# Critical edition of the Goladīpikā (Illumination of the sphere) by Parameśvara, with translation and commentaries

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# Critical edition of the *Goladīpikā* (Illumination of the Sphere) by Parameśvara, with translation and commentaries

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**Titre:** Edition critique de *«Goladīpikā»* (L'illumination de la sphère) par Parameśvara, avec une traduction et des Résumé

**Résumé:** Le *Goladīpikā* (L'illumination de la sphère) est un traité composé par Parameśvara. Il existe deux versions de ce texte: l'une a été édité avec une traduction anglaise et l'autre n'est qu'une édition utilisant trois manuscrits. Cette thèse donne une nouvelle édition de la deuxième version en utilisant onze manuscrits dont un commentaire anonyme nouvellement trouvé. Elle se compose aussi d'une traduction anglaise et de notes explicatives.

Pour l'essentiel, le *Goladīpikā* est une collection de procédures pour déterminer la position des objets célestes. Cette thèse décrit les outils mathématiques qui sont utilisées dans ces procédures, en particulier les Règles de trois, et discute de la manière dont Parameśvara les fonde. Il y a une description d'une sphère armillaire au debut du *Goladīpikā.* Donc ce doctorat examine aussi comment cet instrument a pu être utilisé pour expliquer ces procédures. Ce travail tente aussi de positionner le *Goladīpikā* au sein du corpus des oeuvres Parameśvara et d'autres auteurs.

**Mots clés:** Inde, Kérala, sphère armillaire, histoire de l'astronomie, sanskrit

**Title:** Critical edition of the *Goladīpikā* (Illumination of the Sphere) by Parameśvara, with translation and commentaries

**Abstract:** The *Goladīpikā* (Illumination of the Sphere) is a Sanskrit treatise by Parameśvara, which is extant in two distinctly different versions. One of them has been edited with an English translation and the other has only an edition using three manuscripts. This dissertation presents a new edition of the latter version using eleven manuscripts, adding a newly found anonymous commentary. It further consists of an English translation of the base text and the commentary as well as explanatory notes.

The main content of the *Goladīpikā* is a collection of procedures to find the positions of celestial objects in the sky. This dissertation highlights the mathematical tools used in these procedures, notably Rules of Three, and discusses how the author Parameśvara could have grounded the steps. There is a description of an armillary sphere at the beginning of the *Goladīpikā*, and the dissertation also examines how this instrument could have been involved in explaining the procedures. In the course of these arguments, the dissertation also attempts to position the *Goladīpikā* among the corpus of Parameśvara's text as well as in relation to other authors.

**Keywords:** India, Kerala, armillary sphere, history of astronomy, Sanskrit

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In August 2013, I have examined a manuscript (No. 13259) at the Kerala University Oriental Research Institute and Manuscripts Library which turned out to have been wrongly labeled. Out of curiosity I tried to identify the text correctly, and very luckily I found that it was the *Goladīpikā* by Parameśvara since the critical edition by T. Gaṇapati Sāstrī was available at the library of the Kyoto University Faculty of Letters. This was how, by the chance discovery of a mislabeled manuscript, my dissertation on the *Goladīpikā* began. But I was even more fortunate to have reached the end thanks to the aid and advice of many people.

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I also extend my thanks to the developers of softwares. The entire dissertation is written in XeLaTeX, and I have used the ledmac package for the critical edition. Every figure has been drawn with Autodesk® Graphic. I have also used StellaNavigator® 10 by AstroArts Inc. for simulating the positions and motions of celestial objects.

Last but not least, I thank my families: Marc Schoenauer and Michèle Sebag, who have been indeed my father and mother in France, and Nobuaki and Keiko, my parents in Japan, who have always been on my side.

# Contents

## List of abbreviations

### Primary sources

***Ābh*** The *Āryabhaṭīya* of Āryabhaṭa

***GD1*** The *Goladīpikā I* of Parameśvara,
edition and translation by K.V. Sarma

***GD2*** The *Goladīpikā II* of Parameśvara,
edition by T. Gaṇapati Śāstrī

***MBh*** The *Mahābhāskarīya* of Bhāskara I

### Institute name

**KOML** Kerala University Oriental Research Institute and Manuscripts Library

# Introduction

The *Goladīpikā* ("Illumination of the Sphere") is a Sanskrit treatise by Parameśvara, which is extant in two distinctly different versions. One of them (hereafter *GD1*) has been edited with an English translation (K. V. Sarma (1956–1957)) and the other (*GD2*) has only an edition using three manuscripts (Sāstrī (1916)). This work is a new edition of *GD2* with eleven manuscripts, adding a newly found anonymous commentary. I have also translated the base text and the commentary into English and added my explanatory notes in an attempt to highlight the mathematical and astronomical tools used in this treatise, and position it among the corpus of Parameśvara's text as well as in relation to other authors.

## 0.1 The author Parameśvara

### 0.1.1 Dates of Parameśvara

Typically, Parameśvara's date is given as *c.* 1360-1455 CE (K. V. Sarma (1972, p. 52)). His birth date is estimated using his own words that he wrote the *Dṛggaṇita* in the Śaka year 1353[1] (1431-32 CE) and the following words by Nīlakaṇṭha, a student of Parameśvara's son, in his commentary on the *Āryabhaṭīya*:

> Then Parameśvara, having well understood the reasoning of mathematics and the Sphere already in his youth indeed from experts on the Sphere such as Rudra, Parameśvara's son Nārāyaṇa and Mādhava, having been acquainted with the practices formed from them having disagreements with observation and of their causes, having perceived it in may treatises, having made observations for fifty-five years, having examined eclipses, planetary conjunctions and the like, and made the entire *Dṛggaṇita*.[2]

The statement suggests that Parameśvara should have started his observations by the Śaka year 1298 (1376-77 CE). Assuming that Parameśvara should have been in his late teens when he began observing, this puts his birth date around 1360. However, Parameśvara himself says in his commentary on *MBh* 5.77 that he started observing eclipses from the Śaka year 1315 (1393-1394 CE). This might imply that Parameśvara was observing astronomical phenomena other than eclipses before Śaka 1315.

As we will see in section 0.1.7, Parameśvara's grandfather was a student of Govinda, who died on a date corresponding to October 24th 1314 CE according to popular tradition (Raja (1995, p. 15))[3]. This suggests that Parameśvara's grandfather must have been born at least before 1300 CE, and it is reasonable that his grandson would be born 60 years later.

As for the date of Parameśvara's death, the reference by Nīlakaṇṭha (born 1444 CE) to him as "our master (*asmad ācārya*)" in his commentary on *Ābh* 4.11 (Pillai (1957b, p. 27)) is often quoted

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[1] *Dṛggaṇita* 2.26 (K. V. Sarma (1963, p. 26)). The Śaka years are counted in expired years (by contrast to the common era where the first year would is counted as year 1 and the second year after one year has expired is year 2) starting from the spring equinox in 78 CE.

[2] *parameśvaras tu rudraparameśvarātmajanārāyaṇamādhavādibhyo golavidbhyo gaṇitagolayuktīr api bālya eva samyag gṛhītvā tebhya eva kriyamāṇaprayogasya dṛgvisaṃvādaṃ tatkāraṇaṃ cāvadhārya śāstrāṇy api bahūny ālocya pañcapañcāśad varṣakālaṃ nirīkṣya grahaṇagrahayogādiṣu parīkṣya samadṛggaṇitaṃ karaṇaṃ cakāra* / (Pillai (1957b, p. 154))

[3] This is represented by the phrase *kālindīpriyatuṣṭaḥ* which is 1,612,831 in *Kaṭapayādi* notation. This is the number of days since the beginning of the *Kali-yuga*. However Raja (1995) does not provide any reliable source for this information and it must be treated with caution. Moreover, he converts this date wrongly to 1295 CE.

as an evidence that Nīlakaṇṭha had learned directly from Parameśvara and that Parameśvara must have been still alive around 1455-60 CE. However, addressing someone as one's "master" does not necessarily indicate a direct mentorship. Nīlakaṇṭha even refers to Āryabhaṭa (476-c.550 CE) as his master (Pillai (1957b, p. 1)). Parameśvara's son, Dāmodara, was indeed Nīlakaṇṭha's teacher, and thus Nīlakaṇṭha more often calls Parameśvara his "grand-teacher (*paramaguru*)" (K. V. Sarma (1977a, p. xxxii)). Thus Parameśvara's death could have been earlier than usually admitted. The earliest limit would be the Śaka year 1365 (1443-1444 CE) when he composed the *Goladīpikā* I. But we must take into account that he seems to have written an auto-commentary on this treatise in response to students finding it difficult to understand, as we will see below.

Therefore, I estimate that Parameśvara was born between 1360-75 CE and died between 1445-60 CE.

### 0.1.2 Where Parameśvara lived

Parameśvara provides abundant information on his location. In several of his works such as *GD1* 1.2 (see chapter 1), he refers to his place of dwelling as the northern bank at the mouth of a river called Nilā. This is another name of the river Bhāratappuz ha which flows through central Kerala. On the north bank at its mouth with the Arabic sea is the village of Purathur in the Malappuram district. Parameśvara himself refers to his place as the village of Aśvattha, for example in *Grahaṇamaṇḍana* 14cd (K. V. Sarma (1965, pp. 6-7)).

Parameśvara also mentions the geographic longitude and latitude of his location occasionally, such as in *GD1* 4.91:

> Living in a village at a distance of eighteen *yojana*s west to the geographic prime meridian and at a latitude of six hundred and forty-seven, in the Śaka year thirteen hundred and sixty-five, ...[4]

The geographic prime meridian (*samarekhā*) is considered to go through the city of Ujjain (see section 11.2). However, the line of longitude passing through modern Ujjain goes into the Arabic sea at the latitude of Kerala, and we do not know how Parameśvara measured his longitude[5]. As for the geographic latitude, the value 647 is the Sine, and the corresponding arc is $10°51'$. This falls exactly on the modern village of Purathur at the mouth of river Bhāratappuzha. Parameśvara also uses the value 647 in his examples, including *GD2*.

The village of Aśvattha is the reference point for the geographic longitude and latitude in Parameśvara's texts. This resembles the role of Ujjain, and invokes the question whether Aśvattha was a place of scholarship and center of astronomy, as Ujjain is alleged to have been such location. Parameśvara and his son Dāmodara probably lived in Aśvattha, but we have no information about scholars prior to Parameśvara living in the same spot, nor any evidence of educational institutions in the village. Thus this hypothesis is very uncertain.

### 0.1.3 The variants in his name

In *GD2* 68, the author calls himself *paramādi īśvara*, separating the two words. This is the only occurrence of his name in the text, including the colophons of the manuscript. The same form

[4] *samarekhāyāḥ paścād aṣṭādaśayojanāntare grāme |*
*svarakṛtaṣaṭtulitākṣe vasatā śāke 'kṣaṣaṭtricandramite ||4.91||* (K. V. Sarma (1956–1957, p. 68))

[5] It is very unlikely that he measured the longitude of Ujjain directly by himself, and it is possible that the value of "18 *yojana*s west" had simply been handed down to him. Contrarily, he defines the prime meridian in reference to his own location by saying that the prime meridian "is eastward 18 *yojana*s from a village called Aśvattha (*aśvatthākhyād grāmād aṣṭādaśayojane*)" in *Grahaṇamaṇḍana* 14cd (K. V. Sarma (1965, pp. 6-7)).

can be seen in the concluding verse of his commentary on the *Sūryasiddhānta* (Shukla (1957, p. 144)). Meanwhile the compounded form *paramādīśvara* is found in the *Dṛggaṇita* (verse 2.46, K. V. Sarma (1963, p. 26)) and the commentary on the *Ābh* (opening and closing verses, Kern (1874, pp. 1, 100))[6]. These are variations of the name *Parameśvara* and not another author. We can identify him from his reference to other texts of his own as well as remarks on his location.

Furthermore, Nīlakaṇṭha quotes *Dṛggaṇita* 2.46 in his commentary on *Ābh* 4.48 right after referring to the treatise as the "*Dṛggaṇita* taught by Parameśvara [7]". Other informations in the text, including the reference to the location, also support the author's identity. This indicates that at Nīlakaṇṭha's days, people were well aware that *parameśvara* and *paramādi īśvara* were references to the same author.

### 0.1.4 Works by Parameśvara

Pingree (1981, pp. 187-192) enumerates 25 extant works of Parameśvara which is based on the identifications by K. V. Sarma (1972). Among the list we have not counted his auto-commentary on *Goladīpikā 1* (No.8 in Pingree's list) as an independent work, included the "expanded version on the second *Goladīpikā* (No.16)" in *Goladīpikā 2* and taken the *Vivāhānukūlya* (No.24) as part of the *Ācārasaṃgraha*. This leaves 22 works in our list.

In the concluding verses of his *Karmadīpikā*, Parameśvara names 8 of his treatises ending with *dīpikā*[8]. Among them, the *Muhūrtāṣṭakadīpikā*, *Vākyadīpikā* and the *Bhādīpikā* have no extant manuscripts bearing their names, and could be works that are yet to be recovered.

All of his known works are in Sanskrit, but Kṛṣṇadāsa (1756-1812 CE) quotes a Malayalam passage attributed to Parameśvara. Whether this is only a view of Parameśvara expressed in Malayalam or really an unknown Malayalam work by Parameśvara is an open question (K. V. Sarma (ibid., pp. 74-75)).

#### Commentaries on *Siddhānta*s

The longitude of planets including the sun and moon is involved in almost every topic in Sanskrit astronomy. It was a central theme in the so-called "standard *Siddhānta* (treatise)" (Plofker (2009)) texts, comprehensive works which compute planetary longitudes from a very early epoch (the beginning of the *Kalpa* or *Kali-yuga*).

There is no *Siddhānta* attributed to Parameśvara. In other words, all his original astronomical works known today are focused on a specific area. However, he frequently commented on *Siddhānta*s, and these commentaries are essential to understand his general ideas in his more specialized texts including *GD2*.

**1. Commentary on Bhāskara I's *Laghubhāskarīya* (Lesser/Short/Easy [treatise] of Bhāskara)** Critical edition by B. Āpṭe (1946).

The *Laghubhāskarīya* is the last known text composed by Bhāskara I after his *Mahābhāskarīya* and the commentary on the *Āryabhaṭīya*, and is considered an abridged word of his former treatise for younger readers (Shukla (1976, pp. xxx-xxxii)).

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[6] The editor Kern calls him "Paramâdîçvara".

[7] *parameśvarācāryapraṇītadṛggaṇita* (Pillai (1957b, p. 151))

[8] "Parameśvara made the illuminations (*dīpikā*) of the *Muhūrtāṣṭaka*, *Siddhānta*, *Vākya*, *Bhā*, *Nyāya* and the *Karma*, as well as those of the *Gola* and *Bhaṭa*."
*muhūrtāṣṭakasiddhāntavākyabhānyāyakarmaṇām* /
*dīpikāṃ golabhaṭayoś cākarot parameśvaraḥ* // (Kale (1945, p. 92))

On the other hand, Parameśvara's commentary on this treatise was probably composed relatively earlier than his other works. In his commentary on *Laghubhāskarīya* 2.16 (B. Āpṭe (1946, p. 22)), he computes the amount of trepidation of the solstitial points in the ecliptic for the Śaka year 1330 (1408/1409 CE)[9] which is more than 20 years earlier than the *Siddhāntadīpikā* or *GD2*. Shukla (1976, p. cv) adds that Parameśvara was still a student at this moment on the basis of the following passage in his concluding verses:

> Thus, for the benefit of novices, I, serving at the lotus-like foot of my teacher, have explained the meaning of the *Bhāskarīya* concisely.[10]

I am not sure whether this really means that he was still a student, or whether he is only being modest. If Parameśvara was born around 1360 CE, he is already near 50 at this point.

The glosses are short in general, using paraphrasing. But some verses are followed by articulating Rules of Threes that ground the computations, quotations of related texts or less frequently, examples. Commentaries on some verses in the fifth (solar eclipse) and sixth (visibility and phase of the moon) chapters (B. Āpṭe (1946, pp. 58-82)) are conspicuously detailed.

Parameśvara himself does not refer to the *Laghubhāskarīya* very often in his later texts. I have found no trace of it in his *Bhaṭadīpikā* and the *Goladīpikā*s. Nonetheless, a detailed study on this commentary would provide us with good information on Parameśvara's earlier theories and its development. In addition, this commentary was read by Parameśvara's successors; Nīlakaṇṭha mentions or quotes from it occasionally in his commentary on the *Āryabhaṭīya* (Śāstrī (1931, p. 63), Pillai (1957b, pp. 79,81)).

**2. *Bhaṭadīpikā* (Illumination of [Ārya]bhaṭa['s work]) on Āryabhaṭa's *Āryabhaṭīya***
Critical edition by Kern (1874).

The *Āryabhaṭīya* (composed 499 CE or later by Āryabhaṭa[11]) is among the oldest extant Sanskrit treatises on mathematical astronomy and has been influential in southern India (Pingree (1978)). Parameśvara's *Bhaṭadīpikā* was composed in 1432 CE or later.

Kern (1874) is the earliest critical edition in Parameśvara's corpus. But at the same time, this was also the first edition containing the entire text of the *Āryabhaṭīya*. Therefore Parameśvara's commentary has been used to interpret the base text itself, but not much attention has been payed to the commentator himself and his background (this tendency can be observed in the English translation of the *Āryabhaṭīya* by Clark (1930)) before the works by K.V. Sarma.

Parameśvara's commentary has the reputation of being brief (cf. K. V. Sarma and Shukla (1976, p. xl)), especially compared to other famous commentators such as Bhāskara I[12], Sūryadeva[13] and Nīlakaṇṭha[14]. This might be one reason why studies on this commentary are relatively scarce. Nonetheless, K. V. Sarma (1972, p. 53) has pointed out that this commentary contains "the enunciation of some of his new findings, theories and interpretations". I have located some

[9] *triṃśadguṇacandramite śakakāle*

[10] *mandabuddhihitāyaivaṃ gurupādābjasevinā |*
*mayārtho bhāskarīyasya saṃkṣepeṇa pradarśitaḥ ||* (B. Āpṭe (1946, p. 92))

[11] This Āryabhaṭa (476-c.550 CE) is sometimes called Āryabhaṭa I to distinguish him from his namesake Āryabhaṭa II (c.950 CE). Hereafter we shall constantly address the former Āryabhaṭa without the numbering.

[12] Critical edition by Shukla (1976).

[13] Critical edition by K. V. Sarma (1976).

[14] Nīlakaṇṭha only wrote commentaries on chapters 2-4. Their critical editions are Śāstrī (1930), Śāstrī (1931) and Pillai (1957b).

important discussions which are associated with topics in *GD2* and which are crucial for understanding some of the steps or reasonings that are omitted in *GD2*. Parameśvara himself links some of his statements with Āryabhaṭa and even quotes some verses from the *Āryabhaṭīya*.

**3. *Siddhāntadīpikā* (Illumination of the treatise) on Govindasvāmin's commentary of Bhāskara I's *Mahābhāskarīya* (Great/Extensive [treatise] of Bhāskara)** Critical edition by T. Kuppanna Sastri (1957).

The Mahābhāskarīya was composed before 629 CE[15] by Bhāskara I, and Govindasvāmin's commentary was composed around 800-850 CE (T. Kuppanna Sastri (ibid., p. xlxvii)). Govindasvāmin holds the view that the *Mahābhāskarīya* is a gloss on the *Āryabhaṭīya*, and Parameśvara has the same opinion (cf. T. Kuppanna Sastri (ibid., p. xxii)). Parameśvara seems to have composed this super-commentary in 1432 CE, as he refers to his observation of a solar eclipse that occurred in February 1432, and his commentary on the *Sūryasiddhānta*, estimated to be composed in 1432-33 CE, refers to the *Siddhāntadīpikā*.

Under each verse of the *Mahābhāskarīya*, Parameśvara glosses Govindasvāmin's commentary passage by passage, but occasionally adds his own ideas extensively. Parameśvara refers to the *Siddhāntadīpikā* in *GD2* 69 and hints that its content overlaps with *GD2*. Therefore this super-commentary is not only important to know how Parameśvara relates to Āryabhaṭa and Bhāskara I, but also to understand some of his original rules in *GD2*.

**4. *Karmadīpikā* (Illumination of the method) on Bhāskara I's *Mahābhāskarīya*** Critical edition by Kale (1945).

In this work, Parameśvara comments directly on the *Mahābhāskarīya*. He mentions the *Siddhāntadīpikā* in his conclusion and therefore we know that the *Karmadīpikā* was composed after it. Parameśvara keeps his glosses very short. Unlike his previous super-commentary, he hardly goes beyond the content of the base text. There are no quotations and no examples are provided, apart from those given in the *Mahābhāskarīya* itself.

**5. Commentary on the *Sūryasiddhānta* (Treatise of the sun)** Critical edition by Shukla (1957).

The *Sūryasiddhānta*, ascribed to the mythical character Maya, was stabilized around the 9th century. Parameśvara's commentary was probably composed around 1432-33 AD, according to one of his examples in his text [16].

The commentaries are short in general, and Parameśvara does not go often into details. According to the editor Shukla (ibid., pp. 67-68), Parameśvara points out some difference in the astronomical constants with those used by Bhāskara I, notes some variant readings and suggests some corrections to the longitudes of planets at the beginning of the *Kali-yuga*. On the other hand, Parameśvara does not add any significant remarks on the computational rules that contradict those in *GD2*[17].

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[15] Bhāskara I uses a date corresponding to 629 CE as an example in his commentary on *Ābh* 1.9 (Shukla (1976, p. 34)), which suggests that the commentary was also composed around that period. On the other hand, he frequently quotes the *Mahābhāskarīya* in the commentary which indicates that the *Mahābhāskarīya* was composed earlier.

[16] In an example under *Sūryasiddhānta* 3.11cd-12ab (Shukla (1957, p. 44)), Parameśvara takes 4533 as the years elapsed since the beginning of the *Kali-yuga*, which corresponds to 1432-33 AD.

[17] For example, the computation of the Sine of sight-motion (*dṛggatijyā*). See section 21.6.1

**Texts on eclipses**

Parameśvara has composed three texts that are fully dedicated to Eclipses (both lunar and solar). Their major goal is to find the possibility of eclipses and compute their duration. Meanwhile the three texts differ from each other in their styles and details; the *Grahaṇamaṇḍana* is an extensive set of computational procedures, the *Grahaṇanyāyadīpikā* omits some rules but adds more grounding and the *Grahaṇāṣṭaka* is an extremely short text which provides a minimum set of approximate rules.

**6. *Grahaṇamaṇḍana* (Ornament of eclipses)** Critical edition and translation by K. V. Sarma (1965).

Parameśvara uses a date corresponding to July 15th 1411 as the epoch[18], and he probably composed the treatise itself around this period.

This treatise gives a set of computations for solar and lunar eclipses. Parameśvara mentions in verse 4 (K. V. Sarma (ibid., pp. 2-3)) that he composes this treatise because previous methods do not agree with the results of eclipses, and in his conclusions (K. V. Sarma (ibid., pp. 32-35)) he justifies adding new corrections that are not included in previous texts. 20 years later in his *Dṛggaṇita*, he gives additional corrections to be applied to his *Grahaṇamaṇḍana* (K. V. Sarma (1963, p. 26), K. V. Sarma (1965, pp. 36-37)), which shows how meticulous he is on this topic.

**7. *Grahaṇanyāyadīpikā* (Illumination on the methods for eclipses)** Critical edition and translation by K. V. Sarma (1966).

Parameśvara refers to the *Grahaṇamaṇḍana* at the beginning of this treatise. As the word *nyāya* (literally "rule", "method" and also the name of a philosophical system which developed logics and methodology), this work supplies groundings for computational rules. For instance, the *Grahaṇamaṇḍana* only gives the conditions when a certain equation is to be added or subtracted, but the *Grahaṇanyāyadīpikā* also mentions why it is so[19]. The *Grahaṇanyāyadīpikā* can be read as an independent treatise without the *Grahaṇamaṇḍana*, but it does not contain some topics such as the corrections applied to the celestial longitude on account of the terrestrial longitude , equation of the center and the ascensional difference.

**8. *Grahaṇāṣṭaka* (Octad on eclipses)** Critical edition and translation by K. V. Sarma (1958–1959).

As the name suggests, this is a very short text in eight verses (excluding the opening and concluding stanzas). Some corrections are omitted, and as Parameśvara himself mentions in the opening, this is a crude / approximate calculation for eclipses (*sthuloparagagaṇita*).

**Treatises on other astronomical topics**

**9. *Dṛggaṇita* (Observation and computation)** Critical edition by K. V. Sarma (1963).

As aforementioned, the *Dṛggaṇita* was composed in 1431-32 CE. This treatise focuses on finding the days elapsed since the beginning of the *Kali-yuga* and computing the longitude of planets, which are major topics that *GD2* does not cover. One striking feature is that their are two parts in the texts where much of the second part is a restatement of the first part in an easier

[18] This is 1,648,157 days since the beginning of the *Kali-yuga*, which we find in verse 5 (K. V. Sarma (1965, pp. 2-3)).

[19] Compare *Grahaṇamaṇḍana* 73cd-76ab (K. V. Sarma (ibid., pp. 26-27)) and *Grahaṇanyāyadīpikā* 65-71 (K. V. Sarma (1966, pp. 20-23)) on deflections (*valana*, K. V. Sarma translates "deviations") due to geographic latitude and to the "course" of the moon.

language, notably using the *Kaṭapayādi* instead of word-numerals for stating numbers. There are manuscripts that only contain either one of the two parts, indicating that they could have been read as separate texts. Parameśvara mentions at the first verse in the second part that he will give a clearer version of the *Dṛgganita* for "the benefit of studies during childhood"[20]. Therefore we can see Parameśvara's attitude in this text to present the same topic in different ways for different readers.

The *Dṛgganita* is probably the best known work by Parameśvara today, due to its reputation to have introduced a new set of parameters in order to make "the results of computation accord with observation (K. V. Sarma (1972, p. 9))". However, we must be cautious with this statement for two reasons. One is that it makes us focus too much on the numbers and disregard the computational rules. The second is that all we know about Parameśvara's observations is his records of eclipses, but eclipses are not the topic of *Dṛgganita*.

**10. *Goladīpikā 1* (Illumination of the Sphere)** Parameśvara has also written an auto-commentary on this work. We will discuss its content in section 0.2.8.

**11. *Goladīpikā 2*** This treatise is the main subject of our work.

**12. *Candracchāyāgaṇita* (Computation of the moon's shadow)** No critical edition.

K. V. Sarma (ibid., p. 115) attributes this text which is extant in only one manuscript to Parameśvara. We have examined the manuscript[21] but could not find the authorship of this text. Moreover, the title of this text given at the beginning is *Himaraśmicchāyāgaṇita* (*himaraśmi* is a synonym of *candra*, moon). Sarma does not explain how he identified this text, and its status is dubious at the moment.

**13. *Vākyakaraṇa* (Making [astronomical] sentences)** No critical edition.

Only one manuscript[22] is available for this text. There are 69 verses in total. As already quoted above, Parameśvara mentions his name and his teacher Rudra in this text.

This treatise is different from the *Vākyakaraṇa* of anonymous authorship which is edited by T. S. Kuppanna Sastri and K. V. Sarma (1962), but deals with the same topic: a set of rules for composing *Vākya*s (literally "sentence") which are versified mnemonic tables which give the periodically recurring positions of celestial objects.

#### Commentaries on other mathematical and astronomical treatises

**14. Commentary on Mañjula's *Laghumānasa* (Easy thinking)** Critical edition by B. Āpṭe (1952). Also used in the English translation and commentary on the *Laghumānasa* by Shukla (1990).

The *Laghumānasa* is a treatise of 60 verses that is categorized today in the genre of *karaṇa*s (literally "making"), texts that use a recent epoch for the ease of computation (Plofker (2009, pp. 105-106)). The epoch in the *Laghumānasa* corresponds to 932 CE. Parameśvara uses a date corresponding to March 17th 1409 CE (see Shukla (1990, p. 30)) which suggests that he wrote his commentary around this date.

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[20] *spaṣṭīkartuṃ dṛgganitaṃ vakṣye … bālābhyāsahitaṃ* (K. V. Sarma (1963, p. 14))

[21] 475 I of KOML. This comes right before the folios of *GD2* in 475 J.

[22] T.166 A of KOML. This is a notebook written in year 1039 of the Kollam Era (1863-64 CE), and C.133 A which is likely the original manuscript (K. V. Sarma (1972, p. 164), Pingree (1981, p. 189)) was lost when we investigated the manuscripts in September 2014.

In general, the commentaries by Parameśvara expand the concise verses and explains the rules in detail. Parameśvara interprets that the concluding remark[23] claims the correctness of the work because it "follows other treatises and agrees with observations (*śāstrāntarānusāritvād dṛṣṭisāmyāc ca*)".

**15. *Parameśvarī* on Bhāskara II's *Līlāvatī* (Beautiful)** No critical edition of the commentary.

The *Līlāvatī* is the first part of the *Siddhāntaśiromaṇi* (composed 1149-50 CE) by Bhāskara II. Parameśvara does not make reference to the other three parts, namely the *Bījagaṇita*, the *Grahagaṇitādhyāya* and the *Golādhyāya*. The last two deal with several topics that overlap with *GD2*, and we can even find resemblance in some of the rules by Bhāskara II and Parameśvara (cf. section 11.3). The fact that Parameśvara has left a commentary on the *Līlavatī* suggests the possibility that he also had access to the other parts of the *Siddhāntaśiromaṇi* which could have influenced him.

This commentary is also interesting because this is the only base text that deals exclusively with mathematics. Its content is yet to be studied. At the moment, we know that Parameśvara comments extensively on each verse and occasionally inserts verses of his own[24].

**16. Commentary on the *Vyatīpātāṣṭaka* (Octad on the *Vyatīpāta*)** No critical edition.

A *vyatīpāta* is a moment when the sun and moon have the same declination while their change in declination are in different directions (i.e. if one is moving northward, the other must be moving southward). Although it is an astrological concept, it involves the computation of the moon's latitude and is thus discussed in astronomical treatises, sometimes in a whole chapter[25]. The *Vyatīpātāṣṭaka* is likely to be a treatise of such kind, but the original text is lost. Not much is known about Parameśvara's commentary and there is just a brief discussion by K. V. Sarma (1972).

### Treatises on astrology

**17. *Ācārasaṃgraha* (Summary of good conducts)** Critical edition by Amma (1981).

This treatise deals with various types of divinations, especially those related to timings (*muhūrta*). Parameśvara refers to Govinda, the teacher of his grandfather, and implies that the *Ācārasaṃgraha* summarizes his teachings.

The edition counts 367 verses in 34 sections marked by Parameśvara himself. There are several manuscripts that only contain the section *Vivāhānukūlya* (Suitableness of marriage). K. V. Sarma (1972) and Pingree (1981) treat it as an individual work.

**18. *Ṣaḍvargaphala* (Result from the six categories)** No critical edition.

This work only remains in one paper manuscript[26]. It is a list of divinatory results from six categories in astrology: lunar mansions (*nakṣatra*), days of the week (*vāra*), lunar days (*tithi*) half lunar days (*karaṇa*), time division according to the sun and moon's longitudes (*yoga*) and

[23] "Those who will imitate it (this treatise) or find fault with it shall earn a bad reputation.", translation by Shukla (1990, p. 192)

[24] I would like to thank Takao Hayashi for providing me with information on the manuscripts.

[25] For example, chapter 11 of the *Sūryasiddhānta* (Shukla (1957, pp. 102-108)). See also discussions under *GD2* 163-164 (section 10.6) on the true declination.

[26] T.166 B of KOML. This is in the same notebook as the *Vākyakaraṇa*, and was probably copied from C.133 which is now lost.

zodiacal signs (*rāśi*). However the name of the author is not given in the text. We do not know why K. V. Sarma (1972, p. 172) identified this text as Parameśvara's work.

**19. *Jātakapaddhati* (Manual on nativity)** Edition by Menon (1926).

This treatise gives a set of computational rules that may be used for making horoscopes in 44 verses. According to K. V. Sarma (1972, pp. 119-120), there is only one commentary in Sanskrit by an anonymous commentator, but there are 7 commentaries in Malayalam which suggests that Parameśvara's text was very popular in the vernacular tradition of astrologers.

#### Commentaries on astrological treatises

**20. *Bālaprabodhinī* (Awakening of the young) on Śrīpati's *Jātakakarmapaddhati* (Manual on methods of nativity)** No critical edition.

The *Jātakakarmapaddhati* by Śrīpati was edited and translated into English under the title *Śrīpatipaddhati* by Sastri (1937). Parameśvara calls the work *Jātakapaddhati* in the concluding verse of his commentary (Pingree (1981, p. 192)), which is the same as his own treatise (see 19. above). However we do not know the relation between his commentary *Bālaprabodhinī* and his treatise *Jātakapaddhati*; whether one was influenced by the other or not.

It is noteworthy that Parameśvara refers to Śrīpati in *GD1* 3.62 in the context of cosmology (see introduction in chapter 3). His statements on cosmography in *GD1* might be affected by Śrīpati's treatise on astronomy, the *Siddhāntaśekhara*. However we could not find any prominent influence of Śrīpati in *GD2*.

**21. *Parameśvarī* on Pṛthuyaśas' *Praśnaṣaṭpañcāśikā* (Fifty-six [verses] on astrological inquiries)** No critical edition.

Pṛthuyaśas (fl. c. 575 CE) is the son of Varāhamihira, and his *Praśnaṣaṭpañcāśikā* was very popular and survives in numerous manuscripts, chiefly from northern India (Pingree (ibid., pp. 212-221)). Only three of them are in KOML, all of which include the commentary by Parameśvara. They are yet to be examined.

**22. Commentary on Govinda's *Muhūrtaratna* (Jewel of the *Muhūrta*)** Neither the *Muhūrtaratna* nor the commentary has been published.

Govinda (1236-1314 CE) is the teacher of Parameśvara's grandfather. K. V. Sarma (1972, p. 49) says that the *Muhūrtaratna* "has been very popular", but all we know today is that there are nine extant manuscripts[27].

### 0.1.5 Mutual relation and order of texts

In the following we shall focus on treatises and commentaries on astronomy and investigate the order of their composition.

Parameśvara has given the date[28] of the work in only two treatises:

***Dṛggaṇita*** 1431-32 CE

***Goladīpikā 1*** 1443-44 CE

[27] According to Pingree (1971, p. 143). Excluding recent transcriptions.

[28] Dates in the texts themselves are given in days or years since the beginning of the *Kali-yuga* or in Śaka years, but we will convert them to dates in the Julian calendar of the common era.

The texts below make reference to a date which suggests the period of the text itself:

**Commentary on the *Laghubhāskarīya*** Uses 1408-09 CE in one of its examples

**Commentary on the *Laghumānasa*** Epoch is March 17th 1409 CE

***Grahaṇamaṇḍana*** Epoch is July 15th 1411 CE

***Siddhāntadīpikā*** Last eclipse mentioned is on February 2nd 1432

**Commentary on the *Sūryasiddhānta*** Uses 1432-33 CE in one of its examples

Some texts refer to or quote from other titles, which is useful for determining their order:

***Grahaṇamaṇḍana*** The commentary on the *Laghubhāskarīya*

***Dṛggaṇita*** The *Grahaṇamaṇḍana*

***Bhaṭadīpikā*** The *Siddhāntadīpikā* and commentaries on the *Laghubhāskarīya*, the *Laghumānasa* and the *Līlāvatī*

**Commentary on the *Sūryasiddhānta*** The commentary on the *Laghubhāskarīya*, the *Siddhāntadīpikā*, the commentary on the *Līlāvatī* and other texts, in this order[29]

***Grahaṇanyāyadīpikā*** Works beginning with the *Grahaṇamaṇḍana*[30] and the *Siddhāntadīpikā*

***Goladīpikā 2*** The *Siddhāntadīpikā*

***Karmadīpikā*** The *Siddhānta-*, [*Grahaṇa*]*nyāya-*, *Gola-* and *Bhaṭa- dīpikā*s

In addition, we have found the following in relation to the contents of the *Goladīpikā 2*:

- Govindasvāmin's commentary on *MBh* 5.4 quotes *Ābh* 4.14 with the reading "at its quarter (*taccaturaṃśe*)" and Parameśvara's *Siddhāntadīpikā* follows it. But in his own commentary on *Ābh* 4.14, Parameśvara refers to a variant reading "at a fifteenth (*pañcadaśāṃśe*)". He quotes *Ābh* 4.14 with this variant as *GD2* 38 (section 4.1). This suggests that the order of composition was the *Siddhāntadīpikā*, then the *Bhaṭadīpikā*, and finally the *Goladīpikā 2*.

- Statements in *GD2* 51 and 53 on the order of rising signs in polar regions are wrong. *GD1* 3.54 on the same topic is correct (4.7). This suggests that *GD2* was composed before *GD1*.

From these evidences, we propose the order of texts as given in table 0.1. Dates that can be inferred from evidence within the texts are given next to the title. Horizontal lines indicate that we are confident about the order of the texts above and below.

The commentary on the *Sūryasiddhānta* refers to another text with a "subject on the motion of planets (*grahagativiṣaya*)" after *Līlāvatī*. The *Bhaṭadīpikā* the *Karmadīpikā*, or a yet

[29] "By whom the *Laghubhāskarīya*, after that the *Mahābhāskarīya* with the commentary, later the *Līlāvatī* and some other subject on the motion of planets were commented upon, ..."
*vyākhyātaṃ bhāskarīyaṃ laghu tadanu mahābhāskarīyaṃ sabhāṣyaṃ /*
*paścāl līlāvatī ca grahagativiṣayaṃ kiñcid anyac ca yena //* (Shukla (1957, p. 1))
Shukla (ibid., introduction, p. 69) claims that this list also includes his commentary on the *Mahābhāskarīya* (*Karmadīpikā*), but we do not think so.

[30] "But the steps of methods there (= *Grahaṇanyāyadīpikā*) have been explained before in those beginning with the [*Grahaṇa*]maṇḍana."
*karmakramas tu tatra prāṅ maṇḍanādau pradarśitaḥ //1//* (K. V. Sarma (1966, p. 1))

Table 0.1: Deduced order of texts ("c." stands for "commentary on")

| Date (CE) | Title |
|---|---|
| 1408-09 | c. *Laghubhāskarīya* |
| 1409 | c. *Laghumānasa* |
| 1411 | *Grahaṇamaṇḍana* |
| 1431-32 | *Dṛggaṇita* |
| 1432 | *Siddhāntadīpikā* |
| 1432-33 | c. *Līlāvatī* |
| | *Bhaṭadīpikā* |
| 1432-33 | c. *Sūryasiddhānta* |
| | *Grahaṇanyāyadīpikā* |
| | *Goladīpikā 2* |
| | *Karmadīpikā* |
| 1443-44 | *Goladīpikā 1* |

unknown treatise would correspond to this, but we think that it is most likely the *Bhaṭadīpikā*. The *Goladīpikā 2* must have been composed after *Bhaṭadīpikā*, and the *Karmadīpikā* after one *Goladīpikā*[31], which we assume is the *Goladīpikā 2*. If the commentary on the *Sūryasiddhānta* had been composed after the *Karmadīpikā*, given that this commentary seems to be composed around 1432-33 CE, we will have to assume that Parameśvara composed 6 texts beginning with the *Siddhāntadīpikā* in one year, which is an unprecedented pace. Therefore I assume that it was composed after the *Bhaṭadīpikā*. The *Grahaṇanyāyadīpikā* was composed between the *Siddhāntadīpikā* and the *Karmadīpikā*. It refers to other works that deal with similar methods (i.e. methods on eclipses), which could be either the *Bhaṭadīpikā*, the commentary on the *Sūryasiddhānta*, the *Goladīpikā 1* or the *Grahaṇāṣṭaka*. But we have no clue for the date of *Grahaṇāṣṭaka*, and neither for the *Candracchāyāgaṇita* and the *Vākyakaraṇa*.

### 0.1.6 Parameśvara and observation

Parameśvara is well known for his astronomical observations, which we have seen above in Nīlakaṇṭha's testimonies. I would like to discuss two aspects of how observations are involved in Parameśvara's works on astronomy: (1) Efforts on integrating observation and computation, and (2) keeping observational records.

Observation in Indian astronomy has been a controversial topic (see Plofker (2009, pp. 113-120) for a general discussion). Opinions range from one extreme that every Sanskrit astronomical text was strictly based on observation (Billard (1971)) to the other that no serious observation was done in ancient India (Pingree (1978, p. 629)). The reason why discussions tend to be overheated is because it inevitably involves the problem of origins, and also to some degree because of the value judgment that astronomy without observation is inferior. These are often done in comparison with Greek astronomy; see reflections by Pingree (1992). In this work, we will only focus on Parameśvara, and discuss not whether he observed or how accurate his observations were, but what he states about observations (*dṛk*).

It is debatable whether the Sanskrit term *dṛk* is the exact equivalent of the English term "observation", or "astronomical observation" in a modern sense. Hereafter we have interpreted *dṛk* in a narrow sense: To see the sky directly or indirectly (with instruments like gnomons) to acquire

[31]The *Karmadīpikā* refers to the [*dīpikā*s] "of the *Gola* and *Bhaṭa* (*golabhaṭayoś*)" in the dual. This means that there is only one *Goladīpikā*.

the position of celestial objects. In *GD2*, the derivative *darśana* appears in *GD2* 199, and there are many occurrences of the verb *dṛś*. On the other hand, *dṛk* itself only appears in compounds used as technical terms such as *dṛgvṛtta* (circle of sight) which may not necessarily be linked with observation itself. We do not rule out the possibility that further studies on Parameśvara's texts will change our understanding, or that other authors use *dṛk* with a different nuance.

At the beginning of his *Dṛggaṇita* (Observation and computation), Parameśvara claims that his aim is to make computation agree with observation (K. V. Sarma (1963)). K. V. Sarma (1972, p. 9) calls the set of parameters introduced in this treatise the "*Dṛk* system" which revises the previous system. K. V. Sarma mentions that "no new methodology is enunciated here", but we may raise the question whether Parameśvara has only modified astronomical constants as a result of his observations, and not the computational rules themselves.

Direct evidence of Parameśvara's observations comes from his versified records of eclipses in the *Siddhāntadīpikā* under *MBh* 5.77 (T. Kuppanna Sastri (1957, pp. 329-331)). There are 8 solar eclipses and 5 lunar eclipses (including one that was expected but not observed) that occurred between 1398 and 1432 CE in this list, with additional information such as his locations or totality of the eclipses (Montelle (2011, pp. 279-283)). Parameśvara himself mentions that he observed more than he included in the list. He also writes extensively on computations of eclipses and has left three treatises on this subject (see page 6). Many topics in *GD2* are also related to eclipses. We will investigate how he treats observation in *GD2* later.

Another important piece of information included in Parameśvara's list of eclipse observations is that he records the "foot-shadow (*padabhā*)" when some of the eclipses occurred. This indicates a shadow of a gnomon with a given height for measuring the altitude of the illuminating body at a given moment. S. R. Sarma (2008, p. 246) points out that the shadows in Parameśvara's lists are those of a gnomon with 6 "feet (*pada*)". Usually, the gnomon in Sanskrit astronomical texts, including *GD2*, have a height of 12 *aṅgula*s (literally "fingers" or "digits"). On the other hand, Islamic texts refer to gnomons in "feet (*qadam*)" besides "digits (*iṣba'*)", and their astrolabes typically have shadow squares (scales for finding the altitude of a celestial object) in both units at their back (S. R. Sarma (ibid., p. 186)). Thus S. R. Sarma (ibid., p. 246) concludes that Parameśvara could be using an astrolabe, and that his knowledge of the instrument is likely based on a tradition different from those prevailing in western and northern India, because Sanskrit astrolabes usually have shadow squares for gnomons of 7 *aṅgula*s and 12 *aṅgula*s. This raises the question whether some characteristics in the works of Parameśvara, including his emphasis on observations, are the result of influence from Arabic or Persian sources.

### 0.1.7 Pedagogical lineage

#### History of Indian astronomy and "schools"

Studies on the history of Indian astronomy are also often studies on "schools". The word "school" has been associated with the Sanskrit term *pakṣa* (literally "wing, side") to indicate groups of astronomers, but historians use the term in different nuances.

The 19th century scholar Colebrooke uses "school", "sect" and "system" as synonyms (Colebrooke (1817, p. viii)). Thus he gives a foretaste of the multitude of meanings "school" takes today in the literature of astronomy in South Asia . However, he uses three terms to indicate only three groups (either people or their doctrines) that count the day from sunrise (*audayaka*), from midnight (*ārdharātrika*) or from noon (*mādhyandina*).

T. S. Kuppanna Sastri (1969) argues that it "is possible to classify early Hindu astronomers and astronomical works into specific schools on the strength of certain peculiarities of each." He gives, for example, the division of the *caturyuga* into four equal parts, number of cycles of planetary motions in a given period and the computational rule for the equation of the center as peculiarities in the "school of Āryabhaṭa". In his arguments, "school" is no more an actor's category

Pingree (1978) focuses on the parameters for categorizing "schools". This usage of "school" is popular today. For example, Plofker (2009, pp. 69-70) states: "different schools or *pakṣa*s, which are distinguished from one another mostly by the values of the parameters they use for the main divisions of time and the cycles of the heavens."

Under this definition, Pingree (1981, p. 613) asserts that Parameśvara is in the "school of the *Sūrya* (*Saurapakṣa*)" because his *Dṛggaṇita* uses parameters that are close to the *Sūryasiddhānta*. But in the case of Parameśvara, there is yet another "school" to be discussed - the "Kerala school".

### The "Kerala school"

Parameśvara is often seen as a member of the "Kerala school". This term came to be well known after the book titled "A History of the Kerala School of Hindu Astronomy" by K. V. Sarma (1972). However, K.V. Sarma rarely uses the term "school" in the content of this book and refers to "Kerala astronomy" or "Kerala astronomers" instead. This refers to any astronomer or their work in the region of Kerala. We may interpret that "school" in this case is defined by a geographical location.

However, the expression "Kerala school" tends to be used in a narrower sense – a "'chain of teachers' originating with Mādhava in the late fourteenth century and continuing at least into the beginning of the seventeenth" (Plofker (2009, p. 217)). In this sense, it is also called the "Mādhava school"[32]. This "school" has been noticed especially for their mathematical achievements. Whish (1834) made an early discovery on the usage of power series by astronomers or mathematicians in Kerala. Later studies showed that these scholars often refer to Mādhava (Gupta (1973)), and hence Mādhava came to be acknowledged as the founder of this knowledge.

Not much is known about Mādhava himself, and few of his own works are extant[33]. On the other hand, Parameśvara, who has been acknowledged as the student of Mādhava by Nīlakaṇṭha, and as such his only known student, has become an important "link" in the chain of scholars. Whether the mathematical and astronomical achievements of Parameśvara are really linked with Mādhava and with his pedagogical descendants like Nīlakaṇṭha or not needs to be carefully examined. In our study, we shall focus chiefly on the computational rules in *GD2* and see whether they echo with those of other authors.

### Parameśvara's own remarks

As quoted above, Nīlakaṇṭha mentions three names as the teachers of Parameśvara when he was young: Rudra, Nārāyaṇa and Mādhava. But Parameśvara himself only refers to Rudra. He states in the opening verse of his *Vākyakaraṇa*:

> This student of the honorable Rudra, Parameśvara, composes the Vākyakaraṇa to establish the parts of an [astronomical] sentence (*vākya*). [34]

[32] See Plofker (2009, pp. 217-253) for more details on scholars identified in this group and their works.

[33] See K. V. Sarma (1972, pp. 51-52) for more information on Mādhava and his works.

[34] *pūjyapādasya rudrasya śiṣyo 'yaṃ parameśvaraḥ |*
*karoti vākyakaraṇaṃ vākyāvayavasiddhaye ||* (Vākyakaraṇa 1, from manuscript T.166 A of KOML)

Parameśvara also refers to himself as a student of Rudra in the opening of his commentary on the *Sūryasiddhānta* (Shukla (1957, p. 1)) and in the conclusion of his *Siddhāntadīpikā* (T. Kuppanna Sastri (1957, p. 395))[35].

Another scholar that Parameśvara refers to is Govinda (1236-1314 CE, also called Govindabhaṭṭa or Govinda bhaṭṭatiri), who was a teacher of his grandfather. The following is Parameśvara's remark in *Ācārasaṃgraha* 279:

> What was said by the teacher of my father's father, a brahman named Govinda who is celebrated in the world, reached me through the chain of teachers, and it stands here as the Ācāra[saṃgraha].[36]

K. V. Sarma (1974) reports that an old palmleaf document records a line of tradition beginning with Govinda, followed by Parameśvara's grandfather, Parameśvara, Parameśvara's son Dāmodara, his student Nīlakaṇṭha, his student Jyeṣṭhadeva, and his student Acyuta. If we can rely on this manuscript, this means that Govinda and Parameśvara's grandfather were considered more important in the lineage of scholars than Rudra, let alone Mādhava who we will discuss in the next section.

### Mādhava and Parameśvara

Parameśvara is believed to have studied under Mādhava. No other student of Mādhava is known, and therefore the lineage of the "Kerala school = Mādhava school" cannot be constructed without Parameśvara. However, the only evidence of their master-disciple relationship comes from the above mentioned statement of Nīlakaṇṭha. Parameśvara himself has left no remark.

K. V. Sarma (1966, pp. 26-27) claims that the penultimate verse of the *Grahaṇanyāyadīpikā* refers to Mādhava as *golavid* (expert on the Sphere). His translation is as follows:

> There is another method (to compute the solar eclipse) without finding the parallax at new moon etc. This has been explained (by me) in the *Siddhāntadīpikā*, as given by (Mādhava) 'the Golavid' (lit. 'expert in sperics').[37]

*Siddhāntadīpikā* is a super-commentary on the *Mahābhāskarīya* by Parameśvara. Sarma points that the method referred to is given in the commentaries to *MBh* 5.68-71 (T. Kuppanna Sastri (1957, pp. 314-317)). However, what Parameśvara states there is different. *MBh* 5.68-70 itself claims that the parallax of the moon and related elements are necessary in lunar eclipse computations, too. This is an unnecessary statement (they are only relevant in solar eclipses) and Parameśvara attempts to save Bhāskara I by saying that he is giving the opinion of some other astronomers (T. Kuppanna Sastri (ibid., p. civ)).

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[35] Another case where he might be referring to Rudra is in the concluding verse of his commentary on the *Līlāvatī*, according to Pingree (1981, p. 190). However, I have only examined three manuscripts (5783, 10614 B, T.295 of the KOML), all of which had corrupt readings of this passage.

[36] *pituḥ pitur me gurur agrajanmā*
*govindanāmo bhuvi viśruto yaḥ |*
*tenodito yo gurupaṅktito māṃ*
*prāptaḥ sa ācāra iha pratiṣṭhaḥ ||279||* (Amma (1981, p. 54))

[37] *upāyāntaram apy asti parvalambādibhir vinā |*
*siddhāntadīpikāyāṃ tal likhitaṃ golavitsmṛtam ||84||*

> In that case, those other than some experts on the essence of the Sphere desire the parallax even in the case of a lunar eclipse.[38]
> ... The experts on the Sphere state that this is all inapplicable.[39]

Parameśvara refers to "experts" in the plural, which may be interpreted as an honorific expression to address a single person. But furthermore he adds "some (*kecit*)", which gives an indefinite sense. Therefore I argue that the *golavid* in the *Siddhāntadīpikā* is more likely a reference to multiple astronomers including Bhāskara I and not Mādhava alone. In addition, there are four occurrences of the word *golavid* in *GD2*, but all of them indicate people working on the field of spheres collectively and not a single person[40]. To conclude, it is highly questionable whether the word *golavid* in the *Grahaṇanyāyadīpikā* is a reference to Mādhava.

Therefore, the only unambiguous link between Parameśvara and Mādhava is the short remark by Nīlakaṇṭha. Our study on *GD2* will further show that Parameśvara uses several computational rules that are not found in previous authors and even differ from those attributed to Mādhava. This shows that Parameśvara does not seem to acknowledge Mādhava, at least explicitly, as his teacher.

## 0.2 The treatise: *Goladīpikā* (*GD2*)

### 0.2.1 Overview and previous studies

The *Goladīpikā* (literally "illumination of the Sphere", hereafter *GD2*), as the author calls the treatise in its final verse (*GD2* 302), is a fully versified treatise in 302 stanzas. As its name suggests, it deals with spheres in a broad sense in astronomy.

As discussed in section 0.1.5, *GD2* was composed after 1432 CE, and probably before 1443 CE. There are eleven extant manuscripts as listed later in section 0.3.1.

T. Gaṇapati Sāstrī edited the text as a "*Goladīpikā*" in the Trivandrum Sanskrit Series (Sāstrī (1916)). Sāstrī was not an expert on astronomy and did not discuss the contents of the texts apart from saying that "it has neither commentary or illustrations". He states that he published the treatise "in the hope that it might be of some use to students of Hindu Astronomy". He only used three manuscripts, and the edition is heavily influenced by their corruptions. The verse numbers in our critical edition follow the numbers alloted by Sāstri. However, many verses have been left unnumbered[41] which has caused some problems (for example in *GD2* 244; see section 18.13).

As we will see in section 0.2.8, Parameśvara has composed another treatise with the same name. This *Golādīpikā* (hereafter *GD1*) was published by K. V. Sarma (1956–1957) where it was stated for the first time that Parameśvara composed two *Goladīpikā*s. Sarma also remarks that "there is a unique manuscript" in London; this is the Indian Office Sanskrit 3530 ($I_1$) which we have used in our critical edition. He knew that the text was *GD1* and that it contained quotations from other treatises, but did not indicate the commentaries. Later, K. V. Sarma (1972, p. 53) stated that "there are three works on spherics, being the *Goladīpikā*s I-III", which

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[38] *tatra kecid golatattvavidbhyo 'nye candragrahaṇe 'pi lambanam icchanti* | (T. Kuppanna Sastri (1957, p. 314))

[39] *etat sakalam anupapannam iti golavida āhuḥ* | (T. Kuppanna Sastri (ibid., p. 315))

[40] The *golavid* in *GD2* 35 and *GD2* 65 represent people who share the same view on cosmography as Parameśvara. The cosmography dealt with in these verses are general and very unlikely to be opinions that are attributed to a single astronomer. *GD2* 246 is an example where Parameśvara challenges the reader by saying "if you are an expert on the Sphere". Lastly in *GD2* 302, Parameśvara links the reader with "experts" again, saying "may the reader be enumerated among the experts on the Sphere".

[41] Apparently, Sāstri has skipped the number when there is no space after the last line of the verse.

led to some misunderstanding. Pingree (1981, p. 191) calls it "an expanded version of the second *Goladīpikā*" and counts it as an individual work.

The content of *GD2* has not yet been studied in detail, and the current work provides an extensive research on its topics for the first time.

### 0.2.2 Authorship

Evidence of Parameśvara's authorship on this text comes directly from *GD2* 68 where his name is given (chapter 5). The six examples use the value 647 as the Sine of geographic latitude, which is also the location of Parameśvara. In addition, parallels between other texts attributed to Parameśvara, notably the commentary on the *Āryabhaṭīya* and the super-commentary *Siddhānta-dīpikā* on the *Mahābhāskarīya* can be found in almost every part of the treatise, which also support the identity of the author.

### 0.2.3 Structure and style

As a whole, the 302 verses in *GD2* are continuous. *GD2* 68-69 summarize the previous contents and mentions what will be presented in the following, thereby indicating a transition in the topic (see chapter 5). Non-versified short preambles precede the six examples (*GD2* 209, 212, 231, 232, 245 and 246) and two sets of procedures (*GD2* 210-211, 213-217). There are no other statements that divide the text, and no chapters are specified by Parameśvara.

*GD2* 244 has an extra half-verse while *GD2* 247 only has a half-verse. These two verses could be a sign of corruption. The total number of verses, 302, suggest the possibility that two extra verses have slipped in. In every manuscript with verse numbers written, the last verse is numbered 300. Each of these manuscripts have overlaps or omissions of numbers in different places. The fact that they nonetheless end up in 300 suggests that the verses was expected to be exactly this number. In addition, some astronomical treatises are composed in multiples of hundred verses. The *Sūryasiddhānta* has exactly 500 verses which is probably not by pure chance. Parameśvara composed his *Grahaṇamaṇḍana* initially in 89 verses but later added 11 stanzas to make this number 100 (K. V. Sarma (1965, pp. xvii-xvii)). But contradictorily, the case of *Grahaṇamaṇḍana* could actually support that 302 is the right number of verses in *GD2*; Parameśvara himself remarks in the last verse of the *Grahaṇamaṇḍana* that there are 100 verses, but this does not count the opening and concluding stanzas. Therefore, it could also be the case that he composed *GD2* in exactly 300 verses without counting both ends. Therefore the strange numbering in the manuscripts might indicate that scholarly descendants of Parameśvara knew that this *Goladīpikā* had 300 verses but misunderstood how to enumerate them.

Concerning the meter, almost all verses are in *Gīti*. Apart from the 6 examples (*GD2* 209, 212, 231, 232, 245 and 246) and 3 quoted verses (*GD2* 37, 38, 44), only 5 verses (*GD2* 56, 84, 132, 137, 172) are in a different meter (all 5 are *Ārya* verses).

Every number in *GD2* is described in word numerals (*Bhūtasaṃkhyā*). See appendix A.1 for an exhaustive list.

### 0.2.4 Contents

There are no chapters or any other explicit sectioning in *GD2*, but we have divided the verses in our commentary to make it easier to read. Some of our divisions are made on the basis of Parameśvara's wordings, some according to the different procedures contained in the verses, and few others are arbitrary.

**1 *GD2* 1** Invocation.

**2** ***GD2*** **2-17** An introduction on various names of circles, their mutual positions and their meaning. The circles are largely divided into two groups, the stellar sphere and the celestial sphere. Descriptions in this section can also be read as an introduction to the armillary sphere.

**3** ***GD2*** **18-36** This part deals with miscellaneous topics on cosmography, especially those concerning the motion of celestial objects. Parameśvara takes views that are mainly from the Purāṇas and either refutes them or reconciles them with his own opinions.

**4** ***GD2*** **37-67** Arguments on cosmography continue. In these verses Parameśvara discusses the different locations of different entities and defines the "days" from their viewpoints. Some have very long timescales.

**5** ***GD2*** **68-69** These two verses give the authorship of the treatise and also summarize the previous and upcoming contents.

**6** ***GD2*** **70-88** Segments and arcs with variable lengths produced in the stellar sphere and celestial sphere are introduced. All of them depend on only two factors, the geographic latitude and the celestial longitude of the sun.

**7** ***GD2*** **89-102** Rules on the time it takes for given longitudes or signs in the ecliptic to rise above the horizon. Effectively, it explains how to find a length of arc on the celestial equator that corresponds to an arc in the ecliptic.

**8** ***GD2*** **103-124** New sets of segments and arcs that are produced from yet another factor: the time of the day. The most important among them is the great gnomon.

**9** ***GD2*** **125-152** The rule to compute the celestial latitude and supplementary explanations. To ground the rule, Parameśvara discusses the deviation of a planet in its set of orbits. This involves a drawing of planetary orbits.

**10** ***GD2*** **153-194** Discussion on celestial latitudes as seen from the observer, its relation with the declination, and its effect on the rising or setting time of the planet. The set of computations for finding this timing is called the visibility operation. Parameśvara explains the two different factors in the visibility equation, then gives a unified method. In the procedure he introduces the concept of "sight-deviation" which represents the distance of the ecliptic from the zenith.

**11** ***GD2*** **195-208** A set of three corrections to the longitude of a planet at the moment of sunrise. These are the corrections for the geographic longitude of the observer, for the sun's equation of center and for the ascensional difference.

**12** ***GD2*** **209-211** Example 1. We compute the sun's longitude from its shadow when the sun is on the prime vertical.

**13** ***GD2*** **212-219** Example 2: This time we use its shadow at midday.

**14** ***GD2*** **220-230** A procedure for finding the length of a shadow when the longitude of the sun and its direction in the sky is known. This is practiced in examples 3 and 4.

**15** ***GD2*** **231** Example 3.

**16** ***GD2*** **232** Example 4.

**17** ***GD2* 233-234** Supplementary remark on the previous procedure and examples, focusing on the "without-difference" method (iterative method) used therein. Here Parameśvara (and the commentator) seem to discuss its convergence.

**18** ***GD2* 235-244** Another procedure using a gnomon in two steps. In the first step, the longitude of the sun is computed from the length of a shadow in an intermediate direction and the time of the day. The second step uses a "without-difference" method to find the geographic latitude. Examples 5 and 6 can be solved with this procedure.

**19** ***GD2* 245** Example 5.

**20** ***GD2* 246-247** Example 6.

**21** ***GD2* 248-276** Rules to compute the geocentric parallax and its longitudinal and latitudinal components. Parameśvara discusses extensively how the rules are grounded.

**22** ***GD2* 277-301** A few topics on eclipses, including the size of objects, the difference between a solar and a lunar eclipse and the computation of the Earth's shadow. Parameśvara does not integrate these topics with previous subjects that are also relevant to eclipses, and the reader would probably have had to learn from other treatises.

**23** ***GD2* 302** Concluding remark.

### 0.2.5 Questions running through *GD2*

Some of the topics listed above share some questions in common. We can also find issues and subjects that run through the entire text and are not confined to a single section.

From our modern viewpoint, the issues can be divided into those of mathematics and those of astronomy. The Sanskrit term that is commonly translated into "mathematics" is *gaṇita* (literally "counted" or "reckoned"). Parameśvara's commentary on the *Āryabhaṭīya* suggests what he puts under this word.

*Āryabhaṭa* enumerates three of his chapters, namely *gaṇita*, *kālakriyā* (reckoning of time) and *gola (sphere)*, in *Ābh* 1.1 (Kern (1874, pp. 1-2)). Parameśvara first enumerates what he considers as *gaṇita*:

> In that case, that called mathematics has many forms beginning with "heaps (*saṃkalita*", "mixture (*miśra*)", "series (*śreḍhī*)", "knowledge of seeing (? *darśadhī*)", "pulverizers (*kuṭṭākāra*)[42]", "shadows (*chāyā*)" and "figures (*kṣetra*)".[43]

I do not know what *darśadhī* (or *darśa* and *dhī*) refers to, as Parameśvara uses the term nowhere else. Otherwise, the topics mentioned do indeed have a corresponding part in the *gaṇita* chapter. However, according to Parameśvara, mathematics (*gaṇita*) is also relevant in the other two chapters; he comments that *kālakriyā* stands for "the mathematics of planets consisting of methods on subdivisions of time [44]" and that the *gola* (sphere) is "the realm where special mathematics is performed because it is a circular figure and because it supports the making of many figures beginning with the quadrilateral"[45]. In other words, the subjects in the *kālakriyā*

[42]Indeterminate analysis.

[43]*tatra gaṇitaṃ nāma saṃkalitamiśraśreḍhīdarśadhīkuṭṭākāracchāyākṣetrādyanekavidham* / (Kern (1874, p. 2), but I have amended *gaṇitannāma* to *gaṇitaṃ nāma*, *saṅkalita* to *saṃkalita* and *śreḍī* to *śreḍhī*)

[44]*kālaparicchedopāyabhūtaṃ grahagaṇitaṃ kālakriyety arthaḥ* / (Kern (ibid.))

[45]*sa ca vṛttakṣetratvāc caturaśrādyanekakṣetrakalpanādhāratvāc ca gaṇitaviśeṣagocara eva / (Kern (ibid.))*

chapter (which includes calendrics and computations of true planets) are themselves a type of mathematics, while the *gola* is a place where a special type of mathematics is applied. Here the word *gola* (sphere) is taken as an object, but I assume that the statement can be applied to some extent to the *gola* as a topic.

*GD2* shares many topics with the *gola* chapter in the *Āryabhaṭīya*. Parameśvara gives various methods for locating a celestial object in the sphere, finding the length of a certain arc or length et cetera, but he never refers to an entire method as mathematical. In *GD2* 218, he contrasts the longitude of the sun derived from the "shadow" and from "mathematics", where the former is a reference to the method for finding the sun's longitude from the length of the shadow at midday as explained in *GD2* 213-217 whereas "mathematics" might refer to "mathematics of planets", the true planet computation as explained in the *kālakriyā* chapter or other texts.

Meanwhile, mathematics are relevant when we focus on the steps within each method. Most notable are the Rules of Three and the Pythagorean theorem. We may also add Sine computation (see also appendix B) here, although we do not know for sure what Parameśvara includes in his category of "special mathematics".

### Mathematical issues

**The sphere as an object** In *GD2* 33 he refers to the surface area and the volume of a sphere (the Earth), which is a subject that can be found in mathematical texts. Parameśvara does not explain how the area and volume are to be computed, but gives their approximate values. This is an interesting case where mathematical knowledge is used in the context of cosmography.

**Rules and their groundings** Computational rules in *GD2* are often followed by explanations as to why the computation is necessary or why the computation is correct. A typical way to answer why the rule is required is with the aid of diagrams as we will see in the next section. Expressions that suggest the usage of armillary spheres can also be seen.

Meanwhile, the grounding of a computational rule is frequently done by bringing to light the form of the Rule of Three in the previous procedure. The Rules of Three are often associated with a pair of similar right triangles, and armillary spheres could have been used in the explanation, too.

The Pythagorean theorem is also used in the rules, and in such cases the grounding is done by showing a right triangle and listing its base, upright and hypotenuse.

**Rule of Three** The Rule of Three (*trairāśika*) has been frequently used for solving astronomical problems since its first appearance in *Ābh* 2.26[46]. The computational rules in *GD2* are in line with this general trend, but the way that Parameśvara presents them are very characteristic. He gives both the computation and the Rule of Three behind it; typically the Rule of Three comes after the computation in a separate verse. By "computation" or "computational rule" I refer to statements that use expressions meaning "multiply" and "divide", for instance:

> The Sine of declination **multiplied by (*hata*)** the Radius and **divided by (*vihṛta*)** the [Sine of] co-latitude is the solar amplitude. (*GD2* 84ab)

The Rule of Three corresponding to this rule is:

[46]See S. R. Sarma (2002) for a history of the Rule of Three in India and its applications, including its usage in mathematics outside astronomy.

> If the Radius is the hypotenuse **of the upright (*koṭyāḥ*)** that is the [Sine of] co-latitude, what is the hypotenuse **of the upright** that is the [Sine of] declination? Thus the Rule of Three should be known for attaining the solar amplitude. (*GD2* 87)

The expression articulates the correspondence between the pairs of values using the genitive case in this example. Sometimes the instrumental, ablative or locative can be used instead. Mostly, the rules are based on a pair of similar triangles. This is stressed in *GD2* 106 which uses the word "proportion (*anupāta*)":

> With the base and so forth produced in one figure, here, with proportion, another figure is established ...

Therefore Parameśvara often adds "upright", "base" or "hypotenuse" in the statement of the Rule of Three which could have helped the reader locate the segments.

In the example above, Parameśvara mentions the word "Rule of Three". In some other cases, he uses the word "grounding (*yukti*)" instead (cf. *GD2* 188). Repeating the Rule of Three after the computation is indeed the structure of reasoning in *GD2*. This feature cannot be found in other treatises by Parameśvara, even in the *Grahaṇanyāyadīpikā* or *GD1* which put emphasis on reasoning. To be precise, both texts do have statements of Rules of Three as in *GD2* 87, but they stand alone and do not have the corresponding computations.

Lastly, I would like to mention that an unusual mode of statement can be seen in *GD2* 119cd.

> In this case, the grounding is because the Sine of geographic latitude is as the gnomonic amplitude for the [Sine of] co-latitude which is as the [great] gnomon.

This statement does not use special cases to link the corresponding segments (Sine of geographic latitude : gnomonic amplitude, and Sine of co-latitude : great gnomon); one pair is simply put in a compound and the other is only a juxtaposition of nominatives. Furthermore, the sentence does not use the conditional to connect the two pairs. This peculiar structure might have come from a tradition outside typical Sanskrit mathematical and astronomical texts.

### Astronomical issues

There is no term in *GD2* that corresponds to the modern notion of "astronomy". Instead, Parameśvara uses the word *gola* (Sphere) as a reference to the entirety of the subject that is being dealt with in *GD2*. The Sanskrit word "*gola*" as in *Goladīpikā* can refer to all kinds of spheres such as spheres as solid objects, celestial spheres, heavenly bodies with the form of a sphere, or even the name of a topic in astronomy or cosmography concerning them. In this section we shall focus on subjects that may be considered as astronomical from our viewpoint, that is, topics concerning the location of celestial objects.

**Armillary sphere** The term *gola* can also refer to an armillary sphere which is used for instructions. In *GD2*, various circles in the sky are used for locating heavenly objects. Without knowing their names, positions and motions as given in the beginning of the text, the rest of the treatise is incomprehensible. The wordings give the impression that an armillary sphere is being used. A name of a specific ring in the instrument is also used to address the corresponding celestial circle.

On the other hand, treatises often refer to an extremely complex system of "*gola*", such as the system with 51 moving circles in Brahmagupta's *Brāhmasphuṭasiddhānta* 21.49-58,67-69 (Ikeyama (2002, pp. 130-140,154-155)), which is unlikely to have been actually built in a complete form. Meanwhile, whenever the word *gola* is used in combination with *yantra* (instrument), the object described is much simpler. For example, the *Śiṣyadhīvṛddhidatantra* of Lalla, the *Siddhāntaśekhara* of Śrīpati and the *Siddhāntaśiromaṇi* of Bhāskara II each have chapters titled *Golabandha* (binding or constructing the sphere) and *Yantra*, and the system described in the former is very complex[47] while the description in the *Yantra* chapters are brief (Ôhashi (1994, pp. 268-271)).

The word *yantra* does not appear in *GD2*, while there is only one place that explicitly refers to a material of the instrument ("piece of wood or clay" in *GD2* 6). Among other texts by Parameśvara, *GD1* describes almost the same set of circles/rings as *GD2* in its chapter 1 titled "Method of constructing the sphere (*golabandhavidhi*)". The auto-commentary on this chapter (K. V. Sarma (1956–1957, p.11)) mentions that the rings should be made "with pieces of bamboo and the like (*vaṃśaśalākādinā*)". He uses the same expression in *PĀbh* 4.18 (Kern (1874, p.82)) where he also presents an armillary sphere. Although there is no explicit statement on the material of the rings in *GD2*, one example (*GD2* 212) refers to parts of the prime meridian as "bamboo-pieces (*śalākā*)", which hints that an armillary sphere made of bamboo might have been used. However, if this text were actually a description of an instrument, information on the size of the armillary sphere, including the ratios between each part of the instrument, are missing. Thus it is a question to know whether what is being presented here is an actual armillary sphere, mental object or just a description of the cosmos.

*GD2* 2-17 refers to rings in the armillary sphere, and they are stated as if the instrument was under the author or readers' eyes. Elsewhere, Parameśvara does not refer explicitly to the armillary sphere (*gola*), but there are several passages that could be interpreted as traces of the instrument being used. For instance, *GD2* 155 refers to a "hole (*vedha*)", which suggests a hole pierced in a ring of the armillary sphere (section 10.2). We have read and interpreted the reasonings given by Parameśvara in the *GD2* with the hypothesis that the armillary sphere was used as a tool. This could include mental configurations of the sphere without the physical object. Some of the groundings seem to require the projection of the configuration on a plane, but this could also have been done by looking at the instrument from a specific position.

A typical case where the armillary sphere might be involved is *GD2* 75-77 which locate various segments such as the Sine of declination in the sphere. The verses follow *GD2* 73-74 which give the set of computations for finding the length of these segments. *GD2* 75-77 might also serve as grounding for the rules, since they not only explain the segments themselves but also point out the right triangles that they shape.

Parameśvara makes wrong statements concerning the rising of signs in polar regions in *GD2* 51 and 53. The statement is corrected in *GD1* 3.54, where he says that this "should be explained completely on a sphere" (section 4.7). This shows that the armillary sphere could have been used for examining and correcting rules.

*GD2* begins with a description of the armillary sphere, and continues with various topics on astronomy that could be explained with it. From this viewpoint, this is a treatise whose entirety is devoted to an instrument – a category that is known to have appeared in Sanskrit literature after the contact with Islamic astronomy (S. R. Sarma (2008, p. 21)).

**Using diagrams** Parameśvara gives instructions to draw diagrams in two places; one to show the three orbits of a planet and its corrections (section 9.7) and another to explain the geocentric parallax (section 21.4). Both cases involve multiple circles, and one of their goals is to demon-

[47] *Śiṣyadhīvṛddhidatantra* 15.31-32 (Chatterjee (1981, 1, p. 205)) and *Siddhāntaśekhara* 16.38-39 (Miśra (1947, p. 216)) both enumerate the same 51 rings as in *Brāhmasphuṭasiddhānta* 21.68-69.

strate the apparent position of a planet by projecting its position from another circle to a great circle around the observer.

While the diagrams visualize how the position of a planet changes and necessitates a correction, they do not always explain how their values can be computed.

**Observation** The contrast between results derived from computation and those derived from observation is a topic in the commentaries on the examples. It remains a question whether this is important for Parameśvara too in *GD2*. As discussed above, his diagrams show the difference between the position computed with equations and the position observed. For the latter, Parameśvara uses the term *sākṣāt* (literally "with the eyes") in *GD2* 145 and 148 which has a very strong nuance of actual observation. However, while there seems to be an aim at making observation and computation agree, most of Parameśvara's instructions in *GD2* are how to compute and not how to observe. The only object that is evidently observed is the shadow of a gnomon.

The six examples in *GD2* link observation with computation, which is done in two directions. Examples 1, 2, 5 and 6 compute parameters such as the sun's longitude from the observed shadow length, and examples 3 and 4 find the expected length of the shadow from the given parameters. Parameśvara does not explain the reason for this procedure in detail, but *GD2* 218cd suggests that part of the motivation to compute the sun's longitude from observation is to find the motion of the solstice, or in modern notion, precession (section 13.5).

**Importance of the celestial longitude** The English terms longitude and latitude are considered as a pair of coordinates. But in *GD2*, we find that the celestial longitude is the most important parameter of a planet, which can also be seen from the fact that the word for "planet (*graha*, etc.)" can also mean its longitude. Meanwhile, the celestial latitude (*kṣepa*) is only a deviation[48] for which we must correct the longitude as in the visibility operation (chapter 10). The declination is only a parameter which follows the longitude, and what we call the right ascension is chiefly used to measure the timing of rising or setting of the body, or the time corresponding to the motion of a given arc of longitude on the ecliptic.

Parameśvara does not discuss the celestial longitude in particular in *GD2*, nor is he alone in Sanskrit astronomical literature to treat the longitude in this way. Yet this is a recurring topic that we modern readers must keep in mind upon interpreting his words and reconstructing the computational rules or their groundings in *GD2*. We discuss how *GD2* treats the celestial longitude in sections 6.2 and 9.1.

**Astronomical constants and other values** *GD2* gives many constants and values related to cosmology and chronology, notably those related to long time periods in *GD2* 55-64 (section 4.8, table 4.1), longitudes of planetary nodes and inclinations of orbits (section 9.5, table 9.1) and the sizes and distances of the sun and moon (section 22.1, table 22.1). However, the treatises lack some constants that are required for the computational methods introduced in it. For example, the apparent celestial latitude of planets cannot be computed without their distances from the Earth. *GD2* 89-102 states how measures or rising times of signs can be computed, but do not give the rising times of signs at the terrestrial equator which are given as constants in other treatises like the *Mahābhāskarīya* (see section 7.5). Rising times of signs are needed to compute visibility equations (*GD2* 169, 177, 193). These facts show that the methods in *GD2* were assumed to be operated using other treatises or tables that contain the relevant values.

[48]The Sanskrit term *kṣepa* (or *vikṣepa*) itself means "to hurl" or "deviation".

If the main purpose of *GD2* was to provide groundings than to serve as a manual, we may question why *GD2* provides some constants in the first place. One possibility is that they are also part of the reasoning and not for actual usage. For example, Parameśvara gives the circumference of the Earth (3299 *yojana*s) in *GD2* 201 from which the observer's circumference (the length measured on the Earth along the geographic latitude of the observer) is computed. But *Grahaṇamaṇḍana* skips this value as well as the rule and directly gives the circumference for an observer at Aśvattha (the village where Parameśvara lived).

### 0.2.6 Influence of other authors

We shall compare the computational rules and other statements in *GD2* with other authors under each section in our commentary[49]. The following is an overview of some important sources that have already been suggested or that have emerged in our study.

#### Āryabhaṭa and his *Āryabhaṭīya*

Āryabhaṭa appears to be an important authority in *GD2*. Passages from the *Āryabhaṭīya* are quoted three times in *GD2* (*GD2* 38=*Ābh* 4.14, *GD2* 39ab=*Ābh* 4.12ab and *GD2* 44≈*Ābh* 4.13) in the context of debates on cosmography. All constants in the *GD2* except for the four parts of the *caturyuga* agree with the *Āryabhaṭīya*. Parameśvara's commentary on the *Āryabhaṭīya* was often helpful to interpret some difficult passages in *GD2*, which also suggests that Parameśvara might have borne the *Āryabhaṭīya* in his mind when he composed those verses.

The title *Goladīpikā* itself has an echo with the fourth chapter "*Golapāda* (quarter on the Sphere)" of the *Āryabhaṭīya*. The order of topics in the *Golapāda* could also have inspired Parameśvara, since both texts deal with cosmography at an earlier stage and put the topic of eclipses at the end.

On the other hand, rules in *GD2* often go beyond the *Āryabhaṭīya*. There are corrections that are not mentioned by Āryabhaṭa and rules that would give much more accurate results than his. Parameśvara never refers to Āryabhaṭa after *GD2* 69 where he switches the topic and focuses to computational rules than statements on static configurations.

We can only speculate how Parameśvara related such rules with the *Āryabhaṭīya*, but his commentary on *Ābh* 4.36 might be a clue. At the conclusion of his remarks on *Ābh* 4.35-36 which deal with the visibility methods for the "course" and for the geographic latitude (see chapter 10), Parameśvara states:

> The twofold correction on visibility having a crude form has been explained here by the master [Āryabhaṭa]. It should be known however that it is not the exact form. The sense is that: The exact form is established from this crude form with grounding.[50]

Parameśvara is aware that the Āryabhaṭa's are approximate, but seems to think that he must build his own methods from them. This explains his statements on the Sine of sight-motion (*dṛggati*) in *GD2* 270 where he strictly follows Āryabhaṭa's rule while giving it a new explanation. By contrast, other authors such as Brahmagupta, Bhāskara II and Nīlakaṇṭha discarded the rule (see section 21.6).

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[49] I am deeply indebted to the notes by Chatterjee (1981) on the *Śiṣyadhīvṛddhidatantra* which lists the corresponding verses in other treatises for each topic.

[50] *ācāryeṇa sthūlarūpaṃ dṛkphaladvayam iha pradarśitam / na tu sūkṣmarūpam iti vedyam / asmāt sthūlarūpāt sūkṣmarūpaṃ yuktyā siddhatīti bhāvaḥ /* (Kern (1874, p. 94))

### Bhāskara I and his *Mahābhāskarīya*

Parameśvara refers to the *Mahābhāskarīya* and his super-commentary *Siddhāntadīpikā* in *GD2* 69. Every major topic after *GD2* 70 (which I have listed in section 0.2.4) can also be found in the *Mahābhāskarīya*, and the discussions in *Siddhāntadīpikā* often complement the succinct verses in *GD2*. However, while such discussions were certainly inspired by Bhāskara I, Parameśvara does not necessarily follow him. The order of the subjects in *GD2* are completely different from those in the *Mahābhāskarīya*, and computational rules that are unique to Bhāskara I, such as the usage of two nodes to find the deviation (see section 9.11) cannot be found.

Any similarity that we find between the *Mahābhāskarīya* and Parameśvara's statements can usually be explained by saying that they both follow the *Āryabhaṭīya*. This is especially the case concerning astronomical constants. Such attitude toward Bhāskara I may be because Parameśvara views the *Mahābhāskarīya* as a sort of commentary on the *Āryabhaṭīya*. This is first mentioned in the concluding verse of Govindasvāmin's commentary, and is followed by Parameśvara in one of his concluding verses of the *Siddhāntadīpikā*:

> Master Āryabhaṭa composed the work (*tantra*) on Brahma's doctrine, then Bhāskara made an extensive commentary (*vṛtti*) on it. And then Govinda[svāmin] [made] a commentary (*bhāṣya*) on it. But its meaning is far from good understanding; thus an easier commentary (*vyākhyā*) on it was composed by me with the help of Rudra.[51]

Parameśvara's discussions are strongly inspired by Govindasvāmin, and he even quotes a passage from the commentary in *GD2* 47. Meanwhile, some of Parameśvara's statements differ from Govindasvāmin. One example is the description of the Sine of sight-motion (section 21.6).

The *Mahābhāskarīya* might not have been the only source authored by Bhāskara I used by Parameśvara. Parameśvara's arguments on cosmology have many parallels in Bhāskara I's commentary on the *Āryabhaṭīya*, which suggests the possibility that he also had access to this work (see chapter 3).

### Brahmagupta and his *Brāhmasphuṭasiddhānta*

Parameśvara quotes the *Brāhmasphuṭasiddhānta* a few times in his commentary on the *Āryabhaṭīya*, and might have been influenced by Brahmagupta upon choosing one of its variant readings (section 4.1). However, not much similarity between Brahmagupta and Parameśvara could be found in their computational rules.

There is one case where Parameśvara's rule resembles Brahmagupta's. In the visibility methods, Parameśvara's and Brahmagupta's rules involve the Sine of a planet's longitude whereas those of Āryabhaṭa and Bhāskara I use the versed Sine (section 10.10).

Another similarity can be found outside *GD2*, in Parameśvara's second order interpolation method introduced in his commentary on the *Laghubhāskarīya* and in the *Siddhāntadīpikā* (appendix B.5). This case is of particular interest because this method agrees with some of the Sine values computed in the commentaries on the examples of *GD2* (appendix B.6.1).

However, these two cases can also be explained as an influence of Bhāskara II, and it is debatable whether Parameśvara's methods were established based upon Brahmagupta directly.

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[51] *ācāryāryabhaṭo 'karod vidhimataṃ tantraṃ punar bhāskaro*
*vṛttiṃ tasya ca vistarāt punar atho bhāṣyaṃ ca tasyās tathā |*
*govindo 'sya ca dūram ety asudhiyām arthas tv idānīm iti*
*vyākhyā tasya mayā kṛtā laghutarā rudraprasādād iti ||* (T. Kuppanna Sastri (1957, p. 395))

#### The *Sūryasiddhānta*

Pingree (1981, p. 613) claims that Parameśvara's *Dṛggaṇita* uses parameters that are close to the *Sūryasiddhānta*, and puts him in the *Saurapakṣa*, a school which derives its name from this treatise. Identifying an author in a specific school is a difficult task, and the same author could write different texts devoted to different schools. We shall leave the question whether the *Dṛggaṇita* indeed belongs to the *Saurapakṣa*. But when we turn to *GD2*, influences from the *Sūryasiddhānta* are not conspicuous. Astronomical constants like the the sizes of the Sun, Moon and Earth are different (section 22.1, table 22.1). Computational methods and their reasonings in *GD2* look very different from those in the *Sūryasiddhānta*. For example, the explanations for the celestial latitude (section 9.2) and the definitions of the Sine of sight-motion (section 21.6.1).

On the other hand, when it comes to topics on cosmography, *GD2* does not differ very often from the *Sūryasiddhānta*. In the description of Mount Meru as an axis piercing the Earth (*GD2* 36), he might even be influenced by the treatise (section 3.7). But the similarities are not strong enough compared to other texts to claim that the *Sūryasiddhānta* was the main source for Parameśvara on the subject of cosmography.

#### Bhāskara II and his *Siddhāntaśiromaṇi*

Computational rules in *GD2* often go beyond Āryabhaṭa and Bhāskara I, by taking new factors into account and adding new steps. As a result, some of them resemble the methods of Bhāskara II very much. Most notable is that both Parameśvara and Bhāskara II add steps for moving from the celestial equator to the ecliptic in order to compute equations (section 10.9.1 and 11.3). Another case is the correction applied to the celestial latitude for finding the true declination (section 10.3). The use of Sines instead of versed Sines in visibility operations is another feature that might come from Bhāskara II (section 10.10). T. Kuppanna Sastri (1957, p. 338) has already suggested that Parameśvara is making reference to Bhāskara II when he says that versed Sines should not be used in his super-commentary on *MBh* 6.3.

In most of these cases, Parameśvara implies that his rules or ideas come from another source by introducing them as opinions of "some (*kecit*)" (*GD2* 157) or "others (*anye*)" (*GD2* 204). He never refers explicitly to Bhāskara II or his works in *GD2*. Meanwhile, we know that he commented upon the *Līlāvatī*, which is a mathematical chapter in the *Siddhāntaśiromaṇi* by Bhāskara II. Whether Parameśvara read the other chapters which deal with astronomy is an open question. Our study suggests that the answer could be yes; but while Parameśvara could have been influenced by Bhāskara II, he did not openly profess to follow him.

#### Mādhava

We have found two cases in *GD2* that can be compared with computational rules in astronomy attributed to Mādhava by Nīlakaṇṭha. They are the rule to compute the true declination (section 10.6.1) and the method for the Sine of sight-deviation (section 10.16.2). In both cases, there is a distinctive difference between Mādhava and Parameśvara (and as a result, also between Nīlakaṇṭha and Parameśvara). If Nīlakaṇṭha's attribution is correct, we must conclude that *GD2* hardly shows any influence of Mādhava.

### 0.2.7 Commentary on *GD2*

Previously discovered manuscripts only contained the base text of *GD2*, and it has long been thought that *GD2* does not have a commentary.

However we have found that manuscript Indian Office Sanskrit 3530 of the British Library, whose text has been previously recognized as a version of *GD2* expanded with quotations, also includes commentaries on *GD2* 209-246. Manuscript 13259 of KOML, which contains an uncommented version of *GD2*, also has an excerpt of *GD2* 209-246 with commentaries. The two texts agrees in general, and comes from a common source.

The commentaries are inserted after the following verses:

***GD2* 211** Solution of example 1 in *GD2* 209, following the procedure in *GD2* 210-211.

***GD2* 217** Solution of example 2 in *GD2* 212, following the procedure in *GD2* 213-217.

***GD2* 218** Clarifies the passage, as well as adding an example.

***GD2* 219** Clarifies and expands the passage.

***GD2* 231** Solution of example 3 in *GD2* 231 with a preamble to *GD2* 232.

***GD2* 232** Solution of example 4 in *GD2* 232.

***GD2* 233** Clarifies and expands the passage.

***GD2* 234** Some statement concerning the previous examples (?), and a preamble to *GD2* 235.

***GD2* 245** Solution of example 5 in *GD2* 245

***GD2* 246** Solution of example 6 in *GD2* 246. The last sentence is identical to *GD2* 247.

As we can see, most of the commentary is on the 6 examples. Solutions for the examples are given by providing the intermediate values one by one. In this way, the commentaries show the steps to be followed, but there are no details on how the computations are carried out, or on the rules in *GD2* which are to be used. Some steps are not stated, including those that are mentioned in *GD2* itself. We shall discuss the procedures under each chapter for the examples.

So far, we have no information on who could have written these commentaries. The manuscripts containing the commentary come from an early branch in the stemma (see figure 0.12 in section 0.3.2). Therefore, it is possible that this could be an auto-commentary. *GD2* 247 is too short for an independent verse, and could be the last sentence of the commentary that was accidentally left in the copy of a manuscript when the scribe tried to copy the verses without the commentary (section 20.2). However I consider it unlikely that the commentator was Parameśvara himself because the numbers are written in numerals. By contrast, the auto-commentary on *GD1* always uses word numerals, even in the solution of an example. The numbers in the commentary also suggest the possibility that there were multiple commentators, since the way that fractions are expressed are very different among the examples (appendix A.3).

### 0.2.8 The other *Goladīpikā*

Parameśvara has composed another treatise with the title *Goladīpikā* (hereafter *GD1*). The two *Goladīpikā*s share many common topics, but their structure is different.

*GD1* has 267 verses divided into four chapters. The segmentation was obviously intended by the author himself, as can be seen from the fact that every manuscript has a colophon giving the titles of the chapters at each end and that Parameśvara composed an auto-commentary indicating the same division. The critical edition of *GD1* and its auto-commentary as well as an English translation of the verses were published by K. V. Sarma (1956–1957).

Chapter 1 (15 verses), called “Rule for constructing the sphere (*golabandhavidhi*)” is an introduction devoted to the armillary sphere. In chapter 2 (50 verses) “Rule of planetary motion (*grahacāravidhi*)” the motion of planets along the circles given in the previous chapter, as well as the nature of the Earth, sun and moon, are explained. Chapter 3 (110 verses) “Thoughts on the Earth and the like (*bhūmyādicintana*)” deals with the shape and size of the Earth with a detailed explanation of traditional cosmography in Hinduism integrated into the theory of a spherical Earth. Finally, the untitled chapter 4 (92 verses) mentions a variety of topics in astronomy that require computation, including the gnomon, parallax, eclipses and precession.

Table 0.2 lists the topics in each chapter as well as their correspondence with *GD2*. Note that some of these corresponding verses can be completely identical while others can be very different in appearance. We can see that most of the contents in chapters 1-3 correspond to *GD2* 2-67. Subjects dealt with after *GD2* 70 are concentrated in chapter 4. The following topics that involve many steps of computations and advanced knowledge do not appear in *GD1*: Orbits of planets and their deviation (*GD2* 125-152), celestial latitude and visibility methods (*GD2* 153-178) and corrections to the mean planet at sunrise (*GD2* 195-201). *GD1* 1.7cd-8ab briefly refers to the inclined circle (*vikṣepamaṇḍala*, the path of a planet that is inclined against the ecliptic), but there is no further explanation on the celestial latitude itself.

Meanwhile, the extensive descriptions on purāṇic cosmology and geography in *GD1* 3.62-110 have no parallel in *GD2*. Some instructions on drawings using the gnomon and its shadow can be found in *GD1* 4.27-36, but there is no corresponding passage in *GD2*.

Table 0.2: Contents of *GD1* and correspondence with *GD2*

| *GD1* | Topic | *GD2* |
|---|---|---|
| 1.1 | Benediction | - |
| 1.2-14 | Constructing the armillary sphere | 2-6, 10-15ab, 126 |
| 1.15 | Geographic latitude and co-latitude | 88 |
| 2.1-4 | Diurnal motion | 7-9 |
| 2.5-6 | Geocentric parallax | 249 |
| 2.7-13 | Diurnal motion of planets | 15cd-16 |
| 2.14-17 | Definition of solar amplitude | 75cd,84-87 |
| 2.18-19 | Diurnal motion of sun in different latitudes | none |
| 2.20-28 | Daily motion of planets | 18-21 |
| 2.29-34ab | That the Sun’s orbit is higher than the Moon | 66,67 |
| 2.34cd-37 | Source of moonlight | 22-24, 283 |
| 2.38-45 | Cause of eclipses: denying myths | none |
| 2.46-50 | Spherity of planets, size of Sun and Moon | 22-24,277,279 |
| 3.1-5 | Stability and immobility of the Earth | 25-27 |
| 3.6-18 | Size of the Earth | 30-35,37,70-72 |
| 3.19-24ab,30-35 | Compromising with purāṇic cosmology | 31,36,39,66-67 |
| 3.24cd-29,36-42 | Defining directions, geography | 34-35,38,41,43-44 |
| 3.43-57 | Length of day and night at various latitudes | 41-54 |
| 3.58-61 | Very long units of time | 56-65 |
| 3.62-110 | Purāṇic cosmology and geography | none |
| 4.1-6 | Defining a great gnomon and related segments | 103-115 |
| 4.7-22 | Computing the great gnomon | 121-124,220-230,233-234 |

(continued from previous page)

| *GD1* | Topic | *GD2* |
|---|---|---|
| 4.23 | Example | ≈232 |
| 4.24-26 | The twelve *aṅgula* gnomon | 116-120 |
| 4.27-36 | Drawings for the gnomon and shadow | none |
| 4.37-51 | Computing the sun's longitude and geographic latitude from the shadow | 235-244 |
| 4.52-53ab | Midday shadow | 213-217 |
| 4.53cd | Prime vertical shadow | 210-211 |
| 4.54-58 | Earth's shadow | 286-301 |
| 4.59 | Computing apparent sizes of discs | 280 |
| 4.60-61 | Difference between solar and lunar eclipse | 281,282 |
| 4.62-78 | Computing parallaxes | 248-276 |
| 4.79-84 | Rising time of zodiac signs | 89-102 |
| 4.85-90 | Motion of solstitial points | 218-219 |
| 4.91-92 | Conclusion | - |

Table 0.3: Contents of *GD2* and correspondence with *GD2*

| *GD2* | Topic | *GD1* |
|---|---|---|
| 1 | Invocation | - |
| 2-17 | Parts of the armillary sphere and their meaning | 1.2-14, 2.1-4,7-13 |
| 18-21 | Motion of the stars and planets | 2.20-28 |
| 22-24 | Forms of the sun and moon | 2.34cd-37,46-50 |
| 25-27 | Stability and immobility of the Earth | 3.1-5,20 |
| 28-36 | Surface of the Earth | 3.6-19,22,30-32 |
| 37-39 | Mount Meru and Laṅkā | 3.11,26-29 |
| 40-54 | Day and night at various places | 3.43-58 |
| 55-65 | Very long timescales | 3.52-61 |
| 66-67 | Contradicting statements on the distances of the sun and moon | 2.29-34ab |
| 68-69 | Authorship and summary | - |
| 70-72 | Geographic latitude and co-latitude | 3.8-11 |
| 73-83 | Computing the ascensional difference | (2.15) |
| 84-87 | Sine amplitude | 2.14-17 |
| 88 | Another description for Latitude and co-latitude | 1.15 |
| 89-102 | Rising time of zodiac signs | 4.79-84 |
| 103-115 | The great gnomon | 4.1-6 |
| 116-120 | Great gnomon and the twelve *aṅgula* gnomon | 4.7-22 |
| 121-124 | The prime vertical gnomon | 4.10-11 |
| 125-152 | Orbits of planets and their deviation | none |
| 153-178 | Celestial latitude and visibility methods | none |
| 195-201 | Corrections to the mean planet at sunrise | none |
| 209-211 | Example 1 | 4.53cd |
| 212-217 | Example 2 | 4.52-53ab |
| 218-219 | Motion of solstitial points | 4.85-90 |
| 220-230 | Length of shadow when the sun is in a given direction | 4.7-22 |

(continued from previous page)

| *GD2* | Topic | *GD1* |
|---|---|---|
| 231-232 | Example 3,4 | ≈4.23 |
| 233-234 | Speed of "without-difference" method | 4.21-22ab |
| 235-244 | Finding the sun and geographic latitude from the shadow in an intermediate direction | 4.37-51 |
| 245-247 | Example 5,6 | (4.37-51) |
| 248-276 | Parallax | 2.5-6, 4.62-78 |
| 277-280 | Distance and size of Sun and Moon | 2.46-50 |
| 281-282 | Difference between solar and lunar eclipse | 4.60,61 |
| 283-301 | The shadow of the Earth | 4.54-59 |
| 302 | Conclusion | - |

There are 9 known manuscripts of *GD1* and two of its auto-commentary (see page 36 for the list). This suggests that the *GD1* and *GD2* (11 extant manuscripts) were both popular, more or less to the same extent. However, in contrast to *GD2*, of which we could not find any quotations in later literature, verses from *GD1* are quoted by Nīlakaṇṭha in his commentary on the *Āryabhaṭīya* (cf. Pillai (1957b, p. 27)). One would wonder whether *GD1*, which seems to have been composed after *GD2*, had replaced it in some milieus, but this remains very speculative. However, the difference in focus of *GD2* and *GD1* suggests that they could have been prepared for different readers. Notably, the first chapter on the armillary sphere in *GD1* proceeds as if one were building the instrument, but the description in *GD2* ignores the order of construction, which suggests that the reader was expected to have better access (either physical or mental) to the armillary sphere (Hirose (2016)). Topics that are not included in *GD1* require good knowledge of circles and segments within the sphere, which also supports the possibility that the *GD2* was intended for more advanced learners.

#### Order of the *Goladīpikā*s

K. V. Sarma (1956–1957) was the first to reflect on the two versions of the *Goladīpikā*. He does not mention whether one is a revision of the other, but he seems to think that *GD2* was composed later, as he writes "In the *Goladīpikā* published in the Trivandrum Sanskrit Series (=*GD2*), ... some topics like cosmogony are left out; others, like the conception of the *yuga-s* and calculation of the latitudes of planets, are newly introduced" (K. V. Sarma (ibid., p. 3)). Probably this is the reason why he numbered them *GD1* and *GD2* in his survey (K. V. Sarma (1972))[52]. Pingree (1981, p. 191) comments that *GD2* refers to *GD1*[53], but this is not correct.

We follow the numberings of *GD1* and *GD2* since they are already widely used. However, as discussed previously in section 0.1.5, what we call *GD1* seems to have been composed after *GD2*.

### 0.2.9 Concluding remarks

What can we say about Parameśvara in relation to other authors, and what can we say about *GD2* in relation to other texts by Parameśvara?

[52] Meanwhile he first numbered the texts in reverse order (K. V. Sarma (1963), K. V. Sarma (1965) and K. V. Sarma (1966)), possibly due to the order the editions in which were published

[53] "A Goladīpikā in 302 verses in which Parameśvara refers to his first *Goladīpikā* and his *Karmadīpikā* on the *Mahābhāskarīya*." (page 191) However, *GD2* 69 refers to the *Siddhāntadīpikā*, Parameśvara's super-commentary to the *Mahābhāskarīya*, but not to the *Karmadīpikā* which is his direct commentary on the treatise.

Our study on *GD2* shows that Parameśvara connects himself with his predecessors in two ways. His attitude toward Āryabhaṭa, and to some extent toward Bhāskara I (who is viewed as a commentator of Āryabhaṭa), is different from his treatment of other authors. Even when he finds that the rules in the *Āryabhaṭīya* or the *Mahābhāskarīya* are inaccurate and must be replaced, Parameśvara still acknowledges their work and keeps some of their elements in his reasonings: A typical case is his explanation of the two visibility methods (*GD2* 165-177) before giving the unified method (*GD2* 178-194). In other words, the *Āryabhaṭīya* is the foundation on which Parameśvara must build his theories. It is at the point when he constructs his rules that he makes use of other authors. Parameśvara does acknowledge such influence, but he keeps distance by merely calling them "others" or the like. We assume that Bhāskara II is a representative of this case.

Given this difference in Parameśvara's usage of previous authors and the resulting stratum in his work, it is impossible to categorize Parameśvara in a single "school" - whether it be a "school" of people that share the same idea, or use the same parameter. As for the "Kerala school", we have found evidence in *GD2* that denies influence of Mādhava, and as Parameśvara himself does not refer to him, we must reconsider the position of Parameśvara in this pedagogical lineage.

As for the nature of *GD2* itself, my feeling is that it puts emphasis on grounding the rules rather than giving a handy set of methods that can be used right away. This is in contrast to other treatises that only include the rules, such as the *Grahaṇamaṇḍana*. Yet, this does not mean that *GD2* was for an elementary reader. A comparison with *GD1* shows that the contents of *GD2* are advanced, and that it requires some expertise on the armillary sphere or the configuration of celestial circle that it represents.

## 0.3 Manuscripts of *Goladīpikā 2*

### 0.3.1 Description of manuscripts used in the critical edition

We have used 11 manuscripts labeled $K_1$-$K_8$ and $I_1$-$I_3$ for editing the verses of *GD2*. One of them, $I_1$, contained commentaries, and another one $K_5$ had extra folios (which we label $K_5^+$) with commentaries. Thus for editing the commentary we have used $I_1$ and $K_5^+$.

Every extant manuscript is in palm leaves with Malayalam script. We have acquired digital copies for all of them, and examined each of them directly at least once.

**$K_1$** MS. No. 475 J (Catalog No. 5054 in Pillai (1957a)) of the Kerala University Oriental Research Institute and Manuscripts Library (ORI & MSS)[54]. 16 unnumbered folios, 30cm × 5cm. 8-10 lines per page and about 70 letters per line.

The bundle 475 includes: (A) *Āryabhaṭīya* of Āryabhaṭa, (B) *Mahābhāskarīya* of Bhāskara I, (C) *Laghubhāskarīya* of Bhāskara I, (D) *Siddhāntadarpaṇa* of Nīlakaṇṭha Somayājin, (E) *Tantrasaṅgraha* of Nīlakaṇṭha Somayājin, (unlabeled) *Candracchāyāgaṇita* of Nīlakaṇṭha Somayājin, (F) *Līlāvatī* of Bhāskara II, (G) *Pañcabodha*, (H) *Laghumānasa* of Muñjala, (I) *Candracchāyāgaṇita* of Parameśvara, (J) *Goladīpikā 2* of Parameśvara, (K) *Grahaṇāṣṭaka* of Parameśvara.

The colophon of 475A gives the date of transcription as 1,699,817 days after the beginning of the *Kali Yuga*, which amounts to December 23rd, 1552[55]. There is a passage after 475F that

[54] Address: Oriental Research Institute and Manuscript Library, University of Kerala, Kariavattom, Thiruvananthapuram - 695 581, Kerala, India. Website: http://www.keralauniversity.ac.in/departments/ori/

[55] The material of the folios and the handwriting are almost consistent throughout the whole bundle, which suggest that most or all of the folios were written by the same scribe. It might have taken a considerable time to write the entire bundle, but we assume that its period is not very far off from the date written here.

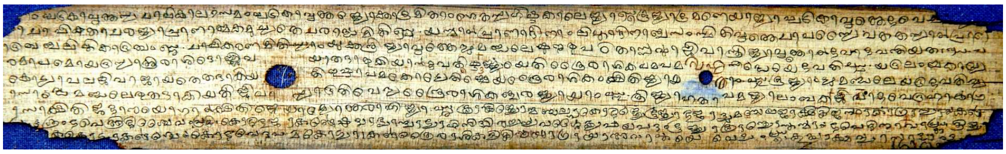

Figure 0.1: Manuscript 475 J ($K_1$), folio 4 verso

says "this manuscript is written and owned by Nīlakaṇṭha of Vaṭaśreṇyā[56]". Vaṭaśreṇyā was also where Parameśvara lived.

**K**$_2$ MS. No. 5867 A (Catalog No. 5058 in Pillai (1957a)) of ORI & MSS. 45 folios numbered 101 to 145 (in the letter-numeral system beginning with *na-nna-nya*[57]), 18cm × 4cm. 7 lines per page and about 30 letters per line. Formerly property of a Brahman, Haridasan Tuppan Namboodirippadu Ponnorkkod Mana.

The bundle 5867 includes: (A) *Goladīpikā 2* of Parameśvara, (B) *Golasāra* of Nīlakaṇṭha Somayājin, (C) *Siddhāntadarpaṇa* of Nīlakaṇṭha Somayājin.

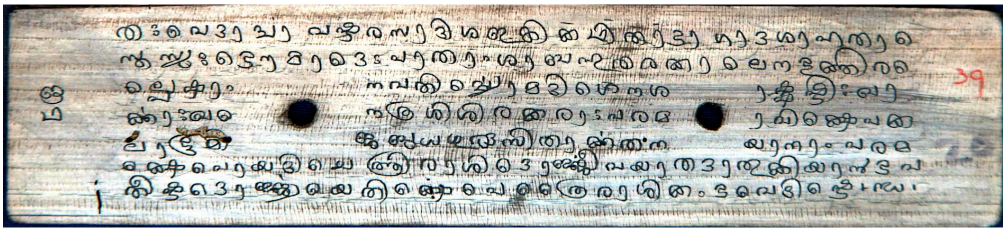

Figure 0.2: Manuscript 5867 A ($K_2$), folio 120 recto

**K**$_3$ MS. No. 8327 A (Catalog No. 5059 in Pillai (ibid.)) of ORI & MSS. 27 folios numbered 2 to 28 (in *na-nna-nya* letter numerals; folio 1 missing), 17cm × 4cm. Badly damaged. 9-11 lines per page and about 35 letters per line. Formerly property of Chirakkal palace Library.

The bundle 8327 includes: (A) *Goladīpikā 2* of Parameśvara, (B) and (C) *Horāsāroccaya* of Acyuta with Malayalam commentary.

**K**$_4$ MS. No. 10583 A (Catalog No. 24883 in Bhaskaran et al. (1988)) of ORI & MSS. 15 folios numbered 1 to 15 (in Grantha Malayalam numerals[58]), 17cm × 3.5cm. 8-10 lines per page and about 60 letters per line. Formerly property of Eḍappaḷḷy palace Library.

The bundle 10583 includes: (A) *Goladīpikā 2* of Parameśvara, (B) *Golasāra* of Nīlakaṇṭha Somayājin, (C) *Siddhāntadarpaṇa* of Nīlakaṇṭha Somayājin.

**K**$_5$ MS. No. 13259 A (Catalog No. 1840 in Pillai (1957a)) of ORI & MSS. 49 folios numbered 4 to 57 (in *na-nna-nya* letter numerals; folios 1-3, 14, 15, 43-45 completely missing), 20cm ×

[56] *vaṭaśreṇyākhyena nīlakaṇṭhena likhitam idaṃ pustakaṃ svīyaṃ ca*

[57] See Grünendahl (2001, p. 94) for the full list of numerals and Bendall (1896) for additional information on this system.

[58] See Grünendahl (2001, p. 93) for a full list.

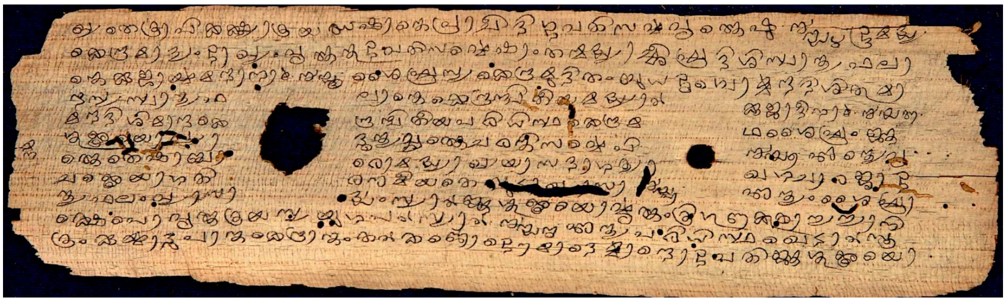

Figure 0.3: Manuscript 8327 A ($K_3$), folio 11 recto

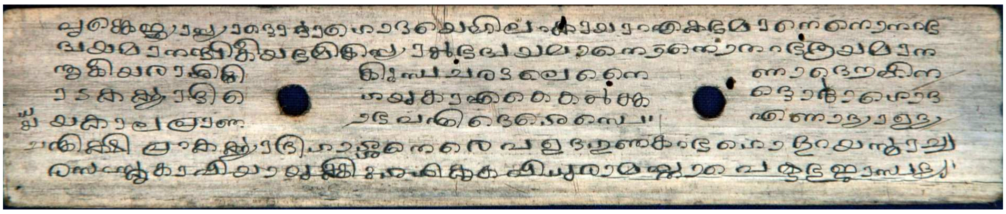

Figure 0.4: Manuscript 10583 A ($K_4$), folio 14 recto

3.5cm. Many folios are only left in fragments and every folio is badly damaged. 6 lines per page and about 30 letters per line. Origin unidentified. Wrongly identified as "Bhāṣya [commentary] by Bhāskarācārya of the *Āryabhaṭīya*" in the catalogue.

Considering the frequent lacunae and discontinuity, this bundle appears to be a copy of a manuscript which was already damaged or fragments of manuscripts. For example, the text is cut abruptly in the middle of *GD2* 109 at folio 19 recto. 19 verso is blank. Folio 20 recto starts from the middle of *GD2* 103. Thus there is an overlap.

The bundle 13259 includes the *Goladīpikā 2* of Parameśvara, a fragment of an identified text on the nodes and latitude of the moon, a commentary on *Goladīpikā 2* ($K_5^+$), an unidentified text on astral science throughout folios 80 to 109, and (B) *Āryabhaṭīya* of Āryabhaṭa.

Figure 0.5: Manuscript 13259 A ($K_5$), folio 11 verso

**$K_5^+$** Additional folios in MS. No. 13259 A containing verses 209 to 247 with commentaries. Readings of the verses are sometimes different from those in $K_5$, and therefore we shall treat $K_5$ and $K_5^+$ as different samples. 19 folios numbered 59 to 80 (folios 76-78 missing).

**K$_6$** MS. No. 17945 B (Catalog No. 24884 in Bhaskaran et al. (1988)) of ORI & MSS. 15 folios numbered 1 to 15 (in Grantha Malayalam numerals), 4.5cm × 35cm. Formerly property of a Brahman, Tharayil Kuzhikkattillam Agnisarman Bhattathiri.

The bundle 17945 includes: (A) *Śeṣasamuccaya* (tantrism), (B) *Goladīpikā 2* of Parameśvara, (C) *Pañcākṣaramantravidhi* (mantras), (D) *Tālaprastāra* (musicology). K$_6$ is the only manuscript in our list that comes from such a variegated codex.

Figure 0.6: Manuscript 17945 B (K$_6$), folio 2 recto

**K$_7$** MS. No. C.224 F (Catalog No. 5060 in Pillai (1957a)) of ORI & MSS. 11 folios numbered 54 to 64 (in *na-nna-nya* letter numerals), 33cm × 4cm. 10-13 lines per page and about 80 letters per line. Formerly property of Eḍappaḷḷy palace Library.

The folios are fairly well preserved and the letters are neatly inscribed, but the text includes numerous scribal errors that have been both inherited and newly caused.

The bundle C.224 includes: (A) *Āryabhaṭīya* of Āryabhaṭa with commentary of Sūryadeva Yajvan, (B) *Laghubhāskarīya* of Bhāskara I, (C) *Tantrasaṅgraha* of Nīlakaṇṭha Somayājin, (D) *Mahābhāskarīya* of Bhāskara I, (E) *Sūryasiddhānta*, (F) *Goladīpikā 2* of Parameśvara, (G) *Siddhāntaśekhara* of Śrīpati. K. V. Sarma (1976, p. xvii) gives detailed information on this manuscript. According to him, a colophon in (C) gives the date of transcription as Kollam era 928, which corresponds to 1752-53 CE.

Figure 0.7: Manuscript C.224 F (K$_7$), folio 64 verso

**K$_8$** MS. No. C.1024 D (Catalog No. 5061 in Pillai (1957a)) of ORI & MSS. 38 folios numbered 1 to 38 (in Grantha Malayalam numerals), 32 × 4cm. 8 lines per page and about 30 letters per line. Formerly property of the Rājā of Cirakkal.

The bundle C.1024 includes: (A) *Āryabhaṭīya* of Āryabhaṭa, (B) *Sūryasiddhānta*, (C) *Sūryasiddhānta*, (D) *Goladīpikā 2* of Parameśvara, (E) *Golasāra* of Nīlakaṇṭha Somayājin, (F) *Siddhāntadarpaṇa* of Nīlakaṇṭha Somayājin.

**I$_1$** Indian Office Sanskrit 3530 (Catalog No. 6297 in Eggeling (1887)) of the British Library[59]. 56 folios numbered 1 to 56 (in Grantha Malayalam numerals), 19 × 4 cm. 7-8 lines per page and about 40 letters per line. A slit of paper included in the bundle reads “Found in Silmory”

[59] Address: The Asian & African Studies Reading Room, The British Library, 96 Euston Road, London, NW1 2DB, United Kingdom. Website: http://www.bl.uk/reshelp/inrrooms/stp/rrbysubj/aasrr/aasrr.html

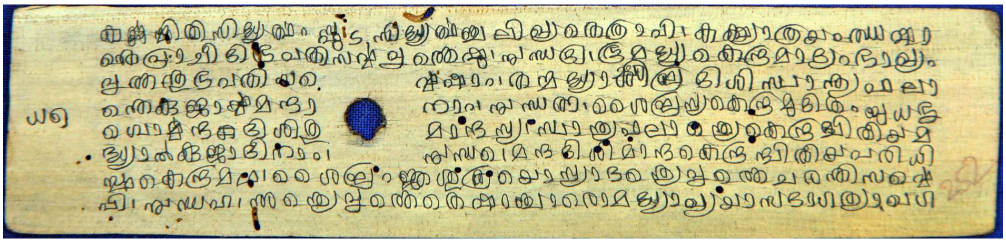

Figure 0.8: Manuscript C.1024 D ($K_8$), folio 17 recto

in English, but we could not find the corresponding location. The catalog dates this manuscript to the 18th century.

This is the only text in the bundle, but 37 blank folios are included after the *Goladīpikā 2*.

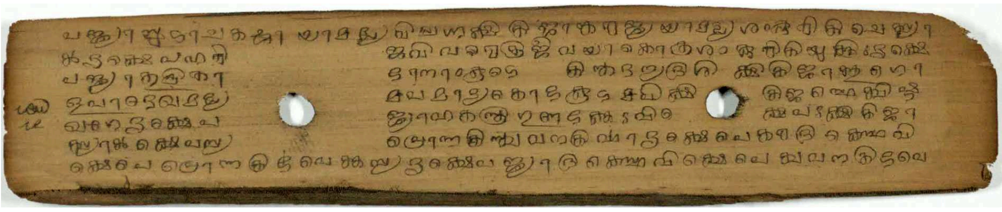

Figure 0.9: Manuscript Indian Office Sanskrit 3530 ($I_1$), folio 33 recto

$I_1$ includes many quotations from other astronomical texts. The full list is as follows (in order of verse number in *GD2* and quotations following that verse or half-verse):

- **1** *SŚe* 15.1-6, *BSS* 21.1
- **4ab** *Ābh* 4.1
- **6** *Ābh* 4.2
- **8**[60] *SŚe* 15.52
- **13** *Ābh* 4.18-19
- **21** *Ābh* 3.15, 13 and 14
- **23** *SŚe* 10.1-13
- **25** *Ābh* 4.7, 6, and 8, *BSS* 21.2, *PS* 13.1, *BSS* 21.2cd, *SŚe* 15.7-19
- **26** *SŚe* 15.20-23
- **30** *Ābh* 4.11
- **36** *SŚe* 15.24-26

[60] 8abc followed by 8b, probably due to dittography. 8cd follows the quotation.

- **37**[61] *SŚe* 15.27-72, 2.69-70
- **301** *BṛS* 5.1-15, *SŚe* 17.15, *SŚi.G* 11.10

In addition, the manuscript gives commentaries on the examples (section 0.2.7).

**$\mathrm{I}_2$** Indian Office Burnell 107b (Catalog No. 6298 in Eggeling (1887)) of the British Library. 13 folios numbered 1 to 13 (in Grantha Malayalam numerals that have not yet been inked), 37 × 4 cm. 9-10 lines per page and about 70 letters per line. Acquired by Arthur Coke Burnell in the 1860s, but it is uncertain whether the manuscript was newly copied for him. The initial writings are blackened but numerous corrections have been inscribed later without blackening. Perfectly preserved.

The bundle Burnell 107b includes: (A) *Sūryasiddhānta* with commentary of Parameśvara, (B) *Goladīpikā 2* of Parameśvara, (C) *Āryabhaṭīya* of Āryabhaṭa with commentary of Parameśvara, (D) *Āryabhaṭīya*.

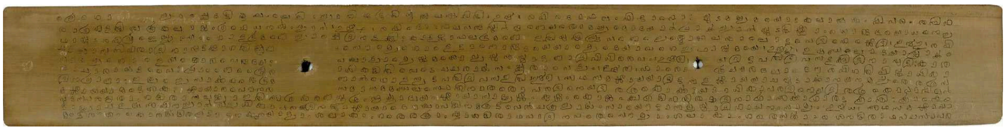

Figure 0.10: Manuscript Indian Office Burnell 107b ($\mathrm{I}_2$), folio 9 recto

**$\mathrm{I}_3$** Indian Office Burnell 17c (Catalog No. 6299 in Eggeling (ibid.)) of the British Library. 23 folios numbered 1 to 23 (in Grantha Malayalam numerals), 22 × 4 cm. 8-9 lines per page and about 50 letters per line. The entire volume was "written for Burnell (Eggeling (ibid., p. 774))", in the 1870s. Well preserved without fragmentation.

The bundle Burnell 17c includes: (A) *Sūryasiddhānta* (B) *Sūryasiddhānta*, (C) *Goladīpikā 2* of Parameśvara, (D) *Goladīpikā 1* of Parameśvara, (E) *Golasāra* of Nīlakaṇṭha Somayājin, (F) *Siddhāntadarpaṇa* of Nīlakaṇṭha Somayājin. This is almost identical with $\mathrm{K}_3$; the difference is that $\mathrm{K}_3$ has the *Āryabhaṭīya* at the beginning and does not contain *Goladīpikā 1*. This suggests that the two bundles are closely related, but it is unlikely that one is the direct descendant of the other.

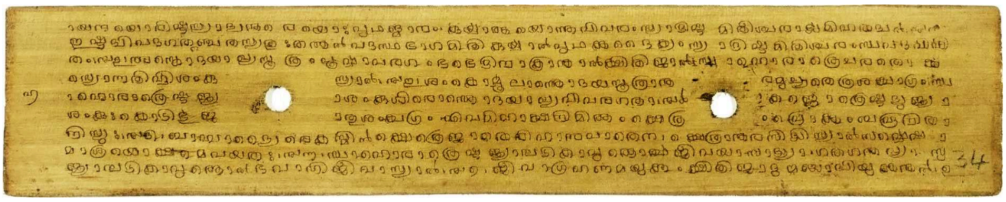

Figure 0.11: Manuscript Indian Office Burnell 17c ($\mathrm{I}_3$), folio 8 recto

[61] *GD2* 37 is repeated again after the quotations. The second occurrence, labeled $\mathrm{I}_1^+$ in the critical edition, reads differently from the first one.

**Sāstrī** Sāstrī's critical edition (Sāstrī (1916)). Sāstrī mentions that he used three manuscripts "obtained from the Raja of Idappalli", but gives no more information on their background. He labels them *ka*, *kha* and *ga*. We identify *ka* and *kha* as $K_4$ and $K_7$ that come from the Eḍappaḷḷy palace Library. However, Sāstrī seems to have made a confusion between the two. In his critical apparatus, *ka* follows the variants of $K_4$ and *kha* that of $K_7$ until verse 126, and later on they are exchanged. There are also many variants that are not given. Sāstrī remarks that *ka* "contains fewer mistakes than the other two manuscripts", and judging from his reading, here he is referring to $K_7$. Therefore many of the corruptions in $K_7$, including those unique to this manuscript[62], are left in his edition. The remaining *ga* cannot be identified with any other extant manuscript (see $K_9$ below). Its variants suggest that it is a descendant of Q*, in the same group with *ka* ($K_7$).

**($K_9$)** MS. No. L.1313 A (Catalog No. 5063 in Pillai (1957a)) of ORI& MSS. A loaned manuscript that included both versions of the *Goladīpikā*, but its location could not be traced when we requested for its information at ORI& MSS in August 2013. It could be one of the manuscripts used by Sāstrī which he labeled *ga*.

**Manuscripts of *GD1*** There are nine known manuscripts of *GD1*, which we shall list below so as not to be confused with those of *GD2*.

- MS. No. 762 E (Catalog No. 5062 in Pillai (ibid.)) of ORI & MSS: Manuscript "B" in the edition of K. V. Sarma (1956–1957). 762 F is Parameśvara's auto-commentary.
- MS. No. 5864 A (Catalog No. 5055 in Pillai (1957a)) of ORI & MSS: Manuscript "C" in Sarma's edition.
- MS. No. 8358 B (Catalog No. 5056 in Pillai (ibid.)) of ORI & MSS: Manuscript "D" in Sarma's edition.
- MS. No. 13719 (Catalog No. 174 of Jyotiṣa section in Śiromaṇi (1999)) of the Maharaja Sayajirao University of Baroda Oriental Institute: Pingree (1981, p. 191) counts this as a manuscript of *GD2*.
- Indian Office Burnell 17d (Catalog No. 6300 in Eggeling (1887)) of the British Library
- MS. No. L.1313 B (Catalog No. 5057 in Pillai (1957a)) of ORI & MSS: Manuscript "E" in Sarma's edition. Lost.
- MS. No. T.341 of ORI & MSS: Manuscript "F" in Sarma's edition. Lost.
- MS. No. R.5192 of the Government Oriental Manuscripts Library, Madras: Manuscript "A" in Sarma's edition. We have not confirmed this manuscript.
- Manuscript "G" in Sarma's edition, "a transcript by Sri G. Harihara Sastri, Madras". We have not confirmed this manuscript[63].

[62] Variants in 10.d, 12.d, 43.d, 53.c, 184.b, 194.b, 218.b, 236.d, 238.d, 292.d occur only in $K_7$ but are adopted in Śāstri's edition.

[63] This is probably a copy of 13719 Baroda which was sold to the institute by the same "G. Harihara Sastri" and contains the same variant readings.

### 0.3.2 Stemma and genealogy of manuscripts

Figure 0.12 is a stemma showing the relationship between the manuscripts extant judged from their variants, with their archetype (the hypothetical lowest common ancestor of every known manuscript) and hyparchetypes (the hypothetical common ancestor for a subgroup)[64]. There are conspicuous sets of variants that enable us to identify their genealogy relatively easily. On the other hand, there are traces of contamination involved. Therefore we have chosen to construct the stemma manually (without using computer programs).

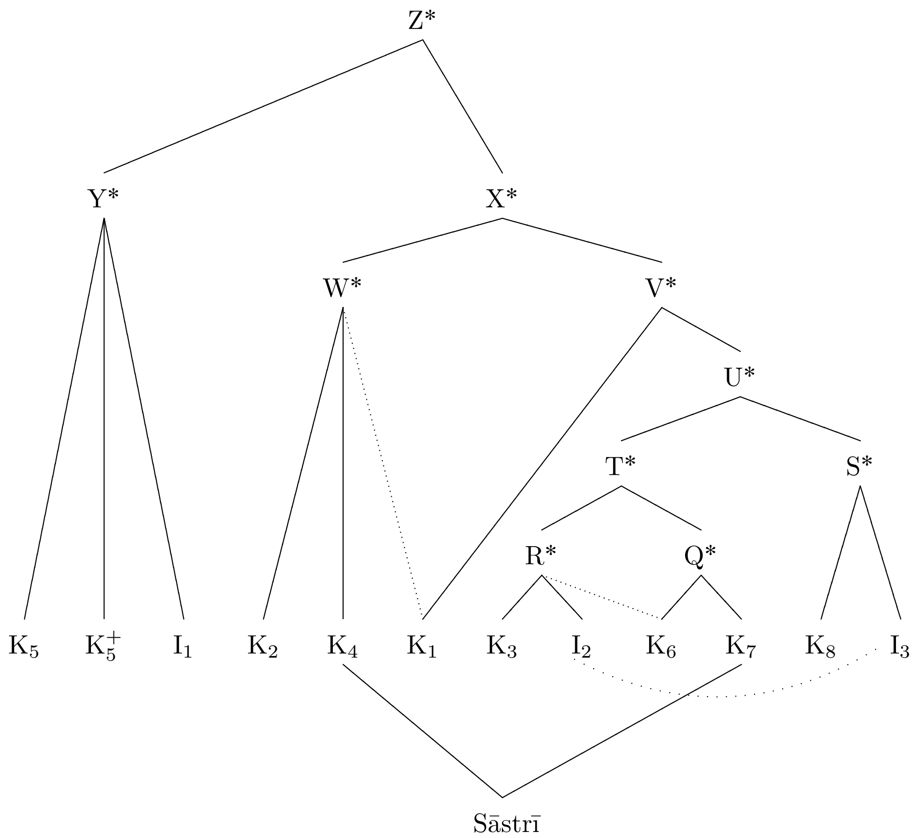


Figure 0.12: Stemma for manuscripts of *GD2*

**Z* (Archetype)** The stemma is constructed with the assumption that there was only one manuscript prepared by Parameśvara which became the common ancestor. Our archetype is probably very close to the autograph itself, as there is no significant corruption common to every manuscript. There are only three amendments that we have made which appear in none of our manuscripts. *yatime* and *tatime* which we have corrected to *yatame* and *tatame* in *GD2* 263 could be conventional notations. *tadviguṇa* in *GD2* 286 is uninterpretable; Sāstrī reads *taddviguṇa* which we have also adopted, but even this reading is problematic. The autograph could have been very different here.

[64]Here we follow the terminology in West (1973).

**Y*** We have identified this hyparchetype with three manuscripts ($K_5$, $K_5^+$ and $I_1$) as the closest to our archetype, and evaluated the readings of its manuscripts higher than others. Y* does have some corruptions of its own; the variant *māsa* in place of *rāśi* (*GD2* 41) and the repetition of *GD2* 249-250ab are decisive upon identifying this subgroup. The most significant feature in this subgroup is that it contains a commentary. We do not know when and by whom the commentary was added.

**X*** This is a hyparchetype of W* and V* combined. There are 11 common variants among them while there is only one betwen Y* and W* and three between Y* and V*. The omission of *saṃjñita* in *GD2* 273 is most distinctive, but otherwise the variants do not stand out as clearly as the variants in the two subgroups W* and V*, which suggests that they were divided at an early period.

**W*** $K_2$ and $K_4$ belong to this hyparchetype. There are 43 common variants. Some of them change the meaning of the verses significantly, such as *sphuṭa* instead of *śruti* in *GD2* 128 and *karṇa* instead of *śaṅku* in *GD2* 288. $K_1$ follows unique variants of W* in 7 places. We have put $K_1$ under the hyparchetype V* due to its common variants, but it is likely that there is some contamination from W* in $K_1$.

**V*** This hyparchetype combines $K_1$ with hyparchetype U*. There are 23 common variants in V* while there are 42 within U*. $K_1$ is an oddity in this group which may be explained to some extent by contaminations from W*.

**U*** Many conspicuous corruptions characterize this hyparchetype. The omissions of *GD2* 93cd, *GD2* 216d-217a and *GD2* 264b are especially prominent. There are numerous corruptions below this hyparchetype that not only distort the meaning but even break the meter. We deem manuscripts under this hyparchetype relatively unreliable.

**T*** Most of the variants under this hyparchetype are simple elisions or mis-transcriptions and are hardly useful. They are occasionally corrected by second hand in its descendants.

**S*** Not only are the variants in this hyparchetype numerous but also unique. For example, *dyumaṇḍala* is often written *dyunmaṇḍala*. The variants often show some traces of efforts to make the phrases meaningful rather than being simple mis-transcription. Such is the case for *khaga* instead of *kalā* (possibly affected by *kheṭa* nearby) in *GD2* 156. This is probably why S* sometimes show correct readings where its supposed hyparchetype is wrong; for instance, *copaikyaṃ* in X* is back to *cāpaikyaṃ* in *GD2* 164 and V* has *smate* in *GD2* 235 where S* correctly gives *smṛte*. Among the two manuscripts, $K_8$ has very few variants of its own while $I_3$ has been corrected frequently by a second hand. We assume that $I_3$ has been copied from $K_8$ itself or another manuscript not far from it.

**R*** We estimate that this hyparchetype is not very far from T* as there are only 14 unique variants. Furthermore, 8 of them are common with $K_6$. As $K_6$ and $K_7$ have 60 variants in common, there is no doubt that these two had a common ancestor, but there could have been some contamination. The two manuscripts in this group, $K_3$ and $I_2$, have many variants of their own and it is not very likely that one was a copy of the other. $I_2$ has been frequently corrected by a later hand. It has 19 common variants with S* which suggests that the scribe referred either $I_3$ ($I_2$ and $I_3$ were both copied for Burnell) or its direct parent.

**Q*** This hyparchetype is discernible because $K_6$ and $K_7$ have 60 variants in common.

# Part I

# Critical edition

# Notes on the edition

## Variants to be ignored

The text in this edition is presented in Roman alphabets with diacritic marks in the International Alphabet of Sanskrit Transliteration scheme (Monier-Williams, 1899, p. xxx). Sandhis are separated whenever the borders between words are distinct (table 0.4. Words in compounds are kept separated, and whenever a word or compound extends over two *pāda*s, its continuity is marked with a hyphen at the end of the line.

Table 0.4: Examples of sandhis and how they are presented in this edition

| Before Sandhi | After Sandhi | In our text |
|---|---|---|
| *bhavet + hi* | *bhaveddhi* | *bhaved dhi* |
| *ādau + ante* | *ādāvante* | *ādāv ante* |
| *bāhus+ced+śaṅku-* | *bāhuścecchaṅku-* | *bāhuś cec chaṅku-* |
| *tasya + api* | *tasyāpi* | *tasyāpi* (cannot be separated) |
| *ca + eva* | *caiva* | *caiva* (cannot be separated) |

We have systematically ignored some of the variants which merely comes from scribal conventions or typical errors and do not affect our decision. The peculiarities that we have located in the manuscripts and listed below had already been included in a more detailed and exhaustive list by Esposito (2012). The corpus of her list are South Indian drama manuscripts, but the variants are not necessarily associated with the genre of texts and can be applied to our examples too.

- Doubling of consonants after *r* and before *y*.
- Nasals instead of *anusvāra* or vice versa.
- *anusvāra* instead of *m* at end of (half-)verse.
- The assimilation of a *visarga* before a sibilant. When the double sibilant is followed by another consonant, one of the two sibilants can be dropped. We assume that this has happened in *GD2* 165, where we exceptionally noted the reading of the manuscripts as-is.
- Intervocalic ള (*ḷ*) instead of ല (*l*).
- *dṛkṣepa* instead of *dṛkkṣepa*. This can be explained by a more general phenomenon where a consonant can be dropped if it is geminated and further followed by another consonant. The case with *dṛkkṣepa* is very frequent and yet so obviously an error that we have decided to systematically ignore it.
- Voiceless consonant word-endings left as they are when they should become voiced as a result of Sandhi with the following voiced consonant. For example, every manuscript reads *bhramaṇā**t** goḷasya* instead of *bhramaṇā**d** golasya* in *GD2* 208. This happens because we only identify prepausal consonant characters for voiceless consonants (Grünendahl (2001, p. 92)). It is arguable whether they were actually pronounced voicelessly.
- Non-existence of *avagraha*s. Apart from Sastri's edition, the manuscripts never write *avagraha*s. Therefore variants in this edition will be given without *avagraha*s.

- Texts missing due to breakage in the manuscript, unless the missing part is longer than a *pada* or is in a difficult place (especially when we adopt readings from few or no manuscripts).

In addition, we shall ignore some scribal errors as long as they appear only in one manuscript and do not affect the decision. We ourselves too have difficulty in distinguishing some letters from one another; such cases are left unnoted as long as the reading can be easily decided from the context. The following is a list of similar sets of letters and ligatures which can be a source of errors.

- പ (*p*) and വ (*v*), and in some manuscripts ച (*c*), ല (*l*) and ഖ (*kh*).
- The right side of ൌ (vowel -*au*) and ന (*n*).
- ഹി (hi) and എ (e).
- Dropping one letter in ൈ (vowel -*ai*) makes it read -*e*.
- ഗ (*g*) and ശ (*ś*).
- ദ (*d*) and ഭ (*bh*).
- In some manuscripts, ാ (-*ā*) and ഃ (*ḥ*) .

### Notations in the apparatus

When every manuscript in the same group has the same variant reading, the siglum for their common archetype will be given in the apparatus, instead of individual manuscript. However, if there are diversities within a group that can be explained as a result of modification from the same variant reading of their archetype, the variant and siglum of the archetype will be followed by those of individual manuscripts or sub-archetypes.

**br.** The manuscript is broken in the corresponding part or an entire folio containing the text is missing.

- *kṣatra…ca* ] br. $K_1$: "The passage *kṣatra…ca* is broken in $K_1$"

*+* A space of one *akṣara* (letter) is broken. This will be indicated in the order it appears in the manuscript, but due to the nature of Malayalam scripts, the missing element change its position in an alphabetical transcription. പ+ത (*pa+ta*) could be any among പതിത (*patita*), പാത (*pāta*) or പതേ (*pate*).

- *koṭir api ca tajjīvā* ] *+++pi ca tajjīv+* $K_5$: "For the lemma കോടിരപി ച തജ്ജീവാ, $K_5$ is broken and only has പി ച തജ്ജീവ"

**om.** The lemma is omitted in the manuscript. If no lemma is indicated, it means that the whole *pāda* is omitted.

- *madhyagata* ] om. $I_3$: "The passage *madhyagata* is omitted in $I_3$"
- **28**.d om. $K_4$: "The *pāda* 28.d is omitted in $K_4$"

**lacuna** The lemma itself is omitted, but some space roughly corresponding to the number of omitted letters is left in the manuscript.

- *gacchanty…evaṃ* ] lacuna $K_6$: "The passage *gacchanty…evaṃ* is omitted with space left in $K_6$"

**corr.** The reading has been corrected to the text in the critical edition.

- *saṃdhyā* ] *bandhyā* corr. $K_8$: "The passage *saṃdhyā* was initially *bandhyā* in $K_8$, but was corrected to *saṃdhyā*"
- *śuklāṣṭamyardhā* ] *śuklāṣṭamyārdhā* T* (corr. $K_7$): "The passage *śuklāṣṭamyardhā* reads *śuklāṣṭamyārdhā* in descendants of archetype T*, but $K_7$ was corrected from the initial reading *śuklāṣṭamyārdhā* to *śuklāṣṭamyardhā*"

**corr.**$_{\text{sec.m.}}$ The correction is apparently by a second hand (*secunda manu*)[65].

- *bhavati* ] *bhavanti* T* (corr.$_{\text{sec.m.}}$ $I_2$): "The passage *bhavati* reads *bhavanti* in descendants of archetype T*, but $I_2$ was corrected by a later hand from the initial reading *bhavanti* to *bhavati*"

**corr.**$_{\text{sec.m.}}$ **to** The reading has been corrected to the following text.

- *kiyatī* ] *kayati* T* (corr.$_{\text{sec.m.}}$ to *kiyati* $I_2$): "The passage *kiyatī* reads *kayati* in descendants of archetype T*, but $I_2$ was corrected by a later hand from the initial reading *kayati* to *kiyati*"

**del.** The lemma or reading has been deleted (crossed out) without replacement.

- *gacchanty…evaṃ* ] *saṃyoga 21 maṇḍalam arkādīnā* del.$_{\text{sec.m.}}$ $K_7$: "In place of *gacchanty…evaṃ*, $K_7$ had the reading *saṃyoga 21 maṇḍalam arkādīnā* which was crossed out by a later hand without replacement"

***X/Y*** The manuscript can be read as either X or Y and cannot be decided from syntax.

- *aikyapadaṃ* ] *aikyat padaṃ/aikyalpadaṃ* $K_4$: "In place of *aikyapadaṃ*, $K_4$ has a reading which could be either *aikyat padaṃ* or *aikyalpadaṃ* (The Malayalam letter ൽപ could be either *tp* or *lp*)"

### Abbreviation of sources

Titles of other texts are abbreviated as follows in the apparatus.

***Ābh*** The *Āryabhaṭīya* of Āryabhaṭa (Kern (1874))

***BṛS*** The *Bṛhatsaṃhitā* of Varāhamihira (Tripāṭhī (1968))

***BSS*** The *Brāhmasphuṭasiddhānta* of Brahmagupta (Dvivedī (1902), Ikeyama (2002))

***GD1*** The *Goladīpikā I* of Parameśvara (K. V. Sarma (1956–1957))

***GMBh*** *Mahābhāskarīyabhāṣya* of Govindasvāmin, his commentary on the *Mahābhāskarīya* of Bhāskara I (T. Kuppanna Sastri (1957))

***PS*** The *Pañcasiddhāntikā* of Varāhamihira (T. S. Kuppanna Sastri (1993))

***SŚe*** The *Siddhāntaśekhara* of Śrīpati (Miśra (1932) and Miśra (1947))

[65] Scripts are scratched on palm leaves and black powder with oil is applied afterwards for reading (Kumar, Sreekumar, and Athvankar (2009)). Newly made corrections have none or less powder rubbed in the scratches and are easily recognizable.

### Line numbers

Each verse is separated into four lines corresponding to the four *pāda*s in the meter. The lines are marked from a to d. The exceptions are *GD2* 244 which has an extra half-verse and *GD2* 247 which is only half a verse. Numbers are allocated to lines in the prose parts (both for the base text and commentary). None of these lines reflect the actual appearance in the manuscripts.

### Commentary

Commentaries that are written in $K_5^+$ and $I_1$ are inserted after the relevant verses, as they appear in these manuscripts. Not every prose in this edition is part of a commentary; some preambles (such as those before *GD2* 209 and 210) are included in every manuscript, and is therefore considered part of the original work. Horizontal lines are inserted before and after the commentary to distinguish it from the base text.

# *Goladīpikā*

a *vighneśaṃ vāgdevīṃ*
b *gurūn dineśādikān grahān natvā /*
c *vakṣye bhagolam asmai*
d *kṣoṇīmānādikaṃ ca laghumataye // **1** //*

a *adha-ūrdhvayāmyasaumyagam*
b *iha vṛttaṃ dakṣiṇottarākhyaṃ syāt /*
c *adha-ūrdhvābhyāṃ ghāṭikam*
d *akṣāgre saumyayāmyayor lagnam // **2** //*

a *tasyāpy adha-ūrdhvābhyāṃ*
b *tadvat paramāpame 'pamaṃ lagnam /*
c *ghāṭikamadhye tiryag*
d *raśanāvartasya vṛttam aparaṃ syāt // **3** //*

a *etad viṣuvatsaṃjñaṃ*
b *ghāṭikam api dakṣiṇottaraṃ ca tathā /*
c *apamaṇḍalākhyavṛtte*
d *pūrvābhimukho raviḥ sadā carati // **4** //*

a *ghāṭikamadhyagaviṣuvad-*
b *yāmyottaravṛttayor mithoyogāt /*
c *svastikayugmaṃ yat syāt*
d *tatproto golamadhyagatadaṇḍaḥ // **5** //*

a *samavṛttām api bhūmiṃ*
b *bhagoladaṇḍasya madhyagāṃ kuryāt /*
c *kāṣṭhena vā mṛdā vā*
d *prāṇinivāsādi kalpayet tasyām // **6** //*

a *pravahamarutprakṣipto*
b *bhagola urvīṃ pradakṣiṇīkṛtya /*
c *aparābhimukhaṃ ṣaṣṭyā*
d *ghaṭikābhir bhramati bhūyo 'pi // **7** //*

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**1**.a–**13**.d br. K$_3$ **1**.a–**22**.b br. K$_5$ **1**.b *gurūn*] *guruṃ* S*K$_1$K$_6$ **2**.a–b om. I$_3$ **2**.d *saumyayāmyayor*] *yāmyasaumyayor* K$_4$K$_7$ Sāstrī (corr.$_{\text{sec.m.}}$ K$_4$), *saumyayor* I$_1$ **3**.b *paramāpame 'pamaṃ lagnam*] *param apamaṇḍalaṃ* K$_4$ **3**.d *vartasya*] *vṛttasya* Q* Sāstrī **5**.d *madhyagata*] om. I$_3$ **6**.c *kāṣṭhena vā*] *kāṣṭhena* I$_1$ **6**.c *mṛdā vā*] om. S*I$_2$ **7**.a *prakṣipto*] *prakṣiptaṃ* W* **7**.b *bhagola*] *bhagolam* W*

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**1**. K$_1$ begins with *++++++taye namaḥ avi(gh)na+(s)tu*, K$_2$, K$_4$ and K$_7$ with *hariḥ śrī gaṇapataye namaḥ avighnamastu*, K$_6$, I$_1$ and I$_2$ with *śrī gaṇapataye namaḥ avighnamastu* and K$_8$ and I$_3$ with *hariḥ*. K$_3$ and K$_5$ are broken at the beginning.

**1**. I$_1$ adds *śrīpatiḥ* followed by *SŚe* 15.1-6 and *brahmaguptaḥ* followed by *BSS* 21.1

**4**. I$_1$ adds (between b and c) *āryabhaṭa* followed by *Ābh* 4.1

**6**. I$_1$ adds *āryaḥ* followed by *Ābh* 4.2

**7**. = *GD1* 2.2. *Āryā* metre.

a *bhūpṛṣṭhād upari marud*
b *raviyojanasaṃmitāntare pravahaḥ* /
c *niyatagatir aparagaḥ syād*
d *bhūvāyur adhaś ca tasya bhinnagatiḥ* // **8** //

a *ghāṭikaṣaṣṭyaṃśasya*
b *bhramaṇe kālo 'tra nāḍikety uditā* /
c *na tu divasaṣaṣṭibhāgo*
d *golabhramaṇād yato 'dhiko divasaḥ* // **9** //

a *ghaṭikāmaṇḍalapārśve*
b *ghāṭikavṛttānusāri yad vṛttam* /
c *sūryasya bhramaṇastham*
d *svāhorātrākhyavṛttam uditaṃ tat* // **10** //

a *tāni bahūni bhavanti ca*
b *divase divase yato 'rkagatibhedaḥ* /
c *nakṣatragola eṣa hi*
d *bāhye 'sya ca niścalaḥ khagolaḥ syāt* // **11** //

a *pūrvāparādha-ūrdhvagam*
b *uditaṃ samamaṇḍalaṃ khagolastham* /
c *yāmyottarādha-ūrdhvagam*
d *asminn api dakṣiṇottarākhyaṃ syāt* // **12** //

a *pūrvāparayāmyodag-*
b *gatam iha bhūpārśvasaṃsthitaṃ kṣitijam* /
c *tasminn udayāstamayau*
d *sarveṣāṃ bhagrahāṇāṃ staḥ* // **13** //

a *yāmye 'dhaścordhvam udak*
b *kṣitijād akṣāṃśakāntare lagnam* /
c *prāgaparayoś ca lagnaṃ*
d *vidyād unmaṇḍalaṃ khagolastham* // **14** //

a *unmaṇḍalayāmyodak-*
b *svastikayātaś ca goladaṇḍo 'yam* /
c *unmaṇḍalordhvabhāge*
d *bhramaṇaṃ golasya khāgnināḍībhiḥ* // **15** //

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**9**.a *aṃśasya*] *aṃśatasya* $\mathrm{I}_3$ **9**.c *bhāgo*] *bhā* $\mathrm{I}_3$ **10**.a *ghaṭikā*] *ghāṭika* W* Sāstrī **10**.d *svāhorātrākhyavṛttam*] *svāhorātrākhyam* S*$\mathrm{I}_2$, *svāhorātrārdhavṛttam* $\mathrm{K}_7$ Sāstrī **11**.b *divase divase*] *divase* $\mathrm{K}_1$ **11**.b *bhedaḥ*] *bhedāt* $\mathrm{K}_7$ **11**.c–d *kṣatra…ca*] br. $\mathrm{K}_1$ **11**.c *gola eṣa hi*] *golam etad* W* **11**.d *niścalaḥ khagolaḥ*] *niścalaṃ khagolaṃ* W* **12**.d *asminn api*] *api tasmin* $\mathrm{K}_7$ Sāstrī **12**.d *kṣiṇo…syāt*] *br.* $\mathrm{K}_1$ **13**.b *iha*] *iva* S* **13**.d *bhagrahāṇāṃ*] *hi grahāṇāṃ* Q* Sāstrī **14**.a *yāmye*] *yāmyo* U*, *yāmyā* $\mathrm{K}_7$ **14**.b *āṃśakā*] *āntaśakā* S*$\mathrm{I}_2$ **14**.c *aparayoś ca*] *aparayos tu* W*

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**8**. = *GD1* 2.3. $\mathrm{I}_1$ repeats 8b after 8c. Then it adds *śrīpatiḥ* followed by *SŚe* 15.52, after which 8c is written again, this time followed correctly by 8d.

**9**. = *GD1* 2.4.

**10**. Corresponds to *GD1* 1.6cd-7ab.

**13**. $\mathrm{I}_1$ adds *āryaḥ* followed by *Ābh* 4.18-19

a *unmaṇḍalād adhaḥstham*
b *saumye yāmye tadūrdhvagaṃ kṣitijam* /
c *tasmāt saumyagate 'rke*
d *dinam adhikaṃ yāmyage niśā hy adhikā* // **16** //

a *kṛtvā vā prāgaparaṃ*
b *ghāṭikam anyac ca tadvaśāt kṛtvā* /
c *unmaṇḍalayāmyodak-*
d *svastikaniṣprotadaṇḍakaṃ kuryāt* // **17** //

a *acalāni bhāni teṣām*
b *adhaḥ kramān mandajīvakujadinapāḥ* /
c *bhṛgubudhaśaśinaś caite*
d *prāggatayo golavegato 'paragāḥ* // **18** //

a *yojanasaṃkhyā tulyā*
b *teṣāṃ divase gatau kalā bhinnāḥ* /
c *kakṣyā mahaty uparigā*
d *yasmāl liptāḥ samāś ca sarvāsu* // **19** //

a *mandagatir indur ārkiḥ*
b *śīghragatis tārakās tu śīghratarāḥ* /
c *gacchanty aparābhimukhaṃ*
d *sarve 'py evaṃ vadanti kila kecit* // **20** //

a *etan na yuktam iti hi*
b *bruvanti gole kṛtaśramā gaṇakāḥ* /
c *vakragavihagasya yataḥ*
d *svapaścimāśāgatarkṣasaṃyogaḥ* // **21** //

a *maṇḍalam arkādīnāṃ*
b *golākāraṃ smṛtaṃ gaṇakavaryaiḥ* /
c *taijasam arkasya tu tac*
d *candrasyāpyaṃ svataḥ prakāśonam* // **22** //

a *darpaṇavṛttākāraṃ*
b *maṇḍalam icchanti ye tu te mugdhāḥ* /
c *śauklyasya kramavṛddhir*
d *ghaṭate yasmād vidhor na tatpakṣe* // **23** //

a *salilamaye śaśini raver*
b *dīdhitayo mūrchitās tamo naiśam* /
c *kṣapayanti darpaṇagatā*
d *mandiragam iveti cāryajanavākyam* // **24** //

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**16**.b *ūrdhvagaṃ*] *ūrdhvajaṃ* W* **16**.d *yāmyage*] *yāmyagate* Q*$K_1$ **16**.d *niśā hy*] *niśāpy* $K_2K_7$ Sāstrī **17**.a *kṛtvā vā*] *kṛtvā* $K_4$ **17**.d *kuryāt*] *kāryāt* $I_2$ **18**.b *dinapāḥ*] *dinavāraḥ* $I_1$ **18**.c *caite*] *caiva te* $K_7$ **19**.a–b *tulyā teṣāṃ divase*] *teṣāṃ divase tulyā* W*$I_1$ **20**.b *tārakās*] *śīghras tārakās* $K_6$ **20**.c–d *gacchanty…evaṃ*] lacuna $K_6$, *saṃyoga 21 maṇḍalam arkādīnā* del.$_{\text{sec.m.}}$ $K_7$ **21**.a *etan*] *evan* W* **22**.d *candra*] *cāndra* $K_7$ **23**.b *mugdhāḥ*] *tammuddhāḥ* $I_3$ **23**.c *śauklyasya*] *śauklasya* $K_1K_2K_6I_3$ **23**.c *vṛddhir*] *vṛddhīḥ* $K_5$ **23**.d *vidhor*] *vidhau* $K_6$ *vidher* $K_7$ **24**.d *mandiragam*] *mandiram* Q* **24**.d *iveti*] *iheti* $K_7$

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**21**. $I_1$ adds *āryabhaṭaḥ* followed by *Ābh* 3.15, 13 and 14

**23**. ab = *PS* 13.36ab. cd is very close to *PS* 13.36cd. $I_1$ adds *śrīpatiḥ* followed by *SŚe* 10.1-4

a *golākārā pṛthivī*
b *khe tiṣṭhati sarvadā svaśaktyaiva* /
c *sthalabahulam ūrdhvagārdhaṃ*
d *jalabahulam adho 'bdhayo 'tra ca dvīpāḥ* // **25** //

a *bhūmir anantena dhṛtety*
b *eke 'nye diggajair iti bruvate* /
c *ādhārasya ca kalpyo*
d *'trādhāro 'to 'navasthitis teṣām* // **26** //

a *pūrvābhimukhaṃ bhramati*
b *kṣoṇī nāsti bhramaḥ khagarkṣāṇām* /
c *iti kila vadanti kecin*
d *nābhimataṃ tad api cāryabhaṭabudhasya* // **27** //

a *adha-uparipārśvabhāgeṣv*
b *asyā niyataṃ vasanti vasudhāyāḥ* /
c *ditisutadevanarādyāḥ*
d *prāṇiviśeṣās tathā saridagādyāḥ* // **28** //

a *bhūmadhyagataṃ cakraṃ*
b *sarveṣāṃ prāṇinām adhaḥsthānam* /
c *bhūpṛṣṭhe sarvatra*
d *prāṇijalādeḥ sthitis tato ghaṭate* // **29** //

a *yojanasaṃkhyā gaditā*
b *bhūvṛttasyāṅkarandhrayamalaguṇāḥ* /
c *āryabhaṭena tathoktaṃ*
d *yojanamātro bhavec ca merur iti* // **30** //

a *bhūmer yojanamānaṃ*
b *bahukoṭimitaṃ vadanti sudhiyo 'nye* /
c *naitad gaṇakābhimataṃ*
d *yato 'nyathā mānasiddhir akṣavaśāt* // **31** //

a *samayāmyodagdeśa-*
b *dvayapalabhāgāntaroddhṛtā tu tayoḥ* /
c *vivaragabhūmiś cakrā-*
d *ṃśatāḍitā syād bhuvaḥ paridhimānam* // **32** //

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**25**.a *golākārā*] *golakārā* $K_4$ **25**.b *tiṣṭhati*] *tiṣṭati* S* **25**.d *ca*] om. W*$K_1$ **26**.d *'trādhāro 'to 'navasthitis*] *trātosthitis* $K_4$ **28**.b *vasanti*] *vadanti* Q* **28**.d om. $K_4$ **29**.a *bhū*] *trū* $I_1$ **29**.b *adhaḥsthāna*] *adhavasthāna* S* **31**.c *gaṇakābhimataṃ*] *gaṇikābhimataṃ* $I_1$, *gaṇakābhimātaṃ* $I_3$ **31**.d *yato*] *yatho* S* **32**.b *tu tayoḥ*] *kṛtayoḥ* U*, *uta bhayoḥ* $K_5$ **32**.d *tāḍitā syād*] *tāḍitāsya* $K_5$

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**25**. $I_1$ adds *ārya* followed by *Ābh* 4.7, 6 and 8, *brahmagu* followed by *BSS* 21.2, *varāhamihiraḥ* followed by *PS* 13.1 and *BSS* 21.2cd and *śrīpatiḥ* followed by *SŚe* 15.7-19

**26**. $I_1$ adds *śrīpatiḥ* followed by *SŚe* 15.20-23

**27**. Sāstrī adds : '*bhaṭṭasya*' *iti vṛttānuguṇaṃ* '*bhaṭakasya*' *iti vā*. However this *pāda* has 18 syllables and needs no metrical correction.

**30**. $I_1$ adds *āryabhaṭa* followed by *Ābh* 4.11

a *yojanamitaphalasaṃkhyā*
b *bhūpṛṣṭhe ced anekalakṣamitā* /
c *bhūgolāntaryojana-*
d *phalasaṃkhyā ced anekakoṭimitā* // **33** //

a *prāṇinivāso hy antaḥ*
b *pātāleṣv api ca bhavati medinyāḥ* /
c *vākyāvirodha evaṃ*
d *vicintya sudhiyāṃ sudhībhir iha neyaḥ* // **34** //

a *atyunnatiś ca meror*
b *na cintyate golavidbhir iha gaṇakaiḥ* /
c *yasmād dhruvasya saumye*
d *prāggāminyo bhavanti khe tārāḥ* // **35** //

a *kecid vadanti bhūmer*
b *ūrdhvaṃ cādhaḥ praviṣṭa iti meruḥ* /
c *āryabhaṭenātroktaṃ*
d *bhūgolāt tasya mānam ūrdhvagatam* // **36** //

a *laṅkāyām upari gato*
b *golānte 'rko dhruvaḥ sadā kṣitije* /
c *merau so 'rkaḥ kṣitije*
d *dhruva upari yato 'nayoḥ svabhūmir adhaḥ* // **37** //

a *sthalajalamadhyāl laṅkā*
b *bhūkakṣyāyā bhavec caturbhāge* /
c *ujjayinī laṅkāyāḥ*
d *pañcadaśāṃśe samottarataḥ* // **38** //

a *svarmerū sthalamadhye*
b *narako baḍavāmukhaś ca jalamadhye* /
c *eṣā sārdhā tv āryā*
d *bhaṭena gaditātra likhyate 'smābhiḥ* // **39** //

a *sthalamadhyagamerusthā*
b *devās tadadhojalasthagā danujāḥ* /
c *śaśimaṇḍalamadhyasthāḥ*
d *pitaro manujāḥ kugolapārśvagatāḥ* // **40** //

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**33**.a *phala*] *pala* $I_1$ Sāstrī **33**.b *lakṣamitā*] *lakṣaṇamitāḥ* $I_1$ **33**.d *phala*] *pala* $K_2$ Sāstrī **33**.d *mitā*] *mitāḥ* $I_1$ **34**.a *hy antaḥ*] *hantaḥ* $I_3$ **34**.b *pātāleṣv api*] *pātāle pi* $I_3$ **34**.b *bhavati*] *bhavanti* T* (corr.$_{\text{sec.m.}}$ $I_2$) **34**.b *medinyāḥ*] *medhinyāḥ* $K_8$ **34**.d *sudhiyāṃ*] *sudhiyā* $I_1$ Sāstrī **34**.d *iha*] *īha* $K_8I_2$ **35**.b *cintyate*] *vidyate* Q* **35**.b *gaṇakaiḥ*] *nipuṇaiḥ* Q* **36**.b *praviṣṭa*] *praviṣṭam* V* **37**.b *'rko*] *rkād* W* **37**.c *'rkaḥ kṣitije*] *rkakṣitije* T*$I_1^+$ (corr.$_{\text{sec.m.}}$ $I_2$), *rkaṃ kṣitije* $K_5$ **37**.d *yato*] *gato* $I_1^+$ **37**.d *svabhūmir adhaḥ*] *saṃbhūmidha* $K_5$, *svabhūmir ataḥ* S*, *svabhūmidharaḥ* $I_1^+$ **38**.a *madhyāl*] *madhyā* Q*$K_4$ Sāstrī **38**.c *ujjayinī*] *ujjayanī* W*$K_1K_8I_1$ **39**.b *baḍavāmukha*] *vaḍabāmukha* Sāstrī **39**.c *sārdhā tv āryā*] *sardharthāryā* $I_1$ **39**.c–d *āryā bhaṭena*] *āryabhaṭena* $K_7$ **39**.d *bhaṭena*] *kaṭena* $I_1$ **40**.a *madhyaga*] *madhya* $K_4$

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**36**. $I_1$ adds *śrīpatiḥ* followed by *SŚe* 15.24-26

**37**. $I_1$ adds *śrīpatiḥ* followed by *SŚe* 15.27-72, 2.69-70, then repeats this verse. The two writings are slightly different, and only the second occurence (labled $I_1^+$) contains variant readings.

**38**. = *Ābh* 4.14. *Ārya* verse.

**39**. ab = *Ābh* 4.12ab

a *uttaragolagam arkaṃ*
b *paśyanty amarāḥ sadānyagaṃ ditijāḥ |*
c *meṣādirāśiṣaṭkaṃ*
d *dinam amarāṇāṃ niśā tad asurāṇām ||* **41** *||*

a *proktaṃ dinaṃ pitṝṇāṃ*
b *kṛṣṇāṣṭamyardhakālam ārabhya |*
c *śuklāṣṭamyardhāntaṃ*
d *paśyanti yataḥ sadaiva te dinapam ||* **42** *||*

a *laṅkādyanakṣadeśe*
b *triṃśadghaṭikā dinaṃ tathaiva niśā |*
c *akṣābhāvāt sthalajala-*
d *saṃdhau sthānāni cāha tatra bhaṭaḥ ||* **43** *||*

a *udayo yo laṅkāyāṃ*
b *so 'stamayaḥ savitur eva siddhapure |*
c *madhyāhno yavakoṭyāṃ*
d *romakaviṣaye 'rdharātram iti ||* **44** *||*

a *dinarātrikālayoge*
b *ṣaṣṭir ghaṭikāḥ syur akṣayutadeśe |*
c *tatrodaggole 'rke*
d *dinasya vṛddhir niśādhikā yāmye ||* **45** *||*

a *paramāpamena tulyā*
b *yasmin deśe 'valambakajyā syāt |*
c *tatra yamāntagate 'rke*
d *nāḍīṣaṣṭyā dinaṃ tad uktaṃ ca ||* **46** *||*

a *yatra toyanidhimekhalātale*
b *nāstam eti mithunāntasaṃsthitaḥ |*
c *taptahāṭakanibho divākaras*
d *tatra bho 'kṣaparimāṇam ucyatām ||* **47** *||*

a *iti tatra palajyā syāt*
b *paramāpamakoṭisaṃmitā tasmāt |*
c *pañcadaśa syuś caradala-*
d *ghaṭikāḥ ṣaṣṭir dine 'py ato ghaṭikāḥ ||* **48** *||*

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**41**.b *sadānyagaṃ*] *sādānyagaṃ* S* **41**.c *rāśi*] *māsa* Y* **41**.d *niśā*] *diśā* S*$I_2$ **42**.b *kṛṣṇāṣṭamyardha*] *kṛṣṇāṣṭamyārdha* T*$K_1K_5$ (corr. $K_7$, corr.$_{\text{sec.m.}}$ $I_2$) **42**.b *kāla*] om. $K_4$ **42**.c *śuklāṣṭamyardhā*] *śuklāṣṭamyārdhā* T* (corr. $K_7$, corr.$_{\text{sec.m.}}$ $I_2$) **42**.d *te*] om. $K_4$, *tan* $K_5$ **43**.a *anakṣa*] *anakṣatra* $I_3$ **43**.c *akṣābhāvāt*] *akṣābhāvā* $K_4$, *akṣābhāgāt* $I_1$ **43**.d *tatra*] om. $K_7$ Sāstrī **44**.c *madhyāhno*] *madhyāhne* S* **44**.d *rātram*] *rātra* V*$I_1$ **45**.a *kālayoge*] *yogakāle* $K_5$ **45**.b *akṣayuta*] *akṣayute* Q* **46**.b *'valambaka*] *valambakā* Q* **47**.d *ucyatām*] *ucyatām iti* $K_7$ **48**.c *pañcadaśa syuś cara*] *pañcadaśasya dvira* Q*

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**44**. = *GD1* 3.41 = *Ābh* 4.13 except for *iti* which is originally *syāt*. *Ārya* verse.

**47**. = *GD1* 3.33 = *GMBh* 3.53. *Rathoddhatā* verse.

a *tatpūrvāparadivasās*
b *tasmān nyūnāḥ krameṇa taddeśe /*
c *cāpānte 'rke tu niśā*
d *tadvat tatpārśvagā niśāś ca tathā //* **49** *//*

a *rāśidvayāpamasamā*
b *lambajyā yatra tatra cāpamṛgau /*
c *yāto nodayam astaṃ*
d *karkiyamau yānti harijam anye 'ṣṭau //* **50** *//*

a *vṛṣabhānantaralagnaṃ*
b *siṃhaḥ korpyūrdhvalagnam api kumbhaḥ /*
c *vīṇaiṇakarkidhanuṣāṃ*
d *lagnatvaṃ tatra vidyate naiva //* **51** *//*

a *ekarkṣāpamatulyā*
b *lambajyā cen na yānti vṛṣabhādyāḥ /*
c *catvāro 'staṃ vṛścika-*
d *dhanureṇaghaṭās tathā na yānty udayam //* **52** *//*

a *mīno meṣaḥ kanyā*
b *tulādharaś ceti tatra lagnāni /*
c *catvāry eva kramaśo*
d *nānyeṣāṃ harijasaṃgatir yasmāt //* **53** *//*

a *meṣādyāḥ ṣaṇ nāstaṃ*
b *merau yānty udayam api ca jūkādyāḥ /*
c *dṛśyādṛśyavibhāgau*
d *kalpyau vyatyāsato 'surasurāṇām //* **54** *//*

a *dvādaśarāśiṣu bhānoś*
b *cārād iha mānuṣaṃ bhaved varṣam /*
c *divyaṃ tad ahorātraṃ*
d *divyābdaḥ kharasavahnibhiḥ svadinaiḥ //* **55** *//*

a *divyair varṣasahasrair*
b *dvādaśabhiḥ syāc caturyugaṃ tv ekam /*
c *divyaṃ yugam iti kathitaṃ*
d *caturyugaṃ caikam ācāryaiḥ //* **56** *//*

a *ahivedā rasarāmāḥ*
b *kṛtadasrā dvīndavaś ca śatanihatāḥ /*
c *divyābdāḥ santi kṛte*
d *tretāyāṃ dvāpare kalau kramaśaḥ //* **57** *//*

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**49**.d *pārśvagā*] *pārśvagatā* X* (corr.$_{\text{sec.m.}}$ $I_2$) *pārśvagā* S* **49**.d *niśāś*] *niśaś* T* (corr.$_{\text{sec.m.}}$ $I_2$) **51**.b *lagna*] om. V* (corr.$_{\text{sec.m.}}$ $K_1$) **51**.d *lagnatvaṃ*] *lagnaṃ* V* (corr.$_{\text{sec.m.}}$ $K_1$) **51**.d *vidyate*] *vidyatena* $K_1$ **52**.b *yānti*] *yāti* S* **53**.b *dharaś*] *dhanuś* $K_5$ **53**.c *eva*] *evaṃ* $K_7$ Sāstrī **53**.d *saṃgatir yasmāt*] *saṃgatismāt* $K_8$, *saṃgatisyāt* $I_3$ **54**.a *ṣaṇ nāstaṃ*] *ṣaṇḍāstaṃ* S* **54**.c *dṛśyādṛśya*] *dṛśyadṛśya* W* **54**.d *'surasurāṇām*] *surāsurāṇām* $I_1$ **55**.c *tad ahorātraṃ*] *tāhorātraṃ* T* (corr.$_{\text{sec.m.}}$ $I_2$), *cāhorātraṃ* $K_7$ **55**.d *divyābdaḥ*] *divyābdaiḥ* $I_1$ **56**.b *dvādaśabhiḥ*] *dvādaśabhiś ca* $K_1$ **57**.a *rasarāmāḥ*] *rasarāmā* $I_1$ **57**.b *kṛtadasrā*] *kṛtadasrāḥ* R*$K_6$ **57**.b *śatanihatāḥ*] *śatanihatā* $I_1$

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**56**. *Ārya* verse.

a _divase caturyugānāṃ_
b _vidheḥ sahasraṃ bhavet tathā rātrau /_
c _sṛṣṭiḥ sthitiś ca divase_
d _lokasya vināśa eva cāsya niśi //_ **58** //

a _dinam idam uditaṃ kalpaś_
b _caturdaśa syur dine vidher manavaḥ /_
c _manvantare yugānāṃ_
d _saikā syāt saptatiḥ paraṃ saṃdhyā //_ **59** //

a _kalpasyādāv ante_
b _manuvivareṣv api ca pañcadaśa saṃdhyāḥ /_
c _ṣaṇṇāṃ caturyugānāṃ_
d _pañcadaśāṃśaḥ smṛto 'tra saṃdhyeti //_ **60** //

a _manuvivare saṃdhyāyāḥ_
b _pūrvāparabhāgayoḥ kramāt saṃjñā /_
c _saṃdhyāṃśaḥ saṃdhyeti ca_
d _kālavibhāgaḥ kṛto budhaiḥ kaiścit //_ **61** //

a _pañcāśat svā abdā_
b _vidher gatā ādya eva śeṣasya /_
c _kalpyo 'smin manavaḥ ṣaḍ_
d _gatāḥ parasyāpi bhair mitayugāni //_ **62** //

a _aṣṭāviṃśe 'pi yuge_
b _kṛtādayo 'smin gatās trayaḥ pādāḥ /_
c _śeṣo 'yaṃ kalipādaḥ_
d _pravartate pūrvasūrivacanam iti //_ **63** //

a _atidūragaṃ dineśaṃ_
b _paśyati kalpe sadā kamalayoniḥ /_
c _pralaye raver abhāvād_
d _brahmāpi raviṃ nirīkṣate naiva //_ **64** //

a _ekenaiva hi raviṇā_
b _daivaṃ pitryaṃ ca mānuṣaṃ brāhmam /_
c _dinam iti caturvidhaṃ syād_
d _golavidāṃ tāni golagamyāni //_ **65** //

a _sūryoparīndur iti yair_
b _uktaṃ teṣāṃ hi saṃsthitir merau /_
c _bhānām ūrdhvaṃ munayaḥ_
d _sarveṣāṃ ca dhruvo yatas teṣām //_ **66** //

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**59**.a _uditaṃ_] _uditaḥ_ X*, _udimasya_ K$_5$, _uditaṃ_ K$_7$ **59**.a _kalpaś_] _kalpaṃ_ W* **59**.c _yugānāṃ_] _yugāṃ_ R*K$_6$ (corr.$_{\text{sec.m.}}$ I$_2$) **59**.d _saikā_] _sekā_ T* (corr.$_{\text{sec.m.}}$ K$_6$I$_2$) **59**.d _saṃdhyā_] _bandhyā_ corr.$_{\text{sec.m.}}$ K$_8$ **60**.a _kalpasyādāv ante_] _kalpasyābhāvante_ K$_6$ **60**.b _saṃdhyāḥ_] _saṃdhyā_ V*I$_1$ **60**.d _pañcadaśāṃśa_] _pañcadaśāyāṃśa_ K$_6$ **61**.b _bhāga_] _kāla_ X* Sāstrī **61**.c _saṃdhyāṃśaḥ saṃdhyeti_] _saṃdhyāṃśas tulyeti_ K$_7$ **61**.c _ca_] om. I$_1$ **61**.d _vibhāgaḥ_] _vibhāge_ Y*W*, _vibhāgo_ R*K$_6$ (corr.$_{\text{sec.m.}}$ I$_2$), _vibhāgā_ K$_1$ **61**.d _kṛto_] _kṛtā_ Y*R*K$_4$K$_6$ (corr.$_{\text{sec.m.}}$ I$_2$) **62**.a _svā abdā_] _svābdānāṃ_ K$_7$ **62**.a _svā_] _sva_ S* **62**.b _gatā ādya_] _gatādya_ K$_7$ **62**.b _eva_] _eṣa_ Y* **62**.c _kalpyo_] _kalpo_ S*I$_1$, _kalpe_ K$_7$ Sāstrī **62**.d _mitayugāni_] _mitāni yugāni_ W* **63**.c _śeṣo_] _manavo_ K$_7$ **63**.d _vacanam iti_] _vacanan tat_ I$_1$ **65**.a _ekenaiva_] _ekenaivaṃ_ Q* **65**.a _hi_] om. K$_7$ **65**.d _golavidāṃ_] _golavidhāṃ_ K$_4$K$_8$ **66**.c _bhānām_] _bhām_ R*K$_6$ (corr.$_{\text{sec.m.}}$ K$_6$I$_2$)

a *tatrodagvikṣiptaḥ*
b *śaśy upari ca dṛśyate yamānte 'rkāt |*
c *tasmāt tathoktir eṣāṃ*
d *tatrānyad vāsti daivataṃ saumyam //* **67** *//*

a *paramādinoktam evaṃ*
b *saṃkṣepād īśvareṇa golasya |*
c *saṃsthānaṃ laghumataye*
d *vaktavyaṃ cānyad asti golagatam //* **68** *//*

a *yuktiḥ pradarśitā prāṅ*
b *mayā mahābhāskarīyabhāṣyasya |*
c *siddhāntadīpikāyāṃ*
d *vivṛtau vakṣye tathāpi śaṅkvādeḥ //* **69** *//*

a *ghaṭikāpamamaṇḍalayor*
b *yogasthārkasya yā mahācchāyā |*
c *dinamadhye sākṣajyā*
d *lambakajīvātha tasya śaṅkuḥ syāt //* **70** *//*

a *yāmyottarākhyavṛtte*
b *ghaṭikāsamamaṇḍalāntaraṃ hy akṣaḥ |*
c *avalambakas tu tasmin*
d *ghaṭikākṣitijākhyavṛttayor vivaram //* **71** *//*

a *kṣitijadhruvayor vivare*
b *jātā jīvāthavākṣajīvā syāt |*
c *vyomno madhyadhruvayor*
d *vivarabhavā jyā tu lambakajyā syāt //* **72** *//*

a *sphuṭadorjyā saptanava-*
b *tryekair nihatā trirāśiguṇavihṛtā |*
c *krāntiḥ syāt tattrijyā-*
d *kṛtivivarapadaṃ bhaved dyudalajīvā //* **73** *//*

a *akṣajyāghnā krāntir*
b *lambakajīvoddhṛtā kṣitijyā syāt |*
c *bhūjyā trijyānighnā*
d *dyudalajyābhājitā carajyā syāt //* **74** *//*

a *unmaṇḍalārkayogāj*
b *jīvā yāmyottarāpamajyā syāt |*
c *svāhorātrārdhajyā*
d *dyujyāvṛttasya yo 'rdhaviṣkambhaḥ //* **75** *//*

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**67**.b *ca*] om. V* (corr.$_{\text{sec.m.}}$ $K_1$) **69**.c *siddhāntadīpikāyāṃ*] *siddhānte dīpikāyāṃ* $I_1$ **70**.a *āpama*] *āpa* $I_3$ **70**.b *yogasthārkasya yā*] *yogasthasyāt svayā* T* (corr.$_{\text{sec.m.}}$ to *yogasthāsyāt svayā* $I_2$), *yogasthāsyāt svayā* S* **70**.d *jīvātha tasya*] *jīvāta ca sya* $I_1$ **71**.b *ghaṭikā*] *ghaṭighaṭikā* W* **71**.c *avalambakas*] *avalaṃbasakas* Q* **71**.d *ghaṭikākṣitijā*] *ghaṭikākṣatijā* $K_3K_8I_2$ **72**.b *jīvāthavā*] *vāthavā* $I_3$ **73**.a *jyā*] *jya* S* **73**.b *vihṛtā*] *visūtā* Q* **74**.a–b om. $I_3$ **74**.a *akṣa*] *akṣana* $K_4$ **74**.c–d om. $I_1$ **75**.a *unmaṇḍalārkayogāj*] *unmaṇḍale rkayogāj* X* Sāstrī, *unmaṇḍalārkayogāj* S* **75**.c *svāhorātrā*] *sāhorātrā* S*

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**75**. ab is simillar to *GD1* 2.15abc which uses the expression *unmaṇḍalārkayoga*

a *kṣitijonmaṇḍalavivare*
b *dyumaṇḍalajyā smṛtā kṣitijyeti* /
c *trijyākarṇasya bhujā*
d *krāntiḥ koṭir dyumaṇḍalārdhajyā* // **76** //

a *bhramaṇaṃ dyumaṇḍalānāṃ*
b *ghaṭikāvṛttasya cāpi kālasamam* /
c *ghaṭikāvṛttajyoktā*
d *bhramitāṃśe tasya hīṣṭakāle jyā* // **77** //

a *bhūjyā bhramaṇe yā jyā*
b *ghaṭikāvṛtte bhavec carajyā sā* /
c *cāpīkṛtā carajyā*
d *prāṇātmakam ucyate carārdham iti* // **78** //

a *yasmāt prāṇādīnāṃ*
b *liptādīnāṃ ca saṃsthitir vṛtte* /
c *cāpasyaiva tataḥ syāt*
d *prāṇāditvaṃ ca liptikāditvam* // **79** //

a *cāpīkaraṇaṃ yuktaṃ*
b *trijyāvṛtte dyumaṇḍaleṣu na tu* /
c *paṭhitāḥ sarvā jīvās*
d *trijyāvṛttodbhavā bhavanti yataḥ* // **80** //

a *paramāpamo yadi syāt*
b *trirāśidorjīvayā tadā tu kiyān* /
c *bhavatīṣṭadorjyayeti*
d *trairāśikam apamasiddhaye bhavati* // **81** //

a *yadi lambakākhyakoṭyā*
b *palajīvā jāyate tadā kiyatī* /
c *iṣṭāpamakoṭyeti*
d *jñeyaṃ trairāśikaṃ kṣitijyāyām* // **82** //

a *bhūjyā dyumaṇḍale yadi*
b *bhavati vyāsārdhamaṇḍale tu tadā* /
c *kiyatī jīvā syād iti*
d *vedyaṃ trairāśikaṃ carajyāyām* // **83** //

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**76**.b *kṣitijyeti*] *kṣitijeti* K$_4$, *kṣitijyoti* K$_6$ **76**.d *koṭir*] *koṭi* Y*T*K$_1$ Sāstrī (corr.$_{\text{sec.m.}}$ I$_2$), *ko* W* **77**.b om. I$_1$ **77**.b–**89**.b *cāpi…kheṭasya*] br. K$_5$ **77**.d *bhramitāṃśe*] *bhramitā eśa* K$_7$ **78**.d *ātmakam*] *ātmam* S* **79**.b *liptādīnāṃ*] *liptādīnāś* Q* **79**.b–d *saṃsthitir…ca*] om. I$_3$ **79**.d *liptikāditvam*] *liptikātvaditvam* K$_6$, *liptikātvaditva* K$_7$ **80**.a *karaṇaṃ*] *karaṇam iti syāṃ* V* (corr.$_{\text{sec.m.}}$ K$_1$) **80**.b *dyumaṇḍaleṣu*] *dyunmaṇḍaleṣu* S* **80**.c *paṭhitāḥ*] *pavitāḥ* S* **81**.b *kiyān*] *kiyāt* S* **81**.c *īṣṭadorjya*] *īṣṭajyā* T* (corr.$_{\text{sec.m.}}$ I$_2$), *īṣṭarjya* corr.$_{\text{sec.m.}}$ K$_1$ **82**.a *ākhya*] *ākhyā* K$_8$ **82**.b *jīvā*] *jīvāya* I$_3$ **82**.b *kiyatī*] *kayati* T*K$_1$ (corr.$_{\text{sec.m.}}$ K$_1$, corr.$_{\text{sec.m.}}$ to *kiyati* I$_2$), *kiyati* S* **83**.a *dyumaṇḍale*] *dyunmaṇḍale* S* **83**.b *vyāsārdha*] *vyāsārdhe* U* **83**.d *jyāyām*] *jyāyāt* K$_6$, *jyā syāt* K$_7$

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**76**. ab is almost identical to *GD1* 2.17ab

a *trijyāhatāpamajyā*
b *lambakavihṛtā bhaved ihārkāgrā* /
c *sā kṣitijabhānuyogāt*
d *kṣitije yāmyottarā hi jyā* // **84** //

a *krāntijyonmaṇḍalagā*
b *koṭir bhūjyā bhujā dyumaṇḍalajā* /
c *kṣitijasthārkāgrā syāt*
d *karṇas tryaśraṃ bhavet tribhiś caivam* // **85** //

a *koṭibhujākarṇeṣu*
b *dvābhyāṃ dvābhyāṃ hi siddhir anyasya* /
c *vargaikyapadaṃ bhūjyā-*
d *krāntyos tasmād bhaved ināgrā vā* // **86** //

a *trijyā lambakakoṭyāḥ*
b *karṇaś cet ko bhaved apamakoṭyāḥ* /
c *karṇas trairāśikam iti*
d *sūryāgrāyā avāptaye vedyam* // **87** //

a *kṛtvākṣavyāsārdhaṃ*
b *dyumaṇḍalaṃ daṇḍanābhiharijānte* /
c *tanmadhyagapalalambau*
d *tathāsya paridhisthatacchrutiś cohyā* // **88** //

a *golāntāt kheṭāntaṃ*
b *kheṭasya bhujādhanur bhujā tajjyā* /
c *ayanāntād vihagāntaṃ*
d *koṭidhanuḥ koṭir api ca tajjīvā* // **89** //

a *bāhuḥ krāntir abhīṣṭā-*
b *bhīṣṭabhujajyā śrutiś ca koṭis tu* /
c *svāhorātre 'bhīṣṭā*
d *jīvā tryaśraṃ bhaved amībhiś ca* // **90** //

a *paramadyujyā śaśikṛta-*
b *vidhurāmās taddhatā bhujajyeṣṭā* /
c *trijyābhaktā svāho-*
d *rātre jīvā bhaved abhīṣṭākhyā* // **91** //

**84**.b *vihṛtā*] *jīvā* V* Sāstrī (corr.$_{\text{sec.m.}}$ K$_1$), *bhajitā* I$_1$ **84**.b *ihārkāgrā*] *ihārkṣāgrā* K$_6$, *ihākṣāgrā* K$_7$ **84**.c *kṣitija*] *kṣiti* Q* **85**.b *dyumaṇḍalajā*] *dyunmaṇḍalajā* S*, *dyumaṇḍalagā* I$_1$ **85**.c *ārkāgrā*] *ārkagrā* Q* **85**.d *tryaśraṃ*] *tryaṃśaṃ* S*, *tryaṃśraṃ* K$_7$I$_2$ **86**.c *aikyapadaṃ*] *aikyat padaṃ/aikyalpadaṃ* K$_4$ **86**.d *ināgrā*] *inā* R*K$_1$ (corr.$_{\text{sec.m.}}$ K$_1$, corr.$_{\text{sec.m.}}$ to *dināgrā* I$_2$), *dināgrā* S* **87**.a *koṭyāḥ*] *koṭyā* T*K$_1$K$_2$ (corr. K$_2$, corr.$_{\text{sec.m.}}$ I$_2$) **87**.c *karṇas trairāśikam*] *karṇatrairāśikam* X* **87**.d *avāptaye*] *avāstaye* K$_8$ **87**.d *vedyam*] *vedyat* Q* **88**.b *dyumaṇḍalaṃ*] *dyunmaṇḍalaṃ* S*I$_2$ **89**.c–d *yanāntād...koṭir a*] br. K$_5$ **89**.d *koṭir api ca tajjīvā*] *koṭiracitatajjīvā* W*, *kotiracitajjīvā* V*, *koṭiracitajjīvāḥ* U*, *koṭiracitatajjīvāḥ* Sāstrī **90**.a *bāhuḥ krāntir*] *bāhukrāntir* S* **90**.d *tryaśraṃ*] *tryaṃśraṃ* K$_6$, *tryaṃśaṃ* K$_7$ **91**.c *trijyābhaktā*] *trijyā bhakto* X* (corr.$_{\text{sec.m.}}$ I$_2$), *trijyābhaktā* S*, *trijyā bhakte* Sāstrī

**84**. *Ārya* verse.

a *koṭiḥ paramadyujyā*
b *trijyāyāś ced abhīṣṭadorjyāyāḥ* /
c *keti dyumaṇḍaleṣṭa-*
d *jyāyās trairāśikaṃ vicintyaṃ syāt* // **92** //

a *iṣṭāpamadorjīvā-*
b *kṛtyor vivarasya mūlam athavā syāt* /
c *svāhorātreṣṭajyā*
d *rāśīnāṃ mānasiddhaye kathitāḥ* // **93** //

a *svāhorātreṣṭajyā*
b *trijyāghnā svadyuśiñjinībhaktā* /
c *cāpīkṛtā syur asavas*
d *taddorbhāgodaye hi laṅkāyām* // **94** //

a *iyatī dyujyāvṛtte*
b *jyā ced vyāsārdhamaṇḍale kiyatī* /
c *iti ghaṭikāvṛtte jyā*
d *syād dorbhāgodaye hi laṅkāyām* // **95** //

a *ekabhamānenonaṃ*
b *bhadvayamānaṃ dvitīyabhamitiḥ syāt* /
c *bhadvayamānenonaṃ*
d *bhatrayamānaṃ tṛtīyarāśimitiḥ* // **96** //

a *svacaradalenaiṇādau*
b *hīnāḥ karkyādige yutā ete* /
c *tattaddorbhāgodaya-*
d *kālaprāṇā bhavanti deśe sve* // **97** //

a *eṇādyā udyanti*
b *kṣipraṃ karkyādikāḥ śanair eva* /
c *udagunnataṃ bhagolaṃ*
d *yasmāc carasaṃskṛtāv iyaṃ yuktiḥ* // **98** //

a *śaśikṛtavidhurāmaghnā*
b *veṣṭabhujā svadyuśiñjinībhaktā* /
c *cāpīkṛtāḥ syur asavo*
d *laṅkāyām iṣṭabāhudhanurudaye* // **99** //

a *trairāśikayugasiddhā*
b *bhamitir ihādye haras trirāśijyā* /
c *anyatra sā guṇo ’tas*
d *taddvayahīnaṃ ca karmayuktam idam* // **100** //

**92**.a *koṭiḥ*] *koṭi* T* (corr.$_{\text{sec.m.}}$ $\mathrm{I}_2$) **92**.c *keti*] *koṭi* U* **92**.c *dyumaṇḍale*] *dyunmaṇḍale* S*$\mathrm{I}_2$ **92**.d *vicintyaṃ*] *vicintya* Q* **93**.c–d om. U* **94**.b *śiñjinī*] *śiktiñjinī* $\mathrm{K}_6$, *śiktijinī* $\mathrm{K}_7$ **94**.c *cāpīkṛtā syur asavas*] *cāpīkṛtasya rasavas* S*$\mathrm{I}_2$ **94**.d *hi*] *tu* $\mathrm{I}_1$ **95**.b *jyā*] *jyāś* X*$\mathrm{K}_5$ (corr. $\mathrm{K}_5$, corr.$_{\text{sec.m.}}$ $\mathrm{I}_2$), *jyā* S* **95**.d *hi*] *tu* $\mathrm{K}_5$ **96**.c *mānenonaṃ*] *mānonenaṃ* S* **97**.a *dalenaiṇādau*] *dalenainodau* Q* **97**.b *ādige*] *ādi+na* $\mathrm{K}_5$ **97**.c *dorbhāgodaya*] *dogāgodaya* Q* **97**.d *sve*] *sye* Q*$\mathrm{I}_2$ (corr.$_{\text{sec.m.}}$ $\mathrm{I}_2$) **98**.b *śanair*] *śaner* X* **98**.c *udag*] *ivadag* S*, *deg* corr.$_{\text{sec.m.}}$ $\mathrm{I}_2$ **98**.c *unnataṃ*] *annataṃ* Q* **98**.d *saṃskṛtāv iyaṃ*] *saskṛtāniyaṃ* S* **98**.d *iyaṃ*] *iyā* W* **100**.d *dvaya*] *dvadvaya* Q*

**98**. c = *GD1* 4.84a. *GD2* 98 and *GD1* 4.84 are identical in general.

a *saty ayane sāyanayor*
b *iṣṭasyādyantayoḥ pṛthaṅ mānam* /
c *kuryāt tayos tu vivaraṃ*
d *syād iṣṭamitiś carārdham iha tadvat* // **101** //

a *iṣṭaṃ dvipadagataṃ cet*
b *tasya tu tattatpadasthabhāgamitim* /
c *kuryāt pṛthak tadaikyaṃ*
d *syād iṣṭamitiś caraṃ svapadavihitam* // **102** //

a *astodayākhyasūtraṃ*
b *pūrvāparagaṃ bhaved ināgrāntāt* /
c *kṣitijāt svāhorātre*
d *carato 'rkasyonnatir hi śaṅkuḥ syāt* // **103** //

a *śaṅkor mūlāstodaya-*
b *sūtrāntaram ucyate 'tra śaṅkvagram* /
c *svāhorātreṣṭajyā*
d *śaṅkuśirostodayākhyavivaragatā* // **104** //

a *karṇo 'treṣṭadyujyā*
b *śaṅkuḥ koṭir bhujā tu śaṅkvagram* /
c *evam ihākṣanimittaṃ*
d *kṣetraṃ proktaṃ bahūni tāni syuḥ* // **105** //

a *bāhvādyair ekasmin*
b *kṣetre jātair ihānupātena* /
c *kṣetrāntarasiddhiḥ syāt*
d *sarveṣām āśrayo 'kṣam eva yataḥ* // **106** //

a *svāhorātreṣṭajyā*
b *ghaṭikāvṛttotthajīvayā sādhyā* /
c *gatagantavyāsujyā*
d *ghaṭikāvṛttodbhavā hi jīvā syāt* // **107** //

a *jīvāgrahaṇam ayuktaṃ*
b *kṣitijād unmaṇḍalād dhi yuktaṃ tat* /
c *unmaṇḍalam eva syād*
d *bhagolamadhyasthitaṃ yato nānyat* // **108** //

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**101**.b *iṣṭasyādyantayoḥ*] *eṣṭasyāntaṃ* $K_5$ **101**.b *antayoḥ*] *antareyāḥ* U* (corr.$_{\text{sec.m.}}$ to *antareyoḥ* $K_8$), *antareyoḥ* $K_6I_3$ **102**.a *iṣṭaṃ*] *iṣṭa* U*$K_6$ **102**.a *taṃ dvipadagata*] br. $K_1$ **102**.a–b *cet tasya*] *cetasya* S*$I_2$ **102**.b *stha*] *sva* $K_2$, *sya* $K_7$, *sta* corr.$_{\text{sec.m.}}$ to *sya* $I_2$ **103**.b *pūrvāparagaṃ*] *pūrvāparaṃ* W* **103**.b *bhaved ināgrāntāt*] *bhabhedinākrāntāt* S* **105**.b *koṭir*] *koṭī* W*$K_1K_3$, *koṭi* $K_6I_1I_2$ (corr.$_{\text{sec.m.}}$ $I_2$) **106**.d *yataḥ*] *yatoḥ* $I_3$ **108**.d *yato*] *no* $K_5$ (both sections: see below)

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**101**. $K_5$ writes verse number "100" after 101b.

**103**. There is an overlapping in $K_5$ beginning from *v(e)d ināgrāntāt* until verse 108 (Folios 18r to 19r and 20r to 21r, 19v being blank). The latter section is severly damaged but whatever readings remaining on both sections are the same.

**108**. $K_5$ starts from *syād bhagola…* until the end, then puts *saumye ca* (beginning of verse 109) then returns to the beginning of this verse, *jīvagrahaṇam…* until *unmaṇḍalam eva*. Here folio 19r ends. 19v is blank, and 20r starts from *v(e)d ināgrāntāt* in verse 103. The overlapping section continues until folio 21r where verse 108 ends.

a *saumye carahīnānāṃ*
b *gole yāmye carārdhayuktānām |*
c *gatagantavyāsūnāṃ*
d *jīvā hy unmaṇḍalordhvagā bhavati ||* **109** *||*

a *iṣṭadyuvṛttabāhye*
b *ghaṭikāvṛtte prakalpite jñeyā |*
c *yuktiś carasaṃskāre*
d *dyugate carabhūjyayoḥ sarūpaṃ vā ||* **110** *||*

a *sonmaṇḍalordhvagā jyā*
b *svāhorātrāhatā triguṇabhaktā |*
c *unmaṇḍalordhvabhāge*
d *svāhorātreṣṭajīvakā bhavati ||* **111** *||*

a *iyatī ghaṭikāvṛtte*
b *jyā cet kiyatī tadā dyumaṇḍalajā |*
c *trairāśikam iti vedyaṃ*
d *svāhorātreṣṭajīvakānayane ||* **112** *||*

a *bhūjyārahitā yāmye*
b *saumye bhūjyānvitā ca sā dyujyā |*
c *kṣitijordhvabhāgajātā*
d *svāhorātreṣṭajīvakā bhavati ||* **113** *||*

a *sā jyā lambakanihatā*
b *trijyābhaktā bhaven mahāśaṅkuḥ |*
c *tattrijyākṛtibhedān*
d *mūlaṃ chāyā ca tasya śaṅkoḥ syāt ||* **114** *||*

a *yadi lambakakoṭiḥ syāt*
b *trijyākarṇena kā tadā koṭiḥ |*
c *iṣṭadyujīvayā syāc*
d *chaṅkau trairāśikaṃ bhaved evam ||* **115** *||*

a *ravinihatā sā mahatī*
b *chāyā bhaktā ca śaṅkunā mahatā |*
c *arkāṅgulaśaṅkoḥ syāc*
d *chāyā trairāśikād iyaṃ cāptā ||* **116** *||*

a *dyujyārkaghnā kṣitijāt*
b *palakarṇahṛtāthavā mahāśaṅkuḥ |*
c *dyujyā sā krāntighnā*
d *sūryāgrahṛtāthavā mahāśaṅkuḥ ||* **117** *||*

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**110**.b *jñeyā*] *kreyā* $I_1$ **110**.d *cara*] *caraṃ* S* **110**.d *sarūpa*] *svarūpaṃ* Y*U*, br. $K_1$ **111**.d *bhavati*] *bhavanti* S* **112**.b *dyumaṇḍala*] *dyunmaṇḍala* S*$I_2$ **112**.b *jā*] *gā* $I_1$ **113**.b *saumye*] *om.* corr.$_{\text{sec.m.}}$ to *gole* $K_5$ **113**.b *bhūjyānvitā ca*] *bhūjyānvitāya* $K_4$ **113**.b *sā dyujyā*] *saumyajyā* $K_5$ **114**.a *sā*] *syāj* S* **115**.d *chaṅkau*] *chaṅko* S*$I_2$ **116**.b *mahatā*] *mahantā* $K_5$ **116**.c *arkāṅgula*] *akṣāṅgula* $K_5$ **116**.d *cāptā*] *prāptā* $K_7$ **117**.a *ghnā*] *ghnāt* X* Sāstrī **117**.a *kṣitijāt*] *kṣitijā* $K_5$ **117**.b *palakarṇa*] *calakarṇa* W*$K_1$ **117**.c–d om. $K_5I_3$

a *saumyāyatakarṇavaśāc*
b *cordhvāyatakoṭisādhanam ihoktam* /
c *tad yuktam eva yasmāj*
d *jātaṃ tad dvandvam akṣato bhavati* // **118** //

a *akṣajyāghnaḥ śaṅkur*
b *lambakabhajito bhavec ca śaṅkvagram* /
c *yasmāl lambakaśaṅkoḥ*
d *śaṅkvagraṃ palaguṇo 'tra yuktir iti* // **119** //

a *athavā śaṅkvagraṃ syāt*
b *palāṅgulaghno 'rkabhājitaḥ śaṅkuḥ* /
c *bhūjyāghno vā śaṅkuḥ*
d *krāntijyābhājitaś ca śaṅkvagram* // **120** //

a *akṣajyālpākrāntiḥ*
b *saumyā trijyāhatā palajyāptā* /
c *samamaṇḍalasthaśaṅkuḥ*
d *pūrvāparasūtrage ravau bhavati* // **121** //

a *samamaṇḍalage bhānau*
b *śaṅkvagram ināgrayā samaṃ hi bhavet* /
c *syāt krānteś cārkāgrā*
d *tasmācchaṅkvagram iha bhavet krānteḥ* // **122** //

a *krānteḥ śaṅkvagraṃ syād*
b *anupātāc chaṅkur api ca śaṅkvagrāt* /
c *trairāśikayugmaṃ syāt*
d *samamaṇḍalaśaṅkusiddhaye 'treti* // **123** //

a *hara iha lambaka ādye*
b *sa tūpari guṇo 'tha naṣṭayos tu tayoḥ* /
c *trijyā tu guṇo 'kṣajyā*
d *hāraḥ krānteḥ phalaṃ tu samaśaṅkuḥ* // **124** //

a *cāraś candrādīnāṃ*
b *sve sve vikṣepamaṇḍale kathitaḥ* /
c *apamaṇḍale tu teṣāṃ*
d *caranti pātā vilomagās te syuḥ* // **125** //

a *apamaṇḍale svapāte*
b *tasya ca katame vimaṇḍalaṃ lagnam* /
c *paramakṣepāntaritaṃ*
d *pādāntaṃ tasya saumyayāmyadiśoḥ* // **126** //

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**118**.c *tad*] *tasmād* $K_5$ **118**.d *tad*] *yad* $I_3$ **119**.b *bhajito*] *bhajite* W*$I_2$ Sāstrī *bhajjito* $K_5$ **119**.d *palaguṇo*] *palagaṇo* $K_4$ **119**.d *yuktir*] *yattir* $K_5$ **121**.c *stha*] *sva* Q*$I_2$ (corr.$_{\text{sec.m.}}$ $I_2$) **122**.b *samaṃ*] *samamaṃ* W* (corr. $K_2$), *saṃ* S* **122**.d *agram*] *agraham* S* **123**.d *treti*] *trayeti* S*$I_2$ **124**.a *lambaka*] *laṅka* S* **124**.b *tūpari*] *rūpari* Y*T* (corr.$_{\text{sec.m.}}$ $K_6$) **124**.b *'tha naṣṭayos*] *vinaṣṭayos* Y* **124**.c *tu guṇo*] *guṇato* Y* **126**.b *katame*] *kāme* Y*V*

a *mandasphuṭāt svapātāḥ*
b *śodhyāḥ śīghroccatas tu budhasitayoḥ |*
c *pātonabhujā parama-*
d *kṣepaghnā trijyayoddhṛtā kṣepaḥ // **127** //*

a *sa punar vyāsārdhahato*
b *mandaśrutibhājitaḥ sphuṭaḥ kathitaḥ |*
c *so 'pi vyāsārdhahato*
d *bhaumādeḥ syāt svaśīghrakarṇahṛtaḥ // **128** //*

a *vedā dvāv aṣṭa rasā*
b *diśa iti bhāgā daśāhatās te syuḥ |*
c *bhaumādeḥ pātāṃśā*
d *bahutarakālena bhuktir alpaiṣām // **129** //*

a *navatir vyomadineśāḥ*
b *ṣaṣṭiḥ khārkāḥ khanetraśiśirakarāḥ |*
c *paramā vikṣepakalā*
d *bhūmijabudhagurusitārkatanayānām // **130** //*

a *paramakṣepo yadi cet*
b *trirāśidorjīvayā tadā tu kiyān |*
c *bhavatīṣṭadorjyayeti*
d *kṣepe trairāśikaṃ bhaved iṣṭe // **131** //*

a *karṇe svalpe vṛddhis*
b *tāsāṃ hrāso bhavet tathā mahati |*
c *dūrādūraviśeṣaiḥ*
d *kṣetrasya hi liptikābhedaḥ // **132** //*

a *śaighrān māndāc coccād*
b *bhaumādeḥ syād adho gatiś cordhvam |*
c *karṇadvayena tasmād*
d *grahabhūmyor antarālamitisiddhiḥ // **133** //*

a *bhaumeḍyamandapātāḥ*
b *śodhyāḥ svāt svāt sphuṭād iti bruvatām |*
c *śīghrajyāsaṃskāro*
d *grahavat pāte nije bhavet pakṣe // **134** //*

a *karṇasthitisiddhyartham*
b *sphuṭasiddhyarthaṃ ca likhyate 'trāpi |*
c *kakṣyātrayaṃ jhaṣānte*
d *prācī dig bhavati sarvavṛtteṣu // **135** //*

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**127**.a *pātāḥ*] *pātoḥ* R*$K_6$ (corr.$_{\text{sec.m.}}$ $I_2$) **127**.d *trijyayo*] *trijyāyo* T* *trajyāyo* Q* **128**.b *mandaśruti*] *mandasphuṭa* W* Sāstrī **128**.d *svaśīghra*] *svataghra* Q* **128**.d *karṇa*] *kaṇṭaka* $K_8$ **129**.b *bhāgā*] *bhābhāgā* Q* **129**.c *pātāṃśā*] *pātāṃśāḥ* U* *pādāṃśāḥ* S* **129**.d *tara*] *tanu* $K_1$, br. $K_3$, lacuna $K_6$, om. $K_7$, *taṇa* $K_8$, *ranu* $I_2$ **130**.a *dineśāḥ*] *digenaśāḥ* W* (corr.$_{\text{sec.m.}}$ $K_2$) **130**.b *ṣaṣṭiḥ*] *ṣaḍbhiḥ* U* Sāstrī **131**.a *kṣepo*] *vikṣepo* T* **132**.b *tathā*] *tadā* S* **132**.b *mahati*] *mahatī* Sāstrī **132**.d *hi*] *tu* $I_1$ **133**.a *māndāc*] *mandāc* U*$I_1$ Sāstrī (corr. $I_2$) **134**.a *eḍya*] *esya* U* (corr.$_{\text{sec.m.}}$ $K_6$), *eḍya* $K_7$, *ebhya* $I_1$ **134**.a *pātāḥ*] *bhāvatāḥ* U* **135**.b *'trāpi*] *tatrāpi* $K_3$ **135**.c *trayaṃ*] *traya* X*, *trayaṃ* $K_8$, *tra* corr.$_{\text{sec.m.}}$ $I_2$

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**132**. *Ārya* verse.

a *bhūmadhyakendram ādyaṃ*
b *bhākhyaṃ vṛttaṃ tu bhavati sarveṣām |*
c *tanmadhyāc chīghradiśi*
d *svāntyaphalānte kujāryamandānām* // **136** //

a *śaighrasya kendram uditaṃ*
b *budhabhṛgvor mandadiśi tu māndasya |*
c *svāntyaphalānte kendraṃ*
d *dvitīyamadhyāt kujādīnām* // **137** //

a *mandadiśi māndakendraṃ*
b *dvitīyaparidhisthabhānukendram atha |*
c *śaighraṃ jñaśukrayoḥ syād*
d *antye vṛtte caranti sarve 'pi* // **138** //

a *antye vṛtte teṣāṃ*
b *cāro madhyākhyayā sadā gatyā |*
c *khagacārajā bhacakre*
d *yā gatir anumīyate sphuṭākhyā sā* // **139** //

a *antyaṃ śaighrāntyaphala-*
b *vyāsārdhaṃ syāj jñaśukrayor vṛttam |*
c *triguṇakṛtāny anyāni*
d *kṣepo vṛttatrayasya yugapat syāt* // **140** //

a *antyaparidhisthakheṭāt*
b *sūtraṃ kuryād upāntyakendrāntam |*
c *tatkarṇo bhaumāder*
d *māndo bhavati jñaśukrayoḥ śaighraḥ* // **141** //

a *śrutimārgageṣṭasūtraṃ*
b *dvitīyaparidhau tu yatra tatra bhavet |*
c *mandasphuṭaḥ kujādes*
d *tatra tu śīghrasphuṭo jñabhṛgusūnvoḥ* // **142** //

a *mandasphuṭāt kujāder*
b *budhabhṛgvoḥ śīghrajāt sphuṭāt sūtram |*
c *kuryād bhacakrakendrā-*
d *ntam etad uktā śrutiḥ kujādīnām* // **143** //

a *śaighrānyayos tu māndā*
b *śrutimārgagasūtrabhākhyaparidhiyutau |*
c *śaighrasphuṭaḥ kujādes*
d *tatra tu mandasphuṭo jñabhṛgusūnvoḥ* // **144** //

**136**.a *ādyaṃ*] *adyaṃ* Q* **137**.b *diśi*] *niśi* $K_5$ **137**.b *māndasya*] *mandasya* $I_1I_2$ **137**.c *ānte*] *āntye* $K_8I_2$ **137**.c *kendraṃ*] *kendra* Q*S* Sāstrī **138**.b *bhānu*] *om.* V* **140**.a *phala*] *phalaṃ* U*, *phalana* $K_1$ **140**.b *vṛttam*] *vṛtte* Y* (corr.$_{\text{sec.m.}}$ $K_5$) **140**.c *anyāni*] *ānyāni* Q* **141**.b *upāntya*] *upānta* X* (corr.$_{\text{sec.m.}}$ $I_2$), *upāntya* S* **142**.d *tu*] om. T*$K_1$ **144**.b *bhākhya*] om. $K_5$ **144**.c *śaighra*] *śaighraḥ* S*$I_2$

**137**. *Ārya* verse.

a *dvyuccānāṃ sphuṭayugalaṃ*
b *bhavati bhaparidhau gataḥ sphuṭo hi khagaḥ* /
c *bhedas tasya kadācit*
d *sākṣāt sphuṭakhecarād bhaved alpaḥ* // **145** //

a *mandaśrutiś ca śaighraṃ*
b *phalaṃ kujādes tu bhedahetuḥ syāt* /
c *śīghraśrutiś ca māndaṃ*
d *phalaṃ vibhede sitajñayor hetuḥ* // **146** //

a *jīvāphalārdhasaṃskṛta-*
b *madhyān māndaṃ phalaṃ tataḥ kriyate* /
c *sarveṣāṃ budhasitayoḥ*
d *kramabhedo 'py atra kalpitas tasmāt* // **147** //

a *antyaparidhisthakheṭād*
b *ihādyakendrāntam api kṛte sūtre* /
c *tatsūtrādyaparidhyor*
d *yoge sākṣāt sphuṭagraho bhavati* // **148** //

a *madhyāntagate karṇe*
b *kṣepo madhyāntyavṛttayor iṣṭaḥ* /
c *yadi cet trijyākarṇe*
d *kaḥ syād iti madhyaparidhigaḥ kṣepaḥ* // **149** //

a *prathamadvitīyayoś cet*
b *karṇe madhyāntage tv ayaṃ kṣepaḥ* /
c *trijyākarṇe kaḥ syād*
d *iti vikṣepaḥ sphuṭo bhacakre syāt* // **150** //

a *sphuṭayugasiddhasya yathā*
b *dṛgbhedo 'lpo grahasya bhavati tathā* /
c *karṇadvayasiddhasya*
d *kṣepasyāpīti kasyacic cintā* // **151** //

a *arkendvor dve vṛtte*
b *bhavṛttakendrān nijoccadiśi māndam* /
c *vṛttaṃ svāntyaphalānte*
d *sphuṭakarmaikaṃ bhaved yathā svoccam* // **152** //

a *vikṣepāpamadhanuṣos*
b *tulyadiśor bhinnayor yutir viyutiḥ* /
c *proktaṃ svakrāntidhanus*
d *tasya jyā svasphuṭāpamajyā syāt* // **153** //

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**145**.a *dvyuccānāṃ*] *dyuccānāṃ* U* **145**.b *hi*] *e* $K_8$ *eva* $I_3$ **145**.d *khecarād*] *kecarād* S* **146**.c–d *śīghra...bhede si*] br. $K_5$ **146**.d *phalaṃ vibhede*] *phalam api bhede* $I_1$ **147**.b *madhyān māndaṃ*] *madhyānāndaṃ* $I_1$ **148**.b *ādya*] *āntya* X* Sāstrī, *ānya* corr.$_{\text{sec.m.}}$ to *āntya* $K_5$ **148**.d *yoge*] *yogo* $I_1$ **149**.a *ānta*] *āntya* Y* **149**.b *po madhyāntyavṛt*] br. $K_5$ **149**.b *āntya*] *ānta* X* **150**.b *āntage*] *āntanate* $K_5$ **151**.a *yathā*] br. $K_5$, *yadā* $I_1$ **151**.b *dṛgbhedo*] *dṛggedo* $I_1$ **151**.b *'lpo*] *tra lpo* $K_5$ *lpe* $I_3$ **152**.b *bhavṛtta*] *bhavṛtte* $K_4$ **153**.c *proktaṃ*] *prokta* Y*

a *yāmyottaravṛtte 'pama-*
b *yogād rāśitrayāntare vedhau |*
c *kāryau sarvarkṣāṇāṃ*
d *saṃpātād rāśikūṭasaṃjñau tau* // **154** //

a *golasya dakṣiṇodak-*
b *svastikayugmād yathā ghaṭīvalayam |*
c *cakraturīyāṃśe syād*
d *bhakūṭayugmāt tathāpamākhyaṃ ca* // **155** //

a *yāmyodagāyatā syāt*
b *kheṭasthakalā bhakūṭayugmāntā |*
c *kheṭasthaliptikāyāṃ*
d *kṣepas tasyāpamāt sadā yāti* // **156** //

a *kṣepasyordhvādhogatir*
b *unmaṇḍalato 'sty ato bhakūṭavaśāt |*
c *kṣepāpakramadhanuṣor*
d *ato 'tra yogādy ayuktam iti kecit* // **157** //

a *lagne 'yanāntage syād*
b *unmaṇḍalagaṃ bhakūṭayugalam atha |*
c *golānte 'dhaś cordhvaṃ*
d *koṭivaśāt syāt tadunnatir ato 'tra* // **158** //

a *ayanāntasphuṭakhecara-*
b *vivarajalaṅkodayāsuguṇanihatā |*
c *paramakrāntis trijyā-*
d *vihṛtā syād unnatir bhakūṭasya* // **159** //

a *saumyonnatir eṇādau*
b *vihage yāmyonnatiḥ kulīrādau |*
c *vihagasyodaya evaṃ*
d *vyastaṃ syād unnatis tadastamaye* // **160** //

a *laṅkodayakālasamaṃ*
b *golabhramaṇaṃ tato bhakūṭasya |*
c *golabhramajonnatir api*
d *laṅkodayakālajīvayā sādhyā* // **161** //

a *khagakoṭir vāntyāpama-*
b *nihatā sthūlonnatis triguṇabhaktā |*
c *sthūlāpi nāpradarśyā*
d *laghutā yadi karmaṇo bhavet tatra* // **162** //

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**154**.b *yogād*] om. T*K$_1$(corr.$_{\text{sec.m.}}$ to *yo* K$_7$, corr.$_{\text{sec.m.}}$ I$_2$) **154**.b–d *rāśitrayāntare …saṃpātād*] om. I$_3$ **155**.d–**156**.a om. I$_3$ **156**.b *kalā bha*] *khagāpa* S* **157**.a *kṣepasyo*] *kṣepasvo* S* *kṣepastho* Sāstrī **158**.d *ato*] *atro* W* **159**.a *ayanānta*] *ayana* I$_1$ **160**.d *vyastaṃ*] *vyaktaṃ* S* **162**.a *koṭir*] *koṭibhir* S*I$_2$ **162**.c *sthūlāpi*] corr.$_{\text{sec.m.}}$ to *sthūlādhi* K$_5$

a *vikṣepaghnā trijyā-*
b *bhaktā yā connatir bhakūṭasya /*
c *tatkṣepavargavivarāt*
d *padaṃ sphuṭakṣepa īritaḥ krāntyām //* **163** //

a *tatkrāntyoś cāpaikyaṃ*
b *tulyadiśor bhinnayor dhanurbhedaḥ /*
c *apamadhanuḥ syāt spaṣṭaṃ*
d *spaṣṭā bhūjyādayo 'pi tajjyātaḥ //* **164** //

a *ūrdhvādhogamanāt syāt*
b *kṣepasyonmaṇḍalād udayabhedaḥ /*
c *apamād api yāmyodak-*
d *sthityā dṛkkarmaṇī grahe 'taḥ staḥ //* **165** //

a *vikṣepeṇābhihatā*
b *triguṇena hṛtonnatir bhakūṭasya /*
c *kṣepasyonnatir athavā*
d *tasyaivonmaṇḍalād avanatiḥ syāt //* **166** //

a *kṣepo yadi rāśīnāṃ*
b *kūṭonnatibhāgagas tadā tasya /*
c *kṣepasyonnatir uditā*
d *viparītadigāśritasya cāvanatiḥ //* **167** //

a *kṣeponnatir bhujā syāt*
b *karṇaḥ kṣepo 'sya bhavati yā koṭiḥ /*
c *sonmaṇḍalagaḥ kṣepaḥ*
d *kriyate krāntes tu dhanuṣi yac cāpam //* **168** //

a *kṣeponnatis trijīvā-*
b *guṇitā dyudaloddhṛtā ca yā tasyāḥ /*
c *cāpaṃ bhaliptikāghnaṃ*
d *kheṭagatarkṣāsubhājitaṃ svarṇam //* **169** //

a *ṛṇam unnatāv avanatau*
b *dhanam udaye tadvad eva vāstamaye /*
c *unnatir udayabhavā yadi*
d *sāstabhavā ced dhanādi viparītam //* **170** //

a *kheṭāstarkṣaprāṇā*
b *hāraḥ syād astadṛkphalāv āptau /*
c *rāśeḥ kālo 'stamaye*
d *svasaptamarkṣodayāsutulita iti //* **171** //

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**164**.a *cāpaikyaṃ*] *copaikyaṃ* X* (corr.$_{\text{sec.m.}}$ $K_6I_2$), *cāpaikyaṃ* S* **164**.c–d *spaṣṭaṃ spaṣṭā*] *sphaṣṭaṃ sphaṣṭā* S* **164**.d *tajjyā*] *tajyā* W*Y*$K_6K_8$ **165**.a *gamanāt*] *gamanaṃ* $K_5$ **165**.c–d br. $K_3$, om. $K_8$ **165**.d *karmaṇī*] *karmaṇi* $K_1K_7$ **165**.d *grahe 'taḥ staḥ*] *grahe ta staḥ* Z*, *grahe tantaḥ* T* (corr.$_{\text{sec.m.}}$ $I_2$), *grahetastat* Sāstrī **166**.b *triguṇena*] *triguṇe* S* **167**.b *kūṭonnati*] *kūṭonnatir* $K_7$ **167**.b *tadā*] *tasyadā* Q* **168**.a–b *kṣeponnatir … 'sya*] om. $I_3$ **168**.b *karṇaḥ kṣepo*] *karṇakṣepo* $K_8$ Sāstrī **169**.b *guṇitā*] *guṇitada* corr.$_{\text{sec.m.}}$ to *guṇitāda* $K_6I_2$ **169**.c *ghnaṃ*] *graṃ* $K_5$ **169**.d *rkṣā*] *rkṣe* $K_5$ **170**.a *unnatā*] *unnato* W* (corr. $K_2$) **170**.a *avanatau*] *anatau* S*$K_7I_2$ **170**.b *dhanam*] *dhanum* Sāstrī **171**.b *hāraḥ syād*] *harasya (ce)d* Sāstrī **171**.b *hāra*] *hara* X* (corr.$_{\text{sec.m.}}$ $I_2$), *hāra* $I_3$ **171**.b *āptau*] *āstau* $K_6$ **171**.c *rāśeḥ*] *rāśau* $I_1$ **171**.c *kālo*] *kalo* R*$K_1K_6$ (corr.$_{\text{sec.m.}}$ $I_2$) **171**.d *tulita*] *tulitami* W* (corr. $K_2$), *tulitasya* $K_5$

a *khābhrāhīndukalā yadi*
b *labhyante svāsubhir vilagnasya* /
c *syur dṛkphalāsubhiḥ kā*
d *bhavati trairāśikam itīha* // **172** //

a *laṅkodayāsuharaṇaṃ*
b *ye tv atrecchanti dṛkphalāvāptyai* /
c *sudhiyas te gaṇakāḥ syuḥ*
d *kiṃ tv iha golaikadeśavettāraḥ* // **173** //

a *svāstamaye kālasya*
b *svasaptamarkṣodayāsutulyatvam* /
c *bhānāṃ bhavati carasya*
d *vyastatvād udayakālato 'stamaye* // **174** //

a *vikṣepasaṃskṛtā yā*
b *krāntijyā kevalā ca yātra tayoḥ* /
c *vivaraṃ vikṣepabhavā*
d *krāntiḥ syād akṣadṛkphalaṃ tu tataḥ* // **175** //

a *apamo vikṣepabhavas*
b *tv akṣahato lambakajyayā vihṛtaḥ* /
c *trijyāghno dyudalāptas*
d *tasya dhanuḥ kṣepakṛtacarāṃśaḥ syāt* // **176** //

a *kṣepacaraṃ bhakalāghnaṃ*
b *kheṭastharkṣāsubhājitaṃ śodhyam* /
c *udaye kṣepe saumye*
d *deyaṃ yāmye 'nyathā khagasyāste* // **177** //

a *dṛkkarmadvayam etat*
b *proktaṃ kheṭodayāstalagnāptyai* /
c *na tu tatsphuṭāṅgam etad*
d *dvitayaṃ vaikena karmaṇā sidhyet* // **178** //

a *apamasyārdhaṃ hy uditaṃ*
b *sarvatrārdhaṃ tathā sadāstagatam* /
c *uditāṃśasya tu madhye*
d *dṛkkṣepākhyaṃ sadā sthitaṃ lagnam* // **179** //

a *uditāṃśasya ca madhyaṃ*
b *lagnāstavilagnayor hi madhye syāt* /
c *dṛkkṣepalagnam uditaṃ*
d *prāglagnaṃ bhatrayeṇa hīnam ataḥ* // **180** //

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**172**.a *kalā*] *khalā* S* **173**.b *ye tv atre*] *yatvatre* S*$I_2$ **173**.b *āptyai*] *āptyaiḥ* S* **174**.b *saptama*] *saptame* W*$K_1$ (corr. $K_1$) **174**.b *tulyatvam*] *tulyaś ca* corr.$_{\text{sec.m.}}$ $K_6I_2$, *tulyartham* $I_3$ **174**.c *bhānāṃ*] *dhānāṃ* $K_7I_2$ (corr.$_{\text{sec.m.}}$ $I_2$) **174**.d *tvād*] *syād* $K_7$ **176**.b–a *bhavas tv akṣa*] *bhavaḥ pakṣa* S* **176**.b *vihṛtaḥ*] *vihṛtiḥ* R*$K_6$ (corr.$_{\text{sec.m.}}$ $I_2$) **177**.a *kṣepacaraṃ*] *kṣepakcaraṃ* $K_8$, *kṣeparkaraṃ* $I_3$ **177**.a *kalāghnaṃ*] *kalārdhaṃ* $K_7$ **177**.b *kheṭastha*] *kheṭasta* S*$I_2$ (corr.$_{\text{sec.m.}}$ $I_2$) **177**.d *deyaṃ*] *yā* Q* **178**.b *kheṭodayā*] *kheṭomayā* U*, *kheṭodayā* $K_7$ **178**.d *dvitayaṃ*] *dvitīyaṃ* U* **179**.b *tathā*] *tadā* Y* **179**.b *gatam*] *gate* $K_7$ **179**.c *tu*] om. $K_6$ **180**.a *ca*] *tu* $I_1$

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**172**. *Ārya* verse.

a *dṛkkṣepajyā coktā*
b *khamadhyadṛkkṣepalagnavivarajyā |*
c *dṛkkṣepalagnage 'rke*
d *dṛkkṣepajyā smṛtā mahācchāyā ||* **181** *||*

a *yāmyottaravṛtte 'pama-*
b *bhāgo madhyākhyalagnam iti kathitam |*
c *tad dhy arko madhyāhne*
d *natalaṅkāmitivaśāc ca sādhyaṃ tat ||* **182** *||*

a *udayaviparītam aste*
b *rāśeś carasaṃskṛtir yatas tasmāt |*
c *na syāt khamadhyage sā*
d *laṅkāmitir eva madhyamānam ataḥ ||* **183** *||*

a *madhyavilagnakrāntyāḥ*
b *palajīvāyāś ca cāpayoḥ samayoḥ |*
c *yogād vidiśor vivarāj*
d *jātā jīvātra madhyajīvoktā ||* **184** *||*

a *ghāṭikakhamadhyaghaṭikā-*
b *dyuvṛttavivare palāpamau hi staḥ |*
c *tābhyāṃ dyumaṇḍalanabho-*
d *madhyāntarajīvakā tataḥ sādhyā ||* **185** *||*

a *trijyāmadhyajyākṛti-*
b *vivarapadaṃ madhyaśaṅkur iti kathitaḥ |*
c *madhyavilagnonodaya-*
d *lagnabhujajyā tu madhyaśaṅkubhujā ||* **186** *||*

a *madhyākhyaśaṅkunihataṃ*
b *vyāsārdhaṃ madhyaśaṅkubhujayāptam |*
c *dṛkkṣepaśaṅkur ukto*
d *dṛkkṣepajyā sphuṭā ca tacchāyā ||* **187** *||*

a *madhyavilagnakṣitijā-*
b *ntarajyayā madhyaśaṅkur iha cet syāt |*
c *dṛkkṣepaharijavivare*
d *trijīvayā ko 'tra śaṅkur iti yuktiḥ ||* **188** *||*

a *dṛkkṣepajyā tulitā*
b *bhānāṃ kūṭonnatis tadanyadiśi |*
c *kṣitijāt tu golapāde*
d *khamadhyam apamād yato bhakūṭam api ||* **189** *||*

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**181**.b *madhyadṛkkṣepa*] *madhyamakṣepa* $\mathrm{K}_1$ **182**.d *nata*] *nati* S*$\mathrm{I}_2\mathrm{K}_7$ (corr.$_{\mathrm{sec.m.}}$ $\mathrm{I}_2$) **182**.d *vaśāc*] *vaśāś* S* **183**.c *na syāt*] om. $\mathrm{K}_6$ **184**.b *pala*] *para* $\mathrm{K}_7$ Sāstrī **184**.c *yogād vidiśor*] *yogādiśor* $\mathrm{I}_1$ **184**.c *vivarāj*] *++rajāñ* $\mathrm{K}_5$ **184**.d *jīvātra*] *jīvo tra* S*, *jīvā ca* Sāstrī **185**.a *ghāṭika*] *ekādika* $\mathrm{K}_7$ **185**.c *dyumaṇḍala*] *dyunmaṇḍala* S*$\mathrm{I}_2$ **186**.a *madhya*] *masya* T*(corr. $\mathrm{K}_6$, corr.$_{\mathrm{sec.m.}}$ $\mathrm{I}_2$) **186**.a *kṛti*] *kṛti* $\mathrm{K}_3\mathrm{K}_6$ **187**.d *tacchāyā*] *tajjāyā* S* **188**.b *jyayā*] *jyā* $\mathrm{K}_4$ **188**.b *iha*] *iti* Y* **188**.d *tri*] om. V* (corr.$_{\mathrm{sec.m.}}$ $\mathrm{K}_1$) **188**.d *ko*] *to* W* **189**.b *tadanya*] *tadantya* U*, *tadānya* $\mathrm{K}_3$ **189**.c *gola*] *golacakra* W*, om. V*(corr.$_{\mathrm{sec.m.}}$ to *cakra* $\mathrm{K}_1$) **189**.d *api*] *iti* S*

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**181**. $\mathrm{K}_7$ and Sāstrī transverse 181 and 182.

a *kṣitijasthe tv iṣṭakhage*
b *dṛkkṣepajyāhatas triguṇabhaktaḥ* /
c *vikṣepaḥ kṣitijāt syāt*
d *kṣepasya pronnatis tv avanatir vā* // **190** //

a *dṛkkṣepetaradiksthe*
b *vikṣepe pronnatir bhavet tasya* /
c *dṛkkṣepajyādiksthe*
d *vikṣepe tv avanatir bhavet tasya* // **191** //

a *kṣepasyonnatir athavā-*
b *vanatis trijyāhatāvalambahṛtā* /
c *trijyāghnāṃ dyudalāptā*
d *yā taccāpaṃ hi dṛkphalaprāṇāḥ* // **192** //

a *khakhadhṛtinihatā lagna-*
b *prāṇāptā dṛkphalād ihonnatijāt* /
c *liptāḥ śodhyā udaye*
d *kṣepyāś cāste 'nyathāvanatijāc cet* // **193** //

a *palaguṇamadhyavilagna-*
b *krāntyor adhikasya yā tu dik saiva* /
c *madhyajyādṛkkṣepa-*
d *jyayor bhavet sakaladṛkphalam ihoktam* // **194** //

a *samarekhāyāṃ madhyama-*
b *bhānor unmaṇḍalodaye hi budhaiḥ* /
c *uditā vihagās tasmāt*
d *saṃskārās teṣu deśajādyāḥ syuḥ* // **195** //

a *samarekhānijabhūmyor*
b *antarajair yojanair hatā bhuktiḥ* /
c *nijabhūvṛttahṛtā svaṃ*
d *rekhāyāḥ paścime tv ṛṇaṃ prācyām* // **196** //

a *samarekhāyāḥ prācyāṃ*
b *prāgudayaḥ paścime raveḥ paścāt* /
c *deśagatir ataḥ prācyāṃ*
d *viśodhyate dīyate tathā paścāt* // **197** //

a *nijabhūvṛttabhramaṇe*
b *dinabhuktir yadi bhavet tadā kiyatī* /
c *samarekhānijabhūmyor*
d *vivarabhramaṇe 'tra yuktir iti cintyā* // **198** //

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**190**.a *sthe*] *ste* $K_3K_5K_6$ **191**.a *dṛkkṣepe*] *vikṣepe* U* **192**.b *trijyāhatā*] *trijyāhṛtā* U*, *trijyāhatā* $I_2$ **192**.d *prāṇāḥ*] *pramāṇāḥ* $I_1$ **193**.a *nihatā*] *nihatāl* $K_5$, *hi hatā* $I_1$ **193**.a *lagna*] *lagnaṃ* $K_5$ **193**.b *prāṇāptā*] *prāṇā* Q* **194**.a *pala*] *phala* $I_1$ **194**.b *adhikasya*] *akakasya* $K_6$, *akṣasya* $K_7$ Sāstrī **194**.d *ihoktam*] *iti hoktam* S*Q* **195**.a *madhyama*] *madhya* $K_2$, *madhye ma* S*$K_5I_2$ **196**.c *hṛtā*] *hatā* Y*V* **197**.d *tathā*] om. S*$I_1I_2$ **198**.b *tadā*] *tathā* Y* **198**.d *cintyā*] *cintyāt* T*, br. $K_7$

a *pūrvābhimukhaṃ gacchan*
b *nijabhūvṛtte sadā naro gacchet* /
c *darśanam arkasya yato*
d *nijabhūvṛttānusāri dik cārkāt* // **199** //

a *palayoḥ sāmyaṃ ca yayoḥ*
b *pūrvāparasaṃsthitau hi deśau tau* /
c *nijabhūvṛtte hy eva ca*
d *tat sāmyaṃ hārako 'ta iha tat syāt* // **200** //

a *trijyālambe 'nakṣe*
b *bhūvṛttaṃ randhragośviguṇatulitam* /
c *syāc ced abhīṣṭalambe*
d *kiṃ syān nijabhūmivṛttalabdhir iti* // **201** //

a *ravidoḥphalaṃ hi bhānoḥ*
b *sphuṭamadhyamayoḥ kalātmakaṃ vivaram* /
c *tannihatā grahabhuktiś*
d *cakrakalāptaṃ grahe dhanarṇaṃ syāt* // **202** //

a *ravidoḥphalavat tasminn*
b *ṛṇe yato madhyamodayāt prāk syāt* /
c *sphuṭatīkṣṇāṃśor udayo*
d *dhane 'nyathāsphuṭaravir vrajed dhy udayam* // **203** //

a *golabhramaṇe syāc ced*
b *dinabhuktiḥ kā bhujāphalabhramaṇe* /
c *iti yuktiṃ bruvate 'nye*
d *doḥphalakālo bhaved ihecchetí* // **204** //

a *ravicaradalāsunihatā*
b *dināsubhaktā gatis tv ṛṇaṃ saumye* /
c *gole bhānor udaye*
d *yāmye deyā khage 'nyathāstamaye* // **205** //

a *unmaṇḍalodayāt prāk*
b *saumye gole yato raver udayaḥ* /
c *paścād yāmye 'stamayo*
d *vyastaṃ tasmād ṛṇādividhir evam* // **206** //

a *yadi bhavati divasabhuktir*
b *dināsubhiḥ kā tadā carārdhabhavaiḥ* /
c *prāṇais trairāśikam iti*
d *kheṭe ca carārdhasaṃskṛtau vedyam* // **207** //

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**199**.a *pūrvābhimukhaṃ*] br. $K_7$ **199**.b *gacchet*] *gacchan* Q* **199**.c *darśanam*] *diśanam* $K_1$ **200**.b *saṃsthitau*] *saṃjñitau* X* Sāstrī **201**.a *lambe 'nakṣe*] *lambonarkṣe* U* Sāstrī, *laṃbonakṣe* $I_1$ **201**.b *vṛttaṃ*] *vṛtto* $K_1$ **201**.d *bhūmi*] *bhūmir* U* **202**.a *ravidoḥ*] *ravindoḥ* $K_5$ **202**.d–**206**.d br. $K_5$ **203**.d *dhane*] *dhanye* $K_7$ **204**.b *phala*] *pala* Sāstrī **204**.c *yuktiṃ*] *yukti* W*T* (corr.$_{\text{sec.m.}}$ $I_2$), *yuktir* $K_6 I_1$ **207**.b *dināsu*] *dinādi* S* **207**.d *saṃskṛtau*] *saṃskṛtā* T* (corr.$_{\text{sec.m.}}$ $I_2$) **207**.d *vedyam*] *vedyat* Q*

a *hāro 'tra caradalādau*
b *ravigatiliptādhikā dinaprāṇāḥ* /
c *ity anye sārkagater*
d *bhramaṇād golasya bhavati divasa iti* // **208** //

*atha samacchāyayā dinadalacchāyayā ca sphuṭārkānayanam* / *tatra samacchāyāyām uddeśakaḥ* /

a *chāyā ravau narasamā samamaṇḍalasthe*
b *hīnā tato 'paradine yadi tatra ko 'rkaḥ* /
c *yad vādhikāparadine yadi tatra ko vā*
d *vidvan vada svarakṛtāṅgamitā palajyā* // **209** //

*iti* / *atra karaṇasūtram āryādvayam* /

a *chāyāsādhyaḥ śaṅkuḥ*
b *śaṅkoḥ śaṅkvagram iha hi tadināgrā* /
c *arkāgrātaḥ krāntiḥ*
d *krānter dorjyā ca taddhanur inaḥ syāt* // **210** //

a *yady adhikāparadinajāc*
b *chāyā doścāpahīnam atra bhavet* /
c *cakrasyārdhaṃ sāyana-*
d *bhānur yasmād ihāyanaṃ yāmyam* // **211** //

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*atra cchāyākarṇād anupātenānītaḥ śaṅkuḥ 2431* / *śaṅkvagram 466* / *etat tu pādahīnaṃ grāhyam* / *etad ināgrā ca* / *arkāgrāto vyastavidhinānītā krāntiḥ 457* / *etat tu sārdhaṃ grāhyam* / *krānteḥ siddhabhujajyāyāś cāpam 1147* / *arkaḥ 0 19 7* / *dvitīyo 'rkaḥ 5 10 53* / *krāntisiddhatvād etau sāyanau* //

3. *0 19 7*] *1 9 7* $K_5^+$ 3. *5 10 53*] *5 10 5iṃ* $K_5^+$ (ഇം instead of ൩.)

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*atha madhyacchāyāyām uddeśakaḥ* /

a *śaṅkor ardhamitā prabhā dinakare yāmyāṃ śalākāṃ gate*
b *tatrāṣṭāṃśamitāthavātha dinape saumyāṃ śalākāṃ gate* /
c *saptāṃśena mitā ca sāparadine sarvā mahatyo 'thavā*
d *hīnā brūhi kave ravī nagacatuṣṣaḍbhiḥ palajyā samā* // **212** //

**preamble**.1 *dinadala*] *didaṃnala* Q*$K_3$, *bhinnala* S*$I_2$ (corr.$_{\text{sec.m.}}$ to *bhannala* $I_2$), *digdala* Sāstrī **209**.a *ravau*] *rava* $K_5^+$ **209**.a *samamaṇḍala*] *samaṇḍala* S* **209**.b–c *tatra ko 'rkaḥ yad vādhikāparadine*] om. $K_3$ **210**.d *ca taddhanur inaḥ*] *caturdhanuvina* T* (corr.$_{\text{sec.m.}}$ to *caturdhanur ina* $I_2$), *caturdhanur ina* S* **211**.c *sāyana*] *sāyau* U* (corr.$_{\text{sec.m.}}$ $I_2$) **211**.d *yāmyam*] *sāmyam* U* (corr.$_{\text{sec.m.}}$ to *saumyam* $I_2$), *saumyam* S*, *yāmyam iti* $K_5^+$ **preamble**.1 *madhya*] om. W* Sāstrī, *sama* V* (corr. $K_1$), *madhyaiś* $K_5^+$ **212**.a *prabhā dinakare*] *prabhādikare* S*, *prabhāne kare* R*$K_6$ (corr.$_{\text{sec.m.}}$ $I_2$) **212**.b *āṣṭāṃśa*] *āṃṣṭāśa* R* **212**.b *āthavātha*] *āthavā* U* **212**.b *dinape*] *dinapate* T* (corr.$_{\text{sec.m.}}$ to *dinapatena* $I_2$), *dinapatena* S* **212**.c *mitā ca*] *mitāthava ca* U*, *mitātha* S*, *mitātha ca* $K_3$, corr.$_{\text{sec.m.}}$ to *mitātha* $I_2$, *mitāthavā* $K_7$ **212**.d *ravī*] *ravīṃ* S*

**209**. *vasantatilakā* verse.
**212**. *śārdūlavikrīḍita* verse.

*iti / atra karaṇasūtram āryāpañcakam /*

a *divasadale mahatī yā*
b *cchāyā sā procyate natajyeti /*
c *natapaladhanuṣor vivaraṃ*
d *krāntidhanur yāmyage ravau madhyāt //* **213** *//*

a *saumye 'rke natapalayor*
b *aikyaṃ krāntis tadā tu golam udak /*
c *pūrvatra nate tv adhike*
d *yāmyaṃ golaṃ pale 'dhike saumyam //* **214** *//*

a *yāmye khamadhyato 'rke*
b *chāyāvṛddhau tu yāmyam ayanaṃ syāt /*
c *taddhānyām udagayanaṃ*
d *vyastaṃ saumye khamadhyato 'rke syāt //* **215** *//*

a *krānter dorjyā sādhyā*
b *cāpaṃ tasyā ravir bhaved gole /*
c *saumye 'yane ca saumye*
d *yāmye tv ayane tadūnacakradalam //* **216** *//*

a *bhānuḥ saṣaḍbhacāpaṃ*
b *yāmye gole 'yanaṃ ca yadi yāmyam /*
c *saumye 'yane 'tra cakraṃ*
d *cāponaṃ sāyano ravir bhavati //* **217** *//*

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*atra prathamacchāyayā tatkarṇena ca siddhā mahācchāyā 1537 / eṣaiva natajyā ca / atra sūryasya madhyād yāmyagatatvān natapaladhanuṣor vivaram apakramadhanuḥ 943 / atra natasyādhikyād dakṣiṇaṃ golam / krāntijyāto labdhabhujāyāś cāpam 2509 / dakṣiṇagolagatatvād etac cāpaṃ ṣaḍrāśiyutaṃ cchāyāvṛddhau sūryaḥ 7 11 49 / aparadinacchāyāyāṃ hīnāyāṃ saumyam ayanaṃ syāt / atas tadbhujācāpahīnaṃ dvādaśarāśyātmakaṃ cakraṃ sūryaḥ 10 18 11 //*
*atha dvitīye cchāyāṅgulam 1 30 / mahācchāyā 426 / atrāpi sūryasya madhyād yāmyagatatvāt palanatadhanuṣor vivaraṃ krāntidhanuḥ 224 / atra palasyādhikyāt golaṃ saumyam / krānteḥ siddhabhujācāpam 553 / saumyagolagatasūryasya madhyād yāmyagatatvāc chāyāvṛddhau yāmyam ayanaṃ syāt / ata etac cāpahīnaṃ rāśiṣaṭkaṃ sūryaḥ 5 20 47 / aparadinacchāyāyāṃ svalpāyāṃ bhujācāpam eva sūryaḥ 0 9 13 //*
*atha tṛtīye cchāyāṅgulam 1 43 / mahācchāyā 487 / arkasya madhyāt saumyagatatvān natapaladhanuṣor yogaḥ krāntidhanuḥ 1140 / bhujācāpam 3194 / atrārkasya saumyagolagatatvāc chāyāvṛddhāv idaṃ cāpam eva sūryaḥ 1 23 14 / chāyāhānyāṃ cāponarāśiṣaṭkam arkaḥ 4 36 46 // krāntisiddhatvād ete sāyanāravayaḥ //*

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1. *prathamacchāyayā*] *prathamacchāyā* $I_1$  2. *943*] *94* corr.$_{sec.m.}$ $I_1$  3. *2509*] *259* $K_5^+$ $I_1$  3. *ṣaḍrāśi*] *ṣaddhrāśi* $K_5^+$  5. *10*] *1* $K_5^+$  5. *18*] *1* corr.$_{sec.m.}$ to *12* $I_1$  6. *30*] *3* $I_1$  8. *saumya*] *saumye* $K_5^+$  8. *madhyād yāmya*]

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**213**.c *pala*] *phala* $K_1$  **213**.d *yāmyage*] *yāmyate* T* (corr.$_{sec.m.}$ $I_2$)  **214**.a *pala*] *phala* $K_1$  **214**.d *yāmyaṃ*] *yāmyā* R* (corr.$_{sec.m.}$ $I_2$), *yāmya* Q*  **214**.d *'dhike*] *dhite* S*  **214**.d *saumyam*] *saumye* corr.$_{sec.m.}$ $I_2$  **215**.a *kha*] om. U* (corr.$_{sec.m.}$ $I_2$)  **215**.a *madhyato*] *madhyagate* $K_7$  **215**.d *vyastaṃ*] *vyaktaṃ* S*  **215**.d *kha*] *bala* U* (corr.$_{sec.m.}$ $I_2$)  **215**.d *madhyato*] *madhyagato* S*  **216**.c *saumye*] *sāmye* $K_5$  **216**.d–**217**.a om. U*  **217**.c *saumye 'yane 'tra cakraṃ*] *Lacuna* Sāstrī *madhyāddhyāmya* K$_5^+$  9. *20*] *3* K$_5^+$  10. *0*] om. K$_5^+$  11. *tṛtīye cchāyā*] *tṛtīyacchāyā* K$_5^+$  12. *1140*] *114* K$_5^+$  13. *chāyā*] *chāpa* K$_5^+$  13. *14*] om. I$_1$  13. *4 36 46*] *4646* K$_5^+$, *46 46 14* I$_1$

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a *krāntinatacāpayoḥ syād*
b   *vivaraṃ samayor yutis tu bhinnadiśoḥ* |
c *paladhanur antaram ayanaṃ*
d   *chāyāgaṇitāptayos tu ravyoḥ syāt* // **218** //

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*ekadiggatayor apakramanatacāpayor vivaram akṣadhanuḥ syāt* | *bhinnadiggatayos tayos tu yogo 'kṣacāpaṃ bhavati* | *evaṃ chāyārkābhyām akṣaḥ sādhyaḥ* | *pūrvodāharaṇe prathamacchāyādhanuḥ 1594* | *krāntidhanuḥ 943* | *dakṣiṇagatayor anayor vivaram akṣadhanuḥ 651* | *atha dvitīyanatadhanuḥ 427* | *krāntidhanuḥ 224* | *atra krāntiḥ saumyā nataṃ yāmyam* | *ato 'nayor aikyam akṣadhanuḥ 651* //
*yat punar madhyacchāyānītagaṇitatantrānītayor arkayor antaraṃ tad ayanacalanaṃ bhavati* | *evaṃ madhyacchāyāvaśād ayanacalanaṃ ca siddhyati* //

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a *ekasmin sthiraśaṅku-*
b   *cchāyāgraṃ kālayor yayor bindau* |
c *patati tayor madhyasthe*
d   *kāle 'rkaḥ sāyano 'yanānte syāt* // **219** //

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*sarvadā niścalīkṛtasya śaṅkoḥ stambhārohaṇādibhūtasya niścalakāṣṭhasya vāgrādyavayavabhedān niṣpannaṃ chāyāgraṃ yadābhīṣṭabindau patati punaḥ kālāntare ca yadā tacchāyāgraṃ tasminn eva bindau patati tayoḥ kālayor madhyagatakāle sāyanārko 'yanāntagato bhavati* | *evaṃ cāyanacalanaṃ jñeyam* //

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a *iṣṭāśāsthe bhānau*
b   *chāyā sādhyā viśeṣavidhinātra* |
c *āśāvṛtte kalpyā*
d   *chāyā sūtreṇa vṛttam iha kāryam* // **220** //

a *samayoḥ śaṅkvagrārkā-*
b   *grayor yutir bhinnayos tayor vivaram* |
c *chāyākarṇakṣetre*
d   *digbāhur bhavati yāmyasaumyaśirāḥ* // **221** //

a *sārdharkṣasya hi jīvā*
b   *digjīvā koṇage ravau bhavati* |
c *taddalajīvā madhye*
d   *surapāgnyor ūhyam evam aparam api* // **222** //

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**218**.a *nata*] *gata* U*  **218**.b *yutis tu*] *yuti* K$_5^+$  **218**.b *diśoḥ*] *diśo* K$_7$ Sāstrī  **218**.c *antaram*] *antam* K$_1$  **219**.a–d om. K$_1$  **219**.a *śaṅku*] *śaṅko* K$_5^+$I$_1$  **219**.b *yayor*] om. U*  **219**.b *bindau*] *vidhoḥ patatindoḥ* K$_7$  **219**.d *'yanānte*] *nayānte* I$_1$  **220**.a *iṣṭāśā*] *iṣṭāṃśā* K$_5^+$  **221**.b *yutir*] *yuti* Y*R*K$_6$ (corr.$_{\text{sec.m.}}$ I$_2$)  **222**.b–c *digjīvā …dalajīvā*] om. I$_3$  **222**.d *surapāgnyor*] *surapāśyor* T* (corr.$_{\text{sec.m.}}$ I$_2$)  **222**.d *ūhyam evam aparam*] corr. to *ūhyaṃ mavamaparam* K$_8$, *ūhyaṃ mavamaparam* I$_3$

a *digjyeṣṭacchāyāghnā*
b *trijyāptā sādhyabāhur iti kathitaḥ* /
c *digbāhusādhyabāhū*
d *tulyau ced iṣṭadiśi gato 'rkaḥ syāt* // **223** //

a *digbāhusādhyabāhvoḥ*
b *samayor vivarād vidikkayor aikyāt* /
c *guṇanihatād dhārāptaṃ*
d *chāyāyām ṛṇam uta svam iṣṭāyām* // **224** //

a *digbāhau sādhyākhyād*
b *yāmyagate svaṃ viśodhyam atha saumye* /
c *vyastaṃ saumyanate syāc*
d *chāyādvandve kṛtaṃ tathā kāryam* // **225** //

a *mahati pale saumyanate*
b *yady adhikā digguṇād ināgrā syāt* /
c *ekasyām eva diśi*
d *cchāye dve sto yato gatir vṛtte* // **226** //

a *digbāhāv alpe svaṃ*
b *chāyāyāṃ phalam ihādhike śodhyam* /
c *prathamaprabhārtham evaṃ*
d *kāryaṃ vyastaṃ dvitīyabhāvāptyai* // **227** //

a *natadiśy udaye hāras*
b *trijyāsūryāgrayor bhaved vivaram* /
c *yogo 'nyathāviśeṣe*
d *trijyāmadhyāhnabhāntaraṃ tu guṇaḥ* // **228** //

a *atroktau guṇahārau*
b *digbhir bhaktau śatena veṣṭena* /
c *tau vā guṇahārau sto*
d *na hy aviśeṣe 'lpabhedato doṣaḥ* // **229** //

a *chāyātaḥ śaṅkuḥ syāc*
b *chaṅkvagram ato bhujādvayaṃ ca tayoḥ* /
c *vivarāt prabhā ca bhūyo*
d *'py evaṃ bāhvos tu sāmyam iha yāvat* // **230** //

1 *atrodāharaṇam* /

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**223**.a *digjyeṣṭacchāyā*] *digjyeṣṭā chāyā* Sāstrī **223**.b *kathitaḥ*] *kalitaḥ* $K_5$ **224**.b *samayor*] om. U* **224**.d *uta*] *ataḥ* V*$K_5$ **225**.a *bāhau*] *bāhū* $K_4K_5$ **225**.c *nate*] *natau* X*$K_5$ Sāstrī **226**.a *saumya*] *saumye* T* **226**.b–**241**.c br. $K_5$ **226**.b *digguṇād ināgrā*] *digguṇādhināgrā* $K_4$ **227**.a–b *svaṃ chāyāṃ*] *svaṃ jāyāṃ* Q*, *svacchāyāṃ* $K_3$, *svacchāyāyāṃ* $K_5K_5^+$ Sāstrī, *svajyāyāṃ* corr.$_{\text{sec.m.}}$ $I_2$ **227**.c *prabhārtham*] *prabhām* $K_5^+$ **227**.d *āptyai*] *āptyaiḥ* S* **228**.d *bhāntaraṃ tu*] *bhantu* $K_4$, *bhāntarantu* Sāstrī

a *korpyāntage 'rke dahanasya diksthite*
b *vṛṣāntagene śivadiksthite prabhe |*
c *ke brūhi śaṅkos tulitasya bhāskarair*
d *vidvan palajyā nagavedaṣaṇmitā ||* **231** *||*

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*atrobhayatra krāntiḥ 1210 | arkāgrā 1232 || atra prathame kalpitā chāyā trijyātulyā | ato 'rkāgraiva digbāhuḥ | trijyātaḥ siddhā sādhyabāhuḥ 2431 | ekadikkayor anayor antaram 1199 | etat guṇyam | atrodaye madhyāhne ca sūryasya yāmyadiggatatvāt trijyārkāgrayor vivaraṃ hāraḥ 2206 | madhyāhnabhā 1795 | trijyāmadhyacchāyāntaraṃ guṇaḥ 1643 | aviśeṣakarmaṇi sadaivaṃ guṇahārau bhavataḥ | guṇyāt guṇanihatād dhārakeṇa labdham 893 | etat sādhyabāhuto digbāhor alpatvāt saumyagatvāt pūrvānītāyāṃ trijyātulitacchāyāyām ṛṇaṃ bhavati | tathā kṛte siddhā cchāyā 2545 | eṣātreṣṭacchāyā | ataḥ śaṅkuḥ sādhyaḥ | śaṅkoḥ śaṅkvagraṃ ca | śaṅkvagrārkāgrayos tulyadiktvād yogaś chāyākarṇavṛtte dakṣiṇottarāyato digbāhuḥ 1675 | cchāyātaḥ siddhaḥ sādhyabāhuḥ 1800 | anayor antaraṃ 125 | asmād guṇanihatād dhārākeṇa vibhajya labdham 93 | etad atrāpi digbāhor alpatvāt saumyagatvāt pūrvacchāyāyām 2545 viśodhyam | tathā kṛte cchāyā 2452 | punar ato 'pi śaṅkvādikṛtvāviśiṣṭācchāyā 2407 | eṣā vahnikoṇagate 'rke mahācchāyā bhavati | ataḥ siddhā dvādaśāṅgulaśaṅkoś chāyā* $\begin{matrix}11\\46\end{matrix}$ *|| atha dvitīya udayakāle madhyāhnakāle 'pi sūryasya saumyadiggatatvāt trijyāsūryāgrayor vivaraṃ hāraḥ pūrvasiddha eva 2206 | atra madhyāhnacchāyā 584 | madhyāhnacchāyātrijyayor antaraṃ guṇaḥ 2854 | atra cchāyām abhīṣṭāṃ prakalpya tataḥ śaṅkuśaṅkvagradigbāhusādhyabāhūn pūrvavad ānīya bāhvantarāt guṇahārābhyāṃ phalaṃ cānīya svakalpitapūrvacchāyāyām ṛṇaṃ dhanaṃ vā yathāvidhi kṛtvā aviśiṣṭāṃ cchāyām ānayet | aviśiṣṭā sā 840 | eṣaiśakoṇagate 'rke cchāyā | arkāṅgulaśaṅkoś chāyā* $\begin{matrix}3\\1\end{matrix}$ *|| yadārkaḥ saumyadiśy udito yāmyadiśi madhyaṃ gacchati tadā trijyāsūryāgrayor yogo hāraḥ ||*

*anyat pūrvavad udāharaṇam |*

1. *1210*] *121* $K_5^+$ 2. *chāyā trijyātulyā*] *chāyās trijyātulyāḥ* $K_5^+$ 2. *'rkāgraiva*] *'rkāgraivātra* $K_5^+$ 3. *2431*] *2432* $K_5^+$ $I_1$ 4. *2206*] *226* $K_5^+$ 9. *1800*] *180* $K_5^+$ 9. *125*] *13 5* $K_5^+$, *1325* $I_1$ 11. *2545*] *3545* $K_5^+$ 11. *'pi śaṅkvādikṛtvā*] *śaṅkvādikṛtvāpi* $K_5^+$ 12. *46*] om. $K_5^+$ 14. *2206*] *226* $K_5^+$ 17. *840*] *84* $K_5^+$ 18. *śaṅkoś*] *śaṅkoñ* $K_5^+$ 18. *1*] om. $I_1$

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a *vahner āśāṃ meṣamadhyasthite 'rke*
b *yāte vīṇāmadhyage cendraśambhvoḥ |*
c *āśāmadhyaṃ naḥ pṛthag brūhi vidvañ*
d *chāyāṃ prāgvac chaṅkur akṣo 'pi cātra ||* **232** *||*

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**231**.a *āntage 'rke*] *antagārke* Y* **231**.a *ānta*] *anta* S*, corr.$_{\text{sec.m.}}$ to *anta* $I_2$ **231**.a *dahanasya*] *hanasya* S* **231**.a *diksthite*] *diksthe* W* **231**.d *vidvan*] *vidvān* S* **232**.a *vahner*] *vahnir* T* (corr.$_{\text{sec.m.}}$ $I_2$) **232**.b *yāte*] *yānte* Q* **232**.b *śambhvoḥ*] *śambhoḥ* S* **232**.c *madhyaṃ*] *madhyān/madhyāṃ* Q* **232**.c *pṛthag*] *pṛtha* T* (corr.$_{\text{sec.m.}}$ $I_2$) **232**.d *'pi*] *hi* V*, *ha* S*

**231**. *upajāti* verse; acd in *indravaṃśa* and b in *vaṃśasthavila*

**232**. *śālinī* verse.

*atha prathame 'rkāgrā saumyā 368 / yāmyā dinārdhabhā 289 / anayor bhinnadiktvād atra trijyāsūryāgrayor yogo hāraḥ 3806 / guṇaḥ 3149 / kalpiteṣṭacchāyā 2977 / śaṅkvagrahīnārkāgrā 39 / eṣa digbāhuḥ saumyaḥ / atra sādhyabāhur yāmyaḥ 2104 / vidiśor anayor yogād guṇanihatād dhārāptam 1773 / etad digbāhoḥ saumyagatvāt pūrvacchāyāyāṃ śodhyam / tatra jātācchāyā 1204 / punar apy evaṃ kṛtvāviśiṣṭacchāyā 405 //*
*atha dvitīye 'rkāgrā 1373 / eṣā saumyā / saumyadinārdhabhā 731 / hāraḥ 2065 / guṇaḥ 2707 / atra digjyā 1315 / kalpitacchāyā 3438 / atrārkāgraiva digbāhuḥ / digjyaiva sādhyabāhuḥ / bāhvantarāt phalaṃ 76 / etad digbāhor adhikatvāt prathamacchāyāsiddhyartham iṣṭacchāyāyāṃ śodhyaṃ bhavati / yadā digbāhur alpā syāt tadā kṣepyam / atrāviśiṣṭā cchāyā 3422 / eṣā rudrapuraṃdarayor madhyabhāgaṃ gate sūrye mahācchāyā syāt / atraiva dvitīyācchāyā ca bhavati / tatsiddhyarthaṃ prathamasiddheṣṭadikcchāyām iṣṭasaṃkhyāhīnām iṣṭabhāṃ prakalpya karmaṃ kāryam / tatra sahasrahīnā pūrvacchāyā 2422 / digbāhuḥ 906 / sādhyabāhuḥ 926 / bāhvantaraphalam 26 / etad digbāhor alpatvād dvitīyacchāyāsiddhyarthaṃ śodhyam / atrāviśiṣṭā cchāyā 2318 / eṣā dvitīyeṣṭadikcchāyā //*
*ābhyām arkāṅgulaśaṅkoś chāyādvayaṃ sādhyaḥ //*

2. *3806*] *3826* $I_1$ 3. *saumyaḥ*] *saumyā* $K_5^+$ 4. *etad digbāhoḥ*] *etadiścāhos* $I_1$ 6. *saumyā*] om. $K_5^+ I_1$ 6. *hāraḥ*] *haraḥ* $K_5^+$ 8. *digbāho*] *digbāhya* corr. $I_1$ 11. *karmaṃ kāryam*] *karmakāryam* $K_5^+ I_1$ 12. *906*] *96* $K_5^+$ 15. *śaṅkoś*] *śaṅkoñ* $K_5^+$

a *svārdhādiyutaṃ grāhyaṃ*
b *phalam aviśeṣe śanair yadāsaktiḥ /*
c *ūrdhvādhogamanaṃ cec*
d *chaighryād yuktyā tadā dalādyūnam //* **233** *//*

*aviśeṣakarmaṇi yadā sādhyasyāsaktiḥ śanair bhavati tadā tatra tatra labdhaṃ phalaṃ yuktyā svārdhena yutaṃ vā ekaghnaphalayutaṃ vā dviguṇayutaṃ vā gater māndyavaśāt grāhyam / yadā gateḥ śaighryāt sādhyam ekadordhvagatam ekadādhogataṃ ca bhavati tadā phalaṃ svārdhena vā tribhāgadvayena vā caturbhāgatrayeṇa vā śaighryavaśād dhīnaṃ kāryam / evaṃ kṛte sādhyasiddhiḥ śīghraṃ bhavati / etat sarvatrāpy aviśeṣavidhau cintyam //*

1. *sādhyasyā*] *sādhyasyaṃ* $I_1$ 3. *vā tribhāga*] *vātra bhāga* $I_1$

a *antaram athavā bāhvoḥ*
b *kevalam athavā dvinighnam api dalitam /*
c *chāyāyāṃ svarṇaṃ syād*
d *aviśiṣṭaphalaṃ prasādhyam iha yasmāt //* **234** *//*

*yadā koṇādidiggatacchāyayāpakramādisādhyate tadā laṃbādibhiḥ sādhanaiḥ sāvayavaiḥ sūkṣmācchāyā sādhyā / vṛṣāntagene śivadiksthite ity atra tathā sādhitāviśiṣṭā cchāyā 838 //*

**233**.a *svārdhādi*] *svārdhādidi* T*, *svādhyādi* $I_2$ **233**.b *aviśeṣe*] *api śeṣe* Sāstrī **233**.b *āsaktiḥ*] *āsaktaḥ* Q* **233**.d *chaighryād yuktyā*] *chaighrāṃ dyuktā* Q* **233**.d *chaighryād*] *chaighyād* X* Sāstrī **233**.d *tadā*] om. X* Sāstrī **234**.b *nighnam*] *guṇam* X* Sāstrī **234**.b *dalitam*] *dalitaṃ/dalitā* $K_7$, *dalitā* Sāstrī **234**.c–d repeated twice in $I_3$ **234**.d *aviśiṣṭa*] *avaśiṣṭa* Sāstrī **234**.d S* adds *yatasmānmānantadā bhujākoṭyāḥ* *atha koṇagatārkacchāyayā tatkālajātena natakālena ca sūryānayanaṃ tadapakramādinā palajyānayanaṃ ca pradarśyate* /

1. *sādhanaiḥ*] *sārdhanaiḥ* $I_1$  1. *sāvayavaiḥ*] *sādhanaiḥ* $K_5^+$

a *koṇagate 'rke chāyā-*
b *karṇasya same smṛte bhujākoṭī* /
c *chāyāvargārdhapadaṃ*
d *tasmān mānaṃ tadā bhujākoṭyoḥ* // **235** //

a *pūrvāparāyatā syāt*
b *koṭir yāmyodagāyatātra bhujā* /
c *chāyākoṭisamā syāt*
d *pūrvāparagā dyuvṛttakoṭir api* // **236** //

a *khārkāntarakālajyā*
b *pūrvāparagā bhaved ghaṭīvṛtte* /
c *natasaṃjñitanāḍīnāṃ*
d *jīveti ca kathyate tadā saiva* // **237** //

a *natajīvayā yadi syād*
b *dyuvṛttakoṭis tadā bhavet kiyatī* /
c *tribhajīvayeti bhavati*
d *dyujyāvṛttasya cārdhaviṣkambhaḥ* // **238** //

a *svāhorātrārdhād iha*
b *sādhyā krāntir bhujādhanuḥ krānteḥ* /
c *taddhanur iha bhānuḥ syāt*
d *tadrahitaṃ maṇḍalārdham athavārkaḥ* // **239** //

a *atha yāmye gole syāc*
b *cakrārdhaṃ taddhanuryutaṃ bhānuḥ* /
c *taddhanurūnaṃ cakraṃ*
d *vārko divasadvayaprabhāmānāt* // **240** //

a *aviśeṣakarmaṇākṣa-*
b *jyātra ca sādhyā prabhābhujādivaśāt* /
c *krāntiḥ kenāpi yutā*
d *sūryāgrety atra kalpyate prathamam* // **241** //

a *samavidiśoḥ sūryāgra-*
b *cchāyābāhvoḥ kramād viyogayutī* /
c *śaṅkvagraṃ tacchaṅkor*
d *vargaikyapadaṃ dyumaṇḍalajyeṣṭā* // **242** //

**235**.a–**238**.a *'rke…yadi*] lacuna $K_5^+$  **235**.b *smṛte*] *smate* V* (corr.$_{\text{sec.m.}}$ $I_2$), *smṛte* S*  **235**.d *koṭyoḥ*] *koṭyāḥ* T* (corr.$_{\text{sec.m.}}$ $I_2$)  **236**.b *koṭir*] *koṭī* $I_1$  **236**.d *dyuvṛtta*] *dvivṛtta* $K_7$ Sāstrī  **238**.d *cārdha*] *cātra* $K_7$ Sāstrī  **239**.b *krāntir*] *krānti* T*$I_1$ (corr.$_{\text{sec.m.}}$ $I_2$)  **239**.d *athavā*] *adhavā* S*  **241**.a *aviśeṣa*] *aviśeṣe* Q*  **241**.d *kalpyate*] *kalpite* S*  **241**.d *prathamam*] *prathamaḥ* $K_4I_1$  **242**.a *vidiśoḥ*] *dvidiśoḥ* W*, *viśadoḥ* S*$K_6$ (corr.$_{\text{sec.m.}}$ $K_6$)  **242**.b *kramād*] *bhramād* $K_7$  **242**.b *yutī*] *yuktiḥ* $K_5$, *yutiḥ* $K_7$  **242**.c *chaṅkor*] *chaṅkvo* S*, *chaṅko* corr.$_{\text{sec.m.}}$ to *chaṅkvo* $I_2$  **242**.d *pada*] om. X* Sāstrī, Sāstrī puts *ca* after *aikyaṃ* in parenthesis **242**.d *maṇḍalajye*] *mandale* $K_5$

a *trijyā śaṅkvagrahatā*
b *bhakteṣṭadyujyayā palajyā syāt /*
c *palato lambajyā syāl*
d *lambāpamato bhavet sphuṭārkāgrā // **243** //*

a *punar api kuryāc chāyā-*
b *bāhudineśāgrayor viyogādim /*
c *śaṅkvagreṣṭadyujye*
d *palajīvālambajīvake 'rkāgrā //*
e *aviśeṣāntam ihaivaṃ*
f *sphuṭāviśiṣṭā bhavet palajyātra // **244** //*

*atrodāharaṇam /*

a *chāyādryaṅgarasaikasaṃmitanarasyoktā navaikābdhibhis*
b *tulyā rudradiśaṃ gate dinapatau vyomārkayoś cāntare /*
c *prāṇā bhūdharavedabāṇanayanair abdhyaṃśakaiḥ saṃmitā*
d *vācyo 'rkaś ca palaṃ tvayā gaṇitavid gole kṛtaś cec chramaḥ // **245** //*

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*atra śaṅkuḥ 1667 / tacchāyā 419 / ābhyāṃ svakarṇam ānīya karṇacchāyābhyāṃ trijyākarṇe siddhā mahācchāyā 838 / tacchaṃkuḥ 3334 / cchāyāvargārdhapadam 592 / tadavayavaviliptāḥ 33 / anena mūlena samā tadānīṃ chāyākarṇakṣetre bhujā tathā koṭiś ca / etat koṭisamā tadānīṃ dyumaṇḍale pūrvāparāyatā koṭijyāpi yataś chāyākoṭir dyuvṛttakoṭyām avatiṣṭhate / khamadhyārkayor antarālagatanatāsavaś caturguṇitā 2547 / ete 'bdhyaṃśakatvāc caturbhir hartavyāḥ / tathā kṛte prāṇāḥ 636 / tadavayavāḥ ṣaṣṭyaṃśāḥ 45 / eṣāṃ jīvāḥ 633 / avayavāś ca 4 / eṣā ghaṭikāmaṇḍale pūrvāparāyatā jyā / chāyākoṭisamā dyuvṛttakoṭijyā trijyāhatā natajyābhaktā kiṃcid ūnā 3218 / etat cāhorātrārdham / asmāt siddhāpamaḥ 1210 / asya bhujā kiṃcid ūnā 2978 / asya dhanur ekaliptāsahitaṃ rāśidvayam / etat sūryaḥ / tadūnaṃ rāśiṣaṭkaṃ vā sūryaḥ / aparadinacchāyā cet prathamaḥ / pūrvadinacchāyādhikā ced dvitīyaḥ //*
*athākṣasiddhyartham iṣṭāpame 1210 iṣṭasaṃkhyā prakṣepyā / tatra daśabhir yutā krāntiḥ 1220 / eṣārkāgreti kalpyate / cchāyākarṇabāhuḥ 593 / samadiśor anayor antaraṃ 627 / etac chaṃkvagram / śaṅkuḥ 3334 / bhujākoṭirūpayor anayor vargayogamūlaṃ 3392 / etat karṇarūpā svāhorātreṣṭajyā / punaḥ śaṅkvagranihatāṃ trijyām anayā svāhorātreṣṭajyayā vibhajet / tatra labdhaṃ 636 / etat palajyeti kalpyā / palajyātrijyākṛtyor viśleṣamūlaṃ 3379 / etad avalaṃbakajyā / punas trijyānihatāṃ krāntiṃ laṃbakajyayānayā vibhajet / tatra labdhaṃ sphuṭārkāgrā 1231 / punar apy arkāgrācchāyābāhor viśleṣaṃ śaṅkvagraṃ prakalpya proktavidhināviśiṣṭām akṣajyām ānayet / tatrāviśiṣṭā sphuṭākṣajyā 647 //*

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1. *atra ...*] Entire part of this commentary is broken in $K_5^+$ 6. *4*] *tva* $I_1$ (ത്വ instead of ൪) 13. *anayor*] *anayor anayor* $I_1$ 13. *3392*] *3394* $I_1$ 16. *1231*] *12131* $I_1$

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**243**.b *palajyā*] *palayajyā* S*, *palajyayā* $K_5$ **244**.a *kuryāc*] *kāryāc* U*, *karyāc* $K_1$ **244**.b *ādim*] *ādi* U*, *ādim* $K_3$ **244**.d–**246**.a *ālamba...śaṅko*] br. $K_5^+$ **244**.d *jīvake*] *jīvako* Q* **244**.f *viśiṣṭā*] *vidhiṣṭā* T*, *vidhīṣṭā* S* **245**.a *chāyādryaṅgara*] *chāyādvaṅgara* $K_5$ **245**.a–c *saṃmitanarasyoktā ...abdhyaṃśakaiḥ*] om. S* **245**.a *navaikābdhibhis*] *navaikābhis* $K_5$ **245**.b *diśaṃ*] *graśaṃ* $K_5$

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**244**. Every descendant of V* ($K_1$, $K_3$, $K_6$, $K_7$, $K_8$, $I_2$, $I_3$) and Sāstrī put verse number after d. There is no verse number after f in any manuscript.

**245**. *śārdūlavikrīḍita* verse.

*atha yāmyagole udāharaṇam |*

a *śaṅkor ekadaśāṃśakaṃ rasaviyaccandrāṃśakaṃ ca tyajec*
b *chaṅkoḥ śeṣa iha prabhā dinapatau yāte kṛśānor diśam |*
c *prāṇāś cārkanatodbhavā rasadharārandhrakṣamābhiḥ samā*
d *brūhi prājña divākaraṃ palam api tvaṃ golavit syād yadi ||* **246** *||*

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*atra svamatikalpitaśaṅkuḥ 2454 | ṣaṣṭyaṃśāś ca 28 | asmād ekottaraśatena labdham 24 | ṣaṣṭyaṃśāḥ 18 | punar api tasmāt ṣaḍuttaraśatena labdham 23 | ṣaṣṭyaṃśāḥ 9 | etat phaladvayaṃ svakalpitapūrvaśaṅkos tyajyet | tatra śiṣṭam 2407 | ṣaṣṭyaṃśaḥ 1 | etat tasya śaṅkoś chāyā bhavati | ābhyāṃ śaṅkucchāyābhyāṃ siddhaḥ karṇas trijyāsamaḥ syāt | ata ete evātra mahāśaṅkumahācchāye bhavataḥ | natāsavaḥ 1916 | tajjyā 1818 | ṣaṣṭyaṃśāḥ 17 | cchāyākoṭisamo dyukhaṇḍaḥ 1702 | ṣaṣṭyaṃśaḥ 1 | atra labdhā dyujyā 3217 | ṣaṣṭyaṃśāḥ 54 | apamaḥ 1209 | ṣaṣṭyaṃśāḥ 38 | asmāt siddhā bhujajyā | prāyo dvirāśijyā samā | rāśidvayaṃ taccāpam | tanmaṇḍalārdhayutaṃ bhānuḥ | tadūnaṃ maṇḍalaṃ vā bhānuḥ | akṣas tu pūrvāvat ||*

1. *ṣaṣṭyaṃśāś ca*] *ṣaṣṭyaṃśāścaṃśāḥ* $K_5^+$, *ṣaṣṭyaṃśāścaṃś ca* +(space) *śāḥ* $I_1$ 1. *28*] *18* $K_5^+$, *38* $I_1$ 1. *asmād …18*] transversed with *punar api …ṣaṣṭyaṃśāḥ 9* $K_5^+$, om. $I_1$ 1. *24*] *2* $K_5^+$ 2. *18*] *9* $K_5^+$ 2. *9*] *28* $K_5^+$ 2. *phala*] *pala* $K_5^+ I_1$ 5. *17*] *54* corr. $K_5^+$

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a *iṣṭadiksthe savitary apy*
b *anena nyāyena sakalaṃ sādhyam ||* **247** *||*

a *vṛtte kumadhyakendre*
b *nijakakṣyāsaṃmite bhramanti khagāḥ |*
c *draṣṭā kupṛṣṭhagaḥ syāt*
d *kupṛṣṭhamadhyaṃ tato 'sya dṛgvṛttam ||* **248** *||*

a *kṣitijād bhūmadhyagatād*
b *bhūvyāsārdhāntare bhaved ūrdhvam |*
c *draṣṭuḥ svīyaṃ kṣitijaṃ*
d *yasmāt tatrodayo 'sya cāstamayaḥ ||* **249** *||*

a *bhūmadhyāt kṣitijastho*
b *vihago draṣṭur bhaved adhaḥ kṣitijāt |*
c *bhūvyāsārdhamitādho-*
d *gatiś ca sā tasya lambanam ihoktam ||* **250** *||*

a *bhūmadhyasyordhvagataṃ*
b *vihagaṃ draṣṭā ca paśyati svordhvam |*
c *tasmāt khamadhyasaṃsthe*
d *vihage na tu lambanaṃ bhavet tasya ||* **251** *||*

**preamble**.1 *yāmya*] *yo* U*, *yā* $K_1$ **246**.c *prāṇāś*] *ghrāṇāś* T*, *pāṇāś* S* **247**.b *sakalaṃ*] *sarvaṃ* $K_5^+$, *sarvaṃ sarvaṃ sarva* $I_1$ **248**.a *vṛtte ku*] *vṛttaika* U*, *vṛtte ka* Q* **248**.c *draṣṭā*] *diṣṭā* $I_1$ **248**.d *'sya*] om. S* **249**.a–**250**.b *kṣitijād …adhaḥ*] repeated twice Y* **250**.c *mitādho*] *mito dho* Y*, *mito* V* **251**.b *draṣṭā*] *dṛṣṭā* $I_3$ **251**.b *svordhvam*] *sordhaṃ* R* (corr.$_{\text{sec.m.}}$ $I_2$), *sārdhaṃ* $K_6$, *sordhvaṃ* $K_7$, *svordhaṃ* $I_3$

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**246**. *śārdūlavikrīḍita* verse.

**247**. $K_5^+$ and $I_1$ include this verse in the commentary on *GD2* 246.

a *na syān nabhaso madhye*
b *kṣitije syāl lambanaṃ paraṃ yasmāt /*
c *dṛgjyātaḥ sādhyaṃ syād*
d *anupātāl lambanaṃ khagasya tataḥ //* **252** *//*

a *trijyāntare khamadhyād*
b *bhūvyāsārdhaṃ yadi svakakṣyāyām /*
c *dṛgjyāntare tadā kiṃ*
d *syād iti tatkālalambanaṃ bhavati //* **253** *//*

a *lambanayojanamāne*
b *tulye 'py atraikaliptikāsthānāt /*
c *lambanaliptā bhinnāḥ*
d *kakṣyābhedād bhavanti vihagānām //* **254** *//*

a *lambanayojanamānaṃ*
b *nijakakṣyāyām iyat khagasya yadi /*
c *trijyāvṛtte syāt kiyad*
d *iti lambanaliptikāmitir bhavati //* **255** *//*

a *trijyāmaṇḍalam uditaṃ*
b *liptāsamayojanaṃ tato 'trāptam /*
c *yojanaphalam api liptā-*
d *phalaṃ bhaven nāmabheda eva yataḥ //* **256** *//*

a *ekakalāsthān vihagān*
b *paśyati tasmāt kupṛṣṭhago draṣṭā /*
c *bhinnasthānāc chīghras*
d *tatrādhaḥstho 'lpabhuktir ūrdhvagataḥ //* **257** *//*

a *nijalambanāntarasamaṃ*
b *grahayor vivaraṃ tadādha-ūrdhvagatam /*
c *draṣṭā paśyati yasmād*
d *ubhayor api lambanaṃ nijaṃ bhavati //* **258** *//*

a *nijanijalambanaliptā*
b *svāt svāc chaṅkor viśodhya śiṣṭaṃ tu /*
c *bhūpṛṣṭhe sphuṭaśaṅkuḥ*
d *svīyaḥ syād iti ca siddham atra bhavet //* **259** *//*

a *chedyakadṛśyam idaṃ syād*
b *vilikhed vṛttaṃ bhuvo 'tha tanmadhyam /*
c *kendraṃ kṛtvā svaṃ svaṃ*
d *kakṣyāvṛttaṃ likhet sadiksūtram //* **260** *//*

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**252**.d *anupātāl*] *anupātā* Q* **253**.d *kāla*] *kalā* $K_6$, *kadā* $K_7$ **254**.b *sthānāt*] *sthānāṃ* $I_1$ **257**.b *pṛṣṭha*] *pṛṣṭa* Sāstrī **257**.c *sthānāc chīghras*] *sthānā śīghras* $I_1$ **258**.c *draṣṭā*] *dṛṣṭā* $K_4$ **259**.a *liptā*] *liptāt* U* Sāstrī **259**.d *iti ca*] *atra* $I_1$ **259**.d *atra*] *eva* $I_1$ **260**.d *vṛttaṃ*] *sūtraṃ* X* Sāstrī

a *kendraṃ kṛtvā yāmyo-*
b *daksūtrakuparidhiyogam atha vilikhet* /
c *vṛttaṃ trijyāsūtreṇ-*
d *aitad dṛṅmaṇḍalaṃ sadiksūtram* // **261** //

a *bhāgair aṅkitam athavā*
b *ghaṭikābhiḥ sarvavṛttam iha kāryam* /
c *yāmyodaksūtram iha*
d *prakalpyam adha-ūrdhvayātasūtram iti* // **262** //

a *kakṣyāvṛtte svīye*
b *yatame bhāge grahas tadā carati* /
c *dṛṅmaṇḍale 'pi tatame*
d *bhāge kuryāt khagarkṣabindum iha* // **263** //

a *kakṣyāparidhigakhecara-*
b *dṛṅmaṇḍalakendragasya sūtrasya* /
c *dṛṅmaṇḍalaparidhiyutau*
d *binduṃ grahasaṃjñitaṃ punaḥ kuryāt* // **264** //

a *anayoḥ khagarkṣakhecara-*
b *saṃjñitabindvor yad antarālaṃ syāt* /
c *lambanaliptāmānaṃ*
d *tad bhavati hi khecarasya tadā* // **265** //

a *kakṣyāvyāsasame dve*
b *sūtre dṛgvṛttamadhyato neye* /
c *bindudvayage ca tayoḥ*
d *śirontaraṃ lambayojanasya mitam* // **266** //

a *dṛṅmaṇḍala eva syād*
b *vihagābhimukhe vilambanaṃ satatam* /
c *lambanam iti dṛgbhedo*
d *dṛṣṭir draṣṭuḥ khagānugā ca yataḥ* // **267** //

a *karṇātmakam uktam ato*
b *lambanam apamānugā tu tasya gatiḥ* /
c *bāhus tad itaragā syāt*
d *koṭir grahaṇe hi lambananatī te* // **268** //

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**261**.b *sūtrakuparidhi*] *sūtrakaparidhi* Sāstrī **261**.d *aitad*] *aika tad* T* (corr.$_{\text{sec.m.}}$ $I_2$) **262**.d *prakalpyam*] *kalpyam* $K_5$ **262**.d *adha*] *atha* $K_4$ **263**.a–b om. $I_3$ **263**.b *yatame*] *yatime* Z* **263**.b *tadā*] *sadā* $K_8$ **263**.c–**264**.c *'pi …dṛṅmaṇḍala*] om. $I_2$ (Omitted text was inserted by later hand but still lacks 264.b) **263**.c *tatame*] *tatime* Z* **263**.d *bhāge*] om. V* except $I_2$ where it is part of a long insertion **263**.d *bindum*] *vindum* T* except $I_2$, ditto **264**.b om. U* **264**.d *binduṃ*] *vinduṃ* T* **265**.a *anayoḥ*] *anuyoḥ* $K_5$ **265**.b *saṃjñita*] om. $K_4$ **265**.b *bindvor*] *vindor* T*, *bindor* $K_1K_5$ Sāstrī **266**.a *kakṣyāvyāsa*] *kakṣyākhyāsa* U*, *kakṣyāsa* $K_6$, *kakṣyāyāsa* $I_2$ **266**.b *dṛgvṛtta*] *digvṛtta* X* Sāstrī **266**.b *madhyato*] *maddhato* S* **266**.c *bindu*] *vindu* V* **266**.d *mitam*] *miti* $I_1$ **267**.a *maṇḍala*] *maṇḍa* Q* **267**.a *eva syā*] om. S* **267**.c *dṛg*] *dig* $K_7$ **267**.d *dṛṣṭir*] *dṛṣṭair* $K_7$ **267**.d *khagānugā*] *khagānugāya* S*$I_2$ **268**.b *apamānugā tu tasya*] *apamānugasya* $K_4$ **268**.b *gatiḥ*] *natiḥ* X* Sāstrī **268**.d *koṭir*] *koṭi* S*Q* Sāstrī **268**.d *hi lambana*] *vilambana* X* Sāstrī

a *lambanam ity apamagatir*
b *grahaṇe vihagasya kalpyate gaṇakaiḥ* /
c *natir iti ca svād apamād*
d *vikṣepas te tato bhujākoṭī* // **269** //

a *dṛkkṣepaguṇāt sādhyā*
b *koṭir bāhus tu dṛggatijyātaḥ* /
c *dṛgjyādṛkkṣepajyā-*
d *kṛtivivarapadaṃ hi dṛggatijyoktā* // **270** //

a *śūnye sati dṛkkṣepe*
b *lambanam apamaṇḍale sthitaṃ sarvam* /
c *apamaṇḍalam eva tadā*
d *yasmād dṛṅmaṇḍalaṃ grahābhimukham* // **271** //

a *triguṇasame dṛkkṣepe*
b *lambanam apamasya pārśvagaṃ nikhilam* /
c *dṛṅmaṇḍalasya madhye*
d *yasmād raśanāvad apamavṛttam iha* // **272** //

a *dṛkkṣepābhidhakoṭyā*
b *vṛddhivaśāt syād ato 'tra nativṛddhiḥ* /
c *dṛggatisaṃjñitabāhor*
d *vṛddhivaśāl lambanasya vṛddhir api* // **273** //

a *bhūmivyāsārdhahatād*
b *dṛkkṣepād dṛggateś ca ye labdhe* /
c *trijyābhidhakarṇena*
d *kramaśo natilambayojanamitī te* // **274** //

a *yojanakarṇe yojanam*
b *etāvac cet kiyat triguṇakarṇe* /
c *iti natilambanayor iha*
d *sādhyā liptātmikā mitiś cāpi* // **275** //

a *dṛggatidṛkkṣepajye*
b *dṛgjyākarṇasya bāhukoṭī cet* /
c *lambanakarṇasya tu ke*
d *iti vā grahaṇoktalambananatī staḥ* // **276** //

a *yojanakarṇo bhānoḥ*
b *pañcāhīṣvaṅkabāṇajaladhisamaḥ* /
c *indor yojanakarṇaḥ*
d *parvatanagarāmavedadahanasamaḥ* // **277** //

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**269**.a–b *apamagatir grahaṇe*] *apamagatigrahaṇe* Sāstrī **270**.a–b *dṛk…koṭir*] om. $K_4$ **271**.d *yasmād*] *yasya* Q* **271**.d *maṇḍalaṃ grahā*] *maṇḍalāgraṃ vā* $K_4$ **272**.c *maṇḍala*] *maṇḍa* S*, *mala* $K_3K_6$ **273**.a *ābhidha*] *āpama* $K_2$, *ama* $K_4$ **273**.b *nati*] *gati* $I_1$ **273**.c *dṛggatisaṃjñita*] *++dṛggati* Sāstrī **273**.c *saṃjñita*] om. X* **275**.b *kiyat tri*] *kiyati* $K_5$ **276**.d *grahaṇokta*] *grahaṇoktana* W* (corr. $K_2$) **276**.d *lambananatī*] *lambanatī* S* **276**.d *staḥ*] *sta* T* (corr.$_{\text{sec.m.}}$ $I_2$) **277**.a *karṇo*] *karṇe* T* Sāstrī (corr.$_{\text{sec.m.}}$ $I_2$) **277**.d *nagarāma*] *karṇāma* $K_7$

a *aviśeṣakarṇanihatau*
b *trijyābhaktāv imau sphuṭau bhavataḥ |*
c *nīcoccabhāgago 'smād*
d *adha upari cared yato grahaḥ sthānāt ||* **278** *||*

a *vyomendūdadhivedais*
b *tulito bhānor vidhos tithijvalanaiḥ |*
c *kheṣukhavidhubhir bhūmer*
d *vyāso bimbasya yojanaiḥ proktaḥ ||* **279** *||*

a *bimbavyāsāv uditau*
b *raviśaśinos triguṇatāḍitau ca tayoḥ |*
c *sphuṭayojanakarṇābhyāṃ*
d *vihṛtau liptātmakau sphuṭau bhavataḥ ||* **280** *||*

a *svādhaḥsthitena śaśinā*
b *chādanam uditaṃ raver nijaṃ grahaṇam |*
c *kakṣyābhedād anayoḥ*
d *pratideśaṃ chādanaṃ raver bhinnam ||* **281** *||*

a *nijamārgagabhūcchāyā-*
b *praveśa indor nijaṃ grahaṇam uktam |*
c *tamasi praviṣṭa induḥ*
d *sarvatraikaprakāra eva bhavet ||* **282** *||*

a *tamasā bādhyaś candraḥ*
b *katham iti ced ucyate tamohantā |*
c *bhānoḥ karā hi śaśinaḥ*
d *karās tatas tamasi te kathaṃ syur iti ||* **283** *||*

a *tejaḥsūtraṃ yasmin*
b *patati sthānaṃ hi tat prakāśayutam |*
c *tejaḥsūtravihīnaṃ*
d *sthānaṃ tamasāvṛtaṃ bhaven nikhilam ||* **284** *||*

a *yatra ravir bhūchannas*
b *tatrasthatamo bhavet kṣiticchāyā |*
c *tasyā mānaṃ sādhyaṃ*
d *chāyāyuktyā pradarśyate cātra ||* **285** *||*

a *śaṅkur ināṅgulatulyas*
b *taddviguṇasamonnatiḥ pradīpasya |*
c *śaṅkupradīpavivare*
d *bhūḥ śaṅkumitātra cintyate chāyā ||* **286** *||*

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**278**.a *nihatau*] *nītau* Q* (corr.$_{\text{sec.m.}}$ K$_6$) **278**.c *bhāgago*] *bhāgo* S* (corr. K$_8$) **280**.b *tāḍitau*] *tāhitau* K$_7$ *tāpi tau* Sāstrī **280**.b *ca*] *rca* W* (corr. K$_2$) **281**.b *uditaṃ*] *ucitaṃ* K$_5$ **283**.a *bādhyaś*] *madhyaś* Q* **283**.d *te*] om. K$_5$ **284**.d *vṛtaṃ*] *mṛtaṃ* K$_2$ **285**.a *channas*] *chāyānnas* K$_6$ **285**.b *sthatamo*] *sthamato* Q* **286**.a *śaṅkur inā*] *śaṅkuvinā* T* (corr.$_{\text{sec.m.}}$ I$_2$) **286**.b *taddviguṇa*] *tadviguṇa* Z* **286**.b *pradīpasya*] *pradīpa* Q*

a *dīpāt pravṛttasūtraṃ*
b *śaṅkuśiraḥspṛk patet kṣitau yatra* /
c *chāyāgraṃ tatra bhavec*
d *chaṅkoḥ sūtraṃ ca karṇasaṃjñaṃ tat* // **287** //

a *chāyāgraśaṅkumūlā-*
b *ntarabhūr bāhus tu śaṅkukoṭyā syāt* /
c *chāyāgradīpamūlā-*
d *ntarabhūr bāhuḥ pradīpakoṭyāś ca* // **288** //

a *śaṅkūnadīpakoṭyā*
b *bāhuḥ śaṅkvagradīpavivaragataḥ* /
c *karṇas tu bāhukoṭyor*
d *agradvayavivaragaṃ bhavet sūtram* // **289** //

a *śaṅkūnadīpakoṭyā*
b *bāhuś cec chaṅkudīpavivarabhuvā* /
c *tulito 'tra śaṅkukoṭyāḥ*
d *ko bāhur iti prabhā bhavec chaṅkoḥ* // **290** //

a *ravibimbavyāsārdhaṃ*
b *dīpo bhūvyāsadalam iha tu śaṅkuḥ* /
c *sphuṭayojanakarṇaḥ syād*
d *bhānoḥ śaṅkupradīpavivarajabhūḥ* // **291** //

a *atroditasya śaṅkor*
b *yā chāyā sā bhavet kṣiticchāyā* /
c *vṛttā sā bhūmisamā*
d *mūle 'lpā śirasi pucchavat sā goḥ* // **292** //

a *raviparidhinirgatānāṃ*
b *sūtrāṇāṃ yatra bhūparidhigānām* /
c *saṃyogo vyomni bhaved*
d *bhūcchāyāyā bhaved dhi tatrāgram* // **293** //

a *sphuṭayojanakarṇo 'to*
b *bhānor bhūvyāsatāḍito 'rkabhuvoḥ* /
c *vyāsāntareṇa bhakto*
d *bhūcchāyādairghyayojanamitiḥ syāt* // **294** //

a *icchārāśer atra*
b *dvaiguṇyāj jāyate na phalabhedaḥ* /
c *yasmād dvābhyāṃ nighnaḥ*
d *pramāṇarāśiś ca parigṛhīto 'tra* // **295** //

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**287**.b *patet*] *pate* Q*, *caret* $K_5$ **288**.a *śaṅku*] *karṇa* W* Sāstrī **288**.c *dīpamūlā*] *mūladīpā* $K_7 I_1$ **289**.b *gataḥ*] *gataṃ* Y*V* **289**.c *karṇas*] *karṇās* Q* **290**.c *koṭyāḥ*] *koṭyoḥ* W*$K_1$ (corr. $K_1$) **291**.c *karṇa*] *karṇā* Q* **292**.c *samā*] *sthamā* $K_4$ **292**.d *śirasi*] om. $K_5$ **292**.d *sā goḥ*] *sāgrauḥ* T*, *sāgroḥ* $K_1$, *sāgre* $K_7$ Sāstrī, *sagauḥ* corr.$_{\text{sec.m.}}$ $I_2$ **294**.a *'to*] *tā* Q*, *rkatā* corr.$_{\text{sec.m.}}$ to *rke tā* $I_2$ **294**.b *bhuvoḥ*] *bhavoḥ* Q* **295**.b *guṇyāj*] *guṇyāñ* $K_5$

a *sphuṭayojanakarṇonād*
b *indor bhūvyāsatāḍitāl labdham* /
c *chāyādairghyāc chāyā-*
d *dairghyeṇa vyāsamānam iha tamasaḥ* // **296** //

a *tamaso vyāsas trijyā-*
b *nihataś candrasya yojanaśrutyā* /
c *vihṛtas tamaso bimbaṃ*
d *kalātmakaṃ bhavati śiśirakaramārge* // **297** //

a *chāyādairghyaṃ śaśinaḥ*
b *sphuṭayojanakarṇavivararahitaṃ yat* /
c *śaśimārgād ūrdhvagatac-*
d *chāyābhāgasya dairghyamānaṃ tat* // **298** //

a *chāyāgrāt taddairghyā-*
b *ntare kutulyo hi bhavati tadvyāsaḥ* /
c *śaśimārgordhvagatāśā-*
d *ntare tadā syāt ka iti tamovyāsaḥ* // **299** //

a *yadi śaśikakṣyāyāṃ syād*
b *etāvān kas tadā triguṇavṛtte* /
c *iti tamaso bimbaṃ syāt*
d *kalātmakaṃ śiśiradīdhiter mārge* // **300** //

a *chādyacchādakavivara-*
b *kṣetraṃ tadbimbadalayuter ūnam* /
c *yāvat tāvad grahaṇaṃ*
d *tato 'dhike dṛśyate grahaḥ sakalaḥ* // **301** //

a *ity uditā saṃkṣepād*
b *asmābhir goladīpikā ya imām* /
c *puruṣaḥ paṭhet sa loke*
d *golavidāṃ gaṇyate nṛṇāṃ madhye* // **302** //

1 *iti goladīpikā samāptā* //

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**296**.c *dairghyāc*] *dairghya* $K_7$, *dairghyaṃ* Sāstrī **296**.d *vyāsamānam*] *samānam* $K_5$ **298**.b *vivararahitaṃ*] *virahitaṃ* $K_5$, *vivarahitaṃ* $K_7$ **298**.b *yat*] *yāt* $I_1$ **298**.c *gatac*] *gataṃ* Y*, *gataḥ* S* **299**.d *ntare*] *ntareṇa* $K_3$ **300**.a–b *syād etāvān*] *syād detāvān* T*, *syād etāvān* $K_7$ **300**.b *kas*] *kadas* S* **300**.c *tamaso*] *tamo* T* (corr.$_{\text{sec.m.}}$ $I_2$) **302**.b *imām*] *imā* T*$K_1$ (corr.$_{\text{sec.m.}}$ $I_2$) **colophon**. *iti goladīpikā samāptā*] om. $K_5$ **colophon**. *iti*] om. W*$K_1I_1$

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**301**. $I_1$ adds *varāhamihirasaṃhitāyāṃ* followed by *BṛS* 5.1-15 and *śrīpatiḥ* followed by *SŚe* 17.17 and *SŚi.G* 11.10
**302**. $I_1$ adds:
*doṣo 'py eko yadi bahuguṇās tatra muktvā guṇaughān* /
*doṣagrāhī bhavati hi khalas sallikātulyadharmā* //
*doṣaṃ muktvā guṇam anubhavan svatvam apy eti tṛptim* /
*sādhur loke salilamilitakṣirapāyīva haṃsaḥ* // (*kṣira* in the last line should actually be *kṣīra*)

**colophon**. $K_1$ adds: *śrīgurubhyo namaḥ*, $K_2$ adds: *śiva*, $K_3$ adds: *hariḥ gam śivam astu*, $K_4$ adds: *śiva*, $K_7$ adds: *karakṛtam aparādhaṃ kṣantum arhanti santaḥ śubhaṃ*, $K_8$ adds: *śubham astu hariḥ*, $I_1$ adds: *nārāyaṇāya namaḥ śivam astu*, $I_2$ adds: *śubham astu*, $I_3$ adds: *śubham astu hariḥ*

# Part II

# Translation

# Notes on the translation

**Technical terms** Whenever Sanskrit terms have their counterparts in the English vocabulary of astronomy, the English word is used even if it is different from the literal sense of the Sanskrit. Otherwise, words are chosen with respect to both the literal meaning of the Sanskrit term and its concept in astronomy. For consistency, even terms that have only been transliterated in previous conventions are given an English form; e.g. "slow"-apogee instead of *manda-ucca*.

As for measuring units, transliterations of the original Sanskrit words are used, with the exception of arc lengths (signs, minutes, etc).

**Numerical values** Every value is translated into English words, followed by Arabic numerals when there are more than three digits; e.g. twelve, three hundred and fifteen (315). Usages of the *Bhūtasaṃkhyā* system, i.e. specific nouns instead of numerals such as "eye" for two or "mountain" for seven, are listed in Appendix A.1.

**Commentary** Translations for the commentaries are given in the same order as in the critical edition, including the lines for separating them from the base text. Most of the commentaries are solutions for the examples, but some of them gloss the base text by paraphrasing or supplying words. For the latter case, every word that is cited from the base text is indicated by "quotations". This is only for clarification and does not reflect the appearance in the manuscripts. Paraphrases are also shown by giving the Sanskrit words in the base text and the commentary.

# *Goladīpikā*

**1** Having bowed to Gaṇeśa (*vighneśa*; remover of obstacles), Sarasvatī (*vāgdevī*; goddess of speech), teachers and the planets beginning with the sun, I shall state the stellar sphere, the size of the Earth and so forth for this novice.

**2** This circle going below, above, south and north is what is called the "solstitial colure". The celestial equator is touching at the tip of the geographic latitude north and south from below and above respectively.

**3** Again from this [celestial equator's points] below and above, the ecliptic is touching at [the tip of the] greatest declination likewise (north and south). A girdle at the middle of the celestial equator, transverse to the rotation, is another circle.

**4** This is known as the equinoctial colure [or equal division circle]. The celestial equator and the solstitial colure are also [called] likewise. The sun always moves eastward on the circle called the "ecliptic".

**5** Since the equinoctial colure going through the middle of the celestial equator and the solstitial colure are connected to one another there is a pair of crosses. The axis going through the middle of the sphere pierces them.

**6** One should make a uniformly round Earth, located at the middle of the axis of the stellar sphere, out of either a piece of wood or clay. The dwelling of living beings and so forth are assumed to be within it.

**7** The stellar sphere hurled by the *pravaha* wind goes clockwise around the Earth and rotates continuously toward the west in sixty *ghaṭikā*s.

**8** The *pravaha* wind should have a constant movement toward the west above the Earth's surface at a distance of twelve *yojana*s. The wind of Earth having a different movement is below it.

**9** Here, the time in which a sixtieth of the celestial equator rotates is indicated as a *nāḍikā* (i.e. *ghaṭikā*), not the sixtieth of a day, because a day is longer than a revolution of the [stellar] sphere.

**10** On the side of the celestial equator is a circle that is a companion of the celestial equator. It is indicated as the diurnal circle that is the place of the sun's revolution.

**11** Many of them exist, because for each day there is a difference in the motion of the sun. This is the stellar sphere. The celestial sphere outside it should be immovable.

**12** The prime vertical situated on the celestial sphere is indicated as going through the east, west, below and above. What is called the "prime meridian" on it (the celestial sphere) too should go through the south, north, below and above.

**13** Here, the horizon situated on the side of the Earth goes through the east, west, south and north. The rising and setting of all the stars and planets takes place on it.

**14** One should know that the six o'clock circle which is situated on the celestial sphere is touching at a distance in degrees which is the geographic latitude below the south and above the north, and is touching to the east and west.

**15** This axis of the sphere goes through the crosses of the six o'clock circle and the prime meridian. In the portion above the six o'clock circle the revolution of the sphere takes thirty *nāḍī*s (i.e. *ghaṭikā*s).

**16** The horizon is situated below the six o'clock circle in the north and goes above it in the south. Therefore when the sun is to the north [of the celestial equator] the daylight is long and when to the south it is the night that is long.

**17** Or, having made the celestial equator in the east-west direction, and having made another one according to it, one should make an axis piercing the crosses of the six o'clock circle and the prime meridian.

**18** The stars are immobile. Below them in order are Saturn, Jupiter, Mars, the sun, Venus, Mercury and the moon. They have an eastward motion, [but also] move to the west because of the impetuosity of the [stellar] sphere.

**19** They (the motion of planets) are equal in terms of motion per day counted in *yojana*s. [They are] different [in terms of motion per day] counted in arc minutes. This is because those having large orbits are located above and arc minutes are equal among them all.

**20** The moon has a slow motion, Saturn has a swift motion and the stars are swifter. Moreover, all move toward the west. Some people reportedly say so.

**21** Calculators who have worked hard on the Sphere state that this is not suitable, because of the retrograding planets' conjunction with stars on their west side.

**22** It is remembered by excellent calculators that the orb of the sun and others has the shape of a sphere. [The orb] of the sun is bright but that of the moon is made of water and lacks light on its own.

**23** Those who are foolish want the orb to have the form of a round mirror, because the gradual increase in whiteness of the moon does not occur in their school.

**24** The rays of the sun reflected on the moon made of water destroy the nightly darkness, as [rays] leaving a mirror [destroy the darkness] in a house. Thus is the opinion of a noble person.

**25** The Earth with the shape of a sphere stands in space at all times just by its own power. The upper half is abundant with soil, the lower abundant with water. Here are the oceans and continents.

**26** Some say that the Earth is supported by *Ananta*, others by elephants in cardinal directions. Here, a support of a support is to be assumed, hence they are endless.

**27** The Earth rotates toward the east and there is no revolution of stars going in the sky, thus some reportedly say. This is not the wise Āryabhaṭa's intention.

**28** Demons, gods and human beings always stay in the bottom, top and side parts of this Earth respectively. Likewise other creatures, rivers, mountains and the like.

**29** A circle going through the middle of the Earth stands below all creatures. Therefore it happens that creatures, water and so forth abide everywhere on the Earth's surface.

**30** It is said that the circumference of the Earth counted in *yojana*s is three thousand two hundred and ninety-nine (3299). It is also said by Āryabhaṭa that Mount Meru should be the size of a *yojana*.

**31** Other wise people say that the measure of the Earth in *yojana*s is measured as many crores (tens of millions). This is not the calculators' intention, because the measure is established in another way from the geographic latitude.

**32** [The length of] the ground along the gap between the two which are the locations of the same [longitude] north and south divided by [their] difference in degrees of geographic latitude and multiplied by the degrees in a circle should be the measure of the circumference of the Earth.

**33** The resulting number measured in *yojana*s is several lakhs (hundreds of thousands) when it is on the Earth's surface, and several crores (tens of millions) when it is the resulting number inside the Earth's sphere.

**34** The habitat of creatures exists everywhere, even in the nether regions inside the Earth. The conflicting statements of the wise ones should be considered in this way and should be managed by the wise ones in this case.

**35** Here, the calculators who are experts on the Sphere do not think that Mount Meru has an exceeding elevation, because there exist stars going eastward in the sky north of the pole star.

**36** Some say that Mount Meru goes into the Earth at the top and bottom. In this respect Āryabhaṭa said that it is measured from the top of the Earth's sphere.

**37** At Laṅkā, the sun comes to the top [of the sky] when it is at an equinoctial point. The pole star is always on the horizon. At Mount Meru, this sun is on the horizon and the pole star is at the top. Because both (the sun at equinoctial point and the pole star) have their own spot (i.e. Laṅkā and Mount Meru, respectively) below.

**38** Laṅkā should be at a quarter of the Earth's circumference from the middle of the land or of the water. Ujjain is at a fifteenth [of the Earth's circumference] due north from Laṅkā.

**39** Heaven and Mount Meru are at the middle of the land and hell and the "mare's mouth" at the middle of the water. This *Ārya* verse and a half, spoken by Āryabhaṭa, is written by us here.

**40** The gods stand on Mount Meru located in the middle of the land and the demons are located in the spot of water below that. The manes stand on the middle of the disk of the moon and human beings are situated at the side of the Earth's sphere.

**41** The gods always see the sun located in the northern celestial hemisphere. The demons [always see the sun] located on the other side. [When the sun is on] the six signs beginning with Aries, it is the divine day and that is the demonic night.

**42** The day of the manes is said to start at the time in the middle of the eighth day of the dark [half-month] and ends in the middle of the eighth day of the bright [half-month], because [during this period] they always see the sun.

**43** At a location with no geographic latitude such as Laṅkā and the like, a day is thirty *ghaṭikā*s and a night is just as much. Āryabhaṭa states sites that are on the border of land and water since they have no geographic latitude:

**44** The very sun that rises at Laṅkā sets at Siddhapura. [At the same moment] it is midday at Yavakoṭi and midnight at the region of Romaka.

**45** When daytime and nighttime are added, [the sum] should be sixty *ghaṭikā*s at a location with geographic latitude. There, the day is increased when the sun is in the northern celestial hemisphere and night exceeds when in the south.

**46** At a location where the Sine of co-latitude is equal to the [Sine of] greatest declination, there, when the sun is on the end of Gemini, the day is sixty *nāḍī*s (i.e. *ghaṭikā*s), and this is said:

**47** Ho, say the measure of the latitude where the sun situated on the end of Gemini, like red hot gold on the horizon of the ocean, does not set.

**48** In that case, the Sine of geographic latitude should have the same measure as the "upright" [Sine] of the greatest declination. Therefore, the ascensional difference in *ghaṭikā*s should be fifteen, and hence there are sixty *ghaṭikā*s during daytime.

**49** From thereon, the previous and later daytimes diminish in due order at that location. When the sun is at the end of Sagittarius the night is likewise [sixty *ghaṭikā*s] and the nights on its sides [diminish] in the same manner.

**50** Where the Sine of co-latitude is equal to the declination [corresponding to a longitude of] two signs, Sagittarius and Capricorn appear not to rise, Cancer and Gemini [appear not to] set, and the other eight appear on the horizon.

**51** Leo is the ascendant after Taurus and Aquarius is the ascendant subsequent to Scorpio. Gemini, Capricorn, Cancer or Sagittarius is never known as an ascendant there.

**52** If the Sine of co-latitude is equal to the declination [corresponding to a longitude of] one sign, the four [signs] beginning with Taurus do not appear to set. Likewise Scorpio, Sagittarius, Capricorn and Aquarius do not appear to rise.

**53** Pisces, Aries, Virgo and Libra; thus are the four ascendants there, [rising] in this order, because the others do not reach the horizon.

**54** The six [signs] beginning with Aries appear not to set at Mount Meru, and those beginning with Libra [appear not to] rise. The two sections of visible and invisible [signs] are to be assumed inversely for the demons and gods.

**55** Due to the motion of the sun in the twelve signs, the human year exists here [on Earth]. This is a divine daylight and night. A divine year [is measured] by three hundred and sixty of their days.

**56** One *caturyuga* should be [measured] by twelve thousand divine years. And, masters have called the *caturyuga* a divine *yuga*.

**57** Forty-eight, thirty-six, twenty-four and twelve [each] multiplied by a hundred should be, in order, the divine years in a *Kṛta*, *Tretā*, *Dvāpara* and *Kali*[-*yuga*].

**58** There should be one thousand *caturyuga*s in a daylight of Brahmā, likewise in a night. The creation and maintenance of the world takes place in a day and its destruction in a night.

**59** This daylight is indicated as a *kalpa*. There should be fourteen *manu*s in a daylight of Brahmā. There should be seventy-one *yuga*s during a *manu*. After that is a twilight.

**60** There are fifteen twilights, at the beginning and the end of a *kalpa* and in between *manu*s. It is remembered in this case that six fifteenths of a *caturyuga* is [the length of] a twilight.

**61** Within a twilight in between *manu*s, the former and latter potions are known as the "portion of twilight" and "twilight" respectively. The division of time has been done by some intelligent ones.

**62** Fifty of our own years of Brahmā have past. The very first of the remaining is to be assumed. Within this, six *manu*s have past, as well as twenty-seven *yuga*s in what follows.

**63** Even in the twenty-eighth *yuga*, three in four parts beginning with *Kṛta* have past. This remaining part, the *Kali*, is going on. Thus are the words of an ancient sage.

**64** Brahmā constantly sees the sun exceedingly far away during a *kalpa*. Since the sun does not exist during the destruction [of the world], even Brahmā does not see the sun.

**65** With only one sun, the four kinds of daylights, which are those of gods, of the manes, of humans and of Brahmā, exist. They should be understood with spheres for experts on the Sphere.

**66** Those who say that the moon is above the sun are situated on Mount Meru, where the sages are above the stars, and [above] them all is the pole star.

**67** There, the moon with northern latitude is seen above the sun at the end of Gemini. Therefore they state it like that in such case. Otherwise, it is another moon deity.

**68** Thus the configuration of the sphere is stated concisely by Parameśvara. For the novice, there is more to be said concerning the Sphere.

**69** The grounding of gnomons and so forth, which I have explained previously in the *Siddhānta-dīpikā*, a super-commentary on a commentary of the *Mahābhāskarīya*, shall nevertheless be spoken of.

**70** The great shadow of the sun, when it is at the intersection of the celestial equator and ecliptic at midday, is the Sine of geographic latitude. And its [great] gnomon should be the Sine of co-latitude.

**71** The distance between the celestial equator and the prime vertical on the circle called the "prime meridian" is the geographic latitude. Then the co-latitude is the gap between the two circles called the "celestial equator" and the horizon on that [prime meridian].

**72** Otherwise, the Sine produced in the gap between the horizon and the pole star should be the Sine of geographic latitude. Then the Sine produced in the gap between the middle of the sky (zenith) and the pole star should be the Sine of co-latitude.

**73** The "base" Sine of the true [longitude] multiplied by one thousand three hundred and ninety seven (1397) and divided by the Radius should be the [Sine of] declination. The square root of the difference of the squares of this [declination] and the Radius will be the diurnal "Sine".

**74** The [Sine of] declination multiplied by the Sine of geographic latitude and divided by the Sine of co-latitude should be the Earth-Sine. The Earth-Sine multiplied by the Radius and divided by the diurnal "Sine" should be the Sine of ascensional difference.

**75** The Sine [of the arc] from the intersection of the six o'clock circle and the sun [to the east or west crossing][66] should be the Sine of declination south or north. The diurnal "Sine" is the half-diameter of the diurnal circle.

**76** A Sine in the diurnal circle in the gap between the horizon and the six o'clock circle is declared to be the Earth-Sine. The base for the hypotenuse, which is the Radius, is the declination and the upright is the diurnal "Sine".

**77** A revolution of diurnal circles and that of the celestial equator are the same in terms of time. It is stated that a Sine in the celestial equator when it is revolved is the Sine of this [celestial equator] in a given time.

[66] Supplied from the wordings in *GD1* 2.14 (*unmaṇḍalārkayogaprāgaparasvastikāntarālajyā krāntijyā*).

**78** The Sine of ascensional difference is the Sine in the celestial equator [formed in] a revolution corresponding to the Earth-Sine. The Sine of ascensional difference made into an arc in *prāṇa*s is called the "ascensional difference".

**79** Since there is coexistence of [time units] beginning with *prāṇa*s and [arc lengths] beginning with minutes on a circle, an arc should be in [units] beginning with *prāṇa*s and beginning in minutes.

**80** It is suitable to compute an arc on a great circle, not on a diurnal circle, because all the Sines mentioned arise from a great circle.

**81** When there is the greatest declination with the "base" Sine of three signs, then how much with the given "base" Sine? Thus is the Rule of Three for producing the declination.

**82** When with the upright that is called the co-latitude the Sine of the geographic latitude is produced, then how much with the upright that is the [Sine of] given declination? Thus the Rule of Three should be known in the case of the Earth-Sine.

**83** When the Earth-Sine is in a diurnal circle, then how much is the Sine in the great circle? Thus the Rule of Three should be known in the case of the Sine of ascensional difference.

**84** The Sine of declination multiplied by the Radius and divided by the [Sine of] co-latitude is the solar amplitude. This is the Sine southward or northward [corresponding to the arc in] the horizon from the intersection of the horizon and the sun [to due east or due west].

**85** The Sine of declination in the six o'clock circle is the upright, the Earth-Sine produced in the diurnal circle is the base [and] the solar amplitude situated in the horizon is the hypotenuse. A trilateral is formed with the three.

**86** With any two among the upright, base and hypotenuse the other one is produced. Therefore the square root of the sum of the squares of the Earth-Sine and [Sine of] declination should be the solar amplitude.

**87** If the Radius is the hypotenuse of the upright that is the [Sine of] co-latitude, what is the hypotenuse of the upright that is the [Sine of] declination? Thus the Rule of Three should be known for attaining the solar amplitude.

**88** Having made a diurnal circle that has the [Sine of] geographic latitude as half-diameter on the central axis and at the end of the horizon, it should be conveyed that the [Sine of] geographic latitude and [Sine of] co-latitude are on its middle and that their hypotenuse is situated at its circumference.

**89** A planet's "base" arc is [the arc] from the equinoctial point to the end of the planet['s longitude]. Its Sine is the "base". An "upright" arc is [the arc] from the solstitial point to the end of the planet['s longitude]. Moreover, its Sine is the "upright".

**90** The given [Sine of the] declination is the base and the given "base" Sine is the hypotenuse. As for the upright, it is the given Sine in the diurnal circle. They should form a trilateral.

**91** Three thousand one hundred and forty-one (3141) is the diurnal "Sine" [when the declination is] greatest. The given "base" Sine multiplied by this (3141) and divided by the Radius should be called the "given Sine in the diurnal circle".

**92** A Rule of Three should be considered: [If] the diurnal "Sine" [when the declination is] greatest is the upright for the given "base" Sine when it is a Sine of three [signs] (Radius) , what [is the upright] for the given Sine in the diurnal circle?

**93** Or, it should be the square root of the difference between the squares of the given declination and the "base" Sine. The given Sine in the diurnal circle has been described in order to establish the measure of signs.

**94** The given Sine in the diurnal circle, multiplied by the Radius, divided by the diurnal "Sine", made into an arc, will be the *asu*s (i.e. *prāṇa*s) when those degrees of the "base" rise at Laṅkā.

**95** When the Sine is this much in the diurnal circle, how much is it in the great circle? Thus should be the Sine in the celestial equator when the degrees of the "base" rise at Laṅkā.

**96** The measure of two signs minus the measure of one sign should be the measure of the second sign. The measure of three signs minus the measure of two signs is the measure of the third sign.

**97** These (the amount of time) are decreased by the ascensional difference when [the rising point] is in [the six signs] beginning with Capricorn and increased when it is in [the six signs] beginning with Cancer. This becomes the time *prāṇa*s when each of those degrees of the "base" rise at one's location.

**98** [Signs] beginning with Capricorn rise quickly, and those beginning with Cancer slowly, because the stellar sphere is elevated at the north. This is the grounding in the correction of the ascensional difference.

**99** Or, the given "base" multiplied by three thousand one hundred and forty-one (3141) divided by the radius of the diurnal circle and then made into a chord should be the *asu*s (i.e. *prāṇa*s) it takes for a given arc of "base" to rise at Laṅkā.

**100** The measure of a sign is established by joining Rules of Three. Here, the Radius is the divisor at first and elsewhere it is the multiplier. Thus these two are excluded. This is a suitable method.

**101** When there is passage, one should make the measure of the beginning and end of a given [sign] with passage separately. Their difference should be the measure of a given [sign]. Here, [the correction of] ascensional difference is likewise.

**102** If the given [sign] goes through two quadrants, one should separately make measures in degrees situated in each quadrant. The given measure should be their sum. The ascensional difference is determined in each quadrant.

**103** A line called the "rising-setting" should go in the east-west [direction], from the end of the solar amplitude. The elevation of the sun moving on the diurnal circle from the horizon is the [great] gnomon.

**104** The distance between the foot of the [great] gnomon and the rising-setting line is then called the "gnomonic amplitude". The given "Sine" in the diurnal circle goes through the gap between the tip of the [great] gnomon and [the line] called the "rising-setting".

**105** In this case, the given "Sine" in the the diurnal circle is the hypotenuse, the [great] gnomon is the upright and the gnomonic amplitude is the base. In this manner, here is a figure caused by the geographic latitude. It is mentioned that there should be many of them.

**106** With the base and so forth produced in one figure, here, with proportion, another figure is established, since it is the geographic latitude that all are based on.

**107** The given "Sine" in the diurnal circle is established with a "Sine" arising in the celestial equator. The "Sine" arising in the celestial equator should be a "Sine" [of an arc measured in] *asu*s (i.e. *prāṇa*s), elapsed [since sunrise] or to come [before sunset].

**108** The expression "Sine" is unsuitable [for a segment extending] from the horizon, but it is suitable for that from the six o'clock circle. Because it is the six o'clock circle that goes through the middle of the stellar sphere, not the other one.

**109** The Sine of the *asu*s (i.e. *prāṇa*s), elapsed [since sunrise] or to come [before sunset], decreased by the ascensional difference when [the sun is] in the northern [celestial hemisphere] and increased by the ascensional difference when in the southern celestial hemisphere, becomes [a Sine] in the portion above the six o'clock circle.

**110** When the celestial equator is assumed to be outside the given diurnal circle, the grounding concerning the correction of the ascensional difference within the [time] past in a day should be known, or that the [Sine of] ascensional difference and the Earth-Sine have the same form [should be known].

**111** This Sine in the portion above the six o'clock circle multiplied by [the radius of] a given diurnal circle divided by the Radius becomes the given Sine in the diurnal circle in the portion above the six o'clock circle.

**112** When the Sine on the celestial equator is this much, then how much should be the [Sine] produced in the diurnal circle? Thus the Rule of Three must be known when computing the given Sine in the diurnal circle.

**113** The Sine in the diurnal circle, having the Earth-Sine subtracted when [the sun is] in the south [of the celestial equator] and having the Earth-Sine added when in the north, becomes the given "Sine" in the diurnal circle that arises in the portion above the horizon.

**114** This [given] "Sine" multiplied by the [Sine of] co-latitude and divided by the Radius should be the great gnomon. The square root of the difference between the squares of this (great gnomon) and the Radius should be the [great] shadow of this [great] gnomon.

**115** If with the Radius as the hypotenuse the [Sine of] co-latitude is the upright, then what should be the upright with the given “Sine” in the diurnal circle [as the hypotenuse]? Thus should be the Rule of Three concerning the [great] gnomon.

**116** This great shadow multiplied by twelve and divided by the great gnomon is the shadow of the twelve *aṅgula* gnomon. This is obtained from the Rule of Three.

**117** Or beginning from the horizon, the [given] “Sine” in the diurnal circle multiplied by twelve and divided by the hypotenuse at equinoctial midday is the great gnomon. Or else, this [given] “Sine” in the diurnal circle multiplied by the [Sine of] declination and divided by the solar amplitude is the great gnomon.

**118** The establishment of the upright extending upward by the effect of the hypotenuse extending northward is stated here. This is suitable, because this pair arises from the geographic latitude.

**119** The [great] gnomon multiplied by the Sine of geographic latitude divided by the [Sine of] co-latitude should be the gnomonic amplitude. In this case, the grounding is because the Sine of geographic latitude is as the gnomonic amplitude for the [Sine of] co-latitude which is as the [great] gnomon.

**120** Or, the [great] gnomon multiplied by the *aṅgula*s of the [shadow at] equinoctial midday and divided by twelve should be the gnomonic amplitude. Or else, the [great] gnomon multiplied by the Earth-Sine and divided by the Sine of declination is the gnomonic amplitude.

**121** The [Sine of] declination, which is smaller than the Sine of geographic latitude and in the northern direction, multiplied by the Radius and divided by the [Sine of] geographic latitude is the [great] gnomon situated in the prime vertical when the sun is on the east-west line.

**122** When the sun is on the prime vertical, the gnomonic amplitude should be the same as the solar amplitude. The solar amplitude should be [established] from the [Sine of] declination. Therefore here, the gnomonic amplitude should be [established] from the [Sine of] declination.

**123** The gnomonic amplitude should be [established] from the [Sine of] declination with proportion and the [great] gnomon [should] also [be established] from the gnomonic amplitude. The pair of Rules of Three should be for establishing the prime vertical gnomon here.

**124** Here the co-latitude is the divisor at first, then it is the multiplier afterward, and then they both disappear. The Radius is the multiplier of the [Sine of] declination, the Sine of geographic latitude the divisor, and the result is the prime vertical gnomon.

**125** The motion of [celestial objects] beginning with the moon is described in each of their own inclined circles. Their nodes move on the ecliptic. They should be going retrograde.

**126** The inclined circle touches where its own node is on the ecliptic. Its quadrant’s end has a distance which is the greatest deviation [from the ecliptic, inclined towards] the north and south directions.

**127** [The longitudes of] their own nodes should be subtracted from the “slow” corrected [longitude of the planet], and from the “fast” apogee in case of Mercury and Venus. The “base” [of the longitude] diminished by the node multiplied by the greatest deviation and divided by the Radius is the deviation.

**128** Then, this multiplied by the half-diameter and divided by the “slow” radial distance is the corrected [deviation] that has been described. And in the case of those beginning with Mars, this is also multiplied by the half-diameter and divided by its own “fast” radial distance.

**129** Four, two, eight, six and ten multiplied by ten degrees should be the degrees of the nodes of those beginning with Mars. They have a small motion over a long time.

**130** Ninety, one hundred twenty, sixty, one hundred twenty, one hundred twenty are the greatest deviation in minutes of Mars, Mercury, Jupiter, Venus and Saturn.

**131** If with a “base” Sine of the Radius the greatest deviation [is produced], then how much is produced with a given “base” Sine? Thus should be the Rule of Three when a given deviation [is sought].

**132** When the radial distance is small their [deviation] should be increased. Likewise, when [the radial distance is] big [their deviation] should be decreased, because there is a difference in minutes of the figure due to the difference of far and near.

**133** The motion of [the planets] beginning with Mars should be below and above because of the “fast” and “slow” apogees. Therefore the measure of the intermediate space between a planet and the Earth is established with two radial distances.

**134** Let them state that the nodes of Mars, Jupiter and Saturn should be subtracted from each of their true positions. In their own school, there should be a correction with the Sine of the “fast” [anomaly] on the node as [done with a] planet.

**135** But in order to establish the situation of radial distances and to establish the true [planet], three orbits are drawn here. Within all circles, the eastern direction is at the end of Pisces.

**136-138** The first circle for all [planets] is called the “zodiac” whose center is the middle of the Earth. It is indicated that the center of the “fast” [circle] for Mars, Jupiter and Saturn is in the direction of the “fast” [apogee] at the distance of [the Sine of] its greatest equation starting from that middle [of the Earth]. As for Mercury and Venus, the center of the “slow” [circle] is in the direction of the “slow” [apogee] at the distance of [the Sine of] its greatest equation. The center of the “slow” [eccentric circle] for those beginning with Mars is in the direction of the “slow” [apogee] starting from the middle of the second (i.e. “fast” circle). Now, the “fast” [circle] for Mercury and Venus should have as its center the sun, located on the second circumference. All [planets] move on the last circle.

**139** Their movement on the last circle is always with a motion called “mean”. The motion produced by the movement of a planet on the zodiac which is inferred is called “true”.

**140** The last circle for Mercury and Venus should have the "fast" greatest equation as its half-diameter. The other [circles] have the Radius [as its half-diameter]. The triad of circles should have an interlocked deviation.

**141** One should put a line starting from the planet situated on the last circumference and having the center of the penultimate [circle] as its end. It is the "slow" radial distance of those beginning with Mars and the "fast" [radial distance] of Mercury and Venus.

**142** Where the given line going through the path of the radial distance should be on the second circumference is the "slow" corrected [planet] of those beginning with Mars and the "fast" corrected [planet] of Mercury and Venus.

**143-144** One should put a line, starting from the "slow" corrected [planet] in the case of those beginning with Mars and from the "fast" corrected [planet] in the case of Mercury and Venus, having the center of the zodiac as its end. It is mentioned that [the length of] this [line] is the "fast" radial distance of those beginning with Mars and the "slow" [radial distance] of the other two [planets]. The "fast" corrected [planet] of those beginning with Mars is on the intersection of the line going through the path of the radial distance and the circumference of the zodiac. However, that is where the "slow" corrected [planet] of Mercury and Venus is.

**145** The true planet on the circumference of the zodiac is the pair of corrections of the two apogees. Sometimes there should be a small difference with the observed true planet.

**146** The "slow" radial distance and the "fast" equation should be the cause of difference in the case of those beginning with Mars. The "fast" radial distance as well as the "slow" equation should be the cause concerning the difference in the case of Mercury and Venus.

**147** Thus, for all [planets], the "slow" equation is calculated from the mean [planet] corrected by half the Sine equation. In addition, a difference in steps for Mercury and Venus is assumed in this case.

**148** In this case, when a line is also made from a planet situated on the last circumference with the first center as its end, the observed true planet is on the intersection of this line and the first circumference.

**149** If there is a given deviation within the radial distance between the middle and the end of the middle (i.e. second) and the last circle [respectively], how much is there within a radial distance [equal to] the Radius? Thus is the deviation on the middle (i.e. second) circumference.

**150** If when the radial distance is between the middle and the end of the first and second [circles respectively], there is this much of deviation, [then] how much is there when the radial distance [is equal to] the Radius? Thus is the true deviation on the zodiac.

**151** Some think that: "In the same manner that the difference in sight of a planet established with a pair of true [planets] becomes small, [the difference] of deviation established with two radial distances [becomes small]".

**152** There are two circles for the sun and moon. There is a "slow" circle [whose center is] in the direction of its own apogee at a distance of its own greatest equation from the center of the zodiac. There should be a single correction method since it has a [single] apogee of its own.

**153** The sum [or] difference of [a planet's] latitude and declination when they are in a same direction [or] in a different [direction] respectively, is said to be the arc of its own declination. Its Sine should be the Sine of its own corrected declination.

**154** Two holes made in the solstitial colure at a distance of three signs from the conjunction with the ecliptic are known as the ecliptic poles (literally "summit of signs") because they are the conjunction of all signs.

**155** Just as the celestial equator is at a quarter of a circle from the sphere's pair of south and north crosses, that which is called the "ecliptic" [is at a quarter of a circle] from the pair of ecliptic poles.

**156** The arc minute where a planet is situated should extend south and north with the pair of ecliptic poles as its end. The latitude in the arc minute where a planet is situated always proceeds from its declination.

**157** Thus, in accordance with the ecliptic pole, the latitude has a motion going above and below the six o'clock circle. Some [say] that joining and [subtracting] arcs of latitude and declination is unsuitable in this case.

**158** When the solstitial point is touching [the six o'clock circle], the pair of ecliptic poles should be on the six o'clock circle. When the equinoctial point [is touching the six o'clock circle], it should be below or above [the six o'clock circle] according to the "upright". Thus is the elevation of those [ecliptic poles] in this case.

**159** The [Sine of] greatest declination multiplied by the Sine corresponding to the *asu*s (i.e. *prāṇa*s) it takes for the gap between the solstitial point and the true planet to rise at Laṅkā divided by the Radius is the elevation of ecliptic pole.

**160** It is an elevation in the north when a planet is [in the six signs] beginning with Capricorn and an elevation in the south when beginning with Cancer. Thus is the elevation when a planet rises. It should be the opposite when it sets.

**161** The revolution of the sphere is the same as the rising time at Laṅkā. Thus the elevation of ecliptic pole caused by the revolution of the sphere is also established from the Sine of the rising time at Laṅkā.

**162** Otherwise, the "upright" of a planet multiplied by the [Sine of] greatest declination and divided by the Radius is the crude elevation. Though crude, if the method would become simple in that case, it is not to be unexplained.

**163** The elevation of ecliptic pole is multiplied by the [Sine of] latitude and divided by the Radius. The square root of the difference between the squares of this and the [Sine of] latitude is called the "[Sine of] corrected latitude on the declination".

**164** When this (corrected latitude) and the declination are in the same direction, the sum of the arcs, and when different, the difference of the arcs should be the true arc of declination. The true Earth-Sine and so forth are also [computed] from its Sine.

**165** There should be a difference in rising because of the latitude going above or below the six o'clock circle. [There is] also [a difference] because of the [planet's] situation south or north of the ecliptic. Thus there are two methods on visibility for a planet.

**166** The elevation of ecliptic pole multiplied by the [Sine of] latitude and divided by the Radius is the elevation of latitude, or its depression from the six o'clock circle.

**167** If a latitude is on the portion where the ecliptic pole is elevated, then it is indicated that this latitude has an elevation. And [a latitude] based on the opposite direction has a depression.

**168** The elevation of latitude should be the base, the [Sine of] latitude is the hypotenuse, and its upright is [the Sine of] the latitude set on the six o'clock circle, whose arc is on the arc of the declination.

**169** The elevation of latitude is multiplied by the Radius and divided by the diurnal "Sine". Its arc multiplied by the arc minutes in a sign and divided by the *asu*s (i.e. *prāṇa*s) of the sign where the planet has gone is additive or subtractive.

**170** It is subtractive when [the latitude has] an elevation, and additive when [it has] a depression when [the planet] rises. Or, when it sets, it is indeed the same if the elevation [or depression] is produced upon rising. If it is produced upon setting, additive and so forth is inverted.

**171** The *prāṇa*s of the sign in which the planet sets should be the divisor when obtaining the visibility equation upon setting. The time within which the sign sets is equal to the *asu*s (i.e. *prāṇa*s) within which its seventh sign rises.

**172** If one thousand eight hundred minutes of arc are obtained with the *asu*s (i.e. *prāṇa*s) of nothing else but the ascendant (rising sign), how much with the *asu*s of the visibility equation? Thus is the Rule of Three in this case.

**173** Those who desire to divide by the *asu*s (i.e. *prāṇa*s) rising [time] at Laṅkā in this case to obtain the visibility equation should be wise calculators. However, [they] are those who know [only] one location on the sphere in this case.

**174** The time within which [the sign] itself sets is equal to the *asu*s (i.e. *prāṇa*s) within which the seventh sign [from] it rises, because the ascensional difference of the signs upon setting is the opposite of the time they rise.

**175** The difference between the Sine of declination corrected by the celestial latitude and [the Sine of declination] itself in this case should be the declination produced by the celestial latitude. From there the visibility equation for the geographic latitude [is established].

**176** The declination produced by the celestial latitude is multiplied by the [Sine of] geographic latitude, divided by the Sine of co-latitude, multiplied by the Radius and divided by the diurnal "Sine". Its arc should be the portion of the ascensional difference made by the celestial latitude.

**177** The ascensional difference [made by] the celestial latitude, multiplied by the arc minutes in a sign, and divided by the *asu*s (i.e. *prāṇa*s) of the sign where the planet is situated is to be subtracted upon its rising when the celestial latitude is in the north, and is to be added when in the south. Reversely when the planet sets.

**178** This pair of visibility methods has been mentioned to obtain the ascending and descending points, but this is not its true subdivision. Instead, the two could be established with one method.

**179** Half of the ecliptic is risen at all times and likewise half is always set. Now, in the middle of the risen portion is always situated an ecliptic point called the "sight-deviation".

**180** The middle of the risen portion should be in the middle of the ascending and descending [points]. Therefore it is indicated that the ecliptic point of sight-deviation is the ascending point in the east decreased by three signs.

**181** The Sine in the gap between the zenith and the ecliptic point of sight-deviation is called the "Sine of sight-deviation". When the sun is on the ecliptic point of sight-deviation, the Sine of sight-deviation is remembered as be the great shadow.

**182** The portion of the ecliptic on the prime meridian is described as the ecliptic point called the "midheaven", because it is [the position of] the sun at midday. This [longitude of midheaven] should be established according to the hour angle and the measure at Laṅkā.

**183** The correction of ascensional difference when the signs set is opposite of when they rise, thus this [ascensional difference] should not exist at the middle of the sky. Therefore the measure at Laṇkā is indeed the measure of midheaven.

**184** The Sine produced from the sum of the arcs of the midheaven ecliptic point's declination and the geographic latitude when they are in the same [direction or] their difference when in the opposite direction is said to be the midheaven Sine.

**185** The two gaps[, one between] the celestial equator and the zenith [and the other between] the celestial equator and the diurnal circle are the geographic latitude and declination [respectively]. Thereupon, from these two, the Sine [of the arc] between the diurnal circle and the zenith should be established.

**186** The square root of the difference between the squares of the Radius and the midheaven Sine is declared to be the midheaven gnomon. Then the "base" Sine of the ascending point decreased by the midheaven ecliptic point is the "base" of the midheaven gnomon.

**187** The Radius multiplied by the gnomon called the "midheaven" and divided by the "base" of the midheaven gnomon is mentioned as the gnomon of sight-deviation. Its [great] shadow is the true Sine of sight-deviation.

**188** If, in this case, the midheaven gnomon should be [established] with the Sine [of an arc in the ecliptic] between the midheaven ecliptic point and the horizon, then what with the Radius [which is the Sine of an arc in the ecliptic] in the gap between the ecliptic point of sight-deviation and the horizon? [This is the] gnomon [of sight-deviation] in this case. Thus is the grounding.

**189** The elevation of ecliptic pole [from the horizon] is equal to the Sine of sight-deviation, in the direction opposite to it. This is because the zenith is at a quarter of the sphere from the horizon, and so is the ecliptic pole from the ecliptic.

**190** When a given planet is situated on the horizon, the latitude multiplied by the Sine of sight-deviation and divided by the Radius should be the elevation or depression of latitude.

**191** When the latitude is situated in a direction other than the [Sine of] sight-deviation, it should be its elevation. When the latitude is situated in the direction of the Sine of sight-deviation, however, it is its depression.

**192** The latitude's elevation or depression is multiplied by the Radius, divided by the [Sine of] co-latitude, multiplied by the Radius and divided by the diurnal "Sine". Its arc is the visibility equation in *prāṇa*s.

**193** The visibility equation, which is the elevation in this case, is multiplied by one thousand eight hundred and divided by the *prāṇa*s of the ascendant (rising sign). The arc minutes should be subtracted when [the planet] is rising, and added when it is setting. Reversely when [the visibility equation] is a depression.

**194** The direction of the larger between the Sine of geographic latitude and the declination of the midheaven ecliptic point should be that of the midheaven Sine and the Sine of sight-deviation. In this case, the entire visibility equation has been stated.

**195** When the mean sun rises above the six o'clock circle at the geographic prime meridian, planets corrected from this [moment] are indicated by intelligent ones, among which there should be those due to [the motion corresponding to the observer's] location and so forth.

**196** The daily motion, multiplied by the *yojana*s produced in the distance between the geographic prime meridian and one's spot and divided by one's circumference, is additive when in the west and subtractive when in the east.

**197** The rising of the sun is early in the east of the geographic prime meridian and late in the west. Thus the motion [due to] location should be subtracted in the east and should be added in the west.

**198** When a daily motion occurs in a revolution along one's circumference, how much then [occurs] in a revolution along the gap between the geographic prime meridian and one's spot? Thus is the grounding to be considered in this case.

**199** A man going toward the east should always go on one's circumference, because the observation of the sun follows one's circumference and the directions [come] from the sun.

**200** Two locations that have the same geographic latitude are situated east and west. This [geographic latitude] is the same on one's circumference indeed. Therefore this [circumference] should be the divisor in this case.

**201** When the circumference of the Earth is three thousand two hundred and ninety-nine (3299) [at a place] where the [Sine of] co-latitude is a Radius and there is no geographic latitude, then what would it be [at a place] with a given [Sine of] co-latitude? Thus one's circumference is obtained.

**202** The sun's equation of center is the difference between the true and mean suns in minutes. The daily motion of a planet multiplied by this and divided by [the number of] minutes in a circle should be additive or subtractive against the planet.

**203** As the sun's equation of center, when this [correction] is subtractive, the rising of the true sun should be before the rising of the mean [sun]. When it is additive, the true sun should rise in the reversely.

**204** When a daily motion is produced in a revolution of the [stellar] sphere, what [is produced] then in a revolution corresponding to the equation of center? Thus the grounding is said by others. Here, the time corresponding to the equation of center should be the desire [quantity].

**205** The [daily] motion multiplied by the *asu*s (i.e. *prāṇa*s) of the sun's ascensional difference and divided by the *asu*s in a day is subtractive against the planet when the sun rises in the northern celestial hemisphere and additive when [it rises] in the southern [celestial hemisphere]. It is reverse when [the sun] sets.

**206** The rising of the sun [above the horizon occurs] before it rises above the six o'clock circle when it is in the northern celestial hemisphere and after when it is in the southern [celestial hemisphere], and reversed for the setting, therefore the rule for subtractive and so forth is like this.

**207** When there is a daily motion with the *asu*s (i.e. *prāṇa*s) in a day, then what is with the *prāṇa*s in the ascensional difference? Thus a Rule of Three should be known for the correction of the ascensional difference against [the longitude of] a planet.

**208** Others say that in this case, the divisor for the ascensional difference and the other (daily motion) is the minutes of the sun's [daily] motion added to the *prāṇa*s in a day, [because] a day arises from the revolution of the sphere together with the motion of the sun.

Now, the computation of the true sun from the prime vertical shadow and from the midday shadow.

Here is an example of the prime vertical shadow.

**209** If the shadow of the sun on the prime vertical, is the same [length] as the gnomon, and then shorter on the next day, what is the sun['s longitude]. Or, if it is longer on the next day, then what is it, say, o learned one! The Sine of geographic latitude is measured as six hundred and forty-seven (647).

Here is the procedural rule in two *āryā* verses.

**210** The [great] gnomon is established from the shadow, the gnomonic amplitude from the [great] gnomon, and in this case that [gnomonic amplitude] indeed is the solar amplitude. The [Sine of] declination from the solar amplitude, the "base" Sine from the [Sine of] declination, and the sun['s longitude] should be its arc.

**211** If the shadow produced on the next day is longer, in this case [the longitude of] the sun with passage should be half a circle decreased by the "base" arc, because in this case the course is southward.

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**(Commentary)** In this case, the [great] gnomon computed from the hypotenuse of the shadow with proportion is 2431. The gnomonic amplitude is 466. However this should be understood as lessened by a quarter. This is the solar amplitude. The [Sine of] declination computed from the solar amplitude by a rule to reverse is 457. However this should be understood as increased by a half. The arc of the "base" Sine established from the declination is 1147. The sun['s longitude] is 0 19 7. The second sun['s longitude] is 5 10 53. Since they are established from the declination, these two [are the positions of the sun] with passage.

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Now an example on the midday shadow.

**212** The shadow of the gnomon is measured half when the sun is on the southern bamboo-piece, or in that circumstance [the shadow is] measured one eighth. When the sun is on the northern bamboo-piece, it is measured one seventh. All (shadows) are longer or shorter on the next day. Say o wise, the two [longitudes of the] sun [in each situation]. The Sine of geographic latitude is equal to six hundred and forty-seven (647).

Here is the procedural rule in five *āryā* verses.

**213** The great shadow at midday is called the Sine of meridian zenith distance [of the sun]. The arc of declination is the gap between the arcs of meridian zenith distance and geographic latitude when the sun is located to the south of the zenith.

**214** When the sun is to the north [of the zenith], the sum of the meridian zenith distance and the geographic latitude is the declination. In that case, [the sun] is in the northern celestial hemisphere. In the preceding case (i.e. when the sun is to the south of the zenith), if the meridian zenith distance is larger [the sun is in] the southern celestial hemisphere, if the geographic latitude is larger [it is in] the northern [celestial hemisphere].

**215** When the sun is to the south of the zenith and the shadow is growing, [the sun] should be on the southward course. If [the shadow] is shrinking, [the sun is on] the northward course. It should be reversed when the sun is to the north of the zenith.

**216** The "base" Sine is established from the declination, its arc should be the sun['s longitude] when it is in the northern celestial hemisphere and on the northward course. When on the southward course, [the sun's longitude] is half a circle diminished by [the "base" Sine].

**217** The [established longitude of] the sun is increased by an arc of six signs when it is in the southern celestial hemisphere and if on the southward course. When on the northward course, a circle diminished by the arc produces the [longitude of] the sun with passage.

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**(Commentary)** In this case, the great shadow established from the first shadow and its hypotenuse is 1537. This is also the Sine of meridian zenith distance [of the sun]. In this case, since the sun is to the south of the zenith, the difference between the arcs of meridian zenith distance and geographic latitude is the arc of declination, 943. In this case, since the meridian zenith distance is larger, [the sun] is in the southern celestial hemisphere. The arc of the "base" Sine obtained from the Sine of declination is 2509. Since it is in the southern celestial hemisphere, this arc increased by six signs is [the longitude of] the sun when the shadow is growing, 7 11 49. When the shadow on the next day is shrinking, [the sun] should be on the northward course. Therefore, a circle made of twelve signs, decreased by this "base" arc, is [the longitude of] the sun, 10 18 11.
Now in the second case, the shadow in *aṅgula*s is 1 30. The great shadow is 426. In this case too, since the sun is to the south of the zenith, the difference between the arcs of geographic latitude and meridian zenith distance is the arc of declination, 224. In this case, since the geographic latitude is larger, [the sun] is in the northern celestial hemisphere. The arc of the "base" Sine established from the [Sine of] declination is 553. Since the sun located in the northern celestial hemisphere is to the south of the zenith, it should be on the southward course when the shadow is growing. Therefore, six signs decreased by this arc is [the longitude of] the sun, 5 20 47. When the shadow on the next day is shorter, the "base" Sine itself is [the longitude of] the sun, 0 9 13.
Now in the third case, the shadow in *aṅgula*s is 1 43. The great shadow is 487. Since the sun is to the north of the zenith, the sum of the arcs of the meridian zenith distance and the geographic latitude is the arc of declination, 1140. The "base" arc is 3194. In this case, since the sun is located in the northern celestial hemisphere, when the sun is growing, this arc itself is [the longitude of] the sun, 1 23 14. When the shadow is shrinking, six signs decreased by the arc is [the longitude of] the sun, 4 36 46.
Since they are established from the declination, these [are the positions of the sun] with passage.

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**218** The gap between the arcs of declination and meridian zenith distance when they are in the same [direction], or their sum when they are in different directions, should be the arc of geographic latitude. The distance between the two [longitudes of] the sun obtained from shadow and mathematics is the [motion of] the solstice.

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**(Commentary)** When the two are in one direction, the "gap (difference)" between the "arcs" of declination (*krānti* paraphrased to *apakrama*) and "meridian zenith distance" should be the "arc" of geographic latitude (*pala* paraphrased to *akṣa*). And when the two are in "different directions", their sum is the arc of geographic latitude (*paladhanus* paraphrased to *akṣacāpa*). In this manner, the geographic latitude is established from the shadow and the sun. In the previous example, the arc of the [great] shadow in the first case is 1594. The arc of declination is 943. Both being in the south, their difference is the arc of geographic latitude, 651. Now, the arc of meridian zenith distance in the second case is 427. The arc of declination is 224. In this case, the declination is in the north and the meridian zenith distance in the south. Therefore their sum is the arc of geographic latitude, 651.

Now, the “distance” among the two [longitudes of the] sun (*ravi* paraphrased to *arka*) computed from the meridian “shadow” and computed from a treatise on “mathematics” is the motion of the “solstice”. In this manner, the motion of the solstice is established according to the midday shadow.

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**219** When the extremity of the shadow of a fixed gnomon falls on one [and the same] dot at two [moments in] time, the sun with passage should be on a solstitial point at the [moment in] time situated in the middle of these two [moments of time].

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**(Commentary)** At any time, when the “extremity of the shadow”, produced by a prominent part like the extremity of a “gnomon” made immovable, something like a post or mountain, or an unmoving piece of wood, “falls” on a given “dot”, and then when at another “[moment of] time” the “extremity of” that “shadow” “falls” on this very “dot”, the “sun with passage” is on a “solstitial point” at the “[moment of] time” in “the middle of these two [moments of] time”. The motion of solstice is to be known in this manner.

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**220** In this case, the shadow of the sun situated in a given direction is to be established with a specific rule. The [great] shadow is to be assumed in a circle of direction. The circle should be made here with a string.

**221** The sum of the gnomonic amplitude and the solar amplitude in the same [direction, or] their difference when in different [directions] is the “base of direction”, heading south or north, in the figure that has the [great] shadow as its hypotenuse.

**222** The Sine of one and a half sign is the “Sine of direction” when the sun is in an intermediate direction. The Sine of half of that [is the Sine of direction] when in the middle of east and south-east. [The Sines for] other [arcs] are also to be found likewise.

**223** It is described that: “The ‘base to be established’ is the Sine of direction multiplied by the given [great] shadow and divided by the Radius”. If the base of direction and the base to be established are equal, the sun should be in the given direction.

**224** The quotient of the difference between the base of direction and the base to be established when they are in the same [direction], and their sum when in different directions multiplied by a multiplier with a divisor, is subtractive or additive against the given [great] shadow.

**225** In this respect, when the base of direction is located south of that called “the [base] to be established” it is additive and should be subtracted when in the north. It is reversed when the meridian zenith distance is in the north. When there is a pair of [great] shadows, what is done should be done in this way.

**226** If the geographic latitude is large and the meridian zenith distance is in the north, the solar amplitude could be larger than the Sine of direction. There are two [great] shadows in one same direction because the motion [of the sun] is in a circle.

**227** Here, when the base of direction is small [compared to the base to be established], the result is additive against the [great] shadow and when bigger [the result] should be subtracted. It should be done in this way for the sake of the first [great] shadow, and reversed to obtain the second [great] shadow.

**228** When the sun rises in the direction of the meridian zenith distance, the divisor should be the difference between the Radius and the solar amplitude. Otherwise it is the sum. The multiplier is the difference between the Radius and the [great] shadow at midday in the "without-difference" [method].

**229** The multiplier and divisor mentioned here, divided by tens or a given [number of] hundreds, [can] also be a multiplier and divisor, since there is no fault in the "without-difference" [method] because the difference is small.

**230** From the [great] shadow, the [great] gnomon should be [computed]. From that, the gnomonic amplitude and the two bases. Then from the difference between these two, the [great] shadow. It is repeated again in this manner until the two bases here are the same.

Here is an example.

**231** When the sun at the end of Scorpio is situated in the southeast direction, [and] when [the sun] at the end of Taurus is situated in the northeast direction, say o wise one, what are the [lengths of] the two shadows for a gnomon equal to twelve. The Sine of geographic latitude is measured as six hundred and forty-seven (647).

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**(Commentary)** In both cases, the [Sine of] declination is 1210. The solar amplitude is 1232. In the first case, the shadow is assumed to be equal to the Radius. Then the solar amplitude itself is the base of direction. From the Radius, the base to be established is established as 2431. The difference of these two in one [same] direction is 1199. This is the multiplicand. In this case, since the sun is in the southern direction at sunrise and at midday, the difference between the Radius and the solar amplitude is the divisor, 2206. The midday shadow is 1795. The difference between the Radius and the midday shadow is the multiplier, 1643. These two will always be the multiplier and divisor in the "without-difference" method. The quotient [of the division] of the multiplicand multiplied by the multiplier by the divisor is 893. Since the base of direction is smaller than the base to be established [and thus] to the north [of it], this is subtractive against the shadow equal to the Radius that has been previously computed. When done in this way, the shadow is established as 2545. In this case, this is the given shadow. Thus the [great] gnomon is established, and the gnomonic amplitude from the [great] gnomon. Since the gnomonic amplitude and the solar amplitude are in the same direction, their sum is the base of direction, extended north and south in the circle that has the shadow as its hypotenuse, 1675. From the shadow, the base to be established is established as 1800. The difference between these two is 125. Having divided this multiplied by the multiplier by the divisor, the quotient is 93. In this case again, one should subtract this from the previously [established] shadow, 2545, since the base of direction is smaller than the base to be established [and thus] to the north [of it]. Having done in that manner, the shadow is 2452. Thus again, having done the [great] gnomon and so forth, the shadow without difference is 2407. This is the great shadow when the sun is in the southeast direction. Thus the shadow of the twelve *aṅgula* gnomon is established as $\begin{matrix} \mathit{11} \\ \mathit{46} \end{matrix}$.

Now in the second case, since the sun is in the northern direction at the time of sunrise and at the time of midday too, the difference between the Radius and the solar amplitude is the divisor, that has been indeed previously established, 2206. In this case, the midday shadow is 584. The difference between the midday shadow and the Radius is the multiplier, 2854. In this case, having assumed a given [great] shadow, having computed the [great] gnomon, the gnomonic amplitude, the base of direction and the base to be established from it as before, and having computed the result of the difference between the [two] bases with the multiplier and divisor and having shaped [the result] against the shadow assumed previously by oneself, subtractive or additive according to the rule, the [great] shadow without difference should be computed. This [great shadow] without difference is 840. This is the [great] shadow when the sun is in the northeast direction. The shadow of the twelve *aṅgula* gnomon is $\begin{matrix}3\\1\end{matrix}$.
When the sun risen in the northern direction goes to the meridian in the southern direction, then the sum of the Radius and the solar amplitude is the divisor.

Another example like the previous one:

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**232** When the sun situated at the middle of Aries goes to the southeast direction, and when [the sun] at the middle of Gemini [goes to] the middle direction of east and northeast, tell us each shadow o wise one, here the gnomon and geographic latitude are as previously.

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**(Commentary)** Now in the first case, the solar amplitude in the north is 368. The midday shadow in the south is 289. Since these two are in different directions, in this case the sum of the Radius and the solar amplitude is the divisor, 3806. The multiplier is 3149. The given assumed [great] shadow is 2977. The solar amplitude decreased by the gnomonic amplitude is 39. This is the base of direction in the north. In this case, the base to be established in the south is 2104. The sum of these two in different directions multiplied by the multiplier and divided by the divisor is 1773. Since the base of direction is in the north, this should be subtracted from the previous [great] shadow. In that case, the [great] shadow produced is 1204. Having done again in this way, the [great] shadow without difference is 405.
Now in the second case, the solar amplitude is 1373. This is northward. The midday shadow in the north is 731. The divisor is 2065. The multiplier is 2707. In this case, the Sine of direction is 1315. The assumed [great] shadow is 3438. In this case, the solar amplitude itself is the base of direction. The Sine of direction itself is the base to be established. From the difference between the bases, the result is 76. This should be subtracted from the given shadow in order to establish the first [great] shadow, since the base of direction is larger. When the base of direction is smaller, then it should be added. In this case, the [great] shadow without difference is 3422. This should be the great shadow when the sun is at the midpoint between the northeast and east. In this very case, there is a second [great] shadow. In order to establish it, having assumed a given [great] shadow decreased by a given number from the [great] shadow in the given direction established in the first case, the computation is to be carried out. In that case, the previous [great] shadow decreased by a thousand is 2422. The base of direction is 906. The established shadow is 926. The result from the difference between the bases is 26. This should be subtracted in order to establish the second [great] shadow, since the base of direction is smaller. In this case, the [great] shadow without difference is 2318. This is the second [great] shadow in the given direction.
From these two, the two shadows of the twelve *aṅgula* gnomon are established.

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**233** It should be understood that the result be increased by half or the like of itself when the approach in an "without-difference" [method] is slow. When it is going upward and downward (i.e. oscillating) due to the quickness, half or the like is subtracted with reason.

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**(Commentary)** "In an 'without-difference"' method, "when the approach" of what is to be established "is slow", then in each of these cases "it should be understood" "with reason" that the obtained "result be increased by half of itself", increased by the result multiplied by one or increased by twice [of itself], according to the slowness of progress. When, "due to the quickness" of the progress, the establishment goes "upward" once and then "downward" once [and so on], "then" the result must be subtracted (*ūna* paraphrased to *hīna*) by half (*dala* paraphrased to *ardha*) of itself, two thirds or three quarters according to the fastness. Thus done, the establishment becomes fast. This must also be considered for every "without-difference" method.

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**234** The difference itself between the two bases, or twice, or even half should be additive or subtractive against the [great] shadow, so that the result without difference is established in this case.

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**(Commentary)** If the declination and so forth are established by the shadow in a direction like the intermediate, then by the co-latitude and so forth established with fractions the exact shadow is established. Thus in the case of [the example beginning with] "when [the sun] at the end of Taurus is situated in the northeast direction", the [great] shadow without difference established in that manner is 838.

Now, the computation of [the longitude of] the sun with the shadow of the sun in an intermediate direction produced at that time and with the hour angle, and the computation of the Sine of geographic latitude with that [sun's] declination and so forth is explained.

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**235** When the sun is at the intermediate direction, it is remembered that the base and upright of the shadow as hypotenuse is equal. Therefore the root of half of the squared shadow is the measure of the base and upright.

**236** The upright should be extended east and west, and the base should be extended south and north here. The "upright" in the diurnal circle, going east and west, should be equal to the [great] shadow's upright.

**237** The Sine corresponding to the time difference between [the middle of] the sky and the sun in the east or west should be produced in the equator. Then this is described as the Sine of the *nāḍī*s (i.e. *ghaṭikā*s) called the hour angle.

**238** If with the Sine of hour angle the "upright" in the diurnal circle [is established], then how much [is established] with the Radius? Thus is the half-diameter of the diurnal circle.

**239** The [Sine of] declination is established from the half[-diameter] of the diurnal circle in this case. The "base" arc [is established] from the [Sine of] declination. This arc should be [the longitude of] the sun in this case. Otherwise that decreased by half a circle is [the longitude of] the sun.

**240** In this respect, when it is in the southern [celestial] hemisphere, the arc of the ["base"] increased by half a circle should be [the longitude of] the sun. Or, its arc decreased by a circle is [the longitude of] the sun, [decided] from the [change in] measure of the shadow on two days.

**241** In this case, the Sine of geographic latitude is to be established with an "without-difference" method according to the base of [great] shadow and so forth. It is first assumed in this case that some amount added to the [Sine of] declination is the solar amplitude.

**242** When the solar amplitude and the base of [great] shadow are in the same or different direction, respectively, their difference or sum is the gnomonic amplitude. The square root of the sum of the squares of this (gnomonic amplitude) and the [great] gnomon is the given "Sine" in the diurnal circle.

**243** The Radius multiplied by the gnomonic amplitude and divided by the given "Sine" in the diurnal circle should be the Sine of geographic latitude. From the [Sine of] geographic latitude, the Sine of co-latitude should be [obtained]. From the [Sines of] co-latitude and declination, the corrected solar amplitude should be [obtained].

**244** Again, the difference of the base of [great] shadow and solar amplitude and so forth should be done. The gnomonic amplitude and given "Sine" in the diurnal circle, the Sine of geographic latitude and Sine of co-latitude, [and] the solar amplitude [should be computed as well]. Thus here at the end of such "without-difference" [method], should be the corrected "without-difference" Sine of geographic latitude in this case.

Here is an example:

**245** The shadow of the gnomon measuring one thousand six hundred and sixty-seven (1667) is said to be equal to four hundred and nineteen (419) when the sun goes to the northeast direction. The *prāṇa*s between [the middle of] the sky and the [current] sun are measured as two thousand five hundred and forty-seven (2547) fourths. The sun and the geographic latitude are to be said by you, o knower of mathematics, if [studies have been made] with exertion on the Sphere.

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**(Commentary)** In this case, the gnomon is 1667. Its shadow is 419. Having computed their hypotenuse from these two, and then, when the Radius is the hypotenuse, the great shadow established from the hypotenuse and the shadow is 838. Its gnomon is 3334. The square root of half the [great] shadow's square is 592. Its fraction in seconds is 33. Then, the base in the figure that has the [great] shadow as hypotenuse is the same as this root. Then, likewise for the upright. Then, the "upright" Sine extending east and west in the diurnal circle is also the same as this upright, because the upright of the [great] shadow is situated on the "upright" in the diurnal circle. The hour angle in *asu*s (i.e. *prāṇa*s) going between the zenith and the sun multiplied by four is 2547. Since there are fourths, these [*asu*s] are to be divided by four. The *prāṇa*s thus made are 636. Their fraction which is the sixtieth is 45. Their Sine is 633. And the fraction is

4 [sixtieths]. This is the Sine extending east and west in the celestial equator. The "upright" Sine in the diurnal circle, that is the same as the upright of the [great] shadow, multiplied by the Radius and divided by the Sine of hour angle is somewhat less than 3218. And this is the diurnal "Sine". The [Sine of] declination established from it is 1210. Its [corresponding] "base" [Sine] is somewhat less than 2978. Its arc is two signs increased by one minute. This is [the longitude of] the sun. Or else, six signs decreased by this is [the longitude of] the sun. If the shadow on the next day [is larger], the first [is the answer]. If the shadow on the previous day is larger, the second.
Now, in order to establish the geographic latitude, a given number is to be added to the given [Sine of] declination, 1210. In that case, the Sine of declination increased by ten is 1220. This is to be assumed as the solar amplitude. The base [in the trilateral] where the [great] shadow is hypotenuse is 593. The difference between these two in the same direction is 627. This is the gnomonic amplitude. The [great] gnomon is 3334. The square root of the sum of the squares of these two that have the forms of the base and upright is 3392. This is the given "Sine" in the diurnal circle that has the form of a hypotenuse. Then, the Radius multiplied by the gnomonic amplitude should be divided by this given "Sine" in the diurnal circle. In that case, the quotient is 636. This should be assumed as the Sine of geographic latitude. The square root of the difference between the squares of the Sine of geographic latitude and the Radius is 3379. This is the Sine of co-latitude. Then, the [Sine of] declination multiplied by the Radius should be divided by this Sine of co-latitude. In that case, the quotient is the corrected solar amplitude, 1231. Then again, having assumed that the difference between the solar amplitude and the base of [great] shadow is the gnomonic amplitude, the Sine of geographic latitude without difference is to be computed with the method that has been mentioned. Then, the corrected Sine of geographic latitude without difference is 647.

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Here is an example in the southern celestial hemisphere:

**246** One one hundred and oneth (1/101) and one one hundred and sixth (1/106) should be subtracted from the gnomon. The remainder of the gnomon here is the shadow of the sun in the southeast direction. The number of *prāṇa*s arising from the midday sun are one thousand nine hundred and sixteen (1916). Say, o wise one, the sun['s longitude] and the geographic latitude too, if you are an expert on the Sphere.

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**(Commentary)** In this case, the gnomon assumed by one's own wit is 2454. And the sixtieths are 28. The quotient [of the division] of this by one hundred and one is 24. The sixtieths are 18. Then again, the quotient [of the division] of this by one hundred and six is 23. The sixtieths are 9. These two quotients are to be subtracted from the previous gnomon assumed with one's own wit. Then the remainder is 2407. The sixtieth is 1. This is a shadow of this gnomon. From these two, the gnomon and shadow, the hypotenuse that is the same as the Radius should be established. Thus in this case, these two are indeed the great gnomon and great shadow. The hour angle in *asu*s (i.e. *prāṇa*s) is 1916. Its Sine is 1818. The sixtieths are 17. The segment in the diurnal circle is the same as the upright of the [great] shadow, 1702. The sixtieth is 1. In this case, the quotient is the diurnal "Sine", 3217. The sixtieths are 54. The [Sine of] declination is 1209. The sixtieths are 38. The "base" Sine is established from it. It is almost the same as a Sine of two signs. Its arc is two signs. This increased by half a circle is [the longitude of] the sun. Or else, a circle decreased by this is [the longitude of] the sun. As for the geographic latitude, it is as previously.

**247** Even when the sun is in any direction, everything is established with this method.

**248** Planets revolve on a circle which has the middle of the Earth as its center and has the measure of its own orbit. The observer should be on the Earth's surface. Therefore his circle of sight has the Earth's surface as its center.

**249** The observer's own horizon should be above the horizon going through the middle of the Earth by a difference of the Earth's half-diameter, because his [sight of] rising and setting [occurs] there (at his own horizon).

**250** A planet situated on the horizon from the middle of the Earth should be below the horizon of an observer. Here, the downward motion [of a planet] having a measure of the Earth's half-diameter is called its parallax.

**251** The observer sees a planet located above the middle of the Earth above himself, too. Therefore, when a planet is situated on the zenith, its parallax should not exist.

**252** Since there should be no [parallax] on the middle of the sky and the parallax should be greatest on the horizon, the parallax of a planet should be established from the Sine of sight with proportion.

**253** If a half-diameter of the Earth is [obtained] on [a planet's] own orbit when [the planet is] at a distance [whose Sine corresponds to] the Radius from the middle of the sky, then what at the Sine of sight? Thus is the parallax at that time.

**254** Even if the parallax measured in *yojana*s are equal in this case due to being situated in one [and the same] minute of arcs, parallaxes in minutes become different due to difference in orbits of planets.

**255** If the parallax measured in *yojana*s on the planet's own orbit is this much, how much on a great circle? Thus is the parallax measured in minutes.

**256** In this case, it is indicated that the great circle obtains the same *yojana*s as minutes, because even an equation in *yojana*s is an equation in minutes with merely a different name.

**257** Therefore, an observer on the Earth's surface sees planets situated on one [and the same] minute. Because the locations [of planets] are different, those situated below are quick and those located above have a small daily motion.

**258** An observer sees the gap between two planets located below and above that is equal to the difference between their own parallaxes, because they both indeed have their own parallax.

**259** Each of their own minutes of parallax should be subtracted from each of their own [arc of great] gnomon. The remainder should be its own corrected [arc of great] gnomon [as seen] at the Earth's surface. Thus should be established in this case.

**260** This should be instructed with a drawing. One should draw a circle of the Earth. Then having set its middle as center, each of [the planets'] own orbital circle should be drawn with the lines of direction.

**261** Having set the intersection of the north-south line and the circumference of the Earth as center, one should then draw a circle with a string [having the length] of the Radius. This is the circle of sight with the lines of direction.

**262** One should make every circle marked with degrees or *ghaṭikā*s here. In this case, the north-south line is to be assumed as a line extending below and above.

**263** In this case, on that very degree in the circle of sight, which is the degree on its own orbital circle that the planet is moving at that time, one should make a dot [called] the "star in space".

**264** One should again make a dot called the "planet" on the conjunction of the circumference of the circle of sight and a line that goes through the planet moving on the circumference of the orbit and the center of the circle of sight.

**265** What exists in the intermediate space between these two dots called the "star in space" and the "planet" is the parallax measured in minutes of the planet at that time.

**266** One should draw two lines equal to the [half-]diameter of the orbit from the middle of the circle of sight going through the two dots. The distance between their tips is the measure of the parallax in *yojana*s.

**267** The parallax should always be on the circle of sight directed toward the planet. The difference in sight is the parallax because the view of the observer follows the planet.

**268** Hence the parallax is said to have the nature of a hypotenuse. Meanwhile, the motion of this [planet] follows the ecliptic. This is the base, the other should be the upright. These two are the longitudinal parallax and latitudinal parallax in an eclipse.

**269** It is assumed by calculators that the longitudinal parallax is a planet's motion on the ecliptic in an eclipse. [It is assumed that] the latitudinal parallax is [its] deviation from its own ecliptic. Therefore the two are base and upright.

**270** The upright is established from the Sine of sight-deviation while the base [is established] from the Sine of sight-motion. The square root of the difference between the squares of the Sine of sight and the Sine of sight-deviation is called the Sine of sight-motion.

**271** When the [Sine of] sight-deviation is zero, the whole parallax is situated on the ecliptic, because at that time it is the ecliptic that happens to be the circle of sight directed toward the planet.

**272** When the [Sine of] sight-deviation is equal to the Radius, the entire parallax goes through the side of the ecliptic, because in this case the ecliptic is like a girdle in the middle of the circle of sight.

**273** By the effect of the increase in the upright called the "[Sine of] sight-deviation", there should be an increase in latitudinal parallax in this case. By the effect of the increase in the base called the [Sine of] sight-motion, there is also increase in longitudinal parallax.

**274** The [Sine of] sight-deviation and the [Sine of] sight-motion, each multiplied by the half-diameter of the Earth and divided by the hypotenuse called the "Radius", are the latitudinal parallax and longitudinal parallax, respectively, measured in *yojana*s.

**275** If there is this much *yojana*s in the radial distance of *yojana*s, how much in the radial distance of a Radius? Thus also the measures of the latitudinal parallax and longitudinal parallax having the nature of minutes are established here.

**276** Or, If the Sines of sight-motion and sight-deviation are the base and upright of the hypotenuse which is the Sine of sight, then what two [are the base and upright] of the parallax as hypotenuse? Thus are the longitudinal parallax and latitudinal parallax stated in eclipses.

**277** The [mean] radial distance in *yojana*s of the sun is equal to four hundred fifty-nine thousand five hundred and eighty-five (459,585). The [mean] radial distance in *yojana*s of the moon is equal to thirty-four thousand three hundred and seventy-seven (34,377).

**278** These two multiplied by the radial distance without difference and divided by the Radius becomes the true [distance in *yojana*s], because a planet on the degrees of the perigee and apogee would move below and above respectively from this location.

**279** Four thousand four hundred and ten (4,410) for the sun, three hundred and fifteen (315) for the moon, one thousand fifty (1,050) for the Earth. The diameter of the orb in *yojana*s has been mentioned.

**280** The diameters of the orbs of the sun and moon that have been indicated, multiplied by the Radius and divided by their true radial distance in *yojana*s, are the true [sizes] in minutes.

**281** The obscuring of the sun by the moon situated below it is called its eclipse. Because the orbits of the two (the sun and moon) are different, the obscuring of the sun is different in each location.

**282** The entrance of the moon into the Earth's shadow on its own path is called its eclipse. The moon that has entered into the umbra should have a single shape everywhere.

**283** If the moon is obscured by the umbra, then why is it called the "destroyer of darkness"? Because the rays of the moon are the rays of the sun. Therefore, how can they be in the umbra?

**284** A place where a string of light falls is provided with brightness. A place without a string of light should be entirely covered with darkness.

**285** The shadow situated at the place where the sun is obscured by the Earth should be the shadow of the Earth. Its established measure is explained here with the grounding belonging to the "shadows".

**286** [The height of] a gnomon is equal to twelve *aṅgula*s. The height of a lamp is equal to twice that amount. In this case, the ground in the space between the gnomon and the lamp is considered in the measuring [units] of the gnomon [and likewise for] the shadow.

**287** The extremity of the gnomon's shadow should be the place where a string, starting from the lamp and touching the tip of the gnomon, falls on the ground. That string is known as the hypotenuse.

**288** The ground between the extremity of the shadow and the foot of the gnomon should be the base with the gnomon as upright. The ground between the extremity of the shadow and the foot of the lamp is the base of the upright which is the lamp.

**289** The base belonging to the lamp decreased by the gnomon as upright is located in the space between the extremity of the gnomon and the lamp. Then the hypotenuse for the base and upright should be the string between the two extremities.

**290** If the base [produced] from the upright, which is the lamp decreased by the gnomon, is equal to the ground in the gap between the gnomon and the lamp in this case, what is the base [produced] from the upright which is the gnomon? Thus the shadow of the gnomon should be produced.

**291** Here the lamp is the half-diameter of the sun's orb and the gnomon is the Earth's half-diameter. The ground in the space between the gnomon and lamp should be the corrected radial distance of the sun in *yojana*s.

**292** In this case, the shadow of the indicated gnomon should be the Earth's shadow. Its circle is equal to the Earth [in size] at the foot, small at the head. It is [cusped] like a cow's tail.

**293** The place in the space where the strings that departed from the sun's circumference and went through the Earth's circumference join together should be the tip of the Earth's shadow.

**294** Thus, the corrected radial distance of the sun in *yojana*s multiplied by the Earth's diameter and divided by the difference of the diameters of the sun and the Earth should be the measure of the length of the Earth's shadow in *yojana*s.

**295** In this case, there is no difference in the result when doubling the desire quantity, because here it is understood that the measure quantity is multiplied by two.

**296** The quotient of [the division of] the shadow's length decreased by the true radial distance of the moon in *yojana*s multiplied by the Earth's diameter by the shadow's length is the measure of the umbra's diameter in this case.

**297** The diameter of the umbra multiplied by the Radius and divided by the [true] radial distance of the moon in *yojana*s is the disk of the umbra in minutes on the path of the moon.

**298** The length of the shadow decreased by the gap [corresponding to] the true radial distance of the moon in *yojana*s is the measure of the length of the shadow's portion that has gone above the path of the moon.

**299** [Concerning the Earth's shadow,] at a distance of its length from the shadow's tip, its diameter becomes equal to the Earth. Then what would it be at the distance above the path of the moon? Thus is the diameter of the umbra [in *yojana*s].

**300** When it is this much (the diameter of the umbra in *yojana*s) on the orbit of the moon, then how much is it on a great circle? Thus should be the disk of the umbra in minutes on the path of the moon.

**301** An eclipse [occurs] as long as the figure in the gap between the eclipsed and the eclipsing is smaller than the sum of their half-diameters. When [the figure] is bigger than that, the whole planet is seen.

**302** Thus the *Goladīpikā* has been proclaimed by us concisely. May the reader be enumerated among the experts on the Sphere.

# Part III

# Commentary

# Notes on the commentary

We do not have a fully extant commentary on *GD2*, and the following commentaries are my interpretation of the verses. Our goal is not to examine the accuracy or validity of the contents in comparison with modern astronomy, but to reconstruct Parameśvara's intentions and reasonings behind his words. Therefore I shall rely on other texts by Parameśvara, notably his commentary on the *Āryabhaṭīya* and super-commentary *Siddhāntadīpikā* on the *Mahābhāskarīya*. Other authors and treatises shall also be quoted to reflect on his sources of ideas. Sources (critical editions) of the texts shall be mentioned each time. Unless indicated otherwise, the English translations are of my own, for the sake of uniformity in the expressions. However I am deeply indebted to preexisting translations, especially those accompanying the critical editions.

Diagrams shall be used frequently for our explanations, but apart from a few exceptions where I follow Parameśvara's verbal instructions, they are my interpretations. None of our manuscripts contain diagrams. I have drawn most of the diagrams three-dimensionally under the hypothesis that an armillary sphere could have been used for the explanation. Unless noted otherwise, north is to the left as they are expressed in the same word (*uttara*, etc.) in Sanskrit. I shall also use projections on planes and images as viewed from an observer inside the sphere whenever necessary.

Formulas are used for simplifying the expressions. Numerous arcs and segments are introduced and named by Parameśvara, and I have assigned a symbol (basically Roman or Greek letters, with indexes or suffixes whenever needed) for each of them. I have tried not only to be consistent within this treatise but also with previous historians. Nonetheless there are many cases where I had to introduce an original symbol. See appendix D for a full list.

Numbers are written in decimal notation, but fractional parts may also be written in sexagesimals whenever necessary. In this case, the integer and fractional parts are separated with a semicolon (;) and lower places are placed after commas (,). For example, $633;15,35 = 633+\frac{15}{60}+\frac{35}{3600}$.

# 1 Invocation (*GD2* 1)

In *GD2* 1.ab, Parameśvara pays homage to the god Gaṇeśa (the deity of knowledge and remover of obstacles), the goddess Sarasvatī (the goddess of speech and learning), teachers and planets. It is more usual for him to praise Śiva or the sun (table 1.1), and the only other exceptions (apart from those which do not have invocation verses) are *GD1* and the commentary on the *Līlāvatī*.

*GD1* 1.1 is fully dedicated to Gaṇeśa:

> I bow to the child Gaṇeśa (*gajānana*; elephant-faced) child settling in the lap of Pārvatī[1], intent upon drinking milk under the wishing tree (*kalpadruma*). (*GD1* 1.1)[2]

The opening verse in *Parameśvarī*, a commentary on the *Līlāvatī* by Bhāskara II, resembles both *Goladīpikā*s.

> I bow to Gaṇeśa (*gaṇeśāna*) settling in the lap of Pārvatī, also to "the god of speech (*vāgīśvara*)" and holy Śiva (*rudra*[3]), the ocean of compassion (*kṛpānidhi*). (*Parameśvarī* opening verse 1)[4]

If *vāgīśvaram* was actually read *vāgīśvarīm* (which causes no metrical problem), it would refer to Sarasvatī like *GD2*.

Next, he announces what will follow *GD2* 1.cd as "the stellar sphere, the size of the Earth and so forth". Interestingly, he does not mention the celestial sphere (*khagola*), which forms an armillary sphere together with the stellar sphere and a miniature Earth. There will be some reference to the armillary sphere including the celestial sphere in the following verses, but indeed the main subjects in *GD2* 2-67 are celestial objects that can be demonstrated on the stellar sphere and the Earth. Parameśvara sums up these topics in *GD2* 68 as "the nature of the Sphere (*golasya saṃsthāna*)".

Here again, it is worth comparing this half-verse with the second verses of *GD1* and the commentary on the *Līlāvatī*.

> Parameśvara, belonging to the lineage of Bhṛgu, situated at the seashore in the northern bank of the Nilā river, states briefly the nature of the Sphere for the young. (*GD1* 1.2)[5]

> I, Parameśvara, standing on the shore of the Nilā river and also of the sea, make the commentary of the *Līlāvatī* for this young one. (*Parameśvarī* opening verse 2)[6]

---

[1] Mother of Gaṇeśa and wife of Śiva.

[2] *vande kiśoraṃ pārvatyā aṅkasaṃsthaṃ gajānanam |*
*stanyapānarataṃ kalpadrumasyādho vināyakam ||1.1||* (K. V. Sarma (1956–1957, p. 11))

[3] Parameśvara had a teacher named Rudra, and we cannot rule out the possibility that this word is addressing him.

[4] *praṇamāmi gaṇeśānaṃ pārvatyā aṅkasaṃsthitam |*
*vāgīśvaram api tathā śrīrudraṃ ca kṛpānidhim ||*
Text from an unpublished critical edition in progress presented in Narayanan (2014). I have added here my own translation based on this text.

[5] *nilāyāḥ saumyatīre 'bdheḥ kūlasthaḥ parameśvaraḥ |*
*saṃkṣepād golasaṃsthānaṃ vakti bālāya bhārgavaḥ ||1.2||* (K. V. Sarma (1956–1957, p. 11))

[6] *nilāyāḥ sāgarasyāpi tīrasthaḥ parameśvaraḥ |*
*vyākhyānam asmai bālāya līlāvatyāḥ karomy aham ||*

Table 1.1: Objects of dedication in invocation verses of treatises and commentaries by Parameśvara ("c." stands for "commentary on")

| Title | Dedicated to |
|---|---|
| *Bhaṭadīpikā* (c. *Āryabhaṭīya*) | Śiva (*śaśibhūṣaṇa*) |
| *Karmadīpikā* (c. *Mahābhāskarīya*) | Śiva (*hari*) |
| c. *Laghubhāskarīya* | Śiva (*śaśāṅkārdhabhūṣaṇa*) |
| c. *Sūryasiddhānta* | Śiva? (*jagatas mahas*) |
| *Ācārasaṅgraha* | sun (*aruṇa*) |
| *Grahaṇanyāyadīpikā* | sun (*savitṛ*) |
| *Grahaṇamaṇḍana* | sun (*dineśa*) |
| *Grahaṇāṣṭaka* | sun (*bhāskara*) |
| *Dṛgganita* | sun (*sahasrāṃśu*) |
| c. *Laghumānasa* | sun (*aruṇa*) |
| *Siddhāntadīpikā* (super-c. *Mahābhāskarīya*) | sun (*khagapati*) |
| *Goladīpikā 1* | Gaṇeśa (*vināyaka*) |
| *Goladīpikā 2* | Gaṇeśa (*vighneśa*), Sarasvatī (*vāgdevī*), teachers (*guru*), planets (*graha*) |
| *Parameśvarī* (c. *Līlāvati*) | Gaṇeśa (*gaṇeśāna*), Sarasvatī? (*vāgīśvara*), god Śiva (*rudra*) or teacher Rudra |
| *Parameśvarī* (c. *Praśnaṣaṭpañcāśikā*) | *Gaṇeśa* |
| *Bālaprabodhinī* (c. *Jātakakarmapaddhati*) | Planets? (*keśajārkaniśākarān kṣitijavijjīvāpnujitsūryajān*) |
| c. *Goladīpikā 1* | none |
| *Candracchāyāgaṇita* | none |
| *Vākyakaraṇa* | none |
| *Ṣaḍvargaphala* | none |
| *Jātakapaddhati* | uninvestigated |
| c. *Muhūrtaratna* | uninvestigated |
| c. *Vyatīpātāṣṭaka* | uninvestigated |

*GD1* and *Parameśvarī* are strikingly resembling, especially in the structure of the first half and the usage of *bālaya* (for the young). *GD2* uses *laghumataye* (for the novice, or literally "light-minded") which is not far in meaning, and the occurrence of the dative demonstrative pronoun *asmai* is common between *Parameśvarī* and *GD2*.

Nothing sure can be said about what this similarity signifies. The dates of *Parameśvarī* and *GD1* are separated by more than a decade, and several treatises which have very different invocations are composed between this period (see introduction 0.1.5). Therefore something other than the proximity in their dates of composition seems to be behind this.

# 2 Parts of the armillary sphere and their meaning (*GD2* 2-17)

## 2.1 Description of an actual armillary sphere

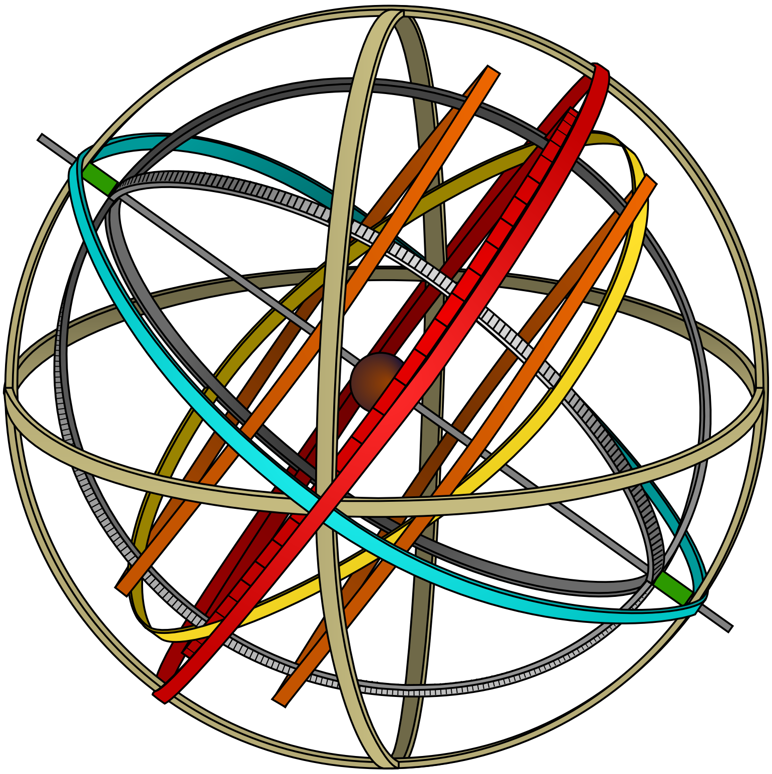

Figure 2.1: A representation of the entire armillary sphere. Colors are added for distinction, and do not represent their actual appearance.

The armillary sphere as described by Parameśvara consists of two layers of rings connected by an axis (figure 2.1). The inner set of rings showing the coordinates of stars and planets revolves on the axis while the outer set of rings are fixed and represent the observer's horizontal coordinate. This double-layered armillary sphere appears to have been common, and can be seen in older texts such as the commentary on the *Āryabhaṭīya* by Bhāskara I (629 CE), the *Śiṣyadhīvṛddhidatantra* (8th century) by Lalla, the later *Sūryasiddhānta* (c. 800 CE), the *Siddhāntaśekhara* (1039) by Śrīpati and the *Siddhāntaśiromaṇi* (1150) by Bhāskara II.

The inner set of rings called the "stellar sphere (*bhagola*[1])" (figure 2.2) contains three rings representing the equatorial coordinates: celestial equator (*ghāṭika*), solstitial colure (*dakṣiṇottara*) and equinoctial colure (*viṣuvat*). A fourth ring tilted 24 degrees against the celestial equator

[1]Each part of the armillary sphere is often called by different Sanskrit terms in different texts and even within *GD2*. The Sanskrit words given here are those used in the first appearance.

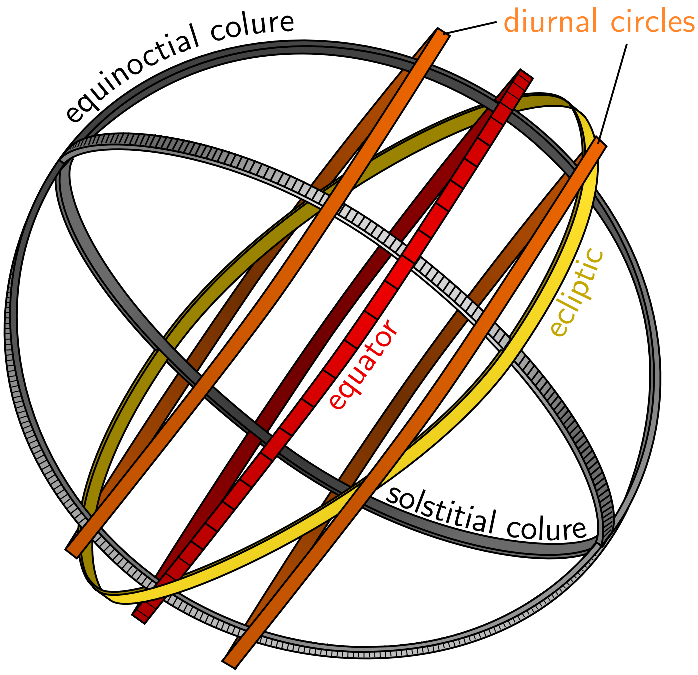


Figure 2.2: Stellar sphere

represents the ecliptic (*apama*), the path of the sun in a solar year. Optionally, diurnal circles (*svāhorātra*) parallel with the celestial equator that are approximations of the path of the sun on a given day[2] can be added. An axis (*daṇḍa*) pierces the stellar sphere in the two celestial poles, i.e. the intersections of the two colures, so that the whole sphere can rotate to represent the geocentric motion of heavenly objects. A miniature Earth made of wood or clay is placed in the middle of the axis. Explanations on the stellar sphere and its parts including the axis are in *GD2* 2-11c.

The outer set of rings, or the "celestial sphere (*khagola*)" (figure 2.3) represents the horizontal coordinates with the prime vertical (*samamaṇḍala*), the prime meridian (*dakṣiṇottara*) and the horizon (*kṣitija*). The polar axis carrying the stellar sphere is attached to the prime meridian, tilted so that the celestial north pole is elevated against the horizon by an angle corresponding to the local latitude. Finally a fourth ring is attached to the celestial sphere so that it goes through the horizon at the east and west and through the two tips of the axis. This is the six o'clock circle (*unmaṇḍala*). *GD2* 11d-17 are related to the celestial sphere and its rings.

In the following sections, we shall look at the descriptions in *GD2* while also comparing them with those in *GD1*.

[2] An approximation in the sense that the sun is assumed not to move along the ecliptic in the course of that day. Otherwise it could not form a single closed loop.

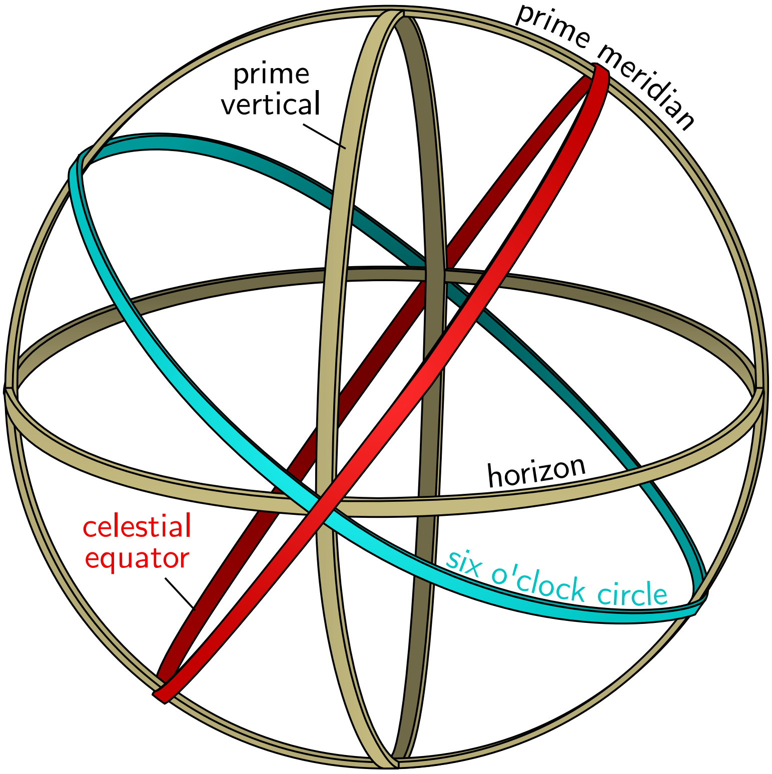


Figure 2.3: Celestial sphere

## 2.2 The three equal division circles and the ecliptic (*GD2* 2-4)

The solstitial colure is the first circle to be introduced in *GD2* 2ab. It is mentioned with the four directions which the circle goes through (figure 2.4). The words south (*yāmya*) and north (*saumya*) are also words which mean right and left[3]. Therefore this can also be read as an explanation of the ring in an armillary sphere.

The Sanskrit word *dakṣiṇottara* also means south-north (*dakṣiṇa-uttara*), but since the stellar sphere rotates, the circle does not always go through the directions of due north and south. In this case, "south" and "north" may be referring to the celestial poles or hemispheres.

The celestial equator is introduced (*GD2* 2cd) by referring to two points in the solstitial colure to which it adheres. One is point A separated toward the north from the bottom point of the solstitial colure by a distance of the geographic latitude $\varphi$[4] and the other point A′ is separated likewise from above toward the south. We cannot determine the position of the celestial equator from *GD2* 2 since it can move around the two points A and A′. The circle is perpendicular against

[3]Likewise, east (*pūrva*) also means "forward" and west (*apara*) "backward", in this case

[4]Parameśvara does not mention whether this is the arc of the geographic latitude or its Sine. If it were the arc, we can measure it along the solstitial colure. If it were the Sine, the linear distance between the line going through above and below and the point of conjunction would be taken into account. Both interpretations are possible: In *GD2* 14 we can find the expression "adhering at a distance in degrees which is the geographic latitude" which is in favor for the arc, while in *GD1* 1.11, the latitude is introduced by placing the axis at "the tip of the Sine of geographic latitude" from the horizon.

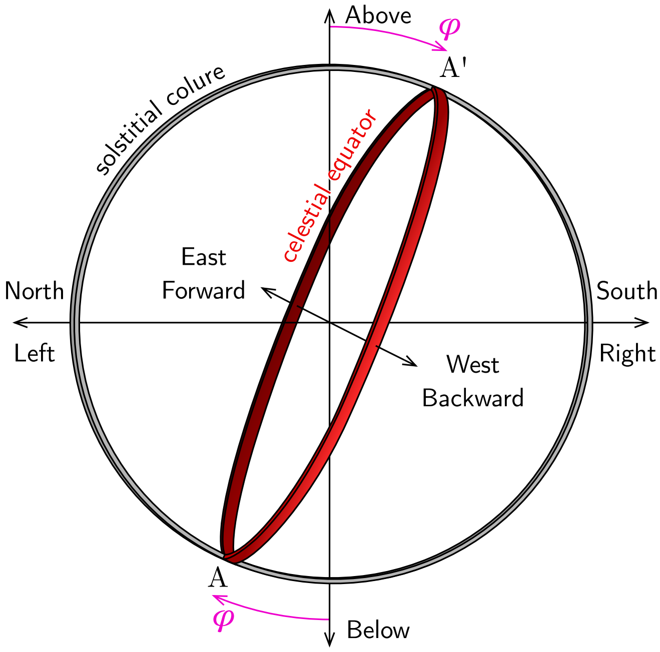


Figure 2.4: Solstitial colure and celestial equator

the solstitial colure and therefore goes through the east and west, but Parameśvara mentions nothing on this point.

The ecliptic is also introduced (*GD2* 3ab) by giving the two points to which it is fixed on. They are point C in the solstitial colure which is separated northward from A by the greatest declination $\varepsilon$ and point C′ separated southward from A′. C and C′ are the summer and winter solstitial points respectively. This circle should also be orthogonal against the solstitial colure, but there is no reference to this in Parameśvara's text.

The equinoctial colure (*GD2* 3cd-4a) is referred to as a girdle (*raśanā*) at the middle (*madhya*) of the celestial equator. Here the word "middle" seems to indicate the points at the east and west on the celestial equator, which are at the middle between above and below. "Girdle" might be an expression for showing the orthogonality of the circle, which is further explained as being transverse to the rotation.

The term *viṣuvat*, literally "in the middle", can stand for the equinoctial colure and also collectively for the three circles, i.e. the solstitial colure, the celestial equator and the equinoctial colure. In the latter sense, I translate *viṣuvat* as "equal division circle", taking into account that the three circles intersect each other in the middle. This term might be an expression for indicating the orthogonality of the circles, which was lacking in the case for the celestial equator against the solstitial colure.

*GD2* 4cd refers to the motion of the sun along the ecliptic. However it is stated nowhere in *GD2* that this motion is annual[5]. The reader of *GD2* is expected to know the rate of the sun's revolution around the Earth in advance[6].

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[5] *GD2* 55 states that the year of human beings (solar year) exists due to the motion of the sun, but not that the motion of the sun takes a year.

[6] We can compare this with *Ābh* 4.2, which mentions the motion of the sun, moon and planets along the

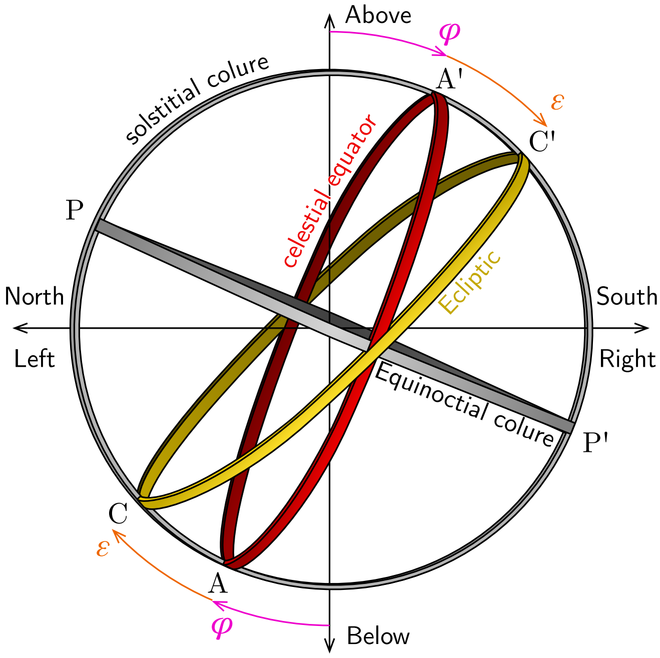


Figure 2.5: Ecliptic and equinoctial colure

### 2.2.1 Description in *GD1*

*GD1* does not take into account the local latitude at the beginning, as if the observer were on the equator. It first describes the three orthogonal rings of the stellar sphere with their six conjunctions facing below, above and the four cardinal directions. Unlike *GD2*, four fixed points are given for each ring, thereby unambiguously determining their positions.

> Here, a circle passing below, above, south and north is to be called the solstitial colure. There is also a circle inside it [attached to it at] the below and top, [passing through] the east and west, called the celestial equator. Outside them both horizontally should be another circle [producing] crosses in the four quarters. (*GD1* 1.3-4ab)[7]

In this situation the "another circle" (the equinoctial colure) is placed parallel to the horizon, and so is the polar axis which will pierce it at the north and south. Then the celestial sphere is introduced, aligned with the stellar sphere. After that, the stellar sphere and the axis is tilted against the celestial sphere to represent the geographic latitude as in the following passage.

> Thus should be the state of the sphere at a latitude-less location (equator). However for a given location, one should make two holes in the celestial sphere down and up from the

ecliptic without reference to their speed. The number of revolutions that each of these seven celestial objects perform in a *yuga* is given in *Ābh* 1.3.

[7] *adha-ūrdhvayāmyasaumyagam iha vṛttaṃ dakṣiṇottarākhyaṃ syāt |*
*tanmadhye 'py adha-ūrdhvaṃ vṛttaṃ pūrvāparaṃ tu ghaṭikākhyam ||1.3||*
*bahir anayos tiryak syāc caturāśāsvastikaṃ paraṃ vṛttam |* (K. V. Sarma (1956–1957, p.11))

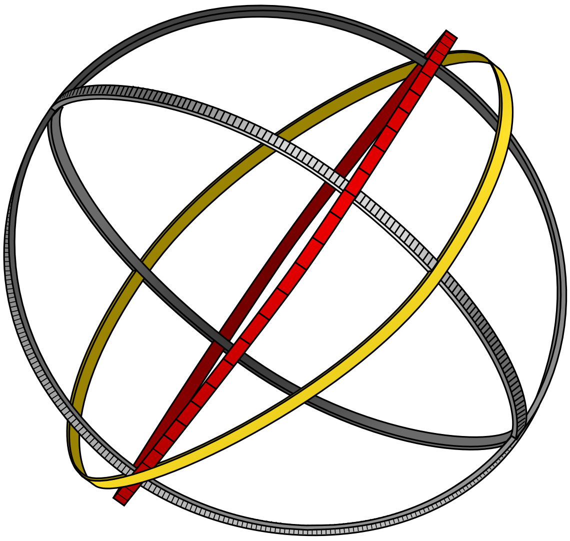

Figure 2.6: The rings graduated as stated in *GD1*

south and north crosses [respectively] at the distance of the Sine of latitude and then make the axis of the celestial sphere pierce them. (*GD1* 1.11-12ab)[8]

*GD1* also describes how the three rings are graduated.

Here the celestial equator has 60 divisions.
Here the other two [circles] have 360 divisions. One should attach yet another circle called the ecliptic, likewise [having 360 divisions], passing through the east and west crosses, to the solstitial colure at 24 degrees north and south [respectively] from the [crosses at] the below and the top. (*GD1* 1.4d-6ab)[9]

The auto-commentary explains the meanings of the gradations as follows:

... the celestial equator is marked with 60 lines. The use of marks is for knowing that it is the celestial equator (*ghaṭikā*)[10]. ...the other two circles are marked with 360 lines. The use

[8] *golasthitir evaṃ syāt nirakṣadeśe hy abhīṣṭadeśe tu |*
*adha ūrdhvaṃ ca khagole yāmyodaksvastikāt palajyānte ||1.11||*
*kṛtvā vedhadvitayaṃ tatprotaṃ goladaṇḍakaṃ kuryāt |* (K. V. Sarma (1956–1957, p.13))

[9] *...kharasāṅkam atra ghaṭikākhyam ||1.4||*
*kharasāgnyaṅkam ihānyad dvitayaṃ tadvat punaḥ paraṃ vṛttam |*
*pūrvāparasvastikagam adha-ūrdhvābhyāṃ ca saumyadakṣiṇayoḥ ||1.5||*
*jinabhāge badhnīyād apamākhyaṃ dakṣiṇottare vṛtte |* (K. V. Sarma (ibid., p.12))

[10] In *GD1* the word *ghaṭikā* refers to the time unit as well as the celestial equator. I shall explain the relation between the time unit and the circle in section 2.5.

> of marks with these two is to know the units of 30 degrees[11,12].

The gradation for degrees in the solstitial colure could immediately be used in the next step for tilting the ecliptic 24 degrees against the celestial equator. Thus this passage, especially with the commentary, would have helped the reader assemble the rings, whether it be with his hands or in his mind.

In contrast, *GD2* mentions nothing about gradations on the rings. The inclination of the ecliptic is only mentioned as the "greatest declination". Furthermore, the ecliptic is introduced after the solstitial colure and the celestial equator, without waiting for the third orthogonal ring (the equinoctial colure). This might be due to the fact that the ecliptic is far more important than the equinoctial colure. In *GD1*, the equinoctial colure plays a role in introducing the ecliptic: it produces two crosses in the east and west with the celestial equator, which are the points that the ecliptic has to pass through.

## 2.3 The polar axis (*GD2* 5)

Parameśvara refers to the intersections of the two colures, P and P′ (figure 2.5). They correspond to the two celestial poles, but Parameśvara only refers to them as a pair of crosses (*svastikayugma*) of the two colures. Another word for "celestial pole" is *dhruva*, literally "fixed". It refers to the pole star. The term *dhruva* in *GD2* is used for the celestial pole as seen by an observer. *svastika* might hint that an armillary sphere is behind the explanation. This is also true when it is used later in *GD2* 155.

An "axis" can refer to the hypothetical polar axis as well as a physical axis in the armillary sphere. However the word *prota* (fixed, piercing) in *GD2* 5 gives the impression that there is an actual object. There is a detailed description which even refers to the material with which the axis is made in *PĀbh* 4.19:

> Then, having put a smooth and straight iron rod into punctures in the two crosses south and north of the sphere, ... [13]

Therefore, if the armillary sphere described in the *Goladīpikā*s were to be actually constructed, the axis would have been made with iron.

## 2.4 Miniature Earth (*GD2* 6)

As aforementioned, this is the only place in *GD2* which refers to the material in a part of the instrument is made. Yet in the same verse, Parameśvara goes on to explain what this miniature Earth is supposed to represent, namely the dwelling of living beings (*prāṇinivāsa*) and so forth. This expression may be comparable with *GD2* 28 where Parameśvara refers to rivers and mountains as being on the Earth alongside creatures. *GD2* 29 stresses that creatures abide everywhere on the Earth's surface.

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[11] Here a circle is divided into 12 signs each consisting of 30 degrees.

[12] *...rekhāṇāṃ ṣaṣṭyā aṅkitaṃ ghaṭikāmaṇḍalam / ghaṭikājñānārtham aṅkavidhiḥ / ...rekhāṇāṃ ṣaṣṭyuttaraśatatrayeṇāṅkitam anyat maṇḍaladvayam / triṃśāṃśakaparijñānārthaṃ tayor aṅkavidhiḥ /* (K. V. Sarma (1956–1957, p.12))

[13] *punaḥ ślakṣṇām ṛjvīm ayaḥśalākāṃ golasya dakṣiṇottarasvastikadvayābhivedhināṃ nidhāya...* (Kern (1874, p.83))

## 2.5 Rotation of the stellar sphere (*GD2* 7-9)

The motion of the stellar sphere, corresponding to the diurnal motion in modern astronomy, is explained in *GD2* 7-9. This motion is constant and clockwise (*pradakṣiṇīkṛt*, literally "towards the right") according to *GD2* 7. For this to be true we need to look at the stellar sphere from the direction of the celestial north pole (assuming that the armillary sphere is being used for explanation), but Parameśvara is implicit on this point. The cause of this motion is a cosmological wind or moving force (*vāyu*) called the *pravaha*, which "blows" at a constant rate outside the Earth. There is a layer of twelve *yojana*s above the Earth surface where the *pravaha* does not blow, but is instead dominated by the wind of Earth.

The speed of the rotation is once every sixty *ghaṭikā*s, which, as explained in *GD2* 9, is shorter than one day. In this case, a "day" is a civil day, measured from sunrise to sunrise. *Ābh* 3.5 differentiates the civil (*sāvana*)[14] day from the sidereal (*nākṣatra*) day, i.e. one revolution of the stellar sphere. *GD2* does not refer to the two measures strictly, and in *GD2* 43-49 we can even find statements implying that sixty *ghaṭikā*s do make one civil day. Nonetheless we could interpret that the *ghaṭikā* in *GD2* 9 is a sidereal *ghaṭikā* and those in *GD2* 43-49 are "civil *ghaṭikā*s" (see also section 4.5).

The term "stellar sphere" appears for the first time in *GD2* 7. However, Parameśvara does not specify what he means by this term. Only later in *GD2* 11c, he states that "this is the stellar sphere", referring to the set of circles that has been described. Why does *GD2* 7-9 refer to the stellar sphere without locating it in the armillary sphere?

### 2.5.1 Description in *GD1*

In this respect, it is worth comparing the three verses with *GD1* 2.2-4 since they are identical apart from a small paraphrasing[15]. *GD1* completely separates cosmological explanation (chapters 2 and 3) from the description of the instrument (chapter 1), whereas *GD2* tends to blend them. *GD2* 4cd on the motion of the sun is another example for the latter. The ambiguity of the term "stellar sphere" in *GD2* 7-9 could be explained if they were initially composed as *GD1* 2.2-4 and later rearranged for *GD2* with the intention to guide the reader to cosmology together with the rings.

## 2.6 Diurnal circles (*GD2* 10-11ab)

The diurnal circle is first introduced in *GD2* 10 as a singular noun. This could have drawn the reader's attention to its function, which is to represent the revolution of the sun on a given day. To be precise, the diurnal circle represents the revolution of a point in the sky where the sun is located at a given moment. This is stated more clearly in *GD1*:

> The portion [of the sky] where the sun is situated revolves on a circle which is called the diurnal circle. (*GD1* 2.16cd)[16]

The sun changes its declination in the course of a day and therefore its actual trajectory in the sky would not be a closed circle. The expression "companion of the celestial equator (*ghāṭikavṛttānusārin*)" is probably a way to express that it is parallel to the celestial equator.

[14] Literally act of pressing [the juice of the soma], and derivatively "duty" or "daily action".

[15] *ghāṭikaṣaṣṭyaṃśasya bhramaṇe* in *GD2* 9 and *ghaṭikākhyaṣaṣṭibhāgabhramaṇe* in *GD1* 2.4, both meaning "in which a sixtieth of the celestial equator rotates".

[16] *yasmin vṛtte sūryasthitabhāgo bhramati tad dyuvṛttākhyam //2.16//* (K. V. Sarma (1956–1957, p.17))

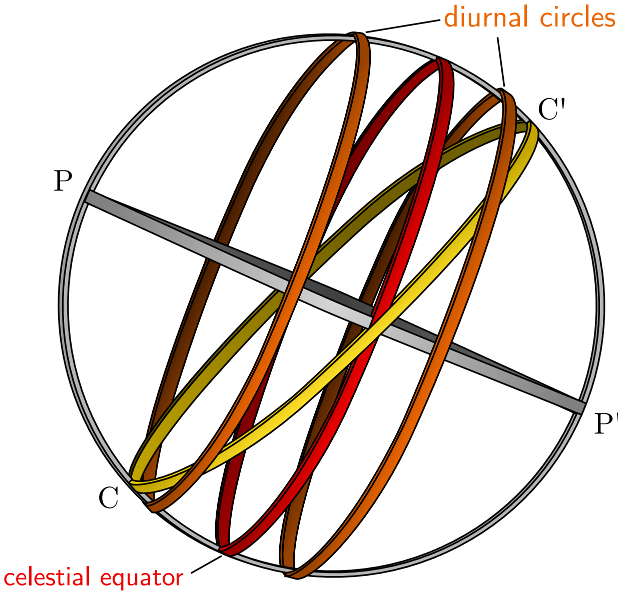


Figure 2.7: Diurnal circles attached to the stellar sphere

Parameśvara then mentions that there can be multiple diurnal circles corresponding to different days (figure 2.7). Since he articulates that they are related to the revolutions of the sun, diurnal circles thus defined should always intersect the ecliptic, and cannot be to the north of the summer solstice C nor to the south of the winter solstice C′. However there is an exceptional case in *GD2* 88 (section 6.7) which makes use of a "diurnal circle" that is unrelated with the sun's motion.

### 2.6.1 Description in *GD1*

> One should attach, on both sides of the celestial equator, at a distance of a given declination from it, likewise, circles called diurnal [circles] of unequal [sizes]. (*GD1* 1.6cd-7ab)[17]

Here the multiplicity of diurnal circles is stated from the beginning. There is a reference to their sizes which is not in *GD2*. Meanwhile there is no association with the sun in this verse. Like the previous cases, *GD1* focuses on the appearance of the rings on the armillary sphere while *GD2* also stresses its function or related cosmology.

## 2.7 Two layers of spheres (*GD2* 11cd)

The latter half of *GD2* 11 tells us that the celestial sphere is outside the stellar sphere and that the celestial sphere does not move. We have already seen in *GD2* 7-9 that the stellar sphere rotates at a constant rate. There is no reference to the ratio of their sizes[18]. *GD1* 1.13 instructs

[17] *ghaṭikākhyobhayapārśve 'bhīṣṭakrāntyantare tatas tadvat //1.6//*
*svāhorātrākhyāni ca badhnīyān maṇḍalāny atulyāni /* (K. V. Sarma (1956–1957, p.12))

[18] According to *Ābh* 3.12, a planet would take 60 solar years to make one revolution if it were on the circumference of [the orbit of] fixed stars, and a *yuga* (4,320,000 solar years) if it were on the circumference of space. It

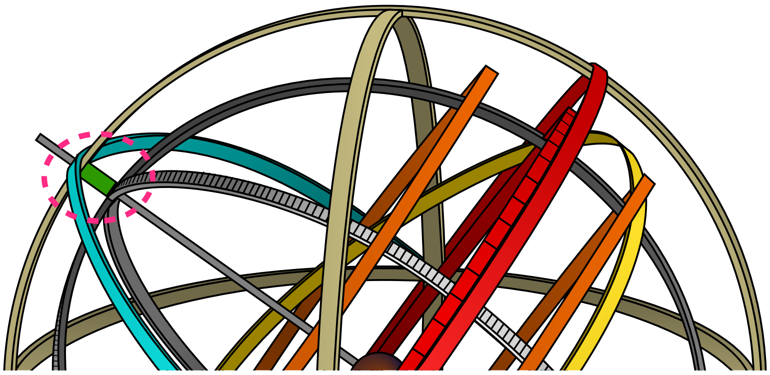

Figure 2.8: Reed attached to the axis (indicated by dotted circle)

to attach two pieces of reed (*śaradaṇḍikā*) to the axis to separate the stellar sphere and the celestial sphere (figure 2.8), but this is not mentioned in *GD2*.

## 2.8 Prime vertical, prime meridian and horizon (*GD2* 12-13)

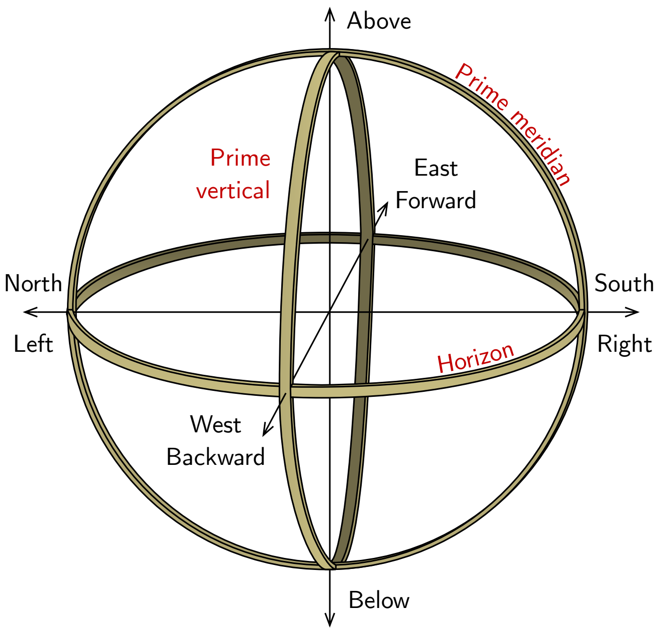


Figure 2.9: Three orthogonal circles in the celestial sphere

The three orthogonal circles in the celestial sphere are named in *GD2* 12-13, each of them with four directions which determine their orientation.

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is unlikely that this cosmology would have been taken into account if this were the discription of the armillary sphere.

The prime meridian in the celestial sphere and the solstitial colure in the stellar sphere are both called *dakṣiṇottara* (literally "south-north") in Sanskrit, and thus the word "too (*api*)" in *GD2* 12cd draws attention that the term as well as the directions (south, north, below and above) are being repeated.

Parameśvara supplies some additional explanation for the horizon in *GD2* 13cd. The rising time and ascensional difference are one of the central topics in *GD2* (especially *GD2* 90-102 and *GD2* 153-194). Therefore the role of the horizon might have been considered important here.

## 2.9 Six o'clock circle (*GD2* 14-16)

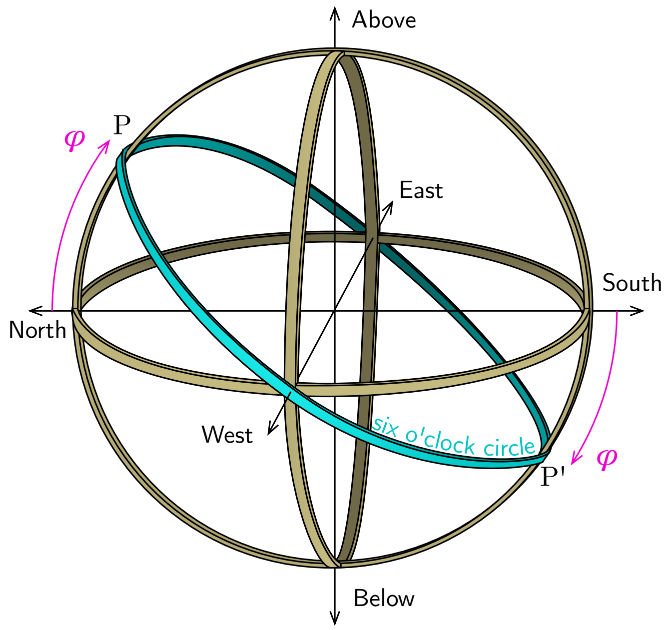


Figure 2.10: The six o'clock circle

The expression used for locating the six o'clock circle (figure 2.10) resembles the way that the celestial equator was introduced in *GD2* 2. Both are tilted in accordance with the geographic latitude. Here in *GD2* 14, the geographic latitude is measured in degrees. This implies that the prime meridian could have been graduated with 360 degrees, but neither *GD1* nor *GD2* refers to gradations of the rings in the celestial sphere.

As stated in *GD2* 15, the circle cuts the stellar sphere so that any point in the sky will take 30 *ghaṭikā*s to revolve above (and below) the six o'clock circle. In other words, every diurnal circle is cut into equal halves by the six o'clock circle (figure 2.11). The time of the day when the sun on any diurnal circle crosses the six o'clock circle (points $O_1$ and $O_2$) corresponds to the moment of sunset or sunrise on an equinoctial day (six o'clock AM or PM in modern notation). The time difference between this and the actual sunset or sunrise of the day is the ascensional difference, which will be dealt with later in *GD2* 74 and onwards. *GD2* 16 refers to the ascensional difference by the length of daylight or night. As we can see in figure 2.11, the ascensional difference can be visualized with the six o'clock circle and the horizon. When the diurnal circle is to the north of

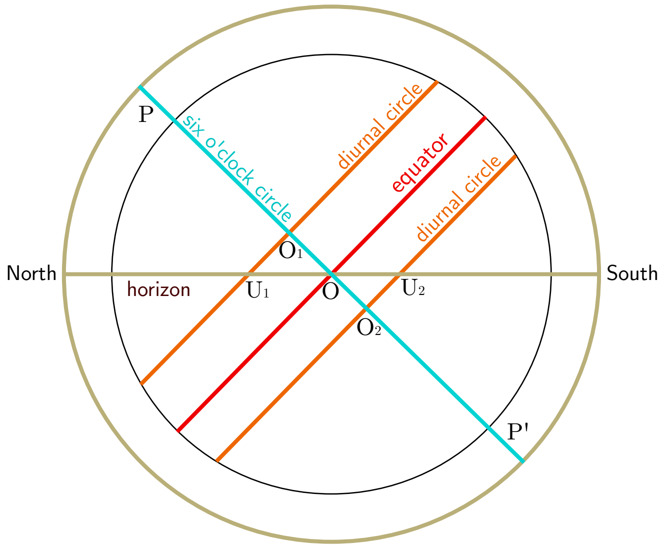


Figure 2.11: The six o'clock circle dividing the diurnal circles and the celestial equator, as seen from the west towards due east. The prime vertical is omitted.

the celestial equator, the daylight is longer due to the ascensional difference $U_1O_1$ and when to the south $U_2O_2$ shortens daylight and increases the length of the night.

According to the previous instructions, the horizon is supposed to be level without being tilted above or below. However, *GD2* 16ab evokes it as being below and above with reference to the six o'clock circle. This point of view can be used for reasonings concerning the ascensional difference (see section 7.5).

## 2.10 Outer celestial equator (*GD2* 17)

The meaning of *GD2* 17 is ambiguous, but it most likely describes another ring, the representation of the celestial equator on the celestial sphere (figure 2.12). The verse uses the expression "or (*vā)*". This implies that the ring is optional, and not necessarily included in the armillary sphere described in *GD2*. Such a ring is not mentioned in *GD1*. Some authors such as Bhāskara II (*Siddhāntaśiromaṇi Golādhyāya* 6.4[19]) do describe a ring for the celestial equator being added to the celestial sphere.

[19]D. Āpṭe (1943–1952, p.201)

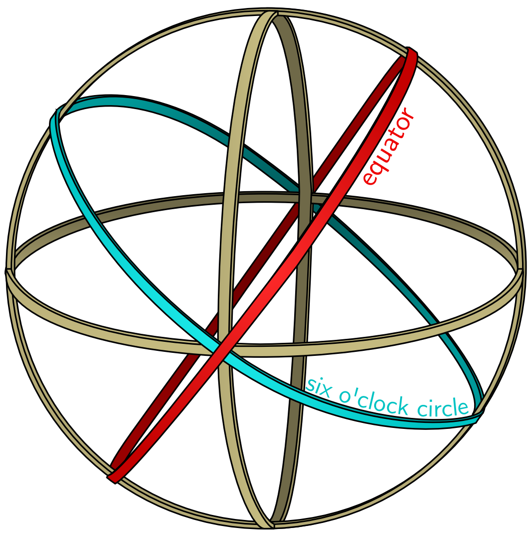


Figure 2.12: The equator on the celestial sphere.

# 3 Arguments on cosmology (*GD2* 18-36)

The topic in *GD2* 18-36 is the existence of conflicting views on cosmology. Parameśvara's position in each discussion is clear. He often refers to the opinions that he agrees upon as those of "calculators (*gaṇaka*)". They are opposed against other views, mainly from the Purāṇas (groups of sacred texts on Hinduism), which are either refuted or reconciled with the cosmology that Parameśvara supports. Several texts gather arguments similar to those found in *GD2*, and among them, four are of special interest: the *Pañcasiddhāntikā* (c.550, hereafter *PS*) of Varāhamihira, Bhāskara I's commentary (629 CE) on the *Āryabhaṭīya* of Āryabhaṭa (hereafter *BhĀbh*), *Śiṣyadhīvṛddhidatantra* (c.748 CE[1], hereafter *ŚDh*) of Lalla and the *Siddhāntaśekhara* (c.1050, hereafter *SŚe*) of Śrīpati.

*PS* contains some criticism on views of the Purāṇas and the Jains and furthermore a refutation on the notion of the Earth's rotation. One of its verses (13.36) is incompletely quoted in *GD2* 23.

*BhĀbh* has detailed discussions on cosmological topics, and some of them resemble the arguments developed by Parameśvara more than other texts, as we will see. Although Parameśvara never quotes *BhĀbh*, the similarities give the impression that he knew the texts.

*ŚDh* was the first text to deal exhaustively with the cosmological tradition of the Purāṇas and was followed by many texts including the *SŚe* (Pingree (1990)). Parameśvara quotes the *ŚDh* in his commentary on the *Āryabhaṭīya* and probably refers to it indirectly in *GD2* 134 (see section 9.6).

*SŚe* is the most prominent treatise that is referred to in our sources of Parameśvara concerning cosmology. In *GD1*, he refers to Śrīpati in the context of cosmology as follows:

> On the other hand, the seven continents and so forth on the spheric Earth have also been mentioned by Śrīpati. Thus we also write, for the young, on some of this subject. (*GD1* 3.62)[2]

This is followed by an extensive description of cosmography in accordance with the Purāṇas (*GD1* 3.63-110) which does not exist in *GD2*. Here in *GD2*, Parameśvara concentrates on geographical descriptions that are strictly based on the spheric Earth model.

In addition, manuscript $I_1$ frequently quotes verses from *SŚe* in between these verses[3]. This shows that at least this reader must have been associating these verses with *SŚe*.

*GD2* 25 and onwards deal with the shape of the Earth and geography. This topic continues into the next subject, the "daylights" of human beings, manes, gods and Brahmā. My sectioning between *GD2* 36 and *GD2* 37 is purely expedient.

## 3.1 Motion of the stars and planets (*GD2* 18-21)

According to *GD2* 18, the fixed stars are in the outer layer of the cosmos and the orbit of planets are located inside them[4]. However the expressions in *GD2* 18ab require attention. Parameśvara

[1] According to Chatterjee (1981, 2, p. xii).

[2] *śrīpatinā tu proktāḥ saptadvīpādayo 'pi bhūgole |
tadviṣayam ataḥ kiṃcid vilikhyate 'smābhir api ca bālebhyaḥ ||3.62|| (K. V. Sarma (1956–1957, p. 36))*

[3] To give an exhaustive list: *SŚe* 10.1-13 after *GD2* 23, *SŚe* 15.7-19 after *GD2* 25, *SŚe* 15.20-23 after *GD2* 26, *SŚe* 15.24-26 after *GD2* 36 and *SŚe* 15.27-72, 2.69-70 after *GD2* 37 (*GD2* 37 is repeated again after the quotations).

[4] The same order of stars and planets are given in *Ābh* 3.15.

refers to the "stars" in plural (*bhāni*) and not as a "stellar sphere (*bhagola*)". Nor does he refer to the orbits of planets at this point. The cosmological structure seems to be described without any link to an armillary sphere. The order of the planets itself is not argued for in *GD2* 18-21. Some comments on views of the Purāṇas concerning the order of the sun and moon can be seen later in *GD2* 66-67.

Parameśvara describes that each planet has an eastward and westward motion (except for the stars which do not have an eastward motion). The westward motion is due to the rotation of the stellar sphere (*GD2* 18cd), which has been described in detail in *GD2* 7-9, and therefore affects every planet (including the stars) equally. This corresponds to the diurnal motion. The eastward motion, which corresponds to the mean motion of planets in their orbits[5], is described in detail in *GD2* 19. Every planet moves an equal distance of *yojana*s along their orbit[6] but their motion in arc minutes as observed from the Earth is different. Parameśvara gives his reasoning in *GD2* 19, which I have visualized in figure 3.1 (note that Parameśvara does not use diagrams in his own explanation). When a planet moves from A to A′ while a planet outside it moves from B to B′, the lengths of $\widehat{\text{AA'}}$ and $\widehat{\text{BB'}}$ are equal when measured in *yojana*s. However, both orbits are equally segmented and thus should have an equal number of arc minutes (21600 minutes in a revolution). Since the outer orbit is larger, there are fewer minutes within $\widehat{\text{A'B'}}$ compared to $\widehat{\text{AB}}$. A similar discussion can be found in *Ābh* 3.14.

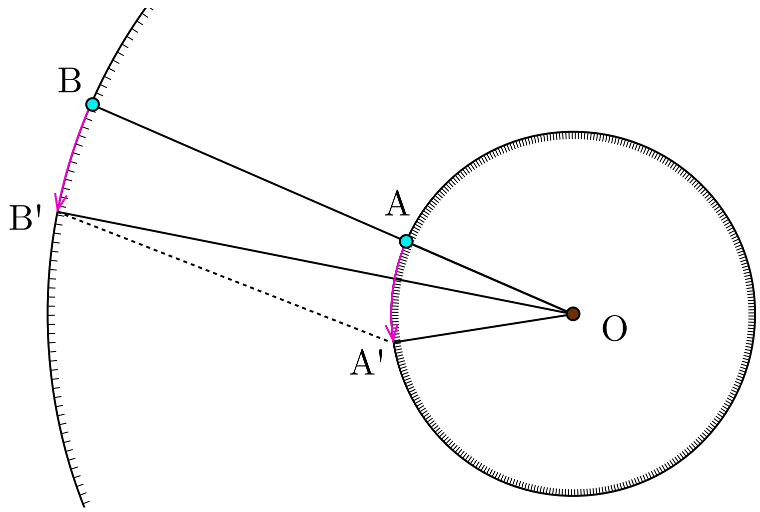


Figure 3.1: Planets on different orbits, having the same daily motion in *yojana*s but different in arc minutes (this diagram shows gradations in degrees).

[5]When Parameśvara is talking about a constant eastward motion, the true motion is not taken into account, since it would cause the motion of the planets to vary, and sometimes even make them move westward by retrograding.

[6]Parameśvara makes no reference to the *yojana*s of a planetary daily motion. We can compute its value from the *Āryabhaṭīya*: The moon revolves 577,533,336 times in a yuga (*Ābh* 1.3) and one arc minute in the moon's orbit is 10 *yojana*s (*Ābh* 1.6). Therefore the moon moves $10 \times 21,600 \times 57,533,336 = 12,474,720,576,000$ *yojana*s in a yuga. Meanwhile the number of civil days in a *yuga* is the number of conjunctions of the Earth with the sun (*Ābh* 3.6), the Earth rotates (or according to those who refute this reading including Parameśvara, the stars rotate) 1,582,237,500 times in a *yuga* and the sun revolves 4,320,000 times (*Ābh* 1.3), thus there are $1,582,237,500 - 4,320,000 = 1,577,917,500$ civil days in a *yuga*. Therefore, the moon moves $12,474,720,576,000 \div 1,577,917,500 = 7905;48,\cdots$ *yojana*s per day. And as *Ābh* 3.12 states, paraphrased in *GD2* 19, this is the same for every planet.

By following *GD2* 18-19, one can conclude that the moon has the largest eastward motion in arc minutes, followed by Mercury, Venus and so forth and Saturn has the smallest eastward motion. When this is combined with the westward diurnal motion, the moon moves westward slower than the other planets because it is dragged eastward, and the fixed stars which do not have an eastward motion appear to move westward more quickly than the planets. Parameśvara introduces a theory in *GD2* 20 which claims this is the result of a single westward motion, slowest for the moon and fastest for the stars, and not of a combination with an eastward motion. This could be an opinion raised from the Purāṇas, as they only refer to a single driving force for the motions of celestial bodies. For example, the *Viṣṇupurāṇa* says that the "orbs of all the planets, asterisms, and stars are attached to Dhruva, and travel accordingly in their proper orbits, being kept in their places by their respective bands of air" (*Viṣṇupurāṇa* 2.12.24-25, translation by Wilson (1840, p. 240)). Neither *ŚDh* nor *SŚe* refer to a single-motion theory, but Bhāskara I introduces it in *BhĀbh* 3.15, in the context of order of planetary orbits:

> Others think that: "The stars, Saturn, Jupiter, Mars, the sun, Venus, Mercury and the moon are located on one same orbit. However they have a swifter motion in this order. Therefore [a planet] having a slightly slower motion is slightly beaten by the asterisms which have a quick motion, and [a planet] having a very slow motion [is beaten] by a large margin. Saturn is slightly beaten because it has a slightly slow motion and the moon [is beaten] by a large margin because it has a very slow motion."[7]

Parameśvara refutes this theory in *GD2* 21 by referring to the retrograde motion (*vakra*, literally "crooked"). His argument is repeated in *GD1*. *GD1* 2.27 is exactly identical with *GD2* 20, and *GD1* 2.28 paraphrases *GD2* 21 with a specific example:

> I think that this is not suitable, because a retrograding planet situated in the asterism of the deity *Anala* (=the lunar mansion *Kṛttikā*[8]) is seen on another day in *Bharaṇī*[9] [which is the lunar mansion] to its west, not in the eastern direction. (*GD1* 2.28)[10]

The lunar mansion *Bharaṇī* is to the west of *Kṛttikā*, so if a planet is first seen in *Kṛttikā* and then later observed in *Bharaṇī*, it would indicate that it had moved westward relatively against the stars. I do not understand how this works as a reasoning, since one could argue back that the planet is not "retrograding toward the east" but "accelerating toward the west" in such case. Nonetheless an identical argument had been made by Bhāskara I more than 800 years earlier. He first states that the stars and planets cannot be moving eastward altogether, and then denies that they are moving in a single westward motion.

> Here in this case too[11], if the planets and the like were facing the east, then [a planet], beaten [in terms of speed] by the asterisms which have a swift motion and face the east,

[7] *anye manyante | tulyakakṣyāsthā eva bhagaṇaśanaiścarabṛhaspatikujaravisitabudhaniśākarāḥ | kin tu yathā-krameṇa śīghragatayaḥ | ato drutagatibhir nakṣatrair īṣamandagatir īṣaj jīyate, atimandagatis tu dūrād iti | īṣan mandagatitvāc chanaiścara īṣaj jīyate, atimandagatitvāc candramā dūram iti |* (Shukla (1976, p. 214))

[8] Third lunar mansion when counted eastward from *Aśvinī*.

[9] Second lunar mansion counted from *Aśvinī*.

[10] *manye tad api na yuktaṃ yasmād vakrigraho 'nalarkṣasthaḥ | tatpaścimagabharaṇyāṃ dināntare dṛśyate na pūrvadiśi ||2.28||* (K. V. Sarma (1956–1957, p. 19))

[11] Prior to this statement, Bhāskara I refutes another theory that places the stars closest to the Earth and the moon at the outermost.

observed in *Aśviṇī*[12] would be seen [later] in *Revatī*[13], not in *Bharaṇī*. Moreover, at times of retrograding, due to its backward motion, [a planet] observed in *Aśviṇī* would indeed be seen [later] in *Bharaṇī*. Now, if these planets and the like are assumed to face the west, even so, at times of retrograding, [a planet] observed in *Aśviṇī* would be seen [later] in *Bharaṇī* due to its backward motion.[14]

Although their reasoning is not obvious to us, we can see that Parameśvara and Bhāskara I share the same argument. Whether Parameśvara had borrowed directly from Bhāskara I is uncertain. This topic does not appear in the two other works of Bhāskara I, namely the *Mahābhāskarīya* and the *Laghubhāskarīya*. Parameśvara never cites *BhĀbh* in his texts, but this resemblance makes us believe that he was familiar with its content.

Some authors after Parameśvara have dealt with this topic, although their relation is yet to be studied. For example, The *Siddhāntasaṃhitāsārasamuccaya* (1583 CE) of Sūryadāsa attempts to find passages in the Purāṇas that support the existence of two motions (Minkowski (2004)).

## 3.2 Forms of the sun and moon (*GD2* 22-24)

In *GD2* 22, Parameśvara defends the idea of "excellent calculators" that heavenly objects beginning with the sun are all spheric. This includes the moon, and probably the five planets too. There might be several sources corresponding to "excellent calculators", but one of them is doubtlessly Varāhamihira. Parameśvara quotes *PS* 13.36 (T. S. Kuppanna Sastri (1993, p. 258)) as *GD2* 24[15]. This verse reasons that the moon can illuminate the darkness during the night by reflecting the rays of the sun by comparing it to a mirror. Meanwhile *GD2* 22 roughly corresponds to *PS* 13.35 (T. S. Kuppanna Sastri (ibid.)). It can also be compared with *Ābh* 4.5 which states that the Earth, planets and stars are spherical, half of the sphere being illuminated by the sun while the other half stays dark. The same notion and reasoning can also be found in *ŚDh* 16.39-41 (Chatterjee (1981, 1, p. 221).

Meanwhile in *GD2* 23, Parameśvara refers to an opposing theory which claims that the objects have the form of a round mirror. "Round (*vṛtta*)" refers to a flat circle, thereby contrasted with "sphere (*gola*)". The "gradual increase of the whiteness of the moon" is a reference to the waxing of the moon. If the moon were flat, the entire surface must be illuminated at the same time when it faces the sun. Therefore it could not appear as a half-moon or crescent. Parameśvara attributes this opinion to some other point of view (*pakṣa*), but I could not trace the origin of this interpretation[16]. Neither *PS* nor *ŚDh* refers to this theory.

*GD2* 22-24 has many parallels with the sequence of discussions (Shukla (1976, pp. 250-251)) provided by Bhāskara I in his commentary on *Ābh* 4.5. This includes reference to the sun and

[12]The lunar mansion which is typically counted as the first in order.

[13]The twenty-second and last lunar mansion. It is to the west of *Aśviṇī*.

[14]*atrāpi yadi prāṅmukhā grahādayas tadā prāṅmukhair drutagatibhir nakṣatrair jīyamāno 'śvinyāṃ dṛṣṭo revatyām upalakṣyeta, na bharaṇyām / vakrakāle 'pi ca, pratilomagatitvād aśvinyāṃ dṛṣṭo bharaṇyām evopalakṣyeta / athaite grahādayo 'parābhimukhāḥ kalpyante, tathāpi vakrakāle 'śvinyāṃ dṛṣṭaḥ pratilomagatitvād bharaṇyām upalakṣyeta /* (Shukla (1976, p. 214))

[15]*PS* 13.36ab and *GD2* 24ab are identical. *GD2* 24cd has been probably modified from *PS* 13.36cd to mention that this is a quotation. Varāhamihira is referred to as a noble person (*āryajana*). Parameśvara must have been aware that the verse was indeed composed by Varāhamihira, as he quotes *PS* 13.12 (T. S. Kuppanna Sastri (1993, p. 250)) in *PĀbh* 4.17 (Kern (1874, p. 82)), referring to the author as Varāhamihira.

[16]Purāṇas are not explicit on the shape of the moon. Nonetheless their explanations on the wax and wane of the moon did not require it to be spheric. For example, the *Viṣṇupurāṇa* explains that the moon waxes as it is fed by the sun, and then it wanes as its ambrosia is drunk by the immortals and the progenitors (Wilson (1840, p. 236)).

moon as having the shape of a round mirror, although it is not specifically referred as an opinion of somebody else.

> How can one understand that these planets and the like have a body with a spheric shape? As for the Earth, others think of the shape of a cart or the shape of a round mirror.
>
> This is not so. I shall speak later so that one understands that the Earth has a spheric shape[17].
>
> But how can one understand in this case that these planets have a spheric shape? Rather, the sun and moon are perceived as having the shape of a round mirror. Likewise for other [planets] too. ...
>
> This is not so. These planets and the like, though having spheric bodies, are perceived as having the shape of a round mirror because they revolve at a distant place.[18]

Interestingly, Bhāskara I juxtaposes the discussion on the shape of the Earth with that on the shape of other celestial bodies. Parameśvara seems to separate the discussion in *GD2*, and there is no explicit reference to opposing opinions claiming that the Earth is flat. The refutation of the false notion that the earth is flat is a common topic in *ŚDh* and subsequent treatises (Pingree (1990)), while notions that other bodies are flat are rarely cited, as we have seen.

## 3.3 The Earth and its support (*GD2* 25-26)

In *GD2* 25, Parameśvara claims that the Earth is a sphere and that it stands in space without support. There is no further debate on the first point, and for the second point, Parameśvara cites Puraṇic theories concerning the supporters of the Earth and refutes them.

*Ananta* is the name of a serpent who is referred to as the supporter of the Earth in the Purāṇas[19]. The concept of elephants in cardinal directions (*diggaja*) as supporters of the Earth is not as conspicuous[20], but are frequently cited by astronomers as theories to be refuted. *ŚDh* 20.7 is a typical example. Parameśvara's reasoning for refuting these theories follows the typical form of pointing out that such ideas lead to an infinite regress of supporting and supported bodies (Plofker (2005)).

## 3.4 Rotation of the Earth (*GD2* 27)

In this verse Parameśvara refutes the notion of the Earth's rotation, which is usually attributed to Āryabhaṭa (Chatterjee (1974)). *PS* 13.6-7 (T. S. Kuppanna Sastri (1993, pp. 249-250)) is the first text arguing against this theory without specifying its source. Brahmagupta quotes the phrase *prāṇenaiti kalāṃ bhūḥ* (the Earth [rotates] one arc minute in one *prāṇa*), which is a

[17] This probably refers to his commentary on *Ābh* 4.6, but the corresponding part is not extant.

[18] *katham ete grahādayo golākāraśarīrāṇi pratipadyante / bhūvaṃ tāvad anye śakaṭākārāṃ darpaṇavṛttākārāṃ ca manyante /*
*naitad evam / yathā golākārā bhūḥ pratipadyate tathottarato vakṣyāmi /*
*kathaṃ punar atrāmī grahāḥ golākārāḥ pratipadyante / atha ca darpaṇavṛttākārau sūryācandramasau lakṣyete, evam anye 'pi / ...*
*naitad asti / ete grahādayo golaśarīrāpi santo dūradeśavartitvād darpaṇavṛttākārā upalakṣyante /* (Shukla (1976, p. 250))

[19] e.g. *Viṣṇupurāṇa* 5.17.12 (Annangaracharya (1972, p. 340), translation in Wilson (1840, p. 541))

[20] The *diggaja*s appear in *Viṣṇupurāṇa* 2.9.15 (Annangaracharya (1972, p. 156)) but they are not referred to as supporters of the Earth.

quotation from *Ābh* 1.6, in *BSS* 11.17 (Dvivedī (1902, p. 152)). Other verses in the *Āryabhaṭīya* which concern this topic are *Ābh* 1.3, *Ābh* 3.5 and *Ābh* 4.9.

Parameśvara insists that Āryabhaṭa did not claim that the Earth was rotating. This can also be seen in his commentaries on the aforementioned verses.

In *Ābh* 1.6, he quotes *prāṇenaiti kalāṃ bhaṃ* (the zodiac[21] [rotates] one arc minute in one *prāṇa*) instead of *prāṇenaiti kalāṃ bhūḥ*. Likewise his reading of *Ābh* 3.5 (Kern (1874, p. 55)) includes *bhāvarta* (revolution of the zodiac) instead of *kvāvarta* (rotation of the Earth). He does not refer to variant readings in both cases[22].

*Ābh* 1.3 includes the passage *ku ṅiśibuṇḷkhṣṛ*[23] *prāk* which can be translated as "the Earth [rotates] eastward one billion five hundred eighty-two million two hundred thirty seven thousand five hundred times [in a *yuga*]". In his commentary, Parameśvara explains:

> Since the zodiac moves westward due to the hurl of *pravaha* wind, the rotation of the Earth is recognized due to false conception. Having agreed upon this, the rotation of the Earth is stated here. However in reality, the rotation of the Earth does not exist. Therefore it should be known that the description of the Earth's rotation in this case is above all for pointing out the revolution of the zodiac.[24]

After this passage, he quotes *Ābh* 4.9 by saying that "the false conception will be spoken thus[25]". *Ābh* 4.9 itself compares the apparent motion of stars to the landscape as seen from a boat:

> Just as one standing in a boat with a prograde motion sees immobile [objects] going retrograde, [one] at Laṅkā sees immobile stars moving uniformly westward.[26]

Parameśvara introduces this as a false conception, and concludes:

> However, the highest truth is that the Earth is indeed fixed. Thus is the meaning [of the verse].[27]

Parameśvara's attitude in *GD2* is consistent with these commentaries on *Ābh*.

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[21] Parameśvara paraphrases *bha* as *jyotiścakra* in his commentary (Kern (1874, p. 9)). He uses *bha* in the sense of zodiac in *GD2* too. See glossary "*bha* (2)" and "*bhacakra*" for details.

[22] Every commentator included in the critical edition of *Āryabhaṭa* by K. V. Sarma and Shukla (1976) chooses the same reading for *Ābh* 1.6 and *Ābh* 3.5. The reading *bhūḥ* in *Ābh* 1.6 can be seen in Pṛthūdaka's commentary on the *Brāhmasphuṭasiddhānta* and Udayadivākara's commentary on the *Laghubhāskarīya* (Chatterjee (1974)). The reading *kvāvarta* is mentioned as a variant reading in the commentaries of Bhāskara I (Shukla (1976, p. 187)) and of Raghunātharāja (according to K. V. Sarma and Shukla (1976)).

[23] *ṅiśibuṇḷṣkhṛ* in K. V. Sarma and Shukla (ibid.). This is the alphanumeric encoding system used uniquely by Āryabhaṭa (see Plofker (2009, pp. 73-75) for a detailed explanation).

[24] *pravahakṣepāt paścimābhimukhaṃ bhramato nakṣatramaṇḍalasya mithyājñānavaśād bhūmer bhramaṇaṃ pratīyate / tadaṅgīkṛtyeha bhūmer bhramaṇam uktam / vastutas tu na bhūmer bhramaṇam asti / ato nakṣatramaṇḍalasya bhramaṇapradarśanaparam atra bhūbhramaṇakathanam iti vedyam /* (Kern (1874, p. 5))

[25] *vakṣyati ca mithyājñānaṃ* (Kern (ibid.))

[26] *anulomagatir nausthaḥ paśyaty acalaṃ vilomagaṃ yadvat /*
*acalāni bhāni samapaścimagāni laṅkāyām //4.9//* (K. V. Sarma and Shukla (1976, p. 119))

[27] *paramārthatas tu sthiraiva bhūmir ity arthaḥ /* (Kern (1874, p. 76))

## 3.5 Life on the surface of the Earth (*GD2* 28-29)

What Parameśvara intends to explain or support in *GD2* 28 and *GD2* 29 is unclear to me.

*GD2* 28abc explains the positions of demons, gods and human beings on the Earth, but this is also mentioned in *GD2* 40 with more details: for example, *GD2* 28abc only says that the gods stay at the top of the Earth, but *GD2* 40 specifies that they stand on Mount Meru. In addition, *GD2* 40 is followed by verses on the sun and the zodiac as seen from different locations on the Earth, and we can link their contents. Whereas the statements in *GD2* 28abc has almost nothing to do with the surrounding verses. *GD2* 28d adds other creatures, rivers and mountains and the like to the list. "Likewise (*tathā*)" probably indicates that they are in the same position with the human beings.

One possible role of *GD2* 28 is that it serves as a reasoning for *GD2* 27. The verse stresses that all these entities "always stay (*nityam vasanti*)" at their locations. Parameśvara might be arguing that if the Earth rotated, everything on the Earth would move together with it too. There are two difficulties with this interpretation: the rotation would not change the fact that these beings are at "the bottom, top and side", and moreover, the typical reasoning for refuting the notion of the Earth's rotation are different. For example, *PS* 13.6cd says "if so, eagles and the like would not come back again from the sky to their own resting-places[28]". Even Parameśvara uses a similar argument in *GD1* 3.4cd: "In this case, how can birds that went out of [their] nests go [back to their] nests?[29]"

*GD2* 29 is even more problematic. I do not have a definitive interpretation for the "circle" mentioned here. If we interpret that the "middle of the Earth" refers to any zone between the north and south poles and not its center, it could be the terrestrial equator. This can be linked to *GD2* 30 which gives the circumference of the Earth. However the additional remark that the circle "stands below all creatures" makes the verse difficult. Be it in the sense the circle (terrestrial equator) is to the south of all creatures or under them, it contradicts the statement in *GD2* 34 that life exists everywhere including underground. *GD2* 29cd juxtaposes "creatures (*prāṇin*)" with "water (*jala*)" while *Ābh* 4.7 talks of "water-born (i.e. aquatic creatures) and land-born (i.e. land creatures) (*jalajaiḥ sthalajaiś ca*)" being everywhere on the spheric Earth. *GD1* 3.36 also lists "creatures, plants and water[30]". Perhaps Parameśvara might be referring to aquatic animals with the word "water" and representing land creatures by simply saying "creatures". This might explain the first half of *GD2* 29 since the northern terrestrial hemisphere is considered to be covered mostly by land. Yet the inconsistency with *GD2* 34 persists.

Another possible explanation for these conflicts is that some of these verses are quotations or paraphrases of other texts. Currently, I do not have a definite candidate for their sources.

## 3.6 Size of the Earth (*GD2* 30ab, 31-34)

The size of Mount Meru stated in *GD2* 30cd, which we will discuss in the next section, is clearly attributed to Āryabhaṭa. It is ambiguous whether he is also the source for the Earth's circumference given in *GD2* 30ab, which is 3299 *yojana*s. The value itself is not given in the *Āryabhaṭīya*, but we can derive it from Āryabhaṭa's statements. According to *Ābh* 1.7, the diameter of the Earth $d_\oplus$ is 1050 *yojana*s. Since the circumference of a circle with a diameter of 20,000 is approximately 62832 (*Ābh* 2.10), we can (approximately) compute the circumference of the Earth $c_\oplus$ with a Rule of Three:

[28] *yady evaṃ śyenādyā na svāt punaḥ svanilayam upeyuḥ* T. S. Kuppanna Sastri (1993, p. 249)

[29] *katham atrāgaccheyur nīḍaṃ nīḍād bahirgatā vihagāḥ* (K. V. Sarma (1956–1957, p. 25))

[30] *prāṇino drumāś cāpaḥ* (K. V. Sarma (ibid., p. 31))

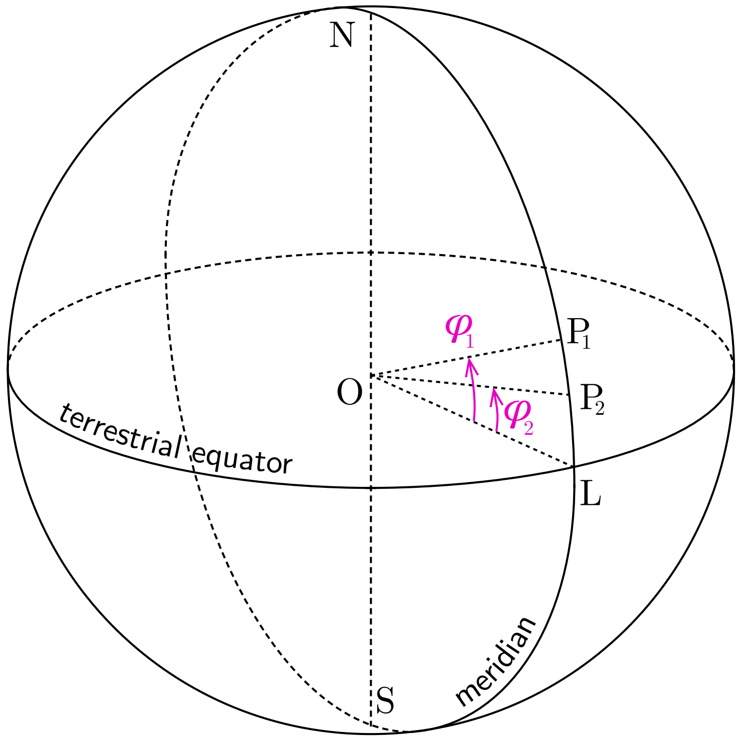


Figure 3.2: Two points $P_1$ and $P_2$ on the same meridian.

$$\begin{aligned} c_\oplus &= \frac{62832 \cdot d_\oplus}{20000} \\ &= \frac{62832 \cdot 1050}{20000} \\ &\sim 3299 (\textit{yojanas}) \end{aligned} \tag{3.1}$$

Parameśvara's claim (*GD2* 31cd) is that this value must have been established with another method. The computation is given in *GD2* 32, which comes from a Rule of Three involving the arc length between two terrestrial locations $P_1$ and $P_2$ with the same longitude (figure 3.2). When L is the intersection of the terrestrial equator with the meridian which goes through points $P_1$ and $P_2$, $\widehat{LP_1}$ and $\widehat{LP_2}$ are their latitudes $\varphi_1$ and $\varphi_2$, respectively, when measured in degrees. $\varphi_1 - \varphi_2$ is the difference in degrees of geographic latitude between $P_1$ and $P_2$. Meanwhile, the distance $\mathcal{D}_{P_1P_2}$ between the two points can be measured in *yojana*s. Since there are 360 degrees in a circle, the circumference of the Earth $c_\oplus$ in *yojana*s is

$$c_\oplus = \frac{\mathcal{D}_{P_1P_2}}{\varphi_1 - \varphi_2} \cdot 360 \tag{3.2}$$

in which we recognize the computation evoked in *GD2* 32. Meanwhile, the measure of the Earth according to wise people (*sudhī*) is in the order of crores, or tens of millions[31] of *yojana*s. This fits the scale of the flat Earth appearing in Purāṇas. For example, the *Viṣṇupurāṇa* describes that mountains called the Lokāloka surround the concentric rings of oceans and continents, and according to Wilson (1840, p. 207) the diameter of its outer rim is five crores ten lakhs and ten thousand (51,010,000) *yojana*s. The *Śivatantra* mentions that the golden land (the outermost continent) is ten crores of *yojana*s (Wilson (ibid.)).

Parameśvara tries to solve this conflict by claiming that great numbers are referring to the surface area or volume[32] of the Earth (*GD2* 33). He does not give specific values of the area and volume, but if he had actually done some computation, then he could have used the rules given in *Līlāvatī* 201[33] (K. V. Sarma (1975, p. 393)). According to the verse, the area $A$ of a circle with a circumference $c$ and diameter $d$ is $A = \frac{cd}{4}$, the surface area $A'$ of a sphere with the same diameter is $A' = 4A$ and its volume is $V = \frac{dA'}{6}$. Since the circumference of the Earth $c_{\oplus}$ is 3299 *yojana*s and its diameter $d_{\oplus}$ is 1050 *yojana*s, its surface area $A'_{\oplus}$ is

$$\begin{aligned} A'_{\oplus} &= 4 \cdot \frac{c_{\oplus} d_{\oplus}}{4} \\ &= 3299 \cdot 1050 \\ &= 3,463,950 \ (yojanas) \end{aligned} \tag{3.3}$$

or roughly 35 lakh *yojana*s, while its volume $V_{\oplus}$ is

$$\begin{aligned} V_{\oplus} &= \frac{d_{\oplus} A'_{\oplus}}{6} \\ &= \frac{1050 \cdot 3463950}{6} \\ &= 606,191,250 \ (yojanas) \end{aligned} \tag{3.4}$$

which is roughly 61 crore *yojana*s. I have decided to use the words “lakh” and “crore” in my translations since claiming that $A'_{\oplus}$ (larger than three million) is “hundreds of thousands” or that $V_{\oplus}$ (approximately six hundred million) is “tens of millions” seemed unnatural.

*GD2* 34 reasons why the numbers may be interpreted as the surface area or volume of the Earth by saying that creatures live everywhere including the nether regions, or the Pātālas[34]. The same argument can be found in *GD1* 3.12-18 (K. V. Sarma (1956–1957, pp. 26-27)), but not in any other text that we have compared with *GD2* in this section. The most similar statement is *ŚDh* 20.33.

[31] The Sanskrit word for ten million is *koṭi*, which entered the English vocabulary through Hindi as “crore”. Likewise for *lakṣa* = lakh = hundred thousand.

[32] To be precise, Parameśvara does not use words for “area (*kṣetraphala*)” or “volume (*ghaṇaphala*)”, and instead uses the expression “resulting number (*phalasaṃkhyā*)” on the surface (for the surface area) or inside the sphere (for the volume).

[33] The *Āryabhaṭa* does not give a rule for the surface area of a sphere and the rule for its volume in *Ābh* 2.7 is wrong. Parameśvara has written a commentary on the *Līlāvatī* and would have been able to apply its rules.

[34] According to the *Viṣṇupurāṇa* there are seven Pātālas layered below the Earth (Wilson (1840, p. 204)). There is no copious description of the Pātālas in any of the Purāṇas, but various texts do refer to their inhabitants (Wilson (ibid., pp. 204-205 footnote)).

> [Even] if it appears to be immense or have many *yojana*s by the effect of it being round, yet this very [Earth] has such sort or circumference and measure [as given before] and not another [value].[35]

But here the nuance is that one can measure the length along the Earth infinitely because it is round, and not that one can use the surface area or volume. Lalla gives the surface area of the Earth in *ŚDh* 17.11[36], but does not compare it with other views.

### 3.6.1 Removing the contradiction (*virodha*)

Parameśvara's approach toward the problem of the size of the Earth is different from those toward the previous ones (motion of celestial objects, their form, the support of the Earth and its rotation).

When he deals with the issues of size, he refers to the opposing side as "wise ones (*sudhī*)". By contrast, he only used normal expressions like "others" or derogatory expressions like "foolish (*mugdhāḥ*)" (*GD2* 23) in the previous cases. This could be a way of acknowledging the authorities of the Purāṇas. In both cases, Parameśvara's side is represented by "calculators (*gaṇaka*)".

Furthermore, he does not reject the views of the wise people, but tries to find an explanation for them. By claiming that the "measure of the Earth" of the calculators is its circumference while that of the wise people is its surface area or volume, he tries to defend both views. At this point, he diverges from previous authors who simply rejected larger sizes for the Earth[37].

In *GD2* 34, Parameśvara uses the word contradiction (*virodha*) to indicate the difference between the two views. This recalls the *virodhaparihāra* or "removal of contradiction" approach starting with Jñānarāja's *Siddhāntasundara*[38] (c.1503 CE), where astronomers tried to find a reconciliation with the Purāṇas without refuting their cosmological elements (Minkowski (2004)). I do not consider Parameśvara as a precursor to this trend, as he follows the manner of refusals by previous authors in many points, and also because later authors do not follow Parameśvara's idea of using the Earth's surface area and volume.

Several questions remain on this subject. Why did Parameśvara differentiate some cosmological topics in the Purāṇas from the others and defend them? What were his sources of the Purāṇas? Does he have a predecessor or did he come up with the idea of the surface area and volume on his own? Can the same argument be found in works after his generation? I would like to pursue them in later research.

## 3.7 Size of Mount Meru (*GD2* 30cd, 35-36)

In *GD2* 30cd, Parameśvara says that the size of Mount Meru is one *yojana* according to Āryabhaṭa. This is mentioned twice in the *Āryabhaṭīya*. The first is *Ābh* 1.7 (*ka meroḥ*). Parameśvara supplies "The measure in *yojana*s of Mount Meru's height is one[39]". The other is in *Ābh* 4.11

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[35] *yadi vṛttavaśena gacchatām amitā bhāty atha bhūriyojanā /*
*paritas tu tadā tathāvidhā parimāṇaṃ tv idam eva nāparam //20.33//* (Chatterjee (1981, 1, p. 236))

[36] The area is 2,856,338,557 [square] *yojana*s, which is based on a wrong computation and far off the right value, as is pointed out by Bhāskara II (Chatterjee (ibid., 2, p. 250)).

[37] The typical reasoning is that the celestial sphere and celestial objects would not be able to revolve around the Earth if it were too large. Examples are *ŚDh* 20.30 (Chatterjee (ibid., 1, p. 236)) and *SŚe* 15.24 (Miśra (1947, p. 148).

[38] Critical edition, translation and explanatory notes including discussions on the *virodhaparihāra* issue by Knudsen (2014).

[39] *meror vyāsayojanapramāṇaṃ ka* (Kern (1874, p. 10))

Figure 3.3: A lotus flower and its cylindrical ovary.

(*merur yojanamātraḥ*) on which Parameśvara glosses "Mount Meru has a height measuring a *yojana* and has a width of that much[40]". *GD1* 3.65 compares its shape to a ovary of a lotus[41], which is cylindrical, and therefore we can figure that the supposed shape of Mount Meru is a cylinder with a diameter and height of one *yojana*.

This is contrasted with the theory that Mount Meru is exceedingly high. The typical height given in the Purāṇas is 84,000 *yojana*s, for example in *Viṣṇupurāṇa* 2.2.8 (Annangaracharya (1972, p. 115)). *GD1* 3.30 refers[42] to this value too.

Parameśvara argues that Mount Meru cannot be excessively high by referring to stars in the northern sky moving to the east. This is true for stars which move between the northern horizon and the pole star (figure 3.4). In such situation, the diurnal motion takes them from the west to the east. If Mount Meru were very high, it should be seen in the northern direction and therefore obstruct these stars.

Neither *PS*, *ŚDh* nor *SŚe* deal with this problem. *ŚDh* focuses on another "false notion" which is that Mount Meru causes the night by hiding the sun (*ŚDh* 20.4, 20.10-13[43]). The section of *BhĀbh* concerning the height of Mount Meru is not extant, but Someśvara, whose commentary summarizes that of Bhāskara I (Shukla (1976, p. cix)), has left a relatively long discussion under *Ābh* 4.11. We can find an argument which resembles the claim by Parameśvara.

> Moreover, if Meru had a great measure, stars in the north would not be seen because they are hidden by the summit of Mount Meru.[44]

In *GD2* 36ab, Parameśvara introduces the theory that Mount Meru pierces the Earth like an axis at the north and south poles. This does not appear in *ŚDh* and *SŚe* but can be found in the

[40] *merur yojanamātrocchritas tāvad vistṛtaś ca* (Kern (1874, p. 76))

[41] *bhūpadmasyāsyāsau madhyasthaḥ karṇikākāraḥ* (K. V. Sarma (1956–1957, p. 36))

[42] "The height of Mount Meru is said to be the measure of eighty-four thousand *yojana*s (*meror ucchritir uktā caturaśītisahasrayojanamiteti*)" (K. V. Sarma (ibid., p. 29))

[43] Chatterjee (1981, 1, pp. 232-233)

[44] *kim ca yadi mahāpramāṇaḥ meruḥ syāt meruśikharāntaritatvāt bhāvāt uttareṇa tārakāḥ na dṛśyeran* (Shukla (1976, p. 262))

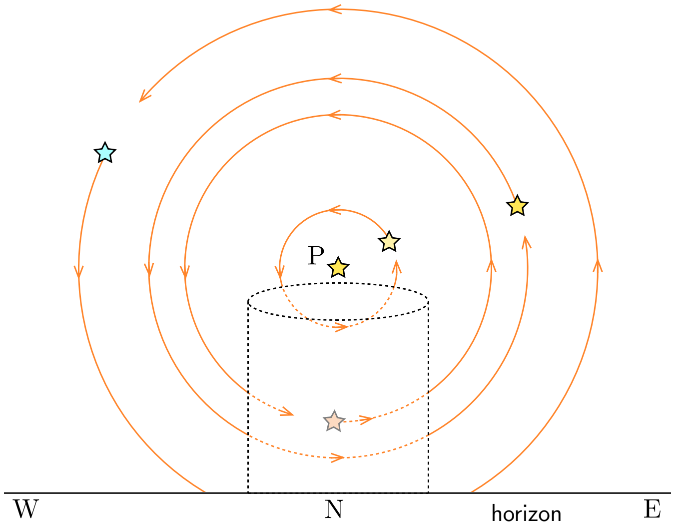



Figure 3.4: An excessively high Mount Meru obstructing the stars moving below the pole star P.

*Sūryasiddhānta* (12.33cd-34ab). Parameśvara explains it here as if it were the opinion of other people, but *GD1* 3.23 suggests that he supports this view.

> Mount Meru should have a pair of tips. One of them is above the middle of the land [and the other] is situated below[45] the middle of water. They are inhabited by gods and demons, respectively. (*GD1* 3.23)[46]

Whether this theory is to claim that Mount Meru is actually very long and thereby solve the contradiction with the Purāṇas[47] is uncertain. Meanwhile Parameśvara avoids the conflict between Āryabhaṭa's view (cited in *GD2* 30cd) that the size of Mount Meru is only a *yojana* by adding that the measurement should be done from the level of the Earth's sphere.

## 3.8 Conclusion: comparison with previous texts

We have seen that Parameśvara's topics or arguments are often different from those in *ŚDh* or *SŚe*, two typical texts that dealt with cosmological contradictions. While we cannot rule out the possibility that they could have inspired Parameśvara in some subjects, we must look at different places to find the sources for his discussions. The similarities between the discussions of Parameśvara and Bhāskara I are striking, and this is certainly a promising direction for further studies.

[45] In this verse, "above" and "below" is from the viewpoint of someone at the north pole (middle of the land). This is stated in *GD1* 3.24ab. Therefore, "below the middle of water" means that Mount Meru sticks out from the south pole.

[46] *meror agrayugaṃ syāt sthalamadhyād ūrdhvagaṃ tayor ekam |*
*jalamadhyāc cādhaḥsthaṃ śiṣṭaṃ devāsuraiḥ kramāt sevyam ||3.23||* (K. V. Sarma (1956–1957, p. 28))

[47] Mount Meru would still be only $1050 + 1 + 1 = 1052$ *yojanas* long and far too short compared to 84000 *yojanas*.

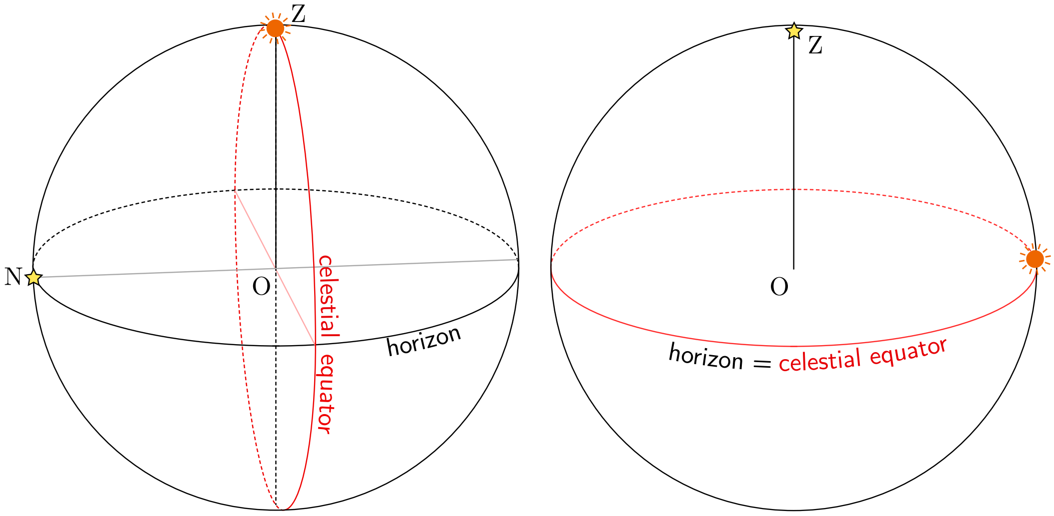
Another unique feature of Parameśvara's arguments is that he frequently represents his views on cosmology as those of calculators (*gaṇaka*). This is very rare for any other authors. Notably, *ŚDh* and *SŚe* never refer to other supporters or advocates of the author's opinion on cosmology. Parameśvara's attitude gives the impression that he is building his opinions and reasoning on top of previous authors, or at least that he is placing himself among other "calculators" who share the same view.

# 4 Geography and long timescales (*GD2* 37-65)

There are two main topics in *GD2* 37-65, tightly related to each other. The first is geography, concerning the sphericity of the Earth. This subject is continued from the arguments on conflicting cosmologies that we have seen in the previous section. The second topic is units of long timescales, notably the four types of "days" (human days, days of the manes, divine days and days of Brahmā) which are periods when the sun is visible to each of these four entities located in different places[1]. Therefore the subject is strongly tied to cosmography and also involves the sphericity of the Earth.

## 4.1 Mount Meru and Laṅkā (*GD2* 37-39)

I have included *GD2* 37-39 in this section and not in the previous one (Arguments on cosmology), since they no longer refer to opposing theories. Parameśvara himself makes no distinct segmentation. Manuscript $I_1$ quotes 48 verses from the *Siddhāntaśekhara*, mainly from chapter 15 on purāṇic geography, after *GD2* 37. Since these quotes are related to the topics in the previous verses, the scribe of this manuscript (or its ancestor) might have intended to insert a division here[2].




(a) Laṅkā (terrestrial equator) (b) Mount Meru (north pole)

Figure 4.1: Positions of the sun on an equinoctial point and the pole star.

*GD2* 37 explains the appearances of the sun on an equinoctial point and the pole star as seen from two locations; Laṅkā on the terrestrial equator and Mount Meru which is the north

[1] One day followed by one night makes one full day. Any Sanskrit word for "day" can also indicate a "full day". In general, we can distinguish one from the other from context. The only place in *GD2* with ambiguity is *GD2* 65 (using *dina*) which concludes this topic (see section 4.12).

[2] *GD2* 37 is repeated twice before and after the quotations. Therefore it is possible that the first is a mistranscription and that the intended segmentation is after *GD2* 36.

pole (figure 4.1). To be precise, we must assume that the sun is culminating in the sky at Laṅkā (this assumption is unnecessary for Mount Meru). At Laṅkā, the sun is on the zenith while the pole star is fixed on the northern horizon (figure 4.1(a)). Meanwhile, the sun is on the horizon and the pole star is on the zenith at Mount Meru (figure 4.1(b))[3]. There is no reference to the celestial equator in *GD2* 37, but I have added them in my diagrams. It goes through the zenith at Laṅkā and coincides with the horizon at Mount Meru. Later in the treatise, the geographic latitude and co-latitude are defined using the sun on an equinoctial point (*GD2* 70), the celestial equator (*GD2* 71) and the pole star (*GD2* 72).

Parameśvara quotes *Ābh* 4.14 as *GD2* 38 and *Ābh* 4.12ab as *GD2* 39ab. In the cosmology that they share, the northern terrestrial hemisphere mainly consists of land while there is more seawater in the southern hemisphere. Thus the expressions "middle of the land" and "middle of the water" indicates the north pole and south pole, respectively. Laṅkā is at a distance of a quarter of the Earth's circumference, i.e. 90 degrees, from both points. *GD2* 38cd=*Ābh* 4.14cd then refers to the geographic latitude of Ujjain (*Ujjayinī*)[4], the city which is associated with the terrestrial prime meridian. According to *GD2* 38cd, it is "at a fifteenth (*pañcadaśāṃśe*) [of the Earth's circumference] due north from Laṅkā", corresponding to 24° north.

However, in his commentary on *Ābh* 4.14 (Kern (1874, p. 79)) Parameśvara reads *taccaturaṃśe* instead of *pañcadaśāṃśe*. This would be translated to "its quarter" where "it" refers to "the quarter of the Earth's circumference" mentioned in the previous half-verse. A quarter of a quarter, i.e. a sixteenth of the Earth's circumference, amounts to 22°30′. Subsequently he introduces the reading "fifteenth" as mentioned by "someone". Furthermore he quotes *Brāhmasphuṭasiddhānta* 21.9cd which states that the distance is a fifteenth of the Earth's circumference. He does not discuss whether the variant reading is correct. Which was his initial knowledge, and when did he change his reading?

Further evidence comes from Govindasvāmin's commentary on the *Mahābhāskarīya* (*GMBh*) and Parameśvara's super-commentary, *Siddhāntadīpikā* (*SD*). *GMBh* 5.4 quotes *Ābh* 4.14 with the reading *taccaturaṃśe* and *SD* 5.4 follows it. Neither of them refer to variant readings. Since Parameśvara's commentary on the *Āryabhaṭīya* mentions his *Siddhāntadīpikā*[5], the *Siddhāntadīpikā* was composed earlier. Thus it is likely that Parameśvara first understood that *taccaturaṃśe* was the correct reading, and later adopted *pañcadaśāṃśe*. If we are right, this suggests that Parameśvara composed *GD2* after his commentary on the *Āryabhaṭīya*. The next question is why he decided to choose *pañcadaśāṃśe* as the correct reading. As aforementioned, he quotes Brahmagupta's *Brāhmasphuṭasiddhānta* 21.9cd. *Pañcasiddhāntikā* 13.10 by Varāhamihira also hints that Ujjain was separated from Laṅkā by 24°, the fifteenth of the Earth's circumference[6]. These two authors could have been Parameśvara's authorities on this topic. Parameśvara's grand-student Nīlakaṇṭha asserts that *pañcadaśāṃśe* is the correct reading and refutes the reading *taccaturaṃśe* by quoting *Brāhmasphuṭasiddhānta* 21.9cd and *Pañcasiddhāntikā* 13.10 (Pillai (1957b, pp. 29-30)). He might be following Parameśvara's decision, but at this moment, I shall just point it out as a possibility.

[3]Notice that in this figure, the sun could be in any direction. Parameśvara seems to think that cardinal directions could be defined on Meru, as he states in *GD1* 3.28: "Laṅkā, Romaka, Siddhapurī and Yavakoṭi. Those cities are by the sea in the southern, western, northern and eastern directions from Mount Meru (*laṅkā ca romakākhyā siddhapurīsaṃjñitā ca yavakoṭiḥ / yāmyāparasaumyaprāgdikṣu nagaryo 'bdhigā imā meroḥ //3.28//*, K. V. Sarma (1956–1957, p. 29))". Some authors deny that Mount Meru has directions, such as Lalla in *Śiṣyadhīvṛddhidatantra* 20.5 (Chatterjee (1981, 1, p. 232)).

[4]Other texts sometimes call the city *Avantī*, but I shall also use the name Ujjain when referring to those occurrences.

[5]For example in his commentary on *Ābh* 2.10 (Kern (1874, p. 26)).

[6]T. S. Kuppanna Sastri (1993, p. 250). See also discussion in Neugebauer and Pingree (1971, p. 84)

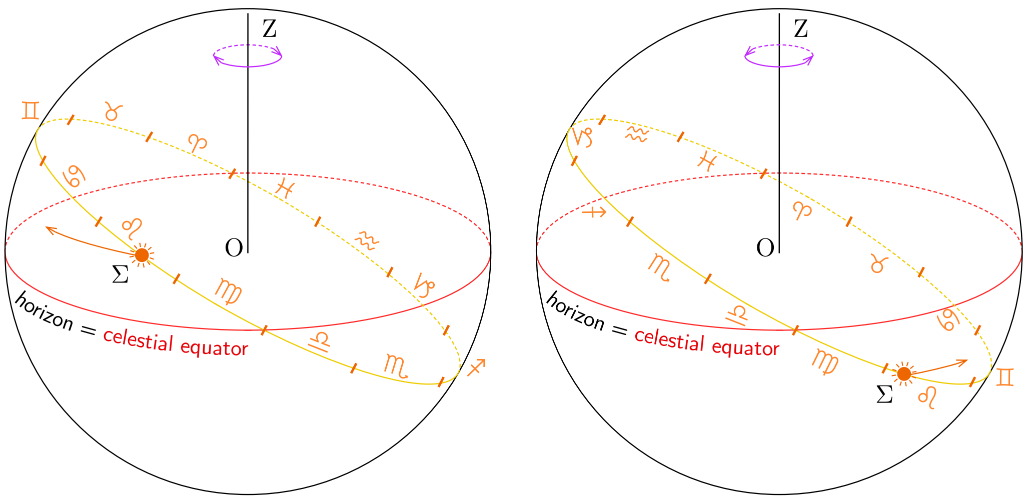
*GD2* 39ab=*Ābh* 4.12ab tells us that heaven (*svar*) and Mount Meru are at the north pole while hell (*naraka*) and its entrance called the "mare's mouth (*baḍavāmukha*)" is at the south pole. There is no information concerning the mutual positions of heaven and Mount Meru or hell and the mare's mouth. *Ābh* 4.12c continues "gods (*amara*) and demons (*mara*)..." to which Parameśvara comments: "Gods live in heaven. Demons live in hell[7]." In the following verses of *GD2*, Parameśvara states that gods live on Mount Meru. It seems that he does not strictly differentiate between heaven and Mount Meru, and likewise, between hell and the "mare's mouth".

## 4.2 Positions of the gods, demons, manes and human beings (*GD2* 40)

*GD2* 40 repeats what has been said in *GD2* 28, and the only new information here is the location of the manes. The difference is that *GD2* 28 was stated in the context of arguments on cosmology and geography, whereas *GD2* 40 is at the beginning of a new topic, "days" of various entities.

According to Parameśvara's descriptions, a day is the period of time that the sun is visible, and night is when the sun is hidden. This can change greatly depending on the observer's location. Parameśvara explains divine days, demonic days, days of the manes and human days in the following verses, which follows the order of his statement in *GD2* 40: gods, demons, manes and human beings.

## 4.3 Divine and demonic day and night (*GD2* 41)




(a) Gods (north pole)

(b) Demons (south pole)

Figure 4.2: Visibility of the sun from the locations of the gods and demons.

From the viewpoint of the gods at the north pole (figure 4.2(a)), the northern celestial hemisphere is always visible, and therefore the same half of the ecliptic can be constantly seen moving

[7] *amarāḥ svargavāsinaḥ / marā narakavāsinaḥ /* (Kern (1874, p. 77))

from left to right. The six visible signs are Aries (♈), Taurus (♉), Gemini (♊), Cancer (♋), Leo (♌) and Virgo (♍). During the half of a solar year when the sun is in these six signs (i.e. from vernal equinox to autumn equinox), the sun will never set. Therefore this half year is a divine day.

During the same half year, the sun is below the horizon when seen from the south pole where the demons are situated (figure 4.2(b)). Thus this period is the demonic night, as stated in *GD2* 41cd. Conversely, when the sun is in the six signs of Libra (♎), Scorpio (♏), Sagittarius (♐), Capricorn (♑), Aquarius (♒) and Pisces (♓), the sun will always be visible from the demons and hidden from the gods. This is the demonic day and the divine night. Parameśvara only refers to the divine day and the demonic night in *GD2*, but he gives a full description in *GD1* 3.43-45 (K. V. Sarma (1956–1957, p. 32)).

## 4.4 Ancestral day and night (*GD2* 42)

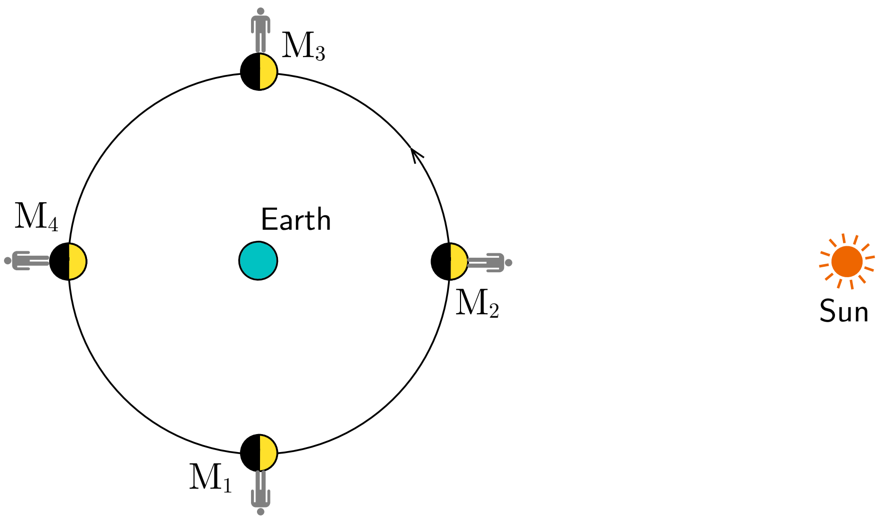



Figure 4.3: The moon's revolution around the Earth causing the day and night of the manes who stand on the back of the moon.

According to *GD2* 40, the manes stand on the "middle of the disk of the moon". *GD2* 23 denies that the moon is flat, and therefore this "disk (*maṇḍala*)" must be a reference to its shape as seen from the Earth. *GD1* 3.58 (K. V. Sarma (ibid., p. 35)) mentions that the manes are "above the orb of the moon (*śaśibimbasya-ūrdhva*)". Since "above" is often used in the sense of "far" from the center of the Earth, we may conclude that the manes are located on the back of the moon as seen from the Earth (figure 4.3). In this situation, the sun becomes visible to the manes when the moon is half and waning (M$_1$). It rises to the zenith at new moon (M$_2$) and sets when the moon is half and waxing (M$_3$). This is the day as seen from the manes. The sun cannot be seen from the manes after M$_3$ until M$_1$ including the moment of full moon (M$_4$). This period is the night of the manes. The dark (*kṛṣṇa*) half-month is from full moon to new moon, and the middle of its eighth day is the midpoint, i.e. waning half moon (M$_1$). Likewise, the bright (*śukla*) half-month is from new moon to full moon, and the middle of its eighth day refers to the waxing half moon ($M_3$). Other treatises, such as the *Brāhmasphuṭasiddhānta*[8], the *Sūryasiddhānta*[9], the *Siddhāntaśekhara*[10] and the *Siddhāntaśiromaṇi*[11] give the same definition.

However, this does not agree with the following statement in the *Mānavadharmaśāstra*.

> The night and day of the manes is a month divided into two half-months. The dark [half-month] is the day for performing activities and the bright [half-month] is the night for sleeping.[12]

In this definition, the day of the manes begins at new moon and ends at full moon. None of the astronomical treatises listed above refer to this discrepancy, let alone argue on it.

## 4.5 Day and night on Earth (*GD2* 43-45)

The day and night at various places on Earth are the main topics in the following verses. The description begins from the terrestrial equator. Unless the geographic latitude is exceedingly large, one day and night equals 60 *ghaṭikā*s. This is the day and night of human beings who "are situated at the side of the Earth's sphere" as stated in *GD2* 40.

### 4.5.1 Two measures of *ghaṭikā*s

According to *GD2* 43ab, the day and night are both 30 *ghaṭikā*s on a location with no geographic latitude, i.e. the terrestrial equator. *GD2* 45 adds that days and nights vary in length at a location other than the equator, but that their sum will always be 60 *ghaṭikā*s. In both cases, one full day is equal to 60 *ghaṭikā*s. This seems inconsistent with what has been mentioned in *GD2* 9 ("the time in which a sixtieth of the celestial equator rotates is proclaimed to be a *nāḍikā*, not the sixtieth of a day"), but Parameśvara is using two different measures (civil and sidereal) for a *ghaṭikā*. He is explicit on this point in *GD1* 2.9-10:

> The sun on the six o'clock circle at the east side reaches the six o'clock circle at the west side in thirty *ghaṭikā*s, and then from there, [reaches the six o'clock circle] at the east side in that much amount of time.
> But in this case, the word "*ghaṭikā*" is said to express a sixtieth part of a day, because this is indeed used in practice except for the rotation of the sphere.[13]

Hereafter in this section, we will interpret *ghaṭikā* as a sixtieth of a full day on the terrestrial equator, or a mean civil day.

[8] *Brāhmasphuṭasiddhānta* 21.8 (Ikeyama (2002, pp. 49-50))

[9] *Sūryasiddhānta* 14.14cd-15ab (Shukla (1957, p. 140))

[10] *Siddhāntaśekhara* 15.61 (Miśra (1947, p. 169))

[11] *Siddhāntaśiromaṇi Golādhyāya* 7.13-14 (Chaturvedi (1981, pp. 408-409))

[12] *pitrye rātryahanī māsaḥ pravibhāgas tu pakṣayoḥ |*
*karmaceṣṭāsv ahaḥ kṛṣṇaḥ śuklaḥ svapnāya śarvarī ||1.66||* (Olivelle (2005, p. 394))

[13] *prāgunmaṇḍalago 'rkas triṃśadghaṭikābhir eti paścimagam |*
*unmaṇḍalaṃ tato 'pi ca tāvat kālena pūrvagatam ||2.9||*
*atra tu ghaṭikāśabdo dinaṣaṣṭyaṃśasya vācakaḥ proktaḥ |*
*vyavahāro hy anayaiva syād golabhramaṇato 'nyatra ||2.10||* (K. V. Sarma (1956–1957, p. 16))

### 4.5.2 Places of human beings

*GD2* 43-45 also adds some information on geography. Some of the previous verses have implied that the northern terrestrial hemisphere is mainly covered by land whereas much of the southern hemisphere is water. This is stressed by Parameśvara's statement in *GD2* 43cd that the four cities on the terrestrial equators are on the border of land and water. Furthermore, he mentions that the day is longer when the sun is in the northern celestial hemisphere. This is only true if the observer is in the northern terrestrial hemisphere. Apparently, Parameśvara does not take human activities in the southern terrestrial hemisphere into consideration. This applies elsewhere in *GD2*.

## 4.6 Midnight sun and polar night (*GD2* 46-49)

From hereon, Parameśvara describes regions with extremely high latitudes where the sun does not set or rise during some period. This is the polar region in modern terminology. *GD2* 46-49 focuses on the place where a midnight sun can be seen at summer solstice and a polar night occurs at winter solstice (i.e. a place on the arctic circle), while *GD2* 50-54 introduces areas with higher geographic latitudes, including the north pole.

This topic first appears in Varāhamihira's *Pañcasiddhāntikā* 13.21-25 (T. S. Kuppanna Sastri (1993, pp. 254-255)), and has been repeated by many texts, such as Lalla's *Śiṣyadhīvṛddhidatantra* 16.20 (Chatterjee (1981, 1, p. 208)), Śrīpati's *Siddhāntaśekhara* 16.56-57 (Miśra (1947, pp. 231-232)) and Bhāskara II's *Siddhāntaśiromaṇi Golādhyāya* 7.25, 7.28-30 (Chaturvedi (1981, pp. 411, 413)). Neither Āryabhaṭa nor Bhāskara I deals with this subject.

Figure 4.4 illustrates the situation described in *GD2* 46-48. The arc distance $\widehat{\mathrm{ZP}}$ between the zenith Z and the celestial north pole P is the co-latitude $\bar{\varphi}$, and the arc distance $\widehat{\mathrm{M\Sigma}}$ of the summer solstice point on the ecliptic (in this case also the place of the sun $\Sigma$) from the celestial equator is the greatest declination $\varepsilon$. At this location, the Sine of co-latitude $\mathrm{Sin}\,\bar{\varphi}$ is equal to the Sine of greatest declination $\mathrm{Sin}\,\varepsilon$. If the sun is at the end of Gemini, i.e. on the summer solstice, the entire diurnal circle would be above the horizon with only one intersection at due north.

*GD2* 47 is a quotation from Govindasvāmin's commentary on *Mahābhāskarīya* 3.53 (T. Kuppanna Sastri (1957, p. 167)). Govindasvāmin himself attributes this verse to Āryabhaṭa and quotes it to refute that Mount Meru is very high, because the mountain would hide the sun in that case (cf. section 3.7). In his super-commentary *Siddhāntadīpikā*, Parameśvara mentions that this verse was composed by Bhāskara [I]. In *GD1*, Parameśvara quotes the same verse as *GD1* 3.33 (K. V. Sarma (1956–1957, p. 30)) to argue against views that Mount Meru is high, as did Govindasvāmin. Here in *GD2*, Parameśvara does not link the quote with Mount Meru.

*GD2* 47 has the form of a question, and Parameśvara gives the answer in *GD2* 48ab. The Sine of geographic latitude $\mathrm{Sin}\,\varphi$ is equal to the upright Sine, i.e. the Cosine of greatest declination $\mathrm{Cos}\,\varepsilon$. *GD2* 48 then states the ascensional difference $\omega$ at this moment. When the sun $\Sigma$ is on the horizon at due north and the point on the celestial equator corresponding to its longitude is M (figure 4.4), $\omega = \widehat{\mathrm{ME}} = \widehat{\mathrm{MW}} = 15$ *ghaṭikā*s. In this situation (summer solstice), the day is 60 *ghaṭikā*s which is as long as it can be and the night does not fall.

Parameśvara links the geographic latitude and the greatest declination with the ascensional difference in this verse, but it is doubtful that Parameśvara intended the reader to actually compute the ascensional difference from them[14]. The situation could be easily visualized with an armillary sphere, but we have no further clue to Parameśvara's actual intention.

[14] If he really did, the required steps would be to compute the radius $r$ of the diurnal circle from the Sine of declination $\mathrm{Sin}\,\varepsilon$ (*GD2* 73cd), the Earth-Sine $k$ from $\mathrm{Sin}\,\varepsilon$, $\mathrm{Sin}\,\varphi$ and $\mathrm{Sin}\,\bar{\varphi}$ (*GD2* 74ab) and the Sine of ascensional

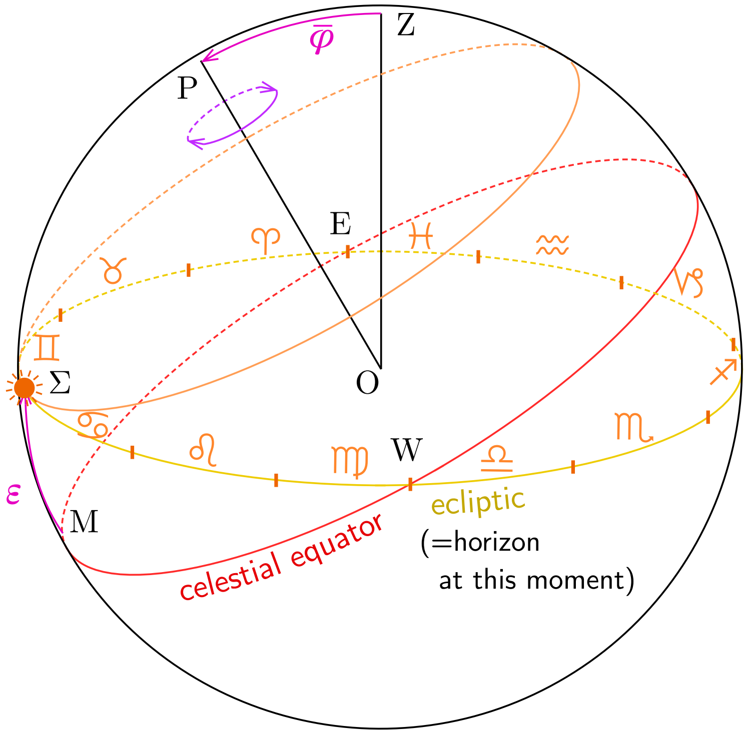



Figure 4.4: The sky when the co-latitude $\bar{\varphi}$ is equal to the greatest declination $\varepsilon$ and the sun $\Sigma$ is on the summer solstice.

*GD2* 49ab refers to days before and after the summer solstice. Days closer to the summer solstice have a longer daytime, and the daytime diminishes when the day is further from the summer solstice (either it be before or after). When the sun is on the other side, which is the end of Sagittarius or winter solstitial point (figure 4.5), the diurnal circle will be under the horizon, touching it at due south. Therefore on this day, which is the winter solstice, the observer will see a polar night of sixty *ghaṭikā*s (*GD2* 49cd).

## 4.7 Ascending signs at polar regions (*GD2* 50-54)

When the geographic latitude is even larger (and the co-latitude smaller) than the situation described in *GD2* 46-49, there is a section on the ecliptic that will always be visible in the course of the day, and another section that will never rise above the horizon. *GD2* 50-51 describe a location where the co-latitude $\bar{\varphi}$ is equal to the declination $\delta_2$ corresponding to a longitude of two signs from the vernal equinox (figure 4.6). The point on the ecliptic with such longitude is the beginning of Gemini (G). It will touch the horizon but never set at this location. This is the same for the end of Cancer (K), which is two signs away from the autumn equinox. Meanwhile, the beginning of Sagittarius (D) and the end of Capricorn (C), which are two signs away from the equinoxes toward the winter solstitial point, touch the horizon but do not rise above it in

difference from $k$ and $r$ (*GD2* 74cd).

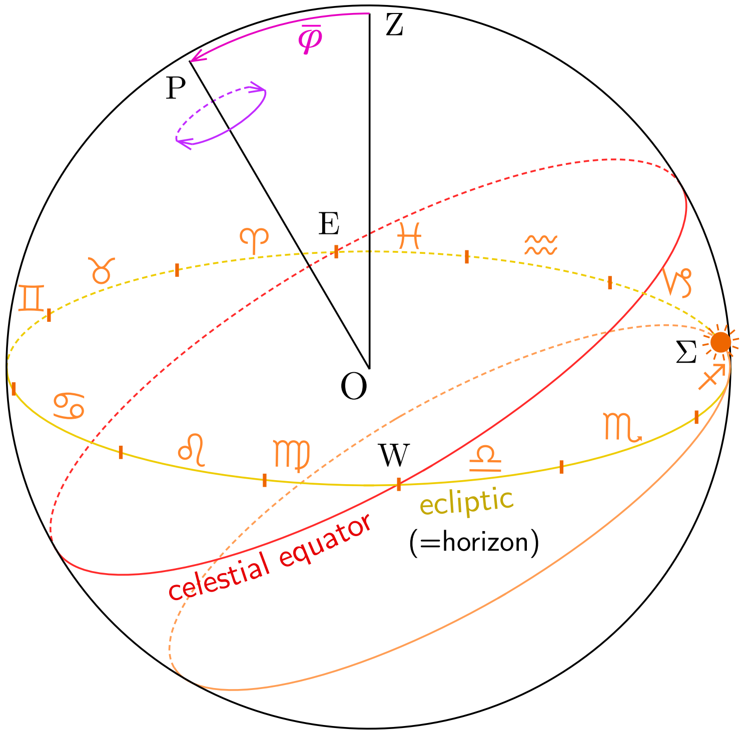



Figure 4.5: The sky when the co-latitude $\bar{\varphi}$ is equal to the greatest declination $\varepsilon$ and the sun $\Sigma$ is on the winter solstice.

the course of the day. Therefore Gemini and Cancer are always above the horizon, and the sun will not set while it is in these two signs. Sagittarius and Capricorn never rise, and neither will the sun in these signs. Parameśvara only refers to the visibility of the signs themselves and does not relate it to the sun.

The remaining eight signs may rise and set. This is what Parameśvara means by "appear (*yānti*) on the horizon" or "become an ascendant (*lagna*, literally adhere or touch; the point of the ecliptic that is on the horizon in the east)". In *GD2* 51 he also refers to the order in which the signs become ascendants, and at this point Parameśvara gives a wrong statement. The sign which rises after Taurus is actually Aries and not Leo as Parameśvara says. Taurus, not Aquarius, is the ascendant subsequent to Scorpio. Let us look at the moment when Taurus rises after Scorpio (figure 4.7). Before this moment, Leo, Virgo, Libra and Scorpio rise in the normal order, while the ascendant in the horizon shifts from north to south. Scorpio rises near due south as Taurus sets near due north (figure 4.7(a)) until their ends touch the horizon (figure 4.7(b)). Subsequently, Taurus will begin rising in the eastern half of the horizon as Scorpio sets in the western half (figure 4.7(c)). Now the order of ascendants is backwards, and Aries will rise after Taurus, followed by Pisces and Aquarius, as the ascendant shifts from north to south again . At the same time, Scorpio, Libra, Virgo and Leo will set in this reversed order. The descendant constantly shifts from south to north. Leo rises again after the beginning of Aquarius (its border with Capricorn) touches the horizon. This reversal of the ascendant will not occur outside the polar region where the ecliptic does not intersect with the horizon at due north.

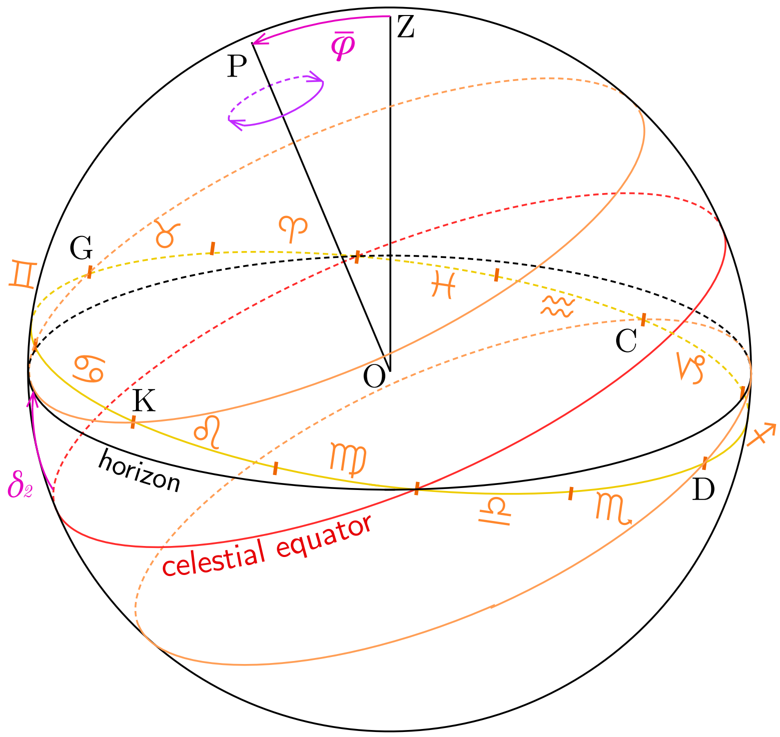



Figure 4.6: The sky when the co-latitude $\bar{\varphi}$ is equal to the declination $\delta_2$ corresponding to a longitude of two signs.

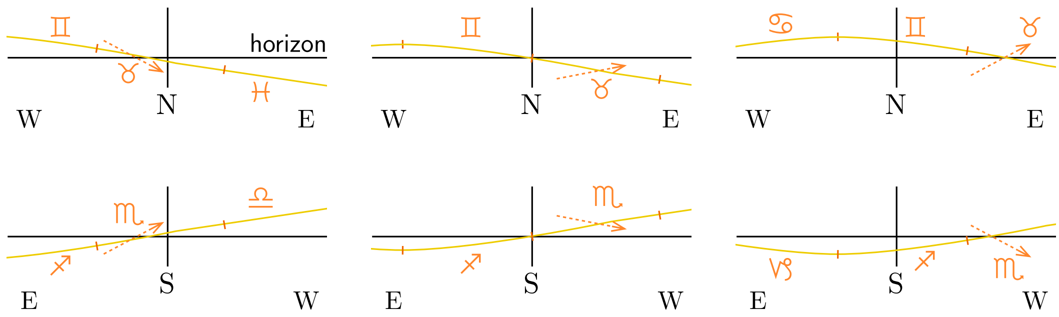
In *GD2* 52-53, Parameśvara describes the sky as seen from a location where the co-latitude $\bar{\varphi}$ is equal to the declination $\delta_1$ on the ecliptic where the longitude is one sign from the vernal equinox (figure 4.8). The corresponding point is the beginning of Taurus (T). This point, as well as the end of Leo (L) which is one sign from the autumn equinox, touch the horizon in the north but do not set. The beginning of Scorpio (V) and the end of Aquarius (A) touch the horizon in the south but do not rise. This agrees with Parameśvara statement in *GD2* 52. However he makes the same mistake as previously for the order of rising signs in *GD2* 53. It should be Aries, Pisces, Virgo and Libra.

Parameśvara was apparently unaware at this moment that signs could rise in reverse order in polar regions. Other treatises which could have been available to him do not deal with this topic. However, he acknowledges this phenomenon in *GD1*. This is an evidence that *GD1* must have been composed after *GD2*. Parameśvara's expression in *GD1* 3.54 hints that he might have reflected upon this topic with the usage of an armillary sphere.

> Wherever the Sine of co-latitude is smaller than the greatest declination, there, some of the signs should rise in reverse order. This should be explained completely on a sphere.[15]

*GD2* 54 is essentially repeating what has been stated in *GD2* 41 but in a different context. Mount Meru, or the north pole, is a location where the co-latitude $\bar{\varphi}$ is zero. It gives the

[15] *paramāpakramato 'lpā lambajyā yatra tatra rāśīnām |*
*keṣāṃcid utkramāt syād udayo gole pradṛśyam akhilaṃ tat ||3.54||* (K. V. Sarma (1956–1957, p. 34))




(a) Scorpio (♏) rising (b) Ends touch horizon (c) Taurus (♉) rising

Figure 4.7: The ecliptic in the north and south directions when the ascendant changes from Scorpio to Taurus. The diurnal motion moves the ecliptic from west to east in the north and from east to west in the south.

Table 4.1: Long time periods appearing in *GD2* 55-64.

| Introduced period | Relation with previous units | Verse |
|---|---|---|
| Full divine day | 1 human year | 55 |
| Divine year | 360 full divine days | 55 |
| *Caturyuga* | 12,000 divine years | 56 |
| Divine *Yuga* | 1 *caturyuga* | 56 |
| *Kṛtayuga* | 4,800 divine years | 57 |
| *Tretāyuga* | 3,600 divine years | 57 |
| *Dvāparayuga* | 2,400 divine years | 57 |
| *Kaliyuga* | 1,200 divine years | 57 |
| Day of Brahmā | 1,000 *caturyuga*s | 58 |
| Night of Brahmā | 1,000 *caturyuga*s | 58 |
| *Kalpa* | Day of Brahmā | 58 |
| *Manu* | 14 *manu*s = 1 day of Brahmā | 59 |
| Twilight | $\frac{6}{15}$ *caturyuga*s | 60 |
| Year of Brahmā | (360 full days of Brahmā) | 62 |

impression that there is a continuity in the subject with *GD2* 46-49 (where $\bar{\varphi}$ is equal to the declination corresponding to a longitude of three signs from an equinox), *GD2* 50-51 (equal to the declination corresponding to a longitude of two signs) and *GD2* 52-53 (one sign).

## 4.8 Divine day and year (*GD2* 55)

*GD2* 55 mentions the annual motion of the sun which was also implied in the previous verses (*GD2* 46-54). In this verse, it is referred to as the cause of the "human year" which amounts to a solar year. This, in turn, is stated as the equivalent of a "divine day and night". 360 full divine days (day and night combined), i.e. 360 solar years, amount to a divine year. From here on, time periods exceeding human timescales are given, as listed in table 4.1.

*Ābh* 3.1ab is a general statement on the relation between a day and a year.

> Twelve months are a year, and this month should be thirty days.[16]

[16] *varṣaṃ dvādaśa māsās triṃśad divaso bhavet sa māsas tu* / (Kern (1874, p. 51))

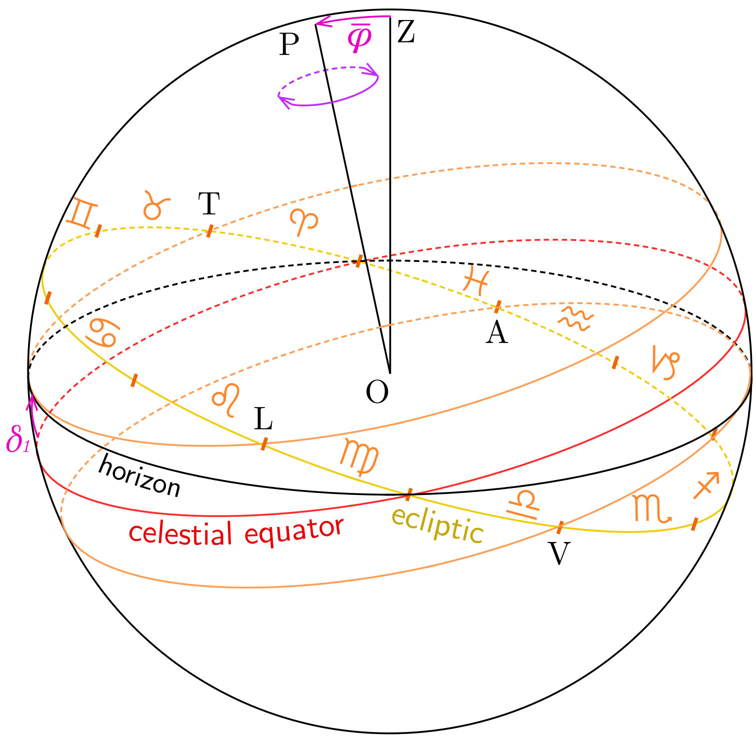



Figure 4.8: The sky when the co-latitude $\bar{\varphi}$ is equal to the declination $\delta_1$ corresponding to a longitude of one sign.

Āryabhaṭa does not specify the definition of a "day" or "year" in this verse. In his commentary (Kern (1874, pp. 51-52)), Parameśvara says that this division applies to 9 different measures of time, and quotes the *Sūryasiddhānta* 14.1:

> The nine measures are indeed [those of] Brahmā, manes, divine, [of the] lord of creatures, Jovian, solar, civil, lunar and sidereal.[17]

Among these nine measures, those of Brahmā, the manes, divine and civil (i.e. human) are enumerated in *GD2* 65. According to *Sūryasiddhānta* 14.21cd (Shukla (1957, p. 142)), the "measure of the lord of creatures" refers to the time unit *manu*, which is treated in *GD2* 59. The "Jovian measure" indicates the Jupiter cycle of sixty years (see Burgess and Whitney (1858, p. 179) and Srinivasan (1979, pp. 144-146)). This measure does not appear in *GD2*.

## 4.9 The *caturyuga* and its division (*GD2* 56-57)

*GD2* 56 introduces the *caturyuga*, literally "four *yuga*s", which is further divided into four parts as explained in *GD2* 57. Table 4.2 lists the length of these four parts in solar years, comparing

[17] *brāhmaṃ pitryaṃ tathā divyaṃ prājāpatyaṃ ca gauravam / sauraṃ ca sāvanaṃ cāndram ārkṣaṃ mānāni vai nava //14.1//* (Kern (1874, p. 52), matches with the critical edition of Shukla (1957, p. 138))

Table 4.2: Lengths of each *yuga* according to different texts (in solar years)

| Name of period | *GD2* | *Manu* | *Āryabhaṭīya* |
|---|---|---|---|
| *Kṛta-yuga* | 1,728,000 | 4,000 | 1,080,000 |
| Twilight | | 400×2 | |
| *Tretā-yuga* | 1,296,000 | 3,000 | 1,080,000 |
| Twilight | | 300×2 | |
| *Dvāpara-yuga* | 864,000 | 2,000 | 1,080,000 |
| Twilight | | 200×2 | |
| *Kali-yuga* | 432,000 | 1,000 | 1,080,000 |
| Twilight | | 100×2 | |
| Total (*caturyuga*) | 4,320,000 | 12,000 | 4,320,000 |

them with the years according to the *Mānavadharmaśāstra* (denoted "*Manu*" in the table) and the *Āryabhaṭīya*[18].

The four parts are unequal in length with a ratio of 4:3:2:1, which resembles the *Mānavadharmaśāstra*. However *Mānavadharmaśāstra* 1.69-71 (Olivelle (2005, p. 394)) defines that the *Kṛta-yuga* itself is 4,000 years (normal years, and not the divine years). Twilights of 400 years are placed before and after the *Kṛta-yuga*. The *Tretā-yuga* is 3,000 years with twilights of 300 years, and so on. The total for each part including the twilight in solar years are 4,800, 3,600, 2,400, 1,200 respectively. The same values occur in *GD2* 57, except that they are the divine years and not solar years. *Mānavadharmaśāstra* 1.71 concludes that the *caturyuga*, with a total of 12,000 years, is the "divine *yuga*". This resembles the statement in *GD2* 56cd.

On the other hand, Āryabhaṭa is believed to have divided the *caturyuga* into four equal parts. He uses the expression *yugapāda* in *Ābh* 1.5 and *Ābh* 3.10 which could be translated to a "quarter of a *yuga*". Bhāskara I comments: "Meanwhile for us, every quarter of a *yuga* is indeed of equal timespan (Commentary on *Ābh* 3.8)[19]". Āryabhaṭa had very few followers after Bhāskara I; Vaṭeśvara is one of them[20]. Other treatises adopt a system with *yuga*s of 4:3:2:1, as is the case with *GD2*.

## 4.10 Day of Brahmā (*GD2* 59-61)

Another unique feature in Āryabhaṭa's system is that 1,008 *caturyuga*s make up a day of Brahmā (*Ābh* 3.8). *GD2* 58 states that it is 1,000 *caturyuga*s. According to *Ābh* 1.5, a day of Brahmā is further divided into 14 *manu*s and each period consists of 72 *caturyuga*s. Hence $14 \times 72 = 1,008$. *GD2* 59 also defines that there are 14 *manu*s in a day of Brahmā, but each has only 71 *caturyuga*s. $14 \times 71 = 994$, and the remaining 6 *caturyuga*s are divided into 15 parts, distributed at the beginning and end of a day of Brahmā and in between *manu*s. This is called the twilight (*saṃdhyā*), each lasting $\frac{6}{15}$ *yuga*s (*GD2* 60). Many astronomers, apart from Āryabhaṭa and his followers, explain the same system[21]. However, Parameśvara makes a peculiar statement in *GD2* 61. He further divides the twilight of a *manu* into two parts. It resembles the structure of the

[18] This investigation was inspired by Yano (1980) which compares the *yuga-kalpa* (day of Brahmā) system in the *Āryabhaṭīya*, the "traditional system (represented by the *Brāhmasphuṭasiddhānta*)" and the *Mānavadharmaśāstra*.

[19] *asmākaṃ tu yugapādāḥ sarva eva ca tulyakālāḥ* / (Shukla (1976, p. 197))

[20] *Vaṭeśvarasiddhānta* 1.14 (Shukla (1985, pp. 147-148)) is an objection to *Brāhmasphuṭasiddhānta* 11.4 which criticized Āryabhaṭa. Not every time unit in the *Vaṭeśvarasiddhānta* agrees with the *Āryabhaṭīya*, but it does divide the *caturyuga* into equal parts.

[21] For example in *Sūryasiddhānta* 1.18-20 (Shukla (1957, pp. 4-5))

two "twilights" allocated before and after the four *yuga*s in the *Mānavadharmaśāstra*, which are also called the "portion of twilight (*saṃdhyāṃśa*)" and "twilight". But no other treatise divides the twilight of a *manu* in this manner. Whether there was a confusion by Parameśvara himself or during the transmission is yet to be studied.

## 4.11 Elapsed time in the life of Brahmā (*GD2* 62-63)

Parameśvara does not explicitly state the length of a "year of Brahmā" which appears in *GD2* 62, but it may be inferred from his commentary on *Ābh* 3.1 (see previous statement in section 4.8) that 360 full days of Brahmā make one year of Brahmā. This unit of time does not appear in the *Mānavadharmaśāstra*, nor is it mentioned in the *Āryabhaṭīya*. The purāṇic system developed this cycle, and further added that 100 years of Brahmā was his life span (González-Reimann (2009, p. 420)). Later astronomical treatises, such as the *Siddhāntaśekhara*[22], adopt this system. The elapsed years of Brahmā, *manu*s and *yuga*s as stated in *GD2* 62 also match the descriptions in this purāṇic system. The expression "the very first of the remaining is to be assumed (*ādya*[23] *eva śeṣasya kalpyo*)" is problematic; what is expected here is a reference to the fact that we are in the first day of Brahmā of what remains. $K_7$ (followed by Sāstrī (1916)) reads *kalpe* instead of *kalpyo*, which changes the translation to "in the very first *kalpa* (= day of Brahmā) of the remaining". This looks suitable, but this phrase does not contain a nominative[24]. Moreover it cannot connect grammatically with the previous or following phrase, and must be a standalone sentence. Therefore I have rejected this reading.

According to the standard cosmology shared by the Purāṇas and astronomical texts, the *Kṛta-*, *Tretā-* and *Dvāpara-yuga*s in the current *caturyuga* have already elapsed, and we are now in the *Kali-yuga* (cf. Kirfel (1920)). *GD2* 63 agrees with this view, except that he uses the words *trayaḥ pādāḥ*, which would be normally translated to "three quarters", to refer to the three past *yuga*s. The same expression is used in the *Āryabhaṭīya* which, according to later astronomers such as Bhāskara I, divides the *caturyuga* into four equal parts. This is clearly contradictory to what Parameśvara stated in *GD2* 57. Probably, he is using the word *pāda* to refer to four unequal parts and not exact quarters. In his commentaries on *Ābh* 1.5 (Kern (1874, pp. 7-8)) and *Ābh* 3.10 (Kern (ibid., p. 58)), he does not problematize this expression nor say that the four parts are of equal length. Therefore it could be possible that Parameśvara interprets that even Āryabhaṭa thought the four *yuga*s were of unequal length.

## 4.12 Concluding remark (*GD2* 64-65)

Previously in *GD2* 58, Parameśvara mentioned that the world is created and maintained during the day of Brahmā and is destroyed during the night. Therefore the sun would only exist during the day of Brahmā as stated in *GD2* 64. There is no reference to the location of Brahmā elsewhere in our text, but Parameśvara seems to think that his position is far enough for the sun to be always visible (as long as it exists) without being obscured.

The four types of days (table 4.3) are all defined by the visibility of the sun, as is stated in *GD2* 65. In this verse, the word *dina* can be interpreted as both "daytime" or "full day (day and night)". However, elsewhere in *GD2* (and also in *GD1*), Parameśvara is explicit whether he is

[22] *Siddhāntaśekhara* 1.20 (Miśra (1932, p. 13)). The *Sūryasiddhānta* seems to refer to the same notions, but there is some ambiguity in its expression and requires a commentary for its full interpretation (Burgess and Whitney (1858, p. 155)).

[23] The non-euphonized form is *ādyas*.

[24] In this reading, the non-euphonized form of *ādya* is *ādye*. Otherwise it is nonsensical.

Table 4.3: "Days" as seen from four points of view

| | location | length of a day | day and night |
|---|---|---|---|
| human | side of the Earth | sunrise to sunset | 60 *ghaṭikā*s |
| manes | other side of the moon | waning half moon to waxing half moon | 1 lunar month |
| divine | north pole of the Earth | vernal equinox to autumn equinox | 1 solar year |
| Brahmā | remote from the sun | creation of the sun to its destruction | 2 *kalpa*s |

referring only to a day or to a full day and night. He never refers to a full day of the manes or a full day of Brahmā, and only once to a full divine day in *GD2* 55. Therefore it is more possible that *dina* in *GD2* 65 refers to the daytime, but the English word "day" should keep the same ambiguity in the original Sanskrit.

*GD2* 65cd also adds that "spheres (*gola*)" should be used for understanding the different days. This could be a reference to spheres as a solid such as the Earth and the moon, or the celestial spheres which represent the motion of heavenly bodies, both of which could be within an armillary sphere. In either case, the sphere is used for explaining four different locations from where the same sun is viewed, resulting in four kinds of days. This passage also seems to emphasize that these time units are indeed to be dealt with in the topic called the "Sphere".

## 4.13 Contradicting statements on the distances of the sun and moon (*GD2* 66-67)

After the series of statements on various time units, Parameśvara turns back to contradictions in cosmology. I cannot find an explanation for why he separated *GD2* 66-67 far from the previous arguments on cosmology (*GD2* 18-36).

In *GD2* 66 he introduces the opinion that the moon is above the sun, which conflicts with his previous statements that the moon has the lowest orbit. This is based on a typical cosmological model in the Purāṇas: the orbits of celestial bodies are situated above the Earth's disk, the sun is on a low orbit, the moon revolves above it, above every planet and star are the "seven sages (*saptarṣi*)" or the seven stars of the Big Dipper, and above them is the pole star[25]. This is a common target in astronomical treatises. The statement is refuted by pointing out that the moon would always be near full moon if it were above the sun's orbit, or that eclipses would not occur[26]. Parameśvara makes the same argument in *GD1* 2.32cd-34ab, but unlike previous authors, he also justifies that the statements of the Purāṇas are true at the same time in *GD1* 2.29-32ab (K. V. Sarma (1956–1957, pp. 19-20)). In *GD1*, he gives two solutions for removing the contradiction. This is also stated in *GD2* 66-67.

The first solution is to assume that the observer is at the north pole (figure 4.9). If the moon (M) has a northward celestial latitude, it will always be higher the sun (Σ) in the course of their diurnal motion. Additionally, the seven sages (S) would be above them and on top of all, on the zenith (Z), the pole star would be situated. In *GD2* 67, Parameśvara states that the sun is at the end of Gemini (summer solstice), but the statement in *GD2* 66 will be fulfilled as long as the

[25]e.g. *Viṣṇupurāṇa* 2.7.3-11 (Annangaracharya (1972, pp. 136-137))

[26]e.g. *Śiṣyadhīvṛddhidatantra* 20.28 (Chatterjee (1981, 1, p. 234))

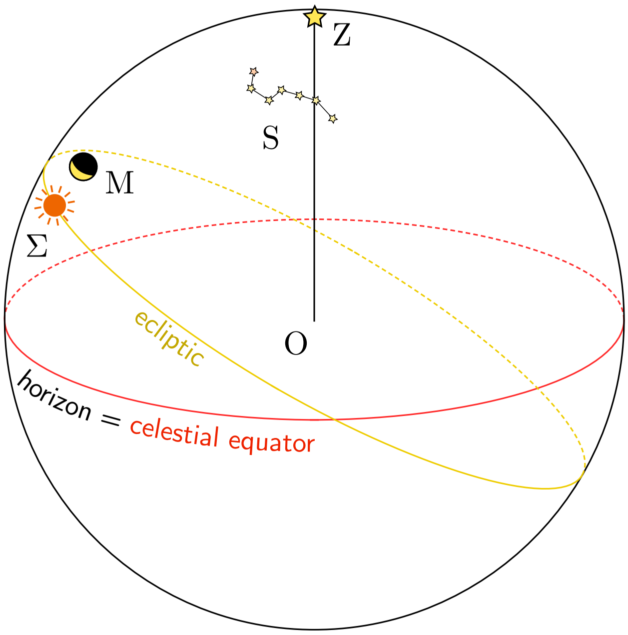



Figure 4.9: An observer O at the north pole, seeing the moon M above the sun Σ

sun is visible (i.e. on the ecliptic from the vernal equinox to the autumn equinox) and the moon near conjunction has a northward latitude.

The other solution is to consider that there is another deity bearing the name of the moon above the sun. *GD1* 2.29 states it more explicitly.

> In the school of wise ones who say that the moon should be above the sun, it is not this moon which is present before the eyes but another deity of the moon that is being assumed there.[27]

Neither of the solutions could be found in other texts. Authors working on the removal of contradiction (*virodhaparihāra*, see page 3.6.1) tried to defend the Purāṇas but with different reasonings. For example, Sūryadasa (born 1507/1508 CE) thinks that the sages of the Purāṇas had known that the moon must be below the sun, and seeks texts which support his claim (Minkowski (2002, p. 367)).

[27] *arkād upari śaśī syād iti kavayo ye vadanti tatpakṣe |*
*nendur ayaṃ pratyakṣas tatrānyac candradaivataṃ kalpyam ||2.29||* (K. V. Sarma (1956–1957, p. 19))

# 5 Authorship and summary (*GD2* 68-69)

The only place in *GD2* where Parameśvara gives his name is *GD2* 68[1]). This is unusual, since we would normally expect authorships to be stated in the opening or concluding verses. *GD2* 68 itself not only concludes the previous set of verses but also suggests that more should be said. Therefore it is unlikely that Parameśvara had initially composed this treatise with only 68 verses, and added the remaining later. Whatever his intention might have been, the wordings of *GD2* 68 and 69 give a strong impression that there is a transition in the topic. Previous verses have dealt with topics such as names of celestial circles and time periods, which are themselves static or constant. From *GD2* 70 and onward, Parameśvara turns to segments and arcs formed within these circles that change in the course of time or according to the observer's location. This contrast of constancy and variance is embodied in the word *saṃsthāna*, as we will see. There is no other segmentation in *GD2* by the author which is as explicit as *GD2* 68-69.

*GD2* 68 refers to the previous contents as "the configuration (*saṃsthāna*) of the sphere". The word *saṃsthāna* appears 8 times in his commentary on the *Āryabhaṭīya*, all of them in the 4th chapter "*gola*".

> Then he states the configuration of the ecliptic. (Introduction to *Ābh* 4.1)[2]

*Ābh* 4.1 is on the inclination of the ecliptic.

> He states the configuration of the inclined circle. (Introduction to *Ābh* 4.3)[3]

> Thus the configuration of the inclined circle supporting the moon has been proclaimed. (Commentary on *Ābh* 4.3)[4]

> Thus is the configuration of the inclined circle which is the supporter of Jupiter, Mars and Saturn. (Commentary on *Ābh* 4.3)[5]

*Ābh* 4.3 describes that the moon and five planets deviate from the ecliptic north and south and pass the nodes. Parameśvara paraphrases the verse in detail in his commentary, but it is interesting that he does not refer to these statements as motion of planets but as the configuration of inclined circles on which they move.

> He states the configuration of the orbits and the configuration of the Earth. (Introduction to *Ābh* 4.6)[6]

> [This is] a repeated statement on the configuration of the Earth established in [the verse] beginning with "below the stars" (*Ābh* 3.15)[7]. (Commentary on *Ābh* 4.6) [8]

[1] See introduction 0.1.3 for explanation on the form of his name in this verse, *paramādi īśvara*.

[2] *tatrāpamaṇḍalasaṃsthānam āha* / (Kern (1874, p. 70))

[3] *vikṣepamaṇḍalasya saṃsthānam āha* / (Kern (ibid., p. 71))

[4] *evaṃ candrādhārasya vikṣepamaṇḍalasya saṃsthānam uditam* / (Kern (ibid., p. 72))

[5] *evaṃ gurukujamandānām ādhārabhūtasya vikṣepamaṇḍalasya saṃsthānam* / (Kern (ibid.))

[6] *kakṣyāsaṃsthānaṃ bhūsaṃsthānaṃ cāha* / (Kern (ibid., p. 74))

[7] Below the [orbit of] stars are [the orbits of] Saturn, Jupiter, Mars, the sun, Venus, Mercury and Moon. And below them is the Earth as the [central] pillar standing in the middle of space.
*bhānām adhaḥ śanaiścarasuragurubhaumārkaśukrabudhacandrāḥ* /
*teṣām adhaś ca bhūmir medhībhūtā khamadhyasthā //3.15//* (Kern (ibid., p. 61))

[8] *bhānām adha ity ādisiddhasya bhūsaṃsthānasya punarvacanaṃ* / (Kern (ibid., p. 75)). I have interpreted this as an independent sentence and added a *daṇḍa* at its end.

*Ābh* 4.6 refers to the Earth's position in the middle of every planetary orbit and also its shape and composition. However Parameśvara focuses on its position, which is emphasized by his last statement as quoted above.

> This is named the celestial sphere. There is also the stellar sphere situated in its interior. Now its configuration is: (Commentary on *Ābh* 4.19)[9]

Before this passage, Parameśvara mentions the names, positions and orientations of rings in the celestial sphere. He does the same thing for the stellar sphere.

In every case, *saṃsthāna* refers to a description concerning the positions and orientations of celestial circles and objects, which stay constant through time (including constant rotations or revolutions). Meanwhile Parameśvara does not use *saṃsthāna* when referring to verses in the *Āryabhaṭīya* which involve arcs and segments created by their combination, whose lengths change with time or place. Broadly speaking, we find the same tendency when comparing verses before and after *GD2* 68. Nonetheless, the distinction made by Parameśvara under the word *saṃsthāna* is not strict, such as the days of various beings which do not fit into this categorization, or the inclined circle which appears in *GD2* 125-126.

In *GD2* 69, Parameśvara refers to his super-commentary on Govindasvāmin's commentary of Bhāskara I's *Mahābhāskarīya*, the *Siddhāntadīpikā*. Indeed, many of the contents after *GD2* 70 overlap with what we can find in the *Siddhāntadīpikā*. The word *yukti* in this verse could be understood as "application" or "usage", such as computations and maybe even observations using the gnomon, but none of the instances later on in *GD2* fit this interpretation. Instead, I chose to translate it "grounding". In *GD2* 119, 188, 198 and 204, *yukti* refers to a proportion[10] which grounds a given rule, and the case in *GD2* 233 (the word is in the instrumental, *yuktyā*) might also imply some proportionality (see chapter 17). Meanwhile the cases in *GD2* 98 and 110 might refer to a visual "grounding", possibly using an armillary sphere (see section 7.5).

[9] *etat khagolaṃ nāma bhavati / asyāntargataṃ nakṣatragolam apy asti / tatsaṃsthānaṃ tu /* (Kern (1874, p. 83))

[10] *GD2* 119 gives a proportion in a peculiar manner, while the rest are in the standard form of a Rule of Three.

# 6 Segments and arcs produced in the stellar sphere and celestial sphere (*GD2* 70-88)

Just before this section, *GD2* 68 refers to the previous statements as those on "configurations (*saṃsthāna*)", indicating constant states. From hereon Parameśvara introduces segments and arcs which change their lengths according to various conditions. Some of these segments and arcs can be given as initial parameters, whereas others have to be computed. In *GD2* 70-88, every value is computed from the geographic latitude $\varphi$ and the longitude of the sun $\lambda_\Sigma$ which is assumed to be constant for a given day. Any variance that occurs in the course of the day, which will be introduced after *GD2* 103, is not taken into account in these verses.

$\varphi$ is explained in detail together with the co-latitude in *GD2* 70-72 while there is no description of $\lambda_\Sigma$ itself. Instead, *GD2* 73 abruptly mentions the "base" Sine of the sun's longitude without explanation. *GD2* 73-74 are a series of computations that give the Sine of declination $\operatorname{Sin}\delta$, diurnal "Sine" $r$, Earth-Sine $k$ and Sine of ascensional difference $\operatorname{Sin}\omega$. Explanations or groundings behind these computations are supplied in *GD2* 75-83. *GD2* 84-88 states two rules for computing the solar amplitude $\operatorname{Sin}\eta$ with additional explanations.

## 6.1 Geographic latitude and co-latitude (*GD2* 70-72)

The geographic latitude (*akṣa* or *pala*) $\varphi$ has already been mentioned in *GD2* 2 and the co-latitude (*lambaka* or *avalambaka*) $\bar{\varphi}$ was first referred to in *GD2* 46, but these verses do not specify the meaning of the terms. The geographic latitude and co-latitude, either as arcs or segments, are described for the first time in *GD2* 70-72 in three different ways.

I will argue below that each of the three descriptions might have had different roles. On the other hand, having multiple definitions itself might have been important too. *GD2* 105-106 explains that many figures (triangles) are caused by the geographic latitude as a reasoning for their similarity. The three different descriptions of the geographic latitude and co-latitude might be for highlighting their omnipresence.

*GD2* 70 describes the situation when the sun is at an equinoctial point and culminating in the south at midday (figure 6.1). The great gnomon (*mahāśaṅku*) at this moment, which is the elevation of the sun against the horizon B*Σ*, is the Sine of co-latitude $\operatorname{Sin}\bar{\varphi}$ while the great shadow (*mahācchāyā*), which is the distance OB* from the center of the sphere to the foot of the great gnomon, is the Sine of geographic latitude $\operatorname{Sin}\varphi$. However, the proper definition of the great gnomon comes much later in *GD2* 103 and the great shadow is introduced even later in *GD2* 114. Yet Parameśvara seems to assume that the reader knows them already.

Theoretically, the great gnomon and the great shadow can be computed with a gnomon $g =$ XO and its shadow $s =$ OC*. The hypotenuse $h^* =$ C*X formed by this gnomon and shadow[1] is computed from the Pythagorean theorem ($h^* = \sqrt{g^2 + s^2}$). Assuming that the sun is infinitely far away, $\angle \mathrm{OC^*X} = \angle \mathrm{B^*O\Sigma^*}$, $\angle \mathrm{XOC^*} = \angle \mathrm{\Sigma^*B^*O} = 90°$ and therefore $\triangle \mathrm{XOC^*} \sim \triangle \mathrm{\Sigma^*B^*O}$. The hypotenuse OΣ* of the great gnomon and the great shadow is the Radius $R$. Thus

$$\begin{aligned} \mathrm{B^*O} &= \frac{\mathrm{OC^*} \cdot \mathrm{O\Sigma^*}}{\mathrm{C^*X}} \\ \operatorname{Sin}\varphi &= \frac{sR}{h^*} \end{aligned} \tag{6.1}$$

[1] Later in *GD2* 117ab, this hypotenuse is given the name *palakarṇa*.

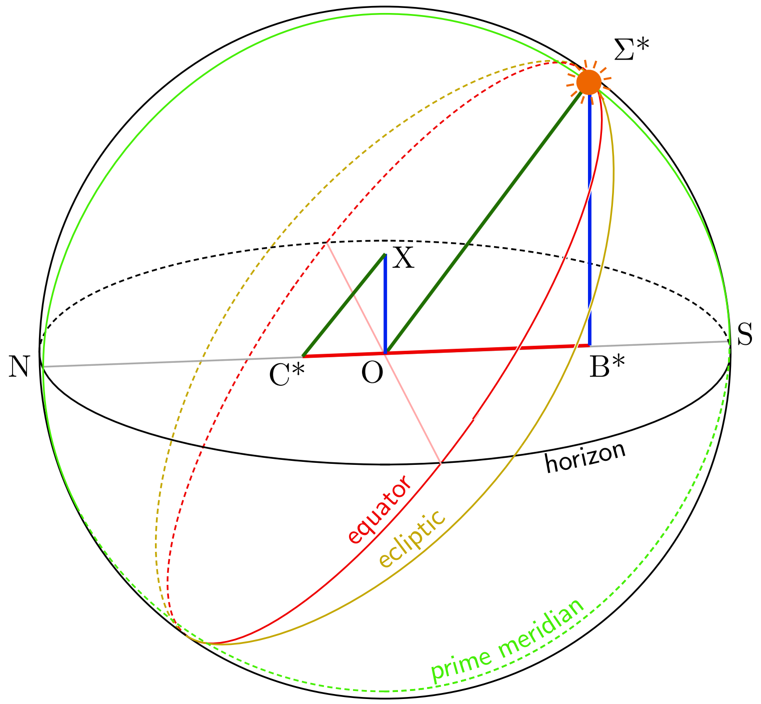



Figure 6.1: Sine of geographic latitude $\mathrm{Sin}\,\varphi = \mathrm{B^*O}$ and Sine of co-latitude $\mathrm{Sin}\,\bar{\varphi} = \Sigma^*\mathrm{B}^*$ according to *GD2* 70. Here the sun $\Sigma^*$ is on an equinoctial point and at its culmination.

$$\begin{aligned}\Sigma^*\mathrm{B}^* &= \frac{\mathrm{XO}\cdot\mathrm{O}\Sigma^*}{\mathrm{C^*X}}\\ \mathrm{Sin}\,\bar{\varphi} &= \frac{gR}{h^*}\end{aligned} \tag{6.2}$$

This very method is explained in *MBh* 3.4-5 (T. Kuppanna Sastri (1957, pp. 107-109)). In fact, this is the only rule given in the *Mahābhāskarīya* to find the geographic latitude, which is why Parameśvara could have stated this before the other two descriptions. The gnomon, shadow and its hypotenuse at midday on an equinoctial day is used in *GD2* 117-118 where Parameśvara makes reference to the geographic latitude (section 8.7) and suggests the connection between these verses.

By describing the Sine of geographic latitude as a great shadow, Parameśvara could also be suggesting its direction. Many of the examples in *GD2* presuppose that the direction of the sun is also the direction of the great shadow, and the commentary on *GD2* 232 even refers explicitly to a "[great] shadow in the given direction (*iṣṭadikcchāyā*)". If the direction of the great shadow at midday on an equinoctial day is also the direction of $\mathrm{Sin}\,\varphi$, it must be *southward* as long as the observer is in a northern hemisphere. Later in *GD2* 184 (section 10.14.2), we will see that the arc of geographic latitude $\varphi$ is indeed treated as being southward[2].

[2]Additionally, Parameśvara also states that the geographic latitude is southward in *Grahaṇāṣṭaka* 3 (K. V.

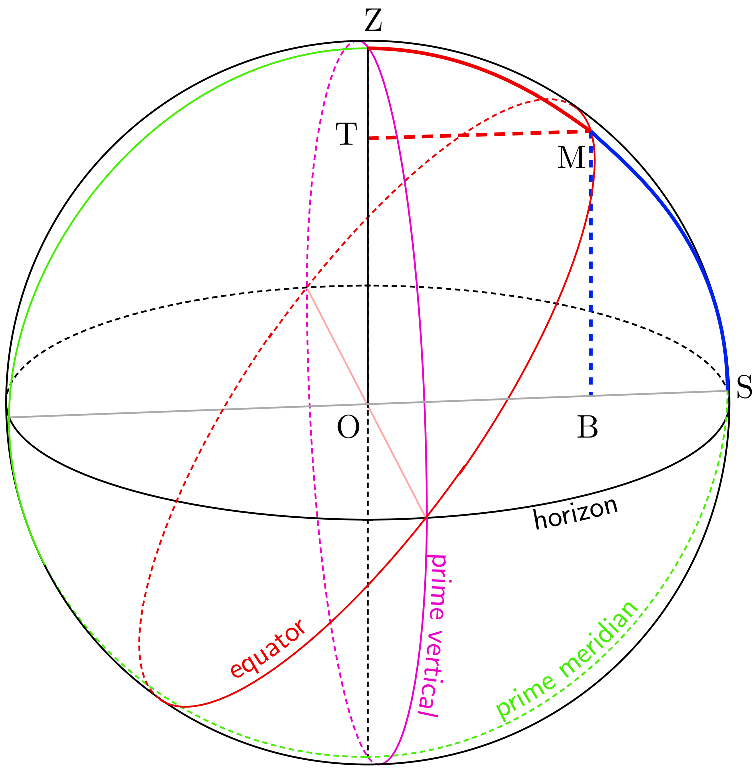



Figure 6.2: Latitude $\varphi = \widehat{\mathrm{MZ}}$ and co-latitude $\bar{\varphi} = \widehat{\mathrm{SM}}$ according to *GD2* 71

*GD2* 71 describes the geographic latitude and co-latitude as arcs which are the "distance (*antara*)" or "gap (*vivara*)" on the south-north circle (figure 6.2). However there is room for consideration, especially on the co-latitude, which is called the *lambaka* or *avalambaka*, both of which can mean "hanging down" or the "perpendicular". Unlike the previous case, we cannot observe the geographic latitude and co-latitude defined in this way. Nonetheless, we can easily find them in an armillary sphere. I assume that *GD2* 71 could have been added for the purpose of explanation with an instrument.

The description in *GD2* 72 uses the pole star (*dhruva*). This is an expression which evokes the viewpoint of an observer, compared with the word "cross" as in *GD2* 154 that imply an armillary sphere, but I think that this rule can be interpreted as both a way of finding the geographic latitude from observation and locating it on the armillary sphere.

In this configuration, we can find a right triangle $\triangle \mathrm{OB'P}$ which has the polar axis PO as its hypotenuse. This could help explain the etymology of *akṣa* (geographic latitude), literally "axis".

## 6.2 "Celestial longitudes" in *GD2*

The longitude of the sun is another important parameter for computing other segments and arcs, but unlike the geographic latitude, Parameśvara does not evoke it directly. Later in *GD2* 89, we can find an explanation of the "base" and "upright" which are distances in longitudes measured

Sarma (1958–1959, pp. 55,58)).

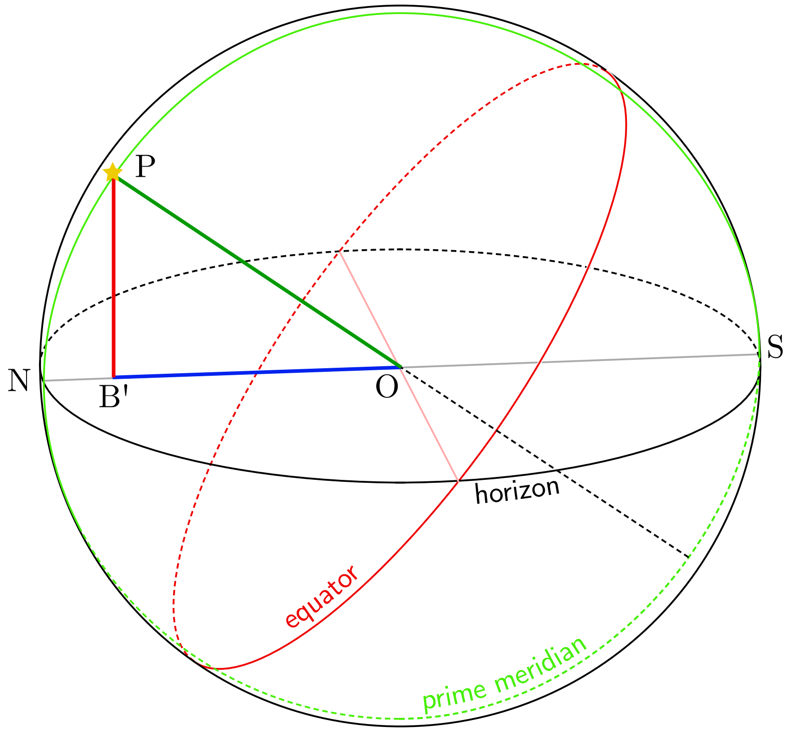



Figure 6.3: Sine of geographic latitude $\varphi = \mathrm{B'P}$ and Sine of co-latitude $\bar{\varphi} = \mathrm{OB'}$, described in *GD2* 72

from the equinoctial and solstitial points, respectively, but even for understanding this verse, the reader must have the notion of a "celestial longitude" beforehand.

What we call "longitude" or "celestial longitude" (to distinguish it from a terrestrial longitude) here is an arc measured[3] along the ecliptic, westward from the vernal equinox. Actually, there is no Sanskrit word that corresponds to celestial longitudes in general, and the name of the celestial object itself (e.g. sun, planet) signifies its longitude[4]. The lack of explanation for the "celestial longitude" in general is not unique to Parameśvara.

In the following verses, we will only be dealing with the sun which is always on the ecliptic. Thus, there is no necessity to distinguish the position of the object (sun) and its longitude. However, the situation is complicated for other planets which have celestial latitudes. I will argue later in section 9.1 that Parameśvara occasionally uses words for "planet" in the sense of its longitude on the ecliptic and not for the object itself which is separated from it by its celestial latitude.

[3]Unless specified, it could be of any arc unit. In our scope, it would be either signs, degrees or minutes.

[4]This has been pointed out by Whitney (1866, pp. 30-31), but has never been discussed in more detail ever since. Whitney, and also Colebrooke (1807, p. 327) before him, mention that *dhruva* or *dhruvaka* is used for the longitude of fixed stars, but this dealt with in *GD2*.

## 6.3 Computing the ascensional difference (*GD2* 73-83)

*GD2* 73-74 introduces four new segments in four sets of computations: Sine of declination $\mathrm{Sin}\,\delta$, diurnal "Sine" $r$, Earth-Sine $k$ and Sine of ascensional difference $\mathrm{Sin}\,\omega$. Parameśvara stops here and supply additional explanations that locate the newly introduced segments in the sphere or give the Rule of Three behind the computations, before going further to the solar amplitude in *GD2* 84-88. The verses also deal with the arc of ascensional difference $\omega$. These might indicate that he considered the ascensional difference an important waypoint in the procedure. The other three segments are also crucial, but the ascensional difference plays a central role in the upcoming topic of the measure of signs. $\mathrm{Sin}\,\delta$ and $r$ only depend on the longitude of the celestial object, but $k$ and $\omega$ also depend on the geographic latitude. Consequently, the ascensional difference comes into play whenever one needs to deal with the motion of a celestial body at a location with geographic latitude.

The explanations have a structure corresponding to the order of the computations in *GD2* 73-74. *GD2* 75-77 locate the positions of the new segments in relation to other circles or segments. *GD2* 78-80 is a discussion on converting the Sine of ascensional difference to an arc. *GD2* 81-83 are three sets of Rules of Three which ground the computations. In the following subsections we shall look at the verses for the computation and explanation for each segment together for convenience.

### 6.3.1 Sine of declination (*GD2* 73ab, 75ab, 81)

The word "declination (*apama*, *krānti*)" in *GD2* usually indicates its Sine ($\mathrm{Sin}\,\delta$) than the arc ($\delta$) itself. This is also the case in *GD2* 73ab. As a Sine, the declination is the distance of a celestial object from the plane of the celestial equator, and as an arc, it is the arc distance from the celestial equator.

*GD2* 75ab refers to the Sine of declination in relation to the position of the sun, and therefore I assume that his descriptions in this section are basically for the sun. Technically, they could be applied for any given point on the ecliptic, which we will see in *GD2* 89-102 (chapter 7). As for planets with celestial latitude, *GD2* 163-164 introduces the concept of "true declination" which one could use instead (section 10.6).

Figure 6.4 is a reconstruction of how Parameśvara could have explained his computational rule in *GD2* 73ab on the basis of the Rule of Three in *GD2* 81. O is the observer in the center of the sphere, surrounded by two great circles, the celestial equator and the ecliptic. Σ is the position of the sun in the ecliptic. *GD2* 73ab only refers to the position of the celestial object as the "'base' Sine of the true (*sphuṭadorjyā*)". Here, a "base" Sine (*dorjyā*) $\lambda_{B(\Sigma)}$ is the Sine of an arc in the ecliptic between a given point and the nearest equinoctial point Q, as defined in *GD2* 89. Parameśvara seems to stress that the longitude must be corrected from its mean position beforehand by adding the word "true (*sphuṭa*)". This is not a topic in *GD2* (see appendix C).

L and K are the foots of perpendiculars dropped from Σ to the plane of the celestial equator and on OQ, respectively. Thus $\mathrm{L\Sigma} = \mathrm{Sin}\,\delta$ and $\mathrm{\Sigma K} = \mathrm{Sin}\,\lambda_{B(\Sigma)}$. As LΣ is perpendicular to the plane of the celestial equator and $\mathrm{\Sigma K} \perp \mathrm{OQ}$, $\mathrm{KL} \perp \mathrm{OQ}$ from the theorem of three perpendiculars.

On the other hand, S is a solstitial point and T is the foot of the perpendicular dropped to the plane of the celestial equator. Thus TS is the Sine of greatest declination $\mathrm{Sin}\,\varepsilon$. *GD2* 73ab gives the value $\mathrm{Sin}\,\varepsilon = 1397$. This indicates that Parameśvara uses $\varepsilon = 24°$ and computes the Sine with Āryabhaṭa's Sine table and linear interpolation (see appendix B.3). Later in *GD2* 159, he also refers to this value as the "[Sine of] greatest declination (*paramakrānti*)". $\mathrm{SO} = R$ is the Radius of a great circle, but *GD2* 81 also refers to it as the "base" Sine of 90 degrees.

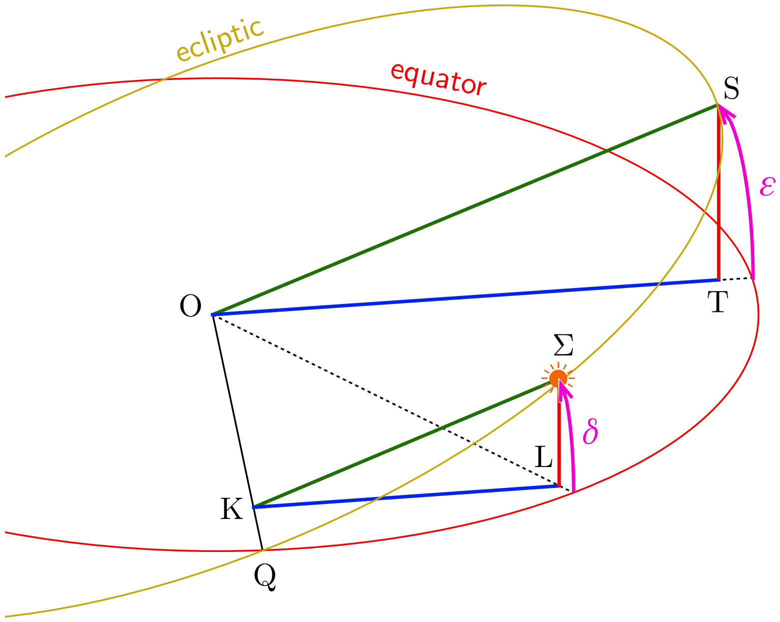



Figure 6.4: Sines of declination $\mathrm{L}\Sigma = \mathrm{Sin}\,\delta$ and greatest declination $\mathrm{TS} = \mathrm{Sin}\,\varepsilon$.

$\mathrm{SO} \perp \mathrm{OQ}$ and therefore $\Sigma\mathrm{K} \parallel \mathrm{SO}$. $\mathrm{OT} \perp \mathrm{OQ}$ and therefore $\mathrm{KL} \parallel \mathrm{OT}$. Since the two pairs of segments forming angles are parallel, $\angle\Sigma\mathrm{KL} = \angle\mathrm{SOT}$. $\angle\mathrm{KL}\Sigma = \angle\mathrm{OTS} = 90°$. Thus $\triangle\mathrm{KL}\Sigma \sim \triangle\mathrm{OTS}$ and:

$$\begin{aligned} \mathrm{L}\Sigma &= \frac{\Sigma\mathrm{K} \cdot \mathrm{TS}}{\mathrm{SO}} \\ \mathrm{Sin}\,\delta &= \frac{\mathrm{Sin}\,\lambda_{B(\Sigma)} \cdot 1397}{R} \end{aligned} \tag{6.3}$$

There is no reference to the measuring unit, but since 1397 is a value which supposes that $R = 3438$ so that one unit of a segment corresponds to one minute of arc, we can assume that $\mathrm{Sin}\,\delta$ is also measured in the same unit of segment length (see appendix A.2). The same can be said for almost every computation given without measuring units in the rest of *GD2*.

*GD2* 75ab locates the Sine of declination within the sphere (figure 6.5). The half-verse itself is very terse and we have relied on *GD1* 2.14 (K. V. Sarma (1956–1957, p. 17)) which includes many words in common to interpret *GD2* 75ab. Parameśvara assumes that the sun $\Sigma$ is on the six o'clock circle; this implies that he considers the sun's longitude and its declination as fixed in the course of a day, and that the declination for any moment can be described by moving the sun along the diurnal circle up to the six o'clock circle. E is one of the two intersections of the celestial equator and the six o'clock circle, which should also intersect with the horizon (not drawn in the figure). This is due east or west. $\widehat{\mathrm{E}\Sigma}$ is the arc of declination, and its Sine may be either $\mathrm{L}\Sigma$ or $\mathrm{OO}'$ where L is the foot of the perpendicular dropped from $\Sigma$ to OE and $\mathrm{O}'$ is the center of the diurnal circle. I assume that $\mathrm{OO}'$ is used in the description and grounding for the diurnal "Sine" which we will see in the next section.

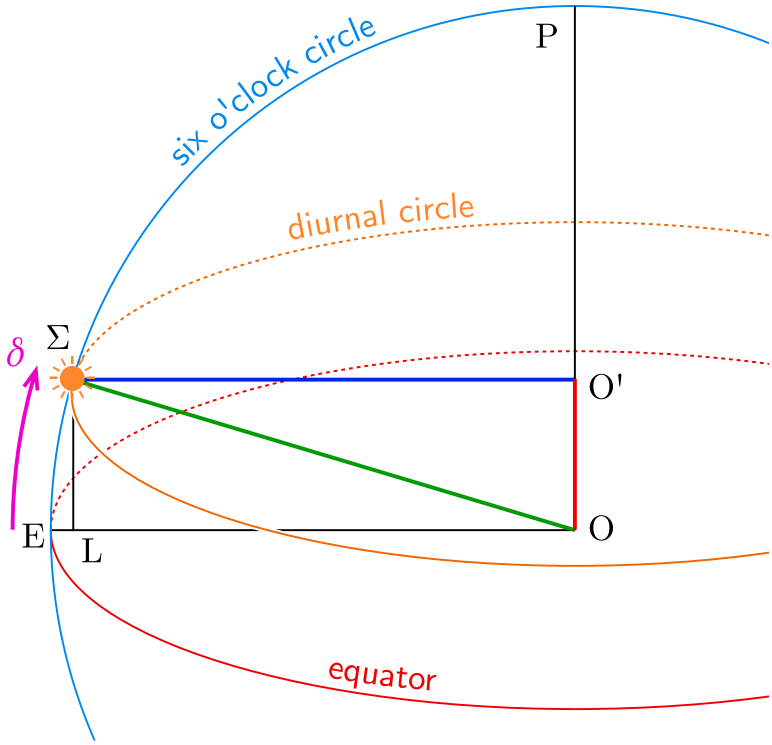



Figure 6.5: The Sine of declination located in the six o'clock circle. The celestial north pole P is at the top.

## 6.4 Diurnal "Sine" (*GD2* 73cd, 75cd, 76cd)

The diurnal "Sine" (*dyudalajīvā* in *GD2* 73, *svāhoratrārdhajyā* in *GD2* 75) refers to the radius of the diurnal circle, as is explicated in *GD2* 75cd[5]. *GD2* 76cd suggests that the diurnal "Sine" forms a right triangle by referring to it as an upright. In our previous diagram (figure 6.5), $\Sigma\mathrm{O}' = r$ is the diurnal "Sine". Then the Radius $\mathrm{O}\Sigma = R$ is the hypotenuse, and the Sine of declination $\mathrm{O}'\mathrm{O} = \mathrm{Sin}\,\delta$ is the base. *GD2* 73cd uses this configuration to compute the diurnal "Sine" with the Pythagorean theorem.

$$\begin{aligned} \Sigma\mathrm{O}' &= \sqrt{\mathrm{O}\Sigma^2 - \mathrm{O}'\mathrm{O}^2} \\ r &= \sqrt{R^2 - \mathrm{Sin}^2\,\delta} \end{aligned} \tag{6.4}$$

This relation enables us to move from a diurnal circle, on which the sun moves, to a great circle, on which time is measured as an arc length. Parameśvara gives more explanation on this point later, with the introduction of the ascensional difference.

[5]We notate "Sine" in quotation marks because it is not a Sine in a great circle. See *dyudalajīvā* in glossary for details.

## 6.5 Earth-Sine (*GD2* 74ab, 76ab, 82)

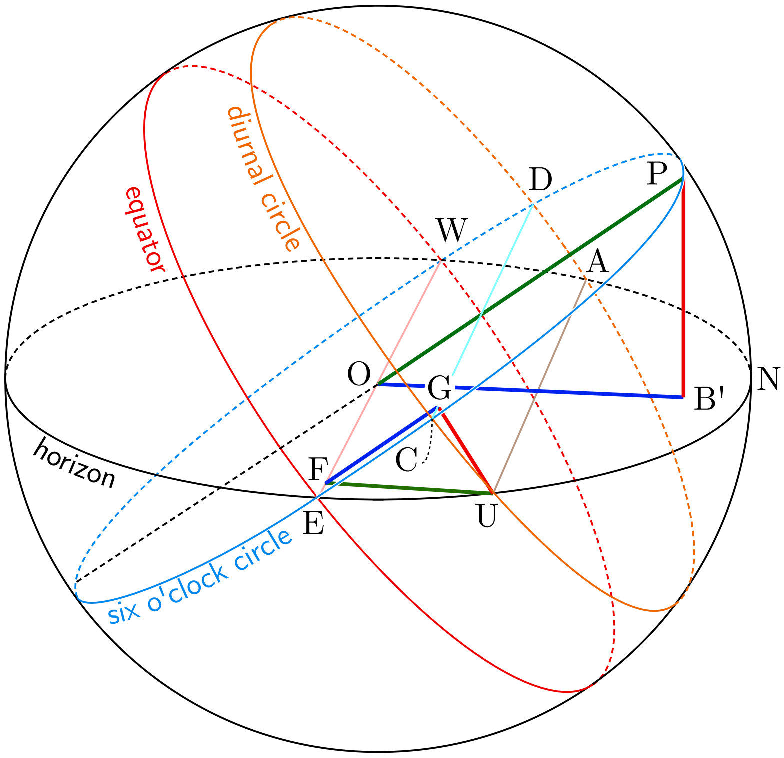



Figure 6.6: The Earth-Sine GU $= k$ when the declination is northward. North is to the right.

As shown in figure 6.6, the horizon, celestial equator and the six o'clock circle intersect at the same two points (due east E and west W), but the pair of intersections of the diurnal circle and the six o'clock circle (rising point U and setting point A) does not coincide with the intersections of the diurnal circle and the horizon (C and D). The arc $\overparen{\mathrm{UC}}$ or $\overparen{\mathrm{AD}}$ is what *GD2* 76ab refers to as the "gap between the horizon and the six o'clock circle" in the diurnal circle. The six o'clock circle cuts the diurnal circle in half (see section 2.9), and therefore, if CD is above the horizon (which is when the declination is northward), $\overparen{\mathrm{UC}}$ and $\overparen{\mathrm{AD}}$ are the additional motion of the sun after sunrise and before sunset compared with an equinoctial day, and if CD is below the horizon (when the declination is southward as in figure 6.7), the arcs represent the shortening of the daylight.

Since CD is the diameter of the diurnal circle, the distance between CD and UA (hereafter we choose GU where G is the foot of the perpendicular dropped from U on CD) is the "sine" corresponding to $\overparen{\mathrm{UC}}$ or $\overparen{\mathrm{AD}}$. This is the Earth-Sine $k$. However, the diurnal circle is not a great circle, and the Earth-Sine is not a Sine in the strict sense.

When F is the foot of the perpendicular dropped from G to EW, FG is the Sine of declination corresponding to its arc $\overparen{\mathrm{EC}}$ because they are both in the plane of the six o'clock circle. GU is in the plane of the diurnal circle which is parallel to the celestial equator and FG is in the plane of the six o'clock circle which is perpendicular to the equator. Therefore the two segments form a triangle $\triangle$FGU where $\angle$FGU $= 90°$.

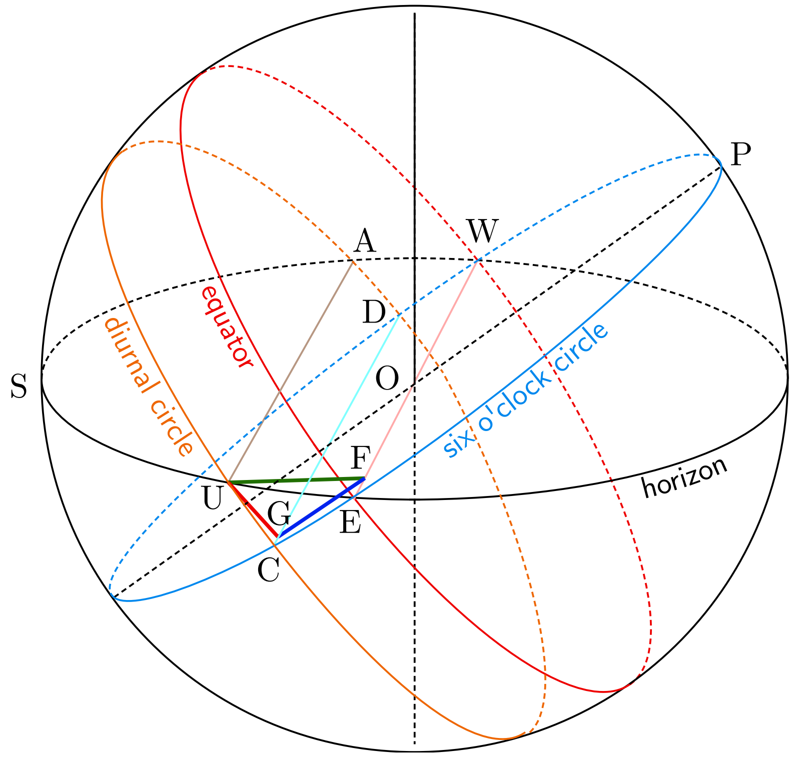



Figure 6.7: The Earth-Sine GU $= k$ when the declination is southward. North is to the right.

FG $\parallel$ PO (the axis between the celestial north pole P and the observer O) since they are both in the same plane and are perpendicular to the same line EW. Exactly for the same reason, UF $\parallel$ OB′ where B′ is the foot of the perpendicular dropped from P to the plane of the horizon. Thus $\angle$UFG $= \angle$POB′. $\angle$FGU $= \angle$OB′P $= 90°$. Therefore $\triangle$FGU $\sim \triangle$OB′P. As discussed in *GD2* 72, B′P is the Sine of geographic latitude ($\mathrm{Sin}\,\varphi$) and OB′ is the Sine of co-latitude ($\mathrm{Sin}\,\bar{\varphi}$). Hence the Rule of Three in *GD2* 82, which gives the computation in *GD2* 74ab:

$$\begin{aligned} \mathrm{GU} &= \frac{\mathrm{B'P}\cdot\mathrm{FG}}{\mathrm{OB'}} \\ k &= \frac{\mathrm{Sin}\,\varphi\,\mathrm{Sin}\,\delta}{\mathrm{Sin}\,\bar{\varphi}} \end{aligned} \tag{6.5}$$

## 6.6 Sine and arc of ascensional difference (74cd, 77-80, 83)

The change in the diurnal motion of the sun caused by the geographic latitude and the celestial longitude is represented by the Earth-Sine or its arc. The next step is to measure the time corresponding to this difference. *GD2* 77ab tells us that a revolution (*bhramaṇa*) of the celestial equator and diurnal circles are the same in terms of time. Or to reformulate the expression, the circles revolve once in the same amount of time (i.e. one day). This statement might be to evoke that portions of revolutions also correspond (figure 6.8). *GD2* 77cd links the Sine produced in the celestial equator by a motion to the Sine produced in the given time. Thus, *GD2* 77 suggests

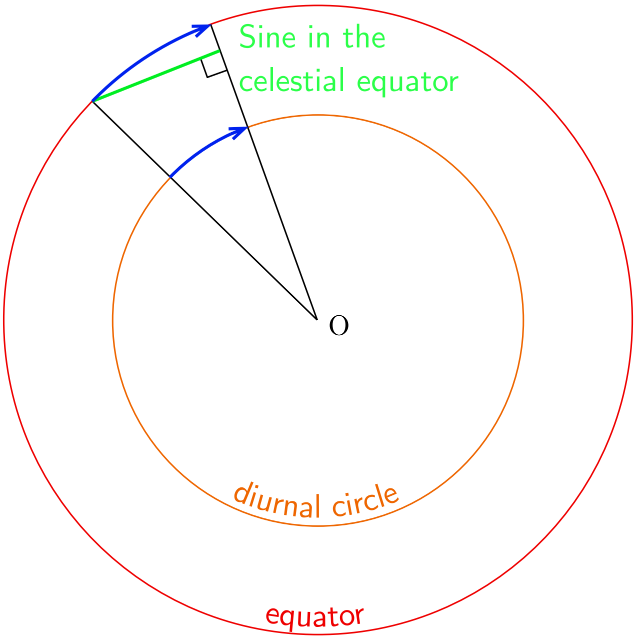



Figure 6.8: Corresponding revolution of the celestial equator and diurnal circle.

that we should find a Sine in the celestial equator corresponding to the Earth-Sine in the diurnal circle, which will represent the time it takes for the sun to move between the horizon and the six o'clock circle. This is explicated in *GD2* 78ab, and a Rule of Three based on the correspondence between the celestial equator and the diurnal circle is formulated in *GD2* 83, which gives the computation in *GD2* 74cd. But before looking into the computation itself, the following questions may be raised: Why do we need to move from the diurnal circle to the celestial equator, and why cannot we measure the time using the diurnal circle instead?

*GD2* 79-80 can be read as responses to such questions. *GD2* 79 adds more explanation on the correspondence between units of time and units of arc length. One *prāṇa*, or its synonym *asu*, is equivalent to the time in which a stellar sphere revolves one minute of arc[6]. Therefore we need the arc and not its Sine to measure the time. But *GD2* 80 says that the arc can only be computed on a great circle and not on a diurnal circle. I assume that Parameśvara has the Sine table in his mind when he makes this statement. A Sine table will only give a set of Sines for a circle with a certain radius. In Parameśvara's case, the Sine table assumes a great circle with the Radius of 3438 (see appendix B.3). Therefore, we must find the Sine in the celestial equator (which is a great circle) that corresponds to the Earth-Sine, and then find the corresponding length of arc.

[6]According to *Ābh* 1.6c (Kern (1874, pp. 8-9)): "The celestial sphere [revolves] one minute in a *prāṇa* (*prāṇe-naiti kalāṃ bhaṃ*)". Here we have used Parameśvara's gloss on the word *bha* (literally "star") that it is used in the sense of "stellar sphere (*jyotiścakra*)".

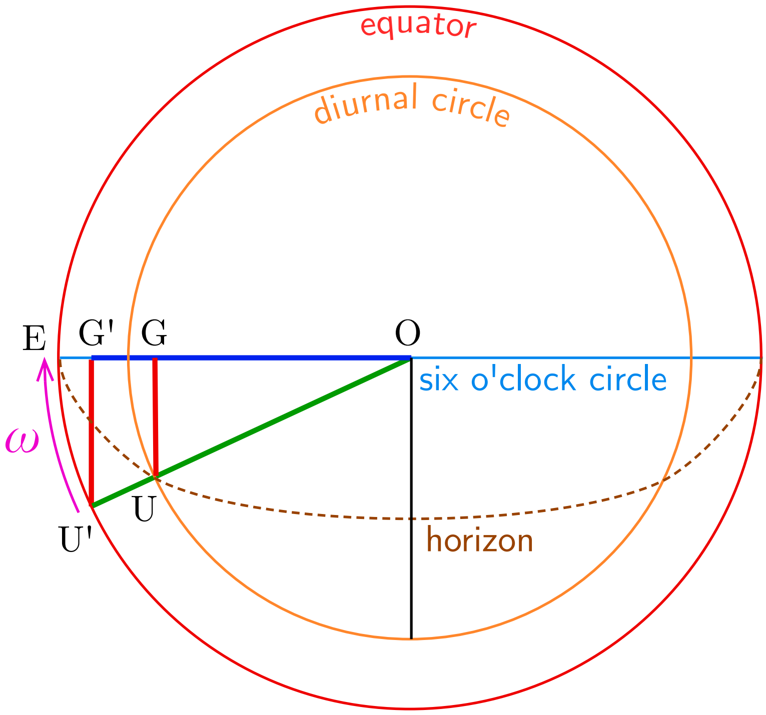



Figure 6.9: Earth-Sine GU and Sine of ascensional difference G′U′.

The Sine in the celestial equator that corresponds to the Earth-Sine is called the Sine of ascensional difference (*cara*) $\mathrm{Sin}\,\omega$. The clue for establishing a computational rule can be found in *GD2* 110 where the celestial equator is assumed to be outside the diurnal circle (section 8.4). This could also be visualized by looking at the armillary sphere from the direction of the celestial pole (figure 6.9). Here, GU is the Earth-Sine and G′U′ is the Sine of ascensional difference. OU′ is the radius of the celestial equator, which is the Radius $R$ of the great circle, and OU is the diurnal "Sine" $r$. △OGU and △OG′U′ are similar because they are right triangles sharing one acute angle. Therefore the Rule of Three in *GD2* 83 can be established, which gives the computation in *GD2* 74cd:

$$\begin{aligned} \mathrm{G'U'} &= \frac{\mathrm{GU} \cdot \mathrm{OU'}}{\mathrm{OU}} \\ \mathrm{Sin}\,\omega &= \frac{kR}{r} \end{aligned} \tag{6.6}$$

*GD2* 78cd adds that the ascensional difference $\omega$ is the arc corresponding to this Sine, and that it is in the unit of *prāṇa*s. As discussed in *GD2* 79, one minute of arc in the celestial equator is equivalent to one *prāṇa*, so we can use the value of the arc, converted from the Sine using a Sine table, without modification.

I would like to add some words on the expression "beginning with (*ādi*)" in *GD2* 79 which suggests an enumeration of measuring units. The list of time and arc units would be either longer or shorter than the *prāṇa* and the arc minute. Sanskrit astronomical treatises do not use time units shorter than the *prāṇa*; *Sūryasiddhānta* 1.11ab (Shukla (1957, p. 2)) distinguishes "real (*mūrta*)" time units beginning with the *prāṇa* and shorter ones that are "unreal (*amūrta*)" (see also Burgess and Whitney (1858, pp. 149-150)). Thus we would expect longer units. The time units following a *prāṇa* are the *vighaṭikā* and *ghaṭikā* where 1 *vighaṭikā* = 6 *prāṇa*s and 1 *ghaṭikā* = 60 *vighaṭikā*s. Likewise, 1 degree = 60 arc minutes. Thus 1 degree = 10 *vighaṭikā*s and 6 degrees = 1 *ghaṭikā*, which means that there is no one-to-one correspondence, but the word "coexistence (*saṃsthiti*)" in *GD2* 79 need not be taken in such narrow sense.

## 6.7 Solar amplitude (*GD2* 84-88)

*GD2* 84 introduces another segment, the solar amplitude $\mathrm{Sin}\,\eta$. This is the Sine in the plane of the horizon, corresponding to the arc distance between its conjunction with the diurnal circle and the point due east or west. The description in *GD2* 84cd is short and does not refer to the ending point of the arc which is due east or west. This was also the case for the Sine of declination in *GD2* 75ab. The two half-verses have in common the fact that the "sun" is used in place of the diurnal circle. Another remark to be made is that although *GD2* 84cd refers to the "conjunction" of the horizon with the sun (diurnal circle) in the ablative case (*kṣitijabhānuyogāt*), thereby suggesting that the direction of the solar amplitude is from this conjunction toward the east-west line, computational methods on gnomons imply that it is the opposite (section 14.3 and 18.8).

We have already seen that the Sine of declination $\mathrm{Sin}\,\delta$ and the Earth-Sine $k$ form a right triangle (figure 6.6). The solar amplitude happens to be its hypotenuse. This is emphasized in *GD2* 85 where the Sine of declination is labeled the upright and the Earth-Sine the base.

The solar amplitude is separated from the other four segments whose computational rules were put together in *GD2* 73-74. Part of the reason might be because the solar amplitude itself is indeed important. It appears frequently in the solving procedures of the six examples in *GD2* 209-247. But another purpose could be to stress the importance of the right triangle $\triangle$FGU where FG = $\mathrm{Sin}\,\delta$, GU = $k$ and UF = $\mathrm{Sin}\,\eta$ (figure 6.6). We have already seen that this is similar with $\triangle$OB′P where B′P is the perpendicular on the horizon going through the celestial north pole P. Therefore the computation in *GD2* 84 holds, which is also grounded by the Rule of Three in *GD2* 87:

$$\begin{aligned} \mathrm{UF} &= \frac{\mathrm{OP}\cdot\mathrm{FG}}{\mathrm{B'O}} \\ \mathrm{Sin}\,\eta &= \frac{R\,\mathrm{Sin}\,\delta}{\mathrm{Sin}\,\bar{\varphi}} \end{aligned} \tag{6.7}$$

$\triangle$FGU can also be used to find the solar amplitude with the Pythagorean theorem, as stated in *GD2* 86:

$$\begin{aligned} \mathrm{UF} &= \sqrt{\mathrm{FG}^2+\mathrm{GU}^2} \\ \mathrm{Sin}\,\eta &= \sqrt{k^2+\mathrm{Sin}^2\,\delta} \end{aligned} \tag{6.8}$$

However, elsewhere in *GD2* we only find evidences of formula 6.7 being used. *GD2* 85 seems sufficient for drawing attention to the right triangle, and Parameśvara's intention in *GD2* 86 is questionable.

### 6.7.1 Another description for the geographic latitude and co-latitude (*GD2* 88)

*GD2* 88 might be for providing further reasoning for the Rule of Three. It gives a special situation where the diurnal circle, having a radius equal to the Sine of geographic latitude $\mathrm{Sin}\,\varphi$, touches the horizon at one point (figure 6.10)[7]. The segment between the center of the diurnal circle and the horizon (O′N) is equivalent to the Sine of the geographic latitude while that between the center of the diurnal circle and the observer (OO′) has the length of the Sine of co-latitude. These two form a right triangle △OO′N with the hypotenuse in the plane of horizon, extending from the observer to the circumference of the diurnal circle (NO). The three segments can also be seen as the Earth-Sine (O′N), Sine of declination (OO′) and the solar amplitude (NO), thus explaining their correspondence in *GD2* 87.

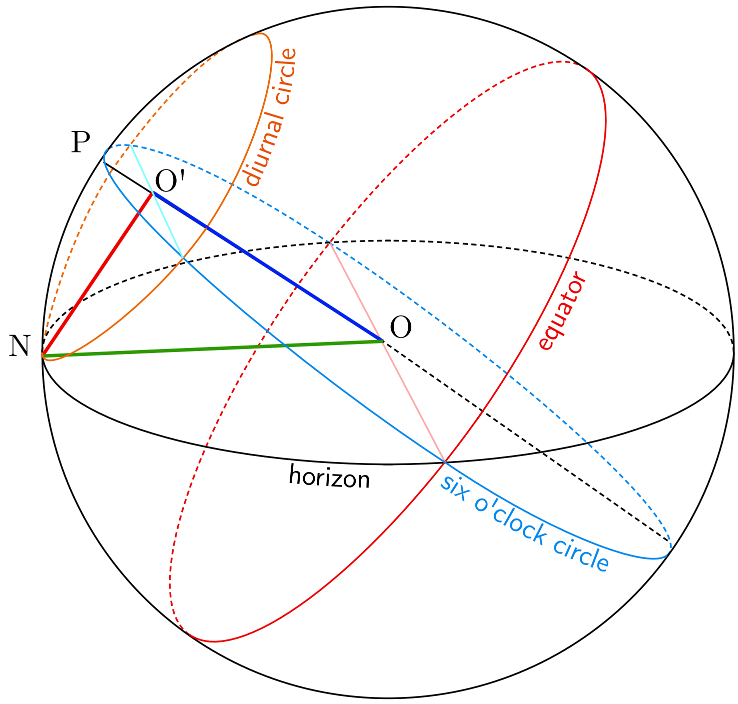



Figure 6.10: Diurnal circle with a radius equal to $\mathrm{Sin}\,\varphi$

At the same time, this situation can be understood as yet another way of defining the geographic latitude and co-latitude. This is clearer in *GD1* 1.15 which resembles *GD2* 88. Note that this is the last verse in the first chapter of *GD1*, "Rule for binding the sphere (*golabandhavidhi*)".

*lambākṣajñānārthaṃ prakalpyate daṇḍanābhiharijānte |*
*anyad dyuvṛttam anyair bhūjyākṣajyeha lambakaḥ krāntiḥ ||1.15||*

[7]Here the diurnal circle is above the horizon and touches it at the northern point, but we can also think of a case where the declination is southward and the diurnal circle being below the horizon touching it at the southern point.

> Another diurnal circle having the axis as center and the horizon as its end is prepared by others, in order to know the co-latitude and geographic latitude. Here the Earth-Sine is the Sine of geographic latitude and the [Sine of] declination is the co-latitude. (*GD1* 1.15)

The correspondence between the segments is made explicit in this verse by mentioning the Earth-Sine and the declination.

# 7 Rising of the signs (*GD2* 89-102)

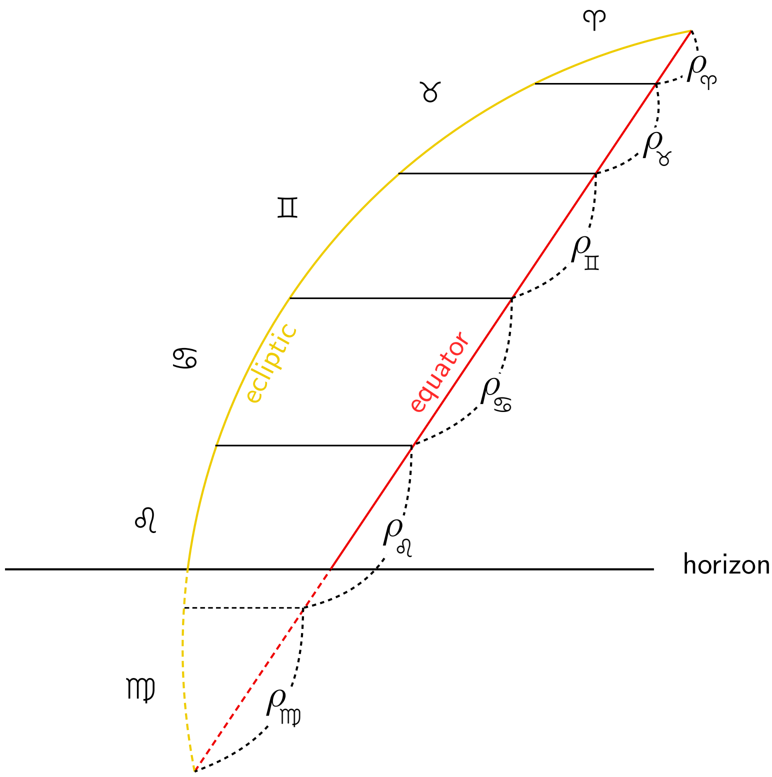



Figure 7.1: Measures of signs ($\rho$)

The subject in these verses is the ascensional difference corresponding to a zodiacal sign, i.e. the time it takes for a entire sign to rise above the horizon. This is called the "measure (*māna* / *miti*) of a sign", and is equivalent to the corresponding length of arc in the equator (figure 7.1). Their relations will be used occasionally later on in the treatise, whenever we need to move from an arc in the ecliptic to the celestial equator or vice versa.

Parameśvara's steps can be described as follows: First, he defines the "base" and "upright", which are two ways to describe an arc of longitude or its corresponding Sine in the ecliptic (*GD2* 89). Then he explains how to find the arc in the celestial equator corresponding to a given "base" arc. This is done in two steps (*GD2* 90-93 and *GD2* 94-95), each containing a Rule of Three. The measure of a sign as seen from the terrestrial equator is obtained by taking the difference between two arcs (*GD2* 96). This corrected by the ascensional difference gives the measure at a given geographic latitude (*GD2* 97-98). In *GD2* 99-100, Parameśvara gives an alternative rule for *GD2* 90-95 which combines the two Rules of Three into one. Last of all, he discusses the effect by the motion of equinoxes and solstices (*GD2* 101-102).

## 7.1 "Base" and "upright" on the ecliptic (*GD2* 89)

*GD2* 89 defines the "base" and "upright" of a planet, which are its longitudes measured from equinoctial or solstitial points on the ecliptic. We have seen previously that the "base" of the sun appears unexplained in *GD2* 73 (section 6.2). The reason why Parameśvara placed this verse here instead of before *GD2* 73 could be explained that he considered these notions relevant to the succeeding topic. The "base" arc and its Sine are indeed important in the process of computing the measures of signs. However the "upright" remains unused until *GD2* 158.

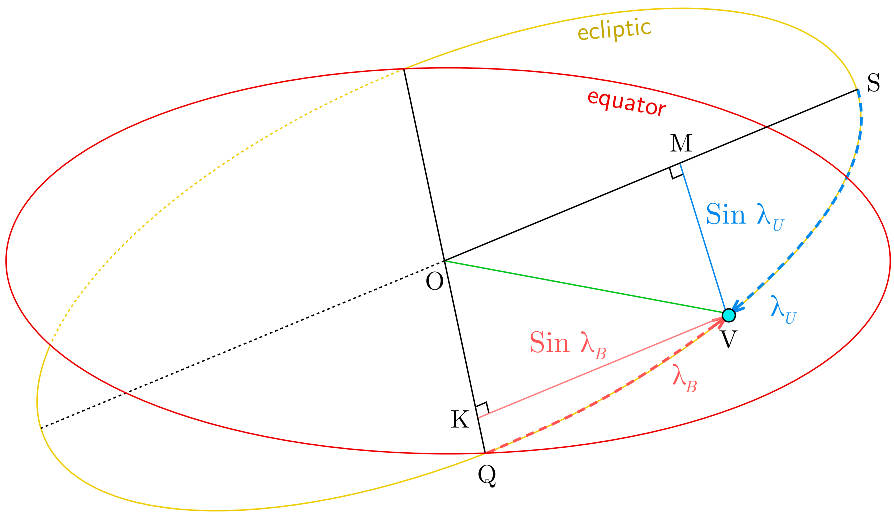



Figure 7.2: "Base" KV = Sin $\lambda_B$, "upright" MV = Sin $\lambda_U$ and their arcs of a planet on the ecliptic

In figure 7.2, Q is an equinoctial point (*golānta*) and S is a solstitial point (*ayanānta*). If the planet is located at point V in the ecliptic, $\overset{\frown}{\mathrm{QV}}$ is the arc of its "base" $\lambda_B$ while the corresponding Sine KV is the "base" Sin $\lambda_B$ , or sometimes referred to as the "base" Sine. Meanwhile, $\overset{\frown}{\mathrm{SV}}$ is the arc of its "upright" $\lambda_U$ and MV is the "upright" Sin $\lambda_U$ itself. Their names probably come from the fact that one can draw a right triangle where the two segments really are the base and upright, with the distance from the observer to the planet (OV) as its hypotenuse (△OKV where KV is the base and OK = MV is the upright, or △OMV where OM = KV is the base and MV is the upright). However, the names "base" and "upright" can be used to address these segments even when they are in triangles other than △OKV or △OMV. For example, the "base" appearing in *GD2* 91 is actually the hypotenuse of a right triangle.

In modern notation, the "base" and "upright" corresponds to the absolute value of the Sine and Cosine of a planet's longitude. Or alternatively, for a longitude $\lambda$, the "base" Sine (Sin $\lambda_B$) and the "upright" Sine (Sin $\lambda_U$) are:

$$\begin{array}{llrl}
\text{When} & 0^\circ \leqslant \lambda < \ \ 90^\circ & \operatorname{Sin}\lambda_B &= \operatorname{Sin}\lambda \\
 & & \operatorname{Sin}\lambda_U &= \operatorname{Sin}(90^\circ - \lambda) \\
\text{When} & 90^\circ \leqslant \lambda < 180^\circ & \operatorname{Sin}\lambda_B &= \operatorname{Sin}(180^\circ - \lambda) \\
 & & \operatorname{Sin}\lambda_U &= \operatorname{Sin}(\lambda - 90^\circ) \\
\text{When} & 180^\circ \leqslant \lambda < 270^\circ & \operatorname{Sin}\lambda_B &= \operatorname{Sin}(\lambda - 180^\circ) \\
 & & \operatorname{Sin}\lambda_U &= \operatorname{Sin}(270^\circ - \lambda) \\
\text{When} & 270^\circ \leqslant \lambda < 360^\circ & \operatorname{Sin}\lambda_B &= \operatorname{Sin}(360^\circ - \lambda) \\
 & & \operatorname{Sin}\lambda_U &= \operatorname{Sin}(\lambda - 270^\circ)
\end{array}$$

A few comments on Parameśvara's wordings in this verse are to be added. It is notable that each point on the circle includes the word *anta* (end) in Sanskrit. The word *ayanānta* (literally "end of the course [of the sun in the northward or southward direction]") for a solstitial point is common in astronomical texts. *golānta* (literally "end of the celestial hemisphere") for an equinoctial point is rarely seen in other texts[1], but not difficult to interpret. On the other hand, Parameśvara also adds *anta* to words for "planet" (*kheṭa*, *vihaga*). An "end of the planet" is a strange expression if we interpret the planet as a celestial object or point in the sky. However, we have discussed in section 6.2 that the name of a celestial object can also signify its longitude. If we take a "longitude" as an arc on the ecliptic starting from the vernal equinox and ending at the planet, then the "end of the planet" can signify a specific point on the ecliptic where the planet is located. Therefore I have translated the compounds *kheṭānta* and *vihagānta* as an "end of the planet['s longitude]". This is still a hypothesis and more studies on the notion of "longitudes" and "planets", both by Parameśvara and in Sanskrit astronomical texts in general, are required.

## 7.2 Given Sine in the diurnal circle (*GD2* 90-93)

The first step is to compute the length of a segment called the "given Sine in the diurnal circle" $j_\lambda$. The Sanskrit term either uses the locative of "diurnal circle" (e.g. *svāhorātre 'bhīṣṭā jīvā*) or a single compound (e.g. *svāhorātreṣṭajyā*). Two computations are given in *GD2* 90-93, and *GD2* 93cd refers to its purpose, which is to establish the measure of signs.

The same triangle △KLΣ as in figure 6.4 could be used here, but shifting the Sine of declination from LΣ to L′K so that the given Sine ΣL′ is "in the diurnal circle" should be a better representation (figure 7.3). Likewise, the Sine of greatest declination TS is shifted to T′O and ST′ is the diurnal "Sine" when the declination is greatest (*paramadyujyā*), i.e. the radius of the diurnal circle at solstice ($r_\epsilon$). Its value $r_\epsilon = 3141$ is given in *GD2* 91 without explanation, but is probably derived from the Pythagorean theorem given in 73cd ($r = \sqrt{R^2 - \operatorname{Sin}^2\delta}$)[2]. Therefore, the given Sine $\Sigma L' = j_\lambda$ is

[1]The only other instance that I have found so far is in Parameśvara's commentary on *Ābh* 4.48 (Kern (1874, p. 99)). More research is required to confirm whether this term is unique to Parameśvara.

[2]The Cosine gives a different value: $\operatorname{Cos} 24^\circ = \operatorname{Sin}(90^\circ - 24^\circ) = \operatorname{Sin} 66^\circ = \operatorname{Sin} 63^\circ 45' + (\operatorname{Sin} 67^\circ 30' - \operatorname{Sin} 63^\circ 45') \cdot \frac{67^\circ 30' - 66^\circ}{225'} = 3084 + (3177 - 3084) \cdot \frac{3}{5} = 3139;48$, rounded to 3140.

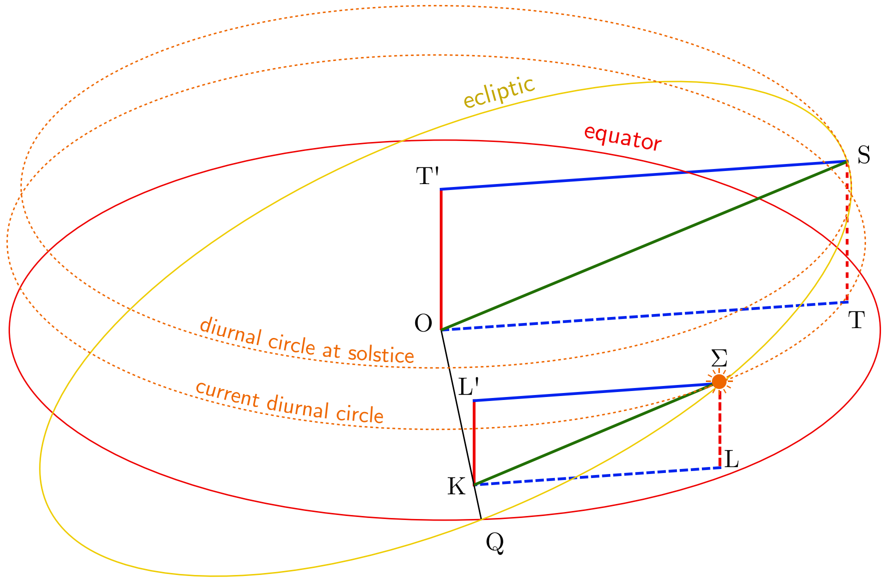



Figure 7.3: Given Sine in the diurnal circle ΣL′ and diurnal "Sine" of greatest declination ST′.

$$
\begin{aligned}
\Sigma\mathrm{L}' &= \frac{\mathrm{ST}' \cdot \mathrm{K}\Sigma}{\mathrm{OS}} \\
j_\lambda &= \frac{3141 \operatorname{Sin} \lambda_B}{R}
\end{aligned}
\tag{7.1}
$$

This computation and Rule of Three are given respectively in *GD2* 91 and 92. *GD2* 93 provides an alternative computation using the Pythagorean theorem.

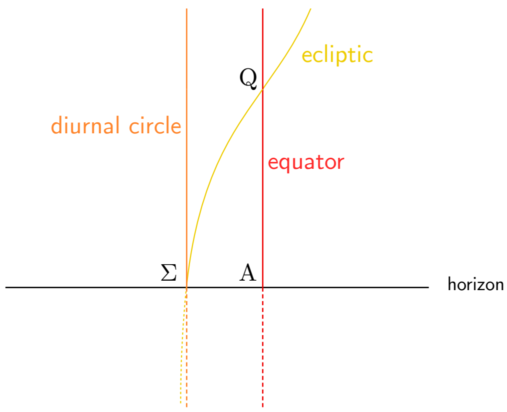
$$
\begin{aligned}
\Sigma\mathrm{L}' &= \sqrt{\mathrm{K}\Sigma^2 - \mathrm{L}'\mathrm{K}^2} \\
j_\lambda &= \sqrt{\operatorname{Sin}_B^2 \lambda - \operatorname{Sin}^2 \delta}
\end{aligned}
\tag{7.2}
$$

## 7.3 Rising time at the terrestrial equator (*GD2* 94-95, 99-100)

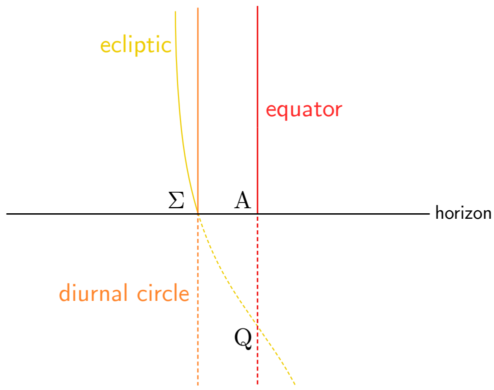
The next step is to compute the time it takes for a given length of arc on the ecliptic to rise from the horizon when observed from the terrestrial equator, represented by Laṅkā. This is equivalent to find the corresponding length of arc on the celestial equator that rises at the same time with this arc. As has been mentioned in *GD2* 77-79, the minutes of arc measured on the celestial equator is equivalent to the time (in units of *prāṇa*s or *asu*s) it takes for that proportion of the stellar sphere to revolve.

Let Σ be a given point on the ecliptic and Q be the nearest equinoctial point. If point A on the celestial equator rises at the same time with Σ, $\overset{\frown}{\mathrm{AQ}}$ corresponds to the arc of "base" $\overset{\frown}{\Sigma\mathrm{Q}}$ (figure 7.4).

The arc of "base" has been converted to the given Sine in the diurnal circle $j_\lambda = \Sigma\mathrm{M}$ in the previous step. The corresponding Sine on the equator AB (figure 7.5) is computed in the same




(a) When Q is above horizon.




(b) When Q is below horizon.

Figure 7.4: Arc of "base" $\widehat{\Sigma Q}$ and its corresponding arc on the celestial equator $\widehat{AQ}$ as seen from an observer on the terrestrial equator.

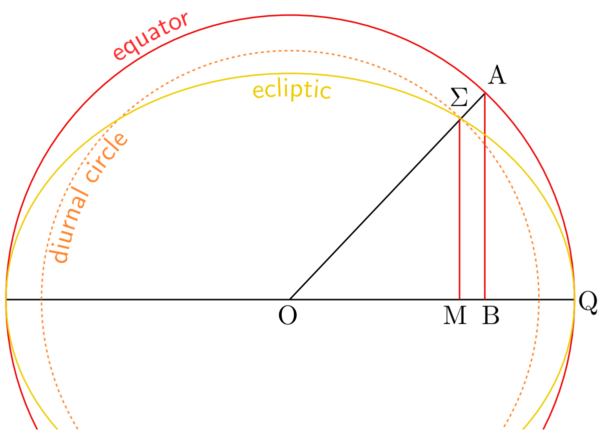



Figure 7.5: Sine in the diurnal circle ΣM to Sine in the equator AB.

way the Sine of ascentional difference was derived from the Earth-Sine in *GD2* 74cd. Since $\mathrm{O\Sigma}$ is the radius of the diurnal circle $r$ and OA is the Radius of the great circle $R$, the Sine $\mathrm{AB} = \operatorname{Sin}\alpha$ is:

$$\begin{aligned} \mathrm{AB} &= \frac{\Sigma\mathrm{M}\cdot\mathrm{OA}}{\mathrm{O\Sigma}} \\ \operatorname{Sin}\alpha &= \frac{j_\lambda R}{r} \end{aligned} \tag{7.3}$$

The Rule of Three is given in *GD2* 95, and the computation in *GD2* 94. *GD2* 94 further refers to converting the Sine $\mathrm{AB} = \operatorname{Sin}\alpha$ to the arc $\widehat{\mathrm{AQ}} = \alpha$. This is the rising time, i.e. the time it takes for $\widehat{\Sigma\mathrm{Q}}$ to rise above the horizon at Laṅkā (the terrestrial equator). This is the equivalent of the modern right ascension of point $\Sigma$.

The two Rules of Three (equations 7.1 and 7.3) can be combined together, eliminating $R$ as mentioned in *GD2* 100.

$$\begin{aligned} \operatorname{Sin}\alpha &= \frac{3141\operatorname{Sin}\lambda_B}{R}\cdot\frac{R}{r} \\ &= \frac{3141\operatorname{Sin}\lambda_B}{r} \end{aligned} \tag{7.4}$$

This resulting computation is given a little bit later in *GD2* 99, but the "*vā* (or)" in *GD2* 99b is obviously intended for giving an alternative for *GD2* 94. In fact, all that is necessary for the following steps is the single computation in *GD2* 99. *GD2* 90-95 is redundant in this sense, but Parameśvara might be intending a step-by-step grounding for the final result. *GD1* also provides these steps: Rule of Three (7.1) in *GD1* 4.80, Rule of Three (7.3) in *GD1* 4.81 and computation (7.4) in *GD1* 4.82, with the auto-commentary supplying the grounding for combining the two Rules of Three as *GD2* 100 did. In contrast, treatises such as *Ābh* (verse 4.25), *MBh* (verse 3.9) and *SūS* (verses 3.42-43) only give the last computation (7.4). Parameśvara supplies the two Rules of Three upon commenting on *Ābh*, and does so too following Govindasvāmin's commentary on the *Mahābhāskarīya* in his super-commentary *Siddhāntadīpikā*, but gives no explanation when he directly comments on the *Mahābhāskarīya*[3] and neither in his commentary on the *Sūryasiddhānta*.

## 7.4 Measure of signs at the terrestrial equator (*GD2* 96)

Within each of the four quadrants[4] in the ecliptic (figure 7.6), one can compute the measure (i.e. rising time) of the first sign from the equinox observed from the equator ($\alpha_1 = \widehat{\mathrm{A_1Q}}$) directly using the previous procedure. The measures of the second and third signs ($\alpha_2$, $\alpha_3$) are given as differences of arcs in *GD2* 96.

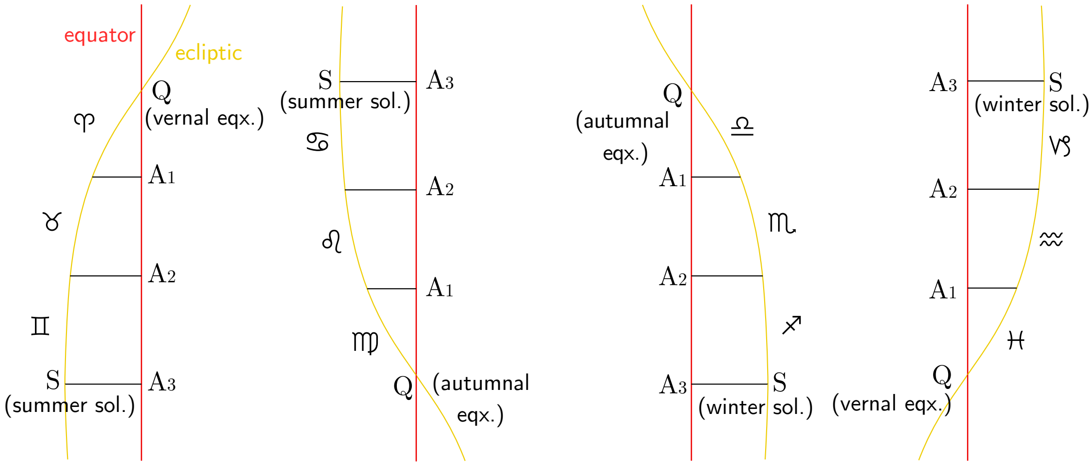
$$\alpha_1 = \widehat{\mathrm{A_1Q}} \tag{7.5}$$

$$\alpha_2 = \widehat{\mathrm{A_2A_1}} = \widehat{\mathrm{A_2Q}} - \widehat{\mathrm{A_1Q}} \tag{7.6}$$

$$\alpha_3 = \widehat{\mathrm{A_3A_2}} = \widehat{\mathrm{A_3Q}} - \widehat{\mathrm{A_2Q}} \tag{7.7}$$

[3] His commentary on the *Mahābhāskarīya* (*Karmadīpikā*) refers to the *Siddhāntadīpikā* and is thus a later work.

[4] Though the word "quadrant (*pada*)" does not appear in *GD2* 96, it is evident that Parameśvara is giving this explanation for each quadrant from the fact that he only mentions three signs and also from the usage of *pada* in *GD2* 102.




(a) *
1st

(b) *
2nd

(c) *
3rd

(d) *
4th

Figure 7.6: The measure of signs ($\overset{\frown}{QA_1}$, $\overset{\frown}{A_1A_2}$, $\overset{\frown}{A_2A_3}$) in each quadrant

Table 7.1: Measure of signs at the terrestrial equator

| Quadrant | | Sign | Measure | Quadrant | | Sign | Measure |
|---|---|---|---|---|---|---|---|
| 1st | ♈ | Aries | $\alpha_1$ | 3rd | ♎ | Libra | $\alpha_1$ |
| | ♉ | Taurus | $\alpha_2$ | | ♏ | Scorpio | $\alpha_2$ |
| | ♊ | Gemini | $\alpha_3$ | | ♐ | Sagittarius | $\alpha_3$ |
| 2nd | ♋ | Cancer | $\alpha_3$ | 4th | ♑ | Capricorn | $\alpha_3$ |
| | ♌ | Leo | $\alpha_2$ | | ♒ | Aquarius | $\alpha_2$ |
| | ♍ | Virgo | $\alpha_1$ | | ♓ | Pisces | $\alpha_1$ |

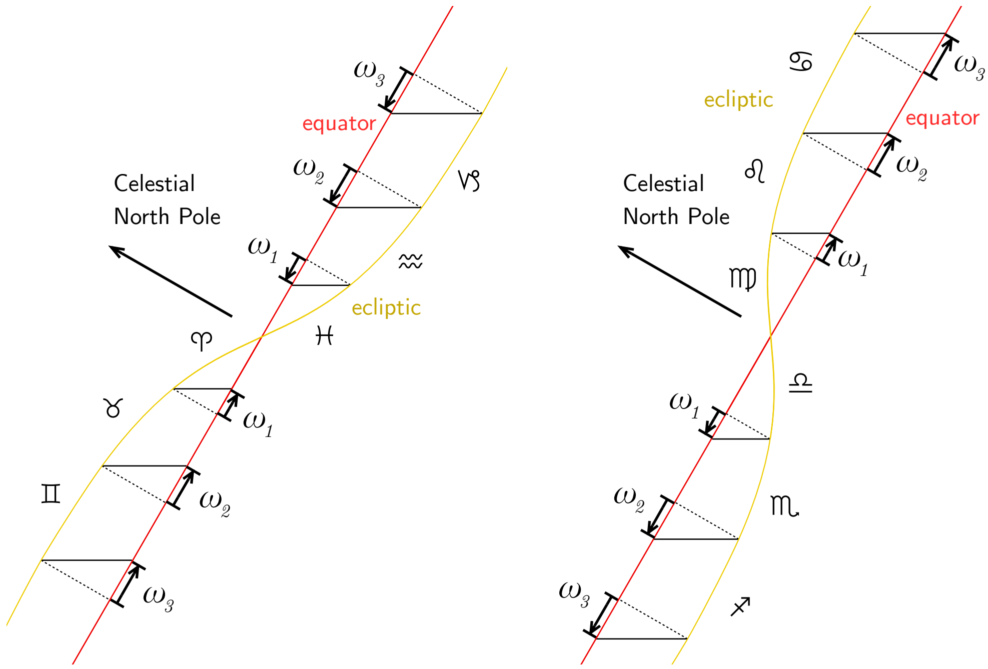
The measures for all twelve signs are given in table 7.1.

## 7.5 Measure of signs at a given location (*GD2* 97-98)

As for locations other than the terrestrial equator (*svadeśa*, or one's own location), Parameśvara only considers the northern hemisphere, as he says in *GD2* 98 that "the stellar sphere is elevated at the north". In this case, the rising time of a "base" arc in the 4th and 1st quadrants (beginning with Capricorn) decreases (figure 7.7(a)) and those in the 2nd and 3rd (beginning with Cancer) increases (figure 7.7(b)) with the amount equivalent to their ascensional difference, as stated in *GD2* 97 .

The grounding (*yukti*) according to Parameśvara in *GD2* 98 is that those beginning with Capricorn rise quickly and those beginning with Cancer slowly because the stellar sphere is elevated at the north. In the auto-commentary on *GD1* 4.84 which is identical in content with *GD2* 98, he adds:

The meaning is, due to the horizon being low at the north, those having their ends in the




(a) 4th and 1st quadrants

(b) 2nd and 3rd quadrants

Figure 7.7: The measure of signs as seen from an observer in the northern hemisphere

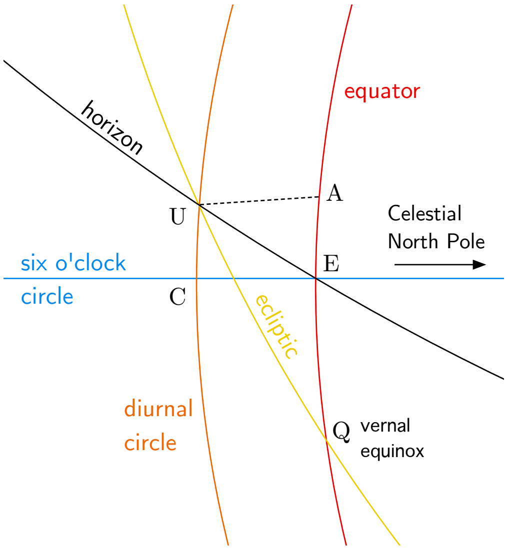
north are fast and other signs are slow.[5]

In this explanation, the geographic latitude is represented by the tilt of the horizon, keeping the six o'clock circle level and the celestial equator perpendicular. This is not what an observer on the Earth would normally perceive, and it might be an instruction using an armillary sphere, where we are free to tilt the instrument to our needs. The six o'clock circle represents the horizon as seen from the terrestrial equator, and by keeping it level, we can maintain the rotation of the stellar sphere as it was in the previous explanations, and introduce the geographic latitude by the inclination of a single ring, the horizon. A similar description of the horizon against the six o'clock circle can be seen in *GD2* 16.

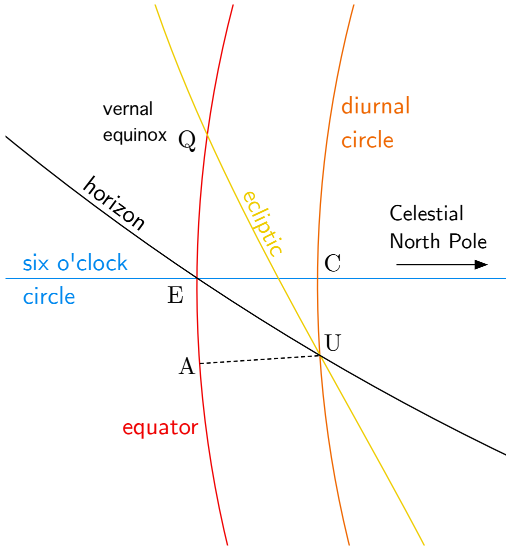
This is visualized in figure 7.8 where U is a point on the ecliptic that is on the horizon, Q is the nearest equinoctial point and E is the due east and also the point on the equator that rises at the same time with U. C is the intersection of the six o'clock circle with diurnal circle of U, and thus $\overset{\frown}{\mathrm{CU}}$ is the arc of the Earth-Sine. $\overset{\frown}{\mathrm{AE}}$ is the arc corresponding to $\overset{\frown}{\mathrm{CU}}$ on the equator, i.e. the ascensional difference $\omega$.

For the given arc of "base" $\overset{\frown}{\mathrm{QU}}$, $\overset{\frown}{\mathrm{QA}}$ is the rising time if the observer were on the terrestrial equator. But here the horizon is not level, and lower at the north. Thus, when U is on the 4th or 1st quadrant (top row in figure 7.8), where the ecliptic runs from south to north, the ascensional difference $\overset{\frown}{\mathrm{AE}} = \omega$ is subtractive ($\overset{\frown}{\mathrm{QE}} = \overset{\frown}{\mathrm{QA}} - \overset{\frown}{\mathrm{AE}}$), and in the 2nd or 3rd quadrant (bottom row in figure 7.8) it is additive($\overset{\frown}{\mathrm{QE}} = \overset{\frown}{\mathrm{QA}} + \overset{\frown}{\mathrm{AE}}$).

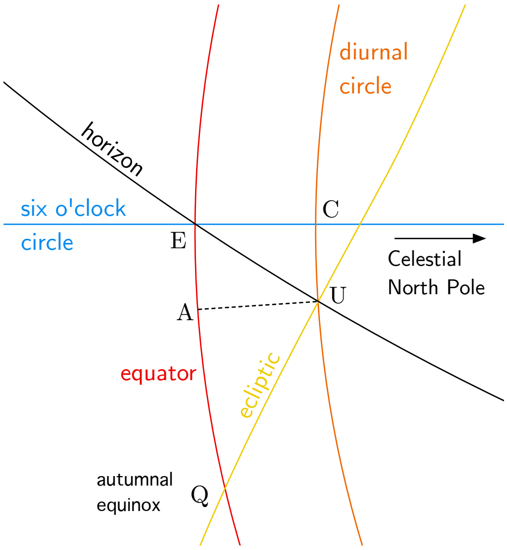
[5] *kṣitijasya udaṅnīcatvād udagantāḥ śīghram anye rāśayaḥ śanair ity arthaḥ* / (K. V. Sarma (1956–1957, p. 66))



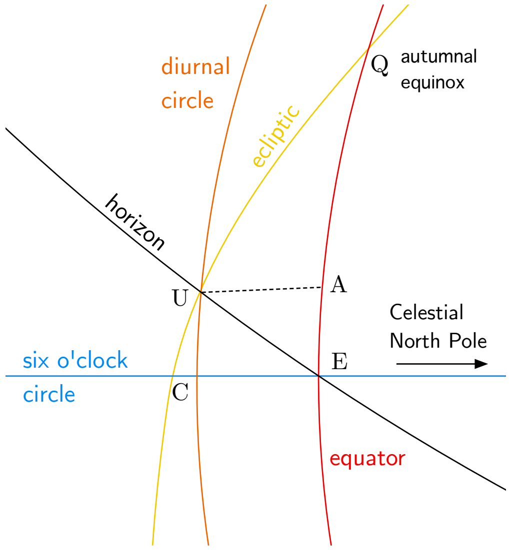

(a) *
4th




(b) *
1st




(c) *
2nd




(d) *
3rd

Figure 7.8: The ascensional difference in each quadrant, as viewed from outside an armillary sphere with the horizon inclined to the north

In the text, Parameśvara only mentions the corrections for a single point and not for the measure of signs. This is done by computing the ascensional difference at the ends of the first, second and third signs ($\omega_1$, $\omega_2$, $\omega_3$) in each quadrant, and then taking their difference $\Delta\omega$:

Table 7.2: Measure of signs at a given latitude

| Quad. | Sign | Measure | Quad. | Sign | Measure |
|---|---|---|---|---|---|
| 1st | Aries | $\rho = \alpha_1 - \Delta\omega_1$ | 3rd | Libra | $\rho = \alpha_1 + \Delta\omega_1$ |
| | Taurus | $\rho = \alpha_2 - \Delta\omega_2$ | | Scorpio | $\rho = \alpha_2 + \Delta\omega_2$ |
| | Gemini | $\rho = \alpha_3 - \Delta\omega_3$ | | Sagittarius | $\rho = \alpha_3 + \Delta\omega_3$ |
| 2nd | Cancer | $\rho = \alpha_3 + \Delta\omega_3$ | 4th | Capricorn | $\rho = \alpha_3 - \Delta\omega_3$ |
| | Leo | $\rho = \alpha_2 + \Delta\omega_2$ | | Aquarius | $\rho = \alpha_2 - \Delta\omega_2$ |
| | Virgo | $\rho = \alpha_1 + \Delta\omega_1$ | | Pisces | $\rho = \alpha_1 - \Delta\omega_1$ |

$$\Delta\omega_1 = \omega_1 \tag{7.8}$$

$$\Delta\omega_2 = \omega_2 - \omega_1 \tag{7.9}$$

$$\Delta\omega_3 = \omega_3 - \omega_2 \tag{7.10}$$

Other treatises such as *MBh* 3.8 give these values for a specific latitude, which can be easily converted for other latitudes.

These are subtractive for signs in the 4th and 1st quadrant and additive for those in the 2nd and 3rd. Table 7.2 lists the measure of signs $\rho$ in a given latitude. Parameśvara calls this rule of subtraction or addition the "correction of ascensional difference (*carasaṃskṛti* or *carasaṃskāra*)" in *GD2* 98, and refers to it later in *GD2* 110 and *GD2* 183.

## 7.6 Taking the motion of equinoxes and solstices into consideration (*GD2* 101, 102)

Sanskrit astrological traditions usually use the *nirayaṇa* (without passage) system, where the twelve zodiacal signs are aligned with the fixed stars. In contrast, a system where the "passage" or motion of the equinoxes and solstices against the stars are taken into account and the signs shift according to them is called the *sāyana* (with passage) system.

Parameśvara considers that this passage oscillates; i.e. it is not a precession in the modern sense but trepidation. For example, he mentions in *GD1* 90cd that "it is assumed to be subtractive or additive by those who know the grounding of mathematics[6]". The notion of trepidation can also be found in his commentary on *Ābh* 3.10 (Pingree (1972)).

Table 7.2 works for a *sāyana* system, but not in a *nirayaṇa* system where the signs move their positions against the equinoxes. *GD2* 101 explains the computation in such case. First, the longitudes of the beginning and end of a sign must be shifted to the *sāyana* system, after which their measures (i.e. the rising time for the arc between that point and the nearest equinox) can be computed in the same manner as explained. The difference of the two measures is the measure for that sign, but as stated in *GD2* 102, a sign could straddle a border of quadrants. In such case, The two measures (1) between the beginning of the sign and the border and (2) between the border and the end of the sign should be computed separately and added later.

[6] *ṛṇam athavā dhanam iti ca prakalpyate tad dhi gaṇitayuktividā* //

# 8 The great gnomon (*GD2* 103-124)

*GD2* 103-106 introduce the great gnomon (*mahāśaṅku*), together with the gnomonic amplitude (*śaṅkvagra*) and given "Sine" (*iṣṭajyā*) in the diurnal circle which form a right triangle. From here on, the time of the day becomes an important parameter. *GD2* 107-113 is on computing a segment called the given "Sine" (*iṣṭajyā*) in the diurnal circle when the time of the day is known, and *GD2* 114-115 explain the computation of the great gnomon and the great shadow (*mahācchāyā*) from this given "Sine". We can find computations that relate the great gnomon with the gnomon as an instrument and its actual shadow in *GD2* 116-120. Finally, *GD2* 121-124 explains a special case when the sun is in the prime vertical.

## 8.1 Rising-setting line (*GD2* 103ab)

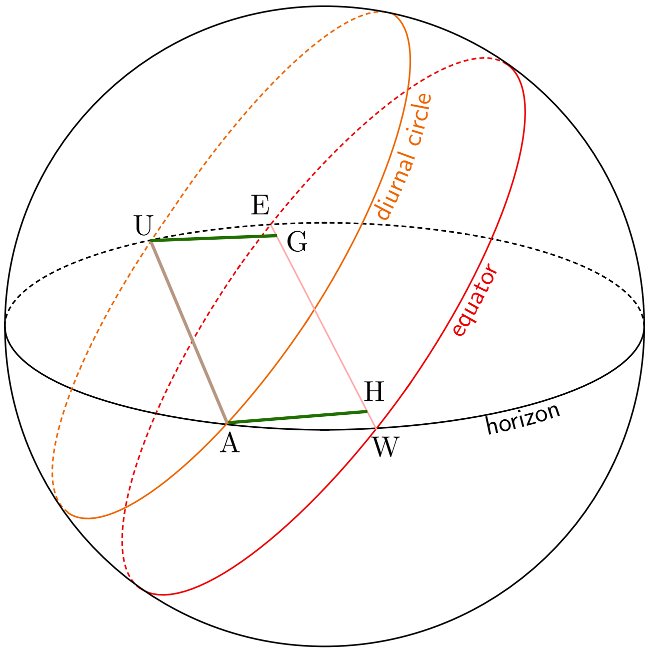



Figure 8.1: The rising-setting line AU

*GD2* 103ab describes the rising-setting line (*astodayasūtra*). It is defined as a line extending to the east and west from the tip of the solar amplitude (figure 8.1 where UF or HA is the solar amplitude). Why is it expressed in such way while one could simply refer to the rising point U and setting point A of the sun? Maybe Parameśvara's intention is to provide continuity with the topics dealt in *GD2* 70 to 88 (ending with the solar amplitude). This can also explain why the rising-setting line comes right before the great gnomon to which it is not directly linked, instead of the great shadow which indeed uses the rising-setting line in its definition.

In *GD1*, the solar amplitude and the rising-setting line are introduced together in one verse (*GD1* 2.14):

> The Sine [starting] from where the sun meets the horizon and having the east-west line as its end is the solar amplitude. The rising-setting line [extends] east and west from its tip. [1]

The first part (*GD1* 2.14abc) corresponds to *GD2* 84cd and the rest (*GD1* 2.14cd) to *GD2* 103ab.

The rising-setting line is the intersection of the plane of the horizon with the plane of the diurnal circle, and although the description with the solar amplitude draws our attention to the horizon, it is also important that this line exists in the plane of the diurnal circle, as will be seen later.

## 8.2 Definition of the great gnomon (*GD2* 103cd)

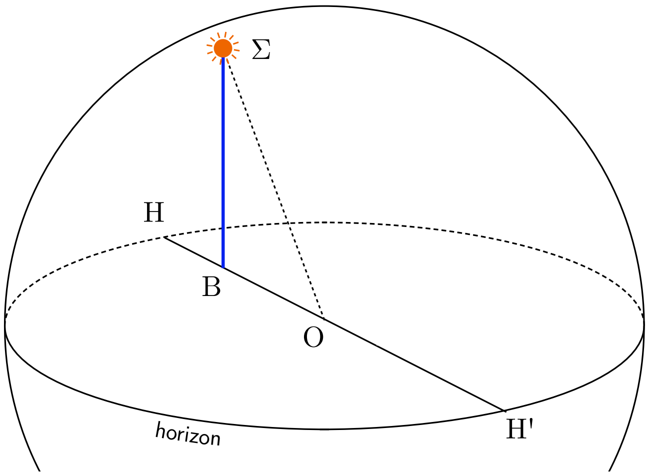



Figure 8.2: The great gnomon ΣB

The great gnomon is defined in *GD2* 103cd as the elevation (*unnati*) of the sun above the horizon. *GD1* 4.1 also defines the great gnomon, but there are some interesting differences.

> The line of Earth is connected with a [point on the] horizon and the opposite [point on the] horizon and goes through the Earth's center. The line hanging down from the Sun and having the Earth-line as its end shall be the [great] gnomon. [2]

Parameśvara adds in his auto-commentary:

> Here [in the half verse beginning with] "[The line] hanging down", the line of Earth is assumed in order to understand a common flat surface on the horizon. The meaning is that a [great] gnomon is the measure of elevation from the flat surface to the sun.[3]

[1] *kṣitije yatrārkayutis tasmāt pūrvāparākhyasūtrāntā /*
*jīvārkāgrā 'stodayasūtraṃ pūrvāparaṃ tadagrāc ca //2.14//* (K. V. Sarma (1956–1957, p. 17))

[2] *kṣitijāparakṣitijayor baddhaṃ bhūmadhyagaṃ ca bhūsūtram /*
*avalambitaṃ hi sūtraṃ sūryāc chaṅkur bhavet kusūtrāntam //4.1//* (K. V. Sarma (ibid., p. 43))

[3] *kṣitijasamānasamatalajñānārtham atra bhūsūtraṃ prakalpyate - avalambitaṃ hīti / samatalād raver unnatimānaṃ śaṅkur ity arthaḥ /* (K. V. Sarma (ibid.))

*GD1* introduces a line called the line of Earth (HH′ in figure 8.2), which is a given diameter in the horizon, just for the sake of defining the great gnomon. It seems that Parameśvara supposes the reader understands the horizon as a circle and not a plane. In the auto-commentary he mentions that the great gnomon is actually the elevation (*unnati*) from a flat surface. Meanwhile, he obviously puts "the horizon" in place of "flat surface" in *GD2* 103cd.

Another difference to be mentioned is that *GD2* 103cd spares some words to say that the sun is moving on the diurnal circle. Of course this is no new information (it has already been stated in *GD2* 10), but again, this might be for the sake of continuity. The diurnal circle has been very important in the previous verses, and will still be in the following verses.

## 8.3 Gnomonic amplitude and given "Sine" in the diurnal circle (*GD2* 104-106)

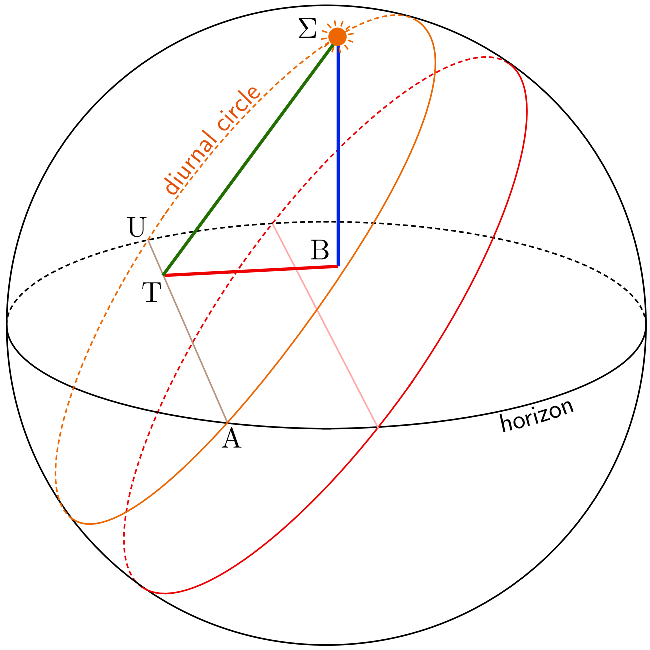



Figure 8.3: The gnomonic amplitude BT and given "Sine" in the diurnal circle TΣ

The gnomonic amplitude (*śaṅkvagra*) is the distance between the foot (*mūla*, literally "root") of the great gnomon and the rising-setting line, while the given "Sine" in the diurnal circle (*svāhorātreṣṭajyā*) is the distance between its tip (*śiras*, literally "head") and the rising-setting line (figure 8.3).

Although Parameśvara uses the word *jyā* (and later *jīva* and *jīvaka*), the given "Sine" is neither a Chord nor a Sine (figure 8.4). Therefore, in order to respect the Sanskrit wording I have translated it "Sine" in quotation marks. The **given** "Sine" **in** the diurnal circle is different from the "Sine" **of** diurnal circle (i.e. its radius) that first appeared in *GD2* 73, as the former can take multiple values for a given diurnal circle while there is only one value for the latter. It is also different from the given Sine in the diurnal circle stated in *GD2* 90, which was actually a Sine (though not of a great circle).

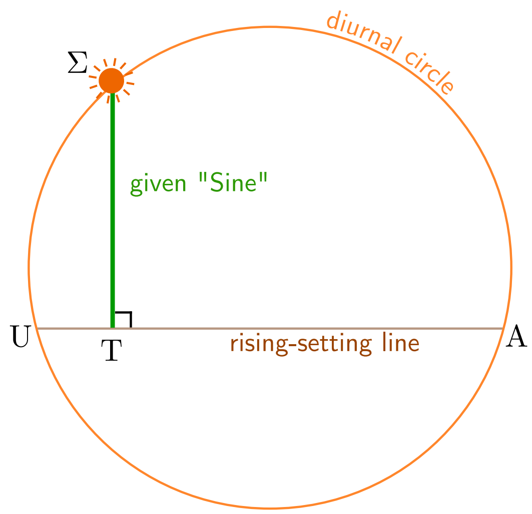



Figure 8.4: The plane of the diurnal circle and the given "Sine"

As can be seen in figure 8.3, the given "Sine" TΣ, great gnomon ΣB and gnomonic amplitude BT are the hypotenuse, upright and base of a right triangle △ΣBT. *GD2* 105 mentions that this is a figure (*kṣetra*) caused by (*nimitta*) the geographic latitude, and that there are many of them. My interpretation is that this refers to triangles that are similar to the right triangle formed with the Radius and the Sines of geographic latitude and co-latitude (*GD2* 72). K. V. Sarma and Shukla (1976, pp. 130-132) remarks that *Ābh* 4.23 is a statement on this triangle, and adds that Āryabhaṭa II and Bhāskara II have given a list of triangles that are similar to it. Parameśvara's commentary on *Ābh* 4.23 (Kern (1874, pp. 85-86)) does not refer to similar triangles, but the remark in *GD2* 105 suggests that he is indeed conceiving a group of similar triangles. Parameśvara does not refer to other examples, but concerning the computations involved in *GD2*, three triangles are important for us: △ΣBT formed from the given "Sine" in the diurnal circle, great gnomon and gnomonic amplitude (figure 8.3), △OB′P formed from the Radius PO, Sine of co-latitude OB′ and the Sine of latitude B′P, and △FGU formed from the Earth-Sine GU, the solar amplitude UF and the Sine of declination FG (see section 6.5, figure 6.6 for △OB′P and △FGU). *GD2* 106 mentions that the segments of one triangle can be used to establish another triangle by means of proportion, which comes as a conclusion from their similarity.

There is nothing equivalent of a modern "proofs" for their similarities in *GD2*, but it can be done as follows. First, we look at the armillary sphere from due east or make a projection. It will look like figure 8.5 when the declination is to the north and figure 8.6 when to the south. In both cases, SN is the horizon, PP′ is the polar axis and also represents the six o'clock circle projected as a line. The diurnal circle projected as a line goes through the sun Σ and intersects with the polar axis at Q and with the horizon at T. QT is the distance between the six o'clock circle and horizon measured along the diurnal circle, which is the Earth-Sine as stated in *GD2*

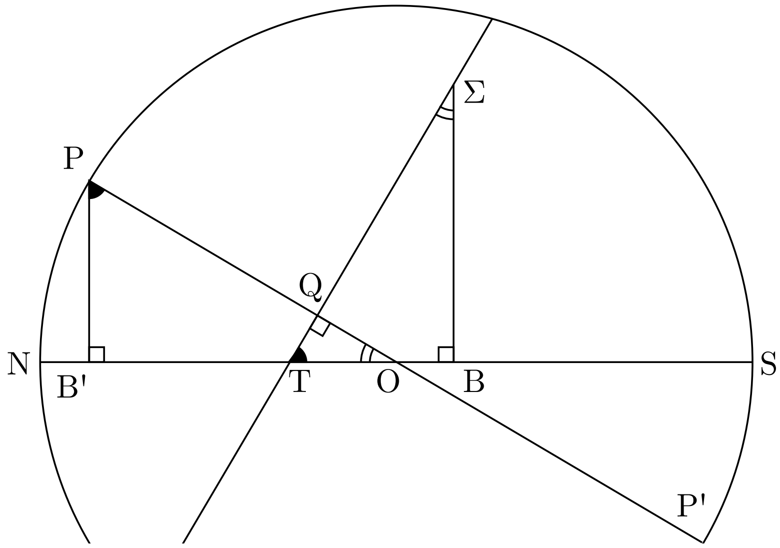



Figure 8.5: Three triangles "caused by the geographic latitude" projected on a plane when the declination is to the north.

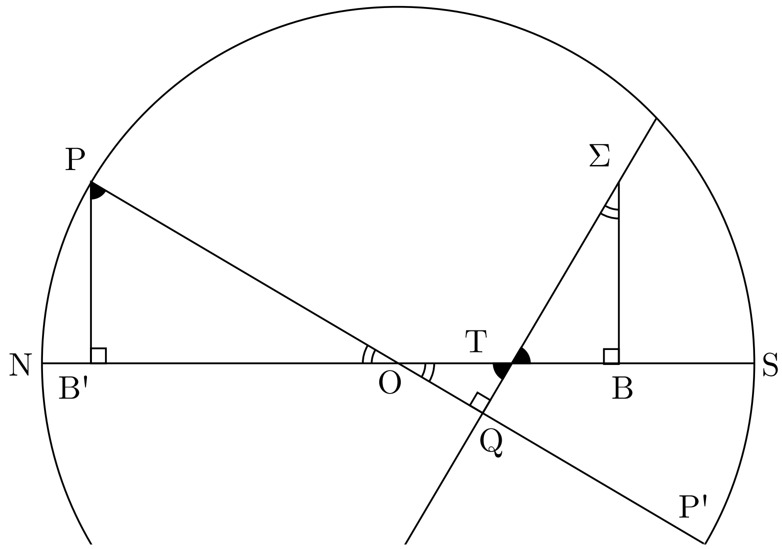



Figure 8.6: Ditto, when the declination is to the south.

76. Likewise, TO is the solar amplitude and OQ the Sine of declination. Therefore △OQT (figure 8.5) is exactly the same with △FGU (figure 6.6) which we have used in the previous discussions. Now B is the foot of the great gnomon and ∠ΣBT is a right angle. B′ is the foot of the perpendicular drawn from P to SN. The diurnal circle is parallel to the celestial equator and the celestial equator is perpendicular to the polar axis, therefore ∠OQT is a right angle. When the declination is to the north, △OB′P and △OQT are both right triangles sharing one acute angle ∠POB′ = ∠TOQ, and are therefore similar. When the declination is to the south, ∠POB′

and ∠TOQ are corresponding angles and equal, thus the right triangles △OB′P and △OQT are similar. Likewise, ∠QTO = ∠BTΣ when the declination is in either direction, and thus △OQT and △ΣBT are similar. Therefore we can conclude that △OB′P ∼ △OQT ∼ △ΣBT.

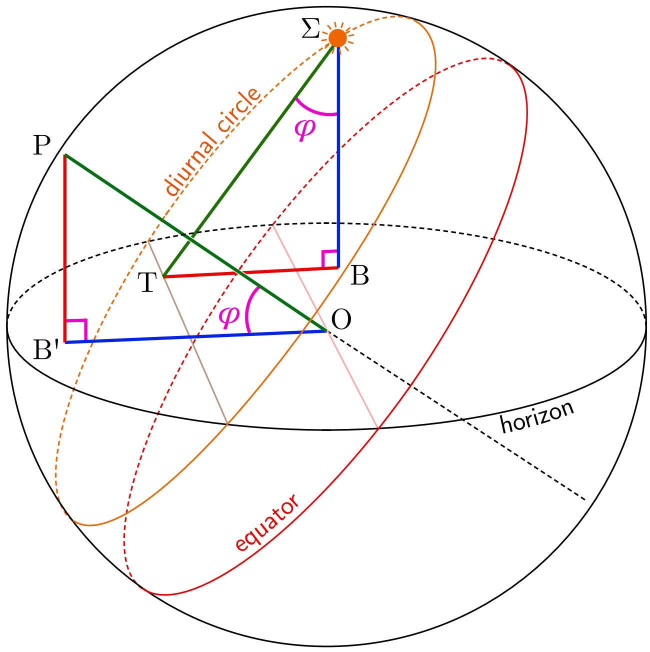



Figure 8.7: The two triangles caused by the geographic latitude, △ΣBT and △OB′P

The similarity between △ΣBT and △OB′P is utilized later in *GD2* 114ab to derive the great gnomon from the given "Sine" in the diurnal circle. The preceding verses *GD2* 107-113 concern the derivation of this "Sine" when the time is given.

## 8.4 The given "Sine" in the diurnal circle (*GD2* 107-113)

*GD2* 107-113 are on the procedure for computing the given "Sine" in the diurnal circle when the time of the day is known. *GD2* 114ab uses this given "Sine" to compute the great gnomon.

### 8.4.1 The two shifts (*GD2* 107-108)

The given "Sine" in the diurnal circle $j_t$ cannot be computed from the time with one Rule of Three or any simple computation. We have to make two "shifts", which is implied in *GD2* 107-108. First, it is the celestial equator whose arc is linked with time, as mentioned in *GD2* 107, and not the diurnal circle. Second, the chord measured from the horizon is not a Sine in the strict sense, as Parameśvara says in *GD1*, "the Sine is assumed to be in a quadrant"[4]. The six o'clock circle always goes through the middle of the stellar sphere and cuts every diurnal circle into half, therefore forming the necessary quadrant (figure 8.8).

[4] *jīvā hi vṛttapāde kalpyate* (auto-commentary on *GD1* 4.5)

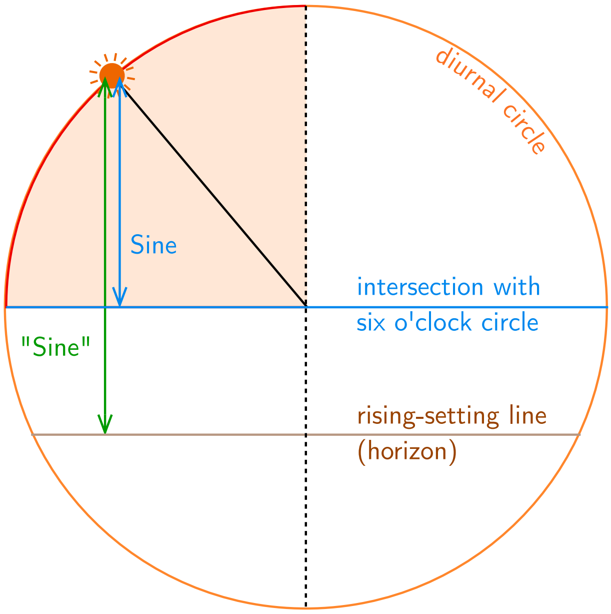



Figure 8.8: Sine from the six o'clock circle and "Sine" from the horizon. Only the six o'clock circle cuts the diurnal circle to form a quadrant and makes a Sine. However it is to be noted that this is not a Sine in a great circle and has to be shifted to the celestial equator so as to compute the Sine from the arc.

Parameśvara argues in *GD2* 108ab that the expression (*grahaṇa*) "Sine" is suitable (*yukta*) only when the segment has its end on the plane of the six o'clock circle and not on the horizon. Yet, as we have seen, there is no difference between the wordings he uses for a Sine from the six o'clock circle and a "Sine" from the horizon.

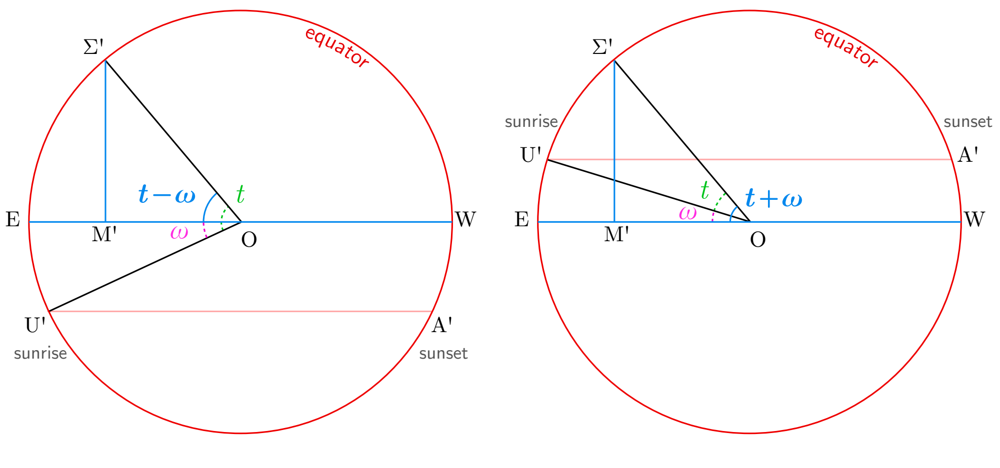
### 8.4.2 Sine in the equator measured from the six o'clock circle (*GD2* 109)

Parameśvara begins the procedure in *GD2* 109 by computing a Sine in the equator measured from the six o'clock circle $J'_t$. He uses the expression "in the portion above the six o'clock circle (*unmaṇḍalordhvabhāge*)", indicating that cases when the sun is above the horizon but below the six o'clock circle are ignored.

Figure 8.9 shows how $J'_t$ is derived. The time $t$ is measured in units of *asu*s, and is counted along the celestial equator from sunrise U′ when it is before noon and counted backward from sunset A′ in the afternoon (Only the former situation is shown in figure 8.9). The ascensional difference $\omega$ is the arc between sunrise U′ and due east E or between sunset A′ and due west W.

The computation is different depending on whether the declination of the sun is to the north (figure 8.9(a)) or to the south (figure 8.9(b)). Parameśvara describes these situations as "when in the northern (*saumye*)" and "when in the southern celestial hemisphere (*gole yāmye*" without specifying the subject. I have supplied "the sun", but this is still open to discussion. Another possibility is "the diurnal circle" supposing that an armillary sphere is being used for explanation.

When the declination is northward, the arc measured from the six o'clock circle $\widehat{\text{E}\Sigma'}$ or $\widehat{\text{W}\Sigma'}$ is




(a) Northward declination (b) Southward declination

Figure 8.9: Computing the given Sine in the equator from the six o'clock circle $J'_t = \mathrm{M}'\Sigma'$

$t-\omega$ (figure 8.9(a)), and when it is southward it is $t+\omega$. Thus the corresponding Sine $\mathrm{M}'\Sigma' = J'_t$ is:

$$J'_t = \begin{cases} \mathrm{Sin}(t-\omega) & \text{Northward declination} \\ \mathrm{Sin}(t+\omega) & \text{Southward declination} \end{cases} \tag{8.1}$$

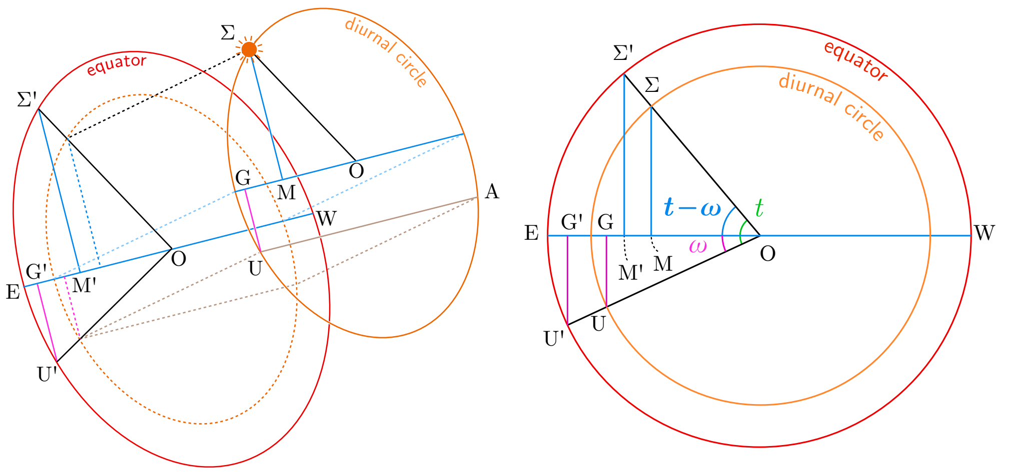



Figure 8.10: Moving two segments from the celestial equator to the diurnal circle. Only the situation where the declination is northward and the time is before noon is shown in these diagrams.

### 8.4.3 Sine in the diurnal circle measured from the six o'clock circle (*GD2* 110-111)

*GD2* 110 refers to the computation for moving from the equator to the diurnal circle, presenting a situation where they are placed concentrically (figure 8.10). The Sine measured from the six o'clock circle in the equator $J'_t = \mathrm{M}'\Sigma'$ moves to that in the diurnal circle $j'_t = \mathrm{M}\Sigma$ and the Sine of ascensional difference $\mathrm{Sin}\,\omega = \mathrm{G}'\mathrm{U}'$ moves to the Earth-Sine $k = \mathrm{GU}$. The expression "outside (*bāhya*)" suggests that this could be a visual reasoning where one has to look at the armillary sphere from the celestial north pole so that the celestial equator appears to be outside the diurnal circle.

The "grounding concerning the correction of the ascensional difference (*yuktiś carasaṃskāre*)" is apprently linked with *GD2* 98 which uses the same phrase. *GD2* 98 itself grounds the rule in *GD2* 97 where the ascensional difference $\omega$ is subtracted or added to the measure of signs depending on the quadrant that they are located in (see section 7.5). The rule there was that $\omega$ is additive in the 2nd and 3rd quadrants of the ecliptic, and subtractive when in the 4th and 1st. However in the current case, the relevant rule is in *GD2* 109 where $\omega$ is additive when the sun is in the 1st or 2nd quadrant and subtractive when in the 3rd or 4th. Therefore it is not the rules themselves that we must compare, but their groundings. We have discussed in section 7.5 that the grounding in *GD2* 98 might be using the armillary sphere, moving the horizon against the six o'clock circle. As seen in figure 7.8, the ascensional difference is produced in the distance between these two circles, and we can visualize whether it must be added or subtracted to find the length of time that the sun is above the horizon.

"Within the [time] past in a day (*dyugate*)" could only refer to a case before noon, since for the afternoon, we would measure the time "to be passed" from that moment in the day until sunset. The expression "passed or to be passed (*gatagantavya*)" in *GD2* 107 covers both cases, and would also be preferred here in *GD2* 110.

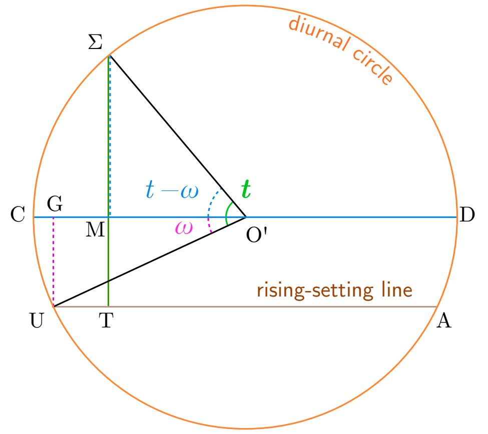
The last part of *GD2* 110 refers to the relation between the Sine of ascensional difference and the Earth-Sine explained in *GD2* 74cd. Parameśvara uses the expression "having the same form*sarūpa*" which indicates their similarity.

Following *GD2* 110, *GD2* 111 gives the computation to obtain the given Sine measured from the six o'clock circle in the diurnal circle $j'_t$ using the Rule of Three given in *GD2* 112. This can be deduced from the similarity between $\triangle\mathrm{OM}'\Sigma'$ and $\triangle\mathrm{OM}\Sigma$ (figure 8.10) implied by *GD2* 110.

$$\begin{aligned} \mathrm{M}\Sigma &= \frac{\mathrm{M}'\Sigma' \cdot \Sigma\mathrm{O}}{\Sigma'\mathrm{O}} \\ j'_t &= \frac{J'_t r}{R} \end{aligned} \tag{8.2}$$

### 8.4.4 The given "Sine" (*GD2* 113)

The next procedure, explained in *GD2* 113, is illustrated in 8.11. $\mathrm{M}\Sigma$ is the given Sine measured from the six o'clock circle in the diurnal circle $j'_t$, $\mathrm{UG} = \mathrm{TM}$ is the Earth-Sine $k$. The given "Sine" in the diurnal circle $j_t = \mathrm{T}\Sigma$ is their sum when the sun is in the north of the celestial equator (figure 8.11(a)) and their difference when the sun is in the south (figure 8.11(b)).




(a) Northward declination (b) Southward declination

Figure 8.11: Computing the given "Sine" above the horizon

$$\mathrm{T\Sigma} = \begin{cases} \mathrm{M\Sigma} + \mathrm{TM} & \text{Northward declination} \\ \mathrm{M\Sigma} - \mathrm{TM} & \text{Southward declination} \end{cases}$$

$$j_t = \begin{cases} j'_t + k & \text{Northward declination} \\ j'_t - k & \text{Southward declination} \end{cases} \tag{8.3}$$

### 8.4.5 Comparing the steps with *GD1*

Equations 8.1, 8.2 and 8.3 could be combined (though not mentioned in *GD2*) into the following equation.

$$j_t = \begin{cases} \frac{r}{R}\,\mathrm{Sin}(t-\omega) + k & \text{Northward declination} \\ \frac{r}{R}\,\mathrm{Sin}(t+\omega) - k & \text{Southward declination} \end{cases} \tag{8.4}$$

*GD1* also deals with this topic, but from a different approach. First, in *GD1* 4.4:

> The Sine produced in the diurnal circle is established by the "Sine" of time (*kālajyā*) with proportion: "When there is this much in a great circle, then how much in a diurnal circle?"[5]

This verse states the relation between the given "Sine" in the diurnal circle $j_t$ and the "Sine" of time, i.e. "Sine" in the celestial equator $J_t$, both measured from the horizon:

$$j_t = \frac{J_t r}{R} \tag{8.5}$$

Then in *GD1* 4.6:

[5] *kālajyayā hi sādhyā dyuvṛttajājyāanupātena |*
*iyatī trijyāvṛtte yadi kiyatī syāt tadā dyuvṛtta iti ||4.4||* (K. V. Sarma (1956–1957, p. 44))

> Therefore the "Sine" of time when [the sun is] in the two hemispheres is the Sine of the *asus* to come [before sunset] or elapsed [after sunrise] in the day, subtracted by or added with the ascensional difference, added with or subtracted by the Sine of ascensional difference. [6]

That is to say:

$$J_t = \begin{cases} \mathrm{Sin}(t-\omega) + \mathrm{Sin}\,\omega & \text{Northward declination} \\ \mathrm{Sin}(t+\omega) - \mathrm{Sin}\,\omega & \text{Southward declination} \end{cases} \tag{8.6}$$

Since $k = \frac{r}{R}\,\mathrm{Sin}\,\omega$ (from formula 6.6), equations 8.5 and 8.6 combined are also equivalent with formula 8.4. The reason why Parameśvara took two different approaches is yet to be solved.

## 8.5 Great gnomon and great shadow (*GD2* 114-115)

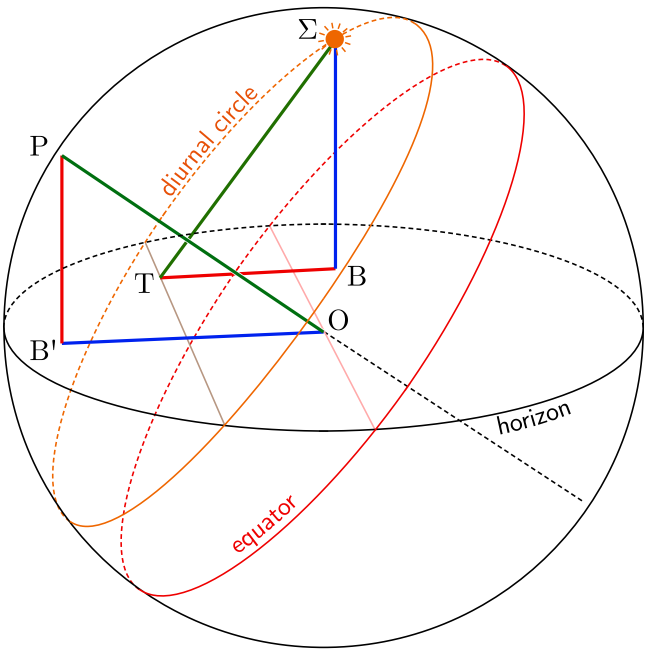



Figure 8.12: $\triangle\Sigma\mathrm{BT}$ and $\triangle\mathrm{OB'P}$

As we have already seen in *GD2* 105-106, the great gnomon[7] $\mathcal{G} = \Sigma\mathrm{B}$ can be computed from the given "Sine" in the diurnal circle $j_t = \mathrm{T}\Sigma$, using the fact that they form a right triangle $\triangle\Sigma\mathrm{BT}$ which is similar to $\triangle\mathrm{OB'P}$ where P is the celestial north pole, B′ its foot on the plane of the horizon and therefore OB′ is the Sine of co-latitude (figure 8.12). The rule of three is given in *GD2* 115 and the computation is in *GD2* 114ab.

[6] *caradalahīnayutānām ato dinasyaiṣyayātajāsūnām |*
*jīvā carajyayāpi ca yutahīnā golayos tu kālajyā ||4.6||* (K. V. Sarma (1956–1957, p. 44))

[7] I have decided not to follow the custom of denoting the great gnomon as a Sine (such as $\mathrm{Sin}\,a$), since it would give the false impression that Parameśvara is associating the great gnomon with a specific arc, which he actually does not.

$$\Sigma\mathrm{B} = \frac{\mathrm{T}\Sigma \cdot \mathrm{OB}'}{\mathrm{PO}}$$
$$\mathcal{G} = \frac{j_t \,\mathrm{Sin}\, \bar{\varphi}}{R} \tag{8.7}$$

*GD2* 114cd gives a rule for computing its shadow (*chāyā*), i.e. the great shadow corresponding to the great gnomon. However, he does not describe what a great shadow is, or where it is located in the sphere in relation to other segments and circles. Meanwhile it is mentioned briefly in *GD1* 4.2ab:

> The [great] shadow, having the center of the Earth as its end and [starting] from the root of the [great] gnomon,... *bhūmadhyāntaṃ śaṅkor mūlāc chāyā* ... (K. V. Sarma (1956–1957, p. 43))

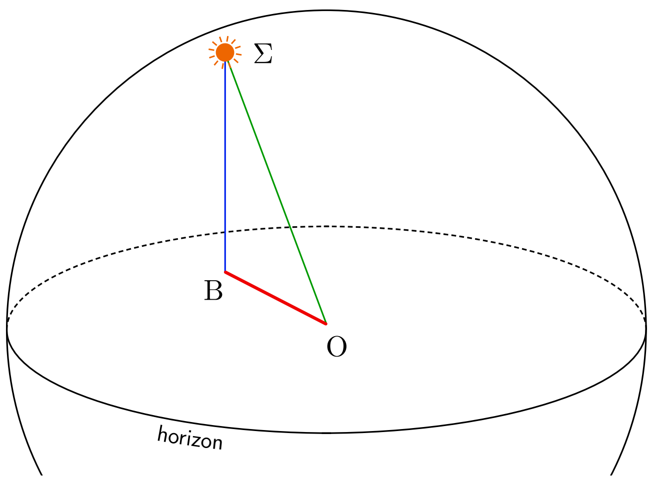



Figure 8.13: The great shadow BO

If we ignore the parallax (which is introduced late in *GD2* 248-276), the great shadow is the distance between the foot B of the great gnomon ΣB and the observer O (figure 8.13). The great shadow BO, the great gnomon ΣB and the Radius OΣ form a right triangle △ΣBO. This is explicated in *GD1* 4.2cd:

> These two (the great shadow and the great gnomon) are the base and the upright. The Radius is the hypotenuse of these two. With these three a trilateral [is formed]. [8]

While *GD2* does not, only giving the computation (i.e. deriving the great shadow $\mathcal{S}$ with the Pythagorean theorem) in *GD2* 114ab:

$$\mathrm{BO} = \sqrt{\mathrm{O}\Sigma^2 - \Sigma\mathrm{B}^2}$$
$$\mathcal{S} = \sqrt{R^2 - \mathcal{G}^2} \tag{8.8}$$

[8] *doḥkoṭī te dve staḥ karṇas trijyā tayos tribhis tryaśram //4.2//* (K. V. Sarma (1956–1957, p. 43))

*GD2* itself does not refer to the computation in the other direction, i.e.:

$$\mathcal{G} = \sqrt{R^2 - \mathcal{S}^2} \tag{8.9}$$

although it is actually required in the methods and their examples appearing later in the treatise.

## 8.6 From the great shadow to the shadow (*GD2* 116)

The gnomon as an instrument appears for the first time in *GD2* 116. Within *GD2*, it is consistently distinguished from the great gnomon using the expression "twelve *aṅgula*s" which refers to its length, with the exception of the six computational examples that use the word "gnomon" (*śaṅku* or *nara*) without any modifier. Parameśvara does not mention in any of his treatises or commentaries whether this "*aṅgula*", literally "width of finger", refers to the actual length or is an arbitrary unit. However we will see in *GD2* 245 that he refers to a gnomon having a length other than twelve units.

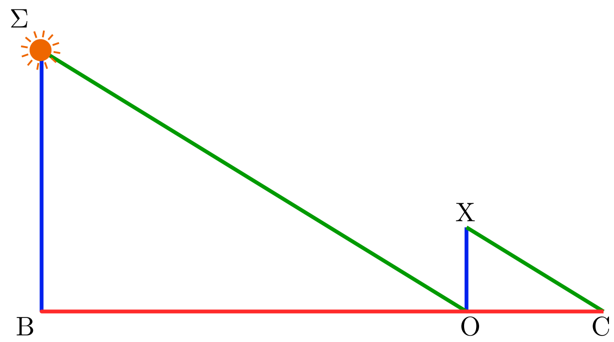



Figure 8.14: The great gnomon ΣB and the gnomon XO

*GD2* 116 mentions the relation between the shadows of the great gnomon and the gnomon. Figure 8.14 illustrates the two triangles involved. When the Σ is the sun projected on the sphere, ΣB the great gnomon, XO the twelve *aṅgula* gnomon and C the tip of its shadow, assuming that the light-source is infinitely far, OΣ and CX are parallel. Thus $\triangle\Sigma\mathrm{BO} \sim \triangle\mathrm{XOC}$, and

$$\begin{aligned}\mathrm{OC} &= \frac{\mathrm{BO}\cdot\mathrm{XO}}{\Sigma\mathrm{B}}\\ s &= \frac{12\mathcal{S}}{\mathcal{G}}\end{aligned} \tag{8.10}$$

where $s$ and $\mathcal{S}$ are the lengths of the shadows of the 12 *aṅgula* gnomon and great gnomon, respectively.

## 8.7 From the given "Sine" in the diurnal circle to the great gnomon (*GD2* 117-118)

*GD2* 117 gives two computations for deriving the great gnomon from the given "Sine" in the diurnal circle[9], just like *GD2* 114ab. The difference is the triangle paired with $\triangle\Sigma$BT.

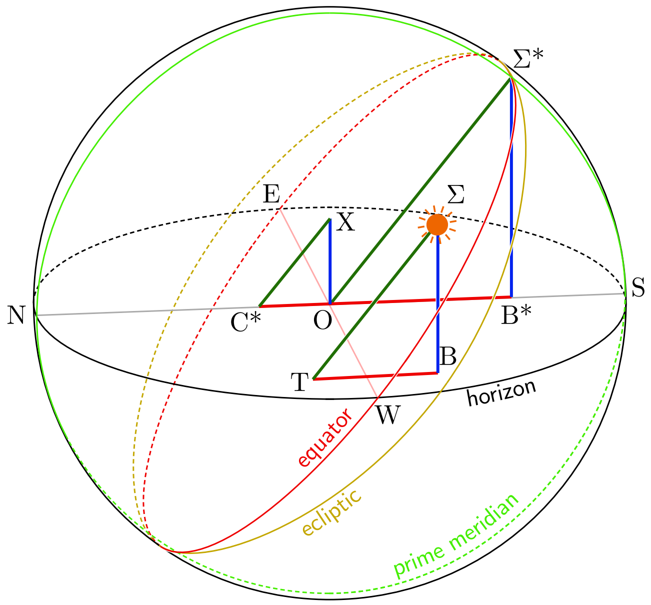



Figure 8.15: Computation with the gnomon XO at equinoctial midday.

The gnomon XO and its shadow on an equinoctial midday OC* form $\triangle$XOC*. As we have seen, this is similar to $\triangle\Sigma^*$B*O where $\Sigma^*$B* is the great gnomon $\mathcal{G}$ at that moment. Since the diurnal circle is equal to the celestial equator, the east-west line EW is also the rising-setting line, and hence the great shadow B*O is also the gnomonic amplitude. Therefore $\triangle\Sigma^*$B*O is similar to $\triangle\Sigma$BT formed by the great gnomon $\Sigma$B and gnomonic amplitude BO at any given moment. Thus $\triangle\Sigma\mathrm{BT} \sim \triangle\mathrm{XOC}^*$, and

$$\begin{aligned} \Sigma\mathrm{B} &= \frac{\mathrm{XO} \cdot \mathrm{T}\Sigma}{\mathrm{C}^*\mathrm{X}} \\ \mathcal{G} &= \frac{12 j_t}{h^*} \end{aligned} \tag{8.11}$$

Here $h^*$ is the *palakarṇa*, a term for indicating the hypotenuse C*X formed by the gnomon at midday on an equinoctial day.

*GD2* 117cd involves $\triangle$FGU formed by the Sine of declination FG, Earth-Sine GU and solar amplitude UF (figure 8.16). *GD2* 118 mentions that $\triangle\Sigma$BT and $\triangle$FGU have a different orien-

[9]The Sanskrit word used here is *dyujyā* without *iṣṭa* (given), and could have various meanings in itself, but the word *kṣitijāt* (from the horizon) makes it clear that it is indeed the given "Sine" in the diurnal circle measured from the horizon.

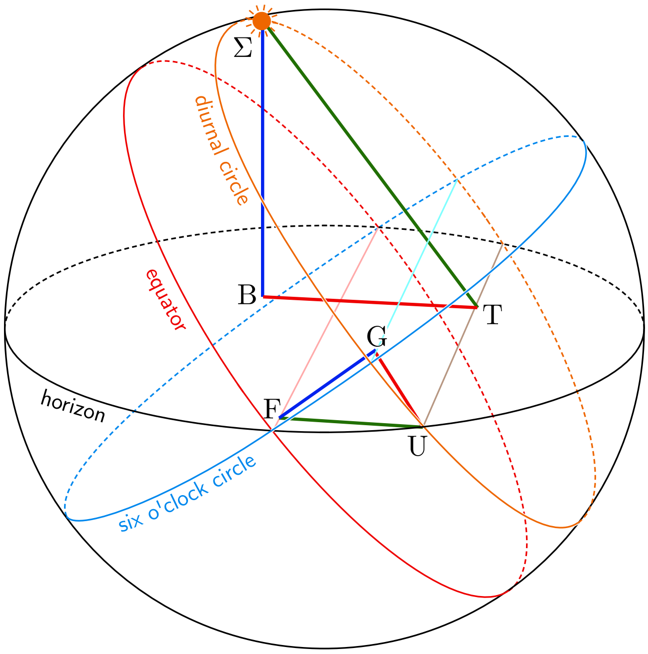



Figure 8.16: Computation with the triangle formed by the Sine of declination, Earth-Sine and solar amplitude. North is to the right.

tation ; the upright of △ΣBT (great gnomon $\Sigma B = \mathcal{G}$), which is the value to be established, extends upward, and the hypotenuse of △FGU (solar amplitude $FU = \mathrm{Sin}\,\eta$) extends northward, but the fact that they both arise from the geographic latitude assures their similarity and the computation:

$$\begin{aligned} \Sigma B &= \frac{FG \cdot T\Sigma}{UF} \\ \mathcal{G} &= \frac{j_t \,\mathrm{Sin}\,\delta}{\mathrm{Sin}\,\eta} \end{aligned} \tag{8.12}$$

## 8.8 From the great gnomon to the gnomonic amplitude (*GD2* 119-120)

The three computations in *GD2* 119, *GD2* 120ab and *GD2* 120cd each use the same pair of triangles with *GD2* 114ab, *GD2* 117ab and *GD2* 117cd. The difference is that this time the gnomonic amplitude $\mathcal{A}$ is going to be computed from a given value of the great gnomon $\mathcal{G}$.

The structure of the sentence in *GD2* 120 resembles *GD2* 117, notably the repeating of “or (*athavā* / *vā*)”. In *GD2* 120, the first “or” clearly follows *GD2* 119. But in the case of *GD2* 117, it might be referring back to *GD2* 114ab.

*GD2* 119 uses △OB′P formed by the Sine of geographic latitude B′P and the Sine of co-latitude OB′ and its similarity with △ΣBT (figure 8.12):

$$\begin{aligned} \mathrm{BT} &= \frac{\mathrm{B'P} \cdot \Sigma\mathrm{B}}{\mathrm{OB'}} \\ \mathcal{A} &= \frac{\mathcal{G}\,\mathrm{Sin}\,\varphi}{\mathrm{Sin}\,\bar{\varphi}} \end{aligned} \tag{8.13}$$

*GD2* 120ab uses $\triangle$XOC* formed by the gnomon XO and its shadow on an equinoctial midday (*palabhā*) OC* $= s^*$ (figure 8.15):

$$\begin{aligned} \mathrm{BT} &= \frac{\mathrm{OC}^* \cdot \Sigma\mathrm{B}}{\mathrm{XO}} \\ \mathcal{A} &= \frac{s^* G}{12} \end{aligned} \tag{8.14}$$

*GD2* 120cd uses $\triangle$FGU formed by the Earth-Sine GU and the Sine of declination FG (figure 8.16):

$$\begin{aligned} \mathrm{BT} &= \frac{\mathrm{GU} \cdot \Sigma\mathrm{B}}{\mathrm{FG}} \\ \mathcal{A} &= \frac{k\mathcal{G}}{\mathrm{Sin}\,\delta} \end{aligned} \tag{8.15}$$

## 8.9 The prime vertical gnomon (*GD2* 121-124)

Normally, the length of the great gnomon cannot be derived straightforward when the time of the day is unknown and only the direction of the sun is given. One of the few exceptions is when the sun is in the due east or west, in other words when it is on the prime vertical. The great gnomon at this moment is called the "[great] gnomon situated in the prime vertical (*samamaṇḍalasthaśaṅku*)", or abbreviated "prime vertical gnomon (*samamaṇḍalaśaṅku*, *samaśaṅku*)".

In *GD2* 121, Parameśvara uses a strange expression "when the sun is on the east-west line (*pūrvāparasūtrage ravau*)". This would usually refer to a line drawn in this direction or a line connecting the due east and west on the horizon. In the situation being dealt with, the sun should be above the horizon; "When the sun is on the prime vertical" as mentioned in *GD2* 122 is more precise. The same expression appears in *PĀbh* 4.31[10]. Perhaps this describes a situation when the armillary sphere is viewed from above, or when this is drawn as a diagram.

Parameśvara first gives the computation for the prime vertical gnomon $G_{\mathrm{EW}}$ as follows in *GD2* 121:

$$\mathcal{G}_{\mathrm{EW}} = \frac{R\,\mathrm{Sin}\,\delta}{\mathrm{Sin}\,\varphi} \tag{8.16}$$

Here, according to Parameśvara, the declination $\delta$ must meet two conditions. First, it must be smaller than the geographic latitude,[11], since otherwise the diurnal circle would not intersect with the prime vertical. Second, it must be in the north if the observer is in the northern hemisphere, which is Parameśvara's assumption. If the declination were southward, the sun would rise south of the prime vertical and never goes through it.

[10] *tatra labdhaṃ pūrvāparasūtragate 'rke śaṅkur bhavati* [Kern (1874, p. 91)] (Then the quotient is the [great] gnomon when the sun is on the east-west line)

[11] In this respect, Parameśvara compares their Sines and not their arcs.

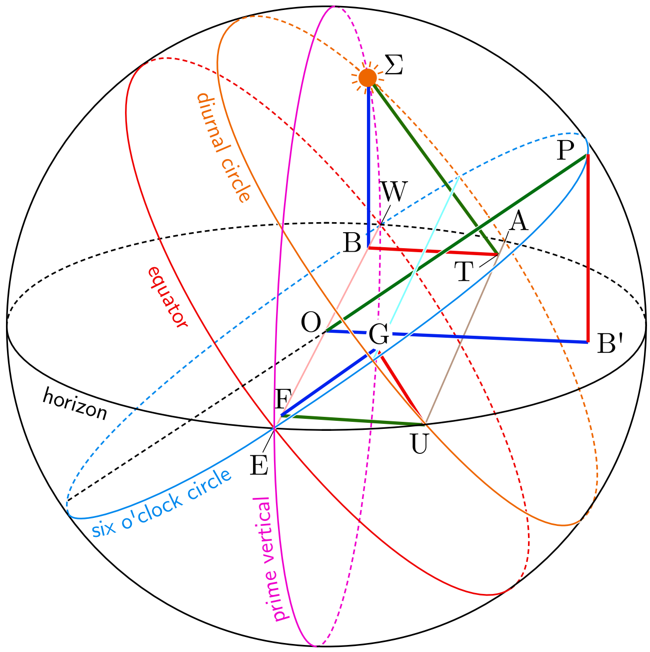



Figure 8.17: The prime vertical gnomon ΣB. North is to the right.

The next three verses provide the grounding for this computation. Since the foot of the gnomon B is on the east-west line, the gnomonic amplitude BT $= A_{\text{EW}}$ is equal to the solar amplitude UF $= \text{Sin}\,\eta$. Therefore from *GD2* 84 (formula 6.7) we obtain:

$$\mathcal{A}_{\text{EW}} = \text{Sin}\,\eta = \frac{R\,\text{Sin}\,\delta}{\text{Sin}\,\bar{\varphi}} \tag{8.17}$$

Meanwhile, since $\triangle\Sigma\text{BT} \sim \triangle\text{OB}'\text{P}$,

$$\begin{aligned} \Sigma\text{B} &= \frac{\text{OB}' \cdot \text{BT}}{\text{B}'\text{P}} \\ \mathcal{G}_{\text{EW}} &= \frac{\mathcal{A}_{\text{EW}}\,\text{Sin}\,\bar{\varphi}}{\text{Sin}\,\varphi} \end{aligned} \tag{8.18}$$

This is the first Rule of Three mentioned in *GD2* 123, and formula 8.17 is repeated as the second Rule of Three. The co-latitude $\text{Sin}\,\bar{\varphi}$ appears as the divisor in the first Rule of Three and as the multiplier in the second and can be reduced, and the result is formula 8.16.

We will see an example for computing the prime vertical gnomon in *GD2* 209, but until then, the treatise turns to a totally different direction — latitude of planets.

# 9 Orbits of planets and their deviation (*GD2* 125-152)

Discussions concerning the celestial latitude begins in *GD2* 125. This is a fundamental topic that later develops into the visibility equation (*GD2* 153-194) and parallaxes (*GD2* 248-276). Computations for the celestial latitude involve theories of planetary motions. In the geocentric configuration underlying *GD2*, orbits of planets are inclined, which causes the planets to deviate from the plane of the ecliptic. This deviation can be observed from the Earth as the celestial latitude from the ecliptic. Parameśvara uses the same word *kṣepa* / *vikṣepa* (literally "throwing") for inclination $i$ and deviation $b$. *GD2* 128 gives the celestial latitude $\beta$ by correcting the deviation for the planet's distance, but Parameśvara does not give a name to this result. From *GD2* 153 onwards, he also refers to the celestial latitude as *kṣepa* and *vikṣepa*.

Parameśvara starts in *GD2* 125-126 by introducing a simple situation where a planet moves on a circle which is inclined against the ecliptic. *GD2* 127 is the rule to find the planet's deviation from its longitude, and *GD2* 128 gives the correction for its radial distance. Essentially, these two verses are the core of this section which gives the latitude as seen from the observer, but this is not emphasized by Parameśvara. *GD2* 129-130 give the longitudes of the nodes and the inclinations of orbits for the five planets Mars, Mercury, Jupiter, Venus and Saturn; these values are used in the previous computations. *GD2* 131-133 states some brief groundings for *GD2* 127-128 and *GD2* 134 introduces an alternative rule for the deviation according to another school. From *GD2* 135 onward, Parameśvara starts a long description of planetary orbits. Three circles for each of the five planets are drawn (*GD2* 135-140), and the corrected longitudes and radial distances are shown by drawing lines or strings (*GD2* 141-145ab). *GD2* 145cd-148 discusses the discrepancy between the planet thus corrected and its observed position. Next, Parameśvara applies the same method (as used for longitudes) to the planet's deviation in *GD2* 149-151. These could be considered as reasonings for *GD2* 128. Additionally, *GD2* 152 refers to the sun and moon which have only two circles.

Many terms appear without explanation, and Parameśvara seems to assume that the reader has already studied this topic through other treatises. Therefore I have added some explanation based on the *Āryabhaṭīya* and Parameśvara's commentary in Appendix C.

## 9.1 Celestial longitude and latitude

We have previously discussed in section 6.2 that words for "planet" can signify the celestial longitude of the planets. With the introduction of celestial latitudes, we must be even more cautious. *GD2* 151 compares the correction of a "planet (*graha*)" with that of the "deviation". Here the word "planet" must be interpreted as the "longitude (of the planet)". This is the way I make sense of the following verses, especially those starting from *GD2* 135. In the following, I will distinguish whenever I interpret "planet" as its "longitude", but we must keep in mind that the terms are not conflated in Parameśvara's texts. In my translations, I have kept the word "planet" and have only supplied "longitude" in brackets when the passage would be otherwise incomprehensible.

In *GD2* 153-194 especially, a "planet" refers exclusively to the point on the ecliptic and not the celestial object itself. On the other hand, the "celestial latitude (*kṣepa*)" is more likely to indicate the position of the body. Yet at the end of this section, Parameśvara only uses the celestial latitude to find the corresponding longitude on the ecliptic that rises at the same time with the object; this is what he calls the "visibility methods of a planet (*graha*)" in *GD2* 165. In this case the celestial latitude is only secondary to the longitude, which might also be reflected in the terms *graha* and *kṣepa*. The English terms longitude and latitude refer to a system of coordinates. In Sanskrit, the *graha* is the essential coordinate while *kṣepa* serves as its correction.

## 9.2 Inclined circle (*GD2* 125-126)

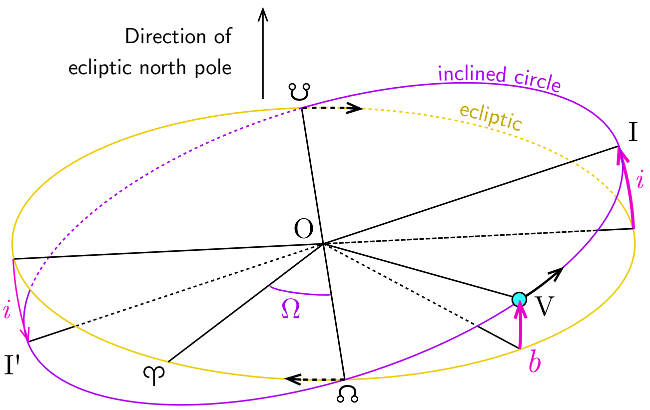



Figure 9.1: The inclined circle with inclination $i$ and deviation $b$ for a given longitude.

*GD2* 125-126 describes an "inclined circle (*vikṣepamaṇḍala*, *vimaṇḍala*)", inclined against the ecliptic (figure 9.1). ☊ and ☋ are the ascending and descending nodes respectively. At a quadrant's distance, on points I and I′, the distance of the circle from the ecliptic is equal to its greatest deviation, in other words the inclination of the orbit $i$. This explanation is applied to the moon as well as the five planets[1]. The descriptions suggest that the inclined circles are great circles like the ecliptic, and this will become even more evident in *GD2* 127cd. However, as we will see later in section 9.10, the actual configuration with inclined orbits are much more intricate and most notably those for Mercury and Venus involve an inclined orbit which is not a great circle.

The term "inclined circle" only appears in *GD2* 125-126. The same word (in the form *kṣepa-maṇḍala*) can be seen in the first chapter of *GD1* which is dedicated to the armillary sphere:

> The inclined circle of [each planet] beginning with the moon goes through the two nodes on the ecliptic and is separated by their greatest latitude north and south at [the two points] three signs from there. (*GD1* 1.7cd-8ab)[2]

The explanation in *GD2* 125-126 might also have involved armillary spheres. The inclined circle also appears in the description of the Armillary sphere by Bhāskara I in his commentary on

[1] The seven "planets" (including the sun and the moon) are always enumerated in the order of the weekdays beginning with the sun. Therefore "beginning with the moon (*candrādi)*" refers to the moon, Mars, Mercury, Jupiter, Venus and Saturn. And to refer to the five planets, Parameśvara says those "beginning with Mars (*bhaumādi*)" as in *GD2* 128 and 129.

[2] *apamagapātadvayagaṃ saumye yāmye tataś ca bhatritaye ||1.7||*
*paramakṣepāntaritaṃ candrādeḥ kṣepamaṇḍalaṃ bhavati* (K. V. Sarma (1956–1957, p. 12))

the *Āryabhaṭīya* (Lu (2015))[3]. *Brāhmasphuṭasiddhānta* 21.53cd-54ab (Ikeyama (2002, p. 135)) also refers to inclined circles for each planet, but the configuration of the *gola* described therein is too complex to physically construct it in a complete form. The same can be said for the descriptions in the *Golabandha* chapters of *Śiṣyadhīvṛddhidatantra* 15.9 (Chatterjee (1981, 1, p. 202)), *Siddhāntaśekhara* 16.34-35 (Miśra (1947, pp. 211-212)) and *Siddhāntaśiromaṇi Golādhyāya* 6.13-26 (Chaturvedi (1981, pp. 397-403)). The *Yantra* (instrument) chapter of these three texts do not refer to inclined circles[4]. The *Sūryasiddhānta* stands out as it does not describe an inclined circle. Instead, in 13.11cd-12ab the planets are stated to be "drawn away from the ecliptic by the nodes based on the ecliptic[5]". Parameśvara comments nothing significant on this passage, and does not even mention the term "inclined circle".

*GD2* 125 further adds that the two nodes are actually moving on the ecliptic, retrograde against the revolution of the planet. The rates of revolutions are not given in *GD2*; most probably, Parameśvara follows Āryabhaṭa. The corresponding passages from the *Āryabhaṭīya* with Parameśvara's commentaries are as follows.

> The retrograding node is *buphinaca*. ... (*Ābh* 1.4c)
> "***buphinaca***" is the revolutions of the **node**, [i.e.] the moon's node, which has the nature of **retrograding**. ***bu***, two hundred thirty thousand. ***phi***, two thousand two hundred. ***na***, twenty. ***ca***, six. He will state the revolutions of the nodes of those beginning with Mars [later].[6]

> Mercury, Venus, Mars, Jupiter [and] Saturn, *na-va-ra-ṣa-ha*. Having moved [these] degrees, [their] first nodes [are placed]. (*Ābh* 1.9ab)
>
> **Mercury**'s node in degrees is ***na***, twenty. [That] of **Venus *va***, sixty. Of **Mars *ra***, forty. Of **Jupiter *ṣa***, eighty. Of **Saturn *ha***, one hundred. **Having moved degrees, first nodes**. Having moved these very degrees that have been stated from the beginning of Aries, the first nodes of those beginning with Mercury should be placed. With the word "first", it is indicated that there is also a second node[7]. And this should be situated at a distance of half a circle from the first node. The intersecting place of the inclined circle and the ecliptic is stated with the word "node". But this is on both sides. From the statement "having moved", the motion of these nodes is intended. And the motion is retrograde. With this [passage] "retrograding node" (*Ābh* 1.4c), it has been stated that the nodes have a retrograde movement. It is said that the nodes are settled in our time.[8]

---

[3] Lu points out that Bhāskara I does not explain how to add the orbit rings (*vimaṇḍala*) of Mercury and Venus according to their scale of the "fast" epicycle. My suggestion is that Bhāskara I might be simply assuming an inclined circle equal in size with the ecliptic, as is the case for the moon and other planets (but with the position of the "fast" apogee being tracked instead of the planet). Adding epicycles to armillary spheres would have been physically difficult, and they could be explained separately in diagrams, as is the case with Parameśvara.

[4] See also section 2.1 for the descriptions of the *gola* in these texts.

[5] *candrādyāś ca svakaiḥ pātair apamaṇḍalam āśritaiḥ ||13.11||*
*tato 'pakṛṣṭā dṛśyante vikṣepāgreṣv apakramāt |* ((Shukla (1957, p. 133)))

[6] *buphinaca pātavilomā ... (1.4c)*
*buphinaca iti pātasya candrapātasya vilomātmakabhagaṇāḥ | bu ayutānāṃ trayoviṃśatiḥ | phi śatadvayādhikasahasradvayam | na viṃśatiḥ | ca ṣaṭ | kujādīnāṃ pātabhagaṇān vakṣyati |* (Kern (1874, pp. 6-7))

[7] The first node refers to the ascending node, and the second is the descending node.

[8] *budhabhṛgukujaguruśani navaraṣahā gatvāṃśakān prathamapātāḥ |(1.9ab)*
*budhasya pātāṃśāḥ na viṃśatiḥ | bhṛgoḥ va ṣaṣṭiḥ | kujasya ra catvāriṃśat | guroḥ ṣa aśītiḥ | śaneḥ ha śatam | gatvāṃśakān prathamapātāḥ | uktān etān evāṃśakān meṣādito gatvā vyavasthitā budhādīnāṃ prathamapātāḥ syuḥ | prathamaśabdena dvitīyo 'pi pāto 'stīti sūcitam | sa ca prathamapātāc cakrārdhāntare sthitaḥ syāt |*

According to Parameśvara's interpretation, the nodes of the moon and the five planets all have a retrograde motion, but only the moon's node has a significant rate of 232,226 revolutions per *yuga* and the others can be regarded as still within our timespan. *GD2* 129 repeats the numbers for the positions of the five planets' ascending nodes as given in *Ābh* 1.9ab as well as mentioning that they have a small motion.

## 9.3 Deviation from the ecliptic (*GD2* 127, 131)

The deviation depends on the arc distance from the node to a specific point (depending on the planet). From hereon we shall call this arc the "argument" of the celestial latitude (Parameśvara does not use a specific term). As we have seen previously, words for celestial objects themselves can signify their longitudes along the ecliptic. The same can be said in *GD2* 127 for words like "node" or "slow" corrected [planet], and therefore the argument is an arc measured along the ecliptic. Parameśvara does not specify which node is to be taken, but as he lists the positions of the ascending nodes in *GD2* 129, it would be natural to take the longitude of the ascending node ($\overset{\frown}{♈☊} = \Omega$). Concerning the point which completes the argument, Parameśvara first mentions in *GD2* 127ab that the longitude of the "slow" corrected planet $\overset{\frown}{♈\mathrm{L}_\mu} = \lambda_\mu$ is used without specifying the planet. Since Mercury and Venus are mentioned in the next case, this applies to the moon, Mars, Jupiter and Saturn (figure 9.2 and 9.3). The moon has only one apogee (which is "slow" as mentioned in *GD2* 152) and thus the "slow" corrected longitude is already its true longitude $\overset{\frown}{♈\mathrm{L}_\mathrm{T}} = \lambda_\mathrm{T}$. For Mars, Jupiter and Saturn, the "slow" corrected longitude $\lambda_\mu$ without the "fast" correction applied is to be used. In the case of Mercury and Venus, the longitude $\lambda_{\mathrm{U}_\sigma}$ of its "fast" apogee $\mathrm{U}_\sigma$ is used (figure 9.4).

Thus the argument for each case is:

$$\lambda - \Omega = \begin{cases} \lambda_\mathrm{T} - \Omega & \text{Moon} \\ \lambda_\mu - \Omega & \text{Mars, Jupiter, Saturn} \\ \lambda_{\mathrm{U}_\sigma} - \Omega & \text{Mercury, Venus} \end{cases} \tag{9.1}$$

*GD2* 127cd gives the rule for computing the deviation $b$ from the "base" Sine of the argument, expressed as the "[longitude] diminished by the node (*pātona*)". Here, the "base" refers to the distance starting from the nearest node and not from the equinoctial points. If the planet is closer to the descending node, the difference between their longitudes should be taken as the "base" arc. Hereafter I shall denote the "base" Sine $\mathrm{Sin}(\lambda - \Omega)_B$ for every case.

The rule in *GD2* 127cd resembles *GD2* 73ab (formula 6.3) which gives the Sine of declination from the "base" Sine and the Sine of greatest declination. The corresponding Rule of Three, given later in *GD2* 131, resembles *GD2* 81 very well (*GD2* 131bc and *GD2* 81bc are exactly the same). Our visual explanation in figure 9.5 also looks like what we used for *GD2* 73ab (figure 6.4). Yet there are two differences between the case for the declination and the case for the deviation. First, although the celestial object is on the inclined circle, the "base" must be measured on the ecliptic (in *GD2* 73, both were on the ecliptic). Second, Parameśvara refers to the deviation itself and not its Sine. The Sine of deviation can be approximated with its arc because it is very small[9].

*vikṣepamaṇḍalāpamaṇḍalayoḥ saṃpātasthānaṃ pātaśabdenocyate / tad dhy ubhayatra bhavati / gatvetivacanāt teṣāṃ pātānāṃ gatir abhipretā / gatiś ca vilomā / pātavilomā ity anena pātānāṃ vilomagatvam uktam / asmin kāle pātānāṃ sthitir evam ity uktaṃ bhavati /* (Kern (1874, p. 12))

[9]The greatest deviation of the moon, which is the largest of all planets, is 4°30' (*Ābh* 1.8) which is 270'. Its Sine is 269;48, rounded to the same 270.

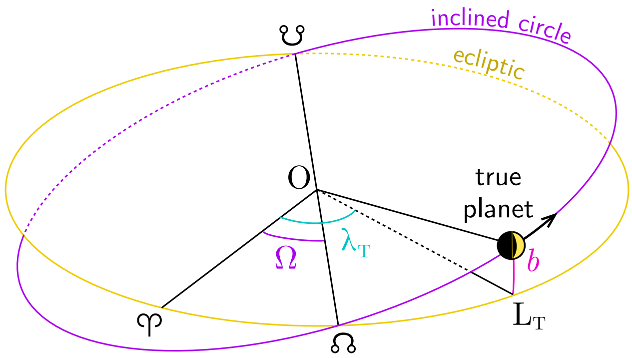



Figure 9.2: Argument of latitude for the moon

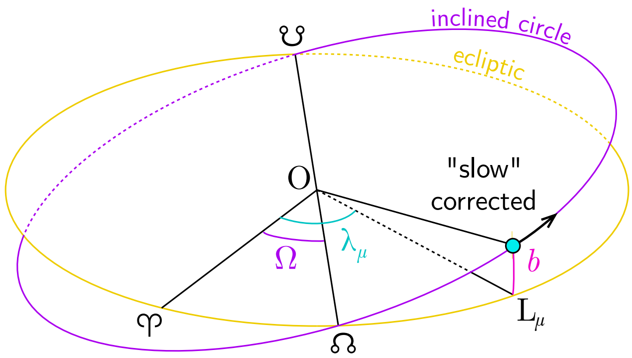



Figure 9.3: Argument of latitude for Mars, Jupiter and Saturn

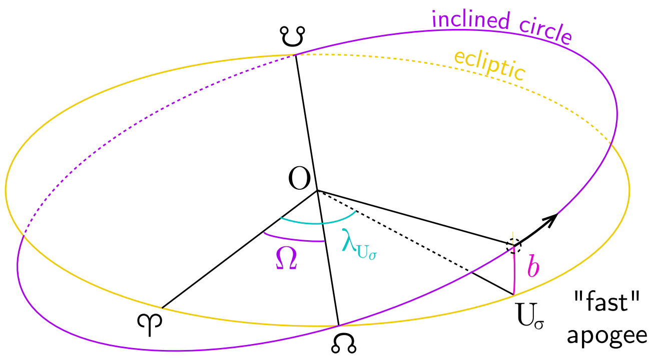



Figure 9.4: Argument of latitude for Mercury and Venus

In figure 9.5, L represents the longitude of the true planet, "slow" corrected planet or "fast" apogee, LM is the Sine of deviation Sin $b(\sim b)$ and WT is the Sine of greatest deviation Sin $i(\sim i)$. △LMK and △WTO are similar, where KL is the "base" Sine $\text{Sin}(\lambda - \Omega)_B$ and OW is its largest value, the Radius $R$. Therefore,

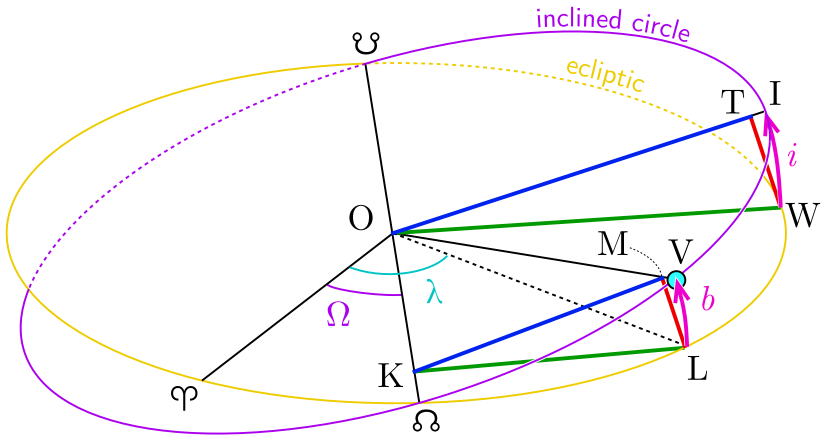



Figure 9.5: Deviation $b$ at a given argument

$$\mathrm{LM} = \frac{\mathrm{WT} \cdot \mathrm{MK}}{\mathrm{TO}}$$
$$\mathrm{Sin}\, b = \frac{\mathrm{Sin}\, i\, \mathrm{Sin}(\lambda - \Omega)_B}{R} \tag{9.2}$$

And approximating the Sines with arcs,

$$b = \frac{i\, \mathrm{Sin}(\lambda - \Omega)_B}{R} \tag{9.3}$$

The arc $b$ should be in minutes, as *GD2* 130 gives the greatest deviation for each planet in minutes too.

The Rule of Three corresponding to *GD2* 127 is given later in *GD2* 131. This suggests that Parameśvara is stating *GD2* 127-128 as a single procedure. *GD2* 129-130 supply the constants to be used in this procedure, and *GD2* 131-133 add the reasonings.

## 9.4 Distance correction (*GD2* 128, 132-133)

The deviation computed so far is the length of arc on a great circle as seen from its center. In order to find the true deviation as seen from the Earth, i.e. the celestial latitude, the variance in radial distance caused by the "slow" and "fast" apogees must be taken into account. When a planet without the "slow" correction is at a distance of the Radius $R$, the "slow" radial distance $\mathcal{R}_\mu$ is its distance after the correction. Likewise, the "fast" correction changes the planet's distance from $R$ to a "fast" radial distance of $\mathcal{R}_\sigma$. Parameśvara explains the relation between the radial distances, corrections and the apogees later in *GD2* 134-150.

*GD2* 128ab gives the correction for the "slow" radial distance $\mathcal{R}_\mu$. This pertains to all planets including the moon. Figure 9.6 illustrates how this rule could be explained. V is the uncorrected position of the planet and $\mathrm{V}_\mu$ is the "slow" corrected planet on a great circle whose center is

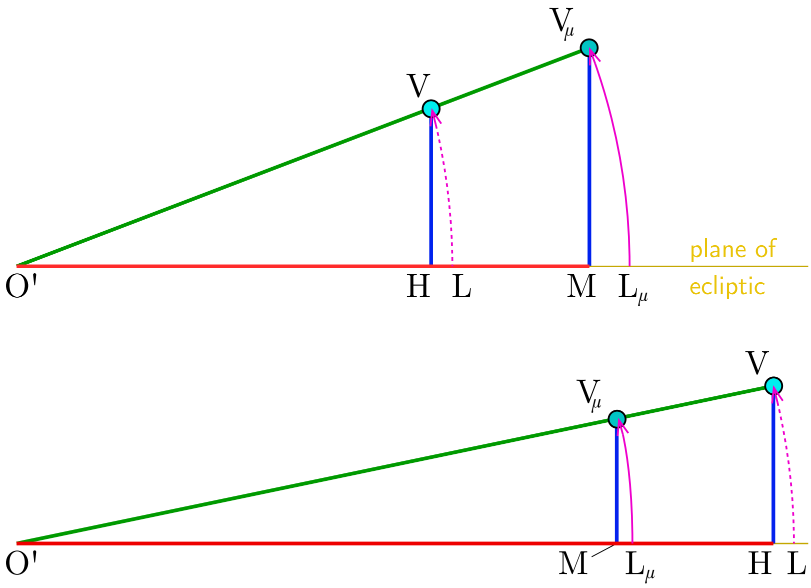



Figure 9.6: The "slow" radial distance $O'V = \mathcal{R}_\mu$ and the corrected deviation $L_\mu V_\mu \simeq b_\mu$, when it is closer (above) or further (below) than the Radius.

$O'$[10]. The diagram shows a segment going through these three points and perpendicular to the ecliptic. $O'V_\mu = R$ while $O'V = \mathcal{R}_\mu$ is the "slow" radial distance[11]. HV is the Sine of the uncorrected deviation $\text{Sin}\, b$ while $V_\mu M$ is the Sine of the "slow" corrected deviation $\text{Sin}\, b_\mu$. As both Sines of deviations are very small, they can be approximated with their arcs $\widehat{LV} = b$ and $\widehat{L_\mu V_\mu} = b_\mu$. Since $\triangle O'HV$ and $\triangle O'MV_\mu$ are right triangles sharing one acute angle, they are similar. Therefore,

$$
\begin{aligned}
V_\mu M &= \frac{VH \cdot V_\mu O'}{VO'} \\
\text{Sin}\, b_\mu &= \frac{\text{Sin}\, b \cdot R}{\mathcal{R}_\mu}
\end{aligned}
\tag{9.4}
$$

And by approximating the Sines with their arcs,

$$
b_\mu = \frac{bR}{\mathcal{R}_\mu}
\tag{9.5}
$$

*GD2* 132 supplies some explanation for this rule. As shown in figure 9.6, the deviation projected at a distance of the Radius becomes larger when the radial distance is shorter, and

[10] We will see later that for Mars, Jupiter and Saturn, this corresponds to the "fast" eccentric circle with center $O_\sigma$, while for Mercury and Venus it is the zodiac with center O.

[11] Alternatively, if we are to comply with the notion that a "planet" is always on the ecliptic, we may use the corresponding segments $O'L_\mu = R$ and $O'L = \mathcal{R}_\mu$ on the plane of the ecliptic. But this does not change the result.

becomes smaller when the radial distance is longer. What the phrase "difference in minutes of the figure (*kṣetrasya liptikābheda*)" refers to is unclear. One possibility is that "figure" stands for a right triangle, and therefore refers to the similarity involved in the computation. Interestingly, Parameśvara does not use the word "deviation" in *GD2* 132. Perhaps this reasoning could have been applied to other rules, such as the apparent size of an object (cf. *GD2* 280).

The moon has only one apogee ("slow"), and therefore $b_\mu$ is the true deviation. Meanwhile, planets beginning with Mars, i.e. Mars, Mercury, Jupiter, Venus and Saturn have a "fast" apogee in addition which causes a difference in radial distance on its own. This is mentioned in *GD2* 133, where "below and above" refers to being closer to or further from the Earth. The "fast" corrected deviation $b_\sigma$ can be computed in exactly the same way as the "slow" correction. When the uncorrected deviation is $b$ and the "fast" radial distance is $\mathcal{R}_\sigma$:

$$\mathrm{Sin}\, b_\sigma = \frac{\mathrm{Sin}\, b \cdot R}{\mathcal{R}_\sigma} \tag{9.6}$$

The Sines can be approximated with arcs:

$$b_\sigma = \frac{bR}{\mathcal{R}_\sigma} \tag{9.7}$$

We will see later that for Mars, Jupiter and Saturn, the "fast" correction is applied after the "slow" correction. This corresponds to using $b_\mu$ instead of $b$ in formula 9.7. On the other hand, the order is reversed for Mercury and Venus. This is equivalent to using $b_\sigma$ in place of $b$ in formula 9.5. In both cases, the twice-corrected deviation $b_\mathrm{T}$ is

$$b_\mathrm{T} = \frac{bR^2}{\mathcal{R}_\mu \mathcal{R}_\sigma} \tag{9.8}$$

which we can also find from *GD2* 128. Parameśvara does not give a name to this twice-corrected deviation. Later in *GD2* 150 he uses the expression "true deviation (*vikṣepa sphuṭa*)". We may conclude that this is the celestial latitude as seen from the Earth.

Formula 9.8 is equivalent to correcting a deviation of $b$ once when its distance is $\frac{\mathcal{R}_\mu \mathcal{R}}{R}$. This has a parallel with *Ābh* 3.25ab, which states the distance of a planet with two apogees:

> The distance between the Earth and a star-planet (the five planets) is the product of its radial distances divided by the half-diameter.[12]

However, *Ābh* 3.25ab is incorrect (see appendix C.6) and so is *GD2* 128. The error comes from treating the two corrections as if they were independent from each other. *GD2* 151 (section 9.11) might be a reference to this fact.

## 9.5 Values of the nodes and inclinations (*GD2* 129-130)

*GD2* 129 lists the "degrees of the nodes" $\Omega$ (table 9.1), which is the longitude of the ascending node ☊ measured from the vernal equinox ♈ (figure 9.1). The greatest deviations, or the inclination $i$ of their orbits are given in *GD2* 130. The values of $\Omega$ and $i$ for the five planets are exactly

[12] *bhūtārāgrahavivaraṃ vyāsārdhahṛtaḥ svakarṇasaṃvargaḥ* / (Kern (1874, p. 69))

Table 9.1: Parameters of inclined circles as given in *GD2* 129 and 130

| | | Mars | Mercury | Jupiter | Venus | Saturn |
|---|---|---|---|---|---|---|
| Ascending node | $\Omega$ | 40° | 20° | 80° | 60° | 100° |
| Greatest deviation | $i$ | 90′ | 120′ | 60′ | 120′ | 120′ |

those mentioned in *Ābh* and *MBh* but not with the same measurement unit for $i$ ($\Omega$ given in *Ābh* 1.9 and *MBh* 7.10, and $i$ in *Ābh* 1.8 and *MBh* 7.9[13]). As previously mentioned in section 9.2, Parameśvara considers that every node moves retrograde, but that those of the five planets are slow enough that they can be considered as constant. The moon's node has a significant motion, and therefore it makes sense that *GD2* 129 does not refer to the moon. However, the greatest deviation of the moon does not change (half of nine degrees i.e. 4°30′according to *Ābh* 1.8) , but yet it is excluded from *GD2* 130. The *Āryabhaṭīya* gives in the same verse the value of $i$ for the moon and those of the five planets while the *Mahābhāskarīya* omits it as is the case with *GD2*.

## 9.6 Alternative computation for the argument (*GD2* 134)

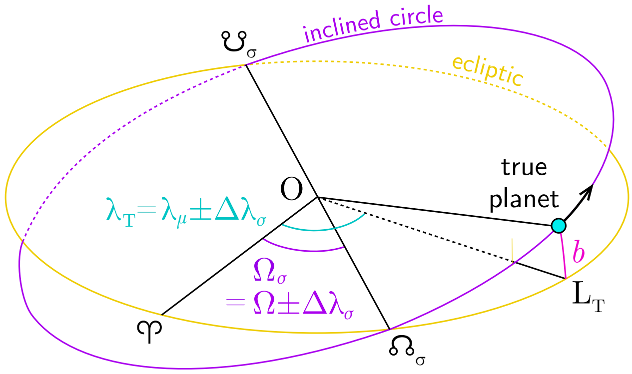



Figure 9.7: Argument of the true planet $\lambda_T - \Omega_\sigma$

An alternative computation for the deviation using a different argument in the case of Mars, Jupiter and Saturn is given in *GD2* 134 (figure 9.7), where the true longitude on the ecliptic $L_T$ (i.e. the position after both "slow" and "fast" corrections are applied) is used instead of the "slow" corrected longitude $L_\mu$. In this case, the "fast" correction is also applied to the node. As explained in appendix C.4, the correction is done by deriving the equation from the "base" Sine of the "fast" anomaly (*śīghrakendrabhujajyā*). The word *śīghrajyā* in *GD2* 134 is most likely its abbreviation, meaning that the same equation $\sigma$ should be added to or subtracted from both the planet and the node. Since we take their difference, the equation is canceled out and the same value $\lambda_\mu - \Omega$ is obtained as the argument.

$$\begin{aligned} &\lambda_{L_T} - \Omega_\sigma \\ &= \left(\lambda_{L_\mu} \pm \sigma\right) - (\Omega \pm \sigma) \\ &= \lambda_\mu - \Omega \end{aligned} \tag{9.9}$$

[13]Both *Ābh* and *MBh* give the values for $i$ in degrees.

Parameśvara introduces this as a method practiced by another school (*pakṣa*), which probably refers here to Lalla. There is a detailed discussion in his commentary on *Ābh* 4.3, where he quotes Lalla's *Śiṣyadhīvṛddhidatantra* as an example.

> Some masters, having made the equation for the "fast" apogee of Jupiter, Mars and Saturn on their node as with the planet, having subtracted their node thus made from the true planet, make the computation of the deviation. And in the case of Mercury and Venus, however, [the masters] having made their "slow" equation on their node, having subtracted that node from the "fast" apogee, make the deviation. And likewise, the master Lalla [states]:
>
> The nodes of Mars, Jupiter and Saturn have their own "fast" (*cala*) equation subtracted from or added to them accordingly. For Mercury and Venus, the degrees of their own nodes corrected by their own "slow" (*mṛdu*) equation should be true. (*Śiṣyadhīvṛddhidatantra* 10.6)
>
> In this school, the node is subtracted from the true planet of Mars, Jupiter and Saturn.[14]

Chatterjee (1981, 2, p. 182) has already pointed out that astronomers differ from one another in calculating the argument.

## 9.7 Diagram of orbits (*GD2* 135-140)

From *GD2* 135 onward, Parameśvara turns to a description of a diagram which is first used to show the corrected longitudes of planets, then for the radial distance and ultimately the grounding for the correction on deviations as given in *GD2* 128. Parameśvara deals exclusively with the five planets in *GD2* 135-150, and refers to the moon later in *GD2* 151.

A similar set of instructions can be seen in his *Siddhāntadīpikā* under the commentary on *Mahābhāskarīya* 4.54 (T. Kuppanna Sastri (1957, pp. 233-238)), following the method for computing the true planet using the "slow" and "fast" equations. There are 32 verses in total, beginning with the following:

> The reasoning for the rule of correction cannot be established without a diagram of the planets. Therefore the method of their drawing is explained here concisely.[15]

Parameśvara gives a similar but more concise description in 12 verses under his commentary on *Ābh* 4.24 (Kern (1874, pp. 68-69))[16]. The first verse is almost identical with the verse quoted above. In both cases, Parameśvara tries to give the reasoning for combining the "slow" and "fast" equations in a specific method (as explained in Appendix C). In *GD2*, the same type of diagram is used for explaining the deviation.

---

[14] *kecid ācāryā gurukujaśanīnāṃ śīghroccaphalaṃ svapāte 'pi grahavat kṛtvā tathākṛtaṃ svapātaṃ sphuṭagrahād viśodhya vikṣepānayanaṃ kurvanti budhaśukrayos tu svamandaphalaṃ svapāte kṛtvā taṃ pātaṃ śīghroccād viśodhya vikṣepaṃ kurvanti | tathā ca lallācāryaḥ |*
*kṣitisutagurusūryasūnupātāḥ svacalaphalonayutā yathā tathaiva |*
*śaśisutasitayoḥ svapātabhāgāḥ svamṛduphalena ca saṃskṛtāḥ sphuṭāḥ syuḥ ||*
*iti asmin pakṣe kujaguruśanīnāṃ sphuṭagrahāt pātonam ||* (Kern (1874, p. 73), textual corruption in the quote amended using Chatterjee (1981, 1, p. 147))

[15] *sphuṭavidhiyuktiḥ sidhyet*
*naiva vinā chedyakena vihagānām |*
*tasmād iha saṃkṣepāt*
*chedyakakarma pradarśyate teṣām ||1||* (T. Kuppanna Sastri (1957, p. 233))

[16] This is explained in Sriram, Ramasubramanian, and Srinivas (2002, pp. 91-94)

*GD2* 135 begins with drawing three circles called "orbits (*kakṣyā*)" for each of the five planets. The term "orbital circle (*kakṣyāmaṇḍala*)" is normally used (such as in *Ābh* 3.18) for the geocentric great circle, as opposed to eccentric circles and epicycles, but Parameśvara seems to use *kakṣyā* merely as a synonym of "circle (*vṛtta*)" that appears in the same verse. As stated in *GD2* 140, one of the circles for Mercury and Venus is not even a great circle. Meanwhile, he explains in *GD2* 135 that the vernal equinox (referred to as "the end of Pisces") points towards the front of the person who draws (figure 9.8). The "front" direction is expressed by the word "east (*prāñc*)". Longitudes can be defined and measured on every circle as if they were geocentric great circles.

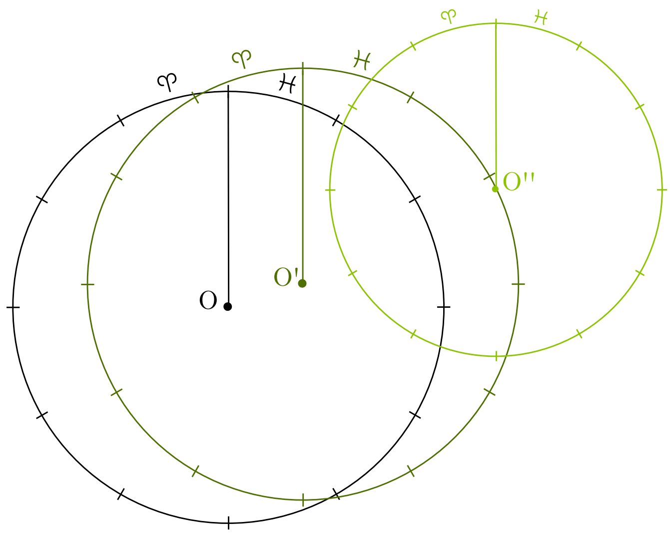



Figure 9.8: Three circles for a planet (Mercury or Venus in this example). All circles have the vernal equinox (end of Pisces ♓ or beginning of Aries ♈) in the same direction.

*GD2* 136-138 explain the configuration of the three circles, which are different among two groups; (1) Mars, Jupiter and Saturn and (2) Mercury and Venus. In both cases, the first circle is called the *bha*. I have adopted the translation "zodiac" for *bha* and its synonym *bhacakra* or *bhavṛtta* to differentiate them from the "ecliptic (*apamaṇḍala*)". The zodiac is not only the great circle on which the true planet is projected (*GD2* 145), but also the zone or belt on which its true deviation (latitude) is to be measured (*GD2* 150).

The three circles for Mars, Jupiter and Saturn are drawn in figure 9.9. Their second circle is the "fast" eccentric circle whose center $O_\sigma$ is on $OU_\sigma$ where O is the Earth's center and $U_\sigma$ is the direction of the "fast" apogee (*GD2* 136d-137a). $OO_\sigma$ is equivalent to the Sine of the greatest possible equation (*antyaphala*) in the planet's "fast" correction, which is also the radius of the "fast" epicycle. The third circle is the "slow" eccentric circle, and this time its center $O_\mu$ is in the direction of the "slow" apogee $U_\mu$ when seen from $O_\sigma$. $O_\sigma O_\mu$ is the Sine of the greatest possible "slow" equation, or the radius of the "slow" epicycle.

Figure 9.10 shows the three circles for Mercury and Venus. This time the second circle is the "slow" eccentric circle having $O_\mu$ as its center, $OO_\mu$ being the greatest "slow" equation. The last circle is not a great circle, as mentioned in *GD2* 140. It is the "fast" epicycle with its center $\Sigma$ on the "slow" eccentric circle. Parameśvara mentions in *GD2* 138 that this center of the "fast" epicycle is the sun; this cannot mean that Mercury and Venus have a heliocentric orbit, since Parameśvara follows, in *GD2* 18, Āryabhaṭa's description of planetary orbits where

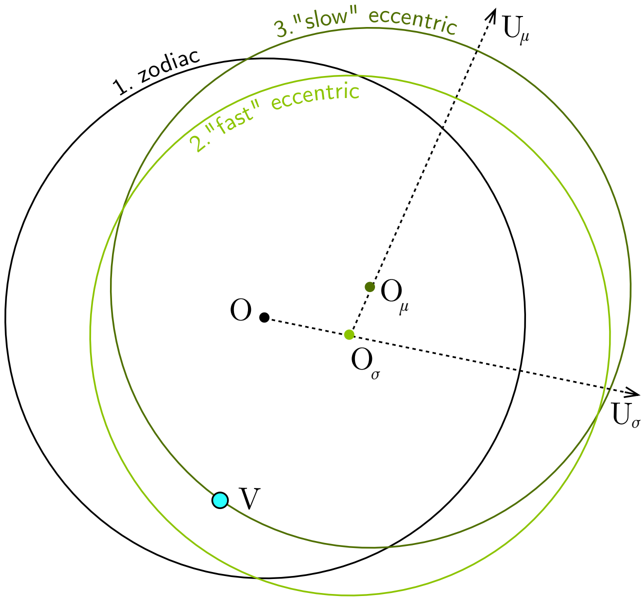



Figure 9.9: Three circles for Mars, Jupiter and Saturn

Mercury, Venus and the sun revolve on separate geocentric orbits, closer to the Earth in this order. Rather, we should take the word "sun" as a reference to the sun's longitude.

It is remarkable that Parameśvara explains in *GD2* 139 that the mean motion takes place on the last circle. This is in contrast with the *Āryabhaṭīya*, where the mean planet revolving with mean motion is located on the geocentric orbital circle (i.e. the "first circle" in Parameśvara's explanation). In the case of Mercury and Venus it is even contradictory to the *Āryabhaṭīya*, because if we take Parameśvara's statement in *GD2* 140, the mean motion should be on the "fast" epicycle. In Āryabhaṭa's model, it is the motion of the "fast" apogee that occurs on the "fast" epicycle.

This statement could be anticipating Nīlakaṇṭha who replaced the "fast" apogee with the mean position for Mercury and Venus in his *Tantrasaṅgraha* (Ramasubramanian and Sriram (2011, p.508-509)), but other than this succinct passage in *GD2* 139, Parameśvara's explanations agree with the *Āryabhaṭīya*.

In *GD2* 139cd, Parameśvara remarks that the true motion on the zodiac is "inferred (*anumīyate*)". He might also here be a precursor to Nīlakaṇṭha who discusses fundamental topics in astronomy using philosophical concepts, such as inference (*anumāna*) in his *Jyotirmīmāṃsā* (K. V. Sarma (1977a)). The verb *anu-mā* itself does appear in previous treatises, such as in *Ābh* 3.11cd:

> This time which has neither beginning nor end is inferred from planets and stars in the field.[17]

Yet Parameśvara's commentary accentuates the nuance of "infer":

> Time which has neither beginning nor end is inferred from the planets and the stars too situated on the field, the sphere. This is what is stated: Even though time has neither

[17] *kālo 'yam anādyanto grahabhair anumīyate kṣetre //3.11//* (Kern (1874, p. 59))

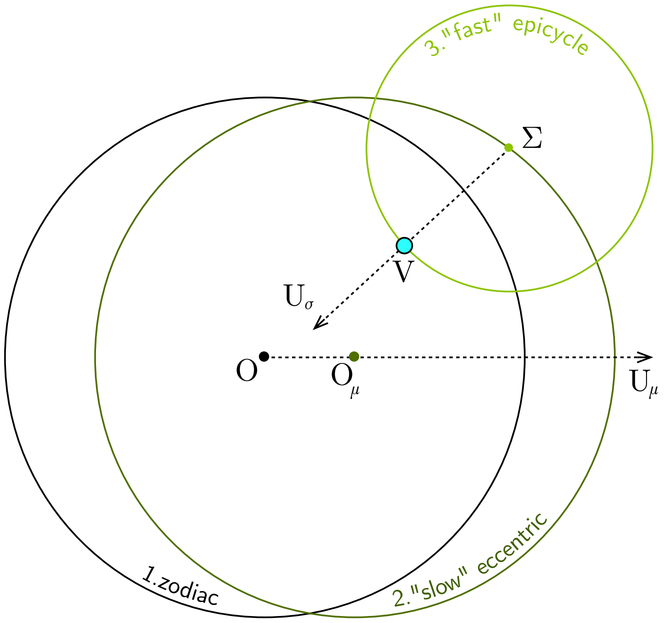



Figure 9.10: Three circles for Mercury and Venus

> beginning nor end, it is separated in the form of *kalpa*, *manu*, *yuga*, year, month, day and so forth with conditioning existences (*upādhi-bhūta*) situated in the stellar sphere.[18]

*upādhi* is another word that appears frequently in philosophical arguments, such as "anything which may be taken for or has the mere name or appearance of another thing, appearance, phantom, disguise (Monier-Williams (1899))" or as "a 'condition' which must be supplied to restrict a too general term (Cowell and Gough (1882, p. 275))" in logics. I assume that such logical concepts underlies the word "infer" in *GD2* 139. The term also contrasts with the "observed / directly perceived (*sākṣāt*)" true planet mentioned in *GD2* 145.

*GD2* 140cd refers to the *kṣepa* (inclination/latitude) among the three circles. Such reference to the inclination of the set of rings collectively is very rare[19]. Yet the expression is very ambiguous, and the meaning can change depending on how we interpret the word *kṣepa*. If we take it in the sense of "inclination", it could either mean that all three circles are inclined in the same way or that their inclinations are different but interlocked (*yugapad*). It is impossible to reproduce the rule in *GD2* 128 if the circles are uniformly inclined. The latter interpretation does not fit with the Sanskrit where *kṣepa* is in the singular. My interpretation is that *kṣepa* means "deviation" and that the configuration of the three circles produce a single value for the deviation. We shall examine this configuration in detail in section 9.10. Nonetheless, Parameśvara's true intention is still an open question.

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[18] *anādyantaḥ kālaḥ kṣetre gole sthitair grahair bhair apy anumīyate / etad uktaṃ bhavati / yady apy anādyantaḥ kālas tathāpi jyotiścakrasthair upādhibhūtaiḥ kalpamanvantarayugavarṣamāsadivasādirūpeṇa paricchidyate iti //* (Kern (1874, pp. 59-60))

[19] Nīlakaṇṭha gives a detailed description of how each circle should be inclined in his commentary on *Ābh* 4.3 (Pillai (1957b, pp. 13-14)). However his configuration of orbits are different from previous theories (Ramasubramanian and Sriram (2011, pp. 511-512)).

## 9.8 Corrected positions of planets (*GD2* 141-148)

*GD2* 141-145 explain how the corrections of planetary longitudes can be displayed in these three circles. This is done by systematically drawing lines.

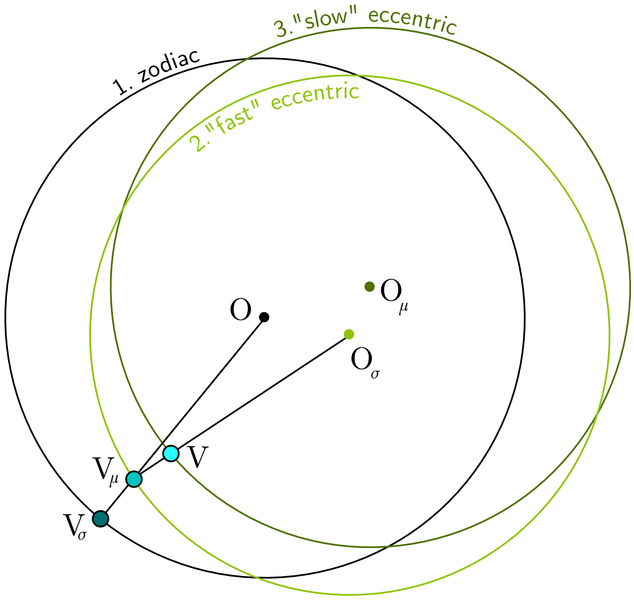



Figure 9.11: Corrected positions of Mars, Jupiter and Saturn.

In the case of Mars, Jupiter and Saturn (figure 9.11), we start with the mean planet V on the last circle, i.e. "slow" eccentric. Then we draw a line between the center $O_\sigma$ of the second circle and V; its length is the "slow" radial distance (*GD2* 141). The intersection of line $O_\sigma V$ with the circumference of the second circle is the "slow" corrected planet $V_\mu$ (*GD2* 142). The difference in longitude between V and $V_\mu$ corresponds to the "slow" equation.

Another line is drawn between $V_\mu$ and the center of the first circle O. The length of $OV_\mu$ is the "fast" radial distance, and its intersection with the circumference of the first circle is the "fast" corrected planet $V_\sigma$ (*GD2* 143-144). This corresponds to applying a "fast" equation to the "slow" corrected planet.

The procedure is almost the same for Mercury and Venus (figure 9.12). The length of the first segment $O_\mu V$ drawn between the center of the "slow" eccentric circle $O_\mu$ and the true planet V is the "fast" radial distance (*GD2* 141) and its intersection with the circumference of the second circle is the "fast" corrected planet $V_\sigma$ (*GD2* 142). Its distance from the center of the zodiac O is the "slow" radial distance and the intersection of $OV_\sigma$ with the zodiac is the "slow" corrected planet $V_\mu$. Here, the sequence is equivalent to applying a "fast" equation to the mean planet, followed by a "slow" equation.

As a result, in both cases, we shall obtain the position of the planet which is corrected once for each of the "slow" and "fast" apogees. This is probably what is mentioned in *GD2* 145ab by saying that the true planet (*sphuṭa khaga*) is obtained with a pair of corrections (*sphuṭayugala*). The word *dvyuccānāṃ* ("of the two apogees") is in the plural and not in the dual, which suggests that Parameśvara is explaining the situation for all planets collectively. Then in *GD2* 145cd Parameśvara remarks that the "true planet" thus computed is different with its "observed

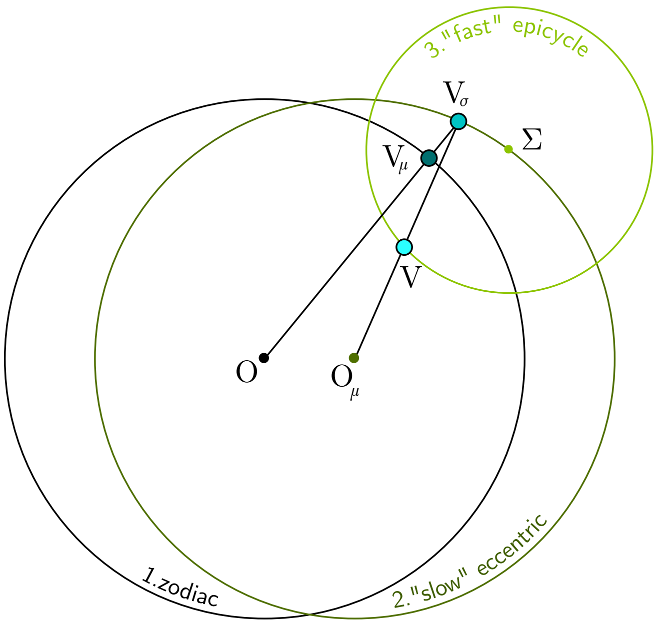
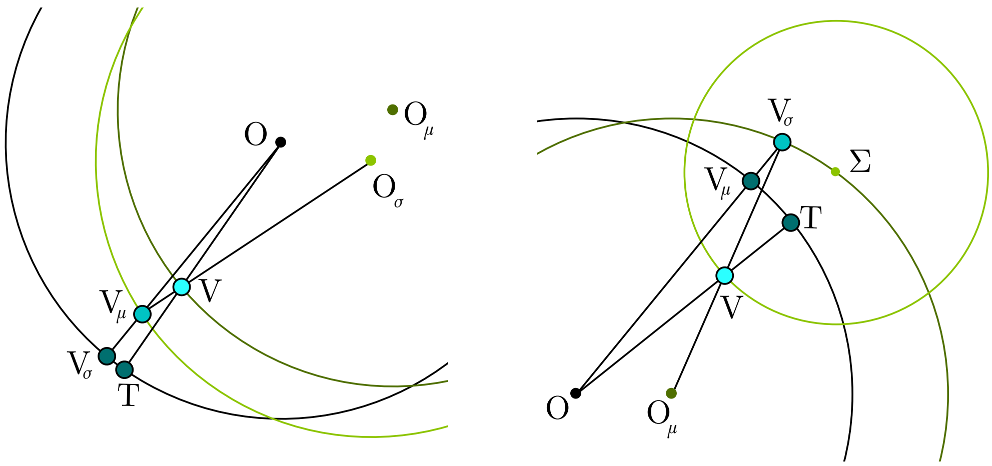



Figure 9.12: Corrected positions of Mercury and Venus.

position", literally "before one's eyes (*sākṣa*)". Indeed, the accurate longitude of a planet cannot be obtained by simply applying the two equations one by one (appendix C.5).




(a) Mars, Jupiter and Saturn

(b) Mercury and Venus

Figure 9.13: The observed true position T of planets.

The observed position T is the intersection of line OV and the first circle (figure 9.13), as explained in *GD2* 148. The difference with the once-computed position is $\widehat{TV_\sigma}$ for Mars, Jupiter and Saturn, and $\widehat{TV_\mu}$ for Mercury and Venus. *GD2* 146 explains where this difference comes from.

The "fast" equation for computing $V_\sigma$ of Mars, Jupiter and Saturn was erroneous because it assumed that the planet was on $V_\mu$ and not on V. $V_\mu$ is at a distance of the Radius from $O_\sigma$, whereas V is at the "slow" radial distance.

With Mercury and Venus, the error is in the "slow" equation which assumes that the planet is on $V_\sigma$, separated by the Radius from $O_\mu$, instead of V, separated by the "fast" radial distance.

Thus, astronomical texts such as the *Āryabhaṭīya* give additional steps where half of the equations are applied for reducing this difference (appendix C.5). *GD2* 147 briefly refers to this procedure, including the fact that the steps for Mercury and Venus are different from those for Mars, Jupiter and Saturn.

## 9.9 Inclined circle and the configuration of circles

Parameśvara turns back to the corrections for the deviations in *GD2* 149-150; they can be read as reasonings for *GD2* 128. Before looking at these verses, let us first consider the position of the inclined circle among the set of three circles described in *GD2* 135-148. Apart from the brief statement in *GD2* 140, Parameśvara says nothing about the three dimensional configuration. The following is a hypothetical model that may explain the statements in *GD2*, but Parameśvara's actual conception is yet to be examined.

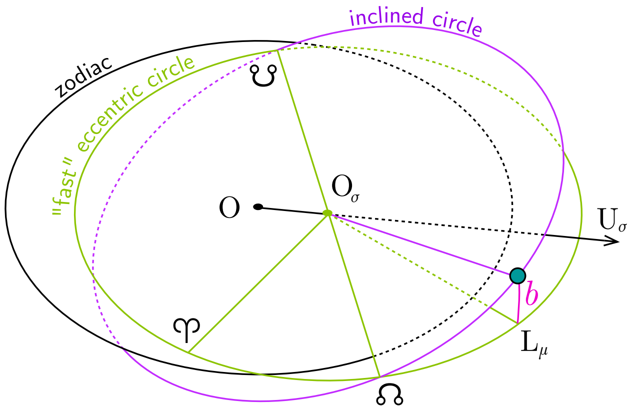



Figure 9.14: Inclined circle of Mars, Jupiter and Saturn

In the case of Mars, Jupiter and Saturn, the argument of the deviation involves the longitude of the "slow" corrected planet $L_\mu$, as mentioned in *GD2* 127ab. $L_\mu$ is located on the "fast" eccentric circle, which suggests that the inclined circle should be located on that circle (figure 9.14). To be precise, there are two possibilities: One is that there is an independent inclined circle connected to the "fast" eccentric circle at the two nodes, and the other is that the eccentric circle itself is inclined. Our diagram depicts the first situation, but Parameśvara's expressions allow both possibilities. The same can be said for Mercury and Venus explained later.

Another problem with Mars, Jupiter and Saturn is the position of the "slow" eccentric circle. The "fast" eccentric circle is the second circle in the configuration, but *GD2* 149 suggests that the given deviation $b$ should be found on the end (i.e. circumference) of the last circle, which is the "slow" eccentric circle. My interpretation is as follows: The deviation $b$ is computed according to the longitude $L_\mu$ on the "fast" eccentric circle, but the actual locus of this deviation is on the "slow" eccentric circle, at the mean longitude L (figure 9.15). It is also questionable whether the

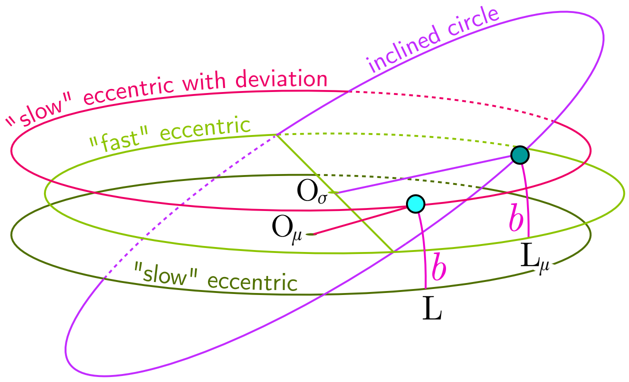



Figure 9.15: “Slow” eccentric circle and planet with deviation $b$.

“slow” eccentric circle should be considered as elevated in accordance with this deviation, but in my diagrams I shall keep it in the same plane with the zodiac and the “fast” eccentric circle.

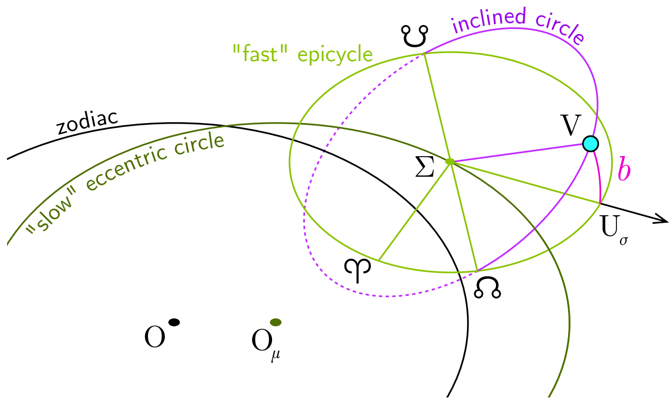



Figure 9.16: Inclined circle of Mercury and Venus

Meanwhile, the longitude of the “fast” apogee $U_\sigma$ is taken for the argument for Mercury and Venus according to *GD2* 127ab. The direction of $U_\sigma$ gives the position of the planet V in the “fast” epicycle, and therefore the inclined circle should also be situated there (figure 9.16). Since the “fast” eccentric epicycle is the last of the three circles in the case of Mercury and Venus, the given deviation $b$ as stated in *GD2* 149 is the deviation in this inclined circle. The “slow” eccentric circle and the zodiac stay on the plane of the ecliptic.

## 9.10 Grounding the rules for the deviation (*GD2* 149-150)

Two Rules of Three concerning the deviation are given in *GD2* 149 and 150. *GD2* 149 starts from the deviation $b$ on the last circle which we have discussed in the previous section, and gives the once-corrected deviation on the second circle. The use of “middle (*madhya*)” to refer to the second circle (which is in the middle of the three circles) in this verse is peculiar. *GD2* 150 produces the true deviation $b_{\mathrm{T}}$ (which is also the celestial latitude $\beta$) on the zodiac from the once-corrected deviation.

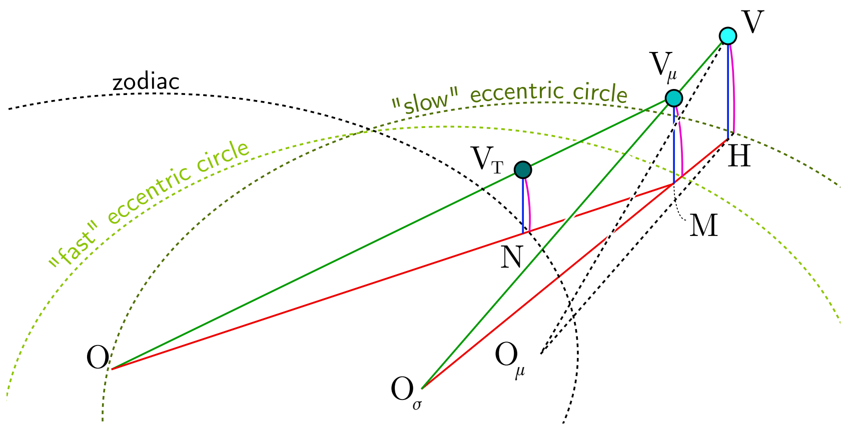



Figure 9.17: Computing the latitude of Mars, Jupiter and Saturn

In the case of Mars, Jupiter and Saturn (figure 9.17), the Sine of the given deviation $\mathrm{Sin}\, b$ is VH on the "slow" eccentric circle. $\mathrm{V}_\mu\mathrm{M} = \mathrm{Sin}\, b_\mu$ is the Sine of deviation of the "slow" corrected planet on the "fast" eccentric circle. Note that this is not the deviation on the inclined circle (figure 9.14); that deviation $b$ has been moved to the "slow" eccentric circle figure 9.15), and this time we are correcting this deviation for the "slow" radial distance $\mathrm{O}_\sigma\mathrm{V}_\mu$. Finally, $\mathrm{V}_{\mathrm{T}}\mathrm{N} = \mathrm{Sin}\, b_{\mathrm{T}}$ is the true deviation on the zodiac. All three circles are assumed to be on the plane of the ecliptic in this diagram.

We have already seen in section 9.4 that $\triangle\mathrm{O}_\sigma\mathrm{HV} \sim \triangle\mathrm{O}_\sigma\mathrm{MV}_\mu$, and therefore:

$$\begin{aligned} \mathrm{V}_\mu\mathrm{M} &= \frac{\mathrm{VH} \cdot \mathrm{V}_\mu\mathrm{O}_\sigma}{\mathrm{VO}_\sigma} \\ \mathrm{Sin}\, b_\mu &= \frac{\mathrm{Sin}\, b \cdot R}{\mathcal{R}_\mu} \end{aligned} \tag{9.10}$$

which corresponds to the first Rule of Three (*GD2* 149). Likewise, $\triangle\mathrm{OMV}_\mu \sim \triangle\mathrm{ONV}_{\mathrm{T}}$ and thus we have the second Rule of Three (*GD2* 150):

$$\begin{aligned} \mathrm{V}_{\mathrm{T}}\mathrm{N} &= \frac{\mathrm{V}_\mu\mathrm{M} \cdot \mathrm{V}_{\mathrm{T}}\mathrm{O}}{\mathrm{V}_\mu\mathrm{O}} \\ \mathrm{Sin}\, b_{\mathrm{T}} &= \frac{\mathrm{Sin}\, b_\mu R}{\mathcal{R}_\sigma} \end{aligned} \tag{9.11}$$

From formulas 9.10 and 9.11, we obtain the computation in *GD2* 128 (formula 9.8).

As for Mercury and Venus (figure 9.18), $\mathrm{VH} = \mathrm{Sin}\, b$ is the Sine of the given deviation on the "fast" epicycle, $\mathrm{V}_\sigma\mathrm{M} = \mathrm{Sin}\, b_\sigma$ is the Sine of the "fast" corrected deviation on the"slow" eccentric circle and $\mathrm{V}_{\mathrm{T}}\mathrm{N} = \mathrm{Sin}\, b_{\mathrm{T}}$ is the true deviation on the zodiac.

Since $\triangle\mathrm{O}_\mu\mathrm{HV} \sim \triangle\mathrm{O}_\mu\mathrm{MV}_\sigma$, we obtain the first Rule of Three, and

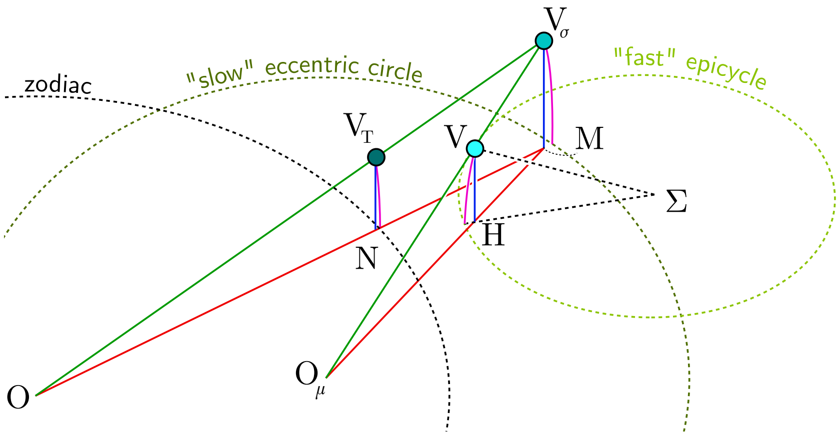



Figure 9.18: Computing the latitude of Mercury and Venus

$$\mathrm{V}_\sigma\mathrm{M} = \frac{\mathrm{VH} \cdot \mathrm{V}_\sigma\mathrm{O}_\mu}{\mathrm{VO}_\mu}$$

$$\mathrm{Sin}\, b_\sigma = \frac{\mathrm{Sin}\, b \cdot R}{\mathcal{R}_\sigma} \tag{9.12}$$

Next, we use the second Rule of Three from $\triangle\mathrm{OMV}_\sigma \sim \triangle\mathrm{ONV}_\mathrm{T}$:

$$\mathrm{V}_\mathrm{T}\mathrm{N} = \frac{\mathrm{V}_\sigma\mathrm{M} \cdot \mathrm{V}_\mathrm{T}\mathrm{O}}{\mathrm{V}_\sigma\mathrm{O}}$$

$$b_\mathrm{T} = \frac{b_\sigma R}{\mathcal{R}_\mu} \tag{9.13}$$

Here again, from formulas 9.10 and 9.11, we obtain formula 9.8. As a result, the rule in *GD2* 128 can be applied to all five planets.

## 9.11 Computation for a more accurate latitude? (*GD2* 151)

*GD2* 151 introduces another opinion, which argues that the twofold correction for the deviation be applied in the same manner with the two corrections for the longitude. Indeed, the rule in *GD2* 128 is incorrect because multiplying the two radial distances does not yield the actual distance of the planet from the Earth (see appendix C.6). As for what is being suggested in *GD2* 151 itself, it is probably a procedure similar to those of the true longitude correction with half the "slow" and "fast" equations. However I could not find any other text which derives the planetary latitude with an extra "half-value" computation procedure.

The peculiar method used by Bhāskara I in *MBh* 7.28cd-33 (T. Kuppanna Sastri (1957, pp. 382-383)), which he attributes to the *Ārdharātrika* (midnight-reckoning) system of Āryabhaṭa, could be related to Parameśvara's rule. In this method, two sets of nodes, the "slow" node and "fast" node are defined for each planet, and the deviation caused by each of them is to be combined to obtain the latitude. But nevertheless, Bhāskara I only combines them by simply adding or subtracting. Furthermore, Parameśvara comments here on deviations established with two “radial distances” and not “nodes”.

## 9.12 Deviation of the moon (*GD2* 152)

*GD2* 152 mentions that the sun and the moon only have two circles, the zodiac and the “slow” eccentric circle. We can apply the explanations for the other planets in this case; the mean positions of the sun and the moon revolve on their “slow” eccentric circles, and their “slow” corrected position, which is also the true position, on the zodiac.

The argument for computing the deviation of the moon concerned its true position, and therefore its inclined circle is on the zodiac.

# 10 Celestial latitude and visibility methods (*GD2* 153-194)

The celestial latitude of a planet as seen from the Earth has been established in the previous step, and the next goal in *GD2* is to compute the "visibility equation (*dṛkphala*)". This is a value added to or subtracted from the longitude of a planet with a given celestial latitude to obtain its corresponding ascendant (*udayalagna*) or descendant (*astalagna*), i.e. the point on the ecliptic which rises or sets at the same moment as the planet. Parameśvara mentions nothing about the purpose of this computation. One possible application is to find whether a planet is visible above the horizon when it is close to the sun.

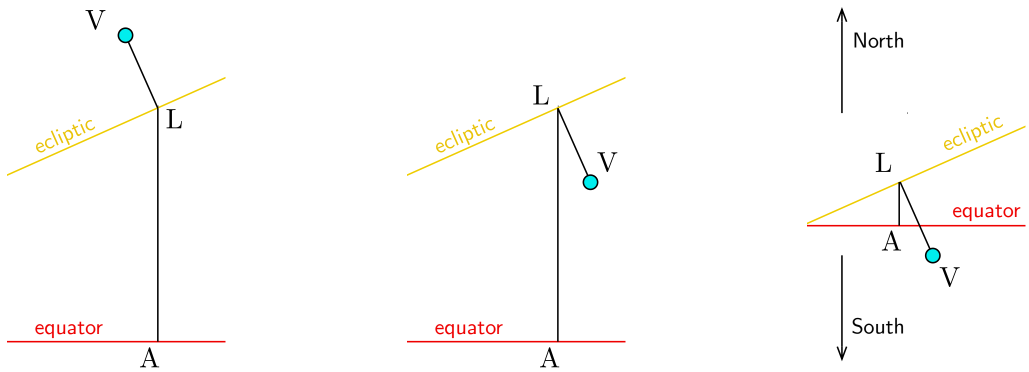
The sets of computations involved in computing the visibility equation and applying it to the longitude is called a "visibility method". Parameśvara demonstrates two different approaches. First he uses a pair of equations: the equation corresponding to the "visibility method for the 'course' (*āyanaṃ dṛkkarma*)" (*GD2* 169-174) and the equation corresponding to the "visibility method for the geographic latitude (*ākṣaṃ dṛkkarma*)" (*GD2* 175-178). The second is a unified method where only one visibility equation is used (*GD2* 192-194).

Many new arcs and segments are introduced to explain these methods. Among them, the elevation (*unnati*) or depression (*avanati*) of the planet's latitude is most crucial for the visibility equations. This is first described in *GD2* 156-157, 166-168 as the distance of the planet with a latitude above or below the six o'clock circle when the corresponding longitude on the ecliptic is on the six o'clock circle. This is a parameter in the visibility equation for the "course". Later in *GD2* 190-191, the elevation or depression is restated as the distance from the horizon, which is then used for the unified visibility equation. Other new concepts include the composition of the declination and the celestial latitude (*GD2* 153, 163-164), the ecliptic pole and its elevation (*GD2* 154-155, 158-162, 189), the points of sight-deviation (*dṛkkṣepa*) and midheaven (*madhya*) on the ecliptic and their gnomons (*GD2* 179-188). The point of sight-deviation appears again in the section on parallaxes (*GD2* 248-276) where it plays a central role in finding the longitudinal and latitudinal parallaxes.

## 10.1 Corrected declination (*GD2* 153)

Previous verses in *GD2* have only dealt with the declination $\delta$ of a point on the ecliptic, which is simply its distance from the celestial equator. The definition is not so simple for the declination of a planet which is separated from the ecliptic by its latitude. Unlike modern astronomy, where the declination is merely part of the equatorial coordinate system, a "declination" of a planet V in Sanskrit sources involves its corresponding longitude on the ecliptic L (figure 10.1). This may be related to the importance of the celestial longitude over the latitude which is visible from the fact that words for "planet" can signify its celestial longitude (section 6.2). Parameśvara explains two ways of combining the latitude $\beta$ with the declination of its corresponding point on the ecliptic $\delta$; he calls them the "corrected (*sphuṭa*)" declination and the "true (*spaṣṭa*)" declination, respectively. The corrected declination, which is also referred to as the planet's "own declination (*svakrānti*)", is given in *GD2* 153, while the true declination comes after a series of computations in *GD2* 164.

The corrected declination is simply the sum or difference of $\beta$ and $\delta$, ignoring the fact that they are not in one straight line (figure 10.1). Parameśvara states explicitly two cases, where the latitude $\overset{\frown}{\mathrm{LV}} = \beta$ and declination $\overset{\frown}{\mathrm{AL}} = \delta$ are in the same direction (figure 10.1(a)) or opposite directions (figure 10.1(b)). He does not specify that the latitude should be subtracted from the




(a) Same direction (b) Opposite direction (c) Larger latitude

Figure 10.1: Computing the "corrected" declination

declination, and therefore the statement could allow cases where the latitude is larger than the declination. For example, in figure 10.1(c), the northward and smaller declination is subtracted from the southward and larger latitude, resulting in a southward corrected declination. Thus the corrected declination $\delta^*$ in different cases is as follows.

$$\delta^* = \begin{cases} \delta + \beta & \text{(a) Same direction} \\ \delta - \beta & \text{(b) Opposite direction} \\ \beta - \delta & \text{(c) Opposite and latitude is larger} \end{cases} \tag{10.1}$$

Parameśvara comments nothing on the exactness or validity of this corrected declination, nor does he even refer to its usage. The "true declination" defined later in *GD2* 164 is essentially a refinement of this approximative method, but Parameśvara makes no comparison between the two. I assume that Parameśvara only uses the true declination in his visibility methods, and that the corrected declination is mentioned only because his predecessors such as Bhāskara I have used it.

*MBh* 6.8 states that the sum or difference of the moon's latitude and declination is used for computing its ascensional difference. In his commentary *Karmadīpikā* (Kale (1945, p. 70)), Parameśvara only paraphrases this verse and gives no further information. He says almost nothing in his super-commentary *Siddhāntadīpikā* (T. Kuppanna Sastri, 1957, p. 344). *Sūryasiddhānta* 2.57 gives the same rule, calling the result a "true (*spaṣṭa*) declination ". Although Parameśvara adds no further information in his commentary (Shukla, 1957, p. 36), he constantly paraphrases it as "corrected (*sphuṭa*)", suggesting the possibility that he may have had the differentiation in his mind. Neither the *Mahābhāskarīya*, *Sūryasiddhānta* nor their commentaries by Parameśvara refer to the true declination as defined in *GD2* 164.

## 10.2 Ecliptic poles (*GD2* 154-155)

The ecliptic poles in modern terminology are two points in the stellar sphere separated from the ecliptic by 90 degrees. They are first introduced in *GD2* 154 as *vedha*s, which I have translated as "hole". The Sanskrit word is derived from the verb root *vyadh* "pierce", and it suggests the usage of an armillary sphere as an object (figure 10.2). Two holes (K and K′) are made in the

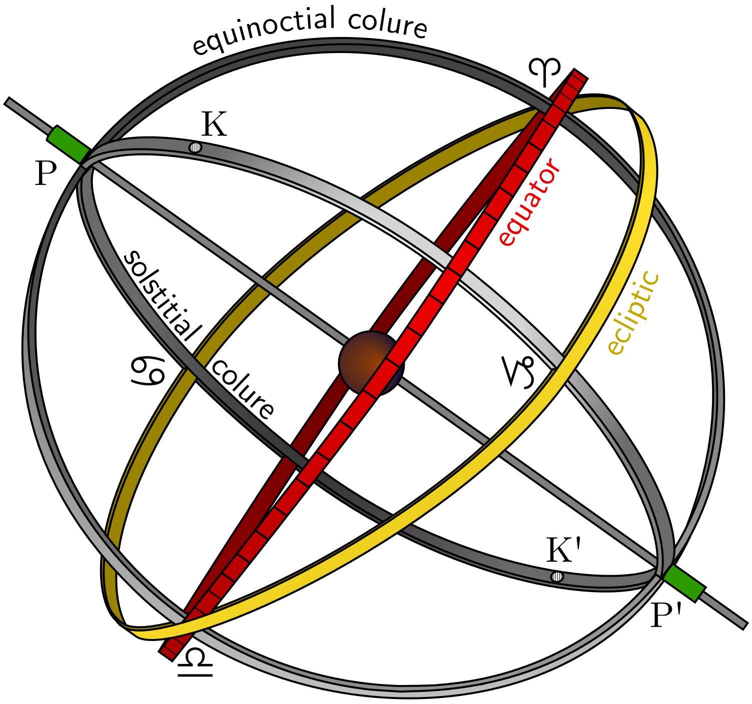



Figure 10.2: The ecliptic poles K and K′ as holes in the armillary sphere

solstitial colure, 90 degrees from its conjunction with the ecliptic (which are the solstitial points, ♋ and ♑). In *GD2* 155, the separation of the ecliptic from the ecliptic poles is compared with the celestial equator which is separated from celestial poles (P and P′) by 90 degrees (a quarter of a circle). Here the celestial pole is described as a "cross (*svastika*)" [of the solstitial and equinoctial colures] as in *GD2* 5 and not "pole star (*dhruva*)" as in *GD2* 35 etc. "Cross" suggests an armillary sphere while "pole star" implies the viewpoint of an observer on the Earth.

Parameśvara uses the expression "three signs" in *GD2* 154 for 90 degrees. In this case, a "sign (*rāśi*)" is a measurement of arc along a great circle.

These two holes are named *rāśikūṭa*, "summit of signs" in the same verse. Parameśvara adds that these "summit of signs" are named so because they are the conjunction of all signs. This can be understood by dividing the stellar sphere into twelve sections as in figure 10.3. Here we interpret that a "zodiacal sign" is not only a division of the ecliptic but of the entire stellar sphere[1]. This is necessary for defining the sign or longitude of a planet with a latitude. Since here the *rāśi* or its synonym no more refers to the measurement unit of a "sign" but to segments of the stellar sphere, hereafter I shall use "ecliptic pole" as a translation of *rāśikūṭa* and its synonyms.

[1]Parameśvara himself does not explicitly refer to this point, but we can find the idea of measurement units as divisions of the stellar sphere in other texts. For example, *Ābh* 3.2 states that the field (*kṣetra*) is divided in the same way that the time is divided. It further states that the units begin with *bhagaṇa*. Practically this term is translated as "revolution" but literally it is a "multitude/group of stars", which gives us the impression that it refers to the entire sky and not only the zodiac. Bhāskara I paraphrases "field" with "stellar sphere (*bhagola*)" in his commentary (Shukla (1976, p. 176)). He also refers to the units, starting with "twelve signs are a 'revolution' (*dvādaśarāśayo bhagaṇaḥ*)".

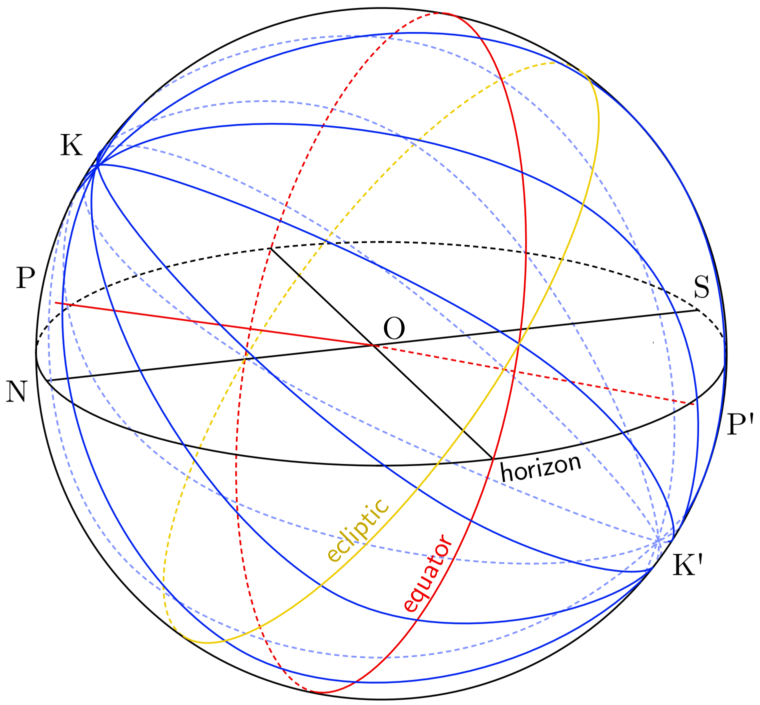



Figure 10.3: The ecliptic poles K and K′ and the borders of zodiacal signs. N and S are due north and south on the horizon, P and P′ are the celestial poles.

## 10.3 The direction of the celestial latitude (*GD2* 156-157)

*GD2* 156 refers to the direction of the celestial latitude. Its nuance depends on how we interpret the phrase "the arc minute where a planet is situated (*kheṭasthakalā*)". One possibility is to take it as the measurement of the latitude $\widehat{\mathrm{LV}}$, interpreting the word planet (*kheṭa*) as the actual celestial body V (figure 10.4). But in *GD2* 156cd, Parameśvara uses the expression "latitude" (*kṣepas*, nominative) "in the arc minute where a planet is situated" (*kheṭasthaliptikāyāṃ*, locative) which does not make sense if we consider that "the arc minute where a planet is situated" is the latitude itself.

My suggestion is that the "arc minute" should be that of the celestial longitude. The word *kheṭa* may indicate the body itself (V) or the corresponding point on the ecliptic (L). In this case, the arc minute "extending south and north" is a reference to $\widehat{\mathrm{KLK'}}$ (or very narrow zone with a breadth of one minute) which is the "line of longitude" with the two ecliptic poles at its end. We have already seen that the signs can be understood as zones extending towards the "summit of signs (*rāśikūṭa*) = ecliptic pole". I could not find other cases in Sanskrit texts where a line of longitude is expressed in this way, and my interpretation is still a hypothesis that needs to be examined. However it does explain the wordings in *GD2* 156cd well. The latitude $\widehat{\mathrm{VL}}$ can indeed be in the line of longitude $\widehat{\mathrm{KLK'}}$.

The word *apama* in *GD2* 156cd could be either "ecliptic" or "declination". The difficulty with "ecliptic" is the genitive *tasya* (its) added to this word. Without *tasya*, we could interpret that *GD2* 156cd states that the distance of the celestial point V from the ecliptic circle is the

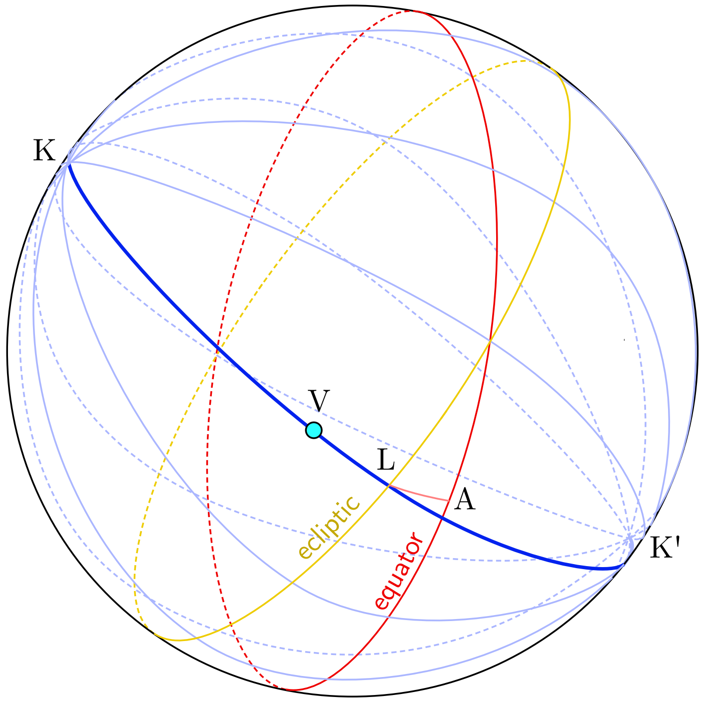
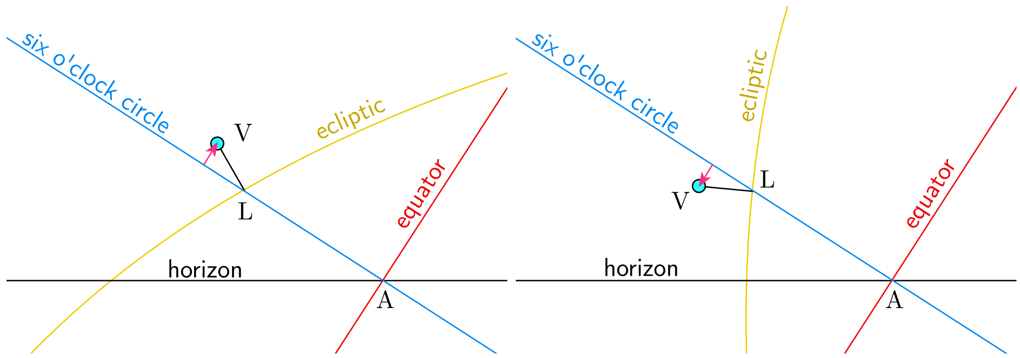



Figure 10.4: The latitude $\widehat{\mathrm{LV}}$ measured along the line of longitude.

latitude. With *tasya*, the nuance would be "its [point on] the ecliptic" (referring to L), but we have no other instance where the word ecliptic is used for signifying a single point. Therefore I have adopted "declination" for *apama*. This still leaves some ambiguity: it may signify "point L which is separated from the equator by the declination", or it could be "the arc of declination $\widehat{\mathrm{AL}}$". In the latter interpretation, the verb *yāti* could mean "go away from"; the arc of latitude is not aligned with the arc of declination and "goes away" from it. This makes a good link with *GD2* 157, but there is still room for discussion.

*GD2* 157ab describes a situation where the point on the ecliptic L corresponding to the planet's longitude is on the six o'clock circle (figure 10.5). When the arc of latitude $\widehat{\mathrm{LV}}$ is not aligned with the arc of declination $\widehat{\mathrm{AL}}$, the planet V at the end of the latitude goes above or below the six o'clock circle. It is remarkable that the word "latitude" indicates the position of the planet itself. This becomes more distinct later when the distance of V from the six o'clock circle is given the term "elevation / depression of latitude" (*GD2* 166). The expression "in accordance with the ecliptic pole (*bhakūṭavaśāt*)" is probably a reference to the "elevation of ecliptic pole" described in *GD2* 158-160. The elevation or depression of latitude is computed from the elevation of ecliptic pole.

In *GD2* 157cd, Parameśvara turns back to the concept of the "corrected declination" given in *GD2* 153. As we have seen in section 10.1, this is an approximate method because the declination and celestial latitude are not in a straight line. Parameśvara refers to "some (*kecit*)" who point this out. This might be Bhāskara II or his followers, as *Siddhāntaśiromaṇi* is the only




(a) Above six o'clock circle.

(b) Below six o'clock circle.

Figure 10.5: The position of a planet at the tip of latitude $\widehat{LV}$

major treatise before Parameśvara that criticizes the approximation[2]. In the *Golādhyāya* of the *Siddhāntaśiromaṇi* he states:

> Brahmagupta and others did not make the correction [to the latitude] because the difference is small. (9.11 ab)[3]
> Those who think that the latitude is on the line of the declination are stupid. (9.13ab)[4]

Bhāskara II combines the component of the latitude which is aligned with the declination to obtain the "true declination"[5]. Here in *GD2*, this method is introduced and explained later in *GD2* 163-164, but Parameśvara still keeps the older methods and introduces them first (*GD2* 153). Furthermore, he does not even evoke this criticism in his commentaries on the *Mahābhāskarīya* and *Sūryasiddhānta*. Thus the influence from Bhāskara II on this point is debatable.

## 10.4 Elevation of ecliptic pole (*GD2* 158-161)

As was the case with a planet with a latitude, the ecliptic pole can also be above or below the six o'clock circle. This depends on the point where the ecliptic intersects the six o'clock circle (figure 10.6). When it is the summer solstice (figure 10.6(b)) or winter solstice (figure 10.6(d)), the northern ecliptic pole K is on the six o'clock circle. Otherwise it is not. The distance with the six o'clock circle, called the "elevation (*unnati*)[6] of the ecliptic pole", depends on the "upright (*koṭi*)" of the point on the six o'clock circle, i.e. its distance along the ecliptic from a solstitial point (c.f. *GD2* 89, section 7.1). The elevation is largest when the "upright" is largest, that is

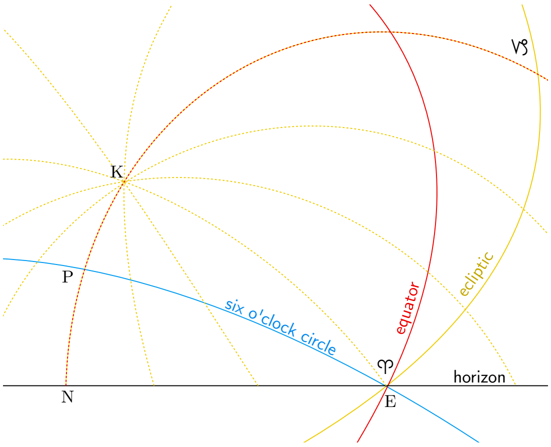
[2] *MBh* 5.21 (T. Kuppanna Sastri (1957, p. 274)), *MBh* 6.8 (T. Kuppanna Sastri (ibid., p. 344)), *Brāhmasphuṭasiddhānta* 7.5 (Dvivedī (1902, p. 101), *Sūryasiddhānta* 2.57 (Shukla (1957, p. 36)), *Śiṣyadhīvṛddhidatantra* 9.2 (Chatterjee (1981, 1, p. 132)) and *Siddhāntaśekhara* 10.7 (Miśra (1932, p. 439)) simply add the arcs of the declination and celestial latitude.

[3] *brahmaguptādibhiḥ svalpāntaratvān na kṛtaḥ sphuṭaḥ* / (Chaturvedi (1981, p. 434))

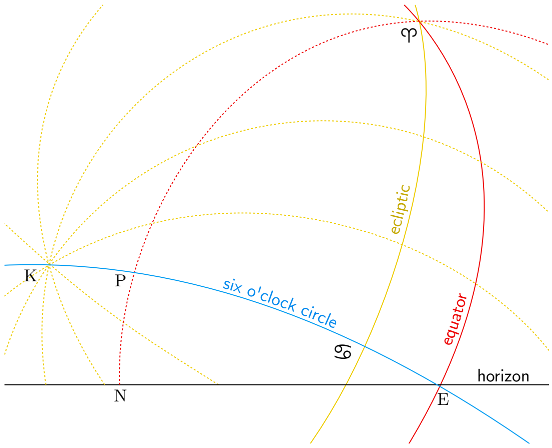
[4] *krāntisūtre śaraṃ kecin manyate te kubuddhayaḥ* / (Chaturvedi (ibid.))

[5] *Siddhāntaśiromaṇi Grahagaṇitādhyāya* 7.2 and 7.13 (Chaturvedi (ibid., pp. 276-278,282))

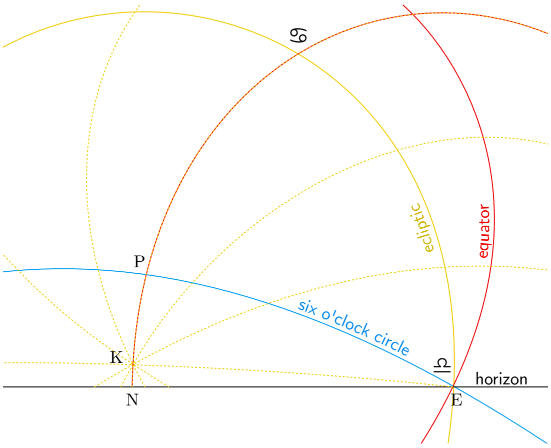
[6] As we will see in the following verses, this term refers to the Sine corresponding to the arc distance between the ecliptic pole and the six o'clock circle. The same word "elevation" is used even when the northern ecliptic pole is below the six o'clock circle.



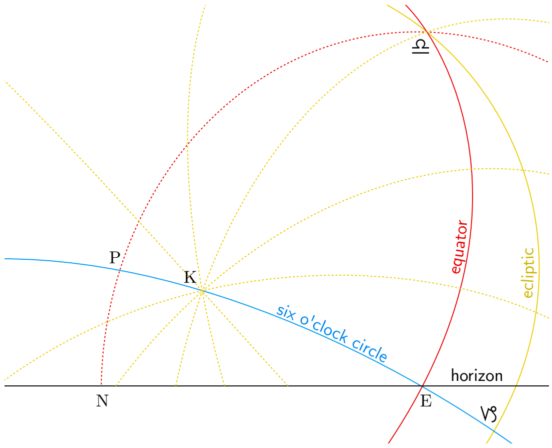

(a) Vernal equinox ♈




(b) Summer solstice ♋




(c) Autumn equinox ♎




(d) Winter solstice ♑

Figure 10.6: The ecliptic pole K when the equinoxes and solstices are on the six o'clock circle (from the viewpoint of an observer on the Earth).

when an equinoctial point is on the six o'clock circle. K is above at its greatest distance when the vernal equinox is on the six o'clock circle (figure 10.6(a)) and below when it is the autumn equinox (figure 10.6(c)). We will see later that the elevation of ecliptic pole is used to find the elevation or depression of the planet itself from the six o'clock circle, which in turn is crucial for computing the visibility equation.

The word *lagna* is usually translated "ascendant" and indicates the point where the ecliptic intersects the horizon. However, under this interpretation the rule above is invalid when the observer is on a location with geographic latitude (figure 10.7). Most probably, Parameśvara is describing the situation on the terrestrial equator where horizon and six o'clock circle overlap. We may assume that this premise is applied to *GD2* 158 too. Nonetheless, we cannot rule out the possibility that *lagna*, literally "touch", is used in a wider sense. For example, *lagna* in *GD2* 179 indicates a point on the ecliptic remote from the horizon. Therefore, I have translated *lagna*

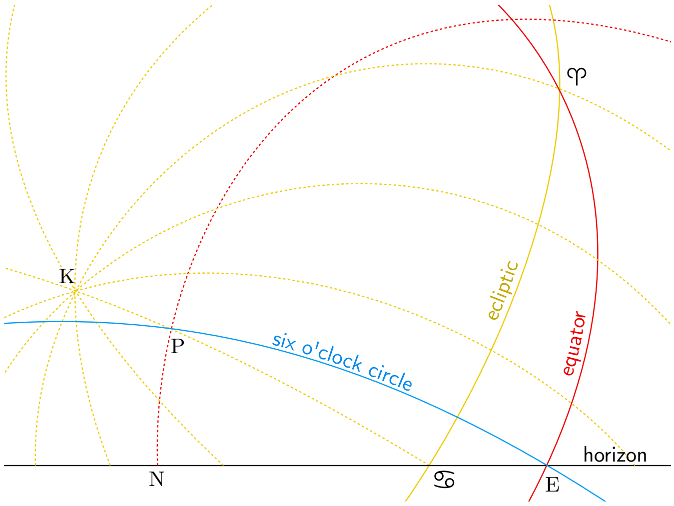
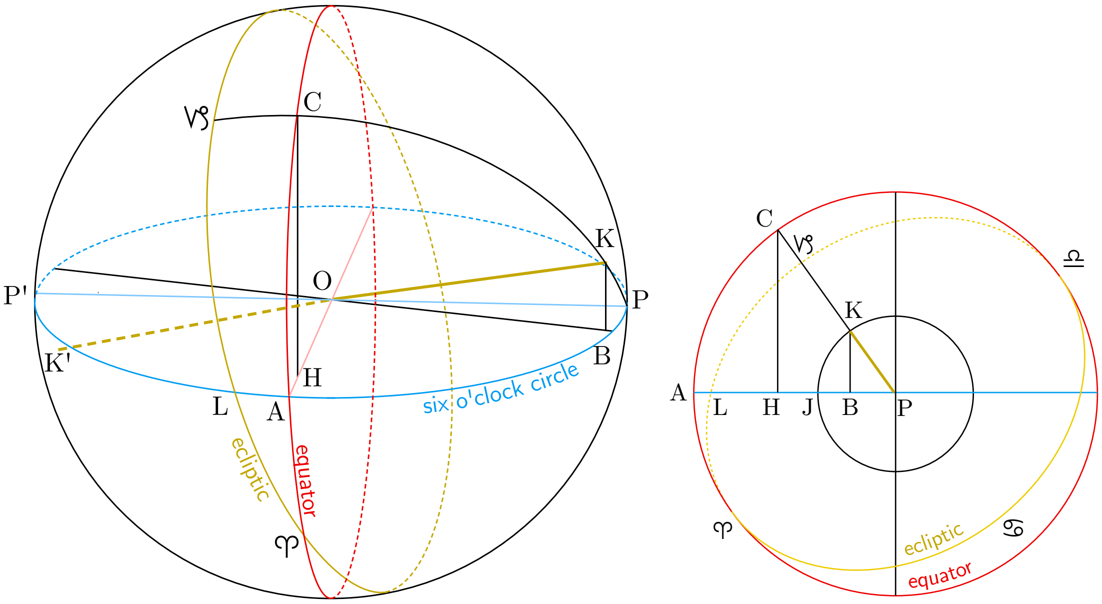



Figure 10.7: The moment when the summer solstice ♋ is the ascendant. The ecliptic pole is not on the six o'clock circle.

in *GD2* 158 as "adhering [to the six o'clock circle]".

The rule for computing the value of this elevation is given in *GD2* 159. Four segments, all of which are Sines of the great circle, are involved. First is the [Sine of] greatest declination $\operatorname{Sin}\varepsilon$, which is expressed in words and not by its actual value ($\operatorname{Sin}24° = 1397$) as in *GD2* 73. The second is the Sine (*guṇa*) corresponding to the difference in time for a planet on the six o'clock circle to rise and a solstitial point to rise $\operatorname{Sin}\bar{\alpha}$. The time difference (in *prāṇa*s) is an arc measured on the celestial equator, as is dealt with in *GD2* 89-102. There the "rising time at the terrestrial equator" $\alpha$ is measured as a distance from an equinoctial point, but here the reference is the point corresponding to a solstitial point. The other two are the Radius of the great circle $R$ and the elevation of ecliptic pole $\operatorname{Sin}\zeta_{\mathrm{K}}$.

Parameśvara does not explain how the rule is obtained, but we can understand it as follows. Let us assume that the observer O is on the celestial equator where the six o'clock circle is the horizon (figure 10.8(a)). L, which is between the winter solstice ♑ and the vernal equinox ♈ in this case, is the ascendant. A and C are the points on the celestial equator corresponding to L and ♑, respectively. Therefore $\overset{\frown}{\mathrm{AC}} = \bar{\alpha}$. H is the foot of the perpendicular dropped from C to the plane of the six o'clock circle and $\mathrm{HC} = \operatorname{Sin}\bar{\alpha}$. P and P′ are the northern and southern celestial pole. K and K′ are the northern and southern ecliptic pole. B is the foot of the perpendicular dropped from K to the plane of the six o'clock circle, and BK is the elevation of ecliptic pole $\operatorname{Sin}\zeta_{\mathrm{K}}$. $\overset{\frown}{\mathrm{CP}}$ is part of the solstitial colure. From *GD2* 154 we know that ♑ and K are on the solstitial colure too. Furthermore, from *GD2* 155, $\overset{\frown}{♑\mathrm{K}} = \overset{\frown}{\mathrm{CP}} = 90°$. Thus $\overset{\frown}{\mathrm{KP}} = \overset{\frown}{♑\mathrm{C}} = \varepsilon$, and as the stellar sphere revolves, K draws a circle around a point on OP with a radius of $\operatorname{Sin}\varepsilon$. This circle is drawn in figure 10.8(b) where the sphere is projected from the direction of the northern celestial pole. The circle is concentric with the celestial equator, and their center is projected here on point P. Since △PBK and △PHC are right triangles sharing an acute angle, △PBK ∼ △PHC, and therefore

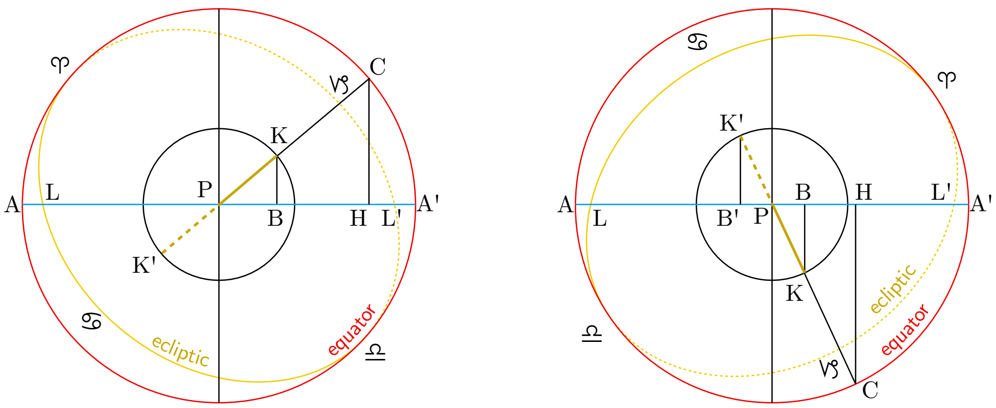



(a) In a sphere. North is to the right.

(b) Its projection seen from the northern celestial pole.

Figure 10.8: Elevation of ecliptic pole BK $=$ Sin $\zeta_{\mathrm{K}}$ when L is the ascendant.

$$
\begin{aligned}
\mathrm{BK} &= \frac{\mathrm{KP} \cdot \mathrm{HC}}{\mathrm{CP}} \\
\operatorname{Sin}\zeta_{\mathrm{K}} &= \frac{\operatorname{Sin}\varepsilon \cdot \operatorname{Sin}\bar{\alpha}}{R}
\end{aligned}
\tag{10.2}
$$




(a) Elevation in the north BK.

(b) Elevation in the south B′K′.

Figure 10.9: Elevation of ecliptic for an ascendant L or descendant L′.

The northern ecliptic pole K is above the six o'clock circle as long as the ascendant L is on the ecliptic between the winter solstice ♑ and the summer solstice ♋ including the vernal equinox ♈ (figure 10.9(a)). When L is on the other side of the ecliptic, i.e. from ♋ to ♑ including the autumn equinox ♎, K is below the six o'clock circle and the southern ecliptic pole K′ goes above (figure 10.9(b)). Its elevation B′K′ (where B′ is its foot) is equal to BK which is now below the six o'clock circle. Parameśvara distinguishes the two situations in *GD2* 160 by calling them the "elevation in the north (*saumyonnati*)" and "elevation in the south (*yāmyonnati*)". Furthermore, *GD2* 160d adds that the elevation can also be defined when a point of the ecliptic L′ is the descendant, i.e. at the moment when it sets below the six o'clock circle. In this case, the northern ecliptic pole is elevated when L′ is between ♋ and ♑ including ♎ (figure 10.9(b)) and the southern ecliptic pole is elevated otherwise (figure 10.9(a)).

*GD2* 161 seems to be a reasoning for using $\bar{\alpha}$ along the celestial equator, which is in units of time (*prāṇas*) but corresponds to an amount of revolution of the stellar sphere in arc minutes. Indeed by contrast, an arc in the ecliptic is not the revolution of the sphere itself. Parameśvara's intention might be to compare this with the approximate method using the longitude appearing in the next verse.

## 10.5 Crude elevation (*GD2* 162)

An arc degree or arc minute along the ecliptic does not exactly correspond to an arc degree or arc minute of revolution by the stellar sphere. Therefore, if we use the distance from a solstitial point to the point on the ecliptic, i.e. its "upright" $\lambda_U$ instead of the corresponding arc on the celestial equator, the result is only approximate. This corresponds to taking the Sine of $\overset{\frown}{\mathrm{L}♑}$ instead of $\overset{\frown}{\mathrm{AC}}$ in figure 10.8. Yet Parameśvara gives this as an alternative rule to obtain the "crude" value of elevation $\mathrm{Sin}\,\tilde{\zeta_{\mathrm{K}}}$.

$$\mathrm{Sin}\,\tilde{\zeta_{\mathrm{K}}} = \frac{\mathrm{Sin}\,\lambda_U \cdot \mathrm{Sin}\,\varepsilon}{R} \tag{10.3}$$

Parameśvara justifies this rule on the ground that the method becomes simple. Indeed, the process to find a point on the celestial equator that corresponds to a given longitude can be cumbersome (*GD2* 89-102). We do not know whether Parameśvara actually preferred using the crude elevation in practice. Hereafter in our interpretations, we will stick to the accurate elevation $\mathrm{Sin}\,\zeta_{\mathrm{K}}$ but technically it could have been replaced with $\mathrm{Sin}\,\tilde{\zeta_{\mathrm{K}}}$.

## 10.6 Corrected latitude and true declination (*GD2* 163-164)

The "suitable" way to combine the latitude with the declination that has been implied in *GD2* 157 is explained in *GD2* 163-164. Here we take the component of the latitude in the direction of declination, instead of the latitude itself, as the "corrected latitude (*sphuṭakṣepa*)" (figure 10.10). When F is the intersection of the planet's diurnal circle with $\overset{\frown}{\mathrm{AL}}$ extended, $\overset{\frown}{\mathrm{LF}}$ is the corrected latitude. This added to or subtracted from the declination $\overset{\frown}{\mathrm{AL}}$ is the "true (*spaṣṭa*) declination" $\overset{\frown}{\mathrm{AF}}$ which is the actual arc distance of a planet from the celestial equator[7]. This can be compared with the "corrected (*sphuṭa*) declination" in *GD2* 153 which simply adds or

[7]Parameśvara never suggests that the arc of true declination should have the actual position of the planet V at its end, as would be expected in modern astronomy.

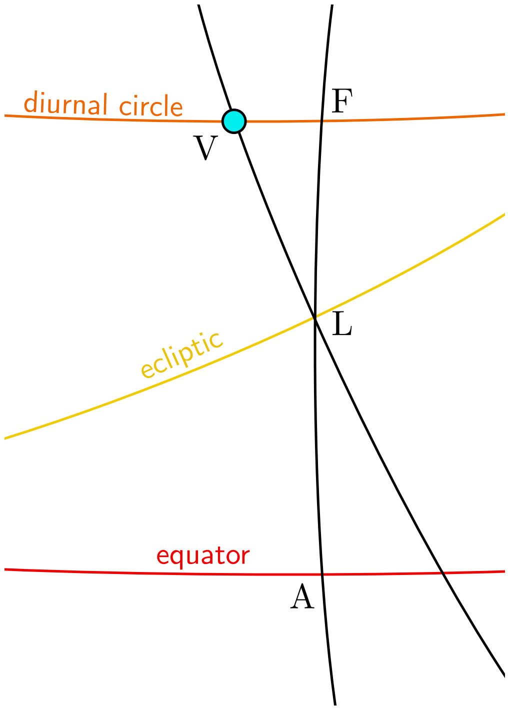



Figure 10.10: Corrected latitude $\widehat{\mathrm{LF}}$ and true declination $\widehat{\mathrm{AF}}$.

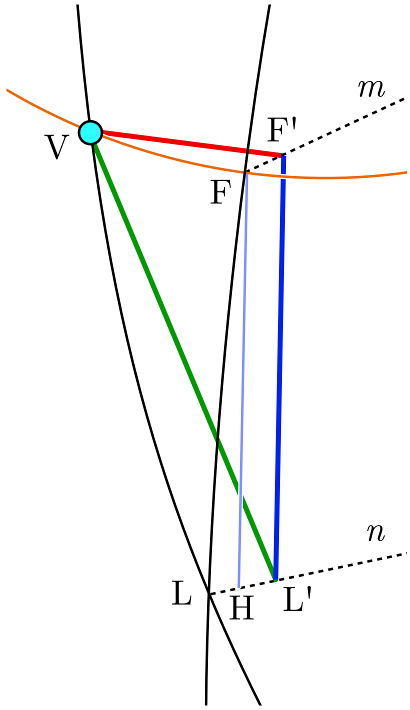



Figure 10.11: Spherical $\triangle$LFV and plane $\triangle$L′F′V.

subtracts the latitude itself. Parameśvara rarely uses the word *spaṣṭa*[8], and in *GD2*, this is the only occurrence. Therefore he might be making a distinction between a correction which is only approximate and something which is more "true". Other texts use *sphuṭa* and *spaṣṭa* differently for the latitude and the declination: for example, *Sūryasiddhānta* 2.57 (Shukla (1957, p. 36)) uses *spaṣṭa* to refer to what we understand as the approximate corrected declination.

In figure 10.10 the latitude $\widehat{\mathrm{LV}} = \beta$ and the corrected latitude $\widehat{\mathrm{LF}} = \beta^*$ look as if they form a triangle with $\widehat{\mathrm{FV}}$. This $\triangle$LFV is a spherical triangle, and Parameśvara might be approximating it with a plane right triangle, as he states a Pythagorean theorem in *GD2* 163cd that treats the "latitude" and "corrected latitude" as segments. However it is also possible that he could be abbreviating the word "Sine" here. My interpretation is the latter, because Parameśvara refers to the "arc" of this corrected latitude in *GD2* 164. There are other cases in *GD2* where Parameśvara makes a distinction between an arc and its Sine (appendix B.1).

We can draw a plane triangle including the Sine of latitude $\mathrm{Sin}\,\beta$ as shown in figure 10.11. The circle going through A, L and F represents the six o'clock circle when L is on it. $\widehat{\mathrm{LV}}$ is on a circle which represents the longitude of the planet. Line $m$ is the intersection of the planes of the six o'clock circle and the diurnal circle and $n$ is the intersection of the planes of the six o'clock circle and the circle of the longitude. $m$ goes toward the center of the diurnal circle while $n$ passes the center of the great circle, and the two lines are not parallel. F′ and L′ are the feet of the perpendiculars drawn from V to $m$ and $n$, respectively. Since the diurnal circle and the six o'clock circle are orthogonal, F′V $\perp$ L′F′ and $\triangle$L′F′V is a plane right triangle. L′V is the Sine of latitude $\mathrm{Sin}\,\beta$. Meanwhile the Sine of the corrected latitude $\mathrm{Sin}\,\beta^*$ is HF where H is the foot

[8]See glossary entry *spaṣṭa* for a general discussion on the difference between *sphuṭa* and *spaṣṭa*.

of the perpendicular drawn from F to $n$. Let us approximate that L′F′ is equal to HF = $\text{Sin}\,\beta*$, which may be justified because $\overset{\frown}{\text{LF}}$ is very small and thus FF′ and LL′ are extremely minute.

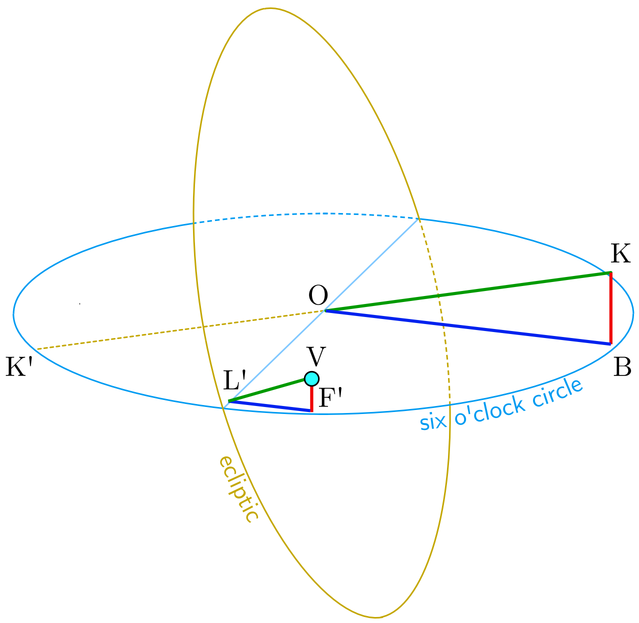



Figure 10.12: The elevation of latitude F′V = $\text{Sin}\,\zeta_\beta$ and the elevation of ecliptic pole BK = $\text{Sin}\,\zeta_{\text{K}}$. North to the right.

Parameśvara does not mention how the rule for computing $\text{Sin}\,\beta*$ (*GD2* 163) is derived, but we can explain it as follows (figure 10.12). BK is the elevation of ecliptic pole and O is the observer. ∠VL′F′ = ∠KOB since they both complement the angle formed by the ecliptic and the six o'clock circle. ∠L′F′V = ∠OBK = 90° and thus △L′F′V ∼ △OBK. We can compute the length of segment F′V with a Rule of Three:

$$\begin{aligned} \text{F}'\text{V} &= \frac{\text{BK} \cdot \text{VL}'}{\text{KO}} \\ &= \frac{\text{Sin}\,\zeta_{\text{K}} \cdot \text{Sin}\,\beta}{R} \end{aligned} \tag{10.4}$$

Thus from the Pythagorean theorem, the Sine of corrected latitude $\text{Sin}\,\beta*$ is

$$\begin{aligned} \text{L}'\text{F}' &= \sqrt{\text{VL}'^2 - \text{F}'\text{V}^2} \\ \text{Sin}\,\beta* &= \sqrt{\text{Sin}^2\beta - \left(\frac{\text{Sin}\,\zeta_{\text{K}} \cdot \text{Sin}\,\beta}{R}\right)^2} \end{aligned} \tag{10.5}$$

As given in *GD2* 164, the sum or difference of its arc $\beta*$ and the declination $\delta$, according to their directions, is the true declination $\delta_T$. The cases are exactly the same with what we saw in *GD2* 153 (formula 10.1):

$$\delta_T = \begin{cases} \delta + \beta^* & \text{(a) Same direction} \\ \delta - \beta^* & \text{(b) Opposite direction} \\ \beta^* - \delta & \text{(c) Opposite and corrected latitude is larger} \end{cases} \quad (10.6)$$

Here again, there is no reference to case (c) in Parameśvara's description.

*GD2* 164d further refers to its usage: to compute the "true Earth-Sine" and so forth. We can find a resemblance between this passage and *MBh* 6.8d which explains the usage of the Sine of "corrected declination": "The method for the moon's ascensional difference in *nāḍikā*s [is established] with this[9]". By using the rules in *GD2*, we can compute the moon's "true" Earth-Sine (*GD2* 74ab) and also the radius of the moon's diurnal circle with its true declination (*GD2* 73cd). From the true Earth-Sine and radius of the diurnal circle, the Sine of the moon's ascensional difference can be obtained (*GD2* 74cd). *MBh* 6.8d is followed by rules for computing the great gnomon (i.e. elevation from the ground) of the moon (this can be done by applying *GD2* 107-114ab) and other parameters to find the "elevation of the moon's horn (*śṛṅgonnati*)", i.e. the orientation of the lunar crescent (*MBh* 6.9-42, T. Kuppanna Sastri (1957, pp. 345-363)). We can thus draw a dialog between *GD2* and the *Mahābhāskarīya* concerning the visibility of the moon.

Another case where the corrected or true declination would be used is for computing the occurrence of a *vyatīpāta*, the moment when the declinations of the sun and the moon become equal[10]. Parameśvara makes no reference to the *vyatīpāta* here, which can be contrasted with Nīlakaṇṭha who devoted a whole chapter on the *vyatīpāta* in his *Tantrasaṅgraha* (Ramasubramanian and Sriram (2011, pp. 357-384)).

*GD2* 175 refers to a Sine of declination "corrected by the celestial latitude (*vikṣepasaṃskṛtā*)". Which is most likely the true declination, as we will see later in section 10.11. Otherwise there is no explicit reference to either the corrected declination or the true declination, but the visibility methods involve the diurnal circle of the planet with a latitude. I assume that its radius must have been computed by using the true declination.

### 10.6.1 "Mādhava's rule" for the true declination

Nīlakaṇṭha quotes, in his commentary on on *Ābh* 4.46, two verses which he attributes to Mādhava[11]:

> Having multiplied the Sine of latitude with the "upright" [Sine] of the greatest declination, [and having multiplied] a given [Sine of] declination with the "upright" [Sine] of that [latitude], the two divided by the Radius are suitable for adding or subtracting.
>
> When these two are in the same direction, [their] sum, and when in different directions,

[9] *tena candracaranāḍikāvidhiḥ //6.8//* (T. Kuppanna Sastri, 1957, p. 344)

[10] On this topic, see the commentary notes on *Tantrasaṅgraha* chapter 6 by Ramasubramanian and Sriram (2011, pp. 357-384) which includes discussions on the moon's declination which is specific to Nīlakaṇṭha but otherwise gives a detailed overview. Burgess and Whitney (1858, pp. 379-386) on *Sūryasiddhānta* chapter 11 is also useful, despite its claim that "of all the chapters in the treatise, this is the one which has least interest and value".

[11] I am deeply indebted to the Kyoto Seminar for the History of Science in India for this section. My understanding of Nīlakaṇṭha's commentary comes from the Japanese translation and notes prepared by Setsuro Ikeyama for the seminar.

[their] difference is [the Sine of] true declination. The "upright" [Sine] of true declination is the diurnal "Sine" of those staying on the inclined circle.[12]

The "upright" (*koṭi*) [Sine] corresponds to the Cosine of an arc[13]. In this rule, the Sine of true declination $\mathrm{Sin}\,\delta_T$ is computed from the celestial latitude $\beta$, greatest declination $\varepsilon$ and declination $\delta$ as follows:

$$\mathrm{Sin}\,\delta_T = \begin{cases} \dfrac{\mathrm{Sin}\,\beta\,\mathrm{Cos}\,\varepsilon}{R} + \dfrac{\mathrm{Sin}\,\delta\,\mathrm{Cos}\,\beta}{R} & \text{(a) Same direction} \\ \left|\dfrac{\mathrm{Sin}\,\beta\,\mathrm{Cos}\,\varepsilon}{R} - \dfrac{\mathrm{Sin}\,\delta\,\mathrm{Cos}\,\beta}{R}\right| & \text{(b) Opposite direction} \end{cases} \tag{10.7}$$

This quotation is followed by a long explanation for deriving this rule (Pillai (1957b, pp. 108-114)). It should suffice for us to say that this approach is very different from what we have seen in formulas 10.5 and 10.6[14]. Nīlakaṇṭha also uses these verses in the chapter on *vyatīpāta* in his own treatise (*Tantrasaṅgraha* 6.4-5[15]) without mentioning that they are quotations. If this rule had indeed come from Mādhava, it left no trace in Parameśvara's works. On the other hand, Parameśvara's rule for the true declination was not adopted by Nīlakaṇṭha.

## 10.7 Two visibility methods (*GD2* 165)

The term "visibility method (*dṛkkarman*)" appears for the first time in *GD2* 165. This term refers to the method to find the point on the ecliptic which rises at the same time as the planet. As the verse states, there are two of them. Parameśvara does not give their individual names in *GD2*, but in his commentaries on *Ābh* 4.36 (Kern (1874, pp. 93-94)) and *Ābh* 4.35 (Kern (ibid., p. 93)) where basically the same methods appear, he calls them the "visibility method for the 'course' (*āyana-dṛkkarman*[16])" and the "visibility method for the geographic latitude (*ākṣa-dṛkkarman*[17])" respectively. The core of these methods are to add or subtract a "visibility equation (*dṛkphala*)" to the longitude of a planet.

Only the visibility method for the "course" is necessary when the observer is at the terrestrial equator and the horizon is the six o'clock circle (figure 10.13). As stated in *GD2* 157, the planet with celestial latitude V goes above or below the six o'clock circle (which is also the horizon at the equator) when its longitude L is the ascendant. When L* is the point on the ecliptic which rises at the same time as V at the terrestrial equator (i.e. L* and V have the same right ascension), $\widehat{\mathrm{LL}^*}$ is the visibility equation to be applied to the longitude. We have seen that the amount of elevation or depression depended on the elevation of ecliptic pole, which in turn

[12] *paramāpakramakoṭyā vikṣepajyāṃ nihatya tatkoṭyā |*
*iṣṭakrāntiṃ cobhe trijyāpte yogavirahayogye staḥ ||*
*sadiśoḥ saṃyutir anayor viyutir vidiśor apakramaḥ spaṣṭaḥ |*
*spaṣṭāpakramakoṭir dyujyā vikṣepamaṇḍale vasatām ||* (Pillai (1957b, p. 108))

[13] The same expression can be found in *GD2* 48.

[14] See also Plofker (2002) for further discussions on the true declination methods of Bhāskara II and Nīlakaṇṭha as well as those inspired by Islamic astronomy.

[15] Ramasubramanian and Sriram (2011, pp. 359-362). It includes the derivation of this rule which is different from Nīlakaṇṭha's procedure in his commentary on *Ābh* 4.46.

[16] Today, historians tend to call this method the *ayanadṛkkarma* (for example, Pingree (1978)).

[17] More often called the *akṣadṛkkarma*.

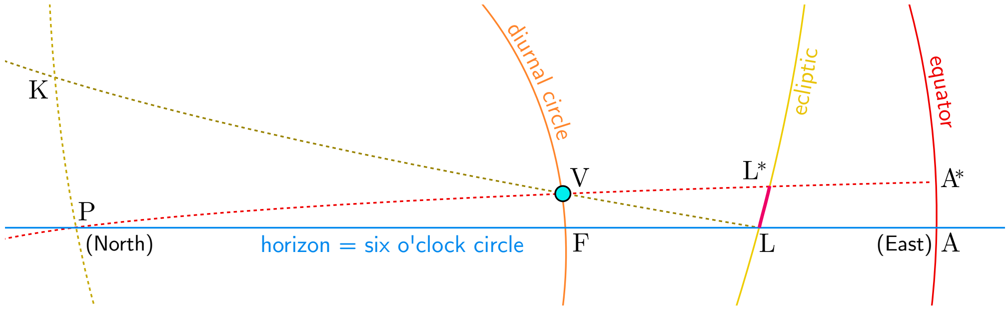



Figure 10.13: Visibility equation for the "course" $\widehat{\mathrm{LL}^*}$ of planet V rising at a place on the terrestrial equator, from the viewpoint of an observer on the Earth.

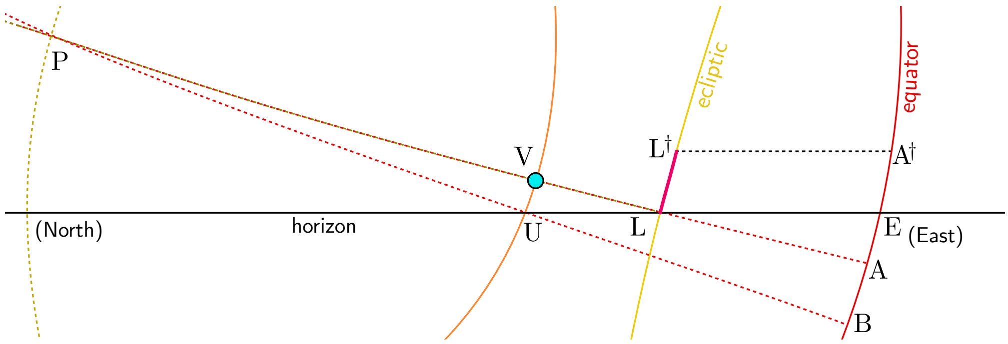



Figure 10.14: Visibility equation for the geographic latitude $\widehat{\mathrm{LL}^\dagger}$ of planet V whose longitude is on a solstitial point.

changed according to the ascendant's distance from a solstitial point (*ayanānta*, literally "end of course [towards solstice]"). Hence this method is associated with the course (*ayana*). The steps for the visibility method for the "course" are given in *GD2* 169-171.

There is no correction for the "course" if the planet's longitude L coincides with a solstitial point. In this situation, if the observer is at a location other than the terrestrial equator, the second method for the geographic latitude alone is required (figure 10.14). The point on the ecliptic $\mathrm{L}^\dagger$ which rises with V cannot be drawn as easily as the previous case[18]. The visibility equation $\widehat{\mathrm{LL}^\dagger}$ is found by computing the time difference between the rising of the planet and its longitude on the ecliptic measured along the celestial equator ($\widehat{\mathrm{BA}} = \widehat{\mathrm{EA}^\dagger}$). The steps for the visibility method for the geographic latitude are stated in *GD2* 175-177.

*GD2* 175 refers explicitly to the name "visibility equation for the geographic latitude (*akṣa-dṛkphala*)". However *GD2* 165cd refers to its cause as the planet's situation south or north of the ecliptic. This is probably a reference to the fact that the equation becomes additive or subtractive depending on the latitude's direction, as we will see in *GD2* 177. By contrast, the equation for the "course" is additive or subtractive depending on the planet's "course" northward or southward on the ecliptic. In situations where the planet's longitude is not on a solstitial point and the observer is at a place with geographic latitude, the two equations are combined.

[18]Nevertheless, we do not know whether Parameśvara used a diagram or an armillary sphere to describe the two equations.

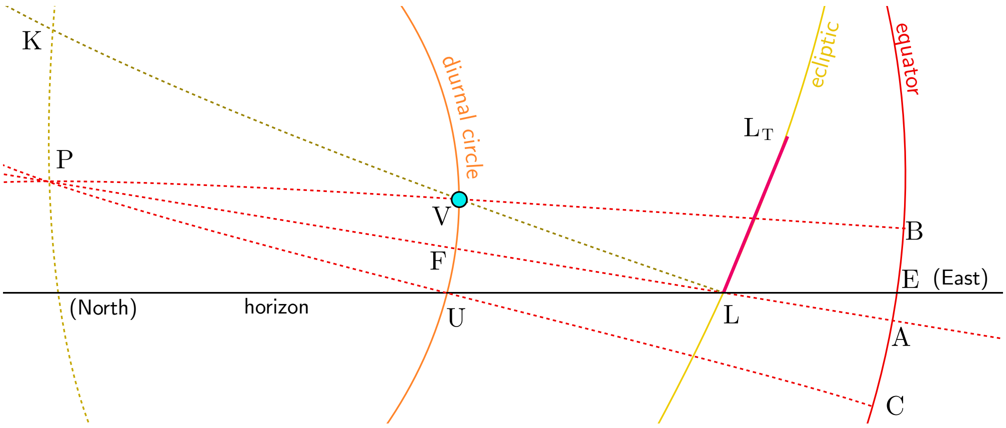



Figure 10.15: A situation where both visibility equations are to be applied. The two corrections cannot be drawn in the same diagram.

Parameśvara does not explain how the two equations must be combined when both visibility operations are required (figure 10.15). We can locate the arc in the celestial equator $\overset{\frown}{\mathrm{AB}}$ corresponding to the visibility equation for the “course” or $\overset{\frown}{\mathrm{AC}}$ for the geographic latitude together in our diagram, but not their equations in the ecliptic. Whether both equations should be computed from the longitude of L and simply combined, or whether one should be applied first and the second should be computed from the once-corrected longitude is unknown. Eventually, Parameśvara denies that the visibility method should be subdivided in *GD2* 178 and suggests a unified method instead.

The last phrase in *GD2* 165 (ഗ്രഹേ ƒതഃ സ്ഥഃ / *grahe 'taḥ staḥ*) is corrupted in many manuscripts and the critical edition of Sāstrī (1916, p. 17) gives an uninterpretable reading. *visarga*s preceding sibilants are often omitted in Malayalam manuscripts, and therefore the phrase is written ഗ്രഹേതസ്ഥഃ (*grahetastaḥ*), which should have lead to the confusion.

## 10.8 Elevation and depression of latitude (*GD2* 166-168)

The computation of the corrected latitude in *GD2* 163 involved an unnamed segment whose length is given in *GD2* 163ab. *GD2* 166 repeats this rule, and now this segment is called the elevation (*unnati*) or depression (*avanati*) of the latitude $\zeta_\beta$, depending on whether the planet is above or below the six o'clock circle.

$$\operatorname{Sin}\zeta_\beta = \frac{\operatorname{Sin}\zeta_{\mathrm{K}}\operatorname{Sin}\beta}{R} \tag{10.8}$$

*GD2* 167 gives the conditions for determining whether $\zeta_\beta$ is an elevation or depression, which can be reformulated as follows:

- The northern ecliptic pole is elevated
  - Celestial latitude is northward: $\zeta_\beta$ is an elevation
  - Celestial latitude is southward: $\zeta_\beta$ is a depression

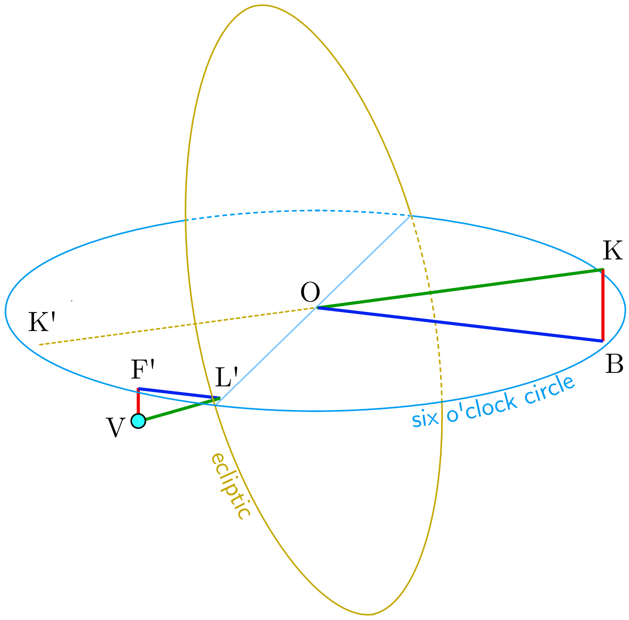



Figure 10.16: The depression of latitude F′V and the elevation of the northern ecliptic pole BK. North to the right.

- The southern ecliptic pole is elevated
  - Celestial latitude is northward: $\zeta_\beta$ is a depression
  - Celestial latitude is southward: $\zeta_\beta$ is an elevation

Figure 10.16 shows a case where the northern ecliptic pole is elevated and the latitude is southward. In this case $\zeta_\beta$ is a depression.

*GD2* 168 tells us that the elevation (or depression) of latitude (F′V in figure 10.16), the Sine of latitude (VL′) and the Sine of corrected latitude (L′F′) form a right triangle by naming them the base, hypotenuse and upright. The verse further adds that the arc of the corrected latitude is on the same arc with the declination. These remarks look like groundings for *GD2* 163cd and *GD2* 164, but what Parameśvara intended by mentioning them here is uncertain.

## 10.9 Visibility method for the "course" (*GD2* 169-174)

*GD2* 169-171 gives the set of computations within the visibility method for the "course" with its conditions, while *GD2* 172-174 supply groundings and explanations for some of the steps.

Figure 10.18 illustrates a situation when point L on the ecliptic which represents the longitude of the planet V is on the horizon as seen from an observer at a location with geographic latitude.

In order to isolate the visibility equation for the "course" from that for the geographic latitude, let us first consider a situation at the terrestrial equator (figure 10.17). The horizon and the six o'clock circle coincide. P and K are the celestial and ecliptic poles respectively. Since P and K are separated, the planet V is separated from the horizon at the moment when its corresponding point L on the ecliptic is rising. F and A are the intersections of the six o'clock circle with the planet's diurnal circle and the celestial equator. $\widehat{\mathrm{FV}}$ is the extra diurnal motion due to the

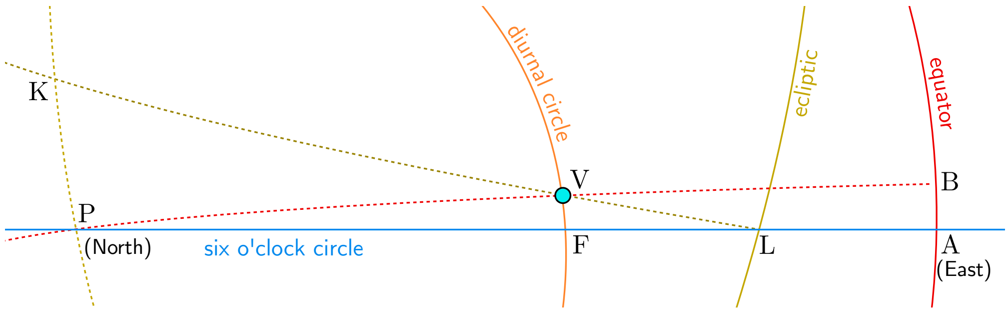



Figure 10.17: A planet rising from the viewpoint of an observer at the terrestrial equator.

elevation or depression of the celestial latitude; therefore the visibility equation can be computed by finding the time it takes for the planet to move along $\overset{\frown}{\mathrm{FV}}$ and find the amount of longitude the ecliptic moves in the same time. The time is measured on the celestial equator, and therefore if B is the point which rises with V, $\overset{\frown}{\mathrm{AB}}$ is the time difference corresponding to $\overset{\frown}{\mathrm{FV}}$.

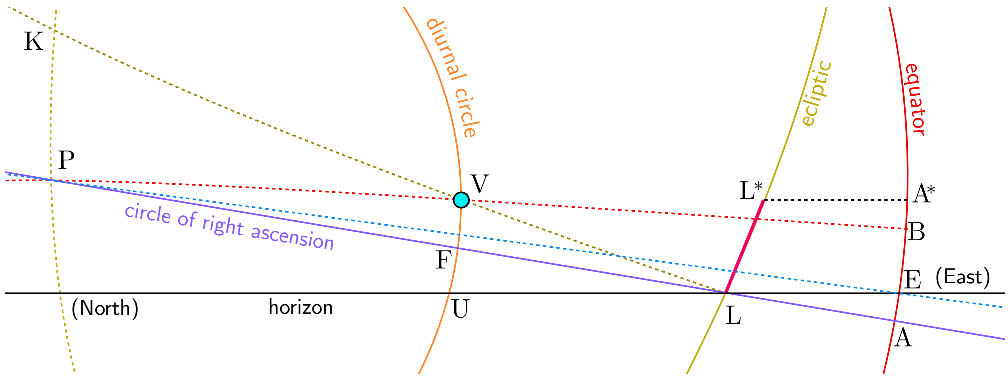



Figure 10.18: Visibility equation for the "course" $\overset{\frown}{\mathrm{LL^*}}$ of planet V at a location with geographic latitude. $\overset{\frown}{\mathrm{PE}}$ is the six o'clock circle and $\overset{\frown}{\mathrm{PLA}}$ is what we shall call the "circle of right ascension".

Now let us introduce the geographic latitude to this situation by lifting P while fixing L on the horizon(figure 10.18). The six o'clock circle (connecting P with due east on the horizon E) no longer goes through L, but F, L and A will still be on the same circle. In modern terms, these three points have the same right ascension. Therefore let us call this circle the "circle of right ascension"[19]. The length of $\overset{\frown}{\mathrm{FV}}$, caused by the elevation or depression of the celestial latitude, remains unchanged. Therefore the visibility equation for the "course" should be computed from it. Meanwhile, $\overset{\frown}{\mathrm{UF}}$ is the additional path of the planet caused by the geographic latitude, and therefore should considered later in the visibility method for the geographic latitude.

To measure the time difference corresponding to FV, we use the same arc AB on the celestial equator. B and V have the same right ascension, so we may also say that they are on another circle of right ascension of their own. But hereafter, I shall use the term "circle of right ascension" exclusively for the circle which includes L.

[19]Note that this is not Parameśvara's terminology. He does not even use this circle in his explanation.

The point on the celestial equator that rises with L is not A, but E. Therefore to find the arc in the ecliptic that has risen since the planet V rose, or is yet to rise before V rises, we need to move from $\widehat{\mathrm{AB}}$ to $\widehat{\mathrm{EA*}}$ which is an arc in the celestial equator with the same length and has the horizon at its end. A* can be above or below the horizon depending on whether the celestial latitude has an elevation or depression. Finally, we find the arc $\widehat{\mathrm{LL*}}$ on the ecliptic which rises with $\widehat{\mathrm{EA*}}$, and this is the visibility equation for the "course". *GD2* 169 is the rule for computing this arc, but the term "visibility equation" itself appears in *GD2* 172.

### 10.9.1 Steps to move between circles

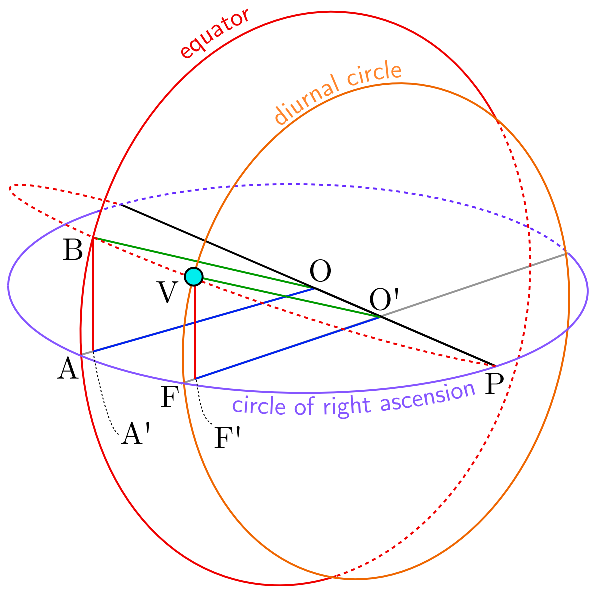



Figure 10.19: Moving from segment FV′ in the diurnal circle to segment A′B in the celestial equator.

As we have seen previously in section 10.6 (figure 10.11), the elevation or depression of the latitude $\mathrm{F'V} = \mathrm{Sin}\,\zeta_\beta$ corresponds to $\widehat{\mathrm{FV}}$ in the diurnal circle of the planet. O′ is its center. Meanwhile, since A is the point on the celestial equator whose right ascension is equal to V, $\angle\mathrm{BOA'} = \angle\mathrm{VO'F}$. Therefore, when A′ is the foot of the perpendicular drawn from B onto AO, the two right triangles $\triangle\mathrm{OA'B}$ and $\triangle\mathrm{O'F'V}$ are similar. Thus, when the radius of the diurnal circle is $r$:

$$\begin{aligned}\mathrm{A'B} &= \frac{\mathrm{F'V}\cdot\mathrm{BO}}{\mathrm{VO'}}\\ &= \frac{\mathrm{Sin}\,\zeta_\beta R}{r}\end{aligned} \tag{10.9}$$

No other description or Rule of Three is given in *GD2* for this computation.

Parameśvara states explicitly that “the arc ($\overparen{\mathrm{AB}}$) of this ($\mathrm{A'B}$)” must be taken. Actually, $\mathrm{A'B}$ is no larger than the order of the celestial latitude[20], and could be small enough to be approximated by $\overparen{\mathrm{AB}}$. We will see later that many of Parameśvara's predecessors have essentially done so without even mentioning the approximation. Why is Parameśvara referring to this step when it could be skipped while computing? My hypothesis is that this is part of an educational instruction, where the aim is to teach the students to understand how rules can be grounded. As we will see later, the twofold visibility method is discarded later in place of a unified method, and therefore the aim of this verse itself is for grounding the theories and not to give a practical computational rule. The previous step (formula 10.9) can be demonstrated in an armillary sphere. In that case, it is essential to distinguish the arc and the segment.

$$\begin{aligned}\overparen{\mathrm{AB}} &= \operatorname{arcSin} \mathrm{A'B} \\ &= \operatorname{arcSin}\left(\frac{\operatorname{Sin}\zeta_\beta R}{r}\right) \qquad (10.10)\end{aligned}$$

For a place with geographic latitude, we need to consider $\overparen{\mathrm{EA^*}}$ with the same length as $\overparen{\mathrm{AB}}$, as we have discussed previously (figure 10.18).

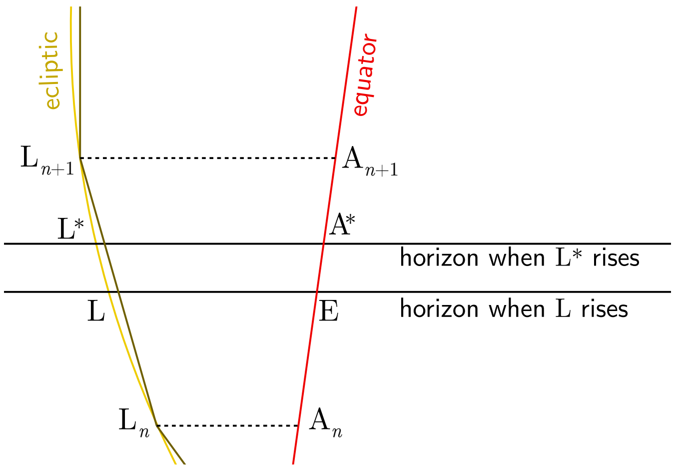



Figure 10.20: Linear approximation of the ecliptic within a zodiacal sign $\overparen{\mathrm{L_n L_{n+1}}}$.

The last step for the equation is to find $\overparen{\mathrm{LL^*}} = l_{v(\mathrm{c})}$ in the ecliptic corresponding to $\overparen{\mathrm{EA^*}} = \overparen{\mathrm{AB}}$. Parameśvara supplies a Rule of Three for this computation in *GD2* 172, from which we can reconstruct the situation as in figure 10.20. The term *vilagna* refers to the sign that is rising at the moment. The inclination of the ecliptic against the horizon is different at every longitude, but here we assume that it is constant from the beginning $\mathrm{L}_n$ of a zodiacal sign to its end $\mathrm{L}_{n+1}$. Parameśvara does not mention this approximation. $\overparen{\mathrm{L_n L_{n+1}}}$ has the length of one sign which is 1800 arc minutes. The section on the celestial equator that rises with this sign, $\overparen{\mathrm{A_n A_{n+1}}}$,

[20] The elevation or depression of latitude $\operatorname{Sin}\zeta_\beta$ is shorter than the Sine of celestial latitude $\operatorname{Sin}\beta$ itself, because $\operatorname{Sin}\zeta_\beta$ is the base of the right triangle where $\operatorname{Sin}\beta$ is the hypotenuse (*GD2* 168). Its corresponding segment in the celestial equator is slightly larger by the factor of $\frac{R}{r}$, but this is not significant; $\frac{R}{r} = \frac{3438}{3141}$ when the declination is 24°. Moreover, $\operatorname{Sin}\zeta_\beta$ is smaller when the declination increases, and is 0 when the longitude is on a solstitial point.

represents the rising time or measure (*māna* / *miti*) of the sign $\rho_n$ which can be found from the rules in *GD2* 89-102 (section 7.1). Since $\widehat{\mathrm{L_nL_{n+1}}} : \widehat{\mathrm{A_nA_{n+1}}} = \widehat{\mathrm{LL^*}} : \widehat{\mathrm{EA^*}}$,

$$\widehat{\mathrm{LL^*}} = \frac{\widehat{\mathrm{EA^*}} \cdot \widehat{\mathrm{L_nL_{n+1}}}}{\widehat{\mathrm{A_nA_{n+1}}}}$$
$$l_{v(\mathrm{c})} = \frac{\mathrm{arcSin}\left(\frac{\mathrm{Sin}\,\zeta_\beta R}{r}\right) \cdot 1800}{\rho_n} \tag{10.11}$$

Unless the observer is on the terrestrial equator, the divisor must be the measure of the sign $\rho_n$ which takes into account the ascensional difference and not the rising time at Laṅkā (i.e. right ascension), $\alpha_n$. This is probably what Parameśvara states in *GD2* 173, although it is unclear what he means by addressing those who divide by $\alpha_n$ as “wise calculators (*sudhiyaḥ gaṇakāḥ*)”. The expression “those who know one location of the sphere (*golaikadeśavettāraḥ*)” can be taken in the sense of “those who consider only one location on the Earth's sphere”, “those who consider only one state of the armillary sphere”, or even “those who understand only one part in the discipline of the Sphere”. We will see in the following section that Bhāskara II has actually stated that the rising time at the equator should be taken as the divisor.

*GD2* 169 remarks that the equation $l_{v(\mathrm{c})}$ is additive or subtractive and *GD2* 170 explains this in further detail. Parameśvara does not specify where it is added to or subtracted from, but we can assume that it is the longitude $\lambda$ of the planet. The verse begins with the case when the planet is rising.

$$\lambda' = \begin{cases} \lambda - l_{v(\mathrm{c})} & \text{Celestial latitude has an elevation} \\ \lambda + l_{v(\mathrm{c})} & \text{Celestial latitude has a depression} \end{cases} \tag{10.12}$$

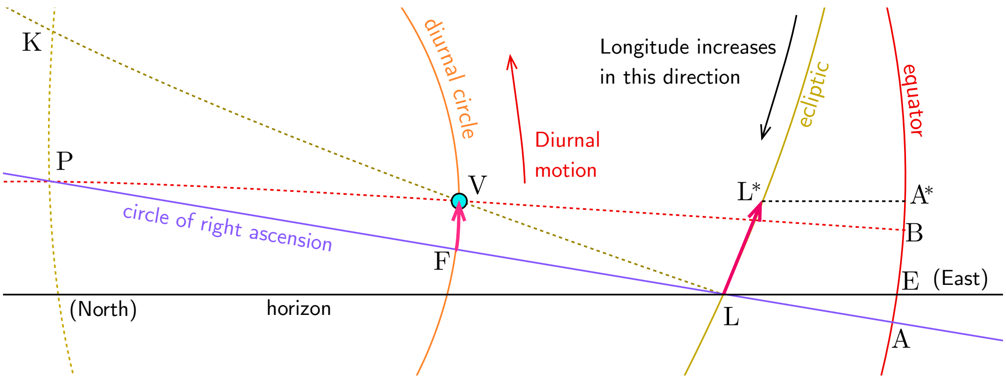



Figure 10.21: $l_{v(\mathrm{c})} = \widehat{\mathrm{LL^*}}$ is subtractive upon rising when the celestial latitude has an elevation.

In our previous discussions, we have used diagrams where the celestial latitude has an elevation, which I represent again in figure 10.21. In this case, assuming that the circle of right ascension is fixed in the sky as a reference, the planet V with celestial latitude will cross this circle earlier than its corresponding longitude L. Therefore, the point in the ecliptic L* that crosses the circle of right ascension at the same time with V should also rise earlier than L. Since the direction that the longitude in the ecliptic increases is from west to east, opposite of the diurnal motion, the longitude of L* should be smaller than L. This means that the visibility equation $\mathrm{LL}^* = l_{v(\mathrm{c})}$ should be subtracted from the original longitude of L in order to find L*.

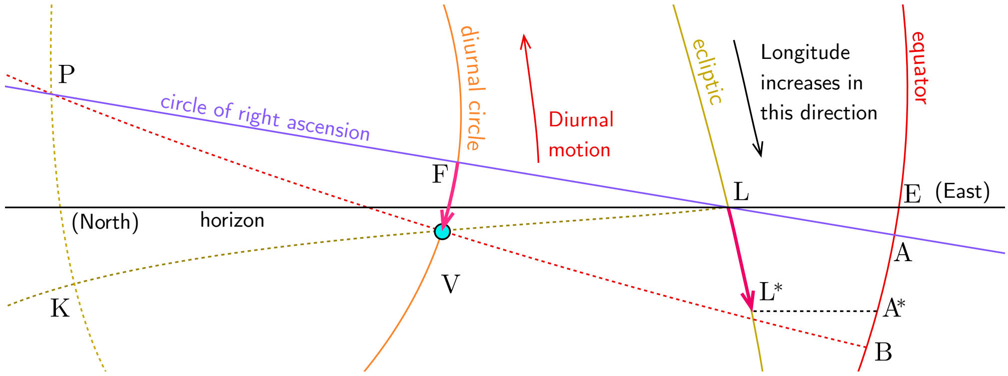



Figure 10.22: $l_{v(\mathrm{c})} = \widehat{\mathrm{LL}^*}$ is additive upon rising when the celestial latitude has a depression.

The situation is different when the celestial latitude of a planet has a depression (figure 10.22). V crosses the circle of right ascension after its corresponding longitude L. The depression of the celestial latitude corresponds to the arc $\widehat{\mathrm{FV}}$ below the circle of right ascension that the planet has yet to move on. The time required for this motion is measured on the celestial equator $\widehat{\mathrm{AB}}$. We must then find the arc $\widehat{\mathrm{EA}^*}$ which has the same length but starts from the intersection with the horizon and goes downward. Then we find the corresponding longitude $\widehat{\mathrm{LL}^*}$ where L* is the point on the ecliptic that will touch the horizon at the same time the planet V will reach the circle of right ascension. Contrary to the previous case, L* is in the direction that the celestial longitude increases, and thus $\mathrm{LL}^* = l_{v(\mathrm{c})}$ should be added.

*GD2* 170 also adds the cases when the planet is setting. The expression "the elevation (*unnatir*) is produced upon rising(*udayabhavā*)" is difficult to understand alone, and I have interpreted that the word "elevation" alone refers to whether the latitude has an elevation or depression. Therefore the passage deals with a situation where the planet and its longitude is setting but its elevation or depression has been measured at the moment of its rising. The last part refers to when its elevation or depression is also taken at the moment of the plane's setting.

$$\lambda' = \begin{cases} \lambda - l_{v(\mathrm{c})} & \text{(a). Latitude has an elevation when planet rises} \\ \lambda + l_{v(\mathrm{c})} & \text{(b). Latitude has a depression when planet rises} \\ \lambda + l_{v(\mathrm{c})} & \text{(c). Latitude has an elevation when planet sets} \\ \lambda - l_{v(\mathrm{c})} & \text{(d). Latitude has a depression when planet sets} \end{cases} \tag{10.13}$$

If a planet with celestial latitude is elevated above the circle of right ascension when its corresponding longitude on the ecliptic rises in the east, it would be below the circle of right ascension when the same longitude sets in the west. This is so because their motion should be symmetrical about the prime meridian[21]. Thus, saying that "the celestial latitude of a planet had an elevation when it was rising, and now it is setting" and "a planet is setting and its celestial latitude has a depression" would describe the same situation (figure 10.23). This corresponds

[21] At least if we ignore the diurnal motion of the planet. *GD2* 170 is uninterpretable if the diurnal motion has to be taken into account.

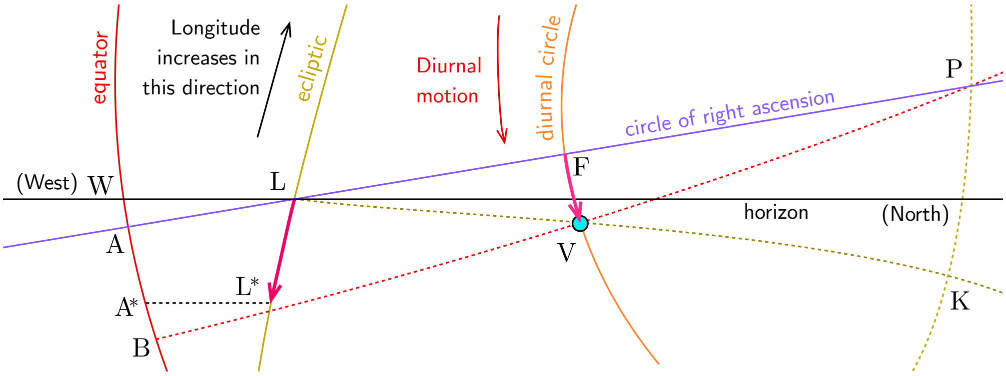



Figure 10.23: $l_{v(c)} = \widehat{\mathrm{LL}^*}$ is subtractive upon setting when the celestial latitude has a depression at that moment.

to case (a) or case (d) in formula 10.13. The planet V is below the circle of right ascension that goes through its corresponding longitude L, which means that the planet has traversed it in advance. $\widehat{\mathrm{FV}}$ represents the extra motion of the planet, and its corresponding time is measured by $\widehat{\mathrm{AB}}$ on the celestial equator. After moving this arc to $\widehat{\mathrm{WA}^*}$ with the same length that touches the horizon, we find the corresponding arc $\widehat{\mathrm{LL}^*}$ on the ecliptic. L* is the point on the ecliptic that touches the horizon at the moment that the planet V is on the circle of right ascension. It must be in the direction which sets before L, which is also the direction in which the longitude decreases, and therefore $\mathrm{LL}^* = l_{v(c)}$ is subtractive.

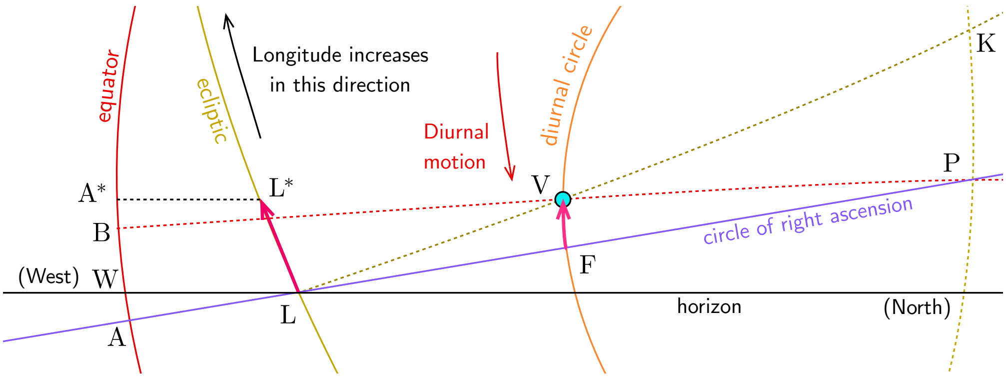



Figure 10.24: $l_{v(c)} = \widehat{\mathrm{LL}^*}$ is additive upon setting when the celestial latitude has an elevation at that moment.

Likewise, saying that "the celestial latitude of a planet had a depression when it was rising, and now it is setting" and "a planet is setting and its celestial latitude has an elevation" would describe the same situation (figure 10.24). This corresponds to case (b) or case (c) in formula 10.13. In this case the planet V passes the circle of right ascension after its longitude L, and therefore the point on the ecliptic L* that sets under the horizon when V is on the circle of right ascension should be above L. This is in the direction that the longitude increases, and thus $\mathrm{LL}^* = l_{v(c)}$ should be added to the initial longitude.

*GD2* 171 supplies some explanation concerning the rule in *GD2* 169 (formula 10.11). The

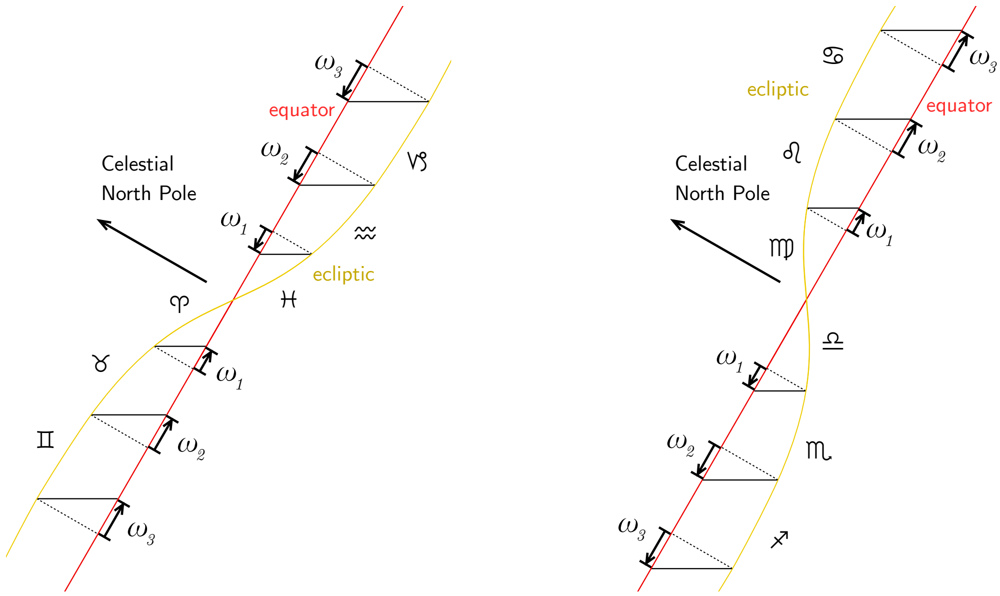



Figure 10.25: Ascensional differences at the borders of signs.

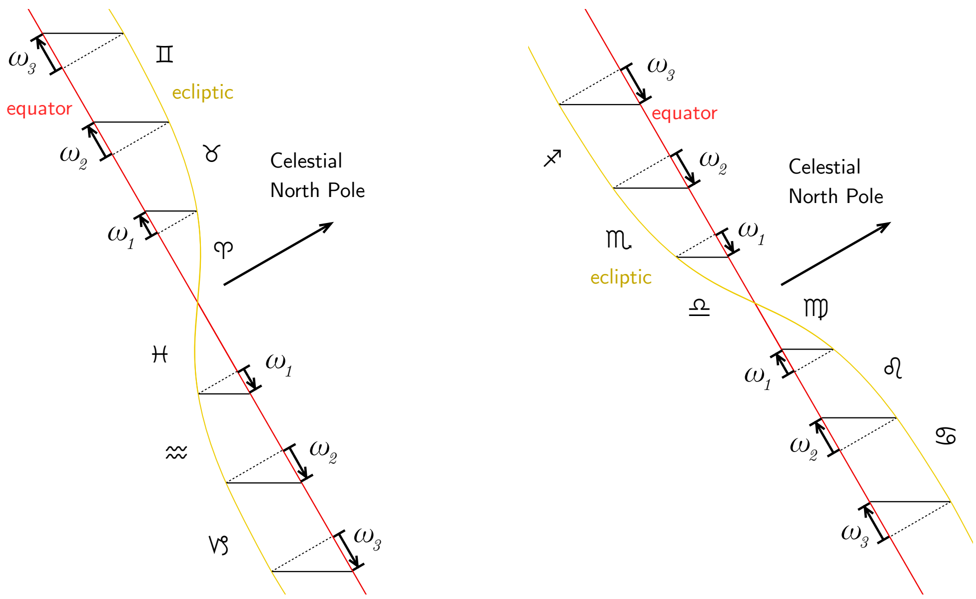



Figure 10.26: Descensional differences at the borders of signs.

computation uses the measure of sign $\rho_n$, but this is only described as the time it takes for a sign to rise in *GD2* 89-102. *GD2* 171 explains that the time it takes for a sign to set is equal to the rising time of "its seventh sign". This can be understood as the seventh sign along the zodiac counting itself as the first, i.e. the sign in its opposition. The reason, as explained in *GD2* 174, is because the ascensional difference of a sign when it sets (i.e. descensional difference) is in the opposite direction from when it rises. We can see from figures 10.25, 10.26 that the descensional difference at the border of each sign is opposite of its ascensional difference. Since the ascensional difference of the signs themselves is the difference between the ascensional difference at its beginning and end (formula 7.10), the descensional difference of the signs themselves should also be in the opposite direction (and same value) from their ascensional difference. The measure of the sign at the terrestrial equator $\alpha_n$ itself does not change regardless of when it rises or sets.

## 10.10 Characterizing Parameśvara's method

The visibility method in *GD2* is different in many respects compared to other treatises, most notably the *Āryabhaṭīya* and the *Mahābhāskarīya*. According to Parameśvara's commentary on *Ābh* 4.36 [22], the visibility equation for the "course" can be expressed as follows.

$$l_{v(c)} = \frac{\text{verSin}\,\lambda_U\,\text{Sin}\,\beta\,\text{Sin}\,\varepsilon}{R^2} \tag{10.14}$$

Where verSin $\theta$ is the "versed Sine (*utkramajyā*)" and verSin $\theta = R - \text{Sin}(90° - \theta)$. *Ābh* 4.36 only says "versed (*utkramaṇa*)" which Parameśvara paraphrases "versed Sine of the 'upright' (*koṭyā utkramajyā*)" verSin $\lambda_U$.

*MBh* 6.2cd-3 [23] give the following rule:

$$l_{v(c)} = \frac{\text{verSin}(\lambda - 90°)\,\text{Sin}\,\beta\,\text{Sin}\,\varepsilon}{R^2} \tag{10.15}$$

Since Sin $\lambda_U = \text{Sin}(\lambda - 90°)$, the two are equivalent.

On the other hand, Brahmagupta in his *Brāhmasphuṭasiddhānta* 6.3 [24] gives a different form:

$$l_{v(c)} = \frac{\text{Sin}\,\beta\,\text{Sin}\,\delta_{\lambda+90°}}{R} \tag{10.16}$$

Where Sin $\delta_{\lambda+90°}$ is the Sine of declination corresponding to a longitude of $\lambda + 90°$. Since Sin $\delta_{\lambda+90°} = \frac{\text{Sin}\,\lambda_B\,\text{Sin}\,\varepsilon}{R}$, formula 10.16 is different from Āryabhaṭa and Bhāskara I in the sense that it uses the Sine in place of the versed Sine.

These are comparable with Parameśvara's rule for the elevation or depression of latitude which in formula 10.8. By assigning the elevation of the celestial pole as in formula 10.2, we obtain:

---

[22] "The versed [Sine] multiplied by the latitude and the [Sine of greatest] declination divided by the square of the Radius are subtractive and additive when the [latitude] is northward and southward [respectively] during a northward 'course'; additive and subtractive in a southward 'course'."
*vikṣepāpakramaguṇam utkramaṇaṃ vistarārdhakṛtibhaktam /*
*udagṛṇadhanam udagayane dakṣiṇage dhanam ṛṇaṃ yāmye //36//* (Kern (1874, p. 94))

[23] "The versed [Sine] of the moon diminished by three signs, the [Sine of greatest] declination and the latitude should be multiplied. Experts say [that this] divided by the square of the Radius should be subtracted from the moon when the directions of the 'course' and the inclined circle are the same. In the opposite case, this equation is always additive against the moon."
*varjitatribhavanasya śītagor utkramāpamavisaṃhatiṃ haret //6.2//*
*vyāsavarganicayena śodhayet candramo 'yanavimaṇḍalāśayoḥ /*
*tulyayor dhanam uśanti tadvido vyatyaye śaśini tatphalaṃ sadā //6.3//* (T. Kuppanna Sastri (1957, p. 334))

[24] "The arc minutes, which are the product of the latitude and the [Sine of] declination [of the planet's latitude] with three signs divided by the Radius, should be subtracted if these two are in the same direction and if these two are in different directions they should be added."
*vikṣepasatrirāśikrāntivadho vyāsadalahṛto liptāḥ /*
*śodhyās tayoḥ samadiśor yady anyadiśos tayoḥ kṣepyāḥ //6.3//* (Dvivedī (1902, pp. 93-94))

$$\operatorname{Sin}\zeta_\beta = \frac{\operatorname{Sin}\bar{\alpha}\operatorname{Sin}\beta\operatorname{Sin}\varepsilon}{R^2} \tag{10.17}$$

Alternatively, if we use the crude elevation of ecliptic pole (formula 10.3):

$$\operatorname{Sin}\zeta_\beta = \frac{\operatorname{Sin}\lambda_U\operatorname{Sin}\beta\operatorname{Sin}\varepsilon}{R^2} \tag{10.18}$$

Since we do not know how Brahmagupta and Parameśvara derived their rules, we cannot assert that they belong to the same group. Nonetheless, it is obvious that Parameśvara departs from Āryabhaṭa and Bhāskara I who use the versed Sine in this method. In his super-commentary on *MBh* 6.3, Parameśvara cites 19 verses that give a method that are almost the same as those in *GD2* (T. Kuppanna Sastri (1957, pp. 338-339)).

In addition, Parameśvara applies three steps of corrections in *GD2* 169 (moving from the diurnal circle to the celestial equator, changing the segment to an arc and moving to the ecliptic) while Āryabhaṭa, Bhāskara I and Brahmagupta all skip these processes.

Śrīpati remarks in his *Siddhāntaśekhara* 9.6 [25] that the true (*spaṣṭa*) visibility equation can be obtained by multiplying the initial correction by 1800 and dividing by the rising time of the sign $\rho_n$. This only corresponds to Parameśvara's third and last step for moving from the celestial equator to the ecliptic.

The visibility equation for the "course" according to Bhāskara II is the closest to Parameśvara[26]. However, his steps are distinctly different. Bhāskara II's rule involves an arc which is called the deflection (*valana*)[27] of the "course". *Siddhāntaśiromaṇi Grahagaṇitādhyāya* 5.21cd-22ab[28] gives the rule for this deflection $\gamma_c$ which can be described in the following formula:

$$\gamma_c = \operatorname{arcSin}\left(\frac{\operatorname{Sin}\lambda_U\operatorname{Sin}\epsilon}{r}\right) \tag{10.19}$$

Then *Siddhāntaśiromaṇi Grahagaṇitādhyāya* 7.4 [29] gives the rule for the visibility equation $l_{v(c)}$:

$$l_{v(c)} = \frac{\gamma_c\operatorname{Sin}\beta}{r}\cdot\frac{1800}{\alpha_n} \tag{10.20}$$

I would like to leave the full analysis of this equation by Bhāskara II in comparison with Parameśvara for another occasion. What can be said right away is that Bhāskara II does resemble

[25] "The first visibility equation by the name 'course' multiplied by one thousand eight hundred and divided by the rising time of the sign where the diurnal circle touches is reproduced as the true [correction] in this case." *khanabhodhṛtibhiḥ samāhataṃ prathamaṃ dṛkphalam āyanāhvayam / dyucaraśritabhodayāsubhir vihṛtaṃ spaṣṭam iha prajāyate //9.6//* (Miśra (1932, p. 426))

[26] This has been first pointed out by T. Kuppanna Sastri (1957, p. 338).

[27] The Sanskrit word *valana* means "turning" or "moving round in a circle", but as an astronomical term it has rarely been translated in English except for Burgess and Whitney (1858) who attempted to call it "deflection".

[28] "The 'upright Sine of the moon with the portion [of longitude due to] the motion [of solstices] (i.e. longitude with precession taken into account) is multiplied by the Sine of twenty-four degrees (= greatest declination) and divided by the diurnal 'Sine'. The arc of the obtained result should be the [deflection of] the 'course in the direction of the moon's 'course."
*yutāyanāṃśoḍupakoṭiśiñjinī jināṃśamaurvyā guṇitā vibhājitā //5.21//*
*dyujīvayā labdhaphalasya kārmukaṃ bhavec chaśāṅkāyanadikkam āyanam /* (Chaturvedi (1981, p. 247))

[29] "The deflection of the 'course' multiplied by the non-corrected latitude, divided by the diurnal 'Sine, multiplied by one thousand eight hundred and divided by the rising [time at a place] without geographic latitude of the sign where the planet is based on."
*āyanaṃ valanam asphuṭeṣuṇā saṃguṇaṃ dyuguṇabhājitaṃ hatam /*
*pūrṇapūrṇadhṛtibhir grahāśritavyakṣabhodayahṛd āyanāḥ kalāḥ //7.4//* (Chaturvedi (ibid., p. 278))

Parameśvara in the sense that they are both aware of the difference between the arcs in the diurnal circle, celestial equator and ecliptic, and also that they involve a step for changing the Sine to its arc, but their order of steps are apparently different. Furthermore, Bhāskara II uses the rising time of the sign at a place without geographic latitude $\alpha_n$ in place of the rising time at a given geographic latitude $\rho_n$. I infer from *GD2* 173, which refers to "those who desire to divide by the rising time at Laṅkā", that Parameśvara had been aware of the *Siddhāntaśiromaṇi Grahagaṇitādhyāya* and its rules but did not see it as a text to follow and build his theories upon.

To conclude, no known predecessor of Parameśvara has given the same computation for the visibility method for the "course". But there is still room to consider whether Parameśvara's rules were stated with the same aim as the other treatises. We have discussed that Parameśvara's statements could be educational instructions. It is clear that he did not expect the reader to use the visibility method for the "course" in actual situations, as he denies this method later in *GD2* 179. Meanwhile, other authors might be simply keeping the rule short and approximate for practical reasons.

## 10.11 Visibility method for the geographic latitude (*GD2* 175-177)

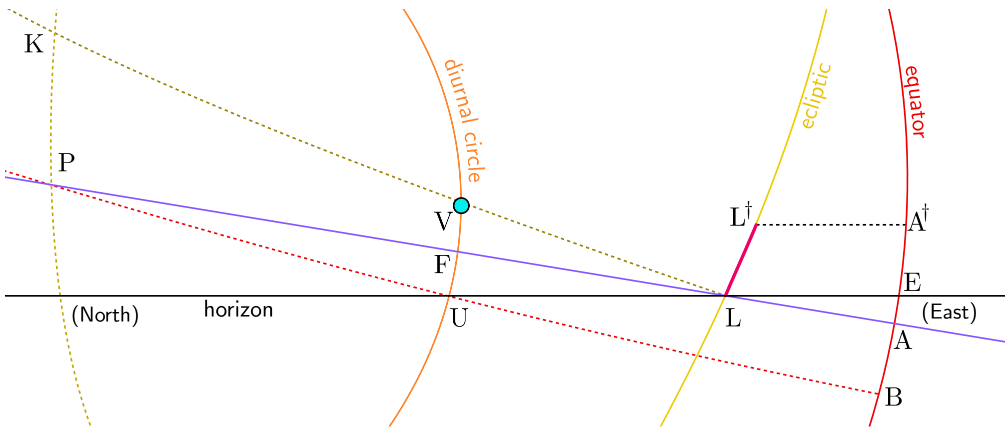



Figure 10.27: Visibility equation for the geographic latitude $\widehat{\mathrm{LL}^\dagger}$ for planet V as seen from inside the sphere.

The term "visibility equation for the geographic latitude (*akṣadṛkphala*)" appears in *GD2* 175. This is the second equation to be applied to the planet's longitude. The steps involved in *GD2* 176-177 resemble those for the visibility equation for the "course"; we move from the diurnal circle of the planet V to the celestial equator, and then to the ecliptic (figure 10.27). L is the planet's longitude on the ecliptic and A is the right ascension of L on the celestial equator. The circle of right ascension, which we have introduced previously for our understanding, goes through A, L and also the celestial pole P. The intersection of this circle with the planet's diurnal circle is F. U is the intersection of the horizon and the diurnal circle that represents the moment when it rises. We have already seen that $\widehat{\mathrm{FV}}$ is the motion of the planet corresponding to the visibility equation for the "course". $\widehat{\mathrm{UF}}$ represents the extra diurnal motion caused by the geographic latitude. $\widehat{\mathrm{BA}}$ is the corresponding time measured on the celestial equator; B is on the same circle of right ascension with U, just like A which is on the circle of right ascension going through F. Then we construct $\widehat{\mathrm{EA}^\dagger}$ with the same arc length such that E is on the horizon.

$\widehat{\mathrm{LL}^{\dagger}}$ is the resulting equation. No reasonings are provided by Parameśvara for the computations involved in this method. Let us first see how the first steps to find the segment corresponding to $\widehat{\mathrm{UF}}$ could have been explained.

### 10.11.1 The computation with the "declination produced by the celestial latitude" and its error

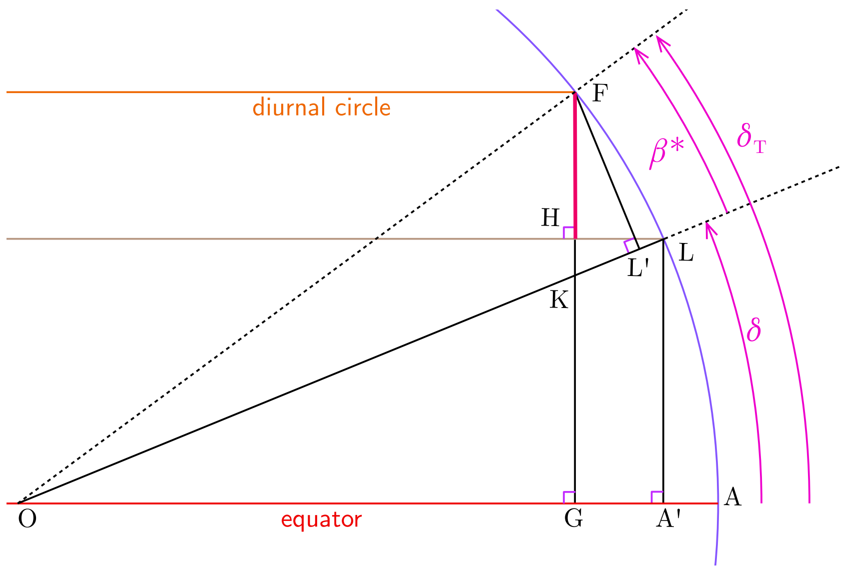



Figure 10.28: The declination produced by the celestial latitude HF as the difference between two Sines $\mathrm{FG} = \mathrm{Sin}\,\delta_{\mathrm{T}}$ and $\mathrm{LA}' = \mathrm{Sin}\,\delta$.

*GD2* 175 begins with a preliminary step where a segment called the "declination produced by the celestial latitude (*vikṣepabhavā-krānti*)" is computed. Despite the name "declination", it is neither an arc nor a Sine of the great circle, but a difference between two Sines (figure 10.28). The diagram shows the circle of right ascension going through F, L and A. The Sine of declination $\mathrm{Sin}\,\delta$ is the perpendicular $\mathrm{LA}'$ drawn from L to OA. On the other hand, I assume that the Sine of declination "corrected by the celestial latitude" refers to the Sine of true declination $\mathrm{FG} = \mathrm{Sin}\,\delta_{\mathrm{T}}$, where G is the foot of the perpendicular drawn from F to OA[30]. The declination produced by the celestial latitude $J'_{\delta}$ is:

$$\begin{aligned} \mathrm{FH} &= |\mathrm{FG} - \mathrm{LA}'| \\ J'_{\delta} &= |\,\mathrm{Sin}\,\delta^* - \mathrm{Sin}\,\delta| \end{aligned} \tag{10.21}$$

I presume that Parameśvara's idea is to use a plane triangle corresponding to the spherical triangle $\triangle$LFU (figure 10.27), where $\angle\mathrm{LFU} = 90°$ and $\angle\mathrm{ULF} = \varphi$ (since the angle of the six o'clock circle against the horizon is $\varphi$ and the circle of right ascension is parallel with the six o'clock circle). If the angles in the plane triangle are the same, we can find the segment

[30]We cannot find any possible explanation for using the "corrected declination $\delta$*", which is the sum or difference of the declination and the uncorrected latitude.

corresponding to $\widehat{\mathrm{UF}}$ from that corresponding to $\widehat{\mathrm{LF}}$, using the Sine of geographic latitude and the Sine of co-latitude and the Radius. But if this is indeed how Parameśvara constructed his rules, his assumption that $\mathrm{FH} = J'_\delta$ is the segment corresponding to $\widehat{\mathrm{LF}}$ is wrong. The correct segment in figure 10.28 is KF, where K is the intersection of FG and OL.

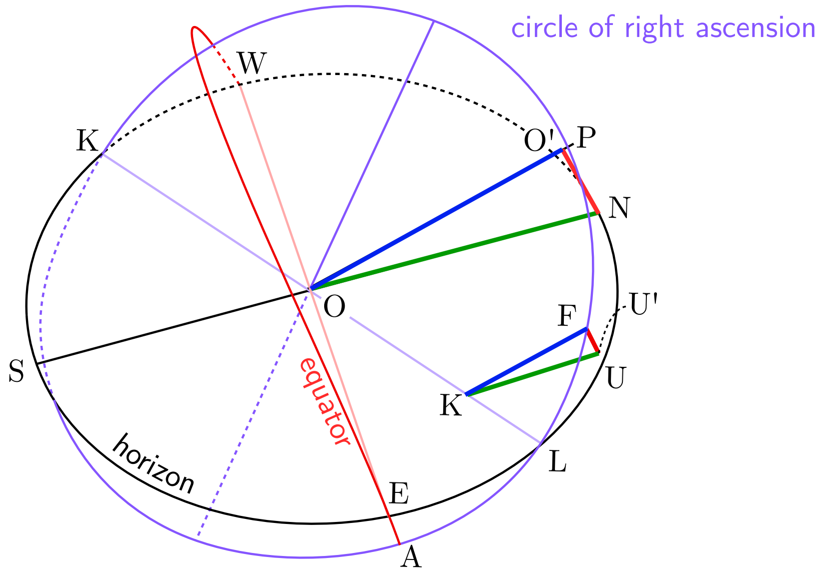



Figure 10.29: The circle of right ascension and $\triangle\mathrm{KFU'}$, the plane triangle corresponding to $\triangle\mathrm{LFU}$.

Figure 10.29 shows the circle of right ascension corresponding to L, going through the celestial north pole P. The intersection of its plane with the plane of the horizon is OL. FU′ is the Sine-like segment in the diurnal circle corresponding to $\widehat{\mathrm{FU}}$; therefore it is perpendicular to the circle of right ascension because the plane of diurnal circle is parallel to the celestial equator. Consequently, $\triangle\mathrm{KFU'}$ is a right triangle.

Meanwhile, when N is due north on the horizon and O′ is the center of a hypothetical diurnal circle with O′N as its radius, as in *GD2* 88 (section 6.7.1), $\triangle\mathrm{OO'N}$ forms a right triangle where O′N is the Sine of geographic latitude $\mathrm{Sin}\,\varphi$ and OO′ is the Sine of co-latitude $\mathrm{Sin}\,\bar{\varphi}$. The planes of $\triangle\mathrm{OO'N}$ and $\triangle\mathrm{KFU'}$ are parallel, $\angle\mathrm{NOO'} = \angle\mathrm{U'KF}$ and therefore $\triangle\mathrm{OO'N} \sim \triangle\mathrm{KFU'}$. Then we find that

$$\begin{aligned}\mathrm{FU'} &= \frac{\mathrm{KF}\cdot\mathrm{O'N}}{\mathrm{OO'}}\\ &= \frac{\mathrm{KF}\cdot\mathrm{Sin}\,\varphi}{\mathrm{Sin}\,\bar{\varphi}}\end{aligned} \tag{10.22}$$

Parameśvara uses $\mathrm{LF} = J'_\delta$ in place of KF (*GD2* 176). From FU′ thus computed, we compute the corresponding Sine in the celestial equator as we will see in the next step.

No other author has used $J'_\delta$ in the visibility equation for the geographic latitude. For example, *Ābh* 4.35 [31] gives the following rule for the visibility equation for the geographic latitude $l_{v(\varphi)}$:

[31] "The Sine of geographic latitude multiplied by the celestial latitude and divided by the [Sine of] co-latitude

$$l_{v(\varphi)} = \frac{\operatorname{Sin}\varphi \operatorname{Sin}\beta}{\operatorname{Sin}\bar{\varphi}} \tag{10.23}$$

Therefore I conclude that this method, although inexact, shows Parameśvara's effort to improve or ground the method. This is remarkable, especially given the fact that he discards the visibility equation for the geographic latitude itself soon after in the same treatise.

### 10.11.2 Steps to move between circles

The steps to find the visibility equation for the geographic latitude $\overset{\frown}{\mathrm{LL}^\dagger} = l_{v(\varphi)}$ (figure 10.27) is identical with those of the visibility equation for the "course" $l_{v(\mathrm{c})}$. However, while Parameśvara put all the steps for $l_{v(\mathrm{c})}$ in one sentence (*GD2* 169), the expression for $l_{v(\varphi)}$ looks different. In the previous case, the segment in the diurnal circle was explicitly referred to as the elevation or depression of latitude. Here, the computation to find FU′ in the diurnal circle (formula 10.22) is integrated with the computation to find the corresponding Sine in the celestial equator and also with the step for changing from the Sine from the arc:

$$\Delta\omega_\beta = \operatorname{arcSin}\left(\frac{J'_\delta \operatorname{Sin}\varphi}{\operatorname{Sin}\bar{\varphi}} \cdot \frac{R}{r}\right) \tag{10.24}$$

Parameśvara stops here and calls this intermediate arc $\Delta\omega_\beta$ (corresponding to $\overset{\frown}{\mathrm{BA}}$ or $\overset{\frown}{\mathrm{EA}^\dagger}$ in figure 10.27) the "ascensional difference made by the celestial latitude (*kṣepakṛtacarāṃśa*)". This is not the case in the visibility method for the "course" (*GD2* 169), where Parameśvara puts all the steps in one sentence without explicating the intermediary segments or arcs.

*GD2* 177 gives the final step for moving from $\Delta\omega_\beta$ in the celestial equator to $l_{v(\varphi)}$ in the ecliptic.

$$l_{v(\varphi)} = \frac{\Delta\omega_\beta \cdot 1800}{\rho_n} \tag{10.25}$$

The same verse gives the rules for whether the equation is additive or subtractive. This depends on whether the celestial latitude is northward or southward.

Figures 10.30 and 10.31 show the situations when the longitude of the planet on the ecliptic L is rising on the horizon. If the celestial latitude is northward, the diurnal circle of the planet would be between the celestial north pole P and the ecliptic (figure 10.30). In this case, the intersection of the diurnal circle and the circle of right ascension F is above the horizon U. Thus the planet gains an extra motion $\overset{\frown}{\mathrm{UF}}$ above the horizon before L rises. Hence the corrected longitude $\mathrm{L}^\dagger$ must be above the horizon too. But $\overset{\frown}{\mathrm{LL}^\dagger}$ is in the direction that the longitude decreases, and therefore the visibility correction for the geographic latitude $l_{v(\varphi)} = \overset{\frown}{\mathrm{LL}^\dagger}$ is subtractive. If the celestial latitude is southward, the diurnal circle is on the other side of the ecliptic from P (figure 10.31). The planet has yet to make an extra motion $\overset{\frown}{\mathrm{FU}}$ below the horizon after L rises, and thus the corrected position of the longitude $\mathrm{L}^\dagger$ is below the horizon. This is also in the direction that the longitude increases, and $l_{v(\varphi)} = \overset{\frown}{\mathrm{LL}^\dagger}$ is additive.

should be subtractive upon rising and additive upon setting when the moon is situated to the north [of the ecliptic], and additive and subtractive when it is situated to the south." *vikṣepaguṇākṣajyā lambakabhaktā bhaved ṛṇam udaksthe |*
*udaye dhanam astamaye dakṣiṇage dhanam ṛṇaṃ candre ||35||* (Kern (1874, p. 93))

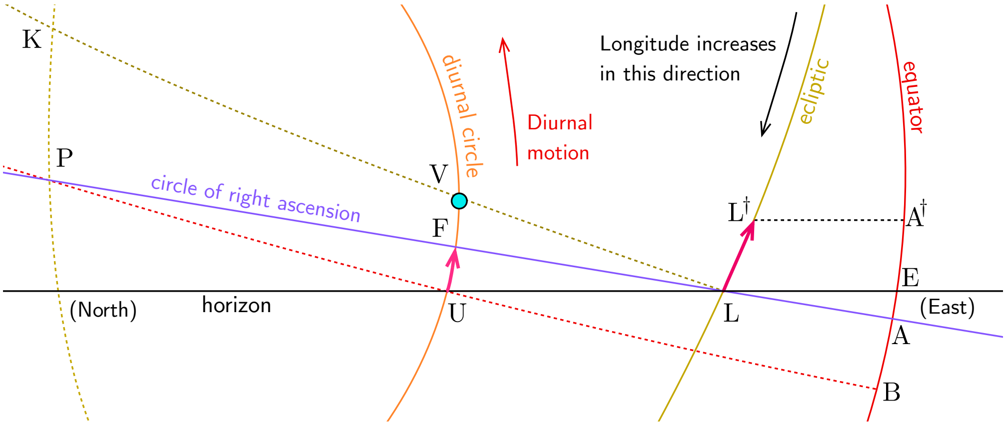



Figure 10.30: $l_{v(\varphi)} = \widehat{\mathrm{LL}^\dagger}$ is subtractive upon rising when the celestial latitude is northward.

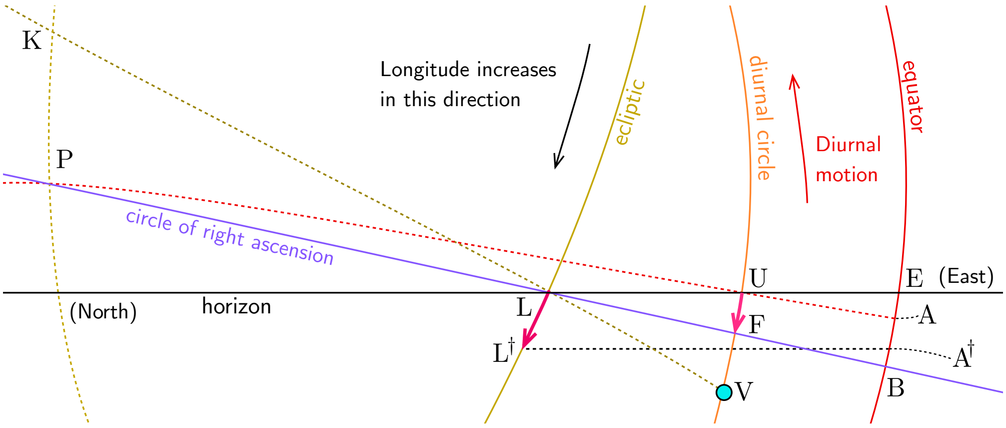



Figure 10.31: $l_{v(\varphi)} = \widehat{\mathrm{LL}^\dagger}$ is additive upon rising when the celestial latitude is southward.

When the planet is setting, we only need to take into account that the direction that the longitude increases is reversed. $l_{v(\varphi)} = \widehat{\mathrm{LL}^\dagger}$ is additive when the celestial latitude is northward (figure 10.32), and subtractive when it is southward (figure 10.33).

Therefore, the longitude $\lambda'$ corrected by the visibility equation for the geographic latitude is:

$$\lambda' = \begin{cases} \lambda - l_{v(\varphi)} & \text{Celestial latitude is northward when planet rises} \\ \lambda + l_{v(\varphi)} & \text{Celestial latitude is southward when planet rises} \\ \lambda + l_{v(\varphi)} & \text{Celestial latitude is northward when planet sets} \\ \lambda - l_{v(\varphi)} & \text{Celestial latitude is southward when planet sets} \end{cases} \tag{10.26}$$

This is as stated in *GD2* 177.

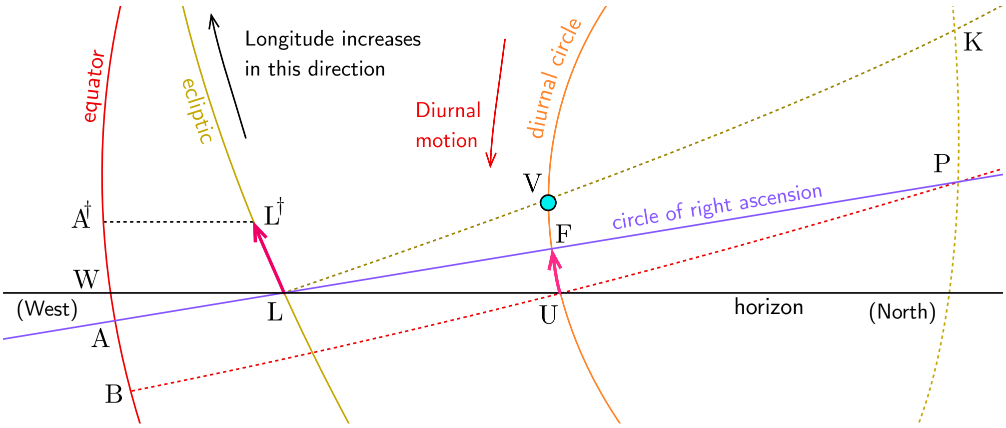



Figure 10.32: $l_{v(\varphi)} = \widehat{\mathrm{LL}^{\dagger}}$ is additive upon setting when the celestial latitude is northward.

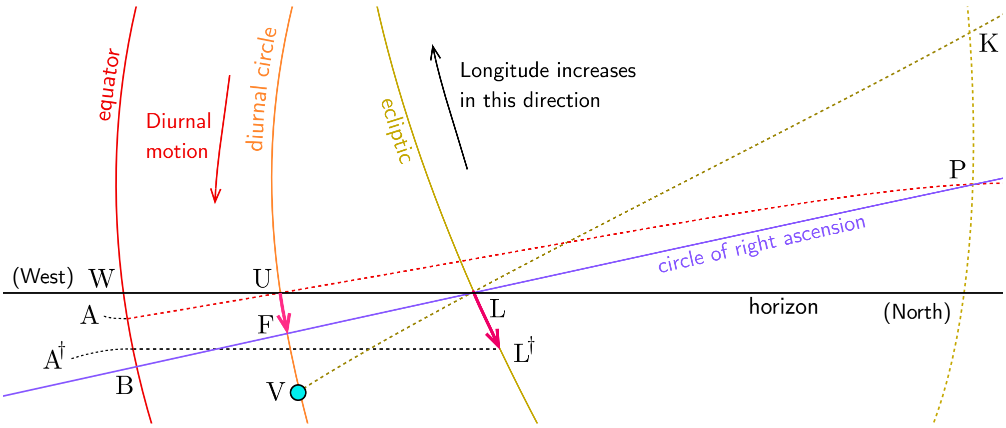



Figure 10.33: $l_{v(\varphi)} = \widehat{\mathrm{LL}^{\dagger}}$ is subtractive upon rising when the celestial latitude is southward.

## 10.12 Unified visibility method (*GD2* 178)

This verse is the beginning of an alternative method where one method is done in place of two. That is to say, only one "visibility equation" will be added to or subtracted from the longitude of the planet in this method. The rule itself is explained much later after the introduction of new segments involved in the computation. In *GD2* 178, there are two things to be considered. What does Parameśvara mean when he says that the visibility methods for the "course" and the geographic latitude are not the true subdivision (*sphuṭāṅga*)? If only one unified method is required, why did he mention the two methods in the first place?

Concerning the first question, I have two hypotheses. One is because each of the two methods involve some approximation. However we do not know whether Parameśvara was aware of those individual approximations. The second is that we cannot visually divide the unified equation on the ecliptic into two arcs which can be exclusively called the equations for the "course" and for the geographic latitude. The distinction was possible on the diurnal circle (FV and FU), but upon moving them to the ecliptic, both had been treated as equations on the position of the planet's longitude (point L) itself. This is problematic when we are to apply both equations, as the second equation should be applied on the once-corrected longitude and not on the initial longitude of the planet. The situation is analogous to the computation of the true planet (appendix C), where two equations for the "slow" and "fast" apogees have to be applied to the mean planet.

As for the second point, Parameśvara might be explaining the rules for the two methods for the reader to locate this subject in relation to other treatises. Authors before Parameśvara, including Āryabhaṭa and Bhāskara I do not use a unified method. Another reason could be to show that there are two causes behind a single equation.

Nīlakaṇṭha's *Tantrasaṅgraha* explains the two visibility equations in 7.1-4ab (Ramasubramanian and Sriram (2011, p. 385)) and then gives an unified method in 7.8-9 (Ramasubramanian and Sriram (ibid., p. 394)). This style resembles *GD2*, but as we will see, their computations differ. Their relation and the origin of these unified methods are yet to be studied.

### 10.12.1 Process of the unified method

Parameśvara does not emphasize what the essential steps in the procedure are. In between the rules, Parameśvara inserts what may be the grounding for the computation, or introduces new points and arcs. We can summarize the steps as follows:

- Longitude of the sun at midday [$\lambda_\Sigma$] and hour angle [$H$] → Longitude of midheaven [$\lambda_M$] (*GD2* 182)
- ($\lambda_M$ → Declination of midheaven [$\delta_M$])
- $\delta_M$ and geographic latitude $\varphi$ → Midheaven Sine [$\operatorname{Sin} z_M$] (*GD2* 184)
- $\operatorname{Sin} z_M$ → Midheaven gnomon [$\mathcal{G}_M$] (*GD2* 186)
- Longitude of midheaven [$\lambda_M$] and longitude of ascendant point [$\lambda_{Asc}$] → "Base" of midheaven gnomon [$\mathcal{B}_{\mathcal{G}_M}$] (*GD2* 186)
- $\mathcal{G}_M$ and $\mathcal{B}_{\mathcal{G}_M}$ → Gnomon of sight-deviation [$\mathcal{G}_D$] (*GD2* 187)
- $\mathcal{G}_D$ → Sine of sight-deviation [$\operatorname{Sin} z_D$] (*GD2* 187)
- $\operatorname{Sin} z_D$ and Sine of latitude [$\operatorname{Sin}\beta$] → Elevation or depression of latitude [$\operatorname{Sin}\zeta_{\varphi\beta}$] (*GD2* 190-191)
- $\delta_M$ → diurnal "Sine" [$r$] (no reference)
- $\operatorname{Sin}\zeta_{\varphi\beta}$, Sine of co-latitude [$\operatorname{Sin}\bar{\varphi}$] and $r$ → Visibility equation along the equator [$l'_v$] (*GD2* 192)
- $l'_v$ and rising time of the ascending sign [$\rho_n$] → Visibility equation on the ecliptic [$l_v$] (*GD2* 193)

The main point of this procedure is to redefine the elevation of ecliptic pole and the elevation or depression of latitude. In the visibility method for the "course", they are measured from the six o'clock circle, whereas here the horizon at the given geographic latitude is the reference. Therefore, starting with the elevation or depression of latitude $\operatorname{Sin}\zeta_{\varphi\beta}$, we can follow the same steps as in the visibility method for the "course" to find the visibility equation which takes into account the geographic latitude. What is new in the procedure is the additional steps to find the redefined elevation.

In general, the segments or arcs involved in the procedure are stated in the order that they should be computed. But there is one exception: Segments related to a point called the "sight-deviation" is used later in the procedure at *GD2* 187, but Parameśvara defines the point of sight-deviation in *GD2* 179-181 before starting with the procedure itself.

## 10.13 The ecliptic point of sight-deviation and its Sine (*GD2* 179-181)

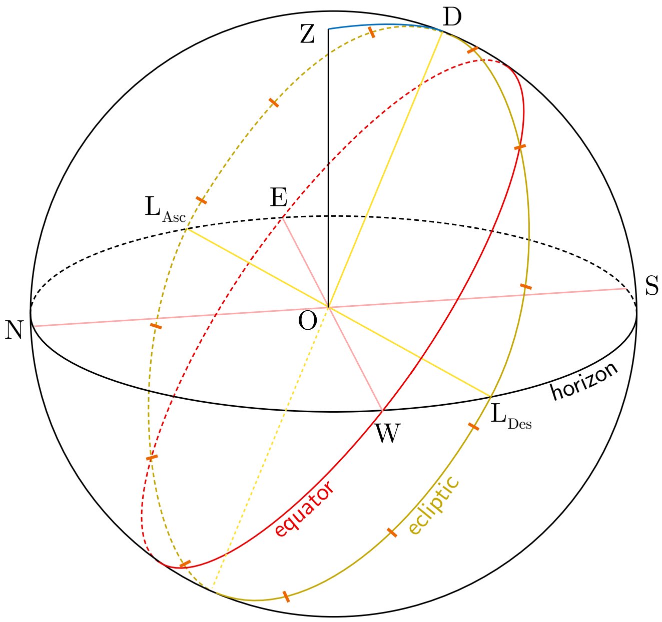



Figure 10.34: The ecliptic point of sight-deviation D. The ecliptic is graduated with signs.

According to *GD2* 179, the term "sight-deviation (*dṛkkṣepa*)" refers to the midpoint on the ecliptic above the horizon (figure 10.34, 10.35). Parameśvara supplies that half of the ecliptic is always above the horizon and half is always below. We can understand this as an intersection of two great circles (ecliptic and horizon). Since an arc length of six signs is above the horizon, the distance from the ascending point $L_{Asc}$ to the ecliptic point D should be three signs. The longitude on the ecliptic decreases from east to west, and therefore the longitude of sight-deviation $\lambda_D$ is the longitude of the ascendant $\lambda_{Asc}$ decreased by three signs, as stated in *GD2* 180.

$$\lambda_D = \lambda_{Asc} - 3^s \tag{10.27}$$

In *GD2* 181, Parameśvara uses the same word *dṛkkṣepa* in the form of *dṛkkṣepajyā* (Sine of sight-deviation). This is described as the Sine corresponding to the arc distance $z_D$ of the ecliptic point of sight-deviation from the zenith (figure 10.36). In this case, the word *dṛkkṣepa* might refer to the arc $z_D$ rather than the point. Actually, the latter interpretation is more common (cf. Bhattacharya (1987, p. 50)) and other authors rarely use *dṛkkṣepa* to signify the ecliptic point of sight-deviation, although what this term alone means for Parameśvara and others remains a question[32].

[32] See glossary entries *dṛkkṣepa* (1) and *dṛkkṣepa* (2) for more discussion.

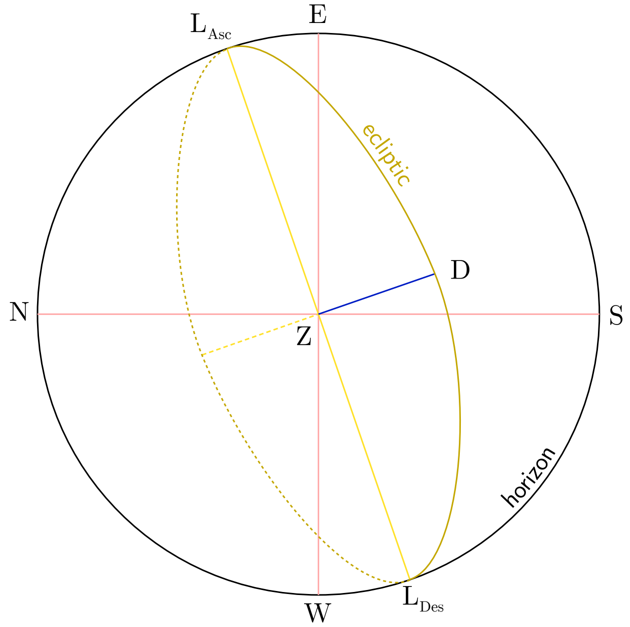



Figure 10.35: The ecliptic point of sight-deviation D as seen from above, with the zenith Z as center.

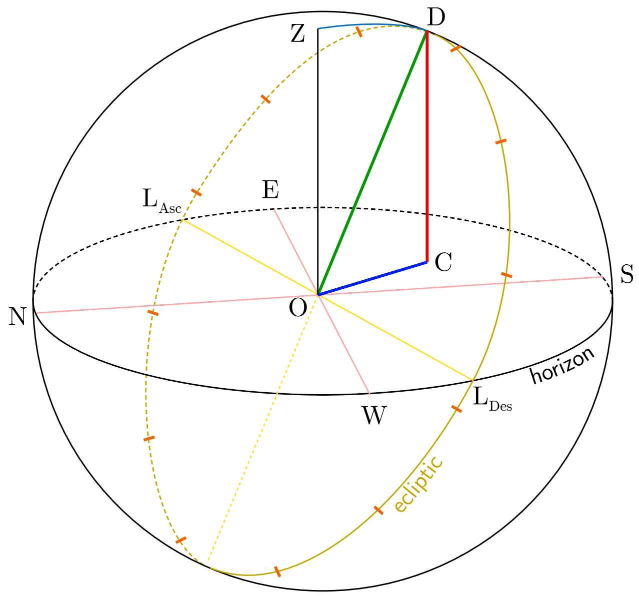



Figure 10.36: The Sine of sight-deviation $\text{Sin}\, z_D$ = OC corresponding to the distance of the ecliptic point of sight-deviation from the zenith $\widehat{\text{ZD}}$.

The statement on the sight-deviation in *GD2* 179-181 is separated from its actual usage to find the unified visibility method in *GD2* 187-191. My hypothesis for explaining this inconsistency is that Parameśvara wants to make the sight-deviation stand out among other miscellaneous segments and arcs involved in the process. Prior to Parameśvara, the sight-deviation was used only for computing the parallax, and not for the visibility method. Parameśvara introduces the sight-deviation in both situations. Therefore the Sine of sight-deviation appears again in *GD2* 270-276 to find the longitudinal and latitudinal parallaxes. If someone concentrating on that section had to review the topic of the sight-deviation, *GD2* 179-181 would serve as a marker, and the reader could easily find the description in these verses, or advance up to *GD2* 187 to understand the steps to find the value of the Sine of sight-deviation.

## 10.14 The midheaven ecliptic point and its Sine (*GD2* 182-185, 194)

### 10.14.1 Finding the longitude of midheaven

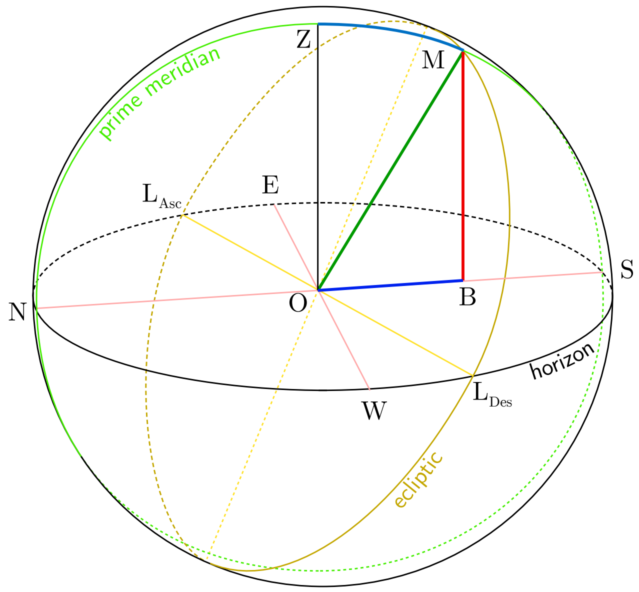



Figure 10.37: The midheaven M in the ecliptic.

Effectively, the unified visibility method starts with finding a point on the ecliptic called "midheaven (*madhyavilagna*)", which is the intersection of the ecliptic and the prime meridian (figure 10.37). As was the case with the point of sight-deviation, its arc distance from the zenith ZM $= z_M$ forms a Sine OB $= \text{Sin}\, z_M$ called the midheaven Sine (*madhyajīvā*) which appears later in *GD2* 184. *GD2* 182c explains the word *madhya* as derived from midday (*madhyāhna*). At midday, the position of the sun is the midheaven. However, the midday can be defined at any time of the day, regardless of the sun's position.

Not only is *GD2* 182c an explanation for the etymology, but together with *GD2* 182d, it implies how the longitude of midheaven must be computed; with the longitude of the sun, the hour angle (*nata*) and the measure at Laṅkā (*laṅkāmiti*). Parameśvara does not describe the process in *GD2*, but has stated it at the end of his commentary on *Ābh* 4.33. Before looking at the computation itself, let us first see why the measure at Laṅkā, i.e. the point or length of arc in the celestial equator that rises simultaneously with a given longitude or sign in the ecliptic as explained in *GD2* 89-102 (chapter 7) is involved. Parameśvara has spared *GD2* 183 for this reasoning.

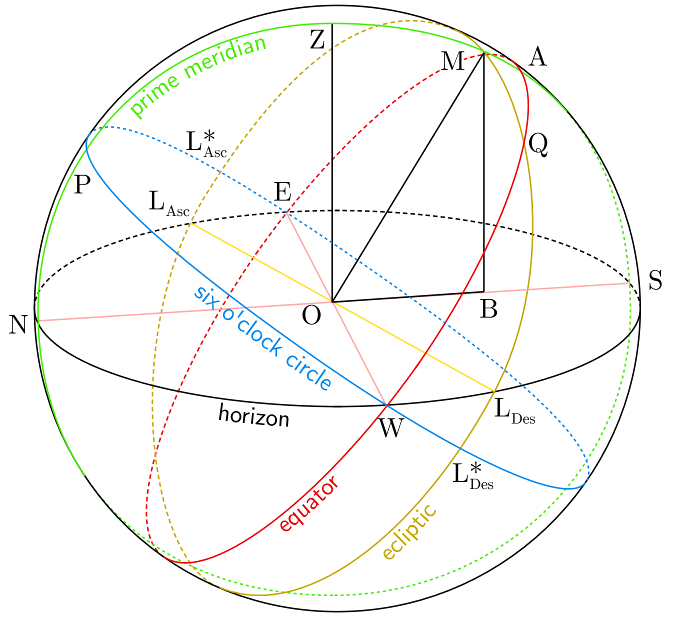



Figure 10.38: The ascending and descending points change in accordance with the geographic latitude, but the correspondence between midheaven M and culminating point A in the celestial equator remains unchanged.

Parameśvara argues in *GD2* 183 that the measure of midheaven (*madhyamāna*) which I interpret as the arc distance $\overset{\frown}{\mathrm{QA}}$ along the celestial equator between an equinoctial point Q and the point A that culminates with midheaven[33], is equal to the measure or "rising time" $\alpha_M$ observed at Laṇkā (section 7.3).

The correspondence between midheaven M and point A can be visualized as in figure 10.38 and 10.39. The six o'clock circle corresponds to the horizon as seen from Laṅkā. The geographic latitude causes the ascending point on the ecliptic (corresponding to E in the celestial equator) to move from $\mathrm{L}^*_{\mathrm{Asc}}$ to $\mathrm{L}_{\mathrm{Asc}}$ and the descending point (corresponding to W) from $\mathrm{L}^*_{\mathrm{Des}}$ to $\mathrm{L}_{\mathrm{Des}}$, but M and A remain on the prime meridian.

Meanwhile, Parameśvara uses the ascensional difference of the signs for his reasoning in *GD2* 183. His argument resembles *GD2* 174 (figure 10.25 and 10.26). The logic seems to be that the ascensional difference and descensional difference of a given longitude is the same value in opposite direction and therefore should be zero at the middle.

I imagine that these explanations could be done easily by moving the armillary sphere. The instrument would also demonstrate that the two points M and A rise simultaneously above the horizon at the terrestrial equator. Therefore the measure of midheaven is equal to the rising time at Laṅkā $\alpha_M$.

[33] This is almost identical with the "measure" as stated in *GD2* 89-102; the only difference is that we are taking the prime meridian instead of the horizon as the reference.

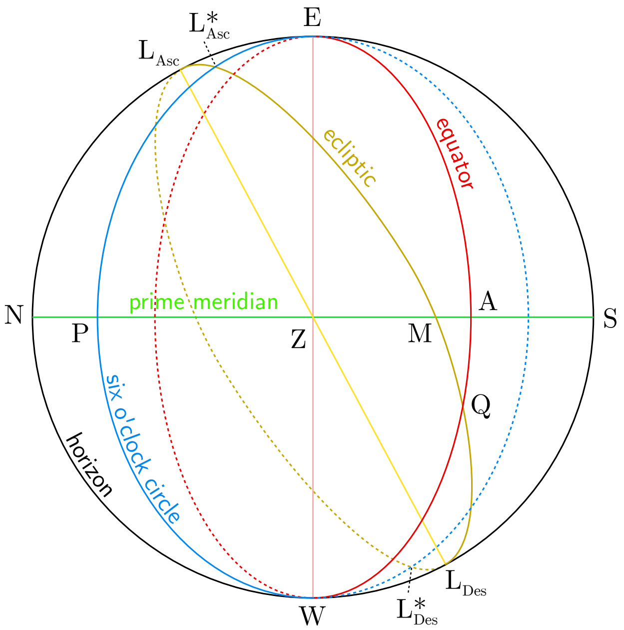



Figure 10.39: The configuration of figure 10.38 seen from above.

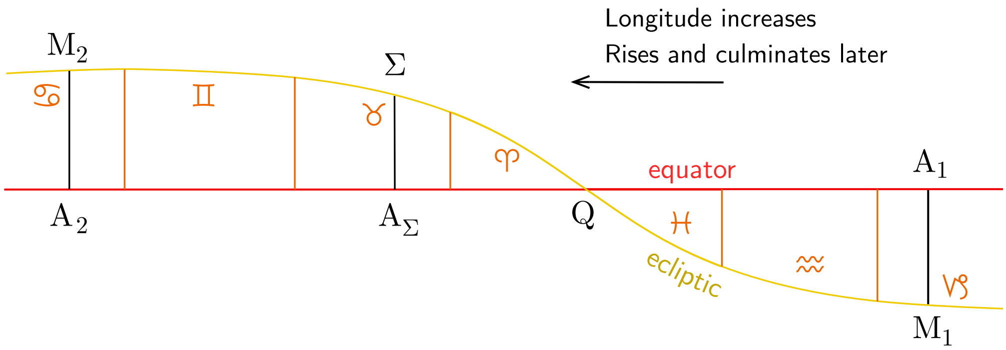



Figure 10.40: Points on the celestial equator corresponding to the sun $\Sigma$ and the midheaven, before noon ($M_1$) or afternoon ($M_2$)

This correspondence between the midheaven and a point on the celestial equator enables us to easily find the longitude of midheaven for a given moment. The hour angle $H$ is the time difference between the culmination of a celestial body (in our case, the sun) and a given moment (see also section 18.4), which corresponds to an arc along the celestial equator. Therefore, to find the midheaven $M_1$ at a moment before noon, we should subtract the longitude $\widehat{\Sigma M_1}$ corresponding to the hour angle $\widehat{A_\Sigma A_1}$ and for the midheaven $M_2$ in the afternoon, the longitude $\widehat{\Sigma M_2}$ corresponding to the hour angle $\widehat{A_\Sigma A_2}$ should be added. This seems to be what Parameśvara describes in his commentary on *Ābh* 4.33:

As for the midheaven ecliptic point: When it is before noon, one should subtract the rising

> [time] at Laṅkā in *asu*s (i.e. *prāṇa*s) [of signs] in reverse order beginning with the portion of the sign where the sun is situated from the hour angle in *asu*s, subtract the corresponding signs from the sun and establish [the midheaven longitude]. Meanwhile in the afternoon, one should subtract the rising [time] at Laṅkā in *asu*s in order beginning with the portion where the sun is situated from the hour angle in *prāṇa*s, add the corresponding signs to the sun and establish [the midheaven longitude].[34]

The computation starts with two numbers, the hour angle in *prāṇa*s and the sun's longitude, probably in signs and minutes[35]. The measure of signs are subtracted from the hour angle in reverse order if the moment concerned is before noon. In our example in figure 10.40, we start with Taurus ♉ where the sun is located and go backward to Aries ♈, Pisces ♓ and so on until no *prāṇa*s are left. Each time we subtract a measure of sign, we subtract one sign (1800 minutes) from the longitude, as we are going in the direction which the longitude decreases. Parameśvara does not mention what to do with fractions of signs, but I assume that this was managed with linear interpolation, as we have seen in *GD2* 172. Thus in our example, we should first find the minutes of arc between the longitude of the sun and the beginning of Taurus, multiply the number with the measure of Taurus $\alpha_{♉}$ and divide it with 1800 to find the corresponding *prāṇa*s to subtract from the hour angle. Likewise, after reaching the beginning of Aquarius ♒ we must multiply the remaining number of *prāṇa*s with 1800 and divide it with the measure of Capricorn $\alpha_{♑}$ to find the minutes of arc to subtract from the longitude and locate the longitude of midheaven $\mathrm{M}_1$ inside Capricorn. As for the case in the afternoon, we must add the signs and minutes since we are going in the direction in which the longitude increases.

The longitude of midheaven $\lambda_M$ itself is used later in *GD2* 186cd, but the next step in *GD2* 184 requires the declination of midheaven $\delta_M$. If we are to follow the rules in *GD2* strictly, we need to compute the "base" Sine of the longitude, $\mathrm{Sin}\,\lambda_{B(M)}$ compute the Sine of declination $\mathrm{Sin}\,\delta_M$ using *GD2* 73ab (formula 6.3) and convert it into an arc. However, we cannot rule out the possibility that tables were used for direct conversion (see appendix B.6.2).

### 10.14.2 Computing the midheaven Sine

*GD2* 184 gives the rule to find the arc corresponding to the meridian zenith distance of the midheaven $z_M$ from the two arcs, the declination of midheaven $\delta_M$ and the geographic latitude $\varphi$, depending on whether they are in the same or opposite direction. The Sine of this arc $\mathrm{Sin}\,z_M$ is the midheaven Sine.

$$z_M = \begin{cases} \delta_M + \varphi & \text{(a) Same direction} \\ |\delta_M - \varphi| & \text{(b) Opposite direction} \end{cases} \tag{10.28}$$

There is one important piece of information which can be derived from this simple rule. Parameśvara is assuming that the direction of the geographic latitude is southward (figure 10.41),

[34] *madhyalagnaṃ tu pūrvāhṇe natāsubhyo ravisthitarāśibhāgād utkrameṇa laṅkodayāsūn viśodhya tāvato rāśīn ravau viśodhya sādhyam / aparāhṇe tu nataprāṇebhyo ravisthitabhāgāt krameṇa laṅkodayāsūn viśodhya tāvato rāśīn ravau prakṣipya sādhyam //* (Kern, 1874, p. 92)

[35] We can see that the longitude shorter than one sign is measured in minutes of arc from rules that correlate a measure of sign with "one thousand eight hundred minutes of arc" such as in *GD2* 172. Whether degrees of arc were involved is debatable. The commentator on *GD2* 209 shows us one possibility: only arc minutes appear in intermediate steps, but to denote the final value of the longitude, minutes are converted to signs, degrees and minutes.

contrary to the modern notion that the geographic latitude is northward for those in the northern hemisphere. This confirms our inference from *GD2* 70 (section 6.1). The direction of the midheaven Sine is stated oddly later in *GD2* 194. It goes from the zenith towards midheaven, and thus it is southward if both $\delta_M$ and $\varphi$ are southward (figure 10.41 (a)), southward when $\delta_M$ is northward but $\delta_M < \varphi$ (figure 10.41 (b) i) and northward if $\delta_M > \varphi$ (figure 10.41 (b) ii).

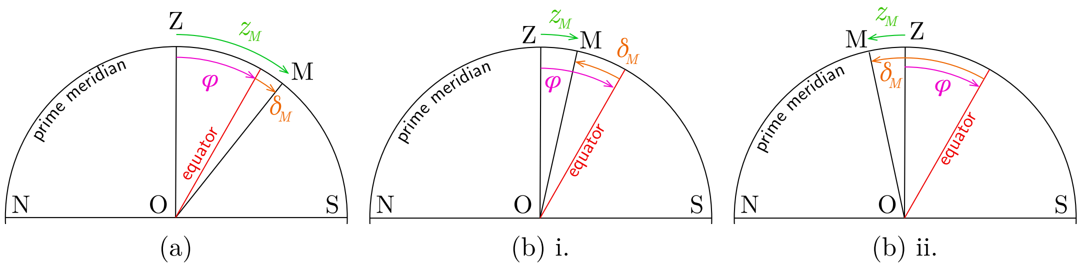



Figure 10.41: Zenith distance of midheaven $z_M$ when (a) its declination $\delta_M$ is southward, (b) i. when $\delta_M$ is northward but smaller than $\varphi$ and (b) ii. when it is larger. To be consistent with Parameśvara's expression in *GD2* 184, the geographic latitude $\varphi$ has to be southward.

*GD2* 185 adds some explanation, describing the geographic latitude as an arc in the "gap between the celestial equator and the zenith (*ghāṭikakhamadhyavivara*)" and the declination as in the "gap between the celestial equator and the diurnal circle (*ghaṭikādyuvṛttavivara*)". However, the word order does not agree with the direction of the arcs. The "diurnal circle" should be that corresponding to the declination of the midheaven (figure 10.42).

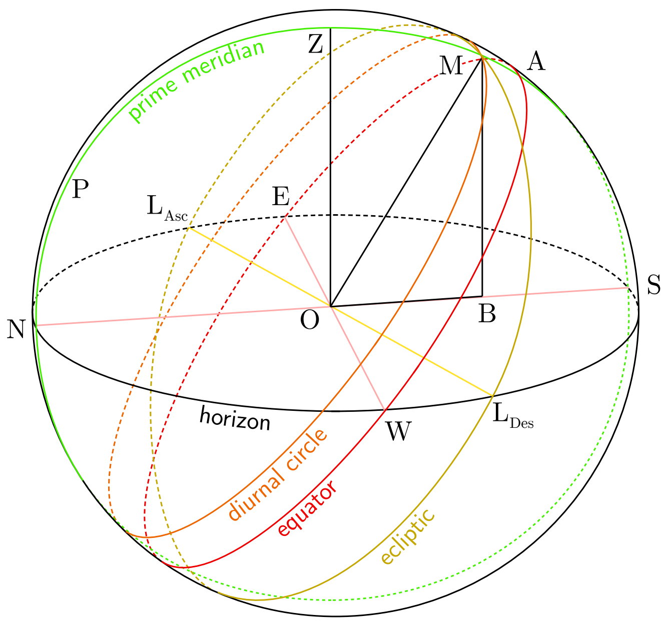



Figure 10.42: The geographic latitude $\overset{\frown}{\mathrm{ZA}}$ between the zenith and the celestial equator, and the declination $\overset{\frown}{\mathrm{AM}}$ between the celestial equator and the diurnal circle of the midheaven.

## 10.15 Midheaven gnomon and its "base" (*GD2* 186)

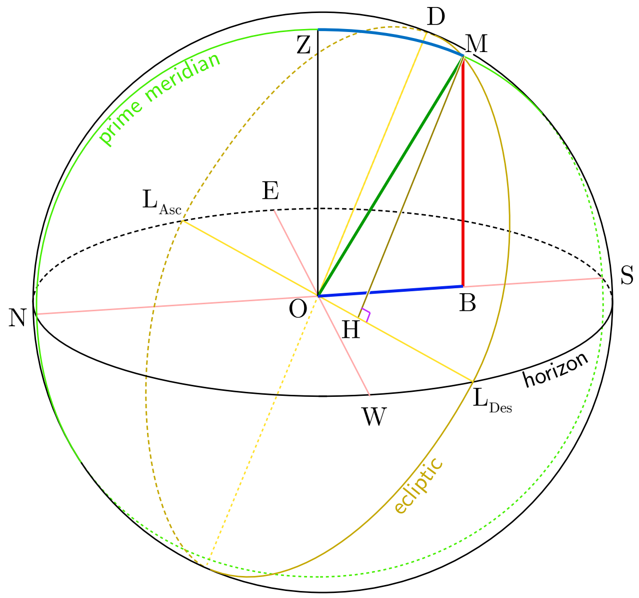



Figure 10.43: Midheaven gnomon BM $= \mathcal{G}_M$ and its "base" MH $= \mathcal{B}_{(\mathcal{G}_M)}$.

The midheaven gnomon (*madhyaśaṅku*) $\mathcal{G}_M$ is the elevation of the midheaven BM (figure 10.43). Since △OBM is a right triangle, the following rule in *GD2* 186ab can be obtained:

$$\begin{aligned} \mathrm{BM} &= \sqrt{\mathrm{MO}^2 - \mathrm{OB}^2} \\ \mathcal{G}_M &= \sqrt{R^2 - \mathrm{Sin}^2\, z_M} \end{aligned} \tag{10.29}$$

*GD2* 187cd gives the rule for another segment which is called the "'base' of the midheaven gnomon (*madhyaśaṅkubhujā*)". This is the "base" Sine of the ascending longitude $\lambda_{Asc}$ decreased by the longitude of the midheaven $\lambda_M$.

$$\mathcal{B}_{(\mathcal{G}_M)} = \mathrm{Sin}(\lambda_{Asc} - \lambda_M)_B \tag{10.30}$$

In this case, the references for the "base" are not the equinoctial points but the ascending and descending points. That is, $\mathrm{Sin}(\lambda_{Asc} - \lambda_M)_B = \mathrm{Sin}(\lambda_{Asc} - \lambda_M)$ while the arc is smaller than $90°$, but when it is larger, the descending longitude $\lambda_{Des}$ is used instead and $\mathrm{Sin}(\lambda_{Asc} - \lambda_M)_B = \mathrm{Sin}(\lambda_M - \lambda_{Des})$.

In figure 10.43, $\mathcal{B}_{(\mathcal{G}_M)}$ is the perpendicular MH drawn from the midheaven M to $\mathrm{L_{Asc}L_{Des}}$, the line between the ascending and descending points. The reason why this segment is associated with the midheaven gnomon is uncertain, but it might be because this segment is the midheaven gnomon projected on the plane of the ecliptic. In addition, calling this segment the "base" of the midheaven would cause confusions with $\mathrm{Sin}\,\lambda_M$, the "base" Sine with reference to the equinoctial points.

## 10.16 Gnomon of sight-deviation and Sine of sight-deviation (*GD2* 187-188, 194)

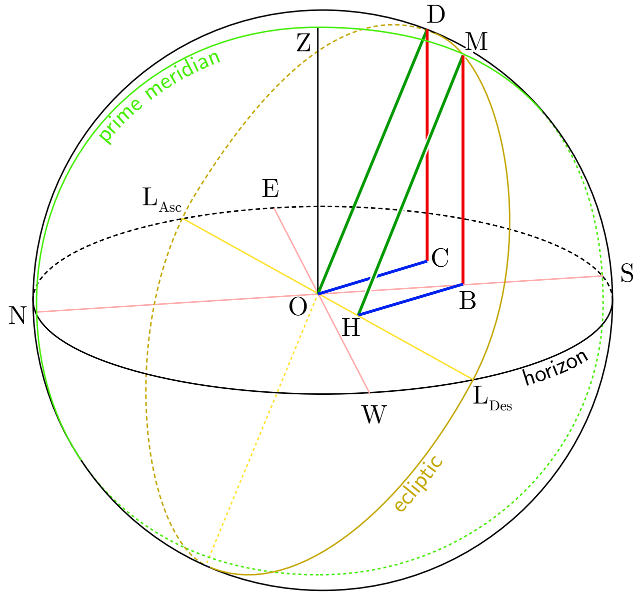



Figure 10.44: Two similar triangles, △HBM formed with the midheaven gnomon and △OCD with the gnomon of sight-deviation.

The "base" of the midheaven gnomon $\mathcal{B}_{(\mathcal{G}_M)}$ takes its largest value $R$ when the argument is 90 degrees, that is, when the midheaven coincides with the point of sight-deviation. Even when they are separated, $\mathcal{B}_{(\mathcal{G}_M)} = \text{MH}$ is parallel with the Radius DO with the point of sight-deviation at its end. *GD2* 188 stresses that they are both Sines in the ecliptic. CD corresponds to the midheaven gnomon BM, and is called the gnomon of sight-deviation (*dṛkkṣepaśaṅku*) $\mathcal{G}_D$. The two gnomons are also parallel because they are both perpendicular against the horizon. Thus $\angle\text{CDO} = \angle\text{BMH}$ and $\angle\text{OCD} = \angle\text{HBM} = 90°$. Therefore $\triangle\text{OCD} \sim \triangle\text{HBM}$. *GD2* 188 states the Rule of Three to find the length of $\mathcal{G}_D$. The computation, as stated in *GD2* 187, is:

$$\text{CD} = \frac{\text{DO} \cdot \text{BM}}{\text{MH}}$$
$$\mathcal{G}_D = \frac{R\mathcal{G}_M}{\mathcal{B}_{(\mathcal{G}_M)}} \tag{10.31}$$

*GD2* 187 briefly adds that the great shadow OC corresponding to the gnomon of sight-deviation is the Sine of sight-deviation Sin $z_D$. This suggests that we can use *GD2* 114ab (formula 8.8), which is the rule for computing the great shadow with the Pythagorean theorem:

$$\text{OC} = \sqrt{\text{DO}^2 - \text{CD}^2}$$
$$\text{Sin}\, z_D = \sqrt{R^2 - \mathcal{G}_D^2} \tag{10.32}$$

Since OC ∥ HB, the points of sight-deviation and midheaven are always on the same side from the zenith. Thus the Sine of sight-deviation and the midheaven Sine are in the same direction and *GD2* 194 states the rule to find their direction collectively. This direction is needed right afterward in *GD2* 189, and it is strange that Parameśvara has added this rule at the very end of the procedure.

### 10.16.1 The "true" Sine of sight-deviation and the approximative method

*GD2* 187 refers to the Sine of sight-deviation computed in this procedure as "true (*sphuṭa*)", which suggests that there must be an "untrue" Sine of sight-deviation. I assume that the following method by Āryabhaṭa, presented in *Ābh* 4.33, is in Parameśvara's mind.

> The product of the midheaven Sine and the rising Sine is divided by the Radius. The square root of the difference between the squares of this and the midheaven Sine is [the planet's] own Sine of sight-deviation.[36]

The "rising Sine (*udayajīvā*)" $\mathrm{Sin}\,\upsilon$ is the Sine corresponding to the arc distance between due east on the horizon and the ascending ecliptic point. Parameśvara does not give any reasonings for this rule in his commentary on *Ābh* 4.33. Meanwhile, Govindasvāmin's commentary on *MBh* 5.23 quotes this verse and explains it in an instruction of a drawing (*chedyaka*) (T. Kuppanna Sastri (1957, pp. 276-277)). Parameśvara adds further comments on this instruction in his *Siddhāntadīpikā*. The following is my interpretation of the grounding based on the commentaries of Govindasvāmin and Parameśvara[37].

Figure 10.45 shows the ecliptic projected on the plane of the horizon with the cardinal directions N, S, E and W. The intersection of the two circles, $\mathrm{L_{Asc}}$ and $\mathrm{L_{Des}}$, are the ascending and descending ecliptic points. The distance of the ascending point from the east-west line EW, $\mathrm{PL_{Asc}}$ is the rising Sine. The intersection of the ecliptic with the north-south line NS, B, is the foot of the midheaven gnomon. OF is a Radius in the horizon that is perpendicular to $\mathrm{L_{Asc}L_{Des}}$. Its intersection with the ecliptic, C, is the foot of the gnomon of sight-deviation. OB is the midheaven Sine and OC is the Sine of sight-deviation. OBC is the projection of the spherical triangle △ZDM on the plane of the horizon (figure 10.46): the spherical angle ∠ZDM is a right angle but ∠OBC is not, as we will see below.

A circle is drawn around O with $\mathrm{OB} = \mathrm{Sin}\, z_M$ as its radius, and its intersection with OF is G (thus $\mathrm{OG} = \mathrm{Sin}\, z_M$). Q and H are the foots of the perpendiculars drawn from F and G on NS.

Comparing $\triangle\mathrm{OPL_{Asc}}$ and $\triangle\mathrm{OQF}$, $\angle\mathrm{L_{Asc}OP} = 90° - \angle\mathrm{POF} = \angle\mathrm{FOQ}$, $\angle\mathrm{OPL_{Asc}} = \angle\mathrm{OQF} = 90°$ and $\mathrm{L_{Asc}O} = \mathrm{OQ} = R$. The hypotenuse and an acute angle is equal, and thus $\triangle\mathrm{OPL_{Asc}} \equiv \triangle\mathrm{OQF}$. Therefore $\mathrm{QF} = \mathrm{PL_{Asc}} = \mathrm{Sin}\,\upsilon$. △OQF and △OHG are right triangles sharing one acute angle and thus △OQF ∼ △OHG. Therefore,

$$\begin{aligned}\mathrm{HG} &= \frac{\mathrm{QF}\cdot\mathrm{OG}}{\mathrm{OF}}\\ &= \frac{\mathrm{Sin}\,\upsilon\,\mathrm{Sin}\,z_M}{R}\end{aligned} \tag{10.33}$$

[36] *madhyajyodayajīvāsaṃvarge vyāsadalahṛte yat syāt / tanmadhyajyākṛtyor viśeṣamūlaṃ svadṛkkṣepaḥ //4.33//* (Kern (1874, p. 92))

[37] Govindasvāmin's commentary contains several difficult compounds, and Parameśvara does not explain them literally but often substitutes them with his own words. Govindasvāmin proceeds to illustrate the Sine of sight-deviation and Sine of sight-motion, which is rejected by Parameśvara (see section 21.6). Therefore there seems to be a gap between the notions of Govindasvāmin and Parameśvara. Due to this complexity in their commentaries, I have decided not to translate and interpret them literally here.

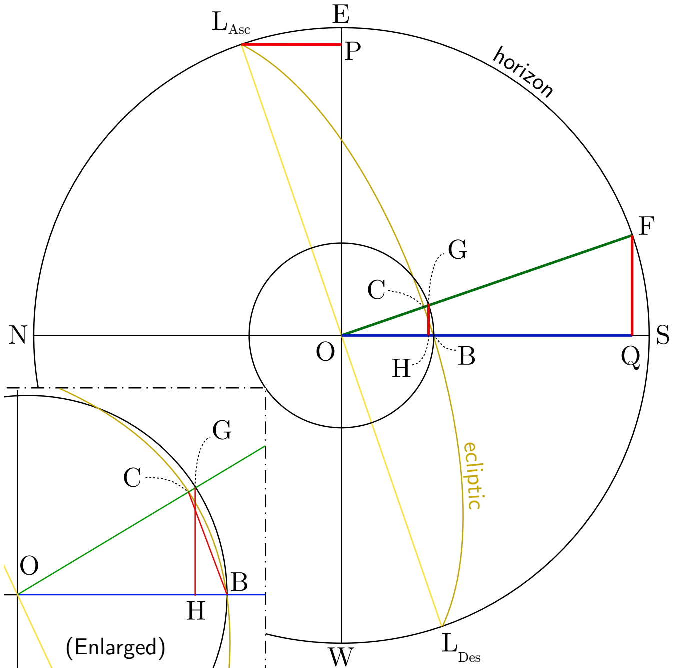



Figure 10.45: The midheaven B and the ecliptic point of sight-deviation C projected on the plane of horizon, and the rising Sine $L_{Asc}G$.

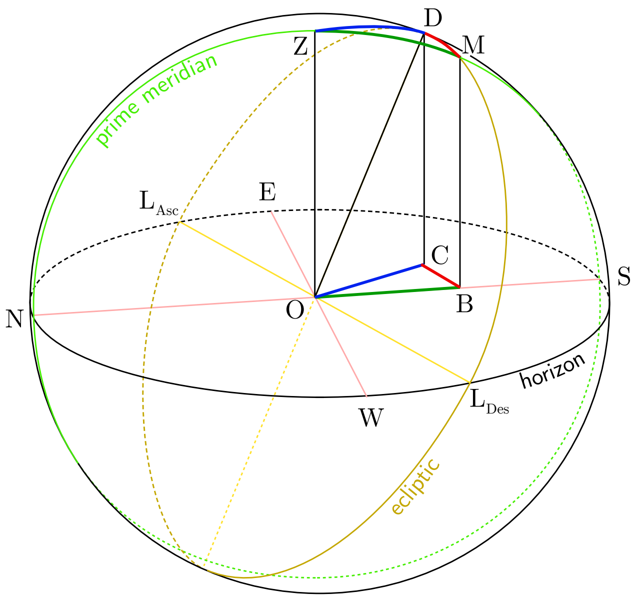



Figure 10.46: $\triangle$OCB and spherical triangle $\triangle$ZDM.

It is assumed that the distance between the foots of the midheaven gnomon and the gnomon of sight-deviation BC is equal to HG but this is incorrect. If BC were a tangent of the ecliptic, BC $\perp$ OG, and therefore $\triangle$OBC $\equiv$ $\triangle$OHG and BC = HG, but BC is not a tangent and BCO < 90°. Yet the Pythagorean theorem is used to find OC = $\mathrm{Sin}\, z_D$ on the premise that BCO = 90° and BC = HG. Neither Govindasvāmin nor Parameśvara mention that this is an approximation.

$$\begin{aligned} \mathrm{OC} &\sim \sqrt{\mathrm{BO}^2 - \mathrm{BC}^2} \\ \mathrm{Sin}\, z_D &\sim \sqrt{\mathrm{Sin}^2 z_M - \left(\frac{\mathrm{Sin}\, \upsilon\, \mathrm{Sin}\, z_M}{R}\right)^2} \end{aligned} \tag{10.34}$$

The method in *GD2* is different not only because it does not involve this approximation but also because it does not use the rising Sine.

Methods identical with *Ābh* 4.33 can be found in *MBh* 5.19 (T. Kuppanna Sastri (1957, p. 274)), in *Sūryasiddhānta* 5.5cd-6ab (Shukla (1957, p. 67)) and in *Śiṣyadhīvṛddhidatantra* 6.5 (Chatterjee (1981, 1, p. 111)). As is the case with the *Āryabhaṭīya*, none of them spell out the approximation. But there is one significant difference: Āryabhaṭa does not mention why the Sine of sight-deviation is required, while the three treatises introduce this Sine in the chapter on solar eclipses, and use it for computing the latitudinal parallax which has to be considered during a solar eclipse.

Brahmagupta criticizes that the Sine of sight-deviation stated by Āryabhaṭa is wrong (*asat*) and leads to a wrong result in a solar eclipse (*Brāhmasphuṭasiddhānta* (hereafter *BSS*) 11.29-30, Dvivedī (1902, p. 160))[38]. Brahmagupta himself computes the eclipse in a distinctly different method in *BSS* chapter 5; in *BSS* 5.2-3 (Dvivedī (ibid., p. 79)) he first gives the rule for finding the altitude of the ecliptic point of sight-deviation (which he only calls "elevation (*avanati*)"), using the time it takes for the point to rise. This altitude corresponds to the "gnomon of sight-deviation $\mathcal{G}_M$" in *GD2* 188, but the approach is very different and it is unlikely that Parameśvara followed Brahmagupta for finding his rule. The rule in *BSS* 5.11ab (Dvivedī (ibid., pp. 82-83)) is the equivalent of formula 10.32, but Brahmagupta does not call it the Sine of sight-deviation, and proceeds in *BSS* 5.11-12 to find the latitudinal parallax. As a result, the term "Sine of sight-deviation" does not appear in his method. I consider his approach too different to compare with *GD2* and the *Āryabhaṭīya*[39]. The methods in *Siddhāntaśekhara* chapter 6 (Miśra (1932, pp. 382-401)) and *Siddhāntaśiromaṇi Grahagaṇitādhyāya* chapter 6 (Chaturvedi (1981, pp. 258-274)) also start with the altitude of the ecliptic point.

### 10.16.2 The method by Mādhava and Nīlakaṇṭha

Gupta (1985a) points out that Nīlakaṇṭha, in his commentary on *Ābh* 4.33, quotes two verses attributed to Mādhava which gives a mathematically correct method for finding the Sine of sight-deviation. The following is my translation of the verses:

> The ecliptic point of sight-deviation is the ascendant decreased by three signs. The squared Sine of the [arc] distance between this and the midheaven ecliptic point should be subtracted

[38]Sengupta (1935, pp. xxxviii-xxxix) states that the mistake for the rule was perhaps first pointed out by Pṛthūdakasvāmin in his commentary on *BSS* 11.27. Pṛthūdakasvāmin explains how the rule gives totally wrong values in extreme situations when the ecliptic point of sight-deviation is on the zenith and on the horizon.

[39]See Pingree (1978, pp. 574-575) for an overview. Yano (1982) gives a detailed discussion on Brahmagupta's method.

from the square of the midheaven Sine and from the square of the Radius. The square roots of these two are the multiplier and divisor, respectively. The complete [Sine of] sight-deviation is always produced from these two with the Radius.[40]

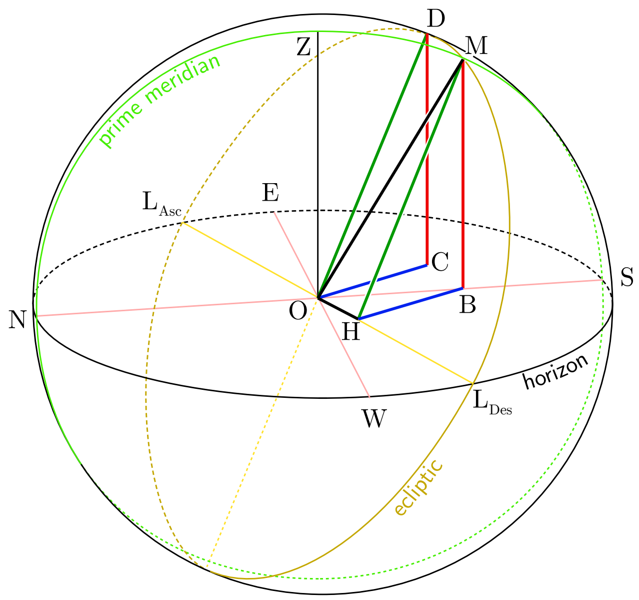



Figure 10.47: Representation of Mādhava's method for finding the Sine of sight-deviation OC .

Figure 10.47 represents this method in the sphere. H is the foot of the perpendicular drawn from midheaven M to $\mathrm{L_{Asc}L_{Des}}$, as was the case in figure 10.43. OH is the Sine of $\overparen{\mathrm{DM}} = \lambda_D - \lambda_M$, the arc distance between the ecliptic point of sight-deviation and midheaven. The multiplier $p$ is computed by a Pythagorean theorem, and we can see that it corresponds to HB in figure 10.47:

$$\begin{aligned} \mathrm{HB} &= \sqrt{\mathrm{OB}^2 - \mathrm{OH}^2} \\ p &= \sqrt{\mathrm{Sin}^2 z_M - \mathrm{Sin}^2(\lambda_D - \lambda_M)} \end{aligned} \tag{10.35}$$

On the other hand, the divisor $q$ corresponds to MH:

$$\begin{aligned} \mathrm{MH} &= \sqrt{\mathrm{OM}^2 - \mathrm{OH}^2} \\ q &= \sqrt{R^2 - \mathrm{Sin}^2(\lambda_D - \lambda_M)} \end{aligned} \tag{10.36}$$

as discussed previously in section 10.16, $\triangle\mathrm{HBM} \sim \triangle\mathrm{OCD}$. Therefore, with $\mathrm{DO} = R$ as the multiplicand,

[40] *lagnaṃ tribhonaṃ dṛkkṣepalagnaṃ tanmadhyalagnayoḥ |*
*vargīkṛtyāntarālajyāṃ madhyajyāvargatas tyajet ||*
*trijyākṛteś ca tanmūle kramaśo guṇahārakau |*
*tābhyāṃ dṛkkṣepasaṃsiddhiḥ trijyāyā jāyate sadā ||* (Pillai (1957b, p. 75))

$$
\begin{aligned}
\mathrm{OC} &= \frac{\mathrm{DO} \cdot \mathrm{HB}}{\mathrm{MH}} \\
&= \frac{pR}{q}
\end{aligned}
\tag{10.37}
$$

This method gives the same result as *GD2* 182-187 and both use the similar triangles △HBM ∼ △OCD, but otherwise the two procedures are distinctly different and it is unlikely that Parameśvara developed his method on the basis of Mādhava.

Gupta (1985a) also mentions that Nīlakaṇṭha gives a method similar to Mādhava's in *Tantrasaṅgraha* 5.5-7. In these verses (Ramasubramanian and Sriram (2011, p. 309)), he calls the value corresponding to OH the "'base' Sine (*bāhumaurvikā*)"[41]. This means that the "base" arc of midheaven is being measured from the ecliptic point of sight-deviation. On the other hand, Parameśvara calls MH the "'base' of the midheaven gnomon" in *GD2* 187, which suggests that the "base" arc starts from the ascending or descending ecliptic point. Hence I conclude that Parameśvara and Nīlakaṇṭha are looking at the same configuration from different views, and that it is doubtful that their theories are directly connected. If the verses quoted by Nīlakaṇṭha indeed belong to Mādhava, then I assume that there is a thread between these two authors that do not go through Parameśvara.

Parenthetically, Mādhava's rule states that the result is a "complete (*saṃsiddhi*) Sine of sight-deviation". This resembles Parameśvara's expression in *GD2* 187, the "true (*sphuṭa*) Sine of sight-deviation". Related to this point, Gupta (1985a) remarks that Nīlakaṇṭha interprets the word "own (*sva*)" in *Ābh* 4.33 is added to indicate that the "Sine of sight-deviation" computed in the verse is an intermediary value. It does indeed correspond to the multiplier $p$ in Mādhava's rule, and can be corrected using his method.

However, Parameśvara's interpretation on "own" seems to be no more than indicating the individual planets, as he states in his commentary:

> The meaning is: It is the Sine of sight-deviation of the planet, [i.e.] the sun or the moon, whose midheaven has been taken.[42]

This might be an echo of *MBh* 5.12 which stresses the difference in the lengths of the Sines between the sun and the moon.

> The difference in Sines of the moon and the sun are proclaimed because of the difference in orbit. And [this] is taught in the words of the master beginning with "own Sine of sight-deviation".[43]

I assume that Bhāskara I interprets "own" as an expression to stress the difference between the sun and the moon and that Parameśvara is following him while being aware that Āryabhaṭa's method is approximative. Parameśvara and Nīlakaṇṭha's differ on this point, too.

## 10.17 Elevation of ecliptic pole from the horizon (*GD2* 189)

In *GD2* 189, Parameśvara introduces the elevation (*unnati*) of the ecliptic pole again. He does not say clearly that the definition of "elevation" has changed; the reference for the elevation had

[41] Nīlakaṇṭha's computation for finding this value is different from Mādhava.

[42] *yasya grahasya raveḥ śaśino vā madhyalagnaṃ parigṛhītaṃ tasya dṛkkṣepajyā bhavatīty arthaḥ* / (Kern (1874, p. 92))

[43] *kakṣyābhedāc chaśībhānvor jīvābhedaḥ prakīrtyate* /
*jñāpakaṃ ca svadṛkkṣepa ityādivacanaṃ prabhoḥ* //5.12// (T. Kuppanna Sastri (1957, p. 158))

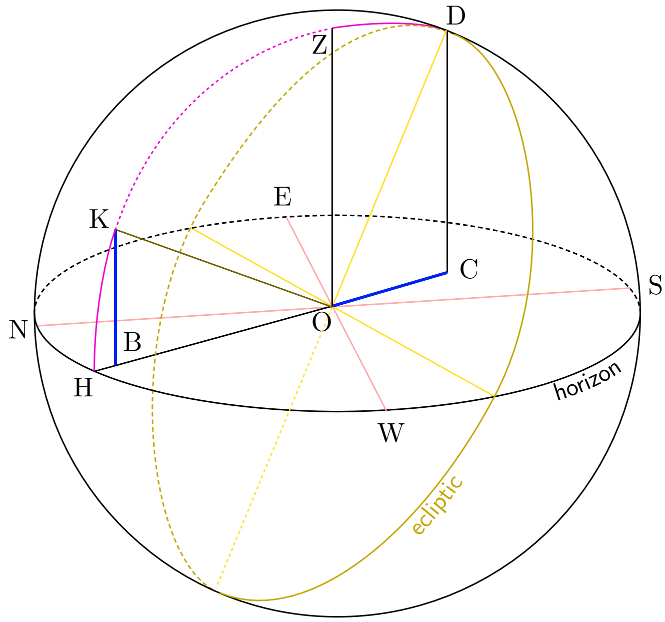



Figure 10.48: The Sine of sight-deviation OC and the elevation of ecliptic pole BK. Here the elevation is in the north.

previously been the plane of the six o'clock circle, whereas now the plane of horizon is involved[44]. Nonetheless, the new sense of "elevation" is clear from *GD2* 189cd, especially when we visualize the situation (figure 10.48).

To follow Parameśvara's reasoning in *GD2* 189cd, the zenith Z in the sky is at a distance of 90°from the horizon, and so is the ecliptic pole K from the ecliptic. This statement resembles *GD2* 155 where the ecliptic and its pole was compared with the celestial equator. We have argued that the armillary sphere could have been involved there (section 10.2), and it is also possible that the instrument is used for explaining *GD2* 189 too. If the ecliptic pole K is on point H on the horizon, the point of sight-deviation D should be on the zenith Z. As we lift K, D will move to the south of the prime vertical and if K is below the horizon, D will be to the north. $\widehat{\mathrm{HK}} = 90° - \widehat{\mathrm{KZ}} = \widehat{\mathrm{ZD}}$, and therefore their Sines $\mathrm{BK} = \mathrm{Sin}\,\zeta_{\varphi \mathrm{K}}$ and $\mathrm{OC} = \mathrm{Sin}\,z_D$ should also be equal:

$$\mathrm{Sin}\,\zeta_{\varphi \mathrm{K}} = \mathrm{Sin}\,z_D \tag{10.38}$$

As was the case with the elevation from the six o'clock circle (figure 10.9), the elevation is "in the north" when the northern ecliptic pole is above the horizon, and "in the south" when the northern ecliptic pole is below the horizon and the southern ecliptic pole is elevated instead. Thus *GD2* 189 also tells us the following rule.

- Nonagesimal is to the north: the southern ecliptic pole is elevated.
- Nonagesimal is to the south: the northern ecliptic pole is elevated.

[44]Since the new definition takes into account the geographic latitude $\varphi$, I shall denote the new "elevation of ecliptic pole" $\mathrm{Sin}\,\zeta_{\varphi \mathrm{K}}$ in contrast to $\mathrm{Sin}\,\zeta_{\mathrm{K}}$ whose reference is the six o'clock circle or the horizon at the terrestrial equator. Likewise for the elevation / depression of latitude $\mathrm{Sin}\,\zeta_{\varphi\beta}$

## 10.18 Elevation or depression of latitude from the horizon (*GD2* 190-191)

*GD2* 190-191 gives the rules for the elevation or depression of latitude $\mathrm{Sin}\,\zeta_{\varphi\beta}$. As was the case with the ecliptic pole $\mathrm{Sin}\,\zeta_{\varphi\mathrm{K}}$ in *GD2* 189, the reference for the elevation or depression here is the horizon. $\mathrm{Sin}\,\zeta_{\varphi\beta}$ is linked directly with the Sine of sight-deviation in these verses without any reasoning, but they could be explained by first considering the relation between $\mathrm{Sin}\,\zeta_{\varphi\beta}$ and $\mathrm{Sin}\,\zeta_{\varphi\mathrm{K}}$.

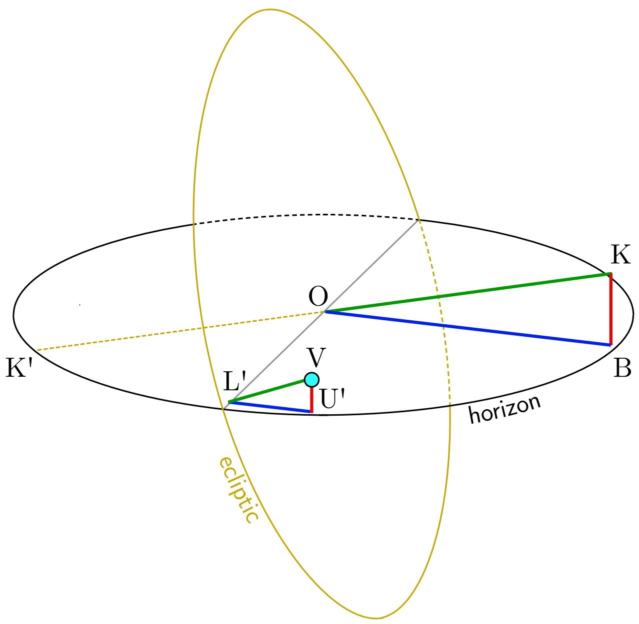



Figure 10.49: The elevation of latitude $\mathrm{F'V} = \mathrm{Sin}\,\zeta_{\varphi\beta}$ and the elevation of ecliptic pole $\mathrm{BK} = \mathrm{Sin}\,\zeta_{\varphi\mathrm{K}}$ from the horizon. North to the right.

Exactly the same argument as in *GD2* 163-164 (section 10.6) and *GD2* 166-168 (section 10.8) can be used here. Figure 10.49 is a modified version of figure 10.12 with the six o'clock circle replaced by the horizon. B and U′ are foots of the perpendiculars drawn to the plane of the horizon from the ecliptic pole K and the planet V with latitude VL′, and therefore $\mathrm{BK}\,\mathrm{Sin}\,\zeta_{\varphi\mathrm{K}}$ and $\mathrm{F'V} = \mathrm{Sin}\,\zeta_{\varphi\beta}$. $\angle\mathrm{VL'U'} = \angle\mathrm{KOB}$ since they both complement the angle formed by the ecliptic and the horizon. $\angle\mathrm{L'F'V} = \angle\mathrm{OBK} = 90^\circ$ and thus $\triangle\mathrm{L'U'V} \sim \triangle\mathrm{OBK}$. Therefore,

$$\mathrm{U'V} = \frac{\mathrm{BK}\cdot\mathrm{VL'}}{\mathrm{KO}}$$

$$\mathrm{Sin}\,\zeta_{\varphi\beta} = \frac{\mathrm{Sin}\,\zeta_{\varphi\mathrm{K}}\cdot\mathrm{Sin}\,\beta}{R} \tag{10.39}$$

and from formula 10.38,

$$\mathrm{Sin}\,\zeta_{\varphi\beta} = \frac{\mathrm{Sin}\,z_D\,\mathrm{Sin}\,\beta}{R} \tag{10.40}$$

As in *GD2* 167, the following condition holds:

- The northern ecliptic pole is elevated
  - Celestial latitude is northward: $\zeta_{\varphi\beta}$ is an elevation
  - Celestial latitude is southward: $\zeta_{\varphi\beta}$ is a depression
- The southern ecliptic pole is elevated
  - Celestial latitude is northward: $\zeta_{\varphi\beta}$ is a depression
  - Celestial latitude is southward: $\zeta_{\varphi\beta}$ is an elevation

Combining this with *GD2* 189, we find the following rule as reformulated from *GD2* 191:

- Nonagesimal is to the north of zenith
  - Celestial latitude is northward: $\zeta_{\varphi\beta}$ is a depression
  - Celestial latitude is southward: $\zeta_{\varphi\beta}$ is an elevation
- Nonagesimal is to the south of zenith
  - Celestial latitude is northward: $\zeta_{\varphi\beta}$ is an elevation
  - Celestial latitude is southward: $\zeta_{\varphi\beta}$ is a depression

## 10.19 Unified visibility equation (*GD2* 192-194)

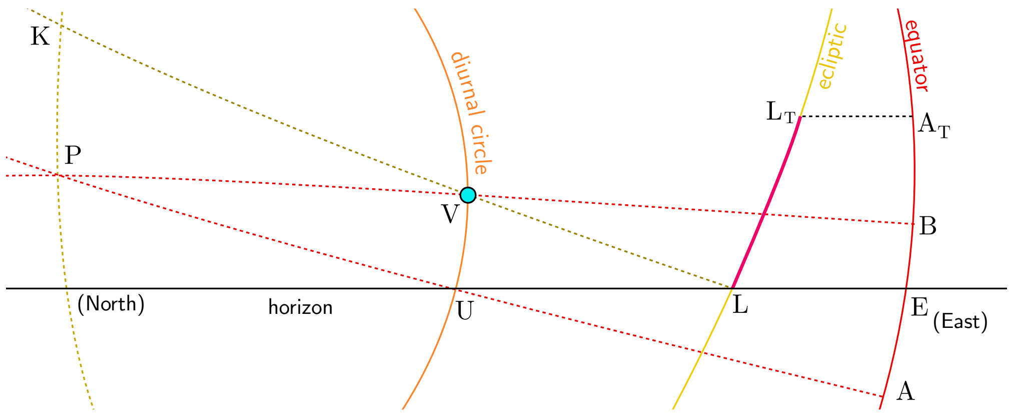



Figure 10.50: Unified visibility equation $l_v = \widehat{\mathrm{LL_T}}$.

The elevation or depression of latitude U′V from the horizon is a segment that corresponds to the arc $\widehat{\mathrm{UV}}$ in the diurnal circle from the horizon to the planet when its corresponding longitude L is on the horizon (figure 10.50). Thus, the rest of the steps in the unified method is the same as the previous ones: we find the segment in the equator corresponding to U′V, compute its arc $\widehat{\mathrm{AB}} = \widehat{\mathrm{EA_T}}$ and move to the ecliptic $\widehat{\mathrm{LL_T}}$, which is the unified visibility equation $l_v$. Parameśvara separates the steps in two rules as was the case with the method for the geographic latitude. However, in the previous case he called the arc in the celestial equator “the portion of the ascensional difference" whereas in *GD2* 192 he calls $\widehat{\mathrm{AB}} = \widehat{\mathrm{EA_T}} = l'_v$ the "visibility equation in *prāṇa*s (*dṛkphalaprāṇa*)".

$$l'_v = \operatorname{arcSin}\left(\frac{\operatorname{Sin}\zeta_{\varphi\beta}R}{\operatorname{Sin}\bar{\varphi}} \cdot \frac{R}{r}\right) \tag{10.41}$$

The transfer from the celestial equator to the ecliptic is done by linear interpolation within the sign as previously.

$$l_v = \frac{l'_v \cdot 1800}{\rho_n} \tag{10.42}$$

The conditions for whether the equation is additive or subtractive follows the rule in the visibility method for the "course" as in *GD2* 169-170. Here the expression is simplified, and unlike *GD2* 170, the elevation or depression is defined independently upon the rising and setting of the planet.

$$\lambda' = \begin{cases} \lambda - l_v & \text{Planet is rising and celestial latitude has an elevation} \\ \lambda + l_v & \text{Planet is setting and celestial latitude has an elevation} \\ \lambda + l_v & \text{Planet is rising and celestial latitude has a depression} \\ \lambda - l_v & \text{Planet is setting and celestial latitude has a depression} \end{cases} \tag{10.43}$$

# 11 Corrections to the planet at sunrise (*GD2* 195-208)

The following verses explain three types of corrections that are to be applied to the longitude of a planet. Parameśvara does not specify whether he is dealing with the mean longitude or the true longitude. The rules involve the daily motion of the planet, but this could also be the mean motion or true motion. Theoretically, all options are possible, and I shall leave the ambiguity in Parameśvara's words as it is[1]. But in this chapter I shall use the mean longitude and mean daily motion to simplify the explanation.

Parameśvara uses the word "correction (*saṃskṛti*)" only once in this section (*GD2* 207). Two of the three corrections do not even have a specific name, and are only mentioned as something additive or subtractive against the planet's longitude. Hereafter, I shall refer to all of them as "corrections".

## 11.1 Three corrections for correcting the time of sunrise (*GD2* 195)

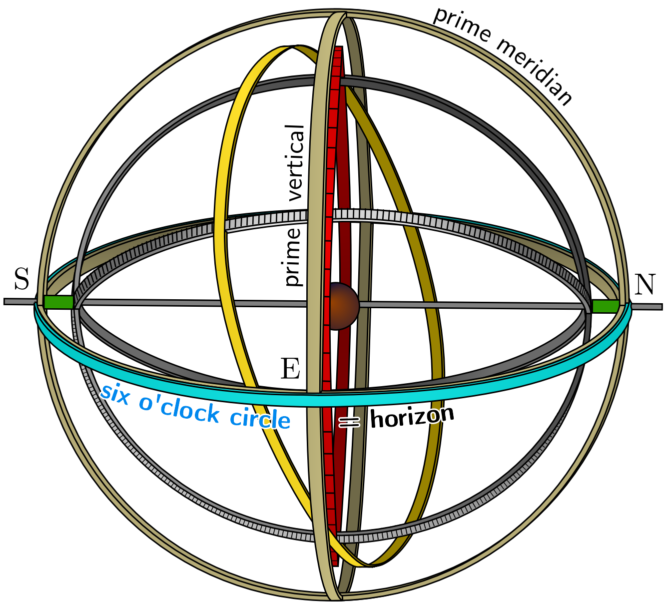



Figure 11.1: An armillary sphere adjusted for an observer at the terrestrial equator. The horizon and the six o'clock circle overlap.

The mean longitude of a planet at the beginning of the day can be approximately computed by multiplying its mean daily motion $v$ by the number of days elapsed since a given epoch (especially the beginning of the *yuga*). As Parameśvara is following the *Āryabhaṭīya* and the

[1] I would like to avoid confusion that could occur from discrepancies among commentators and interpreters. To give an example: The rules for correcting longitudes also appear in the *Mahābhāskarīya*, where we have the same ambiguity. The words for "daily motion" are unspecified in *MBh* 4.24-27, and Shukla (1960, pp. 126-128) supplies "mean" while T. Kuppanna Sastri (1957, p. xc) explains that they are "true".

*Mahābhāskarīya*, the beginning of the day is the moment of sunrise[2]. In order to derive the precise mean longitude, we need to know the exact moment of sunrise at the given location of the observer. *GD2* 195 refers to a "standard" moment of sunrise, which is when the mean sun rises above the horizon at zero latitude and zero longitude. The six o'clock circle represents the horizon as seen from the terrestrial equator (figure 11.1). Therefore, the following three factors must be taken into account.

**Correction for the geographic longitude** This corresponds to the "time difference" in modern notation. It is explained in *GD2* 196-201.

**Sun's equation of center (*doḥphala*)** This correction is applied to the mean sun to find the true sun which affects the time of sunrise. *GD2* 202-204 is on this topic.

**Ascensional difference (*caradala*)** This correction is caused by the geographic latitude. It is dealt with in *GD2* 205-208.

In every case, a specific amount of longitude is added when the observer's sunrise is earlier than the standard and subtracted when it is later. The amount is a portion of the planet's daily motion, which Parameśvara will explain with Rules of Three.

## 11.2 Correction for geographic longitude (*GD2* 196-201)

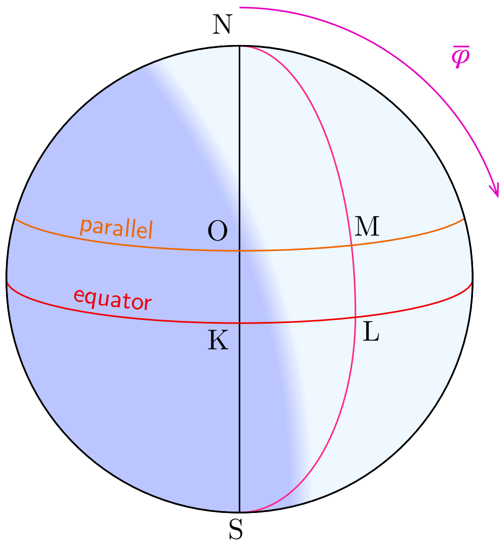



Figure 11.2: Distance OM $= \mathcal{D}_\theta$ along the circumference of an observer at O from the prime meridian $\widehat{\text{NMLS}}$. The border of daylight and a terrestrial meridian line do not overlap except on an equinoctial day.

[2] Āryabhaṭa also established a system called the *Ārdharātrika* which chose midnight as the beginning of the day, but his treatise based on this system has not been extensively transmitted to us (see Pingree (1978, pp. 602-608) for details). *MBh* 7.21-35 (T. Kuppanna Sastri (1957, pp. 380-385)) introduce the parameters in the *Ārdharātrika* system, but elsewhere the *Mahābhāskarīya* defines sunrise as the beginning of the day.

The first of the three factors is the time difference caused by the geographic longitude, or the distance from the geographic prime meridian (where the longitude is zero). Parameśvara repeatedly uses the word "geographic prime meridian (*samarekhā*)" but never specifies where it is. In general, Indian astronomers consider that the prime meridian passes through Ujjain and intersect the terrestrial equator at Laṅkā (Plofker (2009, p. 78)). This might have been a common knowledge for the readers of *GD2*. *MBh* 2.1-2 (T. Kuppanna Sastri (1957, p. 92)) gives an extensive list of places on the prime meridian, which could be one of the sources for Parameśvara and his readers. Indeed the second chapter of the *Mahābhāskarīya* deals with topics related to the geographic longitude, suggesting that this could be possible. But only the computation described in its last verse, *MBh* 2.10 (T. Kuppanna Sastri (ibid., p. 100)), appears as *GD2* 196, and the other verses have no equivalent passages in *GD2*.

Parameśvara first gives the rule in *GD2* 196. The situation is illustrated in figure 11.2. When the observer is at a distance of $\mathcal{D}_\theta$ *yojana*s from the prime meridian along his circumference (equivalent to the modern term "parallel" or "line of latitude") and the entire circumference is $c_\varphi$ *yojana*s, the correction applied to the longitude of a planet $\lambda$ whose daily motion is $v$ shall be as follows:

$$\lambda_\theta = \begin{cases} \lambda + \dfrac{v\mathcal{D}_\theta}{c_\varphi} & \text{(west from prime meridian)} \\ \lambda - \dfrac{v\mathcal{D}_\theta}{c_\varphi} & \text{(east from prime meridian)} \end{cases} \tag{11.1}$$

$c_\varphi$ is implicitly in *yojana*s. Parameśvara does not specify the measurement units for the remaining values, but the corrected longitude $\lambda_\theta$, $\lambda$ and $v$ should have the same unit of arcs.

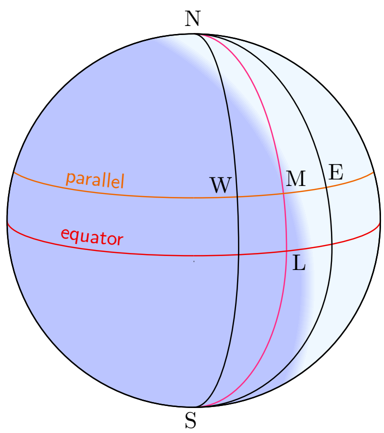



Figure 11.3: Day and night at the east (E) and west (W) of a point M on the prime meridian.

The reasonings for this computation is given in the following verses. *GD2* 197 refers to the fact that the sun rises earlier for an observer to the east of the prime meridian and later for one to the west (figure 11.3) as the reason for adding or subtracting the correction. *GD2* 198 is the Rule of Three which gives the amount of correction. Parameśvara calls it the grounding (*yukti*) for this case. The word revolution (*bhramaṇa*) most likely refers to the revolution of the stellar sphere which will appear in *GD2* 204. To be precise, it should be the revolution of the sphere and the sun's daily motion combined as shall be stated later in *GD1* 208. But in *GD2* 198, one revolution around the observer's circumference is compared to one day and the portion of this circumference corresponds to the portion of a day.

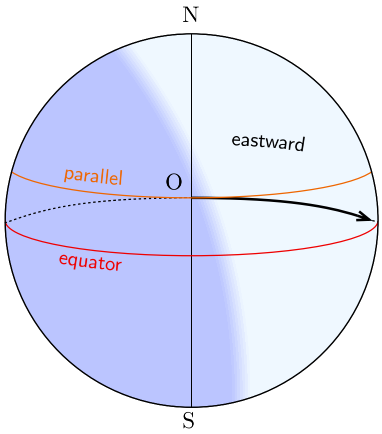



Figure 11.4: True eastward direction from the observer at point O.

*GD2* 199 links the observer's circumference with the observation of the sun. As Parameśvara states, one can find the cardinal directions at a given spot by observing the sun (with a gnomon, for example). However, if one makes only one observation and continues walking towards the "east" determined at the initial spot, the person would diverge from the circle parallel to the celestial equator (figure 11.4). This can be explained from the viewpoint of someone looking at the sphere of the Earth from outside. A line extended north and south from any spot O on the surface of the Earth will go through the north and south poles, drawing a great circle. For the observer at O, a line drawn east and west should be perpendicular against this line N – O – S. But from a larger viewpoint, a perpendicular drawn on the surface of a sphere against a great circle should also be part of a great circle (dotted line in figure 11.4). There is no evidence that Parameśvara was aware of this fact. For the same reason *GD2* 200ab is wrong from the point of view of an observer in one of the locations. How *GD2* 200cd connects to the previous statement is ambiguous to me.

A Rule of Three for computing the observer's circumference $c_\varphi$ is given in *GD2* 201. "The circle of the Earth where the [Sine of] co-latitude is a Radius" refers to the terrestrial equator. We have already seen in *GD2* 30 (section 3.6) that the circumference of the Earth at the equator $c_\oplus$ is 3299 *yojana*s. Meanwhile, the radius corresponding to the observer's circumference is the Sine of co-latitude $\mathrm{Sin}\,\bar{\varphi}$ (figure 11.5). Hence the Rule of Three compares the two radii (C′O and CK) and the two circumferences. As a result, the circumference $c_\varphi$ at a geographic latitude of $\varphi$ is

$$\begin{aligned} c_\varphi &= \frac{c_\oplus \,\mathrm{Sin}\,\bar{\varphi}}{R} \\ &= \frac{3299\,\mathrm{Sin}\,\bar{\varphi}}{R} \ (\textit{yojanas}) \end{aligned} \tag{11.2}$$

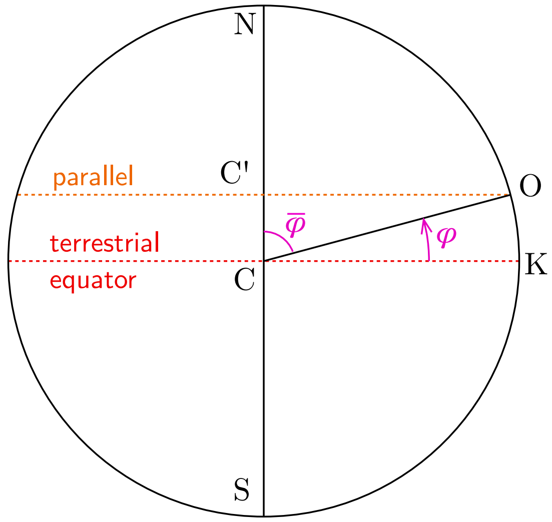



Figure 11.5: Segment of the Earth cut at the meridian passing the observer O. C′O is the radius corresponding to his circumference.

## 11.3 Correction for the sun's equation of center (*GD2* 202-204)

The next correction is for adjusting the mean solar time to the true solar time. This is known as the "equation of time" in modern terminology[3]. We will see later that many Indian astronomers, including Parameśvara, treat it approximately (probably unknowingly). The "equation of time" is not to be confused with an "equation" in general, which refers to the difference in longitude between a mean position and true position for any planet (see appendix C.4). In *GD2* 202-204, Parameśvara focuses on the sun's *doḥphala*, literally "equation of base", which I translate as the "equation of center"[4]. When the longitudes of the true sun and mean sun are $\lambda_{T\odot}$ and $\lambda_{M\odot}$ (in arc minutes), the sun's equation of center $q_\Sigma$ is

$$q_\Sigma = \lambda_{T\odot} - \lambda_{M\odot} \text{ (arcminutes)} \tag{11.3}$$

Since there are 21,600 minutes in a circle, the rule for correcting a planet's longitude $\lambda$ to $\lambda_q$ for the sun's equation of center is, according to *GD2* 202cd:

$$\lambda_q = \begin{cases} \lambda - \dfrac{vq_\Sigma}{21600} & \text{(true sun rises before mean sun)} \\ \lambda + \dfrac{vq_\Sigma}{21600} & \text{(true sun rises after mean sun)} \end{cases} \tag{11.4}$$

The same rule is given in *MBh* 4.7 (T. Kuppanna Sastri (1957, p. 185)). Brahmagupta calls this correction *bhujāntara* (literally "difference of base") in his *Brāhmasphuṭasiddhānta* 2.29

[3]See Pedersen (2011, pp. 154-158) and Ramasubramanian and Sriram (2011, pp. 464-465) for general explanations on this topic.

[4]Parameśvara uses the term *phala* alone to refer to equations of any planets, but when he adds *doḥ* or any synonym of "base" in *GD2*, it always refers to the sun. I shall follow his distinction by translating *phala* as "equation" and *doḥphala* as "equation of center".

(Dvivedī (1902, p. 35)). The rules in *Śiṣyadhīvṛddhidatantra* 2.16 (Chatterjee (1981, 1, p. 34)), *Siddhāntaśekhara* 3.46 (Miśra (1932, p. 178)) are also equivalent, although they use different units for arc measurements. *Sūryasiddhānta* 2.45 states the same rule, and Parameśvara comments that this verse is on the *bhujāntara* correction (Shukla (1957, p. 32)).

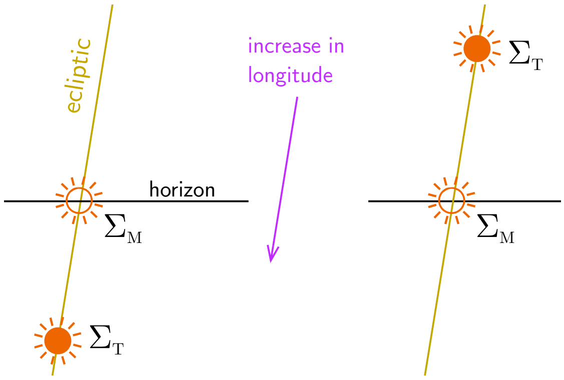



Figure 11.6: When the correction for the equation of center is additive (left) and subtractive (right)

*GD2* 203 explains when the correction is additive or subtractive. Celestial objects rise earlier for an observer in the east when their celestial longitude is smaller. Therefore, the true sunrise is earlier than mean sunrise when the equation of center is subtractive against the sun's mean longitude, and later when additive. On the other hand, an earlier sunrise will result in a subtractive correction to a planet's longitude as it advances less, and when it is later it will be additive.

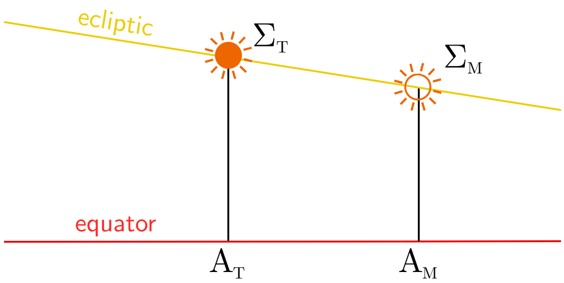



Figure 11.7: The sun's equation of center $q_\Sigma = \widehat{\Sigma_M \Sigma_T}$ and its corresponding arc on the equator $\widehat{A_M A_T}$.

The correction itself can be derived from a Rule of Three which compares a full cycle on the ecliptic to a full daily motion and the equation of center, a portion of the ecliptic, to a portion of the daily motion. However, the Rule of Three given in *GD2* 204 is slightly different. It uses the revolution of the stellar sphere instead of the ecliptic, and the "time corresponding to the equation of center" instead of the equation of center itself. This time is represented by the arc on the celestial equator corresponding to the equation of center (figure 11.7). Parameśvara refers to this as the grounding (*yukti*) of other people, but it is uncertain who he is referring to. The only major treatise before *GD2* which used the time instead of measurements on the ecliptic was the *Siddhāntaśiromaṇi* (*Grahagaṇitādhyāya*) of Bhāskara II[5].

> The sun's equation, multiplied by the rising [time] of the sign with the sun when there is no geographic latitude, divided by one thousand eight hundred, multiplied by the [daily] motion of a planet and divided by the *asus* (i.e. *prāṇas*) in a day and night is additive or subtractive against the planet, as the sun['s equation is additive or subtractive]. This is called the *bhujāntara*. (*Siddhāntaśiromaṇi Grahagaṇitādhyāya* 2.61)[6]

Bhāskara II approximates that the rising time at the terrestrial equator (cf. section 7.3) for a given longitude changes linearly within each zodiacal sign. Therefore the equation of center $q_\Sigma$ multiplied by the rising time $\alpha_n$ of the sign where it is located and divided by 1800, the number of arc minutes in a zodiacal sign, is approximately the "time corresponding to the equation of center" which Parameśvara mentioned in *GD2* 204. However there is a significant difference with this method given in *GD2* 204. According to Bhāskara II's auto-commentary, the *prāṇas* in a day and night are 21659, which is 21600 sidereal *prāṇas* ("the revolution of the [stellar] sphere" in *GD2* 204) plus 59 *prāṇas* for the sun's daily motion. Therefore, Bhāskara II's rule can be represented as follows.

$$\lambda_{q'} = \begin{cases} \lambda + \dfrac{q_\Sigma \alpha_n}{1800} \cdot \dfrac{v}{21659} & \text{(sun's equation of center is additive)} \\ \lambda - \dfrac{q_\Sigma \alpha_n}{1800} \cdot \dfrac{v}{21659} & \text{(sun's equation of center is subtractive)} \end{cases} \tag{11.5}$$

This correction for moving from the ecliptic to the celestial equator resembles the visibility methods, where an arc in the ecliptic corresponding to the arc in the celestial equator was computed by multiplying by 1800 and dividing by the local rising time of the sign (formula 10.11 in section 10.9, formula 10.25 in 10.11 and formula 10.42 in 10.17). Bhāskara II is also the first known author to apply this step in visibility methods. Parameśvara applies this correction, without even discussing the possibility of ignoring it, in the case of those methods; here for the equation of center, he chooses the approximate method as his standard rule and only suggests the correction as an alternative. This contrast is an interesting case to be further studied for considering the influence from Bhāskara II on Parameśvara.

We must also be aware of another difference between the two authors; that is, there is another element in the equation of time which is recognized by Bhāskara II but not by Parameśvara. So far, we have only dealt with the correction due to the eccentricity of the sun's orbit. This was represented as the difference between the longitudes of the mean and true suns (as in *GD2* 202) or their right ascensions (as in the *Siddhāntaśiromaṇi*). On the other hand, even the mean sun on the ecliptic does not rise at the same moment on each day, because the ecliptic is inclined against the celestial equator. Figure 11.8 describes this situation. $\mathrm{L_M}$ is a hypothetical sun moving with a constant daily motion on the celestial equator. Meanwhile, the mean sun $\Sigma_\mathrm{M}$ moves uniformly if measured along the ecliptic, but its corresponding right ascension on the equator $\mathrm{A_M}$ does not change linearly. Thus we need to correct the arc length $\overset{\frown}{\mathrm{L_M A_M}}$ to obtain the full equation

[5] I have relied on the translation and commentary by Arkasomayaji (1980, pp. 203-208) on *Grahagaṇitādhyāya* 2.61-63 for the discussion on the rules by Bhāskara II.

[6] *bhānoḥ phalaṃ guṇitam arkayutasya rāśer vyakṣodayena khakhanāgamahīvibhaktam |*
*gatyā grahasya guṇitaṃ dyuniśāsubhaktaṃ svarṇaṃ grahe 'rkavad idaṃ tu bhujāntarākhyam ||61||* (Chaturvedi (1981, p. 133))

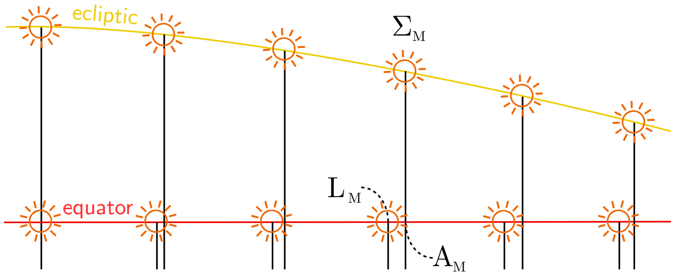



Figure 11.8: The mean sun moving uniformly on the ecliptic $\Sigma_M$, its corresponding right ascension on the equator $A_M$ and a mean sun moving uniformly along the equator $L_M$.

of time. This correction is also explained by Bhāskara II[7] who calls it the *udayāntara* (literally "difference in rising"), but we cannot find any trace of it in Parameśvara's works including *GD2*.

Since $L_M$ moves along the celestial equator with the same daily motion as $\Sigma_M$ does along the ecliptic, its right ascension measured from the vernal equinox is equivalent to the longitude of the mean sun, $\lambda_{M\odot}$. If $\alpha_{M\odot}$ denotes the right ascension corresponding to the mean sun on the ecliptic and $\alpha_{T\odot}$ that of the true sun, the first correction by Bhāskara II (*bhujāntara*) can be represented as $\alpha_{T\odot} - \alpha_{M\odot}$ and the second correction *udayāntara* as $\alpha_{M\odot} - \lambda_{M\odot}$. Therefore the full equation of time $E$ is

$$\begin{aligned} E &= (\alpha_{T\odot} - \alpha_{M\odot}) + (\alpha_{M\odot} - \lambda_{M\odot}) \\ &= \alpha_{T\odot} - \lambda_{M\odot} \end{aligned} \tag{11.6}$$

Nīlakaṇṭha, in his *Tantrasaṅgraha*, describes a set of rules which effectively gives the same equation of time[8](Ramasubramanian and Sriram (2011, p. 82)). However the two corrections that he mentions are different from those of Bhāskara II. The first is called *liptāprāṇāntara* or *prāṇakalāntara* (both literally "difference between the *prāṇa*s and arc minutes"), referring to the difference between the right ascension and the longitude of the true sun, and the second is the equation of center (*Tantrasaṅgraha* 2.28-32). The first can be represented by $\alpha_{T\odot} - \lambda_{T\odot}$ while the other is $\lambda_{T\odot} - \lambda_{M\odot}$, and therefore

$$\begin{aligned} E &= (\alpha_{T\odot} - \lambda_{T\odot}) + (\lambda_{T\odot} - \lambda_{M\odot}) \\ &= \alpha_{T\odot} - \lambda_{M\odot} \end{aligned} \tag{11.7}$$

This approach is different from what Parameśvara refers to in *GD2* 204, and thus it is unlikely that Nīlakaṇṭha's method for computing the equation of time has its origins in Parameśavra's theories, at least at the moment of *GD2*.

## 11.4 Correction for ascensional difference (*GD2* 205-208)

The third and last correction to be applied to a planet's longitude is the correction due to the sun's ascensional difference, which in turn is produced by the geographic latitude of the observer.

[7] *Siddhāntaśiromaṇi Grahagaṇitādhyāya* 2.62-63 (Chaturvedi (1981, p. 134))

[8] Note that neither Bhāskara II nor Nīlakaṇṭha use a specific term corresponding to "equation of time".

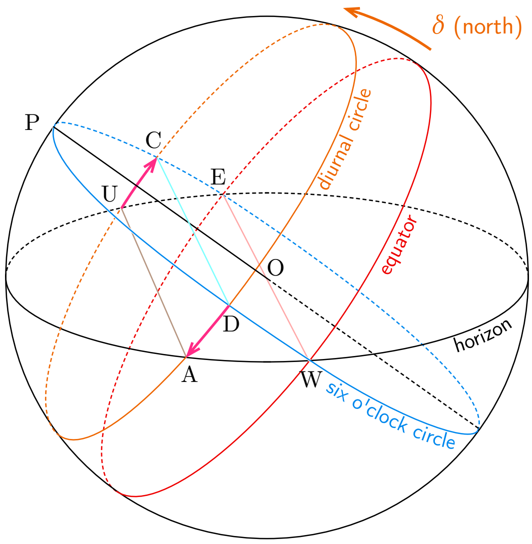
Unlike the other two rules which were only given for correcting the longitude a the moment of sunrise, this rule also explains how it should be applied at the moment of sunset. This mentioning to the sunrise and sunset occurs in previous treatises[9]. There is no explanation why the rule for sunset is necessary, but it was probably mentioned because this is the only rule where the correction is added or subtracted differently for sunrise and sunset.

The computation is stated in *GD2* 205. As we will see later in *GD2* 208, the "day" mentioned here is actually a sidereal day, or one revolution of the stellar sphere. The number of *prāṇa*s is equal to the arc minutes in a circle, which is 21600. When the sun's ascensional difference is $\omega$, the corrected longitude of the planet at sunrise $\lambda_\omega$ is

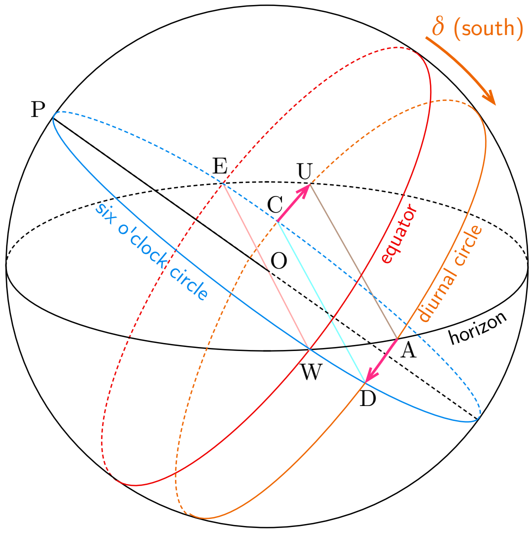
$$\lambda_\omega = \begin{cases} \lambda - \dfrac{v\omega}{21600} & \text{(sunrise in northern celestial hemisphere)} \\ \lambda + \dfrac{v\omega}{21600} & \text{(sunrise in southern celestial hemisphere)} \end{cases} \tag{11.8}$$

and the corrected longitude at sunset $\lambda'_\omega$ is

$$\lambda'_\omega = \begin{cases} \lambda + \dfrac{v\omega}{21600} & \text{(sunset in northern celestial hemisphere)} \\ \lambda - \dfrac{v\omega}{21600} & \text{(sunset in southern celestial hemisphere)} \end{cases} \tag{11.9}$$




(a) Northward declination




(b) Southward declination

Figure 11.9: The sun's rising and setting points (U and A) and its intersections with the six o'clock circle (C and D). The arrows represent the direction of the sun's diurnal motion.

[9]For example, *MBh* 4.26-27 (T. Kuppanna Sastri (1957, p. 214))

The explanations in *GD2* 206-207 can be visualized as in figure 11.9. Figure 11.9(a) represents the situation when the sun is in the northern celestial hemisphere, i.e. when its declination is northward, and figure 11.9(b) is when it is in the southern celestial hemisphere and its declination is southward. The sun's rising point (moment when it touches the horizon) is U, its intersection with the six o'clock circle in the east is C, its setting point (when it touches the horizon again) is A and the intersection with the six o'clock circle in the west is D.

- For sunrise:
  - if the declination is northward, sunrise is earlier → correction is subtractive
  - if the declination is southward, sunrise is later → correction is additive
- For sunset:
  - if the declination is northward, sunset is later → correction is additive
  - if the declination is southward, sunset is earlier → correction is subtractive

The ratio of $\overset{\frown}{\mathrm{UC}}$ or $\overset{\frown}{\mathrm{AD}}$ against the circumference of the diurnal circle is equal to the ratio of the ascensional difference $\omega$ against the celestial equator, and therefore the proportion of the corresponding time in a whole day.

However, Parameśvara mentions in *GD2* 208 that the divisor in rules 11.8 and 11.9 are different according to different people. In this verse the words for "day" refer to two different measures of days. When Parameśvara refers to the *prāṇa*s in a day, this is linked to the revolution of the stellar sphere, and hence is a sidereal day. The number of *prāṇa*s are equal to the number of arc minutes in a revolution, 21,600. When the sun's daily motion in minutes is added, this becomes the *prāṇa*s of a civil day. As we have seen, Bhāskara II was aware of this distinction and used the number of *prāṇa*s in a civil day[10], but we have no other evidence that *GD2* 208 is referring to Bhāskara II or his followers.

[10]The previous case was for the equation of center, but Bhāskara II uses the same divisor in the correction for the ascensional difference stated in *Siddhāntaśiromaṇi Grahagaṇitādhyāya* 2.53 (Chaturvedi (1981, p. 130)).

# 12 Example 1: Prime vertical shadow (*GD2* 209-211)

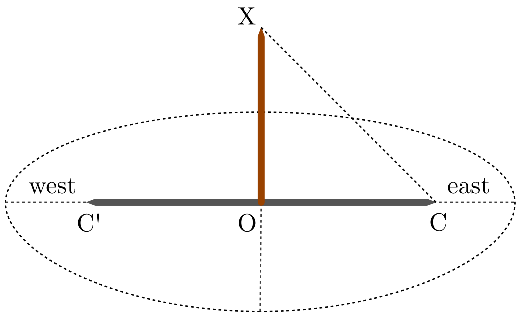



Figure 12.1: Situation in *GD2* 209. The shadow of the gnomon OX points either due east (OC) or due west (OC′)

*GD2* 209 is an example of computations related to the prime vertical shadow, i.e. the shadow of the sun when it is on the prime vertical. A rule for deriving the prime vertical gnomon from the sun's declination was explained in *GD2* 121-124, but this example goes the other way round; the prime vertical gnomon is given, and we have to compute the sun's longitude via its declination.

The situation described in *GD2* 209 is as follows:

- The sun is on the prime vertical.
- The length of a gnomon's shadow is equal to the gnomon itself.
- The Sine of geographic latitude is 647.
- On the next day, the shadow is
  1. shorter.
  2. longer.
- The sun's longitude is to be computed for the two cases.

*GD2* 209 itself does not articulate whether the "gnomon" is a gnomon with twelve *aṅgula*s and not a great gnomon, but the commentary hints that we are dealing with a twelve *aṅgula* gnomon and its shadow (see first paragraph in section 12.2).

## 12.1 Procedure (*GD2* 210-211)

*GD2* 210 describes the procedure of the solution by naming the segment or arc to be computed at each step. The computations themselves (indicated by arrows in the scheme below) are not given in detail.

> Shadow $s$ → Great gnomon $\mathcal{G}$ → Gnomonic amplitude $\mathcal{A}$ = Solar amplitude $\mathrm{Sin}\,\eta$ → Sine of declination $\mathrm{Sin}\,\delta$ → "Base" Sine $\mathrm{Sin}\,\lambda_B$ → "Base" arc $\lambda_B$ → Sun's longitude $\lambda$

In the last step, if the sun were in the first quadrant of the ecliptic (from vernal equinox to summer solstice), in which case the length of the shadow would be shorter on the next day, the arc corresponding to the "base" Sine itself is the sun's longitude, as mentioned in *GD2* 210. However if it were in the second (summer solstice to autumn equinox), when the shadow is longer on the next day, the arc has to be subtracted from a semicircle, i.e. 6 signs. This is stated in *GD2* 211. The sun cannot be on the third or fourth quadrant, since in such case it would never traverse the prime meridian in the course of the day (assuming that the observer is in the north of the terrestrial equator).

## 12.2 Solution

The steps in the commentary are parallel to those of *GD2* 210-211 as given in the previous section. I shall quote each passage in the commentary followed by my remarks which include reconstructing the silent steps in the procedure followed, finding the computation used by the commentary in the process, accounting for the numbers appearing in the commentary, especially when there is a discrepancy with the reconstructed process and comparing the steps and computations with the statements by Parameśvara.

**"In this case, the [great] gnomon computed from the hypotenuse of the shadow with proportion is 2431." (Shadow → Great gnomon)**

The computation done here might be equivalent to what we can see in *PĀbh* 4.28 (Kern (1874, p. 89)), which refers to a twelve *aṅgula* gnomon and its shadow:

> *dvādaśāṅgulaśaṅkunā trijyāṃ nihatyesṣṭacchāyākarṇena vibhajya labdhaṃ mahāśaṅkur bhavati* /
>
> Having multiplied the Radius with a twelve *aṅgula* gnomon, having divided it by the **hypotenuse of a given shadow**, the quotient which is the great gnomon is produced.

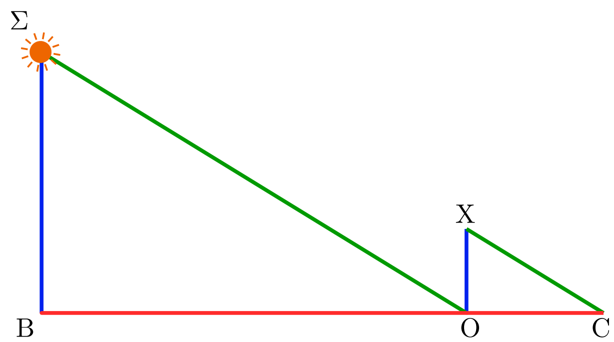



Figure 12.2: The great gnomon ΣB and the gnomon XO

This refers to a rule of three involving two triangles (figure 12.2). Σ is the sun in a great circle, ΣB the great gnomon, XO the twelve *aṅgula* gnomon and C the tip of its shadow. Assuming that the light-source is infinitely far, OΣ and CX are parallel. Thus △ΣBO ∼ △XOC, and

$$\Sigma\mathrm{B} = \frac{\mathrm{O}\Sigma \cdot \mathrm{XO}}{\mathrm{CX}}$$
$$\mathcal{G} = \frac{R \cdot 12}{\sqrt{12^2 + s^2}} \tag{12.1}$$

This computation does not appear in *GD2*, but *GD2* 116 (formula 8.10) uses the same set of triangles.

Our commentary refers to the "hypotenuse of the shadow", which is $\mathrm{CX} = \sqrt{12^2 + s^2}$. It also refers to the "proportion" which we have seen above. Since the length of shadow $s$ is equal to the gnomon, 12, the great gnomon $\mathcal{G}$ is

$$\mathcal{G} = \frac{3438 \cdot 12}{\sqrt{12^2 + 12^2}}$$
$$= 2431; 1, \cdots \tag{12.2}$$

This is rounded to $\mathcal{G} = 2431$.

**"The gnomonic amplitude is 466. However this should be taken as lessened by a quarter." (Great gnomon → Gnomonic amplitude)**

We use *GD2* 119 (formula 8.13) to compute the gnomonic amplitude $\mathcal{A}$:

$$\mathcal{A} = \frac{\mathcal{G} \operatorname{Sin} \varphi}{\operatorname{Sin} \bar{\varphi}} \tag{12.3}$$

The Sine of latitude $\operatorname{Sin} \varphi = 647$ is given in the verse. The Sine of co-latitude $\operatorname{Sin} \bar{\varphi}$ can be derived from the Pythagorean theorem:

$$\operatorname{Sin} \bar{\varphi} = \sqrt{R^2 - \operatorname{Sin} \varphi^2}$$
$$= \sqrt{3438^2 - 647^2}$$
$$= 3376; 34, \cdots \tag{12.4}$$

We have no definitive clue for what the commentator(s) of the examples in *GD2* used as the value of $\operatorname{Sin} \bar{\varphi}$ corresponding to $\operatorname{Sin} \varphi = 647$. Parameśvara in his auto-commentary on *GD1* 4.23 uses the rounded value 3377 (K. V. Sarma (1956–1957, p. 49)). The results given in the commentary for the two following computations (formulas 12.4 and 12.6) can be explained slightly better with the rounded value, and therefore I have opted to use 3377.

Putting these values in formula 12.3, we obtain:

$$\mathcal{A} = \frac{\mathcal{G} \operatorname{Sin} \varphi}{\operatorname{Sin} \bar{\varphi}}$$
$$= \frac{2431 \cdot 647}{3377}$$
$$= 465; 45, 20, \cdots \tag{12.5}$$

The value given in the commentary, 466 lessened by a quarter (465; 45), can be explained as a result of the second order sexagesimal being rounded. If the non-rounded value for Sine of co-latitude $\operatorname{Sin} \bar{\varphi} = 3376; 34$ were used, we obtain $\mathcal{A} = 465; 48, 55, \cdot$, which is different in the first order sexagesimal.

**"This is the solar amplitude." (Gnomonic amplitude = Solar amplitude)**

From *GD2* 122 we know that the gnomonic amplitude $\mathcal{A}$ and the solar amplitude $\mathrm{Sin}\,\eta$ are equal when the sun is on the prime vertical.

**"The [Sine of] declination computed from the solar amplitude by a rule to reverse is 457. However this should be taken as increased by a half." (Solar amplitude → Sine of declination)**

The computation of the solar amplitude from the Sine of declination is given in *GD2* 84ab ($\mathrm{Sin}\,\eta = \frac{R\,\mathrm{Sin}\,\delta}{\mathrm{Sin}\,\bar{\varphi}}$). A "rule to reverse (*vyastavidhi*)", which is explained in *Ābh* 2.28, is to convert multiplications to divisions and vice versa when reversing a rule. Thus by reversing the formula, we obtain the Sine of declination $\mathrm{Sin}\,\delta$ from the solar amplitude.

$$\begin{aligned}\mathrm{Sin}\,\delta &= \frac{\mathrm{Sin}\,\eta\,\mathrm{Sin}\,\bar{\varphi}}{R}\\ &= \frac{465;45\cdot 3377}{3438}\\ &= 457;29,10,\cdots\end{aligned}\tag{12.6}$$

The value in the commentary, 457 and a half (457; 30)", can be obtained if we round up the second order sexagesimal. In this case, if the non-rounded value for Sine of co-latitude $\mathrm{Sin}\,\bar{\varphi} = 3376;34$ were used, we obtain $\mathcal{A} = 457;25,45,\cdot$, which is again different in the first order sexagesimal.

**"The arc of the 'base' Sine established from the declination is 1147." (Sine of declination → "Base" Sine → "Base" arc)**

Here we see a discrepancy between Parameśvara's steps and the commentary, as the former mentions the "base" Sine as one step while the latter appears to jump immediately to its arc.

If we were to follow Parameśvara's steps, the "base" Sine $\mathrm{Sin}\,\lambda_B$ can be computed by reversing the rule in *GD2* 73ab ($\mathrm{Sin}\,\delta = \frac{1397\,\mathrm{Sin}\,\lambda_B}{R}$)[1].

$$\begin{aligned}\mathrm{Sin}\,\lambda_B &= \frac{\mathrm{Sin}\,\delta\cdot R}{1397}\\ &= \frac{457;30\cdot 3438}{1397}\\ &= 1125;54,\cdots\end{aligned}\tag{12.7}$$

If we use Āryabhaṭa's Sine series and linear interpolation (the same applies hereafter), this value is between $\mathrm{Sin}\,1125' = 1105$ and $\mathrm{Sin}\,1350' = 1315$ and therefore the corresponding arc $\lambda_B$ in minutes is approximately

$$\begin{aligned}\lambda_B &= 1125 + \frac{1125;54 - 1105}{1315 - 1105}\cdot 225\\ &= 1147;23\end{aligned}\tag{12.8}$$

[1] Parameśvara gives the value 1397 without mentioning that it is in fact the Sine of greatest declination $\mathrm{Sin}\,\varepsilon = \mathrm{Sin}\,24^\circ$.

If we round off $\mathrm{Sin}\,\lambda_B$ to 1125 we obtain $\lambda_B = 1146;25,\cdots \sim 1146$ and if we raise it to 1126, $\lambda_B = 1147;30 \sim 1148$. The commentary gives 1147, suggesting that the fractional part of the Sine was taken into consideration.

Another possibility is that the commentator computed the arc $\lambda_B$ directly from $\mathrm{Sin}\,\delta$ using a table. *Khaṇḍakhādyaka* 3.7[2] is an example of such table, but uses a different value for the Radius $R$ and thus unlikely to have been used here.

### "The sun['s longitude] is 0 19 7."("Base" arc → Sun's longitude)

The first case in *GD2* 209 is when the shadow is shorter on the next day. The sun is in the first quadrant of the ecliptic, and therefore $\lambda = \lambda_B$. The value in signs, degrees and minutes are $0^s\ 19^\circ\ 7'$. Manuscript $\mathrm{I}_1$ gives the values in one line and marking the different units by putting a space in between. The units themselves (sign, degree and minute) are not specified. Manuscript $\mathrm{K}_5^+$ is apparently a descendant of one with the same notation, but includes scribal errors. This is likewise for the next case.

### "The second sun['s longitude] is 5 10 53."

The second case is when the shadow is longer, and the sun is in the second quadrant. In this case $\lambda = 6^s - \lambda_B$. The result is $5^s\ 10^\circ\ 53'$.

### "Since they are established from the declination, these two [are the positions of the sun] with passage."

A longitude "with passage (*sāyana*)" refers to a coordinate where we take into account the motion of the equinoxes and solstices (see section 7.6). In such coordinate, a point on the zodiac would always stay at the same declination.

We see a similar remark at the end of the commentary on the next example, which seems to be connected to *GD2* 218cd-219.

[2] This is the verse number according to commentaries by Bhaṭṭotpala (Chatterjee (1970, 2. p. 34)) and Pṛthūdaka (Sengupta (1941, p. 83)). The verse number is 3.11 in Āmarāja's commentary (Misra (1925, p. 103)).

# 13 Example 2: Midday shadow and motion of solstices (*GD2* 212-219)

The midday shadow (*madhyacchāyā*) is the shadow or great shadow of the sun at midday, when it is on the prime meridian. *GD2* 212 is an example of a computation using the midday shadow, *GD2* 213-217 are a general explanation of the procedure and *GD2* 218-219 are some remarks related to this topic.

The situation described in *GD2* 212 is as follows.

- The sun is on the prime meridian.
- The length of a gnomon's shadow is

  **Case 1.** half the gnomon, and the sun is to the south of the zenith.
    i. On the next day the shadow is longer.
    ii. On the next day the shadow is shorter.

  **Case 2.** 1/8 of the gnomon, and the sun is to the south of the zenith.
    i. On the next day the shadow is longer.
    ii. On the next day the shadow is shorter.

  **Case 3.** 1/7 of the gnomon, and the sun is to the north of the zenith.
    i. On the next day the shadow is longer.
    ii. On the next day the shadow is shorter.

- The Sine of geographic latitude is 647.
- The sun's longitude is to be computed for each case.

See figure 13.1 for descriptions of the three cases given above.

The side of the prime meridian in which the sun is located is expressed by saying "southern/northern bamboo-piece (*yāmyā/saumyā śalākā*)", which recalls an armillary sphere whose rings are made of bamboo. The prime meridian is literally called "south-north (*dakṣiṇottara*", and hence the expression "southern bamboo-piece" indicates the southern side of the prime meridian (i.e. south of the zenith) and likewise for "northern bamboo-piece".

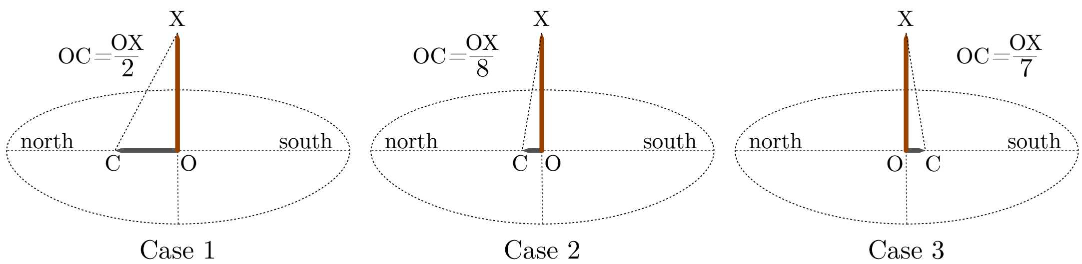
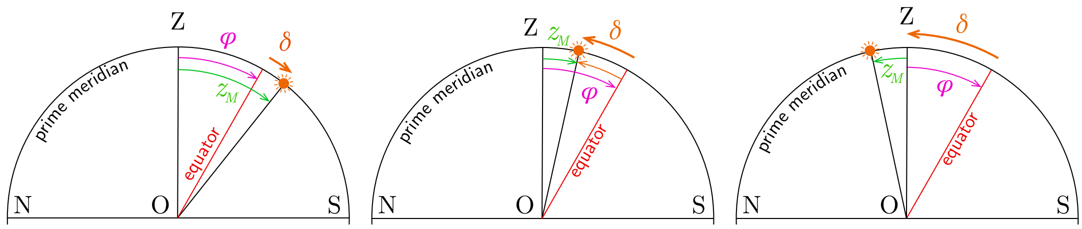



Figure 13.1: The three cases of midday shadows given in example 2.




(a) i. (a) ii. (b)

Figure 13.2: The three different positions of the sun explained in *GD2* 213-217, corresponding to the three cases in the example.

## 13.1 Procedure (*GD2* 213-217)

Parameśvara states the procedure in five verses (*GD2* 213-217). It is significantly longer than that of example 1 which was in two verses (*GD2* 210-211). This is mostly due to the fact that there are many different cases regarding the sun's location. The sun can be to the north or south of the zenith, and it can be in the northern or southern celestial hemisphere. The case when the sun is to the north of the zenith and in the southern celestial hemisphere is unmentioned, since it would require that the geographic latitude is to the south of the geographic equator, which is a case that is usually not examined in Sanskrit astronomical texts. Thus there are three possible cases, which is covered by example 2.

The steps of the procedure as given by Parameśvara are as follows:

1. The great shadow $\mathcal{S}$ at midday is equal to the Sine of meridian zenith distance of the sun $\text{Sin}\, z_M$. (*GD2* 213ab)

2. The declination $\delta$ is computed from $z_M$ and geographic latitude $\varphi$. There are three different cases (see figure 13.2)[1]:

   a) The sun is to the south of the zenith: $\delta = |z_M - \varphi|$. (*GD2* 213cd)

      i. If $z_M > \varphi$, the sun is in the southern celestial hemisphere (*GD2* 214cd). This implies $\delta = z_M - \varphi$

      ii. If $\varphi > z_M$, the sun is in the northern celestial hemisphere (*GD2* 214cd). This implies $\delta = \varphi - z_M$

   b) The sun is to the north of the zenith: $\delta = z_M + \varphi$. The sun is in the northern celestial hemisphere. (*GD2* 214ab)

3. Whether the sun is in the northward course (moving from winter solstice to summer solstice in the ecliptic) or in the southward course (summer solstice to winter solstice):

   a) The sun is to the south of the zenith

      i. The shadow-length increases on the next day: southward course. (*GD2* 215ab)

      ii. The shadow-length decreases on the next day: northward course. (*GD2* 215c)

   b) The sun is to the north of the zenith: contrary to above (*GD2* 215d), i.e.

      i. The shadow-length increases on the next day: northward course.

[1] Note that the geographic latitude has been drawn as a southward arc, as can be inferred from *GD2* 184 (see section 10.14.2).

ii. The shadow-length decreases on the next day: southward course.

4. $\delta$ (or from $\mathrm{Sin}\,\delta$)[2] $\rightarrow$ "base" Sine $\mathrm{Sin}\,\lambda_B$ (*GD2* 216a)

5. $\mathrm{Sin}\,\lambda_B \rightarrow$ "base" arc $\lambda_B$, and the sun's longitude $\lambda$ is:

- Sun in northern celestial hemisphere / northward course: $\lambda = \lambda_B$ (*GD2* 216bc)
- Sun in northern celestial hemisphere / southward course: $\lambda = 6^s - \lambda_B$ (*GD2* 216d)
- Sun in southern celestial hemisphere / southward course: $\lambda = \lambda_B + 6^s$ (*GD2* 217ab)
- Sun in southern celestial hemisphere / northward course: $\lambda = 12^s - \lambda_B$ (*GD2* 217cd)

## 13.2 Solution

The solution provided by the commentary after *GD2* 217 explains each of the three cases in almost the same process. The values for the great shadow $\mathcal{S}$, arc of declination $\delta$, "base" arc $\lambda_B$ and the longitudes of the sun $\lambda$ are always provided. The only difference in terms of values is that only the second and third case have the length of the twelve *aṅgula* gnomon's shadow. The values for the meridian zenith distance of the sun $z_M$ and the arc of geographic latitude $\varphi$ are not mentioned. The commentary on *GD2* 218abc refers to them, but only for the first and second cases. In this respect, I assume that they are only given as examples and not for supplementing the commentary on *GD2* 217.

Hereafter I shall proceed by quoting the commentary on *GD2* 217.

### Case 1

**"In this case, the great shadow established from the first shadow and its hypotenuse is 1537."**

Unlike the next two cases, the commentary does not give the value for the length of the twelve *aṅgula* gnomon, and just refers to it as the "first shadow". According to the verse it is half the gnomon, and we can easily compute its value $s = 6$. Then we can derive the great shadow $\mathcal{S}$ at midday from this shadow and its "hypotenuse" as we did in example 1 (formula 12.1).

$$\begin{aligned}\mathcal{S} &= \frac{Rs}{\sqrt{12^2+s^2}}\\ &= \frac{3438\cdot 6}{\sqrt{12^2+6^2}}\\ &= \frac{20628}{\sqrt{180}}\end{aligned} \tag{13.1}$$

If we extract $\sqrt{180}$ without approximation, the result is $\mathcal{S} = 1537;31,\cdots \approx 1538$ whereas the commentary gives 1537. However, if we round off its second order sexagesimal ($\sqrt{180} = 13;24,59,\cdots \approx 13;25$) we obtain $\mathcal{S} = 1537;29,\cdots \approx 1537$. Meanwhile, if we stop at $13;24$, the result is $1539;24,\cdots$. This suggests that the square root was computed up to the second order and then rounded. It is also possible that some sort of approximative method (cf. Gupta (1985b)) was the cause. See Appendix A.4.1 for a discussion on square roots in *GD2*.

[2] *GD2* 216 mentions "declination (*krānti*)", which could either be the arc of the declination $\delta$ or its Sine, $\mathrm{Sin}\,\delta$.

**"This is also the Sine of meridian zenith distance [of the sun]."**

This corresponds to *GD2* 213ab (step 1 in section 13.1).

$$\mathrm{Sin}\, z_M = \mathcal{S} = 1537 \tag{13.2}$$

**"In this case, since the sun is to the south of the zenith, the difference between the arcs of meridian zenith distance and geographic latitude is the arc of declination, 943. In this case, since the meridian zenith distance is larger, [the sun] is in the southern celestial hemisphere."**

Here the commentary refers to the arcs of meridian zenith distance $z_M$ and geographic latitude $\varphi$, but neither Parameśvara nor the commentator refers to the steps for computing them. The following steps are my reconstruction for computing $z_M$ and $\varphi$.

$\mathrm{Sin}\, z_M = 1537$ is between $\mathrm{Sin}\, 1575' = 1520$ and $\mathrm{Sin}\, 1800' = 1719$. Thus $z_M$ is approximately

$$\begin{aligned} z_M &= 1575 + \frac{1537 - 1520}{1719 - 1520} \cdot 225 \\ &= 1594; 13, \cdots \end{aligned} \tag{13.3}$$

The commentary on *GD2* 218abc gives the value 1594, although it refers to it as the arc corresponding to the great shadow $\mathcal{S}$.

Next, since the given Sine of geographic latitude $\mathrm{Sin}\, \varphi = 647$ is between $\mathrm{Sin}\, 450' = 449$ and $\mathrm{Sin}\, 675' = 671$, its arc $\varphi$ in minutes is approximately

$$\begin{aligned} \varphi &= 450 + \frac{647 - 449}{671 - 449} \cdot 225 \\ &= 650; 40, \cdots \end{aligned} \tag{13.4}$$

and the commentary on *GD2* 218abc gives the rounded value 651. Hereafter I assume that $\varphi = 651$ is always being used by the commentator(s) of the examples in *GD2*.

Now let us come back to the commentary. The expression "southern bamboo-piece" in *GD2* 212 refers to the south of the zenith. From *GD2* 213cd,

$$\begin{aligned} \delta &= |z_M - \varphi| \\ &= |1594 - 651| \\ &= 943 \end{aligned} \tag{13.5}$$

Furthermore, since $z_M > \varphi$, the sun is in the southern celestial hemisphere. At this point we find that case 1 corresponds to (a) i. that we listed in section 13.1.

**"The arc of the 'base' Sine obtained from the Sine of declination is 2509."**

This corresponds to step 4 in section 13.1. Using $\mathrm{Sin}\, 900' = 890$ and $\mathrm{Sin}\, 1125' = 1105$, the Sine of declination $\mathrm{Sin}\, \delta$ is

$$\begin{aligned} \mathrm{Sin}\, \delta &= 890 + (1105 - 890) \cdot \frac{943 - 900}{225} \\ &= 931; 5, \cdots \end{aligned} \tag{13.6}$$

If we round this off to 931, the "base" Sine is

$$\begin{aligned}\mathrm{Sin}\,\lambda_B &= \frac{\mathrm{Sin}\,\delta \cdot R}{1397} \\ &= \frac{931 \cdot 3438}{1397} \\ &= 2291;10,\cdots \end{aligned} \tag{13.7}$$

which can be rounded to 2291 and is between $\mathrm{Sin}\,2475' = 2267$ and $\mathrm{Sin}\,2700' = 2431$. Thus the "base" arc in minutes is approximately

$$\begin{aligned}\lambda_B &= 2475 + \frac{2291 - 2267}{2431 - 2267} \cdot 225 \\ &= 2507;55,\cdots \end{aligned} \tag{13.8}$$

This rounds to 2508 and not 2509 as in the commentary. This still holds true even when we take fractional parts into account in the intermediary steps. As was the case in example 1 (page 12.2), this might be due to a direct computation from $\mathrm{Sin}\,\delta$ to $\lambda_B$ using a table.

**"Since it is in the southern celestial hemisphere, this arc increased by six signs is [the longitude of] the sun when the shadow is growing, 7 11 49."**

Since the sun on the meridian is to the south of the zenith, the sun is on its southward course if the shadow-length increases on the next day (*GD2* 215ab), but this is unmentioned in the commentary. Since the sun is in the southern celestial hemisphere, from *GD2* 217ab,

$$\begin{aligned}\lambda &= \lambda_B + 6^s \\ &= 2509' + 6^s \\ &= 7^s\ 11^\circ\ 49' \end{aligned} \tag{13.9}$$

**"When the shadow on the next day is shrinking, [the sun] should be on the northward course. Therefore, a circle made of twelve signs, decreased by this 'base' arc, is [the longitude of] the sun, 10 18 11."**

When the shadow-length decreases on the next day, the sun is on its northward course (*GD2* 215c), and since the sun is in the southern celestial hemisphere, from *GD2* 217cd,

$$\begin{aligned}\lambda &= 12^s - \lambda_B \\ &= 12^s - 2509' \\ &= 10^s\ 18^\circ\ 11' \end{aligned} \tag{13.10}$$

**Case 2**

**"Now in the second case, the shadow in *aṅgula*s is 1 30."**

This time the commentary starts by stating the shadow of the twelve *aṅgula* gnomon. As it is one eighth of the gnomon's length, $s = \frac{12}{8} = 1;30$. The unit, *aṅgula* is also given. This is in contrast with other arcs and segments that are conceived in the great circle and are unitless in the commentary.

**"The great shadow is 426."**

As in case 1, the great shadow $\mathcal{S}$ is computed from formula 12.1:

$$\begin{aligned}\mathcal{S} &= \frac{Rs}{\sqrt{12^2+s^2}} \\ &= \frac{3438 \cdot 1;30}{\sqrt{12^2+(1;30)^2}} \\ &= 426;25,\cdots \end{aligned} \tag{13.11}$$

which can be rounded to 426.

Contrary to case 1, there is no mentioning that this is equal to the Sine of the sun's meridian zenith distance $\text{Sin}\, z_M$, but it is implied.

Next, as in case 1, the values for the arcs of meridian zenith distance $z_M$ and geographic latitude $\varphi$ are expected but not apparent.

Since $\text{Sin}\, z_M = 426$ is between $\text{Sin}\, 225' = 225$ and $\text{Sin}\, 450' = 449$, the arc is approximately

$$\begin{aligned} z_M &= 225 + \frac{426-225}{449-225} \cdot 225 \\ &= 426;53,\cdots \end{aligned} \tag{13.12}$$

The commentary on *GD2* 218abc gives 427. $\varphi = 651$ as in the previous case. This is also mentioned in the commentary on *GD2* 218abc.

**"In this case too, since the sun is to the south of the zenith, the difference between the arcs of geographic latitude and meridian zenith distance is the arc of declination, 224. In this case, since the geographic latitude is larger, [the sun] is in the northern celestial hemisphere."**

As was in case 1, the sun is to the south of the zenith. From *GD2* 213cd,

$$\begin{aligned} \delta &= |z_M - \varphi| \\ &= |427 - 651| \\ &= 224 \end{aligned} \tag{13.13}$$

In this case, $\varphi > z_M$ and the sun is in the northern hemisphere. This is the situation (a) ii. in section 13.1.

**"The arc of the 'base' Sine established from the [Sine of] declination is 553."**

The arc of declination is smaller than 225 arc seconds, therefore by linear approximation it is equal to its Sine ($\text{Sin}\, \delta = 224$).

Then the "base" Sine is

$$\begin{aligned} \text{Sin}\, \lambda_B &= \frac{\text{Sin}\, \delta \cdot R}{1397} \\ &= \frac{224 \cdot 3438}{1397} \\ &= 551;15,\cdots \end{aligned} \tag{13.14}$$

which can be rounded to 551 and is between $\mathrm{Sin}\,450' = 449$ and $\mathrm{Sin}\,675' = 671$. The "base" arc in minutes is approximately

$$\begin{aligned}\lambda_B &= 450 + \frac{551 - 449}{671 - 449} \cdot 225 \\ &= 553; 22, \cdots \end{aligned} \tag{13.15}$$

which can be rounded off to $\lambda_B = 553'$.

**"Since the sun located in the northern celestial hemisphere is to the south of the zenith, it should be on the southward course when the shadow is growing. Therefore, six signs decreased by this arc is [the longitude of] the sun, 5 20 47."**

Since the sun on the meridian is to the south of the zenith, the sun is on its southward course if the shadow-length increases on the next day (*GD2* 215ab), and since the sun is in the northern celestial hemisphere, from *GD2* 216d,

$$\begin{aligned}\lambda &= 6^s - \lambda_B \\ &= 6^s - 553' \\ &= 5^s\ 20^\circ\ 47' \end{aligned} \tag{13.16}$$

**"When the shadow on the next day is shorter, the 'base' Sine itself is [the longitude of] the sun, 0 9 13."**

Implicitly, when the shadow-length decreases on the next day, the sun is on its northward course (*GD2* 215c). Since the sun is in the northern celestial hemisphere, from *GD2* 216bc,

$$\begin{aligned}\lambda &= \lambda_B \\ &= 553' \\ &= 0^s\ 9^\circ\ 13' \end{aligned} \tag{13.17}$$

**Case 3**

**"Now in the third case, the shadow in *aṅgula*s is 1 43."**

This time the shadow of the twelve *aṅgula* gnomon is one seventh its length. $s = 1; 42, 51, \cdots \approx 1; 43$ *aṅgula*s.

**"The great shadow is 487."**

$$\begin{aligned}\mathcal{S} &= \frac{Rs}{\sqrt{12^2 + s^2}} \\ &= \frac{3438 \cdot 1; 43}{\sqrt{12^2 + (1; 43)^2}} \\ &= 486; 52, \cdots \end{aligned} \tag{13.18}$$

which can be rounded to 487. Yet again, implicitly, the sun's meridian zenith distance $\mathrm{Sin}\, z_M$ and its arc $z_M$ are derived.

$$\mathrm{Sin}\, z_M = \mathcal{S} = 487 \tag{13.19}$$

This is between $\mathrm{Sin}\, 450' = 449$ and $\mathrm{Sin}\, 675' = 671$. Thus the arc of meridian zenith distance is approximately

$$\begin{aligned} z_M &= 450 + \frac{487 - 449}{671 - 449} \cdot 225 \\ &= 488;30,\cdots \end{aligned} \tag{13.20}$$

For case 3, *GD2* 218abc does not refer to the values of $z_M$ and $\varphi$. As it is obvious that we use $\varphi = 651$ again, considering the next computation, $z_M$ is rounded to 489.

**"Since the sun is to the north of the zenith, the sum of the arcs of the meridian zenith distance and the geographic latitude is the arc of declination, 1140."**

This time the sun is to the north of the zenith, thus from *GD2* 214ab,

$$\begin{aligned} \delta &= z_M + \varphi \\ &= 489 + 651 \\ &= 1140 \end{aligned} \tag{13.21}$$

We are in situation (b) of section 13.1. Unlike cases 1 and 2, i.e. situation (a), where the celestial hemisphere had to be considered, the sun is always in the northern hemisphere. The commentary is silent about it at this stage.

**"The 'base' arc is 3194."**

Using $\mathrm{Sin}\, 1125' = 1105$ and $\mathrm{Sin}\, 1400' = 1315$, the Sine of declination $\mathrm{Sin}\, \delta$ is

$$\begin{aligned} \mathrm{Sin}\, \delta &= 1105 + (1315 - 1105) \cdot \frac{1140 - 1125}{225} \\ &= 1119 \end{aligned} \tag{13.22}$$

Therefore the "base" Sine is

$$\begin{aligned} \mathrm{Sin}\, \lambda_B &= \frac{\mathrm{Sin}\, \delta \cdot R}{1397} \\ &= \frac{1119 \cdot 3438}{1397} \\ &= 2753;50,\cdots \end{aligned} \tag{13.23}$$

which can be rounded to $\mathrm{Sin}\, \lambda_B = 2754$. This is between $\mathrm{Sin}\, 3150' = 2728$ and $\mathrm{Sin}\, 3375' = 2859$, and thus the "base" arc in minutes is approximately

$$
\begin{aligned}
\lambda_B &= 3150 + \frac{2754 - 2728}{2859 - 2728} \cdot 225 \\
&= 3194; 39, \cdots \qquad (13.24)
\end{aligned}
$$

which would be rounded to 3195, and here again we have a discrepancy from the value 3194 given in the commentary.

**"In this case, since the sun is located in the northern celestial hemisphere, when the sun is growing, this arc itself is [the longitude of] the sun, 1 23 14."**

The fact that the sun is in the northern celestial hemisphere is emphasized by using "in this case (*atra*)". Meanwhile, the commentator says nothing about the northward/southward courses of the sun in case 3. We can find from *GD2* 215d that it is northward when the shadow-length increases on the next day and southward if it decreases. In the former case, since the sun is in the northern celestial hemisphere, from *GD2* 216bc

$$
\begin{aligned}
\lambda &= \lambda_B \\
&= 3194' \\
&= 1^s\ 23^\circ\ 14' \qquad (13.25)
\end{aligned}
$$

**"When the shadow is shrinking, six signs decreased by the arc is [the longitude of] the sun, 4 36 46."**

From *GD2* 216d,

$$
\begin{aligned}
\lambda &= 6^s - \lambda_B \\
&= 6^s - 3194' \\
&= 4^s\ 36^\circ\ 46' \qquad (13.26)
\end{aligned}
$$

Our two manuscripts give wrong values for this final result: "4646" in $K_5^+$ and "46 46 14" in $I_1$. This can be explained by a common ancestor which omitted 3 and put "46 46". In the case of $I_1$, "14" could have moved from the previous value "1 23 14" for some reason.

**"Since they are established from the declination, these [are the positions of the sun] with passage."**

After finishing all three cases, the commentator repeats the concluding remark in example 1. This makes a connection with the following discussion in *GD2* 218cd-219.

## 13.3 Geographic latitude, declination and meridian zenith distance (*GD2* 218abc)

*GD2* 218abc refers to the arc of geographic latitude $\varphi$, depending on whether the arcs of declination $\delta$ and meridian zenith distance $z_M$ are in the same or different directions ("direction" referring to northward or southward):

$$\varphi = \begin{cases} |\delta - z_M| & \text{(Same direction)} \\ \delta + z_M & \text{(Different directions)} \end{cases} \tag{13.27}$$

Unlike the commentaries after *GD2* 211 and *GD2* 217, which focused on solving the example, the commentary here starts by paraphrasing and suppling words in a very typical style of glossing. It mentions that this rule enables one to compute the geographic latitude from the "shadow and the sun", likely referring to the shadow of a gnomon at noon and the sun's longitude.

## 13.4 Comparison with the *Mahābhāskarīya*

This rule might be traced back to some verses in the third chapter of the *Mahābhāskarīya* (T. Kuppanna Sastri (1957, pp. 124-128)), where verses 13-15 is on the computation of the sun's declination from its meridian zenith distance and geographic latitude (roughly corresponding to *GD2* 213-215), verse 16 on obtaining the sun's longitude from its declination (likewise similar to *GD2* 216-217) and verse 17 follows:

> When the sun is in the northern [celestial hemisphere, the declination and meridian zenith distance] should be added. When in the southern [celestial hemisphere], the difference between the declination and meridian zenith distance is remembered. [Thus] should be the geographic latitude [established] from the shadow [at midday]. (*MBh* 3.17) [3]

This is close to what we see in *GD2* 218abc except that the condition here is whether the sun is in the northern or southern celestial hemisphere and not the directions of $\delta$ and $z_M$. In fact, if the sun is in the southern celestial hemisphere, the meridian zenith distance will also be to the south, and thus $\varphi = z_M - \delta$ (see figure 13.2 (a) i.). However, if the sun is in the northern celestial hemisphere, the rule in *MBh* 3.17 holds true only when the meridian zenith distance is to the south. In such case $\delta$ and $z_M$ are in different directions and therefore $\varphi = \delta + z_M$ (figure 13.2 (a) ii.). Both Govindasvāmin and Parameśvara supply the other situation in their commentary and super-commentary (T. Kuppanna Sastri (ibid., p. 128)): if the shadow of a twelve *aṅgula* gnomon at midday is extending towards the south (in which case the meridian zenith distance of the sun $z_M$ is to the north), the difference between $\delta$ and $z_M$ should be taken. Here $\varphi = \delta - z_M$ (figure 13.2 (b)).

Whether or not Bhāskara I had intended to include the third case[4], Parameśvara interprets that all three cases are included. In the *Siddhāntadīpikā* he refers to a variant reading (see footnote 3) that could change the meaning of the verse to:

> When the sun is in the northern [celestial hemisphere, the declination and meridian zenith distance] should be added. When [the sun is] in the southern [celestial hemisphere] and also when the shadow [of a gnomon at midday is to the south], difference between the declination and meridian zenith distance is remembered. [Thus] should be the geographic latitude.

---

[3] *uttare yojayet sūrye viśleṣo dakṣiṇe smṛtaḥ |*
*apakramanatāṃśānāṃ chāyāyāś ca palaṃ bhavet ||*
In his *Siddhāntadīpikā*, Parameśvara mentions the variant *chāyāyāṃ* (locative) for *chāyāyāś* (ablative / genitive). In the *Karmadīpikā* he adopts it as the proper reading.

[4] According to Shukla (1976, p. xxv), Bhāskara I might have lived and taught the region of Surāṣṭra (today Saurashtra). The Tropic of Cancer goes through this region, and at a geographic latitude to the north to the Tropic of Cancer the Sun is always to the south of the zenith at midday.

and furthermore he adopts it as the proper reading in his *Karmadīpikā* composed later.

The similarity between *MBh* 3.13-17 and *GD2* 213-218abc not only suggests that the latter might have been influenced by the former, but also confirms that *GD2* 218abc is indeed linked to the previous verses, although not counted among "the procedural rule in five *āryā* verses". The commentary also makes a connection by bringing instances of values from the previous example. However *GD2* 218abc itself is never used in the examples, and in fact it is questionable whether this computation itself is valid or not.

### 13.4.1 Practicality of the rule

The longitude of the sun, from which we compute its declination according to the commentary, is either derived from the "shadow" or "mathematics" as we can see in *GD2* 218d. In order to compute the sun's longitude from the length of a shadow we need to know the geographic latitude in advance, and if we use the sun's longitude computed from mathematics, we will be using an erroneous value for the declination as the motion of the solstice is not taken into account (see next section), and thus end up with a wrong value for the geographic latitude.

Bhāskara I, who might be the original author of this rule, negates the motion of the solstice in his commentary on *Ābh* 3.5 (Shukla (1976, p. 183)). For him, this rule (*MBh* 3.17) would have indeed been valid. Whether Parameśvara was aware of this but had other intentions is uncertain. In any case, the same rule is included in treatises by authors both before and after Parameśvara, including Nīlakaṇṭha in his *Tantrasaṅgraha* 3.35 (Ramasubramanian and Sriram (2011, p. 175)).

*GD2* 213cd-214 was on computing $\delta$ from $z_M$ and $\varphi$, and *GD2* 218abc explains the determination of $\varphi$ from $z_M$ and $\delta$. There is also a computation to obtain $z_M$ from $\delta$ and $\varphi$ too, but is given in a different context in *GD2* 184-185 (section 10.14.2, formula 10.28). Unlike *GD2* 218abc, this third rule seems to have been used for the method given in *GD2* 220-230, where the sun's longitude and the geographic latitude is given and the value for the sun's meridian zenith distance is required in the process. Bhāskara I has placed the same rule in *MBh* 3.11, close to the other two.

## 13.5 "Passage" or motion of solstice (*GD2* 218cd-219)

*GD2* 218cd explains how one can find the motion of the solstice[5] using the shadow of a gnomon, and *GD2* 219 tells us how to find the solstitial point itself. The bulk of the commentaries on these verses are paraphrases of the sentences, and many words are supplied.

*GD2* 218cd refers to the sun's longitude obtained from the shadow and from mathematics (*gaṇita*). The former must be a reference to the process that has been described in the previous examples. The commentary stresses this by saying that it is computed from the *meridian* shadow. The longitude obtained from mathematics probably indicates the procedure which does not involve observation but computation using the motion of the planet and the current time; finding the mean position, computing the true position as well as applying the corrections that are described in *GD2* 195-208 (chapter 11). The commentary supplies that it is the *gaṇitatantra*, *tantra* of mathematics. This Sanskrit word could refer to "doctrines" in general, but it could refer to a specific "treatise". I have chosen "treatise", following the usages by Parameśvara in his auto-commentary on *GD1*:

> He (Parameśvara himself) states that the discrepancy on the measure of the Earth, the measure of the radial distance and so forth seen among different treatises on mathematics (*gaṇi-*

[5] Parameśvara does not recognize this as a precession. See section 7.6.

*tatantra*) are [caused] by the assumption on the measure of a *yojana*. (Auto-commentary on *GD1* 3.7)[6]

Seven lands, seven oceans, different parts and so forth on this Earth have been mentioned by a master going by the name of Śrīpati in his own treatise on mathematics (*gaṇitatantra*). (Auto-commentary on *GD1* 3.62)[7]

In the latter case, we can identify that this treatise is the *Siddhāntaśekhara* of Śrīpati (see chapter 3). It is also clear that the word *gaṇita* does not necessarily stand for mathematics in the narrow sense, but for treatises on astronomy in general. Although the commentator may not be Parameśvara himself, the nuance of *gaṇitatantra* is probably the same.

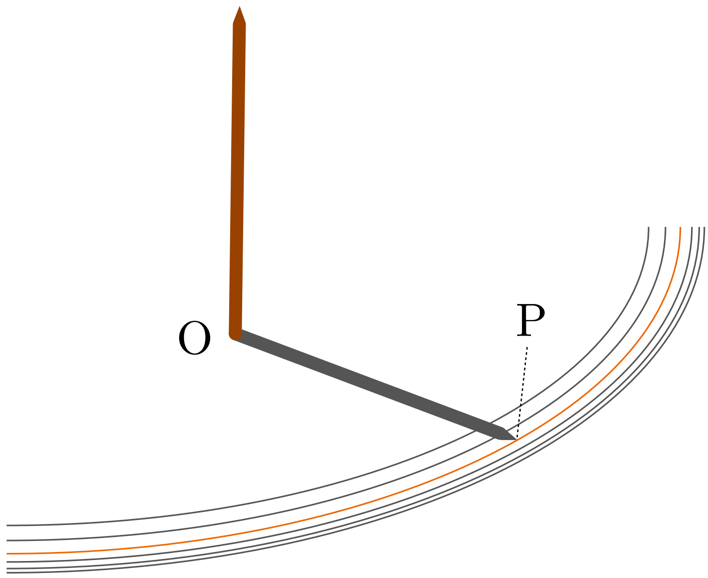



Figure 13.3: The trace of the shadow's tip changing each day.

*GD2* 219 is a method to find the exact moment of the solstice. First, one may choose any moment when the shadow of a gnomon can be observed and record the position of its tip, P. The trace of the shadow's tip in the course of daytime will gradually change each day (figure 13.3). This depends on the sun's declination, and when after some time the sun returns to the initial declination, the tip of the gnomon's shadow will fall on the same point P. The two moments when the shadow's tip fall on the same point are separated from the moment of solstice (this could be either summer or winter) by the same amount of time. Therefore the moment of time in the middle of these two moments should be the summer solstice or winter solstice according to Parameśvara.

To be precise, the sun's declination changes continuously and the traces of the shadow in figure 13.3 are not exactly parallel with each other. Unless the sun reaches the same declination at exactly the same time on another day, the tip of the shadow will not fall on the same point.

[6] *gaṇitatantrabhedeṣu dṛśyamāno bhūmānakarṇamānāder bhedo yojanamānaklptyety āha* / (K. V. Sarma (1956–1957, p. 25))

[7] *śrīpatināmnā ācāryeṇa svakṛtagaṇitatantre sapta dvīpāḥ sapta samudrāś ca bhūmeḥ khaṇḍabhedādayaś coktāḥ* / (K. V. Sarma (ibid., p. 36))

Parameśvara seems to be aware of this, and in his auto-commentary for *GD1* 4.87cd-90, he explains that the second moment of time can be found by interpolating observations on two consequtive days.

The commentary adds that the motion of solstice can be known with this method too. This could be done by finding the "without passage (*nirayaṇa*)" longitude of the sun at the moment of the solstice found in the above procedure. Another intresting feature in the commentary is the reference to all kinds of objects that could be used instead of a gnomon. It is comparable to the following passage in Parameśvara's auto-commentary on *GD1* 4.87b.

> A very high lamp post or flagpole, a new peak settled on the upper part of a temple, or a cane settled on the ground is to be assumed as a gnomon, and then its shadow should be observed.[8]

[8] *atyunnataṃ dīpastambhaṃ dhvajastambhaṃ vā devālayasyordhvabhāgasthāpitabālakūṭaṃ vā bhūmau sthāpitaveṇvādiṃ vā śaṅkum iti prakalpya tasya chāyām īkṣeta* / (K. V. Sarma (1956–1957, p. 67)): Amended *devālayo* to *devālayasyo* which is the reading in MS. No.762 F of the Kerala University Oriental Research Institute and Manuscripts Library.

# 14 Length of shadow when the sun is in a given direction (*GD2* 220-230)

## 14.1 Summary of the method

This method, as summarized in *GD2* 220ab, is to find the length of a shadow when the direction of the sun is known. The verses do not articulate the values needed for this computation, which are:

- The longitude of the sun
- The direction of the sun
- The geographic latitude

The entire procedure is an iterative method, which Parameśvara calls the "without-difference" (*aviśeṣa*) method. It starts with assuming that the great shadow is an arbitrary value, and then computes two values called the "base of direction (*digbāhu*)" and "base to be established (*sādhyabāhu*)". The two are equal if the assumption for the great shadow is correct. Otherwise, the assumed value is corrected using the difference between the two bases, and the procedure is iterated until there is no difference, hence the name "without-difference".

The explanatory verses *GD2* 220cd-230 consistently use the word "shadow (*chāyā* or its synonyms)" and not "great shadow (*mahācchāyā* etc.)". However, considering the segments involved in the computation, every instance of "shadow" actually refers to the great shadow. There is no reference to a shadow of an ordinary gnomon, as if the great shadow was the final goal in this procedure. Meanwhile, the goal of examples 3 (*GD2* 231) and 4 (*GD2* 232) is the shadow of a twelve *aṅgula* gnomon.

## 14.2 Initial assumption (*GD2* 220)

*GD2* 220cd explains that a "shadow" should be assumed inside a "circle of direction", "made" using a string.

This implies that the procedure is carried out with the aid of diagrams. This "shadow" is actually a "great shadow", as is clear from the procedures that follow, and is also confirmed by the commentary on the examples.

What is referred to as a circle of direction is probably a circle with two lines oriented north-south and east-west[1].

Parameśvara's auto-commentary on *GD1* 4.12-13ab appears to be a more detailed explanation of what is intended here. The only difference is that the goal in *GD1* is to compute the great gnomon and not the (great) shadow.

> Now, in order to compute the [great] gnomon in a given direction, the base of the figure having the [great] shadow as its hypotenuse and the upright are shown [with the verse (*GD1* 4.12) beginning with] "the east-west line as its end". Having drawn a great circle, two lines of direction should be made. In this circle, the great shadow is indeed the distance from the center to where the great gnomon's foot is at that moment. The base of the hypotenuse,

[1] Such a line is called a line of direction (*diksūtra*) in *GD2* and also appears in the auto-commentary of *GD1*.

which is the great shadow, [departs] from this [great] gnomon's foot, has the east-west line as its end, and extends north and south. The upright of this hypotenuse, which is the [great] shadow, [departs] from this [great] gnomon's foot, has the north-south line as its end, and extends east and west. Thus the base and upright is always in the circle of the [great] shadow. With these base, upright and hypotenuse the [great] gnomon of the sun located in a given direction is established.[2]

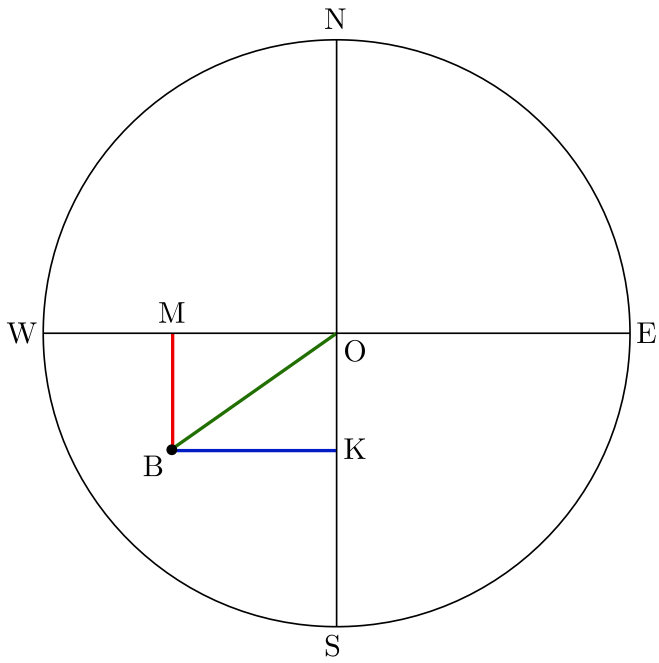



Figure 14.1: The foot of a great gnomon B in a great circle with directions of line

Presuming that the procedure in *GD2* is the same, we have to draw a diagram with the foot of a great gnomon located on a great circle (figure 14.1). OB is the great shadow, BM is its base and BK its upright. The base of direction $\mathcal{B}_d$ and base to be established $\mathcal{B}_s$ are essentially this base of shadow BM computed in two different ways, and should be equal if the assumed value of the great shadow OB is correct, as indicated in *GD2* 223cd.

## 14.3 Base of direction (*GD2* 221)

The "base of direction" $\mathcal{B}_d$ is the sum or difference between the gnomonic amplitude $\mathcal{A}$ and the solar amplitude $\text{Sin}\,\eta$. According to *GD2* 221, the sum is taken when they are the "same" and the difference is taken when they are "different", probably referring to their direction. The Sanskrit

[2] *atheṣṭāśāsthaśaṅkvānayanārthaṃ chāyākarṇakṣetrabhujāṃ koṭiṃ ca pradarśayati pūrvāpararekhāntam iti / trijyāvṛttam ālikhya diksūtre ca kuryāt / tasmin vṛtte yatra mahāśaṅkor mūlaṃ tatkāle bhavati kendrān mahācchāyāntare hi tad bhavati / tasmāc chaṅkumūlāt* (Amended from *tasmāt śaṅkumūlāt*) *pūrvāpararekhāntā yāmyodagāyatā mahācchāyākarṇasya bhujā bhavati / tasmāc chaṅkumūlāt yāmyottararekhāntā pūrvāparāyatā chāyākarṇasya koṭir bhavati / evaṃ sadā chāyāvṛtte bhujākoṭī bhavataḥ / karṇas tu mahācchāyā / etaiḥ bhujākoṭikarṇaiḥ iṣṭadiksaṃsthe savitari śaṅkuḥ sādhyaḥ /* (K. V. Sarma (1956–1957, p. 46))

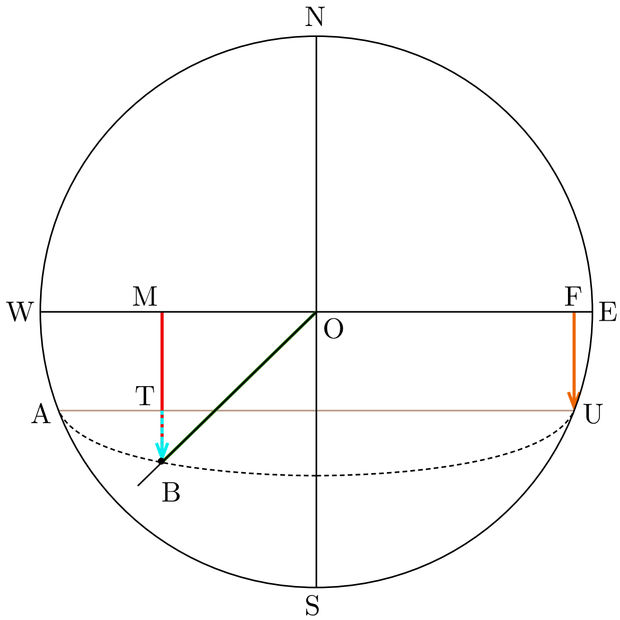



Figure 14.2: The base of direction BM when the gnomonic amplitude TB and the solar amplitude FU are in the same direction.

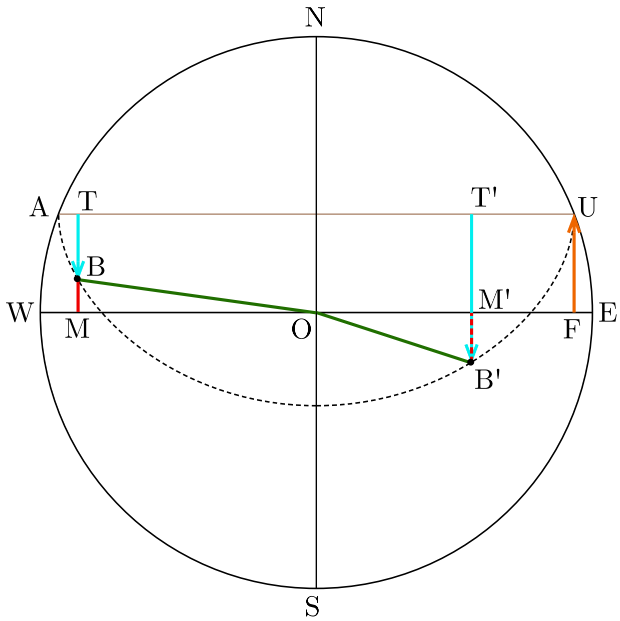



Figure 14.3: When in different directions. BM is the base of direction when the gnomonic amplitude TB is smaller than the solar amplitude. B′M′ is the base of direction when the gnomonic amplitude T′B′ is larger.

term for gnomonic amplitude, *śaṅkvagra*, can be interpreted as "that which has the gnomon as its extremity", and thus I assume that its direction is from the rising-setting line toward the foot of the great gnomon. Likewise, *arkāgrā* (solar amplitude) can be interpreted as "that which has the sun as its extremity", implying that the point on the horizon where the sun rises or sets is the extremity. The description in *GD2* 103 that the rising-setting line extends from the tip of the solar amplitude (section 8.1) also supports this idea.

Figure 14.2 shows the situation when the gnomonic amplitude and the solar amplitude are in the same direction, and figure 14.3 when they are different. The base of direction $\mathcal{B}_d$ is obtained as follows:

$$\mathcal{B}_d = \begin{cases} \mathcal{A} + \operatorname{Sin}\eta & \text{(Same direction)} \\ |\mathcal{A} - \operatorname{Sin}\eta| & \text{(Different directions)} \end{cases} \tag{14.1}$$

## 14.4 Base to be established (*GD2* 222-223)

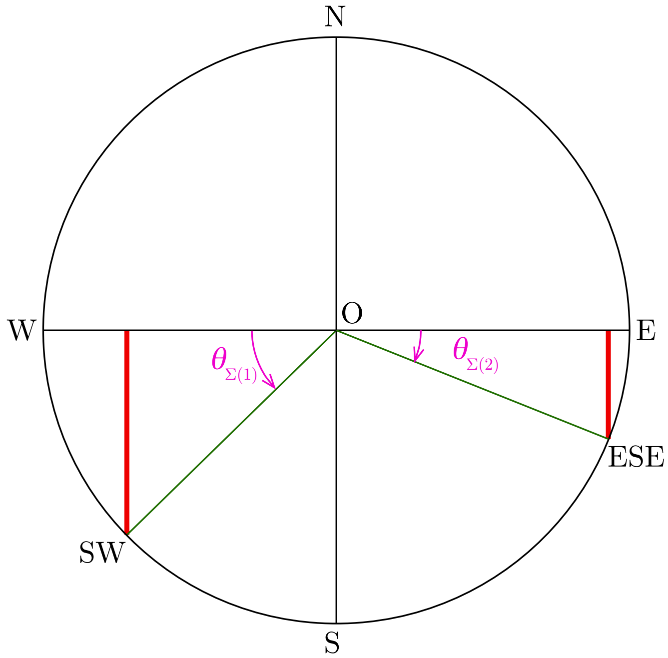



Figure 14.4: Sines of direction (in bold lines) when the sun is in an intermediate direction (southwest in this diagram) $\theta_{\Sigma(1)}$ and east-southeast $\theta_{\Sigma(2)}$.

The "base to be established" $\mathcal{B}_s$ is the component of the great shadow in the north-south direction. In order to derive it, the Sine corresponding to the direction of the sun, or the "Sine of direction (*digjīvā*)" $\operatorname{Sin}\theta_\Sigma$ is first stated in *GD2* 222.

Figure 14.4 shows the two examples of the Sine of direction given in *GD2* 222. When the sun is between east and south-east (i.e.
east-southeast), the arc $\theta_{\Sigma(2)}$ corresponding to the Sine of direction is half the arc $\theta_{\Sigma(1)}$ in an

intermediate direction. This tells us that the arc of direction $\theta_\Sigma$ in general is measured from due east or due west.

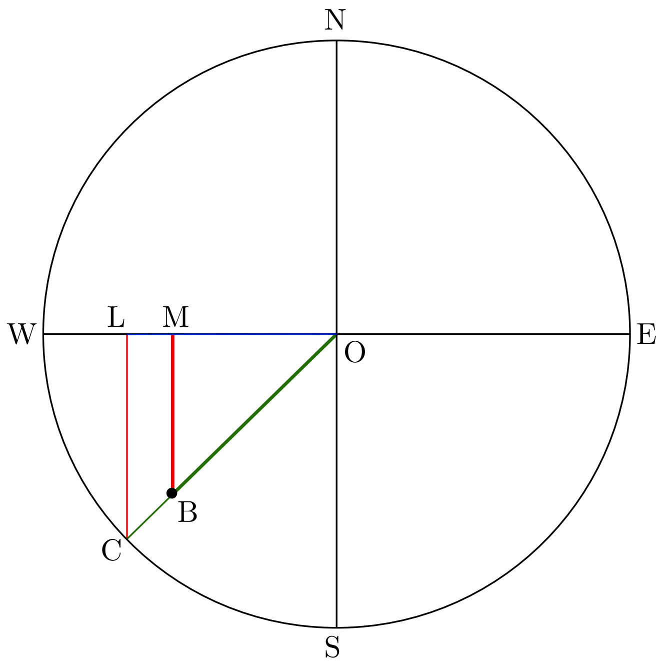



Figure 14.5: Computing the base to be established BM from the Sine of direction CL

The computation in *GD2* 223ab, which is that the "'base to be established' is the Sine of direction multiplied by the given [great] shadow and divided by the Radius", can be explained as follows. In figure 14.5, B is the foot of the gnomon, OB is the great shadow $\mathcal{S}$ and $\overset{\frown}{\mathrm{CW}}$ is the arc of direction $\theta_\Sigma$. BM is the north-south component of OB, i.e. the base to be established $\mathcal{B}_s$, and CL is the Sine of direction $\mathrm{Sin}\,\theta_\Sigma$. Since $\triangle$BMO and $\triangle$CLO are both right triangles and share one angle, $\triangle\mathrm{BMO} \sim \triangle\mathrm{CLO}$, and

$$\begin{aligned} \mathrm{BM} &= \frac{\mathrm{CL}\cdot\mathrm{OB}}{\mathrm{OC}} \\ \mathcal{B}_s &= \frac{\mathrm{Sin}\,\theta_\Sigma \cdot \mathcal{S}}{R} \end{aligned} \tag{14.2}$$

If the base of direction and base to be established are equal, *GD2* 223cd mentions that the guess is correct. Parameśvara does not state the case when they are unequal, but we can interpret that the "without-difference" method using their difference, whose explanation begins from *GD2* 224 is to be applied.

## 14.5 Correction of the great shadow (*GD2* 224-225, 228-229)

In each step of the "without-difference" method, while the base of direction $\mathcal{B}_d$ and base to be established $\mathcal{B}_s$ are unequal, the great shadow $\mathcal{S}$ is corrected as given in *GD2* 224:

$$\mathcal{S}_{i+1} = \mathcal{S}_i \pm \frac{|\mathcal{B}_d \pm \mathcal{B}_s| \cdot p}{q} \tag{14.3}$$

Concerning $|\mathcal{B}_d \pm \mathcal{B}_s|$, the difference is taken when the two are in the same direction (i.e. both extending northwards or both extending southwards from the east-west line) and the sum is taken when they are in opposite directions. The latter case occurs only if an extreme value is assumed for the great shadow when the sun rises in the north and culminates in the south and is relatively rare. Perhaps for this reason, other passages such as *GD2* 230 and *GD2* 234, only refer to their difference.

The multiplier $p$ and divisor $q$ are specified in *GD2* 228.

The multiplier $p$ is the Radius minus the midday shadow (great shadow at midday). As stated in *GD2* 213, the midday shadow is equal to the Sine of meridian zenith distance of the sun $\mathrm{Sin}\, z_\Sigma$. Parameśvara gives no instruction for computing $z_\Sigma$ in this section, but the rule to find the midheaven Sine $\mathrm{Sin}\, z_M$ from the declination[3] $\delta$ and the geographic latitude $\varphi$ in *GD2* 184-185 (formula 10.28) must have been used. *GD2* 182 supplies that the position of the sun at midday is the midheaven, and therefore $\mathrm{Sin}\, z_\Sigma = \mathrm{Sin}\, z_M$. The direction of the midheaven Sine in accordance with $\delta$ and $\varphi$ is stated in *GD2* 194.

The divisor $q$ is the Radius minus the solar amplitude $\mathrm{Sin}\, \eta$ when the sun rises and culminates at the same side of the prime vertical (north or south), and is the sum of the Radius and $\mathrm{Sin}\, \eta$ when the sun traverses the prime vertical.

$$p = R - \mathrm{Sin}\, z_\Sigma \tag{14.4}$$

$$q = R \mp \mathrm{Sin}\, \eta \tag{14.5}$$

There are no reasonings given by the author or commentator for these values. They do not correspond to any geometrical element except for a very special case, which is when the sun is on the prime meridian and the guess for the great shadow is $\mathcal{S} = R$, the correction in formula 14.3 with the multiplier $p$ and divisor $q$ will give the exact value of the great shadow. My hypothesis is that they approximately reduce $|\mathcal{B}_d \pm \mathcal{B}_s|$ to $|\mathcal{B} - \mathcal{B}_s|$, where $\mathcal{B}$ is the true base of the shadow when $\mathcal{S}$ is correct. I would like to come back to this point in my future research.

Parameśvara mentions in *GD2* 229 that $p$ and $q$ may be reduced by a common number as it only makes a small difference.

Furthermore, he adds in *GD2* 233 that the whole correction may be multiplied by one and a half if the convergence is slow, and by half or smaller if the value oscillates. In *GD2* 234, he even mentions that the difference between the base of direction and base to be established itself can be used for correction, without $p$ and $q$.

The entire correction $\frac{|\mathcal{B}_d \pm \mathcal{B}_s| \cdot p}{q}$, which is called the result (*phala*) [of division], is either additive or subtractive, depending on the cases given in *GD2* 225. By saying "the base of direction is located south of that called the established", Parameśvara is comparing the end which is not on the east-west line for each of the two bases. The expression "the meridian zenith distance is in the north" means that the sun is to the north of the zenith at midday.

1. The sun is to the south of the zenith at midday

   a) $\mathcal{B}_d$ is to the south of $\mathcal{B}_s$: additive

   b) $\mathcal{B}_d$ is to the north of $\mathcal{B}_s$: subtractive

[3] Computed in the process of deriving the solar amplitude from the sun's longitude.

2. The sun is to the north of the zenith at midday
   a) $\mathcal{B}_d$ is to the south of $\mathcal{B}_s$: subtractive
   b) $\mathcal{B}_d$ is to the north of $\mathcal{B}_s$: additive

## 14.6 Situation with two great shadows as solutions (*GD2* 226-227)

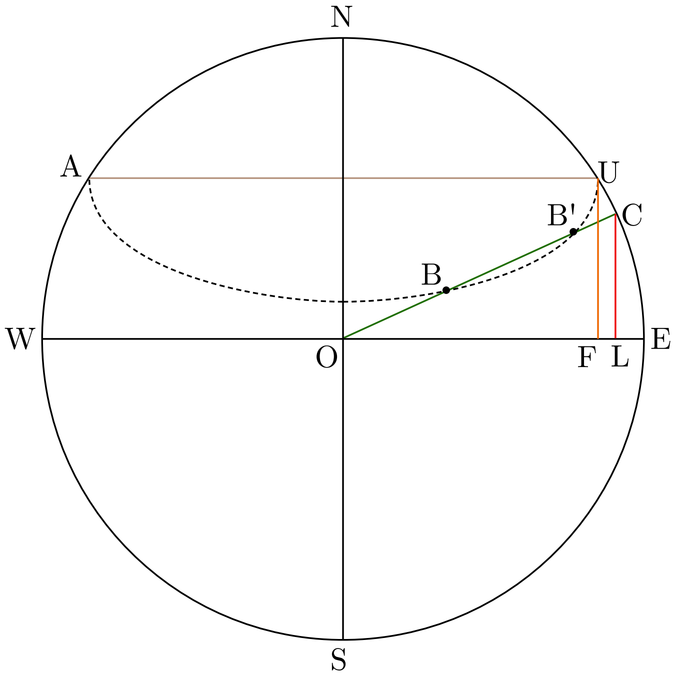



Figure 14.6: Situation with two great shadows OB and OB′

*GD2* 226 mentions that there could be a special case where two great shadows could be possible as solutions in the given direction (figure 14.6). According to Parameśvara, this happens when (1) the geographic latitude is sufficiently large and (2) the sun is to the north of the zenith at midday.

In fact, (2) is the only condition necessary. We can explain it as follows. The trail of the great shadow's foot in the course of a day, which is the projection of the diurnal circle on the plane of horizon as seen from the zenith, is always convex towards the south. Therefore, as long as the trail does not traverse the east-west line, we can always find a straight segment OC (which is the radius of the great circle) that intersects with the trail at two points B and B′. The condition for such trail is that the Sine of direction CL is smaller than the solar amplitude UF. This is indicated in *GD2* 226.

The two solutions (lengths of OB and OB′) are approached from opposite directions. That is, the entire correction in formula 14.3 is additive when $\mathcal{B}_d < \mathcal{B}_s$ and subtractive when $\mathcal{B}_d > \mathcal{B}_s$ to approach the "first" great shadow, and the rule is reversed to approach the "second" great shadow.

## 14.7 Steps in the "without-difference" method (*GD2* 230)

*GD2* 230 gives the order of computation in the "without-difference" method. The order is marked by the use of the ablative, and I have indicated it with arrows in the following list. Relevant verse numbers and sections/formulas in my explanatory notes are added in brackets.

1. Great shadow $\mathcal{S}$ → great gnomon $\mathcal{G}$ [*GD2* 114cd / section 8.5]

2. $\mathcal{G}$ → gnomonic amplitude $\mathcal{A}$ [*GD2* 119 / formula 8.13], base of direction $\mathcal{B}_d$ [*GD2* 221 / formula 14.1] and base to be established $\mathcal{B}_s$ [*GD2* 222-223 / formula 14.2]

3. The difference between $\mathcal{B}_d$ and $\mathcal{B}_s$ → $\mathcal{S}$ [*GD2* 224 / formula 14.3]. Repeat until $\mathcal{B}_d = \mathcal{B}_s$.

*GD2* 230 does not refer to the initial guess for $\mathcal{S}$. It also does not mention the computation of values that are only computed once and are fixed throughout the scheme, namely the solar amplitude $\mathrm{Sin}\,\eta$ and the multiplier $p$ and divisor $q$ of the correction. The commentary compute them at different places in the procedure. For example 3, which has two different cases, $\mathrm{Sin}\,\eta$ is computed at the very beginning since the "base" of the sun's longitude happens to be the same for both cases. Meanwhile $p$ and $q$ are computed at different places in the two cases. In the first case, it is at the very moment when they are applied to the difference between the two bases to compute the correction, but in the second case it is at the very beginning. The commentary on example 4 computes all of them before giving the initial guess.

# 15 Example 3 (*GD2* 231)

This is an example of the method explained in *GD2* 220-230. The situation described in this verse is as follows:

- Case 1
  - The sun's longitude is at the end of Scorpio ($\lambda = 8^s$).
  - The sun is in the southeast direction.
- Case 2
  - The sun's longitude is at the end of Taurus ($\lambda = 2^s$).
  - The sun is in the northeast direction.
- The Sine of geographic latitude is 647.
- The shadow-length of a gnomon with twelve *aṅgula*s is to be computed for the two cases.

## 15.1 Solution

**Initial values**

Before starting with the individual cases, the commentary computes the values for the Sine of declination and solar amplitude.

**"In both cases, the [Sine of] declination is 1210."**

In both cases, the "base" arc $\lambda_B$ is 2 signs, whose Sine is 2977 according to Bhāskara II and 2978 according to *Ābh* 1.12 (see Appendix B.4). If we use the former, *GD2* 73ab (formula 6.3) gives the Sine of declination $\mathrm{Sin}\,\delta$:

$$\begin{aligned} \mathrm{Sin}\,\delta &= \frac{1397\,\mathrm{Sin}\,\lambda_B}{R} \\ &= \frac{1397 \cdot 2978}{3438} \\ &= 1209;40,\cdots \end{aligned} \tag{15.1}$$

This can be rounded off to 1210. Āryabhaṭa's value 2978 gives $\mathrm{Sin}\,\delta = 1210;4,\cdots$, resulting in the same rounded value.

**"The solar amplitude is 1232."**

From *GD2* 84ab (formula 6.7), the solar amplitude is

$$\begin{aligned} \mathrm{Sin}\,\eta &= \frac{R\,\mathrm{Sin}\,\delta}{\mathrm{Sin}\,\bar{\varphi}} \\ &= \frac{3438 \cdot 1210}{3377} \\ &= 1231;51,\cdots \end{aligned} \tag{15.2}$$

which is rounded to 1232. The commentary is silent on the difference between the two cases, which is the direction of the solar amplitude as measured from the east-west line: It extends towards the south in case 1 and towards the north in case 2. The commentator implicitly uses this fact in the following passages.

### Case 1

**"In the first case, the shadow is assumed to be equal to the Radius."**

The commentary assumes that the great shadow $\mathcal{S}_1$ is equal to the radius $R$, 3438.

**"Then the solar amplitude itself is the base of direction."**

If we were to follow Parameśvara's instruction in *GD2* 230, we have to find the values for the great gnomon $\mathcal{G}_1$ is 0 and the gnomonic amplitude $\mathcal{A}_1$. The assumption $\mathcal{S}_1 = R$ puts the sun on the horizon and thus both $\mathcal{G}_1$ and $\mathcal{A}_1$ are 0. Thus the base of direction, which is the difference between the solar amplitude and the gnomonic amplitude, is equal to the solar amplitude ($\mathcal{B}_{d1} = \operatorname{Sin}\eta = 1232$). However, the commentary skips all these intermediate steps and goes immediately to the last point, as if it were self-evident.

**"From the Radius, the base to be established is established as 2431."**

The sun is in the southeast direction, which is an intermediary direction. *GD2* 222 tells us that the Sine of direction $\operatorname{Sin}\theta_\Sigma$ in this case is the Sine of one and a half sign.

$$\begin{aligned}\operatorname{Sin}\theta_\Sigma &= \operatorname{Sin}(1^s\ 15^\circ)\\ &= 2431\end{aligned} \tag{15.3}$$

which comes straightforward from the Sine series of the *Āryabhaṭīya*. Since the great shadow is equal to the Radius ($\mathcal{S} = R$ in formula 14.2), this Sine of direction itself is the base to be established, $\mathcal{B}_{s1}$.

Both manuscripts read 2432 instead of 2431, which must be a scribal error, since 2431 is being used in the next step.

**"The difference of these two in one [same] direction is 1199. This is the multiplicand."**

Both bases extend southward, thus their difference is taken as the multiplicand of the correction.

$$\begin{aligned}\mathcal{B}_{s1} - \mathcal{B}_{d1} &= 2431 - 1232\\ &= 1199\end{aligned} \tag{15.4}$$

**"In this case, since the sun is in the southern direction at sunrise and at midday, the difference between the Radius and the solar amplitude is the divisor, 2206."**

As the declination is southward, the sun rises at the south of due east. Since the observer is in the northern hemisphere, the diurnal circle is inclined to the south, and thus the sun also culminates in the south. Therefore from formula 14.5 the divisor $q$ is

$$\begin{aligned} q &= R - \operatorname{Sin} \eta \\ &= 3438 - 1232 \\ &= 2206 \end{aligned} \tag{15.5}$$

**"The midday shadow is 1795."**

The next value mentioned is the midday shadow, which from our reconstruction involves several steps of computation.

Since $\operatorname{Sin} \delta = 1210$ is between $\operatorname{Sin} 1125' = 1105$ and $\operatorname{Sin} 1350' = 1315$, the arc of declination $\delta$ is approximately:

$$\begin{aligned} \delta &= 1125 + \frac{1210 - 1105}{1315 - 1105} \cdot 225 \\ &= 1237;30 \end{aligned} \tag{15.6}$$

which can be rounded off to 1238. The declination is in the southern direction, opposite of the geographic latitude $\varphi = 651'$ (see page 294 for its derivation). Here we can use the rule mentioned in *GD2* 184-185 (formula 10.28) to obtain the meridian zenith distance $z_\Sigma$:

$$\begin{aligned} z_\Sigma &= \delta + \varphi \\ &= 1238 + 651 \\ &= 1889 \end{aligned} \tag{15.7}$$

Using $\operatorname{Sin} 1800' = 1719$ and $\operatorname{Sin} 2025' = 1910$, the midday shadow $\operatorname{Sin} z_\Sigma$ is approximately:

$$\begin{aligned} \operatorname{Sin} z_\Sigma &= 1719 + (1910 - 1719) \cdot \frac{1889 - 1800}{225} \\ &= 1794;33,\cdots \end{aligned} \tag{15.8}$$

which is rounded to 1795 as in the commentary.

**"The difference between the Radius and the midday shadow is the multiplier, 1643."**

From formula 14.4 the multiplier $p$ is

$$\begin{aligned} p &= R - \operatorname{Sin} z_\Sigma \\ &= 3438 - 1795 \\ &= 1643 \end{aligned} \tag{15.9}$$

**"These two will always be the multiplier and divisor in the 'without-difference' method."**

It might be worth remarking that this is the only place in the commentary which refers to the multiplier $p$ and divisor $q$ as being constant throughout the "without-difference" method. This is also the only case where $p$ and $q$ are computed in the middle of the "without-difference" method (i.e. after the initial guess has been given). The commentaries on case 2 of this example and on the two cases in example 4 compute $p$ and $q$ before the "without-difference" method.

**"The quotient [of the division] of the multiplicand multiplied by the multiplier by the divisor is 893."**

The entire correction is

$$\frac{(\mathcal{B}_{s1} - \mathcal{B}_{d1}) \cdot p}{q} = \frac{1199 \cdot 1643}{2206} \\ = 892; 59, \cdots \tag{15.10}$$

rounded to 893.

**"Since the base of direction is smaller than the base to be established [and thus] to the north [of it], this is subtractive against the shadow equal to the Radius that has been previously computed."**

Concerning the two bases, $\mathcal{B}_{s1} > \mathcal{B}_{d1}$. The commentary does not refer to their orientations, but we have seen that they are both southwards, and thus $\mathcal{B}_{d1}$ is to the north of $\mathcal{B}_{s1}$. We already know that the sun is to the north of the zenith at midday, and therefore the whole correction is subtractive.

**"When done in this way, the shadow is established as 2545."**

From formula 14.3,

$$\begin{aligned} \mathcal{S}_2 &= \mathcal{S}_1 - 893 \\ &= 3438 - 893 \\ &= 2545 \end{aligned} \tag{15.11}$$

**"In this case, this is the given shadow."**

There is no reference to cycles in the iteration method, but the second iteration starts here, by using the corrected value $\mathcal{S}_2$ in place of the initial guess for the great shadow.

**"Thus the [great] gnomon is established, and the gnomonic amplitude from the [great] gnomon."**

Unlike the first cycle, there is reference to the great gnomon and gnomonic amplitude. However their values are not given.

From the Pythagorean theorem (formula 8.9), the great gnomon $\mathcal{G}_2$ is

$$\begin{aligned} \mathcal{G}_2 &= \sqrt{R^2 - \mathcal{S}_2^2} \\ &= \sqrt{3438^2 - 2545^2} \\ &= 2311; 27, \cdots \end{aligned} \tag{15.12}$$

which can be rounded to 2311. Then using formula 8.13, we obtain the gnomonic amplitude $\mathcal{A}_2$:

$$
\begin{aligned}
\mathcal{A}_2 &= \frac{\mathcal{G}_2 \operatorname{Sin} \varphi}{\operatorname{Sin} \bar{\varphi}} \\
&= \frac{2311 \cdot 647}{3377} \\
&= 442; 45, \cdots
\end{aligned} \tag{15.13}
$$

which is likely rounded off to 443.

**"Since the gnomonic amplitude and the solar amplitude are in the same direction, their sum is the base of direction, extended north and south in the circle that has the shadow as its hypotenuse, 1675."**

The gnomonic amplitude always extends to the south, and as we have seen, the solar amplitude is also southward. Thus from formula 14.1, the base of direction in the second cycle is

$$
\begin{aligned}
\mathcal{B}_{d2} &= \mathcal{A}_2 + \operatorname{Sin} \eta \\
&= 443 + 1232 \\
&= 1675
\end{aligned} \tag{15.14}
$$

There is reference to a "circle that has the shadow as its hypotenuse (*chāyākarṇavṛtta*)", which is probably a reference to the circle of direction as seen in *GD2* 220 (section 14.2). This might indicate that the commentator was also using diagrams in the course of this procedure.

**"From the shadow, the base to be established is established as 1800."**

From formula 14.2, the base to be established in the second cycle is

$$
\begin{aligned}
\mathcal{B}_{s2} &= \frac{2431 \cdot \mathcal{S}_2}{R} \\
&= \frac{2431 \cdot 2545}{3438} \\
&= 1799; 33, \cdots
\end{aligned} \tag{15.15}
$$

which is rounded to 1800.

**"The difference between these two is 125."**

The two bases are in the same direction and we take their difference $\mathcal{B}_{s2} - \mathcal{B}_{d2} = 125$.

**"Having divided this multiplied by the multiplier by the divisor, the quotient is 93."**

Using the values of $p$ and $q$ as obtained previously, the whole correction is:

$$
\begin{aligned}
\frac{(\mathcal{B}_{s2} - \mathcal{B}_{d2}) \cdot p}{q} &= \frac{125 \cdot 1643}{2206} \\
&= 93; 5, \cdots
\end{aligned} \tag{15.16}
$$

which is rounded off to 93.

**"In this case again, one should subtract this from the previously [established] shadow, 2545, since the base of direction is smaller than the base to be established [and thus] to the north [of it]. Having done in that manner, the shadow is 2452."**

Again, $\mathcal{B}_{d2}$ is to the north of $\mathcal{B}_{s2}$ and the sun is to the north of the zenith at midday, therefore the whole correction is subtractive:

$$\begin{aligned}\mathcal{S}_3 &= \mathcal{S}_2 - 93\\ &= 2545 - 93\\ &= 2452\end{aligned} \tag{15.17}$$

**"Thus again, having done the [great] gnomon and so forth, the shadow without difference is 2407. This is the great shadow when the sun is in the southeast direction."**

The commentary tells us to carry on with the iteration method, but gives no more values except for the final result. I have ran a program using the software SAGE (The Sage Developers (2016)) to examine how the value would converge. Values are rounded off after each computation. The result is shown in table 15.1.

Table 15.1: Example 3 case 1 computed with SAGE

| Cycle | $\mathcal{S}$ | $\mathcal{B}_d$ | $\mathcal{B}_s$ | Correction |
|---|---|---|---|---|
| 1 | 3438 | 1232 | 2431 | 893 |
| 2 | 2545 | 1675 | 1800 | 93 |
| 3 | 2452 | 1694 | 1734 | 30 |
| 4 | 2422 | 1699 | 1713 | 10 |
| 5 | 2412 | 1701 | 1706 | 4 |
| 6 | 2408 | 1702 | 1703 | 1 |
| 7 | 2407 | 1702 | 1702 | 0 |

We arrive at the same value 2407 after 6 cycles, and can confirm that this is the final value in the 7th cycle.

**"Thus the shadow of the twelve *aṅgula* gnomon is established as $\genfrac{}{}{0pt}{}{11}{46}$."**

The commentary goes from the great shadow to the shadow of the twelve *aṅgula* gnomon without explanation, but we can find a rule for this in *GD2* 116. First we compute the great gnomon $\mathcal{G}$:

$$\begin{aligned}\mathcal{G} &= \sqrt{R^2 - \mathcal{S}^2}\\ &= \sqrt{3438^2 - 2407^2}\\ &= 2454;49,\cdots\end{aligned} \tag{15.18}$$

rounded to 2455, and then from *GD2* 116 (formula 8.10):

$$\begin{aligned} s &= \frac{12\mathcal{S}}{\mathcal{G}} \\ &= \frac{12 \cdot 2407}{2455} \\ &= 11; 45, 55, \cdots \end{aligned} \tag{15.19}$$

which is rounded to 11;46. Here the manuscript $I_1$ gives the value in a column, placing the integer 11 over the sexagesimal 46 (figure 15.1). Manuscript $K_5^+$ omits 46, but probably the original form was the same as $I_1$, since $K_5^+$ follows the same style to write 3;1 in the next case. These are the only occurrences of fractional parts notified in the form of a column.

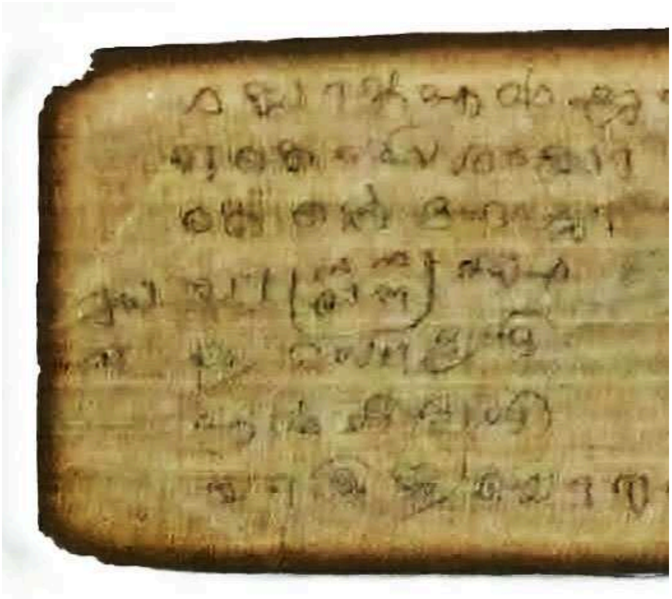


Figure 15.1: Part of Manuscript Indian Office Sanskrit 3530 ($I_1$), folio 41 recto. 11;46 in a column surrounded by a line can be seen at the middle of the image. The digital image acquired had been greatly distorted, and I have enhanced it here to clarify the letters.

### Case 2

**"Now in the second case, since the sun is in the northern direction at the time of sunrise and at the time of midday too, the difference between the Radius and the solar amplitude is the divisor, that has been indeed previously established, 2206."**

This time the commentary starts by computing the multiplier and divisor before the "without-difference" method.

The Sine of declination is 1210 as in the previous case, but this time it is northward. Thus sunrise occurs at the north of the prime vertical. As we have seen in section 14.5, there is no direct clue in *GD2* to find the direction of culmination, but from *GD2* 214ab we can derive the fact that if both $\delta$ and $\varphi$ are northward and $\delta > \varphi$, $z_\Sigma$ is northward and its value is $\delta - \varphi$. In the previous case we obtained $\delta = 1238$ and $\varphi = 651$. This time they are both northward, and thus the meridian zenith distance is northward and

$$\begin{aligned} z_\Sigma &= \delta - \varphi \\ &= 1238 - 651 \\ &= 587 \end{aligned} \tag{15.20}$$

Hence the computation for the divisor $q$ is to subtract the solar amplitude $\operatorname{Sin}\eta$, whose value we have already obtained, from the Radius.

$$\begin{aligned} q &= R - \operatorname{Sin}\eta \\ &= 3438 - 1232 \qquad = 2206 \end{aligned} \tag{15.21}$$

as was in case 1.

**"In this case, the midday shadow is 584."**

We have computed the meridian zenith distance of the sun $z_\Sigma$ in the previous step[1] (formula 15.20). Using $\operatorname{Sin} 450' = 449$ and $\operatorname{Sin} 675' = 671$, $\operatorname{Sin} z_\Sigma$ is approximately:

$$\begin{aligned} \operatorname{Sin} z_\Sigma &= 449 + (671 - 449) \cdot \frac{587 - 450}{225} \\ &= 584; 10, \cdots \end{aligned} \tag{15.22}$$

which is rounded to 584. This is equal to the midday shadow.

**"The difference between the midday shadow and the Radius is the multiplier, 2854."**

$$\begin{aligned} p &= R - \operatorname{Sin} z_\Sigma \\ &= 3438 - 584 \\ &= 2854 \end{aligned} \tag{15.23}$$

**"In this case, having assumed a given [great] shadow, having computed the [great] gnomon, the gnomonic amplitude, the base of direction and the base to be established from it as before, and having computed the result of the difference between the [two] bases with the multiplier and divisor and having shaped [the result] against the shadow assumed previously by oneself, subtractive or additive according to the rule, the [great] shadow without difference should be computed."**

Here the style of the commentary is very different compared with the previous cases. Instead of giving specific values for the great shadow and the following steps, the commentator focuses on the procedure itself. The contents of *GD2* 230 are given here with more specification. In

[1] Of course we do not know whether the commentator himself has actually computed the value of $z_\Sigma$ when he says "the sun is in the northern direction at the time of sunrise and at the time of midday" or just compared $\delta$ and $\varphi$.

addition to the sequence of segments involved, the fact that the great shadow is assumed at the beginning is mentioned, and the computation to obtain the correction with the multiplier and divisor is given in detail. The expression "subtractive or additive according to the rule" further adds the impression that this is a general commentary rather than dealing with a specific case.

**"This [great shadow] without difference is 840. This is the [great] shadow when the sun is in the northeast direction."**

Table 15.2 is the result of the "without-difference" method for this case, computed with a SAGE program. I have given $\mathcal{S}_1 = 3438$ as the initial guess. $\mathcal{S}$ converges to 839 instead of 840 as in the manuscripts[2] in 5 cycles. If we try to take the steps backwards and start from $\mathcal{S} = 840$, $\mathcal{B}_d = 593$ from formula 14.1 and $\mathcal{B}_s = 594$ from formula 14.2, and we still have a difference between the two bases. Furthermore, we will see that in the commentary after *GD2* 234, another value $\mathcal{S} = 838$ is given as an answer for this case. I cannot explain where these differences in the result come from.

Table 15.2: Example 3 case 2 computed with SAGE

| Cycle | $\mathcal{S}$ | $\mathcal{B}_d$ | $\mathcal{B}_s$ | Correction |
|---|---|---|---|---|
| 1 | 3438 | 1232 | 2431 | 1551 |
| 2 | 1887 | 681 | 1334 | 845 |
| 3 | 1042 | 604 | 737 | 172 |
| 4 | 870 | 595 | 615 | 26 |
| 5 | 844 | 593 | 597 | 5 |
| 6 | 839 | 593 | 593 | 0 |

**"The shadow of the twelve *aṅgula* gnomon is $\begin{matrix}\mathit{3}\\ \mathit{1}\end{matrix}$."**

If we follow the manuscript and use 840 as the great shadow, the great gnomon is

$$\begin{aligned}\mathcal{G} &= \sqrt{R^2 - \mathcal{S}^2}\\ &= \sqrt{3438^2 - 840^2}\\ &= 3333;48,\cdots \end{aligned} \tag{15.24}$$

rounded to 3334, and thus the shadow of the twelve *aṅgula* gnomon is

$$\begin{aligned} s &= \frac{12\mathcal{S}}{\mathcal{G}}\\ &= \frac{12\cdot 840}{3334}\\ &= 3;1,24,\cdots \end{aligned} \tag{15.25}$$

If we use $\mathcal{S} = 839$ and follow the same procedure, we obtain $s = 3;1,11,\cdots$. In both cases, the value can be rounded off to 3;1, corresponding to the value in manuscript $\mathrm{K}_5^+$, given in the form of a column. Manuscript $\mathrm{I}_1$ omits the sexagesimal 1.

[2] To be exact, manuscript $\mathrm{K}_5^+$ reads 84, but since the omission of 0 occurs frequently in this manuscript, this suggests that the original reading must have been 840 too, and not 839.

**“When the sun risen in the northern direction goes to the meridian in the southern direction, then the sum of the Radius and the solar amplitude is the divisor.”**

The commentary on example 3 (*GD2* 231) ends with a reference to a situation that is not covered by this example. However it does appear right afterwards as the first case in example 4 (*GD2* 232). Whether this passage was meant for supplementing information for readers just dealing with example 3 or as a connector to the next example is questionable.

# 16 Example 4 (*GD2* 232)

This is another example of the method given in *GD2* 220-230. Case 2 provides a situation where there are two possible shadow lengths, as mentioned in *GD2* 226-227.

- Case 1
  - The sun's longitude is at the middle of Aries ($\lambda = 0^s\ 15°$).
  - The sun is in the southeast direction
- Case 2
  - The sun's longitude is at the middle of Gemini ($\lambda = 2^s\ 15°$).
  - The sun is midway between east and northeast
- The Sine of geographic latitude is 647.
- The shadow-length of a gnomon with twelve *aṅgula*s is to be computed for the two cases.

## 16.1 Solution

### Case 1

#### "Now in the first case, the solar amplitude in the north is 368."

Unlike example 3, the "base" arc of the sun is different in the two cases, and therefore the solar amplitude is computed for both cases. Another difference is that the value for the Sine of declination is unmentioned. We assume that the Sine of declination is computed from the given longitude and then the solar amplitude is derived from the Sine of declination.

In the first case, the "base" arc $\lambda_B$ is $0^s\ 15° = 900'$, whose Sine is 890. From *GD2* 73ab (formula 6.3), the Sine of declination $\mathrm{Sin}\,\delta$ is

$$\begin{aligned}\mathrm{Sin}\,\delta &= \frac{1397\,\mathrm{Sin}\,\lambda_B}{R}\\ &= \frac{1397 \cdot 890}{3438}\\ &= 361;38,\cdots \end{aligned} \tag{16.1}$$

which is expected to be rounded off to 362. However, considering the values of the solar amplitude $\mathrm{Sin}\,\eta$ (368) and the midday shadow $\mathrm{Sin}\,z_\Sigma$ (289) which appear in the text, this has to be rounded off to 361. Indeed, if $\mathrm{Sin}\,\delta$ were rounded to 362, $\mathrm{Sin}\,\eta = 369$ and $\mathrm{Sin}\,z_\Sigma = 288$ after rounding. I have examined the possibility of other Sine tables and interpolation methods being used. Table 16.1 shows the results using those of Govindasvāmin, Mādhava and Nīlakaṇṭha[1], all of which end up being rounded to 362 or larger. This suggests the possibility of a table linking $\lambda_B$ directly with $\mathrm{Sin}\,\delta$ being used, as we have discussed in example 1 (section 12.2).

[1] Here I have only used the combination of each table with their corresponding interpolation method (e.g. Govindasvāmin's table with his interpolation method). For Nīlakaṇṭha I have used the table reconstructed from his second recursion method. It is safe to say that other combination of tables and methods will give no significantly different result, as Sin 24° is never smaller than 1397 and $R$ is never larger than 3438. See also appendix B.6.1.

Table 16.1: Using other Sines for computing $\mathrm{Sin}\,\delta$. $\mathrm{Sin}\,24°$ substitutes the value 1397 in formula 16.1.

| | $\mathrm{Sin}\,24°$ | $\mathrm{Sin}\,15°$ | $R$ | $\mathrm{Sin}\,\delta$ |
|---|---|---|---|---|
| Govindasvāmin | 1400;58,33 | 889;45,8 | 3437;44,19 | 362;35,··· |
| Mādhava | 1398,16,01 | 889;45,15 | 3437;44,48 | 361;53,··· |
| Nīlakaṇṭha | 1398;15,27 | 889;45,16 | 3437;44,47 | 361;53,··· |

Assuming $\mathrm{Sin}\,\delta = 361$, from *GD2* 84ab (formula 6.7), the solar amplitude $\mathrm{Sin}\,\eta$ is

$$\begin{aligned}\mathrm{Sin}\,\eta &= \frac{R\,\mathrm{Sin}\,\delta}{\mathrm{Sin}\,\bar{\varphi}}\\ &= \frac{3438 \cdot 361}{3377}\\ &= 367;31,\cdots \end{aligned} \tag{16.2}$$

which is rounded to 368.

### "The midday shadow in the south is 289."

The Sine of declination $\mathrm{Sin}\,\delta = 361$ is between $\mathrm{Sin}\,225' = 225$ and $\mathrm{Sin}\,450' = 449$. Thus the arc of declination $\delta$ is approximately

$$\begin{aligned}\delta &= 225 + \frac{361-225}{449-225} \cdot 225\\ &= 361;36,\cdots \end{aligned} \tag{16.3}$$

which can be rounded to 362. This declination is in the northern direction, as is the geographic latitude $\varphi$ (whose value is 651 from formula 13.4). The sun is to the south of the zenith, as in figure 16.1. Thus from *GD2* 184-185 (formula 10.28) the meridian zenith distance $z_\Sigma$ is

$$\begin{aligned}z_\Sigma &= \varphi - \delta\\ &= 651 - 362\\ &= 289 \end{aligned} \tag{16.4}$$

This is already equal to the value given in the commentary for the midday shadow, which is the Sine of this meridian zenith distance. We can confirm that the Sine and arc are approximately the same by linear interpolation. Using $\mathrm{Sin}\,225' = 225$ and $\mathrm{Sin}\,450' = 449$, the midday shadow $\mathrm{Sin}\,z_\Sigma$ is approximately:

$$\begin{aligned}\mathrm{Sin}\,z_\Sigma &= 225 + (289-225) \cdot \frac{449-225}{225}\\ &= 288;42,\cdots \end{aligned} \tag{16.5}$$

rounded to 289.

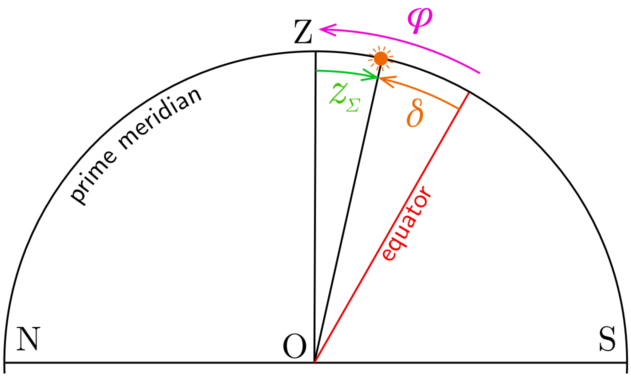



Figure 16.1: Meridian zenith distance $z_\Sigma$

**"Since these two are in different directions, in this case the sum of the Radius and the solar amplitude is the divisor, 3806."**

"These two" refers to the directions of the solar amplitude (northward) and the midday shadow (southward). Thus from *GD2* 228ab (formula 14.5) the divisor $q$ is

$$\begin{aligned} q &= R + \mathrm{Sin}\,\eta \\ &= 3438 + 368 \\ &= 3806 \end{aligned} \tag{16.6}$$

**"The multiplier is 3149."**

From *GD2* 228cd (formula 14.4) the multiplier $p$ is

$$\begin{aligned} p &= R - \mathrm{Sin}\,z_\Sigma \\ &= 3438 - 289 \\ &= 3149 \end{aligned} \tag{16.7}$$

**"The given assumed [great] shadow is 2977."**

The guess for the great shadow $\mathcal{S}_1$ is 2977, which is the equivalent of $\mathrm{Sin}\,60°$ given by Bhāskara II [2]. There is no explanation why it was not 3438, as is the case with every other guess for the great shadow throughout the commentaries in *GD2*. According to my computation with a SAGE program, 2977 as an initial guess requires 9 cycles of iteration to obtain the final result while it will converge in 8 cycle if 3438 were given.

A plausible explanation is that the commentator is demonstrating that the initial guess could be any value and not just 3438. At least, it is not the case that he chose an assumption that would work out the problem in a neat way.

[2] Bhāskara II gives 2977 instead of 2978 as in the *Āryabhaṭīya*. See Appendix B.4 for details.

**"The solar amplitude decreased by the gnomonic amplitude is 39. This is the base of direction in the north."**

This corresponds to the beginning of the first cycle of the "without-difference" method. However, the commentator does not refer to the values of the great gnomon and the gnomonic amplitude. This is the same with what we saw in example 3.

The great gnomon $\mathcal{G}_1$ could either be derived from the Pythagorean theorem (which gives $1719;41,\cdots \sim 1720$) or from the co-Sine ($\mathrm{Cos}\,60° = \mathrm{Sin}(90° - 60° = \mathrm{Sin}\,30° = 1719$). The final result of this step is in favor of the latter, 1719.

Then from *GD2* 119 (formula 8.13), the gnomonic amplitude $\mathcal{A}_1$ is

$$\begin{aligned}\mathcal{A}_1 &= \frac{\mathcal{G}_1 \,\mathrm{Sin}\,\varphi}{\mathrm{Sin}\,\bar{\varphi}} \\ &= \frac{1719 \cdot 647}{3377} \\ &= 329;20,\cdots \end{aligned} \tag{16.8}$$

which must have been rounded to 329. This gnomonic amplitude is southward while the solar amplitude is northward. Thus from formula 14.1, the base of direction is northward, its value is computed as follows:

$$\begin{aligned}\mathcal{B}_{d1} &= \mathrm{Sin}\,\eta - \mathcal{A}_1 \\ &= 368 - 329 \\ &= 39 \end{aligned} \tag{16.9}$$

**"In this case, the base to be established in the south is 2104."**

For the "base to be established" $\mathcal{B}_{s1}$, we first need to find the Sine of direction $\mathrm{Sin}\,\theta_\Sigma$. As we are dealing with an intermediate direction ($\theta_\Sigma = 1^s15° = 2700'$), $\mathrm{Sin}\,\theta_\Sigma = 2431$ as we computed in the previous example. Then from formula 14.2,

$$\begin{aligned}\mathcal{B}_{s1} &= \frac{\mathrm{Sin}\,\theta_\Sigma \cdot \mathcal{S}_1}{R} \\ &= \frac{2431 \cdot 2977}{3438} \\ &= 2105;1,\cdots \end{aligned} \tag{16.10}$$

Here we have used $\mathrm{Sin}\,60° = 2977$ according to Bhāskara II. If we use Āryabhaṭa's value, $\mathrm{Sin}\,60° = 2978$, the result is $2105;44,\cdots$ and the difference from "2104" as given in the text becomes larger. This discrepancy cannot be explained by replacing numbers[3], nor is it a scribal error (the results of the following steps show that $\mathcal{B}_{s1} = 2104$ is indeed being used). There seems to be an error in the computation itself.

[3] $\mathrm{Sin}\,\theta_\Sigma$ is the value of $\mathrm{Sin}\,45°$ or $\mathrm{Sin}\,2700'$, and the smallest value found in other tables is $2430;45,41$ according to Nīlakaṇṭha's first recursion method (see appendix B.6). The value for Sin 60 is between 2977 and 2978 in other tables, and $R$ is always smaller than 3438. None of these values can make $\mathcal{B}_{s1}$ smaller than 2104;30.

**"The sum of these two in different directions multiplied by the multiplier and divided by the divisor is 1773."**

$\mathcal{B}_{d1}$ is northward and $\mathcal{B}_{s1}$ southward, thus they should be added. From formula 14.3, the correction is

$$\begin{aligned}\frac{(\mathcal{B}_{d1}+\mathcal{B}_{s1})\cdot p}{q} &= \frac{(39+2104)\cdot 3149}{3806} \\ &= 1773;4,\cdots \end{aligned} \tag{16.11}$$

which is rounded to 1773.

**"Since the base of direction is in the north, this should be subtracted from the previous [guess] shadow."**

The commentary does not mention one of the conditions for determining whether the correction is additive or subtractive, which is the direction of the sun at midday. In this case, it is to the south of the zenith. Therefore, from *GD2* 225 we subtract the correction from the initial guess.

**"In that case, the [great] shadow produced is 1204."**

$$\begin{aligned}\mathcal{S}_2 &= \mathcal{S}_1 - 1773 \\ &= 2977 - 1773 \\ &= 1204 \end{aligned} \tag{16.12}$$

**"Having done again in this way, the [great] shadow without difference is 405."**

The "without-difference" method with a SAGE program converges as in table 16.2. Here I have used the values $\operatorname{Sin}\eta = 368$ and $\operatorname{Sin} z_\Sigma = 289$, and taken into account that the value $\mathcal{B}_{s1} = 2104$ is used in the first cycle. Interestingly, if we assume that every computation and rounding is performed as expected, and thus that the values $\operatorname{Sin}\eta = 369$ and $\operatorname{Sin} z_\Sigma = 288$ were used, the "without-difference" method will converge to a different value (table 16.3)[4]. This suggests that the final value for the great shadow is indeed the outcome of the "without-difference" method whose first steps have been shown here.

The commentary ends with the value of the great shadow, despite the fact that the example is asking for the shadow-length of a twelve *aṅgula* gnomon. Let us reconstruct the final answer.

The great gnomon $\mathcal{G}$ is computed from the Pythagorean theorem:

$$\begin{aligned}\mathcal{G} &= \sqrt{R^2 - \mathcal{S}^2} \\ &= \sqrt{3438^2 - 405^2} \\ &= 3414;3,\cdots \end{aligned} \tag{16.13}$$

rounded to 3414, and then from *GD2* 116 (formula 8.10):

[4] What matters for the result is the values for $\operatorname{Sin}\eta$ and $\operatorname{Sin} z_\Sigma$. Whether $\mathcal{B}_{s1}$ is 2104 or 2105 does not affect the computation.

Table 16.2: Example 4 case 1 computed with SAGE. $\mathrm{Sin}\,\eta = 368$, $\mathrm{Sin}\,z_\Sigma = 289$ and $\mathcal{B}_{s1} = 2104$ as in the commentary

| Cycle | $\mathcal{S}$ | $\mathcal{B}_d$ | $\mathcal{B}_s$ | Correction |
|---|---|---|---|---|
| 1 | 2977 | 38 | 2104 | 1773 |
| 2 | 1204 | 249 | 851 | 498 |
| 3 | 706 | 277 | 499 | 184 |
| 4 | 522 | 283 | 369 | 71 |
| 5 | 451 | 285 | 319 | 28 |
| 6 | 423 | 286 | 299 | 11 |
| 7 | 412 | 286 | 291 | 4 |
| 8 | 408 | 286 | 288 | 2 |
| 9 | 406 | 286 | 287 | 1 |
| 10 | 405 | 286 | 286 | 0 |

Table 16.3: Example 4 case 1, using $\mathrm{Sin}\,\eta = 369$, $\mathrm{Sin}\,z_\Sigma = 288$ and $\mathcal{B}_{s1} = 2105$.

| Cycle | $\mathcal{S}$ | $\mathcal{B}_d$ | $\mathcal{B}_s$ | Correction |
|---|---|---|---|---|
| 1 | 2977 | 39 | 2105 | 1774 |
| 2 | 1203 | 248 | 851 | 499 |
| 3 | 704 | 276 | 498 | 184 |
| 4 | 520 | 282 | 368 | 71 |
| 5 | 449 | 284 | 317 | 27 |
| 6 | 422 | 285 | 298 | 11 |
| 7 | 411 | 285 | 291 | 5 |
| 8 | 406 | 285 | 287 | 2 |
| 9 | 404 | 285 | 286 | 1 |
| 10 | 403 | 285 | 285 | 0 |

$$\begin{aligned} s &= \frac{12\mathcal{S}}{\mathcal{G}} \\ &= \frac{12 \cdot 405}{3414} \\ &= 1;25,24,\cdots \end{aligned} \tag{16.14}$$

Thus we would expect 1;25 as the shadow-length of a twelve *aṅgula* gnomon, rounded to the first sexagesimal.

### Case 2

This is a situation with two solutions for the shadow. However, the commentary says nothing on how we can conclude so, and goes on as if this fact was known from the beginning.

### "Now in the second case, the solar amplitude is 1373. This is northward."

The "base" arc $\lambda_B$ is $2^s\ 15°$, whose Sine is 3321. From *GD2* 73ab (formula 6.3), the Sine of declination $\mathrm{Sin}\,\delta$ is

$$\begin{aligned}\operatorname{Sin}\delta &= \frac{1397 \operatorname{Sin}\lambda_B}{R} \\ &= \frac{1397 \cdot 3321}{3438} \\ &= 1349; 27, \cdots \end{aligned} \tag{16.15}$$

which is probably rounded to 1349.

From *GD2* 84ab (formula 6.7), the solar amplitude $\operatorname{Sin}\eta$ is

$$\begin{aligned}\operatorname{Sin}\eta &= \frac{R \operatorname{Sin}\delta}{\operatorname{Sin}\bar{\varphi}} \\ &= \frac{3438 \cdot 1349}{3377} \\ &= 1373; 22, \cdots \end{aligned} \tag{16.16}$$

rounded to 1373 as expected. Since the sun is in Gemini, its declination and the solar amplitude are northward. I have supplied the word "northward (*saumyā*)" which does not appear in the manuscripts, but is required for the reading to make sense[5].

### "The midday shadow in the north is 731."

The Sine of declination $\operatorname{Sin}\delta = 1349$ is between $\operatorname{Sin} 1350' = 1315$ and $\operatorname{Sin} 1575' = 1520$. Thus the arc of declination $\delta$ is approximately

$$\begin{aligned}\delta &= 1350 + \frac{1349 - 1315}{1520 - 1315} \cdot 225 \\ &= 1387; 19, \cdots \end{aligned} \tag{16.17}$$

which would be expected to be rounded to 1387. This declination is in the northern direction, as is the geographic latitude $\varphi$, thus from *GD2* 184-185 (formula 10.28) the meridian zenith distance $z_\Sigma$ is

$$\begin{aligned}z_\Sigma &= \delta - \varphi \\ &= 1387 - 651 \\ &= 736 \end{aligned} \tag{16.18}$$

The sun is to the north of the zenith, as in figure 16.2. Using $\operatorname{Sin} 675' = 671$ and $\operatorname{Sin} 900' = 890$, the midday shadow $\operatorname{Sin} z_\Sigma$ is approximately

$$\begin{aligned}\operatorname{Sin} z_\Sigma &= 671 + (890 - 671) \cdot \frac{736 - 675}{225} \\ &= 730; 22, \cdots \end{aligned} \tag{16.19}$$

which would be rounded off to 730, but the value given here and used in the following step is 731. We have no clue to why.

[5]The omission of *saumyā* can be explained as a haplology. Without it, the word "this *eṣā*" would be joined with next sentence to read "This is the midday shadow in the north, 731 (*eṣā saumyadinārdhabhā 731*)" where "this" becomes meaningless.

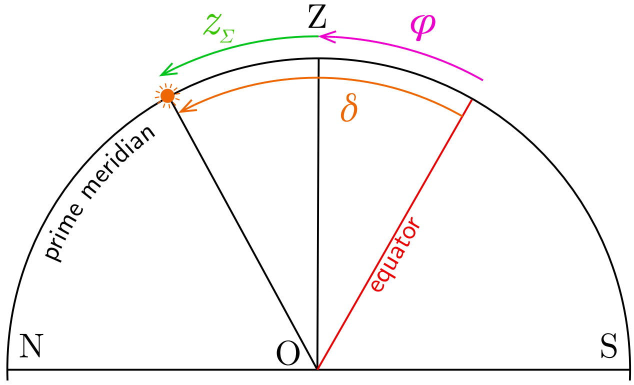



Figure 16.2: Meridian zenith distance $z_\Sigma$

### "The divisor is 2065."

Both sunrise and the culmination of the sun occur in the north. Thus from formula 14.5 the divisor $q$ is

$$\begin{aligned} q &= R - \operatorname{Sin}\eta \\ &= 3438 - 1373 \\ &= 2065 \end{aligned} \tag{16.20}$$

### "The multiplier is 2707."

From formula 14.4 the multiplier $p$ is

$$\begin{aligned} p &= R - \operatorname{Sin} z_\Sigma \\ &= 3438 - 731 \\ &= 2707 \end{aligned} \tag{16.21}$$

### "In this case, the Sine of direction is 1315."

Here the commentary refers to the Sine of direction and its value for the first time throughout the solutions of example 3 and 4. This might be related to the fact that in all the previous cases the sun was in an intermediate direction while here, the direction is between east and northeast, i.e. $22^\circ\ 30'$ north from due east. The Sine of direction $\operatorname{Sin}\theta_\Sigma$ is $\operatorname{Sin} 22^\circ\ 30' = 1315$.

At this point, we can find out from *GD2* 226 that there should be two solutions for the great shadow, since the solar amplitude is larger than the Sine of direction in the north. However the commentary says nothing on this point.

### "The assumed [great] shadow is 3438."

In order to approach the two solutions from one initial guess following *GD2* 226-227, the guess should fall between the two final values of the great shadow. By chance, the assumption $\mathcal{S}_1 = 2977$ that was used in the first case matches this condition, but here the commentary assumes

$\mathcal{S}_1 = 3438$. We have discussed in the previous case that the commentator's intention seems not to be to give a smart solution, and such tendency can be seen here too.

We might also be able to justify the commentator's assumption by the fact that the first of the two great shadow is the longer one. $\mathcal{S}_1 = 3438$ is the largest value possible as a guess, and will lead to the first great shadow with certainty. The commentator's strategy appears to be to find the first great shadow in this way, and then use a value smaller than the established first great shadow, which in turn will lead to the second great shadow.

### "In this case, the solar amplitude itself is the base of direction."

As we have already seen in example 3, the assumption that the great shadow is equal to the Radius leads to the conclusion the base of direction is equal to the solar amplitude (in this case, $\mathcal{B}_{d1} = \mathrm{Sin}\,\eta = 1373$).

### "The Sine of direction itself is the base to be established."

We have also seen in the previous example that the base to be established is equal to the Sine of direction (in this case, $\mathcal{B}_{s1} = \mathrm{Sin}\,\theta_\Sigma = 1315$) when the initial guess is the Radius.

### "From the difference between the bases, the result is 76."

Both $\mathcal{B}_{d1}$ and $\mathcal{B}_{s1}$ are northward, and $\mathcal{B}_{d1} > \mathcal{B}_{s1}$. From formula 14.3, the correction is

$$\begin{aligned}\frac{(\mathcal{B}_{d1} - \mathcal{B}_{s1}) \cdot p}{q} &= \frac{(1373 - 1315) \cdot 2707}{2065} \\ &= 76; 1, \cdots \end{aligned} \tag{16.22}$$

which is rounded off to 76.

### "This should be subtracted from the given shadow in order to establish the first [great] shadow, since the base of direction is larger."

It is at this point that the commentary explicitly makes the reader aware that there are two solutions. It informs us that the correction 76 has to be subtracted since $\mathcal{B}_{d1} > \mathcal{B}_{s1}$. This rule comes from *GD2* 227.

### "When the base of direction is smaller, then it should be added."

The commentator refers to the other situation, which is that the correction should be added if $\mathcal{B}_{d1} < \mathcal{B}_{s1}$. There is no specific instruction to iterate the procedure, but at least it has provided every information necessary to do so.

### "In this case, the [great] shadow without difference is 3422. This should be the great shadow when the sun is at the midpoint between the northeast and east."

There are problems in both the "without-difference" method and the final value given in the commentary. The iteration carried on with a SAGE program resulted in an oscillation, as shown in table 16.4. It might be possible that Parameśvara was aware that this could happen, since

*GD2* 233cd refers precisely to when an oscillation occurs in an "without-difference" method. We may follow his instruction and subtract the correction 17 by half of itself ($17 \div 2 \sim 9$), which reduces the correction to 8. By chance, if we adopt this value in the third cycle and subtract it from $\mathcal{S}_3 = 3429$, we obtain 3421 which gives $\mathcal{B}_d = \mathcal{B}_s = 1308$ and the "without-difference" method is immediately finished.

Table 16.4: Example 4 case 2 (first shadow) computed with SAGE

| Cycle | $\mathcal{S}$ | $\mathcal{B}_d$ | $\mathcal{B}_s$ | Correction |
|---|---|---|---|---|
| 1 | 3438 | 1373 | 1315 | 76 |
| 2 | 3362 | 1235 | 1286 | 67 |
| 3 | 3429 | 1325 | 1312 | 17 |
| 4 | 3412 | 1292 | 1305 | 17 |
| 5 | 3429 | 1325 | 1312 | 17 |
| 6 | 3412 | 1292 | 1305 | 17 |
| | ... | | | |

However, the value we obtain is 3421 and not 3422 as in the commentary. $\mathcal{S} = 3422$ gives $\mathcal{B}_d = 1310$ and $\mathcal{B}_s = 1309$ after rounding, and we still have a difference between the two bases. This value 3422 is used for creating the initial guess in the next step, and cannot be a scribal error. In any case, if we take this value as the great shadow, the great gnomon is

$$\begin{aligned}\mathcal{G} &= \sqrt{R^2 - \mathcal{S}^2}\\ &= \sqrt{3438^2 - 3422^2}\\ &= 331; 18, \cdots \end{aligned} \tag{16.23}$$

rounded to 331, and thus the shadow of the twelve *aṅgula* gnomon is

$$\begin{aligned}s &= \frac{12\mathcal{S}}{\mathcal{G}}\\ &= \frac{12 \cdot 3422}{331}\\ &= 124; 3, 37, \cdots \end{aligned} \tag{16.24}$$

which would be rounded to either 124 or 124;3, but the commentator makes no reference to its value. If we choose $\mathcal{S} = 3421$, we obtain $s = 120; 23, 13, \cdots$, which makes a significant difference.

### "In this very case, there is a second [great] shadow."

Again the commentator draws attention to the existence of the second solution, although it has been already mentioned in the course of the previous solution.

### "In order to establish it, having assumed a given [great] shadow decreased by a given number from the [great] shadow in the given direction established in the first case, the computation is to be carried out."

If use the initial guess 3438 as in the first great shadow and follow *GD2* 227, the correction will now be additive, leading to an impossible value (larger than the Radius) in the next step. As we have already discussed, we need to start with a value smaller than the first solution. This is a procedure which Parameśvara has not mentioned.

### "In that case, the previous [great] shadow decreased by a thousand is 2422."

The commentator subtracts 1000 from the first great shadow as the starting point ($\mathcal{S}_1 = 2422$) for the second great shadow. Any value would work, and we cannot find a specific reason for the choice of 1000.

### "The base of direction is 906."

We already know the values for the solar amplitude, the multiplier and divisor. If we were to follow Parameśvara's steps, we have to compute the great gnomon and the gnomonic amplitude, but they are unmentioned here. In any case, we need them to compute the base of direction.

From the Pythagorean theorem, the great gnomon $\mathcal{G}_1$ is

$$\begin{aligned}\mathcal{G}_1 &= \sqrt{R^2 - \mathcal{S}_1^2}\\ &= \sqrt{3438^2 - 2422^2}\\ &= 2440; 1, \cdots \end{aligned} \tag{16.25}$$

which can be rounded to 2440.

Using formula 8.13, the gnomonic amplitude $\mathcal{A}_1$ is

$$\begin{aligned}\mathcal{A}_1 &= \frac{\mathcal{G}_1 \operatorname{Sin} \varphi}{\operatorname{Sin} \bar{\varphi}}\\ &= \frac{2440 \cdot 647}{3377}\\ &= 467; 28, \cdots \end{aligned} \tag{16.26}$$

which can be rounded to 467.

The solar amplitude is northward and the gnomonic amplitude southward. Thus from formula 14.1, the base of direction is northward and its value is

$$\begin{aligned}\mathcal{B}_{d1} &= \operatorname{Sin} \eta - \mathcal{A}_1\\ &= 1373 - 467\\ &= 906 \end{aligned} \tag{16.27}$$

### "The established shadow is 926."

From formula 14.2,

$$\begin{aligned}\mathcal{B}_{s1} &= \frac{\operatorname{Sin} \theta_\Sigma \cdot \mathcal{S}_1}{R}\\ &= \frac{1315 \cdot 2422}{3438}\\ &= 926; 23, \cdots \end{aligned} \tag{16.28}$$

which is rounded to 926.

### "The result of the difference between the bases is 26."

Both $\mathcal{B}_{d1}$ and $\mathcal{B}_{s1}$ are northward, and $\mathcal{B}_{d1} > \mathcal{B}_{s1}$. From formula 14.3, the correction is

$$\begin{aligned}\frac{(\mathcal{B}_{s1} - \mathcal{B}_{d1}) \cdot p}{q} &= \frac{(926 - 906) \cdot 2707}{2065} \\ &= 26; 13, \cdots \end{aligned} \tag{16.29}$$

rounded to 26.

### "This should be subtracted in order to establish the second [great] shadow, since the base of direction is smaller."

Since we are computing the second great shadow and $\mathcal{B}_{d1} > \mathcal{B}_{s1}$, following *GD2* 227, the correction is to be subtracted from the guessed great shadow.

### "In this case, the [great] shadow without difference is 2318. This is the second [great] shadow in the given direction."

The "without-difference" method computed with SAGE proceeds as in table 16.5. This time the convergence is slow, and we can see again a connection with *GD2* 233, although neither the commentary nor Parameśvara refers to this point. The final value in our computation is 2320 and not 2318 as in the commentary. If we reverse the computation and start from $\mathcal{S} = 2318$ we obtain $\mathcal{B}_d = \mathcal{B}_s = 887$ after rounding. Therefore 2318 is another value which fits the condition. The fact that the commentary gives this number could be explained by increasing the correction at some point, as *GD2* 233 instructs to do when the "without-difference" method is converging slowly.

Table 16.5: Example 4 case 2 (second shadow) computed with SAGE

| Cycle | $\mathcal{S}$ | $\mathcal{B}_d$ | $\mathcal{B}_s$ | Correction |
|---|---|---|---|---|
| 1 | 2422 | 906 | 926 | 26 |
| 2 | 2396 | 901 | 916 | 20 |
| 3 | 2376 | 897 | 909 | 16 |
| 4 | 2360 | 894 | 903 | 12 |
| 5 | 2348 | 892 | 898 | 8 |
| 6 | 2340 | 890 | 895 | 7 |
| 7 | 2333 | 889 | 892 | 4 |
| 8 | 2329 | 888 | 891 | 4 |
| 9 | 2325 | 888 | 889 | 1 |
| 10 | 2324 | 888 | 889 | 1 |
| 11 | 2323 | 888 | 889 | 1 |
| 12 | 2322 | 887 | 888 | 1 |
| 13 | 2321 | 887 | 888 | 1 |
| 14 | 2320 | 887 | 887 | 0 |

Let us reconstruct the answer required by the example, which is the shadow-length of a twelve *aṅgula* gnomon. If we choose $\mathcal{S} = 2318$, the great gnomon $\mathcal{G}$ computed from the Pythagorean theorem is

$$\begin{aligned}\mathcal{G} &= \sqrt{R^2 - \mathcal{S}^2} \\ &= \sqrt{3438^2 - 2318^2} \\ &= 2539; 2, \cdots \end{aligned} \tag{16.30}$$

rounded to 2539, and then from *GD2* 116 (formula 8.10):

$$\begin{aligned}s &= \frac{12\mathcal{S}}{\mathcal{G}} \\ &= \frac{12 \cdot 2318}{2539} \\ &= 10; 57, 19, \cdots \end{aligned} \tag{16.31}$$

which would be rounded to 10;57 as the shadow's length.

### "From these two, the two shadows of the twelve *aṅgula* gnomon are established."

Last of all the commentary does mention that we need to compute the shadow-length of the twelve *aṅgula* gnomon but does not give its value. Here, it is ambiguous whether "these two" refer to the two solutions in case 2 or to the two cases in this example.

# 17 Speed of "without-difference" method (*GD2* 233-234)

In example 4, we came across a case where the convergence of the "without-difference" method was slow, and also a case where the value oscillated and did not converge. *GD2* 233 is an instruction on what to do in such situations. Whether the two cases in example 4 and/or its solution[1] were designed to cause such peculiarity in its convergence is uncertain, but even if it were not, it is reasonable that Parameśvara put this verse at this position, since *GD2* 220-230 is the first appearance of an "without-difference" method in this treatise. He has made a similar statement in *GD1* 4.21-22 (see quotation later in this section), right after an explanation of an "without-difference" method. This comes before an example (*GD1* 4.23), and should thus be understood as a general rule and not as an instruction limited to a specific example. The commentary confirms the generality of this rule in its last sentence.

"Result" refers to the correction produced from the two bases, the multiplier and divisor (formula 14.3). The statement of the verse is repeated in the commentary in an expanded style, referring to more values than in the verse (table 17.1). It is remarkable that both Parameśvara and the commentator speaks of "adding" and "subtracting" values against the correction and not of multiplying or dividing it. Instead of saying "double or triple the result", the commentary uses a lengthy expression "add with the result multiplied by one or added by twice".

In *GD2* 234, Parameśvara states that the difference between the base of direction and the base to be established itself can be used as the correction, without applying the multiplier and divisor. This time he suggests doubling or halving the amount, contrary to what we have just seen. This mixture of expressions (adding / subtracting and multiplying / dividing) can also be found in *GD1*:

> The result to be subtracted and added should be assumed to be increased by half or multiplied by two in the rule of the "without-difference" method, in accordance with the slowness of approach toward the desired value.
> When [the approach is] too fast, in like manner, [the result] should be assumed to be lessened by a third or halved. (*GD1* 4.21-22ab)[2]

Plofker (2004, pp. 581-582) explains Parameśvara's procedure as "multiplying their difference by a scale factor" which is 1.5, 2, $\frac{2}{3}$ or $\frac{1}{2}$. However, considering Parameśvara's expressions, it is questionable whether he is introducing a scale factor or relaxation factor as in iterative methods used today. One clue is the word *yuktyā* used in *GD2* 233 and its commentary which I have translated "with reason". This is the instrumental of *yukti*, which is used in the sense of "grounding" almost elsewhere in *GD2*. *GD2* 119, 188, 198 and 204 use *yukti* to refer to a

[1] By "solution" I refer to the choice of the initial value. However, choosing a different guess did not change the process very often, especially in the case with slow convergence.

[2] *śodhyaṃ kṣepyaṃ ca phalaṃ sārdhaṃ dviguṇaṃ tathāviśeṣavidhau |*
*āsatter māndyavaśād abhīṣṭarāśeḥ sadā kalpyam ||4.21||*
*atiśaighrye tryaṃśonaṃ dalitaṃ vā tadvad eva kalpyaṃ syāt* | (K. V. Sarma (1956–1957, p. 48))

Table 17.1: Corrections to be applied in an "without-difference" method when the original value is $x$

| | *GD2* 233 | Commentary |
|---|---|---|
| Slow convergence | $x+\frac{x}{2}$ | $x+\frac{x}{2}$, $x+x$, $x+2x$ |
| Oscillation | $x-\frac{x}{2}$ | $x+\frac{x}{2}$, $x+\frac{2x}{3}$, $x+\frac{3x}{4}$ |

proportion or Rule of Three that grounds a specific rule. If *yuktyā* in *GD2* 233 is also conveying the sense of "proportion", we may say that some idea of scaling is behind the rule, even when Parameśvara refers to adding or subtracting.

The commentary after *GD2* 234 is apparently unrelated with the verse itself. The text is difficult to interpret, and we cannot even rule out the possibility of the text being corrupted. One interpretation is that this statement is for taking into account the motion of the solstice. In the previous examples, the longitude was given by the zodiacal sign, i.e. a sidereal coordinate. The shadow length thus computed would be different from observation. Meanwhile, if we compute the sun's longitude and declination using an observed shadow, with a method such as the one expressed in *GD2* 213-217 or even the method in the next section, *GD2* 235-244. However, it is questionable whether it is meaningful to compute the shadow again, and nothing can be said about what "by the co-latitude and so forth established with fractions (*laṃbādibhiḥ sādhanaiḥsāvayavaiḥ*)" stands for.

The commentary then turns back to case 2 in example 3. There is a suggestion of a "without-difference" method performed to obtain the great shadow, probably using different values as the declination and so forth. Its value given here is 838, different from 840 which was given in the solution or 839 that we derived. However this is another correct answer for the example without modifying any of the given values; 839 as the great shadow gives the same value 593 for the base of direction and the base to be established.

We shall discuss the contents of the next paragraph in chapter 18, since it is related to verses *GD2* 235-244.

# 18 Finding the sun and geographic latitude from the shadow in an intermediate direction (*GD2* 235-244)

## 18.1 Summary of the method

### Outline according to the commentary

Parameśvara explains a new method in *GD2* 235-244, but unlike the previous method where *GD2* 220ab gave a summary, we have no explanation on its goals. The commentary provides us with its outline before *GD2* 235. According to it, there are two steps.

In the first step, we find the longitude of the sun when the sun is in an intermediate direction, from (1) the length of a shadow[1] and (2) the hour angle, i.e. the time left before the sun reaches culmination or elapsed after its culmination.

In the second step, we start with the sun's declination (this is obtained in the course of the previous step, but the commentary makes no remark on this point) and compute the Sine of geographic latitude. We will see later that this is done by an "without-difference" method. The sun' declination is obtained in the course of the previous step, and hence bridges the two steps. However the declination is not essentially the starting point of this computation, as we will discuss later.

### The substeps

We can summarize the entire method with its two steps and substeps as follows.

- Step 1
    1. The shadow's base $\mathcal{B}$ and upright $\mathcal{U}$ are computed. (*GD2* 235)
    2. The "upright" in the diurnal circle $u$ is equal to $\mathcal{U}$. (*GD2* 236cd)
    3. The Sine of the hour angle $\mathrm{Sin}\, H$ is computed. (*GD2* 237)
    4. The radius of the diurnal circle $r$ is computed from $\mathrm{Sin}\, H$, $u$ and $R$ with a Rule of Three (*GD2* 238)
    5. $r \rightarrow$ [Sine of] declination $\mathrm{Sin}\, \delta$ (*GD2* 239ab)
    6. $\mathrm{Sin}\, \delta \rightarrow$ "base" arc $\lambda_B$ (*GD2* 239b)
    7. $\lambda_B \rightarrow$ longitude $\lambda$ (*GD2* 239cd-240)
- Step 2
    1. Some amount added to $\mathrm{Sin}\, \delta$ is the first assumption for the solar amplitude $\mathrm{Sin}\, \eta$ (*GD2* 241)
    2. $\mathrm{Sin}\, \eta$ and $\mathcal{B} \rightarrow$ gnomonic amplitude $\mathcal{A}$ (*GD2* 242ab)
    3. $\mathcal{A}$ and great gnomon $\mathcal{G} \rightarrow$ given "Sine" in the diurnal circle $j_t$ (*GD2* 242cd)
    4. $R, \mathcal{A}$ and $j_t \rightarrow$ Sine of geographic latitude $\mathrm{Sin}\, \varphi$ (*GD2* 243ab)
    5. $\mathrm{Sin}\, \varphi \rightarrow$ Sine of co-latitude $\mathrm{Sin}\, \bar{\varphi}$ (*GD2* 243c)
    6. $\mathrm{Sin}\, \bar{\varphi}$ and $\mathrm{Sin}\, \delta \rightarrow \mathrm{Sin}\, \eta$ (*GD2* 243d)
    7. Repeat 2-6. The result is $\mathrm{Sin}\, \varphi$ without difference. (*GD2* 244)

[1]Here the commentary does not say whether this is a shadow of a gnomon or a great shadow.

**The intermediate direction**

It may be worth remarking that Parameśvara only explains the case when the sun is in an intermediate direction (northeast, southeast, southwest or northwest). In fact, we could easily generalize the rule[2] so that it would be applicable to the sun in any direction, as was the case with the previous method (*GD2* 220-230). The problem of finding the great gnomon when the sun is in an intermediate direction is a popular topic in Sanskrit astronomical treatises although its motivation is unknown (Plofker (2004)), and Parameśvara's choice is most likely in line with this tradition.

*GD2* 247 might be a reference to this matter, indicating that the method is applicable when the sun is in any direction. We will see this later in section 20.2.

## 18.2 Base and upright of the shadow in an intermediate direction (*GD2* 235-236ab)

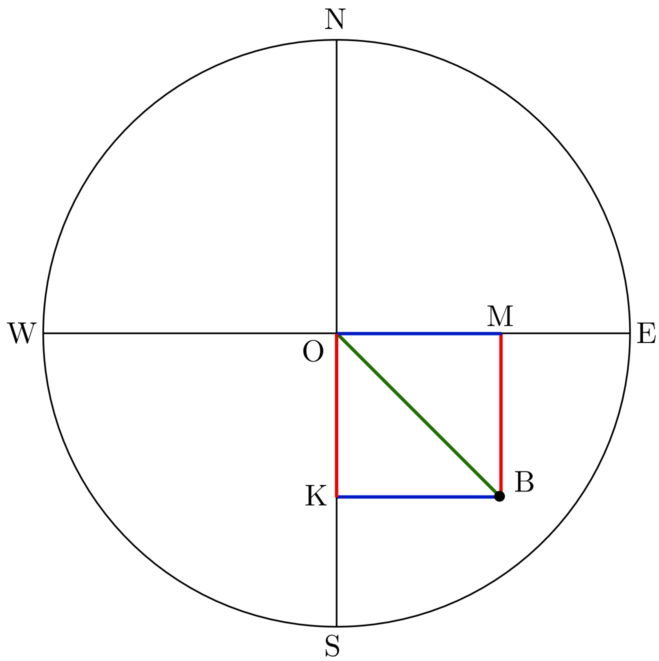



Figure 18.1: The base MB / OK and upright KB / OM of the great shadow OB when the great gnomon is in an intermediate direction (here southeast)

As was the case with the previous method (see section 14.2), *GD2* 235-244 only refer to the "shadow (*chāyā*)" without adding "great (*mahā*)", and the statement in *GD2* 235 is valid for both the great shadow and the shadow of a gnomon. Therefore I have translated this word as "shadow" without supplying "great". Meanwhile the commentary starts by computing the great

[2] To be specific, we would only need to change *GD2* 235. The base of a shadow can be computed with the "Sine of direction" as we did for the "base to be established" in the previous method, and then the upright can be obtained with a Pythagorean theorem.

shadow, and therefore in the following explanatory notes I shall treat what Parameśvara states "shadow" as a great shadow.

The two components of the great shadow, extending north-south and east-west respectively, are equal in length if the sun is in an intermediate direction (figure 18.1). *GD2* 236ab tells us that the north-south component is called the base of the shadow while the east-west component is the upright. The base is fully utilized in the previous method (*GD2* 220-230), and as quoted in section 14.2, the auto-commentary on *GD1* 4.12-13ab describes the base and upright in a similar manner. The difference is that if we follow the auto-commentary, the base and upright have to be segments which have the foot of the great gnomon as one end. Thus in figure 18.1, only MB could be called the base and KB the upright. However, *GD2* 236ab allows for a loose interpretation, since it does not refer to the foot of the great gnomon. In figure 18.1, we can also take OK as the base and OM as the upright. If this is really what Parameśvara intended, it might be because we can form a right triangle with the great shadow as hypotenuse in this way. This is also an isosceles triangle, and therefore the length of the base or upright is the hypotenuse divided by the square root of two. Or to formulate what we have in *GD2* 235, the base MB = OK = $\mathcal{B}$ and upright KB = OM = $\mathcal{U}$ of the great shadow $\mathcal{S}$ are

$$\mathcal{B} = \mathcal{U} = \sqrt{\frac{\mathcal{S}^2}{2}} \tag{18.1}$$

## 18.3 The upright in the diurnal circle (*GD2* 236cd)

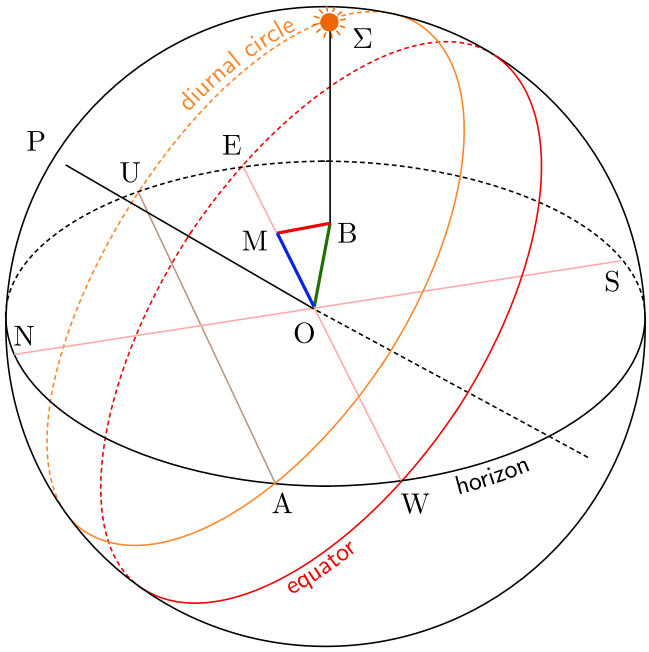



Figure 18.2: The upright of the shadow OM and the celestial sphere

If we choose OM as the upright of the shadow, it is in the plane of the celestial equator and not in the plane of the diurnal circle (figure 18.2). However, by looking at this situation from

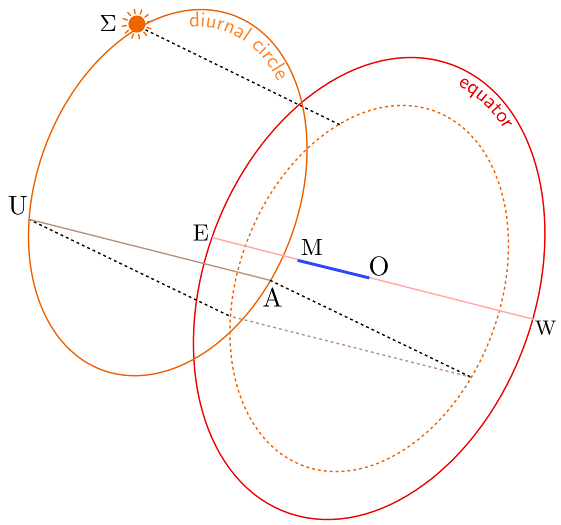



Figure 18.3: Projecting the diurnal circle

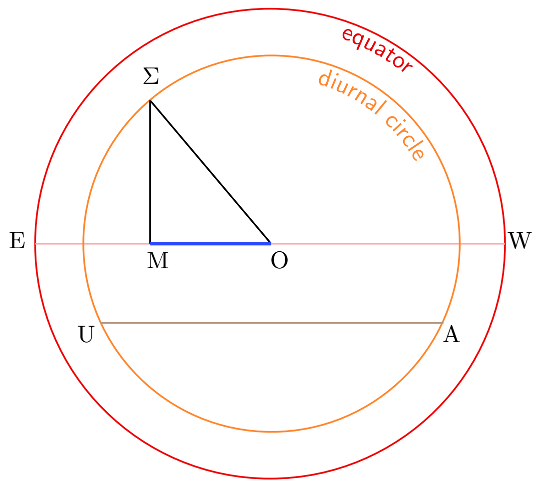



Figure 18.4: △OMΣ in the diurnal circle

the celestial north pole so that the celestial equator and the diurnal circle appear as concentric circles, we can project the diurnal circle to the plane of the celestial equator (figure 18.3). As a result, OM now forms a right triangle △OMΣ with the point of the sun Σ (figure 18.4). This can be easily visualized with an armillary sphere. My interpretation of what Parameśvara calls the "upright in the diurnal circle" is this projected segment OM. I will come back to the reason why he refers to it as an upright in section 18.5.

This situation is comparable with what has been discussed in *GD2* 110 (section 8.4, page 196), although Parameśvara does not make the connection. In *GD2* 110, the aim was to move from a segment in the celestial equator to a segment in the diurnal circle with the use of Rules of Three. Meanwhile, the procedure in *GD2* 236cd itself is different in the sense that the segment OM has been moved to the diurnal circle without changing its length. But this OM shall be used right afterward in *GD2* 238 to form a Rule of Three which will relate segments of different lengths in the celestial equator and the diurnal circle.

## 18.4 Hour angle (*GD2* 237)

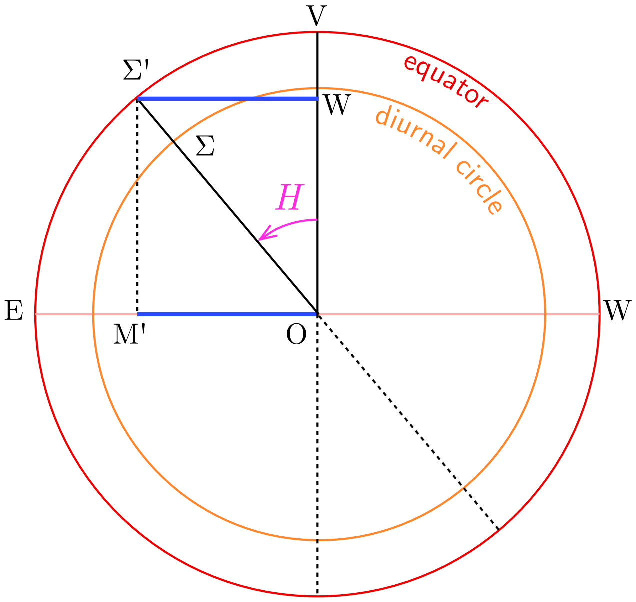



Figure 18.5: The hour angle $H = \widehat{\mathrm{V}\Sigma'}$ (for a moment of time in the morning) and its Sine W$\Sigma'$ or OM$'$

*GD2* 237 describes the Sine of an arc which corresponds to an "hour angle (*nata*)". This is stated as the time difference between the sky (*kha*) and the sun. The same expression occurs in *GD2* 245, where the commentary paraphrases "sky" with "zenith (*khamadhya*, literally 'middle of sky')". If we imagine two great circles going through the celestial poles, one passing the zenith and one passing the sun, we have the same situation with the above definition. Alternatively, we can interpret that the words "[middle of the] sky" and "sun" each refer to the rising time of the two points in the stellar sphere. Their difference is an arc measured on the celestial equator.

Let us look at the armillary sphere from the celestial north pole again (figure 18.5). Here the northern celestial pole overlaps with the observer O. $\Sigma'$ is the intersection of the celestial equator with the great circle passing the celestial pole and the sun $\Sigma$, and V is that of the prime meridian with the celestial equator[3]. $\widehat{\mathrm{V}\Sigma'}$ is the hour angle $H$ as stated in *GD2* 236.

Today, the hour angle is usually measured westward from the meridian zenith, but here in Parameśvara's explanation, it can be in both directions. The hour angle of the sun in the morning is measured eastward and that in the afternoon westward. Another difference is the unit: modern astronomy uses either hours or degrees, but here Parameśvara uses *nāḍī*s (1/60 of a day, synonym *ghaṭikā*). Interestingly, both examples 5 and 6 (*GD2* 245, 246) give them in *prāṇa*s (1/21600 of a day, synonym *asu*). The latter is more convenient for computation, as one

[3] The zenith is not shown in this figure. It would be somewhere between V and O, depending on the geographic latitude.

*prāṇa* corresponds to one minute of arc in the celestial equator. The usage of *nāḍī*s might be a reference to the measuring of time with a water clock (*nāḍī*, *nāḍikā* or *ghaṭikā*), which is the etymology of this time unit. In *GD1* 4.37, Parameśvara says explicitly that the hour angle is measured with a water clock.

> *koṇastho 'rko yasmin kāle tasmād dinārdhaparyantam |*
> *kālaṃ vidyād ghaṭikāyantreṇa natāhvayaḥ sa kālaḥ syāt ||4.37||*
>
> The time starting from when the sun is situated in the intermediate direction and having midday as its end should be known by a water clock (*ghaṭikāyantra*). This time should be called the hour angle.

It is remarkable that the hour angle is being measured from the given point towards midday and not the other way round as in *GD2*.

$\mathrm{W}\Sigma'$ is the Sine of the hour angle ($\mathrm{Sin}\, H$) in figure 18.5. I would like to shift $\mathrm{W}\Sigma'$ to $\mathrm{OM}'$, $\mathrm{M}'$ being the foot of the perpendicular drawn from $\Sigma'$ to EW, to make the discussion in the next section easier.

## 18.5 Computing the sun's longitude (*GD2* 238-240)

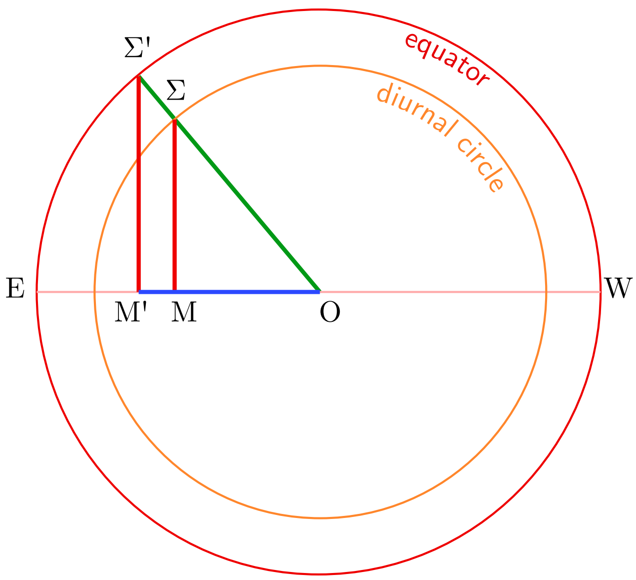



Figure 18.6: Upright in the diurnal circle OM and Sine of the hour angle $\mathrm{OM}'$, with the radius of the diurnal circle $\Sigma\mathrm{O}$ and the Radius $\Sigma'\mathrm{O}$.

The length of a shadow in an intermediate direction and the hour angle are the initial parameters in this method. We have seen that they are converted to the "upright" in the diurnal circle and the Sine of the hour angle, respectively. Figure 18.6 shows the two segments drawn in one diagram. The "upright" in the diurnal circle $\mathrm{OM} = u$ (equal to the upright of the great shadow

$\mathcal{S}_u$) forms a right triangle $\triangle\mathrm{OM}\Sigma$ with the radius of the diurnal circle $\Sigma\mathrm{O} = r$, and the Sine of the hour angle $\mathrm{OM}' = \operatorname{Sin} H$ forms another right triangle $\triangle\mathrm{OM}'\Sigma''$ with the Radius $\Sigma'\mathrm{O} = R$. The two right triangles share one acute angle and are thus similar. This is how we can interpret the rule of three given in *GD2* 238. To represent it in a formula,

$$\Sigma\mathrm{O} = \frac{\Sigma'\mathrm{O} \cdot \mathrm{OM}}{\mathrm{OM}'}$$
$$r = \frac{Ru}{\operatorname{Sin} H} \tag{18.2}$$

This set of triangles is the same with those used in *GD2* 110-111 (formula 8.2). There, the aim was to move from what was called a Sine in the celestial equator measured in the equator $\mathrm{M}'\Sigma'$ to that in the six o'clock circle $\mathrm{M}\Sigma$. Now we can see why Parameśvara might have named OM the "upright" in the diurnal circle: if we consider the segments $\mathrm{M}'\Sigma'$ and $\mathrm{M}\Sigma$ as the "base" Sines, then the corresponding segments $\mathrm{OM}'$ and OM are the "upright" Sines.

Furthermore, *GD2* 238 refers to the radius of the diurnal circle as "half-diameter (*ardha-viṣkambha*)" and not "diurnal 'Sine"' as we have often seen previously. This may be to avoid confusion with the term "diurnal circle (*dyujyāvṛtta*)", literally the "circle of the diurnal 'Sine"'[4].

*GD2* 239ab tells us that we can compute the declination of the sun from the radius of the diurnal circle, and the "base" arc from the declination. Considering the possible computation here and later in *GD2* 241, I have supplied "Sine of" in my translation. Let us reproduce the actual computation.

First, we can use *GD2* 76cd which states that the Radius $R$, Sine of declination $\operatorname{Sin}\delta$ and the radius of the diurnal circle $r$ form a right triangle (section 6.4). From the Pythagorean theorem,

$$\operatorname{Sin}\delta = \sqrt{R^2 - r^2} \tag{18.3}$$

I assume that the next step is the same as what we saw in the previous examples. We compute the "base" Sine $\operatorname{Sin}\lambda_B$ by reversing the rule in *GD2* 73ab:

$$\operatorname{Sin}\lambda_B = \frac{\operatorname{Sin}\delta \cdot R}{1397} \tag{18.4}$$

This is converted to the "base" arc $\lambda_B$. It is remarkable that Parameśvara does not mention the "base" Sine (contrary to *GD2* 210 and *GD2* 216). We have seen in the previous examples that discrepancies occur frequently at this step, which might have been caused because the commentator was using tables to compute "base" arcs directly from the declination. However, we have no more clues to discuss whether this is relevant here.

*GD2* 239cd-240 explain how to compute the longitude of the sun from its "base" arc. This is essentially the same rule with what is given in *GD2* 215-217, but explained far more succinctly. Four cases are given, and the only conditions mentioned are that the latter two are when the sun is in the southern celestial hemisphere and that the cases depend on the "measure of the shadow on two days". The measure of the shadow refers to the change in shadow-length in two consecutive days, from which we find whether the sun is in the northward course (moving from winter solstice to summer solstice in the ecliptic) or in the southward course (summer solstice to winter solstice).

[4] See also entry for *ardhaviṣkambha* in the glossary. The word *svāhorātrārdha* appearing in *GD2* 239 is also debatable; see its glossary entry.

## 18.6 Without-difference method for computing the Sine of geographic latitude (*GD2* 241ab)

*GD2* 241ab states that the Sine of geographic latitude is computed with an "without-difference" method. The method involves various segments, but Parameśvara emphasizes the base of the great shadow $\mathcal{S}_b$. Meanwhile the commentary before *GD2* 235 only mentioned the sun's declination. The declination, or its Sine ($\mathrm{Sin}\,\delta$) to be precise, is one of the later values obtained in the previous set of computations and also the first value appearing in the course of this method (*GD2* 241). Why did Parameśvara refer to $\mathcal{S}_b$ instead?

## 18.7 Initial assumption: solar amplitude (*GD2* 241cd)

Perhaps the answer is because we do not necessarily need to start with the Sine of declination in this method. *GD2* 241cd tells us that we first assume that some amount (let us notate $c$) added to the Sine of declination is the solar amplitude $\mathrm{Sin}\,\eta_1$.

$$\mathrm{Sin}\,\eta_1 = \mathrm{Sin}\,\delta + c \tag{18.5}$$

Essentially, we could just say "assume that the solar amplitude is some amount". In this sense, the Sine of declination is not strictly our starting point. Nonetheless, we can think of a good reason for the Sine of declination to be included. From *GD2* 84ab the solar amplitude is

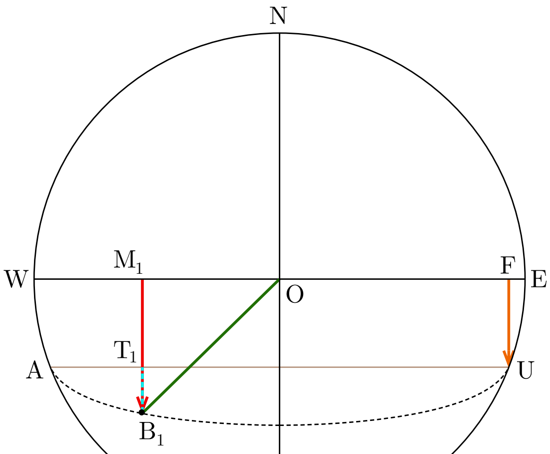
$$\mathrm{Sin}\,\eta = \frac{R\,\mathrm{Sin}\,\delta}{\mathrm{Sin}\,\bar{\varphi}} \tag{18.6}$$

where $\mathrm{Sin}\,\bar{\varphi}$ is the Sine of co-latitude, and in the localities of Parameśvara which is close to the equator, $\mathrm{Sin}\,\bar{\varphi}$ is only slightly smaller than the Radius $R$. Thus we would expect that $\mathrm{Sin}\,\eta$ is slightly smaller than $\mathrm{Sin}\,\delta$, and it is reasonable to start by adding a small value.

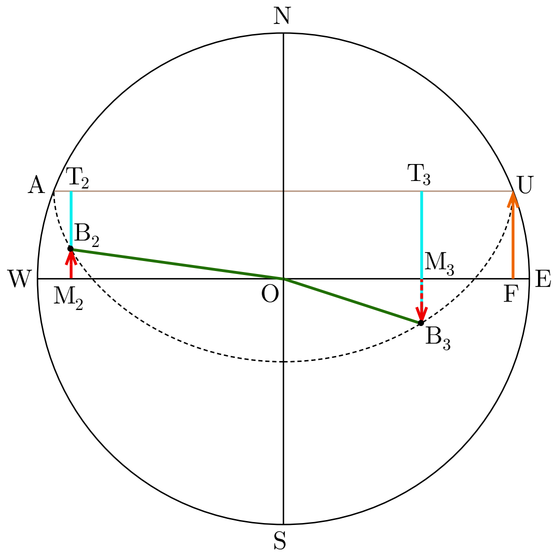
## 18.8 Solar amplitude and base of great shadow → gnomonic amplitude (*GD2* 242ab)

We have seen in *GD2* 221-223 that the base of the great shadow $\mathcal{B}$ can be represented in two ways, namely the "base of direction" and "base to be established". The base of direction is the sum or difference of the gnomonic amplitude and the solar amplitude, based on their directions as explained in *GD2* 221 (section 14.3). By reversing this rule, we can derive the gnomonic amplitude $\mathcal{A}_1$ from the base of the great shadow and the solar amplitude (figure 18.7). I interpret that the direction of the solar amplitude FU is measured from the east-west line toward the rising-setting line and that the direction of the great shadow's base MB is from the east-west line to the foot of the great gnomon. Then we have three cases as in figure 18.7: (1) both the solar amplitude and the great shadow's base are southward, (2) both are northward and (3) the solar amplitude is northward and the great shadow's base southward. $\mathrm{Sin}\,\eta_1$ and $\mathcal{B}$ are in the same direction in cases (1) and (2) and $\mathcal{A}_1$ shall be their difference. In case (3) they are in different directions and $\mathcal{A}_1$ is their sum. To summarize the result,

$$\mathcal{A}_1 = \begin{cases} |\,\mathrm{Sin}\,\eta_1 - \mathcal{B}| & \text{(Same direction)} \\ \mathrm{Sin}\,\eta_1 + \mathcal{B} & \text{(Different directions)} \end{cases} \tag{18.7}$$




(a) Southward solar amplitude




(b) Northward solar amplitude

Figure 18.7: Gnomonic amplitude TB as the sum or difference of the base of direction MB and the solar amplitude FU. Case numbers are represented by subscripts.

As $\mathcal{B}$ is constant throughout the "without-difference" method and the direction of Sin $\eta$ is also determined (it follows the declination whose value and direction is already known), the "sum" or "difference" will remain unchanged during the iteration. To say it in other words, if for example the difference is taken in the first cycle, it will always be the difference in the next cycles and never the sum.

## 18.9 Gnomonic amplitude and great gnomon → given "Sine" in the diurnal circle (*GD2* 242c)

Next we compute the given "Sine" in the diurnal circle $j_{t1}$. This is the same segment that appeared first in *GD2* 104, and as mentioned in *GD2* 105, it forms a right triangle ΣBT with the great gnomon $\Sigma B = \mathcal{G}$ and the gnomonic amplitude $BT = \mathcal{A}_1$ (figure 18.8). Thus from the Pythagorean theorem,

$$\mathrm{T\Sigma} = \sqrt{\mathrm{BT}^2 + \Sigma\mathrm{B}^2}$$
$$j_{t1} = \sqrt{{\mathcal{A}_1}^2 + \mathcal{G}^2} \tag{18.8}$$

$\mathcal{A}_1$ has been derived in the previous step, and $\mathcal{G}$ can be computed from the great shadow using the Pythagorean theorem (formula 8.9).

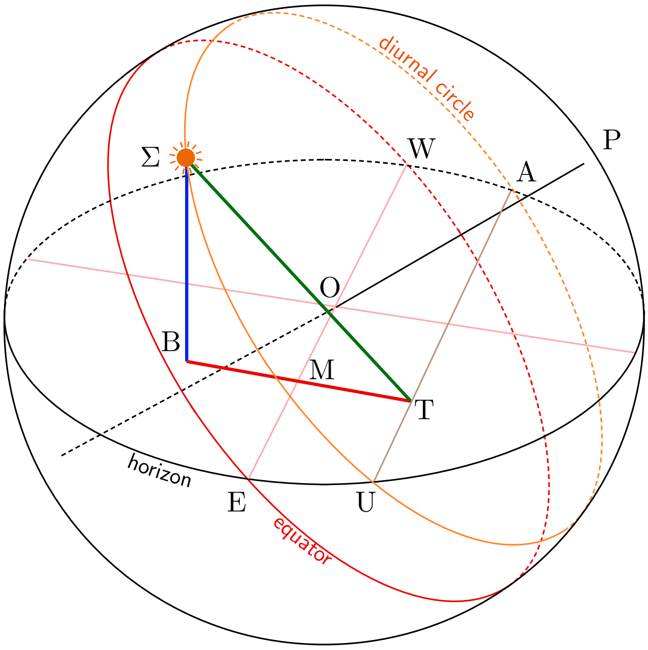



Figure 18.8: The given "Sine" in the diurnal circle TΣ with the gnomonic amplitude BT and the great gnomon ΣB. North is to the right.

## 18.10 Given "Sine" in the diurnal circle → Sine of geographic latitude (*GD2* 243ab)

ΣBT is similar to $\triangle$OB′P, the right triangle formed from the Radius PO $= R$, Sine of co-latitude OB′ $= \operatorname{Sin}\bar{\varphi}_1$ and the Sine of latitude B′P $= \operatorname{Sin}\varphi_1$ (figure 18.9, see also section 8.3). Therefore using the proportion, we can compute the Sine of latitude using the gnomonic amplitude BT $= \mathcal{A}_1$ and given "Sine" in the diurnal circle TΣ $= j_{t1}$ as stated in *GD2* 243ab:

$$\mathrm{B'P} = \frac{\mathrm{PO}\cdot\mathrm{BT}}{\mathrm{T\Sigma}}$$
$$\operatorname{Sin}\varphi_1 = \frac{R\mathcal{A}_1}{j_{t1}} \tag{18.9}$$

## 18.11 Sine of geographic latitude → Sine of co-latitude (*GD2* 243c)

Parameśvara only mentions that the next step is to go from the Sine of geographic latitude $\operatorname{Sin}\varphi_1$ to the Sine of co-latitude $\operatorname{Sin}\bar{\varphi}_1$. This seems to suggest that we should use the Pythagorean theorem, as does the commentary on example 5 (*GD2* 245).

$$\operatorname{Sin}\bar{\varphi}_1 = \sqrt{R^2 - \operatorname{Sin}^2\varphi_1} \tag{18.10}$$

Parameśvara could have reduced one step by computing the Sine of co-latitude directly from the great gnomon $\mathcal{G}$ and the given "Sine" in the diurnal circle

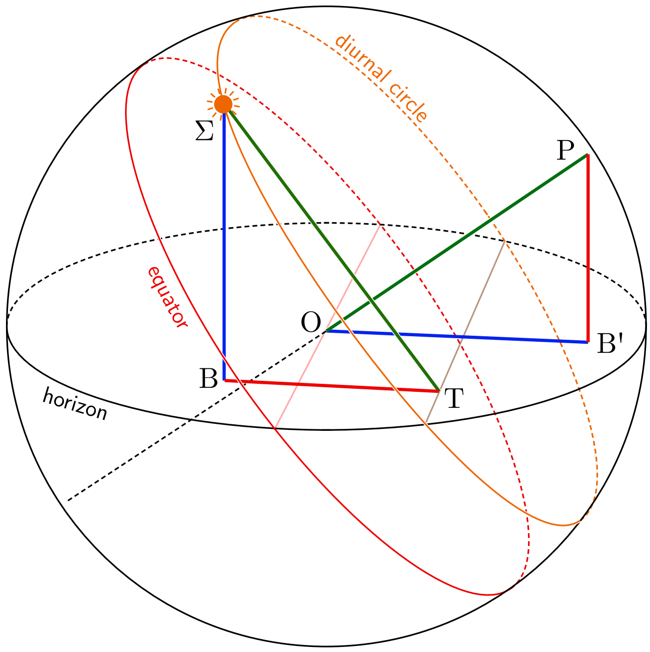



Figure 18.9: Similar triangles $\triangle \Sigma \mathrm{BT}$ and $\triangle \mathrm{OB'P}$. The Given "Sine" in the diurnal circle is $\mathrm{T}\Sigma$ and the Sine of geographic latitude $\mathrm{B'P}$. North is to the right.

$$\mathrm{Sin}\,\bar{\varphi}_1 = \frac{R\mathcal{G}}{j_{t1}} \tag{18.11}$$

in which case we could iterate the steps until another value (such as the solar amplitude) remains unchanged in two consecutive steps, and then compute the Sine of geographic latitude. Parameśvara does not explicitly say when to finish the computation, but his choice of including the Sine of geographic latitude in each cycle suggests that we should check its value at each cycle with the previous one and end when it is the same.

## 18.12 Sine of co-latitude → solar amplitude (*GD2* 243d)

We come back to the solar amplitude again from the Sine of co-latitude and the Sine of declination $\mathrm{Sin}\,\delta$. This time, $\mathrm{Sin}\,\delta$ is no more part of a guess and we need its exact value. To complement Parameśvara's brief explanation is brief, we use the similarity between $\triangle \mathrm{OB'P}$ and $\triangle \mathrm{FGU}$ which consists of the Sine of declination FG, the Earth-Sine GU and the solar amplitude UF (figure 18.10). A Rule of Three concerning these triangles can be found in *GD2* 87, and *GD2* 84ab is a statement for computing the solar amplitude (formula 6.7,). Using this, the corrected solar amplitude $\mathrm{Sin}\,\eta_2$ is

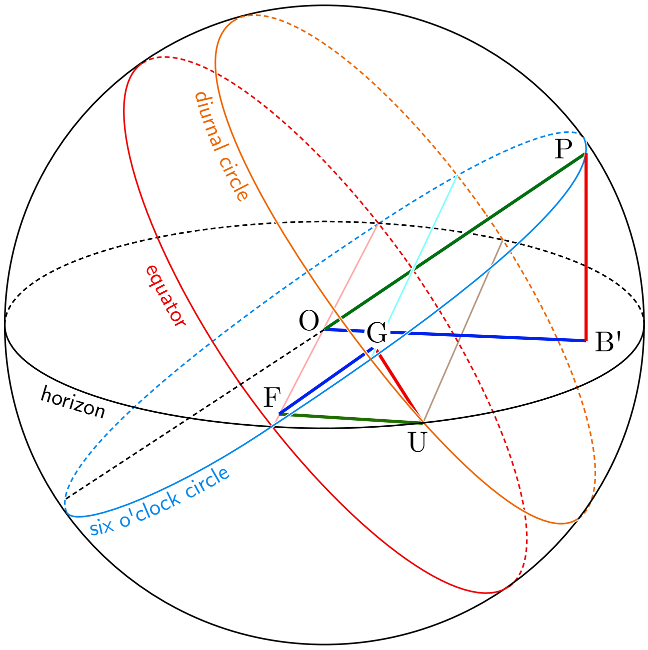



Figure 18.10: Similar triangles $\triangle$OB′P and $\triangle$FGU. The solar amplitude is UF. North is to the right.

$$\mathrm{UF} = \frac{\mathrm{OP} \cdot \mathrm{FG}}{\mathrm{B'O}}$$

$$\mathrm{Sin}\,\eta_2 = \frac{R\,\mathrm{Sin}\,\delta}{\mathrm{Sin}\,\bar{\varphi}_1} \tag{18.12}$$

## 18.13 Repeating the process (*GD2* 244)

The verse mentions that the Sine of geographic latitude will be obtained at the end. As we have discussed, the decision to end the iteration is probably made when the Sine of geographic latitude computed at each step is unchanged.

There is some peculiarity with the structure of *GD2* 244. I have included one and a half verse in the same number, but the critical edition by Sāstrī (1916) ends *GD2* 244 with cd and leaves the remaining half-verse unnumbered. This is also the case with 7 of the manuscripts. It is extremely difficult to tell whether they counted the half-verse as number 245, because none of them do not give verse numbers to the two examples (enumerated *GD2* 245 and 246 in my edition) and to the half-verse following them (*GD2* 247 in my edition). The remaining manuscripts are unhelpful as they do not write numbers around these verses.

I have included the half-verse in *GD2* 244 as parts ef, since it seemed unnatural to leave this verse unnumbered. This is a statement concluding the “without-difference” method and therefore constitutes an indispensable part of the text. Meanwhile, the verse still makes sense if we take away *GD2* 244cd as follows:

> Again, the difference of the base of [great] shadow and solar amplitude and so forth should be done.
> Thus here at the end of such "without-difference" method, the Sine of geographic latitude should become corrected without difference in this case.

This relies on how we understand the word *viyogādim* (the difference and so forth) in *GD2* 244b. One interpretation is that it stands for the difference and sum (*viyogayutī*), as in *GD2* 242ab. However it is unusual that *ādi* ("and so forth" or "those beginning with") is used for counting only two things[5], and its usage does not help with the meter (*viyogādim* and *viyogayutī* have the same number of syllable lengths). My interpretation is that Parameśvara has omitted the case for adding the two values, as he did in *GD2* 230 and *GD2* 234 with reference to the two bases (see section 14.5)[6], and that the *ādi* refers to the values computed in the steps after computing the difference (or sum). This would make *GD2* 244cd redundant.

Furthermore, there is a grammatical peculiarity with *GD2* 244cd. It consists of two compounds in the dual nominative / accusative and one word in the singular nominative:

***śaṅkvagreṣṭadyujye*** "gnomonic amplitude and given diurnal 'Sine"': Dual nominative / accusative

***palajīvālambajīvake*** "Sine of geographic latitude and Sine of co-latitude": Dual nominative / accusative

***'rkāgrā*** "solar amplitude": Singular nominative

Elsewhere in *GD2*, such sequence of steps are described by repeating pairs of an ablative and a nominative (cf. *GD2* 210, *GD2* 230). Therefore it is possible that *GD2* 244cd was inserted by someone else who felt it necessary to repeat the steps. Nonetheless I have left it in the critical edition since we do not have a decisive evidence to rule out the possibility of Parameśvara's own authorship.

[5]The grouping of planets in *GD2* 127-147 is a good example. Mercury and Venus are always addressed in the dual compound form, while the other three (Mars, Jupiter and Saturn) are often referred to as "those beginning with Mars".

[6]However, unlike the cases in *GD2* 230 and *GD2* 234 where the possibility of adding the two bases were rare, there is no special reason to think that adding the base of great shadow and the solar amplitude is less likely.

# 19 Example 5 (*GD2* 245)

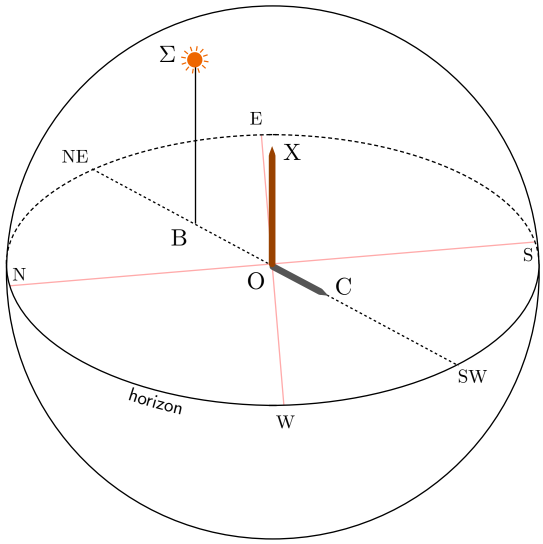



Figure 19.1: Situation in *GD2* 245. The gnomon is OX and its shadow OC when the sun Σ is in the northeast.

*GD2* 245 is an example for the method in *GD2* 234-244. The peculiarity with this example is that it involves a gnomon which is neither a great gnomon nor a twelve *aṅgula* gnomon. The length of the gnomon is 1667 and its shadow is 419, both without units. It turns out during the computation that those are half the values of the great gnomon and the great shadow. It is unreasonable to think that these were numbers involved in an actual observation, and as a whole, *GD2* 245 gives us the impression that this is a situation constructed as an example. Perhaps the numbers were chosen to make the situation more complex, and this can also be said for the hour angle given in the example which is not an integer. Another possibility is that they might have been computed backward from a specific longitude of the sun (exactly 2 signs) and Sine of geographic latitude (647). We will discuss this in my notes on the solution by the commentary.

Figure 19.1 illustrates the situation in example 5. The sun Σ is in the northeast, and if we assume that the gnomon in the example really is a gnomon as an instrument XO, its shadow OC should be extending towards the southwest. Parameśvara says nothing about the direction of the shadow, and it is irrelevant in the solution as given by the commentary. We can summarize example 5 as follows:

- The length of a gnomon is 1667.
- The length of its shadow is 419.
- The sun is in the northeast direction.
- The hour angle in *prāṇas* is 2547 divided by 4.
- The longitude of the sun and the geographic latitude are to be computed.

## 19.1 Solution

An important feature of this commentary is that fractional parts are often mentioned, either as a sexagesimal or by the expression "somewhat less than". This may be explained as the result of trying to follow the precision of the problem itself, where we have the number "2547 divided by 4" being involved. Meanwhile, it is noticeable that fractional parts are no longer taken into account in the second part where we compute the Sine of geographic latitude. This part involves an "without-difference" method, where higher precision in the intermediary values do not significantly affect the final result. Both the values and the method seem to be taken into account by the commentator upon deciding whether to include fractional parts in the computation.

**"In this case, the gnomon is 1667. Its shadow is 419."**

The commentary starts by repeating the values given in the verse, which was not the case in the previous examples. The numbers in *GD2* 245 are given in word numerals (*bhūtasaṃkhyā*) while they are written in decimal place value notations here, and therefore we can interpret that the commentator is trying to clarify the verse for the reader.

**"Having computed their hypotenuse from these two, and then, when the Radius is the hypotenuse, the great shadow established from the hypotenuse and the shadow is 838."**

The similarity between the right triangles △XOC and △ΣBO in figure 19.1 is used. This step resembles the first steps in examples 1 and 2, except for the length of the gnomon. The hypotenuse CX in △XOC is

$$\begin{aligned} \mathrm{CX} &= \sqrt{\mathrm{XO}^2 + \mathrm{OC}^2} \\ &= \sqrt{1667^2 + 419^2} \\ &= 1718;51,\cdots \end{aligned} \tag{19.1}$$

In the previous examples, we have assumed that numbers are rounded off to integers and that the value of the Radius is 3438. We may apply it here too, in which case the hypotenuse is rounded to 1719, exactly half the Radius. However, if we consider the sexagesimal part and double this value, we obtain approximately 3437;42. This is close to the values of the Radius used by Govindasvāmin, Mādhava and Nīlakaṇṭha (approximately 3437;45). In either case, it is most likely that values have been chosen so that the hypotenuse CX is half the length of the Radius OΣ. Since △XOC ∼ △ΣBO,

$$\begin{aligned} \mathrm{BO} &= \frac{\mathrm{OC} \cdot \mathrm{O\Sigma}}{\mathrm{CX}} \\ \mathcal{S} &= \frac{419 \cdot R}{R/2} \\ &= 838 \end{aligned} \tag{19.2}$$

**"Its gnomon is 3334."**

Likewise the great gnomon $\mathcal{G}$ is

$$\begin{aligned}\Sigma\mathrm{B} &= \frac{\mathrm{XO}\cdot\mathrm{O}\Sigma}{\mathrm{CX}}\\ \mathcal{G} &= \frac{1667\cdot R}{R/2}\\ &= 3334 \end{aligned} \tag{19.3}$$

Thus we can see that the given values of the gnomon and shadow were half the values of the great gnomon and the great shadow.

The great gnomon is not required for computing the sun's longitude, and Parameśvara does not mention it between *GD2* 235-240. However, we do need it in the "without-difference" method for computing the Sine of geographic latitude (*GD2* 242c, formula 18.8). It is reasonable to compute it at this point, which might explain why its value is mentioned here, although it will be repeated later.

**"The square root of half the [great] shadow's square is 592. Its fraction in seconds is 33. Then the base in the figure that has the [great] shadow as hypotenuse is the same with this root. Likewise for the upright."**

Using *GD2* 235 (formula 18.1),

$$\begin{aligned}\mathcal{B} = \mathcal{U} &= \sqrt{\frac{838^2}{2}}\\ &= 592;33,19,\ldots\end{aligned} \tag{19.4}$$

The commentary rounds off the second order. The first order sexagesimal is referred to as *viliptā*, which is usually used in the sense of "second" or "arc second". Apart from *aṅgula*s, this is the only place in the commentaries on *GD2* where we find a unit for a segment. Here the commentator might be implicitly using "minutes" as the basic unit of a segment when the great circle has a radius of 3438, as one minute of arc and one "minute" of segment would be approximately equal in length.

**"Then, the "upright" Sine extending east and west in the diurnal circle is also the same as this upright, because the upright of the [great] shadow is situated on the "upright" in the diurnal circle."**

The first half of this statement is equivalent to *GD2* 236cd, but here the commentator further adds some reasoning. The verb *avatiṣṭhate*, which we have translated "be situated on", might be a reference to how the upright of the great shadow appears when viewed from the northern celestial pole (figure 18.4). If so, this indicates that the commentary is using an armillary sphere or a projected diagram, mentally if not physically.

**"The hour angle in *asu*s (i.e. *prāṇa*s) going between the zenith and the sun multiplied by four is 2547. Since there are fourths, these [*asu*s] are to be divided by four."**

Here again, values stated by word numerals in the verse are repeated. This time, 2547 is given in decimal place values while four is given as a numeral. The commentator has also spared many words to clarify the word *aṃśaka* (denominator). Furthermore, he paraphrases the time unit *prāṇa* to *asu*. These make a contrast with the commentaries on examples 1 to 4 which concentrated on explaining steps and values but not the meaning of the verse itself.

**"The *prāṇa*s thus made are 636. Their fraction which is the sixtieth is 45."**

$H = 2547 \div 4 = 636;45$. This time the fraction is referred to as a *ṣaṣṭyaṃśa*, literally "having sixty as denominator". Since one *prāṇa* along the celestial equator is equal to one minute of arc, we can compute its Sine.

**"Their Sine is 633. And the fraction is 4 [sixtieths]. This is the Sine extending east and west in the celestial equator."**

The Sine of the hour angle Sin $H$ is computed. It has a fractional part. The reading of manuscript $I_1$ corresponding to the fraction[1] is *avayavaś ca tva*, which does not make sense. We presume that *tva* (ത്വ) is a mistranscription of a number. The best candidate is 4 (ൻ), but other single digit numbers cannot be ruled out.

We have computed the Sine for $H = 636'45''$ with various Sine tables and interpolation methods (table 19.1). The alphabets of the Sine table indicate:

a. Āryabhaṭa

b. Āryabhaṭa with corrections

c. Govindasvāmin

d. Mādhava

e. Nīlakaṇṭha (first recursion method)

f. Nīlakaṇṭha (second recursion method)

g. Vaṭeśvara

The interpolation methods are:

1. Linear interpolation
2. Nīlakaṇṭha's second order interpolation
3. Mādhava's second order interpolation
4. Brahmagupta and Bhāskara II's second order interpolation. Parameśvara gives the same method in his commentary on the *Laghubhāskarīya*
5. Govindasvāmin's second order interpolation
6. Another second order interpolation by Parameśvara
7. Parameśvara's third order interpolation

We also use two methods that do not use tables:

- Formula by Bhāskara I

[1] Folios corresponding to the entire commentary on *GD2* 245 is missing in the other manuscript, $K_5^+$.

- Power series expansion according to Śaṅkara and Jyeṣṭhadeva

The alphabets and numbers follow Hayashi, 2015 except for interpolation methods 6 and 7 which we have added. See appendix sections B.5 and B.6.1 for details of these tables and methods.

Table 19.1: Sin $H$ computed with various methods, up to the second order sexagesimal (arc thirds).

| | | Sine tables | | | |
|---|---|---|---|---|---|
| | | a.Ābh. | b.Ābh.cor. | c.Gov. | d.Mādh. |
| Inter- | 1. Linear | 633;15,35 | 633;15,35 | 632;56,14 | 632;56,19 |
| polation | 2. Nīlakaṇṭha | 633;26,41 | 633;26,41 | 633;06,50 | 633;06,55 |
| methods | 3. Mādhava | 633;26,28 | 633;26,28 | 633;06,47 | 633;06,52 |
| | 4. Brahmagupta | 633;24,03 | 633;24,03 | **633;04,22** | **633;04,27** |
| | 5. Govindasvāmin | 633;28,17 | 633;28,17 | 633;08,26 | 633;08,31 |
| | 6. Parameśvara 2 | 633;26,38 | 633;26,38 | 633;06,47 | 633;06,52 |
| | 7. Parameśvara 3 | 633;26,42 | 633;26,42 | 633;06,51 | 633;06,56 |
| Bhāskara I's formula | | 638;44,40 | 638;44,40 | 638;41,46 | 638;41,51 |
| Mādhava's power series | | 633;06,57 | 633;06,57 | 633;06,55 | 633;06,55 |
| | | Sine tables | | | |
| | | e.Nīl.1 | f.Nīl.2 | g.Vaṭ. | |
| Inter- | 1. Linear | 632;55,17 | 632;56,19 | 633;05,48 | |
| polation | 2. Nīlakaṇṭha | 633;05,49 | 633;06,55 | 633;07,02 | |
| methods | 3. Mādhava | 633;05,48 | 633;06,52 | 633;07,02 | |
| | 4. Brahmagupta | 633;03,25 | **633;04,27** | 632;56,59 | |
| | 5. Govindasvāmin | 633;07,29 | 633;08,31 | 633;02,58 | |
| | 6. Parameśvara 2 | 633;05,46 | 633;06,52 | 633;07,02 | |
| | 7. Parameśvara 3 | 633;05,50 | 633;06,56 | 633;07,02 | |
| Bhāskara I's formula | | 638;38,41 | 638;41,51 | 638;41,42 | |
| Power series (third order) | | 633;06,53 | 633;06,55 | 633;06,55 | |

Values which can be rounded off to 633;4 are indicated with bold fonts in the table. Only the second order interpolation according to Brahmagupta and Bhāskara II give the expected value when combined with tables of higher order (Govindasvāmin, Mādhava and Nīlakaṇṭha's second recursion method). The result is not surprising if we consider that Parameśvara cites a method that is equivalent to Brahmagupta's in his works (appendix B.3).

The combination of Nīlakaṇṭha's second recursion method with the second order interpolations of Nīlakaṇṭha, Mādhava or Parameśvara's other second order interpolation method and third order interpolation give approximately 633;7. Meanwhile Āryabhaṭa's table and linear interpolation gives approximately 633;16. I shall examine the following computation using these three results for Sin $H$ in order to conclude which value must have been used.

**"The 'upright' Sine in the diurnal circle, that is the same as the upright of the [great] shadow, multiplied by the Radius and divided by the Sine of hour angle is somewhat less than 3218. This is the diurnal 'Sine'."**

The commentary repeats that the "upright" Sine in the diurnal circle $u$ is equal to the upright of the great shadow $\mathcal{U}$. Its value is 592;33, as computed previously. From *GD2* 238 (formula 18.2), the radius of the diurnal circle (here expressed as diurnal "Sine") $r$ is computed using

Table 19.2: Radius of diurnal circle $r$ computed from different values.

| $\mathrm{Sin}\,H$ \ $R$ | 3438 | 3437;45 | 3437;28 |
|---|---|---|---|
| 633;4 | 3217;58 | 3217;44 | 3217;27 |
| 633;7 | 3217;43 | 3217;29 | 3217;13 |
| 633;16 | 3216;57 | 3216;43 | 3216;27 |

$u$, $\mathrm{Sin}\,H$ and $R$. Table 19.2 shows the result of formula 18.2 ($r = \frac{Ru}{\mathrm{Sin}\,H}$) using different values for $\mathrm{Sin}\,H$ and $R$. The three values for $\mathrm{Sin}\,H$ are those mentioned in the previous paragraph. $R = 3438$ is Āryabhaṭa's value (and also the greatest value among the candidates), $R = 3437;45$ is an approximation of Govindasvāmin, Mādhava and Nīlakaṇṭha (second method)'s values and $R = 3437;28$ is Nīlakaṇṭha (first method)'s value approximated (this is the smallest value).

The statement "somewhat less than 3218" suggests that the result should be at least within a range of 3217;30 to 3218. Therefore we can rule out $\mathrm{Sin}\,H = 633;16$ as derived from Āryabhaṭa's Sine table and linear interpolation. $\mathrm{Sin}\,H = 633;7$ fits the statement only if we choose $R = 3438$. $\mathrm{Sin}\,H = 633;4$ works for both $R = 3438$ and 3437;45.

We have already seen that the values of the gnomon and the shadow might have been chosen so that the hypotenuse will be half of $\sim 3437;45$ instead of 3438. Moreover it seems inconsistent to use $R = 3438$ when using a Sine whose value was computed within a system that uses another value for $R$. However, we will see that the next computation must be using $R = 3438$, and we cannot rule out this possibility. The combination of $R = 3438$ and $\mathrm{Sin}\,H = 633;4$ gives the most suitable value for the statement "somewhat less than 3218".

To conclude, it is likely that $\mathrm{Sin}\,H = 633;4$ as indicated from the manuscript was used in this computation.

**"The [Sine of] declination established from it is 1210."**

If we round $r$ to 3218, the Sine of declination $\mathrm{Sin}\,\delta$ is obtained using *GD2* 239ab (formula 18.3).

$$\begin{aligned}\mathrm{Sin}\,\delta &= \sqrt{R^2 - r^2}\\ &= \sqrt{3438^2 - 3218^2}\\ &= 1210; 5, \cdots \end{aligned} \tag{19.5}$$

Here we have used $R = 3438$. Values of the Radius with fractional parts do not reproduce a value that can be approximated to 1210. For example, if $R = 3437; 44, 48$ as with Mādhava, the result is $\mathrm{Sin}\,\delta = 1209; 22, \cdots$ which is approximated to 1209. We have presupposed in our previous cases that $R = 3438$ is being used whenever the arc or Sine is computed in the order of minutes (without sexagesimal parts), but this might not be the case here. It is likely that the commentator prefers the value of $R$ with a higher precision, or at least used multiple Sine tables with different values for $R$.

There is no reference to the direction of the declination. Assuming that the observer is to the north of the equator, the sun can be to the north of the prime vertical only when it is in the northern celestial hemisphere. Therefore this declination is northward, and this fact will be used later in the procedure.

**"Its [corresponding] 'base' [Sine] is somewhat less than 2978."**

The commentaries on examples 1 to 4 have never referred to the "base" Sine $\mathrm{Sin}\,\lambda_B$, suggesting that a table could have been used to obtain the "base" arc directly from the Sine of declination. Here we have the reference to the "base" Sine[2] as well as its value.

It is debatable whether *GD2* 73ab was involved in this computation, since it uses the value for the Sine of greatest declination $\mathrm{Sin}\,24° = 1397$ as obtained from Āryabhaṭa's Sine table and linear interpolation, which was not the case for $\mathrm{Sin}\,H$. However, we have seen that $R = 3438$ has been used in the previous step. Furthermore, any value for $\mathrm{Sin}\,24°$ obtained with other methods fail to produce the value of the "base" Sine (somewhat less than 2978) as stated here. Therefore we assume that *GD2* 73ab, or to be precise its reversed rule as in formula 18.4, is indeed being used:

$$\begin{aligned}\mathrm{Sin}\,\lambda_B &= \frac{\mathrm{Sin}\,\delta \cdot R}{1397} \\ &= \frac{1210 \times 3438}{1397} \\ &= 2977;47,42,\cdots \end{aligned} \tag{19.6}$$

which is indeed approximately, but smaller than, 2978.

**"Its arc is two signs increased by one minute."**

According to *Ābh* 2.12, 2978 is the Sine for $3600' = 60° = 2^s$. Therefore the statement that an arc of a Sine smaller than 2978 is larger than two signs implies that Āryabhaṭa's Sine table is not used. Example 4 involved the value 2977, which is probably the value for $\mathrm{Sin}\,2^s$ in Bhāskara II's table (page 324. See also appendix B.4). However, this too does not fit here if we use linear interpolation. Assuming $\mathrm{Sin}\,3600' = 2977$ and $\mathrm{Sin}\,3825' = 3084$, and rounding $\mathrm{Sin}\,\lambda_B$ to 2977,48, the "base" arc $\lambda_B$ by linear interpolation is $3601;40,\cdots$, which is rounded to two signs and two minutes.

Meanwhile, any Sine table with fractional parts can produce the result. For example, $\mathrm{Sin}\,3600' = 2977;10,34$ and $\mathrm{Sin}\,3825' = 3083;13,17$ according to Mādhava, and from $\mathrm{Sin}\,\lambda_B = 2977;47,42$ we obtain $\lambda_B = 3601;18,\cdots$ by linear interpolation which is approximately two signs and a minute. Second order interpolation could have been used, but it would not change anything concerning the precision of this result.

We have examined the values so far in accordance with the commentary, but let us turn to what Parameśvara could have intended. As discussed in appendix B.3, Parameśvara himself seems to be using Āryabhaṭa's Sine table where 2978 corresponds to exactly 2 signs of an arc. Can it be that Parameśvara has created this example by computing backward from a longitude of 2 signs?

Unfortunately, we could only compute backward easily up to the radius of the diurnal circle ($r = 3218$). We could not find a value for the Sine of hour angle $\mathrm{Sin}\,H$ that can be computed from $H = \frac{2547}{4}$ (using Āryabhaṭa's Sine table with linear interpolation) and gives $r = 3218$ from formula 18.2 with whatever rounding. Nonetheless, given the inconsistency in the value of the Radius $R$ in our reconstructed computations, there is still room left to consider.

[2]The word "Sine" does not appear in the text but is easily inferred from the statement "its arc" in the next passage.

**"This is [the longitude of] the sun. Or else, six signs decreased by this is [the longitude of] the sun. If the shadow on the next day [is larger], the first [is the answer]. If the shadow on the previous day is larger, the second."**

From the statement in *GD2* 245, the sun to the north of the prime vertical. According to *GD2* 215, the sun is on its northward course if the sun is to the north and the shadow-length increases on the next day, and southward if the shadow-length decreases. Therefore from *GD2* 216-217, $\lambda = \lambda_B$ in the first case and $\lambda = 6^s - \lambda_B$ in the second. Here we need to know that the Sun is in the northern celestial hemisphere. To conclude,

$$\lambda = \begin{cases} 2^s 1^\circ & \text{(shadow-length increasing)} \\ 3^s 59^\circ & \text{(shadow-length decreasing)} \end{cases} \tag{19.7}$$

The commentary does not give the value for the second case.

**"Now, in order to establish the geographic latitude, a given number is to be added to the given [Sine of] declination, 1210."**

The commentator refers to the next goal, the (Sine of) geographic latitude. This will be done by a "without-difference" method, as stated in *GD2* 241ab. The first sub-step is to add a number to the Sine of declination as stated in *GD2* 241cd.

**"In that case, the Sine of declination increased by ten is 1220. This is to be assumed as the solar amplitude."**

As we have discussed in section 18.7, the solar amplitude $\mathrm{Sin}\,\eta$ is expected to be slightly smaller than the Sine of declination $\mathrm{Sin}\,\delta$. The commentator has chosen a relatively small number ten (given in the text by an ordinary numeral) to be added to the Sine of declination. This is the first guess for the solar amplitude $\mathrm{Sin}\,\eta_1$.

$$\begin{aligned} \mathrm{Sin}\,\eta_1 &= \mathrm{Sin}\,\delta + 10 \\ &= 1210 + 10 \\ &= 1220 \end{aligned} \tag{19.8}$$

**"The base [in the trilateral] where the [great] shadow is hypotenuse is 593."**

The base of the great shadow $\mathcal{B}$ has been previously obtained together with the upright (formula 19.4).

**"The difference between these two in the same direction is 627. This is the gnomonic amplitude."**

The sun is in the north-east direction and therefore the base of its great shadow $\mathcal{B}$ is northward. We have also seen that the commentator is silently using the fact that the declination is northward. The solar amplitude $\mathrm{Sin}\,\eta_1$ will then be in the same direction, northward. Therefore $\mathcal{B}$ and $\mathrm{Sin}\,\eta_1$ are in the same direction, and from *GD2* 242ab (formula 18.7), the gnomonic amplitude $\mathcal{A}_1$ is

$$\begin{aligned}\mathcal{A}_1 &= |\operatorname{Sin}\eta_1 - \mathcal{B}| \\ &= |1220 - 593| \\ &= 627\end{aligned} \tag{19.9}$$

**“The [great] gnomon is 3334.”**

The great gnomon $\mathcal{G}$ has been previously computed (formula 19.3).

**“The square root of the sum of the squares of these two that have the forms of the base and upright is 3392. This is the given ‘Sine’ in the diurnal circle that has the form of a hypotenuse.”**

The “Sine” in the diurnal circle $j_{t1}$ is computed with the Pythagorean theorem (formula 18.8). It is remarkable that the commentator not only refers to the computation itself as stated in *GD2* 242c, but also draws the reader’s attention to the right triangle been involved by pointing to its three sides.

$$\begin{aligned}j_{t1} &= \sqrt{{\mathcal{A}_1}^2 + \mathcal{G}^2} \\ &= \sqrt{627^2 + 3334^2} \\ &= 3392; 26, \cdots\end{aligned} \tag{19.10}$$

This can be rounded off to 3392. The manuscript $\mathrm{I}_1$ gives 3394 instead, but the result of the next computation can only be explained with $j_{t1} = 3392$. Therefore this is probably a scribal error.

**“Then, the Radius multiplied by the gnomonic amplitude should be divided by this given ‘Sine’ in the diurnal circle. In that case, the quotient is 636. This should be assumed as the Sine of geographic latitude.”**

*GD2* 243ab (formula 18.9) can be used for computing the Sine of geographic latitude $\operatorname{Sin}\varphi_1$. We have assumed that $R = 3438$ has been used in previous sub-steps (formulas 19.5 and 19.6), and we apply it here too.

$$\begin{aligned}\operatorname{Sin}\varphi_1 &= \frac{R\mathcal{A}_1}{j_{t1}} \\ &= \frac{3438 \cdot 627}{3392} \\ &= 635; 30, \cdots\end{aligned} \tag{19.11}$$

which can be rounded to 636. Using $j_{t1} = 3394$ instead gives $635; 7, \cdots$ which is rounded to 635 and does not match the statement. Using a value for $R$ smaller than 3438 will also reduce the result, and we can confirm that $R = 3438$ must have been indeed used here. We shall continue using this value for the rest of the procedure.

**"The square root of the difference between the squares of the Sine of geographic latitude and the Radius is 3379. This is the Sine of co-latitude."**

From *GD2* 243c (formula 18.10), the Sine of geographic latitude $\operatorname{Sin}\bar{\varphi}_1$ is

$$\begin{aligned}\operatorname{Sin}\bar{\varphi}_1 &= \sqrt{R^2 - \operatorname{Sin}^2\varphi_1}\\ &= \sqrt{3438^2 - 636^2}\\ &= 3378;39,\cdots \end{aligned} \tag{19.12}$$

which can be rounded to 3379.

**"Then, the [Sine of] declination multiplied by the Radius should be divided by this Sine of co-latitude. In that case, the quotient is the corrected solar amplitude, 1231."**

From *GD2* 243d (formula 18.12), the corrected solar amplitude $\operatorname{Sin}\eta_2$ is

$$\begin{aligned}\operatorname{Sin}\eta_2 &= \frac{R\operatorname{Sin}\delta}{\operatorname{Sin}\bar{\varphi}_1}\\ &= \frac{3438\cdot 1210}{3379}\\ &= 1231;7,\cdots \end{aligned} \tag{19.13}$$

which can be rounded to 1231.

**"Then again, having assumed that the difference between the solar amplitude and the base of [great] shadow is the gnomonic amplitude, the Sine of geographic latitude without difference is to be computed with the rule that has been mentioned."**

We are to repeat the computation starting from $\mathcal{A}_2 = |\operatorname{Sin}\eta_2 - \mathcal{B}|$. The commentary gives no more numbers, but mentions that the Sine of geographic latitude *without difference* should be computed, implying the "without-difference" method.

**"Then, the corrected Sine of geographic latitude without difference is 647."**

In the second cycle, $\operatorname{Sin}\eta_2 = 1231$ gives $\operatorname{Sin}\varphi_2 = 646$, which in turn gives $\operatorname{Sin}\eta_3 = 1232$. Then $\operatorname{Sin}\varphi_3 = 647$, which gives $\operatorname{Sin}\eta_4 = 1232$ and $\operatorname{Sin}\varphi_4 = 647$ again. Therefore the "without-difference" method ends with only 3 cycles (or 4 for confirmation). We obtain $\operatorname{Sin} = \varphi = 647$ as the Sine of geographic latitude, which is the value that has been used repeatedly in the previous examples, and also the value which Parameśvara mentions as the geographic latitude of his village (see introduction 0.1.2).

# 20 Example 6 (*GD2* 246-247)

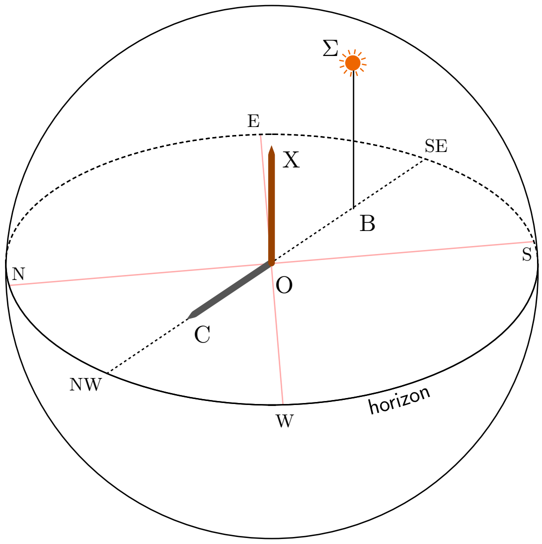



Figure 20.1: Situation in *GD2* 246. The gnomon is OX and its shadow OC when the sun Σ is in the southeast.

*GD2* 246 is another example, following *GD2* 245, for computing the sun's longitude and Sine of geographic latitude from a shadow in the intermediary direction. As was the case with the previous example, the gnomon is not specified to be a great gnomon or a twelve *aṅgula* gnomon. However, this time the length of the gnomon itself is unspecified. The difference between the lengths of the gnomon and its shadow is given as a proportion. This is unlikely to be the result of an actual observation, and is most likely a constructed example with numbers chosen so that the computation is more precise than in *GD2* 245.

The situation is shown in figure 20.1. This time the sun Σ is in the southeast and the shadow OC of the gnomon XO extends towards the northwest. The information given in *GD2* 246 is as follows:

- The length of a gnomon's shadow is $\frac{1}{101}$ and $\frac{1}{106}$ shorter than the gnomon itself. $\left[s = \left(1 - \frac{1}{101} - \frac{1}{106}\right) g\right]$
- The sun is in the southeast direction.
- The hour angle is 1916 *prāṇa*s.
- The longitude of the sun and the geographic latitude are to be computed.

By following the procedures in *GD2* 235-240, we can compute the Sine of the sun's declination. However, there is nothing that tells us whether it is northward or southward. We need to determine its direction to compute the sun's longitude and the Sine of geographic latitude. It is the introductory sentence[1] that tells us that the sun is in the southern celestial hemisphere,

[1] The introductory sentence appears in every manuscript and is not to be confused with the commentary, which can only be found in two manuscripts.

and hence that the declination is southward[2]. Therefore we can consider that the introductory sentence is also part of this example. If this is indeed Parameśvara's intention, *GD2* 246 (as well as the other examples) might have been designed to be fixed in the treatise and not for being used separately.

## 20.1 Solution

The commentary on this example refers to fractional parts more frequently than in the commentary on example 5. For example, even the Sine of declination which had been rounded off to an integer previously is given with its fractional part. The notation for the fractional part itself is also different from the previous case. The commentary on example 5 uses *avayava* ("fraction", literally "limb" or "portion") in combination with other words, but here the format is fixed as *ṣaṣṭyaṃśāḥ* + number "the sixtieths are ..." or *ṣaṣṭyaṃśaḥ 1* "the sixtieth is 1". Such difference in style might be due to different authorships. However, the last statement in this commentary ("As for the geographic latitude, it is as previously.") indicates a continuity with the previous example.

**"In this case, the gnomon assumed by one's own wit is 2454. And the sixtieths are 28."**

This "assumption" here turns out later to be the value of the great gnomon itself, and the following steps essentially confirm this. At first glance, there is no explanation on how one should obtain this value without great intuition. However, this procedure of confirmation can also be read as an instruction for arriving to this value beginning with a random guess. In any case, this is a step that does not appear in *GD2* itself.

We will first look at the values, and then come back to see the procedure itself. We start from $g = 2454;28$. The sexagesimal value 28 is different from the readings in both manuscripts. There are corruptions in the next two values (24;18 and 23;9) too. We have used the value of the shadow, 2407;1, to compute backward and correct these values. This will be explained later in the corresponding passage.

**"This divided by one hundred and one is 24. The sixtieths are 18."**

$$\frac{2454;28}{101} = 24;18,5,\cdots \tag{20.1}$$

This can be rounded off to 24;18.

**"Then again, this divided by one hundred six is 23. The sixtieths are 9."**

$$\frac{2454;28}{106} = 23;9,19,\cdots \tag{20.2}$$

This can be rounded off to 23;9.

[2] We will see later that this gives the Sine of geographic latitude $\mathrm{Sin}\,\varphi = 647$, which is the same value seen in every other example and also the latitude of Parameśvara's village. If it were northward, the result would be $\mathrm{Sin}\,\varphi = 2935$, corresponding to a latitude of more than 58° north.

**"These two results are to be subtracted from the previous gnomon assumed by one's own wit. Then the remainder is 2407. The sixtieth is 1. This is the shadow of this gnomon."**

The gnomon's shadow $s$ is

$$2454;28 - 24;18 - 23;9 = 2407;1 \tag{20.3}$$

Here, the "sixtieth" is in a singular (*ṣaṣṭyaṃśaḥ*). Therefore we are more certain that the number 1 is correct and not a scribal error. We have used this as a firm starting point to correct the corruptions in the previous values.

**"From these two, the gnomon and shadow, the hypotenuse that is the same as the Radius should be established. Thus in this case, these two are indeed the great gnomon and great shadow."**

The hypotenuse $h$ of the gnomon and shadow is computed with the Pythagorean theorem.

$$\begin{aligned} h &= \sqrt{g^2 + s^2} \\ &= \sqrt{2454;28^2 + 2407;1^2} \\ &= 3437;45,5,\cdots \end{aligned} \tag{20.4}$$

This is very close to the values of the Radius $R$ used by Govindasvāmin (3437;44,19), Mādhava (3437;44,48) and Nīlakaṇṭha (3437;44,47)[3]. We cannot make $h$ closer to these values by changing the first order sexagesimals of $g$ and $s$, and we can conclude that they are indeed values of the great gnomon $\mathcal{G}$ and great shadow $\mathcal{S}$ (approximated to the first sexagesimal). It is also significant that the value of $R$ with fractional parts are being used, and obviously not 3438. Unlike example 5, which sometimes seems to use 3438, the commentator is consistent in using $R$ with fractions, as we will see.

Now let us go back to the first step and see how we can read the text to understand the procedure to find the great gnomon and great shadow beginning with a pure guess.

Instead of following the value given by the commentator, we choose another number $x$ as "the gnomon assumed by one's own thought".

"This divided by one hundred and one is" $\frac{x}{101}$.

"Then again, this divided by one hundred six is" $\frac{x}{106}$.

"These two results are to be subtracted from the previous gnomon assumed by one's own thought." $x - \frac{x}{101} - \frac{x}{106}$. "This is the shadow of this gnomon."

"From these two, the gnomon and shadow, the hypotenuse" ... "should be established."

Now we can use the values of the shadow, the hypotenuse and the Radius to compute the great shadow with a Rule of Three, as we have done in the previous example. Thus it is possible to use the previous statements as instructions for the procedure beginning with a guess of any value.

The next substep in the commentary on example 5 was to compute the base and upright of the great shadow. In this example it is missing, and only the upright of the great shadow is mentioned later in the procedure.

[3]See appendix B.6.1 for details. Nīlakaṇṭha's value is reconstructed from his second incursion method.

**"The hour angle in *asus* (i.e. *prāṇas*) is 1916."**

$H = 1916$ *asu*s. The time unit *prāṇa* is paraphrased to *asu*, as was the case with the commentary on example 5. This time, the value is an integer without a sexagesimal part. However, its corresponding Sine will be given with a sexagesimal fraction as we will see in the next substep.

**"Its Sine is 1818. The sixtieths are 17."**

Here again the Sine of the hour angle $\operatorname{Sin} H$ is given with its sexagesimal fraction. In example 5, we only had one manuscript with a corrupted reading for the value. This time, we have two manuscripts which give the same values in an unmistakable script.

I have computed the Sine for $H = 1916$ with the same methods as in the previous example. See appendix sections B.5 and B.6.1 for details.

Table 20.1: $\operatorname{Sin} H$ computed with various methods, up to the second order sexagesimal (arc thirds).

| | | Sine tables | | | |
|---|---|---|---|---|---|
| | | a.Ābh. | b.Ābh.cor. | c.Gov. | d.Mādh. |
| Inter- | 1. Linear | 1817;28,15 | 1817;28,15 | 1817;21,32 | 1817;21,47 |
| polation | 2. Nīlakaṇṭha | 1818;25,26 | 1818;25,26 | 1818;19,49 | 1818;20,05 |
| methods | 3. Mādhava | 1818;24,31 | 1818;24,31 | 1818;18,55 | 1818;19,10 |
| | 4. Brahmagupta | 1818;28,12 | 1818;20,42 | **1818;16,41** | **1818;16,56** |
| | 5. Govindasvāmin | 1818;28,12 | 1818;20,42 | **1818;16,41** | **1818;16,56** |
| | 6. Parameśvara 2 | 1818;24,31 | 1818;24,31 | 1818;18,54 | 1818;19,10 |
| | 7. Parameśvara 3 | 1818;25,53 | 1818;25,53 | 1818;20,16 | 1818;20,32 |
| Bhāskara I's formula | | 1817;43,17 | 1817;43,17 | 1817;35,00 | 1817;35,15 |
| Mādhava's power series | | 1818;21,37 | 1818;21,37 | 1818;20,44 | 1818;20,46 |

| | | Sine tables | | |
|---|---|---|---|---|
| | | e.Nīl.1 | f.Nīl.2 | g.Vaṭ. |
| Inter- | 1. Linear | 1817;18,16 | 1817;21,47 | 1818;19,23 |
| polation | 2. Nīlakaṇṭha | 1818;16,25 | 1818;20,05 | 1818;20,16 |
| methods | 3. Mādhava | 1818;15,34 | 1818;19,10 | 1818;20,16 |
| | 4. Brahmagupta | 1818;13,25 | **1818;16,56** | 1818;24,38 |
| | 5. Govindasvāmin | 1818;13,25 | **1818;16,56** | 1818;24,38 |
| | 6. Parameśvara 2 | 1818;15,30 | 1818;19,10 | 1818;20,16 |
| | 7. Parameśvara 3 | **1818;16,52** | 1818;20,32 | 1818;20,16 |
| Bhāskara I's formula | | 1817;26,16 | 1817;35,15 | 1817;34,50 |
| Mādhava's power series | | 1818;19,49 | 1818;20,46 | 1818;20,43 |

Here again, Brahmagupta's second order interpolation with the reconstructed tables of Govindasvāmin, Mādhava and Nīlakaṇṭha (second recursion method) reproduces the value 1818; 17 (table 20.1). Govindasvāmin's interpolation method gives the same method as Brahmagupta's, since they are mathematically equal when the arc is between 30° and 60° (Gupta (1969, p. 92)). The combination of Parameśvara's third order interpolation and Nīlakaṇṭha's first recursion method can also be rounded to 1818; 17, but it is very unlikely that this was the method that was actually used.

Both this and the previous case in *GD2* 245 (633;4 as the value of Sin 636; 45), suggest that these computations were done by the second order interpolation stated by Brahmagupṭa and Bhāskara II and cited by Parameśvara. However these are only two examples, and depending on hidden errors, this conclusion may change. Further examples of Sine computation in astronomical problems are yet to be examined.

**“The segment in the diurnal circle is the same as the upright of the [great] shadow, 1702. The sixtieth is 1.”**

Using *GD2* 235 (formula 18.1),the base $\mathcal{B}$ and upright $\mathcal{U}$ of the great shadow are

$$\begin{aligned}\mathcal{B} = \mathcal{U} &= \sqrt{\frac{2407;1^2}{2}} \\ &= 1702;1,4,\ldots \end{aligned} \tag{20.5}$$

which can be rounded to 1702;1. There is no reference to $\mathcal{B}$ in the commentary, but it is required later.

The “upright” in the diurnal circle $u$ is referred to here as the *khaṇḍa*, literally “fragment” or “segment”. In the context of Sines, this term is often used in the sense of “Sine difference”, i.e. the difference between two consecutive values of Sines in a table. However we need to understand it here as a reference to an entire segment.

**“In this case, the quotient is the diurnal ‘Sine’, 3217. The sixtieths are 54.”**

From *GD2* 238 (formula 18.2), the radius of the diurnal circle $r$ is

$$\begin{aligned} r &= \frac{Ru}{\operatorname{Sin} H} \\ &= \frac{3437;44,48 \times 1702;1}{1818;17} \\ &= 3217;55,35,\cdots \end{aligned} \tag{20.6}$$

leaving a small discrepancy with the text. Here we have chosen Mādhava’s value for $R$, but choosing other values between 3437;44 and 3437;45 does not fully account for the difference ($r = 3217;54,50,\cdots$ when $R = 3437;44$ and $r = 3217;55,46,\cdots$ when $R = 3437;45$). The discrepancy seems to have originated in the computation itself.

**“The [Sine of] declination is 1209. The sixtieths are 38.”**

From *GD2* 239ab (formula 18.3), the Sine of declination $\operatorname{Sin}\delta$ is

$$\begin{aligned} \operatorname{Sin}\delta &= \sqrt{R^2 - r^2} \\ &= \sqrt{3437;44,47^2 - 3217;54^2} \\ &= 1209;38,10\cdots \end{aligned} \tag{20.7}$$

which can be approximated to 1209;38. Here we use Mādhava’s value for $R$, but assuming that $r \sim 3217;54$ and $\operatorname{Sin}\delta \sim 1209;38$ are correct, we can examine the value of $R$ used by the commentator from this computation. If $r = 3217;54$ and $109;37,30 < \operatorname{Sin}\delta < 1209;38,30$, then $3437;44,33 < R < 3437;44,53$. It is remarkable that values approximated to the first sexagesimal like 3437;44 or 3437;45 cannot reproduce the result. Among the Sine tables that we have listed in appendix B.6.1, only those of Mādhava ($R = 3437;44,47$) and Nīlakaṇṭha (reconstructed from his second recursion method, $R = 3437;44,48$) fit this condition.

**“From it the “base” Sine is established. It is almost the same as a Sine of two signs. Its arc is two signs.”**

There is an explicit reference to the “base” Sine $\mathrm{Sin}\,\lambda_B$, and we can see that the commentator does not jump directly from the Sine of declination to the “base” arc. However, its value is given only approximatively. Compared with all the previous statements, where the values up to the first sexagesimal were given, this is a striking difference.

Whether *GD2* 73ab was used for computing $\mathrm{Sin}\,\lambda_B$ is yet again a problem. If the commentator were consistent, we would expect him to use the value $R = 3437;44,47$ or $R = 3437;44,48$, and use $\mathrm{Sin}\,24^\circ$ computed from either Mādhava’s or Nīlakaṇṭha’s Sine table with a second order interpolation (which is 1398;12,28 in either case) fas the Sine of greatest declination. In this case (using Mādhava’s value for $R$),

$$\begin{aligned}\mathrm{Sin}\,\lambda_B &= \frac{\mathrm{Sin}\,\delta \cdot R}{\mathrm{Sin}\,24^\circ}\\ &= \frac{1219;38 \times 3437;44,48}{1398;12,28}\\ &= 2973;59,\cdots \end{aligned} \tag{20.8}$$

which is close to the Sine of two signs (2977;10,34 according to Mādhava’s table), but with all the precision in the previous passages, it is strange that the commentator concludes that “its arc is two signs”. If we use $R = 3438$ and $\mathrm{Sin}\,24^\circ = 1397$ instead, the result is $\mathrm{Sin}\,\lambda_B = 2976;53,\cdots$ and close to Bhāskara II’s value for the Sine of two signs, 2977. However we are still left with a great inconsistency.

One hypothesis is that the commentator has expected that the “base” arc would be exactly two signs, and finding that he could not reproduce the value, left the statement ambiguous. In any case, the attitude is different with the commentary on example 5 which gives the value of the “base” arc up to its degrees.

**“This increased by half a circle is [the longitude of] the sun. Or else, a circle decreased by this is [the longitude of] the sun.”**

These are the computations to obtain the sun’s longitude from the “base” arc. According to *GD2* 216-217, they correspond to the cases when the sun is in the southern celestial hemisphere and on its southward course, and when the sun is in the southern celestial hemisphere and on its northward course, respectively. Here we need the statement in the introductory sentence of the verse to know that the sun is in the southern celestial hemisphere, but the commentary makes no remark on this point.

**“As for the geographic latitude, it is as previously.”**

The commentator does not explain the “without-difference” method in detail. All we can see is that there is a reference to the previous example by saying “previously”, but it is ambiguous whether this refers to the final value for the Sine of geographic latitude, or to the method itself.

I have simulated the “without-difference” method beginning with $\mathrm{Sin}\,\eta_1 = \mathrm{Sin}\,\delta + 10$ with a SAGE program. Three cases were examined, using different values for the Radius $R$, base of great shadow $\mathcal{B}$, great gnomon $\mathcal{G}$ and Sine of declination $\mathrm{Sin}\,\delta$, and with a difference in rounding.

a. $R = 3438$, $\mathcal{B} = 1702$, $\mathcal{G} = 2454$, $\mathrm{Sin}\,\delta = 1210$. Values rounded to integers at every step.

b. $R = 3437;44,48$, $\mathcal{B} = 1702;1$, $\mathcal{G} = 2454;28$, $\mathrm{Sin}\,\delta = 1209;38$. Values rounded to integers at every step.

c. $R = 3437;44,48$, $\mathcal{B} = 1702;1$, $\mathcal{G} = 2454;28$, $\mathrm{Sin}\,\delta = 1209;38$. No rounding. Iteration stopped when $\mathrm{Sin}\,\varphi_{N-1} - \mathrm{Sin}\,\varphi_N < 0;0,1$.

Table 20.2: Without-different method in Example 6 computed with SAGE.

(a) Rounded values, rounding at each step.

| Cycle (N) | $\mathrm{Sin}\,\eta_N$ | $\mathrm{Sin}\,\varphi_N$ | $\mathrm{Sin}\,\varphi_{N-1} - \mathrm{Sin}\,\varphi_N$ |
|---|---|---|---|
| 1 | 1220 | 662 | – |
| 2 | 1233 | 645 | -17 |
| 3 | 1232 | 646 | 1 |
| 4 | 1232 | 646 | 0 |

(b) Parameters with fraction, rounding at each step.

| Cycle (N) | $\mathrm{Sin}\,\eta_N$ | $\mathrm{Sin}\,\varphi_N$ | $\mathrm{Sin}\,\varphi_{N-1} - \mathrm{Sin}\,\varphi_N$ |
|---|---|---|---|
| 1 | 1220 | 663 | – |
| 2 | 1232 | 646 | -17 |
| 3 | 1231 | 647 | 1 |
| 4 | 1231 | 647 | 0 |

(c) Parameters with fraction, no rounding

| Cycle (N) | $\mathrm{Sin}\,\eta_N$ | $\mathrm{Sin}\,\varphi_N$ | $\mathrm{Sin}\,\varphi_{N-1} - \mathrm{Sin}\,\varphi_N$ |
|---|---|---|---|
| 1 | 1219;38,00 | 662;56,44 | – |
| 2 | 1232;46,23 | 645;31,57 | -18;35,13 |
| 3 | 1231;32,26 | 647;10,05 | 1;38,07 |
| 4 | 1231;39,17 | 647;00,59 | -1;50,54 |
| 5 | 1231;38,39 | 647;01,50 | 0;00,50 |
| 6 | 1231;38,43 | 647;01,45 | 0;00,05 |
| 7 | 1231;38,42 | 647;01,46 | 0;00,00 |

The results are shown in table 20.2. If we expect that $\mathrm{Sin}\,\varphi = 647$ as in the other examples, we need to use the parameters with their fractional parts as computed in the previous step. This is different from example 5, where we could obtain $\mathrm{Sin}\,\varphi = 647$ with values rounded in each computation.

Examples 1 to 4 could be solved with rounding done in every computation, but examples 5 and 6 require computations with fractions. While some steps of example 5 could be done with rounding, example 6 seems to involve fractional numbers at every step. If this was Parameśvara's intention, he might have arranged the examples to be in the order of difficulty.

## 20.2 Expanding the method to any direction (*GD2* 247)

The method explained in *GD2* 235-244 with its examples in *GD2* 245 and *GD2* 246 was limited to the case when the sun was in an intermediary direction. The statement in *GD2* 247 is probably a reference that the same procedure[4] can be applied to any case, regardless of the sun's direction. We assume that "everything" refers to whatever value obtained in this method, such as the sun's longitude and the Sine of geographic latitude.

I have numbered this half-verse *GD2* 247, but its status as an independent verse may be questioned. Only two manuscripts ($K_2$ and $K_4$) note its verse number, and two manuscripts ($K_5^+$ and $I_1$) that contain commentaries include *GD2* 247 in the commentary on *GD2* 246[5].

One possibility is that the archetype(s) of every extant manuscript contained commentaries including this text. At one point a copyist decided to copy the verses without the commentaries, but *GD2* 247 was kept because it fitted the half-*gīti* meter by chance. In this case, *GD2* 247 concludes the commentary on *GD2* 246 by referring to situations which are not covered by this example. This is similar to what we can see at the end of the commentary on *GD2* 231 (page 321).

Alternatively, we can explain the position and length of *GD2* 247 by considering that it originally formed a full verse with *GD2* 244ef. This also accounts for the fact that *GD2* 244 consists of one verse and a half. However, we will then have to explain why *GD2* 245 and *GD2* 246 were inserted in this position. We have also seen in section 18.13 that it is probable that *GD2* 244cd had been inserted later. Therefore we think that this hypothesis is not convincing enough.

Although we cannot completely rule out these two possibilities, we have decided to keep *GD2* 247 at this position, assuming that this was Parameśvara's intention. We may account for its position in the treatise as follows. Not only *GD2* 235-244 but also the two examples in *GD2* 245-246 limit the situation to when the sun is in the intermediary direction. Therefore it is reasonable that *GD2* 247, a statement which goes out of this boundary, is placed after the examples.

[4] Except that *GD2* 235 needs to be modified. See footnote 2 in section 18.1.

[5] This can be seen from the decorative segmentation mark put after *GD2* 247. Elsewhere in the manuscripts, the same mark is always put at the end of a commentary.

# 21 Parallax (*GD2* 248-276)

The section starts in *GD2* 248 by contrasting the geocentric orbit of planets with the "circle of sight" around the observer on the Earth. *GD2* 249-253 continues how the observer's position on the Earth causes the (geocentric) parallax[1] (*lambana*). The measurement unit of this parallax is not specified, but it shall turn out that this is parallax in *yojana*s, measured in the orbit. *GD2* 254-259 compare this with the parallax measured in minutes inside the circle of sight. Parameśvara uses a drawing for visualizing the difference between the parallax in *yojana*s and in minutes in *GD2* 260-266. The subject shifts in *GD2* 267-269, where the geocentric parallax is divided into its longitudinal and latitudinal components (*lambana* and *nati*). These components are linked in *GD2* 270-273 with two Sines formed with the ecliptic called the Sine of sight-motion (*dṛggati*) and the Sine of sight-deviation (*dṛkkṣepa*). *GD2* 274-276 deals with the rules for computing these components with measuring units of *yojana*s and minutes.

In general, Sanskrit texts on astronomy do not deal with the entire geocentric parallax. For example, *Brāhmasphuṭasiddhānta* chapter 5 on solar eclipses mentions the longitudinal and latitudinal parallax at the very beginning (5.1-3) and deals with their computation throughout the chapter, but hardly any reference is made to their combined amount (Yano (1982))[2]. Likewise, *MBh* 5.24-27 (T. Kuppanna Sastri (1957, pp. 280-282)), *Sūryasiddhānta* 5.1-9 (Shukla (1957, pp. 66-68)), *Śiṣyadhīvṛddhidatantra* 6.6-7 (Chatterjee (1981, 1, p. 112)), *Siddhāntaśekhara* 6.1-3 (Miśra (1932, pp. 382-384)) and *Siddhāntaśiromaṇi Grahagaṇitādhyāya* 6.1-4 (Chaturvedi (1981, pp. 258-261)) also deal only with the two components[3]. All of these verses are in a chapter titled "solar eclipse" where the two components and not the entire parallax were necessary.

In *GD2*, Parameśvara does occasionally refer to the longitudinal and latitudinal parallax or its element as something "in an eclipse" (*GD2* 268, 269, 276). Yet he does not explain how the parallax is actually applied to eclipses, such as in finding the possibility of a solar eclipse or computing its timing. There is no reference to parallaxes in *GD2* 277-301 on eclipses. Parameśvara seems to focus on explaining the principle of the parallax in general, rather than giving the practical rules. The reader would have to advance to other texts to find instructions on such computations involving parallaxes.

## 21.1 The circle of sight (*GD2* 248-249)

*GD2* 248 begins with describing the circle of sight (*dṛṅmaṇḍala*), a circle centered on the location of the observer on the surface of the Earth, as opposed to circles of planets which are concentric with the Earth. This contrast evokes the description of planetary orbits where two great circles, the concentric orbital circle and the eccentric circle, are separated by a given distance (appendix C.1). We may apply the same interpretation for Parameśvara's description in *GD2* 248 and

[1] In modern astronomy, the term "parallax" has many meanings. Hereafter we shall use this word in the sense of "geocentric parallax".

[2] The longitudinal and latitudinal parallax also appear in *Brāhmasphuṭasiddhānta* 21.65 (Ikeyama (2002, p. 149)), again without the entire parallax.

[3] Meanwhile, there is no reference to the longitudinal and latitudinal parallax in the *Āryabhaṭīya*, but *Ābh* 4.34 seems to be related with this topic. K. V. Sarma and Shukla (1976, p. 147) asserts that the word *dṛkchāya* in this verse means parallax. On the other hand, Parameśvara's commentary (Kern (1874, pp. 92-93)) does not gloss this term, but uses other words in *Ābh* 4.34 to formulate two rules which give the latitudinal and longitudinal parallaxes in *yojana*s (this is equivalent to *GD2* 274). Then, in a shorter sentence he mentions that the [unified] parallax can be obtained likewise (*evam*). Therefore, it is unlikely that Parameśvara considers *Ābh* 4.34 as a reference exclusively to a united parallax.

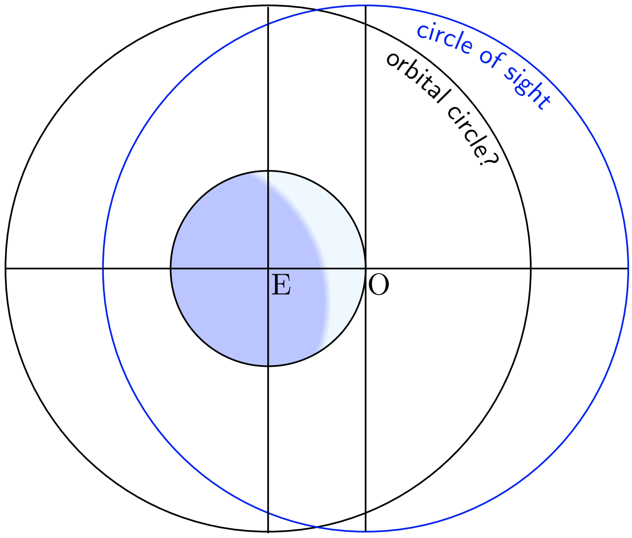



Figure 21.1: A wrong model for *GD2* 248: A concentric circle around the Earth's center E and the circle of sight around the observer O.

depict two circles with the same size (figure 21.1), but I think that this is incorrect. *GD2* 248 refers to the planets in the plural, suggesting that there should be different circles for each planet. Meanwhile, there is only one circle of sight. Therefore it is unlikely that the size of this circle of sight is not linked with the orbits of circles.

From *GD2* 254 onward, arcs in the orbits of planets are measured in *yojana*s. Meanwhile, Parameśvara explains later in the text that a *yojana* in a great circle is equal to an arc minute (*GD2* 256) and draws the circle of sight as a great circle (*GD2* 261). Since orbits of planets are much larger in *yojana*s[4], the circle of sight will be always inside them.

Parameśvara's statements in *GD2* suggest that the circle of sight is only used for describing the parallax in a plane diagram. Meanwhile, we do not know whether the armillary sphere could also be used for explaining this topic. Bhāskara II describes a "sphere of sight (*dṛggola*)" put outside the stellar sphere and the celestial sphere in his *Siddhāntaśiromaṇi Golādhyāya* 6.8-9 (Chaturvedi (1981, p. 315)), but this sphere is not associated with the parallax and serves only as a place for projecting the circles in the two inner spheres together (Ôhashi (1994, p. 269)).

In the following verses, Parameśvara refers to planets on the horizon and on the zenith. To interpret all his statements in *GD2*, we must assume that the circle of sight goes through the direction of the observer's zenith Z (figure 21.2). The problem with this model, however, is that the orbits of planets do not necessarily go through the zenith. *GD2* 248 itself does not call the circles of planets their "orbit", but only state that the measure of these circles are equal to their orbit. Parameśvara might be distinguishing the orbit of a planet that does not always go through the horizon from a circle which goes through the planet at a given moment and the zenith of the observer. However, in *GD2* 260-266 Parameśvara draws a circle in the same plane with the circle of sight and calls it an "orbital circle (*kakṣyāvṛtta*)". Furthermore, even in *GD2* 248 he says that Planets "revolve (*bhramanti*)" on the circles, suggesting that this circle itself is the orbit of the planet.

[4]For example, 10 *yojana*s on the moon's orbit is equal to an arc of minute according to *Ābh* 1.6 (Kern (1874, p. 8)), and therefore the circumference of the moon's orbit should be 10 times larger than a great circle in *yojana*s.

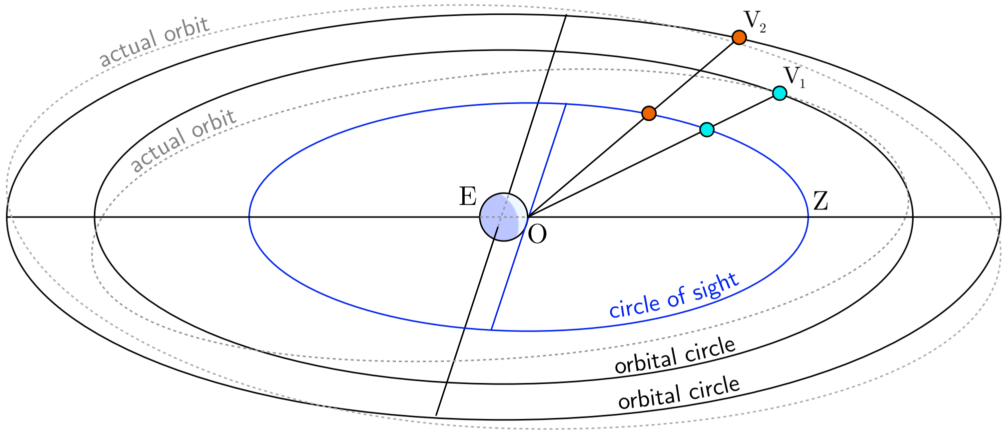



Figure 21.2: Orbital circle of planets with the circle of sight. The zenith is to the right.

Hereafter, I shall use the term "orbit" or "orbital circle" for a circle which which goes through the planet at a given moment and through the direction of the observer's zenith, whose center is the observer and whose radius is the distance between the observer and the planet at the given moment. The situation becomes more complex when we try to build the configuration with multiple planets because they are not in the same plane. Parameśvara makes no remark on this problem at all and draws the orbits of planets in *GD2* 260-266 as if they were in the same plane. My hypothesis is that the word "planet" refers to the longitude of the planet and not the celestial body itself, as we have discussed in sections 6.2 and 9.1. In this case, all the "planets" will be in the plane of the ecliptic. The concept of dividing the parallax into components, as we will see later, also assume that the planets are on the ecliptic. In addition, the parallax is computed for eclipses; the longitudes of the planets considered (to be precise, the sun and the moon) would be the same or very close, and therefore we could assume that the planets and the circle of sight as being in the same plane.

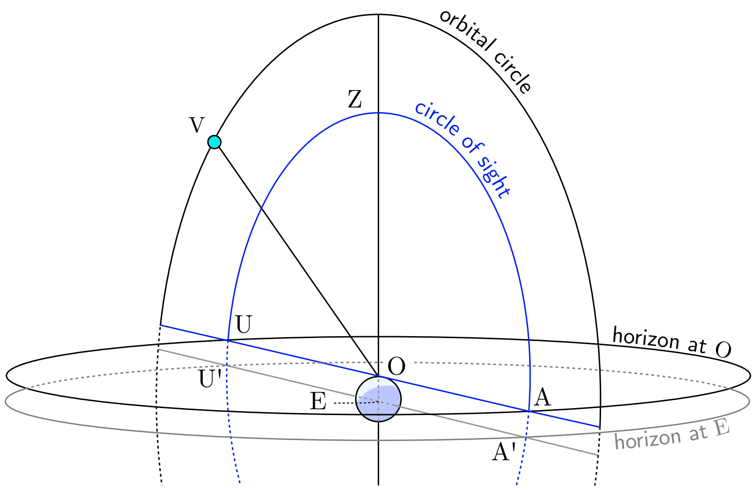



Figure 21.3: The horizon seen from the observer O and from the center of the Earth E.

*GD2* 249 describes the observer's own horizon which is above the horizon seen from the center of the Earth (figure 21.3). It is uncertain whether Parameśvara is referring to the horizons as lines (UOA for the observer and U′EA′ for the center of the Earth) or as planes, but his intention is probably to emphasize the difference in the moment of rising and setting due to the difference in the horizon.

## 21.2 Parallax in *yojanas* (*GD2* 250-253)

The observer's distance from the center of the Earth, as described in *GD2* 249, causes what is called today the geocentric parallax. *GD2* 250 describes the case when the planet is on the horizon when seen from the center of the Earth, which is when the geocentric parallax is the largest according to *GD2* 252. *GD2* 251 explains that there is no parallax when the planet is on the zenith and *GD2* 252 concludes that the amount of the parallax is proportional with the planet's distance from the zenith. *GD2* 253 gives a Rule of Three, but Parameśvara does not explain how this rule is formulated.

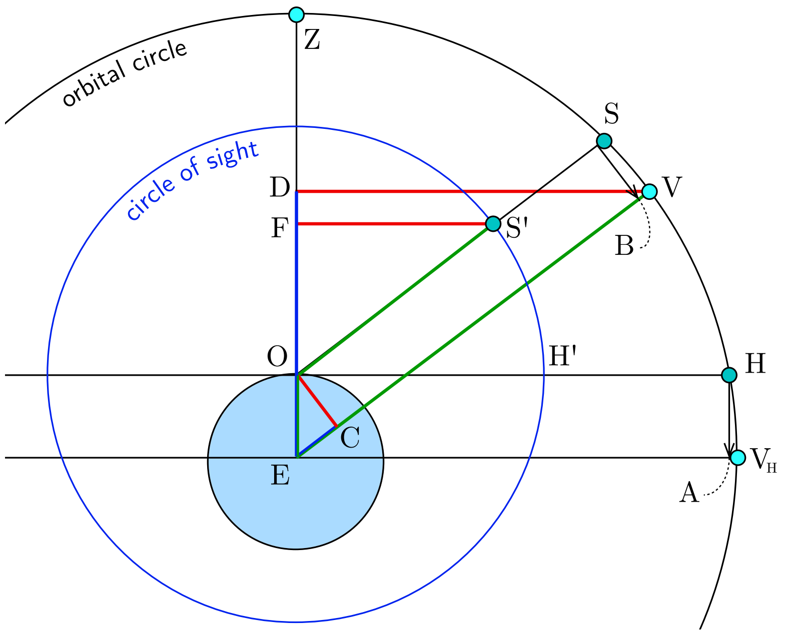



Figure 21.4: Geocentric parallax in *yojanas*. $p_{\max} = \widehat{\mathrm{HV_H}}$ when an observer at the center of the Earth E sees the planet $\mathrm{V_H}$ on the horizon, 0 when it is on the zenith Z and $p = \widehat{\mathrm{SV}}$ when it is on V. $\mathrm{FS}' = \operatorname{Sin} z_V$ is the Sine of sight in the last case.

My understanding is that what Parameśvara refers to as a "parallax" is the difference of the planet's position measured in *yojanas* in the orbital circle (figure 21.4). This is because the statements in *GD2* 250-253 associate the parallax with the half-diameter (i.e. radius)[5] of the Earth $\frac{d_\oplus}{2}$; later in *GD2* 279, Parameśvara explains that the diameter of the Earth is 1050 *yojanas*, and

[5]Parameśvara constantly uses the word half-diameter (*vyāsārdha*) when referring to the radius of the Earth, and in *GD2* 279 he gives the value of its diameter $d_\oplus = 1050$ (*yojanas*) and not of its radius. Therefore I follow him by using the English term "half-diameter" and the expression $\frac{d_\oplus}{2}$ in my formulas.

therefore the parallax must also be in units of *yojana*s. I would also like to emphasize at this point that Parameśvara is not explicit whether a parallax is an arc or a segment. The computational rules suggest that it should be a segment, while Parameśvara's graphical representation in *GD2* 260-266 suggest that they should be arcs. I assume that a parallax in *GD2* is essentially an arc, but that it is small enough to be approximated by its Sine[6]. This resembles the case with the deviation and the celestial latitude of planets.

In our diagram, $\mathrm{V_H}$ represents the position of the planet which is on the horizon when seen from the center of the Earth E. The observer O does not see this planet in the direction of his horizon $\mathrm{O - H' - H}$ but below it. What Parameśvara calls the "downward motion (*adhogati*)" in *GD2* 250 is the arc $\widehat{\mathrm{HV_H}}$ or its corresponding Sine HA. We have drawn the Earth very largely in figure 21.4, but actually it is very small compared to the orbital circle and thus $\widehat{\mathrm{HV_H}} \sim \mathrm{HA}$. This is the greatest parallax $p \sim \mathrm{Sin}\, p_{\max}$, and apparently $\mathrm{HA} = \mathrm{OE} = \frac{d_\oplus}{2}$.

*GD2* 251 states that a planet on the zenith Z as seen from E stays at the same position when seen from the observer O. Thus $p = 0$ in this case.

V is the position of a planet which is between the horizon and the zenith when seen from E. Parameśvara does not describe what the parallax of V is, and only mentions in *GD2* 252 that it should be established from the "Sine of sight (*dṛgjyā*)" by "proportion (*anupāta*)". I have reconstructed the situation in figure 21.4. S is a point on the orbital circle such that $\mathrm{OS} \parallel \mathrm{EV}$ and represents the direction of the planet as seen from the observer O if there were no parallax. The arc $\widehat{\mathrm{SV}}$, or approximately its Sine SB, is what I understand as the parallax in *yojana*s $p \sim \mathrm{Sin}\, p$.

The "Sine of sight" is never defined in *GD2*, but its name suggests that it is related to the circle of sight. My understanding is that it represents the distance of the planet from the horizon when the parallax is neglected. The position of the planet without parallax in the circle of sight is its intersection with OS which is S′ and thus $\mathrm{FS'} = \mathrm{Sin}\, z_V$ is the Sine of sight in this situation. When the planet without parallax is on the horizon, it reaches its maximum $\mathrm{OH'} = R$.

Elsewhere in *GD2*, the word "proportion" implies a pair of similar figures from which a Rule of Three can be established. *GD2* 253 is indeed a Rule of Three. We can formulate this rule in the following way, although I am not sure if this was indeed how Parameśvara grounded the computation: $C$ is the foot of the perpendicular drawn from O to EV. Since $\mathrm{OS} \parallel \mathrm{EV}$, $\mathrm{OC} = \mathrm{SB} = \mathrm{Sin}\, p$. $\triangle\mathrm{ECO}$ and $\triangle\mathrm{EDV}$ are right triangles sharing an acute angle $\angle\mathrm{OEC} = \angle\mathrm{VED}$, and thus $\triangle\mathrm{ECO} \sim \triangle\mathrm{EDV}$. On the other hand, comparing $\triangle\mathrm{EDV}$ and $\triangle\mathrm{OFS'}$: $\angle\mathrm{VED} = \angle\mathrm{S'OF}$ since they are corresponding angles and $\angle\mathrm{DEV} = \angle\mathrm{FOS'}$. Thus $\triangle\mathrm{EDV} \sim \triangle\mathrm{OFS'}$, so $\triangle\mathrm{ECO} \sim \triangle\mathrm{EDV} \sim \triangle\mathrm{OFS'}$. Therefore

$$\mathrm{OC} = \frac{\mathrm{FS'} \cdot \mathrm{OE}}{\mathrm{S'O}}$$

$$\mathrm{Sin}\, p = \frac{\mathrm{Sin}\, z_V \cdot \frac{d_\oplus}{2}}{R} \tag{21.1}$$

By approximating the Sine with its arc,

$$p = \frac{\mathrm{Sin}\, z_V \cdot \frac{d_\oplus}{2}}{R} \tag{21.2}$$

As discussed previously, the measuring unit is not specified at this moment. However we can infer from *GD2* 254 that it is in *yojana*s.

[6]For example, the greatest value of the parallax in *yojana*s is 1050. The planet with the smallest circumference is the moon, and *Ābh* 1.6 (Kern (1874, p. 8)) implies that 10 *yojana*s on the moon's orbit is equal to an arc of minute. 1050 *yojana*s amount to 105 minutes, but Āryabhaṭa's Sine table suggests that arcs smaller than 225 can be approximated by its Sine.

## 21.3 Parallax in minutes (*GD2* 254-259)

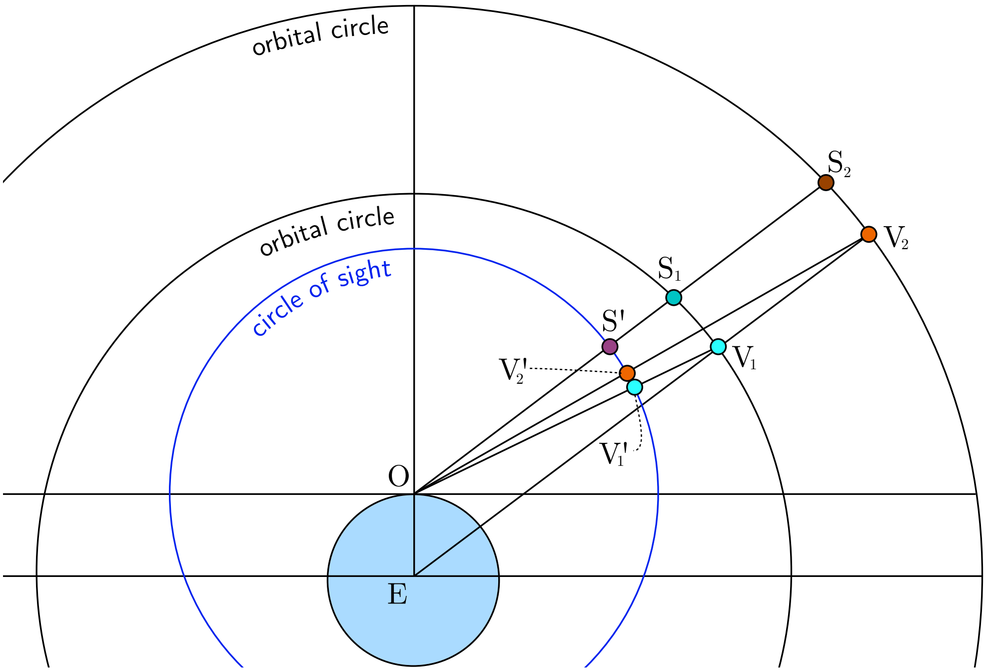



Figure 21.5: Planets whose parallaxes are the same in *yojana*s ($\widehat{S_1V_1} = \widehat{S_2V_2}$) but different in minutes ($\widehat{S'V'_1} \neq \widehat{S'V'_2}$).

*GD2* 254 contrasts the parallax measured in *yojana*s along the orbits of planets with the parallax in arc minutes (figure 21.5). Planets $V_1$ and $V_2$ are in the same direction when seen from the center of the Earth E. S′ is the position of the planets in the same direction as seen from the observer O if there was no parallax. $S_1$ and $S_2$ are the intersections of the orbits and the observer's eyesight OS′. $\widehat{S_1V_1}$ and $\widehat{S_2V_2}$ are the parallaxes in *yojana*s. If we approximate them with their corresponding Sines, as we have done in the previous section, $\widehat{S_1V_1} = \widehat{S_2V_2}$. However, their positions in the circle of sight, which are the intersections of the circle with the lines of the observer's eyesight toward their actual positions, are separated ($V'_1$ and $V'_2$). In other words, their parallax in the circle of sight, $\widehat{S'V'_1}$ and $\widehat{S'V'_2}$, are different. Parameśvara only refers to a great circle and not the circle of sight in *GD2* 254-256, but I assume that his statements apply to the circle of sight.

*GD2* 256 suggests that a *yojana* on the great circle is equivalent to an arc minute. The latter half of this verse gives a reasoning, but I do not understand what Parameśvara means by "equation (*phala*)". He does not refer to a parallax as an equation elsewhere, and I suppose that the "equation" in *GD2* 256 does not refer to a parallax. In any case, $\widehat{S'V'_1}$ and $\widehat{S'V'_2}$ are parallaxes in arc minutes.

*GD2* 255 indicates a rule for converting the parallax in *yojana*s to a parallax in minutes, but the multiplier and divisor are not specified. Let us first look at the possible configuration for explaining this rule (figure 21.6). BV is the Sine of the parallax in *yojana*s $\widehat{SV} = p$ and B′V′ is the Sine of the parallax in minutes $\widehat{S'V'} = \pi$. Both Sines approximate their arcs. △OBV and

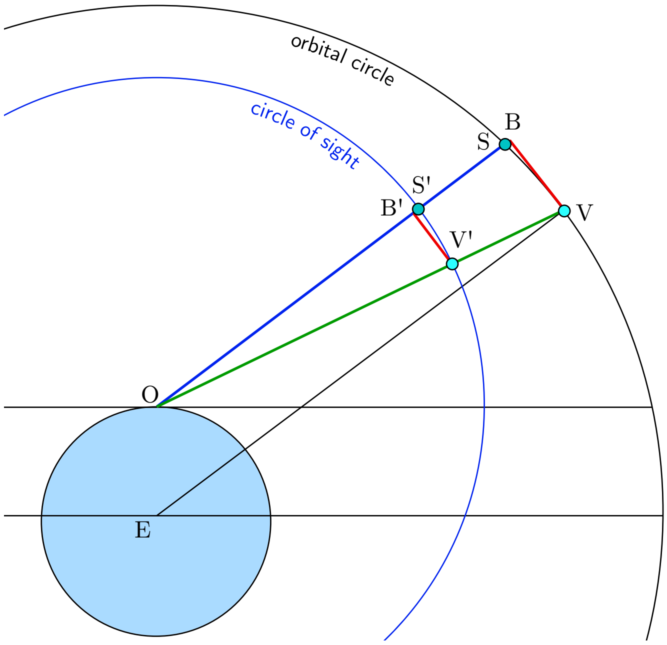



Figure 21.6: The parallax in *yojana*s ($p = \widehat{\mathrm{SV}} \sim \mathrm{BV}$) and in minutes ($\pi = \widehat{\mathrm{S'V'}} \sim \mathrm{B'V'}$).

$\triangle\mathrm{OB'V'}$ are right triangles that share one acute angle and thus

$$\mathrm{B'V'} = \frac{\mathrm{BV}\cdot\mathrm{V'O}}{\mathrm{VO}}$$
$$\mathrm{Sin}\,\pi = \frac{\mathrm{Sin}\,p\cdot R}{\mathrm{VO}} \tag{21.3}$$

I assume that Parameśvara approximates the distance from the observer to the planet VO with the radial distance from the center of the Earth $\mathrm{VE} = \mathcal{D}$ (*yojana*s). This is possible because $\mathrm{VE} \gg \mathrm{OE}$, and later in *GD2* 275 $\mathcal{D}$ is indeed involved in the computation. Thus, applying this approximation as well as approximating the Sines with their arcs,

$$\pi = \frac{pR}{\mathcal{D}}\ \text{(minutes)} \tag{21.4}$$

The next three verses seem to stress the difference in parallax between planets in different orbits. *GD2* 257ab, especially the word "because (*tasmāt*)" does not make sense. *GD2* 257cd is even more strange, as it turns to the daily motion of planets which is not the cause of parallaxes itself. It is merely resembles the parallax in the sense that the daily motion of planets in *yojana*s are supposed to be the same while their apparent motion in arc minutes are different due to the difference in their distance. This has been already stated in *GD2* 19 (section 3.1). "Below (*adhas*)" means that the planet is closer to Earth and "above (*ūrdhva*)" indicates that it is further. *GD1* 4.62 has some words in common with *GD2* 257, and I believe that it represents what Parameśvara wants to state in *GD2* 257:

> Even if the sun and moon are situated on one [and the same] minute, the two situated above and below are seen [separated] in the east and west directions by people standing on the surface of the Earth.[7]

This repeats the statement in *GD2* 254 (figure 21.5) on the difference between the parallax in *yojana*s and parallax in minutes, and further stresses the viewpoint of the observer(s) as well as the location of the planets above and below. *GD1* 4.62 focuses on the sun and moon while *GD2* 257 refers to planets in the plural. *GD1* tends to refer to the sun and moon in the context of parallaxes (*GD1* 4.62, 65-67, 74) while *GD2* does not. As a result, *GD2* gives us the impression that the author is trying to make a general statement without tying the parallax exclusively to eclipses. We can also see that *GD1* 4.62 refers to people in plural, which might be for indicating the difference in parallax between observers in different locations.

The "own parallax (*nijalambana*)" of planets in *GD2* 258 seem to refer to the parallax in minutes which are different among planets. Together with *GD2* 257ab, this verse states how planets in the same direction as seen from the center of the Earth appear in different positions when seen from an observer (figure 21.5). $\widehat{\mathrm{S'V'_1}}$ is the parallax of a planet situated below and $\widehat{\mathrm{S'V'_2}}$ is the parallax of a planet above it. The difference in their positions as seen from the observer O is $\widehat{\mathrm{S'V'_1}} - \widehat{\mathrm{S'V'_2}} = \widehat{\mathrm{V'_2V'_1}}$.

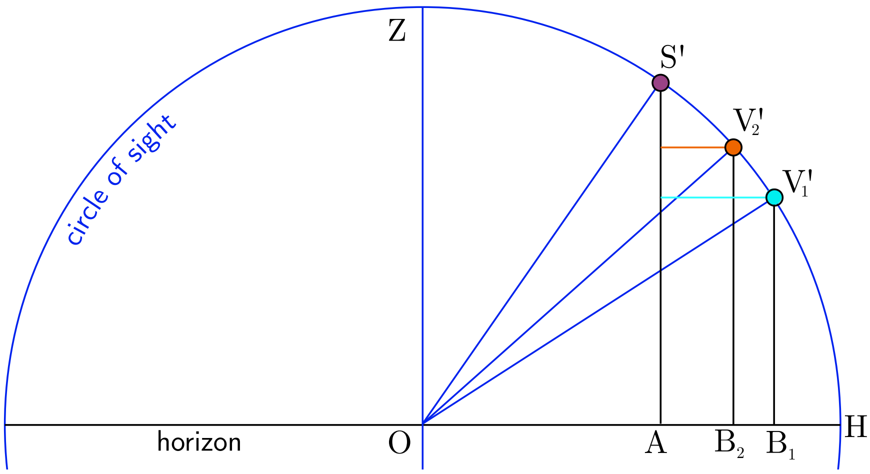



Figure 21.7: Uncorrected (S′A) and corrected great gnomons ($\mathrm{B_1V'_1}$, $\mathrm{B_2V'_2}$) seen from the observer O.

What Parameśvara intends to say in *GD2* 259 is not clear to me. He states that the gnomons of planets are corrected by the parallax in minutes, but if this means that the parallax is the difference between the uncorrected great gnomon as seen from the center of the Earth and the great gnomon as seen from the observer (figure 21.7), he is wrong. Neither the arc of the parallax nor its Sine correspond to the difference of the great gnomons. Therefore I interpret that the word gnomon (*śaṅku*) in this verse refers to the arc of the great gnomon. In our figure, S′ is the position of planets $\mathrm{V_1}$ and $\mathrm{V_2}$ as seen from the observer O if there were no parallax, S′A is its great gnomon and $\widehat{\mathrm{S'H}}$ is the arc of the great gnomon. When the apparent positions of the planets

[7] *raviśaśināv ekakalāsthāv api bhūpṛṣṭhasaṃsthitair manujaiḥ /*
*ūrdhvādhaḥsthau prācyāṃ paścimadiśi ca pradṛśyete //4.62//* (K. V. Sarma (1956–1957, p. 61))

are $V'_1$ and $V'_2$, $\widehat{S'V'_1}$ and $\widehat{S'V'_2}$ are their parallax, and $\widehat{S'H} - \widehat{S'V'_1} = \widehat{V'_1H}$ and $\widehat{S'H} - \widehat{S'V'_2} = \widehat{V'_2H}$ are the arcs of their respective great gnomons.

However I do not know any other case where Parameśvara refers to an arc of a gnomon. And even if this interpretation were correct, I cannot figure out its application.

## 21.4 Explanation with a diagram (*GD2* 260-266)

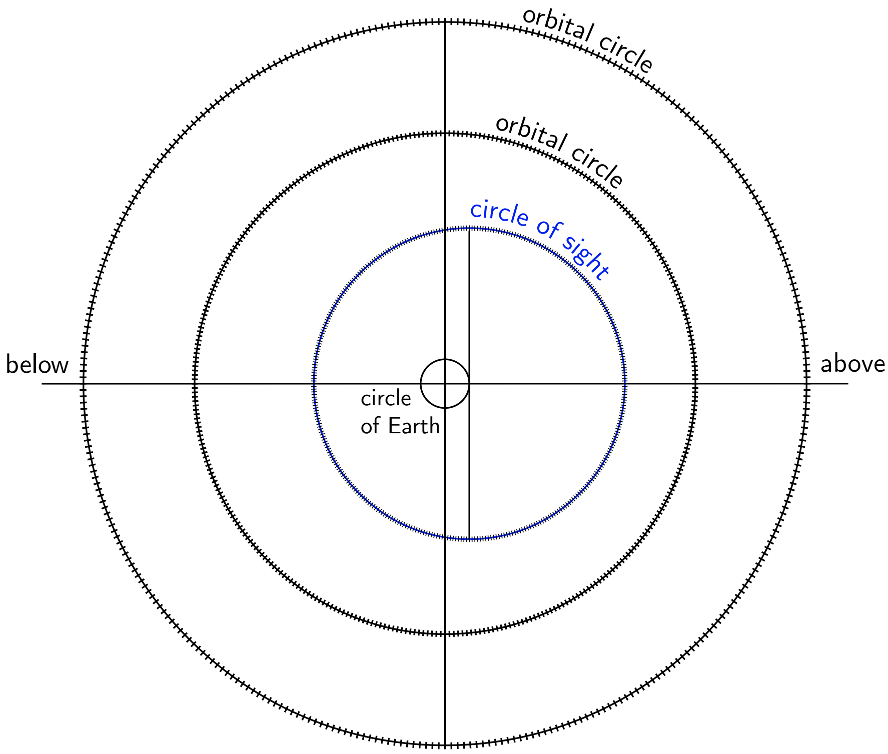



Figure 21.8: Circle of the Earth, orbital circles and the circle of sight drawn with lines of direction and graduations of degrees.

The next seven verses are instructions with a drawing (*chedyaka*) in which most of the previous statements are repeated. There are several circles in this drawing; the circle of the Earth (*GD2* 260b) and orbital circles (*GD2* 260cd)[8] which are concentric, as well as the circle of sight (*GD2* 261) whose center is on the circumference of the Earth (figure 21.8). Parameśvara adds that the orbital circles and the line of sight should have "lines of directions (*diksūtra*)", which probably refers to a pair of lines in the four cardinal directions that divide a circle into quadrants. *GD2* 262cd refers to a north-south line (*yāmyodaksūtra*) which should be one of the lines of directions. However in this situation the directions of the lines have nothing to do with the directions as seen from observers in this diagram themselves. *GD2* 262cd mentions that this line represents the directions below and above, probably from the observer on Earth. Concerning this point, *yāmyodaksūtra* can also be translated "right-left line", referring to the direction of the line as

[8]By repeating *svaṃ* here, Parameśvara emphasizes that there are multiple planets.

seen from the person drawing the diagram. Parameśvara makes no further remark to the other line going east and west, but they could also be given a meaning: the east-west line of the orbital circles represents the horizon as seen from the circumference of the Earth while that of the circle of sight corresponds to the horizon of the observer.

*GD2* 262ab instructs the reader to graduate every circle. If we take this literally, the circle of the Earth must also be graduated, which is meaningless and very unlikely. The units of gradations are degrees (*bhāga*) or *ghaṭikā*s. Parallaxes in *ghaṭikā*s do not appear elsewhere in *GD2*. However it is very common in Sanskrit astronomical treatises to compute the parallax, especially its longitudinal component, in *ghaṭikā*s since it is related to the timing of eclipses[9]. Therefore this statement might be made on the premise that the readers know the application of parallaxes to some extent or that they will learn it soon. Hereafter, Parameśvara only refers to degrees in his diagram.

There is no instruction concerning the size of the circles, although ideally the circle of the Earth and the orbital circles should have been to scale. However the orbital circles are very large compared to the circle of the Earth[10], and it is uncertain whether the ratio was really kept. In our reconstructed diagram the Earth is drawn considerably larger than it should be. The circle of sight is drawn with a string with the length of the Radius (*trijyā*). As discussed in *GD2* 256, a *yojana* in the circumference of the great circle is one minute. Therefore its radius is 3438 *yojana*s, which puts this circle between the circle of the Earth and the orbits of planets.

The next set of instructions in *GD2* 263-265 locate the parallax in minutes. Only one orbital circle is used hereafter (figure 21.9). There is an assumption that the longitude of the planet V on the orbital circle in degrees is already given. *GD2* 263 states that we should put a dot S′ on the same degree on the circle of sight. The gradations are probably used for this step. *GD2* 264 gives the second dot on the circle of sight which is its intersection V′ with a line (*sūtra*, this could also mean “string”) going through V and the center of the circle O. Parameśvara calls S′ a “star in space (*khagarkṣa*)” and V′ a “planet (*graha*)”, and *GD2* 265 mentions that the space between the two points, which could be either the line segment S′V′ or arc $\overset{\frown}{\mathrm{S'V'}}$, is the parallax in minutes. The two Sanskrit terms *khagarkṣa* and *graha* look like arbitrary labels, but if so, Parameśvara is mixing up these labels: V on the orbital circle is called *graha* in *GD2* 263 and *khecara* (literally “that which moves in the sky”) in *GD2* 264. Furthermore, V′ is called *khecara* in *GD2* 265. S′ is constantly called *khagarkṣa*, which may have a special nuance, but if so I feel that this is a strange choice. Elsewhere in *GD2*, especially in relation with the celestial latitude, we have seen that the word “planet” refers to the invisible point on the ecliptic which represents its longitude while the visible position of the object is marked by another word like “latitude”. Here, the invisible point representing the degrees is given a different name while the visible position, which is actually deviated by the parallax, is called the “planet”.

The parallax in *yojana*s is drawn in *GD2* 266. Two lines which both start from O and go through S′ and V′ are drawn. Their length is stated as “equal to the *vyāsa* (diameter) of the orbit”, but apparently this is too long. I assume that either a word for “half” is omitted (this is my interpretation for the translation) or that the text originally read *kakṣyākarṇa* (radial distance of the orbit) but was corrupted in the archetype. In either case, my understanding is that the lengths of the two lines are equal to the radius of the orbital circle. Then the ends of the two lines, P and Q, will be slightly outside the orbital circle. Parameśvara calls their distance the parallax in *yojana*s. There is nothing in his words that infer an arc between P and Q, and

[9]cf. Pingree (1978). Parameśvara too computes longitudinal parallaxes in *ghaṭikā*s in his texts on eclipses such as *Grahaṇanyāyadīpikā* 31-32ab (K. V. Sarma (1966, pp. 10-11)) or even in *GD1* 4.74 (K. V. Sarma (1956–1957, p. 64).

[10]The nearest planet is the moon and the radius of its orbital circle is 34,377 *yojana*s (*GD2* 277) whereas the diameter of the Earth is only 1050 (*GD2* 279).

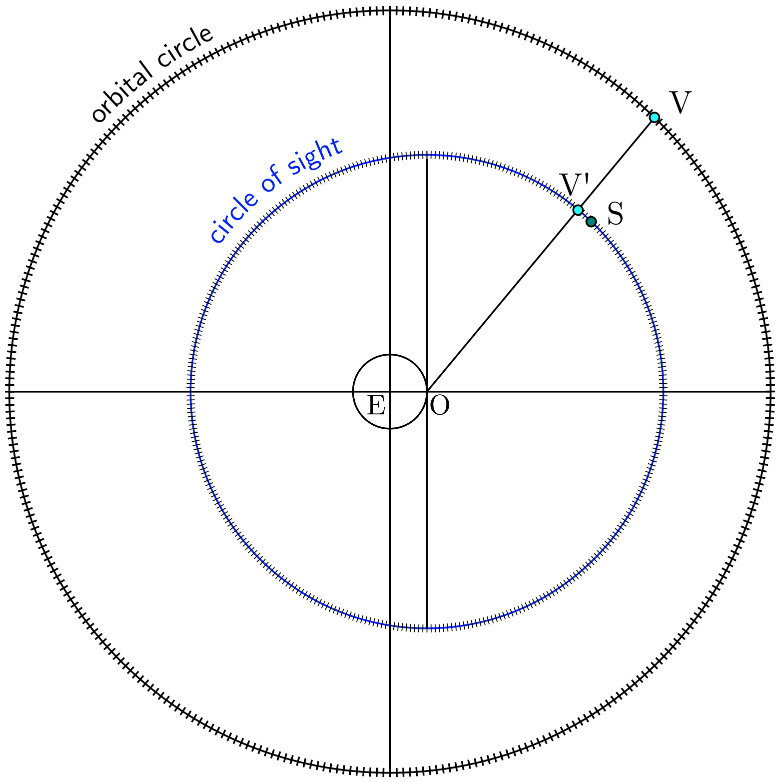



Figure 21.9: The "star in space" S′ and the "planet" V′ on the circle of sight.

therefore I presume that this "distance" is the segment PQ. We can notice in the process of drawing that this parallax is not exactly on the orbital circle and that it is a segment. On the other hand, this construction indicates that the ratio between the two parallaxes are equal to the radii of the circle of sight and the orbital circle. Thus the drawing could indicate that there is an approximation in the previous rules, but Parameśvara does not touch this point.

There is a significant difference between *GD2* 260-266 and the previous statements. *GD2* 253 (formula 21.2) establishes the parallax in *yojana*s and *GD2* 255 (formula 21.4) converts this to the parallax in minutes. The order is reversed in his instructions for drawing. Therefore it is unlikely that the drawing is for grounding these rules. It remains a question what he refers to in *GD2* 260 by saying "'This (*idam*)' should be instructed with a drawing". The verses *GD2* 260-266 themselves give the impression that Parameśvara is explicating the difference between the two types of parallaxes.

## 21.5 Longitudinal and latitudinal parallaxes (*GD2* 267-269)

Starting from *GD2* 267, the subject shifts to the components of the parallax, and hereafter Parameśvara describes configurations that cannot be represented in a plane diagram. I presume that armillary spheres could have been used for explaining such cases. For example, *GD2* 267ab not only states that the parallax of a planet is on the circle of sight, but also that the circle of sight is directed toward (*abhimukha*) the planet (figure 21.11). Elsewhere in *GD2*, *abhimukha* is used in combination with "east" or "west" and thus the expression stresses the orientation of the

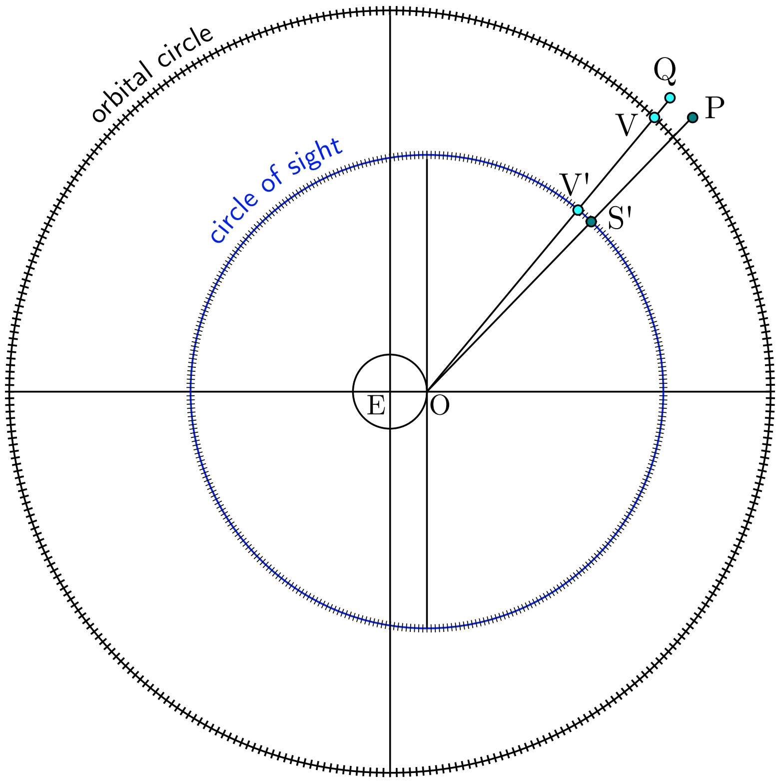



Figure 21.10: Points P and Q at the tip of two lines whose lengths are equal to the radius of the orbital circle. PQ is the parallax in *yojana*s.

circle, which could be represented by its intersection with the horizon H or its angle with other circles like the prime meridian.

*GD2* 267cd gives the impression that there another shift is made; so far the parallax has been described from a viewpoint outside the Earth, but now the focus is on the view of the observer. In this half-verse, the word "parallax" and the motion of the planet are what have been dealt with before, and they are linked with the "difference in sight (*dṛgbheda*)" and the "view of the observer (*dṛṣṭir draṣṭuḥ*)".

*GD2* 268 introduces the two components of the parallax as the base and upright when the entire parallax is the hypotenuse (figure 21.12). According to Parameśvara, the [entire] parallax has "the nature of a hypotenuse (*karṇātmaka*)"; this might be a way to state that it can be divided into two components. *GD2* 268 is also the first reference to an eclipse (*grahaṇa*) in *GD2*. The word is repeated in the following verses, and emphasizes that this is the purpose for dividing the parallax.

The reference for dividing the components is the ecliptic. Parameśvara seems to compare the entire parallax, which arises from the difference in the observer's line of sight following the planet (*GD2* 267d), with the longitudinal component which arises from the difference in the planet's position following the ecliptic (*GD2* 268-269). Parameśvara uses "motion (*gati*)" to indicate this difference, as he did to describe the geocentric parallax in *GD2* 250. If these similarities in his expressions are intentional, it may be for reasoning why the entire parallax and the longitudinal parallax are addressed with the same Sanskrit word (*lambana* or *vilambana*, literally "hanging

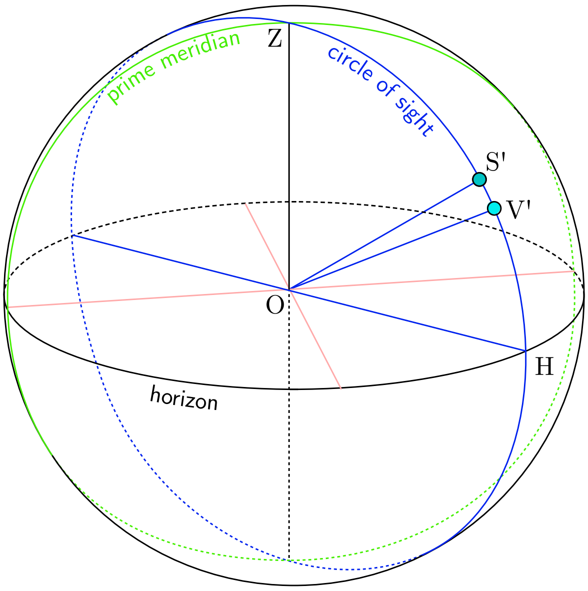



Figure 21.11: The circle of sight in a sphere in the direction of the planet OH.

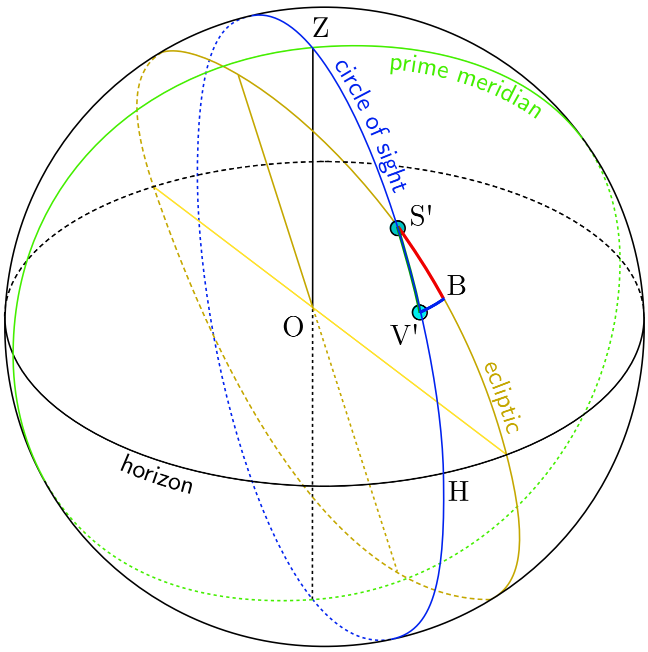



Figure 21.12: The entire parallax $\widehat{S'V'} \sim S'V'$, longitudinal parallax $\widehat{S'B} \sim S'B$ and latitudinal parallax $\widehat{BV'} \sim BV'$.

down").

The latitudinal parallax (*nati*) is described as the deviation (*kṣepa*) from the ecliptic. The word *kṣepa* may also be taken in the sense of "celestial latitude". This clarifies that Parameśvara assumes the planet without parallax to be on the ecliptic.

In our previous discussions on the unified parallax, we have seen that the distinction between an arc and a segment is not clear. *GD2* 268-269 seems to claim that we should treat the components as segments. Stating that the longitudinal parallax follows the ecliptic might imply that it is ultimately an arc, but the terms hypotenuse, base and upright indicate very strongly that they are segments. It seems that the arcs themselves are approximated as segments, and that the spherical triangle $\triangle$V′BS′ is taken as a plane triangle.

## 21.6 Sines of sight, sight-deviation and sight-motion (*GD2* 270-273)

*GD2* 270 introduces two new segments called the Sine of sight-deviation (*dṛkkṣepajyā*) and the Sine of sight-motion (*dṛggatijyā*). The verse follows *GD2* 269 by saying that the upright (latitudinal parallax) and the base (longitudinal parallax) should be established from these two Sines respectively. The term *kṣepa* ("deviation" or "celestial latitude") in "sight-deviation" itself suggests the link with the latitudinal parallax, and the name "sight-motion" brings to our mind that the longitudinal parallax was associated with a motion (*gati*) on the ecliptic.

*GD2* 270 also suggests that the two Sines themselves are also an upright and base. This is explicitly stated in *GD2* 273, and *GD2* 276 even tells us that the corresponding hypotenuse is the Sine of sight. Thus we have two trios of upright, base and hypotenuse: the latitudinal parallax, the longitudinal parallax and the whole parallax on one hand, and the Sine of sight-deviation, the Sine of sight-motion and the Sine of sight on the other.

As shown in figure 21.13, the Sine of sight ($\mathrm{Sin}\, z_V$) is the Sine OB corresponding to the arc distance of the planet without parallax S′ from the zenith. The Sine of sight-deviation ($\mathrm{Sin}\, z_D$) is the Sine OC corresponding to the arc distance of the midpoint D in the ecliptic above the sky from the zenith, as stated in *GD2* 179-181. But Parameśvara never mentions in *GD2* where the Sine of sight-motion ($\mathrm{Sin}\, \Pi_\lambda$) is. Instead, he gives the following computational rule in *GD2* 270cd.

$$\mathrm{Sin}\, \Pi_\lambda = \sqrt{\mathrm{Sin}^2 z_V - \mathrm{Sin}^2 z_D} \tag{21.5}$$

This suggests that the three Sines should form a right triangle in which we can apply the Pythagorean theorem. But if we connect the tips of the Sine of sight and Sine of sight-deviation to form a new segment BC (figure 21.14), $\triangle$OBC in the plane of horizon is not a right triangle because BC will always be longer than the distance between B and OC due to the curvature of the ecliptic projected on the horizon. Additionally, the spherical triangle $\triangle$ZDS′ where $\widehat{\mathrm{DS'}}$ corresponds to BC looks similar to the triangle of parallax $\triangle$V′FS′ when the ecliptic point of sight-deviation D is high (figure 21.13), but this is not the case when it is low (figure 21.15)[11].

Parameśvara rejects the idea of taking BC as the Sine of sight-motion, as we will see next.

The rule in *Ābh* 4.34ab and *MBh* 5.23 for finding the Sine of sight-motion is the equivalent of formula 21.5. The two texts do not specify the locus of the Sine of sight-motion, but Govindasvāmin's commentary on *MBh* 5.23 (T. Kuppanna Sastri (1957, pp. 275-277)) does. He starts by quoting *Ābh* 4.34, followed by an instruction for drawing a diagram which represents the plane of the horizon on which the ecliptic and the Sines are projected (see section 10.16.1). The

[11]From the viewpoint of spherical trigonometry, we know that a pair of triangles on the same sphere cannot be similar unless they are congruent.

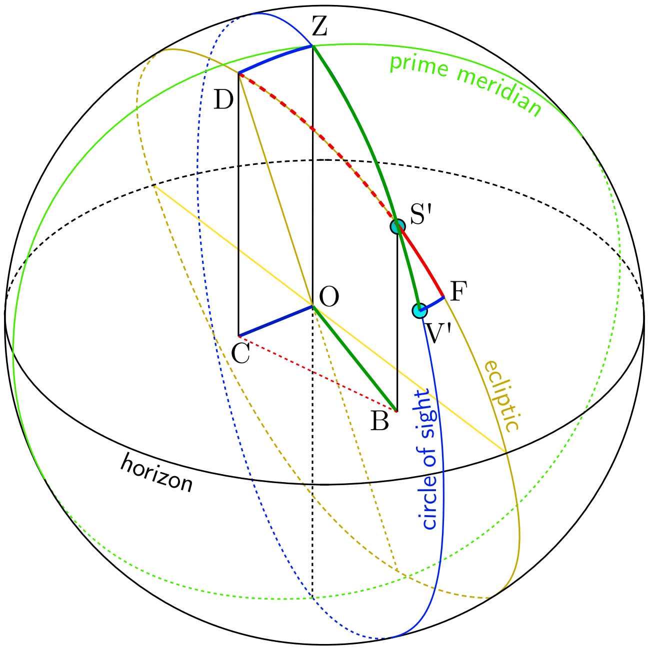



Figure 21.13: The arcs corresponding to the Sines of sight $\widehat{ZS'}$ and sight-deviation $\widehat{ZD}$. Whether $\widehat{DS'}$ corresponds to the Sine of sight-motion is a problem.

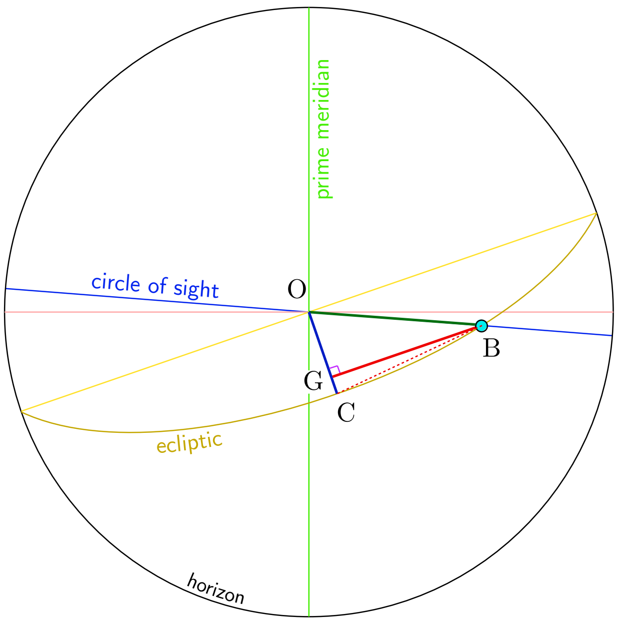



Figure 21.14: The segment BC between the tips of the Sine of sight and Sine of sight-deviation.

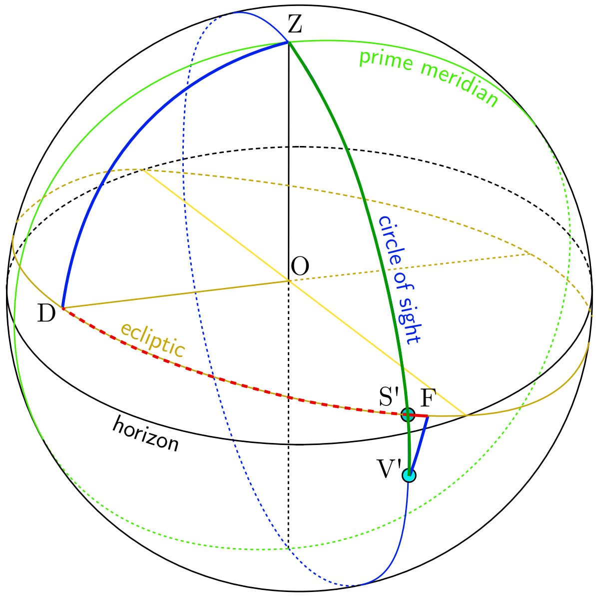



Figure 21.15: When the ecliptic point of sight-deviation D is low. Clearly $\triangle \mathrm{ZDS}' \not\sim \triangle \mathrm{V}'\mathrm{FS}'$.

following is his statement at the end of this section, where "center" is the center of the circle of horizon O, "second dot" is the ecliptic point of sight-deviation projected on the plane C and "sun" represents the position of the planet without parallax B.

> The distance between the center and the second dot is the Sine of sight-deviation. The distance between the sun and the center is the Sine of sight. The distance between the sun and the second dot is the Sine of sight-motion. Thus is the configuration of the Sines.[12]

Thus Govindasvāmin states explicitly that OC is the Sine of sight-deviation, BO is the Sine of sight and BC is the Sine of sight-motion. But Parameśvara's commentary is against his last remark, and proposes an alternative way to draw the triad of Sines.

> Moreover, what has been stated here [in the statement] "The distance between the sun and the second dot is the Sine of sight-motion" is improper, because the base produced with the [Sine of] sight-deviation as upright goes transversely against the path. Therefore, having set the middle of a great circle as center, having drawn a circle with a radial distance (*karṇa*: also "hypotenuse") of the Sine of sight, having stretched out a string that starts from the tip of the Sine of sight-deviation as upright, follows its base and ends at the circumference of the circle [whose radius is] the Sine of sight, a line should be made. This is the Sine of sight; thus is to be seen. [13]

[12] *kendradvitīyabindvantaraṃ dṛkkṣepaḥ / ravikendrāntaraṃ dṛgjyā / ravidvitīyabindvantaraṃ dṛggatiḥ / iti jyāsaṃsthānam* / (T. Kuppanna Sastri (1957, p. 277))

[13] *yat punar iha ravidvitīyabindvantaraṃ dṛggatir ity uktaṃ tan na ghaṭate / dṛkkṣepakoṭisaṃbhūtabhūjāyā mārgatas tiryaggatatvāt / ato vyāsārdhamaṇḍalamadhyaṃ kendraṃ kṛtvā dṛgjyākarṇena vṛttam ālikhya dṛkkṣepakoṭyagrāt tadbhujānusāreṇa dṛgjyāvṛttaparidhyantaṃ sūtraṃ prasārya rekhāṃ kuryāt / sa dṛggatijyā bhavatīti draṣṭavyam* / (T. Kuppanna Sastri (ibid.))

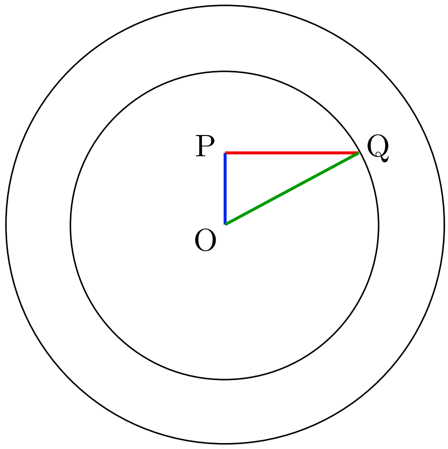
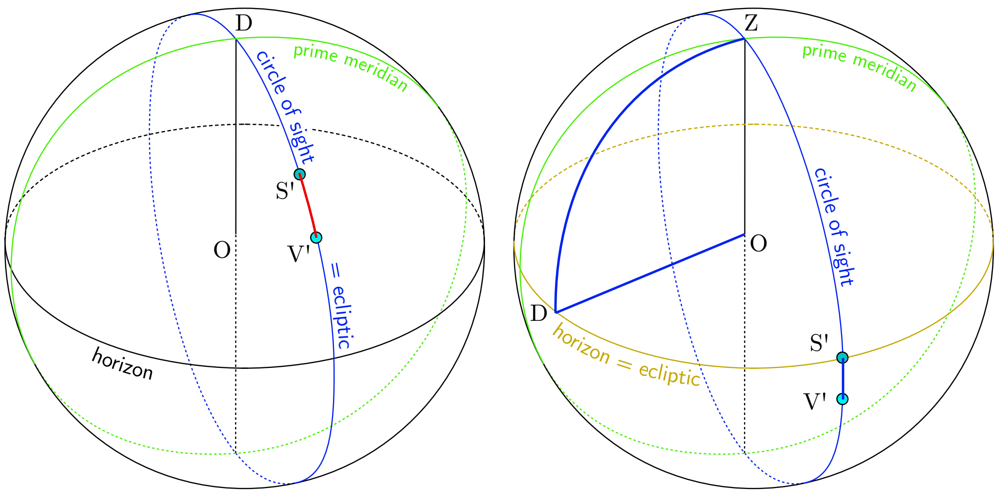



Figure 21.16: Drawing the right triangle of the three Sines.

Parameśvara draws a completely new diagram (figure 21.16), which suggests that he might have concluded that the "Sine of sight-motion" cannot be located in the existing configuration. We draw a great circle, then draw a circle around the same center O with the Sine of sight as radius, locate a point P that is separated from the center with a distance of the Sine of sight-deviation, and draw a line from P in the direction of the base, i.e. perpendicular to the upright OP. With its intersection Q with the circle of the Sine of sight, we have a right triangle △OPQ where the upright OP is the Sine of sight-deviation, the base PQ the Sine of sight-motion and the hypotenuse QO the Sine of sight. This might be how Parameśvara would ground the rule in *GD2* 270cd (formula 21.5).




(a) $\mathrm{Sin}\, z_D = 0$ (Z = D) (b) $\mathrm{Sin}\, z_D = \mathrm{OD} = R$

Figure 21.17: The ecliptic and the Sine of sight-deviation $\mathrm{Sin}\, z_D$ in extreme cases.

On the other hand, Parameśvara seems to have no problem with saying that the Sine of sight-deviation (Sin $z_D$) can be represented in the configuration of the horizon and ecliptic and that it corresponds to the latitudinal parallax, but does not give a thorough grounding. Instead, he gives two extreme cases (figure 21.17): when the ecliptic point of sight-deviation is on the zenith and Sin $z_D = 0$ (*GD2* 271), and when the ecliptic point is on the horizon and Sin $z_D = R$ (*GD2* 272). In *GD2* 272, Parameśvara uses the word girdle (*raśanā*) that he has also used in *GD2* 3 (section 2.2). As in the previous case, this expresses that a circle is orthogonal against another circle and intersecting at the middle, i.e. midpoints between the zenith and nadir.

*GD2* 271 and 272 are parallel to *GD2* 251 and 252 which located the planet on the zenith and horizon and related the parallax to the Sine of sight.

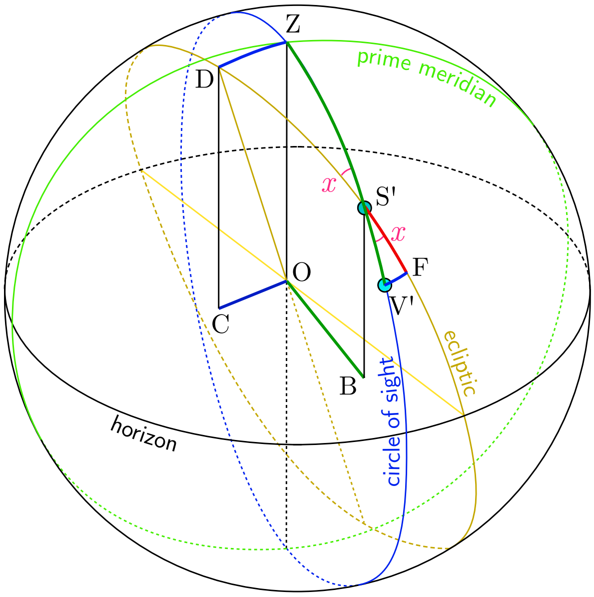



Figure 21.18: Modern interpretation of the relation between the zenith distance of sight-deviation $\widehat{\mathrm{ZD}}$ and the latitudinal parallax $\widehat{\mathrm{V'F}}$.

We have no clue how Parameśvara grounded this link between the Sine of sight-deviation and latitudinal parallax. It can be explained easily with spherical trigonometry as follows, but it is doubtful whether Parameśvara has used it[14].

The spherical angles $\angle \mathrm{DS'Z}$ and $\angle \mathrm{FS'V'}$ are alternate angles and therefore equal. Let their values be $x$. $\angle \mathrm{ZDS'} = \angle \mathrm{V'FS'} = 90°$. From the sine rule in $\triangle \mathrm{ZDS'}$:

[14]Versions of Islamic rules for correcting the declination with the celestial latitude using spherical trigonometry can be found in North India as early as 1370, and Nīlakaṇṭha's *Tantrasaṅgraha* in 1500 includes a similar rule (Plofker (2002)). But as we have discussed in section 10.6.1, Parameśvara's rule for the true declination is very different from Nīlakaṇṭha's, and can be explained with plane trigonometry.

$$
\begin{aligned}
\frac{\sin x}{\sin \overparen{\mathrm{ZD}}} &= \frac{\sin 90^\circ}{\sin \overparen{\mathrm{S'Z}}} \\
\sin x &= \frac{\sin \overparen{\mathrm{ZD}}}{\sin \overparen{\mathrm{S'Z}}} \\
&= \frac{\operatorname{Sin} z_D}{\operatorname{Sin} z_V}
\end{aligned} \tag{21.6}
$$

Likewise in $\triangle\mathrm{V'FS'}$:

$$
\begin{aligned}
\frac{\sin x}{\sin \overparen{\mathrm{V'F}}} &= \frac{\sin 90^\circ}{\sin \overparen{\mathrm{S'V'}}} \\
\sin x &= \frac{\sin \overparen{\mathrm{V'F}}}{\sin \overparen{\mathrm{S'V'}}} \\
&= \frac{\operatorname{Sin} \pi_\beta}{\operatorname{Sin} \pi}
\end{aligned} \tag{21.7}
$$

where $\pi_\beta$ and $\pi$ are the latitudinal parallax and entire parallax in the same arc unit with $z_D$ and $z_V$.

From formulas 21.6 and 21.7,

$$
\frac{\operatorname{Sin} z_D}{\operatorname{Sin} z_V} = \sin x = \frac{\operatorname{Sin} \pi_\beta}{\operatorname{Sin} \pi} \tag{21.8}
$$

Hence it follows that a right triangle with $\operatorname{Sin} z_D$ and $\operatorname{Sin} z_V$ as its upright and hypotenuse is similar to a right triangle with corresponding segments $\operatorname{Sin} \pi_\beta$ and $\operatorname{Sin} \pi$. The latter is very small and the spherical triangle $\triangle\mathrm{V'FS'}$ can be approximated with this plane triangle. Thus the Sine of sight-deviation $\operatorname{Sin} z_D$ corresponds to the latitudinal parallax, the Sine of sight $\operatorname{Sin} z_V$ to the entire parallax, and the remaining base, the Sine of sight-motion $\operatorname{Sin} \Pi_\lambda$, corresponds to the longitudinal parallax.

*GD2* 273 emphasizes the correspondence between the two uprights and the two bases.

### 21.6.1 The Sine of sight-motion in other texts

As already mentioned, Parameśvara's description of the Sine of sight-motion is in accordance with *Ābh* 4.34 and *MBh* 5.23. *Śiṣyadhīvṛddhidatantra* 6.6ab (Chatterjee (1981, 1, p. 112)) gives the same rule, although it is limited to the Sine of sight-motion of the sun at the moment of new moon. Meanwhile, the *Brāhmasphuṭasiddhānta* does not use the term "Sine of sight-motion"; it does not even refer to the Sine of sight-deviation, as we saw in section 10.16.1. The chapters on solar eclipses in the *Siddhāntaśekhara* (chapter 6, Miśra (1932, pp. 382-401)) and *Siddhāntaśiromaṇi Grahagaṇitādhyāya* (chapter 6, Chaturvedi (1981, pp. 258-274)) do not refer to the Sine of sight-deviation, too.

Meanwhile, *Sūryasiddhānta* 5.6cd gives a different definition for the Sine of sight-motion.

> The square root from the difference between the squares of that (Sine of sight-deviation) and the Radius is the gnomon. This is the Sine of sight-motion.[15]

[15] *tattrijyāvargaviśleṣān mūlaṃ śaṅkuḥ sa dṛggatiḥ //5.6//* (Shukla (1957, p. 67), *śaṅkuḥ sa* amended from *śaṅkussa*)

Thus in the treatise, the altitude of the ecliptic point of sight-deviation, or what Parameśvara called the gnomon of sight-deviation (*dṛkkṣepaśaṅku*), is the Sine of sight-motion. This is followed by later author such as Jñānarāja in *Siddhāntasundara Grahagaṇitādhyāya* 6.7d-8a(Knudsen (2014, pp. 233,372)).

Parameśvara's commentary on the *Sūryasiddhānta* makes no remark on how it differs from the *Āryabhaṭīya* or other texts, and we see no trace of the *Sūryasiddhānta* in the verses on parallaxes in *GD2*. By contrast, Nīlakaṇṭha shows great interest. He quotes *Sūryasiddhānta* 5.3-7ab in his commentary on *Ābh* 4.33 (Pillai (1957b, p. 78)) and emphasizes the phrase "true (*sphuṭa*) [Sines of] sight-deviation and sight-motion" in verse 7ab. At the beginning of his commentary on *Ābh* 4.34, he states that the *Sūryasiddhānta* gives the "greatest (*parama*) Sine of sight-motion", followed by a passage from Parameśvara's commentary on *Laghubhāskarīya* 5.11-12 (B. Āpṭe (1946, p. 65)) which computes the longitudinal parallax using a value which is the square root of the difference between the squares of the Radius and the Sine of sight-deviation, claiming that "master Parameśvara explains the computation of the given [Sine of] sight-deviation of the moon and so forth". However Parameśvara himself has quoted these verses as the statement of someone (*kaścid*), and the verses themselves do not use the word "sight-motion". Whether there is a connection between Parameśvara and Nīlakaṇṭha on this point is debatable. In any case, it seems that Nīlakaṇṭha understood the Sine of sight-motion in two ways. Ramasubramanian and Sriram (2011, p. 334) points out that both versions of the Sine of sight-motion appear in his 5th chapter of the *Tantrasaṅgraha*.

## 21.7 Longitudinal and latitudinal parallaxes in *yojanas* and in minutes (*GD2* 274-276)

Table 21.1: Correspondence between parallax and Sine

| | Hypotenuse | Base | Upright |
|---|---|---|---|
| Parallax | Whole | Longitudinal | Latitudinal |
| (*yojanas*) | $p$ | $p_\lambda$ | $p_\beta$ |
| (minutes) | $\pi$ | $\pi_\lambda$ | $\pi_\beta$ |
| Sine of | sight | sight-motion | sight-deviation |
| | $\mathrm{Sin}\, z_V$ | $\mathrm{Sin}\, \Pi_\lambda$ | $\mathrm{Sin}\, z_D$ |

*GD2* 274-276 are the computational methods for finding the longitudinal and latitudinal parallaxes. Parameśvara's descriptions are brief, but we can explain his rules using the correspondence between the parallaxes and the Sines that were stated in the previous verses (table 21.1).

*GD2* 253 (formula 21.2) gives the rule for the whole parallax in *yojanas*. By replacing the parallax and the Sine of sight with their respective components, we obtain the latitudinal parallax $p_\beta$ and longitudinal parallax $p_\lambda$ in *yojanas*:

$$p_\beta = \frac{\mathrm{Sin}\, z_D \cdot \frac{d_\oplus}{2}}{R} \ (yojanas) \tag{21.9}$$

$$p_\lambda = \frac{\mathrm{Sin}\, \Pi_\lambda \cdot \frac{d_\oplus}{2}}{R} \ (yojanas) \tag{21.10}$$

*GD2* 274 refers to this pair of computations in one sentence. Here we have assumed that the arcs of parallaxes are approximated as segments, and the same holds in the following formulas.

The next statement in *GD2* 275 that gives the components of parallaxes in arc minutes from their *yojana* counterparts is in the form of a Rule of Three. This rule is parallel with *GD2* 255. In *GD2* 275, the contrast between an orbital circle with the radius in *yojana*s and the great circle with the Radius is visible. Parameśvara uses the term radial distance (*karṇa*) to indicate the radii of the circles in this verse. This is its first appearance in the context of parallaxes, and it might have been introduced to connect the current subject with the following verses on eclipses (*GD2* 277-301) where the radial distance is frequently mentioned. Substituting the parallax and Sine of sight in formula 21.4, we find the latitudinal parallax $\pi_\beta$ and longitudinal parallax $\pi_\lambda$ in minutes:

$$\pi_\beta = \frac{p_\beta R}{\mathcal{D}} \text{ (minutes)} \tag{21.11}$$

$$\pi_\lambda = \frac{p_\lambda R}{\mathcal{D}} \text{ (minutes)} \tag{21.12}$$

The last rule in *GD2* 276 links the parallax with its components directly with a Rule of Three. The terms base, upright and hypotenuse seem to emphasize that a pair of right triangles are behind this rule. The measuring units for the parallax are not given, and the unit of the whole parallax will be the unit of its computed components. However, Parameśvara adds that they are the parallaxes "stated in eclipses (*grahaṇokta*)". Between the parallaxes in *yojana*s and in minutes, only the latter would be practical in computations of eclipses. Thus it is more likely that *GD2* 276 indicates the latitudinal parallax $\pi_\beta$ and longitudinal parallax $\pi_\lambda$ in minutes:

$$\pi_\lambda = \frac{\pi \operatorname{Sin} \Pi_\lambda}{\operatorname{Sin} z_V} \text{ (minutes)} \tag{21.13}$$

$$\pi_\beta = \frac{\pi \operatorname{Sin} z_D}{\operatorname{Sin} z_V} \text{ (minutes)} \tag{21.14}$$

We can also apply other measuring units here. For instance, *GD2* 262 hints that *ghaṭikā*s can be used[16]. Nonetheless, I assume that this rule is for elucidating the relation between the entire parallax and its components. Different methods seem to have been used in actual eclipses; in his *Grahaṇamaṇḍana*, Parameśvara gives procedures for finding the longitudinal parallax without using the Sine of sight-motion.

[16]The *ghaṭikā* is used especially for the longitudinal parallax. See also footnote 9.

# 22 Eclipse (*GD2* 277-301)

In the following verses, Parameśvara explains some topics that are directly linked to eclipses (*grahaṇa*). These include the distances (*GD2* 277-278) and sizes (*GD2* 279-280) of celestial objects involved in eclipses, the contrast between solar and lunar eclipses (*GD2* 281-282), an explanation that the Earth's shadow is the cause of lunar eclipses (*GD2* 283-285), a comparison of the Earth's shadow with that of a gnomon followed by an actual computation (*GD2* 286-295), the rule to obtain the size of the umbra (*GD2* 296-300) and last of all, the occurrence of eclipses (*GD2* 301). The last verse is a very brief statement, and practical rules to find when and where an eclipse can be seen (to give examples of the rules: finding the moment of syzygies, applying the parallax, computing the distance between the obscuring and obscured bodies, and so on) are not included in *GD2*. These are treated in his other works on eclipses, namely the *Grahaṇamaṇḍana*, the *Grahaṇanyāyadīpikā* and the *Grahaṇāṣṭaka*.

## 22.1 Distance of the sun and moon (*GD2* 277-278)

Conversions between lengths in *yojana*s and lengths in arc units, which have been dealt with in relation to parallaxes, continue to be a topic in the following verses.

*GD2* 277 gives the [mean] radial distances (*karṇa*) of the sun and the moon, i.e. radii of their orbital circles, in *yojana*s. Their values ($\overline{\mathcal{D}_{\odot}} = 459,585$ *yojana*s for the sun and $\overline{\mathcal{D}_{☾}} = 34,377$ *yojana*s for the moon) are exactly the same with those given in *MBh* 5.2 (T. Kuppanna Sastri (1957, pp. 250-251)), which is most likely Parameśvara's sources of them.

Table 22.1 compares the values with those found in other treatises[1]. The treatises can be roughly divided into two groups. Those including *GD2* which have smaller values follow the *Āryabhaṭīya* where one arc minute in the moon's orbit is claimed to be 10 *yojana*s (therefore the circumference of the moon's orbit is 216,000 *yojana*s) and others following the *Brāhmasphuṭa-siddhānta* which gives 15 *yojana*s[2] for an arc minute in the moon's orbit. Parameśvara is aware of this difference, as he states in *GD1* 3.7:

> The measure of the Earth, radial distances and so forth spoken by Āryabhaṭa are mentioned half as large again by others. This is due to a different assumption of the scale of a *yojana*.[3]

*GD2* 278 is a rule to find the true radial distance of the sun and the moon in *yojana*s. The "radial distance without difference" refers to their radial distance corrected by the "slow" apogee $\mathcal{R}_{\mu}$ (see appendix C.4.1). $\mathcal{R}_{\mu}$ is the true distance when the orbital circle is a great circle with Radius $R$, and therefore the true radial distances in *yojana*s for the sun ($\mathcal{D}_{\odot}$) and moon ($\mathcal{D}_{☾}$) are

[1] Verses (and references) which include these values are: *Ābh* 1.7, *MBh* 5.2 and 4, *Brāhmasphuṭasiddhānta* 21.32 (Ikeyama (2002, p. 115)), *Śiṣyadhīvṛddhidatantra* 1.43, 5.4 and 6 (Chatterjee (1981, 1, pp. 27,93-94)), *Sūryasiddhānta* 1.58, 4.1 (Shukla (1957, pp. 19,58)), *Siddhāntaśekhara* 2.94, 5.3 and 7 (Miśra (1932, pp. 125,344,347)) and *Siddhāntaśiromaṇi Grahagaṇitādhyāya* 1.7.1, 1.5.3 and 5cd (Chaturvedi (1981, pp. 93,230,232)).

[2] Brahmagupta himself does not give the values for the mean radial distances. Pṛthūdakasvāmin comments under *Brāhmasphuṭasiddhānta* 21.31ab (Ikeyama (2002, pp. 113-114)) that the radial distance of the sun and moon (in *yojana*s) are 685,018 and 51,240.

[3] *āryabhaṭena yad uktaṃ bhūkarṇādeḥ pramāṇam anyais tat |*
*ardhādhikaṃ tu paṭhitaṃ yojanamānasya bhedakḷptyā tat ||3.7||* (K. V. Sarma (1956–1957, p. 25))

Table 22.1: Measures concerning eclipses from various sources (in *yojana*s)

| | Radial distance | | Diameter | | |
|---|---|---|---|---|---|
| Treatise | Sun | Moon | Sun | Moon | Earth |
| *Goladīpikā 2* | 459,585 | 34,377 | 4,410 | 315 | 1,050 |
| *Āryabhaṭīya* | - | - | 4,410 | 315 | 1,050 |
| *Mahābhāskarīya* | 459,585 | 34,377 | 4,410 | 315 | 1,050 |
| *Brāhmasphuṭasiddhānta* | - | - | 6,522 | 480 | 1,581 |
| *Śiṣyadhīvṛddhidatantra* | 459,585 | 34,377 | 4,410 | 315 | 1,050 |
| *Sūryasiddhānta* | - | - | 6,500 | 480 | 1,600 |
| *Siddhāntaśekhara* | 684,870 | 51,299 | 6,522 | 480 | 1,581 |
| *Siddhāntaśiromaṇi* | 689,377 | 51,566 | 6,522 | 480 | 1,581 |

$$\mathcal{D}_{\odot} = \frac{\overline{\mathcal{D}_{\odot}}\mathcal{R}_{\mu(\odot)}}{R} \tag{22.1}$$

$$\mathcal{D}_{☾} = \frac{\overline{\mathcal{D}_{☾}}\mathcal{R}_{\mu(☾)}}{R} \tag{22.2}$$

*GD2* 278cd mentions that that the perigee (*nīca*) and apogee (*ucca*) causes the difference in distance. By definition, the celestial object on the perigee is at is furthest distance (“above” the orbital circle as seen from the Earth) and closest (“below” the orbital circle) when on the apogee. See also appendix C.1.

## 22.2 Diameters of orbs (*GD2* 279-280)

The diameters in *yojana*s of the sun ($d_{\odot}$), the moon ($d_{☾}$) and the Earth ($d_{\oplus}$) as stated in *GD2* 279 are also listed in table 22.1. Here again, Parameśvara follows the *Āryabhaṭīya* and the *Mahābhāskarīya*. Not only does he give the same values, but he also puts them in a single verse as in *Ābh* 1.7 and *MBh* 5.4. The only other text which does the same in table 22.1 is the *Brāhmasphuṭasiddhānta* (21.32). The rest place the diameter of the Earth in a different chapter. Parameśvara calls $d_{\odot}$, $d_{☾}$ and $d_{\oplus}$ diameters of the “orb (*bimba*)”. The word *bimba* may be interpreted as a “disk” or “orb”. It is also used in *GD1* 3.58 to refer to a location in the moon, while *GD2* 40 uses *maṇḍala* (which I have translated into “disk”) in the same context (section 4.4).

In the case of the moon or the sun, both “disk” and “orb” are valid interpretations. In the former case, it is the disk as the appearance of the spheres as seen from the Earth. On the other hand, the Earth may only be taken as an orb in astronomical texts, and therefore I have chosen the word “orb” for translating *GD2* 279-280. However, in addition, *bimba* is also used to indicate the size of an umbra, which is the projection of the Earth’s shadow that has a conic shape[4]. In this case I choose “disk” as a translation, but it is worth noting that the two *bimba*s of the Earth and the umbra are linked by Rules of Three (*GD2* 294, 297).

The computation to find the apparent size of the umbra from the radius of the Earth’s disk is explained in detail later in the treatise. On the other hand, the rule to find the apparent size

[4]For example, *tamaso bimba* (disk of the umbra) in *GD2* 297 and 300, *rāhubimba* (disk of Rāhu; a mythical entity that devours the sun and moon and used here in the sense of umbra) in *MBh* 5.7 (T. Kuppanna Sastri (1957, p. 265)) and *prabhayā bhuvo ... bimba* (disk of the Earth’s shadow) in *Śiṣyadhīvṛddhidatantra* 5.7 (Chatterjee (1981, 1, p.94)).

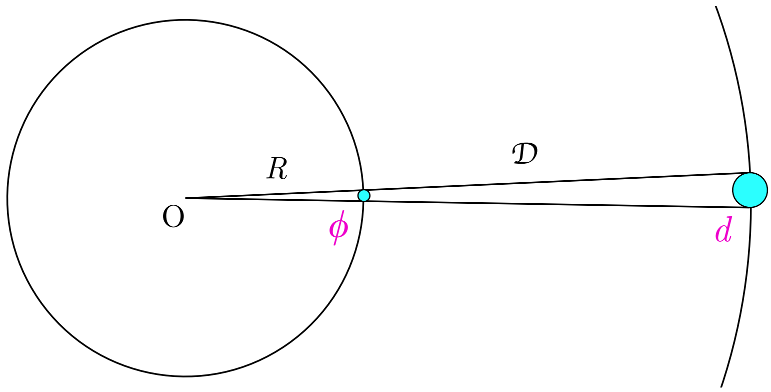



Figure 22.1: A disk with diameter $d$ at a distance $\mathcal{D}$ and its apparent size $\phi$ on a great circle.

of the sun and moon is given briefly in *GD2* 280. Its derivation may be explained as follows. The apparent diameter $\phi$ of a celestial body's disk in arc minutes corresponds to the arc length that it occupies in a great circle of Radius $R = 3438$. This arc length is small enough to be approximated by a segment. Then the ratio of $\phi$ to $R$ is equal to the ratio of the actual size $d$ to the actual distance $\mathcal{D}$. Therefore the apparent size of the sun ($\phi_{\odot}$) and the moon ($\phi_{☾}$) in arc minutes are:

$$\phi_{\odot} = \frac{d_{\odot} R}{\mathcal{D}_{\odot}} \tag{22.3}$$

$$\phi_{☾} = \frac{d_{☾} R}{\mathcal{D}_{☾}} \tag{22.4}$$

The apparent diameters of the disks of the sun and the moon, together with the diameter of the umbra's disk for which Parameśvara spends most of the remaining verses (*GD2* 283-300), is involved in the rule for the occurrence of eclipses (*GD2* 301).

## 22.3 Solar eclipse and lunar eclipse (*GD2* 281-282)

*GD2* 281 is a description of a solar eclipse's mechanism. The verse looks as if it is intended for someone who is not familiar with the subject. The same can be said for the description of lunar eclipses in *GD2* 282. Both verses may also be read as the definitions of both types of eclipses, as Parameśvara uses the word "called (*ukta* / *udita*)". The word *nija* (own) is very peculiar in these verses. I have translated the expression *nijaṃ grahaṇam* as "its eclipse", but there it is not clear why Parameśvara did not use the expression *tadgrahaṇam* (its eclipse) or more straightforwardly *raver grahaṇam* (solar eclipse) and the like. The expression *svādhas* in *GD2* 281 has also been interpreted as "below it (= the sun)".

The causes of the eclipses are also linked to their visibility. The appearance of a solar eclipse (probably referring to the shape of the eclipsed sun) is different when viewed from different locations on the Earth, because the sun (which is hidden) and the moon (which hides the sun) are at different distances from the Earth. Parameśvara does not repeat the previous statements (*GD2* 268, 269, 276) that the parallax is involved in eclipses. Meanwhile, as specified in *GD2* 282, the moon enters the umbra (Earth's shadow) regardless of the observer's location, and its appearance is the same for every observer. To be precise, the moon must be above the horizon for the lunar eclipse to be observed, but Parameśvara makes no remark on this point.

Eclipses are not always described as explicitly in other treatises. For example, Bhāskara I in the fifth chapter of the *Mahābhāskarīya* goes directly to the computations without ever mentioning what a solar eclipse or lunar eclipse is. Among the three treatises on eclipses by Parameśvara, the *Grahaṇāṣṭaka* says nothing about the cause of eclipses. The *Grahaṇamaṇḍana* inserts the following remark after some rules concerning solar and lunar eclipses have already been stated:

> The sun is hidden by the moon, just like a pot [hidden] by another pot. The obscuring of the moon by the umbra should be like enter into the water. (*Grahaṇamaṇḍana* 36) [5]

In the *Grahaṇanyāyadīpikā*, the first occurrence of "eclipse (*grahaṇa*)" apart from the invocation verse is as follows:

> The sun's eclipse is as long as the moon and the sun are on one line of sight. The moon's [eclipse] should be as long as [it] is situated in the umbra.
> The moon should be situated in the middle of the umbra at the end of a lunar period[6]. The conjunction of the sun and the moon should be before or after the end of a lunar period because of the parallax. (*Grahaṇanyāyadīpikā* 13-14) [7]

The description in *Grahaṇanyāyadīpikā* 13-14 is closest to what we have in *GD2* 281-282, although to be strict, it is explaining the duration of the eclipse and not the definition of the word or the mechanism. The three treatises on eclipses contain detailed rules to find the appearance of the eclipses, most of which cannot be found in the *GD2*. This comparison suggests that verses concerning eclipses in *GD2* were written as an introduction on this topic.

## 22.4 The cause of lunar eclipses (*GD2* 283-285)

The rest of the treatise focuses on lunar eclipses, which follows the statement in *GD2* 282 that the eclipse occurs when the moon enters the Earth's shadow (or umbra). *GD2* 283 seems to take the form of a response to *GD2* 282, but this verse is difficult to interpret.

*iti cet* is usually used in the sense of "if this objection is raised, then" where the objection precedes *iti cet* and the response follows it (Tubb and Boose (2007, p. 244)). If we respect the order of words in *GD2* 282ab, the translation would be

> If [one were to ask] how the moon is obscured by the shadow, it is said: the destroyer of darkness.

The response does not make sense. In addition, the sentence must be cut after "destroyer of darkness (*tamohantā*)" because it is in the singular whereas the first phrase in *GD2* 282cd is a

---

[5] *kumbhāntareṇa kumbho yathā tathā chādyate raviḥ śaśinā |*
*vāripraveśavat syāt candrasya chādanaṃ tamasā ||36||* (K. V. Sarma (1965, p. 15))

[6] The word *parvan*, literally "knot" or "joint", refers to the day of new moon or full moon (when the sun and the moon are in conjunction or opposition). Here I have borrowed the translation "lunar period" from Burgess and Whitney (1858, p. 412).

[7] *ekadṛksūtragau yāvac candrārkau grahaṇaṃ raveḥ |*
*tāvan niśākṛto yāvat tāvat syāt tamasi sthitiḥ ||13||*
*sthitir indos tamomadhye parvānte syāt śaśīnayoḥ |*
*yutiḥ parvāntataḥ prāg vā paścād vā lambanād bhavet ||14||*

genitive + plural (rays of the sun / *bhānoḥ karā*). Therefore I have interpreted that *katham* (why) is part of the response. In this interpretation, *GD2* 282ab are the words of a supposed opponent and *GD2* 282cd is Parameśvara answering back. The latter half contains another *katham* and therefore Parameśvara is asking back, which makes the communication look strange, but I cannot think of a better interpretation.

The word *tamohantṛ* appears in *Sūryasiddhānta* 12.17 (Shukla (1957, p. 111)) as a reference to the sun. I could not find a case where this Sanskrit word is being used for the moon. The compound *tamoghna*, where *ghna* is derived from the same root (*han*) as *hantṛ*, appears as a synonym of "moon" in some lexicons (Monier-Williams (1899)).

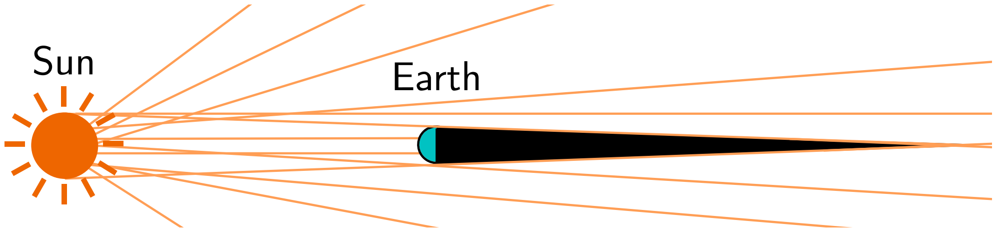



Figure 22.2: The rays of the sun as strings and the Earth's shadow.

In our interpretation, *GD2* 283 says that moonlight originates from the rays of the sun, and that therefore the moon cannot shine in the umbra where the sunbeam is blocked. This may be linked with the notion explained in *GD2* 284-285ab. Here Parameśvara describes a ray of the sun as a string of light (*tejaḥsūtra*). This could evoke a graphical representation (figure 22.2): we can draw straight lines from all over the surface of the sun, and wherever the line reaches is illuminated, while a specific area is always blocked by the Earth. This is the shadow of the Earth. However, we have no further evidence that Parameśvara intended to perform a graphical representation here. Furthermore, he mentions in *GD2* 285cd that the measure (i.e. length) of the Earth's shadow is to be explained with the "grounding of the shadows". "Grounding (*yukti*)" refers to the Rule of Three which is to be used for establishing the length of the shadow. "Shadows (*chāyā*)" is probably a reference to a category in mathematics. Texts treating arithmetics (*pāṭīgaṇita*) often enumerate eight practical problems (*vyavahāra*) where shadows are commonly included (Hayashi (2008)). Parameśvara also enumerates "shadows" as a topic in mathematics (*gaṇita*) in his commentary on *Ābh* 1.1 (Kern (1874, pp. 1-2))[8].

As we will see in the following section, Parameśvara's explanation follows the rule on shadows in the mathematical chapter of the *Āryabhaṭīya* (*Ābh* 2.15).

## 22.5 Comparing the Earth's shadow with a gnomon's shadow (*GD2* 286-295)

### 22.5.1 The gnomon's shadow

Following his declaration in *GD2* 285cd, Parameśvara explains the computation of the Earth's shadow by comparing it with a gnomon's shadow. Figure 22.3 illustrates his description in *GD2* 286. XO is a gnomon whose height is $g = 12$ *aṅgula*s, L is a lamp (*pradīpa* or *dīpa*) placed at a given height of LB $= h$, and they are separated by a distance of BO $= \mathcal{D}$. Concerning the height of the lamp, every manuscript reads *tad-viguṇa-samā* (equal to the *viguṇa* of that). No English

[8] See also introduction 0.2.5 on Parameśvara's categorization of mathematics.

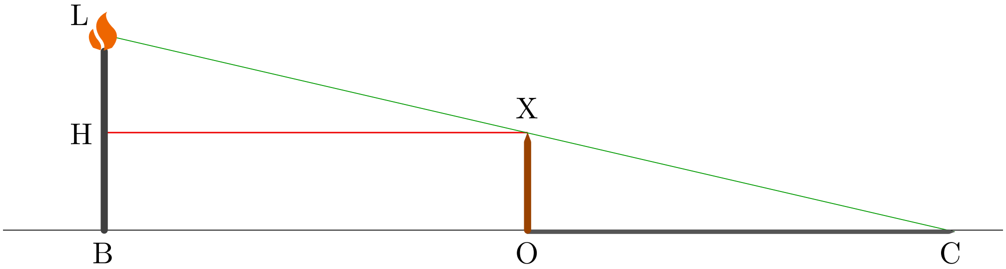



Figure 22.3: Gnomon XO and its shadow OC produced by lamp L.

word corresponding to *viguṇa* (deficient, unsuccessful, adverse, void of qualities, etc.) makes sense. Meanwhile, Śāstri's edition has supplied an extra *d* to read *tad-dviguṇa-samā* (equal to twice that amount), which I have adopted in my edition. However there is no necessity for the lamp to be twice the height of the gnomon (i.e. $h = 2g$) for this explanation. Furthermore, the diameter of the sun (which is later compared with the height of the lamp) is approximately four times that of the Earth (compared with the gnomon) and therefore such statement would be even misleading. Yet I cannot find a better interpretation. The word *śaṇkumitā* in *GD2* 286d is also problematic. If we take it as an predicate adjective of *bhū* (ground), the translation would be "the ground in the space between the gnomon and the lamp is the measure of the gnomon" ($\mathcal{D} = g$), which is another unnecessary assumption. Instead, I read *śaṇkumitā* as the modifier of *chāyā* (shadow), thereby interpreting the passage in the sense of "the [measure (i.e. measuring units) of the] shadow is considered in the measure (i.e. *aṅgula*s) of the gnomon".

In *GD2* 287, Parameśvara uses the word "string (*sūtra*)" again. We may interpret that it refers to the ray of light cast from the lamp L and going past the head of the gnomon X towards the ground C. *sūtra* can also be translated as "line". Here the string or line in question is XC, which is the extension of LX. This is the hypotenuse of the right triangle △XOC whose base is the shadow from its foot to the end OC and upright the gnomon XO (*GD2* 288ab). The distance from the foot of the lamp to the end of the shadow BC and the height of the lamp LB constitute the base and upright of another right triangle △LBC (*GD2* 288cd). △LBC is not involved in the following Rule of Three, and the statement in *GD2* 288cd may be to help the reader understand the correspondence of the segments and avoid confusion. Even another right triangle △LHX is formed by the lamp's excess in height over the gnomon LH as upright, the distance between the head of the gnomon and the lamp-post HX as base and the string / line between the lamp and the head of the gnomon LX as hypotenuse (*GD2* 289). Since HX ∥ BC and ∠LHX = ∠XOC = 90°, therefore △LHX ∼ △XOC. Hence the rule of three in *GD2* 290. Parameśvara does not state the corresponding computation, which we can express as follows:

$$\begin{aligned}\mathrm{OC} &= \frac{\mathrm{HX}\cdot\mathrm{XO}}{\mathrm{LH}}\\ &= \frac{\mathrm{BO}\cdot\mathrm{XO}}{\mathrm{LB}-\mathrm{XO}}\\ s &= \frac{\mathcal{D}g}{h-g}\end{aligned} \tag{22.5}$$

The result $s$ is in *aṅgula*s, in accordance with our interpretation for *GD2* 286d.

This computation resembles *Ābh* 2.15 which states:

> The gap between the gnomon and the base, multiplied by the gnomon and divided by the difference between the gnomon and the base; the quotient should be known as the shadow of the gnomon indeed from its foot.[9]

Parameśvara's commentary paraphrases "base" with "lamp-pole (*dīpayaṣṭi*)". Interestingly, he states in *GD2* 288cd that the lamp is the upright. Perhaps he might not have intended the reader to compare the two texts. His commentary on *Ābh* 2.15 mentions nothing on comparing the gnomon's shadow with the Earth's shadow. Yet we can affirm that he was aware of the connection: *MBh* 5.71 gives the rule to find the Earth's shadow, which is equivalent to *GD2* 294 that we will see soon, and Govindasvāmin quotes *Ābh* 2.15 in his commentary on this verse. Parameśvara, in his super-commentary *Siddhāntadīpikā*, glosses the quoted verse and compares it with the situation for the Earth's shadow (T. Kuppanna Sastri (1957, pp. 314-316)).

### 22.5.2 The Earth's shadow

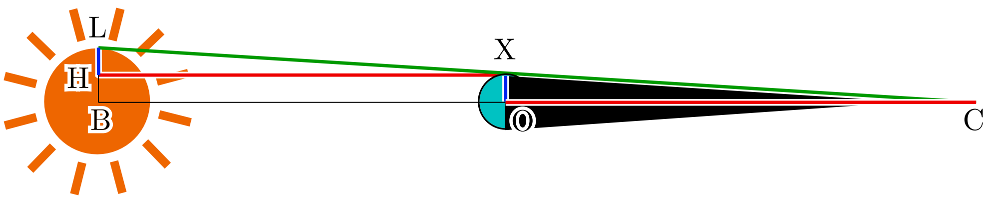



Figure 22.4: Length of the Earth's shadow OC.

Figure 22.4 shows the configuration of the lamp and gnomon projected on the sun, the Earth and its shadow. If we ignore the sphericity of the sun and the Earth and assume that CL tangents the two bodies at L and X[10], the situation is identical with figure 22.3. *GD2* 291-292ab lists the corresponding segments (table 22.2).

Table 22.2: Comparing the shadows of the gnomon and the Earth

| *GD2* 290 (figure 22.3) | | *GD2* 291-292ab (figure 22.4) | | Segment |
|---|---|---|---|---|
| Height of lamp | $h$ | Sun's half-diameter | $\frac{d_\odot}{2}$ | LB |
| Height of gnomon | $g$ | Earth's half-diameter | $\frac{d_\oplus}{2}$ | XO |
| Distance between gnomon and lamp | $\mathcal{D}$ | Corrected radial distance of the sun in *yojana*s | $\mathcal{D}_\odot$ | OB |
| Length of gnomon's shadow | $s$ | Length of Earth's shadow | $l_\bullet$ | OC |

*GD2* 292cd-293 draws our attention to the three dimensional shape of the Earth's shadow. Parameśvara uses the word string / line (*sūtra*) again. Together with the expression "like a tail of a cow (*pucchavat ... goḥ*)", it evokes a visual image.

*GD2* 294 gives the rule for computing the Earth's shadow, which can be derived from the Rule of Three in *GD2* 290 considering segments in *yojana*s. In addition, *GD2* 294 uses the diameters

[9] *śaṅkuguṇaṃ śaṅkubhujāvivaraṃ śaṅkubhujayorviśeṣahṛtam / yallabdhaṃ sā chāyā jñeyā śaṅkoḥ svamūlāddhi //2.15//* (Kern (1874, p. 33))

[10] In reality, the tangential line should go through points L′ and X′ on their circumferences which are slightly closer to C, so that BL′ ⊥ L′C and OX′ ⊥ L′C. Parameśvara's expression in *GD2* 299ab (the diameter of the Earth's shadow at its root is equal to the Earth's diameter) suggests that he is unaware of the approximation.

in place of half-diameters. This is justified in *GD2* 295 by pointing out that this is equivalent of doubling both the desire quantity (*icchārāśi*: in this case the Earth's half-diameter) and the measure quantity (*pramāṇarāśi*: the difference between the sun and Earth's half-diameters), which would be canceled out. This can be formulated as follows:

$$
\begin{aligned}
\mathrm{OC} &= \frac{\mathrm{HX} \cdot \mathrm{XO}}{\mathrm{LH}} \\
&= \frac{\mathrm{HX} \cdot 2\mathrm{XO}}{2\mathrm{LH}} \\
l_{\bullet} &= \frac{\mathcal{D}_{\odot} d_{\oplus}}{d_{\odot} - d_{\oplus}} \qquad (22.6)
\end{aligned}
$$

The Earth's shadow $l_{\bullet}$ is in *yojana*s.

## 22.6 The diameter of the umbra (*GD2* 296-300)

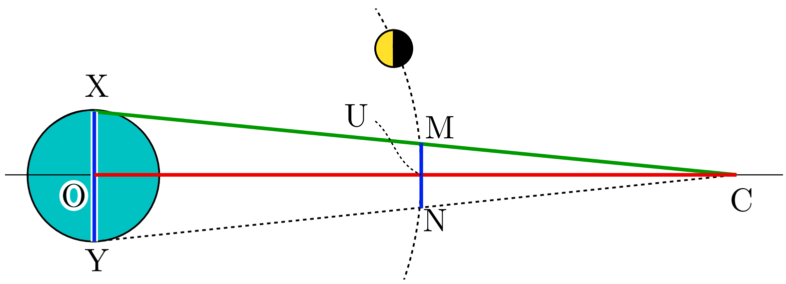



Figure 22.5: Diameters of the Earth $d_{\oplus} = \mathrm{XY}$ and the umbra $d_{\bullet} = \mathrm{MN}$.

By contrast to the Earth's shadow (*bhūcchāyā*) which has been described as similar to a cow's tail, the umbra (*tamas*) refers to a segment of this shadow, which appears as a disk if its outline could be seen from the Earth. As it is a shadow in the middle of darkness, the umbra itself is imperceptible unless the moon enters it to be obscured. Its diameter may vary depending on where the shadow is cut, but in the computation of a lunar eclipse, the only relevant point is its intersection with the path of the moon (figure 22.5). The term "path (*mārga*)" may have been used for distinguishing the moon's true distance, which is important here, from its mean distance on the orbit (*kakṣyā*). The rule for computing the umbra's diameter in *yojana*s is given in *GD2* 296. In *GD2* 297, it is converted to arc minutes. *GD2* 298-299 and *GD2* 300 ground the rules with a Rule of Three for each of them.

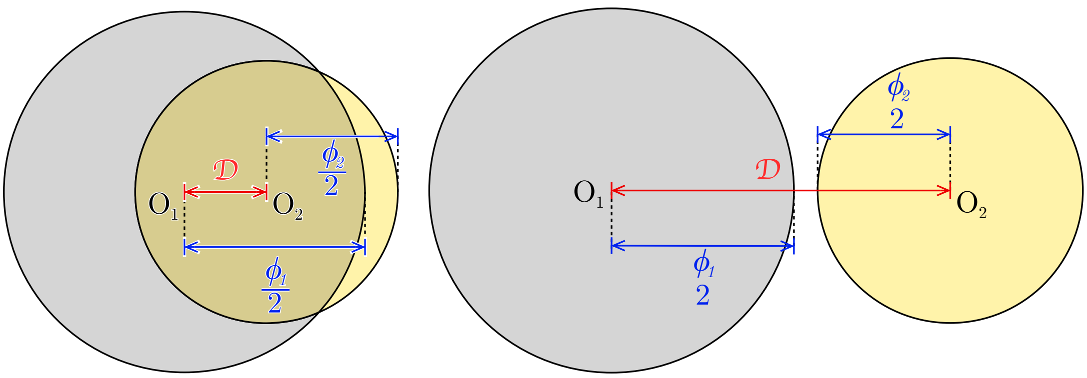
Let us assume that C is the tip of the Earth's conic shadow and that X and Y are on the circumference of its base. If we follow Parameśvara's statement in *GD2* 299ab, this base goes through the center of the Earth O and therefore XY is equivalent to the Earth's diameter $d_{\oplus}$ (as previously mentioned, this is an approximation which Parameśvara seems to be unaware of). U is a point on the central line of the Earth's shadow OC such that OU is the radial distance of the moon at a given moment $\mathcal{D}_{\leftmoon}$ in *yojana*s. The moon itself does not have to be on U at this moment. MN is the segment of the Earth's shadow cut at U, parallel with XY. Its length is the diameter of the umbra $d_{\bullet}$ in *yojana*s. Another segment used for computing $\mathrm{MN} = d_{\bullet}$ is UC, the distance from the tip of the shadow to the center of the umbra. This is described in *GD2* 298 as the "shadow's portion that has gone above the path of the moon (*śaśimārgād* *ūrdhvagatacchāyābhāga*)". Here, "above" is used in the sense of "further from the Earth", i.e. in the direction from U toward C. As stated in *GD2* 298, UC = OC−OU. Since △CMN and △CXY are isosceles triangles which share their apex, △CMN ∼ △CXY. Therefore by comparing their heights CU and CO, we can find the computation given in *GD2* 296:

$$
\begin{aligned}
\mathrm{MN} &= \frac{\mathrm{UC}\cdot\mathrm{XY}}{\mathrm{OC}}\\
&= \frac{(\mathrm{OC}-\mathrm{OU})\cdot\mathrm{XY}}{\mathrm{OC}}\\
d_{\bullet} &= \frac{(l_{\bullet}-\mathcal{D}_{\leftmoon})d_{\oplus}}{l_{\bullet}}
\end{aligned}
\tag{22.7}
$$

The last rule (*GD2* 297) for computing the diameter in arc minutes $\phi_{\bullet}$ is identical with that to find the arc minutes of the sun's and moon's diameters in *GD2* 280. Parameśvara gives the corresponding Rule of Three in *GD2* 300. We can apply the same explanation that was used in *GD2* 280 (section 22.2).

$$
\phi_{\bullet} = \frac{d_{\bullet}R}{\mathcal{D}_{\leftmoon}}
\tag{22.8}
$$

## 22.7 Occurrence of eclipses (*GD2* 301)




(a) Eclipse (b) Non-eclipse

Figure 22.6: When a body with center $O_1$ eclipses a body with center $O_2$ when their distance is $\mathcal{D}$.

*GD2* 301 states when an eclipse occurs and when it does not (figure 22.6). When the distance between the centers of the two celestial objects is $\mathcal{D}$, the diameter of the object that may cause the eclipse is $\phi_1$ and that of the object that may be eclipsed is $\phi_2$, the condition can be easily found as follows:

$$\begin{cases} \mathcal{D} < \dfrac{\phi_1}{2} + \dfrac{\phi_2}{2} & \text{Eclipse} \\ \\ \mathcal{D} > \dfrac{\phi_1}{2} + \dfrac{\phi_2}{2} & \text{Non-eclipse} \end{cases} \tag{22.9}$$

The previous statements in *GD2* 277-300 are sufficient for computing $\phi_1$ and $\phi_2$. As for $\mathcal{D}$, there is no other clue in *GD2*. Many of the topics that have already appeared in this treatise are relevant to find $\mathcal{D}$, but for the actual computation the reader would have had to learn from other treatises.

# 23 Concluding the treatise (*GD2* 302)

The final verse in *GD2* contains no information on Parameśvara himself, whereas he usually mentions his name at the end of other treatises or commentaries[1]. Let us note that Parameśvara would also give more information such as the year or his location in his final remarks (see introduction 0.1.2), but this is not the case here.

The word "concisely (*saṃkṣepād*)" suggests that he had more detailed contents in his mind. This might include his *Siddhāntadīpikā* that he mentions in *GD2* 69.

[1]Among the texts available to me, there were only three other texts where Parameśvara did not give his name in the concluding verses: the *Karmadīpikā* (commentary on the *Mahābhāskarīya*), the *Vākyakaraṇa* and the commentary on the *Laghumānasa*.

# Part IV

# Appendices

# A Numbers in *GD2*

*GD2* and its commentary contain various numbers in different forms. In this appendix we shall look at how numbers are formatted and presented in the texts.

## A.1 Numbers in words

A *Bhūtasaṃkhyā*, or word numeral, is a number represented by specific words. The number can be a single digit (e.g. "eye" - 2) or two digits (e.g. "sun" - 12), and for large numbers the words are listed in compounds, starting from the lowest place. All of the numbers that have more than three figures and some numbers in double figure are described with word numerals in the base text of *GD2*.

In the following list, the Sanskrit form (compounds are decomposed) is followed by its literal meaning, word-by-word replacement of numerals and finally the actual number in Arabic numerals. There are cases where some or all words in a compound are simple numerals, and not a *Bhūtasaṃkhyā*, but they have been listed here nonetheless. Simple non-compound numerals are ignored.

***GD2* 8** *ravi* = "sun" for "twelve" 12

***GD2* 15** *kha-agni* = "sky-fire" for "zero-three" 30

***GD2* 30** *aṅka-randhra-yamala-guṇa* = "numeral-hole-twin-quality" for "nine-nine-two-three" 3299

***GD2* 55** *kha-rasa-vahni* = "sky-taste-fire" for "zero-six-three" 360

***GD2* 57** *ahi-veda* = "snake-Veda" for "eight-four" 48
*rasa-rāma* = "taste-Rāma (Name of mythical character)" for "six-three" 36
*kṛta-dasra* = "dice-Aśvin (Name of twin deity)" for "four-two" 24
*dvi-indu* = "two-moon" for "two-one" 12

***GD2* 62** *bha* = "asterisms (lunar mansions)" for "twenty-seven" 27

***GD2* 73** *sapta-nava-tri-eka* = "seven-nine-three-one" 1397

***GD2* 91** *śaśin-kṛta-vidhu-rāma* = "moon-dice-moon-Rāma" for "one-four-one-three" 3141

***GD2* 99** *śaśin-kṛta-vidhu-rāma* = "moon-dice-moon-Rāma" for "one-four-one-three" 3141

***GD2* 116** *ravi* = "sun" for "twelve" 12
*arka* = "sun" for "twelve"-*aṅgula* 12

***GD2* 117** *arka* = "sun" for "twelve" 12

***GD2* 120** *arka* = "sun" for "twelve" 12

***GD2* 129** *veda* = "Veda" for "four" 4
*rasa* = "taste" for "six" 6
*diś* = "direction" for "ten" 10

***GD2* 130** *vyoman-dineśa* = "sky-sun" for "zero-twelve" 120
*kha-arka* = "sky-sun" for "zero-twelve" 120
*kha-netra-śiśirakara* = "sky-eye-moon" for "zero-two-one" 120

***GD2* 172** *kha-abhra-ahi-indu* = "sky-sky-snake-moon" for "zero-zero-eight-one" 1800

***GD2* 193** *kha-kha-dhṛti* = "*sky-sky-Dhṛti* (name of meter)" for "zero-zero-eighteen" 1800

***GD2* 201** *randhra-go-aśvin-guṇa* = "hole-cow-Aśvin-quality" for "nine-nine-two-three" 3299

***GD2* 209** *nara* = "man (referring to a gnomon with twelve *aṅgula*s)" for "twelve" 12
*svara-kṛta-aṅga* = "*Svara* (name of meter)-dice-numeral" for "seven-four-six" 647

***GD2* 212** *naga-catur-ṣaṭ* = "mountain-four-six" for "seven-four-six" 647

***GD2* 229** *diś* = "direction" for "ten" 10

***GD2* 231** *bhāskara* = "sun" for "twelve" 12

***GD2* 245** *adri-aṅga-rasa-eka* = "mountain-limb-taste-one" for "seven-six-six-one" 1667
*nava-eka-abdhi* = "nine-one-ocean" for "nine-one-four" 419
*bhūdhara-veda-bāṇa-nayana* = "mountain-Veda-arrow-eye" for "seven-four-five-two" 2547
*abdhi* = "ocean" for "four" 4

***GD2* 246** *eka-daśa* = "one-ten" 101
*rasa-viyat-candra* = "taste-sky-moon" for "six-zero-one" 106

***GD2* 246** *rasa-dharā-randhra-kṣama* = "taste-Earth-hole-Earth" for "six-one-nine-one" 1916

***GD2* 277** *pañca-ahi-iṣu-aṅka-bāṇa-jaladhi* = "five-snake-arrow-numeral-arrow-ocean" for "five-eight-five-nine-five-four" 459585
*parvata-naga-rāma-veda-dahana* = "mountain-mountain-Rāma-Veda-fire" for "seven-seven-three-four-three" 34377

***GD2* 279** *vyoman-indu-udadhi-veda* = "sky-moon-ocean-Veda" for "zero-one-four-four" 4410
*tithi-jvalana* = "lunar day-fire" for "fifteen-three" 315
*kha-iṣu-kha-vidhu* = "sky-arrow-sky-Earth" for "zero-five-zero-one" 1050

***GD2* 286** *ina* = "sun" for "twelve" 12

## A.2 Measuring units

*GD2* 19 compares arcs measured in *yojana*s and in minutes on orbits of planets. Here, the arc minutes can be located on orbits with different sizes just like the modern definition of angles. But when dealing with parallaxes, from *GD2* 254 onward, Parameśvara distinguishes the parallax in *yojana*s measured on a planet's orbit with the parallax in minutes that is measured on a great circle. Effectively, the arcs in *yojana*s are converted to arcs in minutes by projecting them on the "circle of sight" which is a great circle.

Arcs other than the *yojana*, especially arc minutes (*kāla*, *liptā*, *liptikā*), are linked with the great circle. This must be related with the correspondence between the lengths of segments and arcs in a great circle: the Radius 3438 is chosen so that the circumference is 21600, which is the number of minutes in a circle. However, the lengths of segments are never mentioned with their units in *GD2* and in the commentaries. *GD2* 80 stresses the correspondence between the arc and its Sine in a great circle. This is used as a reasoning for why arcs are not measured in non-great circles such as the diurnal circle.

The time unit *prāṇa* is the time that the celestial equator revolves one arc minute. Thus the *prāṇa* is also a type of arc minute measured on the celestial equator. This relation is explained in *GD2* 79. Parameśvara distinguishes arc lengths on the celestial equator measured in *prāṇa*s and arc lengths on the ecliptic measured in minutes. His particularity on their difference is noticeable in his rules concerning equations of longitudes (chapters 10 and 11) where he explicits the steps for converting minutes to *prāṇa*s or vice versa, where many of his predecessors have simply approximated the *prāṇa*s on the celestial equator and the corresponding minutes on the ecliptic as equal.

Degrees are not involved very often in the rules, but is always used when Parameśvara instructs a drawing or when his explanations suggest the usage of an armillary sphere. Arc minutes are too small to be drawn, and degrees might have been used instead in such cases.

## A.3 Fractional parts

In general, numbers appearing in the base text of *GD2* are whole numbers, but there are a few cases, notably in some of the six examples, where values are given in the form of fractions. For instance, *GD2* 38 refers to a "fifteenth" of the Earth's circumference, *GD2* 212 (example 2) to a to a "seventh" and "eighth" of a gnomon's length and *GD2* 245 (example 5) to a time length in units of *prāṇa*s as "two thousand five hundred and forty-seven (2547) fourths". All of these are either in the form of $\frac{1}{x}$ or $\frac{y}{x}$ (where $x$ and $y$ are integers and $y$ can be larger than $x$). On the other hand, numbers with fractions in the form of $z + \frac{y}{x}$ ($z$ is another integer) appear only in commentaries. Commentaries on different examples use different styles for expressing fractions.

**Word expressions.** The commentary on example 1 gives "466 …lessened by a quarter (*pādahīna*)" (i.e. $466 - 1/4$) and "457 …increased by a half (*sārdha*)" (i.e. $457 + 1/2$).

**Sexagesimals.** The answers for examples 1 and 2 are given in signs, degrees and minutes, but they are denoted by simple spacing without the units (e.g. "7 11 49" for "7 signs, 11 degrees and 49 minutes"). Example 2 also indicates a fraction of an *aṅgula* by spacing (e.g. "1 30" for "1;30"). By contrast, examples 3 and 4 give the answers in columns (e.g. $\begin{matrix}11\\46\end{matrix}$ for 11;46 *aṅgula*s). Meanwhile, the commentaries on examples 5 and 6 use *ṣaṣṭyaṃśa* frequently for indicating a sexagesimal fraction.

## A.4 Rounding

Parameśvara makes no explicit reference to approximations in his base text of *GD2*. However, the methods that he presents include many divisions and square root computations, and rounding between the steps are inevitable. Commentators on the examples sometimes mention that intermediary steps are not exact integers and thereby infer that rounding is being done. The commentary on *GD2* 245 (example 5) uses "somewhat less than (*kiṃcid ūna*)" twice, suggesting that the value is rounded up in the next step. The commentator on *GD2* 246 (example 6) says "almost (*prāyas*) the same as a Sine of two signs" near the end of the procedure. But in most cases, intermediary values or final results are presented without explanation on how they were approximated. I assume that rounding off[1] was preferred over uniformly rounding down or rounding up[2] the lower fractional part. In my explanatory notes, I have computed such fractional parts in sexagesimals but whether the commentators actually did so is yet to be reflected upon.

We do not know whether the rounding in the commentary reflects Parameśvara's intention. His own notion of rounding is yet to be studied through other texts that include solved examples with rounding.

### A.4.1 Square roots

Computational rules in *GD2* use the Pythagorean theorem frequently. As a result, one needs to compute square roots to carry out the methods. We can see this in the commentaries on the six examples. This raises the question how square roots are actually extracted.

Parameśvara himself mentions nothing about square root computations. *Ābh* 2.4 (Kern (1874, p. 20)) deals with square roots, but it can only be directly applied to integers in decimal place notation (Keller (2015)). Most of the root extractions in the examples involve numbers with fractions, possibly in sexagesimals. *Brāhmasphuṭasiddhānta* 12.64-65 (Dvivedī (1902, p. 213)) gives a rule that computes the square root of the sum or difference of two squares, where one is the square of an integer and another the square of a number with a sexagesimal fraction (Plofker (2008)). *Śiṣyadhīvṛddhidatantra* 4.52 (Chatterjee (1981, 1, p. 85)) gives a short rule for finding the square root of a number with a sexagesimal fraction. Since Parameśvara knew both texts, he or his followers could have used these methods.

Upon examining the commentaries, we have simply calculated square roots with computers. We found no discrepancy with the approximated results in most cases except one instance in example 2 (page 293, formula 13.1). This could be explained by assuming that the calculator computed the square root up to the second order sexagesimal and then rounded off. But since the square root in question is that of a relatively simple number ($\sqrt{180}$), we do not rule out other types of approximative methods for the square root (Gupta (1985b)).

[1] To "round off" is to round down the fractional part when it is smaller than a half and to round it up when it is a half or larger. For example, 1537;29 is rounded down to 1537 and 1537;31 is rounded up to 1538.

[2] Cases where we have to round down or round up to obtain the value as given in the commentary are rare, and such situations also suggest the possibility that our assumptions on how the computation itself was performed might be wrong.

# B Sine computations

Sine computations appear repeatedly in mathematical astronomy. Therefore, Sanskrit texts on astronomy often include rules or tables for finding the Sine, such as the "Sine production (*jyotpatti*)" chapter in the *Siddhāntaśiromaṇi Golādhyāya* of Bhāskara II (Chaturvedi (1981, pp. 526-528)). Meanwhile, *GD2* makes no reference to Sine computation itself, but yet Sines are relevant in almost every computational method given in the text. Hereafter we shall examine how Parameśvara and the commentator(s) on the examples use Sines and also how they could be computing these Sines.

## B.1 Distinction of an arc and its Sine

Parameśvara does not always distinguish an arc and a Sine. In *GD2*, the term for an arc could also refer to its corresponding Sine. For example, any word for "declination" could mean both the arc $\delta$ or the Sine of declination $\mathrm{Sin}\,\delta$. Usually, we can identify whether it is an arc or a segment from the context. Sometimes a word for "Sine" could be added either to the genitive of a word or in a compound; likewise for an arc, but in *GD2*, the Sine is significantly more often mentioned than the arc. Adding the word "Sine" or "arc" might be for avoiding ambiguity in some cases, but more often than not it could be for metric reasons.

Sometimes, omitting words for "Sine" or "arc" could imply that a small arc is being approximated as a segment. This is prominent in the case of a planet's "deviation (*kṣepa* / *vikṣepa*)" which is always used alone (see chapter 9). The same could be said when the same Sanskrit words are used in the sense of "celestial latitude". However if we look carefully at the rules given by Parameśvara, there are cases such as the visibility equation (see section 10.9) where he strictly distinguishes the arc from its Sine even if they could practically be equal. His rule on the visibility equation for the geographic latitude in *GD2* 175-176 involves a segment called the "declination produced by the celestial latitude" which is the difference between the Sines of the true declination and the declination. The computation would have been much easier (and in fact even more correct) if he had used the arc of the corrected celestial latitude instead, but I think that this cumbersome method reflects Parameśvara's intention to differentiate the arc from its Sine.

## B.2 "Sines" that are not in great circles or not half chords

In general, what Parameśvara calls Sines (*jyā*, *jīva* or *guṇa*) is a half chord corresponding to an arc of a great circle. He even emphasizes in *GD2* 80 that a Sine corresponding to an arc can only be computed when the circle is a great circle (section 6.6) and in *GD2* 108 that the end of a Sine has to be on a line going through the center of the circle (which cuts the chord into halves). Yet, Parameśvara occasionally uses these Sanskrit terms to indicate segments that are not in a great circle or not a half chord. We use "Sine" in quotation marks for segments that are not half chords. The diurnal "Sine" may be interpreted as a type of half chord, but taken into account the peculiarity of this word I put it in quotation marks too. In the following we list the "Sines" that are not in a great circle or not half chords:

### Not in a great circle

**Diurnal "Sine" (*dyudalajīvā*)** The radius (sine of 90°) of a diurnal circle.

**Earth-Sine (*kṣitijyā*)** Segment in a diurnal circle.

**Sine of sight-motion (*dṛggatijyā*)** In Parameśvara's interpretation, this is a "base" Sine in a circle whose radius is the Sine of sight. Some of his predecessors claimed that there is a corresponding arc in the ecliptic (see 21.6.1).

**Given Sine [in the diurnal circle] (*iṣṭajyā* (1))** Distance from the sun to the intersection of the planes of the diurnal circle and the equinoctial colure.

**Given Sine [in the diurnal circle] (*iṣṭajyā* (3))** Distance from the sun to the intersection of the planes of the diurnal circle and the horizon.

#### Not a half chord

There is no case in *GD2* where a segment that is not a half chord but is in a great circle is called a "Sine". There is one case in *GD1* 4.4, which is the "'Sine' of time (*kālajyā*)" (see section 8.4.5).

#### Neither a half chord nor in a great circle

**Given "Sine" [in the diurnal circle] (*iṣṭajyā* (3))** Distance from the sun to the intersection of the planes of the diurnal circle and the six o'clock circle.

The reason for using the word "Sine" is probably different in each case, but it is remarkable that many of the "Sines" are associated with an arc in some way, especially an arc representing the motion of a celestial object in the sky. In this regard, we may compare them with segments that are Sines in a great circle but not named a Sine. One example is the solar amplitude (*arkāgrā*) that is a Sine corresponding to an arc on the horizon, but the sun does not actually move along this arc. Although Parameśvara recognizes the solar amplitude as a Sine and indicates the arc in *GD2* 84 (section 6.7), the segment is never addressed as a Sine. The solar amplitude is treated as a segment that separates the rising-setting line and the east-west line in *GD2* 103 (section 8.1).

## B.3 Parameśvara's Sine computation in *GD2*

Parameśvara does not mention how one should compute Sines from arcs, or arcs from Sines within the rules of *GD2*. However, considering the adherence to the *Āryabhaṭīya* and the *Mahābhāskarīya* which can be seen throughout *GD2* (see introduction 0.2.6), it is likely that he follows these two treatises. From the Sine differences stated in *Ābh* 1.12, we can reconstruct a table of Sines for every $3°45'(225')$ between $0°$ and $90°$ (table B.1). *MBh* 7.16 refers to this verse, implying that the same Sine table should be used[1]. A rule for using this table to find the Sine for a given arc by linear interpolation can be found in *MBh* 4.3-4. Thus, I assume that Parameśvara is using a Sine table reconstructed from *Ābh* 1.12 with linear interpolation.

Parameśvara's usage of 1397 as the value for the Sine of greatest declination $(24°)$[2] supports this assumption. Hayashi (2015, 608, Table 2) has computed the value of $\mathrm{Sin}\,24°$ by using several types of Sine difference table (including those according to Āryabhaṭa, Govindasvāmin,

[1] Right after this statement, *MBh* 7.17-18 introduces a formula which computes Sines for a given arc without tables. We will also take this into account when examining other methods for Sine computation.

[2] The value 1397 appears in *GD2* 73 while he does not refer to the measure of arc $24°$. However $24°$ is a standard value for the greatest declination in texts that Parameśvara has quoted upon, and he also mentions in *GD1* 1.6 that the separation of the equator and the ecliptic is $24°$.

Table B.1: List of Sine differences ($\Delta$ Sine) given in *Ābh* 1.12 and the accumulated values.

| arc length | | | $\Delta$ | Sine |
|---|---|---|---|---|
| $3^\circ\ 45'$ | $=$ | $225'$ | 225 | 225 |
| $7^\circ\ 30'$ | $=$ | $450'$ | 224 | 449 |
| $11^\circ\ 15'$ | $=$ | $675'$ | 222 | 671 |
| $15^\circ$ | $=$ | $900'$ | 219 | 890 |
| $18^\circ\ 45'$ | $=$ | $1125'$ | 215 | 1105 |
| $22^\circ\ 30'$ | $=$ | $1350'$ | 210 | 1315 |
| $26^\circ\ 15'$ | $=$ | $1575'$ | 205 | 1520 |
| $30^\circ$ | $=$ | $1800'$ | 199 | 1719 |
| $33^\circ\ 45'$ | $=$ | $2025'$ | 191 | 1910 |
| $37^\circ\ 30'$ | $=$ | $2250'$ | 183 | 2093 |
| $41^\circ\ 15'$ | $=$ | $2475'$ | 174 | 2267 |
| $45^\circ$ | $=$ | $2700'$ | 164 | 2431 |
| $48^\circ\ 45'$ | $=$ | $2925'$ | 154 | 2585 |
| $52^\circ\ 30'$ | $=$ | $3150'$ | 143 | 2728 |
| $56^\circ\ 15'$ | $=$ | $3375'$ | 131 | 2859 |
| $60^\circ$ | $=$ | $3600'$ | 119 | 2978 |
| $63^\circ\ 45'$ | $=$ | $3825'$ | 106 | 3084 |
| $67^\circ\ 30'$ | $=$ | $4050'$ | 93 | 3177 |
| $71^\circ\ 15'$ | $=$ | $4275'$ | 79 | 3256 |
| $75^\circ$ | $=$ | $4500'$ | 65 | 3321 |
| $78^\circ\ 45'$ | $=$ | $4725'$ | 51 | 3372 |
| $82^\circ\ 30'$ | $=$ | $4950'$ | 37 | 3409 |
| $86^\circ\ 15'$ | $=$ | $5175'$ | 22 | 3431 |
| $90^\circ$ | $=$ | $5400'$ | 7 | 3438 |

Mādhava and Nīlakaṇṭha) in combination with different interpolation methods (linear and second order by Mādhava, Brahmagupta and Govindasvāmin) and furthermore by Bhāskara I's rational approximation formula and a power series expansion without using tables[3]. Only Āryabhaṭa's reconstructed table with linear interpolation produced the value 1397, and every other method resulted in 1398 or higher after rounding.

## B.4 Corrected value for $\mathrm{Sin}\,60^\circ$

The value for $\mathrm{Sin}\,60^\circ$ deserves special mentioning among others.

Bhāskara II, in his *Grahagaṇitādhyāya* 2.3-4 of the *Siddhāntaśiromaṇi*, gives Āryabhaṭa's Sine table with only one modification, which is $\mathrm{Sin}\,60^\circ = 2977$ instead of 2978. This does not affect the value $\mathrm{Sin}\,24^\circ = 1397$ (Hayashi (2015, p. 603)). There is no evidence that Parameśvara used $\mathrm{Sin}\,60^\circ = 2977$, which is closer to the true value, and he says nothing in his commentary on the Sine table in *Āryabhaṭīya* (Kern (1874, p. 17)), nor in his commentary on *Sūryasiddhānta* 2.17-22ab (Shukla (1957, pp. 27-28)) which gives $\mathrm{Sin}\,60^\circ = 2978$.

In the commentary on example 4 (*GD2* 232), the value 2977 is used as an assumed value in an "without-difference" computation (commentary section 16.1, page 324). This implies that Bhāskara II's Sine table might have been used by the commentator. Meanwhile, we have a case in the commentary on example 5 (*GD2* 245. Commentary section 19.1, page 356) where the

[3]There is also reference to Vateśvara, who gives the value of $\mathrm{Sin}\,24$ itself in the *Vateśvarasiddhānta*.

arc of a Sine that is "somewhat less than 2978" is calculated as $2^s\ 1°$, which clearly shows that $\mathrm{Sin}\,60°$ is not 2978. In this case, further examination shows that a Sine value with fractions is behind this value, and not the integer 2977.

A Sine table with $\mathrm{Sin}\,60° = 2977$ appears in some manuscripts of the *Grahaṇamaṇḍana* (K. V. Sarma (1965, pp. 10-11)), but they cannot be part of Parameśvara's original text. In this case too, the author's intention is unclear but at least some of his readers are using the value 2977.

## B.5 Second order interpolation method by Parameśvara

Evidences in the base text of *GD2* suggest that Parameśvara uses linear interpolations, but this does not mean he never used other methods. Gupta (1969, pp. 94-96) points out that Parameśvara refers to two methods of second order interpolations.

The first appears in his commentary on the *Laghubhāskarīya* (B. Āpṭe (1946, p. 16)) and also in his *Siddhāntadīpikā* (T. Kuppanna Sastri (1957, p. 204)). The method can be represented by the following formula, where $\theta_i$ is the $i$th value of an arc in the Sine table, $k$ the interval of arcs in the table, $\Delta_i$ the $i$th Sine difference ($\Delta_i = \mathrm{Sin}\,\theta_i - \mathrm{Sin}\,\theta_{i-1}$) and $\epsilon$ is an elemental arc such that $0 < \epsilon < k$.

$$\mathrm{Sin}(\theta_i + \epsilon) = \mathrm{Sin}\,\theta_i + \frac{\epsilon}{k}\Delta_{i+1} + \frac{\epsilon(k-\epsilon)\cdot\frac{1}{2}(\Delta_i - \Delta_{i+1})}{k^2} \tag{B.1}$$

Parameśvara's statement also covers the versed Sine ($\mathrm{verSin}\,\theta$), in which case the order of $\theta_i$ and $\Delta_i$ in the table are to be reversed:

$$\mathrm{verSin}(\theta_i + \epsilon) = \mathrm{verSin}\,\theta_i + \frac{\epsilon}{k}\Delta_{i+1} - \frac{\epsilon(k-\epsilon)\cdot\frac{1}{2}(\Delta_{i+1} - \Delta_i)}{k^2} \tag{B.2}$$

Gupta compares this method with Govindasvāmin, but I consider that we must compare Parameśvara's method with Brahmagupta and Bhāskara II whose rules are explained in the same article (Gupta (1969, pp. 87-90)). Not only do their methods give the same results (by contrast, Govindasvāmin's method only gives the same values when $30° < \theta < 60°$), but they are also similar in the fact that they cover versed Sines. Parameśvara attributes his method to some others (*kecit*) in the *Siddhāntadīpikā*. This is exactly the same way he cites an opinion in favor of the corrected celestial latitude for computing the true declination in *GD2* 157cd, which we argue could be a reference to Bhāskara II and his followers (section 10.3). Therefore, it is possible that Parameśvara inherited this method from Bhāskara II. Of course, the influence could also be directly from Brahmagupta, as Parameśvara quotes his *Brāhmasphuṭasiddhānta* (see section 4.1).

Parameśvara refers to another second order interpolation method in the *Siddhāntadīpikā* (T. Kuppanna Sastri (1957, pp. 204-205), verses 7-12ab). This is done by computing the "upright Sine[4] resulting from the middle of the residual arc (*cāpakhaṇḍasya madhyotthā yā koṭijyā*)" $\mathrm{Cos}(\theta_i + \frac{\epsilon}{2})$ as an intermediate step. According to the interpretation by Gupta (1969, p. 96)[5], the rule for finding the Sine can be expressed as follows:

$$\begin{aligned} \mathrm{Sin}(\theta_i + \epsilon) - \mathrm{Sin}\,\theta_i &= \frac{\mathrm{Cos}(\theta_i + \frac{\epsilon}{2})\cdot\epsilon}{R} \\ \mathrm{Cos}(\theta_i + \frac{\epsilon}{2}) &= \mathrm{Cos}\,\theta_i - \frac{\mathrm{Sin}\,\theta_i\cdot\epsilon}{2R} \end{aligned} \tag{B.3}$$

[4]For clarity, we shall denote the upright Sine (*koṭi*) with a Cosine ($\mathrm{Cos}\,\theta$).

[5]Note that the letters used in the formulas by Gupta are different from ours.

As Gupta (1969) and Plofker (2001, pp. 285-286) point out, the formulas can be combined in the following form:

$$\mathrm{Sin}(\theta_i + \epsilon) = \mathrm{Sin}\,\theta_i + \mathrm{Cos}\,\theta_i \cdot \frac{\epsilon}{R} - \frac{Sin\theta_i}{2} \cdot \left(\frac{\epsilon}{R}\right)^2 \tag{B.4}$$

which is the equivalent of the Taylor series approximation up to the second order. Plofker (ibid.) further states that this rule is exactly equivalent to Nīlakaṇṭha's interpolation method given in *Tantrasaṅgraha* 2.10-14ab (Ramasubramanian and Sriram (2011, pp. 64-65)) and cited as Mādhava's method in his commentary on *Ābh* 2.12 (Pillai (1957b, p. 55)). However, the expressions are distinctively different. The method begins by preparing a certain value[6] $q_1$ as follows:

$$q_1 = \frac{13751}{2\epsilon} \tag{B.5}$$

It follows that 13751 is an approximation of $4R$. In other words, the rule presupposes that $R \approx 3437;45$.

The Sine is expressed by the following rule:

$$\mathrm{Sin}(\theta_i + \epsilon) = \mathrm{Sin}\,\theta_i + \frac{2}{q_1}\left(\mathrm{Cos}\,\theta_i - \frac{\mathrm{Sin}\,\theta_i}{q_1}\right) = \tag{B.6}$$

This can be transformed to formula B.4 if we use $4R$ in place of 13751, but it is difficult to say whether this was really the source of Parameśvara's method.

Gupta (1974) remarks that Parameśvara even gives a third order interpolation method in the *Siddhāntadīpikā* (T. Kuppanna Sastri (1957, p. 205)). The rule uses a divisor defined by $q_2 = \frac{R}{\epsilon}$. Then the Sine difference is:

$$\mathrm{Sin}(\theta_i + \epsilon) - \mathrm{Sin}\,\theta_i = \frac{\mathrm{Cos}\,\theta_i - \frac{\mathrm{Sin}\,\theta_i + \frac{\mathrm{Cos}\,\theta_i}{2q_2}}{2q_2}}{q_2} \tag{B.7}$$

which can be transformed to:

$$\mathrm{Sin}(\theta_i + \epsilon) = \mathrm{Sin}\,\theta_i + \mathrm{Cos}\,\theta_i \cdot \frac{\epsilon}{R} - \frac{\mathrm{Sin}\,\theta_i}{2} \cdot \left(\frac{\epsilon}{R}\right)^2 - \frac{\mathrm{Cos}\,\theta_i}{4} \cdot \left(\frac{\epsilon}{R}\right)^3 \tag{B.8}$$

As Gupta points out, this is close to the third order approximation in the Taylor series except that the divisor in the third order term must be $3! = 6$ instead of 4[7].

Whether Parameśvara's interpolation method is related to Mādhava or other authors is yet to be discussed. In the next section, we will use Parameśvara's three methods along with other possible interpolation methods to examine two values appearing in the commentaries on *GD2*.

[6] This is called the divisor (*hāraka*) by the commentator Śaṅkara Vāriyar (Ramasubramanian and Sriram (2011, p. 66)).

[7] See Plofker (2001) for a proposed reconstruction of Parameśvara's approximation method that accounts for his error.

## B.6 Sine computations by the commentator(s) in *GD2*

The 6 examples in *GD2* involve computations of Sines from arcs or arcs from Sines, and the commentaries on the examples often note their value. However, the commentators do not tell us how the values were actually computed. Most of these values can be derived from the Sine table of *Ābh* 1.12 and linear interpolation[8]. Meanwhile, there are two cases (examples 5 and 6) where the value cannot be accounted for with this computation: when the values of the Sine and arc are given with fractions, and when the longitude is computed from the declination or vice versa.

### B.6.1 Sine and arc with fractional parts

The commentaries on examples 5 and 6 include the computation of a Sine, whose result is given with a sexagesimal fraction. Āryabhaṭa's Sine table which only uses integers fail to produce the values, and it is most likely that other tables using seconds or even thirds of arcs, along with other interpolation techniques, have been used. Therefore I have computed the Sines in these examples using methods that appear in Hayashi (2015), which are:

- Sine tables

  a. Āryabhaṭa, reconstructed from *Ābh* 1.12. We assume that Parameśvara used this in *GD2*.

  b. Āryabhaṭa with minor corrections as given in Hayashi (1997). This corrects some values in *Ābh* 1.12 that are larger or smaller than the true values rounded. There is no case in *GD2* where these corrections seem to have been applied except for $\text{Sin}\, 60° = 2977$ in the commentaries. We will see this in a separate section below.

  c. Govindasvāmin in his commentary on *MBh* 4.22 (T. Kuppanna Sastri (1957, pp. 200-201)), correcting Āryabhaṭa's table. Parameśvara comments on this table in his *Siddhāntadīpikā* and knew it when he composed *GD2*.

  d. Mādhava's table cited in Nīlakaṇṭha's commentary on *Ābh* 2.12 (Śāstrī (1930, p. 55)) and in Śaṅkara's commentary on *Tantrasaṅgraha* 2.10ab (Ramasubramanian and Sriram (2011, p. 63)). This appears nowhere in Parameśvara's corpus, but commentators of later generations could have known it well.

  e. Nīlakaṇṭha, reconstructed from his first recursion method in *Tantrasaṅgraha* 2.3cd-6ab (Ramasubramanian and Sriram (ibid., p. 56)). This could have been used by commentators after the period of Nīlakaṇṭha.

  f. Nīlakaṇṭha, reconstructed from his second recursion method in *Tantrasaṅgraha* 2.6cd-10ab (Ramasubramanian and Sriram (ibid., pp. 60-61)). Same as above.

  g. Vaṭeśvara, who gives the Sine for every $56'15''$ ($90°$ divided into 96)[9]. There is no trace of Vaṭeśvara's works in Parameśvara, and it is not very likely that commentators on *GD2* could have used this table, but we shall examine its result for comparison.

- Interpolation methods

[8] In such cases I give the reconstructed computation in my explanatory notes without further remarks.

[9] Hayashi (2015) does not use this Sine table itself but the values of $\text{Sin}\, 24°$ and $R^2$ which are given in *Vaṭeśvarasiddhānta* (hereafter *VS*) 2.1.50. The Sine table is given in *VS* 2.1.2-27a, linear interpolation is explained in *VS*2.1.58-62 and second order interpolation in *VS*2.1.63-80. There are 9 different forms given for second order interpolation, all of which can be reduced to the same formula (Shukla (1985, p. 179)) and is ultimately equivalent to Brahmagupta's method (Shukla (ibid., p. 174))

1. Linear interpolation. We assume that this was how Parameśvara made his interpolations in *GD2*.
2. Nīlakaṇṭha's interpolation according to *Tantrasaṅgraha* 2.17-20 (Ramasubramanian and Sriram (2011, p. 74)). There is a good chance that those using tables e or f above would use this method as they appear in the same treatise.
3. Mādhava's second order interpolation cited by Nīlakaṇṭha's commentary on *Ābh* 2.12 (Śāstrī (1930, p. 55)). This also appears in *Tantrasaṅgraha* 2.10cd-13 (Ramasubramanian and Sriram (2011, pp. 64-65)). Table d is more likely to be used with this interpolation.
4. Brahmagupta's second order interpolation according to *Brāhmasphuṭasiddhānta* 25.17 (Dvivedī (1902, p. 418)) = *Khaṇḍakhādyaka* II, 1.4 (Chatterjee (1970, 2. p. 177)). Bhāskara II gives the same method in *Siddhāntaśiromaṇi Grahagaṇita* 2.16 (Chaturvedi (1981, p. 104)). As discussed above, the same method is also cited by Parameśvara.
5. Govindasvāmin's second order interpolation in his commentary on *MBh* 4.22 (T. Kuppanna Sastri (1957, pp. 201-202)). It is unlikely that Parameśvara himself adopted this rule since he remarks that this method is not very accurate and gives his own method instead (see previous section).

- Bhāskara I's approximation formula in *MBh* 7.17-18 (T. Kuppanna Sastri (ibid., p. 378)) which also appears in a number of other texts (see Hayashi (1991)). Parameśvara comments on this method, but it is uncertain how much he used it.

- Power series expansion without using tables mentioned by Śaṅkara in his Yuktidīpikā 440-443 (K. V. Sarma (1977b, p. 118)) and by Jyeṣṭhadeva in his Yuktibhāṣa 7.5.5. (Sriram (2010, pp. 102-103,232-233,426-427)). Technically, Mādhava's table can be computed with this method.

In addition, I have also used the second and third order interpolation methods stated in the *Siddhāntadīpikā* as numbers 6 and 7.

The result for the Sine computation in example 5 (*GD2* 245) is given in page 353 and the case in example 6 (*GD2* 246) in page 363. In both cases, the combination that reproduces the number given in the manuscript is a second order interpolation method equivalent to Brahmagupta's with a Sine table by either Govindasvāmin, Mādhava or Nīlakaṇṭha. However, it is difficult to conclude that they had been actually used, since we cannot evaluate possible errors in the computations. We have only looked at two cases in *GD2*, and further examples are to be studied to understand what the practice was.

### B.6.2 Declination and longitude

The only rule in *GD2* that refers directly to the relation between the declination and the longitude is *GD2* 73ab (formula 6.3), which computes the Sine of declination from a given "base" Sine. Yet, procedures in *GD2*, especially in the 6 examples, involve their arcs. Parameśvara himself seems to suggest that the computations always involve the two Sines and rule *GD2* 73ab; in *GD2* 210 (section 12.1) and *GD2* 216 (section *GD2* 213), he mentions that to find the arc of longitude, one must first compute the "base" Sine from the declination and then convert it to an arc. Meanwhile, commentaries on examples 1-4 do not refer to the "base" Sine nor its value. Moreover, the values of the arcs stated in these commentaries are often different from what would be expected if we used *GD2* 73ab. I have examined example 4 case 1 (see page 16.1) and found that even using different Sine tables cannot account for the discrepancy. The most probable explanation is that a table to find the “base” arc directly from the declination (or vice versa) is being used.

# C Orbits of planets according to the *Āryabhaṭīya*

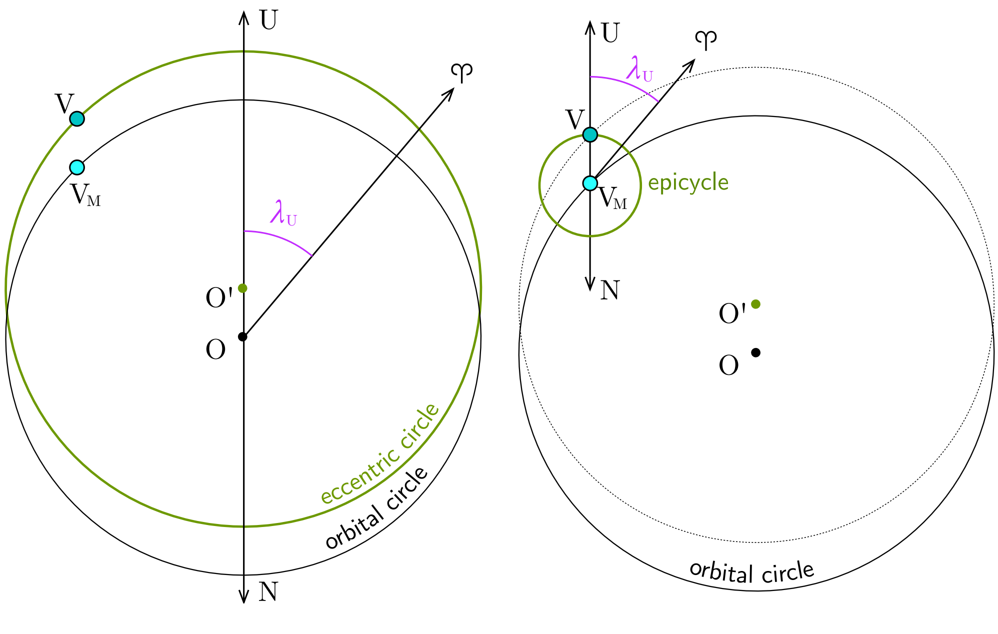
The latitude of planets is a major topic in *GD2*, but the verses cannot be read without prior knowledge of planetary orbits. Parameśvara must have assumed that the reader had already studied other treatises, notably the *Āryabhaṭīya*.

The following is a brief explanation of the planetary theory in *Ābh* 3.17-25[1], based on Parameśvara's commentary.

## C.1 Eccentric circle and epicycle

> All planets revolve on the orbital (*kakṣyā-*) and eccentric (*prati-*) circles (*maṇḍala*) with their own motion (*cāra*). From the "slow" apogee (*mandocca*) it is prograde and retrograde from the "fast" apogee (*śīghrocca*). (*Ābh* 3.17)[2]

> Each of their own eccentric circle is equal to the orbital circle [in size]. The center of the eccentric circle is outside the center of the solid Earth. (*Ābh* 3.18)[3]




(a) Eccentric circle (b) Epicycle

Figure C.1: Two models of the true planet

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[1] Sanskrit text from Kern (1874) with my translation. Words are supplied from Parameśvara's commentary whenever necessary.

[2] *kakṣyāpratimaṇḍalagā bhramanti sarve grahāḥ svacāreṇa |*
*mandoccād anulomaṃ pratilomaṃ caiva śīghroccāt ||3.17||*

[3] *kakṣyāmaṇḍalatulyaṃ svaṃ svaṃ pratimaṇḍalaṃ bhavaty eṣām |*
*pratimaṇḍalasya madhyaṃ ghanabhūmadhyād atikrāntam ||3.18||*

According to *Ābh* 3.17ab-18 (figure C.1(a)), the mean (*madhya*) planet $V_M$ revolves with a constant mean motion[4] on the "orbital circle (*kakṣyāmaṇḍala*)" which has the Earth O as its center. The corrected or true (*sphuṭa*) planet V moves with the same mean motion on an eccentric circle (*pratimaṇḍala*) whose center $O'$ is separated from O at a certain distance, in a direction[5] which is called the apogee (*ucca*) U, separated from the vernal equinox ♈ by an angular distance of $\lambda_U$. The opposite side is the perigee (*nīca*) N. The size of an eccentric circle is equal to the orbital circle, and both are great circles with a circumference of 12 signs, 360 degrees or 21600 minutes.

Alternatively, we can assume that V is revolving in an epicycle (*uccanīcavṛtta*, literally "circle of apogee and perigee") as stated in *Ābh* 3.19 (figure C.1(b)).

> The half-diameter of its own epicycle (*uccanīcavṛtta*) is the gap between the [centers of] the eccentric circle and the Earth. These planets revolve with a mean motion (*madhyamacāra*) on the circumference of the epicycle (*vṛtta*). (*Ābh* 3.19)[6]

The radius of the epicycle is equal to the distance $OO'$. Longitudes can be conceived in the epicycle as it is done on an orbital circle, with $V_M$ being in the center instead of O. V is in the direction of the apogee separated from ♈ by $\lambda_U$. The circumferences of epicycles are given in *Ābh* 1.10-11 in a very peculiar manner. First, Āryabhaṭa supposes that epicycles change their size depending on the anomaly (the distance in longitude between the mean planet and the apogee), and gives two values for each epicycle; one is the circumference when the anomaly is at the end of the first or third quadrant, and the other is when it is at the end of the second or fourth. Values in between are linearly interpolated[7]. The second peculiarity is that each value given in *Ābh* 1.10-11 must be multiplied by "half of nine" (= 4;30) — likely a means to keep the expression short. Last of all, the value thus computed is the circumference of the epicycle when the circumference of the orbital circle is 360°. For example, the given values of the "slow" epicycle (corresponding to the "slow" apogee as explained in the next section) of Jupiter is seven at the end of the first and third quadrant, and eight at the end of the second and fourth. The actual circumferences are those multiplied by 4;30, i.e. 31;30 and 36 respectively, and for example, if the anomaly were 45°, the circumference would be their average 33;45.

Parameśvara seems to interpret that *Ābh* 3.25cd also refers to the equivalence of an eccentric circle and an epicycle.

> The speed of the planet on the "slow" epicycle is that on the orbital [circle]. (*Ābh* 3.25cd)[8]

Clark (1930) remarks: "The second half of the stanza [=*Ābh* 3.25] is uncertain. This same statement was made in unmistakable terms in III, 19. ... [Parameśvara] explains that the meaning may be that the radius of the epicycle is equal to the greatest distance by which the mean orbit lies inside or outside of the eccentric circle".

---

[4] Parameśvara paraphrases motion (*cāra*) with mean motion (*madhyamagati*).

[5] Neither Āryabhaṭa nor Parameśvara declares whether the apogee is a direction or a point in the orbit, but as it is always measured in degrees, it should be better to treat it as a direction in our explanation.

[6] *pratimaṇḍalabhūvivaraṃ vyāsārdham svoccanīcavṛttasya |*
*vṛttaparidhau grahās te madhyamacāraṃ bhramanty eva ||3.19||*

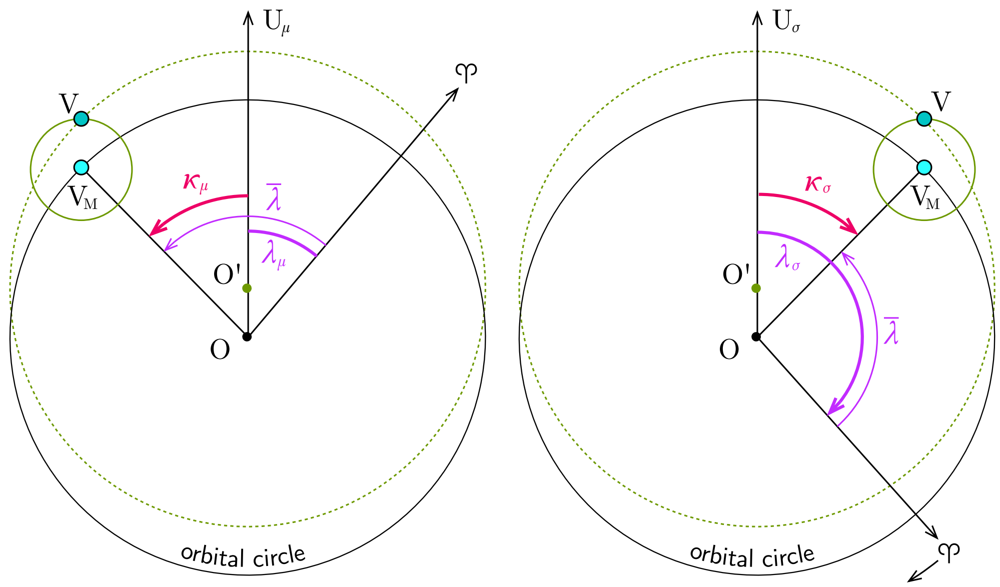
[7] Āryabhaṭa himself only states the values without clear instructions, and here I follow Parameśvara's commentary.

[8] *kakṣyāyāṃ grahavego yo bhavati sa mandanīcocce ||3.25||*

## C.2 “Slow” and “fast” apogees

The inequality of the planet’s motion is decomposed into two individual elements caused by two apogees: the “slow” apogee (*mandocca*) and “fast” apogee (*śīghrocca*). The moon’s “slow” apogee revolves at a slow rate compared to the moon itself, and the “slow” apogees for other planets including the sun are regarded as fixed. Meanwhile, the “fast” apogee is always faster than the mean motion of a planet.

The goal in this procedure is to combine the inequalities caused by the “slow” and “fast” apogees on the mean planet to find the longitude of the true planet.




(a) “Slow” (*mandocca*)    (b) “Fast” (*śīghrocca*)

Figure C.2: The two types of apogees

*Ābh* 3.17cd contrasts the difference between these two apogees with respect to the motion of the mean planet against the apogee (figure C.2). *Ābh* 3.21 states the same thing, but the center of the epicycle is mentioned in place of the mean planet.

> Epicycles have a prograde motion from the “slow” [apogee] and have a retrograde motion from the “fast” [apogee]. The mean planet in the middle of its own epicycle is adhering to the orbital circle. (*Ābh* 3.21)[9]

The “slow” apogee $U_\mu$ of the moon revolves very slowly compared to the mean position, and with the other planets it is almost fixed (figure C.2(a))[10]. The motion of the mean planet is prograde, and its longitude $\bar{\lambda}$ increases constantly. Meanwhile the longitude of the “slow”

[9] *anulomagāni mandāc chīghrāt pratilomagāni vṛttāni |*
*kakṣyāmaṇḍalalagnaḥ svavṛttamadhye graho madhyaḥ ||3.21||*

[10] We will see in section C.4 that the position of the planet V in this diagram is slightly modified for the correction due to the “slow” apogee.

apogee $\lambda_\mu$ does not change, therefore the mean planet can be considered as having a prograde motion against the "slow" apogee, and their angular distance increases constantly. This angular distance is usually referred to as the "slow" anomaly (*kendra*) $\kappa_\mu$, which is treated as a "base" arc; here, the starting point of the "base" arc is not the two equinoctial points (cf. commentary section 7.1) but the "slow" apogee and perigee (*nīca*, opposite side of the apogee).

The "fast" apogee $\mathrm{U}_\sigma$ moves prograde against ♈. Therefore, if we draw a diagram with $\mathrm{U}_\sigma$ fixed (figure C.2(b)), ♈ moves retrograde. The mean motion is slower than the motion of the "fast" apogee, and although the mean longitude $\bar{\lambda}$ keeps increasing, the separation $\lambda_\sigma$ of ♈ from $\mathrm{U}_\sigma$ increases faster in the opposite direction. As a result, the angular distance decreases, and therefore the motion of the mean planet is retrograde against the "fast" apogee. The "fast" anomaly $\kappa_\sigma$ is computed as a "base" arc of the planet's longitude with the "fast" apogee and perigee as the reference.

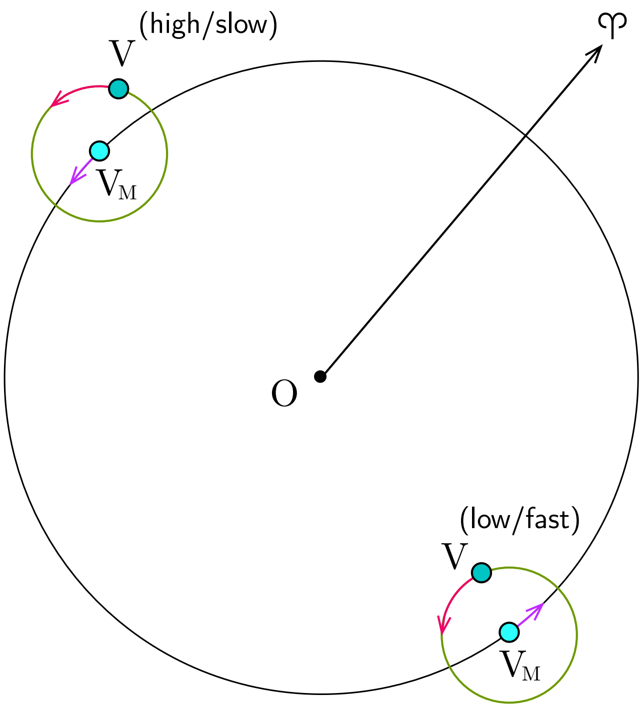



Figure C.3: Retrograde motion caused by the "fast" apogee

The "fast" apogee accounts for retrograde motion, and Parameśvara seems to interpret *Ābh* 3.20 as an explanation for this phenomenon[11].

> [A planet] that has a fast motion due to its own apogee has a retrograde motion on its own orbit [called] the epicycle. A planet that has a slow motion revolves [with] a prograde

[11] The wording of *Ābh* 3.20 is very ambiguous and interpretations differ among commentators. For example, Parameśvara's grand-student Nīlakaṇṭha comments that this verse tells how the planets in the "slow" and "fast" apogees rotate differently (Śāstrī (1931, p. 38))

motion on the epicycle. (*Ābh* 3.20)[12]

An apogee can cause the true planet to be lower or higher (closer to or further from the Earth) than the mean planet. Since the actual speed on the eccentric circle is consistent, its apparent speed is faster while the planet is low and slower while it is high. Parameśvara further explains that the direction of a true planet's motion on the epicycle is prograde (in the same direction with the mean planet) when it is higher than the orbital circle, and retrograde when it is lower (figure C.3). Parameśvara does not mention whether he is talking about the "slow" apogee or "fast" apogee, but it could only be about the "fast" apogee, since the motion of the true planet on the "slow" apogee would be in the opposite direction of the mean planet when it is higher than the orbital circle and in the same direction when it is lower.

Retrograde motion is not a significant topic in *GD2*, and it only appears in *GD2* 21.

## C.3 Two categories of planets

The "fast" apogees of Mars, Jupiter and Saturn, and the mean positions of Mercury and Venus are always in the same direction with the mean position of the sun. From the viewpoint of modern astronomy, this can be explained by the heliocentric motion of planets where the superior planets Mars, Jupiter and Saturn revolve outside the orbit of Earth and Mercury and Venus are inferior planets revolving closer to the sun than the Earth.

In the tradition followed by Parameśvara, the notion of "superior" and "inferior" itself does not exist and nor do the two groups of planets have a specific name. However, computations are often different between the two categories, and in such cases they will be distinguished by saying "Mars, Jupiter or Saturn" (or "those beginning with Mars"[13]) and "Mercury and Venus".

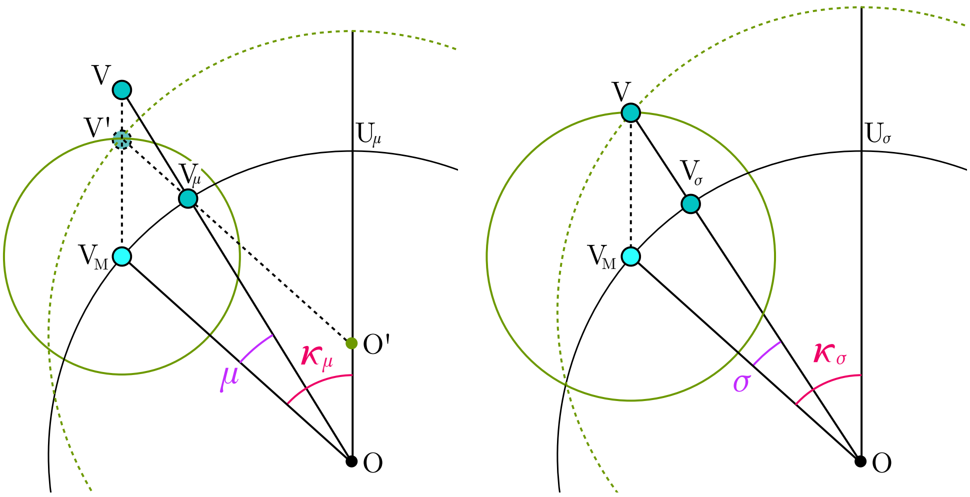
## C.4 True planet and equation

Commentators of the *Āryabhaṭīya* refer to the difference in longitude between the mean planet and the true planet as *phala*, literally "result". I adopt the English translation "equation". Āryabhaṭa explains how the equations occurring from the "slow" and "fast" apogees are combined, but without using a specific term for it. We also have to rely on commentators for how equations themselves are derived.

The "slow" equation (*mandaphala*) $\mu$ is derived in a peculiar way (figure C.4(a)). Despite the fact that Parameśvara describes that a true planet is on the eccentric circle or epicycle (position $V'$ in the diagram), his actual computation for the equation and for the distance to the true planet implies that it is slightly drawn towards the direction of the "slow" apogee (position V). Parameśvara does not draw a diagram for this explanation, but we can reproduce it by first drawing a line towards $V'$ from the center of the eccentric circle $O'$, and marking the intersection of $O'V'$ with the eccentric circle as the longitude of the "slow" corrected (*mandasphuṭa*) planet $V_\mu$ in the zodiac. The actual location of the planet V is the intersection of the extended line of sight $OV_\mu$ with line $V_MV'$. Ôhashi (2009, p. 32) mentions that the accuracy of this method is slightly less than a simple eccentric model. As we will see in equation C.1, the Sine of equation $\mathrm{Sin}(\mu)$

[12] *yaḥ śīghragatiḥ svoccāt pratilomagatiḥ svavṛttakakṣyāyām /*
*anulomagatir vṛtte mandagatir yo graho bhramati //3.20//* (*bhramati* reads *bhavati* in the critical edition by K. V. Sarma and Shukla (1976))

[13] Parameśvara also uses the same expression to refer to the planets in weekday order enumerated from Mars, i.e. the five planets excluding the sun and moon. This applies to *GD2* 128. The distinction between the two meanings are obvious from the context in general.




(a) "Slow" equation $\mu$

(b) "Fast" equation $\sigma$

Figure C.4: Equations resulting from the apogees

will be a simple function of the Sine of anomaly $\mathrm{Sin}\,(\kappa_\mu)_B$ in this method. Ôhashi speculates that an idea of a "kind of physical force" originating from the apogee was behind this model, but claims that further investigation is required.

The depiction of the "fast" equation (*śīghraphala*) is simple (figure C.4(b)); the actual planet V is on the eccentric circle or epicycle, and the "fast" corrected (*śīghrasphuṭa*) position of the planet on the zodiac $V_\sigma$ is the intersection of OV and the orbital circle. $\widehat{V_M V_\sigma}$ is the "fast" equation $\sigma$.

Parameśvara explains the procedure for computing the two equations in his commentary after *Ābh* 3.24 as follows:

> Now, a way of computing the equation. Having multiplied the "base" Sine of the "slow" anomaly (*mandakendra*) by the corrected "slow" epicycle (*mandasphuṭavṛtta*), having divided by eighty, the arc corresponding to the quotient which is the "slow" equation is produced. Likewise, having multiplied the "base" Sine of the "fast" anomaly (*śīghrakendra*) by the corrected "fast" epicycle (*mandaśīghravṛtta*), having divided by eighty, having multiplied the quotient by the Radius, having divided it by the "fast" radial distance (*śīghrakarṇa*), the arc corresponding to the quotient which is the "fast" equation is produced[14].

Here, by the expression *vṛtta* for epicycle, Parameśvara is referring to its circumference, and more precisely, its value without the coefficient $\frac{9}{2}$ as given in *Ābh* 1.10-11.

In the case of the "slow" equation $\mu$, Parameśvara's explanation can be represented as follows, where $c_\mu$ is the circumference of the "slow" epicycle without coefficient and $\mathrm{Sin}\,(\kappa_\mu)_B$ the "base" Sine of the "slow" anomaly:

[14] *phalānayanaprakāras tu | mandakendrabhujājyāṃ mandasphuṭavṛttena nihatyāśītyā vibhajya labdhasya cāpaṃ mandaphalaṃ bhavati | tathā śīghrakendrabhujajyāṃ śīghrasphuṭavṛttena nihatyāśītyā vibhajya labdhaṃ vyāsārdhena nihatya śīghrakarṇena vibhajya labdhasya cāpaṃ śīghraphalaṃ bhavati* // (Kern (1874, p. 67))

$$\operatorname{Sin}\mu = \frac{c_\mu \cdot \operatorname{Sin}(\kappa_\mu)_B}{80} \tag{C.1}$$

The "fast" equation $\sigma$ is obtained from a similar rule that involves the circumference $c_\sigma$ of the "slow" epicycle without coefficient and the "base" Sine of the "fast" anomaly $\operatorname{Sin}(\kappa_\sigma)_B$. The difference is that there is a divisor $\mathcal{R}_\sigma$ called the "fast" radial distance (*śīghrakarṇa*).

$$\operatorname{Sin}\sigma = \frac{c_\sigma \operatorname{Sin}(\kappa_\sigma)_B}{80\mathcal{R}_\sigma} \tag{C.2}$$

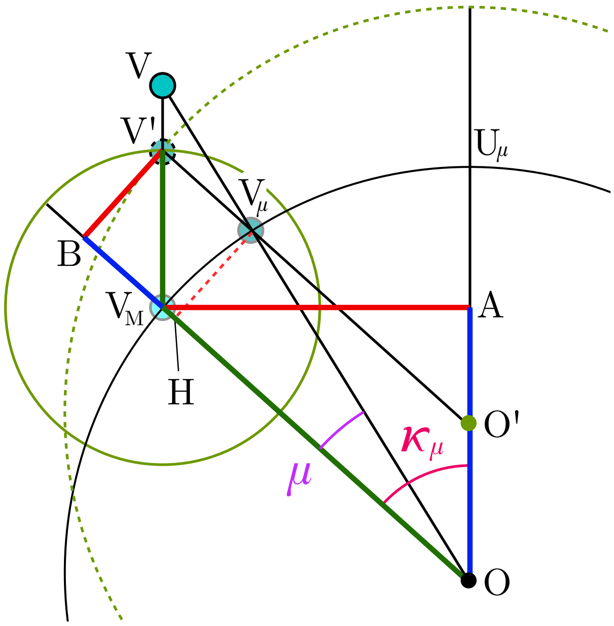



Figure C.5: Computing the "slow" equation

Figure C.5 illustrates how this computation can be derived. A perpendicular is drawn from the mean planet $\mathrm{V_M}$ to $\mathrm{OU}_\mu$ where $\mathrm{U}_\mu$ is the direction of the "slow" apogee. Let A be its foot. Likewise another perpendicular is drawn from $\mathrm{V}'$ to $\mathrm{OV_M}$ (extended) and B is its foot. Since $\mathrm{VV_M} \parallel \mathrm{U}_\mu\mathrm{O}$ while $\mathrm{BV_M}$ and $\mathrm{OV_M}$ are in one line, corresponding angles $\angle\mathrm{BV_MV}'$ and $\angle\mathrm{AOV_M}$ are equal. Furthermore, $\angle\mathrm{V'BV_M} = \angle\mathrm{V_MAO} = 90°$, therefore $\triangle\mathrm{V'BV_M} \sim \triangle\mathrm{V_MAO}$. Thus

$$\mathrm{V'B} : \mathrm{V_MA} = \mathrm{V_MV'} : \mathrm{OV_M} \tag{C.3}$$

Here, the two hypotenuses $\mathrm{V_MV'}$ and $\mathrm{OV_M}$ are also the radius of the epicycle and orbital circle, respectively. The proportion of the two radii are equal to their circumferences, which are $\frac{9}{2}c_\mu$ and 360 respectively.

$$\mathrm{V_M V'} : \mathrm{OV_M} = \frac{9}{2} c_\mu : 360 \tag{C.4}$$

From formulas C.3 and C.4,

$$\begin{aligned}
\mathrm{V'B} : \mathrm{V_M A} &= \frac{9}{2} c_\mu : 360 \\
\mathrm{V'B} &= \frac{\frac{9}{2} c_\mu \cdot \mathrm{V_M A}}{360} \\
&= \frac{c_\mu \cdot \mathrm{V_M A}}{80}
\end{aligned} \tag{C.5}$$

Since the Sine of "slow" equation $\mathrm{V}_\mu\mathrm{H} = \mathrm{Sin}\,\mu$ is equal to $\mathrm{V'B}$ and $\mathrm{V_M A}$ is the "base" Sine of "slow" anomaly, we obtain formula C.1.

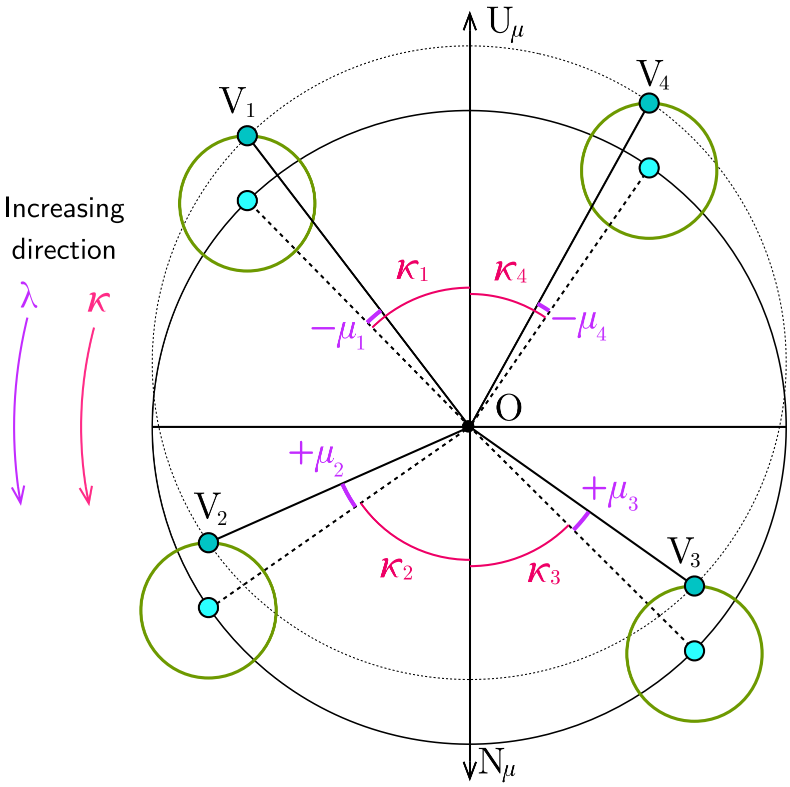



Figure C.6: Equations in the four quadrants starting from the "slow" apogee $\mathrm{U}_\mu$. $\mathrm{N}_\mu$ is the "slow" perigee. Here, the change in anomaly $\kappa$ occurs in the same direction with the longitude $\lambda$.

The Sine of the equation is reduced to an arc, and then added to or subtracted from the longitude of the mean planet depending on the quadrant (with reference to the "slow" apogee) of the mean planet. This is stated in *Ābh* 3.22ab:

> [The equation] from the "slow" apogee should be subtractive, additive, additive and subtractive [in the four quadrants respectively], and the opposite from the "fast" apogee. (*Ābh* 3.22ab)[15]

As shown in figure C.6, the corrected planet is behind the mean planet in the first quadrant ($V_1$) and the second quadrant ($V_2$), and ahead in the third ($V_3$) and fourth ($V_4$). Meanwhile, the anomaly $\kappa$ is a "base" arc, measured from the apogee $U_\mu$ in the first and fourth quadrant and from the perigee $N_\mu$ in the second and third. As a result, the equation $\mu$ is subtractive in the first and fourth quadrant and additive in the second and third.

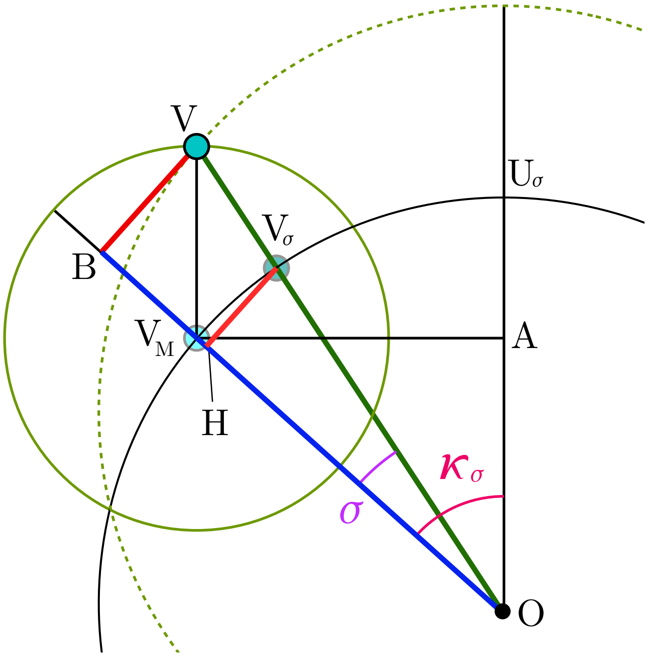



Figure C.7: Computing the "fast" equation

The first steps for the "fast" equation is equivalent to the previous procedure. This time the actual planet V is in place of V′, and when $\kappa_\sigma$ is the "fast" anomaly and $\frac{9}{2}c_\sigma$ the circumference of the epicycle,

$$
\begin{aligned}
\mathrm{VB} : \mathrm{V_M A} = \mathrm{V_M V'} : \mathrm{OV_M} &= \frac{9}{2}c_\sigma : 360 \\
\mathrm{VB} &= \frac{\frac{9}{2}c_\sigma \cdot \mathrm{V_M A}}{360} \\
&= \frac{c_\sigma \cdot \mathrm{V_M A}}{80} \qquad \text{(C.6)}
\end{aligned}
$$

[15] *ṛṇadhanadhanakṣayāḥ syur mandoccād vyatyayena śīghroccāt* / (*ṛṇadhana* reads *kṣayadhana* in K. V. Sarma and Shukla (1976))

Unlike the case with the "slow" equation, the Sine of "fast" equation $\mathrm{V}_\sigma\mathrm{H} = \mathrm{Sin}\,\sigma$ is slightly smaller than VB (figure C.7). $\angle\mathrm{VBO} = \angle\mathrm{V}_\sigma\mathrm{HO} = 90°$, therefore $\triangle\mathrm{VBO} \sim \triangle\mathrm{V}_\sigma\mathrm{HO}$, and

$$\begin{aligned}
\mathrm{V}_\sigma\mathrm{H} &= \frac{\mathrm{VB}\cdot\mathrm{OV}_\sigma}{\mathrm{OV}} \\
\mathrm{Sin}\,\sigma &= \frac{\mathrm{VB}\cdot R}{\mathcal{R}_\sigma} \\
\mathrm{Sin}\,\sigma &= \frac{c_\sigma R\,\mathrm{Sin}\,(\kappa_\sigma)_B}{80\mathcal{R}_\sigma}
\end{aligned} \tag{C.7}$$

Hence we obtain formula C.2. The "fast" radial distance $\mathrm{OV} = \mathcal{R}_\sigma$ is yet to be computed. The length of $\mathrm{BV_M}$ is computed from $\mathrm{V_MV}$ and VB with the Pythagorean theorem, which is added to $\mathrm{OV_M} = R$ to obtain OB, and again with the Pythagorean theorem, OV is obtained from VB and OB. To summarize,

$$\mathrm{OV} = \sqrt{\mathrm{VB}^2 + \left(\mathrm{OV_M} + \sqrt{\mathrm{V_MV}^2 - \mathrm{VB}^2}\right)^2} \tag{C.8}$$

As a result, the relation between the "fast" equation and the "fast" anomaly is not as simple as the "slow" ones.

Planets move retrograde from the "fast" apogee, and therefore its increase or decrease in anomaly occurs in the opposite direction in comparison with the case of the "slow" apogee. Therefore the four quadrants are placed in reverse order (figure C.8). The "fast" equation is subtractive against the "fast" anomaly in the first and fourth quadrant and additive in the second and third, as it was with the "slow" equation. However, since the "fast" anomaly itself is a subtractive value against the longitude as it changes in the opposite direction, the computation is reversed when they are applied to $\bar{\lambda}$. Thus, whether the "fast" equation is additive of subtractive depending on the quadrant is opposite from the case of the "slow" equation, as stated in *Ābh* 3.22ab.

### C.4.1 The "slow" radial distance

Following his instructions on the equations, Parameśvara also explains (Kern (1874, pp. 67-68)) how to compute radial distances (*karṇa*), which are the distances of the "slow" or "fast" corrected planet from the Earth. We have already seen that the "fast" radial distance $\mathcal{R}_\sigma$ can be easily computed. Meanwhile, the "slow" radial distance $\mathcal{R}_\mu$ (OV in figure C.9) cannot be found straightforwardly. He uses what can be interpreted as an iterative method, or to use his vocabulary, computations repeated until there is no difference (*aviśeṣa*). I shall summarize his method, adding my geometrical interpretations[16].

The initial guess is that the true planet is $\mathrm{V}_1$ on the "slow" epicycle. The radial distance $\mathcal{R}_{\mu(1)}$ for this guess can be computed by the same method for the "fast" radial distance (formula C.8). However, the true planet should be on the line of sight $\mathrm{OV}_\mu$. Thus we locate point $\mathrm{M}_1$ on the extension of $\mathrm{OV}_\mu$ so that $\mathrm{OM}_1 = \mathrm{OV}_1$. Next, we draw lines from $\mathrm{V}_\mu$ and $\mathrm{M}_1$ which are parallel with $\mathrm{V}_1\mathrm{V_M}$. Let their intersections with $\mathrm{OV_M}$ be S and $\mathrm{S}_1$. $\triangle\mathrm{OV}_\mu\mathrm{S}$ and $\triangle\mathrm{OM}_1\mathrm{S}_1$ share one angle and have corresponding angles and are therefore similar. Thus

[16] My description follows the explanation by Shukla (1960, pp. 122-125) on what he calls the eccentric theory as interpreted from *MBh* 4.19-20.

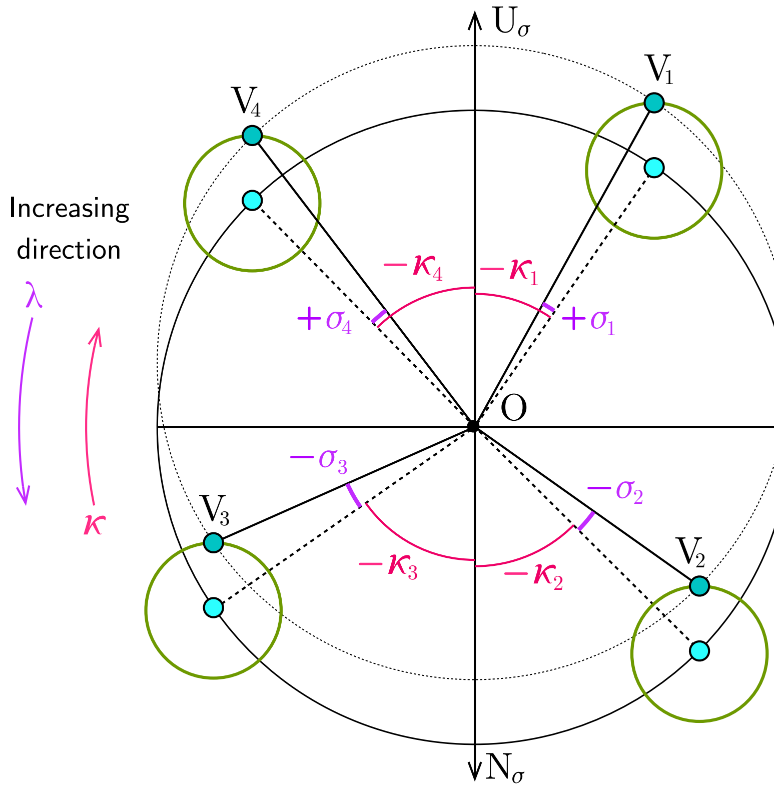



Figure C.8: Equations in the four quadrants starting from the "fast" apogee $U_\sigma$. $N_\sigma$ is the "fast" perigee. While the longitude $\lambda$ is measured anticlockwise in this diagram, the change in anomaly $\kappa$ occurs clockwise.

$$\begin{aligned}
M_1S_1 &= \frac{V_\mu S \cdot OM_1}{OV_\mu} \\
&= \frac{V_1V_M \cdot OV_1}{OV_\mu} \\
&= \frac{\frac{9}{2}c_\mu \mathcal{R}_{\mu(1)}}{360} \\
&= \frac{c_\mu \mathcal{R}_{\mu(1)}}{80}
\end{aligned} \tag{C.9}$$

Here I used the ratio $V_1V_M : OV_\mu = \frac{9}{2}c_\mu : 360$ since they are the radii of the epicycle and great circle. We next find $V_2$ on the same line with $V_1V_M$ so that $V_2V_M = M_1S_1$, find the corresponding radial distance $OV_2 = \mathcal{R}_{\mu(2)}$, and continue the process until there is no difference in the values between two successive steps. Parameśvara calls the result $\mathcal{R}_\mu$ the "radial distance without difference (*aviśeṣakarṇa*)".

The sun and the moon only have the "slow" apogee, and thus their true radial distance is the "slow" radial distance. Parameśvara also refers to them as the "radial distance without difference" in *GD2* 278.

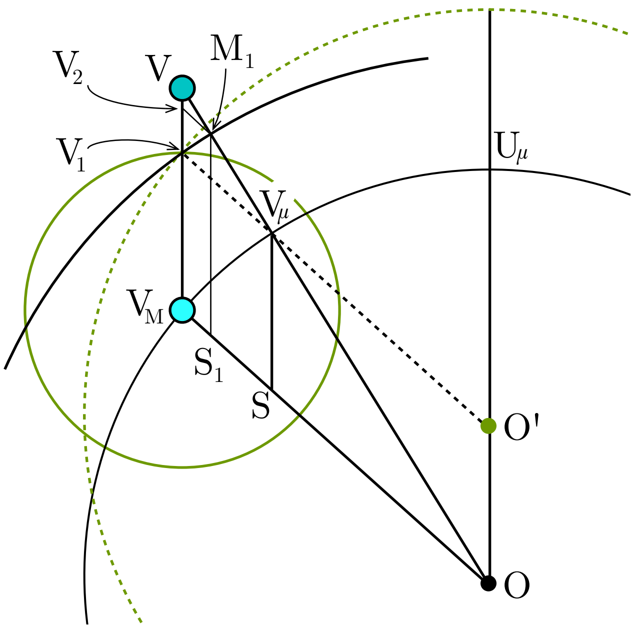



Figure C.9: Finding the "slow" radial distance OV starting from $OV_1$ as an initial guess.

## C.5 Combining the two equations

Both "slow" and "fast" equations take the longitude of the mean planet as their input and their output is the longitude of the corrected planet. This is equivalent to assuming that they both stay on the orbital circle without changing their distance from the Earth's center. This causes an error when we combine the two equations. *GD2* 145-148 refers to this error, by showing the positions of the "observed true planet (*sākṣātsphuṭakhecara*)" and the twice-corrected planet in his diagram of three circles. Let us first see how this can be explained in the configuration of the *Āryabhaṭīya* and then examine how Parameśvara's diagram displays the same error.

### C.5.1 Error explained in the configuration of epicycles

Figure C.10 illustrates how the "slow" and "fast" epicycles can be combined together on the orbital circle in Āryabhaṭa's configuration. $V'$ shows the position of the "slow" corrected planet when $V_M$ is the mean planet[17]. $V_\mu$ represents the "slow" corrected longitude on the orbital circle, and therefore $\mu = \overset{\frown}{V_M V_\mu}$ is the "slow" equation. Applying the "fast" correction to the "slow" corrected longitude corresponds to drawing the "fast" epicycle around $V_\mu$, locating the planet F in the direction of the "fast" apogee, and then finding the intersection of OF with the orbital circle B. $\sigma_{V_\mu} = \overset{\frown}{V_\mu B}$ is the "fast" equation. However, the actual position of the planet V should be on a circle centered at $V'$ with the same radius as the "fast" epicycle, in the direction of the "fast" apogee as seen from $V'$. Let us temporarily call this circle the "actual" epicycle

[17]Here, we have approximated that the "slow" corrected planet $V'$ is on its epicycle. This is because Parameśvara's configuration involves the same simplification, as we will see later. The error by combining the two epicycles exists nonetheless, and this approximation alters none of our conclusions.

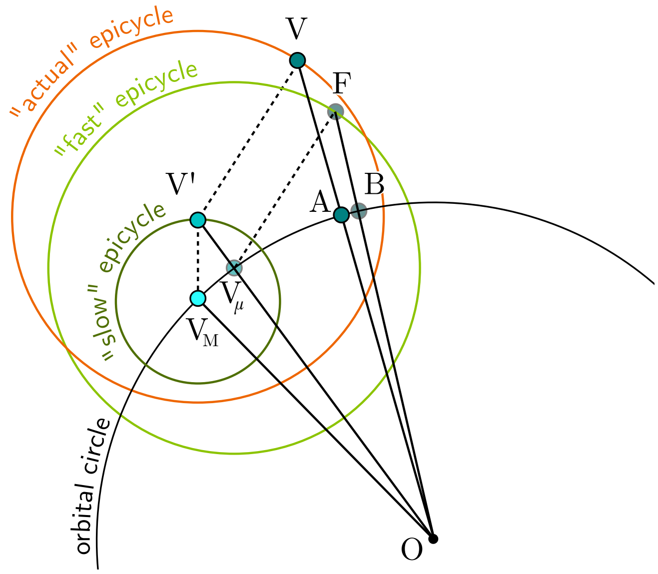



Figure C.10: The observed position of a planet V (projected at A in the orbital circle) and its false position F (projected at B) obtained by simply adding the two corrections.

since it represents the path of the actual planet. The observed longitude on the orbital circle should be A. Thus we have an error in longitude $\widehat{AB}$.

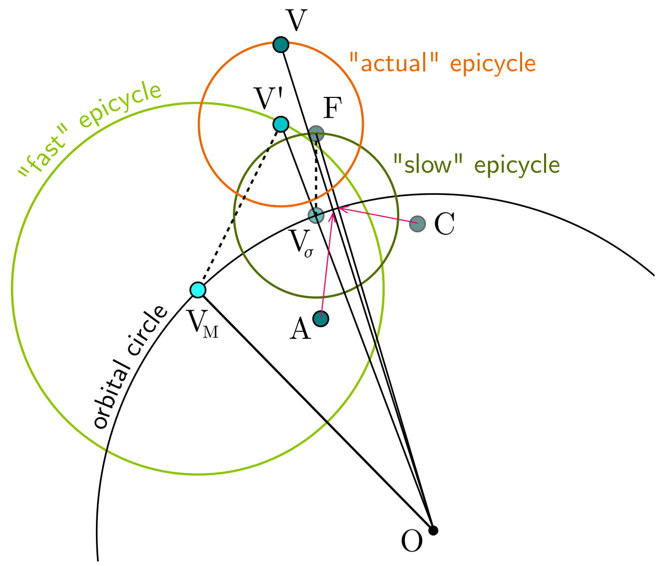



Figure C.11: Combining the two epicycles by applying the "fast" epicycle first.

Likewise, an error occurs even if the "fast" correction $\sigma = \widehat{V_M V_\sigma}$ is applied first (figure C.11).

The amount of the error in longitude $\widehat{\mathrm{AC}}$ itself is different from the previous case[18].

### C.5.2 Parameśvara's configuration for Mars, Jupiter and Saturn

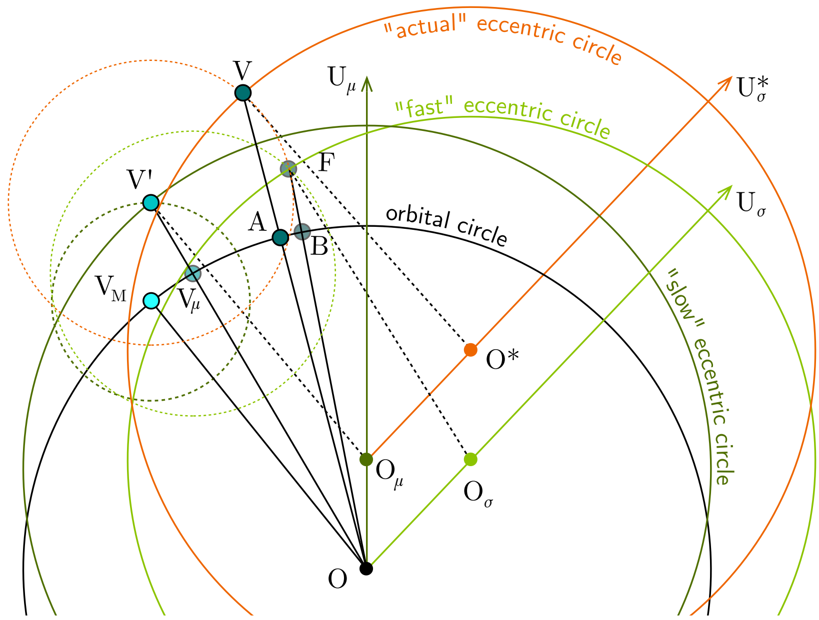



Figure C.12: Configuration in figure C.10 replaced with eccentric circles.

Since epicycles and eccentric circles are equivalent, a configuration which causes the same error can be demonstrated with two eccentric circles (figure C.12). The center of the "slow" eccentric circle $\mathrm{O}_\mu$ is in the direction of the "slow" apogee $\mathrm{U}_\mu$ from the center of the Earth O at a distance equivalent to the radius of the "slow" epicycle. On the other hand, the "fast" eccentric circle is in the direction of the "fast" apogee $\mathrm{U}_\sigma$ at a distance of its epicycle's radius from O. If we measure this distance from $\mathrm{O}_\mu$ instead, we obtain the center of the "actual" eccentric circle O* corresponding to the "actual" epicycle around the "slow" corrected planet V′ (The direction of the "fast" apogee on the "slow" and "fast" eccentric circles is denoted $\mathrm{U}^*_\sigma$ to avoid confusion).

In this configuration, the correction can be described as follows: we locate V′ on the "slow" eccentric circle such that its anomaly $\widehat{\mathrm{U}_\mu\mathrm{V}'}$ is equal to the "slow" anomaly of the mean planet $\widehat{\mathrm{U}_\mu\mathrm{V}_\mathrm{M}}$ on the orbital circle. The intersection of OV′ with the orbital circle $\mathrm{V}_\mu$ is the "slow" corrected longitude. Then we find F on the "fast" eccentric circle whose anomaly $\widehat{\mathrm{U}_\sigma\mathrm{F}}$ is equal to the "fast" anomaly of the "slow" corrected longitude $\widehat{\mathrm{U}^*_\sigma\mathrm{V}_\mu}$. The intersection of OF with the orbital circle is the twice-corrected longitude B. On the other hand, the actual position of the planet V is on the "actual" eccentric circle, separated from $\mathrm{U}^*_\sigma$ with the same "fast" anomaly.

It is to be noted that the position of V is equivalent to the mean position of $\mathrm{V}_\mathrm{M}$ on the orbital circle. $\mathrm{OV}_\mathrm{M} \parallel \mathrm{O}_\mu\mathrm{V}'$ because $\angle \mathrm{V}_\mathrm{M}\mathrm{OU}_\mu = \angle \mathrm{V}'\mathrm{O}_\mu\mathrm{U}_\mu$ and $\mathrm{O}_\mu\mathrm{V}' \parallel \mathrm{O}^*\mathrm{V}$ because $\angle \mathrm{V}'\mathrm{O}_\mu\mathrm{U}^*_\sigma = \angle \mathrm{VO}^*\mathrm{U}^*_\sigma$. Therefore the longitude of V on the "actual" eccentric circle is always equal to the mean longitude.

[18] In our diagram, the error occurring when the "fast" correction is applied first looks smaller than when the "slow" correction is first, but this depends on the sizes of the epicycles and directions of the apogees.

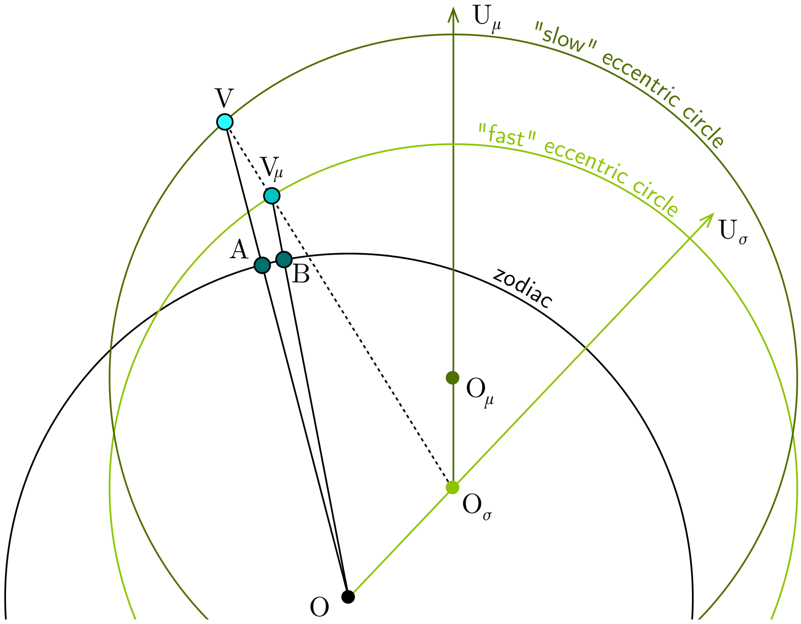



Figure C.13: Parameśvara's configuration for Mars, Jupiter and Saturn.

Parameśvara's configuration with three great circles for Mars, Jupiter and Saturn as described in *GD2* 135-146 can be reproduced by sliding the "slow" eccentric circle to the position of what we have been calling the "actual" eccentric circle (figure C.13). The center of the "slow" eccentric circle $O_\mu$ replaces O*; it is in the direction of the "slow" apogee from $O_\sigma$ separated by the distance of the "slow" epicycle's radius. The "slow" corrected planet $V_\mu$ replaces what was F. We have already discussed that V moves with a mean motion. This is also stated in *GD2* 139. The first circle is no more the place where the mean motion takes place, but only the circle on which the longitude of the planet as seen from the Earth is projected; Parameśvara calls it the "zodiac (*bhacakra*)" instead of "orbital circle". The two corrections can be represented in the same manner as stated in commentary section 9.8. Here again, B represents the twice-corrected longitude of the planet, and A the actual longitude as seen from the Earth. The same error $\overparen{AB}$ that occurred in Āryabhaṭa's configuration can be represented here.

### C.5.3 Parameśvara's configuration for Mercury and Venus

By keeping the "slow" eccentric circle in our initial model and replacing the "actual" epicycle with the "fast" epicycle, we can reproduce Parameśvara's configuration for Mercury and Venus (figure C.14). This time, the correction according to *GD2* 141-144 corresponds to applying the "fast" equation and then the "slow" equation. As we have seen previously, the resulting twice-corrected position C is different from B when the "slow" equation is applied first, but we still have a difference from the actual longitude A on the zodiac.

### C.5.4 Reducing the error

*GD2* 145-148 mostly deals with this error itself and not with the methods for reducing it, but some explanation is required for *GD2* 147ab which mentions a correction by "half the Sine

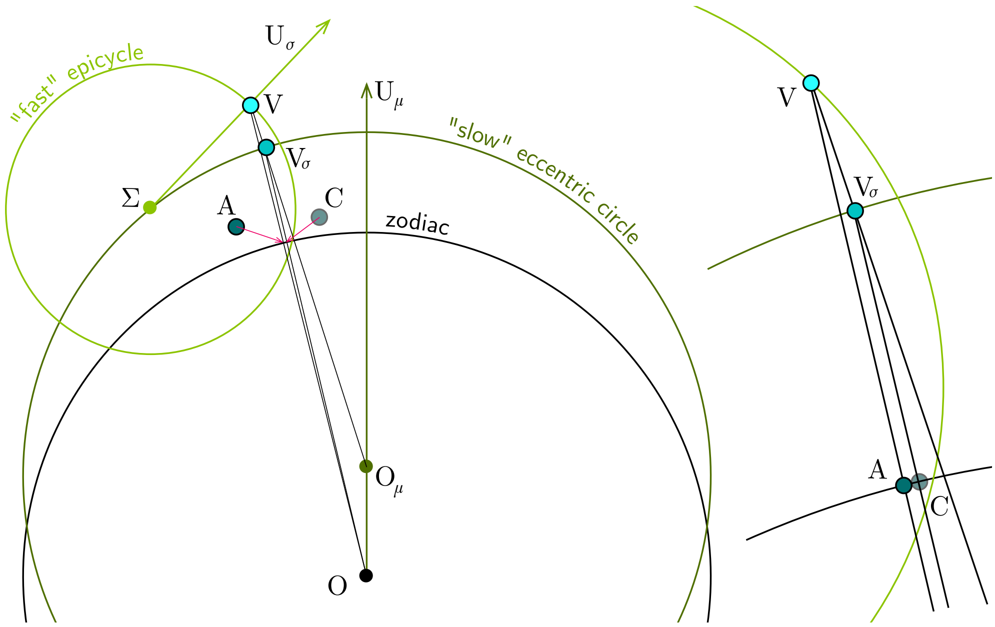



Figure C.14: Parameśvara's configuration for Mercury and Venus. Corrected positions are magnified in the right.

equation (*jīvāphalārdha*)"[19]. This refers to additional steps to find the true planet seen in *Ābh* 3.22-24, where half the values of the equations are applied in order to decrease the error[20].

> In the case of Saturn, Jupiter and Mars half the "slow" [equation computed] from the "slow" apogee is subtractive or additive [against the mean planet] at first. Half [the "fast" equation computed] from the "fast" apogee is subtractive or additive against the "slow" [corrected] planet. [This corrected by the "slow" equation computed] from the "slow" apogee is the corrected-mean (*sphuṭamadhya*) [planet]. And [this further corrected by the "slow" equation computed] from the "fast" apogee is to be known as the true [planet]. (*Ābh* 3.22cd-23)[21]

> [In the case of Venus and Mercury, the "fast" equation computed] from the "fast" apogee decreased by half [of itself] should be made subtractive or additive against its own "slow" apogee. [The mean planets corrected by the "slow" equation computed] from the established "slow" apogee are the corrected-mean [positions of] Venus and Mercury. [By applying the "fast" equation] they become true [planets]. (*Ābh* 3.24)[22]

[19]Hereafter we shall focus on the meaning of "half". As for "Sine", this probably refers to the fact that the "slow" and "fast" equations are computed from the Sine of anomaly, as we have previously seen.

[20]See Neugebauer (1956) for a discussion on how the procedures in the *Āryabhaṭīya* (based on Parameśvara's commentary), *Sūryasiddhānta* and the *Khaṇḍakhādyaka* make the error small. However his argument that this is a compromise in an arithmetical procedure requires further discussion.

[21]*śanigurukujeṣu mandād ardham ṛṇadhanaṃ bhavati pūrve //3.22//*
*mandoccāc chīghroccād ardham ṛṇadhanaṃ graheṣu mandeṣu /*
*mandoccāt sphuṭamadhyāḥ śīghroccāc ca sphuṭā jñeyāḥ //3.23//*

[22]*śīghroccād ardhonaṃ kartavyam ṛṇaṃ dhanaṃ svamandocce /*
*sphuṭamadhyau tu bhṛgubudhau siddhān mandāt sphuṭau bhavataḥ //3.24//*

The verses are extremely terse and allows various interpretations. We shall follow Parameś-vara's commentary in the following explanation[23].

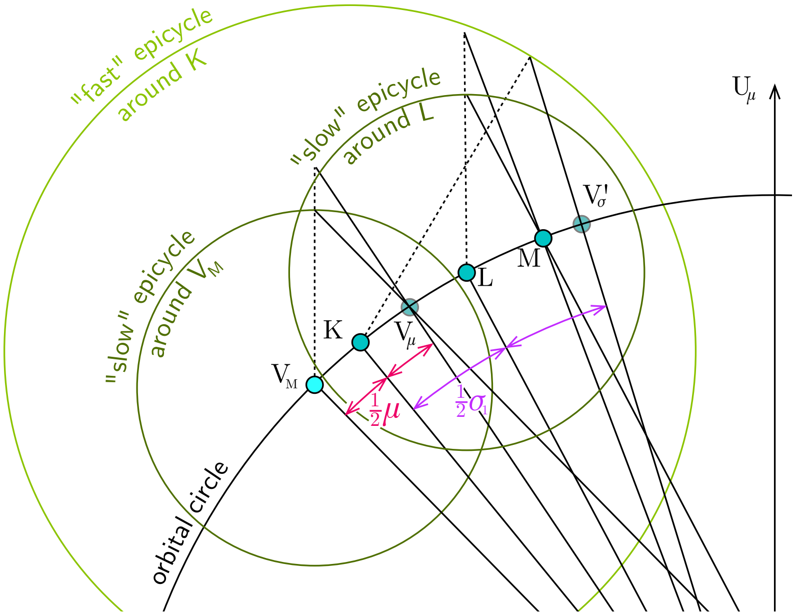



Figure C.15: Positions K after half the "slow" correction and L after half the "fast" correction, and the new "slow" equation $\widehat{\mathrm{LM}}$.

For Mars, Jupiter and Saturn, there are four corrections (*Ābh* 3.22cd-23). In the first step, the "slow" equation $\mu = \widehat{\mathrm{V_M V_\mu}}$ is computed normally but only half its value is applied to the mean longitude $\bar{\lambda}$. In figure C.15, this amounts to finding the point K on the orbital circle between $\mathrm{V_M}$ and $\mathrm{V_\mu}$. The correction is subtractive in figure C.15, but depending on the anomaly, it may be additive (see section C.4).

$$\lambda_1 = \bar{\lambda} \pm \tfrac{1}{2}\mu \tag{C.10}$$

The second is to apply half the "fast" correction to $\widehat{♈\mathrm{K}} = \lambda_1$. This is equivalent to drawing a "fast" epicycle around K, finding the "fast" corrected position $\mathrm{V}'_\sigma$ and locating the point L in the middle of $\widehat{\mathrm{KV}'_\sigma}$. When the "fast" equation $\widehat{\mathrm{KV}'_\sigma}$ is $\sigma_1$, the second corrected longitude $\widehat{♈\mathrm{L}} = \lambda_2$ is

$$\lambda_2 = \lambda_1 \pm \tfrac{1}{2}\sigma_1 \tag{C.11}$$

The third step begins with computing the "slow" equation $\mu_2$ from the "slow" anomaly of L, $\widehat{\mathrm{U_\mu L}} = \kappa_2$. Then we apply the entire equation to the mean longitude $\bar{\lambda}$. In our figure, this amounts to finding the "slow" corrected position M corresponding to L, then finding the point N on the orbital circle such that $\widehat{\mathrm{LM}} = \widehat{\mathrm{V_M N}}$ (figure C.16).

[23] Neugebauer (1956) described the procedures in formulas using the English translation by Clark (1930). This was repeated by Yano (1980). I have also used their interpretations together with Parameśvara's commentary itself.

$$\lambda_3 = \bar{\lambda} \pm \mu_2 \tag{C.12}$$

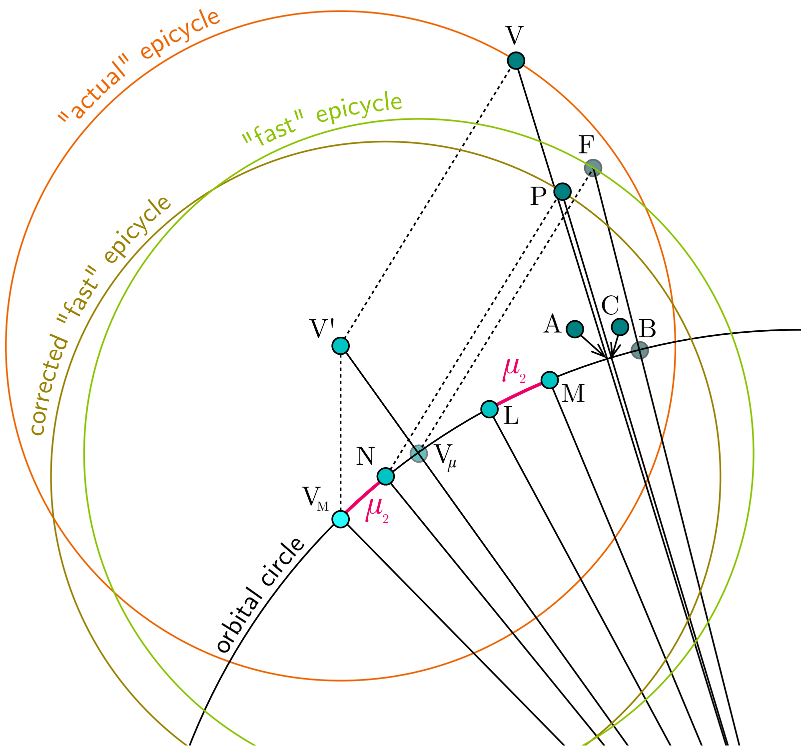



Figure C.16: The longitude C of a planet after the full procedure.

Last of all, we compute the "fast" equation $\sigma_3$ from the anomaly of $\lambda_3$ and apply it to $\lambda_3$. This corresponds to drawing a "fast" epicycle around N, locating the planet P in the direction of the "fast" anomaly and finding its longitude on the orbital circle, C.

$$\lambda_T = \lambda_3 \pm \sigma_3 \tag{C.13}$$

Thus we find the true longitude $\lambda_T$ for Mars, Jupiter and Saturn in four steps (formulas C.10, C.11, C.12 and C.13).

For Venus and Mercury, Āryabhaṭa gives a different procedure in *Ābh* 3.24. According to Parameśvara's commentary, we skip the first "slow" correction[24] and find the "fast" equation $\sigma$ from the anomaly of the mean planet. But instead of applying this to the mean planet, we correct the longitude of the "slow" apogee $\lambda_\mu$.

$$\lambda'_\mu = \lambda_\mu \mp \tfrac{1}{2}\sigma \tag{C.14}$$

[24] Yano (1980, pp. 62-63) suggests that this is because the "slow" epicycles of Venus and Mercury are much smaller than their "fast" epicycles and thus $\frac{1}{2}\sigma_1$ can be ignored.

Parameśvara does not explain why the “slow” apogee is corrected instead of the mean planet. However he remarks in his commentary that the addition or subtraction of the equation is reversed: “the meaning is that [the computation is done] with the rule of the ‘fast’ correction reversed[25]”. As a result, the two different approaches (correcting the apogee or the mean planet) give the same value for the “slow” anomaly $\kappa'_\mu$ of the mean planet:

$$\begin{aligned}\kappa'_\mu &= |\bar{\lambda} - (\lambda_\mu \mp \tfrac{1}{2}\sigma)| \\ &= |(\bar{\lambda} \pm \tfrac{1}{2}\sigma) - \lambda_\mu| \end{aligned} \tag{C.15}$$

With this anomaly $\kappa'_\mu$ we compute the “slow” equation $\mu'$ and apply it to the mean planet.

$$\lambda' = \bar{\lambda} \pm \mu' \tag{C.16}$$

The last step is the same as the case with the other three planets.

$$\lambda_T = \lambda' \pm \sigma' \tag{C.17}$$

Formulas C.14, C.16 and C.17 represent the three corrections for Venus and Mercury.

## C.6 Distance from the Earth

> The distance between the Earth and a star-planet (the five planets) is the product of its radial distances divided by the half-diameter. The speed of the planet on the “slow” epicycle is that on the orbital [circle]. (*Ābh* 3.25)[26]

The last verse in the third chapter of the *Āryabhaṭīya* consists of two parts. We have already seen (section C.1) that Parameśvara interprets the second half, *Ābh* 3.25cd, as a statement on the equivalence of an eccentric circle and an epicycle. Meanwhile, *Ābh* 3.25ab is on the distance $\mathcal{D}$ of a “star-planet (*tārāgraha*)” from the Earth. A star-planet refers to the five planets with a “slow” and “fast” apogee. When the “slow” radial distance caused by the “slow” apogee alone is $\mathcal{R}_\mu$ and the “fast” radial distance is $\mathcal{R}_\sigma$, The statement can be formulated as follows:

$$\mathcal{D} = \frac{\mathcal{R}_\mu \mathcal{R}_\sigma}{R} \tag{C.18}$$

This is incorrect, as we can see in figure C.17. $\mathrm{V_M}$ is the mean planet, $\mathrm{V'}$ is the “slow” corrected planet and $\mathrm{OV'}$ is the “slow” radial distance $\mathcal{R}_\mu$. $\mathrm{O}_\sigma$ is a point such that $\mathrm{OO}_\sigma = \mathrm{V_M V'}$, V is the true planet on the “actual” epicycle, and $\mathrm{O}_\sigma\mathrm{V}$ is the “fast” radial distance $\mathcal{R}_\sigma$ while OV is the distance $\mathcal{D}$ between the planet and the Earth. Formula C.18 is equivalent to

$$\mathrm{OV} = \frac{\mathrm{OV'} \cdot \mathrm{O}_\sigma \mathrm{V}}{\mathrm{O}_\sigma \mathrm{V'}} \tag{C.19}$$

which requires $\triangle \mathrm{OV'O}_\sigma \sim \triangle \mathrm{OVO}_\sigma$. This is not true because OV and OV′ are not aligned and therefore $\angle \mathrm{V'OO}_\sigma \neq \angle \mathrm{VOO}_\sigma$.

[25] *śīghravidhivyatyayenety arthaḥ* / (Kern (1874, p. 67))

[26] *bhūtārāgrahavivaraṃ vyāsārdhahṛtaḥ svakarṇasaṃvargaḥ /*
*kakṣyāyāṃ grahavego yo bhavati sa mandanīcocce //3.25//*

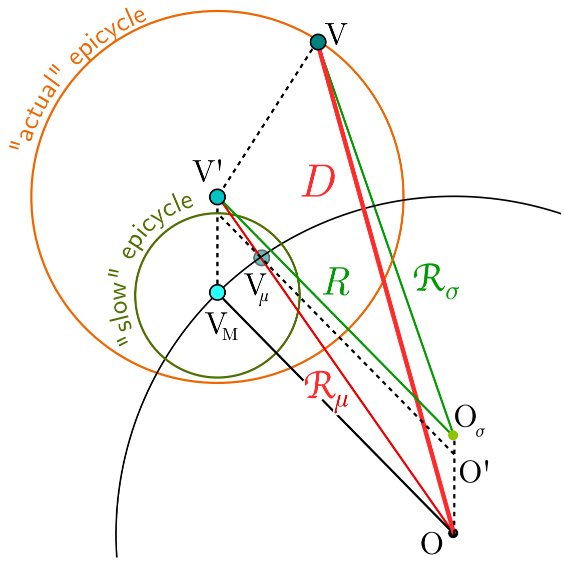



Figure C.17: The distance of a planet and its two radial distances.

Parameśvara makes no remark on how this rule could have been derived. One possible key is that Parameśvara turns to the computation of the celestial latitude in his commentary on *Ābh* 3.25ab. As we can see in *GD2* 128, 132-133 (commentary section 9.4), the latitude is computed by first correcting the deviation of a planet as seen from one of the radial distances, and then finding how this once-corrected deviation appears from the other radial distance. The result (see formula 9.8) is equal to correcting the deviation as seen from a distance of $\frac{\mathcal{R}_\mu \mathcal{R}_\sigma}{R}$. *GD2* 151 suggests that Parameśvara might have been aware of the error in this rule. Yet he makes no remark on the validity of *Ābh* 3.25ab.

# D List of letters used in the formulas

Listed in alphabetical order, starting with the Roman alphabet followed by the Greek alphabet. $\Delta$ is exceptionally ignored upon sorting. Each letter is accompanied with a brief description. If there is a corresponding entry in the glossary a Sanskrit term in brackets are added.

$\mathcal{A}$ Gnomonic amplitude. [*śaṅkvagra* (1)]

$\mathcal{B}$ Base of great shadow. [*chāyābāhu*]

$\mathcal{B}_d$ Base of direction. [*digbāhu*]

$\mathcal{B}_{(\mathcal{G}_M)}$ "Base" of the midheaven gnomon. [*madhyaśaṅkubhujā*]

$\mathcal{B}_s$ Established base. [*sādhyabāhu*]

$_B$ Subscript to indicate that the arc is a "base" arc. Its Sine shall be a "base" Sine. [*dorjīvā*]

$b$ Deviation of a planet in the inclined circle. [*kṣepa* (1)]

$b_{\mathrm{T}}$ True deviation, i.e. celestial latitude. [*kṣepa* (2)]

$c$ Circumference of a circle. [*paridhi*]

$c_\oplus$ Circumference of the Earth. [*bhūvṛtta*]

$c_\phi$ Circumference of a parallel (line of latitude) with a geographic latitude $\phi$. [*nijabhūvṛtta*]

$\mathcal{D}$ Distance in general. [*antara*]

$\mathcal{D}_\theta$ Distance along the parallel (line of latitude) from the prime meridian, in *yojana*s.

$\mathcal{D}_\odot$ Radial distance of the sun in *yojana*s. [*karṇa* (2)]

$\overline{\mathcal{D}_\odot}$ Mean radial distance of the sun in *yojana*s which is 459,585. [*karṇa* (2)]

$\mathcal{D}_☾$ Radial distance of the moon in *yojana*s. [*karṇa* (2)]

$\overline{\mathcal{D}_☾}$ Mean radial distance of the moon in *yojana*s which is 34,377. [*karṇa* (2)]

$d$ Diameter of any circle or sphere. [*vyāsa*]

$d_\oplus$ Diameter of the Earth in *yojana*s.

$d_\odot$ Diameter of the Sun in *yojana*s.

$d_☾$ Diameter of the Moon in *yojana*s.

$E$ Equation of time.

$_{\mathrm{EW}}$ Subscript for values when the sun is situated on the prime vertical (*samamaṇḍala*).

$\mathcal{G}$ Great gnomon. [*mahāśaṅku*]

$g$ Length of a twelve *aṅgula* gnomon, i.e. 12. [*śaṅku* (1)]

$H$ Hour angle. [*nata* (2)]

$\boldsymbol{h^*}$ The hypotenuse formed by a twelve *aṅgula* gnomon on an equinoctial midday. [*palakarṇa*]

$\boldsymbol{i}$ Inclination or greatest separation of a planetary orbit. [*paramakṣepa*]

$\boldsymbol{J_t}$ "Sine" in the celestial equator, which is a segment related to the arc (but not a true Sine) between the point corresponding to the given moment and the point of sunrise or sunset (depending on whether it is in the morning or in the afternoon).

$\boldsymbol{J'_t}$ Sine in the celestial equator, which is the distance between the point corresponding to the given moment and the six o'clock circle.

$\boldsymbol{j_t}$ Given "Sine" in the diurnal circle, which is the distance between the sun and the horizon. [*iṣṭajyā* (2)]

$\boldsymbol{j'_t}$ Given Sine in the diurnal circle, which is the distance between the sun and the six o'clock circle. [*iṣṭajyā* (3)]

$\boldsymbol{J'_\delta}$ Portion of declination produced by the latitude. This is a difference of two Sines. [*vikṣepabhava*]

$\boldsymbol{k}$ Earth-Sine. [*kṣitijyā*]

$\boldsymbol{l_v}$ Unified visibility equation. [*dṛkphala*]

$\boldsymbol{l'_v}$ Visibility equation in *prāṇa*s, i.e. measured along the celestial equator. [*dṛkphala*]

$\boldsymbol{l_{v(\mathrm{c})}}$ Visibility equation for the "course".

$\boldsymbol{l_{v(\varphi)}}$ Visibility equation for the geographic latitude. [*akṣadṛkphala*]

$\boldsymbol{l_\bullet}$ Length of the Earth's shadow. [*bhūcchāyā*]

$\boldsymbol{p}$ Multiplier.

$\boldsymbol{p}$ Parallax in *yojana*s. [*lambana* (1)]

$\boldsymbol{p_{\mathrm{max}}}$ Greatest parallax in *yojana*s.

$\boldsymbol{p_\lambda}$ Longitudinal parallax in *yojana*s. [*lambana* (2)]

$\boldsymbol{p_\beta}$ Latitudinal parallax in *yojana*s. [*nati*]

$\boldsymbol{q}$ Divisor.

$\boldsymbol{q_\Sigma}$ The sun's equation of center. [*doḥphala*]

$\boldsymbol{R}$ Radius of a great circle. [*trijyā*]

$\boldsymbol{\mathcal{R}_\mu}$ "Slow" radial distance. [*mandaśruti*]

$\boldsymbol{\mathcal{R}_\sigma}$ "Fast" radial distance. [*śīghraśruti*]

$\boldsymbol{r}$ Radius of a non-great circle, especially (but not limited to) the radius of a diurnal circle (*dyudalajīvā*). [*ardhaviṣkambha*]

$\boldsymbol{\mathcal{S}}$ Great shadow. [*mahācchāyā*]

$\boldsymbol{s}$ Shadow of a twelve *aṅgula* gnomon. [*chāyā* (1)]

$\boldsymbol{s^*}$ Shadow of a twelve *aṅgula* gnomon on an equinoctial midday. [*chāyā* (1)]

**Sin** Sine in a great circle with Radius $R$. $\text{Sin}\,\theta$ stands for $R\sin\theta$. [*jyā*]

$_{\mathbf{T}}$ Subscript for corrected or "true" positions of planets and related values. [*sphuṭa*]

$\boldsymbol{t}$ Time for a given moment of the day, elapsed since sunrise if the moment is in the morning, and left until sunset if it is in the afternoon.

$\boldsymbol{\mathcal{U}}$ Upright of great shadow. [*chāyākoṭi*]

$_{\boldsymbol{U}}$ Subscript to indicate that the arc is an "upright" arc. Its Sine shall be an "upright" Sine. [*koṭi* (2)]

$\boldsymbol{u}$ Upright in the diurnal circle.

$\boldsymbol{v}$ Daily motion of a planet. [*dinabhukti*]

$\boldsymbol{z}$ Zenith distance of a specific point (denoted by the subscript).

$\boldsymbol{z_D}$ Zenith distance of the ecliptic point of sight-deviation (*dṛkkṣepa* (1)). $\text{Sin}\,z_D$ is the Sine of sight-deviation. [*dṛkkṣepajyā*]

$\boldsymbol{z_M}$ Zenith distance of the midheaven ecliptic point (*madhyavilagna*). $\text{Sin}\,z_M$ is the midheaven Sine. [*madhyajyā*]

$\boldsymbol{z_V}$ Zenith distance of a planet. $\text{Sin}\,z_V$ is the Sine of sight. [*dṛgjyā*]

$\boldsymbol{z_\Sigma}$ Meridian zenith distance of the sun at midday. $\text{Sin}\,z_\Sigma$ is the midday shadow. [*dinadalacchāyā*]

$\boldsymbol{\alpha}$ Rising time of a celestial point or arc at Laṅkā, i.e. its distance from an equinoctial point along the celestial equator. Effectively its right ascension. [*laṅkodaya*]

$\boldsymbol{\bar{\alpha}}$ The distance of a celestial point from a solstitial point along the celestial equator.

$\boldsymbol{\beta}$ Celestial latitude of a planet as observed from the Earth. [*kṣepa* (2)]

$\boldsymbol{\beta^*}$ Corrected latitude [*sphuṭakṣepa*]

$\boldsymbol{\gamma_c}$ Deflection for the "course" (*āyanavalana*, does not appear in *GD2*).

$\boldsymbol{\delta}$ Declination. [*apama* (1)]

$\boldsymbol{\delta^*}$ Corrected declination. [*sphuṭāpama*]

$\boldsymbol{\delta_T}$ True declination. [*spaṣṭa*]

$\boldsymbol{\varepsilon}$ Greatest declination (24°). [*paramāpama*]

$\boldsymbol{\zeta_K}$ Elevation of ecliptic pole from the plane of the six o'clock circle, in the form of a Sine ($\text{Sin}\,\zeta_\text{K}$). [*bhakūṭonnati*]

$\boldsymbol{\tilde{\zeta}_K}$ Crude elevation of ecliptic pole, in the form of a Sine ($\text{Sin}\,\tilde{\zeta}_\text{K}$). [*sthūlonnati*]

$\boldsymbol{\zeta_\beta}$ Elevation or depression of celestial latitude from the plane of the six o'clock circle, in the form of a Sine ($\text{Sin}\,\zeta_\beta$). [*unnati* / *avanati*]

$\boldsymbol{\zeta_{\varphi K}}$ Elevation of ecliptic pole from the plane of the horizon, in the form of a Sine ($\mathrm{Sin}\,\zeta_{\varphi \mathrm{K}}$). [*bhakūṭonnati*]

$\boldsymbol{\zeta_{\varphi\beta}}$ Elevation or depression of celestial latitude from the plane of the horizon, in the form of a Sine ($\mathrm{Sin}\,\zeta_{\varphi\beta}$). [*unnati* / *avanati*]

$\boldsymbol{\eta}$ Solar amplitude, always in the form of a Sine ($\mathrm{Sin}\,\eta$) [*arkāgrā*]

$\boldsymbol{\theta_\Sigma}$ Direction of the sun. $\mathrm{Sin}\,\theta_\Sigma$ is the Sine of direction. [*digjīvā*]

$\boldsymbol{\kappa}$ Anomaly of a planet's longitude (*kendra*, only in appendix).

$\boldsymbol{\lambda}$ Longitude in general.

$\boldsymbol{\lambda_{Asc}}$ Longitude of the ascendant. [*lagna* (1)]

$\boldsymbol{\lambda_D}$ Longitude of the sight-deviation ecliptic point. [*dṛkkṣepalagna*]

$\boldsymbol{\lambda_M}$ Longitude of the meridian ecliptic point. [*madhyavilagna*]

$\boldsymbol{\lambda_q}$ Longitude of a planet at sunrise corrected for the sun's equation of center.

$\boldsymbol{\lambda_\theta}$ Longitude of a planet at sunrise corrected for the geographic longitude.

$\boldsymbol{\lambda_\omega}$ Longitude of a planet at sunrise corrected with the ascensional difference.

$\boldsymbol{\lambda'_\omega}$ Longitude of a planet at sunset corrected with the ascensional difference.

$\boldsymbol{\mu}$ "Slow" equation. [*phala* (1) (2)]

$_{\boldsymbol{\mu}}$ Subscript for positions and values related to the "slow" apogee. [*manda* (1)]

$\boldsymbol{\Pi_\lambda}$ As a Sine ($\mathrm{Sin}\,\Pi_\lambda$), the Sine of sight-motion. [*lambana* (2)]

$\boldsymbol{\pi}$ Parallax in arc minutes. [*lambana* (1)]

$\boldsymbol{\pi_\lambda}$ Longitudinal parallax in arc minutes. [*lambana* (2)]

$\boldsymbol{\pi_\beta}$ Latitudinal parallax in arc minutes. [*nati*]

$\boldsymbol{\rho}$ Measure of a sign. [*bhamiti*]

$\boldsymbol{\Sigma}$ Position of the sun in the ecliptic. [*arka*]

$\boldsymbol{\sigma}$ "Fast" equation. [*śīghra* (1)]

$_{\boldsymbol{\sigma}}$ Subscript for positions and values related to the "fast" apogee. [*śīghra* (1)]

$\boldsymbol{\upsilon}$ Its Sine $\mathrm{Sin}\,\upsilon$ is the rising Sine (*udayajīvā*). Does not appear in *GD2*.

$\boldsymbol{\phi}$ Apparent size of an object in arc minutes.

$\boldsymbol{\phi_\odot}$ Apparent size of the sun in arc minutes.

$\boldsymbol{\phi_☾}$ Apparent size of the moon in arc minutes.

$\boldsymbol{\phi_\bullet}$ Apparent size of the umbra in arc minutes. [*tamas* (2)]

$\boldsymbol{\varphi}$ Geographic latitude. [*akṣa*]

$\bar{\varphi}$ Geographic co-latitude. [*avalambaka*]

$\Omega$ Longitude of ascending node. [*pāta*]

$\omega$ Ascensional difference. [*cara*]

$\Delta\omega_{\beta}$ Portion of the ascensional difference made by the (celestial) latitude. [*kṣepakṛtacarāṃṣa*]

# Glossary of Sanskrit terms

**Introduction** The following is a list of Sanskrit terms in *GD2*. Numbers at the end of each entry indicate the verse number in which they appear. If a term appears in a preamble of a verse, it shall be counted as an occurence in the verse itself. Terms appearing in the commentary (proses which are only seen in manuscripts $K_5^+$ and $I_1$) or in quotations within my explanatory notes are not included, and descriptions for each term are limited to their meaning within *GD2* unless indicated otherwise.

All Sanskrit words are given here in their dictionary forms (i.e. stems) regardless of their appearance in the text. Entries are given in Sanskrit alphabetical order.

Compounds are kept as far as they refer to a single object, figure or value as a whole and are not coordinate compounds. For example, *krāntijyā* (Sine of declination) will be counted as one term, and not enumerated as occurences of *krānti* (declination) or *jyā* (Sine). *dakṣiṇottara* consistently refers to the solstitial colure or prime meridian in *GD2*, and therefore will not be decomposed into *dakṣiṇa* (south) and *uttara* (north), but *dakṣiṇodak* in *GD2* 155 gives entries for *dakṣiṇa* and *udañc* because it is used in the sense of "south and north".

Whenever a compound is decomposed in the verse but the individual words do not convey any meanings on their own, the original compound is entered in the glossary. For example, *bhānāṃ kūṭonnatis* in *GD2* 189 is counted as an entry for *bhakūṭonnati* (elevation of an ecliptic pole) as *bha* no longer has the sense of "sign" or "star" here.

The morpheme *ākhya* (called), which is frequently integrated in compounds for introducing a new term, is omitted in the entries because it has nothing to do with the meaning of a term itself. However, the situation is complex if it is in the middle of a compound. In general, the words before and after *ākhya* are taken as two entries; for example, *apamaṇḍalākhyavṛtta* (circle called the ecliptic) in *GD2* 4 gives the two entries *apamaṇḍala* (ecliptic) and *vṛtta* (circle), as *maṇḍala* also means circle and the form *apamaṇḍalavṛtta* would not be used. Meanwhile for *yāmyottarākhyavṛtta* (circle called the prime meridian) in *GD2* 71, *yāmyottara* alone means "north and south" and is never used without *vṛtta* in the sense of "prime meridian" within *GD2*, where there are 3 occurrences of the compound *yāmyottaravṛtta*. Therefore I have counted it as an occurrence of *yāmyottaravṛtta*, while also adding entries for *yāmyottara* and *vṛtta* to avoid confusion.

**a**

***aṃśa* (1)** Degree of arc. The 360th part of a full circle or revolution. 32, 129, 155

***aṃśa* (2)** Portion. It can also follow a number to indicate that it is a denominator. 9, 38, 60, 77, 179, 180, 212

***aṃśaka* (1)** Degree. 14, *see* *aṃśa* (1)

***aṃśaka* (2)** Portion or denominator. 245, 246, *see* *aṃśa* (2)

***akṣa*** Geographic latitude as a measurement of the arc or as the length of its Sine. "Geographic" is added to the English translation to avoid confusion with the latitude of a planet from the ecliptic (*kṣepa* (1), *vikṣepa* (1)). *GD2* only considers situations when the observer is in the northern hemisphere. In modern terminology, this would mean that the geographic latitude is northward. However, Parameśvara seems to regard the direction of the geographic latitude as southward. 2, 14, 31, 43, 45, 47, 71, 88, 105, 106, 118, 176, 232

***akṣajīvā*** Sine of geographic latitude. 72

***akṣajyā*** Sine of geographic latitude. 70, 74, 119, 121, 124, 241, *see akṣa*

***akṣadṛkphala*** Visibility equation for the geographic latitude. 175, *see dṛkkarman*

***agni*** Southeast. The god Agni is regarded as the guardian of this direction, and therefore any of his names can stand for southeast. 222

***agra*** Extremity, the highest or furthest point of a segment. 219, 287–289, 293, 299

***aṅgula*** Measuring unit of length, literally "finger". Twelve *aṅgula*s is the standard length of a gnomon as instrument. 120, 286

***adhas*** Downward direction, below, bottom. 2, 3, 8, 12, 14, 16, 18, 25, 28, 29, 36, 37, 40, 157, 158, 165, 233, 250, 257, 258, 262, 278, 281

***anakṣa*** Not having geographic latitude, i.e. be on the terrestrial equator. 201

***anakṣadeśa*** Location with no geographic latitude, i.e. on the terrestrial equator. 43

***anupāta*** Proportion. Used in the ablative (*anupātāt*) or instrumental (*anupātena*) cases to state that the length of a specific segment is computed from / by proportion. This term indicates that there is a set of similar figures which gives a Rule of Three. 106, 123, 252

***antara*** Distance, or difference between two values. 14, 32, 59, 71, 104, 185, 188, 196, 218, 234, 237, 245, 249, 253, 258, 266, 288, 294, 299

***antarāla*** Distance. 133, 265, *see antara*

***antarita*** Distance. 126, *see antara*

***antyaphala*** "Greatest equation" possible for an apogee (*ucca*). Its Sine is equal to the radius of the epicycle or the distance between the centers of the eccentric circle and orbital circle. Literally "last result". 136, 137, 140, 152

***antyāpama*** Greatest declination, literally "last declination". 162, *see paramāpama*

***apakramadhanus*** Arc of declination. 157, *see apama* (1)

***apama* (1)** Declination. The distance of a specific point on the ecliptic from the celestial equator. 46, 50, 52, 81, 82, 87, 93, 153, 176, 179, 182, 185, 189, 243, 268, 269, 272

***apama* (2)** Ecliptic. 3, 154–156, 165, *see apamaṇḍala*

***apamajyā*** Sine of declination. 75, 84, *see apama* (1)

***apamaṇḍala*** Ecliptic. A great circle inclined 24 degrees against the celestial equator. Longitudes are measured along the ecliptic and latitudes are measured as the distance from the ecliptic. 4, 125, 126, 271

***apamadhanus*** Arc of declination. 164, *see apama* (1)

***apamamaṇḍala*** Ecliptic. Literally "circle of declination". 70, *see apamaṇḍala*

***apamavṛtta*** Ecliptic. 272, *see apamaṇḍala*

***apara* (1)** Western direction. 7, 8, 12–14, 17, 18, 103, 200, 236, 237

***apara* (2)** Later in time. 49, 61

***abda*** Year. 62

***abhīṣṭā*** Given [Sine in the diurnal circle]. 90, 91, *see iṣṭajyā* (1)

***ayana* (1)** Northward or Southward "course" (both in the sense of motion and pathway), usually of the sun, towards a solstitial point. 211, 215–217

***ayana* (2)** "Passage" or motion of the solstices and equinoxes against the fixed stars. Corresponds to precession in modern astronomy, but Parameśvara considers this motion to be a trepidation. 101, 218

***ayanānta*** Solstitial point. Literally "end of the course" of the sun in the northward or southward direction. 89, 158, 159, 219

***arka*** The sun, often referring to its position in the sky or longitude in the ecliptic, rather than the object itself. *arka* and its synonyms can also represent the number twelve, in which case it is not included in the following verses. 11, 16, 22, 37, 41, 45, 46, 49, 67, 70, 75, 103, 152, 181, 182, 199, 208, 209, 214, 215, 219, 223, 231, 232, 235, 237, 239, 240, 245, 246, 294

***arkatanaya*** The planet Saturn. Literally "son of the sun". 130

***arkāgrā*** Solar amplitude. Literally "tip of sun", and indicates the Sine corresponding to the arc distance between the rising point of the sun and due east on the horizon. Burgess and Whitney (1858, p. 242) translates this term "measure of amplitude" in the *Sūryasiddhānta* and K. V. Sarma (1956–1957) uses "sin. amplitude" for *GD1*. Considering the fact that *arkāgrā* or any of its synonyms are unrelated to "Sine", I have consistently translated this term "solar amplitude". Meanwhile Parameśvara does describe it as a Sine in *GD2* 84, so for mathematical representation I use the form $\mathrm{Sin}\,\eta$. 84, 85, 122, 210, 221, 243, 244

***arkāṅgulaśaṅku*** Gnomon of twelve *aṅgula*s, i.e. the instrument and not the great gnomon. 116

***ardharātra*** Midnight. 44

***ardhaviṣkambha*** Half-diameter, indicating a radius of a circle which is not a great circle. There are two ways to refer to a radius, which is "half of a diameter" as in this case, or "the Sine of three signs (*trijyā* etc.)". The latter is exclusively used for the Radius of a great circle. Therefore I use the literal translation "half-diameter" for marking the usage of this term for non-great circles. The distinction is significant in *GD2*, and *GD1* is also consistent in using "half-diameter" for non-great circles. Meanwhile the *Āryabhaṭīya* uses both "half-diameter" and "Sine of three signs" for the Radius, and Parameśvara too mixes both expressions in his commentary. 75, 238

***avanati*** Depression. The distance of a celestial point below a given level (especially the planes of the six o'clock circle or the horizon). Antonym of *unnati* (elevation). 166, 167, 170, 190–193

***avalamba*** Co-latitude. 192, *see avalambaka*

***avalambaka*** Co-latitude as a measure of arc or its Sine. To be precise, it is the *geographic* co-latitude, but since *planetary* co-latitudes are undefined, the translation shall always be abbreviated. 71

***avalambakajyā*** Sine of co-latitude. 46, *see avalambaka*

***aviśiṣṭa*** Without difference. An adjective for a value established as the result of an "without-difference" method. Synonym of *aviśeṣa*, but unlike *aviśeṣa* (1) Parameśvara does not use this term to indicate the computation itself. 234, 244

***aviśeṣa* (1)** Without-difference method. 228, 229, 233, 244, *see aviśeṣakarman*

***aviśeṣa* (2)** Without difference. 278, *see aviśiṣṭa*

***aviśeṣakarman*** "Without-difference" method. An iterative method, where computations are repeated until there is no more difference between two specific values. Parameśvara does not use the synonym *asakṛtkarman* (not-once method), which is close in nuance to "iterative method". I have chosen the literal translation for *aviśeṣakarman* to emphasize Parameśvara's choice of terminology. 241

***asu*** *Asu* as time unit, literally "respiration", corresponding to the time it takes for the stellar sphere to revolve one minute of arc. Therefore it is also referred to as "sidereal asu (*ṛkṣāsu*". Four sidereal seconds in modern notation. Apart from metrical reasons, there is no distinction with its synonym *prāṇa* and the two are sometimes used in the same verse. In order to avoid confusion, I have added (i.e. *prāṇa*s) in parenthesis to every first appearance of *asu* in a verse or commentary passage, following the convention used in the translation of K. V. Sarma (1956–1957). 94, 99, 107, 109, 159, 171–174, 177, 205, 207

***asta*** Setting of a heavenly object. 50, 52, 54, 170, 171, 177, 179, 183, 193

***astama*** Setting. 47, *see astamaya*

***astamaya*** Setting of an heavenly body beneath the horizon. If the subject is a length of arc, such as a sign, it refers to the time it takes for the whole arc to set below the horizon. 13, 44, 160, 170, 171, 174, 205, 206, 249

***astalagna*** Descending point. The intersection of the ecliptic with the horizon in the west. 178

***astavilagna*** Descendant. 180, *see astalagna*

***astodaya*** Rising-setting [line]. 104, *see astodayasūtra*

***astodayasūtra*** Rising-setting line. The line connecting the two intersections of the diurnal circle and the horizon. 103, 104

***ahorātra*** Day and night, i.e. one full day. 55

**ā**

***ārki*** The planet Saturn. Literally "[produced] from the sun". 20

***ārya*** The planet Jupiter. 136

***āśā*** Direction as seen from the center of a circle, or side as seen from a specific heavenly object. 21, 220, 232, 299, *see also diś*

***āśāvṛtta*** Circle of direction. A circle, probably drawn on the ground, with lines of direction (*diksūtra*) indicating the cardinal directions. 220

***āsakti*** Adherence. When an adherence is slow in a "without-difference" method, the difference of a specific value decreases slowly. When it is fast, the value oscillates. Parameśvara also uses *āsatti* (reaching) in *GD1* 4.21. 233

**i**

***iḍya*** The planet Jupiter. 134

***ina*** The sun. 210, *see* *arka*

***ināgrā*** Solar amplitude. 86, 103, 122, 210, 226, *see* *arkāgrā*

***indu*** The moon. 20, 66, 152, 277, 282, 296

***indra*** East. The god Indra is regarded as the guardian of this direction, and therefore any of his names can stand for east. 232

***iṣṭajyā* (1)** Given Sine [in the diurnal circle], which is the distance from the sun to the line of intersection with the plane of the **equinoctial colure**. The verse numbers given below include the occurrences of every synonym (same for definitions 2 and 3). 91–94

***iṣṭajyā* (2)** Given “Sine” [in the diurnal circle], which is the distance from the sun to the line where the planes of the diurnal circle and **horizon** intersect. It is not precisely a Sine (half-Chord), but is the result of one Sine being added to or subtracted from another. 104, 105, 107, 113–115, 117, 242–244

***iṣṭajyā* (3)** Given Sine [in the diurnal circle], which is the distance from the sun to the line where the planes of the diurnal circle and **six o’clock circle** intersect. Used for computing *iṣṭajyā* (2). 111–113

***iṣṭadyujīvā*** Given “Sine” in the diurnal circle. 115, *see* *iṣṭajyā* (2)

***iṣṭadyujyā*** Given “Sine” in the diurnal circle. 105, 243, 244, *see* *iṣṭajyā* (2)

**u**

***ucca*** Apogee. The direction of the center of an eccentric circle as seen from the center of the Earth. Alternatively, it is the direction of the true planet as seen from the center of the epicycle. In both cases, the word refers to its longitude measured from the vernal equinox rather than a specific point. 133, 145, 152, 278

***ujjayinī*** City of Ujjain, located on the prime meridian of the Earth. 38

***uttara*** Northern direction, northern side. 12, 38, 41, 75, 84

***udañc*** Northern direction, northern side. 13–15, 17, 45, 67, 98, 155, 156, 165, 214, 215, 236

***udaya*** Rising of a heavenly body from the horizon. If the subject is a length of arc, such as a sign, it refers to the time it takes for the whole arc to appear above the horizon. 13, 44, 50, 52, 54, 94, 95, 97, 99, 160, 165, 170, 171, 174, 177, 178, 183, 193, 197, 203, 205, 206, 228, 249

***udayalagna*** Ascendant. 178, 186, *see* *lagna* (1)

***unnati*** Elevation. The height of an object, or the Sine corresponding to an arc distance of a point in the stellar or celestial sphere above a given level (especially the horizon or six o’clock circle). 35, 103, 158–161, 163, 166, 167, 170, 189, 192, 193, 286

***unmaṇḍala*** Six o'clock circle. Literally "up-circle" or "rising circle". A great circle going through the celestial poles and the east and west crossings, fixed on the celestial circle in an armillary sphere. It divides every diurnal circle into two equal parts, and the modern name comes from the fact that the mean sun is always on this circle at six o'clock AM and PM. This circle represents the horizon as seen from the terrestrial equator. 14–17, 75, 76, 85, 108, 109, 111, 157, 158, 165, 166, 168

***unmaṇḍalodaya*** Rising above the six o'clock circle. In *GD2*, it only refers to the moment when the sun crosses the six o'clock circle in the morning; thus in other terms it is six o'clock AM. It also corresponds to the moment of sunrise at the terrestrial equator on the same line of longitude as the observer. I have avoided using the term "mean sunrise" which dates back at least to Warren (1825), since it has no corresponding Sanskrit word and is also confusing, because it is not the "rising of the mean sun". 195, 206

***upari* (1)** Above. Basically in the sense of "further from Earth", but it could also mean "higher in the sky". Parameśvara takes advantage or this ambiguity in *GD2* 66-67. 8, 19, 37, 66, 67, 124, 278, *see also* *ūrdhva* (1)

***upari* (2)** Top. Used to refer to the north pole of the Earth. 28

***urvī*** The Earth. 7, *see* *bhūmi*

**ū**

***ūrdhva* (1)** Above, upward. Like *upari* (1), it can be used in the sense of "further from Earth" or "higher in the sky", but Parameśvara uses it most frequently when describing configurations of circles, possibly with the actual armillary sphere in mind, to refer to the direction above. 2, 3, 12, 14–16, 25, 36, 66, 109, 111, 113, 118, 157, 158, 165, 233, 249, 251, 257, 258, 262, 298, 299

***ūrdhva* (2)** Subsequent to. 51

**ṛ**

***ṛkṣa*** Star, zodiacal sign or both. 21, 52, 154, 169, 171, 174, 177, 222, *see* *bha* (1)

***ṛṇa*** Subtractive. Indicates that a computed value (usually an equation) should be subtracted from another given value. 169, 170, 202, 203, 205, 206, 224, 234

**e**

***eṇa*** The zodiacal sign Capricorn. 51, 52, 97, 98, 160

**k**

***kakṣyā* (1)** Orbital [circle]. Literally "girth" or "girdle". A circle with the center of Earth as its center on which the position of planets are measured. Its radius differs among planets. In *GD2*, it is used in contrast with the circle of sight (*dṛgvṛtta*) where the descriptions often suggests that all circles should be drawn in the same plane. However, planets do not necessarily revolve on a circle which goes above the horizon. Therefore, my interpretation of what Parameśvara calls an "orbit" or "orbital circle" in the context of parallaxes is a circle which can only be defined at a given moment, which goes through the planet and through the direction of the observer's zenith and whose radius is the distance between the observer and the planet at the given moment. 248, 253–255, 264, 266, 281, 300

***kakṣyā* (2)** Orbit of planets. Referring to any circle on which a planet (mean, corrected or true) revolves. The usage in *GD2* 135 includes even eccentric circles or epicycles, which are usually contrasted against the concentric orbital circle. 19, 135

***kakṣyāvṛtta*** Orbital circle. 260, 263, *see kakṣyā* (1)

***kanyā*** The zodiacal sign Virgo. 53

***kamalayoni*** Brahmā. Literally "born from a lotus". 64, *see brahmā*

***karki*** The zodiacal sign Cancer. 50, 51, 97, 98

***karṇa* (1)** Hypotenuse of a right triangle. 76, 85–87, 105, 115, 118, 168, 221, 235, 268, 274–276, 287, 289

***karṇa* (2)** Radial distance. A line joining a celestial object or point with the center of Earth, or the length of this line. 128, 132, 133, 135, 141, 149–151, 275, 277, 278, 297, 298

***kalā*** Minute of arc, i.e. one sixtieth of a degree. There are 21600 minutes of arc in a circumference, and the measure of the Radius 3438 is chosen so that one unit of a segment is approximately the same length as a minute of arc. Thus arcs corresponding to Sines are usually given in minutes. 19, 130, 156, 172, 177, 202, 257, 297, 300

***kali*** *Kali-yuga*, the last and present subdivision of the *caturyuga*. 120,000 solar years. 57, 63

***kalpa*** A time period of 4,320,000,000 solar years, or one thousand *caturyuga*s. 59, 60, 62, 64

***kāla*** Time. Either "point of time" or "timespan". 9, 45, 61, 77, 97, 129, 161, 171, 174, 204, 219, 237, 253

***ku*** The Earth. 299, *see bhūmi*

***kugola*** Earth's sphere. 40, *see bhūgola*

***kuja*** The planet Mars. The expression "beginning with Mars (*kujādi*)" and its synonyms refer to either the five planets Mars, Mercury, Jupiter, Venus and Saturn, or the three planets among them known today as outer planets, Mars, Jupiter and Saturn. 18, 136, 137, 142–144, 146

***kuparidhi*** Earth's circumference. 261, *see bhūparidhi*

***kupṛṣṭha*** Earth's surface. 248, 257, *see bhūpṛṣṭha*

***kumadhya*** Earth's center. 248, *see bhūmadhya*

***kumbha*** The zodiacal sign Aquarius. 51

***kulīra*** The zodiacal sign Cancer. 160

***kṛta*** *Kṛta-yuga*, the first in the four subdivisions of the *caturyuga*. 480,000 solar years. 57, 63

***kṛśānu*** Southeast. 246, *see agni*

***kṛṣṇa*** Dark half of a lunar month, from full moon to new moon. 42

***kendra*** Center of a circle. 136–138, 141, 148, 152, 248, 260, 261, 264

***koṭi* (1)** Upright of a right triangle. 76, 82, 85–87, 90, 92, 105, 115, 118, 168, 235, 236, 268–270, 273, 276, 288–290

***koṭi* (2)** “Upright” [Sine]. The Sine corresponding to the arc between a given point and the nearest solstitial point. This concept can be expanded to other set of points or circles and may be also interpreted as a “Cosine”. 48, 89, 158, 162

***koṭi* (3)** A “crore”, or ten million. 31, 33

***koṭidhanus*** Arc of “upright”. An arc between a given point on the ecliptic and the nearest solstitial point. 89

***koṇa*** Intermediate direction. The four directions between the four cardinal directions, namely northeast, northwest, southwest and southeast. 222, 235

***korpi*** The zodiacal sign Scorpio. 51, 231

***krama*** Step. A set of computations with a certain order. 147

***krānti*** Declination. 73, 74, 76, 86, 90, 117, 121–124, 163, 164, 168, 175, 184, 194, 210, 214, 216, 218, 239, 241, *see apama* (1)

***krāntijyā*** Sine of declination. 85, 120, 175, *see apama* (1)

***krāntidhanus* (1)** Arc of declination. 213, *see apama* (1)

***krāntidhanus* (2)** Arc of [a celestial point’s own] declination. 153, *see svakrāntidhanus*

***kṣiti*** Ground. 287, *see bhū* (1)

***kṣiticchāyā*** Earth’s shadow. 285, 292, *see bhūcchāyā*

***kṣitija*** Horizon. A circle in the celestial circle connecting the four cardinal directions and representing the horizon of the observer. Literally “produced from the Earth”. 13, 14, 16, 37, 71, 72, 76, 84, 85, 103, 108, 113, 117, 188–190, 249, 250, 252

***kṣitijyā*** Earth-Sine. The Sine corresponding to an arc in the diurnal circle (therefore not a Sine of a great circle) between its intersection with the six o’clock circle and that with the horizon. 74, 76, 82

***kṣetra*** Figure, especially in the sense of a figure in a plane. 105, 106, 132, 221, 301

***kṣepa* (1)** Deviation of a planet from the plane of the ecliptic, or the inclination of its orbit which causes the deviation. Literally “throwing”. In general, previous translators and commentators of Sanskrit texts have not differentiated it with “latitude”, with the exception of Ramasubramanian and Sriram (2011) who use “deflection”. However, since deflection is a term in modern astronomy (as in “deflection of light by gravity”), I shall avoid it and use the word “deviation” as in the survey of the *Almagest* by Pedersen (2011). 127, 131, 140, 149–151, 167, 177, 190, 192, 193

***kṣepa* (2)** Latitude. The effect of *kṣepa* (1) as seen from the observer. Not to be confused with “geographic latitude” (*akṣa*). I use the term “celestial latitude” if clarification is required. 156, 157, 163, 165, 166, 168

***kṣepakṛtacarāṃṣa*** Portion of the ascensional difference made by the (celestial) latitude. The length of arc along the celestial equator which corresponds to the time difference between the rising of a planet with a latitude and of that of the point with the same longitude on the ecliptic. This is used for computing the visibility equation for the geographic latitude (*akṣadṛkphala*). 176

***kṣepacara*** [Portion of] the ascensional difference [made by] the latitude. 177, *see* *kṣepakṛtacarāṃṣa*

***kṣeponnati*** Elevation of latitude. The distance of a planet with celestial latitude above the plane of the six o'clock circle (or rarely the horizon), when its corresponding longitude on the ecliptic is on the six o'clock circle (or horizon). Its antonym is the depression (*avanati*) of latitude. 168, 169

***kṣoṇī*** The Earth. 1, 27, *see* *bhūmi*

**kh**

***kha*** **(1)** The sky as seen from the observer, or the space in which the Earth and planets are situated. 25, 35

***kha*** **(2)** Probably short for *khamadhya* (middle of the sky), and indicating the prime meridian. 237, *see* *vyoman* (2)

***khaga*** Planet. Literally "that which goes in the sky". 139, 145, 162, 177, 190, 205, 248, 252, 255, 267, *see* *graha*

***khagarkṣa*** Star in space. To translate more literally, "star going in the sky/space". This is an unusual term, as just "*khaga* (that going in the sky" would mean "planet". In *GD2* 27, this term is used in the context of discussing the revolution of planets and stars in the sky. In this case, it could be interpreted as a compound of *khaga* (planet) and *ṛkṣa* (star/asterism). Meanwhile in *GD2* 263 and 265 it is a single word that indicates a point in the circle of sight, corresponding to the position of a planet that would have been observed if the parallax did not exist. 27, 263, 265

***khagola*** Celestial sphere. 11, 12, 14

***khamadhya*** Zenith. Literally "middle of the sky". 181, 183, 185, 189, 215, 251, 253

***khecara*** Planet. Literally "that which moves in the sky". 145, 264, 265, *see* *graha*

***kheṭa*** Planet. 89, 141, 148, 156, 169, 171, 177, 178, 207, *see* *graha*

**g**

***gaṇaka*** Calculator, i.e. one who calculates. Parameśvara uses this term to refer to astronomers well versed in their field. 21, 22, 31, 35, 173, 269

***gati*** Motion, especially the motion of planets along their orbits. Sometimes this term is used in a wider sense to describe the difference in the position of a celestial object even when the object itself is not moving, such as the geocentric parallax which is caused by the position of the observer. 8, 11, 19, 20, 139, 157, 197, 205, 208, 226, 250, 268, 269

***guṇa*** Sine. The term *guṇa* is often used in the sense of "multiplier", but here it is literally "bow-string" or "chord". *see jyā*

***guru*** The planet Jupiter. 130

***gola* (1)** Armillary sphere. 5, 15, 155

***gola* (2)** Sphere, either in the sense of the stellar sphere (*bhagola*) or celestial sphere (*khagola*). There is only one case where it is clearly used in the sense of the latter (*GD2* 189). It is frequently used with the word "rotation (*bhramaṇa*)", in which case it clearly refers to the stellar sphere. However there is ambiguity between a sphere as an instrument and a celestial entity, although it is uncertain whether this ambiguity was intended by Parameśvara or not. 9, 15, 18, 161, 189, 204, 208

***gola* (3)** Sphere as something to work on. Either the armillary sphere or "Sphere" as a subject (in this case I avoid "spherics" which can mean "spherical trigonometry" and use "Sphere" with a capital "S"). I have consistently translated it as a singular, but we cannot rule out the possibility that in compounds it could refer to plural "spheres", e.g. the stellar sphere, the celestial sphere, etc. 21, 65, 68, 173, 245, *see also golavid*

***gola* (4)** Sphere as a shape. 22, 25

***gola* (5)** Celestial hemisphere. The stellar sphere divided north and south by the celestial equator. 41, 45, 109, 205, 206, 214, 216, 217, 240, 246

***goladaṇḍa*** Polar axis. The axis around which the stellar sphere rotates and causes the diurnal motion. In an armillary sphere, it is a rod which connects the inner stellar sphere with the outer celestial sphere, fixed at the two celestial poles. 15

***golavid*** An expert on the Sphere. Someone who is well acquainted with the types of subjects that appear in the *Goladīpikā*. 35, 65, 246, 302

***golānta*** Equinoctial point. Here *gola* stands for "celestial hemisphere", and thus *golānta* is the point when the sun is at the end (*anta*) of both hemispheres. The same term is used in Parameśvara's commentaries on *Ābh* 4.24 and *Ābh* 4.48 (Kern (1874, pp. 86,99)), but is rarely seen in other texts. 37, 89, 158

***graha*** Planet, including the sun and moon. This term often refers to a planet's longitude on the ecliptic rather than the body itself. When Parameśvara explains parallaxes, he uses this term to refer to the observed position, as opposed to the hypothetical position (*khagarkṣa*) where the object would have been if there were no parallax. 1, 13, 133, 134, 151, 165, 202, 258, 263, 264, 271, 278, 301

***grahaṇa*** Eclipse. 268, 269, 276, 281, 282, 301

***grahabhukti*** Daily motion of a planet. 202, *see dinabhukti*

**gh**

***ghaṭa*** The zodiacal sign Aquarius. 52

***ghaṭikā*** One sixtieth of a day. Parameśvara mentions that it should be a sixtieth of the time it takes for the celestial equator to rotate once (i.e. a sidereal day), but most of its usage seems to refer to a sixtieth of a solar day. Literally, *ghaṭikā* refers to a bowl, and bowls with a hole placed in water tanks were used as water clocks. 7, 43, 45, 48, 185, 262

***ghaṭikāmaṇḍala*** Celestial equator. 10, 70, 71, *see ghāṭika*

***ghaṭikāvṛtta*** Celestial equator. 71, 77, 78, 95, 107, 110, 112, *see ghāṭika*

***ghaṭikāvṛttajyā*** Sine in the celestial equator. Locative *tatpuruṣa* compound; its separated form (*ghaṭikāvṛtte jyā*) can be seen in verses 78 and 95. 77

***ghaṭīvalaya*** Celestial equator. 155, *see ghāṭika*

***ghaṭīvṛtta*** Celestial equator. 237, *see ghāṭika*

***ghāṭika*** Celestial equator. The circle on which time is measured. In the armillary sphere it is graduated with 60 marks representing the 60 *ghaṭikā*s in a day. To distinguish it from the terrestrial equator, I have consistently added "celestial" in the translation. It is omitted in the commentary whenever it is evident. 2–5, 9, 17, 185

***ghāṭikavṛtta*** Celestial equator. 10, *see ghāṭika*

**c**

***cakra*** Circle, especially in the sense of 360 degrees. 29, 32, 155, 202, 211, 216, 240

***caturyuga*** A long time period of 4,320,000 solar years. Literally "four *yuga*s", i.e. the *Kṛta-yuga*, *Tretā-yuga*, *Dvāpara-yuga* and *Kali-yuga* combined. 56, 58, 60

***candra*** The moon. 22, 125, 283, 297

***cara*** Ascensional difference. The time difference between sunrise or sunset on a given day and that on an equinoctial day. It can also be defined as "half" the difference between daytime within a full day and exactly half a day. The synonyms *carārdha* or *caradala* are compounds with the Sanskrit word for this "half". In a wider sense, the ascensional difference refers to the time difference which is an arc along the celestial equator, between the moment a celestial object touches the horizon and when it touches the six o'clock circle. It accounts for the geographic latitude of the observer and the declination of the celestial object. In English, sometimes the "ascensional difference" and the "descensional difference" are distinguished, while there is no such nuance in Parameśvara's text. Therefore I shall constantly translate the term "ascensional difference". 102, 109, 110, 174

***carajyā*** Sine of ascensional difference. 74, 78, 83

***caradala*** Ascensional difference. 48, 97, 205, 208, *see cara*

***carasaṃskāra*** Correction of ascensional difference. Refers to the addition or subtraction of the ascensional difference against the measure of signs, depending on their quadrants. 110

***carasaṃskṛti*** Correction of ascensional difference. 98, 183, *see carasaṃskāra*

***carārdha*** Ascensional difference. 78, 101, 109, 207, *see cara*

***carārdhasaṃskṛti*** Correction of ascensional difference. 207, *see carasaṃskāra*

***cāpa*** Arc. The notion of "angles" do not appear, and the corresponding arc, especially that of a great circle, is used instead. 49, 50, 79, 164, 168, 169, 184, 192, 216–218

***cāpin*** Arc. 78, 80, 94, 99

***cāra*** Motion, movement. 55, 125, 139, *see gati*

**ch**

***chādaka*** Eclipsing object. Literally "covering". 301

***chādya*** Eclipsed object. Literally "to be covered". 301

***chāyā* (1)** Shadow in general, caused by some body blocking the ray of light, especially the ray of sunlight. 116, 187, 209–211, 215, 218, 219, 232, 235, 245, 285–288, 292, 296, 298, 299

***chāyā* (2)** [Great] shadow. Sometimes *mahā* (great) is added separately in the verse as an adjective, but it is often completely omitted. 114, 116, 187, 213, 220, 221, 223–227, 230, 234, *see mahācchāyā*

***chāyākoṭi*** Upright of [great] shadow. The component of a great shadow in the east-west direction. 236

***chāyābāhu*** Base of [great] shadow. The component of a great shadow in the north-south direction. 242, 244

***chedyaka*** Drawing. Drawing a diagram on the ground, perhaps in contrast with demonstration on the armillary sphere or mental reproduction. 260

**j**

***jīva*** The planet Jupiter. 18

***jīvā*** Sine. 72, 80, 83, 89–91, 107–109, 147, 161, 184, 185, 222, 237, *see jyā*

***jūka*** The zodiacal sign Libra. 54

***jña*** The planet Mercury. 138, 140–142, 144, 146

***jyā*** Sine. Capitalized to indicate that it is the actual length of a segment in a given circle, and not the modern sine (length of a half chord in a circle whose radius is 1). Unless indicated otherwise, it is the Sine of a great circle with a Radius of 3438. 72, 77, 78, 84, 89, 95, 107, 111, 112, 114, 153, 164, 181, 188, 237

**jh**

***jhaṣa*** The zodiacal sign Pisces. 135

**t**

***tamas* (1)** Darkness. A situation or zone which is devoid of light. 24, 283–285

***tamas* (2)** Umbra. The section of the Earth's shadow (*bhūcchāyā*) at the level of the moon's path (*śaśimārga*), with the form of a disk when seen from the Earth and which causes a lunar eclipse. 282, 283, 296, 297, 299, 300

***tamohantṛ*** The moon. Literally "destroyer of darkness". 283

***tāra*** (Fixed) star as opposed to planets. It refers to individual stars and not asterisms. 35

***tāraka*** (Fixed) star. 20, *see tāra*

***tiryañc*** Transverse. Indicating that a ring is not parallel with others. 3

***tulādhara*** The zodiacal sign Libra. 53

***tejaḥsūtra*** String of light, or ray. 284

***triguṇa*** Radius of a great circle. Literally "Sine of three [signs]". *guṇa* is literally "bow-string" or "chord", hence a synonym of *jyā* (Sine). 111, 140, 162, 166, 190, 272, 275, 280, *see trijyā*

***triguṇavṛtta*** Great Circle. Literally "Circle [whose radius is] a Sine of three [signs]". 300

***trijīvā*** Radius of a great circle. Literally "Sine of three [signs]". 169, 188, *see trijyā*

***trijyā*** Radius. Literally "Sine of three [signs]". Unlike words like *vyāsārdha* (1) which could also refer to radii of circles that are not great circles, *trijyā* and its synonyms always indicate the Radius of a great circle. Among its synonyms, *trijyā* is by far most frequently used. 73, 74, 76, 84, 87, 91, 92, 94, 114, 115, 121, 124, 127, 149, 150, 159, 163, 176, 186, 192, 201, 223, 228, 243, 253, 261, 274, 278, 297

***trijyāmaṇḍala*** Great circle, literally "circle with the Sine of three [signs as radius]". 256

***trijyāvṛtta*** Great circle, literally "circle with the Sine of three [signs as radius]". 80, 255

***tribhajīvā*** Radius of great circle. Literally "Sine of three signs". 238, *see trijyā*

***trirāśi*** Radius of a great circle, literally "[Sine of] three signs". 81, 131, *see trijyā*

***trirāśiguṇa*** Radius of a great circle. *guṇa* is literally "bow-string" or "chord", hence a synonym of *jyā* (Sine), and as a whole the compound means "Sine of three signs". 73, *see trijyā*

***trirāśijyā*** Radius of great circle. Literally "Sine of three signs". 100, *see trijyā*

***tretā*** *Tretā-yuga*, the second in the four subdivisions of the *caturyuga*. 360,000 solar years. 57

***tryaśra*** Trilateral, used exclusively to refer to a right triangle. 85, 90

**d**

***dakṣiṇa*** Southern direction. Literally "right" (because the right side of a man facing east will be in the southern direction). 155

***dakṣiṇottara* (1)** Solstitial colure. Literally "south-north", a circle in the stellar sphere going through the north and south celestial poles. The same expression is used for the prime meridian (see below). 2, 4

***dakṣiṇottara* (2)** Prime meridian. Literally "south-north", a circle in the celestial sphere going through the due north and south. Terminologically, no distinction is made with the solstitial colure. In *GD2* 12, Parameśvara mentions that there is a prime meridian in the celestial sphere "too (*api*)" as the same word *dakṣiṇottara* had been used to refer to the solstitial colure. 12

***daṇḍa*** Polar axis. 5, 6, 88, *see goladaṇḍa*

***daṇḍaka*** Polar axis. 17, *see goladaṇḍa*

***dahana*** Southeast. 231, *see agni*

***diksūtra*** Line of direction. This term appears only in the form "*sadiksūtra* (with *diksūtra*)" as an adjective, which does not tell us how many lines there are. Nor is there any further explanation on this word in *GD2*. However there is an occurrence in Parameśvara's commentary on *GD2* 2.11 which explicitly refers to two lines: "*tajjye diksūtrayugmānte* (these two Sines have the pair of lines of direction [respectively] as their ends)", which is a reference to the "base" Sine (*dorjīvā*) and the "upright" Sine (*koṭi* (2)) in a quadrant. I assume that the line of direction in *GD2* is also the pair of lines that divide a circle into quadrants. To be precise, there should be two lines of direction drawn inside a circle in the north-south (or left-right) and east-west (or front-back) directions and intersecting each other at the center of the circle. 260, 261

***digguṇa*** Sine of direction. *guṇa* is literally "bow-string" or "chord", hence a synonym of *jyā* (Sine). 226, *see digjīvā*

***digjīvā*** Sine of direction. The Sine corresponding to the sun's direction, measured from due east or west. Not to be confused with the base of direction (*digbāhu*). 222

***digjyā*** Sine of direction. 223, *see digjīvā*

***digbāhu*** Base of direction. The sum of the gnomonic amplitude and the solar amplitude (when they are in opposite directions) or their difference (when they are in the same direction). If the values are correct, the base of direction should coincide with the base to be established (*sādhyabāhu*), and therefore one of the strategies in "without-difference" methods is to repeat the computation until there is no difference between the base of direction and the base to be established. 221, 223–225, 227

***dina* (1)** [Full] day, i.e. day and night. A full day of human beings is from sunrise to sunrise. This is known as a civil day, but in *GD2* it could also refer to a sidereal day, or one rotation of the stellar sphere. "Days" for the manes, gods and Brahmā are defined differently, and in each of these cases *dina* and its synonyms can often be interpreted both as a "full day" or only the "day" excluding the night. 16, 55, 65, 205, 207–209, 211, 212

***dina* (2)** Day, from sunrise to sunset as opposed to night. In an expanded definition, it is the period during which the sun is visible. This allows for various measures of "days" corresponding to different locations of entities. 41–43, 45, 46, 48, 59

***dinakara*** The sun. 212, *see arka*

***dinadalacchāyā*** Midday shadow, the great shadow at midday. It may also be interpreted as the great shadow corresponding to the midheaven gnomon (*madhyaśaṅku*), but the two are used in different contexts in *GD2*. The midday shadow appears in problems concerning the sun, while the midheaven gnomon is measured regardless of the sun. 209

***dinapa*** The sun. Literally "lord of day". 18, 42, 212, *see arka*

***dinapati*** The sun. Literally "lord of day". 245, 246, *see arka*

***dinabhukti*** Daily motion. The change in longitude of a planet as observed from the Earth within one day. Parameśvara does not specify whether it is the mean daily motion or true daily motion, but I assume that it is the former throughout *GD2*. Its unit does not need to be specified, but some verses in *GD2* mention that it should be measured in arc minutes. 198, 204

***dinamadhya*** Midday. 70, *see madhyāhna*

***dineśa*** The sun. 1, 64, *see arka*

***dineśāgrā*** Solar amplitude. 244, *see arkāgrā*

***divasa* (1)** Day. 9, 11, 19, 49, 208, 240, *see dina* (1)

***divasa* (2)** [Full] day. 58, *see dina* (2)

***divasadala*** Midday. 213, *see madhyāhna*

***divasabhukti*** Daily motion. 207, *see dinabhukti*

***divākara*** The sun. 47, 246, *see arka*

***divya*** Divine. Adjective for time unit. A full divine year is one solar year and a divine year is 360 solar years. 55, 56

***divyābda*** Divine year. One divine day and night is one solar year, and therefore one divine year is 360 solar years. 55, 57

***diś*** Direction. As a word-numeral it can mean 10, in which case it refers to the 4 cardinal directions, 4 intermediate directions and up and down. In *GD2*, apart from its usage as a numeral, *diś* either refers to a horizontal direction, or is used for mentioning whether two segments are in the same or opposite "directions". 26, 126, 135–138, 152, 153, 164, 167, 189, 191, 194, 199, 218, 223, 224, 226, 228, 231, 245–247

***dīpa*** Lamp. 287–291, *see pradīpa*

***dṛkkarman*** Visibility method. A computation to find the point on the ecliptic which rises or sets simultaneously with a planet on account of its celestial latitude. Astronomers before Parameśvara divides the method in two; one due to geographic latitude (normally called *akṣadṛkkarma* by modern historians) and the other for the "course", i.e. distance from a solstitial point (*ayanadṛkkarma*). Both of them are computed by adding or subtracting a value called the visibility equation (*dṛkphala*). Parameśvara first states this twofold method and then gives a unified method with only one visibility equation. 165, 178

***dṛkkṣepa* (1)** Ecliptic point of sight-deviation, as an abbreviation of *dṛkkṣepalagna*. The midpoint of the ecliptic above the horizon at a given moment. This point is often left unnamed in other treatises, such as the *Āryabhaṭīya*, *Mahābhāskarīya* or the *Sūryasiddhānta*. Brahmagupta uses the expression "*vitribhalagna* (ascending point minus three signs)" to refer to its longitude in *Khaṇḍakhādyaka* 1.5.1, and the commentator Bhaṭṭotpala glosses it as the name of the point itself (Chatterjee (1970, 2, p. 120-121)). Bhāskara II uses the word *vitribha* in his auto-commentary on *Siddhāntaśiromaṇi Grahagaṇitādhyāya* 4.3cd-5 (Chaturvedi (1981, p. 228)). Raṅganātha, a commentator on the *Sūryasiddhānta*, uses the synonym *tribhonalagna* (Burgess and Whitney (1858, p. 286)). All of these authors and commentators use the term *dṛkkṣepa* in the sense of an arc or Sine (see *dṛkkṣepa* (2)).

Therefore Parameśvara's wording is exceptional, but he does have at least one predecessor. Nīlakaṇṭha quotes a verse which he attributes to Mādhava in his commentary on *Ābh* 4.33 which begins with *lagnaṃ tribhonaṃ drikṣepalagnaṃ* (The ecliptic point of sight-deviation is the ascending point minus three signs).

Incidentally, the corresponding English term “nonagesimal” comes from the Latin *nonagesimus* (ninetieth) since it is ninety degrees from the ascending point. Its nuance is close to the Sanskrit *vitribhalagna* or *tribhonalagna*. Nonetheless, I have translated *dṛkkṣepa* literally to respect this difference in meaning. The etymology of this term has not been studied in detail, but I surmise that it comes from the fact that this point represents the inclination (*kṣepa*) of the ecliptic in the sky, or its deviation (*kṣepa*) from the zenith. I have chosen “deviation” since it matches the nuance in *dṛkkṣepa* (2) 179, 188

***dṛkkṣepa* (2)** [Sine of] sight-deviation. Abbreviation of *dṛkkṣepajyā*. The length of arc between the zenith and the ecliptic point of sight-deviation, or its Sine. However there is no instance of this term in *GD2* that exclusively refers to the arc. Most authors and commentators before Parameśvara use *dṛkkṣepa* in this sense, as an arc or its Sine, and not as a point. 191, 271–274

***dṛkkṣepaguṇa*** Sine of sight-deviation. *guṇa* is literally “bow-string” or “chord”, hence a synonym of *jyā* (Sine). 270, *see* *dṛkkṣepa* (1)

***dṛkkṣepajyā*** Sine of sight-deviation. 181, 187, 189–191, 194, 270, 276, *see* *dṛkkṣepa* (2)

***dṛkkṣepalagna*** Ecliptic point of sight-deviation. 180, 181, *see* *dṛkkṣepa* (1)

***dṛkkṣepaśaṅku*** Gnomon of sight-deviation. Elevation of ecliptic point of sight-deviation against the plane of horizon. 187

***dṛkphala*** Visibility equation. An additive or subtractive value applied to the longitude of a planet with a given celestial latitude to find the point on the ecliptic which rises or sets with the planet. 171–173, 192–194, *see also* *dṛkkarman*

***dṛggati*** [Sine of] Sight-motion. This is the name of an arc or its Sine, but Parameśvara only defines its Sine as “the square root of the difference between the squares of the Sine of sight () and the Sine of sight-deviation (*dṛkkṣepa* (2))”. Computationally, it corresponds with the longitudinal parallax (*lambana* (2)). His remarks in the *Siddhāntadīpikā* (T. Kuppanna Sastri (1957, p. 277)) indicate that the Sine of sight-motion cannot be located in a configuration with the horizon and the ecliptic. Hence his notion with other texts that consider that this Sine corresponds to the distance between a planet and the ecliptic point of sight-deviation (*dṛkkṣepa* (1)) like Govindasvāmin’s commentary on the *Mahābhāskarīya*. Parameśvara also differs from many other authors who define the Sine of sight-motion as the elevation of ecliptic point of sight-deviation. 273, 274, 276

***dṛggatijyā*** Sine of sight-motion. 270, *see* *dṛggati*

***dṛgjyā*** Sine of sight. The Sine corresponding to the arc between the zenith and a given planet. 252, 253, 270, 276

***dṛgbheda*** Difference in sight, which occurs when the observer is not in the center of the circle where the planet is. 151, 267

***dṛgvṛtta*** Circle of sight. A great circle having the observer on the surface of the Earth as its center. 248, 266

***dṛṅmaṇḍala*** Circle of sight. 261, 263, 264, 267, 271, 272, *see* *dṛgvṛtta*

***dṛṣṭi*** View. This term is widely used in Sanskrit literature in the sense of “seeing”, “view”, or even “theory”. Parameśvara seems to use it with the nuance of the “line of sight”, representing the angles of direction and elevation of an observer’s viewing. 267

***dairghya*** Length. In *GD2*, it is only used for the length of the Earth's cone-shaped shadow. 294, 296, 298, 299

***daiva*** Divine. 65, *see divya*

***doḥcāpa*** "Base" arc. 211, *see bhujādhanus*

***doḥphala*** Equation of center. It may refer to any planet, but in *GD2* Parameśvara uses it exclusively for the distance of arc between the true sun and mean sun. Literally "base result", referring to its derivation where a right triangle inside the epicycle is drawn with its radius as hypotenuse, and its base is the Sine of the equation of center. 204

***dorjīvā*** "Base" Sine. According to Parameśvara's explanation, this is a Sine in the ecliptic corresponding to an arc between a given point and the nearest equinoctial point. However, depending on the context, other points are used in place of the equinoctial points; e.g. the apogee and perigee or the ascending and descending nodes. 81, 93, 131

***dorjyā*** "Base" Sine. 73, 81, 92, 131, 210, 216, *see dorjīvā*

***dorbhāga*** Degrees of the "base". Referring to a given "base" arc in the ecliptic and its length in degrees. 94, 95, 97

***dyu*** Day. 110, *see dina* (1)

***dyujyā* (1)** [Given] "Sine" in the diurnal circle. 117, *see iṣṭajyā* (2)

***dyujyā* (2)** [Given] Sine in the diurnal circle. 113, *see iṣṭajyā* (3)

***dyujyāvṛtta*** Diurnal circle. Literally "circle with the *dyujyā* (diurnal "Sine") [as radius]". 75, 95, 238, *see dyumaṇḍala*

***dyudala*** Diurnal "Sine". Probably short for *dyudalajyā* or *dyudalajīvā*. 169, 176, 192

***dyudalajīvā*** Diurnal "Sine". The radius of a diurnal circle. I notate "Sine" in quotation marks since it is not a segment whose length changes in accordance with the choice of an arc length (it is always the Sine of a 90 degree arc), but with the size of the circle itself. Synonyms such as *dyumaṇḍalajyā* suggest that *dyu* is an abbreviation of *dyumaṇḍala* (diurnal circle), but at the same time, Parameśvara uses the word *dyujyāvṛtta* (circle of *dyujyā*) to indicate a diurnal circle. Therefore I have opted to translate *dyudalajīvā* and its synonyms without the word "circle". 73

***dyudalajyā*** Diurnal "Sine". 74, *see dyudalajīvā*

***dyumaṇḍala*** Diurnal circle. An imaginary circle representing the diurnal motion of the sun in a given longitude, with the assumption that its longitude (and therefore its declination) does not change. 77, 80, 83, 85, 88, 112, 185

***dyumaṇḍalajyā*** Diurnal "Sine". 76, *see dyudalajīvā*

***dyumaṇḍalajyeṣṭā*** Given "Sine" [in the diurnal circle]. 242, *see iṣṭajyā* (2)

***dyumaṇḍalārdhajyā*** Diurnal "Sine". 76, *see dyudalajīvā*

***dyumaṇḍaleṣṭajyā*** Given Sine in the diurnal circle. 92, *see iṣṭajyā* (1)

***dyuvṛtta*** Diurnal circle. 110, 185, *see dyumaṇḍala*

***dyuvṛttakoṭi*** Upright in the diurnal circle. A segment corresponding to an arc in the diurnal circle between the current position of the sun and its position at midday. It is proportional to the Sine of hour angle in the celestial equator. 236, 238

***draṣṭṛ*** Observer, standing on the surface of the Earth. The observer is the center of the circle of sight (*dṛgvṛtta*). 248–251, 257, 258, 267

***dvāpara*** *Dvāpara-yuga*, the third in the four subdivisions of the *caturyuga*. 240,000 solar years. 57

**dh**

***dhana*** Additive. Indicates that a computed value (usually an equation) should be added to another given value. 170, 202, 203

***dhanus* (1)** Arc. 153, 164, 168, 176, 210, 213, 239, 240, *see cāpa*

***dhanus* (2)** The zodiacal sign Sagittarius. 51, 52

***dhruva*** Pole star. It could also refer to an imaginary star on the celestial south pole, but in *GD2* it always refers to the northern celestial pole. 35, 37, 66, 72

**n**

***nakṣatragola*** Stellar sphere. 11

***nata* (1)** Meridian zenith distance. The arc or Sine of a planet at culmination, usually the sun, measured from the zenith. 213, 214, 218, 225, 226, 228, 246

***nata* (2)** Hour angle, the time left before a heavenly body reaches culmination or elapsed after its culmination. In modern astronomy, the hour angle is defined as "the angle between an observer's meridian (a great circle passing over his head and through the celestial poles) and the hour circle (any other great circle passing through the poles) on which some celestial body lies (Encyclopædia Britannica, *Hour Angle* (2016))". In this definition, the hour angle takes a negative value before culmination. Meanwhile, the hour angle in *GD2* is not expressed as being "negative" or "subtractive". Furthermore, it is expressed in *prāṇa*s and not modern hours. The number of *prāṇa*s are equal to the minutes of arc in the celestial equator corresponding to the body's actual motion in the diurnal circle. 182, 237

***natajīvā*** Sine of hour angle. 238, *see nata* (2)

***natajyā*** Sine of meridian zenith distance. 213, *see nata* (1)

***nati*** Latitudinal parallax. The component of a parallax that is perpendicular to the ecliptic, in other words in the direction of the celestial point's latitude. 268, 269, 273–276

***nabhas*** Sky. 252, *see kha* (1)

***nabhomadhya*** Zenith. Literally "middle of the sky". 185

***nara*** Gnomon. 209, 245, *see śaṅku* (1)

***naraka*** Hell. 39

***nāḍikā*** Time unit, synonym of *ghaṭikā*. Upon its appearance in the translation, I have added "(i.e. *ghaṭikā*s)" for clarity. Literally "hollow tube or stalk" used as a water clock. 9

***nāḍī*** Time unit, synonym of *ghaṭikā*. Upon its appearance in the translation, I have added "(i.e. *ghaṭikā*s)" for clarity. Literally "hollow tube or stalk" used as a water clock. 46, 237

***nābhi*** Center, central point. 88

***nija*** Own. 134, 152, 248, 255, 258, 259, 281, 282, *see sva* (1)

***nijabhūmi*** One's spot, or observer's location, where the geographic longitude and latitude can take any value, as opposed to Laṅkā. 196, 198

***nijabhūmivṛtta*** One's circumference. 201, *see nijabhūvṛtta*

***nijabhūvṛtta*** One's circumference. Equivalent to the modern term "parallel" or "line of latitude" (a circle encompassing the Earth which is parallel to the equator) on which the observer is located. 196, 198–200, *see also bhūvṛtta*

***niś*** Night, from sunset to sunrise. 58

***niśā*** Night. 16, 41, 43, 45, 49, *see niś*

***nīca*** Perigee. The opposite side of the apogee (*ucca*) with 180 degrees of difference in longitude. 278

**p**

***pakṣa*** School or side. A group of people who share the same idea, with the implication that there is an opposing side. It could also be used in the sense of "theory" or "opinion" on a specific matter. In this case, the group of people sharing the same opinion need not be in the same scholarly lineage. 23, 134

***pada*** Quadrant. The four quadrants of a circle in which a given point or arc is situated. 102

***paramakrānti*** Greatest declination. 159, *see paramāpama*

***paramakṣepa*** Greatest deviation of a planet from the ecliptic, i.e. the inclination of its orbit. 126, 127, 131

***paramadyujyā*** Diurnal "Sine" (*dyujyā*) [when the declination is] greatest. Its value is 3141 for a great circle with $R = 3438$. I have interpreted this word as a compound of *paramāpama* and *dyujyā*. This takes into account the synonym *paramāpamasiddhāhorātrātrārdha* (diurnal half[-chord] established by the greatest declination) used by Parameśvara in his commentary on *Ābh* 4.25 (Kern (1874, p. 86)). 91, 92, *see dyudalajīvā* & *paramāpama*

***paramāpama*** Greatest declination, which is the distance of a solstitial point from the equator. 24 degrees. 3, 46, 48, 81

***paridhi*** Circumference of a circle or sphere. 32, 88, 138, 141, 142, 144, 145, 148, 149, 264

***pala*** Geographic latitude. 32, 88, 120, 185, 200, 214, 226, 243, 245, 246, *see akṣa*

***palakarṇa*** Hypotenuse at equinoctial midday. The distance between the tips of a gnomon and its shadow at midday on an equinoctial day. 117

***palaguṇa*** Sine of geographic latitude. *guṇa* is literally "bow-string" or "chord", hence a synonym of *jyā* (Sine). 119, 194, *see akṣa*

***palajīvā*** Sine of geographic latitude. 82, 184, 244

***palajyā*** Sine of geographic latitude. 48, 121, 209, 212, 231, 243, 244, *see akṣa*

***paladhanus*** Arc of geographic latitude. 213, 218, *see akṣa*

***paśca*** Be in the west. 197

***paścāt*** After or later in time. 206

***paścima*** Western direction or west side. 21, 196, 197

***pāta*** Node of planet, especially the ascending node. 125–127, 129, 134

***pātāla*** Nether region. The interior of the Earth. 34

***pāda*** One portion of something divided into four parts. Without context, it usually refers to an exact quarter and in the case of a circle, *pāda* could be translated into quadrant. However, there are cases where four *pāda*s could be unequal, such as the division of of a *caturyuga*. 63, 126, 189

***pārśva*** Side, in the sense of the zone which is neither top nor bottom. 10, 13, 28, 40, 49, 272

***pitṛ*** Manes. Deceased ancestors who are assumed to be on the surface of the moon, at the side which does not face the Earth. One lunar month is equal to a full day of the manes. 42

***pitrya*** Ancestral. 65, *see pitṛ*

***pūrva* (1)** Eastern direction. 4, 12, 13, 27, 103, 199, 200, 236, 237, *see prāñc (1)*

***pūrva* (2)** Before in time, previously. 49, 61, 63

***pūrvāparasūtra*** East-west line. The intersection of the plane of the prime vertical with the plane of the horizon. 121

***pṛthivī*** The Earth. 25, *see bhūmi*

***pṛṣṭha*** Surface. *see bhūpṛṣṭha*

***pradakṣiṇīkṛt*** Clockwise. Literally "towards the right", as seen from the north pole. 7

***pradīpa*** Lamp. A source of light used in place of the sun for explaining the cause of eclipses and their computation. 286, 288, 291

***prabhā* (1)** Shadow. 212, 231, 240, 246, 290, *see chāyā (1)*

***prabhā* (2)** [Great] shadow. 227, 230, *see mahācchāyā*

***prabhābhujā*** Base of [great] shadow. 241, *see chāyābāhu*

***pravaha*** A wind or moving force which makes the stellar sphere rotate constantly around the Earth. 7, 8

***prāñc* (1)** Eastern direction, eastern side or as an adjective, be in the east. Sometimes it may be used in the sense of "front". 14, 17, 18, 35, 135, 180, 196, 197, 203

***prāñc* (2)** Before in time. 69, 206, 232

***prāṇa*** Measurement unit of time. Literally "respiration". 78, 79, 97, 171, 192, 193, 207, 208, 245, 246, *see* *asu*

***pronnati*** Elevation. 190, 191, *see* *unnati*

**ph**

***phala* (1)** Result of a computation, especially a Rule of Three. Sometimes translated "quotient". 33, 124, 227, 233, 234, 256, 295

***phala* (2)** Equation. The correction applied to the longitude of a planet in general, especially on account of the angular difference between the true planet and mean planet due to the apogee. There are two equations for planets which have two apogees ("slow" and "fast"). 146, 147

**b**

***baḍavāmukha*** "Mare's mouth", the entrance to the underworld imagined to be located at the south pole. The entry in Monier-Williams' dictionary (Monier-Williams (1899)) is *vaḍabāmukha*, but under the entry *vaḍaba* (mare), he lists *baḍava* as one of its variants. 39

***bāhu* (1)** Base of a right triangle. 90, 106, 268, 270, 273, 276, 288–290

***bāhu* (2)** Either or both of the two bases involved in an "without-difference" method, i.e. base of direction (*digbāhu*) and base to be established (*sādhyabāhu*). 230, 234

***bāhudhanus*** "Base" arc. 99, *see* *bhujādhanus*

***bindu*** Dot. A point drawn in a diagram. 219, 263–266

***bimba*** Orb or disk. Referring to the appearance of a body as a circular shape, including spheres. 279, 280, 291, 297, 300

***bimbadala*** Half-[diameter] of an orb or a disk. 301, *see also* *ardhaviṣkambha*

***budha*** The planet Mercury. 18, 127, 130, 137, 143, 147

***brahmā*** Brahmā, creator of the world. A day and night of Brahmā each consist of a thousand *caturyuga*s, i.e. 4,320,000,000 solar years and is also called a *kalpa*. 64

***brāhma*** of Brahmā. 65, *see* *brahmā*

**bh**

***bha* (1)** (Fixed) stars as opposed to planets. From the verb *bhā* (to shine). However, it is unclear in general whether Parameśvara intends to distinguish "stars"from "asterisms" when he uses this word. Sometimes it is obvious from the context that *bha* refers to a zodiacal sign, but otherwise I translate the term to "star". 13, 18, 66, 96, 169, 174, 180, 217

***bha* (2)** Zodiac. Short for *bhacakra* or *bhavṛtta*. 136, 144, 145, *see* *bhacakra*

***bhakūṭa*** Ecliptic pole. The two points that are separated from the ecliptic by 90 degrees. Literally "summit of zodiacal signs". Parameśvara also mentions that they are the "conjunction of all signs (*sarvarkṣāṇāṃ saṃpāta*)", suggesting that zodiacal signs could be seen as segments of the stellar sphere, resembling the segments of an orange. 155–159, 161, 163, 166, 189

***bhakūṭonnati*** Elevation of ecliptic pole. Its distance from the plane of the six o'clock circle (rarely the horizon) at a given time. The "ecliptic pole" (*bhakūṭa* or *rāśikūṭa*) is in the singular whenever the compound is decomposed, and refers to either one of the northern ecliptic pole or the southern ecliptic pole which is elevated. Words like "in the north" or "in the south" may be added to signify which pole is above the six o'clock circle (or horizon). 189

***bhagola*** Stellar sphere. A set of rings in the armillary sphere attached to an axis inclined at an angle corresponding to the local latitude. Its rotation represents the diurnal motion of celestial objects, and the term itself can be used to refer to the diurnal motion by saying "rotation (*bhramaṇa*) of the stellar sphere". 1, 6, 7, 98, 108

***bhacakra*** Zodiac. The zone around the ecliptic on which the motion of planets are measured. It is geocentric as opposed to "fast" (*śaighra*) and "slow" (*māndа*) orbits (*kakṣyā* (2)). This term is consistently used in the context of latitude, and therefore must be differentiated from the ecliptic (*apamaṇḍala*). A planet with a latitude deviating from the ecliptic can still be considered as moving on the zodiac. 139, 150

***bhacakrakendra*** Center of the zodiac, which by definition, is the center of the Earth. 143, *see also* *bhacakra*

***bhamāna*** Measure of a sign or measure of signs. 96, *see* *māna* (2)

***bhamiti*** Measure of a sign or measure of signs. 96, 100, *see* *māna* (2)

***bhavṛtta*** Zodiac. 152, *see* *bhacakra*

***bhā* (1)** Shadow. *see* *chāyā* (1)

***bhā* (2)** [Great] shadow. 227, *see* *mahācchāyā*

***bhāga* (1)** Degree. 32, 102, 129, 182, 262, 263, 278, *see*

***bhāga* (2)** Portion. 15, 28, 38, 61, 111, 113, 167, 298, *see* *aṃśa* (2)

***bhānu*** The sun. 55, 84, 122, 138, 195, 202, 205, 211, 217, 220, 239, 240, 277, 279, 283, 291, 294, *see* *arka*

***bhukti* (1)** Daily motion. 196, 257, *see* *dinabhukti*

***bhukti* (2)** Motion of a celestial point in general. It is unclear whether it refers to the motion measured in minutes like the daily motion, or motion in general as in *gati*. 129

***bhujajyā*** "Base" Sine. 90, 91, 186, *see* *dorjīvā*

***bhujā* (1)** Base of a right triangle. 76, 85, 86, 105, 168, 235, 236, 269

***bhujā* (2)** "Base" [Sine]. 89, 99, 127, *see* *dorjīvā*

***bhujā* (3)** Two bases, always in the form *bhujādvaya* (3). 230

***bhujādvaya* (3)** Two bases, i.e. the base of direction (*digbāhu*) and base to be established (*sādhyabāhu*). 230

***bhujādhanus*** "Base" arc. An arc between a given point on the ecliptic and the nearest equinoctial point. 89, 239

***bhujāphala*** Equation of center. 204, *see doḥphala*

***bhū* (1)** The Earth. 13, 32, 249, 250, 253, 260, 285, 291, 294, 296, *see bhūmi*

***bhū* (2)** Ground, especially referring to a distance measured along the ground. 286, 288, 290, 291

***bhūkakṣyā*** Circumference of the Earth. 38, *see kakṣyā* (3)

***bhūgola*** Earth's sphere. Referring to the Earth itself, but stressing its form as a sphere. 33, 36

***bhūcchāyā*** Earth's shadow, extending towards the opposite side of the sun from the Earth in a conic form. Its segment at the level of the moon's path is the umbra (*tamas* (2). 282, 293, 294

***bhūjyā*** Earth-Sine. 74, 78, 83, 85, 86, 110, 113, 120, 164, *see kṣitijyā*

***bhūparidhi*** Earth's circumference. This term and its synonym is only used when drawing a diagram, unlike *bhūpṛṣṭha* (Earth's surface). 293

***bhūpṛṣṭha*** Earth's surface. Used in the context of parallaxes to indicate the observer's position, separated from the Earth's center (*bhūmadhya*) by its radius. In this case it is a single point. It can also be used to describe the surface as a zone, or its area. 8, 29, 33, 259

***bhūmadhya*** Earth's center, as a single point. 29, 136, 249–251

***bhūmi*** The Earth. *Bhūmi* and its synonyms are used for referring to the Earth as a spherical body unless it appears in a compound, in which case it can have other nuances such as "ground". 6, 26, 31, 32, 36, 133, 274, 279, 292

***bhūmija*** The planet Mars. 130, *see kuja*

***bhūvāyu*** Wind of Earth. It blows at surface level, below the *pravaha* wind and in a different direction. 8

***bhūvṛtta*** Circumference of the Earth. It is contrasted with *nijabhūvṛtta* (one's circumference), and in this context it is the circumference of the terrestrial equator. 30, 201

***bhṛgu*** The planet Venus. 18, 137, 143

***bhṛgusūnu*** The planet Venus. 142, 144

***bhauma*** The planet Mars. 128, 129, 133, 134, 141, *see kuja*

***bhrama*** Revolution of objects in space. 27

***bhramaṇa*** Revolution, often used as a counting unit. It can also refer to a fractional part of a revolution. 9, 10, 15, 77, 78, 161, 198, 204, 208

**m**

***maṇḍala* (1)** Circle. Used alone as a synonym of 360 degrees, and used in compounds for various circles that can be located on the armillary sphere. 10, 12, 239

***maṇḍala* (2)** Disk. 22, 23, 40, *see bimba*

***madhya* (1)** Middle. For a segment or arc, it is the midpoint; For a circle or sphere it is usually either the exact center (*kendra*) or any point inside it. Depending on the context, it could also be a point on the circumference or surface. For cases where it is used with words meaning "sky", see *khamadhya*. 3, 5, 6, 38–40, 72, 88, 108, 136, 137, 149, 150, 179, 180, 219, 222, 232, 252, 260, 266, 272

***madhya* (2)** Mean as opposed to true/corrected (*sphuṭa*), referring to the mean planet, its motion or orbit. A mean motion is the constant revolution of the mean planet. 139, 147, 149

***madhya* (3)** Midheaven. Short for *madhyavilagna*. 182, 183

***madhya* (4)** Midheaven gnomon. Used only once in *GD2* in the form *madhyākhyaśaṅku* with *ākhya* (called), and therefore may be considered as a variation of *madhyaśaṅku* rather than *madhya* alone conveying this meaning. 187

***madhya* (5)** Middle [of sky], i.e. zenith. 213

***madhyacchāyā*** Midday shadow. 212, *see dinadalacchāyā*

***madhyajīvā*** Midheaven Sine. 184, *see madhyajyā*

***madhyajyā*** Midheaven Sine. The Sine corresponding to the distance of the midheaven from the zenith. 186, 194

***madhyama*** Mean. 195, 202, 203, *see madhya* (2)

***madhyavilagna*** Midheaven. The intersection of the ecliptic and the prime meridian above the horizon. Today it is typically translated the "meridian ecliptic point" (e.g. Bhattacharya (1987, p. 63)) but I choose the word "mid"heaven which corresponds to *madhya*, which Parameśvara relates to *madhyāhna*. 184, 186, 188, 194

***madhyaśaṅku*** Midheaven gnomon. The distance between the plane of horizon and the intersection of the prime meridian and the diurnal circle. Parameśvara relates the word *madhya* with midday (*madhyāhna*), but this segment is used in the visibility method which is not directly linked with the sun itself. 186–188

***madhyaśaṅkubhujā*** "Base" of the midheaven gnomon. Parameśvara explains that his is the "base" Sine of the ascending longitude decreased by the midheaven's longitude. The word "base" is not being used in the sense of measuring the arc from an equinoctial point (the intersections of the ecliptic and the celestial equator), but from the ascending or descending point (the intersections of the ecliptic and the horizon). If the midheaven is closer to the descending point, the difference between their longitudes should be taken instead. Visually, this segment is the midheaven gnomon projected on the plane of the ecliptic. 186, 187

***madhyāhna*** Midday, i.e. noon, the moment when the sun culminates at due south. 44, 182

***madhyāhnabhā*** Midday shadow. 228, *see dinadalacchāyā*

***manu*** *Manu* as a time period of 71 *yuga*s (30,672,000,000 solar years). *Manu* refers to 14 mythical progenitors who rule the world in order, each for a period of *manu*. 59–62

***manda* (1)** “Slow”. Referring to the “slow” apogee (*mandocca*) which moves slower than the mean planet along the ecliptic or the planet whose position has been corrected by this apogee. In a compound it can be used adjectively to indicate anything related to the “slow” apogee. 137, 138

***manda* (2)** The planet Saturn. Literally “slow”, referring to its motion along the ecliptic. 18, 134, 136

***manda* (3)** Literally, slow. 20

***mandaśruti*** “Slow” radial distance. For Mars, Jupiter and Saturn it is the distance between the planet on the “slow” eccentric circle and the center of the “fast” eccentric circle. For Mercury and Venus it is the distance between the planet on the “slow” eccentric circle and the center of the Earth. 128, 146

***mandasphuṭa*** “Slow” corrected [planet]. The longitude of a planet after the “slow” equation has been applied. 127, 142–144

***marut*** Wind or moving force. It is consistently used with *pravaha* in *GD2*. 7, 8

***mahācchāyā*** Great shadow. The distance between the foot of the great gnomon and the observer. 70, 181

***mahāśaṅku*** Great gnomon. The elevation of the sun against the horizon, expressed as a Sine in a great circle. 114, 117

***māna* (1)** Measure. Referring to a value, especially length, of a given segment, arc, etc. Sometimes it is used together with a measurement unit for specification. 1, 31, 32, 36, 96, 235, 240, 254, 255, 265, 285, 296, 298

***māna* (2)** Measure in a narrower sense, referring to a point or arc in the celestial equator that corresponds to a specific longitude or arc in the ecliptic. Basically, their rising above the horizon is concerned: The measure of a given longitude is the time it takes for an arc in the ecliptic between that longitude and the nearest equinoctial point to traverse the horizon in the east. The measure of a zodiacal sign is the time since it starts ascending above the horizon until it completely rises. Sometimes the measure could be taken when the celestial points or arcs set below the western horizon, or even more rarely, when they culminate at the prime meridian. 93, 101, 183

***mānuṣa*** Human. Adjective for time units, referring to the regular solar day and year in contrast to time units of the manes, gods and Brahmā. 55, 65

***mānda*** “Slow”, an adjective used for anything related to the “slow” (*manda* (1)) apogee. 133, 137, 141, 144, 146, 147, 152

***māndakendra*** Center of the “slow” [eccentric circle]. 138

***mārga*** Path, especially that of the moon, referring to its path in space. This term is used in the context of lunar eclipses, where the actual location of the moon with its true radial distance (and not its mean distance on its “orbit”) is concerned. 282, 300

***mita*** Measure in general. 31, 33, 62, 209, 212, 250, 266, 286, *see māna* (1)

***miti* (1)** Measure in general. 133, 255, 274, 275, 294, *see* *māna* (1)

***miti* (2)** Measure of an arc. 101, 102, 182, 183, *see* *māna* (2)

***mithuna*** The zodiacal sign Gemini. 47

***mīna*** The zodiacal sign Pisces. 53

***muni*** Sages, in plural (*munayaḥ*). It appears once in *GD2*, where it refers to the seven sages that can also be identified with the seven stars of the big dipper. 66

***mṛga*** The zodiacal sign Capricorn. 50

***medinī*** The Earth. 34, *see* *bhūmi*

***meru*** Mount Meru. It is imagined to be on the north pole, and thus it is often used in the sense of "terrestrial north pole". 30, 35–37, 39, 40, 54, 66

***meṣa*** The zodiacal sign Aries. Its starting point corresponds to the vernal equinox if no precession or trepidation is taken into account. 41, 53, 54, 232

**y**

***yama*** The zodiacal sign Gemini. 46, 50, 67

***yavakoṭi*** Yavakoṭi, an imaginary place on the equator, 90 degrees east of Laṅkā. 44

***yāmya*** Southern direction, southern side. Literally "[direction] of the god Yama". 2, 12–17, 45, 75, 84, 113, 126, 156, 160, 165, 177, 205, 206, 211–217, 221, 225, 236, 240, 246

***yāmyottara*** Prime meridian. Used only once in *GD2* in the form *yāmyottarākhyavṛtta* with *ākhya* (called), and therefore may be considered as a form of *yāmyottaravṛtta*, rather than *yāmyottara* itself conveying the meaning of "prime meridian". 71, *see* *dakṣiṇottara* (2)

***yāmyottaravṛtta* (1)** Solstitial colure. Literally "south-north circle". 5, 154, *see* *dakṣiṇottara* (1)

***yāmyottaravṛtta* (2)** Prime meridian. Literally "south-north circle". 71, 182, *see* *dakṣiṇottara* (2)

***yāmyodaksūtra*** North-south line. One of the two lines of direction drawn inside a circle, extending north and south and going through the center of the circle. The word order in Sanskrit is south (*yāmya*) - north (*udak*), but I follow the natural order in our English translation. The direction of the line does not necessarily reflect the actual cardinal directions. This term can also be translated right (*yāmya*) - left (*udak*), which refers to the direction of the line as seen from the person drawing the diagram. 261, 262

***yukti*** Grounding or Reasoning. Parameśvara often uses this word to refer to a proportion or Rule of Three behind a given computation. 69, 98, 110, 119, 188, 198, 204, 233

***yuga*** Either a *caturyuga* or any of its subdivisions. Every occurrence in *GD2* is the former. 56, 59, 62, 63

***yojana*** A measure of distance. Often used in contrast with divisions of a circle such as *liptā*. 8, 19, 30, 31, 33, 196, 254–256, 266, 274, 275, 277, 279, 294, 297

**r**

***ravi*** The sun. 4, 8, 24, 64, 65, 121, 197, 205, 206, 208, 209, 212, 213, 216–218, 222, 280, 281, 285, 291, *see arka*

***ravidoḥphala*** Sun's equation of center. 202, 203, *see doḥphala*

***raviparidhi*** Sun's circumference. Term used during the drawing of a diagram. 293

***rātri*** Night. 45, 58, *see niś*

***rāśi*** Zodiacal sign. One of the twelve zodiacal signs, or an arc length of a zodiacal sign, i.e. 30 degrees. Unlike other synonyms, this word is never used in the sense of individual stars or asterism other than zodiacal signs. 41, 50, 55, 93, 96, 154, 171, 183

***rāśikūṭa*** Ecliptic pole. 154, *see bhakūṭa*

***rāśikūṭonnati*** Elevation of ecliptic pole. 167, *see bhakūṭonnati*

***rāśimiti*** Measure of a sign or measure of signs. 96, *see māna* (2)

***rudra*** Northeast. 245, *see śiva*

***rekhā*** Geographic prime meridian, abbreviation of *samarekhā*. 196

***romaka*** Romaka, an imaginary place on the equator likely inspired by the Roman empire, 90 degrees west of Laṅkā. 44

**l**

***lakṣa*** A "lakh", or a hundred thousand. 33

***lagna* (1)** Ascending point. The rising point of the ecliptic, i.e. its intersection with the horizon in the east. In some contexts the same term can refer to the ascendant, i.e. the zodiacal sign (i.e. an arc instead of a point) which is rising. Literally "adhering". 51, 53, 180–182, 193

***lagna* (2)** Adhering point or ecliptic point. Any given point on the ecliptic which is its intersection with another circle, including the ascending point (also *lagna* or *udayalagna*) and descending point (*astalagna*) which are the intersections of the ecliptic with the horizon in the east and west. 158, 179

***laghumati*** Novice. Literally "light-minded" or "one having weak understanding". 1, 68

***laṅkā*** Laṅkā, an imaginary place on the Earth which is the intersection of equator and prime meridian. 37, 38, 43, 44, 94, 95, 99, 182, 183, *see also laṅkodaya*

***laṅkodaya*** Rising [time of a celestial point or arc] at Laṅkā. The length of arc in the celestial equator between a reference point (usually the vernal equinox) and the point on the equator which rises at the same moment with the given celestial point. It corresponds to the right ascension in modern astronomy. 159, 161, 173

***lamba* (1)** Co-latitude. 88, 201, 243, 266, *see avalambaka*

***lamba* (2)** Longitudinal parallax. Abbreviated form for *lambana* (2), probably for metric purpose. 274

**lambaka** Co-latitude. 82, 84, 87, 114, 115, 119, 124, 176, *see avalambaka*

**lambakajīvā** Sine of co-latitude. 70, 74, *see avalambaka*

**lambakajyā** Sine of co-latitude. 72, 176, *see avalambaka*

**lambajīvakā** Sine of co-latitude. 244, *see avalambaka*

**lambajyā** Sine of co-latitude. 50, 52, 243, *see avalambaka*

**lambana (1)** Parallax. Literally "hanging down". The geocentric parallax of a celestial point due to the observer being on the surface of the Earth and not at its center. It is resolved into the longitudinal parallax and latitudinal parallax, where the former element can also be called *lambana*. Perhaps for avoiding confusion, Parameśvara sometimes uses the expression "whole (*sarva*) or entire (*nikhila*) parallax". 250–255, 258, 259, 265, 267, 268, 271, 272, 276

**lambana (2)** Longitudinal parallax. The component of a parallax that is in the direction of the ecliptic. 268, 269, 273, 275, 276

**liptā** Minute of arc. 19, 79, 193, 208, 254, 256, 259, 265, 275, 280, *see kalā*

**liptikā** Minute of arc. 79, 132, 156, 169, 254, 255, *see kalā*

**v**

**vakra** Retrograde motion. Literally "crooked", "not straight". 21

**varṣa** Year. 55, 56

**vasudhā** The Earth. 28, *see bhūmi*

**vahni** Southeast. 232, *see agni*

**vikṣipta** Having a latitude. Literally "thrown away". 67

**vikṣepa (1)** Deviation of a planet from the ecliptic or the inclination. 130, 150, 175, 190, 191, 269, *see kṣepa (1)*

**vikṣepa (2)** Latitude (celestial). 150, 153, 163, 166, *see kṣepa (2)*

**vikṣepabhava** Portion of a planet's declination "produced by the celestial latitude". The difference between the Sine of declination itself and the corrected Sine of declination. To be precise, this value is neither an arc nor a Sine of a specific arc. 175, 176

**vikṣepamaṇḍala** Inclined circle. The orbit of a planet that is inclined against the ecliptic and causes latitude, and possibly a ring on the armillary sphere used for its demonstration. 125

**vidhi (1)** Brahmā. 58, 59, 62, 220, *see brahmā*

**vidhi (2)** Rule. Within the usages in *GD2*, it refers to conditions when a given value becomes additive or subtractive. 206

**vidhu** The moon. 23, 219, 279

**vimaṇḍala** Inclined circle. 126, *see vikṣepamaṇḍala*

**vilagna** Ascendant. 172, 186, *see lagna (1)*

***vilambana*** Parallax. 267, *see* *lambana* (1)

***vilomaga*** Go retrograde. *vilomaga*, literally "against the hair", is used here in the sense of "opposite direction against the revolution of planets". 125

***vivara*** Gap or difference. "Gap" is a literal translation and adopted when the word is used for referring to the distance between points, lines or planes. However the word can also be used for the difference between two values. 32, 60, 61, 71–73, 76, 93, 101, 104, 175, 181, 185, 186, 188, 198, 202, 213, 218, 221, 224, 228, 230, 258, 270, 286, 289–291, 298, 301

***viṣuvat*** Literally "in the middle" or "central". The equinoctial colure alone, or collectively the three equal division circles, i.e. the equinoctial colure, solstitial colure and the equator. 4, 5

***vihaga*** Planet. Literally "that which goes in the sky". 21, 89, 160, 195, 250, 251, 254, 257, 267, 269, *see* *graha*

***vīṇā*** The zodiacal sign Gemini. 51, 232

***vṛtta*** Circle in general, often used in compound with *ākhya* ("called") to signify the name of a specific circle. At times it can refer to the circumference. 2–6, 10, 23, 71, 79, 135, 136, 138–140, 149, 152, 220, 226, 248, 260–262, 292, *see also* *maṇḍala* (1)

***vṛścika*** The zodiacal sign Scorpio. 52

***vṛṣa*** The zodiacal sign Taurus. 231

***vṛṣabhā*** The zodiacal sign Taurus. 51, 52

***vega*** Impetuosity which causes the diurnal motion of the planets. From the verb *vij* (to move quickly, to speed, to tremble). 18

***vyāsa*** Diameter. 266, 279, 280, 294, 296, 297, 299

***vyāsadala*** Half-diameter. 291, *see* *ardhaviṣkambha*

***vyāsārdha* (1)** Half-diameter. The radius of a circle of sphere with any size. 88, 140, 249, 250, 253, 274, 291, *see* *ardhaviṣkambha*

***vyāsārdha* (2)** Half-diameter as the Radius of a great circle. It is more common to use *trijyā* (Sine of three [signs]) or its synonyms in *GD2*, and the choice of this Sanskrit term in a verse suggests a strong influence from the *Āryabhaṭīya*. 128, 187

***vyāsārdhamaṇḍala*** Great circle. Here the word *vyāsārdha* refers to the Radius of a great circle. 83, 95

***vyoman* (1)** Sky or space. 72, 293, *see* *kha* (1)

***vyoman* (2)** In its sole usage, the commentary paraphrases it as *khamadhya* (middle of sky, i.e. zenith). From the context, it is likely that it further indicates the circle that goes through the zenith and the celestial poles, i.e. the prime meridian. The intersection of the prime meridian and the celestial equator is the point from where the hour angle is measured. 245

**ś**

***śaṅku* (1)** Gnomon, an instrument for measuring the shadow. 212, 231, 232, 246, 286–288, 290–292

***śaṅku* (2)** [Great] gnomon. Sometimes *mahā* (great) is added separately in the verse as an adjective, but it is often completely omitted. 69, 70, 103–105, 114–116, 119–121, 123, 188, 210, 219, 230, 242, 259, *see mahāśaṅku*

***śaṅkvagra* (1)** Gnomonic amplitude, the distance between the foot of a great gnomon and the rising-setting line. 104, 105, 119, 120, 122, 123, 210, 221, 230, 242–244

***śaṅkvagra* (2)** Used literally, "the extremity (i.e. tip) of a gnomon". 289

***śambhu*** Northeast. 232, *see śiva*

***śaśin*** The moon. 18, 24, 40, 67, 280, 281, 283, 298, 300

***śaśimārga*** Path of the moon. 298, 299, *see mārga*

***śiras*** Tip. Especially the tip of something pointed. Literally "head". 104, 221, 266, 287, 292

***śiva*** Northeast. The god Śiva is regarded as the guardian of this direction, and therefore any of his names can stand for northeast. 231

***śiśirakaramārga*** Path of the moon. 297, *see mārga*

***śiśiradīdhiti*** The moon. 300

***śīghra* (1)** "Fast". Referring to the "fast" apogee (*śīghrocca*) which moves faster than the mean planet along the ecliptic or the planet whose position has been corrected by this apogee. In a compound it can be used adjectively to indicate anything related to the "fast" apogee. 128, 136, 143

***śīghra* (2)** Literally, fast. 20, 257

***śīghrajyā*** ["Base"] Sine of the "fast" [anomaly]. Short for *śīghrakendrabhujajyā*. This is the parameter for computing the equation caused by the "fast" apogee. 134

***śīghraśruti*** "Fast" radial distance. For Mars, Jupiter and Saturn it is the distance between the planet on the "fast" eccentric circle and the center of the Earth. For Mercury and Venus it is the distance between the planet on the "fast" epicycle and the center of the "slow" eccentric circle. 146

***śīghrasphuṭa*** "Fast" corrected [planet]. 142, *see śaighrasphuṭa*

***śīghrocca*** "Fast" apogee, probably referring to the fact that it moves faster than the mean planet along the ecliptic. 127

***śukra*** The planet Venus. 138, 140, 141

***śukla*** Bright half of a lunar month, from new moon to full moon. 42

***śaighra*** "Fast", an adjective used for anything related to the "fast" (*śīghra* (1)) apogee. 133, 137, 138, 140, 141, 144, 146

***śaighrasphuṭa*** “Fast” corrected [planet]. The longitude of a planet after the “fast” equation has been applied. There is one case in *GD2* where Parameśvara uses the augmented form *śaighra* together with *sphuṭa*, and another case where he uses the ordinary form *śīghra* (1). By contrast, for the “slow” corrected planet he always uses *mandasphuṭa* but never *māndasphuṭa* with the augmented form. This trend can be seen in his *Dṛggaṇita* where he mixes *śaighrasphuṭa* and *śīghrasphuṭa* but is consistent with *mandasphuṭa*. 144

***śruti* (1)** Hypotenuse of a right triangle. 88, 90

***śruti* (2)** Radial distance. 143, 297, *see* *karṇa* (2)

***śrutimārga*** Path of the radial distance. A line drawn from the Earth’s center to a planet (either mean or true). The distance between these two points is the radial distance. However, according to Parameśvara’s usage, the “path” itself may be extended beyond the planet. 142, 144, *see also* *karṇa* (2)

**s**

***saṃdhyā*** Twilight as a period of time, literally “junction”. It occurs at the beginning and end of a *kalpa* and between *manu*s, and has a timespan of six fifteenth of a *caturyuga*, or 1,728,000 solar years. Each twilight consists of two parts, and the latter half is also called a twilight. 59–61

***saṃdhyāṃśa*** Portion of twilight. The first half of a twilight. 61, *see* *saṃdhyā*

***saṃmita*** Measure in general. 48, 245, 248, *see* *māna* (1)

***saṃsthāna*** Configuration. Literally “standing together”. Refers to the position and orientation of an object, circle or their combination. 68

***samacchāyā*** Prime vertical shadow. The great shadow when the sun is located on the prime vertical (*samamaṇḍala*). 209

***samamaṇḍala*** Prime vertical. A circle in the celestial sphere going through the due east, zenith, due west and nadir. Literally “even-circle”. 12, 71, 121, 122, 209

***samamaṇḍalaśaṅku*** Prime vertical gnomon. The great gnomon when the sun is situated on the prime vertical (*samamaṇḍala*). 123

***samarekhā*** Geographic prime meridian, literally “equal line”. 195–198

***samaśaṅku*** Prime vertical gnomon. 124, *see* *samamaṇḍalaśaṅku*

***savitṛ*** The sun. 44, 247, *see* *arka*

***sādhya*** Established [base], with *bāhu* (base) being implied. Used only once in this sense in the passage “when the base of direction is ... from that called the established *digbāhau sādhyākhyād* ...”. 225, *see* *sādhyabāhu*

***sādhyabāhu*** Base to be established. The north-south component of the great shadow. It is established by multiplying the great shadow by the Sine of direction (*digjīvā*) and dividing by the Radius. If the values are correct, the base to be established should coincide with the base of direction (*digbāhu*). 223–225

***sāyana*** With passage. As a noun, it refers to a system which takes into account the "passage" (*ayanac*). As an adjective, it signifies that a given longitude is measured in this system, where the starting point is the vernal equinox and not 0°of Aries. In modern terminology, it is the tropical longitude as opposed to sidereal. 101, 211, 217, 219

***siṃha*** The zodiacal sign Leo. 51

***sita*** The planet Venus. 127, 130, 146, 147

***siddhapura*** Siddhapura, an imaginary place on the equator, at the opposite side of Laṅkā. 44

***sudhī*** Wise one. Contrasted with calculators (*gaṇaka*). Probably refers to the authorities of the Purāṇas. 31, 34

***surapa*** East. 222, *see indra*

***sūtra*** Line or string. This term often appears in relation to diagrams. Sometimes the word is used explicitly in the sense of "string" with the length of a given radius for drawing a circle. Elsewhere, the word could indicate a line drawn in a diagram or a string place in the diagram; my interpretation is that it refers to a drawn line. The word*sūtra* does not appear alone in the context of explanations involving three dimensional configurations where armillary spheres could have been involved, and only rarely in compounds (*astodayasūtra* and *pūrvāparasūtra*). 141–144, 148, 220, 261, 262, 264, 266, 287, 289, 293

***sūrya*** The sun. 10, 66, *see arka*

***sūryāgrā*** Solar amplitude. 87, 117, 228, 241, 242, *see arkāgrā*

***saumya* (1)** Northern direction, northern side. Literally "related to the Soma ritual". 2, 16, 35, 109, 113, 118, 121, 126, 160, 177, 205, 206, 212, 214–217, 221, 225, 226

***saumya* (2)** Related to the moon (*soma*). 67

***sthūlonnati*** Crude elevation, here for the elevation of ecliptic pole. It is an approximate value which uses the approximates an arc along the celestial equator with an arc along the ecliptic for making the procedure easy. 162

***spaṣṭa*** True. Conveys the sense of apparent / true to observation. Sometimes it can be contrasted with *sphuṭa*, which refers to a true value as the result of correction with computation. In *GD2* the only occurrence of *spaṣṭa* is as an adjective in "true declination", which refers to the actual distance of a celestial point from the equator, whereas the "corrected (*sphuṭa*) declination" is a simple sum or difference of the declination and latitude without taking into account their alignment. In Sanskrit astronomical texts in general, *sphuṭa* and *spaṣṭa* are usually treated as synonyms (cf. Bhattacharya (1987)), but some differences are observable. Most notably, *spaṣṭa* occurs significantly less often. Sewell, Dīkshita, and Schram (1896, p. 11) argues that "apparent" is a suitable translation for *spaṣṭa* but does not refer to *sphuṭa*. Michio Yano (personal communication, 2016) points out that Bhaṭṭotpala's commentary on Brahmagupta's *Khaṇḍakhādyaka* contains some cases worth inspection. For example, verse 2.18 refers to the planet's longitude after every required correction has been applied as "*spaṣṭa*", but Bhaṭṭotpala paraphrases this with "*sphuṭa-graha* (planet)" (Chatterjee (1970, 2, p. 82)). A thorough survey and reflection on these two words are wanting. 164

***sphuṭa*** Corrected or true. Used as an adjective, indicating that the position or distance of a planet has been corrected, in contrast to "mean" (*madhya* (1)). Often the word "planet" or "sun" is omitted and needs to be supplied. For intermediate states where some equations (*phala* (1)) have been applied to the mean planet but further equations are to be applied, I use the translation "corrected". When there is no more correction to be applied, I use "true". In addition, there are cases where *sphuṭa* conveys the meaning of "apparent", referring to something observed instead of computed. The usage of the word *sphuṭa* requires more inspection, especially in comparison with *spaṣṭa*. 73, 128, 134, 135, 139, 143, 145, 150, 151, 178, 187, 202, 243, 244, 278

***sphuṭakarman*** Correction method. Applying an equation to the uncorrected or intermediary corrected planet. 152

***sphuṭakṣepa*** Corrected latitude. The distance of a celestial point from the diurnal circle corresponding to its (uncorrected) declination. 163

***sphuṭakhecara*** True planet. 159, *see sphuṭagraha*

***sphuṭagraha*** True planet. Referring to the longitude of the mean planet corrected for the "slow" and "fast" apogees. 148

***sphuṭatīkṣṇāṃśu*** True sun or its longitude. 203, *see sphuṭaravi*

***sphuṭayojanakarṇa*** Corrected radial distance in *yojana*s. This compound is used when referring to the exact distance of the sun or moon from the Earth, in the context of eclipses. 280, 291, 294, 296, 298

***sphuṭaravi*** True sun or its longitude. A true sun can be established by correcting the mean sun with the equation of center, but it can be also be computed from observation, using a gnomon. 203

***sphuṭaśaṅku*** Corrected [great] gnomon. The great gnomon of a given planet reduced by its parallax, which represents its great gnomon as seen from the center of the Earth. 259

***sphuṭāpamajyā*** Sine of corrected declination. 153, *see sphuṭāpama*

***sphuṭārka*** True sun or its longitude. 209, *see sphuṭaravi*

***sva* (1)** Own. When this prefix is added to a term, it emphasizes the fact there are multiple things/values indicated by the term, but only one of them which is related to the subject in question should be used. For example, *svakakṣyā* (own orbit) would refer to the orbit of the planet that is being dealt with, and not other planets, which would have a different radius when measured in *yojana*s. There are three cases where I have where I have identified *sva* and its synonym *nija* as part of the term and have not enumerated them under this entry: *nijabhūmi*, *svakrāntidhanus* and *svāhorātra* (1). 21, 25, 55, 62, 97, 102, 110, 121, 126–128, 134, 136, 137, 152, 153, 171, 172, 174, 233, 251, 253, 259, 260, 269, 281

***sva* (2)** Additive. 169, 196, 224, 225, 227, 234, *see dhana*

***svakrāntidhanus*** Arc of [a celestial point's] own declination. From context it refers to the corrected declination (*sphuṭāpama*). Perhaps the prefix *sva* (1) might be adding the nuance that it it is an intrinsic, true value. 153

***svadyuśiñjinī*** Diurnal "Sine". *śiñjinī* is a synonym of *jyā* (bow-string, hence Sine). 94, 99, *see dyudalajīvā*

***svabhūmi*** One's spot. 37, *see* *nijabhūmi*

***svar*** Heaven. 39

***svastika*** A crossing formed in the intersection of two perpendicular rings. 5, 15, 17, 155

***svāhorātra* (1)** Diurnal circle. Literally "own day and night", emphasizing the fact that it is the diurnal circle of a specific day and night among all other possibilities. It is always added before *ahorātra* when Parameśvara refers to a diurnal circle, and therefore I have chosen to interpret *svāhorātra* as a whole in the sense of "diurnal circle", and not to decompose *sva*. 10, 90, 91, 103, *see also* *dyumaṇḍala*

***svāhorātra* (2)** Diurnal "Sine". 111, *see* *dyudalajīvā*

***svāhorātrārdha*** Half[-diameter] of the diurnal circle. The expression *dyujyāvṛttasya ... ardhaviṣkambha* (half diameter of the diurnal circle) appears in *GD2* 238, and therefore I interpret that this term, occurring in the next verse, is an abbreviation of *svāhorātrārdhaviṣkambha* and not *svāhorātrārdhajyā* (literally half chord of the diurnal, i.e. diurnal "Sine"), although both interpretations refer to the same thing. 239, *see* *dyudalajīvā*

***svāhorātrārdhajyā*** Diurnal "Sine". 75, *see* *dyudalajīvā*

***svāhorātreṣṭajīvakā* (1)** Given "Sine" in the diurnal circle. 113, *see* *iṣṭajyā* (2)

***svāhorātreṣṭajīvakā* (2)** Given Sine in the diurnal circle. 111, 112, *see* *iṣṭajyā* (3)

***svāhorātreṣṭajyā* (1)** Given Sine in the diurnal circle. 93, 94, *see* *iṣṭajyā* (1)

***svāhorātreṣṭajyā* (2)** Given "Sine" in the diurnal circle. 104, 107, *see* *iṣṭajyā* (2)

**h**

***harija*** Horizon. 50, 53, 88, 188, *see* *kṣitija*

# List of English translations for Sanskrit terms

The following list provides a list of English terms used in my translation with the original Sanskrit words. Proper names or measurement units transliterated exactly the same in the edition and translation are not included in this list. An underlined Sanskrit word indicates that a detailed explanation is given under that entry in the glossary of Sanskrit terms.

**A**

**above** *upari* (1), *ūrdhva* (1).

**additive** *dhana*, *sva* (2).

**adherence** *āsakti*.

**apogee** *ucca*.

**Aquarius** *kumbha*, *ghaṭa*.

**arc** *cāpa*, *dhanus* (1).

**arc of (own) declination** *krāntidhanus* (2), *svakrāntidhanus*.

**arc of declination** *apakramadhanus*, *apamadhanus*, *krāntidhanus* (1).

**arc of geographic latitude** *paladhanus*.

**Aries** *meṣa*.

**armillary sphere** *gola* (1).

**ascendant** *lagna* (1), *vilagna*.

**ascending point** *udayalagna*, *lagna* (1), *vilagna*.

**ascensional difference** *cara*, *caradala*, *carārdha*.

**B**

**base** *bāhu* (1), *bāhu* (2), *bhujā* (1), *bhujā* (3).

**"Base" arc** *doḥcāpa*, *bāhudhanus*, *bhujādhanus*.

**base of direction** *digbāhu*.

**base of [great] shadow** *chāyābāhu*, *prabhābhujā*.

**"base" Sine** *dorjīvā*, *dorjyā*, *bhujajyā*, *bhujā* (2).

**base to be established** *sādhya*, *sādhyabāhu*.

**below** *adhas*.

**bottom** *adhas*.

**Brahmā, of Brahmā** *kamalayoni*, *brahmā*, *brāhma*, *vidhi* (1).

**bright [half-month]** *śukla*.

**C**

**calculator** *gaṇaka*.

**Cancer** *karki*, *kulīra*.

**Capricorn** *eṇa*, *mṛga*.

**celestial equator** *ghaṭikā*, *ghaṭikāmaṇḍala*, *ghaṭikāvṛtta*, *ghaṭīvalaya*, *ghaṭīvṛtta*, *ghāṭika*, *ghāṭikavṛtta*.

**celestial hemisphere** *gola* (5).

**celestial sphere** *khagola*, *gola* (2).

**center** *kendra*, *nābhi*.

**center of the "slow" [eccentric circle]** *māndakendra*.

**center of the zodiac** *bhacakrakendra*.

**circle** *cakra*, *maṇḍala* (1), *vṛtta*. Often abbreviated in compounds.

**circle of direction** *āśāvṛtta*.

**circle of sight** *dṛgvṛtta*, *dṛṅmaṇḍala*.

**circumference** *kakṣyā* (3), *paridhi*. 1, *see also* one's circumference

**circumference of the Earth** *kakṣyā* (3), *bhūkakṣyā*, *bhūvṛtta*.

**clockwise** *pradakṣiṇīkṛt*.

**co-latitude** *avalamba*, *avalambaka*, *lamba* (1), *lambaka*.

**configuration** *saṃsthāna*.

**corrected** *sphuṭa*.

**corrected declination** *sphuṭāpama*.

**corrected [great] gnomon** *sphuṭaśaṅku*.

**corrected latitude** *sphuṭakṣepa*.

**corrected radial distance in *yojanas*** *sphuṭayojanakarṇa*.

**correction method** *sphuṭakarman*.

**correction of ascensional difference** *carasaṃskāra*, *carasaṃskṛti*, *carārdhasaṃskṛti*.

**course** *ayana* (1).

**crossing** *svastika*.

**crude elevation** *sthūlonnati*.

**D**

**daily motion** *dinabhukti*, *divasabhukti*, *bhukti* (1).

**daily motion of a planet** *grahabhukti*.

**dark [half-month]** *kṛṣṇa*.

**darkness** *tamas* (1).

**day** *ahorātra*, *dina* (1), *divasa* (1), *dyu*.

**day and night** *ahorātra*.

**daytime** *dina* (2), *divasa* (2).

**declination** *apama* (1), *krānti*.

**declination produced by the celestial latitude** *see* produced by the celestial latitude.

**degree** *aṃśa* (1), *aṃśaka* (1), *bhāga* (1).

**degrees of the "base"** *dorbhāga*.

**denominator** *aṃśa* (2), *aMzakaf*.

**depression** *avanati*.

**descending point** *astalagna*, *astavilagna*.

**deviation** *kṣepa* (1), *vikṣepa* (1).

**diameter** *vyāsa*.

**difference in sight** *dṛgbheda*.

**direction** *āśā*, *diś*.

**disk** *bimba*, *maṇḍala* (2).

**distance** *antara*, *antarāla*, *antarita*.

**diurnal "Sine"** *dyudala*, *dyudalajīvā*, *dyudalajyā*, *dyumaṇḍalajyā*, *dyumaṇḍalārdhajyā*, *svadyuśiñjinī*, *svāhorātra* (2), *svāhorātrārdhajyā*.

**diurnal "Sine" [when the declination is] greatest** *paramadyujyā*.

**diurnal circle** *dyujyāvṛtta*, *dyumaṇḍala*, *dyuvṛtta*, *svāhorātra* (1).

**divine** *divya*, *daiva*.

**divine year** *divyābda*.

**dot** *bindu*.

**downward** *adhas*.

**drawing** *chedyaka*.

**E**

**Earth** *urvī*, *ku*, *kṣoṇī*, *pṛthivī*, *bhū* (1), *bhūmi*, *medinī*, *vasudhā*.

**Earth's center** *kumadhya*, *bhūmadhya*.

**Earth's circumference** *kuparidhi*, *bhūparidhi*.

**Earth's shadow** *kṣiticchāyā*, *bhūcchāyā*.

**Earth's sphere** *kugola*, *bhūgola*.

**Earth's surface** *kupṛṣṭha*, *bhūpṛṣṭha*.

**Earth-Sine** *kṣitijyā*, *bhūjyā*.

**east, eastern or eastward** *indra*, *pūrva* (1), *prāñc* (1), *surapa*.

**east-west line** *pūrvāparasūtra*.

**eclipse** *grahaṇa*.

**eclipsed object** *chādya*.

**eclipsing object** *chādaka*.

**ecliptic** *apama* (2), *apamaṇḍala*, *apamamaṇḍala*.

**ecliptic point** *lagna* (2).

**ecliptic point of sight-deviation** *dṛkkṣepalagna*.

**ecliptic pole** *bhakūṭa*, *rāśikūṭa*.

**elevation** *unnati*, *pronnati*.

**elevation of ecliptic pole** *bhakūṭonnati*, *rāśikūṭonnati*.

**elevation of latitude** *kṣeponnati*.

**equal division circle** *viṣuvat*.

**equation** *phala* (2).

**equation of center** *doḥphala*, *bhujāphala*.

**equator** *See* celestial equator *or* terrestrial equator.

**equinoctial colure** *viṣuvat*.

**equinoctial point** *golānta*.

**extremity** *agra*.

**extremity of a gnomon** *śaṅkvagra* (2).

**F**

**fast** *śīghra* (2).

**"fast"** *śīghra* (1), *śaighra*.

**G**



**H**

**hemisphere** *gola* (5).

**horizon** *kṣitija*, *mekhalātala*, *harija*.

**hour angle** *nata* (2).

**human beings, human** *mānuṣa*.

**hypotenuse** *karṇa* (1), *śruti* (1).

**hypotenuse at equinoctial midday** *palakarṇa*.

**I**

**impetuosity** *vega*.

**inclination** *kṣepa* (1), *vikṣepa* (1).

**inclined circle** *vikṣepamaṇḍala*, *vimaṇḍala*.

**intermediate direction** *koṇa*.

**J**

**Jupiter** *ārya*, *iḍya*, *guru*, *jīva*.

**L**

**lamp** *dīpa*, *pradīpa*.

**later** *apara* (2).

**latitude (celestial)** *kṣepa* (1), *vikṣepa* (1).

**latitude (geographic)** *See* geographic latitude.

**latitudinal parallax** *nati*.

**length** *dairghya*.

**Leo** *siṃha*.

**Libra** *jūka*, *tulādhara*.

**line** *sūtra*.

**line of direction** *diksūtra*.

**location with no geographic latitude** *anakṣa*, *anakṣadeśa*.

**longitude** No specific Sanskrit word. In general, the longitude of a celestial object or point is simply referred to by its name. For example, the word *graha* (planet) can refer to its longitude rather than the object itself. See discussions under commentary sections 6.2, 7.1 and 9.1.

**longitudinal parallax** *lamba* (2), *lambana* (2).

**M**

**manes, of the manes (time unit)** *pitṛ*, *pitrya*.

**mare's mouth** *baḍavāmukha*.

**Mars** *kuja*, *bhūmija*, *bhauma*.

**mean** *madhya* (2), *madhyama*.

**measure** *māna* (1), *māna* (2), *mita*, *miti* (1), *miti* (2), *saṃmita*.

**measure of a sign/signs** *māna* (2), *bhamāna*, *rāśimiti*.

**Mercury** *jña*, *budha*.

**meridian zenith distance** *nata* (1).

**midday** *dinamadhya*, *divasadala*, *madhyāhna*.

**midday shadow** *dinadalacchāyā*, *madhyacchāyā*, *madhyāhnabhā*.

**middle** *madhya* (1).

**middle of the sky** *kha* (2), *vyoman* (2).

**midheaven** *madhya* (3), *madhyavilagna*.

**midheaven gnomon** *madhya* (4), *madhyaśaṅku*.

**midheaven Sine** *madhyajīvā*, *madhyajyā*.

**midnight** *ardharātra*.

**minute (of arc)** *kalā*, *liptā*, *liptikā*.

**moon** *indu*, *candra*, *tamohantṛ*, *vidhu*, *śaśin*, *śiśiradīdhiti saumya* (2).

**motion** *gati*, *cāra*, *bhukti* (2).

**movement** *cāra*.

**N**

**nether region** *pātāla*.

**night** *niś*, *niśā*, *rātri*.

**node** *pāta*.

**north, northern or northward** *uttara*, *udañc*, *saumya* (1).

**northeast** *rudra*, *śambhu*, *śiva*.

**north-south line** *yāmyodaksūtra*.

**novice** *laghumati*.

**O**

**observer** *draṣṭṛ*.

**one's (own) spot** *nijabhūmi*, *svabhūmi*.

**one's circumference** *nijabhūmivṛtta*, *nijabhūvṛtta*.

**orb** *bimba*.

**orbit** *kakṣyā* (1).

**orbital circle** *kakṣyā* (1), *kakṣyāvṛtta*.

**own** *nija*, *sva* (1).

**P**

**parallax** *lambana* (1), *vilambana*.

**passage** *ayana* (2).

**path of radial distance** *śrutimārga*.

**path, path of the moon** *mārga*, *śaśimārga*, *śiśirakaramārga*.

**perigee** *nīca*.

**Pisces** *jhaṣa*, *mīna*.

**planet** *khaga*, *khecara*, *kheṭa*, *graha*, *vihaga*.

**polar axis** *goladaṇḍa*, *daṇḍa*, *daṇḍaka*.

**pole star** *dhruva*.

**portion** *aṃśa* (2), *aṃśaka* (2), *bhāga* (2).

**portion of the ascensional difference made by the latitude** *kṣepakṛtacarāṃṣa*, *kṣepacara*.

**portion of twilight** *saṃdhyāṃśa*.

**previous, previously** *pūrva* (2), *prāñc* (2).

**prime meridian** *dakṣiṇottara* (2), *yāmyottaravṛtta* (2).

**prime meridian (geographic)** *see* geographic prime meridian.

**prime vertical** *samamaṇḍala*.

**prime vertical gnomon** *samamaṇḍalaśaṅku*, *samaśaṅku*.

**prime vertical shadow** *samacchāyā*.

**produced by the celestial latitude** *vikṣepabhava*.

***proportion*** *anupāta*.

**Q**

**quadrant** *pada*, *pāda*.

**R**

**radial distance** *karṇa* (2), *śruti* (2).

**Radius (of great circle)** *triguṇa*, *trijīvā*, *trijyā*, *tribhajīvā*, *trirāśi*, *trirāśiguṇa*, *trirāśijyā*, *vyāsārdha* (2).

**radius of diurnal circle** *see* diurnal "Sine".

**result** *phala* (1).

**retrograde** *vakra*, *vilomaga*.

**revolution** *bhrama*, *bhramaṇa*.

**right ascension** This notion is represented by the term "rising at Laṅkā (*laṅkodaya*)".

**rising** *udaya*.

**rising above the six o'clock circle** *unmaṇḍalodaya*.

**rising at Laṅkā** *laṅkodaya*.

**rising-setting line** *astodaya*, *astodayasūtra*.

**rule** *vidhi* (2).

**S**

**sage** *muni*.

**Sagittarius** *dhanus* (2).

**Saturn** *arkatanaya*, *ārki*, *manda* (2).

**school** *pakṣa*.

**Scorpio** *korpi*, *vṛścika*.

**setting** *asta*, *astama*, *astamaya*.

**side** *pārśva*, *āśā*.

**sight-deviation** *dṛkkṣepa* (1).

**sight-motion** *dṛggati*.

**sign** *ṛkṣa*, *bha* (1), *rāśi*.

**Sine** *guṇa*, *jīvā*, *jyā*. Often abbreviated in compounds.

**Sine in the celestial equator** *ghaṭikāvṛttajyā*.

**Sine of ascensional difference** *carajyā*.

**Sine of co-latitude** *avalambakajyā*, *lambakajīvā*, *lambakajyā*, *lambajīvakā*, *lambajyā*, *see also* *avalambaka*.

**Sine of corrected declination** *sphuṭāpamajyā*.

**Sine of declination** *apamajyā*, *krāntijyā*, *see also* *apama* (1).

**Sine of direction** *digguṇa*, *digjīvā*, *digjyā*.

**Sine of geographic latitude** *akṣajīvā*, *akṣajyā*, *palaguṇa*, *palajīvā*, *palajyā*, *see also* *akṣa*.

**Sine of hour angle** *natajīvā*.

**Sine of meridian zenith distance** *natajyā*.

**Sine of sight** *dṛgjyā*.

**Sine of sight-deviation** *dṛkkṣepaguṇa*, *dṛkkṣepajyā*, *see also* *dṛkkṣepa* (2).

**Sine of sight-motion** *dṛggatijyā*.

**Sine of the "fast"** *śīghrajyā*.

**six o'clock circle** *unmaṇḍala*.

**sky** *kha* (1), *nabhas*, *vyoman* (1).

**slow** *manda* (3).

**"slow"** *manda* (1), *mānda*.

**"slow" corrected [planet]** *mandasphuṭa*.

**"slow" radial distance** *mandaśruti*.

**solar amplitude** *arkāgrā*, *ināgrā*, *dineśāgrā*, *sūryāgrā*.

**solstitial colure** *dakṣiṇottara* (1), *yāmyottaravṛtta* (1).

**solstitial point** *ayanānta*.

**south, southern or southward** *dakṣiṇa*, *yāmya*.

**southeast** *agni*, *kṛśānu*, *dahana*, *vahni*.

**space** *kha* (1), *vyoman* (1).

**sphere** *gola* (1).

**spot** *see* one's (own) spot.

**star** *ṛkṣa*, *tāra*, *tāraka*, *bha* (1).

**star in space** *khagarkṣa*.

**stellar sphere** *gola* (2), *nakṣatragola*, *bhagola*.

***step*** *krama*.

**string** *sūtra*.

**string of light** *tejaḥsūtra*.

**subtractive** *ṛṇa*.

**sun** *arka*, *ina*, *dinakara*, *dinapa*, *dinapati*, *dineśa*, *divākara*, *bhānu ravi*, *savitṛ*, *sūrya*.

**sun's circumference** *raviparidhi*.

**sun's equation of center** *ravidoḥphala*.

**surface** *pṛṣṭha*.

**T**

**Taurus** *vṛṣa*, *vṛṣabhā*.

**terrestrial equator** *anakṣa*, *anakṣadeśa*. Translated literally as location with no geographic latitude.

**time** *kāla*.

**tip** *śiras*.

**transverse** *tiryañc*.

**trilateral** *tryaśra*.

**true** *spaṣṭa*, *sphuṭa*.

**true planet** *sphuṭakhecara*, *sphuṭagraha*.

**true sun** *sphuṭatīkṣṇāṃśu*, *sphuṭaravi*, *sphuṭārka*.

**twilight** *saṃdhyā*.

**U**

**Ujjain** *ujjayinī*.

**umbra** *tamas* (2).

**upright** *koṭi* (1).

**"upright"** *koṭi* (2).

**upright in the diurnal circle** *dyuvṛttakoṭi*.

**upright of [great] shadow** *chāyākoṭi*.

**upward** *ūrdhva* (1).

**V**

**Venus** *bhṛgu*, *bhṛgusūnu*, *śukra*, *sita*.

***view*** *dṛṣṭi*.

**Virgo** *kanyā*.

**visibility equation** *dṛkphala*.

**visibility equation for the geographic latitude** *akṣadṛkphala*.

**visibility method** *dṛkkarman*.

**W**

**west, western or westward** *apara* (1), *paśca*, *paścima*.

**wind** *marut*.

**wind of Earth** *bhūvāyu*.

**with passage** *sāyana*.

**without difference** *aviśiṣṭa*, *aviśeṣa* (2).

**"without-difference" method** *aviśeṣa* (1), *aviśeṣakarman*.

**Y**

**year** *abda*, *varṣa*.

**Z**

**zenith** *khamadhya*, *nabhomadhya*, *madhya* (5).

**zodiac** *bha* (2), *bhacakra*, *bhavṛtta*.

**zodiacal sign** *see* sign.

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