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An experimental study of preferences for information

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Introduction

In most real-life choice situations, decision makers cannot predict the outcome of their action with certainty. However, the uncertainty inherent to the bet on the flip of a fair coin is different from the uncertainty associated with the bet on the results of a football match. The former bet consists of a risky lottery where the probabilities of the different events are objectively known, whereas, in the latter bet, the agent is confronted with uncertainty regarding the probabilities. Hence, as opposed to the situation of risk where probabilities can be theoretically inferred, ambiguity arises when the agent fails to report a unique probability distribution over outcomes. Drawing on the analysis of Frisch and Baron (1988), who describe ambiguity as being caused by the lack of information, a standard definition due to Camerer and Weber (1992) states that:

> "Ambiguity is uncertainty about probability, created by missing information that is relevant and could be known." (Camerer and Weber, 1992, p.330)

Thus, risk and ambiguity can be conceptualized as two extreme positions along the same spectrum, based on the level of certainty regarding the outcome probabilities. However, most everyday decision situations do not fit this typical classification into risk and ambiguity. Indeed, agents are often provided with partial information consisting of past observations. Reconsider the previous example of a bet on the results of a football match: before placing a bet, the decision maker may be willing to collect data on the past performances of the players of the football teams, on the results of their potential previous

meetings, etc. This example illustrates the notion of inductive reasoning that is at the heart of the knowledge accumulation process: the agent observes occurrences which lead him to generate predictions about future observations by using similarity judgments (Hume, 1748).

Although there is ample experimental evidence that uncertainty affects decision making, there are comparatively few papers addressing the issue of choice behavior when situations are informed by data. Moreover, the vast majority of the literature on decision theory considers uncertainty and data separately.

This thesis aims to provide new insights into the understanding of decision making under uncertainty when data are available. Different strategies can be implemented to gather data. Since data resulting from randomized statistical sampling provides objective information about the underlying probability distribution governing outcomes, the present analysis focuses on this type of data collection. Hence, the first goal of this research is to investigate the perception of ambiguity and the attitudes towards risk and ambiguity when agents are provided with statistical information. The second axis of research explores the individual valuation for additional information. These issues are considered from an experimental perspective.

Literature review

The distinction between measurable uncertainty, in which the probability distribution of outcomes is known, and unmeasurable uncertainty, in which the informational context does not allow such objective probability judgment, dates back to Knight (1921). In both circumstances, the agents are confronted with a choice between different acts yielding uncertain consequences. Each decision maker is characterized by a preference relation over the set of all acts. In order to predict which act will be chosen by the agent, economists have proposed various theoretic frameworks accounting for the individual's attitude towards

uncertainty.

The benchmark model of decision making under uncertainty is the Expected Utility (EU) theory, originally introduced by Bernoulli (1738) and then formalized by Von Neumann and Morgenstern (1944). This model provides the relevant and usually agreed upon framework to study preferences over risky prospects. Under risk, each available act yields an outcome with a well-specified objective probability. According to the EU representation theorem, the preferences of a rational decision maker can be described by a cardinal utility function whose curvature captures the individual's attitude towards risk. The preferred act is the one that results in the highest expected utility. This prediction requires the Independence axiom to hold. This central – but controversial – axiom states that the preferences between two risky alternatives are invariant to mixing each of the alternatives with a third one in the same proportions [1].

There has been extensive debate about the nature of probabilities as the numerical expression of beliefs (Hacking, 1975). In particular, a relevant question asks whether objective probabilities derived from logical analysis or empirical frequencies should be treated differently than subjective probabilities based on other type of (more disputable) evidence. According to Savage (1954) and De Finetti (1937), the distinction between both types of probabilities is inconsequential in normative decision theory. Savage's characterization of Subjective EU combines the von-Neumann-Morgenstern EU framework with subjective probabilities. In this model, the decision maker has a single additive subjective probability measure over events, which reflects his beliefs about relative likelihoods. The preferred act is the one that yields the highest subjective expected utility. Anscombe and Aumann (1963) propose an alternative approach of Subjective EU which involves both objective and subjective uncertainty.

The models based on subjective probabilities postulate that the type and the amount of

1. Although the EU theory has remained the standard decision criterion to study preferences over objective lotteries, its descriptive power has been challenged by Allais (1953) who outlines situations leading to systematic violations of the Independence axiom. Alternative models of preferences, accounting for the Allais paradox, have been lately proposed to describe preferences under risk (Machina, 2017).

evidence supporting the individual probability measure does not influence decision making. Ellsberg (1961) instead identifies patterns of choice that cannot be explained exclusively by the desirability of outcomes and the likelihood of events. Indeed, he suggests that an appropriate framework should also account for:

> "[...] the nature of one's information concerning the relative likelihood of events. What is at issue might be called the ambiguity of this information, a quality depending on the amount, type, reliability and "unanimity" of, information, and giving rise to one's degree of "confidence" in an estimate of relative likelihoods." (Ellsberg, 1961, p.657)

Ellsberg presents thought-experiments to illustrate his statement. One of them posits a pair of urns containing 100 balls each. Each ball can be either black or red. The proportions of black and red balls in Urn I are unknown and Urn II contains exactly 50 black and 50 red balls. A ball is randomly drawn and the decision maker is asked to express preferences between different bets. Table 1 displays the four bets offered to the agent.

	Urn I		Urn II	
Act	Black	Red	Black	Red
a	\$100	\$0		
b			\$100	\$0
a'	\$0	\$100		
b'			\$0	\$100

Table 1 – Ellsberg's two-color urns experiment

Ellsberg argues that decision makers are typically indifferent between betting on black or on red within each urn ($a \sim b$ and $a' \sim b'$) but they are likely to exhibit a strict preference for Urn I over Urn II to bet on black and to bet on red ($a \succ a'$ and $b \succ b'$). The reason is that agents usually feel uneasy with making predictions in the absence of information. Consequently, they may be reluctant to bet on unknown probabilities. The resulting twofold choice is termed ambiguity aversion – although less likely, the opposite

pattern ($a' \succ a$ and $b' \succ b$) is accordingly termed ambiguity loving. However, preferring a over a' implies $\Pr(\textit{black in Urn I}) < .5$ and preferring b over b' implies $\Pr(\textit{red in Urn I}) < .5$. Hence, $\Pr(\textit{black in Urn I}) + \Pr(\textit{red in Urn I}) < 1$. Such beliefs cannot be resumed in a unique additive probability measure. As a result, the models relying on subjective probabilities fail to explain this pattern of preferences, which is referred to as the Ellsberg paradox.

The empirical validity of the Ellsberg paradox has been supported by ample experimental evidence [2]. Thus, over the last decades, a large number of alternative models have been proposed to account for Ellsberg-type preferences [3]. Schmeidler (1989) extends the Subjective EU theory to non-additive probabilities. He proposes the Choquet EU model where beliefs are characterized by a single capacity which weighs the different events without necessarily summing up to 1. In this model, the acts are evaluated by a Choquet integral with respect to the capacity. As in the EU theory, the preferred act is the one that results in the highest weighted sum of the utilities of its outcomes. However, the weights used to evaluate each of the acts depend on the act itself, which allows to accommodate ambiguity aversion.

The multiple-prior approach assumes that the lack of information leads the decision maker to consider a set of probabilities as feasible. Inspired by the Wald's (1950) maximin criterion, the Maxmin EU model of Gilboa and Schmeidler (1989) adapts the Anscombe-Aumann framework to the case where the agent formulates multiple probabilistic priors and evaluates each available act by its minimal expected utility over the set of priors. The axiom of Uncertainty Aversion states that the decision maker has a preference for a mixture of two different acts rather than each of these acts. Hence, ambiguity-averse agents exhibit a preference for hedging. In contrast, when mixing with a certain act, such hedging motives are absent. The Independence axiom is thus restricted to only apply

2. Early empirical literature is reviewed in Camerer and Weber (1992), most recent experiments are surveyed in Trautmann and Van De Kuilen (2015). See also Table 1 in Oechssler and Roomets (2015).

3. For a comprehensive theoretical assessment of the Ellsberg paradox, see Machina and Siniscalchi (2014).

to such mixtures. As a result, the representation allows to accommodate the Ellsberg paradox.

Initiated by Segal (1987), the multiple-stage models posit that ambiguous decision situations can be represented by compound risk. For instance, in the Ellsberg's two-color example, the first lottery determines the composition of Urn I which is drawn among the set of feasible urns, and, in the subsequent lottery, the agent bets on the color of the ball. In particular, the Smooth Ambiguity model of Klibanoff *et al.* (2005) represents preferences by a functional of the double expectational form. If the agent has probabilistic beliefs on the set of urns and reduces compound lotteries (i.e., is indifferent between a compound lottery and its reduction to a simple one), the decision problem can be solved with Subjective EU. In contrast, an ambiguity-sensitive decision maker first evaluates the expected utility of each lottery by a utility function which reflects his ambiguity-attitude and then uses a second-order prior over lotteries to compute a second-order expected utility. Differently from the other models, this model separates perceived ambiguity from ambiguity attitudes and incorporates the multiple prior model as a special case.

Although these popular approaches provide tractable representations to model decision making [4], they remain silent about the informational structure of the decision situation. In particular, these models rely on subjective beliefs without explicitly incorporating the objective information which may be at the disposal of the agent. Consequently, these models do not allow to separate the objective features of the available information from the subjective individual perception of this information that both lead to the formation of beliefs. Gajdos *et al.* (2008) consider a decision problem in which the information does not allow to point-identify probabilities but enables to define a range of possible values instead. Hence, their representation of preferences builds on Gilboa and Schmeidler's (1989) multiple-prior framework and the relation between the objective information and the subjective beliefs is made explicit. In this model, the agent is able to compare acts

4. See Etner *et al.* (2012) for an extensive review of the recent models in decision theory under ambiguity.

under different informational structures. Then, they define aversion towards imprecision as the preference to act in the informational environment that yields the smaller set of probabilities.

Relying on Hume's (1748) principle of inductive reasoning, Gilboa and Schmeidler (1995, 2001) propose a formal model of decision making under uncertainty, called the Case-Based Decision Theory, in which the agents decide by drawing analogies with past cases in memory. Hence, the available acts are evaluated according to their previous performance in similar decision situations. This approach appears to be particularly salient and realistic to model decision making when the agent has only imperfect knowledge about the decision situation [5]. Combining the case-based approach and an adaptation of the Maxmin EU model, Eichberger and Guerdjikova (2013) present a decision model that allows to distinguish between the perception of ambiguity and the individual attitude towards it. Their representation can be used to study preferences for additional information. In particular, they show that ambiguity averse agents prefer to decide with more precise sets of information.

The theoretical literature on updating describes how decision makers form beliefs given statistical data. When the agent is able to establish a probability distribution over the states of nature, Savage's (1954) representation of preferences implies that the agent will update beliefs according to Bayes rule. Another crucial implication of Savage's theory is dynamic consistency which requires that the ex ante contingent plan coincides with the actual choice made after obtaining the information. However, such ideal statistical information allowing subjects to formulate correct probabilistic beliefs is not always available. Most frequently, the informational context describes only partially the decision situation at hand. Blackwell (1951, 1953) provides a criterion to rank different informative structures. In his framework, the agent conducts experiments (i.e., sampling procedures) to

5. Camerer (1998) emphasizes: "When you choose a movie, house, or colleague, do you think about possible consequence states and weigh their likelihood? Or do you instinctively compare each movie, restaurant, or colleague to others you have seen and liked or disliked? You almost surely do *some* of the latter." (Camerer, 1998, p.176).

address decision problems in which the loss that is obtained depends on the true state of the world. Experiment I is said to be more informative than experiment II if the former is more valuable than the latter for all Subjective EU maximizers. This implies that experiment I yields lower expected loss than experiment II for any subjective prior.

As reviewed above, Ellsberg (1961) points out paradoxical choices which provide evidence against the descriptive validity of the Subjective EU theory. The variety of models of decision making based on non-probabilistic beliefs leads to multiple interpretations with regard to the way beliefs and ambiguity-sensitive preferences are updated to incorporate new information. Indeed, while non-EU theories explain observed behaviors, giving up the Independence axiom comes at a cost. First, the non-EU models do not provide a unique way of generalizing the Bayes updating rule to non-additive beliefs. Gilboa and Schmeidler (1993) consider a Maxmin EU maximizer and axiomatize a class of updating rules which generalize the Bayesian rule to the case of multiple priors. Eichberger *et al.* (2007) characterize Bayesian updating in the context of the Choquet EU model. A second problem related to the weakening of the Independence axiom is the violation of dynamic consistency (e.g., Siniscalchi, 2011). The lack of dynamic consistency poses further challenges for the definition of appropriate updating rules under ambiguity. In particular, Epstein and Schneider (2003) axiomatize the recursive multiple-priors model, an intertemporal version of the Maxmin EU model that preserve dynamic consistency. Dynamic inconsistency has also led researchers to consider alternative approaches to updating. Hanany and Klibanoff (2007, 2009) allows the update rule to depend on the event that is observed. They propose a weak version of dynamic consistency according to which the ex ante optimal act remains conditionally optimal. Siniscalchi (2011) draws on the consistent-planning approach (Strotz, 1955) and characterizes the decision maker as sophisticated if he has correct beliefs regarding his own future decisions. His analysis provides a theoretic framework to study decision making with dynamically inconsistent preferences.

Another important issue related to updating concerns statistical learning. A common view is that ambiguity should resolve if decision makers are repeatedly facing the same data generation process. In the Ellsberg's urn, this argument implies that increasing the number of random draws with replacement lessens ambiguity. Consequently, it is in principle possible for the decision maker to learn the exact composition of the urn by observing an infinite number of draws. The standard models of Bayesian inference predict that the subjective estimates (absolutely continuous with respect to the truth) of the EU maximizers will indeed converge to the event's true probability distribution as the number of i.i.d. observations gets large. Marinacci (1999, 2002) derives a Law of Large Numbers which allows statistical learning under ambiguity when beliefs functions are modelled through capacities. Epstein and Schneider (2007) study the long-term behavior of beliefs within the Maxmin EU model and provides conditions under which multiple posteriors concentrate at the true objective probability distribution. Until now, however, the theoretical literature fails to provide conditions applying to a large class of non-EU preferences under which ambiguity would systematically vanish in the long run.

Finally, the literature on the value of information under ambiguity is sparse. A notable exception is the analysis of Snow (2010) who shows that the value of information reducing ambiguity is positive (negative) for ambiguity averse (loving) decision makers. In the absence of dynamic consistency, models of ambiguity does not allow for a direct application of Blackwell's (1951) results. A dynamically inconsistent agent may be willing to pay to avoid information (Wakker, 1988). Assuming that the decision maker can commit to a strategy contingent on the signal observed, Li and Zhou (2016) show that most classes of ambiguity-sensitive preferences admit a partial ranking of informative structures equivalent to Blackwell's ranking. Consequently, the increase in informativeness is valuable for both ambiguity averse and ambiguity loving decision makers under suitable technical assumptions.

In a nutshell, ambiguity raises theoretical issues when studying updating and learn-

ing. Therefore, the experiments presented in this thesis are designed to address dynamic decision making with particular care [6]. Moreover, updating and learning depend on the model used to describe choice behaviors and beliefs. Since there is no canonical model of decision making under ambiguity and it is not straightforward to deduce which model is implemented by experimental subjects, this thesis adopts a model-free approach.

Contribution and outline of the thesis

In this thesis, I explore choice behaviors under "partial ambiguity", defined by situations where sets of observations are available to the agents. More precisely, this research experimentally investigates individual attitudes towards uncertainty when prospects can be informed by statistical data. By providing decision makers with datasets, this study is in line with the case-based approach and is intended to simulate a tractable and realistic process of accumulation of information.

This thesis consists of three chapters: In the first and the second chapters, I examine individual preferences for uncertain prospects when ambiguous options are described by datasets containing more or less observations. In other words, Chapters 1 and 2 describe decision making in the presence of information, i.e., when prospects are informed *ex ante*. On the other hand, the third chapter addresses choice behaviors when agents can acquire statistical information. In particular, this chapter aims at measuring the value of additional information in uncertain environments.

The experiments presented in this thesis draw on the Ellsberg's (1961) urns which provide a straightforward and easily implementable design. Moreover, this enables me to directly relate to the extensive empirical literature based on Ellsberg's experiments. In this thesis, the original two-color setting is modified by allowing the subject to observe random draws with replacement so that he can learn about the composition of the urn. Datasets

6. The implications of dynamic inconsistency on empirical analysis are discussed in Chapter 3 especially.

differ with respect to two objective dimensions: their precision and their frequencies. The empty dataset (i.e., the one containing 0 observation) corresponds to the extreme case of full ambiguity, as originally considered by Ellsberg. Increasing the number of observations (i.e., the length of the dataset) while keeping the frequencies unchanged improves the precision of the information, which in turn lessens ambiguity in an objective sense.

In Chapter 1, I report the results of an experimental study which examines whether individual choices can be reconciled with the Subjective EU theory when both ambiguous prospects are informed by more or less precise datasets. This experiment departs from the classic Ellsberg two-color setting by confronting subjects with pairs of urns whose exact compositions are all unknown. However, in each question, the two urns to be compared are described by datasets which can be either relatively precise (100 draws) or imprecise (10 draws) with almost identical frequencies. Consequently, choosing the most precise dataset for both color-bets is interpreted as ambiguity aversion in this framework (Eichberger and Guerdjikova, 2013). The experimental design allows to distinguish between neutral and non-neutral ambiguity attitudes where the former consists of frequentist and Bayesian choices and the latter includes ambiguity averse and ambiguity loving preferences.

In this experiment, most answers (58%) are consistent with the EU hypothesis but a significant proportion of choices (29%) can still be interpreted as an expression of non-neutral ambiguity attitude – the remaining answers (13%) violate a standard property of monotonicity. The results suggest that the frequency of the winning color in the datasets does not significantly impact the distribution of answers across these three categories. From a within-subject perspective, I calculate an interpersonal score of ambiguity-sensitivity that allows to estimate the extent to which individual preferences deviate from ambiguity neutrality. This leads me to conclude that the non-negligible share of non-neutral answers is not due to a small number of extremely ambiguity-sensitive subjects but is rather due to a general tendency among subjects to make ambiguity averse choices in several decision situations.

In Chapter 2, I present an experiment designed to measure the effect of information precision on the perception of ambiguity. The Ellsberg's original two-urns experiment is adapted so that the subjects are provided with sets of observations informing on the composition of the ambiguous urn. Depending on the experimental treatment, the number of observations spans from 0 (no-information treatment) to 500 (highly-informative treatment). Individual preferences are elicited in comparative (via binary choices between urns) and non-comparative (via Certainty Equivalents – CEs – measurements) contexts. Hence, I am able to address the occurrence of preference reversals that have been widely documented under risk (Lichtenstein and Slovic, 1971; Grether and Plott, 1979) but relatively less discussed under ambiguity. The central feature of the design consists in the symmetry of draws reported in the datasets, i.e., in each dataset, the frequencies equal 1/2-1/2. Without information, the agent is likely to adopt a symmetric prior regarding the composition of the ambiguous urn. Moreover, according to most of the standard updating rules, the symmetry of draws implies the symmetry of the revised beliefs. Consequently, the comparison between the subject's valuations of the risky urn and of the ambiguous urn depends only on the individual's perception of ambiguity and his attitude towards ambiguity.

The experimental results suggest that the availability of information does not eliminate Ellsberg-type preferences since most subjects (61%) prefer the risky urn to the ambiguous urn for both color-bets. However, the differences between the CEs for the risky and ambiguous prospects are not statistically significant on average. When comparing direct choices and valuations, a non-negligible share of participants (24%) exhibit preference reversals with CE measurements, contrary to what was found by Trautmann *et al.* (2011) in a fully ambiguous framework. Between treatments, I do not find evidence that the increase in information precision is associated with a higher valuation of the ambiguous prospect. These findings lead me to conclude that ambiguity-averse preferences are only weak since ambiguity aversion deduced from direct choices does not translate into signifi-

cant differences in valuation. I relate this result to the fact that decision makers might also experience difficulties in pricing bets whereas it may be easier to deal with comparative tasks.

In Chapter 3, as opposed to the previous chapters, uncertain prospects are not informed *ex ante* but the decision maker has the opportunity to obtain statistical information. In this chapter, I propose an experimental design to measure the value of information under compound risk and ambiguity. The second objective of this research is to provide operating instructions to study subjects' ranking of informative structures in the light of Blackwell's (1951) equivalence result and its extension (Li and Zhou, 2016). Moreover, the proposed design allows to investigate the link between valuation of information and ambiguity attitudes. Hence, I formulate three experimental hypotheses that can be examined through the proposed setting: the value of information is strictly positive and strictly increasing in the informativeness of the signal under both compound risk (hypothesis 1) and ambiguity (hypothesis 2) and the value of information increases with ambiguity-aversion (hypothesis 3).

Drawing on the Ellsberg's two-color experiment, the subject faces three different urns corresponding to three types of choice situations: simple risk, compound risk and ambiguity. Without information, the participant is asked to state his reservation price for a color-bet in each of these urns. By enabling the agent to obtain random draws with replacement, it is then possible to investigate how agents value informative signals under compound risk and ambiguity. I show that increasing the number of draws in the dataset improves its informativeness in Blackwell's sense. Hence, the subject's reservation price for different lengths of datasets is elicited which allows an experimental assessment of Blackwell's equivalence result. Special caution must be taken when studying decision making under ambiguity in dynamic frameworks since ambiguity-sensitive preferences are prone to dynamic inconsistency. Hence, delaying the choices after the observation of the draws might lead to information-aversion in the absence of commitment. Conse-

quently, one key feature of the proposed design consists in asking subjects to announce their signal-contingent strategy before the realization of the draws. The implementation of the experiment is left for future research.

This thesis is divided into three self-contained chapters. Thus, certain concepts may be defined several times. The instructions and other experimental materials are collected in an appendix at the end of each chapter.

Methodological issues

Every economic experiment on decision making under ambiguity has to address three methodological concerns: (*i*) eliciting preferences, (*ii*) generating ambiguity in the lab and (*iii*) incentivizing decisions.

The standard methods to elicit individual preferences consist of direct choices between alternatives or contingent valuations in which alternatives are priced in isolation. Each strategy has its own advantages and drawbacks. The direct choices method is straightforward and easy to implement but it usually requires confronting the subject with a large number of alternatives which can reduce the tractability of the device. On the other hand, valuation tasks are time-saving but cognitively demanding. Although *procedure invariance* predicts that both strategies yield the same results, there is overwhelming empirical evidence of systematic divergence in risky frameworks (Seidl, 2002). Hence, in this thesis, both procedures are examined. In particular, I elicit preferences through direct choices in Chapter 1, and through CEs [7] in Chapter 3. In Chapter 2, both elicitation methods are combined to assess the robustness of preferences: direct choices are primarily used to evaluate the incidence of Ellsberg-type preferences, and CE measurements provide an estimation of the strength of the elicited preferences.

Simulating ambiguity in the lab is, by definition, challenging. In most incentivized

7. The experimental method used to elicit CEs is discussed in detail in Chapter 2.

studies, the experimental instructions remain silent about the method to generate ambiguity and the description of the ambiguous urn simply replicates Ellsberg's (1961) characterization: "Urn I contains 100 red and black balls, but in a ratio entirely unknown [...]; there may be from 0 to 100 red balls." (Ellsberg, 1961, p.650). This method is theoretically sound but, in practice, some have argued that the asymmetry of information can yield subjects to disproportionately avoid the ambiguous prospect as a result of comparative ignorance effects (Fox and Tversky, 1995) and suspicion [8] (Dominiak and Duersch, 2015). Hence, ideally and whenever possible, nobody (i.e., neither the participant nor the experimenter) should know how ambiguity is generated. Although this condition is easily attainable for hypothetical choices, real experiments require the definition of a transparent and reliable device which is not straightforward.

So far, there is no consensus regarding the best strategies for generating ambiguity in the lab. A common approach approximates ambiguity by two-stage probabilities. However, in theory, this method appears to be inappropriate since it allows probability calculus for decision makers reducing compound lotteries. Moreover, empirical studies report significant differences between compound risk attitudes and ambiguity attitudes (Chow and Sarin, 2002; Abdellaoui *et al.*, 2015). Another way to generate ambiguity consists in involving participants in the urn generation process. E.g., in Trautmann and Zeckhauser (2013) and in Abdellaoui *et al.* (2015), each subject fills his own ambiguous urn by drawing balls from another urn. The composition of the individual's urn is kept secret by covering the balls or by having subjects wear a blindfold so that participants cannot observe the color of the balls. Alternative methods relying on sophisticated machines have been proposed: the Bingo Blower (Hey *et al.*, 2010), i.e., a transparent box containing balls in continuous motion in such a way that the balls of the different colors cannot be counted, and the ir-

8. Hey *et al.* (2010) stress that "[...] the possibility of distrust in the experimenter can change the entire problem for the subjects. [...] If they assume that the experimenter wants to spend as little money as possible on the experiment, they will naturally try and imagine ways that the experimenter could have rigged Urn I to save money. Urn I becomes the 'suspicious urn' – and not the ambiguous/uncertain urn – whether or not there are grounds for suspicion." (Hey *et al.*, 2010, p.84).

regular Galton Box (Oechssler and Roomets, 2015), i.e., a mechanism that arranges balls in various bins according to an unknown distribution, are two of them [9].

In this thesis, the design of the experiments in Chapter 1 and Chapter 2 requires controlling for the frequencies reported in the datasets. Due to time and practical constraints, I did not generate datasets in the presence of subjects and the urns are hence simulated before the experimental sessions. The instructions of both experiments do not contain an explicit description of the implemented mechanism as in the original Ellberg's experiments and the information provided to subjects is formulated as follows: "X balls have been randomly drawn from the urn" which is actually true. In Chapter 3, the composition of the urns (except the one whose exact composition is known in the case of simple risk) results from a two-stage procedure that involve the participants: one of them blindly draws a numbered chip at random and tosses a coin. The result of the tossing of the coin determines if the number of the drawn chip corresponds to the quantity of black balls or the quantity of red balls in the urn. The composition of the urn remains secret to the participants and the experimenter until the end of the session.

Another important issue in experimental economics deals with the proper way to incentivize decisions in the lab. Most of experiments involve several choices and paying every one of them might yield income effects. Popularized by Savage (1954), the Random-Incentive-System (RIS) addresses this problem by randomly selecting, at the end of the experiment, only one of the decisions to be implemented for real. The RIS presents other advantages: it is easy to understand, easy to implement, and, given budget constraints, it allows to collect a large number of observations while keeping the face values of the stakes unchanged. For these reasons, the RIS is the most commonly used incentive mechanism to study individual choices (Grether and Plott, 1979; Holt and Laury, 2002).

Although, in theory, the RIS yields unbiased measurement of EU preferences, the

9. These machines can be seen in action at `https://www.youtube.com/watch?v=z16_H5kM26E` (the Bingo Blower) and `https://www.youtube.com/watch?v=Kq7e6cj2nDw` (the regular Galton Board that arranges balls in bins according to a normal distribution).

procedure has been criticized for its inability to induce truthfull-telling for agents with non-EU preferences. Indeed, subjects may perceive the whole experiment (i.e., the decision situations together with the random device) as a single lottery. If so, the RIS might fail to elicit the true preferences of non-EU maximizers due to violations of the Independence axiom (Holt, 1986). Hence, uncertainty, and ambiguity in particular, raises challenging theoretical issues with regard to the implementation and the efficiency of the RIS. Empirical assessments, however, suggest that decision makers treat each experimental decisions in isolation when confronted with simple binary choices, providing support for the use of the RIS in simple frameworks [10] (Starmer and Sugden, 1991; Cubitt *et al.*, 1998). Consequently, in this thesis, the decisions in the experiment related to Chapter 1 are incentivized through the RIS procedure.

Most recently, Johnson *et al.* (2015) propose an original incentive mechanism, called Prince, which allows to overcome the inherent drawbacks of the RIS by minimizing violations of isolation. A key feature of this innovative device consists in selecting at random the question that serves for payment *at the beginning* of the experiment. The real choice question is not observed by the participant but provided to him in a concrete form (e.g., in a sealed envelope). This procedure encourages subjects to consider each choice situation in isolation and hence to truthfully answer the experimental questions. Therefore, the experiments in Chapter 2 and Chapter 3 use the Prince mechanism to incentivize decisions [11].

10. See Bardsley *et al.* (2010, §6.5) for a detailed discussion on incentive mechanisms in experiments.

11. The Prince mechanism had not been presented yet at the moment I implemented the experiment of Chapter 1, which explains why Prince has been only used in the experiment of Chapter 2.

Chapter 1

Precise versus imprecise datasets: revisiting ambiguity attitudes in the Ellsberg paradox

1.1 Introduction

1.1.1 Motivation

In line with Knight's (1921) distinction between risk and uncertainty, the famous Ellsberg paradox (1961) was a first experimental attempt to illustrate the failure of the Expected Utility hypothesis to predict individual behavior in an uncertain environment. Consider two urns: one urn is filled with 50 red and 50 black balls while the other urn is filled with 100 black and red balls in unknown proportions. Whether the bet is on black or on red, most people prefer betting on the urn with known composition. It is then impossible to infer additive probabilities from these choices. This behavior, known as the Ellsberg paradox, provides evidence of ambiguity aversion since decision makers (henceforth, DMs) are reluctant to bet on events with unknown probabilities.

While the classic Ellsberg's thought-experiment has been replicated several times (Camerer and Weber, 1992 provide an overview), the present research deals with the question of decision making in ambiguous situations when information about the urns is provided *in the form of datasets*. In the classic version of the experiment, exact probabilistic information is provided for one of the urns (the risky urn) whereas no information is given for the other urn (the ambiguous urn). However, this type of information might not be readily available in real-life situations. Indeed, DMs usually observe data generated by the process at hand and have to make a decision based on more or less precise datasets. For instance, in traditional societies, farmers are urged to adapt to climate change by adopting new technologies, for which data are often scarce. Eichberger and Guerdjikova (2012) show that the availability of information on returns of the new technology is crucial for motivating them to modify their practices. In the absence of precise and relevant information and if the share of ambiguity-averse agents is too high, most agents prefer the known prospect (the traditional method) to the unknown alternative (the new technology), hence innovation in the society is slow, resulting in an inefficient equilibrium. Financial trading provides also a relevant example to illustrate the consequences of lack and imprecision of information on portfolio choices. Indeed, market participants have only partial information regarding expected returns of traded assets. As a result, they might engage in imitative behavior if they believe that other DMs have some important additional information. In particular, Ford *et al.* (2013) prove that herding can be rational for agents with Choquet Expected Utility preferences. However, such behavior contributes to the formation of bubbles, and to the subsequent crisis resulting from the burst of bubbles.

In an attempt to describe decision making in ambiguous environments, I draw on the Ellsberg's experiment with two urns and two colors of balls and I describe both urns by sets of data (Gilboa and Schmeidler, 2001). I investigate the Ellsberg paradox in this set-up of "partial ambiguity". In particular, I ask whether there is still a sizable proportion of ambiguity-averse DMs in this context of partial ambiguity. Consider for instance the

following set-up : there are two urns containing 200 balls each. Each of the urns contains an unknown proportion of blue and red balls. Both urns are described by datasets of equal frequencies but different number of observations (i.e., different information precision). In urn 1, 5 blue balls and 5 red balls have been randomly drawn with replacement. In urn 2, 50 blue balls and 50 red balls have been randomly drawn with replacement. Which urn is preferred when betting on blue? On red? A frequentist (or a Bayesian with a prior 50/50 on the composition of the urn) would be indifferent between both urns. In contrast, an ambiguity-averse subject might prefer to bet on the urn with more draws regardless of the color of the ball. Similarly, subjects who choose the least precise urn for both bets exhibit ambiguity-loving preferences. Such preferences are axiomatized in Eichberger and Guerdjikova (2013). The preferences for information precision are investigated for different frequencies of blue and red balls in the datasets describing the urns. There, I ask whether ambiguity attitudes change with the proportion of balls of the winning color in the dataset, which ranges from 0.1 to 0.9 in the experiment.

I conducted a lab experiment to give insights into the individual preferences for information precision. The goal is two-fold: First, I describe individual decision making in the context of partial ambiguity. In particular, I examine whether individual choices can be reconciled with Savage's Subjective Expected Utility (SEU) model or whether non-neutral ambiguity attitudes are needed to explain preferences for data-sources with various degrees of precision. Indeed, the experiment is designed in such a way as to allow a classification of subject' answers into 4 classes of behavior: frequentist, Bayesian, ambiguity-averse and ambiguity-lover (Section 1.2.2 describes these patterns of preferences in detail). Second, I estimate the extent to which agents' preferences deviate from the SEU predictions by calculating an interpersonal score of ambiguity-sensitivity.

This experiment yields two key findings:

1. Among experimental answers satisfying a standard property of monotonicity, around 2/3 can be explained by the SEU model. The remaining 1/3 contradict the SEU and

can be interpreted as an expression of non-neutral ambiguity attitude. Among them, 2/3 displays ambiguity-aversion and 1/3 of answers can be classified as ambiguity-loving.

2. The average score of ambiguity-sensitivity in the experiment is slightly biased towards ambiguity-aversion.

To summarize, when both prospects are partially described, most of experimental choices are compatible with SEU maximization. Nevertheless, non-neutral ambiguity attitudes are still required to explain preferences in the described context of partial ambiguity since I observe a small but non-negligible share of ambiguity-sensitive agents.

The remainder of the chapter is organized as follows. The relevant research is reviewed next. Section 1.2 presents the experimental design and details the different ambiguity attitudes in the current set-up. The results of the experiment are discussed in section 1.3. Section 1.4 concludes. Appendix 1.A provides supplementary material for the experiment. Appendix 1.B contains complementary tables and figures.

1.1.2 Related literature

The present research builds on the seminal work of Ellsberg (1961), which illustrates the difference between risk and ambiguity. Here, I depart from the classic experiment and describe both urns by sets of observations. The aim of this study is to characterize decision making in ambiguous situations when the information is provided in the form of datasets. In particular, individual decisions in such realistic frameworks have been modelled by Gilboa and Schmeidler (2001) in the Case-Based Decision Theory (CBDT), designed to explain the effect of the available observations (cases) on agents' evaluation of actions. Drawing on Hume's principles (1748), the authors state that:

> "[...] the main reasoning technique that people use is drawing analogies between past cases and the one at hand.
>
> Applying this idea to decision making, we suggest that people choose acts

based on their performance in similar problems in the past." (Gilboa and Schmeidler, 2001, pp.32-33)

Indeed, the key idea that underlies the CBDT is the following: information arrives in the form of data, which might be more or less precise and more or less relevant for the decision to be made. As a result, the CBDT provides an original paradigm to model decision making when the probabilities of outcomes are not salient and cannot be easily constructed. Furthermore, Eichberger and Guerdjikova (2013) axiomatize an original version of the alpha-maxmin decision model (Ghirardato *et al.*, 2004) combining the case-based approach with the theoretical literature on decision under ambiguity. In their framework, DMs are characterized by a representation of preferences described by a von-Neumann-Morgenstern subjective utility function over outcomes and non-additive beliefs which associate a set of priors with each data set. More specifically, my experimental design is inspired by their example of betting on a draw from an urn (Eichberger and Guerdjikova, 2013, Example 1, pp.1437-1438), which provides a tractable testing setup of the Ellsberg's two-colors experiment when information about both urns consists of a set of observations. In particular, when datasets exhibit identical frequencies but differ in precision, they predict that ambiguity-averse (ambiguity-loving) DMs prefer the more (less) precise dataset, regardless of the bet. This chapter is meant to provide experimental assessment of preferences under this framework.

Although numerous experiments have evidenced the existence of Ellsberg paradox (Camerer and Weber, 1992), the authors have principally focused on risk versus full ambiguity, and little is known about individual behavior with intermediate stages of knowledge. Therefore, this research proposes an original version of the Ellsberg's two-colors experiment which builds upon CBDT by providing DMs with samples of observations. Recent research has shown that CBDT provides a relevant framework to model decision making in experimental settings under ambiguity (Grosskopf *et al.*, 2015; Bleichrodt *et al.*, 2017).

Besides, several lab studies have suggested that DMs are ambiguity averse when faced

with imprecise information. Arad and Gayer (2012) estimate the degree of confidence in observations that are imprecise. In their experiments, there are as many observations in the precise set as observations in the imprecise set, but some cases in the imprecise dataset are irrelevant for the decision problem. They deduce from experimental results that imprecision of information is a source of ambiguity aversion whereas subjects act as if there was no ambiguity with precise information. The present study departs from their analysis in two main respects: First, in my experiment, all observations in both the precise and imprecise sets are relevant for the choice to be made. The precision of information is determined by the length of the dataset and hence, the precise dataset contains more observations than the imprecise one. Second, they consider only short sequences of observations (8 cases), which is comparable to the imprecise sets in my experiment, whereas the precise sets here contain significantly more observations (100 cases).

In Baillon *et al.* (2017), the authors study the effect of learning new information on decisions to trade options with payoffs dependant on stock prices. They use Initial Public Offerings (for which no prior information on returns is available) that provide an adequate natural framework to study the effect of information on beliefs and ambiguity attitudes. Although they report only little ambiguity-aversion, they find that it does not decrease with information received, yielding them to conclude that ambiguity-aversion is a stable characteristic of DM's preferences. As opposed to their study, I present a lab experiment which allows to control for the frequencies of good/bad outcomes in the datasets. Consequently, it is possible to investigate the influence of reported frequencies on preferences.

Lastly, Chew *et al.* (2017) describe attitudes towards variants of partial ambiguity: two-point ambiguity (two possible compositions of red and black cards in a deck) and disjoint ambiguity (union of disjoint intervals). They observe aversion to increasing size of ambiguity in terms of the number of possible compositions of decks. Although their study provides relevant intuitions, their definition of partial ambiguity differs significantly from

the one considered in the present chapter. Indeed, they provide partial description about the underlying probabilities of the prospects whereas participants in my experiment have to learn them from observations. Description and statistical inference might induce different behaviors, as evidenced in the literature on the description-experience gap (Barron and Erev, 2003; Hau *et al.*, 2010; Dutt *et al.*, 2014).

In these three experimental papers, the ambiguity attitudes are investigated via Certainty Equivalent measurements. By contrast, participants are confronted with binary choices between prospects in my experiment. Although procedure invariance predicts that normatively equivalent procedures should give the same ranking between options, preference reversals have been widely reported by experimental psychologists and economists (Lichtenstein and Slovic, 1971, 1973; Grether and Plott, 1979). Hence, this chapter aims to complement the study of ambiguity attitudes and precision of information with an alternative method for eliciting preferences.

1.2 Experimental design

1.2.1 Stimuli

In the spirit of Ellsberg's two-colors urns, I design a short experiment of binary questions on preferences over pairs of bags containing balls. Each bag in the experiment contains 200 balls which can be either blue or red. Each bag is described by a set of observations. The datasets inform participants on previous random draws with replacement from bags and can be either relatively *Precise* (100 draws) or *Imprecise* (10 draws). Keeping the precision (i.e., the length) of the datasets constant makes it simpler for participants to mentally represent frequencies along the experiment. For the sake of brevity, in the following, the bags described by a precise (imprecise) dataset are denominated the precise (imprecise) bags [12]. Given the datasets, the participant is asked to choose his preferred

12. Note that, during the experimental sessions, the experimenter referred to bags with neutral capital letters (bag A, bag B, bag C...).

bag to bet on blue and his preferred bag to bet on red.

The proportions of blue balls in datasets (p_B) range approximately from 0.1 to 0.9. In the experiment, p_B can take 5 values: $0.1, 0.3, 0.5, 0.7, 0.9$. These questions deal with simple probabilities that participants can easily mentally represent. For each proportion p_B, there are two pairs of bets. In each pair of bets, datasets describing bags exhibit similar but different frequencies. The imprecise datasets display identical frequencies in both pairs: the number of blue draws is simply given by $p_B * 10$ and the number of red draws is equal to $(1 - p_B) * 10$. On the other hand, the number of blue balls in the precise datasets is given by $p_B * 100 + 1$ in one pair of questions and by $p_B * 100 - 1$ in the second pair of bets [13].

The detailed questionnaire is presented in Table 1.1. The experiment consists of 10 pairs of questions presented in random order so as to avoid potential order effects. The bags differ in each pair of questions. Therefore, there are 20 bags in this experiment: 10 precise bags and 10 imprecise bags. In questions where the proportion p_B is indexed by a (+), the frequency of blue balls is higher in the precise dataset than in the imprecise one and in questions indexed by a (-) the precise dataset displays a smaller frequency of blue balls. For instance, consider the pairs of bets 0.1^+ and 0.1^-. In q1, the participant is asked to choose between two bags to bet on a blue draw. The precise bag is described by a dataset containing 100 draws, among which 11 are blue and 89 are red; and the imprecise bag is described by a dataset containing 10 draws, among which 1 is blue and 9 are red. In q2, the participant faces the same pair of bags but the winning ball is now red. In q3 and q4, the respondent considers a different pair of bags. The precise dataset contains 9 blue draws and 91 red draws; and the imprecise dataset displays 1 blue draw and 9 red draws. Choosing the precise (imprecise) bag in the 4 bets provides convincing evidence of strict preference for information precision (imprecision). Therefore,

13. The frequency of blue balls in the precise datasets is different but set as close as possible to the frequency of blue balls in the imprecise datasets. The smallest variation is given by 1 observation since the number of balls can only be described by an integer.

p_B	Question	Number of draws				Bet
		Precise bag		Imprecise bag		
		Blue	Red	Blue	Red	
0.1^+	q1	11	89	1	9	Blue
	q2	11	89	1	9	Red
0.1^-	q3	9	91	1	9	Blue
	q4	9	91	1	9	Red
0.3^+	q5	31	69	3	7	Blue
	q6	31	69	3	7	Red
0.3^-	q7	29	71	3	7	Blue
	q8	29	71	3	7	Red
0.5^+	q9	51	49	5	5	Blue
	q10	51	49	5	5	Red
0.5^-	q11	49	51	5	5	Blue
	q12	49	51	5	5	Red
0.7^+	q13	71	29	7	3	Blue
	q14	71	29	7	3	Red
0.7^-	q15	69	31	7	3	Blue
	q16	69	31	7	3	Red
0.9^+	q17	91	9	9	1	Blue
	q18	91	9	9	1	Red
0.9^-	q19	89	11	9	1	Blue
	q20	89	11	9	1	Red

Table 1.1 – Questionnaire

the combination of the two pairs of questions per proportion p_B is used as a robustness test for the elicited preferences. Consequently, an agent whose preferences switch from the precise (imprecise) bag to the other bag for one out of the 4 bets would be interpreted as having only weak preferences for information precision (imprecision). The different attitudes towards ambiguity are described in detail in Section 1.2.2. Moreover, the slight difference in frequencies allows to constrain subjects to choose between bags, without including an indifference option. Enabling subjects to express indifference would raise technical problems when implementing one choice for real for payment [14].

1.2.2 Attitudes towards ambiguity

Ambiguity neutrality

Ambiguity-neutral attitude is revealed when the answers of a subject can be explained by SEU maximization. Frequentist and Bayesian DMs fall into this category.

A frequentist sets his beliefs equal to the frequency of observations in the dataset. He is therefore indifferent between any two datasets with different precisions but identical frequencies. A frequentist is thus insensitive to the precision of information. For two datasets with different frequencies, he will always chose to bet on the bag with the highest frequency of the winning color whatever the length of the dataset. Table 1.2 details all the possible combinations of answers for each p_B in the experiment. For the analysis, the questions are here combined by color: for instance, the bets on blue for the proportion 0.1, where the precise dataset displays 11 blue balls in q1 and 9 blue balls in q3, are presented together. Below, the bet on red in q2 is coupled with the bet on red in q4. To bet on blue, a frequentist prefers the precise bag (P) in q1 and the imprecise bag (I) in q3, whereas, to bet on red, he prefers the imprecise bag in q2 and the precise bag in q4.

14. It would require the experimenter to randomize between bags. However, as first noticed by Raiffa (1961), an ambiguity-averse agent may exhibit a strict preference for randomization and then use the randomization device to hedge. Theoretical debate is still ongoing (see, e.g., Epstein, 2010; Eichberger *et al.*, 2016).

A Bayesian subject is characterized by an updating belief function given by:

$$\lambda = \delta\mu + (1-\delta)f \tag{1.1}$$

The updated belief (λ) is a convex combination of the prior regarding the composition of the bag (μ) and the observed frequencies in the dataset (f), weighted by a parameter (δ) that depends on and decreases in the number of observations. The shorter the dataset, the more one relies on his prior: hence, $\delta_I > \delta_P$. A Bayesian DM starts the experiment with a prior regarding the proportion of blue balls in the precise bag ($\mu_P(Blue)$) and a prior regarding the proportion of blue balls in the imprecise bag ($\mu_I(Blue)$). Consider for instance a DM with symmetric prior on the composition of both bags, which are natural and plausible beliefs without information on the composition of bags (Gilboa and Marinacci, 2016, p392). Hence, he assigns the same subjective probabilities to both colors in both bags, i.e., $\mu_P(Blue) = \mu_P(Red) = \mu_I(Blue) = \mu_I(Red) = 0.5$. Intuitively, with equal frequencies in both datasets:

$$\lambda_I(Blue) - \lambda_P(Blue) = (\delta_I - \delta_P)(\mu(Blue) - f(Blue)) \tag{1.2}$$

$$\lambda_I(Red) - \lambda_P(Red) = (\delta_I - \delta_P)(\mu(Red) - f(Red)) \tag{1.3}$$

The statistical information corresponds to a negative signal for blue balls if the frequency of blue balls in the datasets is less than the prior, i.e., $f(Blue) < \mu(Blue)$ in Eq. (1.2). Since $\delta_I > \delta_P$, the updated belief is greater in the imprecise bag than in the precise bag, i.e., $\lambda_I(Blue) > \lambda_P(Blue)$. Thus, the DM prefers the imprecise bag to bet on blue. If the frequency of blue balls is less than the prior, the frequency of red balls in the datasets is necessarily higher than the prior. Hence, a negative signal for blue balls translates to a positive signal for red balls for Bayesian DMs. Since $f(Red) > \mu(Red)$ in Eq. (1.3), the updated belief is higher in the precise bag than in the imprecise bag, i.e., $\lambda_P(Red) > \lambda_I(Red)$. Thus, to bet on red, he prefers the precise bag. This reasoning can

be extended to the present design with *roughly* equal frequencies. Therefore, to bet on Blue in questions q1 and q3, a Bayesian DM with symmetric prior prefers the imprecise bag [15] (column Bay1 in Table 1.2). On the other hand, he chooses the precise bag to bet on red in q2 and q4.

In Table 1.2, the three columns (Freq, Bay1, Bay2) contain all the neutral answers. Column Bay1 displays the answers of Bayesians with priors greater than p_B and Column Bay2 contains the answers of Bayesians with priors smaller than p_B. Note that it is not possible to differentiate between a frequentist and a Bayesian with a prior *equal* to p_B. Indeed, they both prefer to bet on the dataset which displays the highest frequency of the winning color (whatever the precision of the datasets) and hence, their answers coincide in this case. This yields Bayesians to be counted as frequentists and consequently, the measure of frequentist answers might be biased upwards. However, this is of minor importance since both patterns of choice fall into the global category of ambiguity-insensitive preferences and hence it does not affect the partitioning between neutral and non-neutral ambiguity attitudes.

Ambiguity non-neutrality

The columns Pess, WP1, WP2, Opti, WO1 and WO2 in Table 1.2 describe the non-neutral ambiguity attitudes. Ambiguity non-neutral preferences contradict SEU maximization, meaning that no additive probabilities can be deduced from these patterns of choices. This is the case when, given two particular datasets with (almost) identical observed frequencies, the DM selects the same bag to bet on blue and to bet on red. For instance, choosing the bag associated to the dataset that contains more (less) draws for both bets indicates preference for information precision (imprecision). This is particularly salient because in each pair of questions, one dataset displays a higher frequency of blue balls while the frequency of red balls is higher in the other dataset. Hence, the slight dif-

15. Formally, for q1: $\lambda_I(Blue) = \delta_I 0.50 + (1-\delta_I)0.10$ and $\lambda_P(Blue) = \delta_P 0.50 + (1-\delta_P)0.11$. $\lambda_I(Blue) - \lambda_P(Blue) = (\delta_I - \delta_P)0.40 - (1-\delta_P)0.01$ is positive for all $\delta_I \in [0;1]$ and all $\delta_P \in [0;1]$ such that $\delta_I > \delta_P$.

ference in frequencies does not compensate for the difference in the lengths of the datasets. The combination of two pairs of bets with proportions p_B allows to measure the strength of preferences. Pessimistic choices (Pess) consist of strict preferences for information precision such that, given a particular p_B, the respondent prefers the precise bag for the 4 bets. WP1 and WP2 stand for weakly-pessimistic choices in the following sense: the subject chooses 3 times the most precise dataset out of 4 bets. Thus, pessimistic and weakly-pessimistic answers compose the general class of ambiguity-averse preferences. On the other hand, optimistic choices (Opti) are interpreted as strict preferences for information imprecision (4 choices in favour of the imprecise bag) and weakly-optimistic choices (WO1 and WO2) consist of weak preferences for information imprecision (3 choices in favour of the imprecise bag). Consequently, optimistic and weakly-optimistic answers define the general class of ambiguity-loving preferences.

Non-monotonic preferences

The last two columns of Table 1.2 (NM1 and NM2) gather the choices that do not satisfy the property of monotonicity. For instance, monotonicity requires that a DM who prefers the precise bag to bet on blue in q3 should also prefer it in q1 since 11/100 evidence for blue (q1) is always at least as good as 9/100 evidence for blue (q3). If the DM prefers the imprecise bag in q1, a symmetrical argument implies preference for the imprecise bag in q3. Hence, a DM with preference for the imprecise bag in q1 and preference for the precise bag in q3 (as described in the column NM1) violates monotonicity of preferences.

p_B	Q.	Bet	Precise bag		Imprecise bag		Types of answers										
			Blue	Red	Blue	Red	Freq	Bay1	Bay2	Pess	WP1	WP2	Opti	WO1	WO2	NM1	NM2
0.1	q1	Blue	11	89	1	9	P	I	P	P	P	P	I	P	I	I	
	q3	Blue	9	91	1	9	I	I	P	P	I	P	I	I	I	P	
	q2	Red	11	89	1	9	I	P	I	P	P	I	I	I	I		P
	q4	Red	9	91	1	9	P	P	I	P	P	P	I	I	P		I
0.3	q5	Blue	31	69	3	7	P	I	P	P	P	P	I	P	I	I	
	q7	Blue	29	71	3	7	I	I	P	P	I	P	I	I	I	P	
	q6	Red	31	69	3	7	I	P	I	P	P	I	I	I	I		P
	q8	Red	29	71	3	7	P	P	I	P	P	P	I	I	P		I
0.5	q9	Blue	51	49	5	5	P	I	P	P	P	P	I	P	I	I	
	q11	Blue	49	51	5	5	I	I	P	P	I	P	I	I	I	P	
	q10	Red	51	49	5	5	I	P	I	P	P	I	I	I	I		P
	q12	Red	49	51	5	5	P	P	I	P	P	P	I	I	P		I
0.7	q13	Blue	71	29	7	3	P	I	P	P	P	P	I	P	I	I	
	q15	Blue	69	31	7	3	I	I	P	P	I	P	I	I	I	P	
	q14	Red	71	29	7	3	I	P	I	P	P	I	I	I	I		P
	q16	Red	69	31	7	3	P	P	I	P	P	P	I	I	P		I
0.9	q17	Blue	91	9	9	1	P	I	P	P	P	P	I	P	I	I	
	q19	Blue	89	11	9	1	I	I	P	P	I	P	I	I	I	P	
	q18	Red	91	9	9	1	I	P	I	P	P	I	I	I	I		P
	q20	Red	89	11	9	1	P	P	I	P	P	P	I	I	P		I

Table 1.2 – The 11-groups classification of choices

Note: Freq: Frequentist, Bay1: Bayesian with prior $> p_B$, Bay2: Bayesian with prior $< p_B$, WP1 and WP2: Weakly Pessimistic preferences, WO1 and WO2: Weakly Optimistic preferences, NM1 and NM2: Non-Monotonic preferences.

1.2.3 Participants [16]

I ran 6 experimental sessions during January and February, 2014 in the LEEP (Laboratoire d'Economie Expérimentale de Paris [17]). 91 participants were recruited using ORSEE (Greiner, 2015): females represent ⅔ of the pool of subjects, the average age is above 29 and a majority of people completed higher education. The average payment amounts to 12.86€ for a 30 minutes experiment, compared to the french minimum hourly wage which is less than 7.50€ in 2014.

1.2.4 Procedures

The experiment was programmed and conducted with the experiment software z-Tree [18] (Fischbacher, 2007). The script was written in French. There were no particular requirements for participation and there was no time limit to answer the questions.

The timing of the experiment is composed of 6 stages:

1. First, I welcome the participants in the lab. Before entering the room, they randomly pick a number that determines where they sit in the room. They are also asked to sign a participation consent before the start of the session.
2. Second, I gave the instructions in the form of an oral presentation with slides [19].
3. Next, the participants are asked to answer a short comprehension test to check their understanding of the experiment [20]. No one can reach the next stage before I have made sure that everybody has answered correctly.
4. Stage 4 consists of filling out the questionnaire on choices of bags.

16. I am grateful to undergraduate and PhD Economic students from the University of Cergy-Pontoise for their participation to non-incentivized pilot sessions. Preliminary experimental data were also collected through Qualtrics online platform, these participants are acknowledged for their valuable help.
17. `http://leep.univ-paris1.fr/accueil.htm`. Maison des Sciences Economiques, 106 - 112 boulevard de l'Hôpital, 75647 Paris cedex 13, France.
18. `http://www.ztree.uzh.ch/en.html`
19. Appendix 1.A.1 provides a full set of the instructions.
20. See Appendix 1.A.2 for a screenshot of the comprehension questionnaire.

5. In a next stage, participants are asked to fill out a complementary questionnaire on socio-economic characteristics.

6. Lastly, participants are paid according to a Random-Lottery Incentive System (see section 1.2.5).

Regarding the technical aspect of the composition of the bags, I wrote a 3-step program on MATLAB: first, I generate 199 bags with composition ranging from (1 blue, 199 red), (2 blue, 198 red), ... to (199 blue, 1 red); second, for a random half of them, I randomly draw 10 balls with replacement, for the second half, 100 balls are randomly drawn with replacement; lastly, I keep the bags for which I obtain datasets with frequencies of interest. Detailed bags composition is provided in Appendix B (Table 1.6).

1.2.5 Incentives

The payment scheme includes a show-up fee of 5€ and in addition each participant plays out one of his choices for real according to a Random Incentive System (RIS). In concrete terms, when the subject has completed the whole questionnaire, one question is randomly selected and displayed on his computer. Hence, the question used for payment is not necessarily the same for all participants. The answer of the respondent is reminded and the subject is asked to reach the experimenter's office. There, he faces the pair of bags corresponding to the selected question and he has to randomly draw a ball from the bag chosen during the experiment. If he wins the bet, he is paid 18€ (including show-up fee); if he loses, he gets 6€ (including show-up fee). Participants implement real draws with real bags in order to persuade subjects that the procedure is truly not-manipulated by experimenters. This has an organisational cost since I display as many real bags as questions, i.e., 20 different bags in total.

Popularized by Savage (1954), the use of the RIS induces subjects to consider all questions of the experiment to be equally relevant while only paying one of them, which avoids potential portfolio effects resulting from the payment of all questions in the experiment.

Hence, for a given research budget, this method allows to collect a large number of observations from each subject. Moreover, the RIS is easy to explain, to understand and to implement in the lab. For these reasons, it has been extensively used to incentivize experimental choices. The RIS requires that subjects perceive each decision as a single real choice (i.e., in isolation). Rather, they may perceive the whole experiment as a single choice problem involving compound lotteries, in which case the RIS does not allow to elicit true preferences (Holt, 1986; Karni and Safra, 1987; Bade, 2015). However, several experimental studies have concluded that isolation is verified when decision problems consist of simple binary choices (Starmer and Sugden, 1991; Hey and Lee, 2005a, 2005b). In the present experiment, the subject answers 20 binary questions, therefore I use the RIS which provides an appropriate mechanism to incentivize decisions.

1.3 Results

1.3.1 Neutral and non-neutral ambiguity attitudes

The percentages of answers falling in the 4 general classes of ambiguity attitudes are given in Table 1.3. The detailed frequencies of answers within the 11-groups classification are displayed in Table 1.7 in Appendix B.

	Neutral		**Non-neutral**		
p_B	Freq	Baye	AA	AL	NM
0.1	33	26	19	10	12
0.3	28	29	25	6	12
0.5	42	12	19	15	12
0.7	27	33	17	5	18
0.9	33	23	21	9	14
Total	*33*	*25*	*20*	*9*	*13*
	58		***29***		***13***

Table 1.3 – Choices by categories (in %)

Note: Baye contains Bay1 and Bay2 subgroups; AA stands for ambiguity-averse and contains Pess, WP1 et WP2 subgroups; AL stands for ambiguity-loving and contains Opti, WO1 and WO2 subgroups; NM contains NM1 and NM2 subgroups.

On average, 87% of answers satisfy the standard property of monotonicity. Among them, 2/3 of choices are ambiguity-neutral and can be explained by SEU maximization. They are distributed among frequentist and Bayesian answers. The remaining 1/3 of answers satisfying monotonicity contradict the Expected Utility hypothesis and can be interpreted as an expression of non-neutral ambiguity attitude. Among them, 2/3 displays ambiguity-aversion and only 1/3 of the answers can be classified as ambiguity-loving.

Regarding non-neutral ambiguity attitudes, a share of 20% of the whole sample of answers displays ambiguity-aversion (pessimistic and weakly-pessimistic answers), suggesting that the difference in frequencies does not compensate for the precision of the datasets. For half of them, the bag associated with the most precise dataset is chosen for the two complementary bets even when the frequencies of the datasets are slightly changed (pessimistic). The other half prefers the precise dataset for 3 bets out of 4 (weakly-pessimistic answers). On the other side, only 9% of answers can be classified as ambiguity-loving (optimistic and weakly-optimistic answers).

Non-monotonic answers amount to 13% of response patterns in the experiment: 60.4% of respondents do not violate monotonicity in any of the 5 pairs of bets, 18.7% violate monotonicity only once, 13.2 % twice and 7.7% thrice.

The shares of respondents falling into the 5 reported categories are pretty stable across the proportions p_B apart from the questions related to frequencies 1/2–1/2: except for the proportion 0.5, where they amount to 42%, frequentist answers represent approximately a share of 30% of the total answers. At 0.5, only 12% of answers correspond to Bayesian preferences, while they represent at least 23% in the other proportions. This may be due to the fact that a significant share of subjects behave as Bayesians with a prior on p_B equals to 1/2, because, as stated previously, it is not possible to distinguish between frequentists and Bayesians with prior *equal* to p_B. This is confirmed by the fact that significant percentages of answers fall into Bay1, i.e., prior greater than p_B, for questions related to proportions smaller than 0.5 and the tendency is reversed for proportions greater than 0.5

with a substantial share of answers falling in Bay2, i.e., prior less than p_B (see Table 1.7 in Appendix B). Apart from this difference due to classification of neutral answers, it is not possible to conclude on a particular effect of frequencies of blue and red balls in the datasets on ambiguity attitudes.

1.3.2 Score of ambiguity-sensitivity

From a within-subject perspective, I calculate an individual score S of ambiguity-sensitivity: for a given color in a given proportion p_B, it takes the value 1 if the DM exhibits ambiguity-averse preferences, 0 if neutral and -1 if ambiguity-loving. The total score for a given individual is obtained by summing up the scores for the 10 pairs of questions. Hence, a score of 10 refers to extreme ambiguity-aversion and -10 denotes extreme ambiguity-loving. Formally, $S = \sum_{p_B,j} s_{p_B,j}$, where $p_B \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, $j \in \{B, R\}$ denotes the color of the bet and

$$s_{p_B,j} = \begin{cases} +1 & \text{if the precise bag is chosen twice } (PP), \\ 0 & \text{if if each bag is chosen once } (IP \text{ or } PI), \\ -1 & \text{if the imprecise bag is chosen twice } (II). \end{cases}$$

In the data, scores range from -4 to 10 (see Table 1.4 and Figure 1.1). 43% of participants have a positive score, 38% have a null score and the remaining 19% have a score below 0. The average score in the experiment is .96, significantly different from 0 (one-sample $t(90) = 3.2,\ p < 0.01$). This number includes non-monotonic answers for which $s_{p_B,j}$ takes value 0 (since each dataset is chosen once). Excluding non-monotonic answers, the average score equals 1.04.

Since null scores can be obtained with ambiguity-neutral answers as well as with combinations of ambiguity-loving and ambiguity-averse answers, further investigations are needed to distinguish between scores resulting from unstable ambiguity-sensitive prefer-

S	N	Freq	CumFreq
-4	2	2.2	2.2
-3	6	6.6	8.8
-2	3	3.3	12.1
-1	6	6.6	18.7
0	35	38.4	57.1
1	16	17.6	74.7
2	5	5.5	80.2
3	7	7.7	87.9
4	0	0.0	87.9
5	4	4.4	92.3
6	1	1.1	93.4
7	2	2.2	95.6
8	1	1.1	96.7
9	0	0.0	96.7
10	3	3.3	100.0
Total	*91*	*100.0*	

Table 1.4 – Score of ambiguity-sensitivity

Note: N gives the number of subjects, Freq stands for Frequency, CumFreq for Cumulative Frequency.

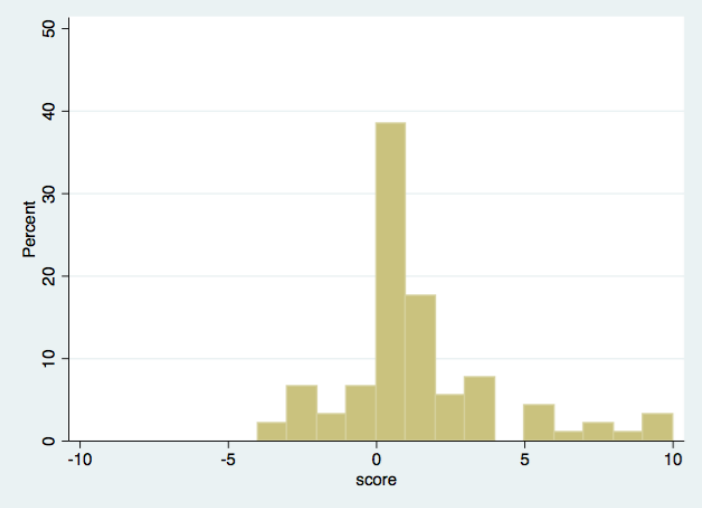


Figure 1.1 – Histogram of individual score

ences and scores due to regular decision patterns. For $p_B \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, $s_{p_B,.}$ is the individual score in proportion p_B obtained by summing up the score for bets on blue and bets on red: $s_{p_B,.} = s_{p_B,B} + s_{p_B,R}$ hence $s_{p_B,.} \in [-2; 2]$. Excluding non-monotonic

answers, the maxima and minima $s_{p_B,\cdot}$ are given in Table 1.5 [21]. The individuals on the bold diagonal have the same decision mode along the experiment since their *per proportion* score is constant. For instance, the 4 subjects in group (2,2) choose the precise dataset at each question. They exhibit extreme ambiguity-aversion. The farther from the diagonal, the more the DM exhibits different choice behaviors. The largest group is the (0,0) group with 35 subjects: such participants obtain a zero score at each proportion, this denotes clear ambiguity-neutrality. Looking more in detail at the data: almost half of them (16) behave as pure frequentists and the rest (19) exhibits a mix of frequentist and Bayesian answers. The second largest group is the (0,1) group. These two findings are in line with the previous result showing a weak bias towards ambiguity-aversion. It is worth noting that only one subject obtains a negative maximum score -1 and no one stands in the other groups with negative maximum scores. This suggests that no regularity can be found in the patterns of ambiguity-loving answers.

$s_{p_B,\cdot}$ min \ $s_{p_B,\cdot}$ max	-2	-1	0	1	2	*Total*
-2	**0**	1	2	8	4	*15*
-1	-	**0**	5	2	6	*13*
0	-	-	**35**	13	8	*56*
1	-	-	-	**1**	2	*3*
2	-	-	-	-	**4**	*4*
Total	*0*	*1*	*42*	*24*	*24*	*91*

Table 1.5 – Maxima and minima *per* p_B individual scores

Note: Excluding non-monotonic answers.

1.3.3 Discussion

In order to keep the experiment as simple as possible, the precision of the datasets is kept constant in this study and the imprecise (precise) dataset contains always 10 (100) draws. Ert and Trautmann (2014) propose an experiment where participants can learn

21. Keeping the non-monotonic answers does not significantly change the reported frequencies in the table.

about an ambiguous prospect by sampling unlimitedly at no cost. A sample returns the result of an independent draw from the relevant distribution. They report that median numbers of samples are small and lie between 11 and 15 (depending on the decision problem). Hence, the reported neutral ambiguity attitudes in my experiment might result from the fact that DMs consider the observation of 10 draws to be sufficient to form a confident judgment regarding the composition of the bag. Consequently, agents would not process differently the dataset containing 10 draws and the dataset containing 100 draws.

This finding is also is in line with the "law of small numbers" of Tversky and Kahneman (1971). According to them, DMs have a tendency to regard limited samples of observations as highly representative. The reason is that they believe random samples to be very similar to one another and to the population from which they are drawn. Hence, agents make confident inference about the true distribution based on the results of short samples, as if the law of large numbers applied to them. The results of Experiment 1 in Arad and Gayer (2012) support this prediction. Participants are asked to consider betting on a draw from an opaque two-color Ellsberg's urn. In the control group, participants know the composition of the urn whereas subjects of the treatment group are provided with a sample of observations from the urn. The urn in the treatment group contains 90 balls and the dataset consists of 8 random draws with replacement. The distribution of Certainty Equivalents do not differ significantly across treatments. Therefore, the authors conclude that there should be no great difference between the beliefs in both informative conditions, yielding agents to value the prospects equally.

1.4 Conclusion

I present an adaptation of the Ellsberg's two-colors experiment allowing to explore decision making under partial ambiguity. In this study, one bag is described by a precise set of observations containing a large number of draws and the other bag is described by an

imprecise set of observations containing a small number of draws. I report that a majority of choices in the experiment are consistent with the SEU model when datasets are available for both bags. Indeed, most answers correspond to frequentist and Bayesian preferences. On the other hand, non-neutral ambiguity attitudes cannot be removed from the analysis. More precisely, ambiguity-non-neutral choices consist of a majority of ambiguity-averse answers whereas ambiguity-loving attitudes can be interpreted as a marginal and unstable trait of preferences. This is confirmed by the estimation of a score of ambiguity-sensitivity which is slightly biased towards ambiguity-aversion on average.

In order to reach a general assessment of preferences under partial ambiguity, it is necessary to conduct additional experiments with different lengths of datasets. In particular, it might be worthwhile to compare the case of full ambiguity to partial ambiguity when very few observations are available since there is empirical evidence of ambiguity-aversion under full ambiguity and of ambiguity-neutrality when ambiguous prospects are described by short sequences of information. Both findings suggest that ambiguity-averse DMs consider small samples informative enough to resolve the ambiguity characterizing the decision situation. This question will be addressed in the subsequent chapters.

Appendix

1.A Supplementary material for the experiment

1.A.1 Instructions (translated from French)

Note: The sentences in brackets are not included in the instructions but inform the reader on the moment an action is performed.

— Start of Instructions —

Hello everybody, and thank you for accepting our invitation to participate in this experiment. My name is Roxane Bricet and I am pursuing my research at the Economics department of the University of Cergy-Pontoise. I am going to briefly present the outline of the experiment, please pay close attention to this short presentation.

First, some important rules which must be respected: It is important that the experiment takes place in silence, you are not allowed to communicate with other participants. The use of mobile phones and calculators is forbidden. No particular preliminary knowledge is needed to participate. As soon as the experiment starts, if you have any questions, please raise your hand and I will assist you.
Thank you in advance for your cooperation.

What do we do?

You are invited to participate in an economic experiment on decision theory.

Formally, in each question you will consider two opaque bags containing balls. In each bag, balls can be either blue or red.

Consider the following example:



(a) Bag Y. Bag Y contains 200 balls, which can be either blue or red.



(b) Bag Z. Bag Z contains 200 balls, which can be either blue or red.

The exact proportions of blue and red balls in each bag are unknown. However, we have partial information on the content of the bags. Indeed, we have randomly drawn one ball from bag Y and reported its color, **then we replaced it in bag Y**. This operation has been replicated several times in bag Y and in bag Z. Note that the number of draws from bag Y may be different from the number of draws in bag Z.

[At this point, we use real bags Y and Z to randomly draw with replacement 2 balls from one of the two bags.] These **random draws with replacement** are given for your information.

Example:

From bag Y, we have drawn a blue ball 15 times and a red ball 85 times.

From bag Z, we have drawn a blue ball 2 times and a red ball 8 times.

Your choices of bags:

In the experiment, you will be asked to answer questions of the following type:

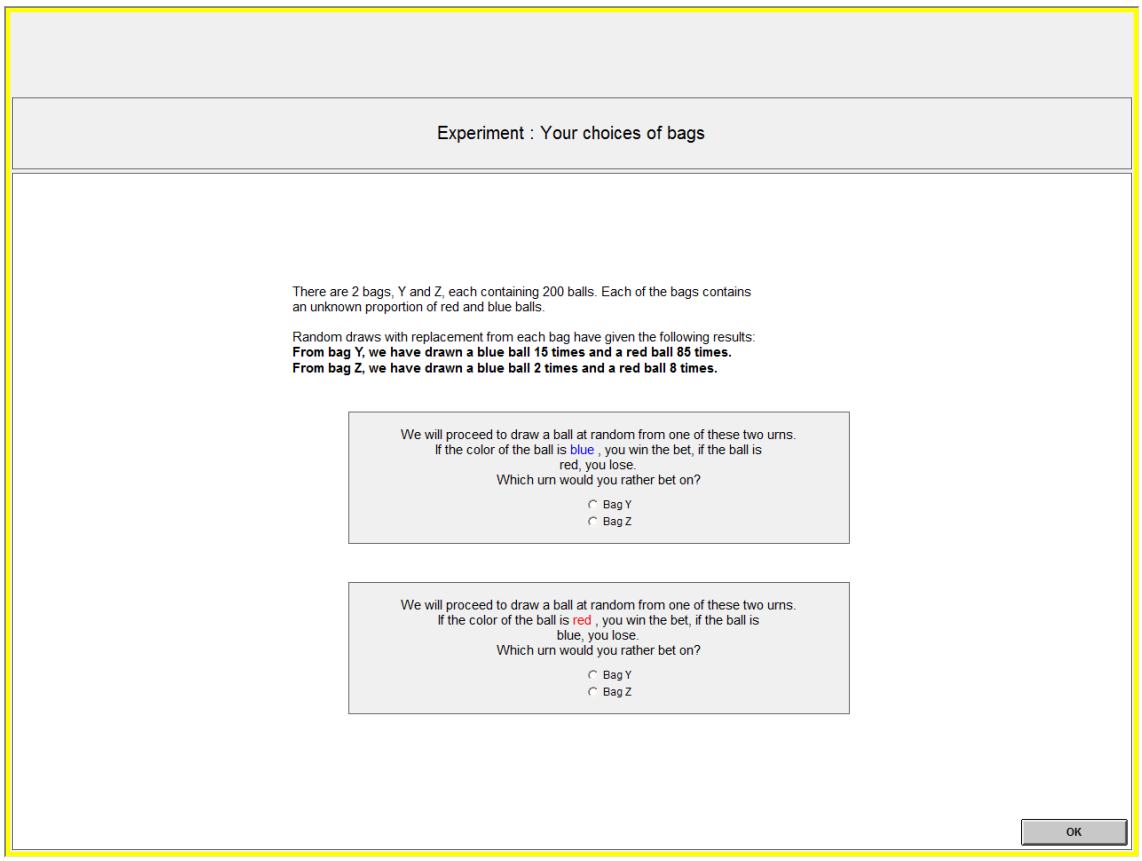
We will proceed to draw a ball at random from one of these two bags. If the color of the ball is blue, you win the bet, if the ball is red, you lose.

Which bag would you rather bet on?

☐ Bag Y ☐ Bag Z

Important: There are no "right" or "wrong" answers in this experiment, it is only a matter of preferences.

Consider the following example of screen that you will face during the experiment:



Your payment:

— For your participation in the experiment, each of you will receive **5 euros**.

— Moreover, you have the opportunity to win **an additional 13 euros**. Indeed, at the end of the session, one of the questions in the experiment will be randomly selected. Each participant will be rewarded depending on his choice to the selected question. Let us return to the previous example: suppose that you prefer to draw a new ball from bag Y, you will proceed to draw a ball at random from bag Y. If the ball is blue, you win the bet and you obtain 13 additional euros; if the ball is red, you lose the bet and you only receive one extra euro. Therefore, it is in your own interest to make choices according to your true preferences.

Altogether, you can win up to 18 euros if you draw the right ball from the selected bag!

Plan of the experiment:

In practice, you will answer the questions of the experiment using the computer in front of you.

— The first screen will remind you briefly of the outline of the experiment.

— Then, you will be asked 5 comprehension questions to check your understanding of the instructions. Your answers to these questions do not affect your payment.

— Next, the experiment on your choices of bags will start. **The question that determines your payment will be randomly selected among them**. This part will consist of 10 consecutive screens for a total of 20 questions.

— Finally, you will be asked to fill out a short complementary questionnaire to get to know you better. Your answers to these questions do not affect your payment.

— The last screen will inform you of the randomly selected question and you will be reminded of your answer to this question.

Your answers will be kept strictly confidential and anonymous, henceforth, feel free to

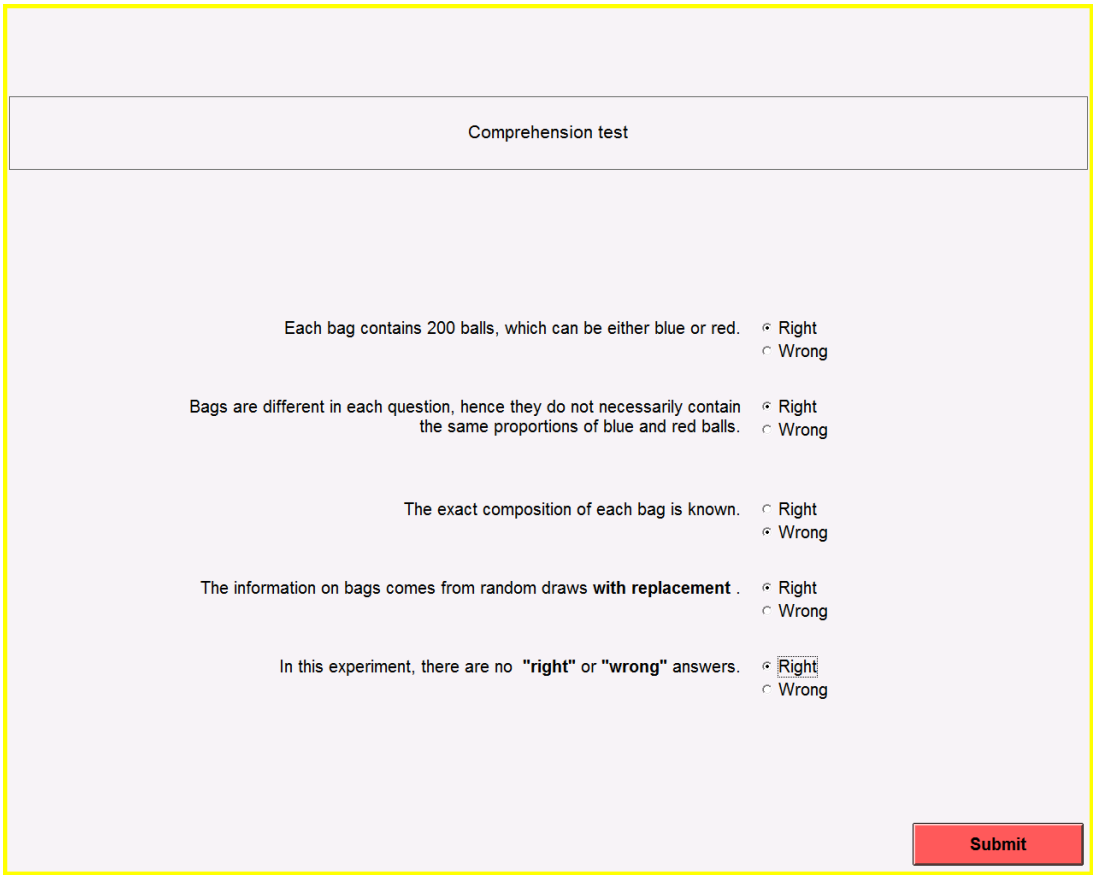
answer as you like. Moreover, there is no time limit, so take all the time you need to read the instructions and answer the questions.

Do you have any questions?

If everything is clear, you can now start the experiment!

— End of Instructions —

1.A.2 Comprehension test



1.B Complementary tables and figures

p_B	Question	Bag	Dataset		Bag	
			Blue	Red	Blue	Red
0.1^+	q1, q2	P	11	89	38	162
		I	1	9	98	102
0.1^-	q3, q4	P	9	91	26	174
		I	1	9	61	139
0.3^+	q5, q6	P	31	69	62	138
		I	3	7	58	142
0.3^-	q7, q8	P	29	71	68	132
		I	3	7	50	150
0.5^+	q9, q10	P	51	49	88	112
		I	5	5	93	107
0.5^+	q1, q12	P	49	51	102	98
		I	5	5	106	94
0.7^+	q13, q14	P	71	29	133	67
		I	7	3	105	95
0.7^-	q15, q16	P	69	31	145	55
		I	7	3	158	42
0.9^+	q17, q18	P	91	9	188	12
		I	9	1	163	37
0.9^-	q19, q20	P	89	11	175	25
		I	9	1	170	30

Table 1.6 – Bags composition

Note: See Section 1.2.4 for the description of the procedure.
Lecture: In the pair of questions (q1,q2), participants are informed that 100 balls have been randomly drawn with replacement from the precise bag, among them 11 were blue and 89 were red; 10 balls have been randomly drawn with replacement from the imprecise bag, among them 1 was blue and 9 were red (public information). The precise bag contains actually 38 blue balls and 162 red balls and the imprecise bag contains actually 98 blue balls and 102 red balls (non-public information).

p_B	Freq	Bay1	Bay2	Pess	WP1	WP2	Opti	WO1	WO2	NM1+NM2
0.1	32.9	20.9	5.5	8.8	4.4	5.5	3.3	3.3	3.3	12.1
0.3	28.5	19.8	8.8	15.4	7.7	2.2	2.2	2.2	1.1	12.1
0.5	41.7	7.7	4.4	9.9	1.1	7.7	7.7	6.6	1.1	12.1
0.7	27.4	9.9	23.1	7.7	4.4	4.4	2.2	2.2	1.1	17.6
0.9	32.9	6.6	16.5	11.0	4.4	5.5	4.4	3.3	1.1	14.3
Total	*32.7*	*11.6*	*13.0*	*10.5*	*4.4*	*5.1*	*4.0*	*3.5*	*1.5*	*13.6*

Table 1.7 – Detailed frequencies of choices

Note: Freq: Frequentist, Bay1: Bayesian with prior $> p_B$, Bay2: Bayesian with prior $< p_B$, WP1 and WP2: Weakly Pessimistic preferences, WO1 and WO2: Weakly Optimistic preferences, NM1 and NM2: Non-Monotonic preferences.

Chapter 2

Preferences for information precision under ambiguity

2.1 Introduction

In real-life decision situations, exact probabilistic information about the outcomes of actions is usually unavailable. However, most decision makers are reluctant to chose ambiguous prospects and hence, are more likely to bet on events with known rather than unknown probabilities. This pattern of choices, known as ambiguity aversion, has found considerable empirical support since the pioneer paper by Ellsberg (1961) [22]. In this article, Ellsberg describes the following thought-experiment: 2 opaque urns are filled with 100 balls each and balls can be either black or red. Urn I contains exactly 50 black and 50 red balls while the composition of Urn II is unknown. What is the preferred urn to bet on black? And to bet on red? Most agents strictly prefer the known urn (Urn I) for both bets. These choices cannot be reconciled with Savage's (1954) axiomatization of the Subjective Expected Utility theory and consequently, are known as the Ellsberg paradox. Subsequent influential works have provided various theoretical frameworks to account for

22. Surveyed by Camerer and Weber (1992) and most recently by Trautmann and Van De Kuilen (2015).

this type of preferences (Gilboa and Schmeidler, 1989; Ghirardato *et al.*, 2004; Klibanoff *et al.*, 2005).

However, compared to the limit cases of risk (where probabilites are objectively known) and ambiguity (where no information on probabilites is known), decision makers often own partial information about the decision process at hand: they refer to weather forecasts before setting an outside meeting, they compare customer ratings and reviews before an expensive investment, they purchase medical trials before undergoing an innovative treatment... In our modern societies, information has become one of the most traded good and, from global interests to micro motives, the issue of information acquisition is paramount in decision under uncertainty. Indeed, learning allows to mitigate ambiguity (Marinacci, 2002; Epstein and Schneider, 2007; Zimper and Ma, 2017). In particular, regarding the proportions of colored balls in Ellsberg urns, Nicholls *et al.* (2015) observe that "subjective estimates converge to the true proportions of differently colored balls (=‘objective’ probabilities) if the respondents can observe large data samples from multivariate Bernoulli trials." (pp.103–104). Consequently, information that reduces ambiguity has a positive value for ambiguity-averse decision makers (Snow, 2010; Attanasi and Montesano, 2012). In an experiment, Ambuehl and Li (2018) elicit subjects' Willingness-to-Pay (WTP) for useful information (a single observation) using uncertainty equivalents. They especially find that agents have a strong preference for information that resolves uncertainty.

In the present article, I report the results of an experimental investigation on the role of information in ambiguous settings. This chapter explores the following question: how do agents value partially ambiguous bet, i.e., when some information describing the decision situation is available? Since information allows to mitigate ambiguity, additional information may render the gamble more attractive for the ambiguity-averse decision maker. Moreover, I investigate the extent to which preferences are robust to different elicitation procedures. Considering two choice options, preference reversals describe situations where an individual provides contradictory preference orders under different elicitation mecha-

nisms. As initially demonstrated by Lichtenstein and Slovic (1971), it is pretty common that one gamble, characterized by a high probability of winning a small gain (usually called the P-bet), is strictly preferred to a second gamble, featuring a small probability of winning a high gain (usually called the $-bet). However, the preferred gamble is often assigned a lower selling price. Both decisions appear to be mutually inconsistent and this is known as the standard preference reversal phenomenon. Indeed, most of the standard theories of rational choice do not admit the existence of preference reversals. This raises the question of what true preferences are since choices may depend on how the task is framed.

While preference reversals have been widely studied in riskless and risky environments [23], there are comparatively few papers addressing the occurrence of such paradox in ambiguous frameworks. Maafi (2011) investigates standard preference reversals and finds that agents do reverse their preferences under ambiguity, and the effect is even more salient than under risk. In her experiment, although the share of subjects preferring the P-bet is roughly constant across the risky and the ambiguous conditions, ambiguity increases the gap between the prices of the ambiguous $-bets and their corresponding ambiguous P-bets, so that reversals occur more frequently. Closely related to my research is the Trautmann *et al.* (2011) study of preference reversals within only one attribute: contrary to standard design, the payments of the 2 lotteries are identical and the lotteries only differ in their likelihood. In several Ellsberg 2-color experiments, they report a minor but significant share of preference reversals under ambiguity when the price is elicited via WTP; however, this finding is mitigated when gambles are evaluated via Certainty Equivalents (CE). They show that WTP measurements entail a general overestimation of ambiguity aversion due to a reference point effect and the resulting loss aversion. Therefore, this study asks whether preference reversals occur under ambiguity. My design focuses on preference reversals exclusively within the likelihood attribute as in Trautmann *et al.* (2011), but I

23. Surveyed by Seidl (2002).

enable agents to observe informative signals whereas they focus on complete ambiguity.

In a lab experiment, subjects are asked (1) to provide their CE for an unambiguous (hence, risky) bet and an ambiguous bet and (2) to select their most preferred bet between the two. The experimental design draws on Ellsberg's two-color urns and the bets consist of gambling on the color of the ball to be drawn from an urn. In the risky bets, the participant is asked to consider gambling on blue and gambling on red in the known urn containing as much blue balls as red balls; while in the ambiguous bets, the exact composition of the urn is unknown but the urn is described by a dataset containing random draws with replacement. Hence, the degree of precision of information depends on the number of reported draws. This experiment consists of 5 treatments for 5 distinct ambiguous urns. Depending on the treatment, the number of random draws performed in the ambiguous urn spans from 0 to 500.

One key feature of the design relies on the symmetry of draws reported in the dataset: half of them are blue, the other half being red. Without information, the agent is likely to adopt a symmetric prior about the composition of the ambiguous urn (Gilboa and Marinacci, 2016, p.392). Thereafter, all commonly used updating rules predict that symmetric information about the color of draws result in symmetric revised beliefs. Hence, the symmetry of draws allows for a direct comparison of the CEs and the preferences between the risky urn and the ambiguous urn, since the risky urn contains as much blue balls as red balls. Indeed, if the symmetry of prior and posterior beliefs is verified, the comparison between the CEs depends only on the individual perception of ambiguity and the individuals' attitude towards ambiguity. Besides, the symmetry of signals allows to relate this study to other experimental replications of Ellsberg's two-urns problem in similar symmetrical frameworks.

Further, this experiment can be used to investigate the strength of Ellsberg-type preferences: the direct choices between urns are used to evaluate the incidence of ambiguity aversion in the case of partial ambiguity and the CE measurements provide an estimation

of the magnitude of the preference for one urn compared to the other (high differences in CE indicate strong preferences whereas small differences suggest weak preferences). In the last stage of the experiment, the participant is asked to explicitly state his individual beliefs on the composition of the ambiguous bag. More specifically, his estimation of the number of blue balls in bag B is collected and the respondent has to provide lower and upper bounds for the proportion of blue balls in the bag. This non-incentivized task serves to study how the precision of information influences beliefs and perceived ambiguity.

In terms of payment device, the choices are incentivized via the Prince mechanism, recently proposed by Johnson *et al.* (2015). Compared to standard methods, the Prince mechanism allows to clarify consequences of decisions and makes incentive compatibility completely transparent to subjects. To the best of my knowledge, this chapter is the first experimental study to implement this method to incentivize preferences for information precision in an ambiguous framework.

This experiment yields three key findings:

1. Within treatments, despite the fact that the majority of participants prefer the risky urn to the ambiguous urn for both bets (in conformity with the Ellsberg paradox), they do not place a higher value on the risky gamble compared to the ambiguous one (even in the no-information treatment).
2. Between treatments, although one would have expected a positive correlation between the precision of information and the value attributed to the ambiguous bet (the more information, the more appealing the ambiguous gamble), there is no significant relationship between the number of draws in the dataset and the CE of the ambiguous bet. Moreover, the comparison of standard deviations of the distributions of CE by treatments does not indicate more dispersion in the less informative treatments.
3. On average, 1/4 of the subjects exhibit preference reversals. Hence, CE measurements do not eliminate preference reversals in this framework, as opposed to the

findings of Trautmann *et al.* (2011).

I can conclude from these results that ambiguity-averse preferences expressed by the subjects are relatively weak since ambiguity-aversion deduced from direct choices does not translate into significant CE differences between the risky and the ambiguous gambles. The reason is that it might be easier to evaluate attributes jointly than separately. In particular, my finding can be related to the "coherent arbitrariness" explanation proposed by Ariely *et al.* (2003). The authors argue that DMs experience difficulties to value goods and they show in a series of experiments that subjects' valuations are notably arbitrary and unstable since individual prices can be manipulated by normatively irrelevant factors (such as framing and anchors). Furthermore, the similarity of valuations between treatments suggests that the increase in informativeness does not significantly modify the perception of ambiguity and this tends to prove that subjects form a confident "fifty–fifty" probability judgment for the composition of the ambiguous urn even when no or very little information is available. This is confirmed by individual estimates on the composition of the ambiguous bag.

The type of information considered here is fundamentally different from Eliaz and Schotter (2010) who study demand for non-instrumental information (information that may not change the final decision). In a risky environment, Eliaz and Schotter report that experimental subjects assign a positive value to non-instrumental information. This may be due to what they call the "confidence effect", i.e., an intrinsic preference for being confident in choosing the right decision. This study departs also from Ambuehl and Li's (2018) experiment for two main reasons: First, while they estimate WTP for information, I choose to focus on CE since WTP measurements have been blamed for artificially inflating the number of ambiguity averse agents and subsequent estimates of preference reversals (Trautmann *et al.*, 2011). Second, their sets of observations contain only one draw from the urn whereas, in this chapter, the datasets describing the ambiguous bag are of different lengths depending on the treatment. Hence, I am able to generate a wide range of degrees

of information precision, ranging from 2 observations (weakly-informative treatment) to 500 observations (highly-informative treatment), whereas the respondents of the control group (no-information treatment) do not receive any statistical feedback. Contrary to Trautmann *et al.* (2011) who impose that CEs lie between the minimum payoff and the Expected Value of the risky lottery (or the mean of the payoffs of the ambiguous lottery), the CE elicited in my experiment can take any value between the minimum and the maximum payoffs of the gamble. Indeed, 1/3 of estimated CE in this experiment are higher than the Expected Value of the lottery.

This chapter proceeds as follows. Section 2.2 presents the experimental design. Results of the experimental sessions are discussed in section 2.3. Section 2.4 concludes. Appendix 2.A provides supplementary material for the experiment (including the instructions of the experiment). Additional tables and figures appear in Appendix 2.B.

2.2 Experimental Design

2.2.1 Stimuli

The experimental design draws on Ellsberg's two-color urns experiment. Subjects are shown two opaque bags: bag A and bag B. They are informed that both bags contain 100 balls each and that balls can be either blue or red. The composition of bag A is perfectly known: it contains exactly 50 blue balls and 50 red balls. The proportions of blue and red balls contained in bag B are unknown but some information on its composition is sometimes available: subjects are told that previous random draws with replacement from bag B have been been carried out and are informed of the results of the draws.

The gambles presented to the participant are of the following type: if you draw a ball of a prespecified color, you win 15€, otherwise you win 5€[24]. There are 4 different possible

24. Given that there is a 5€ show-up fee, the minimum reward in this experiment does amount to 5€. This fee is included in the gambles payoff because agents seem to pay more attention to their decisions and make less errors when the monetary incentives are high (see for instance Lévy-Garboua *et al.*, 2012).

gambles: betting on a blue ball from bag A and betting on a red ball from bag A (risky bets), betting on a blue ball from bag B and betting on a red ball from bag B (ambiguous bets). For each of these gambles, the subject's Certainty Equivalents (CE, defined as the sure amount equally desirable as the gamble) are elicited.

There exist two alternative experimental strategies to elicit prices such as CE: direct matching and choice-based procedures. The direct matching method consists of determining the CE by directly asking the subject to find the monetary outcome that represents the switching point [25]. In contrast, choice-based procedures refer to strategies based on outright choices, where the value of CE is deduced from a series of choices. The Multiple-Price List (MPL) technique belongs to this last category [26]: it consists of binary decisions between the prospect and (usually equally spaced) sure amounts (e.g., Bruhin *et al.*, 2010, Experiment 4 in Trautmann *et al.*, 2011, Chew *et al.*, 2017). The seminal paper of Holt and Laury (2002) popularized the MPL procedure. Thereafter several recent experimental studies on decision theory estimate prices through MPL. Indeed, this method presents several advantages. First, it is the easiest method to deal with: (*i*) for experimenters, because instructions and implementation are clear and straightforward; (*ii*) for participants, because it is easy to understand. Although direct matching is undoubtedly the most time-saving procedure, it is cognitively demanding since subjects are asked to find their indifference point which is not a natural inquiry. In contrast, the MPL method interrogates subjects about their preferences, which is more intuitive. Second, the simplicity of the MPL procedure (together with the Prince incentive method, see Section 2.2.6 on incentives) makes it relatively obvious for participants that truthful revelation of preferences is in their best interest. Third, experimental evidence shows that choice-based procedures are more consistent than direct matching. Indeed, preference reversals occur more often with matching techniques (Bostic *et al.*, 1990). Since there is a particular concern with the

25. For instance, individual Willingness-To-Accept is elicited via direct matching procedure in Maafi (2011).

26. Along with the bisection method (Abdellaoui, 2000). See also pages 555-556 of Glimcher and Fehr (2013) for a discussion of the different elicitation techniques.

prevalence of preference reversals in this chapter, I selected the method that is the least prone to such paradoxical choices to avoid artificial inflation of preference reversals occurrences. For a review on the most recent experimental literature on Ellsberg-urns tasks and on the different valuation and incentive methods, see Trautmann and Van De Kuilen (2015, Table 1).

Therefore, in this experiment, the CEs are reported using MPL. Precisely, for the 4 gambles, subjects are asked to consider 10 choices between playing the gamble and receiving a sure amount. In each row of the MPL, the participant has to single out his preferred option between: "Bet on *color* in *bag*" or "x€", with *color* being replaced by blue or red, *bag* being replaced by bag A or bag B and x taking the 10 equally spaced values between 5.50 and 14.50. In each MPL, the monetary amounts rise moving down the list while the bet remains the same. By linear interpolation, the CE is taken as the mid-point of the two sure amounts for which the subject switched preferences. However, several experiments show that people often report multiple switching points, which violates monotonicity of preferences. For instance, Lévy-Garboua *et al.* (2012) replicate Holt and Laury's (2002) seminal procedure for measuring risk aversion, they report a substantial rate of inconsistencies under various frames (in their experiment, 30% of subjects violate monotonicity when presented with 10 simultaneous ranked choices). In Cohen *et al.* (2011), most participants violate monotonicity at least once in their online experiment. To avoid this type of issues in my experiment, I wrote a computer program that enforces the monotonicity of revealed preferences by allowing at most one switching point from the gamble to the sure amount. More concretely, once the subject ticks a given option on the MPL, the computer fills in the lines above and below so as to ensure monotonicity (see the Instructions in Appendix 2.A for an exhaustive description of the mechanism). This convenient technique has been implemented in numerous cases since the influential paper of Gonzalez and Wu (1999)[27]. Moreover, in an experimental study comparing different MPL filling

27. See recently for instance: Csermely and Rabas (2016), Gerhardt *et al.* (2017), Ambuehl and Li

designs, Andersen *et al.* (2006) finds no systematic effect of monotonicity-enforcement on subjects responses. Finally, this method allows to keep the experiment as short as possible to maintain subjects aware and concentrated since the tedious task of filling rows one by one can be boring.

In a second step, the participant is asked to select his most preferred bag to bet on blue and his most preferred bag to bet on red.

Table 2.1 summarizes the 6 different tasks performed by the participant.

Task 1	Fill out the MPL for "Bet on blue in bag A".
Task 2	Fill out the MPL for "Bet on red in bag A".
Task 3	Fill out the MPL for "Bet on blue in bag B".
Task 4	Fill out the MPL for "Bet on red in bag B".
Task 5	Choose between "Bet on blue in bag A" and "Bet on blue in bag B".
Task 6	Choose between "Bet on red in bag A" and "Bet on red in bag B".

Table 2.1 – Incentivized tasks

Valuation tasks (Tasks 1-4 in Table 2.1) are run before choice tasks (Tasks 5-6) [28]. The two types of tasks are performed one after the other. Indeed, in the literature on preference reversals, there is a substantial number of works arguing that valuation tasks and choice tasks call forth different heuristics and different cognitive processes [29] (see recently Loomes and Pogrebna, 2016). Thus, I decide to not randomize between types of tasks to make the experiment easy to understand and to avoid confusion. Similarly, since it is much more straightforward to mentally represent risky prospects than ambiguous ones, the risky bag is presented first in the instructions and the description of the ambiguous bag follows. Then, when performing the tasks, the participant evaluates first the bets on blue/red in bag A (Tasks 1 and 2) and second, the bets on blue/red in bag B (Tasks 3 and 4).

(2018). Cubitt *et al.* (2014) proposes an alternative but similar framework: the participants can freely complete the choice-list but the computer program only accepts answers with a single switching point.

28. There is no particular agreement relative to the presentation order of tasks: for instance, in Maafi (2011), participants answer valuation questions before choice questions whereas in Trautmann *et al.* (2011) the participants perform choice tasks first.

29. Besides, Grether and Plott (1979) show that the presentation order between valuation and choice tasks does not significantly influence the pattern of reversals under risk.

2.2.2 Treatments

There are 5 different treatments for 5 different bags B. For the sake of clarity, these 5 distinct bags are labelled bag B1, bag B2, bag B3, bag B4, bag B5 in this chapter but note that the partially ambiguous bag is always named bag B during the experimental sessions. In each treatment, the participant faces only 2 bags: the risky bag (bag A) and the ambiguous bag (bag B). The treatments differ in the number of draws contained in the dataset describing the ambiguous bag. Indeed, different lengths of datasets correspond to different degrees of information precision. One key feature of the design relies on the symmetry of draws reported in the dataset: in each treatment, half of the draws are blue, the other half being red. Since the alternative bag (bag A) contains as much blue balls as red balls, the symmetry of draws in the dataset describing bag B greatly simplifies the comparison of preferences for bets on the ambiguous bag B to the situation of objective risk in bag A. Indeed, if initial beliefs on the composition of bag B are symmetric and if the symmetry of draws implies the symmetry of revised beliefs, the comparison between CEs depends only on the individual's perception of ambiguity and his attitude towards ambiguity.

Table 2.2 describes the different datasets associated to the 5 ambiguous bags. For example, consider treatment 1: in this treatment, the subject is informed that 500 random draws with replacement have been carried out in bag B. We have observed 250 blue balls and 250 red balls. Depending on the treatment, the number of previous draws spans from 0 to 500 where the no-information treatment (treatment 5) mimics the classical Ellsberg's experiment and serves as a benchmark for the impact of information precision on ambiguity perception.

2.2.3 Non-incentivized task

After the valuation and choice questions, an additional belief task is added to the experiment: the subject is asked about his individual belief about the composition of the

Treatment	Bags	Dataset		
		Total number of draws (d)	Blue draws	Red draws
1	B1	500	250	250
2	B2	50	25	25
3	B3	10	5	5
4	B4	2	1	1
5	B5	0	0	0

Table 2.2 – Number of draws describing the ambiguous bag

ambiguous bag. In particular, he has to explicitly report his estimation of the number of blue balls in bag B. Then, he reports his lower and upper bounds for the proportion of blue balls in the bag which provides a measure of his degree of confidence in his estimate [30].

This task allows to study the influence of precision of information on learning. Learning occurs when the subjective estimates converge to the objective probabilities. In my framework, given the symmetry of draws reported in the datasets, it is likely that respondents state equal proportions for both colors. Moreover, increasing the number of observations raises the precision of information which might lead the agent to behave as if he knew the true composition of the bag in the limit. For instance, one might feel more confident in one's 50-50 probability judgment with the observation of 250 blue draws and 250 red draws (treatment 1) than with the realization of 1 red draw and 1 blue draw (treatment 4). Therefore, subjects belonging to the most informative treatments are expected to report less dispersed estimates as compared to subjects in the less informative treatments. The control group is given by the no-information treatment (treatment 5).

30. The questions are framed as follows: "In your opinion, how many blue balls are contained in bag B? Indicate also your minimal estimate and your maximal estimate of blue balls contained in bag B.".

2.2.4 Participants [31]

The experiment was conducted at LEEP (Laboratoire d'Economie Expérimentale de Paris [32]) and consisted of 10 sessions in July, 2017. For each of the 5 treatments, 40 participants were recruited using ORSEE (Greiner, 2015) and no one was allowed to participate in more than one treatment. None of the subjects had previously participated in a similar economic experiment. 181 participants showed up and took part in the experiment: 60% of them are female, ages vary between 18 and 77 (median age being 28), and half of them have completed at least 3 years of higher education. Table 2.3 details the sample size and the average gain by treatment (see also table 2.10 in Appendix B for additional figures on participants). The experiment lasted about 50 minutes (including payment). The average payment amounts to 10.32€ (including the 5€ show-up fee), compared to the french minimum hourly wage which is less than 7.60€ in 2017.

Treatment	Number of participants	Average gain
1	37	10.55€
2	37	10.32€
3	34	10.02€
4	36	9.90€
5	37	10.56€
Total	*181*	*10.32€*

Table 2.3 – Sample size and average gain by treatment

2.2.5 Procedures

The experiment was programmed and conducted with the experiment software z-Tree [33] (Fischbacher, 2007). No particular skill is required to participate, except the

31. I am grateful to undergraduate Economic students from the University of Cergy-Pontoise for their participation to non-incentivized pilot sessions.

32. `http://leep.univ-paris1.fr/accueil.htm`. Maison des Sciences Economiques, 106 - 112 boulevard de l'Hôpital, 75647 Paris cedex 13.

33. `http://www.ztree.uzh.ch/en.html`

understanding of french language. There is no time limit to answer the questions.
A typical session is structured as follows: after an oral presentation of the experiment [34], the participant is asked to perform the 6 tasks using the computer in front of him. Then the subject reaches the experimenter's office for payment: one question of the experiment is played for real according to the Prince incentive mechanism (see section 2.2.6). If the preferred option is a sure amount, he receives the sure gain; if the preferred option is a bet, he randomly draws one ball from the bag involved in the bet. Besides, the subject may implement real draw in real bag in order to persuade him that the procedure is truly not-manipulated by experimenters. The detailed script of the experiment is provided in Appendix 2.A.2.

The experimental design is as follows: the participant is presented with only 2 different bags. Both bags are fully described in the instructions so that when evaluating one bag, the subject knows about the existence of the other. There are only 6 tasks to perform. This short and easily understandable design allows mainly for between-subject analysis.

Regarding the technical aspects of the composition of the bags, bag A contains 50 blue balls and 50 red balls as announced. The composition of bags B1-B4 follows from a 3-steps program on MATLAB: first, I generate all the possible bags containing 100 (blue or red) balls, this yields 101 different bags with composition ranging from (0 blue, 100 red), (1 blue, 99 red), ... to (100 blue, 0 red); second, I randomly draw d balls with replacement from all the bags ($d \in \{2, 10, 50, 500\}$) and I keep the bags for which I obtain the dataset with frequencies of interest, i.e., $d/2$ blue and $d/2$ red balls; third, for each d, I randomly select one of these bags to be presented to the participants. Lastly, since no balls have been drawn from bag B5, bag B5 is randomly drawn among the 101 possible bags. The detailed bags composition is provided in Appendix B (Table 2.11).

34. See Appendix 2.A.1 for a full set of instructions.

2.2.6 Incentives

One commonly known challenge of experimental economics lies in the choice of the adequate method to incentivize decisions made in the laboratory. Studies usually involve more than one decision and paying every choice results in wealth effects. Thus, in most experiments, only one round is chosen by a specified random device at the end of the experiment (*ex post*) and implemented for real. Such design, popularized by Savage (1954) under the name Random-Incentive-System (RIS), produces valid results only if participants consider each experimental decision in isolation (i.e., as a single real choice). Otherwise, the method violates incentive-compatibility (IC). An experimental design is said to be IC when it ensures that *all* participants act according to their true preferences.

Most experimental studies in the literature on decisions under uncertainty uses RIS, often combined with the Becker-DeGroot-Marschak (BDM) mechanism (1964), to elicit preferences for ambiguous lotteries. However, these devices have been criticized for their inability to motivate truthfull-telling for non-expected-utility maximizers. It has been shown that BDM does not ensure IC in experiments involving risky lotteries (Karni and Safra, 1987), and even in experiments involving non-random goods (Horowitz, 2006). Besides, Bade (2015) argues that without the assumption of expected utility preferences, a participant's behavior in the RIS needs not reflect his behavior in single choice experiments. The reason for this is that ambiguity-averse agents can use random device to hedge across experimental tasks and hence may not act according to their true preferences. Baillon *et al.* (2015) also demonstrate that usual implementations of the RIS yield violations of IC for ambiguity-averse DMs. The underlying argument is that, in case of ex-post situation determination, agents may represent the choice problem as a meta-lottery, leading to violations of isolation. They show how IC may be restored with a slight modification of the device in which the randomization takes place before the decisions are made [35].

35. See also Oechssler and Roomets (2014) and Kuzmics (2017) for further discussions on hedging and misclassifications of ambiguity-sensitive subjects in experiments.

I thus choose to incentivize the decisions in this experiment using the innovative Prince mechanism proposed by Johnson *et al.* (2015). Compared to standard methods, this mechanism allows to clarify consequences of decisions and makes IC completely transparent to subjects. In concrete terms, this device relies on the following main features contained in the acronym PRINCE:

> "[...] (1) the choice question implemented for real is randomly selected PRior to the experiment; (2) subjects' answers are framed as INstructions to the experimenter about the real choice to be implemented at the end; (3) the real choice question is provided in a Concrete form, e.g., in a sealed envelope; (4) the Entire choice situation, rather than only one choice option, is described in that envelope." (Johnson *et al.*, 2015, p.3)

Prince allows to overcome the inherent drawbacks of other standard techniques by enhancing isolation between experimental tasks. Consequently, participants are ensured that truthful revelation is in their best interest and the internal validity of the experiment is improved. Finally, in an experiment, in which Johnson *et al.* (2015) elicit the Willingness-to-Accept for a mug, Prince has been found to produce similar results under choice and matching strategies. Here, I elicit the CE of several gambles which are more complex to mentally represent than a concrete object such as a mug [36]. Moreover, there is no empirical evidence that both strategies are equivalent to estimate CE of lotteries under Prince. For this reason and the ones surveyed in section 2.2.1, I chose to provide subjects with MPLs.

In my experiment, before performing the tasks, each subject receives a closed envelope containing the question that will be implemented for real. Each question describes two choice options (e.g., a lottery versus a certain amount of money, see Figure 2.1). The goal of the participant is to obtain the most preferred option. For each session, there are 60 envelopes, numbered 1-60 in random order. Besides, each of the 6 tasks is associated

36. Proposing a matching strategy here is equivalent to asking subjects to determine the amount that makes them indifferent between playing the lottery and receiving the amount for sure, which is not a straightforward task.

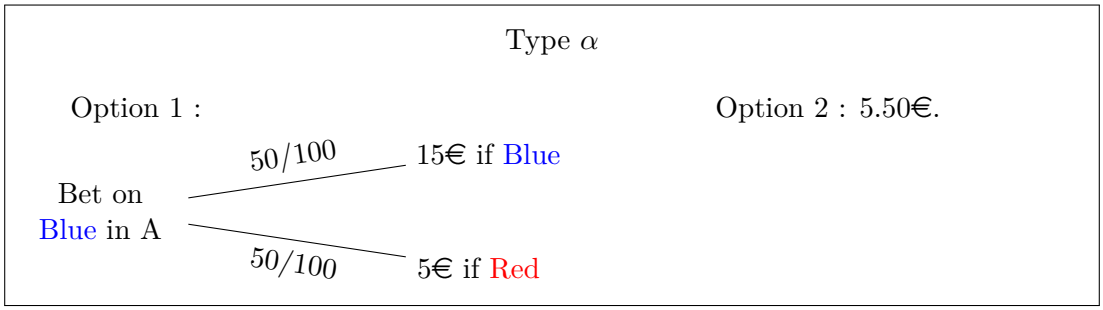


Figure 2.1 – An example of envelope content

to a type and referred to by a Greek letter, which helps improving and speeding up the payment phase. There are 10 envelopes of each type (e.g., envelope contents of type α correspond to the rows of the MPL of Task 1 [37]). Hence, each question of the experiment is contained in one envelope at least. The type of the envelope remains secret until the opening of the envelope. During the experimental session, all the possible contents of envelopes are presented to the subjects. Each of these choices is made particularly salient with the envelope at hand. The subject is informed about all the possible contents of the envelopes in the experiment but does not know the content of his own in particular. At the end of the session, the envelope is opened and the choice of the subject is performed recalling the instructions he gave on the computer: If the preferred option was a sure amount, the subject receives the sure gain; if the preferred option was a bet, he randomly draws one ball from the bag involved in the bet and receives the gain associated with the color of the drawn ball.

The rules of the experiment are made explicitly clear from the beginning of the experiment: as soon as they get into the room, the participant is asked to sign a consent form where it is specified that the gain is conditional upon the respect of the instructions and especially upon the envelope remaining closed until the end of the experiment (See Appendix 2.A.4). Moreover, to render the experiment completely transparent, the participant gets verification lists at the end of the session. These lists describe the contents of all

37. See the table in the instructions (p.82) for the complete list of the types of envelopes.

the envelopes in the experiment and each participant can check that they were truthfully described, avoiding deception (See Appendix 2.A.3).

2.3 Results

2.3.1 Certainty Equivalents

Descriptive statistics on CEs are reported in table 2.4. As in all subsequent tables, the numbers of draws describing the ambiguous bag (in parentheses) are listed next to the treatment. N gives the sample size. In the following, the CE for the bet on blue (red) in bag A is denoted $CE_A(blue)$ ($CE_A(red)$) and similar notation applies for bag B. Normal distributions of CEs cannot be rejected [38]. However, whenever possible, I favour nonparametrical analysis over parametrical tests because the sample sizes are relatively small (less than 40 subjects in each treatment).

First, the experimental data do not provide evidence on substantial difference of valuation of bets depending on the color of the ball to be drawn. The differences between the CEs in bag A and the CEs in bag B are not statistically significant (paired Wilcoxon Signed-rank test [39], all $p > .05$), except for the treatment 5 where $CE_B(blue)$ differs significantly from $CE_B(red)$ (paired Wilcoxon Signed-rank test, $z = -1.970$ and $p = .0488$).

Apart from treatment 1, the means and the medians of CEs on bag A are between 9 and 10 euros and are not significantly different from 10, which is the expected value of the risky prospect (one-sample t-tests: all $p > .05$ and one-sample Wilcoxon tests: all $p > .05$) [40]. This finding suggests, on average, relative risk-neutrality for participants

38. I performed a series of statistical tests for normality of variables that preclude the rejection of the null hypothesis that CEs are drawn from a normal distribution: Shapiro-Wilk (all $p > .05$), Shapiro-Francia (all $p > .05$ except for $CE_A(blue)$ in treatment 2: $z = 1.667$ and $p = 0.04779$), Jarque-Bera (all $p > .05$).

39. Under the assumptions that (1) data are paired and come from the same population, (2) paired observations are independent and (3) paired differences can be ranked, the paired Wilcoxon-Signed rank test (non-parametric) tests the null hypothesis that the median of the paired differences is zero.

40. Under the assumption of the variables being approximately normally distributed, the one-sample t-test is used to test the null hypothesis that the mean is equal to 10. In the absence of such normal pattern, one-sample Wilcoxon test allows to test the hypothesis of equality of median with 10.

		Treatment				
		1 (500) $N = 37$	2 (50) $N = 37$	3 (10) $N = 34$	4 (2) $N = 36$	5 (0) $N = 37$
Bag A (risky)						
$CE_A(blue)$	mean	10.84	9.76	9.86	9.58	9.95
	med	10	10	10	10	9
	var	7.08	7.13	7.28	7.91	9.83
	std	2.66	2.67	2.70	2.81	3.14
$CE_A(red)$	mean	10.54	9.51	9.24	9.78	9.73
	med	10	10	9.5	10	10
	var	7.81	6.26	8.37	6.92	8.70
	std	2.79	2.50	2.89	2.63	2.95
Bag B (ambiguous)						
$CE_B(blue)$	mean	10.32	9.68	9.44	9.36	9.21
	med	10	10	10	9	9
	var	8.95	5.95	11.10	7.89	7.73
	std	2.99	2.44	3.33	2.81	2.78
$CE_B(red)$	mean	10.57	9.27	9.29	9.31	9.95
	med	10	10	9	9.5	10
	var	7.53	6.09	8.82	5.99	7.72
	std	2.74	2.47	2.97	2.45	2.78

Table 2.4 – CEs - Descriptive statistics

Note: med, var and std stand for median, variance and standard deviations, respectively.

in treatments 2-5. In treatment 1, $CE_A(blue)$ differs significantly from 10 (one-sample Wilcoxon, $z = 1.99$, $p = .0458$) but the null hypothesis of equality of mean with 10 cannot be rejected with a one-sample t-test ($p > .05$). Identical analysis on $CE_A(red)$ suggests that it is not significantly different from 10.

Figure 2.2 depicts the proportions of choices in favour of the sure gain in bag A and in bag B. It shows that the percentage choosing the sure gain falls as the sure amount increases. There is little graphical evidence of stochastic dominance of one distribution over the other. Indeed, for each treatment, the differences between $CE_A(blue)$ and $CE_B(blue)$ are not statistically significant (Wilcoxon-Signed rank test: all $p > .05$). The same results apply for the differences between $CE_A(red)$ and $CE_B(red)$. These findings are summarized in Result 1.

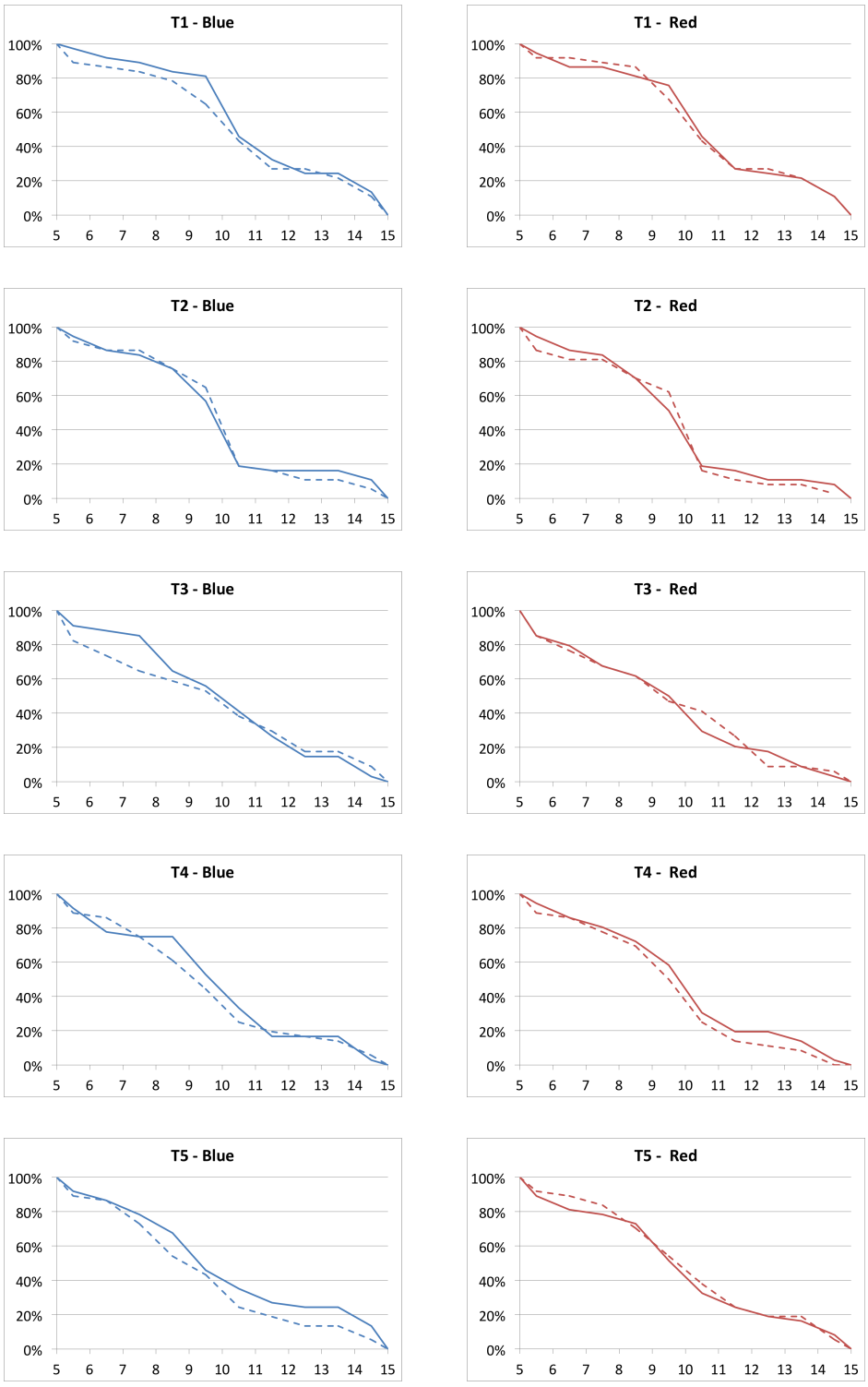


Figure 2.2 – Proportions of choices in favour of the sure amount, by treatment

Note: bag A (solid line) versus bag B (dashed line). x-axis: sure amount, y-axis: percentage of respondents.

Result 1 *Within treatments, for both color bets, there is no significant difference between the valuation of bets on the risky bag and the valuation of bets on the ambiguous bag.*

This result is in line with Maafi's (2011) study of preference reversals in Ellsberg's two-color framework. My design is closely related to a particular condition in her experiment where the probability of winning from the risky urn equals .5 while the probability of winning from the ambiguous urn is unknown (Maafi, 2011: $-bet of Pair V in Table 1). In case of success, the participant wins 12€ (+ 5€ of show-up fee) and 0€ (+ 5€) otherwise, which is similar to the stakes in my experiment. The Willingness-To-Accept for both bets is not significantly different and is close to the Expected Value of the risky urn, as reported in the present chapter with CE measurements. Maafi's analysis falls within the framework of Prospect Theory due to Kahneman and Tversky (1979, 1992), and she argues that her results may be due to the particular form of the probability-weighting function, which is S-shaped and has an inflexion point in the middle region. According to this explanation, while DMs tend to overweigh small probabilities and underweigh large probabilities, they are relatively insensitive to probability changes in the middle zone. This implies ambiguity-aversion for likely-events and ambiguity-loving for rare events, whereas DMs exhibit ambiguity-neutrality for the events that are assigned intermediate probabilities of occurrence. This explains hence that the risky and the ambiguous prospects are valued almost equally.

Between treatments, the valuation of the bet on blue in the ambiguous bag is increasing in the number of observations describing the bag. This is in line with the intuition that more information renders the bet more attractive. The valuation of the bet on red does not exhibit such trend. However, Kruskal-Wallis (K-W) tests [41] suggest that, for both bets, the CE differences *between* treatments is statistically insignificant (all $p > .05$). Moreover, the dispersion of the results across treatments is never statistically significant according

41. Under the assumptions that the observations across groups are independent and that the scale of measurement is at least ordinal, the Kruskal-Wallis test (non-parametric) tests the null hypothesis that the medians of all groups are equal.

to a Levene's test [42] (all $p > .05$). This yields to formulate Result 2.

Result 2 *Between treatments, more information does not induce agents to value significantly more the ambiguous bet.*

This is confirmed by the analysis of the difference between the CEs for the bets on blue and the difference between the CEs for the bets on red in the risky bag and in the ambiguous bag. Table 2.5 provides descriptive statistics of both differences (histograms are plotted in Figures 2.3 and 2.4 in Appendix B).

		Treatment				
		1 (500) $N = 37$	2 (50) $N = 37$	3 (10) $N = 34$	4 (2) $N = 36$	5 (0) $N = 37$
$CE_A(blue) - CE_B(blue)$	mean	0.51	0.08	0.41	0.22	0.73
	med	0	0	0	0	0
	var	4.98	6.47	12.01	4.98	14.65
	std	2.64	2.54	3.47	2.23	3.82
$CE_A(red) - CE_B(red)$	mean	-0.03	0.24	-0.06	0.47	-0.22
	med	0	0	0	0	0
	var	4.42	5.19	7.39	3.80	10.23
	std	2.10	2.28	2.72	1.95	3.20

Table 2.5 – CE differences - Descriptive statistics

Note: med, var and std stand for median, variance and standard deviations, respectively.

Since CEs lie in the interval $[5; 15]$, the differences can take values from -10 to 10. As explained previously, the experimental design allows to examine agent's attitude towards ambiguity by directly comparing the CEs in both bags because: (i) bag A contains as much blue balls as red balls, and (ii) the symmetry of draws in the datasets describing bag B should induce the symmetry of beliefs. Hence, positive differences are associated to ambiguity-averse preferences (henceforth denoted *CE-ambiguity-aversion*) and negative differences reflect ambiguity-loving preferences (*CE-ambiguity-loving*). All means and medians are not significantly different from 0 (one-sample t-tests: all $p > .05$, one-sample

42. Under the following assumptions: (1) observations across groups are independent, (2) the scale of measurement is at least ordinal and (3) the samples are approximately normally distributed, the Levene's test (parametric) tests the null hypothesis of equality of sample variances (homoscedasticity).

Wilcoxon tests: all $p > .05$). Moreover, for both colors, I cannot reject the hypothesis that all differences originate from the same distribution (K-W tests: all $p > .05$). At last, although one would have expected more dispersion among the less informative treatments, this is not supported by experimental data since the variances do not significantly differ across treatments according to Levene's tests (all $p > .05$).

2.3.2 Direct choices

Table 2.6 reports the decisions of subjects in the direct-choice tasks.

	Treatment					
	1 (500) $N = 37$	2 (50) $N = 37$	3 (10) $N = 34$	4 (2) $N = 36$	5 (0) $N = 37$	*Total* $N = 181$
Bet on blue						
Bag A (risky)	76	65	71	61	73	*69*
Bag B (ambiguous)	24	35	29	39	27	*31*
Bet on red						
Bag A (risky)	70	78	79	75	81	*77*
Bag B (ambiguous)	30	22	21	25	19	*23*
Bet on blue and on red						
Bag A (risky)	59	59	65	52	68	*61*
Bag B (ambiguous)	14	16	15	17	14	*15*

Table 2.6 – Direct choices (in %)

To bet on a particular color, there are, on average, more than 2/3 of subjects that prefer the risky bag over the ambiguous one. When combining the two direct choices, 61% of participants exhibit Ellsberg-type behavior and prefer the risky bag for both bets (henceforth denoted *choice-ambiguity-aversion*). This is even more salient in the no-information treatment where the share of ambiguity-averse agents is maximum (68%), although the differences between treatments are not statistically significant (K-W test: $p > .05$). This finding is in line with the experimental evidence on Ellsberg-type problems. Oechssler and Roomets (2015) surveyed 40 Ellsberg's experiments and they report that, on average, slightly more than half of subjects are classified as ambiguity averse.

On the other hand, there is little evidence of *choice-ambiguity-loving* in my experiment since, on average, only 15% of subjects choose the ambiguous bag twice. The remaining subjects (24%) whose preferences switched from one bag to the other are classified as ambiguity-neutral. These effects are summarized in Result 3.

Result 3 *In all treatments, the majority of subjects exhibit Ellsberg-type preferences by preferring the risky bag to the ambiguous bag to bet on both colors.*

Hence, the information on the ambiguous bag does not allow to remove the Ellsberg paradox from the analysis of patterns of preferences as ambiguity-aversion is the behavior most often reported in all information conditions. I report ambiguity-averse preferences when subjects are comparing the bets (Result 3), but the valuations are not significantly different when prospects are priced in isolation (Result 2). Therefore, choice-ambiguity-aversion seems to be an unstable trait of preferences since it does not translate into CE-ambiguity-aversion. One rationale for this finding can be found in Ariely *et al.* (2003) argument that subjects' absolute valuations of goods are arbitrary and manipulable. In my experiment, DMs may have a hard time pricing gambles in isolation whereas comparative tasks might be easier to deal with. Henceforth, the Ellsberg paradox prevails especially when the DM directly expresses preferences between risky and ambiguous prospects.

2.3.3 Preference reversals

For each color bet, two types of preference reversals can occur: either bag A is preferred to bag B but the value attributed to the bet on bag B exceeds the value associated with the bet on bag A (choice-ambiguity-aversion and CE-ambiguity-loving), or the converse, i.e., when bag B is preferred to bag A but the value attributed to the bet on bag A exceeds the value associated with the bet on bag B (choice-ambiguity-loving and CE-ambiguity-aversion). Table 2.7 reports the proportions of choices that reverse preferences.

As it has been previously found in Trautmann *et al.* (2011) with WTP measurements,

	Treatment					
	1 (500)	2 (50)	3 (10)	4 (2)	5 (0)	*Total*
	$N = 37$	$N = 37$	$N = 34$	$N = 36$	$N = 37$	$N = 181$
Choice-ambiguity-aversion and CE-ambiguity-loving						
Blue	14	8	15	17	27	*16*
Red	16	19	26	14	22	*19*
Total	*15*	*14*	*19*	*15*	*24*	*17*
Choice-ambiguity-loving and CE-ambiguity-aversion						
Blue	11	5	6	19	5	*9*
Red	5	5	3	8	3	*5*
Total	*8*	*5*	*4*	*14*	*4*	*7*

Table 2.7 – Preference reversals (in %)

I report more preference reversals that result from the combination of choice-ambiguity-aversion and CE-ambiguity-loving (17%) than choice-ambiguity-loving and CE-ambiguity-aversion (7%). In particular, the results of their Experiment 4 can be compared to the control group (treatment 5): in their experiment, the agents consider one risky bag containing half winning chips and one ambiguous bag with unknown composition. The gamble consists in choosing the color on which to bet for the ball to be drawn from the bag, and the individual valuation for the bet is given by a CE measurement. They observe a majority of ambiguity-averse subjects (2/3 of subjects prefer the risky bag to bet on the color of a random draw). However, they find no credible evidence of preference inconsistencies with CE estimations (8%), whereas 28% of subjects reverse preferences in my no-information condition. Moreover, in my experiment, when more information is provided about the ambiguous bag, the frequency of preference reversals tends to decrease without disappearing, although the differences between treatments do not reach statistical significance (K-W tests: all $p > .05$). This yields to formulate Result 4.

Result 4 *In all treatments, a small but statistically significant share of subjects exhibit preference reversals with CE measurements.*

2.3.4 Estimates on the composition of the ambiguous bag

Descriptive statistics of the individual beliefs of the balls' proportions are reported in Table 2.8 (histograms are plotted in Figures 2.5-2.7 in Appendix B). The individual estimate (*Estim_Blue*) is bounded by a lower limit (*Min_Blue*) and an upper limit (*Max_Blue*), all reported by the participant. The estimates are not significantly different from 50 (one-sample Wilcoxon tests: all $p > .05$) and not different across treatments (K-W test: $p > .05$).

		Treatment				
		1 (500) $N = 37$	2 (50) $N = 37$	3 (10) $N = 34$	4 (2) $N = 36$	5 (0) $N = 37$
Min_Blue	mean	38.3	26.8	26.5	24.6	28.3
	med	40	25	25	27.5	30
	var	464.4	130.8	263.2	300.8	361.2
	std	21.6	11.4	16.2	17.3	19.0
Estim_Blue	mean	53.5	46.2	51.7	47.5	51.5
	med	50	50	50	50	50
	var	440.7	265.8	405.3	492.1	308.2
	std	21.0	16.3	20.1	22.2	17.6
Max_Blue	mean	63.4	57.7	64.5	60.5	68.1
	med	60	60	60	60	65
	var	529.4	358.2	382.1	567.3	643.2
	std	23.0	18.9	19.6	23.8	25.4

Table 2.8 – Estimates and limit bounds of the number of blue balls in bag B - Descriptive statistics

Note: med, var and std stand for median, variance and standard deviations, respectively.

On average, the mean of the lower bounds is the highest in the most informative condition (treatment 1) and exceeds the mean in the control group by 10 points (K-W test: $p > .05$). Regarding the upper bounds, the difference between the means of treatment 1 and treatment 5 is weaker and equals -4.7 points (K-W test: $p > .05$). Moreover, I measure the size of the range of beliefs (dispersion) by calculating the difference between the limit bounds: *Max_Blue* - *Min_Blue*. Descriptive statistics are reported in Table 2.9. As expected, the difference is the largest in the control group (treatment

5) and the weakest in the treatment 1 [43]. This denotes higher confidence in the 50-50 belief regarding the composition of the bag in the most-informative condition than in the less-informative treatments although the differences between all treatments do not reach statistical significance (K-W test: $p > .05$).

	Treatment				
	1 (500) $N = 37$	2 (50) $N = 37$	3 (10) $N = 34$	4 (2) $N = 36$	5 (0) $N = 37$
mean	25.1	30.9	38	35.9	39.8
med	20	25	37.5	30	40
var	557.15	609.0	263.2	775.1	850.7
std	23.6	21.8	24.7	27.8	29.2

Table 2.9 – Difference between *Max_Blue* and *Min_Blue*

Note: med, var and std stand for median, variance and standard deviations, respectively.

This result can be related to Nicholls *et al.* (2015) who observe statistical learning in a 3-colors urn framework. Their urn contained 60 blue, 20 red and 20 yellow balls. In the absence of information, the respondents of the control group report roughly equal proportions for all colors (36-33-31 respectively, on average), whereas answers of participants who receive statistical feedback (54-25-21) converge to the true proportions. In my experiment, the subjects of the control group report 50-50 probability judgment as a reflection of their ignorance. This prior is confirmed by the information and as the number of draws rises, the size of the range of beliefs decreases which suggests that subjects feel more confident about their estimation. These findings are summarized in Result 5.

Result 5 *Increasing the precision of information tends to reduce the dispersion of beliefs on the composition of the ambiguous bag (without reaching statistical significance).*

43. According to (two-sample) K-W tests, the differences between dispersions are statistically significant at the 5% level for treatment 1 and treatment 5 and for treatment 1 and treatment 3, at the 10% level for treatment 1 and treatment 4, and not significant for treatment 1 and treatment 2.

2.4 Conclusion

In this chapter, I present an experiment designed to measure the effect of information precision on ambiguity attitudes and preference reversals. To this end, subjects are provided with sets of observations that inform on the ambiguous alternative. The informative signals differ within only one attribute: their precision (the number of observations varies across treatments), while the reported frequencies are kept constant.

Although most subjects exhibit Ellsberg-type preferences (Result 3), the valuations of the risky and ambiguous prospects do not differ significantly (Result 1), suggesting only weak ambiguity-aversion. An alternative explanation might be that DMs experience difficulties to provide absolute valuation for prospects whereas comparing alternatives is an easier task. Moreover, I did not find evidence that increasing the information precision increases the attractiveness of the ambiguous prospect (Result 2) although respondents seem to feel more confident in their estimation as the number of observations rises (Result 5). Finally, contrary to what was found by Trautmann *et al.* (2011), preference reversals are not eliminated even when using CE estimations (Result 4).

Several tests reported in this experiment do not provide statistically significant results. This has to be seen in the context of the small sample sizes of each treatment. Therefore, it may be worthwhile to conduct additional experiments with a larger pool of participants to gain further insights into the link between information precision and ambiguity attitudes. Moreover, the increase of information considered here is not valuable for frequentist DMs. Indeed, a frequentist is indifferent between any two datasets with different precision but identical frequencies. Therefore, the study of the relation between ambiguity perception and the informativeness of signals that differ within the two attributes (precision and frequencies) might exhibit different and more significant effects. This question will be addressed in Chapter 3.

Appendix

2.A Supplementary material for the experiment

2.A.1 Instructions (Treatment 1, translated from French)

Note: The sentences in brackets are not included in the instructions but inform the reader on the moment an action is performed.

— Start of Instructions —

Hello everybody, and thank you for accepting our invitation to participate in this experiment. My name is Roxane Bricet and I am working at the Economics lab of the University of Cergy-Pontoise. I am going to briefly present the outline of the experiment, please pay close attention to this short presentation.

First, some important rules which must be respected: It is important that the experiment takes place in silence, you are not allowed to communicate with other participants. The use of mobile phones and calculators is forbidden. No particular preliminary knowledge is needed to participate. As soon as the experiment starts, if you have any questions, please raise your hand and I will assist you.
Thank you in advance for your cooperation.

What do we do?

You are going to participate in an economic experiment on decision theory.
In this experiment, there are two opaque bags, A and B, each containing 100 balls. These balls can be either blue or red.

The composition of bag A is perfectly known. It contains exactly 50 blue balls and 50 red balls.

The composition of bag B is not known. However, partial information on the content of this bag is provided. Indeed, we have randomly drawn one ball from bag B and reported its color, then we have replaced it in bag B. This operation has been replicated 500 times and we have observed: 250 blue balls and 250 red balls. [This phrase was adjusted to reflect the information in the specific treatment (2, 3 and 4). In treatment 5, the last sentences are replaced by: "The composition of bag B is unknown.".]

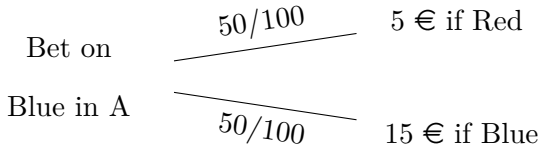
Note that nothing indicate that bag B contains as many blue balls as red balls!

The gambles:

During the experiment, you will be asked to consider gambles on bags A and B. Here is an example of gamble on bag A:

You are going to proceed to draw a ball at random from bag A. If the color of the ball is blue, you win 15€, if the ball is red you win 5€.

The composition of bag A being known, these gambles are presented in the following way:

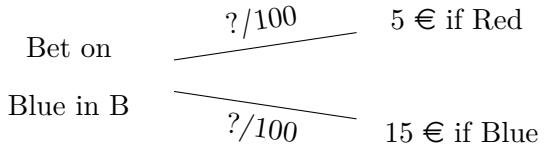


In other words, since 50 of 100 balls contained in bag A are blue, you have a chance of 50 out of 100 to win 15 € and a chance of 50 out of 100 to win 5 €.

Here is an example of a gamble on bag B:

You are going to proceed to draw a ball at random from bag B. If the color of the ball is blue, you win 15€, if the ball is red you win 5€.

The composition of bag B being unknown, these gambles are presented in the following way:

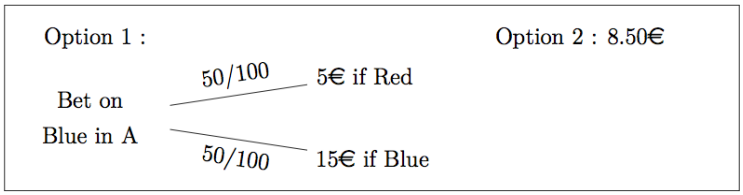


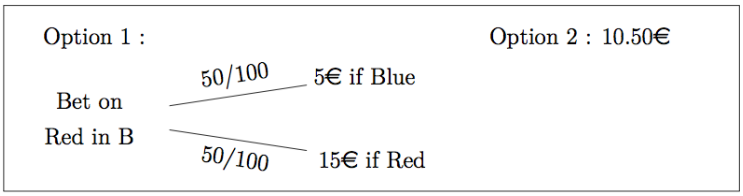
The envelopes:

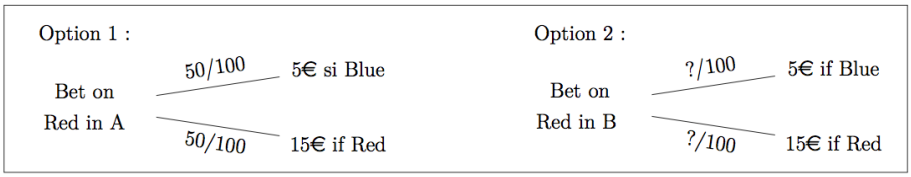
Before the beginning of the experiment, each of you will receive a **closed** envelope. Each envelope contains two options. These options differ across the envelopes. Your goal is to tell us which of the two options you prefer for each envelope in the experiment. At the end of the experiment, you will be rewarded depending on the content of your own envelope and your choices of options.

Important: There are no "right" or "wrong" answers in this experiment, it is only a matter of preferences.

Here are some examples of envelope content:





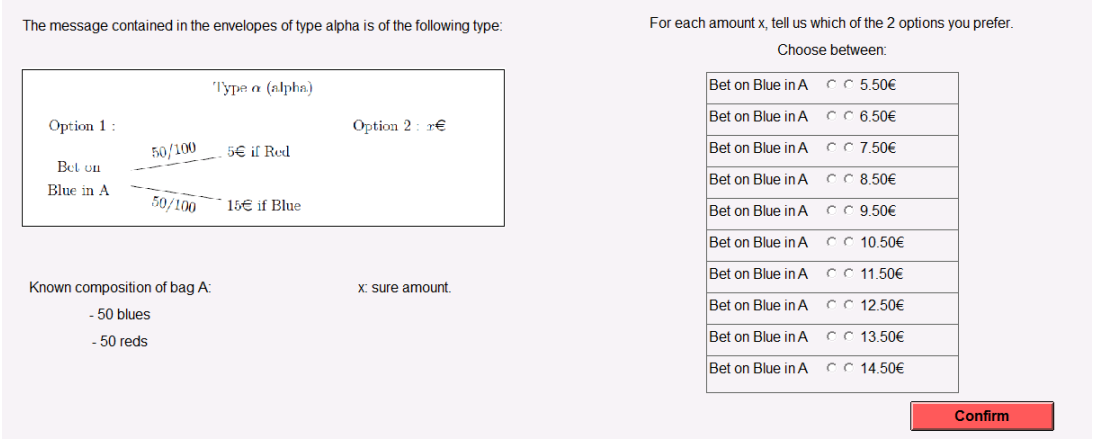


In concrete terms, there are 6 types of envelopes: type α (alpha), type β (beta), type γ (gamma), type δ, type ϵ and type ϕ (phi), corresponding to 6 different questions.

The options contained in the different types of envelopes are given in the following table:

envelopes	option 1	option 2
α	bet on Blue in A	x€
β	bet on Red in A	x€
γ	bet on Blue in B	x€
δ	bet on Red in B	x€
ϕ	bet on Blue in A	bet on Blue in B
ϵ	bet on Red in A	bet on Red in B

During the experiment, the contents of the envelopes will be presented to you as follows:



This is the message contained in the envelopes of type alpha. Option 1 corresponds to the bet on the draw of a blue ball from bag A. If you draw a blue ball, you win 15€, if you draw a red ball, you win 5€. Option 2 corresponds to a sure amount x in euros.

For each amount x, tell us which of the 2 options you prefer. To this end, fill out the following table. In this table, each line describes the content of an envelope.

The filling of this table is automated. When you tick a box of a line of the table, option 1 is automatically ticked for the lines above and option 2 is automatically ticked for the lines below.

Thus, it is enough for you to tick a single box of the table to fill out every lines of the table.

Indeed, if for the first line of the table you choose option 1 "Bet on Blue in A", tick option 1 on the first line of the table. Then, option 2 will be automatically ticked for the rest of the decisions, that is to say for the lines 2 to 10. If for the first line of the table, you choose option 2 "5.50€", tick option 2 on the first line of the table. Then, option 2 will be automatically ticked for the rest of the decisions, that is to say for the lines 2 to 10. [Each pattern of answers described in the instructions is illustrated with an example of a filled table.]

You may also choose option 1 for the second line of the table. In this case, tick option 1 on the second line of the table. Option 1 will be automatically ticked for the line 1

of the table, and option 2 will be automatically ticked for the lines 3 to 10 of the table. Otherwise, you may choose option 2 for the second line of the table. In this case, tick option 2 on the second line of the table. Option 1 will be automatically ticked for the line 1 of the table, and the option 2 will be automatically ticked for the lines 3 to 10 of the table.

Again, you may choose option 1 for the third line of the table. In this case, tick option 1 on the third line of the table. Option 1 will be automatically ticked for the lines 1 and 2 of the table, and option 2 will be automatically ticked for the lines 4 to 10 of the table. Otherwise, you may choose option 2 for the third line of the table. In this case, tick option 2 on the third line of the table. Option 1 will be automatically ticked for the lines 1 and 2 of the table, and option 2 will be automatically ticked for the lines 4 to 10 of the table.

And so on until the end of the table. You can choose option 1 for the tenth line of the table. In this case, tick option 1 on the tenth line of the table. Option 1 will be automatically ticked for the rest of the decisions, that is to say for the lines 1 to 9.

Note that you can revise your choice as many times as needed before confirming your choice and proceeding to the following question.

There are 60 envelopes, 10 of each type.

These envelopes are numbered from 1 to 60. Three of you are now asked to check their numbering.

With this established, I am going to walk among you and each of you will randomly draw one of the envelopes.

DO NOT OPEN YOUR ENVELOPE!

Anyone who opens his envelope will be immediately excluded from the experiment and will not receive any financial reward.

YOUR PAYMENT:

In this experiment, the minimum gain is 5 euros and the maximum gain is 15 euros.

Reminder: There are no "right" or "wrong" answers in this experiment, it is only a matter of preferences.

Plan of the experiment:

In practice, you will answer the questions of the experiment using the computer in front of you.

First, you will be asked some comprehension questions to check your understanding of the instructions. *Your answers to these questions do not affect your payment.*

Then, the experiment on your choices of options will start. The experiment consists of 6 successive screens for the 6 types of envelopes (6 types of different questions).

Finally, you will be asked to fill out a short complementary questionnaire to get to know you better. *Your answers to these questions do not affect your payment.*

The last screen will invite you to reach the experimenter's office to proceed to the payment.

In the office of the experimenter:

I will open the envelope in front of you and I will just implement what you chose during the session. If the option chosen during the session is a bet: you will proceed to draw a ball at random from the bag involved in the bet. If the drawn ball matches the winning color, you win 15€; otherwise, you only win 5€. If the option is a sure amount x: you get the x€ gain.

Because you do not know the content of your envelope and because I will implement your choices, it is in your best interest to tell us your preferred option at each question. Indeed, if you tell us what you want, your preferred option will be the one implemented!

Verification:

At the end of the experiment, you will get a list describing the contents of all envelopes.

This list describes the content of each envelope: number of the envelope, type of the envelope, option 1, option 2.

You will then check that our description of the numbered envelopes was truthful.

You will also be able to check that the list does contain the 6 mentioned tasks.

Your answers will be kept strictly confidential and anonymous, henceforth, feel free to answer as you like. Moreover, there is no time limit, so take all the time you need to read the instructions and answer the questions.

Do you have any questions?

If everything is clear, you can now start the experiment!

— End of Instructions —

2.A.2 Detailed script

0. *Preliminaries:* The 60 envelopes are grouped in 20-tuples (1-20; 21-40; 41-60). The envelopes and the opaque bags A and B are placed on the table in the front of the room.
1. The participants are welcome at the doorstep of the lab. Before entering the room, they randomly pick a number that determines where they sit in the room. They are also asked to sign a participation consent before the start of the session.
2. The experimenter gives the instructions in the form of an oral presentation with slides. At the relevant points during the explanations:
   - The bags A and B are showed.
   - Each 20-tuple of envelopes is checked for the presence of every number by a volunteering subject, and then thrown into an opaque bag. Each participant then randomly picks one envelope from the bag.
3. *Computerized:* The participants are asked to answer a short comprehension test to

check their understanding of the experiment. No one can reach the next stage before everybody has answered correctly.

4. *Computerized:* The participants perform the 6 tasks of the experiment.
5. *Computerized:* Stage 5 consists of filling out a short complementary questionnaire. This form consists of:
   — individual estimations of the composition of bag B;
   — socio-demographic variables: sex, age, level of education.
6. When they are done, the participants reach the office of the experimenter. The payment phase proceeds individually:
   6.1 The experimenter opens the envelope of the subject and reads aloud the note contained inside.
   6.2 The experimenter recalls the answer of the participant via the computerized experimental data collection.
   6.3 The option chosen by the participant is implemented: if it is a sure amount, the participant gets the sure gain; if it is a bet in a bag, the participant proceeds to draw a ball at random from the bag in question.
   6.4 The participant receives the corresponding payment and signs a receipt.
   6.5 The participant gets the verification lists describing all envelopes in order to check that the envelopes were truthfully described.
7. The participants leave the room.

2.A.3 Verification lists (Treatment 1, session 1)

List of envelopes by type

Envelope nr.	Type	Option 1	Option 2
45	Type α (alpha)	Bet on Blue in A	5.50 €
57	Type α (alpha)	Bet on Blue in A	6.50 €
24	Type α (alpha)	Bet on Blue in A	7.50 €
1	Type α (alpha)	Bet on Blue in A	8.50 €
28	Type α (alpha)	Bet on Blue in A	9.50 €
16	Type α (alpha)	Bet on Blue in A	10.50 €
32	Type α (alpha)	Bet on Blue in A	11.50 €
41	Type α (alpha)	Bet on Blue in A	12.50 €
27	Type α (alpha)	Bet on Blue in A	13.50 €
25	Type α (alpha)	Bet on Blue in A	14.50 €
58	Type β (bêta)	Bet on Red in A	5.50 €
59	Type β (bêta)	Bet on Red in A	6.50 €
26	Type β (bêta)	Bet on Red in A	7.50 €
29	Type β (bêta)	Bet on Red in A	8.50 €
17	Type β (bêta)	Bet on Red in A	9.50 €
18	Type β (bêta)	Bet on Red in A	10.50 €
13	Type β (bêta)	Bet on Red in A	11.50 €
30	Type β (bêta)	Bet on Red in A	12.50 €
39	Type β (bêta)	Bet on Red in A	13.50 €
14	Type β (bêta)	Bet on Red in A	14.50 €
3	Type γ (gamma)	Bet on Blue in B	5.50 €
11	Type γ (gamma)	Bet on Blue in B	6.50 €
20	Type γ (gamma)	Bet on Blue in B	7.50 €
5	Type γ (gamma)	Bet on Blue in B	8.50 €
42	Type γ (gamma)	Bet on Blue in B	9.50 €
36	Type γ (gamma)	Bet on Blue in B	10.50 €
22	Type γ (gamma)	Bet on Blue in B	11.50 €
7	Type γ (gamma)	Bet on Blue in B	12.50 €
40	Type γ (gamma)	Bet on Blue in B	13.50 €
44	Type γ (gamma)	Bet on Blue in B	14.50 €
49	Type δ (delta)	Bet on Red in B	5.50 €
50	Type δ (delta)	Bet on Red in B	6.50 €
4	Type δ (delta)	Bet on Red in B	7.50 €
31	Type δ (delta)	Bet on Red in B	8.50 €
46	Type δ (delta)	Bet on Red in B	9.50 €
55	Type δ (delta)	Bet on Red in B	10.50 €
34	Type δ (delta)	Bet on Red in B	11.50 €
23	Type δ (delta)	Bet on Red in B	12.50 €
8	Type δ (delta)	Bet on Red in B	13.50 €
21	Type δ (delta)	Bet on Red in B	14.50 €
38	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
54	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
12	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
43	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
47	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
6	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
35	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
60	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
53	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
2	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
10	Type θ (theta)	Bet on Red in A	Bet on Red in B
52	Type θ (theta)	Bet on Red in A	Bet on Red in B
56	Type θ (theta)	Bet on Red in A	Bet on Red in B
48	Type θ (theta)	Bet on Red in A	Bet on Red in B
19	Type θ (theta)	Bet on Red in A	Bet on Red in B
51	Type θ (theta)	Bet on Red in A	Bet on Red in B
15	Type θ (theta)	Bet on Red in A	Bet on Red in B
33	Type θ (theta)	Bet on Red in A	Bet on Red in B
9	Type θ (theta)	Bet on Red in A	Bet on Red in B
37	Type θ (theta)	Bet on Red in A	Bet on Red in B

List of envelopes by number

Envelope nr.	Type	Option 1	Option 2
1	Type α (alpha)	Bet on Blue in A	8.50 €
2	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
3	Type γ (gamma)	Bet on Blue in B	5.50 €
4	Type δ (delta)	Bet on Red in B	7.50 €
5	Type γ (gamma)	Bet on Blue in B	8.50 €
6	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
7	Type γ (gamma)	Bet on Blue in B	12.50 €
8	Type δ (delta)	Bet on Red in B	13.50 €
9	Type θ (theta)	Bet on Red in A	Bet on Red in B
10	Type θ (theta)	Bet on Red in A	Bet on Red in B
11	Type γ (gamma)	Bet on Blue in B	6.50 €
12	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
13	Type β (bêta)	Bet on Red in A	11.50 €
14	Type β (bêta)	Bet on Red in A	14.50 €
15	Type θ (theta)	Bet on Red in A	Bet on Red in B
16	Type α (alpha)	Bet on Blue in A	10.50 €
17	Type β (bêta)	Bet on Red in A	9.50 €
18	Type β (bêta)	Bet on Red in A	10.50 €
19	Type θ (theta)	Bet on Red in A	Bet on Red in B
20	Type γ (gamma)	Bet on Blue in B	7.50 €
21	Type δ (delta)	Bet on Red in B	14.50 €
22	Type γ (gamma)	Bet on Blue in B	11.50 €
23	Type δ (delta)	Bet on Red in B	12.50 €
24	Type α (alpha)	Bet on Blue in A	7.50 €
25	Type α (alpha)	Bet on Blue in A	14.50 €
26	Type β (bêta)	Bet on Red in A	7.50 €
27	Type α (alpha)	Bet on Blue in A	13.50 €
28	Type α (alpha)	Bet on Blue in A	9.50 €
29	Type β (bêta)	Bet on Red in A	8.50 €
30	Type β (bêta)	Bet on Red in A	12.50 €
31	Type δ (delta)	Bet on Red in B	8.50 €
32	Type α (alpha)	Bet on Blue in A	11.50 €
33	Type θ (theta)	Bet on Red in A	Bet on Red in B
34	Type δ (delta)	Bet on Red in B	11.50 €
35	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
36	Type γ (gamma)	Bet on Blue in B	10.50 €
37	Type θ (theta)	Bet on Red in A	Bet on Red in B
38	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
39	Type β (bêta)	Bet on Red in A	13.50 €
40	Type γ (gamma)	Bet on Blue in B	13.50 €
41	Type α (alpha)	Bet on Blue in A	12.50 €
42	Type γ (gamma)	Bet on Blue in B	9.50 €
43	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
44	Type γ (gamma)	Bet on Blue in B	14.50 €
45	Type α (alpha)	Bet on Blue in A	5.50 €
46	Type δ (delta)	Bet on Red in B	9.50 €
47	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
48	Type θ (theta)	Bet on Red in A	Bet on Red in B
49	Type δ (delta)	Bet on Red in B	5.50 €
50	Type δ (delta)	Bet on Red in B	6.50 €
51	Type θ (theta)	Bet on Red in A	Bet on Red in B
52	Type θ (theta)	Bet on Red in A	Bet on Red in B
53	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
54	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B
55	Type δ (delta)	Bet on Red in B	10.50 €
56	Type θ (theta)	Bet on Red in A	Bet on Red in B
57	Type α (alpha)	Bet on Blue in A	6.50 €
58	Type β (bêta)	Bet on Red in A	5.50 €
59	Type β (bêta)	Bet on Red in A	6.50 €
60	Type ϵ (epsilon)	Bet on Blue in A	Bet on Blue in B

2.A.4 Participation consent form







CONSENTEMENT A PARTICIPER A UNE SESSION EXPERIMENTALE EN ECONOMIE

Nom, Prénom : .....

Adresse : .....

Adresse Email * : .....

(dénommé par la suite « le participant »)

Le participant consent librement à prendre part à la session expérimentale suivante :
Dossier N°2016-2017 n° 18

Date de la session expérimentale : 20/07/2017.....
Durée maximum (en heures) : 1h.....

Le montant de la compensation reçue à l'issue de la session expérimentale par le participant dépend des résultats obtenus durant cette expérience. Ce montant est au minimum de 5 € (Euros) à condition de respecter les règles de l'expérience (notamment ne pas ouvrir l'enveloppe fermée reçue au début de la session).
Les données issues de cette expérience seront traitées de façon anonyme

Fait à Paris, le 20/07/2017

Signature du participant contractant

*** Votre adresse Email et adresse postale ne seront pas communiquée. Elles ont pour but de nous permettre de vous contacter en cas de contrôle afin que vous nous confirmiez le montant reçu au titre de l'expérience.**

2.B Complementary tables and figures

Treatment	Female	Age	Education
1	54%	26	3 years
2	65%	27	3 years
3	53%	25	2 years
4	72%	31.5	3 years
5	59%	30	3 years

Table 2.10 – Socio-demographic variables

Note: *Age* and *Education* correspond to median values. *Education* stands for the number of completed years in higher education.

Treatment	Bag	Draws			Bag composition	
		Total	Blue	Red	Blue	Red
1	B1	500	250	250	47	53
2	B2	50	25	25	43	57
3	B3	10	5	5	38	62
4	B4	2	1	1	10	90
5	B5	0	0	0	90	10

Table 2.11 – Bags composition

Note: See Section 2.2.5 for the description of the procedure.
Lecture: Participants of treatment 1 are informed that 500 balls have been randomly drawn with replacement from bag B1, 250 were blue and 250 were red (public information). The bag B1 contains actually 47 blue balls and 53 red balls (non-public information).

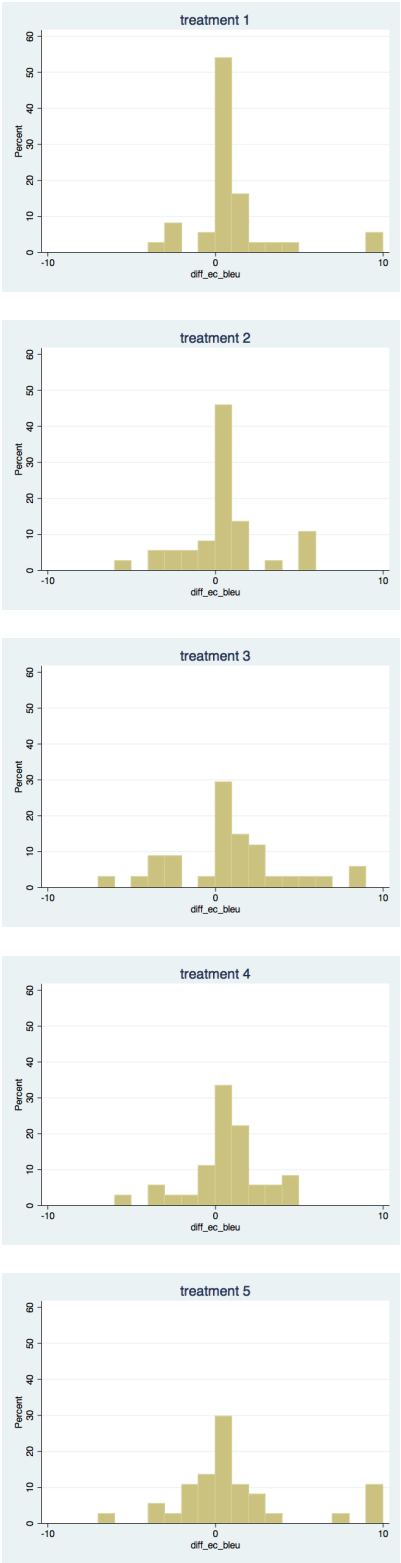


Figure 2.3 – Histograms of the differences between CEs on blue ($CE_A(blue) - CE_B(blue)$), by treatment

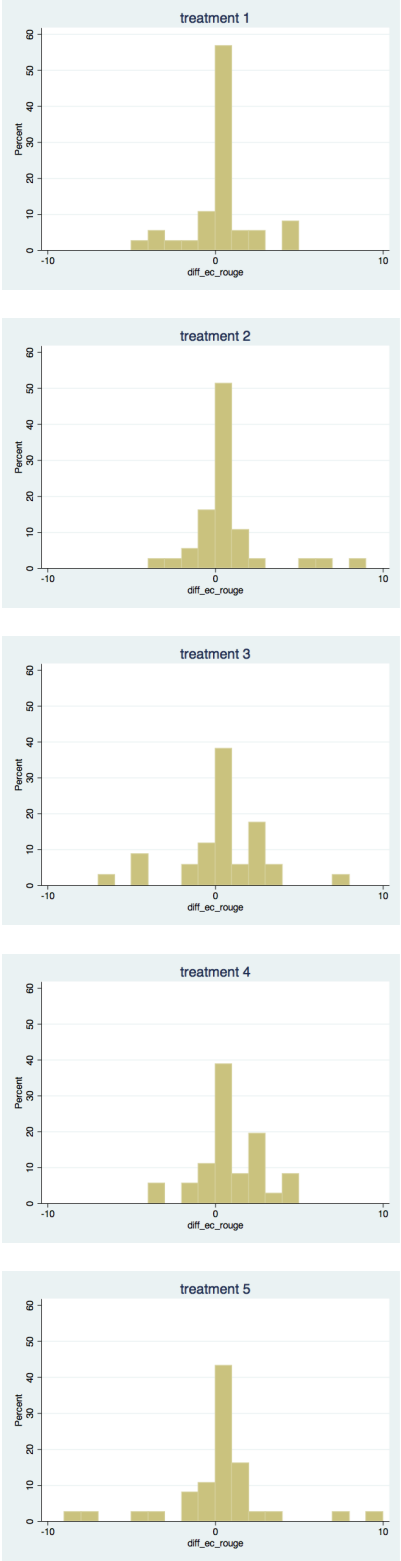


Figure 2.4 – Histograms of the differences between CEs on red ($CE_A(red) - CE_B(red)$), by treatment

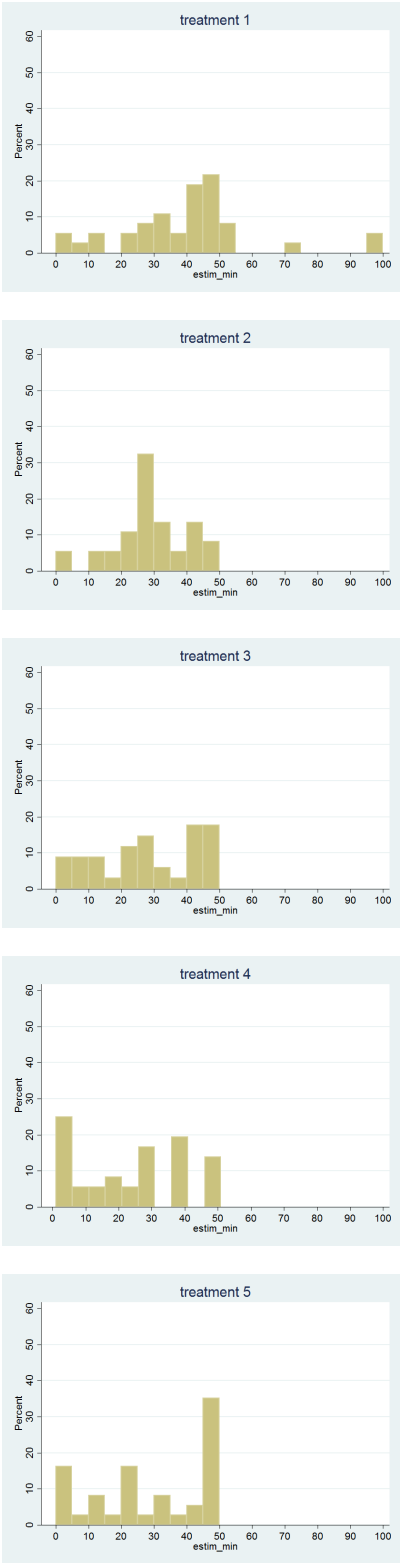


Figure 2.5 – Histograms of the estimates on the proportion of blue balls in the ambiguous bag, by treatment

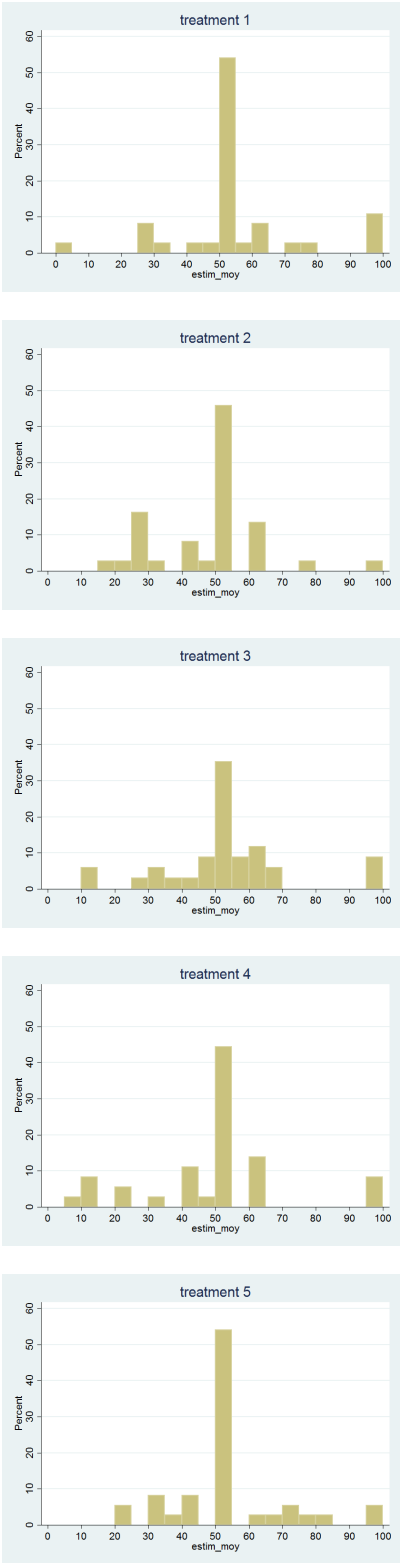


Figure 2.6 – Histograms of the estimates on the proportion of blue balls in the ambiguous bag, by treatment

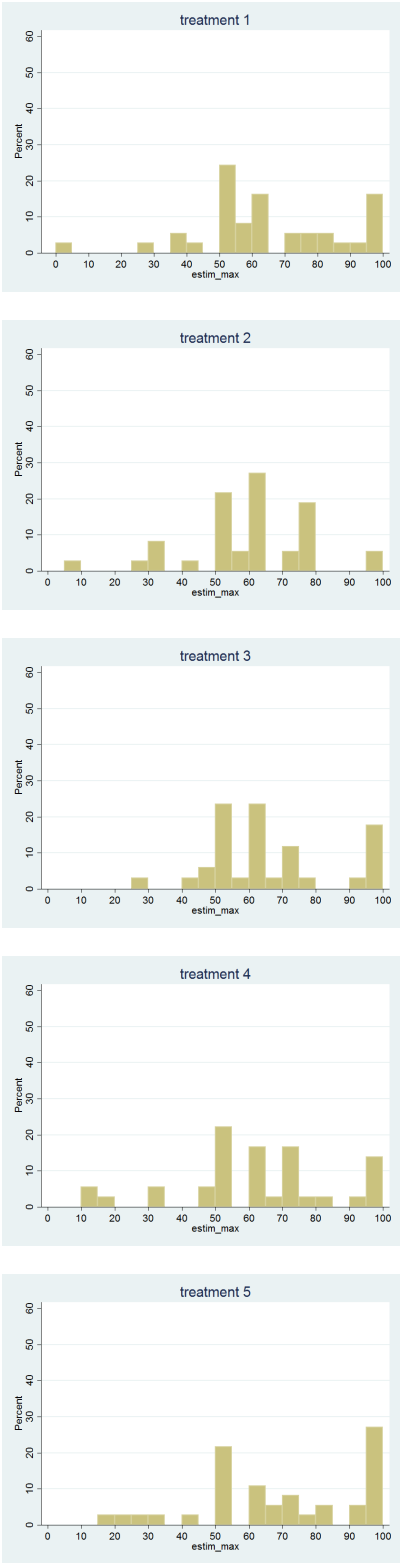


Figure 2.7 – Histograms of the estimates on the proportion of blue balls in the ambiguous bag, by treatment

Chapter 3

The value of instrumental information

3.1 Introduction

3.1.1 Motivation

In most decision situations, the available information is insufficient to allow agents to predict the outcome of an action with full confidence. However, a Decision Maker (henceforth, DM) who can observe statistical information in the form of data generated by the true probabilities will become increasingly confident in his personal estimates. Hence, subjective beliefs may converge to objective probabilities through statistical learning. In this chapter, I focus on the subsequent question: How do agents value instrumental information?

While statistical observations are theoretically not valuable for simple risky prospects (e.g., betting on the result of the toss of a fair coin), it might inform decisions under compound risk. Consider the decision situation in which the state of the world is unknown but drawn from a known and well-defined distribution function. For instance, suppose that

you have to bet on the result of the toss of a coin which is chosen uniformly at random between a fair one and a biased coin that comes up tails with objective probability 9/10. Since data collection might inform on the true state of the world (the coin that is tossed), the value of information under compound risk is a worthwhile question. However, empirical evidence on the valuation of informative signals in such risky framework is missing.

As opposed to risk, a decision situation is characterized by ambiguity when the agents are not able to derive a unique and additive probability measurement from the informational context (Knight, 1921). This is the case when the probability distribution over the states of the world cannot be inferred from the available data. Suppose, for instance, that you are asked to bet on the result of the toss of a coin but now, you have no clue on the fairness of the coin. While the study of choice under ambiguity has been one of the main topics in decision theory over the last decades, the related question of the value of information that mitigates or resolves ambiguity has attracted seemingly less attention.

Therefore, this chapter is intended to further investigate how DMs value instrumental information under *uncertain* decision contexts — the word 'uncertainty' is used, here, as a general concept that encompasses both compound risk and ambiguity. To answer this research question, I provide an operational experimental design which allows to estimate the reservation price of agents for informative signals on the objective probability structure of uncertain prospects. Discriminating between different relevant signals requires the definition of a sorting criterion depending on their informativeness. This is provided by Blackwell's (1951, 1953) seminal papers in which he proposes a ranking of information structures independent of beliefs. According to his equivalence result, experiment I (i.e., a sampling procedure) is more "Blackwell-informative" than experiment II if the former is more valuable than the latter for all expected-utility maximizers. Li and Zhou (2016) extend Blackwell's equivalence result to a general class of ambiguity-averse preferences. It can be inferred from their analysis that both ambiguity-averse and ambiguity-loving DMs should place higher value on informative data as its degree of informativeness increases.

This chapter aims at improving understanding about the pricing of informative signals by ambiguity-sensitive agents. For ambiguity-neutral DMs who behave in accordance with Subjective Expected Utility (SEU) theory, it provides also experimental assessments of the Blackwell result, which lacks empirical evidence.

The main contribution of this research is to provide a unified Blackwell's experimental framework to measure the value of information in uncertain settings. Next, the effect of the informativeness on the value of the signal can be estimated and examined in the light of Blackwell equivalence result and its extension. With my design, it is also possible to gain insight into the link between the value of information and ambiguity attitudes, which has not been addressed in the literature so far to the best of my knowledge. At last, this research is conducted in a model-free set-up. Since I do not restrain the analysis to a particular class of non-EU preferences, it provides a general testing context.

The design proposed here is an extension of Ellsberg's (1961) two-color experiment when samples of draws with replacement are available. In a laboratory experiment, the subject is asked to consider gambles on three different urns containing 100 (blue or red) balls each. The first urn contains 50 blue and 50 red balls (simple risk); the number of blue balls in the second urn is uniformly distributed between 0 and 100 (compound risk); the composition of the third urn cannot be deduced from an objective distribution function (ambiguity). The gambles offer the participant to bet on his preferred color without information (for all urns) and with information (for the last two urns). Especially, the Certainty Equivalents (CEs) are estimated for different number of random draws, where weakly-informative signals contain small number of draws and highly-informative signals consist of large number of draws. This design intends to simulate a realistic process of accumulation of information. The first urn serves as a benchmark: it allows to distinguish between attitudes toward risk and ambiguity and to study agents' ability to reduce compound lotteries. While controlling for ambiguity attitudes, this design enables experimental assessments of: (*i*) the value attributed to informative signals and (*ii*) the

individual ranking of signals of different informativeness. One key feature of the design relies on the sequence of the bet: the subject is asked to announce his signal-contingent strategy *before* observing the draws. Indeed, Li et al.'s (2016) statement requires full commitment to hold. It is motivated by the fact that an ambiguity-averse DM might be dynamically inconsistent and the inner conflict between ex-ante and ex-post preferences could make him reject information in the absence of commitment (Siniscalchi, 2011).

The novelty of the approach also lies in the payment procedure: the choices of participants are incentivized according to the Prince mechanism, recently proposed by Johnson *et al.* (2015). Therefore, I depart from the related literature that incentivizes decisions with the standard techniques (Random-Incentive-System, Becker-DeGroot-Marschak) even though these mechanisms have been criticized for their inability to induce truthful revelation of preferences in ambiguous contexts (see, e.g., Bade, 2015; Baillon *et al.*, 2015). The features of Prince allow to overcome inherent drawbacks of other techniques and, thus, ensure incentive-compatibility of experimental choice questions.

The remainder of the chapter proceeds as follows. Related literature is reviewed next. Section 3.2 presents the experimental design. Section 3.3 discusses the theoretical background and its requirements. Section 3.4 concludes. Appendix 3.A explains how the described signals can be compared with respect to their informativeness in Blackwell's sense. Appendix 3.B displays a full set of the instructions of the experiment.

3.1.2 Related literature

Marinacci (1999) presents a relevant mathematical framework for learning under ambiguity and derives a Law of Large Number that applies to non-additive probabilities in the context of capacities. In 2002, he studies the particular case of learning by sampling with replacement from the same ambiguous urn and shows that ambiguity resolves asymptotically. Most recently, Marinacci and Massari (2017) generalize the previous result to the case in which the DM have misspecified prior beliefs over the decision problem and

they prove that ambiguity may still fade away under standard assumptions. Besides, Epstein and Schneider (2007) and Zimper and Ma (2017) generalize the Bayesian learning model within the decision-theoretic framework of Gilboa and Schmeidler (1989) Maxmin Expected Utility (MEU) preferences. So far, the theoretical literature fails to provide suitable conditions that apply to a general class of ambiguity preferences and thus it does not ensure that ambiguity would systematically resolve with experience.

Blackwell (1951, 1953) defines an informativeness ranking of data structures. According to his main theorem, structure I is more "Blackwell-informative" than structure II if II is a garble [44] of I. For any decision problem and subjective prior, the expected loss supported by the DM from choices based on I is at least as low as the expected loss resulting from choices based on II. Hence, I is preferred to II. However, Wakker (1988) shows that violation of the Independence Axiom (as formulated by Von Neumann and Morgenstern, 1944) might lead to the existence of decision situations in which information-aversion is exhibited, in contradiction to Blackwell's theorem. The rationale for this phenomenon lies in the typical non-compliance with the commutativity principle of non-EU maximizers and their subsequent violations of dynamic consistency. Moreover, Siniscalchi (2011) and Epstein and Ji (2017) study the dynamics of information acquisition under ambiguity. The resulting optimal stopping problem is solved and they show that rejection of learning opportunities can be optimal for an ambiguity averse agent given a small cost of information (Epstein and Ji, 2017) or even when the information is freely available (Siniscalchi, 2011). Çelen (2012) was the first to apply Blackwell's ranking to ambiguity-sensitive preferences in the case of MEU preferences. Further, Heyen and Wiesenfarth (2015) draw on Çelen's framework and provide a definition of the value of information that is compatible with recursive utility and thus respects dynamic consistency. Lately, Li and Zhou (2016) show that Blackwell's equivalence result holds for a general class of ambiguity-averse preferences, under reduction and commitment.

44. A garble is a transformation from one informative structure to another which adds noise to the original structure. See Appendix 3.A for a formal definition of a garble.

The literature on the value of information and ambiguity is limited. Using Machina's (2004) concept of almost-objective acts, Quiggin (2007) shows that ambiguity aversion may be defined in terms of the value of information. He states that, for expected utility preferences, the value of information with respect to almost-objective acts is asymptotically equal to zero. Snow (2010) studies the value of information in a particular adaptation of the smooth ambiguity model (Klibanoff *et al.*, 2005). His analysis relies on a particular definition of greater ambiguity in terms of mean-preserving spreads. Given this framework, he proves that ambiguity averters (lovers) are willing to pay to obtain (avoid) information that lessens ambiguity while leaving the decision maker's expectations unchanged [45]. Similarly, Eichberger and Guerdjikova (2013) study preferences for information precision in an adaptation of the Maxmin Expected Utility model. Increasing the number of observations while keeping frequencies constant enhances the precision of information in an objective sense and therefore the informativeness of the data. Controlling for the frequency of observations, they show that the value attributed to precise sets of observations rises with greater ambiguity aversion. Although this theoretical literature provides compelling insights, the experiment presented in this chapter leads to a substantially different analysis since additional information affects both the ambiguity perceived by the DM and his expectations.

Reviewing the most recent experimental research on ambiguity, Trautmann and Van De Kuilen (2015) point out that there are only few experimental studies on how learning influences ambiguity attitude, and how ambiguity attitude may affect the decision to experiment and learn. Regarding the effect of information on beliefs and ambiguity attitudes, the existing studies display particularly mixed evidence. Nicholls *et al.* (2015) find that statistical learning does not reduce the number of violations of the basic Savage's sure-thing principle compared to a control group. On the other hand, Baillon *et al.*

45. Besides, Snow (2010) also states that the value of information which *resolves* ambiguity increases with greater ambiguity and with greater ambiguity aversion. Note that, in this chapter, I will only consider information that *reduces* ambiguity.

(2017) observe that subjects' beliefs move towards Expected Utility with more information although substantial deviations remain even in their maximum information condition. Moreno and Rosokha (2016) estimate a behavioral model of belief updating in a three-urns design closed to mine. They note that learning behavior in the compound urn is consistent with Bayesian updating whereas participants tend to overweigh additional information in ambiguous frameworks. As predicted by Prospect Theory, Abdellaoui *et al.* (2016) report that subjects tend to be more ambiguity-averse for likely events and more ambiguity-loving for rare events when subjects can learn about the composition of an ambiguous urn. Moreover, they find that participants exhibit less ambiguity-sensitive behavior as the sample size increases although statistical learning does not allow to preclude ambiguity-nonneutral preferences. On the contrary, Ert and Trautmann (2014) show that sampling completely reverses traditional patterns of preferences: they observe ambiguity-aversion with low frequencies and ambiguity-seeking with high frequencies.

Regarding the preferences for information, Trautmann and Zeckhauser (2013) report that subjects forgo opportunities to learn about an ambiguous urn in favour of the risky urn, even in an extreme learning experience (complete resolution of ambiguity). Ambuehl and Li (2018) find that subjects underreact to the increase of informativeness of data and they have a strong preference for information that entirely resolves ambiguity. Further, Eliaz and Schotter (2010) report that experimental subjects are willing to pay for information that will not affect their decision (non-instrumental information). This result contrasts with Hoffman (2016) who finds that agents exhibit significant overconfidence about their knowledge, inducing low demand information.

So far, some recent papers have examined the influence of information on ambiguity attitudes, but experimental evidence regarding the effects of ambiguity attitudes on the willingness to pay for additional information is clearly missing. This research aims at providing a relevant framework to study this open question.

3.2 Experimental design

3.2.1 Stimuli

The experimental design draws on Ellsberg's (1961) two-color urns experiment. In this experiment, the participants are shown three opaque bags. They are informed that each of the three bags contain 100 balls and that balls can be either blue or red. The subjects could win a fixed amount of money by betting on the color of their choice to be blindly drawn from a bag by themselves. More specifically, the gambles on the color of the ball pay 15€ if the color of the drawn ball matches the color of the bet, 5€ otherwise. The composition of the first bag is perfectly known to the subjects or equivalently *certain*: it contains exactly 50 blue balls and 50 red balls. The exact proportions of balls in the second and third bags remain unknown but they differ with respect to the delivered description of their composition. The participants know that the composition of the second bag is determined using a uniform distribution over the 101 possible bags containing 100 balls, the composition of this bag is then *risky*. The composition of the third bag, however, is *ambiguous* in the sense that there is no such objective distribution device that defines the proportions of balls: the number of blue (or red) balls in the bag is determined by a number drawn from a pool of 200 numbers ranging from 0 to 100, with no hints on the frequencies of each number in the pool except that each number appears at least once. For the sake of brevity, in the rest of the chapter, the three bags are referred to as the certain bag, the risky bag and the ambiguous bag respectively [46]. Subsection 3.2.2 provides a detailed description of the procedure to generate the bags. Since the composition of the certain bag is known, objective probabilities can be easily inferred and betting on a color from this bag relates to simple risky prospects.

The risky and ambiguous bags refer to different ways to generate uncertainty in the lab. The former have been substantially used in the experimental research (e.g., Becker

46. During the experimental sessions, note that one should refer to bags with neutral labels such as bag A, bag B and bag C.

and Brownson, 1964; Maafi, 2011). Although this transparent technique enhances the trust of participants in the experimental design, this condition may not be perceived as real ambiguity. Indeed, the prospects related to this bag result from a compound lottery which allows subjects who satisfy the Reduction Of Compound Lotteries (ROCL) axiom to use second-order objective probabilities to inform their beliefs. Recent empirical studies display contradictory findings on the link between ambiguity attitudes and multiple-stage lotteries. For instance, Halevy (2007) finds equivalence between ROCL and ambiguity-neutrality, whereas ambiguity-sensitive agents generally fail to reduce compound lotteries. Moreover, Halevy *et al.* (2008) and Seo (2009) provide axiomatizations which explicitly relate attitudes towards ambiguity and two-stage lotteries while accomodating Halevy's (2007) findings. On the other hand, Abdellaoui *et al.* (2015) report contrasting empirical evidence: they find that a substantial share of ambiguity-neutral agents violates the ROCL axiom while most of the subjects who successfully reduce two-stage lotteries exhibit non-neutral ambiguity attitudes [47]. Finally, the strategy used to generate the third bag (ambiguous bag) described previously can be viewed as an attempt to create an ambiguous condition with a tractable and credible device that prevents subjects from using probability reduction.

Such "three-urns" design has been previously implemented in two other articles, Yates and Zukowski (1976) and Chow and Sarin (2002), who study standard Ellsberg-type preferences without learning. They found evidence that the two-stage lottery occupies an intermediate position: it is usually preferred to the ambiguous prospect but less attractive than the simple one-stage objective lottery. Therefore, combining both strategies to generate uncertainty in this experiment allows me to estimate the differential effects of this distinct types of uncertainty on the value of information while relating to existing sparse research.

47. See also Attanasi *et al.* (2014) who conduct an experimental study on ambiguity attitudes using bets with unknown probabilities, as well as compound lotteries, when preferences are specified within the smooth ambiguity model (Klibanoff *et al.*, 2005).

The experiment consists of three stages: in the first stage, the participant's attitude towards risk is estimated, this preliminary step is necessary to disentangle risk attitude from ambiguity attitude in the subsequent data analysis; in the second stage, the individual reservation price for information about the uncertain bag is elicited for varying degrees of informativeness; the third stage replicates the questions of the second stage on the ambiguous bag. All subjects participate to each stage in that order.

Stage 1. In this stage, the participant deals with the certain bag. First, he has to choose the color of the ball on which to bet to be drawn from this bag. Then, he is asked to express preferences between the bet on the color of the ball previously chosen and different sure amounts. He is presented with a series of 10 binary choices where the sure amounts is increased in order to elicit the subject's Certainty Equivalent (CE, defined as the sure amount equally desirable as the gamble) for such risky prospect (CE_C). The CE is reported using a Multiple-Price List (MPL). More precisely, in each row of the MPL, the participant has to single out his preferred option between Option 1: "Bet on the color of the ball" or Option 2:"x€", with x taking the 10 equally spaced values between 5.50 and 14.50. The monetary amounts rise moving down the table while the bet remains the same. Figure 3.1 provides an example of such MPL. By linear interpolation, the CE is taken as the mid-point of the two sure amounts for which the subject switched preferences. Moreover, to avoid inconsistent answers resulting from multiple switching points, a computer program enforces the monotonicity of revealed preferences by allowing at most one switching point from the gamble to the sure amount. More specifically, the filling of the table is automated: by ticking a box of a line of the table, the boxes corresponding to the bet are automatically ticked for the lines above and the boxes corresponding to the sure amount are automatically ticked for the lines below [48].

48. See Chapter 2 for a thorough discussion on the advantages of the MPL as compared to other valuation methods and on the automatic filling of the table.

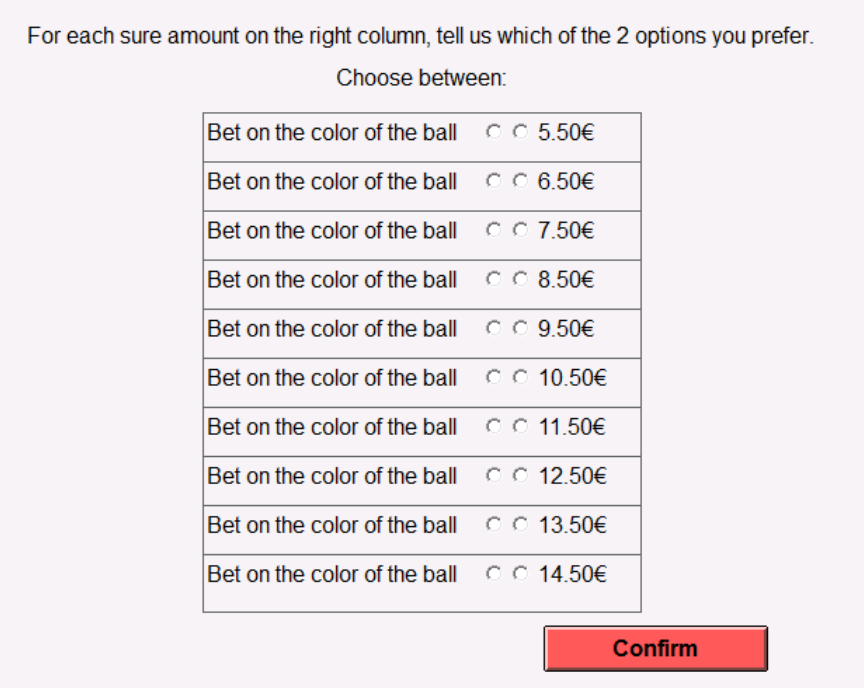


Figure 3.1 – Example of MPL

Stage 2. In the second stage, the participant has to consider bets on the risky bag. This stage consists of two successive phases:

Non-informational phase. First, the subject is asked to select a color on which to bet for a draw from the risky bag. Then, the CE for the related prospect is elicited when no information is available ($CE_{R,0}$). The CE is estimated via a similar procedure as in stage 1: the subject has to fill out a MPL with 10 binary choices between a bet on the color of the ball to be drawn from the risky bag and a sure gain.

Informational phase. Next, the participant is asked to consider different informational situations in which a sample of random draws with replacement from the risky bag is available. This phase consists of 5 informational situations corresponding to datasets of 5 different lengths ranging from weakly-informative for small number of draws to highly-informative for large number of draws. The number of draws (d) are fixed and $d \in \mathcal{D} = [1; 5; 20; 50; 200]$. For each informational condition: First, the subject has to an-

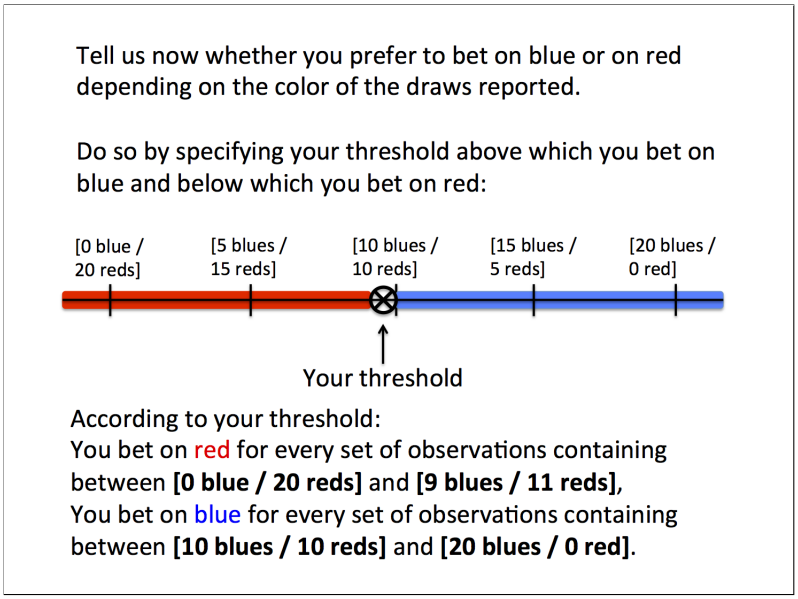


Figure 3.2 – Example of a slider when the dataset contains 20 draws

nounce his color-betting strategy conditional on the resulting draws of the dataset. There, he specifies the particular signal, i.e., the minimum number of blue draws out of the d draws and the corresponding maximum number of red draws, above which he chooses to bet on a blue ball to be drawn and below which his bet goes on red. The subject selects this threshold ($t_{R,d}$) using an horizontal slider as proposed in Figure 3.2. Then, the subject is asked to fill out a MPL with 10 binary choices between a bet on the color of the ball to be drawn from the risky bag conditional on the information and a sure gain. Hence, the CE ($CE_{R,d}$) is elicited via the aforementioned procedure.

Stage 3. The last stage replicates the phases of stage 2 on the ambiguous bag. Thus, this stage starts with the non-informational phase which serves to elicit the CE for the ambiguous prospect without information ($CE_{A,0}$). The signal-contingent strategies ($t_{A,d}$) and the CEs associated to the informative signals ($CE_{A,d}$) are collected for the 5 different lengths of datasets ($d \in \mathcal{D}$) specified above in the subsequent informational phase.

Table 3.1 summarizes the structure of the experiment, the tasks performed by the participant and the experimental variables collected. The informational phases proceed by increasing the degree of informativeness of the dataset and the participant never receives information on color of the draws nor feedback about his decisions until the end of the experiment. Indeed, delivering information on the draws throughout the course of the experiment would progressively alter the experimental variables as the participant learns about the composition of the risky and ambiguous bags. Since this experiment is not meant to study the dynamics of learning under ambiguity, each decision has to be taken in isolation. Therefore, subjects make all choices before having observed the signals and the ex-post strategies implemented in the payment stage correspond to the ex-ante optimal strategies (*full commitment*). Indeed, the generalization of Blackwell equivalence result by Li and Zhou (2016) requires full commitment to all signal-contingent strategies (this requirement is discussed in Subsection 3.3.2).

Bag	Information	Tasks	Variables
Stage 1			
Certain	No	Select the color of the bet & Fill out the MPL	CE_C
Stage 2			
Non-informational phase			
Risky	No	Select the color of the bet & Fill out the MPL	$CE_{R,0}$
Informational phase			
Risky	$d = 1$	State the threshold & Fill out the MPL	$t_{R,1}$; $CE_{R,1}$
	$d = 5$	State the threshold & Fill out the MPL	$t_{R,5}$; $CE_{R,5}$
	$d = 20$	State the threshold & Fill out the MPL	$t_{R,20}$; $CE_{R,20}$
	$d = 50$	State the threshold & Fill out the MPL	$t_{R,50}$; $CE_{R,50}$
	$d = 200$	State the threshold & Fill out the MPL	$t_{R,200}$; $CE_{R,200}$
Stage 3			
Non-informational phase			
Ambiguous	No	Select the color of the bet & Fill out the MPL	$CE_{A,0}$
Informational phase			
Ambiguous	$d = 1$	State the threshold & Fill out the MPL	$t_{A,1}$; $CE_{A,1}$
	$d = 5$	State the threshold & Fill out the MPL	$t_{A,5}$; $CE_{A,5}$
	$d = 20$	State the threshold & Fill out the MPL	$t_{A,20}$; $CE_{A,20}$
	$d = 50$	State the threshold & Fill out the MPL	$t_{A,50}$; $CE_{A,50}$
	$d = 200$	State the threshold & Fill out the MPL	$t_{A,200}$; $CE_{A,200}$

Table 3.1 – Structure of the experiment

3.2.2 Procedures

Composition of the bags

Regarding the technical aspect of the composition of the bags, the certain bag contains 50 blue balls and 50 red balls as announced. The risky and ambiguous bags contain 100 balls each, balls being either blue or red, but in unknown proportions. This experimental design relies on the results of Li and Zhou (2016) which require a fully supported prior on the set of balls. If the set of priors of the DM reduces to a singleton, i.e., if the DM is certain about the composition of the bag, the information might be of no value to him (as, e.g., is the case for a max-min EU maximizer). Hence, the method for generating the bags has to make explicitly clear that the risky and ambiguous bags in the experimental sessions can be any bag out of the 101 possible bags containing 100 blue or red balls. To do so, the composition of the risky bag is determined as follows: 101 chips numbered from 0 to 100 are presented so that the subject can check for the presence of every number. After verification, they are thrown in an opaque bag. A participant blindly draws one numbered chip out of the 101 chips of the bag. Next, a fair external device (toss of a coin) is used to determine if the number of the drawn chip corresponds to the quantity of blue balls or the quantity of red balls in the risky bag. Hence, each color can be selected with an objective probability of 50%. With this two-stage technique, the probability to draw a blue ball is the same as the probability to draw a red ball which equals 50% — although nothing ensures that the subjects achieve to calculate them.

The composition of the ambiguous bag follows a different procedure: A second set of 101 chips numbered from 0 to 100 are presented to the subjects. After checking, they are thrown in an opaque bag. Another 99 numbered chips are added to the previous pool of chips but the chips of this second set can take any number between 0 and 101 and multiple chips can have the same number. The numbers of this second set of chips are not observed by the subject. Both pools of chips are shuffled jointly and a participant blindly draws

one numbered chip out of the 200 chips of the bag. Next, a coin is tossed to determine if the number of the drawn chip corresponds to the quantity of blue balls or the quantity of red balls in the ambiguous bag.

This method allows to generate uncertainty about the composition of the risky and ambiguous bags while inducing the subject to consider the entire set of possible bags. It is pretty straightforward to implement a transparent procedure to determine the composition of the risky bag. On the other hand, the authors usually choose to remain silent about the method to generate the ambiguous bag in the lab and describe it only as being filled with blue and red balls *in unknown proportions* (e.g., Halevy, 2007; Chow and Sarin, 2002). However, when the experimenter holds private information on the composition of the bag, the participants are more inclined to distrust the experimenter, suspecting him to have generated the bag in a way to save them money. The participation of subjects in the composition of the bag allows to minimize suspicion by ensuring that the bag is truly not-manipulated by the experimenter. For instance, in Trautmann and Zeckhauser (2013), each subject filled his own bag by blindly drawing balls from a bag whose composition is known. Ideally, the composition of the bag is unknown to everyone, reducing potential "comparative ignorance" effects (Fox and Tversky, 1995) in which the presence of an informed agent reduces the desirability of the prospect. Therefore, in this experiment, the number of the drawn chips and the result of the tossing of the coins remain secret to the participants and the experimenter until the end of the session.

Incentive device

At the end of the session, one question of the experiment, corresponding to one row out of the 13 MPLs of the experiment, is played for real according to the Prince incentive mechanism (Johnson *et al.*, 2015). The key component of this mechanism design relies on the timing of the random selection of the question that serves for payment: contrary to most standard techniques, the question is selected at the beginning of the experiment and

provided to the participant in a sealed envelope. Each subject knows about the content of all envelopes in the session but does not know the content of his own. Two examples of envelope content are provided in Figures 3.3 and 3.4. At the end of the session, the envelope is opened and the choice of the subject is performed. There are 3 different types of options: (*i*) if the preferred option was a sure amount, the subject receives the sure gain; (*ii*) if the preferred option was a bet without information, he randomly draws one ball from the bag involved in the bet and if the color of the ball matches the color chosen before, he receives 15€, otherwise he gets 5€; (*iii*) if the preferred option was a bet with information, the computer generates a sample of d random draws with replacement from the ambiguous bag; the result of the draws is compared to the threshold t_d announced by the subject and the winning color of the bet is deduced; then the participant randomly draws one ball from the ambiguous bag and if the color of the ball matches the color stated before, he receives 15€, otherwise he gets 5€. While the usual Random-Incentive-System (RIS, first introduced by Savage, 1954) and Becker-DeGroot-Marschak (BDM, 1964) mechanisms have been blamed for their inability to induce truthfull-telling for non-expected-utility maximizers and for the subsequent violations of incentive-compatibility in ambiguous contexts (Bade, 2015), the Prince device allows to overcome inherent drawbacks of existing techniques and ensures subjects that truthful revelation of preferences is in their best interest [49].

In this experiment, there are 13 MPLs, each containing 10 rows. In sum, this experiment consists of 130 binary choices. Hence, there are 130 envelopes numbered 1-130 in random order, such that each envelope contains one of the 130 questions. At the beginning of the experiment, these envelopes are randomly distributed to subjects without replacement. At the very end of the experiment, each participant receives a list describing all envelopes which avoids deception by allowing him to verify that his envelope has correct content.

49. see Chapter 2 for a complete description of the Prince mechanism in a similar framework.

Option 1 : Bet on the color of the ball to be drawn from **bag A**.	Option 2 : **8.50€**.

Figure 3.3 – An example of envelope content (certain bag)

Option 1 : Bet on the color of the ball to be drawn from **bag B** with a sample of **200 random draws** with replacement.	Option 2 : **12.50€**.

Figure 3.4 – An example of envelope content (risky bag)

Proceedings

The experiment starts with an oral presentation of the instructions [50]. When the explanation reaches the relevant points, the composition of the risky and ambiguous bag are determined through the method described before and each subject randomly picks one sealed envelope that serves for payment. After the instructions, a comprehension questionnaire is implemented to test subject's understanding of the experiment. Then, the participant performs the tasks of the experiment using the computer in front of him. After that, he is asked to complete a short form including basic socio-demographic variables (age, gender, studies). The last screen of the experiment invites him to reach the experimenter's office with his envelope to proceed for payment.

Each experimental session should not last more than 1 hour including payment. The maximum payoff is 15€ and the minimum payoff equals 5€. The show-up fee (usually 5€) is included in the gambles payoff because agents seem to pay more attention to their decisions and make less errors when the monetary incentives are high (see for instance Lévy-Garboua *et al.*, 2012). There is no particular subject requirement to participate. The experiment can be easily programmed with the experiment software z-Tree (Fischbacher, 2007). Finally, the design allows within-subject as well as between-subject analysis.

50. See Appendix 3.B for a full set of instructions.

3.2.3 Experimental measures

Symmetric prior

With the elicitation of the signal-contingent strategies $(t_{R,d})_{d\in\mathcal{D}}$ and $(t_{A,d})_{d\in\mathcal{D}}$, it is possible to test for the existence of individual symmetric priors regarding the composition of the risky bag and the composition of the ambiguous bag. In this framework, the symmetry of beliefs seems to be pretty natural. Especially, the DM may think about the bags as follows: "The number of red balls in it can be any number between 0 and 100. My information is completely symmetric, and there is no reason to believe that there are more red balls than [blue] balls or vice versa. Hence, if I were to adopt a prior probability over the composition of the [bag], from [0:100] to [100:0], I should choose a symmetric prior. That is, the probability that there are 3 red balls should be equal to the probability that there are 97 red balls, and so forth." (Gilboa and Marinacci, 2016, p392). Moreover, since the composition of the risky bag is objectively drawn from a uniform distribution over the entire range of possible bag composition, subjects who successfully perform ROCL should postulate that the probability to draw a blue ball is precisely 50%. Therefore, a DM who reduces compound lotteries or has a symmetric prior should bet on blue for any set of observations containing more than 50% of blue draws and bet on red otherwise (with indifference at 50%). The same strategy is optimal for the ambiguous bag, as long as the DM has a symmetric prior. Given the transparency and credibility of the random drawing experimental procedures, any other signal-contingent strategy can be taken to imply an asymmetric prior belief.

Ambiguity aversion

The aversion to ambiguity is estimated within the standard Ellsberg framework where the DM is confronted with a binary choice between a prospect on the known urn and a prospect on the ambiguous urn. Thus, the following ambiguity-aversion index based on

certainty equivalents (Sutter *et al.*, 2013) can be calculated for each subject:

$$I_{AA} = \frac{CE_C - CE_{A,0}}{CE_C + CE_{A,0}} \tag{3.1}$$

I_{AA} is the normalized measure of the difference between the CE for the certain bag and the CE for the ambiguous bag without information. For instance, $CE_C > CE_{A,0}$ means that the ambiguous bag is less attractive than the certain bag for a color-bet, denoting ambiguity-aversion. Since CE_C and $CE_{A,0}$ can take values in the interval $[5; 15]$, I_{AA} ranges from -$\frac{1}{2}$ to $\frac{1}{2}$, where $\frac{1}{2}$ refers to extreme ambiguity-aversion and $-\frac{1}{2}$ indicates extreme ambiguity-loving. Hence, the participants are classified according to the sign of their index: ambiguity-averse for strictly positive I_{AA}, ambiguity-neutral for I_{AA}=0 and ambiguity-lover for strictly negative I_{AA} [51]. The denominator of the index accounts for the fact that a 1€-difference should weigh more heavily for those who report low CEs than for those who announce high CEs.

Second, it is possible to study the ability of participants to reduce compound lotteries, although the detailed description of the connection between reduction/non-reduction and ambiguity neutrality/non-neutrality is an issue outside the scope of this chapter [52]. There, the following index can be calculated :

$$I_{ROCL} = \frac{CE_C - CE_{R,0}}{CE_C + CE_{R,0}} \tag{3.2}$$

which is the normalized measure of the difference between the CE for the certain bag and the CE for the risky bag without information. In the spirit of Abdellaoui *et al.* (2015) and

51. Note that one cannot distinguish between an ambiguity-lover and a neutral agent with an asymmetric prior regarding the composition of the ambiguous bag. Indeed, both agents might strictly prefer the ambiguous bag and thus get a strictly positive I_{AA}. However, this appears to be only a minor disadvantage of this tractable index because symmetric beliefs are the most reasonable class of beliefs given the setting of the experiment. Moreover, the design allows to test for the symmetry of prior and provides hence a further insight on the beliefs.

52. Notwithstanding, the calculus of these two indexes allows to relate and compare the results of this experiment with Halevy (2007) and Abdellaoui *et al.* (2015, see Table 5).

Halevy (2007), if the subject's preferences satisfy ROCL, he should treat both associated prospects as equivalent and value them equally, then $I_{ROCL} = 0$. On the contrary, if participants fail to reduce compound lotteries, the reported CE for the certain bag differs from the CE for the risky bag, which yields $I_{ROCL} \neq 0$. Moreover, this index informs on the distance between both CEs which is a relevant question. It allows to discriminate between participants who value both prospects almost equally and those whose valuations for these prospects differ significantly.

To summarize, if the DM's preferences and beliefs satisfy: (*i*) ROCL, (*ii*) the symmetry of priors on the composition of the bags, and (*iii*) ambiguity-neutrality, the theory predicts that the agent will be indifferent between the three bags, which implies: $CE_C = CE_{R,0} = CE_{A,0}$. If one of these 3 conditions is relaxed, one equality does not hold as described above.

Value of information

The value of information can be broadly defined as the amount a DM would be willing to pay to obtain information prior to making a decision. There is a particular amount below which he accepts to pay for information and above which he rejects the information. This threshold corresponds to the DM's reservation price for information which is the amount that makes him indifferent between making the decision with information (and hence paying the price) and making the decision without information. Although any DM should be able to announce his price in theory, such task is not easily comprehensible in practice and can be misunderstood. This relates concretely to answering the following question: "What is the price for information that makes you indifferent between the bet on the bag without information and the bet on the bag with information?", which is far from intuitive. Hence, a crucial operational issue related to this type of design concerns the elicitation of this price. Alternatively, the DM can be confronted with a series of binary choices between the bet without information and the bet with information at price

p. Indeed, asking subjects to express such preferences is less cognitively demanding than asking them to reveal their indifference point. The price is varied so that the reservation price can be elicited, and the same process is replicated for different informative signals. I find this technique particularly heavy, potentially confusing and difficult to implement in the lab. Thus, it is not straightforward to find a convenient and tractable device that easily fits the intended purpose. In Eliaz and Schotter (2010) for instance, subjects, endowed with money, are asked if they want to pay a fee to obtain information on the bet at hand. In their experiment, the fee can take only 3 values ($.50; $2; $4) which does not allow to precisely identify the individual indifference point. Attanasi and Montesano (2012) proposes an operational design where the agent's reservation price for information is estimated through the BDM mechanism. However, the use of the BDM device to incentivize choices has met numerous critics for its opaqueness and its failure to induce truthful revelation of preferences [53]. In Ambuehl and Li (2018), the value of the informative structure is approximated via uncertainty equivalents. In particular, they define the agent's Willingness-To-Pay for an ambiguous prospect with information as the probability mixture over the gamble's best outcome and zero that generates indifference.

To summarize, a consensus on the method to estimate the DM's reservation price for information has not yet emerged. In this experiment, the value of information is estimated through CE measurements. The value of the prospect without information is elicited in the non-informational phase. The effect of information on the valuation of the prospect is estimated during the subsequent informational phase. Hence, it is possible to build the following individual measures:

$$V_R(d, 0) = CE_{R,d} - CE_{R,0} \tag{3.3}$$

$V_R(d, 0)$ is the value of the signal s_d containing d draws that informs on the risky bag. A

53. Bardsley *et al.* (2010) stresses the complexity of the BDM mechanism in Section 6.5. See also Karni and Safra (1987) and Bade (2015) for theoretical discussions.

strictly positive $V_R(d,0)$ denotes an increase in the attractiveness of the risky prospect, which means that the information is valuable to the DM. If the value of information is positive, is it increasing in the number of observations d? If Blackwell's theorem holds, then $V_R(d,d')$ defined by $V_R(d,d') = CE_{R,d} - CE_{R,d'}$ is positive for any $d > d'$.

$V_A(d,0)$ corresponds to the value of information in the ambiguous bag. It is given by:

$$V_A(d,0) = CE_{A,d} - CE_{A,0} \tag{3.4}$$

As reviewed in the Introduction, theoretical results predict positive value for ambiguity-averse and ambiguity-loving agents (this last point is discussed more carefully in 3.3.3). As for $V_R(d,0)$, twofold analysis applies here: (i) the sign of $V_A(d,0)$, and (ii) the comparison with the value of signals containing more/less draws.

This may be also of interest to study the difference:

$$V_A(d,0) - V_R(d,0) = (CE_{A,d} - CE_{A,0}) - (CE_{R,d} - CE_{R,0}) \tag{3.5}$$

which measures the differential effect of signals of the same length on the ambiguous and risky bags. This question is especially relevant for those who perceive the risky bag as an intermediate condition between the certain bag and the ambiguous bag (Yates and Zukowski, 1976; Chow and Sarin, 2002). How does this difference depend on ambiguity-aversion as captured by the index I_{AA} calculated previously?

3.3 Theoretical background

3.3.1 Blackwell's framework

The design presented previously falls into Blackwell's framework in the sense that the different information structures used in this experiment are Blackwell-comparable. Indeed, the signal s_{d+1} which contains $d+1$ random draws with replacement from an unknown bag

is more Blackwell-informative than the signal s_d which consists of d draws. As stressed by Gollier (2004, §24.3.2), such signal s_d can be obtained from signal s_{d+1} by using a 'garbling machine', which adds a noise uncorrelated with the true state of nature (i.e., the composition of the bag). This is equivalent to showing that there exists a garble of s_{d+1} which preserves the posterior probabilities of s_d on the states of the world. A short explanation is sketched here and the detailed discussion is relegated to Appendix 3.A.

Consider now the more general problem in which the bag contains N balls, each ball being either blue or red. The proportions of balls are unknown and the composition of the bag can be parameterized by the number of blue balls, $k \in \{0, 1, ..., N\}$. Without information, the $N + 1$ compositions are objectively equally likely inducing a uniform distribution on the states of the world. Hence, each composition is assumed to occur with a prior probability of $\frac{1}{N+1}$.

A signal is a sequence of random draws with replacement. For instance, the signal corresponding to the following sequence of draws: blue, red, blue, is denoted (brb). Assume that the DM obtains a signal of d draws from the bag. Among the d draws, a blue ball has been drawn d_b times and it follows that $d_b \in \{0, 1, ..., d\}$ and that a red ball has been drawn $d - d_b$ times. The joint probability of the state of the world k and the particular signal $(b...br...r)$ is given by:

$$\Pr\{\text{Bag } k, (b...br...r)\} = \frac{k^{d_b}(N-k)^{d-d_b}}{N^d(N+1)} \tag{3.6}$$

The conditional probability for this signal structure is hence:

$$\Pr\{\text{Bag } k \mid (b...br...r)\} = \frac{k^{d_b}(N-k)^{d-d_b}}{\sum_{k=0}^{N} k^{d_b}(N-k)^{d-d_b}} \tag{3.7}$$

To show that this information structure is less informative than the one with $d + 1$ draws, consider a DM, who obtains now a signal of $d + 1$ draws, but a garbling machine replaces the last information of the sequence by b with probability $\frac{1}{2}$ and by r with proba-

bility $\frac{1}{2}$. For instance, reconsider the signal $(b...br...r)$ containing d draws described before for which a noisy information is added. This yields:

$$\Pr\left\{\text{Bag } k,(\underbrace{b...br...r}_{d \text{ draws}}\, b)\right\} = \Pr\left\{\text{Bag } k,(\underbrace{b...br...r}_{d \text{ draws}}\, r)\right\} = \frac{1}{2}\frac{k^{d_b}(N-k)^{d-d_b}}{N^d(N+1)} \tag{3.8}$$

The conditional probabilities are given by:

$$\Pr\left\{\text{Bag } k \mid (\underbrace{b...br...r}_{d \text{ draws}}\, b)\right\} = \Pr\left\{\text{Bag } k \mid (\underbrace{b...br...r}_{d \text{ draws}}\, r)\right\} = \frac{k^{d_b}(N-k)^{d-d_b}}{\sum_{k=0}^{N} k^{d_b}(N-k)^{d-d_b}} \tag{3.9}$$

Hence the conditional probabilities are preserved with the adding of a noisy information and the signal structure obtained with the garbling on the $(d+1)$th draws is equivalent to the signal structure with d draws.

3.3.2 Commitment

It is well-known that non-EU preferences, especially ambiguity-sensitive preferences, might not comply with the Independence Axiom which implies dynamically inconsistent behavior (Siniscalchi, 2011). Dynamic consistency requires that ex-ante contingent choices coincide with ex-post preferences. However, in this experiment, there is no reason to assume that preferences before the realization of the draws match preferences after the observation of the signal. Within the MEU framework for instance, Epstein and Schneider (2003) axiomatize a recursive structure for utility which requires that each prior is updated in a Bayesian way and that the set of priors satisfy the restrictive *rectangularity* condition. Wakker (1988), Hilton (1990) and Safra and Sulganik (1995) prove that delaying the choices after the realization of the signal might lead to the existence of choice situations in which non-EU DMs exhibit information-aversion, inducing the Blackwell theorem to fail. In particular, Li's (2016) extension of Blackwell informativeness ranking does not apply in general. Hence, the theoretical framework requires full-commitment to all signal-

contingent strategies. Since the experimenter cannot exert control over the way subjects form beliefs and the way they update their beliefs, dynamic consistency might not be achieved in this setting and experimental results are more robust with full-commitment.

3.3.3 Mixed strategies

From the proof of Theorem 1 in Li and Zhou (2016), it can be inferred that: if the signal s_d is more Blackwell-informative than the signal $s_{d'}$, then s_d is more valuable than $s_{d'}$ for all (ambiguity-neutral and ambiguity-nonneutral) DMs [54]. When the DM has the opportunity to observe an informative signal, he determines his optimal signal-contingent strategy. A necessary condition for the result of Li is the possibility to replicate this optimal strategy when observing a more Blackwell-informative signal. Hence, the value of information is always positive since one can always choose to not take into account the additional information. This is especially relevant for ambiguity-lovers who may be willing to pay to avoid information (Snow, 2010; Attanasi and Montesano, 2012). One way to proceed is to allow the agents to randomize their strategies, i.e., to play mixed strategies. Consider the following example: The DM observes a signal containing 5 draws. Assume that his optimal strategy is to bet on blue if the signal contains 3 or more blue balls and to bet on red otherwise. Hence, he bets on blue starting from the signal containing (3 blue / 2 red). He is then confronted with a more informative signal containing 6 balls but he wants to replicate his 5-draws optimal strategy. To this end, he rejects the additional information by randomly eliminating one draw from the 6-draws signal. His optimal strategy to bet on blue now starts at (3 blue / 3 red) with probability .5 and at (4 blue / 2 red) with probability .5.

Although appealing in theory, mixed strategies are difficult to implement in practice and it is not straightforward to find a tractable mechanism to elicit mixed strategies in the lab. Moreover, it is not judicious to offer subjects a random device to reduce the

54. The converse is not true and requires convexity of preferences to hold.

length of signals, since Bade (2015) has proved that ambiguity-averse agents may also use randomization devices to hedge, yielding contamination of the experimental data. In my experiment, subjects who want to avoid additional information should be able to *mentally* randomize. Therefore, my design departs slightly from the theoretical predictions since it does not allow subjects to explicitly state mixed strategies. This is motivated by the fact that adding mixed strategies would excessively complicate the instructions while reducing the tractability of the design. Moreover, it would expose the experimenter to misclassification of subjects.

3.3.4 Predictions

Two hypotheses are derived from the theoretical literature reviewed in the previous sections and can be examined through the presented experimental setting:

H1. *Blackwell's equivalence result under compound risk* (Blackwell, 1951). To bet on the risky bag, the value of information is strictly positive and strictly increasing in the informativeness of the signal.

H2. *Blackwell's equivalence result under ambiguity* (Li and Zhou, 2016). To bet on the ambiguous bag, the value of information is strictly positive and strictly increasing in the informativeness of the signal.

The prediction H1 translates to the following experimental hypothesis: For all DM i, $0 < V_R^i(d,0) < V_R^i(d',0)$, for any $d, d' \in \mathcal{D}$ such that $d < d'$. H2 is equivalent to: For all DM i, $0 < V_A^i(d,0) < V_A^i(d',0)$, for any $d, d' \in \mathcal{D}$ such that $d < d'$. It is of interest to investigate further the effect of the introduction of ambiguity in the composition of the bag on the valuation of the signal. To this end, the difference $V_A^i(d,0) - V_R^i(d,0)$ provides the relevant measure.

Moreover, it is also possible to examine an additional hypothesis:

H3. *Value of information and attitudes towards ambiguity.* The value of information increases with ambiguity-aversion.

Thus, H3 is equivalent to the following statement: For any DM i and DM j ($i \neq j$), if i is more ambiguity-averse than j ($I^i_{AA} > I^j_{AA}$), then $V^i_A(d,0) > V^j_A(d,0)$, for all $d \in \mathcal{D}$. For ambiguity-averse DMs, some authors have argued that the value attributed to informative signals which *resolves* ambiguity increases with ambiguity-aversion (Snow, 2010; Attanasi and Montesano, 2012). H3 is intented to test the existence of such increasing relation when information *reduces* ambiguity.

3.4 Conclusion

Thanks to technological innovations, the mass of available information has been considerably increasing in the last decades, hence the individual valuation of information is an issue of particular interest in our modern societies. Drawing on Ellsberg's two-color experiment, I have proposed an experimental design to measure the value of information under compound risk and ambiguity in a model-free setup. The second objective of this chapter is to provide an operational framework to test Blackwell equivalence result, which lacks empirical evidence. Moreover, once subjects' reservation price for information is elicited, it can be related to ambiguity attitudes. Finally, the 3-urns design proposed here allows to study the relation between reduction of compound lotteries and ambiguity attitudes which makes it possible to conduct further comparative analysis with Halevy (2007) and Abdellaoui *et al.* (2015).

Appendix

3.A Proof: Blackwell's garbling

In this Appendix, I show that the information structure containing d random draws with replacement from the bag is a garble of the information structure with $d+1$ draws.

Formally, there are $N+1$ possible compositions of the bag containing N balls, and $\forall k \in \{0, N\}$, $\frac{k}{N}$ of balls are blue and $\frac{N-k}{N}$ of balls are red in Bag k. An information structure is a tuple (S, P) where $S := \{s_1, ..., s_{|S|}\}$ is a set of signals and $P_{|S|\times(N+1)}$ a matrix. In particular, p_{sk} is the probability that bag k be realized if signal s is observed, i.e., $p_{sk} := \Pr(k|s)$. Vector $p_{s.} = (p_{s1}, ..., p_{sN+1})$ gives the posterior probabilities conditional to signal s.

The information structure (S', P'), such that $S' := \{s'_1, ..., s'_{|S'|}\}$ and $P' = [p'_{s'k}]_{|S'|\times(N+1)}$, is a garble of information structure (S, P) if and only if there exists a Markov matrix [55] M such that:

$$P' = MP \tag{3.10}$$

$M = [m_{s's}]_{|S'|\times|S|}$ is the garbling matrix. The posterior distribution of the garbled information structure is given by:

$$p_{s'.} = \sum_{s=1}^{S} m_{s's} p_{s.} \tag{3.11}$$

55. M is a Markov matrix if and only if it is nonnegative and row-stochastic, i.e., $m_{ij} \geq 0$ and $\sum_j m_{ij} = 1$ for all i.

Thus, all posterior probabilities in structure (S', P') are a convex combination of the posterior probabilities in structure (S, P).

Showing that the information structure containing d draws (S_d) is a garble of the information structure with $d+1$ draws (S_{d+1}) is equivalent to proving that there exists a garble of S_{d+1} which preserves the posterior probabilities of S_d on the states of the world. It then follows from Blackwell (1951) that the structure given by d draws with replacement is less informative than that containing $d+1$ draws.

Assumption: Each composition k occurs with a prior probability of $\frac{1}{N+1}$.

d draws

Suppose that the DM obtains a signal s_d of d draws from the bag. Among the d draws, a blue ball has been drawn d_b times and a red ball has been draws d_r times. It follows that $d_b \in \{0, 1, ..., d\}$ and $d_r = d - d_b$.

For instance,

$$\Pr\left\{\text{Bag } k, (\underbrace{b...br...r}_{d \text{ draws}})\right\} = \frac{1}{N+1}\underbrace{\frac{k}{N}...\frac{k}{N}}_{d_b \text{ draws}}\underbrace{\frac{N-k}{N}...\frac{N-k}{N}}_{d-d_b \text{ draws}} = \frac{k^{d_b}(N-k)^{d-d_b}}{N^d(N+1)}$$

With d draws, the set of possible signals s_d is S_d and $|S_d| = \sum_{d_b=0}^{d}\binom{d}{d_b} = 2^d$.

The joint probability distribution of the state of the world (i.e., the composition of the bag) and the signal (i.e., the d draws from the bag) is:

	$(w...w)$	$(w...wb)$	...	$(w...wb...b)$	...	$(b...b)$	Total
Bag k	$\frac{k^d}{N^d(N+1)}$	$\frac{k^{d-1}(N-k)}{N^d(N+1)}$	...	$\frac{k^{d_b}(N-k)^{d-d_b}}{N^d(N+1)}$	...	$\frac{(N-k)^d}{N^d(N+1)}$	$t_k = \frac{1}{N+1}$
Total	$\frac{\sum_{k=0}^{N}k^d}{N^d(N+1)}$	$\frac{\sum_{k=0}^{N}k^{d-1}(N-k)}{N^d(N+1)}$	...	$\frac{\sum_{k=0}^{N}k^{d_b}(N-k)^{d-d_b}}{N^d(N+1)}$	...	$\frac{\sum_{k=0}^{N}(N-k)^d}{N^d(N+1)}$	$T = 1$

The conditional probabilities for the signal structure (S_d, P_d), where $P_d := [p_{s_d k}]_{|S_d|\times(N+1)}$ and $p_{s_d k} := \Pr(\text{Bag } k | \text{ signal } s_d)$, are given by:

	$(w...w)$	$(w...wb)$	...	$(w...wb...b)$	...	$(b...b)$
$p_{s_d k}$	$\frac{k^d}{\sum_{k=0}^{N} k^d}$	$\frac{k^{d-1}(N-k)}{\sum_{k=0}^{N} k^{d-1}(N-k)}$	...	$\frac{k^{d_b}(N-k)^{d-d_b}}{\sum_{k=0}^{N} k^{d_b}(N-k)^{d-d_b}}$	...	$\frac{(N-k)^d}{\sum_{k=0}^{N}(N-k)^d}$
Total	1	1	...	1	...	1

$d+1$ draws

Suppose that the DM obtains a signal s_{d+1} of $d+1$ draws from the urn. Among the $d+1$ draws, a blue ball has been drawn d_b times and a red ball has been draws d_r times. It follows that $d_b \in [0, d+1]$ and $d_r = d+1-d_b$.

With $d+1$ draws, the set of possible signals s_{d+1} is S_{d+1} and $|S_{d+1}| = 2^{d+1}$.

The joint probability distribution of the state of the world and the signal is:

	$(w...ww)$	$(w...wb)$	...	$(w...wb...b)$	...	$(b...b)$	Total
Bag k	$\frac{k^{d+1}}{N^{d+1}(N+1)}$	$\frac{k^d(N-k)}{N^{d+1}(N+1)}$	...	$\frac{k^{d_b}(N-k)^{d+1-d_b}}{N^{d+1}(N+1)}$	...	$\frac{(N-k)^{d+1}}{N^{d+1}(N+1)}$	$\frac{1}{N+1}$
Total	$\frac{\sum_{k=0}^{N} k^{d+1}}{N^{d+1}(N+1)}$	$\frac{\sum_{k=0}^{N} k^d(N-k)}{N^{d+1}(N+1)}$	...	$\frac{\sum_{k=0}^{N} k^{d_b}(N-k)^{d+1-d_b}}{N^{d+1}(N+1)}$	...	$\frac{\sum_{k=0}^{N}(N-k)^{d+1}}{N^{d+1}(N+1)}$	1

The conditional probabilities for the signal structure (S_{d+1}, P_{d+1}), where $P_{d+1} := [p_{s_{d+1}k}]_{|S_{d+1}|\times(N+1)}$ and $p_{s_{d+1}k} := \Pr(\text{Bag } k | \text{ signal } s_{d+1})$, are given by:

	$(w...w)$	$(w...wb)$	...	$(w...wb...b)$	...	$(b...b)$
$p_{s_{d+1}k}$	$\frac{k^{d+1}}{\sum_{k=0}^{N} k^{d+1}}$	$\frac{k^d(N-k)}{\sum_{k=0}^{N} k^d(N-k)}$	...	$\frac{k^{d_b}(N-k)^{d+1-d_b}}{\sum_{k=0}^{N} k^{d_b}(N-k)^{d+1-d_b}}$	...	$\frac{(N-k)^{d+1}}{\sum_{k=0}^{N}(N-k)^{d+1}}$
Total	1	1	...	1	...	1

Garbling machine

A garbling machine replaces the last signal by b with probability $\frac{1}{2}$ and by r with probability $\frac{1}{2}$. Such garbled signal is denoted s^g_{d+1}. Then:

	$(w...ww)$	$(w...wb)$	...	$(w...wb...bw)$	$(w...wb...bb)$	...	$(b...bw)$	$(b...bb)$	Total
Bag k	$\frac{k^d}{2N^d(N+1)}$	$\frac{k^d}{2N^d(N+1)}$	...	$\frac{k^{d_b}(N-k)^{d-d_b}}{2N^d(N+1)}$	$\frac{k^{d_b}(N-k)^{d-d_b}}{2N^d(N+1)}$	...	$\frac{(N-k)^d}{2N^d(N+1)}$	$\frac{(N-k)^d}{2N^d(N+1)}$	$t'_k = \frac{1}{N+1}$
Total	$\frac{\sum_{k=0}^N k^d}{2N^d(N+1)}$	$\frac{\sum_{k=0}^N k^d}{2N^d(N+1)}$	...	$\frac{\sum_{k=0}^N k^{d_b}(N-k)^{d-d_b}}{2N^d(N+1)}$	$\frac{\sum_{k=0}^N k^{d_b}(N-k)^{d-d_b}}{2N^d(N+1)}$	...	$\frac{\sum_{k=0}^N (N-k)^d}{2N^d(N+1)}$	$\frac{\sum_{k=0}^N (N-k)^d}{2N^d(N+1)}$	$T' = 1$

The conditionals for the signal structure (S^g_{d+1}, P^g_{d+1}) are given by:

	$(w...ww)$	$(w...wb)$	...	$(w...wb...bw)$	$(w...wb...bb)$	...	$(b...bw)$	$(b...bb)$
$p_{s^g_{d+1}k}$	$\frac{k^d}{\sum_{k=0}^N k^d}$	$\frac{k^d}{\sum_{k=0}^N k^d}$	...	$\frac{k^{d_b}(N-k)^{d-d_b}}{\sum_{k=0}^N k^{d_b}(N-k)^{d-d_b}}$	$\frac{k^{d_b}(N-k)^{d-d_b}}{\sum_{k=0}^N k^{d_b}(N-k)^{d-d_b}}$	...	$\frac{(N-k)^d}{\sum_{k=0}^N (N-k)^d}$	$\frac{(N-k)^d}{\sum_{k=0}^N (N-k)^d}$
Total	1	1	...	1	1	...	1	1

which is equivalent to the signal structure obtained with d draws from the bag.

Note that the conditional probability of obtaining signal s^g_{d+1} from the garbling machine given that the machine received signal s_{d+1}, $m_{s^g_{d+1}s_{d+1}}$ is given by:

$m_{s^g_{d+1}s_{d+1}}$		Signal received by the machine							
		$(w...ww)$	$(w...wb)$	...	$(w...wb...bw)$	$(w...wb...bb)$	...	$(b...bw)$	$(b...bb)$
Signal delivered by the machine	$(w...ww)$	$\frac{1}{2}$	$\frac{1}{2}$	...	0	0	...	0	0
	$(w...wb)$	$\frac{1}{2}$	$\frac{1}{2}$	...	0	0	...	0	0
	...	...	...	...	...	...	...	...	...
	$(w...wb...bw)$	0	0	...	$\frac{1}{2}$	$\frac{1}{2}$	...	0	0
	$(w...wb...bb)$	0	0	...	$\frac{1}{2}$	$\frac{1}{2}$	...	0	0
	...	...	...	...	...	...	...	...	...
	$(b...bw)$	0	0	...	0	0	...	$\frac{1}{2}$	$\frac{1}{2}$
	$(w...wb)$	0	0	...	0	0	...	$\frac{1}{2}$	$\frac{1}{2}$

It is then possible to deduce the relationship between the structure with d useful draws and one garbled draw, the structure with $d+1$ draws and $m_{s^g_{d+1}s_{d+1}}$:

$$\begin{aligned}
\Pr\left\{\text{Urn } k; \underbrace{w...wb...b}_{d \text{ draws}}\right\} &= \Pr\left\{\text{Urn } k; \underbrace{w...wb...b}_{d \text{ draws}} w\right\}_{\text{garbling}} + \Pr\left\{\text{Urn } k; \underbrace{w...wb...b}_{d \text{ draws}} b\right\}_{\text{garbling}} \\
&= \frac{k^{d_b}(N-k)^{d-d_b}}{N^d(N+1)}
\end{aligned}$$

Moreover:

$$\begin{aligned}
\Pr\left\{\text{Urn } k; \underbrace{w...wb...b}_{d \text{ draws}}\right\} &= \frac{1}{2}_{\text{garble w}} \Pr\left\{\text{Urn } k; \underbrace{w...wb...b}_{d \text{ draws}} w\right\} + \frac{1}{2}_{\text{garble w}} \Pr\left\{\text{Urn } k; \underbrace{w...wb...b}_{d \text{ draws}} b\right\} \\
&\quad + \frac{1}{2}_{\text{garble b}} \Pr\left\{\text{Urn } k; \underbrace{w...wb...b}_{d \text{ draws}} w\right\} + \frac{1}{2}_{\text{garble b}} \Pr\left\{\text{Urn } k; \underbrace{w...wb...b}_{d \text{ draws}} b\right\} \\
&= 2\frac{1}{2}\frac{k^{d_b+1}(N-k)^{d+1-(d_b+1)}}{2N^{d+1}(N+1)} + 2\frac{1}{2}\frac{k^{d_b}(N-k)^{d+1-d_b}}{2N^{d+1}(N+1)} \\
&= \frac{1}{N^{d+1}(N+1)}\left[k^{d_b+1}(N-k)^{d-d_b} + k^{d_b}(N-k)^{d+1-d_b}\right] \\
&= \frac{1}{N^{d+1}(N+1)}\left[k^{d_b}(N-k)^{d-d_b}(k+(N-k))\right] \\
&= \frac{k^{d_b}(N-k)^{d-d_b}}{N^d(N+1)}
\end{aligned}$$

which is verified for all k and for all d_b. Hence, the information structure obtained with the garbling on the $(d+1)$th draws is equivalent to the signal structure with d draws. Thus, the structure S_{d+1} is more informative than the structure S_d.

■

3.B Instructions

Note: The sentences in brackets are not included in the instructions but inform the reader on the moment an action is performed.

— Start of Instructions —

Welcome everybody, and thank you for accepting our invitation to participate in this

experiment. I am going to present the outline of the experiment, please pay close attention to this presentation.

First, some important rules which must be respected: It is important that the experiment takes place in silence, you are not allowed to communicate with other participants. The use of mobile phones and calculators is forbidden. No particular preliminary knowledge is needed to participate. As soon as the experiment starts, if you have any questions, please raise your hand and I will assist you.
Thank you in advance for your cooperation.

What do we do?

You are going to participate in an economic experiment on decision theory.
In this experiment, you will have to consider three opaque bags, A, B and C, each containing 100 balls. Each ball can be either blue or red.

The bags:

Bag A contains 100 balls and its composition is perfectly known. It contains exactly 50 blue balls and 50 red balls.

Bag B contains 100 balls and the proportions of blue and red balls are given by the following procedure: There are 101 chips numbered 0 to 100. [The chips are presented to the audience in increasing order so that everybody can easily check for the presence of every number.] These chips are thrown and shuffled in a bag. One of you is now asked to blindly draw one chip from this bag. [The chip is drawn and placed in an opaque box on the table in front of the room.]
Here is a coin. One of you is going to toss this coin and if it comes up heads, the number of the drawn chip determines the quantity of blue balls in bag B, the rest of the bag being filled with red balls. If the coin comes up tails, the number of the drawn chip determines

the quantity of red balls in bag B, the rest of the bag being filled with blue balls. For instance, if the drawn chip is numbered 12 and the coin comes up tails, bag B is filled with 12 red balls and 88 blue balls; if the the coin comes up heads, bag B is filled with 12 blue balls and 88 red balls. The number of this chip and the result of the coin toss are kept secret until the end of the experiment. [At this point, the coin is tossed in an opaque box and placed on the table in front of the room.]

Bag C contains 100 balls and the proportions of blue and red balls are given by the following procedure: Here again, there are 101 chips numbered 0 to 100. [The chips are presented to the audience in increasing order so that everybody can easily check for the presence of every number.] These chips are thrown and shuffled in a bag. 99 secret numbered chips are added to the bag. These chips can take any number between 0 and 100 and multiple chips can have the same number. [The second set of numbered chips is thrown in the bag, while their numbering cannot be checked by the subjects.] One of you is now asked to blindly draw one chip from this bag. [The chip is drawn and placed in an opaque box on the table in front of the room.]

Here is a coin. one of you is going to toss this coin and if it comes up heads, the number of the drawn chip determines the quantity of blue balls in bag C, the rest of the bag being filled with red balls. If the coin comes up tails, the number of the drawn chip determines the quantity of red balls in bag C, the rest of the bag being filled with blue balls. The number of this chip and the result of the coin toss are kept secret until the end of the experiment. [At this point, the coin is tossed in an opaque box and placed on the table in front of the room.]

The gambles:

During the experiment, you will be asked to consider gambles on bags A, B and C. Especially, you will have to place bets on the color of the ball to be randomly drawn from these bags. If the color of the drawn ball matches your selected color, you win 15€, otherwise

you win 5€.

In some cases, you will be able to condition your decision on the revelation of observations from the bag. In such cases, a random ball is drawn from the bag, the color of the draw is reported and the ball is replaced in the bag. This operation, called a random draw with replacement, may be replicated.

The sets of observations provide information about bag B and bag C. They contain 1, 5, 20, 50 or 200 random draws with replacement.

The envelopes:

Before the beginning of the experiment, each of you will receive a **closed** envelope. Each envelope contains two options. These options differ depending on the envelopes. Your goal is to tell us which of the two options you prefer for each envelope of the experiment. At the end of the experiment, you will be rewarded depending on your envelope content and your choices of options.

Important: There are no "right" or "wrong" answers in this experiment, it is only a matter of preferences.

Here are some examples of envelope content:

Option 1 : Bet on the color of the ball to be drawn from **bag A**.	Option 2 : **8.50€**.
Option 1 : Bet on the color of the ball to be drawn from **bag B** with a sample of **50 random draws** with replacement.	Option 2 : **12.50€**.
Option 1 : Bet on the color of the ball to be drawn from **bag C** with a sample of **200 random draws** with replacement.	Option 2 : **10.50€**.

Each envelope is of a particular type indicated by a greek letter (alpha, beta, gamma...)

in the envelope. There are 13 types of envelopes corresponding to 13 different types of questions.

The options contained in the different types of envelopes are given in the following table:

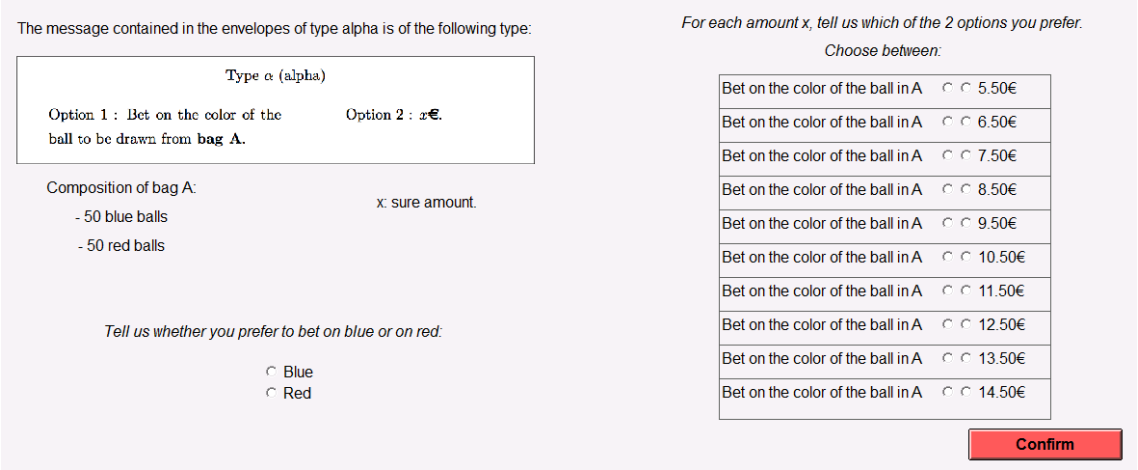
envelopes	option 1	option 2
α	bet on a ball from A	x€
β	bet on a ball from B	x€
γ	bet on a ball from B with 1 observation	x€
δ	bet on a ball from B with 5 observations	x€
ϵ	bet on a ball from B with 20 observations	x€
θ	bet on a ball from B with 50 observations	x€
ι	bet on a ball from B with 200 observations	x€
κ	bet on a ball from C	x€
λ	bet on a ball from C with 1 observation	x€
π	bet on a ball from C with 5 observations	x€
σ	bet on a ball from C with 20 observations	x€
ϕ	bet on a ball from C with 50 observations	x€
ω	bet on a ball from C with 200 observations	x€

We have 130 envelopes, 10 of each type. These envelopes are numbered from 1 to 130. Five of you are now asked to check their numbering.

With this established, I am going to walk among you and each of you will randomly draw one of the envelopes.

DO NOT OPEN YOUR ENVELOPE! Anyone who opens his envelope will be immediately excluded from the experiment and will not receive any financial reward.

During the experiment, the contents of the envelopes of type alpha will be presented to you as follows:



This question concerns bag A. Option 1 corresponds to the bet on the color of the ball to be drawn from bag A. If the color of the drawn ball matches the selected color, you win 15€, otherwise, you win 5€. Option 2 corresponds to a sure amount x in euros.

First, tell us whether you prefer to bet on a blue draw or on a red draw from bag A.

Second, fill out the table on the right. Here, for each amount x, tell us which of the two options you prefer. To this end, fill out the following table. In this table, each line describes the content of an envelope. For your convenience, the filling of this table is automated. When you tick a box of a line of the table, option 1 is automatically ticked for the lines above and option 2 is automatically ticked for the lines below. Thus, it is enough for you to tick a single box of the table to fill out every lines of the table.

Indeed, if for the first line of the table you choose option 1 "Bet on the color of the ball in A", tick option 1 on the first line of the table. Then, option 2 will be automatically ticked for the rest of the decisions, that is to say the lines 2 to 10. [During the oral explanations with slides, each pattern of answers should be illustrated with an example of a filled table.] If for the first line of the table, you choose option 2 "5.50€", tick option 2 on the first line of the table. Then, option 2 will be automatically ticked for the rest of the decisions, that is to say for the lines 2 to 10.

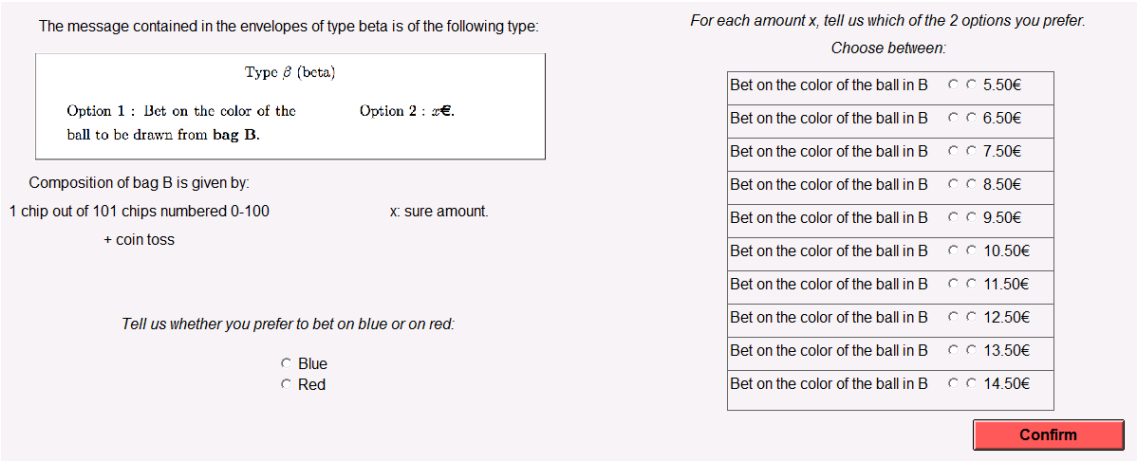
You may also choose option 1 for the second line of the table. In this case, tick option

1 on the second line of the table. Option 1 will be automatically ticked for the line 1 of the table, and option 2 will be automatically ticked for the lines 3 to 10 of the table. Otherwise, you may choose option 2 for the second line of the table. In this case, tick option 2 on the second line of the table. Option 1 will be automatically ticked for the line 1 of the table, and option 2 will be automatically ticked for the lines 3 to 10 of the table.

And so on until the end of the table. You can choose option 1 for the tenth line of the table. In this case, tick option 1 on the tenth line of the table. Option 1 will be automatically ticked for the rest of the decisions, that is to say for the lines 1 to 9.

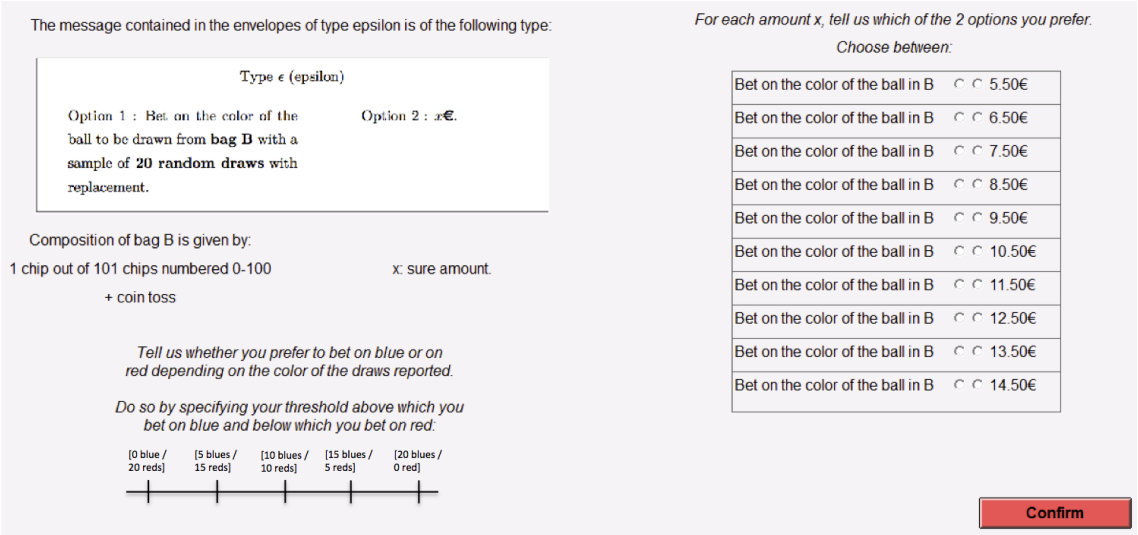
Note that you can revise your choice as many times as needed before confirming your choice and proceeding to the following question.

The contents of the envelopes of type beta will be presented to you as follows:

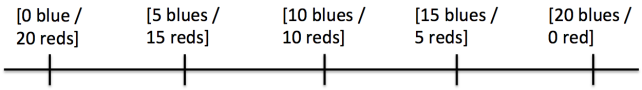


This question concerns bag B. Option 1 corresponds to the bet on the color of the ball to be drawn from bag B. Option 2 corresponds to a sure amount x in euros. First, tell us whether you prefer to bet on a blue draw or on a red draw from bag A. Second, for each amount x, tell us which of the two options you prefer. Do so by filling out the table on the right as described previously.

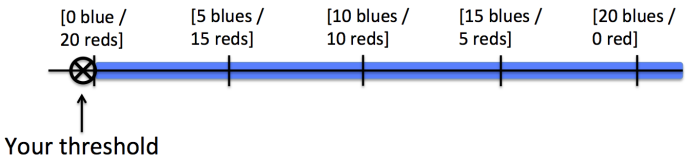
When observations from bag B are available, the contents of the envelopes will be presented to you a bit differently, for instance:



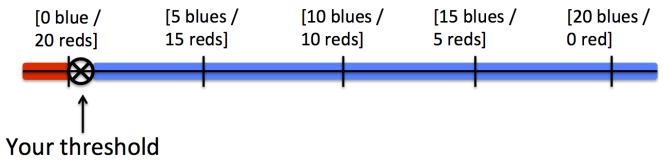
This is the content of envelope of type epsilon. Option 1 corresponds to the bet on the color of the ball to be drawn from bag B with a sample of 20 random draws with replacement. Option 2 corresponds to a sure amount x in euros. First, you announce whether you prefer to bet on blue or on red depending on the color of the draws reported. Here, you have to specify the threshold above which you bet on blue and below which you bet on red. You answer this question using a horizontal slider as below:



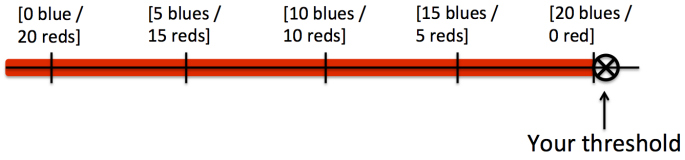
You are asked to place your threshold on this horizontal line. If you place your threshold below [0 blue / 20 reds], it means that you will bet on blue for every sets of observations containing 20 balls.



If you place your threshold between [0 blue / 20 reds] and [1 blue / 19 reds], it means that you will bet on red for a set of observations containing no blue ball while you will bet on blue for every sets of observations containing 1 and more blue balls.

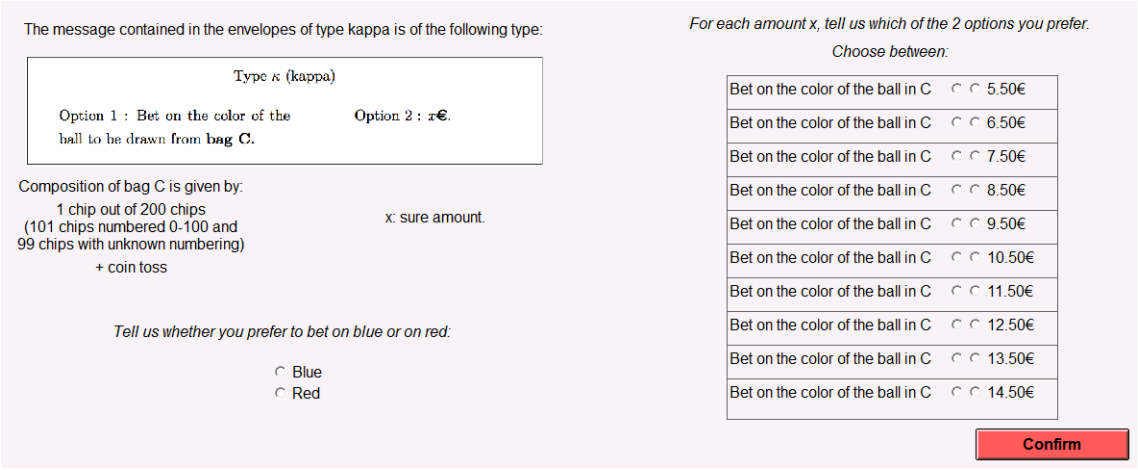


And so on until the last threshold: if your threshold is above [20 blues / 0 red], it means that you will bet on red for every sets of observations containing 20 balls.

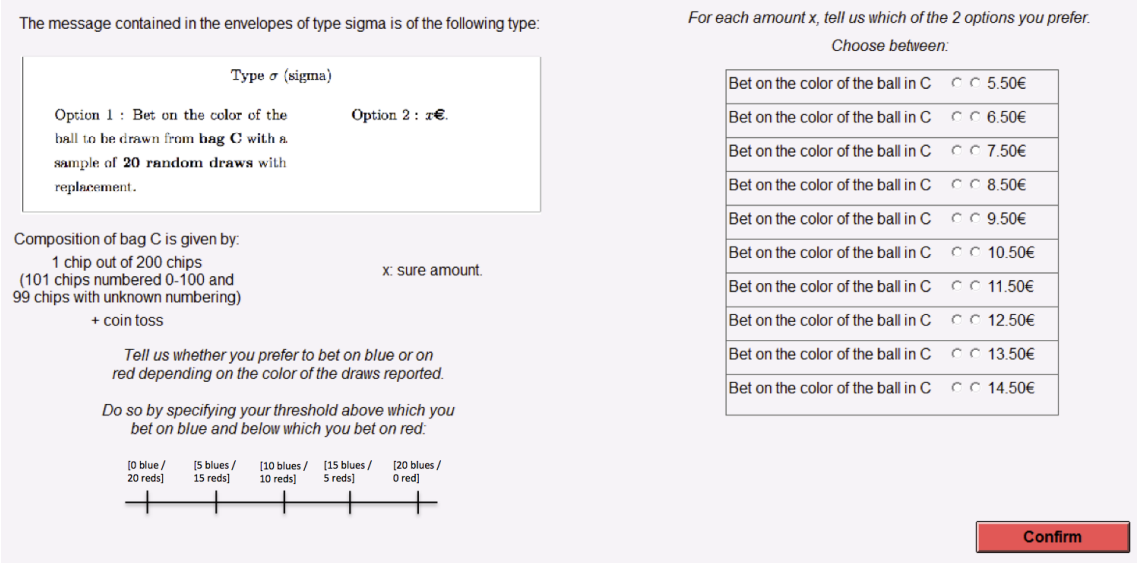


After that, fill out the table on the right. Here, for each amount x, tell us which of the 2 options you prefer.

Regarding bag C, the contents of the envelope of type kappa will be presented as follows:



When observations are available, you will face for instance:



YOUR PAYMENT:

In this experiment, the minimum gain is 5 euros and the maximum gain is 15 euros.

Reminder: There are no "right" or "wrong" answers in this experiment, it is only a matter of preferences.

PLAN OF THE EXPERIMENT:

Practically, you will answer the questions of the experiment using the computer in front of you.

First, you will be asked some comprehension questions to check your understanding of the instructions. Your answers to these questions do not affect your payment.

Then, the experiment on your choices of options will start. The experiment consists of 13 successive stages for 13 types of envelopes (13 different types of questions).

Finally, you will be asked to fill out a short complementary questionnaire to get to know you better. Your answers to these questions do not affect your payment.

The last screen will invite you to reach the experimenter's office to proceed for payment.

In the office of the experimenter:

I will open the envelope in front of you and I will just implement what you chose during the session.

If the option chosen during the session is a bet without observation of draws: the color that you selected for this specific question will be recalled and you will proceed to draw a ball at random from the bag involved in the bet. If the drawn ball matches the winning color, you win 15€; otherwise, you only win 5€.

If the option is a bet with observation of draws: the computer will generate a set of random draws with replacement from the bag involved in the bet. The threshold you announce for this particular question will be recalled and compared to the set of random draws to determine your winning color. Then you will proceed to draw a ball at random from the bag involved in the bet. If the drawn ball matches the winning color, you win 15€; otherwise, you only win 5€.

If the option is a sure amount x: you get the x€ gain.

Because you do not know the content of your envelope and because I will implement your choices, it is in your best interest to tell us your preferred option at each question. Indeed, if you tell us what you want, your preferred option will be the one implemented!

Verification:

At the end of the experiment, the two numbered chips and the two results of coin tosses used to generate the bag B and the bag C will be publicly displayed. You will be free to check the content of all bags.

Besides, you will get a list describing the contents of all envelopes. This list describes the content of each envelope: number of the envelope, type of the envelope, option 1, option 2. You will then check that our description of the numbered envelopes was truthful. You will also be able to check that the list does contain the 13 mentioned tasks.

Your answers will be kept strictly confidential and anonymous, henceforth, feel free to answer as you like. Moreover, there is no time limit, so take all the time you need to read the instructions and answer the questions.

Do you have any questions?

If everything is clear, you can now start the experiment!

— End of Instructions —

Conclusion

This thesis investigates decision making under uncertainty when agents are provided with statistical information. To address this issue, I draw on the theoretical literature that provides models of ambiguity sensitive preferences and I examine the question from an experimental viewpoint. More precisely, in all three experiments presented in this thesis, the Ellsberg's (1961) two-color experiment is adapted so that the subject can observe random draws which enable him to learn the composition of the urn.

In Chapters 1 and 2, I study preferences for information precision when decision makers are informed ex ante. In the first chapter, I show that most experimental answers can be explained by the Subjective Expected Utility model when both urns are partially ambiguous, i.e., described by more or less precise sets of observations. However, experimental results suggest weak but significant ambiguity aversion at the individual level. In the second chapter, datasets are defined in such a way that posterior beliefs regarding the composition of the ambiguous urn coincide with the actual composition of the risky urn. Thus, the design of the experiment allows to directly assess individual's perception of ambiguity and his attitude towards it. I report that most subjects behave in conformity with the Ellsberg paradox when preferences are elicited through direct choices. However, the valuations of the bets on the risky urn and on the ambiguous urn do not differ significantly, regardless of the precision of information.

The experimental findings of Chapter 1 and Chapter 2 question the robustness of ambiguity aversion. On one hand, my results on the Ellsberg two-color setting with a risky

urn and a (fully or partially) ambiguous urn replicate the proportion of ambiguity aversion usually reported in the literature (Oechssler and Roomets, 2015). Hence, increasing the number of observations on the ambiguous urn does not result in an increase in its attractiveness in comparison to the risky urn, which suggests robust ambiguity aversion. On the other hand, I find that valuation tasks elicit lower ambiguity aversion than choice tasks. Moreover, when the decision situation involves two partially ambiguous prospects, most choices can be reconciled with Subjective Expected Utility maximization. Therefore, this research is in line with the experimental studies challenging the consensus on universal ambiguity aversion and according to which ambiguity aversion among decision makers is mitigated as soon as the design moves away from the standard Ellsberg two-color design: e.g., in the loss domain or with low likelihoods (Kocher *et al.*, 2015), in the Ellsberg three-color design (Charness *et al.*, 2013) and with natural sources of ambiguity such as the variation in stock indexes (Baillon *et al.*, 2017).

In Chapter 3, I consider the situation in which decision makers are uninformed and have the opportunity to obtain information. I provide operating instructions for the elicitation of the reservation price for datasets that contain more or less observations and, hence, differ in their informativeness in Blackwell's (1951) sense. The proposed design allows to experimentally assess: (1) the value of additional information; (2) the Blackwell's ranking of informative structures; and (3) the relation between the valuation of informative signals and ambiguity attitudes. The implementation of the experiment is the natural future development of this research.

In this thesis, I focus on individual choices. In order to further explore the effects of information on decision making under uncertainty, it would be of interest to study the implications of information in strategic games when players have ambiguity sensitive preferences. In game theory, informative signals can consist of sets of observations of previous actions taken by the players. It can also refer to communication between players, which is more or less reliable depending on the type of the message (e.g., cheap talk or

certified information). Hence, this topic raises many stimulating questions that have not been discussed yet.

Many economic problems can be represented as decision making under uncertainty. An interesting research question asks about the influence of ambiguity on job search. In the standard search-and-matching models of unemployment, unemployed workers and unfilled vacancies all know the probability distribution of the match specific productivity. In real life, however, agents usually do not have such precise knowledge and form beliefs about the productivity distribution instead. Since productivity determines the wage earned by the worker and the surplus collected by the firm, the ambiguity regarding its distribution and the attitudes of agents towards ambiguity might have large impacts on the job search process. The communication of information (e.g., via résumés and interviews) allows to learn about the match specific productivity. In future studies, it would be of particular interest to investigate the effects of ambiguity and information transmission on job search and job creation decisions from both theoretical and empirical perspectives.

Another domain that provides a relevant framework to study information and choice behaviors under uncertainty is health economics. When undergoing a medical treatment, the patient has only partial knowledge regarding the future effects of the drug on his condition. How agents process and value additional information? The choice of the health insurance contract can also be conceptualized as decision making under uncertainty in strategic games. Several works have attempted to model health decision making under the realistic context of ambiguity but empirical evidence is sparse. Indeed, experimental studies in health economics are challenging because non-monetary outcomes raises the issue of incentives. However, hypothetical experiments have gained popularity among economists recently. Further analysis in this direction must be considered.

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An experimental study of preferences for information

Abstract: In most real-life choice situations, decision makers cannot predict the outcome of their action with certainty. Decision theory generally distinguishes between the typical case of risk, in which the probabilities of the different events are objectively known, and the situation of ambiguity, in which the informational context does not allow such objective probability judgment. However, agents are often provided with partial information consisting of past observations, leading to situations beyond the traditional known-unknown probabilities dichotomy. This thesis empirically investigates decision-making under uncertainty when statistical information is available. The first two chapters focus on choice behaviors in the presence of information, i.e., when prospects are described by datasets ex ante. These chapters study the influence of information precision on the perception of ambiguity and on the attitudes towards risk and ambiguity. In the third chapter, I consider the situation in which agents are uninformed and have the opportunity to acquire information. This chapter proposes an experiment designed to explore the individual valuation for additional information.

Key words: Ambiguity, Decision Making, Experimental Economics, Preferences for Information, Ellsberg Paradox.

Une étude expérimentale des préférences pour l'information

Résumé: Dans la plupart des situations concrètes de choix, les décideurs ne peuvent pas prédire le résultat de leur action avec certitude. La théorie de la décision distingue généralement entre le cas typique du risque, où les probabilités des différents événements sont connus objectivement, et la situation de l'ambiguïté, où le contexte informationnel ne permet pas d'établir de telles probabilités objectives. Cependant, les agents détiennent souvent de l'information partielle sous la forme d'ensembles d'observations passées, conduisant à des situations qui dépassent la dichotomie traditionnelle entre probabilités connues et inconnues. Cette thèse fournit une analyse empirique des comportements de décision en situation d'incertitude lorsque de l'information statistique est disponible. Les deux premiers chapitres portent sur les comportements de choix en présence d'information, c'est-à-dire lorsque les différentes options sont décrites par des ensembles de données ex ante. Ces chapitres étudient l'influence de la précision de l'information sur la perception de l'ambiguïté et sur les attitudes vis-à-vis du risque et de l'ambiguïté. Dans le troisième chapitre, je considère le cas dans lequel les agents ne sont pas informés et ont l'opportunité d'acquérir de l'information. Ce chapitre propose une expérience dont l'objectif est d'étudier la manière dont les individus évaluent l'information additionnelle.

Mots clés: Ambiguïté, Prise de Décision, Economie Expérimentale, Préférences pour l'Information, Paradoxe d'Ellsberg.