
\end{align}  \tag{3.36}.

System \eqref{3.36} represents a classical mathematical problem, which corresponds geometrically to the intersection of the three spheres of centers \( A_1, A_2, \) and \( A_3' \) (of coordinates \( A_1', A_2, \) and \( A_3' \)), and with radii \( L_1, L_2, \) and \( L_3 \), respectively. Omitting details, we get in general two possible solutions for \( B_1 \). Let us denote them by \( B_{11} \) and \( B_{12} \). Then, we still need to get \( B_3 \equiv B_4 \). For this purpose, we consider the following relations:

\begin{align}
A_3B_3^2 &= L_3^2, \text{ with: } B_3 \equiv B_4 \\
A_4B_4^2 &= L_4^2
\end{align}  \tag{3.37}.

But we know that \( z_{b1} \equiv z_{b4} = z_{b1} - a \), then \eqref{3.37} can be rewritten after incorporating the aforementioned relation as:

\begin{align}
A_1B_1^2 &= L_1^2 \\
A_4B_4^2 &= L_4^2
\end{align}  \tag{3.38}.

The problem expressed by \eqref{3.38} is described geometrically as the intersection of two circles in the horizontal plane of equation \( z = z_{b1} = z_{b1} - a \). It can be easily solved for each possible solution of \( z_{b1} \). For \( z_{b1} = z_{b1}^{(1)} \) (corresponding to the \( z \)-component of \( B_{11} \)), we get generally two possible solutions for \( B_1 \), denoted by \( B_{11} \) and \( B_{11}^{(2)} \). Likewise, for \( z_{b1} = z_{b1}^{(2)} \) (corresponding to the \( z \)-component of \( B_{12} \)), we get another two possible solutions, say: \( B_{12}^{(1)} \) and \( B_{12}^{(2)} \).

In brief, we generally obtain a set of four possible solutions, denoted by \( S = \{ S_1, S_2, S_3, S_4 \} \), with \( S_1 = \langle B_{11}^{(1)}, B_{11}^{(2)} \rangle, \ S_2 = \langle B_{11}^{(1)}, B_{11}^{(2)} \rangle, \ S_3 = \langle B_{12}^{(1)}, B_{12}^{(2)} \rangle, \ S_4 = \langle B_{12}^{(1)}, B_{12}^{(2)} \rangle \). Among these four solutions, there is only one \( S \in S' \) that satisfies the assembly mode condition described by \eqref{3.33}. Hence, we have determined uniquely \( B_1 \) through \( B_6 \). It remains to calculate \( P \) and \( \theta_z \).

Defining \( \eta = (\eta_z, \eta_y, 0)^T = B_3B_1 - (B_3B_3^T e_z)e_z \), the rotation \( \theta_z \) is calculated, thanks to the equation below:

\[ \theta_z = \arctan 2(\eta_z, \eta_y) \]  \tag{3.39}.

As for \( P \), it is given by:
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

\[ P = \begin{pmatrix} x & y & z \end{pmatrix}^T = B_i - R B_i^{\text{plat}}, \text{ with } R = \text{Rot}_z(\theta_z) \]  
(3.40).

As a result, the end-effector pose \( x = (x \ y \ z \ \theta_z)^T \) is computed and the DGM is established.

### 3.3.4- Kinematic Model

Let us now derive the relation between \( \dot{q} \) and \( \dot{x} \). For this purpose, we proceed in the very same manner as in §3.2.4. First, we get the velocities \( v_{B_i} \) and \( v_{A_i} \) in terms of \( \dot{x} \) and \( \dot{q} \), respectively. This is described via the equations:

\[
\begin{align*}
\begin{cases}
\dot{v}_{B_i} &= v + \omega \times PB_i = v - \dot{\theta}_z (PB_i \times e_z) = [1_{3\times3} \ -(PB_i \times e_z)] \dot{x} \\
&\text{with:} \\
v &= \dot{P} \quad \text{and} \quad \omega = \dot{\theta}_z e_z \\
\forall i = 1...6
\end{cases}
\end{align*}
\]

and

\[
v_{A_i} = \dot{q}_i e_x, \quad \forall i = 1...6 \tag{3.42}.
\]

As \( A_i B_i \) is a rigid body for all \( i = 1...6 \), then we have:

\[
A_i B_i^T v_{A_i} = A_i B_i^T v_{B_i}, \quad \forall i = 1...6 \tag{3.43}.
\]

Based on (3.43), and substituting (3.41) and (3.42), we get the following kinematic relation:

\[
\begin{align*}
\begin{cases}
J_q \dot{q} &= J_x \dot{x}, \quad \text{with:} \\
J_q &= \text{diag}(n_1^T e_x, ..., n_6^T e_x) \\
J_x &= \begin{bmatrix}
n_1^T & -n_1^T (PB_1 \times e_z) & & \\
& \ddots & \ddots & \\
n_6^T & -n_6^T (PB_6 \times e_z)
\end{bmatrix} \\
n_i = \frac{\bar{A}_i \bar{B}_i}{\left\| \bar{A}_i \bar{B}_i \right\|}, \quad \forall i = 1...6
\end{cases}
\end{align*}
\]

(3.44).

Notice that \( J_q \) is a 6×6 square matrix as compared with \( J_x \), which is rectangular of 6×4 dimension. This is due to actuation redundancy. Thus, we can get naturally the inverse kinematic model (IKM), based on the inverse Jacobian, \( J_m \). This is given by:

\[
\text{IKM: } \dot{q} = J_m \dot{x}, \quad \text{with } J_m = J_q^{-1} J_x \tag{3.45}.
\]

As for the direct kinematic model (DKM), it requires a pseudo-inversion procedure and it can be expressed as follows:

\[
\text{DKM: } \dot{x} = J \dot{q} = J_m^T \dot{q}, \text{ provided that } \dot{q}^T \left[ \text{null}(J_m^T) \right] = 0_{6 \times 2} \tag{3.46}.
\]
The relations (3.45) and (3.46) are valid as long as $J_q$ and $J_x$ are non-singular.

### 3.3.5- Singularity Analysis

Similarly to MachLin5, the singularity analysis of ARROW V1 can be partitioned into separate steps. The first step is concerned with the constraint singularities and mainly focuses on the pair of parallelograms. As for the second step, it deals with the classical singularities based on (3.44).

#### A- Constraint Singularities

Following the same reasoning as in §3.2.5.A, there is no constraint singularity since it is avoided by construction. Actually, it would occur if and only if the parallelograms are collinear. Due to the assembly condition (3.33), this cannot happen unless:

$$2 L_y = L_3 + L_4$$

(3.47) is satisfied. As this singularity occurs for a specific set of geometric parameters, those satisfying (3.47), then it is of architectural nature.

Nevertheless, (3.47) is avoided by having $L_3 + L_4 > 2 L_y$. In fact, having (3.47) fulfilled, the PKM is rendered useless ($B_3 \equiv B_4$ maintains always fixed $y$ and $z$ components).

It remains then to consider (3.44) and its related singularities. These will be discussed in what follows.

#### B- Series-Type Singularities

These occur when $\det(J_q) = 0$, which can be mathematically described as:

$$\det(J_q) = 0 \iff \exists i, \in \{1, \ldots, 6\}; \ n_i^T e_x = 0 \iff n_i \perp e_x$$

(3.48)

From (3.48), this occurs when one or more arms are completely stretched and laid in the $yz$ plane. This cannot happen except on the boundary of the geometric workspace, provided the pose is accessible.

#### C- Parallel-Type Singularities

Such singularities occur when $J_x$ becomes rank deficient. To study these singularities, some linear operations are performed on $J_x$ to arrive at a simpler matrix $T_1$, possessing the same rank as $J_x$. Adding $(n_i^T (PB_1 \times e_z) \cdots n_i^T (PB_1 \times e_z))$ to the fourth column of $J_x$ yields $T_1$, defined by:
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

\[
T_1 = \begin{bmatrix}
    n_1^T & 0 \\
    n_2^T & 0 \\
    n_3^T & -n_3^T (r \times e_z) \\
    n_4^T & -n_4^T (r \times e_z) \\
    n_5^T & 0 \\
    n_6^T & 0
\end{bmatrix}, \quad \text{with } r = B_1B_3 = B_1B_4 \tag{3.49}
\]

Then, we can guarantee that \( T_1 \) (equivalently \( J_x \)) is of full rank if the following two submatrices:

\[
TS_1 = \begin{bmatrix} n_1 & n_2 & n_5 & n_6 \end{bmatrix}^T \tag{3.50}
\]

and

\[
TS_2 = \begin{bmatrix} -n_3^T (r \times e_z) \\
    -n_4^T (r \times e_z) \end{bmatrix} \tag{3.51}
\]

are non-singular.

\( TS_1 \) becomes singular when the four vectors \( n_1, n_2, n_5, \) and \( n_6 \) are coplanar. Due to (3.33) and as we have chosen \( L_z \neq a \) (particularly \( L_z > a \)), the only possibility of coplanarity is having all the concerned arms in the \( yz \) plane. Hence, the configuration is also confounded with series-type singularity. Therefore, this situation if to occur, it will be on the boundary of the geometric workspace and not inside it.

On the other hand, \( TS_2 \) becomes rank deficient in the case where:

\[
n_3^T (r \times e_z) = n_4^T (r \times e_z) = 0 \tag{3.52}
\]

The condition (3.52) implies that we should have \( n_1, r, \) and \( e_z \) coplanar on one side, and we should have \( n_4, r, \) and \( e_z \) coplanar on the other. But \( r, \) \( e_z, \) and \( r \times e_z \) are always non-zero. So based on this latter idea and the assembly condition (3.33), the aforementioned coplanarity is possible if and only if \( n_1, n_4, \) and \( r \) reside in the \( yz \) plane. This means being confounded with series-type singularity that cannot occur except on the boundary of the geometric workspace.

Therefore, we deduce that if parallel-type singularities are to exist, then they are restricted to the boundary of the workspace and form no problem.

**D- Results' Briefing**

Based on the above analyses, we brief the results as follows. ARROW V1 does not admit any type of singularities within the geometrically accessible workspace excluding its boundary. This is provided that \( L_z \neq a \) and \( L_3 + L_4 \neq 2 L_z \). These latter two conditions are always satisfied by construction since having them otherwise renders the PKM functionless in the first place.
3.3.6- Simplified Dynamic Model (SDM)

In this section, we present the SDM of ARROW V1. The simplification done here is embodied by the following hypotheses:

1. The frictional forces of all types, whether dry or viscous, are neglected.
2. The arms are considered massless and their influence is compensated by partitioning their real masses into two halves. The first half-mass of each arm is added on the corresponding linear actuator, whereas the second half is placed on the platform at the articulation point. This is actually justifiable due to the small mass of the arms in comparison with the other components.

Applying the law of motion on the actuators, we get:

$$
M_a \ddot{\mathbf{q}} = \mathbf{\tau} + J_q^T (-\mathbf{f})
$$

$$
M_a = \text{diag}(m_{a1}, m_{a2}, m_{a3}, m_{a4}, m_{a5}, m_{a6})
$$

$$
\mathbf{\tau} = (\tau_1 \cdots \tau_6)^T
$$

$$
\mathbf{f} = (f_1 \cdots f_6)^T
$$

with

- $m_{ai}$: the mass of the actuator, moving cart, and the half-mass of the simple arm (chain (I), (II), (V) or (VI));
- $m_{ap}$: the mass of the actuator, moving cart, and the half-mass of the parallelogram arm (chain (III) or (IV));
- $\tau_i$: the actuation force of the $i$-th actuator, for all $i = 1...6$;
- and $f_i$: the force due to the acceleration and the gravitational force acting on the platform. The force $f_i$ is applied on the arm at point $A_i$, for all $i = 1...6$.

On the other hand, applying Newton-Euler method on the platform, we get:

$$
M_p \ddot{\mathbf{x}} + A_c \dot{\mathbf{x}} = J_x^T \mathbf{f} + m_p \mathbf{g}
$$

with:

- $M_p$: the inertia matrix of the platform;
- $A_c$: the matrix of Coriolis and centrifugal effects;
- $m_p$: the total mass of the platform including the half-masses from the different arms;
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

- $b$: the $x$-component of the center of mass of the assembly formed by the platform, and the point masses placed at $B_i$, for $i=1...6$ (the point mass at $B_i$ being equal to the half-mass of the corresponding arm);
- $I_{pz}$: the total moment of inertia about the $z$-axis passing through the TCP of the assembly formed by the platform and the point masses at $B_i$, for $i=1...6$;
- $g = (0 \ 0 \ -G \ 0)^T$: the gravity wrench acting on the platform per kg mass, where $G = 10\ N/\ kg = 10\ m/\ s^2$.

Then, merging (3.53) and (3.54) after substituting $^1$:

$$\dot{q} = J_m \dot{x} + \dot{J}_m \dot{x}$$

we get:

$$\dot{x} = H \tau - A \dot{x} + a^{offset}_g$$

$$H = (M_p + J_m^T M_a J_m)^{-1} J_m^T$$

$$A = H M_a J_m + (M_p + J_m^T M_a J_m)^{-1} A_c$$

$$a^{offset}_g = (M_p + J_m^T M_a J_m)^{-1} m_p g,$$

$$\dim(H) = 4 \times 6, \quad \dim(a^{offset}_g) = 4 \times 1$$

$$\dim(A) = 4 \times 4$$

The relation (3.56) represents the direct dynamic model (DDM), which is unique. However, the inverse dynamic model (IDM) is not unique and is generally given by:

$$\tau = C \left( \dot{x} + A \dot{x} - a^{offset}_g \right) + \left[ \text{null}(H) \right] \lambda,$$

with:

$$C = H^+ : \text{pseudo-inverse of } H$$

$$\lambda \in \mathbb{R}^2 \text{ (arbitrary)}$$

Yet, in our case, we consider the particular solution of the IDM with zero components along the null space of $H$ (which corresponds also to the null space of $J_m^T$)$^2$, meaning $\lambda = 0$. This is mathematically referred to as the minimum norm solution and can be achieved by control means (see §4.2 in Chapter 4). Hence, the IDM is reduced to:

$$\tau = C \left( \dot{x} + A \dot{x} - a^{offset}_g \right)$$

---

$^1$ The matrix $J_m$ can be easily derived from $J_m$. Its expression is not given here being lengthy. Yet, it can be easily obtained using the symbolic MATLAB toolbox.

$^2$ Since $H = S^{-1} J_m^T$, where $S = \left(M_p + J_m^T M_a J_m \right)$ is supposedly non-singular square matrix; if $S$ is singular, $S^{-1}$ does not exist in the first place.
As a result, the simplified direct and inverse dynamic models have been easily derived. In the upcoming section, we present the dimensional synthesis based on two primary criteria. One of these criteria is related to the dynamic performance. The evaluation of this performance requires only the knowledge of IDM, which is expressed by (3.58).

3.3.7- Dimensional Synthesis Based on Dual Criteria: Precision and Dynamics

Dimensional synthesis is undeniably one of the most prominent steps in the design of any PKM. Therefore, it is essential to choose properly the optimization criteria as to fit the intended application, and its direct requirements. Moreover, we should be very careful about how we perform this optimization, especially if it is of multi-objective nature. In §2.5, we have made some suggestions in this matter, in which we recommended the use of Pareto-based methods. Here, we abide by the aforementioned guidelines and exploit our knowledge of the mechanism to the extreme to facilitate the optimization further.

In the case of ARROW V1, the targets have been clearly stated since the early beginning. These are precision and rapidity. This is in addition to being capable of supporting large external wrenches. Under these global goals, we can notice the involvement of several masked sub-criteria, such as singularity-related performance, accuracy, repeatability, resolution, rigidity, twist, wrench, and acceleration capacities. Nonetheless, as the rotational performances are less interesting and expected to be well, we concentrate, therefore, on the translational aspects of the above-mentioned requirements. Still, if we consider all these requirements as they are, we would end up with an optimization based on a huge number of criteria. As a result, it would become difficult not only to visualize the problem based on Pareto method\(^3\), but also to properly make a decision. That is why, in what follows, we approach the problem by first simplifying the targets.

In fact, the rigidity of the PKM can be worked on via the proper dimensioning of the components (cross section of the arms, etc.), and its resulting errors can be even compensated by means of elasto-geometric calibration\(^4\), if needed. Thus, rigidity can be omitted. Regarding precision terms, having the actuators of same nature, then all translational precision requirements can be considered simultaneously through \(TPAF\) (the translational precision amplification factor defined in §2.2.2). Regarding singularity, we have studied it explicitly, and verified that no singular poses within the geometric accessible workspace excluding the boundary. Also, an investigation based on \(FVI_s\), for some samples of geometric parameters, has demonstrated that the quality of singularity performance is acceptable. Therefore, singularity aspect can be omitted as well. As for the kinetostatic translational performance, we

\(^3\) In general, when the number of criteria exceeds three, it becomes difficult to have a good visualization of the problem, even with the use of the parallel coordinates’ concept.

\(^4\) If elasto-geometric calibration is needed, it will not be more complex than the geometric one. This is since it will be sufficient to consider the deformation due to pure tension/compression forces in the arms.
have investigated it via $FVI_p$, defined in §2.2.3.B, for several different sets of geometric parameters. We have found that it is most often satisfactory for the requirements being $v_{req} = 0.25 \hat{q}_{\text{max}}$, and $f_{req} = 0.25 \tau_{\text{max}}$. Hence, we can omit $FVI_p$ as primary criterion, provided we verify if later in a secondary step. In what concerns acceleration capabilities, we consider the isotropic linear acceleration starting from rest in the absence of any external non-gravitational wrench. This can give us a sufficient idea about the PKM acceleration capability. In particular, considering acceleration capacity in the absence of an external non-gravitational wrench is completely justifiable for contactless applications (e.g. laser cutting). This is since it complies with reality. In what concerns other contact machining applications, external wrench is important only in the machining phase. This latter phase is, however, characterized by being rather quasi-static than dynamic. In fact, in a contact application, the interest in elevated acceleration exists in the transient phase\(^5\), which is characterized by the absence of any external non-gravitational wrench.

So in brief, the dimensional synthesis that we are going to present here is based on the dual criteria of precision and dynamics. The precision criterion is the worst $TPAF$, denoted by $WTPAF$, over the desired workspace, $DWS$. As for the dynamic criterion, it is the worst isotropic linear acceleration, denoted by $WILA$, over $DWS$. Yet, a verification on the secondary criterion $FVI_p$ will be done on the chosen set of geometric parameters.

The following subsection (§3.3.7.A) discusses the optimization process and its results. Afterwards, §3.3.7.B presents the workspace analysis, and the detailed performance evaluation.

**A- Synthesis Procedure and Its Results**

Based on the spindle to be assembled on the platform, the latter’s parameters (shown in Fig. 3-8) are set to the values given in Table 3-2. In this same table, the actuator characteristics are given as well. Regarding the inertia parameters, they are provided in Table 3-3.

Concerning $DWS$, it is defined as follows:

$$
DWS : \begin{cases} 
-0.15 \text{ m} \leq y \leq +0.15 \text{ m} \\
-0.15 \text{ m} \leq z \leq +0.15 \text{ m} \\
-45^\circ \leq \theta_z \leq +45^\circ 
\end{cases} \tag{3.59}
$$

Notice that the workspace requirement regarding $x$-motion is not specified in (3.59), as it can be fulfilled by choosing the appropriate stroke length for the linear actuators.

Regarding the parameters to optimize, they are $L_y$, $L_z$, and $L_p = L_p$. These parameters must remain within the following ranges:

---

\(^5\) It is the phase where no machining is done, but the TCP is moved rapidly from the current location to nearby the working piece.
The lower bounds in (3.60) are to assure accessibility to $DWS$, defined in (3.59). As for the upper bounds in (3.60), they are based on the consideration of stiffness and compactness of ARROW V1. We mention also that imposing $L_s = L_p$ is done for the benefit of manufacturing.

The Pareto distribution is constructed by taking five equally spaced values for each interval of parameters, then evaluating the different compromises between the two major criteria: $WTPAF = \max_{x \in DWS} \left( TPAF(x) \right)$ and $WILA = \min_{x \in DWS} \left( ILA(x) \right)$, where:

- $TPAF(x)$ is the value of $TPAF$ for a pose $x$, defined as in §2.2.2;
- $ILA(x)$ is a special case of the specific isotropic linear acceleration, $\dot{a}_{iso}^p$, defined in §2.2.4, in which we set: $\dot{v}_{req} = 0$, $\dot{\omega}_{req} = 0$, $\dot{\alpha}_{req} = 0$, $\dot{f}_{req} = 0$, and $\dot{m}_{req} = 0$.

Note that the expression of $ILA(x)$ is reduced to what follows:

\[
\begin{align*}
0.35 \text{ m} & \leq L_s \leq 0.5 \text{ m} \\
0.314 \text{ m} & \leq L_{z} \leq 0.5 \text{ m} \\
0.69 \text{ m} & \leq L_s = L_p \leq 1 \text{ m}
\end{align*}
\]

(3.60)
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

The Pareto diagram is provided in Fig. 3-9. As it can be noticed from the figure, we did not only plot the Pareto-Front (limiting curve), but all the compromises. This allows us to visualize all possible options, in case we needed to reconsider our choice after verifying the secondary criterion. Based on the shown dual performances, we have decided to set the limits of acceptable solutions, or more precisely the trade-off in performances, to having $WTPAF \leq 4.5$, and $WILA \geq 8 \, g$. This trade-off is satisfied by three candidates, all of which have almost the same value of $WTPAF = 4.5$. So, we choose the one with the higher $WILA$, being equal to 8.5 g (it is encircled in red in Fig. 3-9). The investigation of the secondary criterion $FVI_p$, for $v_{req} = 0.25 \dot{q}_{max}$ and $f_{req} = 0.25 \tau_{max}$, shows that $FVI_p > 1.5$ over DWS. Hence, $FVI_p$ is satisfied and therefore, the following geometric parameters are adopted (same as those shown in Fig. 3-9, but after some numerical adjustment):

$$ILA(x) = \min_{i=1...6} (e_i), \quad \text{with:}$$

$$e_i = \begin{cases} 0, & \text{if } (-\tau_{max} + t_i^{off}) > 0 \text{ or } (\tau_{max} + t_i^{off}) < 0 \\ \min \left( \left| -\tau_{max} + t_i^{off} \right|, \left| \tau_{max} + t_i^{off} \right| \right), & \text{if } \left( (-\tau_{max} + t_i^{off}) < 0 , (\tau_{max} + t_i^{off}) > 0 \right) \\ \infty, & \text{otherwise} \end{cases}$$

$t_i^{off}$: $i$-th component of $t^{off} = C \, a^{off}$

$c_{pr}$: $i$-th row vector of $C_g$, the translational part of $C$

\forall i = 1...6

(3.61)
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

\[
L_s = L_p = 0.93 \text{ m} \\
L_y = 0.5 \text{ m} \\
L_z = 0.41 \text{ m}
\] (3.62).

Having synthesized the geometric parameters of ARROW V1 based on Pareto method, the following subsection provides detailed performance evaluation and workspace analysis.

B- Workspace and Performance Evaluation of ARROW V1

We first proceed by presenting the evaluation relative to the dual criteria used in the dimensional synthesis, namely TPAF and ILA. Also, we present the kinetostatic performance relative to \(v_{iso}^p\) and \(f_{eiso}^p\), the pure isotropic linear speed and static force respectively. The plots of \(v_{iso}^p\) and \(f_{eiso}^p\) are more expressive than \(FVI_p\), which is defined as

\[
FVI_p = \min \left( \frac{v_{iso}^p}{v_{req}^p}, \frac{f_{eiso}^p}{f_{ereq}^p} \right) \text{ for } v_{req}^p = 0.25 \bar{q}_{max} \text{ and } f_{ereq}^p = 0.25 \tau_{max}. \]

The presentation is done as function of \(y\) and \(z\), for \(\theta_z = 0^\circ\) on one hand, and for \(\theta_z \in [-45^\circ; 45^\circ]\) on the other.

In the presentation with \(\theta_z \in [-45^\circ; 45^\circ]\), we highlight what follows. For each \((y, z)\) couple, the performance index is calculated for several values of \(\theta_z\), then the worst value of the index is associated with the corresponding \((y, z)\). In all these figures, we consider the legend defined below:

- Dotted black box represents the \(yz\) projection of the physical limits of the PKM, i.e. the floor, walls, and ceiling.
- Solid red box represents the limits that the TCP should not exceed in order to avoid collisions.
- Dotted magenta box represents DWS for which the PKM has been optimized.

TPAF and ILA are depicted in Fig. 3-10 and Fig. 3-11, respectively. We recall here that TPAF holds not only a precision significance, but also an indication regarding peak linear speed, \(v_{peak}\). As the actuators are identical, we have \(v_{peak}/\bar{q}_{max} = TPAF\). Based on this, we can say that the theoretical \(v_{peak}\) of ARROW V1 is 22.5 to 31.5 m/s. However, this speed occurs only along a particular direction and therefore, it is of least interest from the perspective of machine tools. Regarding ILA, we can notice that on the majority of DWS, the value of ILA is greater than 9 g, and reaches 10 g for some region in the case of \(\theta_z = 0^\circ\). This is intriguing in comparison with the reported acceleration capabilities for some already available PKMs (refer to Chapter 1).

In Fig. 3-12 and Fig. 3-13, \(v_{iso}^p\) and \(f_{eiso}^p\) of the optimized PKM are mapped into the workspace. The results show that on the majority of DWS, we have \(v_{iso}^p \geq 0.5 \bar{q}_{max} = 2.5 \text{ to } 3.5 \text{ m/s} \) and \(f_{eiso}^p \geq 0.84 \tau_{max} = 2100 \text{ N}\). These are quite remarkable, not due to the values in particular but more importantly the associated isotropy significance with them.

119
We went further in performance evaluation, in which we have investigated the peak linear acceleration, $PLA$, starting from rest and in the absence of an external non-gravitational wrench. The mathematical formulation of $PLA$ is given hereafter:

$$PLA_{\text{max}} = \max_{\tau \in Z_r} \left( \left\| H_p \tau + a_{gp}^{\text{offset}} \right\| \right)$$

$$Z_{r_{\text{min}}} = \{ \tau \in \mathbb{R}^6, \tau^T \left[ \text{null}(H) \right] = 0_{6 \times 2} \text{ and } |\tau_i| \leq \tau_{\text{max}}, \forall i = 1...6 \}$$

$$H_p : \text{ translational part of } H$$

$$a_{gp}^{\text{offset}} = \left( a_{g_s}^{\text{offset}} a_{g_z}^{\text{offset}} a_{g_s}^{\text{offset}} \right)^T : \text{ the first three components of } a_g^{\text{offset}}$$

**Fig. 3-10:** $TPAF$ of ARROW V1: case of $\theta_z = 0^\circ$ (left) and case of $|\theta_z| \leq 45^\circ$ (right).

**Fig. 3-11:** $ILA$ of ARROW V1: case of $\theta_z = 0^\circ$ (left) and case of $|\theta_z| \leq 45^\circ$ (right).
Fig. 3-12: $V_{iso}^p$ of ARROW V1: case of $\theta_\parallel = 0^\circ$ (left) and case of $|\theta_\parallel| \leq 45^\circ$ (right).

Fig. 3-13: $f_{iso}^p$ of ARROW V1: case of $\theta_\parallel = 0^\circ$ (left) and case of $|\theta_\parallel| \leq 45^\circ$ (right).

Fig. 3-14: PLA of ARROW V1: case of $\theta_\parallel = 0^\circ$ (left) and case of $|\theta_\parallel| \leq 45^\circ$ (right).
We emphasize that (3.63) can be computed analytically, in the very similar way as we have done for the operational resolutions in §2.2.2. The atlases of PLA are represented in Fig. 3-14. These show that PLA varies between 22.5 g and 24.5 g over DWS, which is interesting. In fact, we have $ILA/PLA = 9/22.5 = 40\%$ over the majority of DWS; this is noteworthy taking into account the isotropic nature of ILA and the direction-dependency of PLA. It is important to highlight here that the value of PLA associated with $(y, z)$ in the case where $\theta_z \in [-45^\circ, +45^\circ]$ is the largest value of PLA attained for different orientations; i.e. the best value unlike the remaining performance indices where we assumed their worst values.

In addition to what we have mentioned, it is worth highlighting the smooth variations of the different performances over the global workspace. This rather homogenous aspect of performances is attributable to actuation redundancy.

Finally and for the sake of completeness, we provide a sample of the spatial workspaces of ARROW V1 in Fig. 3-15 and Fig. 3-16. These are obtained for the following strokes of linear motors:

$$\begin{align*}
&\text{Actuators 1, 2, 5 and 6: Stroke Length } = 1.49 \text{ m} \\
&\text{Actuators 3 and 4: Stroke Length } \approx 1.53 \text{ m}
\end{align*}$$

These workspaces correspond for the case of having $\theta_z = 0^\circ$ and $\theta_z \in [-45^\circ, +45^\circ]$. Their plots demonstrate that the $x$-dimensions of the spatial workspaces are relatively acceptable in comparison with the maximum actuator stroke.
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

3.3.8- Synopsis

In §3.3, we have introduced ARROW V1 PKM that is redundantly actuated and capable of four-dof (3T-1R) motion. This PKM is intended to serve as a branch in a left-hand right-hand based machine tool with five dofs (3T-2R). In this latter machine, the second rotational dof is provided by means of a turntable. The detailed analysis of this PKM, starting from its modeling and ending with its performance evaluation, has revealed undeniable characteristics, such as:

1. Large workspace and tilting capacity;
2. Design and modeling simplicities;
3. Absence of singularities within its workspace excluding its boundary;
4. Having arms under tension/compression and therefore, amelioration of precision thanks to the reduced deformation;
5. Performance homogeneity over the workspace;
6. Elevated precision, kinetostatic, and dynamic performances (\( WTPAF = 4.5 \), \( \nu_{iso}^p \geq 0.5 \dot{q}_{max} = 2.5 \text{ to } 3.5 \text{ m/s} \), \( f_{iso}^p \geq 0.84 \tau_{max} = 2100 \text{ N} \), \( WILA = 8.5 \text{ g} \), and \( PLA = 24.5 \text{ g} \) over \( DWS \));
7. The relatively good ratio between the \( x \)-dimension of the spatial workspaces and the maximum available stroke length for the actuators.

In comparison with MachLin5 presented in §3.2, ARROW V1 has the advantage of having only two parallel walls on which the actuators are placed. This is not to mention the enhanced structural symmetry of the architecture and its simplicity. In this scope, the design can be made

![Spatial Workspace of ARROW V1 (Case \( \theta \leq 45^\circ \) and for stroke lengths: \( SL_x = 1.49 \text{ m} \) (actuators 1, 2, 5 and 6) and \( SL_p = 1.53 \text{ m} \) (actuators 3 and 4))](image_url)
even simpler for implementation by considering the suggestions given in §3.2.1 regarding the execution of the spherical joints. Nevertheless, the main feature of having independent \( x \)-motion is compromised when adding the turntable. In fact, there are two main options for adding the turntable to have the eventual five-dof machine tool. These are described as follows:

1. First option: It consists of adding the turntable with its axis of rotation along the \( y \)-axis of the base frame of ARROW V1. This obviously does not make use of the independent \( x \) motion of the PKM. Therefore, it is not encouraged.

2. Second option: This consists of adding the turntable along the \( x \)-axis of the base frame of ARROW V1. However, this results in inevitable collisions between the turntable and the parallel module. Eventually, this not only reduces the exploitable workspace but also complicates motion planning.

In conclusion, ARROW V1, despite of its features, it is still not adequate to use with a turntable to achieve the five-dof (3T-2R) machine. Nonetheless, with a slight modification, we can arrive at a more feasible architecture. This modification is simply placing all the actuators on one wall and in pairs. The outcome is the ARROW V2 concept, the subject of upcoming part.

3.4- ARROW V2: From Theoretical Concept to Prototype Execution

3.4.1- ARROW V2: An Enhanced Version of ARROW V1

ARROW V2 is an evolution of ARROW V1, aiming at overcoming the latter’s drawbacks. The CAD views of the PKM and its platform are shown in Fig. 3-17 and Fig. 3-18, respectively. Regarding the graph diagram, it is the same as that of ARROW V1 (see Fig. 3-7). Moreover, regarding the execution of the spherical joints, the suggestions given in §3.2.1 are applicable here as well.

In Fig. 3-19, the five-dof (3T-2R) machine tool based on ARROW V2 is shown. It obeys left-hand right-hand paradigm, in which a turntable is added facing the parallel module. The turntable has its axis aligned along the \( x \)-direction of the base frame. With this configuration, several benefits are obtained in comparison with ARROW V1:

1. All the actuators lay in the same plane. Thus, we only need to perfect the planarity of one surface instead of two.

2. Having each pair of actuators share a common track is profitable in three aspects. These are enumerated below:
   a) We only need to control the parallelism of three linear guides instead of six.
   b) The number of geometric parameters is decreased, which facilitates geometric calibration.
   c) The overall costs are reduced.
3. Though there is still a possibility of collision between the turntable and the parallel module, it is not that severe.

As the principle of functioning of ARROW V2 is the same as its precursor and as its models are derivable in the very same manner, we are going to directly give these models in what follows and just concentrate on the singularity analysis. However, it is worth emphasizing that for ARROW V2 to function properly, still the condition of having \( L_z \neq a \) must be satisfied. In fact, practicality necessitates having \( L_z > a \).

**A- Inverse Geometric Model (IGM)**

Knowing the pose \( x = (x, y, z, \theta)^T \), the joint positions \( q = (q_1, \ldots, q_6)^T \) are given by:
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

Fig. 3-18: The platform of ARROW V2 and its corresponding notations.

Fig. 3-19: The five-dof (3T-2R) machine tool: ARROW V2 with turntable.

\[ q_i = x_i = x_{bi} + (-1)^i \sqrt{L_i^2 - (y_{bi} - y_i)^2 - (z_{bi} - z_i)^2} \]  \tag{3.65} \\

with:

\[
\begin{align*}
B_i &= (x_{bi} \ y_{bi} \ z_{bi}) = P + R B_i^{plat} \\
P &= (x \ y \ z)^T, \quad R = \text{Rot}_z(\theta_i) \\
A_i &= (x_i \ y_i \ z_i)^T = (q_i \ y_i \ z_i)^T, \quad y_i = 0 \\
    &= \begin{cases} 
      L_i, & \text{if } i = 1, 2 \\
      0, & \text{if } i = 3, 4 \\
      -L_i, & \text{otherwise}
    \end{cases} \\
\forall i &= 1...6
\end{align*}
\]

and \( B_i^{plat} \ (\forall i = 1...6) \) representing the coordinates of the points \( B_i \) in the platform frame of reference.
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

Note that (3.65) is based on the assembly mode condition described by:

\[ x_i \leq x_{bi} \equiv x_{b(i+1)} \leq x_{i+1}, \quad \forall i = 1, 3, 5 \]

(3.67).

B- Direct Geometric Model (DGM)

The DGM can be derived exactly as described in §3.3.3. However, we are going here to benefit from the structural symmetry \((L_1 = L_2 = L_3 = L_4 = L_p)\) of ARROW V2 and compute directly the \(x\) components of points \(B_i\) through \(B_6\) as follows:

\[
\begin{cases}
(x_{b1} \equiv x_{b2}) = (x_{b5} \equiv x_{b6}) = \frac{x_1 + x_2 + x_3 + x_4}{2} = \frac{y}{2}, \\
x_{b3} \equiv x_{b4} = \frac{x_1 + x_4}{2}, \quad \text{with: } x_i = q_i, \quad \forall i = 1...6
\end{cases}
\]

(3.68).

As result, we can calculate the two possible solutions for \(\theta_z \in \left[ -\pi; +\pi \right]\), denoted by \(\theta_z^{1}\) and \(\theta_z^{2}\), and the unique solution of \(x\) via:

\[
\theta_z^{1} = \begin{cases}
\sin \left( \frac{x_{b3} - x_{b1}}{d} \right), & \text{or} \\
-\pi - \sin \left( \frac{x_{b3} - x_{b1}}{d} \right), & \text{if } (x_{b3} - x_{b1}) \geq 0
\end{cases}
\]

\[
\theta_z^{2} = \begin{cases}
\pi - \sin \left( \frac{x_{b3} - x_{b1}}{d} \right), & \text{otherwise}
\end{cases}
\]

(3.69)

and

\[
x = x_1 - \left( t_p - \frac{d}{2} \right) \sin (\theta_z) = x_1 - \left( t_p - \frac{d}{2} \right) \left( \frac{x_{b3} - x_{b1}}{d} \right)
\]

(3.70).

To get \(y\) and \(z\), we only need to determine the intersection of the two circles (in the vertical plane \(x = x_{b1}\)) described by:

\[
\begin{align*}
A_i B_i^1 &= L_i^1 = L_i^2 \\
A_i B_i^2 &= L_i^3 = L_i^4, \quad \text{with: } A_i' = A_i + 2a e_z
\end{align*}
\]

(3.71).

From (3.71), two possible solutions exist, say: \(B_i^{1}\) and \(B_i^{2}\). The acceptable solution is the one with \(y_{b1} \geq 0\). Hence, we get a unique solution for \(B_i\). Based on \(B_i\), we get:

\[
\begin{align*}
\begin{cases}
z = (z_{b3} \equiv z_{b4}) = z_{b1} - a \\
(z_{b5} \equiv z_{b6}) = z_{b1} - 2a
\end{cases}
\end{align*}
\]

(3.72).
Knowing $x_{b3} \equiv x_{b4}$ and $z_{b3} \equiv z_{b4}$, the term $y_{b3} \equiv y_{b4}$ can be computed by using $A_jB_j^T = L_j = L_j^T$, which yields:

$$y_{b3} \equiv y_{b4} = y_3 \pm \sqrt{L_j^2 - (x_{b3} - x_3)^2 - (z_{b3} - z_3)^2} = y_3 \pm \sqrt{L_j^2 - \left(\frac{x_4 - x_3}{2}\right)^2 - \left(z_{b3} - z_3\right)^2} \quad (3.73).$$

But the condition $y_{b3} \equiv y_{b4} \geq y_3 = y_4 = 0$ must be satisfied. Therefore, we only accept the following solution for $y_{b3} \equiv y_{b4}:

$$y_{b3} \equiv y_{b4} = y_3 + \sqrt{L_j^2 - \left(\frac{x_4 - x_3}{2}\right)^2 - \left(z_{b3} - z_3\right)^2} = \sqrt{L_j^2 - \left(\frac{x_4 - x_3}{2}\right)^2 - \left(z_{b3} - z_3\right)^2} \quad (3.74).$$

Based on the sign of $(y_{b4} - y_{b3})$, we can then identify the valid solution for $\theta_z$ among $\{\theta_z^{(1)}, \theta_z^{(2)}\}$, which is:

$$\theta_z = \begin{cases} 
\theta_z^{(1)}, & \text{if } (y_{b4} - y_{b3}) \geq 0 \\
\theta_z^{(2)}, & \text{otherwise}
\end{cases} \quad (3.75).$$

Hence, we easily derived the DGM of ARROW V2 in a slightly different fashion, in which we exploited the structural symmetry of the robot.

C- Kinematic Model

Similarly to what have been done in §3.3.3, we can effortlessly establish the relation between $\dot{q}$ and $\dot{x}$. This kinematic relation is:

$$\begin{bmatrix} J_q \dot{q} \\ J_x \dot{x} \end{bmatrix} = \begin{bmatrix} J_x \end{bmatrix}, \quad \text{with:}$$

$$J_q = \text{diag} \left( n_1^T e_i, \ldots, n_6^T e_i \right), \quad \text{dim} (J_q) = 6 \times 6$$

$$J_x = \begin{bmatrix} n_1^T & -n_1^T (PB_i \times e_i) \\ \vdots & \vdots \\ n_6^T & -n_6^T (PB_i \times e_i) \end{bmatrix}, \quad \text{dim} (J_x) = 6 \times 4 \quad (3.76).$$

The inverse kinematic model (IKM) is given based on the inverse Jacobian, $J_m$, as below:

$$\text{IKM: } \dot{q} = J_m \dot{x}, \quad \text{with: } J_m = J_q^{-1} J_x \quad (3.77).$$

On the other hand, the direct kinematic model (DKM) is established by performing a pseudo inversion procedure as follows:

$$\text{DKM: } \dot{x} = J \dot{q} = J_m^{-1} \dot{q}, \quad \text{provided that } \dot{q}^T \left[ \text{null} (J_m^T) \right] = 0_{6 \times 2} \quad (3.78).$$

As we have highlighted in §3.3.3, the relations (3.77) and (3.78) hold as long as $J_q$ and $J_x$ are non-singular.
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

D- Singularity Analysis

In analogy with ARROW V1, we proceed by analyzing constraint singularities, then the classical ones.

Constraint Singularities

In ARROW V1, the only constraint singularity has been identified as an architectural one. This is since it occurred for a particular set of geometric parameters. Nevertheless, in ARROW V2 it is not any more the case.

The constraint singularity is still related to the capability of the pair of parallelograms to restrict the undesired rotations (those about any axis laying in the \(xy\) plane of the base frame). Yet, it can occur for any general set of geometric parameters. In particular, it occurs when the two parallelograms become collinear. This would happen only in two configurations of \(A_3B_3\) and \(A_4B_4\). These are described below:

1. The first configuration is described by having \(A_3B_3\) and \(A_4B_4\) in the \(yz\) plane. This cannot happen except on the boundary of the geometric workspace. Additionally, it exists necessarily confounded with series-type singularity since it implies \(n_i^T e_x = n_i^T e_x = 0 \iff \det(J_q) = 0\) (refer to following part). This would not practically occur as inter-collisions between the third and fourth actuators will precede.
2. The second case is when \(A_3B_3//A_4B_4//e_x\); i.e. the two arms are aligned along the \(x\) axis.

This also cannot practically befall as collisions antecede that.

Consequently, constraint singularities not only exist on the limits of the geometric workspace, but also cannot practically happen.

Series-Type Singularities

Such singularities exist when the \(J_q\) (in (3.76)) is rank deficient, which is equivalent to:

\[
\det(J_q) = 0 \iff \exists i_o \in \{1,...,6\}; \ n_i^T e_x = 0 \iff n_i \perp e_x
\]  

Fortunately, this not only cannot happen except on the boundary of the geometrically accessibly workspace, but is also preceded by inter-collisions between the actuators on the same linear track. This is since if \((2j-1)\)-th arm (with \(j=1,2,3\)) is in the \(yz\) plane, the corresponding \((2j)\)-th limb is necessarily laying in the aforementioned plane as well, and vice versa.

Parallel-Type Singularities

As usual, these happen when \(J_x\) becomes rank deficient. Simplifying \(J_x\) by linearly operating on it, we arrive at the following matrix \(T\) of same rank as \(J_x\):
Based on (3.80), we can assure that $T$ (equivalently $J_x$) is non-singular if the two submatrices, $T_{S_1}$ and $T_{S_2}$, defined by:

$$T_{S_1} = \begin{bmatrix} n_1^T & n_2^T & n_3^T & n_4^T & n_5^T & n_6^T \end{bmatrix}$$

$$T_{S_2} = \begin{bmatrix} -n_3^T (r \times e_z) \\ -n_4^T (r \times e_z) \end{bmatrix}$$

are of full rank.

$T_{S_1}$ is rank deficient when the four simple arms are co-planar. Since $L_z \neq a$, this would happen only in two configurations:

1. The simple arms are in the $yz$ plane (i.e. arms completely stretched). This occurs confoundedly with a series-type singularity, and only on the boundary of the geometrically accessible workspace. In reality, it is not possible as collisions transpire prior to that.
2. The simple arms are in the $xz$ plane, which is practically impossible being preceded by collisions as well.

On the other hand, the rank deficiency of $T_{S_2}$ necessitates having the parallelogram arms and $r$ in the same vertical plane, which corresponds to two theoretically possible situations. The first case is having $n_3$, $n_4$, and $r$ in the $xz$ plane. Regarding the second situation, it corresponds to having the aforementioned three vectors in the $yz$ plane (confounded with series-type singularities). In both cases, we have the same argument as discussed for $T_{S_1}$, summarized as being preceded by collisions.

Therefore, we can assure that ARROW V2 does not admit any parallel type singularity within the feasible workspace.

**Results’ Briefing**

As a conclusion of the above analysis, we can assure that no singularities of any type exist within the feasible workspace. In fact, series singularities not only occur on the boundary of the geometric workspace, but also are practically infeasible due to prior arising of collisions. The same applies for parallel-type singularities. These have the possibility to occur on the boundary.
of the geometric workspace, or in the situation where the arms and platform lay in the $xz$ plane. Moreover, we highlight that unlike ARROW V1, the constraint singularities here are not of architectural nature. Nonetheless, their occurrence is limited to having the parallelograms in the $yz$ plane or aligned along the $x$-axis of the base frame. Both cases are not to be worried about being practically infeasible.

### 3.4.2- Manufacturability Study of ARROW V2 and Its Mutated Versions

The simplicity of ARROW V2 and its brought in benefits have made it surpass the theoretical study into the implementation stage. In this phase, the manufacturability of each part, as well as the plans of fabrication and assembly, has been deeply investigated. Within the scope of this detailed examination, possible collisions between the arms have been put into attention.

In fact, based on the preliminary CAD presented in Fig. 3-17 and due to the possibility of having inter-arm collisions for some poses within the workspace, a reconsideration of the design has been made. This reconsideration is particularly at the level of the U-joints that connect the arms to the spindle. To overcome the aforementioned conflict, several suggestions have been made. Among these proposals, only two have been considered and we refer to them by mutations, as they resulted in a topological change of the architecture, though the performances have not been noticeably compromised.

These mutations are described by replacing each of the concerned U-joints by two orthogonal but non-concurrent revolute ones (R-joints). The distance between these R-joints is kept small enough in order not to dramatically alter the global performances of the initial conceptual design. The new common graph diagram for the resulting mutated versions, called ARROW V2 M1/M2\(^6\), is given in Fig. 3-20.

In Fig. 3-21, we depict the R-joints at the level of the platform in the case of ARROW V2 M1/M2. In the first version, ARROW V2 M1, the R joints are offset one from another while satisfying: $r_p = 0$ and $r \neq 0$ (see their interpretation in Fig. 3-21). On the other hand, in ARROW V2 M2, both offset distances, $r_p$ and $r$, are non-zero. Though ARROW V2 M1 is a special case of ARROW V2 M2, it has been distinguished since its models are simpler to obtain. Nevertheless, the M2 version is better regarding collision avoidance. This can be clearly noticed from Fig. 3-21. Therefore, the decision has been made on the implementation of ARROW V2 M2.

However, we should emphasize that this mutation has resulted in some adverse effects. These are detailed as follows:

1. The establishment of the DGM is difficult, if not impossible.

---

\(^6\) M1 and M2 stand for mutation type 1 and type 2, respectively.
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

Fig. 3-20: Graph diagram of ARROW V2 M1/M2, the mutated versions of ARROW V2.

Fig. 3-21: Visualization of the offsets between the revolute joints in ARROW V2 M1/M2 versions (top view): ARROW V2 M1 corresponds to having $r_p = 0$ and $r \neq 0$, whereas ARROW V2 M2 corresponds to the situation where $r_p \neq 0$ and $r \neq 0$.

2. The other models, such as the inverse geometric and kinematic models, are not as straightforward as before.
3. Singularity analysis is more complex to perform. Thus, it is indispensable to discuss the geometric and kinematic models as well as the singularity analysis of ARROW V2 M2. All these points, in addition to the dynamic model and the dimensional synthesis, will be discussed in the upcoming section.

3.4.3- ARROW V2 M2: The Implemented PKM

The CAD drawings of ARROW V2 M2 with some notations are given in Fig. 3-22 and Fig. 3-23. Fig. 3-24 presents a schematic of a kinematic chain of this PKM with additional symbols. These will be used later in modeling. Note that in the case of complex chains (i.e. chains (III) and (IV) with P-(SRR)2 structure), the representation in Fig. 3-24 corresponds to the virtual equivalent
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

PURR chain in the mediator plane of P-(SRR)\textsubscript{2}. The lengths $L_i$ ($\forall i = 1, 2, 5, 6$) of the simple arms are equal to $L_s$. Regarding those of the complex ones (i.e. $L_i$ with $i = 3, 4$), they are equal to $L_p$. Having clarified the different symbols and their geometric interpretation, in what follows, we present the models of ARROW V2 M2.

We emphasize that the modeling is based on the idea that the complex limbs maintain the same constraints as a classical pair of parallelograms. The validity of this assumption is proved later by investigating constraint singularities.

A- Inverse Geometric Model (IGM)

We denote the base-frame coordinates of the points $A_i$, $B_i$, $C_i$, and $D_i$ by:

$A_i = (x_i, y_i, z_i)^T$, $B_i = (x_{bi}, y_{bi}, z_{bi})^T$, $C_i = (x_i, y_{ci}, z_{ci})^T$, and $D_i = (x_{di}, y_{di}, z_{di})^T$, respectively and for all $i = 1...6$. To derive the IGM, we use the following properties ($\forall i = 1...6$):

1. Points $A_i$, $B_i$, and $C_i$ are in the same vertical plane. This means that their projections on the $xy$ plane are collinear.
2. Points $B_i$, $C_i$, and $D_i$ lay in the same horizontal plane. Thus, they all have the same $z$ component in the base frame.

Then, knowing the end-effector pose $x = (x, y, z, \theta)^T$ and $p_i^{plat} = PD_i^{plat}$ (the coordinates of $D_i$ in the platform frame of reference), we get $D_i$ as follows:

$$
\begin{align*}
D_i &= (x_{di}, y_{di}, z_{di})^T = P + R p_i^{plat}, \\
P &= (x, y, z)^T, \\
R &= \text{Rot}_z(\theta), \\
p_i^{plat} &= PD_i^{plat}
\end{align*}
$$

Then, we proceed by getting the length $L_{xyi}$ ($\forall i = 1...6$) of the $xy$ projection of $L_i = A_iB_i$, which is:

$$
\begin{align*}
L_{xyi} &= \sqrt{L_i^2 - (z_i - z_{bi})^2}, \\
L_i &= \begin{cases} 
L_p, & \text{if } i = 3, 4 \\
L_s, & \text{otherwise}
\end{cases}, \\
L_s &= \begin{cases} 
L_s, & \text{if } i = 1, 2 \\
-L_s, & \text{otherwise}
\end{cases}, \\
z_i &= \begin{cases} 
0, & \text{if } i = 3, 4 \\
-L_s, & \text{otherwise}
\end{cases}, \\
z_{bi} &= z_{di}
\end{align*}
$$

The length $n_{xyi}$ ($\forall i = 1...6$) of the projection of $n_i = A_iC_i$ on the $xy$ plane is therefore computable and given by:
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

\[
\begin{align*}
L_{xyi} &= L_{xyi} + r, \quad \text{with:} \\
L_{xyi} &= \|B_i C_i\| = r, \quad \forall i = 1...6
\end{align*}
\]  

(3.85).

As the projections of \( A_i, C_i, \) and \( D_i \) on \( xy \) plane form a right triangle at \( C_i \) (for all \( i = 1...6 \)), we can then compute the \( \text{x}-\text{projected length} \) \( m_{xyi} \) of \( m_i = A_i D_i \), thanks to the relation hereafter:

\[
\begin{align*}
m_{xyi} &= \sqrt{n_{xyi}^2 + r^2} = \sqrt{(L_{xyi} + r)^2 + r_p^2}, \quad \text{with:} \\
r_p &= \|r_p\| = \|C_i D_i\| = r_p, \quad \forall i = 1...6
\end{align*}
\]  

(3.86).

Based on (3.86), we can easily derive \( q_i = x_i \) (\( \forall i = 1...6 \)) by solving the following equation:

\[
m_{xyi} = \sqrt{(x_{di} - x_i)^2 + (y_{di} - y_i)^2} = \sqrt{(x_{di} - q_i)^2 + (y_{di} - y_i)^2}, \quad \forall i = 1...6
\]

(3.87), which yields:

\[
q_i = x_i = x_{di} \pm \sqrt{m_{xyi}^2 - (y_{di} - y_i)^2}, \quad \forall i = 1...6
\]

(3.88).

But the assembly mode condition, described clearly in \textbf{Fig. 3-17}, dictates having:

\[
q_i = x_i \leq x_{di} \equiv x_{d(i+1)} \leq q_{i+1} = x_{i+1}, \quad \forall i = 1,3,5
\]

(3.89).

Hence, the IGM of ARROW V2 M2 is expressed by:

\[
q_i = x_i = x_{di} + (-1)^{i-1} \sqrt{m_{xyi}^2 - (y_{di} - y_i)^2}, \quad \forall i = 1...6
\]

(3.90).

\section*{B- Kinematic Model}

Unlike the previously discussed architectures, the input-output kinematic relation is not as easy to get here, at least not in the usual compact form. Of course, it is still simple to get the inverse kinematic model (IKM) by differentiating the symbolic expression of the IGM. But this does not result in a suitable form for the Jacobians, \( J_q \) and \( J_x \), as to facilitate their analyses.

So, to establish the kinematic model in a more compact form, we start by defining the following quantities: \( v = \dot{P} \), \( \omega = \dot{\theta}_z e_z \), \( \phi = \langle \phi_1 \ldots \phi_6 \rangle^\top \), and \( \psi = \langle \psi_1 \ldots \psi_6 \rangle^\top \). For the definition of angles \( \phi \) and \( \psi \), refer to \textbf{Fig. 3-24}. The absolute angular velocities of parts \( B_i C_i D_i \) and \( A_i B_i \) are respectively given by:

\[
\omega_p = \psi_i e_z, \quad \forall i = 1...6
\]

(3.91) and

\[
\omega_n = \phi_i \mu_i + \psi_i e_z, \quad \forall i = 1...6
\]

(3.92).

Based on the aforementioned, we can then write the respective linear velocities of \( D_i \) and \( B_i \) as follows:
\[ v_{D_i} = v + \omega \times p_i = v - (p_i \times e_z) \dot{z} = \left[ I_{3x3} - (p_i \times e_z) \right] \dot{x}, \ \forall i = 1...6 \] (3.93)

and

\[ v_{B_i} = v_{D_i} + \omega \times D_i B_i = \left[ I_{3x3} - (p_i \times e_z) \left( (r_i + r_p) \times e_z \right) \right] \left( \dot{x} \right), \ \forall i = 1...6 \] (3.94).

As \( A_i B_i \) is supposedly a rigid body, we have:

\[ L_i^T v_A = L_i^T v_{B_i}, \ \forall i = 1...6 \] (3.95),

with the velocity of \( A_i \) being:

\[ v_A = \dot{q}_i e_x, \ \forall i = 1...6 \] (3.96).

Substituting (3.94) and (3.96) in (3.95) yields the following relation:

\[ L_i^T e_x \dot{q}_i = L_i^T v - L_i^T (p_i \times e_z) \dot{\theta}_z + L_i^T (r_p \times e_z) \dot{\theta}_i, \ \forall i = 1...6 \] (3.97).

As it is obvious from (3.97), \( \dot{q} \) and \( \dot{x} \) are coupled with \( \dot{\theta}_i \). Therefore, we need to find other relations. The aim from getting new relations is not only isolating \( \dot{q} \) and \( \dot{x} \), but also determining the angular velocities of the passive joints, in particular \( \dot{\phi}_i \) and \( \dot{\psi}_i \) for all \( i=1...6 \).

To do that, we first write \( v_A \) in terms of \( v_{B_i} \) and \( \omega_{B_i} \) as hereafter:

\[
\begin{align*}
v_A &= v_{B_i} + \omega_{B_i} \times (-L_i) \\
&= v - (p_i \times e_z) \dot{\theta}_i + ((r_i + r_p) \times e_z) \dot{\theta}_i + L_i (\psi_i e_z + \phi_i \mu_i) \\
&= \left[ I_{3x3} - (p_i \times e_z) \left( m_i \times e_z \right) \left( L_i \times \mu_i \right) \right] \begin{pmatrix} \dot{x} \\ \dot{\psi}_i \\ \dot{\phi}_i \end{pmatrix}, \ \forall i = 1...6
\end{align*}
\] (3.98).

Then, the projection of (3.98) along \( e_z \) and \( \mu_i \) respectively leads to:

\[ e_z^T v_A = 0 = e_z^T v + e_z^T \left( L_i \times \mu_i \right) \phi_i, \ \forall i = 1...6 \] (3.99)

and

\[ \mu_i^T v_A = \mu_i^T e_x \dot{q}_i = \mu_i^T v - \mu_i^T (p_i \times e_z) \dot{\theta}_z + \mu_i^T \left( m_i \times e_z \right) \dot{\psi}_i, \ \forall i = 1...6 \] (3.100).

From (3.99), we get the following relation:
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

\[
J_q \dot{\phi} = J_{xp} \dot{x} \Rightarrow \dot{\phi} = J_{qm} \dot{x}, \text{ with:}
\]
\[
J_q = \text{diag}(e_z^T (L_x \times \mu_1), \ldots, e_z^T (L_6 \times \mu_6))
\]
\[
J_{xp} = \begin{bmatrix}
-e_z^T & 0 \\
\vdots & \vdots \\
-e_z^T & 0
\end{bmatrix}, \quad J_{qm} = J_{q}^{-1} J_{xp}
\]
\[
\text{dim}(J_q) = 6 \times 6, \quad \text{dim}(J_{xp}) = \text{dim}(J_{qm}) = 6 \times 4
\]

The system of equations of (3.97) and (3.100) can be written under the following matrix form (for each \(i = 1 \ldots 6\)):

\[
M_{qp,i} \begin{bmatrix} \dot{q}_i \\ \dot{\psi}_i \end{bmatrix} = M_{x,i} \dot{x}, \text{ with:}
\]
\[
M_{qp,i} = \begin{bmatrix}
L_i^T e_x & -L_i^T (r_p \times e_z) \\
\mu_i^T e_x & -\mu_i^T (m_i \times e_z)
\end{bmatrix}, \quad \text{dim}(M_{qp,i}) = 2 \times 2
\]
\[
M_{x,i} = \begin{bmatrix}
L_i^T & -L_i^T (p_i \times e_z) \\
\mu_i^T & -\mu_i^T (p_i \times e_z)
\end{bmatrix}, \quad \text{dim}(M_{x,i}) = 2 \times 4
\]

(3.102)

Resolving (3.102) for \(\dot{q}_i\) and \(\dot{\psi}_i\), we get:

\[
\begin{bmatrix} \dot{q}_i \\ \dot{\psi}_i \end{bmatrix} = M_{qp,i}^{-1} M_{x,i} \dot{x}, \text{ with:}
\]
\[
M_{qp,i}^{-1} = \frac{1}{\det(M_{qp,i})} \begin{bmatrix}
-\mu_i^T (m_i \times e_z) & L_i^T (r_p \times e_z) \\
-\mu_i^T e_x & L_i^T e_x
\end{bmatrix}, \quad \forall i = 1 \ldots 6
\]
\[
\det(M_{qp,i}) = (m_i^T e_x) (e_z^T (L_i \times \mu_i))
\]

(3.103)

which is valid provided that \(\det(M_{qp,i}) = (m_i^T e_x) (e_z^T (L_i \times \mu_i)) \neq 0\).

Based on (3.103) and some simplifications, we arrive at the uncoupled relations between \(\dot{q}\) and \(\dot{x}\) on one hand, and between \(\dot{\psi}\) and \(\dot{x}\) on the other. These are given below as:

\[
J_q \dot{q} = J_x \dot{x} \Rightarrow \dot{q} = J_m \dot{x}
\]
\[
J_q = \text{diag}\left((m_1^T e_x)(e_z^T (L_1 \times \mu_1), \ldots, (m_6^T e_x)(e_z^T (L_6 \times \mu_6))\right)
\]
\[
J_x(i,1) = (e_z^T (L_i \times \mu_1)) (e_z^T m_1), \quad J_x(i,2) = (e_z^T (L_i \times \mu_2)) (e_z^T m_2)
\]
\[
J_x(i,3) = (e_z^T (m_i \times \mu_1)) (e_z^T m_1), \quad J_x(i,4) = -(e_z^T (L_i \times \mu_2)) (e_z^T (m_i \times p_i))
\]
\[
\forall i = 1 \ldots 6
\]
\[
J_m = J_q^{-1} J_x
\]
\[
\text{dim}(J_q) = 6 \times 6, \quad \text{dim}(J_x) = 6 \times 4, \quad \text{dim}(J_m) = 6 \times 4
\]

and
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

$$J_y \psi = J_{xy} \dot{x} \Rightarrow \psi = J_{ym} \dot{x}$$

$$J_y = \text{diag} \left( \left( m_1^T e_x \right) \left( e_z^T \left( L_1 \times \mu_1 \right) \right), ..., \left( m_6^T e_x \right) \left( e_z^T \left( L_6 \times \mu_6 \right) \right) \right)$$

$$J_{xy} = \begin{bmatrix}
0 & e_z^T \left( L_1 \times \mu_1 \right) & -e_x^T \mu_1 & e_z^T \left( L_1 \right) & e_z^T \left( p_1 \right) & e_z^T \left( L_1 \times \mu_1 \right) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & e_z^T \left( L_6 \times \mu_6 \right) & -e_x^T \mu_6 & e_z^T \left( L_6 \right) & e_z^T \left( p_6 \right) & e_z^T \left( L_6 \times \mu_6 \right)
\end{bmatrix}$$

(3.105),

$$J_{ym} = J_y^{-1} J_{xy}$$

\[ \dim \left( J_y \right) = 6 \times 6, \ \dim \left( J_{xy} \right) = \dim \left( J_{ym} \right) = 6 \times 4 \]

with \( J_x \left( i, j \right) \) denoting the element of the \( i \)-th row and \( j \)-th column of \( J_x \).

Hence, we have established all the necessary kinematic relations, not only the input-output one. The IKM is uniquely defined and given by \( q = J_m \dot{x} \), with \( J_m \) being defined in (3.104). On the other hand, the direct kinematic model (DKM) is obtainable by means of a pseudo-inversion, as we are used to with redundantly actuated PKMs. We then get:

\[
\text{DKM}: \ \dot{x} = J_\text{DKM} \dot{q} = J_m \dot{q}, \ \text{provided that} \ \dot{q}^T \left[ \text{null} \left( J_m^T \right) \right] = 0_{1 \times 2} \quad (3.106).
\]

The upcoming step is to analyze the singularities of the PKM at hand. This is crucial as to assure that the topological mutation has not adversely resulted in unanticipated singular configurations.

C- Singularity Analysis

Constraint Singularities

The main target here is to investigate whether the pair of complex chains, namely (III) and (IV), fulfills its premise in prohibiting undesired rotations of the platform (i.e. imposing \( \omega_x = \omega_y = 0 \): the \( x \) and \( y \) components of the angular velocity, \( \omega_z \), of the platform).

For this purpose, we consider the full twist, \( t = \left( v^T \omega^T \right)^T = \left( v_x \ v_y \ v_z \ \omega_x \ \omega_y \ \omega_z \right)^T \), of the end-effector and ignore all components of the PKM except the two limbs: (III) and (IV). Besides, we modify the point \( P \) (the TCP) to be confounded with \( D_3 \equiv D_4 \) for simplification purposes. In addition to the notation presented in Fig. 3-24 that corresponds to the virtual equivalent chains of limbs (III) and (IV), we define similar vectorial and scalar quantities with subscripts \( ij \) (with \( i = 3, 4 \) and \( j = 1, 2 \)) (refer to Fig. 3-25).
In nominal conditions, we have:

\[
\begin{align*}
L_{i} &= A_{i}B_{i}, \quad r_{i} = B_{i}C_{i}, \quad r_{p_{i}} = C_{i}D_{i} \\
L_{j} &= A_{j}B_{j}, \quad r_{j} = B_{j}C_{j}, \quad r_{p_{j}} = C_{j}D_{j} \\
n_{i} &= A_{i}C_{i} = L_{i} + r_{i}, \quad m_{i} = A_{i}D_{i} = L_{i} + r_{i} + r_{p_{i}} \\
p_{i} &= PD_{i}, \quad \mu_{i} = \frac{r_{i} \times e_{z}}{||r_{i} \times e_{z}||}, \quad \varphi_{i} = (B_{i}C_{i}, B_{i}A_{i}) = (r_{i} - L_{i}) \\
\psi_{i} &= (e_{z}, D_{i}C_{i}) = (e_{z}, -r_{p_{i}}), \quad \psi_{i} = (e^{*}_{p_{i}}, D_{i}C_{i}) = (e^{*}_{p_{i}}, -r_{p_{i}})
\end{align*}
\]

with \( u \) representing the unit vector along the direction of the axes of the first revolute joints placed at \( D_{ij} \) (\( \forall i = 3, 4 \) and \( \forall j = 1, 2 \)).

Let us denote the absolute angular velocities of the rigid bodies, \( B_{ij}C_{ij}D_{ij} \) and \( A_{ij}B_{ij} \), by \( \omega_{p_{ij}} \) and \( \omega_{a_{ij}} \), respectively. Their expressions are given as:

\[
\omega_{p_{ij}} = \omega + \psi_{ij} u = \omega + \psi_{ij} e_{z}, \quad \forall i = 3, 4; \quad \forall j = 1, 2
\]  

(3.108)

and

\[
\omega_{a_{ij}} = \omega_{p_{ij}} + \phi_{ij} \quad H_{ij} = \omega + \psi_{ij} e_{z} + \phi_{ij} \mu_{ij}, \quad \forall i = 3, 4 \text{ and } \forall j = 1, 2
\]  

(3.109).

Exploiting (3.108) and (3.109), we derive the respective velocities of \( D_{ij}, C_{ij}, B_{ij}, \) and \( A_{ij} \) as demonstrated below:

\[
v_{D_{ij}} = v - p_{ij} \times \omega = \begin{bmatrix} 1_{3 \times 3} & -p_{ij} \end{bmatrix} \quad t, \quad \forall i = 3, 4 \text{ and } \forall j = 1, 2
\]  

(3.110),
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

\[
\begin{align*}
\begin{bmatrix}
    v_{B_0} & = & v_{D_0} + \omega_{a_0} \times (-r_{ij} - r_{ji}) \\
    & = & \begin{bmatrix} 1 \times 3 \times 3 \\
    \left(-\hat{p}_{ij} + \hat{r}_{ij} + \hat{r}_{ji} \right) (r_{ij} + r_{ji}) \times e_z \end{bmatrix} \begin{bmatrix} t \\
    \psi_{ij} \end{bmatrix}
\end{bmatrix} \\
\end{align*}
\]

(3.111)

and

\[
\begin{align*}
\begin{bmatrix}
    v_{A_i} & = & v_{A_i} = \dot{q}_i e_x = v_{B_0} + \omega_{a_0} \times (-L_{ij}) \\
    & = & \begin{bmatrix} I \ (\left(-\hat{p}_{ij} + \hat{m}_i \right) m_i \times e_z \ L_i \times \mu_i \end{bmatrix} \begin{bmatrix} t \\
    \psi_{ij} \end{bmatrix}
\end{bmatrix} \\
\end{align*}
\]

(3.112).

\forall i = 3, 4 \ ; \ \forall j = 1, 2

Note that in (3.110) through (3.112), we have \( p_{i_2} = h e_z \) and \( p_{i_2} = -h e_z \), where \( h \) is the semi-height of the complex chain (refer to Fig. 3-25).

To inspect constraint singularities, we must analyze (3.112) after regrouping it in the form below:

\[
C_E \ \dot{\Theta} = V_A \ ; \ \dim(C_E) = 12 \times 14, \ \dim(\dot{\Theta}) = 14 \times 1, \ \text{and} \ \dim(V_A) = 12 \times 1 \quad (3.113),
\]

with:

\[
\dot{\Theta} = \begin{bmatrix} t^T & \psi_{31} & \psi_{32} & \psi_{33} & \psi_{41} & \psi_{42} & \psi_{43} \end{bmatrix}^T
\]

(3.114),

\[
V_A = \begin{bmatrix} v_{A_{1x}} & v_{A_{1y}} & v_{A_{1z}} & \cdots & v_{A_{3x}} & v_{A_{3y}} & v_{A_{3z}} \end{bmatrix}^T
\]

(3.115)

and

\[
C_E = \begin{bmatrix}
    1 \times 3 \times 3 & \left(-\hat{p}_{ij} + \hat{m}_i \right) m_i \times e_z & L_i \times \mu_i & 0_{3 \times 1} & 0_{3 \times 1} & 0_{3 \times 1} & 0_{3 \times 1} & 0_{3 \times 1} & 0_{3 \times 1} \\
    0_{3 \times 3} & 0_{3 \times 1} & 0_{3 \times 1} & m_i \times e_z & L_i \times \mu_i & 0_{3 \times 1} & 0_{3 \times 1} & 0_{3 \times 1} & 0_{3 \times 1} \\
    0_{3 \times 3} & 0_{3 \times 1} & 0_{3 \times 1} & 0_{3 \times 1} & 0_{3 \times 1} & m_i \times e_z & L_i \times \mu_i & 0_{3 \times 1} & 0_{3 \times 1} \\
    0_{3 \times 3} & 0_{3 \times 1} & 0_{3 \times 1} & 0_{3 \times 1} & 0_{3 \times 1} & 0_{3 \times 1} & m_i \times e_z & L_i \times \mu_i & 0_{3 \times 1}
\end{bmatrix} \quad (3.116).
\]

Note that in (3.115), the terms, \( v_{A_{1x}}, v_{A_{1y}}, \) and \( v_{A_{1z}} \), correspond to the \( x, y, \) and \( z \) components of \( v_{A_i} \), with \( i = 3, 4 \) and \( j = 1, 2 \). Also, recall that \( v_{A_i} = v_{A_{1x}} = \dot{q}_i e_x, \ \forall i = 3, 4 \).

In the absence of any constraint singularity, we should always have \( \omega_x = \omega_y = 0 \). Utilizing MATLAB R2010a symbolic toolbox, we have determined the following general solution for \( \dot{\Theta} \) (in the general case of \( V_A \), i.e. before substituting \( v_{A_1} = v_{A_2} = \dot{q}_i e_x, \ \forall i = 3, 4 \):
(or \( \dot{z} \)) and \( \omega_z \) are arbitrary

\[
\begin{align*}
\dot{v}_x &= \frac{f_1(V_A, v_z, \omega)}{2 h (m_{3x} m_{4y} - m_{3y} m_{4x}) (L_{3x} \mu_{3y} - L_{3y} \mu_{3x})^2 (L_{4x} \mu_{4y} - L_{4y} \mu_{4x})^2}, \\
\dot{v}_y &= \frac{f_2(V_A, v_z, \omega)}{2 h (m_{3x} m_{4y} - m_{3y} m_{4x}) (L_{3x} \mu_{3y} - L_{3y} \mu_{3x})^2 (L_{4x} \mu_{4y} - L_{4y} \mu_{4x})^2}, \\
\dot{\omega}_x &= \frac{f_3(V_A, v_z, \omega)}{2 h (m_{3x} m_{4y} - m_{3y} m_{4x}) (L_{3x} \mu_{3y} - L_{3y} \mu_{3x}) (L_{4x} \mu_{4y} - L_{4y} \mu_{4x})}, \\
\dot{\omega}_y &= \frac{f_4(V_A, v_z, \omega)}{2 h (m_{3x} m_{4y} - m_{3y} m_{4x}) (L_{3x} \mu_{3y} - L_{3y} \mu_{3x}) (L_{4x} \mu_{4y} - L_{4y} \mu_{4x})}.
\end{align*}
\]

(3.117)

\[
\begin{align*}
\dot{\psi}_{31} &= \frac{f_1(V_A, v_z, \omega)}{2 h (m_{3x} m_{4y} - m_{3y} m_{4x}) (L_{3x} \mu_{3y} - L_{3y} \mu_{3x})^2 (L_{4x} \mu_{4y} - L_{4y} \mu_{4x})^2}, \\
\dot{\psi}_{32} &= \frac{f_2(V_A, v_z, \omega)}{2 h (m_{3x} m_{4y} - m_{3y} m_{4x}) (L_{3x} \mu_{3y} - L_{3y} \mu_{3x})^2 (L_{4x} \mu_{4y} - L_{4y} \mu_{4x})^2}, \\
\dot{\psi}_{41} &= \frac{f_3(V_A, v_z, \omega)}{2 h (m_{3x} m_{4y} - m_{3y} m_{4x}) (L_{3x} \mu_{3y} - L_{3y} \mu_{3x})^2 (L_{4x} \mu_{4y} - L_{4y} \mu_{4x})^2}, \\
\dot{\psi}_{42} &= \frac{f_4(V_A, v_z, \omega)}{2 h (m_{3x} m_{4y} - m_{3y} m_{4x}) (L_{3x} \mu_{3y} - L_{3y} \mu_{3x})^2 (L_{4x} \mu_{4y} - L_{4y} \mu_{4x})^2}.
\end{align*}
\]

(3.118)

and

\[
\begin{align*}
\phi_{31} &= \frac{f_7(V_A, v_z, \omega)}{2 h (L_{3x} \mu_{3y} - L_{3y} \mu_{3x})}, \quad \phi_{32} = \frac{f_6(V_A, v_z, \omega)}{2 h (L_{3x} \mu_{3y} - L_{3y} \mu_{3x})}, \\
\phi_{41} &= \frac{f_9(V_A, v_z, \omega)}{2 h (L_{4x} \mu_{4y} - L_{4y} \mu_{4x})}, \quad \phi_{42} = \frac{f_{10}(V_A, v_z, \omega)}{2 h (L_{4x} \mu_{4y} - L_{4y} \mu_{4x})}.
\end{align*}
\]

(3.119)

with vect, \( \text{vect}_y \), and vect, designating respectively the \( x \), \( y \), and \( z \) components of vector vect, and with \( f_1 \) through \( f_{10} \) representing polynomial functions of \( V_A \), \( v_z \), and \( \omega_z \) (these expressions are not given explicitly here being quite lengthy and not necessarily for the study).

Substituting \( v_{Ai} = v_{A2} = \dot{q}_i \), \( e_i \), \( \forall i = 3, 4 \), we get \( f_3(V_A, v_z, \omega) = 0 \). So, provided that none of the denominators in (3.117) through (3.119) is null, the chains (III) and (IV) prohibit the undesired rotations by imposing \( \omega_x = \omega_y = 0 \). Moreover, substituting \( V_A = 0 \) (i.e. locking the actuators in place) yields additionally \( f_i(V_A, v_z, \omega) = 0 \); meaning \( v_z = 0 \) and which is intuitively expected.

Therefore, to assure the absence of constraint singularities, we must investigate whether the following condition:

\[
\left( m_{3x} m_{4y} - m_{3y} m_{4x} \right) \left( L_{3x} \mu_{3y} - L_{3y} \mu_{3x} \right) \left( L_{4x} \mu_{4y} - L_{4y} \mu_{4x} \right) \neq 0
\]

(3.120)
is satisfied or not.

Then, three main cases lead to the degeneration of (3.117) through (3.119), which are:

1. \( h = 0 \): This is avoided by construction since \( h \neq 0 \). Such constraint singularity can be coined an architectural one since it occurs for a particular set of geometric parameters.

2. \( \left( m_3, m_4, -m_3, m_4 \right) = 0 \): This means the \( xy \) projections of \( m_3 \) and \( m_4 \) are collinear.

3. \( \left( L_3, \mu_3, -L_3, \mu_3 \right) = 0 \) or \( \left( L_4, \mu_4, -L_4, \mu_4 \right) = 0 \): This corresponds to having the \( xy \) projections of \( L_3 \) (\( L_4 \)) and \( \mu_3 \) (correspondingly \( \mu_4 \)) collinear. But due to the structural symmetry of the mechanism, we always have both terms, \( \left( L_3, \mu_3, -L_3, \mu_3 \right) \) and \( \left( L_4, \mu_4, -L_4, \mu_4 \right) \), simultaneously zero or non-zero.

The case of having \( \left( m_3, m_4, -m_3, m_4 \right) = 0 \) implies that either \( m_3 = m_4 = 0 \) (\( m_3 \) and \( m_4 \) in the \( xz \) plane) or \( m_3 = m_4 = 0 \) (\( m_3 \) and \( m_4 \) in the \( yz \) plane). Both not only occur on the boundary of the geometric workspace, but also are practically preceded by collisions (collision between platform and sliders’ wall in the first case, and between the actuators in the second). Similarly, \( \left( L_3, \mu_3, -L_3, \mu_3 \right) = \left( L_4, \mu_4, -L_4, \mu_4 \right) = 0 \) cannot theoretically happen except in the case where \( L_3 // L_4 // e_z \). In addition to being confounded with classical-type singularity (as we are going to see later), this singularity is circumvented by prior collisions.

Based on the above, we deduce that within the geometric accessible region excluding its boundary, we can guarantee the absence of constraint singularities. In what follows, we discuss the remaining classical singularities.

**Classical Singularities**

For studying these singularities, we only need to consider (3.101), (3.104), and (3.105). We recall that singularity occurs in two cases:

1. The first case is described as follows: knowing \( \dot{q} \), we cannot definitely determine \( \dot{x} \), \( \dot{\phi} \) or \( \dot{\psi} \). Mathematically, this occurs when \( J_q, J_\phi, \) or \( J_\psi \) is singular.

2. The second case is described as follows: knowing \( \dot{x} \), we cannot definitely determine \( \dot{q} \), \( \dot{\phi} \), or \( \dot{\psi} \). In terms of mathematics, this occurs when \( J_q, J_\phi, \) or \( J_\psi \) is singular.

Let us then inspect the configurations for which one or more of the four matrices, \( J_q, J_x, J_\phi, \) and \( J_\psi \), become rank deficient.

**i- Rank Deficiency of \( J_q = J_x \) and \( J_\psi \):**

We have \( J_q = J_\psi \) singular if and only if:

\[
\det(J_q) = \det(J_\psi) = 0 \iff \exists i_0 \in \{1, \ldots, 6\}: e^T_x m_{i_0} = 0 \text{ or } e^T_x (L_{i_0} \times \mu_{i_0}) = 0
\]

(3.121)

As for \( J_\phi \), its singularity is equivalent to:

142
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

Fig. 3-26: Singularity analysis: case \( e_z^T m_b = 0 \).

Fig. 3-27: Singularity analysis: case \( e_z^T (L_0 \times \mu_0) = 0 \)

\[
\det \left( J_\phi \right) = 0 \iff \exists i_0 \in \{1,...,6\} : e_z^T (L_0 \times \mu_0) = 0
\]

which is already embedded in (3.121).

The case \( e_z^T m_b = 0 \) (see Fig. 3-26) corresponds to having \( m_b \) in the \( yz \) plane. Again, this not only occurs on the boundary of the geometric workspace, but also provokes prior collisions.

The case where \( e_z^T (L_0 \times \mu_0) = 0 \) is analogous to what have been discussed in the study of constraint singularities. It dictates having \( L_0 \parallel e_z \) and leads to prior collisions between the platform and the sliders’ plane (refer to Fig. 3-27). Notice that in the case where \( i_0 \neq 3,4 \), the complex chains, namely (III) and (IV), maintain the constraints on the platform. This means that the revolutes at \( D_i \) (\( \forall i = 1...6 \)) sustain their vertical axes and hence, \( \phi_b = 90^\circ \) is imposed. This latter idea is clarified in the same aforementioned figure.
**Chapter 3: The Novel Synthesized Architectures and ARROW PKM**

**ii- Rank Deficiency of \( J_x \):**

To study the rank of \( J_x \), it is necessary to perform some linear operations to achieve a simpler form. We start by changing the TCP to be confounded with \( D_1 \) (note that this corresponds to a linear operation on \( J_x \)). As a result, we get the matrix \( T \) defined by:

\[
T = \begin{bmatrix}
(e_x^T (L_x \times \mu_1)) (e_x^T m_1) & (e_x^T (L_x \times \mu_2)) (e_x^T m_2) & (e_x^T (L_x \times \mu_3)) (e_x^T m_3) \\
(e_x^T (L_2 \times \mu_1)) (e_x^T m_2) & (e_x^T (L_2 \times \mu_2)) (e_x^T m_3) & (e_x^T (L_2 \times \mu_3)) (e_x^T m_4) \\
(e_x^T (L_3 \times \mu_1)) (e_x^T m_4) & (e_x^T (L_3 \times \mu_3)) (e_x^T m_5) & (e_x^T (L_3 \times \mu_4)) (e_x^T m_6) \\
\end{bmatrix}
\]

(3.123)

with:

\[
t_{34} = -(e_x^T (L_3 \times \mu_3)) (e_x^T (m_3 \times r_3)) \quad t_{44} = -(e_x^T (L_4 \times \mu_4)) (e_x^T (m_4 \times r_4))
\]

\[
r_c = D_1 D_3 = D_4
\]

We can assure that \( T \) (equivalently \( J_x \)) is of full rank if the two matrices, \( T_1 \) and \( T_2 \), defined by:

\[
T_1 = \begin{bmatrix}
(e_x^T (L_1 \times \mu_1)) (e_x^T m_1) & (e_x^T (L_1 \times \mu_2)) (e_x^T m_2) & (e_x^T (L_1 \times \mu_3)) (e_x^T m_3) \\
(e_x^T (L_2 \times \mu_1)) (e_x^T m_2) & (e_x^T (L_2 \times \mu_2)) (e_x^T m_3) & (e_x^T (L_2 \times \mu_3)) (e_x^T m_4) \\
(e_x^T (L_3 \times \mu_1)) (e_x^T m_4) & (e_x^T (L_3 \times \mu_3)) (e_x^T m_5) & (e_x^T (L_3 \times \mu_4)) (e_x^T m_6) \\
\end{bmatrix}
\]

(3.124)

and

\[
T_2 = \begin{bmatrix}
-(e_x^T (L_3 \times \mu_3)) (e_x^T (m_3 \times r_3)) \\
-(e_x^T (L_4 \times \mu_4)) (e_x^T (m_4 \times r_4))
\end{bmatrix}
\]

(3.125)

are non-singular.

Starting with the simplest matrix \( T_2 \), its singularity necessitates having:

\[
(e_x^T (L_3 \times \mu_3)) (e_x^T (m_3 \times r_3)) = (e_x^T (L_4 \times \mu_4)) (e_x^T (m_4 \times r_4)) = 0
\]

(3.126)

Due to the structural symmetry with respect to the mediator plane of segment \([A_3 A_4]\), the relation (3.126) can be reduced to:
Chapter 3: The Novel Synthesized Architectures and ARROW PKM

\[
\begin{align*}
\begin{cases}
(e_\zeta^T (L_4 \times \mu_i)) = (e_\zeta^T (L_4 \times \mu_i)) = 0 \\
(e_\zeta^T (m_4 \times r_\zeta)) = (e_\zeta^T (m_4 \times r_\zeta)) = 0
\end{cases}
\end{align*}
\]  
(3.127).

The first case of (3.127) has been discussed in the previous paragraphs (refer to Fig. 3-27). We have proven that it is non-problematic. This is because it is prevented by prior collisions. Regarding the second case (i.e. \( e_\zeta^T (m_4 \times r_\zeta) = e_\zeta^T (m_4 \times r_\zeta) = 0 \)), it is geometrically translated to having the four vectors, \( e_\zeta, m_3, m_4, \) and \( r_\zeta \), in the same plane, which is either the \( xy \) or the \( yz \) plane. Once more, both situations are practically inapplicable because of collisions between the sliders’ plane and the platform, or between the third and fourth actuators. Thus, singularity of \( T_2 \) is not to be concerned about.

It remains to discuss the singularity of \( T_1 \). Doing some linear operations on \( T_1 \) and considering the symmetry of each pair of chains, (I & II) and (V & VI), relative to the mediator planes of the segments \([A_1A_2]\) and \([A_3A_4]\) respectively, we get matrix \( TS_1 \) of same rank as \( T_1 \). \( TS_1 \) is given by:

\[
TS_1 = \begin{bmatrix}
(e_\zeta^T (L_4 \times \mu_i)) (m_1^T e_x) & 0 & 0 \\
0 & (e_\zeta^T (L_4 \times \mu_i)) (m_1^T e_y) & (e_\zeta^T (n_1 \times \mu_i)) (L_4^T e_z) \\
0 & (e_\zeta^T (L_5 \times \mu_i)) (m_5^T e_x) & 0 \\
0 & (e_\zeta^T (L_5 \times \mu_i)) (m_5^T e_y) & (e_\zeta^T (n_5 \times \mu_i)) (L_5^T e_z)
\end{bmatrix}
\]  
(3.128).

\( TS_1 \) (equivalently \( T_1 \)) is of full rank provided that:

\[
\begin{align*}
\bullet & \left( (e_\zeta^T (L_4 \times \mu_i)) (m_1^T e_x) \neq 0 \text{ or } (e_\zeta^T (L_5 \times \mu_i)) (m_5^T e_x) \neq 0 \right) \\
\text{and } \quad \bullet D_1 = (e_\zeta^T (L_4 \times \mu_i)) (m_1^T e_y) (e_\zeta^T (n_1 \times \mu_i)) (L_4^T e_z) \\
& \quad - (e_\zeta^T (L_5 \times \mu_i)) (m_5^T e_y) (e_\zeta^T (n_5 \times \mu_i)) (L_5^T e_z) \neq 0
\end{align*}
\]  
(3.129).

While the practical inapplicability of \( (e_\zeta^T (L_4 \times \mu_i)) (m_1^T e_x) = (e_\zeta^T (L_5 \times \mu_i)) (m_5^T e_x) = 0 \) has been demonstrated in the previous parts, proving that \( D_1 \neq 0 \) over the whole geometric workspace is not an easy task. However, in order to avoid collisions between the platform and the frame’s upper and lower limits, the feasible workspace is restricted to the region defined by: \( |z| \leq H = L_z - a \), where \( L_z > a \). Therefore, it is enough to prove that \( D_1 \) remains strictly positive over this feasible workspace. For this purpose, we need first to emphasize that \( m_1^T e_y = m_5^T e_y \) because of the structural symmetry. Furthermore, we always have \( m_1^T e_y = m_5^T e_y \neq 0 \) except for the practically impossible situation described by having \( m_1 \) and
in the $xz$ plane. This is physically prevented by prior collisions. This latter idea enables us replacing the study of $D_t$ by another one on $D_{tr}$, where:

$$
D_{tr} = (e_z^T (L_z \times \mu_t)) (e_z^T (n_z \times \mu_s)) \left( L_z^T e_z \right) - (e_z^T (L_z \times \mu_s)) (e_z^T (n_z \times \mu_t)) \left( L_z^T e_z \right) \tag{3.130}.
$$

In the feasible workspace (i.e. $L_z = m_z \leq 0$ and $L_z = m_z \geq 0$), the performed sign analysis on the composing terms of $D_{tr}$ shows that $D_{tr}$ not only is positive, but also equal to the sum of two positive terms. This is clarified below:

$$
D_{tr} = \begin{cases}
\left( e_z^T (L_z \times \mu_t) \right) \left( e_z^T (n_z \times \mu_s) \right) \left( L_z^T e_z \right) & \frac{\leq 0}{\geq 0} \\
\left( e_z^T (L_z \times \mu_s) \right) \left( e_z^T (n_z \times \mu_t) \right) \left( L_z^T e_z \right) & \frac{\leq 0}{\geq 0} \\
\end{cases} \tag{3.131}.
$$

Thus, $D_{tr} = t_1 + t_2 \geq 0$ with:

$$
t_1 = (e_z^T (L_z \times \mu_t)) (e_z^T (n_z \times \mu_s)) \left( L_z^T e_z \right) \geq 0 \tag{3.132}
$$

and

$$
t_2 = - (e_z^T (L_z \times \mu_s)) (e_z^T (n_z \times \mu_t)) \left( L_z^T e_z \right) \geq 0 \tag{3.133},
$$

over the feasible workspace. Based on (3.132) and (3.133), we can assert that $D_{tr} = 0$ $\iff$ $t_1 = t_2 = 0$ and hence, we only need to investigate the case where:

$$
( e_z^T (L_z \times \mu_t)) (e_z^T (n_z \times \mu_s)) \left( L_z^T e_z \right) = ( e_z^T (L_z \times \mu_s)) (e_z^T (n_z \times \mu_t)) \left( L_z^T e_z \right) = 0 \tag{3.134}.
$$

Relation (3.134) can be put in the following simplified form:

$$
\begin{cases}
( e_z^T (L_z \times \mu_t)) = 0 \\
( e_z^T (n_z \times \mu_s)) = 0 \\
( L_z^T e_z) = 0
\end{cases}
$$

and

$$
\begin{cases}
( e_z^T (L_z \times \mu_s)) = 0 \\
( e_z^T (n_z \times \mu_t)) = 0 \\
( L_z^T e_z) = 0
\end{cases} \tag{3.135}.
$$

The practical impossibility of $e_z^T (L_z \times \mu_t) = 0$ or $e_z^T (L_z \times \mu_s) = 0$ has been discussed earlier. As for the condition $e_z^T (n_z \times \mu_t) = 0$ or $e_z^T (n_z \times \mu_s) = 0$, it can be translated as having the three concerned vectors in each case laying in the same vertical plane. But as $\mu \perp n$ and $\mu = 0$, the only possibility is having $n_z // e_z$ or $n_z // e_z$, respectively. Nevertheless, this is practically impossible as collisions befall prior to that. Regarding having $L_z^T e_z = 0$ or $L_z^T e_z = 0$, it cannot occur except on the boundary of the feasible workspace. Also, since we have $H = L_z - a \neq 0$, it is not possible to have simultaneously: $L_z^T e_z = 0$ and $L_z^T e_z = 0$.

Therefore, on the feasible workspace, we can guarantee that $TS_1$ (equivalently $T_1$) is of full rank. Besides, as $T_z$ has been proven already to be of full rank over the broader geometrically
accessible workspace, we can then assert that $T$ (equivalently $J_x$) is non-singular over the feasible region.

**Results’ Briefing**

In §3.4.3.C, we have detailed the constraint and classical singularity analyses. The results are that within the feasible workspace, we can assure the absence of all types of singularities. Nevertheless, as it can be obviously noticed in comparison with all previous studies, this investigation has not been straightforward but rather complex.

**D- Simplified Dynamic Model (SDM)**

In brief, to obtain the SDM, we follow the same maneuver and hypotheses described in §3.3.6. We get the direct dynamic model (DDM) as below:

$$
\begin{align*}
\dot{x} &= H \tau - A \dot{x} + a_g^{\text{offset}} \\
H &= (M_p + J_m^T M_a J_m)^{-1} J_m^T \\
A &= H M_a J_m + (M_p + J_m^T M_a J_m)^{-1} A, \\
a_g^{\text{offset}} &= (M_p + J_m^T M_a J_m)^{-1} m_p g, \\
\dim(H) &= 4 \times 6, \quad \dim(a_g^{\text{offset}}) = 4 \times 1 \\
\dim(A) &= 4 \times 4
\end{align*}
$$

(3.136)

with all symbols carrying the same significances and definitions as given in §3.3.6. Choosing the inverse dynamic model (IDM) based on zero components along the null space of $H$, we get:

$$
\begin{align*}
\tau &= C \left( \dot{x} + A \dot{x} - a_g^{\text{offset}} \right), \quad \text{with:} \\
C &= H^+ : \text{pseudo-inverse of } H
\end{align*}
$$

(3.137)

Hence, as the IDM has been obtained, we can proceed with the dimensional synthesis of ARROW V2 M2, which will be covered in the upcoming section.

**E- Dimensional Synthesis**

The dimensional synthesis of ARROW V2 M2 follows the same reasoning and approach described in §3.3.7. It is based on the two primary criteria of precision and dynamics, namely $WTPAF$ and $WILA$. Also, it is followed by a secondary verification based on $FVI_p$, with the values of $v_{req}^p$ and $f_{req}^p$ being identical to those in §3.3.7 (i.e. $v_{req}^p = 0.25 \dot{q}_{\text{max}}$ and $f_{req}^p = 0.25 \tau_{\text{max}}$).
For this synthesis procedure, the actuators’ characteristics are exactly those given in Table 3-2. As for the inertia and fixed geometric parameters, they are respectively presented in Table 3-4 and Table 3-5.
The target parameters to optimize are the following: $L_z$, $L_p$, and $y_o$, where $y_o$ is the $y$ offset of the center of the desired workspace, $DWS$, defined by:

$$
DWS : \begin{cases}
-0.15 \text{ m} \leq z \leq +0.15 \text{ m} \\
-45^\circ \leq \theta \leq +45^\circ
\end{cases}
$$

As for the $x$ length of $DWS$, it is desired to be surely the largest possible value obtainable for:

$$q_{\text{min}} = -1.5 \text{ m} \leq q_i \leq q_{\text{max}} = +1.5 \text{ m}, \ \forall i = 1...6
$$

but after fulfilling the primary and secondary criteria. The chosen ranges for $L_z$, $L_p$, and $y_o$ are as follows:

$$
\begin{cases}
0.84 \text{ m} \leq L_z \leq 1 \text{ m} \\
0.47 \text{ m} \leq L_p \leq 1 \text{ m} \\
y_o^{\text{min}} \leq y_o \leq y_o^{\text{max}}
\end{cases}
$$

with $y_o^{\text{min}}$ and $y_o^{\text{max}}$ representing the lower and upper bounds on $y_o$ that should be respected to allow the accessibility to $DWS$. These latter terms, $y_o^{\text{min}}$ and $y_o^{\text{max}}$, are dependent on $L_z$ and $L_p$.

The Pareto diagram is constructed by considering five equally spaced values for each

![Fig. 3-28: Pareto diagram of ARROW V2 M2 with the two potential candidates encircled in red.](image)

149
parameter in its valid range, then computing \( WTPAF \) and \( WILA \) characteristics. The results are depicted in Fig. 3-28. Based on the available options, we decided to consider the trade-offs described by having \( WTPAF \leq 3.5 \) and \( WILA \geq 7.4 \text{ g} \), meaning those two solutions encircled in red in Fig. 3-28. Among these two sets of geometric parameters, we have selected the one with the largest value of \( WILA \). This solution is interesting because although it has \( WTPAF = 3.4 \), the majority of \( DWS \) is characterized by \( TPAF \leq 3 \). Besides, it admits both: a compact overall structure and a large \( x \)-length of \( DWS \), which is about 0.9 m.

The values of the geometric parameters corresponding to this optimal solution are as follows (after slight numerical modifications):

\[
\begin{cases}
L_s &= 0.96 \text{ m} \\
L_p &= 0.61 \text{ m} \\
y_o &= 0.55 \text{ m}
\end{cases}
\] (3.141).

For this set of parameters, described in (3.141), we have performed an analysis of \( FVI_p \), the result of which has been satisfactory. In fact, we went further than that and performed an intensive study relative to other performance indices, as well.

In Fig. 3-29, Fig. 3-30, Fig. 3-31, Fig. 3-32, and Fig. 3-33, we represent the performance atlases of the optimal ARROW V2 M2 PKM with respect to \( ILA, TPAF, y''_{iso}, f''_{iso}, \) and \( PLA \), respectively. These performances are mapped into the \( yz \) workspace of the PKM, for \( \theta_z = 0^\circ \) on one side and \( \theta_z \in [-45^\circ; 45^\circ] \) on the other.

**Fig. 3-29:** ARROW V2 M2 PKM - \( ILA \) capacity: case \( \theta_z = 0^\circ \) (left) and case \( \theta_z \in [-45^\circ; 45^\circ] \) (right).
Fig. 3-30: ARROW V2 M2 - TPAF performance: case $\theta_z = 0^\circ$ (left) and case $\theta_z \in [-45^\circ; +45^\circ]$ (right).

Fig. 3-31: ARROW V2 M2 PKM - $v_{iso}$ capacity: case $\theta_z = 0^\circ$ (left) and case $\theta_z \in [-45^\circ;+45^\circ]$ (right).

In all the aforementioned figures, the legend (i.e. the significance of the red line and the magenta box) and the methodology of computation for the case of $\theta_z \in [-45^\circ;+45^\circ]$ are the same as clarified in §3.3.7.
Among the interesting results in this analysis, we mention the $p_{iso}$ capacity that exceeds $0.55q_{\max}$, and $PLA$ that reaches $22\,g$, which is absolutely intriguing. This is not to mention the $f_{iso}^p$ capacity that exceeds $0.8\,\tau_{\max}$ over $DWS$.

Fig. 3-32: ARROW V2 M2 PKM - $f_{iso}^p$ capacity: case $\theta_z = 0^\circ$ (left) and case $\theta_z \in [-45^\circ;+45^\circ]$ (right).

Fig. 3-33: ARROW V2 M2 PKM - $PLA$ capacity: case $\theta_z = 0^\circ$ (left) and case $\theta_z \in [-45^\circ;+45^\circ]$ (right).
Actually, this study reveals not only an outstanding performance, but also a rather

Fig. 3-34: ARROW V2 M2: illustrations of the spatial workspaces in the case of $\theta_z = 0^\circ$ (top) and $\theta_z \in [-45^\circ, +45^\circ]$ (bottom).

Fig. 3-35: ARROW V2 M2 PKM with turntable (rendered CAD drawing).

Actually, this study reveals not only an outstanding performance, but also a rather
homogeneous one.

To end this section, we provide in Fig. 3-34 illustrations of the spatial workspaces of ARROW V2 M2 for the stroke lengths given in (3.139). Also, in Fig. 3-35, we show the total assembly of ARROW V2 M2 PKM with the turntable. Notice that the turntable has its axis parallel to the $x$ direction and passing through the point $P_0$ of coordinates $P_0 = (0, y_0, 0)^T$.

3.5- Conclusion

In this chapter, we have discussed some novel architectures and detailed their modeling, singularity analysis, and workspace evaluation. All this has been done while highlighting their merits and drawbacks. Even for some, such as ARROW V1 and ARROW V2, an additional thorough study has been carried out. In particular, ARROW V2 has eventually exceeded the theoretical stage into the scope of practical implementation. Nevertheless, this implementation has put into attention the possibility of inter-arm collisions and necessitated some modifications. With the intention of sustaining the interesting manufacturability features of the initial concept, a slight change in the topology of ARROW V2 has been proposed and validated. The validation has been mainly concerned with assuring that the topological mutation does not result in any type of unexpected singularities. Fortunately, the altered design, namely ARROW V2 M2, has proved to preserve the main characteristics from singularity perspective. However, the mutation has resulted in adverse influence regarding modeling simplicity. An analytical form of the DGM is no more available, and neither the kinematic nor the singularity analysis is a straightforward problem. Ultimately, the dimensional synthesis of ARROW V2 M2 has been presented based on two primary criteria: precision ($WTPAF$) and dynamics ($WILA$). The optimized PKM is characterized by isotropic and peak linear accelerations of about 7.5 g and 22 g, respectively. As for precision, its translational amplification factor does not exceed $WTPAF = 3.4$ over the desired workspace. Moreover, its isotropic linear speed and static force capacities are respectively more than $0.55 \dot{q}_{max} = 2.75$ to $3.85$ m/s and $0.8 \tau_{max} = 2000$ N. Regarding peak linear speed, it can reach $WTPAF \dot{q}_{max} = 3.4 \dot{q}_{max} = 17$ to $23.8$ m/s.

Based on the above, ARROW V2 M2 demonstrates to be worth holding its name. It is really an accurate and rapid PKM with a large operational workspace.

It remains to discuss the preliminary performance analysis of the implemented prototype and provide some insights on the possible future amelioration. These constitute the essence of the last chapter of this dissertation.
In this chapter:

This chapter is dedicated for discussing the preliminary precision evaluation of the ARROW prototype and the possible future enhancements. It starts by giving a general overview on the basic implemented control and the redundancy treatment. Then, the dynamic precision evaluation of the prototype, based on proprioceptive sensors, is presented, and some observations and remarks are accentuated. Afterwards, the general guidelines of geometric calibration and error compensation are provided. In this scope, the main points to deal with the absence of an explicit inverse geometric model, which embraces all the geometric parameters, are detailed. Furthermore, some insights on possible compensation of elastic deformations are discussed, in case a further improvement is needed to attain the anticipated precision.
4.1- Introduction

Within the time allocated for this doctoral thesis and due to the delay in the delivery of the prototype, we could perform neither a detailed performance analysis by exteroceptive means nor a geometric calibration. Nevertheless, we had the opportunity to assess the preliminary performances on the level of precision and dynamics, the results of which are quite promising.

This chapter addresses the aforementioned results and provides an overview on the possible future improvements. These refinements are related to the precision aspects of the PKM, as rapidity and dynamical requirements have been almost fulfilled.

Finally, the current chapter is outlined as follows. First, few words on the current basic control methodology and the treatment of redundancy are discussed. Afterwards, the dynamic precision performance along a sample trajectory is provided. Also, some of the apparent sources of the observed precision errors are accentuated. Based on the presented arguments, a general overview on the geometric calibration and error compensation is presented. Moreover, insights on the compensation of errors due to elastic deformations are exposed. The chapter eventually ends with conclusions.

4.2- ARROW V2 M2 Prototype: The Basic Control Strategy and Treatment of Redundancy

The currently implemented control is depicted in Fig. 4-1. Though it is generally based on the classical PID law, perhaps, few points are worth emphasizing, particularly regarding the treatment of redundancy. In fact, the control of RAMs is usually impeded by the possible occurrence of antagonistic control forces that might lead to system deformation, or even damage if not well treated.

This treatment is done by means of adding the regularization matrix, $R_m$, at the entry as well as the output of the PID block (refer to Fig. 4-1). This method has been proposed in the confidential Ph.D thesis of (YANG, 2012). A similar approach has been also suggested by (HUFNAGEL & MULLER, 2012) and applied to PD control law. In this latter work, $R_m$ is called antagonism filter (AF). The matrix $R_m$ is defined as follows:

$$ R_m = J_m^T J_m^T $$

where $J_m$ is the inverse Jacobian at the reference pose.
In fact, multiplying the joint error vector, \( \delta q \), by \( R_m \) is to assure that only the consistent part of \( \delta q \) is considered by the PID. Then, one might ask: what are the sources of the inconsistent part of \( \delta q \)? The answer is quite simple. Generally speaking, there are two main sources for having \( \delta q^T \left[ \text{null} \left( J_m^T \right) \right] \neq 0 \) (or in other words \( \delta q - R_m \delta q \neq 0 \)). These are the inaccuracy of the measurement device, and the geometric errors. In our case, as we have very precise encoders with nanometric resolution, the incertitude in measurement has negligible effect. Contrarily to that, geometric errors are most often the primary source of the inconsistent portion of \( \delta q \). Actually, as these errors increase, the value \( \delta q - R_m \delta q \) increases as well. That is why the value \( \| \delta q - R_m \delta q \| \) can be a good indicator on how well the geometric calibration has been done.

On the other hand, the regularization at the output of the PID block assures that only the actuation-force vector \( \tau \), with zero components along the null space of \( J_m^T \) (i.e. \( \tau^T \left[ \text{null} \left( J_m^T \right) \right] = 0 \)), is being considered and commanded for the actuators. This methodology fulfills the assumption we have made in the establishment of the inverse static and dynamic models for this redundantly actuated PKM. Nevertheless, we might have \( \tau^T \left[ \text{null} \left( J_m^T \right) \right] = 0 \) violated in one case, which is when electric current saturation occurs.

\[ \text{Note that electric current is directly proportional the force in an electric actuator.} \]
A final remark to make is regarding the anti-windup strategy. The main role of this part is to maintain control stability and avoid oscillatory behavior of the PKM when saturation occurs. In particular, its target is to overcome the problem of integral term accumulation in the mentioned situation. Within the same scope, we highlight that a slightly modified approach to manage integration has been adopted as well, as to avoid numerical overflow. This modified management is that proposed by (YANG, 2012); it is not depicted in Fig. 4-1 for simplicity.

Having presented briefly the implemented control strategy and clarified the main points concerning redundancy, the following section will present some of the acquired preliminary performances of ARROW V2 M2 PKM.

### 4.3- Preliminary Precision Evaluation of ARROW V2 M2 PKM

In this section, we provide some dynamic precision analysis of the prototype for a commanded sample trajectory. We highlight here that due to a noticed defect in some of the joints in addition to power supply limitations, we did not exceed $10 \text{ g}$ linear acceleration and $4 \text{ m/s}$ linear speed. Several trajectories have been tested, but we are going to limit our discussion to the most prominent one. This is given in Fig. 4-2. We clarify that in this figure, the capitalized notations, $X$, $Y$, $Z$, and $B$, are the names of the axes of motion of the PKM following the machine tool conventions; they correspond respectively to the notations: $x$, $z$, $(-y)$, and $\theta_z$, which are used in modeling and which we adopt throughout this report.

Both the non-regularized and regularized joint tracking errors, namely $E = \delta q$ and $E_{\text{reg}} = R_m \delta q$ in Fig. 4-1, have been registered in realtime and are depicted in Fig. 4-3. In Fig. 4-4, an estimation of the tracking errors in the operational space have been computed and visualized. Their calculation has been based on the following relation:

$$
\delta x = \begin{pmatrix} \delta x \\ \delta y \\ \delta z \\ \delta \theta \end{pmatrix}^T = J_m^* \delta q
$$

Several points can be made based on Fig. 4-3 and Fig. 4-4. These are summarized in the enumerated list below:

1. All tracking errors peak at the same instant (about $t = 64 \text{ s}$) and this is due to the high loading involved on the defected joints, particularly those of the third and fourth complex limbs. Notice that at this instant, we have $10 \text{ g}$ linear acceleration in the vertical direction (refer to the green curve in Fig. 4-2). Currently, the design of these joints is being reconsidered. This will probably help overcome the observed phenomenon.
2. The translational tracking errors, except for the aforementioned case, seem to be rather interesting, in which they do not exceed $0.1\, \text{mm}$. Moreover, in statics, it has been noticed that such tracking errors go below $20\, \mu\text{m}$, which is quite remarkable.

3. The controlled orientation error is interesting as well. Except for $t \approx 64\, \text{s}$, the estimated error is less than $0.01^\circ$.

Fig. 4-2: The tested trajectory profile.
Analyzing Fig. 4-3 shows that the regularized errors are much less than the non-regularized ones. This indicates the presence of non-negligible geometric errors. This can be supported by observing the non-regularized tracking errors after $t = 73$ s (static phase), in which they settle to about 1 mm. On the other hand, the small-regularized tracking errors show that though the control is of basic nature, it is still rather effective.
and well tuned. Thus, we can expect much better performance based on a more advanced control strategy.

Finally, we highlight that in what concerns geometric errors, we have already some knowledge of these discrepancies. In fact, some of them have been mentioned by the manufacturing company in its quality control report. However, while compensating for some of these errors is a straightforward process (e.g. the simple arm lengths), it is not the case for others (e.g. lengths of the rods in the complex chains, the direction of the revolute joints, etc.).

Therefore, it is indispensable to explore the calibration of ARROW PKM and the possible method of error compensation. These are the essence of the upcoming section.

4.4- Geometric Calibration and Methodology of Error Compensation

While repeatability is attributable to non-deterministic sources, accuracy is of deterministic nature and therefore subject for enhancement. Among the main sources of accuracy degradation are geometric errors. These result at the time of manufacturing of the different components (manufacturing tolerances) or at their assembly. Other causes of inaccuracy are deformations; a machine tool though is usually very stiff, it remains non-infinitely rigid and consequently elastic deformations are inevitable.

This section is dedicated to deal with geometric errors and their compensation, in which we brief the main guidelines. The goal is to present the general approach, as slight modifications exist in comparison to what we are classically used to.
Chapter 4: ARROW PKM: Preliminary Performances, Geometric Sensitivity, and Possible Error Compensation

First, we mention that a proper calibration should consider simultaneously: the parallel module, the turntable, the measurement frame on the end-effector, and the proper frame of the measurement device. This is clarified in the schematic given in Fig. 4-5. In this figure, the measurement frame corresponds to that of the measurement device, such as the Metris camera, Laser Tracker, etc. Regarding the measurement probes, they correspond to the means by which the end-effector pose is determined, for instance: LEDs, spherical mirrors, etc. Therefore, in practice, we need to identify not only the parallel module and turntable geometric parameters, namely $\xi_{PM}$ and $\xi_{TP}$, but also those that determine the transformation matrices: $T^{MD}$, $T^{MPE}$, and $T^{MPT}$.

However, our aim here is just to give an overview on the calibration approach to deal with the parallel module only. Actually, one of the key issues in this scope is how to account for the influence of the complex chain geometric errors. Obtaining an analytic inverse geometric model (IGM) embracing all the possible geometric parameters, particularly the aforementioned ones, is very difficult, if not impossible. So, how can we overcome this barrier?

Perhaps, the solution is to consider the strategy proposed by (SAVOURE, et al., 2006) for parallelogram-based robots. In the following subsection (§4.4.1), we exploit this method to perform a sensitivity analysis on the parallel module of the ARROW machine. Afterwards, §4.4.2 presents the methodology of error compensation, and §4.4.3 provides some general remarks.

4.4.1- Geometric Sensitivity of ARROW V2 M2

The geometric parameters that are considered for calibration are clarified in Fig. 4-6 and Fig. 4-7. In these figures, we highlight the following designations:

1. $e_{xp}$ corresponds to the unit vector along the $x$-direction of the platform frame.
2. $\text{vect}_{\text{plat}}$ signify the components of vector $\text{vect}$ in the platform frame.
3. $\text{proj}_{xy} (...)$ and $\text{proj}_{xyz} (...)$ denote the projection of a vector into the $xy$ planes of the base and platform frames, respectively.
4. $c_\theta$ and $s_\theta$ denote $\cos(\theta)$ and $\sin(\theta)$, respectively.
5. $x_c = \begin{pmatrix} P^T & \theta^T \end{pmatrix}^T = (x \ y \ z \ \theta_x \ \theta_y \ \theta_z)^T$ is the complete pose of the end-effector, with $P$ being the coordinates of $P \equiv TCP$ and $\theta$ corresponding to the vector composed of Euler angles.
6. $e_x$ corresponds to the direction of the k-th linear track. We have $k = 1$ for $i = 1, 2$, $k = 2$ for $i = 3, 4$ and $k = 3$ for $i = 5, 6$, where $i$ is the number of the kinematic chain.
Chapter 4: ARROW PKM: Preliminary Performances, Geometric Sensitivity, and Possible Error Compensation

7. The points $B_i$, $B_j$, $D_i$, and $D_j$ have the following coordinates in their proper frames:

\[
B_i^{BCD} = \begin{pmatrix}
    x_{BCD}^i \\
    y_{BCD}^i \\
    z_{BCD}^i
\end{pmatrix}^T, \quad
B_j^{BCD} = \begin{pmatrix}
    x_{BCD}^j \\
    y_{BCD}^j \\
    z_{BCD}^j
\end{pmatrix}^T,
\]

\[
D_i^{plat} = \begin{pmatrix}
    x_{plat}^i \\
    y_{plat}^i \\
    z_{plat}^i
\end{pmatrix}^T, \quad
D_j^{plat} = \begin{pmatrix}
    x_{plat}^j \\
    y_{plat}^j \\
    z_{plat}^j
\end{pmatrix}^T,
\]

with $BCD_i$ and $BCD_j$ being the proper frames of origins $D_i$ and $D_j$, respectively. Note that $BCD_i$ and $BCD_j$ are shorthands for $B_iC_iD_i$ and $B_jC_jD_j$, respectively.
Then, to establish the relation between the geometric errors of the parallel module, $\delta \xi_{PM}$, and the complete end-effector pose error, $\delta_\varepsilon = (\delta x, \delta y, \delta z, \delta \theta_x, \delta \theta_y, \delta \theta_z)^T$, a classical approach would be to determine the IGM, then differentiate it with respect to $q, x_\varepsilon$, and $\xi_{PM}$. But as mentioned earlier, such IGM is difficult to establish. Thus, the solution is to consider the basic constraint equations, $F(q, x_\varepsilon, \xi_{PM}) = 0$, as suggested by (SAVOURE, et al., 2006).

Following the lead of (SAVOURE, et al., 2006), we start by considering the following constraints:

$$A_i B_i^T = \bar{L}_i^2, \ \forall i = 1, 2, 5, 6$$

and

$$A_j B_j^T = \bar{L}_j^2, \ \forall i = 3, 4 \text{ and } \forall j = 1, 2$$

Differentiating (4.3) and (4.4), we get:

$$A_i B_i^T dA_i B_i = A_i B_i^T dB_i - dA_i = L_i dL_i, \ \forall i = 1, 2, 5, 6$$

and

$$A_j B_j^T dA_j B_j = A_j B_j^T dB_j - dA_j = L_j dL_j, \ \forall i = 3, 4 \text{ and } \forall j = 1, 2$$

The idea now is to replace $dA_i, dB_i, dA_j,$ and $dB_j$ in terms of $dx_\varepsilon$ (the differential of $x_\varepsilon$), $dq$ (the differential of $q$), and $d\xi_{PM}$ (the differential of $\xi_{PM}$). This is done by considering the following expressions:

$$A_i = A_{i0} + (q_i + x_\varepsilon) e_k \Rightarrow dA_i = dA_{i0} + (dq_i + dx_\varepsilon) e_k + (q_i + x_\varepsilon) de_k$$

and

$$A_j = A_{j0} + (q_j + x_\varepsilon) e_k \Rightarrow dA_j = dA_{j0} + (dq_j + dx_\varepsilon) e_k + (q_j + x_\varepsilon) de_k, \ \text{with:}$$

$$A_{i0} = A_{i0} + A_{i0}^{bw}$$

$$B_i = P + R B_i^{plat} = P + R (D_i^{plat} + R^{BCD} B_i^{BCD}), \ \text{which implies:}$$

$$dB_i = dP + dR D_i^{plat} + R dB_i^{plat} + dR^{BCD} B_i^{BCD} + R^{BCD} dB_i^{BCD}$$

and

$$B_j = P + R B_j^{plat} = P + R (D_j^{plat} + R^{BCD} B_j^{BCD}), \ \text{which implies:}$$

$$dB_j = dP + dR D_j^{plat} + R dB_j^{plat} + dR^{BCD} B_j^{BCD} + R^{BCD} dB_j^{BCD}$$

with:

- $R$: the rotation matrix of the platform with respect to the base frame in terms of Euler angles $(\theta_x, \theta_y, \text{and } \theta_z)$;
Chapter 4: ARROW PKM: Preliminary Performances, Geometric Sensitivity, and Possible Error Compensation

- \( N^{frame} \): the coordinates of a point \( N = (B_i, B_{ij}, D_i, D_{ij}) \) expressed in the frame of reference, \( frame \ (frame = plat, BCD_i, BCD_{ij}) \);
- \( R_{plat}^{BCD_i} (R_{plat}^{BCD_{ij}}) \): the rotation matrix of the part \( BCD_i \) (correspondingly \( BCD_{ij} \)) with respect to the platform;
- and \( R_{plat}^{BCD_i} = R \ R_{plat}^{BCD_{ij}} (R_{plat}^{BCD_{ij}} = R \ R_{plat}^{BCD_i}) \): the rotation matrix of \( BCD_i \) (correspondingly \( BCD_{ij} \)) with respect to the base frame.

Note that we have \( R_{plat}^{BCD} = \text{Rot} (u_i, \psi_r) \) and \( R_{plat}^{BCD_{ij}} = \text{Rot} (u_{ij}, \psi_{r_{ij}}) \), where \( \psi_r \) and \( \psi_{r_{ij}} \) are the rotation angles about \( u_i \) and \( u_{ij} \), respectively. For nominal conditions of assembly and geometric parameters, we have \( u_i = u_{ij} = e_z \) and consequently, \( \psi_r \) and \( \psi_{r_{ij}} \) are the same as those clarified in Fig. 3-24 and Fig. 3-25.

In brief, we proceed by developing (4.7) through (4.10) in terms of \( dq \), \( dx \), \( d\xi_{PM} \), \( d\psi_r \), and \( d\psi_{r_{ij}} \). Then, we substitute the latter results in (4.5) and (4.6) to get the following general relation:

\[
M_q \ dq + M_x \ dx + M_{\psi_r} \ d\psi_r + M_{\xi_{PM}} \ d\xi_{PM} = 0 \quad (4.11),
\]

with:
- \( M_q \), \( M_x \) and \( M_{\psi_r} \): being \( 8 \times 6 \) matrices;
- \( M_{\xi_{PM}} \): being \( 8 \times \text{dim}(\xi_{PM}) \) matrix;
- and \( d\psi_r = (d\psi_{r_1} \ d\psi_{r_2} \ d\psi_{r_{31}} \ d\psi_{r_{32}} \ d\psi_{r_{41}} \ d\psi_{r_{42}} \ d\psi_{r_{5}} \ d\psi_{r_{6}})^\text{T} \).

Now, we should get rid of \( d\psi_r \) from (4.11). This can be easily done by considering the nominal conditions, in which we can write \( d\psi_r \) in terms of \( dx \) by utilizing (3.105) and knowing that:

\[
\begin{align*}
&d\psi_{r_1} = \psi_{r_1} \ dt, \ \forall i = 1, 2, 5, 6 \\
&d\psi_{r_2} = \psi_{r_2} \ dt, \ \forall i = 3, 4 \text{ and } \forall j = 1, 2 \\
&\dot{\psi}_{r_{31}} = \psi_{r_{32}} = \psi_{r_{41}}, \ \forall i = 3, 4 \\
&\dot{\psi}_{r_{5}} = \psi_{r_{6}} - \theta_z, \ \forall i = 1...6
\end{align*}
\quad (4.12).
\]

As a result, we get the following general form\(^2\):

\[
J_q^F \ dq + J_x^F \ dx + J_{\xi_{PM}}^F \ d\xi_{PM} = 0 \quad (4.13),
\]

\(^2\) Details are omitted and only general forms are given here. Yet, the explicit expressions can be established symbolically by hand or by using the symbolic toolbox of MATLAB (following the described steps).
Chapter 4: ARROW PKM: Preliminary Performances, Geometric Sensitivity, and Possible Error Compensation

\[ \mathbf{J}_q^F \quad \text{and} \quad \mathbf{J}_s^F \quad \text{being} \quad 8 \times 6 \quad \text{matrices} \quad \text{and} \quad \mathbf{J}_{\xi_{\text{ref}}}^F \quad \text{being} \quad 8 \times \dim(\xi_{\text{PM}}) \quad \text{matrix.} \]

As we are only considering geometric errors, we set \( \mathbf{dq} = 0 \). This implies:

\[ \mathbf{dx}_e = \mathbf{S}_g^e \, \mathbf{d} \xi_{\text{PM}} = -\mathbf{J}_{s_e^F} \, \mathbf{J}_{\xi_{\text{ref}}}^F \, \mathbf{d} \xi_{\text{PM}} \quad \text{(4.14)} \]
where $S_g$ is the geometric sensitivity matrix. Hence, for small geometric errors, $\delta \xi_{PM}$, the estimated pose error is:

$$\delta x_c = S_g \delta \xi_{PM}$$

(4.15)
As the procedure guidelines have been described, it is essential to discuss the geometric parameters and their proper choice. In fact, we can consider some realistic and practical assumptions to reduce the number of these parameters. These assumptions are:

\[ u_1 \equiv u_2, \quad u_{31} \equiv u_{41}, \quad u_{32} \equiv u_{42}, \quad u_5 \equiv u_6, \quad D_1 \equiv D_2, \quad D_{31} \equiv D_{41}, \quad D_{32} \equiv D_{42}, \quad \text{and} \quad D_5 \equiv D_6. \]

Based on the above, we have 84 geometric parameters (i.e. \( \dim(\mathbf{g}_{PM}) = 84 \)), namely:

- \( L_1, L_2, L_{31}, L_{32}, L_{41}, L_{42}, L_5 \) and \( L_6 \) (i.e. 8 parameters);
- \( e_k \) (\( \forall k = 1, 2, 3 \)) (i.e. \( \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2 \) and \( \beta_3 \); 6 parameters);
- \( x_{r1} \ldots x_{r6} \) (i.e. 6 parameters);
- \( A_{ij} = (0 \quad y_{ai} \quad z_{ai})^T, \forall i = 1, 2, 5, 6 \) (i.e. 8 parameters);
- \( A_{ij} = (x_{ai} \quad y_{ai} \quad z_{ai})^T, \forall i = 3, 4 \) and \( \forall j = 1, 2 \) (i.e. 12 parameters);
- \( u_i \) (\( \forall i = 1, 2, 5, 6 \)) and \( u_j \) (\( \forall i = 3, 4 \) and \( \forall j = 1, 2 \)) (i.e. \( \alpha_i^{a_1} \equiv \alpha_i^{a_2}, \beta_i^{b_1} \equiv \beta_i^{b_2}, \alpha_{31}^{a_1} \equiv \alpha_{41}^{a_2}, \beta_{31}^{b_1} \equiv \beta_{32}^{b_2}, \alpha_i^{a_2} \equiv \alpha_i^{a_3}, \text{and} \beta_i^{b_2} \equiv \beta_i^{b_3} \); 8 parameters);
- \( D_1 \equiv D_2, \quad D_{31} \equiv D_{41}, \quad D_{32} \equiv D_{42}, \quad \text{and} \quad D_5 \equiv D_6 \) (i.e. 12 parameters);
- \( B_i \) (\( \forall i = 1, 2, 5, 6 \)) and \( B_j \) (\( \forall i = 3, 4 \) and \( \forall j = 1, 2 \)) (i.e. 24 parameters).

For these geometric parameters and for their nominal values given in \( \S 3.4.3.E \), we have investigated the mean and standard deviation of the absolute sensitivity over the desired workspace, \( DWS \), defined in (3.138). The results are depicted in Fig. 4-8 and Fig. 4-9 for a representative subset of the aforementioned parameters (particularly the parameters of the simple chain (I) and the complex chain (III)). As it can be clearly noticed from the figures, there are several parameters of negligible influence as compared with the others, particularly: \( y_{ai}^{BCD} \) (\( \forall i = 1, 2, 5, 6 \)), \( y_{ai}^{BCD} \) (\( \forall i = 3, 4 \) and \( \forall j = 1, 2 \)), and the directions of the revolute joints, \( u_i \) (\( \forall i = 1, 2, 5, 6 \)) and \( u_j \) (\( \forall i = 3, 4 \) and \( \forall j = 1, 2 \)). These have not only low mean absolute sensitivity, but also a small standard deviation. Hence, we can mainly consider the remaining 68 parameters for the PKM calibration. This number might be even reduced upon performing identifiability study, which is not our concern here.

As we have derived the geometric sensitivity matrix, the upcoming section focuses on the latter’s implementation for the purpose of error compensation.

---

3 Note that we have \( A_{ij} = (x_{ai} \quad y_{ai} \quad z_{ai})^T = A_{00} + A_{i0}^{\text{v}} \). We considered \( A_{ij} \) instead of \( (A_{00} + A_{i0}^{\text{v}}) \) to avoid obvious redundancy in some geometric parameters.

4 The results of sensitivity relative to the geometric parameters of the other chains are analogous to those presented in the mentioned figures. This is due to the structural symmetry of the PKM on one hand, and the symmetry of the desired workspace, \( DWS \), with respect to the \( xy \) and \( yz \) planes on the other hand.
4.4.2- Compensation of Geometric Errors of ARROW PKM

In this part, we focus on the compensation of the geometric errors of the ARROW machine, which is composed of the parallel module, ARROW V2 M2, and the turntable. In §4.4.1, we have focused on the parallel module only, as the turntable modeling and calibration study are trivial. Nevertheless, in the scope of precision performance improvement of the whole machine, it is indispensable to put the turntable into the play, the importance of which will become clear by the end of the current section.

In fact, if we consider ARROW V2 M2 separately, we cannot compensate except for four components of \( \delta x_c \), specifically those corresponding to the controllable dofs. Similarly, if we consider the treatment of the turntable in isolation from the parallel module, we cannot compensate except for one orientation error. Yet, considering both constituting modules at once, allows for better error compensation where only the insignificant orientation error about the spindle axis is not compensated. This latter technique is what we are going to discuss here.

Fig. 4-10 presents the turntable with its geometric parameters, \( x_o, y_o, z_o, \alpha_t, \beta_t, \theta_{t1}, \) and \( \theta_{t2} \); these parameters are consolidated in the term \( \delta T_T \). In nominal conditions, \( e_t \) is parallel to \( e_x \); i.e. parallel to the \( x \)-axis of the base frame.

Now, we suppose that for \( \delta T_T \), we have estimated \( \delta \theta \), the orientation error of the turntable corresponding to the desired orientation, \( \theta_t \). Also, we presume that starting from
\[ \delta \xi_{PM} \], we have computed \( \delta x = (\delta x \ \delta y \ \delta z \ \delta \theta_x \ \delta \theta_y \ \delta \theta_z)^T = S_g(x_d) \delta \xi_{PM} \), where
\[ x_d = (x_d \ y_d \ z_d \ \theta_d) \]^T is the corresponding desired controllable pose of the parallel module.

Based on the above, we have the infinitesimal rotational error vector of the spindle with respect to the base frame given by:

\[
\begin{align*}
\epsilon_{PM}^{\text{rot}} &= \begin{pmatrix} 1 & 0 & 0 & \delta \theta_x & \cos(\theta_x) & \sin(\theta_x) \\ 0 & 1 & 0 & \delta \theta_y & -\sin(\theta_x) \cos(\theta_x) & \cos(\theta_x) \\ 0 & 0 & 1 & \delta \theta_z & \sin(\theta_x) \cos(\theta_x) & \cos(\theta_x) \end{pmatrix}^{\text{T}} \delta \xi_{PM} = \Omega \delta \theta, \\
\Omega &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin(\theta_x) & \cos(\theta_x) \\ 0 & -\sin(\theta_x) \cos(\theta_x) & \cos(\theta_x) \end{pmatrix}, \ 
\delta \theta &= \begin{pmatrix} \delta \theta_x \\ \delta \theta_y \\ \delta \theta_z \end{pmatrix}
\end{align*}
\]

As \( \theta_x \) and \( \theta_y \) are practically very small and tend to zero (nominally \( \theta_x = \theta_y = 0 \)), we have \( \Omega \equiv 1_{3 \times 3} \). Thus, relation (4.16) becomes:

\[
\epsilon_{PM}^{\text{rot}} = \delta \theta
\] (4.17).

Regarding the turntable, we have the infinitesimal rotational error vector given by:

\[
\epsilon_{TT}^{\text{rot}} = \delta \theta, \ e_t
\] (4.18).

Hence, we have the main ingredients that allow us to proceed with error compensation. But before describing this procedure, the reader should keep in mind that our interest is to get rid of the relative errors between the parallel module and the turntable, not the base frame. In fact, to work with the ARROW machine, the trajectory generation should be done in the turntable frame rather than the base one. That is why the relative position and orientation of

![Fig. 4-11: Illustration of the impact of the rotation \( e_t^{\text{T}} \epsilon_{PM}^{\text{rot}} \approx \delta \theta_x \) of the turntable.](image-url)
the spindle with respect to the turntable are the ones to be compensated. This is crucial to understand and make sense of what follows.

We start by considering the rotational errors about \( e_t \). The relative rotational error between the spindle and the turntable, \( \delta \vartheta_{TT}^{PM} \), is given by:\(^5\)

\[
\delta \vartheta_{TT}^{PM} = e_t^T e_{PM}^{rot} - \delta \vartheta_t = e_t^T \delta \vartheta - \delta \vartheta_t = \delta \vartheta_z - \delta \vartheta_t
\]  

(4.19).

Then, we compensate for \( \delta \vartheta_{TT}^{PM} \) by commanding the following rotation of the turntable:

\[
\theta_{comp} = \theta_t + \delta \vartheta_{TT}^{PM} = \theta_t + \delta \vartheta_z - \delta \vartheta_t
\]  

(4.20) instead of \( \theta_t \).

On the other hand, the relative orientation error between the spindle and the turntable about \( e_z \), denoted \( \delta \varepsilon_{TT}^{PM} \), is given by:

\[
\delta \varepsilon_{TT}^{PM} = e_t^T e_{PM}^{rot} - e_z^T e_{TT}^{rot} = \delta \theta_z - (e_z^T e_t) \delta \vartheta_t = \delta \theta_z
\]  

(4.21).

Therefore, to compensate for \( \delta \varepsilon_{TT}^{PM} \), we need to command the following corrected rotation of the spindle about the \( z \) axis:

\[
\theta_{z\text{comp}} = \theta_{zd} - \delta \varepsilon_{TT}^{PM} = \theta_{zd} - \delta \theta_z
\]  

(4.22).

It remains then to consider the compensated position of \( P \equiv TCP \), i.e. the tool center point of the spindle. This compensation is divided into parts. The first part accounts for the positional error of the parallel module, \( \delta P = (\delta x \quad \delta y \quad \delta z)^T \), whereas the second considers the positional error, \( \delta P_{Tt} \), due to the rotation added to compensate \( e_t^T e_{PM}^{rot} = \delta \vartheta_t \). To clarify this latter point, an illustration is provided in Fig. 4-11. In this figure, the point \( P_t \) is the center of the turntable, and \( P_t \) is the position of \( P \) in the case where no rotation is made to compensate \( e_t^T e_{PM}^{rot} = \delta \vartheta_t \). When the turntable rotates an additional \( e_t^T e_{PM}^{rot} = \delta \vartheta_t \) angle, the spindle’s TCP remains in its place at \( P_t \). But as we said earlier, we are interested in maintaining the same relative position between the TCP and \( P_t \). This implies the need to move the TCP from \( P_t \) to \( P_t' \), where \( P_t' \) is obtained via rotating the segment \( P_t P_t' \) an angle equal to \( e_t^T e_{PM}^{rot} = \delta \vartheta_t \). As \( e_t^T e_{PM}^{rot} = \delta \vartheta_t \) is sufficiently small, we have:

\[
\begin{aligned}
\delta P_{n} &= -\delta \vartheta_t \ e_t \times P_t P_d, \\
P_n P_d &= (x_d - x_t \quad y_d - y_t \quad z_d - z_t)^T
\end{aligned}
\]  

(4.23),

\(^5\) Note that we have \( e_t^T \delta \vartheta = \cos(\alpha_t) \cos(\beta_t) \delta \vartheta_t + \sin(\alpha_t) \cos(\beta_t) \delta \vartheta_t + \sin(\beta_t) \delta \vartheta_t = \delta \vartheta_t \), where \( \alpha_t \) and \( \beta_t \) are supposedly very small (they are nominally equal to zero).
where \( P_d = (x_d, y_d, z_d)^T \) represents the coordinates of the desired position of \( P \equiv TCP \), and \( P_o = (x_o, y_o, z_o)^T \) corresponds to the coordinates of the turntable center.

As a result, the compensated desired position of the parallel module should be:

\[
P_{d_{\text{comp}}} = (x_{d_{\text{comp}}}, y_{d_{\text{comp}}}, z_{d_{\text{comp}}})^T = P_d - \delta P - \delta P_u \tag{4.24}.
\]

Eventually, the compensated poses of the parallel module and the turntable, which allow maintaining their desired relative configuration, are given by:

\[
\begin{align*}
x_{d_{\text{comp}}} &= \left( P_{d_{\text{comp}}} \right) = \left( P_d - \delta P - \delta P_u \right) \\
\theta_{d_{\text{comp}}} &= \theta_d - \delta \theta_d - \delta \theta_z
\end{align*}
\tag{4.25}.
\]

As the compensation methodology of geometric errors has been clarified, it remains to stress some essential remarks concerning classical calibration and its execution. This is the subject of the following subsection.

4.4.3- General Remarks on Geometric Calibration

In general, the classical calibration can be carried out with the target of minimizing some cost function related to the pose errors, such as: the root mean square of the different pose errors, the maximal pose error, etc. However, it is important to know that with the current PKM, there are some constraints to be respected due to actuation redundancy.

In particular, one should take into account the minimization of \( \left\| \delta q - R_m \delta q \right\| \) for all configurations. In fact, having \( \left\| \delta q - R_m \delta q \right\| \) large implies not only an inconsistency in the geometric parameters, but also a possible increase in deformations. The inclusion of \( \left\| \delta q - R_m \delta q \right\| \) can be done by means of adding the equality constraint \( \left\| \delta q - R_m \delta q \right\| = 0 \) to the classical optimization, or considering the minimization of \( \max \left( \left\| \delta q - R_m \delta q \right\| \right) \) as one of the objectives in the optimization problem. In this aspect, we would recommend the former option.

4.5- Compensation of Elastic Deformation

The ARROW machine being well manufactured and having its geometric parameters precisely monitored, we think that geometric calibration will be sufficient to achieve the anticipated precision performance. Nonetheless, if needed, a compensation of the elastic deformations can be done as well. This section summarizes the main guidelines in this aspect.

As we have the joints very stiff, especially after the redesign of the defected ones, their stiffness can be neglected. We can only consider the stiffness of the individual rods under tension/compression forces. These are denoted by \( K_i \ (\forall i = 1, 2, 5, 6) \) and
Chapter 4: ARROW PKM: Preliminary Performances, Geometric Sensitivity, and Possible Error Compensation

\( K_{ij} \) \(( \forall i = 3, 4 \text{ and } \forall j = 1, 2)\); they correspond respectively to the rods of the simple chains (I, II, V, VI), and the complex ones (III, IV). Good estimates of \( K_i \) and \( K_{ij} \) can be obtained by applying finite element method. It then remains to describe the compensation methodology.

For the compensation of deformation errors, we are going to follow the lead of (DEBLAISE, 2006), but in the general dynamic case instead of the static one. The essence of the aforementioned procedure can be described as follows. First, the forces in the rods are computed. Then, the deformations and the resulting pose error are determined and compensated for in the very same manner as described in \( \S 4.4.2 \).

Regarding the determination of the forces, it is straightforward. For the simple arms, it is done by considering the dynamic equations of the corresponding actuators, and knowing their respective actuation forces, thanks to the simplified inverse dynamic model (\( \S 3.4.3.D \)). As for the forces in the individual rods of the complex chains (III) and (IV), they are computed by considering the equations of motion of the corresponding actuators in addition to the constraint relations on the platform (i.e. the relation between these forces and the torques about the \( x \) and \( y \) axes of the base frame). Omitting the details and sticking to the key points, we get the forces’ values in these rods, say: \( F_i \) \( (\forall i = 1, 2, 5, 6) \) for the simple limbs (I, II, V, and VI), and \( F_{ij} \) \( (\forall i = 3, 4 \text{ and } \forall j = 1, 2) \) for the complex ones (III and IV), with the convention that positive values indicate tension.

The influence of these forces is then translated as variations of the lengths \( L_i \) and \( L_{ij} \), which are denoted by \( \delta L_i^{\text{def}} \) \( (\forall i = 1, 2, 5, 6) \) and \( \delta L_{ij}^{\text{def}} \) \( (\forall i = 3, 4 \text{ and } \forall j = 1, 2) \), respectively. Their values are simply given by:

\[
\begin{align*}
\delta L_i^{\text{def}} &= \frac{F_i}{K_i}, \ \forall i = 1, 2, 5, 6 \\
\delta L_{ij}^{\text{def}} &= \frac{F_{ij}}{K_{ij}}, \ \forall i = 3, 4 \text{ and } \forall j = 1, 2
\end{align*}
\]

Knowing \( \delta L_i^{\text{def}} \) and \( \delta L_{ij}^{\text{def}} \), the compensation of their induced pose error can be achieved through applying the same strategy described in \( \S 4.4.2 \). More precisely, it is performed by adding \( \delta L_i^{\text{def}} \) and \( \delta L_{ij}^{\text{def}} \) to the corresponding geometric errors, \( \delta L_i \) and \( \delta L_{ij} \).

Hence, we have went an additional step in the precision improvement of the ARROW machine. Perhaps, it is worth mentioning here that although we have considered the rods’ deformations only, still we can follow the same technique and include the stiffness matrices of the joints. Nevertheless, the gained accuracy not only might be unworthy compared to the added complexity, but also its online implementation can be infeasible. This is due to the computational time expense.
4.6- Conclusion

In this chapter, several aspects have been discussed. Starting by the exposition of the basic control strategy, some points have been emphasized, specifically concerning the treatment of actuation redundancy. Following that, the preliminary dynamic precision analysis of ARROW V2 M2 prototype has been carried out, founded on the data provided by the proprioceptive sensors. The results have been generally insightful and promising. Actually, they are expected to be better upon the redesign of the defected joints, performing geometric calibration, and applying a more advanced control strategy. In this scope, the geometric calibration and the methodology of compensation have been discussed. Also, the pose sensitivity of the parallel architecture with respect to the different geometric parameters has been presented. It has demonstrated the possibility of considering a reduced list of 68 parameters for ARROW V2 M2 in a first step, which can then be ensued by a further reduction based on identifiability study. Moreover, if geometric error compensation proved to be insufficient for accomplishing the target precision, the possibility of compensating elastic deformation has been highlighted and its main principles have been described.

This was in brief the core of the current chapter and the plan for the performance enhancement of the ARROW machine. Yet, several interesting topics can be investigated in the future. Among these, we mention the possibility of auto-calibration. Having ARROW V2 M2 possessing two extra sensors is a feature worth exploiting via auto-calibration. The purpose behind that should not be bypassing the classical calibration procedure, but rather compensating for very slight parameter modifications during the working phase. These slight modifications could be due to temperature drift, replacement of a damaged component and the alike. Another idea is to exploit auto-calibration as means of signaling when a classical geometric identification is required.
General Conclusions and Perspectives

In this doctoral thesis, we have focused on parallel manipulators (PMs) and their implementation as machine tools. In fact, PMs have been there for more than half a century and attracted a lot of research. Yet, their real exploitation in industry has been shy and limited, not exceeding the research stages at university laboratories and industrial enterprises. This is mainly due to workspace limitations, design complexities, singularities, and lack of good compromise between rapidity and precision. In the attempt to change this harsh reality, we have addressed two main problematics in this field. The first is related to the design itself, in which we have exposed some novel topologies that fulfill the yearned for manufacturability features, while maintaining large workspace, rapidity, and precision. The second problematic that we have focused on is performance assessment regarding precision, kinetostatics, and dynamics. In particular, we have been interested in having the new suggested indices applicable to general manipulators and free of the inherent limitations of the classical measures, regarding redundancy and heterogeneity conflicts. We have considered all of this, while sustaining a physical interpretation and having it directly linked to what the roboticist or end-user look for in such machines. In the light of the above, a new rapid and precise prototype, namely ARROW V2 M2, has been designed. It has been dimensionally synthesized based on the dual criteria of precision and dynamics. The initial performances of this PKM have been quite intriguing, though based on proprioceptive means and not exteroceptive ones.

Actually, we do not claim that the so-far presented work is the perfect approach to acquire rapidity and precision, but perhaps, it is a solid one-step towards the aforementioned goals. In addition, we stress that still the preliminary assessment is not sufficient to verify the acquirement of the preset targets, as a more intensive exteroceptive precision investigation as well as geometric calibration should follow.

In what follows, we emphasize some points that are interesting to research in the future. These fall in the scope of complementary work regarding the suggested performance indices, and the possible enhancements of the implemented PKM.

Possible Advancements Regarding the Suggested Performance Measures

In the currently presented work, we did not deal with two types of manipulators, namely: the task redundant ones, and those with mixed simultaneous kinematic and actuation redundancies (AR and KR, respectively).

The treatment of task redundancy (TR) is rather simple, as long as no AR or KR is involved. Otherwise, its intricacy is similar to the case of mixed-redundancy manipulators (MRMs). For
MRMs, the treatment sophistication is that of obtaining the direct and inverse kinetostatic models. As mentioned earlier, MRMs, especially of rigid type, are not being researched according to our knowledge. Nevertheless, it seems it will not remain the same for cable-driven parallel robots. More precisely, marine robotic designs, based on active cables and thrusters, are currently subject of research at LIRMM. Such designs involve both AR and KR simultaneously. Therefore, it is essential to inspect such situations; especially that one of our targets has been the general applicability of the proposed indices. Assuring that the latter idea holds is important. This is since it aids in evaluating and comparing designs of different types, without invoking any disputes regarding the need for some common basis.

Eventually, as means of promoting the current evaluation methodology, the development of a software package that includes the required functionalities is necessary. This step, in spite of being of technical nature, is undeniably influential for the sake of popularizing the approach and advancing it, based on the feedback and inspirations supplied by the users.

**Possible Improvements Regarding ARROW Machine**

With respect to the currently executed ARROW prototype, several possible advancements can be made. These are enumerated below:

1. Geometric or elasto-geometric calibration must be done, at least incorporating the identified geometric errors given in the quality control report. This can be sufficient to effortlessly promote the precision performance further, and reduce the disagreement between the regularized and non-regularized joint errors. In this aspect, it would be also worthy to investigate the possibility of performing calibration based on the quality of machined parts, as suggested by (CHANAL, 2006). Such approach is interesting due to having the cost function directly related to the target, which is the agreement between the real machined piece and the desired one.

2. Auto-calibration would be interesting to investigate and exploit for improving the precision performance, via the compensation of some marginal geometric errors (e.g. those arising from temperature drift, component replacement, etc.).

3. In the current thesis, we have done the dimensional synthesis without including the parallel module and turntable inter-collision checks. This has been based on the assumption that it is possible to account for it at the design stage. Yet, it is preferable to consider this point in future optimizations, especially that we have recently developed an advanced collision detection module for this purpose. This module can be used in the optimization procedure on one hand, and for offline collision pre-check in the industrial controller on the other. Nonetheless, it is not recommendable for use in realtime, as it might be time consuming.

These were the main points regarding the current ARROW machine, in its current right-hand left-hand structure. Nevertheless, it is not completely disfavored to consider a hybrid version consisting of ARROW V2 M2 in series with a one-dof (1R) wrist, especially if the application
does not require a massive rotary actuator. At the end, this remains another compromise to make between elevated dynamical capabilities and simple motion planning.
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Bibliography


Appendices

Appendix A: Precision-Related Performances of Some Rapid Industrial Robots and Prototypes at LIRMM

A.1- Generalities and Definitions

This appendix details the precision evaluation of several prototypes and industrial robots that has been done at LIRMM. This assessment considered accuracy and repeatability not only for static poses, but also in dynamics; i.e. as the robot follows a pre-specified trajectory. Moreover, multi-directional variation of accuracy, in the case of statics, has been investigated as well. The norm adopted for all the aforementioned tests is [ISO 9283; 1998 (F)].

The study considered the following robots: Quattro, Par2, DUAL V, and Veloce (the prototype). However, regarding dynamic performance (path following), not all robots were possible to evaluate due to technical issues and constraints.

In what follows, we clarify all the aforementioned terms, in which we limit ourselves to generic definitions and clarifications. As for the formulae, they are easy to compute based on the provided definitions. Yet, for more insights and details, the reader may refer to [ISO 9283; 1998 (F)].

Accuracy

Pose accuracy expresses the deviation between the commanded pose and the mean of the attained ones when approaching the former (i.e. desired/commanded pose) from the same direction. In other words, it is the distance between the mean of the attained poses and the

Fig. Ap-1: Static accuracy and repeatability illustration.
desired one. The closer it is to zero the better, indicating an accurate robot. In general, we can
distinguish between positional and orientation accuracies.

**Repeatability**

It expresses the closeness of agreement between the attained poses after \( n \) repetitive visits
to the same commanded pose, and along the same direction. We speak generally of positional
and orientation repeatabilities. This repeatability is also referred to as uni-directional one, as it
considers single direction. Geometrically, it is the radius of the sphere centered at the mean
pose and embracing all the attained ones.

Assuming that the attained poses are normally distributed about the mean (i.e. Gaussian
distribution\(^1\)), the repeatability is \( 3 \bar{\sigma} \), with \( \bar{\sigma} \) being the estimated standard deviation (this is
applied for positional and orientation repeatability). The closer the value is to zero the better,
indicating a very repeatable robot.

An illustration of the above two terms is depicted in **Fig. Ap-1**. Based on the
aforementioned, a robot can be accurate but not repeatable, or vice-versa. In fact, the
precision as referred to in several places is more related to repeatability than accuracy. Here,
we will use the words “precision” and “precise” to refer globally to all terms such as accuracy,
repeatability, and resolution (operational resolution\(^2\)) that has been described in **Chapter 2**.
Therefore, when we say a “precise” robot, we mean a one that is both accurate and repeatable.

**Multi-Directional Variation of Accuracy**

Briefly, this expresses how the pose accuracy (in position and orientation) varies with the
change of direction of approach to the desired pose. Mathematically, it is the largest difference
between the means of the actual poses obtained for the same commanded pose, but along
different directions. An illustration of this term is shown in **Fig. Ap-2**.

Note that in all the aforementioned static precision evaluations, the attained pose is
measured after a sufficient settling time.

---

\(^1\) In Gaussian or normal distribution, the region containing the vast majority (99.99%) of the cases are those
within a distance of three standard deviations from the mean. Notice that we used the notation \( \bar{\sigma} \) to denote
estimated value of this deviation, \( \sigma \). For more on this matter, the reader may refer to [ISO 9283; 1998 (F)]
and any statistical article in this subject (estimators of standard deviations).

\(^2\) Operational resolution is the smallest detectable step the robot can do in the operational space. It is similar to
the measurement resolution of a scale (ruler). If the ruler is marked with 1 mm divisions, it means we cannot use it
to measure an object with a length smaller than 1 mm; in fact, any measurement is done with rough ±1 mm
error.
Also, note that the most important of these measures are repeatability and multi-directional variation of accuracy. This is due to the fact that the reasons behind these mentioned terms are rather of non-deterministic random nature (due for example to backlashes, joint clearances, stick-slip effects, control and sampling time, encoder resolution, disturbances, thermal fluctuations, hysteresis, etc.). As for accuracy, it is due to deterministic sources (such as geometric errors, assembly errors, elasticity, incompleteness of geometric model, etc.). Note also that a part from the non-deterministic influences, specifically their mean, contributes in a secondary part to the accuracy. Theoretically speaking, while accuracy related errors are possible to compensate, repeatability errors are rather impossible to circumvent. That is why having less accurate but repeatable robot is less problematic than the opposite case. Based on Fig. Ap-1 and Fig. Ap-2, it is clear that when repeatability and multi-directional variation of accuracy are both very close to zero, the attained poses for the same commanded destination are tightly clustered, which implies improved multi-directional repeatability as well.

So far, we have defined static precision-related terms. It remains to define the dynamic precision-related measures that quantify the robot performance in trajectory following.

Considering a trajectory that is described several times, then one can define the path accuracy and path repeatability (in position and orientation).

**Trajectory or Path Accuracy**

Consider the robot moving along a prescribed path several times in the same direction. Then, the trajectory positional accuracy is defined as the maximum distance between the desired path and the barycenters of the different positions attained at different steps of the controlled trajectory. Similar definition is considered regarding orientation accuracy along a desired path. For better understanding, refer to Fig. Ap-3.
Appendices

196

Trajectory or Path Repeatability

It expresses the closeness of agreement between the several trajectories traveled for the same commanded path. It also utilizes the six-sigma rule, assuming a Gaussian distribution. For clarification, see Fig. Ap-3.

A.2- Briefing on the Procedure and Main Results

Evaluation Procedure in Brief

Having defined the different terms, we can proceed to discuss the evaluation done. First, it is worth mentioning here that the pose of the tested robot has been measured by the use of Metris K600-CMM system (shown in Fig. Ap-4). According to its technical specifications, within the first zone of measurement (1.5 to 3 m), the uncertainty is \(90 \mu m + (25 \mu m/m) d_m\), where \(d_m\) is the distance between the LEDs and the camera. Yet, it might be slightly different due to the temperature variations, especially that the calibration of the machine is not possible during experimentation. As for the measurement repeatability, we have estimated it by checking several measurements of fixed LEDs (for one second duration), and it was in the vicinity of 10 \(\mu m\).

\[^3\text{We have done our measurements in this zone.}\]
The complete pose of the end-effector is measured by means of fixing three LEDs on the platform as close as possible to the TCP, and some techniques were used to account for the impact of rotational errors on the estimation of the real TCP position –when possible. Also, some estimation technique was used to determine the transformation matrix between the measuring frame and the proper base frame of the tested robot. We will not get further in details into these aspects as being rather technical, but we will highlight later the impact of some of these issues. The reader should keep in mind that the intention behind this evaluation is having insights on the order of magnitude of the precision measures, and not their exact values.

The schematic illustrating the experimentation is depicted in Fig. Ap-5. Notice that in this figure, three LEDs are fixed on the moving platform and are measured by the Metris device to estimate the complete end-effector pose. Also, other three LEDs -when possible- are mounted on the fixed base frame as to inspect the vibrational effects and any inconsistencies. When using six LEDs, the measuring frequency is 500 Hz. In the case of using three LEDs, the measuring frequency is 1000 Hz.
Finally, for measurement acquiring, an automatic triggering signal was channeled from the robot controller, allowing for automated procedure—except when it was impossible (due to compatibility issue, as it was the case with DUAL V).

**Static Precision Results**

![Diagram](image)

**Fig. Ap-6:** The largest inscribed cube or rectangular parallelepiped within the accessible workspace, the diagonal plane, and the test points ($P_1$...$P_9$).

**Fig. Ap-7:** Illustration of the possible cycle schemes.

<table>
<thead>
<tr>
<th>Cycle No.</th>
<th>Measurement Scheme</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$P_3$</td>
<td>Nothing Measured at Cycle No. 0</td>
</tr>
<tr>
<td>1</td>
<td>$P_2$ → $P_1$ → $P_6$ → $P_2$ → $P_1$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$P_2$ → $P_1$ → $P_6$ → $P_2$ → $P_1$</td>
<td>Measurement Done (At All Poses)</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>$P_2$ → $P_1$ → $P_6$ → $P_2$ → $P_1$</td>
<td></td>
</tr>
</tbody>
</table>

OR

<table>
<thead>
<tr>
<th>Cycle No.</th>
<th>Measurement Scheme</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$P_3$ → $P_1$ → $P_2$ → $P_3$ → $P_1$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$P_3$ → $P_1$ → $P_2$ → $P_3$ → $P_1$</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>$P_3$ → $P_1$ → $P_2$ → $P_3$ → $P_1$</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. Ap-7:** Illustration of the possible cycle schemes.
According to [ISO 9283; 1998 (F)], the static tests may be carried by determining the largest cube or rectangular parallelepiped inscribed within the accessible or most frequently utilized workspace. Then, any of the four possible diagonal planes can be used (see Fig. Ap-6). Within this plane, five points $P_i$ (with $i = 1...5$) are considered being the center, and the points along the diagonals, with the latter ones offset from the vertices a pre-specified distance for safety.

Regarding the way each of the five points (or poses) is approached for static accuracy evaluation, it can be done in one of the two manners depicted in Fig. Ap-7. We have chosen the second scheme. As for multi-directional variation of accuracy, the test is only done at $P_1$, $P_2$, and $P_4$, in the way shown in Fig. Ap-8. For each of the three poses, $P_1$, $P_2$, and $P_4$, the approach is done along three directions parallel to the axes of the base frame, and each is repeated 30 times.

Having clarified the main points regarding the procedure, we proceed by presenting the static precision results of Quattro (see Fig. 1-45), with the articulated platform (four dofs of 3T-1R nature) and the rigid one (three dofs of 3T type) (see Fig. Ap-9). Note that in the case of using rigid platform, the robot becomes redundantly actuated with one degree of redundancy. The static accuracies and repeatabilities for position and orientation of Quattro, in its two versions, are presented in Table Ap-1, Table Ap-2, Table Ap-3, Table Ap-4, Table Ap-5, and Table Ap-6.
Fig. Ap-9: Adept Quattro platforms: articulated (four dofs 3T-1R on the left) and rigid (three dofs 3T on the right) (http://www.adept.com/products/robots/parallel/quattro-s650h/downloads).

Table Ap-1: Static positional accuracy and repeatability of Adept Quattro (case of articulated platform).

<table>
<thead>
<tr>
<th>Angle</th>
<th>Orientation Accuracy (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_x$</td>
<td>$P_1$</td>
</tr>
<tr>
<td>$\theta_y$</td>
<td>0.0029</td>
</tr>
<tr>
<td>$\theta_z$ (controlled)</td>
<td>-0.1218</td>
</tr>
</tbody>
</table>

Table Ap-2: Static positional accuracy and repeatability of Adept Quattro (case of rigid platform).

<table>
<thead>
<tr>
<th>Angle</th>
<th>Orientation Accuracy (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_x$</td>
<td>$P_1$</td>
</tr>
<tr>
<td>$\theta_y$</td>
<td>0.0029</td>
</tr>
<tr>
<td>$\theta_z$ (controlled)</td>
<td>-0.1218</td>
</tr>
</tbody>
</table>

Table Ap-3: Static orientation accuracy of Adept Quattro (case of articulated platform).

<table>
<thead>
<tr>
<th>Angle</th>
<th>Pose</th>
<th>Orientation Accuracy (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_x$</td>
<td>$P_1$</td>
<td>$P_2$</td>
</tr>
<tr>
<td>$\theta_y$</td>
<td>0.0029</td>
<td>-0.2842</td>
</tr>
<tr>
<td>$\theta_z$ (controlled)</td>
<td>-0.1218</td>
<td>1.4040</td>
</tr>
</tbody>
</table>
Appendices

Table Ap-4: Static orientation repeatability of Adept Quattro (case of articulated platform).

<table>
<thead>
<tr>
<th>Angle\Pose</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_x$</td>
<td>0.0088</td>
<td>0.0134</td>
<td>0.0108</td>
<td>0.0147</td>
<td>0.0096</td>
</tr>
<tr>
<td>$\theta_y$</td>
<td>0.0029</td>
<td>0.0036</td>
<td>0.0027</td>
<td>0.0049</td>
<td>0.0027</td>
</tr>
<tr>
<td>$\theta_z$ (controlled)</td>
<td>0.0115</td>
<td>0.0221</td>
<td>0.0102</td>
<td>0.0176</td>
<td>0.0194</td>
</tr>
</tbody>
</table>

Table Ap-5: Static orientation accuracy of Adept Quattro (case of rigid platform).

<table>
<thead>
<tr>
<th>Angle\Pose</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_x$</td>
<td>-0.0014</td>
<td>0.0744</td>
<td>-0.2785</td>
<td>-0.1257</td>
<td>0.0499</td>
</tr>
<tr>
<td>$\theta_y$</td>
<td>-0.0053</td>
<td>-0.1882</td>
<td>-0.0469</td>
<td>0.1339</td>
<td>0.1723</td>
</tr>
<tr>
<td>$\theta_z$</td>
<td>-0.0352</td>
<td>-0.0341</td>
<td>-0.0826</td>
<td>-0.1585</td>
<td>-0.1067</td>
</tr>
</tbody>
</table>

Table Ap-6: Static orientation repeatability of Adept Quattro (case of rigid platform).

<table>
<thead>
<tr>
<th>Angle\Pose</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_x$</td>
<td>0.0393</td>
<td>0.0413</td>
<td>0.0179</td>
<td>0.0127</td>
<td>0.0134</td>
</tr>
<tr>
<td>$\theta_y$</td>
<td>0.0048</td>
<td>0.0064</td>
<td>0.0041</td>
<td>0.0032</td>
<td>0.0028</td>
</tr>
<tr>
<td>$\theta_z$</td>
<td>0.0227</td>
<td>0.0333</td>
<td>0.0207</td>
<td>0.0102</td>
<td>0.0103</td>
</tr>
</tbody>
</table>

As for the multi-directional variation of accuracy in position, it is given in Table Ap-7. The results show that while Quattro is undeniably rapid, yet its precision performances are still far from the requirements for high-speed and high-precision applications. It is worth mentioning here that the precision evaluation, based on the poses calculated via the feedback sensors, showed a better accuracy (for both versions) and better repeatability (only for rigid platform). In fact, we inferred the problem to be in addition to the use of articulated platforms, to some issues at control level and the implemented direct geometric model (numerical model). However, the control being closed, no further inspection was possible to make at this level. Also, the way in which the parallelograms are executed, by means of ball-socket connections and springs, is an additional contributor to this lack of precision. This is not to mention the geometric errors and calibration related sources. In fact, in another project, called PRADA (executed by Tecnalia France), noticeable improvements on accuracy have been made reaching about 55% (thanks to reconsidering the geometric models and performing adequate
Another four-dof (3T-1R) robot whose static precision has been investigated is Veloce (the prototype). Its static accuracy, repeatability, and multi-directional variation of accuracy are given in Table Ap-8, Table Ap-9, and Table Ap-10. The poor performance regarding Veloce prototype has been expected and it is due to two main reasons. The first is that the robot has never been calibrated, whereas the second is that the rotational errors were large. In fact, the high rotational errors affected the positional accuracy. This is due to the fact that the influence of the offset between the real TCP and the estimated one has not been compensated because of technical issues (note that the rotational errors are sufficiently high to have the influence very noticeable; see the values in Table Ap-8 and Table Ap-9 for $P_4$ and $P_5$). Regarding repeatability, it is similar somehow to Quattro. Also, the orientation errors, especially for constrained rotations, are rather small and repeatable. These are due to assembly and geometry errors. As a matter of fact, controlled rotation always has more degraded accuracy compared with constrained rotations, but still acceptably repeatable. In Veloce prototype, the main contributor of its diminished accuracy (especially rotational one) is the uncompensated geometrical errors as well as the presence of the screw-nut mechanism, which also has its impact on repeatability. 

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**Table Ap-7: Multi-directional variation of positional accuracy of Adept Quattro.**

<table>
<thead>
<tr>
<th>Version\Pose</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quattro (4 dofs, 3T-1R)</td>
<td>0.1007</td>
<td>0.0886</td>
<td>0.2292</td>
</tr>
<tr>
<td>Quattro (3 dofs, 3T)</td>
<td>0.0978</td>
<td>0.2253</td>
<td>0.1066</td>
</tr>
</tbody>
</table>

**Table Ap-8: Static positional accuracy, repeatability, and multi-directional variation of accuracy of Veloce prototype.**

<table>
<thead>
<tr>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static Accuracy (mm)</td>
<td>0.1162</td>
<td>3.4294</td>
<td>1.8707</td>
<td>8.3440</td>
</tr>
<tr>
<td>Static Repeatability (mm)</td>
<td>0.0963</td>
<td>0.0959</td>
<td>0.1053</td>
<td>0.0664</td>
</tr>
<tr>
<td>Multi-Directional Variation of Accuracy (mm)</td>
<td>0.1207</td>
<td>0.1175</td>
<td>Not Done</td>
<td>0.1772</td>
</tr>
</tbody>
</table>
Table Ap-9: Static orientation accuracy of Veloce prototype.

<table>
<thead>
<tr>
<th>Angle\Pose</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_x$</td>
<td>-0.0496</td>
<td>-0.0505</td>
<td>-0.0819</td>
<td>-0.0985</td>
<td>-0.1793</td>
</tr>
<tr>
<td>$\theta_y$</td>
<td>-0.0188</td>
<td>-0.0573</td>
<td>0.0090</td>
<td>-0.0044</td>
<td>0.1019</td>
</tr>
<tr>
<td>$\theta_z$ (controlled)</td>
<td>0.1110</td>
<td>-2.2557</td>
<td>1.9811</td>
<td>9.6003</td>
<td>7.0918</td>
</tr>
</tbody>
</table>

Table Ap-10: Static orientation repeatability of Veloce prototype.

<table>
<thead>
<tr>
<th>Angle\Pose</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_x$</td>
<td>0.0121</td>
<td>0.0070</td>
<td>0.0065</td>
<td>0.0062</td>
<td>0.0265</td>
</tr>
<tr>
<td>$\theta_y$</td>
<td>0.0089</td>
<td>0.0029</td>
<td>0.0030</td>
<td>0.0024</td>
<td>0.0062</td>
</tr>
<tr>
<td>$\theta_z$ (controlled)</td>
<td>0.0187</td>
<td>0.0131</td>
<td>0.0422</td>
<td>0.0096</td>
<td>0.0322</td>
</tr>
</tbody>
</table>

It is worth mentioning that the general observations regarding orientation errors are almost the same in all the tested robots. In what follows, we just present the interesting positional accuracy and repeatability of Par2 and DUAL V.

Par2 (see Fig. Ap-10) is a two-dof (2T) robot developed for pick-and-place applications. In fact, the coupled two passive chains that characterize this robot help in increasing its transversal stiffness, which constitutes an interesting feature as compared with other planar robots. Moreover, its Delta-like structure allowed it to be the most rapid manipulator with two dofs, having an acceleration capability up to $43 g$ and up to $260 \text{ cycle/min}$ capacity (Adept cycle of 700 mm length and 25 mm height) (COMPANY, et al., 2011). As for DUAL V (see Fig. Ap-10), it is a rapid three-dof (2T-1R) planar motion manipulator characterized by actuation redundancy that not only helps eliminating singularities, but also homogenizing performances. An interesting feature of this robot is the possibility of having it dynamically balanced and thus, reducing the influence of inertial forces and moments. The balanced case has therefore the advantage regarding precision. For more information, refer to (WIJK, et al., 2013). In our experimentation, we considered the unbalanced version and yet we had interesting results.
The static accuracy and repeatability of Par2 and DUAL V are given in Table Ap-11 and Table Ap-12, respectively. In fact, though the accuracies are not that better than the previous discussed robots, their repeatabilities are quite interesting being between 10 µm and 25 µm. This is promising, especially that the repeatability of the measurement device is about 10 µm.

Furthermore, we mention that regarding accuracy measurement, there have been several problems that would have highly influenced the results, particularly for these two aforementioned robots (compatibility between Metris K600 and the robots). The latter issue was severely critical in the case of DUAL V and forced us to use manual control for measurement triggering.

An important result concluded from DUAL V experimentation is that actuation redundancy is not a problem, especially if well treated. In fact, based on the results of Quattro with rigid platform, such a conclusion is not possible to make. Also, the use of revolute joints has been an

Table Ap-11: Static positional accuracy and repeatability of Par2.

<table>
<thead>
<tr>
<th></th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static Accuracy (mm)</td>
<td>0.0330</td>
<td>2.0793</td>
<td>2.1001</td>
<td>1.1723</td>
<td>1.3427</td>
</tr>
<tr>
<td>Static Repeatability (mm)</td>
<td>0.0129</td>
<td>0.0128</td>
<td>0.0086</td>
<td>0.0253</td>
<td>0.0252</td>
</tr>
</tbody>
</table>

Table Ap-12: Static accuracy and repeatability of DUAL V.

<table>
<thead>
<tr>
<th></th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static Accuracy (mm)</td>
<td>0.0302</td>
<td>1.0369</td>
<td>0.7291</td>
<td>0.9414</td>
<td>0.3777</td>
</tr>
<tr>
<td>Static Repeatability (mm)</td>
<td>0.0244</td>
<td>0.0091</td>
<td>0.0098</td>
<td>0.0103</td>
<td>0.0154</td>
</tr>
</tbody>
</table>
essential factor in promoting DUAL V precision and particularly its repeatability -as compared with the joint technology used for parallelogram spherical joints (e.g. in Quattro and Veloce). We add to this, the use of direct drive that eliminates the backlash problems coming from the use of gear trains.

So far, we have exposed some static precision results. In the upcoming part, we present some dynamic precision measures concerning trajectory following.

**Dynamic Precision Results**

For assessing path accuracy and repeatability, [ISO 9283; 1998 (F)] suggests several types of trajectories such as straight lines, rectangular paths, and circles. All these trajectories must be inside the chosen diagonal plane. In our case, we have experimented with linear, triangular, and circular paths. However, we only present the main results of two robots, namely Quattro and Par2, for the linear trajectories along $P_1P_2$ (refer to Fig. Ap-6), while focusing solely on positional errors. We emphasize that according to the aforementioned norm, the trajectory is repeated 10 times for 100%, 50%, and 10% of robot’s maximal speed.

The positional path accuracy and repeatability of Quattro and Par2 are presented in Table Ap-13.

<table>
<thead>
<tr>
<th></th>
<th>10% Maximum Speed</th>
<th>50% Maximum Speed</th>
<th>100% Maximum Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quattro (4 DoFs, 3T-1R Version)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Path Positional Accuracy (mm)</td>
<td>0.2447</td>
<td>0.7820</td>
<td>1.2430</td>
</tr>
<tr>
<td>Path Positional Repeatability (mm)</td>
<td>0.4740</td>
<td>0.7516</td>
<td>1.9674</td>
</tr>
<tr>
<td>Quattro (4 DoFs, 3T Version)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Path Positional Accuracy (mm)</td>
<td>0.8025</td>
<td>1.0941</td>
<td>1.6241</td>
</tr>
<tr>
<td>Path Positional Repeatability (mm)</td>
<td>1.4066</td>
<td>2.2855</td>
<td>4.1633</td>
</tr>
<tr>
<td>Par2 (2 DoFs, 2T)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Path Positional Accuracy (mm)</td>
<td>0.2396</td>
<td>0.3222</td>
<td>0.3905</td>
</tr>
<tr>
<td>Path Positional Repeatability (mm)</td>
<td>0.6354</td>
<td>0.6907</td>
<td>0.8588</td>
</tr>
</tbody>
</table>
Appendices

**Ap-13.** As it can be noticed and expected, both path accuracy and repeatability are degraded with the increase of speed. This is due in one part to the decreased number of sampled points and in another part to the increase of the other dynamic influences. The values clearly show that rapidity is accompanied by rather poor dynamic precision.

In brief, this section has proven that we are still far from having parallel robots with at least a good compromise between rapidity and precision.
Appendix B: Derivation of Lower Bounds for the Dynamic Specific Isotropic Values in the Case of Rigid Manipulators

B.1- The Derivation Procedure

This appendix details the procedure of obtaining lower bounds for the specific isotropic values in the case of dynamics. For this purpose, we assume the requirements to be: \(d_a\), \(d_{\alpha}\), \(d_v\), \(d_{\omega}\), \(d_f\), and \(d_m\), with their significance being as defined in §2.3.4.

We start by considering the following relations:
\[
K \xi + h + h_\alpha + h_v + \tau_g = \tau \quad \text{(Ap.1)}
\]
\[
\left| \dot{q}_i \right| \leq q_i^\text{max}, \ \forall i = 1...m \quad \text{(Ap.2)}
\]
and
\[
\left| \tau_i \right| \leq \tau_i^\text{max}, \ \forall i = 1...m \quad \text{(Ap.3)}.
\]

In addition to the above relations, we recall the inverse kinematic equation that is also necessary for the study. It is given by:
\[
\dot{q} = J_m t \quad \text{(Ap.4)}.
\]

The first step is replacing (Ap.2) and (Ap.4) by deriving their imposed upper bounds on \(d_{V_{iso}}\) and \(d_{\omega_{iso}}\). This is equivalent to getting the specific isotropic linear and angular speeds, \(k_{V_{iso}}\) and \(k_{\omega_{iso}}\), as discussed in §2.2.3.C, in which we set \(\omega_{req} = d_{\omega_{req}}\) and \(v_{req} = d_{v_{req}}\), respectively. Thus, (Ap.2) and (Ap.4) can be replaced by:
\[
d_{V_{iso}} \leq k_{V_{iso}} \quad \text{(Ap.5)}
\]
and
\[
d_{\omega_{iso}} \leq k_{\omega_{iso}} \quad \text{(Ap.6)},
\]
with:
1. \( \max \) \( \min \), such that:

\[
A_i = \begin{cases} 
0 , & \text{if } a_i < 0 \\
\frac{a_i}{\|j_{mpr}\|} , & \text{if } \|j_{mpr}\| \neq 0 \text{ and } a_i \geq 0 \\
\infty , & \text{otherwise} 
\end{cases}
\] (Ap.7),

\[\forall i = 1 \ldots m\]

and

\[
B_i = \begin{cases} 
0 , & \text{if } b_i < 0 \\
\frac{b_i}{\|j_{mor}\|} , & \text{if } \|j_{mor}\| \neq 0 \text{ and } b_i \geq 0 \\
\infty , & \text{otherwise} 
\end{cases}
\] (Ap.8).

\[
\forall i = 1 \ldots m
\]

If \( k v_{iso}^{m} < d v_{req} \) (or \( k \omega_{iso}^{m} < d \omega_{req} \))\(^4\), the robot cannot initially fulfill velocity requirements and consequently, all dynamic specific isotropic values, except those related to velocity, must be set to zero by convention (i.e. \( d a_{iso} = d f_{iso} = d m_{iso} = 0 \)). In such a case, one only needs to investigate \( d v_{iso}^{m} \) and \( d \omega_{iso}^{m} \). Aside from that, we can proceed by deriving the dynamic measures.

However, in what follows, we are going only to elaborate the derivation of two dynamic specific isotropic terms, namely \( d a_{iso}^{m} \) and \( d v_{iso}^{m} \), being slightly different from each other and as the remaining can be derived in a similar fashion.

For this purpose, we consider (Ap.1) and (Ap.3) that yield the following system of double inequalities:

\[
-\tau_{ij}^{max} \leq \left( k_{r}^{a} \right)^{T} a + \left( k_{r}^{f} \right)^{T} f_{r} + \left( k_{r}^{m} \right)^{T} m_{r} + v^{T} H^{f} v + \omega^{T} H^{m} \omega + 2 v^{T} H^{m} \omega + \tau_{ij}^{max} \leq +\tau_{ij}^{max}
\] (Ap.9),

\[
\forall i = 1 \ldots m
\]

\(^4\) Note that it is enough to do either test of the two, as we have a mutuality influence (meaning: \( k v_{iso}^{m} < d v_{req} \Leftrightarrow k \omega_{iso}^{m} < d \omega_{req} \)).
Appendices

with \( k^a_i \) standing for the part of \( i \)-th row vector of \( K \) corresponding to the acceleration \( a \), and similar meaning regarding the others. As for \( \tau_{e_i} \), it is the \( i \)-th component of \( \tau_{e} \). Then, isolating \((k^a_i)^T a\), we get:

\[
\begin{aligned}
\forall i = 1 \ldots m: & \\
-\tau_{i}^\text{max} - (k^a_i)^T a - (k^{f_i}_a)^T f_e - (k^{m_a}_r)^T m_e - v^T H_i^e v - \omega^T H_i^\omega \omega - \tau_{e_i} \leq (k^a_i)^T a
\end{aligned}
\]  

(Ap.10)

and

\[
\begin{aligned}
\forall i = 1 \ldots m: & \\
(k^a_i)^T a \leq & + \tau_{i}^\text{max} - (k^a_i)^T a - (k^{f_i}_a)^T f_e - (k^{m_a}_r)^T m_e - v^T H_i^e v - \omega^T H_i^\omega \omega - \tau_{e_i}
\end{aligned}
\]  

(Ap.11).

Then, let us consider (Ap.10) and the worst case, which can be described as maximizing the left-hand side and minimizing the right-hand one. This worst case is expressed as having:

\[
\begin{aligned}
\forall i = 1 \ldots m: & \\
& \left( k^a_i \right)^T a = \max_{\|k^a_i\|} \left( -v^T H_i^e v - \omega^T H_i^\omega \omega - 2 v^T H_i^\omega \omega - \tau_{e_i} \right) - \left( k^a_i \right)^T a
\end{aligned}
\]  


In general, obtaining \( \max_{\|k^a_i\|} \left( -v^T H_i^e v - \omega^T H_i^\omega \omega - 2 v^T H_i^\omega \omega \right) \) can be done numerically by classical constrained optimization. However, in the case of homogeneous-dof robot, we can always get the term \( \max_{\|k^a_i\|} \left( -v^T H_i^e v \right) \) or \( \max_{\|k^a_i\|} \left( -\omega^T H_i^\omega \omega \right) \) analytically by the use of Lagrange multipliers (see Appendix B.2). In this case, we will have the maximum expressed in terms of the eigenvalues of \( H_i \equiv H_i^e \) or \( H_i \equiv H_i^\omega \). Also, in the case of having \( H_i^\omega = 0 \) (uncoupled influence of \( v \) and \( \omega \)), we can determine

\[
\max_{\|k^a_i\|} \left( -v^T H_i^e v - \omega^T H_i^\omega \omega \right) = \max_{\|k^a_i\|} \left( -v^T H_i^e v \right) + \max_{\|k^a_i\|} \left( -\omega^T H_i^\omega \omega \right)
\]  

(Ap.13).

Thus, each term of the right-hand side of (Ap.13) can be expressed in terms of the eigenvalues of \( H_i^e \) and \( H_i^\omega \).

So, the only conflict is when \( H_i^\omega = 0, \forall i = 1 \ldots m \). In this case, we either do the optimization, or we avoid its use and its possible time expense by means of utilizing a simple analytical upper bound on \( \max_{\|k^a_i\|} \left( -v^T H_i^e v - \omega^T H_i^\omega \omega - 2 v^T H_i^\omega \omega \right) \). This upper bound is calculated as follows:
Here, we are going to give the approach for the most general case based on bounding the velocity-dependent terms (the used bounds become exact values in the case of robots with homogeneous dofs or when $H_i = 0, \forall i = 1...m$).

Thus, we can rewrite (Ap.12) in the general case as:

\[
\begin{align*}
\mathbf{a}^i & = -\tau_i \max + \|k_{r_i}^a\| \mathbf{d} \alpha^i + \|k_{f_i}^a\| \mathbf{d} f^i + \|k_{m_i}^a\| \mathbf{d} m^i \\
& = -\varepsilon_{\max, i} \left( d \mathbf{v} \right)^2 - \varepsilon_{\min, i} \left( d \mathbf{\omega} \right)^2 + 2 \sigma_{\max, i} d \mathbf{v} d \mathbf{\omega} - \tau_i \leq -\|k_{r_i}^a\| a
\end{align*}
\]

(Ap.15),

where $\varepsilon_{\max, i}$ and $\varepsilon_{\min, i}$ denote the maximum and minimum singular values of the matrices $H_i$, respectively.

A final remark is that as $-\|k_{r_i}^a\| a \leq 0$, $d \varepsilon_i^a$ must be less than or equal to zero as well. Otherwise, we cannot assure there exists $d \mathbf{a}^i_{\text{iso}}$.

From (Ap.15), we can write:

\[
\begin{cases}
\mathbf{a} \leq a \mathbf{E}_i, \text{ with } (\forall i = 1...m): \\
\text{0, if } a \varepsilon_i^a > 0 \\
a \mathbf{E}_i = -\|k_{r_i}^a\| a \varepsilon_i^a, \text{ if } a \varepsilon_i^a \leq 0 \text{ and } \|k_{r_i}^a\| \neq 0 \\
\infty, \text{ otherwise}
\end{cases}
\]

(Ap.16),

where $\varepsilon_i^a$ indicates a lower bound for $\varepsilon_i^a$.

Similar manner is adopted to deal with (Ap.11), in which we obtain:

\[
\begin{align*}
\mathbf{a}^i & \leq a \mathbf{E}_i^+ + \tau_i \max - \|k_{r_i}^a\| \mathbf{d} \alpha^i - \|k_{f_i}^a\| \mathbf{d} f^i - \|k_{m_i}^a\| \mathbf{d} m^i \\
& = -\varepsilon_{\max, i} \left( d \mathbf{v} \right)^2 - \varepsilon_{\max, i} \left( d \mathbf{\omega} \right)^2 - 2 \sigma_{\max, i} d \mathbf{v} d \mathbf{\omega} - \tau_i
\end{align*}
\]

(Ap.17),

\[
\begin{cases}
\forall i = 1...m, \\
\left( k_{r_i}^a \right)^T a \leq a \varepsilon_i^a + \tau_i \max - \|k_{r_i}^a\| \mathbf{d} \alpha^i - \|k_{f_i}^a\| \mathbf{d} f^i - \|k_{m_i}^a\| \mathbf{d} m^i \\
& - \varepsilon_{\max, i} \left( d \mathbf{v} \right)^2 - \varepsilon_{\max, i} \left( d \mathbf{\omega} \right)^2 - 2 \sigma_{\max, i} d \mathbf{v} d \mathbf{\omega} - \tau_i
\end{cases}
\]

We cannot assure that $d \mathbf{a}^i_{\text{iso}}$ exists because we are using an upper bound on $a \varepsilon_i^a$ and not its exact value.

210
Appendices

with \( \xi^v_{\text{max}} = \max \{ \text{eigs} \left( H_i^v \right), 0 \} \) and \( \xi^a_{\text{max}} = \max \{ \text{eigs} \left( H_i^a \right), 0 \} \).

As a result, we get:

\[
\begin{align*}
\max \{ \text{eigs} \left( H_i^v \right) \}, & \quad 0, \quad \text{if } e_i^+ < 0 \\
\max \{ \text{eigs} \left( H_i^v \right) \}, & \quad \text{if } e_i^+ \geq 0 \quad \text{and} \quad \kappa_i^v \neq 0 \\
\infty, & \quad \text{otherwise}
\end{align*}
\]

\[
\begin{align*}
\max \{ \text{eigs} \left( H_i^a \right) \}, & \quad 0, \quad \text{if } e_i^+ < 0 \\
\max \{ \text{eigs} \left( H_i^a \right) \}, & \quad \text{if } e_i^+ \geq 0 \quad \text{and} \quad \kappa_i^a \neq 0 \\
\infty, & \quad \text{otherwise}
\end{align*}
\]

\[
\begin{align*}
\{K_i^v\}, & \quad 0, \quad \text{if } k_i^v \neq 0 \quad \text{and} \quad \kappa_i^v \neq 0 \\
\infty, & \quad \text{otherwise}
\end{align*}
\]

\[
\begin{align*}
\{K_i^a\}, & \quad 0, \quad \text{if } k_i^a \neq 0 \quad \text{and} \quad \kappa_i^a \neq 0 \\
\infty, & \quad \text{otherwise}
\end{align*}
\]

\[
\begin{align*}
\{\kappa_i^v\}, & \quad 0, \quad \text{if } \kappa_i^v \neq 0 \\
\infty, & \quad \text{otherwise}
\end{align*}
\]

\[
\begin{align*}
\{\kappa_i^a\}, & \quad 0, \quad \text{if } \kappa_i^a \neq 0 \\
\infty, & \quad \text{otherwise}
\end{align*}
\]

\[
\begin{align*}
\xi^v_{\text{max}}, & \quad \xi^a_{\text{max}}, \quad \text{and} \quad \kappa_{\text{max}}.
\end{align*}
\]

Merging (Ap.16) and (Ap.18) together with the condition \( v_{\text{iso}} < d v_{\text{req}} \) (or \( \omega_{\text{iso}} < d \omega_{\text{req}} \)), we obtain:

\[
\begin{align*}
d a_{\text{iso}}^d & = \min_{i=1...m} \left( K_i^v, a E_i^-, a E_i^+ \right), \quad \text{with:} \\
K_i^v & = \begin{cases} 
0, & \text{if } K_i^v < d v_{\text{req}} \quad \text{and} \quad \omega_{\text{req}} \quad \text{or} \quad \omega_{\text{req}} \\
\infty, & \text{otherwise}
\end{cases} \\
a E_i^- & = \begin{cases} 
\left. - \frac{a e_i^+}{k_i^v} \right|_{k_i^v \neq 0}, & \text{if } a e_i^+ \leq 0 \quad \text{and} \quad \kappa_i^v \neq 0 \\
\infty, & \text{otherwise}
\end{cases} \\
a E_i^+ & = \begin{cases} 
\left. - \frac{a e_i^+}{k_i^v} \right|_{k_i^v \neq 0}, & \text{if } a e_i^+ \geq 0 \quad \text{and} \quad \kappa_i^v \neq 0 \\
\infty, & \text{otherwise}
\end{cases}
\end{align*}
\]

The result in (Ap.19) can be interpreted as follows: the manipulator can achieve an acceleration at least up to \( d a_{\text{iso}}^d \) in all directions while allowing simultaneously angular acceleration, linear and angular velocities, force, and moment to attain their demanded requirements in all directions. Actually, in (Ap.19) we had established a lower bound for \( d a_{\text{iso}}^d \), which is sufficient as no harm in having \( d a_{\text{iso}}^d \) higher than \( d a_{\text{iso}}^d \). Nonetheless, if one would like to know an upper bound for \( d a_{\text{iso}}^d \), a similar approach can be used, but this time considering \( e_i^- \) and \( e_i^+ \).
Similarly, we get regarding \( d \alpha_{isp} \), \( d f_{isp} \), and \( d m_{isp} \). The corresponding relations are given hereafter:

\[
\begin{split}
\frac{d \alpha_{isp}}{d \alpha_{isp}} & \geq \min_{i=1..m} \left( K_C, \frac{\alpha_{isp}}{\alpha_{isp}} \right), \\
K_C & = \begin{cases} 
0, \text{ if } k_{v_{iso}}^{isp} < d v_{req} \text{ (or } k_{\omega_{iso}}^{isp} < d \omega_{req} \text{)} \\
\infty, \text{ otherwise}
\end{cases} \\
\frac{\alpha_{isp}}{\alpha_{isp}} & = \begin{cases} 
0, \text{ if } \frac{\alpha_{isp}}{\alpha_{isp}} > 0 \\
\infty, \text{ otherwise}
\end{cases} \\
\frac{\alpha_{isp}}{\alpha_{isp}} & = \begin{cases} 
0, \text{ if } \frac{\alpha_{isp}}{\alpha_{isp}} < 0 \\
\infty, \text{ otherwise}
\end{cases}
\end{split}
\]

\( \forall i = 1..m \) (Ap.20),

\[
\begin{split}
\frac{d f_{isp}}{d f_{isp}} & \geq \min_{i=1..m} \left( K_C, \frac{f_{isp}}{f_{isp}} \right), \\
K_C & = \begin{cases} 
0, \text{ if } k_{v_{iso}}^{isp} < d v_{req} \text{ (or } k_{\omega_{iso}}^{isp} < d \omega_{req} \text{)} \\
\infty, \text{ otherwise}
\end{cases} \\
\frac{f_{isp}}{f_{isp}} & = \begin{cases} 
0, \text{ if } \frac{f_{isp}}{f_{isp}} > 0 \\
\infty, \text{ otherwise}
\end{cases} \\
\frac{f_{isp}}{f_{isp}} & = \begin{cases} 
0, \text{ if } \frac{f_{isp}}{f_{isp}} < 0 \\
\infty, \text{ otherwise}
\end{cases}
\end{split}
\]

\( \forall i = 1..m \) (Ap.21),
and

\[
\begin{align*}
\frac{d m^p_{\text{iso}}}{d m_{\text{iso}}} & \geq \frac{d m^p_{\text{iso}}}{d m_{\text{iso}}} = \min \left( K_c, m^t E^-_, m^t E^+ \right), \text{ with:} \\
K_c &= \begin{cases} 
0, & \text{if } k v_{\text{iso}}^p < k v_{\text{req}} \text{ or } k \omega_{\text{iso}}^p < k \omega_{\text{req}} \\
\infty, & \text{otherwise}
\end{cases} \\
m^t E^-_i &= \frac{m^t E^-_i}{\left\| k_{r^i}^m \right\|}, \text{ if } m^t E^-_i \leq 0 \text{ and } \left\| k_{r^i}^m \right\| \neq 0, \quad m^t E^+_i = \frac{m^t E^+_i}{\left\| k_{r^i}^m \right\|}, \text{ if } m^t E^+_i \geq 0 \text{ and } \left\| k_{r^i}^m \right\| \neq 0 \\
\infty, & \text{otherwise}
\end{align*}
\]

\[
\begin{align*}
\forall i = 1...m:
& m^t e^-_i = -\tau_i^m + \left\| k_{r^i}^a \right\| d \alpha_{\text{req}} + \left\| k_{r^i}^e \right\| d f_{\text{req}} - \zeta_{\text{min}}^- \left( d v_{\text{req}} \right)^2 - \zeta_{\text{min}}^- \left( d \omega_{\text{req}} \right)^2 \\
& + 2 \alpha_{\text{max}}^v d v_{\text{req}} - d \omega_{\text{req}} - \tau_{g_i}^e \\
& m^t e^+_i = +\tau_i^m - \left\| k_{r^i}^a \right\| d \alpha_{\text{req}} - \left\| k_{r^i}^e \right\| d f_{\text{req}} - \zeta_{\text{max}}^v \left( d v_{\text{req}} \right)^2 - \zeta_{\text{max}}^- \left( d \omega_{\text{req}} \right)^2 \\
& - 2 \alpha_{\text{max}}^v d v_{\text{req}} - d \omega_{\text{req}} - \tau_{g_i}^e
\end{align*}
\]


It remains to get \( \frac{d v_{\text{iso}}^p}{d m_{\text{iso}}} \) and \( \frac{d \omega_{\text{iso}}^p}{d m_{\text{iso}}} \). We will consider \( \frac{d v_{\text{iso}}^p}{d m_{\text{iso}}} \) only, as \( \frac{d \omega_{\text{iso}}^p}{d m_{\text{iso}}} \) can be determined in the same way. For this purpose, we re-examine (Ap.9) that can be rewritten as below, after isolating the velocity terms, \( v \) and \( \omega \):

\[
\begin{align*}
\forall i = 1...m: \\
-\tau_i^m - \left( k_{r^i}^a \right)^T a - \left( k_{r^i}^e \right)^T f_e \\
- \left( k_{r^i}^m \right)^T m_e - \tau_{g_i}^e \leq v^T H_i^r v + \omega^T H_i^{\omega} \omega + 2 v^T H_i^{\omega} \omega
\end{align*}
\]

(Ap.23),

and

\[
\begin{align*}
\forall i = 1...m: \\
v^T H_i^r v + \omega^T H_i^{\omega} \omega + 2 v^T H_i^{\omega} \omega \leq +\tau_i^m - \left( k_{r^i}^a \right)^T a - \left( k_{r^i}^e \right)^T f_e \\
- \left( k_{r^i}^m \right)^T m_e - \tau_{g_i}^e
\end{align*}
\]


Let us study (Ap.23) by considering the worst case to get an upper bound for \( v = \| v \| \) attainable in all directions, while all other dynamic requirements are satisfied. This corresponds to maximizing left-hand side of (Ap.23) and minimizing its right-hand one. Thus, we have:

\[
\begin{align*}
\forall i = 1...m: \\
-\tau_i^{\text{max}} + \left\| k_{r^i}^a \right\| d \alpha_{\text{req}} + \left\| k_{r^i}^e \right\| d f_{\text{req}} \\
+ \left\| k_{r^i}^m \right\| d m_{\text{req}} - \tau_{g_i}^e \leq \min \left\{ v^T H_i^r v + \omega^T H_i^{\omega} \omega \right\}
\end{align*}
\]

(Ap.25).
Replacing \( \min_{K^{\omega_{eq}}} \left( v^T H^r_i v + \omega^T H^o_i \omega + 2 v^T H^{req}_i \omega \right) \) by its lower bound,
\[
\zeta^{r}_{\min} v^2 + \zeta^{\omega}_{\min} \left( d \omega_{req} \right)^2 - 2 \sigma^{\omega}_{\max} d \omega_{req} v, \]
yields:
\[
\begin{align*}
\forall i = 1 \ldots m : \\
-\tau_i^{\max} + \| k'_i\| d \omega_{req} + \| k''_i\| d f_{req} + \| k'''_i\| d m_{req} - \zeta^{r}_{\min} \left( d \omega_{req} \right)^2 - \tau_{\epsilon} \leq -\zeta^{r}_{\min} v^2 - 2 \sigma^{\omega}_{\max} d \omega_{req} v \\
\end{align*}
\] (Ap.26).

Note that \( \zeta^{r}_{\min} \leq 0 \) and \(-2 \sigma^{\omega}_{\max} d \omega_{req} \leq 0 \) due to having: \( \zeta^{r}_{\min} = \min \left( \text{eigs} \left( H^r \right) \right), \sigma^{\omega}_{\max} \geq 0, \) and \( d \omega_{req} \geq 0 \). We can rewrite (Ap.26) as:
\[
\begin{align*}
\forall i = 1 \ldots m : \\
-\zeta^{r}_{\min} v^2 + 2 \sigma^{\omega}_{\max} d \omega_{req} v \leq -\overline{e}_i, \text{ with:} \\
\overline{e}_i = -\tau_i^{\max} + \| k'_i\| d \omega_{req} + \| k''_i\| d f_{req} + \| k'''_i\| d m_{req} - \zeta^{\omega}_{\min} \left( d \omega_{req} \right)^2 - \tau_{\epsilon}, \\
\end{align*}
\] (Ap.27).

Thus \( \overline{e}_i \) must be negative; otherwise, we cannot assure that \( d v^{\omega}_{iso} \) exists and thus, \( d v^{\omega}_{iso} = 0 \) by convention. Since \( f(v) = -\zeta^{r}_{\min} v^2 + 2 \sigma^{\omega}_{\max} d \omega_{req} v \) is increasing over \([0, +\infty[\), the maximum of \( f(v) \) over \([0, v_o]\) occurs at \( v_o \). Therefore, if we have \( \overline{e}_i \leq 0 \), we get an upper bound, \( v_o \), for \( d v^{\omega}_{iso} \) by solving:
\[
-\zeta^{r}_{\min} v^2 + 2 \sigma^{\omega}_{\max} d \omega_{req} v + \overline{e}_i = 0, \forall i = 1 \ldots m \] (Ap.28).

In \( \zeta^{r}_{\min} = 0 \), we have:
\[
v_o = \begin{cases} 
-\overline{e}_i & \text{if } \sigma^{\omega}_{\max} d \omega_{req} \neq 0 \\
2 \sigma^{\omega}_{\max} d \omega_{req} & \text{if } \sigma^{\omega}_{\max} d \omega_{req} = 0 
\end{cases} \] (Ap.29).

On the other hand, if \( \zeta^{r}_{\min} \neq 0 \), then (Ap.28) yields:
\[
v_o = \begin{cases} 
-\sigma^{\omega}_{\max} d \omega_{req} - \sqrt{\left( \sigma^{\omega}_{\max} d \omega_{req} \right)^2 + \zeta^{r}_{\min} \overline{e}_i} \leq 0 \\
-\zeta^{r}_{\min} \end{cases} \] \text{or:} \] (Ap.30).
Yet, as the first solution in (Ap.30) is always negative, it should be rejected and only the second solution must be considered after checking its positiveness. If \( -\sigma_{\text{max}}^{\omega_{\text{req}}} + \sqrt{(\sigma_{\text{max}}^{\omega_{\text{req}}})^2 + \zeta_{\text{min}}^v e_i^v} \left/ \left( \zeta_{\text{min}}^v \right) \right. < 0 \), we cannot assure that there exists \( d_v^{\text{req}} \) and we set \( d_v^{\text{req}} = 0 \) by convention. Based on (Ap.29) and (Ap.30), we have:

\[
\begin{align*}
&v_{o1}^+ = \frac{-\sigma_{\text{max}}^{\omega_{\text{req}}} d_{\omega_{\text{req}}} \alpha_{\text{req}}^v d_{\omega_{\text{req}}} e_i^v}{2 \sigma_{\text{max}}^{\omega_{\text{req}}} d_{\omega_{\text{req}}}}, \\
&v_{o2}^+ = \frac{-\sigma_{\text{max}}^{\omega_{\text{req}}} d_{\omega_{\text{req}}} \alpha_{\text{req}}^v d_{\omega_{\text{req}}} e_i^v}{\zeta_{\text{max}}^v}, \\
&v_{o1}^- = \frac{-\sigma_{\text{max}}^{\omega_{\text{req}}} d_{\omega_{\text{req}}} \alpha_{\text{req}}^v d_{\omega_{\text{req}}} e_i^v}{\zeta_{\text{min}}^v}, \\
&v_{o2}^- = \frac{-\sigma_{\text{max}}^{\omega_{\text{req}}} d_{\omega_{\text{req}}} \alpha_{\text{req}}^v d_{\omega_{\text{req}}} e_i^v}{\zeta_{\text{min}}^v}.
\end{align*}
\]

\( \forall i = 1...m \)

We proceed in the same manner regarding (Ap.24) to get:

\[
\begin{align*}
&v_{o1}^+ = \frac{\alpha_{\text{max}}^{\omega_{\text{req}}} d_{\omega_{\text{req}}} e_i^v}{2 \sigma_{\text{max}}^{\omega_{\text{req}}}}, \\
&v_{o2}^+ = \frac{\alpha_{\text{max}}^{\omega_{\text{req}}} d_{\omega_{\text{req}}} e_i^v}{\zeta_{\text{max}}^v}, \\
&v_{o1}^- = \frac{\alpha_{\text{max}}^{\omega_{\text{req}}} d_{\omega_{\text{req}}} e_i^v}{\zeta_{\text{max}}^v}, \\
&v_{o2}^- = \frac{\alpha_{\text{max}}^{\omega_{\text{req}}} d_{\omega_{\text{req}}} e_i^v}{\zeta_{\text{max}}^v}.
\end{align*}
\]

\( \forall i = 1...m \)

Thus, we obtain based on (Ap.31), (Ap.32), and the kinematic condition (Ap.5):
\begin{equation}
\begin{aligned}
\left\{ \begin{array}{ll}
d_{v_{iao}}^{sp} \geq d_{v_{iao}}^{sp} = \min_{i=1\ldots m} \left( k v_{v_{iao}}^{sp} v E_{-i}, v E_{+i} \right), \text{ with:} \\
\left\{ \begin{array}{ll}
v_{\text{ot1}}^-, \text{ if } \frac{v}{e_i} \leq 0, \sigma_{\text{max}}^{\omega}, d \omega_{\text{req}} \neq 0, \text{ and } \zeta_{\text{min}}^{v} = 0 \\
\infty, \text{ if } v e_i^+ \geq 0, \sigma_{\text{max}}^{\omega}, d \omega_{\text{req}} \neq 0, \text{ and } \zeta_{\text{max}}^{v} = 0 \\
0, \text{ otherwise} \\
\end{array} \right.
\end{array} \right.
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
\left\{ \begin{array}{ll}
v_{\text{ot1}}^+, \text{ if } \frac{v}{e_i} \leq 0, \sigma_{\text{max}}^{\omega}, d \omega_{\text{req}} \neq 0, \text{ and } \zeta_{\text{max}}^{v} = 0 \\
\infty, \text{ if } v e_i^+ \geq 0, \sigma_{\text{max}}^{\omega}, d \omega_{\text{req}} \neq 0, \text{ and } \zeta_{\text{max}}^{v} = 0 \\
0, \text{ otherwise} \\
\end{array} \right.
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
\left\{ \begin{array}{ll}
v_{\text{ot1}}^-, \text{ if } \frac{v}{e_i} \geq 0, \sigma_{\text{max}}^{\omega}, d \omega_{\text{req}} \neq 0, \text{ and } \zeta_{\text{min}}^{v} = 0 \\
\infty, \text{ if } v e_i^+ \geq 0, \sigma_{\text{max}}^{\omega}, d \omega_{\text{req}} \neq 0, \text{ and } \zeta_{\text{max}}^{v} = 0 \\
0, \text{ otherwise} \\
\end{array} \right.
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
\left\{ \begin{array}{ll}
v_{\text{ot1}}^+, \text{ if } \frac{v}{e_i} \geq 0, \sigma_{\text{max}}^{\omega}, d \omega_{\text{req}} \neq 0, \text{ and } \zeta_{\text{max}}^{v} = 0 \\
\infty, \text{ if } v e_i^+ \geq 0, \sigma_{\text{max}}^{\omega}, d \omega_{\text{req}} \neq 0, \text{ and } \zeta_{\text{max}}^{v} = 0 \\
0, \text{ otherwise} \\
\end{array} \right.
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
\left\{ \begin{array}{ll}
v_{\text{ot2}}^- \geq \frac{-e_i}{2 \sigma_{\text{max}}^{\omega} d \omega_{\text{req}}}, v_{\text{ot2}}^+ \geq \frac{\sigma_{\text{max}}^{\omega} d \omega_{\text{req}} - \sqrt{(\sigma_{\text{max}}^{\omega} d \omega_{\text{req}})^2 + \zeta_{\text{min}}^{v} e_i^2}}{\zeta_{\text{min}}^{v}} \\
\end{array} \right.
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
\left\{ \begin{array}{ll}
v_{\text{ot2}}^- \geq \frac{-e_i}{2 \sigma_{\text{max}}^{\omega} d \omega_{\text{req}}}, v_{\text{ot2}}^+ \geq \frac{\sigma_{\text{max}}^{\omega} d \omega_{\text{req}} + \sqrt{(\sigma_{\text{max}}^{\omega} d \omega_{\text{req}})^2 + \zeta_{\text{max}}^{v} e_i^2}}{\zeta_{\text{max}}^{v}} \\
\end{array} \right.
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
\left\{ \begin{array}{ll}
v_{\text{ot1}}^- \geq \frac{v_e^+}{2 \sigma_{\text{max}}^{\omega} d \omega_{\text{req}}}, v_{\text{ot1}}^+ \geq \frac{-\sigma_{\text{max}}^{\omega} d \omega_{\text{req}} + \sqrt{(\sigma_{\text{max}}^{\omega} d \omega_{\text{req}})^2 + \zeta_{\text{max}}^{v} e_i^2}}{\zeta_{\text{max}}^{v}} \\
\end{array} \right.
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
\left\{ \begin{array}{ll}
v_{\text{ot1}}^- \geq \frac{v_e^+}{2 \sigma_{\text{max}}^{\omega} d \omega_{\text{req}}}, v_{\text{ot1}}^+ \geq \frac{-\sigma_{\text{max}}^{\omega} d \omega_{\text{req}} + \sqrt{(\sigma_{\text{max}}^{\omega} d \omega_{\text{req}})^2 + \zeta_{\text{max}}^{v} e_i^2}}{\zeta_{\text{max}}^{v}} \\
\end{array} \right.
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
\forall i = 1 \ldots m
\end{aligned}
\end{equation}

Likewise, we get a lower bound $d_{\omega_{iao}}^{sp}$ for $d_{\omega_{iao}}^{sp}$. The result is given hereafter:
Hence, we have established lower bounds for specific isotropic values in dynamics. These bounds become the exact values for the corresponding specific isotropic values if we have homogeneous-dof robots, \( h^{\omega} = 0, \quad d_{\text{req}} = 0, \) or \( \omega_{\text{req}} = 0. \)

**B.2- Optimization Using Lagrange Multipliers**

Consider the following optimization problem:

\[
\min_{x \in \mathbb{R}^n} \left\{ -x^T A x \right\}, \text{ with:} \\
\dim(A) = n \times n, \quad x \in \mathbb{R}^n, \quad \text{and} \quad b > 0
\]

(Ap.35),

where \( A \) is a real symmetric homogeneous matrix.

Note that as \( A \) is a real symmetric matrix, all of its eigenvalues are real. The optimization in (Ap.35) can be written as the following minimization problem:

\[ \min_{x} \left\{ -x^T A x \right\}, \text{ with:} \]

\[ \dim(A) = n \times n, \quad x \in \mathbb{R}^n, \quad \text{and} \quad b > 0 \]
Several cases exist:

- \(A \geq 0\) (positive semi-definite); i.e. for every real \(x \neq 0\), we have \(x^T A x \geq 0\);
- \(A \leq 0\) (negative semi-definite); i.e. for every real \(x \neq 0\), we have \(x^T A x \leq 0\);
- \(A\) (indefinite); i.e. neither of the above is true.

If we have \(A \leq 0\), then the function \(f(x) = x^T A x\) is with upward concavity\(^7\). Hence, its minimum is at \(x_o = 0\) and \(\max_{\|x\| \leq b} (-x^T A x) = 0\).

If we have \(A \leq 0\), then \(f(x) = x^T A x\) is with downward concavity, and its minimum is at the boundary of the allowable region of \(x\). In our case, the \(x\) region is defined by \(\|x\| \leq b\) and hence, its boundary is the surface of the \(n\)-dimensional sphere described by \(\|x\| = b\).

Therefore, (Ap.36) becomes:

\[
\min_{\|x\| \leq b} (x^T A x), \text{ with } A \leq 0
\]


The problem expressed by (Ap.37) can be solved by using Lagrange multipliers as follows:

\[
\min_{x, \lambda} \left( x^T A x + \lambda \left( x^T x - b^2 \right) \right)
\]

(Ap.38), with \(\lambda\) being an arbitrary real number.

The local minimum of \(g(x) = x^T A x + \lambda \left( x^T x - b^2 \right)\) is at \((x_o, \lambda_o)\), where:

\[
\nabla g(x_o, \lambda_o) = \begin{pmatrix}
\frac{\partial g}{\partial x}(x_o, \lambda_o) \\
\frac{\partial g}{\partial \lambda}(x_o, \lambda_o)
\end{pmatrix}
= \begin{pmatrix}
2A x_o + 2 \lambda_o 1_{n \times n} x_o \\
x_o x_o - b^2
\end{pmatrix} = 0
\]


Based on \(2A x_o + 2 \lambda_o 1_{n \times n} x_o = 0\), we get:

\[
(A + \lambda_o 1_{n \times n}) x_o = 0
\]

(Ap.40),

which is nothing other than the definition of eigenvalues. So, let \(\xi_i\) (with \(i = 1...n\)) be the \(n\) eigenvalues of \(A\), which are necessarily negative (since \(A \leq 0\)). Then, \(x_o\) should be along the direction of the unit eigenvector, \(e_{\min}\), of the corresponding minimum eigenvalue, \(\xi_{min} = \min_{i=1...n}(\xi_i) \leq 0\) and hence, we get:

\[
x_o = \pm \|x_o\| e_{\min}
\]

(Ap.41).

---

\(^7\)Since the Hessian of \(f\) is \(2A\) where \(A \geq 0\).
But we have $\|x_o\| = b$, thus we obtain:

$$\max_{\|x\| \leq b} (-x^T A x) = \max_{\|x\| \leq b} (-x^T A x) = -\xi_{\min} b^2 \quad \text{(Ap.42)}.$$ 

Finally, in the case of having $A$ indefinite matrix, we consider its eigenvalues, $\xi_i$ (with $i = 1...n$). These eigenvalues are not all positive. To prove that still (Ap.42) is valid, we consider the optimization:

$$\min_{\|e\| = 1} (e^T A e) \quad \text{(Ap.43)}.$$

The minimization in (Ap.43) can be solved using Lagrange multipliers and yields (similar to (Ap.37)):

$$e = e_{\min} \quad \text{(Ap.44)},$$

where $e_{\min}$ is the unit eigenvector corresponding to $\xi_{\min} = \min_{i=1...n} (A) < 0$.

Thus, we have $\forall r > 0$:

$$\max_{\|x\| \leq r} (-x^T A x) = \max_{\|x\| \leq r} (-x^T A x) = -\xi_{\min} r^2 \quad \text{(Ap.45)}.$$

Varying $r$ between zero and $b$, we get:

$$\max_{\|x\| \leq b} (-x^T A x) = \max_{0 \leq r \leq b} (-\xi_{\min} r^2) = -\xi_{\min} b^2 \quad \text{(Ap.46)}.$$

Thus, we can generalize, for any real symmetric homogeneous square matrix, $A$, the following relations:

$$\begin{cases}
\max_{\|x\| \leq b} (x^T A x) = \vartheta_{\max} b^2 \\
\min_{\|x\| \leq b} (x^T A x) = \vartheta_{\min} b^2
\end{cases} \quad \text{(Ap.47)},$$

with:

$$\begin{align*}
\vartheta_{\min} &= \min (\xi_{\min}, 0) \\
\vartheta_{\max} &= \max (\xi_{\max}, 0) \\
\xi_{\min} &= \min_{i=1...n} (\xi_i) \\
\xi_{\max} &= \max_{i=1...n} (\xi_i) \\
\xi_i &: \text{i-th eigenvalue of } A, \ \forall i = 1...n
\end{align*} \quad \text{(Ap.48)}.$$
B.3-Case Study: DUAL V

This part presents the dynamic evaluation of DUAL V based on the notion of specific isotropic values. The different required geometric parameters are depicted in Fig. 2-3 and Fig. Ap-11. In the latter figure, the positions of the center of masses of all the parts are illustrated. Additionally, the values of the inertia-related parameters are provided in Table Ap-14\(^8\) and the actuators' capabilities, as well as the dynamic performance requirements, are given in Table Ap-15.

The DM in the required form can be established as described by (WIJK, et al., 2013). In this aspect, the only difference is that we have assumed the presence of an external force, \(f_e = (f_{ex}, f_{ey})^T\), and moment, \(m_e = (m_{ez})\), applied on the platform at the TCP. This can be easily introduced into the formulation given in the aforementioned reference.

Based on all the above and applying the concepts of specific isotropic values in the case of dynamics, lower bounds of these latter values have been established over the xy workspace, while fixing \(\theta = 0^\circ\). These results are graphed in Fig. Ap-12.

---

\(^8\) The values of these parameters are exactly those given in (WIJK, et al., 2013) for the unbalanced situation.
Table Ap-14: The inertia parameters of DUAL V

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{i1} = |A \text{CoM}_{i1}| )</td>
<td>0.0737 m</td>
<td>( p_{i2} = |B \text{CoM}_{i2}| )</td>
<td>0.1279 m</td>
</tr>
<tr>
<td>( m_{i1} )</td>
<td>1.169 kg</td>
<td>( m_{i2} )</td>
<td>0.606 kg</td>
</tr>
<tr>
<td>( I_{i1} )</td>
<td>0.012967 kg m(^2)</td>
<td>( I_{i2} )</td>
<td>0.006417 kg m(^2)</td>
</tr>
<tr>
<td>( m_p )</td>
<td>0.899 kg</td>
<td>( I_{pz} )</td>
<td>0.008168 kg m(^2)</td>
</tr>
<tr>
<td>( I_{ma} )</td>
<td>0.0041 kg m(^2)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \text{CoM}_{i1} \) is the center of mass of the arm \( AB_{i1} \), with \( m_{i1} \) and \( I_{i1} \) being its corresponding mass and moment of inertia relative to the axis passing through \( \text{CoM}_{i1} \). Similar significances are to be considered regarding the arm \( BC_{i} \), and its characteristics: \( \text{CoM}_{i2} \), \( m_{i2} \), and \( I_{i2} \). The terms \( m_p \) and \( I_{pz} \) are the mass and moment of inertia of the platform, with its center of mass being at \( P \). Finally, \( I_{ma} \) is the moment of inertia of the individual actuator with respect to its axis of rotation.

Table Ap-15: Actuators’ capacities and required dynamic performances for DUAL V

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_{\text{max}} )</td>
<td>Maximum actuator’s speed</td>
<td>100 rpm = 10.47 rad/s</td>
</tr>
<tr>
<td>( \tau_{\text{max}} )</td>
<td>Maximum actuator’s torque</td>
<td>60 N m</td>
</tr>
<tr>
<td>( a_{\text{req}} )</td>
<td>Required linear acceleration</td>
<td>30 m/s(^2)</td>
</tr>
<tr>
<td>( \alpha_{\text{req}} )</td>
<td>Required angular acceleration</td>
<td>50 rad/s(^2)</td>
</tr>
<tr>
<td>( v_{\text{req}} )</td>
<td>Required linear speed</td>
<td>0.25 m/s</td>
</tr>
<tr>
<td>( \omega_{\text{req}} )</td>
<td>Required angular speed</td>
<td>3 rpm = 0.31 rad/s</td>
</tr>
<tr>
<td>( f_{\text{req}} )</td>
<td>Required force capacity</td>
<td>200 N</td>
</tr>
<tr>
<td>( m_{\text{req}} )</td>
<td>Required moment capacity</td>
<td>15 N m</td>
</tr>
</tbody>
</table>
Appendices

Fig. Ap-12: DUAL V dynamic analysis: ratios of specific isotropic values' lower bounds relative to their corresponding requirements in the case of zero rotation.

Fig. Ap-13: DUAL V: the regions with specific isotropic values' lower bounds greater or equal to their corresponding requirements in the case of zero rotation.
Moreover, the regions of satisfactory dynamic performances based on the different dynamic indices have been investigated (see Fig. Ap-13). These are confounded as it has been expected, based on the property of equivalence in §2.2.4.B.
Appendix C: The Establishment of the Operational Wrench Zonotope of Cable-Driven Parallel Robots (CDPRS) in Statics

Consider the following equation that relates the actuators’ torques, $\tau$, to the operational wrench, $w$, by:

$$ w = \Gamma \tau $$  \hspace{1cm} (Ap.49),

with $\Gamma = -J_m^T$.

Also, assume that we have the following constraints on $\tau$:

$$ 0 \leq \tau_i^{\text{min}} \leq \tau_i \leq \tau_i^{\text{max}} \hspace{0.1cm}, \forall i = 1...m $$  \hspace{1cm} (Ap.50).

Then, to determine the operational wrench zonotope, we follow the methodology of (BOUCHARD, et al., 2008) that is summarized in what follows.

First, we define the following terms:

$$ \tau^{\text{min}} = (\tau_1^{\text{min}} \ldots \tau_m^{\text{min}})^T, \tau^{\text{max}} = (\tau_1^{\text{max}} \ldots \tau_m^{\text{max}})^T, $$

$$ \Delta \tau = \frac{1}{2}(\tau^{\text{max}} - \tau^{\text{min}}), \hspace{0.1cm} N = \text{diag} (\Delta \tau), $$

$$ \text{and} \hspace{0.1cm} \hat{\tau} = \frac{1}{2}(\tau^{\text{min}} + \tau^{\text{max}}) $$  \hspace{1cm} (Ap.51).

Applying the following change of variables:

$$ \tau = N \bar{\tau} + \hat{\tau}, \text{ with}: $$

$$ -1 \leq \bar{\tau}_i \leq +1, \forall i = 1...m $$  \hspace{1cm} (Ap.52),

$$ \bar{\tau}_i : \text{i-th component of } \bar{\tau} $$

we get:

$$ \tilde{w} = \tilde{\Gamma} \bar{\tau}, \text{ with:} $$

$$ \tilde{\Gamma} = \Gamma N, \hspace{0.1cm} \tilde{\tau} = \tilde{\Gamma} \hat{\tau} $$

$$ \text{and} \hspace{0.1cm} \tilde{w} = w - \tilde{w} $$  \hspace{1cm} (Ap.53).

Following the lead of (BOUCHARD, et al., 2008), we get the zonotope boundaries of $\tilde{w}$ by taking all the possible combinations of $d = n-1$ linearly independent column vectors, $\{\tilde{\gamma}_i, \ldots, \tilde{\gamma}_d\}$, of $\tilde{\Gamma}$, with $i_k \in \{1, \ldots, m\}, \forall k = 1...d$. Then, the unit normal to the boundary is calculated via the cross product defined in the $n$-dimensional space as follows:
In fact, there are two parallel boundaries of the same normal, \( n \), but that differ in the points they pass through. These points are determined by the complementary set of vectors defined by:

\[
\{ \mathbf{\tilde{\gamma}}_h, \ldots, \mathbf{\tilde{\gamma}}_m \} = \{ \mathbf{\gamma}_1, \ldots, \mathbf{\gamma}_n \} \ominus \{ \mathbf{\gamma}_h, \ldots, \mathbf{\gamma}_i \}
\]  
(Ap.55),

with \( \ominus \) denoting set subtraction operation\(^\text{9}\). The two aforementioned points are given by:

\[
\begin{align*}
\mathbf{p}^+ &= \sum_{k=1}^{d} \left( n^T \mathbf{\gamma}_h \right) n \\
\mathbf{p}^- &= \sum_{k=1}^{d} \left( -n^T \mathbf{\gamma}_h \right) n
\end{align*}
\]  
(Ap.56).

Thus, in the \( \mathbf{w} \) space, the pair of parallel planes is defined using a point and unit outward normal as follows:

\[
\left\{ \begin{array}{l}
\left( \mathbf{p}^+ \right) = \text{plane} \left( \mathbf{p}^+, n \right) \\
\left( \mathbf{p}^- \right) = \text{plane} \left( \mathbf{p}^-, -n \right)
\end{array} \right.
\]  
(Ap.57),

in which:

\[
\begin{align*}
\mathbf{p}^+ &= \mathbf{\tilde{p}}^+ + \mathbf{\hat{w}} \\
\mathbf{p}^- &= \mathbf{\tilde{p}}^- + \mathbf{\hat{w}}
\end{align*}
\]  
(Ap.58).

Defining \( n_p \) as the total number of combinations of \( d \) linearly independent \( d \) column vectors of \( \mathbf{\Gamma} \), we obtain \( n_p \) pairs of boundary planes defined as in(Ap.57). The intersection of the latter half-spaces, positioned on the negative side compared to the corresponding outward normal, form the sought zonotope. Note that the number \( n_p \) is upper-bounded by \( \binom{d}{m} \), which is defined as:

\[
\binom{d}{m} = \frac{m!}{d!(m-d)!}
\]  

So, the operational-wrench zonotope and its boundary hyperplanes are now known. For the sake of readability, let us say that these hyperplanes are defined each by a point, \( \mathbf{w}_{0k} \), and outward-unit normal, \( \mathbf{\eta}_k \), with \( k = 1 \ldots 2n_p \). Thus, a given wrench, \( \mathbf{w} \), is feasible if and only if:

\[
\mathbf{\eta}_k^T \mathbf{w} \leq \mathbf{\eta}_k^T \mathbf{w}_{0k}, \quad \forall k = 1 \ldots 2n_p
\]  
(Ap.60).

---

\(^9\) \( C = A \ominus B = \{ x; x \in A \text{ and } x \notin B \} \) with \( A \) and \( B \) being two sets.
The system of inequalities (Ap.60) is all what we need to compute the specific isotropic force and moment in statics.
Appendix D: Dynamic Performance Evaluation of CDPRs

D.1-Formulation of the Dynamic Model for CDPRs

Neglecting friction, we can write the DM of a CDPR under the following generalized form:

\[ M'(x) \rho + C'(x,t) t - w_{geq}(x) - w_e = J_m^T \tau \]  \hspace{1cm} (Ap.61),

with $M'$, $C'$, $w_{geq}$, and $w_e$ corresponding to the generalized inertia matrix, the generalized Coriolis and centrifugal effects matrix, the total gravitational wrench as seen from the platform, and the external non-gravitational wrench acting on the platform, respectively. As for the other symbols, they hold the same meanings as defined earlier\(^{10}\).

In addition to (Ap.61), we need to consider the relation that allows the computation of cable tensions, $f_c$. This can be easily obtained via applying the law of motion on the individual actuators to get:

\[ I_a \ddot{q} - w_{ga} = \tau - \Psi f_c \]  \hspace{1cm} (Ap.62),

with $I_a$ and $w_{ga}$ corresponding to the $m \times m$ diagonal inertia matrix of the actuators and the gravitational wrenches acting on the actuators due to their proper weight, respectively. As for $\Psi$, it is an $m \times m$ diagonal matrix. Note that: $I_a = I_a(x)$, $w_{ga} = w_{ga}(x)$, and $\Psi = \Psi(x)$.

In CDPRs with $m \geq n$, we always have the inverse kinematic relation, which is given by:

\[ \dot{q} = J_m t \]  \hspace{1cm} (Ap.63).

Differentiating (Ap.63) with respect to time, we get:

\[ \ddot{q} = J_m \rho + \dot{J}_m t \]  \hspace{1cm} (Ap.64).

Substituting (Ap.64) in (Ap.62) yields:

\[ I_a J_m \rho + I_a \dot{J}_m t - w_{ga} = \tau - \Psi f_c \]  \hspace{1cm} (Ap.65).

Merging (Ap.61) and (Ap.65) gives:

\[ \rho = (a^T \ a^T)^T = (\dot{v}^T \ \omega^T)^T \] (linear and angular operational accelerations), and $t = (v^T \ \omega^T)^T$ (operational twist: linear and angular velocities).

\(^{10}\) Meaning: $\rho = (a^T \ a^T)^T = (\dot{v}^T \ \omega^T)^T$ (linear and angular operational accelerations), and $t = (v^T \ \omega^T)^T$ (operational twist: linear and angular velocities).
Appendices

\[
K \ddot{\xi} + h - \mathbf{w}_g = \Phi \gamma,
\]
with:

\[
K = \begin{bmatrix}
    M' & -I_{\text{non}} \\
    I_a J_m & 0_{\text{non}}
\end{bmatrix},
\]
\[
\xi = \begin{bmatrix}
    \rho \\
    \mathbf{w}_e
\end{bmatrix},
\]
\[
h = \begin{bmatrix}
    C' \\
    I_a \dot{J}_m
\end{bmatrix},
\]
\[
t = \begin{bmatrix}
    t^T H_1(x) \\
    \vdots \\
    t^T H_{n+m}(x)
\end{bmatrix}
\]

\[\mathbf{w}_g = \begin{bmatrix}
    \mathbf{w}_{\text{eq}} \\
    \mathbf{w}_e
\end{bmatrix}, \quad \Phi = \begin{bmatrix}
    J_m^T & 0_{\text{non}} \\
    I_{\text{non}} & \Psi
\end{bmatrix}, \quad \text{and } \gamma = \begin{bmatrix}
    \tau \\
    f_c
\end{bmatrix}
\]

where:

\[
\begin{align*}
    h &= h(x,t) = \begin{bmatrix} t^T H_1(x) \vdots t^T H_{n+m}(x) \end{bmatrix}^T \\
    H_i &= H_i(x) = \begin{bmatrix} H_i^v(x) & H_i^{\omega v}(x) \\
    (H_i^{\omega v}(x))^T & H_i^{\omega \omega}(x) \end{bmatrix}, \quad \forall i = 1\ldots(n+m)
\end{align*}
\]

Further reforming of (Ap.67) leads to the favorable form below:

\[
K \ddot{\xi} + \mathbf{h}^r + \mathbf{h}^{\omega v} + \mathbf{h}^{\omega \omega} - \mathbf{w}_g = \Phi \gamma
\]

with:

\[
\begin{align*}
    \mathbf{h}^r &= \mathbf{h}^r(x,v) = \begin{bmatrix} v^T H_1^v \vdots v^T H_{n+m}^v \end{bmatrix}^T \\
    \mathbf{h}^{\omega v} &= \mathbf{h}^{\omega v}(x,\omega) = \begin{bmatrix} \omega^T H_1^{\omega v} \vdots \omega^T H_{n+m}^{\omega v} \end{bmatrix}^T \\
    \mathbf{h}^{\omega \omega} &= \mathbf{h}^{\omega \omega}(x,t) = \begin{bmatrix} 2 v^T H_1^{\omega \omega} \vdots 2 v^T H_{n+m}^{\omega \omega} \end{bmatrix}^T
\end{align*}
\]

D.2- Case Study: A Fully-Constrained CDPR

This study is carried out on the same CDPR depicted in Fig. 2-9. The characteristics of the actuators, the inertia parameters, and the required dynamic performances are those given in Table Ap-16. To simplify the dynamic modeling, we neglect the mass of the cables in addition to friction.

The cable tensions are related to the actuators’ torques, \( \tau \), by:

\[
\begin{align*}
    I_a \ddot{q} &= \tau - \Psi f_c, \quad \text{with:} \\
    I_a &= \text{diag}(I_{ma}, I_{ma}, I_{ma}, I_{ma}) \\
    \Psi &= \text{diag}(r_c, r_c, r_c, r_c) \\
    f_{c_i} &\geq 0, \quad \forall i = 1\ldots4 \text{ (cables should be under tension)}
\end{align*}
\]

Regarding the matrices \( M' \) and \( C' \), they are given by:
Table Ap-16: Actuators’ characteristics, inertia parameters, and required dynamic performances for the CDPR in Fig. 2-9.*

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^{\max}$</td>
<td>Maximum actuator’s speed</td>
<td>$100 \text{ rpm} = 10.47 \text{ rad/s}$</td>
</tr>
<tr>
<td>$\tau^{\max}$</td>
<td>Maximum actuator’s torque</td>
<td>$60 \text{ N m}$</td>
</tr>
<tr>
<td>$f_c^{\min}$</td>
<td>Minimum cable tension</td>
<td>$10 \text{ N}$</td>
</tr>
<tr>
<td>$f_c^{\max}$</td>
<td>Maximum cable tension</td>
<td>$500 \text{ N}$</td>
</tr>
<tr>
<td>$d a_{\text{req}}$</td>
<td>Required linear acceleration</td>
<td>$30 \text{ m/s}^2$</td>
</tr>
<tr>
<td>$d \alpha_{\text{req}}$</td>
<td>Required angular acceleration</td>
<td>$50 \text{ rad/s}^2$</td>
</tr>
<tr>
<td>$d v_{\text{req}}$</td>
<td>Required linear speed</td>
<td>$0.25 \text{ m/s}$</td>
</tr>
<tr>
<td>$d \omega_{\text{req}}$</td>
<td>Required angular speed</td>
<td>$3 \text{ rpm} = 0.31 \text{ rad/s}$</td>
</tr>
<tr>
<td>$d f_{\text{req}}$</td>
<td>Required force capacity</td>
<td>$200 \text{ N}$</td>
</tr>
<tr>
<td>$d m_{\text{req}}$</td>
<td>Required moment capacity</td>
<td>$15 \text{ N m}$</td>
</tr>
<tr>
<td>$I_{ma}$</td>
<td>Actuator’s Moment of Inertia</td>
<td>$4.92 \times 10^{-3} \text{ kg m}^2$</td>
</tr>
<tr>
<td>$m_p$</td>
<td>Platform’s mass</td>
<td>$5 \text{ kg}$</td>
</tr>
<tr>
<td>$I_{pzz}$</td>
<td>Platform’s Moment of Inertia</td>
<td>$4.3 \times 10^{-3} \text{ kg m}^2$</td>
</tr>
</tbody>
</table>

*All actuators are identical. The positive sense of the actuator torque is the same as the positive sense of rotation (which tends to shorten or wind up the cable). The cables are assumed massless.

\[
\begin{aligned}
M' &= M_p + J_m^T \, I_a \, J_m \\
M_p &= \text{diag} \left( m_p, m_p, I_{pzz} \right) \\
C' &= C_p + J_m^T \, I_a \, \dot{J}_m \\
C_p &= 0_{3 \times 3}
\end{aligned}
\]  

(Ap.71)
with \( C_p \) representing the Coriolis and centrifugal effects on the platform\(^{11}\). Regarding the external non-gravitational wrench, \( \mathbf{w}_e \), it is:

\(^{11}\) Note that the point \( P \) (the TCP) is confounded with the center of mass of the platform.
Utilizing all the aforementioned data, the dynamic evaluation, as described in §2.3.4, has been done. The ratios of the dynamic specific isotropic values’ lower bounds to their corresponding requirements are shown in Fig. Ap-14, for $\theta = 0^\circ$. Notice that the lower bounds for the specific isotropic angular acceleration and speed are quite interesting. They are large in comparison with their preset requirements. Finally, the regions of satisfactory dynamic performance are depicted in Fig. Ap-15.

\[ w_e = \begin{pmatrix} f_x^T \\ f_y^T \\ m_e^T \end{pmatrix} = \begin{pmatrix} f_{cx} \\ f_{cy} \\ m_e \end{pmatrix}^T \] (Ap.72).
Appendix E: Photos of ARROW Machine

In Fig. Ap-16 and Fig. Ap-17, two photos of the implemented ARROW V2 M2 prototype are shown. In Fig. Ap-18, a close-up side view of the turntable is depicted. Notice the spring assembly in this latter figure. This is used to counteract the gravitational effect. More precisely, it allows maintaining the static turntable orientation without the need of any actuation torque. This is beneficial regarding energy saving.
Fig. Ap-18: ARROW machine CAD drawing: close view on the turntable showing the springs used to counteract the gravity effect.
Publications


