New Analytical Methods for the Analysis and Optimization of Energy-Efficient Cellular Networks by Using Stochastic Geometry

Lam Thanh Tu

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New Analytical Methods for the Analysis and Optimization of Energy-Efficient Cellular Networks by Using Stochastic Geometry

Thèse de doctorat de l’Université Paris-Saclay préparée à Université Paris-Sud

Ecole doctorale n° 580:
Sciences et Technologies de l’Information et de la Communication
Spécialité de doctorat: Réseaux, Information et Communications

Thèse présentée et soutenue à Gif-sur-Yvette, le 18 Juin 2018, par

Lam-Thanh TU

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Titre: Nouvelles méthodes d’analyse et d’optimisation des réseaux cellulaires à haute efficacité énergétique en utilisant la géométrie stochastique.

Mots clés: Géométrie stochastique, Efficacité énergétique, Probabilité de couverture

Résumé: L’analyse et l’optimisation au niveau de système sont indispensables pour la progression de performance des réseaux de communication. Ils sont nécessaires afin de faire fonctionner de façon optimale des réseaux actuels et de planifier des réseaux futurs. La modélisation et l’analyse au niveau de système des réseaux cellulaires ont été facilitées grâce à la maîtrise de l’outil mathématique de la géométrie stochastique et, plus précisément, la théorie des processus ponctuels spatiaux. Du point de vue de système, il a été empiriquement validé que les emplacements des stations cellulaires de base peuvent être considérés comme des points d’un processus ponctuel de Poisson homogène dont l’intensité coïncide avec le nombre moyen de stations par unité de surface. Dans ce contexte, des contributions de ce travail se trouvent dans le développement de nouvelles méthodologies analytiques pour l’analyse et l’optimisation des déploiements de réseaux cellulaires émergents. Le lecteur découvrira alors trois contributions principales dans ce manuscrit.

La première contribution consiste à introduire une approche pour évaluer la faisabilité de réseaux cellulaires multi-antennes, dans lesquels les dispositifs mobiles à faible énergie décodent les données et récupèrent l’énergie à partir d’un même signal reçu. Des outils de géométrie stochastique sont utilisés pour quantifier le taux d’information par rapport au compromis de puissance captée. Les conclusions montrent que les réseaux d’antennes à grande échelle et les déploiements ultra-denses de stations base sont tous deux nécessaires pour capter une quantité d’énergie suffisamment élevée et fiable. En outre, la faisabilité de la diversité des récepteurs pour l’application aux réseaux cellulaires descendants est également étudiée. Diverses options basées sur la combinaison de sélection et la combinaison de taux maximal sont donc comparées. En s’appuyant sur l’inégalité de Frechet, nous mettons en évidence des avantages et des limites de chaque schéma en fonction du taux de transmission et de la puissance captée. Ces derniers sont requis strictement au niveau des dispositifs à basse énergie.

Notre analyse montre qu’aucun système n’est plus performant que les autres pour chaque configuration de système: les dispositifs à basse énergie doivent fonctionner de manière adaptative, en choisissant le schéma de diversité des récepteurs en fonction des exigences imposées.

La deuxième contribution consiste à introduire une nouvelle approche pour la modélisation et l’optimisation de l’efficacité énergétique des réseaux cellulaires. Contrairement aux approches analytiques actuellement disponibles qui fournissent des expressions analytiques trop simples ou trop complexes de la probabilité de couverture et de l’efficacité spectrale des réseaux cellulaires, l’approche proposée est formulée par une solution de forme fermée qui se révèle en même temps simple et significative. Une nouvelle expression de l’efficacité énergétique du réseau cellulaire descendant est proposée à partir d’une nouvelle formule de l’efficacité spectrale. Cette expression est utilisée pour l’optimisation de la puissance d’émission et la densité des stations cellulaires de base. Il est prouvé mathématiquement que l’efficacité énergétique est une fonction unimodale et strictement pseudo-concave de la puissance d’émission en fixant la densité des stations de base, et de la densité des stations de base en fixant la puissance d’émission. La puissance d’émission optimale et la densité des stations de base s’avèrent donc être la solution des équations non linéaires simples. La troisième contribution consiste à introduire une nouvelle approche pour analyser les performances des réseaux cellulaires hétérogènes équipés des sources d’énergie renouvelables, telles que les panneaux solaires. L’approche proposée permet de tenir compte de la distribution spatiale des stations de base en utilisant la théorie des processus ponctuels, ainsi que l’apparition aléatoire et la disponibilité de l’énergie en utilisant la théorie des chaînes de Markov. En utilisant l’approche proposée, l’efficacité énergétique des réseaux cellulaires peut être quantifiée et l’interaction entre la densité des stations de base et le taux d’énergie d’apparition peut être quantifiée et optimisée.

Key words: Poisson Point Process, Stochastic Geometry, Energy Efficiency, Coverage Probability

Simultaneous Wireless Information and Power Transfer, Renewable Energy, Markov chains

Abstract In communication networks, system-level analysis and optimization are useful when one is interested in optimizing the system performance across the entire network. System-level analysis and optimization, therefore, are relevant for optimally operating current networks, and for deploying and planning future networks. In the last few years, the system-level modeling and analysis of cellular networks have been facilitated by capitalizing on the mathematical tool of stochastic geometry and, more precisely, on the theory of spatial point processes. It has been empirically validated that, from the system-level standpoint, the locations of cellular base stations can be abstracted as points of a homogeneous Poisson point process whose intensity coincides with the average number of based stations per unit area. In this context, the contribution of the present Ph.D. thesis lies in developing new analytical methodologies for analyzing and optimizing emerging cellular network deployments. The present Ph.D. thesis, in particular, provides three main contributions to the analysis and optimization of energy-efficient cellular networks.

The first contribution consists of introducing a tractable approach for assessing the feasibility of multiple-antenna cellular networks, where low-energy mobile devices decode data and harvest power from the same received signal. Tools from stochastic geometry are used to quantify the information rate vs. harvested power tradeoff. Our study unveils that large-scale antenna arrays and ultra-dense deployments of base stations are both necessary to harvest, with high reliability, a sufficiently high amount of power. Furthermore, the feasibility of receiver diversity for application to downlink cellular networks is investigated. Several options that are based on selection combining and maximum ratio combining are compared against each other. By capitalizing on the Frechet inequality, we shed light on the advantages and limitations of each scheme as a function of the transmission rate and harvested power that need to be fulfilled at the low-energy devices. Our analysis shows that no scheme outperforms the others for every system setup.

It suggests, on the other hand, that the low-energy devices need to operate in an adaptive fashion, by choosing the receiver diversity scheme as a function of the imposed requirements.

The second contribution consists of introducing a new tractable approach for modeling and optimizing the energy efficiency of cellular networks. Unlike currently available analytical approaches that provide either simple but meaningless or meaningful but complex analytical expressions of the coverage probability and spectral efficiency of cellular networks, the proposed approach is conveniently formulated in a closed-form expression that is proved to be simple and meaningful at the same time. By relying on the new proposed formulation of the spectral efficiency, a new tractable closed-form expression of the energy efficiency of downlink cellular network is proposed, which is used for optimizing the transmit power and the density of cellular base stations. It is mathematically proved, in particular, that the energy efficiency is a unimodal and strictly pseudo-concave function in the transmit power, given the density of the base stations, and in the density of the base stations, given the transmit power. Under these assumptions, therefore, a unique transmit power and density of the base stations is proved to exist and to maximize the energy efficiency, regardless of the specific system setup. The optimal transmit power and density of base stations are proved to be the solution of simple non-linear equations.

The third contribution consists of introducing a new tractable approach for analyzing the performance of multi-tier cellular networks equipped with renewable energy sources, such as solar panels. The proposed approach allows one to account for the spatial distribution of the base stations by using the theory of point processes, as well as for the random arrival and availability of energy by using Markov chain theory. By using the proposed approach, the energy efficiency of cellular networks can be quantified and the interplay between the density of base stations and energy arrival rate can be quantified and optimized. The proposed approaches have been validated with the aid of extensive Monte Carlo simulations.

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L’analyse et l’optimisation au niveau de système sont indispensables pour la progression de performance des réseaux de communication. Ils sont nécessaires afin de faire fonctionner de façon optimale des réseaux actuels et de planifier des réseaux futurs. La modélisation et l’analyse au niveau de système des réseaux cellulaires ont été facilitées grâce à la maîtrise de l’outil mathématique de la géométrie stochastique et, plus précisément, la théorie des processus ponctuels spatiaux. Du point de vue de système, il a été empiriquement validé que les emplacements des stations cellulaires de base peuvent être considérés comme des points d’un processus ponctuel de Poisson homogène dont l’intensité coïncide avec le nombre moyen de stations par unité de surface. Dans ce contexte, des contributions de ce travail se trouvent dans le développement de nouvelles méthodologies analytiques pour l’analyse et l’optimisation des déploiements de réseaux cellulaires émergents. Le lecteur découvrira alors trois contributions principales dans ce manuscrit.

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1.4.5 Aggregate Other-cell Interference ........................................... 15
1.4.6 Performance Metrics ............................................................ 18
1.4.7 State-of-the-art on system level modeling ............................... 26

1.5 Wireless Power Transfer (WPT) - Simultaneous Wireless Information and Power Transfer (SWIPT) ......................................................... 27
1.5.1 SWIPT Architectures ............................................................. 28
1.5.2 SWIPT with multiple-input multiple-output (MIMO) ............... 29
1.5.3 Interference management in SWIPT-enabled networks .............. 30
1.5.4 State-of-the-art on SWIPT-enabled networks ............................ 30

1.6 Energy Efficiency in Cellular Network ....................................... 31
1.6.1 Definition of Energy Efficiency .............................................. 32
1.6.2 Improving the Energy Efficiency ............................................ 33
1.6.3 State-of-the-art on Energy Efficiency Analysis .......................... 35

2 SWIPT-Enabled Cellular Networks with MIMO 37
2.1 Abstract ...................................................................................... 37
2.2 Introduction ................................................................................. 37
2.3 System Model ............................................................................. 39
   2.3.1 Cellular Network Modeling .................................................. 39
   2.3.2 SWIPT Based on Power Splitting ........................................... 39
   2.3.3 Channel Modeling ............................................................... 39
   2.3.4 Cell Association ................................................................. 40
2.4 MIMO Cellular Networks .......................................................... 40
2.5 System-Level Analysis ............................................................... 42
   2.5.1 Exact Mathematical Framework .......................................... 43
   2.5.2 Approximated Mathematical Framework ............................... 44
   2.5.3 Asymptotic (Large-Scale) Mathematical Framework ............. 45
2.6 Numerical and Simulation Results .............................................. 46

3 SWIPT-Enabled Cellular Networks with Receiver Diversity 49
3.1 Abstract ...................................................................................... 49
3.2 Introduction ................................................................................. 50
3.3 System Model ........................................... 52
  3.3.1 Cellular Networks Modeling .......................... 52
  3.3.2 Channel Modeling ....................................... 54
  3.3.3 Cell Association ......................................... 55
  3.3.4 Directional Beamforming at the BSs ....................... 55
  3.3.5 SWIPT and Receiver Diversity at the LEDs ............. 55
3.4 Problem Statement ........................................ 57
  3.4.1 Motivation: On the Benefits of Receiver Diversity ....... 57
  3.4.2 Problem Formulation ..................................... 59
3.5 Performance Comparison and Trends ......................... 59
  3.5.1 SAR versus ID-SC ....................................... 60
  3.5.2 SAR versus EH-SC ....................................... 61
  3.5.3 ID-SC versus EH-SC ..................................... 61
  3.5.4 PS-MRC versus A-SC .................................... 62
  3.5.5 PS-MRC versus PS ....................................... 63
  3.5.6 Main Performance Trends ................................ 65
3.6 System-Level Analysis ..................................... 66
3.7 Numerical and Simulation Results .......................... 70
3.8 Conclusion ................................................. 76
3.9 Appendix: Exact J-CCDF of SAR, ID-SC and EH-SC SWIPT Schemes .... 77

4 System-Level Energy Efficiency of Cellular Networks ............ 79
  4.1 Abstract .................................................. 79
  4.2 Introduction ............................................... 80
    4.2.1 Fundamental Limitations of Current Approaches for System-Level
    EE Optimization .............................................. 82
    4.2.2 State-of-the-Art on System-Level EE Optimization ........ 84
    4.2.3 Research Contribution and Novelty ........................ 85
  4.3 System Model ............................................... 86
    4.3.1 Cellular Network Modeling .............................. 86
    4.3.2 Channel Modeling ....................................... 87
4.3.3 Cell Association Criterion ........................................ 87
4.3.4 Load Modeling ......................................................... 88
4.3.5 Power Consumption Modeling ........................................ 89
4.4 A New Analytical Formulation of the PSE ................................ 89
  4.4.1 Analytical Formulation of the PSE ................................ 93
  4.4.2 Closed-Form Expressions of PSE and $P_{\text{grid}}$ .................. 96
4.5 System-Level EE Optimization: Formulation and Solution ........... 98
  4.5.1 Preliminaries ....................................................... 99
  4.5.2 Optimal Transmit Power Given the Density of the BSs ............ 101
  4.5.3 Optimal Density Given the Transmit Power of the BSs ............ 101
  4.5.4 On the Dependency of Optimal Transmit Power and Density of the
        BSs ............................................................... 102
  4.5.5 Joint Optimization of Transmit Power and Density of the BSs .... 104
4.6 Numerical Results ....................................................... 108
4.7 Conclusion ............................................................. 114
4.8 Appendix A – Proof of Proposition 6 .................................. 115
4.9 Appendix B – Proof of Theorem 1 ...................................... 116
4.10 Appendix C – Proof of Theorem 2 ..................................... 117

5 Cellular Networks with Renewable Energy Sources ................. 121
  5.1 Introduction .......................................................... 121
  5.2 System Model .......................................................... 124
    5.2.1 Cellular Networks Modeling ..................................... 124
    5.2.2 Channel Modeling ............................................... 125
    5.2.3 Cell Association ................................................ 126
    5.2.4 Load Model ...................................................... 126
    5.2.5 Power Consumption Modeling .................................. 127
    5.2.6 Energy Harvesting Modeling ................................... 128
    5.2.7 Probability of the off-grid BSs’ activation ...................... 134
  5.3 Performance Analysis ................................................ 134
    5.3.1 Power Consumption of Macro BSs ............................. 134
5.3.2 Potential Spectral Efficiency (PSE) ......................... 137
5.3.3 Energy Efficiency ............................................... 138
5.4 Numerical Results ............................................... 138
5.5 Conclusion ....................................................... 142

6 Conclusions and Future Works 143

6.1 Conclusions ...................................................... 143
6.2 Future Works .................................................... 144
  6.2.1 Beyond PPP modeling ...................................... 144
  6.2.2 UAV-aided cellular networks .............................. 145
  6.2.3 Network Slicing (NS) ..................................... 145
  6.2.4 Cellular Networks with Decoupled Architecture ......... 145

Bibliography 147
## Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3GPP</td>
<td>The 3rd Generation Partnership Project</td>
</tr>
<tr>
<td>A2G</td>
<td>Air-to-Ground</td>
</tr>
<tr>
<td>ADC</td>
<td>Analog-to-Digital Converter</td>
</tr>
<tr>
<td>AS</td>
<td>Antenna Switching</td>
</tr>
<tr>
<td>A-SC</td>
<td>Adaptive Selection Combining</td>
</tr>
<tr>
<td>ASE</td>
<td>Area Spectral Efficiency</td>
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<tr>
<td>BER</td>
<td>Bit Error Rate</td>
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<tr>
<td>BSs</td>
<td>Base Stations</td>
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<tr>
<td>CAPEX</td>
<td>CAPital EXpenditure</td>
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<tr>
<td>CCDF</td>
<td>Complementary Cumulative Distribution Function</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>CF</td>
<td>Characteristic Function</td>
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<tr>
<td>D2D</td>
<td>Device-to-Device Communications</td>
</tr>
<tr>
<td>DL</td>
<td>Downlink</td>
</tr>
<tr>
<td>DUD</td>
<td>Decoupled Uplink and Downlink</td>
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<tr>
<td>EE</td>
<td>Energy Efficiency</td>
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<tr>
<td>EH</td>
<td>Energy Harvesting</td>
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<tr>
<td>EH-SC</td>
<td>EH-Prioritized Selection Combining</td>
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<tr>
<td>EM</td>
<td>Electromagnetic</td>
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<tr>
<td>ICT</td>
<td>Information and Communications Technology</td>
</tr>
<tr>
<td>ID</td>
<td>Information Decoding</td>
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<tr>
<td>ID-SC</td>
<td>ID-Prioritized Selection Combining</td>
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<tr>
<td>IoTs</td>
<td>Internet of Things</td>
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<tr>
<td>J-CCDF</td>
<td>Joint Complementary Cumulative Distribution Function</td>
</tr>
<tr>
<td>LEDs</td>
<td>Low-Energy Devices</td>
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<tr>
<td>LOS</td>
<td>Line-Of-Sight</td>
</tr>
<tr>
<td>MANET</td>
<td>Mobile Ad Hoc Network</td>
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<tr>
<td>MBSs</td>
<td>Macro Base Stations</td>
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<td>MGF</td>
<td>Moment Generating Function</td>
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<tr>
<td>MIMO</td>
<td>Multiple-Input-Multiple-Output</td>
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<tr>
<td>MRC</td>
<td>Maximum Ratio Combining</td>
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<tr>
<td>Acronym</td>
<td>Full Form</td>
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<tr>
<td>MRT</td>
<td>Maximum Ratio Transmission</td>
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<tr>
<td>MTs</td>
<td>Mobile Terminals</td>
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<td>NB IoT</td>
<td>Narrow Band IoT</td>
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<td>NLOS</td>
<td>Non-Line-Of-Sight</td>
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<td>NS</td>
<td>Network Slicing</td>
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<td>OPEX</td>
<td>OPerating EXpense</td>
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<td>PA</td>
<td>Power Amplifier</td>
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<tr>
<td>Pcov</td>
<td>Coverage Probability</td>
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<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<td>PGFL</td>
<td>Probability Generating Functional</td>
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<tr>
<td>PP</td>
<td>Point Process</td>
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<tr>
<td>PPP</td>
<td>Poisson Point Process</td>
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<td>PS</td>
<td>Power Switching</td>
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<tr>
<td>PSE</td>
<td>Potential Spectral Efficiency</td>
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<tr>
<td>PS-MRC</td>
<td>Power Splitting with Maximum Ratio Combining</td>
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<tr>
<td>QoS</td>
<td>Quality of Service</td>
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<tr>
<td>RAN</td>
<td>Radio Access Network</td>
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<tr>
<td>RF</td>
<td>Radio Frequency</td>
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<tr>
<td>RV</td>
<td>Random Variable</td>
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<tr>
<td>SAR</td>
<td>Separate Antenna Receiver</td>
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<tr>
<td>SBSs</td>
<td>Small Cell Base Stations</td>
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<tr>
<td>SC</td>
<td>Selection Combining</td>
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<tr>
<td>SE</td>
<td>Spectral Efficiency</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
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<tr>
<td>SINR</td>
<td>Signal to Interference plus Noise Ratio</td>
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<tr>
<td>SIR</td>
<td>Signal to Interference Ratio</td>
</tr>
<tr>
<td>SWIPT</td>
<td>Simultaneous Wireless Information and Power Transfer</td>
</tr>
<tr>
<td>TS</td>
<td>Time Switching</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Aerial Vehicle</td>
</tr>
<tr>
<td>UL</td>
<td>Uplink</td>
</tr>
<tr>
<td>WIT</td>
<td>Wireless Information Transfer</td>
</tr>
<tr>
<td>WPT</td>
<td>Wireless Power Transfer</td>
</tr>
<tr>
<td>WSN</td>
<td>Wireless Sensor Network</td>
</tr>
</tbody>
</table>
Notations and Functions

$(\cdot)^*$  Conjugate operator

$j = \sqrt{-1}$  Imaginary unit

$\mathbb{E}\{\cdot\}$  Expectation operator

$\Pr\{\cdot\}$  Probability measure

$\cup$  Union of sets

$\text{Im}\{\cdot\}$  Imaginary part operator

$\text{min}\{\cdot,\cdot\}$  Minimum operator

$\text{max}\{\cdot,\cdot\}$  Maximum operator

$1(\cdot)$  Indicator function

$\delta(\cdot)$  Dirac delta function

$\mathcal{H}(\cdot)$  Heaviside function

$\mathcal{H}(x) = 1 - \mathcal{H}(x)$  Complementary Heaviside function

$pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; \cdot)$  Generalized hypergeometric function

$\Gamma(\cdot, \cdot)$  Upper-incomplete gamma function

$f_X(\cdot)$  PDF of Random Variable (RV) $X$

$F_X(\cdot)$  CDF of RV $X$

$F_X(\cdot)$  CCDF of RV $X$

$\Phi_X(\cdot)$  CF of RV $X$

$\mathcal{M}_X(\cdot)$  MGF of RV $X$

$\lfloor \cdot \rfloor$  Floor function

$\Gamma(\cdot)$  Gamma function
List of Figures

1.1 BSs deployments and its voronoi cell, the circles denote the locations of BSs, the solid lines are the voronoi boundary: (a) hexagonal model, (b) actual locations of O2 (until 2012) in downtown London, (c) PPP distributed BSs. 7

1.2 An illustration of a dual-state path-loss model. 9

1.3 Pcov vs. Rcell with the single- and dual-state unbounded path-loss models. These curves are plotted by using Monte Carlo simulation with 250 thousand realizations. Noise figure is 10 dB, noise spectral density is -174 dBM/Hz; omnidirectional antenna; $\kappa = \left( \frac{4\pi}{v} \right)^2$, $v = \frac{c}{f_c}$; c is the speed of light and $f_c = 2.1$ GHz is the carrier frequency. Rayleigh fading with unit mean and no shadowing. The path-loss exponent of dual-state are $\beta_{\text{LOS}} = 2.5$, and $\beta_{\text{NLOS}} = 3.5$. The probability of a link is in LOS and NLOS followed the one-ball model in Table 1.1 with $D = 109.8517$, $q_{\text{LOS}}^{[0,D]} = 0.7195$, and $q_{\text{LOS}}^{(D,\infty)} = 0.0002$. The path-loss exponent of single-state model is $\beta = 3.5$. 11

1.4 Path-loss inversion vs. transmission distance of the single-state unbounded and bounded path-loss models in section 1.4.2. The unbounded curve is plotted by using Eq. (1.3); the bounded curves 1, 2 and 3 are plotted by Eq. (1.4), (1.5), and (1.6), respectively. 12
1.5 An illustration of cell association: (a) Based on the smallest distance, (b) Based on the average highest received power. $P_1$ and $P_2$ are the transmit power of MBS and SBS; $d_1$ and $d_2$ are distances from the SBS and MBS to the typical user, respectively.

1.6 Coverage Probability vs. $R_{cell}$ with various values of $\gamma_1$. These curves are plotted by using Monte Carlo simulation with 250 thousand realizations. Noise figure is 10 dB, noise spectral density is $-174$ dBm/Hz; without beamforming (omnidirectional antenna); mean and standard deviation of shadowing are 2 dB and 1 dB, respectively; Rayleigh fading with unit mean; $\kappa = \left(\frac{4\pi}{v}\right)^2; v = \frac{c}{f_c}$; $c$ is the speed of light and $f_c = 2.1$ GHz is the carrier frequency.

1.7 Coverage Probability vs. $R_{cell}$ with various values of path-loss exponents $\beta$. These curves are plotted by using Monte Carlo simulation with 250 thousand realizations. Noise figure is 10 dB, noise spectral density is $-174$ dBm/Hz; without beamforming (omnidirectional antenna); mean and standard deviation of shadowing are 2 dB and 1 dB, respectively; Rayleigh fading with unit mean; $\kappa = \left(\frac{4\pi}{v}\right)^2; v = \frac{c}{f_c}$; $c$ is the speed of light and $f_c = 2.1$ GHz is the carrier frequency.

1.8 Coverage Probability vs. $R_{cell}$ with different kinds of small-scale fadings. These curves are plotted by using Monte Carlo simulation with 250 thousand realizations. Noise figure is 10 dB, noise spectral density is $-174$ dBm/Hz; without beamforming (omnidirectional antenna); mean and standard deviation of shadowing are 2 dB and 1 dB, respectively; $\kappa = \left(\frac{4\pi}{v}\right)^2; v = \frac{c}{f_c}$; $c$ is the speed of light and $f_c = 2.1$ GHz is the carrier frequency. All curves are unit mean.
1.9 Coverage Probability vs. Rcell with and without consider directional beamforming. These curves are plotted by using Monte Carlo simulation with 250 thousand realizations. Noise figure is 10 dB, noise spectral density is -174 dBm/Hz; mean and standard deviation of shadowing are 2 dB and 1 dB, respectively; Rayleigh fading with unit mean; \( \kappa = \left( \frac{4\pi}{v} \right)^2, v = \frac{c}{f_c} \); \( c \) is the speed of light and \( f_c = 2.1 \) GHz is the carrier frequency. The array gain of without beamforming is 1. The array gain of beamforming is followed (flat top antenna pattern, section 1.4.4): \( G_{max} = 10 \) dB; \( G_{min} = -10 \) dB; main lobe is 30 degree.

1.10 Coverage Probability vs. Rcell with various value of the transmit power. These curves are plotted by using Monte Carlo simulation with 250 thousand realizations. Noise figure is 10 dB, noise spectral density is -174 dBm/Hz; without beamforming (omnidirectional antenna); mean and standard deviation of shadowing are 2 dB and 1 dB, respectively; Rayleigh fading with unit mean; \( \kappa = \left( \frac{4\pi}{v} \right)^2, v = \frac{c}{f_c} \); \( c \) is the speed of light and \( f_c = 2.1 \) GHz is the carrier frequency.

1.11 Coverage Probability vs. Rcell with and without small-scale fading. These curves are plotted by using Monte Carlo simulation with 250 thousand realizations. Noise figure is 10 dB, noise spectral density is -174 dBm/Hz; without beamforming (omnidirectional antenna); mean and standard deviation of shadowing are 2 dB and 1 dB, respectively; Rayleigh fading with unit mean; \( \kappa = \left( \frac{4\pi}{v} \right)^2, v = \frac{c}{f_c} \); \( c \) is the speed of light and \( f_c = 2.1 \) GHz is the carrier frequency.

1.12 SWIPT-enabled receiver: (a) Separated Receiver; (b) TS Receiver; (c) PS Receiver and (d) AS Receiver.

2.1 Contour lines of J-CCDF vs. \( N_t (\rho = 0.5, R_{cell} = 83.4122 \) m [5], \( G_M = 1, G_m = 1, \omega_M = 360 \) degrees (omnidirectional antennas) [7]). They show the pairs \( (R_*, Q_*) \) so that \( F_c(R_*, Q_*) = 0.75 \). Solid lines show exact and approximated frameworks in (2.10) and (2.18). Dashed lines show the asymptotic framework in (2.20). Markers show Monte Carlo simulations.
2.2 Harvested power $Q_*$ vs. $N_t$ and $R_{\text{cell}}$ so that $F_c(R_*, Q_*) = 0.90$ and $R_* = \text{100 kbits/sec}$. $G_M = 32.4195$, $G_m = 0.4363$, $\omega_M = 12$ degrees (directive antennas) [7]. The curves are obtained by using (2.18) and each point corresponds to the optimal $\rho$ that maximizes the JCCDF.

3.1 Contour lines of the J-CCDF of PS and PS-MRC schemes as a function of $\rho$. The curves show the pairs $(R_*, Q_*)$ so that $\mathcal{F}(R_*, Q_*) = 0.75$. Markers: Monte Carlo simulations. Solid lines: Proposition 4.

3.2 Contour lines of the J-CCDF of SAR, ID-SC and EH-SC schemes. The curves show the pairs $(R_*, Q_*)$ so that $\mathcal{F}(R_*, Q_*) = 0.75$. Markers: Monte Carlo simulations. Solid, dotted and dashed lines: Proposition 5.

3.3 Contour lines of the J-CCDF of OPS, OPS-MRC, A-SC and A-SWIPT schemes. The curves show the pairs $(R_*, Q_*)$ so that $\mathcal{F}(R_*, Q_*) = 0.75$. The setups PS-Ideal and PS-MRC-Ideal refer to the PS and PS-MRC schemes where $\rho_{ID} = 0$ and $\rho_{EH} = 1$ simultaneously, e.g., ID and EH are assumed to be performed on the same signal without the need of PS. $\rho_{opt}$ is computed by using Proposition 4. Markers: Monte Carlo simulations. Solid lines: Proposition 4 for OPS, OPS-MRC and A-SWIPT schemes and in Proposition 5 for the A-SC scheme. In the considered setup, the OPS-MRC scheme coincides with the A-SWIPT scheme.

3.4 Highest harvested power $Q_*$ of OPS-MRC and A-SC so that $\mathcal{F}(R_*, Q_*) = 0.90$ and $R_* = 100$ kbps, as a function of $R_{\text{cell}}$. $\rho_{opt}$ is computed by using Proposition 4. Markers: Monte Carlo simulations for the A-SC scheme. Solid lines: Proposition 4 for the OPS-MRC scheme. Dashed lines: Proposition 5 for the A-SC scheme.
4.1 Illustration of the interplay between $P_{tx}$ and $\lambda_{BS}$. For simplicity, only a cluster of seven BSs is represented by keeping the size of the region of interest (square box) the same. The inter-site distance of the BSs (represented as red dots), i.e., the size of the hexagonal cells, is determined by $\lambda_{BS}$. The shape of the cells depends on the cell association in (4.2). The circular shaded disk (in light yellow) represents the actual coverage region of the BSs that is determined by $P_{tx}$: i) a MT inside the disk receives a sufficiently good signal to detect the BS and to get associated with it, ii) a MT outside the disk cannot detect the BS and is not in coverage. The sub-figures (a)-(c) are obtained by assuming the same $\lambda_{BS}$ but a different $P_{tx}$. The sub-figures (d) and (e) are obtained by considering a $\lambda_{BS}$ greater than that of sub-figures (a)-(c) but keeping the same $P_{tx}$ as sub-figures (a) and (b), respectively. The sub-figure (f) is obtained by considering a $\lambda_{BS}$ smaller than that of sub-figure (c) but keeping the same $P_{tx}$ as it. We observe that, for a given $\lambda_{BS}$, the transmit power $P_{tx}$ is appropriately chosen in sub-figures (a), (e) and (f). $P_{tx}$ is, on the other hand, under-provisioned in sub-figure (b) and over-provisioned in sub-figures (c) and (d). In the first case, the MTs are not capable of detecting the BS throughout the entire cell, i.e., a high outage probability is expected. In the second case, the BSs emit more power than what is actually needed, which results in a high power consumption.

4.2 Optimal transmit power (a) and energy efficiency (b) versus $R_{cell}$. Solid lines: Optimum from Theorem 1. Markers: Optimum from a brute-force search of (4.15). Special case with $\beta = 6.5$ and $\lambda_{MT} = 21$ MTs/km$^2$.

4.3 Energy efficiency versus the transmit power for Load Model 1 (a) and Load Model 2 (b). Solid lines: Framework from (4.14). Markers: Monte Carlo simulations.

4.4 Energy efficiency versus $R_{cell}$ for Load Model 1 (a) and Load Model 2 (b). Solid lines: Mathematical Framework from (4.14). Markers: Monte Carlo simulations.
4.5 Optimal transmit power (a) and energy efficiency (b) versus $R_{\text{cell}}$. Solid lines: Optimum from Theorem 1. Markers: Optimum from a brute-force search of (4.15). LM-1: Load Model 1 and LM-2: Load Model 2.

4.6 Optimal density of BSs ($R_{\text{cell}}$) (a) and energy efficiency (b) versus the transmit power. Solid lines: Optimum from Theorem 2. Markers: Optimum from a brute-force search of (4.17). LM-1: Load Model 1, LM-2: Load Model 2.

4.7 Optimal transmit power (a), density of BSs ($R_{\text{cell}}$) (b), and energy efficiency (c) versus the density of MTs ($R_{\text{MT}}$). Solid lines: Optimum from the algorithm in Table 4.3. Markers: Optimum from a brute-force search of (4.21). LM-1: Load Model 1 and LM-2: Load Model 2.

4.8 Optimal transmit power (a), density of BSs ($R_{\text{cell}}$) (b), and energy efficiency (c) versus the reliability thresholds ($\gamma_D = \gamma_A$). Solid lines: Optimum from the algorithm in Table 4.3. Markers: Optimum from a brute-force search of (4.21). LM-1: Load Model 1 and LM-2: Load Model 2.


4.10 Number of iterations of the algorithm in Table 4.3 as a function of $\epsilon > 0$. The number of iterations is averaged (15000 trials) over the initial guess $\lambda_{\text{BS}} = \lambda_{\text{BS}}^{(\text{opt})} \in [\lambda_{\text{BS}}^{(\text{min})}, \lambda_{\text{BS}}^{(\text{max})}]$. (a) Load Model 1 and (b) Load Model 2. Setup: $R_{\text{cell}}^{(\text{min})} = 10$ m, $R_{\text{cell}}^{(\text{max})} = 2000$ m, $P_{\text{tx}}^{(\text{min})} = -20$ dBm, $P_{\text{tx}}^{(\text{max})} = 60$ dBm.

5.1 Coverage Probability vs. $R_{\text{cell}}$ of full-duplex mode of load model 1 and 2 with various value of $\gamma_D$. Solid lines are plotted by Eq. (5.14) and Eq. (5.16). Marker are Monte Carlo simulation.
5.2 PSE vs. $\gamma_D$ of full-duplex mode of load model 1 and 2 with various scenarios. Solid lines are plotted by Eq. (5.14) and Eq. (5.16). Marker are Monte Carlo simulation. Case study 1: $T_{ON}, T_{OFF}, \alpha_{ON}, \alpha_{OFF} = 3, 1, 1, 3$. Case study 2: $T_{ON}, T_{OFF}, \alpha_{ON}, \alpha_{OFF} = 3, 1, 3, 1$. Case study 3: $T_{ON}, T_{OFF}, \alpha_{ON}, \alpha_{OFF} = 1, 3, 3, 1$. Case study 4: $T_{ON}, T_{OFF}, \alpha_{ON}, \alpha_{OFF} = 1, 3, 1, 3$.

5.3 EE vs. $P_{tx}^{ON}$ of half-duplex mode of load model 1 and 2 with various value of $\gamma_D$. Solid lines are plotted by Eq. (5.17). Marker are Monte Carlo simulation.

5.4 EE vs. $R_{cell}$ of full and half-duplex mode of load model 2 with various value of $\gamma_D$. Solid lines are plotted by Eq. (5.17). Marker are Monte Carlo simulation.

5.5 EE vs. $\gamma_D$ of full and half-duplex mode of load model 1 with various value of $\lambda_{EP}$. Solid lines are plotted by Eq. (5.17). Marker are Monte Carlo simulation.
1.1 [7, Table. II], Commonly used link state models. The parameters $a_s$, $b_s$, and $c_s$ are environmental-dependent. Here $p_s(r)$ is probability in state $s \in \{\text{LOS, NLOS}\}$ of transmission distance $r$; $p_{\text{OUT}}(r)$ is probability of transmission distance $r$ in outage. .................................................. 12

3.1 Main notation and mathematical symbols. .................................................. 53

3.2 Definition and distribution of $U_z^{(0)}$, $\gamma_z^{(i)}$ and $\rho_z$ in (3.4) for the five SWIPT schemes in Section 3.3.5. $\gamma_r^{(0)} \sim \mathcal{E}(1)$ and $\gamma_r^{(i)} \sim \mathcal{E}(1)$ are the channel power gains of intended and $i$th interfering BSs at the $r$th receive antenna; $\gamma_{\text{MRC}}^{(i)}$, $\gamma_{\text{max}}^{(i)}$, and $\gamma_{\text{min}}^{(i)}$ are the channel power gains of the $i$th interfering BS after applying MRC and impinging on the best and worst (as far as the the probe link is concerned) receive antennas, respectively. Short-hands: psc = power splitting circuit, 2rx = two receive antennas, asc = antenna switching circuit, imp = implementation. .................................................. 58

3.3 Definitions of $(a_{\text{ID}}, b_{\text{ID}}, c_{\text{ID}})$ and $(a_{\text{EH}}, b_{\text{EH}}, c_{\text{EH}})$ according to the PDFs in Table 3.2. .................................................. 69
3.4 Optimum power splitting ratio for different values of $Q_*$ and $R_*$ expressed in dBm and kbps. The “Exact” values are obtained by using Proposition 4. The “Upper-bound” values are obtained by using Proposition 5. The “Mixed” values are obtained by using Proposition 5 to compute $\rho_{\text{opt}}$ and by obtaining the corresponding J-CCDF from Proposition 4. Error is the relative error (in %) of the resulting J-CCDF with respect to that obtained with the exact framework. .................................................. 74

3.5 J-CCDF of PS and PS-MRC schemes as a function of $Q_*$ and $R_*$ expressed in dBm and kbps. Setup: $\rho = 0.001$ for PS and $\rho = 0.999$ for PS-MRC. 74

3.6 Comparison of the time (in seconds) for computing the J-CCDF via Monte Carlo simulations and Proposition 4, for different pairs $(Q_*, R_*)$ in (dBm, kbps). As for Monte Carlo simulations, the number of spatial and channel realizations is set equal to $10^5$ for all setups, while the simulation area, Area, (in square meters) is chosen so that the relative error, Error, between the J-CCDF functions computed with Proposition 4 and estimated via simulation have a relative error equal to around 0.1%. For completeness, the relative error (in %) is reported as well. The time for computing $\rho_{\text{opt}}$ is not taken into account. .................................................. 75

4.1 Summary of main symbols and functions used throughout this chapter. 86

4.2 Summary of main auxiliary functions used throughout the chapter. 99

4.3 Alternating optimization of the EE. 104

5.1 Summary of main symbols and functions used in this chapter. $k \in \{\text{ON, OFF}\}, d \in \{L1, L2\}$, and $o \in \{H, F\}$. .................................................. 125

5.2 Transition probability matrix of full-duplex battery. The entries of the matrix are $r_{i,j}$, $i, j \in \{1, 2, \ldots, L\}$ and $R_i$, $i \in \{1, 2, \ldots, L\}$, is the $i$-th row of the matrix with size $1 \times L$ and $R_i = [r_{i,1} \ r_{i,2} \ \ldots \ r_{i,L-2}]$, is the reduce of the $i$-th row with size $1 \times L - 2$. $p_i = (\frac{\lambda_{EP}}{i!})^i \exp(-\frac{\lambda_{EP}}{i})$, $i \in \{0, \ldots, L - 2\}$; $p_{L-1} = 1 - \sum_{i=0}^{L-2} p_i$; $q_e$ and $y_e$ are given in Eq. (5.4), and (5.5). 129
5.3 Transition probability matrix of half-duplex battery. The entries of the matrix are \( r_{i,j} \), \( i, j \in \{1, 2, \ldots, \mathcal{L}\} \) and \( \mathbf{R}_i \), \( i \in \{1, 2, \ldots, \mathcal{L}\} \), is the \( i \)-th row of transition probability matrix with size \( 1 \times \mathcal{L} \) and \( \mathbf{R}_i^{-} = [ r_{i,1} \ r_{i,2} \ \ldots \ r_{i,\mathcal{L} - 2} ] \), is the reduce of the \( i \)-th row with size \( 1 \times \mathcal{L} - 2 \). 

\[
p_i = \frac{(\lambda_{EP})^i \exp(-\lambda_{EP})}{i!}, i \in \{0, \ldots, \mathcal{L} - 2\}; \quad p_{\mathcal{L} - 1} = 1 - \sum_{i=0}^{\mathcal{L} - 2} p_i; \quad q_e \text{ and } y_e \text{ are given in Eq. (5.4) and (5.5).}
\]

5.4 Summary of main auxiliary functions used throughout the chapter. \( k \in \{\text{ON, OFF}\} \), \( d \in \{L_1, L_2\} \), and \( o \in \{H, F\} \).

5.5 Setup of parameters (unless otherwise stated).
CHAPTER 1

Introduction

1.1 Background

As the era of the Internet of Things (IoT) is approaching, lots of Small cell Base Stations (SBSs), i.e., micro, pico and femto cells, are deployed on the top of power-hungry Macro Base Stations (MBSs) in order to serve networks with a massive number of devices and wide heterogeneity. Such networks, however, can not be modeled by using the conventional hexagonal grid model due to the irregular BSs’ deployment, especially for SBSs. Moreover, these dense networks deployments require time-consuming and memory-consuming system-level simulations to evaluate the network performance, as is usually done in the 3rd Generation Partnership Project (3GPP). In this thesis, we exploit the promising mathematical tool of Stochastic Geometry (SG) to model cellular networks. This is due to its mathematical tractability and adequate accuracy compared with the regular grid model [1].

In addition, with approximately 3 millions of BSs, the BSs’ power consumption has become one of the biggest issues in the Information and Communications Technology (ICT) field, which accounts for about 2% of the world-wide CO₂ emissions [2, 3]. Furthermore, around 0.2-0.4 GW per year are consumed by approximately 3 billions mobile devices in
the world. Reducing the power consumption and increasing the Energy Efficiency (EE) have become among the main targets of the next generation’s cellular networks [4], along with other conventional requirements such as improving the Spectral Efficiency (SE), the network reliability and coverage area.

In this dissertation, motivated by these considerations, we focus our attention on developing new analytical methodologies for analyzing and optimizing emerging cellular networks deployments with special focus on enhancing their EE by using renewable energy sources, and the emerging concept of Simultaneous Wireless Information and Power Transfer (SWIPT) for reducing their power consumption and carbon footprint. To this end, we will use the mathematical tool of SG.

1.2 Major Contributions

The contributions of this dissertation are summarized as follows:

- In Chapter 2 and 3, a tractable mathematical framework for computing the joint complementary cumulative distribution function (JCCDF) of harvested power and data rate is provided instead of analyzing them separately as in other literature. The mathematical frameworks are validated with experimental data [5–7]. In particular, the BSs location and building footprints are taken from OFCOM and Ordnance Survey dataset in downtown London, United Kingdom. Directional beamforming with simple flat-top radiative pattern is exploited to both reduce the interference and increase the array gain. The specific contributions of each chapter are as follows.

  - In Chapter 2, the joint performance of information rate and harvester power is evaluated with multiple antennas at both MTs and BSs. In particular, Maximum Ratio Transmission (MRT) and Maximum Ratio Combining (MRC) schemes at both BSs and Low-Energy Devices (LEDs) are considered. Three mathematical frameworks are proposed, which provide the exact, approximated, and large-scale asymptotic expressions of the JCCDF of information rate and harvested power. Our study shows that large-scale Multiple-Input-Multiple-Output (MIMO) and ultra-dense deployments of BSs are both necessary to
1.2. Major Contributions

harvest, with high reliability, an amount of power with the order of a milliwatt.

- In Chapter 3, we pay attention to the practical implementations where one or two receive antennas are available at the LEDs (only one or two Radio Frequency (RF) front-ends are needed). Two options based on Selection Combining (SC) and MRC schemes are investigated, and their achievable performance versus implementation complexity trade-off is discussed. In particular, the results show that no scheme outperforms the others for every system setup. It suggests, on the other hand, that the LEDs need to operate in an adaptive fashion, by choosing the receiver diversity scheme to be used as a function of the performance requirements. The new mathematical frameworks are also introduced to adaptively optimize the SWIPT-enabled cellular networks.

- In Chapter 4, a new closed-form mathematical formulation of the Potential Spectral Efficiency (PSE) for interference-limited cellular networks (data transmission), which depends on the transmit power and density of the BSs is proposed. The new expression of the PSE is obtained by taking the power sensitivity of the receiver into account, not only for data transmission but also for cell association. Based on the new expression of the PSE, a new system-level EE optimization problem is formulated and comprehensively studied. It is mathematically proved that the EE is a unimodal and strictly pseudo-concave function in the transmit power given the fixed BSs’ density and in the BSs’ density given the fixed transmit power. The dependency of the optimal power as a function of the density, and that of the optimal density as a function of the power are discussed. A first-order optimal pair of transmit power and density of the BSs is obtained by using a simple alternating optimization algorithm. Numerical evidence of the global optimality of this approach is provided as well. Two load models for the BSs are analyzed and compared with each other. It is shown that they provide the same PSE but have different network power consumption. Hence, the optimal transmit power and density of the BSs that maximize their EEs are different. Their optimal EEs and PSEs are studied and compared with each other.

- In Chapter 5, a two-tier (MBSs and SBSs) downlink (DL) heterogeneous cellular
networks is considered where the SBSs use green energy sources (renewable energy sources) and the MBSs are connected to the power grid. The battery of the SBSs is modeled as a discrete Markov chain with finite capacity, and study two operation models: full-duplex and half-duplex transmission. Bias factor is taken into account for offloading the MTs from the MBSs to the SBSs. A new mathematical framework is proposed, which integrates Markov chains into SG. Monte Carlo simulations are provided to verify the correctness of the mathematical framework. The obtained approach is used to study the performance trends and to gain insights on the impact of renewable energy sources on the performance of cellular networks. The results show that the EE and PSE are unimodal functions of the transmit power given the density of BSs and vice versa. Finally, it is shown that the full-duplex operating mode provides better performance compared to the half-duplex operating mode at the expense of a higher installation cost.

1.3 Publications and Awards

The publications and awards originated from this thesis are as follows.

1.3.1 Journals


1.3.2 Conferences


1.3.3 Awards

- The 2017 IEEE SigTelCom Best Paper Award.

1.4 System-Level Modeling

Recently, the design, modeling, and optimization of mobile networks are shifting from using the conventional hexagonal grid model to more promising abstraction models based on the mathematical tool called SG in order to be able to handle the enormous growth of both mobile terminals and small cell BSs [8]. Different from MBSs which are usually deployed based on well-defined networks planning, the SBSs are often deployed randomly in the same coverage area of the MBSs. SG and the theory of Poisson Point Processes (PPPs) are suitable tools for modeling the random locations of SBSs. During the recent years, however, it has been shown that they can be used for modeling the location of MBSs as well.

In fact, Andrews et al. in [1] proved that the idealized hexagonal grid model provides upper bound estimates of the coverage probability while the model based on PPPs provides a lower bound of the coverage probability. Thus, both approaches have strengths and weaknesses, and none of them outperform the other in terms of accuracy, while the PPP-based approach being more analytically tractable.

An illustration of the BS locations according to the hexagonal model, to the actual locations in downtown London, UK [5], and to the PPP model are shown in Fig. 1.1. The figure shows that the BSs locations based on the hexagonal model are quite different from the actual BSs locations, while the PPP-based modeling approach is capable of taking into account the irregularity of the cell distribution.

Background material on system-level modeling and performance evaluation of cellular networks based on stochastic geometry is given in the next section. In particular, this section begins with the definition of PPP and proceeds by introducing the concepts of cell association, directional beamforming, aggregate other-cell interference, and relevant performance metrics.
Figure 1.1: BSs deployments and its voronoi cell, the circles denote the locations of BSs, the solid lines are the voronoi boundary: (a) hexagonal model, (b) actual locations of O2 (until 2012) in downtown London, (c) PPP distributed BSs.

1.4.1 Cellular Networks Modeling

Let us consider a two-dimensional downlink (DL) cellular networks, where the BSs are modeled according to a homogeneous Poisson Point Process denoted by $\Psi$ whose spatial intensity is $\lambda_{BS}$. The MTs are modeled as another homogeneous PPP with intensity $\lambda_{MT}$, which is independent of $\Psi$. The definition of homogeneous Poisson point process (or uniform PPP) is given in [9, Definition 2.8]:

**Definition 1** The homogeneous PPP, with intensity $\lambda$, is a point process in $\mathbb{R}^d$ such that

- for every compact set $B$, $N(B)$ has a Poisson distribution with mean $\lambda|B|$;
- if $B_1, B_2, \ldots, B_m$ are disjoint bounded sets, then $N(B_1), N(B_2), \ldots, N(B_m)$ are independent random variables.

In this chapter, we assume $\lambda_{MT} \gg \lambda_{BS}$, i.e., fully-loaded scenario. In contrast, Chapter 4 and 5 release this assumption and we observe a significant difference between fully-loaded and lightly-loaded scenarios. In addition, single antenna is considered at both BSs and MTs in this chapter, while multiple-antenna scenario is investigated in Chapter 2 and 3.

Without loss of generality, the performance of the typical MT, which is located at the origin, is studied. It is emphasized that the results obtained based on such user can be
extended to arbitrary users at any locations [9] owing to the property of PPP (motion-invariant Point Process (PP)). Here, we choose PPP due to its mathematical tractability and sufficient accuracy compared to other PPs [10].

1.4.2 Signal to Interference plus Noise Ratio (SINR) - Path-loss Models

The signal to interference plus noise ratio (SINR) is the most important metric in any wireless networks. Mathematically, it is defined by the ratio between the power of useful signal and the power of destructive elements, interference and noise, and it is given by

\[
\text{SINR} = \frac{S}{I + N},
\]

(1.1)

Here, \(S\) and \(N\) are the received power and thermal noise at the typical/probe user/MT\(^1\) while \(I\) is the aggregate interference from other on-going transmissions. The aggregate interference comes only from other cells since there is no intra-interference at the typical cell by assuming orthogonal resource allocation inside each cell.

The received power at the probe user can be written as follows

\[
S = \frac{P_{tx}G^{(0)}|h^{(0)}|^2\chi^{(0)}}{L^{(0)}},
\]

(1.2)

where \(P_{tx}\) is the transmit power; \(h^{(0)}, \chi^{(0)}, G^{(0)}\) are the small-scale fading, the shadowing and the antenna gain of the intended link; and \(L^{(0)}\) is the path-loss or the large scale fading from serving BS to probe user.

Typically, \(h\) follows Rayleigh fading due to its mathematical tractability compared to other general small-scale fading models, i.e., Nakagami fading, Rician fading, which do not allow one to represent the coverage probability in closed-form expressions even in the interference-limited regime [11, 12]\(^2\). In addition, Rayleigh fading provides the worst performance among all small-scale fading models, hence, the systems are designed based on Rayleigh fading are capable of working with others as shown in Fig. 1.8 and [13].

\(^1\)In this thesis, the term typical user/MT and probe user/MT are exchangeable.

\(^2\)Please see section 1.4.6 for more detail.
is worth noting that the impact of small-scale fading is a minor factor compared with the path-loss in the system-level performance.

Moreover, $\chi$ is the shadowing and typically follows a log-normal distribution \cite{14}. The impact of shadowing can be studied implicitly via path-loss with blockage consideration \cite{7}, due to its high mathematical complexity.

In system-level performance evaluation, in fact, the most important role is played by the path-loss (see Fig. 1.11). In the literature, there are several path-loss models, some of them are summarized as follows.

**Single-State Unbounded Path-loss Model**

The single-state unbounded path-loss model is formulated as

$$L = \kappa r^\beta,$$

where $\kappa$ is the path-loss constant; $r$ stands for the Euclidean distance between the typical user and the BSs, and $\beta$ is the path-loss exponent.

The advantages of using this path-loss model are mathematical tractability and low

![Figure 1.2: An illustration of a dual-state path-loss model.](image-url)
computational complexity. Nevertheless, there is a unavoidable singularity issue when the BSs are close to the typical user, i.e., in Eq. (1.3), \( \lim_{r \to 0} \left( \frac{1}{\kappa r^\beta} \right) = \infty \). Another drawback of this model is the inaccuracy of the model itself, as shown in Fig. 1.2, where the impact of blockage makes the transmission either in Light-of-Sight (LOS) or Non-LOS (NLOS) resulting in different path-loss exponents [15–17]. It means that the path-loss exponent of LOS and NLOS is different, i.e., \( \beta_{\text{LOS}} \neq \beta_{\text{NLOS}} \), instead of having one value like Eq. (1.3), \( \beta_{\text{LOS}} = \beta_{\text{NLOS}} = \beta \).

The performance between the single-state and dual-state path-loss models is shown in Fig. 1.3. Specifically, in single-state model, increasing BSs density does not provide any benefits while in dual-state model, a maximum value of BSs density exists or network densification does not always give better performance. Indeed, the performance degrades significantly if the number of BS is greater than a given density level. This result can be explained by noting that the interferers change from NLOS to LOS as the density of BSs increases, while the intended link is almost stable, and therefore the performance becomes worse [7].

**Single-State Bounded Path-loss Model**

The bounded path-loss model overcomes the singularity issue in the unbounded counterpart by either using the max function or adding a small factor, e.g., \( \varepsilon \), to the distance between the BSs and the typical user. The two models have the same accuracy when the distance between BSs and MT is sufficiently large as illustrated in Fig. 1.4. Theoretically, there are several bounded path-loss models and which are given by

\[
L = \kappa \max\left(1, r^\beta\right) \quad (1.4)
\]
\[
L = \kappa (1 + r)^\beta \quad (1.5)
\]
\[
L = \kappa \left(1 + r^\beta\right) \quad (1.6)
\]

Fig. 1.4 reveals that with bounded path-loss model, the singularity problem is avoided when the transmission distance is close to zero. It also shows that path-loss in Eq. (1.5) is the lower bound of the others while the one in Eq. (1.6) approaches the unbounded
Figure 1.3: Pcov vs. Rcell with the single- and dual-state unbounded path-loss models. These curves are plotted by using Monte Carlo simulation with 250 thousand realizations. Noise figure is 10 dB, noise spectral density is -174 dBm/Hz; omnidirectional antenna; $\kappa = \left(\frac{4\pi}{\nu}\right)^2$, $\nu = \frac{c}{f_c}$; $c$ is the speed of light and $f_c = 2.1$ GHz is the carrier frequency. Rayleigh fading with unit mean and no shadowing. The path-loss exponent of dual-state are $\beta_{\text{LOS}} = 2.5$, and $\beta_{\text{NLOS}} = 3.5$. The probability of a link is in LOS and NLOS followed the one-ball model in Table 1.1 with $D = 109.8517$, $q_{\text{LOS}}^{[0,D]} = 0.7195$, and $q_{\text{LOS}}^{[D,\infty)} = 0.0002$. The path-loss exponent of single-state model is $\beta = 3.5$.

Dual-State Unbounded Path-loss Model

The dual-state unbounded path-loss model is given as follows

$$L = \begin{cases} \kappa_L r^{\beta_L}, & \text{in LOS state} \\ \kappa_N r^{\beta_N}, & \text{in NLOS state} \end{cases},$$

(1.7)

where $\kappa_s$ is path-loss constant of state $s \in \{\text{LOS, NLOS}\}$. The probability of a link in state $s$ is a function of distance $r$ and is summarized in Table 1.1.

Table 1.1 shows that the longer transmission distance corresponds to higher probability that the link is in NLOS for all models, which is consistent with practical measurement.
Figure 1.4: Path-loss inversion vs. transmission distance of the single-state unbounded and bounded path-loss models in section 1.4.2. The unbounded curve is plotted by using Eq. (1.3); the bounded curves 1, 2 and 3 are plotted by Eq. (1.4), (1.5), and (1.6), respectively.

| 3GPP [18] | min \( \frac{a_{3G}}{r}, c_{3G} \) \( 1 - e^{-\frac{r}{b_{3G}}} \) + e \( -\frac{r}{b_{3G}} \) | 1 - \( p_{LOS} (r) \) | 0 |
| Random Shape [19] | \( a_{RS} \exp \left(-b_{RS} r\right) \) | 1 - \( p_{LOS} (r) \) | 0 |
| Linear [20] | \( 1 - p_{PNLOS} (r) \) | min \( \left\{ a_{L} r + b_{L}, c_{L} \right\} \) | 0 |
| Empirical mmWave [21] | \( (1 - p_{OUT} (r)) \exp (-a_{mm} r) \) | 1 - \( p_{OUT} (r) - p_{LOS} (r) \) | max \( \left\{ 0, 1 - e^{-b_{mm} r} \right\} \) |
| Two-ball mmWave [21] | see (1.8) with \( S = 3, s = \text{LOS, NLOS, OUT} \), \( B = 2 \) |

Table 1.1: [7, Table. II], Commonly used link state models. The parameters \( a_{(\cdot)} \), \( b_{(\cdot)} \) and \( c_{(\cdot)} \) are environmental-dependent. Here \( p_{s} (r) \) is probability in state \( s \in \{\text{LOS, NLOS}\} \) of transmission distance \( r \); \( p_{OUT} (r) \) is probability of transmission distance \( r \) in outage.

\[ p_{S} (r) = \sum_{b=1}^{B+1} q_{s}^{\left[D_{b-1},D_{b}\right]} 1_{\left[D_{b-1},D_{b}\right]} (r), \quad (1.8) \]

where \( \sum_{s \in S} q_{s}^{\left[D_{b-1},D_{b}\right]} = 1 \); \( B \) is the number of balls and \( D \) is the radius of the \( b \)th ball with \( D_{0} = 0 \) and \( D_{B+1} = \infty \), \( q_{s}^{\left[D_{b-1},D_{b}\right]} \) is the probability that the link is in state \( s \) if \( r \in \left[D_{b-1},D_{b}\right] \); \( 1_{\left[D_{b-1},D_{b}\right]} (r) \) is indicator function defined by \( 1_{\left[D_{b-1},D_{b}\right]} (r) = 1 \) if \( r \in \left[D_{b-1},D_{b}\right] \) and 0 otherwise.

The dual-state path-loss model is especially significant in mmWave communications.

[15, 22].
due to the high carrier frequency. An illustration of the dual-state path-loss model is given in Fig. 1.2.

**Dual-State Bounded Path-loss Model**

The dual-state bounded path-loss model is a combination of the bounded path-loss and dual-state models. In this dissertation, the two-state unbounded path-loss model is studied in Chapter 2 and Chapter 3 while the single-state unbounded path-loss is used in Chapter 4 and Chapter 5. Research based on the bounded path-loss model is left for future work.

### 1.4.3 Cell Association Modeling

In cellular networks, the MT is typically served by one BS for both downlink and uplink (UL) transmission. The decoupled uplink and downlink (DUD) architecture will be discussed thereafter (section 6.2.4). In this section, some well-known association criterion to select the serving BS are discussed. For the illustration purpose, the single-state unbounded path-loss in section 1.4.2 is considered in this section.

![Illustration of cell association](image)

**Figure 1.5:** An illustration of cell association: (a) Based on the smallest distance, (b) Based on the average highest received power. $P_1$ and $P_2$ are the transmit power of MBS and SBS; $d_1$ and $d_2$ are distances from the SBS and MBS to the typical user, respectively.
Cell Association Based on the Smallest Distance

The serving BS based on the smallest distance association is the BS which has the minimum physical distance to the probe user. It should be noted that in single-state path-loss model, the association based on the smallest distance is exactly the same as the smallest path-loss and is given by

\[
L^{(0)} = \min_{i \in \Psi} \left( \kappa r_i^\beta \right),
\]

where \(L^{(i)}\) is the path-loss from the generic BS to the typical MT.

Cell Association Based on the Highest Received Power

The highest received power is the association based on the minimum of both path-loss and small-scale fading from all BSs and computed as

\[
L^{(0)} = \min_{i \in \Psi} \left( \frac{\kappa r_i^\beta}{|h^{(i)}|^2} \right),
\]

where \(h^{(i)}\) is the small-scale fading from the generic BS to the typical user. Compared with the smallest distance association, this approach comes up with better serving BS as it takes both large and small-scale fading into account with the price of simultaneous channel state information (CSI) requirements from all BSs.

General speaking, the association based on the smallest distance requires less handover than the highest received power because the distance between the MT and the BS is almost stationary compared with fading. In addition, apart from these above-mentioned associations, there are also other kinds of associations, i.e, the average highest received power by taking into account the shadowing, \(L^{(0)} = \min_{i \in \Psi} \left( \frac{\sigma r_i^\beta}{\chi^{(0)}} \right)\), or the smallest path-loss where the path-loss is no longer single-state model, i.e., section 1.4.2, [23]. However, these associations typically do not allow one to obtain closed-form expressions for the coverage probability and average rate.

Fig. 1.5 illustrates an example of cell association based on the smallest distance (the smallest path-loss) and the average highest received power. From Fig. 1.5(b), we observe that the serving BS is MBS rather than the SBS in spite of the longer distance to MT.
1.4.4 Directional Beamforming

Directional beamforming is an essential technique of 5G cellular networks as it not only provides higher array gain but also reduces the inter-cell interference. It is especially important in mmWave communications due to high path-loss. In this section, some wide applied beamforming techniques are introduced.

Flat-Top Antenna Pattern

The array gain of this antenna pattern attains its maximum within the beamwidth of the main lobe and the minimum for the rest. The advantage of this antenna pattern is its simplicity and low computational complexity [23, 24].

Sinc Antenna Pattern

The sinc antenna pattern provides tight lower bound of an actual antenna pattern. The accuracy of this tight lower bound is shown in [25, Appendix I and II].

Cosine Antenna Pattern

The cosine antenna pattern achieves a good approximation for the gain of the main lobe at the expense of low accuracy for the side lobe [26].

Although both sinc and cosine antenna pattern provide better approximation compared with the flat-top approach, they require more sophisticated mathematical frameworks. This thesis and other works, e.g., [23], as a result, employ the flat-top antenna. In particular, the flat-top antenna is applied in Chapter 2 and 3, to both reduce the inter-cell interference and boost the antenna gain.

1.4.5 Aggregate Other-cell Interference

The aggregate inter-cell interference in cellular networks is created by the set of active transmitters operating on the same resource block (time and frequency) as the serving BS of the typical user. It can be formulated as follows

\[
I = \sum_{i \in \Psi \setminus \{0\}} \frac{P_{tx} G^{(i)} |h^{(i)}|^2 \chi^{(i)}}{L^{(i)}} 1\{L^{(i)} > L^{(0)}\}. \tag{1.11}
\]
Here $\Psi(0)$ is the set of active transmitters from other cells. It is emphasized that in this section, all BSs are active since we assume that $\lambda_{MT} \gg \lambda_{BS}$. Other load scenarios are considered in Chapter 4 and 5.

Theoretically, the set of interferers can approach infinity if the whole networks is considered. Consequently, it can not be computed by using the traditional regular grid model. The main drawback of the regular grid model is that it does not lead to a tractable model for the aggregate interference. It requires to calculate a large number of integrations over the distance between the probe user and interferers when evaluating the coverage probability; and these integrations primarily may not be calculated efficiently even with numerical methods. Thus, in order to evaluate the performance of the typical user, the networks operators typically use the system-level Monte Carlo simulation instead [27]. However, even with Monte Carlo simulation, they only consider the interference up to the first two “rings” BSs (6 BSs in the first ring, 12 BSs in the second ring) around the typical cell [28, 29]. This allows to save significantly memory requirement and running time by constraint the region to the “19-cell wraparound region” [28].

These issues can be overcome by using SG and the theory of PPP. In this case, the aggregate other-cell interference in Eq. (1.11), $I$, can be easily characterized by using the Campbell’s theorem and the Probability Generating Functional (PGFL) of PPP, which are reported as follows.

**Definition 2** Campbell’s Theorem:
Let $\Psi$ be a 2-D PPP of density $\lambda$ and $f(x): \mathbb{R}^2 \rightarrow \mathbb{R}^+$

$$
\mathbb{E} \left\{ \sum_{x \in \Psi} f(x) \right\} = \lambda \int_{\mathbb{R}^2} f(x) \, dx. \tag{1.12}
$$

**Definition 3** Probability Generating Functional:
Let $\Psi$ be a 2-D PPP of density $\lambda$ and $f(x): \mathbb{R}^2 \rightarrow [0, 1]$ be a real value function. Then:

$$
\mathbb{E} \left\{ \prod_{x \in \Psi} f(x) \right\} = \exp \left( -\lambda \int_{\mathbb{R}^2} (1 - f(x)) \, dx \right). \tag{1.13}
$$

With the help of Definition 2 and 3, we are able to compute the mean interference
of the aggregate other-cell interference via Campbell’s theorem or evaluate the coverage probability of SINR with only one integration\(^3\). For example, the mean interference of bounded path-loss model with omnidirectional antenna in Eq. (1.4) is given in the closed-form expression in [30, Section. 3.2.1] while the moment generating function (MGF) of unbounded path-loss model with and without fading of wireless networks are given in [30, Section. 3.2.2 and 3.2.3], [31].

In particular, by utilizing the PGFL theorem in Definition 3, the MGF of the aggregate other-cell interference in Eq. (1.11) with the single-state unbounded path-loss model, no shadowing and omnidirectional antenna, conditioned on the path-loss of serving BS is calculated as

\[
\mathcal{M}_I \left( s; L^{(0)} \right) = E_{\Psi,|h^{(i)}|} \left\{ \exp \left( -s \sum_{i \in \Psi} \frac{P_{tx}|h^{(i)}|^2}{L^{(i)}} 1 \left( L^{(i)} > L^{(0)} \right) \right) \right\} \\
= E_{\Psi,|h^{(i)}|} \left\{ \prod_{i \in \Psi} \exp \left( -s \frac{P_{tx}|h^{(i)}|^2}{L^{(i)}} 1 \left( L^{(i)} > L^{(0)} \right) \right) \right\} \\
\overset{(a)}{=} \exp \left( E_{y|h^{(i)}|} \left\{ \int_{x=L^{(0)}}^{\infty} \left( \exp \left( -s \frac{P_{tx}y}{x} \right) - 1 \right) \left( \frac{2\pi\lambda}{\beta} \left( \frac{1}{\kappa} x^{\frac{2}{\beta}-1} \right) \right) dx \right\} \right) \\
\overset{(b)}{=} \exp \left( \int_{y=0}^{\infty} \left[ \pi\lambda \left( \frac{L^{(0)}}{\kappa} \right)^{\frac{2}{\beta}} \left( 1 - \mathcal{F}_1 \left( \frac{2}{\beta}, 1 - \frac{2}{\beta}, -sP_{tx}y \right) \right) \right] \left( \frac{1}{\Omega} \exp \left( -\frac{y}{\Omega} \right) \right) dy \right) \\
\overset{(c)}{=} \exp \left( \pi\lambda \left( \frac{L^{(0)}}{\kappa} \right)^{\frac{2}{\beta}} \left( 1 - \mathcal{F}_1 \left( 1, -\frac{2}{\beta}, 1 - \frac{2}{\beta}, -sP_{tx} \right) \right) \right) .
\] (1.14)

Here \((a)\) holds by using the PGFL of inhomogeneous PPP [32, Lemma 1]; \((b)\) is attained by borrowing the result in [23, Appendix A] as follows

\[
\int_{a}^{\infty} \left( \exp \left( \frac{b}{x} \right) - 1 \right) x^{v-1} dx = \frac{1}{v} a^v \left( 1 - \mathcal{F}_1 \left( -v, 1 - v, \frac{b}{a} \right) \right) ;
\] (1.15)

and \((c)\) is obtained by calculating the RV \(y\) with unit mean, i.e., \(\Omega = 1\). Here \(\mathcal{F}_1 (a, b, z)\) and \(\mathcal{F}_1 (a, b, c, z)\) are Kummer confluent hypergeometric and Gaussian hypergeometric functions defined in [33, Eq. 13.1.2, and 15.1.1], and \(E \{.\} \) is the expectation operator.

Although Eq. (1.14) provides the exact closed-form expression of the MGF of \(I\), the

---

\(^3\)Further discussions are given in section 1.4.6.
exact closed-form expressions of both probability density function (PDF) and cumulative distribution function (CDF), however, usually do not exist for general value of path-loss exponent, \( \beta \) [30]. In addition, the mean interference of unbounded path-loss model is also not convergent [30].

Even though it is not possible to represent the PDF of \( I \) in closed-form expression, it is, however, possible to compute the coverage probability in closed-form expression for some specific case studies based on the MGF of \( I \). In the next section, some performance metrics of interest are introduced and discussed.

### 1.4.6 Performance Metrics

#### Coverage Probability

Let’s start this section with the most essential metric, the distribution of SINR in Eq. (1.1) or the coverage probability (Pcov) of the typical user, which is written as:

\[
P_{\text{cov}} = \Pr \{ \text{SINR} \geq \gamma_I \},
\]

where \( \Pr \{ . \} \) is the probability operator and \( \gamma_I \) is the data rate threshold. If the SINR drops below this threshold, the MT will not successfully decode the information from the serving BS.

The coverage probability by assuming fully-loaded scenario, no shadowing and omni-directional antenna can be written as

\[
P_{\text{cov}} = \Pr \left\{ \frac{S}{I + N} = \left. \frac{P_{\text{tx}} |h(0)|^2 / L(0)}{\sum_{i \in \Psi \setminus (0)} P_{\text{tx}} |h(i)|^2 / L(i)} + N \right \} \geq \gamma_I \right\}
\]

\[
\overset{(a)}{=} \int_{x=0}^{\infty} \int_{o=0}^{\infty} \exp \left( -\frac{x \gamma_I \sigma_N^2}{P_{\text{tx}}} \right) \exp \left( -\frac{x \gamma_I o}{P_{\text{tx}}} \right) f_I (o) f_{L(0)} (x) dx do
\]

\[
\overset{(b)}{=} \int_{x=0}^{\infty} \exp \left( -\frac{x \gamma_I \sigma_N^2}{P_{\text{tx}}} \right) \mathcal{M}_I \left( \frac{x \gamma_I}{P_{\text{tx}}}; x \right) f_{L(0)} (x) dx,
\]

where (a) holds because \( |h(0)|^2 \) follows an exponential distribution with unit mean (Rayleigh fading); (b) is obtained by using the definition of MGF function, \( \mathcal{M}_X (s) = \mathbb{E} \{ \exp (-sX) \} = \int_0^\infty \exp (-sx) f_X (x) dx \). \( f_{L(0)} (x) \) is the PDF of the smallest path-loss of serving BS which
1.4. System-Level Modeling

can be computed by using the void probability of PPP as [1, 23]:

\[
    f_{L(0)}(x) = \frac{2\pi \lambda}{\beta} \left( \frac{1}{\kappa} \right)^{\frac{2}{\beta} - 1} \exp \left( -\pi \lambda \left( \frac{x}{\kappa} \right)^{2} \right),
\]

while the MGF of the aggregate inter-cell conditioned on \( L(0), \mathcal{M}_I(s;x) \), is given in Eq. (1.14). \( f_I(o), \sigma_N^2 \) are the PDF of the aggregate other-cell interference and the noise variance at the probe receiver, respectively.

In spite of being able to represent the coverage probability by means of a single integration rather than a large number of integrations as for the regular grid model, it is still impossible to attain an exact closed-form expression for general path-loss exponent except for some specific cases, i.e., \( \beta = 4 \) [1].

Indeed, there are several approaches to compute/approximate Eq. (1.17) by either using Laplace approximation [34], saddle point approximation [35] or inverse Laplace transform (Gil-Pelaez theorem) [36]. However, these works generally require numerical evaluation to compute the coverage probability.

Nevertheless, if the interference-limited regime is considered, a closed-form expression of Eq. (1.17) can be obtained and given as

\[
    P_{\text{cov}} = \int_{x=0}^{\infty} \mathcal{M}_I \left( \frac{x\gamma_I}{P_{tx}}; x \right) f_{L(0)}(x) \, dx
\]

\[
    \overset{(a)}{=} \int_{x=0}^{\infty} \exp \left( \pi \lambda \left( \frac{x}{\kappa} \right)^{\frac{2}{\beta}} \left( 1 - _2F_1 \left( 1, -\frac{2}{\beta}; 1 - \frac{2}{\beta}, -\gamma_I \right) \right) \right)
\]

\[
    \times \left( \frac{2\pi \lambda}{\beta} \left( \frac{1}{\kappa} \right)^{\frac{2}{\beta} - 1} \exp \left( -\pi \lambda \left( \frac{x}{\kappa} \right)^{2} \right) \right) \, dx
\]

\[
    \overset{(b)}{=} \frac{1}{_2F_1 \left( 1, -\frac{2}{\beta}, 1 - \frac{2}{\beta}, -\gamma_I \right)},
\]

where (a) is obtained by substituting the MGF and PDF of \( I \) and \( L(0) \) in Eq. (1.14) and Eq. (1.18); (b) holds by computing the integration.

**Remark 1** Under interference-limited regime with fully-loaded and Rayleigh fading scenario, the coverage probability is a function of the data threshold, \( \gamma_I \), and the path-loss exponent, \( \beta \). It is totally independent of the transmit power and the density of BS. It,
Figure 1.6: Coverage Probability vs. Rcell with various values of $\gamma_I$. These curves are plotted by using Monte Carlo simulation with 250 thousand realizations. Noise figure is 10 dB, noise spectral density is -174 dBm/Hz; without beamforming (omnidirectional antenna); mean and standard deviation of shadowing are 2 dB and 1 dB, respectively; Rayleigh fading with unit mean; $\kappa = \left( \frac{4\pi}{v} \right)^2$, $v = \frac{c}{f_c}$; $c$ is the speed of light and $f_c = 2.1$ GHz is the carrier frequency.

However, is not true in practical cellular networks as illustrated in Fig. 4.1. In addition, the fully-loaded assumption does not always hold in realistic network deployment especially in rural area. This dissertation, as a result, is to propose a new definition of the coverage probability in Chapter 4 and Chapter 5 which is able to take into account not only the relation between the transmit power and the density of BS but also the threshold during the association phase.

We study the coverage probability versus the density of BSs (via the inter-site distance, $R_{cell} = \frac{1}{\sqrt{\pi \lambda}}$) with various setup in Figs. 1.6 to 1.11.

Fig. 1.6 confirms the above statement that in interference-limited regime, the coverage probability is independent of the density of BSs. In addition, it is not difficult to recognize that increasing the data threshold, $\gamma_I$, will decrease the coverage probability.
Fig. 1.7: Coverage Probability vs. Rcell with various values of path-loss exponents $\beta$. These curves are plotted by using Monte Carlo simulation with 250 thousand realizations. Noise figure is 10 dB, noise spectral density is -174 dBm/Hz; without beamforming (omnidirectional antenna); mean and standard deviation of shadowing are 2 dB and 1 dB, respectively; Rayleigh fading with unit mean; $\kappa = \left(\frac{4\pi}{v}\right)^2$, $v = \frac{c}{f_c}$; $c$ is the speed of light and $f_c = 2.1$ GHz is the carrier frequency.

Fig. 1.7 studies the $P_{cov}$ with different path-loss exponents. It is obvious that the larger path-loss exponent is the higher $P_{cov}$. The reason is that with larger path-loss exponent, the interference from BSs which are away from the typical MT is almost zero, thereby getting better performance. The performance of $P_{cov}$ with different kinds of small-scale fading is given in Fig. 1.8. The figure reveals and validates the above comments, where Rayleigh fading presents worse performance than others, i.e., Nakagami-$m$ fading. Moreover, with Nakagami case, increasing shape parameter, $m$, will lead to better performance; this confirms the statement that when $m \to \infty$, there is no longer small-scale fading and gets better results as shown in Fig. 1.11.

The comparison between with and without the directional beamforming is given in Fig. 1.9. With the directional beamforming, $P_{cov}$ is improved dramatically compared
Figure 1.8: Coverage Probability vs. Rcell with different kinds of small-scale fadings. These curves are plotted by using Monte Carlo simulation with 250 thousand realizations. Noise figure is 10 dB, noise spectral density is -174 dBm/Hz; without beamforming (omnidirectional antenna); mean and standard deviation of shadowing are 2 dB and 1 dB, respectively; $\kappa = \left(\frac{4\pi}{v}\right)^2, v = \frac{c}{f_c}$; c is the speed of light and $f_c = 2.1$ GHz is the carrier frequency. All curves are unit mean.

to omnidirectional antenna. Once again, Pcov of SINR is independent of the density of BS and equal to Pcov of Signal to Interference Ratio (SIR) when $\lambda \rightarrow \infty$. Furthermore, when the density of BS decreases, the Pcov is moving from the interference-limited to noise-limited regime. It can be interpreted that with smaller BSs density or larger Rcell, there is almost no active BSs, thereby reducing the interference.

The impact of the transmit power on the performance of Pcov is illustrated in Fig. 1.10. It shows that increasing the transmit power will shift the Pcov to interference-limited regime faster$^4$.

Fig. 1.11 indicates the minor impact of small-scale fading in system-level performance. In particular, without small-scale fading, the coverage probability only improves around

---

$^4$In this figure, the circuitry power consumption in BS is not considered. More general power consumption models are provided in Chapter 4 and Chapter 5.
Figure 1.9: Coverage Probability vs. Rcell with and without consider directional beam-forming. These curves are plotted by using Monte Carlo simulation with 250 thousand realizations. Noise figure is 10 dB, noise spectral density is -174 dBm/Hz; mean and standard deviation of shadowing are 2 dB and 1 dB, respectively; Rayleigh fading with unit mean; $\kappa = \left(\frac{4\pi}{\nu}\right)^2$, $\nu = \frac{c}{f_c}$; $c$ is the speed of light and $f_c = 2.1$ GHz is the carrier frequency. The array gain of without beamforming is 1. The array gain of beamforming is followed (flat top antenna pattern, section 1.4.4): $G_{\text{max}} = 10$ dB; $G_{\text{min}} = -10$ dB; main lobe is 30 degree.

0.05 compared with taking the small-scale fading into account.

**Average Rate**

The average rate of the typical user is calculated as

$$R = \mathbb{E}\{\log_2 (1 + \text{SINR})\}.$$  \hspace{1cm} (1.20)

In general, there are two approaches to compute the average rate. The first one utilizes the distribution of SINR [1] and another uses MGF function [37]. These approaches require the computation of at least one integral, even when the interference-limited regime is considered. However, the same trends as the coverage probability can be observed in the
Figure 1.10: Coverage Probability vs. Rcell with various value of the transmit power. These curves are plotted by using Monte Carlo simulation with 250 thousand realizations. Noise figure is 10 dB, noise spectral density is -174 dBm/Hz; without beamforming (omnidirectional antenna); mean and standard deviation of shadowing are 2 dB and 1 dB, respectively; Rayleigh fading with unit mean; \( \kappa = \left( \frac{4\pi}{v} \right)^2 \), \( v = \frac{c}{f_c} \); \( c \) is the speed of light and \( f_c = 2.1 \) GHz is the carrier frequency.

interference-limited regime: the average rate of the typical user is independent of both the transmit power and the density of BS [1, 37, 38].

The above-mentioned performance metrics are from the point view of the end user. In the next section, the performance metrics from the network point of view are discussed.

**Area Spectral Efficiency**

The area spectral efficiency (ASE) provides the number of bits per second per square meter (bits/s/m²) and is given as follows

\[
ASE = \lambda_{MT} \mathbb{E} \left\{ \log_2 (1 + \text{SINR}) \right\}.
\] (1.21)
Figure 1.11: Coverage Probability vs. Rcell with and without small-scale fading. These curves are plotted by using Monte Carlo simulation with 250 thousand realizations. Noise figure is 10 dB, noise spectral density is -174 dBm/Hz; without beamforming (omnidirectional antenna); mean and standard deviation of shadowing are 2 dB and 1 dB, respectively; \( \kappa = \left( \frac{4\pi}{v} \right)^2 \), \( v = \frac{c}{f_c} \); c is the speed of light and \( f_c = 2.1 \) GHz is the carrier frequency.

Area Potential Spectral Efficiency (APSE)

The area potential spectral efficiency or simply potential spectral efficiency (PSE) is similar to the ASE. The difference between the two metrics is that the ASE is computed based on the average rate while PSE is computed based on the coverage probability, which is given by

\[
PSE = \lambda_{MT} \log_2 (1 + \gamma_I) \Pr \{ \text{SINR} \geq \gamma_I \}. \tag{1.22}
\]

Both ASE and PSE provide the throughput of whole networks. Nevertheless, the network operator needs to consider both the throughput and the power consumption (or energy efficiency) in ultra-dense cellular networks owing to the extremely high BSs and MTs deployments. The detailed discussions about the power consumption and energy
efficiency are provided in Section 1.6 and Chapter 4 and 5.

1.4.7 State-of-the-art on system level modeling

Seminal work on system-level modeling and analysis was conducted by Andrew et al. in [1]. Specifically, this paper studied the so-called standard modeling assumptions\(^5\). After that, several works were examined based on the fully-loaded assumption, i.e., \(\lambda_{\text{MT}} \gg \lambda_{\text{BS}}\), [13, 39–44].

In [39], the error probability of wireless networks in interference-limited was studied. In particular, uniform approximation of SIR was proposed and, based on such approximation, the error probability was computed. The comprehensive error performance of downlink cellular networks was investigated in [13] with arbitrary number of antennas at both transmitter and receiver. On the other hand, the uplink coverage probability and average rate were addressed in [43] with maximum ratio combining and Optimum Combining (OC). The performance of relay-aided cellular networks was investigated in [42]. However, all aforementioned works assumed that the density of mobile user is very large compared to the density of BS. This is not always true especially in rural area, as proven in Chapter 4. In fact, significant different conclusions are obtained if the condition \(\lambda_{\text{MT}} \gg \lambda_{\text{BS}}\) does not hold anymore. The performance analysis based on vulnerability regions was addressed in [45, 46] while works in [47, 48] provided the bound based on the \(n\)th nearest interferers.

Recently, the Meta distribution of SIR of device-to-device (D2D) communications was studied in [49, 50]. The meta distribution of the SIR is formulated from the distribution of the conditional success probability given the point process. While the application of PPP to wireless sensor networks (WSNs) and cognitive radio (CR) were provided in [51, 52]. These results, however, can not directly be applied to cellular networks due to different radio interface, protocol et al.

In the next chapters of this thesis, we will propose new approaches for taking into account several important modeling factors, which include different load models, impact of MTs’ density in Chapter 4 and 5 or considering multiple-antenna at MT and/or BSs, general link state model (two states) with practical simulated data in Chapter 2 and 3,

\(^5\)The detail discussion about this model is provided in Chapter 4.
Moreover, in next generation mobile networks, the power consumption is a compelling priority due to the exponential growth of wireless devices and power-consuming applications. In the next section, we introduce the concepts of energy efficiency (EE) and wireless power transfer (WPT), which are relevant in this context.

1.5 Wireless Power Transfer (WPT) - Simultaneous Wireless Information and Power Transfer (SWIPT)

Nowadays, mobile terminals have become an important part of human society as they are able to connect people and to improve human life in several aspects, i.e., entertainment, healthcare, disaster-alert et al. However, the phone battery only has limited capacity with current technology and becomes a bottleneck of mobile industry as the power consumption is obviously increasing. As a result, lots of researchers from both industry and academia have recently put emphasis on energy harvesting (EH) technologies in order to allow low energy mobile devices to increase the duration of their battery life.

EH is a process of capturing, transferring, and making useless energy such as heat, wind, and radio frequency signals into useful electricity. Nonetheless, EH based on natural resources can not provide ultra-reliable operation owing to its irregular and intermittent nature. In this context, Wireless Power Transfer (WPT) is a technology that tackles the above-mentioned limitations [53], where the device battery is charged from intended electromagnetic (EM) radiation. Basically, WPT can be employed by using either near-field electromagnetic induction, i.e., magnetic resonant coupling and inductive coupling, or far-field EM radiation via rectennas (rectifying antennas) [54]. While the near-field EM induction can only support short distance (up to several meters) and is vulnerable by the misalignment between the transmitter and receiver; the far-field EM is able to support a larger coverage area, hence can be deployed in cellular networks [55].

Simultaneous Wireless Information and Power Transfer (SWIPT) is one kind of WPT where the RF signals are able to convey information as the conventional communications systems and replenish the battery of mobile devices concurrently [53]. Theoretically speak-
SWIPT can provide many benefits. Firstly, SWIPT allows the MT to harvest energy while receiving data, thereby prolonging their lifetime. With SWIPT, the interference is kept under control and it can even be beneficial for dedicated receiver. Nevertheless, the information decoding (ID) and EH cannot be generally performed on the same received signal because of practical hardware constraint. In the next section, some well-known SWIPT architectures are introduced and discussed.

1.5.1 SWIPT Architectures

Separated Receiver

In separated receiver, an ID receiver and an EH receiver are deployed at two separate receivers with different antennas [56] as shown in Fig. 1.12(a). This can be easily implemented by using off-the-rack components for the two individual receivers. The advantage of this structure is that it permits the receiver to utilize both EH and ID independently and simultaneously without any performance lost. However, the high deployment cost makes it unsuitable for practical implementation.

Time Switching (TS) Receiver

Time switching receiver is a co-located receiver architecture which consists of an RF energy harvester, information decoder, and a switcher. Specifically, the receive antenna is switched between the EH and ID receiver periodically based on a time switching sequence.

Power Splitting (PS) Receiver

The incoming RF signal is split into two parts with the aid of power splitter in power splitting (PS) receiver (Fig. 1.12(c)). Particularly, a part of the received power is sent to the rectennas circuit of EH receiver while the remaining part is converted to baseband for ID receiver. Theoretically, the PS receiver provides better performance than the TS receiver at the expense of higher installation cost [23].
1.5. Simultaneous Wireless Information and Power Transfer

Antenna Switching (AS) Receiver

Different from PS and TS receiver, the antenna switching structure requires at least two receive antennas in order to make SWIPT feasible. In general, the AS receiver splits the incoming RF signal in spatial domain instead of time or frequency domain of TS/PS scheme. In addition, antenna switching may be considered as a distinct case of power splitting scheme with binary PS ratio at each receive antenna. Even though the performance of PS scheme is better than TS scheme, the performance between PS and AS scheme is not clear since it depends on many factors such as power splitting ratio, number of antennas, as shown and proved in Chapter 3.

Fig. 1.12 illustrates a typical SWIPT-enabled receiver, TS, PS, and AS receiver, respectively.

![Figure 1.12: SWIPT-enabled receiver: (a) Separated Receiver; (b) TS Receiver; (c) PS Receiver and (d) AS Receiver.](image)

1.5.2 SWIPT with multiple-input multiple-output (MIMO)

Multiple-input multiple-output (MIMO) is a well-known technique for improving both transmission reliability and system capacity by virtue of the so-called diversity and spatial multiplexing gains. Combining SWIPT with MIMO technology will offer several benefits. At first, the use of the additional antennas at the receiver side can yield more harvested energy as a result of the broadcast nature of wireless propagation. Next, with multiple
antennas, directional beamforming can be easily applied to improve the performance of both information decoding and energy transfer [57].

1.5.3 Interference management in SWIPT-enabled networks

Interference is detrimental for conventional wireless systems as it disrupts useful signals significantly if it is not properly managed. As a consequence, one of the most important design objectives of conventional cellular networks is to reduce as much as possible the interference.

In SWIPT-enabled cellular networks, nevertheless, the same interference is a natural source of power for energy harvesting. As a result, the development of interference management techniques that exploit interference for EH and counteract it for ID play a fundamental role in SWIPT-enabled networks [58].

1.5.4 State-of-the-art on SWIPT-enabled networks

Most of the prior works on SWIPT either studied small-scale networks or are focused on non-cellular networks, like WSNs, D2D, where the corresponding results cannot be directly applied to cellular networks. This is because the distance between the serving BS and the typical MT in cellular networks is a random variable instead of being deterministic.

Relevant works are summarized as follows.

• In [59], the outage probability and ergodic capacity of two transmission modes, delay-limited and delay-tolerant were proposed and studied via two EH schemes, TS and PS. However, it only investigated the link-level performance without large-scale interference consideration.

• The authors in [60] studied EH with Amplify and Forward (AF) and Decode and Forward (DF) relaying in both half- and full-duplex modes. The results showed that the performance of AF relaying was slightly worse than DF relaying but AF scheme seemed to be more efficient if taking the energy processing cost into account. It, once again, only focused on small-scale networks.
• WPT with multiple antennas were studied in [61]. The results pointed out that the optimal DL energy harvesting time, which maximized the system throughput, was obtained at high Signal-to-Noise Ratio (SNR) regime.

• The performance of WSNs was addressed in [61, 62] by assuming that the nodes were distributed according to either Ginibre point process or repulsive point process. The results, however, can not be applied to cellular networks.

• The authors in [63] addressed the performance of large-scale networks with and without relaying with single antenna at both BSs and MTs and fixed transmitter and receiver distance while the D2D communications were studied in [64] where all nodes are modeled as PPP.

The most closely work to this thesis is [23] but it only considered single antenna at both transmitter and receiver. In this thesis, on the other hand, multiple antennas are considered at MT and/or BSs in both Chapter 2 and 3.

1.6 Energy Efficiency in Cellular Network

Energy consumption has become a major concern for mobile networks and it is predicted that there will be more than 50 billion devices connected to Internet by 2020 [65]. Furthermore, cellular networks are already on the top of energy consumers (for example, Telecom Italia is the second largest energy consumer in Italy [2]) within the ICT field.

Moreover, in cellular networks, it has been reported that more than 50% of total operation energy consumption is in the BSs [66–68]. Approximately, there are around 3 million BSs worldwide that consume 4.5 GW per year [3]. From mobile terminals side, there are roughly 3 billion devices that consume around 0.2–0.4 GW per year [69]. The high energy consumption also contributes to high electronic pollution and heat dissipation [70]. Consequently, it has raised the major financial and environmental concerns for both service provider and end user.

Due to the aforementioned concerns, it is significant to shift from pursuing optimal capacity and spectral efficiency to energy efficiency in mobile networks.
1.6.1 Definition of Energy Efficiency

The most general definition of the efficiency is that of benefit-cost ratio. It is the ratio between the outcome by utilizing a given resource and the corresponding incurred cost. This definition is applied to all science fields, from finances to physics, and wireless communications is not exceptional.

In specific, EE is commonly defined as the number of information bits per unit-energy consumption and is measured in bits-per-Joule. The EE is given by

\[
EE = \frac{R}{P_{\text{con}}}. \tag{1.23}
\]

From Eq. (1.23), EE is dependent on the power consumption, \( P_{\text{con}} \), and the networks outcome \( R \). It is noted that the outcomes of the networks typically are PSE, average rate, throughput, etc., which are introduced in section 1.4.6.

Power Consumption Models at BSs

There are several power consumption models, which are proposed in the literature and can be categorized as follows:

1. Load independent and no idle, circuitry and others power consumption

\[
P_{\text{con}} = P_{\text{tx}}, \tag{1.24}
\]

where \( P_{\text{tx}} \) is the transmit power. This model is the most popular one due to its simplicity, however, it is also the least accuracy as it ignores the power consumption of other BSs’ elements such as cooling systems, signal processing unit \textit{et al.}

2. Load independent with idle, circuitry and others power consumption \cite{71}

\[
P_{\text{con}} = \begin{cases} 
P_{\text{tx}} + P_{\text{cir}} & \text{if BS is active} \\
P_{\text{idle}} & \text{if BS is inactive}
\end{cases}, \tag{1.25}
\]

where \( P_{\text{cir}} \) is the circuitry power which includes all kinds of power consumption except for the transmit power and \( P_{\text{idle}} \) is the idle power when BS is inactive. By
taking into account the power consumption of other elements and the consumption in sleeping mode, this model overcomes the limitations of the above-mentioned model. Nevertheless, the drawback of this model is the independence of the transmit power with load, which makes the total power consumption approach infinity if the number of user goes to infinity.

3. Load dependent and no idle, circuitry and others power consumption [72]

\[ P_{\text{con}} = \rho P_{\text{tx}}, \]  

(1.26)

where \( \rho \) is the system traffic load density. On the one hand, this model overcomes the disadvantage of load independent model, on the other hand, it also inherits the drawback of the first model.

4. Load dependent with idle, circuitry and others power consumption [72]

\[ P_{\text{con}} = \begin{cases} 
\rho P_{\text{tx}} + P_{\text{cir}} & \text{if BS is active} \\
P_{\text{idle}} & \text{if BS is inactive} 
\end{cases}, \]  

(1.27)

The last model has the benefits of both the first and the third models.

It is noted that all above-mentioned models are linear in the parameters of interest, e.g., the transmit power. More sophisticated models which are able to capture the complicated relation between BS components can be found in [73]. This thesis uses linear models in Chapter 4 and Chapter 5 due to their simplicity.

In next section, we discuss some approaches to improve EE.

1.6.2 Improving the Energy Efficiency

Resource allocation

Improving the EE by optimizing the resource allocation in order to maximize the energy efficiency rather than the network throughput is the most essential approach. Different from the traditional resource allocation schemes, EE maximization requires new mathematical frameworks. This is because it is the ratio of the network throughput and
its power consumption instead of only the network throughput as assumed by conventional resource allocation schemes. This approach provides substantial energy efficiency gains at the price of a moderate loss of network throughput [74].

**Network planning and deployment**

Another approach is to deploy the BSs in a way to maximize the coverage area per consumed energy rather than only the coverage area. Specifically, by optimizing the BSs’ density combined with offloading technique, massive MIMO, mmWave, it is possible to improve both spectral efficiency and energy efficiency of the whole networks.

**Energy harvesting and power transfer**

The third approach is to exploit the benefits of energy harvesting technique which is described in section 1.5. This approach obviously plunges the networks power consumption while still keeping the same networks throughput, therefore, improves EE.

**Hardware solutions**

The last method mentioned in this thesis is to design the hardware so that it explicitly reduces the energy consumption [67], and to apply major network architectures changes, i.e., the cloud-based implementation of the radio access network (RAN) [75].

In particular, increasing the EE based on hardware solutions refers to a set of strategies that include green design of RF chain, i.e., single RF chain, power amplifier (PA), simplified transmitter/receiver structures. In fact, most of works are concentrated on improving EE of power amplifier since it accounts for 65% (included feeder) of the total power consumption of MBSs [76]. Some works, on the other hand, focus on simplifying the transceiver, i.e., applying coarse signal quantization (e.g. one bit quantization [77]) and hybrid analog/digital beamformers [78]. These designs, are especially applicable in systems with multiple antennas such as massive MIMO and mmWave communications.
1.6.3 State-of-the-art on Energy Efficiency Analysis

In this section, some related works on EE are provided. Typically, these studies either focus on hardware improvement, other non-cellular kinds of communication networks (D2D, WSN) or are focused on link-level performance metrics.

- The energy consumption of power amplifier was investigated in [79] with the consideration of both transmit energy and dissipated energy.

- The SE–EE tradeoff region was studied in [68, 80]. The paper showed that the curve is a convex function with the monotonically decreasing property. A few theoretical studies on the SE–EE tradeoff had been presented recently in [81]. In [82, 83], envelope tracking architecture was proposed to improve the power amplifier efficiency in various wireless transmitters.

- A mixed Analog-to-Digital Converter (ADC) architecture for massive MIMO systems was proposed in [78]. The paper revealed that its proposed structure with a fairly small number of high-resolution ADCs was able to attain a large proportion of the channel capacity of the conventional architecture, and to decrease the energy consumption dramatically compared with antenna selection strategies, for both single-user and multi-user scenarios.

- A Hybrid A/D beamforming structure was proposed to reduce the complexity and energy consumption of large-scale antenna elements in mmWave communications [84].

- The investigations on EE in D2D communications were given in [85–87]. Specifically, in [85], the power allocation and channel resources in D2D communications underlay cellular networks were studied by using game theory. While the tradeoff between SE and EE was addressed in [86] as a non-cooperative game whereas each MT is self-interested and aims at maximizing its own EE.

- The performance of EE in WSNs was investigated in [88, 89]. In [88], the minimization of energy consumption of both MIMO and cooperative MIMO was investigated by optimizing the modulation and transmission schemes while in [89], an adaptive
algorithm was proposed to minimize the energy consumption in order to satisfy the designated Bit Error Rate (BER).

This dissertation studies and optimizes the energy efficiency from the system-level point of view, i.e., by taking into account the spatial distribution of the network elements, by jointly optimizing BSs density and transmit power in Chapter 4. In addition, the trade-off between SE and EE is also studied with several load models and power consumption.
2.1 Abstract

In this chapter, we introduce a tractable approach for studying the feasibility of multiple-antenna cellular networks, where low-energy devices decode information data and harvest power simultaneously. Tools from stochastic geometry are used to quantify the information rate vs. harvested power tradeoff. Our study unveils that large-scale antenna arrays and ultra-dense deployments of base stations are both necessary to harvest, with high reliability, an amount of power of the order of a milliwatt.

2.2 Introduction

SWIPT is a technology where the same radio frequency signal is used for data transmission and for replenishing the battery of LEDs [53]. The design of SWIPT cellular networks [90] poses new research challenges. Cellular networks are designed based on the assumption that the interference has a negative impact on Wireless Information Transfer (WIT) [37]. The same interference, on the other hand, is a natural source of power for improving WPT [91]. As a result, the development of interference management techniques
that exploit interference for WPT and counteract it for WIT plays a fundamental role.

Multiple-Input-Multiple-Output (MIMO) systems constitute a promising solution to manage and exploit the interference at once [56]. At the receiver, multiple antennas enhance data reliability and increase the harvested power via spatial diversity. At the transmitter, multiple antennas improve information and power transfer via spatial beamforming. Spatial beamforming, however, results in interference isolation that reduces the harvested power. Using MIMO, thus, introduces several tradeoffs in SWIPT cellular networks, which have not been quantified yet [56]. This is the objective of this chapter.

Most works on SWIPT are focused on small-scale networks [56]. Its potential in large-scale networks is, on the other hand, less investigated. In [62, 63, 92, 93], relay-aided networks and ad hoc networks are studied. These papers, however, consider single-antenna transmission. In [11], ad hoc networks with multiple-antenna transmitters are studied. The analysis, however, is not applicable to cellular networks. SWIPT cellular networks are investigated in [94] and [95] for legacy and millimeter-wave frequencies. In both cases, directional antennas are taken into account but MIMO is not.

We introduce a tractable approach to quantify the potential of MIMO in SWIPT cellular networks. MRT and MRC at base stations and LEDs are considered. The locations of the BSs are modeled as points of a PPP and stochastic geometry is used for system-level analysis. Three mathematical frameworks are proposed, which provide exact, approximated, and large-scale asymptotic expressions of the Joint Complementary Cumulative Distribution Function (J-CCDF) of information rate and harvested power.

Our feasibility study shows that large-scale MIMO and ultra-dense deployments of BSs are both necessary to harvest, with high reliability, an amount of power of the order of a milliwatt. In the large-scale MIMO regime, also, the J-CCDF depends only on the average strength of the intended link.

**Notation:** Uppercase and lowercase boldface symbols denote matrices and vectors. $\mathbb{C}^{M \times N}$ is the field of $M \times N$ complex matrices. $X \sim \mathcal{CN} (\mu, \sigma^2)$ is a complex Gaussian Random Variable (RV) with mean $\mu$ and variance $\sigma^2$. $X \sim \mathcal{E} (\Omega)$ is an exponential RV with mean $\Omega$. $(\cdot)^*$ is the conjugate transpose. $j = \sqrt{-1}$ is the imaginary unit. $\mathbb{E} \{ \cdot \}$ is the expectation operator. $(\cdot)!$ is the factorial operator. $\| \cdot \|_F$ is the Frobenius norm. $\mathbb{1} (\cdot)$ and $\mathcal{H} (\cdot)$ are indicator and Heaviside functions. $\overline{\mathcal{H}} (x) = 1 - \mathcal{H} (x)$. $p_{q_r} (\cdot; \cdot; \cdot)$,
δ(·), Γ(·) and Γ(·, ·) are generalized hypergeometric, delta, gamma and upper-incomplete gamma functions. \( f_X(·) \) and \( \Phi_X(·) \) denote the Probability Density Function (PDF) and the Characteristic Function (CF) of RV \( X \).

2.3 System Model

2.3.1 Cellular Network Modeling

A downlink MIMO cellular network is considered. BSs and LEDs are equipped with \( N_t \) and \( N_r \) antennas, respectively. The BSs are modeled as points of a homogeneous PPP, denoted by \( \Psi \), of intensity \( \lambda \). Their transmit power is \( P \). The analysis is performed for the typical LED located at the origin [37].

2.3.2 SWIPT Based on Power Splitting

The typical LED is equipped with information and energy receivers that operate according to the PS scheme [53]. The received power is split into two parts, according to the power splitting ratio \( 0 \leq \rho \leq 1 \), and is used for Energy Harvesting (EH) and Information Decoding (ID).

2.3.3 Channel Modeling

The channel model accounts for Line-of-Sight (LOS) and Non-LOS (NLOS) links due to spatial blockages, as well as for path-loss and fast-fading [7]. Shadowing is implicitly accounted for via the LOS and NLOS link model [5].

LOS/NLOS Links

Let \( r \) be the distance from a BS to the typical LED. The probability to be in LOS and NLOS as a function of \( r \), \( p_s(·) \) for \( s \in \{\text{LOS}, \text{NLOS}\} \), is as follows:

\[
p_s(r) = \begin{cases} 
q_s^{[0,D]} & \text{if } r \in [0, D) \\
q_s^{[D,\infty]} & \text{if } r \in [D, +\infty) 
\end{cases}
\] (2.1)
where $q_{[a,b]}^{[a,b]} + q_{[a,b]}^{[a,b]} = 1$, $q_{s}^{[a,b]}$ is the probability that a link of length $r \in [a, b]$ is in state $s$, and $D$ takes into account that LOS and NLOS probabilities are different for short and long distances [5]. Assuming no spatial correlation, $\Psi$ can be split into two independent and non-homogeneous PPPs, $\Psi_{\text{LOS}}$ and $\Psi_{\text{NLOS}}$, such that $\Psi = \Psi_{\text{LOS}} \cup \Psi_{\text{NLOS}}$. From (2.1), the density of $\Psi_s$ is $\lambda_s(r) = \lambda p_s(r)$ for $s \in \{\text{LOS, NLOS}\}$.

**Path-Loss**

The path-loss of LOS and NLOS links is $l_s(r) = \kappa_0 r^{\beta_s}$ for $s \in \{\text{LOS, NLOS}\}$, where $\kappa_0 = (4\pi/\nu)^2$, $\nu$ is the wavelength and $\beta_s$ is the path-loss exponent.

**Fast-Fading**

All channels are assumed to be independent and identically distributed (i.i.d.) $CN(0, 1)$ RVs.

**2.3.4 Cell Association**

The typical LED is served by the BS providing the smallest path-loss. The other BSs act as interferers. The smallest path-loss is denoted by $L^{(0)} = \min \left\{ L_{\text{LOS}}^{(0)}, L_{\text{NLOS}}^{(0)} \right\}$, where, for $s \in \{\text{LOS, NLOS}\}$, $L_s^{(0)}$ is defined as follows:

$$L_s^{(0)} = \min_{n \in \Psi_s} \left\{ l_s(r^{(n)}) \right\},$$

(2.2)

and $r^{(n)}$ is the distance between a BS and the typical LED.

**2.4 MIMO Cellular Networks**

At the BSs, MRT with directive antennas is used. Two-lobe antennas are considered, where $G_M$ and $G_m$ are the gains of main and secondary lobes, and $\omega_M$ is the width of the main lobe [7]. Each BS steers its main lobe towards the LED associated to it, hence the unintended links are randomly oriented with respect to each other and uniformly
distributed in $[0, 2\pi)$. Thus, the received vector, $\mathbf{y} \in \mathbb{C}^{N_r \times 1}$, is:

$$\mathbf{y} = \mathbf{U} + \mathbf{I}_{\text{LOS}} + \mathbf{I}_{\text{NLOS}} + \mathbf{n},$$

(2.3)

where $\mathbf{U} \in \mathbb{C}^{N_r \times 1}$ is the intended signal and $\mathbf{I}_s \in \mathbb{C}^{N_r \times 1}$ for $s \in \{\text{LOS}, \text{NLOS}\}$ is the other-cell interference:

$$\mathbf{U} = \sqrt{P G(0)} / L(0) H(0) w_t(0) x(0),$$

$$\mathbf{I}_s = \sum_{i \in \Psi_s} \sqrt{P G(i) / l_s(r(i))} H(i) w_t(i) x(i) 1(l_s(r(i)) > L(0)),$$

(2.4)

and $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$ is the Gaussian noise with $\mathbf{n}(\tau) \sim \mathcal{CN}(0, \sigma_n^2)$ for $\tau = 1, 2, \ldots, N_r$. The indicator function in (2.4) is due to the cell association. The notation is as follows.

- $H(0) \in \mathbb{C}^{N_r \times N_t}$ and $H(i) \in \mathbb{C}^{N_r \times N_t}$ are the channel matrices of serving and $i$th interfering BSs. Their elements are i.i.d., with $H(0) (\tau, t) \sim \mathcal{CN}(0, 1)$ and $H(i) (\tau, t) \sim \mathcal{CN}(0, 1)$ for $\tau = 1, 2, \ldots, N_r$ and $t = 1, 2, \ldots, N_t$.

- $x(0)$ and $x(i)$ are the data symbols of serving and $i$th interfering BSs. They are i.i.d. with zero mean and unit power, i.e., $\mathbb{E} \left\{ |x(0)|^2 \right\} = \mathbb{E} \left\{ |x(i)|^2 \right\} = 1$.

- $w_t(0) \in \mathbb{C}^{N_t \times 1}$ with $\|w_t(0)\|^2 = 1$ is the beamforming vector of the serving BS. It is the eigenvector corresponding to the largest eigenvalue of $F(0) = (H(0))^* H(0) \in \mathbb{C}^{N_t \times N_t}$, which is $\chi(0) = \left\| (H(0) w_t(0))^* H(0) w_t(0) \right\| [96, \text{Eq. (33)}]$. Its PDF is as follows [96, Eq. (9)]:

$$f_{\chi(0)}(\xi) = K_{p,q} \sum_{u=1}^{q} \sum_{v=1}^{(p+q-2v)u} c_{v,u} \xi^u \exp(-v\xi),$$

(2.5)

where $p = \max\{N_t, N_r\}$, $q = \min\{N_t, N_r\}$, $K_{p,q} = (\prod_{a=1}^{q} (q-a)!(p-a)!)^{-1}$, $c_{v,u}$ follows from [96].

- $w_t(i) \in \mathbb{C}^{N_t \times 1}$ with $\|w_t(i)\|^2 = 1$ is the beamforming vector of the $i$th interfering BS. It is the eigenvector corresponding to the largest eigenvalue of $F(i) = (G(i))^* G(i) \in \mathbb{C}^{N_t \times N_t}$, where $G(i) \in \mathbb{C}^{N_r \times N_t}$ is the channel matrix of the LED served by the $i$th BS.
• $G^{(0)}$ and $G^{(i)}$ are the directivity gains of intended and interfering links, where $G^{(0)} = G_M$ and $f_{G^{(i)}} (g) = (\omega_M/2\pi) \delta (g - G_M) + (1 - \omega_M/2\pi) \delta (g - G_m)$. 

At the LEDs, MRC with omnidirectional antennas is used. The demodulation vector is $w^{(0)}_r = H^{(0)} w^{(0)}_t / \|H^{(0)} w^{(0)}_t\|$. The signals at the input of ID and EH receivers are:

$$z_{\text{ID}} = \sqrt{1 - \rho} (w^{(0)}_r)^* y + m_{\text{ID}}; \quad z_{\text{EH}} = \sqrt{\rho} (w^{(0)}_r)^* y + m_{\text{EH}}, \quad (2.6)$$

where $m_{\text{ID}} \sim \mathcal{CN}(0, \sigma_{\text{ID}}^2)$ and $m_{\text{EH}} \sim \mathcal{CN}(0, \sigma_{\text{EH}}^2)$ are the additive noises of ID and EH receivers [94], respectively.

From (2.6), the Shannon rate (in bits/sec), $R$, and harvested power (in Watts), $Q$, of ID and ED receivers are as follows:

$$R = B_w \log_2 \left( 1 + \frac{PG^{(0)} \chi^{(0)} / L^{(0)}}{PI_{\text{ID}} + \sigma_N^2 + \sigma_{\text{ID}}^2/(1 - \rho)} \right),$$

$$Q = \rho \zeta \left( PG^{(0)} \chi^{(0)} / L^{(0)} + PI_{\text{EH}} \right), \quad (2.7)$$

where $B_w$ is the bandwidth, $0 \leq \zeta \leq 1$ is the EH conversion efficiency, and $I = I_{\text{ID}} = I_{\text{EH}}$ is the other-cell interference:

$$I = \sum_{i \in \Psi_{\text{LOS}}} (1/l_{\text{LOS}}(r^{(i)})) G^{(i)} \gamma^{(i)} 1(l_{\text{LOS}}(r^{(i)}) > L^{(0)})$$

$$+ \sum_{i \in \Psi_{\text{NLOS}}} (1/l_{\text{NLOS}}(r^{(i)})) G^{(i)} \gamma^{(i)} 1(l_{\text{NLOS}}(r^{(i)}) > L^{(0)}), \quad (2.8)$$

where $\gamma^{(i)} = \left\| (H^{(0)} w^{(0)}_t)^* H^{(i)} w^{(i)}_t \right\|^2 / \chi^{(0)} \sim \mathcal{E}(1)$ is independent of $\chi^{(0)}$. Further details are available in [96].

### 2.5 System-Level Analysis

In SWIPT cellular networks, the tradeoff between information rate and harvested power is quantified in terms of the J-CCDF of $R$ and $Q$ defined in (2.7). The J-CCDF is [94]:

$$F_c(R_*, Q_*) = \Pr \{ R \geq R_*, Q \geq Q_* \}, \quad (2.9)$$
where $R_*$ and $Q_*$ are the minimum data rate and harvested power needed for the LED to perform its tasks.

### 2.5.1 Exact Mathematical Framework

**Proposition 1** The J-CCDF in (2.9) can be formulated as:

$$F_c (R_*, Q_*) = K_{p,q} \sum_{v=1}^{q} \sum_{u=p-q}^{(p+q-2v)} c_{v,u} (J_{v,u}^{(1)} - J_{v,u}^{(2)}), \quad (2.10)$$

where $r_* = (2R_*/B_w - 1)^{-1}$, $\sigma_*^2 = \sigma_N^2 + \sigma_{ID}^2 (1 - \rho)^{-1}$, $q_* = Q_* (p\zeta)^{-1}$, $T_* = (q_* + \sigma_*^2)/(r_* + 1)$, $J_{v,u}^{(1)}$ and $J_{v,u}^{(2)}$ are given in Eq. (2.11)

$$J_{v,u}^{(1)} = \frac{1}{\pi \omega} \int_0^{\infty} \int_0^{\infty} \frac{1}{\pi \omega} \text{Im} \left\{ \exp \left( -j\omega \frac{q_*}{PG_M} \right) \left( v - j\omega \right)^{-1} \right\} \frac{f_{L(0)} (y)}{y} d\omega dy$$

$$J_{v,u}^{(2)} = \frac{1}{\pi \omega} \int_0^{\infty} \int_0^{\infty} \frac{1}{\pi \omega} \text{Im} \left\{ \exp \left( j\omega \frac{\sigma_*^2}{PG_M} \right) \left( v + j\omega r_* \right)^{-1} \right\} \frac{f_{L(0)} (y)}{y} d\omega dy,$$  

and $f_{L(0)} (\cdot)$ is the PDF of $L(0)$:

$$f_{L(0)} (x) = \Lambda (0, x) \exp (-\Lambda (0, x)), \quad (2.12)$$

$\Lambda (0, x) = \Lambda_{\text{LOS}} (0, x) + \Lambda_{\text{NLOS}} (0, x)$, $\Lambda (0, x) = \Lambda_{\text{LOS}} (0, x) + \Lambda_{\text{NLOS}} (0, x)$, and $\Lambda_s ([\cdot, \cdot])$, $\Lambda_{\text{NLOS}} ([\cdot, \cdot])$ are:

$$\Lambda_s ([0, x]) = \pi \lambda q_s^{[0,D]} (x \kappa_{0})^2 \mathcal{H} (x - \kappa_0 D^{\beta_s})$$

$$+ \pi \lambda \left( \left( \frac{x}{\kappa_0} \right)^{2/\beta_s} q_s^{[D,\infty]} + D^2 \left( q_s^{[0,D]} - q_s^{[D,\infty]} \right) \right) \mathcal{H} (x - \kappa_0 D^{\beta_s})$$

$$\Lambda_{\text{NLOS}} ([0, x]) = (2\pi \lambda/\beta_s) q_s^{[0,D]} \kappa_0^{-2/\beta_s} x^{(2/\beta_s - 1)} \mathcal{H} (x - \kappa_0 D^{\beta_s})$$

$$+ (2\pi \lambda/\beta_s) q_s^{[D,\infty]} \kappa_0^{-2/\beta_s} x^{(2/\beta_s - 1)} \mathcal{H} (x - \kappa_0 D^{\beta_s}), \quad (2.13)$$
Chapter 2. SWIPT-Enabled Cellular Networks with MIMO

and \( \Phi_I (\cdot | L(0)) \) is the CF of \( I \) conditioned on \( L(0) \):

\[
\Phi_I (\omega | L(0)) = \Phi_I (\omega | L(0); \text{LOS}) \Phi_I (\omega | L(0); \text{NLOS}),
\]

where \( \Phi_I (\cdot | L(0); s) \) is in (2.15):

\[
\Phi_I (\omega | L(0); s) = \exp \left( \lambda \pi q_s^{D,\infty} \max \left\{ D^2, \left( \frac{L(0)}{\kappa_0} \right)^{2/\beta_s} \right\} \left( 1 - \Upsilon_s (\omega, \max \left\{ \kappa_0 D^\beta_s, L(0) \right\}) \right) \right)
\times \exp \left( \pi \lambda q_s^{[0,D]} \left[ \left( \frac{L(0)}{\kappa_0} \right)^{2/\beta_s} \left( 1 - \Upsilon_s (\omega, L(0)) \right) - D^2 \left( 1 - \Upsilon_s (\omega, \kappa_0 D^\beta_s) \right) \right] \right)
\times H \left( L(0) - \kappa_0 D^\beta_s \right)
\]

(2.15)

\( \Upsilon_s (\omega, Z) = (\omega M / 2 \pi) \ _2F_1 \left( 1, -2/\beta_s, 1 - 2/\beta_s, j \omega G_M / Z \right) \\
+ (1 - \omega M / 2 \pi) \ _2F_1 \left( 1, -2/\beta_s, 1 - 2/\beta_s, j \omega Z / Z \right). \)

Proof: It follows by using an approach similar to [94]. □

The J-CCDF in (2.10) is exact but the number of addends of the summations in (2.10) increases with \( N_t \) or \( N_r \).

2.5.2 Approximated Mathematical Framework

To increase the computational efficiency of (2.10) and the insight that can be gained from it, we approximate the PDF of \( \chi^{(0)} \) in (2.5) with that of a gamma RV \( \tilde{\chi}^{(0)} \), i.e.,

\( \chi^{(0)} \approx \tilde{\chi}^{(0)} \):

\[
f_{\tilde{\chi}^{(0)}} (\xi) = \xi^{m_a - 1} / (\Gamma (m_a) \theta_a^{m_a}) \exp (-\xi / \theta_a),
\]

(2.16)

where \( \mu_{\tilde{\chi}^{(0)}} = \mathbb{E} \{ \tilde{\chi}^{(0)} \} = m_a \theta_a \) and \( \eta_{\tilde{\chi}^{(0)}} = \mathbb{E} \{ \left( \tilde{\chi}^{(0)} \right)^2 \} - (\mathbb{E} \{ \tilde{\chi}^{(0)} \})^2 = m_a \theta_a^2 \) are the mean and the variance of \( \tilde{\chi}^{(0)} \). The pair \( (\mu_{\tilde{\chi}^{(0)}}, \eta_{\tilde{\chi}^{(0)}}) \) is obtained from the moment matching method, i.e., by solving the system of equations \( \mathbb{E} \{ \chi^{(0)} \} = \mu_{\tilde{\chi}^{(0)}} \) and \( \mathbb{E} \{ \left( \chi^{(0)} \right)^2 \} - (\mathbb{E} \{ \chi^{(0)} \})^2 = \eta_{\tilde{\chi}^{(0)}} \), which yields:

\[
m_a = \frac{\left( \mathbb{E} \{ \chi^{(0)} \} \right)^2}{\mathbb{E} \{ (\chi^{(0)})^2 \} - (\mathbb{E} \{ \chi^{(0)} \})^2}, \quad \theta_a = \frac{\mathbb{E} \{ (\chi^{(0)})^2 \} - (\mathbb{E} \{ \chi^{(0)} \})^2}{(\mathbb{E} \{ \chi^{(0)} \})^2},
\]

(2.17)
where $\mathbb{E}\{\chi^{(0)}\} = K_{p,q} \sum_{u=1}^{q} \sum_{v=p-q}^{(p+q-2)v} (C_{v,u}/v^{\alpha+\eta+1}) \times \Gamma(u + \eta + 1)$ is the $\eta$th moment of $\chi^{(0)}$ obtained from (2.5).

**Proposition 2** The J-CCDF in (2.9) can be approximated as:

$$F_{c}(R^*, Q^*) \approx \frac{1}{\Gamma(m_a)} \theta_{ma} (J^{(1)}_{a}(m_a, \theta_a) - J^{(2)}_{a}(m_a, \theta_a)),$$

(2.18)

where $J^{(1)}_{a}(\cdot, \cdot)$, $J^{(2)}_{a}(\cdot, \cdot)$ are in (2.19):

$$J^{(1)}_{a}(m_a, \theta_a) = \int_{0}^{+\infty} \int_{0}^{+\infty} \frac{1}{\pi \omega} \text{Im} \left\{ \exp \left( -j \omega \frac{q_{s}}{P_{G}M} \right) \left( \frac{1}{\theta_a} - \frac{j \omega}{y} \right)^{-m_a} \right\} \times \Gamma \left( m_a, \frac{T_{s}}{P_{G}M} \left( \frac{y}{\theta_a} - j \omega \right) \right) \Phi \left( \omega_{G}M \left| y \right. \right) f_{L^{(0)}}(y) \, d\omega dy$$

$$J^{(2)}_{a}(m_a, \theta_a) = \int_{0}^{+\infty} \int_{0}^{+\infty} \frac{1}{\pi \omega} \text{Im} \left\{ \exp \left( j \omega \frac{\sigma_{s}^{2}}{P_{G}M} \right) \left( \frac{1}{\theta_a} + \frac{j \omega r_{s}}{y} \right)^{-m_a} \right\} \times \Gamma \left( m_a, \frac{T_{s}}{P_{G}M} \left( \frac{y}{\theta_a} + j \omega r_{s} \right) \right) \Phi \left( \omega_{G}M \left| y \right. \right) f_{L^{(0)}}(y) \, d\omega dy.$$  

(2.19)

Proof: It is similar to the proof of Proposition 1. 

Compared with the approach in [95], Proposition 2 is more accurate, as it is applicable for any $m_a$ and not just for $m_a \gg 1$. Thanks to the single-ball blockage model in (2.1), it is more tractable as well [7]. As a result, it is useful for analyzing both centimeter- and millimeter-wave cellular networks.

### 2.5.3 Asymptotic (Large-Scale) Mathematical Framework

**Proposition 3** In the large-scale MIMO regime, i.e., $N_t \gg 1$ or $N_r \gg 1$, the J-CCDF in (2.9) is asymptotically equal to:

$$F_{c}(R^*, Q^*) \to \int_{0}^{+\infty} \int_{0}^{\tilde{y}_{M}} (\pi \omega)^{-1} f_{L^{(0)}}(y) \times \text{Im} \left\{ J_{\infty}(\mu_{\tilde{\chi}^{(0)}}, \omega, y) \Phi \left( \omega_{G}M \left| y \right. \right) \right\} d\omega dy,$$

(2.20)

where $\tilde{y}_{M} = \mu_{\tilde{\chi}^{(0)}} P_{G}M / T_{s}$ and $J_{\infty}(\cdot, \cdot, \cdot)$ is defined as:

$$J_{\infty}(\mu_{\tilde{\chi}^{(0)}}, \omega, y) = \exp \left( -j \omega \left( q_{s}/P_{G}M - \mu_{\tilde{\chi}^{(0)}} / y \right) \right)$$

$$- \exp \left( -j \omega \left( -\sigma_{s}^{2}/P_{G}M + r_{s} \mu_{\tilde{\chi}^{(0)}} / y \right) \right).$$

(2.21)
Proof: It follows from (2.18), setting $\theta_a = \mu_{\tilde{\chi}(0)}/m_a$, noting, from (2.17), that $m_a \to \infty$ if $N_t \to \infty$ or $N_r \to \infty$, and:

\[
\begin{align*}
\lim_{m_a \to +\infty} \Gamma(m_a, z)/\Gamma(m_a) &= 1 \quad (\text{Re} \{z\} \leq m_a) \\
\lim_{m_a \to +\infty} (1 + z/m_a)^{-m_a} &= \exp(-z),
\end{align*}
\]

which hold for any $z \in \mathbb{C}$ whose real part is non-negative. \(\square\)

From the Gil-Pelaez theorem [97], (2.20) can be written as:

\[
F_c(R_*, Q_*) \to \mathbb{E}_{L(0)} \left\{ \Pr \left\{ I \geq \left( q_s/P_G M - \mu_{\tilde{\chi}(0)}/L(0) \right) | L(0) \right\} \right\} \\
- \mathbb{E}_{L(0)} \left\{ \Pr \left\{ I \geq \left( -\sigma_s^2/P_G M + r_s \mu_{\tilde{\chi}(0)}/L(0) \right) | L(0) \right\} \right\},
\]

for $L(0) \leq \mu_{\tilde{\chi}(0)} P G_M / T_s$ and $F_c(R_*, Q_*) \to 0$ otherwise.

As for the impact of $N_t$ and $N_r$, the following conclusions can be drawn, in the large-scale antenna regime, from (2.23).

- The J-CCDF depends only on the average strength of the intended link: $\mu_{\tilde{\chi}(0)} = \mathbb{E} \{ \tilde{\chi}(0) \} = \mathbb{E} \{ \chi(0) \} = \mu_{\chi}(0)$.
- The J-CCDF increases as $\mu_{\tilde{\chi}(0)}$ increases, since, by definition, the first and the second addend of (2.23) increases and decreases, respectively, as a function of $\mu_{\tilde{\chi}(0)}$.

From its definition in (2.17), $\mu_{\chi}(0)$ increases as either $N_t$ or $N_r$ increases. We conclude that, for any $\mathcal{R}_*$ and $\mathcal{Q}_*$, $F_c(\mathcal{R}_*, \mathcal{Q}_*)$ can be arbitrary close to one if either $N_t$ or $N_r$ are sufficiently large. Due to the limited size of the LEDs, this implies that the BSs need to be equipped with many antennas.

2.6 Numerical and Simulation Results

Considered setup: $\nu = c_0/f_c$, where $c_0$ is the speed of light in m/sec and $f_c = 2.1$ GHz is the carrier frequency; $\sigma_{ID}^2 = -70$ dBm; $\sigma_N^2 = -174 + 10 \log_{10} (B_w) + \mathcal{F}_N$ dBm, where $B_w = 200$ KHz and $\mathcal{F}_N = 10$ dB is the noise figure; $P = 30$ dBm; $\zeta = 0.8$; $\lambda = 1/(\pi R_{cell}^2)$ where $R_{cell}$ is the average cell radius. The channel model is [5]: $D = 109.8517$ m; $q_{\text{LOS}}^{[0,D]} =$
Figure 2.1: Contour lines of J-CCDF vs. $N_t$ ($\rho = 0.5$, $R_{\text{cell}} = 83.4122$ m [5], $G_M = 1$, $G_m = 1$, $\omega_M = 360$ degrees (omnidirectional antennas) [7]). They show the pairs $(R^*, Q^*)$ so that $F_c(R^*, Q^*) = 0.75$. Solid lines show exact and approximated frameworks in (2.10) and (2.18). Dashed lines show the asymptotic framework in (2.20). Markers show Monte Carlo simulations.

$0.7195; q_{\text{LOS}}^{D,\infty} = 0.0002; \beta_{\text{LOS}} = 2.5; \beta_{\text{NLOS}} = 3.5$. $N_r = 2$ is assumed, which may be applicable to smart-watches, as the average circumference of a human wrist is 14-20 cm. Simulations (details can be found in [5]) and frameworks are obtained with Matlab and Mathematica.

In Fig. 2.1, we validate the accuracy of the mathematical frameworks as a function of $N_t$. The exact and approximated frameworks in (2.10) and (2.18) are indistinguishable from each other. The asymptotic framework in (2.20) becomes tighter as $N_t$ increases. In the considered setup, it is accurate enough for $N_t = 4$. In Fig. 2.2, we perform a feasibility study of SWIPT cellular networks. The curves are obtained from (2.18) by computing $Q_*$ that corresponds to the optimal $F_c(\cdot, \cdot)$ as a function of $\rho$ and such that $R_* = 100$ kbits/sec and $F_c(R_*, Q_*) = 0.90$. The existence of an optimal $\rho$ originates from the fact that $Q$ increases and $R$ decreases with $\rho$. Fig. 2.2 shows that different results are obtained if
Figure 2.2: Harvested power $Q_*$ vs. $N_t$ and $R_{\text{cell}}$ so that $F_c(R_*, Q_*) = 0.90$ and $R_* = 100$ kbits/sec. $G_M = 32.4195$, $G_m = 0.4363$, $\omega_M = 12$ degrees (directive antennas) [7]. The curves are obtained by using (2.18) and each point corresponds to the optimal $\rho$ that maximizes the JCCDF.

LOS and NLOS links are neglected. This justifies the adoption of the considered blockage model. Fig. 2.2 shows, in addition, that network densification and large-scale MIMO are both essential for enabling SWIPT cellular networks harvest an amount of power of the order of a milliwatt, while still guaranteeing a sufficient rate for LEDs applications. It is worth noting that the performance trends shown in Figs. 2.1 and 2.2 assume that the input-output response of the EH receiver is linear with the input power. The analysis of non-linear EH models [98] may, on the other hand, result in different performance trends.
3.1 Abstract

In this chapter, we study the feasibility of receiver diversity for application to downlink cellular networks, where low-energy devices are equipped with information decoding and energy harvesting receivers for simultaneous wireless information and power transfer. We compare several options that are based on selection combining and maximum ratio combining, which provide different implementation complexities. By capitalizing on the Frechet inequality, we shed light on the advantages and limitations of each scheme as a function of the transmission rate and harvested power that need to be fulfilled at the low-energy devices. Our analysis shows that no scheme outperforms the others for every system setup. It suggests, on the other hand, that the low-energy devices need to operate in an adaptive fashion, by choosing the receiver diversity scheme as a function of the imposed requirements. With the aid of stochastic geometry, we introduce mathematical frameworks for system-level analysis. We show that they constitute an important tool for system-level optimization and, in particular, for identifying the diversity scheme that optimizes wireless information and power transmission as a function of a sensible set of parameters. Monte Carlo simulations are used to validate our findings and to illustrate
the trade-off that emerge in cellular networks with simultaneous wireless information and power transfer.

3.2 Introduction

The Internet of Things (IoT) is expected to connect billions of LEDs by 2020 [99]. One of the main challenges of the IoT is how to provide enough energy for the electronics of the LEDs, in order to have them operational over a reasonable amount of time and without making their battery too large or the device itself too bulky. For several applications, it may not be even possible to (re-)charge some kinds of LEDs.

In this context, the emerging concept of SWIPT constitutes a suitable solution for prolonging the battery life of the LEDs and, in a foreseeable future, for making them energy-neutral, i.e., operational in a complete self-powered fashion. SWIPT is a technology where the same radio frequency signal is used for information transmission and for replenishing the battery of the LEDs [53]. SWIPT may find application in the emerging market of cellular-enabled IoT, where the LEDs, e.g., smart watches [100], receive notifications from their cellular connection [90] and, simultaneously, re-charge their battery.

The recent decision to standardize NarrowBand IoT (NB-IoT), a new narrow-band radio technology that addresses the requirements of the IoT, confirms the wish of capitalizing on the ubiquitous coverage offered by the cellular network infrastructure for IoT applications [101].

The design of SWIPT-enabled cellular networks introduces, however, new research challenges and never observed trade-offs. Conventional cellular networks are designed based on the assumption that the interference has a negative impact on ID, since it reduces the coverage and rate [37]. The same interference, on the other hand, is a natural source of power for EH [91]. As a result, the development of interference management techniques that exploit interference for EH and counteract it for ID plays a fundamental role. In this context, receiver diversity is considered to be a promising solution for enhancing the reliability of data transmission and for increasing the amount of harvested power [56]. The size of the LEDs, in fact, is expected to be larger than that of sensor nodes, and, hence, multiple radiating (antenna) elements may be available. On the other hand, their size,
cost and power consumption requirements may still limit the number of radio frequency front-ends to be used [102]. Receiver diversity constitutes a practical solution for taking advantage of the available antenna elements by using fewer radio frequency front-ends. It provides, in fact, the possibility of optimizing performance, cost and power consumption for a given size of the LEDs.

The potential of receiver diversity for application to SWIPT-enabled systems has recently been analyzed in [103] and [104]. In these papers, in particular, it has been shown that receiver diversity based on selection combining and antenna switching constitutes a promising alternative to typical approaches based on power splitting and time switching [105]. As elaborated in [104], in fact, power splitting and time switching need dedicated hardware components (power splitters and time switches), which may increase the complexity and cost of the LEDs and may be subject to efficiency losses. Time switching, in addition, necessitates dedicated time slots and synchronization circuits for EH, which results in the discontinuous transmission of information data. Receiver diversity, on the other hand, is a mature technology that may overcome these limitations. It requires, however, the availability of multiple antenna elements at the LEDs. This leads to new performance versus implementation complexity trade-offs (further details are available in Section 3.4.1) that, to the best of our knowledge, are not totally understood.

Motivated by these considerations, we study the potential of receiver diversity for application to SWIPT-enabled cellular networks. In particular, we focus our attention on practical implementations where one or two receive antennas are available at the LEDs. This implies that only one or two radio frequency front-ends are needed. This case study may find concrete application to LEDs such as smart watches, since the typical circumference of a human wrist is 14-20 cm, and, thus, two compact integrated antennas and radio frequency front-ends may be accommodated at typical transmission frequencies. More antenna elements may be used, by still employing one or two radio frequency front-ends, for LEDs of larger size, e.g., for relay nodes [56]. We study various options based on selection combining and maximum ratio combining schemes, and discuss their achievable performance versus implementation complexity trade-off. Our analysis, in particular, shows that no scheme outperforms the others for every system setup. It suggests, on the other hand, that the LEDs need to operate in an adaptive fashion, by choosing the receiver diversity
scheme to be used as a function of the performance requirements that need to be fulfilled.

Against the state-of-the-art of research on performance evaluation of wireless networks with SWIPT, our contribution is twofold. Compared to [103] and [104], we focus our attention on system-level analysis and optimization rather than on link-level optimization. More specifically, we take into account the impact of large-scale network deployments and introduce new mathematical frameworks for the adaptive optimization of SWIPT-enabled cellular networks. This is performed by exploiting the mathematical tool of stochastic geometry and by modeling the locations of cellular Base Stations (BSs) as points of a Poisson Point Process (PPP). Compared to recent papers that have exploited similar mathematical tools for large-scale analysis of wireless networks with SWIPT, e.g., [11, 62, 63, 92–94], our work is the first that investigates the potential of receiver diversity for application to cellular networks. In [11, 62, 63, 92, 93], on the other hand, decentralized (ad hoc) networks without receiver diversity are studied. In [94], cellular networks are analyzed but single-antenna LEDs are considered. The latter paper constitutes, however, the benchmark against which the potential benefits of receiver diversity studied in this chapter are quantified.

This chapter is organized as follows. In Section 4.3, the system model is introduced. In Section 3.4, the research problem is first motivated and then formulated in terms of the Joint Complementary Cumulative Distribution Function (J-CCDF) of information rate and harvested power. In Section 3.5, several SWIPT schemes are compared against each other with the aid of the Frechet inequality. In Section 3.6, mathematical frameworks for system-level performance evaluation and for the adaptive optimization of SWIPT-enabled cellular networks are introduced. In Section 3.7, analysis and findings are validated with the aid of numerical simulations. Section 3.8 concludes this chapter.

Notation: Notation and definitions are reported in Table 3.1.

3.3 System Model

3.3.1 Cellular Networks Modeling

A downlink cellular network is considered. The BSs are modeled as points of a homogeneous PPP, denoted by Ψ, of density λ. The transmit power of the BSs is assumed to
### Table 3.1: Main notation and mathematical symbols.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\cdot)^*$</td>
<td>Conjugate operator</td>
</tr>
<tr>
<td>$j = \sqrt{-1}$</td>
<td>Imaginary unit</td>
</tr>
<tr>
<td>$\mathbb{E}{\cdot}$</td>
<td>Expectation operator</td>
</tr>
<tr>
<td>$\operatorname{Pr}{\cdot}$</td>
<td>Probability measure</td>
</tr>
<tr>
<td>$\cup{\cdot}$</td>
<td>Union of sets</td>
</tr>
<tr>
<td>$\operatorname{Im}{\cdot}$</td>
<td>Imaginary part operator</td>
</tr>
<tr>
<td>$\min{\cdot, \cdot}, \max{\cdot, \cdot}$</td>
<td>Minimum and maximum operators</td>
</tr>
<tr>
<td>$1(\cdot)$</td>
<td>Indicator function</td>
</tr>
<tr>
<td>$\delta(\cdot)$</td>
<td>Dirac delta function</td>
</tr>
<tr>
<td>$H(\cdot)$</td>
<td>Heaviside function</td>
</tr>
<tr>
<td>$H(x) = 1 - H(x)$</td>
<td>Complementary Heaviside function</td>
</tr>
<tr>
<td>$pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; \cdot)$</td>
<td>Generalized hypergeometric function</td>
</tr>
<tr>
<td>$\Gamma(\cdot, \cdot)$</td>
<td>Upper-incomplete gamma function</td>
</tr>
<tr>
<td>$f_X(\cdot)$</td>
<td>Probability Density Function (PDF) of Random Variable (RV) $X$</td>
</tr>
<tr>
<td>$F_X(\cdot)$</td>
<td>Cumulative Distribution Function (CDF) of RV $X$</td>
</tr>
<tr>
<td>$\Phi_X(\cdot)$</td>
<td>Characteristic Function (CF) of RV $X$</td>
</tr>
<tr>
<td>$X \sim E(\mu)$</td>
<td>$X$ is a RV whose PDF is $f_X(\xi) = \mu \exp(-\mu \xi)$</td>
</tr>
<tr>
<td>$X \sim E_{\max}(1)$</td>
<td>$X$ is a RV whose PDF is $f_X(\xi) = 2 \exp(-\xi) - 2 \exp(-2\xi)$</td>
</tr>
<tr>
<td>$X \sim G(2, 1)$</td>
<td>$X$ is a RV whose PDF is $f_X(\xi) = \xi \exp(-\xi)$</td>
</tr>
<tr>
<td>$X \overset{d}{=} Y$</td>
<td>The RVs $X$, $Y$ are equivalent in distribution, i.e., their CFs and MGFs are the same</td>
</tr>
<tr>
<td>$\mathcal{R}, \mathcal{Q}$</td>
<td>Information rate, harvested power</td>
</tr>
<tr>
<td>$\mathcal{R}_s, \mathcal{Q}_s$</td>
<td>Information rate, energy harvesting requirements</td>
</tr>
<tr>
<td>$q_{s, n, m}$</td>
<td>J-CCDF of $\mathcal{R}$ and $\mathcal{Q}$</td>
</tr>
<tr>
<td>$D$</td>
<td>Line-of-Sight (LOS) / Non-LOS (NLOS) ball</td>
</tr>
<tr>
<td>$L(0)$</td>
<td>Smallest path-loss</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>Poisson Point Process (PPP) of Base Stations (BSs)</td>
</tr>
<tr>
<td>$\Psi_s$</td>
<td>PPP of BSs in state $s$</td>
</tr>
<tr>
<td>$\lambda, \lambda_s$</td>
<td>Density of BSs, density of BSs in state $s$</td>
</tr>
<tr>
<td>$P$</td>
<td>Transmit power of BSs</td>
</tr>
<tr>
<td>$\beta_s, \kappa_0$</td>
<td>Path-loss exponent of links in state $s$, free-space path-loss constant</td>
</tr>
<tr>
<td>$G_M, G_S$</td>
<td>Beamforming gains of main (M) and side (S) lobes</td>
</tr>
<tr>
<td>$b_M$</td>
<td>Beamwidth of main lobe</td>
</tr>
<tr>
<td>$0 &lt; \rho &lt; 1$</td>
<td>Power splitting ratio</td>
</tr>
<tr>
<td>$B_w$</td>
<td>Transmission bandwidth</td>
</tr>
<tr>
<td>$\sigma_N^2$</td>
<td>Thermal noise power</td>
</tr>
<tr>
<td>$\sigma_{ID}^2$</td>
<td>Noise power due to the signal conversion from RF to baseband</td>
</tr>
<tr>
<td>$0 \leq \zeta \leq 1$</td>
<td>Efficiency of energy harvesting conversion</td>
</tr>
<tr>
<td>$r_s = (2^{\kappa_0} / B_w - 1)^{-1}, \tilde{r}_s = r_s$</td>
<td>Short-hands used in Proposition 4 and Proposition 5</td>
</tr>
<tr>
<td>$\sigma_s^2 = \sigma_N^2 + \sigma_{ID}^2 (1 - \rho)^{-1}$</td>
<td>Short-hands used in Proposition 4 and Proposition 5</td>
</tr>
<tr>
<td>$\tilde{\sigma}<em>s^2 = \sigma_N^2 + \sigma</em>{ID}^2 / \rho_{ID}$</td>
<td>Short-hands used in Proposition 4 and Proposition 5</td>
</tr>
<tr>
<td>$q_s = Q_s(\kappa)^{-1}$</td>
<td>Short-hands used in Proposition 4 and Proposition 5</td>
</tr>
<tr>
<td>$\tilde{q}<em>s = Q_s(\rho</em>{EH}\zeta)^{-1}$</td>
<td>Short-hands used in Proposition 4 and Proposition 5</td>
</tr>
<tr>
<td>$T_s = (q_s + \sigma_s^2)^{-1} / (r_s + 1)$, $\tilde{T}_s = (\tilde{q}_s + \tilde{\sigma}_s^2)^{-1} / (\tilde{r}_s + 1)$</td>
<td>Short-hands used in Proposition 4 and Proposition 5</td>
</tr>
</tbody>
</table>
be fixed and is denoted by $P$. Without loss of generality, the analysis is performed for the typical LED located at the origin \[37\].

### 3.3.2 Channel Modeling

The channel model accounts for Line-of-Sight (LOS) and Non-LOS (NLOS) links due to spatial blockages, for the path-loss, and for the fast-fading. Shadowing is implicitly taken into account via the LOS and NLOS link model \[5\].

#### LOS/NLOS Links

Let $r$ be the distance from a BS to the typical LED. The probability of LOS and NLOS as a function of $r$, $p_s(\cdot)$ for $s \in \{\text{LOS}, \text{NLOS}\}$, is formulated as follows:

$$p_s(r) = \begin{cases} q_s^{[0,D]} & \text{if } r \in [0,D) \\ q_s^{[D,\infty]} & \text{if } r \in [D, +\infty) \end{cases}$$

(3.1)

where $q_{\text{LOS}}^{[a,b]} + q_{\text{NLOS}}^{[a,b]} = 1$, $0 \leq q_s^{[a,b]} \leq 1$ is the probability that a link of length $r \in [a,b)$ is in state $s$, and $D$ takes into account that LOS and NLOS probabilities are different for short and long distances \[5\]. Assuming no spatial correlation among the links, $\Psi$ can be split in two independent and non-homogeneous PPPs, $\Psi_{\text{LOS}}$ and $\Psi_{\text{NLOS}}$, such that $\Psi = \Psi_{\text{LOS}} \cup \Psi_{\text{NLOS}}$. From (3.1) and the thinning theorem of PPPs, the density of $\Psi_s$ is $\lambda_s(r) = \lambda p_s(r)$ for $s \in \{\text{LOS}, \text{NLOS}\}$.

#### Path-Loss

The path-loss of LOS and NLOS links is $l_s(r) = \kappa_0 r^{\beta_s}$ for $s \in \{\text{LOS}, \text{NLOS}\}$, where $\kappa_0 = (4\pi/\nu)^2$, $\nu$ is the transmission wavelength, and $\beta_s$ is the path-loss exponent.

#### Fast-Fading

The channel gains are independent and identically distributed (i.i.d.) complex Gaussian RVs with zero mean and unit variance, \textit{i.e.}, Rayleigh fading is considered.
3.3.3 Cell Association

The typical LED is served by the BS providing the smallest path-loss. The other BSs act as interferers. The smallest path-loss can be formulated as \( L^{(0)} = \min \left\{ L^{(0)}_{\text{LOS}}, L^{(0)}_{\text{NLOS}} \right\} \), where, for \( s \in \{ \text{LOS, NLOS} \} \), \( L^{(0)}_s \) is the smallest path-loss of \( \Psi_s \), which is defined as follows:

\[
L^{(0)}_s = \min_{n \in \Psi_s} \left\{ l_s \left( r^{(n)} \right) \right\},
\]

where \( r^{(n)} \) is the distance between the \( n \)th BS of \( \Psi_s \) and the typical LED.

3.3.4 Directional Beamforming at the BSs

At the BSs, to enhance the efficiency of information transmission and energy transfer over long distances, directional beamforming is used. Compared with more sophisticated beamforming schemes [11], it has the advantage of not necessitating channel information at the BSs. Directional beamforming can be implemented by using, e.g., uniform linear arrays [94, Sec. II-C]. We consider a two-lobe model for the radiation pattern, where \( \theta_M \) is the beamwidth of the main lobe, and \( G_M \) and \( G_S \) are the beamforming gains of main and side lobes, respectively. The triplet \((\theta_M, G_M, G_S)\) satisfies the unit power constraint, i.e., \( \theta_M G_M + (2\pi - \theta_M) G_S = 2\pi \). Due to their small size, the LEDs are assumed to use omnidirectional antennas with a unit gain.

The typical LED and its serving BS estimate the angles of arrival and adjust their antenna steering orientations accordingly. Thus, the antenna gain of the typical intended link is \( G^{(0)} = G_M \). From the perspective of the typical LED, on the other hand, the beams of all interfering BSs are randomly oriented, i.i.d., and uniformly distributed in \([0, 2\pi]\). Thus, the PDF of the antenna gain of the \( i \)th interfering link, \( G^{(i)} \), is:

\[
f_{G^{(i)}}(g) = \frac{\theta_M}{2\pi} \delta(g - G_M) + \left( 1 - \frac{\theta_M}{2\pi} \right) \delta(g - G_S).
\]

3.3.5 SWIPT and Receiver Diversity at the LEDs

Due to their small form factor, the LEDs cannot accommodate many receive antennas. Hence, we analyze the case studies where the number of receive antennas and radio
frequency front-ends, $N_r$, is either $N_r = 1$ or $N_r = 2$. These two setups find practical application to wrist-worn LEDs, e.g., smart watches, since the average circumference of a human wrist is about 14-20 cm. The LEDs are equipped with separate units for ID and EH. To shed light on the impact of receiver diversity, five schemes for SWIPT are studied and compared.

- **Power Splitting (PS).** $N_r = 1$ is assumed and the received power, $P_{RX}$, is split in two parts, according to a power splitting ratio $0 \leq \rho \leq 1$: $P_{EH} = \rho P_{RX}$ is used for EH and $P_{ID} = P_{RX} - P_{EH} = (1 - \rho) P_{RX}$ is used for ID.

- **Power Splitting with Maximum Ratio Combining (PS-MRC).** $N_r = 2$ is assumed and the signals of the two receive antennas are combined according to the MRC scheme. The power after combining, $P_{RX}$, is split in two parts, according to a power splitting ratio $0 \leq \rho \leq 1$: $P_{EH} = \rho P_{RX}$ is used for EH and $P_{ID} = P_{RX} - P_{EH} = (1 - \rho) P_{RX}$ is used for ID.

- **Separate Antenna Receiver (SAR).** $N_r = 2$ is assumed and the received power of the first and second receive antenna is sent, without loss of generality, to the input of the ID and EH unit, respectively. The two antennas can be used for ID and EH interchangeably.

- **ID-Prioritized Selection Combining (ID-SC).** $N_r = 2$ is assumed and the received power of the antenna providing the best and the worst channel power gain is sent to the input of the ID and EH unit, respectively.

- **EH-Prioritized Selection Combining (EH-SC).** $N_r = 2$ is assumed and the received power of the antenna providing the best and the worst channel power gain is sent to the input of the EH and ID unit, respectively.

The proposed study can be generalized for application to SWIPT implementations based on the time switching scheme [94]. For brevity, this case study is not analyzed in this chapter.
3.4 Problem Statement

Considered individually, the performance of ID and EH units is usually quantified in terms of information rate and harvested power, respectively. Let $R$ and $Q$ denote the Shannon rate (in bits/sec) of the ID unit and the harvested power (in Watts) of the EH unit, respectively. As for the five SWIPT schemes introduced in Section 3.3.5, $R$ and $Q$ can be formulated as follows:

$$
R = B_w \log_2 \left( 1 + \frac{PG(0)U_{ID}(0)}{PL_{ID}(L(0)) + \sigma^2 + \sigma^2_{ID}/\rho_{ID}} \right),
$$

$$
Q = \rho_{EH} \zeta \left( PG(0)U_{EH}(0)/L(0) + PI_{EH}(L(0)) \right),
$$

where the notation in Table 3.1 is used, and, for $z \in \{\text{ID, EH}\}$, $0 \leq \rho_z \leq 1$ accounts for the amount of power at the input of ID and EH units, $U_{z}(0)$ is the power gain of the intended link, and $I_z(\cdot)$ is the aggregate other-cell interference defined as follows:

$$
I_z(L(0)) = \sum_{s \in \{\text{LOS, NLOS}\}} \sum_{i \in \Psi_s} G_{z}(i) \gamma_{z}(i) I_s(r^{(i)}) \mathbb{1}(r^{(i)}>L(0)),
$$

where $\gamma_{z}(i)$ is the power gain of the $i$th interfering BS. The definition and the distribution of the parameters $U_{z}(0)$, $\gamma_{z}(i)$ and $\rho_z$ are summarized in Table 3.2.

Remark 2 Based on Table 3.2, $I_{ID}(\cdot) = I_{EH}(\cdot)$ for PS and PS-MRC schemes, but $I_{ID}(\cdot) \neq I_{EH}(\cdot)$ for SAR, ID-SC and EH-SC schemes. As far as the latter three SWIPT schemes are concerned, however, $I_{ID}(\cdot) \overset{d}{=} I_{EH}(\cdot)$. More precisely, $I_{ID}(\cdot)$ and $I_{EH}(\cdot)$ are partially correlated RVs because the locations of the interfering BSs are the same but the power channel gains are related to different receive antennas. For all SWIPT schemes, nevertheless, the distribution of the aggregate other-cell interference is the same, i.e., $\Phi_{I_{ID}}(\cdot) = \Phi_{I_{EH}}(\cdot) = \Phi_{I}(\cdot)$.

3.4.1 Motivation: On the Benefits of Receiver Diversity

To better motivate our research, let us compare PS and SAR schemes based on (3.4). From Table 3.2 and Remark 2, we evince that the power gains of the intended link and the
Table 3.2: Definition and distribution of $U_{z}^{(0)}$, $\gamma^{(i)}$ and $\rho$ in (3.4) for the five SWIPT schemes in Section 3.3.5. $\gamma^{(0)}\sim \mathcal{E}(1)$ and $\gamma^{(i)}\sim \mathcal{E}(1)$ are the channel power gains of intended and $i$th interfering BSs at the $r$th receive antenna; $\gamma_{\text{MRC}}^{(i)}$, $\gamma_{\text{max}}^{(i)}$, and $\gamma_{\text{min}}^{(i)}$ are the channel power gains of the $i$th interfering BS after applying MRC and impinging on the best and worst (as far as the the probe link is concerned) receive antennas, respectively. Short-hands: psc = power splitting circuit, 2rx = two receive antennas, asc = antenna switching circuit, imp = implementation.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>$U_{z}^{(0)}$</th>
<th>$U_{\text{EH}}^{(0)}$</th>
<th>$\gamma^{(i)}_{\text{ID}}$</th>
<th>$\gamma^{(i)}_{\text{EH}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS</td>
<td>$\gamma_{1}^{(0)} \sim \mathcal{E}(1)$</td>
<td>$\gamma_{1}^{(0)} \sim \mathcal{E}(1)$</td>
<td>$\gamma^{(i)}_{\text{ID}} \sim \mathcal{E}(1)$</td>
<td>$\gamma^{(i)}_{\text{EH}} \sim \mathcal{E}(1)$</td>
</tr>
<tr>
<td>PS-MRC</td>
<td>$\gamma_{1}^{(0)} + \gamma_{2}^{(0)} \sim \mathcal{G}(2,1)$</td>
<td>$\gamma_{1}^{(0)} + \gamma_{2}^{(0)} \sim \mathcal{G}(2,1)$</td>
<td>$\gamma^{(i)}_{\text{MRC}} \sim \mathcal{E}(1)$</td>
<td>$\gamma^{(i)}_{\text{MRC}} \sim \mathcal{E}(1)$</td>
</tr>
<tr>
<td>SAR</td>
<td>$\gamma_{1}^{(0)} \sim \mathcal{E}(1)$</td>
<td>$\gamma_{2}^{(0)} \sim \mathcal{E}(1)$</td>
<td>$\gamma^{(i)}_{\text{ID}} \sim \mathcal{E}(1)$</td>
<td>$\gamma^{(i)}_{\text{EH}} \sim \mathcal{E}(1)$</td>
</tr>
<tr>
<td>ID-SC</td>
<td>$\max{\gamma_{1}^{(0)}, \gamma_{2}^{(0)}} \sim \mathcal{E}_{\text{max}}(1)$</td>
<td>$\min{\gamma_{1}^{(0)}, \gamma_{2}^{(0)}} \sim \mathcal{E}(2)$</td>
<td>$\gamma^{(i)}_{\text{id}} \sim \mathcal{E}(1)$</td>
<td>$\gamma^{(i)}_{\text{min}} \sim \mathcal{E}(1)$</td>
</tr>
<tr>
<td>EH-SC</td>
<td>$\min{\gamma_{1}^{(0)}, \gamma_{2}^{(0)}} \sim \mathcal{E}(2)$</td>
<td>$\max{\gamma_{1}^{(0)}, \gamma_{2}^{(0)}} \sim \mathcal{E}_{\text{max}}(1)$</td>
<td>$\gamma^{(i)}_{\text{min}} \sim \mathcal{E}(1)$</td>
<td>$\gamma^{(i)}_{\text{max}} \sim \mathcal{E}(1)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scheme</th>
<th>$\rho_{\text{ID}}$</th>
<th>$\rho_{\text{EH}}$</th>
<th>Imp</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS</td>
<td>$1 - \rho$</td>
<td>$\rho$</td>
<td>psc</td>
</tr>
<tr>
<td>PS-MRC</td>
<td>$1 - \rho$</td>
<td>$\rho$</td>
<td>psc + 2rx</td>
</tr>
<tr>
<td>SAR</td>
<td>1</td>
<td>1</td>
<td>2rx</td>
</tr>
<tr>
<td>ID-SC</td>
<td>1</td>
<td>1</td>
<td>2rx + asc</td>
</tr>
<tr>
<td>EH-SC</td>
<td>1</td>
<td>1</td>
<td>2rx + asc</td>
</tr>
</tbody>
</table>

aggregate other-cell interferences of both schemes are equivalent in distribution, e.g., they have the same PDF and CF. As for the PS scheme, only a fraction of the received power is used by ID and EH units. As for the SAR scheme, on the other hand, the complete received power is available at the input of both ID and EH units. As a result, $R^{(\text{SAR})} \geq R^{(\text{PS})}$ and $Q^{(\text{SAR})} \geq Q^{(\text{PS})}$ simultaneously, i.e., the SAR scheme always outperforms the PS scheme. This is obtained because $N_r = 1$ and $N_r = 2$ for PS and SAR schemes, respectively. From the implementation standpoint, the PS scheme requires appropriate circuits for splitting the received power while the SAR scheme avoids them by leveraging the availability of two receive antennas. This example shows the potential of using, whenever possible, multiple antennas at the LEDs. It highlights, in addition, the associated performance versus implementation trade-off, e.g., the possibility of replacing power splitters with an additional antenna element and radio frequency front-end. As far as the computational (signal processing) complexity of PS and SAR schemes is concerned, it is apparent that it is the same, since ID and EH receivers perform the same operations in both cases.

Unlike the PS and SAR schemes, the comparison of the other schemes deserves more attention because of the different distribution of the channel power gain of the intended link, of the different power at the input of ID and EH units, and of the partial correlation...
of the aggregate other-cell interference. Quantifying the performance gain of each scheme compared to the others is, however, important because of the different implementation complexities, which are briefly summarized in Table 3.2. Assessing these trade-offs is the ultimate objective of this chapter.

### 3.4.2 Problem Formulation

As far as the LEDs as a whole are concerned, the trade-off between information rate and harvested power is quantified in terms of the J-CCDF of $R$ and $Q$ defined in (3.4) [94]:

$$F(R_*, Q_*) = \Pr \{ R \geq R_*, Q \geq Q_* \}, \quad (3.6)$$

where $R_*$ and $Q_*$ are the minimum bit rate and harvested power, respectively, needed for the LEDs to perform their tasks.

In the next sections, we compare the five SWIPT schemes introduced in Section 3.3.5 in terms of their J-CCDF and provide mathematical frameworks that allow us to optimize them and to decide the best scheme to use as a function of $R_*$ and $Q_*$.}

### 3.5 Performance Comparison and Trends

To facilitate the comparison among the SWIPT schemes, we start by introducing three remarks.

**Remark 3** By using the Frechet inequality of the probability of logical conjunctions [106], the J-CCDF is upper-bounded as:

$$F(R_*, Q_*) \leq F_{UB}(R_*, Q_*)$$

$$= \min \{ \Pr \{ R \geq R_* \}, \Pr \{ Q \geq Q_* \} \}. \quad (3.7)$$

By direct inspection, it is apparent that the upper-bound in (3.7) is asymptotically tight for every $Q_*$ if $R_* \to 0$ or $R_* \to \infty$ and for every $R_*$ if $Q_* \to 0$ or $Q_* \to \infty$. If $R_* \ll 1$ or $Q_* \gg 1$, the system operates in the EH-limited regime, i.e., $F(R_*, Q_*) \approx \Pr \{ Q \geq Q_* \}$. If $R_* \gg 1$ or $Q_* \ll 1$, the system operates in the ID-limited regime, i.e., $F(R_*, Q_*) \approx$
Remark 4 The J-CCDF of PS and PS-MRC schemes depends on $\rho$, i.e., $\Pr\{R \geq R_*\} = \chi_{\text{ID}}(\rho)$ and $\Pr\{Q \geq Q_*\} = \chi_{\text{EH}}(\rho)$. In particular, $\chi_{\text{ID}}(\cdot)$ and $\chi_{\text{EH}}(\cdot)$ are monotonically decreasing and increasing functions of $\rho$, respectively, and, by definition, $\chi_{\text{ID}}(0) \leq 1$, $\chi_{\text{ID}}(1) = 0$ and $\chi_{\text{EH}}(0) = 0$, $\chi_{\text{EH}}(1) \leq 1$. Based on these properties and on the upper-bound in (3.7), we evince that an optimal value of $\rho$, $\rho_{\text{opt}}$, that maximizes the J-CCDF exists and that it is the unique solution of the equation $\chi_{\text{ID}}(\rho_{\text{opt}}) = \chi_{\text{EH}}(\rho_{\text{opt}})$. Since (3.7) is a upper-bound, however, the optimal value of $\rho$ that maximizes the exact J-CCDF may be different from the solution of the latter equation. The optimal power splitting ratios computed by using the exact J-CCDF and the upper-bound in (3.7) are compared in Section 3.7 for some relevant case studies. Still based on (3.7), we evince that the J-CCDF satisfies the property that, as a function of $\rho$, the equation $\chi(\rho) = \tau$, where $\chi(\rho) = \min\{\chi_{\text{ID}}(\rho), \chi_{\text{EH}}(\rho)\}$ and $0 \leq \tau \leq 1$ is a constant value, has at least one solution if $\tau \leq \chi(\rho_{\text{opt}})$ and no solution if $\tau > \chi(\rho_{\text{opt}})$.

Remark 5 Based on Remark 4, the upper-bound of the J-CCDF of PS and PS-MRC schemes attains its maximum if $\rho = \rho_{\text{opt}}$, where $\rho_{\text{opt}}$ is the solution of the equation $\chi_{\text{ID}}(\rho_{\text{opt}}) = \chi_{\text{EH}}(\rho_{\text{opt}})$. Since $\chi_{\text{ID}}(\cdot)$ and $\chi_{\text{EH}}(\cdot)$ are monotonically decreasing and increasing functions of $\rho$, respectively, this implies that $\chi_{\text{EH}}(\rho) \leq \chi_{\text{ID}}(\rho)$ if $\rho \leq \rho_{\text{opt}}$ and $\chi_{\text{ID}}(\rho) \leq \chi_{\text{EH}}(\rho)$ if $\rho \geq \rho_{\text{opt}}$. Thus, the system operates in the EH-limited regime if $\rho \leq \rho_{\text{opt}}$, i.e., $\chi(\rho) = \min\{\chi_{\text{ID}}(\rho), \chi_{\text{EH}}(\rho)\} = \chi_{\text{EH}}(\rho)$ and in the ID-limited regime if $\rho \geq \rho_{\text{opt}}$, i.e., $\chi(\rho) = \min\{\chi_{\text{ID}}(\rho), \chi_{\text{EH}}(\rho)\} = \chi_{\text{ID}}(\rho)$, respectively.

In Section 3.4.1, we have shown that the SAR scheme always outperforms the PS scheme. In the next sub-sections, we explicitly compare the other SWIPT schemes. For ease of presentation, the following notation is used: $\mathcal{F}_{\text{ID}}(R_*) = \Pr\{R \geq R_*\}$ and $\mathcal{F}_{\text{EH}}(Q_*) = \Pr\{Q \geq Q_*\}$. Our derivations and conclusions are based on the upper-bound of the J-CCDF in (3.7).

### 3.5.1 SAR versus ID-SC

#### Lemma 1
Let us assume that $R_*$ is given. The ID-SC scheme outperforms the SAR scheme, i.e., $\mathcal{F}_{\text{UB}}^{(\text{ID-SC})}(R_*, Q_*) \geq \mathcal{F}_{\text{UB}}^{(\text{SAR})}(R_*, Q_*)$, if $Q_* < Q_*^{(c)}$, where $Q_*^{(c)}$ is the unique
solution of the equation $F_{\text{ID}}^{(\text{SAR})}(R_*) = F_{\text{EH}}^{(\text{ID}-\text{SC})}(Q_*^{(c)})$. On the other hand, the SAR scheme outperforms the ID-SC scheme if $Q_* > Q_*^{(c)}$. Let us assume that $Q_*$ is given. The ID-SC scheme outperforms the SAR scheme if $R_* > R_*^{(c)}$, where $R_*^{(c)}$ is the unique solution of the equation $F_{\text{EH}}^{(\text{ID}-\text{SC})}(Q_*) = F_{\text{ID}}^{(\text{SAR})}(R_*^{(c)})$. On the other hand, the SAR scheme outperforms the ID-SC scheme if $R_* < R_*^{(c)}$.

Proof: It follows from (3.7), since: i) $F_{\text{ID}}^{(\text{ID}-\text{SC})}(R_*) \geq F_{\text{ID}}^{(\text{SAR})}(R_*)$ for every $R_*$, since max $\{a,b\} \geq a$ and max $\{a,b\} \geq b$ for every $(a,b)$; ii) $F_{\text{EH}}^{(\text{ID}-\text{SC})}(Q_*) \leq F_{\text{EH}}^{(\text{SAR})}(Q_*)$ for every $Q_*$, since min $\{a,b\} \leq a$ and min $\{a,b\} \leq b$ for every $(a,b)$; iii) $F_{\text{ID}}(\cdot)$ is independent of $Q_*$ and monotonically decreasing with $R_*$; and iv) $F_{\text{EH}}(\cdot)$ is independent of $R_*$ and monotonically decreasing with $Q_*$. □

From Lemma 1, we conclude that the ID-SC scheme outperforms the SAR scheme for large values of $R_*$ and for small values of $Q_*$, i.e., if the system operates in the ID-limited regime.

### 3.5.2 SAR versus EH-SC

**Lemma 2** Let us assume that $R_*$ is given. The EH-SC scheme outperforms the SAR scheme, i.e., $F_{\text{UB}}^{(\text{EH}-\text{SC})}(R_*, Q_*) \geq F_{\text{UB}}^{(\text{SAR})}(R_*, Q_*)$, if $Q_* > Q_*^{(c)}$, where $Q_*^{(c)}$ is the unique solution of the equation $F_{\text{EH}}^{(\text{ID}-\text{SC})}(Q_*) = F_{\text{ID}}^{(\text{SAR})}(R_*^{(c)})$. On the other hand, the SAR scheme outperforms the EH-SC scheme if $Q_* < Q_*^{(c)}$. Let us assume that $Q_*$ is given. the EH-SC scheme outperforms the SAR scheme if $R_* < R_*^{(c)}$, where $R_*^{(c)}$ is the unique solution of the equation $F_{\text{EH}}^{(\text{SAR})}(Q_*) = F_{\text{EH}}^{(\text{ID}-\text{SC})}(R_*^{(c)})$. On the other hand, the SAR scheme outperforms the EH-SC scheme if $R_* > R_*^{(c)}$.

Proof: It follows from the upper-bound in (3.7), by using the same line of thought as that of the proof of Lemma 1. □

From Lemma 2, we conclude that the EH-SC scheme outperforms the SAR scheme for large values of $Q_*$ and for small values of $R_*$, i.e., if the system operates in the EH-limited regime.

### 3.5.3 ID-SC versus EH-SC

**Lemma 3** Let us assume that $R_*$ is given. The ID-SC scheme outperforms the EH-SC scheme, i.e., $F_{\text{UB}}^{(\text{ID}-\text{SC})}(R_*, Q_*) \geq F_{\text{UB}}^{(\text{EH}-\text{SC})}(R_*, Q_*)$, if $Q_* < Q_*^{(c)}$, where $Q_*^{(c)}$ is the unique
solution of the equation $F_{\text{ID}}^{(EH-SC)}(R_*) = F_{\text{EH}}^{(ID-SC)}(Q_*^{(c)})$. On the other hand, the EH-SC scheme outperforms the ID-SC scheme if $Q_* > Q_*^{(c)}$. Let us assume that $Q_*$ is given. The ID-SC scheme outperforms the EH-SC scheme if $R_* > R_*^{(c)}$, where $R_*^{(c)}$ is the unique solution of the equation $F_{\text{EH}}^{(ID-SC)}(Q_*) = F_{\text{ID}}^{(EH-SC)}(R_*^{(c)})$. On the other hand, the EH-SC scheme outperforms the ID-SC scheme if $R_* < R_*^{(c)}$.

Proof: It follows from the upper-bound in (3.7), by using the same line of thought as that of the proof of Lemma 1 and by noting that $\max\{a, b\} \geq \min\{a, b\}$ for every $(a, b)$. □

From Lemma 3, we conclude that the ID-SC scheme is to be preferred to the EH-SC scheme if the system operates in the ID-limited regime. If the system operates in the EH-limited regime, on the other hand, the EH-SC scheme is to be preferred.

Remark 6 From Lemmas 1-3, three main conclusions can be drawn: 1) there is no SWIPT scheme among the SAR, ID-SC and EH-SC schemes that outperforms the others for every pair $(R_*, Q_*)$; 2) the SAR scheme is a special case of a generalized SWIPT scheme that is obtained by choosing, as a function of $(R_*, Q_*)$, the best scheme between the ID-SC and the EH-SC schemes; and 3) given the reliability constraints $(R_*, Q_*)$, the performance of SWIPT-enabled cellular networks can be optimized by enabling the LEDs to adaptively use, as a function of $(R_*, Q_*)$, either the ID-SC scheme or the EH-SC scheme.

Based on Remark 6, we introduce a new SWIPT scheme that is referred to as Adaptive Selection Combining (A-SC), which subsumes SAR, ID-SC and EH-SC schemes and foresees that the LEDs use the best SWIPT scheme, between ID-SC and EH-SC, as a function of $(R_*, Q_*)$. More specifically, the LEDs operate by using either the ID-SC or the EH-SC scheme according to the switching points $Q_*^{(c)}$ and $R_*^{(c)}$ introduced in Lemmas 1-3. The practical implementation of this adaptive scheme is elaborated at the end of Section 3.5.6.

### 3.5.4 PS-MRC versus A-SC

Lemma 4 Let us assume that the PS-MRC scheme operates at its optimum $\rho_{\text{opt}}$, which is obtained by maximizing the J-CCDF as a function of $\rho$. The PS-MRC scheme outperforms the A-SC scheme if they both operate either in the ID-limited regime or in the EH-limited...
3.5. Performance Comparison and Trends

Proof: In the ID-limited regime and EH-limited regime, we have $F_{UB}^{(PS-MRC)}(R_*, Q_*) \approx F_{ID}^{(PS-MRC)}(R_*)$ and $F_{UB}^{(PS-MRC)}(R_*, Q_*) \approx F_{EH}^{(PS-MRC)}(Q_*)$, respectively. This implies that $\rho_{opt} = 0$ and $\rho_{opt} = 1$ in the ID-limited regime and in the EH-limited regime, respectively. In addition, the A-SC scheme reduces to the ID-SC scheme and to the EH-SC scheme in the ID-limited regime and in the EH-limited regime, respectively. The proof follows from the upper-bound in (3.7), by noting that $a + b \geq \max\{a, b\}$ for every non-negative $(a, b)$.

□

Lemma 5 Let us assume that the PS-MRC scheme operates at its optimum $\rho_{opt}$, which is obtained by maximizing the J-CCDF as a function of $\rho$. For a given pair $(R_*, Q_*)$, the PS-MRC scheme outperforms the A-SC scheme if, as a function of $\rho$, the equation $F_{UB}^{(PS-MRC)}(R_*, Q_; \rho) = F_{UB}^{(A-SC)}(R_*, Q_*)$ admits at least one solution for $0 \leq \rho \leq 1$. If the equation admits no solution, on the other hand, the A-SC scheme outperforms the PS-MRC scheme.

Proof: It follows from (3.7) and Remark 4. □

3.5.5 PS-MRC versus PS

Lemma 6 Let us assume that the PS-MRC scheme and the PS scheme operate by using the same $\rho$. The PS-MRC scheme outperforms the PS scheme for every $(R_*, Q_*)$.

Let us assume, on the other hand, that the PS-MRC scheme and the PS scheme operate at their respective optima $\rho_{opt}^{(PS-MRC)}$ and $\rho_{opt}^{(PS)}$, which are obtained by finding the maximum, as a function of $\rho$, of $F_{UB}^{(PS-MRC)}(R_*, Q_; \rho)$ and $F_{UB}^{(PS)}(R_*, Q_; \rho)$, respectively. The PS-MRC scheme outperforms the PS scheme for every $(R_*, Q_*)$.

Proof: Let us assume that $(R_*, Q_*)$ is given. Let us use the same notation as in Remark 4, i.e., $F_{UB}^{(PS-MRC)}(R_*, Q_; \rho) \leq \chi^{(PS-MRC)}(\rho) = \min\{\chi^{(PS-MRC)}_{ID}(\rho), \chi^{(PS-MRC)}_{EH}(\rho)\}$ and $F_{UB}^{(PS)}(R_*, Q_; \rho) \leq \chi^{(PS)}(\rho) = \min\{\chi^{(PS)}_{ID}(\rho), \chi^{(PS)}_{EH}(\rho)\}$. Since $a + b \geq a$ and $a + b \geq b$ for every non-negative $(a, b)$, we obtain $\chi^{(PS-MRC)}_{ID}(\rho) \geq \chi^{(PS)}_{ID}(\rho)$ and $\chi^{(PS-MRC)}_{EH}(\rho) \geq \chi^{(PS)}_{EH}(\rho)$ for every $0 \leq \rho \leq 1$. This proves the first part.

From the upper-bound in (3.7) and Remark 4, $\rho_{opt}^{(PS-MRC)}$ and $\rho_{opt}^{(PS)}$ satisfy the equalities $\chi^{(PS-MRC)}_{ID}(\rho_{opt}^{(PS-MRC)}) = \chi^{(PS-MRC)}_{EH}(\rho_{opt}^{(PS-MRC)})$ and $\chi^{(PS)}(\rho_{opt}^{(PS)}) = \chi^{(PS)}(\rho_{opt}^{(PS)})$. 

respectively. Accordingly, the following holds:

\[
F^{(\text{PS-MRC})}(\mathcal{R}_*, Q_*; \rho_{\text{opt}}^{(\text{PS-MRC})}) \\
\leq \chi^{(\text{PS-MRC})}(\rho_{\text{opt}}^{(\text{PS-MRC})}) \\
= \chi_{\text{ID}}^{(\text{PS-MRC})}(\rho_{\text{opt}}^{(\text{PS-MRC})}) \\
\geq \chi_{\text{EH}}(\rho) \geq \chi_{\text{ID}}(\rho) \\
= \chi^{(\text{PS})}(\rho) \geq F^{(\text{PS})}(\mathcal{R}_*, Q_*; \rho),
\]

where (a) follows from the equality \( \chi^{(\text{PS-MRC})}(\rho_{\text{opt}}^{(\text{PS-MRC})}) = \chi_{\text{EH}}(\rho) \), (b) holds for \( \rho \leq \rho_{\text{opt}}^{(\text{PS-MRC})} \) since \( \chi_{\text{EH}}(\cdot) \) is a monotonically increasing function of \( \rho \), (c) follows from the first part of the proof, and (d) holds for \( \rho \leq \rho_{\text{opt}}^{(\text{PS})} \) since \( \chi^{(\text{PS})}(\rho) \leq \chi^{(\text{PS})}_{\text{ID}}(\rho) \) if \( \rho \leq \rho_{\text{opt}}^{(\text{PS})} \) (see Remark 5 for the details). By using a similar line of thought, the following holds:

\[
F^{(\text{PS-MRC})}(\mathcal{R}_*, Q_*; \rho_{\text{opt}}^{(\text{PS-MRC})}) \\
\leq \chi^{(\text{PS-MRC})}(\rho_{\text{opt}}^{(\text{PS-MRC})}) \\
= \chi_{\text{ID}}^{(\text{PS-MRC})}(\rho_{\text{opt}}^{(\text{PS-MRC})}) \\
\geq \chi_{\text{ID}}^{(\text{PS})}(\rho) \geq \chi_{\text{ID}}^{(\text{PS})}(\rho) \\
= \chi^{(\text{PS})}(\rho) \geq F^{(\text{PS})}(\mathcal{R}_*, Q_*; \rho),
\]

for \( \rho \geq \rho_{\text{opt}}^{(\text{PS-MRC})} \) and \( \rho \geq \rho_{\text{opt}}^{(\text{PS})} \).

From (3.8) and (3.9), in conclusion, we obtain the inequality \( F^{(\text{PS-MRC})}(\mathcal{R}_*, Q_*; \rho_{\text{opt}}^{(\text{PS-MRC})}) \geq F^{(\text{PS})}(\mathcal{R}_*, Q_*; \rho) \) if \( \rho \leq \min \left\{ \rho_{\text{opt}}^{(\text{PS-MRC})}, \rho_{\text{opt}}^{(\text{PS})} \right\} \) or \( \rho \geq \max \left\{ \rho_{\text{opt}}^{(\text{PS-MRC})}, \rho_{\text{opt}}^{(\text{PS})} \right\} \). This implies that, for every \( \left( \rho_{\text{opt}}^{(\text{PS-MRC})}, \rho_{\text{opt}}^{(\text{PS})} \right) \) and \( (\mathcal{R}_*, Q_*) \), the PS-MRC scheme outperforms the PS scheme if they operate at their respective optima as a function of \( \rho \).

Lemma 6 establishes that PS-MRC is superior to PS if either they use the same power splitting ratio or they operate at their respective optimal power splitting ratios. For arbitrary values of their power splitting ratios, on the other hand, no general conclusion can be drawn. Numerical examples are illustrated in Section 3.7 for some relevant case studies.
3.5.6 Main Performance Trends

From the comparative analysis of the SWIPT schemes, three main conclusions can be drawn.

1. There exists a unique value of $\rho$ that optimizes the performance of PS and PS-MRC schemes. PS and PS-MRC schemes that operate at their respective optima are referred to as Optimum PS (OPS) and Optimum PS-MRC (OPS-MRC) schemes.

2. For every pair $(R_*, Q_*)$, i) the A-SC scheme outperforms the SAR scheme, ii) the SAR scheme outperforms the PS scheme, iii) the PS-MRC scheme outperforms the PS scheme if $\rho$ is the same, iv) and the OPS-MRC scheme outperforms the OPS scheme. This implies that A-SC outperforms PS.

3. The OPS-MRC scheme outperforms or underperforms the A-SC scheme depending on the pair $(R_*, Q_*)$ and on the specific setup of parameters being considered (see Lemma 5).

In order to optimize the J-CCDF, our analysis suggests that the LEDs need to operate in an adaptive fashion by choosing the SWIPT scheme to use as a function of $(R_*, Q_*)$. Depending on the application, in general, each LED may have different $(R_*, Q_*)$ requirements. Concretely, an adaptive SWIPT scheme can be implemented as follows. Assume that mathematical frameworks for the J-CCDF of ID-SC, EH-SC and PS-MRC schemes are available in a tractable and computable form. For any $(R_*, Q_*)$ of interest, either the BSs or the LEDs (depending on their computational complexity capabilities) adaptively choose the best SWIPT scheme to use as follows: 1) the power splitting coefficient that optimizes the J-CCDF of the PS-MRC scheme is estimated and the related optimal J-CCDF is computed, 2) the J-CCDF of the ID-SC and EH-SC schemes are computed, 3) the LEDs use the SWIPT scheme that provides the best J-CCDF among the three. If the BSs perform these tasks, they need to forward the related information to their intended LEDs. This usually requires just a few control bits (two in the considered setup). A system that operates according to this adaptive policy is referred to as Adaptive SWIPT (A-SWIPT). To implement the A-SWIPT scheme in practice, mathematical expressions
of the J-CCDF of all SWIPT schemes studied in this chapter are provided in the next section.

It is worth emphasizing that, by definition, the J-CCDF in (3.6) is obtained by computing the average with respect to the spatial topology of the cellular network and with respect to the channel statistics. This implies that the considered (system-level) optimization can be performed off-line with a minimum overhead, since it depends on system parameters that are slowly-varying, e.g., that usually change on a geographical scale. For example, it can be performed at the beginning of each communication round and if the performance requirements \((R_*, Q_*)\) change.

### 3.6 System-Level Analysis

The following two propositions provide mathematical expressions of the J-CCDF for the SWIPT schemes introduced in Section 3.3.5. **Proposition 4** is exact and is applicable only to PS and PS-MRC schemes. **Proposition 5** is based on the upper-bound in (3.7) and is applicable to all SWIPT schemes. An exact mathematical framework for SAR, ID-SC and EH-SC schemes may be obtained by using the multi-dimensional inversion theorem in [107]. The resulting framework, however, would be formulated in terms of multi-fold integrals with limited mathematical and numerical tractability. For this reason, it is not explicitly considered in this chapter. Some hints on how to exploit the multi-dimensional inversion theorem for computing the J-CCDF are reported in Appendix 3.9. The mathematical complexity of studying SAR, ID-SC and EH-SC schemes compared with PS and PS-MRC schemes originates from the fact that the aggregate other-cell interferences of ID and EH units are dissimilar and are only equivalent in distribution (see Remark 2).

**Proposition 4** Let \(f_{L^{(0)}}(\cdot)\) be the PDF of the smallest path-loss, \(L^{(0)}\), defined in (3.2):

\[
f_{L^{(0)}}(x) = \hat{\Lambda}([0,x]) \exp(-\Lambda([0,x])),
\]

where \(\Lambda([0,x]) = \Lambda_{\text{LOS}}([0,x]) + \Lambda_{\text{NLOS}}([0,x])\) is the intensity measure of the PPP of the path-losses [94], \(\hat{\Lambda}([0,x]) = \hat{\Lambda}_{\text{LOS}}([0,x]) + \hat{\Lambda}_{\text{NLOS}}([0,x])\) is its first derivative computed with respect to \(x\), and, for \(s \in \{\text{LOS, NLOS}\}\), \(\Lambda_s([\cdot, \cdot])\) and \(\hat{\Lambda}_s([\cdot, \cdot])\) are the intensity
measure and its first derivative, respectively, of the PPP of the path-losses in state \( s \) defined as:

\[
\Lambda_s ([0, x]) = \pi \lambda q_s^{0, D} \left( \frac{x}{\kappa_0} \right)^{2/\beta_s} \mathcal{H} (x - \kappa_0 D^{\beta_s}) \\
+ \pi \lambda \left( \frac{x}{\kappa_0} \right)^{2/\beta_s} q_s^{D, \infty} + D^2 (q_s^{0, D} - q_s^{D, \infty})
\times \mathcal{H} (x - \kappa_0 D^{\beta_s}),
\]

(3.11)

\[
\hat{\Lambda}_s ([0, x]) = \left( \frac{2 \pi \lambda}{\beta_s} q_s^{0, D} \kappa_0^{-2/\beta_s} x^{(2/\beta_s - 1)} \mathcal{H} (x - \kappa_0 D^{\beta_s}) \right) \\
+ \left( \frac{2 \pi \lambda}{\beta_s} q_s^{D, \infty} \kappa_0^{-2/\beta_s} x^{(2/\beta_s - 1)} \mathcal{H} (x - \kappa_0 D^{\beta_s}) \right).
\]

(3.12)

Let \( \Phi_I (\cdot | L^{(0)}) \) be the CF of the other-cell interference, \( I \), conditioned on \( L^{(0)} \) given in (3.5):

\[
\Phi_I (\omega | L^{(0)}) = \Phi_I (\omega | L^{(0)}; \text{LOS}) \Phi_I (\omega | L^{(0)}; \text{NLOS}),
\]

(3.13)

where, for \( s \in \{ \text{LOS, NLOS} \} \), \( \Phi_I (\cdot | L^{(0)}; s) \) is, conditioned on \( L^{(0)} \), the CF of the other-cell interference of all links in state \( s \) is given as

\[
\Phi_I (\omega | L^{(0)}; s) = \exp \left( \pi \lambda q_s^{D, \infty} \max \left\{ D^2, \left( \frac{L^{(0)}}{\kappa_0} \right)^{2/\beta_s} \right\} (1 - \Upsilon_s (\omega, \max \{ \kappa_0 D^{\beta_s}, L^{(0)} \})) \right) \\
\times \exp \left( \pi \lambda q_s^{0, D} \left[ \left( \frac{L^{(0)}}{\kappa_0} \right)^{2/\beta_s} (1 - \Upsilon_s (\omega, L^{(0)})) - D^2 (1 - \Upsilon_s (\omega, \kappa_0 D^{\beta_s})) \right] \right) \\
\times \mathcal{H} (L^{(0)} - \kappa_0 D^{\beta_s}),
\]

(3.14)

and \( \Upsilon_s (\cdot, \cdot) \) is the following short-hand:

\[
\Upsilon_s (\omega, Z) = \frac{\theta_M}{2 \pi} 2F_1 \left( 1, -\frac{2}{\beta_s}, 1 - \frac{2}{\beta_s}, \frac{j \omega}{Z} \mathcal{G}_{M} \right) \\
+ \left( 1 - \frac{\theta_M}{2 \pi} \right) 2F_1 \left( 1, -\frac{2}{\beta_s}, 1 - \frac{2}{\beta_s}, \frac{j \omega}{Z} \mathcal{G}_{S} \right).
\]

(3.15)

Then, the J-CCDF for both the PS and PS-MRC schemes is:

\[
\overline{F} (R_s, Q_s) = \int_0^\infty \int_0^\infty \frac{1}{\pi \omega} \text{Im} \left\{ J (\omega, y) \Phi_I \left( \omega | G^{(0)} \right) \right\} f_{L^{(0)}} (y) d \omega dy,
\]

(3.16)
where, for simplicity, the following short-hand is introduced:

\[
J(\omega, y) = \exp\left(-j\omega \frac{q_y}{PG(0)}\right) \left(1 - \frac{j\omega}{y}\right)^{-1+u} \\
\times \Gamma \left(1 + u, \frac{T_y}{PG(0)} (y - j\omega)\right) \\
- \exp\left(j\omega \frac{\sigma_y^2}{PG(0)}\right) \left(1 + \frac{j\omega r_y}{y}\right)^{-1+u} \\
\times \Gamma \left(1 + u, \frac{T_y}{PG(0)} (y + j\omega r_y)\right),
\]

(3.17)

and \(u = 0\) for PS and \(u = 1\) for PS-MRC schemes, respectively.

Proof: Since \(U(0)^{ID} = U(0)^{EH} = U(0)^{PS}\) (see Table 3.2), from (3.4) the J-CCDF can be formulated as in (3.18) as follows

\[
F(R^*, Q^*) = \Pr\{R \geq R^*, Q \geq Q^*\} \\
= 0 \quad \text{if } G(0)^{U(0)} r_y/L(0) - \sigma_y^2/P \leq I \leq G(0)^{U(0)} r_y/L(0) - \sigma_y^2/P \quad \text{if } G(0)^{U(0)} \geq (T_y/P) L(0) \]

\[
= E_{L(0)} \left\{ \int_{(T_y/P)(L(0)/G(0))}^{\infty} F_I \left(G(0)^{x} r_y/L(0) - \sigma_y^2/P \right| L(0)) f_{L(0)}(x) dx \right\} \\
- E_{L(0)} \left\{ \int_{(T_y/P)(L(0)/G(0))}^{\infty} F_I \left(-G(0)^{x}/L(0) + q_y/P \right| L(0)) f_{L(0)}(x) dx \right\},
\]

(3.18)

where \(F_I(x) = \Pr\{I < x\}\) is the CDF of \(I\). With the aid of the Gil-Pelaez theorem [36, Eq. (16)], \(F_I(\cdot)\) can be formulated in terms of the CF of \(I\). The proof follows by computing the PDF of \(L(0)\) and the CF of \(I\) as in [94, Lemma 1] and [94, Lemma 2], respectively, by inserting them in (3.18) and by using some algebra. \(\square\)

**Proposition 5** Let \(f_{L(0)}(\cdot)\) be the PDF in (3.10) and \(\Phi_I(\cdot | \cdot)\) be the CF in (3.13). Let \(\widehat{M}_I(z) = dM_I(z)/dz\) be the first derivative of the MGF of the aggregate other-cell interference, i.e., \(M_I(z) = \Phi_I(-jz)\). Then, the J-CCDF of the SWIPT schemes in Table 3.2 can be upper-bounded as \(F(R^*, Q^*) \leq \min \{F_{ID}(R^*), F_{EH}(Q^*)\}\), where \(F_{ID}(R^*) = \Pr\{R \geq R^*\}\) and \(F_{EH}(Q^*) = \Pr\{Q \geq Q^*\}\) can be formulated, respectively,
Table 3.3: Definitions of \((a_{ID}, b_{ID}, c_{ID})\) and \((a_{EH}, b_{EH}, c_{EH})\) according to the PDFs in Table 3.2.

<table>
<thead>
<tr>
<th></th>
<th>(a_{ID})</th>
<th>(b_{ID})</th>
<th>(c_{ID})</th>
<th>(a_{EH})</th>
<th>(b_{EH})</th>
<th>(c_{EH})</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PS-MRC</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>SAR</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ID-SC</td>
<td>2</td>
<td>-2</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>EH-SC</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>-2</td>
<td>0</td>
</tr>
</tbody>
</table>

as follows:

\[
F_{ID} (\mathcal{R}_*) = \int_0^\infty \mathcal{J}_{ID} (y) f_{L(0)} (y) \, dy, 
\]

\[
F_{EH} (\mathcal{Q}_*) = \frac{1}{2} + \int_0^\infty \int_0^\infty \frac{1}{\pi \omega} \text{Im} \left\{ \exp \left( -j \omega \tilde{q}_* \right) \right\} \times \mathcal{J}_{EH} \left( \omega, \frac{G^{(0)}}{y} \right) \Phi_I (\omega | y) \right\} f_{L(0)} (y) \, d\omega \, dy, 
\]

where, for simplicity, the following short-hands are introduced:

\[
\mathcal{J}_{ID} (y) = (a_{ID} + c_{ID}) \exp \left( -\frac{\tilde{\sigma}_*^2 y}{PG^{(0)} \tilde{r}_*} \right) M_I \left( \frac{y}{G^{(0)} \tilde{r}_*} \right) 
+ \frac{b_{ID}}{2} \exp \left( -\frac{2\tilde{\sigma}_*^2 y}{PG^{(0)} \tilde{r}_*} \right) M_I \left( \frac{2y}{G^{(0)} \tilde{r}_*} \right) 
+ \frac{c_{ID} \tilde{\sigma}_*^2 y}{PG^{(0)} \tilde{r}_*} \exp \left( -\frac{\tilde{\sigma}_*^2 y}{PG^{(0)} \tilde{r}_*} \right) M_I \left( \frac{y}{G^{(0)} \tilde{r}_*} \right) 
- c_{ID} \frac{y}{G^{(0)} \tilde{r}_*} \exp \left( -\frac{\tilde{\sigma}_*^2 y}{PG^{(0)} \tilde{r}_*} \right) \hat{M}_I \left( \frac{y}{G^{(0)} \tilde{r}_*} \right), 
\]

\[
\mathcal{J}_{EH} (\omega, z) = a_{EH} (1 - j\omega z)^{-1} + b_{EH} (2 - j\omega z)^{-1} 
+ c_{EH} (1 - j\omega z)^{-2},
\]

and the two triplets of coefficients \((a_{ID}, b_{ID}, c_{ID})\) and \((a_{EH}, b_{EH}, c_{EH})\) are defined in Table 3.3.

Proof: \(F_{ID} (\cdot)\) is obtained by using the Pcov-based approach in [37, Sec. III-G] and \(F_{ID} (\cdot)\) is obtained by using the Gil-Pelaez theorem in [36]. The triplets \((a_{ID}, b_{ID}, c_{ID})\) and \((a_{EH}, b_{EH}, c_{EH})\) are obtained based on the PDFs in Table 3.2.

The J-CCDFs in Proposition 4 and Proposition 5 are formulated in terms of two-fold integrals that can be efficiently computed with the aid of state-of-the-art computational
software programs and have the advantage of avoiding lengthy Monte Carlo simulations. For brevity, the explicit expression of the first derivative of the MGF of the aggregate other-cell interference is not reported. It can be computed and formulated in closed-form from (3.13) and (3.14). The numerical complexity associated with the computation of Proposition 4 and Proposition 5 instead of using Monte Carlo simulations is discussed in Section 3.7.

3.7 Numerical and Simulation Results

In this section, we use Monte Carlo simulations to validate our findings and mathematical frameworks, as well as to assess the potential of the proposed A-SWIPT scheme for application to cellular networks. Monte Carlo simulations are obtained by using the approach discussed in [5]. Unless otherwise stated, the following setup is considered: \( \nu = c_0/f_c \), where \( c_0 \) is the speed of light in m/s and \( f_c = 2.1 \) GHz is the carrier frequency; \( \sigma^2_{\text{ID}} = -70 \) dBm; \( \sigma^2_N = -174 + 10 \log_{10}(B_w) + \mathcal{F}_N \) dBm, where \( B_w = 200 \) kHz and \( \mathcal{F}_N = 10 \) dB is the noise figure; \( P = 30 \) dBm; \( \zeta = 0.8; \omega_M = 25.6 \) degrees; \( G_M = 7.47 \); and \( G_S = 0.5 \). The channel model and the density of BSs, \( \lambda \), are chosen in agreement with [5]: \( D = 109.8517 \) m, \( q^{[0,D]}_{\text{LOS}} = 0.7195 \), \( q^{[D,\infty]}_{\text{LOS}} = 0.0002 \), \( \beta_{\text{LOS}} = 2.5 \), \( \beta_{\text{NLOS}} = 3.5 \), \( \lambda = 1/(\pi R_{\text{cell}}^2) \) where \( R_{\text{cell}} = 83.4122 \) m is the average cell radius. In Figs. 3.1-3.3, without loss of generality, we analyze the case study \( F(R_*, Q_*) = 0.75 \). Usually, imposing higher values of the J-CCDF results in lower values of \( R_\star \) and \( Q_\star \) that satisfy them. The setup \( F(R_*, Q_*) = 0.75 \) is considered only as an illustrative example. Our frameworks can be applied to arbitrary parameters and setups. In Fig. 3.4, we analyze a case study where \( F(R_*, Q_*) = 0.9 \), which corresponds to an application scenario where the imposed requirements of information rate and harvested power need to be achieved with high reliability.

In Figs. 3.1 and 3.2, we validate the correctness of Proposition 4 and Proposition 5, respectively, against Monte Carlo simulations. In particular, Fig. 3.1 confirms that the J-CCDF in Proposition 4 is exact and Fig. 3.2 highlights that the Frechet inequality in Proposition 5 provides a upper-bound of the J-CCDF, which is asymptotically tight as the system operates either in the ID-limited or in the EH-limited regimes. From the
3.7. Numerical and Simulation Results

![Contour lines of the J-CCDF of PS and PS-MRC schemes as a function of ρ. The curves show the pairs (R∗, Q∗) so that F (R∗, Q∗) = 0.75. Markers: Monte Carlo simulations. Solid lines: Proposition 4.](image)

engineering standpoint, Figs. 3.1 and 3.2 show that information rate and harvested power highly depend on the choice of ρ for PS and PS-MRC schemes, as well as that there is no scheme among SAR, ID-SC and EH-SC that outperform all the others for every pair (R∗, Q∗). All lemmas and remarks in Section 3.5 are, in particular, confirmed. This motivates the need of the proposed adaptive schemes for system-level optimization.

In Fig. 3.3, we investigate the performance of the proposed adaptive schemes and compare them against the ideal setups where ID and EH can be performed without any practical implementation constraints (denoted by “Ideal” in the figure). We observe that the proposed A-SWIPT scheme outperforms all the other schemes, as well as that its J-CCDF is not far from the corresponding ideal benchmark. In the considered setup, in particular, we note that the J-CCDF of the A-SWIPT scheme coincides with the J-CCDF of the OPS-MRC scheme. This implies that, in the considered setup, the OPS-MRC scheme outperforms the A-SC scheme for every pair (R∗, Q∗).

In Fig. 3.4, we leverage the proposed mathematical frameworks for computing the highest power that can be harvested, Q∗, for some given requirements of achievable rate,
Figure 3.2: Contour lines of the J-CCDF of SAR, ID-SC and EH-SC schemes. The curves show the pairs \((R_\ast, Q_\ast)\) so that \(F(R_\ast, Q_\ast) = 0.75\). Markers: Monte Carlo simulations. Solid, dotted and dashed lines: \textit{Proposition 5}.

\(R_\ast\), and reliability formulated in terms of J-CCDF. The figure, in particular, highlights the impact of the density of BSs via \(R_{\text{cell}}\) and of directional beamforming. More precisely, \(N_t\) is the number of antennas of the directional beamformer in [94]. The values of \(\omega_M, G_M\) and \(G_S\) used in Figs. 3.1-3.3 can be obtained by setting \(N_t = 4\). The figure proves that the densification of BSs and antenna elements increases the amount of harvested power remarkably. We note, in particular, that \(Q_\ast\) increases almost linearly with the logarithm of \(N_t\). In addition, two important performance trends can be identified: i) OPS-MRC and A-SC provide similar performance as \(N_t\) increases, which highlights that SC may be a low-complexity option with minimal performance degradation with respect to MRC (especially if the system operates in the EH-limited regime) and ii) the mathematical framework in \textit{Proposition 5} is in good agreement with Monte Carlo simulations. It is worth mentioning that the values of \(N_t\) for which the J-CCDF does not reach 0.9 are not shown.

In Table 3.4, we analyze the accuracy of computing the optimum power splitting ratio, \(\rho\), by using the upper-bound in \textit{Proposition 5}. This study is motivated by the comment in \textit{Remark 4}. The numerical results confirm that, even though the upper-bound may provide
3.7. Numerical and Simulation Results

Figure 3.3: Contour lines of the J-CCDF of OPS, OPS-MRC, A-SC and A-SWIPT schemes. The curves show the pairs \((R_*, Q_*)\) so that \(F(R_*, Q_*) = 0.75\). The setups PS-Ideal and PS-MRC-Ideal refer to the PS and PS-MRC schemes where \(\rho_{ID} = 0\) and \(\rho_{EH} = 1\) simultaneously, e.g., ID and EH are assumed to be performed on the same signal without the need of PS. \(\rho_{opt}\) is computed by using Proposition 4. Markers: Monte Carlo simulations. Solid lines: Proposition 4 for OPS, OPS-MRC and A-SWIPT schemes and in Proposition 5 for the A-SC scheme. In the considered setup, the OPS-MRC scheme coincides with the A-SWIPT scheme.

slightly different values of \(\rho_{opt}\), its accuracy is usually acceptable and the corresponding values of the J-CCDF are sufficiently close to those obtained by using the exact optimum power splitting ratio.

In Table 3.5, we provide numerical examples for substantiating the comment made right after Lemma 6, i.e., PS may outperform PS-MRC for values of \(\rho\) that do not satisfy the conditions stated in the lemma. The reason of this performance trend, which may be considered to be unexpected at the first sight, is related to the fact that PS and PS-MRC schemes may be configured to operate in different regimes. For small and large values of the power splitting ratios, for example, PS and PS-MRC may operate close to the ID-limited and EH-limited regimes, respectively. This implies that their J-CCDFs have different mathematical expressions and, thus, the impact of MRC cannot be predicted based on
Table 3.4: Optimum power splitting ratio for different values of $Q_s$ and $R_s$ expressed in dBm and kbps. The “Exact” values are obtained by using Proposition 4. The “Upper-bound” values are obtained by using Proposition 5. The “Mixed” values are obtained by using Proposition 5 to compute $\rho_{opt}$ and by obtaining the corresponding J-CCDF from Proposition 4. Error is the relative error (in %) of the resulting J-CCDF with respect to that obtained with the exact framework.

<table>
<thead>
<tr>
<th>$(Q_s, R_s) \rightarrow$</th>
<th>(-60, 100)</th>
<th>(-60, 200)</th>
<th>(-60, 300)</th>
<th>(-60, 400)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td>$\rho_{opt} = 0.9568$</td>
<td>$\rho_{opt} = 0.9108$</td>
<td>$\rho_{opt} = 0.8926$</td>
<td>$\rho_{opt} = 0.7825$</td>
</tr>
<tr>
<td></td>
<td>$T_{opt} = 0.7447$</td>
<td>$T_{opt} = 0.7401$</td>
<td>$T_{opt} = 0.7334$</td>
<td>$T_{opt} = 0.7243$</td>
</tr>
<tr>
<td>Upper-bound</td>
<td>$\rho_{opt} = 0.9640$</td>
<td>$\rho_{opt} = 0.9036$</td>
<td>$\rho_{opt} = 0.8006$</td>
<td>$\rho_{opt} = 0.6352$</td>
</tr>
<tr>
<td></td>
<td>$T_{opt} = 0.7465$</td>
<td>$T_{opt} = 0.7452$</td>
<td>$T_{opt} = 0.7425$</td>
<td>$T_{opt} = 0.7375$</td>
</tr>
<tr>
<td></td>
<td>Error = 0.2417</td>
<td>Error = 0.6891</td>
<td>Error = 1.2408</td>
<td>Error = 1.8224</td>
</tr>
<tr>
<td>Mixed</td>
<td>$\rho_{opt} = 0.9640$</td>
<td>$\rho_{opt} = 0.9036$</td>
<td>$\rho_{opt} = 0.8006$</td>
<td>$\rho_{opt} = 0.6352$</td>
</tr>
<tr>
<td></td>
<td>$T_{opt} = 0.7446$</td>
<td>$T_{opt} = 0.7401$</td>
<td>$T_{opt} = 0.7327$</td>
<td>$T_{opt} = 0.7213$</td>
</tr>
<tr>
<td></td>
<td>Error = 0.0134</td>
<td>Error = 0</td>
<td>Error = 0.0954</td>
<td>Error = 0.4142</td>
</tr>
</tbody>
</table>

Table 3.5: J-CCDF of PS and PS-MRC schemes as a function of $Q_s$ and $R_s$ expressed in dBm and kbps. Setup: $\rho = 0.001$ for PS and $\rho = 0.999$ for PS-MRC.

<table>
<thead>
<tr>
<th>$(Q_s, R_s) \rightarrow$</th>
<th>(-60, 800)</th>
<th>(-60, 900)</th>
<th>(-60, 944.144)</th>
<th>(-60, 950)</th>
<th>(-60, 1050)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS</td>
<td>0.020811</td>
<td>0.0207128</td>
<td>0.0206576</td>
<td>0.0206493</td>
<td>0.0204861</td>
</tr>
<tr>
<td>PS-MRC</td>
<td>0.020809</td>
<td>0.02072655</td>
<td>0.0207265</td>
<td>0.0207265</td>
<td>0.0207265</td>
</tr>
</tbody>
</table>

conventional arguments. This motivates the need and relevance of the comparison and findings summarized in Lemma 6. The values reported in the third column of Table 3.5, in particular, correspond to the pair $(Q_s, R_s)$ for which PS and PS-MRC schemes provide almost the same J-CCDF.

In Table 3.6, we compare the proposed mathematical frameworks and Monte Carlo
Table 3.6: Comparison of the time (in seconds) for computing the J-CCDF via Monte Carlo simulations and Proposition 4, for different pairs \((Q_*, R_*)\) in (dBm, kbps). As for Monte Carlo simulations, the number of spatial and channel realizations is set equal to \(10^5\) for all setups, while the simulation area, Area, (in square meters) is chosen so that the relative error, Error, between the J-CCDF functions computed with Proposition 4 and estimated via simulation have a relative error equal to around 0.1%. For completeness, the relative error (in %) is reported as well. The time for computing \(\rho_{opt}\) is not taken into account.

<table>
<thead>
<tr>
<th>((Q_<em>, R_</em>))</th>
<th>((-63, 200))</th>
<th>((-66, 300))</th>
<th>((-69, 400))</th>
<th>((-75, 400))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monte Carlo</td>
<td>Time = 1761</td>
<td>Time = 294</td>
<td>Time = 1782</td>
<td>Time = 304</td>
</tr>
<tr>
<td>(\bar{F} (\cdot))</td>
<td>0.7540</td>
<td>0.75999</td>
<td>0.7534</td>
<td>0.7617</td>
</tr>
<tr>
<td>Area</td>
<td>2.1858e9</td>
<td>2.1858e8</td>
<td>2.1858e9</td>
<td>2.1858e8</td>
</tr>
<tr>
<td>Error</td>
<td>0.1049</td>
<td>0.0996</td>
<td>0.0904</td>
<td>0.1156</td>
</tr>
<tr>
<td>Framework</td>
<td>Time = 83</td>
<td>Time = 63</td>
<td>Time = 71</td>
<td>Time = 65</td>
</tr>
<tr>
<td>(\bar{F} (\cdot))</td>
<td>0.754792</td>
<td>0.759144</td>
<td>0.7527194</td>
<td>0.7606211</td>
</tr>
<tr>
<td>(\rho_{opt})</td>
<td>0.843616</td>
<td>0.60452875</td>
<td>0.3198705</td>
<td>0.106576097</td>
</tr>
</tbody>
</table>

Figure 3.4: Highest harvested power \(Q_*\) of OPS-MRC and A-SC so that \(\bar{F} (R_*, Q_*) = 0.90\) and \(R_* = 100\) kbps, as a function of \(R_{cell}\). \(\rho_{opt}\) is computed by using Proposition 4. Markers: Monte Carlo simulations for the A-SC scheme. Solid lines: Proposition 4 for the OPS-MRC scheme. Dashed lines: Proposition 5 for the A-SC scheme.

Simulations in terms of the computation time that is necessary for estimating the J-CCDF for a single pair \((Q_*, R_*)\). The time for computing \(N\) pairs is, in fact, equal to \(N\) times that needed for computing a single pair. Since the computation time of Monte Carlo simulations
highly depends on the simulation area, which determines the accuracy of the result, the study is conducted by setting the simulation area so that the relative error between the J-CCDF computed with the mathematical framework and that estimated via Monte Carlo simulations is around 0.1%. It is worth mentioning that Table 3.6 does not account for the time needed to compute the optimal power splitting ratio, $\rho_{\text{opt}}$. This computation is, in fact, not affordable by using Monte Carlo simulations due to the many possible values that need to be analyzed. Even neglecting it, we note that the proposed mathematical frameworks are faster than Monte Carlo simulations for values of the simulation area that provide sufficiently accurate estimates of the J-CCDF. This justifies even further the usefulness of the proposed mathematical frameworks: they are not only insightful and make our numerical illustrations easier to be reproduced (reproducible research), but are more numerically tractable as well.

### 3.8 Conclusion

In this chapter, we have analyzed SWIPT-enabled cellular networks that employ several receiver diversity schemes. We have shown that receiver diversity has the potential of enhancing the information rate and of increasing the harvested power simultaneously. We have proved, in addition, that the system-level performance can be improved by adaptively choosing the receiver diversity scheme as a function of the information rate and harvested power requirements that need to be fulfilled. With the aid of stochastic geometry, we have introduced mathematical frameworks that enable one to perform this system-level and adaptive optimization. All findings and performance trends have been validated with the aid of Monte Carlo simulations.

As far as the SWIPT schemes that use selection combining are concerned, the analysis has been based on a upper-bound that exploits the Frechet inequality. This proposed bound provides several important design guidelines for system-level optimization and it is asymptotically tight in the ID-and EH-limited regimes. In general, however, a gap between the exact J-CCDF and its estimate based on the upper-bound exists (see Figs. 3.2 and 3.3). The authors are currently working on the development of more accurate, but still tractable, bounds and approximations for arbitrary information rate and harvested power requirements.
3.9 Appendix: Exact J-CCDF of SAR, ID-SC and EH-SC SWIPT Schemes

In Section 3.6, it is argued that an exact expression for the J-CCDF of SAR, ID-SC and EH-SC schemes may be obtained with the aid of the multi-dimensional inversion theorem [107], which, however, would result in a multi-fold integral expression that is much less tractable than the mathematical framework reported in Proposition 5. In this appendix, we provide further details on how this framework can be developed. Due to space limitations, no formulas are reported but only the approach is briefly summarized.

The approach is along the same lines as the proof of Proposition 4. More precisely, the J-CCDF can be formulated as shown in the first line of (3.18). In this case, however, the other-cell interferences of ID and EH receiver are different as discussed in Remark 2. As a result, the equality in the second line of (3.18) does not hold anymore, since it assumes that the other-cell inferences are the same. Nevertheless, the joint CF of the other-cell interferences of ID and EH receivers can be computed by using the same steps as in [94, Lemma 2] and the associated joint CDF can be formulated in terms of the joint CF by using [107, Eq. (11)]. The resulting expression, however, necessitates the computation of a two-fold integral. Since the computation of the expectation with respect to the smallest path-loss (see (3.18)) requires another integral, the final expression of the J-CCDF may require the computation of at least a three-fold integral. This makes the resulting mathematical framework less tractable and less numerically stable than the upper-bound in Proposition 5. This motivates the use of the Frechet bound in (3.7).
4.1 Abstract

In this chapter, we analyze and optimize the energy efficiency of downlink cellular networks. With the aid of tools from stochastic geometry, we introduce a new closed-form analytical expression of the potential spectral efficiency (bit/sec/m$^2$). In the interference-limited regime for data transmission, unlike currently available mathematical frameworks, the proposed analytical formulation depends on the transmit power and deployment density of the base stations. This is obtained by generalizing the definition of coverage probability and by accounting for the sensitivity of the receiver not only during the decoding of information data, but during the cell association phase as well. Based on the new formulation of the potential spectral efficiency, the energy efficiency (bit/Joule) is given in a tractable closed-form formula. An optimization problem is formulated and is comprehensively studied. It is mathematically proved, in particular, that the energy efficiency is a unimodal and strictly pseudo-concave function in the transmit power, given the density of the base stations, and in the density of the base stations, given the transmit power. Under these assumptions, therefore, a unique transmit power and density of the base stations exist, which maximize the energy efficiency. Numerical results are illustrated in order to
confirm the obtained findings and to prove the usefulness of the proposed framework for optimizing the network planning and deployment of cellular networks from the energy efficiency standpoint.

### 4.2 Introduction

The EE is regarded as a key performance metric towards the optimization of operational cellular networks, and the network planning and deployment of emerging communication systems [108]. The EE is defined as a benefit-cost ratio where the benefit is given by the amount of information data per unit time and area that can be reliably transmitted in the network, i.e., the network spectral efficiency, and the cost is represented by the amount of power per unit area that is consumed to operate the network, i.e., the network power consumption. Analyzing and designing a communication network from the EE standpoint necessitate appropriate mathematical tools, which are usually different from those used for optimizing the network spectral efficiency and the network power consumption individually [74]. The optimization problem, in addition, needs to be formulated in a sufficiently simple but realistic manner, so that all relevant system parameters appear explicitly and the utility function is physically meaningful.

Optimizing the EE of a cellular network can be tackled in different ways, which include [108]: the design of medium access and scheduling protocols for optimally using the available resources, e.g., the transmit power; the use of renewable energy sources; the development of innovative hardware for data transmission and reception; and the optimal planning and deployment of network infrastructure. In this chapter, we focus our attention on optimizing the average number of BSs to be deployed (or to be kept operational) per unit area and their transmit power. Henceforth, this is referred to as “system-level EE” optimization, i.e., the EE across the entire (or a large portion of the) cellular network is the utility function of interest.

System-level analysis and optimization are useful when the network operators are interested in optimizing the average performance across the entire cellular network. Hence, they are relevant for optimally operating current networks, and for deploying and planning future networks. In the first case, given an average number of BSs per unit area...
already deployed, they may provide information on the average number of BSs that can be switched off based on the average load of the network, and on their optimal transmit power to avoid coverage holes. In the second case, they may guide the initial deployment of cellular infrastructure that employs new types of BSs (e.g., powered by renewable energy sources), new transmission technologies (e.g., large-scale antennas), or that operate in new frequency bands (e.g., the millimeter-wave spectrum).

In the last few years, the system-level modeling and analysis of cellular networks have been facilitated by capitalizing on the mathematical tool of stochastic geometry and, more precisely, on the theory of spatial point processes [1]-[7]. It has been empirically validated that, from the system-level standpoint, the locations of the BSs can be abstracted as points of a homogeneous PPP whose intensity coincides with the average number of BSs per unit area [5]. A comprehensive survey of recent results in this field of research is available in [109].

A relevant performance metric for the design of cellular networks is the PSE, which is the network’s information rate per unit area (measured in bit/sec/m²) that corresponds to the minimum signal strength for reliable transmission. Under the PPP modeling assumption, the PSE can be obtained in two steps: i) first by computing the PSE of a randomly chosen MT and by assuming a given spatial realization for the locations of the BSs and ii) then by averaging the obtained conditional PSE with respect to all possible realizations for the locations of the BSs and MTs. In the interference-limited regime, this approach allows one to obtain a closed-form expression for the PSE under the (henceforth called) standard modeling assumptions, i.e., single-antenna transmission, singular path-loss model, Rayleigh fading, fully-loaded BSs, cell association based on the highest average received power [1]. Motivated by these results, the PPP modeling approach for the locations of the BSs has been widely used to analyze the trade-off between the network spectral efficiency and the network power consumption, e.g., [71], as well as to minimize the network power consumption given some constraints on the network spectral efficiency or to maximize the network spectral efficiency given some constraints on the network power consumption [110]. The PPP modeling approach has been applied to optimize the EE of cellular networks as well. Notable examples for this field of research are [111]-[112]. A general study of the energy and spectral efficiencies of multi-tier cellular networks can be found in [29].
In our opinion, however, currently available approaches for modeling and optimizing the system-level EE of cellular networks are insufficient and/or unsuitable for mathematical analysis. This is elaborated in the next section.

**4.2.1 Fundamental Limitations of Current Approaches for System-Level EE Optimization**

We begin with an example that shows the limitations of the available analytical frameworks. In the interference-limited regime, under the standard modeling assumptions, the PSE is:

\[
PSE = \frac{\lambda_{BS} B_W \log_2 (1 + \gamma_D)}{2 F_1 (1, -2/\beta, 1 - 2/\beta, -\gamma_D)}, \tag{4.1}
\]

where \(\lambda_{BS}\) is BSs’ density, \(B_W\) is the transmission bandwidth, \(\gamma_D\) is the threshold for reliable decoding, \(\beta > 2\) is the path-loss exponent, \(2 F_1 (\cdot, \cdot, \cdot, \cdot)\) is the Gauss hypergeometric function, \(P_{cov} (\cdot)\) is the coverage probability defined in [1, Eq. (1)], and (a) follows from [1, Eq. (8)].

The main strength of (4.1) is its simple closed-form formulation. This is, however, its main limitation as well, especially for formulating meaningful system-level EE optimization problems. Under the standard modeling assumptions, in fact, the network power consumption (Watt/m²) is\(^1\) \(P_{grid} = \lambda_{BS} (P_{tx} + P_{circ})\), where \(P_{tx}\) is the transmit power of the BSs and \(P_{circ}\) is the static power consumption of the BSs, which accounts for the power consumed in all hardware blocks, e.g., analog-to-digital and digital-to-analog converters, analog filters, cooling components, and digital signal processing [108]. The system-level EE (bit/Joule) is defined as the ratio between (4.1) and the network power consumption, i.e., \(EE = \frac{PSE}{P_{grid}}\). Since the PSE in (4.1) is independent of the transmit power of the BSs, \(P_{tx}\), and the network power consumption, \(P_{grid}\), linearly increases with \(P_{tx}\), we conclude that any EE optimization problems formulated based on (4.1) would result in the trivial optimal solution consisting of turning all the BSs off (the optimal transmit power is zero). In the context of multi-tier cellular networks, a similar conclusion has been obtained in some early papers on system-level EE optimization, e.g., [71], where it is shown

\(^1\)In this chapter, this holds true for Load Model 1 that is introduced in Section 4.3.4.
that the EE is maximized if all macro BSs operate in sleeping mode. A system-level EE optimization problem formulated based on (4.1) would result, in addition, in a physically meaningless utility function, which provides a non-zero benefit-cost ratio, i.e., a strictly positive EE while transmitting zero power \((EE(P_{tx} = 0) = PSE/(\lambda_{BS}P_{circ}) > 0)\). In addition, the EE computed from (4.1) is independent of the density of BSs. We briefly mention here, but will detail it in Section 4.4, that the load model, i.e., the fully-loaded assumption, determines the conclusion that the EE does not depend on \(\lambda_{BS}\). This assumption, however, does not affect the conclusion that the optimal \(P_{tx}\) is zero. This statement is made more formal in the sequel (Proposition 6, Corollary 1). It worth nothing that the conclusion that the PSE is independent of \(P_{tx}\) is valid regardless of the specific path-loss model being used\(^2\). It depends, on the other hand, on the assumptions of interference-limited operating regime and of having BSs that emit the same \(P_{tx}\).

Based on these observations, we conclude that a new analytical formulation of the PSE that explicitly depends on the transmit power and density of the BSs, and that is tractable enough for system-level EE optimization is needed. From an optimization point of view, in particular, it is desirable, that the PSE is formulated in a closed-form expression and that the resulting EE function is unimodal and strictly pseudo-concave in the transmit power (given the density) and in the density (given the transmit power) of the BSs. This would imply, e.g., that the first-order derivative of the EE with respect to the transmit power of the BSs (assuming the density given) would have a unique zero, which would be the unique optimal transmit power that maximizes the EE [74]. Similar conclusions would apply to the optimal density of the BSs for a given transmit power. Further details are provided in Section 4.5. In this regard, a straightforward approach to overcome the limitations of (4.1) would be to abandon the interference-limited assumption and to take the receiver noise into account. In this case, the PSE would be formulated in terms of a single-integral that, in general, cannot be expressed in closed-form [1], [113, Eq. (9)]. This integral formulation, in particular, results in a system-level EE optimization problem that is not easy to tackle. This approach, in addition, has the inconvenience of formulating the optimization problem for an operating regime where cellular networks are unlikely to

\(^2\)The reader may verify this statement by inspection of (4.4), where \(P_{tx}\) cancels out for any path-loss models.
operate in practice.

### 4.2.2 State-of-the-Art on System-Level EE Optimization

We briefly summarize the most relevant research contributions on energy-aware design and optimization of cellular networks. Due to space limitations, we discuss only the contributions that are closely related to ours. A state-of-the-art survey on EE optimization is available in [74].

In [71], the authors study the impact of switching some macro BSs off in order to minimize the power consumption under some constraints on the coverage probability. Since the authors rely on the mathematical framework in (4.1), they conclude that all macro BSs need to be switched off to maximize the EE. In [110], the author exploits geometric programming to minimize the power consumption of cellular networks given some constraints on the network coverage and capacity. The EE is not studied. A similar optimization problem is studied in [114] and [113] for two-tier cellular networks but the EE is not studied either. As far as multi-tier cellular networks are concerned, an important remark is necessary. In the interference-limited regime, optimal transmit powers and densities for the different tiers of BSs may exist if the tiers have different thresholds for reliably decoding the data. The PSE, otherwise, is the same as that of single-tier networks, i.e., it is independent of the transmit power and density of the BSs. In [115], the authors study the EE of small cell networks with multi-antenna BSs. For some parameter setups, it is shown that an optimal density of the BSs exists. The EE, however, still decreases monotonically with the transmit power of the BSs, which implies that the EE optimization problem is not well formulated from the transmit power standpoint. More general scenarios are considered in [111], [116–121] but similar limitations hold. In some cases, e.g., [122], the existence and uniqueness of an optimal transmit power and density of the BSs are not mathematically proved or, e.g., in [123], the problem formulation has a prohibitive numerical complexity as it necessitates the computation of multiple integrals and infinite series. It is apparent, therefore, that a tractable approach for system-level EE optimization is missing in the open technical literature. In this chapter, we introduce a new definition of PSE that overcomes these limitations.
4.2.3 Research Contribution and Novelty

In the depicted context, the specific novel contributions made by this chapter are as follows:

- We introduce a new closed-form mathematical formulation of the PSE for interference-limited cellular networks (data transmission), which depends on the transmit power and density of the BSs. The new expression of the PSE is obtained by taking into account the power sensitivity of the receiver not only for data transmission but for cell association too.

- Based on the new expression of the PSE, a new system-level EE optimization problem is formulated and comprehensively studied. It is mathematically proved that the EE is a unimodal and strictly pseudo-concave function in the transmit power given the BSs’ density and in the BSs’ density given the transmit power. The dependency of the optimal power as a function of the density and of the optimal density as a function of the power is discussed.

- A first-order optimal pair of transmit power and density of the BSs is obtained by using a simple alternating optimization algorithm whose details are discussed in the sequel. Numerical evidence of the global optimality of this approach is provided as well.

- Two load models for the BSs are analyzed and compared against each other. It is shown that they provide the same PSE but have different network power consumptions. Hence, the optimal transmit power and density of the BSs that maximize their EEs are, in general, different. Their optimal EEs and PSEs are studied and compared against each other.

The chapter is organized as follows. In Section 4.3, the system model is presented. In Section 4.4, the new definition of PSE is introduced. In Section 4.5, the EE optimization problem is formulated and studied. In Section 4.6, numerical results are shown. Section 4.7 concludes this chapter.

Notation: The main symbols and functions used in this chapter are reported in Table 4.1.
Table 4.1: Summary of main symbols and functions used throughout this chapter.

<table>
<thead>
<tr>
<th>Symbol/Function</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[\cdot], Pr[\cdot]$</td>
<td>Expectation operator, probability measure</td>
</tr>
<tr>
<td>$\lambda_{BS}, \lambda_{MT}$</td>
<td>Density of base stations, mobile terminals</td>
</tr>
<tr>
<td>$\Psi_{BS}, \Psi_{MT}$</td>
<td>PPP of base stations, mobile terminals</td>
</tr>
<tr>
<td>$\Psi_{BS}^{(I)}$</td>
<td>PPP of interfering base stations</td>
</tr>
<tr>
<td>$BS_0, BS_i, BS_n$</td>
<td>Serving, interfering, generic base station</td>
</tr>
<tr>
<td>$P_{tx}, P_{circ}$</td>
<td>Transmit, circuits power consumption of base stations</td>
</tr>
<tr>
<td>$P_{idle}$</td>
<td>Idle power consumption of base stations</td>
</tr>
<tr>
<td>$r_n, g_n$</td>
<td>Distance, fading power gain of a generic link</td>
</tr>
<tr>
<td>$L(\cdot), L_n$</td>
<td>Path-loss, shorthand of path-loss</td>
</tr>
<tr>
<td>$L_0$</td>
<td>Path-loss of intended link</td>
</tr>
<tr>
<td>$\kappa, \beta &gt; 0$</td>
<td>Path-loss constant, slope (exponent)</td>
</tr>
<tr>
<td>$B_W, N_0$</td>
<td>Transmission bandwidth, noise power spectral density</td>
</tr>
<tr>
<td>$\sigma_N^2 = B_WM_0, I_{agg}(\cdot)$</td>
<td>Noise variance, aggregate other-cell interference</td>
</tr>
<tr>
<td>$\gamma_D, \gamma_A$</td>
<td>Reliability threshold for decoding, cell association</td>
</tr>
<tr>
<td>$\mathcal{L}(x) = 1 - (1 + x/\alpha)^{-\alpha}, \alpha = 3.5$</td>
<td>Probability that a base station is in transmission mode</td>
</tr>
<tr>
<td>$f_X(\cdot)$, $F_X(\cdot)$</td>
<td>Probability density/mass function of $X$, cumulative distribution/mass function of $X$</td>
</tr>
<tr>
<td>$\mathbb{1}(\cdot)$</td>
<td>Indicator function</td>
</tr>
<tr>
<td>$2F_1(\cdot, \cdot, \cdot, \cdot), \Gamma(\cdot)$</td>
<td>Gauss hypergeometric function, gamma function</td>
</tr>
<tr>
<td>$\max{x, y}, \min{x, y}$</td>
<td>Maximum, minimum between $x$ and $y$</td>
</tr>
<tr>
<td>$\Upsilon = 2F_1(-2/\beta, 1, 1 - 2/\beta, -\gamma_D) - 1 \geq 0$</td>
<td>Shorthand</td>
</tr>
<tr>
<td>$Q(x, y, z) = 1 - \exp\left(-\pi x(y/\eta)^{2/\beta}(1 + \Upsilon \mathcal{L}(z))\right)$</td>
<td>Shorthand with $\eta = \kappa \sigma_N^2 \gamma_A$</td>
</tr>
<tr>
<td>SIR</td>
<td>Signal-to-interference-ratio</td>
</tr>
<tr>
<td>SNR</td>
<td>Average signal-to-noise-ratio</td>
</tr>
<tr>
<td>$P_{cov}, PSE$</td>
<td>Coverage, potential spectral efficiency,</td>
</tr>
<tr>
<td>$P_{grid}$</td>
<td>Network power consumption</td>
</tr>
<tr>
<td>$z_x(x, y), \tilde{z}_x(x, y)$</td>
<td>First-order, second-order derivative with respect to $x$</td>
</tr>
</tbody>
</table>

4.3 System Model

In this section, the network model is introduced. With the exception of the load model, we focus our attention on a system where the standard modeling assumptions hold. One of the main aims of this chapter is, in fact, to highlight the differences between currently available analytical frameworks and the new definition of PSE that is introduced. The proposed approach can be readily generalized to more advanced system models such as that recently adopted in [7].

4.3.1 Cellular Network Modeling

A downlink cellular network is considered. The BSs are modeled as points of a homogeneous PPP, denoted by $\Psi_{BS}$, of density $\lambda_{BS}$. The MTs are modeled as another homogeneous PPP, denoted by $\Psi_{MT}$, of density $\lambda_{MT}$. $\Psi_{BS}$ and $\Psi_{MT}$ are independent of
each other. The BSs and MTs are equipped with a single omnidirectional antenna. Each BS transmits with a constant power denoted by \( P_{tx} \). The analytical frameworks are developed for the typical MT, denoted by \( \text{MT}_0 \), that is located at the origin (Slivnyak theorem \cite[Th. 1.4.5]{10}). The BS serving \( \text{MT}_0 \) is denoted by \( \text{BS}_0 \). The cell association criterion is introduced in Section 4.3.3. The subscripts 0, \( i \) and \( n \) identify the intended link, a generic interfering link, and a generic BS-to-MT link. The set of interfering BSs is denoted by \( \Psi^{(i)}_{\text{BS}} \). As for data transmission, the network operates in the interference-limited regime, i.e., the noise is negligible compared with the inter-cell interference.

### 4.3.2 Channel Modeling

For each BS-to-MT link, path-loss and fast-fading are considered. Shadowing is not explicitly taken into account because its net effect lies in modifying the density of the BSs \cite{7}. All BS-to-MT links are assumed to be mutually independent and identically distributed (i.i.d.).

#### Path-Loss

Consider a generic BS-to-MT link of length \( r_n \). The path-loss is \( l(r_n) = \kappa r_n^\beta \), where \( \kappa \) and \( \beta \) are the path-loss constant and the path-loss slope (exponent). For simplicity, only the unbounded path-loss model is studied in this chapter. The analysis of more general path-loss models is an interesting but challenging generalization that is left to future research \cite{124}.

#### Fast-Fading

Consider a generic BS-to-MT link. The power gain due to small-scale fading is assumed to follow an exponential distribution with mean \( \Omega \). Without loss of generality, \( \Omega = 1 \) is assumed. The power gain of a generic BS-to-MT link is denoted by \( g_n \).

### 4.3.3 Cell Association Criterion

A cell association criterion based on the highest average received power is assumed. Let \( \text{BS}_n \in \Psi_{\text{BS}} \) denote a generic BS of the network. The serving BS, \( \text{BS}_0 \), is obtained as
follows:
\[
BS_0 = \arg \max_{BS_n \in \Psi_{BS}} \{1/l(r_n)\} = \arg \max_{BS_n \in \Psi_{BS}} \{1/L_n\},
\]
(4.2)
where the short-hand \(L_n = l(r_n)\) is used. As for the intended link, \(L_0 = \min_{r_n \in \Psi_{BS}} \{L_n\}\) holds.

### 4.3.4 Load Modeling

Based on (4.2), several or no MTs can be associated to a generic BS. In the latter case, the BS transmits zero power, i.e., \(P_{tx} = 0\), and, thus, it does not generate inter-cell interference. In the former case, on the other hand, two load models are studied and compared against each other. The main objective is to analyze the impact of the load model on the power consumption and EE of cellular networks. Further details are provided in the sequel. Let \(N_{MT}\) denote the number of MTs associated to a generic BS and \(B_W\) denote the transmission bandwidth available to each BS. If \(N_{MT} = 1\), for both load models, the single MT associated to the BS is scheduled for transmission and the entire bandwidth, \(B_W\), and transmit power, \(P_{tx}\), are assigned to it.

**Load Model 1: Exclusive Allocation of Bandwidth and Power to a Randomly Selected MT**

If \(N_{MT} > 1\), the BS randomly selects, at each transmission instance, a single MT among the \(N_{MT}\) associated to it. Also, the BS allocates the entire transmission bandwidth, \(B_W\), and the total transmit power, \(P_{tx}\), to it. The random scheduling of the MTs at each transmission instance ensures that, in the long term, all the MTs associated to a BS are scheduled for transmission.

**Load Model 2: Equal Allocation of Bandwidth and Power Among All the MTs**

If \(N_{MT} > 1\), the BS selects, at each transmission instance, all the \(N_{MT}\) MTs associated to it. The BS equally splits the available transmission bandwidth, \(B_W\), and evenly spreads the available transmit power, \(P_{tx}\), among the \(N_{MT}\) MTs. Thus, the bandwidth and power are viewed as continuous resources by the BS’s scheduler: each MT is assigned a bandwidth equal to \(B_W/N_{MT}\) and the power spectral density at the detector’s (i.e., the typical MT,
4.4 A New Analytical Formulation of the PSE

MT\(_0\) is equal to P\(_{tx}/B_W\).

In the sequel, we show that the main difference between the two load models lies in the power consumption of the BSs. In simple terms, the more MTs are scheduled for transmission the higher the static power consumption of the BSs is. The analysis of general load models, e.g., based on a discrete number of resource blocks [7], is left to future research due to space limits.

4.3.5 Power Consumption Modeling

In the considered system model, the BSs can operate in two different modes: i) they are in idle mode if no MTs are associated to them and ii) they are in transmission mode if at least one MT is associated to them. The widespread linear power consumption model for the BSs is adopted [108], [125], which accounts for the power consumption due to the transmit power, P\(_{tx}\), the static (circuit) power, P\(_{circ}\), and the idle power, P\(_{idle}\). If the BS is in idle mode, its power consumption is equal to P\(_{idle}\). If the BS is in transmission mode, its power consumption is a function of P\(_{tx}\), P\(_{circ}\), and depends on the load model. Further details are provided in the sequel. In this chapter, based on physical considerations, the inequalities 0 \leq P_{idle} \leq P_{circ} are assumed.

4.4 A New Analytical Formulation of the PSE

In this section, we introduce and motivate a new definition of coverage probability, P\(_{cov}\), and PSE, which overcomes the limitations of currently available analytical frameworks and is suitable for system-level optimization (see Section 4.2.1). All symbols are defined in Table 4.1.

**Definition 4** Let \(\gamma_D\) and \(\gamma_A\) be the reliability thresholds for successfully decoding the information data and for successfully detecting the serving BS, BS\(_0\), respectively. The coverage probability, P\(_{cov}\), of the typical MT, MT\(_0\), is defined as follows:

\[
P_{cov} (\gamma_D, \gamma_A) = \begin{cases} 
\Pr \{ \text{SIR} \geq \gamma_D, \text{SNR} \geq \gamma_A \} & \text{if MT}_0 \text{ is selected} \\
0 & \text{if MT}_0 \text{ is not selected}
\end{cases}
\]  

(4.3)
where the Signal-to-Interference-Ratio (SIR) and the average Signal-to-Noise-Ratio ($\overline{\text{SNR}}$) can be formulated, for the network model under analysis, as follows:

$$\text{SIR} = \frac{P_{tx}g_0/L_0}{\sum_{\text{BS}_i \in \Psi} P_{tx}g_i/L_i} \mathbb{1}(L_i > L_0)$$

$$\overline{\text{SNR}} = \frac{P_{tx}/L_0}{\sigma^2_N}. \quad (4.4)$$

**Remark 7** The definition of $P_{\text{cov}}$ in (4.3) reduces to the conventional one if $\gamma_A = 0$ [1].

**Remark 8** The average $\overline{\text{SNR}}$, in (4.4) is averaged with respect to the fast fading. The SIR depends, on the other hand, on the fast fading. The reason of this choice is discussed below.

**Remark 9** The new definition of coverage probability, $P_{\text{cov}}$, in (4.3) is in agreement with the cell selection criterion specified by the 3rd generation partnership project [126, Sec. 5.2.3.2].

**Motivation for the New Definition of $P_{\text{cov}}$**

The motivation for the new definition of coverage probability originates from the inherent limitations of the conventional definition (obtained by setting $\gamma_A = 0$ in (4.3)), which prevents one from taking into account the strong interplay between the transmit power and the density of the BSs for optimal cellular networks planning. In fact, the authors of [1] have shown that, in the interference-limited regime, $P_{\text{cov}}$ is independent of the transmit power of the BSs. If, in addition, a fully-loaded model is assumed, i.e., $\lambda_{\text{MT}}/\lambda_{\text{BS}} \gg 1$, then $P_{\text{cov}}$ is independent of the density of BSs as well. This is known as the invariance property of $P_{\text{cov}}$ as a function of $P_{tx}$ and $\lambda_{\text{BS}}$ [7]. The tight interplay between $P_{tx}$ and $\lambda_{\text{BS}}$ is, on the other hand, illustrated in Fig. 4.1, where, for ease of representation, an hexagonal cellular layout is considered. Similar conclusions apply to the PPP-based cellular layout studied in this chapter. In Fig. 4.1, it is shown that, for a given $\lambda_{\text{BS}}$, $P_{tx}$ needs to be appropriately chosen in order to guarantee that, for any possible location of $\text{MT}_0$ in the cell, two conditions are satisfied: i) the MT receives a sufficiently good signal quality, i.e., the average SNR is above a given threshold, $\gamma_A$, that ensures a successful cell
Figure 4.1: Illustration of the interplay between $P_{tx}$ and $\lambda_{BS}$. For simplicity, only a cluster of seven BSs is represented by keeping the size of the region of interest (square box) the same. The inter-site distance of the BSs (represented as red dots), i.e., the size of the hexagonal cells, is determined by $\lambda_{BS}$. The shape of the cells depends on the cell association in (4.2). The circular shaded disk (in light yellow) represents the actual coverage region of the BSs that is determined by $P_{tx}$: i) a MT inside the disk receives a sufficiently good signal to detect the BS and to get associated with it, ii) a MT outside the disk cannot detect the BS and is not in coverage. The sub-figures (a)-(c) are obtained by assuming the same $\lambda_{BS}$ but a different $P_{tx}$. The sub-figures (d) and (e) are obtained by considering a $\lambda_{BS}$ greater than that of sub-figures (a)-(c) but keeping the same $P_{tx}$ as sub-figures (a) and (b), respectively. The sub-figure (f) is obtained by considering a $\lambda_{BS}$ smaller than that of sub-figure (c) but keeping the same $P_{tx}$ as it. We observe that, for a given $\lambda_{BS}$, the transmit power $P_{tx}$ is appropriately chosen in sub-figures (a), (e) and (f). $P_{tx}$ is, on the other hand, under-provisioned in sub-figure (b) and over-provisioned in sub-figures (c) and (d). In the first case, the MTs are not capable of detecting the BS throughout the entire cell, i.e., a high outage probability is expected. In the second case, the BSs emit more power than what is actually needed, which results in a high power consumption.

association, i.e., to detect the presence (pilot signal) of the serving BS and ii) the BSs do not over-provision $P_{tx}$, which results in an unnecessary increase of the power consumption.

It is expected, therefore, that an optimal value of $P_{tx}$ given $\lambda_{BS}$ and an optimal value of $\lambda_{BS}$ given $P_{tx}$ that optimize EE exist [127].

Advantages of the New Definition of $P_{cov}$

The new definition of $P_{cov}$ allows one to overcome the limitations of the conventional definition and brings about two main advantages. The first advantage originates from
direct inspection of (4.4). In the conventional definition of $P_{cov}$, only the SIR is considered and the transmit power of the BSs, $P_{tx}$, cancels out between numerator and denominator. This is the reason why $P_{cov}$ is independent of $P_{tx}$. In the proposed new definition, on the other hand, $P_{cov}$ explicitly appears in the second constraint and does not cancel out. The density of the BSs, $\lambda_{BS}$, appears implicitly in the distribution of the path-loss of the intended link, $L_0$. The mathematical details are provided in the sequel. The second inequality, as a result, allows one to explicitly account for the interplay between $P_{tx}$ and $\lambda_{BS}$ (shown in Fig. 4.1). If $\lambda_{BS}$ increases (decreases), in particular, $L_0$ decreases (increases) in statistical terms. This implies that $P_{tx}$ can be decreased (increased) while still ensuring that the average SNR is above $\gamma_A$. The second advantage is that the new definition of $P_{cov}$ is still mathematically tractable and the PSE is formulated in a closed-form expression. This is detailed in Proposition 6.

Remark 10 The new definition of $P_{cov}$ in (4.3) is based on the actual value of $L_0$ because a necessary condition for the typical MT to be in coverage is that it can detect the pilot signal of at least one BS during the cell association. If the BS that provides the highest average received power cannot be detected, then any other BSs cannot be detected either. The second constraint on the definition of $P_{cov}$, in addition, is based on the average SNR, i.e., the SNR averaged with respect to the fast fading, because the cell association is performed based on long-term statistics, i.e., based on the path-loss in this chapter, in order to prevent too frequent handovers. □

Remark 11 Compared with the conventional definition of coverage based on the Signal-to-Interference+Noise-Ratio (SINR) [1], the new definition in (4.3) is conceptually different. Equation (4.3) accounts for the signal quality during both the cell association and data transmission phases. The definition of coverage based on the SINR, on the other hand, accounts for the signal quality only during the data transmission phase. In spite of this fundamental difference, $P_{cov}$ in (4.3) may be interpreted as an approximation for the coverage probability based on the SINR, and, more precisely, as an alternative method to incorporate the thermal noise into the problem formulation. Compared with the coverage based on the SINR, however, the new definition in (4.3) accounts for the impact of thermal noise when it is the dominant factor, i.e., during the cell association phase when the
inter-cell interference is negligible as orthogonal pilot signals are used.

Remark 12 Figure 4.1 highlights that the new definition of coverage in (4.3) is not only compliant with [126] but it has a more profound motivation and wider applicability. In PPP-based cellular networks, in contrast to regular grid-based network layouts, the size and shape of the cells are random. This implies that it is not possible to identify a relation, based on pure geometric arguments, between the cell size and the transmit power of the BSs that makes the constraint on $\text{SNR}$ in (4.3) ineffective in practice. In equivalent terms, in this case, the threshold $\gamma_A$ may turn out to be sufficiently small to render the constraint on $\text{SNR}$ ineffective. This is, for example, the approach employed in [127, Eq. (1)], where the relation between the transmit power and density of the BSs is imposed a priori based on the path-loss. In practice, however, cellular networks are irregularly deployed, which makes the optimal relation between the transmit power and density of the BSs difficult to identify because of the coexistence of cells of small and large sizes. The constraint on $\text{SNR}$ in (4.3) allows one to take into account the interplay between the transmit power and density of the BSs in irregular (realistic) cellular network deployments.

4.4.1 Analytical Formulation of the PSE

In this section, we provide the mathematical definitions of the PSE for the two load models introduced in Section 4.3.4. They are summarized in the following two lemmas, which constitute the departing point for obtaining the closed-form analytical frameworks derived in Section 4.4.2.

Remark 13 The PSE is defined from the perspective of the typical MT, MT₀, rather than from the perspective of the typical cell (or BS). This implies that the proposed approach allows one to characterize the PSE of the so-called Crofton cell, which is the cell that contains MT₀. This approach is commonly used in the literature and is motivated by the lack of results on the explicit distribution of the main geometrical characteristics of the typical cell of a Voronoi tessellation. Further details on the Crofton and typical cells are available in [128] and [129].
Let $\bar{N}_{\text{MT}}$ be the number of MTs that lie in the cell of the typical MT, MT$_0$, with the exception of MT$_0$. $\bar{N}_{\text{MT}}$ is a discrete random variable whose probability mass function in the considered system model can be formulated, in an approximated closed-form expression, as [130, Eq. (3)]:

$$f_{\bar{N}_{\text{MT}}} (u) = \Pr \{ \bar{N}_{\text{MT}} = u \} \approx \frac{3.5^{4.5} \Gamma (u + 4.5) (\lambda_{\text{MT}}/\lambda_{\text{BS}})^u}{\Gamma (4.5) \Gamma (u + 1) (3.5 + \lambda_{\text{MT}}/\lambda_{\text{BS}})^{u+4.5}}. \quad (4.5)$$

**Remark 14** The probability mass function in (4.5) is an approximation because it is based on the widely used empirical expression of the probability density function of the area of the Voronoi cells in [131, Eq. (1)]. A precise formula for the latter probability density function is available in [132]. It is, however, not used in this chapter due to its mathematical intractability, as recently remarked in [112]. Throughout the rest of this chapter, for simplicity, we employ the sign of equality (“=”) in all the analytical formulas that rely solely on the approximation in (4.5). This is to make explicit that our analytical frameworks are not based on any other hidden approximations. □

Based on (4.5), a formal mathematical formulation for the PSE is given as follows.

**Lemma 7** Let Load Model 1 be assumed. The PSE (bit/sec/m$^2$) can be formulated as follows:

$$\text{PSE} (\gamma_D, \gamma_A) = \mathbb{E}_{\bar{N}_{\text{MT}}} \{ \text{PSE} (\gamma_D, \gamma_A | \bar{N}_{\text{MT}}) \}$$

$$= (a) \lambda_{\text{MT}} B W \log_2 (1 + \gamma_D) \Pr \{ \text{SIR} \geq \gamma_D, \overline{\text{SNR}} \geq \gamma_A \} \Pr \{ \bar{N}_{\text{MT}} = 0 \}$$

$$+ \sum_{u=1}^{+\infty} \lambda_{\text{MT}} B W \log_2 (1 + \gamma_D) \frac{1}{u + 1} \Pr \{ \text{SIR} \geq \gamma_D, \overline{\text{SNR}} \geq \gamma_A \} \Pr \{ \bar{N}_{\text{MT}} = u \}$$

$$= \lambda_{\text{MT}} B W \log_2 (1 + \gamma_D) \Pr \{ \text{SIR} \geq \gamma_D, \overline{\text{SNR}} \geq \gamma_A \} \sum_{u=0}^{+\infty} \frac{\Pr \{ \bar{N}_{\text{MT}} = u \}}{u + 1}.$$

$$\quad (4.6)$$

Proof: It follows from the definition of PSE [7], where (a) originates from the fact that MT$_0$ is scheduled for transmission with unit probability if it is the only MT in the cell, while it is scheduled for transmission with probability $1/(u + 1)$ if there are other $u$ MTs in the cell. □
Lemma 8 Let Load Model 2 be assumed. The PSE (bit/sec/m$^2$) can be formulated as follows:

$$
\text{PSE} (\gamma_D, \gamma_A) = \mathbb{E}_{N_{MT}} \left\{ \text{PSE} \left( \gamma_D, \gamma_A \mid N_{MT} \right) \right\} \\
= \sum_{u=0}^{+\infty} \lambda_{MT} \frac{B_W}{u+1} \log_2 (1 + \gamma_D) \Pr \{ \text{SIR} \geq \gamma_D, \overline{\text{SNR}} \geq \gamma_A \} \Pr \{ N_{MT} = u \} \\
= \lambda_{MT} B_W \log_2 (1 + \gamma_D) \Pr \{ \text{SIR} \geq \gamma_D, \overline{\text{SNR}} \geq \gamma_A \} \sum_{u=0}^{+\infty} \frac{\Pr \{ N_{MT} = u \} }{u + 1}.
$$

(4.7)

Proof: It follows from the definition of PSE [7], where (b) originates from the fact that $N_{MT}$ is scheduled for transmission with unit probability but the bandwidth is equally allocated among the MTs in the cell, i.e., each of the $u+1$ MTs is given a bandwidth equal to $B_W/(u + 1)$.

Remark 15 By comparing (4.6) and (4.7), we note that the same PSE is obtained for both load models. This originates from the fact that $P_{\text{cov}}$ in (4.3) is independent of the number of MTs in the cell. This property follows by direct inspection of (4.4) and has been used in the proof of Lemma 7 and Lemma 8. As far as the first load model is concerned, this property originates from the fact that a single MT is scheduled at every transmission instance. It is, however, less intuitive for the second load model. In this latter case, as mentioned in Section 4.3.4, $P_{\text{tx}}$ and $B_W$ are viewed as continuous resources by the BS’s scheduler. The transmit power per unit bandwidth of both intended and interfering links is equal to $P_{\text{tx}}/B_W$. Regardless of the number of MTs available in the interfering cells, $N_{MT}$ “integrates” this transmit power per unit bandwidth over the bandwidth allocated to it, which depends on the total number of MTs in its own cell. Let the number of these MTs be $u + 1$. Thus, the receiver bandwidth of $N_{MT}$ is $B_W/(u + 1)$. This implies that the received power (neglecting path-loss and fast-fading) of both intended and interfering links is $P_{\text{rx}} = (P_{\text{tx}}/B_W)(B_W/(u + 1)) = P_{\text{tx}}/(u + 1)$. As a result, the number of MTs, $u + 1$, cancels out in the SIR of (4.4). Likewise, the received average SNR (neglecting the path-loss) is equal to $P_{\text{rx}}/(N_0 B_W/(u + 1)) = (P_{\text{tx}}/(u + 1))/(N_0 B_W/(u + 1)) = P_{\text{tx}}/\sigma_N^2$, which is independent of the number of MTs, $u + 1$, and agrees with the definition of average SNR.
in (4.4). In the next section, we show that the load models are not equivalent in terms of network power consumption.

4.4.2 Closed-Form Expressions of PSE and $P_{\text{grid}}$

In this section, we introduce new closed-form analytical frameworks for computing the PSE. We provide, in addition, closed-form expressions of the network power consumption for the two load models under analysis. These results are summarized in the following three propositions.

Let $N_{MT}$ be the number of MTs that lie in an arbitrary cell. The probability that the BS is in idle mode, $P_{BS}^{(idle)}$, and in transmission mode, $P_{BS}^{(tx)}$, can be formulated as follows [130, Prop. 1]:

\[
P_{BS}^{(idle)} = \Pr \{N_{MT} = 0\} = 1 - \mathcal{L}(\lambda_{MT}/\lambda_{BS})
\]

\[
P_{BS}^{(tx)} = \Pr \{N_{MT} \geq 1\} = 1 - P_{BS}^{(idle)} = \mathcal{L}(\lambda_{MT}/\lambda_{BS}),
\]

where $\mathcal{L}(\cdot)$ is defined in Table 4.2. Using (4.8), PSE and $P_{\text{grid}}$ are given in the following propositions.

**Proposition 6** Consider either Load Model 1 or Load Model 2. Assume notation and functions given in Tables 4.1 and 4.2. The PSE (bit/sec/m$^2$) can be formulated, in closed-form, as follows:

\[
PSE(\gamma_D, \gamma_A) = B_W \log_2 (1 + \gamma_D) \frac{\lambda_{BS} \mathcal{L}(\lambda_{MT}/\lambda_{BS})}{1 + \Upsilon \mathcal{L}(\lambda_{MT}/\lambda_{BS})} Q(\lambda_{BS}, P_{tx}, \lambda_{MT}/\lambda_{BS}).
\]  

(4.9)

Proof: See Appendix A.

**Corollary 1** If $\gamma_A = 0$, i.e., the conventional definition of $P_{\text{cov}}$ is used, the PSE in (4.9) simplifies as follows:

\[
PSE(\gamma_D, \gamma_A = 0) = B_W \log_2 (1 + \gamma_D) \frac{\lambda_{BS} \mathcal{L}(\lambda_{MT}/\lambda_{BS})}{1 + \Upsilon \mathcal{L}(\lambda_{MT}/\lambda_{BS})}.
\]

(4.10)

If, in addition, $\lambda_{MT}/\lambda_{BS} \gg 1$, the PSE in (4.9) reduces to (4.1).

Proof: It follows because $Q(\cdot,\cdot,\cdot) = 1$ if $\gamma_A = 0$ and $\mathcal{L}(\lambda_{MT}/\lambda_{BS} \gg 1) \rightarrow 1$.

□
Remark 16 Corollary 1 substantiates the comments made above in this section about the need of a new definition of PSE, as well as the advantages of the proposed analytical formulation. In particular, (4.10) confirms that the PSE is independent of $P_{tx}$ if $\gamma_A = 0$ and that the PSE is independent of both $P_{tx}$ and $\lambda_{BS}$ if fully-loaded conditions hold, i.e., $\lambda_{MT}/\lambda_{BS} \gg 1$. □

Proposition 7 Let Load Model 1 be assumed. $P_{\text{grid}}$ (Watt/m$^2$) can be formulated as follows:

$$P_{\text{grid}}^{(1)} = \lambda_{BS}(P_{tx} + P_{\text{circ}}) \mathcal{L}(\lambda_{MT}/\lambda_{BS}) + \lambda_{BS}P_{\text{idle}}(1 - \mathcal{L}(\lambda_{MT}/\lambda_{BS})). \quad (4.11)$$

Proof: The network power consumption is obtained by multiplying the average number of BSs per unit area, i.e., $\lambda_{BS}$, and the average power consumption of a generic BS, which is $P_{tx} + P_{\text{circ}}$ if the BS operates in transmission mode, i.e., with probability $\mathcal{L}(\lambda_{MT}/\lambda_{BS})$, and $P_{\text{idle}}$ if the BS operates in idle mode, i.e., with probability $1 - \mathcal{L}(\lambda_{MT}/\lambda_{BS})$. This concludes the proof. □

Proposition 8 Let Load Model 2 be assumed. $P_{\text{grid}}$ (Watt/m$^2$) can be formulated as follows:

$$P_{\text{grid}}^{(2)} = \lambda_{BS}P_{tx} \mathcal{L}(\lambda_{MT}/\lambda_{BS}) + \lambda_{MT}P_{\text{circ}} + \lambda_{BS}P_{\text{idle}}(1 - \mathcal{L}(\lambda_{MT}/\lambda_{BS})). \quad (4.12)$$

Proof: It is similar to the proof of Proposition 7. The difference is that the power dissipation of a generic BS that operates in transmission mode is, in this case, equal to $P_{tx} + P_{\text{circ}} \sum_{u=1}^{+\infty} u \Pr\{N_{MT} = u\} = P_{tx} + P_{\text{circ}}(\lambda_{MT}/\lambda_{BS})$, where $N_{MT}$ is the number of MTs in the cell and the last equality follows from [130, Lemma 1]. This concludes the proof. □

Remark 17 The power consumption models obtained in (4.11) and (4.12), which account for the transmit, circuits, and idle power consumption of the BSs, have been used, under some simplifying assumptions, in previous research works focused on the analysis of the EE of cellular networks. Among the many research works, an early paper that has adopted this approach under the assumption of fully-loaded BSs and of having a single active MT per cell is [71]. □
**Remark 18** Since $L(\lambda_{MT}/\lambda_{BS}) \leq \lambda_{MT}/\lambda_{BS}$ for every $\lambda_{MT}/\lambda_{BS} \geq 0$, we conclude that $P_{\text{grid}}^{(2)} \geq P_{\text{grid}}^{(1)}$ by assuming the same $P_{\text{tx}}$ and $\lambda_{BS}$ for both load models. This originates from the fact that, in this chapter, we assume that the circuits power consumption increases with the number of MTs that are served by the BSs. It is unclear, however, the best load model to be used from the EE standpoint, especially if $P_{\text{tx}}$ and $\lambda_{BS}$ are optimized for maximizing their respective EEs. In other words, the optimal $P_{\text{tx}}$ and $\lambda_{BS}$ that maximize the EE of each load model may be different, which may lead to different optimal EEs. The trade-off between the optimal PSE and the optimal EE is analyzed numerically in Section 4.6 for both load models. □

### 4.5 System-Level EE Optimization: Formulation and Solution

In this section, we formulate a system-level EE optimization problem and comprehensively analyze its properties. For convenience of analysis, we introduce the following auxiliary function:

$$M(\lambda_{MT}/\lambda_{BS}) = \begin{cases} 0 & \text{if Load Model 1 is assumed} \\ \lambda_{MT}/\lambda_{BS} - L(\lambda_{MT}/\lambda_{BS}) & \text{if Load Model 2 is assumed.} \end{cases}$$  

(4.13)

A unified formulation of the EE (bit/Joule) for the cellular network under analysis is as follows:

$$\text{EE}(P_{\text{tx}}, \lambda_{BS}) = \frac{\text{PSE}}{P_{\text{grid}}} \frac{B_{W} \log_2 (1 + \gamma_D) L(\lambda_{MT}/\lambda_{BS}) Q(\lambda_{BS}, P_{\text{tx}}, \lambda_{MT}/\lambda_{BS})}{[1 + Y L(\lambda_{MT}/\lambda_{BS})] [L(\lambda_{MT}/\lambda_{BS})(P_{\text{tx}} + P_{\text{circ}} - P_{\text{idle}}) + P_{\text{idle}} + M(\lambda_{MT}/\lambda_{BS}) P_{\text{circ}}]}.$$  

(4.14)

where the parameters of interest from the optimization standpoint, i.e., $P_{\text{tx}}$ and $\lambda_{BS}$, are explicitly highlighted. In the rest of this chapter, all the other parameters are assumed to be given.
Table 4.2: Summary of main auxiliary functions used throughout the chapter.

<table>
<thead>
<tr>
<th>Function Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{L}(\lambda_{MT}/\lambda_{BS}) = 1 - (1 + (1/\alpha) \lambda_{MT}/\lambda_{BS})^{-\alpha} )</td>
</tr>
<tr>
<td>( \mathcal{M}(\lambda_{MT}/\lambda_{BS}) = \lambda_{MT}/\lambda_{BS} - \mathcal{L}(\lambda_{MT}/\lambda_{BS}) )</td>
</tr>
<tr>
<td>( \mathcal{Q}(\lambda_{BS}, P_{tx}, \lambda_{MT}/\lambda_{BS}) = 1 - \exp\left(-\pi \lambda_{BS}(P_{tx}/\eta)^{2/\beta} (1 + 2\mathcal{Y} \mathcal{L}(\lambda_{MT}/\lambda_{BS}))\right) )</td>
</tr>
<tr>
<td>( \dot{\mathcal{Q}}<em>{P</em>{tx}}(\lambda_{BS}, P_{tx}, \lambda_{MT}/\lambda_{BS}) = \pi \lambda_{BS}(1/\eta)^{2/\beta} (2/\beta) (1 + 2\mathcal{Y} \mathcal{L}(\lambda_{MT}/\lambda_{BS})) P_{tx}^{2/\beta - 1} \times \exp\left(-\pi \lambda_{BS}(P_{tx}/\eta)^{2/\beta} (1 + 2\mathcal{Y} \mathcal{L}(\lambda_{MT}/\lambda_{BS}))\right) )</td>
</tr>
<tr>
<td>( \dot{\mathcal{Q}}<em>{P</em>{tx}}(\lambda_{BS}, P_{tx}, \lambda_{MT}/\lambda_{BS}) = \pi \lambda_{BS}(1/\eta)^{2/\beta} (2/\beta) (1 + 2\mathcal{Y} \mathcal{L}(\lambda_{MT}/\lambda_{BS})) P_{tx}^{2/\beta - 1} \times \left[-\dot{\mathcal{Q}}<em>{P</em>{tx}}(\lambda_{BS}, P_{tx}, \lambda_{MT}/\lambda_{BS}) \right] + \pi \lambda_{BS}(1/\eta)^{2/\beta} (2/\beta) (2/\beta - 1) \times \left(1 + 2\mathcal{Y} \mathcal{L}(\lambda_{MT}/\lambda_{BS})\right) P_{tx}^{2/\beta - 2} \times \exp\left(-\pi \lambda_{BS}(P_{tx}/\eta)^{2/\beta} (1 + 2\mathcal{Y} \mathcal{L}(\lambda_{MT}/\lambda_{BS}))\right) )</td>
</tr>
<tr>
<td>( \dot{\lambda}<em>{BS}(\lambda</em>{MT}/\lambda_{BS}) = - (\lambda_{MT}/\lambda_{BS}^2) (1 + (1/\alpha) \lambda_{MT}/\lambda_{BS})^{-(\alpha + 1)} )</td>
</tr>
<tr>
<td>( \dot{\lambda}<em>{MBS}(\lambda</em>{MT}/\lambda_{BS}) = - (\lambda_{MT}/\lambda_{BS}^2) \left[ 1 - (1 + (1/\alpha) \lambda_{MT}/\lambda_{BS})^{-(\alpha + 1)} \right] )</td>
</tr>
<tr>
<td>( \dot{\lambda}<em>{MBS}(\lambda</em>{MT}/\lambda_{BS}) = \pi (P_{tx}/\eta)^{2/\beta} \left[ 1 + 2\mathcal{Y} \mathcal{L}(\lambda_{MT}/\lambda_{BS}) + \lambda_{BS} \dot{\lambda}<em>{BS}(\lambda</em>{MT}/\lambda_{BS}) \right] \times \exp\left(-\pi \lambda_{BS}(P_{tx}/\eta)^{2/\beta} (1 + 2\mathcal{Y} \mathcal{L}(\lambda_{MT}/\lambda_{BS}))\right) )</td>
</tr>
<tr>
<td>( \dot{\lambda}<em>{BS}(\lambda</em>{MT}/\lambda_{BS}) = (\lambda_{MT}/\lambda_{BS}^3) (1 + (1/\alpha) \lambda_{MT}/\lambda_{BS})^{-(\alpha + 1)} \times \left[ 2 - (1 + \alpha) (1/\alpha) \lambda_{MT}/\lambda_{BS} (1 + (1/\alpha) \lambda_{MT}/\lambda_{BS})^{-1} \right] )</td>
</tr>
<tr>
<td>( \dot{\lambda}<em>{MBS}(\lambda</em>{MT}/\lambda_{BS}) = 2 (\lambda_{MT}/\lambda_{BS}) \left[ 1 - (1 + (1/\alpha) \lambda_{MT}/\lambda_{BS})^{-(\alpha + 1)} \right] \quad + \quad (1 + \alpha) (1/\alpha) (\lambda_{MT}/\lambda_{BS}^2) (1 + (1/\alpha) \lambda_{MT}/\lambda_{BS})^{-(\alpha + 2)} )</td>
</tr>
<tr>
<td>( S_P(P_{tx}) = \left( \frac{\lambda_{MT}}{\lambda_{BS}} \right)^{p_{circ}} \left[ \frac{\mathcal{Q}(\lambda_{BS}, P_{tx}, \lambda_{MT}/\lambda_{BS})}{P_{tx}} - (P_{tx} + \Delta P) \right] - P_{circ} \mathcal{M}(\lambda_{MT}/\lambda_{BS}) )</td>
</tr>
<tr>
<td>( S_D(\lambda_{BS}) = \mathcal{L}(\lambda_{BS}) - \mathcal{L}(\lambda_{BS}) (P_{tx} + \Delta P) + P_{circ} \mathcal{L}(\lambda_{BS}) + \mathcal{L}(\lambda_{BS}) (P_{tx} + \Delta P) \times \left( \frac{\lambda_{MT}}{\lambda_{BS}} \right)^{p_{circ}} \mathcal{M}(\lambda_{MT}/\lambda_{BS}) )</td>
</tr>
<tr>
<td>( - \mathcal{L}(\lambda_{BS}) (P_{tx} + \Delta P) \times \left( \frac{\lambda_{MT}}{\lambda_{BS}} \right)^{p_{circ}} \mathcal{M}(\lambda_{MT}/\lambda_{BS}) )</td>
</tr>
</tbody>
</table>

4.5.1 Preliminaries

For ease of presentation, we report some lemmas that summarize structural properties of the main functions that constitute (4.14). Some lemmas are stated without proof because they are obtained by simply studying the sign of the first-order and second-order derivatives of the function with respect to the variable of interest and by keeping all the other variables fixed. Functions of interest for this section are given in Table 4.2. Also, we define \( \Delta P = P_{circ} - P_{idle} \geq 0 \).

**Lemma 9** The function \( \mathcal{L}(\lambda_{MT}/\lambda_{BS}) \) fulfills the following properties with respect to \( \lambda_{BS} \) (assuming \( \lambda_{MT} \) fixed): i) \( \mathcal{L}(\lambda_{MT}/\lambda_{BS}) \geq 0 \) for \( \lambda_{BS} \geq 0 \); ii) \( \mathcal{L}(\lambda_{MT}/\lambda_{BS}) = 1 \) if \( \lambda_{BS} \to 0 \); iii) \( \mathcal{L}(\lambda_{MT}/\lambda_{BS}) = 0 \) if \( \lambda_{BS} \to \infty \); iv) \( \dot{\lambda}_{BS}(\lambda_{MT}/\lambda_{BS}) \leq 0 \) for \( \lambda_{BS} \geq 0 \);
\vspace{0.5cm}
\begin{itemize}
\item[v)] \( \tilde{L}_{\lambda_{\text{BS}}} (\lambda_{MT}/\lambda_{\text{BS}}) \leq 0 \) for \( \lambda_{MT}/\lambda_{\text{BS}} \geq 2\alpha/(\alpha - 1) = 2.8 \); and
\item[vi)] \( \tilde{L}_{\lambda_{\text{BS}}} (\lambda_{MT}/\lambda_{\text{BS}}) \geq 0 \) for \( \lambda_{MT}/\lambda_{\text{BS}} \leq 2\alpha/(\alpha - 1) = 2.8 \).
\end{itemize}

Lemma 10 As far as Load Model 2 is concerned, the function \( \mathcal{M} (\lambda_{MT}/\lambda_{\text{BS}}) \) fulfills the following properties with respect to \( \lambda_{\text{BS}} \) (assuming \( \lambda_{MT} \) fixed): i) \( \mathcal{M} (\lambda_{MT}/\lambda_{\text{BS}}) \geq 0 \) for \( \lambda_{\text{BS}} \geq 0 \); ii) \( \mathcal{M} (\lambda_{MT}/\lambda_{\text{BS}}) \rightarrow \infty \) if \( \lambda_{\text{BS}} \rightarrow 0 \); iii) \( \mathcal{M} (\lambda_{MT}/\lambda_{\text{BS}}) = 0 \) if \( \lambda_{\text{BS}} \rightarrow \infty \); iv) \( \mathcal{M}_{\lambda_{\text{BS}}} (\lambda_{MT}/\lambda_{\text{BS}}) \leq 0 \) for \( \lambda_{\text{BS}} \geq 0 \); and v) \( \mathcal{M}_{\lambda_{\text{BS}}} (\lambda_{MT}/\lambda_{\text{BS}}) \geq 0 \) for \( \lambda_{\text{BS}} \geq 0 \).

Lemma 11 The function \( \mathcal{Q}(\lambda_{\text{BS}}, P_{tx}, \lambda_{MT}/\lambda_{\text{BS}}) \) fulfills the following properties with respect to \( P_{tx} \): i) \( \mathcal{Q}(\lambda_{\text{BS}}, P_{tx}, \lambda_{MT}/\lambda_{\text{BS}}) \geq 0 \) for \( P_{tx} \geq 0 \);
\item[ii)] \( \mathcal{Q}(\lambda_{\text{BS}}, P_{tx}, \lambda_{MT}/\lambda_{\text{BS}}) = 0 \) if \( P_{tx} \rightarrow 0 \);
\item[iii)] \( \mathcal{Q}(\lambda_{\text{BS}}, P_{tx}, \lambda_{MT}/\lambda_{\text{BS}}) = 1 \) if \( P_{tx} \rightarrow \infty \);
\item[iv)] \( \mathcal{P}_{P_{tx}} (\lambda_{\text{BS}}, P_{tx}, \lambda_{MT}/\lambda_{\text{BS}}) \geq 0 \) for \( P_{tx} \geq 0 \); and v) \( \mathcal{P}_{P_{tx}} (\lambda_{\text{BS}}, P_{tx}, \lambda_{MT}/\lambda_{\text{BS}}) \leq 0 \) for \( P_{tx} \geq 0 \).

Proof: The result in v) follows from \( \mathcal{P}_{P_{tx}} (\cdot, \cdot, \cdot) \) in Table 4.2, since iv) and \( \beta > 2 \) hold.
\[ \square \]

Lemma 12 The function \( \mathcal{Q}(\lambda_{\text{BS}}, P_{tx}, \lambda_{MT}/\lambda_{\text{BS}}) \) fulfills the following properties with respect to \( \lambda_{\text{BS}} \) (assuming \( \lambda_{MT} \) fixed): i) \( \mathcal{Q}(\lambda_{\text{BS}}, P_{tx}, \lambda_{MT}/\lambda_{\text{BS}}) \geq 0 \) for \( \lambda_{\text{BS}} \geq 0 \);
\item[ii)] \( \mathcal{Q}(\lambda_{\text{BS}}, P_{tx}, \lambda_{MT}/\lambda_{\text{BS}}) = 0 \) if \( \lambda_{\text{BS}} \rightarrow 0 \);
\item[iii)] \( \mathcal{Q}(\lambda_{\text{BS}}, P_{tx}, \lambda_{MT}/\lambda_{\text{BS}}) = 1 \) if \( \lambda_{\text{BS}} \rightarrow \infty \);
\item[iv)] \( \mathcal{Q}_{\lambda_{\text{BS}}} (\lambda_{\text{BS}}, P_{tx}, \lambda_{MT}/\lambda_{\text{BS}}) \geq 0 \) for \( \lambda_{\text{BS}} \geq 0 \); and v) \( \mathcal{Q}_{\lambda_{\text{BS}}} (\lambda_{\text{BS}}, P_{tx}, \lambda_{MT}/\lambda_{\text{BS}}) \leq 0 \) for \( \lambda_{\text{BS}} \geq 0 \).

Proof: The result in iv) follows from \( \mathcal{Q}_{\lambda_{\text{BS}}} (\cdot, \cdot, \cdot) \) in Table 4.2 because, for \( \lambda_{\text{BS}} \geq 0 \), \( L (\lambda_{MT}/\lambda_{\text{BS}}) + \lambda_{\text{BS}} \mathcal{Q}_{\lambda_{\text{BS}}} (\lambda_{MT}/\lambda_{\text{BS}}) \geq 0 \). This latter inequality holds true because \( 1 + x (1 + 1/\alpha) \leq (1 + x/\alpha)^{(\alpha + 1)} \) for \( x \geq 0 \). The result in v) follows without explicitly computing \( \mathcal{Q}_{\lambda_{\text{BS}}} (\cdot, \cdot, \cdot) \) because \( \mathcal{Q}_{\lambda_{\text{BS}}} (\cdot, \cdot, \cdot) \) in Table 4.2 is the composition of two increasing and concave functions in \( \lambda_{\text{BS}} \), i.e., the function in the square brackets in the first row and the exponential function in the second row.
\[ \square \]

Lemma 13 The EE in (4.14) fulfills the following properties with respect to \( P_{tx} \) and \( \lambda_{\text{BS}} \):
\item[i)] \( \text{EE}(P_{tx}, \lambda_{\text{BS}}) = 0 \) if \( P_{tx} \rightarrow 0 \) or \( \lambda_{\text{BS}} \rightarrow 0 \); and
\item[ii)] \( \text{EE}(P_{tx}, \lambda_{\text{BS}}) = 0 \) if \( P_{tx} \rightarrow \infty \) or \( \lambda_{\text{BS}} \rightarrow \infty \).

Proof: This immediately follows from Lemmas 9-12.
4.5.2 Optimal Transmit Power Given the Density of the BSs

In this section, we analyze whether there exists an optimal and unique transmit power, $P_{\text{tx}}^{(\text{opt})}$, that maximizes the EE formulated in (4.14), while all the other parameters, including $\lambda_{\text{BS}}$, are fixed and given. In mathematical terms, the optimization problem can be formulated as follows:

$$\max_{P_{\text{tx}}} \text{EE} (P_{\text{tx}}, \lambda_{\text{BS}}) \quad \text{subject to } P_{\text{tx}} \in \left[ P_{\text{tx}}^{(\text{min})}, P_{\text{tx}}^{(\text{max})} \right],$$

(4.15)

where $P_{\text{tx}}^{(\text{min})} \geq 0$ and $P_{\text{tx}}^{(\text{max})} \geq 0$ are the minimum and maximum power budget of the BSs, respectively. One may assume, without loss of generality, $P_{\text{tx}}^{(\text{min})} \to 0$ and $P_{\text{tx}}^{(\text{max})} \to \infty$.

The following theorem completely characterizes the solution of (4.15).

**Theorem 1** Let $S_P (\cdot)$ be the function defined in Table 4.2. The EE in (4.14) is a unimodal and strictly pseudo-concave function in $P_{\text{tx}}$. The optimization problem in (4.15) has a unique solution given by $P_{\text{tx}}^{(\text{opt})} = \max \left\{ P_{\text{tx}}^{(\text{min})}, \min \left\{ P_{\text{tx}}^*, P_{\text{tx}}^{(\text{max})} \right\} \right\}$, where $P_{\text{tx}}^*$ is the only stationary point of the EE in (4.14) that is obtained as the unique solution of the following equation:

$$\text{EE}_{P_{\text{tx}}} (P_{\text{tx}}^*, \lambda_{\text{BS}}) = \text{P}_{\text{idle}} - S_P (P_{\text{tx}}^*) = 0 \leftrightarrow S_P (P_{\text{tx}}^*) = \text{P}_{\text{idle}}.$$

(4.16)

Proof: See Appendix B.

4.5.3 Optimal Density Given the Transmit Power of the BSs

In this section, we analyze whether there exists an optimal and unique density of BSs, $\lambda_{\text{BS}}^{(\text{opt})}$, that maximizes the EE formulated in (4.14), while all the other parameters, including $P_{\text{tx}}$, are fixed and given. In mathematical terms, the optimization problem can be formulated as follows:

$$\max_{\lambda_{\text{BS}}} \text{EE} (P_{\text{tx}}, \lambda_{\text{BS}}) \quad \text{subject to } \lambda_{\text{BS}} \in \left[ \lambda_{\text{BS}}^{(\text{min})}, \lambda_{\text{BS}}^{(\text{max})} \right],$$

(4.17)

where $\lambda_{\text{BS}}^{(\text{min})} \geq 0$ and $\lambda_{\text{BS}}^{(\text{max})} \geq 0$ are the minimum and maximum allowed density of the BSs, respectively. One may assume, without loss of generality, $\lambda_{\text{BS}}^{(\text{min})} \to 0$ and $\lambda_{\text{BS}}^{(\text{max})} \to \infty$. 

Proof: See Appendix B.
The following theorem completely characterizes the solution of (4.17).

**Theorem 2** Let $S_D(\cdot)$ be the function defined in Table 4.2. The EE in (4.14) is a unimodal and strictly pseudo-concave function in $\lambda_{BS}$. The optimization problem in (4.17) has a unique solution given by $\lambda_{BS}^{(\text{opt})} = \max \left\{ \lambda_{BS}^{(\text{min})}, \min \left\{ \lambda_{BS}^{*}, \lambda_{BS}^{(\text{min})} \right\} \right\}$, where $\lambda_{BS}^{*}$ is the only stationary point of the EE in (4.14) that is obtained as the unique solution of the following equation:

$$\hat{E}_E(\lambda_{BS}^{*}, P_{tx}) = S_D(\lambda_{BS}^{*}) - P_{idle} = 0 \iff S_D(\lambda_{BS}^{*}) = P_{idle}. \quad (4.18)$$

**Proof:** See Appendix C. \[\square\]

### 4.5.4 On the Dependency of Optimal Transmit Power and Density of the BSs

The optimal transmit power and BSs’ density that maximize the EE are obtained from the unique solutions of (4.16) and (4.18), respectively. These equations, however, cannot be further simplified and, therefore, explicit analytical expressions for $P_{tx}^{(\text{opt})}$ and $\lambda_{BS}^{(\text{opt})}$ cannot, in general, be obtained. This is an inevitable situation when dealing with EE optimization problems, and, indeed, a closed-form expression of the optimal transmit power for simpler EE optimization problems does not exist either [108]. In some special cases, the transmit power can be implicitly expressed in terms of the Lambert-W function, which, however, is the solution of a transcendental equation [74]. Notable examples of these case studies include even basic point-to-point communication systems without interference [133]. Based on these considerations, it seems hopeless to attempt finding explicit analytical expressions from (4.16) and (4.18), respectively. However, thanks to the properties of the EE function, i.e., unimodality and strict pseudo-concavity, proved in **Theorem 1** and **Theorem 2**, $P_{tx}^{(\text{opt})}$ and $\lambda_{BS}^{(\text{opt})}$ can be efficiently computed with the aid of numerical methods that are routinely employed to obtain the roots of non-linear scalar equations, e.g., the Newton’s method [134]. For example, the unique solutions of (4.16) and (4.18) may be obtained by using the functions **FSo1ve** in Matlab and **NSo1ve** in Mathematica. **Theorem 1** and **Theorem 2** are, however, of paramount importance, since they
state that an optimum exists and is unique.

Even though explicit analytical formulas for $P_{tx}^{(opt)}$ and $\lambda_{BS}^{(opt)}$ cannot be obtained, it is important to understand how these optimal values change if any other system parameter changes. For instance, two worthwhile questions to answer are: “How does $P_{tx}^{(opt)}$ change as a function of $\lambda_{BS}$?” and “How does $\lambda_{BS}^{(opt)}$ change as a function of $P_{tx}$?” These questions are relevant to optimize the deployment of cellular networks from the EE standpoint, since they unveil the inherent interplay between transmit power and density discussed in Section 4.4 and illustrated in Fig. 4.1. A general answer to these two questions is provided in the following two propositions.

**Proposition 9** Let $P_{tx}^*$ be the unique solution of (4.16) if $\lambda_{BS} = \bar{\lambda}_{BS}$. Let the optimal $P_{tx}$ according to Theorem 1 be $P_{tx}^{(opt)} = \max \{P_{tx}^{(min)}, \min \{P_{tx}^{*}, P_{tx}^{(opt)}\}\}$. Let $\bar{\lambda}_{BS} \leq \lambda_{BS}$ be another BSs’ density. Let $\bar{\lambda}_{BS}$ be the first-order derivative in (4.16). The following holds:

$$P_{tx}^{(opt)} \preceq P_{tx}^{*} \quad \Leftrightarrow \quad \bar{\lambda}_{BS} \preceq \lambda_{BS}^{(opt)}.$$  \hspace{1cm} (4.19)

Proof: Theorem 1 states that the EE function has a single stationary point that is its unique global maximizer. In mathematical terms, this implies $\bar{\lambda}_{BS}^{EE}(P_{tx}^{*}, \lambda_{BS}) > 0$ if $P_{tx} < P_{tx}^*$ and $\bar{\lambda}_{BS}^{EE}(P_{tx}, \lambda_{BS}) < 0$ if $P_{tx} > P_{tx}^*$ for every $\lambda_{BS} \geq 0$. Therefore, the optimal transmit power needs to be increased (decreased) if the first-order derivative of the EE is positive (negative). Based on this, (4.19) follows because $\min \{\cdot, \cdot\}$ and $\max \{\cdot, \cdot\}$ are increasing functions. \hfill $\square$

**Proposition 10** Let $\bar{\lambda}_{BS}$ be the unique solution of (4.18) if $P_{tx} = \bar{P}_{tx}$. Let the optimal $\lambda_{BS}$ according to Theorem 2 be $\lambda_{BS}^{(opt)} = \max \{\lambda_{BS}^{(min)}, \min \{\lambda_{BS}^{*}, \lambda_{BS}^{(opt)}\}\}$. Let $\bar{P}_{tx} \leq \bar{P}_{tx}$ be another transmit power. Let $\bar{\lambda}_{BS}^{EE}(\cdot, \cdot)$ be the first-order derivative in (4.18). The following holds:

$$\lambda_{BS}^{(opt)} \preceq \lambda_{BS}^{(opt)} \quad \Leftrightarrow \quad \bar{\lambda}_{BS}^{EE}(P_{tx}^{(opt)}, \lambda_{BS}) \preceq 0.$$  \hspace{1cm} (4.20)

Proof: It follows from Theorem 2, similar to the proof of Proposition 9. \hfill $\square$

**Remark 19** It is worth mentioning that the approach utilized to prove Proposition 9 and Proposition 10 is applicable to study the dependency of $P_{tx}^{(opt)}$ and $\lambda_{BS}^{(opt)}$, respectively, with...
Table 4.3: Alternating optimization of the EE.

<table>
<thead>
<tr>
<th>Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Let $P_{tx} \in \left[ P_{tx}^{(\min)}, P_{tx}^{(\max)} \right]$; $\lambda_{BS} \in \left[ \lambda_{BS}^{(\min)}, \lambda_{BS}^{(\max)} \right]$;</td>
</tr>
<tr>
<td>Set $\lambda_{BS} = \lambda_{BS}^{(\text{opt})} \in \left[ \lambda_{BS}^{(\min)}, \lambda_{BS}^{(\max)} \right]$ (initial guess); $V = 0$; $\epsilon &gt; 0$;</td>
</tr>
<tr>
<td>Repeat</td>
</tr>
<tr>
<td>$V_0 = V$;</td>
</tr>
<tr>
<td>$P_{tx}^{(\text{opt})} = \max \left{ P_{tx}^{(\min)}, \min \left{ P_{tx}^{(\text{opt})}, P_{tx}^{(\max)} \right} \right}$; (4.16)</td>
</tr>
<tr>
<td>$\lambda_{BS}^{(\text{opt})} = \max \left{ \lambda_{BS}^{(\min)}, \min \left{ \lambda_{BS}^{(\text{opt})}, \lambda_{BS}^{(\max)} \right} \right}$; (4.18)</td>
</tr>
<tr>
<td>$V = \EE \left( P_{tx}^{(\text{opt})}, \lambda_{BS}^{(\text{opt})} \right)$; (4.14)</td>
</tr>
<tr>
<td>Until $</td>
</tr>
<tr>
<td>Return $P_{tx}^{(\text{opt})} = P_{tx}^{(\text{opt})}$; $\lambda_{BS}^{(\text{opt})} = \lambda_{BS}^{(\text{opt})}$.</td>
</tr>
</tbody>
</table>

respect to any other system parameters. The findings in Proposition 9 and Proposition 10 are especially relevant for cellular network planning. Let us consider, e.g., (4.19). By simply studying the sign of the first-order derivative $\EE_{P_{tx}} (\cdot, \cdot)$, one can identify, with respect to an optimally deployed cellular network, the set of BSs’ densities that would require to increase or decrease the transmit power while still operating at the optimum. In Section 4.6, numerical examples are shown to highlight that $P_{tx}^{(\text{opt})}$ may either decrease or increase as $\lambda_{BS}$ increases or decreases. □

### 4.5.5 Joint Optimization of Transmit Power and Density of the BSs

In Sections 4.5.2 and 4.5.3, either $\lambda_{BS}$ or $P_{tx}$ are assumed to be given, respectively. In practical applications, however, it is important to identify the optimal pair $\left( P_{tx}^{(\text{opt})}, \lambda_{BS}^{(\text{opt})} \right)$ that jointly maximizes the EE in (4.14). This joint optimization problem can be formulated as follows:

$$
\max_{P_{tx}, \lambda_{BS}} \EE (P_{tx}, \lambda_{BS}) \quad \text{subject to } P_{tx} \in \left[ P_{tx}^{(\min)}, P_{tx}^{(\max)} \right] \text{ and } \lambda_{BS} \in \left[ \lambda_{BS}^{(\min)}, \lambda_{BS}^{(\max)} \right],
$$

(4.21)

where a notation similar to that used in (4.15) and (4.17) is adopted.

In Theorem 1 and Theorem 2, we have solved the optimization problem formulated in (4.21) with respect to $P_{tx}$ for a given $\lambda_{BS}$ and with respect to $\lambda_{BS}$ for a given $P_{tx}$,
respectively. By leveraging these results, a convenient approach for tackling (4.21) with respect to $P_{tx}$ and $\lambda_{BS}$ is to utilize the alternating optimization method, which iteratively optimizes $P_{tx}$ for a given $\lambda_{BS}$ and $\lambda_{BS}$ for a given $P_{tx}$ until convergence of the EE in (4.14) within a desired level of accuracy [135, Proposition 2.7.1]. The algorithm that solves (4.21) based on the alternating optimization method is reported in Table 4.3. Its convergence and optimality properties are summarized as follows.

**Proposition 11** Let $P_{tx}^{(opt)}(n)$, $\lambda_{BS}^{(opt)}(n)$, and $EE(n)$ be $P_{tx}$, $\lambda_{BS}$ and EE obtained from the algorithm in Table 4.3 at the $n$th iteration, respectively. The sequence $EE(n)$ is monotonically increasing and converges. In addition, every limit point of the sequence $(P_{tx}^{(opt)}(n), \lambda_{BS}^{(opt)}(n))$ fulfills the Karush-Kuhn-Tucker (KKT) first-order optimality conditions of the problem in (4.21).

**Proof:** At the end of each iteration of the algorithm in Table 4.3, the value of EE does not decrease. The sequence $EE(n)$, hence, converges, because the EE in (4.14) is a continuous function over the compact feasible set of the problem in (4.21) and, thus, it admits a finite maximum by virtue of the Weierstrass extreme value theorem [135]. From [135, Proposition 2.7.1], the alternating optimization method fulfills the KKT optimality conditions, provided that i) the objective and constraint functions are differentiable, ii) each constraint function depends on a single variable, and iii) each subproblem has a unique solution. The first and second requirements follow by direct inspection of (4.21). The third requirement is ensured by Theorem 1 and Theorem 2. \square

**Remark 20** The optimization problems in Theorem 1 and Theorem 2 can be efficiently solved by using the Newton's method, which allows one to find the root of real-valued objective functions via multiple iterations of increasing accuracy and at a super-linear (i.e., quadratic if the initial guess is sufficiently close to the actual root) convergence rate [134]. The properties of convergence of the alternating maximization algorithm in Table 4.3 to a stationary point of the objective function in (4.21) are discussed in [135, Proposition 2.7.1]. Under mild assumptions that hold for the specific problem at hand, the algorithm in Table 4.3 is locally $q$-linearly convergent to a local maximizer of the objective function provided that the initial guess is sufficiently close to the actual root [136, Section 2]. Further details can be found in [136]. \square
TABLE IV: Setup of parameters (unless otherwise stated). It is worth noting that the setup $\gamma_D = \gamma_A$ constitutes just a case study and that the main findings of the present chapter hold true for every $\gamma_A > 0$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>3.5</td>
</tr>
<tr>
<td>$\kappa$ = $(4\pi f_c / 3 \cdot 10^8)^2$</td>
<td>$f_c = 2.1$ GHz</td>
</tr>
<tr>
<td>$N_0$</td>
<td>-174 dBm/Hz</td>
</tr>
<tr>
<td>$B_W$</td>
<td>20 MHz</td>
</tr>
<tr>
<td>$P_{\text{circ}}$</td>
<td>51.14 dBm [71]</td>
</tr>
<tr>
<td>$P_{\text{idle}}$</td>
<td>48.75 dBm [71]</td>
</tr>
<tr>
<td>$P_{\text{tx}}$</td>
<td>43 dBm [71]</td>
</tr>
<tr>
<td>$\lambda_{\text{BS}} = 1 / (\pi R_{\text{cell}}^2)$ BSs/m$^2$</td>
<td>$R_{\text{cell}} = 250$ m</td>
</tr>
<tr>
<td>$\lambda_{\text{MT}} = 1 / (\pi R_{\text{MT}}^2) = 121$ MTs/km$^2$</td>
<td>$R_{\text{MT}} = 51.29$ m</td>
</tr>
<tr>
<td>$\gamma_D = \gamma_A$</td>
<td>5 dB</td>
</tr>
</tbody>
</table>

Figure 4.2: Optimal transmit power (a) and energy efficiency (b) versus $R_{\text{cell}}$. Solid lines: Optimum from Theorem 1. Markers: Optimum from a brute-force search of (4.15). Special case with $\beta = 6.5$ and $\lambda_{\text{MT}} = 21$ MTs/km$^2$.

In Section 4.6, numerical evidence of the global optimality of the algorithm in Table 4.3 is given as well. In addition, numerical results on the average (with respect to the initial guess) number of iterations as a function of the tolerance of convergence, $\epsilon > 0$, are illustrated.
4.5. System-Level EE Optimization: Formulation and Solution

Figure 4.3: Energy efficiency versus the transmit power for Load Model 1 (a) and Load Model 2 (b). Solid lines: Framework from (4.14). Markers: Monte Carlo simulations.

Figure 4.4: Energy efficiency versus $R_{\text{cell}}$ for Load Model 1 (a) and Load Model 2 (b). Solid lines: Mathematical Framework from (4.14). Markers: Monte Carlo simulations.
4.6 Numerical Results

In this section, we show numerical results to validate the proposed mathematical framework for computing the PSE and EE, as well as to substantiate the findings originating from the analysis of the system-level EE optimization problems as a function of the transmit power and density of the BSs. Unless otherwise stated, the simulation setup is summarized in Table IV. For ease of understanding, the BSs’ density is represented via the inter-site distance ($R_{\text{cell}}$) defined in Table IV. A similar comment applies to the density of the MTs that is expressed in terms of their average distance ($R_{\text{MT}}$). As far as the choice of the setup of parameters is concerned, it is worth mentioning that the power consumption model is in agreement with [71] and [125]. The density of the MTs coincides with the average density of inhabitants in France.

Validation Against Monte Carlo Simulations

In Figs. 4.3 and 4.4, we validate the correctness of (4.14) against Monte Carlo simulations. Monte Carlo results are obtained by simulating several realizations, according to the
4.6. Numerical Results

Figure 4.6: Optimal density of BSs ($R_{\text{cell}}$) (a) and energy efficiency (b) versus the transmit power. Solid lines: Optimum from Theorem 2. Markers: Optimum from a brute-force search of (4.17). LM-1: Load Model 1, LM-2: Load Model 2.

PPP model, of the cellular network and by empirically computing the PSE according to its definition in (4.6) and (4.7), as well as the power consumption based on the operating principle described in the proofs of Proposition 7 and Proposition 8. It is worth mentioning that, to estimate the PSE, only the definitions in the first line of (4.6) and (4.7) are used. The results depicted in Figs. 4.3 and 4.4 confirm the good accuracy of the proposed mathematical approach. They highlight, in addition, the unimodal and pseudo-concave shape of the EE as a function of the transmit power, given the BSs’ density, and of the BSs’ density, given the transmit power. If the same transmit power and BSs’ density are assumed for both load models, we observe, as expected, that the first load model provides a higher EE than the second one.

Validation of Theorem 1 and Theorem 2

In Figs. 4.5 and 4.6, we compare the optimal transmit power and BSs’ density obtained from Theorem 1 and Theorem 2, i.e., by computing the unique zero of (4.16) and (4.18), respectively, against a brute-force search of the optimum of (4.15) and (4.17), respectively.
Figure 4.7: Optimal transmit power (a), density of BSs ($R_{\text{cell}}$) (b), and energy efficiency (c) versus the density of MTs ($R_{\text{MT}}$). Solid lines: Optimum from the algorithm in Table 4.3. Markers: Optimum from a brute-force search of (4.21). LM-1: Load Model 1 and LM-2: Load Model 2.

We observe the correctness of Theorem 1 and Theorem 2 for the load models analyzed in this chapter. Figures 4.5 and 4.6, in addition, confirm two important remarks that we have made throughout this chapter. The first is that a joint pair of transmit power and BSs’ density exists. This is highlighted by the fact that the EE evaluated at the optimal transmit power, given the BSs’ density, and at the optimal BSs’ density, given the transmit power, is still a unimodal and pseudo-concave function. This motivates one to use the alternating optimization algorithm proposed in Section 4.5.5. The second is related to the difficulty of obtaining an explicit closed-form expression of the optimal transmit power as a function of the BSs’ density and of the BSs’ density as a function of the transmit power. Figure 4.6(a), for example, clearly shows that the behavior of the optimal transmit power is not monotonic as a function of the BSs’ density. This is in contrast with heuristic optimization criteria based on the coverage probability metric [127]. Figure 4.5(a), on the other hand, provides more intuitive trends according to which the optimal transmit power increases as the density of the BSs decreases. This is, however, just a special case that
4.6. Numerical Results

Figure 4.8: Optimal transmit power (a), density of BSs ($R_{\text{cell}}$) (b), and energy efficiency (c) versus the reliability thresholds ($\gamma_D = \gamma_A$). Solid lines: Optimum from the algorithm in Table 4.3. Markers: Optimum from a brute-force search of (4.21). LM-1: Load Model 1 and LM-2: Load Model 2.

is parameter-dependent. A counter-example is, in fact, illustrated in Fig. 4.2, where, for a different set of parameters, it is shown that the optimal transmit power may increase, decrease and then increase again as a function of the average inter-site distance of the BSs ($R_{\text{cell}}$). In this case, the density of the MTs coincides with the average density of inhabitants in Sweden and a large path-loss exponent is assumed to highlight the peculiar performance trend. These numerical examples clearly substantiate the importance of Theorem 1 and Theorem 2, and highlight the complexity of the optimization problem that is analyzed and successfully solved in this chapter.

Validation of the Alternating Optimization Algorithm in Table 4.3

In Figs. 4.7 and 4.8, we provide numerical evidence of the convergence of the alternating optimization algorithm introduced in Section 4.5.5 towards the global optimum of the optimization problem formulated in (4.21). The study is performed by computing the joint optimal transmit power and BSs’ density as a function of the density of the MTs.
Figure 4.9: Analysis of the EE vs. PSE trade-off. Solid lines: Optimum from the algorithm in Table 4.3. Markers: Optimum from a brute-force search of (4.21). LM-1: Load Model 1 and LM-2: Load Model 2.

(Fig. 4.7) and of the reliability thresholds (Fig. 4.8). We observe a very good agreement between the algorithm in Table 4.3 and a brute-force search of the optimum of (4.21).

**Comparison Between Load Model 1 and 2**

With the exception of Figs. 4.3 and 4.4, all the figures reported in this section illustrate the achievable EE of the two load models analyzed in this chapter when they operate at their respective optima. Based on the obtained results, we conclude that, for the considered system setup, the first load model outperforms the second one in terms of EE. Figures 4.7 and 4.8 show, for example, that this may be obtained by transmitting a higher power but, at the same time, by reducing the deployment density of the BSs. It is worth mentioning that, even though both load models provide the same PSE and serve, in the long time-horizon, all the MTs of the network, they have one main difference: the MTs under the first load model experience a higher latency (i.e., the MTs experience a longer delay before being served, since they are randomly chosen among all the available MTs in the cell), since a single MT is served at any time instance. We evince, as a result, that the higher
4.6. Numerical Results

Figure 4.10: Number of iterations of the algorithm in Table 4.3 as a function of $\epsilon > 0$. The number of iterations is averaged (15000 trials) over the initial guess $\lambda_{BS} = \lambda_{BS}^{(opt)} \in \left[\lambda_{BS}^{(min)}, \lambda_{BS}^{(max)}\right]$. (a) Load Model 1 and (b) Load Model 2. Setup: $R_{cell}^{(min)} = 10$ m, $R_{cell}^{(max)} = 2000$ m, $P_{tx}^{(min)} = -20$ dBm, $P_{tx}^{(max)} = 60$ dBm.

EE provided by the first load model is obtained at the price of increasing the MTs’ latency. The analysis and optimization of energy-efficient cellular networks with latency constraints is, therefore, an important generalization of the study conducted in this chapter.

Analysis of the EE vs. PSE Trade-Off

In Fig. 4.9, we illustrate the trade-off between EE and PSE, which is obtained by setting the transmit power and density of the BSs at the optimal values that are obtained by solving the optimization problem in (4.21) with the aid of the algorithm in Table 4.3. Figure 4.9 provides a different view of the comparison between Load Model 1 and 2 introduced in Section 4.3.4. The Load Model 1 is a suitable choice for obtaining a high EE at low-medium PSEs, while the Load Model 2 is a more convenient option for obtaining a good EE at medium-high PSEs. Based on these results, the optimization of the EE vs. PSE trade-off constitutes an interesting generalization of the study carried out in this chapter.
thesis.

**Convergence Analysis of the Maximization Algorithm in Table 4.3**

Motivated by *Remark 20*, Fig. 4.10 shows the average number of iterations of the alternating optimization algorithm in Table 4.3 as a function of the convergence accuracy \( \epsilon \). We observe that the algorithm necessitates more iterations for Load Model 1. In general, however, we observe that the number of iterations that are required to converge within the required convergence accuracy is relatively small.

### 4.7 Conclusion

In this chapter, we have introduced a new closed-form analytical expression of the potential spectral efficiency of cellular networks. Unlike currently available analytical frameworks, we have shown that the proposed approach allows us to account for the tight interplay between transmit power and density of the base stations in cellular networks. Therefore, the proposed approach is conveniently formulated for the optimization of the network planning of cellular networks, by taking into account important system parameters. We have applied the new approach to the analysis and optimization of the energy efficiency of cellular networks. We have mathematically proved that the proposed closed-form expression of the energy efficiency is a unimodal and strictly pseudo-concave function in the transmit power, given the density, and in the density, given the transmit power. Under these assumptions, as a result, a unique transmit power and density of the base stations exist, which can be obtained by finding the unique zero of a simple non-linear function that is provided in a closed-form expression. All mathematical derivations and findings have been substantiated with the aid of numerical simulations. We argue that the applications of the proposed approach to the system-level modeling and optimization of cellular networks are countless and go beyond the formulation of energy efficiency problems.

Extensions and generalizations of the mathematical and optimization frameworks proposed in this chapter, include, but are not limited to, the system-level analysis and optimization of i) the energy efficiency versus spectral efficiency trade-off, ii) uplink cellular

4.8 Appendix A – Proof of Proposition 6

Under the assumption that $\text{MT}_0$ is selected, from (4.3) and (4.4), we have:

$$P_{\text{cov}}(\gamma_D, \gamma_A) = \Pr \left\{ \frac{g_0}{L_0} \frac{L_0}{L_0} \frac{L_i}{L_i} \left( L_i > \frac{L_0}{\gamma_A \sigma^2_N} \right) \geq \gamma_D \right\} = \int_0^{\text{Ptx}/(\gamma_A \sigma^2_N)} \Pr \left\{ \frac{g_0}{x} \frac{g}{L_i} \left( L_i > x \right) \geq \gamma_D \right\} f_{L_0}(x) \, dx, \quad (4.22)$$

where $f_{L_0}(x) = 2\pi \lambda_{\text{BS}}(\kappa^{2/\beta} \beta^{-1} x^{2/\beta-1} \exp \left( -\pi \lambda_{\text{BS}}(x/\kappa)^{2/\beta} \right)$ is the probability density function of $L_0$ that is obtained by applying the displacement theorem of PPPs [7, Eq. (21)]. It is worth mentioning that (4.22) is exact if the Crofton cell is considered, while it is an approximation if the typical cell is considered (see Remark 13 for further details).

The probability term, $G(\cdot; \cdot)$, in the integrand function of (4.22) can be computed as follows:

$$G(\gamma_D; x) \overset{(a)}{=} \exp \left( -\int_x^{+\infty} \left( 1 + \frac{y}{x \gamma_D} \right)^{-1} 2\pi \lambda_{\text{BS}}(\kappa^{2/\beta} \beta^{-1} x^{2/\beta-1} \exp \left( -\pi \lambda_{\text{BS}}(x/\kappa)^{2/\beta} \right) dy \right) \overset{(b)}{=} \exp \left( -\pi \lambda_{\text{BS}}(\text{tx}) (x/\kappa)^{2/\beta} \right), \quad (4.23)$$

where (a) follows from the probability generating functional theorem of PPPs [1] by taking into account that, based on (4.8), the interfering BSs constitute a PPP of intensity equal to $\lambda_{\text{BS}}(\text{tx}) = \lambda_{\text{BS}} \mathcal{L}(\lambda_{\text{MT}}/\lambda_{\text{BS}})$, and (b) follows by solving the integral. The intensity of the interfering PPP, $\lambda_{\text{BS}}(\text{tx})$, is obtained by taking into account that only that BSs that are in transmission mode contribute to the inter-cell interference. The analytical expression of $\lambda_{\text{BS}}(\text{tx})$ is, in particular, obtained with the aid of the independent thinning theorem of PPPs, similar to [7] and [137]. The impact of the spatial correlation that exists among the BSs that operate in transmission mode [138], is, on the other hand, postponed to a
future research work.

By inserting (4.23) in (4.22) and by applying some changes of variable, we obtain:

\[ P_{\text{cov}}(\gamma_D, \gamma_A) = \pi \lambda_{\text{BS}} \kappa^{-2/\beta} \int_0^{(P_{\text{tx}}/(\gamma_A \sigma^2))^{2/\beta}} \exp \left( -\pi \lambda_{\text{BS}} \kappa^{-2/\beta} \left( 1 + \Upsilon \mathcal{L} \left( \lambda_{\text{MT}}/\lambda_{\text{BS}} \right) \right) z \right) dz. \]  

(4.24)

The proof follows from (4.6) and (4.7) with the aid of some simplifications and by using the identity \( \sum_{u=0}^{+\infty} (u+1)^{-1} \Pr \{ \bar{N}_{\text{MT}} = u \} = (\lambda_{\text{MT}}/\lambda_{\text{BS}})^{-1} \mathcal{L} (\lambda_{\text{MT}}/\lambda_{\text{BS}}) \) [130, Proposition 2].

4.9 Appendix B – Proof of Theorem 1

In this section, we are interested in the functions that depend on \( P_{\text{tx}} \). For ease of writing, we adopt the simplified notation: \( P_{\text{tx}} \to P, \mathcal{L}(\cdot) \to \mathcal{L}, \mathcal{M}(\cdot) \to \mathcal{M}, \mathcal{Q}(\cdot, P_{\text{tx}}, \cdot) \to Q(P), \hat{Q}_{P_{\text{tx}}} (\cdot, P_{\text{tx}}, \cdot) \to \hat{Q}(P), P_{\text{circ}} = P_c, P_{\text{idle}} = P_i, \text{EE} (P_{\text{tx}}, \cdot) \to \text{EE} (P), \) and \( \hat{\text{EE}}_{P_{\text{tx}}} (P_{\text{tx}}, \cdot) \to \hat{\text{EE}} (P) \). A similar notation is adopted for higher-order derivatives with respect to \( P \).

The stationary points of (4.14) are the zeros of the first-order derivative of \( \text{EE}(\cdot) \) with respect to \( P \). From (4.14), we obtain \( \hat{\text{EE}} (P) = 0 \iff P_i - S_P (P) = 0 \), which can be re-written as follows:

\[ \frac{Q(P)}{\hat{Q}(P)} - P = \frac{\Delta P + P_i/\mathcal{L} + P_c \mathcal{M}/\mathcal{L}}{W_{\text{left}}(P)}. \]  

(4.25)

With the aid of some algebraic manipulations and by exploiting Lemmas 9-12, the following holds: i) \( W_{\text{right}} \geq 0 \) is a non-negative function that is independent of \( P \), ii) \( W_{\text{left}} (P) \geq 0 \) is a non-negative and increasing function of \( P \), i.e., \( W_{\text{left}} (P) \geq 0 \), since \( Q(P) \geq 0 \) and \( \hat{Q}(P) \leq 0 \) from Lemma 11, iii) \( W_{\text{left}} (P \to 0) = 0 \) and \( W_{\text{left}} (P \to \infty) = \infty \). This implies that \( W_{\text{left}} (\cdot) \) and \( W_{\text{right}} \) intersect each other in just one point. Therefore, a unique stationary point, \( P^* \), exists. Also, \( \hat{\text{EE}} (P) > 0 \) for \( P < P^* \) and \( \hat{\text{EE}} (P) < 0 \) for \( P > P^* \). Finally, by taking into account the constraints on the transmit power, it follows that the unique optimal maximizer of the EE is \( P^{(\text{opt})} = \max \{ P^{(\min)}, \min \{ P^*, P^{(\max)} \} \} \), since \( P \in [P^{(\min)}, P^{(\max)}] \). This concludes the proof.
4.10 Appendix C – Proof of Theorem 2

In this section, we are interested in the functions that depend on $\lambda_{BS}$. For ease of writing, we adopt the simplified notation: $\lambda_{BS} \rightarrow \lambda$, $\mathcal{L}(\cdot/\lambda_{BS}) \rightarrow \mathcal{L}(\lambda)$, $\mathcal{M}(\cdot/\lambda_{BS}) \rightarrow \mathcal{M}(\lambda)$, $\mathcal{Q}(\lambda_{BS}, \cdot/\lambda_{BS}) \rightarrow \mathcal{Q}(\lambda)$, $\dot{\mathcal{Q}}_{\lambda_{BS}}(\lambda_{BS}, \cdot/\lambda_{BS}) \rightarrow \dot{\mathcal{Q}}(\lambda)$, $P_{circ} = P_c$, $P_{idle} = P_i$, $EE(\cdot, \lambda_{BS}) \rightarrow EE(\lambda)$, $\dot{EE}_{\lambda_{BS}}(\cdot, \lambda_{BS}) \rightarrow \dot{EE}(\lambda)$, $P_{tx} \rightarrow P$. Similar notation applies to higher-order derivatives.

The proof is split in two parts: i) $\lambda_{MT}/\lambda \geq 2.8$ and ii) $\lambda_{MT}/\lambda \leq 2.8$. This is necessary because, from Lemma 9, $\mathcal{L}(\cdot)$ is concave in $\lambda$ if $\lambda_{MT}/\lambda \geq 2.8$ and convex in $\lambda$ if $\lambda_{MT}/\lambda \leq 2.8$.

**Case Study** $\lambda_{MT}/\lambda \geq 2.8$

The stationary points of (4.14) are the zeros of the first-order derivative of $EE(\cdot)$ with respect to $\lambda$. From (4.14), we obtain $\dot{EE}(\lambda) = 0 \iff S_D(\lambda) - P_1 = 0$. This stationary equation can be re-written as follows ($W_{right}(\lambda) = \sum_{\ell=1}^{5} W_{\ell}(\lambda)$):

$$
\begin{align*}
\underbrace{P_i}_{W_{left}} = & \underbrace{-(\mathcal{L}(\lambda)/\dot{\mathcal{L}}(\lambda))}_{W_1(\lambda)} \underbrace{[\dot{\mathcal{Q}}(\lambda)/\mathcal{Q}(\lambda)]}_{W_2(\lambda)} \underbrace{[1 + \gamma \mathcal{L}(\lambda)]}_{W_3(\lambda)} \underbrace{[\mathcal{L}(\lambda)(P + \Delta P) + P_1 + P_c \mathcal{M}(\lambda)]}_{W_4(\lambda)} \\
& + \underbrace{P_{circ}(\dot{\mathcal{M}}(\lambda) \mathcal{L}(\lambda)/\dot{\mathcal{L}}(\lambda) - \mathcal{M}(\lambda))}_{W_5(\lambda)} \underbrace{[\mathcal{L}^2(\lambda)(P + \Delta P) + \gamma P_c \mathcal{L}^2(\lambda) \dot{\mathcal{M}}(\lambda)/\dot{\mathcal{L}}(\lambda)]}_{W_6(\lambda)}.
\end{align*}
\quad (4.26)
$$

With the aid of some algebraic manipulations and by exploiting Lemmas 9-12, the following holds: i) $W_{left} \geq 0$ is a non-negative function that is independent of $\lambda$, ii) $W_{right}(\lambda) \geq 0$ is a non-negative function of $\lambda$, since $W_{\ell}(\lambda) \geq 0$ for $\ell = 1, \ldots, 5$ if $\lambda_{MT}/\lambda \geq 2.8$. In particular, $W_{3}(\lambda) \geq 0$ if $\lambda_{MT}/\lambda \geq 1.4$ and $W_{\ell}(\lambda) \geq 0$ for $\ell \geq 0$ if $\ell = 1, 2, 4, 5$, iii) $W_{right}(\lambda \rightarrow 0) = \infty$ and $W_{right}(\lambda \rightarrow \infty) = 0$. This implies that $W_{left}$ and $W_{right}(\cdot)$ would intersect each other in just a single point if $W_{right}$ is a decreasing function in $\lambda$, i.e., $\dot{W}_{right}(\lambda) \leq 0$ for $\lambda_{MT}/\lambda \geq 2.8$. A sufficient condition for this to hold is that $W_{\ell}(\cdot)$ for $\ell = 1, \ldots, 5$ are decreasing functions in $\lambda$, i.e., $\dot{W}_{\ell}(\lambda) \leq 0$ for $\lambda_{MT}/\lambda \geq 2.8$. This holds to be true and can be proved as follows. $\dot{W}_{2}(\lambda) \leq 0$ for $\lambda \geq 0$ and $\dot{W}_{4}(\lambda) \leq 0$ for $\lambda \geq 0$ because $\mathcal{L}(\cdot)$ and $\mathcal{M}(\cdot)$ are decreasing functions in $\lambda$ (see Lemma 9 and Lemma
\( \hat{W}_3 (\lambda) \leq 0 \) for \( \lambda \geq 0 \) and \( \hat{W}_5 (\lambda) \leq 0 \) for \( \lambda \geq 0 \) immediately follow by inserting into them the first-order derivatives of \( L (\cdot) \) and \( M (\cdot) \) with respect to \( \lambda \) and with the aid of simple algebraic manipulations. Less evident is the behavior of \( W_1 (\cdot) \) as a function of \( \lambda \).

Using some algebra, the first-order derivative satisfies the following:

\[
\dot{W}_1 (\lambda) = \frac{Q (\lambda)}{L_r (\lambda) + P_c M (\lambda)} \cdot \left( \lambda^2 \right) = -\frac{L_r (\lambda)}{L_r (\lambda) + P_c M (\lambda)} \cdot Q (\lambda).
\]

A sufficient condition for \( W_1 (\cdot) \) to be a decreasing function in \( \lambda \) is that \( A_\ell (\lambda) \leq 0 \) for \( \ell = 1, \ldots, 4 \). From Lemmas 9-12, this can be readily proved. In particular, \( A_\ell (\lambda) \leq 0 \) for \( \lambda \geq 0 \) if \( \ell = 1, 2, 3 \) and \( A_4 (\lambda) \leq 0 \) for \( \lambda_{MT}/\lambda \geq 2.8 \). Therefore, a unique stationary point, \( \lambda^* \), exists. Also, \( \hat{E}E (\lambda) > 0 \) for \( \lambda < \lambda^* \) and \( \hat{E}E (\lambda) < 0 \) for \( \lambda > \lambda^* \). Finally, by taking into account the constraints on the density of BSs, it follows that the unique optimal maximizer of the EE is \( \lambda^{(opt)} = \max \{ \lambda^{(min)}, \min \{ \lambda^*, \lambda^{(max)} \} \} \), since \( \lambda \in [\lambda^{(min)}, \lambda^{(max)}] \).

**Case Study** \( \lambda_{MT}/\lambda \leq 2.8 \)

As for this case study, we leverage a notable result in fractional optimization [74]: the ratio between a i) non-negative, differentiable and concave function, and a ii) positive, differentiable and convex function is a pseudo-concave function. It is, in addition, a unimodal function with a finite maximizer if the ratio vanishes when the variable of interest (i.e., the BSs’ density) tends to zero and to infinity. As for the case study under analysis, the EE in (4.14) can be re-written, by neglecting unnecessary constants that are independent of \( \lambda \) and do not affect the properties of the function, as follows:

\[
EE (\lambda) = \frac{Q (\lambda)}{[1 + \lambda L (\lambda)] \left( (P + \Delta P) + P_c M (\lambda) + P_r M (\lambda) / L (\lambda) \right)}.
\]

From Lemma 12, the numerator of (4.28) is a non-negative, differentiable, increasing and concave function for \( \lambda \geq 0 \). From Lemma 13, the EE in (4.28) tends to zero if \( \lambda \to 0 \) and \( \lambda \to \infty \). Therefore, a sufficient condition to prove the unimodality and pseudo-concavity of the EE is to show that the denominator of (4.28) is a positive, differentiable
and convex function in \( \lambda \) for \( \lambda_{MT}/\lambda \leq 2.8 \). From \textit{Lemma 9} and \textit{Lemma 10}, the first two properties are immediately verified. To complete the proof, the convexity of the denominator of (4.28) needs to be analyzed.

Let \( \text{Den}(\cdot) \) be the denominator of (4.28). Let us introduce the function \( \mathcal{K}(\lambda) = \frac{2\lambda^2}{\lambda} - \bar{L}(\lambda) \). The second-order derivative of \( \text{Den}(\cdot) \), as a function of \( \lambda \), is as follows:

\[
\frac{\text{Den}(\lambda)}{\lambda} = \frac{\Upsilon(P + \Delta P)\bar{L}(\lambda)}{D_1(\lambda)} + \frac{\Upsilon P_c\bar{L}(\lambda)}{D_2(\lambda)} + \frac{(P_c/L^2(\lambda))(2\lambda_{MT}/\lambda^3)(L(\lambda) + \lambda\bar{L}(\lambda))}{D_3(\lambda)} + \frac{P_c(M(\lambda)/L^2(\lambda))\mathcal{K}(\lambda)}{D_5(\lambda)} + \frac{(P_c/L(\lambda))\mathcal{K}(\lambda)}{D_6(\lambda)} + \frac{(P_c/L^2(\lambda))\mathcal{K}(\lambda)}{D_7(\lambda)}.
\]

(4.29)

A sufficient condition for proving that \( \text{Den}(\cdot) \) is a convex function in \( \lambda \) is to show that \( D_\ell(\lambda) \geq 0 \) for \( \ell = 1, 2, \ldots, 7 \) and \( \mathcal{K}(\lambda) \geq 0 \) if \( \lambda_{MT}/\lambda \leq 2.8 \). This can be proved as follows. \( D_1(\lambda) \geq 0 \) for \( \lambda_{MT}/\lambda \leq 2.8 \) follows from \textit{Lemma 9}. \( D_\ell(\lambda) \geq 0 \) for \( \ell = 2, 5 \) if \( \lambda \geq 0 \) follows from \textit{Lemma 10}. \( D_\ell(\lambda) \geq 0 \) for \( \ell = 3, 6, 7 \) if \( \lambda \geq 0 \) follows from \textit{Lemma 9}. \( D_4(\cdot) \) and \( \mathcal{K}(\cdot) \) require deeper analysis. Define \( \xi = \lambda_{MT}/\lambda \). \( D_4(\cdot) \) and \( \mathcal{K}(\cdot) \) are positive functions in \( \xi \) if:

\[
\begin{align*}
D_4(\xi) &= 0 \iff D_4(\xi) = 1 - (1 + \xi/\alpha)^{-\alpha} - x(1 + \xi/\alpha)^{-\alpha + 1} \geq 0, \\
\mathcal{K}(\xi) &= 0 \iff \mathcal{K}(\xi) = (1 + \xi/\alpha)^{-\alpha} + [2 + (1 + 1/\alpha)x][2 - (1 - 1/\alpha)x]^{-1} \geq 1.
\end{align*}
\]

(4.30)

By direct inspection of (4.30), it is not difficult to prove the following: i) \( D_4(\xi \to 0) = 0 \) and \( D_4(\xi) \geq 0 \) for \( \xi \geq 0 \), and ii) \( \mathcal{K}(\xi \to 0) = 1 \) and \( \mathcal{K}(\xi) \geq 0 \) for \( \xi \leq 2.8 \). These two conditions imply \( D_4(\lambda) \geq 0 \) for \( \lambda \geq 0 \) and \( \mathcal{K}(\lambda) \geq 0 \) for \( \lambda_{MT}/\lambda \leq 2.8 \). This concludes the proof.
CHAPTER 5

Cellular Networks with Renewable Energy Sources

5.1 Introduction

With the rapid increase of the numbers of mobile terminal (MT) and small cell base stations (SBSs), the power consumption has become one of the biggest issues in cellular networks design. To tackle this issue, one of the easiest way is to feed the networks by utilizing green and cost-effective energy sources. A viable option is that the BSs are powered by solar and/or wind power instead of the power grid. Such networks can achieve several benefits. Firstly, by using renewable energy sources, we reduce the CO$_2$ emission thereby overcoming the global warming problem as more than 80% of energy is generated from fossil fuel [139]. Secondly, with renewable energy sources enabled cellular networks, the operating expense (OPEX) of the network operator goes down since around 80% of the electricity bill comes from the BSs [2, 3]. Thirdly, the coverage area can be expanded to remote and isolated places by employing the small cell BSs powered by renewable energy sources. Moreover, the SBSs are typically deployed randomly and overlaid the coverage area of macro BSs (MBSs). As a consequence, it is not easy to provide power sources to these BSs, hence, the use of solar and/or wind power is more practical. Finally, as shown in [140], the electromagnetic signal severely affects the human health compared with wind
and light signals.

Although cellular networks with the help of renewable energy sources provide lots of benefits, the performance of such networks, however, is still unclear especially from the point view of system-level performance. The system-level performance is the performance which accounts for the whole networks instead of focusing on some specific network realizations. System-level performance study usually requires time-consuming and memory-consuming simulations [27]. This is even exacerbated in the context of using renewable energy sources, since the temporal dynamics of the energy arrivals need to be taken into account. As mentioned in the previous chapters of this thesis, we exploit stochastic geometry tools to avoid lengthly simulations. More precisely, we propose an approach that allows us to take into consideration both spatial dynamics (the locations of the network elements) and temporal dynamics (the arrival of the energy sources that is not deterministic).

In the literature, there are some works which study the performance of wireless networks with renewable energy sources. In [141], the maximization of networks throughput of a mobile ad-hoc network (MANET) by using stochastic geometry and random walk theory was investigated. The results showed that the throughput is proportional to the optimal transmission probability. On the other hand, the performance of a point-to-point transmission was studied in [142]. Particularly, by assuming the energy packet arrivals follow the Poisson counting process, the maximization of the average number of delivered bits was provided. These above-mentioned works have either investigated the performance of mobile ad-hoc networks or cognitive radio networks which can not be used in the cellular networks since the radio interface, protocol of these networks are different compared with the cellular networks.

The performance of cellular networks with green energy was investigated in [143] where the minimum grid consumption is derived with dynamic resource allocation. This study, however, does not consider multi-tier heterogeneous networks (HetNets). In [144], the utility enhancement of HetNets was addressed by taking into consideration both the offloading technique and renewable energy SBSs. The trade-off between the throughput and associated power cost of the small cell BSs was studied in [145] where the dynamic activation of energy harvesting BS was applied. The upper and lower bound of association probability of HetNets was provided in [146] where all BSs are solely powered by green
energy sources. The average rate of the multi-tier cellular networks was addressed in [147] with random walk theory, fixed point analysis and stochastic geometry. In addition, by modeling the temporal energy level as a birth-death process, the energy utilization rate was calculated. Energy-aware traffic offloading schemes were proposed in [148], where user associations, ON–OFF states of the SBSs, and power control were jointly optimized based on the statistical information of energy arrival and traffic.

These works, however, focus on minimizing the network power consumption while we are interested in maximizing both the potential spectral efficiency (PSE) and energy efficiency (EE). Moreover, not all of the above-mentioned works consider multi-tier cellular networks by allowing to offload MTs from the MBSs to the SBSs and taking into account the sleeping mode of the MBSs. In this chapter, we overcome these system models and assumptions. In particular, we consider a two-tier cellular networks where the MBSs are connected to the power grid and the SBSs are fed by green energy sources. To minimize the network power consumption, two operation modes of the MBSs are considered: transmission mode and sleeping mode. Moreover, in order to increase the EE without sacrificing other network metrics, i.e., coverage probability (Pcov) and PSE, we only consider the SBSs with enough energy to serve at least one MT instead of all SBSs.

The contributions of this chapter are summarized as follows.

- A two-tier (MBSs and SBSs) downlink (DL) heterogeneous cellular networks is considered where the SBSs use green energy sources and the MBSs are connected to the power grid. Furthermore, we only consider the SBSs which are activated instead of all SBSs. Offloading from the MBSs to the SBSs is taken into account via a bias factor.

- Two operation modes for the MBSs are considered, i.e., transmission mode and idle mode, which lead to different power consumption. In addition, apart from the transmit power, we also take into account the power consumption from other MBSs’ elements such as the cooling systems, processing units, etc.

- The battery of the SBS is modeled as a discrete Markov chain with finite capacity. The battery can operate in one of two modes, namely, full-duplex and half-duplex. The steady state vector of the Markov chain are obtained via numerical method.
• The energy arrivals are modeled as Poisson random variables (RVs).

• The networks power consumption is studied by considering two load models, i.e., exclusive resource allocation to some MTs and equal distribution of the available resources to some MTs.

• New analytical frameworks for Pcov, PSE, and EE are derived, which are shown to be tractable and easy to compute.

• Monte Carlo simulations are provided to verify the correctness of our framework. Insights and findings are drawn based on both the frameworks and numerical results.

The organization of this chapter is as follows. In Section 5.2, the system models and assumptions are provided. The network power consumption, PSE and EE of different load models are studied in Section 5.3. In Section 5.4, Monte Carlo simulations are given to confirm the correctness of our mathematical framework. Finally, Section 5.5 concludes this chapter.

Table 5.1 provides all notations and definitions in this chapter.

5.2 System Model

5.2.1 Cellular Networks Modeling

We consider a two-tier downlink heterogeneous cellular networks with MBSs and SBSs which are distributed according to homogeneous Poisson point processes (PPP) denoted by, $\Psi_{\text{MBS}}, \Psi_{\text{SBS}}$, whose densities are $\lambda_{\text{MBS-ON}} = \alpha_{\text{ON}}\lambda_{\text{BS}}$ and $\lambda_{\text{SBS-OFF}} = \alpha_{\text{OFF}}\lambda_{\text{BS}}$, $\alpha_{\text{ON}}$ and $\alpha_{\text{OFF}}$ are constant numbers and $\lambda_{\text{BS}}$ is a reference density of BS. The MBSs are connected to the power grid, and called on-grid BS while the SBSs are powered solely by renewable energy sources (solar and wind power), and called off-grid BS. The MTs are modeled as another homogeneous PPP, $\Psi_{\text{MT}}$, with density $\lambda_{\text{MT}}$, which is independent of $\Psi_{\text{MBS}}$ and $\Psi_{\text{SBS}}$. Both BSs and MTs are equipped with single omnidirectional antenna. The total transmit power of MBSs and SBSs are $P_{\text{tx}}^{\text{ON}}$ and $P_{\text{tx}}^{\text{OFF}}$, respectively. Without loss of generality, the performance is studied based on the MT located at the origin, MT$^0$ (Slivnyak theorem [10, Th. 1.4.5]).
Table 5.1: Summary of main symbols and functions used in this chapter. $k \in \{\text{ON, OFF}\}$, $d \in \{\text{L1, L2}\}$, and $o \in \{\text{H, F}\}$.

<table>
<thead>
<tr>
<th>Symbol/Function</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E{\cdot}$, $\Pr{\cdot}$</td>
<td>Expectation operator, probability measure</td>
</tr>
<tr>
<td>$\lambda_{\text{BS}}, \lambda_{\text{MT}}$</td>
<td>Density of base stations, mobile terminals</td>
</tr>
<tr>
<td>$\lambda_k = \alpha_k \lambda_{\text{BS}}$</td>
<td>Density of the $k$-tier BSs, $\alpha_k$ is the constant number</td>
</tr>
<tr>
<td>$\Psi_k, \Psi_{\text{MT}}$</td>
<td>PPP of the $k$-tier base stations, mobile terminals</td>
</tr>
<tr>
<td>$\Psi_k^{(A)}$</td>
<td>PPP of the $k$-tier available BSs</td>
</tr>
<tr>
<td>$\Psi_k^{(I,d)}$</td>
<td>PPP of interference BSs of load model $d$</td>
</tr>
<tr>
<td>$P_{k,\text{tx}}, P_{k,\text{cir}}$</td>
<td>Transmit and circuit (static) power consumption of $k$-tier BSs</td>
</tr>
<tr>
<td>$P_{\text{idle}}$</td>
<td>Idle power consumption of on-grid BS</td>
</tr>
<tr>
<td>$r_n, h_n$</td>
<td>Distance, fading power gain of a generic link</td>
</tr>
<tr>
<td>$l(\cdot), L_k^{(0)}$</td>
<td>Path-loss, path-loss of $k$-tier serving BS</td>
</tr>
<tr>
<td>$L^{(0)}$</td>
<td>Path-loss of the intended link</td>
</tr>
<tr>
<td>$T_k$</td>
<td>Bias factor of the $k$-tier BS</td>
</tr>
<tr>
<td>$\varepsilon_{d,o}$</td>
<td>Probability of off-grid BS not activated in $o$ mode and $d$ load model</td>
</tr>
<tr>
<td>$\kappa, \beta &gt; 0$</td>
<td>Path-loss constant, slope (exponent)</td>
</tr>
<tr>
<td>$BW, N_0$</td>
<td>Transmission bandwidth, noise power spectral density</td>
</tr>
<tr>
<td>$BW_{\text{RB}}$</td>
<td>Transmission bandwidth per resource block in load model 1</td>
</tr>
<tr>
<td>$N_{k,d}^{\text{max}}$</td>
<td>Maximum MT served by $k$-tier BS in load model $d$</td>
</tr>
<tr>
<td>$N_{\text{RB}}$</td>
<td>Number of resource block in load model 1</td>
</tr>
<tr>
<td>$N_{k,L}^{\text{Load}}$</td>
<td>Maximum MT served by $k$-tier BS defined by network operator</td>
</tr>
<tr>
<td>$\sigma_N^2 = BW N_0$</td>
<td>Noise variance</td>
</tr>
<tr>
<td>$\lambda_{\text{EP}}, L$</td>
<td>Density of energy arrival, maximum energy level of battery</td>
</tr>
<tr>
<td>$(c)$</td>
<td>State $(c)$ in the Markov chain</td>
</tr>
<tr>
<td>$P^o$</td>
<td>Transition probability matrix of Markov chain in $o$ mode</td>
</tr>
<tr>
<td>$p_i$</td>
<td>Probability of $i$-th energy packet arrival</td>
</tr>
<tr>
<td>$q_e$</td>
<td>Probability has $e$ MTs in off-grid BS</td>
</tr>
<tr>
<td>$m_d^a$</td>
<td>Energy requirements to activate off-grid BS in load model $d$</td>
</tr>
<tr>
<td>$m_d^s$</td>
<td>Energy requirements to serve one MT in load model $d$</td>
</tr>
<tr>
<td>$\nu^o, v_s^o$</td>
<td>Steady state vector, probability of Markov chain in $o$ mode</td>
</tr>
<tr>
<td>$\gamma_D, \gamma_A$</td>
<td>Reliability threshold for decoding, cell association</td>
</tr>
<tr>
<td>$A P_k$</td>
<td>Association probability of the $k$-tier BS</td>
</tr>
<tr>
<td>$f_X(\cdot)$</td>
<td>Probability density/mass function of $X$</td>
</tr>
<tr>
<td>$F_X(\cdot)$</td>
<td>Cumulative distribution/mass function of $X$</td>
</tr>
<tr>
<td>$F_X(\cdot)$</td>
<td>Complementary cumulative distribution function of $X$</td>
</tr>
<tr>
<td>$\mathbb{1}(\cdot)$</td>
<td>Indicator function, floor function</td>
</tr>
<tr>
<td>$\binom{2}{2} F_1(\cdot, \cdot, \cdot), I(\cdot)$</td>
<td>Gauss hypergeometric function, gamma function</td>
</tr>
<tr>
<td>$\max{x, y}, \min{x, y}$</td>
<td>Maximum, minimum between $x$ and $y$</td>
</tr>
<tr>
<td>$\text{SIR}, \text{SNR}$</td>
<td>Signal-to-interference-ratio, average signal-to-noise-ratio</td>
</tr>
<tr>
<td>$P_{\text{cov}}^{\text{d}}(\gamma_D, \gamma_A, d)$</td>
<td>Coverage probability of $k$-tier in load model $d$</td>
</tr>
<tr>
<td>$P_{\text{SE}}^{\text{d}}, P_{\text{EE}}^{\text{d}}$</td>
<td>Potential spectral efficiency and energy efficiency of load model $d$</td>
</tr>
<tr>
<td>$P_{\text{grid}}^{d}$</td>
<td>Network power consumption in load model $d$</td>
</tr>
<tr>
<td>$P_{\text{Ac}}^{k,d}, P_{\text{Inc}}^{k,d}$</td>
<td>Activation and Inactivation probability of $k$-th tier BS in load model $d$</td>
</tr>
</tbody>
</table>

### 5.2.2 Channel Modeling

#### Fast-Fading

The fast fading follows an independent and identically distributed (i.i.d.) complex Gaussian random variable with zero mean and unit variance, i.e., $CN(0, 1)$.
Path-Loss

The path-loss of a generic BS-to-MT link of length, \( r_n \), is computed by using the single-state unbounded path-loss model (Section 1.4.2) as \( l(r_n) = \kappa r_n^\beta \), where \( \beta \) and \( \kappa \) are the path-loss exponent and the path-loss constant, which is computed as \( \kappa = (\frac{4\pi}{\lambda})^2 \), where \( \lambda \) is the transmission wavelength.

5.2.3 Cell Association

The typical MT\(_0\) is served by the BS providing the smallest path-loss multiplied by a bias factor different for each tier of BSs:

\[
L^{(0)} = \min \left\{ T_{ON}^{(0)} L_{ON}^{(0)}, T_{OFF}^{(0)} L_{OFF}^{(0)} \right\},
\]

where \( T_k, k \in \{ON, OFF\} \), is bias factor of \( k \)-th tier. If \( T_k = 1 \), the association reduces to the smallest distance association (Section 1.4.3). \( L_k^{(0)} = \min_{n \in \Psi_k} \{ l(r_n) \} \) is the path-loss of the serving BS of the \( k \)-th tier. \( \Psi_k \) is the set of the \( k \)-tier available BSs with density \( \lambda_k^{(A)} \), i.e., \( \lambda_k^{(A)} = \lambda_{ON} \) and \( \lambda_k^{(A)} = (1 - \varepsilon_{d,o}) \lambda_{OFF} \), where \( \varepsilon_{d,o}, o \in \{H, F\} \) (H = half-duplex, F = full-duplex), \( d \in \{L1, L2\} \), is the probability that a SBS is inactive due to insufficient energy availability. Further details on \( \varepsilon_{d,o} \) are provided in Section 5.2.7.

5.2.4 Load Model

Two load models are considered in this chapter.

Load Model 1 (L1): Power and bandwidth are viewed as discrete resources by the scheduler

In this load model, each BS randomly selects up to \( N_{k,L1}^{ON} \) MTs, \( N_{max}^{L1} = \min \{ N_{RB}, N_{Load}^{k} \} \), where \( N_{RB} \) is the number of available (discrete) resource blocks (time or frequency resources) of the BS\(^1\), and \( N_{Load}^{k} \) is the largest number of MTs that can be served by a BS, which is setup by the network operator. Each MT is allocated \( \frac{P_{tx}^k}{N_{RB}} \) transmit power and

---

\(^1\)In this chapter, we assume that both MBSs and SBSs have the same number of RBs. However, it is straightforward to extend to case \( N_{ON}^{RB} \neq N_{OFF}^{RB} \).
bandwidth, which are fixed amounts that are independent of the actual number of MTs in the cell. BW is the transmission bandwidth. If there are no MTs inside the cell, the corresponding BS is not transmitting. The power consumption of both the MBSs and SBSs is introduced in Section 5.3.1. The random selection of users ensures that all the MTs are able to participate in the transmission in the long term.

Load Model 2 (L2): Power and bandwidth are viewed as continuous resources by the scheduler

In this case, the BS concurrently serves at most $N_{\text{L2 max}}^k$ out of all MTs associated to it, where $N_{\text{L2 max}}^k$ is setup by the network operator. In particular, each BS equally splits its total transmit power and bandwidth among the actual number of MTs that are available in its cell. If this number is greater than $N_{\text{L2 max}}^k$, then only $N_{\text{max}}^k$ MTs are served and power and bandwidth are equally shared among them. In addition, the MBSs and SBSs do not transmit if at least one MT is not associated to them. In this load model, the BS always consumes all the transmit power and uses all the transmission bandwidth even if only one MT is located in its coverage area. In load model 1, on the other hand, some resource blocks may not be used and thus less power and bandwidth are used.

5.2.5 Power Consumption Modeling

Power Consumption Modeling of Macro BS

As for the MBSs, there are two operating modes: i) the BS is in idle mode and consumes $P_{\text{ON idle}}^\text{MBS}$ power provided that no MTs are associated to it and ii) the BS is in transmission mode if at least one MT is tagged to it. In the transmission mode, the BS consumes both the transmit and the circuitry (static) power, $P_{\text{ON cir}}^\text{MBS}$. The static power takes into account all the power consumption except for the transmit power such as the cooling systems, processing units. Moreover, we also assume that the idle power is smaller than the circuitry power, e.g., $0 \leq P_{\text{ON idle}}^\text{MBS} \leq P_{\text{ON cir}}^\text{MBS}$.

Power Consumption Modeling of Small Cell BS

As for the SBSs, we assume that the BS only operates in transmission and OFF modes. The idle mode is not considered or equivalently $P_{\text{idle}}^\text{SBS} = 0$. Hence, the power consumption
of the SBSs comes from the transmit and static power, $P_{\text{off}}$.

For simplicity, we denote the idle power of the MBSs as $P_{\text{idle}}^\text{ON} = P_{\text{idle}}$ since $P_{\text{idle}}^\text{OFF} = 0$.

### 5.2.6 Energy Harvesting Modeling

We assume that the energy packet arrival at the SBSs is modeled as a i.i.d. discrete Poisson random variable with density $\lambda_{\text{EP}} \geq 0$ [149].

#### Energy Storage and Usage Modeling

In the off-grid BSs, the amount of energy harvested is stored in a battery with finite capacity, i.e., $\mathcal{L} < \infty$ levels. On the contrary, the on-grid BSs, which are connected to the power grid, directly use the energy without storage. The energy level of the battery is modeled by using a discrete Markov chain [149]. In particular, each energy level is represented by one state of the chain. For example, the battery has $\mathcal{L}$ levels corresponding to $\mathcal{L}$ states of the Markov chain, which are from state (0) to state ($\mathcal{L} - 1$). It is noted that state (0) and ($\mathcal{L} - 1$) are equivalent to the empty and full energy of the battery. Furthermore, we assume that each energy level is equal to an energy packet.

In the next section, two operational modes of the off-grid BS's battery are investigated, i.e., full-duplex and half-duplex modes.

#### Full-Duplex (F) Mode

In full-duplex mode, at each time-slot, the battery is able to both harvest and consume energy concurrently. It can be implemented by using two batteries, one is for harvesting and another for consuming energy. At the end of each transmission instance, the harvested battery transfers the harvested energy to the other battery. Specifically, the probability that the SBS gathers $i$ energy packet in one time-slot is as follows

$$p_i = \Pr (i \text{ energy packet arrival at one time-slot}) = \frac{(\lambda_{\text{EP}})^i \exp(-\lambda_{\text{EP}})}{i!}, \quad (5.2)$$

where $\lambda_{\text{EP}}$ is the density of energy packet arrival.

On the other hand, an amount of energy is spent by the consume battery, which depends on the number of MTs associated to the SBSs and given by Eq. (5.4) and
Table 5.2: Transition probability matrix of full-duplex battery. The entries of the matrix are \( r_{i,j}, i,j \in \{1,2, \ldots, \mathcal{L} \} \) and \( R_i, i \in \{1,2, \ldots, \mathcal{L} \} \), is the \( i \)-th row of the matrix with size \( 1 \times \mathcal{L} \) and \( R_i^- = [ r_{i,1} \ r_{i,2} \ \ldots \ r_{i,\mathcal{L}-2} \] \) is the reduce of the \( i \)-th row with size \( 1 \times \mathcal{L} - 2 \). \( p_i = \frac{(\lambda_{EP})^e \exp(-\lambda_{EP})}{e!} \), \( i \in \{0, \ldots, \mathcal{L} - 2 \} \); \( p_{\mathcal{L}-1} = 1 - \sum_{i=0}^{\mathcal{L}-2} p_i \); \( q_e \) and \( y_e \) are given in Eq. (5.4), and (5.5).

<table>
<thead>
<tr>
<th>Transition probability matrix of full-duplex battery, ( P^F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set ( R_1 = \begin{bmatrix} p_0 &amp; p_1 &amp; \ldots &amp; p_{\mathcal{L}-1} \end{bmatrix} )</td>
</tr>
<tr>
<td>For ( i = 2 : m^d )</td>
</tr>
<tr>
<td>( R_i = \begin{bmatrix} 0 &amp; R_{i-1}^- &amp; r_{i-1,\mathcal{L}-1} + r_{i-1,\mathcal{L}} \end{bmatrix} )</td>
</tr>
<tr>
<td>End</td>
</tr>
<tr>
<td>For ( e = 1 : N^B_{\max} )</td>
</tr>
<tr>
<td>( R_{m^d+1+(e-1)m^d} = \begin{bmatrix} r_{m^d+1+(e-1)m^d,1} &amp; \ldots &amp; r_{m^d+1+(e-1)m^d,\mathcal{L}-1} &amp; r_{m^d+1+(e-1)m^d,\mathcal{L}} \end{bmatrix} )</td>
</tr>
<tr>
<td>( r_{m^d+1+(e-1)m^d,j} = y_e p_{j-1} + \sum_{u=1}^{e-1} q_e u p_{j-um^d-1} + q_0 p_{j-m^d_1-1-(e-1)m^d}, j \in {1, \ldots, \mathcal{L} - 1 } )</td>
</tr>
<tr>
<td>( r_{m^d+1+(e-1)m^d,\mathcal{L}} = y_e p_{\mathcal{L}-1} + \sum_{u=1}^{e-1} q_e u \left( 1 - \sum_{b=0}^{\mathcal{L}-2} p_b \right) + q_0 \left( 1 - \sum_{b=0}^{\mathcal{L}-2} p_b \right) )</td>
</tr>
<tr>
<td>If ( e \neq N^B_{\max} )</td>
</tr>
<tr>
<td>For ( c = 1 : m^d - 1 )</td>
</tr>
<tr>
<td>( R_{m^d+1+(e-1)m^d+c} = \begin{bmatrix} 0 &amp; R_{m^d+1+(e-1)m^d+(c-1)}^- &amp; r_{m^d+1+(e-1)m^d+(c-1),\mathcal{L}-1} &amp; +r_{m^d+1+(e-1)m^d+(c-1),\mathcal{L}} \end{bmatrix} )</td>
</tr>
<tr>
<td>End</td>
</tr>
<tr>
<td>Else</td>
</tr>
<tr>
<td>For ( c = 1 : \mathcal{L} - (m^d + 1 + (N^B_{\max} - 1) m^d) )</td>
</tr>
<tr>
<td>( R_{m^d+1+(e-1)m^d+c} = \begin{bmatrix} 0 &amp; R_{m^d+1+(e-1)m^d+(c-1)}^- &amp; r_{m^d+1+(e-1)m^d+(c-1),\mathcal{L}-1} &amp; +r_{m^d+1+(e-1)m^d+(c-1),\mathcal{L}} \end{bmatrix} )</td>
</tr>
<tr>
<td>End</td>
</tr>
<tr>
<td>End</td>
</tr>
<tr>
<td>End</td>
</tr>
</tbody>
</table>

Eq. (5.5). In detail, if the MTs are less than \( N^B_{\max} = \min \left\{ N^\text{OFF.d}_{\max}, \left\lfloor \frac{L - 1 - m^d}{m^d} \right\rfloor + 1 \right\} \), \( d \in \{L1, L2\} \), all MTs are served simultaneously. Otherwise, only \( N^B_{\max} \) MTs are selected randomly. Here \( N^B_{\max} \) is the maximum number of MTs that can be served by the off-grid BS which takes into consideration both the battery capacity and \( N^\text{OFF.d}_{\max} \), where \( N^\text{OFF.d}_{\max} \) is defined in section 5.2.4; \( \lfloor . \rfloor \) is the floor function; \( m^d \) is the energy requirements of load model \( d \) in order to activate the off-grid BS (or to serve the 1st MT), and \( m^d \) is the energy demand for serving one MT of load model \( d \) provided that the BS is activated already.

The mathematical representation of the transition probability matrix of full-duplex mode, \( P^F \), is given in Table 5.2.

**Remark 21** The energy requirements for turning on the SBS are different for the two load models, i.e., \( m^{L1}_1 \neq m^{L2}_1 \). In particular, we have \( m^{L1}_1 = P^{\text{OFF.cir}} + \frac{P^{\text{OFF}}}{N_{\text{RB}}} \leq m^{L2}_1 = P^{\text{OFF}} + P^{\text{OFF.cir}} \). By contrast, the energy needed to serve one MT, \( m^d \), of load model 1 is
greater than the other, \( m^{L1} = P_{c1}^{OFF} + \frac{P_{c2}^{OFF}}{N_{RB}} \geq m^{L2} = P_{c2}^{OFF} \). The difference originates from the distinct resource allocation of the two load models.

\[
P^F = \begin{bmatrix}
p_0 & p_1 & p_2 & p_3 & p_4 & p_5 & 1 - \sum_{b=0}^{5} p_b \\
0 & p_0 & p_1 & p_2 & p_3 & p_4 & 1 - \sum_{b=0}^{5} p_b \\
0 & 0 & p_0 & p_1 & p_2 & p_3 & 1 - \sum_{b=0}^{3} p_b \\
y_{1p0} & y_{1p1} & y_{1p2} & y_{1p3} & y_{1p4} & y_{1p5} & y_1 \left( 1 - \sum_{b=0}^{5} p_b \right) \\
y_{2p0} & y_{2p1} & y_{2p2} & y_{2p3} & y_{2p4} & y_{2p5} & y_2 \left( 1 - \sum_{b=0}^{5} p_b \right) \\
0 & y_{2p0} & y_{2p1} & y_{2p2} & y_{2p3} & y_{2p4} & y_2 \left( 1 - \sum_{b=0}^{4} p_b \right) \\
0 & 0 & y_{1p0} & y_{1p1} & y_{1p2} & y_{1p3} & y_1 \left( 1 - \sum_{b=0}^{4} p_b \right) \\
0 & 0 & 0 & y_{1p0} & y_{1p1} & y_{1p2} & y_1 \left( 1 - \sum_{b=0}^{3} p_b \right)
\end{bmatrix}
\]

(5.3)

Let’s define the probabilities that the off-grid BS has exactly \( e \) MTs, \( q_e \), and more than \( e \) MTs, \( y_e \), as follows

\[
q_e = \Pr \left[ \text{MTs} = e, \text{OFF} \right] = \frac{\Gamma(e + 3.5)(\lambda_{\text{MT}}A_{\text{OFF}})^e}{\Gamma(3.5)} \frac{1}{e! \left( \lambda_{\text{MT}}A_{\text{OFF}} + 3.5 \right)^{e+3.5}} e^{-l}.
\]

(5.4)

\[
y_e = \Pr \left[ \text{MTs} \geq e, \text{OFF} \right] = 1 - \sum_{i=0}^{e-1} \Pr \left[ \text{MTs} = i, \text{OFF} \right]
\]

\[
= 1 - \sum_{i=0}^{e-1} \frac{(3.5)^e \Gamma(i + 3.5)(\lambda_{\text{MT}}A_{\text{OFF}})^i}{\Gamma(3.5) i! \left( \lambda_{\text{MT}}A_{\text{OFF}} + 3.5 \right)^{i+3.5}} e^{-l}
\]

(5.5)

\[
A_{\text{OFF}} = \frac{A_{\text{OFF}}}{(1 - \varepsilon_{d,o}) \alpha_{\text{OFF}} A_{\text{BS}}}
\]

Here \( A_{\text{k}} \), \( k \in \{ \text{ON}, \text{OFF} \} \), is the probability that the MT is associated to the \( k \)-tier BS, which can be computed as follows

\[
A_{\text{k}} = \Pr \left( T_k L_k^{(0)} > T_k L_k^{(0)} \right) = \int_0^{\infty} F_{k,L_k^{(0)}} \left( \frac{T_k}{T_k} x \right) f_{L_k^{(0)}} (x) \, dx = \left( 1 + \frac{\lambda_k}{\lambda_k} \left( \frac{T_k}{T_k} \right)^{3/2} \right)^{-1}
\]

(5.6)
where \( k, \tilde{k} \in \{\text{ON, OFF}\} \), \( \tilde{k} \neq k \); \( F_X(x) \) and \( f_X(x) \) are the complementary CDF (CCDF) and PDF of RV \( X \).

As an example, Eq. (5.3) illustrates the transition probability matrix of full-duplex mode for the case \( \mathcal{L} = 7 \), \( m^d_1 = 3 \), and \( m^d = 2 \).

In Eq. (5.3), the first three rows correspond to case that the battery is not activated, hence, it only receives the energy packets. The next two rows are the case that the battery can serve one MT, thus, its level of energy can either go up or down. Finally, in the 6–7-th rows, two MTs can be served and the battery can move to any state.

**Battery Operation**

The battery is modeled as a discrete Markov chain with \( \mathcal{L} - 1 \) states, \( ((0), (1), \ldots, (\mathcal{L} - 1)) \), which are equivalent to \( \mathcal{L} - 1 \) energy levels. The battery operation below can be applied to both load models.

1. If the current state, \( (c) \), is less than state \( (m^d_1) \), i.e., \( (0) \leq (c) \leq (m^d_1 - 1) \), \( d \in \{L1, L2\} \), the battery will go up to any state based on the energy arrival at that time instance.

2. If the current state \( (c) \) is between \( (m^d_1) \) and \( (m^d_1 + m^d - 1) \), i.e., \( (m^d_1) \leq (c) \leq (m^d_1 + m^d - 1) \), the battery is able to serve at most one MT and harvests energy at the same time. Thus, it can either move to a higher or lower state.

3. Provided the current state \( (c) \) is between \( (m^d_1 + m^d) \) and \( (m^d_1 + 2m^d - 1) \), the battery can serve up to two MTs and harvests energy simultaneously. Hence, it can either go to a higher or lower state.

4. The same explanation can be applied until state \( (m^d_1 + (N^B_{\max} - 1) m^d - 1) \).

5. Unless the current state \( (c) \) is not between \( (m^d_1 + (N^B_{\max} - 1) m^d) \) and \( \mathcal{L} - 2 \), the battery can serve the maximum MTs, \( N^B_{\max} \), and move to any state.

6. Finally, if the current state \( (c) \) is \( \mathcal{L} - 1 \), the battery can serve all MTs and is not able to move to a higher state.
Half-Duplex (H) Mode

In half-duplex mode, at each time-instant, the battery is only able to either harvest or consume energy. As a result, this mode can be employed with only one battery rather than two. All definitions and notations of full-duplex battery can directly apply to this mode. The battery operation is described as follows.

- If the current state \((c)\) is less than state \((m^d_1)\), i.e., \((0) \leq (c) \leq (m^d_1 - 1)\), the battery will go to any higher state based on the energy packet arrival at that time instance.

- If the current state \((c)\) is between \((m^d_1)\) and \((m^d_1 + m^d - 1)\), i.e., \((m^d_1) \leq (c) \leq (m^d_1 + m^d - 1)\), there are two possible cases:
  1. If there is no MTs associated to the off-grid BS, it will move to a higher state.
  2. Otherwise, it goes to state \((c - m^d_1)\) provided that at least one MT is associated to it.

- If the current state \((c)\) is between \((m^d_1 + m^d)\) and \((m^d_1 + 2m^d - 1)\), there are three possible cases:
  1. The BS jumps to any higher state providing that no MTs are located in its coverage area.
  2. Unless one MT is tagged into the BS, it will go to state \((c - m^d_1)\).
  3. Otherwise, the BS goes down to state \((c - m^d_1 - m^d)\) if MT \(\geq 2\).

- The other states from \((m^d_1 + 2m^d)\) to \((L - 2)\) can be easily described by following the above steps.

- Finally, if the current state \((c)\) is \((L - 1)\), the battery can serve all MTs and is not able to move to higher state.

The mathematical representation of the transition probability matrix of half-duplex mode, \(P^H\), is given in Table 5.3.
Table 5.3: Transition probability matrix of half-duplex battery. The entries of the matrix are \( r_{i,j} \), \( i, j \in \{1, 2, \ldots, \mathcal{L}\} \) and \( \mathbf{R}_i \), \( i \in \{1, 2, \ldots, \mathcal{L}\} \), is the \( i \)-th row of transition probability matrix with size \( 1 \times \mathcal{L} \) and \( \mathbf{R}_i = \begin{bmatrix} r_{i,1} & r_{i,2} & \ldots & r_{i,\mathcal{L}-2} \end{bmatrix} \), is the reduce of the \( i \)-th row with size \( 1 \times \mathcal{L} - 2 \). \( p_i = \frac{(\lambda \text{BP})^i}{i!} \exp(-\lambda \text{BP}) \), \( i \in \{0, \ldots, \mathcal{L} - 2\} \); \( p_{\mathcal{L}-1} = 1 - \sum_{i=0}^{\mathcal{L}-2} p_i \); \( q_e \) and \( y_e \) are given in Eq. (5.4) and (5.5).

<table>
<thead>
<tr>
<th>Transition probability matrix of half-duplex battery, ( \mathbf{P}^H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set ( \mathbf{R}<em>1 = \begin{bmatrix} p_0 &amp; p_1 &amp; \cdots &amp; p</em>{\mathcal{L}-1} \end{bmatrix} )</td>
</tr>
<tr>
<td>For ( i = 2 : m_y^d )</td>
</tr>
<tr>
<td>( \mathbf{R}<em>i = [0 \ \mathbf{R}<em>i^{-1} \ r</em>{i-1,\mathcal{L}-1} + r</em>{i-1,\mathcal{L}}] )</td>
</tr>
<tr>
<td>For ( e = 1 : N_{\text{max}}^B )</td>
</tr>
<tr>
<td>If ( e \neq N_{\text{max}}^B )</td>
</tr>
<tr>
<td>For ( c = 1 : m_y^d - 1 )</td>
</tr>
<tr>
<td>( \mathbf{R}<em>{m_y^d+1+(e-1)m_y^d+c} = \begin{bmatrix} 0 &amp; \mathbf{R}</em>{m_y^d+1+(e-1)m_y^d+c}^{-1} \end{bmatrix} + r_{m_y^d+1+(e-1)m_y^d+c,\mathcal{L}-1} )</td>
</tr>
<tr>
<td>End</td>
</tr>
<tr>
<td>Else</td>
</tr>
<tr>
<td>For ( c = 1 : \mathcal{L} - (m_y^d + 1 + (N_{\text{max}}^B - 1) m_y^d) )</td>
</tr>
<tr>
<td>( \mathbf{R}<em>{m_y^d+1+(e-1)m_y^d+c} = \begin{bmatrix} 0 &amp; \mathbf{R}</em>{m_y^d+1+(e-1)m_y^d+c}^{-1} \end{bmatrix} + r_{m_y^d+1+(e-1)m_y^d+c,\mathcal{L}-1} )</td>
</tr>
<tr>
<td>End</td>
</tr>
<tr>
<td>End</td>
</tr>
</tbody>
</table>

As an example, Eq. (5.7) illustrates the transition probability matrix of half-duplex mode for case \( \mathcal{L} = 7 \), \( m_y^d = 3 \), and \( m_y^d = 2 \).

\[
\mathbf{P}^H = \begin{bmatrix}
p_0 & p_1 & p_2 & p_3 & p_4 & p_5 & 1 - \sum_{b=0}^{5} p_b \\
0 & p_0 & p_1 & p_2 & p_3 & p_4 & 1 - \sum_{b=0}^{4} p_b \\
0 & 0 & p_0 & p_1 & p_2 & p_3 & 1 - \sum_{b=0}^{3} p_b \\
y_1 & 0 & 0 & q_0 p_0 & q_0 p_1 & q_0 p_2 & q_0 \left( 1 - \sum_{b=0}^{2} p_b \right) \\
y_1 & 0 & 0 & q_0 p_0 & q_0 p_1 & q_0 \left( 1 - p_0 - p_1 \right) \\
y_2 & 0 & q_1 & 0 & 0 & q_0 p_0 & q_0 \left( 1 - p_0 \right) \\
y_2 & 0 & q_1 & 0 & 0 & 0 & q_0 \end{bmatrix}, \quad (5.7)
\]
We observe that the first three rows of Eq. (5.7) are exactly the same as Eq. (5.3). However, the others are different. In particular, in Eq. (5.7), we see that there are several states with zero probability. In addition, for states with non-zero probability, there is only one option that leads to these states rather than several options as for the full-duplex mode.

### 5.2.7 Probability of the off-grid BSs’ activation

The off-grid BS is called activated if its energy level is from $m_1^d, d \in \{L_1, L_2\}$, (or not lower than state $(m_1^d)$ in the Markov chain). In mathematical terms, the activation probability of the off-grid BS is computed as

$$\Pr(\text{Enough Energy}) = 1 - \Pr(\text{Not Enough Energy}) = 1 - \varepsilon_{d,o} = 1 - \sum_{s=0}^{m_1^d-1} v_s^o, \quad (5.8)$$

where $v_s^o, s \in \{0, 1, \ldots, \mathcal{L} - 1\}, o \in \{H, F\}$, is the steady state probability of state $(s)$ of the Markov chain in $o$ mode. The steady state vector $v^o$ is the root of the following system of nonlinear equations:

$$v^o P^o (v^o) = v^o, o \in \{H, F\}. \quad (5.9)$$

It is noted that the transition probability matrix, $P^o, o \in \{H, F\}$, in our considered model is not independent of $v^o$ as the conventional matrix. Particularly, the entries, $r_{i,j}$, of $P^o$ is a function of $v^o$ (via $q_e$ and $y_e$ in Eqs. (5.4) and (5.5)). This dependence originates from the fact that in our system model, we only consider the activated off-grid BSs instead of all SBSs.

Unfortunately, it is impossible to obtain the exact closed-form expression of $v^o$ for the general scenario even with the traditional Markov chain. Consequently, the steady state vector, $v^o$, is computed by using numerical method such as $lsqnonlin$ function in Matlab.

### 5.3 Performance Analysis

#### 5.3.1 Power Consumption of Macro BSs

In this section, mathematical frameworks to compute the average power consumption of cellular networks are provided. It is emphasized that the consumption only comes from
the on-grid BSs since the off-grid BSs are disconnected from the power grid. Specifically,
the power consumption of both load models are provided as follows.

Table 5.4: Summary of main auxiliary functions used throughout the chapter. $k \in \{ \text{ON, OFF} \}, d \in \{ \text{L1, L2} \},$ and $o \in \{ \text{H, F} \}$.

<table>
<thead>
<tr>
<th>Function Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_k = \frac{AP_k}{\lambda_k}$, $AP_k = \left(1 + \left(\frac{\lambda_k}{\lambda}\right) \left(T_k/T_k\right)^{2/\beta}\right)^{-1}$, $\bar{k} \neq k$</td>
</tr>
<tr>
<td>$\Upsilon(x) = 2F_1(1, -2/\beta, 1 - 2/\beta, -x) - 1$</td>
</tr>
<tr>
<td>$c(n) = (3.5)^3 \Gamma(4.5 + n) / (\Gamma(3.5) \Gamma(2 + n))$</td>
</tr>
<tr>
<td>$e(a, b, c, k) = 2F_1(a, b, c, \lambda_{MT}A_k / (\lambda_{MT}A_k + 3.5))$</td>
</tr>
<tr>
<td>$\Delta_k = \left(T_k/T_k\right) \left(P_{tx}^{k}/P_{tx}^{k}\right)$</td>
</tr>
<tr>
<td>$f_1(d, o, i) = \sum_{s=m_i^d+(i-1)m^d} v_s^o$, $f_2(d, o, n) = \sum_{s=m_i^d+(n-1)m^d} v_s^o$</td>
</tr>
<tr>
<td>$u_1(k, d) = 1 - \exp \left(-\pi \left(P_{tx}^{k}/\eta\right)^{2/\beta} \lambda_k u_2(k, d)\right)$, $\eta = \kappa \gamma \sigma \gamma_{\nu}$</td>
</tr>
<tr>
<td>$u_2(k, d) = O(k, d, 1) + \left(\frac{\lambda_k}{\lambda}\right) \left(T_k/T_k\right)^{2/\beta} O \left(k, d, \Delta_k\right)$</td>
</tr>
<tr>
<td>$O(k, d, x) = 1 + P_{\text{Ac}}^{k,d} \Upsilon(x \gamma_d)$</td>
</tr>
<tr>
<td>$u_3(k, n) = 1 - c(n) \left(\frac{\lambda_{MT}A_k}{\lambda_{MT}A_k + 3.5}\right)^{n+4.5} \times \left(1 + n \right) e \left(1, n + 4.5, n + 1, k\right) - n e \left(1, n + 4.5, n + 2, k\right)$</td>
</tr>
<tr>
<td>$u_4(k, i) = 1 - \frac{\lambda_{MT}A_k}{N_{RB}} + \frac{(c(i)/N_{RB})}{(\lambda_{MT}A_k)^{i+1}} \times \left(\frac{\lambda_{MT}A_k}{\lambda_{MT}A_k + 3.5}\right)^{i+4.5} e \left(2, i + 4.5, i + 2, k\right)$</td>
</tr>
<tr>
<td>$u_5(k, n) = c(n) \left(\frac{\lambda_{MT}A_k}{\lambda_{MT}A_k + 3.5}\right)^{n+4.5} \times \left(1 + 4.5 + n/2 + n\right) \left(\lambda_{MT}A_k\right) / \left(\lambda_{MT}A_k + 3.5\right) e \left(2, n + 5.5, n + 3, k\right)$</td>
</tr>
<tr>
<td>$u_6(k, n) = c(n) \left(\frac{\lambda_{MT}A_k}{\lambda_{MT}A_k + 3.5}\right)^{n+4.5} e \left(2, n + 4.5, n + 2, k\right)$</td>
</tr>
<tr>
<td>$P_{\text{cov}}^{k} (\gamma_D, \gamma_A, d) = u_1(k, d) / u_2(k, d)$</td>
</tr>
<tr>
<td>$P_{\text{ON,L1}}^{\text{L1}} (n) = u_3 (\text{ON}, n)$</td>
</tr>
<tr>
<td>$P_{\text{OFF,L1}}^{\text{L1}} (n) = \left(1 - \varepsilon_{d,o}\right)^{-1} \left(\sum_{i=1}^{n-1} u_3 (\text{OFF}, i) f_1 (d, o, i) + u_3 (\text{OFF}, n) f_2 (d, o, n)\right)$</td>
</tr>
<tr>
<td>$P_{\text{ON,L1}}^{\text{Ac}} = 1 - P_{\text{ON,L1}}^{\text{Inc}}$</td>
</tr>
<tr>
<td>$P_{\text{ON,L1}}^{\text{Inc}} = 1 - \frac{(\lambda_{MT}A_{ON})/N_{RB}}{\left(\lambda_{MT}A_{ON}\right) / N_{RB} + \left(\lambda_{MT}A_{ON}\right) / N_{RB}}$ if $N_{\text{MAX,L1}}^{\text{ON}} = N_{RB}$</td>
</tr>
<tr>
<td>$P_{\text{OFF,L1}}^{\text{L1}} = 1 - \left(1 - \varepsilon_{L1,o}\right)^{-1} \times \left(\sum_{i=1}^{n_{\text{OFF}}-1} u_4 (\text{OFF}, i) f_1 (L1, o, i) + u_4 (\text{OFF}, N_{\text{MAX,L1}}^{\text{B}}) f_2 (L1, o, N_{\text{MAX,L1}}^{\text{B}})\right)$</td>
</tr>
<tr>
<td>$P_{\text{OFF,L1}}^{\text{L1}} = 1 - \left(1 + \frac{1}{\beta} \lambda_{MT}A_k\right)^{-3.5}$</td>
</tr>
<tr>
<td>$P_{\text{Ac}}^{\text{k,l,2}} = 1 - \left(1 + \frac{1}{\beta} \lambda_{MT}A_k\right)^{-3.5}$</td>
</tr>
</tbody>
</table>
Power Consumption of Load Model 1

The power consumption of load model 1, $P^{L1}_{grid}$, is given by

$$P^{L1}_{grid} = \alpha_{ON} \lambda_{BS} N_{RB} \left( P^{ON}_{\text{cir}} + \frac{P^{ON}_{\text{tx}}}{N_{RB}} \right) \left( 1 - P^{ON,L1}_{\text{inc}} \right) + \alpha_{ON} \lambda_{BS} P_{\text{idle}} \left( 1 + \frac{\lambda_{MT} A_{ON}}{3.5} \right)^{-3.5},$$

$$A_{ON} = \frac{AP_{ON}}{\alpha_{ON} \lambda_{BS}},$$

where $AP_{ON}$ is given in Eq. (5.6) and $P^{ON,L1}_{\text{inc}}$ is the inactive probability of one RB of load model 1, which is calculated as follows

$$P^{ON,L1}_{\text{inc}} = \sum_{n=0}^{\infty} \left( 1 - \frac{n}{N_{RB}} \right) \Pr [N = n, \text{ON}]$$

where $A_k = \frac{AP_k}{\lambda_k}$, $k \in \{ \text{ON}, \text{OFF} \}$, and $AP_k$ is given in Eq. (5.6), $u_5$ and $u_6$ are provided in Table 5.4.

Power Consumption of Load Model 2

The power consumption of load model 2, $P^{L2}_{grid}$, is given as follows

$$P^{L2}_{grid} = \alpha_{ON} \lambda_{BS} P^{ON}_{\text{tx}} \left( 1 - \left( 1 + \frac{\lambda_{MT} A_{ON}}{3.5} \right)^{-3.5} \right) + \alpha_{ON} \lambda_{BS} P^{ON}_{\text{cir}} \mathbb{E} \{ \text{MTs, ON} \}$$

$$+ \alpha_{ON} \lambda_{BS} P_{\text{idle}} \left( 1 + \frac{\lambda_{MT} A_{ON}}{3.5} \right)^{-3.5}.$$  

Here $\mathbb{E} \{ \text{MTs, ON} \}$ is the average MTs served by the on-grid BSs of load model 2:

$$\mathbb{E} \{ \text{MTs, ON} \} = \sum_{n=0}^{\infty} n \Pr [\text{MTs} = n, \text{ON}] = \lambda_{MT} A_{ON} - c \left( N^{ON,L2}_{\text{max}} \right)$$

$$\times \frac{\left( \lambda_{MT} A_{ON} \right)^{N^{ON,L2}_{\text{max}}+1}}{(3.5 + \lambda_{MT} A_{ON})^{N^{ON,L2}_{\text{max}}+4.5}} e \left( 2, N^{ON,L2}_{\text{max}} + 4.5, N^{ON,L2}_{\text{max}} + 2, \frac{\lambda_{MT} A_{ON}}{3.5 + \lambda_{MT} A_{ON}} \right),$$

where $c(n)$ and $e(a, b, c, k)$ are given in Table 5.4.
5.3. Performance Analysis

5.3.2 Potential Spectral Efficiency (PSE)

The potential spectral efficiency is calculated by using the same definition as in Chapter 4.

PSE of Load Model 1

The potential spectral efficiency of load model 1 is computed as

\[
\text{PSE}^{L1} = \lambda_{MT} \times BW \times \log_2 \left( 1 + \gamma_D \right) \sum_{k, \tilde{k} \in \{ON, OFF\} \atop k \neq \tilde{k}} \text{Pr} \left( \text{SIR}^k_{L1} \geq \gamma_D, \overline{\text{SNR}}^k \geq \gamma_A, T_{\tilde{k}} L^{(0)}_{\tilde{k}} > T_k L^{(0)}_k \right)
\]

\[
= \lambda_{MT} \times BW \times \log_2 \left( 1 + \gamma_D \right) \left[ \text{P}^{ON} (\gamma_D, \gamma_A, L1) P^{ON,L1}_{sel} (N_{max}^{ON,L1}) + \text{P}^{OFF} (\gamma_D, \gamma_A, L1) P^{OFF,L1}_{sel} (N_{max}^B) \right]
\]

(5.14)

where \( P^{cov}_{k} (\gamma_D, \gamma_A, L1) \) and \( P^{L1}_{sel} (n) \) are provided in Table 5.4. Also, \( BW_{RB} = \frac{BW}{N_{RB}} \), \( \gamma_D \) and \( \gamma_A \) are the transmission bandwidth per resource block, the threshold of data phase and association phase respectively. The signal to interference ratio in load model \( d \) and average signal to noise ratio of \( k \)-tier BSs are given as follows

\[
\text{SIR}^k_d = \frac{P^{k}_{tx} h_{i}^{(0)}/L_k^{(0)}}{P^{k}_{tx} \sum_{i \in \Psi_{k,d}^{(i,d)}} h_{i}^{(0)}/L_k^{(0)}} 1 \left( L^{(i)} > L_k^{(0)} \right) + \frac{P^{k}_{tx} h_{j}^{(j)/L_{k}}}{P^{k}_{tx} \sum_{j \in \Psi_{k,d}^{(j)}} h_{j}^{(j)/L_{k}} 1 \left( L^{(j)} > \frac{T_{\tilde{k}} L^{(0)}_{\tilde{k}}}{T_k L^{(0)}_k} \right)}
\]

\[
\overline{\text{SNR}}^k = \frac{P^{k}_{tx}}{L_{k}^{(0)} \sigma_N^2}.
\]

(5.15)

Here \( \Psi_{k,d}^{(i,d)} \), \( k \in \{ON, OFF\} \), \( \tilde{k} \neq k \), \( d \in \{L1, L2\} \), is the set of interferers BS from \( k \)-tier of load model \( d \), \( \sigma_N^2 = \text{BWN}_0 \) is the noise variance at the typical MT; \( N_{0} \) is the noise power spectral density; \( 1(.) \) is the indicator function.

PSE of Load Model 2

The PSE of load model 2 is computed as

\[
\text{PSE}^{L2} = \lambda_{MT} \times BW \times \log_2 \left( 1 + \gamma_D \right) \sum_{k, \tilde{k} \in \{ON, OFF\} \atop k \neq \tilde{k}} \text{Pr} \left( \text{SIR}^k_{L2} \geq \gamma_D, \overline{\text{SNR}}^k \geq \gamma_A, T_{\tilde{k}} L^{(0)}_{\tilde{k}} > T_k L^{(0)}_k \right)
\]

\[
= \lambda_{MT} \times BW \times \log_2 \left( 1 + \gamma_D \right) \left[ \text{P}^{ON} (\gamma_D, \gamma_A, L2) P^{ON,L2}_{sel} + \text{P}^{OFF} (\gamma_D, \gamma_A, L2) P^{OFF,L2}_{sel} \right]
\]

(5.16)
The definition of $\text{SIR}^k_{L1}$ and $\text{SNR}^k$ are given in Eq. (5.15) and $P_{\text{cov}}^k(\gamma_D, \gamma_A, L2)$ and $P_{\text{sel}}^{k,L1}$ are provided in Table 5.4.

Remark 22 The definition of probability of inactivation (activation) of the two load models are different, $P_{\text{Inc}}^{k,L1} \neq P_{\text{Inc}}^{k,L2}$. In load model 1, the probability is defined per resource block while in load model 2 the definition is applied to the whole BS.

### 5.3.3 Energy Efficiency

The EE of both load models are computed as

$$EE^d = \frac{\text{PSE}^d}{P_{\text{grid}}^d}, d \in \{L1, L2\},$$

where $\text{PSE}^d$ and $P_{\text{grid}}^d$, $d \in \{L1, L2\}$, are provided in Eqs. (5.14), (5.16), (5.10), and (5.12).

### 5.4 Numerical Results

In this section, numerical results are provided to verify our mathematical frameworks. The simulation setup is given in Table 5.5. The parameters in this section are in agreement with Chapter 4.

Figs. 5.1 to 5.5 confirm the correctness of our framework if compared against Monte Carlo simulations.

Specifically, the $P_{\text{cov}}$ vs. $R_{\text{cell}}$ of full-duplex mode is provided in Fig. 5.1. We observe that $P_{\text{cov}}$ is monotonically decreasing with $R_{\text{cell}}$ (increasing with $\lambda_{BS}$). In addition, the $P_{\text{cov}}$ of load model 1 decreases dramatically for the whole range of $R_{\text{cell}}$ while the decrease pace of load model 2 is different from fast to slow when $R_{\text{cell}}$ increasing. Moreover, it is obvious that increasing $\gamma_D$ yields worse performance.

Fig. 5.2 illustrates the performance of $\text{PSE}$ vs. $\gamma_D$ of various scenarios. It is not surprising that the $\text{PSE}$ is unimodal as a function of $\gamma_D$ as $\log_2(1 + \gamma_D)$ is an increasing function with $\gamma_D$ while $P_{\text{cov}}^k(\gamma_D)$ is a monotonically decreasing function with $\gamma_D$. Case study 1 considers the scenario where the MTs are likely connected to the SBSs owing to a smaller bias factor and larger density than the MBSs. Case study 2 studies the case...
Table 5.5: Setup of parameters (unless otherwise stated).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>3.5</td>
</tr>
<tr>
<td>$\kappa = \left(4\pi f_c/3 \cdot 10^8\right)^2$</td>
<td>$f_c = 2.1$ GHz</td>
</tr>
<tr>
<td>$N_0$</td>
<td>-174 dBm/Hz</td>
</tr>
<tr>
<td>BW</td>
<td>20 MHz</td>
</tr>
<tr>
<td>$P_{\text{ON}}$</td>
<td>51.14 dBm [71]</td>
</tr>
<tr>
<td>$P_{\text{idle}}$</td>
<td>48.75 dBm [71]</td>
</tr>
<tr>
<td>$P_{\text{tx}}$</td>
<td>43 dBm [71]</td>
</tr>
<tr>
<td>$P_{\text{OFF}}$</td>
<td>23 dBm</td>
</tr>
<tr>
<td>$\lambda_{\text{BS}} = 1/(\pi R_{\text{cell}}^2)$ BSs/m²</td>
<td>$R_{\text{cell}} = 70$ m</td>
</tr>
<tr>
<td>$\lambda_{\text{MT}} = 1/(\pi R_{\text{MT}}^2)$ = 121 MTs/km²</td>
<td>$R_{\text{MT}} = 51.29$ m</td>
</tr>
<tr>
<td>$\gamma_{\text{D}} = \gamma_{\text{A}}$</td>
<td>5 dB</td>
</tr>
<tr>
<td>$L, \lambda_{\text{EP}}$</td>
<td>31, 1.5</td>
</tr>
<tr>
<td>$m^{L1}, m^{L2}$</td>
<td>3, 2</td>
</tr>
<tr>
<td>$m_1^{L1}, m_1^{L2}$</td>
<td>3, 6</td>
</tr>
<tr>
<td>$N_{\text{RB}}$</td>
<td>4</td>
</tr>
<tr>
<td>$N_{\text{Load}}^{L1}, N_{\text{Load}}^{L2}$</td>
<td>8, 5</td>
</tr>
<tr>
<td>$T_{\text{ON}}, T_{\text{OFF}}$</td>
<td>3, 1</td>
</tr>
<tr>
<td>$\alpha_{\text{ON}}, \alpha_{\text{OFF}}$</td>
<td>1, 3</td>
</tr>
</tbody>
</table>

Figure 5.1: Coverage Probability vs. Rcell of full-duplex mode of load model 1 and 2 with various value of $\gamma_{\text{D}}$. Solid lines are plotted by Eq. (5.14) and Eq. (5.16). Marker are Monte Carlo simulation.

where the MBSs are denser than the SBSs and their bias factor is also bigger than for the SBSs. It is not clear, then, whether the MTs are served by the SBSs or the MBSs. Case
study 3 considers the inverse case of scenario 1 where the MTs are primarily served by the MBSs. Finally, case study 4 is the contrary of case 2, hence, we also do not know what BSs typically serve the MTs. The figure shows that the PSE gets better when $\lambda_{\text{ON}} > \lambda_{\text{OFF}}$. It can be explained that with $\lambda_{\text{ON}} > \lambda_{\text{OFF}}$, the MTs are highly served by the MBSs which have larger transmit power.

Fig. 5.3 illustrates the EE as a function of $P_{\text{tx}}^{\text{ON}}$ of half-duplex mode of both load models. From the figure, we conclude that load model 2 provides better EE compared with load model 1 because the PSE of load model 2 is better than load model 1.

Fig. 5.4 compares the performance of full- and half-duplex modes in terms of EE for load model 2. It is obvious that the EE of full-duplex is better since the battery can harvest and consume power simultaneously.

Fig. 5.5 investigates the behavior of EE as a function of $\lambda_{\text{EP}}$. The results show that increasing $\lambda_{\text{EP}}$ will provide a better EE for both full- and half-duplex modes.
5.4. Numerical Results

Figure 5.3: EE vs. $P_{tx}^{ON}$ of half-duplex mode of load model 1 and 2 with various value of $\gamma_D$. Solid lines are plotted by Eq. (5.17). Marker are Monte Carlo simulation.

Figure 5.4: EE vs. $R_{cell}$ of full and half-duplex mode of load model 2 with various value of $\gamma_D$. Solid lines are plotted by Eq. (5.17). Marker are Monte Carlo simulation.
Figure 5.5: EE vs. $\gamma_D$ of full and half-duplex mode of load model 1 with various value of $\lambda_{EP}$. Solid lines are plotted by Eq. (5.17). Marker are Monte Carlo simulation.

5.5 Conclusion

In this chapter, the system-level performance of cellular networks equipped with renewable energy sources was investigated. The battery of SBS was modeled as a discrete Markov chain and two operational models are studied, full- and half-duplex. In addition, the offloading from the MBS to the SBS was taken into account by applying a bias factor. A new framework was proposed to estimate key performance metrics, including the coverage probability, the spectral efficiency, and the energy efficiency. The results showed that full-duplex operation provided better performance compared with half-duplex operation at the expense of a higher installation and maintenance cost.
6.1 Conclusions

In this thesis, new analytical frameworks for the system-level modeling, performance evaluation, and optimization of cellular networks have been introduced by using the mathematical tool of stochastic geometry. The specific contributions made by and the main conclusions drawn in this dissertation can be summarized follows.

- In Chapter 2, three mathematical frameworks were proposed to study SWIPT-enabled MIMO cellular networks, which provided exact, approximated, and large-scale asymptotic expressions of the JCCDF of information rate and harvested power. Our studies showed that large-scale MIMO and ultra-dense deployments of BSs were both necessary to harvest, with high reliability, an amount of power of the order of a milliwatt.

- In Chapter 3, we analyzed SWIPT-enabled cellular networks with various options based on selection combining and maximum ratio combining schemes, and discussed their achievable performance versus implementation complexity trade-off. Our analysis, in particular, showed that no scheme outperforms the others for every system
setup. It suggested, on the other hand, that the devices needed to operate in an adaptive fashion, by choosing the receiver diversity scheme to be used as a function of the performance requirements.

- In Chapter 4, a new expression for the PSE of cellular networks was introduced by taking into account the power sensitivity of the receiver not only for data transmission but also for cell association. Based on the new expression of the PSE, a new system-level EE optimization problem was formulated and comprehensively studied. It was mathematically proved that the EE is a unimodal and strictly pseudo-concave function in the transmit power given the BSs’ density and in the BSs’ density given the transmit power.

- In Chapter 5, the PSE and EE of a two-tier cellular networks equipped with renewable energy sources was studied. It was showed that the performance of full-duplex battery outperform the half-duplex counterpart at the expense of a higher installation cost. Moreover, it was shown that the PSE and EE are unimodal functions of the density of BS given the transmit power and vice versa, like in Chapter 4.

### 6.2 Future Works

There are many extensions that can be made based on the outcomes of this dissertation. Some of them are discussed below.

#### 6.2.1 Beyond PPP modeling

The homogeneous PPP is applied widely in most of the studies due to its mathematical tractability. It is, however, not realistic in practical mobile network deployments. In fact, the BSs are not deployed independently of each other but are spatially correlated, i.e., there exists spatial repulsion between the BSs. In this context, some research have conducted to use repulsive point processes like Matérn hard-core process (MHCP), the Strauss process, the perturbed lattice, and the $\beta$ Ginibre point process to model more realistic BS deployments [150–152]. An interesting research direction is to generalize
the methodologies proposed in the thesis to the analysis and optimization of spatially-correlated point processes.

### 6.2.2 UAV-aided cellular networks

In recent years, Unmanned Aerial Vehicles (UAV) have gained lots of attention for many applications, such as battle field and disaster relief, due to their ease of deployment and flexibility. Several scholars and researchers such as Zeng et al. in [153] discussed the advantages and the drawbacks of UAV application to wireless networks. As pointed out in [153], there are lots of open issues associated with UAV-aided cellular networks. The first issue is that it is necessary to have a comprehensive channel modeling between UAV and ground MTs. Secondly, due to strict energy constraints, the UAVs are not operational for long periods of time. The system-level performance evaluation and optimization of UAV-aided cellular networks is worthy of investigation.

### 6.2.3 Network Slicing (NS)

One of the key elements of 5G mobile networks will be network slicing (NS). The principle idea of network slicing is to allow numerous different services/users to share the same mobile infrastructure. The users/services can be viewed as distinct logical networks. This is considered as the easiest and cost-effective way to satisfy all distinct applications while not too much increase the capital expenditure (CAPEX) and operating expense (OPEX) of network operators [154–156]. However, the system-level performance and optimization of cellular networks with NS are an open research issue.

### 6.2.4 Cellular Networks with Decoupled Architecture

From the 1\textsuperscript{st} to 4\textsuperscript{th} generation of mobile networks, the basic assumption is that the MT is served by the same BS in both directions (uplink and downlink). This, however, needs to be revisited in the next generation due to dense deployment of small cells with different transmit power and coverage range compared with the traditional MBSs, in order to exploit the large transmit power of MBSs and densely-deployed SBSs. Thus, the MT is more likely to be served by macro BS in downlink due to the large transmit power
of MBS. While in uplink transmission, the MT may need to be served by a small cell BS due to power constraint at the MT [157, 158]. This architecture also requires strong synchronization and data connectivity (e.g., via fiber) between the BSs. As a result, it is of much interest to analyze and optimize heterogeneous cellular networks with decoupled uplink and downlink connectivity.


[126] 3GPP TS36.304. *3rd generation partnership project; technical specification group radio access network, evolved universal terrestrial radio access (E-UTRA); user equipment (UE) procedures in idle mode.*


