Implied Volatility Modelling, Tail Risk Premia, and Volatility Arbitrage Strategies

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> *"(...) parce que, [les Anciens] s'étant élevés jusqu'à un certain degré où ils nous ont portés, le moindre effort nous fait monter plus haut, et avec moins de peine et moins de gloire nous nous trouvons au-dessus d'eux. C'est de là que nous pouvons découvrir des choses qu'il leur était impossible d'apercevoir. Notre vue a plus d'étendue, et, quoiqu'ils connussent aussi bien que nous tout ce qu'ils pouvaient remarquer de la nature, ils n'en connaissaient pas tant néanmoins, et nous voyons plus qu'eux."*
>
> Blaise Pascal, *Traîté sur le Vide* (1647)

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Introduction Générale

Depuis les années 1950, la théorie de la valorisation des actifs financiers a motivé les fondements théoriques des indices de marché modernes. Apparus à l'origine en 1896 avec le Dow Jones Industrial Average (DJIA), les indices de marché satisfont usuellement plusieurs attributs clés, exigés des investisseurs institutionnels et particuliers, des régulateurs et des médias. Etant donné qu'ils sont utilisés pour mesurer et capturer précisément la performance aggrégée d'un vaste segment particulier d'actifs financiers, ils ont pour nécessité de remplir le critère de représentativité. Ainsi, les fournisseurs d'indices communiquent généralement en toute transparence la formule de calcul qui définit le schéma de pondération utilisé pour aggréger les constituants individuels d'indices. En pratique, l'indépendance entre les fournisseurs d'indices et les administrateurs d'indices de référence, communément appelés *benchmarks*, servent à réduire les conflits d'intérêts entre fournisseurs et administrateurs d'indices, et à garantir l'intégrité de l'information.

L'importance des indices de marché et de la gestion passive dans l'industrie financière.

Depuis que la mesure de performance a pris une place centrale dans l'industrie de la gestion d'actifs, décomposer et monitorer les décisions d'allocation a suscité un intérêt de recherche majeur, tant pour les investisseurs que pour les chercheurs. Traditionellement, la décomposition s'effectue sous la forme de vastes indices de marché spécifiques, de secteurs économiques, de régions géographiques, ou de stratégies basées sur des règles. Les fonds benchmarkés capturent usuellement la performance d'un *benchmark* en minimisant l'écart de suivi (la *tracking-error*), c'est-à-dire l'écart de performance à l'indice de référence. Par conséquent, le suivi d'indices pourrait être interprété soit comme une contrainte d'investissement, soit comme un objectif d'investissement. En effet, les stratégies passives visent à répliquer fidèlement la performance du *benchmark*, alors que les stratégies actives visent à surperformer la performance du *benchmark* tout en respectant les contraintes définies par le *benchmark*. Par conséquent, la gestion indicielle a conduit à des innovations révolutionnaires dans l'industrie de la gestion d'actifs, puisqu'elle offre des véhicules d'investissement plus accéssibles tant pour les investisseurs institutionnels que particuliers. Les premiers fonds indexés institutionnels et communs de placement (FCP) furent respectivement développés en 1973 et 1976. Ils furent suivis en 1993 par les premiers fonds indiciels, puis par les *Exchange-Traded Funds* (*ETF*), pour capturer la performance de l'indice S&P 500. Les *ETF* s'avèrent en effet être des véhicules d'investissement particulièrement pratiques et peu coûteux pour atteindre les objectifs d'investissabilité et de transparence. Désormais, la demande pour les *ETF* émanant tant des investisseurs institutionnels que particuliers ne s'est jamais démentie au cours de la dernière décennie. Selon Sanford Bernstein, plus de la moitié des actifs sous gestion aux Etats-Unis devraient être passivement gérés à horizon janvier 2018. Selon un rapport publié récemment par Investment Company Institute, l'industrie des *ETF* américains a représenté 1771 fonds et 3,14 trillions de dollars d'actifs sous gestion en septembre 2017, contre 1686 fonds et 2,4 trillions de dollars un an plus tôt. De plus, l'émission nette de parts d'*ETF* américains a atteint un record de 333 milliards de dollars d'actifs sous gestion en 2017, contre seulement 157 en 2016. Ces véhicules permettent aux investisseurs d'accéder à un vaste univers de classes d'actifs et de marchés, autrefois uniquement accéssibles aux investisseurs institutionnels via les instruments dérivés de gré à gré, appelés instruments *OTC*, *out-of-the counter*. En fournissant une liquidité, une transparence, une régulation, et un rapport coût-qualité performant, les *ETF* offrent désormais une alternative à la gestion active. Selon ETFGI, l'industrie des *ETF* a atteint ainsi en août 2017 un record de 4,3 trillions de dollars d'actifs sous gestion à travers le monde, alimentée par les sortie de capitaux de la gestion active vers la gestion passive. L'industrie est dominée par BlackRock, Vanguard et State Street qui représentent 70% des actifs mondiaux.

L'émergence des indices de marché intelligents face aux critiques des indices de marché traditionnels.

Inspirée de la théorie standard de valorisation des actifs financiers, la gestion indicielle offre une alternative efficace, accéssible et peu coûteuse aux stratégies actives d'investissement traditionnelles. Ainsi, les modèles standards d'équilibre tels que le

Modèle d'Evaluation des Actifs Financiers (MEDAF) introduit par Sharpe (1963) dans [105], Lintner (1965) dans [78], et Mossin (1966) dans [91], postulent l'existence d'un unique facteur de risque identifié comme le facteur de marché, et défini comme le rendement sur la richesse aggrégée de l'investisseur. Plus spécifiquement, le MEDAF stipule que les actifs financiers comportent usuellement deux principales sources de risque, systématique ou idiosyncratique. Le risque systématique émane du facteur de marché définissant la sensitivité de l'actif au marché, communément appellé le *beta* de l'actif par rapport à son indice de marché. D'après la Théorie Moderne du Portefeuille (TMP) introduite par Markowitz (1952) dans [81], le risque systématique récompense les investisseurs avec des rendements espérés supérieurs, puisqu'il ne peut être diversifié. Cependant, bien que les indices pondérés par la capitalisation soient devenus l'approche dominante dans la construction de portefeuilles, les praticiens et les chercheurs ont émis des critiques contre leur diversification sous-optimale. En particulier, Arnott, Hsu, and Moore (2005) dans [19], et Treynor (2005) dans [109] entre autres, ont montré qu'ils surpondèrent nécessairement les actifs sur-évalués et sous-pondèrent les actifs sous-évalués dans un portefeuille. En effet, les indices pondérés par la capitalisation allouent les composantes en se basant sur la capitalisation de marché plutôt que sur le critère rendement-risque. De plus, leur optimalité n'est démontrée que sous l'hypothèse du MEDAF, où l'unique facteur de risque, le facteur de marchés est supposé unique.

Ainsi, motivé par la recherche de diversification, le plus grand fonds de pension du monde, California Public Employees' Retirement System (CalPERS), a adopté en 2006 une nouvelle méthodologie d'indexation en suivant de nouveaux indices, pondérés par les fondamentaux. De manière similaire, le fonds souverain Korea Investment Corporation (KIC) construit son allocation d'actifs depuis 2011 selon trois indices de beta actions alternatifs. Récemment, une vaste littérature met largement en avant cette nouvelle génération d'indices de marchés pondérés par les facteurs fondamentaux associés aux entreprises, tels que les ventes, les bénéfices, les dividendes ou les valeurs comptables.

Alternativement au MEDAF, les modèles d'équilibre modernes tels que le Modèle Intertemporel d'Evaluation des Actifs Financiers, ou MEDAF intertemporel introduit par Merton (1973) dans [88], et les modèles d'arbitrage tels que le Modèle d'Evaluation par Arbitrage, ou MEA de Ross (1976) dans [100], stipulent l'existence de multiples sources de risque systématique. Ces nouveaux modèles d'équilibre sont en totale contradiction avec le MEDAF standard qui repose sur l'hypothèse centrale de l'existence d'un unique facteur. Les modèles multi-factoriels définissent les facteurs de risque comme des variables expliquant les rendements des actions et en modélisant le rendement espéré comme une fonction de plusieurs catégories de facteurs. Les sources de facteurs de risque peuvent être macroéconomiques, comme développé par Chen, Ross, Roll (1986) dans [52] ; ou statistiques, comme identifiés par des techniques statistiques telles que l'Analyse en Composantes Principales ou ACP. En théorie, Cochrane (2005) dans [53] et Ang (2014) dans [17] postulent que détenir un risque systématique donné récompense l'investisseur par une prime de risque positive persistente sur le long-terme. Comme suit, l'intuition économique sous-tendant l'existence d'une prime de risque repose sur le principe que le rendement espéré en excès d'un facteur de risque donné est négativement corrélé avec la covariance entre le facteur de risque et le facteur d'actualisation stochastique, communément appelé *Stochastic Discount Factor* (*SDF*).

Considérons l'agent représentatif suivant, modélisé par la fonction d'utilité $U(\cdot)$, ayant une consommation $\tilde{c}_t$ et $\tilde{c}_{t+1}$ à la date t et $t+1$:

$$U(\tilde{c}_t, \tilde{c}_{t+1}) = u(\tilde{c}_t) + \beta E_t [u(\tilde{c}_{t+1})]$$

où β désigne le facteur d'actualisation subjectif, ou *pricing kernel*. Intuitivement, les agents économiques réduisent leur consommation en temps de récession, se sentant moins riches. Le problème d'allocation consiste en un arbitrage à la date t sur la période $[t, t+1]$ entre une consommation et un investissement du montant ξ du gain du facteur, ou *payoff* du facteur noté $x_{t+1} = p_{t+1} + d_{t+1}$, où p_{t+1} et d_{t+1} désignent respectivement le prix et le dividende du facteur de risque. Par conséquent, le problème de l'agent revient à trouver le montant optimal de richesse ξ qui maximise son utilité suivante

$$\max_{\{\xi\}} u\left(\tilde{c}_t\right) + \beta E_t\left[u\left(\tilde{c}_{t+1}\right)\right] \quad \text{s.t.} \tag{1}$$

$$\tilde{c}_t = c_t - \xi p_t$$

$$\tilde{c}_{t+1} = c_{t+1} + \xi x_{t+1}$$

où c_t et c_{t+1} désignent respectivement les niveaux de consommation initiale à la date t et $t+1$. La condition de premier ordre, ou $F.O.C.$, pour la consommation optimale et le choix de portefeuilles donne l'équation de valorisation suivante à la date t

$$p_t = E_t\left[\beta \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} x_{t+1}\right] \tag{2}$$

où $m_{t+1} \equiv \beta \frac{u'(c_{t+1})}{u'(c_t)}$ représente le facteur d'actualisation stochastique, encore appelé le taux marginal En isolant dans l'équation (1) le terme de covariance entre le taux de substitution marginal m_{t+1} et le *payoff* du facteur x_{t+1}, et en injectant le taux sans risque r^f_{t+1}, nous obtenons le prix du facteur à la date t, exprimé comme le flux espéré de liquidités actualisé par le taux sans risque, plus une prime de risque :

$$p_t = \frac{E_t\left[x_{t+1}\right]}{r^f_{t+1}} + \operatorname{cov}_t\left[m_{t+1}, x_{t+1}\right] \tag{3}$$

En réarrangeant l'expression ci-dessous avec $R_{t+1} = \frac{x_{t+1}}{p_t}$ étant le rendement brut du facteur sur $[t, t+1]$ et $r_{t+1} = R_{t+1} - r^f_{t+1}$ étant le rendement net du taux sans risque, nous obtenons le rendement en excès espéré du facteur de risque donné

$$E_t[r_{t+1}] - r^f_{t+1} = -r^f_{t+1} \cdot \mathrm{cov}_t[m_{t+1}, r_{t+1}] \tag{4}$$

$$= -\frac{\mathrm{cov}_t[m_{t+1}, r_{t+1}]}{E_t[m_{t+1}]} \tag{5}$$

L'équation (4) montre que le rendement espéré en excès du facteur de risque donné co-varie négativement avec le terme de covariance entre le rendement du facteur r_{t+1} et le facteur d'actualisation stochastique m_{t+1}. Par la suite, détenir un *payoff* incertain non-corrélé avec le *pricing kernel* ne devrait pas théoriquement récompenser la prise du risque additionel associé au facteur. En effet, si $\mathrm{cov}_t[m_{t+1}, x_{t+1}] = 0$ dans l'équation (3), alors le prix du facteur à la date t ne devrait être seulement égal qu'à l'espérance du *payoff* du facteur pour la date $t+1$, actualisé par le taux sans risque. En d'autres termes, les risques idiosyncratiques ne devraient rémunérer que la détention de l'actif sans risque. Usuellement, la plupart des facteurs de risque performent mal dans les états défavorables de la nature, et les rendements et les *payoffs* associés co-varient négativement avec le facteur d'actualisation, car les agents économiques se sentent appauvris dans les périodes adverses, c'est-à-dire quand leur utilité marginale de consommation augmente. Par conséquent, de tels facteurs de risque génèrent des rendements en excès positifs dans les périodes favorables, payant en moyenne des primes de risque positives sur le long-terme, à travers tous les possibles états de la nature. Le cas inverse de l'assurance de portefeuilles concerne tout particulièrement l'objet de nos travaux de recherche : le *payoff* du facteur co-varie positivement avec le taux marginal intertemporel de substitution, fournissant des rendements en excès positifs dans les mauvaises périodes. Les investisseurs consentent donc à payer une prime de risque positive sur longue période pour compenser la détention de ce facteur de risque, puisqu'il offre un revenu dans des périodes économiques adverses, quand les agents se sentent appauvris.

L'essor des facteurs de risque associé au succès grandissant de l'industrie de la gestion factorielle.

Traditionellement, les facteurs de risque les plus populaires concernent les facteurs de risque fondamentaux capturant les caractéristiques des actions, telles que la valorisation d'entreprise (*Value*), la croissance (*Growth*), la taille (*Size*), ou la tendance (*Momentum*). En particulier, Fama et French (1992, 1993) dans [60] et [61] ont mis en avant un modèle multi-factoriel expliquant les rendements des marchés actions américains par le facteur de marché, le facteur taille (grande versus petite capitalisation), et le facteur valorisation (bas versus haut ratio valeur comptable sur valeur de marché). Consiérons le modèle Fama-French à trois facteurs suivant qui décompose à la date t le rendement espéré $E_t\left[r_{i,t+1}\right]$ de l'actif i :

$$\begin{aligned} E_t\left[r_{i,t+1}\right]-r_{t+1}^f &= \beta_{1,i}\left(E_t\left[r_{M,t+1}\right]-r_{t+1}^f\right) \\ &\quad + \beta_{2,i}E_t\left[SMB_{t+1}\right]+\beta_{3,i}E_t\left[HML_{t+1}\right] \end{aligned} \tag{6}$$

où $E_t\left[r_{M,t+1}\right]$ désigne le rendement espéré du portefeuille de marché, tel que le terme $E_t\left[r_{M,t+1}\right]-r_{t+1}^f$ est associé au rendement espéré du facteur de marché. Le rendement espéré $E_t\left[SMB_{t+1}\right]$ du facteur taille mesure la différence de rendements entre les petites et les grandes capitalisations ; et le rendement espéré $E_t\left[HML_{t+1}\right]$ du facteur valorisation mesure la différence de rendements entre les grands et les petits ratios de valeur comptable par valeur de marché. Par la suite, Carhart (1997) prend en compte le facteur *momentum* mis en évidence par Jegadeesh et Titman (1993) dans [72] en étendant dans [46] le modèle Fama-French à trois facteurs comme décrit par le modèle suivant à quatre facteurs :

$$\begin{aligned} E_t\left[r_{i,t+1}\right]-r_{t+1}^f &= \beta_{1,i}\left(E_t\left[r_{M,t+1}\right]-r_{t+1}^f\right) \\ &\quad + \beta_{2,i}E_t\left[SMB_{t+1}\right]+\beta_{3,i}E_t\left[HML_{t+1}\right] \\ &\quad + \beta_{4,i}E_t\left[WML_{t+1}\right] \end{aligned} \tag{7}$$

où le rendement espéré $E_t\left[WML_{t+1}\right]$ du facteur *momentum* mesure sur l'année précédente la différence de rendements entre les actions qui surperforment et celles qui sousperforment. Plus récemment, Fama et French ont mis en évidence le modèle multi-factoriel suivant à cinq facteurs

$$\begin{aligned} E_t\left[r_{i,t+1}\right] - r^f_{t+1} &= \beta_{1,i}\left(E_t\left[r_{M,t+1}\right] - r^f_{t+1}\right) \\ &\quad + \beta_{2,i}E_t\left[SMB_{t+1}\right] + \beta_{3,i}E_t\left[HML_{t+1}\right] \\ &\quad + \beta_{4,i}E_t\left[RMW_{t+1}\right] + \beta_{5,i}E_t\left[CMA_{t+1}\right] \end{aligned} \tag{8}$$

où RMW et CMA désignent respectivement le facteur profitabilité et le facteur investissement. Le premier facteur mesure la différence de rendements entre les entreprises ayant la plus forte profitabilité opérationnelle et celles ayant la plus faible, tandis que le second facteur mesure la différence de rendements entre les entreprises ayant une politique d'investissement conservative et celles ayant une politique d'investissement aggressive. Pour illustration, les performances des cinq facteurs sur la période entre 1963 et 2016 sont représentées sur la figure suivante.

Désormais, la littérature empirique a donné naissance à une pléthore de facteurs de risque supposés, mettant en évidence un ''Zoo de Facteurs" ou ''Pêche à Facteurs" comme souligné par Cochrane (2005) dans [53]. En effet, de nombreux facteurs pourraient ne pas être interprétés d'un point de vue économique comme une compensation à long-terme pour la détention de risques systématiques additionels. Pour illustration, Harvey, Liu and Zhu (2014) documentent dans [69] l'existence de 314 supposés facteurs de risque à travers la littérature empirique, remettant en question leur rationnel économique. Néanmoins, les modèles multi-

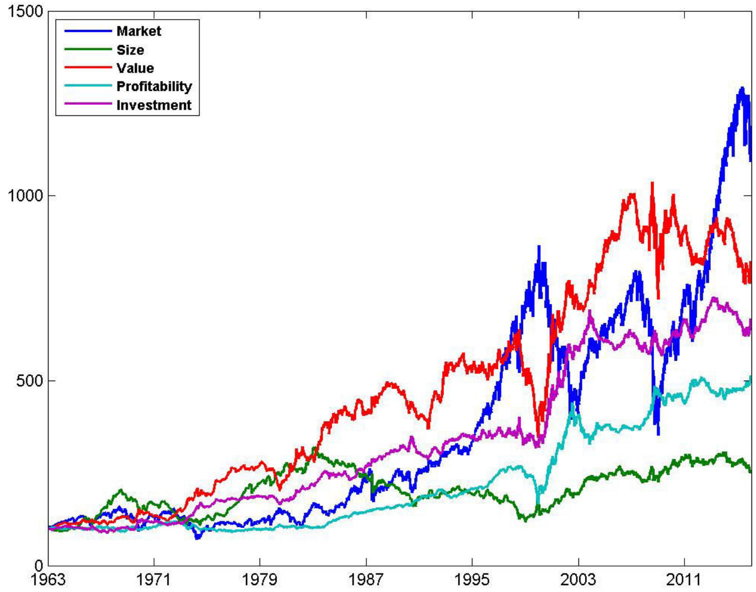

FIGURE 1 – Performances des cinq facteurs Fama-French expliquant les rendements des actions américaines sur la période entre 1963 et 2016.

facteurs sont devenus des standards en finance, donnant naissance aux indices de marchés actions alternatifs ou *smart*, et à l'investissement *beta* actions alternatif ou investissement factoriel actions, communément appelé le *smart beta*. Le Financial Times Lexicon définit ainsi le *smart beta* comme des "stratégies d'investissement basées sur des règles qui n'utilisent pas la convention des poids de la capitalisation de marché". Par la suite, Asness, Frazzini et Pedersen (2012) dans [20] ont récemment étendu le concept de *smart beta* à d'autres classes d'actifs telles que les obligations.

A présent, l'investissement factoriel est devenu l'une des innovations les plus révolutionnaires de l'industrie financière, à la fois pour les gestions passives et actives, puisqu'il décompose les décisions d'allocation en facteurs de risque systématiques, plutôt qu'en classes d'actifs standards. Par conséquent, l'allocation factorielle a donné naissance à un nouveau paradigme d'investissement, car les décisions d'investissement peuvent être désormais exprimées en termes d'allocations de facteurs de risque, par contraste avec les allocations de classes d'actifs. Selon ETFGI, l'industrie des *ETF* de *betas* alternatifs représentait 1279 fonds en 2017, soit près de 630 milliards de dollars à travers le monde, augmentant de 18% par rapport à l'an dernier. A titre de comparaison avec fin 2014, Morningstar Direct et Bloomberg Intelligence estimaient que l'industrie des *betas* alternatifs représentait seulement près de 700 *ETF*, soit 529 milliards de dollars à travers le monde. Selon un sondage publié par FTSE Russell en 2017, 46% des gérants de portefeuilles sondés ont une allocation comprenant des *betas* alternatifs en 2017, contre seulement 26% en 2015. D'ailleurs, 63% des sondés se disent prêts à accroître leurs allocations factorielles. Cette tendance est particulièrement alimentée par les stratégies multi-factorielles puisque 64% des gérants sondés combinent des facteurs, contre seulement 20% en 2015. En outre, 28% des sondés n'emploient actuellement pas de stratégies factorielles mais se disent prêts à en adopter à court ou moyen-terme. Au final, seulement 26% des gérants sondés en 2017 n'emploient actuellement pas de stratégies factorielles et n'envisagent pas de le faire, contre 40% en 2014.

Motivée par la demande croissante pour des *benchmarks* investissables intelligents, l'industrie de la construction d'indices poursuit son développement intense. Les indices factoriels répliquant des facteurs de risque ont pour but soit d'améliorer la diversification de portefeuilles relativement aux indices pondérés par la capitalisation, soit de capturer les primes de risque. Dans ce contexte, l'intuition sous-jacente de ce nouveau paradigme d'allocation factorielle repose sur une approche d'allocation du risque innovante, car la construction de portefeuilles repose sur une budgétisation du risque pour allouer la richesse proportionnellement à des facteurs de risque systématique. Similairement, Roncalli (2013) dans [99] explore la budgétisation du risque où la contribution de chaque constituant du portefeuille au risque global du portefeuille suit un budget cible de risque. Dans un cas spécial, il définit le *Risk Parity* comme une stratégie d'investissement equipondérée pour allouer le budget de risque identiquement pour tous les constituants du portefeuille, de telle sorte qu'ils contribuent de manière égale au risque global.

Le développement des indices de marché a aussi concerné les indices de volatilité traditionnels basés sur les options tels que le VIX.

Par analogie, les indices basés sur les options ont fait leur apparition, tels que le S&P 500 Volatility Index, communément appelé l'indice VIX. Largement interprété par les praticiens et les chercheurs comme le baromètre de la peur des investisseurs, l'indice VIX mesure l'espérance à maturité 30 jours de la volatilité du marché des actions américaines associée à l'indice S&P 500 (SPX). Introduit par Whaley (1993) dans [113], la version originale du VIX – le VXO, inversait à l'origine la formule de *pricing* de Black (1976) dans [31] à partir des prix de marché des options à la monnaie (ATM) sur l'indice S&P 100 (OEX). Néanmoins, le Chicago Board of Options Exchange (CBOE) a révisé en 2003 la méthodologie de calcul de cet indice basé sur les options dans le but de créer un sous-jacent plus approprié pour la réplication d'instruments dérivés de volatilité investissables. La nouvelle approche entièrement sans modèle implémente une moyenne des prix pondérés des options *call* et *put* écrits sur l'indice S&P 500 (SPX) sur un large éventail de prix d'exercice. Ainsi, le nouveau VIX calculé en temps réel comme décrit dans [49] représente désormais convenablement le taux du *variance swap* introduit par Carr and Wu (2009) dans [48]. En outre, combiner une position statique d'un continuum d'options de type européen à une position dynamique de contrats *futures* revient à répliquer les *payoffs* de *swaps* de variance. Plus précisément, comme décrit par la CBOE (2009) dans [49], la formule du nouveau VIX correspond à la règle de calcul suivante :

$$VIX = 10 \left[\frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} \exp\left[r_f T\right] Q\left(K_i\right) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2 \right]^{\frac{1}{2}} \tag{9}$$

où F, T and r_f désignent respectivement le niveau de l'indice *forward* associé aux contrats *futures* SPX, le temps d'expiration, et le taux sans-risque ; K_0 désigne le premier prix d'exercice en dessous du niveau d'indice *forward*, et K_i est le prix d'exercice de la i-ème option SPX en dehors de la monnaie (OTM), c'est-à-dire l'option *call* si $K_i > K_0$ et l'option *put* sinon. Ensuite, l'intervalle entre les prix d'exercice est égal à $\Delta K_i = \frac{1}{2}\left[K_{i+1} - K_{i-1}\right]$, et $Q\left(K_i\right)$ désigne la valeur moyenne de l'écart *bid-ask* pour chaque option SPX associée au prix d'exercice K_i.

Quelques mois après le changement méthodologique, la CBOE a introduit respectivement en 2004, 2006, et 2009 les contrats *futures* de VIX, les options sur VIX écrites sur les *futures* de VIX, et les *ETP* sur VIX. Fournissant l'une des innovations les plus importantes de l'industrie financière, l'indexation au VIX donna naissance aux options sur VIX devenues désormais le second contrat le plus traîté au monde. En effet, ils rencontrent une considérable demande en couverture émanant d'assureurs de portefeuilles contre de futurs chutes du marché actions. Les articles séminaux de Whaley (1993, 2009) dans [113] et [114] vantent ainsi la diversification de portefeuilles et la réduction de risque fournies par les dérivés de VIX, puisqu'ils capturent la corrélation inverse entre les rendements de l'indice actions et la volatilité associée. En particulier, Szado (2009) dans [108] met en évidence l'assurance de portefeuilles fournie par une stratégie passive de détention de *futures* de VIX durant la crise des subprimes. Sur la période s'étendant du 1er août 2008 au 31 décembre 2008, combiner une allocation de 10% en contrats *futures* de

VIX à un portefeuille actions-obligations réduit les pertes annualisées de -15.9% à -0.3%, et réduit la volatilité annualisée de 21.7% à 13.3%. Plus généralement, Chen, Chung, and Ho (2011) dans [51] appliquent le critère moyenne-variance sur la période 1996-2008 pour mettre en évidence une amélioration des ratios de Sharpe *in-sample* dans le cas d'une diversification de portefeuilles traditionnels avec de la volatilité implicite actions. Néanmoins, les bénéfices de diversification des dérivés de VIX sont partiellement incompris par de nombreux investisseurs, puisque la très mauvaise performance a refroidi les attentes des investisseurs sur les bénéfices de l'indexation au VIX. En effet, les *ETP* VIX les plus traîtés – le VXX notamment, ont perdu près de 99.6% de sa valeur sur la période 2009-2014, ruinant les investisseurs non-sophistiqués. Alors que les investisseurs traditionnels paient des dividendes ou des coupons, l'exposition au VIX fournit par contraste une prime d'assurance que l'investisseur consent à payer pour se couvrir contre les risques extrêmes du marché actions. Par conséquent, Whaley (2013) dans [115] suggère que les *ETP* VIX forment des stratégies passives d'investissement inappropriées, puisqu'ils génèrent des coûts de portage appelés coûts de *carry* considérables. Sur la période entre mars 2004 et mars 2012, il identifie une pente moyenne de 2,3% pour la structure par terme des prix de *futures* sur VIX pour une maturité de 30 jours, mettant en évidence un *contango* dans près de 81% des jours de cotation.

L'essor des indices de volatilité intelligents face aux critiques de l'in-

dice de volatilité VIX.

Par analogie avec les indices pondérés par la capitalisation, les indices traditionnels basés sur les options tels que l'indice VIX souffrent des mêmes biais rencontrés par les indices de marchés conventionnels. En effet, Jiang et Tian (2007) dans [74] mettent en évidence que la formule de calcul du VIX génère des erreurs d'approximations conséquentes, en estimant la volatilité espérée par la moyenne des prix des options *call* et *put* de l'indice S&P 500 sur un large éventail de prix d'exercice. Bien qu'en théorie la formule requiert un large continuum de prix d'exercice, l'approximation du VIX comme fournie par la CBOE ne requiert qu'un petit éventail de prix d'exercice traîtés sur le marché. Par conséquent, Jiang and Tian (2007) mettent en évidence que sous de hauts (inversement bas) niveaux de volatilité, la procédure de calcul du VIX sous-estime (inversement sur-estime) la vraie volatilité implicite de près de 198 (inversement 79) points de base d'indice. Le point de base d'indice correspondant à 10 dollars par contrat *futures* sur VIX, les erreurs d'approximation se traduisent donc par des erreurs de *pricing* substantielles allant de 790 à 1980 dollars par contrat *futures*. Désormais, l'industrie de la construction d'indice accorde une attention particulière à la création d'indices intelligents basés sur les options, remplissant les critères de représentativité et d'investissabilité. Pour illustration, le Chicago Board Options Exchange a introduit l'indice SKEW en 2011, implicitant le risque extrême de *crash* à partir des prix d'un portefeuille d'options SPX en dehors de la monnaie (OTM). Comme décrit par le Tableau 1 ci-dessous, la théorie moderne de la valorisation d'actifs bâtit désormais les fondements de l'industrie de la construction d'indices intelligents basés sur les options pour capturer des primes de risque implicites.

	MEDAF	**MEDAF Intertemporel, MEA**
Indices Traditionnels	**Pondération par la capitalisation** *Ex. : Indices S&P 500, Dow Jones*	**Indices Intelligents** *Ex. : Indices Value, Size, Momentum*
Indices Basés Options	**Pondération par les strikes** *Ex. : Indice VIX*	**Indices Intelligents** *Ex. : Indice SKEW*

TABLE 1 – Des indices traditionnels pondérés par la capitalisation boursiére aux indices intelligents implicités par les options.

Théoriquement, la prime de risque calculée à partir des prix de marché des options définit la différence entre les espérances physique et risque-neutre des moments de rendements d'actifs. Carr et Wu (2009) dans [48], et Bollerslev, Tauchen, et Zhou (2009) dans [36] définissent comme suit la prime de risque de variance $VRP_{t,t+\tau}$ mesurée à la date t sur la période τ comme la différence entre la variation du rendement réalisée ex post sur l'intervalle de temps $[t-\tau, t]$ et l'espérance risque-neutre ex ante de la variation du rendement d'actif sur l'intervalle de temps $[t, t+\tau]$:

$$VRP_{t,t+\tau} \equiv E_t^P\left[\sigma_{t,t+\tau}^2\right] - E_t^Q\left[\sigma_{t,t+\tau}^2\right] \quad (10)$$

où $E_t^Q[\cdot]$ et $E_t^P[\cdot]$ désignent l'opérateur d'espérance conditionnelle à la date t, respectivement sous la mesure risque-neutre Q et la mesure physique P. Par la suite, $E_t^P\left[\sigma_{t,t+\tau}^2\right]$ et $E_t^Q\left[\sigma_{t,t+\tau}^2\right]$ sont les valeurs espérées conditionnellement à la date t de la variance réalisée sur la période τ, respectivement sous les mesures de probabilité physique et risque-neutre. En outre, la prime de risque de variance $VRP_{t,t+\tau}$ multipliée par un montant notionnel en dollars définit usuellement le *payoff* à la maturité $t+\tau$ du rendement d'un *swap* de variance. Sous la condition de non-arbitrage, le taux constant du *variance swap* $SW_{t,t+\tau}$ déterminé à la date t et payé à la date $t+\tau$ est égal à l'espérance risque-neutre de la variance future réalisée, qui approxime le VIX.

Puisque la prime de risque de variance est négative de manière persistente sur longue période, elle est usuellement considérée comme une assurance de portefeuilles couvrant le risque de la réalisation de krachs futurs du marché actions. En effet, dans l'équation (4), la covariance durablement positive $\text{cov}_t\left[m_{t+1}, x_{t+1}\right]$ montre que le *pricing kernel* tend à co-varier positivement avec le *payoff* du facteur associé au *swap* de variance, fournissant par la suite des rendements en excès positifs dans les états défavorables de la nature. Par conséquent, les investisseurs consentent à payer une prime de risque positive sur longue période, offrant un gain dans les périodes économiques adverses, quand ils se sentent appauvris. Désormais, l'usage intensif des *swaps* pour capturer la prime de risque de variance sur longue période augure du succès futur de nouveaux contrats de *swaps*, puisqu'ils offrent une assurance de portefeuilles contre les états défavorables de la nature.

Dans ce contexte, mes travaux de thèse investiguent de nouvelles approches d'investissement de l'arbitrage de volatilité et visent à repenser la philosophie de ces stratégies récemment décriées pour leurs très mauvaises performances. En effet, les stratégies d'arbitrage de volatilité, autrefois réservées aux investisseurs sophistiqués tels que les *hedge funds* et les banques d'investissement, se sont récemment popularisées auprès des gérants d'actifs traditionnels et des investisseurs particuliers. L'idée prometteuse d'une stratégie de couverture de portefeuilles était d'exploiter la relation inverse (*implied leverage effect*) entre les indices de marché actions et la volatilité implicite des marchés d'options. Dans le sillage de la création des *futures* et options sur indice VIX, le marché de la réplication passive par des produits dérivés indexés sur les *futures* VIX tels que les *ETP* VIX a connu un essor considérable. Or, leurs performances décevantes, l'*ETP* VXX a perdu près de 99% de sa valeur entre 2009 et 2014, ont totalement remis en question l'approche traditionnelle des stratégies de volatilité basées sur la réplication passive.

Présentation des travaux de recherche structurés en trois chapitres complémentaires.

Sous la forme d'une introduction empirique à visée descriptive, le chapitre 1 diagnostique les stratégies traditionnelles de volatilité basées sur la diversification et la couverture de portefeuilles par l'utilisation des *ETP* VIX. Pour cela, nous examinons l'adéquation de ces instruments dérivés complexes avec le degré d'aversion au risque des investisseurs. L'étude de l'optimalité du choix de portefeuilles en environnement incertain est ainsi appliquée sur des allocations *overlay* d'actions, d'obligations, et d'*ETP* indexés sur *futures* de VIX, dans le cadre de la théorie de l'utilité espérée, simulant le comportement d'un investisseur rationnel. Plus précisément, notre étude empirique fait appel à deux métriques, la première mesurant la surprise de l'investisseur ; la seconde mesurant la prime de risque que l'investisseur consentirait à payer ex-post pour couvrir son portefeuille contre les risques extrêmes. Les résultats empiriques montrent que la couverture de portefeuilles par des *ETP* VIX bat significativement la couverture traditionnelle de portefeuilles. Cependant, ce type de couverture reposant sur l'investissement de long-terme par des instruments de réplication passive apparaît particulièrement inadéquate pour les investisseurs peu averses au risque, dont les décisions d'investissement s'avèrent aléatoires, de type jeux de hasard. Ce chapitre a donc des implications pratiques pour l'industrie de la gestion d'actifs puisqu'il recommande un effort pédagogique soutenu auprès des investisseurs lors de la distribution de telles stratégies.

Faisant suite au diagnostic des stratégies traditionelles de volatilité basées sur la réplication passive, le chapitre 2 pave la voie à une nouvelle génération de stratégies de volatilité, cette fois actives, optionelles, et basées sur l'investissement factoriel.

Inspirée d'une intuition simple, notre étude théorique et empirique part de l'idée suivante. Les prix de marché d'options, c'est-à-dire la volatilité implicite, reflètant l'incertitude et le risque extrême, nous exploitons l'écart entre les distributions de densités de probabilités implicitées des options, et les distributions de densités constatées sur le sous-jacent. Cet écart de valorisation, matérialisé dans le niveau, la pente, et la convexité du *smile* de volatilité implicite, mesure l'écart entre la distribution de probabilités des rendements du sous-jacent et la distribution log-normale à la Black-Scholes. Aussi, notre approche d'investissement ''Smart Vega'' décompose le risque contenu dans le *smile* de volatilité implicite en stratégies optionelles investissables répliquant les primes de risque de volatilité, skewness, et kurtosis, sous la forme de *swap* de divergences. Plus précisément, nous dérivons une représentation analytique de la fonction de *smile* de volatilité implicite exprimée comme une combinaison de primes de risque investissables qui récompensent le portage de risques d'ordres supérieurs. En outre, notre approche est validée sur le plan empirique, en particulier pour des distributions de probabilités fortement asymétriques et leptokurtiques.

Dans cette même continuité, le chapitre 3 teste notre approche active de l'arbitrage de volatilité en la comparant aux stratégies utilisées par les investisseurs sophistiqués tels que les *hedge funds*. Ainsi, nous identifions le risque extrême dans la performance des *hedge funds* sous la forme des stratégies d'investissement optionelles étudiées dans notre approche "Smart Vega". Nous montrons que les stratégies de primes de risques d'assurance constituent des déterminants importants dans la performance des *hedge funds*, tant en analyse temporelle qu'en analyse cross-sectionnelle. En contrôlant des facteurs de risque standards à la Fung-Hsieh communément utilisés dans la littérature, nous mettons en évidence qu'un choc positif de la prime de risque de volatilité est associée à une baisse substantielle de la performance aggrégée des rendements *hedge funds*. En particulier, nous démontrons que les *hedge funds* fortement sensibles à la prime de volatilité (kurtosis) surperforment substantiellement les fonds moins sensibles. Ce résultat suggère dans quelle mesure l'alpha des *hedge funds* provient en réalité de la vente de stratégies d'assurance contre le risque extrême. Ce papier ouvre donc la voie à la réplication de la performance de *hedge funds* sophistiqués par la réplication de stratégies de primes de risque d'assurance.

Chapter 1

***When Gambling is Not Winning*: Exploring Optimality of VIX Trading Under the Expected Utility Theory**

When Gambling is Not Winning:

Exploring Optimality of VIX Trading Under the Expected Utility Theory [1]

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Recently, financial innovations have given rise to complex derivatives within the asset management industry. Although traditional assets pay dividends or coupons, VIX futures contracts have been partly misunderstood by unsophisticated investors, as they only provide portfolio insurance against stock market crashes. Therefore, over the calmer period 2009-2014, the most traded VIX futures exchange-traded product lost practically all of its value, ruining unexperienced investors. Hence, this paper investigates appropriateness of these complex derivatives with investor's risk aversion. We address portfolio-choice optimality under uncertainty, for overlay allocations composed of equities, bonds, and VIX futures. This paper proposes a non-trivial solution based on the Expected Utility (EU) theory to simulate investor's behavior with risk aversion. Furthermore, it derives an investor's surprise metric defined as a welfare criterion measure, and a model-implied risk premium defined as the insurance premium investor pays ex post to hedge. Empirical results show investing in VIX futures significantly beats traditionally diversified portfolios, but they turn to be particularly inappropriate for risk-loving investors. From the asset management perspective, this paper has practical implications since it recommends pedagogical efforts to raise investors' awareness of overlay strategies.

1.1 Introduction

You don't gamble to win. You gamble so you can gamble the next day.

Bert Ambrose, English bandleader and violonist

In the recent years, a multitude of financial innovations designed for a wide variety of investors have flourished within the asset management industry. Learning from the past, risks inherent in complex new financial products prove to be partly misunderstood, especially by unsophisticated investors. For example, this had been the case with regard to the monetization risk associated to capital protection funds, such as constant proportion portfolio insurance (CPPI), especially when risky assets underperform at launch. Similarly, this paper investigates the inherent risks in newly launched complex hedging strategies based on volatility derivatives. Specifically, developed by the Chicago Board Options Exchange (CBOE) in 2004, VIX futures contracts have set an all-time monthly trading volume record in October 2014[1], extoling the diversification effects of implied equity volatility. Following the seminal papers of Whaley (1993, 2009) in [113] and [114], volatility derivatives are assumed to provide portfolio diversification and risk-reduction, capturing the leverage effect, i.e. inverse correlation between stock market index and market volatility. In particular, Szado (2009) in [108] exhibits the portfolio insurance provided by a buy-and-hold VIX futures exposition during the subprime crisis. From August 2008 to December 2008, adding 10% VIX futures contracts to an equity-bond portfolio improves annualized return from -15.9% to -0.3%, and mitigates annualized standard deviation from 21.7% to 13.3%. More generally, Chen, Chung, and Ho (2011) in [51] apply mean-variance spanning tests over the period 1996-2008 to exhibit enhanced in-sample Sharpe ratios when diversifying traditional portfolios with implied equity volatility.

However, over the period from January 2009 to July 2014, the most traded short-term VIX

[1] Totaling an average daily trading volume of 323,761 futures contracts. Source: CBOE Futures Exchange (CFE), as of November 3, 2014.

futures exchange-traded product in the U.S., i.e. the VXX ETP, lost practically all of its value (-99.6%). This outstandingly disappointing performance brought ruin upon many uninformed investors[2], putting into question the benefits of such innovations within the asset management industry. In particular, Whaley (2013) in [115] examines ETPs benchmarked to VIX short-term futures indexes as buy-and-hold investments. From March 2004 to March 2012, he investigates the slope of VIX futures term structure at 30 day to expiration. As costs of carry prove to be painful for rollovers, he finds the average slope at 30 days to expiration is 2.3% over the period 2004-2012, and the prices curve is usually upward-sloping in nearly 81% of trading days. Consequently, his undermining intuition suggests that futures contracts in VIX Index prove to be inappropriate buy-and-hold instruments for risk-loving investors, as these instruments lose money with certainty through time. This arises from the fact that volatility derivatives do not deliver certain cash flows, forming therefore a proper asset class with specific properties. Although traditional asset classes, i.e. equities and bonds, pay either certain dividends or coupons, VIX futures rather consist in an insurance premium that investors consent to pay to hedge their portfolios against scarce stock market crashes.

In this paper, we evaluate the appropriateness of complex volatility derivatives with investor's risk-aversion degree. For this purpose, we examine the optimality of portfolio choice under uncertainty, for an overlay allocation composed of equities, bonds, and VIX futures contracts, from December 31, 2004 to July 4, 2014. Therefore, the issue this paper addresses does not prove to be trivial, as the approaches commonly used within the asset management industry and the existing literature are inappropriate here. On the one hand, Alexander and Korovilas (2011) in [12] apply the standard mean-variance criterion to examine the diversification effects provided by buy-and-hold investments in VIX futures. However, as described by Jondeau and Rockinger

[2]E.g. following VIX ETNs incidents in 2012, legal investigations were initiated against ETNs providers, claiming they fail to inform retail investors of inherent risks, whereas the largest regulator in the U.S. (FINRA) published an alert about VIX trading risks.

(2006) in [75], this common framework pioneered by Markowitz (1959) in [82] inappropriately handles complex derivatives, as it proves ineffective under large departure from normality. Featured by strongly non-normal return distributions, VIX futures contracts require to investigate portfolio optimality under non-quadratic preferences to take into account higher-order moments. On the other hand, the practical issue usually met by asset managers when implementing optimal portfolio strategies consists in mitigating the frequency of portfolio rebalancing, costly for investors. However, this information loss proves to be statistically detrimental for portfolio optimization.

This paper investigates optimality of investment decisions under uncertainty by modeling investor's behavior under the normative decision theory[3]. Inspired by the Expected Utility (EU) theory à la von Neumann-Morgenstern (1947) in [93] and by seminal papers pioneered by Samuelson (1969) in [103] and Merton (1969) in [87], we perform direct numerical optimizations to maximize the EU of investor's terminal wealth. Direct optimization relies specifically on the empirical estimation of joint returns distributions, based on a multivariate block bootstrap procedure described by Kunsch (1989) in [77] to capture the dependence structure of neighbored data, both over time and cross-sectionally. This approach, ensuring robustness among alternative time-settings, consistently addresses the non-quadratic preferences entailed by strongly non-Gaussian returns on VIX futures. Therefore, we implement an asset-allocation strategy for a portfolio composed of equities, bonds, and VIX futures. We address investment-decision optimality under three various criterion measures. As a first step, optimal portfolios are examined under the criterion of risk-adjusted performance measures, especially the Adjusted for Skewness Sharpe ratio (ASSR) introduced by Koekebakker and Zakamouline (2009) in [116], handling risk-preferences at the third order. On the one hand, between traditional portfolios, portfolios

[3] This paper considers the theoretical framework of a purely rational investor, but investment decision optimality could also be derived under the behavioural Cumulative Prospect Theory (CPT).

composed of equities and bonds, and alternative portfolios, i.e. portfolios diversified with VIX futures. On the other hand, between alternative portfolios depending on investor's risk aversion. As a second step, we evaluate the investor welfare gains provided by VIX futures optimal positioning, particularly depending on investor's risk appetite. Inspired by the microeconomics works pioneered by Akerlof and Dickens (1982) in [9], we derive an investor's surprise metric, defined as a welfare criterion measure. More precisely, investors feel satisfaction as the final outcome they obtain ex post exceeds their rational expectations. Conversely, negative investor's surprise corresponds to unfulfilled expectations, generating ex post investor's pain. As a third step, this paper addresses optimality of portfolio insurance by extracting the model-implied risk premium from optimal portfolios. This defines the insurance premium that the rational investor implicitly consented to pay ex post in order to hedge his portfolio against extreme events.

Empirical results provide the three following evidence that proved to be robust, both in-sample and when implementing portfolio strategies, and whatever the time settings. First, under the criteria of risk-adjusted performance measures and investor's welfare, investing in VIX futures significantly beats traditionally diversified allocations, across the relative risk-aversion coefficient γ. In-sample $ASSR$ exhibits that portfolios diversified with VIX futures (4.09) significantly outperforms equity-bond portfolios (1.93), when $\gamma = 7$ for example. Moreover, implemented strategies preserve the notable outperformance of alternatively diversified portfolios (0.24) against traditional portfolios (0.14). Therefore, VIX futures positioning significantly improves the ex post investor welfare, whatever the risk aversion. For example, when $\gamma = 5$, ex post positive surprise is on average 47% higher for portfolios adding VIX futures, suggesting that they significantly exceed investor rational expectations. Second, empirical results confirm that VIX futures contracts are particularly inappropriate buy-and-hold instruments for risk-loving investors. Increasing the relative risk-aversion coefficient from $\gamma = 2$ to $\gamma = 12$ efficiently improves in level our investor welfare metric from 0.17% to 0.51%. This suggests that, when investing in VIX futures, risk-loving investors tend to feel more ex post pain than risk-averse investors. For example, when risk aversion is low for $\gamma = 2$, VIX futures positioning provides notably higher investor disappointment (-1.35%) than traditional asset classes (-0.75%). Besides, higher risk aversion drastically mitigates the volatility of investor surprise and of ex post discomfort, respectively by 32% and 45%. This consistently validates that risk-loving investors inappropriately evaluate the risks inherent in VIX futures contracts. Distorted by gambling attitudes, their decision-process underestimates the painful costs of carry that are paid to maintain portfolio insurance. Third, the model-implied insurance premia extracted ex post from optimal portfolios are relevant with the first empirical findings. The ex post risk premia derived from alternative portfolios, i.e. equity-bonds portfolios diversified with VIX futures, significantly outdo those derived from traditional equity-bonds portfolios. This result proves to be consistent whatever the investor's risk aversion, under the EU framework. For example, when $\gamma = 3$, VIX futures optimal positioning provides far more effective insurance premium (23.13%) than traditional equity-bonds portfolios (8.21%). This result confirms that VIX futures provides better portfolio insurance against tail risks than traditional diversification.

This paper extends the existing literature in the four following ways. The technology undermining our paper upgrades the previous works of Szado (2009) in [108], Chen, Chung, and Ho (2011) in [51], and Alexander and Korovilas (2011) in [12]. The commonly used mean-variance criterion they apply proves to be inappropriate to address portfolio choice optimality when handling derivatives, such as VIX futures contracts. Therefore, we address optimality by performing direct numerical optimizations of agent's EU. This results in handling appropriately non-quadratic agent's preferences, and in mitigating portfolio rebalancing frequency. Therefore, under this theoretical framework, we derive the model-implied risk premium from optimal positioning. This defines the portfolio insurance provided either by traditional asset classes, i.e. equities and bonds, or by VIX futures positioning. Subsequently, when agents are expected-utility maximizers, this paper extends portfolio choice optimality by exploring VIX futures positioning. To the best of our knowledge, this paper is the first examining optimality under the EU framework, for an asset allocation composed of equities, bonds, and VIX futures contracts. Sharpe (2007) in [106] maximizes EU within an asset-allocation composed of equities, bonds, and cash. Similarly, Carr and Madan (2001) in [47] study optimal positioning of European-style options under the EU theory, within a bond-equity portfolio, but they do not consider volatility derivatives. Besides, this paper illustrates the microeconomics theory pioneered by Akerlof and Dickens (1982) in [9]. As decision-makers feel ex post pain if the final outcome does not exceed their rational expectations about future, we propose an original welfare criterion measure to investigate decision-process optimality under uncertainty. More precisely, this investor surprise metric evaluates the welfare gains, i.e. either positive or negative surprise, provided either by traditional asset classes, or by VIX futures positioning. Furthermore, our most decisive contribution consists in validating the intuition undermining Whaley (2013) in [115], both by risk-adjusted portfolio performance measures and by our welfare criterion metric. Empirical results confirm that VIX futures contracts are particularly inappropriate buy-and-hold investments for risk-loving investors. Distorted by gambling attitudes, risk-loving investors do not evaluate appropriately the risks inherent in VIX futures, especially the painful costs of carry.

This paper arises two practical implications, especially within the asset management industry. First, from the perspective of product management, promoting overlay and hedging strategies based on volatility derivatives requires to implement intensive pedagogical efforts to raise investors' awareness of the risks inherent to such complex derivatives. In effect, our empirical evidence proves that VIX futures contracts are particularly inappropriate buy-and-hold investments for risk-loving investors. Therefore, pedagogical efforts should bring to investors the relevant expertise to efficiently benefit from portfolio insurance provided by VIX futures positioning. Second, from the perspective of quantitative asset managers, this paper proposes a consistent alternative approach to the commonly used mean-variance criterion. A direct numerical optimization of the expected utility appropriately handles the non-quadratic preferences related to complex derivative instruments, generalizing the mean-variance framework at higher order-moments. Furthermore, we propose two relevant metrics to examine portfolio choice optimality and portfolio insurance. The welfare criterion measure evaluates model-risk management, and the model-implied risk premium gauges portfolio risk-reduction.

The remainder of the paper is organized as follows. In section 1, we explore the dataset and discuss the statistical properties of asset returns, validating the approach undermining this paper. In section 2, we describe the standard EU asset-allocation problem and its practical implementation within the asset management industry. This section especially defines two criterion measures used to investigate portfolio-choice optimality. First, our investor welfare criterion measure gauges ex post discrepancies between the realized and expected utilities derived from VIX futures positioning. Second, the model-implied risk premium, extracted from optimal portfolios under the EU theory, evaluates portfolio insurance provided either by traditional asset classes or by VIX futures. In section 3, we investigate the empirical patterns related to portfolio-choice optimality, under the three following performance criteria: risk-adjusted portfolio performance measures, welfare criterion metric, and model-implied risk premium. Section 4 exposes some concluding remarks and practical implications within the asset management industry.

1.2 Data

The dataset consists of three time series for equity, bond, and VIX futures indices, composed of 2,246 historical daily closing prices. Data sample is provided by Bloomberg, over the period from December 30, 2005 to July 4, 2014. Equity, bonds, and VIX futures indices are respectively S&P 500 Total Return Index, JPM Global Aggregate Bond Index, and S&P 500 VIX Short-

Term Futures Index. The last index replicates a buy-and-hold strategy that rolls over VIX futures contracts, on a daily basis, from the nearest month to the next month. This results in maintaining a constant one-month rolling long position in the first and second month VIX futures contracts. In the previous literature, the S&P 500 VIX Short-Term Futures Index has been well documented, especially by Whaley (2013) in [115]. As of March 30, 2012, seven of the eight largest VIX ETPs traded in the U.S. are benchmarked to the S&P 500 VIX Short-Term Futures Index, totalling an asset value of nearly $2,985 million.

Figure 1.1 displays the time-varying Pearson correlations of the most traded VIX ETNs in the U.S. with their benchmark, the S&P 500 VIX Short-Term Futures Index, since their inception. For multiple equal to 1, the VXX, VIXY, and VIIX ETNs have average strong positive correlations, respectively equal to 96.3%, 95.8%, and 93.0%. For leveraged ETNs, the TVIX and UVXY are also strongly positively correlated, respectively at 94.2% and 95.4%. However, much less traded VIX ETNs, like the XXV, exhibit weaker time-varying correlations. As the main VIX ETNs tend to be strongly correlated to their associated benchmark, the S&P 500 VIX Short-Term Futures Index proves to be fairly-typical of the widely traded VIX ETNs. Alternatively, our empirical study could be declined with mid-term VIX futures, rebalanced daily to maintain five-month constant maturity. Launched on March 26, 2004 by the CBOE, VIX futures contracts are preeminently characterized by a usual upward-sloping term structure, generating important costs of carry for buy-and-hold strategies. More precisely, Whaley (2013) in [115] calculates that the average slope of VIX futures term structure at 30 day to expiration is 2.3%. In other words, the 30-day futures price tends to decrease on average by 2.3% per day.

Table 1.1 (Panel A) exhibits the outstandingly disappointing performance of VIX futures investing. This puts into question the contribution of such alternative asset within the asset management industry. From 2005 to 2014, a buy-and-hold investment in VIX futures contracts lost practically all of its value (-99.2%), and displayed a considerable annualized volatility

(61.3%). In contrast, traditional asset classes such as equity or bonds achieved impressive annualized returns (respectively 9.7% and 6.5%), with much lower annualized volatilities (respectively 21.2% and 5.3%). However, breaking down the dataset into sub-periods of stock market crises (Panel B) and of calm (Panel C) extols the benefits of VIX futures investing for portfolio diversification and risk reduction. Although time slicing proves to be artificial, especially by violating path-dependency of asset returns, this method exhibits stylized effects characterizing these asset classes. Triggered by the Lehman Brothers bankruptcy, the subprime crisis ranges from August 29, 2008 to November 20, 2008, as VIX index spiked from 20.7% to 80.9%. Therefore, the European sovereign debt crisis ranges from July 11, 2011 to October 3, 2011, whilst the *gauge fear index* spiked from 18.4% to 45.5%. Over periods of stock market turbulence (Panel B), equities achieved negative holding period returns (-52.2%), strongly contrasting with VIX futures (1139.0%). This illustrates clearly the diversification effects exhibited by Szado (2009) in [108] of a buy-and-hold VIX futures exposition during the recent financial crises, by capturing the implied leverage effect between a stock index and its implied volatility. Following Table 1.2, the negative correlation between equities and VIX futures particularly increases during the periods of financial crises (-83%), whilst bonds offered only limited diversification (-17%). This illustrates the previous works of Whaley (1993, 2009) in [113] and in [114] that investing in volatility derivatives could benefit to long equity investors.

Furthermore, the distinct empirical properties exhibited above raises the following statistical issue. VIX futures behave very differently from traditional asset classes, e.g. equities and bonds, as they displayed on average strongly negative returns and high volatility from December 30, 2005 to July 4, 2014. This stems from the fact that VIX futures contracts form distinct securities that do not generate certain cash flows. Although equities and bonds pay certain dividends and coupons, respectively, VIX futures contracts provide an insurance premium against stock market crashes. Consequently, commonly used parametric methods, such as mean-variance frameworks, are inappropriate to explore VIX futures positioning within an equity-bond allocation. Therefore, it is especially true as the Figure 1.2 displays distinct stylized effects characterizing returns on VIX futures, in terms of higher-order moments. As expected in Table 1.1 and Figure 1.2, returns distributions of equity, bonds, and VIX futures are significantly non-normal, asymmetric, peaked and heavy-tailed. However, distinct empirical properties characterize VIX futures returns in terms of higher order moments. From Table 1.1 (Panel A), returns distribution of VIX futures is strongly skewed to the right, more rounded, and less heavy-tailed ($Sk = 0.9$, $\kappa = 6.9$), in comparison to equities ($Sk = -0.1$, $\kappa = 13.8$), and bonds ($Sk = 0.3$, $\kappa = 8.4$). Following Cont (2001), mean-variance approaches are invalidated when stylized effects cannot be modeled appropriately with the first two moments. This is especially the case in this paper, when considering sophisticated instruments such as volatility derivatives. Therefore, VIX futures investing would be always penalized by quadratic-preferences agents, whereas these securities offer efficient equity diversification in times of stock market turmoil. This validates the approach of direct numerical optimization undermining this paper, as it appropriately handles higher-order moments.

1.3 Empirical Methodology

In this section, we describe the methodology implemented to evaluate empirically the appropriateness of VIX futures to investor's risk-aversion degree. For this purpose, we expose the asset-allocation problem that consists in maximizing the agent's expected utility by optimally allocating wealth between equity, bonds, and VIX futures contracts. Subsequently, this section defines two relevant criterion measures to investigate portfolio-choice optimality under uncertainty. First, our investor welfare criterion metric measures the ex post discrepancies between the realized and expected utilities derived from VIX futures positioning. Second, the model-implied risk premium, extracted from optimal portfolios under the EU theory, evaluates portfolio insurance provided either by traditional asset classes or by VIX futures diversification.

1.3.1 Framework

In the literature, the mean-variance framework introduced by Markowitz (1959) in [82] is one of the most commonly used approaches to examine diversification benefits. More specifically, Alexander and Korovilas (2011) in [12] use the mean-variance criterion to examine portfolio diversification with buy-and-hold positions in VIX futures contracts. However, this results in maximizing the investor's expected utility at only order two. Therefore, it does not handle higher-order moments that must be taken in account when investing in alternative assets, characterized by strong non-normally distributed returns and substantial downside tail risk. Subsequently, the following framework that we propose proves to handle more appropriately risk preferences, especially when investing in sophisticated derivatives such as VIX futures contracts.

Pioneered by Samuelson (1969) in [103] and Merton (1969) in [87], the asset-allocation problem we apply in this paper is one of the classic problems of modern finance. Standard formulation consists in an investor's objective to maximize the expected utility $E\left[U\left(W_T\right)\right]$ of end-of-period wealth W_T, by allocating wealth W_{T-1} at time $T-1$ between equities, bonds and VIX futures over the investment period $[T-1,T]$. We assume that his utility function $U\left(.\right)$ exhibits a constant relative risk aversion, as defined below by the so-called isoelastic utility:

$$U\left(W_T\right) \equiv \frac{W_T^{1-\gamma}}{1-\gamma}, \gamma \neq 1, \gamma > 0 \tag{1.1}$$

where γ denotes the coefficient of agent's relative risk aversion. This results in finding the optimal investment policy $\{\omega_{i,T}^{*}\}$ for $i \in \{1,3\}$, i.e. the optimal weights of equities, bonds, and VIX futures, respectively, maximizing the following expected utility over the investment period

$[T-1,T]$:

$$E^{IP}\left[U\left(W_T^*\right)\right]=\max_{\{\omega_{i,T}\}} E^{IP}\left[U\left(W_T\right)\right] \tag{1.2}$$

subject to the following constraints

$$W_T=W_0\left(1+\sum_{i=1}^{3}\omega_{i,T}r_{i,T}\right) \tag{1.3}$$

$$\sum_{i=1}^{2}\omega_{i,T}=1 \tag{1.4}$$

$$\omega_{i,T,\min}\leq\omega_{i,T}\leq\omega_{i,T,\max}, i=\{1,2,3\} \tag{1.5}$$

where $W_T^*=W_0\left(1+\sum_{i=1}^{3}\omega_{i,T}^*r_{i,T}\right)$, defining the end-of-period wealth generated by the optimal investment policy $\{\omega_{i,T}^*\}$. For $i=\{1,2,3\}$, $r_{i,T}$ designate respectively returns on equities, bonds and VIX futures contracts over the investment period $[T-1,T]$. $E^{IP}[\cdot]$ refers to the expectation operator under the real-world probability measure IP. As specified in (1.5), an optimal positioning in VIX futures contracts denoted $\omega_{3,T}^*$ provides diversification for the equity-bond portfolio defined by (1.4).

As follows, we describe the practical implementation of the asset-allocation problem from

(1.2) to (1.4). The portfolio strategy we implement addresses practical issues frequently met within the asset management industry, especially mitigating portfolio rebalancing without information loss. Therefore, we propose a monthly rebalancing frequency based on intra-monthly data to perform a direct numerical optimization. The budget constraints defined below allow for portfolio leverage and short sales on VIX futures contracts:

$$\begin{aligned} \omega_{i,T,\min} = 20\%, \omega_{i,T,\max} = 80\%, i \leq 2, \\ \omega_{3,T,\min} = -20\%, \omega_{3,T,\max} = 20\% \end{aligned} \tag{1.6}$$

At the end of period $[T-1, T]$, portfolio weights solving (1.2)-(1.6) define the optimal investment policy $\{\omega^*_{i,T}\}$ used to rebalance portfolio over the period $[T, T+1]$. Portfolio strategy at time T is then implemented as below, giving the following end-of-period portfolio value $\tilde{W}_{T+1}$ at time $T+1$

$$\tilde{W}_{T+1} = \tilde{W}_T \left(1 + \sum_{i=1}^{3} \omega^*_{i,T} r_{i,T+1} \right) \tag{1.7}$$

where $\tilde{W}_{T+1}$ defines the end-of-period wealth generated by optimal investment policy $\{\omega^*_{i,T}\}$ calculated over $[T-1, T]$. The procedure is repeated at the end of each investment period, at equally spaced time intervals.

As mentioned earlier, one of the main practical issues undermining the investment policy specified in (1.7) consists in mitigating the frequency of portfolio rebalancing, as trading could be costly. However, extending the length T of investment periods is statistically detrimental for portfolio optimizations. For example, information loss generated by a monthly data frequency results from insufficient observations, i.e. nearly 40 monthly returns with only one new observation at each optimization. Therefore, we address this practical issue by using intra-monthly data, i.e. 30 daily historical returns, to perform monthly portfolio rebalancing.

Furthermore, the mean-variance criterion introduced by Markowitz in [82] is equivalent to maximizing the expected utility at order two. This approach inappropriately investigates the diversification effects provided by sophisticated instruments. Therefore, we extend the previous works of Alexander and Korovilas (2011) in [12] by using a maximization of the expected utility at higher order moments to better handle risk preferences. As Equation (1.2) can't be solved exactly, Jondeau and Rockinger (2006) in [75] apply a Taylor series expansion for $U(W_T)$ of order four around $\overline{W_T} \equiv E^{IP}[W_T]$. We could use the specification (1.1) of the isoelastic utility function $U(\cdot)$ to obtain the approximate solution exposed in the Appendix A. However, this paper rather proposes a direct numerical optimization, where the nonlinear programming problem (1.2)-(1.6) is solved with an active-set algorithm, up to the precision associated to termination conditions.

More specifically, the numerical optimization is based on the estimation of historical joint distributions by simulating scenarios of cross-sectional asset returns. To this purpose, we perform a multivariate block bootstrap procedure to estimate numerous trajectories of terminal wealth W_T over each investment horizon T. As described by Kunsch (1989) in [77], the block-bootstrap procedure preserves the dependence structure of asset returns, both in time and cross-sectionally. For each subsample of historical data, fixed-length blocks of cross-sectional returns are selected randomly with replacement, and then put together in a non-overlapping way to simulate a new subsample. The bootstrap procedure is repeated 10^5 times, for 30-day[4] subsamples and 5-day blocks.

Finally, the benchmark used to investigate rational investment decisions is associated to the optimal portfolio solving (1.2)-(1.6) and implementation (1.7) with $i = \{1, 2\}$, i.e. the optimal

[4] Alternative settings for investment-time windows and block lengths are available upon request. Empirical results associated to alternative settings ensure robustness towards the conclusions exposed in this paper.

equity-bond portfolio excluding VIX futures investing.

1.3.2 Welfare Criterion Measure

Under the standard expected utility theory, decision-makers are assumed to be entirely rational machine men, devoid of anticipatory feelings, i.e. positive surprise or disappointment, when facing uncertainty. However, this assumption has been contradicted by behavioral finance theory. As in Akerlof and Dickens (1982) in [9], rational agents make decisions under risk to maximize their welfare, by anticipating the future and forming endogenous beliefs based on their preferences. Furthermore, ex ante welfare provided by anticipatory feelings is gauged by the expected future utility based on the estimation of risk distribution. Individuals feel therefore pain or disappointment if the final outcome does not reach their rational expectations about the future. By analogy, asset managers usually compare ex post the received payoff of the lottery to the anticipated payoff derived by their forecasting models. This issue is directly related to model risk management defined by Rebonato (2001) in [98] that consists in controlling discrepancies between the mark-to-model value of a security, and the market price at which it had been traded. Therefore, asset managers form ex post either pain or pleasure from the comparison between their model price and the market price.

In this paper, asset-allocation problem (1.2) consists of an investor's objective to maximize expected utility $E^{IP}\left[U\left(W_T\right)\right]$ of his end-of-period wealth W_T. We assume that rational investors are expected-utility maximizers who compare at end-of-period T the realized utility $U\left(\tilde{W}_T\right)$ with the anticipated utility $E^{IP}\left[U\left(\tilde{W}_T\right)|I_{T-1}\right]$ derived from implementation (1.7). Therefore, they form either positive or negative surprise denoted $Surprise_T$ as below

$$Surprise_T = \underbrace{\left|U\left(\tilde{W}_T\right)\right|}_{\substack{\text{Ex post welfare,}\\ \text{i.e. realized utility}}} - \underbrace{\left|E^{IP}\left[U\left(\tilde{W}_T\right)|I_{T-1}\right]\right|}_{\substack{\text{Ex ante welfare,}\\ \text{i.e. expected utility}}} \tag{1.8}$$

i.e. ex post pleasure if $Surprise_T > 0$, and ex post pain otherwise, where implemented portfolio $\tilde{W}_T = \tilde{W}_{T-1}\left(1 + \sum_{i=1}^{3} \omega^*_{i,T-1} r_{i,T}\right)$ as specified by Equation (1.7), and $\{I(t)\}_{t\in[0,T-1]} = \{r_{i,T-1}\}$, for $i \in \{1,3\}$. $E^{IP}[\cdot|\cdot]$ is the conditional expectation operator under the real probability measure IP. $\tilde{W}_T$ defines the end-of-period wealth generated by optimal investment policy $\{\omega^*_{i,T-1}\}$, i.e. portfolio weights solving asset-allocation problem (1.2) over investment horizon $[T-2, T-1]$. For isoelastic functions specified in (1.1), negative utility requires to compare realized with anticipated utility in absolute terms. Therefore, for positive $Surprise_T$, investors feel satisfaction as the final outcome they obtained at end-of-period T exceeds their rational expectations. Conversely, negative $Surprise_T$ corresponds to unfulfilled expectations, generating investor's ex post pain[5].

The intuition behind our welfare criterion measure is that, when investing in VIX futures, risk-loving investors tend to feel more ex post pain than experimented and rational investors. Distorted by gambling attitudes, the decision process of risk-loving investors anticipates inappropriately the risks inherent in complex derivatives. In the literature, recent studies consistent with Whaley (2013) in [115] document the ex post welfare costs of risk-loving investors. Therefore, the hypothesis we test stipulates that our utility criterion measure $Surprise_T$ significantly improves when diversifying with VIX futures, and especially when the degree of investor's risk-aversion γ increases.

[5]This investor welfare measure could be related to some extent to the Regret Decision theory, since decision-makers usually anticipate pain when facing an investment decision by incorporating past negative surprises.

1.3.3 Model-Implied Risk Premium

Although equities and bonds generate cash flows with certainty, i.e. either dividends or coupons, VIX futures contracts provide portfolio insurance against stock market crashes. Therefore, the costs of carry consist in financing an insurance premium to hedge portfolios against stock market downside, as investors consent to pay a risk premium to avoid uncertainty. Subsequently, the hypothesis undermining this section stipulates that VIX futures contracts provide more efficient insurance portfolio than traditionally diversified portfolios, i.e. equity-bonds portfolios.

In preference theory, risk-averse decision-makers systematically prefer to exchange a risky lottery for a certain payment, under uncertainty. Described by Mas-Colell, Whinston, and Green (1995) in [84], the risk premium defines the maximum amount of money that the risk-averse agent consents to pay to avoid lotteries riskiness. Therefore, the risk premium Π_T realized at time T corresponds to the amount of money between the maximum expected wealth $E^{IP}\left[W_T^*\right]$ and the certainty equivalent C_T:

$$\Pi_T = E^{IP}\left[W_T^*\right] - C_T \tag{1.9}$$

where $W_T^* = W_0\left(1 + \sum_{i=1}^{3} \omega_{i,T}^* r_{i,T}\right)$, i.e. wealth generated by optimal investment policy $\{\omega_{i,T}^*\}$.

Explicitly, equation (1.9) refers to the additional incentive that risk-averse agents need to take on the risk of the lottery. As specified by the equation below, the certainty equivalent C_T associated to the asset-allocation problem (1.2) defines the lowest amount of money received with certainty at time T for which the rational decision-maker remains indifferent to a lottery.

$$C_T = U^{-1}\left[E^{IP}\left[U\left(W_T^*\right)\right]\right] \tag{1.10}$$

where $U^{-1}[\cdot]$ denotes the inverse function of the agent's utility, γ denotes the relative risk-aversion coefficient of the risk-averse agent, and $E^{IP}[U(W_T^*)] \equiv \max_{\{\omega_{i,T}\}} E^{IP}[U(W_T)]$, as specified by the standard allocation problem (1.2). Therefore, by plugging the Equation (1.1) of the isoelastic utility function and developing the equation of the certainty equivalent C_T, the risk premium Π_T becomes

$$\Pi_T = \underbrace{E^{IP}[W_T^*]}_{\text{Expected wealth}} - \underbrace{\left[(1-\gamma).\max_{\{\omega_{i,T}\}} E^{IP}[U(W_T)]\right]^{\gamma-1}}_{\text{Certainty equivalent } C_T} \tag{1.11}$$

where $E^{IP}[W_T^*]$ designates the maximum expected value, i.e. the objective maximum value related to the lottery, and $E^{IP}[U(W_T)]$ is the expected utility under the real-probability measure, i.e. the subjective value related to the lottery.

Specifically, Equation (1.11) corresponds to the observation that risk-averse agents usually spend money to get rid of a specific risk. Risk-averse agents may like risky lotteries under uncertainty if the expected payoffs that they yield are worth the riskiness. Similarly, risk-averse investors may purchase risky assets if their expected returns exceed the risk-free rate. Theoretically, higher lotteries uncertainty and/or higher agent's degree of risk-aversion γ increase the risk premium Π_T paid to insure portfolios. Furthermore, another direct consequence is that Π_T proves to be nonnegative, when $U(\cdot)$ is concave, i.e. for risk-averse agents. In accord with equation (1.11), we evaluate the model-implied insurance portfolio provided either by traditional asset classes, e.g. equities and bonds, or by VIX futures contracts, when portfolios are optimally allocated. In our intuition, traditional portfolios provide significantly lower model-implied risk premia Π_T than alternative portfolios, i.e. overlay portfolios including VIX futures. This results that VIX futures positioning provides better portfolio insurance and higher incentives to take on the stock market risks, than traditional asset classes.

1.4 Empirical Results

This section examines the empirical patterns related to portfolio choice optimality, under the three performance criteria described previously. In particular, this part tests the appropriateness of VIX futures contracts to investor's risk aversion. For this purpose, optimal portfolios are investigated, first, under the criterion of risk-adjusted portfolio performance measures, handling appropriately higher-order moments; second, under the criterion of our welfare measure *Surprise*; and third, under the criterion of the model-implied risk premium Π, gauging the portfolio insurance offered by optimally diversified portfolios.

1.4.1 Risk-Adjusted Performance Measures

This subsection compares optimal portfolios under the criterion of risk-adjusted portfolio performance measures that appropriately take into account the risks inherent to VIX futures contracts. Therefore, comparisons are twofold: on the one hand, between traditional portfolios and overlay portfolios diversified with VIX futures, both in-sample portfolios and implemented portfolios; on the other hand, between portfolios adding VIX futures in function of the degree of risk aversion.

Figure 1.3 exhibits the optimal investment policy $\{\omega^*_{i,T}\}$ solving the asset allocation problem (1.2)-(1.6), for an overlay portfolio composed of equities, bonds, and VIX futures contracts. Optimal portfolio weights $\{\omega^*_{i,T}\}$ clearly exhibit time-dependency and specific cross-asset relations. Although optimal weights $\omega^*_{3,T}$ allocated to VIX futures tend to be negative on average over the entire dataset, they turn notably positive in times of stock market crashes, especially during the subprime crisis, i.e. from August 29, 2008 to November 20, 2008, and during the European sovereign debt crisis, i.e. from July 11, 2011 to October 3, 2011. Time-dependency of optimal portfolio weights $\{\omega^*_{i,T}\}$ entails in particular time-variable returns distributions for the implemented portfolios $\tilde{W}_T$, as illustrated by Figure 1.4 that breaks down returns distributions into different time-periods, especially the subprime crisis and the European sovereign debt crisis. Therefore, from Figure 1.3, $\omega^*_{3,T} < 0$ generally implies higher $\omega^*_{1,T}$ and vice versa, capturing the inverse relation between the stock index and its implied volatility, i.e. the implied leverage effect. Furthermore, optimal portfolio weights $\{\omega^*_{i,T}\}$ depend on the coefficient γ of relative risk aversion. Increasing the risk-aversion coefficient from $\gamma = 2$ to $\gamma = 12$ mitigates portfolio overweighting and underdiversification. This result consistently follows the Modern Portfolio Theory (MPT) pioneered by Markowitz (1959) in [82], stipulating that portfolio diversification provides risk reduction. In particular, more risk-averse investors typically spread more nonsystematic risk across asset classes.

Following the statistical issue raised by the framework, commonly-used portfolio performance measures, specifically the Sharpe ratio SR, prove to be only valid for quadratic preferences. This is the case for either quadratic utility functions, and/or normally distributed asset returns, e.g. when asset returns can be precisely modelled with the first two moments.

$$SR_T = \frac{R_T - r_{f,T}}{\sigma_T} \tag{1.12}$$

where R_T and $r_{f,T}$ respectively refer to logarithmic returns on the portfolio and on the risk-free asset over the period $[T-1, T]$. σ_T denotes the standard deviation of portfolio logarithmic returns. Consequently, the Sharpe ratio SR does not handle appropriately the properties inherent to sophisticated derivatives, e.g. risk preferences for higher-order moments and strongly non-

Gaussian returns distributions. Therefore, we examine portfolio performances under the Adjusted for Skewness Sharpe Ratio $ASSR$, proposed by Koekebakker and Zakamouline (2009) in [116],

$$ASSR_T = SR_T. \left[1 + \frac{1}{3}\left(1 + \frac{1}{\gamma}\right) Sk_T \times SR_T\right]^{1/2} \tag{1.13}$$

where SR_T and Sk_T refer respectively to the Sharpe ratio and to the skewness of portfolio returns distribution. As specified by (1.13), the Adjusted for Skewness Sharpe Ratio $ASSR$ handles investors' risk preferences at order three, penalizing high third order-moment Sk_T, especially when relative risk aversion γ increases.

Table 1.3 reports the risk-adjusted performance measures related, on the one hand, to the optimal portfolios W_T^* (Panel A), and on the other hand, to the implemented portfolios $\tilde{W}_T$ (Panel B). Empirical results suggest that adding VIX futures to traditional equity-bond allocations significantly improves the risk-adjusted performance measures, both in-sample (Panel A) and following the implementation (Panel B). On the one hand, in the case of optimal portfolios W_T^* (Panel A), in-sample performances are calculated with the optimal investment policy $\{\omega_{i,T}^*\}$, solving portfolio problem (1.2)-(1.6) over $[T-1, T]$. Compared to traditional portfolios (at the left), the Adjusted for Skewness Sharpe Ratio $ASSR$ of portfolios diversified with VIX futures (at the right) significantly outperforms (4.09 versus 1.93 for $\gamma = 7$). Although annualized volatility of alternative portfolios (at the right) is higher (16.94% versus 10.10% for $\gamma = 5$), annualized return significantly outperforms (45.32% versus 16.49% for $\gamma = 5$). Besides, adding VIX futures drastically mitigates maximum drawdown (8.96% versus 13.70% for $\gamma = 10$), whereas returns distribution is left-skewed, more rounded, and less heavy-tailed ($Sk = -0.22$, $Sk = 7.68$ versus $Sk = 0.26$, $Sk = 9.10$, for $\gamma = 7$). On the other hand, in the case of implemented portfolios $\tilde{W}_T$ (Panel B) globally preserves the patterns described above. Performances are calculated with optimal weights $\{\omega^*_{i,T}\}$ solving problem (1.2)-(1.6) over $[T-1,T]$, and implemented over $[T,T+1]$ as specified by (1.7). In comparison to traditional portfolios (at the left), the Adjusted for Skewness Sharpe Ratio $ASSR$ of portfolios diversified with VIX futures (at the right) keeps outperforming (0.24 versus 0.14 for $\gamma=7$). Although annualized volatility of alternative portfolios (at the right) remains higher (18.97% versus 11.32% for $\gamma=5$), annualized return proves to significantly outperform (8.95% versus 5.48% for $\gamma=5$). These empirical results are relevant with Moran and Dash (2007) in [90], or Brière, Burgues and Ombretta (2010) in [42]. They consistently validate the robust portfolio risk-reduction and downside-risk controlling provided by VIX futures optimal positioning.

Furthermore, Table 1.3 and Figure 1.5 suggest that a higher degree of investor's risk-aversion improves notably the risk-adjusted performance measures. In particular, this is especially true for portfolios diversified with VIX futures contracts, both for portfolios W^*_T and $\tilde{W}_T$. As illustrated by Figure 1.5 and Figure 1.6 for implemented portfolios $\tilde{W}_T$, raising the relative risk aversion γ particularly mitigates the standard deviation and generates more peaked returns distributions. More precisely, Table 1.3 exhibits that, when raising the coefficient of relative risk aversion from $\gamma=2$ to $\gamma=12$, annualized volatility decreases from 19.61% to 17.11%, whereas excess kurtosis remains high, as κ ranges from 10.88 to 12.41. Therefore, this consistently mitigates the maximum drawdown from 42.45% to 36.70%. Globally, a higher risk aversion degree significantly improves the risk-adjusted performance measures of implemented portfolios $\tilde{W}_T$, as the Adjusted for Skewness Sharpe ratio $ASSR_T$ strongly increases from 0.18 to 0.25. In comparison to traditional portfolios, the Adjusted for Skewness Sharpe ratio remains stable near 0.15. However, for less risk-averse investors, adding VIX futures proves to be detrimental in terms of risk-adjusted performance measures. For $\gamma=2$ and $\gamma=3$, alternative portfolios (respectively 0.18 and 0.17) are quite similar to traditional portfolios (both 0.15).

Under risk-adjusted performance measures, empirical results validate the hypotheses undermining this paper. First, overlay portfolios diversified with VIX futures significantly outperforms traditionally diversified equity-bond portfolios. Second, they confirm the intuition of Whaley (2013) in [115] that VIX futures are inappropriate investments for risk-lovers and non-sophisticated investors. Furthermore, we investigate these intuitions under our welfare criterion measure *Surprise*.

1.4.2 Welfare Criterion Measure

This subsection investigates VIX futures optimal positioning under the utility criterion measure *Surprise* specified by (1.8). The hypotheses undermining this part are twofold. First, ex post welfare gains measured by quantity *Surprise* would be significantly higher when diversifying a traditional equity-bond portfolio with VIX futures contracts. Second, strongly risk-loving investors tend to feel more ex post pain than experimented and rational investors.

Over the entire period, Table 1.4 (Panel A) exhibits on average positive investor *Surprise* for both traditional portfolios and portfolios adding VIX futures. Furthermore, ex post pleasure increases with risk aversion, from 0.15% to 0.39% for traditional portfolios, and from 0.17% to 0.51% for portfolios adding VIX futures. Finally, ex post elation is significantly higher when including VIX futures, whatever the risk-aversion coefficient. For example, when $\gamma = 5$, ex post positive surprise is 47% higher for portfolios adding VIX futures. These results suggest that adding VIX futures to a traditional asset allocation better exceeds rational expectations, particularly when investors are highly risk-averse, as illustrated by Figure 1.8. From 2005 to 2014, Table 1.4 breaks down investor's surprise into periods of satisfaction (Panel B) and periods of disappointment (Panel C). As defined by Equation (1.8), investor's satisfaction and disappointment correspond respectively to ex post positive and negative *Surprise*. Although preliminary comments suggest only minor differences in the number of periods, notable differences in level of ex post elation (Panel B) or pain (Panel C) shed light on specific patterns. There are approximately as many periods of satisfaction or disappointment between the two portfolios, and across risk-aversion coefficients. For example, ex post discomfort (Panel C) ranges from 34% to 42% of total periods for traditional portfolios, and from 32% to 46% for portfolios adding VIX futures. However, satisfaction and disappointment levels are higher for portfolios including VIX futures. For example, when $\gamma = 3$, satisfaction and disappointment levels are respectively 76% (Panel B) and 78% (Panel C) higher for portfolios including VIX futures.

Furthermore, Table 1.4 reports the impact of risk-aversion degree on investor's surprise when investing in VIX futures. Globally, increasing risk aversion efficiently mitigates the volatility of investor surprise and of ex post discomfort. Rising risk aversion from $\gamma = 2$ to $\gamma = 12$ reduces drastically surprise's volatility and disappointment's volatility respectively by 84% (Panel A) and by 55% (Panel C). However, for strongly risk-loving investors, adding VIX futures is detrimental in terms of welfare criterion measure *Surprise*. For $\gamma = 2$, overlay portfolios (0.17%) are nearly equivalent to traditional portfolios (0.15%), and investor's disappointment (Panel C) notably goes beyond on average (-1.35% versus -0.75%). This strong evidence consistently extends the intuition behind the previous works of Whaley (2013) in [115] and confirms that VIX futures contracts are inappropriate buy-and-hold instruments for risk-loving investors.

1.4.3 Model-Implied Risk Premium

This subsection examines the level of portfolio insurance provided either by traditional portfolios, i.e. equity-bonds portfolios, or overlay portfolios, i.e. traditional portfolios diversified with VIX futures contracts. For this purpose, we estimate the risk premium Π_T realized at time T, as specified by equation (1.11). The hypothesis undermining this part stipulates that VIX futures offer better portfolio insurance and higher incentives to take on the risks of stock market than traditional asset classes.

Table 1.5 reports the certainty equivalent C_T (Panel C) and the model-implied risk premium Π_T (Panel D), derived from traditional portfolios (at the left) and overlay portfolios (at the right). Empirical results exhibit that for a given relative risk-aversion coefficient γ, VIX futures provide significantly higher risk premia Π_T than traditional asset classes. Preliminary comments from Table 1.5, Figure 1.8, and Figure 1.9 are consistent with the theory of decision-making under uncertainty. First, higher coefficients γ of relative risk aversion decreases the certainty equivalent C_T and increases the risk premium Π_T. For example, with regard to overlay portfolios, from $\gamma = 2$ to $\gamma = 5$, certainty equivalent C_T (Panel C) decreases from 95.01% to 54.18%, whereas risk premium Π_T (Panel D) increases from 10.63% to 51.42%. This observation consistently illustrates that a more risk-averse decision-maker consents to pay higher amounts of money to avoid lotteries riskiness. Second, when relative risk-aversion $\gamma \neq 1$ and $\gamma > 0$, empirical results generally exhibit $\Pi_T > 0$. Nevertheless, Figure 1.8 shows the time-varying risk premia Π_T do not remain nonnegative across time. Theoretically, risk-averse agents, i.e. for concave utility function $U(\cdot)$, need an additional incentive to take on the risk of the lottery, under uncertainty. This extra incentive defines the risk premium, i.e. the cost of risk induced by lotteries uncertainty.

Furthermore, according Table 1.5 (Panel D), VIX futures contracts provide on average significantly higher risk premia than traditional asset classes. For example, for $\gamma = 3$, alternative portfolios offer notably higher incentive (23.13%) than traditional portfolios (8.21%), for each of the lotteries. This proves particularly true for more risk-averse investors. For $\gamma = 5$, the cost of risk associated to lotteries uncertainty becomes respectively 51.42% and 17.57%. For illustration, Figure 1.9 exhibits the certainty equivalent for different degrees of investor's risk aversion.

This last empirical result provides twofold major findings. First, VIX futures positioning provides significantly stronger portfolio insurance than traditionally diversified portfolios, as they better remunerate the cost of risk for each of the lotteries. Second, when investors are more risk-averse, VIX futures provide better portfolio protection than traditional asset classes, validating their appropriateness to only strongly risk-averse investors.

1.5 Conclusions

This paper has been motivated by the outstandingly disappointing performance of volatility derivatives. Learning from the past, dampened investors usually turn away from this original asset class, as they misunderstood risks associated to these complex instruments. Subsequently, this paper addresses the appropriateness of VIX futures contracts to investor's risk-aversion, examining portfolio-choice optimality under risk.

Empirical results provide three evidence that proved to be robust both in-sample and when implementing portfolio strategies, whatever the time settings. First, investing in VIX futures significantly beats traditionally diversified portfolios in terms of Adjusted for Skewness Sharpe Ratio and of ex post investor welfare. For example, when $\gamma = 7$, $ASSR$ notably increases both in-sample (from 1.93 to 4.09), and when implementing portfolio strategies (from 0.14 to 0.24). Therefore, VIX futures positioning significantly improves the ex post investor welfare. When $\gamma = 7$, ex post positive surprise is on average 47% higher for portfolios adding VIX futures, suggesting that they significantly exceed investor rational expectations. Second, results confirm that VIX futures contracts are particularly inappropriate buy-and-hold instruments for strongly risk-loving investors. Increasing the relative risk-aversion from $\gamma = 2$ to $\gamma = 12$ efficiently improves in level our investor welfare metric from 0.17% to 0.51%, and drastically mitigates the volatility of investor surprise and of ex post discomfort respectively by 84% and 55%. This suggests that, when diversifying with VIX futures, risk-loving investors tend to feel more ex post pain than risk-averse investors. Third, the ex post risk premia derived from overlay portfolios, i.e. equity-bonds portfolios diversified with VIX futures, significantly outdo those derived from traditional equity-bonds portfolios. When $\gamma = 3$, VIX futures optimal positioning provides far more effective insurance premium (23.13%) than traditional equity-bonds portfolios (8.21%).

This contributes to the existing literature and opens up a range of new perspectives in the three following ways. First, our decisive contribution validates the hypothesis undermining the previous work of Whaley (2013) in [115], i.e. VIX futures are only appropriate buy-and-hold investments for sophisticated and risk-averse investors. Therefore, this raises practical implications from the perspective of the asset management industry, as it requires intensive pedagogical efforts to educate investors about the inherent risks. Furthermore, future extensions suggest declining the exercise with mid-term VIX futures, rebalanced daily to maintain five-month constant maturity. Second, existing literature examined portfolio-choice optimality under the common mean-variance criterion, e.g. Szado (2009) in [108], Chen, Chung, and Ho (2011) in [51], Alexander and Korovilas (2011) in [12]. Nevertheless, as stipulated by Jondeau and Rockinger (2006) in [75], the framework confirms the Markowitz (1959) in [82] approach inappropriately handles complex derivatives, under large departure from normality. To the best of our knowledge, no previous study investigated this issue under the EU framework pioneered by Samuelson (1969) and Merton (1969) in [103] and [87], for overlay allocations composed of equities, bonds, and VIX futures. Therefore, this paper proposes an alternative approach, but deep evaluations of its practicality are left for future research. Third, this paper illustrates the seminal works pioneered by Akerlof and Dickens (1982) in [9], as it derives an original welfare criterion measure to investigate the optimality of portfolio choice.

1.6 Appendix

1.6.1 Approximate Solution for Expected Utility

In this appendix, we detail the Taylor series expansion of wealth utility $U(W_T)$, as described by Jondeau and Rockinger (2006) in [75]. Applying an approximation at order 4 around the expected wealth $E^{IP}[W_T]$, investor's expected utility can be written as below

$$E^{IP}[U(W_T)] \approx E^{IP}\left[U\left(\overline{W_T}\right)\right] + E^{IP}\left[\frac{\partial U(\overline{W_T})}{\partial W_T}\left(W_T - \overline{W_T}\right)\right] + \frac{1}{2!}E^{IP}\left[\frac{\partial^2 U(\overline{W_T})}{\partial W_T^2}\left(W_T - \overline{W_T}\right)^2\right]$$
$$+\frac{1}{3!}E^{IP}\left[\frac{\partial^3 U(\overline{W_T})}{\partial W_T^3}\left(W_T - \overline{W_T}\right)^3\right] + \frac{1}{4!}E^{IP}\left[\frac{\partial^4 U(\overline{W_T})}{\partial W_T^4}\left(W_T - \overline{W_T}\right)^4\right] + E^{IP}\left[o\left(W_T - \overline{W_T}\right)\right] \tag{1.6.1.1}$$

where $\overline{W_T} \equiv E^{IP}[W_T]$, and γ defines the coefficient of relative risk-aversion. Therefore, (1.1) for the isoelastic utility function gives the approximate solution.

$$E^{IP}[U(W_T)] \approx \frac{1}{1-\gamma}\overline{W_T}^{1-\gamma} - \frac{1}{2}E^{IP}\left[\left(W_T - \overline{W_T}\right)^2\right]\gamma\overline{W_T}^{-\gamma-1}$$
$$+\frac{1}{6}E^{IP}\left[\left(W_T - \overline{W_T}\right)^3\right]\gamma(\gamma+1)\overline{W_T}^{-\gamma-2} - \frac{1}{24}E^{IP}\left[\left(W_T - \overline{W_T}\right)^4\right]\gamma(\gamma+1)(\gamma+2)\overline{W_T}^{-\gamma-3} \tag{1.6.1.2}$$

The 3rd and 4th terms contain risk preferences for the 3rd and 4th order of asset returns co-moments.

Table 1.1: Descriptive Statistics of Asset Returns

	Daily Returns		
	Equity	Bonds	VIX Futures
Panel A: All observations			
Nb of observations	2246	2246	2246
Mean	0,04%	0,02%	-0,14%
Median	0,06%	0,00%	-0,43%
Standard deviation	1,34%	0,33%	3,86%
Annualized standard deviation	21,19%	5,28%	61,30%
Holding period return	86,32%	57,55%	-99,15%
Annualized return	9,68%	6,46%	-11,12%
Skewness	-0,08	0,29	0,85
Kurtosis	13,78	8,41	6,94
Jarque-Bera statistic	10822,34***	2757,78***	1714,22***
Panel B: Periods of stock market crises			
Nb of observations	121	121	121
Mean	-0,55%	-0,01%	2,28%
Median	-0,41%	0,00%	2,17%
Standard deviation	3,33%	0,38%	6,05%
Annualized standard deviation	52,90%	6,10%	95,98%
Holding period return	-52,15%	-1,47%	1139,01%
Annualized return	-108,62%	-3,05%	2372,14%
Skewness	0,39	-0,72	0,40
Kurtosis	4,86	6,20	3,93
Jarque-Bera statistic	18,24***	56,18***	6,64**
Panel C: Periods of stock market calm			
Nb of observations	2125	2125	2125
Mean	0,07%	0,02%	-0,28%
Median	0,07%	0,00%	-0,48%
Standard deviation	1,11%	0,33%	3,65%
Annualized standard deviation	17,65%	5,23%	57,99%
Holding period return	289,41%	59,89%	-99,93%
Annualized return	34,32%	7,10%	-11,85%
Skewness	0,00	0,38	0,75
Kurtosis	9,20	8,58	6,96
Jarque-Bera statistic	3389,71***	2790,4***	1581,9***

This table reports the descriptive statistics of historical asset returns, from December 30, 2005 to July 4, 2014. Calculations above are based on daily simple asset returns. Stock market crises are identified as periods of stock market turmoil, ranging from August 29, 2008 to November 21, 2008, i.e. subprime crisis, and from July 11, 2011 to October 5, 2011, i.e. European sovereign debt crisis. Jarque-Bera statistic tests for the rejection of the null hypothesis, i.e. returns normality. Stars *, ** and *** denote statistical significance at the 10%, 5% and 1% level of confidence, respectively.

Table 1.2: Correlations between Asset Returns

	Correlations		
	Equity	**Bonds**	**VIX Futures**
Panel A: All observations			
Equity	1,00	-0,08	-0,76
Bonds		1,00	0,06
VIX Futures			1,00
Panel B: Stock market crises			
Equity	1,00	-0,17	-0,83
Bonds		1,00	0,02
VIX Futures			1,00
Panel C: Stock market calm			
Equity	1,00	-0,07	-0,77
Bonds		1,00	0,07
VIX Futures			1,00

This table reports the cross-asset correlations between equity, bonds, and VIX futures, from December 30, 2005 to July 4, 2014. Calculations above are based on daily simple asset returns. Stock market crises are identified as periods of stock market turmoil, ranging from August 29, 2008 to November 11, 2008, i.e. subprime crisis, and from July 11, 2011 to October 3, 2011, i.e. European sovereign debt crisis.

Table 1.3: Summary Statistics of Portfolio Performances

	Portfolio Choice without VIX Futures						Portfolio Choice with VIX Futures					
	$\gamma = 2$	$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$	$\gamma = 12$	$\gamma = 2$	$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$	$\gamma = 12$
Panel A: Optimal in-sample portfolios												
Annualized return	16,46%	16,49%	16,49%	16,34%	16,20%	16,05%	45,49%	45,45%	45,32%	45,14%	44,24%	43,52%
Annualized volatility	10,37%	10,22%	10,10%	9,77%	9,50%	9,31%	17,45%	17,30%	16,94%	16,55%	15,24%	14,48%
Skewness	0,14	0,17	0,20	0,26	0,32	0,31	-0,24	-0,23	-0,23	-0,22	-0,07	-0,01
Kurtosis	8,32	8,53	8,77	9,10	9,61	9,81	7,02	7,14	7,45	7,68	6,63	6,62
Maximum drawdown	13,70%	13,70%	13,70%	13,70%	13,70%	13,70%	10,87%	10,87%	10,87%	10,87%	8,96%	8,12%
Sharpe ratio	1,43	1,45	1,47	1,50	1,53	1,55	2,51	2,53	2,58	2,63	2,79	2,89
Adjusted Sharpe ratio	1,75	1,80	1,85	1,93	2,00	2,03	3,81	3,85	3,96	4,09	4,62	4,90
Panel B: Implemented portfolios												
Annualized return	5,62%	5,57%	5,48%	5,23%	5,21%	5,27%	9,23%	8,87%	8,95%	9,85%	9,91%	9,87%
Annualized volatility	11,50%	11,40%	11,32%	10,96%	10,55%	10,35%	19,61%	19,53%	18,97%	18,37%	17,62%	17,11%
Skewness	-1,20	-1,22	-1,24	-1,26	-1,26	-1,28	-1,18	-1,19	-1,08	-1,03	-1,10	-1,17
Kurtosis	15,46	15,93	16,31	16,39	16,86	17,59	12,07	12,22	10,93	10,88	11,68	12,41
Maximum drawdown	34,55%	34,55%	34,55%	34,55%	34,32%	33,38%	42,45%	42,45%	42,15%	39,50%	37,14%	36,70%
Sharpe ratio	0,35	0,34	0,34	0,33	0,34	0,35	0,39	0,37	0,38	0,45	0,47	0,48
Adjusted Sharpe ratio	0,15	0,15	0,15	0,14	0,14	0,15	0,18	0,17	0,19	0,24	0,25	0,25

This table reports the summary statistics of portfolio performances, for different coefficients of relative risk aversion, from December 30, 2005 to July 4, 2014. Optimal portfolios (Panel A), denoted W_T^*, are derived from the optimal investment policy $\{\omega_{i,T}^*\}$ solving the asset-allocation problem (1.2)-(1.6). Implemented portfolios (Panel B) $\tilde{W}_T$, are derived from the allocation problem (1.2)-(1.6) and from the portfolio implementation specified by equation (1.7). Coefficients of relative risk aversion are denoted γ.

Table 1.4: Investor's Surprise

	Portfolio Choice without VIX Futures						Portfolio Choice with VIX Futures					
	$\gamma = 2$	$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$	$\gamma = 12$	$\gamma = 2$	$\gamma = 3$	$\gamma = 5$	$\gamma = 7$	$\gamma = 10$	$\gamma = 12$
Panel A: All investment horizons												
Nb investment horizons	74	74	74	74	74	74	74	74	74	74	74	74
Average	0,15%	0,19%	0,25%	0,29%	0,35%	0,39%	0,17%	0,25%	0,37%	0,46%	0,50%	0,51%
Std deviation	1,03%	1,01%	0,99%	0,97%	0,99%	1,02%	1,84%	1,74%	1,58%	1,46%	1,32%	1,25%
Panel B: Investor satisfaction												
Nb investment horizons	43	44	46	48	48	49	40	42	46	48	49	50
% investment horizons	58%	59%	62%	65%	65%	66%	54%	57%	62%	65%	66%	68%
Average	0,79%	0,80%	0,81%	0,80%	0,83%	0,84%	1,47%	1,41%	1,30%	1,25%	1,15%	1,07%
Std deviation	0,77%	0,76%	0,76%	0,78%	0,85%	0,91%	1,27%	1,23%	1,17%	1,12%	1,09%	1,07%
Panel C: Investor disappointment												
Nb investment horizons	31	30	28	26	26	25	34	32	28	26	25	24
% investment horizons	42%	41%	38%	35%	35%	34%	46%	43%	38%	35%	34%	32%
Average	-0,75%	-0,71%	-0,66%	-0,63%	-0,53%	-0,50%	-1,35%	-1,27%	-1,15%	-1,00%	-0,77%	-0,67%
Std deviation	0,57%	0,55%	0,52%	0,49%	0,48%	0,49%	1,08%	0,98%	0,79%	0,68%	0,63%	0,59%

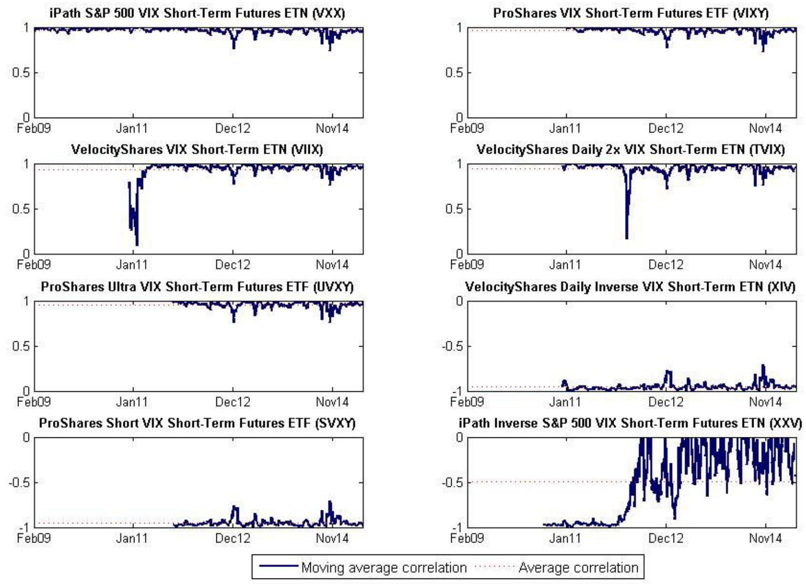
This table reports investor's surprise formed with traditional equity-bonds portfolios (at the left), and with overlay portfolios including VIX futures (at the right), for different coefficients of relative risk aversion, from February 13, 2006 to July 4, 2014. Investor's $Surprise_T$ at time T is derived from (1.8). Panel B investigates the periods of investor's satisfaction, i.e. ex post positive surprise. Panel C investigates the periods of investor's disappointment, i.e. ex post negative surprise. Coefficients of relative risk aversion are denoted γ.

Table 1.5: Certainty Equivalent

	Portfolio without VIX Futures			Portfolio with VIX Futures		
	$\gamma = \mathbf{2}$	$\gamma = \mathbf{3}$	$\gamma = \mathbf{5}$	$\gamma = \mathbf{2}$	$\gamma = \mathbf{3}$	$\gamma = \mathbf{5}$
Panel A: Maximum expected utility						
Mean	-0,9824	-0,4833	-0,2351	-0,9501	-0,4529	-0,2079
Standard deviation	0,0276	0,0272	0,0266	0,0356	0,0336	0,0304
Median	-0,9817	-0,4820	-0,2328	-0,9552	-0,4573	-0,2107
Panel B: Realized utility						
Mean	-0,9809	-0,4815	-0,2325	-0,9483	-0,4504	-0,2042
Standard deviation	0,0276	0,0271	0,0263	0,0362	0,0343	0,0310
Median	-0,9808	-0,4810	-0,2314	-0,9534	-0,4547	-0,2098
Panel C: Certainty equivalent						
Mean	98,24%	93,74%	84,37%	95,01%	82,50%	54,18%
Standard deviation	0,0276	0,1060	0,4135	0,0356	0,1225	0,3435
Median	98,17%	92,94%	75,17%	95,52%	83,64%	50,41%
Panel D: Implied risk premium						
Mean	3,71%	8,21%	17,57%	10,63%	23,13%	51,42%
Standard deviation	0,0563	0,1345	0,4403	0,0761	0,1627	0,3799
Median	3,75%	8,97%	26,74%	9,50%	21,30%	54,31%

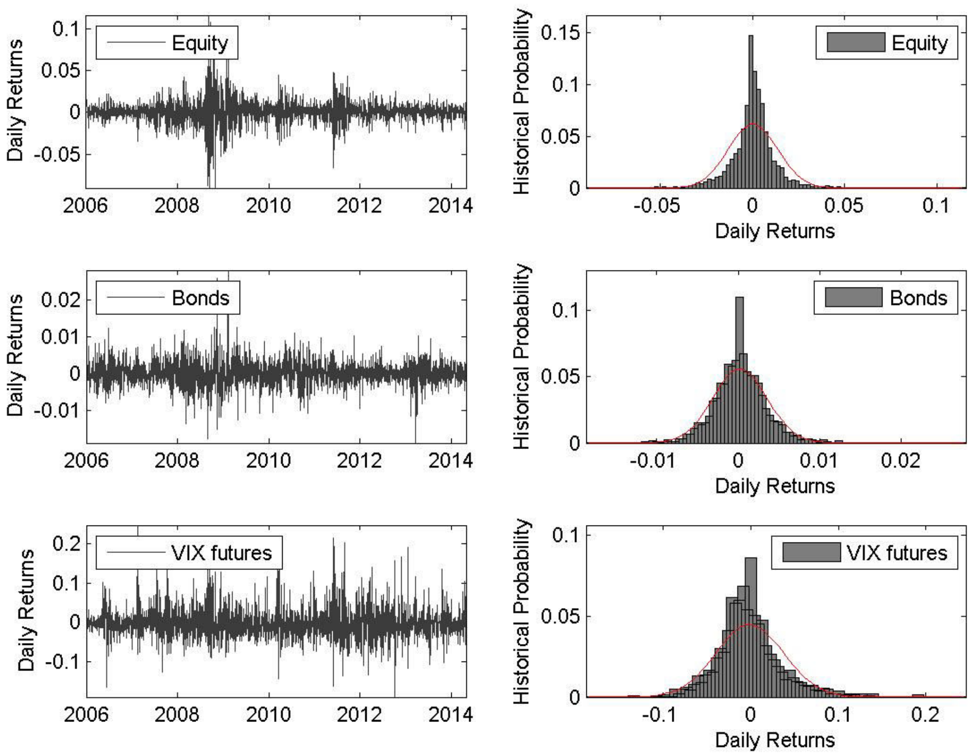
This table reports the suummary statistics of certainty equivalent (Panel C) and model-implied risk premium (Panel D), for different coefficients of relative risk aversion, from February 13, 2006 to July 4, 2014. Certainty equivalent C_T and model-implied risk premium Π_T are expressed as percentage of portfolio values. Coefficients of relative risk aversion are denoted γ. As comparing utility (Panels A and B) between portfolios adding VIX futures and equity-bond portfolios is irrelevant, we rather compare either certainty equivalent (Panel C), or model-implied risk premium (Panel D).

Figure 1.1: Correlations Between VIX ETNs with their Benchmark



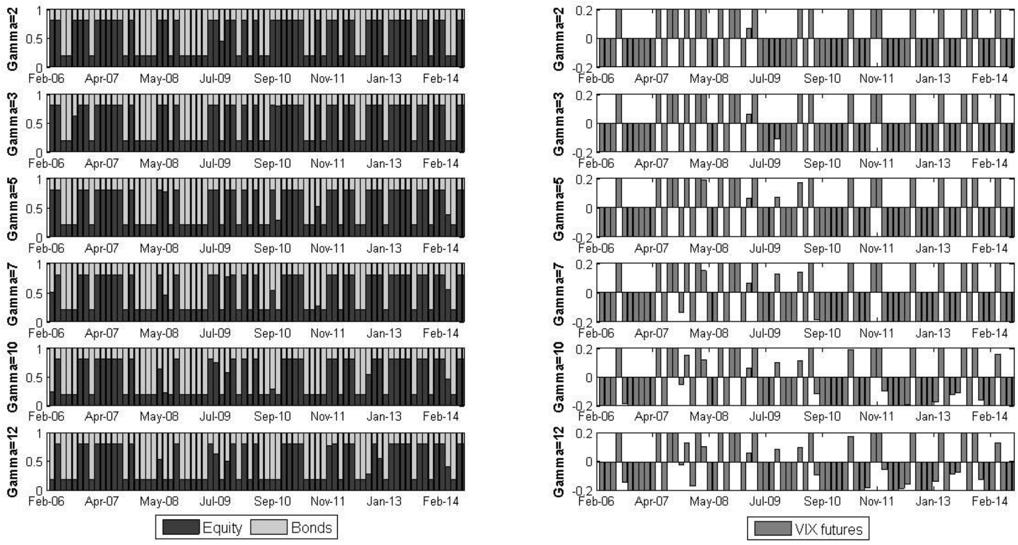
This figure displays the moving average correlations between the most traded VIX ETNs and their benchmark, the S&P 500 VIX Short-Term Futures Index, since their inception. Computations are based on the 20-day rolling Pearson correlations. Multiplier is either 1 for the VXX, VIXY, and VIIX ETNs; or 2 for the TVIX and UVXY ETNs; or -1 for the XIV, SVXY, and XXV ETNs.

Figure 1.2: Historical Asset Returns Distributions



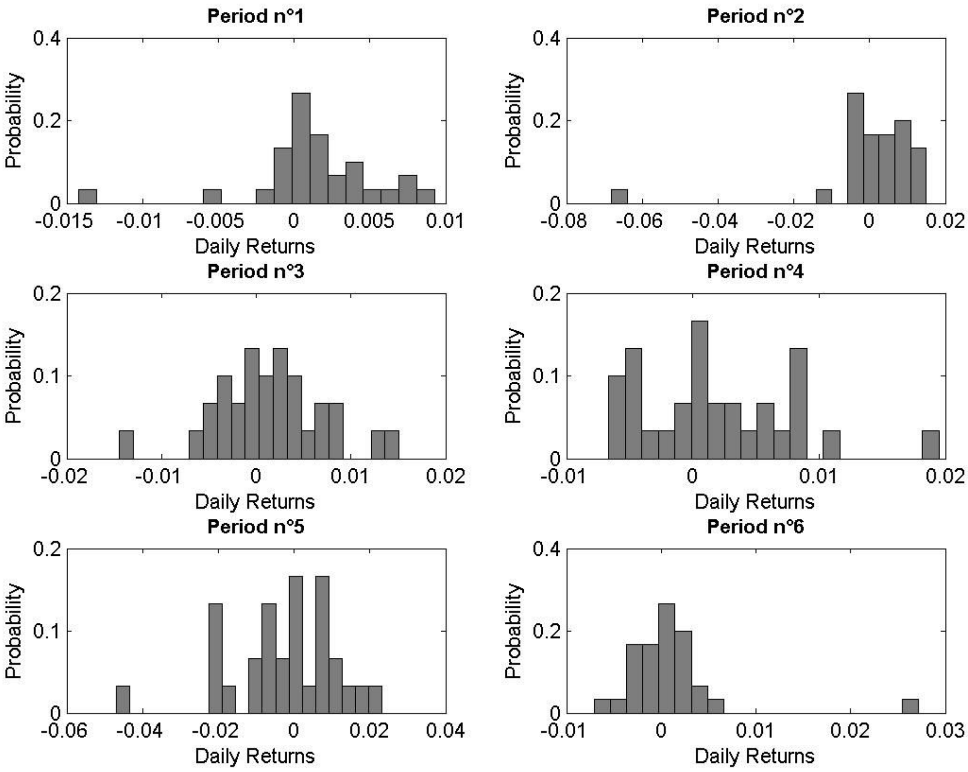
This figure displays the historical returns on equity, bonds, and VIX futures, from December 30, 2005 to July 4, 2014. Calculations above are based on daily simple asset returns. Figures at the left exhibit time-varying historical returns. Figures at the right exhibit historical returns distributions compared to their associated normal probability density function.

Figure 1.3: Optimal Weights



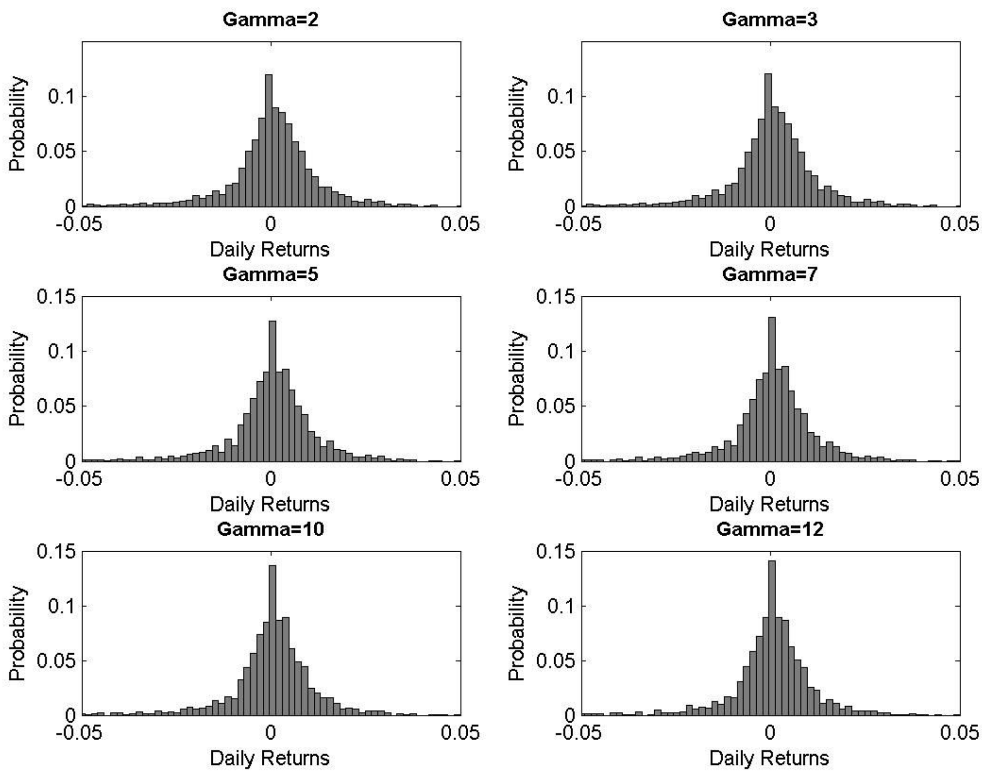
This figure displays the optimal weights for portfolios composed of equity, bonds, and VIX futures, at each investment period, for different coefficients γ of relative risk aversion, from December 30, 2005 to July 4, 2014. Optimal weights correspond to the optimal investment policy $\{\omega_{i,T}^{*}\}$ solving asset-allocation problem (1.2)-(1.6).

Figure 1.4: Returns Distributions of Implemented Portfolios for Different Periods



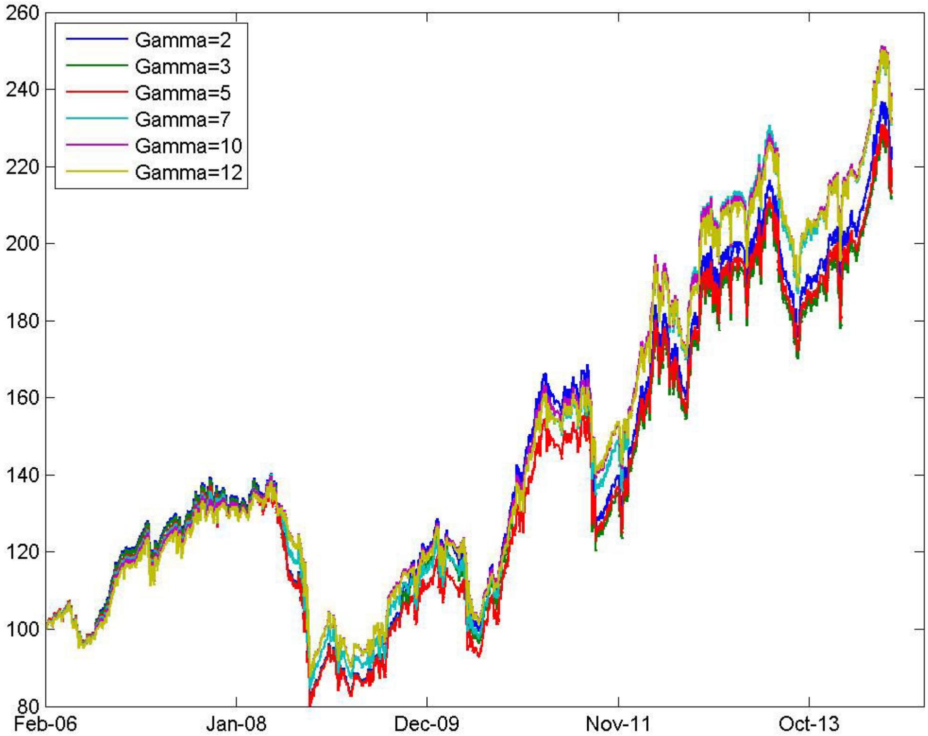
This figure displays the returns distributions of implemented portfolios, for an asset-allocation composed of equity, bonds, and VIX futures, for different periods, and for a constant relative risk-aversion coefficient. Implemented portfolios denoted $\tilde{W}_T$ are derived from equation (1.7). The constant coefficient of relative risk aversion corresponds to $\gamma = 5$. Period 2 and 4 are picked off stock market crises, respectively from October 3, 2008 to November 14, 2008 during the subprime crisis, and from August 19, 2011 to September 30, 2011 during the European sovereign debt crisis. In comparison, the figure also displays standard sub-periods: period 1 (from April 19, 2007 to May 30, 2007), period 3 (from September 4, 2009 to October 16, 2009), period 5 (from May 13, 2013 to June 21, 2013), and period 6 (from February 28, 2014 to April 11, 2014).

Figure 1.5: Returns Distributions of Implemented Portfolios for Different Risk Aversion Degrees



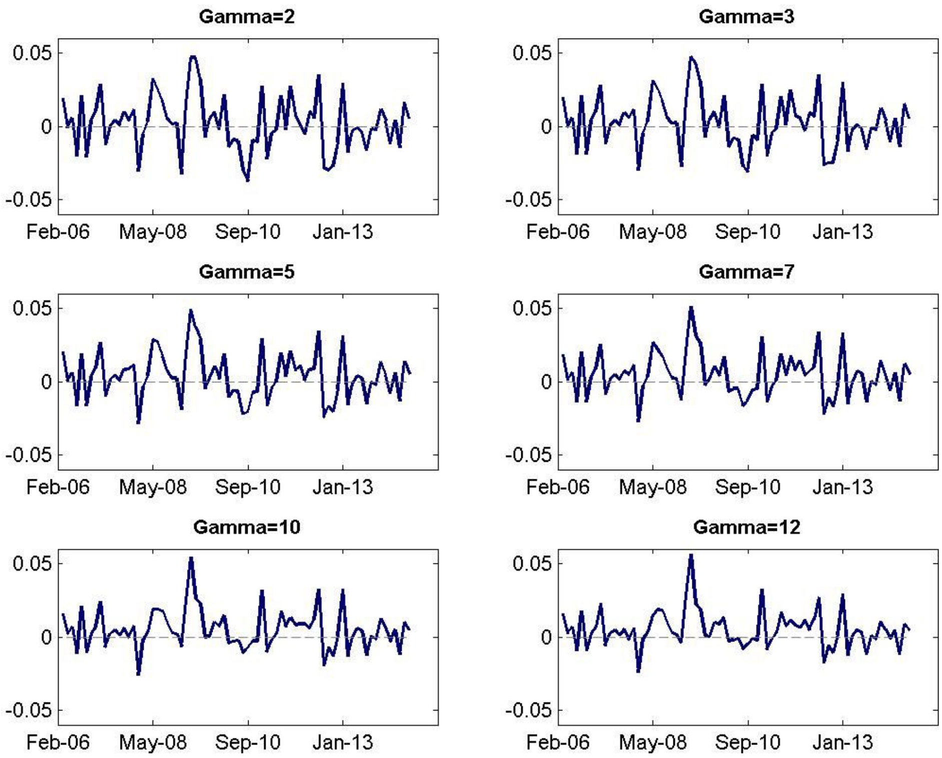
This figure displays the returns distributions of implemented portfolios, composed of equity, bonds, and VIX futures, for different coefficients γ of relative risk aversion, from February 13, 2006 to July 4, 2014. Implemented portfolios $\tilde{W}_T$ are derived from equations (1.2)-(1.6) and implementation (1.7). Coefficients of relative risk aversion are denoted γ.

Figure 1.6: Out-of-Sample Portfolio Performances



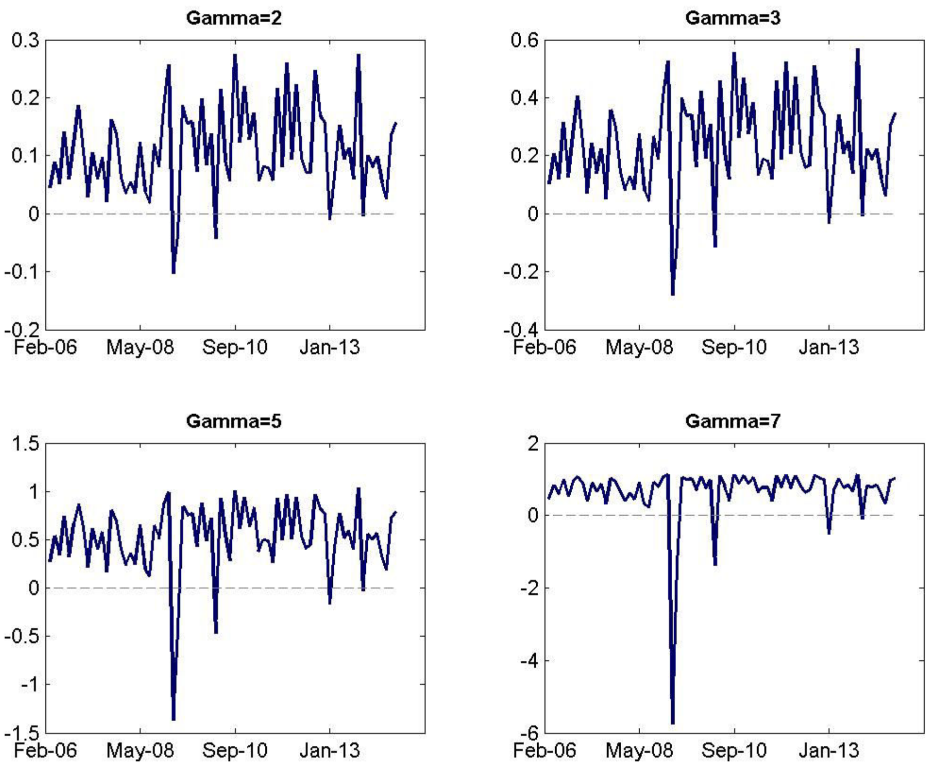
This figure displays the evolution of implemented portfolios values, for different coefficients of relative risk aversion, from February 13, 2006 to July 4, 2014. Implemented portfolios $\tilde{W}_T$ are derived from equation (1.7), allocated between equities, bonds, and VIX futures.

Figure 1.7: Investor's Surprise



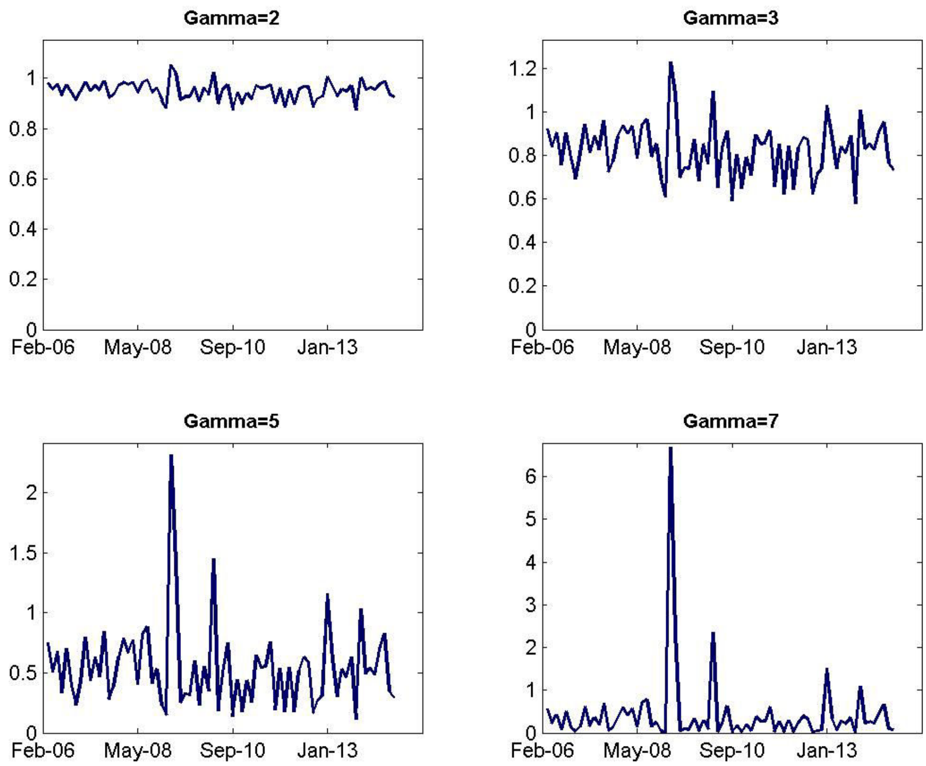
This figure displays the investor's surprise formed with implemented portfolios diversified with VIX futures, for different coefficients of relative risk aversion, from February 13, 2006 to July 4, 2014. Investor's Surprise denoted $Surprise_T$ at time T is derived from (1.8).

Figure 1.8: Model-Implied Risk Premium



This figure displays the model-implied risk premium extracted from implemented portfolios adding VIX futures, for different coefficients of relative risk aversion, from February 13, 2006 to July 4, 2014. Model-implied risk premium, implemented portfolios values and coefficients of relative risk-aversion are denoted Π_T, $\tilde{W}_T$, and γ, respectively.

Figure 1.9: Certainty Equivalent



This figure displays the certainty equivalent extracted from optimal portfolios adding VIX futures, for different coefficients of relative risk aversion, from February 13, 2006 to July 4, 2014. Certainty equivalent, optimal portfolios, and coefficients of relative risk aversion are denoted C_T, W_T^*, and γ, respectively.

Chapter 2

The *Smart Vega* Factor-Based Investing: Disentangling Risk Premia from Implied Volatility Smirk

The *Smart Vega* ™ Factor-Based Investing: Disentangling Risk Premia from Implied Volatility Smirk [1]

[1] This chapter is based on an article that has won the Best Doctoral Paper Award of the Multinational Finance Society at the XXIV$^{\text{th}}$ Annual Conference (2017). I am grateful to Vikas Agarwal, Yacine Aït-Sahalia, Marianne Andries, Kevin Aretz, Eser Arisoy, Steven Clark, Serge Darolles, Matthieu Garcin, Christophe Hurlin, Marcin Kacperczyk, Bryan Kelly, Simone Manganelli, Ilaria Piatti, Todd Prono, Ronnie Sadka, and Fabio Trojani for insightful discussions, and to the participants of the XXI$^{\text{th}}$ International Conference on Computational and Financial Econometrics, the XXIV$^{\text{th}}$ Annual Conference of the Multinational Finance Society, the X$^{\text{th}}$ SoFiE Society for Financial Econometrics Conference, the XV$^{\text{th}}$ INFINITI Conference on International Finance, the Spring 2017 Conference of the Multinational Finance Society, the 25$^{\text{th}}$ Symposium of the Society for Nonlinear Dynamics and EconometricS, the 8$^{\text{th}}$ French Econometrics Conference, the 1$^{\text{st}}$ Portsmouth-Fordham Conference on Banking and Finance, the 1$^{\text{st}}$ ERFIN Econometric Research in Finance, the VI$^{\text{th}}$ FEBS International Conference of the Financial Engineering and Banking Society, the IV$^{\text{th}}$ PhD Workshop in Economics, and the XIII$^{\text{th}}$ Augustin Cournot Doctoral Days.

This paper paves the way for option-based volatility strategies genuinely built on factor-based investing. Since market option prices reflect uncertainty, we exploit the discrepancy between the physical and the risk-neutral distributions, i.e. the fair price of moments. From an economic perspective, the level, slope, and convexity associated to the implied volatility smirk quantify the departure of the returns probability distribution from the lognormal distribution. Subsequently, our so-called ''Smart Vega investing'' proposes option-based replication strategies mimicking the volatility, skewness, and kurtosis risk premia in the form of divergence swap contracts, tradeable at moderate transaction costs in incomplete option markets. Extending the Zhang-Xiang (2008) quadratic approximation, we derive an explicit representation of the implied volatility smirk function, conveniently expressed as a combination of tradeable time-varying risk premia that reward for bearing higher-order risks. Furthermore, we empirically test these theoretical underpinnings on the SPX and the VIX options, under strongly skewed leptokurtic distributions.

2.1 Introduction

Since the S&P 500 index option prices fundamentally incorporate the market's uncertainty, deciphering the fine dynamics of the implied volatility smirk has been of major interest to both academics and practitioners. In this way, the Chicago Board Options Exchange (CBOE) recently introduced in 2011 new options-based indices to benchmark the well-known VIX Index (1993) that approximates the implied volatility associated to SPX options prices. Therefore, the SKEW Index derives the tail risk from the prices of a replicable portfolio of SPX out-of-the-money (OTM) options. Similarly, the VXTH Index tracks the performance of a buy-and-hold SPX portfolio dynamically hedged by VIX call options. Although they propose options-based hedging strategies for buy-and-hold investors, they don't allow to take profit from long-short portfolios replicating option-implied risk premia. Overall, considerable breakthroughs have been achieved concerning volatility modelling in general, notably underpinned by multi-factor models in yield curve modeling. Henceforth, the three-dimensional representation allows to decompose parsimoniously the dynamics of the volatility smirk at any point of time, by accommodating a combination of salient features interpreted primarily as the level, slope, and curvature factors. From an economic motivation, their informational content incorporates agents' risk attitudes and beliefs about the realization of future risks, since they quantify the departure of the returns probability distribution from the lognormal distribution. Specifically, vanished volatility smirk's slope and curvature intuitively reduce the risk-neutral probability distribution to the Black-Scholes lognormal distribution. Hence, vast recent literature including Zhang and Xiang (2008) in [117] and Martin (2013) in [83] among others, disentangles analytically the S&P 500 (SPX) option-implied volatility smirk into the risk-neutral higher-order risks.

In this paper, we explicitly pave the way for new option-based volatility strategies, genuinely built on factor-based investing. By analogy with the "Smart Beta" investing, our so-called

''Smart Vega'' investing generalizes option-based replication strategies, mimicking the volatility, skewness, and kurtosis risk premia contained in swap contracts tradeable at moderate transaction costs in incomplete option markets. Extending the Zhang-Xiang quadratic approximation, we derive an explicit third-way representation of the implied volatility smirk function, expressed as a combination of investable time-varying risk premia reflected in the risk-neutral distribution. Indeed, our analytical decomposition explicitly accommodates the tradeable components of the market price of uncertainty incorporated in the SPX option market prices, exploiting the discrepancies between the physical distribution and the option-implied risk-neutral distribution, i.e. the fair price of moments. From an economic motivation, vanished volatility smirk's slope and curvature reduce the risk-neutral probability distribution to the Black-Scholes lognormal distribution, whereas positive slope and curvature make the risk-neutral density respectively more right-skewed and leptokurtic, i.e. more peaked and heavy tailed. Furthermore, we test empirically our extended Zhang-Xiang quadratic approximation on the VIX options to decompose explicitly the implied volatility smirk into a combination of risk premia. As our analytical approximation particularly accomodates strongly nonlognormal risk-neutral probability distributions, this paper empirically proposes a test on the VIX right-skewed leptokurtic distributions. Since these risk factors define tradeable securities, i.e. volatility, skewness and kurtosis swaps, our decomposition expressly accommodates the components of the market price of the VIX ''fear gauge'', fully priced by the VIX options. Second, to the best of the author's knowledge, this paper is the first to relate analytically the dynamics of the implied volatility smirk of the VIX options to the implied risk-neutral distribution. Although recent papers including Völkert (2014) in [111] and Branger, Kraftschik, and Völkert (2015) in [40] investigate the properties of the risk-neutral moments implied by the VIX option prices, they don't test explicitly the Zhang-Xiang expansion. Overall, this paper covers the two most-actively traded contracts in the U.S. financial markets.

Introduced by Nelson and Siegel (1987) in [92], a large literature covering yield curve modelling documents the dynamic three-factor models, including Litterman and Scheinkman (1991) in [79], Dai and Singleton (2000) in [55], Ang and Piazzesi (2003) in [18], Diebold and Li (2006) in [56], Diebold et al. (2006) in [57], Bianchi et al. (2009) in [30], Aguiar-Conraria et al. (2012) in [7], or Afonso and Martins (2012) in [1]. From an economic motivation, the three salient time-varying risk factors interpreted primarily as the level, slope, and curvature effects, respectively affect the yield curve at different maturities, at short-term, and at medium-term primarily. Extending this approach to volatility modelling, recent literature motivates the three-dimensional representation to capture the dynamics of the implied volatility smirks. From a Karhunen-Loève decomposition of daily implied volatility variations, Cont and da Fonseca (2002) in [54] empirically disaggregate the implied volatility surfaces into a small number of orthogonal factors. Since the first three eigenmodes interpreted as the level, slope, and convexity effects, explain around 98% of the daily variance, a low-dimensional factor model accurately captures the dynamics of the implied volatility surface. Under multi-factor diffusive stochastic volatility models, Bergomi and Guyon (2012) in [29] derive at order two the implied volatility smile in the volatility-of- volatility, exhibiting a quadratic approximation in log-moneyness. Specifically, the Bergomi-Guyon expansion decomposes analytically the implied volatility smirk into the at-the-money (ATM) implied volatility, the ATM skew, and the ATM curvature. These three distinct quantities respectively drive the level, the slope and the convexity effects associated to the implied volatility smile. From an economic perspective, since the implied volatility smirk contains all the information of market option prices, it incorporates the likelihood that investors attach to the realization of future risks. Intuitively, vanished volatility smirk's slope and curvature reduce the risk-neutral probability distribution to the classical Black-Scholes lognormal distribution. Therefore, positive slope and curvature make the risk-neutral density respectively more right-skewed and leptokurtic, i.e. more peaked and heavy tailed. Large literature documents this intuition, including Bates

(2000) in [28], Carr and Wu (2009) in [48], Backus et al. (2011) in [23], Jackworth and Vilkov (2013) in [70], and Andersen, Bondarenko, Todorov, and Tauchen (2014) in [16] among others. They make clear evidence that aggregate S&P 500 option-implied higher moments tend to be fully priced in the market price of variance risk and in the disaster risk. Subsequently, Zhang and Xiang (2008) in [117] derive a second-order polynomial function to relate the dynamics of implied volatility smirks for S&P 500 index options to implied risk-neutral distribution of asset returns. This nearly model-free analytical decomposition doesn't require to assume a dynamics for the implied volatility. They exhibit that the information contained in the risk-neutral variance, skewness, and kurtosis fully describe the level, slope, and curvature of implied volatility smirks. Under the assumption of a continuum of S&P 500 options prices, Martin (2013) in [83] analytically decomposes the VIX squared into the forward-neutral cumulants of the log forward S&P 500 returns. This cumulant expansion of the VIX squared provides clear evidence that the implied volatility smile depends on option-implied higher moments. In the same manner, a very recent literature extends the methodology to VIX options, consisting in European options written on VIX futures contracts. Since VIX option prices incorporate investors' fear and market price of uncertainty, Völkert (2014) in [111] disaggregates the informational content implied in the risk-neutral distribution. Applying a probit model from 2006 to 2011, he finds clear evidence that the risk-neutral variance and skewness contain predictability for the likelihood of upward spikes in the VIX. Prior to upward volatility spikes, average values associated to the risk-neutral variance and skewness lowers respectively from 5.16% to 4.21%, and from 70.71% to 65.51%. Branger, Kraftschik, and Völkert (2015) in [40] observe notable right skewness and excess kurtosis associated to the risk-neutral distribution implied by VIX options. Then, they empirically investigate the time-series properties of the risk-neutral higher moments, and they shed light particularly on remarkable market agents' attitudes towards risk following the U.S. banking crisis in 2008. Persistent upward trends in higher risk-neutral moments indeed suggest investors have become more risk-averse, or expect future stock market crashes.

Subsequently, option-based replication strategies allow mimicking the tradeable option-implied volatility, skewness, and kurtosis risk premia to take bets on the level, slope, and convexity associated to the volatility smirks. Based on the Martin decomposition, Schneider and Trojani (2015) in [104] propose swap trading strategies studied by Bondarenko (2014) in [39] to capture the isolated tradeable compensation for time-varying risks in higher-order moments. Inspired by a new class of divergence trading strategies in Alireza (2005) in [14], they exploit the inconsistency between the option-implied risk-neutral distribution, i.e. the fair price of moments, and the physical distribution of the underlying asset. Similarly, Chang, Zhang, and Zhao (2013) in [50] introduce new derivative contracts, such as skewness and kurtosis swaps, to trade the forward realized third and fourth cumulants. Using S&P 500 index options from 1996 to 2005, they shed light on persistent time-varying properties of higher-order risk premia, offering a justification for such swap strategies. Less recently, Aït-Sahalia, Wang, and Yared (2000) in [8], and Blaskowitz and Schmidt (2002) in [35] document the profitability of skewness and kurtosis trades, exploiting the discrepancies between risk-neutral densities implied by DAX option prices and the historical state-price densities. Similarly, Vazquez (2014) in [110] builds on the Bergomi-Guyon quadratic approximation for the S&P 500 option prices to parametrize the investors' tail risk aversion in terms of the convexity, skew, and kurtosis risk premia. Furthermore, our so-called ''Smart Vega Investing'' addresses naïve long or short equity volatility exposures that consist in allocating volatility derivatives within equity portfolios. For illustration, the most traded VIX exchange-traded note, i.e. the VXX ETN, lost over 99.6% of its value over the period 2009-2014, bringing ruin upon non-sophisticated investors. This paper is particularly motivated by the need to further investigate isolated higher-order risks fully compensated by return premia. Whaley (2013) in [115] suggests that VIX ETPs form inappropriate buy-and-hold investments, as they do not pay certain cash flows, and typically generate positive costs of carry. From March 26, 2004 to

March 30, 2012, he finds a 2.3% average slope for the VIX futures term structure at 30 day to expiration, exhibiting an upward-sloping prices curve, i.e. contango, in nearly 81% of trading days. Under the expected-utility (EU) theory, Alexander and Korovilas (2012) in [13], and Al Wakil (2014) in [10] investigate the risk-reduction provided by VIX futures. Since VIX-related assets exhibit lottery-like features, i.e. high volatility and low skewness, they find that investors' preference for positive skewness, as modelled by risk-aversion, outstandingly affects portfolio diversification. Actually, they economically echo the prudence concept of Menezes et al. (1980) in [86], as characterized in Ebert (2013) in [59] by the aversion to lotteries with negatively-skewed payoff distribution. Differently, Mateus and Konsilp (2014) in [85] investigate stock returns predictability of implied volatility, when decomposed into idiosyncratic and systematic risks. From January 2001 to December 2010, they provide clear evidence of a return premium when carrying only the idiosyncratic part of the implied volatility. Indeed, a one percent rise in the implied idiosyncratic volatility statistically increases future stock returns between 3.06% and 4.26%.

We find analytically an explicit representation of the implied volatility smirk function, expressed as a nonlinear combination of investable time-varying risk premia reflected in the risk-neutral distribution. Subsequently, we find strong evidence that this analytical decomposition is empirically validated for the implied volatility smirks associated either to the SPX or the VIX index options, both accross the strikes and accross the maturities. For illustration, the adjusted R squares remain persistently high, since they range from 17.9% to 45.0% for the SPX put options, and from 11.9% to 31.5% for the VIX call options. Furthermore, empirical results show that the contribution of the higher-order risk premia in the dynamics of the implied volatility smile is more important for the VIX call options than for the SPX put options. Consistently with the prior literature, this empirical finding confirms that the higher-order risks implied by the risk-neutral distribution have considerable informational content in the case of strongly nonlognormal distributions, since the associated market prices fully incorporate expectations about extreme events. Indeed, we find in the case of the VIX options that the kurtosis risk premia incorporate more information than the volatility risk premia, relatively to the SPX options. Subsequently, this paper paves the way for new option-based volatility strategies to capture the compensation for holding the option-implied higher-order risk premia.

This paper extends the prior literature in the three following ways. From the Zhang-Xiang (2008) quadratic approximation in [117] and the decomposition of Martin (2013) in [83], we derive analytically an explicit third-way representation of the implied volatility smirk function, expressed conveniently as a combination of tradeable time-varying risk premia reflected in the risk-neutral distribution. Therefore, to the best of our knowledge, this paper is the first to test these theoretical underpinnings to the VIX right-skewed leptokurtic distribution. Hence, building on Völkert (2014) in [111] and Branger, Kraftschik, and Völkert (2015) in [40], we find clear evidence that the dynamics of the volatility smirk proves to be particularly driven by the higher-order risk factors in the case of the VIX options than for the SPX options. Subsequently, since we propose new option-based replication strategies that allow mimicking the tradeable option-implied volatility, skewness, and kurtosis risk premia to take bets on the level, slope, and convexity of the volatility smirks, this paper extends the prior literature including Schneider and Trojani (2015) in [104], Bondarenko (2014) in [39], Alireza (2005) in [14], Chang, Zhang, and Zhao (2013) in [50], Aït-Sahalia, Wang, and Yared (2000) in [8], and Blaskowitz and Schmidt (2002) in [35] among others.

This paper arises practical implications especially within the industries of the smart indices and the asset management. Since we find clear evidence that our factor-based investing approach may be particularly tailored for trading the implied volatility smile, this paper formally extends the Smart Beta investing to the volatility strategies. Furthermore, our so-called ''Smart Vega Investing'' addresses the controversial performance of long or short equity volatility exposures that consist in allocating naïve volatility derivatives within equity portfolios. Besides, we formally pave the way for the development of new smart investable volatility indices. For illustration, the VIX ''fear gauge' index gave rise to one of the greatest financial innovations, as the VIX options have become the second most actively traded contracts at the CBOE.

The remainder of this paper is organized as follows. In section 1, we provide some descriptive statistics and we describe the methodology applied to clean the options data samples. Furthermore, we discuss some of the well-known empirical properties associated to the SPX and the VIX options. In section 2, we introduce the empirical strategy, i.e. the model setup, by analytically deriving from the Zhang-Xiang quadratic approximation an explicit decomposition of the implied volatility smirk into higher-order risk premia. Therefore, we describe the Bakshi-Kapadia-Madan approach to measure the risk-neutral moments, we document the bias-corrected measures of physical moments based on high-frequency estimation, and we define the tradeable volatility, skewness, and kurtosis risk premia. Hence, we introduce our ''Smart Vega Investing'' framework, that consists in option-based replication strategies mimicking the compensation for time-varying risk factors in higher-order moments. Section 3 presents the empirical results, the time-varying properties of the risk factors we disentangle, and the economic consistency of the replication strategies we propose within our factor-based investing framework. The conclusion resumes the main results, the contributions to the prior literature, the practical implications, and the perspectives.

2.2 Data

The data sample primarily consists in four low-frequency datasets over the period from February 24, 2006 to August 29, 2014, and two high-frequency datasets from January 24, 2008 to December 19, 2012. Provided by OptionMetrics, raw datasets for historical SPX options and VIX options include the closing bid-ask mid prices, expiration dates, strike prices, open interest, and trading volume on a daily frequency, for all the listed maturities. It is particularly noticeable that our empirical study covers the two most-actively traded contracts in the U.S. Therefore, provided by Datastream, historical SPX and VIX futures settlement prices complete the SPX and VIX options datasets on a daily frequency to derive the risk-neutral distributions. The risk-free discount rate provided by Bloomberg on a daily frequency corresponds to the Treasury bill yields for constant maturities ranging from 1-month to 12-month, using linear interpolation across the adjacent maturities when required. Finally, the datasets provided by Bloomberg on an intradaily frequency consist in tick-by-tick historical prices associated to the SPX and the VIX Indexes. The high-frequency datasets allow to derive the model-free physical distribution to compare it to the risk-neutral measures. The data section describes in the one hand, the methodology usually met in the literature that we applied to filter out the market data inconsistencies, and exhibits in the other hand the descriptive statistics related to empirical properties.

2.2.1 SPX Index and Futures

Introduced in 1957 by Standard and Poor's, the S&P 500 Index designates a market capitalization-weighted index, tracking the aggregate and representative performance for the 500 individual stocks listed on the NYSE or the NASDAQ, and associated to the largest U.S. companies. Contrary to strictly rule-based market indices such as the Russell 1000, the S&P 500 companies forming the SPX Index are usually selected by a committee following specific criteria, e.g. market capitalization, liquidity, or public float, to ensure that they appear representative of their economic sector.

Therefore, intraday high-frequency data associated to the SPX Index covers the period from January 24, 2008 to December 19, 2012 with 252 trading days. Each trading day consists in one-minute tick-by-tick prices, such that the trading time spans from 9:30am to 16:00pm.

Commonly considered as the leading indicator of the large-cap companies within the U.S. stock market, the S&P 500 Index gave rise to the S&P 500 index futures contracts. Therefore, in the wake of the successful SPX futures, the Chicago Mercantile Exchange (CME) introduced in 1997 the E-Mini SPX futures, becoming the most traded equity index futures contract in the world. From the cost-of-carry relationship between the SPX Index and the SPX futures, the fair value associated to the futures price corresponds to the discounted index spot. The SPX futures contracts usually expire at each quarter, generally on the third Friday of March, June, September and December. We download the SPX futures daily settlement prices from Datastream, providing quaterly constant maturities for which the future position is continuously rolled into the next contract, at close on the day prior to expiration.

2.2.2 SPX Options

We restrict our empirical analysis to the traditional European index options written on the S&P 500 Index with a.m.-settlement and quarterly near-term expiration time. Introduced by the CBOE in April 1987, the standard SPX index options provide a particularly deep liquidity, a European-style exercise, and a large notional size around $200,000 per contract, with expiration times up to twelve near-term months. Due to cash-settlement, the underlying asset corresponds by convention to the forward S&P 500 index, i.e. SPX futures, at the considered expiration date. Therefore, reverting the Black option pricing formula (1976) in [32], the implied volatilities derived from SPX option mid prices correspond to the volatility parameter such that, plugged into the Black option valuation formula, they give the options market prices. For the purpose of filtering out the pricing anomalies, we apply the standard methodology as described by Neumann and Skiadopoulos (2013) in [94]. Specifically, we restrict the data sample to SPX option quotes for a time to maturity between 5 and 270 days to mitigate the lower liquidity effect. Furthermore, we filter out the abnormal option quotes for zero bid prices, zero open interest, negative bid-ask spreads, and more generally, every time implied volatility can't be computed consistently. Filtered data sample consists in 1,586,801 option quotes, broken down respectively into 634,206 call options quotes, and 952,595 put options quotes.

Table 2.1 displays the descriptive statistics associated to SPX options, over the period from February 24, 2006 to August 29, 2014. It displays the number of observations, the average implied volatilities, open interest, trading volume, and bid-ask spreads, for different log moneyness and maturities buckets. For the purpose of a simple descriptive framework, we define the log moneyness as the logarithm of the strike price divided by the SPX spot. Nevertheless, for the pricing purpose, we consider further the SPX futures prices as the underlying asset. Averaging consists in equally-weighting the call and put options data for each log moneyness-maturity category. Consistently with Skiadopoulos, Hodges, and Clewlow (1999) in [107], our preliminary analysis brings out well-known empirical properties, e.g. the downward-sloping SPX implied volatility smile and the downward-sloping SPX term structure, reflecting that market agents usually trade SPX options to buy long-term portfolio insurance. First, when aggregating the average daily trading volume for SPX options, OTM put options tend to be strongly more actively traded than OTM call options, respectively at 7,503 and 5,558 contracts. In parallel, ITM call options tend to be less actively traded than ITM put options, respectively at 1,729 and 2,089 contracts. Since OTM put options tend to be nearly three times more heavily traded than ITM put options, market agents may usually trade OTM and deep OTM SPX put options to pay insurance portfolio and disaster insurance against stock market crashes. Second, the implied volatility-moneyness function of SPX options tend to be generally downward-sloping, confirming the empirical property that market agents usually trade SPX options to buy portfolio hedges against stock market riskiness. Figure 2.2 displays the average implied volatility smirks for four distinct maturity buckets, over the period from February 24, 2006 to August 29, 2014.

This empirical property clearly shed light by shorter-maturity buckets, is commonly called the volatility skew, characterizing higher implied volatilities for lower strike prices, at a given expiration date. Therefore, the cross-section of implied volatilities as a function of strikes proves to be more roughly V-shaped for shorter maturities, whereas Figure 2.4 exhibits it softens out for long-dated expirations, becoming even flatter and lower. This stylized fact illustrates that OTM put options are relatively even more expensive than ATM and ITM call options when considering shorter maturities. Third, the term structure of SPX implied volatilities tends to be downward sloping, as Figure 2.3 displays. This suggests that specific agents, e.g. mutual funds, may have recourse to longer maturities for OTM SPX put options to buy relatively cheap long-term portfolio protection.

2.2.3 VIX Index and Futures

Introduced by Whaley (1993) in [113], the VIX Index measures the market's expectation of 30-day implied volatility, by initially reverting the Black option pricing formula (1976) in [33] from S&P 100 (OEX Index) option prices, for near-the-money strike prices. Therefore, the CBOE revised the calculation methodology in 2003, switching to an entirely model-free approach based on the S&P 500 (SPX Index), in order to create a suitable underlying for tradable volatility securities. Nowadays, as described by the CBOE (2009) in [49], the real-time VIX starting in January 1990 has been derived from mid bid-ask prices for call and put options on the S&P 500, for the front and the second month expirations, following a model-free approach.

The preliminary analysis puts into evidence some well-known empirical time-series properties of equity implied volatility, in particular upward jumps, heteroskedasticity, i.e. volatility clustering, mean-reversion, and non-normality in probability distribution. Table 2.8 displays the descriptive statistics associated to VIX levels and log returns, over the period from February

24, 2006 to August 29, 2014. Data sample consists in 2,435 historical daily closing VIX spots provided by Bloomberg. Over the entire data sample, the average level of the VIX reaches 20.65% (Panel A), spiking to a historical record high of 80.86% on November 20, 2008 (Panel B). Statistics are then broken down into two distinct subsamples related to stock market stress, either the subprime crisis period (Panel B) from August 29, 2008 to November 20, 2008, or the sovereign debt crisis (Panel C) from July 11, 2011 to October 3, 2011. During the subprime and the sovereign crises, higher volatility in the VIX level (respectively at 18.20% and 8.27%) was materialized by more frequent and stronger spikes, sheding light on volatility clustering. As described by Panel A, the heavily right-skewed and leptokurtic unconditional distribution of the VIX level (respectively 2.24 and 9.50) exhibits that it slowly reverts back to its average level when stock market recovers. Furthermore, the strongly significant Jarque-Bera test statistic (1723.05 in Panel A) associated to the VIX log returns over the entire data sample validates significant departures from normality, specifically the strongly leptokurtic probability distribution characterized by fat tails and peakedness.

Therefore, intraday high-frequency data associated to the VIX Index covers the period from January 24, 2008 to April 26, 2016 with 252 trading days. Each trading day consists in one-minute tick-by-tick prices, such that the trading time spans from 9:30am to 16:00pm.

Extoling the diversification benefits based on the negative correlation between the SPX Index and the VIX Index, the CBOE introduced VIX futures contracts in 2004, just a few month after the revision of the VIX Index calculation methodology. The VIX futures consist in standard futures contracts on the forward 30-day implied volatilities of the S&P 500 Index. Nevertheless, contrary to the SPX futures, there is no cost-of-carry relationship between the VIX Index and the VIX futures. Hence, the fair value associated to VIX futures is derived from the pricing of the forward variance. We download the VIX futures daily settlement prices from Datastream, providing 6 monthly constant maturities for which the future position is continuously rolled into the next-front month contract, on the day prior to expiration.

2.2.4 VIX Options

Since VIX options consist in European-style options written on VIX futures contracts, the underlying asset is not the VIX spot but the forward VIX at expiration date. Specifically, the underlying for a futures contract or an option written on the VIX with maturity $T = t + t_1$ consists in the VIX over the period from T to $T = t + \frac{30}{365}$. Therefore, VIX derivatives are written on the VIX for the next 30 days following the expiration date. By convention, settlement price for the usually six listed maturities is based on expiration date, as defined by the Wednesday that is thirty days prior to the third Friday of the calendar month immediately following the expiring month. Reverting the Black option pricing formula (1976) in [33], implied volatilities are derived from VIX option mid prices, corresponding to the volatility parameter such that, plugged into the Black option valuation formula, they give options market prices.

Due to market data inconsistencies, we apply the standard methodology as described among others by Völkert (2014) in [111] and Branger, Kraftschik, and Völkert (2015) in [40]. Hence, we restrict the data sample to the VIX option quotes for a time to maturity between 7 and 180 days to mitigate pricing anomalies and to take into account the recent contract specifications. Furthermore, we filter out the abnormal option quotes for zero bid prices, zero open interest, negative bid-ask spreads, and more generally, every time implied volatility can't be computed consistently. Filtered data sample consists in 282,314 option quotes, broken down respectively into 146,024 call options quotes, and 136,290 put options quotes.

Table 2.9 displays the descriptive statistics associated to VIX options since their launching, over the period from February 24, 2006 to August 29, 2014. It contains the number of observations, the average implied volatilities, open interest, trading volume, and bid-ask spreads, for different log moneyness and maturities buckets. The preliminary analysis brings out well-known empirical properties, e.g. the upward-sloping implied volatility smile and the downward-sloping term structure, reflecting that market agents usually trade VIX options to buy long-term portfolio insurance. First, when aggregating the average daily trading volume for VIX options, OTM call options tend to be strongly more actively traded than OTM put options, respectively 47,957 and 36,556 contracts. In parallel, ITM call options tend to be nearly twice less actively traded than ITM put options, respectively 15,346 and 26,417 contracts. Since OTM call options tend to be nearly three times more heavily traded than ITM call options, market agents usually trade OTM and deep OTM call options to pay insurance portfolio and disaster insurance against stock market crashes. Second, the implied volatility-moneyness function of VIX options tend to be generally upward-sloping, confirming the empirical property that market agents usually trade VIX options to buy portfolio hedges against stock market uncertainty. Figure 2.10 displays the average implied volatility smirks for four distinct maturity buckets, over the period from February 24, 2006 to August 29, 2014. For shorter maturities, the more pronounced U-shaped functions show that OTM call options are relatively even more expensive than ATM and ITM call options when considering shorter maturities. As illustrated in Figure 2.12, the implied volatility smile flattens out and tends to be lower for longer maturities. Third, the term structure of VIX implied volatilities tends to be strongly downward sloping, as displayed by Figure 2.11. This stylized fact suggests that specific agents, e.g. mutual funds, would have recourse to longer maturities for OTM VIX call options to buy relatively cheap long-term portfolio protection.

2.3 Empirical Strategy

In this section, we analytically extend the Zhang-Xiang quadratic approximation developped in [117] to decompose the implied volatility smirk function into a linear combination of the level, slope, and curvature quantities, and therefore into a nonlinear combination of the option-implied risk-neutral moments. Since options market prices fundamentally incorporate the market price of uncertainty, our underlying intuition consists in explicitly approximating the implied volatility smirk for a certain maturity in terms of tradeable risk premia associated to higher-order risks. The motivation underlying the use of the Zhang-Xiang approximation lies in the fact that this three-dimensional representation allows to decompose parsimoniously the dynamics of the volatility smirk at any point of time, by accommodating a combination of salient features interpreted primarily as the level, slope, and curvature factors. Hence, building on the yield curve modelling, this parsimonious approach relaxes the assumptions about the dynamics of the implied volatility smirk.

2.3.1 Risk-Neutral Moments

We investigate the dynamics of the risk-neutral higher moments extracted from the market prices of options. To this end, we use the model-free approach introduced by Bakshi, Kapadia, and Madan (2003) in [24] detailed in Appendix A. Let $R(t,T) \equiv \ln[S(t+T)] - \ln[S(t)]$ the log return at time t over the time period T. Let define the risk-neutral mean of returns $\mu(t,T)$, volatility $RNVol(t,T)$, skewness $RNSkew(t,T)$, and kurtosis $RNKurt(t,T)$ measured at time t over the time period T by

$$\mu(t,T) \equiv E_t^Q[R(t,T)] \tag{2.3.1.1}$$

$$RNVol(t,T) \equiv \left[E_t^Q\left[R(t,T)^2\right] - \mu(t,T)^2\right]^{\frac{1}{2}} \tag{2.3.1.2}$$

$$RNSkew\left(t,T\right)\equiv\frac{E_t^Q\left[\left(R\left(t,T\right)-E_t^Q\left[R\left(t,T\right)\right]\right)^3\right]}{\left(E_t^Q\left[\left(R\left(t,\tau\right)-E_t^Q\left[R\left(t,T\right)\right]\right)^2\right]\right)^{\frac{3}{2}}} \tag{2.3.1.3}$$

$$RNKurt\left(t,T\right)\equiv\frac{E_t^Q\left[\left(R\left(t,T\right)-E_t^Q\left[R\left(t,T\right)\right]\right)^4\right]}{\left(E_t^Q\left[\left(R\left(t,T\right)-E_t^Q\left[R\left(t,T\right)\right]\right)^2\right]\right)^{2}} \tag{2.3.1.4}$$

From Bakshi and Madan (2000) in [25], any payoff function $H\left[S\right]$ can be spanned algebraically by a continuum of OTM European call and put options. Therefore, let r the risk-free rate, $C\left(t,T;K\right)$ $\left(P\left(t,T;K\right)\right)$ the price of a European call (put) option at time t, with time to expiration T, and strike price K. Let the volatility $V(t,T)$, the cubic $W(t,T)$, and the quartic $X(t,T)$ contracts associated to the payoff function $H\left[S\right]$. As below, Equations (2.3.1.1), (2.3.1.2), (2.3.1.3), and (2.3.1.4) can be expressed in terms of the volatility, cubic, and quartic contracts' fair values under the risk-neutral expectation operator conditional on information at time t :

$$\mu\left(t,T\right)=\exp\left(rT\right)-1-\frac{\exp\left(rT\right)}{2}V\left(t,T\right)-\frac{\exp\left(rT\right)}{6}W\left(t,T\right)-\frac{\exp\left(rT\right)}{24}X\left(t,T\right) \tag{2.3.1.5}$$

$$RNVol\left(t,T\right)=\left[V\left(t,T\right)\exp\left(rT\right)-\mu\left(t,T\right)^2\right]^{\frac{1}{2}} \tag{2.3.1.6}$$

$$RNSkew(t,T) = \frac{\exp(rT)\,W(t,T) - 3\mu(t,T)\exp(rT)\,V(t,T) + 2\mu(t,T)^3}{\left[\exp(rT)\,V(t,T) - \mu(t,T)^2\right]^{\frac{3}{2}}} \quad (2.3.1.7)$$

$$RNKurt(t,T) = \frac{\exp(rT)\,X(t,T) - 4\mu(t,T)\exp(rT)\,W(t,T) + 6\exp(rT)\,\mu(t,T)^2\,V(t,T) - 3\mu(t,T)^4}{\left[\exp(rT)\,V(t,T) - \mu(t,T)^2\right]^2} \quad (2.3.1.8)$$

Furthermore, in Equations (2.3.1.5), (2.3.1.6), (2.3.1.7), and (2.3.1.8), contracts' fair values $V(t,T)$, $W(t,T)$, and $X(t,T)$ can be spanned by a linear combination of OTM European call and put options, the stock and the risk-free asset, requiring a large continuum of traded OTM options. However, since we observe in practice only few option market prices for discretely spaced strike prices, we apply the non-parametric approach of Völkert (2014) in [111] to adress discreteness by applying a cubic smoothing spline to interpolate implied volatilities amongst strike prices. Therefore, we approximate numerically the integral functions of volatility, cubic, and quartic contracts by using trapezoidal approximations.

2.3.2 Realized Moments

The recent literature about high-frequency econometrics, including Bollerslev, Tauchen, and Zhou (2009) in [36], and Neumann and Skiadopoulos (2013) in [94] among others, usually estimates the daily realized variance under a nonparametric approach by summing frequently sampled squared returns. Similarly, Amaya, Christoffersen, Jacobs, and Vasquez (2013) in [15] derive the daily realized skewness and kurtosis from intradaily returns of the S&P 500 Index. Nevertheless, since this standard econometric approach is widely biased by the market microstructure noise on volatility estimation, a naive practice consists in throwing away a lot of available data by sampling less frequently the intradaily underlying asset prices. In this way, we use the model-free approach proposed by Zhang, Mykland, and Aït-Sahalia (2005) in [118] to fully exploit the tick-by-tick data, to correct for the bias of market microstructure noise; and furthermore, to estimate similarly the higher-order realized moments.

According Bollerslev, Tauchen, and Zhou (2009) in [36], the daily realized variance is usually estimated by summing the intradaily returns of the underlying asset. Let $R_{t,i}$ the i-intraday log return calculated on day t and associated to the price index $P_{t,i}$. Then $R_{t,i,T} = \ln\left(P_{t,\frac{i}{N}}\right) - \ln\left(P_{t,\frac{(i-1)}{N}}\right)$, where N denotes the total number of observed intraday log returns in the trading day t. Therefore, the daily realized volatility $RDVol_t^{(all)}$ is usually estimated by summing naively all the n squares of intradaily log returns $R_{t,i}$:

$$RDVol_t^{(all)} = \left(\sum_{i=1}^{N} R_{t,i}^2\right)^{\frac{1}{2}} \tag{2.3.2.1}$$

Similarly, following Amaya, Christoffersen, Jacobs, and Vasquez (2013) in [15], the ex-post realized daily skewness $RDSkew_t$ and kurtosis $RDKurt_{t,T}$ can be expressed as follows, respectively scaled by $N^{\frac{1}{2}}$ and N to ensure they correspond to the daily realized measures:

$$
\begin{aligned}
RDSkew_t^{(all)} &= \frac{N^{\frac{1}{2}} \sum_{i=1}^{N} R_{t,i}^3}{RDVol_t^3}, \\
RDKurt_t^{(all)} &= \frac{n \sum_{i=1}^{N} R_{t,i}^4}{RDVol_t^4}
\end{aligned}
\tag{2.3.2.2}
$$

Nevertheless, Zhang, Mykland, and Aït-Sahalia (2005) in [118] argue that using naively all the tick-by-tick data makes the market microstructure noise totally swamp the estimated realized volatility under the nonparametric case. Suppose the log price process X_t follows a continuous semi-martingale. Then, it is modeled by the stochastic differential equation $dX_t = \mu_t + \sigma_t dW_t$, where μ_t, σ_t, and W_t denote respectively the drift and the volatility of the log return process of X_t at time t, and a standard Brownian motion process. Therefore, the object of interest primarily consists in estimating the integrated variance, i.e. the quadratic variation $\langle X, X\rangle_T = \int_0^T \sigma_t^2 dt$ over the time period $[0, T]$. Indeed, Zhang, Mykland, and Aït-Sahalia (2005) show that $RDVol_T^{(all)}$ in the (2.3.2.1) converges in law to

$$
RDVol_T^{(all)} \overset{L}{\simeq} \langle X, X\rangle_T + 2nE\left[\varepsilon^2\right] + \left[4nE\left[\varepsilon^4\right] + \frac{2T}{n}\int_0^T \sigma_t^4 dt\right]^{\frac{1}{2}} Z_{total} \tag{2.3.2.3}
$$

where $RDVol_T^{(all)}$ is even more positively biased by the market microstructure noise $2nE\left[\varepsilon^2\right]$ when the sample size n of observed intradaily prices increases. Consequently, sampling sparsely either at an arbitrary frequency or even at an optimal frequency by decreasing n are tantamount to ignoring the microstructure noise and to throwing out a large fraction of the available intradaily data. In contrast, Zhang, Mykland, and Aït-Sahalia (2005) propose the following Two-Scales

Realized Volatility estimator $RDVol_T^{(TS)}$ that uses all the available tick-by-tick data but that incorporates subsampling, averaging, and bias correction for the market microstructure noise:

$$RDVol_T^{(TS)} = \frac{1}{K}\sum_{k=1}^{K} RDVol_T^{(k)} - \frac{\overline{n}}{n} RDVol_T^{(all)} \tag{2.3.2.4}$$

where the original grid $G = \{t_k, ..., t_n\}$ of observation times of log prices in a given trading day is partitioned into K non-overlapping and equal subsamples $G^{(k)}$ for $k = \{1, ..., K\}$. The k-th sub-grid is written as $G^{(k)} = \{t_{k-1}, t_{k-1+K} ..., t_{k-1+n_k K}\}$. Therefore, Zhang, Mykland, and Aït-Sahalia (2005) average the estimators $RDVol_T^{(k)}$ obtained on each of the K grids of average size $\overline{n} = \frac{n-K+1}{K}$, giving rise to the estimator $RDVol_T^{(avg)} = \frac{1}{K}\sum_{k=1}^{K} RDVol_T^{(k)}$. Then, bias correction is determined by $K = n^{\frac{2}{3}} \left[12\widehat{E\left[\varepsilon^2\right]}^2 \Big/ \frac{Tn}{3}\int_0^T \sigma_t^4 dt \right]^{\frac{1}{3}}$. Finally, $RDVol_T^{(TS)}$ corrects for the bias $2\overline{n}E\left[\varepsilon^2\right]$ due to the microstructure noise of $RDVol_T^{(avg)}$, since it now increases with the average subsamples size $\overline{n}$.

Similarly, we derive to the higher-order moments the Zhang, Mykland, and Aït-Sahalia (2005) methodology of subsampling, averaging, and bias correction for the market microstructure noise:

$$\begin{aligned} RDSkew_T^{(TS)} &= \frac{1}{K}\sum_{k=1}^{K} RDSkew_T^{(k)} - \frac{\overline{n}}{n} RDSkew_T^{(all)}, \\ RDKurt_T^{(TS)} &= \frac{1}{K}\sum_{k=1}^{K} RDKurt_T^{(k)} - \frac{\overline{n}}{n} RDKurt_T^{(all)} \end{aligned} \tag{2.3.2.5}$$

where $RDSkew_T^{(TS)}$ and $RDKurt_T^{(TS)}$ denote respectively the two-scales realized skewness and kurtosis.

2.3.3 Option-Implied Risk Premia

Option-implied risk premia define the difference between the physical and the risk-neutral expectations of the return moments. We first recall the general definitions of option-implied risk premia, in particular for higher-order moments, and therefore, we derive them for options at different maturities.

Carr and Wu (2009) in [48] and Bollerslev, Tauchen, and Zhou (2009) in [36] define the volatility risk premium as the difference between the realized and the risk-neutral volatilities, i.e. $VRP_{t,t+\tau}$ computed at time t over the period τ as the difference between the ex post realized return volatility over the $[t-\tau, t]$ time interval and the ex ante risk-neutral expectation of the future return volatility over the $[t, t+\tau]$ time interval. Henceforth, since we calculate the premia for different maturities, we note $VRP_{t,t+\tau,T}$ to define the volatility risk premium calculated at time t, over the period τ, as the difference between the realized volatility over $[t-\tau, t]$ and the risk-neutral volatility over $[t, t+\tau]$, associated to options and futures for the given maturity T:

$$VRP_{t,t+\tau,T} \equiv E_t^P \left[\sigma_{t,t-\tau,T}\right] - E_t^Q \left[\sigma_{t,t+\tau}\right] \tag{2.3.3.1}$$

where $E_t^Q[\cdot]$ and $E_t^P[\cdot]$ denote the time-t conditional expectation operator under respectively the risk-neutral measure Q and the physical measure P. Therefore, $E_t^P\left[\sigma_{t,t+\tau,T}\right]$, and $E_t^Q\left[\sigma_{t,t+\tau}\right]$ are the expected values conditional to time t of the volatility realized over time period τ under respectively the physical and the risk-neutral probability measures, associated to options and futures at the given maturity T. Furthermore, the volatility risk premium $VRP_{t,t+\tau,T}$ multiplied by a notional dollar amount usually defines the payoff at maturity $t+\tau$ of a return volatility swap. Under the no-arbitrage condition, the constant volatility swap rate $SW_{t,t+\tau}$ determined at time t and paid at time $t+\tau$ equals the risk-neutral expectation of the future realized volatility. Henceforth, we calculate on a daily frequency the option-implied risk premia for different maturities on a daily frequency, i.e. for $\tau = 1$ day.

In line with Bollerslev, Tauchen, and Zhou (2009) in [36], we estimate $E_t^P\left[\sigma_{t,t+\tau}\right]$ in Equation (2.3.3.1) by the realized volatility denoted $RDVol_t^{(TS)}$ over the day t. For the sake of simplicity, we henceforth drop the subscript τ and we denote $VRP_{t,t+T}$ as the volatility risk premium computed at time t over the period $\tau = 1$ day, associated to options and futures for the given maturity T:

$$VRP_{t,T} = RDVol_t^{(TS)} - RNVol_{t,T} \tag{2.3.3.2}$$

Similarly, the underpinnings of volatility swaps can be theoretically extended to forward contracts written on the third and fourth moments. Indeed, the extensive use of volatility swaps within the financial industry henceforth augures well for the success of higher-order swap contracts. The skewness and the kurtosis swaps have zero net market value at the initiation, since no arbitrage dictates that the skewness and the kurtosis swap rates respectively equals the risk-neutral expected value of the realized skewness $RDSkew_t^{(TS)}$ and kurtosis $RDKurt_t^{(TS)}$. Then, we specify as follows the skewness risk premium $SRP_{t,T}$ associated to the options and the futures at given maturity T

$$SRP_{t,T} = RDSkew_t^{(TS)} - RNSkew_{t,T} \tag{2.3.3.3}$$

Subsequently, the kurtosis risk premium $KRP_{t,T}$ computed at time t over the period $\tau = 1$ day measures the difference between the ex post realized return kurtosis over the $[t - \tau, t]$ time interval and the ex ante risk-neutral expectation of the future return kurtosis over the $[t, t+\tau]$ time interval, associated to the options and futures for the given maturity T:

$$KRP_{t,T} = RDKurt_t^{(TS)} - RNKurt_{t,T} \tag{2.3.3.4}$$

2.3.4 Quadratic Approximation

From Zhang and Xiang (2008) in [117], the three-dimensional representation of the implied volatility smirk IV is assumed to be approximated by a second-order polynomial function in the log-moneyness ξ. For the sake of clarity, we note the implied volatility smirk function $IV_{t,T}$ computed at time t for a certain maturity T as follows

$$IV_{t,T}(\xi) = \gamma_{0,t,T}\left[1 + \gamma_{1,t,T}\xi + \gamma_{2,t,T}\xi^2\right] \tag{2.3.4.1}$$

where the strike K, the forward price F_0, the average volatility $\bar{\sigma}$ of the underlying asset price, and the time to maturity T characterize the log-moneyness $\xi = \frac{\ln(K/F_0)}{\bar{\sigma}\sqrt{T}}$. Then, $\gamma_{0,t,T}$, $\gamma_{1,t,T}$, and $\gamma_{2,t,T}$ respectively designates the level, the slope, and the curvature factors associated to the volatility smirk $IV_{t,T}$ at the maturity T.

From an economic motivation, vanished smirk's slope $\gamma_{1,t,T}$ and curvature $\gamma_{2,t,T}$ reduce the implied volatility smile function to the Black-Scholes model, i.e. a flat at-the-money volatility (ATMV) smile.

$$
\begin{aligned}
IV_{t,T}\left(\xi\right) &= \gamma_{0,t,T}, \\
IV_{t,T}\left(\xi\right) &= \gamma_{0,t,T}\left[1+\gamma_{1,t,T}\xi\right], \\
IV_{t,T}\left(\xi\right) &= \gamma_{0,t,T}\left[1+\gamma_{1,t,T}\xi+\gamma_{2,t,T}\xi^{2}\right]
\end{aligned}
\tag{2.3.4.2}
$$

where the second equation above corresponds to the skewed implied volatility, expressed as a linear function that incorporates the ATMV $\gamma_{0,t,T}$ and the slope $\gamma_{1,t,T}$. Then, the smirked implied volatility designates a quadratic function with both the slope and curvature $\gamma_{2,t,T}$. Hence, the three factors fully quantify the departure of the returns probability distribution from the lognormal distribution, since $\gamma_{1,t,T}$ and $\gamma_{2,t,T}$ incorporate the risk-neutral moments associated to higher-order risks.

Therefore, as briefly detailed in Appendix C, Zhang and Xiang derive from asymptotic expansions the approximations for the level, slope, and curvature $(\gamma_{0,t,T}, \gamma_{1,t,T}, \gamma_{2,t,T})$ associated to the implied volatility smirk $IV_{t,T}$, expressed in terms of the risk-neutral volatility, skewness, and kurtosis $(RNVol_{t,T}, RNSkew_{t,T}, RNKurt_{t,T})$

$$
\begin{aligned}
\gamma_{0,t,T} &\approx \left[1-\frac{1}{24}\left(RNKurt_{t,T}+3\right)\right]RNVol_{t,T}, \\
\gamma_{1,t,T} &\approx \frac{1}{6}RNSkew_{t,T}, \\
\gamma_{2,t,T} &\approx \frac{1}{24}\left[RNKurt_{t,T}+3\right]
\end{aligned}
\tag{2.3.4.3}
$$

Subsequently, we extend the Zhang-Xiang approximations reported in (2.3.4.3) by exhibiting the volatility, skewness, and kurtosis risk premia denoted $(VRP_{t,T}, SRP_{t,T}, KRP_{t,T})$. Then, our following third-way representation of the implied volatility smirk explicitly accommodates the tradeable components of the market price of uncertainty incorporated in options market prices:

$$
\begin{aligned}
\gamma_{0,t,T} &\approx \left[1-\frac{1}{24}\left(RDKurt_t^{(TS)}+3-KRP_{t,T}\right)\right]\left[RDVol_t^{(TS)}-VRP_{t,T}\right],\\
\gamma_{1,t,T} &\approx \frac{1}{6}\left[RDSkew_t^{(TS)}-SRP_{t,T}\right],\\
\gamma_{2,t,T} &\approx \frac{1}{24}\left[RDKurt_t^{(TS)}+3-KRP_{t,T}\right]
\end{aligned}
\tag{2.3.4.4}
$$

where $\left(RDVol_t^{(TS)}, RDSkew_t^{(TS)}, RDKurt_t^{(TS)}\right)$ designate respectively the realized volatility, skewness, and kurtosis associated to the underlying asset prices. Therefore, we analytically decompose as follows the volatility smirk into a convenient combination of the tradeable option-implied risk premia $(VRP_{t,T}, SRP_{t,T}, KRP_{t,T})$,

$$
\begin{aligned}
IV_{t,T}(\xi) &\approx \left[1-\frac{1}{24}\left(RDKurt_t^{(TS)}+3-KRP_{t,T}\right)\right]\left[RDVol_t^{(TS)}-VRP_{t,T}\right]\\
&\quad+\frac{1}{6}\left[1-\frac{1}{24}\left(RDKurt_{t,T}+3-KRP_{t,T}\right)\right]\left[RDVol_t^{(TS)}-VRP_{t,T}\right]\\
&\qquad\left[RDSkew_t^{(TS)}-SRP_{t,T}\right]\xi\\
&\quad+\frac{1}{24}\left[1-\frac{1}{24}\left(RDKurt_t^{(TS)}+3-KRP_{t,T}\right)\right]\left[RDVol_t^{(TS)}-VRP_{t,T}\right]\\
&\qquad\left[RDKurt_t^{(TS)}+3-KRP_{t,T}\right]\xi^2
\end{aligned}
\tag{2.3.4.5}
$$

As suggested by (2.3.4.2), the implied risk premia $SRP_{t,T}$ and $VRP_{t,T}$ that reward for bearing the higher-order risks fully quantify the departure of the implied volatility smile function $IV_{t,T}$ from the Black-Scholes model. Hence, in the case of strongly non lognormal distributions, $SRP_{t,T}$

and $KRP_{t,T}$ should have bigger contributions in the volatility smile approximation (2.3.4.5) when compared to the volatility risk premium $VRP_{t,T}$. Furthermore, we propose later an empirical methodology to test the analytical approximation (2.3.4.5). For this purpose, we perform a Levenberg-Marquardt algorithm to solve the nonlinear least squares minimization problem, respectively in the case of standard distributions, i.e. SPX options, and strongly right-skewed and leptokurtic distributions, i.e. VIX options.

2.3.5 Mimicking Factors

Our factor-based volatility investing framework is theoretically underpinned by capturing the tradeable compensation for time-varying risks in higher-order moments, that we empirically isolate from options market prices. Schneider and Trojani (2015) in [104] propose variance, skewness, and kurtosis swap trading strategies investigated by Bondarenko (2014) in [39]. Based on a new class of divergence trading strategies introduced by Alireza (2005) in [14], these swaps exploit the discrepancy between the option-implied risk-neutral distribution, i.e. the fair price of moments, and the real distribution of the underlying asset. Similarly, Chang, Zhang, and Zhao (2013) in [50] examine the performance of skewness and kurtosis swaps, to trade the forward realized third and fourth cumulants. Using S&P 500 index options from 1996 to 2005, they shed light on salient time-varying properties of higher-order risk premia, offering an economic justification for such swap strategies. Less recently, Aït-Sahalia, Wang, and Yared (2000) in [8], and Blaskowitz and Schmidt (2002) in [35] document the profitability of skewness and kurtosis trades, exploiting the discrepancies between risk-neutral densities implied by DAX option prices and the historical state-price densities.

Specifically, Alireza (2005) in [14] extends divergence trading strategies to arbitrate the SPX implied volatility smile for higher-order moments. Let the option-implied risk- neutral distribution be more skewed to the left than the real distribution of the underlying asset. Then, OTM put options may be relatively overpriced with respect to the OTM call options, since the risk-neutral distribution should reflect the fair price of skewness. Subsequently, trading the skewness consists in selling the OTM SPX put option $P(S_t, K_C)$ and buying the OTM SPX call option $C(S_t, K_C)$, associated to the underlying asset price S_t and the strike price K_C. Therefore, the payoff value Π_{Skew} associated to the corresponding delta-vega-neutral portfolio is expressed as

$$\Pi_{Skew} = C(S_t, K_C) - \frac{\nu_C}{\nu_P} P(S_t, K_C) - \left[\Delta_C - \frac{\nu_C}{\nu_P}\Delta_P\right] S_t \tag{2.3.5.1}$$

where (Δ_C, Δ_P) and (ν_C, ν_P) respectively designate the delta and the vega of the call and put options. Hence, the payoff Π_{Skew} is an increasing function of the underlying asset price S_t, since Δ_P increases more than Δ_C with S_t.

Similarly, let the option-implied risk-neutral distribution has a sharper peak and fatter tails than the real distribution of the underlying asset. Then, OTM options may be relatively overpriced with respect to ATM options. Subsequently, trading the kurtosis consists in selling the OTM call $C(S_t, K_3)$ and OTM put options $P(S_t, K_1)$, and buying the ATM call $C(S_t, K_2)$ and the ATM put option $P(S_t, K_2)$ for strike prices $K_1 > K_2 > K_3$. Hence, the payoff value Π_{Kurt} associated to the corresponding delta-vega-neutral portfolio is

$$\begin{aligned} \Pi_{Kurt} \quad = \quad & C(S_t, K_2) + \frac{\nu_{C_2}}{\nu_{P_2}} P(S_t, K_2) \\ & - C(S_t, K_3) - \frac{\nu_{C_3}}{\nu_{P_1}} P(S_t, K_1) \\ & - \left[\Delta_{C_3} + \frac{\nu_{C_3}}{\nu_{P_1}}\Delta_{P_1} - \Delta_{C_2} - \frac{\nu_{C_2}}{\nu_{P_2}}\Delta_{P_2}\right] S_t \end{aligned} \tag{2.3.5.2}$$

Similarly to a butterfly strategy, the payoff Π_{Kurt} is a decreasing function of the variations range of the underlying asset price S_t. Hence, the skewness and the kurtosis trades are generally interpreted as insurance-selling strategies, but mirror trades may be usually considered as buying insurance. This last approach is particularly motivated by the considerable costs of carry generated by a long exposure to the entire implied volatility. For illustration, the most traded VIX exchange-traded note, i.e. the VXX ETN, lost over 99.6% of its value over the period 2009-2014, bringing ruin upon many non-sophisticated investors. Whaley (2013) in [115] suggests that VIX ETPs form inappropriate buy-and-hold investments, as they do not pay certain cash flows, and typically generate positive costs of carry. From March 26, 2004 to March 30, 2012, he finds a 2.3% average slope for the VIX futures term structure at 30 day to expiration, exhibiting an upward-sloping prices curve, i.e. contango, in nearly 81% of trading days. Hence, deep investigations of option-based strategies built on the factor-based investing are left for future research.

2.4 Empirical Results

2.4.1 Risk-Neutral Moments

In this subsection, we investigate the empirical time-series properties of the option-implied risk-neutral distribution associated to the SPX index options and the VIX options, by extracting the risk-neutral moments related to different time to maturities from options market prices, following the Bakshi-Kapadia-Madan methodology. Our empirical results are consistent with the recent prior literature. In the case of the SPX index options, this includes Panigirtzoglou and Skiadopoulos (2004) in [95], Lynch and Panigirtzoglou (2008) in [80], and Neumann and Skiadopoulos (2013) in [94] among others. And in the case of the VIX options, Völkert (2012) in [111] and Branger, Kraftschik, and Völkert (2015) in [40] are the closest to our results.

SPX Options

Figure 2.5 exhibits the risk-neutral moments for 60-day time to maturity. By convention, the underlying asset of SPX options are the SPX futures associated to the considered time to maturity. Hence, the risk-neutral mean of prices in the upper panel closely coincides with the settlement prices of the SPX futures for 60-day time to maturity. The major jumps in the annualized risk-neutral volatility correspond well to the Lehman Brothers bankruptcy on September 15, 2008 (75.8%), the Flash Crash on May 6, 2010, and the downgrade of the U.S. government credit rating on August 5, 2011, respectively. Therefore, even if we observe strong instability in the estimation of the higher-order SPX risk-neutral moments, their global dynamics follow the prior literature. As described by Neumann and Skiadopoulos (2013) in [94], the risk-neutral skewness is generally negative over the entire sample period, and for any considered maturity and we observe temporary excess kurtosis. Table 2.2 displays summary statistics associated to the risk-neutral moments measured in levels and in first differences. Overall, accross the 60-, 120-, and 180-day time to maturity, the average risk-neutral skewness ranges from -0.065 to -0.413, and the risk-neutral kurtosis ranges from 2.086 to 2.959 on average. Furthermore, we observe considerable excess kurtosis, even reaching 5.059 and 11.768 at the 60-day and the 120-day time to expiration. Hence, quite consistent with the prior literature including Bakshi, Kapadia, and Madan (2003) in [24], our findings validate that the index risk-neutral probability density functions are generally negatively skewed and exhibit some excess kurtosis. In particular, Bates (1991) in [27] and Rubinstein (1994) in [101] observe that equity index probability distributions become risk-neutrally skewed to the left, even more after the crash of 1987. Nevertheless, the risk-neutral kurtosis that we estimated proves to be lower than documented, requiring also some fine-tuning from our methodology.

Moreover, Table 2.2 evaluates the autocorrelations and the stationarity by using respectively the Ljung-Box Q test and the Augmented Dickey-Fuller (ADF) unit root test. Serial autocorrelations exhibit strong persistency of the four risk-neutral moments, measured either in levels or in first differences. Specifically, the Ljung-Box Q test associated to the levels significantly rejects the distribution independency at the 5% level of confidence, as the test statistics ranges from 190.27 to 612.88 for the risk-neutral volatility, for example. Indeed, Neumann and Skiadopoulos (2013) in [94] investigate the higher-order moments predictability, as suggested by the strong negative autocorrelations of the first-differenced risk-neutral skewness and kurtosis. Furthermore, the unit root test results reveal that all the risk-neutral moments are generally integrated of order one, especially when considering the first differences. In particular, Panel B exhibits that the ADF test statistic generally rejects the non-stationarity at the 1% level of confidence in the first differences of the four risk-neutral moments. Overall, the persistent serial autocorrelations and the non-stationarity prove to be consistent with Neumann and Skiadopoulos (2013) in [94].

Subsequently, Table 2.3 displays the Pearson cross-correlations between the option-implied risk-neutral moments. First, the risk-neutral mean and volatility exhibit significant negative correlations, ranging from -0.76 to -0.82 with increasing time to maturity. This finding illustrates the (implied) leverage effect described by Black (1976) in [33], defining the inverse relation between stock prices and (implied) volatility. Besides, the third and fourth risk-neutral moments appear to stay highly positively correlated accross the maturities, since cross-correlations range from 0.54 to 0.95. Third, the negative correlation between the risk-neutral mean and the higher-order risk-neutral moments becomes positive with increasing time to maturity.

VIX Options

Figure 2.13 exhibits the risk-neutral moments for 30-day time to maturity. Since the underlying asset of VIX options is not the spot VIX but the VIX futures prices, the risk-neutral mean of prices in the upper panel closely coincides with the settlement prices of VIX futures. The major jumps in the risk-neutral mean of the VIX respectively correspond to the Lehman Brothers bankruptcy on September 15, 2008, the Flash Crash on May 6, 2010, and the downgrade of the U.S. government credit rating on August 5, 2011. Similarly to the first risk-neutral moment, the risk-neutral volatility strongly spiked during the turbulent times of the recent financial crisis, especially following the Lehman Brothers bankruptcy, the European sovereign debt crisis, and the U.S. credit downgrade. Table 2.10 displays summary statistics associated to the risk-neutral moments in levels. Furthermore, the right skewness and the excess kurtosis suggest that market agents attach a high probability to frequent upward spikes in the VIX. Overall, the risk-neutral skewness and kurtosis respectively exceed 0.464 and 3.797 accross the maturities, illustrating the pronounced right-skewed and heavy-tailed risk-neutral probability distribution. The right-skewed risk-neutral distribution of the VIX consistently validates Bates (1991) in [27] and Rubinstein (1994) in [101], as they observe that equity index probability distributions have become risk-neutrally skewed to the left, even more after the crash of 1987.

Subsequently, Table 2.11 displays the Pearson cross-correlations between the option-implied risk-neutral moments. The first two moments remain positively correlated in levels, ranging from 0.28 to 0.45. Besides, the third and fourth risk-neutral moments appear to stay highly positively correlated accross maturities, since cross-correlations exceed 0.79. Overall, the cross-correlations between the other risk-neutral moments are negative and more moderate, especially when time to expiration decreases. Indeed, the linear association between the risk-neutral volatility and the higher-order risk-neutral moments respectively ranges from -0.53 to -0.36 for the skewness, and from -0.54 to -0.30 for the kurtosis. The risk-neutral volatility slightly more correlates to the higher-order risk-neutral moments, as cross-correlations respectively exceed -0.60 for the skewness, and -0.56 for the kurtosis.

Besides, Table 2.10 examines autocorrelations and stationarity by employing respectively the

Ljung-Box Q test and the Augmented Dickey-Fuller (ADF) unit root test. For all the risk-neutral moments, the ADF test statistic rejects the non-stationarity at the 1% level of confidence in the first differences displayed by Panel B. As clearly evidenced by Neumann and Skiadopoulos (2013) in [94], the considerable negative autocorrelations of the first-differenced risk-neutral skewness and kurtosis suggests higher-order moments predictability. Indeed, the unit root test respectively ranges from -24.78 to -8.46 and from -24.27 to -5.27 for the risk-neutral skewness and kurtosis. Moreover, autocorrelations exhibit even stronger persistency of the four risk-neutral moments for longer time horizons. Specifically, the Ljung-Box Q test rejects the distribution independency at the 5% level of confidence for the four moments, e.g. ranging respectively from 289.98 to 330.71 for the risk-neutral volatility, and from 337.29 to 369.06 for the risk-neutral kurtosis.

Furthermore, Figure 2.13 exhibits a remarkable upward trend both in the risk-neutral skewness and in the kurtosis since the financial turbulence in the fall of 2008, suggesting a significant shift in the investors' risk preferences and beliefs. Henceforth, as described by Branger, Kraftschik, and Völkert (2015) in [40], investors either expect large and frequent upward movements in the VIX, or they become more risk-averse.

2.4.2 Option-Implied Risk Premia

Since option-implied risk premia define the difference between the physical (realized) and the risk-neutral moments, we test the statistical significance of negativity for different time to maturities. Following Chang, Zhang, and Zhao in [50] using the Student test, the t-statistics for the one-tailed test associated to the null-hypothesis $H_0 : \mu_{RP} \geq 0$, where μ_{RP} denotes the average risk premium, is

$$\mathrm{t-stats}_t = \frac{\mu_{RP,t}}{\sigma_{RP,t} \cdot n^{-\frac{1}{2}}} \tag{2.4.2.1}$$

where μ_{RP}, σ_{RP} respectively denotes the average risk premium and its standard deviation, and n is the number of observations over the sample period.

SPX Options

Figure 2.7 plots the time series of option-implied risk premia respectively associated to the volatility, the skewness, and the kurtosis, for 60-days time to maturities, from January 24, 2008, to December 19, 2012. Table 2.6 displays the option-implied risk premia from January 24, 2008, to December 19, 2012. The average volatility risk premia are clearly all significantly negative at the 5% and the 1% confidence level for 60-days and 120-days time to maturities, over the entire sample period. For longer maturities, the volatility risk premia become even more significantly negative since the Student t-stats range from -23.65 to -16.84. Furthermore, the considerable jumps in the volatility risk premia are associated to major financial turmoils, respectively the Lehman Brothers bankruptcy on September 15, 2008 (75.8%), and the downgrade of the U.S. government credit rating on August 5, 2011. We observe that jumps are usually followed by strong declines, what may illustrate Bollerslev, Tauchen, and Zhou in [36] who make clear evidence that high (low) volatility risk premia predict high (low) future returns.

Therefore, from January 24, 2008, to December 19, 2012, Figure 2.7 exhibits on average positive skewness risk premia. Table 2.6 reports that the Student test clearly rejects the null hypothesis H_0 at least at the 5% confidence level across all the maturities, especially for shorter maturities. Indeed, the t-statistics respectively equals 4.52 and 2.21 for 60-day and 120-day maturities. Consistently with the literature, this empirical result confirms that for equity index options, the risk-neutral skewness proves to be more negative than the physical measure, generating usually positive skewness risk premia across the time. Furthermore, Brunnermeier and Parker (2005) in [44], Brunnermeier et al. (2007) in [43], and Barberis and Huang (2008) in [26], make clear evidence that investors' preference for skewness results in overinvesting (underinvesting) in assets with high (low) skewness, leading to lower average returns. In addition, Conrad, Dittmar, and Ghyssels (2013) in [37] show that high (low) skewness risk premia predict low (high) future returns. In the case of the kurtosis risk premia over the period from January 24, 2008, to December 19, 2012, they are statistically negative at the 1% confidence level across all the maturities. Indeed, the Student t-stats respectively equals -6.16 and -6.68 in the case of the 60 and the 120-days time to maturities.

VIX Options

Figure 2.15 plots the time series of option-implied risk premia respectively associated to the volatility, the skewness, and the kurtosis, for 30-days time to maturities, from January 24, 2008 to August 29, 2014. Table 2.14 displays the option-implied risk premia over the same time period. The average risk premia are clearly all significantly negative at the 1% confidence level across the 30, the 60, and the 120-days time to maturities, over the whole sample period. For longer maturities, the volatility risk premia become even more significantly negative since the Student t-stats range from -8.41 to -19.35. As illustrated by Figure 2.15, brief episodes of positive volatility risk premia consistently correspond to equity market turmoils, respectively related to the Lehman Brothers bankruptcy, the European sovereign debt crisis, and the U.S. credit downgrade.

Therefore, Figure 2.15 exhibits on average strongly negative skewness risk premia. Indeed, Figure 2.14 clearly illustrates that physical skewness tends to be strongly more negative than the risk-neutral measure for VIX options. More precisely, Table 2.10 and Table 2.12 show that the minimum skewness respectively equals -3.425 for the physical measure, whereas it ranges from -0.111 to -0.121 for the risk-neutral measure. As reported in Table 2.14, the Student test clearly rejects the null hypothesis H_0 at the 1% confidence level across all the maturities, since the t-statistics ranges from -8.66 to -6.92. Consistently with the literature, these empirical results confirm that for equity index options, the risk-neutral skewness proves to be more negative than the physical measure, generating usually positive skewness risk premia in the case of the SPX options for example. Inversely, skewness risk premia tend to be usually negative in the case of the VIX options, since the leverage effect states a negative relation between the spot and its volatility. Finally, the kurtosis risk premia prove to be statistically significantly negative at the 1% confidence level, across all the maturities over the time period from January 24, 2008 to August 29, 2014, since the Student t-stats ranges from-12.93 to -10.44.

2.4.3 Mimicking Factors

In this subsection, we propose an empirical methodology to test directly the analytical approximation of the implied volatility smile function, as expressed by the equation (2.3.4.5). For this purpose, we perform a Levenberg-Marquardt algorithm to solve the nonlinear least squares (NLS) minimization problem. Subsequently, we test respectively the approximation, first, in the classical case of standard distributions, i.e. SPX index options, and in the case of right-skewed and leptokurtic distributions, i.e. VIX options, and for different times to maturity. The underlying assumption postulates that in the case of strongly non lognormal distributions, higher-order risk premia $SRP_{t,T}$ and $KRP_{t,T}$ should have bigger contributions in the volatility smile approximation when compared to the volatility risk premium $VRP_{t,T}$.

Table 2.7 reports the estimated coefficients, the p-values at the 1% and 5% confidence levels, and the t-statistics, associated to the nonlinear regressions of the SPX implied volatility smile function $IV_{t,T}$ on the option-implied risk premia ($VRP_{t,T}$, $SRP_{t,T}$, $KRP_{t,T}$) for the different maturities $\{60, 120\}$ days. For illustration, Figure 2.8 plots the time series of the implied volatility for 60 days to maturity, respectively derived from the in-the-money, the at-the-money, and the out-of-the money SPX put options, from February 24, 2006, to August 29, 2014. Overall, the estimation results associated to the NLS regressions exhibit significant adjusted R squares, remaining persistently high both accross the strikes, and accross the maturities. For illustration, adjusted R squares range from 17.9% to 45.0%, and statistical significance even more increases for short-term maturities, since they respectively reach 38.8%, 45.0%, and 41.0% for the ATM, the ITM, and the OTM SPX implied volatilities at the 60 days to maturity. Overall, the most proeminent estimated coefficients are associated to the volatility risk premia $VRP_{t,T}$, since they appear all strongly positive and statistically significant at the 1% level of confidence. Furthermore, they contain considerable t-statistics both accross the maturities and the strikes, sheding light particularly on the even more statistically significant volatility risk premia for the shorter maturities. Therefore, the estimated coefficients associated to the higher-order risk premia exhibit in general less statistical significance than the volatility risk premia. The coefficients associated to the skewness risk premia $SRP_{t,T}$ are all negative and statistically significant at least at the 5% level of confidence, especially for shorter maturities. Nevertheless, the coefficients associated to the kurtosis risk premia $KRP_{t,T}$ are much less statistically significant than the volatility risk premia, especially in the cases of the ATM implied volatility and the longer maturities.

Similarly, we test empirically the analytical approximation expressed by (2.3.4.5) for the VIX implied volatility smile. Table 2.15 reports the estimation results associated to the nonlinear least squares regressions of the VIX implied volatility function $IV_{t,T}$ on the option-implied risk premia $VRP_{t,T}$, $SRP_{t,T}$, $KRP_{t,T}$ for the $\{30, 60, 90, 120\}$ days to maturity. For illustration, Figure 2.16 plots the time series of the implied volatility for 30 days to maturity, respectively associated to the ITM, ATM, and the OTM call VIX options, from April 13, 2007, to August 29, 2014. Although statistical significance is quite lower when compared to the SPX smile approximation, the adjusted R squares remain persistently high both accross the strikes and the maturities. Indeed, adjusted R squares are particularly strong in the cases of the ATM and OTM volatilities, and the shorter maturities, since they respectively reach 31.5%, 21.2%, and 24.7% for ATM, ITM, and OTM VIX call options at the 30 days to maturity.

Similarly, the most proeminent coefficients are related to the volatility risk premia $VRP_{t,T}$, since they appear all strongly positive and statistically significant at the 1% level of confidence. Nevertheless, the associated t-statistics are much lower when compared to the SPX smile approximation. Indeed, Student t-stats range from 3.57 to 7.31 for the VIX smile, whereas they range from 5.22 to 13.7 for the SPX smile. Furthermore, although the skewness risk premia $SRP_{t,T}$ do not exhibit statistical significance, the kurtosis risk premia $KRP_{t,T}$ contain relatively much more information when compared to the SPX smile approximation. More precisely, the coefficients associated to $KRP_{t,T}$ are generally all negative and significant at least at the 5% level of confidence in the cases of the ATMV and the OTMV. When they are significant, the t-stats related to $KRP_{t,T}$ range from -2.51 to -4.87, and are of a comparable magnitude with those related to $VRP_{t,T}$. Consequently, the higher-order risk premia associated to the VIX options account for stronger effect on the smile than for the SPX options.

Hence, our results empirically confirm the analytical decomposition of the implied volatility smile function into the option-implied risk premia, since adjusted R squares advocate for a strong statistical significance. Therefore, we show that the contribution of the higher-order risk premia in the dynamics of the implied volatility smile is more important for right-skewed leptokurtic distributions than for standard distributions. Indeed, since VIX options market prices fully incorporate investors' fear and risk aversion, the higher-order risks implied by the risk-neutral distribution have considerable informational content for the market price of uncertainty. Hence, our factor-based investing approach may be particularly tailored for exotic options.

Conclusions

This paper has been motivated by deciphering the fine dynamics of the implied volatility smirk. Since equity index option prices fundamentally incorporate the equity market's uncertainty, this research issue has been of major interest to both academics and practitioners. From the economic perspective, the three-dimensional representation allows to decompose parsimoniously the dynamics of the volatility smirk at any point of time, by accommodating a combination of salient features interpreted primarily as the level, slope, and curvature factors. These factors incorporate agents' risk attitudes and beliefs about the realization of future risks, since they quantify the departure of the returns probability distribution from the lognormal distribution. Building on the modern asset pricing theory, i.e. the APT, the ICAPM and the Fama-French multifactor models, this paper is motivated by decomposing the implied volatility smile function into harvestable persistent option-implied risk premia over long period that theoretically reward the exposure to the systematic risk factors associated to higher-order moments. To the best of our knowledge, this paper is the first to propose this analytical decomposition into risk premia, and to test empirically its validation for standard and non-standard index options. Hence, we formally pave the way for new option-based volatility strategies, genuinely built on factor-based investing. Furthermore, our theoretical approach may be particularly suitable for VIX options, since they fundamentally incorporate the market price of the VIX ''fear gauge''.

This paper provides three results of major interest. First, we derive an explicit analytical third-dimensional representation of the implied volatility smirk function, conveniently expressed as a nonlinear combination of tradeable time-varying risk premia that compensate for bearing the higher-order risks. Furthermore, our analytical decomposition explicitly accommodates the tradeable components of the market price of uncertainty incorporated in the options market prices. Second, the results directly obtained from the estimation of nonlinear least squares (NLS) regressions (Levenberg-Marquardt) empirically validate our analytical decomposition. We find that the adjusted R squares remain persistently high, both accross the maturities and accross the strikes, since they range from 17.9% to 45.0% for the SPX put options, and from 11.9% to 31.5% for the VIX call options. Third, empirical results show that the contribution of the kurtosis risk premia in the dynamics of the implied volatility smile is much more relatively important for right-skewed leptokurtic distributions than for the standard distributions.

This paper contributes to the prior literature in the three following ways. From the Zhang-Xiang (2008) quadratic approximation in [117] and the decomposition of Martin (2013) in [83], we derive analytically an explicit representation of the implied volatility smirk function, expressed conveniently as a combination of tradeable time-varying risk premia reflected in the higher-order risk-neutral moments. Therefore, to the best of our knowledge, this paper is the first to extend this theoretical underpinning to the VIX options. Hence, building on Völkert (2014) in [111] and Branger, Kraftschik, and Völkert (2015) in [40], we find clear evidence that the dynamics of the volatility smirk proves to be particularly driven by the higher-order risk factors in the case of the VIX options. Subsequently, since we propose novative option-based replication strategies that allow mimicking the tradeable option-implied volatility, skewness, and kurtosis risk premia to take bets on the level, slope, and convexity of the volatility smirks, this paper extends the prior literature including Schneider and Trojani (2015) in [104], Bondarenko (2014) in [39], Alireza (2005) in [14], Chang, Zhang, and Zhao (2013) in [50], Aït-Sahalia, Wang, and Yared (2000) in [8], and Blaskowitz and Schmidt (2002) in [35] among others.

Therefore, this paper arises practical implications especially within the industries of the smart indexes and the asset management. Since we find clear evidence that our factor-based investing approach may be particularly tailored for trading the implied volatility smile, this paper formally extends the smart beta investing to the volatility strategies. Furthermore, our so-called "Smart Vega Investing" addresses the controversial performance of long or short equity volatility exposures that consist in allocating naïve volatility derivatives within equity portfolios. Indeed, this paper opens up a wide range of new option-based volatility strategies, genuinely built on factor-based investing. Besides, we formally pave the way for the development of new smart investable volatility indexes. For illustration, the VIX ''fear gauge' index gave rise to one of the greatest financial innovations, since the VIX options have become the second most actively traded contracts at the CBOE.

Nevertheless, this article opens up a wide range of fine-tuning, including the filtering methodology of the options data samples, and the high-frequency estimation of the realized moments that take into account overnight jumps. Second, deep investigations of option-based strategies built on the factor-based investing are left for future research.

2.5 Appendix

2.5.1 Bakshi-Kapadia-Madan Risk-Neutral Moments

From Bakshi and Madan (2000) in [25], any payoff function $H[S]$ can be spanned algebraically by a continuum of OTM European call and put options. Therefore, following Bakshi, Kapadia, and Madan (2003) in [24], let r the risk-free rate, $C(t,T;K)$ ($P(t,T;K)$) the price of a European call (put) option at time t, with time to expiration T, and strike price K, and $R(t,T) \equiv \ln[S(t+T)] - \ln[S(t)]$ the log return at time t over the time period T.

Define the volatility, cubic, and quartic contracts associated to the following payoff functions $H[S]$

$$H[S] = \begin{cases} R(t,T)^2 \text{ volatility contract} \\ R(t,T)^3 \text{ cubic contract} \\ R(t,T)^4 \text{ quartic contract} \end{cases} \tag{2.5.1.1}$$

Under the risk-neutral expectation operator conditional on information at time t, the fair values of the volatility, cubic, and quartic contracts in Equation (2.5.1.1) are respectively $V(t,T) \equiv E_t^Q\left[\exp(-rT)\, R(t,T)^2\right]$, $W(t,T) \equiv E_t^Q\left[\exp(-rT)\, R(t,T)^3\right]$, $X(t,T) \equiv E_t^Q\left[\exp(-rT)\, R(t,T)^4\right]$.

Subsequently, spanning the fair value of the contracts by a linear combination of OTM European call and put options, as well as the stock and the risk-free asset gives for the volatility contract $V(t,T)$

$$
\begin{aligned}
V(t,T) = & \int_{S(t)}^{\infty} \frac{2\left(1-\ln\left(\frac{K}{S(t)}\right)\right)}{K^2} C(t,T,K)\,dK \\
& + \int_{0}^{S(t)} \frac{2\left(1+\ln\left(\frac{S(t)}{K}\right)\right)}{K^2} P(t,T,K)\,dK
\end{aligned}
\tag{2.5.1.2}
$$

Similarly, the price of the cubic contract denoted $W(t,T)$ is given by

$$
\begin{aligned}
W(t,T) = & \int_{S(t)}^{\infty} \frac{6\ln\left(\frac{K}{S(t)}\right) - 3\ln\left(\frac{K}{S(t)}\right)^2}{K^2} C(t,T,K)\,dK \\
& - \int_{0}^{S(t)} \frac{6\ln\left(\frac{S(t)}{K}\right) + 3\ln\left(\frac{S(t)}{K}\right)^2}{K^2} P(t,T,K)\,dK
\end{aligned}
\tag{2.5.1.3}
$$

And the fair value of the quartic contract denoted $X(t,T)$ is specified by

$$
\begin{aligned}
X(t,T) = & \int_{S(t)}^{\infty} \frac{12\ln\left(\frac{K}{S(t)}\right)^2 - 4\ln\left(\frac{K}{S(t)}\right)^3}{K^2} C(t,T,K)\,dK \\
& + \int_{0}^{S(t)} \frac{12\ln\left(\frac{S(t)}{K}\right)^2 + 4\ln\left(\frac{S(t)}{K}\right)^2}{K^2} P(t,T,K)\,dK
\end{aligned}
\tag{2.5.1.4}
$$

2.5.2 Zhang-Xiang Quadratic Approximation

Zhang and Xiang (2008) in [117] derive a two-way representation of the implied volatility smirk function IV specified by (2.3.4.1). From the Black-Scholes (1973) valuation formula in [34], the price of a European call option C_0 is given by

$$C_0 = F_0 \exp\left[-rT\right] N\left(d^* + IV\sqrt{T}\right) - K \exp\left[-rT\right] N\left(d^*\right) \tag{2.5.2.1}$$

where S and r denote respectively the underlying asset price at maturity T and its drift; $d^* = \frac{\ln(F_0/K) - \frac{1}{2}IV^2T}{IV\sqrt{\tau}}$; and $N\left(\cdot\right)$ designate the cumulative normal distribution function defined by

$$N\left(x\right) = \int_{-\infty}^{x} n\left(y\right) dy, \quad n\left(y\right) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}y^2\right] \tag{2.5.2.2}$$

From the Breeden-Litzenberger (1978) formula in [41], we recover the following cumulative probability density function $F\left(S, T; F_0, 0\right)$ for the ATMV

$$F\left(S, T; F_0, 0\right) = 1 + \exp\left[rT\right] \left.\frac{\delta C_0}{\delta K}\right|_{K=S} \tag{2.5.2.3}$$

Subsequently, plugging the European call option price in (2.5.2.1) into the cumulative risk-neutral probability density function in (2.5.2.3):

$$F\left(S, T; F_0, 0\right) = N\left(-d\right) + n\left(d\right) \frac{\gamma_0}{\bar{\sigma}} \left[\gamma_1 + 2\gamma_2 \frac{\ln\left(S/F_0\right)}{\bar{\sigma}\sqrt{T}}\right] \tag{2.5.2.4}$$

where

$$d = -\frac{\ln(S/F_0) + \frac{1}{2}V^2 T}{V\sqrt{T}}, \quad V = \gamma_0 \left[1 + \gamma_1 \frac{\ln(S/F_0)}{\bar{\sigma}\sqrt{T}} + \gamma_2 \left(\frac{\ln(S/F_0)}{\bar{\sigma}\sqrt{T}} \right)^2 \right] \qquad (2.5.2.5)$$

Table 2.1: Descriptive statistics of SPX options

Log Moneyness		SPX Call Options				SPX Put Options			
		Days to Expiration				Days to Expiration			
		5 - 60	60 - 120	120 - 180	180 - 240	5 - 60	60 - 120	120 - 180	180 - 240
[-0,69;-0.29]	Nb. of Obs.	3559	2052	1939	2582	53190	52431	24735	31165
	Avg. IV	93,56	43,93	34,98	33,94	51,25	39,47	35,44	33,05
	Avg. Trading Volume	50	24	8	8	365	217	182	129
	Avg. Open Interest	1966	1765	2348	1554	9166	8604	12613	10163
	Avg. Bid-Ask Spread	0,84	0,81	0,81	0,89	353,17	248,88	145,65	58,32
[-0.29;-0.11]	Nb. of Obs.	27351	15501	10918	12872	176696	70158	23091	26867
	Avg. IV	40,81	28,84	26,45	25,41	34,57	28,01	26,53	25,31
	Avg. Trading Volume	45	25	17	14	728	510	491	325
	Avg. Open Interest	3258	4854	6666	6215	10266	10845	18282	14276
	Avg. Bid-Ask Spread	1,75	1,74	1,76	1,73	144,29	36,45	16,66	11,79
[-0.11;0.00]	Nb. of Obs.	121055	47332	16350	18192	183021	63554	19012	20184
	Avg. IV	20,95	20,20	20,30	20,44	21,43	20,62	20,74	20,97
	Avg. Trading Volume	611	610	165	152	1882	1240	879	553
	Avg. Open Interest	10136	8712	11730	10943	15313	13279	18738	15131
	Avg. Bid-Ask Spread	4,29	3,56	3,07	2,84	25,05	9,14	6,25	5,18
[0.00;0.10]	Nb. of Obs.	147427	62439	17716	19480	86671	34708	12438	13518
	Avg. IV	15,23	16,09	16,61	17,34	17,85	18,08	18,06	18,73
	Avg. Trading Volume	1552	736	585	399	691	544	263	228
	Avg. Open Interest	12466	8954	15627	13400	8684	7857	10532	8949
	Avg. Bid-Ask Spread	75,27	27,00	9,61	6,84	5,35	4,05	3,33	3,02
[0.10;0.22]	Nb. of Obs.	29162	26871	13224	18391	15443	8436	5356	6952
	Avg. IV	24,12	18,31	16,23	16,11	35,27	23,00	19,42	19,53
	Avg. Trading Volume	648	314	303	219	83	82	44	25
	Avg. Open Interest	11863	9101	13231	10057	7952	8004	8048	5468
	Avg. Bid-Ask Spread	306,56	238,51	118,77	47,22	2,24	2,18	1,96	1,83
[0.22;0,69]	Nb. of Obs.	2758	4210	4226	8599	9352	5326	4815	5476
	Avg. IV	45,23	29,98	24,37	21,97	68,14	39,10	32,09	28,65
	Avg. Trading Volume	460	172	99	70	41	56	16	17
	Avg. Open Interest	19553	13792	13060	8551	9798	7418	7056	6391
	Avg. Bid-Ask Spread	585,27	514,06	465,52	362,58	1,01	0,94	0,97	0,93

Descriptive statistics associated to SPX options for different log moneyness and maturities, from February 24, 2006 to August 29, 2014. The average measures are computed by equally-weighting the call and put options for each log moneyness-maturity category. Implied volatilities (in percentage) are derived from the Black (1976) option pricing formula. Log moneyness is defined as the logarithm of the strike price divided by the SPX spot, and bid-ask spreads (in percentage) are computed by dividing the bid-ask spreads by the bid price.

Table 2.2: Descriptive statistics of SPX option-implied risk-neutral moments

	60 Days Maturity				120 Days Maturity				180 Days Maturity			
	RN Mean	RN Vol	RN Skew	RN Kurt	RN Mean	RN Vol	RN Skew	RN Kurt	RN Mean	RN Vol	RN Skew	RN Kurt
Panel A: Levels of Risk-Neutral Measures for Realized Moments												
Nb. Observations	621	621	621	621	373	373	373	373	204	204	204	204
Mean	1345,740	0,220	-0,413	2,086	1296,087	0,357	-0,124	2,779	1234,121	0,454	-0,065	2,959
Median	1332,144	0,191	-0,194	1,655	1301,831	0,318	-0,112	2,781	1252,639	0,427	-0,109	2,817
Max	2000,923	0,758	0,649	5,059	1986,273	0,936	1,370	11,768	1861,664	0,853	0,391	4,229
Min	733,127	0,102	-1,289	0,088	675,200	0,160	-1,499	-0,154	747,395	0,209	-0,306	2,521
Standard Dev.	278,655	0,102	0,442	0,904	281,410	0,142	0,244	0,697	221,299	0,153	0,182	0,402
Skewness	0,225	1,929	-0,340	0,146	0,312	1,628	-2,485	4,749	-0,136	0,741	0,591	1,078
Kurtosis	2,836	8,043	1,630	1,950	2,963	6,356	21,018	79,560	2,740	3,125	2,165	3,200
LBQ	612,88*	572,25*	326,45*	350,94*	363,82*	345,96*	182,35*	63,35*	190,27*	184,62*	149,01*	153,16*
ADF	1,04	-1,5	-7,08**	-3,52**	-0,99	-7,14**	-2,43*	0,74	-3,81**	-0,56	-0,26	-0,4
Panel B: First Differences of Risk-Neutral Measures for Realized Moments												
Nb. Observations	620	620	620	620	372	372	372	372	203	203	203	203
Mean	1,127	0,000	0,001	-0,001	82,891	0,110	0,237	0,432	-62,663	0,211	0,415	0,780
Median	1,674	-0,003	0,001	-0,001	179,739	0,091	-0,028	-0,005	-64,263	0,209	0,041	0,311
Max	106,796	0,303	1,158	3,226	656,674	0,779	1,497	8,916	1070,392	0,661	1,569	3,413
Min	-230,591	-0,206	-1,156	-2,897	-856,204	-0,370	-0,780	-2,336	-719,797	-0,271	-0,391	-1,029
Standard Dev.	26,372	0,029	0,328	0,636	374,448	0,177	0,549	1,108	371,335	0,193	0,676	1,187
Skewness	-2,184	1,895	0,031	-0,026	-0,701	0,986	0,896	1,618	0,219	0,215	0,371	0,301
Kurtosis	20,435	30,939	7,306	9,170	2,616	5,644	2,331	11,186	3,021	3,049	1,366	1,655
LBQ	1,58	5,92*	131,63*	98,24*	366,91*	341,07*	233,71*	150,01*	191,91*	180,23*	159,78*	147,73*
ADF	-26,11**	-27,41**	-40,86**	-37,84**	-0,57	-2,52*	-6,02**	-8,39**	0,85	-1,6	-3,12**	-3,47**

Descriptive statistics associated to SPX option-implied risk-neutral moments for different maturities, from February 24, 2006 to August 29, 2014. Statistics correspond to the risk-neutral expected value of the mean, the volatility, the skewness, and the kurtosis, for 60, 120, and 180 days times to expiration. For the Augmented-Dickey Fuller test, stars * and ** denote statistical significance at respectively 5% and 1% level of confidence. For the Ljung-Box Q test, star * denote statistical significance at respectively 5% level of confidence.

Table 2.3: Cross-correlations of SPX option-implied risk-neutral moments

	RN Mean	RN Volatility	RN Skewness	RN Kurtosis
Panel A: Maturity of 60 Days				
RN Mean	1,00	-	-	-
RN Volatility	-0,76	1,00	-	-
RN Skewness	-0,46	0,55	1,00	-
RN Kurtosis	-0,62	0,62	0,79	1,00
Panel B: Maturity of 120 Days				
RN Mean	1,00	-	-	-
RN Volatility	-0,79	1,00	-	-
RN Skewness	-0,09	0,14	1,00	-
RN Kurtosis	-0,17	0,20	0,54	1,00
Panel C: Maturity of 180 Days				
RN Mean	1,00	-	-	-
RN Volatility	-0,82	1,00	-	-
RN Skewness	0,28	-0,39	1,00	-
RN Kurtosis	0,33	-0,44	0,95	1,00

Cross-correlations associated to SPX option-implied risk-neutral moments for different maturities, from February 24, 2006 to August 29, 2014. Computations are based on the Pearson cross-correlations between the risk-neutral expected value of the mean, the volatility, the skewness, and the kurtosis, for 60, 120, and 180 days to expiration.

Table 2.4: Descriptive statistics of physical moments for intradaily SPX spots

	Realized Moments			
	RD Mean	RD Vol	RD Skew	RD Kurt
Panel A: Levels of Physical Measures for Realized Moments				
Nb. Observations	375	375	375	375
Mean	1187,169	0,141	0,085	1,674
Median	1215,116	0,114	-0,019	2,224
Max	1462,014	1,033	2,464	7,038
Min	754,303	0,027	-2,211	-16,217
Standard Dev.	179,687	0,107	1,100	2,856
Skewness	-0,651	3,299	0,047	-2,698
Kurtosis	2,429	20,616	2,045	13,005
LBQ	368,52*	208,55*	1,35	1,2
ADF	-0,03	-4,35**	-20,36**	-15,54**
Panel B: First Differences of Physical Measures for Realized Moments				
Nb. Observations	374	374	374	374
Mean	0,191	0,000	-0,003	-0,001
Median	2,003	0,000	0,017	-0,054
Max	111,271	0,590	4,002	18,772
Min	-242,182	-0,423	-3,666	-17,277
Standard Dev.	24,685	0,077	1,603	4,157
Skewness	-2,703	0,505	0,088	0,169
Kurtosis	30,828	16,088	2,458	7,826
LBQ	0,5	26,07*	86,88*	95,1*
ADF	-18,59**	-25,25**	-32,57**	-33,51**

Descriptive statistics associated to the physical measures of moments related to intradaily SPX spots, from January 24, 2008 to December 19, 2012. Statistics correspond to the physical measures associated to the realized value of the mean, the volatility, the skewness, and the kurtosis. For the Augmented-Dickey Fuller test, stars * and ** denote statistical significance at respectively 5% and 1% level of confidence. For the Ljung-Box Q test, star * denote statistical significance at respectively 5% level of confidence.

Table 2.5: Cross-correlations between physical measures of moments related to intradaily SPX spots

	RD Mean	RD Volatility	RD Skewness	RD Kurtosis
RD Mean	1,00	-	-	-
RD Volatility	-0,52	1,00	-	-
RD Skewness	-0,06	-0,08	1,00	-
RD Kurtosis	-0,08	0,11	0,04	1,00

Cross-correlations associated to the physical measures of moments related to intradaily SPX spots, from January 24, 2008 to December 19, 2012. Computations are based on the Pearson cross-correlations between the realized value of the mean, the volatility, the skewness, and the kurtosis.

Table 2.6: Descriptive statistics of SPX option-implied risk premia

	60 Days Maturity			120 Days Maturity		
	VRP	**SRP**	**KRP**	**VRP**	**SRP**	**KRP**
Panel A: Levels of Option-Implied Risk Premia						
Nb. Observations	316	316	316	150	150	150
Mean	-0,147	0,291	-1,056	-0,339	0,200	-2,179
Median	-0,144	0,252	-0,517	-0,298	0,134	-0,884
Max	0,785	2,886	4,366	-0,011	3,076	6,619
Min	-0,671	-2,438	-18,857	-0,858	-2,022	-17,129
Standard Dev.	0,155	1,142	3,048	0,176	1,110	3,993
Skewness	0,694	-0,031	-2,293	-1,207	0,109	-1,743
Kurtosis	8,248	2,217	10,394	4,091	2,305	6,071
LBQ Test	223,86*	0,43	3,35	126,12*	0,26	3,02
ADF Test	-3,69**	-17,24**	-17,44**	-1,08	-12,26**	-10,75**
Student Test	-16,84**	4,52**	-6,16**	-23,65**	2,21*	-6,68**
Panel B: First Differences of Option-Implied Risk Premia						
Nb. Observations	315	315	315	149	149	149
Mean	-0,001	-0,004	0,002	-0,182	0,042	-1,408
Median	0,002	0,046	0,000	-0,150	-0,027	-0,513
Max	0,576	4,659	18,744	0,302	3,864	14,286
Min	-0,481	-3,681	-17,339	-0,949	-3,482	-15,930
Standard Dev.	0,086	1,646	4,534	0,256	1,548	4,659
Skewness	0,243	0,083	0,131	-0,699	0,180	-0,686
Kurtosis	13,377	2,618	6,622	3,151	2,544	5,348
LBQ Test	117,23*	0,41	1,23	117,23*	0,41	1,23
ADF Test	-2,53*	-12,81**	-12,09**	-2,53*	-12,81**	-12,09**
Student Test	-8,7**	0,33	-3,69**	-8,7**	0,33	-3,69**

Descriptive statistics associated to SPX option-implied risk premia for different maturities, from January 24, 2008 to December 19, 2012. Option-implied risk premia are computed as the difference between the physical measure and the risk-neutral measure of the expected value of realized moments. Moments estimated under the physical expectation are based on the intradaily prices associated to the SPX spots. Moments estimated under the risk-neutral expectation are based on the SPX option prices associated to different maturities. Stars *, ** denote statistical significance at respectively 5% and 1% level of confidence.

Table 2.7: Nonlinear least squares regressions of SPX implied volatility smiles

	ATM Implied Volatility		ITM Implied Volatility		OTM Implied Volatility	
	60 Days	**120 Days**	**60 Days**	**120 Days**	**60 Days**	**120 Days**
Nb. Obs.	312	150	313	123	316	150
Intercept	0,282**	0,256**	0,371**	0,352**	0,234**	0,203**
	[46,06]	[29,95]	[53,86]	[25,12]	[46,29]	[29,35]
VRP	0,367**	0,131**	0,472**	0,202**	0,32**	0,115**
	[12,94]	[5,71]	[13,7]	[5,22]	[12,62]	[5,93]
SRP			-0,026**	-0,023*	-0,018**	-0,013*
			[-4,23]	[-2,12]	[-3,99]	[-2,34]
KRP	-0,001	0,001	-0,014**	0,007	-0,009**	0,005
	[-0,28]	[0,52]	[-3,29]	[0,92]	[-2,84]	[1,47]
VRP*SRP			-0,132**	-0,066*	-0,096**	-0,036*
			[-4,41]	[-2,36]	[-4,32]	[-2,54]
VRP*KRP	-0,008	0,004	-0,044*	0,036	-0,029	0,022*
	[-0,68]	[0,73]	[-2,15]	[1,62]	[-1,89]	[2,15]
SRP*KRP			0,002	0	0,001	0
			[1]	[-0,05]	[0,86]	[0,03]
VRP*SRP*KRP			0,007	-0,005	0,005	-0,002
			[0,68]	[-0,55]	[0,69]	[-0,58]
KRP2			-0,002**	0,001	-0,001**	0
			[-3,78]	[0,71]	[-3,88]	[1,26]
VRP*KRP2			-0,007**	0,003	-0,005**	0,002
			[-2,99]	[1,22]	[-3,18]	[1,83]
Adj. R2	38,8%	19,9%	45,0%	17,9%	41,0%	20,2%

Estimation results associated to a Levenberg-Marquardt nonlinear least squares (NLS) regression of the SPX implied volatility smiles on the corresponding time-horizon option-implied risk premia, for SPX index options from January 24, 2008 to December 19, 2012, for different maturities. The 3 estimated models for each maturity are derived from the equation (2.3.4.5). Figures in brackets designate the Newey-West t-statistics. Stars *, ** denote statistical significance at respectively 5% and 1% level of confidence.

Table 2.8: Descriptive statistics of VIX spots and log returns

	CBOE Vix Index	
	Spots	**Log returns**
Panel A: All observations		
Nb. of obs.	2435	2435
Mean	20,65	0,00
Min	9,89	-0,35
Max	80,86	0,50
Std. Dev.	10,02	0,07
Skewness	2,24	0,66
Kurtosis	9,50	6,92
Jarque-Bera	6301,18*	1723,05*
Ljung-Box Q	2349,68*	34,17*
Augmented Dickey-Fuller	-2,03*	-55,55*
Panel B: Subprime crisis		
Nb. of obs.	60	60
Mean	49,42	0,02
Min	20,65	-0,28
Max	80,86	0,30
Std. Dev.	18,20	0,11
Skewness	-0,13	-0,16
Kurtosis	1,71	3,59
Jarque-Bera	4,14	0,58
Ljung-Box Q	48,98*	2,39
Augmented Dickey-Fuller	0,73	-8,76*
Panel C: Sovereign debt crisis		
Nb. of obs.	61	61
Mean	32,07	0,02
Min	17,52	-0,31
Max	48,00	0,41
Std. Dev.	8,27	0,11
Skewness	-0,32	0,74
Kurtosis	2,04	5,85
Jarque-Bera	3,28	24,05*
Ljung-Box Q	43,66*	5,91*
Augmented Dickey-Fuller	0,33	-10,25*

Descriptive statistics associated to VIX spots and log returns, from February 24, 2006 to August 29, 2014. Statistics are broken down into subsamples related to stock market turmoils. Panel B corresponds to the subprime crisis period, over the period from August 29, 2008 to November 20, 2008, when VIX Index spiked from 20.65 to 80.86. Panel C corresponds to the European sovereign debt crisis, over the period from July 11, 2011 to October 3, 2011, when VIX Index spiked from 18.39 to 45.45. The Jarque-Bera statistic tests for the rejection of the null hypothesis associated to returns normality. Star * denote statistical significance at the 5% level of confidence, when the test statistic exceeds the critical value.

Table 2.9: Descriptive statistics of VIX options

Log Moneyness		VIX Call Options				VIX Put Options			
		Days to Expiration				Days to Expiration			
		7 - 30	30 - 60	60 - 90	90 - 180	7 - 30	30 - 60	60 - 90	90 - 180
[-0,69;-0.29]	Nb. of Obs.	651	1720	2448	5783	613	1711	2201	5923
	Avg. IV	111,82	79,59	67,60	57,52	103,66	79,30	71,69	59,65
	Avg. Trading Volume	214358	70244	21010	5498	565439	221406	96263	30380
	Avg. Open Interest	2362719	1299597	569288	164058	3659577	2554958	1793299	631058
	Avg. Bid-Ask Spread	5,57	5,74	6,22	8,35	66,02	74,99	82,05	88,14
[-0.29;-0.11]	Nb. of Obs.	1724	2862	3195	7851	1824	2945	3114	7704
	Avg. IV	86,59	70,02	62,05	54,14	85,58	72,55	67,72	57,73
	Avg. Trading Volume	294481	113093	30216	9276	719032	342303	109436	30610
	Avg. Open Interest	2756790	1318874	595224	170373	5403151	3344972	1667047	669578
	Avg. Bid-Ask Spread	6,65	6,45	7,17	9,07	46,16	51,58	53,93	60,65
[-0.11;0.00]	Nb. of Obs.	2089	2588	2642	6628	2156	2628	2679	6559
	Avg. IV	82,36	70,33	63,10	54,83	81,89	74,02	70,13	58,73
	Avg. Trading Volume	479609	222964	58445	15368	906539	442535	151435	40233
	Avg. Open Interest	3537230	1916929	853445	239207	7033456	4175030	2082701	770094
	Avg. Bid-Ask Spread	7,65	6,94	8,10	9,54	25,15	25,99	30,44	37,61
[0.00;0.10]	Nb. of Obs.	2287	2596	2605	6715	2291	2669	2714	6634
	Avg. IV	88,30	75,44	66,89	57,41	88,58	82,29	78,95	63,16
	Avg. Trading Volume	893494	376216	109104	23690	864031	462563	198484	45806
	Avg. Open Interest	5430755	3045758	1329985	348399	7513121	4545858	2229375	798589
	Avg. Bid-Ask Spread	9,17	7,29	8,43	10,16	10,78	11,90	14,97	20,26
[0.10;0.22]	Nb. of Obs.	3119	3623	3675	9538	3110	3756	3757	9162
	Avg. IV	99,87	82,58	71,94	60,81	100,29	91,64	81,50	66,37
	Avg. Trading Volume	1045146	559950	194223	43445	420654	322364	154320	43190
	Avg. Open Interest	7976239	4593112	1946093	546043	6116299	4043339	2052604	679890
	Avg. Bid-Ask Spread	13,68	8,85	9,44	11,50	6,61	7,60	8,13	10,86
[0.22;0,69]	Nb. of Obs.	7908	13419	13878	36480	7825	13087	12630	28598
	Avg. IV	121,21	98,87	83,57	68,73	124,01	108,08	89,65	74,62
	Avg. Trading Volume	775179	487667	224317	63313	59287	32452	25531	12972
	Avg. Open Interest	8481106	5954947	3150625	932040	1638249	1014892	683013	300558
	Avg. Bid-Ask Spread	34,48	27,55	25,02	22,20	3,80	3,84	4,28	5,33

Descriptive statistics associated to VIX options for different log moneyness and maturities, from February 24, 2006 to August 29, 2014. The average measures are computed by equally-weighting the call and put options for each log moneyness-maturity category. Implied volatilities (in percentage) are derived from the Black (1976) option pricing formula. Log moneyness is defined as the logarithm of the strike price divided by the VIX spot, and bid-ask spreads (in percentage) are computed by dividing the bid-ask spreads by the bid price.

Table 2.10: Descriptive statistics of VIX option-implied risk-neutral moments

	30 Days Maturity				60 Days Maturity				90 Days Maturity			
	RN Mean	RN Vol	RN Skew	RN Kurt	RN Mean	RN Vol	RN Skew	RN Kurt	RN Mean	RN Vol	RN Skew	RN Kurt
Panel A: Levels of Risk-Neutral Measures for Realized Moments												
Nb. Observations	426	426	426	426	425	425	425	425	422	422	422	422
Mean	21,997	0,877	0,619	4,883	22,322	1,065	0,554	4,172	22,598	1,178	0,464	3,797
Median	19,875	0,843	0,591	4,555	21,031	1,033	0,504	3,876	21,729	1,151	0,438	3,635
Max	63,990	1,624	2,388	14,355	53,887	1,833	1,789	8,568	49,806	1,760	1,536	6,410
Min	11,307	0,541	-0,121	2,653	12,840	0,762	-0,119	2,688	13,323	0,873	-0,111	2,715
Standard Dev.	9,187	0,167	0,509	1,803	7,554	0,174	0,449	1,092	6,818	0,158	0,372	0,709
Skewness	1,938	1,062	0,751	1,462	1,415	0,917	0,576	1,156	1,284	0,670	0,548	0,971
Kurtosis	7,770	4,744	3,127	6,078	5,541	4,111	2,489	4,143	5,272	3,181	2,631	3,603
LBQ	386,33*	289,98*	314,12*	337,29*	402,44*	314,75*	348,02*	369,06*	403,58*	330,71*	339,56*	366,33*
ADF	-1,23	-1,24	-3,2**	-1,42	-0,94	-2,52*	-0,68	-0,62	-2,7**	-0,43	-0,58	-0,62
Panel B: First Differences of Risk-Neutral Measures for Realized Moments												
Nb. Observations	425	425	425	425	424	424	424	424	421	421	421	421
Mean	0,003	0,000	0,004	0,010	0,306	0,187	-0,059	-0,697	0,510	0,300	-0,146	-1,053
Median	-0,091	-0,010	0,004	0,016	0,448	0,178	-0,027	-0,368	1,024	0,289	-0,076	-0,650
Max	32,892	0,658	1,207	4,623	16,605	0,953	0,976	1,422	24,268	0,976	0,633	0,638
Min	-17,281	-0,658	-1,102	-4,631	-19,987	-0,423	-1,514	-6,834	-25,488	-0,473	-1,510	-8,342
Standard Dev.	2,866	0,099	0,268	0,846	3,820	0,157	0,313	1,088	5,305	0,180	0,354	1,333
Skewness	3,497	0,665	-0,317	-0,392	-0,888	0,253	-0,678	-1,662	-0,218	0,098	-0,875	-1,747
Kurtosis	49,413	13,560	6,248	9,574	11,646	6,623	4,886	7,368	9,214	4,830	3,780	7,295
LBQ	11,62*	18,3*	14,58*	11,56*	148,36*	186,12*	72,91*	159,47*	285,37*	237,78*	183,7*	268,89*
ADF	-24,29**	-25,37**	-24,78**	-24,27**	-10,39**	-5,62**	-12,91**	-8,08**	-6,4**	-3,75**	-8,46**	-5,27**

Descriptive statistics associated to VIX option-implied risk-neutral moments for different maturities, from April 13, 2007 to August 29, 2014. Statistics correspond to the risk-neutral expected value of the mean, the volatility, the skewness, and the kurtosis, for 30, 60, and 90 days times to expiration. For the Augmented-Dickey Fuller test, stars * and ** denote statistical significance at respectively 5% and 1% level of confidence. For the Ljung-Box Q test, star * denote statistical significance at respectively 5% level of confidence.

Table 2.11: Cross-correlations between VIX option-implied risk-neutral moments

	RN Mean	RN Volatility	RN Skewness	RN Kurtosis
Panel A: Maturity of 30 Days				
RN Mean	1,00	-	-	-
RN Volatility	0,45	1,00	-	-
RN Skewness	-0,60	-0,53	1,00	-
RN Kurtosis	-0,58	-0,54	0,95	1,00
Panel B: Maturity of 60 Days				
RN Mean	1,00	-	-	-
RN Volatility	0,28	1,00	-	-
RN Skewness	-0,63	-0,44	1,00	-
RN Kurtosis	-0,61	-0,42	0,95	1,00
Panel C: Maturity of 90 Days				
RN Mean	1,00	-	-	-
RN Volatility	0,31	1,00	-	-
RN Skewness	-0,61	-0,36	1,00	-
RN Kurtosis	-0,56	-0,30	0,88	1,00

Cross-correlations associated to VIX option-implied risk-neutral moments for different maturities, from April 13, 2007 to August 29, 2014. Computations are based on the Pearson cross-correlations between the risk-neutral expected value of the mean, the volatility, the skewness, and the kurtosis, for 30, 60, and 90 days to expiration.

Table 2.12: Descriptive statistics of physical moments for intradaily VIX spots

	Realized Moments			
	RD Mean	RD Vol	RD Skew	RD Kurt
Panel A: Levels of Physical Measures for Realized Moments				
Nb. Observations	329	329	329	329
Mean	24,056	0,708	-0,207	-1,301
Median	21,206	0,651	-0,318	1,959
Max	74,993	2,122	3,024	8,504
Min	11,692	0,041	-3,425	-29,075
Standard Dev.	11,092	0,333	1,393	7,653
Skewness	2,092	1,146	0,058	-1,573
Kurtosis	8,180	5,025	2,139	4,742
LBQ	296,84*	28,14*	1,26	0,63
ADF	-1,34	-4,72**	-18,81**	-18,33**
Panel B: First Differences of Physical Measures for Realized Moments				
Nb. Observations	328	328	328	328
Mean	-0,024	0,000	0,002	0,000
Median	-0,201	0,005	0,024	0,028
Max	38,597	1,503	5,640	30,436
Min	-30,197	-1,425	-5,223	-33,544
Standard Dev.	3,645	0,397	2,033	11,067
Skewness	2,199	0,055	-0,001	-0,046
Kurtosis	55,895	4,136	2,637	3,700
LBQ	2,4	66,31*	69,16*	105,33*
ADF	-19,66**	-29,24**	-29,68**	-34,2**

Descriptive statistics associated to the physical measures of realized moments related to intradaily VIX spots, from January 24, 2008 to April 26, 2016. Statistics correspond to the physical measures associated to the realized value of the mean, the volatility, the skewness, and the kurtosis. For the Augmented-Dickey Fuller test, stars * and ** denote statistical significance at respectively 5% and 1% level of confidence. For the Ljung-Box Q test, star * denote statistical significance at respectively 5% level of confidence.

Table 2.13: Cross-correlations between physical measures of realized moments for intradaily VIX spots

	RD Mean	**RD Volatility**	**RD Skewness**	**RD Kurtosis**
RD Mean	1,00	-	-	-
RD Volatility	0,24	1,00	-	-
RD Skewness	0,00	0,23	1,00	-
RD Kurtosis	-0,07	0,20	0,16	1,00

Cross-correlations associated to the physical measures of realized moments related to intradaily VIX spots, from January 24, 2008 to April 26, 2016. Computations are based on the Pearson cross-correlations between the realized value of the mean, the volatility, the skewness, and the kurtosis.

Table 2.14: Descriptive statistics of VIX option-implied risk premia

	30 Days Maturity			60 Days Maturity			90 Days Maturity		
	VRP	**SRP**	**KRP**	**VRP**	**SRP**	**KRP**	**VRP**	**SRP**	**KRP**
Panel A: Levels of Option-Implied Risk Premia									
Nb. Observations	300	300	300	267	267	267	237	237	237
Mean	-0,193	-0,718	-6,027	-0,411	-0,747	-5,818	-0,487	-0,652	-5,707
Median	-0,242	-0,849	-2,680	-0,447	-0,774	-2,264	-0,515	-0,766	-1,852
Max	1,361	3,031	4,855	1,033	2,645	5,358	0,971	2,507	4,723
Min	-1,280	-4,555	-35,560	-1,205	-5,522	-33,474	-1,258	-4,382	-33,029
Standard Dev.	0,397	1,437	8,073	0,354	1,502	8,056	0,388	1,450	8,420
Skewness	0,736	0,047	-1,437	1,011	-0,157	-1,392	0,662	-0,038	-1,467
Kurtosis	4,017	2,457	4,322	5,091	2,673	4,273	3,559	2,541	4,435
LBQ Test	79,14*	0,01	0,12	40,11*	0,05	0,57	37,21*	0	0
ADF Test	-8,57**	-14**	-12,07**	-6,33**	-13,16**	-10,98**	-5,52**	-12,87**	-11,01**
Student Test	-8,41**	-8,66**	-12,93**	-18,94**	-8,13**	-11,8**	-19,35**	-6,92**	-10,44**
Panel B: First Differences of Option-Implied Risk Premia									
Nb. Observations	299	299	299	266	266	266	236	236	236
Mean	0,000	-0,004	0,025	-0,216	-0,100	0,006	-0,312	-0,013	0,308
Median	-0,016	-0,032	0,055	-0,224	-0,103	0,359	-0,330	-0,223	0,091
Max	1,166	5,826	30,593	1,432	5,615	28,863	1,400	4,908	36,231
Min	-1,443	-5,247	-35,141	-2,003	-6,249	-33,166	-1,966	-5,965	-33,478
Standard Dev.	0,393	2,029	11,543	0,523	2,013	11,314	0,519	1,915	11,922
Skewness	0,051	-0,017	-0,059	-0,030	-0,126	-0,211	0,058	-0,030	-0,062
Kurtosis	3,687	2,687	3,528	3,784	3,179	3,638	3,593	2,930	3,716
LBQ Test	49,88*	61,45*	96,49*	52,91*	0,77	0,58	42,15*	0,89	0,9
ADF Test	-26,52**	-28,05**	-32,7**	-9,03**	-17,1**	-15,53**	-7,95**	-16,3**	-16,26**
Student Test	0	-0,03	0,04	-6,75**	-0,81	0,01	-9,24**	-0,1	0,4

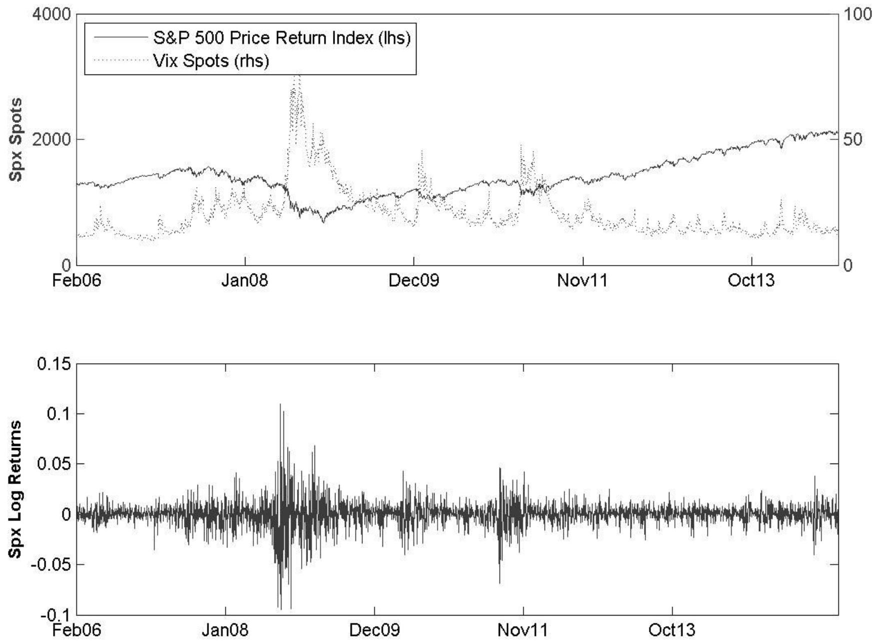
Descriptive statistics associated to VIX option-implied risk premia for different maturities, from January 24, 2008 to August 29, 2014. Option-implied risk premia are computed as the difference between the physical measure and the risk-neutral measure of the expected value of realized moments. Moments estimated under the physical expectation are based on intradaily prices associated to the VIX spots. Moments estimated under the risk-neutral expectation are based on daily VIX option prices for different maturities. Stars *, ** denote statistical significance at respectively 5% and 1% level of confidence.

Table 2.15: Nonlinear least squares regressions of VIX implied volatility smiles

	ATM Implied Volatility				ITM Implied Volatility				OTM Implied Volatility			
	30 Days	60 Days	90 Days	120 Days	30 Days	60 Days	90 Days	120 Days	30 Days	60 Days	90 Days	120 Days
Nb. Obs.	296	262	231	191	246	231	213	179	300	267	237	196
Intercept	0,807**	0,732**	0,669**	0,639**	0,772**	0,667**	0,6**	0,571**	1,069**	0,9**	0,799**	0,748**
	[63,88]	[54,57]	[57,79]	[57,54]	[53,15]	[53,51]	[49,88]	[43,82]	[91,65]	[71,29]	[79,12]	[89,59]
VRP	0,209**	0,17**	0,115**	0,124**	0,147**	0,152**	0,092**	0,072**	0,124**	0,105**	0,075**	0,08**
	[7,31]	[6,65]	[5,9]	[7,08]	[4,43]	[5,97]	[4,28]	[3,57]	[4,73]	[4,24]	[4,18]	[6,21]
SRP					0,011	0,008	0,009	0,009	0,007	0	0,005	0,008
					[1,27]	[1,08]	[1,28]	[1,12]	[0,96]	[-0,05]	[0,83]	[1,6]
KRP	-0,007**	-0,005**	-0,005**	-0,003*	-0,002	-0,002	-0,005	0,001	-0,011**	-0,009**	-0,008**	0
	[-4,87]	[-3,05]	[-3,86]	[-2,51]	[-0,55]	[-0,59]	[-1,52]	[0,2]	[-3,28]	[-2,67]	[-2,72]	[0,07]
VRP*SRP					0,02	0,023	0,033*	0,02	0	0,005	0,006	0,004
					[0,94]	[1,49]	[2,37]	[1,65]	[-0,03]	[0,3]	[0,54]	[0,56]
VRP*KRP	-0,009**	-0,006*	-0,005*	-0,004	-0,01	0,005	0,001	0,003	-0,012	-0,015*	-0,01*	0,001
	[-3,08]	[-2,33]	[-2,42]	[-1,91]	[-1]	[0,78]	[0,24]	[0,54]	[-1,61]	[-2,48]	[-2,37]	[0,23]
SRP*KRP					-0,001	0,001	0,001	0	0	0,001	0	0
					[-0,75]	[0,55]	[0,72]	[0,11]	[-0,18]	[0,67]	[0,02]	[0,46]
VRP*SRP*KRP					0	0,001	0,002	0,001	0	0,001	0	0
					[0,07]	[1,04]	[1,74]	[0,6]	[-0,29]	[0,67]	[0,05]	[0,47]
KRP2					0	0	0	0	0	0*	0	0
					[0,64]	[0,21]	[-0,49]	[0,98]	[-1,32]	[-2,03]	[-1,35]	[0,61]
VRP*KRP2					0	0	0	0	0	-0,001*	0	0
					[-0,25]	[1,16]	[0,25]	[1,09]	[-0,59]	[-2,19]	[-1,71]	[0,43]
Adj. R2	31,5%	26,2%	26,5%	28,8%	21,2%	20,9%	14,5%	11,9%	24,7%	16,1%	19,8%	21,4%

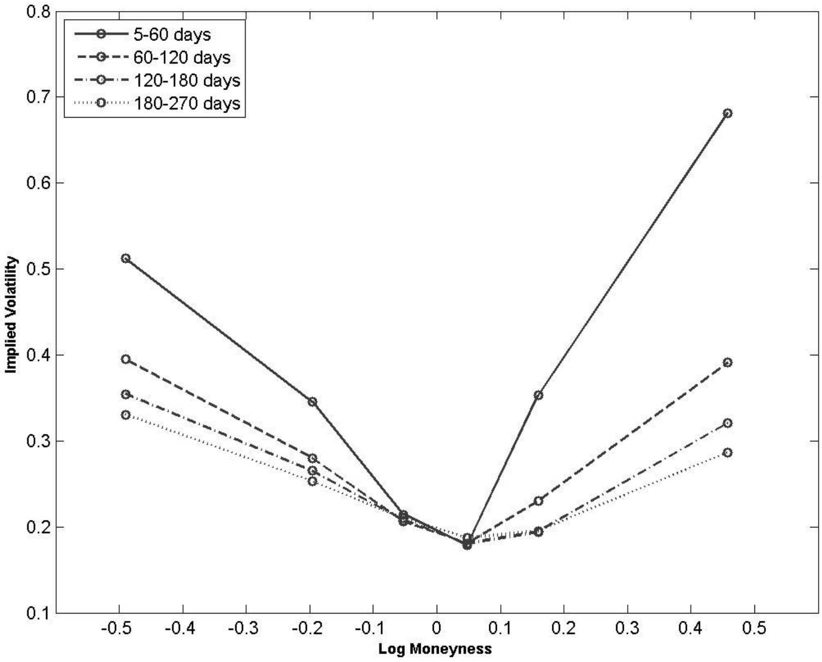
Estimation results associated to a Levenberg-Marquardt nonlinear least squares (NLS) regression of the VIX implied volatility smiles on the corresponding time-horizon option-implied risk premia, for VIX index options from January 24, 2008 to August 29, 2014, for different maturities. The 3 estimated models for each maturity are derived from the equation (2.3.4.5). Figures in brackets designate the Newey-West t-statistics. Stars *, ** denote statistical significance at respectively 5% and 1% level of confidence.

Figure 2.1: Historical Time-Series of the CBOE Volatility Index



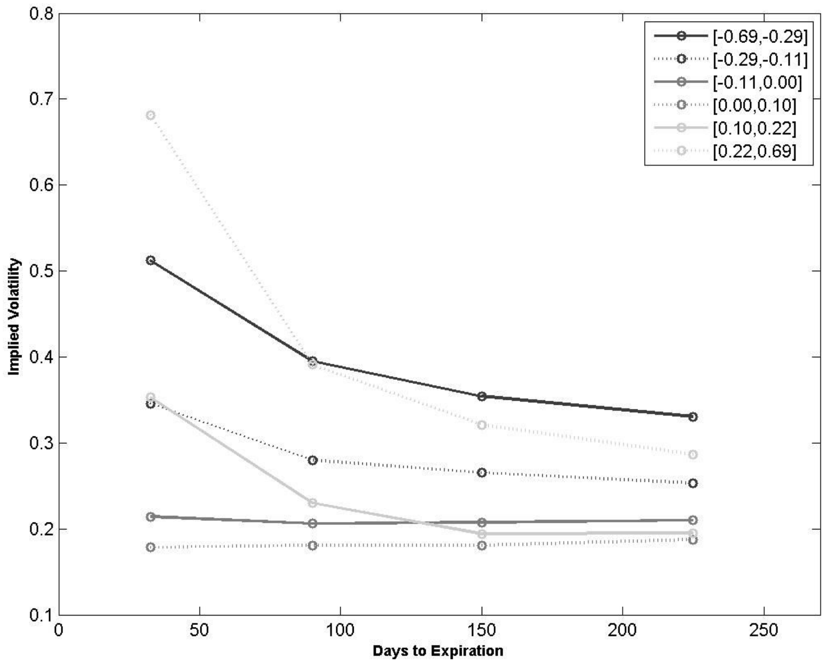
Historical time-series of the CBOE Volatility Index and the SPX Index, from February 24, 2006 to August 29, 2014. The upper and lower panel represents respectively the compared evolutions of the VIX Index and the S&P 500 Price Return Index, and the daily log returns of the VIX Index across time.

Figure 2.2: Average Volatility Smile of SPX Options



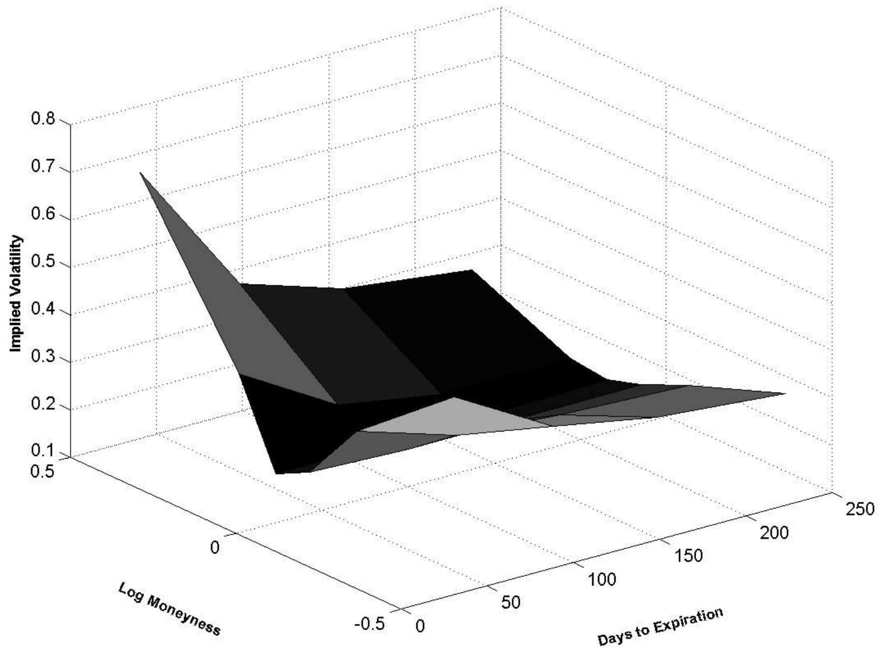
Average volatility smile associated to SPX options for four different maturity buckets, from February 24, 2006 to August 29, 2014. Average implied volatilities are computed in each log moneyness-maturity category, for four maturity buckets (5−60, 60−120, 120−180, 180−270 days), and six log moneyness buckets ([−0.69;−0.29], [−0.29;−0.11], [−0.11;0.00], [0.00;0.10], [0.10;0.22], [0.22;0.69]).

Figure 2.3: Average Term Structure of SPX Options



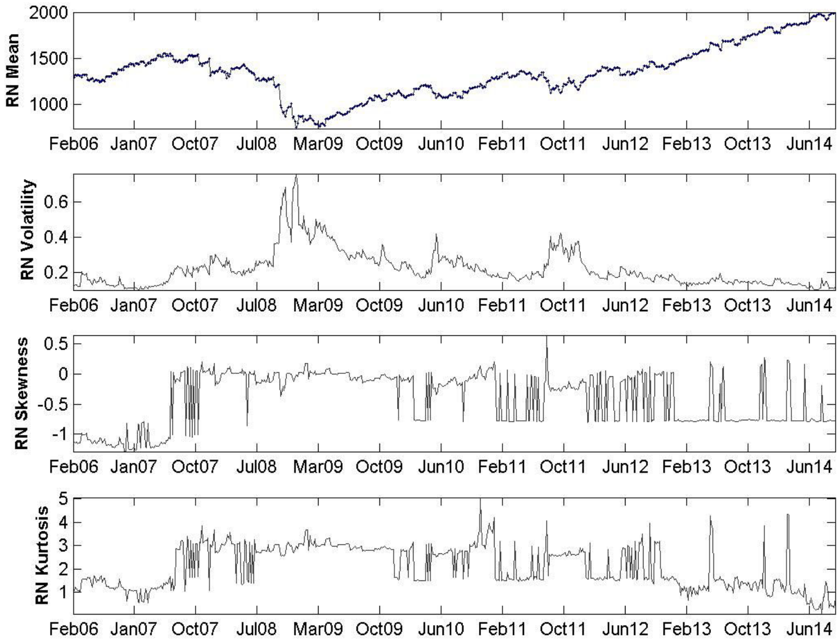
Average term structure associated to SPX options for six different log moneyness buckets, from February 24, 2006 to August 29, 2014. Average implied volatilities are computed in each log moneyness-maturity category, for four maturity buckets (5−60, 60−120, 120−180, 180−270 days), and six log moneyness buckets ([−0.69;−0.29], [−0.29;−0.11], [−0.11;0.00], [0.00;0.10], [0.10;0.22], [0.22;0.69]).

Figure 2.4: Average Volatility Surface of SPX Options



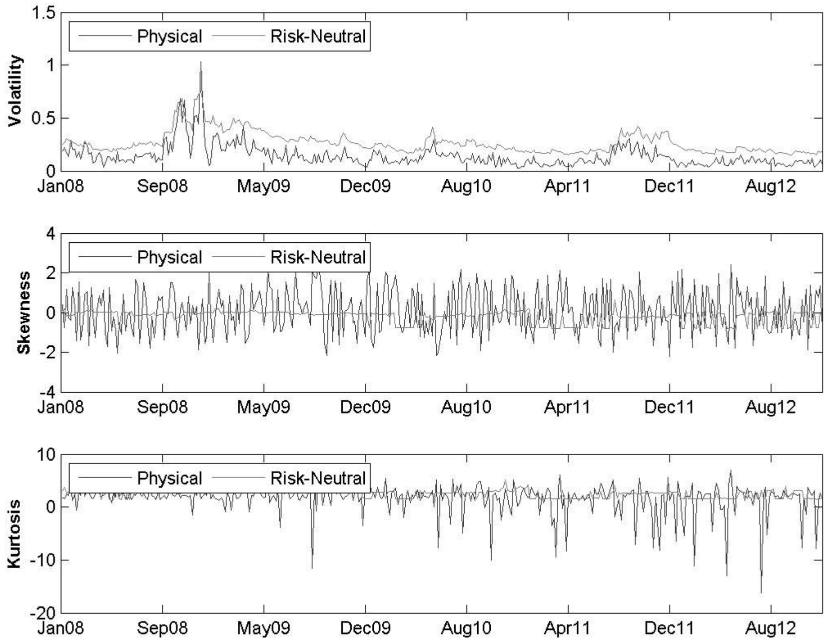
Average implied volatility surface associated to SPX options for six different log moneyness buckets, from February 24, 2006 to August 29, 2014. Average implied volatilities are computed in each log moneyness-maturity category, for four maturity buckets (5−60, 60−120, 120−180, 180−270 days), and six log moneyness buckets ([−0.69;−0.29], [−0.29;−0.11], [−0.11;0.00], [0.00;0.10], [0.10;0.22], [0.22;0.69]).

Figure 2.5: Option-Implied Risk-Neutral Moments of SPX Options



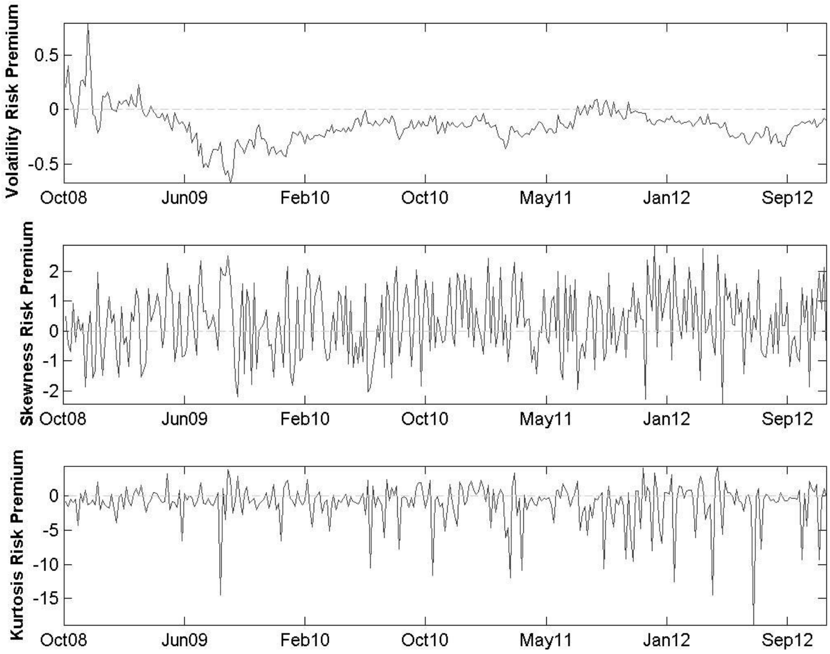
SPX option-implied risk-neutral moments for 60 days time to maturity, from February 24, 2006, to August 29, 2014. The figures plot respectively on a daily basis the levels of the risk-neutral expected value of the mean, the volatility, the skewness, and the kurtosis, for 60 days time to expiration. The blue crosses in the upper figure represent the SPX futures prices for a fixed time to maturity of 60 days.

Figure 2.6: Moments Estimated Under Physical and Risk-Neutral Probability Measures for SPX Options



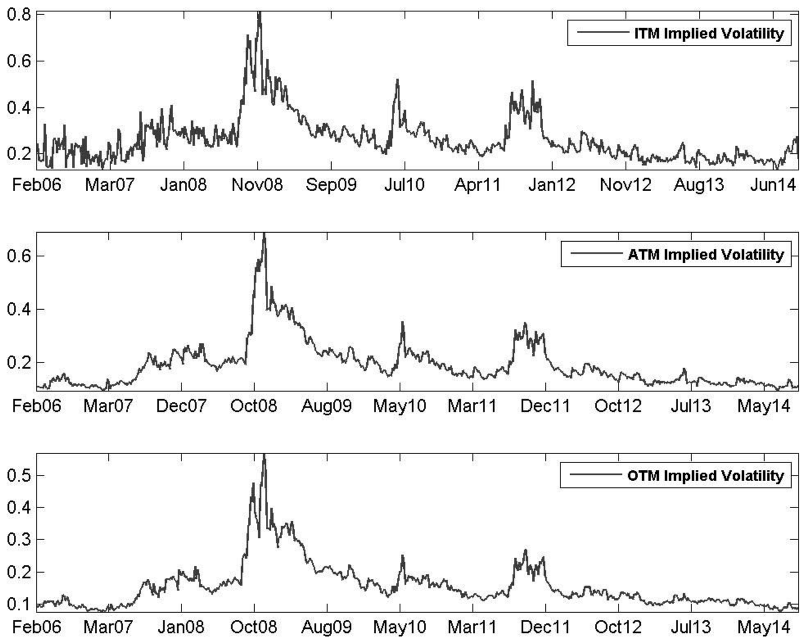
Moments estimated under the physical and the risk-neutral probability measures, associated to the SPX Index from January 24, 2008 to December 19, 2012. Intradaily SPX spots are used to estimate the physical moments, and daily SPX options and futures are used to estimate the risk-neutral moments. The figures plot respectively on a daily basis the levels of the physical and risk-neutral volatility, skewness, and kurtosis, for 60 days time to maturity.

Figure 2.7: Option-Implied Risk Premia of SPX Options



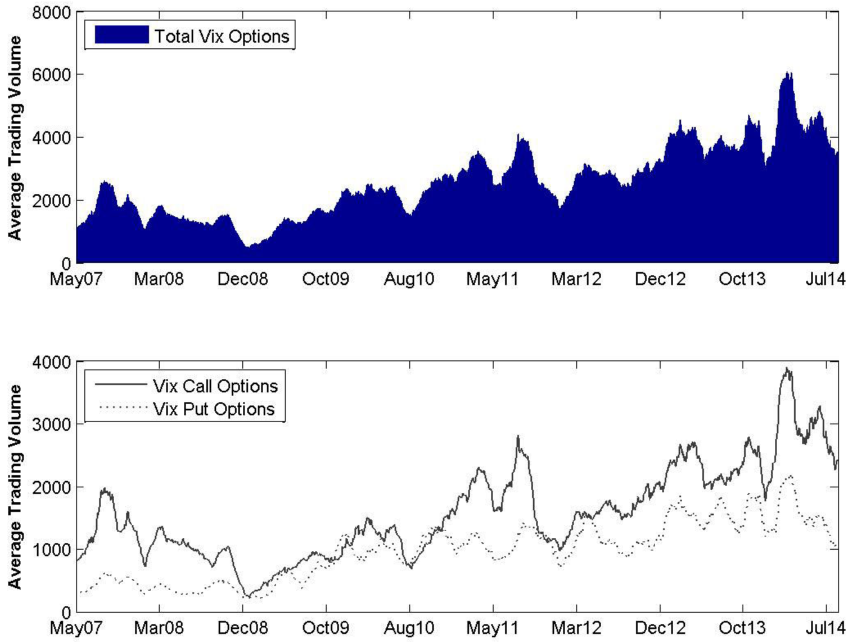
SPX option-implied risk premia for 60 days time to maturity, January 24, 2008 to December 19, 2012. The figures plot respectively on a daily basis the levels of the risk-neutral expected value of the volatility risk premium, skewness risk premium, and kurtosis risk premium, for 60 days time to expiration.

Figure 2.8: Implied Volatility of SPX Options



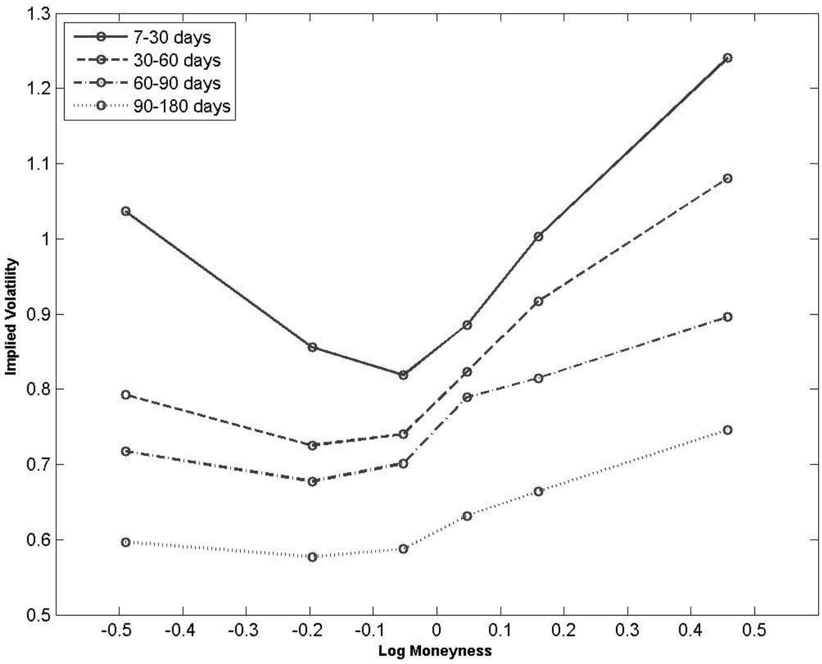
SPX implied volatility for 60 days time to maturity, from February 24, 2006, to August 29, 2014. The figures plot respectively on a daily basis the levels of the implied volatility for ITM, ATM, and OTM SPX options, for 60 days time to expiration.

Figure 2.9: Average Trading Volume of VIX options



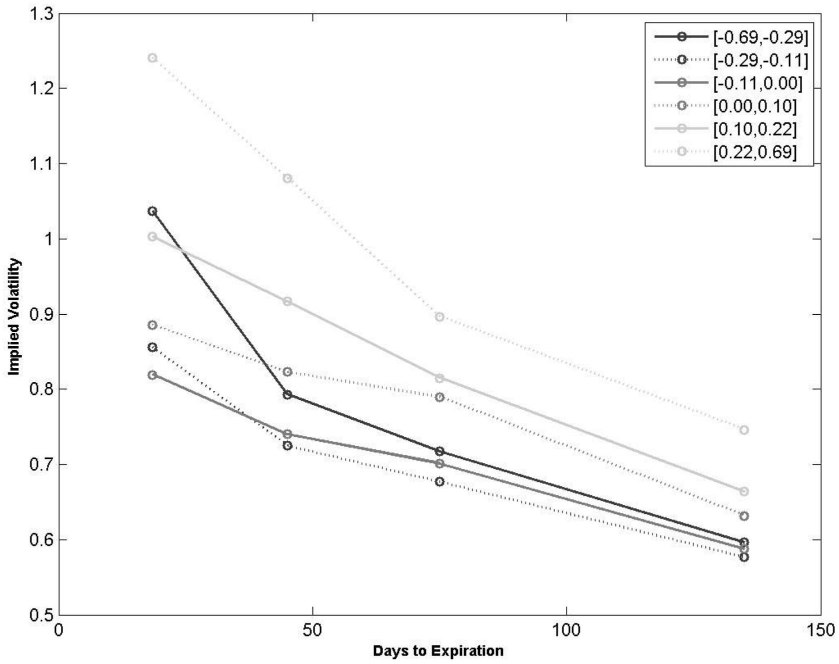
Average daily trading volume of VIX options, from March 26, 2007 to August 29, 2014. The lower panel represents the compared average daily trading volume of VIX call and put options. For clearness, computations are based on the 2-month moving average trading volume.

Figure 2.10: Average Volatility Smile of VIX options



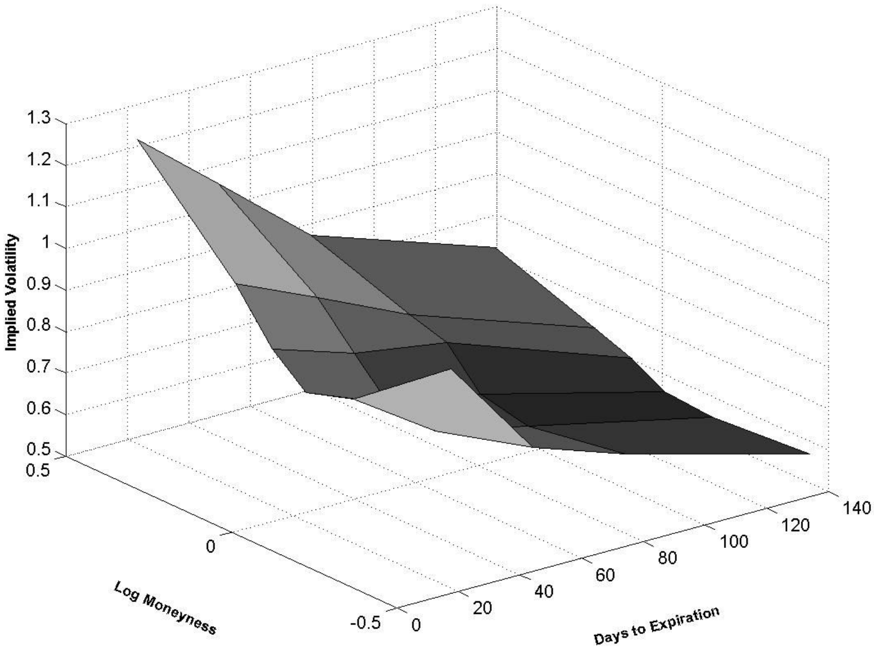
Average volatility smile associated to VIX options for four different maturity buckets, from February 24, 2006 to August 29, 2014. Average implied volatilities are computed in each log moneyness-maturity category, for four maturity buckets (7−30, 30−60, 60−90, 90−180 days), and six log moneyness buckets ([−0.69;−0.29], [−0.29;−0.11], [−0.11;0.00], [0.00;0.10], [0.10;0.22], [0.22;0.69]).

Figure 2.11: Average Term Structure of VIX options



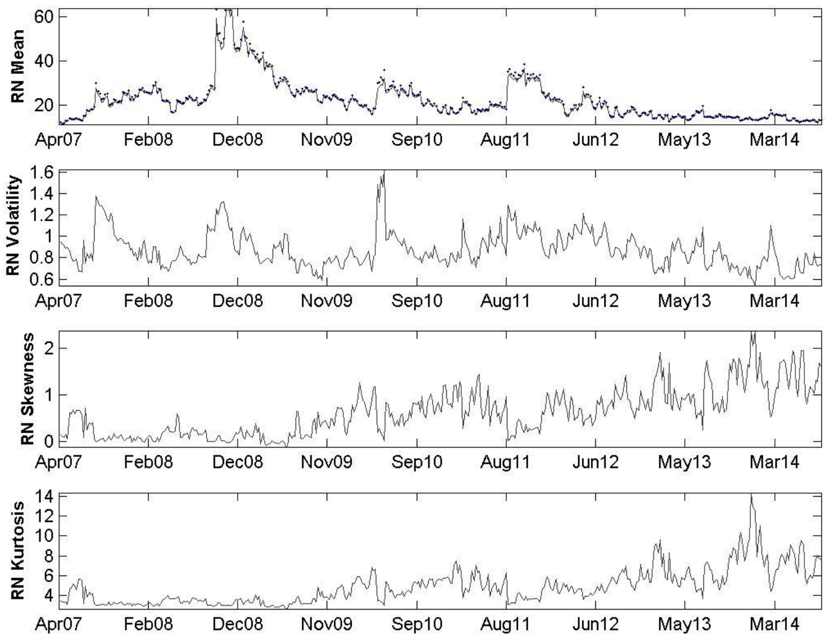
Average term structure associated to VIX options for six different log moneyness buckets, from February 24, 2006 to August 29, 2014. Average implied volatilities are computed in each log moneyness-maturity category, for four maturity buckets (7−30, 30−60, 60−90, 90−180 days), and six log moneyness buckets ([−0.69;−0.29], [−0.29;−0.11], [−0.11;0.00], [0.00;0.10], [0.10;0.22], [0.22;0.69]).

Figure 2.12: Average Implied Volatility Surface of VIX options



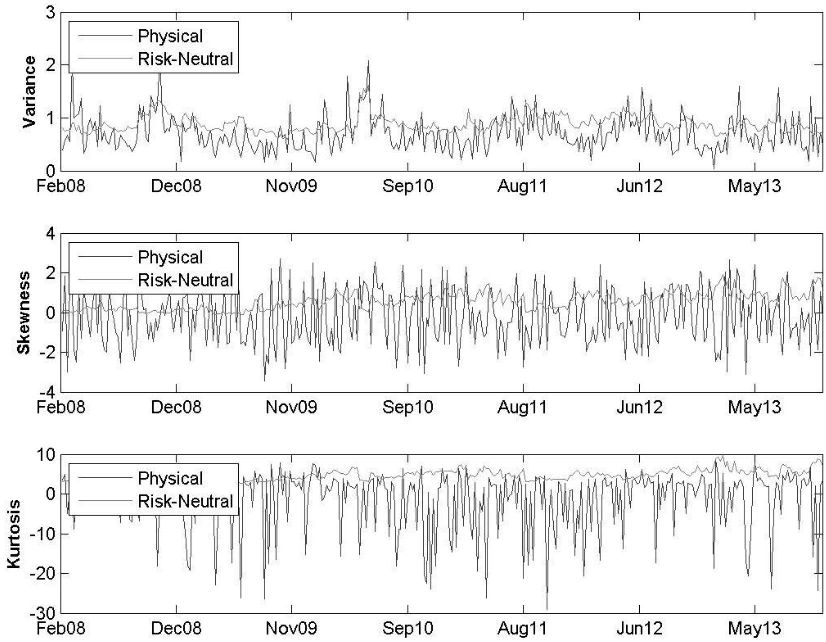
Average implied volatility surface associated to VIX options for six different log moneyness buckets, from February 24, 2006 to August 29, 2014. Average implied volatilities are computed in each log moneyness-maturity category, for four maturity buckets (7−30, 30−60, 60−90, 90−180 days), and six log moneyness buckets ([−0.69;−0.29], [−0.29;−0.11], [−0.11;0.00], [0.00;0.10], [0.10;0.22], [0.22;0.69]).

Figure 2.13: VIX Option-Implied Risk-Neutral Moments



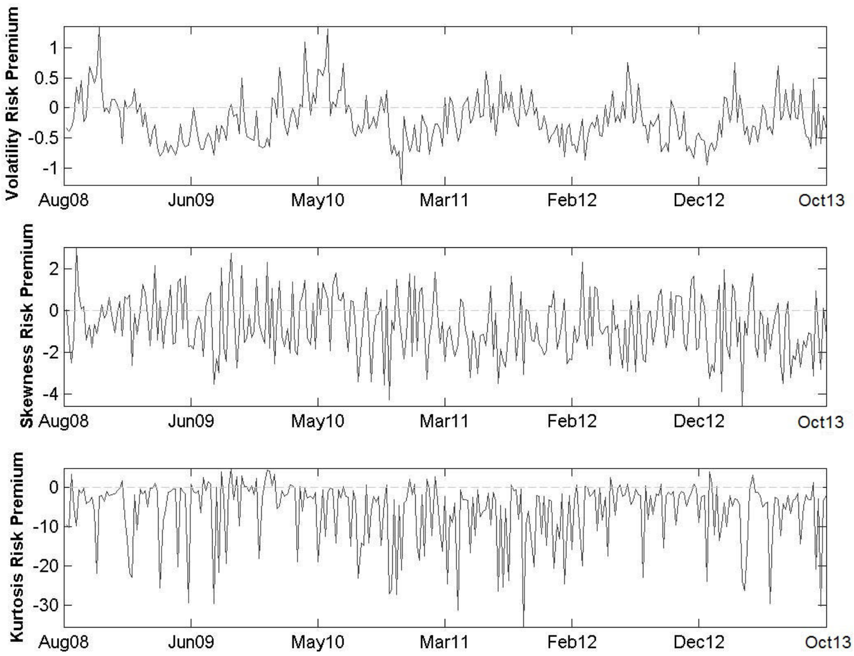
VIX option-implied risk-neutral moments for 30 days time to maturity, from April 13, 2007, to August 29, 2014. The figures plot respectively on a daily basis the levels of the risk-neutral expected value of the mean, the volatility, the skewness, and the kurtosis, for 30 days time to expiration. The blue crosses in the upper figure represent the VIX futures prices for a fixed time to maturity of 30 days.

Figure 2.14: Moments Estimated Under Physical and Risk-Neutral Probability Measures for the VIX



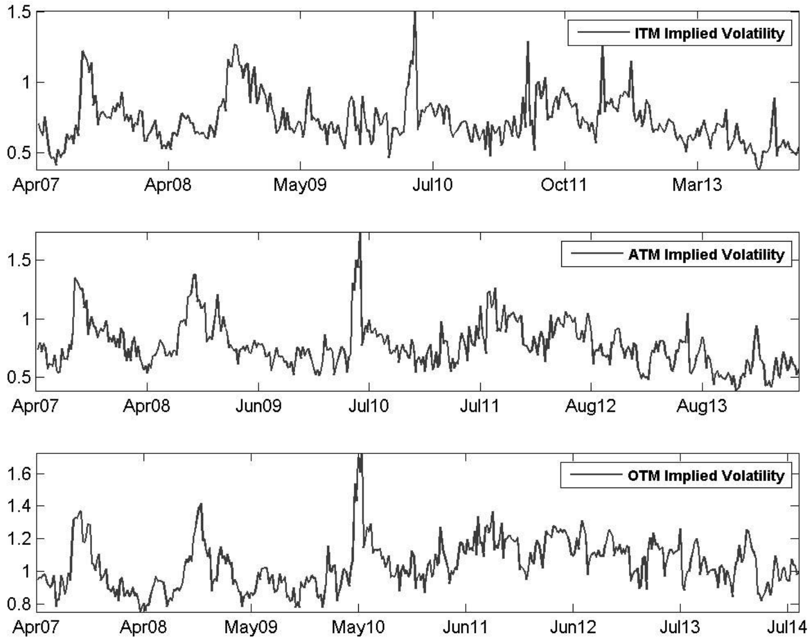
Moments estimated under the physical and risk-neutral probability measures associated to the VIX Index from January 24, 2008 to April 26, 2016. Intradaily VIX spots are used to estimate the physical moments, and daily VIX options and futures are used to estimate the risk-neutral moments. The figures plot respectively on a daily basis the levels of the physical and risk-neutral volatility, skewness, and kurtosis, for 30 days time to expiration.

Figure 2.15: VIX Option-Implied Risk Premia



VIX option-implied risk premia for 30 days time to maturity, from January 24, 2008 to August 29, 2014. The figures plot respectively on a daily basis the levels of the risk-neutral expected value of the second/volatility risk premium, third/skewness risk premium, and fourth/kurtosis risk premium, for 30 days time to expiration.

Figure 2.16: VIX Implied Volatility



VIX implied volatility for 30 days time to maturity, from April 13, 2007, to August 29, 2014. The figures plot respectively on a daily basis the levels of the implied volatility for ITM, ATM, and OTM VIX options, for 30 days time to expiration.

Chapter 3

Do Hedge Funds Hedge? Evidence from Tail Risk Premia Embedded in Options

Do Hedge Funds Hedge? New Evidence from Tail Risk Premia Embedded in Options [1]

[1]This chapter is based on an article jointly written with my PhD supervisor, Pr Serge DAROLLES. The authors are grateful to Vikas Agarwal, Yacine Aït-Sahalia, Eser Arisoy, Matthieu Garcin, Christophe Hurlin, Marcin Kacperczyk, Kevin Mullally, Ilaria Piatti, Todd Prono, Ronnie Sadka, and Fabio Trojani for helpful comments and suggestions. We also appreciate the comments of conference participants at the IX$^{\text{th}}$ French Econometrics Conference, the XIX$^{\text{th}}$ OxMetrics User Conference, and at the II$^{\text{nd}}$ Econometric Research in Finance Conference. Any errors are our own.

This paper deciphers tail risk in hedge funds from option-based dynamic trading strategies. It demonstrates multiple and tradable tail risk premia strategies as measured by pricing discrepancies between real-world and risk-neutral distributions are instrumental determinants in hedge fund performance, in both time-series and cross-section. After controlling for Fung-Hsieh factors, a positive one-standard deviation shock to volatility risk premia is associated with a substantial decline in aggregate hedge fund returns of 25.2% annually. The results particularly evidence hedge funds that significantly load on volatility (kurtosis) risk premia subsequently outperform low-beta funds by nearly 11.7% (8.6%) per year. This finding suggests to what extent hedge fund alpha arises actually from selling crash insurance strategies. Hence, this paper paves the way for reverse engineering the performance of sophisticated hedge fund by replicating implied risk premia strategies.

3.1 Introduction

The recent news[2] of the closure of Eton Park Capital Management, one of the most emblematic figure of the hedge fund industry, came as a shock to the financial community. It has brought the light to the most complicated periods hedge fund industry is experiencing, since liquidations strongly outpaced launches in 2016 according to Hedge Fund Research. In particular, the unexpected outcomes of the Brexit referendum and the U.S. elections have drawn doubts on their ability to manage tail risks. Indeed, hedge funds are often described as *"insurance companies selling earthquake insurance"* (Duarte, Longstaff, and Yu, 2007; Stulz, 2007), since they usually make penny-by-penny gains before incurring substantial losses. Hence, there is scarcely any doubt that hedge funds are particularly sensitive to market crashes, since they replicate short positions on equity index put options (Agarwal and Naik, 2004). Nevertheless, there is only limited literature on sophisticated option-based dynamic trading strategies that secretive hedge funds usually pursue, and how they explain hedge fund performance, risk, and compensation scheme. This research topic has become critical to remunerating hedge fund managers' skills and to understanding to what extent hedge fund alpha actually arises from beta, and specifically from alternative beta and alternative risk premia. Indeed, the highly entrepreneurial hedge fund industry has maintained a strong culture of secret and opaqueness to keep their investment process from fierce competition. Therefore, although U.S. institutional investment managers must report their portfolio holdings on Form 13F to the Securities and Exchange Commission (SEC), section 13(f) securities only concern equities and plain vanilla derivatives. In this way, SEC Form 13F doesn't reflect the highly exotic, out-of-the counter (OTC), and nonlinear payoffs usually hold by hedge fund managers.[3] Hence, we test the following assumptions. First, does crash sensitivity of

[2]Source: Bloomberg, on March 23, 2017.

[3]Our study would not have been possible by exploiting the quarterly holdings as reported by hedge funds to the SEC.

hedge funds arise from the tail risk premia strategies they usually trade? In particular, does tail risk premia investing explain the variation in hedge fund performance, in both the time-series and the cross-section of returns? Second, does crash sensitivity arise from a particular tail risk premia strategy? Specifically, at the hedge fund investment style level, which hedge funds are the most exposed to extreme events? Third, contrary to recent common beliefs, to what extent hedge funds can be simply considered as the *last insurers* against tail risk? In other words, to what extent does hedge fund alpha arise from selling crash insurance?[4]

This paper is the first to explain the time-series and cross-sectional variation in hedge fund performance by tail risk premia. Although existing literature used tail risk measures and simple tail risk strategies, the specificity of the paper rests on: *i/* First, alternative risk premia since divergent swaps are more widely used by hedge fund managers because they fully reflect market price of risk; *ii/* Second, multiple tail risk premia since we decompose implied volatility smirks into three distinct tradable implied risk premia that fully reflect the market price of uncertainty associated to the realization of future extreme events. Tail risk premia usually designate tradable option-based payoffs pricing the market price of tail risk, as measured by the discrepancy between real-world and risk-neutral probability distributions. To that purpose, we derive from risk-neutral distributions and high-frequency data the tradable tail risk premia embedded in VIX options that are widely used by hedge funds, since they become the second most traded contracts at the Chicago Board Options Exchange (CBOE). As evidenced by Al Wakil (2016) in [11], tail risk premia embedded in options incorporate agents' risk attitudes and expectations about higher-order risks, and fully determine the market price of risk embedded in implied volatility surfaces. Therefore, this paper shows tail risk in hedge funds particularly arises from the distinct tradable volatility VRP, skewness SRP, and kurtosis KRP risk premia strategies that hedge funds usually

[4] Furthermore, our study raises the question of the hedge fund manager timing skill ability (alpha) to mitigate tail risk exposure, but this particular issue has been left for future research.

pursue. Thus, they are instrumental determinants in the variation of hedge fund performance, both in the time-series and the cross-section.

This paper finds that exposures of hedge funds to tail risk premia are statistically significant across most investment strategies. Indeed, for the Global Hedge Fund Index, a four-factor model with our tail risk premia has the same explanatory power than the seven-factor model of Fung and Hsieh (2004) over the whole period. In particular, when considering tail events, adjusted R^2 associated to our augmented Fung-Hsieh model significantly increases across all investment styles. First, we exhibit to what extent hedge fund alpha actually arises from selling crash-insurance strategies. After controlling for loadings on Fung-Hsieh seven factors and forming quantile portfolios of cross-sectional hedge fund index returns sorted on the loadings of each of the tail risk premia, we evidence hedge funds that significantly load on volatility (kurtosis) risk premia substantially outperform low-beta funds by nearly 11.7% (8.6%) per year. In other words, when considering cross-sectional exposure to the volatility VRP (kurtosis KRP) risk premium, the high-minus-low portfolio realizes on average an annualized excess return of -11.7% (-8.6%). This finding particularly suggests hedge funds in quantile one are generally selling crash insurance, realizing on average annualized excess returns that compensate for bearing tail risks. Second, we evidence crash sensitivity of hedge funds substantially comes from volatility risk exposure. After controlling for loadings on Fung-Hsieh seven factors, a one-standard deviation increase in the volatility risk premium VRP is associated with a strong decline in aggregate hedge fund returns of 0.10% per day, or 25.20% per year over 2008-2013. Besides, over tail events, a one-standard deviation increase in the volatility risk premium VRP is associated with a substantial decline in aggregate hedge fund returns of 0.32% per day, or 80.64% per year. In particular, at hedge fund investment style level, Relative Value and Equity Hedge are the most negatively exposed strategies to volatility risk, particularly during crises when volatility swap returns are the most profitable. This finding is consistent with literature, since Relative Value hedge funds are usually considered as *the last insurer* against tail risks, executing risk transfer from financial institutions, whereas Equity Hedge managers usually overlay hedge their long positions. Therefore, the associated payoff return profile is equivalent to buying a call option partially hedged by selling realized volatility. Third, at hedge fund investment style level, we evidence Relative Value and Directional hedge funds are the most positively exposed strategies to skewness risk. This result is consistent since they usually profit from underlying's volatility of volatility: Relative Value is commonly long gamma as described by Jaeger (2008) in [71], and trend-followers aim to buying optimally max lookback straddles according to Fung and Hsieh (2001) in [63]. In addition, we show Relative Value hedge funds are not simple insurance sellers, since they partially hedge their volatility risk exposure by buying skewness risk, whereas Global Macro hedge funds are usually negatively exposed to skewness risk. This last result is also consistent since Global Macro managers usually take contrarian bets on tail risks, i.e. selling realized skewness during crises, as their convergence trades are based on mid and long-term macroeconomic trends.

This paper extends the asset pricing literature associated to hedge fund performance for two reasons. First, it provides a new evidence for tail risk in hedge fund performance, showing it is an instrumental determinant in both the time-series and the cross-section of hedge fund returns, and to what extent hedge fund alpha actually arises from selling crash insurance strategies. Specifically, this paper deciphers hedge fund tail risk from multiple option-based dynamic trading strategies, defined as tradable tail risk premia, and decomposed into volatility, skewness, and kurtosis divergent swaps. Hence, we extend among others Asness, Krail, and Liu (2001) in [21], Geman and Kharoubi (2003) in [67], Agarwal and Naik (2004) in [5], Patton (2009) in [96], Agarwal, Ruenzi, and Weigert (2015) in [6], Agarwal, Arisoy and Naik (2017) in [2]. Nevertheless, this vast and recent literature usually deciphers tail risk in hedge funds from nontradable tail risk measures, or from standard and fragmentary option-based strategies. In particular, Agarwal, Green, and Ren (2017) in [4] decompose hedge fund returns into traditional and exotic risk exposures but their approach is parametric using a model horserace, and doesn't reflect the full extent of tail risk premia strategies, since they only consider out-of-the money options and VIX lookback straddles. Second, this paper contributes to the literature by providing a new evidence from multiple tail risk premia strategies that are widely traded by hedge funds, since divergent swap contracts fully incorporate the market price of risk. In particular, we show the volatility, skewness, and kurtosis risk premia, i.e. pricing discrepancies between risk-neutral and physical probability distributions, are distinguishable mimicking portfolios for insurance risk premia, usually harvested by hedge funds. Indeed, this paper evidences most hedge fund styles sell crash insurance, but tail risk exposures across hedge funds are distinct, since they depend on the specific trading strategies hedge fund managers use to arbitrate crash risks. In this sense, we extend the literature among others Aït-Sahalia, Wang, and Yared (2000) in [8], Alireza (2005) in [14], Chang, Zhang, and Zhao (2013) in [50], Bondarenko (2004) in [38], Schneider and Trojani (2015) in [104], and Al Wakil (2016) in [11]. Although this recent literature evidences new profitable divergence trading strategies to monetize compensation for higher-order risks, it generally doesn't explore the issue from hedge fund standpoint.

This paper arises practical implications especially within the industries of hedge funds, asset management, and smart indices. Since we find clear evidence that tradeable tail risk premia explain the variation in hedge fund returns, both in the time-series and the cross-section, this paper paves the way for reverse engineering sophisticated hedge funds by replicating the volatility VRP, skewness SRP, and kurtosis KRP risk premia strategies. Besides, this paper sheds light on the secretive drivers of hedge fund performance, since it disentangles it into real alpha and exotic beta like insurance-crash selling strategies.

The remainder of this paper is organized as follows. Section 1 describes the data used for this study, in particular the data from hedge funds, options, high-frequency trading, and futures.

I document the methodology used to derive the tail risk premia embedded in options, and the Fung-Hsieh trend-following factors on a daily frequency. Section 2 investigates the time-varying exposure of various investment styles to tail risk, while Section 3 extends the analysis to the cross-section of investment styles. Robustness checks are provided in Section 4.

3.2 Literature

There is a vast literature about the instrumental contribution of tail risk in the pricing of hedge fund performance. More generally, this research question falls into the literature investigating the sources of hedge fund performance. In particular, it examines to what extent hedge fund alpha arises actually from market exposure, i.e. beta, and more recently from exotic beta, i.e. alternative beta, since hedge fund managers usually have recourse to sophisticated strategies.

Among others, Geman and Kharoubi (2003) in [67] show hedge funds are particularly sensitive to market distress. Jiang and Kelly (2012) in [68] exhibit a persistently exposure to the left-tail risk, both in the time-series and the cross-section of hedge fund returns. The differential asset pricing relations between dynamic tail and asset prices have been particularly well documented by Gabaix (2011) in [66], Wachter (2012) in [112], Drechsler and Yaron (2011) in [58], and Kelly (2012) in [76], among others. More recently, Agarwal, Ruenzi, and Weigert (2015) in [6] estimate a new tail risk measure from portfolio holdings to investigate the impact of tail risk on hedge fund performance. Specifically, they identify the sources of tail risk in the cross-section of hedge fund returns as tail-sensitive stocks and options.

The underlying assumption postulates hedge funds generally earn extra returns in good states for selling crash insurance, but suffer substantial losses during tail events episodes. Hence, a rich literature has suggested hedge funds are not really hedged, but rather exposed to risk factors, including Asness, Krail, and Liu (2001) in [21], Patton (2009) in [96], and Bali, Brown, and Caglayan (2012) in [22]. Indeed, a vast literature disentangles the sources of hedge fund performance, examining to what extent hedge fund alpha arises from beta. For illustration, the seminal paper of Jensen (1967) in [73] decomposes the mutual fund performance into the market risk exposure and the fund managing skills. This research topic has been particularly determinant since it puts into question the hedge fund compensation scheme, as market exposure (i.e. beta) is cheaper than active performance (i.e. alpha) and manager skills.

Furthermore, the reference papers of Fung and Hsieh (2001, 2004) in [63] and [64] that we extend here decomposes hedge fund performance into a seven-factor model that includes lookback straddles strategies to replicate the dynamics of trend-following hedge funds. Similarly, Mitchell and Pulvino (2001) in [89], and Fung, Hsieh, Naik, and Ramadorai (2008) in [65] evidence hedge fund strategies exhibit option-like payoffs, since systematic risk exposures can be replicated by option-based strategies. Indeed, recent literature examines to what extent hedge fund performance arises by now from complex and exotic beta, i.e. alternative beta, since hedge managers frequently have recourse to sophisticated strategies, using out-of-the counter derivatives and nonlinear payoffs. In particular, Agarwal and Naik (2004) in [5] clearly show left-tail risk in hedge funds arise from replicating short positions on equity index put options, especially for equity-oriented hedge fund styles that bear considerable crash risk. More recently, Agarwal, Arisoy and Naik (2017) in [2] find the uncertainty about equity market volatility is an instrumental determinant of hedge fund performance, both in the cross-section and over time. Specifically, they replicate the volatility of aggregate volatility with tradable lookback straddles on the VIX Index, and evidence a negative risk premium for uncertainty exposure in the cross-section of hedge fund returns.

Subsequently, this paper particularly falls into the recent literature investigating the alternative risk premia strategies usually traded by hedge fund managers to arbitrate the implied volatility smirks. Bondarenko (2004) in [38] estimates the market price of variance risk and clearly evidences that variance swap return is a key determinant in explaining hedge fund performance. Furthermore, he shows hedge fund managers usually sell variance risk, since they are negatively exposed to the variance swap return. More generally, Schneider and Trojani (2015) in [104] propose swap trading strategies studied by Bondarenko (2014) in [39] to capture the isolated tradeable compensation for time-varying risks in higher-order moments. Inspired by a new class of divergence trading strategies in Alireza (2005) in [14], they exploit the inconsistency between the option-implied risk-neutral distribution, i.e. the fair price of moments, and the physical distribution of the underlying asset. Similarly, Chang, Zhang, and Zhao (2013) in [50] introduce new derivative contracts, such as skewness and kurtosis swaps, to trade the forward realized third and fourth cumulants. Using S&P 500 index options from 1996 to 2005, they shed light on persistent time-varying properties of higher-order risk premia, offering a justification for such swap strategies. Less recently, Aït-Sahalia, Wang, and Yared (2000) in [8], and Blaskowitz and Schmidt (2002) in [35] document the profitability of skewness and kurtosis trades, exploiting the discrepancies between risk-neutral densities implied by DAX option prices and the historical state-price densities. Recently, Al Wakil (2016) in [11] evidence implied volatility smirks can be analytically and empirically decomposed into a parsimonious combination of alternative risk premia, mimicking tradable portfolios of option-implied volatility, skewness, and kurtosis risk premia to take bets on the level, slope, and convexity associated to the volatility smirks. These three distinct tail risk premia strategies are usually traded by hedge fund managers to monetize pricing discrepancies reflected in the implied higher-order risks.

Nevertheless, there is limited literature about the detailed tail risk trading strategies usually executed within each hedge fund investment style. Subsequently, we provide thorough understanding from Jaeger (2008) in [71] that sheds light on tail risk strategies implemented by various hedge fund investment styles. In the following paragraphs, we describe three assumptions about hedge funds' exposures that we test in the empirical analysis, in both the time-series and the cross-section of hedge fund returns.

Over 2008-2013, major tail events occured including among others the US Subprime crisis and the Lehman Brothers bankruptcy in 2008, the European sovereign debt crisis in 2010, the US sovereign debt crisis in 2011, and the Taper Tantrum in 2013. Subsequently, the time period had been particularly favourable to insurance-selling strategies, just in the aftermath of tail risk events when central banks envisaged unprecedented bailouts to contain the Global Financial Crisis. Specifically, many hedge funds sold crash insurance when it was expensive in the aftermath of extreme events, earning extra returns over 2008-2013, but making themselves particularly crash sensitive. In particular, Volatility Arbitrage managers increased short positions on expensive realized volatility and went long on cheaper implied volatility, when volatility swap returns were the highest. More globally, hedge funds also usually sold the forward realized third and fourth cumulants, i.e. the skewness and kurtosis via divergent swap contracts. Consequently, we assume hedge funds that substantially loaded on tail risk premia over 2008-2013 should have subsequently outperformed low-beta funds, shedding light to what extent hedge fund alpha arises from selling crash insurance strategies.

At investment style level, although Equity Hedge and in particular Long/Short Equity managers are directional long biased, they partially overlay hedge long positions using short index futures, long OTM puts, and short covered calls. Subsequently, payoff return profile is equivalent to buying a call option hedged by selling realized volatility via volatility swaps. This is particularly true when considering: i/ Equity Market Neutral strategies that try to generate returns uncorrelated to market risk; ii/ Short Selling strategies that partially hedge the short sale bias with OTM call options for example. Similarly, Relative Value strategies are non-equity-directional and they are commonly called arbitrage, spread, or alternative risk premia strategies. In particular, Fixed Income Arbitrage monetizes pricing anomalies associated to global yield curves but fully neutralizes exposure to systematic risk factors. Nevertheless, they are usually considered as the last insurer against market tail risks, as they execute alternative risk transfer strategies from global financial institutions. Hence, payoff return profile is equivalent to shorting put options and realized volatility via volatility swaps. Since available risk premia are small, arbitrageurs have usually recourse to high leverage level, ranging from five to 15 times the asset base, exposing themselves to tail risks. It was particularly true when LTCM (Long Term Capital Management) increased leverage to 30:1 to keep returns targets when assets under management reached USD 4 billion. Consequently, we assume that Relative Value and Equity Hedge strategies are the most negatively exposed hedge fund styles to volatility risk, and we expect this is particularly true risk during crisis periods, when volatility swap returns are the highest.

Considering Relative Value strategies, Fixed Income managers exploit and monetize higher-order risks embedded in the curvature of global yield curves, but neutralize net exposure to yield-curve changes. Spread trades in fixed income usually consist in yield-curve arbitrage, especially butterflies along the yield curve (e.g. long cheap 3-year and 5-year, short expensive 4-year), and related to strong institutional demand. Concerning other Relative Value hedge funds, Convertible and Volatility Arbitrage strategies intensively execute gamma trading to exploit positive convexity of delta hedge ratio function. Specifically, Convertible arbitrageurs are long gamma, i.e. gamma designates delta variation with underlying, since strategies are especially profitable when delta strongly changes, whatever the direction of the move. Since relation between derivative price and underlying price is positively convex, Convertible and Volatility arbitrageurs capture positive gamma by dynamically hedging their delta. Hence, the payoff return profile is equivalent to buying realized skewness, commonly interpreted as an insurance-buying strategy. Concerning Directional strategies, Fung and Hsieh (2001) in [63] show the payoff return profile of Systematic Managed Futures strategies is equivalent to a long straddle position. Indeed, trend-following strategies optimally aim to *buy low and sell high*, corresponding ideally to buying max lookback straddles. Consequently, Directional strategies have usually recourse to buying realized skewness since they generate profit from underlying's volatility of volatility. Alternatively, Global Macro hedge funds identify mid and long-term macro-economic trends, and execute convergence trades to exploit mispricings when market prices substantially deviate from their fair values. Hence, they usually take contrarian bets, maintaining for example a negative exposure to market risk or selling crash risk in crisis periods. Consequently, we assume that Relative Value and Directional strategies are the most positively exposed hedge fund styles to skewness risk, since they are usually buyers of realized skewness via long straddles or positive gamma. Furthermore, contrary to common beliefs, our assumption suggests that Relative Value hedge funds are not completely insurance-sellers strategies, since they partially hedge volatility risk by buying skewness risk. In addition, we assume Global Macro hedge funds can be negatively exposed to skewness risk, since they usually take contrarian bets on tail risk realization in crisis periods.

3.3 Data

Data samples primarily consist in daily hedge fund return provided by HFR and classified into major investment styles; daily VIX options data provided by OptionMetrics, including closing bid-ask mid prices, expiration dates, strike prices, open interest, and trading volume for all the listed maturities - data sample has been filtered following the methodology documented by Al Wakil (2016) in [11]; high-frequency data related to tick-by-tick historical VIX index prices provided by Bloomberg; and a broad range of futures data associated to 15 markets and provided by Datastream and Bloomberg.

3.3.1 Hedge Fund Return Data

Since the time period of our study is restricted by the scarcity of our high-frequency data sample that we use to estimate accurately the tail risk premia, we have recourse to daily hedge fund return data obtained from the HFR Database over the period 2008-2013. HFR indices are constructed to measure the aggregate performance of a wide range of hedge funds grouped by a specific strategy criterion. The hedge fund strategy classification aims to capture pure strategies that reflect the evolution of major trends in the hedge fund industry.

Table 3.1 reports the summary statistics of the hedge fund data sample used for our study. Overall, the sample includes 1,650 daily hedge fund index returns associated to the 5 investment styles and the global index over the period 2008-2013. We restrict the hedge fund data sample to the availability of the tail risk premia that we estimate by using in particular short-length high-frequency data. The average daily hedge fund return is nearly 1 basis point and the daily standard deviation is 0.32%. When comparing the daily returns distribution accross the data sample years over 2008-2013, Panel A exhibits significant disparities between turbulent and calm years. In particular, the returns distribution in 2008 is the only one that shows a negative average daily return of -0.12%. In addition, it exhibits the highest returns dispersion over the time period, including very high and low returns, respectively equal to -2.31% and 2.58%. High returns dispersion is also reflected in the magnitude of the standard deviations: about 0.76% in 2008, when compared to less than 0.28% during 2009-2013. Interestingly, the data sample covers both highly turbulent and calm periods.

[Insert Table 3.1 here]

Panel B associated to Table 3.1 disentangles the descriptive statistics by investment style. We consider the following 5 investment styles: Directional, Equity Hedge, Macro, Merger Arbitrage, and Relative Value (see the Appendix A for further details). Although academic literature points out ambiguity in the hedge fund classification, this strategy classification is currently used by HFR, and by related research, e.g. Patton and Ramadorai (2013) in [97]. Nevertheless, Panel B exhibits some significant disparities between investment styles, what allows to testing the impact of tail risk on hedge fund performance. In particular, Merger Arbitrage and Relative Value exhibits by far the less volatile investment styles (respectively 0.24% and 0.26%) when compared to the Equity Hedge and the Directional strategies (respectively 0.42% and 0.41%). Intuitively, Merger Arbitrage is an event-driven strategy that invests both in long and short positions in the companies that are involved in mergers and acquisitions. Since risk arbitrageurs take risk on deals, Merger Arbitrage strategy typically makes profits when equity markets are up. Hence, it tends to be strongly delta-hedged and lowly volatile. Accordingly, Risk Arbitrage exhibits the highest minimum daily return (-1.25%) over 2008-2013, by contrast with the Directional style (-2.31%) that typically uses trend-following strategies.

[Insert Table 3.1 here]

Figure 3.1 plots the hedge fund investment style performance over the sample period. Overall, they all exhibit negative shocks to highly turbulent and volatile time periods as embodied by the VIX Index that represents the markets fear gauge. Over 2008-2013, major tail events include the US Subprime crisis and the Lehman collapse in 2008, the European sovereign debt crisis in 2010, the US sovereign debt crisis in 2011, and the Taper Tantrum in 2013. Nevertheless, hedge fund investment styles exhibit very distinct dynamics during extreme events. In particular, Merger Arbitrage, and Macro strategies show stronger resilience to the Lehman collapse, the European sovereign debt crisis, and the US sovereign debt crisis, when compared to the Directional and the Equity Hedge strategies. This suggests structural and time-varying tail risk exposures of hedge fund styles.

[Insert Figure 3.1 here]

3.3.2 Fung-Hsieh Factors

In accordance with the hedge fund literature, the paper also includes various factors that appeared to be important in the hedge fund performance, in particular Fung and Hsieh in [62], [63], and [64]. The seven risk factors considered are: MKT-RF and SMB of Fama and French in [61], the change in the term spread (the daily change in the 10-year treasury constant maturity yield), the change in the credit spread (the daily change in Moody's Baa yield less 10-year treasury constant maturity yield), and the Fung-Hsieh trend-following factors, i.e. PTFSBD (bonds), PTFSFX (currencies), and PTFSCOM (commodities). We calculate proxies of the trend-following factors on a daily basis as described by Fung and Hsieh in [64] when modelling the perfect trend-follower strategy (see the Appendix E for further details). When put together, the above seven factors are known in the hedge fund literature as the Fung-Hsieh seven-factor model.

Table 3.3 reports the time-series Pearson pairwise correlations of the Fung-Hsieh seven factors and the tail risk premia. The volatility risk premium VRP appears to be significantly correlated to all the other factors, at least at the 5% level of confidence, whereas the skewness risk premium SRP does not covary significantly with the changes in term spread and credit spread and with the trend-following factors in bonds and commodities. Interestingly, the factor most negatively correlated to both the volatility and the skewness risk premia is the market return, respectively at -0.27% and -0.31%. By construction of the risk premium, this strong negative correlation is consistent with the fact that the realization of tail risks generate stock market crashes. By contrast, the kurtosis risk premium KRP only exhibits significant correlations with the risk premia of volatility VRP and skewness SRP, respectively at 0.19% and 0.21%.

[Insert Table 3.3 here]

3.3.3 Tail Risk Premia

Tail risk premia usually designate disaster insurance that investors pay to hedge against tail events. Intuitively, investors have considerably high marginal utility in such bad states, and they are willing to pay a lot of money to insure extreme event risks. This implies that the market price of tail risk is negative, and thus, tail risk premia generate negative excess returns over long period, but they compensate for paying an insurance by generating income in bad times (see the Appendix D for further details). Since options data reflect agents' attitudes and beliefs towards risk, market option prices incorporate the market price of uncertainty about the realization of future tail risks. Henceforth, tail risks are fully captured by the risk-neutral probability distribution, as market option prices determine the fair price of moments. From an economic motivation, vanished volatility smirk's slope and curvature reduce the risk-neutral probability distribution to the Black-Scholes lognormal distribution, whereas positive slope and curvature make the risk-neutral density respectively more right-skewed and leptokurtic, i.e. more peaked and heavy tailed.

Formally, let $IV_{t,T}$ the implied volatility smirk computed at time t for maturity T associated to moneyness ξ. As specified by Zhang and Xiang (2008) in [117], assume the following three-dimensional representation of the smirk IV approximated by a second-order polynomial function in the log-moneyness ξ. Then:

$$IV_{t,T}(\xi) = \begin{cases} \gamma_{0,t,T} & \text{Black} - \text{Scholes: flat smile} \\ \gamma_{0,t,T}\left[1+\gamma_{1,t,T}\xi\right] & \text{Skewed IV smile} \\ \gamma_{0,t,T}\left[1+\gamma_{1,t,T}\xi+\gamma_{2,t,T}\xi^2\right] & \text{Smirked IV smile} \end{cases} \tag{3.3.3.1}$$

where tail risks are contained in $\gamma_{0,t,T}, \gamma_{1,t,T}, \gamma_{2,t,T}$ that respectively designate the level, the slope, and the curvature effects associated to the shape of the volatility smirk. Subsequently, Zhang and Xiang (2008) in [117] derive asymptotic approximations to clearly evidence the level, slope, and curvature are fully determined by the risk-neutral probability distribution, particularly the risk-neutral volatility $RNVol_{t,T}$, skewness $RNSkew_{t,T}$, and kurtosis $RNKurt_{t,T}$.

$$\begin{aligned}
\gamma_{0,t,T} &\approx \left[1 - \frac{1}{24}\left(RNKurt_{t,T} + 3\right)\right] RNVol_{t,T}, \\
\gamma_{1,t,T} &\approx \frac{1}{6} RNSkew_{t,T}, \\
\gamma_{2,t,T} &\approx \frac{1}{24}\left[RNKurt_{t,T} + 3\right]
\end{aligned} \tag{3.3.3.2}$$

Furthermore, hedge fund managers typically exploit the discrepancies between risk-neutral and real-world distributions, i.e. option-implied risk premia. In particular, Carr and Wu (2009) in [48], and Bollerslev, Tauchen, and Zhou (2009) in [36] define the volatility risk premium as the difference between the realized and the risk- neutral volatilities, i.e. $VRP_{t,t+\tau,T}$ computed at time t over period τ as the difference between the ex post realized return volatility over $[t-\tau, t]$ time interval and the ex ante risk-neutral expectation of the future return volatility over $[t, t+\tau]$, associated to options and futures for the given maturity T.

$$VRP_{t,t+\tau,T} \equiv E_t^P\left[\sigma_{t,t-\tau,T}\right] - E_t^Q\left[\sigma_{t,t+\tau}\right] \tag{3.3.3.3}$$

where $E_t^Q[\cdot]$ and $E_t^P[\cdot]$ denote the time-t conditional expectation operator under respectively risk-neutral Q and physical measure P. Therefore, $E_t^P\left[\sigma_{t,t+\tau,T}\right]$, and $E_t^Q\left[\sigma_{t,t+\tau}\right]$ are the expected values conditional to time t of the volatility realized over time period τ under respectively physical and risk-neutral probability measures. Furthermore, the volatility risk premium $VRP_{t,t+\tau,T}$ multiplied by a notional dollar amount usually defines the payoff at maturity $t+\tau$ of a return volatility swap. Under the no-arbitrage condition, the constant volatility swap rate $SW_{t,t+\tau}$ determined at time t and paid at time $t+\tau$ equals the risk-neutral expectation of the future realized volatility.

In line with Bollerslev et al. (2009) in [36], we estimate $E_t^P\left[\sigma_{t,t+\tau}\right]$ in Equation (3.3.3.3) by the realized volatility $RDVol_t^{(TS)}$ over day t. For the sake of simplicity, we henceforth drop the subscript τ and we denote $VRP_{t,t+T}$ as the volatility risk premium computed at time t over the period $\tau = 1$ day, associated to options and futures for the given maturity T. Similarly, volatility swaps can be theoretically extended to forward contracts written on the third and fourth moments, i.e. swaps respectively associated to skewness risk premium $SRP_{t,T}$ and kurtosis risk premium $KRP_{t,T}$ as follows:

$$Tail\ Risk\ Premia \begin{cases} VRP_{t,T} = RDVol_t^{(TS)} - RNVol_{t,T} & Volatility \\ SRP_{t,T} = RDSkew_t^{(TS)} - RNSkew_{t,T} & Skewness \\ KRP_{t,T} = RDKurt_t^{(TS)} - RNKurt_{t,T} & Kurtosis \end{cases} \tag{3.3.3.4}$$

where risk-neutral moments $RNVol_{t,T}$, $RNSkew_{t,T}$, and $RNKurt_{t,T}$ are extracted from market option prices by using the model-free approach of Bakshi, Kapadia, and Madan (2003) in [24] (see the Appendix B for further details). Realized volatility $RDVol_t^{(TS)}$ designates the Aït-Sahalia, Mykland and Zhang (2005) Two-Scales Realized Volatility measure introduced in [118] that uses subsampling, averaging, and bias correction for the market microstructure noise. This bias-corrected realized measure is then extended to higher moments $RDSkew_t^{(TS)}$ and $RDKurt_t^{(TS)}$ (see the Appendix B for further details).

The three tail risk premia $VRP_{t,T}$, $SRP_{t,T}$, and $KRP_{t,T}$ have been broadly documented by recent literature as mimicking portfolios that harvest the tradeable compensation for time-varying risks in higher-order moments. By construction, these risk premia are generally negative because risk-neutral volatility, skewness, and kurtosis are generally higher than the associated realized moments, since it contains investor's expectations for future non-realized tail risks. Hence, $VRP_{t,T}$, $SRP_{t,T}$, and $KRP_{t,T}$ are similar to insurance-buying strategies that compensate for bearing tail risks by generating positive payoffs in crisis periods. Aït-Sahalia, Wang and Yared (2001) in [8], Blaskowitz and Schmidt (2002) in [35], Alireza (2005) in [14], Chang, Zhang and Zhao (2013) in [50], Bondarenko (2014) in [39], and Schneider and Trojani (2015) in [104] among others, investigate this new class of divergence trading strategies to exploit the discrepancy between risk-neutral and real-world distributions of the underlying asset.

Specifically, Blaskowitz and Schmidt (2002) in [35] and Alireza (2005) in [14] arbitrate implied volatility smile for higher-order moments. Let the option-implied risk-neutral distribution be more skewed to the left than the real distribution of the underlying asset. Then, OTM put options may be relatively overpriced with respect to the OTM call options, since the risk-neutral distribution should reflect the fair price of skewness. Subsequently, trading the skewness consists in selling the OTM put option $P\left(S_t, K_C\right)$ and buying the OTM call option $C\left(S_t, K_C\right)$, associated to the underlying asset price S_t and the strike price K_C. The skewness trade then equals to selling realized skewness. Therefore, the payoff value Π_{Skew} associated to the corresponding delta-vega-neutral portfolio is

$$\Pi_{Skew} = C\left(S_t, K_C\right) - \frac{\nu_C}{\nu_P} P\left(S_t, K_C\right) - \left[\Delta_C - \frac{\nu_C}{\nu_P}\Delta_P\right] S_t \tag{3.3.3.5}$$

where (Δ_C, Δ_P) and (ν_C, ν_P) respectively designate the delta and vega of call and put options. Furthermore, skewness trades are commonly interpreted as long risk reversals or long synthetic stocks. Similarly, let the risk-neutral distribution has a sharper peak and fatter tails than the real-world distribution of the underlying asset. Then, OTM options may be relatively overpriced with respect to ATM options. Subsequently, trading the kurtosis consists in selling the OTM call $C(S_t, K_3)$ and put options $P(S_t, K_1)$, and buying the ATM call $C(S_t, K_2)$ and put option $P(S_t, K_2)$ for strike prices $K_1 > K_2 > K_3$. The kurtosis trade then equals to selling realized kurtosis. Hence, the payoff value Π_{Kurt} associated to the corresponding delta-vega-neutral portfolio can be interpreted as a long modified butterfly

$$\begin{aligned}\Pi_{Kurt} \quad = \quad & C(S_t, K_2) + \frac{\nu_{C_2}}{\nu_{P_2}} P(S_t, K_2) \\ & - C(S_t, K_3) - \frac{\nu_{C_3}}{\nu_{P_1}} P(S_t, K_1) \\ & - \left[\Delta_{C_3} + \frac{\nu_{C_3}}{\nu_{P_1}} \Delta_{P_1} - \Delta_{C_2} - \frac{\nu_{C_2}}{\nu_{P_2}} \Delta_{P_2} \right] S_t \end{aligned} \tag{3.3.3.6}$$

Skewness and kurtosis trades are usually interpreted as insurance-selling strategies, whereas mirror trades are equivalent to buying respectively realized skewness and kurtosis. Typically, hedge fund managers widely trade insurance strategies like volatility, skewness, and kurtosis swaps to arbitrate higher-order risks. To that purpose, VIX options have been widely traded to trade portfolio insurance, since they are European options written on VIX futures. Figure 3.2 plots the trading volume of VIX options and its decomposition into call and put options. As observed, they have become strongly popular since the Lehman Brothers crisis, being by now the second most liquid option contracts listed on CBOE and CFE, as they provide a purer exposure to tail risks than S&P 500 options. More specifically, VIX call options are strongly more actively traded than put options since hedge fund managers usually traded OTM and deep OTM call options to pay tail risk insurance. As documented by Al Wakil (2016) in [11], we derive the tail risk premia $VRP_{t,T}$, $SRP_{t,T}$, and $KRP_{t,T}$ from VIX options on a daily frequency and for different maturities over 2008-2013. We use high-frequency data provided by Bloomberg to estimate the real-world distribution, whereas OptionMetrics provides options data to estimate the risk-neutral distribution.

[Insert Figure 3.2 here]

Figure 3.4 plots the time series of VIX tail risk premia respectively associated to volatility, skewness, and kurtosis, for 30-days time to maturities, over 2008-2013. Overall, the three tail risk premia are generally negative, since favourable states of nature correspond to extreme events. In particular, brief episodes of positive volatility risk premia consistently correspond to the US Subprime crisis and the Lehman collapse in 2008, the European sovereign debt crisis in 2010, the US sovereign debt crisis in 2011, and the Taper Tantrum in 2013.

[Insert Figure 3.4 here]

As observed in Figure 3.3, turmoil periods are associated to realized volatility peaks. Table 3.2 reports summary statistics for tail risk premia. Student t- stats indicate that average risk premia are clearly all significantly negative at the 1% confidence level across the 30, the 60, and the 120-days time to maturities.

[Insert Figure 3.3 here]

[Insert Table 3.2 here]

3.4 Hedge Fund Exposure to Tail Risk Across Time

We investigate the contribution of tail risk premia to the performance of hedge fund investment styles, after controlling for the loadings on the Fung and Hsieh seven factors. Specifically, we perform time-series Ordinary Least Squares (OLS) regressions over the whole 2008-2013 period to evaluate crash sensitivity at hedge fund investment style.

For each hedge fund investment style, Table 3.4 summarizes the results of two time-series OLS regressions: $i/$ a first regression of style index returns on the market factor $MKT-RF$ and the three tail risk premia VRP, SRP, and KRP; and $ii/$ a second regression while adding the Fung-Hsieh seven factors. Overall, t-stats and p-values suggest that Global index is significantly loaded on two of the three tail risk premia after controlling for the loadings on the Fung-Hsieh seven factors. Indeed, a one-standard deviation increase in the volatility risk premium VRP is associated with a strong decline in aggregate hedge fund returns of 0.1% per day, or 25.2% per year. This effect has a statistical significance at the 1% level of confidence (t-stat of -3.40). Comparing regression $i/$ with $ii/$ shows that the four factor model with tail risk factors has the same explanatory power than the Fung-Hsieh seven factor model (adjusted R^2 at 0.41).

[Insert Table 3.4 here]

When analyzing investment styles, we find that the exposure of hedge funds to tail risk is statistically significant across most investment strategies. More specifically, four of the five investment styles (Relative Value, Directional, Equity Hedge, Macro) present a significant loading on at least one of the three tail risk premia, and for at least one of the regression specifications. Only one investment style (Merger Arbitrage) exhibits a statistically insignificant tail risk loading. Precisely, the four styles exhibit all negative and significant loading on the volatility risk premium VRP, at least at the 10% level of confidence. In terms of magnitudes, returns associated to hedge funds that pursue long-short equity strategies, e.g. Equity Hedge (1% level of confidence, t-stat of -2.63) and Relative Value (5% level of confidence, t-stat of -2.25), are especially sensitive to tail risk shocks. A one-standard deviation increase in the volatility risk premium VRP is associated with a drop in returns of respectively 0.13% and 0.10% per day for hedge funds investing in Equity Hedge and Relative Value. This finding is consistent with literature since Relative Value hedge funds are usually considered as the last insurer against tail risks, executing risk transfer from financial institutions, whereas Equity Hedge hedge strategies usually overlay hedge long positions. Therefore, their payoff return profile is equivalent to buying a call option hedged by selling realized volatility.

Therefore, concerning the skewness risk premium SRP, Relative Value (t-stat of 3.47) and Directional (t-stat of 2.68) styles exhibit both the most positive and significant loadings at the 1% level of confidence, whereas Global Macro (t-stat of -2.5) presents rather a negative and significant loading. In other words, Relative Value and Directional hedge funds are the most positively exposed to skewness risk, whereas Global Macro hedge funds are usually negatively exposed to skewness risk. Furthermore, our results show Relative Value hedge funds are not completely insurance sellers, since they partially hedge volatility risk exposure by buying skewness risk. From the economic intuition, these findings generally make sense, since Relative Value and Directional strategies usually profit from underlying's volatility of volatility. As described by Jaeger (2008) in [71], Relative Value style is commonly long gamma, i.e. trading gamma to adjust the delta hedge ratio, whereas trend-followers aim to optimally buying max lookback straddles as documented by Fung and Hsieh (2001) in [63]. Therefore, their payoff return profile is equivalent to buying realized skewness. Alternatively, Global Macro managers usually take contrarian bets, especially on tail risks by selling realized skewness during crises, since they base their convergence trades on mid and long-term macro-trends. For illustration, the loading on the market excess returns $MKT - RF$ associated to Global Macro style is negative (but then nonsignificant in the $ii/$-regression), whereas all the other investment strategies exhibit significant and positive loadings on the market excess returns $MKT - RF$. Finally, none of the hedge fund styles present a significant loading on the kurtosis risk premium KRP.

The findings are robust[5] to the expiration time used to calculate the tail risk premia VRP, SRP, and KRP. Findings clearly show that tail risk loading varies both across investment styles and time, and henceforth, tail risk premia are an instrumental pricing factor in the universe of

[5] Additional tests have been also performed with tail risk premia estimated for the 60, 90, and 120-days to maturity and can be provided on demand.

hedge funds. More precisely, our findings clearly evidence that crash sensitivity associated to hedge funds mainly arises from volatility risk exposure. Indeed, Relative Value and Equity Hedge are the most negatively exposed strategies to volatility risk, even if Relative Value hedge funds partially hedge their volatility risk exposure by buying skewness risk. Conversely, Global Macro managers are the only hedge funds significantly negatively exposed to skewness risk.

3.5 Tail Risk in the Cross-Section of Hedge Funds

In the previous section, we investigate embedded tail risk across time at the hedge fund investment style level. Consistent with the hedge fund literature, the evidence shows that hedge fund returns, in particular for equity-oriented investment strategies, are generally sensitive to tail risk shocks across times, after controlling for commonly used hedge fund risk factors. This finding suggests tail risk premia are instrumental determinants of hedge fund performance. In what follows, we provide now cross-sectional evidence that supports this theory.

Each day, three hedge fund portfolios are formed by sorting the five hedge fund investment styles on their exposures to tail risk. Specifically, at the end of each day, I perform time-series monthly rolling regressions of excess returns associated to hedge fund investment strategies on the market return and on respectively each of the three tail risk premia VRP, SRP, and KRP. In the daily estimation window, the tail risk loading of each investment style is calculated with at least 18 days of data. Nevertheless, the results are robust to running longer rolling windows,and to using longer maturities for the tail risk premia. Therefore, the five investment strategies are sorted into three quantile portfolios based on their tail risk factor loadings. The Fung-Hsieh alpha then designates the intercept associated to the regression of the daily tail-risk beta portfolio excess returns on the seven Fung-Hsieh hedge fund factors. This methodology is consistent with the hedge fund literature about tail risk, including Jiang and Kelly (2012) in [68] among many others, but it investigates tail risk in the cross-section of returns of investment styles rather than of individual hedge funds.

Table 3.5 reports the performance of the three quantile portfolios sorted on hedge fund tail betas, respectively associated to the three tail risk premia VRP, SRP, and KRP. For each of the tail risk factors, it summarizes the average daily tail risk betas, the average annualized excess returns, and the Fung-Hsieh seven factor alpha associated to the three quantile portfolios and to the high minus low portfolio, defined as the return spread between the high-tail-loading and the low-tail-loading portfolios of hedge funds. Results show significant returns dispersion in the investment styles captured by their betas on the tail risk premia.

[Insert Table 3.5 here]

Considering cross-sectional exposure to the volatility (Panel A) risk premia VRP, Table 3.5 shows the low-tail-loading portfolio (average beta of -0.216%) of hedge funds has the highest average annualized excess return (0.57%). Inversely, the high-tail-loading portfolio (average beta of 0.091%) of hedge funds has the lowest average daily excess return (-11.13%). Precisely, hedge funds in quantiles one and two have negative tail risk loadings, respectively of -0.216% and -0.026%. Intuitively, these hedge funds have on average negative returns when tail risk is high, and therefore they are particularly sensitive to tail risk shocks. This suggests that funds in quantiles one and two are generally selling crash insurance, realizing on average higher annualized excess returns of respectively 0.57% and 0.51% that compensate for bearing volatility risk. Inversely, hedge funds in the last quantile have high and positive volatility risk loadings, on average of 0.091%, and are thus generally buyers of crash insurance. Hence, they earn on average significantly lower excess returns of -11.13%. The high-minus-low portfolio realizes on average an annualized return spread of -11.7%, that is significant at the 5% level of confidence, with a t-statistic of -2.38.

Similarly, considering cross-sectional exposure to the kurtosis (Panel C) risk premia KRP, Table 3.5 shows that the low-tail-loading portfolio (average beta of -0.003%) of hedge funds has the highest average annualized excess return (0.28%). Inversely, the high-tail-loading portfolio (average beta of 0.009%) of hedge funds has the lowest average daily excess return (-8.32%). Precisely, hedge funds in quantile one has negative tail risk loadings (-0.003%), and they tend to realize on average negative returns when tail risk is high, and therefore they are particularly sensitive to tail risk shocks. This suggests that funds in quantile one are generally selling crash insurance, realizing on average higher annualized excess returns of 0.28% that compensate for bearing kurtosis risk. Inversely, hedge funds in the second and last quantile have high and positive volatility risk loadings, on average of respectively 0.005% and 0.009%, and are thus generally buyers of crash insurance. Hence, they earn on average significantly lower excess returns of respectively -1.85% and -8.32%. The high-minus-low portfolio realizes on average an annualized return spread of -8.6%, significant at the 5% level of confidence, with a t- statistic of -1.86.

The findings are robust[6] to the expiration time used to calculate the tail risk premia VRP and KRP. Nevertheless, when considering cross-sectional exposure to the skewness (Panel B) risk premia SRP, Table 3.5 shows non-significant extra returns for the high-minus-low portfolio. The average annualized excess return of 0.8% (with a t-statistic of 0.15) suggests the return spread between insurance hedgers and sellers is negligible over the post-Lehman period 2008-2013, and when considering the cross-section of hedge funds at the investment styles level.

Finally, we find clear evidence that hedge funds that significantly load on volatility (kurtosis) risk premia substantially outperform low-beta funds by nearly 11.7% (8.6%) per year. This result sheds light to what extent hedge fund alpha arises actually from selling crash insurance strategies, in particular selling realized volatility and kurtosis via divergent swaps.

[6] Additional tests have been also performed with tail risk premia estimated for the 60, 90, and 120-days to maturity and can be provided on demand.

3.6 Robustness Checks

In this section, we perform various robustness checks[7] to ensure our results are consistent. Specifically, we investigate the contribution of tail risk premia to the performance of hedge fund investment styles, after controlling for the loadings on the Fung and Hsieh seven factors, over tail events to disentangle the time-varying and structural exposures of hedge fund styles.

3.6.1 Tail Risk Periods

Since crises episodes are violent but rare, scarcity implies that above results could be dominated by long non-crisis periods. Hence, we investigate the time-varying and structural exposure of hedge funds to tail risks by considering only financial turmoil. Specifically, we use the VIX *fear gauge* to time-slice the initial data sample by identifying peaks and bottoms associated to realized tail risks. Then, time-sliced sample period includes the US Subprime crisis and the Lehman collapse in 2008, the European sovereign debt crisis in 2010, the US sovereign debt crisis in 2011, and the Taper Tantrum in 2013, providing 71 observation points for data analysis. As expected, Table 3.6 exhibits much stronger adjusted R^2, suggesting that tail risk premia are particularly instrumental in pricing time-varying hedge fund performance. Considering the first regression of style index returns on the market factor $MKT - RF$ and the three tail risk premia VRP, SRP, and KRP, adjusted R^2 increases for all the investment styles, especially for Macro (from 0.03 to 0.19), Relative Value (from 0.10 to 0.18), and Merger Arbitrage (from 0.35 to 0.54); and to a lesser extent, for Equity Hedge (from 0.53 to 0.60) and Directional (from 0.49 to 0.58). Since the loadings on intercept and market factor remain generally unchanged from regression $i/$ to $ii/$, adding tail risk factors appears instrumental to pricing the hedge fund performance.

[7] We are looking for controlling hedge fund risk exposures for other nonlinear risks, e.g. those related to correlation risk by Buraschi, Kosowski, and Trojani (2014) in [45], tail risk by Agarwal, Green, and Ren (2017) in [4], liquidity risk by Sadka (2010) in [102].

[Insert Table 3.6 here]

Overall, t-stats and p-values suggest that Global index is significantly loaded on one tail risk premia after controlling for the loadings on the Fung-Hsieh seven factors. Indeed, a one-standard deviation increase in the volatility risk premium VRP is associated with a considerable drop in aggregate hedge fund returns of 0.32% per day, or 80.64% per year. This effect has a statistical significance at the 1% level of confidence (t-stat of -3.01). Comparing regression $i/$ with $ii/$, the four factor model with tail risk factors has again the same explanatory power than the Fung-Hsieh seven factor model (adjusted R^2 at 0.46). More precisely, t-stats and p-values indicate that now all of the five style indexes present a significant loading on at least one of the three tail risk premia, and for at least one of the regression specifications. Precisely, three investment styles exhibit negative and significant loading on the volatility risk premium VRP, at least at the 10% level of confidence. In terms of magnitudes, it is especially true for Equity Hedge (5% level of confidence, t-stat of -2.46) and Relative Value (5% level of confidence, t-stat of -2.52) that significantly load on tail risk over 2008-2013 period. Precisely, a one-standard deviation increase in the volatility risk premium VRP is associated with a considerable drop in returns of respectively 0.39% and 0.42% per day for hedge funds investing in Equity Hedge and Relative Value. This finding particularly exhibit that Equity Hedge and Relative Value are the most negatively exposed strategies to volatility risk, especially during crises when volatility swap returns are the highest. In other words, they are particularly crash sensitive since they profit from selling realized volatility when it is considered as expensive during crises.

Then, considering the skewness risk premium SRP, Relative Value (t-stat of 2.02) and Directional (t-stat of 1.91) styles exhibit again both a positive and significant loading. This finding exhibits they usually buy realized skewness since they profit from underlying's volatility of volatility. Indeed, Relative Value managers are long gamma, and trend-followers aim to optimally buying max lookback straddles. Their associated payoff return profile is then equivalent to buying realized skewness. Global Macro (t-stat of -1.89) presents again the only negative and now significant loading on skewness risk. This result particularly validates that Global Macro hedge funds are usually selling realized skewness during crises since they base their convergence trades on long-term macro-trends. Interestingly, the Merger Arbitrage investment style presents by now a positive and significant loading on the kurtosis risk premium KRP (t-stat of 2.28). Figure 3.5 illustrates the sensitivities of hedge fund investment styles to the Fung and Hsieh seven factors (without market factor) and the three tail risk premia, where sensitivities are estimated by absolute values of t-statistics after controlling for the market factor loading.

[Insert Figure 3.5 here]

Intuitively, the tail risk embedded in the five hedge fund investment strategies makes sense over crisis periods. All the five investment strategies exhibit generally significant negative loadings on the volatility risk premium VRP, and significant positive loadings on the market excess returns $MKT - RF$.[8] These findings are consistent with Agarwal and Naik (2004) that evidence equity-oriented hedge fund styles generally bear considerable left-tail risk, incurring considerable losses in equity market downward moves. Furthermore, they are also consistent with our previous results: *i/* Relative Value and Equity Hedge are the most negatively exposed strategies to volatility risk, since they are usually volatility sellers; *ii/* Relative Value and Directional are the most positively exposed strategies to skewness risk, and Relative Value managers partially hedge their volatility risk exposure by buying realized skewness; And *iii/* Global Macro hedge funds are usually negatively exposed to skewness risk to take contrarian bets.

[8]Similarly, the loading on the market excess returns $MKT - RF$ associated to Global Macro style is negative but then nonsignificant in the *ii/*-regression. In additional tests with tail risk premia estimated for the 60, 90, and 120-days to maturity, the loading becomes significantly positive.

3.7 Conclusion

This paper has been motivated by filling the gap in the hedge funds and the asset pricing literature. Although there is scarcely any doubt that hedge funds are particularly sensitive to market crashes, there is limited literature on sophisticated option-based dynamic trading strategies that hedge funds usually pursue, and how they explain hedge fund performance. In particular, I address the following assumptions. First, does tail risk in hedge funds come from their tail risk premia strategies? Second, does tail risk premia investing explain the variation in hedge fund performance, in both the time-series and the cross-section? Finally, to what extent does hedge fund alpha arise from managerial skill or from actually selling crash insurance? In particular, do hedge funds can actively time tail risk before market crashes? To our knowledge, this paper is the first to explain the time-series and cross-sectional variation in hedge fund performance by tail risk premia dynamic strategies. Therefore, this paper shows tail risk in hedge funds particularly arises from the tradeable volatility VRP, skewness SRP, and kurtosis KRP risk premia strategies that hedge funds pursue. Thus, they are instrumental determinants in the variation of hedge fund performance, both in the time-series and the cross-section.

This paper finds that exposures of hedge funds to tail risk premia are statistically significant across most investment strategies. Indeed, for the Global Hedge Fund Index, a four-factor model with our tail risk premia has the same explanatory power than the seven-factor model of Fung and Hsieh (2004) over the whole period. In particular, when considering tail events, adjusted R^2 associated to our augmented Fung-Hsieh model significantly increases across all investment styles. First, we exhibit to what extent hedge fund alpha actually arises from selling crash-insurance strategies. After controlling for loadings on Fung-Hsieh seven factors and forming quantile portfolios of cross-sectional hedge fund returns sorted on tail risk loadings, we evidence hedge funds that significantly load on volatility (kurtosis) risk premia substantially outperform low-beta funds by nearly 11.7% (8.6%) per year. In other words, when considering cross-sectional exposure to the volatility VRP (kurtosis KRP) risk premium, the high-minus-low portfolio realizes on average an annualized return spread of -11.7% (-8.6%). This finding particularly suggests hedge funds in quantile one are generally selling crash insurance, realizing on average annualized excess returns that compensate for bearing tail risks. Second, we show crash sensitivity of hedge funds mainly comes from volatility risk exposure. After controlling for loadings on Fung-Hsieh seven factors, a one-standard deviation increase in the volatility risk premium VRP is associated with a strong decline in aggregate hedge fund returns of 0.10% per day, or 25.20% per year over 2008-2013. Over tail events, a one-standard deviation increase in the volatility risk premium VRP is associated with a substantial decline in aggregate hedge fund returns of 0.32% per day, or 80.64% per year. In particular, at HF investment style level, Relative Value and Equity Hedge are the most negatively exposed strategies to volatility risk, particularly during crises when volatility swap returns are the highest. This finding is consistent with literature, since Relative Value hedge funds are usually considered as the last insurer against tail risks, executing risk transfer from financial institutions, whereas Equity Hedge hedge funds usually overlay hedge long positions. Therefore, payoff return profile is equivalent to buying a call option partially hedged by selling realized volatility. Third, at hedge fund investment style level, we evidence Relative Value and Directional hedge funds are the most positively exposed strategies to skewness risk. This result is consistent since they usually profit from underlying's volatility of volatility: Relative Value is commonly long gamma as described by Jaeger (2008) in [71], and trend-followers aim to buying optimally max lookback straddles according Fung and Hsieh (2001) in [63]. Therefore, we show Relative Value hedge funds are not completely insurance sellers, since they partially hedge volatility risk by buying skewness risk, whereas Global Macro hedge funds are usually negatively exposed to skewness risk. This result is also consistent since Global Macro managers usually take contrarian bets on tail risks, i.e. selling realized skewness during crises, as their convergence trades are based on long-term macroeconomic trends.

This paper extends the asset pricing literature of hedge fund performance for two reasons. First, it extends Agarwal and Naik (2004) in [5], and Agarwal, Ruenzi, and Weigert (2015) in [6] that evidence tail risk in hedge funds arise from dynamic strategies replicating short positions in equity index put options. Second, this paper sheds light on Agarwal, Bakshi, and Huij (2010) in [3] that evidence hedge funds are particularly sensitive to market crashes through their exposures to the S&P 500 risk-neutral volatility, skewness, and kurtosis. To that extent, this paper clearly shows tail risk premia strategies that trade the higher-order risks embedded in options are an instrumental determinant in the performance of hedge funds.

This paper arises practical implications especially within the industries of hedge funds, asset management, and smart indices. Since we find clear evidence that tradeable tail risk premia explain the variation in hedge fund returns, both in the time-series and the cross-section, this paper paves the way for reverse engineering sophisticated hedge funds by replicating the volatility VRP, skewness SRP, and kurtosis KRP risk premia strategies. Besides, this paper sheds light on the secretive drivers of hedge fund performance, since it disentangles it into real alpha and alternative beta like insurance-crash selling strategies.

Nevertheless, due to the lack of data about hedge funds, we investigated tail risk sensitivity only at the hedge fund investment style on a daily basis, leaving for future research examination at the individual hedge fund level. Besides, our further research will focus on estimating a new statistical measure to evaluate the timing ability and managerial skills of hedge fund managers to mitigate tail risk exposure. Finally, we have left for future research the question whether tail risk premia can predict future hedge fund returns.

3.8 Appendix

3.8.1 Investment Styles

HFR indices are constructed to track the aggregate performance of a wide range of hedge fund managers grouped by a specific strategy criterion. The hedge fund strategy classification aims to capture pure strategies that reflect the evolution of major trends in the hedge fund industry. The 5 major investment styles used in HFR are based as follows on the definitions provided below by HFR.

- Directional:

  This investment strategy employs quantitative techniques to forecast future price movements and relations between securities. They include in particular Factor-based and Statistical Arbitrage/Trading strategies. Factor-based strategies are based on the systematic analysis of common relationships between securities, while Statistical Arbitrage/Trading strategies consist in exploiting pricing anomalies inherent in security prices. Directional strategies typically maintain time-varying levels of long and short equity market exposure over distinct market cycles.

- Equity Hedge:

  This strategy consists in maintaining positions both long and short in equity stocks and equity derivative instruments. Equity Hedge managers can be either broadly diversified or narrowly concentrated on specific sectors, and they can adjust their net exposure, leverage, holding period, and concentrations. They typically maintain at least 50% exposure to equity, and can be completely invested in, both long and short.

- Macro:

  This investment style covers a broad range of strategies in which investment process is based on the movements in economic variables and their impact these have on various asset classes. Managers use various techniques, both systematic and discretionary, both fundamental and quantitative, and both bottom-up and top-down approaches. Macro strategies usually depart from relative value strategies since they are based on the movements in macroeconomic variables rather than on the discrepancy between securities.

- Merger Arbitrage:

  Merger Arbitrageurs focus on companies that are primarily involved in announced corporate transactions, typically with restricted or no exposure to situations that don't include formal announcement. Since investment process consists typically in going long the stock of the acquired company and going short the stock of the acquirer, deal-failure risk designates the major risk arbitrage risk. These investment strategies typically maintain at least 75% exposure to announced transactions over a given market cycle.

- Relative Value:

  Relative Value arbitrageurs take profit from the realization of a valluation discrepancy between various securities. They use both quantitative and fundamental techniques and a broad range of securities among asset classes to identify attractive risk-adjusted spreads. This investment style can be also involved in corporate transactions, but they depart from Merger Arbitrage since they are based on pricing anomalies between securities, rather than on the outcome of a transaction.

3.8.2 Risk-Neutral Distribution

Following the model-free approach of Bakshi, Kapadia, and Madan (2003) in [24], we extract risk-neutral moments from the market option prices. Let $R(t,T) \equiv \ln[S(t+T)] - \ln[S(t)]$ the log return at time t over the time period T. We define the risk- neutral mean of returns $\mu(t,T)$, volatility $RNVol(t,T)$, skewness $RNSkew(t,T)$, and kurtosis $RNKurt(t,T)$ measured at time t over period T by

$$\mu\left(t,T\right)\equiv E_t^Q\left[R\left(t,T\right)\right] \tag{3.8.2.1}$$

$$RNVol\left(t,T\right)\equiv\left[E_t^Q\left[R\left(t,T\right)^2\right]-\mu\left(t,T\right)^2\right]^{\frac{1}{2}} \tag{3.8.2.2}$$

$$RNSkew\left(t,T\right)\equiv\frac{E_t^Q\left[\left(R\left(t,T\right)-E_t^Q\left[R\left(t,T\right)\right]\right)^3\right]}{\left(E_t^Q\left[\left(R\left(t,\tau\right)-E_t^Q\left[R\left(t,T\right)\right]\right)^2\right]\right)^{\frac{3}{2}}} \tag{3.8.2.3}$$

$$RNKurt\left(t,T\right)\equiv\frac{E_t^Q\left[\left(R\left(t,T\right)-E_t^Q\left[R\left(t,T\right)\right]\right)^4\right]}{\left(E_t^Q\left[\left(R\left(t,T\right)-E_t^Q\left[R\left(t,T\right)\right]\right)^2\right]\right)^{2}} \tag{3.8.2.4}$$

From Bakshi and Madan (2000) in [25], any payoff function $H\left[S\right]$ can be spanned algebraically by a continuum of OTM European call and put options. Therefore, let r the risk-free rate, $C\left(t,T;K\right)$ ($P\left(t,T;K\right)$) the price of a European call (put) option at time t , with time to expiration T, and strike price K . Let the volatility $V(t,T)$, the cubic $W(t,T)$, and the quartic $X(t,T)$ contracts associated to the payoff function $H\left[S\right]$. As below, Equations (3.8.2.1), (3.8.2.2), (3.8.2.3), and (3.8.2.4) can be expressed in terms of the volatility, cubic, and quartic contracts' fair values under the risk-neutral expectation operator conditional on information at time t :

$$\mu\left(t,T\right)=\exp\left(rT\right)-1-\frac{\exp\left(rT\right)}{2}V\left(t,T\right)-\frac{\exp\left(rT\right)}{6}W\left(t,T\right)-\frac{\exp\left(rT\right)}{24}X\left(t,T\right) \quad (3.8.2.5)$$

$$RNVol\left(t,T\right)=\left[V\left(t,T\right)\exp\left(rT\right)-\mu\left(t,T\right)^{2}\right]^{\frac{1}{2}} \quad (3.8.2.6)$$

$$RNSkew\left(t,T\right)=\frac{\exp\left(rT\right)W\left(t,T\right)-3\mu\left(t,T\right)\exp\left(rT\right)V\left(t,T\right)+2\mu\left(t,T\right)^{3}}{\left[\exp\left(rT\right)V\left(t,T\right)-\mu\left(t,T\right)^{2}\right]^{\frac{3}{2}}} \quad (3.8.2.7)$$

$$RNKurt\left(t,T\right)=\frac{\exp\left(rT\right)X\left(t,T\right)-4\mu\left(t,T\right)\exp\left(rT\right)W\left(t,T\right)+6\exp\left(rT\right)\mu\left(t,T\right)^{2}V\left(t,T\right)-3\mu\left(t,T\right)^{4}}{\left[\exp\left(rT\right)V\left(t,T\right)-\mu\left(t,T\right)^{2}\right]^{2}} \quad (3.8.2.8)$$

Furthermore, in Equations (3.8.2.5), (3.8.2.6), (3.8.2.7), and (3.8.2.8), contracts' fair values $V\left(t,T\right)$, $W\left(t,T\right)$, and $X\left(t,T\right)$ can be spanned by a linear combination of OTM European call and put options, the stock and the risk-free asset, requiring a large continuum of traded OTM options. However, since we observe in practice only few option market prices for discretely spaced strike prices, we apply the non-parametric approach of Völkert (2014) in [111] to adress discreteness by applying a cubic smoothing spline to interpolate implied volatilities amongst strike prices. Therefore, we approximate numerically the integral functions of volatility, cubic, and quartic contracts by using trapezoidal approximations.

3.8.3 Real-World Distribution

Recent literature about high-frequency econometrics, including Bollerslev, Tauchen, and Zhou (2009) in [36], and Neumann and Skiadopoulos (2013) in [94] among others, usually estimates the daily realized variance under a nonparametric approach by summing frequently sampled squared returns. Similarly, Amaya, Christoffersen, Jacobs, and Vasquez (2013) in [15] derive the daily realized skewness and kurtosis from intradaily returns. Nevertheless, since this standard econometric approach is widely biased by the market microstructure noise on volatility estimation, a naive practice consists in throwing away a lot of available data by sampling less frequently the intradaily underlying asset prices. In this way, we rather use the model-free approach proposed by Aït-Sahalia, Mykland and Zhang (2005) in [118] to fully exploit the tick-by-tick data, to correct for the bias of market microstructure noise; and furthermore, to estimate similarly the higher-order realized moments.

According Bollerslev, Tauchen, and Zhou (2009) in [36], the daily realized variance is usually estimated by summing the intradaily returns of the underlying asset. Let $R_{t,i}$ the i-intraday log return calculated on day t and associated to the price index $P_{t,i}$. Then $R_{t,i,T} = \ln\left(P_{t,\frac{i}{N}}\right) - \ln\left(P_{t,\frac{(i-1)}{N}}\right)$, where N denotes the total number of observed intraday log returns in the trading day t. Therefore, the daily realized volatility $RDVol_t^{(all)}$ is usually estimated by summing naively all the n squares of intradaily log returns $R_{t,i}$:

$$RDVol_t^{(all)} = \left(\sum_{i=1}^{N} R_{t,i}^2\right)^{\frac{1}{2}} \tag{3.8.3.1}$$

Similarly, following Amaya et al. (2013) in [15], the ex-post realized daily skewness $RDSkew_t$ and kurtosis $RDKurt_{t,T}$ can be expressed as follows, respectively scaled by $N^{\frac{1}{2}}$ and N to ensure they correspond to the daily realized measures:

$$\begin{aligned} RDSkew_t^{(all)} &= \frac{N^{\frac{1}{2}} \sum_{i=1}^{N} R_{t,i}^3}{RDVol_t^3}, \\ RDKurt_t^{(all)} &= \frac{n \sum_{i=1}^{N} R_{t,i}^4}{RDVol_t^4} \end{aligned} \tag{3.8.3.2}$$

Nevertheless, Aït-Sahalia, Mykland and Zhang (2005) in [118] argue that using naively all the tick-by-tick data makes the market microstructure noise totally swamp the estimated realized volatility under the nonparametric case. Suppose the log price process X_t follows a continuous semi-martingale. Then, it is modeled by the stochastic differential equation $dX_t = \mu_t + \sigma_t dW_t$, where μ_t, σ_t, and W_t denote respectively the drift and the volatility of the log return process of X_t at time t, and a standard Brownian motion process. Therefore, the object of interest primarily consists in estimating the integrated variance, i.e. the quadratic variation $\langle X, X \rangle_T = \int_0^T \sigma_t^2 dt$ over the time period $[0, T]$. Indeed, Zhang et al. (2005) show that $RDVol_T^{(all)}$ in the (3.8.3.1) converges in law to

$$RDVol_T^{(all)} \overset{L}{\simeq} \langle X, X \rangle_T + 2nE\left[\varepsilon^2\right] + \left[4nE\left[\varepsilon^4\right] + \frac{2T}{n} \int_0^T \sigma_t^4 dt\right]^{\frac{1}{2}} Z_{total} \tag{3.8.3.3}$$

where $RDVol_T^{(all)}$ is even more positively biased by the market microstructure noise $2nE\left[\varepsilon^2\right]$

when the sample size n of observed intradaily prices increases. Consequently, sampling sparsely either at an arbitrary frequency or even at an optimal frequency by decreasing n are tantamount to ignoring the microstructure noise and to throwing out a large fraction of the available intradaily data. In contrast, Aït-Sahalia, Mykland and Zhang (2005) propose the following Two-Scales Realized Volatility estimator $RDVol_T^{(TS)}$ that uses all the available tick-by-tick data but that incorporates subsampling, averaging, and bias correction for the market microstructure noise:

$$RDVol_T^{(TS)} = \frac{1}{K}\sum_{k=1}^{K} RDVol_T^{(k)} - \frac{\overline{n}}{n} RDVol_T^{(all)} \tag{3.8.3.4}$$

where the original grid $G = \{t_k, ..., t_n\}$ of observation times of log prices in a given trading day is partitioned into K non-overlapping and equal subsamples $G^{(k)}$ for $k = \{1, ..., K\}$. The k-th sub-grid is written as $G^{(k)} = \{t_{k-1}, t_{k-1+K} ..., t_{k-1+n_k K}\}$. Therefore, Zhang et al. (2005) average the estimators $RDVol_T^{(k)}$ obtained on each of the K grids of average size $\overline{n} = \frac{n-K+1}{K}$, giving rise to the estimator $RDVol_T^{(avg)} = \frac{1}{K}\sum\limits_{k=1}^{K} RDVol_T^{(k)}$. Then, bias correction is determined by $K = n^{\frac{2}{3}}\left[12\widehat{E\left[\varepsilon^2\right]}^2 \Big/ \frac{Tn}{3}\int\limits_0^T \sigma_t^4 dt\right]^{\frac{1}{3}}$. Finally, $RDVol_T^{(TS)}$ corrects for the bias $2\overline{n}E\left[\varepsilon^2\right]$ due to the microstructure noise of $RDVol_T^{(avg)}$, since it now increases with the average subsamples size $\overline{n}$.

Similarly, we derive to the higher-order moments the Aït-Sahalia, Mykland and Zhang (2005) methodology of subsampling, averaging, and bias correction for the market microstructure noise:

$$\begin{aligned} RDSkew_T^{(TS)} &= \frac{1}{K}\sum_{k=1}^{K} RDSkew_T^{(k)} - \frac{\overline{n}}{n} RDSkew_T^{(all)}, \\ RDKurt_T^{(TS)} &= \frac{1}{K}\sum_{k=1}^{K} RDKurt_T^{(k)} - \frac{\overline{n}}{n} RDKurt_T^{(all)} \end{aligned} \tag{3.8.3.5}$$

where $RDSkew_T^{(TS)}$ and $RDKurt_T^{(TS)}$ denote respectively the two-scales realized skewness and kurtosis.

3.8.4 Theory of Tail Risk Premia

From Cochrane (2005) in [53], I demonstrate that the expected excess return over long period on tail risk premia VRP, SRP, and KRP is negative, since it is negatively correlated with the negative covariance between factors and stochastic discount factor (SDF).

Let the representative agent modelled by utility function U defined for consumption $\tilde{c}_t$ and $\tilde{c}_{t+1}$:

$$U\left(\tilde{c}_t, \tilde{c}_{t+1}\right) = u\left(\tilde{c}_t\right) + \beta E_t\left[u\left(\tilde{c}_{t+1}\right)\right]$$

where β denotes the subjective discount factor. The intuition underlying tail risk premia makes sense since representative agent feels poorer in bad times, decreasing then their consumption. Hence, he consents to pay a positive risk premium over long period that is compensated by generating positive excess returns in adverse times. Therefore, allocation problem consists in a trade-off at time t over $[t, t+1]$ between consumption and investment in an amount ξ of the factor payoff $x_{t+1} = p_{t+1} + d_{t+1}$, where p_{t+1} and d_{t+1} are respectively price and dividend of the risk factor. Henceforth, agent's problem is to find the optimal amount of wealth ξ that maximizes the utility $U\left(\tilde{c}_t, \tilde{c}_{t+1}\right)$:

$$\max_{\{\xi\}} \left\{u\left(\tilde{c}_t\right) + \beta E_t\left[u\left(\tilde{c}_{t+1}\right)\right]\right\} \quad \text{s.t.} \begin{cases} \tilde{c}_t = c_t - \xi p_t \\ \tilde{c}_{t+1} = c_{t+1} + \xi x_{t+1} \end{cases} \tag{3.8.4.1}$$

By Lagrangean technique, the first-order condition (FOC) for an optimal consumption and portfolio choice gives the pricing equation of the factor at time t

$$p_t = E_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right] \tag{3.8.4.2}$$

where $m_{t+1} \equiv \beta \frac{u'(c_{t+1})}{u'(c_t)}$ is the SDF, i.e. the intertemporal marginal rate of substitution between consumption and investment, written as a function of marginal utility u'. Assuming risk-free asset pays with certainty the payoff $x_{t+1} = 1$, then

$$E_t [m_{t+1}] = \frac{1}{r^f_{t+1}}$$

where r^f_{t+1} is the risk-free rate discounting the payoff x_{t+1} to give the risk-free asset price at t. Exhibiting in (3.8.4.2) the covariance term between SDF m_{t+1} and factor payoff x_{t+1}, factor price at t can now be written as the expected cashflow discounted at risk-free rate plus a risk premium:

$$p_t = \frac{E_t [x_{t+1}]}{r^f_{t+1}} + \mathrm{cov}_t [m_{t+1}, x_{t+1}] \tag{3.8.4.3}$$

Rearranging the expression below where $R_{t+1} = \frac{x_{t+1}}{p_t}$ is the gross return of the factor over $[t, t+1]$ and $r_{t+1} = R_{t+1} - r^f_{t+1}$ is the factor return in excess of the risk-free rate, we obtain the expected excess return on the tail risk factors, i.e. the tail risk premium:

$$E_t [r_{t+1}] = -r^f_{t+1} \cdot \mathrm{cov}_t [m_{t+1}, r_{t+1}] \tag{3.8.4.4}$$

$$= -\frac{\mathrm{cov}_t [m_{t+1}, r_{t+1}]}{E_t [m_{t+1}]} \tag{3.8.4.5}$$

Considering the theoretical underpinnings of a risk premium, Equation (3.8.4.4) provides very straightforward conclusions:

- If the factor excess return r_{t+1} and the SDF m_{t+1} are independent, there is no risk premium for bearing additional risk;

- If the factor excess return r_{t+1} covaries negatively with the SDF m_{t+1}, the risk premium compensates the agent for paying a positive risk premium over long period;

- If the factor excess return r_{t+1} covaries positively with the SDF m_{t+1}, the risk premium generates negative returns over long period, but pays a reward in bad times that compensates the agent for paying an insurance. This is exactly what happens when an investor pays the tail risk premia VRP, SRP, and KRP.

3.8.5 Trend-Following Factors

Since our study is restricted by the size of our tail risk premia data sample, we calculate proxies on a daily basis for the trend-following factors of Fung-Hsieh (2001, 2004) in [63] and [64].

For that purpose, we consider their special case of the perfect trend follower who captures systematically the largest asset price movement over the trading day. Hence, the Primitive Trend-Following Strategy (PTFS) captures the optimal payout $S_{\max} - S_{\min}$, where $S_{\max}$ and $S_{\min}$ respectively designate the maximum and the minimum price of an asset over a trading day. This special case assumes the trend follower can perfectly anticipate asset price movements without incurring trading costs, since he can formally *buy breakouts and sell breakdowns*.

Therefore, we empirically construct returns of the PTFS for each of the following 15 markets by using the futures data provided by Datastream and Bloomberg:

- Bonds: 30-year US bond, 10-year Gilts, 10-year Bund, 10-year French bond, 10-year Australian bond.

- Currencies: British pound, Deutschemark, Japanese yen, Swiss franc.

- Commodities: Corn, wheat, soybean, crude oil, gold, silver.

Finally, returns of the PTFSBD (bonds), PTFSFX (currencies), and PTFSCOM (commodities) used in the Fung-Hsieh seven-factor model are then calculated by equally-weighting the returns of the 15 PTFS associated to each of the 3 asset classes.

Table 3.1: Summary Statistics of Hedge Fund Investment Styles

	N	Min	Pct1	Pct25	Pct50	Pct75	Pct99	Max	Std
Panel A: All investment styles, per year									
2008	20	-0,0231	-0,0229	-0,0042	-0,0012	0,0020	0,0257	0,0258	0,0076
2009	52	-0,0099	-0,0089	-0,0009	0,0004	0,0018	0,0083	0,0118	0,0028
2010	51	-0,0138	-0,0088	-0,0014	0,0000	0,0012	0,0056	0,0105	0,0028
2011	52	-0,0196	-0,0090	-0,0017	0,0000	0,0013	0,0058	0,0072	0,0029
2012	55	-0,0088	-0,0059	-0,0010	0,0001	0,0011	0,0061	0,0071	0,0020
2013	45	-0,0123	-0,0078	-0,0010	0,0000	0,0013	0,0045	0,0068	0,0023
Panel B: Full sample, by investment style									
Aggregate	275	-0,0109	-0,0077	-0,0008	0,0001	0,0011	0,0048	0,0149	0,0022
Directional	275	-0,0231	-0,0184	-0,0017	0,0001	0,0017	0,0073	0,0244	0,0041
Equity Hedge	275	-0,0173	-0,0125	-0,0019	0,0000	0,0018	0,0099	0,0256	0,0042
Macro	275	-0,0160	-0,0088	-0,0022	-0,0003	0,0014	0,0081	0,0118	0,0033
Merger Arbitrage	275	-0,0125	-0,0070	-0,0009	-0,0001	0,0012	0,0087	0,0127	0,0024
Relative Value	275	-0,0188	-0,0071	-0,0007	0,0003	0,0012	0,0045	0,0258	0,0026
Panel C: All investment styles, full sample									
	1650	-0,0231	-0,0097	-0,0013	0,0001	0,0013	0,0077	0,0258	0,0032

This table reports summary statistics for the data of hedge fund investment styles from HFR Database over the period 2008-2013. The data sample that we constructed is restricted to the estimation points associated to the tail risk premia. Statistic N designates either the number of daily returns associated to all the hedge fund investment styles each year (Panel A), or for each investment style over the entire sample period (Panel B). Other statistics consist in the minimum, 1, 25, 50, 75, and 99 percentiles, maximum and standard deviation. Panel C summarizes the total number of daily hedge fund returns in the entire data sample and other statistics.

Table 3.2: Descriptive Statistics of VIX Option-Implied Risk Premia

	30 Days Maturity			60 Days Maturity			90 Days Maturity		
	VRP	**SRP**	**KRP**	**VRP**	**SRP**	**KRP**	**VRP**	**SRP**	**KRP**
Panel A: Levels of Option-Implied Risk Premia									
Nb. Observations	300	300	300	267	267	267	237	237	237
Mean	-0,193	-0,718	-6,027	-0,411	-0,747	-5,818	-0,487	-0,652	-5,707
Median	-0,242	-0,849	-2,680	-0,447	-0,774	-2,264	-0,515	-0,766	-1,852
Max	1,361	3,031	4,855	1,033	2,645	5,358	0,971	2,507	4,723
Min	-1,280	-4,555	-35,560	-1,205	-5,522	-33,474	-1,258	-4,382	-33,029
Standard Dev.	0,397	1,437	8,073	0,354	1,502	8,056	0,388	1,450	8,420
Skewness	0,736	0,047	-1,437	1,011	-0,157	-1,392	0,662	-0,038	-1,467
Kurtosis	4,017	2,457	4,322	5,091	2,673	4,273	3,559	2,541	4,435
LBQ Test	79,14*	0,01	0,12	40,11*	0,05	0,57	37,21*	0	0
ADF Test	-8,57**	-14**	-12,07**	-6,33**	-13,16**	-10,98**	-5,52**	-12,87**	-11,01**
Student Test	-8,41**	-8,66**	-12,93**	-18,94**	-8,13**	-11,8**	-19,35**	-6,92**	-10,44**
Panel B: First Differences of Option-Implied Risk Premia									
Nb. Observations	299	299	299	266	266	266	236	236	236
Mean	0,000	-0,004	0,025	-0,216	-0,100	0,006	-0,312	-0,013	0,308
Median	-0,016	-0,032	0,055	-0,224	-0,103	0,359	-0,330	-0,223	0,091
Max	1,166	5,826	30,593	1,432	5,615	28,863	1,400	4,908	36,231
Min	-1,443	-5,247	-35,141	-2,003	-6,249	-33,166	-1,966	-5,965	-33,478
Standard Dev.	0,393	2,029	11,543	0,523	2,013	11,314	0,519	1,915	11,922
Skewness	0,051	-0,017	-0,059	-0,030	-0,126	-0,211	0,058	-0,030	-0,062
Kurtosis	3,687	2,687	3,528	3,784	3,179	3,638	3,593	2,930	3,716
LBQ Test	49,88*	61,45*	96,49*	52,91*	0,77	0,58	42,15*	0,89	0,9
ADF Test	-26,52**	-28,05**	-32,7**	-9,03**	-17,1**	-15,53**	-7,95**	-16,3**	-16,26**
Student Test	0	-0,03	0,04	-6,75**	-0,81	0,01	-9,24**	-0,1	0,4

Descriptive statistics associated to VIX option-implied risk premia for different maturities, from January 24, 2008 to August 29, 2014. Option-implied risk premia are computed as the difference between the physical measure and the risk-neutral measure of the expected value of realized moments. Moments estimated under the physical expectation are based on intradaily prices associated to the VIX spots. Moments estimated under the risk-neutral expectation are based on daily VIX option prices for different maturities. Stars *, ** denote statistical significance at respectively 5% and 1% level of confidence.

Table 3.3: Pairwise Correlations of Fung-Hsieh Factors and Tail Risk Premia

	MKT-RF	SMB	dTERM	dCREDIT	PTFSBD	PTFSFX	PTFSCOM	VRP	SRP
SMB	0,10 [0,08]								
dTERM	0,37 [0]	0,04 [0,49]							
dCREDIT	-0,33 [0]	-0,05 [0,39]	-0,95 [0]						
PTFSBD	-0,13 [0,04]	-0,06 [0,3]	-0,09 [0,12]	0,11 [0,06]					
PTFSFX	-0,18 [0]	-0,08 [0,21]	0,10 [0,09]	-0,07 [0,24]	0,08 [0,16]				
PTFSCOM	-0,12 [0,05]	-0,13 [0,03]	-0,08 [0,16]	0,17 [0,01]	0,31 [0]	0,12 [0,04]			
VRP	-0,27 [0]	-0,21 [0]	-0,13 [0,04]	0,15 [0,01]	0,18 [0]	0,25 [0]	0,34 [0]		
SRP	-0,31 [0]	-0,16 [0,01]	0,00 [0,95]	-0,02 [0,75]	0,10 [0,1]	0,20 [0]	0,04 [0,47]	0,14 [0,02]	
KRP	-0,04 [0,52]	0,00 [0,95]	-0,05 [0,39]	0,04 [0,48]	-0,02 [0,72]	0,00 [0,98]	-0,06 [0,29]	0,19 [0]	0,21 [0]

This table summarizes the time-series Pearson pairwise correlations of the Fung-Hsieh (2001) seven factors and the tail risk premia in Al Wakil (2016). Sample period is August 2008 to October 2013. The Fung-Hsieh seven factors consist in the market portfolio in excess of the risk-free rate, and the SMB of Fama and French (1993), the term spread change, the credit spread change, and the factors associated to the best trend-following strategies, i.e. PTFSBD (bonds), PTFSFX (currencies), PTFSCOM (commodities). p-values are reported in square brackets.

Table 3.4: Multivariate Regressions Results of Hedge Fund Styles Returns on Fung-Hsieh Factors and Tail Risk Premia over 2008-2013

Investment Style	Nb. Obs.	Intercept	MKT-RF	SMB	dTERM	dCREDIT	PTFSBD	PTFSFX	PTFSCOM	VRP	SRP	KRP	R-Square/ Adj. R-Square
Global	275	0,0000	0,088***							-0,0012***	0,0002**	0,0000	0,41
		[-0,19]	[11,48]							[-4,26]	[2,09]	[0,51]	0,41
		-0,0231	0,0872***	0,0064	-0,0052	-0,0114	0,0024	4,3693	-0,0001**	-0,001***	0,0001*	0,0000	0,43
		[-0,63]	[10,45]	[0,37]	[-0,4]	[-0,63]	[1,6]	[0,63]	[-2]	[-3,4]	[1,8]	[0,36]	0,41
Directionnal	275	0,0003	0,1979***							-0,0009*	0,0004***	0,0000	0,50
		[1,22]	[15,11]							[-1,85]	[3,34]	[1,5]	0,49
		-0,0407	0,1908***	-0,0623**	-0,0027	-0,0206	0,0027	1,2206	-0,0002**	-0,0007	0,0004***	0,0000	0,52
		[-0,66]	[13,52]	[-2,13]	[-0,12]	[-0,67]	[1,06]	[0,1]	[-2,24]	[-1,36]	[2,68]	[1,48]	0,50
Equity Hedge	275	-0,0001	0,1973***							-0,0013***	0,0001	0,0000	0,54
		[-0,22]	[15,54]							[-2,96]	[1,06]	[0,49]	0,53
		0,0096	0,1906***	0,0084	0,0153	0,0051	0,0044*	16,4733	-0,0001*	-0,0013***	0,0001	0,0000	0,55
		[0,16]	[13,88]	[0,29]	[0,71]	[0,17]	[1,79]	[1,43]	[-1,66]	[-2,63]	[0,52]	[0,57]	0,54
Macro	275	-0,0009***	-0,0321**							-0,0011**	-0,0004**	0,0000	0,05
		[-3,2]	[-2,23]							[-2,23]	[-2,5]	[-0,01]	0,03
		-0,0991	-0,0059	0,0866***	-0,0691***	-0,0495	0,0018	-17,9721	-0,0001	-0,0005	-0,0002	0,0000	0,18
		[-1,54]	[-0,4]	[2,83]	[-3]	[-1,54]	[0,69]	[-1,46]	[-1,27]	[-0,95]	[-1,35]	[-0,82]	0,15
Merger Arbitrage	275	0,0002	0,0979***							-0,0001	0,0001	0,0000	0,36
		[1,21]	[11,3]							[-0,47]	[0,62]	[0,88]	0,35
		0,1272***	0,0901***	-0,0272	0,0497***	0,0633***	-0,0001	-8,6250	0,0000	-0,0002	0,0000	0,0000	0,39
		[3,12]	[9,68]	[-1,41]	[3,42]	[3,11]	[-0,06]	[-1,11]	[-0,7]	[-0,53]	[0,5]	[0,98]	0,37
Relative Value	275	0,0004**	0,0424***							-0,0009**	0,0005***	0,0000	0,12
		[2,01]	[3,88]							[-2,4]	[4,14]	[0,06]	0,10
		0,0338	0,0314***	-0,0229	0,0267	0,0168	0,0021	2,7385	-0,0001	-0,001**	0,0004***	0,0000	0,15
		[0,65]	[2,65]	[-0,93]	[1,44]	[0,65]	[1]	[0,28]	[-0,96]	[-2,25]	[3,47]	[0,27]	0,11

This table summarizes the results of time-series OLS regressions of major hedge fund style returns on the Fung-Hsieh (2001) seven factors and the tail risk premia in Al Wakil (2016). Sample period is August 2008 to October 2013. The Fung-Hsieh seven factors consist in the market portfolio in excess of the risk-free rate, and the SMB of Fama and French (1993), the term spread change, the credit spread change, and the factors associated to the best trend-following strategies, i.e. PTFSBD (bonds), PTFSFX (currencies), PTFSCOM (commodities). T-statistics are reported in square brackets and stars *, ** denote statistical significance at respectively 5% and 1% level of confidence.

Table 3.5: Quantile Portfolios Sorted on Hedge Fund Tail Loadings and Return Spread of the High-Minus-Low Portfolio

	Factor Beta Quantiles			
	1 [Low]	**2**	**3 [High]**	**High - Low**
Panel A: Volatility Risk Premia in the Cross-Section				
Average Tail Risk Beta	-0,216%	-0,026%	0,091%	0,307%
	[-18,35]	[-3,37]	[11,47]	[28,88]
Average Excess Return	0,0057	0,0051	-0,1113	-0,117***
	[0,16]	[0,15]	[-2,4]	[-2,38]
Fung-Hsieh Alpha	-11,01%	-3,61%	-1,59%	9,42%
	[-1,97]	[-0,71]	[-0,23]	[1,18]
Panel B: Skewness Risk Premia in the Cross-Section				
Average Tail Risk Beta	-0,019%	0,035%	0,059%	0,078%
	[-6,49]	[14,33]	[18,28]	[16,7]
Average Excess Return	0,001	-0,024	0,008	0,008
	[0,01]	[-0,6]	[0,18]	[0,15]
Fung-Hsieh Alpha	-0,87%	-10,24%	-7,56%	-6,69%
	[-0,15]	[-1,74]	[-1,12]	[-0,84]
Panel C: Kurtosis Risk Premia in the Cross-Section				
Average Tail Risk Beta	-0,003%	0,005%	0,009%	0,012%
	[-9,1]	[17,3]	[21,91]	[31,74]
Average Excess Return	0,0028	-0,0185	-0,0832	-0,086***
	[0,07]	[-0,52]	[-1,79]	[-1,86]
Fung-Hsieh Alpha	-8,91%	-2,50%	-4,01%	4,90%
	[-1,58]	[-0,48]	[-0,59]	[0,68]

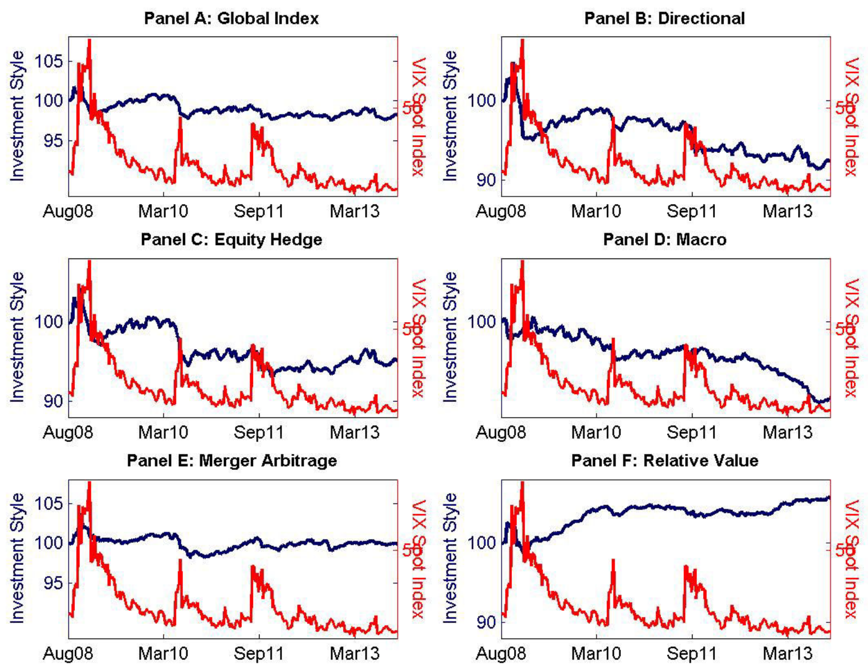
This table reports the average daily tail risk betas, the average annualized excess returns, and the Fung-Hsieh seven factor alpha for hedge fund portfolios sorted on the basis of their loadings on the tail risk premia, respectively associated to the volatility VRP, skewness SRP, and kurtosis KRP. Each day, three quantile portfolios are formed based on the investment styles loadings on the tail risk premia in a regression of returns on the market excess return and the tail risk factor in the past 22 days. Newey-West (1987) t-statistics are reported in square brackets and stars *** denote statistical significance at respectively 5% level of confidence for the average excess returns, and the Fung-Hsieh seven factor alpha.

Table 3.6: Multivariate Regressions Results of Hedge Fund Styles Returns on Fung-Hsieh Factors and Tail Risk Premia over Tail Events

Investment Style	Nb. Obs.	Intercept	MKT-RF	SMB	dTERM	dCREDIT	PTFSBD	PTFSFX	PTFSCOM	VRP	SRP	KRP	R-Square/ Adj. R-Square
Global	71	0,0007	0,0939***							-0,003***	0,0004	0,0000	0,49
		[1,51]	[6,54]							[-3,41]	[1,65]	[0,59]	0,46
		0,0571	0,1044***	-0,0100	0,0136	0,0288	0,0116*	1,6329	-0,0002	-0,0032***	0,0004	0,0000	0,53
		[0,74]	[5,3]	[-0,24]	[0,46]	[0,75]	[1,91]	[0,08]	[-1,58]	[-3,01]	[1,46]	[0,47]	0,46
Directionnal	71	0,0012	0,2318***							-0,0020	0,0011**	0,0000	0,60
		[1,53]	[9,49]							[-1,33]	[2,47]	[0,73]	0,58
		0,1378	0,207***	-0,0845	0,0725	0,0684	0,0146	-32,9534	-0,0004**	-0,0015	0,0009*	0,0000	0,66
		[1,08]	[6,35]	[-1,24]	[1,49]	[1,07]	[1,45]	[-0,97]	[-2,23]	[-0,84]	[1,91]	[0,4]	0,60
Equity Hedge	71	0,0010	0,207***							-0,0034**	0,0005	0,0000	0,62
		[1,4]	[9,47]							[-2,59]	[1,26]	[0,68]	0,60
		0,1467	0,2203***	-0,0033	0,0464	0,0741	0,0182*	5,4085	-0,0002	-0,0039**	0,0005	0,0000	0,66
		[1,26]	[7,4]	[-0,05]	[1,05]	[1,27]	[1,98]	[0,17]	[-1,26]	[-2,46]	[1,08]	[0,69]	0,60
Macro	71	-0,0005	-0,0637***							-0,0028**	-0,0011***	0,0001	0,24
		[-0,73]	[-3,12]							[-2,26]	[-3]	[1,1]	0,19
		-0,0696	-0,0372	-0,0290	-0,0646*	-0,0359	-0,0027	-40,9889	-0,0002	-0,0017	-0,0007*	0,0000	0,41
		[-0,69]	[-1,45]	[-0,54]	[-1,69]	[-0,72]	[-0,34]	[-1,53]	[-1,45]	[-1,22]	[-1,89]	[0,14]	0,32
Merger Arbitrage	71	0,0012**	0,1244***							-0,0009	-0,0002	0,0001**	0,57
		[2,64]	[8,39]							[-0,95]	[-0,66]	[2,3]	0,54
		0,2653***	0,1139***	-0,0527	0,0978***	0,1321***	0,0058	-19,6823	-0,0001	-0,0011	-0,0002	0,0001**	0,67
		[3,62]	[6,1]	[-1,35]	[3,52]	[3,62]	[1,01]	[-1,01]	[-0,78]	[-1,11]	[-0,81]	[2,28]	0,61
Relative Value	71	0,0012*	0,064***							-0,0033**	0,0011***	0,0000	0,23
		[1,68]	[2,8]							[-2,36]	[2,75]	[-0,11]	0,18
		0,0937	0,0662**	0,0199	0,0477	0,0481	0,0191**	31,5392	-0,0001	-0,0042**	0,0009**	0,0000	0,32
		[0,78]	[2,16]	[0,31]	[1,04]	[0,8]	[2,02]	[0,98]	[-0,82]	[-2,52]	[2,02]	[0,32]	0,21

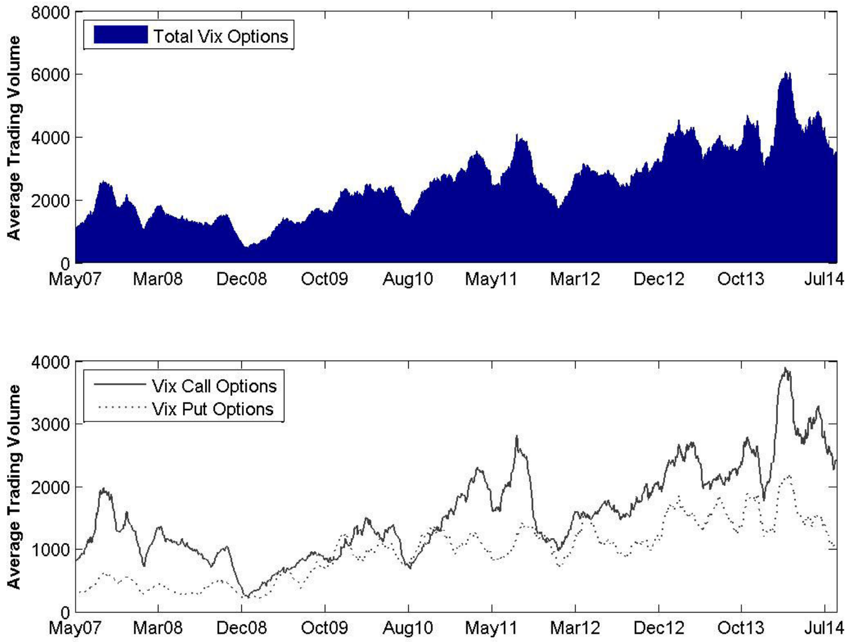
This table summarizes the results of time-series OLS regressions of major hedge fund style returns on the Fung-Hsieh (2001) seven factors and the tail risk premia in Al Wakil (2016). Sample period consists in time-slicing the initial data sample over 2008-2013 to consider major extreme events, including the US Subprime crisis and the Lehman collapse in 2008, the European sovereign debt crisis in 2010, the US sovereign debt crisis in 2011, and the Taper Tantrum in 2013. T-statistics are reported in square brackets and stars *, ** denote statistical significance at respectively 5% and 1% level of confidence.

Figure 3.1: Performances of Hedge Fund Investment Styles



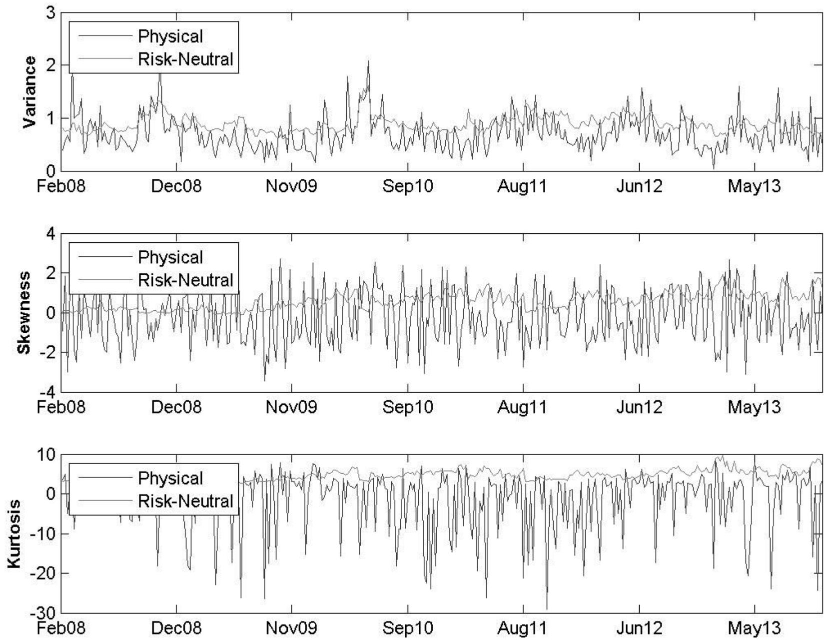
This figure displays the rebased performances of the Aggregate Index, Directional, Equity Hedge, Macro, Merger Arbitrage, and Relative Value strategies and the VIX over 2008-2013. The data sample covers some highly turbulent and volatile periods, including the US Subprime crisis and the Lehman collapse in 2008, the European sovereign debt crisis in 2010, the US sovereign debt crisis in 2011, and the Taper Tantrum in 2013.

Figure 3.2: Average Trading Volume of VIX Options



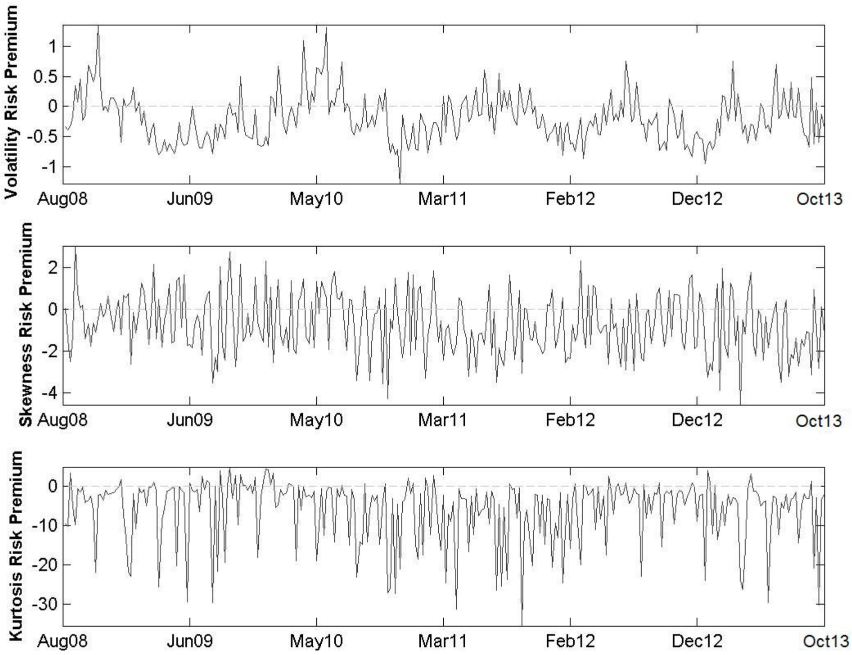
This figure displays the average daily trading volume of VIX options over 2007-2014. The lower panel represents the compared average daily trading volume of VIX call and put options. For clearness, computations are based on the 2-month moving average trading volume.

Figure 3.3: Higher-Order Moments under Real-World and Risk-Neutral Probability Measures



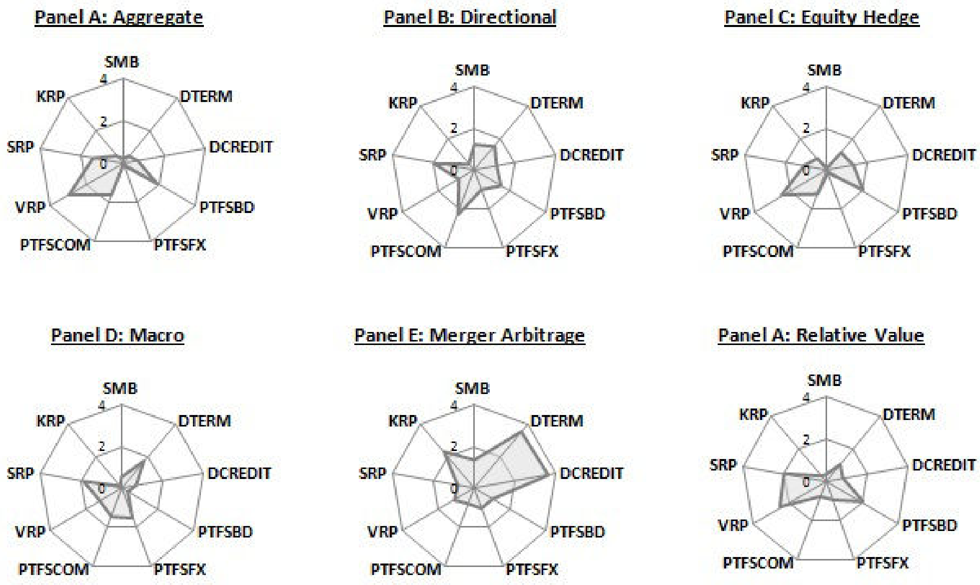
This figure displays the estimates of higher-order moments under real-world and risk-neutral probability measures associated to VIX markets over 2008-2013. Intradaily VIX spots are used to estimate physical moments, and daily VIX options and futures are used to estimate risk-neutral moments. The figures plot respectively on a daily basis the levels of the physical and risk-neutral volatility, skewness, and kurtosis, for 30 days time to expiration.

Figure 3.4: Option-Implied Risk Premia associated to VIX Options



This figure displays the VIX option-implied risk premia for 30 days time to maturity over 2008-2013. On a daily basis, the levels of the risk premia are associated to the volatility, the skewness, and the kurtosis, for 30 days time to expiration.

Figure 3.5: Sensitivity of Hedge Fund Investment Styles to Fung-Hsieh Factors and Tail Risk Premia



This figure displays the sensitivity of hedge fund investment styles to Fung-Hsieh seven factors (without market factor) and tail risk premia in crisis periods over 2008-2013. Option-implied risk premia associated to the volatility, the skewness, and the kurtosis are calculated for VIX options and 30 days time to maturity. Sensitivities are measured by the absolute values of t-statistics.

Conclusion Générale

Mes travaux de thèse s'intéressent à la modélisation de la surface de volatilité implicite dans le cadre des stratégies d'arbitrage de volatilité. Ces stratégies de gestion, très communément utilisées par les investisseurs sophistiqués tels que les *hedge funds* et les banques d'investissement, se sont récemment popularisées suite à la grande crise financière de 2008 auprès des gérants d'actifs traditionnels et des investisseurs particuliers. En effet, exploiter la relation inverse (*implied leverage effect*) entre les indices des marchés actions et la volatilité implicitée des marchés d'options est apparue rapidement comme une idée prometteuse de stratégie de couverture de portefeuille contre les risques extrêmes. Ainsi, dans la continuité de la création des *futures* et options sur indice VIX, le marché de la réplication passive par des produits dérivés indexés sur les *futures* VIX tels que les ETP (*exchange-traded products*) VIX a connu un développement considérable, favorisé par la demande d'investisseurs peu sophistiqués tels que les investisseurs particuliers et les gérants d'actifs traditionnels. Cependant, les performances décevantes, l'ETP VXX a perdu près de 99% de sa valeur entre 2009 et 2014, ont totalement remis en question l'approche traditionnelle des stratégies de volatilité basées sur la réplication passive. Ainsi, cette thèse est motivée par l'objectif de repenser la philosophie des stratégies d'arbitrage de volatilité, en s'inspirant notamment de l'approche pratiquée par les investisseurs sophistiqués tels que les *hedge funds*.

Au travers d'une étude empirique préliminaire et descriptive, le chapitre 1 dresse le diagnostic des stratégies traditionnelles de volatilité basées sur la diversification et la couverture de portefeuille par l'utilisation des ETP VIX. Pour cela, nous investiguons l'adéquation de ces complexes instruments dérivés de couverture avec le degré d'aversion au risque des investisseurs. L'investigation de l'optimalité du choix de portefeuille en environnement incertain est ainsi menée pour des allocations *overlay* composées d'actions, d'obligations, et d'ETP indexés sur *futures* de VIX, dans le cadre théorique de la théorie de l'utilité espérée, simulant le comportement d'un investisseur rationnel doté d'une aversion au risque. Spécifiquement, notre analyse empirique fait appel à deux métriques, l'une mesurant la surprise de l'investisseur, et définie comme une mesure du critère de son bien-être dérivé de ses décisions d'investissement ; l'autre mesurant la prime de risque implicitée par notre modèle, et définie comme la prime d'assurance que l'investisseur consentirait à payer *ex post* pour couvrir totalement son portefeuille *overlay* contre les risques extrêmes. Les résultats empiriques montrent que sous ces deux critères de mesure, la couverture de portefeuille par des ETP VIX bat significativement la couverture traditionnelle de portefeuille, à la fois en *in sample* et en *out-of-sample.* Cependant, ce type de couverture reposant sur l'investissement de long-terme par des instruments de réplication passive apparaît particulièrement inadéquate pour les investisseurs peu averses au risque. En effet, la compréhension des risques complexes liés à ces instruments dérivés est indispensable pour éviter les décisions d'investissement aléatoires de type jeux de hasard. Du point de vue de l'industrie de la gestion d'actifs, ce chapitre a des implications pratiques puisqu'il recommande un effort pédagogique soutenu auprès des investisseurs lors de la distribution de telles stratégies *overlay.* C'est en partie pour cette raison que ce chapitre est en *forthcoming* dans *Journal of Business and Financial Perspectives*, une nouvelle revue académique promue par Thomson Reuters.

Après avoir dressé le diagnostic des stratégies traditionelles de volatilité basées sur la réplication passive, le chapitre 2 ouvre la voie à une nouvelle génération de stratégies de volatilité, car actives, optionelles, et basées sur l'investissement factoriel. En effet, notre étude théorique et empirique est inspirée d'une intuition simple : puisque les prix de marché d'options et donc la volatilité implicite reflètent l'incertitude et le risque extrême, nous exploitons l'écart de valorisation entre les distributions de densités de probabilités neutres au risque, implicitées des marchés d'options et désignant le juste prix des moments de distributions, et les distributions de densités de probabilités physiques, constatées sur les marchés du sous-jacent. D'un point de vue économique, cet écart de valorisation, matérialisé dans le niveau, la pente, et la convexité du *smile* de volatilité implicite, quantifie l'écart entre la distribution de probabilités des rendements du sous-jacent et la distribution log-normale à la Black-Scholes. Par conséquent, notre approche d'investissement "Smart Vega" consiste à décomposer le risque contenu dans le *smile* de volatilité implicite en stratégies optionelles investissables répliquant les primes de risque de volatilité, skewness, et kurtosis, sous la forme de *swap* de divergences. Plus précisément, nous étendons l'approximation quadratique à la Zhang-Xiang (2008) pour dériver une représentation analytique de la fonction de *smile* de volatilité implicite exprimée comme une combinaison de primes de risques investissables qui récompensent le portage de risques d'ordres supérieurs. En outre, comme suggérée par l'intuition, notre approche est validée sur le plan empirique, et d'autant plus sous des distributions de probabilités fortement asymétriques et leptokurtiques. C'est en partie pour cette raison que ce chapitre a été récompensé par le Prix du Meilleur Papier Doctoral par la *Multinational Finance Society* en 2017. De plus, l'approche originale de "Smart Vega" a fait l'objet d'un dépôt de marque et bientôt de brevet auprès de l'INPI.

Dans la continuité du précédent chapitre, le chapitre 3 cherche à valider ou infirmer notre approche active de l'arbitrage de volatilité en la comparant aux stratégies utilisées par les investisseurs sophistiqués tels que les *hedge funds*. Plus précisément, nous décomposons le risque extrême contenu dans la performance des *hedge funds* sous la forme de stratégies d'investissement optionelles et dynamiques, comme suggérées par notre approche "Smart Vega". Ainsi, nous montrons que les stratégies de primes de risques d'assurance, de volatilité, skewness, et kurtosis, sous la forme de *swap* de divergences, constituent des déterminants importants dans la performance des *hedge funds*, que ce soit en analyse temporelle et en analyse cross-sectionnelle. En contrôlant des facteurs de risques standards à la Fung-Hsieh communément utilisés dans la littérature, nous mettons en évidence qu'un choc positif d'un écart-type de la prime de risque de volatilité est associée à une baisse substantielle de 25.2% annuellement de la performance aggrégée des rendements *hedge funds*. En particulier, notre résultat d'importance démontre que les *hedge funds* fortement sensibles à la prime de volatilité (kurtosis) surperforment substantiellement les fonds moins sensibles de près de 11.7% (8.6%) par an. Ce résultat suggère dans quelle mesure l'alpha des *hedge funds* provient en réalité de la vente de stratégies d'assurance contre le risque extrême. Par conséquent, ce papier ouvre la voie à la réplication de la performance de *hedge funds* sophistiqués par la réplication de stratégies de primes de risques d'assurance. Ce chapitre a été co-écrit avec mon superviseur de thèse, le Professeur Serge DAROLLES.

Pour résumer brièvement, mes travaux de thèse investiguent de nouvelles approches d'investissement de l'arbitrage de volatilité et visent à repenser la philosophie d'une stratégie récemment décriée. L'urgence d'un tel diagnostic et de la recherche de nouveaux paradigmes d'investisement est d'autant plus grande que l'environnement actuel de taux bas et de suppression de la volatilité implicite a accéléré le développement de stratégies vendeuses de volatilité mais aussi acheteuses, dans la perspective d'une nouvelle crise financière éventuelle. Le débat entre arbitrageurs de volatilité a pris une telle importance dans l'actualité financière qu'il nous apparait aujourd'hui prometteur de poursuivre nos recherches dans cette voie. Ainsi, mes travaux collectifs s'orientent désormais vers la détection de nouvelles mesures du risque extrême.

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Résumé

Les stratégies de volatilité ont connu un rapide essor suite à la crise financière de 2008. Or, les récentes performances catastrophiques de ces instruments indiciels ont remis en question leurs contributions en couverture de portefeuille.

Mes travaux de thèse visent à repenser, réinventer la philosophie des stratégies de volatilité.

Au travers d'une analyse empirique préliminaire reposant sur la théorie de l'utilité espérée, le chapitre 1 dresse le diagnostic des stratégies traditionnelles de volatilité basées sur la couverture de long-terme par la réplication passive de la volatilité implicite. Il montre que, bien que ce type de couverture bat la couverture traditionnelle, elle s'avère inappropriée pour des investisseurs peu averses au risque.

Le chapitre 2 ouvre la voie à une nouvelle génération de stratégies de volatilité, actives, optionnelles et basées sur l'investissement factoriel. En effet, notre décomposition analytique et empirique du smile de volatilité implicite en primes de risque implicites, distinctes et investissables permet de monétiser de manière active le portage de risques d'ordres supérieurs. Ces primes de risques mesurent l'écart de valorisation entre les distributions neutres au risque et les distributions physiques.

Enfin, le chapitre 3 compare notre approche investissement factoriel avec les stratégies de volatilité employées par les hedge funds. Notre essai montre que nos stratégies de primes de risque d'assurance sont des déterminants importants dans la performance des hedge funds, tant en analyse temporelle que cross-sectionnelle. Ainsi, nous mettons en évidence dans quelle mesure l'alpha provient en réalité de la vente de stratégies d'assurance contre le risque extrême.

Mots Clés

Volatilité implicite, Arbitrage de volatilité, Estimation neutre au risque, Econométrie haute-fréquence, Primes de risque d'assurance, Investissement factoriel, Hedge funds, Evaluation de performance.

Abstract

Volatility strategies have flourished since the Great Financial Crisis in 2008. Nevertheless, the recent catastrophic performance of such exchange-traded products has put into question their contributions for portfolio hedging and diversification.

My thesis work aims to rethink and reinvent the philosophy of volatility strategies.

From a preliminary empirical study based on the expected utility theory, Chapter 1 makes a diagnostic of traditional volatility strategies, based on buy-and-hold investments and passive replication of implied volatility. It exhibits that, although such portfolio hedging significantly outperforms traditional hedging, it appears strongly inapropriate for risk-loving investors.

Chapter 2 paves the way for a new generation of volatility strategies, active, option-based and factor-based investing. Indeed, our both analytical and empirical decomposition of implied volatility smiles into a combination of implied risk premia, distinct and tradeable, enables to harvest actively the compensation for bearing higher-order risks. These insurance risk premia measure the pricing discrepancies between the risk-neutral and the physical probability distributions.

Finally, Chapter 3 compares our factor-based investing approach to the strategies usually employed in the hedge fund universe. Our essay clearly evidences that our tail risk premia strategies are incremental determinants in the hedge fund performance, in both the time-series and the cross-section of returns. Hence, we exhibit to what extent hedge fund alpha actually arises from selling crash insurance strategies against tail risks.

Keywords

Implied Volatility, Volatility Arbitrage, Risk-neutral estimation, High-frequency econometrics, Tail risk premia, Factor-based investing, Hedge funds, Performance evaluation.