Involuntary unemployment and financial frictions in estimated DSGE models

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À mon père.

Contents

Introduction

The use of DSGE models for policy analysis

The reliance on formal models have emerged as the prevalent approach to macroeconomics over the past few decades. One of their merits is the discipline they impose on economic thinking. The mathematical formalism makes the validity of reasoning certain, in the sense that the logical developments from assumptions to conclusions cannot be disputed, so discussions are confined to the relevance of the assumptions used. In that sense, they considerably facilitate critical peer reviews, provided that economists have the necessary technical background. Moreover, unlike other informal approaches, models' predictions are given in quantitative terms, which is almost always required by anyone asking for guidance for the conduct of economic policy.

Their use to explain the business cycle fluctuations of economic time series started with Kydland and Prescott (1982). This paper, assuming flexible prices, has inspired the real business cycle (RBC) theory. The new-Keynesian Dynamic Stochastic General Equilibrium (DSGE) approach, following authors like Rotemberg and Woodford (1999), builds on a framework similar to RBC models, but assumes instead that prices cannot adjust immediately, implying markup variations. The related literature attributes an explicit role to monetary policy and had considerable success with central banks.

DSGE models provide a full description of a fictive world, which is expected to represent a number of relevant features of the real world.[1] They account for resource constraints in the economy and reflect the decisions of optimizing agents who take the future implications of their present choices into consideration. An alternative approach, still widespread among policy institutions, consists in estimating statistical relationships between observed macroeconomic aggregates. But estimations of policy multipliers based on these reduced-form models may be significantly biased. Indeed, there is no assurance that these relationships are stable over time and do not precisely depend on the economic policies that one thinks of evaluating; this idea is known as the 'Lucas critique' (see Lucas (1976)). In addition, expectations indubitably play a role in actual agents decisions; reduced-form models either ignore them, or describe them in an ad hoc manner, not reflecting human rational behaviors. Yet, taking expectations into account is crucial for policy analysis. When private agents are informed about the functioning of the economy and the policy selection process, their reactions

[1] This explains why a large fraction of the recent literature explores the ability of specific modeling assumptions to replicate some typical properties of observed time series.

to policy decisions depend on their forecasts of future policy actions. But these forecasts are affected by current policy decisions. In this regard, Kydland and Prescott (1977) show that policies chosen to maximize a social objective given the current situation without taking the effect of optimizing agents' expectations into consideration may be suboptimal. For these reasons, reduced-form models are definitely not accurate tools for predicting the effects of economic policies. The use of models based on intertemporally optimal behaviors thus comes out, to date, as a more credible way of doing macroeconomic analysis.

Despite their merits, micro-founded dynamic models have difficulty fitting data as much as needed to be fully trusted by policymakers. Pure theoretical behaviors need severe adjustments, along debatable dimensions, to be able to replicate the observed dynamics of the economy. This results, for a large part, from the way agents in the model economy make their predictions about the future: the rational expectations paradigm assumes that agents' predictions are fully consistent, first of all, with the model – though the latter is obviously misspecified – and, second, with the very long term of the economy – which is in reality almost completely unknown. To use the words of Woodford (2012), these are 'heroic assumptions'. Substantial progress remains to be made to model expectations formation in models, and some alternatives to rational expectations, such as learning (see Evans and Honkapohja (1999)), are very promising.

Yet, rational expectations DSGE models are still the standard approach, which serves as a benchmark to other practices. An often cited reason is models' internal consistency requirement: why, in the fictive world of a model economy, would we assume that agents continue making wrong predictions about the future, ever and ever? In addition, the rational expectations assumption is a strong constraint on the model's predictions, which does not leave much space for parametrization or discretionary specifications. This discipline imposed to economists is a typical argument in its favor. Moreover, thanks to recent developments in numerical methods and computing tools, rational expectations DSGE models are suited for a practical utilization by institutions. Models using alternative assumptions for expectations formation are still more tricky to handle.

A number of papers have studied model specifications or parametrizations that best replicate some particular properties of the data. Other authors have rather used formal econometric methods for the quantitative evaluation of their DSGE models. For a review of the quantitative methods used in the DSGE literature, see Fève (2006). They include maximum-likelihood estimation (see Ireland (1997)), methods of moments (see Christiano and Eichenbaum (1992)), methods based on simulations (Jonsson and Klein (1996) or Hairault et al. (1997) for instance use a method of simulated moments, Dupaigne et al. (2005) apply indirect inference), and Bayesian estimation. Recently, the latter has become very popular, partly thanks to authors like Smets and Wouters (2007). By contrast with methods of moments for instance, it is a 'full-information' approach. The parameters and exogenous processes of models are estimated so that they best replicate simultaneously a broad set of dynamic properties of the data. In doing so, the estimates are not influenced by the economist's selection of moments to match. The main advantages of the Bayesian method over the maximum-likelihood, which is also a 'full-information' approach, are the following. First, the data is generally not informative enough to identify all parameters with a sufficient degree of precision; the

Bayesian approach offers the possibility to use other sources of information. Then, from a practical point of view, the Bayesian methodology rules out implausible parameter values, which could be found with the maximum-likelihood approach as a consequence of models misspecification.

What is a relevant size of estimated DSGE models for operational policy analysis?

The new-Keynesian model of Smets and Wouters (2003, 2007) has become the reference for business cycle analysis, forecasting, and policy evaluation in many institutions, especially central banks.

Yet, despite its merits, this standard framework has important shortcomings, among which the absence of international linkages, of credit-market frictions and of labor-market frictions. An abundant literature have proposed model extentions to address these issues, and many papers have emphasized their importance for policy analysis. These extensions have also been successfully embedded in estimated DSGE models. I summarize below these developments.

Extensions of Smets and Wouters framework in the literature

Open economy

The model of Smets and Wouters was originally estimated with euro area data (see Smets and Wouters (2003)). It represents the euro area as a closed economy. This assumption can be misleading from both a positive and a normative point of view. First, disturbances affecting imports or exports, such as world demand or foreign price shocks, are mixed up with government expenditures; yet, foreign and goverment spending shocks may have very different effects in reality. Furthermore, Paoli (2009) shows that the optimal monetary policy in a small open economy is different from the one in a closed economy, because the welfare cost of business cycles is affected by exchange rate fluctuations.

A second limitation of Smets and Wouters (2003) is that the euro area is seen as an homogeneous economy, which is at least a controversial assumption. To begin with, fiscal policies are decided at country-level, and the coordination or non-coordination of these policies is a prevalent concern. Next, in policy experiments involving one country inside the union, it seems important to separate intra-area and extra-area trade flows, because, unlike the former, the latter are affected by exchange rate variations. Moreover, when the welfare criterion is used for such exercises, the fact that a country is part of a monetary union or not is relevant. This is not only because monetary policy is conducted at the area-wide level; as shown by Kollmann (2004), accounting for the monetary union regime in a multi-country model decreases the welfare cost of business cycles, since it eliminates costly shocks to the uncovered interest rate parity. Last, taking into consideration country specificities by allowing structural parameters to take different values in different regions could improve the model's forecasting performance. As an example, Marcellino et al. (2003) find that inflation forecasts of the euro area constructed by country-specific models are more accurate than forecasts based on aggregate data.

Third, Smets and Wouters (2003) ignore oil price fluctuations. Yet, this variable significantly affects inflation and the business cycle, and is for that reason under policymakers' constant observation. A model taking into account oil price dynamics is more relevant to identify appropriate monetary policy responses in periods of tensions in energy markets; in this regard, Fiore et al. (2006) and Natal (2012) put forward evidence in favor of monetary policy reacting specifically to oil prices.

Recently, a number of authors have successfully enriched the standard new-Keynesian framework along these dimensions. Adjemian and Darracq-Pariès (2008) estimate a two-country model of the euro area and the US which takes into account oil price dynamics. In Rabanal (2009), it is a two-country model of the euro area including Spain, while Christoffel et al. (2008) propose a small open economy model of the euro area.

Credit-market frictions

Smets and Wouters (2003, 2007) assume no financial frictions. But many authors, including Fisher (1933) or, later, Bernanke and Lown (1991), argued that deteriorating credit-market conditions are a major factor of real economy crises. More recently, Christiano et al. (2014) identify the so-called 'risk' shock, specific to credit markets, as one of the most important driving forces of the business cycle. Aside from Christiano et al. (2014), estimated models assuming financial frictions include Christensen and Dib (2008) and Villa (2014).

Consistently with these authors' view, there is a large consensus that feedback from credit markets to the real economy played a role in the 2008 great depression (see Christiano et al. (2015)). From a normative perspective, the presence of financial frictions strongly amplifies the response of the economy to interest rate shocks, thus affecting the monetary policy recommendations that may be drawn from a DSGE model. For these reasons, in the aftermath of the crisis, there is a renewed interest in institutions for business cycle models accounting for potential stresses in financial markets.

Labor-market frictions

Standard new-Keynesian DSGE models following the lines of Smets and Wouters only involve voluntary movements in labor, setting aside unemployment − which is paradoxically among the main concerns of European policymakers. They assume that workers who are willing to work, and firms which need work force, immediately enter into flexible agreements with no cost. This Walrasian mechanism is clearly unrealistic.

Blanchard and Galí (2010) argue that incorporating real frictions in the labor market has implications for optimal monetary policy. In a model with search and matching frictions à la Mortensen and Pissarides (1994), they find that strict inflation targeting is inefficient in terms of welfare. Indeed, fully stabilized inflation leads to persistent fluctuations in unemployment in response to technology shocks, whereas it is constant in the constrained efficient allocation (i.e. the central planner problem).

Gertler et al. (2008), Sala et al. (2008) or Christoffel et al. (2009), among others, estimate models with labor-market frictions. Finally, Christiano et al. (2011) estimate a small open economy model of Sweden including both labor-market and financial frictions.

The importance of labor-market and financial frictions for business cycle and policy analysis

Despite the favorable attention received by this research, policy simulations, forecasts and business cycle analyses in institutions are mostly done using models based on Smets and Wouters (2003, 2007). If these tools generally incorporate open economy features, they use an unsophisticated representation of labor and credit markets. For instance, the ECB uses the New Area Wide Model (Christoffel et al. (2008)) for business cycle analysis and forecasts, and the Eagle (Gomes et al. (2010)) for policy evaluations. Other examples are the Ramses used by the Swedish central bank (Adolfson et al. (2008)), The Aino model at the Finnish central bank (Kilponen and Verona (2014)) and the Edo model at the US Federal Reserve (Edge et al. (2010)).[2]

Smets and Wouters' framework has been adopted by institutions mainly owing to the fact that it fits data well. Smets and Wouters (2004) show that its forecasting performance compares to a standard VAR. In fact, this success is partly due to the fact that the theoretical restrictions imposed by the model are limited, whereas the dynamics of seven exogenous shocks, assumed to follow AR or ARMA processes, capture a large part of the variations in the data. But this lessens somewhat their relevance for policy analysis.

Models extended to account for frictions in labor or credit markets are sometimes used in institutions to address specific questions, but they have not supplanted simpler models for operational policy and business cycle analysis, partly because of their accrued complexity. It is thus necessary to question the benefits of including these frictions in theoretical models for operational use. In this thesis, I address this issue and show that microfounded mechanisms specific to labor and credit markets can significantly alter the conclusions based on the use of an estimated DSGE model, from both a positive and a normative perspective. For this purpose, I build a two-country model of France and the rest of the euro area with exogenous rest of the world variables, and estimate it with and without these two frictions using Bayesian techniques. With respect to Smets and Wouters' framework, I add considerable theoretical restrictions: the behavior of the new variables included in the model is strongly constrained by theory, and these variables have strong feedback effects on the dynamics of the 'core' variables. These restrictions inevitably narrow the explanatory power of some exogenous shocks, and reduce the fit of the model. But structural shocks and their propagation are more accurately identified, so the model is able to dispense more refined policy recommendations.

By contrast with the papers cited above that have investigated the relevance of labor and credit-market frictions for policy analysis, I use the recent developments in macro-modeling in the specific context of the euro area in times of financial crisis. In particular, this environment is characterized by a single currency, by concerns about the possible exit of some countries, by labor-

[2] The Finnish central bank currently uses a version of the Aino model without financial frictions, but is working on an extension including a banking sector.

market inefficiencies and by a growing interest in policies aimed at reducing the tax burden on employment.

I undertake a number of analyses to highlight the role of financial and labor market frictions: historical shock decompositions of fluctuations during the crisis, an evaluation of the welfare cost of fluctuations under several monetary policy rules, and counterfactual simulations under the assumption of a flexible exchange rate regime between France and the rest of the euro area. Then I use the model to simulate social VAT scenarios and formulate employment-promoting policy recommendations.

A large-scale DSGE model of France in the euro area

I build and estimate a theoretical model, where agents' short term behaviors in the business cycle are consistent with the long run state of the economy. I elaborate on previous papers by considering a larger number of observable variables, and extending existing DSGE models along several dimensions. The end product is more sophisticated than many of them, and one of the largest models estimated with Bayesian techniques so far.

The basic structure builds on the new-Keynesian model of Smets and Wouters (2003, 2007). It incorporates many open economy features that are relevant for the euro area: it is considered as a small open economy with respect to the rest of the world, it is splitted into two countries representing France and the rest of the euro area, trading with each other, and oil prices impact both production and final consumption.

Financial and labor-market frictions are introduced in the model in the form of two independant options. Put differently, there are four versions of the model: a basic one with none of these frictions, which is hence an open economy version of Smets and Wouters' framework, a model with only financial frictions, a model with only labor market frictions, and a complete version including both.

Financial frictions are captured by a standard financial accelerator mechanism as in Bernanke et al. (1999): entrepreneurs need external financing to invest in productive capital. Credit-market imperfections give rise to a countercyclical external finance premium, which contributes to propagate and amplify aggregate shocks to the economy.

For labor market frictions, I use the standard search and matching framework à la Mortensen and Pissarides (1994): unemployed workers are involved in job search, while firms offer vacant job positions. The number of job seekers and the number of vacancies serve as inputs into an aggregate matching function, which determines the number of jobs actually created. Job matches generate a surplus for the economy. Workers and firms bargain over this surplus to set the wage rate ('Nash bargaining'). The specification of search and matching frictions used in my model follows more specifically Blanchard and Galí (2010)'s new-Keynesian setup. In particular, new jobs contribute immediately to production, whereas the original RBC framework assumes a one-period lag.

Yet, this representation of labor markets has significant shortcomings. First, it does not replicate the slack dynamics of wages that are observed in the data. Second, it ignores the business cycle fluctuations in the size of the labor force, which affect unemployment. I propose two original extensions to address these issues.

Improvements in the specification of labor markets

Wage stickiness

An originality of the proposed model lies in the way wage rigidity is introduced. A widespread view about wage rigidity, promoted in particular by Shimer (2005) and Hall (2005), is that search and matching models generate a lower volatility of employment and vacancies than in the data because bargained wages react strongly and procyclically to exogenous disturbances, thus reducing firms' incentive to post vacancies. On the grounds that wages are very sluggish in the data, they argue that sticky wages can be a solution to match the observed dynamics of both wages and employment.

In the present model, I rather follow Pissarides (2009) who argues that the apparent wage stickiness results from the fact that the average wage rate includes procyclical bargained wages associated with new jobs and, for a large part, inert wages of continuing jobs. Since hiring decisions are based on the expected wage of new jobs, rigidities applying to all jobs are not the right answer to the lack of responsiveness of job creation in traditional search and matching models.

Accordingly, I explicitely define the observed real wage rate as an average between those corresponding to new jobs, which are the outcome of a standard bargain à la Nash, and the other ones, which are simply indexed to the trend in technology. The original aspect of my work in this regard is to incorporate Pissarides (2009) theoretical continuous-time framework into a DSGE model. In the end, this assumption makes the average wage very sluggish, while job creation behaves as if all wages were fully flexible.

To obtain a sufficient degree of responsiveness in job creation, I follow one aspect of the calibration strategy recommended by Hagedorn and Manovskii (2008): firms accounting profits are quantitatively small and therefore very sensitive to economic conditions.[3]

Labor force participation

Modeling the cyclical behavior of the labor force participation rate raises an issue that has been described by several authors, including Tripier (2003), Veracierto (2008) and Ravn (2008): basic models overestimate the volatility and the procyclicality of the participation rate.[4] I contribute to this literature by proposing two joint credible explanations for the dynamic behaviour of participation over the business cycle: heterogeneity in preferences and the presence of unemployment benefits. In order to get a better understanding of the implications of these assumptions, I study them separately from the estimated model of France and the euro area. Instead, they are embedded into a simple DSGE model with search and matching frictions, where the standard perfect insurance assumption is replaced by an imperfect system of consumption allocation. This arrangement makes participation voluntary and unemployment involuntary (in the sense that unemployed workers are

[3]The other recommendation of these authors is to use low values of the bargaining power of workers. It primarily aims at replicating the sluggishness of wages and is thus not needed here.

[4]These papers focus on US data.

worse-off than employed workers). I calibrate this model so that it replicates the volatility and the correlation with output of the US participation rate, as well as many other dynamic properties of the rest of the economy.

I have not included this setup into the estimated model of France and the euro area yet. This work is left for future research.

Next, in building the model, I have carefully studied its main ingredients using separate small-scale frameworks. These analyses have revealed a number of issues, which have guided my modeling choices and the design of one of the policy evaluation exercises proposed in this thesis. These issues are described in what follows.

Modeling issues

Risk shocks and capital utilization

Christiano et al. (2014) put forward the financial 'risk' shock as a major source of fluctuations in the US and the euro area economies. Their model assumes that households owning productive capital incur maintenance costs depending on the capital utilization rate. Models assuming instead that capital depreciation is variable and depends on utilization, like in Greenwood et al. (1988), would probably not attribute such a big role to this shock. Indeed, 'risk' shocks would then imply negative comovements between the capital utilization rate and investment, in contradiction with the data.

The mechanism is the following. Contractionary 'risk' shocks affect negatively credit supply. They decrease the borrowing capacity of entrepreneurs and hence their demand for capital. As a result, the market price of capital also falls, which makes its depreciation relatively less expensive. This translates into a positive reaction of the capital utilization rate, while investment goes down.

Wage stickiness à la Pissarides (2009) and endogenous layoffs

Wage rigidities à la Pissarides (2009) differentiate the wage of newly hired workers and the wages associated with continuing jobs. As previously mentioned, this assumption makes the average wage very sluggish, while not affecting job creation. However, its effects are very contrasted if the model also assumes endogenous job separations as in Den Haan et al. (2000). Indeed, the wages of new jobs vary approximatly as much as productivity, while the other wages remain almost constant. Facing expansionary shocks that increase both labor demand and bargained wages, firms respond by contracting the number of vacancies they post, while also reducing the number of layoffs, in order to match their needs for additional workforce.

Although the replacement of expensive workers by cheaper ones in response to business cycle variations in bargained wages is a mechanism of some relevance, the predictions of such a model are quantitatively at odds with the data.

In addition, assuming a quadratic adjustment cost on layoffs is not able to totally resolve this problem.

Permanent shocks in open economies with incomplete markets

It is well known that open economy models with incomplete markets do not have a unique steady state equilibrium. A common solution to make these models stationary is to assume that the return rate of foreign assets includes a risk premium which depends on the country's net foreign asset position. The steady state equilibrium is then determined by the specification and the calibration of the risk premium function. If DSGE models are primarily designed for the study of business cycle fluctuations, they are also often used to simulate the transition from an initial steady state to another one in response to a permanent stimulus. An example is the simulation of fiscal policies that feature permanent tax changes. With the risk premium assumption, the final steady state is characterized by the same level of foreign debt as in the initial steady state. As a consequence, the short term responses of international trade flows are quantitatively small.

Although this practice is frequent in the literature, preventing any long term effect on foreign debt by using an ad hoc modeling trick such as the risk premium is theoretically questionable. I suggest instead that the final steady state equilibrium after a permanent shock should be determined by the equalization of households' utilities across countries. To justify this condition, I assume that people can migrate between countries, but only a long time after they make the decision to do so.

To illustrate the effects of this alternative assumption, I simulate a permanent social VAT measure in a two-country real business cycle model. I find that it generates larger movements in international trade flows during the transition towards the new steady state equilibrium.

Output gap in the Taylor rule and stochastic growth

Last, standard Taylor rules assume that the nominal interest rate reacts to a measure of output gap. The specification of this variable varies from a model to another; some compute the level of output that would prevail in the absence of nominal frictions, others simply use output or the growth rate of output. When real aggregates include a stochastic trend, simply using the level or the growth rate of detrended output (the cyclical component) instead of the growth rate of actual output decreases the efficiency of the Taylor rule in stabilizing inflation after permanent technology shocks. This raises some difficulties for estimation. A model where the Taylor rule does not respond explicitely to these shocks may strongly exaggerate the volatility of the inflation rate. Conversely, the estimated volatility of the permanent shock may be understated, leaving a unit root in the cyclical components of real aggregates. In the model of chapter 3, I assume that the nominal interest rate responds to the growth rate of actual output.

Estimation methodology

The four versions of the proposed model of France and the euro area are estimated using Bayesian techniques. By contrast with many previous estimations of large-scale DSGE models, I have paid special attention to long run restrictions throughout the estimation process. I have also implemented methodological improvements to satisfy specific constraints implied by this work.

Steady state and estimation

Many models estimated using a Bayesian approach in the literature only partly take into account steady state restrictions. On the one hand, their estimation procedure omits to compute and examine the full steady state for each set of parameters considered. On the other hand, a number of long run properties of the model are fixed. These points are hardly discussed in the literature, apart from Del Negro and Schorfheide (2008) who consider priors for the parameters determining the steady state. A number of studies rather focus on a closely related issue that is parameter identification (see Canova and Sala (2009)).

The standard approach to Bayesian estimation uses a first-order approximation of a DSGE model, which is valid in the neighborhood of its long run equilibrium. Once this linear form is derived analytically, the dynamic model can be manipulated setting aside the steady state. A number of structural parameters of the exact non-linear model may be absent from this linear form and can be calibrated or simply ignored. There are also parameters that are present in the approximated form of the model but that are related to the steady state. A common practice consists in calibrating them too: for instance, Smets and Wouters (2007) calibrate the capital depreciation rate, the steady state markup rate in the labor market and the exogenous spending-to-GDP ratio.

This procedure simplifies the estimation but also imposes restrictions. Indeed, unless identification tests are carried out so that only non-identifiable parameters are calibrated, adjusting the steady state values may improve the dynamic model's fit. That is a first practical reason to estimate them all. A second theoretical reason stems from the Bayesian paradigm. Accordingly, all parameters are random variables characterized by probability density functions, instead of having point values that the estimation aims to reveal. In this view, calibrating parameters amounts to setting their variances a priori to zero, which means full confidence in the chosen values. Using non-degenerated prior densities instead may hence be preferable, even if the data is not informative on them. In such cases, their conditional distributions a posteriori will be identical to their prior distributions. By contrast, posterior estimates of parameters affecting both the steady state and the cyclical properties of the model may be updated.

Rigorously estimating parameters impacting the steady state requires the full static model to be numerically solved when parameters vary, which is very costly in terms of computing time. Of course it is possible to omit this operation, using only the linear form of the dynamic model, but then parameters may take values for which the steady state, at worst, does not exist, or, at least, has some unrealistic properties.

To avoid this problem in estimating the present model, I follow the rigorous approach and solve the steady state equilibrium each time the likelihood of the model's first order approximation is computed. In doing so, I check its existence and impose that a number of long term ratios are always consistent with the data.

Methodological contributions

Macroeconomic datasets often include missing observations (due to delays in the computation of the latest points or simply because some variables were not measured in the past), or timeseries with low frequencies. For the present model, the problem is to observe, within a quarterly business cycle framework, an annual timeseries of hours worked. Indeed, without labor market frictions, the model accounts for fluctuations in total hours of labor, and not properly speeking in employment. If quarterly employment in the euro area has been available with sufficient anteriority for a long time, this was not the case for total hours of labor when I initially built my database: this series was annual and started in 2000.[5] So I describe first a modification of the Kalman filter that makes estimation possible using timeseries with as many missing observations as one wishes.[6]

Then, the estimation of models with many observed variables raises the question of the choice of the structural shocks in which the business cycle is assumed to originate. With full-information estimation methods, the number of the latter needs to be at least equal to the number of observed variables. My experience is that this constraint may sometimes contribute to deteriorate the dynamic properties of estimated large-scale models. This is related to the fact that some shocks have, obviously, counterfactual implications on some variables. Yet, the estimation of misspecified models – i.e. distinct from the data generating process, that is all models – may overestimate their size simply because they capture, like residuals, the gaps between the model predictions and the observed timeseries. I propose two numerical experiments to illustrate this point and support the use of measurement errors for Bayesian estimation.

Last, the estimation of large-scale models on the Dynare platform raises a number of computational issues. The resolution of the steady state is not possible analytically, but a full numerical resolution is unstable and time consuming. In this work, I developped an hybrid approach, in which numerical resolution is limited to small-scale problems handled by compiled programs. Another estimation issue is related to seeking the maximum of a likelihood or a posterior density function depending on many parameters. Often, standard approaches based on a gradient algorithm are unstable and require many “manual” interventions. The technical improvements proposed for this model have increased the stability and the speed of the numerical optimization phase.

Business cycle and policy analysis exercises to highlight the role of frictions

The purpose of this thesis is to show that including labor-market and financial frictions in DSGE models is crucial for business cycle and policy analysis. For that purpose, I use the four versions of the estimated model in various simulation exercises. The complete version is viewed as the most realistic representation of reality among the four considered, since it accounts for imperfections that

[5] It is now available at quarterly frequency and could be used to update my estimations.

[6] This contribution (as well as many developments of the model) results from a joint work with Stéphane Adjemian and has been incorporated to Dynare.

are present in actual credit and labor markets. Hence, the differences with the simulations obtained using the three others reveal the prejudice due to the omission of the frictions.

Historical shock decomposition

I compare the historical shock decompositions over 2007-2013 obtained from the models. First, omitting labor market frictions leads to overstate the weights of foreign trade and of markups in the dynamics of labor and GDP. This is because these shocks act as “reduced-form” parts of the model, capturing developments in observed time series that would be explained differently with fully specified labor markets. The presence of financial frictions reveals the significant contribution of risk shocks to the business cycle fluctuations of investment, GDP, inflation, and, to a lower extent, labor. Otherwise, the persistence of the recent crisis is captured to a larger extent by permanent technology shocks, supporting a “supply-side” interpretation of the recession. Some benefits of the interaction between labor market and financial frictions are also identified thanks to this exercise. For instance, the contribution of risk shocks to French employment only becomes significant in the presence of search and matching frictions.

Monetary policy and welfare

Second, I use the estimated models to compute the welfare cost of business cycles in both regions. I find values between 1 and 2.2% of lifetime consumption, consistently with previous estimates, and much more than suggested by Lucas (1987). Financial frictions and, to a lesser extent, labor market frictions add substancial welfare costs. The results also suggest that the welfare costs added by labor market frictions may be slightly higher in France than in the rest of the euro area. Then I evaluate different calibrations of the model’s Taylor rule using this criterion. This exercise suggests that the monetary authority cannot strongly improve welfare in the euro area by only responding to contemporaneous inflation.

Counterfactual simulations assuming that France has exited the euro

I simulate the effects of a flexible exchange rate regime by comparison with the monetary union. First, I find no significant impact on the welfare cost of fluctuations in all versions of the model. Then, a counterfactual simulation of the recent financial crisis, assuming that France exited from the euro area in 2009Q1, identifies a small appreciation of the French franc against the euro. This would have somewhat deteriorated the French economy. Financial and labor market frictions significantly influence the simulations: the absence of labor market frictions amplifies the short term effects of the exit on GDP and employment, whereas the absence of financial frictions in the model with labor market frictions leads to a larger appreciation of the FF but smaller real effects in France.

Social VAT

The effects of transitory social VAT policies are simulated in the context of the financial crisis of 2009. These policies are assumed to target a reduction in the gap between actual employment and the level of employment that would have prevailed in the absence of nominal frictions in the euro area. Their duration is set to 3 years.

I find that the considered measures yield limited benefits, due to the presence of many frictions in the model, especially labor market imperfections. First, with search and matching frictions, adjustments to employment are protracted. Firms have to pursue a long-term recruitment strategy, similar to investment in productive capital. Therefore, the duration of the measure is determinant for the immediate reaction of labor demand. Second, the estimation identifies a high degree of real inefficiencies in the labor markets of the French and euro area economies, which undermines the transmission to actual job creation.

The efficiency of social VAT could be significantly increased. For that purpose, two policy recommendations can be made on the basis of this analysis. First, social VAT measures should last for a sufficiently long period of time, at least 5 years, this duration being considered credible by private agents. Second, structural reforms of the functioning of the labor market should be implemented first. These reforms should improve the efficiency of the matching process between job seekers and firms. This is a key determinant of the success of policies based on reducing labor costs in the euro area.

Finally, this exercise shows that, in any case, a unilateral social VAT policy in France is preferable to a coordinated implementation across the euro area.

Layout of the thesis

In Chapter 1, I present the resolution and estimation techniques that have been applied to the model, and propose methodological improvements. Chapter 2 investigates the main ingredients of the model, and presents related modeling issues. The model of France and the rest of the euro area is built in Chapter 3. The financial accelerator and search and matching frictions are also described there. Chapter 4 presents the estimation results and some dynamic properties of the complete model. Chapter 5 covers the simulations of the estimated models, and discusses the importance of labor-market and financial frictions for the results. In Chapter 6, the complete model is used to evaluate social VAT policies. Chapter 7 develops a model with endogenous labor force participation. This final chapter can be read as a separate paper.

Chapter 1

Bayesian estimation: methodology and technical developments

1.1 Model resolution

1.1.1 General approach

A DSGE model consists of equilibrium conditions and of equations describing the choices of agents who maximize an objective function under constraints. Once the first order conditions are derived from the optimization program, it reduces to a recursive system of equations of a vector of endogenous variables y_t and of exogenous variables ε_t, which can be written in general terms

$$E_t f(y_{t+1}, y_t, y_{t-1}, \varepsilon_t) = 0, \tag{1.1.1}$$

where E_t is the expectations operator conditional on the information available at date t. Exogenous variables in ε are iid white noises with variance-covariance matrix Σ.

Solving the model consists in finding a form that can be used to compute the present and future values of y using the past values of y and the present values of ε, or more specifically a recursive function h such that

$$y_t = h(y_{t-1}, \varepsilon_t). \tag{1.1.2}$$

A very common approach to solve the model is the (first-order) perturbation method (see Judd and Guu (1997), Gaspar and Judd (1997), Judd (1996)). Christiano (2002) or King and Watson (2002) call it 'linear approximation' and apply it to real business cycle models. It is described below for standard well-designed models. The first step consists in solving for the deterministic steady state of the model $\bar{y}$, which verifies the non-linear system $f(\bar{y}, \bar{y}, \bar{y}, 0) = 0$. Then, the objective of the perturbation approach is to find the linear recursive solution to a linear approximation of the model. It is hence valid only when y_t remains in a neighborhood of its steady state $\bar{y}$. This solution should have the form

$$y_t = \bar{y} + h_1(y_{t-1} - \bar{y}) + h_2\varepsilon_t, \tag{1.1.3}$$

where h_1 and h_2 are unknown matrix. To compute h_1 and h_2, we write a first order Taylor development of the model (1.1.1) for small values of $\hat{y}_{t-1} = y_{t-1} - \bar{y}$:

$$E_t f\left(h(h(\hat{y}_{t-1} + \bar{y}, \varepsilon_t), \varepsilon_{t+1}), h(\hat{y}_{t-1} + \bar{y}, \varepsilon_t), \hat{y}_{t-1} + \bar{y}, \varepsilon_t\right) = 0,$$

$$E_t\left[f(\bar{y}, \bar{y}, \bar{y}, 0) + f_1 h_1 h_1 \hat{y}_{t-1} + f_1 h_1 h_2 \varepsilon_t + f_1 h_2 \varepsilon_{t+1} + f_2 h_1 \hat{y}_{t-1} + f_2 h_2 \varepsilon_t + f_3 \hat{y}_{t-1} + f_4 \varepsilon_t\right] = 0,$$

where f_1 to f_4 represent the known Jocobian matrix of f with respect to arguments 1 to 4. Considering that $f(\bar{y}, \bar{y}, \bar{y}, 0) = 0$ and $E_t \varepsilon_{t+1} = 0$ his equation simplifies to

$$f_1 h_1 h_1 \hat{y}_{t-1} + f_1 h_1 h_2 \varepsilon_t + f_2 h_1 \hat{y}_{t-1} + f_2 h_2 \varepsilon_t + f_3 \hat{y}_{t-1} + f_4 \varepsilon_t = 0.$$

It implies that

$$\begin{cases} f_1 h_1 h_1 + f_2 h_1 + f_3 = 0 \\ f_1 h_1 h_2 + f_2 h_2 + f_4 = 0 \end{cases}$$

The first polynomial matrix equation $f_1 h_1 h_1 + f_2 h_1 + f_3 = 0$ can be solved to find h_1 using different methods. Dynare uses a real QZ decomposition (Klein (2000) and Sims (2001)). Note that, once h_1 and h_2 are computed, equation (1.1.3) defines an exact solution of the approximated model and not an approximation of the exact unknown solution h of the model.

The following subsection presents in further details the resolution as it is done by Dynare.

1.1.2 Resolution in Dynare

Models resolution with a perturbation method can be greatly simplified by using the software Dynare (Collard and Juillard (2001a,b)). Dynare identifies variables with lags as state variables, variables with leads as jumping variables; the other variables are static. For simplicity, I assume that the model only includes m "srictly" forward and p "strictly" backward variables, q variables that are both, and that the maximum leads and lags are one period. Indeed, the model can always be arranged to satisfy those requirement by substituting static variables and using auxiliary variables. The deviations of endogenous variables from their steady state levels are sorted in such a manner that

$$\hat{y}_t = \begin{pmatrix} \hat{k}_t \\ \hat{c}_t \\ \hat{b}_t \end{pmatrix},$$

where k_t denotes the vector of state variables, c_t the vector of jumping variables, and b_t the vector of variables that are both. The total number of endogenous variables is $n = m + p + q$ and the number of exogenous shocks is r. The searched approximated solution of the model associates the value of all endogenous variables with the past value of the state variables and the realization of the shocks in the present period; it has the form

$$\hat{y}_t = H \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \end{pmatrix} + R\varepsilon_t, \tag{1.1.4}$$

where H and R are two unknown matrix with respective sizes $n \times p$ and $n \times r$. I break down H and R as follows:

$$H = \begin{pmatrix} \underset{p\times(p+q)}{M} \\ \underset{(m+q)\times(p+q)}{N} \end{pmatrix} \quad R = \begin{pmatrix} \underset{p\times r}{U} \\ \underset{(m+q)\times r}{V} \end{pmatrix}.$$

The non linear model of equation (1.1.1) can be re-written as

$$E_t g(c_{t+1}, b_{t+1}, k_t, c_t, b_t, k_{t-1}, b_{t-1}, \varepsilon_t) = 0.$$

Its first order Taylor approximation is then

$$g_1 E_t \hat{c}_{t+1} + g_2 E_t \hat{b}_{t+1} + g_3 \hat{k}_t + g_4 \hat{c}_t + g_5 \hat{b}_t + g_6 \hat{k}_{t-1} + g_7 \hat{b}_{t-1} + g_8 \varepsilon_t = 0. \tag{1.1.5}$$

where

$$\underset{n\times(2n+q+r)}{\mathcal{J}} = \begin{pmatrix} g_1 & g_2 & g_3 & g_4 & g_5 & g_6 & g_7 & g_8 \end{pmatrix}$$

is the Jacobian matrix of the model computed analytically and numerically by Dynare. Equation (1.1.5) is arranged into a matrix form

$$AE_t \begin{pmatrix} \hat{k}_t \\ \hat{b}_t \\ \hat{c}_{t+1} \\ \hat{b}_{t+1} \end{pmatrix} = B \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \\ \hat{c}_t \\ \hat{b}_t \end{pmatrix} - g_8 \varepsilon_t, \tag{1.1.6}$$

where

$$\underset{(n+q)\times(n+q)}{A} = \begin{pmatrix} g_3 & g_5 & g_1 & g_2 \\ 0 & I & 0 & 0 \end{pmatrix} \qquad \underset{(n+q)\times(n+q)}{B} = \begin{pmatrix} -g_6 & -g_7 & -g_4 & 0 \\ 0 & 0 & 0 & I \end{pmatrix}.$$

Since A may be singular, Dynare computes a real QZ decomposition of the pencil <A,B> for which the generalized eigenvalues are sorted in ascending order according to their modulus. The output includes Q and Z two unitary matrix, T and S two block upper triangular matrix, with 1×1 or 2×2 blocks on the diagonal, such that $A = QTZ$ and $B = QSZ$. The blocks in S and T correspond to these generalized eigenvalues: 1×1 blocks correspond to either real or infinite eigenvalues S_{ii}/T_{ii}, 2×2 blocks correspond to complex conjugate pairs of eigenvalues. Given that eigenvalues are sorted, matrix S, T and Z can be split horizontally into an upper part which corresponds to eigenvalues with modulus smaller than one and a lower part which corresponds to eigenvalues with modulus strictly greater that one (Klein (2000) and Sims (2001)). If the Blanchard and Kahn (1980) conditions are verified for the model, then the number of eigenvalues larger than one in modulus is equal to the number of forward variables, that is $m+q$. Formally, the matrix S, T and Z are broken down as follows:

$$S = \begin{pmatrix} \underset{(p+q)\times(p+q)}{S_{11}} & \underset{(p+q)\times(m+q)}{S_{12}} \\ \underset{(m+q)\times(p+q)}{0} & \underset{(m+q)\times(m+q)}{S_{22}} \end{pmatrix} \quad T = \begin{pmatrix} \underset{(p+q)\times(p+q)}{T_{11}} & \underset{(p+q)\times(m+q)}{T_{12}} \\ \underset{(m+q)\times(p+q)}{0} & \underset{(m+q)\times(m+q)}{T_{22}} \end{pmatrix}$$

$$Z = \begin{pmatrix} \underset{(p+q)\times(p+q)}{Z_{11}} & \underset{(p+q)\times(m+q)}{Z_{12}} \\ \underset{(m+q)\times(p+q)}{Z_{21}} & \underset{(m+q)\times(m+q)}{Z_{22}} \end{pmatrix}$$

Equation (1.1.6) becomes

$$\begin{pmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{pmatrix} \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} E_t \begin{pmatrix} \hat{k}_t \\ \hat{b}_t \\ \hat{c}_{t+1} \\ \hat{b}_{t+1} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix} \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \\ \hat{c}_t \\ \hat{b}_t \end{pmatrix} - Q' g_8 \varepsilon_t. \tag{1.1.7}$$

Let

$$\hat{s}_t = \begin{pmatrix} Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \\ \hat{c}_t \\ \hat{b}_t \end{pmatrix} \quad \text{and} \quad \hat{u}_t = \begin{pmatrix} Z_{11} & Z_{22} \end{pmatrix} \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \\ \hat{c}_t \\ \hat{b}_t \end{pmatrix}$$

Then

$$\begin{pmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{pmatrix} E_t \begin{pmatrix} \hat{u}_{t+1} \\ \hat{s}_{t+1} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix} \begin{pmatrix} \hat{u}_t \\ \hat{s}_t \end{pmatrix} - Q' g_8 \varepsilon_t. \tag{1.1.8}$$

The lower part of this equation $T_{22} E_t \hat{s}_{t+1} = S_{22} \hat{s}_t + \omega \varepsilon_t$ corresponds to explosive trajectories. The intuition for this statement is the following: if I assume for simplicity that T and S are strictly triangular (only real eigenvalues), then the last line of the matrix equation implies that

$$E_t \hat{s}_{m+q,t+1} = \frac{S_{n+q,n+q}}{T_{n+q,n+q}} \hat{s}_{m+q,t} + \sum_{i=1}^{r} \omega_{m+q,i} \varepsilon_{i,t},$$

where ω_{ij} represent the coefficients of matrix $-Q' g_8$ and $\lambda_{n+q} = S_{n+q,n+q}/T_{n+q,n+q}$ is the $n+q$-th eigenvalue, strictly greater than one. Hence, the equation is solved forward as follows:

$$s_{m+q,t} = \lim_{j\to\infty} \lambda_{n+q}^{-j} E_t \hat{s}_{m+q,t+j} - \sum_{j=0}^{\infty} \lambda_{n+q}^{-j-1} \sum_{i=1}^{r} \omega_{m+q,i} E_t \varepsilon_{i,t+j}$$

$$s_{m+q,t} = -\sum_{i=1}^{r} \frac{\omega_{m+q,i}}{\lambda_{n+q}} \varepsilon_{i,t}$$

Next, the line just above implies

$$E_t \hat{s}_{m+q-1,t+1} = \frac{S_{n+q-1,n+q-1}}{T_{n+q-1,n+q-1}} \hat{s}_{m+q-1,t} - \frac{T_{n+q-1,n+q}}{T_{n+q-1,n+q-1}} E_t \hat{s}_{m+q,t+1} + \frac{S_{n+q-1,n+q}}{T_{n+q-1,n+q-1}} \hat{s}_{m+q,t} + \sum_{i=1}^{r} \omega_{m+q-1,i} \varepsilon_{i,t},$$

so

$$E_t \hat{s}_{m+q-1,t+1} = \lambda_{n+q-1} \hat{s}_{m+q-1,t} + \sum_{i=1}^{r} \left(\omega_{m+q-1,i} - \frac{\omega_{n+q,i}}{\lambda_{n+q}} \frac{S_{n+q-1,n+q}}{T_{n+q-1,n+q-1}} \right) \varepsilon_{i,t},$$

and the trajectory is again explosive. This is continued recursively until the first line of the lower part of equation (1.1.8) is reached. Finally, eliminating explosive trajectories yields

$$\hat{s}_t = \begin{pmatrix} Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \\ \hat{c}_t \\ \hat{b}_t \end{pmatrix} = \Omega \varepsilon_t.$$

Given that the postulated solution of the model includes

$$\begin{pmatrix} \hat{c}_t \\ \hat{b}_t \end{pmatrix} = N \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \end{pmatrix} + V \varepsilon_t,$$

this condition implies that the solution verifies for any value of the initial state and of shocks:

$$\begin{pmatrix} Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I \\ N \end{pmatrix} \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \end{pmatrix} + \begin{pmatrix} Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} 0 \\ V \end{pmatrix} \varepsilon_t = \Omega \varepsilon_t.$$

In particular, provided that matrix Z_{22} is inversible, we get immediately

$$N = -Z_{22}^{-1} Z_{21}.$$

Moving forward, from the postulated solution of the model I get

$$\hat{k}_t = M \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \end{pmatrix} + U \varepsilon_t.$$

In addition,

$$\hat{b}_t = \begin{pmatrix} 0 & I \end{pmatrix} \begin{pmatrix} \hat{c}_t \\ \hat{b}_t \end{pmatrix} = \begin{pmatrix} 0 & I \end{pmatrix} N \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \end{pmatrix} + \begin{pmatrix} 0 & I \end{pmatrix} V \varepsilon_t.$$

So, finally,

$$E_t \begin{pmatrix} \hat{k}_t \\ \hat{b}_t \\ \hat{c}_{t+1} \\ \hat{b}_{t+1} \end{pmatrix} = \begin{pmatrix} I \\ N \end{pmatrix} \begin{pmatrix} \hat{k}_t \\ \hat{b}_t \end{pmatrix} = \begin{pmatrix} I \\ N \end{pmatrix} \begin{pmatrix} M \\ (0 \quad I)N \end{pmatrix} \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \end{pmatrix} + \Gamma\varepsilon_t.$$

The coefficients of ε_t are grouped in Γ; writing them explicitely is not needed for the resolution. Let

$$\underset{(p+q)\times(p+q)}{\tilde{M}} = \begin{pmatrix} M \\ (0 \quad I)N \end{pmatrix}.$$

Plugging into equation (1.1.7) yields:

$$\begin{aligned} &\begin{pmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{pmatrix} \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I \\ N \end{pmatrix} \tilde{M} \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \end{pmatrix} \\ &\qquad = \begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix} \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I \\ N \end{pmatrix} \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \end{pmatrix} + \tilde{\Gamma}\varepsilon_t. \end{aligned}$$

The upper block implies that

$$\left(T_{11}Z_{11} + T_{12}Z_{21} + T_{11}Z_{12}N + T_{12}Z_{22}N\right)\tilde{M} = S_{11}Z_{11} + S_{12}Z_{21} + S_{11}Z_{12}N + S_{12}Z_{22}N$$

Substituting N by $-Z_{22}^{-1}Z_{21}$, we get

$$T_{11}\left(Z_{11} - Z_{12}Z_{22}^{-1}Z_{21}\right)\tilde{M} = S_{11}\left(Z_{11} - Z_{12}Z_{22}^{-1}Z_{21}\right).$$

Now, the fact that Z is unitary implies, after some algebra (assuming that Z_{11} is invertible),

$$Z_{12}Z_{22}^{-1}Z_{21} = Z_{11} - \left(Z_{11}'\right)^{-1}.$$

So

$$T_{11}\left(Z_{11}'\right)^{-1}\tilde{M} = S_{11}\left(Z_{11}'\right)^{-1}. \tag{1.1.9}$$

To compute $\tilde{M}$, the two linear equations of M_1 and M_2 below are solved numerically

$$T_{11}'M_1' = Z_{11} \quad \text{and} \quad Z_{11}M_2' = S_{11}'.$$

Since T_{11} is also invertible by construction, the solutions M_1 and M_2 verify

$$M_2 = S_{11}\left(Z_{11}'\right)^{-1},$$

and

$$M_1 = Z'_{11} T_{11}^{-1}.$$

Therefore

$$T_{11} \left(Z'_{11}\right)^{-1} M_1 M_2 = T_{11} \left(Z'_{11}\right)^{-1} Z'_{11} T_{11}^{-1} S_{11} \left(Z'_{11}\right)^{-1} = S_{11} \left(Z'_{11}\right)^{-1},$$

and $\tilde{M} = M_1 M_2$ solves (1.1.9). M is finally obtained from $\tilde{M}$ by taking its first p rows. To compute the decision rule with respect to exogenous variables R, equation (1.1.5) can be written in the matrix form below:

$$\begin{pmatrix} 0 & g_1 & g_2 \end{pmatrix} E_t \hat{y}_{t+1} + \begin{pmatrix} g_3 & g_4 & g_5 \end{pmatrix} \hat{y}_t + \begin{pmatrix} g_6 & g_7 \end{pmatrix} \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \end{pmatrix} + g_8 \varepsilon_t = 0.$$

Substituting the solution (1.1.4) yields

$$\begin{pmatrix} 0 & g_1 & g_2 \end{pmatrix} H \begin{pmatrix} \hat{k}_t \\ \hat{b}_t \end{pmatrix} + \begin{pmatrix} g_3 & g_4 & g_5 \end{pmatrix} \hat{y}_t + \begin{pmatrix} g_6 & g_7 \end{pmatrix} \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \end{pmatrix} + g_8 \varepsilon_t = 0,$$

$$\begin{pmatrix} 0 & g_1 & g_2 \end{pmatrix} H \begin{pmatrix} I & 0 & 0 \\ 0 & 0 & I \end{pmatrix} \hat{y}_t + \begin{pmatrix} g_3 & g_4 & g_5 \end{pmatrix} \hat{y}_t + \begin{pmatrix} g_6 & g_7 \end{pmatrix} \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \end{pmatrix} + g_8 \varepsilon_t = 0,$$

$$\left[\begin{pmatrix} 0 & g_1 & g_2 \end{pmatrix} H \begin{pmatrix} I & 0 & 0 \\ 0 & 0 & I \end{pmatrix} + \begin{pmatrix} g_3 & g_4 & g_5 \end{pmatrix} \right] \left[H \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \end{pmatrix} + R \varepsilon_t \right]$$
$$+ \begin{pmatrix} g_6 & g_7 \end{pmatrix} \begin{pmatrix} \hat{k}_{t-1} \\ \hat{b}_{t-1} \end{pmatrix} + g_8 \varepsilon_t = 0.$$

So R needs to verify

$$\left[\begin{pmatrix} 0 & g_1 & g_2 \end{pmatrix} H \begin{pmatrix} I & 0 & 0 \\ 0 & 0 & I \end{pmatrix} + \begin{pmatrix} g_3 & g_4 & g_5 \end{pmatrix} \right] R = -g_8,$$

which is solved numerically.

1.2 Bayesian estimation of the approximated model

1.2.1 Likelihood evaluation

Once the model is solved, the transition matrix H can be resized so that it is square by adding zeros in the adequate colums. The recursive solution of the approximated model has then the form

$$x_t = A x_{t-1} + B u_t,$$

where u_t is a vector of exogenous shocks with variance-covariance matrix Q. The observed variables are a subset of x_t, denoted by y_t, such that $y_t = Cx_t$. C is the 'selection' matrix. The model has the following state space representation

$$\begin{cases} x_t = Ax_{t-1} + Bu_t \\ y_t = Cx_t + \eta_t \end{cases}$$

where η_t is a vector of measurement errors. The sample is assumed to include T realizations of the vector of observable variables y_t. It is

$$Y_T = \{y_1, y_2, \ldots, y_T\}.$$

The goal is to compute the density of the sample conditionnally on the value of H, which depends on the values of the deep parameters of the model, denoted by θ. This density, also called likelihood, is formally

$$\mathcal{L}(\theta; Y_T) = p(Y_T|\theta) = \prod_{t=1}^{T} p(y_t|Y_{t-1}, \theta),$$

where by assumption Y_0 is an empty set. The density of y_t conditional on the sample in $t-1$ is also equal to the density of the linear least square projection error of y_t, denoted by

$$\tilde{y}_t = y_t - \hat{E}[y_t|Y_{t-1}].$$

Under the assumption that shocks are Gaussian white noises, the linearity of the model imposes that projection errors are also normally distributed. Remind that the log-density function of a centered random vector k-variate normally distributed with covariance matrix F is

$$\log p(z) = -\frac{k}{2}\log 2\pi - \frac{1}{2}\log|F| - \frac{1}{2}z' F^{-1} z.$$

Therefore, we need to compute for all $t \in [1, T]$ the linear least square projection errors $\tilde{y}_t$ and the covariance matrix

$$F_t = \mathrm{var}(\tilde{y}_t) = E\left[\left(y_t - \hat{E}[y_t|Y_{t-1}]\right)\left(y_t - \hat{E}[y_t|Y_{t-1}]\right)\right].$$

$\tilde{y}_t$ and F_t are obtained from the Kalman filter recursive equations and initial assumptions for $\hat{E}[x_1]$ and Σ_1, following

$$\tilde{y}_t = y_t - C\hat{E}[x_t|Y_{t-1}],$$

$$F_t = C\Sigma_t C' + \mathrm{var}(\eta_t),$$

$$K_t = A\Sigma_t C' F_t^{-1},$$

$$\hat{E}[x_{t+1}|Y_t] = A\hat{E}[x_t|Y_{t-1}] + K_t\tilde{y}_t,$$

and

$$\Sigma_{t+1} = (A\Sigma_t - K_t C\Sigma_t)A' + BQB'.$$

Sections 1.2.2 and 1.2.3 below describe in greater details the Kalman filter and the Kalman smoother, which is used to compute the historical shock decomposition of a sample of observable variables, and present a proof of these equations. They are based on Lindqvist and Sargent (2004) textbook.

Note that these formulas require that F is invertible. The stochastic singularity problem (F is singular) arises in particular when the number of observed variables exceeds the number of shocks (including measurement errors). Indeed, the projection errors are random variables that are included in the state space spanned by (or, put differently, are generated by) the orthogonal shocks of the model. Hence, there exists a linear combination of projection errors which is constant and does not depend on stochastic shocks: $\lambda\tilde{y}_t$ = constant, so $\text{var}(\lambda\tilde{y}_t) = 0 = \lambda F\lambda'$, for λ non zero. Then by definition F is not definite positive. Intuitively, the model cannot find at each date a vector of shocks realization which matches all the observed variables in the data, so the likelihood cannot be evaluated.

Provided that the state space model is set up to avoid stochastic singularity, for all $t \in [1, T]$ we can compute

$$\log p(y_t|Y_{t-1}, \theta) = \log p(\tilde{y}_t|\theta) = -\frac{k}{2}\log 2\pi - \frac{1}{2}\log|F_t| - \frac{1}{2}\tilde{y}_t' F_t^{-1}\tilde{y}_t,$$

where k is the number of observable variables. So finally the log-likelihood of the sample is

$$\log \mathcal{L}(\theta; Y_T) = -\frac{Tk}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\left(\log|F_t| - \frac{1}{2}\tilde{y}_t' F_t^{-1}\tilde{y}_t\right).$$

Conversely, it is possible to compute the likelihood when the number of shocks (including measurement errors) is strictly greater than the number of observed variables. Obviously, in that case, the Kalman filter cannot identify orthogonal chronicles of shocks from the observations, as the filtered shocks are obtained from projections on a smaller number of independant variables. Moreover, it is likely that some parameters are not identifiable on the basis of the likelihood criterion; specifically, different combinations of values of exogenous shocks' variances yield the same variance-covariance matrix F and hence the same likelihood. Yet, in a Bayesian approach, the prior distribution assumed for parameters and shock standard deviations generally fills this gap: the mode of the posterior density may point to a unique vector of parameters. This is the reason why many Bayesian models ignore this issue and tolerate a large number of shocks, particularly when measurement errors are included.

1.2.2 The Kalman filter

Linear least square projections

Definition Let two vectors of random variables x and y, respectively of size n and m. The linear least square projection of x on y is the random variable

$$\hat{E}[x|y] = a + Py$$

where a and P are chosen such that $E\left[\mathrm{tr}(x - \hat{E}[x|y])(x - \hat{E}[x|y])'\right]$ is minimal. In order to find an expression for a and P, we write the first order conditions associated with this problem. First, the trace is

$$\mathrm{tr}(x - \hat{E}[x|y])(x - \hat{E}[x|y])' = \mathrm{tr}(x - a - Py)(x - a - Py)' = \sum_{k=1}^{n}\left(x_k - a_k - \sum_{i=1}^{m} p_{ki}y_i\right)^2.$$

The first order condition with respect to a given a_k is

$$E\left(x_k - a_k - \sum_{i=1}^{m} p_{ki}y_i\right) = 0$$

So when all these equations are stacked into a matrix, we get

$$a = E(x) - PE(y).$$

The first order condition with respect to a given p_{kj} is

$$E\left[\left(x_k - a_k - \sum_{i=1}^{m} p_{ki}y_i\right)y_j\right] = 0,$$

or in a matrix form

$$E[(x - a - Py)y'] = 0.$$

Substituting a yields

$$E[(x - E(x) - P(y - E(y))y'] = 0.$$

$$E[(x - E(x))y'] - PE[(y - E(y))y'] = 0.$$

As $E[(x - E(x))y'] = E[(x - E(x))(y - E(y))'] + E[(x - E(x))]E(y)' = E[(x - E(x))(y - E(y))']$ and the same for $E[(y - E(y))y']$, we have

$$\mathrm{cov}(x, y) - P\mathrm{var}(y) = 0$$

so

$$P = \mathrm{cov}(x, y)\mathrm{var}(y)^{-1},$$

where cov denotes the cross-covariance matrix between two vectors and var the variance-covariance matrix of a vector. The general form of the linear least square projection is thus

$$\hat{E}[x|y] = E(x) + \mathrm{cov}(x,y)\mathrm{var}(y)^{-1}(y - E(y)).$$

Orthogonality The linear least square projection is obviously "unbiaised" as

$$E\hat{E}[x|y] = E(x) + \mathrm{cov}(x,y)\mathrm{var}(y)^{-1}(E(y) - E(y)) = E(x).$$

Thus, from the first order conditions with respect to the coefficients of the matrix P, we get

$$E[(x - \hat{E}[x|y])(y - E(y))'] = 0,$$

so the projection errors are orthogonal to the regressors.

Iterative form Now we assume that the vector y is orthogonal to a vector z of random variables $\{z_i\}_{i=1,\dots,q}$. We write explicitely the expression of the projection of x on $(y', z')'$ and get

$$\begin{aligned}
\hat{E}[x|(y',z')'] &= E(x) + [\mathrm{cov}(x,y) \quad \mathrm{cov}(x,z)] \begin{bmatrix} \mathrm{var}(y) & 0 \\ 0 & \mathrm{var}(z) \end{bmatrix}^{-1} \begin{bmatrix} y - E(y) \\ z - E(z) \end{bmatrix} \\
&= E(x) + [\mathrm{cov}(x,y) \quad \mathrm{cov}(x,z)] \begin{bmatrix} \mathrm{var}(y)^{-1} & 0 \\ 0 & \mathrm{var}(z)^{-1} \end{bmatrix} \begin{bmatrix} y - E(y) \\ z - E(z) \end{bmatrix} \\
&= E(x) + \mathrm{cov}(x,y)var(y)^{-1}(y - E(y)) + \mathrm{cov}(x,z)var(z)^{-1}(z - E(z)) \\
&= \hat{E}[x|y] + \hat{E}[x|z] - E(x).
\end{aligned}$$

The Kalman filter

Consider a basic model of a vector of state variables x, where the exogenous variables in vector u are white noises with a variance-covariance matrix denoted by Q. Without measurement errors, the state-space form is

$$x_{t+1} = Ax_t + Bu_{t+1}$$

$$y_t = Cx_t.$$

The objective is to compute the linear least square projection of x_{t+1} on the information available at date t, $Y_t = \{x_0, y_1, ..., y_t\}$. This projection is denoted by $\hat{E}[x_{t+1}|Y_t]$. It amounts to compute the linear least square projection of x_t on the information Y_t, since

$$\hat{E}[x_{t+1}|Y_t] = A\hat{E}[x_t|Y_t].$$

To continue, we need an orthogonal basis of the linear space spanned by Y_t is the set of projection errors $\tilde{Y}_t = \{x_0, \tilde{y}_1, ..., \tilde{y}_t\}$, such that

$$\tilde{y}_1 = y_1 - \hat{E}[y_1|x_0],$$

$$\forall 1 < j \leq t, \quad \tilde{y}_j = y_j - \hat{E}[y_j|x_0, y_1, ..., y_{j-1}].$$

Linear projections on $\tilde{Y}_t$ are equal to linear projections on Y_t. As the regressors in $\tilde{Y}_t$ are orthogonal,

$$\hat{E}[x_t|Y_t] = \hat{E}[x_t|\tilde{Y}_t] = \hat{E}[x_t|\tilde{Y}_{t-1}] + \hat{E}[x_t|\tilde{y}_t] - E(x_t).$$

From the general form of linear projections,

$$\hat{E}[x_t|\tilde{y}_t] = E(x_t) + \mathrm{cov}(x_t, \tilde{y}_t)\mathrm{var}(\tilde{y}_t)^{-1}\tilde{y}_t.$$

Grouping these results yields

$$\hat{E}[x_{t+1}|Y_t] = A\hat{E}[x_t|Y_{t-1}] + A\ \mathrm{cov}(x_t, \tilde{y}_t)\mathrm{var}(\tilde{y}_t)^{-1}\tilde{y}_t.$$

Now we need to get an expression for $\mathrm{cov}(x_t, \tilde{y}_t)$ and $\mathrm{var}(\tilde{y}_t)$. The projection errors can be written

$$\tilde{y}_t = y_t - \hat{E}[y_t|Y_{t-1}] = C(x_t - \hat{E}[x_t|Y_{t-1}]).$$

So

$$\mathrm{var}(\tilde{y}_t) = E[C(x_t - \hat{E}[x_t|Y_{t-1}])(x_t - \hat{E}[x_t|Y_{t-1}])'C'] \equiv C\Sigma_t C',$$

and

$$\begin{aligned}
\mathrm{cov}(x_t, \tilde{y}_t) &= E[(x_t - E(x_t))(x_t - \hat{E}[x_t|Y_{t-1}])']C' \\
&= E[(x_t - \hat{E}(x_t|Y_{t-1}))(x_t - \hat{E}[x_t|Y_{t-1}])']C' + E[(\hat{E}(x_t|Y_{t-1}) - E(x))(x_t - \hat{E}[x_t|Y_{t-1}])']C' \\
&= E[(x_t - \hat{E}[x_t|Y_{t-1}])(x_t - \hat{E}[x_t|Y_{t-1}])']C' = \Sigma_t C'.
\end{aligned}$$

In this expression, $E[(\hat{E}(x_t|Y_{t-1}) - E(x))(x_t - \hat{E}[x_t|Y_{t-1}])']$ is zero because the projection is a linear application of past information Y_{t-1}, and the projection error is orthogonal to Y_{t-1}. The projection error of y_t is

$$\tilde{y}_t = y_t - \hat{E}[y_t|Y_{t-1}] = y_t - C\hat{E}[x_t|Y_{t-1}],$$

so the Kalman equation is finally

$$\hat{E}[x_{t+1}|Y_t] = A\hat{E}[x_t|Y_{t-1}] + K_t(y_t - C\hat{E}[x_t|Y_{t-1}]) \text{ with } K_t = A\Sigma_t C'(C\Sigma_t C')^{-1}.$$

Now we need a recursive rule for Σ_t. It comes from

$$\begin{aligned}\Sigma_{t+1} &= E[(x_{t+1} - \hat{E}[x_{t+1}|Y_t])(x_{t+1} - \hat{E}[x_{t+1}|Y_t])'] \\ &= E[(Ax_t + Bu_{t+1} - A\hat{E}[x_t|Y_t])(Ax_t + Bu_{t+1} - A\hat{E}[x_t|Y_t])'] \\ &= AE[(x_t - \hat{E}[x_t|Y_t])(x_t - \hat{E}[x_t|Y_t])']A' + BQB'\end{aligned}$$

As shown above,

$$\hat{E}[x_t|Y_t] = \hat{E}[x_t|Y_{t-1}] + \Sigma_t C(C\Sigma_t C')^{-1}\tilde{y}_t,$$

which implies

$$\begin{aligned}E[(x_t - \hat{E}[x_t|Y_t])(x_t - \hat{E}[x_t|Y_t])'] &= E[(x_t - \hat{E}[x_t|Y_{t-1}])(x_t - \hat{E}[x_t|Y_{t-1}])'] \\ &\quad - \Sigma_t C'(C\Sigma_t C')^{-1}\mathrm{cov}(\tilde{y}_t, x_t - \hat{E}[x_t|Y_{t-1}]) \\ &\quad - \mathrm{cov}(x_t - \hat{E}[x_t|Y_{t-1}], \tilde{y}_t)(\Sigma_t C'(C\Sigma_t C')^{-1})' \\ &\quad + \Sigma_t C'(C\Sigma_t C')^{-1}\mathrm{var}(\tilde{y}_t)(\Sigma_t C'(C\Sigma_t C')^{-1})' \\ &= \Sigma_t - \Sigma_t C'(C\Sigma_t C')^{-1}C\Sigma_t - \Sigma_t C'(\Sigma_t C'(C\Sigma_t C')^{-1})' \\ &\quad + \Sigma_t C'(\Sigma_t C'(C\Sigma_t C')^{-1})' \\ &= \Sigma_t - \Sigma_t C'(C\Sigma_t C')^{-1}C\Sigma_t\end{aligned}$$

Here, we have used the following results

$$\mathrm{var}(\tilde{y}_t) = C\Sigma_t C',$$

$$\begin{aligned}\mathrm{cov}(\tilde{y}_t, x_t - \hat{E}[x_t|Y_{t-1}]) &= E(y_t - \hat{E}[y_t|Y_{t-1}])(x_t - \hat{E}[x_t|Y_{t-1}])' \\ &= CE(x_t - \hat{E}[x_t|Y_{t-1}])(x_t - \hat{E}[x_t|Y_{t-1}])' \\ &= C\Sigma_t,\end{aligned}$$

and

$$\begin{aligned}\mathrm{cov}(x_t - \hat{E}[x_t|Y_{t-1}], \tilde{y}_t) &= E(x_t - \hat{E}[x_t|Y_{t-1}])(y_t - \hat{E}[y_t|Y_{t-1}])' \\ &= E(x_t - \hat{E}[x_t|Y_{t-1}])(x_t - \hat{E}[x_t|Y_{t-1}])'C' \\ &= \Sigma_t C'.\end{aligned}$$

Hence, we get

$$\Sigma_{t+1} = A(\Sigma_t - \Sigma_t C'(C\Sigma_t C')^{-1}C\Sigma_t)A' + BQB'$$

Initializing the Kalman filter

Initializing the algorithm requires that we know $\hat{E}[x_1|x_0] = Ax_0$. For that purpose, we generally assume that $x_0 = E(x_t) = 0$ in the linear model above. We also initialize Σ_1 with the inconditional variance-covariance matrix of x_t in the model, that is

$$\Sigma = E[x_t x_t'] = E[(Ax_{t-1} + Bu_t)(Ax_{t-1} + Bu_t)'] = A\Sigma A' + BQB'.$$

The solution of this Lyapunov equation is

$$\text{vec}(\Sigma) = [I - (A \otimes A)]^{-1}\text{vec}(BQB').$$

This approach assumes that the model has been generating the data for a long period of time before the sample starts.

Another possible solution for initialization is to start from the fixed point of the Ricatti equation

$$\Sigma = A(\Sigma - \Sigma C'(C\Sigma C')^{-1}C\Sigma)A' + BQB',$$

assuming that the Kalman filter has been running for a long period of time before the sample starts. This is much faster because the Kalman filter does not need to iterate on the matrix Σ_t. The latter is instead set constant and equal to the fixed point Σ. This does not seem to work when the model includes variables that are zero at order one, such as Calvo price dispersions, but when it does, we suspect that this method provides results close to the standard approach. Further experiments on that topic should be carried on in the future.

A last possibility that should be explored consists in estimating the initial conditions as parameters. This is also left for future research.

1.2.3 The Kalman smoother

Now the objective is to compute the linear least square projection of $x_t,\ t \leq n$ on all the information available, $Y_n = \{x_0, y_1, ..., y_n\}$. This projection is denoted by $\hat{E}[x_t|Y_n]$. Again, we consider the forecast errors

$$\tilde{y}_t = y_t - \hat{E}[y_t|Y_{t-1}],$$

which makes an orthogonal basis of the linear space spanned by Y_n. As $\{\tilde{y}_t, \tilde{y}_{t+1}, ..., \tilde{y}_n\}$ is orthogonal to Y_{t-1},

$$\begin{aligned}\hat{E}[x_t|Y_n] &= \hat{E}[x_t|Y_{t-1}, \tilde{y}_t, ..., \tilde{y}_n] = \hat{E}[x_t|Y_{t-1}] + \hat{E}[x_t|\tilde{y}_t, ..., \tilde{y}_n] - E(x_t) \\ &= \hat{E}[x_t|Y_{t-1}] + \text{cov}[x_t, (\tilde{y}_t', ..., \tilde{y}_n')']\text{var}[(\tilde{y}_t', ..., \tilde{y}_n')']^{-1}(\tilde{y}_t', ..., \tilde{y}_n')'\end{aligned}$$

$$\text{cov}[x_t, (\tilde{y}_t', ..., \tilde{y}_n')'] = E[(x_t(\tilde{y}_t', ..., \tilde{y}_n')] = \begin{bmatrix} E[x_t(y_t - \hat{E}[y_t|Y_{t-1}])'] \\ E[x_t(y_{t+1} - \hat{E}[y_{t+1}|Y_t])'] \\ \vdots \\ E[x_t(y_n - \hat{E}[y_n|Y_{n-1}])'] \end{bmatrix}'$$

In this expression,

$$E[x_t(y_t - \hat{E}[y_t|Y_{t-1}])'] = E[x_t(x_t - \hat{E}[x_t|Y_{t-1}])'C'] = \Sigma_t C',$$

$$\begin{aligned}
E[x_t(y_{t+1} - \hat{E}[y_{t+1}|Y_t])'] &= E[x_t(Ax_t + Bu_{t+1} - \hat{E}[Ax_t + Bu_{t+1}|Y_{t-1}])'C'] \\
&= E[x_t(x_t - \hat{E}[x_t|Y_{t-1}])'A'C'] = \Sigma_t A'C',
\end{aligned}$$

$$\begin{aligned}
E[x_t(y_{t+j} - \hat{E}[y_{t+j}|Y_{t+j-1}])'] &= E\left[x_t\left(A^j x_t + B\sum_{k=1}^{j} A^{k-1}u_{t+j}\right.\right. \\
&\qquad \left.\left.-\ \hat{E}\left[A^j x_t + B\sum_{k=1}^{j} A^{k-1}u_{t+j}|Y_{t-1}\right]\right)' C'\right] \\
&= E[x_t(x_t - \hat{E}[x_t|Y_{t-1}])'\left(A^j\right)'C'] = \Sigma_t\left(A^j\right)'C',
\end{aligned}$$

so finally

$$\operatorname{cov}[x_t, (\tilde{y}_t', ..., \tilde{y}_n')'] = \begin{bmatrix} \Sigma_t C' \\ \Sigma_t A'C' \\ \vdots \\ \Sigma_t\left(A'\right)^{n-t}C' \end{bmatrix}'.$$

$$\operatorname{var}[(\tilde{y}_t', ..., \tilde{y}_n')'] = E[(\tilde{y}_t', ..., \tilde{y}_n')'(\tilde{y}_t', ..., \tilde{y}_n')] = \begin{bmatrix} C\Sigma_t C' & 0 & \dots & 0 \\ 0 & C\Sigma_{t+1}C' & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C\Sigma_n C' \end{bmatrix}.$$

Then

$$\begin{aligned}
\hat{E}[x_t|Y_n] = \hat{E}[x_t|Y_{t-1}] + \Sigma_t &\Big[C'(C\Sigma_t C')^{-1}\left(y_t - C\hat{E}[x_t|Y_{t-1}]\right) \\
&+ A'C'(C\Sigma_{t+1}C')^{-1}\left(y_{t+1} - C\hat{E}[x_{t+1}|Y_t]\right) \\
&+ \ldots + (A')^{n-t}C'(C\Sigma_n C')^{-1}\left(y_n - C\hat{E}[x_n|Y_{n-1}]\right)\Big] \\
\equiv \hat{E}[x_t|Y_{t-1}] + \Sigma_t\Omega_t.&
\end{aligned}$$

$$\begin{aligned}
\Omega_t &= C'(C\Sigma_t C')^{-1}\left(y_t - C\hat{E}[x_t|Y_{t-1}]\right) + A'C'(C\Sigma_{t+1}C')^{-1}\left(y_{t+1} - C\hat{E}[x_{t+1}|Y_t]\right) \\
&\qquad + \ldots + (A')^{n-t}C'(C\Sigma_n C')^{-1}\left(y_n - C\hat{E}[x_n|Y_{n-1}]\right) \\
&= C'(C\Sigma_t C')^{-1}\left(y_t - C\hat{E}[x_t|Y_{t-1}]\right) + A'\Big[C'(C\Sigma_{t+1}C')^{-1}\left(y_{t+1} - C\hat{E}[x_{t+1}|Y_t]\right) \\
&\qquad + \ldots + (A')^{n-t-1}C'(C\Sigma_n C')^{-1}\left(y_n - C\hat{E}[x_n|Y_{n-1}]\right)\Big] \\
&= C'(C\Sigma_t C')^{-1}\left(y_t - C\hat{E}[x_t|Y_{t-1}]\right) + A'\Omega_{t+1}
\end{aligned}$$

This recursion is run backward, starting with

$$\Omega_n = C'(C\Sigma_n C')^{-1}\left(y_n - C\hat{E}[x_n|Y_{n-1}]\right).$$

1.2.4 Bayesian estimation

The Bayesian approach allows economists' beliefs about parameters values, or other sources of information than the observable timeseries, to affect the shape of the likelihood. Its practical motivation is that estimating a DSGE model by maximum likelihood is generally difficult because data is not informative enough and models are misspecified; the shape of the likelihood function in the parameter space is often either strongly irregular, or flat in some directions.

The Bayes theorem stipulates that the probability of parameters conditional to the observed sample verifies

$$p(\theta|Y_T) = \frac{p_0(\theta)p(Y_t|\theta)}{p(Y_T)},$$

where $p(Y_T)$ is the marginal density of the sample, that is

$$p(Y_T) = \int_\Theta p_0(\theta)p(Y_T|\theta)\,d\theta,$$

with Θ the set of parameters admissible values, and $p_0(\theta)$ the prior density. The latter distribution is imposed by the economist on the basis of his or her beliefs or of other sources of information. It is generally specified in the form of standard univariate distributions (gaussian, beta, uniform, gamma, or inverse gamma distributions are the most common), so parameters are assumed to be independant. The density $p(\theta|Y_T)$ is called the parameters posterior density. The Bayes theorem implies that it is proportional to the product of the prior density and the likelihood $p(Y_T|\theta)$, since the denominator does not depend on the parameters value. The object of the Bayesian estimation is thus to find the value of θ which maximises the parameters' posterior density or, equivalently $\log p_0(\theta) + \log \mathcal{L}(\theta; Y_T)$. Once the likelihood is computed as explained in 1.2.1, it is very convenient, since the objective function to maximize is the sum of the log-likelihood and of the log-density of the parameters vector. Morover, in the most common case where parameters prior distributions are orthogonal, the latter is the sum of the univariate log-densities of parameters.

1.3 Estimation with missing observations

Let $\{y_t, t \in \mathbb{N}\}$ be a multivariate $p \times 1$ stochastic process described by the following time invariant state space model:

$$\begin{aligned} y_t &= Z\alpha_t + \eta_t \\ \alpha_{t+1} &= T\alpha_t + R\varepsilon_t \end{aligned} \tag{1.3.1}$$

The first equation (ME) links the variables y_t to the $m \times 1$ state vector α_t through the $p \times m$ selection matrix Z (vertical concatenation of p linearly independent unit row vectors). The second equation

(SE) defines the dynamic of the state vector, T is an $m \times m$ transition matrix, R is an $m \times q$ matrix measuring the marginal effect of the structural innovations ε_t on the state vector. The structural innovations are assumed to be gaussian: $\varepsilon_t \underset{\text{iid}}{\sim} \mathcal{N}(0, Q)$ with Q a $q \times q$ real symmetric positive definite matrix. ME and SE are, for instance, the reduced form solution of a DSGE model. The (optional) gaussian noise $\eta_t \underset{\text{iid}}{\sim} \mathcal{N}(0, H)$, with H a $p \times p$ real symmetric positive definite matrix, stands for any uncorrelated discrepancy between the theoretical variables and the empirical counterparts.

Let $\mathcal{Y}_T \equiv \{y_1^\star, y_2^\star, \dots, y_T^\star\}$, with $y_t^\star$ a $p_t \times 1$ vector and $0 \leq p_t \leq p$ for all $t \in \{1, \dots, T\}$, be the sample. If $p_t = p$ then all the endogenous variables y are observed at time t. If $p_t = 0$ none of the endogenous variables y are observed at time t. We will denote the total number of observations as:

$$\mathscr{N} \equiv \sum_{t=1}^{T} p_t \leq pT$$

Let $\mathscr{S}_t$ be a $p_t \times p$ time varying elimination matrix such that $y_t^\star = \mathscr{S}_t y_t$. If $p_t = p$ then $\mathscr{S}_t$ equals the p-dimensional identity matrix, otherwise $\mathscr{S}_t$ is a subset of the rows of I_p. The state space model for $\mathcal{Y}_T$ is then given by:

$$\begin{aligned} y_t^\star &= \mathscr{Z}_t \alpha_t + \eta_t^\star \\ \alpha_t &= T\alpha_{t-1} + R\varepsilon_t \end{aligned} \tag{1.3.2}$$

with $\mathscr{Z}_t \equiv \mathscr{S}_t Z$, $\eta_t^\star \equiv \mathscr{S}_t \eta_t \underset{\text{iid}}{\sim} \mathcal{N}(0, \mathscr{H}_t)$ and $\mathscr{H}_t \equiv \mathscr{S}_t H \mathscr{S}_t'$. Matrices T, R and Q are still time invariant but the selection matrix and the covariance matrix of the measurement errors are time varying. System (1.3.2) is a standard time varying state space model.

Consequently, the filtering equations of the state space model with missing observations are given by:

$$v_t = \begin{cases} y_t^\star - \mathscr{Z}_t a_t \text{ if } p_t > 0, \\ 0 \text{ otherwise.} \end{cases} \tag{1.3.3a}$$

$$F_t = \begin{cases} \mathscr{Z}_t P_t \mathscr{Z}_t' + \mathscr{H}_t \text{ if } p_t > 0, \\ \infty \text{ otherwise.} \end{cases} \tag{1.3.3b}$$

$$K_t = P_t \mathscr{Z}_t' F_t^{-1} \tag{1.3.3c}$$

$$\alpha_{t+1} = T(a_t + K_t v_t) \tag{1.3.3d}$$

$$P_{t+1} = T(P_t - P_t \mathscr{Z}_t' K')T' + RQR' \tag{1.3.3e}$$

where a_t and P_t are respectively the expectation and variance of the state vector α_t conditional on the sample up to time $t-1$. Note that if, for some t, p_t is equal to zero then the Kalman gain matrix is zero (the sample has nothing to say about the state vector) and equations (1.3.3d) and

(1.3.3e) collapse into

$$a_{t+1} = Ta_t \tag{1.3.4a}$$
$$P_{t+1} = TP_tT' + RQR' \tag{1.3.4b}$$

The state equation in (1.3.2) is time invariant, so the filtering recursion (1.3.3)-(1.3.4) can be initialized as usual with the unconditional expectation and variance of the state vector. Assuming that all the eigenvalues of T are inside the unit circle we have $a_0 \sim \mathcal{N}(0, P_0)$ with P_0 solving $P_0 = TP_0T' + RQR'$. If one of the eigenvalues is greater than one in modulus, then a diffuse Kalman filter may be used.

Finally, the prediction error decomposition of the (log)likelihood is given by:

$$\mathscr{L}(\mathcal{Y}_T) = -\frac{\mathscr{N}}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log|F_t| - \frac{1}{2}\sum_{t=1}^{T} v_t' F_t^{-1} v_t \tag{1.3.5}$$

1.4 Introducing correlation between exogenous processes

In a two-country model, there are many reasons to assume that some exogenous variables of one region are positively correlated with the corresponding ones of the other region. For instance, productivity shocks are likely to be correlated on the grounds that technology and human capital circulate freely inside the euro area. Governement spending shocks may also reveal a degree of mimesis. A desired feature of the estimated model is to allow for positive correlation between country specific shocks. A straightforward manner to do it is to estimate directly the correlation coefficient between some exogenous variables. However, a drawback of this approach is that impulse responses cannot be simulated independantly; the occurence of a shock and the resulting propagation is, with a certain probability, accompanied by another shock and its propagation. Hence, a preferred specification assumes that the variance-covariance matrix of exogenous shocks is diagonal. This section examines different ways to introduce positive correlation between exogenous variables of a model using combinations of uncorrelated shocks only. The best specification put forward is then used in the estimated model of chapter 3. An important issue which is also discussed is the fact that the orthogonal shocks that are introduced to capture this possible correlation cannot be interpreted as having a specific origin (e.g. originating from a specific region, or corresponding to events affecting both regions simultaneously). Rather, they are abstract projections of such interpretable shocks, assuming they exist. But this specification is fully consistent with the orthogonality assumption used in estimation.

1.4.1 Analytical approach

In this paragraph, I consider very basic models of two endogenous variables. Data includes two iid timeseries named x and y, with variance-covariance matrix

$$\Sigma = \begin{bmatrix} \mu^2 & \delta \\ \delta & \nu^2 \end{bmatrix},$$

where $\delta \geq 0$. The first two models include three orthogonal shocks. An intuitive approach consists in simply adding the shocks as follows

$$\begin{aligned} x_t &= \varepsilon_{x,t} + \eta_t, \\ y_t &= \varepsilon_{y,t} + \eta_t, \end{aligned} \tag{1.4.1}$$

where the standard deviation of ε_x, ε_y and η are respectively denoted by σ_x, σ_y and c. This model yields the following variance-covariance matrix for $[x_t \; y_t]'$:

$$E \begin{bmatrix} x_t \\ y_t \end{bmatrix} \begin{bmatrix} x_t & y_t \end{bmatrix} = \begin{bmatrix} \sigma_x^2 + c^2 & c^2 \\ c^2 & \sigma_y^2 + c^2 \end{bmatrix}.$$

Solving implies

$$c = \sqrt{\delta} \qquad \sigma_x^2 = \mu^2 - \delta \qquad \sigma_y^2 = \nu^2 - \delta.$$

These equations do not always have a solution. The cases where $\mu^2 < \delta$ or $\nu^2 < \delta$, that is when x and y are highly correlated but have different variances, has no solution. More precisely this model works if and only if

$$\frac{\delta}{\mu\nu} = \text{corrcoef}(x, y) < \frac{\mu}{\nu} < \frac{1}{\text{corrcoef}(x, y)}.$$

The second model with three shocks considered is

$$\begin{aligned} x_t &= \sigma_x \left(\varepsilon_{x,t} + c\eta_t \right), \\ y_t &= \sigma_y \left(\varepsilon_{y,t} + c\eta_t \right), \end{aligned} \tag{1.4.2}$$

where ε_x, ε_y and η are independant white noises with mean 0 and variance 1. The variance-covariance matrix of $[x_t \; y_t]'$ is in the model

$$E \begin{bmatrix} x_t \\ y_t \end{bmatrix} \begin{bmatrix} x_t & y_t \end{bmatrix} = \begin{bmatrix} \sigma_x^2(1 + c^2) & \sigma_x\sigma_y c^2 \\ \sigma_x\sigma_y c^2 & \sigma_y^2(1 + c^2) \end{bmatrix}.$$

The model can always be estimated by solving

$$\sigma_x^2(1 + c^2) = \mu^2$$

$$\sigma_y^2(1+c^2) = \nu^2$$

$$\sigma_x \sigma_y c^2 = \delta,$$

which yields

$$c = \frac{1}{\sqrt{\frac{\mu\nu}{\delta} - 1}} \qquad \sigma_x = \sqrt{\mu^2 - \frac{\mu\delta}{\nu}} \qquad \sigma_y = \sqrt{\nu^2 - \frac{\nu\delta}{\mu}}.$$

The next two models include two orthogonal shocks. Again, the most simple idea is to consider the model

$$\begin{aligned} x_t &= \varepsilon_{x,t}, \\ y_t &= \varepsilon_{y,t} + \varepsilon_{x,t}, \end{aligned} \tag{1.4.3}$$

with $\varepsilon_{x,t} \sim N(0, \sigma_x^2)$ and $\varepsilon_{y,t} \sim N(0, \sigma_y^2)$, but it implies that the covariance of x and y is equal to the variance of x, so this model is not appropriate unless the data verifies $\delta = \mu^2$.

A more appropriate model with two shocks is the following:

$$\begin{aligned} x_t &= \sigma_x \varepsilon_{x,t}, \\ y_t &= \sigma_y \varepsilon_{y,t} + c\varepsilon_{x,t}, \end{aligned} \tag{1.4.4}$$

where ε_x and ε_y are independant white noises with mean 0 and variance 1. Then the variance-covariance matrix of $[x_t \ y_t]'$ in the model is

$$E \begin{bmatrix} x_t \\ y_t \end{bmatrix} \begin{bmatrix} x_t & y_t \end{bmatrix} = \begin{bmatrix} \sigma_x^2 & \sigma_x c \\ \sigma_x c & \sigma_y^2 + c^2 \end{bmatrix}.$$

The estimation is done analytically and yields

$$\sigma_x = \mu \qquad c = \frac{\delta}{\mu} \qquad \sigma_y = \sqrt{\nu^2 - \frac{\delta^2}{\mu^2}}.$$

To conclude, only models (1.4.2) and (1.4.4) can always be estimated. However, the estimation of model (1.4.2), including three shocks, is subject to a smoothing issue when computing the chronicles of each one over the estimation sample, because the Kalman smoother, based on linear least square projections, can obviously not identify three orthogonal time series ε_x, ε_y and η out of two series of observations for x and y. Specification (1.4.4) should thus be preferred.

1.4.2 Conclusion and implementation in the model

Only model (1.4.4) can always be estimated and allows an identification of orthogonal smoothed shocks, consistent with the estimation assumptions. This is the chosen approach to introduce cross-country correlation between exogenous shocks. This is done for the stationary productivity, investment technology, residual demand, impatience, price and wage markup shocks. When financial and labor market frictions are added in the model, this correlation is also assumed for risk, wealth and wage bargaining shocks. It is explained in detail below for the stationary labor productivity shocks ε^a_F and ε^a_E, who enter in the production function in country F and in country E respectively. These variables are described by the following AR(1) processes:

$$\begin{cases} \varepsilon^a_{F,t} = \left(\varepsilon^a_{F,t-1}\right)^{\rho^a_F} (\bar{\varepsilon}^a_F)^{1-\rho^a_F} \exp\left(\sigma^a_F \eta^a_{F,t} + \varsigma^a_F \eta^a_t\right) \\ \varepsilon^a_{E,t} = \left(\varepsilon^a_{E,t-1}\right)^{\rho^a_E} (\bar{\varepsilon}^a_E)^{1-\rho^a_E} \exp\left(\varsigma^a_E \eta^a_t\right) \end{cases} \tag{1.4.5}$$

where η^a_F and η^a are two independant shocks drawn from a normal distribution with mean 0 and variance 1. The parameters σ^a_F, ς^a_F and ς^a_E are estimated. In practice, the second equation in (1.4.5) is entered in Dynare in the form

$$\varepsilon^a_{E,t} = \left(\varepsilon^a_{E,t-1}\right)^{\rho^a_E} (\bar{\varepsilon}^a_E)^{1-\rho^a_E} \exp\left(\sigma^a_E \eta^a_{E,t} + \varsigma^a_E \eta^a_t\right),$$

with the parameter σ^a_E set to zero, and the shock η^a_E switched off (its variance is zero). This is done in order to keep the symmetry of the two countries. Indeed, country equations are written once inside a Dynare macro-language loop over countries. With this specification, the equations for the shocks are identical and can hence be included in the loop.

The shock η^a_F should not stricto sensu be interpreted as a technology shock specific to France, no more than η^a should be interpreted as a shock common to all countries in the euro area. The occurences of the latter shock represent either events that changed productivity only in the rest of the euro area excluding France, or events that affected productivity in the whole euro area. The former shock reflects events that changed productivity only in France, but is also affected by disturbances common to the whole euro area or specific to the rest of the euro area excluding France.

To make this point clear, assume that the "true" model of country productivities, or data generating process, includes three orthogonal exogenous sources of disturbances, as follows

$$\begin{cases} \varepsilon^a_{F,t} = \left(\varepsilon^a_{F,t-1}\right)^{\rho^a_F} (\bar{\varepsilon}^a_F)^{1-\rho^a_F} \exp\left(a_{1F} \tilde{\varepsilon}_{F,t} + a_{2F} \tilde{\varepsilon}_t\right) \\ \varepsilon^a_{E,t} = \left(\varepsilon^a_{E,t-1}\right)^{\rho^a_E} (\bar{\varepsilon}^a_E)^{1-\rho^a_E} \exp\left(a_{1E} \tilde{\varepsilon}_{E,t} + a_{2E} \tilde{\varepsilon}_t\right) \end{cases}$$

This dgp propagates separately the occurences of disturbances that are country-specific, represented by the shocks $\tilde{\varepsilon}_{F,t}$ and $\tilde{\varepsilon}_{E,t}$, and those that are common to the whole euro area, represented by the shock $\tilde{\varepsilon}_t$. Parameters a_{1F}, a_{2F}, a_{1E} and a_{2E} are assumed to measure the magnitude of the effect of these events on productivity. Then, identification using the model (1.4.5) above implies

$$\eta^a_t = \frac{1}{\varsigma^a_E}(a_{1E} \tilde{\varepsilon}_{E,t} + a_{2E} \tilde{\varepsilon}_t),$$

and

$$\eta^a_{F,t} = \frac{1}{\sigma^a_F}\left(a_{1F} \tilde{\varepsilon}_{F,t} + a_{2F} \tilde{\varepsilon}_t - \frac{\varsigma^a_F}{\varsigma^a_E}(a_{1E} \tilde{\varepsilon}_{E,t} + a_{2E} \tilde{\varepsilon}_t)\right).$$

As demonstrated by these expressions, the shocks η^a and η^a_F cannot be interpreted as originating in

France for the former or hitting all the euro area at the same time for the latter. This specification is just a trick aimed at using two orthogonal shocks to generate positive cross-correlation between two exogenous variables, but does not make shock interpretation more convenient.

1.5 Why using measurement errors?

Estimating models using full information methods such as the Bayesian approach requires that they include at least as many exogenous disturbances as observable variables. This may be an issue for the estimation of large-scale DSGE models including a great number of observable variables. Besides the standard exogenous disturbances that are considered as "structural" in the literature, a natural way to match this constraint consists in using additional wedges in the models equations to get them to the data. For example, in models which aggregate the outputs of many sectors, the aggreagation functions, generally CES, can be assumed to include shocks to their long term input shares, without discussing the corresponding structural interpretation, or possible counterparts in actual economies. These disturbances propagate in the model in the same way as traditional structural shocks such as TFP shocks. In some cases, their presence is justified by the aim of including more timeseries in the dataset, rather than because the economist actually believes that they are likely sources of business cycle fluctuations. This practice may be prejudicial for the estimated model's dynamic properties; the size of a shock with counterfactual second order moments implications may be overestimated by the estimation procedure because it is needed to satisfy the minimum shocks number requirement.

Traditional macroeconometric models avoid this pitfall since they have econometric residuals that are used as exogenous add-factors. They are the source of fluctuations and also propagate in the model, but only through the dynamics of the observed variable to which they are associated. They are in some sort observed and, hence, cannot deteriorate the dynamic behavior of the model. By contrast, shocks introduced as structural ones in DSGE models affect simultaneously many variables, and econometric residuals like those that are found in traditional macroeconometric models cannot be used in DSGE models because equations are not attached to specific variables.

A third way to introduce deviations between models' predictions and observed data consist in using measurement errors. These wedges are attached to specific observed variables, but unlike macroeconometric residuals, they do not propagate in the model. Hence, they can be used as substitutes for structural disturbances to satisfy the minimum shocks number requirement, but they cannot have undesirable effects on a model's dynamic properties. The use of measurement errors is advocated by Ireland (2004); when the standard practice follows Sargent (1989) and assume that they reflect imperfect data collection and are uncorrelated across time and variables, Peter Ireland claims that they reflect both the weaknesses of data collectors and models and allow they to follow a first-order vector autoregression. I am also using measurement errors to estimate the present model, in the form of orthogonal white noise innovations, for two reasons. The first one stems from the ambition of this project, which is to test the ability of large-scale structural models enriched with many frictions to replicate simultaneously many dimensions of observed data, or to use the phrase of Sargent (1987), to "take models seriously econometrically". This leads to using a large number of observed variables, when the literature does not point to such a large number of credible exogenous sources of fluctuations, with acceptable implications on models' dynamic properties. Second, I believe that even large-scale models are misspecified in some directions, and that a proportion of the exogenous disturbances that are identified when taking the model to the data should not be interpreted as actual sources of business cycles, but rather as econometric residuals. As such, they should not propagate in the model. In short, introducing measurement errors is an effort towards disentangling, in the observed fluctuations, the effects that can be interpreted through the lens of a structural model from the residuals about which this approach has nothing to say.

In this section, I conduct two experiments that are intended to provide support to the use of measurement errors. I the first one, I estimate the true model of a simulated economy using a shock that has counterfactul implications and show that the model's unconditional properties are strongly deteriorated. By contrast, using measurement errors instead of this shock mitigates the deterioration. In the second experiment, I esimate a misspecified version of the model on simulated data both with and without measurement errors and show that the model's properties are closer to the data generating model's ones with measurement errors. Although these exercises are very stylized, their purpose is to mimic, in an exaggerated way, the actual situation of modelers, using misspecified models with imperfectly choosen shocks. In such cases, a correct methodology can indeed lead to unexpected side effects on models' properties, which can be hard to identify, whereas measurement errors can help lessen them.

1.5.1 The data generating model: a basic new-Keynesian model with search and matching frictions

The model economy

Identical and perfectly insured households' have an instantaneous utility function $\log C_t$ where C_t is consumption expenditures in period t. In any given period t, they can either be employed and are paid a real wage w_t or unemployed an receive a real benefit z. A lottery allocates randomly all jobs across households in each period. They are faced to the aggregate budget constraint

$$C_t + \frac{B_t}{P_t R_t} \leq w_t N_t + \frac{B_{t-1}}{P_t} + Div_t - T_t,$$

where B/R is nominal risk-free bonds holdings, R is nominal bonds one period remuneration rate, N is the employment rate, P is the price level of consumption goods, Div and T are real dividends and real lump-sum taxes, paid evenly respectively by firms and to the government. There are matching frictions in the labor market; a fraction s of existing jobs are destroyed at the beginning of each period, whereas the number of jobs created and immediately available for production is the outcome of a Cobb-Douglas matching function of the number of job seekers at the beginning of the period $1-(1-s)N_{t-1}$ and the number of vacant positions posted by firms V_t. The number of jobs is governed by

$$N_t = (1-s)N_{t-1} + \Upsilon V_t^{\kappa}(1-(1-s)N_{t-1})^{1-\kappa}.$$

The production of final goods involves firms, intermediate and final retailers. Firms hires N_t workers from the labor market and produces a quantity $a_t N_t$. They incur a unit cost c for keeping open vacancies during one period. A large and constant number of intermediate retailers, indexed by $f \in [0,1]$ purchase the production of firms, differentiate it and sell their output in a monopolistic market. They face nominal frictions à la Calvo (1983): the can reset their price with probability $1-\xi$ at each date; otherwise, their price is automatically indexed to a convex combination of past and long run aggregate inflation rates, $\pi_{t-1}^{\iota}\bar{\pi}^{1-\iota}$. Final retailers aggregate differentiated goods with a Dixit and Stiglitz (1977) technology to produce a quantity Y of final goods, given by

$$Y_t = \left(\int_0^1 y_t(f)^{\frac{\theta}{\theta-1}} df\right)^{\frac{\theta-1}{\theta}}.$$

Wages are the outcome of a standard Nash bargaining process between firms and households, with workers' relative bargaining power being measured by $\delta_t \in [0,1]$. The government absorbs an amount G of final goods, which is financed by lump-sum taxes. Last, monetary policy is represented by a standard Taylor rule, which is

$$\hat{R}_t = \rho \hat{R}_{t-1} + (1-\rho)\left(r_\pi \hat{\pi}_t + r_Y \hat{Y}_t\right) + \varepsilon_{r,t}$$

in its log-linearized version. The exogenous sources of fluctuations are productivity a_t, and workers' bargaining power δ_t, which both follow first-order autoregressive processes, and $\varepsilon_{r,t}$, which is a white noise.

Calibration

The steady state of the following variables is set to standard values found in the literature, or based on observed averages in the US (for more details, please refer to section 7.4.1): the employement rate is 0.57, quarterly inflation is 0.62%, the share of public consumption is 31.9%, the vacancy-filling ratio is 0.9. The job separation rate is 15% per quarter, the unit vacancy cost c is such that vacancy costs represent 1% of output, the time discount rate is 0.99, and both the elasticity of matching with respect to vacancies and workers bargaining power in the steady state are set to 0.5. Next, retailers steady state markup rate is 20%, which corresponds to $\theta = 6$, the Calvo probability is $\xi = 0.75$, indexation to past inflation is $\iota = 0.5$ and the Taylor rule parameters are $\rho = 0.7$, $r_\pi = 1.5$ and $r_Y = 0.125$. Regarding exogenous processes, productivity has persistence $\rho_a = 0.7$ and standard deviation $\sigma_a = 0.0068$, and bargaining power has persistence $\rho_\delta = 0.5$ and standard deviation $\sigma_\delta = 0.3$. The standard deviation of the monetary disturbance is $\sigma_r = 0.0062$.

Simulated data

The model is used to simulate a 1000-period sample of the economy's variables. The first 500 observations are dropped to hedge against initial conditions effects, so the remaining 500 observations are regarded as data. In what follows however, only the log-deviations of output, employment, inflation and the real wage from their steady state values are going to be used.

1.5.2 Estimation with a shock to the matching technology

In this experiment, the true model is known, but not the exogenous disturbances that generated the data. I find that a well-specified model can predict a positive slope of the Beveridge curve, when it is estimated with a shock which has this counterfectual implication. Using measurement errors instead avoids this undesirable situation. This exercise is built so as to illustrate the point stated above, which is that using measurement errors instead of deep shocks in the model may, in some very particular cases, deteriorate the dynamic properties of an estimated model. In particular, the choice of observable variables and of shocks is made on purpose to obtain contrasted results. Note also that I estimate the true dgp (except shocks) in order to pick out the sole effect of the specification of exogenous variables. Implicitely, the economist only observe the data and proposes a model, which is luckily almost the same as the dgp. Therefore, the number of shocks in the dgp does not restricts the number of shocks that are assumed in the estimated models.

Estimation methodology and prior distribution

The model is estimated using a Bayesian approach and observing the log-deviations of employment and inflation from their steady state values. The economist identifies technological shocks to productivity as one credible source of fluctuations. As a second shock is needed to observe both employment and inflation, he assumes that the efficience of the matching technology is also subject to an exogenous AR(1) disturbance, denoted by $\varepsilon_{m,t}$. The persistence ρ_m and the standard deviation σ_m of this process are estimated. I use beta distributions with mean 0.5 and standard deviation 0.2 for parameters that are constrained to be comprises between 0 and 1, that are ξ, ι, ρ, ρ_a and ρ_m. The distributions a priori of the standard deviations of shocks are inverse gamma 2 with mean 0.02 and infinite variance. Normal distributions are assumed for the parameters of the Taylor rule, with means 1.5 and 0.125, standard deviations 0.2 and 0.05 respectively for r_π and r_Y. The model is then estimated with a measurement error for employment, switching off the shock to the matching technology. In that case, the distribution a priori of its standard deviation is inverse gamma 2 with mean 0.001 and infinite variance. We can expect both estimates (with the matching shock or with a measurement error) to be poor, since two shocks are replaced by only one, different; the relevant question is whether the measurement error is a better substitute than the misspecified shock.

Posterior estimates and model properties

The estimation results are summarized in Table 1.1.

Table 1.2 reports the unconditional correlation coefficient between the unemployment rate $U_t = 1 - N_t$ and vacancies V_t, as predicted by the data generating model (i.e. the “true” slope of the Beveridge curve), by the model estimated with shocks to the matching technology and by the model estimated with measurement errors instead. It is important to keep in mind that measurement

Table 1.1: Estimation results for the true model with wrong shocks

parameter	no measurement err. post. mode	post. s.d.	with measurement err. post. mode	post. s.d.
ξ	0.8250	0.0116	0.6955	0.0384
ι	0.3334	0.0490	0.4850	0.0962
r_π	1.6008	0.1378	1.4849	0.1608
r_Y	0.1132	0.0473	0.1262	0.0465
ρ	0.4521	0.1450	0.6371	0.0681
ρ_a	0.6338	0.0331	0.6975	0.0509
ρ_m	0.7709	0.0351	–	–
σ_a	0.0142	0.0008	0.0070	0.0017
σ_m	0.1282	0.0257	–	–
m.e. n	–	–	0.0170	0.0005

errors are only used during the estimation; they are set to zero when the model is simulated and its dynamic properties are computed, since they are not part of the model.

Table 1.2: Theoretical slope of the Beverdige curve

	data generating model	no measurement err.	with measurement err.
corr(U,V)	-0.6431	0.2525	-0.6272

Although the model is well-specified, shocks to the matching technology shift unemployment and vacancies in the same direction. Without all the disturbances used to generate the data being present in the model, the estimation assigns an important weight to this shock as compared to the productivity shock to explain the fluctuations in employment and inflation. As a result, the model's unconditional prediction for the Beveridge curve is reversed. Using a measurement error avoids the need for such shocks; the model's properties are then consistent with responses to technology shocks only. In particular, the model replicates the downward sloping Beveridge curve as previous search and matching models from the literature (see Pissarides (1987)).

Obviously, a more parsimonious estimation using only employment as an observable and a technology shock would be enough to replicate this single property of the true model, since this shock – as well as monetary policy and wage bargaining shocks – implies negative comovements in unemployment and vacancies. Here, the economist wishes to include other variables in his dataset (namely inflation) for independant reasons, and chooses a wrong exogenous source of fluctuations. This is a situation that may occur, in a more indirect and hidden manner, with large-scale models.

1.5.3 Estimation of a model with frictionless labor market

This experiment assumes that the economist ignores the true data generating process and estimates a misspecified model. The estimation considers two to three deep shocks and three observed variables. Again, the exercise is stylized and these choices are made in order to (i) be able to find a well-designed posterior estimate of parameters and (ii) identify contrasted effects of using measurement errors or not. These constraints lead to choose different variables and shocks as compared to the previous exercise. This experiment is therefore totally independant of the previous one.

The misspecified model

This version of the model assumes that the labor market is frictionless. Households contemporaneous utility function is now

$$u_t = \log C_t + \varepsilon_{l,t} \chi \frac{(1 - N_t)^{1-\eta}}{1-\eta},$$

where N_t is households labor supply. Apart from the labor supply curve, the rest of the model's assumptions are unchanged with respect to the data generating model. $\varepsilon_{l,t}$ is an exogenous shock to households preferences for leisure, modeled by an AR(1) with persistence ρ_l and standard deviation σ_l, which is added to the model in order to have three shocks in the model and to be able to observe three variables in the estimation. Indeed, the bargaining power shock δ_t is absent from this version. In this experiment, $\varepsilon_{l,t}$ plays the role of the "dubious" shock, when a_t and $\varepsilon_{r,t}$ are believed to be credible sources of fluctuations.

Estimation methodology and prior distribution

I estimate two versions of the misspecified model using the Bayesian approach and observing the log-deviations of output, inflation and the real wage from their steady state values. In the first version, no measurement error is used but the preference for leisure shock $\varepsilon_{l,t}$ is included in the model and estimated. In the second version, this dubious preference shock is muted but measurement errors are added to all observed variables. This increases the total number of shocks to 5 for only 3 observed variables. Yet, this is what is usually done when estimating with measurement errors. Moreover, the inequitable number of shocks does not give this approach excessive chance to match crucial moments from the data, since measurement errors are muted when the theoretical moments implied by the model are computed; if the estimation relied too much on measurement errors, structural shocks would hardly explain the observed fluctuations and all theoretical standard deviations would not be differentiable from zero.

I use beta distributions with mean 0.5 and standard deviation 0.2 for parameters that are constrained to be comprises between 0 and 1, that are ξ, ι, ρ, ρ_a and ρ_l. The distributions a priori of the standard deviations of shocks are inverse gamma 2 with mean 0.02 and infinite variance. Normal distributions are assumed for the remaining of the parameters, with means 1.5, 0.125 and 1 and standard deviations 0.2, 0.05 and 1 respectively for r_π, r_Y and η. When measurement errors are used, their distributions a priori are inverse gamma 2 with mean 0.001 and infinite variance.

Posterior estimates and model properties

The estimation results are summarized in Table 1.3. Note that the estimated variance of the

Table 1.3: Estimation results for the misspecified model

parameter	no measurement err. post. mode	no measurement err. post. s.d.	with measurement err. post. mode	with measurement err. post. s.d.
ξ	0.7294	0.0076	0.6431	0.0099
ι	0.5361	0.0275	0.7994	0.0453
r_π	1.6151	0.1360	1.7416	0.1578
r_Y	0.1373	0.0343	0.0196	0.0376
η	0.0233	0.0142	0.0840	0.047
ρ	0.7784	0.0248	0.5798	0.0373
ρ_a	0.9790	0.0116	0.8143	0.0232
ρ_l	0.4772	0.0310	–	–
σ_r	0.0048	0.0005	0.0075	0.0007
σ_a	0.5974	0.3584	0.1918	0.1029
σ_l	0.0171	0.0008	–	–
m.e. Y	–	–	0.0003	0.0001
m.e. π	–	–	0.0002	0.0001
m.e. w	–	–	0.0096	0.0003

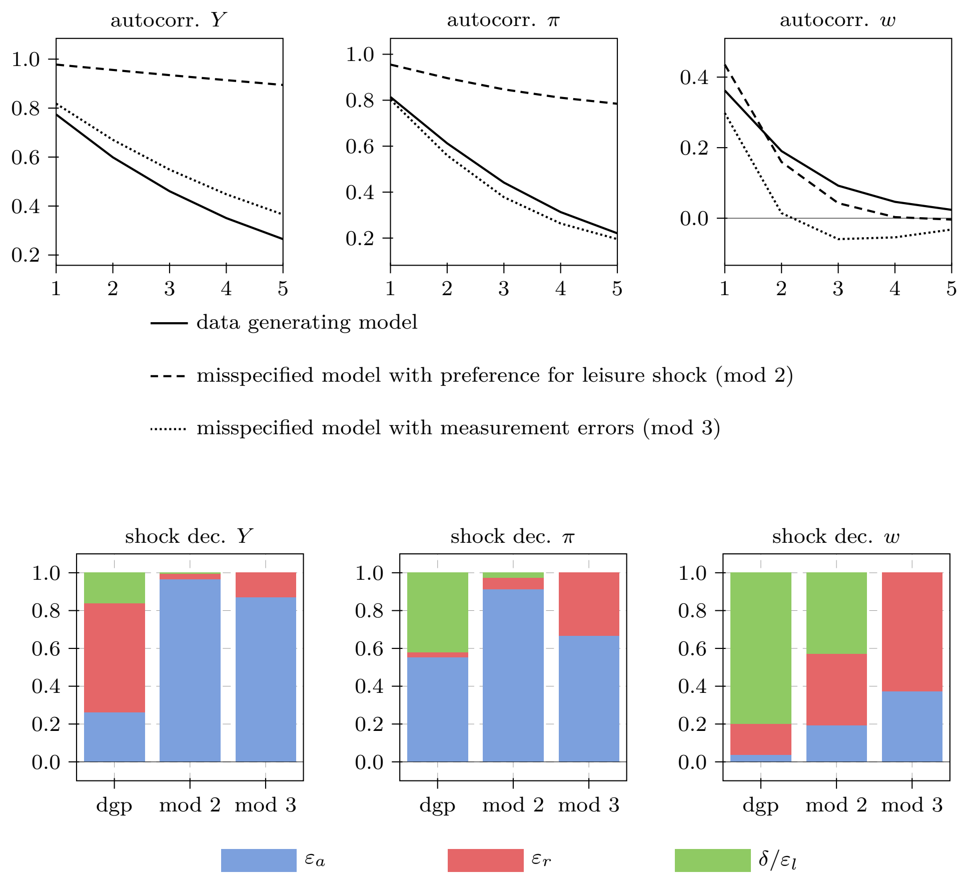
productivity shock is much greater than the “true” one. This is because a positive productivity shock leads to an increase of employment in the model with search and matching frictions, but to a drop in hours worked in the basic new-Keynesian model. As a result, in the latter model, for a given magnitude in output fluctuations, larger increases in productivity are needed to compensate the drops in labor and converserly, whereas productivity and employment variations add up in the presence of search and matching frictions.

Table 1.4 reports the standard deviation of the observable variables and their cross-correlation coefficients implied by the two estimates of the misspecified model and by the data generating model. Figure 1.1 reports their their autocorrelogram in all cases. The dynamic properties of the model estimated with measurement errors are closer to the data generating model's than those obtained without measurement errors. In particular, the standard deviations of the observables are strongly overestimated in that latter case. The negative correlation of output and inflation is also overstated, because the misspecified model underestimates the contribution to output fluctuations of the monetary policy shock relatively to productivity, as shown in the asymptotic shock decompositons in figure 1.1. Last, the persistence of output and inflation are also exaggerated. The size of the measurement error with respect to the real wage suggests that these problems arise because the preference and monetary policy shocks are not suited to replicate the dynamics of the real wage. As a result, the productivity shock is used to “counterbalance” the effects of these shocks on the real wage that are rejected by the data, and a high degree of autocorrelation in the productivity autoregressive process is needed to replicate the persistence of the real wage.

Table 1.4: Moments implied by the misspecified model

	standard deviations			correlation coef.		
model	Y	π	w	(Y,π)	(Y,w)	(w,π)
d.g.m.	0.018	0.006	0.017	-0.504	-0.047	0.609
no m.e.	0.058	0.015	0.024	-0.898	-0.098	0.459
with m.e.	0.021	0.006	0.015	-0.581	-0.048	0.712

Figure 1.1: Autocorrelograms and asymptotic shock decomposition



1.5.4 Concluding remarks

These estimation exercises illustrate that an inappropriate choice of the shocks which are used in a full-information estimation procedure can strongly deteriorate the dynamic properties of a model. It seems preferable to use only trusted shocks with measurement errors to satisfy the constraint that the number of exogenous shocks need to be larger than the number of observables. More, measurement errors can be helpful when the model is misspecified, which is always the case with actual data, as they isolate and neutralize the component in the dynamics of each observable variable that is rejected by the data.

However, this demonstration should not be regarded as an argument in favor of atheoretical models. If measurement errors can be used to improve models' fit and avoid side-effects of model misspecifications, the objective is still to find them as small as possible, so that theory provides insights on the largest part possible of business cycle variations. In that respect, the R-squared criterion is an essential assessment of the quality of estimation results. The claim is rather that measurement errors can help prevent unavoidable mispecifications in some dimensions of the model from contaminating other dimensions. They also represent a good diagnosis tool, since they point to variables and subsamples in the data that are not easily explained by structural shocks.

1.6 Technical improvements

The two country-model of chapter 3 is developed and estimated with Dynare; although this software is convenient for large models, computing time is critical for the estimation of this one, because many parameters are estimated and the steady state is also included in the estimation even though it does not have an analytical expression. For that reason, I have developed some add-ons to the usual version of Dynare. The first one aims at accelerating the computation of the steady state, which is evaluated many times during the estimation process as estimated parameters impact it. The second one is an improvement of the standard gradient optimization routine which stabilizes it in the sense that it prevents unexpected stops in points that are obviously not modes of the posterior distribution. Last, the third allows for prior density on other model's properties than parameter values. In particular, in the model of chapter 3, because of the complexity of the static model, not all relevant steady state ratios or levels can be defined as parameters from which are derived consistently the values of core parameters. It happens then that the values of the parameters found during the estimation with cyclical data imply some irrealistic long run properties of the model, since these properties are simply neither observed nor implied by the joint prior density of parameters. To avoid such a situation, I use priors for steady state ratios in addition to those for parameters.

1.6.1 Acceleration of the numerical computation of the steady state equilibrium

In spite the fact that numerical solvers are used only for small sub-systems of equations, the computing time devoted to the resolution of the steady state in the estimation for such a large model is still too big when using standard Matlab solvers. Therefore, the resolutions of the numerical sub-systems of equations remaining after the partial analytical resolution of the static model are done by specific programs, using the `C` language (embedded in compiled `mex` files). These programs apply a Newton algorithm using analytical expressions for the Jacobian matrix of the systems. Another time-consuming task done in standard user-written steady state files is linking parameter and variable names used in the analytical expressions to the proper location in the vectors of parameter values and steady state levels that are used by Dynare. This is avoided through the use of a specific

Matlab macro which replaces, once and for all, the names of parameters and variables by direct references to the corresponding elements of the parameter and steady state vectors in the final steady state file. This is done before the estimation procedure is started.

1.6.2 Improvements of the gradient algorithm

The shape of large dimensions posterior density functions can be strongly irregular in some areas that are explored by the optimization algorithm. This is particularly true when some estimated parameters strongly affect the steady state equilibrium of the model or its existence; the space spanned by the support of the parameters' prior distribution includes areas where the posterior density is subject to penalties, yielding irregularities in their neighborhood. In such context, the gradient algorithm, that is most often used to find the mode of parameters' posterior density, may show some weaknesses. A usual practice consists in alternating between different optimization algorithm, using in particular a simplex based optimization routine.

Nevertheless, for this project, small changes to the gradient optimization routine of Dynare – which uses Christopher Sim's `csminwel.m` – have been implemented. For large estimations of that kind indeed, this routine very often stops away from a point where the gradient is null. This represents a considerable loss of time when the estimation is launched on external servers and is expected to continue during several hours or days. These changes respond to three observations; the first one is that the routine can often be restarted from the point where it has stopped without changing anything. This is because the approximated hessian matrix evaluated at that point and used to set the relative sizes of the steps in each direction gives too much weight to movements in directions where the density function is problematic. Re-initializing the hessian matrix to the identity matrix may overcome this problem. The reinitialization and restart are hence done once, when the routine fails to improve the target function. It actually stops only if it immediately fails again with the re-initialized hessian matrix. This avoids manual re-lauches of the routine.

A second observation made at points in the space of parameters where the routine stops is that the gradient provide a non-zero value for parameters which deteriorate the posterior density both when they are increased and when they are decreased. This may arise when the point is close to irregularities. Then the gradient provides with a wrong direction and the routine fails to escape from the problematic area. The proposed solution is to include an optional 3 point-gradient function in Dynare, which sets the components of the gradient to zero when the parameters for which the target is deteriorated both on the left and on the right of the intitial value. This function is also helpful to identify and signal the parameters that are concerned by such a problem. Moreover, the routine excludes unusually large values (generally due to penalties) of the target function from the computation of the gradient components; in particular, for a parameter with its initial value just above the frontier of a problematic area, the numerical computation of the derivative only considers the value of the target at the initial point and its value with the parameter incremented by an epsilon ("on the right"), whereas its value "on the left" including penalties is ignored. This avoids large numerical components in the gradient, which generate unstability.

A third observation is that the routine stops at points with non-null gradients at a point of the parameter space while it has found improving points when computing the gradient. This is at odds with the terminating message claiming that the algorithm is unable to progress any further and yields frequent early stops. The idea is to automatically try a mono-directional (moving only one parameter at a time) movement in the space of parameters every time the direction of the full gradient is ineffective. For that purpose, first, the numerical gradient routines of Dynare are amended to return also the vector of parameters implying the best value target function among those considered for the computation of the gradient. The corresponding point is the initial one for all parameters but one. In general cases, the value function found for this point is above the value function at the initial point. Next, the routine `csminit.m` is also amended in such a manner that, every time the gradient fails to improve the target, it attempts to move in the sole direction of the best point returned by the gradient function, testing different step lengths as in the usual case. In the case when this procedure also fails, the last resort consists in moving to this best point and computing a new gradient from there. This ensures that the routine almost never stops until reaching an inflexion point or the maximum number of iterations allowed. In practice, the amended `csminwel.m` has proved successful in the sense that it actually never stops unexpectedly. However, in difficult cases, it seemed that the uninterrupted improvements of the target functions were smaller and smaller, without showing signs that the routine got away from the problematic area. To conclude, this improvement helps to minimize losses of times resulting from too early stops of optimization, but does not exempt from investigating the causes of the irregularities in the posterior density.

1.6.3 Prior on moments

Accounting for such a prior is straightforward although it requires minor changes in Dynare. For any given set of parameters, the static and dynamic models are solved, then the ratios of interest can be easily computed as well as the log density of the distribution a priori for these values. This is done in by a matlab function which has a specific name, that is '[model name]_prior_restriction.m', which returns the sum of log-prior densities and is called by Dynare's 'dsge_likelihood.m' function. This value is then simply added to the posterior density. This device adds another restriction a priori to the parameters values in the estimation procedure. As a result, the complete prior distributions of parameters are not orthogonal any more. An interesting feature of this modification of Dynare is that, since the solved model is an input argument of the 'prior_restriction.m' function, it can be used to include distributions a priori for a large range of moments and model properties in a very simple manner: first order moments like in the estimation presented in what follows, but also correlations, relative variances or impulse responses.

Chapter 2

Disaggregated inspection of the model's ingredients

2.1 The financial accelerator

The canonical new-Keynesian model assumes that conditions in financial and credit market do not affect significantly the real economy. This view is contradicted by a number of episodes of macroeconomic downturns, including the recent depression. Bernanke et al. (1999) (BGG in what follows) argues that incorporating a feedback from credit markets to the real economy into DSGE models is helpful not only to account for such crisis episodes, but also to improve their ability to replicate a number of empirical facts of the business cycle. In particular, credit market frictions can amplify shocks to the economy. Next, authors such as Christiano et al. (2014) claim that shocks specific to financial markets propagate to the real economy but are also major contributors to business cycle fluctuations; this is particularly true for the shock that affects cross-sectional firms' performance, called 'risk' shock.

In this section, I build a very basic new-Keynesian model incorporating credit-market frictions à la BGG, to point up the role of the financial accelerator mechanism in the propagation of standard shocks, as compared to a frictionless credit-model. Next, I describe the dynamics implied by the two main financial shocks that are added to the model by the financial accelerator, that are 'risk shocks' and shocks to firms' net worth. Last, I show the impact of different calibrations of two financial parameters of the model on the model's impulse responses to financial and monetary policy shocks.

2.1.1 The model with frictionless credit markets

Households consuming C_t and working L_t in period t have instantaneous utility

$$U_t = \frac{C_t^{1-\sigma_c}}{1-\sigma_c} + \frac{L_t^{1+\sigma_l}}{1+\sigma_l}.$$

They can trade riskless nominal one-period bonds in an uncomplete market, B_t, remunerated at rate $R_t - 1$. The unit remuneration rate of labor is w_t. They also invest I_t in productive capital.

The capital accumulation technology is

$$K_t = (1-\delta)K_{t-1} + \varepsilon_{i,t}\left(1 - \frac{\varphi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t,$$

where $\varepsilon_{i,t}$ is a shock to investment technology. Capital is remunerated at rate r_t^k by firms. All households receive even (real) payments of dividends div_t. Last, households are perfectly insured against idiosyncratic labor market risk. Their real budget constraint is

$$C_t + I_t + \frac{B_t}{P_t R_t} = w_t L_t + r_t^k K_{t-1} + \frac{B_{t-1}}{P_t} + div_t.$$

The representative household's behavior is summarized by the following equations:

$$\lambda_t = C_t^{-\sigma_c}, \tag{2.1.1}$$

$$1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{R_t}{\pi_{t+1}}, \tag{2.1.2}$$

$$L_t^{\sigma_l} = \lambda_t w_t, \tag{2.1.3}$$

$$\begin{aligned} \frac{1}{\varepsilon_{i,t}} = Q_t &\left(1 - \frac{\varphi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \varphi \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right) \\ &+ \beta\varphi E_t \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1} \frac{\varepsilon_{i,t+1}}{\varepsilon_{i,t}}\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - 1\right), \end{aligned} \tag{2.1.4}$$

where Q_t is the ratio the Lagrange mutliplier associated to the capital accumulation technology and the one associated with the budget constraint, usually called Tobins' Q, and finally

$$Q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left(Q_{t+1}(1-\delta) + r_{t+1}^k\right).$$

Firms use workers and capital to produce goods with technology

$$Z_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}, \tag{2.1.5}$$

where A_t is an AR(1) process that represents exogenous disturbances to technology. They sell their production at relative price x_t to the retail sector in a perfectly competitive market, so

$$x_t = \frac{1}{A_t}\left(\frac{r_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}.$$

They optimally mix labor and capital input, so that their production cost is minimal, subject to technology constraint. The corresonding first order conditions entail

$$\frac{K_{t-1}}{L_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}.$$

The retail sector includes a continuum of monopolistic retailers who set their prices with rigidities à la Calvo (1983). The probability to reset their price is constant and equal to $1-\xi$ in each period. With probability ξ, their price is automatically indexed to a convex combination of past and long run inflation rates, $\pi_{t-1}^{\iota}\bar{\pi}^{1-\iota}$. Differentiated goods are then aggregated with a Dixit and Stiglitz (1977) technology with elasticity of substitution θ. Ignoring the dispersion of prices which does not play any role in a first order approximation of the model, the aggregate inflation rate is governed by the following equations:

$$\tilde{p}_t = \frac{\theta}{\theta-1}\frac{H_{1t}}{H_{2t}} \tag{2.1.6}$$

$$H_{1t} = x_t Y_t + \beta\xi E_t \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\pi_{t+1}}{\pi_t^{\iota}\bar{\pi}^{1-\iota}}\right)^{\theta} H_{1t+1} \tag{2.1.7}$$

$$H_{2t} = Y_t + \beta\xi E_t \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\pi_{t+1}}{\pi_t^{\iota}\bar{\pi}^{1-\iota}}\right)^{\theta-1} H_{2t+1} \tag{2.1.8}$$

$$1 = (1-\xi)\tilde{p}_t^{1-\theta} + \xi\left(\frac{\pi_{t-1}^{\iota}\bar{\pi}^{1-\iota}}{\pi_t}\right)^{1-\theta}. \tag{2.1.9}$$

The presence of monopolists implies that the amount of final goods produced in the economy Y_t is related to production Z_t by

$$d_t Y_t = Z_t,$$

where 'price dispersion' d_t obeys

$$d_t = (1-\xi)\tilde{p}_t^{-\theta} + \xi\left(\frac{\pi_{t-1}^{\iota}\bar{\pi}^{1-\iota}}{\pi_t}\right)^{-\theta} d_{t-1},$$

and plays no role in the first order approximation of the model. Last, general equilibrium entails

$$Y_t = C_t + I_t, \tag{2.1.10}$$

and the monetary authority sets the nominal interest factor according the a standard Taylor rule

$$R_t = R_{t-1}^{\rho}\left(\frac{\bar{\pi}}{\beta}\left(\frac{\pi_t}{\bar{\pi}}\right)^{r_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{r_y}\right)^{1-\rho}\varepsilon_{r,t}, \tag{2.1.11}$$

where ε_r is an exogenous monetary policy shock.

2.1.2 Financial accelerator

A large number of entrepreneurs and a competitive bank serve as intermediaries between households and firms for the service of capital. Households still accumulate capital, but they sell their stock K_t to entrepreneurs in each period at market price $P_t Q_t$, and purchase it back, after it is used in production and depreciated in the following period, at market price $P_{t+1}Q_{t+1}$. By contrast with the basic case without financial frictions, households consider the stock of capital of the previous period as an externality. Since they are atomistic and sell their entire capital stock in each period, they consider the capital stock remaining from the previous period as resulting from the aggregate contribution of all households and not directly related to their past decisions.[1] Entrepreneurs face an idiosyncratic shock to their efficiency in installing capital, denoted by ω. This shock is independently distributed across entrepreneurs and time. Its density function f_{t-1} is assumed to be log-normal with mean 1 and exogeneously varying standard deviation σ_{t-1}. The subscript $t-1$ is used to state that the standard deviation of capital installation productivity in the cross-section of entrepreneurs, called 'risk' in what follows, is known one period in advance. Disturbances to 'risk' or 'risk shocks' are assumed to follow an AR(1) process. This timing assumption implies that the idiosyncratic shock whose distribution is known in period t affects the current period capital stock K_t and hence production. If an entrepreneur who owns a stock of capital K_t is hit by a given ω, she can offer ωK_t to producing firms. The latter use it in the following period for production and remunerate her at rate r^k_{t+1}. Her cash flow in period $t+1$ that results from capital acquisition for $P_tQ_tK_t$ in period t includes the selling of depreciated capital for $Q_{t+1}P_{t+1}(1-\delta)\omega K_t$, the rental from firms $r^k_{t+1}P_{t+1}\omega K_t$. It is written as $\omega R^k_{t+1}Q_tP_tK_t$, with

$$R^k_{t+1} = \pi_{t+1}\frac{Q_{t+1}(1-\delta)+r^k_t}{Q_t}.$$

Entrepreneurs use equity for P_tNt and one period bank loans for D_t (in nominal terms) to purchase capital $P_tQ_tK_t$. Bank loans are remunerated at factor R^D_{t+1}. Therefore, the condition for an entrepreneur to be able to repay her loan in period $t+1$ is

$$\omega R^k_{t+1}Q_tP_tK_t \geq R^D_{t+1}D_t.$$

Otherwise, when an entrepreneur draws an ω below a threshold denoted by $\bar{\omega}_{t+1}$ for which the equation above holds with equality, he faces bankruptcy, and the bank only recover what is left minus a fraction μ assimilated to auditing fees, that is

$$(1-\mu)\omega R^k_{t+1}Q_tP_tK_t.$$

Auditing fees are paid to the governement and then repaid as lump-sum payments to households. On the whole, the bank is paid

$$(1-\mu)\int_0^{\bar{\omega}_{t+1}} \omega P_tQ_tK_tR^k_{t+1}f_t(\omega)\,d\omega + \int_{\bar{\omega}_{t+1}}^{\infty} R^D_{t+1}D_t\,d\omega.$$

The bank refinances her loans from households, and remunerates them at the riskless interest factor of bonds R_t. The interest factor R^D_{t+1} is set in $t+1$ such that bank's profits associated with loans contracted in period t are zero. Therefore, using the accounting equality $P_tQ_tK_t = N_t + D_t$ and the

[1] An equivalent presentation used by Bernanke et al. (1999) or Christiano et al. (2014) assumes a separate capital producing sector. Yet it is equivalent as long as the stock of capital inherited from the previous period is an externality in the optimization program of the concerned agents.

condition that defines the bankruptcy threshold $\bar{\omega}_{t+1}R^k_{t+1}Q_tP_tK_t = R^D_{t+1}D_t$, the equation above can be written as follows in period t:

$$(\bar{\omega}_t\left(1-F_{t-1}(\bar{\omega}_t)\right)+(1-\mu)G_{t-1}(\bar{\omega}_t))\,\frac{R^k_t}{R_{t-1}}\frac{Q_{t-1}K_{t-1}}{N_{t-1}}=\frac{Q_{t-1}K_{t-1}}{N_{t-1}}-1,$$

where F_{t-1} is the c.d.f. of ω in period t, and

$$G_{t-1}(\bar{\omega}_t)=\int_0^{\bar{\omega}_t}\omega f_{t-1}(\omega)d\omega.$$

It defines the credit supply behavior of banks. The program of an entrepreneur consists in maximizing his cash flow expected in $t+1$ from his period t-investment, subject to his expectations of the realization of his idiosyncratic productivity ω, and to a banks credit supply curve for any state of the nature in $t+1$, since the interest rate on loans is revised after the realization of $t+1$-shocks. Formally, entrepreneurs choose a level of leverage ratio Q_tK_t/N_t and a bankrupt threshold $\bar{\omega}(s)$ for any state of the nature s in $t+1$ to maximize

$$Q_tK_t\int_{\mathcal{S}_{t+1}}\left[R^k_{t+1}(s)\int_{\bar{\omega}_{t+1}(s)}^{\infty}(\omega-\bar{\omega}_{t+1}(s))f_t(\omega)d\omega\right]t_{s_t|s}ds,$$

subject to

$$\int_{\mathcal{S}_{t+1}}\nu_{t+1}(s)\left\{\left[\bar{\omega}_t(1-F_{t-1}(\bar{\omega}_t))+(1-\mu)G_{t-1}(\bar{\omega}_t)\right]\frac{R^k_t}{R_{t-1}}\frac{Q_{t-1}K_{t-1}}{N_{t-1}}-\frac{Q_{t-1}K_{t-1}}{N_{t-1}}+1\right\}ds,$$

where $\mathcal{S}_{t+1}$ is the set of the possible states of the nature in period $t+1$, and $t_{s_t|s}$ is the probability density of the transition from the present state s_t to the state s in period $t+1$. This program yields the first order condition

$$\begin{aligned}E_t\Bigg\{&\left(1-\bar{\omega}_{t+1}\left(1-F_t(\bar{\omega}_{t+1})\right)-G_t(\bar{\omega}_{t+1})\right)\frac{R^k_{t+1}}{R_t}\\&+\frac{1-F_t(\bar{\omega}_{t+1})}{1-F_t(\bar{\omega}_{t+1})-\mu G'_t(\bar{\omega}_{t+1})}\left[\left(\bar{\omega}_{t+1}\left(1-F_t(\bar{\omega}_{t+1})\right)+(1-\mu)G_t(\bar{\omega}_{t+1})\right)\frac{R^k_{t+1}}{R_t}-1\right]\Bigg\}=0.\end{aligned}$$

The law of motion of entrepreneurs real equity N_t reflects the accumulation of past profits. To rule out explosive trajectories of entrepreneurs' wealth, a time-varying fraction of entrepreneurs γ_t, chosen randomly, exit the economy in each period and are replaced by new ones. Their accumulated profits at that time are simply paid evenly to households. All non-exiting entrepreneurs also receive a dotation $\mathcal{W}$ from the government, in order to ensure that even new entrants have a strictly positive equity value. Hence, the law of motion of equity is

$$P_tN_t=\gamma_t\left(1-\bar{\omega}_t\left(1-F_t(\bar{\omega}_t)\right)-G_t(\bar{\omega}_t)\right)P_{t-1}Q_{t-1}R^k_tK_{t-1}+\mathcal{W}.$$

The entrepreneur separation rate γ_t is described by an exogenous AR(1) process. Shocks to γ_t are called net worth shocks.

2.1.3 Calibration

I choose the following standard values for the parameters that are common to the models with and without financial frictions:

β	σ_c	σ_l	ϕ	α	θ	ξ	r_π	δ
0.99	1.5	2	2.5	0.3	6	0.75	1.5	0.02

The persistence parameters of the Taylor rule and of the exogenous AR(1) processes are set to 0.7. The values chosen for the financial parameters are very close to those used by Bernanke et al. (1999) or by Christiano et al. (2014). The fraction of auditing costs is 0.27. In the steady state, the exit rate of entrepreneurs is 2.5% and the standard deviation of the distribution of idiosyncratic productivities is 0.26. The level of the dotation $\mathcal{W}$ is arbitrarily set to 0.1. With this parametrization, I find a steady state fraction of bankrupt entrepreneurs of 0.76%.

2.1.4 Simulations

As shown by Figure 2.1, the presence of a financial accelerator dampens the response of investment to productivity shocks. This is because the 'Fisher deflation' channel prevails over the pure financial accelerator channel. The pure financial accelerator channel operates through the increase in entrepreneurial assets resulting from positive shocks to the economy, which makes borrowing even easier. It is described by Bernanke et al. (1999) and magnifies the economic effects of shocks. The 'Fisher deflation' channel is described by Christiano et al. (2003). It results from the fact that banks are committed to paying households a non-state contingent nominal interest rate R_t at time $t+1$ on their deposits D_t. As these banks make no profit ex post, when prices unexpectedly fall after a transitory productivity shock, the real cost of entrepreneurs' debt, payable in the following period, is augmented, for the benefit of households. This mechanism contributes countercyclically to investment. Christiano et al. (2010) also find that the financial accelerator dampens productivity shocks. They demonstrate that this finding is related to the 'Fisher deflation' channel. By contrast, Bernanke et al. (1999) assume that the return on households' deposits is non-contingent in real terms; consistently, they find that the financial accelerator amplifies productivity shocks.

The economic effects of investment technology shocks are also dampened. Indeed, these shocks represent an increase in the supply of capital, so they diminish its price Q_t. As a result, the net asset value of entrepreneurs declines, which raises the cost of external finance for entrepreneurs. This effect is also described by Christiano et al. (2010).

Financial frictions considerably amplify the response of investment to a demand side monetary policy shock. The persistence of the effect on output is also augmented. Figure 2.2 plots the response of the economy to shocks specific to the financial accelerator. The risk shock affects the distribution of idiosyncratic producitivities among entrepreneurs, and thus modifies the appreciation of default probability by banks. It can be viewed as a credit supply shock. The net worth shock directly influence the value of entrepreneurs equity and hence their self-financing capacities. Apart from the higher persistence of the latter, a major difference between those two shocks is the response of credit, which is procyclical after a risk shock and countercyclical after a net worth shock.

Figure 2.3 and 2.4 show the sensitivity of the responses to different calibrations of two key parameters: the size of auditing costs μ and the steady state standard deviation of the distribution of entrepreneurs productivities. The effect of μ is moderate in response to a monetary policy shock, but the response of investment is amplified by larger values of μ in response to financial shocks. The magnitude of the response of credit in response to risk shocks is also an increasing function of this parameter. The response of investment and financial variables are very sensitive to the value of the steady state standard deviation of entrepreneurs' productivities. In fact, the effect of this parameter is strongly linked to the steady state bankruptcy probability $F(\bar{\omega})$. For instance, if the distribution is very tight and the bankruptcy probability is sufficiently low, the bankruptcy threshold $\bar{\omega}$ lies in an area where the distribution is almost flat. Changes in its standard deviation have then almost no effect.

Figure 2.1: Impact of the financial accelerator on impulse response to standard shocks

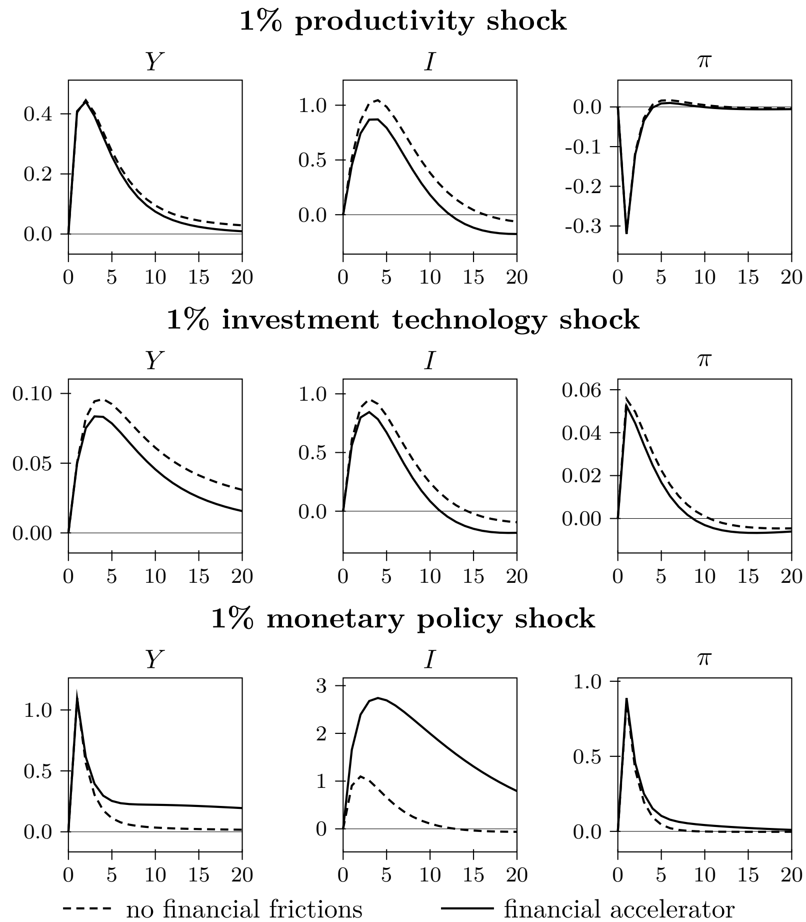
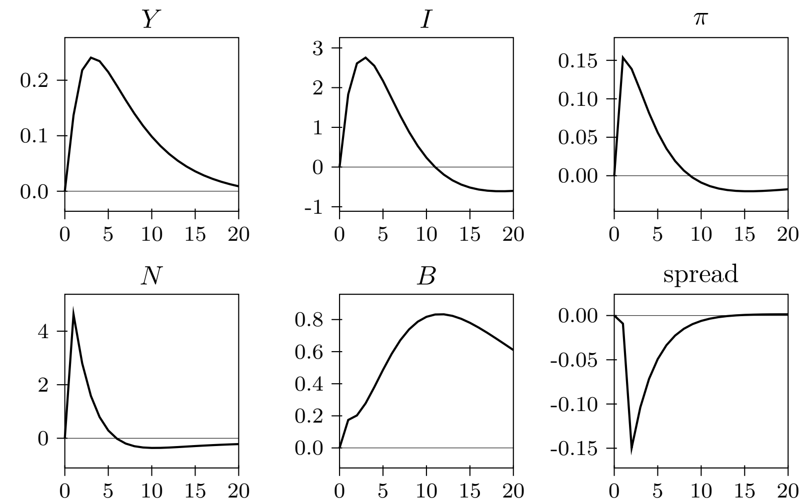


Figure 2.2: Impulse responses to financial shocks

50% risk shock

(i.e. a shock to the standard deviation of the distribution of entrepreneurs idiosyncratic productivities that moves the quarterly bankruptcy rate from 0.76% to 0.69%.)



1% net worth shock

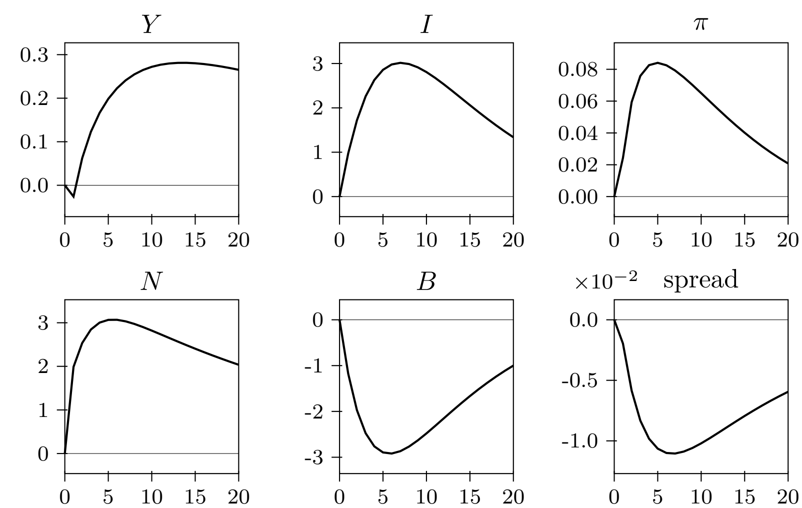


Figure 2.3: Impact of the calibration of auditing costs on the dynamics of the model

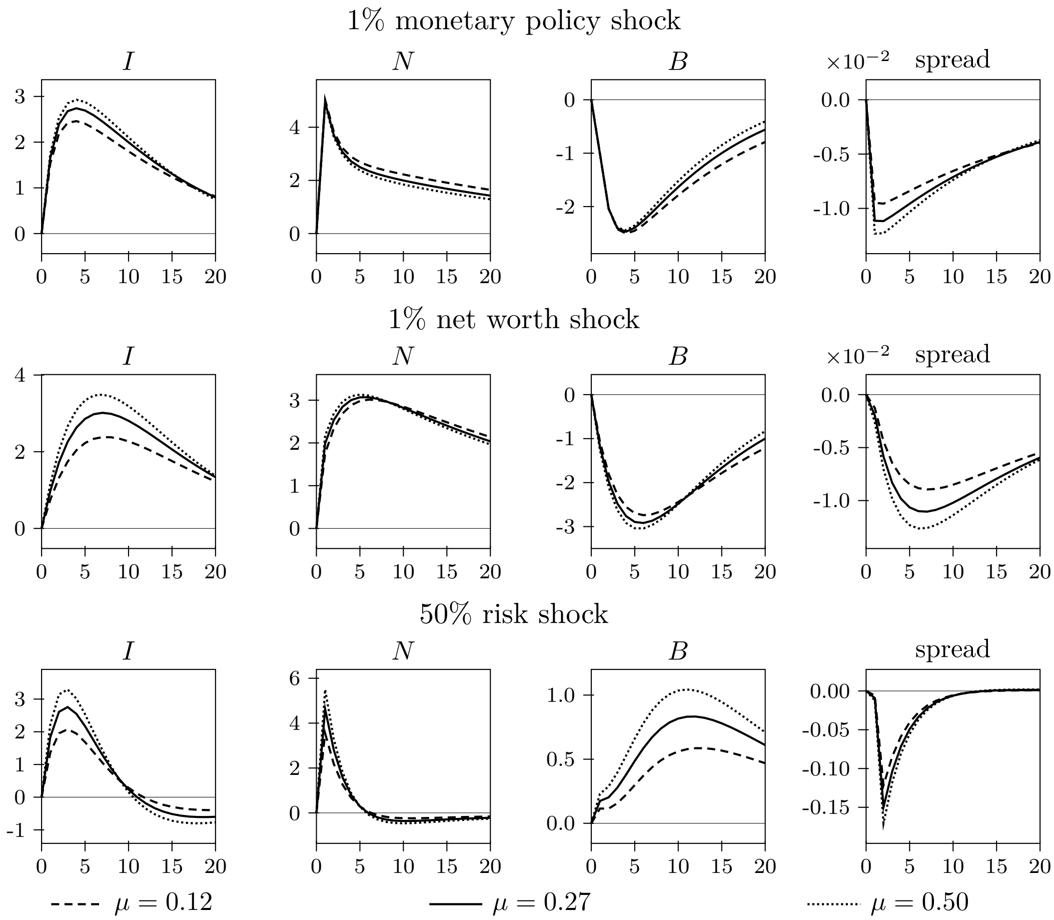
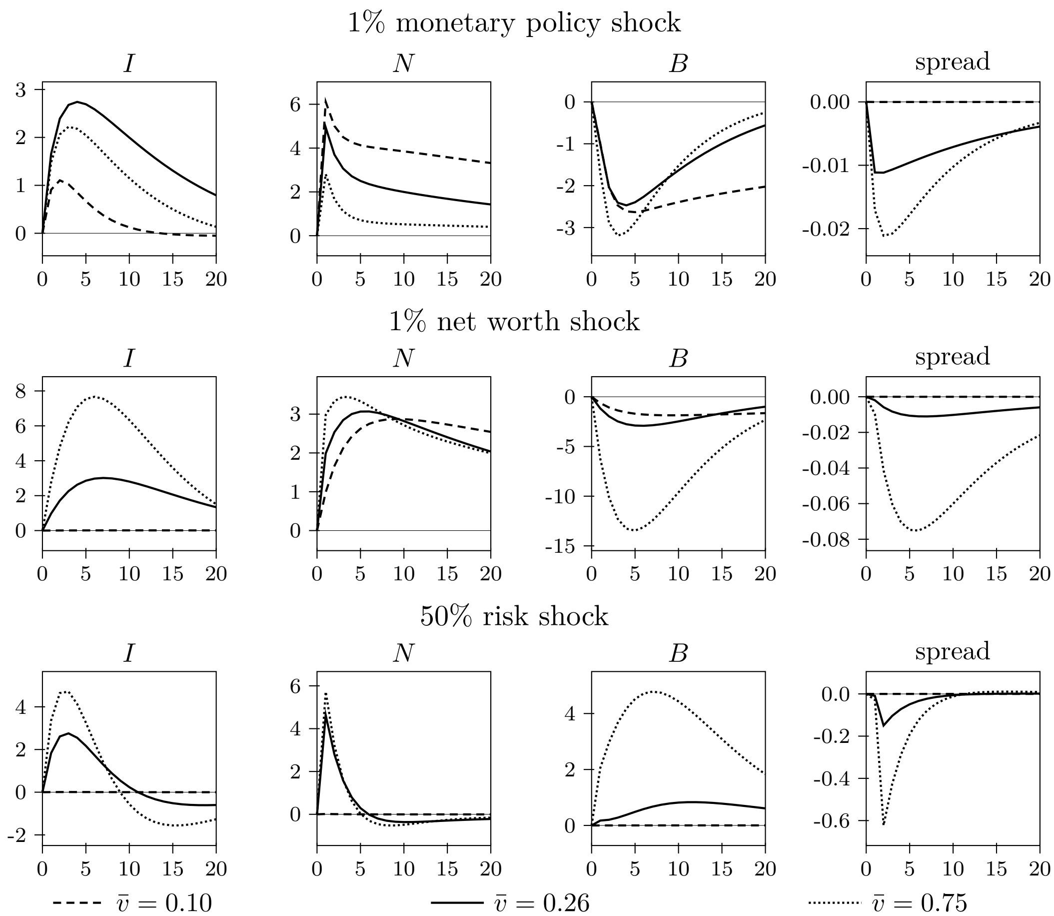


Figure 2.4: Impact of the calibration of the standard deviation of entrepreneurs' productivities on the dynamics of the model



2.2 Capital utilization costs and financial frictions

In this section, I show that a financial accelerator model generates countercyclical capital utilization rate in response to the 'risk' shocks identified by Christiano et al. (2014) as a major source of business cycle fluctuations, if the depreciation rate of capital is a growing function of utilization. Indeed, in response to a negative risk shock, the capital return rate decreases, which would tend to move the CUR downwards. But at the same time, the price of capital also drops, which makes depreciation less costly and stimulates capital utilization. Conversely, a negative technology shock to capital accumulation restricts the supply of capital to entrepreneurs and induces thus a rise in the price of capital, which discourages further capital utilization.

Yet, in French and euro area data, the capital utilization rate is strongly procyclical. Hence, investment technology shocks, which move it procyclically, are likely to be preferred to risk shocks as prime source of business cycle fluctuations. As the capital utilization rate is also procyclical in the US, the findings of Christiano et al. (2014) crucially depend on the assumption that capital utilization costs are not depending on the price of capital.

2.2.1 Introduction

Greenwood et al. (1988) show that variable capacity utilization allows labor productivity and consumption to react positively to shocks to the marginal efficiency of investment in a real business cycle model. This shocks could therefore be an important source of fluctuations since consumption, investment and productivity are positively correlated in the data. In a standard RBC model, the increase in the rate of return of investment drives households to postpone consumption and leisure through an intertemporal substitution effect. Given the fixed stock of installed capital, labor productivity declines. By contrast, variable capacity utilization adds an intratemporal substitution effect towards consumption and away from leisure; in response to a higher rate of return of investment, capacity utilization increases and raises labor productivity, which is also the opportunity cost of current leisure in terms of consumption.

Whereas Greenwood et al. (1988) assume that the capital depreciation rate is variable and depends positively on the capacity utilization rate, many authors, such as Christiano et al. (2005) and Smets and Wouters (2007), consider instead that households incur a cost of capital utilization expressed in units of consumption goods. These two setups are formally pretty close in simple models. Yet, shocks to the marginal efficiency of investment induce a higher response of the capacity utilization rate with variable capital depreciation rather than with simple capital utilization costs, because it allows depreciated capital to be replaced with new, more productive, capital. Hence, variable capital depreciation magnifies the mechanism put forward by Greenwood et al. (1988).

If the model' response to shocks to the marginal efficiency of investment does not clearly discriminate between the two specifications of capital utilization costs, in the presence of financial frictions, there is a marked difference in the response to "risk shocks", i.e. shocks to the variance of entrepreneurs' idiosyncratic productivity, which are identified as the main source of business cycle fluctuations in the US by Christiano et al. (2014) and in the euro area by Tripier and Brand (2014).

Specifically, risk shocks induce positive comovements between the capacity utilization rate and investment in the presence of capacity utilization costs expressed in units of consumption goods, but negative ones with variable capital depreciation. As these two variables are strongly positively correlated in the data for the US, the EA or France, assuming variable capital depreciation could put forward other sources of fluctuations such as investment technology shocks, contrasting with these authors' findings. This issue does not seem to be discussed in the literature, but usual practices suggest that it is well-founded: Christiano et al. (2014) assume capital utilization costs expressed in units of consumption goods, while Sanjani (2014) uses variable depreciation together with a non-conventional specification of the value of capital after depreciation to circumvent this pitfall.

If capital depreciates faster when it is used more intensively, a high price of capital leads agents to reduce the capacity utilization rate to slow down the erosion of the market value of their capital stock. This is clear when looking at the standard linearized capacity utilization equation, which is

$$\omega \hat{z}_t = \frac{\hat{r}_t^k}{\bar{r}^k} - \hat{Q}_t$$

with variable capital depreciation, but

$$\omega \hat{z}_t = \frac{\hat{r}_t^k}{\bar{r}^k}$$

with capacity utilization costs expressed in units of consumption goods. In these equations, z denotes the capacity utilization rate, r^k the rental rate of capital and Q the price of capital. The presence of Q_t in the first one explains the differences in the reaction of the capacity utilization rate in response to shocks to marginal efficiency of investment and to risk shocks. Positive shocks to the marginal efficiency of investment increase the supply of capital so investment increases while the price of capital falls (or, in a model without financial frictions, it relaxes somewhat the capital accumulation constraint so the shadow price of capital or Tobin's Q falls). By contrast, expansionary risk shocks increase the demand of capital by entrepreneurs because financing conditions improve; hence both investment and the price of capital increase. This leads to contrasted responses in the capacity utilization rate. With capital utilization costs expressed in units of consumption, since the rental rate of capital increases in response to both shocks, capacity utilization also moves in the same direction.

2.2.2 Illustration

I illustrate this behavior using a basic new-Keynesian model including financial frictions à la Bernanke et al. (1999), described in section 2.1 and calibrated with standard values. Table 2.1 gives the correlation coefficients implied by both shocks with and without variable capital depreciation, as well as the correlation between capacity utilization and investment in French and EA data.[2] Figure 2.5 shows the corresponding impulse response functions. Future work should include re-estimating the model of Christiano et al. (2014) assuming variable capital depreciation, in order to assess to what extent their finding that risk shocks are the main contributor to the fluctuations

[2] Moments are computed using hp-filtered series from the model's database over 1995Q2-2013Q2.

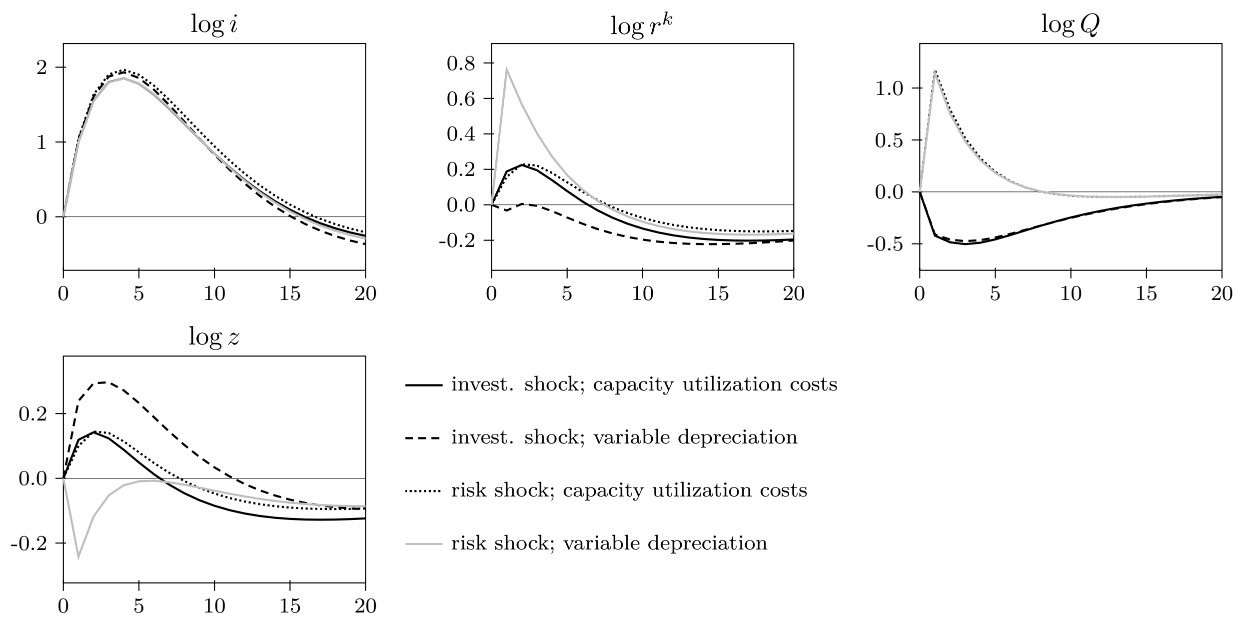
in output is contingent on this particular assumption.

Table 2.1: Effect of variable capital depreciation vs. capital utilization costs

corr with $\log i_t$	invest shock		risk shock		data	
	$\psi(z_t)$	$\delta(z_t)$	$\psi(z_t)$	$\delta(z_t)$	euro area	France
$\log r_t^k$	0.37	0.00	0.50	0.60	–	–
$\log Q_t$	-0.91	-0.87	0.56	0.56	–	–
$\log z_t$	0.37	0.92	0.50	-0.08	0.86	0.73

Notes: the table reports the asymptotic correlation coefficients between investment and the rental rate of capital, the price of capital and the capacity utilization rate, obtained with basic new-keynesian model including financial frictions in response to shocks to the marginal efficiency of investment only and to risk shocks only, under the assumption of variable capital depreciation rate – $\delta(z_t)$ – and capital utilization costs expressed in units of consumption goods – $\psi(z_t)$.

Figure 2.5: Responses to investment technology and risk shocks



2.3 Labor market frictions

In this section, I add search and matching frictions to a basic new-Keynesian model with sticky prices and flexible wages.For the calibration considered, the main difference in the response to technology and monetary policy shocks is that the volatility of wages and inflation is diminished. The response of output to technology shocks is also smoothed.[3]

[3]The large magnitude of the responses observed in the basic model primarily results from the chosen elasticity of labor supply with respect to the wage rate. With $\eta = 2$, this elasticity is 0.38. It is close to the one used by Smets

I also compare the responses to a labor supply preference shock (i.e. a shock to the disutility of labor) in the frictionless model to the responses to a shock to the relative bargaining power of workers in the search and matching model. As compared with a labor supply supply shock, a ten times larger bargaining shock is needed to obtain a response of similar magnitude. Aside from this, the shape of the responses are very similar. This results from the fact that the new-Keynesian economy including unemployment is strongly driven by demand in the goods market. Last, I introduce endogeneous separations as in Den Haan et al. (2000), which induce countercyclical movements in the separation rate. The role of cyclical variations in the separation rate in explaining fluctuations in unemployment is disputed in the literature; while Fujita and Ramey (2009) find a significant contribution, Shimer (2012) argue that fluctuations in the employment exit probability have been quantitatively irrelevant since the 1990s in the US.

Introducing endogeneous separations in an estimated model of the Euro area can be motivated by three reasons: (i) parameter estimation may lead to either a significant or an unimportant role of fluctuations in the separation rate without having to choose, (ii) the separation rate in the Euro area may behave differently from the US, and (iii) the 2008 recession, included in our estimation sample, is presumably characterized by a high exit rate from employment. Yet, the analysis in section 2.5 identifies unwanted effects of this assumption in the presence of a particular form of wage rigidity. Hence, the final version of the model of chapter 3 does not assume endogeneous job separations.

In the small calibrated model developed in what follows, endogeneous job desctructions only marginally alter the dynamics of the economy. The dynamic response of employment is primarily governed by the demand for final goods. So it only marginally depends upon the channels through which the number of jobs may vary.

2.3.1 The model

Frictionless labor market

Households consuming C_t in period t have instantaneous utility

$$U_t = \log C_t + \varepsilon_{l,t}\chi\frac{(1-L_t)^{1-\eta}}{1-\eta},$$

where L_t is the fraction of time they spend working, and $\varepsilon_{l,t}$ is an AR(1) process that represents exogenous disturbances to households disutility of labor. They can trade riskless nominal one-period bonds in an uncomplete market, B_t, remunerated at rate $R_t - 1$. The unit remuneration rate of labor is w_t. All households receive even (real) payments of dividends div_t and government transfers T_t. Last, households are perfectly insured against idiosyncratic labor market risk. Their real budget constraint is

$$C_t + \frac{B_t}{P_t R_t} = w_t L_t + \frac{B_{t-1}}{P_t} + T_t + div_t.$$

and Wouters (2007). With a larger elasticity, the impulse responses would have comparable magnitudes.

The representative household's behavior is summarized by the following equations:

$$\lambda_t = \frac{1}{C_t} \tag{2.3.1}$$

$$1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t}{\pi_{t+1}} \tag{2.3.2}$$

$$\varepsilon_{l,t} \chi (1 - L_t)^{-\eta} = \lambda_t w_t. \tag{2.3.3}$$

Firms use workers to produce goods with technology

$$Z_t = A_t L_t, \tag{2.3.4}$$

where A_t is an AR(1) process that represents exogenous disturbances to technology. They sell their production at relative price x_t to the retail sector in a perfectly competitive market, so

$$x_t = \frac{w_t}{A_t}.$$

The retail sector includes a continuum of monopolistic retailers who set their prices with rigidities à la Calvo (1983). The probability to reset their price is constant and equal to $1 - \xi$ in each period. With probability ξ, their price is automatically indexed to a convex combination of past and long run inflation rates, $\pi_{t-1}^{\iota} \bar{\pi}^{1-\iota}$. Differentiated goods are then aggregated with a Dixit and Stiglitz (1977) technology with elasticity of substitution θ. Ignoring the dispersion of prices which does not play any role in a first order approximation of the model, the aggregate inflation rate is governed by the following equations:

$$\tilde{p}_t = \frac{\theta}{\theta - 1} \frac{H_{1t}}{H_{2t}} \tag{2.3.5}$$

$$H_{1t} = x_t Y_t + \beta \xi E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_{t+1}}{\pi_t^{\iota} \bar{\pi}^{1-\iota}} \right)^{\theta} H_{1t+1} \tag{2.3.6}$$

$$H_{2t} = Y_t + \beta \xi E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_{t+1}}{\pi_t^{\iota} \bar{\pi}^{1-\iota}} \right)^{\theta - 1} H_{2t+1} \tag{2.3.7}$$

$$1 = (1 - \xi) \tilde{p}_t^{1-\theta} + \xi \left(\frac{\pi_{t-1}^{\iota} \bar{\pi}^{1-\iota}}{\pi_t} \right)^{1-\theta}. \tag{2.3.8}$$

The presence of monopolists implies that the amount of final goods produced in the economy Y_t is related to production Z_t by

$$d_t Y_t = Z_t,$$

where 'price dispersion' d_t obeys

$$d_t = (1 - \xi) \tilde{p}_t^{-\theta} + \xi \left(\frac{\pi_{t-1}^{\iota} \bar{\pi}^{1-\iota}}{\pi_t} \right)^{-\theta} d_{t-1},$$

and plays no role in the first order approximation of the model. Last, general equilibrium entails

$$Y_t = C_t + G, \tag{2.3.9}$$

where G is constant government spending, and the monetary authority sets the nominal interest factor according the a standard Taylor rule

$$R_t = R_{t-1}^{\rho} \left(\frac{\bar{\pi}}{\beta} \left(\frac{\pi_t}{\bar{\pi}} \right)^{r_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{r_y} \right)^{1-\rho} \varepsilon_{r,t}, \tag{2.3.10}$$

where ε_r is an exogenous monetary policy shock.

Search and matching frictions with constant separation rate

At the beginning of every period, households are chosen randomly to occupy existing jobs. The number of hours worked by employed households is identical and constant over time. Their instantaneous utility is $U_t = \log C_t - \Gamma$ when they have a job, and $U_t = \log C_t$ when unemployed. Employed workers are paid a real wage w_t per period, while the unemployed receive nothing. All households receive even payments of dividends and government transfers. Last, households are perfectly insured against idiosyncratic labor market risk. The representative household's behavioral equations are the same as with frictionless labor market, excluding the labor supply curve. The number of jobs evolves as described by the following equation

$$N_t = (1 - s)N_{t-1} + m_t, \tag{2.3.11}$$

where the number of new jobs created in period t is given by a standard Cobb-Douglas matching function

$$m_t = \Upsilon v_t^{\kappa} (1 - (1 - s)N_{t-1})^{1-\kappa}. \tag{2.3.12}$$

Firms use workers to produce goods with technology

$$Z_t = A_t N_t, \tag{2.3.13}$$

They can post vacancy with a unit cost c. The optimal number of vacant jobs held by firms is governed by

$$\frac{c}{q_t} = x_t A_t - w_t + \beta(1 - s) E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{c}{q_{t+1}}, \tag{2.3.14}$$

where

$$q_t = \frac{m_t}{v_t} \tag{2.3.15}$$

is the vacancy filling rate. As a result from this condition, the markup of producing firms is no longer zero. The real wage is bargained by workers and firms to share the surplus generated by a successful match. The total surplus is the sum of the marginal values of employment for the firms and the workers, expressed in units of consumption goods. The marginal value of employment for firms is

$$J_t = x_t A_t - w_t + \beta(1-s)E_t \frac{\lambda_{t+1}}{\lambda_t} J_{t+1},$$

and the marginal value of employment for households is

$$W_t = \lambda_t w_t - \Gamma + \beta(1-s)E_t(1-f_{t+1})W_{t+1},$$

where

$$f_t = \frac{m_t}{1-(1-s)N_{t-1}} \tag{2.3.16}$$

is the job finding rate. So the surplus is $S_t = W_t/\lambda_t + J_t$. The outcome of the bargain is assumed to be the real wage rates which maximizes $(W_t/\lambda_t)^\delta J_t^{1-\delta}$, where the parameter δ measures the relative bargaining power of workers. The wage equation is therefore

$$w_t = (1-\delta)\frac{\Gamma_t}{\lambda_t} + \delta\left(x_t A_t + c\beta(1-s)E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{f_{t+1}}{q_{t+1}}\right). \tag{2.3.17}$$

Last, the resource constraint of the economy takes vacancy costs into account, as follows:

$$Y_t = C_t + G + cv_t. \tag{2.3.18}$$

Search and matching frictions with endogeneous separations

Households behavior is unchanged with respect to the model with constant separation rate. After matching for the current period occured, all existing jobs are subject to an idiosyncratic shock ω to their productivity. This shock is independantly distributed across jobs and time, and has a constant log-normal density function f with mean 1 and support $[0, +\infty[$. Any given job that is hit by a shock ω is able to produce ω. However, after observing these shocks, the producing firm can decide to destroy the least productive jobs. Let $\bar{\omega}_t$ be the threshold below which jobs existing at the beginning of period t are destroyed before actually entering in production. Then, the low of motion of employment is changed to

$$N_t = [(1-s)N_{t-1} + m_t](1 - F(\bar{\omega}_t)),$$

where F is the cumulative distribution function of ω, and total production is

$$Z_t = A_t \mathcal{L}_t,$$

with

$$\mathcal{L}_t = ((1-s)N_{t-1} + m_t)\int_{\bar{\omega}_t}^{\infty} \omega f(\omega)d\omega \equiv N_t \frac{G(\bar{\omega}_t)}{1-F(\bar{\omega}_t)}.$$

The optimality conditions of firms are

$$\frac{c}{q_t} = x_t A_t G(\bar{\omega}_t) - (1-F(\bar{\omega}_t))w_t + \beta(1-F(\bar{\omega}_t))(1-s)E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{c}{q_{t+1}}$$

with respect to vacancy posting, and

$$(1 - F(\bar{\omega}_t))x_t A_t \bar{\omega}_t + \frac{c}{q_t} = x_t A_t G(\bar{\omega}_t)$$

with respect to the number of endogeneous layoffs determined by $\bar{\omega}_t$. Last, the wage bargaining condition is changed to

$$w_t = (1 - \delta)\frac{\Gamma_t}{\lambda_t} + \delta \left(x_t A_t \frac{G(\bar{\omega}_t)}{1 - F(\bar{\omega}_t)} + c\beta(1 - s)E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{f_{t+1}}{q_{t+1}} \right). \tag{2.3.19}$$

2.3.2 Calibration

I use the calibration provided in the table below:

parameter	η	$\bar{N}$	s	$\bar{q}$	$\bar{\pi}$	$G/\bar{Y}$	β	$\kappa = \delta$	θ	ξ	ι	r_π	r_y	$F(\bar{\omega})$
no friction	2	0.57	–	–	1.0062	0.319	0.99	–	6	0.75	0.5	1.5	0.125	–
s.m. cst sep.	–	0.57	0.15	0.9	1.0062	0.319	0.99	0.5	6	0.75	0.5	1.5	0.125	–
s.m. var. sep.	–	0.57	0.10	0.9	1.0062	0.319	0.99	0.5	6	0.75	0.5	1.5	0.125	0.05

The autoregressive coefficient is set to 0.7 for all AR(1) and the Taylor rule. In the frictionless version, the value of χ is obtained from the steady state labor supply curve. In the search and matching model, the matching technology scale parameter Υ is set so that the matching and the law of motion of employment are both satisfied in the long run. The labor disutility divided by the marginal utility of consumption (the workers' steady state outside options) $\Gamma/\bar{\lambda}$ needs to verify the wage equation in the steady state. The unit vacancy cost c is set such that recruiting expenditures represent 1% of output in the steady state. Last, in the version with endogeneous job destructions, the cross-sectional standard deviation of workers idiosyncratic productivities is computed numerically such that the first order condition with respect to the layoff threshold $\bar{\omega}_t$ is satisfied in the long run for $F(\bar{\omega}) = 0.05$.

2.3.3 Simulations

With endogenous job desctructions, the dynamics of employment can be broken down into regular separations and layoffs that occur because jobs are not profitable. For the present calibration based on Den Haan et al. (2000), they are presented in figure 2.7. While monetary shocks yield an increase in the demand addressed to firms and only slightly impact employment owing to a decrease in the number of layoffs, productivity and bargaining shocks directly impact the profitability of jobs and hence induce a strong contribution of endogenous job destructions to the dynamics of employment.

Figure 2.6: Effect of search and matching frictions and endogenous job destructions on the dynamics of the model

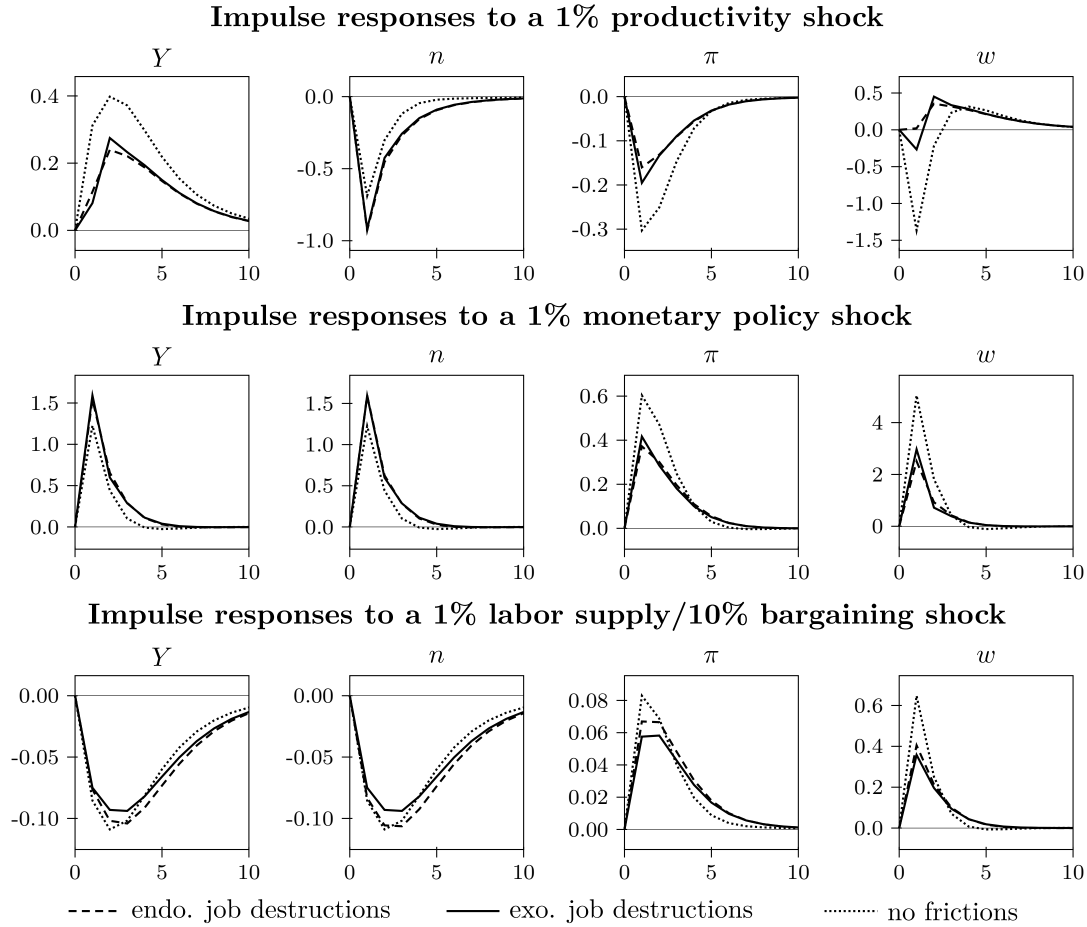
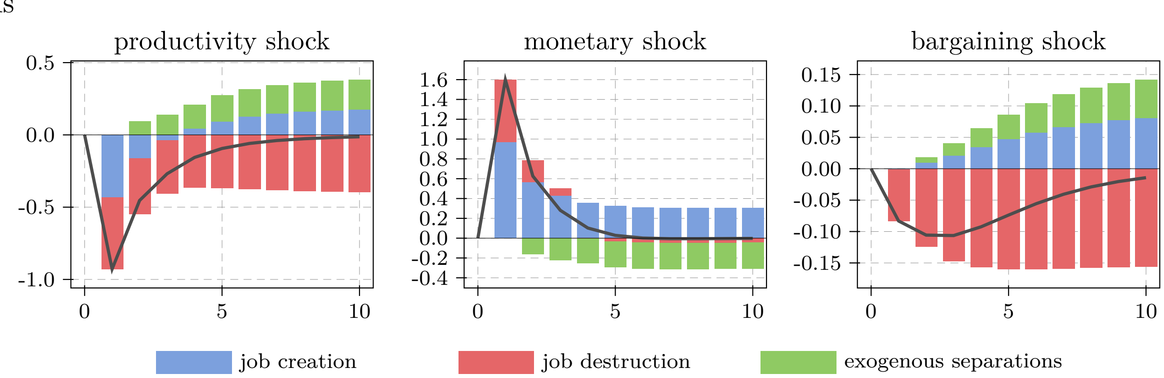


Figure 2.7: Contributions to fluctuations in employment in the model with endogenous job destructions



Notes: since employment is a stock variable and job creations or destructions are flow variables, the histograms picture the cumulated effects of the latter. Hence, these contributions never return to zero in the long run: a transitory increase in job creations ultimately induces an increase in job destructions of the same size, so that employment returns to its steady state level.

2.4 Wages and employment dynamics in new-Keynesian search and matching models

2.4.1 Introduction

In new-Keynesian models with constant hours worked per employee, the "Shimer puzzle" (see Shimer (2005)) only concerns the dynamics of wages, which tend to vary too much over the business cycle. The dynamics of employment are entirely determined by demand in the final goods market, while markups act as a buffer. The transposition of Hagedorn and Manovskii (2008)'s response in such a framework consists in assuming that unemployment benefits are unrealistically close to wages, ruling out dynamic labor supply behaviors. Assuming instead that wages attached to continuing jobs are constant over time, whereas bargaining only applies to new matches, as suggested by Pissarides (2009), is a more realistic answer, in particular without (or with low) unemployment benefits but with dynamic labor supply by workers.

2.4.2 Hagedorn and Manovskii (2008) in a new-Keynesian DSGE model

In this subsection, I investigate how Hagedorn and Manovskii (2008)'s partial equilibrium solution to the Shimer puzzle applies in a general equilibrium basic new-Keynesian model where hours worked per employee are constant over time. The latter assumption is an acceptable approximation especially for the euro area since (i) the intensive margin contributes much less to fluctuations in total labor supply than the extensive margin (see Cho and Cooley (1994)) and (ii) hours per workers are often strictly regulated in European countries. I show that

- the Shimer puzzle – understood as the fact that employment varies too little over the business cycle – is not an issue any more as it is in real business cylce models;
- using the approach advocated by Hagedorn and Manovskii (2008) to match the observed low fluctuations in the real wage requires that outside options are modelled as constant unemployment benefits, ruling out wealth effects in labor supply, and that vacancy costs are low, leaving aside any particular calibration of workers' bargaining power.

Specifically, I test the impact of three assumptions on the model's ability to replicate the high volatility of unemployment and the low volatility of wages: fixed unemployment benefits (no wealth effects on labor supply) vs. fixed disutility of labor; high vs. low vacancy costs (i.e. high vs. low profits for firms); high vs. low bargaining power of workers.

The model

Households consuming C_t in period t have instantaneous utility $U_t = \log C_t - \Gamma$ when they have a job, and $U_t = \log C_t$ when unemployed. They can trade riskless nominal one-period bonds in an uncomplete market, B_t, remunerated at rate $R_t - 1$. Employed are paid a real wage w_t and non-employed are paid a real amount z. All households receive even payments of dividends and government transfers. Last, households are perfectly insured against idiosyncratic labor market risk. The representative household's behavior is summarized by the following equations:

$$\lambda_t = \frac{1}{C_t} \tag{2.4.1}$$

$$1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t}{\pi_{t+1}} \tag{2.4.2}$$

The number of jobs evolves as described by the following equation

$$N_t = (1-s)N_{t-1} + m_t, \tag{2.4.3}$$

where the number of new jobs created in period t is given by a standard Cobb-Douglas matching function

$$m_t = \Upsilon v_t^{\kappa} (1-(1-s)N_{t-1})^{1-\kappa}. \tag{2.4.4}$$

Firms use workers to produce final goods with technology

$$Y_t = A_t N_t, \tag{2.4.5}$$

where A_t is an AR(1) process that represents exogenous disturbances to technology. They can post vacancy with a unit cost c and sell their production at relative price x_t to the retail sector. The optimal number of vacant jobs held by firms is governed by

$$\frac{c}{q_t} = x_t A_t - w_t + \beta(1-s) E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{c}{q_{t+1}}, \tag{2.4.6}$$

where

$$q_t = \frac{m_t}{v_t} \tag{2.4.7}$$

is the vacancy filling rate. The retail sector includes a continuum of monopolistic retailers who set their prices with rigidities à la Calvo (1983). The probability to reset their price is constant and equal to $1-\xi$ in each period. With probability ξ, their price is automatically indexed to a convex combination of past and long run inflation rates, $\pi_{t-1}^{\iota} \bar{\pi}^{1-\iota}$. Differentiated goods are then aggregated with a Dixit and Stiglitz (1977) technology with elasticity of substitution θ. Ignoring the dispersion of prices which does not play any role in a first order approximation of the model, the aggregate inflation rate is governed by the following equations:

$$\tilde{p}_t = \frac{\theta}{\theta-1} \frac{H_{1t}}{H_{2t}} \tag{2.4.8}$$

$$H_{1t} = x_t Y_t + \beta \xi E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_{t+1}}{\pi_t^{\iota} \bar{\pi}^{1-\iota}} \right)^{\theta} H_{1t+1} \tag{2.4.9}$$

$$H_{2t} = Y_t + \beta \xi E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_{t+1}}{\pi_t^{\iota} \bar{\pi}^{1-\iota}} \right)^{\theta-1} H_{2t+1} \tag{2.4.10}$$

$$1 = (1-\xi)\tilde{p}_t^{1-\theta} + \xi \left(\frac{\pi_{t-1}^{\iota} \bar{\pi}^{1-\iota}}{\pi_t} \right)^{1-\theta}. \tag{2.4.11}$$

The real wage is bargained by workers and firms to share the surplus generated by a successful match. The total surplus is the sum of the marginal values of employment for the firms and the workers, expressed in units of consumption goods. The marginal value of employment for firms is

$$J_t = x_t A_t - w_t + \beta(1-s)E_t \frac{\lambda_{t+1}}{\lambda_t} J_{t+1},$$

and the marginal value of employment for households is

$$W_t = \lambda_t(w_t - z) - \Gamma + \beta(1-s)E_t(1-f_{t+1})W_{t+1},$$

where

$$f_t = \frac{m_t}{1-(1-s)N_{t-1}} \tag{2.4.12}$$

is the job finding rate. So the surplus is $S_t = W_t/\lambda_t + J_t$. The outcome of the bargain is assumed to be the real wage rates which maximizes $(W_t/\lambda_t)^{\delta} J_t^{1-\delta}$, where the parameter δ measures the relative bargaining power of workers. The wage equation is therefore

$$w_t = (1-\delta)\left(z + \frac{\Gamma_t}{\lambda_t}\right) + \delta\left(x_t A_t + c\beta(1-s)E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{f_{t+1}}{q_{t+1}}\right). \tag{2.4.13}$$

Last, general equilibrium implies

$$Y_t = C_t + G + cv_t, \tag{2.4.14}$$

where G is constant government spending, and the monetary authority sets the nominal interest factor according the a standard Taylor rule

$$R_t = R_{t-1}^{\rho}\left(\frac{\bar{\pi}}{\beta}\left(\frac{\pi_t}{\bar{\pi}}\right)^{r_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{r_y}\right)^{1-\rho}\varepsilon_{r,t}, \tag{2.4.15}$$

where ε_r is an exogenous monetary policy shock.

Calibration

I use a standard calibration for most parameters, summarized in the table below:

parameter	$\bar{N}$	s	$\bar{q}$	$\bar{\pi}$	$G/\bar{Y}$	β	κ	θ	ξ	ι	ρ	r_π	r_y
value	0.57	0.15	0.90	1.0062	0.319	0.99	0.5	6	0.75	0.5	0.7	1.5	0.125

The steady state employment rate $\bar{N}$ and the quarterly job separation rate s are the ones used by Andolfatto (1996). Using Shimer (2005) values instead would not substantially change the results discussed in what follows. The value of $G/\bar{Y}$ is consistent with the fact that real private consumption represents approximately 70% of real GDP in the US over 1977-2012 (source: OECD), and that

GDP in the model is $Y_t - cv_t$. From a national accounts perspective, G thus includes private investment, foreign trade and changes in inventories. Finally, $\theta = 6$ implies an average markup of monopolists of 6/5-1=20%, which is in the range of [1,1.4] suggested by Rotemberg and Woodford (1995). The matching technology scale parameter Υ is set so that the matching and the law of motion of employment are both satisfied in the long run. Last the sum of unemployment benefits and labor disutility divided by the marginal utility of consumption (the workers' steady state outside options) $z+\Gamma/\bar{\lambda}$ needs to verify the wage equation in the steady state. The autoregressive coefficient of productivity is set to 0.7 and the standard deviation of the innovation is 0.0068. The standard deviation of the monetary shock is 0.0062. Then I consider the cases $z = 0$ and $\Gamma = 0$. For vacancy costs, we assume that they represent either 1% of output, or 3% of output. Last, workers' bargaining power δ takes the values 0.5 and 0.05.

Elements on Hagedorn and Manovskii's partial equilibrium approach

In their partial equilibrium framework, Hagedorn and Manovskii (2008) suggest that (i) outside options are very close to productivity (or equivalently vacancy costs are low) so that firms' profits are small and very sensitive to employment, (ii) workers bargaining power is small so wages do not react much to productivity. These two assumptions can be used almost independantly to replicate (i) the high response of employment and (ii) the low response of wages to productivity shocks. The cross-effects are small. In particular, Table 2.2 shows that the calibration of bargaining power is not crucial to match the high volatility of tightness. In this table, I compute the elasticity of tightness to productivity derived analytically by Hagedorn and Manovskii (2008), that is

$$\frac{d\theta}{dp} = \frac{\theta}{p-z}\frac{r+s+\beta f(\theta)}{(r+s)(1-\eta)+\delta f(\theta)},$$

where r is the interest rate in financial markets, θ is labor market tightness, p is productivity, z is outside options, $f(\theta)$ is the job finding rate, s is the separation rate, δ is workers bargaining power and η is the elasticity of the job finding rate with respect to tightness. I use their weekly calibration: $r = 0.99^{-1/12} - 1$, $s = 0.0081$, $\eta = 0.5462$ and $f(\theta) = 0.1390$, with normalization $p = 1$. Next, they set $\delta = 0.052$ and $z = 0.955$, which are compared to usual values $\delta = 0.455$ and $z = 0.4$. Small $p - z$ is equivalent to setting a small value for the unit vacancy cost c because $(1-\delta)(p-z) = ((r+s)/q(\theta) + \delta\theta)c$ in equilibrium, with $q(\theta)$ the vacancy filling rate.

Table 2.2: Elasticity of tightness with respect to productivity

	$z = 0.400$	$z = 0.955$
$\delta = 0.455$	1.788	23.834
$\delta = 0.052$	2.388	31.837

The fact that the cross-effects of these two calibration assumptions on the dynamics of tightness on the one hand, and of wages on the other hand, can be intuitively understood as follows. Consider the simplified expressions of the wage rate and of firms' profit in the partial equilibrium model of Hagedorn and Manovskii (2008) set out below. Compared with (2.4.13), the differences are that (i) outside options induce no wealth effect in this partial equilibrium, (ii) there are no markups ($x_t = 1$) as there are no nominal frictions, and (iii) savings on future recruiting costs brough by job creations are assumed to be small as compared to their marginal productivity and are ignored. Though, these simplifications are illustrative:

$$w = (1-\delta)z + \delta p \quad \Rightarrow \quad \text{dlog}\, w = \frac{\delta}{w} dp,$$

$$\Pi = p - w = (1-\delta)(p-z) \quad \Rightarrow \quad \text{dlog}\, \Pi = \frac{dp}{p-z}.$$

Increasing outside options z impacts the elasticity of wages only because it raises the value of w. Conversely, lowering bargaining power δ does not impact the elasticity of firms' profits, and its effect on the response of tightness through this channel is moderate.

Simulations of the new-Keynesian model

Wealth effect in general equilibrium models

In general equilibrium, outside options are generally represented by a disutility cost of labor. Hence labor supply involves wealth effects since a lower marginal utility of consumption raises the monetary value of outside options. Put differently, wealth reduces the incentive to work. The presence of wealth effects in labor supply cancels the benefits of Hagedorn and Manovskii's calibration stategy, which is designed in a partial equilibrium framework and hence abstracts from wealth effects. To understand that point, assume that outside options are proportional to productivity ($= zp$ instead of z; zp stands for Γ/λ_t in the general equilibrium model) in the schematic equations above. Then, immediately

$$\text{dlog}\, w = \text{dlog}\, \Pi = \text{dlog}\, p,$$

and wages vary as much as productivity do. Therefore, implementing their calibration in a general equilibrium model imposes that outside options are represented by unemployment benefits instead of labor desutility.

The implication of sticky prices

The Shimer puzzle has mainly been discussed either in partial equilibrium search and matching models, or in real business cycle models, where the only source of fluctuations is shocks to productivity. Assuming a basic new-Keynesian framework, where capital is absent, is likely to modify this problem for two main reasons. First, the sources of fluctuations also include, for a large part, demand side shocks. Since production is $Y_t = A_t N_t$, demand shocks imply that employment vary as gdp, so the conditional relative variance of employment is one. Second, employment reacts negatively to productivity shocks (as pointed to by Galí (1999)). With respect to RBC models, the markup of firms that employ workers may vary a lot; their profit margins adjusts so that job creation is still profitable even if wages increase a lot.

Simulations

To get a comprehensive view of (i) the presence or not of a "Shimer puzzle" in a new-Keynesian model, (ii) the effects of Hagedorn and Manovskii (2008)'s calibration strategy in this framework, I compare in Table 2.3 the same moments, obtained conditionnally to the technology shock only, with sticky prices ($\xi = 0.75$) and with flexible price ($\xi = 0$), to pick out the changes that result specifically from nominal rigidities. Then, I report in Table 2.4 the relative (w.r.t. output) standard deviations of employment N_t and the real wage rate w_t for all the combinations cited in the calibration subsection above, when prices are sticky.

Table 2.3: Effect of price stickiness on the conditional volatility of employment and the real wage

Sticky prices

$z = 0$		$\delta = 0.5$	$\delta = 0.05$	$\Gamma = 0$		$\delta = 0.5$	$\delta = 0.05$
$vc/Y = 0.01$	σ_N/σ_Y	2.26	2.45	$vc/Y = 0.01$	σ_N/σ_Y	0.67	0.67
	σ_w/σ_Y	1.60	1.48		σ_w/σ_Y	0.36	0.02
$vc/Y = 0.03$	σ_N/σ_Y	1.66	1.82	$vc/Y = 0.03$	σ_N/σ_Y	0.75	0.75
	σ_w/σ_Y	3.81	1.06		σ_w/σ_Y	1.86	0.09

Flexible prices

$z = 0$		$\delta = 0.5$	$\delta = 0.05$	$\Gamma = 0$		$\delta = 0.5$	$\delta = 0.05$
$vc/Y = 0.01$	σ_N/σ_Y	0.28	0.37	$vc/Y = 0.01$	σ_N/σ_Y	0.87	0.94
	σ_w/σ_Y	1.38	1.52		σ_w/σ_Y	0.10	0.01
$vc/Y = 0.03$	σ_N/σ_Y	0.10	0.25	$vc/Y = 0.03$	σ_N/σ_Y	0.69	0.85
	σ_w/σ_Y	1.30	1.59		σ_w/σ_Y	0.24	0.02

Table 2.4: Volatility of employment and the real wage for different calibrations with flexible wages

$z = 0$		$\delta = 0.5$	$\delta = 0.05$	$\Gamma = 0$		$\delta = 0.5$	$\delta = 0.05$
$vc/Y = 0.01$	σ_N/σ_Y	1.16	1.17	$vc/Y = 0.01$	σ_N/σ_Y	0.92	0.91
	σ_w/σ_Y	1.77	1.39		σ_w/σ_Y	0.44	0.02
$vc/Y = 0.03$	σ_N/σ_Y	1.11	1.13	$vc/Y = 0.03$	σ_N/σ_Y	0.94	0.94
	σ_w/σ_Y	3.23	1.11		σ_w/σ_Y	2.11	0.11

Comments on Table 2.3

Consistently with the common knowledge about RBC models, in the absence of nominal rigidities, the model generates a small variability of employment and large fluctuations in wages in response to technology shocks when outside options are represented by labor disutility. This setup however differs from a standard RBC model in that there is no capital accumulation and interest rates follows a Taylor rule instead of reflecting the marginal productivity of capital. As a result, employment reacts countercyclically to technology shocks. By comparison, the same model with sticky prices implies larger and hence more realistic fluctuations in employment in response to technology shocks. This is thanks to the reaction of markups, which decrease markedly in response to a positive technology shock, cutting firms' profits and therefore providing them with an incentive to reduce strongly employment.

Second, the simple assumption of outside options materialized by acyclical unemployment benefits, cancelling wealth effects on labor supply, strongly amplifies the response of employment in the model with flexible prices. This is particularly true for small vacancy costs (1% of output). This suggests that the calibration recommanded by Hagedorn and Manovskii (2008) is effective with respect to the volatility of employment in this real economy.[4]

Comments on Table 2.4

Not surprisingly regarding the preceeding results for conditional volatilities, for all the calibrations considered, the new-Keynesian model does not generate insufficient variations in employment. In addition to the large reaction to productivity shocks mentionned above, the dynamics of employment is closely linked to the dynamics of output since demand shocks, leaving productivity constant and hence implying $Y_t = N_t$, are the main drivers of business cycle fluctuations. Again, such an equilibrium is made possible by the response of markups; after a positive demand (or expansionary monetary) shock, firms increase their markup x_t such that new jobs are profitable even if wages strongly increase (see impulse responses in Figures 2.8 and 2.9). Consistently with Hagedorn and Manovskii (2008)'s discussion, in this model, the lower are vacancy costs, the more sensitive in percentage terms firms accounting profits are, so the smaller are the reactions in markups needed to reach the equilibrium.

Concerning the dynamics of wages, Hagedorn and Manovskii (2008)'s response requires, as expected from the schematic reasonning above, no wealth effect in labor supply, that is $\Gamma = 0$ and $z > 0$, in order to successfully replicate their low volatility. In addition, when wealth effects are cancelled, Hagedorn and Manovskii's recommandation to set vacancy costs (and hence profits) to low values (which is a standard calibration used in the literature) is enough to reduce the volatility of wages to realistic values (approximately one half of the volatility of output). A low bargaining power for workers is unnecessary. Indeed, the magnitude of the reaction of markups dictates, to an extent determined by workers' bargaining power, the one of wages; since setting low vacancy costs dampen fluctuations in markups, it also affects the dynamics of wages.

[4]The volatility of employment is already amplified for vacancy costs representing 3% of output because this it is still low vacancy costs relatively to the numerical values used by Hagedorn and Manovskii (2008) – and hence high outside options, that is $z \approx 93\%$ of w, against $z \approx 98\%$ of w when vacancy costs are 1% of output. Using larger vacancy costs, more illustrative, is not possible in this general equilibrium framework since the model with sticky prices would then be explosive.

Figure 2.8: Flexible wage: 1% productivity shock

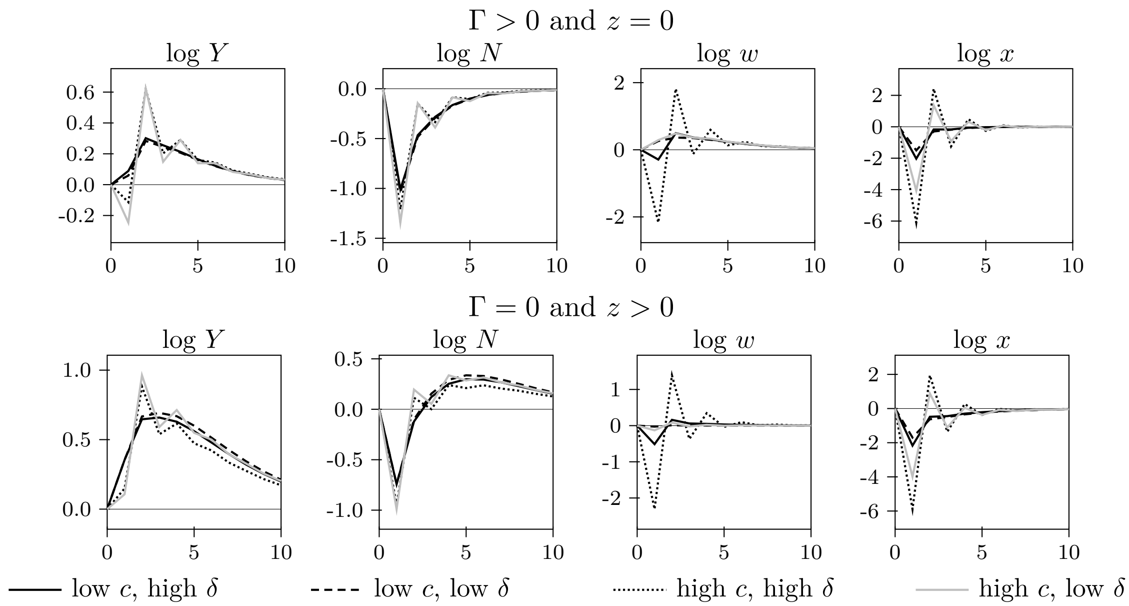
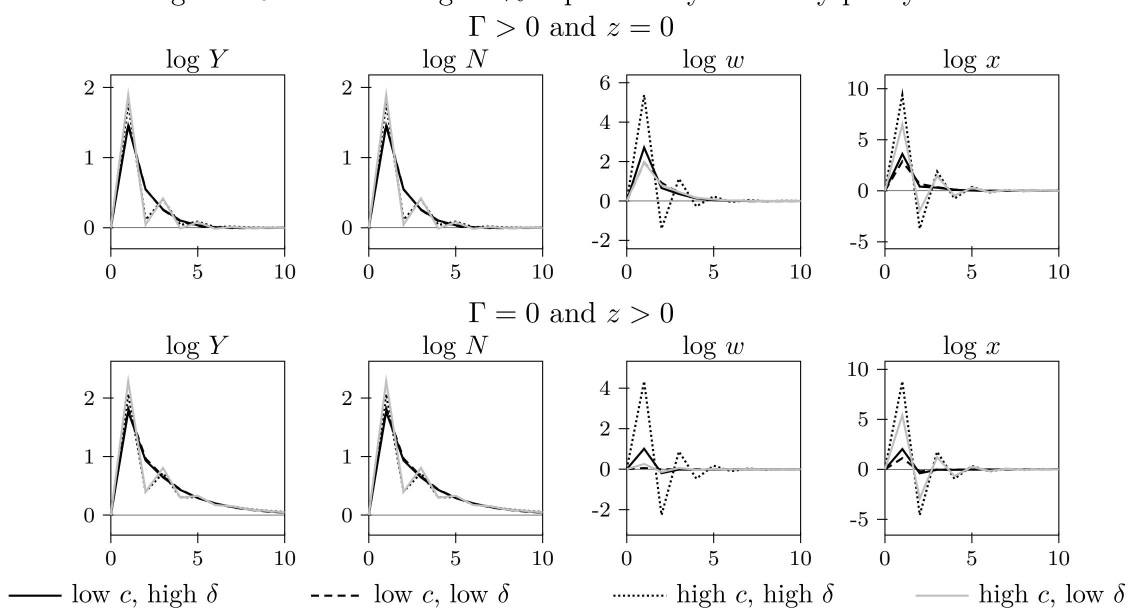


Figure 2.9: Flexible wage: 1% expansionary monetary policy shock



Concluding remarks

Two concluding remarks can be drawn from this exercise:

- replicating the magnitude of employment fluctuations is not an issue in a new-Keynesian model with constant worked hours per worker;

- replicating the magnitude of the variations in wages stemming from standard Nash bargaining requires to assume that workers' outside options include unrealistically high unemployement benefits, instead of labor disutility costs expressed in units of consumption goods.

This leads to consider different ways of modeling wages, leaving the dynamics of employment as it is a standard new-Keynesian model with search and matching frictions.

2.4.3 Constant real wages for ongoing jobs

In this section, I test a new wage setting framework based on the idea that Nash bargained wages only concern new matches, while continuing jobs are paid at a constant real wage rate, following Pissarides (2009). This approach also shares common features with the model proposed by de Walque et al. (2009). Outside options can include labor disutility and yield wealth effects: as shown in the first table above, the only apparent undesirable effect is that bargained wages are more volatile than in the data. But if the observed average wage rate in the data is not compared to the bargained wage rate, but rather to the average wage rate in the model, including a large proportion of constant wages, this is not a problem any more.

Another theoretical issue is that non-renegotiated wages should yet remain inside the bargaining set, to keep away from inefficient allocations of labor for both workers and firms, as discussed by Hall (2005). In the present work, I side-step this question and leave it for future thinking. Still, the decision to create a job and the related wage bargaining by the representants of workers and firms is always efficient since the wage associated with new jobs is inside the bargaining set at the date of their creation.

Changes to the model

New jobs of period t are paid at real wage $\tilde{w}_t$. The continuing jobs are paid at the wage bargained at the time they were created. As s jobs are destroyed randomly during each period of time, the number of jobs created j periods before t that are still alive is $(1-s)^j m_{t-j}$. Hence, the average wage rate w_t is given by

$$w_t N_t = \sum_{j=0}^{\infty} (1-s)^j m_{t-j} \tilde{w}_{t-j} = m_t \tilde{w}_t + (1-s) N_{t-1} w_{t-1}. \tag{2.4.16}$$

At the begin of each period, workers are shuffled randomly across existing jobs: the probability to be unemployed and receive z is $n(z) \equiv 1 - N_t$ and the probability to get a job that is paid $\tilde{w}_{t-j}$ is $n(\tilde{w}_{t-j}) \equiv (1-s)^j m_{t-j}$. All jobs are identically productive although their wages differ. Workers insure perfectly against all labor market idiosyncratic risk: they can purchase a quantity $b_t(x)$ of assets at price $\tau_t(x)$ that deliver each one unit of good when the worker is paid x during the period. The set of possible values for x is $\mathcal{X} = \{\{\tilde{w}_{t-j}\}_{j \geq 0}, z\}$. There are as many budget constraints faced by any household as there are possible revenues x in the labor market lottery; for any $x \in \mathcal{X}$, these constraints are:

$$C_t^x + \frac{B_t^x}{P_t} + \sum_{y \in \mathcal{X}} \tau_t(y) b_t(y) = \frac{B_{t-1} R_{t-1}}{P_t} + x + T_t + div_t,$$

where C^x and B^x are the levels of consumption and savings decided by the household in the event she is paid x in the labor market. The first order conditions associated with the consumption, savings and insurance decisions are, denoting by λ_t^x the Lagrange multipliers associated with the constraints above:

$$\lambda_t^x = \frac{1}{C_t^x}$$

$$\lambda_t^x = \beta E_t \lambda_{t+1}^x \frac{R_t}{\pi_{t+1}}$$

$$n(x)\lambda_t^x(1-\tau_t(x)) = \sum_{y\neq x} n(y)\lambda_t^y \tau_t(x).$$

Next, free entry in the insurance market implies that $\forall x \in \mathcal{X},\ \tau_t(x) = n(x)$. Hence,

$$\lambda_t^x = \lambda_t^x \tau_t(x) + \sum_{y\neq x} \tau_t(y)\lambda_t^y = \sum_{y\in\mathcal{X}} n(y)\lambda_t^y$$

and all λ_t^x are equal. As a result, all households consume and save the same amount in each period, following

$$\lambda_t = \frac{1}{C_t}, \tag{2.4.17}$$

and

$$1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{R_t}{\pi_{t+1}}. \tag{2.4.18}$$

The production sector is unchanged with respect to the baseline version of the model except that firms account for the fact that new jobs are going to be paid at the wage rate decided in the current period unless they are destroyed. Firms choose the number of vacancies v_t and the level of employment N_t that solves the expected discounted value of dividend flows

$$E_t \sum_{j=0}^{\infty} \beta^j \lambda_{t+j}\left(x_{t+j}A_{t+j}N_{t+j} - w_{t+j}N_{t+j} - cv_{t+j}\right),$$

subject to the constraints

$$N_{t+j} = (1-s)N_{t+j-1} + q_{t+j}v_{t+j}$$

and

$$w_{t+j}N_{t+j} = (1-s)N_{t+j-1}w_{t+j-1} + q_{t+j}v_{t+j}\tilde{w}_{t+j}.$$

The optimal vacancy posting condition stemming from this program is

$$\frac{c}{q_t} = x_tA_t - \tilde{w}_t + \beta E_t\frac{\lambda_{t+1}}{\lambda_t}\frac{\partial \mathcal{W}_{t+1}}{\partial N_t} + \beta\frac{\tilde{w}_t - w_t}{N_t}E_t\frac{\lambda_{t+1}}{\lambda_t}\frac{\partial \mathcal{W}_{t+1}}{\partial w_t},$$

where $\mathcal{W}_t$ is the value function of firms in period t. It can be interpreted as follows. The cost of posting a vacancy should compensate for the immediate gains derived from a new match, $x_tA_t - \tilde{w}_t$, the expected discounted value of the created job in the next period, $\beta E_t\lambda_{t+1}/\lambda_t(\partial\mathcal{W}_{t+1}/\partial N_t)$, adjusted to account for the fact the wage rate associated with it in period $t+1$ will be the bargained wage of the present period instead of the average wage rate. This adjustment is represented by the last term in the right hand side of the equation.

Last, firms and workers bargain over the wage rate of newly created job to split the surplus generated by a new match. The surplusses of workers and firms are now the derivative of their respective value functions with respect to matches. They are

$$J_t = x_t A_t - \tilde{w}_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{W}_{t+1}}{\partial N_t} + \beta \frac{\tilde{w}_t - w_t}{N_t} E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{W}_{t+1}}{\partial w_t},$$

for firms, and, for households,

$$W_t = \lambda_t \left(\tilde{w}_t - z\right) - \Gamma + \beta E_t \frac{\partial \mathcal{V}_{t+1}}{\partial N_t} + \beta \frac{\tilde{w}_t - w_t}{N_t} E_t \frac{\partial \mathcal{V}_{t+1}}{\partial w_t},$$

where $\mathcal{V}_t$ is their value function in period t. These surplusses cannot be written recursively any more: they include both present gains obtained from the match and the future marginal value of pre-existing jobs paid at the same wage rate in the next period. The Nash bargaining condition is

$$(1-\delta)\left(1 - \beta \frac{1}{N_t} E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{W}_{t+1}}{\partial w_t}\right) W_t = \delta \left(\lambda_t + \beta \frac{1}{N_t} E_t \frac{\partial \mathcal{V}_{t+1}}{\partial w_t}\right) J_t.$$

Details are provided in Appendix 2.A.

Results

I use the same calibration as for the baseline model. The relative standard deviations of employment N_t and the real wage rate w_t are reported in Table 2.5 for the sets of calibration considered in section 2.4.2, and the impulse responses are plotted in Figures 2.10 and 2.11. As expected, the volatility

Table 2.5: Volatility of employment and the real wage for different calibrations with rigid real wages

$z=0$		$\delta=0.5$	$\delta=0.05$
$vc/Y=0.01$	σ_N/σ_Y	1.16	1.17
	σ_w/σ_Y	0.19	0.19
$vc/Y=0.03$	σ_N/σ_Y	1.11	1.13
	σ_w/σ_Y	0.17	0.16

$\Gamma=0$		$\delta=0.5$	$\delta=0.05$
$vc/Y=0.01$	σ_N/σ_Y	0.92	0.91
	σ_w/σ_Y	0.01	0.00
$vc/Y=0.03$	σ_N/σ_Y	0.94	0.94
	σ_w/σ_Y	0.03	0.00

of the average wage rate is considerably reduced. Yet, the dynamics of employment, markup and output are unchanged with respect to the baseline model. The reason is that job creation decisions are based on the bargained wage, not the average wage, and bargaining is done by agents who know that the decided wage is going to apply during several periods in the future. Their interest is to make a decision that is not harmful for job creation, that is such that firms can satisfy production needs. Put differently, the magnitude of the reaction of the bargained wage is smaller than in the

Figure 2.10: Rigid real wages: 1% productivity shock

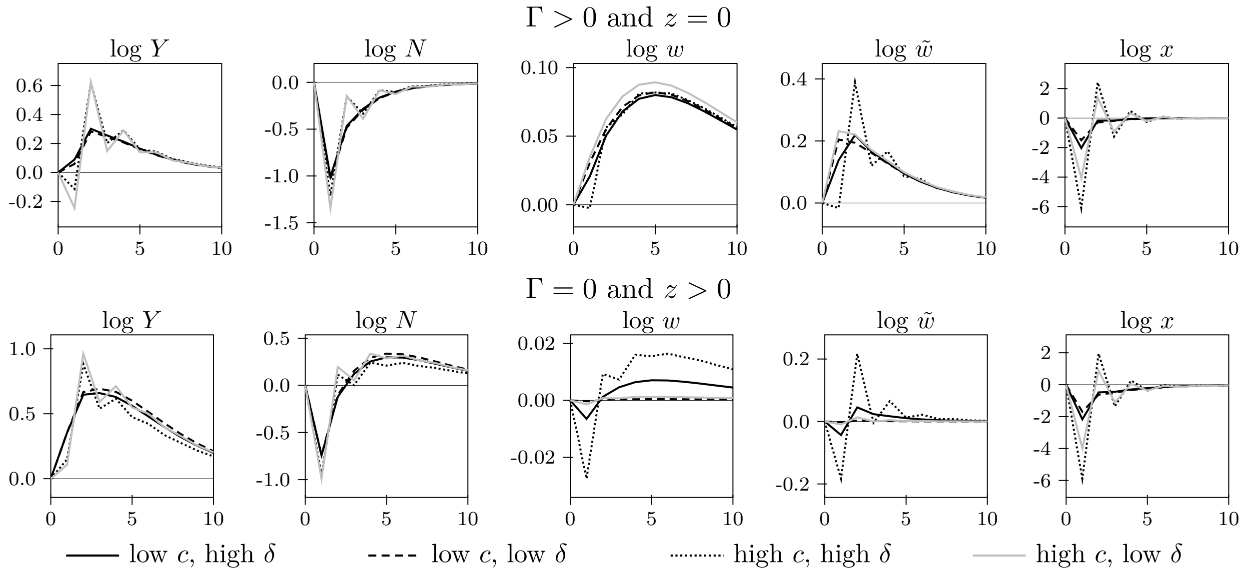
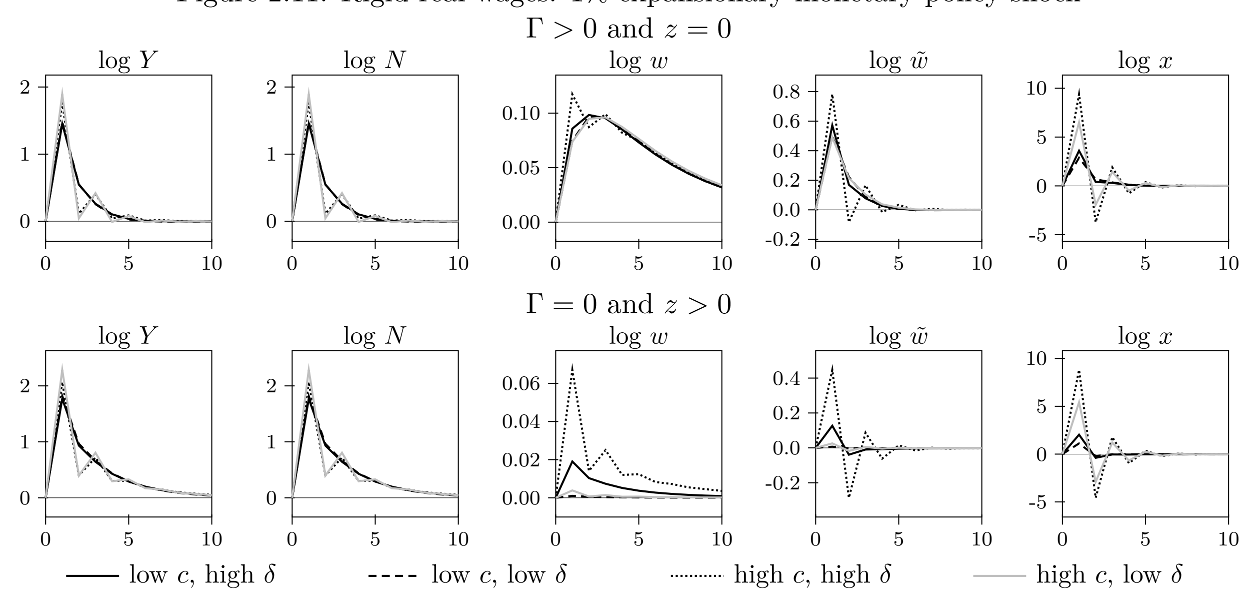


Figure 2.11: Rigid real wages: 1% expansionary monetary policy shock



absence of wage rigidity, so that firms are not discouraged to post vacancies on the grounds that today's bargained wage commits them for several periods in the future. This result is consistent with the findings of Pissarides (2009).

Two other findings drawn from these simulations are striking. First, the volatility of the bargained wage rate in the variant model is much lower than in the baseline model of the previous section. Intuitively, in the former model, the bargained wage is maintained in future periods, which has a much stronger impact on both firms and workers surplusses than when it is only used in the current period. Second, $\Gamma = 0$ implies that wages do not vary enough. This is because the firms' gains from recruiting are much less volatile than in the standard case, so workers' outside options dominate in the dynamics of the wage rate. With $\Gamma = 0$, workers' outside options are represented by z and do not vary in the business cycle. Conversely, the larger is Γ relatively to z, the larger is the procyclical reaction of ouside options materialized in Γ/λ_t. As a result, $z = 0$ implies that wages react positively to productivity. This way of introducing wage rigidity is therefore particularly suited to models where workers' outside options stem from labor disutility rather than from unemployment benefits.

2.4.4 Constant nominal wages for ongoing jobs

In this section I test the same specification in the case where nominal wages are kept constant until job destruction.

Changes to the model

New jobs of period t at paid at nominal wage $P_t\tilde{w}_t$, where $\tilde{w}_t$ is the outcome of a Nash bargain between workers and firms. The jobs created j periods before t that are still alive are paid at nominal wage $P_{t-j}\tilde{w}_{t-j}$. Hence, the real average wage rate in period t verifies

$$w_t N_t = \sum_{j=0}^{\infty} (1-s)^j m_{t-j} \left(\prod_{k=0}^{j-1} \pi_{t-k} \right)^{-1} \tilde{w}_{t-j} = m_t \tilde{w}_t + \frac{1-s}{\pi_t} w_{t-1} N_{t-1}. \tag{2.4.19}$$

The equations describing the behavior of firms are changed to:

$$\frac{\partial \mathcal{W}_t}{\partial w_{t-1}} = \frac{(1-s)N_{t-1}}{\pi_t} \left[-1 + \beta \frac{1}{N_t} E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{W}_{t+1}}{\partial w_t} \right] \tag{2.4.20}$$

$$\frac{\partial \mathcal{W}_t}{\partial N_{t-1}} = (1-s) \left(\frac{c}{q_t} + \tilde{w}_t - \frac{w_{t-1}}{\pi_t} \right) + \beta(1-s) \frac{w_{t-1}/\pi_t - \tilde{w}_t}{N_t} E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{W}_{t+1}}{\partial w_t} \tag{2.4.21}$$

The vacancy posting condition and the expression of the surplus are unchanged:

$$\frac{c}{q_t} = x_t A_t - \tilde{w}_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{W}_{t+1}}{\partial N_t} + \beta \frac{\tilde{w}_t - w_t}{N_t} E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{W}_{t+1}}{\partial w_t}. \tag{2.4.22}$$

$$J_t = x_t A_t - \tilde{w}_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{\partial \mathcal{W}_{t+1}}{\partial N_t} + \frac{\tilde{w}_t - w_t}{N_t} \frac{\partial \mathcal{W}_{t+1}}{\partial w_t} \right]. \tag{2.4.23}$$

Turning to households decisions, the equations become:

$$\frac{\partial \mathcal{V}_t}{\partial w_{t-1}} = \frac{(1-s)N_{t-1}}{\pi_t} \left[\lambda_t + \beta \frac{1}{N_t} E_t \frac{\partial \mathcal{V}_{t+1}}{\partial w_t} \right]. \tag{2.4.24}$$

$$\begin{aligned} \frac{\partial \mathcal{V}_t}{\partial N_{t-1}} = (1-s) \Bigg[& \lambda_t \left(\frac{w_{t-1}}{\pi_t} - f_t \tilde{w}_t - (1-f_t) \left(z + \frac{\Gamma}{\lambda_t} \right) \right) + \beta(1-f_t) E_t \frac{\partial \mathcal{V}_{t+1}}{\partial N_t} \\ & - \beta \frac{f_t(\tilde{w}_t - w_t) + w_t - w_{t-1}/\pi_t}{N_t} E_t \frac{\partial \mathcal{V}_{t+1}}{\partial w_t} \Bigg]. \end{aligned} \tag{2.4.25}$$

The expression of the surplus is unchanged:

$$W_t = \lambda_t(\tilde{w}_t - z) - \Gamma + \beta E_t \left[\frac{\partial \mathcal{V}_{t+1}}{\partial N_t} + \frac{\tilde{w}_t - w_t}{N_t} \frac{\partial \mathcal{V}_{t+1}}{\partial w_t} \right] \tag{2.4.26}$$

Results

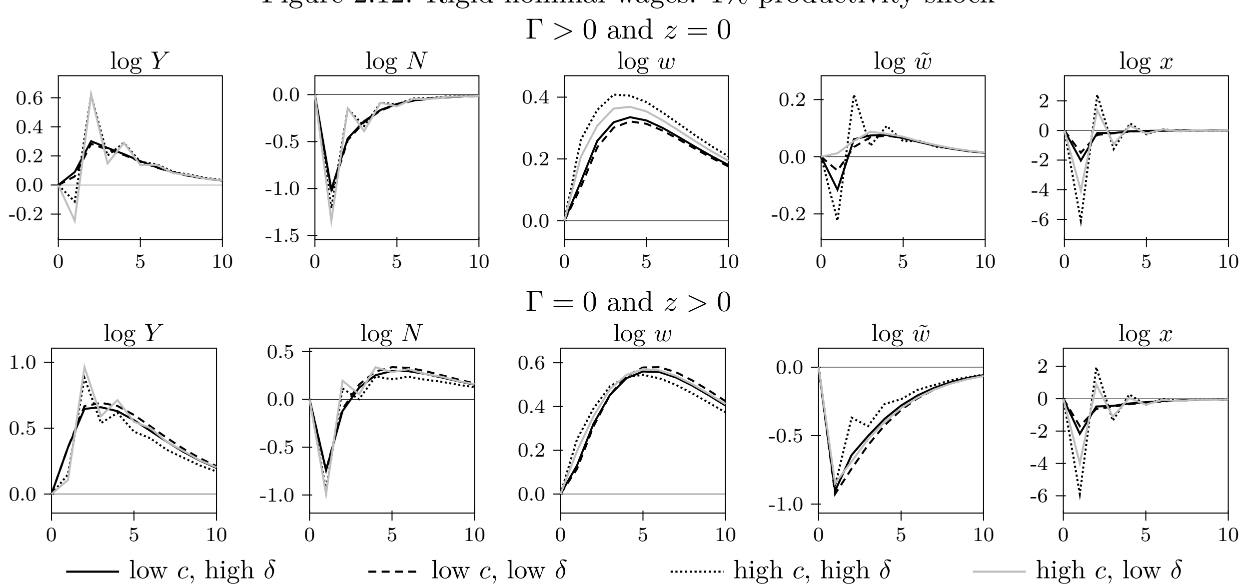
I use the same calibration as previously. By contrast with the case of constant real wages for ongoing jobs, the steady state of the model is slightly changed with respect to the baseline version when it is the nominal wages that are constant. I compute the same statistics and impulse responses (in Table 2.6 and Figures 2.12 and 2.13). The only significant difference with the case of constant real wages

Table 2.6: Volatility of employment and the real wage for different calibrations with rigid nominal wages

$z = 0$		$\delta = 0.5$	$\delta = 0.05$
$vc/Y = 0.01$	σ_N/σ_Y	1.16	1.17
	σ_w/σ_Y	0.74	0.71
$vc/Y = 0.03$	σ_N/σ_Y	1.11	1.13
	σ_w/σ_Y	0.73	0.64

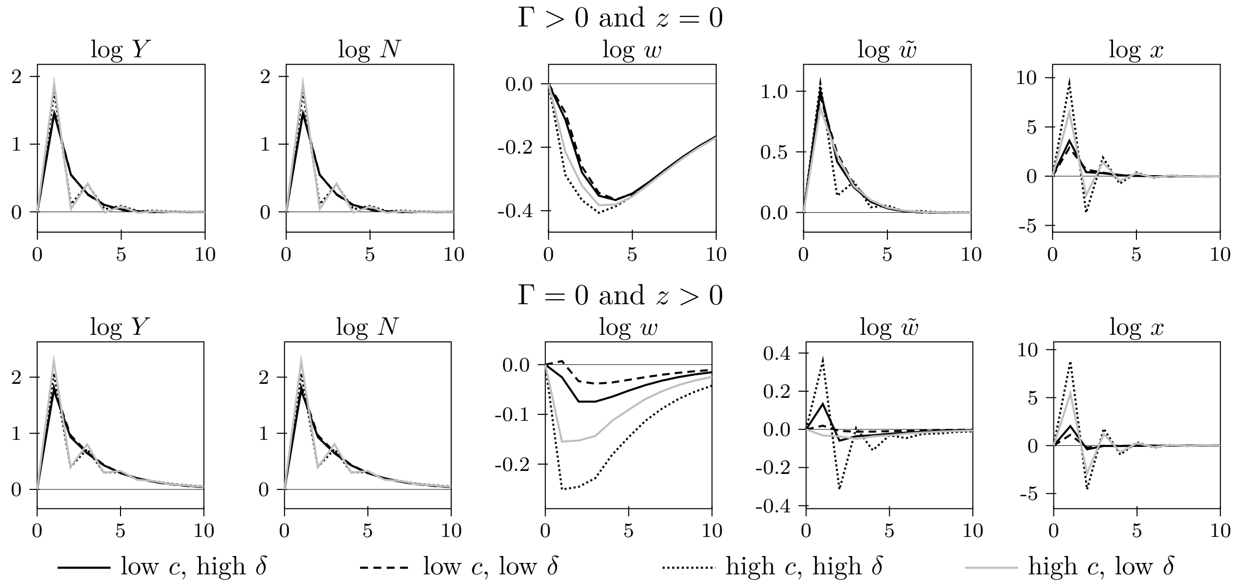
$\Gamma = 0$		$\delta = 0.5$	$\delta = 0.05$
$vc/Y = 0.01$	σ_N/σ_Y	0.92	0.91
	σ_w/σ_Y	0.58	0.57
$vc/Y = 0.03$	σ_N/σ_Y	0.94	0.94
	σ_w/σ_Y	0.58	0.53

Figure 2.12: Rigid nominal wages: 1% productivity shock



is that the magnitude of the fluctuations in the real average wage rate are larger. This is related to the reaction of the price level. In particular, in response to a positive technology shock, the positive reaction of w reflects the positive contribution of the drop in inflation, whereas in response to a demand-side shock, the increase in prices dominates the rise of the real bargained wage rate

Figure 2.13: Rigid nominal wages: 1% expansionary monetary policy shock



and pushes the real average wage downward. The negative reaction of w to expansionary monetary policies is even more marked when $\Gamma > 0$ and $z = 0$ than when $\Gamma = 0$ and $z > 0$ because the price level increases less in the latter case. Indeed, the smaller reaction of bargained wages triggers a smaller increase in markups to keep new matches profitable for the firm, which in turns results in lower inflation in the retail sector. This difference in the reaction of inflation already existed in the model with constant real wages but it did not alter the dynamics of the real average wage rate then. In the case when nominal wages are constant, the result is that the real average wage rate is slightly countercyclical when $\Gamma > 0$ and $z = 0$ (with correlation coefficient of -0.10) and slightly procyclical (with correlation coefficient of 0.18) when $\Gamma = 0$ and $z > 0$.

2.4.5 Concluding remarks

Assuming constant wages of ongoing jobs whereas bargaining only involves new jobs, as suggested by Pissarides (2009), successfully reduces the relative volatility of the average wage rate. It represent an alternative to the calibration strategy suggested by Hagedorn and Manovskii (2008). A key feature of this latter approach is that workers outside options in wage bargaining are represented by acyclical unemployment benefits instead of concave utility of staying at home. This is an important drawback for policy analysis because households labor supply behavior is then invariant to disturbances that affect the marginal utility of consumption (such as a change in consumption tax). By contrast, with wage stickiness à la Pissarides (2009), it is preferable to assume utility of staying at home. Moreover, this specification leaves the dynamic behavior of job creation in the model unchanged with respect to the case where all wages are bargained in every period.

In a new-Keynesian model, the wages of ongoing jobs can be assumed to be constant either in real terms or in nominal terms. In this latter case, the dynamics of the real average wage reflects changes in the price of goods; it may hence be slightly countercyclical for some calibrations, which is not the case in the data.

To conclude this analysis, based on the conclusions above, the estimated model of France and the euro area of chapter 3 incorporates *real* wage rigidity à la Pissarides (2009).

2.5 Endogenous layoffs and wage rigidity

In this section, I include wage rigidity as exposed in section 2.4, according to which only new jobs are concerned with wage bargaining, in a model with endogeneous layoffs à la Den Haan et al. (2000).

The simulations computed in the case of a new-Keynesian model with 'basic' search and matching frictions show that this form of wage rigidity does not affect job creations (as also implicit in Pissarides (2009)). It is therefore a realistic shortcut to describe the dynamics of the average wage rate of an economy – which is the one that is available in most datasets –, with no prejudice for the dynamics of real aggregates. By contrast, when used in a model with endogenous layoffs, this assumption strongly alters the dynamics of job creation. To understand why, I modify the model with endogenous layoffs of subsection 2.3.1 so that only new jobs are concerned with wage bargaining, while others keep a constant wage. I compare the impulse responses to standard technology and monetary policy shocks with those obtained using search and matching models (i) with constant separation rate and flexible wages, (ii) with endogenous layoffs and flexible wages, and (iii) with constant separation rates and rigid wages. These three models are the ones that are described and used in the previous sections of this chapter.

2.5.1 Modifications to the model

First, the model is completed with the law of motion of average wages

$$N_t w_t = (1 - F(\bar{\omega}_t)) \left[m_t \tilde{w}_t + (1-s) N_{t-1} w_{t-1} \right].$$

Next, firms account for the fact that only newly hired workers benefit form the bargained real wage rate, denoted by $\tilde{w}_t$ when they decide the idiosynchratic productivity threshold under which workers are laid off, and the number of vacancies. Their value also depends on the average wage rate in the previous period, which is a state variable.

The real cost of a new job is $\tilde{w}_t$ instead of w_t, and a new hire impacts the firm's next period value not only because it is likely to continue, but also because it modifies the average wage rate, so the job creation condition becomes

$$\frac{c}{q_t} = x_t A_t G(\bar{\omega}_t) - (1 - F(\bar{\omega}_t))\tilde{w}_t + \beta(1 - F(\bar{\omega}_t)) E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\partial \mathcal{V}_{t+1}}{\partial N_t} + \frac{\tilde{w}_t - w_t}{N_t} \frac{\partial \mathcal{V}_{t+1}}{\partial w_t} \right),$$

where $\mathcal{V}_t$ is the value function of firms. In what follows, $\mathcal{W}_t$ denotes the value function of households.

Next, the lay-off decision is changed to

$$x_t A_t \bar{\omega}_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{V}_{t+1}}{\partial N_t} = w_t.$$

Here, conversely, it is the average wage rate w_t that matters for separations, since the idiosyncratic productivity shocks are independant of the distribution of wages across workers.

Finally, the wage bargaining condition is written explicitely as

$$(1-\delta)\left(1 - \frac{\beta}{N_t} E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{V}_{t+1}}{\partial w_t}\right) W_t = \delta \left(\lambda_t + \frac{\beta}{N_t} E_t \frac{\partial \mathcal{W}_{t+1}}{\partial w_t}\right) J_t,$$

where W_t and J_t are respectively the marginal values of matches for workers and firms. The recursive expressions of the marginal values of workers and firms are:

$$\frac{\partial \mathcal{V}_t}{\partial w_{t-1}} = (1 - F(\bar{\omega}_t))(1-s)N_{t-1}\left(-1 + \frac{\beta}{N_t} E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{V}_{t+1}}{\partial w_t}\right),$$

$$\frac{\partial \mathcal{V}_t}{\partial N_{t-1}} = (1-s)\frac{c}{q_t} + (1 - F(\bar{\omega}_t))(1-s)(\tilde{w}_t - w_{t-1}) + \beta \frac{w_{t-1} - \tilde{w}_t}{N_t} E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{V}_{t+1}}{\partial w_t},$$

$$\frac{\partial \mathcal{W}_t}{\partial w_{t-1}} = (1 - F(\bar{\omega}_t))(1-s)N_{t-1}\left(\lambda_t + \frac{\beta}{N_t} E_t \frac{\partial \mathcal{W}_{t+1}}{\partial w_t}\right),$$

$$\begin{aligned}\frac{\partial \mathcal{W}_t}{\partial N_{t-1}} = (1 - F(\bar{\omega}_t))(1-s)\Bigg(& -(1-f_t)\Gamma + \lambda_t\left(-f_t\tilde{w}_t + w_{t-1}\right) \\ & + \beta(1-f_t)E_t \frac{\partial \mathcal{W}_{t+1}}{\partial N_t} - \beta \frac{f_t(\tilde{w}_t - w_t) + w_t - w_{t-1}}{N_t} E_t \frac{\partial \mathcal{W}_{t+1}}{\partial w_t}\Bigg),\end{aligned}$$

$$W_t = (1 - F(\bar{\omega}_t))\left(\lambda_t \tilde{w}_t - \Gamma + \beta E_t\left(\frac{\partial \mathcal{W}_{t+1}}{\partial N_t} + \frac{\tilde{w}_t - w_t}{N_t} \frac{\partial \mathcal{W}_{t+1}}{\partial w_t}\right)\right),$$

$$J_t = \frac{c}{q_t}.$$

2.5.2 Analysis of the response of vacancies in the model with endogenous layoffs and wage rigidities

Impulse responses to a technology and an expansionary monetary policy shocks are plotted in Figures 2.14 and 2.15.

Compared with simpler models, assuming endogenous layoffs and that wage bargaining only concerns new jobs significantly changes the behavior of vacancies, especially in response to a monetary policy shock. More, job creation and job destruction contribute to employment variations in opposite directions, i.e. they compensate each other. This seems inconsistent: firms cut their vacancies although they seek to increase employment, but also reduce strongly layoffs.

Facing upward wage pressures, a firm is induced to reduce expensive new hires, while keeping a larger number of employees with low idiosyncratic productivities, which are expected to be paid at

Figure 2.14: Technology shock

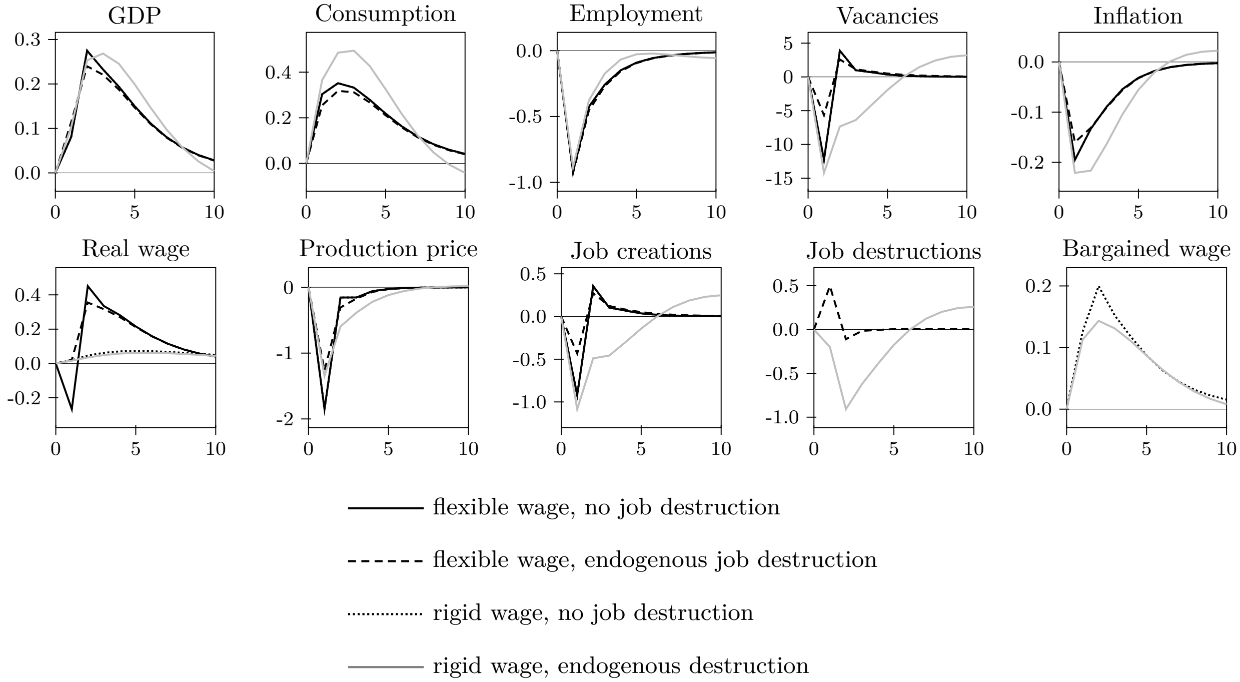
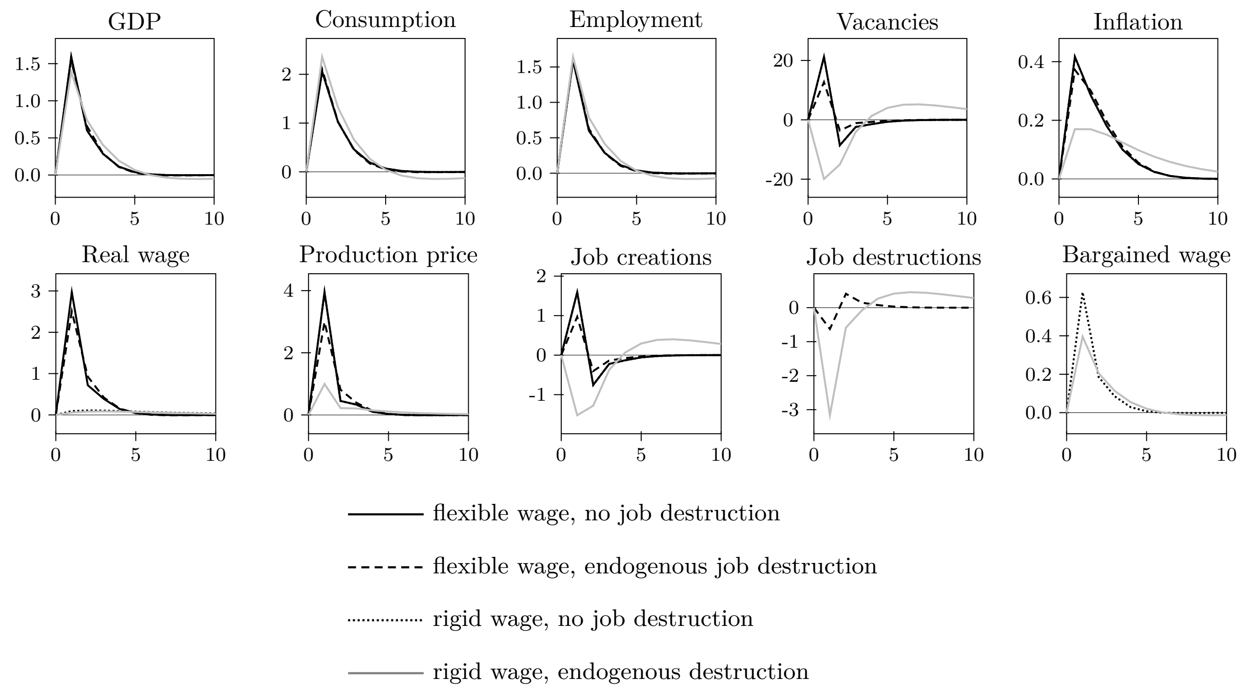


Figure 2.15: Monetary policy shock



the average wage rate. More explicitely, given the marginal value for a firm that can be obtained from holding an additional job at the end of the period, paid at the average wage rate, the marginal cost to be expensed when posting a vacancy is[5]

$$\frac{c}{q_t(1-F(\bar{\omega}_t))}+\tilde{w}_t+\frac{w_t-\tilde{w}_t}{n_t}\frac{\partial W_t}{\partial w_{t-1}},$$

for a marginal profit of

$$mk_t a_t \frac{L_t}{n_t},$$

while the marginal cost to be expensed when keeping the most productive worker among those that would have been fired is

$$w_t,$$

for a marginal profit of

$$mk_t a_t \bar{\omega}_t.$$

Firms are indifferent between the two levers in equilibrium. This translates into

$$mk_t a_t \frac{L_t}{n_t}-\frac{c}{q_t(1-F(\bar{\omega}_t))}-\tilde{w}_t+\frac{\tilde{w}_t-w_t}{n_t}\frac{\partial W_t}{\partial w_{t-1}}=mk_t a_t \bar{\omega}_t-w_t$$

In particular, at the steady state, recruiting expenditures are compensated by the fact that new jobs are expected to deliver higher productivity levels, while keeping low-productivity existing jobs is free:

$$\frac{c}{q(1-F(\bar{\omega}))}=mka\left(\frac{L}{n}-\bar{\omega}\right).$$

Yet, when an aggregate shock hits the economy, the marginal returns of the two options are affected differently. The effects can be broken down as follows:

- the average real wage rate w_t is approximately unchanged;
- the bargained wage's response is procyclical and of a magnitude similar to the one of output, so it implies an even larger increase in the marginal cost of new jobs through vacancy posting, of

  $$\left(1-\frac{1}{n}\frac{\partial W}{\partial w}\right)\tilde{w}_t,$$

  since $\partial W/\partial w$ is obviously negative.
- the impacts of the two options on firms' profits are relatively close to each other, since the level and the variations in the difference between the average individual output and $\bar{\omega}_t$ are quantitatively small with a log-normal distribution with mean 1;
- the variation in $\partial W/\partial w$ is not strongly affected since the wage rate is inertial;

[5] Notations: $q_t = m_t/v_t$ is the vacancy filling rate, $\bar{\omega}_t$ is the layoff threshold for workers idiosyncratic productivities, $F(\bar{\omega}_t)$ is the mass of workers that are laid off before production occurs, w_t is the average real wage, $\tilde{w}_t$ is the bargained real wage, concerning only new matches, $\partial W_t/\partial w_{t-1}$ is firms marginal values of average wages inherited from the past, mk_t is the real price of production, a_t is an aggregate productivity shock and $L_t = \int_{\bar{\omega}_t}^{\infty}\omega f(\omega)d\omega$ is total labor input after layoffs.

- hence, the absence of arbitrage opportunity in equilibrium roughly implies that q_t should increase to cancel the raise of the bargained wage rate. As q_t is proportional to $v_t^{\kappa-1}$, with $\kappa < 1$, this corresponds to a drop in the number of vacancies;

- in that context, decreasing $\bar{\omega}_t$ can support total production without perturbing much the equilibrium condition above.

To conclude, endogenous layoffs and the assumption that only new jobs are concerned with wage bargaining are not compatible. When included in a same model, a new mechanism comes into action: if bargained wages tend to decrease, firms multiply layoffs and issue more vacancies to replace fired workers by new ones. After a negative technology shock, which leads in new-Keynesian frameworks to an increase in employment but a decline in bargained wages, firms unexpectedly increase the number of layoffs, while an upswing in vacancies allows employment to increase. The response to demand shocks, which are the main drivers of business cycle fluctuations in the model economy, is even more problematic, since they alter the sign of the reaction of vacancies. Indeed, after a contractionary monetary policy shock, which results in a reduction of labor demand and bargained wages, the mechanism leads firms to multiply layoffs. This contributes, as awaited, to lowering employment. But this movement is so large that it prompts firms to recruit new workers more intensively than before the shock, in order to replace the fired ones. As a consequence, vacancies respond countercyclically to the shock.

The substitution of early workers by cheaper new workers is certainly a consistent and attested phenomenon. However, its magnitude in actual economies is definitely much lower than predicted by the model, since vacancies are strongly procyclical in the data.

To avoid such behaviour, the final version of the model of chapter 3 does not assume endogenous layoffs.[6]

2.6 Feedback effects of the financial accelerator on job creation

The current crisis and unemployment being one of the main concerns of governments have made it clear that macroeconomic models should account for labor market imperfections on the one hand, and for financial frictions in the other hand. In this perspective, building state-of-the-art DSGE models requires to include both, without necessarily bothering about their interactions. In a recent paper, Petrosky-Nadeau (2014) shows that assuming that vacancy posting requires external financing with imperfections magnifies the dynamics of job creation in response to technology shocks. In this section, I highlight the effects on job creation of the 'usual' way to introduce financial frictions, that is only affecting capital investments, in a simple calibrated new-Keynesian model with search and matching frictions. These effects can be of two kinds: the amplification of standard shocks, and the propagation of financial shocks. As there is no explicit link between borrowing and recruiting,

[6] A previous version of this model used quadradic adjustment costs on layoffs to solve this problem; however, I still found positive comovements between job creations and job destructions once the model was estimated. This attempt is described in Appendix 4.C.

they are channeled by prices in general equilibrium. They may result from labor supply decisions of workers that face changes in the way they value their revenues, from firms' arbitrage between labor and capital, or simply from the impact on the aggregate demand of goods.

I show that financial frictions amplify moderately the response of employment to monetary policy shocks. The effect is passed on through the increase in the aggregate demand of goods, which spreads to the need for production factors. It is neither the fact of substitution effects on labor supply nor of the arbitrage of firms between capital and labor. The volatility of employment relatively to the one of output is not magnified.

Another interaction stems from the propagation of financial shocks, which strongly impact job creation. Accordingly, the main motive for including a financial accelerator in the estimated model of France and the euro area of chapter 3 is shock identification: since they are expected to be important contributors to business cycle fluctuations (see Christiano et al. (2010)), the contributions of financial shocks should not be confused with the ones of the other standard ones already present in Smets and Wouters (2007) framework. Although they significantly alter the dynamic properties of wages, their impact on the main second order moments of employment is modest; thus, the identification of financial shocks does not depend crucially on the observation of labor market variables. This is consistent with previous works suggesting that the observation of financial variables are needed to identify these shocks as big contributors. Yet, including both financial and labor market frictions may also alter business cycle analyses thanks to pure feedback effects, though to a moderate extent as compared to Petrosky-Nadeau (2014)'s findings.

2.6.1 The model

Baseline search and matching

The baseline model used for this exercise is a simple new-Keynesian model with capital accumulation and search and matching frictions. It differs from this of section 2.3 by the presence of capital. Perfectly insured households are have a contemporaneous utility function

$$U_t = \log C_t - \Gamma N_t,$$

where C_t and N_t are real consumption and employment, $\Gamma > 0$ is a parameter. Households can invest I_t to accumulate capital K_t which is rented to firms in the following period at rate r^k_{t+1}, trade nominal bonds B_t remunerated at rate R_t. They receive lump-sum government transfers and dividends for a total of $\mathcal{D}_t$. Their budget constraint is

$$C_t + I_t + \frac{B_t}{P_t R_t} \leq w_t N_t + \frac{B_{t-1}}{P_t} + r^k_t K_{t-1} + \mathcal{D}_t,$$

where w_t is the wage rate. The Lagrange multiplier associated with this constaint is denoted by λ_t. The technology of capital accumulation is subject to a quadratic adjusmtent cost depending on the change in investment:

$$K_t = (1-\delta)K_{t-1} + \left(1 - \frac{\varphi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t.$$

They decide consumption and savings in bonds and capital, in order to maximize their expected intertemporal utility, discouted with factor β, subject to these constraints.

Job creation occurs in the labor market as the result of a matching process between workers seeking for jobs and firms issuing vacancies v_t; the number of jobs created in period t follows a standard Cobb-Douglas technology:

$$m_t = \Upsilon(1-(1-s)N_{t-1})^{1-\kappa} v_t^{\kappa},$$

where Υ and $0 \leq \kappa \leq 1$ are parameters. A constant fraction of existing jobs are exogeneously destroyed at quarterly rate s, so the law of motion of employment is

$$N_t = (1-s)N_{t-1} + m_t.$$

The production sector includes two types of agents: firms of the first category recruit workers and use labor N_t to produce a quantity N_t of an intermediary good. They choose the number of vacant jobs v_t to maximize their intertemporal discounted profits, considering the unit cost per period of a vacancy c and the observed low of motion of employment

$$N_t = (1-s)N_{t-1} + q_t v_t,$$

with $q_t = m_t/v_t$ the job filling rate, taken as given. The second category of firms use capital and the intermediary good to produce the final good with a Cobb-Douglas technology:

$$Y_t = A_t K_{t-1}^{\alpha} N_t^{1-\alpha},$$

where A_t is an exogenous disturbance to technology, modelled as an AR(1) process. They set their demand of capital and intermediary good to minimize their cost $r_t^k K_{t-1} + x_t N_t$, where x_t is the relative price of the intermediary good.

The distribution sector includes two kinds of agents: a continuum of monopolistic distributors purchase the final good, differentiate it and set their price under a standard Calvo lottery. With probability ξ, their price is simply indexed on a convex combination of past and steady state inflation $\pi_t^{\iota}\bar{\pi}^{1-\iota}$. A final distributor aggregates their output using a Dixit-Stiglitz technology with an constant elasticity of substitution θ.

Wages are determined by a standard Nash bargain between firms and workers. Firms relative bargaining power is κ.

Monetary policy is described by a Taylor rule

$$R_t = R_{t-1}^{\rho_R}\left(\frac{\bar{\pi}}{\beta}\left(\frac{\pi_t}{\bar{\pi}}\right)^{r_\pi}\right)^{1-\rho_R} \varepsilon_{R,t}.$$

Last, the governement purchases a fixed amount G of final goods. His budget is balanced in each period by lump-sum transfers to households.

Adding financial frictions

The financial accelerator is introduced as in section 2.1. Entrepreneurs use internal funds, including a fixed subvention $\mathcal{W}_e$ from the government, and bank loans to purchase capital to households. They face an idiosyncratic shock ω, drawn in a distribution with cdf F and exogenously varying variance $\sigma_{v,t-1}$. The latter is known one period in advance and modelled by an AR(1) process. Disturbances in σ_v are called risk shocks. The value of ω determines their level of effective capital. Those with the lowest draws, that is below a threshold denoted by $\bar{\omega}_t$, are bankrupt; the bank seizes all their belongings minus a fraction μ which is paid to the government and defined as auditing cost. The bank sets its interest rate such that it makes no profit, and entrepreneurs make their investment and borrowing decisions to maximize their profits. The entrepreneurs who do not face bankruptcy make profits that form their net worth n_t that is re-invested in the following period. Last, a proportion γ_t of entrepreneurs exit the economy in each period and are replaced by new ones. It is determined by an exogenous AR(1) process, referred to as net worth shocks. The net wealth of exiting entrepreneurs is distributed evenly to households.

Calibration

The baseline model is calibrated as follows:

β	θ	ξ	ι	r_π	$G/\bar{Y}$	s	κ	$\bar{q}$	c	$\bar{n}$	φ	α	δ
0.99	6	0.75	0.5	1.5	0.2	0.1	0.5	0.9	0.01	0.57	0.1	0.4	0.025

The persistence of the Taylor rule and of the AR(1) processes are 0.7. The parameters characterizing the financial accelerator are

μ	$\bar{\gamma}$	$F(\bar{\omega})$	$\mathcal{W}_e/\bar{n}$
0.27	0.975	0.007	0.001

The remaining parameters are obtained from long run constraints. The standard deviations of the productivity, monetary policy, risk and net worth shocks are respectively 1, 1, 30 and 1.

2.6.2 Simulations

Impulse response functions

Figure 2.16 shows the impulse response of the economy to standard shocks without and with financial frictions, and to the new shocks afforded by the financial accelerator. In response to a technology shock, the presence of financial frictions dampens job creation, investment and output, as explained in section 2.1. The response of employment N also becomes slightly negative at the date of the shock. To understand why, a small detour to basic new-Keynesian models is helpful. In Galí (1999), employment reacts countercyclically to productivity shocks. But his model only includes labor as an input in production. If capital is added as in standard RBC models, without assuming further refinements, then the response of employment turns positive again. This is because investment creates an additional market for production, which supports firms demand of production factors. Moreover, it crowds out somewhat consumption, lessening negative wealth effects on labor supply. Consistently with these mechanisms, the inclusion of adjustment costs on investment, or wage stickiness, in such models, can make the response of labor to productivity shocks countercyclical, at least in the short run. Of course, this behavior is influenced by the value of the related parameters. In the present basic model with the calibration used, the size of the adjustment cost on investment is rather small and wages are flexible. As a result, the response of the employment is still positive in the model without financial frictions. But this property crucially depends upon the response of investment. As explained in section 2.1, financial frictions diminish the response of investment to productivity shocks, through a 'debt-deflation' channel. The rise in investment is then insufficient to induce an improvement in employment at the date of the shock.

After an expansionary monetary policy shock, the rise in employment is more marked that without financial frictions. In terms of labor supply, there is no significant effect before 5 quarters, as shown by the unchanged response of λ_t. Thenafter, the contribution of labor supply is even negative through wealth effects. Regarding production factor prices, the findings are identical: the cost of capital relatively to wages is lower after 5 periods, so substitution effects by firms favor capital. The conclusion is that the amplification of job creation results primarily from the rise in final demand, in particular for investment. As a consequence, the variance of employment relatively to output is not magnified, by contrast to Petrosky-Nadeau (2014).

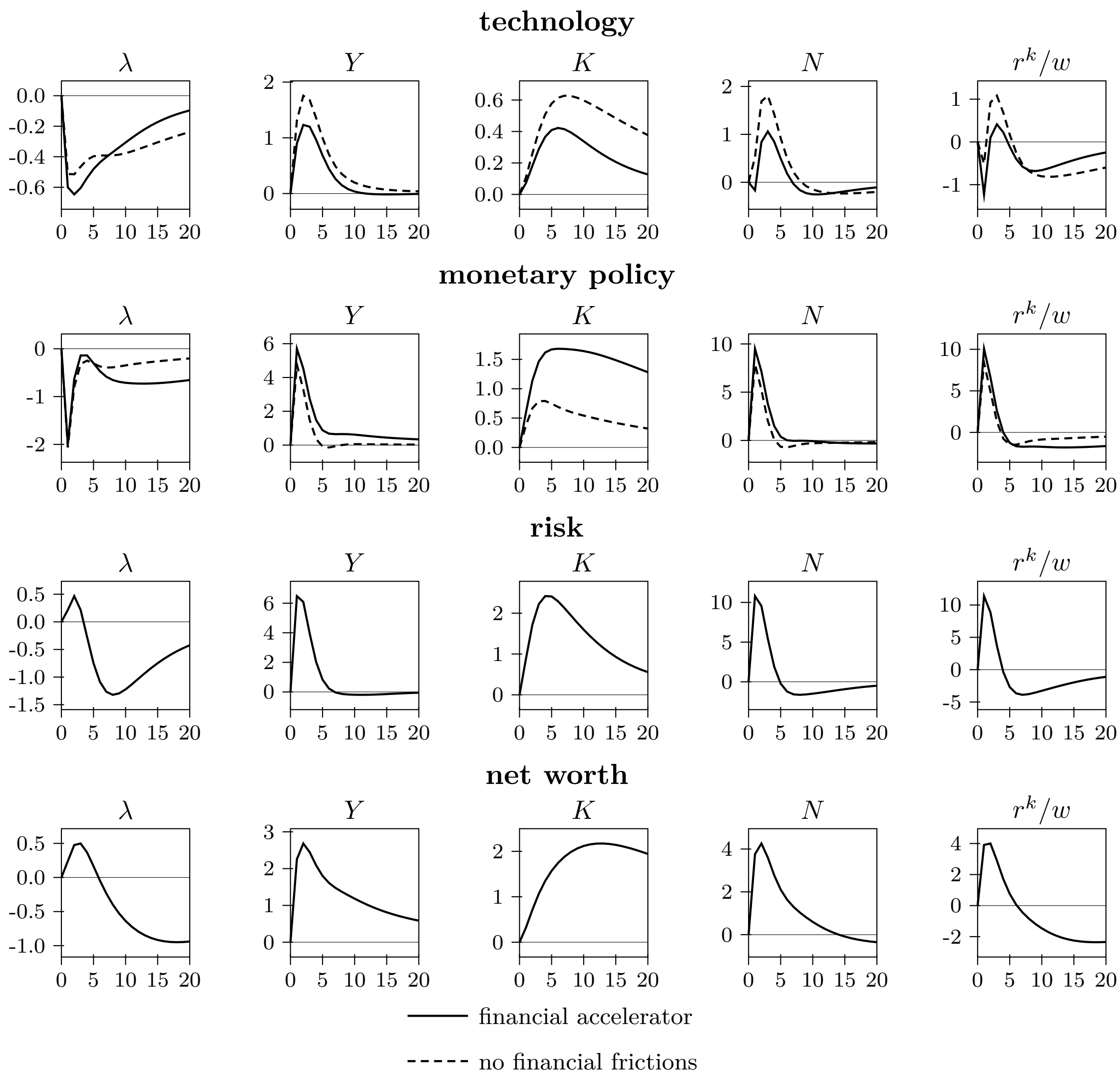
Turning to financial shocks, the response of job creation is significant in both cases. In the short run, all motives go in the same direction: wealth effects contribute positively to labor supply because of the crowding-out of consumption by savings in capital, the relative price of production factors supports employment, and the final demand of goods for investment augments, pulling firms' needs for production factors.

Unconditional second order moments

To assess the relative importance of the sole addition of financial shocks in the total changes brought to the dynamics of job creation by the financial accelerator, I compute a subset of theoretical second order moments implied by (i) the model without financial frictions, (ii) the model with financial frictions but where financial shocks are muted, and (iii) the complete model with financial frictions. They are reported in Tables 2.7 to 2.9. The effects of the financial accelerator when only technology and monetary policy shocks are present are moderate. They include a minor increase in the persistence of employment and wages and a reduction of the procyclicality of real wages. The latter is due to the persistent negative wealth effects on labor supply, which augment workers' outside options in the bargain long after employment has returned to its steady state level.

The activation of financial shocks reinforce these changes: increased persistence of job creation

Figure 2.16: Impulse response with and without financial frictions



and marked reduction of the procyclicality of wages. The correlation of wages with employment is zero. This is because expansionary financial shocks make savings in capital more attractive, so investment crowds out consumption in the short run. The result is positive wealth effects on labor supply, by contrast with monetary policy and technology shocks. Therefore, workers outside options contribute negatively to wages, whereas employment increases to satisfy production needs.

Financial shocks do not increase the relative volatility of employment; it varies in the same proportion relatively to output as in the case of a standard monetary policy shock (approximately with a factor 1.5). This is because changes in employment after financial shocks results from the same channel, that is through the final demand in the goods market. The channel described by Hall (2014) is absent from the model. This author suggests that financial disturbances materialize in fluctuations in discount rates applied to future corporate profits, and that the same discount rate may also apply to the present value of new workers that firms take into account when they decide to invest in job creation. Changes in discount rates applied to sluggish corporate profits translate into large movements in stock market prices. In a similar way, they imply large swing in employment when applied to the present value of new workers.

Last, I find a small reduction in the relative volatility of vacancies, which can be attributed to net worth shocks.

Table 2.7: baseline model

		Y	N	v	w
relative standard deviation		1.00	1.51	25.42	0.50
autocorrelation		0.66	0.59	-0.02	0.60
correlation matrix	Y	1.00	0.95	0.52	0.82
	N	–	1.00	0.59	0.69
	v	–	–	1.00	0.58
	w	–	–	–	1.00

Table 2.8: financial accelerator model without financial shocks

		Y	N	v	w
relative standard deviation		1.00	1.49	23.38	0.55
autocorrelation		0.73	0.65	0.00	0.76
correlation matrix	Y	1.00	0.91	0.47	0.72
	N	–	1.00	0.57	0.42
	v	–	–	1.00	0.47
	w	–	–	–	1.00

Table 2.9: financial accelerator model

		Y	N	v	w
relative standard deviation		1.00	1.50	21.47	0.53
autocorrelation		0.78	0.71	0.09	0.90
correlation matrix	Y	1.00	0.90	0.44	0.36
	N	–	1.00	0.55	-0.02
	v	–	–	1.00	0.16
	w	–	–	–	1.00

2.7 Import prices

In this section, I examine two points related to the modeling of import prices setting. First, I illustrate the effects of assuming local currency pricing vs. producer currency pricing on the pass-through in a very simplified and partial equilibrum framework. Next, I compare quadratic adjustment cost on prices with a Calvo lottery; I show that the adjustment cost gives rise to imperfect exchange rate pass-through as well.

2.7.1 LCP vs PCP

In this economy, the nominal exchange rate S_t is assumed to be a random walk, so the log-linearized dynamics of the real exchange rate $e_t \equiv S_t P_t^\star / P_t$ is

$$\hat{e}_t = \hat{e}_{t-1} + \hat{\pi}_t^\star - \hat{\pi}_t + \varepsilon_t,$$

where $\pi^\star$ is foreign inflation, π is domestic inflation and ε is an iid exogenous disturbance. The local currency pricing assumption implies that firms selling abroad set their price in the foreign currency. For monopolistic intermediate importers, that means that they set their price in the domestic currency, say euro. The competitive final importer aggregates differentiates imported goods $m(i),\ i \in [0,1]$ with CES technology

$$M_t = \left(\int_0^1 m_t(i)^{\frac{s-1}{s}} \, di \right)^{\frac{s}{s-1}},$$

so his demand addressed to the i-th intermediate importer is

$$m_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-s} M_t,$$

with P_t being the aggregate import price in euro. The i-th importer's profits expressed in domestic currency is

$$\left(P_t(i) - S_t P_t^\star \right) m_t(i).$$

Price setting is subject to a Calvo lottery, with probability ξ not to reoptimize and no price indexation for non-optimizers. Intermediate importers who can reoptimize choose the same price $\tilde{P}_t$. Their objective function is

$$E_t \sum_{k=0}^{\infty} (\beta\xi)^k \frac{\lambda_{t+k}}{\lambda_t} \left(\tilde{P}_t - S_{t+k} P_{t+k}^\star \right) \left(\frac{\tilde{P}_t}{P_{t+k}} \right)^{-s} M_{t+k}.$$

The resulting first order condition together with the zero profit condition of the final importer yields the dynamics of domestic import prices

$$\hat{\pi}_t = \frac{(1-\beta\xi)(1-\xi)}{\xi} \hat{e}_t + \beta E_t \hat{\pi}_{t+1}.$$

Conversely, the producer currency pricing means that importers set their price in the foreign currency, say USD. The final importer's demand addressed to the i-th intermediate importer is then

$$m_t(i) = \left(\frac{S_t P_t(i)}{P_t}\right)^{-s} M_t,$$

and the i-th importer's profits expressed in foreign currency is

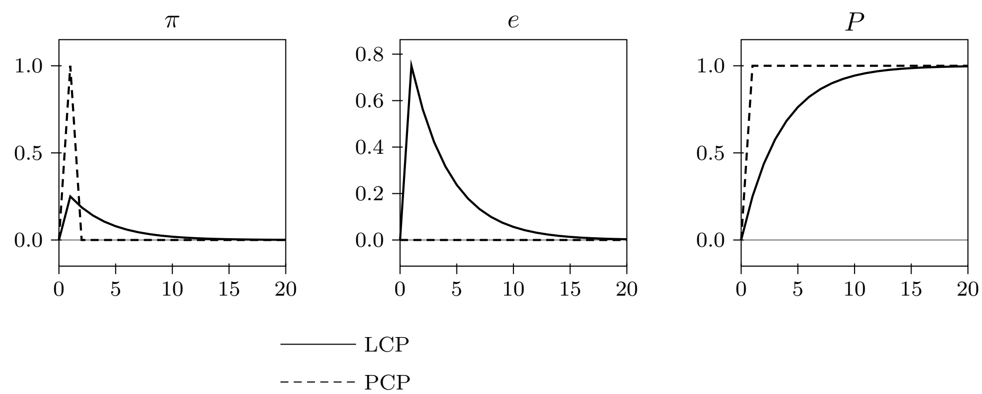
$$\left(P_t(i) - P_t^\star\right) m_t(i).$$

As for local currency pricing, the first order condition together with zero profit by the final importer yields the dynamics of domestic import prices

$$\hat{\pi}_t = \frac{1-\xi}{\xi}\hat{e}_t + \beta\xi E_t\hat{\pi}_{t+1} + \beta(1-\xi)E_t\hat{\pi}^\star_{t+1} - \beta(1-\xi)E_t\hat{e}_{t+1} + \varepsilon_t.$$

The implications of these two assumptions on the exchange rate pass-through are shown in the IRFs plotted in figure 2.17

Figure 2.17: LCP vs. PCP with Calvo – responses to an exchange rate shock



2.7.2 Calvo vs adjustment cost

This paragraph develops the external quadratic adjustment cost under the LCP assumption, which is the option chosen in the model instead of staggered prices à la Calvo (1983), in the simple framework proposed above. IRFs are simulated in order to confirm that this specification can generate a dynamic response of prices in response to exchange rate movements that is similar to the standard approach (LCP with a Calvo lottery). Intermediate importers $i \in [0,1]$ set their prices in euro to maximize their profit

$$(P_t(i) - S_t P_t^\star) m_t(i) - \frac{\phi}{2}\left(\frac{P_t(i)}{\bar{\pi} P_{t-1}} - 1\right)^2 P_t M_t$$

subject to the demand addressed to them

$$m_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-s} M_t.$$

Considering that all importers choose the same price $\tilde{P}_t$, and with $\tilde{p}_t \equiv \tilde{P}_t/P_t$, their objective is to choose $\tilde{p}_t$ to maximize

$$\tilde{p}_t^{1-s} - e_t\tilde{p}_t^{-s} - \frac{\phi}{2}\left(\frac{\pi_t\tilde{p}_t}{\bar{\pi}} - 1\right)^2.$$

The first order condition is

$$(1-s)\tilde{p}_t^{-s} + se_t\tilde{p}_t^{-s-1} - \phi\frac{\pi_t}{\bar{\pi}}\left(\frac{\pi_t\tilde{p}_t}{\bar{\pi}} - 1\right) = 0$$

Then, as from the zero profit condition of the aggregator $\tilde{P}_t = P_t$, this equation simplifies to

$$(1-s) + se_t - \phi\frac{\pi_t}{\bar{\pi}}\left(\frac{\pi_t}{\bar{\pi}} - 1\right) = 0.$$

The log-linearized import price equation is now

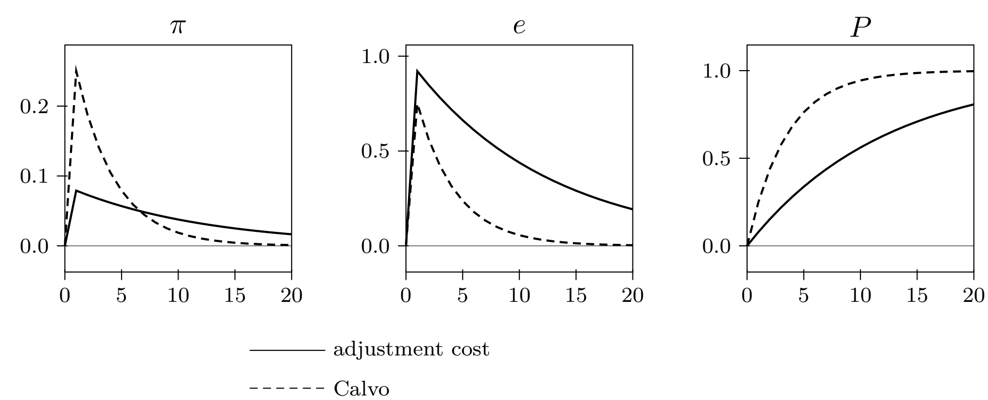
$$(s-1)\hat{e}_t = \phi\hat{\pi}_t.$$

Figure 2.18 shows IRFs simulated for

$$\phi = \frac{(s-1)\xi}{(1-\beta\xi)(1-\xi)},$$

which is a normalisation of the size of the adjustment cost such that the coefficient of the real exchange rate $\hat{e}_t$ in the log-linearized equation for imports price inflation is the same as with a Calvo mechanism, and for the benchmark model including a Calvo lottery.

Figure 2.18: LCP with adjustment costs vs. Calvo – responses to an exchange rate shock



2.8 Small open economy

This section examines the main implications in terms of impulse responses to standard shocks of moving from a closed economy to a small open economy which trades goods and bonds with an exogenous foreign economy, in a new-Keynesian framework. Trade dampens the response of consumption and of inflation to positive technology shocks, although the appreciation of the domestic currency feeds imported deflationary pressures. This is because the increase in foreign demand for domestic goods squeezes domestic consumption and contributes thus positively to domestic inflation. In response to an expansionary monetary policy shock, consumption increases less when the economy is open, while inflation is raised. This results from the marked depreciation of the domestic currency which pushes import prices up.

Then, I analyse the effect of the parametrization of the risk premium rule which is used to make the model stationary (see Schmitt-Grohé and Uribe (2003)); risk premium responds negatively to foreign net asset position and to the expected change in the depreciation rate of domestic currency. First, the dynamics of the model is not changed much when the value of the elasticity with respect to foreign debt changes in a range that is close to usual calibrations, that is between 0 and 0.1, provided of course that it is strictly positive to ensure stationarity. The value of the elasticity with respect to the expected exchange rate depreciation seems to matter more, since higher values are used in the literature (Adjemian et al. (2008) and Adolfson et al. (2008) respectively estimate a value of 0.13 and 0.61). In particular, setting this parameter to 0.5 instead of 0 amplifies the response of the exchange rate to all shocks but productivity, with significant effects on real aggregates. For instance, it changes the sign of the reaction of output and trade flows in response to foreign inflation shocks since it magnifies the resulting appreciation of domestic currency.

2.8.1 The model

Households are identical with instantaneous utility

$$U_t = \log C_t + \chi \frac{(1-N_t)^{1-\eta}}{1-\eta}.$$

where C_t is consumption and N_t labor supply. Their preference for the present is materialized by a constant discount factor β. Their budget constraint is

$$C_t + \frac{B_t}{R_t P_t} + \frac{S_t B_t^\star}{R_t^\star P_t} \leq \frac{B_{t-1}}{P_t} + \frac{S_t B_{t-1}^\star}{P_t} + w_t N_t + div_t + T_t.$$

First order conditions yield

$$\lambda_t = \frac{1}{C_t}, \tag{2.8.1}$$

$$\chi(1-N_t)^{-\eta} = \lambda_t w_t, \tag{2.8.2}$$

$$1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t}{\pi_{t+1}} \tag{2.8.3}$$

$$1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t^\star}{\pi_{t+1}^\star} \frac{s_{t+1}}{s_t}, \tag{2.8.4}$$

where $s_t = S_t P_t^\star / P_t$ is the real exchange rate and $\pi_t^\star$ is world inflation.

The retail sector includes a constant and large number of monopolistic firms who set their prices under a Calvo lottery (with probability $1-\xi$ to reset prices). Firms that cannot optimize update their prices by factor $\bar{\pi}^{1-\iota}(\pi_{t-1}^Y)^\iota$. The optimal relative price (with respect to the aggregate production price) is the same for all optimizing firms and is denoted by $\tilde{p}_t$. The final domestic good Y_t is a Dixit-Stiglitz aggregate of the differentiated outputs of monopolists, with elasticity of substitution ϵ. Its relative price is p_t^Y with respect to the consumption price index. With these assumptions, the dynamics of aggregate inflation of the production price is governed by

$$\tilde{p}_t = \frac{\epsilon}{\epsilon - 1} \frac{H_{1t}}{H_{2t}}, \tag{2.8.5}$$

$$H_{1t} = \frac{w_t Y_t}{A_t} + \beta \xi E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_{t+1}^Y}{\bar{\pi}^{1-\iota}(\pi_t^Y)^\iota} \right)^\epsilon H_{1t+1}, \tag{2.8.6}$$

$$H_{2t} = Y_t p_t^Y + \beta \xi E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_{t+1}^Y}{\bar{\pi}^{1-\iota}(\pi_t^Y)^\iota} \right)^{\epsilon - 1} H_{2t+1}, \tag{2.8.7}$$

$$1 = (1-\xi)\tilde{p}_t^{1-\epsilon} + \xi \left(\frac{\bar{\pi}^{1-\iota}(\pi_{t-1}^Y)^\iota}{\pi_t^Y} \right)^{1-\epsilon}. \tag{2.8.8}$$

The production sector includes identical and perfectly competitive firms. Their output is $A_t N_t$ where A_t is exogenous productivity. Therefore, the quantity of the domestic final good produced is

$$d_t Y_t = A_t N_t, \tag{2.8.9}$$

where d_t is a factor which captures the effect of price dispersion among retailers, and follows

$$d_t = (1-\xi)\tilde{p}_t^{-\epsilon} + \xi \left(\frac{\bar{\pi}^{1-\iota}(\pi_{t-1}^Y)^\iota}{\pi_t^Y} \right)^{-\epsilon} d_{t-1}. \tag{2.8.10}$$

The final production Y_t splits into exports X_t, domestic goods H_t and public expenditures (which are assumed to be constant), so

$$Y_t = H_t + X_t + G. \tag{2.8.11}$$

Domestic goods are combined with imports conforming to CES aggregator with elasticity of substitution θ to form the final consumption good

$$C_t = \left(a^{\frac{1}{\theta}} H_t^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} M_t^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}. \tag{2.8.12}$$

The optimal composition of C_t yields the first order condition

$$\frac{H_t}{M_t} = \frac{a}{1-a}\left(\frac{s_t}{p_t^Y}\right)^{\theta}, \tag{2.8.13}$$

and the zero profit condition of the aggregator writes

$$1 = a(p_t^Y)^{1-\theta} + (1-a)s_t^{1-\theta}. \tag{2.8.14}$$

Exports follow world demand D_t which is exogenous according to the ad hoc rule

$$\frac{X_t}{D_t} = \left(\frac{s_t}{p_t^Y}\right)^{\omega} \tag{2.8.15}$$

Trade is balanced by asset flows such that

$$\frac{s_t b_t^\star}{R_t^\star} - \frac{s_t b_{t-1}^\star}{\pi_t^\star} = p_t^Y X_t - s_t M_t, \tag{2.8.16}$$

where $b_t^\star = B_t^\star / P_t^\star$ is the real net asset position with respect to the rest of the world. The domestic interest rate is determined by a standard Taylor rule

$$R_t = R_{t-1}^{\rho}\left[\frac{\bar{\pi}}{\beta}\left(\frac{\pi_t}{\bar{\pi}}\right)^{r_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{r_Y}\right]^{1-\rho}\varepsilon_{R,t}, \tag{2.8.17}$$

while the interest rate in the rest of the world is an ad hoc decreasing function of the domestic net foreign asset position expressed in unit of domestic consumption goods:

$$R_t^\star = \frac{\bar{\pi}}{\beta}\exp\left(-\psi\frac{s_t b_t^\star}{\bar{Y}}\right)\varepsilon_{s,t}. \tag{2.8.18}$$

Finally come some definitions:

$$p_t^Y = \frac{\pi_t^Y}{\pi_t} p_{t-1}^Y, \tag{2.8.19}$$

and

$$dS_t = \frac{s_t \pi_t}{s_{t-1}\pi_t^\star}. \tag{2.8.20}$$

2.8.2 Calibration

Steady state levels are chosen as follows: the employment rate is 0.57, the annual inflation rate is 2.5%, the share of public expenditure (including also implicitely private investment and changes in inventory) in output is 40%, the markup rate of monopolists is 20% of sales, the shares of import and of exports in output are equal, so that the calibration of the size of residual demand is identical in the closed economy version of the model, and are set to 20%.

A number of parameters usual in real and new-Keynesian models are calibrated with standard values: the time discount factor is 0.99, the labor supply elasticiy is determined by $\eta = 2$, the

Calvo probability is 0.75 which corresponds approximately to price revisions occuring on average at a yearly pace, inflation persistence is 0.5, and the reaction of the nominal interest rate to inflation and output are respectively 1.5 and 0.125, while its persistence is fixed to 0.7. The elasticity of substitution between imports and domestic goods in the consumption bundle is $\theta = 2.5$ and the elasticity of exports is $\omega = 1.5$. Then the leisure preference scale parameter χ and the steady state ratio of the real net asset position with respect to the rest of the world and output are determined by long run restrictions.

Last, the exogenous variables are modelled as AR(1) processes. The persistence of productivity, the interest rate shock, world demand, world inflation and the foreign interest rate shock are respectively 0.7, 0, 0.7, 0.5 and 0.7, and the standard deviations of their innovation are respectively 0.0068, 0.0062, 0.04, 0.002 and 0.01. The size of world demand and foreign interest shocks are chosen so that exports and the change in the nominal exchange rate are approximately 4 times as volatile as domestic output as in data for the euro area including the USD/EUR exchange rate. The size of the world inflation shock almost replicates the volatility of the US GDP delfator. However, these relative shock sizes are unimportant for impulse response comparisons.

2.8.3 Simulations

This section addresses two questions: first, what is the impact of opening the economy on the transmission of domestic shocks? second, how does the calibration of the risk premium function affect the response of the economy to both external and domestic shocks? The answers are shown in the impulse responses in figures 2.19, 2.20 and 2.21: [ht]

Figure 2.19: Effect of opening the economy

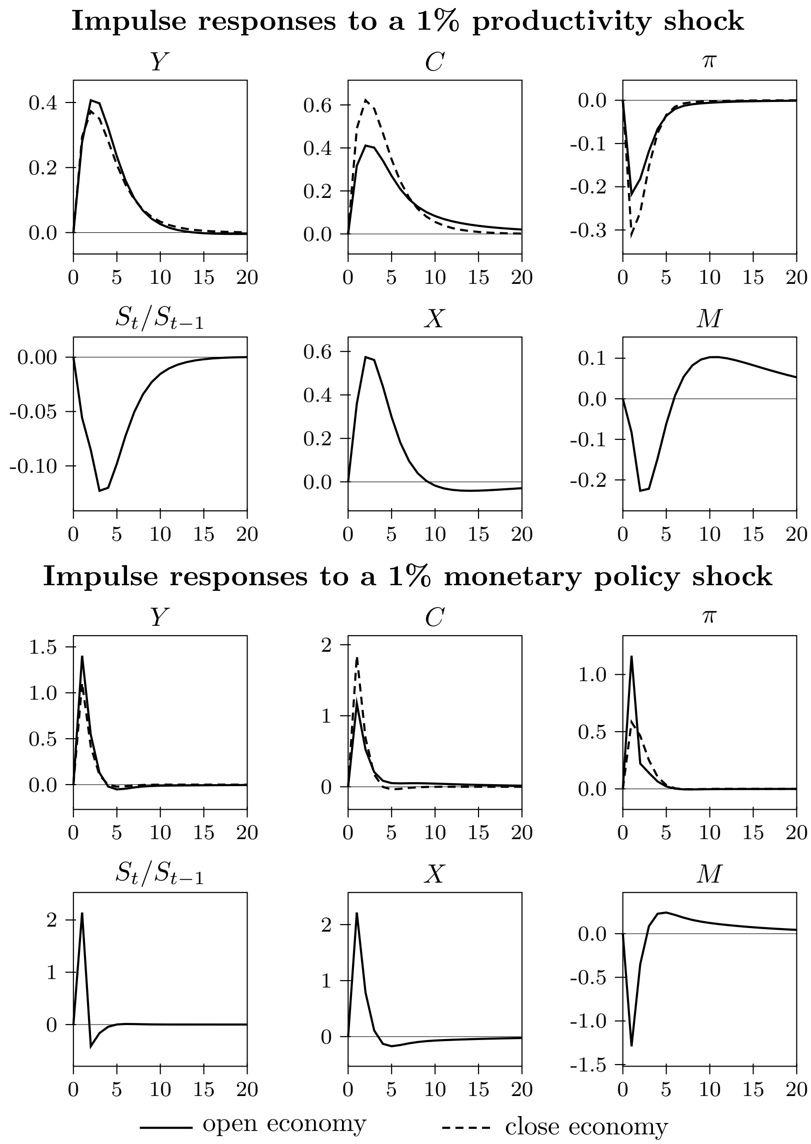
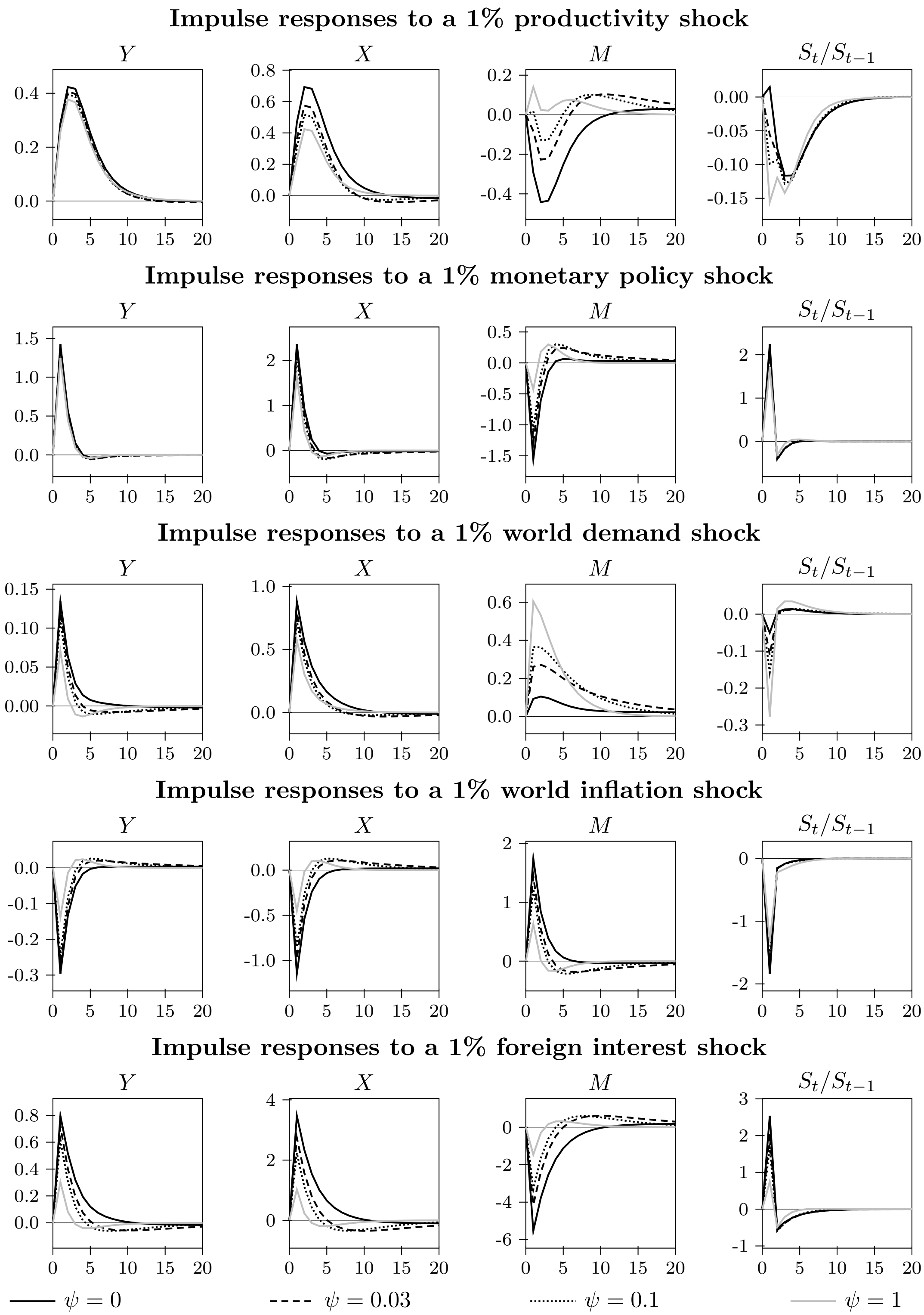


Figure 2.20: Impact of the elasticity of the risk premium to trade imbalance



Now I analyse the effect of assuming that the risk premium (or the interest rate in the rest of the world) depends negatively on the expected change in the nominal exchange rate depreciation, as in Adolfson et al. (2008) and Adjemian et al. (2008). It is

$$R_t^\star = \frac{\bar{\pi}}{\beta}\exp\left(-\psi\frac{s_t b_t^\star}{\bar{Y}} - \gamma\left(\frac{E_t S_{t+1}}{S_{t-1}} - 1\right)\right)\varepsilon_{s,t} = \frac{\bar{\pi}}{\beta}\exp\left(-\psi\frac{s_t b_t^\star}{\bar{Y}} - \gamma\left(dS_t E_t dS_{t+1} - 1\right)\right)\varepsilon_{s,t}. \tag{2.8.21}$$

This specification includes both an argument in real terms (real net foreign assets) and another which is a change in a nominal variable. The first one is needed to make the model stationary and determine the steady state level of the country's net foreign asset position (see section 2.10 for a more extensive discussion of this issue). The justification of the second one is the observation that risk premia are strongly negatively correlated with the expected change in the nominal exchange rate depreciation (see Fama (1984) and Duarte and Stockman (2005)).

The simulations are plotted for different values of γ.

This specification strongly impacts the impulse responses of the model. In particular, persistent appreciations of the exchange rate are either curbed or even changed to depreciations in the short run, which has an impact on international trade flows.

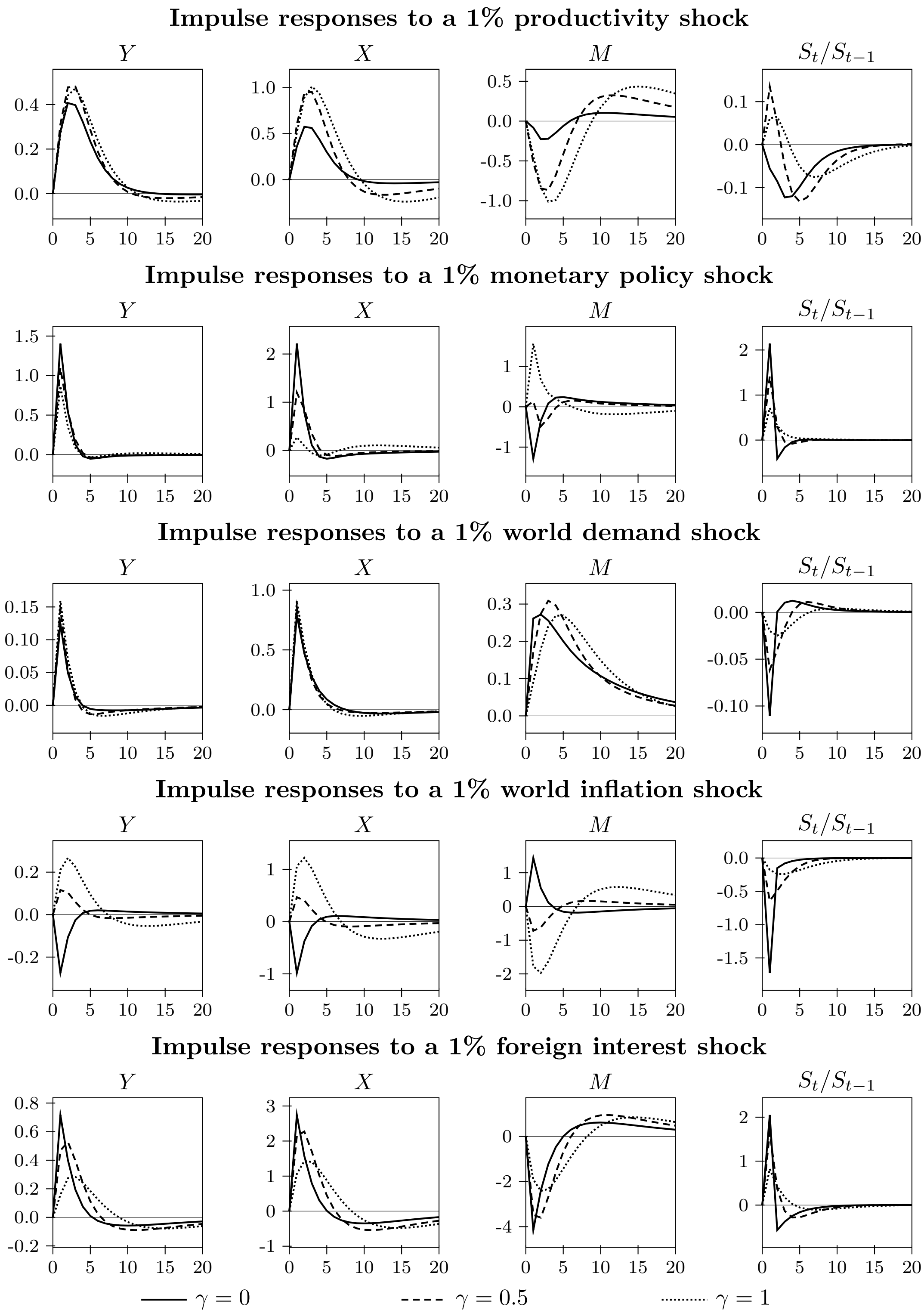
In response to a positive technology shock, domestic production is more attractive, resulting in a rise in exports and a drop in imports. But the domestic currency tends to appreciate, which mitigates these effects on international trade. Positive values of γ thus amplify the response of exports and imports.

An expansionary monetary policy shock entails a depreciation of the domestic currency, pushing exports and eroding imports. This specification hence moderates the reaction of trade flows. More, domestic inflation can make imported goods more competitive if the exchange rate depreciation is almost fully inhibited, which occurs for $\gamma = 1$.

Finally, positive values of γ change the sign of the model's response to world inflation. With a standard risk-premium rule, a rise in foreign prices yields a appreciation of the domestic currency of a larger magnitude. As a result, foreign price expressed in domestic currency decrease at the date of the shock, with a negative effect on competitivity: exports fall, and imports increase. When $\gamma > 0$, the risk-premium mitigates the currency appreciation. The rise in foreign prices is larger than the change in the exchange rate, so we observe an improvement in domestic competitivity. This effect makes sense after a rise in foreign prices.

To conclude, these simulations suggest that the specification (2.8.21) of the risk-premium makes the response of the model to foreign inflation disturbances more sensible. Next, the value of γ seems to have significant effects on the propagation of both domestic and foreigh shocks. It thus adds a valuable degree of freedom when estimating the model. Adolfson et al. (2008) show that this specification is preferred to the standard formulation of the risk premium, according to their estimation results based on Swedish data. For these reasons, it is incorporated in the estimated model of France and the euro area described in chapter 3.

Figure 2.21: Impact of the elasticity of the risk premium to the expected change in the exchange rate



2.9 Monetary union

This section investigates the effect of switching from a floating exchange rate regime to a monetary union, in a basic two-country new-Keynesian model. Only asymmetric technology shocks are considered. In response to transitory shocks in a country, the monetary union imposes the equality of consumption price in both countries, and amplifies the response of inflation in the other one.

2.9.1 The floating exchange rate model

The world economy includes two identical countries $i = 1$ and $i = 2$ with households having instantaneous utility

$$U_{1t} = \log C_{1t} + \chi \frac{(1 - N_{1t})^{1-\eta}}{1-\eta}$$

in country 1, where C_{1t} is consumption and N_{1t} labor supply, and

$$U_{2t} = \log C_{2t} + \chi \frac{(1 - N_{2t})^{1-\eta}}{1-\eta}$$

in country 2. Their preference for the present is materialized by a constant discount factor β. They trade riskless bonds issued in country 1 and bonds issued in country 2, so there is no risk sharing internationally through portfolio diversification). In each country, the return on foreign bonds is affected by a risk premium Ψ which depends negatively on total real foreign assets holdings. The nominal exchange rate S_t represents the value of one unit of country 2's currency expressed in units of country 1's currency. Households budget constraint is

$$C_{1t} + \frac{B_{1t}}{R_{1t}P_{1t}} + \frac{S_t B^\star_{1t}}{\Psi_{1t} R_{2t} P_{1t}} \leq \frac{B_{1t-1}}{P_{1t}} + \frac{S_t B^\star_{1t-1}}{P_{1t}} + w_{1t}N_{1t} + div_{1t} + T_{1t} + F_{1t}$$

in country 1, and

$$C_{2t} + \frac{B_{2t}}{R_{2t}P_{2t}} + \frac{B^\star_{2t}}{S_t \Psi_{2t} R_{1t} P_{2t}} \leq \frac{B_{2t-1}}{P_{2t}} + \frac{B^\star_{2t-1}}{S_t P_{2t}} + w_{2t}N_{2t} + div_{2t} + T_{2t} + F_{2t}$$

in country 2, div, T and F represent respectively dividends from domestic monopolists, transfers from government and dividends from financial intermediaries who collect risk premia. All three variables are externalities and paid evenly to households. For each value of $i \in \{1, 2\}$, j denotes the index of the other country. First order conditions yield

$$\lambda_{it} = \frac{1}{C_{it}}, \tag{2.9.1}$$

$$\chi(1 - N_{it})^{-\eta} = \lambda_{it} w_{it}, \tag{2.9.2}$$

$$1 = \beta E_t \frac{\lambda_{it+1}}{\lambda_{it}} \frac{R_{it}}{\pi_{it+1}} \tag{2.9.3}$$

$$1 = \beta E_t \frac{\lambda_{it+1}}{\lambda_{it}} \frac{\Psi_{it} R_{jt}}{\pi_{jt+1}} \frac{s_{it+1}}{s_{it}}, \tag{2.9.4}$$

where $s_{1t} = S_t P_{2t}/P_{1t}$ is the real exchange rate in country 1 and $s_{2t} = P_{1t}/(S_t P_{2t})$, so

$$s_{1t} = \frac{1}{s_{2t}}. \tag{2.9.5}$$

The retail sector includes a constant and large number of monopolistic firms who set their prices under a Calvo lottery (with probability $1-\xi$ to reset prices). Firms that cannot optimize update their prices by factor $\bar{\pi}^{1-\iota}(\pi^Y_{it-1})^\iota$. The optimal relative price (with respect to the aggregate production price) is the same for all optimizing firms and is denoted by $\tilde{p}_{it}$. The final domestic good Y_{it} is a Dixit-Stiglitz aggregate of the differentiated outputs of monopolists, with elasticity of substitution ϵ. Its relative price is p^Y_{it} with respect to the consumption price index. With these assumptions, the dynamics of aggregate inflation of the production price is governed by

$$\tilde{p}_{it} = \frac{\epsilon}{\epsilon - 1} \frac{J_{it}}{I_{it}}, \tag{2.9.6}$$

$$J_{it} = \frac{w_{it} Y_{it}}{A_{it}} + \beta \xi E_t \frac{\lambda_{it+1}}{\lambda_{it}} \left(\frac{\pi^Y_{it+1}}{\bar{\pi}^{1-\iota}(\pi^Y_{it})^\iota} \right)^\epsilon J_{it+1}, \tag{2.9.7}$$

$$I_{it} = Y_{it} p^Y_{it} + \beta \xi E_t \frac{\lambda_{it+1}}{\lambda_{it}} \left(\frac{\pi^Y_{it+1}}{\bar{\pi}^{1-\iota}(\pi^Y_{it})^\iota} \right)^{\epsilon - 1} I_{it+1}, \tag{2.9.8}$$

$$1 = (1-\xi) \tilde{p}_{it}^{1-\epsilon} + \xi \left(\frac{\bar{\pi}^{1-\iota}(\pi^Y_{it-1})^\iota}{\pi^Y_{it}} \right)^{1-\epsilon}. \tag{2.9.9}$$

The production sector includes identical and perfectly competitive firms. Their output is $A_{it} N_{it}$ where A_{it} is exogenous productivity. Therefore, the quantity of the domestic final good produced is

$$d_{it} Y_{it} = A_{it} N_{it}, \tag{2.9.10}$$

where d_{it} is a factor which captures the effect of price dispersion among retailers, and follows

$$d_{it} = (1-\xi) \tilde{p}_{it}^{-\epsilon} + \xi \left(\frac{\bar{\pi}^{1-\iota}(\pi^Y_{it-1})^\iota}{\pi^Y_{it}} \right)^{-\epsilon} d_{it-1}. \tag{2.9.11}$$

The final production Y_{it} splits into exports M_{jt}, domestic goods H_{it} and public expenditures (which are assumed to be constant), so

$$Y_{it} = H_{it} + M_{jt} + G. \tag{2.9.12}$$

Domestic goods are combined with imports conforming to CES aggregator with elasticity of substitution θ to form the final consumption good

$$C_{it} = \left(a^{\frac{1}{\theta}} H_{it}^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} M_{it}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}. \tag{2.9.13}$$

The optimal composition of C_{it} yields the first order condition

$$\frac{H_{it}}{M_{it}} = \frac{a}{1-a}\left(\frac{s_{it}p_{jt}^Y}{p_{it}^Y}\right)^{\theta}, \tag{2.9.14}$$

and the zero profit condition of the aggregator writes

$$1 = a(p_{it}^Y)^{1-\theta} + (1-a)\left(s_{it}p_{jt}^Y\right)^{1-\theta}. \tag{2.9.15}$$

International trade is balanced by foreign asset flows such that

$$\frac{S_t B_{1t}^\star}{R_{2t}} - S_t B_{1t-1}^\star - \left(\frac{B_{2t}^\star}{R_{1t}} - B_{2t-1}^\star\right) = P_{1t}^Y M_{2t} - S_t P_{2t}^Y M_{1t}.$$

The variation in households 1's wealth held in the form of foreign assets minus the variation in households 2's wealth held in the form of foreign assets, both expressed in the currency of country 1, are equal to the trade surplus of country 1 (or equivalently the trade deficit of country 2) expressed in the same currency. Denoting $b_{it}^\star = B_{it}^\star/P_{jt}^\star$ the real foreign assets holding, it is in real terms

$$\frac{s_{1t}b_{1t}^\star}{R_{2t}} - \frac{s_{1t}b_{1t-1}^\star}{\pi_{2t}} - \frac{b_{2t}^\star}{R_{1t}} + \frac{b_{2t-1}^\star}{\pi_{1t}} = p_{1t}^Y M_{2t} - s_{1t}p_{2t}^Y M_{1t}. \tag{2.9.16}$$

The risk premium on foreign bonds is assumed to be

$$\Psi_{it} = \exp\left(-\psi\frac{s_{it}b_{it}^\star}{\bar{Y}_i}\right). \tag{2.9.17}$$

Interest rates are determined by standard Taylor rules

$$R_{it} = R_{it-1}^{\rho}\left[\frac{\bar{\pi}}{\beta}\left(\frac{\pi_{it}}{\bar{\pi}}\right)^{r_\pi}\left(\frac{Y_{it}}{\bar{Y}}\right)^{r_Y}\right]^{1-\rho}\varepsilon_{R,it}. \tag{2.9.18}$$

Finally come some definitions:

$$p_{it}^Y = \frac{\pi_{it}^Y}{\pi_{it}}p_{it-1}^Y, \tag{2.9.19}$$

and

$$dS_t = \frac{s_{1t}\pi_{1t}}{s_{1t-1}\pi_{2t}}. \tag{2.9.20}$$

2.9.2 The monetary union model

The monetary union imposes a peg regime $S/S_{t-1} = 1$ between country 1 and country 2, and a common interest rate R_t. Monetary policy is described by

$$\frac{s_{1t}}{s_{1t-1}} = \frac{\pi_{2t}}{\pi_{1t}}, \tag{2.9.21}$$

and

$$R_t = R_{t-1}^{\rho} \left[\frac{\bar{\pi}}{\beta} \left(\frac{\pi_{1t}^{0.5} \pi_{2t}^{0.5}}{\bar{\pi}} \right)^{r_\pi} \left(\frac{Y_{1t}^{0.5} Y_{2t}^{0.5}}{\bar{Y}} \right)^{r_Y} \right]^{1-\rho} \varepsilon_{R,t}. \tag{2.9.22}$$

The bonds markets are also modified; households from country 1 can only trade one-period riskless bonds $B^\star$ with households from country 2. These bonds are remunerated at rate R common to the monetary union plus a risk premium as in the floating exchange rate model. The optimal decision for bonds holding yields the unique Euler equation below in each country $i \in \{1, 2\}$:

$$1 = \beta E_t \frac{\lambda_{it+1}}{\lambda_{it}} \frac{\Psi_{it} R_t}{\pi_{it+1}}. \tag{2.9.23}$$

Moreover, the bonds market clearing condition imposes that $S_t B_{1t}^\star = -B_{2t}^\star$, or in real terms

$$s_{1t} b_{1t}^\star = -b_{2t}^\star. \tag{2.9.24}$$

2.9.3 Calibration

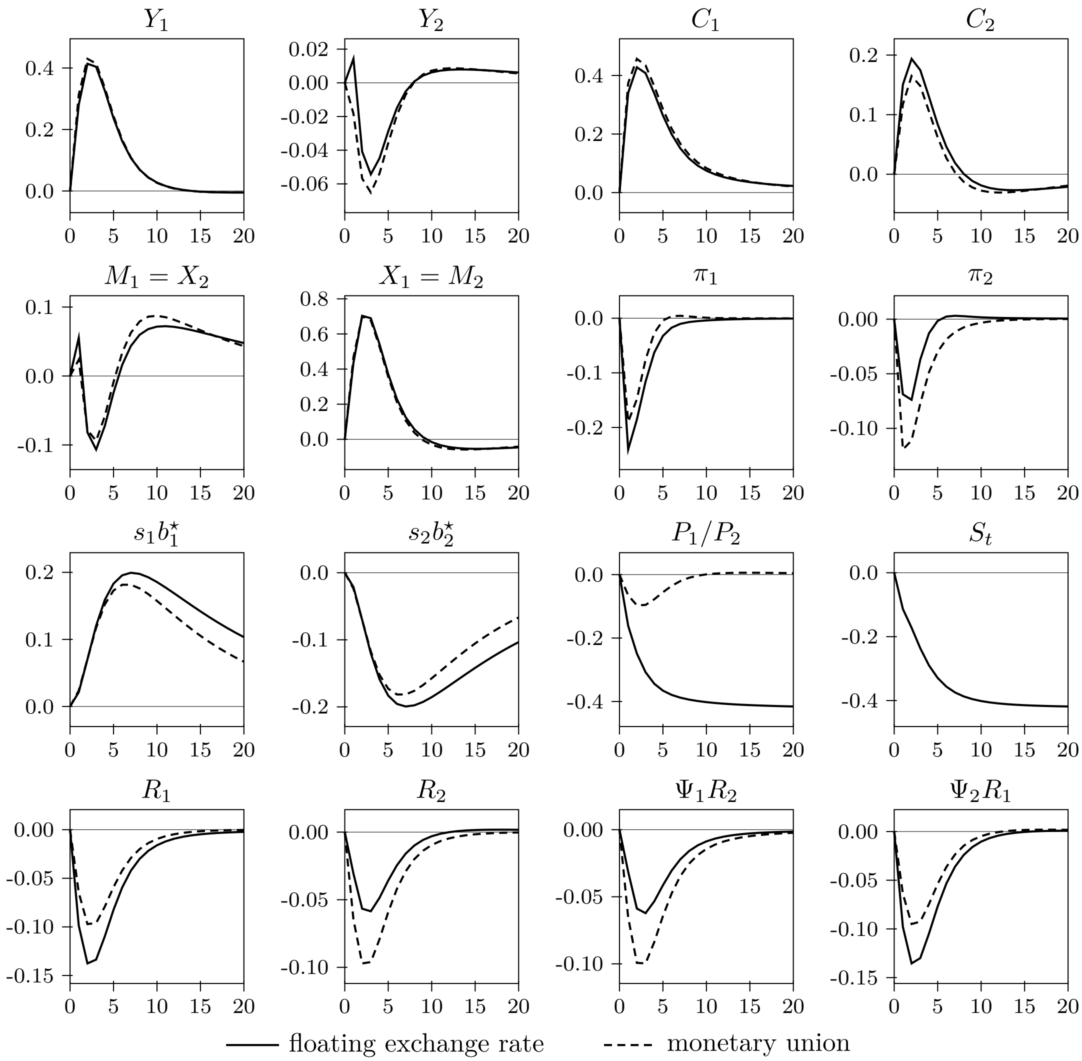
The two countries are assumed to be identical and to have the same size; the parameters that describe the economies have the same value. Steady state levels are chosen as follows: the employment rate is 0.57, the annual inflation rate is 2.5%, the share of public expenditure (including also implicitely private investment and changes in inventory) in output is 40%, the markup rate of monopolists is 20% of sales, the shares of import and of exports in output are equal to 20%.

A number of parameters usual in real and new-Keynesian models are calibrated with standard values: the time discount factor is 0.99, the labor supply elasticiy is determined by $\eta = 2$, the Calvo probability is 0.75 which corresponds approximately to price revisions occuring on average at a yearly pace, inflation persistence is 0.5, and the reaction of the nominal interest rate to inflation and output are respectively 1.5 and 0.125, while its persistence is fixed to 0.7. The elasticity of substitution between imports and domestic goods in the consumption bundle is $\theta = 2.5$. Then the leisure preference scale parameter χ is determined by long run restrictions. Last, labor productivities are independant AR(1) processes with persistence 0.7.

2.9.4 Simulations

The model under a floating exchange rate regime is compared to the monetary union case. The behavior of the economy is simulated for productivity shocks only, since the monetary policy rules, and hence the monetary policy shocks, are different under the two regimes. In response to a transitory technology shock in one country, the monetary union acts to get back to purchasing power parity, that is the equality of price levels in both countries, through financial flows and interest rate movements. Instead, purchasing power parity is obtained when the permanent drift in relative prices is compensated by a permanent drift in the nominal exchange rate under floating exchange rate. Specifically, under the floating exchange rate regime, the productivity shock entails an appreciation of the domestic currency; firms anticipate the possible negative impact on their competitivity with respect to foreign goods and react by further reducing their prices. In a monetary union, this effect is absent. I both case, the price reduction in country 1 impacts inflation in country 2 through import prices. Under a floating exchange rate regime, imported deflation in country 2 is mitigated by currency depreciation. All in all, in the monetary union, deflation is amplified in the other country, while it is slightly dampened in the country where the shock occured. Otherwise, movements in real aggregates are fairly similar.

Figure 2.22: Effect of the monetary union on impulse responses to an asymmetric 1% productivity shock



2.10 Permanent shocks in open economy models with incomplete markets

2.10.1 Exposition of the issue

In open economy models with incomplete asset markets, the steady state equilibrium is not unique; an admissible steady state can be computed for any level of net foreign assets. This can be illustrated immediately with a very simple model with two identical countries $i = 1$ and $i = 2$ that produce and trade one good. Households have instantaneous utility $U_{it} = \log C_{it}$ in country i, where C_{it} is consumption. Their preference for the present is materialized by a constant discount factor β. Output Y_i is exogenous. Households can purchase at price q_t an international bond b_{it} that delivers one unit of good in the subsequent period. Households maximize their lifetime utility subject to $C_{it} + q_t b_{it} = Y_i + b_{it-1}$. With bonds market clearing $b_{1t} = -b_{2t} \equiv b_t$, the full model is

$$q_t = \beta E_t \frac{C_{1t}}{C_{1t+1}},$$

$$q_t = \beta E_t \frac{C_{2t}}{C_{2t+1}},$$

$$Y_1 = C_{1t} + q_t b_t - b_{t-1},$$

$$Y_2 = C_{2t} - q_t b_t + b_{t-1},$$

so the conditions that the steady state equilibrium needs to verify are

$$q = \beta,$$

$$C_1 = Y_1 + (1-\beta)b,$$

$$C_2 = Y_1 + Y_2 - C_1.$$

There are four unknown for three equations in the static model. This proves that the steady state is not unique.

In a stochastic environment, the business cycle dynamics of such a model includes a unit root. This is shown by log-linearizing the two first equations of the model, assuming $\hat{x}_t = \log(x_t/\bar{x})$, where x represent any variable and $\bar{x}$ its steady state level. Indeed, we get immediately the martingale:

$$E_t\left(\hat{C}_{1t+1} - \hat{C}_{2t+1}\right) = \hat{C}_{1t} - \hat{C}_{2t}.$$

Several modifications have however been proposed to induce stationarity: by Schmitt-Grohé and Uribe (2003) for small open economy models, or Bodenstein (2006) in the two-country case. Among those solutions, the most commonly used is to assume that the return rate associated with holding foreign assets ($1/q_t$ in the example above) is altered by an ad hoc 'risk premium' that depends negatively on the country's net foreign asset position. In the example above, the Euler equations could take the form

$$q_t e^{\psi(b_{it}-\bar{b}_i)} = \beta E_t \frac{C_{it}}{C_{it+1}}, \quad i \in \{0,1\},$$

where $\psi > 0$ and $\bar{b}_1 = -\bar{b}_2$ are parameters.[7] With this specification, if a country accumulates trade deficits, the interest rate on its residents' debt would remain above its long run level until they get back to their stationary debt level by decreasing consumption of imported goods and/or improving exports competitiveness. This stationary level is thus fully determined by the assumed risk premium function; it can be calibrated freely. In this regard, the values generally used are either zero or are based on the observed average trade deficits of particular countries. A common practice is also to use small values of the elasticity of the risk premium with respect to net foreign assets, so that it causes barely noticeable variations in the interest rate.

When the model is to be used to simulate the effect of permanent shocks to the economy, such as fiscal policy shocks, the presence of this risk premium implies that the net foreign asset position in the final steady state is identical (in level or in percentage of another variable such as GDP, according to the chosen risk premium rule) to the one of the initial steady state. This is very disputable from a theoretical point of view: fiscal policies, for instance, are part in the degree of competitiveness of a country with respect to others. It is hence sensible to expect that permanent fiscal policies permanently impact international trade flows, which are counterbalanced by assets flows. In this story, it is likely that the final level of net foreign assets owned by domestic households is different from the initial one. There is no micro-foundation for the active role of lenders in financial markets targeting a given and constant level of foreign debt, that is implicit in the risk premium function. In a nutshell, why should a country's steady state external debt be policy-invariant?

In section 2.10.3 below, I compute the simulated effects of a permanent shock, namely a social VAT measure, in such a framework using the standard assumption that foreign debt returns to its initial level. I compare these simulations with those stemming from another assumption for the final steady state level, where the long run level of foreign debt is allowed to vary. I find that this second approach gives rise to larger short term responses of trade flows. The model used is described in section 2.10.2.

2.10.2 The model

I describe below a real business cycle open economy model, which shares some common features with Backus et al. (1994), but with no capital, incomplete markets, and separable utility functions. External habits in consumption produce smooth transitional dynamic responses. The world economy includes two countries $i \in \{1,2\}$, identical except relative population sizes being respectively Σ and $1-\Sigma$. Households have instantaneous utility

$$U_{it} = \log(C_{it} - hC_{it-1}) - \frac{L_{it}^{1+\eta}}{1+\eta},$$

[7] The steady state is uniquely determined by this specification. Indeed, the two first equations of the model do not collapse to $q = \beta$ any more. Instead, considering that $b_1 = -b_2$, we have $qe^{\psi(b_1-\bar{b}_1)} = \beta$ and $qe^{-\psi(b_1-\bar{b}_1)} = \beta$, which yields immediately $b_1 = \bar{b}_1$ and $q = \beta$.

where C_{it} is consumption per head, h is an habit parameter and L_{it} is labor supply per head in country i. Their preference for the present is materialized by a constant discount factor β. Households can trade riskless bonds b_{1t} at price q_t expressed in units of the consumption good of country 1. As discussed above, the price of bonds is affected by an exponential risk premium term, in order to eliminate the unit root in the model. It is $\exp\left[\psi\left(\Sigma b_{1t} - \bar{B}\right)\right]$ in country 1 and $\exp\left[\psi\left((1-\Sigma) b_{2t} + \bar{B}\right)\right]$ is country 2, where $\psi > 0$ and $\bar{B}$ are parameters. Notice that risk-premia depend on the country's total net foreign debt rather than on net foreign debt per head. The difference between the prices with and without risk-premium is collected by financial intermediaries in each country and is repaid evenly to domestic households in the form of dividends. Then, households budget constraints are:

$$(1+\tau_C)C_{1t} + q_t e^{\psi\left(\Sigma b_{1t} - \bar{B}\right)} b_{1t} = w_{1t}L_{1t} + b_{1t-1} + \text{gov. transfers} + \text{dividends},$$

$$C_{2t} + \frac{q_t e^{\psi\left((1-\Sigma) b_{2t} + \bar{B}\right)}}{S_t} b_{2t} = w_{2t}L_{2t} + \frac{b_{2t-1}}{S_t} + \text{dividends},$$

where S_t is the price of the final good in country 2 expressed in units of the final good in country 1 (the numeraire), w_{it} are wage rates expressed in units of domestic consumption goods and τ_C is the VAT rate in country 1. All tax revenues are rebated in the form of lump-sum transfers to country 1 households. Country 2 charges no tax. P_{it} and $P^{\star}_{it}$ are the prices of the good produced in country i and in country j respectively expressed in units of consumption goods of country i. First order conditions are

$$\lambda_{1t} = \frac{1}{(1+\tau_C)(C_{1t} - hC_{1t-1})}$$

$$\lambda_{2t} = \frac{1}{C_{2t} - hC_{2t-1}}$$

$$\text{for } i \in \{1,2\},\ \lambda_{it} w_{it} = L_{it}^{\eta},$$

$$\beta E_t \frac{\lambda_{1t+1}}{\lambda_{1t}} = q_t \exp\left[\psi\left(\Sigma b_{1t} - \bar{B}\right)\right],$$

$$\beta E_t \frac{S_t}{S_{t+1}} \frac{\lambda_{2t+1}}{\lambda_{2t}} = q_t \exp\left[\psi\left((1-\Sigma) b_{2t} + \bar{B}\right)\right].$$

Bonds market clearing implies

$$\Sigma b_{1t} + (1-\Sigma) b_{2t} = 0.$$

Output is obtained from labor following $Y_{it} = L_{it}$. The unit cost incurred by firms in units of final domestic good is $(1+\tau_L)w_{1t}$ in country 1 and w_{2t} in country 2. So zero profit implies

$$P_{1t} = (1+\tau_L)w_{1t},$$

$$P_{2t} = w_{2t}.$$

The final good in country i is used for consumption and is a composite of imports X_{jt} and domestic goods H_{it}:

$$C_{1t} = \left(a^{\frac{1}{\theta}} H_{1t}^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} \left(\frac{1-\Sigma}{\Sigma} X_{2t} \right)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}},$$
$$C_{2t} = \left(a^{\frac{1}{\theta}} H_{2t}^{\frac{\theta-1}{\theta}} + (1-a)^{\frac{1}{\theta}} \left(\frac{\Sigma}{1-\Sigma} X_{1t} \right)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}. \tag{2.10.1}$$

Zero-profit of the final producer of the composite good yields

$$C_{1t} = P_{1t} H_{1t} + P^{\star}_{1t} \frac{1-\Sigma}{\Sigma} X_{2t},$$
$$C_{2t} = P_{2t} H_{2t} + P^{\star}_{2t} \frac{\Sigma}{1-\Sigma} X_{1t}. \tag{2.10.2}$$

The optimal composition of C_{it} is given by

$$\frac{\Sigma}{1-\Sigma} \frac{H_{1t}}{X_{2t}} = \frac{a}{1-a} \left(\frac{P^{\star}_{1t}}{P_{1t}} \right)^{\theta} \tag{2.10.3}$$

The resource constraint is

$$Y_1 = H_{1t} + X_{1t}, \qquad Y_2 = H_{2t} + X_{2t}. \tag{2.10.4}$$

Then perfect mobility of goods implies (law of one price)

$$P_{1t} = P^{\star}_{2t} S_t \tag{2.10.5}$$

and

$$P_{2t} S_t = P^{\star}_{1t} \tag{2.10.6}$$

Terms of trade are defined as

$$TT_{1t} = \frac{P^{\star}_{1t}}{P_{1t}} = \frac{1}{TT_{2t}}.$$

Last, the law of motion of coutry 1's net foreign asset position stems from households budget constraints and is

$$q_t b_{1t} - b_{1t-1} = P_{1t} X_{1t} - P^{\star}_{1t} \frac{1-\Sigma}{\Sigma} X_{2t}.$$

The model is calibrated as follows:

β	θ	η	a	τ_C	τ_L	h	ψ	Σ
0.98	2.5	0.5	0.7	0.15	0.20	0.7	0.03	0.5

Initial conditions for net foreign debts is zero.

2.10.3 Permanent simulations under the usual approach

This variant is conducted using the usual approach assuming that all parameters except tax rates, but including $\bar{b}$, are unchanged. Therefore, the risk-premium acts to restore the initial level of the two countries' net foreign asset position. The simulated policy is a permanent 5-point increase in the VAT rate accompanied by a permanent 7.3-point drop in the employer social contribution factor, keeping government revenues unchanged in the long run. The dynamic response of the economy is the plain line in Figure 2.23.

2.10.4 Alternative anchoring of the steady state equilibrium

The final long run equilibrium determined by the condition that foreign debt returns to its initial level is not satisfactory. Another approach to pick up a final steady state is to assume 'long run' cross-country mobility of households. Specifically, households may decide to move to the other country at no cost, but this migration is only possible after a very long bag-packing time of T periods that can be chosen arbitrarily high. Once they chose to move, they are committed to do so and cannot make any other migration decision until they live in the other country. Otherwise, they can choose again in the next period. Consistently with this program, a household in country 1 would decide in period t to move to country 2 in period $t+T$ if

$$E_t\left[\sum_{j=0}^{T-1}\beta^j U_{1t+j}+\sum_{j=T}^{2T-1}\beta^j U_{2t+j}\right]>E_t\left[\sum_{j=0}^{2T-1}\beta^j U_{1t+j}\right], \tag{2.10.7}$$

and conversely. So in equilibrium, no arbitrage is possible and the economy should verify

$$E_t\sum_{j=T}^{2T-1}\beta^j U_{1t+j}=E_t\sum_{j=T}^{2T-1}\beta^j U_{2t+j}. \tag{2.10.8}$$

In deterministic permanent simulations, all variables converge towards their deterministic steady state levels, and so do their values expected by agents at the present date. In particular

$$\lim_{j\to\infty}E_t\left(U_{1t+j}-U_{2t+j}\right)=\bar{U}_1-\bar{U}_2.$$

Provided that T is chosen sufficiently large, condition (2.10.8) implies that

$$\bar{U}_1=\bar{U}_2. \tag{2.10.9}$$

Indeed, assume that it is not the case and let

$$\delta=\bar{U}_1-\bar{U}_2,$$

strictly positive (otherwise, $\delta = \bar{U}_2 - \bar{U}_1$). From (2.10.7), there exists a T such that for any $\varepsilon > 0$,

$$\forall j \geq T,\ |E_t(U_{1t+j} - U_{2t+j}) - \delta| < \varepsilon.$$

In particular, with ϵ chosen strictly smaller than δ, we can find T such that

$$\forall j \geq T,\ E_t(U_{1t+j} - U_{2t+j}) > \delta - \varepsilon.$$

Therefore

$$E_t \sum_{j=T}^{2T-1} \beta^j (U_{1t+j} - U_{2t+j}) > (\delta - \varepsilon)\beta^T \frac{1-\beta^T}{1-\beta} > 0,$$

and condition (2.10.8) does not hold.

Condition (2.10.9) is hence necessary but it is not sufficient; condition (2.10.8) is more restrictive and only holds exactly for $T \to \infty$. In practice however, I assume that it is reached numerically after T periods if T is sufficiently high (generally between 20 and 50 quarters). Taking steady state levels for expected values at a finite but remote horizon can be viewed as an acceptable assumption; it is usually made when computing numerical simulations using a relaxation algorithm (see Juillard (1996)).

To satisfy condition (2.10.9), the model can be modified without any undesirable effect in such a manner that the initial welfare levels in the two countries are equal. For that purpose, a constant term can be added to one of the utility functions, representing a contributor to well-being associated with the life in a particular country (like weather conditions or geography).

Under the "mobility assumption" described above, the final steady state must then verify

$$\bar{U}_1^{\text{final}} - \bar{U}_1^{\text{initial}} = \bar{U}_2^{\text{final}} - \bar{U}_2^{\text{initial}}. \tag{2.10.10}$$

Given a value of the parameter $\bar{B}$ of the risk premium function, this equality determines the new populations of countries 1 and 2, after migrations occured. Based on the observation that one-sided migrations between similar developed countries are small compared to the fluctuations in macroeconomic aggregates, I assume instead that financial markets immediately adjust risk premia to deter future migrations. In doing so, they anchor agents' long term expectations to a steady state equilibrium in which populations are unchanged. If one is interested in the effects of asymmetric policy decisions between France and Germany for instance, it seems indeed more suited to assume constant populations (and possibly varying foreign debts), than irrealistic population flows. With an adjustment of the foreign debt target level, the equilibrium condition (2.10.10), based on the *possibility* for households to migrate, is verified without any actual migration.

To understand this formally, consider the very basic model introduced in subsection 2.10.1. The only difference is that I take possible differences in populations into account. It is:

$$q_t \exp\left[\psi(\Sigma b_t - \bar{B})\right] = \beta E_t \frac{C_{1t}}{C_{1t+1}},$$

$$q_t \exp\left[-\psi(\Sigma b_t - \bar{B})\right] = \beta E_t \frac{C_{2t}}{C_{2t+1}},$$
$$Y_1 = C_{1t} + q_t b_t - b_{t-1},$$
$$Y_2 = C_{2t} - \frac{\Sigma}{1-\Sigma} q_t b_t + \frac{\Sigma}{1-\Sigma} b_{t-1}.$$

It implies the following long term constraints:

$$q = \beta,$$

$$b = \frac{\bar{B}}{\Sigma},$$

$$C_1 = Y_1 + (1-\beta)\frac{\bar{B}}{\Sigma},$$

$$C_2 = Y_2 - (1-\beta)\frac{\bar{B}}{1-\Sigma}.$$

To verify the equality of initial welfare levels across countries, I assume for simplicity that the initial steady state is characterized by $C_1 = C_2$. Then the risk-premium function should verify

$$\bar{B} = \frac{\Sigma(1-\Sigma)(Y_1 - Y_2)}{1-\beta}.$$

Consider a permanent shock: the economy of country 1 experiences a perpetual shift in its production, which becomes Y_1'. In the standard approach where the target foreign debt level is unchanged and people cannot migrate, the production surplus is entirely consumed in country 1, and the final equilibrium of country 2 is unchanged.

Now, I assume instead that "long term migrations" (as decribed above) are possible for households of countries 1 and 2. Then condition (2.10.10), stemming from the possibility to migrate, implies $C_1' = C_2'$ (the final consumption levels). It translates into

$$\Sigma(1-\Sigma)(Y_1' - Y_2) = (1-\beta)\bar{B}.$$

This may satisfied either through population flows, that is the adjustment of Σ (which possibly reaches one of its bounds, 0 or 1, corresponding to the absorption of a country by the other one), or through asset flows, that is the adjustment of $\bar{B}$. The preferred solution – which is also more consistent with observations for developed countries – is to assume that financial markets adjust $\bar{B}$ to

$$\bar{B}' = \frac{\Sigma(1-\Sigma)(Y_1' - Y_2)}{1-\beta}$$

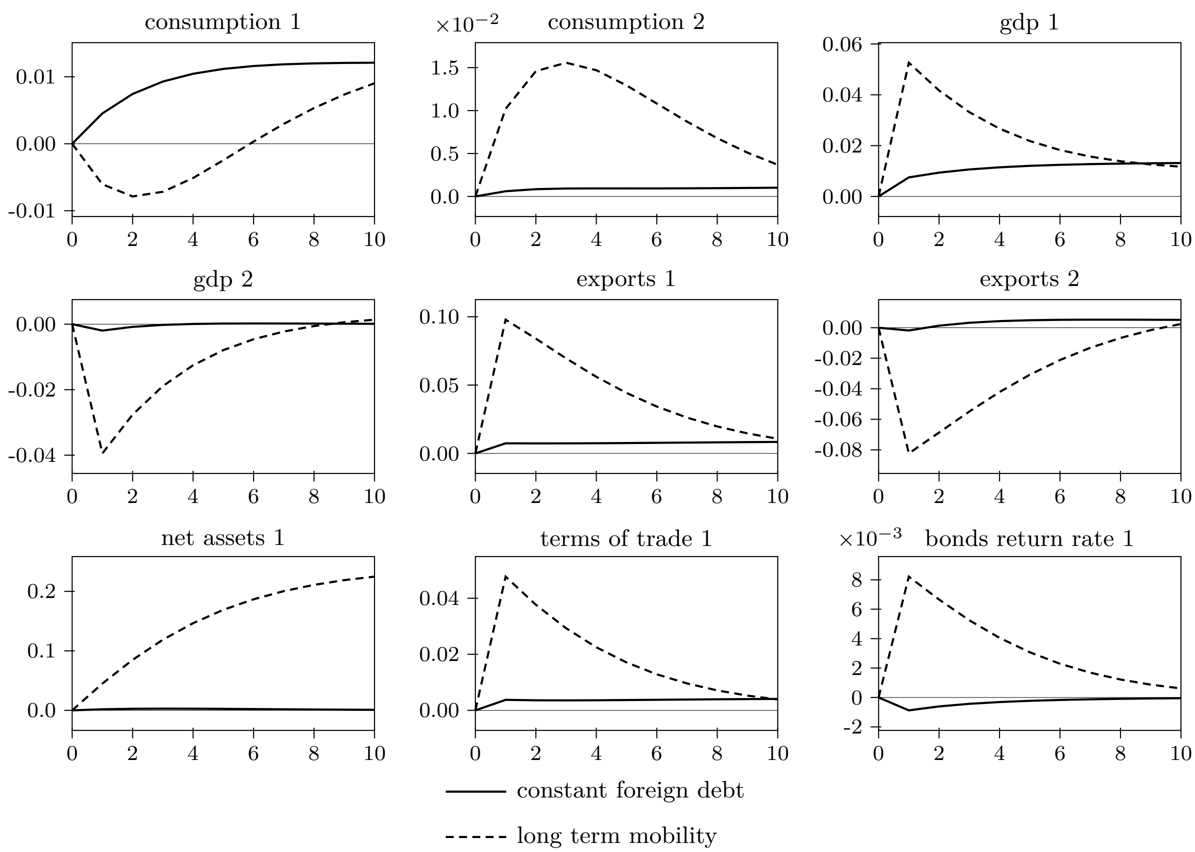
in order to avoid population flows.

Turning back to the more complex model of section 2.10.2, the same variant as in section 2.10.3 is simulated numerically according to this principle. In practice, I adjust the parameter $\bar{B}$ of the risk-premium function at the date of the shock. It is set to the unique value compatible with condition

(2.10.10), which is computed numerically. The dynamic response of the model is represented by the dashed line in Figure 2.23. The impact on the steady state is summarized in Table 2.10.

2.10.5 Simulations and findings

Figure 2.23: Simulation of a permanent social VAT policy



Notes: all variables are shown is percentage deviation from steady state, except net assets, which are given in percentage points of GDP in country 1 (0.01 is 1%). Bonds return rate in country 1 is $\exp\left(-\psi\left(\Sigma b_{1t}-\bar{B}\right)\right)/q_t - 1$.

The most significant difference implied by the long run mobility assumption is that the reaction of trade flows are much larger during the transition towards the new steady state of the economy than with the standard assumption. In that latter case indeed, agents anticipate that any short run improvement in the trade balance will need to be offset by a future deterioration to restore the initial level of foreign debt; the future path of the risk premium they forecast prompts them to make decisions accordingly. Their optimal smooth path is a slight increase in exports in both countries, to reach immediately the neighborhood of the new steady state: exports in country 1 benefit from a minor improvement in the terms of trade, while imports respond positively to the rise in consumption. In that context, the transmission of the policy to the foreign country remains

Table 2.10: Long term effects

	constant foreign debt	long term mobility
consumption 1	+1.20	+1.48
consumption 2	+0.12	-0.12
gdp 1	+1.34	+1.03
gdp 2	+0.00	+0.27
exports 1	+0.90	-0.02
exports 2	+0.46	+1.38
net assets 1	+0.00	+23.92
terms of trade 1	+0.43	+0.06
bonds return rate 1	+0.00	+0.00

subdued. Under the long run mobility assumption, the trade surpluses resulting from the rise in exports and the relative decrease in imports end in an additional accumulation of foreign assets representing more than 20% of GDP in the long term. The country greatly benefits from the tax competitiveness measure, and households lifetime consumption is raised by 1.5%. The magnitude of these effects illustrates that it is essential to allow for possible variations in international trade flows when evaluating a permanent social VAT measure.

Under the long run mobility assumption, domestic consumption is penalized in the short run by the rise in the value added tax. The lower labor tax induces an increase in the wage rate of a smaller magnitude, so the production price of the domestic good decreases. This improves the foreign competitiveness of country 1, and exports grow markedly. The wage increase and wealth effects engendered by lower consumption support labor supply. After a few periods, households in country 1 are wealthier and consume more than before the measure. By contrast, under the standard assumption, the increase in the wage rate almost entirely absorbs the decrease in the labor tax. This considerably limits competitiveness gains, and cancels the negative substitution effects on consumption of the rise in the value added tax.

2.11 Output gap in the interest rate rule

2.11.1 Introduction

This section justifies the inclusion of the non-stationary component of output (stochastic trend) in the measure of output gap used in the Taylor rule of the model of chapter 3. Without this assumption, the monetary policy rule is much less efficient in stabilizing the fluctuations after permanent shocks hit the economy. Technically, these permanent shocks then induce very large cyclical variations in ouput and inflation. Hence, when the model is estimated, the magnitude of stochastic growth is minimized and does not play efficiently its role, which is to capture the unit root components included in observed times series.

2.11.2 Wicksellian interest rate and Taylor rule

Woodford (2001) shows that, in a simple new-Keynesian framework, a Taylor rule should respond to variations in the natural rate of interest, i.e. the real interest rate in the case of perfectly flexible prices, to be consistent with an optimal equilibrium where inflation and the output gap are completely stabilized. In the presence of stochastic growth modelled as a persistent ARIMA process, this "Wicksellian" natural interest rate, defined as $R_t^n = \beta^{-1}\tilde{\lambda}_t / E_t\tilde{\lambda}_{t+1}$ where $\tilde{\lambda}_t$ denotes the marginal utility of consumption that would prevail with flexible prices, is obviously affected positively by permanent technology shocks. This means that the nominal exchange rate should also increase to successfully stabilize inflation and the output gap in that case. In the model, the monetary authority's ability to stabilize inflation after a permanent technology shock depends upon the persistence of technology growth, upon the degree of inertia in the Taylor rule and, obviously upon the value of the coefficient r_Y which is estimated. This mechanism is illustrated below using the basic new-Keynesian model of Woodford (2001), where we assume that the growth factor of the technology shock, denoted by g_t, is governed by an AR(1) process. Using standard notations (y is detrended output, π is inflation, r is the nominal interest factor) and variables in log-deviations from their steady state values, the model is:

$$\begin{cases} y_t = E_t(y_{t+1} + g_{t+1}) - \sigma(r_t - E_t\pi_{t+1}) \\ \pi = \beta E_t\pi_{t+1} + \kappa y_t \\ g_t = \rho_g g_{t-1} + \varepsilon_t \end{cases} .$$

We consider a standard calibration $\sigma = 1$, $\kappa = 0.084$, $\beta = 0.99$ and $\rho_g = 0.6$, and three different interest rate rules: in the first one,

$$r_t = \phi_\pi \pi_t + \phi_y y_t,$$

the monetary authority only reacts to the cyclical variations in output. It is the most simple among standard Taylor rules used in new-Keynesian models, since it assumes that potential output is equal to steady state output instead of computing the level of output that would prevail in the absence of nominal frictions (as in Galí (2008) or Smets and Wouters (2007)). In the second one, the monetary authority follows the "Wicksellian" natural interest rate as suggested by Woodford (2001):

$$r_t = \phi_\pi \pi_t + \phi_y y_t + \frac{1}{\sigma} E_t g_{t+1}.$$

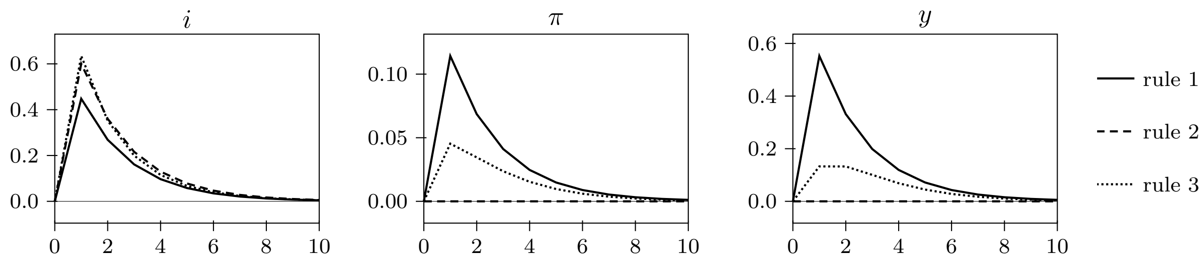
The third one is the one that is used in the model of chapter 3, and is similar to the one used by Christiano et al. (2014). It assumes that the monetary authority reacts to variations in non-detrended output, which is observable, in addition to inflation:

$$r_t = \phi_\pi \pi_t + \phi_y (y_t - y_{t-1} + g_t).$$

It is the most simple extension of rule 1 that accounts for the effect of permanent technology shocks, mitigating their effects on inflation and the cyclical component of output.

Impulse responses are shown in Figure 2.24 for $\phi_\pi = 1.5$ and $\phi_y = 0.5$, and clearly illustrate the discussion above.

Figure 2.24: Responses to a permanent technology shock



2.12 Introducing price markup shocks in a non-linear model

Smets and Wouters (2007) find that price and wage mark-up shocks account for a large part of inflation and wages developments. Therefore, such a mark-up shocks are expected to be useful for the estimation of the present model. If these authors derive manually a log-linearized version of their model before coding it, the introduction of these shocks is problematic when the model has to be written in its non-linear version. The reason is that the first order condition of price setters cannot be written recursively. This section identifies precisely this issue, and proposes a very simple way to introduce it that is strictly equivalent to Smets and Wouters (2003) (which differs from Smets and Wouters (2007) by the use of a less general Dixit-Stiglitz aggregator instead of the one proposed by Kimball (1995)) when the model is approximated at first order (which is the case when the model is estimated).

2.12.1 Variable cost-push shocks

This section derives explicitely the program of wholesalers and intermediate distributors, who face staggered prices à la Calvo (1983), assuming that the elasticity of substitution between intermediate differentiated goods is an exogenous stochastic process.

The production sector includes a continuum of intermediate distributors indexed by $f \in [0, 1]$, producing a differenciated good from an homogeneous input with unit price P_t^Z in period t.[8]. Each firm f sells its production $y(f)$ to the representative wholesaler at a price $P^y(f)$ in a monopolistically competitive market. The later firm aggregates the differenciated goods to produce a quantity of final good, noted Y, which is sold in a perfectly competitive market at a price P^Y. His aggregation

[8]This part uses the notations of the two country model, but applies to the general setup of Smets and Wouters (2003) Hence, variables omit the subscript F.

technology is CES:

$$Y_t = \left(\int_0^1 y_t(f)^{\frac{\theta_{y,t}-1}{\theta_{y,t}}} df \right)^{\frac{\theta_{y,t}}{\theta_{y,t}-1}}$$

where the elasticity of substitution between the differentiated goods θ_y is stochastic.

Y is determined by the demand addressed by consumers. The retailer's program is to chose the optimal mix of goods $y(f)$ in order to satisfy the demand Y, given the prices $P^y(f)$. It yields:

$$y_t(f) = \left(\frac{P_t^y(f)}{P_t^Y} \right)^{-\theta_{y,t}} Y_t$$

The zero-profit condition yields:

$$(P_t^Y)^{1-\theta_{y,t}} = \int_0^1 P_t^y(f)^{1-\theta_{y,t}} df$$

Firms $f \in [0,1]$ set their prices under a Calvo (1983) mechanism: with a constant probability $1 - \xi_y$, a firm can reset optimally her price at date t. Otherwise, they automatically follows the indexation rule:

$$P_t^y(f) = \bar{\pi}^{1-\gamma_y} (\pi_{t-1}^Y)^{\gamma_y} P_{t-1}^y(f) \equiv \Gamma_t P_{t-1}^y(f)$$

where $\pi_t^Y = P_t^Y / P_{t-1}^Y$. Let $V_t(f)$ the value of a firm f that is allowed to reset optimally her price at date t and $W_t(P_{t-1}^y(f))$ the value of a firm that cannot reset her price optimally at date t:

$$V_t(f) = \max_{\tilde{P}_t(f)} \left[\Pi_t(\tilde{P}_t(f)) + \beta E_t \frac{P_t \lambda_{t+1}}{P_{t+1}\lambda_t} \Big((1-\xi_y) V_{t+1}(f) + \xi_y W_{t+1}(\Gamma_{t+1}\tilde{P}_t(f)) \Big) \right]$$

$$W_t(P_t^y(f)) = \Pi_t(P_t^y(f)) + \beta E_t \frac{P_t \lambda_{t+1}}{P_{t+1}\lambda_t} \left((1-\xi_y) V_{t+1}(f) + \xi_y W_{t+1}(\Gamma_{t+1} P_t^y(f)) \right)$$

with $\Pi_t(P_t^y(f))$ the nominal profit of a firm f at date t:

$$\Pi_t(P_t^y(f)) = \left(P_t^y(f) - P_t^Z \right) y_t(f) = \left(\frac{P_t^y(f)}{P_t} - p_t^Z \right) P_t \left(\frac{P_t^y(f)}{P_t^Y} \right)^{-\theta_{y,t}} Y_t$$

The value of a firm does not depend on her past choices and the program associated to the definition of $V(f)$ is identical across firms. Therefore, all firms that can reset their price optimally at a given date t choose the same price $\tilde{P}_t$ and $V_t(f) \equiv V_t$. The first-order condition is:

$$\Pi_t'(\tilde{P}_t) + \beta \xi_y E_t \frac{\lambda_{t+1} P_t}{\lambda_t P_{t+1}} \Gamma_{t+1} W_{t+1}'(\Gamma_{t+1}\tilde{P}_t) = 0$$

with

$$W'_{t+1}(\Gamma_{t+1}\tilde{P}_t) = \Pi'_{t+1}(\Gamma_{t+1}\tilde{P}_t) + \beta\xi_y \frac{\lambda_{t+2}P_{t+1}}{\lambda_{t+1}P_{t+2}}\Gamma_{t+2}W'_{t+2}(\Gamma_{t+1}\Gamma_{t+2}\tilde{P}_t)$$
$$= \Pi'_{t+1}(\Gamma_{t+1}\tilde{P}_t) + E_{t+1}\sum_{k=1}^{\infty}(\beta\xi_y)^k \frac{\lambda_{t+k+1}P_{t+1}}{\lambda_{t+1}P_{t+k+1}}\Gamma_{t+2}\dots\Gamma_{t+k+1}\Pi'_{t+k+1}(\Gamma_{t+1}\dots\Gamma_{t+k+1}\tilde{P}_t)$$

By subsitution into the first order condition:

$$\Pi'_t(\tilde{P}_t) + E_t\sum_{k=1}^{\infty}(\beta\xi_y)^k \frac{\lambda_{t+k}P_t}{\lambda_t P_{t+k}}\Gamma_{t+1}\dots\Gamma_{t+k}\Pi'_{t+k}(\Gamma_{t+1}\dots\Gamma_{t+k}\tilde{P}_t) = 0$$

Let

$$\Gamma_{t,t+k} = \begin{cases} \Gamma_{t+1}\dots\Gamma_{t+k} & \text{if } k \geq 1 \\ 1 & \text{if } k = 0 \end{cases} \qquad \text{and} \qquad \tilde{p}_t = \frac{\tilde{P}_t}{P_t}$$

Given that

$$\Pi'_t(\tilde{P}_t) = (1-\theta_{y,t})\left(\frac{\tilde{P}_t}{P_t^Y}\right)^{-\theta_{y,t}} Y_t + \theta_{y,t}\left(\frac{\tilde{P}_t}{P_t^Y}\right)^{-\theta_{y,t}-1}\frac{p_t^Z}{p_t^Y}Y_t,$$

we have $\forall k \geq 0$

$$\Pi'_{t+k}(\Gamma_{t,t+k}\tilde{P}_t) = (1-\theta_{y,t+k})\left(\frac{\Gamma_{t,t+k}\tilde{P}_t}{P_{t+k}^Y}\right)^{-\theta_{y,t+k}} Y_{t+k} + \theta_{y,t+k}\left(\frac{\Gamma_{t,t+k}\tilde{P}_t}{P_{t+k}^Y}\right)^{-\theta_{y,t+k}-1}\frac{p_{t+k}^Z}{p_{t+k}^Y}$$
$$= (1-\theta_{y,t+k})\left(\frac{\Gamma_{t,t+k}P_t^Y}{P_{t+k}^Y}\right)^{-\theta_{y,t+k}}\tilde{p}_t^{-\theta_{y,t+k}}Y_{t+k} + \theta_{y,t+k}\left(\frac{\Gamma_{t,t+k}P_t^Y}{P_{t+k}^Y}\right)^{-\theta_{y,t+k}-1}\tilde{p}_t^{-\theta_{y,t+k}-1}\frac{p_{t+k}^Z}{p_{t+k}^Y}$$

and the first order condition can be written:

$$E_t\sum_{k=0}^{\infty}(\beta\xi_y)^k \frac{\lambda_{t+k}P_t}{\lambda_t P_{t+k}}\Gamma_{t,t+k}\Pi'_{t+k}(\Gamma_{t,t+k}\tilde{P}_t) = 0$$

$$E_t\sum_{k=0}^{\infty}(\beta\xi_y)^k \frac{\lambda_{t+k}}{\lambda_t}\theta_{y,t+k}\frac{p_{t+k}^Z}{p_{t+k}^Y}Y_{t+k}\left(\Gamma_{t,t+k}\frac{P_t^Y}{P_{t+k}^Y}\right)^{-\theta_{y,t+k}}\tilde{p}_t^{-\theta_{y,t+k}-1}$$
$$= E_t\sum_{k=0}^{\infty}(\beta\xi_y)^k \frac{\lambda_{t+k}}{\lambda_t}(\theta_{y,t+k}-1)Y_{t+k}\left(\Gamma_{t,t+k}\frac{P_t^Y}{P_{t+k}^Y}\right)^{-\theta_{y,t+k}+1}\tilde{p}_t^{-\theta_{y,t+k}}$$

This equation cannot be written recursively because the $\tilde{p}_t$ terms cannot be factorized. Indeed, when θ_y is constant, this equation simplifies to

$$\tilde{p}_t = \frac{\theta_y}{\theta_y - 1}\frac{E_t\sum_{k=0}^{\infty}(\beta\xi_y)^k \frac{\lambda_{t+k}}{\lambda_t}\frac{p_{t+k}^Z}{p_{t+k}^Y}Y_{t+k}\left(\Gamma_{t,t+k}\frac{P_t^Y}{P_{t+k}^Y}\right)^{-\theta_y}}{E_t\sum_{k=0}^{\infty}(\beta\xi_y)^k \frac{\lambda_{t+k}}{\lambda_t}Y_{t+k}\left(\Gamma_{t,t+k}\frac{P_t^Y}{P_{t+k}^Y}\right)^{-\theta_y+1}} \equiv \frac{\theta_y}{\theta_y - 1}\frac{H_{1,t}}{H_{2,t}}$$

And, as

$$\Gamma_{t,t+1+k} = \Gamma_{t+1} \dots \Gamma_{t+k+1} = \Gamma_{t+1}\Gamma_{t+1,t+1+k} = \bar{\pi}^{1-\gamma_y}(\pi_t^Y)_y^\gamma \Gamma_{t+1,t+1+k},$$

we get the recursive forms

$$H_{1,t} = \frac{p_t^Z}{p_t^Y} Y_t + \beta \xi_y E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\bar{\pi}^{1-\gamma_y}(\pi_t^Y)^{\gamma_y}}{\pi_{t+1}^Y} \right)^{-\theta_y} H_{1,t+1}$$

and

$$H_{2,t} = Y_t + \beta \xi_y E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\bar{\pi}^{1-\gamma_y}(\pi_t^Y)^{\gamma_y}}{\pi_{t+1}^Y} \right)^{-\theta_y+1} H_{2,t+1}.$$

Log-linearizing this system yields the following linear equation for inflation:

$$\hat{\pi}_t^Y = \frac{\gamma_y}{1+\beta\gamma_y}\hat{\pi}_{t-1}^Y + \frac{\beta}{1+\beta\gamma_y} E_t \hat{\pi}_{t+1}^Y + \frac{(1-\beta\xi_y)(1-\xi_y)}{\xi_y(1+\beta\gamma_y)} \left(\hat{p}_t^Z - \hat{p}_t^Y \right). \tag{2.12.1}$$

But when $\theta_{y,t}$ is not constant, the equation including infinite sums above is approximated at first order in this form. It yields

$$\hat{\pi}_t^Y = \frac{\gamma_y}{1+\beta\gamma_y}\hat{\pi}_{t-1}^Y + \frac{\beta}{1+\beta\gamma_y} E_t \hat{\pi}_{t+1}^Y + \frac{(1-\beta\xi_y)(1-\xi_y)}{\xi_y(1+\beta\gamma_y)} \left(\hat{p}_t^Z - \hat{p}_t^Y + \frac{1}{1-\bar{\theta}_y}\hat{\theta}_{y,t} \right). \tag{2.12.2}$$

2.12.2 Alternative method to introduce a price markup shock

A clean manner to introduce the markup or cost-push shock is to assume that producers sell the share of their production devoted to the domestic market, which is the domestic good times a dispersion term, denoted by $\nabla_t^y Y_t$ in the model, to another class of monopolistic agents with flexible prices. This group includes a great and constant number of firms indexed by $j \in [0,1]$. The j-th firm differentiate a quantity $z_t(j)$ of goods and sells it at price $P_t^z(j)$ to a great and constant number of competitive firms, which are also another class of agents involved in the production chain. The behavior of the latter firms is represented by the behavior of a representative aggregator with technology

$$\tilde{Z}_t = \left(\int_0^1 z_t(j)^{\frac{\theta_{z,t}-1}{\theta_{z,t}}} dj \right)^{\frac{\theta_{z,t}}{\theta_{z,t}-1}},$$

where the elasticity of substitution between differentiated goods is stochastic. Conversely, the elasticity of substitution θ_y which characterizes the technology of wholesalers, recalled in the previous paragraph is a constant parameter, allowing a recursive formulation of the Phillips curve. The representative aggregator chooses the quantities $z_t(j)$ that minimize his total cost $\int_0^1 P_t^z(j) z_t(j) dj$ subject to technology, which yields

$$\forall j \in [0,1],\ z_t(j) = \left(\frac{P_t^z(j)}{P_t^{\tilde{Z}}} \right)^{-\theta_{z,t}} \tilde{Z}_t,$$

where $P_t^{\tilde{Z}}$ is the price charged to intermediate distributors. The j-th firm purchases $z_t(j)$ out of

$$\nabla_t^y Y_t = \int_0^1 z_t(j)\, dj$$

at price P_t^Z and sets her price $P_t^z(j)$ in order to maximize her profit subject to the demand that is addressed to her

$$\left(P_t^z(j) - P_t^Z\right)\left(\frac{P_t^z(j)}{P_t^{\tilde{Z}}}\right)^{-\theta_{z,t}} \tilde{Z}_t.$$

She repays all profits evenly to households in the form of dividends. This program yields the first order condition

$$P_t^z(j) = \frac{\theta_{z,t}}{\theta_{z,t} - 1} P_t^Z$$

The prices $P_t^z(j)$ and the quantities $z_t(j)$ for $j \in [0, 1]$ are thus equal, so immediately $z_t(j) = \tilde{Z}_t = \nabla_t^y Y_t$ and $P_t^z(j) = P_t^{\tilde{Z}}$. So the only equation that is added to the model is

$$P_t^{\tilde{Z}} = \frac{\theta_{z,t}}{\theta_{z,t} - 1} P_t^Z,$$

where $P_t^{\tilde{Z}}$ is the unit cost incurred by monopolistic intermediate distributors. A straightforward manner to formally introduce the cost-push shock without complicating the structure of the model is hence to replace P_t^Z by $\varepsilon_t^p P_t^Z$ in the Phillips curve equations, where ε_t^p is an exogenous process; an extensive description of the model mentioning the presence of monopolistic agents with flexible prices in the production chain, in addition to those facing staggered prices à la Calvo (1983) is implicit. This is what is done in the model for labor agencies' wage rates, domestic producers and exporters prices.

2.12.3 Calibration of the shock

The variable ε_t^p is the markup factor of the first group of monopolists. It is related to the elasticity of substitution of the differentiated goods $z_t(j)$ by

$$\theta_{z,t} = \frac{\varepsilon_t^p}{\varepsilon_t^p - 1}.$$

When $\theta_{z,t} > 1$, ε_t^p is decreasing of $\theta_{z,t}$; the more substituable the goods are, the smaller are firms' profits. If $\theta_{z,t}$ is smaller than one but positive, then the markup factor becomes negative. Conversely, a situation where the markup factor is lower than one (negative profits) corresponds to a negative elasticity of subsititution. In order to avoid that such case happens, ε_t^p should remain higher than one for likely shocks, that is shocks of small magnitude (a few percents) corresponding to the validity domain of the first order approximation of the model; I assume that the steady state of ε_t is significantly higher that 1, at least 1.1, to reach that goal.

Another constraint is the steady state level of profit margins in the model; following the approach in Smets and Wouters (2003), the total markup rate in the steady state is $1/(\theta_y - 1)$, when it is $(\theta_y + \theta_z - 1)/(\theta_y - 1)/(\theta_z - 1)$ when the shock is introduced via additional monopolistic firms with flexible prices. Considering a particular steady state value, denoted by $\tilde{\theta}_y$, and a variance, denoted by $\tilde{\sigma}_p^2$, desired for θ_y in a model where it is variable. In order to be equivalent to this situation, a model with 2 stacked groups of monopolists as described above should to be calibrated as follows. Consider that the steady state value of ε_p is set to a given value $\bar{\varepsilon}_p$, which corresponds to a steady state θ_z of $\bar{\varepsilon}_p/(\bar{\varepsilon}_p - 1)$. Then the steady state of θ_y is such that $\bar{\varepsilon}_p\theta_y/(\theta_y - 1) = \tilde{\theta}_y/(\tilde{\theta}_y - 1)$, that is

$$\theta_y = \frac{\tilde{\theta}_y}{\tilde{\theta}_y - (\tilde{\theta}_y - 1)\bar{\varepsilon}_p}.$$

Obviously, the existence of such a solution requires that $\bar{\varepsilon}_p$ is chosen lower than $\tilde{\theta}_y/(\tilde{\theta}_y - 1)$.

The variance of ε_t^p should be equal to $1/(1-\tilde{\theta}_y)^2\tilde{\sigma}_p^2$, so that the term $\hat{p}_t^Z - \hat{p}_t^Y$ in equation (2.12.1), which becomes under the specification developed above $\hat{p}_t^{\tilde{Z}} - \hat{p}_t^Y = \hat{p}_t^Z - \hat{p}_t^Y + \hat{\varepsilon}_{p,t}$, is equivalent, in absolute value, to the term $\hat{p}_t^Z - \hat{p}_t^Y + 1/(1 - \bar{\theta}_y)\hat{\theta}_{y,t}$ in equation (2.12.2).

Appendix

2.A Technical appendix to wage rigidity

2.A.1 Derivation of firms program with continuing wages

The value of firms is

$$\begin{aligned}\mathcal{W}_t(N_{t-1}, w_{t-1}) = \max_{v_t, N_t}\Bigg\{ & x_t A_t N_t - q_t v_t \tilde{w}_t - (1-s)N_{t-1}w_{t-1} - c v_t \\ & + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\mathcal{W}_{t+1}\left(N_t, \frac{1}{N_t}\left(q_t v_t \tilde{w}_t + (1-s)N_{t-1}w_{t-1}\right)\right) \\ & + \mu_t((1-s)N_{t-1} + q_t v_t - N_t)\Bigg\}\end{aligned}$$

The first order condition with respect to v_t is

$$-q_t\tilde{w}_t - c + \beta\frac{q_t\tilde{w}_t}{N_t}E_t\frac{\lambda_{t+1}}{\lambda_t}\frac{\partial\mathcal{W}_{t+1}}{\partial w_t} + \mu_t q_t = 0.$$

The first order condition with respect to N_t is

$$x_t A_t + \beta E_t\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\partial\mathcal{W}_{t+1}}{\partial N_t} - \frac{w_t}{N_t}\frac{\partial\mathcal{W}_{t+1}}{\partial w_t}\right) - \mu_t = 0.$$

The enveloppe with respect to N_{t-1} is

$$\frac{\partial\mathcal{W}_t}{\partial N_{t-1}} = -(1-s)w_{t-1} + \beta(1-s)\frac{w_{t-1}}{N_t}E_t\frac{\lambda_{t+1}}{\lambda_t}\frac{\partial\mathcal{W}_{t+1}}{\partial w_t} + \mu_t(1-s).$$

The enveloppe with respect to w_{t-1} is

$$\frac{\partial\mathcal{W}_t}{\partial w_{t-1}} = -(1-s)N_{t-1} + \beta(1-s)\frac{N_{t-1}}{N_t}E_t\frac{\lambda_{t+1}}{\lambda_t}\frac{\partial\mathcal{W}_{t+1}}{\partial w_t}.$$

Rearranging yields

$$\frac{\partial\mathcal{W}_t}{\partial w_{t-1}} = (1-s)N_{t-1}\left[-1 + \beta\frac{1}{N_t}E_t\frac{\lambda_{t+1}}{\lambda_t}\frac{\partial\mathcal{W}_{t+1}}{\partial w_t}\right] \tag{2.A.1}$$

$$\frac{\partial\mathcal{W}_t}{\partial N_{t-1}} = (1-s)\left(\frac{c}{q_t} + \tilde{w}_t - w_{t-1}\right) + \beta(1-s)\frac{w_{t-1}-\tilde{w}_t}{N_t}E_t\frac{\lambda_{t+1}}{\lambda_t}\frac{\partial\mathcal{W}_{t+1}}{\partial w_t} \tag{2.A.2}$$

$$\frac{c}{q_t} = x_t A_t - \tilde{w}_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{W}_{t+1}}{\partial N_t} + \beta \frac{\tilde{w}_t - w_t}{N_t} E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{W}_{t+1}}{\partial w_t}. \tag{2.A.3}$$

The surplus of the firm associated with a successful match is the partial derivative of $\mathcal{W}_t$ with respect to $m_t = q_t v_t$. It is

$$J_t = -\tilde{w}_t + \beta \frac{\tilde{w}_t}{N_t} E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{W}_{t+1}}{\partial w_t} + \mu_t$$

$$J_t = x_t A_t - \tilde{w}_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{\partial \mathcal{W}_{t+1}}{\partial N_t} + \frac{\tilde{w}_t - w_t}{N_t} \frac{\partial \mathcal{W}_{t+1}}{\partial w_t} \right]. \tag{2.A.4}$$

2.A.2 Derivation of households surplus

The value of households is

$$\begin{aligned}
\mathcal{V}_t(B_{t-1}, N_{t-1}, w_{t-1}) = \max_{C_t, B_t} \Bigg\{ & \log C_t - N_t \Gamma + \beta E_t \mathcal{V}_{t+1} \left(B_t, N_t, \frac{1}{N_t} \left[f_t(1-(1-s)N_{t-1}) \tilde{w}_t + (1-s) N_{t-1} w_{t-1} \right] \right) \\
& + \lambda_t \left(\frac{B_{t-1} R_{t-1}}{P_t} + f_t(1-(1-s)N_{t-1}) \tilde{w}_t + (1-s) N_{t-1} w_{t-1} + (1-N_t) z + T_t + div_t \right. \\
& \left. - C_t - \frac{B_t}{P_t} \right) \\
& + \nu_t \left[(1-s) N_{t-1} + f_t(1-(1-s)N_{t-1}) - N_t \right] \Bigg\}.
\end{aligned}$$

The enveloppe with respect to N_{t-1} is

$$\frac{\partial \mathcal{V}_t}{\partial N_{t-1}} = \beta(1-s) \frac{w_{t-1} - f_t \tilde{w}_t}{N_t} E_t \frac{\partial \mathcal{V}_{t+1}}{\partial w_t} + \lambda_t (1-s)(w_{t-1} - f_t \tilde{w}_t) + \nu_t (1-s)(1-f_t).$$

The enveloppe with respect to w_{t-1} is

$$\frac{\partial \mathcal{V}_t}{\partial w_{t-1}} = \beta(1-s) \frac{N_{t-1}}{N_t} E_t \frac{\partial \mathcal{V}_{t+1}}{\partial w_t} + \lambda_t (1-s) N_{t-1}.$$

The first order condition with respect to N_t is

$$-\Gamma + \beta E_t \frac{\partial \mathcal{V}_{t+1}}{\partial N_t} - \beta \frac{w_t}{N_t} E_t \frac{\partial \mathcal{V}_{t+1}}{\partial w_t} - \lambda_t z = \nu_t$$

Rearranging yields

$$\frac{\partial \mathcal{V}_t}{\partial w_{t-1}} = (1-s) N_{t-1} \left[\lambda_t + \beta \frac{1}{N_t} E_t \frac{\partial \mathcal{V}_{t+1}}{\partial w_t} \right]. \tag{2.A.5}$$

$$\begin{aligned}
\frac{\partial \mathcal{V}_t}{\partial N_{t-1}} = (1-s) \Bigg[& \lambda_t \left(w_{t-1} - f_t \tilde{w}_t - (1-f_t) \left(z + \frac{\Gamma}{\lambda_t} \right) \right) + \beta (1-f_t) E_t \frac{\partial \mathcal{V}_{t+1}}{\partial N_t} \\
& - \beta \frac{f_t(\tilde{w}_t - w_t) + w_t - w_{t-1}}{N_t} E_t \frac{\partial \mathcal{V}_{t+1}}{\partial w_t} \Bigg].
\end{aligned} \tag{2.A.6}$$

So the surplus of households associated with a successful match is the partial derivative of $\mathcal{V}_t$ with respect to $m_t = f_t(1 - (1 - s)N_{t-1})$. It is

$$W_t = \beta \frac{\tilde{w}_t}{N_t} E_t \frac{\partial \mathcal{V}_{t+1}}{\partial w_t} + \lambda_t \tilde{w}_t + \nu_t$$

$$W_t = \lambda_t(\tilde{w}_t - z) - \Gamma + \beta E_t \left[\frac{\partial \mathcal{V}_{t+1}}{\partial N_t} + \frac{\tilde{w}_t - w_t}{N_t} \frac{\partial \mathcal{V}_{t+1}}{\partial w_t} \right] \tag{2.A.7}$$

2.A.3 Nash bargaining

The wage $\tilde{w}_t$ is chosen to maximize $(W_t/\lambda_t)^\delta J_t^{1-\delta}$, which implies

$$(1 - \delta) \frac{\partial J_t}{\partial \tilde{w}_t} W_t = -\delta \frac{\partial W_t}{\partial \tilde{w}_t} J_t$$

$$(1 - \delta) \left(1 - \beta \frac{1}{N_t} E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial \mathcal{W}_{t+1}}{\partial w_t} \right) W_t = \delta \left(\lambda_t + \beta \frac{1}{N_t} E_t \frac{\partial \mathcal{V}_{t+1}}{\partial w_t} \right) J_t. \tag{2.A.8}$$

Chapter 3

A DSGE model of France in the euro area

3.1 Introduction

For forecasting and quantitative policy analysis, many institutions either use large macroeconometric models, or structural DSGE models. The former class of models have the advantage of a correct fit to the data, but are strongly disconnected from economic theory and the estimation of their parameters is disputed by the Lucas (1976) critique. The latter models are constrained by economic theory and are internally consistent. Such tools are well-suited to study and understand the main propagation mechanisms of a small number of exogenous disturbances to the economy. But this approach faces serious limitations. First, as economists generally use small focused models to answer to specific questions, the same model cannot be used to simulate a wide range of variants. Any conclusion drawn from such analysis is hence conditional to a set of specific assumptions that need to be examined carefully. Second, such models may fail to replicate some moments of the data when they are calibrated on the basis of long term considerations or microeconomic behaviors, and especially when they have insufficient frictions and lags. These are some of the reasons why many institutions have developed and estimated DSGE models, incorporating real and nominal frictions, for forecasting and policy analysis.

The size of such models and the number of time series used for their estimation are subject to computational limitations and the standard is medium-scale models typified by Christiano et al. (2005), Smets and Wouters (2007) or Christiano et al. (2014). These closed economy models still ignore some important variables and mechanisms, including the interactions with foreign economies, or the fluctuations in unemployment. Moreover, when it represents the economy of a large area, such as the euro zone, it misses the heterogeneity of its constituent regions. Some larger models have been built in that direction. One is the EAGLE of the ECB, which is a multi-country DSGE model (see Gomes et al. (2012)). Another is the BEMOD of the Bank of Spain (see Andrés et al. (2006)), which is also a two-country model of a monetary union. But these tools are calibrated so they are not fully confronted with the data. Rabanal (2009) estimates a DSGE model of the euro area and Spain, but his analysis focuses on inflation dynamics only, and the estimation uses a smaller number of observable variables and involves a smaller number of parameters.

This chapter develops a DSGE model of two symmetrical countries which form a monetary union. By contrast with the works cited above, the present model includes many nominal and real frictions and all parameters are estimated using Bayesian techniques on a large dataset. A baseline version ignores the presence of real imperfections in the financing of firms and in the labor market. A financial accelerator and search and matching in the labor market are then optionally included. The two countries are France and the rest of the euro area, which is assumed to be an homogeneous region. They trade with each other and with the rest of the world. The latter is not modelled explicitely; its variables that affect the economy of the euro area, such as demand of goods and prices, are exogenously determined by simple autoregressive processes. The rest of the world produces and supplies inelastically both non-energy goods and oil. For simplicity, it is assumed to have one currency, namely the USD. Labor productivity includes an exogenous stochastic trend with a deterministic drift, which is common to both regions and real variables of the rest of the world.

The model is estimated using Bayesian technique to fit a set of quarterly series of both France and the euro area. Although the estimation procedure uses a first order approximation of the model, it accounts for all the long term restrictions that arise from the model.

3.2 Notations

The following formal conventions are adopted in equations throughout this chapter. Variables for France are indexed with a F whereas variables for the rest of the euro area are indexed with an E. Flows from France towards the rest of the euro area are indexed with F/E and conversely. Variables of the rest of the world are superscripted with a $\star$. Aggregate variables of the euro area are written without index. Although parameter values in France are obviously different from those in the rest of the euro area, all parameters are written without index unless the region which they refer to is unclear. The specifications of the model that are symmetrical for country F and country E are described only for country F. The model assumes a constant number of infinitely lived households; the population of the euro area is normalized to 1 and those of country F and E are respectively Σ_F and Σ_E, with $\Sigma_F + \Sigma_E = 1$. With this normalization, real variables in the model represent per capita averages in the corresponding areas. Appendix (3.B) gives an exhaustive list of the detrended equations of the model ; again, block equations are only reported for country F and single equations (trade, monetary policy, euro area aggregates and rest of the world) are given separately. Appendix (3.A) summarizes the baseline model and the complete model in diagrams.

3.3 The basic model

3.3.1 Production

Six sectors including each a large and constant number of identical firms are involved in the supply of final goods in the domestic market: equipment providers, intermediate goods producers, intermediate goods distributors, wholesalers, final goods producers and final retailers. Separating these sectors is helpful to incorporate several inputs in production together with nominal and real frictions in a simple way; for that reason, partitioning production of final goods into several sectors has become a standard practice in the modeling literature.

At the bottom of the stack, competitive equipment providers rent capital and purchases oil from the rest of the world in order to provide equipment services to competitive producers. Producers combine this equipment with labor services and sell their output to monopolistic intermediate distributors, who differentiate their production and face nominal frictions à la Calvo. Differentiated goods are aggregated by competitive wholesalers. Then, competitive final good producers use domestic production from wholesalers and imported non-oil goods and provides final goods that are used for investment and government expenditures. Last, on the top of the stack, final retailers aggregate the remaining final goods with imported oil to form the final consumption good, and operate in a competitive market. In addition to entering the final consumption basket, final goods can be directly used for government consumption or in the form of investment by households. At each stage of the production process, production flows are aggregated using CES aggregators.

Oil imports enter in the economy in two differents points: the oil purchased by equipment providers captures fossile energy consumed by the production process in the form of intermediate consumption. It includes in particular transportation costs. The oil entering directly in the consumption basket represents households' fuel and gas expenditures, mainly for transport and heating. The latter flow is useful in particular to capture the immediate impact of oil price movements on the consumption price index, because it is not subject to price rigidity.

Final retailers

Starting from the top of the stack, the representative final retailer assembles the final consumption good. She aggregates a quantity $C_{F,t}^{ne}$ of final (non-energy) goods and a quantity $C_{F,t}^{o}$ of oil using the CES technology

$$C_{F,t} = \left[a_o^{\frac{1}{s_o}} \left(C_{F,t}^{ne} \right)^{\frac{s_o-1}{s_o}} \right.$$
$$\left. + (1-a_o)^{\frac{1}{s_o}} \left(C_{F,t}^{o} \left[1 - \frac{\chi^o}{2} \left(\frac{C_{F,t}^{o} C_{F,t-1}}{C_{F,t-1}^{o} C_{F,t}} - 1 \right)^2 \right] \right)^{\frac{s_o-1}{s_o}} \right]^{\frac{s_o}{s_o-1}}$$

Changes in the share of oil in consumption are subject to a quadratic adjutment cost that is justified by the fact that households use oil with specific equipments (such as cars and boilers), whose replacement may be costly. With adjustment costs in CES aggregators (in the expression above as in the other sectors described in what follows), the model generates realistic hump-shaped responses of inputs to changes in their relative prices. In the expressions of these adjustment costs, the lagged variables are assumed to be externalities for the corresponding firms, so their optimization problem is static. This amounts to assuming that the cost incurred by a specific firm depends on the difference between the share of oil in the final good produced by her in the current period and the share of oil in the aggregated production during the previous period: in the presence of a large number of similar atomistic firms, she ignores the impact of her individual decisions on total production.

She chooses the relative quantities of final non-energy goods and oil that minimize her total cost under technological constraint. She takes as given the demand of final consumption good that is addressed to her by households and the unit prices of her inputs expressed in euro, respectively $P^{ne}_{F,t}$ and $S_t P^{\star}_{o,t}$, with $P^{\star}_{o,t}$ being oil price expressed in USD and S_t the euro/USD nominal exchange rate. The first order condition resulting from this program implies that the ratio of non-energy final goods to oil used in final consumption is a log-linear decreasing function of their relative prices, with $-s_o$ the slope, plus a term that derives from the adjustment cost. The zero-profit condition of final retailers determines the level of the aggregated consumption price as a growing function of the aggregated price of non-energy good and the price of oil.

Final good producers

The representative final good producer also uses a CES technology to aggregate a quantity $Y_{F,t}$ of domestic production and a quantity $M_{F,t}$ of imported non-energy goods to produce a quantity

$$H_{F,t} = \Bigg[(a_y)^{\frac{1}{s_y}} \left(Y_{F,t}\right)^{\frac{s_y-1}{s_y}} + (1-a_y)^{\frac{1}{s_y}} \left(M_{F,t} \left[1 - \frac{\chi^y}{2} \left(\frac{M_{F,t} H_{F,t-1}}{M_{F,t-1} H_{F,t}} - 1 \right)^2 \right] \right)^{\frac{s_y-1}{s_y}} \Bigg]^{\frac{s_y}{s_y-1}}$$

of final goods. Again, changes in the share of imports in non-energy final goods imply a quadratic adjustment cost, which captures inertia in distribution networks and habits in the demand for foreign goods. As for final retailers, the share of imports in non-energy final goods at date $t-1$ in the expression of the adjustment cost refer to an aggregate share and is an externality for the final good producer. Her program consists in optimizing the relative quantities $Y_{F,t}$ and $M_{F,t}$ in function of their unit prices $P^Y_{F,t}$ and $P^M_{F,t}$. Non-energy goods $H_{F,t}$ are purchased by the government of country F, up to $G_{F,t}$, by households in the form of investment good, up to $I_{F,t}$ and by final retailers, up to $C^{ne}_{F,t}$. Perfect competition implies that final good producers make no profit, which determines the price $P^{ne}_{F,t}$.

Wholesalers

The representative wholesaler assemblies the differentiated productions of intermediate distributors indexed by $f \in [0,1]$ with a Dixit and Stiglitz (1977) aggregation technology, into a quantity

$$Y_{F,t} = \left(\int_0^1 y_{F,t}(f)^{\frac{\theta_y - 1}{\theta_y}} \, df \right)^{\frac{\theta_y}{\theta_y - 1}}$$

of homegeneous domestic production, where $y_{F,t}(f)$ denotes the quantity purchased to the f-th intermediate distributor. She chooses the demand addressed to each intermediate distributor $f \in [0,1]$ in order to minimize her total cost

$$\int_0^1 y_{F,t}(f) P^y_{F,t}(f) \, df$$

under technological constraint, given the different prices $P^y_{F,t}(f)$, $f \in [0,1]$ of distributors and the demand $Y_{F,t}$ addressed to her by final good producers. Next, perfect competition implies that the profit of the representative wholesaler is zero, which determines the wholesale price $P^Y_{F,t}$. The demand addressed to the f-th intermediate distributor is found to be a log-linear decreasing function of her relative price $P^y_{F,t}(f)/P^Y_{F,t}$, with $-\theta_y$ the slope, and is proportional to the total quantity of domestic production $Y_{F,t}$.

Intermediate distributors

Monopolistic intermediate distributors are the profit-making agents of the production chain. They set their prices under a Calvo mechanism. The f-th intermediate distributor purchases a quantity $y_{F,t}(f)$ of the homogeneous good produced by producers. She differenciates this production so that it is imperfectly substitutable to other intermediate distributors' productions. Her nominal profit at a date t is

$$\left(P^y_{F,t}(f) - P^Z_{F,t} \right) y_{F,t}(f),$$

where $P^Z_{F,t}$ is the unit price of producers' homogeneous output. During every period t, she is exogenouly informed whether she is able to reoptimize its unit price $P^y_{F,t}$ or not. This possibility is given independantly accross time and distributors with the same constant probability $1 - \xi_y$. With probability ξ_y, she cannot reoptimize her price, which is instead automatically indexed to a convex combination of past aggregated inflation of the wholesale price and long run inflation:

$$\frac{P^y_{F,t}(f)}{P^y_{F,t-1}(f)} = \Gamma^y_{F,t-1/t} \quad \text{where} \quad \Gamma^y_{F,t/t+j} = \begin{cases} 1 & \text{if } j = 0 \\ \left(\prod_{i=0}^{j-1} \pi^Y_{F,t+i} \right)^{\gamma_y} \bar{\pi}^{j(1-\gamma_y)} & \text{if } j \geq 1 \end{cases}$$

When she is able to reoptimize her price, she sets $P^{y}_{F,t}(f) = \tilde{P}^{y}_{F,t}$ that maximizes the discounted sum of her expected profits

$$\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\xi_y)^j \frac{\lambda_{F,t+j} P_{F,t}}{\lambda_{F,t} P_{F,t+j}} \left(\Gamma^{y}_{F,t/t+j} \tilde{P}^{y}_{F,t} - P^{Z}_{F,t+j} \right) y_{F,t+j}(f)$$

subject to the demand addressed by the final good firm

$$y_{F,t+j}(f) = \left(\frac{P^{y}_{F,t+j}(f)}{P^{Y}_{F,t+j}} \right)^{-\theta_y} Y_{F,t+j}.$$

Given that all distributors are identical ex ante, and that the objective function of distributors who are allowed to reoptimize is purely forward looking, the optimal price $\tilde{P}^{y}_{F,t}$ is the same for all of them. This program yields the dynamic equations for the optimal relative price

$$\tilde{p}^{y}_{F,t} = \frac{\tilde{P}^{y}_{F,t}}{P^{Y}_{F,t}}$$

as a function of the production price $P^{Z}_{F,t}$. Then, the zero-profit condition of wholesalers determines the aggregate inflation of wholesale prices $\pi^{Y}_{F,t}$. Finally, wholesale prices are subject to a cost-push shock $\varepsilon^{p}_{F,t}$, introduced as described in section 2.12.

Producers

The representative producer uses labor and equipment services to produce a quantity $Z_{F,t}$ of an homogeneous good with a Cobb-Douglas technology

$$Z_{F,t} = V^{\alpha}_{F,t} \left(A_t \varepsilon^{a}_{F,t} \mathcal{L}_{F,t} \right)^{1-\alpha}$$

where $\mathcal{L}_{F,t}$ is homogeneous labor supplied by labor agencies and $V_{F,t}$ is a flow of equipment services, which corresponds to the provision of facilities, machines and oil to industry. A_t is a stochastic trend in labor productivity. It is described by an ARIMA(1,1,0) process with drift, which transmits a random walk component and a deterministic growth trend to the real variables of the model. The variable $\varepsilon^{a}_{F,t}$ is an exogenous stationary disturbance to labor productivity, described by AR(1) process. The representative producer sells his output to intermediate distributors and to intermediate exporters in perfectly competitive markets, so that

$$Z_{F,t} = \int_0^1 y_{F,t}(f)\,df + \int_0^1 x_{F,t}(f)\,df$$

where $x_{F,t}(f)$ denotes the quantity purchased by the f-th intermediate exporter, which are indexed by $f \in [0,1]$ like intermediate distributors. His program consists in choosing the quantity $V_{F,t}$ of equipment services and the level of labor $\mathcal{L}_{F,t}$ in order to minimize his total cost

$$P^V_{F,t}V_{F,t} + \tau^L W_{F,t}\mathcal{L}_{F,t}$$

under technological constraint. The parameter τ^L refers to the employer social contribution factor in country F. The production price $P^Z_{F,t}$ is determined by the zero-profit condition of producers.

Equipment providers

Finally, the representative equipment provider lies at the bottom of the production chain; she produces equipment services out of capital and oil with a CES technology

$$\begin{aligned} V_{F,t} = \Bigg[& a_v^{\frac{1}{s_v}} \left(z_{F,t}K_{F,t-1}\right)^{\frac{s_v-1}{s_v}} \\ & + (1-a_v)^{\frac{1}{s_v}} \left(O_{F,t}\left[1 - \frac{\chi^v}{2}\left(\frac{O_{F,t}V_{F,t-1}}{O_{F,t-1}V_{F,t}} - 1\right)^2\right]\right)^{\frac{s_v-1}{s_v}} \Bigg]^{\frac{s_v}{s_v-1}}. \end{aligned}$$

Capital $K_{F,t-1}$ is rented to households and oil $O_{F,t}$ is directly imported from the rest of the world. The capital accumulated at the end of period t is used in production during the period $t+1$. Only a fraction $z_{F,t}$ of capital, decided in each period t by households, who are capital owners, is used for production. His program is to choose the optimal quantities of utilized capital $z_{F,t}K_{F,t-1}$ and of oil $O_{F,t}$ that minimize his total cost

$$P_{F,t}r^k_{F,t}z_{F,t}K_{F,t-1} + S_tP^\star_{o,t}O_{F,t},$$

under technological constraint and given the unit price of oil converted to euro ($S_tP^\star_{o,t}$) and the capital return rate demanded by households ($P_{F,t}r^k_{F,t}$ in nominal terms). As previously, in the expression of the adjustment cost that results from changes in the share of energy in equipment services, $O_{F,t-1}$ and $V_{F,t-1}$ refer to aggregate quantities, so they are externalities for the representative equipment provider. Equipment services are sold to the representative producer at price $P^V_{F,t}$ that is determined by her zero-profit condition.

3.3.2 Households

Country F is populated by a large and constant number of households, indexed by h. Under a convenient normalization, the range of possible values for h is the continuum $[0,1]$. In period t, the h-th household consumes a quantity $C_{F,t}(h)$ of an homogeneous good purchased at unit price $P_{F,t}$ and supplies a number of hours worked $L_{F,t}(h)$ remunerated at a real hourly wage rate $w^m_{F,t}$. Her utility is an increasing function of consumption and a decreasing function of hours worked.

Formally, her objective function at date t is

$$U_{F,t}(h) = \mathrm{E}_t \left[\sum_{j=0}^{\infty} \beta^j \varepsilon^{\beta}_{F,t+j} u_{F,t+j}(h) \right],$$

where:

$$u_{F,t}(h) = \frac{\left[C_{F,t}(h) - \eta C_{F,t-1}\right]^{1-\sigma_c}}{1-\sigma_c} \exp\left[\frac{\sigma_c - 1}{1+\sigma_l} \left(L_{F,t}(h)\right)^{1+\sigma_l}\right].$$

$\beta < 1$ is the long run intertemporal discount factor common to the world economy, $\varepsilon^{\beta}_{F,t}$ is an exogenous disturbance which affects the value of present relatively to future consumption. It is called impatience shock and captures unexpected changes in households intertemporal arbitrage decisions. The comtemporaneous utility $u_{F,t}(h)$ belongs to the class of functions that are compatible with steady state growth according to King et al. (1988). It is non-separable in consumption and labor and implies that the marginal utility of consumption is an increasing function of labor supply and the marginal disutility of labor increases with consumption.

Households are subject to external habit formation: they care about the lagged value of aggregate consumption

$$C_{F,t} = \int_0^1 C_{F,t}(h)\, dh.$$

These "catching up with the Joneses" preferences (see Abel (1990)) generate persistence in the dynamics of aggregate consumption and considerably improves the model's empirical performance, in particular because it capture the hump-shaped response of spending and inflation to shocks, as emphasized by Fuhrer (2000). The parameter η measures the importance of past aggregate consumption relatively to current individual consumption in preferences.

In a baseline version of the model without financial frictions, households accumulate capital, which is directly made available to the production sector in exchange for the payment of rents at rate $r^k_{F,t}$. They trade riskless bonds denominated in euro and paying a nominal interest rate $R_t - 1$ with households of country E. They also trade riskless bonds denominated in the currency of the rest of the world (namely the USD) and paying $R^{\star}_t - 1$ with households of the rest of the world. Financial markets are incomplete so that households do not share their country specific risks internationally. Each household owns the same share in monopolistic profit-making firms of the country; therefore, dividends $Div_{F,t}$ are paid evenly to all of them. Likewise, they share evenly a lump-sum payment $T_{F,t}$ from their government. Consumption good is a composite of the non-energy good assembled in country F and oil imported from the rest of the world. Investment $I_{F,t}$ is only made of the non-energy good ; $p^{ne}_{F,t}$ refers to the price of the non-energy good relative to the price of consumption and is therefore also the relative price of investment. Capital is accumulated by the h-th household according to

$$K_{F,t}(h) = (1-\delta) K_{F,t-1}(h) + \varepsilon^{I}_{F,t} \left[1 - \frac{\varphi}{2}\left(\frac{I_{F,t}(h)}{I_{F,t-1}(h)} - (1+g)\right)^2\right] I_{F,t}(h). \tag{3.3.1}$$

This technology includes a stochastic disturbance $\varepsilon^I_{F,t}$, which captures technological changes specific to new investment goods. The presence of this shock is suggested by the fact that sector-neutral productivity changes cannot explain the negative comovements between the relative price of new equipment and new equipment investment. Its importance in business cycle fluctuations is demonstrated by Greenwood et al. (2000) for the US. Capital accumulation also includes a quadratic adjustment cost, which implies that households are incited to avoid abrupt changes in their investment plans, because they generate inefficiencies in capital installation. This specification, popularized by Christiano et al. (2005) in the DSGE litterature, amplifies the persistence of aggregate investment. It is central to models' ability to replicate the hump-shaped responses of investment following shocks. The term $1+g$, where g is the growth rate of the deterministic drift in world technology, is necessary to make the adjustment cost function compatible with a long run equilibrium.

Capital utilization is variable and is chosen by households so that the marginal gains from renting more utilized capital to firms is equal in equilibrium to the marginal costs induced by more intensive use of capital. The literature considers two forms for these costs. Greenwood et al. (1988) plead for a variable capital depreciation rate, being an increasing, convex function of its rate of utilization, and reflecting the user cost of capital. With this specification, they show that investment, labor productivity and also, under some restrictions regarding preferences and technology, consumption, respond positively to investment-specific technological shocks, contrasting with RBC model assuming constant capital utilization. Other authors, as Christiano et al. (2005), Smets and Wouters (2007) or Christiano et al. (2014), assume that households incur a capital utilization cost expressed in units of consumption goods. We follow this route on the grounds that the variable capital depreciation option has undesirable effects when financial frictions are introduced in the model. This issue is extensively discussed in section 2.2.

During period t, the hth household makes consumption, labor supply, savings (bonds and investment in productive capital) and capital utilization rate decisions in order to maximize his intertemporal utility function $U_{F,t}(h)$ subject to both his budget constraint, given below, and the capital accumulation rule:

$$\begin{aligned}\tau^C C_{F,t}(h) + p^{ne}_{F,t} I_{F,t}(h) = \tau^R \Bigg(& \tau^W w^m_{F,t} L_{F,t}(h) + r^k_{F,t} z_{F,t}(h) K_{F,t-1}(h) + \frac{Div_{F,t}}{P_{F,t}} \\ & + \frac{B_{E/F,t-1}(h)}{P_{F,t}} - \frac{B_{E/F,t}(h)}{P_{F,t} R_t \Psi_{F,t}} + \frac{S_t B^\star_{F,t-1}(h)}{P_{F,t}} - \frac{S_t B^\star_{F,t}(h)}{P_{F,t} R^\star_t \Psi^\star_{F,t}} \Bigg) \\ & - p^{ne}_{F,t} a(z_{F,t}(h)) K_{F,t-1} + T_{F,t} + \Xi_{F,t}\end{aligned}$$

τ^C, τ^R and τ^W refer respectively to the consumption tax factor, the income tax factor and the employee social contribution factor. S_t is the euro/USD nominal exchange rate. Real capital utilization costs are represented by $a(z_t(h))K_{F,t-1}$, where a is an increasing convex function. It correspond to the quantity of the final non-oil good that is purchased by households for the upkeep of capital. We choose the following specification:

$$a(z_{F,t}(h)) = \bar{a}\left[\exp(\omega(z_{F,t} - \bar{z}_F)) - 1\right],\ \bar{a} > 0,\ \omega > 0.$$

The parameter ω determines the inverse of the elasticity of the utilization rate with respect to changes in the capital rental rate. $\Psi_{F,t}$ and $\Psi^{\star}_{F,t}$ represent financial intermediation costs paid by households on their bonds revenues. They depend of the aggregate net asset foreign positions in bond markets as follows:

$$\Psi_{F,t} = \bar{\Psi}_F \exp\left[-\psi\left(\frac{B_{E/F,t}}{P_{F,t}\bar{\mathcal{Y}}_F} - \bar{b}\right)\right]$$

$$\Psi^{\star}_{F,t} = \bar{\Psi}^{\star}_F \exp\left[-\psi^{\star}\left(\frac{S_F B^{\star}_{F,t}}{P_{F,t}\bar{\mathcal{Y}}_F} - \bar{b}^{\star}\right)\right].$$

The levels of financial intermediation costs are considered as externalities by households. This specification implies that households have an incentive to get out of debt when the level of the external debt of country F is above its long run level and, conversely, to dispose of excess assets. These costs, amounting to

$$\Xi_{F,t} = \frac{B_{E/F,t}}{P_{F,t}R_t}\left(\frac{1}{\Psi_{F,t}} - 1\right) + \frac{S_t B^{\star}_{F,t}}{P_{F,t}R^{\star}_t}\left(\frac{1}{\Psi^{\star}_{F,t}} - 1\right),$$

are repaid evenly to households in the form of dividends. Their optimal demand for foreign bonds denominated in foreign currency yields an uncovered interest rate parity condition, which determines the dynamics of the real exchange rate with the rest of the world: in equilibrium, differences between the real interest rate (including financial intermediation costs) in the rest of the world and the domestic real interest rate are compensated by exchange rate movements such that foreign and domestic bonds are equally attractive. There is a consensus on the empirical failure of the uncovered interest rate parity condition (see Isard (2006)). However, this assumption is standard in open economy models in the absence of a better theoretical model to link current to expected future exchange rates. In that case, the UIP condition is generally adjusted by an exogenous exchange risk premium which capture unexpected changes in exchange rates. In the present model, the foreign interest variable $R^{\star}$ is assumed to include this premium, which is measured in the data by imposing the observation of changes in the nominal exchange rate and the domestic real interest rate, but leaving the foreign interest rate unobserved.

A well-known issue in New Open Economy Macroeconomics is that International Real Business Cycle models assuming incomplete international financial markets are not stationary; their deterministic steady state is not unique. A number of model specifications have been suggested to induce stationarity (see Schmitt-Grohé and Uribe (2003) for small open economy models or Boileau and Normandin (2008) for two-country models), among which debt-elastic interest rate premia. The latter is the most commonly used and it is also the case in the present model, in the form of the financial intermediation costs postulated above. By increasing the marginal cost of debt to the rest of the world, trade deficits in one country of the monetary union with respect to the rest of the world yield a depreciation of the euro which raises exports price competitiveness and makes imported goods more expensive. This mechanism operates in the opposite direction in the case of trade surpluses, and ensures the stability of Euro area foreign trade. The presence of financial intermediation costs inside the monetary union works similarly: they impact inflation differentials and therefore competitiveness. More precisely, trade deficits in one country with respect to the other one raise financial intermediation costs in the country, which reduces households demand and yields deflationary pressures. Exports are more competitive, while imports from the other region are relatively more expensive, restoring the trade balance. In the present model, the stationarity of the model (i.e. the absence of unit root) requires at least that either ψ_F or ψ_E be strictly positive and that both $\psi_F^\star$ and $\psi_E^\star$ be strictly positive. However, as in Christoffel et al. (2008), the size of ψ and $\psi^\star$ is believed to be very low so that the evolution of net foreign assets has only a small impact on the exchange rate and trade in the short run.

All households are identical and their labor is remunerated at the same hourly rate. The symmetry of the problem implies that they all make the same decisions in equilibrium.

3.3.3 Baseline version of the labor market

A baseline version of the model assumes nominal wage rigidities, as in Smets and Wouters (2007), in a labor market without real frictions, that is where firms needs of labor are satisfied by workers labor supply at each date. In order to introduce nominal frictions, that are essential for the model's empirical properties as emphasized by Christiano et al. (2005), the natural labor supply behavior of the representative households need to be amended. Indeed, workers are not hired directly by producers; labor is supplied to the production sector by a large and constant number of monopolistic unions, indexed by $u \in [0, 1]$. Although each household $h \in [0, 1]$ may have specific skills, she delegates the responsibility of negociating wages to unions, who take advantage of the market power of households that results from their particular skills. In return, they ensure a common real hourly wage rate $w_{F,t}^m$ and a common working time $L_{F,t}$ to all workers. They repay evenly their surplus to them in the form of dividends. The presence of unions is justified by the fact that preferences are non-separable in leisure and consumption. If households themselves made use of their monopoly power, they would have heterogeneous wage rates and labor supplies. With perfect risk-sharing, all households are guaranteed to have the same marginal utility of consumption in equilibrium. With non-separable preferences and heterogeneous labor supply, consumption expenditures would hence also be heterogeneous in equilibrium and the model would be hard to aggregate. When heterogeneity only concerns unions, households are perfectly identical so they make identical decisions. For clarity, the model also assumes that the aggregation of differentiated labor supplies is made by a large and constant number of competitive labor agencies instead of producers.

Several aspects of this approach to the labor market are debatable. If heterogeneity in workers skills is a sensible assumption, the presence of “unions” that differentiate identical workers has no clear analog in actual economies. In additions, these unions apply a “markup” to the wage rate that results from households preferences, that is minus the ratio of marginal disutility of labor to marginal utility of consumption, so the wages paid by firms are not the ones that motivate labor supply.

These limitations are a good reason for building to a more realistic description of the labor market in an extension to this baseline version. It is presented in section 3.5.

Labor agencies

The representative labor agency organizes the differentiated labor times proposed by unions to supply a number $\mathcal{L}_{F,t}$ of homogeneous hours worked to producers. She uses a Dixit and Stiglitz (1977) aggregation technology

$$\mathcal{L}_{F,t} = \left(\int_0^1 l_{F,t}(u)^{\frac{\theta_w - 1}{\theta_w}} \, du \right)^{\frac{\theta_w}{\theta_w - 1}},$$

where $l_{F,t}(u)$ refers to the quantity of differentiated labor supplied by the u-th union at a nominal hourly rate of $W_{F,t}(u)$. Her program is to choose the demand addressed to each union $u \in [0,1]$ in order to minimize her total cost $\int_0^1 l_{F,t}(u) W_{F,t}(u) \mathrm{d}u$ under technological constraint and given labor demand emanating from producers. Next, as it operates in a perfectly competitive market, a zero-profit condition determines the level of the aggregate real hourly wage rate $w_{F,t}$, which is the unit price of hours worked paid by producers. The demand addressed to the u-th union is found to be a log-linear decreasing function of her relative wage rate $W_{F,t}(u)/W_{F,t}$, with $-\theta_w$ the slope, and is proportional to total labor demand $\mathcal{L}_{F,t}$.

Unions

Monopolistic unions set wages under a Calvo mechanism. The u-th union pays a quantity $l_{F,t}(u)$ of labor to households and sell it to labor agencies, at price $W_{F,t}(u)$, including a markup. Her nominal profit at a date t is

$$\left(W_{F,t}(u) - W^m_{F,t} \right) l_{F,t}(u),$$

where $W^m_{F,t}$ is the nominal wage rate paid to households. During every period t, she is exogenouly informed whether she is able to reoptimize its wage rate $W_{F,t}(u)$ or not. This possibility is given independantly accross time and unions with the same constant probability $1-\xi_w$. With probability ξ_w, she cannot reoptimize her wage, which is instead automatically indexed to a convex combination of past consumption price inflation and long run inflation as well as to long run productivity growth:

$$\frac{W_{F,t}(u)}{W_{F,t-1}(u)} = \Gamma^w_{F,t-1/t} \quad \text{where} \quad \Gamma^w_{F,t/t+j} = \begin{cases} 1 & \text{if } j = 0 \\ (1+g)^j \left(\prod_{i=0}^{j-1} \pi_{F,t+i} \right)^{\gamma_w} \bar{\pi}^{j(1-\gamma_w)} & \text{if } j \geq 1 \end{cases}$$

When she is able to reoptimize her wage rate, she sets $W_{F,t}(u) = \widetilde{W}_{F,t}$ that maximizes the discounted sum of her expected profits:

$$\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\xi_w)^j \frac{\lambda_{F,t+j} P_{F,t}}{\lambda_{F,t} P_{F,t+j}} \left(\Gamma^w_{F,t/t+j} \widetilde{W}_{F,t} - W^m_{F,t+j} \right) l_{F,t+j}(u)$$

subject to the demand addressed by labor agencies

$$l_{F,t+j}(u) = \left(\frac{W_{F,t+j}(u)}{W_{F,t+j}} \right)^{-\theta_w} \mathcal{L}_{F,t+j}.$$

Given that all unions are identical ex ante, and that their objective function when they are allowed to reoptimize is purely forward looking, the optimal wage $\widetilde{W}_{F,t}$ is the same for all of them. This program yields the dynamic equations for the optimal real wage rate

$$\widetilde{w}_{F,t} = \frac{\widetilde{W}_{F,t}}{P_{F,t}}$$

as a function of the wage rate paid to households $W^m_{F,t}$. Then, the zero-profit condition of labor agencies determines the aggregate real wage rate $w_{F,t}$. At a given date t, unions may have different wage rates but those which have reoptimized for the last time at the same date $t-s$ apply the same wage rate $\Gamma^w_{F,t-s/t} \widetilde{W}_{F,t-s}$. Given the properties of the Calvo lottery, the dynamics of the aggregate wage rate can be solved regardless of the distribution of wages accross unions. Finally, wages are subject to a cost-push shock $\varepsilon^w_{F,t}$ (also called wage markup shock in the literature), introduced as described in section 2.12.

3.3.4 International trade

Country F exports a fraction of its production to country E and another fraction to the rest of the world. It also imports non-oil goods from both country E and the rest of the world, and oil from the rest of the world.

Export prices are rigid and are set separately from prices for goods aimed at the domestic market, but do not differ depending on their destination. This situation does not correspond to the pricing-to-market assumption with respect to exports of the whole euro area. Pricing-to-market, where export prices are chosen in the currency of the destination market, is standard in two country models. However, Christoffel et al. (2008) show that producer-currency-pricing is preferred by the data on the export side in a small open economy model of the euro area; this option would have also increased the size of the model and computational time, for a doubtful gain. Exports to country E compete with country E's imports from the rest of the world, with which they are imperfectly substitutable. Exports to the rest of the world compete with exports from country E and aimed at the rest of the world, with which they are also imperfectly substitutable. In country F, two sectors including each a large and constant number of identical agents are involved in exports: monopolistic intermediate exporters purchase goods from domestic producers and differentiate them. They face nominal frictions à la Calvo (1983). Differentiated goods are then aggregated by competitive wholesale exporters who sell a fraction of their output to domestic importers in country E and the remaining fraction to final exporters. Final exporters represent a third sector involved in exports, but they are common to countries F and E. They aggregate exports to the rest of the world from both country into a final export good which is consumed abroad.

Import prices are also rigid and are consistent with the pricing-to-market assumption: importers with market power set their prices in the currency of the target market, that is euro. Hence, the pass-through of movements in the exchange rate into import prices and into domestic prices is incomplete in the short run. This is a widespread assumption in the literature (see for example Kollmann (2001)), given the empirical failure of the Law of One Price (see Knetter (1993)). However, import price rigidity results from adjustment costs with lags, whereas it is generally in the form of a Calvo (1983) lottery in medium scale models (see Christoffel et al. (2008)). This is justified by the fact that our data suggests that there may be lags in the transmission of foreign competitors prices to the domestic demand for imported products. The specification chosen in the model allows for a higher degree of freedom in the estimation of the transmission of prices to import movements, while keeping the incomplete pass-through of exchange rate movements property. More precisely, imports transit two sectors common to both countries and located in the rest of the world, and one sector in each country. All sectors include a large and constant number of identical agents. The first two are monopolistic intermediate importers, who purchase goods from non-modelled producers in the rest of the world and incur a price adjustment costs, and competitive final importers. In each country, competitive domestic importers buy imperfectly substitutable goods from the final importer in the rest of the world and from wholesale exporters from the other country. They assemble a final non-energy imported good for final good producers.

Exports

The representative final exporter of the euro area serves as a middleman between wholesale exporters of both countries F and E and the rest of the world. He aggregates exports from F with exports from E, which are supposed to be imperfectly substitutable, using the CES technology

$$X_t = \left[a_x^{\frac{1}{s_x}} \left(\Sigma_E X_{E,t}^{\star} \right)^{\frac{s_x - 1}{s_x}} + (1 - a_x)^{\frac{1}{s_x}} \left(\Sigma_F X_{F,t}^{\star} \right)^{\frac{s_x - 1}{s_x}} \right]^{\frac{s_x}{s_x - 1}} .$$

In this manner, the model accounts for price competition in foreign markets between exporters from country F and exporters from country E. The final importer chooses optimally the demand addressed to each country in function of their relative prices, given the total demand X_t addressed to her by the rest of the world. The latter demand X_t is assumed to be determined by the following ad hoc equation:

$$\frac{D_t \varepsilon_{D,t}}{X_t \left[1 - \Omega_{F,t}\left(\frac{X_t}{D_t}\right)\right]} = \left(\frac{p_{X,t}}{p^\star_{X,t}\left[1 - \Omega_{F,t}\left(\frac{X_t}{D_t}\right) - \frac{X_t}{D_t}\Omega'_{F,t}\left(\frac{X_t}{D_t}\right)\right]}\right)^{\mu^\star}$$

In this equation, X_t depends positively on the final demand for goods and services in the rest of the world D_t, which is exogenous, consistently with the assumption that the euro area is a small open economy. More, X_t depends positively with elasticity $\mu^\star$ on the price competitiveness of euro area exporters with respect to the rest of the world. Price competitiveness is measured as the inverse of the ratio of the aggregated price of euro area exports to the aggregated price of the competitors of european exporters in foreign markets denominated in euro $P^\star_{X,t}$, which is also exogenous. In the expression above, prices are normalized as follows

$$p_{X,t} = \frac{P_{X,t}}{S_t P^\star_t} \text{ and } p^\star_{X,t} = \frac{P^\star_{X,t}}{S_t P^\star_t}.$$

The function

$$\Omega_{F,t}\left(\frac{X_t}{D_t}\right) = \frac{a^\star}{2}\left(\frac{X_t D_{t-1}}{D_t X_{t-1}} - 1\right)^2$$

may be considered like resulting from an adjustement cost. It slows down the response of exports to changes in the demand emanating from the rest of the world or in response to price shocks. This specification is also assumed by Christoffel et al. (2008). $\varepsilon_{D,t}$ is an exogenous disturbance. It captures changes in European market shares in the rest of the world that are not caused by price competitiveness variations.

Next, the representative wholesale exporter aggregates the productions of the intermediate exporters of country F. He buys the quantity $x_{F,t}(f)$ of f-type good at unit price $P^x_{F,t}(f)$ on a monopolistically competitive market. His technology is Dixit-Stiglitz-type so that the differentiated goods produced by the intermediate exporters are imperfectibly substitutable:

$$X_{F,t} = \left(\int_0^1 x_{F,t}(f)^{\frac{\theta_x - 1}{\theta_x}}\, df\right)^{\frac{\theta_x}{\theta_x - 1}}$$

She chooses the demand $x_{F,t}(f)$ addressed to each intermediate firm $f \in [0,1]$ in order to minimize her total cost

$$\int_0^1 P^x_{F,t}(f) x_{F,t}(f)\, df$$

under technological constraint and subject to the demand addressed to her by the final exporter. He sells, at the same unit price, a quantity $X_{F/E,t}$ of his production to the domestic importer of country E, and a quantity $X^\star_{F,t}$ to the final exporter of the euro area.

Last, monopolistic intermediate exporters set their prices under a Calvo mechanism. The f-th intermediate exporter purchases a quantity $x_{F,t}(f)$ of the homogeneous good produced by producers. She differenciates this production so that it is imperfectly substitutable to other intermediate exporters' productions. Her nominal profit at a date t is

$$\left(P^x_{F,t}(f) - P^Z_{F,t}\right) x_{F,t}(f),$$

where $P^Z_{F,t}$ is the unit price of producers' homogeneous output. During every period t, she is exogenouly informed whether she is able to reoptimize its unit price $P^x_{F,t}$ or not. This possibility is given independantly accross time and distributors with the same constant probability $1 - \xi_x$. With probability ξ_x, she cannot reoptimize her price, which is instead automatically indexed to a convex combination of past aggregated inflation of the wholesale price and long run inflation:

$$\frac{P^x_{F,t}(f)}{P^x_{F,t-1}(f)} = \Gamma^x_{F,t-1/t} \quad \text{where} \quad \Gamma^x_{F,t/t+j} = \begin{cases} 1 & \text{if } j = 0 \\ \left(\prod_{i=0}^{j-1} \pi^X_{F,t+i}\right)^{\gamma_x} \bar{\pi}^{j(1-\gamma_x)} & \text{if } j \geq 1 \end{cases}$$

When she is able to reoptimize her price, she sets $P^x_{F,t}(f) = \tilde{P}^x_{F,t}$ that maximizes the discounted sum of her expected profits, which are repaid evenly to domestic households,

$$\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\xi_x)^j \frac{\lambda_{F,t+j} P_{F,t}}{\lambda_{F,t} P_{F,t+j}} \left(\Gamma^x_{F,t/t+j} \tilde{P}^x_{F,t} - P^Z_{F,t+j}\right) x_{F,t+j}(f)$$

subject to the demand addressed by wholesale exporters

$$x_{F,t+j}(f) = \left(\frac{P^x_{F,t+j}(f)}{P^X_{F,t+j}}\right)^{-\theta_x} X_{F,t+j}.$$

Given that all distributors are identical ex ante, and that the objective function of distributors who are allowed to reoptimize is purely forward looking, the optimal price $\tilde{P}^x_{F,t}$ is the same for all of them. This program yields the dynamic equations for the optimal relative price

$$\tilde{p}^x_{F,t} = \frac{\tilde{P}^x_{F,t}}{P^X_{F,t}}$$

as a function of the production price $P^Z_{F,t}$. Then, the aggregate inflation of export prices $\pi^X_{F,t}$ is determined by the zero profit condition of wholesale exporters. Finally, export prices are subject to a cost-push shock $\varepsilon^x_{F,t}$, introduced as described in section 2.12.

Imports

Finally, I describe in what follows the programs of agents involved in imports in the model. In country F, the representative domestic importer aggregates a quantity $\frac{\Sigma_E}{\Sigma_F} X_{E/F,t}$ of goods imported from country E at a price $P^X_{E,t}$ with a quantity $M^\star_{F,t}$ of non oil goods imported from the rest of the world at a price $P_{M,t}$ expressed in euros. $X_{E/F,t}$ is the amount of exports from country E to country F per inhabitant of country E. His technology is CES:

$$M_{F,t} = \left[(a_m)^{\frac{1}{s_m}} \left(\frac{\Sigma_E}{\Sigma_F} X_{E/F,t} \right)^{\frac{s_m-1}{s_m}} \right.$$
$$\left. + (1-a_m)^{\frac{1}{s_m}} \left(M^{\star}_{F,t} \left[1 - \frac{\chi^m}{2} \left(\frac{M^{\star}_{F,t} M_{F,t-1}}{M^{\star}_{F,t-1} M_{F,t}} - 1 \right)^2 \right] \right)^{\frac{s_m-1}{s_m}} \right]^{\frac{s_m}{s_m-1}} .$$

As for other firms involved in the production process, changes in the share of imports from the rest of the world in total imports induce a quadratic adjustment cost, which reflects inertia in distribution networks and habits in the demand for foreign goods. Again, the share of imports from the rest of the world in total imports at date $t-1$ in the expression of the adjustment cost is an externality for the importer. She chooses $\frac{\Sigma_E}{\Sigma_F} X_{E/F,t}$ and $M^{\star}_{F,t}$ in order to minimize her cost under technological constraint. The total imports of country F are sold to the final good producer at a price $P^M_{F,t}$ determined by her zero-profit condition.

Non oil imports from the rest of the world transit through two other sectors located outside the euro area before reaching domestic importers in country F or in country E. These firms are assumed to be owned by the households of the rest of the world. The representative final importer aggregates the differentiated goods sold by intermediate importers into a quantity of non oil goods M_t. He uses a Dixit-Stiglitz aggregation technology

$$M_t = \left(\int_0^1 m_t(i)^{\frac{\theta_{m,t}-1}{\theta_{m,t}}} di \right)^{\frac{\theta_{m,t}}{\theta_{m,t}-1}} ,$$

where $m_t(i)$ is the quantity of i-type good purchased to the i-th intermediate importer, with the index $i \in [0,1]$, in a monopolistically competitive market, and $\theta_{m,t}$ is an exogenous markup shock on import prices. Intermediate importers sell their productions at possibly different prices $P_{M,t}(i)$ expressed in euros. As explained in section 2.7, this *Local Currency Pricing* setting ensures that the exchange rate pass-through is incomplete in the short run. The final importer chooses the demand addressed to each intermediate importer $i \in [0,1]$ in order to minimize its total cost $\int_0^1 m_t(i) P_{M,t}(i)\, di$ under technological constraint and subject to the demand addressed to her, which is the sum of extra euro area imports in country F, $\Sigma_F M^{\star}_F$, and in country E, $\Sigma_E M^{\star}_E$.

Intermediate importers have market power; they can make profits that are redistributed to the households of the rest of the world. The ith intermediate importer purchases a quantity $m_t(i)$ of an homogeneous non oil good directly to producers from the rest of the world at a price $P^{\star}_{M,t}$ expressed in foreign currency. She differentiates it and sells $m_t(i)$ to the final importer at price $P_{M,t}(i)$ expressed in euro. She faces a price adjustment cost which depends on the difference between her particular price chosen for the current period and the lagged value of the aggregate import price, so her nominal profit at date t expressed in foreign currency is[1]

$$\left(\frac{P_{M,t}(i)}{S_t} - P^{\star}_{M,t}\right) m_t(i) - \frac{a_m}{2}\left(\frac{P_{M,t}(i)}{\bar{\pi} P_{M,t-1}} - 1\right)^2 \frac{P_{M,t}}{S_t} M_t.$$

Her program consists in choosing her price $P_{M,t}(i)$ that maximizes her profit given the demand addressed to her

$$m_t(i) = \left(\frac{P_{M,t}(i)}{P_{M,t}}\right)^{-\theta_{m,t}} M_t.$$

The symmetry of the problem implies that all intermediate importers choose the same $P_{M,t}(i)$, which is equal to the price of final importers $P_{M,t}$ thanks to the zero-profit condition of the latter.

3.3.5 Government and monetary authority

Budget balance

Government expenditures in country F include the consumption of a quantity $G_{F,t}$ of non-oil goods assembled by final goods producers, and lump-sum transfers to households $T_{F,t}$. They are financed by distorsive taxes (VAT, households income tax, employee social contributions and employer social contributions). The tax factors are constant over time and government expenditures are exogenous. They are determined by an AR(1) process.

Interest rate rule

The nominal interest factor for the whole euro area (country F and country E) evolves according to the following standard Taylor rule:

$$R_t = R_{t-1}^{\rho_R}\left[\bar{R}\left(\frac{\pi_{t-1}}{\bar{\pi}}\right)^{r_\pi}\left(\frac{\pi_t}{\pi_{t-1}}\right)^{r_{\Delta\pi}}\left(\frac{\mathcal{Y}_t}{\mathcal{Y}_{t-1}}\right)^{r_Y}\right]^{1-\rho_R}\varepsilon_{R,t}.$$

It reflects the reaction of the monetary authority to both inflation and a measure of the output gap that simply consists of the variation in the euro area GDP. With this specification, monetary policy reacts to permanent technology shocks. This avoids strong positive responses of inflation to this shock that would arise if monetary policy reacted to changed in the cyclical component of GDP only.

Last, $\varepsilon_{R,t}$ is an exogenous disturbance to the automatic reaction of the nominal interest rate. It captures discretionary monetary policy decisions as well as any event affecting the determination of

[1]There are several reason for assuming adjustment costs instead of price rigidity a la Calvo. From a computational point of view, the Calvo approach augments the size of the model. Next, both nominal rigidities can replicate the same dynamic behavior of prices; their implications differ mainly from a normative point of view, but that is not an issue here since foreign prices are exogenous. Last, since only import competitors prices are observed ($P^{\star}_M$), and not import prices (P_M), the estimated rigidity is impacted by a number of aspects of the transmission of foreign prices to imported quantities; specifically, the estimation would measure an unrealistically large Calvo probability, in particular because import competitors inflation rate is positively correlated to changed in euro area imports in the data.

the interest rate that is orthogonal to inflation and GDP fluctuations. It is described by an AR(1) equation in the model.

The intra-euro area peg

Second, the monetary authority imposes a peg regime (*i.e.* stability of the nominal exchange rate) between country F and country E so that they make up a monetary union. If

$$\Phi_{F/E,t} = \frac{S_{F/E,t} P_{F,t}}{P_{E,t}}$$

is the real exchange rate between country F and country E, this condition is

$$\frac{\Phi_{F/E,t}}{\Phi_{F/E,t-1}} = \frac{\pi_{F,t}}{\pi_{E,t}} \frac{S_{F/E,t}}{S_{F/E,t-1}} = \frac{\pi_{F,t}}{\pi_{E,t}}.$$

3.3.6 Market clearing and equilibrium

In equilibrium, all markets clear. The market clearing condition in the non-oil final goods markets implies that que production of the final goods producers equals the demand for investment, capital upkeep, government expenditures and non-oil consumption goods, that is

$$H_{F,t} = I_{F,t} + G_{F,t} + C^{ne}_{F,t} + \bar{a} \left[\exp(\omega(z_{F,t} - \bar{z}_F)) - 1 \right] K_{F,t-1}.$$

Production $Z_{F,t}$ equals the total demand of intermediate distributors and of intermediate exporters, so

$$Z_{F,t} = \int_0^1 y_{F,t}(f)\, df + \int_0^1 x_{F,t}(f)\, df$$

The demands of the wholesalers and of the wholesale exporter for f-type goods are respectively given by $y_{F,t}(f) = \left(\frac{P^y_{F,t}(f)}{P^Y_{F,t}} \right)^{-\theta_y} Y_{F,t}$ and $x_{F,t}(f) = \left(\frac{P^x_{F,t}(f)}{P^X_{F,t}} \right)^{-\theta_x} X_{F,t}$. This implies that

$$Z_{F,t} = \nabla^y_{F,t} Y_{F,t} + \nabla^x_{F,t} X_{F,t} \quad \text{with} \quad \nabla^y_{F,t} = \int_0^1 \left(\frac{P^y_{F,t}(f)}{P^Y_{F,t}} \right)^{-\theta_y} df$$
$$\text{and} \quad \nabla^x_{F,t} = \int_0^1 \left(\frac{P^x_{F,t}(f)}{P^X_{F,t}} \right)^{-\theta_x} df$$

The link between aggregate production and demand components is thus affected by the degree of price heterogeneity across intermediate distributors and exporters as measured by $\nabla^y_{F,t}$ and $\nabla^x_{F,t}$. These two variables can be written recursively and cancels out when the model is approximated at order 1 following a perturbation approach.

Households' aggregate supply of utilized physical capital equals equipment provider's demand. In the same way, labor supply from the labor agency equals the labor demand of the producer. Aggregated labor supply from households equals aggregated labor demand from unions. As all households supply the same number of hours worked, we have

$$\int_0^1 L_{F,t}(h)\, dh = L_{F,t} = \int_0^1 l_{F,t}(u)\, du.$$

The demand of the labor agency for u-type of labor is

$$l_{F,t}(u) = \left(\frac{W_{F,t}(u)}{W_{F,t}}\right)^{-\theta_s} \mathcal{L}_{F,t}.$$

Substituting into the market clearing condition yields:

$$L_{F,t} = \nabla^w_{F,t}\mathcal{L}_{F,t} \quad \text{with} \quad \nabla^w_{F,t} = \int_0^1 \left(\frac{W_{F,t}(u)}{W_{F,t}}\right)^{-\theta_s} du$$

Again, the link between households' aggregate labor supply and aggregate labor demand is affected by the degree of wage heterogeneity accross unions as measured by $\nabla^w_{F,t}$. The latter can be written recursively and cancels out when the model is approximated at order 1.

Clearing in the intra-zone bonds market yields

$$\Sigma_F B_{E/F,t} + \Sigma_E B_{F/E,t} = 0.$$

The clearing condition of international financial markets requires that the total trade balance of the country is financed by external debts contracts. Changes in households' bonds holding must thus verify

$$\frac{S_t B^\star_{F,t}}{R^\star_t} - S_t B^\star_{F,t-1} + \frac{B_{E/F,t}}{R_t} - B_{E/F,t-1} = \text{trade surplus.}$$

Finally, the real GDP of country F is defined as the sum of real final demand components, formally

$$\mathcal{Y}_{F,t} = C_{F,t} + I_{F,t} + G_{F,t} + X_{F/E,t} + X^\star_{F,t} - \frac{\Sigma_E}{\Sigma_F} X_{E/F,t} - M^\star_{F,t} - O_{F,t} - C^o_{F,t}.$$

3.3.7 Rest of the world

Variables of the rest of the world's economy are exogenous with respect to the economy of the euro area. This is the case of the foreign interest factor $R^\star_t$, which, as aforementioned, plays the role of a shock to the nominal exchange rate between the euro area and the rest of the world, and is modeled as an AR(1). Other variables of the rest of the world economy which impact the euro area in the model include 4 prices labelled in foreign currency: the "world gdp price" $P^\star_t$, which is the reference level used to compute relative prices and the real exchange rate in the model, the production price of non-oil imported goods $P^\star_{M,t}$, the price of foreign competitors to euro area exporters in foreign markets $P^\star_{X,t}$, and oil price $P^\star_{o,t}$. A condition for stationarity in two country models is that all inflation rates have the same long run level. But price levels are integrated of order 1 and may have different stochastic trends in each country. Hence, the nominal exchange rate is also non-stationary and its stochastic trend is determined by the difference in prices levels in the two countries so that the real exchange rate and the model are stationary. This constrains all foreign prices to share the same stochastic trend.

In the model, the "world gdp price" or, in short, world price, includes a random walk. Put differently, world inflation $\pi_t^\star$ is modeled as a stationary stochastic process. Shocks to non-oil imports price, export competitors price and oil price only affect the corresponding relative prices. The relative prices expressed in foreign currency of non oil imported goods $p^\star_{M,t} = P^\star_{M,t}/P^\star_t$ and of euro area exporters' competitors in foreign markets $p^\star_{X,t} = P^\star_{X,t}/P^\star_t$ are respectively modeled by an ARMA(1,1) and a AR(1) process.[2] The equation for the relative oil price $p^\star_{o,t} = P^\star_{o,t}/P^\star_t$ is detailed below. The causes of any shock hitting specifically one of these three prices are implicitely assumed to be transitory and reversible, so it eventually reverts back to the level it would have had without the shock. All factors that affect permanently world prices are summarized in the shock to world inflation $\eta_{\pi^\star}$.

In order to avoid biased estimates of the role of foreign shocks, some ad hoc relationships are postulated between foreign variables, which are expected to be approximations of the ones that would result from a more structural model of the rest of the world. First, oil prices are impacted by world demand, so that the oil price innovation η_O should be interpreted as an oil supply shock. Next, world inflation $\pi^\star$ is impacted by both oil price shocks and world demand shocks. In particular, this assumption rules out depreciations of the EUR/USD exchange rate that would be caused by positive oil price shocks hitting only the euro area economy. Formally, the equations for the relative price of oil expressed in foreign currency and for world inflation are

$$p^\star_{o,t} = \left(p^\star_{o,t-1}\right)^{\rho_O} \exp\left(\sigma_O \eta_{O,t} + \kappa \eta_{D,t}\right), \qquad \eta_O \sim N(0,1),$$

and

$$\pi^\star_t = \left(\pi^\star_{t-1}\right)^{\rho_{\pi^\star}} \bar{\pi}^{1-\rho_{\pi^\star}} \exp\left(\sigma_{\pi^\star} \eta_{\pi^\star,t} + \varrho \eta_{O,t} + \gamma \eta_{D,t}\right), \qquad \eta_{\pi^\star} \sim N(0,1).$$

Last, the deviation of world demand D_t to world trend technology A_t is an ARMA(1,1):

$$\frac{D_t}{A_t} = \left(\frac{D_{t-1}}{A_{t-1}}\right)^{\rho_D} \bar{D}^{1-\rho_D} \exp\left(\sigma_D \left(\eta_{D,t} + \vartheta_D \eta_{D,t-1}\right)\right), \qquad \eta_D \sim N(0,1).$$

3.3.8 Euro area aggregates

Euro area real variables are the sum of the corresponding variables for country F and country E, weighted by their respective populations. This specification disregards the loss of additivity of volumes that results from chain-linking: in Eurostat's quarterly accounts series used for the estimation, chain-linked volumes for member states do not add up to euro area volumes. The same approximation is made in the model's definition of total imports as the sum of oil and non-oil real imports. This issue is left for future research.

An exception is euro area exports X_t, which is a CES aggregate of country F and country E

[2] I have adopted ARMA processes for import competitors prices and world demand because they better replicate some dynamic properties of the data than AR equations.

exports as explained above.

Because intermediate importers and the final importer are assumed to be located in the rest of the world, euro area imports are defined as the output of the final importer M_t rather than the sum of purchases of monopolistic intermediate importers $\int_0^1 m_t(i)\, di$. It equals the the sum of country F and country E imports.

Finally, euro area consumption and domestic production inflation factors π_t and $\pi_{Y,t}$ are defined as the weighted geometric means of country F and country E corresponding inflation factors.

3.4 Financial frictions

This section introduces financial frictions in the model, in the form of a financial accelerator similar to Bernanke et al. (1999). The decision for capital stock is made by entrepreneurs who face a premium on their cost of external financing; this premium results from banks' imperfect information about entrepreneurs idiosyncratic productivity. It is affected negatively by the accumulated net worth of entrepreneurs and is thus decreased by a succession of positive aggregated shocks to the economy. Therefore, such positive shocks stimulate investment more than in the model without frictions. This mechanism allows financial imperfections to play a role in the transmission of standard shocks, but it also adds financial shocks to the model which could help accounting for observed macro data. Finally, it makes possible to enrich our estimation dataset with additional time series, namely stock prices and outstanding loans.

The model assumes that two classes of agents are involved in financing investment in each country: a representative perfectly competitive bank and a big number of *ex ante* identical entrepreneurs. Households are still accumulating capital from investment but do not rent it directly to equipment providers; instead they supply capital to entrepreneurs. They are also depositing savings ($\tilde{B}_F$ in country F) to the bank. The remuneration rate of bank deposits is the same as the remuneration rate of intra-euro area bonds in equilibrium, that is $R\Psi_F$ in country F. As for bonds, the risk-premium Ψ_F is paid to financial intermediaries and repaid evenly to households in the form of dividends. We also assume no cross-country interactions between financial sectors of the two countries: banks of country F only collects deposits from local households and only lends to local entrepreneurs.

During each period, entrepreneurs may purchase capital accumulated by households, at market price $P_F Q_F$, using both internal funds, that is to say their accumulated profits from previous periods, and external funds borrowed from the bank. At the beginning of the next period, when this stock of capital can be used for production, they face a privately observed idiosyncratic shock ω_F that affects their operative level of capital. Then they decide its optimal utilization rate z_F and rent it to equipment providers who use it for production – which is what households did in the baseline version of the model. Last, they sell it back to households, after depreciation at constant rate δ. The shock ω_F reflects entrepreneurs' performance in installing capital, and results in either an improvement or a deterioration of purchased capital. It is assumed to be log-normally distributed with mean 1 (which corresponds to neither improvement nor deterioration of capital) and time-varying standard deviation σ_F, which is known one period in advance by agents. ω_F is also serially uncorrelated, independant of entrepreneurs' net worth and it is not observed by the bank. In actual economies, such memoriless shocks can be analogized to minor innovations that entrepreneurs add to their production and that can tackle demand at some times and turn out to be unsuccessful at other times, because market needs evolve over time. In what follows, $f_{F,t}$ and $F_{F,t}$ denote respectively the density function and the CDF of the idiosyncratic shock ω_F of period $t+1$ (denoted $\omega_{F,t+1}$) in country F. The following subsections describe the modifications in households decisions resulting from the presence of these new actors, the way capital is financed by entrepreneurs, implying either surpluses or bankruptcies, the role of banks and, last, the optimal choices made by entrepreneurs.

3.4.1 Households

Households' budget constraint in country F at date t is changed to the following:

$$\begin{aligned}\tau^C C_{F,t} + p^{ne}_{F,t} I_{F,t} \leq \tau^R \Bigg(& \tau^W w^m_{F,t} L_{F,t} + \frac{Div_{F,t}}{P_{F,t}} \\ & + Q_{F,t} K_{F,t} - (1-\delta)\, Q_{F,t} K_{F,t-1} + \frac{\tilde{B}_{F,t-1} R_{t-1} \Psi_{F,t-1}}{P_{F,t}} - \frac{\tilde{B}_{F,t}}{P_{F,t}} \\ & + \frac{B_{E/F,t-1}}{P_{F,t}} - \frac{B_{E/F,t}}{P_{F,t} R_t \Psi_{F,t}} + \frac{S_t B^\star_{F,t-1}}{P_{F,t}} - \frac{S_t B^\star_{F,t}}{P_{F,t} R^\star_t \Psi^\star_{F,t}} \Bigg) + T_{F,t} + \Xi_{F,t}\end{aligned} \tag{3.4.1}$$

With respect to the baseline model, now households can have access to remunerated bank deposits, sell and purchase capital stock at market price Q_F, whereas the payment of the rental of capital to productive firms and the capital utilization cost are transferred to entrepreneurs. As explained in section 2.1, they view the stock of capital of the preceding period as an externality in their optimization program. Note that we adopt a different convention for bank deposits from the one adopted for bonds: $\tilde{B}_{F,t}$ is the nominal deposit made at the bank in period t, whereas the nominal quantities of bonds purchased in period t are $\frac{B_{E/F,t}}{R_{F,t}\Psi_{F,t}}$ and $\frac{B^\star_{F,t}}{R^\star_{F,t}\Psi^\star_{F,t}}$. The fact that bank deposits made in period t pays a nominally non-state contingent rate R_t in period $t+1$ (when Bernanke et al. (1999) assume that deposits pay R_{t+1}), together with the fact that banks make no profit (see below) implies that shocks to aggregate inflation involve wealth transfers between households and entrepreneurs and hence Fisher (1933) "debt-deflation" effects: a drop in expected inflation increases the real cost of debt owed by entrepreneurs, who in turn reduce their capital stock.

The first order conditions of households with respect to $C_{F,t}$, $B_{E/F,t}$, $B^\star_{F,t}$ and $L_{F,t}$ are unchanged, and yield the Euler equation, the uncovered interest rate parity condition and the labor supply curve. The first order condition with respect to bank deposits $\tilde{B}_{F,t}$ is the same as the one with respect to intra-euro area bonds $B_{E/F,t}$; as for the other assets, the level of bank deposits will be determined by the needs of funds emanating from banks.

The optimal supply of capital curve is the solution of the problem

$$\max_{I_{F,t}} \Bigg\{ U_{F,t} + \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} \Bigg[\tau^R \Bigg(\tau^W w^m_{F,t+j} L_{F,t+j} + \frac{Div_{F,t+j}}{P_{F,t+j}}$$
$$+ Q_{F,t+j} \left((1-\delta) K_{F,t+j-1} + \varepsilon^I_{F,t+j} \left(1 - \frac{\varphi}{2} \left(\frac{I_{F,t+j}}{I_{F,t+j-1}} - (1+g) \right)^2 \right) I_{F,t+j} \right)$$
$$- (1-\delta) Q_{F,t+j} K_{F,t+j-1} + \frac{\tilde{B}_{F,t+j-1} R_{t-1} \Psi_{F,t+j-1}}{P_{F,t+j}} - \frac{\tilde{B}_{F,t+j}}{P_{F,t+j}}$$
$$+ \frac{B_{E/F,t+j-1}}{P_{F,t+j}} - \frac{B_{E/F,t+j}}{P_{F,t+j} R_t \Psi_{F,t+j}} + \frac{S_t B^\star_{F,t+j-1}}{P_{F,t+j}} - \frac{S_t B^\star_{F,t+j}}{P_{F,t+j} R^\star_t \Psi^\star_{F,t+j}} \Bigg)$$
$$+ T_{F,t+j} + \Xi_{F,t+j} - \tau^C C_{F,t+j} - p^{ne}_{F,t+j} I_{F,t+j} \Bigg] \Bigg\},$$

where the capital accumulation technology has been substituted into households' budget constraint. It is described by the first order condition

$$\frac{p^{ne}_{F,t}}{\tau^R \varepsilon^I_{F,t}} = Q_{F,t} \left[1 - \frac{\varphi}{2} \left(\frac{I_{F,t}}{I_{F,t-1}} - (1+g) \right)^2 - \frac{I_{F,t}}{I_{F,t-1}} \varphi \left(\frac{I_{F,t}}{I_{F,t-1}} - (1+g) \right) \right]$$
$$+ \beta E_t \frac{\lambda_{F,t+1}}{\lambda_{F,t}} Q_{F,t+1} \frac{\varepsilon^I_{F,t+1}}{\varepsilon^I_{F,t}} \left(\frac{I_{F,t+1}}{I_{F,t}} \right)^2 \varphi \left(\frac{I_{F,t+1}}{I_{F,t}} - (1+g) \right).$$

Households increase investment until the marginal gain they make by selling capital to entrepreneurs equals the marginal cost associated with the building of capital (purchase price of investment goods plus adjustment costs). This condition is identical to the first order condition with respect to $I_{F,t}$ in the model without financial frictions, with the Tobin's Q being replaced by the actual price of the capital stock $Q_{F,t}$ times the income tax factor τ^R.

3.4.2 Capital return rate for entrepreneurs

The nominal $t+1$-cash flow associated with the period t-investment of an entrepreneur, for a total amount of $P_{F,t} Q_{F,t} K_{F,t}$, includes the rental to be paid by equipment providers for utilized capital

$$P_{F,t+1} \omega_{F,t+1} z_{F,t+1} K_{F,t} r^k_{F,t+1}$$

and the subsequent sale of the depreciated capital stock to households

$$\omega_{F,t+1} P_{F,t+1} Q_{F,t+1} (1-\delta)) K_{F,t},$$

minus capital utilization costs $P^{ne}_{F,t+1} a(z_{F,t+1}) \omega_{F,t+1} K_{F,t}$. It depends on the idiosyncratic shock $\omega_{F,t+1}$ that he is going to experience at the beginning of $t+1$ as well as on aggregated $t+1$-shocks. Note also that real capital utilization costs are defined in units of non-energy goods (price P^{ne}_F) as investment, whereas $Q_{F,t}$ is the real price of capital in units of final consumption goods. This cash flow can be conveniently written

$$R^k_{F,t+1}P_{F,t}Q_{F,t}\omega_{F,t+1}K_{F,t}$$

with:

$$R^k_{F,t+1} \equiv \frac{z_{F,t+1}P_{F,t+1}r^k_{F,t+1} + P^{ne}_{F,t+1}a(z_{F,t+1}) + P_{F,t+1}Q_{F,t+1}(1-\delta)}{P_{F,t}Q_{F,t}} \tag{3.4.2}$$

$R^k_{F,t+1}$ is common to all entrepreneurs provided that they all choose the same capital utilization rate (which is shown in what follows).

3.4.3 The financing of capital

In order to make net worth accumulation stationary (apart from exogenous technology growth), a strictly positive, time varying fraction $1-\gamma_{F,t}$ of entrepreneurs chosen randomly are assumed to exit the economy during every period. These entrepreneurs are informed at the beginning of the period; after receiving their profits from past investment, they do not enter in any new investment program. They consume a fraction Θ of their net worth and transfer the rest evenly to households. They are immediately replaced by new entrants so that the total number of entrepreneurs in activity is constant over time and normalized to 1. In order to ensure that new entrants (as well as bankrupt entrepreneurs) have strictly positive net worth before borrowing and investing, all non-exiting entrepreneurs are assumed to receive a subsidy W^e_F from the government during every period. Entrepreneurs are risk-neutral; those who do not exit the economy do not consume but rather re-invest their available income (that is the sum of their profits from past investment and of the subsidy they have received from the governement).

Consider an entrepreneur who does not have to exit the economy in period t. In order to purchase a quantity $K_{F,t}$ of capital to be used for production in period $t+1$, he combines his revenue of $t-1$ investment or net worth $N_{F,t}$, with a one period bank loan $\tilde{B}_{F,t}$:

$$\tilde{B}_{F,t} = P_{F,t}Q_{F,t}K_{F,t} - N_{F,t} \tag{3.4.3}$$

We assume that internal entreprenerial funds $N_{F,t}$ are always insufficient to finance the desired level of capital.

The interest rate on a loan contracted in period t and to be paid in period $t+1$ is denoted by $Z_{F,t+1}$. The ability of an entrepreneur to repay his debt to the bank will depend on this cash flow. If:

$$R^k_{F,t+1}P_{F,t}Q_{F,t}\omega_{F,t+1}K_{F,t} \geq Z_{F,t+1}\tilde{B}_{F,t}$$

then the entrepreneur will be able to repay. Conversely, if:

$$R^k_{F,t+1}P_{F,t}Q_{F,t}\omega_{F,t+1}K_{F,t} < Z_{F,t+1}\tilde{B}_{F,t}$$

then he declares himself bankrupt. In that case, the bank seizes all his remaining resourses, after expensing fees for auditing services to discover the value of $\omega_{F,t+1}$. These fees are proportional to the assets of the entrepreneur in liquidation and consume real resources. Specifically, they use non-oil final goods H_F. So in this case the bank receives

$$(1-\mu p^{ne}_{F,t+1})R^k_{F,t+1}P_{F,t}Q_{F,t}\omega_{F,t+1}K_{F,t}.$$

Nevertheless, bankrupt entrepreneurs do not exit the economy; they benefit from the governement subsidy $W^e_{F,t}$ and are able to borrow again in the following periods. For any given entrepreneur, let $\bar{\omega}_{F,t+1}$ be the threshold such that if he draws $\omega_{F,t+1} \geq \bar{\omega}_{F,t+1}$, he will be able to repay and if he draws $\omega_{F,t+1} < \bar{\omega}_{F,t+1}$, he experiencies bankruptcy. Therefore, $\bar{\omega}_{F,t+1}$ is defined by:

$$R^k_{F,t+1}P_{F,t}Q_{F,t}\bar{\omega}_{F,t+1}K_{F,t} = Z_{F,t+1}\tilde{B}_{F,t} \tag{3.4.4}$$

3.4.4 Bank and credit supply

The financial accelerator framework assumes that bank are non-optimizing agents; they are empty shells that collect funds from households and ensures that interest rates on healthy loans are high enough to offset bad debts. A representative bank offers a loan to any entrepreneur seeking to borrow $\tilde{B}_{F,t}$ at the end of period t. Perfect competition in the banking market implies that the expected profit in $t+1$ from this loan is zero:

$$\begin{aligned} E_t\frac{\lambda_{F,t+1}}{\lambda_{F,t}}\bigg(&(1-F_{F,t}(\bar{\omega}_{F,t+1}))\, Z_{F,t+1}\tilde{B}_{F,t} \\ &+ (1-\mu p^{ne}_{F,t+1})R^k_{F,t+1}P_{F,t}Q_{F,t}K_{F,t}\int_0^{\bar{\omega}_{F,t+1}} \omega f_{F,t}(\omega)\,d\omega - R_t\tilde{B}_{F,t}\bigg) = 0 \end{aligned} \tag{3.4.5}$$

As Bernanke et al. (1999), we assume that the parties agree that the interest rate $Z_{F,t+1}$ is revised in period $t+1$ in such a way that the bank makes no profit whatever the realization of period $t+1$ aggregate shocks:

$$\begin{aligned} &(1-F_{F,t}(\bar{\omega}_{F,t+1}))Z_{F,t+1}\tilde{B}_{F,t} \\ &\qquad + (1-\mu p^{ne}_{F,t+1})R^k_{F,t+1}P_{F,t}Q_{F,t}K_{F,t}\int_0^{\bar{\omega}_{F,t+1}} \omega f_{F,t}(\omega)\,d\omega = R_t\tilde{B}_{F,t} \end{aligned} \tag{3.4.6}$$

Or equivalently:

$$\Big(\Gamma_{F,t}(\bar{\omega}_{F,t+1}) - \mu p^{ne}_{F,t+1}G_{F,t}(\bar{\omega}_{F,t+1})\Big)\frac{R^k_{F,t+1}}{R_t}\varrho_{F,t} = \varrho_{F,t} - 1 \tag{3.4.7}$$

with:

$$
\begin{aligned}
G_{F,t}(\bar{\omega}) &= \int_0^{\bar{\omega}} \omega f_{F,t}(\omega) d\omega \\
\Gamma_{F,t}(\bar{\omega}) &= \bar{\omega}\left(1 - F_{F,t}(\bar{\omega})\right) + G_{F,t}(\bar{\omega}) \\
\varrho_{F,t} &= \frac{P_{F,t} Q_{F,t} K_{F,t}}{N_{F,t}}
\end{aligned}
\tag{3.4.8}
$$

This constaint is more restrictive and therefore consistent with the free-entry condition. Anticipating the result shown below that the interest rate $Z_{F,t}$ and the leverage ratio $\varrho_{F,t}$ are common to all entrepreneurs, the arrangement implicit to condition (3.4.7) is that banks (i) deal with the risk related to idiosyncratic shocks ω_F by lending to a sufficient number of entrepreneurs and (ii) are insured by risk-neutral entrepreneurs against aggregate uncertainty.

3.4.5 Entrepreneurs, credit demand and capital utilization

The program of a risk-neutral entrepreneur who does not exit the economy in period t consists in maximizing his cash flow expected in period $t+1$ from present investment and borrowing activities, taking into account that he ignores his future idiosyncratic productivity $\omega_{F,t+1}$ but that he knows its probability distribution, and subject to the credit supply curve (3.4.7) being true for any state of the nature in $t+1$. Banks do not try to identify bad debts, but only pass on the costs of defaulting to the borrowing cost of all entrepreneurs. As the latter internalize banks reactions, it yields a tradeoff for entrepreneurs between increasing or decreasing loan amounts on the one hand, and between increasing or decreasing the interest rate. Indeed, borrowing too much would increase the expected profits of non-bankrupt entrepreneurs at a given interest rate, but it would in fact result in an increase in the interest rate and, therefore, a rise in defaults, deteriorating the unconditional expectation of cash flows in the following period. Conversely, demanding a too low interest rate would have the undesirable effect that banks limit the amounts of loans issuance to balance their books. That is why the optimal borrowing contract is obtained from the optimization problem of entrepreneurs.

Entrepreneurs' expected cash flow is:

$$
P_{F,t} Q_{F,t} K_{F,t} \mathrm{E}_t \left[R^k_{F,t+1} \int_{\bar{\omega}_{F,t+1}}^{\infty} \left(\omega - \bar{\omega}_{F,t+1}\right) f_{F,t}(\omega) d\omega \right].
$$

With the notations introduced above, it can be written

$$
\varrho_{F,t} N_{F,t} \mathrm{E}_t \left[R^k_{F,t+1} \left(1 - \Gamma_{F,t}(\bar{\omega}_{F,t+1})\right) \right]. \tag{3.4.9}
$$

The borrowing contract should define the amount of the loan $\tilde{B}_{F,t}$ and, because the interest rate depends on the ex post realization of aggregate shocks, a set of interest rates $\left\{Z_{F,t}(s)\right\}_{s \in \mathcal{S}_{t+1}}$ for all possible states of the nature in $t+1$, denoted by $s \in \mathcal{S}_{t+1}$. The relationships (3.4.3) and (3.4.4) imply that it is equivalent to choose $\tilde{B}_{F,t}$ and a full set $\left\{Z_{F,t}(s)\right\}_{s \in \mathcal{S}_{t+1}}$ or a level of leverage

$\varrho_{F,t}$ and a full set of bankrupt thresholds $\{\bar{\omega}_{F,t}(s)\}_{s\in\mathcal{S}_{t+1}}$. Hence, to define the optimal borrowing contract, entrepreneurs choose $\varrho_{F,t}$ and $\{\bar{\omega}_{F,t}(s)\}_{s\in\mathcal{S}_{t+1}}$ that maximize (3.4.9) under banks zero profit constaint in every state of the nature in the following period. Formally, the program is

$$\max_{\varrho_{F,t},\,\{\bar{\omega}_{F,t}(s)\}_{s\in\mathcal{S}_{t+1}}} \left\{ \varrho_{F,t} N_{F,t} \int_{\mathcal{S}_{t+1}} R^k_{F,t+1}(s) \left(1 - \Gamma_{F,t}(\bar{\omega}_{F,t+1}(s))\right) t_{s_t|s} \, ds \right.$$
$$\left. + \int_{\mathcal{S}_{t+1}} \nu_{F,t+1}(s) \left(\left(\Gamma_{F,t}(\bar{\omega}_{F,t+1}(s)) - \mu p^{ne}_{F,t+1} G_{F,t}(\bar{\omega}_{F,t+1}(s)) \right) \frac{R^k_{F,t+1}(s)}{R_t} \varrho_{F,t} - \varrho_{F,t} + 1 \right) ds \right\},$$

where $\nu_{F,t+1}(s)$ is the Lagrange multiplier associated with the constraint that banks' profit are nul in period $t+1$ if the state of the economy is s, and $t_{s_t|s}$ is the probability density of the transition from the present state s_t to the state s in period $t+1$. After eliminating the Lagrange multipliers from the first order conditions, it yields the following credit demand curve:

$$\begin{aligned} E_t \Bigg[& \left(1 - \Gamma_{F,t}(\bar{\omega}_{F,t+1})\right) \frac{R^k_{F,t+1}}{R_t} + \frac{\Gamma'_{F,t}(\bar{\omega}_{F,t+1})}{\Gamma'_{F,t}(\bar{\omega}_{F,t+1}) - \mu p^{ne}_{F,t+1} G'_{F,t}(\bar{\omega}_{F,t+1})} \\ & \left(\left[\Gamma_{F,t}(\bar{\omega}_{F,t+1}) - \mu p^{ne}_{F,t+1} G_{F,t}(\bar{\omega}_{F,t+1}) \right] \frac{R^k_{F,t+1}}{R_t} - 1 \right) \Bigg] = 0 \end{aligned} \tag{3.4.10}$$

which determines the bankrupt threshold $\bar{\omega}_{F,t+1}$ and therefore the interest premium as a downward-sloping function of expected aggregated macroeconomic conditions summarized in $R^k_{F,t+1}$, which also depends on the distribution of entrepreneurs idiosyncratic productivity.

This expression also implies that all entrepreneurs have the same $\bar{\omega}_F$ and Z_F. In addition, as the bank supply curve (3.4.7) determines the leverage ratio ϱ_F as a function of $\bar{\omega}_F$ and of aggregated macroeconomic conditions, all entrepreneurs also choose the same ϱ_F. The corollary is that, for a given entrepreneur, the level of capital K_F and the amount of the loan $\tilde{B}_F$ only depend on the level $N_{F,t}$ of his net worth.

In addition, entrepreneurs (exiting or not but with the exception of new entrants) decide the capital utilization rate $z_{F,t}$ for their stock of capital $\omega_{F,t}K_{F,t-1}$ in order to maximize his cash flow from $t-1$-investment. Formally, their program is written

$$\max_{z_{F,t}} \left\{ \varrho_{F,t-1} N_{F,t-1} \frac{z_{F,t} P_{F,t} r^k_{F,t} + P^{ne}_{F,t} a(z_{F,t}) + P_{F,t} Q_{F,t} (1-\delta)}{P_{F,t-1} Q_{F,t-1}} \left(1 - \Gamma_{F,t-1}(\bar{\omega}_{F,t})\right) \right\}.$$

The capital utilization rate $z_{F,t}$ is chosen such the marginal profit from higher capital utilization would be exactly offset by the marginal capital utilization costs, that is

$$r^k_{F,t} = p^{ne}_{F,t} a'(z_{F,t}).$$

This condition is unchanged with respect to the model without financial frictions, although the $z_{F,t}$ is chosen by entrepreneurs instead of households. It implies that $z_{F,t}$ (and therefore $R^k_{F,t}$) is common to all entrepreneurs.

3.4.6 Aggregation and the law of motion of net worth

The real net worth of a non-exiting entrepreneur, excluding new entrants – which represent a mass $\gamma_{F,t}$ – is in period t (*i.e.* after receiving revenues from past investment but before investing again):

$$\frac{N_{F,t}}{P_{F,t}} = \frac{R^k_{F,t}\varrho_{F,t-1}}{P_{F,t}} N_{F,t-1} \max\left\{\omega_{F,t} - \bar{\omega}_{F,t}, 0\right\} + W^e_{F,t}$$

At that time, exiting entrepreneurs have disposed of all their net worth and new entrants – which represent a mass $1-\gamma_{F,t}$ – only own the transfer from government $W^e_{F,t}$. As non-exiting entrepreneurs are chosen independantly of their idiosyncratic productivity and of their past net worth and an entrepreneur's idiosyncratic producitivity is drawn independantly of his past net worth, aggregating across entrepreneurs yields total real net worth in period t as follows:

$$\begin{aligned}
\bar{n}_{F,t} &\equiv \int_0^\infty \frac{N}{P_{F,t}} f_{N,t}(N)\,dN \\
&= \gamma_{F,t}\frac{R^k_{F,t}\varrho_{F,t-1}}{P_{F,t}} \int_0^\infty N f_{N,t-1}(N)\,dN \int_{\bar{\omega}_{F,t}}^\infty \left(\omega - \bar{\omega}_{F,t}\right) f_{F,t-1}(\omega)\,d\omega \\
&\qquad + \gamma_{F,t} W^e_{F,t} + (1-\gamma_{F,t}) W^e_{F,t} \\
&= \gamma_{F,t}\left(1 - \Gamma_{F,t-1}(\bar{\omega}_{F,t})\right) \frac{\varrho_{F,t-1} R^k_t}{\pi_{F,t}} \bar{n}_{F,t-1} + W^e_{F,t}
\end{aligned} \tag{3.4.11}$$

where $f_{N,t}$ (resp. $f_{N,t-1}$) denotes the distribution of wealth across all entrepreneurs in period t (resp. $t-1$).

Total entrepreneurs' consumption in period t is the sum of the consumptions of entrepreneurs who exit the economy. These entrepreneurs represent a mass of $1 - \gamma_{F,t}$. As for aggregated net worth, we have:

$$\begin{aligned}
C^e_{F,t} &= \Theta(1-\gamma_{F,t})\frac{R^k_{F,t}\varrho_{F,t-1}}{P_{F,t}} \int_0^\infty N f_{N,t-1}(N)\,dN \int_{\bar{\omega}_{F,t}}^\infty \left(\omega - \bar{\omega}_{F,t}\right) f_{F,t-1}(\omega)\,d\omega \\
&= \frac{1-\gamma_{F,t}}{\gamma_{F,t}} \Theta\left(\bar{n}_{F,t} - W^e_{F,t}\right)
\end{aligned} \tag{3.4.12}$$

Finally, from (3.4.3) and (3.4.8), aggregated capital and bank loans (or deposits) are:

$$K_{F,t} = \int_0^\infty \frac{\varrho_{F,t}}{P_{F,t} Q_{F,t}} N f_{N,t}(N)\,dN = \frac{\varrho_{F,t}}{Q_{F,t}} \bar{n}_{F,t} \tag{3.4.13}$$

$$\tilde{B}_{F,t} = \int_0^\infty \left(\varrho_{F,t} - 1\right) N f_{N,t}(N)\,dN = \left(\varrho_{F,t} - 1\right) P_{F,t} \bar{n}_{F,t} \tag{3.4.14}$$

3.4.7 General equilibrium

The demand addressed to the final good producer not only includes households' final consumption, but also entrepreneurs' consumption. Therefore, the final good market clearing and zero-profit conditions below hold in equilibrium:

$$\begin{aligned} C_{F,t} + C^e_{F,t} = \Bigg[& \left(a^o_{F,t}\right)^{\frac{1}{s_O}} \left(C^{ne}_{F,t}\right)^{\frac{s_O-1}{s_O}} \\ & + \left(1-a^o_{F,t}\right)^{\frac{1}{s_O}} \left(C^o_{F,t} \left[1 - \frac{\chi^o}{2} \left(\frac{C^o_{F,t} C_{F,t-1}}{C^o_{F,t-1} C_{F,t}} - 1 \right)^2 \right] \right)^{\frac{s_O-1}{s_O}} \Bigg]^{\frac{s_O}{s_O-1}} \end{aligned} \tag{3.4.15}$$

$$C_{F,t} + C^e_{F,t} = p^{ne}_{F,t} C^{ne}_{F,t} + s_{F,t} p^{\star}_{o,t} C^o_{F,t} \tag{3.4.16}$$

In addition, the euro area total consumption is:

$$C_t = \Sigma_F \left(C_{F,t} + C^e_{F,t} \right) + \Sigma_E \left(C_{E,t} + C^e_{E,t} \right). \tag{3.4.17}$$

Auditing fees appear in the market clearing condition of non-oil goods

$$\begin{aligned} H_{F,t} = {} & I_{F,t} + G_{F,t} + C^{ne}_{F,t} + \bar{a} \left[\exp(\omega(z_{F,t} - \bar{z}_F)) - 1 \right] K_{F,t-1} \\ & + \mu p^{ne}_{F,t} G_{F,t-1} \left(\bar{\omega}_{F,t}\right) \frac{R^k_{F,t} Q_{F,t-1} K_{F,t-1}}{\pi_{F,t}}. \end{aligned} \tag{3.4.18}$$

Finally, since the financial accelerator equations include terms related to the distribution of idiosyncratic productivities, a number of transformations are needed in order to write them in a format interpretable by Dynare. They are described in Appendix 3.E. I also show the intra-period chronology of events that underlies the derivations above in Figure 3.D.1 in Appendix 3.D.

3.5 Labor market frictions

This section describes the introduction of labor market frictions in the form of a matching function between job seekers and vacant jobs.

An originality of the model lies in the introduction of real wage rigidity: building on Pissarides (2009), I assume that only the wages of new jobs are negotiated every period. They are the outcome of a Nash bargaining process, as in a standard RBC model with search and matching frictions. The real wages of existing jobs are only indexed to the deterministic trend in productivity. As a result, the magnitude of fluctuations in the average wage rate that is paid to households is small, as it is in the data, whereas job creation decisions are still based on the bargained wage rate. As shown in section 2.4 and consistently with Pissarides (2009) view, this form of wage rigidity does not affect job creation. So it is not used to improve the dynamic properties of the model with respect to employment.

The baseline model without labor-market frictions assumes nominal wage stickiness involving a lottery à la Calvo (1983). By contrast, here, the rigidity concerns the real wage of ongoing jobs. Assuming instead nominal wage rigidity in this framework is possible. As discussed in section 2.4, it would not change the dynamics of job creation in the model, but would alter the dynamics of the average real wage rate, which is fitted to the data. Although the common sense supports the view that nominal wages continue with jobs, real wage stickiness seems to engender better dynamic properties. In particular, the average nominal wage would be so inert that demand shocks that move inflation procyclically would tend to make the average real wage countercyclical.

In the baseline model, identical households supply labor to unions that differentiate it; imperfectly substitutable labor types are aggregated by a representative labor agency that supplies homogeneous labor to firms. The variant model developed here assumes instead that identical households are able to supply a constant quantity of labor hours during every period – so the model ignores changes in labor along the intensive margin. They are employed by a representative intermediate labor-intensive producer. Then, the latter supplies his production $\mathcal{L}_{F,t}$ to intermediate producers, which are unchanged with respect to the baseline model.

Although the introduction of endogenous layoffs à la Den Haan et al. (2000) has been tested in a previous version of the model, in what follows I only consider a constant job separation rate. The reason is that endogenous layoffs are not compatible with wage rigidity à la Pissarides (2009), as shown in section 2.5. The estimation results corresponding to the version with endogenous layoffs, including an adjustment cost in an effort to address this issue, are presented in Appendix 4.C.

3.5.1 The labor market and the matching function

Identical labor intensive intermediate producers in country F, called 'firms' in what follows, hire local workers in order to produce a quantity $\mathcal{L}_{F,t}$ of an homogeneous good that is supplied to intermediate retailers at real price $w_{F,t}$. Employment is the result of both exogenous separations and job creations.

First, a constant fraction s of the $N_{F,t-1}$ jobs that have actually contributed to produce in period $t-1$ are destroyed at the beginning of period t.

Second, the imperfect matching between job seekers and vacant jobs ends in the creation of m_t new jobs. This number m_t of successful matches is assumed to be a Cobb-Douglas function of the number of vacancies issued by the firm on one hand, and of the unemployment at the time of matching. At that time indeed, jobs are not assigned to households so all of them are likely to search. However, the aggregate search effort from households is assumed to depend on the probability for any of them to be unemployed if the lottery only allocated the $(1-s)N_{t-1}$ jobs available before new matches occured. Hence, new matches are given by

$$m_{F,t} = \Upsilon(1-(1-s)N_{F,t-1})^{1-\varphi}v_{F,t}^{\varphi},$$

where $v_{F,t}$ is the number of vacancies posted by firms in period t.

Hence, the number of active jobs used for production in period t is

$$N_{F,t} = (1-s)N_{F,t-1} + m_{F,t}.$$

3.5.2 Households

As in the baseline, the economy is populated by a continuum of households (of mass 1), all identical at the beginning of period. In order to be able to aggregate households' decisions via the assumption of perfect risk sharing across them, we need a separable utility function compatible with exogenous growth. We assume that the contemporaneous utility function of a household in country F at date t is the following growing function of consumption of market goods and home produced goods as:

$$u_{F,t}\left(C_{F,t}\right) = \frac{\left(C_{F,t} - \eta \bar{C}_{F,t-1}\right)^{1-\sigma_c}}{1-\sigma_c} + \frac{\mathcal{C}_t^{1-\sigma_c}}{1-\sigma_c},$$

where $\bar{C}_{F,t-1}$ denotes aggregate consumption in period $t-1$, which is an externality from the point of view of an individual household, and $\mathcal{C}_t$ denotes home production. With this specification, the intertemporal elasticity of substitution of both types of goods and the intratemporal elasticity of substitution of home and market goods are $1/\sigma_c$. The technology for home production is

$$\mathcal{C}_t = A_t \tilde{\Gamma} h_t,$$

with A_t the trend in technology, $\tilde{\Gamma}$ a parameter and h_t the amount of time devoted to home production, which is total time normalized to 1 less worked hours per employee l_t. We assume that $h_t = 1$ for unemployed workers and $h_t = \bar{l}$, with $0 < \bar{l} < 1$, for employed workers. Put differently, we ignore fluctuations in labor along the intensive margin, for the reasons discussed in section 2.4. In what follows, we use the reduced notation

$$\Gamma = \frac{\tilde{\Gamma}^{1-\sigma_c}}{\sigma_c - 1}\left(\bar{l}^{1-\sigma_c} - 1\right) > 0.$$

Every period, households are appointed randomly to available jobs (in quantity $N_{F,t}$) so they ignore at the beginning of any period t whether they are going to be unemployed or not and, if employed, the wage that they are going to get. New jobs yield an identical bargained wage rate $\tilde{w}^m_{F,t}$, while the wage rate of ongoing ones is indexed on average technology growth (at rate g). Hence, the wage rate associated with jobs created j periods before t is $(1+g)^j \tilde{w}^m_{F,t-j}$ in period t. If employed, a household can expect a net-of-income-tax-and-of-social-contributions wage $\tau^R \tau^W w^m_{F,t}$, such that

$$w^m_{F,t} N_{F,t} = m_{F,t} \tilde{w}^m_{F,t} + (1-s)(1+g) w^m_{F,t-1} N_{F,t-1}.$$

In addition, she has the opportunity to insure perfectly against uncertain labor market outcomes by purchasing, for all possible labor market revenue r, a quantity $\tilde{b}_{F,t}(r)$ of bonds that deliver each one unit of currency in the case she eventually gets r. Perfect competition in the insurance market together with first order conditions related to bonds $\tilde{b}_{F,t}(r)$ imply that households make the same consumption and savings decision whatever their income of the current period. Their program reduces to a representative agent's optimization problem. It is summarized by the value function

$$\mathcal{W}_{F,t} = \max_{\{C_{F,t}, B_{E/F,t}, B^{\star}_{F,t}, I_{F,t}, K_{F,t}\}} \left\{ \varepsilon^{\beta}_{F,t} \left(\frac{\left[C_{F,t} - \eta \bar{C}_{F,t-1}\right]^{1-\sigma_c}}{1-\sigma_c} - N_{F,t} A_t^{1-\sigma_c} \Gamma \right) + \beta E_t \mathcal{W}_{F,t+1} \right\},$$

where ε^{β}_{F} is a shock to their subjective discount factor, subject to the aggregated budget constraint:

$$\tau^C C_{F,t} + \text{ other expenses} \leq \tau^R \tau^W N_{F,t} w^m_{F,t} + \text{ other revenues},$$

to the observed law of motion of employment and to the observed low of motion of wages. Workers are atomistic so the matching technology is assumed to be an externality to them; they consider that the number of successful matches is proportional to the number of available workers in excess of existing jobs at the time job search occur, that is $1-(1-s)N_{F,t-1}$. Hence, these law of motions are

$$N_t = (1-s)N_{F,t-1} + (1-(1-s)N_{F,t-1})\Psi_{F,t},$$

and

$$w^m_{F,t} N_{F,t} = (1-(1-s)N_{F,t-1})\Psi_{F,t} \tilde{w}^m_{F,t} + (1-s)(1+g) w^m_{F,t-1} N_{F,t-1},$$

where $\Psi_{F,t}$ is the job finding rate, which is in equilibrium

$$\Psi_{F,t} = \frac{m_{F,t}}{1-(1-s)N_{F,t-1}}.$$

With wage rigidity, the average wage rate of the previous period is a state variable. Hence, we need to account for the fact that $\mathcal{W}_{F,t}$ is a function of $N_{F,t-1}$ and $w^m_{F,t-1}$. The envelope conditions with respect to state variables yield the marginal values of existing jobs and of existing wages for households, which are going to have an impact on wage bargaining. They are

$$\begin{aligned} \frac{\partial \mathcal{W}_{F,t}}{\partial N_{F,t-1}} = (1-s) \Bigg[& \lambda_{F,t} \tau^R \tau^W \left((1+g) w^m_{F,t-1} - \Psi_{F,t} \tilde{w}^m_{F,t} \right) - (1-\Psi_{F,t}) \varepsilon^{\beta}_{F,t} \Gamma + \beta (1-\Psi_{F,t}) E_t \frac{\partial \mathcal{W}_{F,t+1}}{\partial N_{F,t}} \\ & - \frac{\beta}{N_{F,t}} \left(\Psi_{F,t} (\tilde{w}^m_{F,t} - w^m_{F,t}) + w^m_{F,t} - (1+g) w^m_{F,t-1} \right) E_t \frac{\partial \mathcal{W}_{F,t+1}}{\partial w^m_{F,t}} \Bigg], \end{aligned}$$

and

$$\frac{\partial \mathcal{W}_{F,t}}{\partial w^m_{F,t-1}} = (1-s)(1+g) N_{F,t-1} \left[\lambda_{F,t} \tau^R \tau^W + \frac{\beta}{N_{F,t}} E_t \frac{\partial \mathcal{W}_{F,t+1}}{\partial w^m_{F,t}} \right].$$

3.5.3 Firms

Firms production function is simply

$$\mathcal{L}_{F,t} = N_{F,t}.$$

Next, posting vacancies is costly; it consumes final non-oil goods. The real cost of keeping $v_{F,t}$ vacant position during period t is

$$vc_{F,t} = cA_t v_{F,t} p^{ne}_{F,t},$$

where c is a parameter. The value of the firm can then be written as follows:

$$\begin{aligned}
\mathcal{V}_{F,t}(N_{F,t-1}) = \max_{v_{F,t}} & \left\{ w_{F,t}N_{F,t} - w^m_{F,t}N_{F,t} - vc_{F,t} + \beta \mathrm{E}_t \left[\frac{\lambda_{F,t+1}}{\lambda_{F,t}} \mathcal{V}_{F,t+1}(N_{F,t}) \right] \right\} \\
\text{s.t. } & N_{F,t} = (1-s)N_{F,t-1} + \Phi_{F,t}v_{F,t} \\
& N_{F,t}w^m_{F,t} = (1-s)N_{F,t-1}(1+g)w^m_{F,t-1} + \Phi_{F,t}v_{F,t}\tilde{w}^m_{F,t}
\end{aligned} \tag{3.5.1}$$

$\Phi_{F,t} = m_{F,t}/v_{F,t}$ is the apparent probability of filling a vacancy, which is taken as given by the firm. With wage rigidity, the average wage rate of the previous period is a state variable. Hence, we need to account for the fact that $\mathcal{V}_{F,t}$ is a function of $N_{F,t-1}$ and $w^m_{F,t-1}$. The resulting job creation condition is

$$\frac{vc'_{F,t}}{\Phi_{F,t}} = w_{F,t} - \tilde{w}^m_{F,t} + \beta E_t \frac{\lambda_{F,t+1}}{\lambda_{F,t}} \left[\frac{\partial \mathcal{V}_{F,t+1}}{\partial N_{F,t}} + \frac{\tilde{w}^m_{F,t} - w^m_{F,t}}{N_{F,t}} \frac{\partial V_{F,t+1}}{\partial w^m_{F,t}} \right], \tag{3.5.2}$$

with

$$vc'_{F,t} = cA_t.$$

The envelope conditions with respect to state variables yield the marginal values of existing jobs and of existing wages for firms:

$$\frac{\partial \mathcal{V}_{F,t}}{\partial N_{F,t-1}} = (1-s) \left[w_{F,t} - (1+g)w^m_{F,t-1} + \beta E_t \frac{\lambda_{F,t+1}}{\lambda_{F,t}} \left(\frac{\partial \mathcal{V}_{F,t+1}}{\partial N_{F,t}} - \frac{w^m_{F,t} - (1+g)w^m_{F,t-1}}{N_{F,t}} \frac{\partial \mathcal{V}_{F,t+1}}{\partial w^m_{F,t}} \right) \right]$$

$$\frac{\partial \mathcal{V}_{F,t}}{\partial w^m_{F,t-1}} = (1-s)(1+g) \left[-N_{F,t-1} + \beta E_t \frac{\lambda_{F,t+1}}{\lambda_{F,t}} \frac{N_{F,t-1}}{N_{F,t}} \frac{\partial \mathcal{V}_{F,t+1}}{\partial w^m_{F,t}} \right]$$

3.5.4 Effect of financial frictions on job creation

As already discussed in section 2.6, interactions between labor market frictions and financial frictions in the model economy proceed from general equilibrium effects only; developments in credit markets affect the demand of labor intensive goods stemming from producers and hence the number of vacancies posted to satisfy this demand. This contrasts with Petrosky-Nadeau (2014) who assumes that firms borrow from financial markets to pay for vacancy posting. Therefore, credit rationing has in his framework a direct effect on hiring materialized in the job creation equation by an additional coefficient multiplying the expected profits from new jobs, which is absent here.

3.5.5 Wage bargaining

Workers and firms bargain over the surplus generated by a new job. The surplus is the sum of the marginal value of a match for workers and the marginal value of a match for firms. It is in real terms

$$\mathcal{S}_{F,t} = \frac{1}{\lambda_{F,t}} \frac{\partial \mathcal{W}_{F,t}}{\partial m_{F,t}} + \frac{\partial \mathcal{V}_{F,t}}{\partial m_{F,t}} \tag{3.5.3}$$

The worker and the firm bargain over this surplus obtaining shares of $1 - \xi_{F,t}$ and $\xi_{F,t}$ respectively. The relative bargaining power of firms $\xi_{F,t}$ is allowed to vary exogenously during the business cycle, following Sala et al. (2008) and Christoffel et al. (2009), who find that this shock may be an important contributor to the fluctuations in output and inflation in the euro area. The outcome of the bargain is the wage rate $\tilde{w}^m_{F,t}$ such that

$$\left(\frac{1}{\lambda_{F,t}} \frac{\partial \mathcal{W}_{F,t}}{\partial m_{F,t}} \right)^{1-\xi_{F,t}} \left(\frac{\partial \mathcal{V}_{F,t}}{\partial m_{F,t}} \right)^{\xi_{F,t}}$$

is maximal, which implies

$$\xi_{F,t} \left(1 - \frac{1}{N_{F,t}} \beta E_t \frac{\lambda_{F,t+1}}{\lambda_{F,t}} \frac{\partial \mathcal{V}_{F,t+1}}{\partial w^m_{F,t}} \right) \frac{\partial \mathcal{W}_{F,t}}{\partial m_{F,t}} = (1 - \xi_{F,t}) \left(\lambda_{F,t} \tau^R \tau^W + \frac{\beta}{N_{F,t}} E_t \frac{\partial \mathcal{W}_{F,t+1}}{\partial w^m_{F,t}} \right) \frac{\partial \mathcal{V}_{F,t}}{\partial m_{F,t}}$$

This condition determines the dynamics of the wage rate of new jobs $\tilde{w}^m_F$, together with the equations for the marginal values of matches, obtained by derivation of workers and firms value functions respectively:

$$\frac{\partial \mathcal{W}_{F,t}}{\partial m_{F,t}} = \lambda_{F,t} \tau^R \tau^W \tilde{w}^m_{F,t} - \varepsilon^\beta_{F,t} \Gamma + \beta E_t \frac{\partial \mathcal{W}_{F,t+1}}{\partial N_{F,t}} + \beta E_t \frac{\tilde{w}^m_{F,t} - w^m_{F,t}}{N_{F,t}} \frac{\partial \mathcal{W}_{F,t+1}}{\partial w^m_{F,t}}$$

$$\frac{\partial \mathcal{V}_{F,t}}{\partial m_{F,t}} = w_{F,t} - \tilde{w}_{F,t} + \beta E_t \frac{\lambda_{F,t+1}}{\lambda_{F,t}} \left(\frac{\partial \mathcal{V}_{F,t+1}}{\partial N_{F,t}} + \frac{\tilde{w}^m_{F,t} - w^m_{F,t}}{N_{F,t}} \frac{\partial \mathcal{V}_{F,t+1}}{\partial w^m_{F,t}} \right)$$

3.5.6 General Equilibrium

In the baseline model, the dividends of unions (the difference between the wage bill paid by intermediate producers and wages received by households) were repaid evenly to households. Similarly, in the model with labor market frictions the labor intensive producing firm repays all profits evenly to households. In addition, vacancy costs consume real resources, specifically non-oil final goods. Therefore, the goods market clearing condition is changed to

$$H_{F,t} = I_{F,t} + G_{F,t} + C^{ne}_{F,t} + \bar{a} \left[\exp(\omega(z_{F,t} - \bar{z}_F)) - 1 \right] K_{F,t-1} + c A_t v_{F,t}.$$

3.5.7 Euro area aggregates

The average wage rate w^m in the euro area is given by

$$w_t^m = \frac{\Sigma_F N_{F,t}}{N_t} w_{F,t}^m + \frac{\Sigma_E N_{E,t}}{N_t} w_{E,t}^m,$$

and employment in the euro area is

$$N_t = \Sigma_F N_{F,t} + \Sigma_E N_{E,t}.$$

Unemployment rates are:

$$U_{F,t} = 1 - N_{F,t}$$

$$U_t = 1 - N_t$$

3.6 Steady state equilibrium

3.6.1 Introduction

The steady state equilibrium of DSGE models has generally no analytical solution: it is not possible to provide an expression of all endogeneous variables as explicit functions of parameters. That is also true in the case of a basic RBC model. However, it is often possible to consider the steady state of a subset of endogenous variables as parameters, and then to provide an analytical expression for the rest of the steady state values plus a subset of structural parameters, such that the static equations are verified. In the case of the basic RBC model, the steady state share of hours worked in total time is generally chosen, and the scale parameter in the expression of labor disutility is deduced accordingly.

This is not possible any more in the general case for medium-scale multi-country models. The latter generally imply that complex non-linear equations involving parameters and variables must be simultaneously satisfied in the steady stade, so numerical solvers are required. Solving numerically for hundreds of variables at the same time is a huge, time consuming and very capricious task. Not only initial conditions provided by the modeller should be very close to the solution, but the estimation, which involves a very large number of calls to the steady state computation routine at each of its iteration, would be unfeasible. Resolving the steady state equilibrium in that case consists in reducing the size of the system(s) of simultaneous non-linear equations that require a numerical solver, by making as many substitutions as possible.

This task is done using a `_steadystate2.m` file, automatically called by Dynare. In this Matlab program, I provide the analytical expression of the steady state levels of endogenous variables as functions of either parameters or of the steady state values that require numerical solving, update parameter values when necessary, and call numerical solvers.

The approach to the resolution of the present model is summarized in the following subsection (more details are provided in Appendix 3.C). The next subsection explains the coding techniques employed to deal with the different versions of the model (with and without frictions).

3.6.2 Structure of the resolution of the steady state equilibrium

Specifically, the structure of the resolution of the steady state is the following: first, a number of prices are computed, including the rental rate of capital. The latter is different depending on the presence of financial frictions. In that case, a first numerical solver is used to determine the steady state standard deviation of the idiosyncratic shock to entrepreneurs' productivity. Indeed, it enters inside the cumulative density function of the normal distribution ('`normcdf`'), which cannot be inverted analytically.

Then, because of the contribution of import prices to domestic prices, real wages in countries F and E enter in equation of both countries. So they are the unknown of a system of non linear equations, which is solved numerically in all versions of the model.

Once all prices are computed, the structure of the solution diverges according to the presence or not of labor market frictions. Without them, steady state equilibrium labor should be simultaneously consistent with both the goods market clearing and labor supply conditions in the two countries. This problem determines a non-linear system of equations of the GDPs in the two countries. The latter is also solved numerically. Without search and matching frictions, total labor corresponds to employment, the steady state of which does not enter in a convex utility function. It is hence considered as a parameter, and its prior distribution can be directly inferred from the data. The steady state levels of the two countries GDPs are obtained from employments after some calculations. By contrast with the frictionless version of the model, the value of the parameter Γ, which represents the (constant) disutility of labor for a worker, is set such that the labor supply condition is satisfied. So, the presence of search and matching frictions spares a call to a numerical solver.

Once GDPs in both countries are found, the rest of the variables come directly from the remaining equations of the static model.

3.6.3 Technical imbrication of the versions with and without frictions

The model is coded using the Dynare in such a manner that the labor market and financial frictions can be added or removed independantly from each other simply by assigning the values 1 or 0 to option variables. This is straightforward for the dynamic model file (or .mod file) using the **`@#if`**, **`@#else`**, **`@#endif`** commmands of Dynare's macro language (see Adjemian et al. (2011)) and defining the boolean option variables `FinancialAccelerator` and `LaborMarketFrictions` in Dynare using the **`@#define`** command. It is less obvious for the computation of the steady state: not only some variables are present with the frictions and absent otherwise, but the resolution of the steady state has a slightly different structure in the presence of labor market frictions.

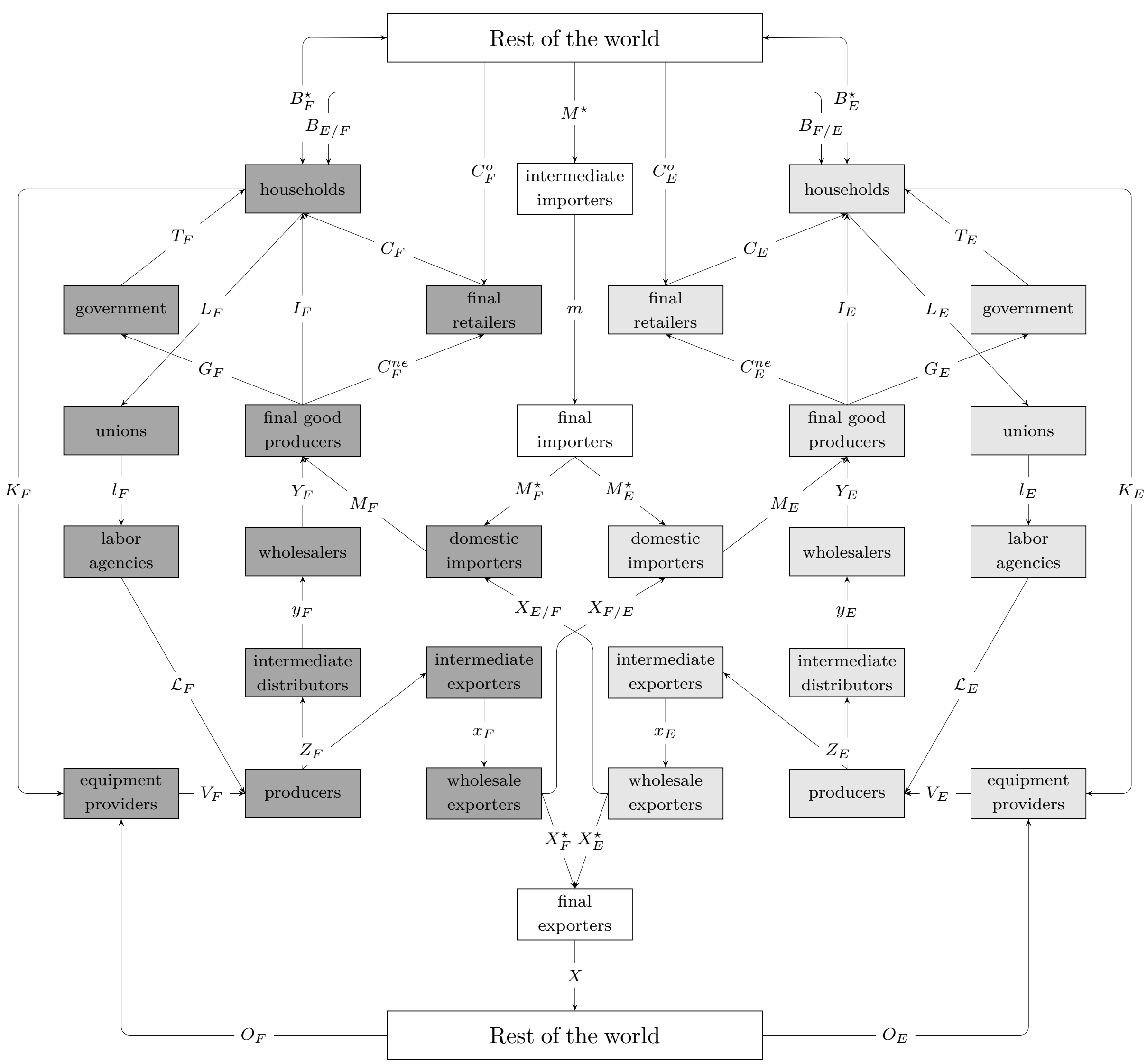
A solution could be to keep 4 different steady state files, corresponding to each situation: no frictions, labor market frictions only, financial frictions only or both frictions. However in that case, any modification of the model that has implications on the computation of the steady state of all versions entails the costly repetition of identical revisions in 4 different files. The solution considered for the present project is to keep only one steady state source file, which is used by a specific program to build the steady state file corresponding to the chosen option. The source file includes blocks that are marked with tags, which indicate in which case the block should be or not included in the final steady state file. The tags used are # BEGIN BLOCK <CASE>, where <CASE> can be <MAIN> to indicate that the block is needed in all versions, <FA > that the block is needed in the presence of financial frictions, <LM > in the presence of labor market frictions, <NOFA> only in the absence of financial frictions and <NOLM> only in the absence of labor market frictions. Blocks end by a # END BLOCK tag. Blocks are positioned inside the source file in the order that they are expected to be executed in all versions. The program that builds the steady state file (make_ss_file.m) is called from the .mod file with the variables FinancialAccelerator and LaborMarketFrictions as options. It reads the source file as a string of characters, identifies the tags, rearrange the code that is inside each block consistently with the options, and then write the full solution of the static equilibrium with the standard function declaration required by Dynare in a matlab _steadystate2.m file.

Appendix

3.A Diagrams

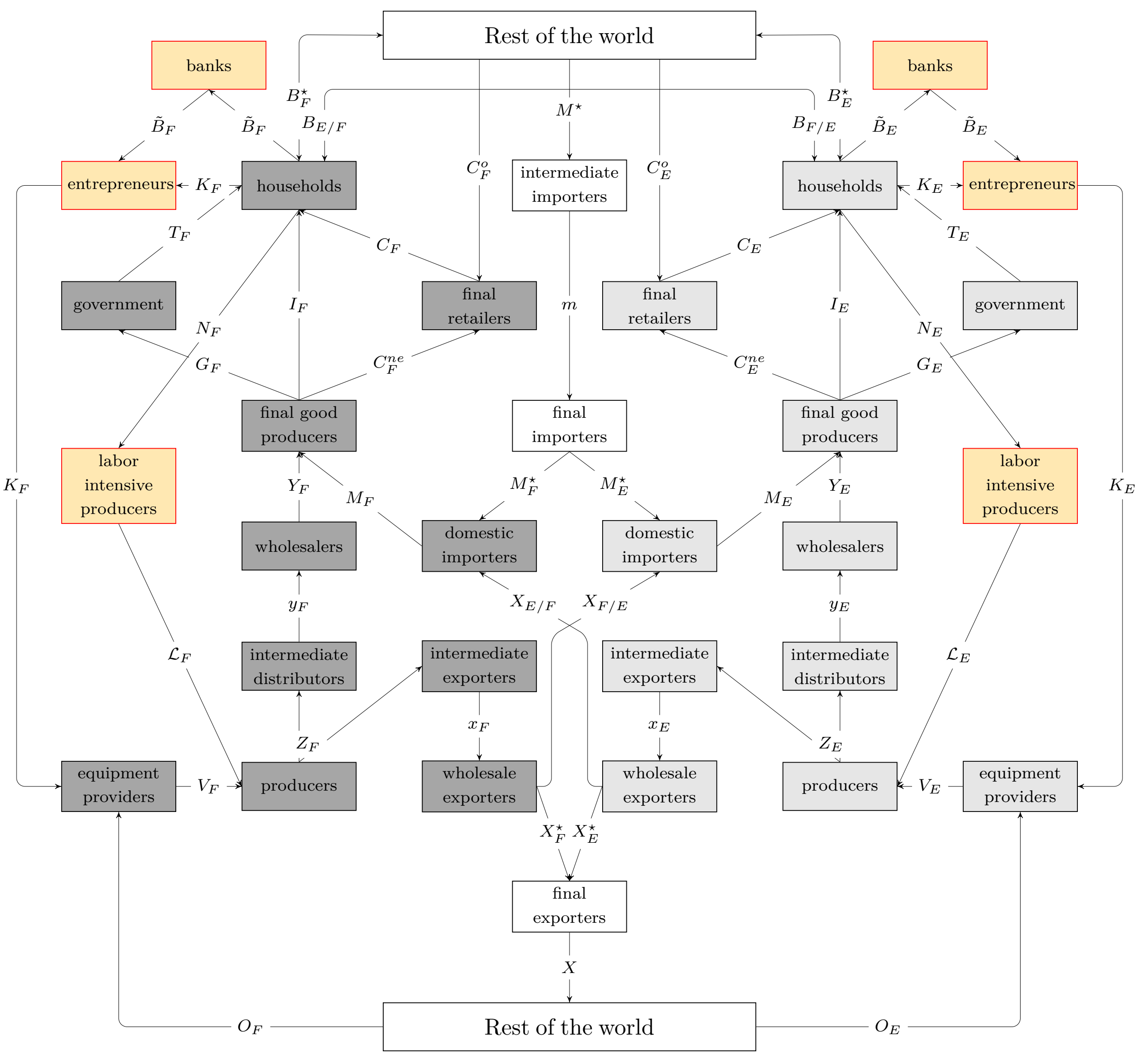
3.A.1 Baseline model

France — Rest of the euro area



3.A.2 Model with financial and labor market frictions

France | Rest of the euro area



3.B The stationary model's equations

3.B.1 Country equations

Households

$$\tau^C \lambda_{F,t} = \varepsilon_{F,t}^{\beta} \left(C_{F,t} - \eta \frac{C_{F,t-1}}{dA_t} \right)^{-\sigma_c} \exp\left[\frac{\sigma_c - 1}{1+\sigma_l} \left(L_{F,t}\right)^{1+\sigma_l} \right] \tag{NOLM-1}$$

$$\tau^C \lambda_{F,t} = \varepsilon_{F,t}^{\beta} \left(C_{F,t} - \eta \frac{C_{F,t-1}}{dA_t} \right)^{-\sigma_c} \tag{LM-1}$$

$$1 = \beta R_t \Psi_{F,t} \mathrm{E}_t \left[\frac{\lambda_{F,t+1}}{\lambda_{F,t} \pi_{F,t+1}} dA_{t+1}^{-\sigma_c} \right] \tag{1}$$

$$\Psi_{F,t} = \bar{\Psi}_F \exp\left[-\psi\left(\frac{b_{E/F,t}}{\bar{\mathcal{Y}}_F} - \bar{b}\right)\right] \tag{2}$$

$$1 = \beta R^\star_t \Psi^\star_{F,t} \mathrm{E}_t\left[\frac{\lambda_{F,t+1}}{\lambda_{F,t}\pi^\star_{t+1}}\frac{s_{F,t+1}}{s_{F,t}} dA_{t+1}^{-\sigma_c}\right] \tag{3}$$

$$\Psi^\star_{F,t} = \bar{\Psi}^\star_F \exp\left[-\psi^\star\left(\frac{s_{F,t}b^\star_{F,t}}{\bar{\mathcal{Y}}_F} - \bar{b}^\star\right)\right] \tag{4}$$

$$\lambda_{F,t}\tau^R\tau^W w^m_{F,t} = \varepsilon^\beta_{F,t}\left(C_{F,t} - \eta\frac{C_{F,t-1}}{dA_t}\right)^{1-\sigma_c} \exp\left[\frac{\sigma_c - 1}{1+\sigma_l}\left(L_{F,t}\right)^{1+\sigma_l}\right] L^{\sigma_l}_{F,t} \tag{NOLM-2}$$

$$\begin{aligned}\frac{p^{ne}_{F,t}}{\varepsilon^I_{F,t}} &= Q_{F,t}\left[1 - \frac{\varphi}{2}(1+g)^2\left(\frac{I_{F,t}dA_t}{(1+g)I_{F,t-1}} - 1\right)^2 - \frac{I_{F,t}dA_t}{I_{F,t-1}}\varphi\left(\frac{I_{F,t}dA_t}{I_{F,t-1}} - 1 - g\right)\right] \\ &+ \beta \mathrm{E}_t\left[\frac{\lambda_{F,t+1}}{\lambda_{F,t}} dA^{-\sigma_c}_{t+1} Q_{F,t+1}\frac{\varepsilon^I_{F,t+1}}{\varepsilon^I_{F,t}}\left(\frac{I_{F,t+1}dA_{t+1}}{I_{F,t}}\right)^2\varphi\left(\frac{I_{F,t+1}dA_{t+1}}{I_{F,t}} - 1 - g\right)\right]\end{aligned} \tag{NOFA-1}$$

$$\begin{aligned}\frac{p^{ne}_{F,t}}{\tau^R\varepsilon^I_{F,t}} &= Q_{F,t}\left[1 - \frac{\varphi}{2}(1+g)^2\left(\frac{I_{F,t}dA_t}{(1+g)I_{F,t-1}} - 1\right)^2 - \frac{I_{F,t}dA_t}{I_{F,t-1}}\varphi\left(\frac{I_{F,t}dA_t}{I_{F,t-1}} - 1 - g\right)\right] \\ &+ \beta \mathrm{E}_t\left[\frac{\lambda_{F,t+1}}{\lambda_{F,t}} dA^{-\sigma_c}_{t+1} Q_{F,t+1}\frac{\varepsilon^I_{F,t+1}}{\varepsilon^I_{F,t}}\left(\frac{I_{F,t+1}dA_{t+1}}{I_{F,t}}\right)^2\varphi\left(\frac{I_{F,t+1}dA_{t+1}}{I_{F,t}} - 1 - g\right)\right]\end{aligned} \tag{FA-1}$$

$$\tau^R r^k_{F,t} = \bar{a}\omega p^{ne}_{F,t}\exp\left(\omega(z_{F,t} - \bar{z}_F)\right) \tag{NOFA-2}$$

$$r^k_{F,t} = \bar{a}\omega p^{ne}_{F,t}\exp\left(\omega(z_{F,t} - \bar{z}_F)\right) \tag{FA-2}$$

$$\begin{aligned}Q_{F,t} &= \beta E_t\frac{\lambda_{F,t+1}}{\lambda_{F,t}} dA^{-\sigma_c}_{t+1}\big(Q_{F,t+1}\left(1-\delta\right) + \tau^R r^k_{F,t+1}z_{F,t+1} \\ &- \bar{a}p^{ne}_{F,t+1}\left(\exp\left(\omega(z_{F,t+1} - \bar{z}_F)\right) - 1\right)\big)\end{aligned} \tag{NOFA-3}$$

$$K_{F,t} = [1-\delta]\frac{K_{F,t-1}}{dA_t} + \varepsilon^I_{F,t}\left[1 - \frac{\varphi}{2}(1+g)^2\left(\frac{I_{F,t}dA_t}{(1+g)I_{F,t-1}} - 1\right)^2\right] I_{F,t} \tag{5}$$

$$\varepsilon^\beta_{F,t} = \left(\varepsilon^\beta_{F,t-1}\right)^{\rho_\beta}\exp\left(\sigma_\beta\eta^\beta_{F,t} + \varsigma_\beta\eta^\beta_t\right) \qquad \eta^\beta_F,\ \eta^\beta \sim N(0,1) \tag{6}$$

$$\varepsilon^I_{F,t} = \left(\varepsilon^I_{F,t-1}\right)^{\rho_I}\exp\left(\sigma_I\eta^I_{F,t} + \varsigma_I\eta^I_t\right) \qquad \eta^I_F,\ \eta^I \sim N(0,1) \tag{7}$$

Wage setting in frictionless labor markets

$$\tilde{w}_{F,t} = \frac{\theta_w}{\theta_w - 1}\frac{\mathcal{H}_{1F,t}}{\mathcal{H}_{2F,t}} \tag{NOLM-3}$$

$$\mathcal{H}_{1F,t} = \lambda_{F,t}\varepsilon^w_{F,t}w^m_{F,t}\mathcal{L}_{F,t}w^{\theta_w}_{F,t} + \beta\xi_w\mathrm{E}_t\left[\left(\frac{\pi_{F,t+1}}{(1+g)\bar{\pi}^{1-\gamma_w}\pi^{\gamma_w}_{F,t}}\right)^{\theta_w}\mathcal{H}_{1F,t+1}dA^{\theta_w+1-\sigma_c}_{t+1}\right] \tag{NOLM-4}$$

$$\mathcal{H}_{2F,t} = \lambda_{F,t}\mathcal{L}_{F,t}w^{\theta_w}_{F,t} + \beta\xi_w\mathrm{E}_t\left[\left(\frac{\pi_{F,t+1}}{(1+g)\bar{\pi}^{1-\gamma_w}\pi^{\gamma_w}_{F,t}}\right)^{\theta_w-1}\mathcal{H}_{2F,t+1}dA^{\theta_w-\sigma_c}_{t+1}\right] \tag{NOLM-5}$$

$$w^{1-\theta_w}_{F,t} = \left(1-\xi_w\right)\tilde{w}^{1-\theta_w}_{F,t} + \xi_w\left(\frac{(1+g)\bar{\pi}^{1-\gamma_w}\pi^{\gamma_w}_{F,t-1}}{\pi_{F,t}}\frac{w_{F,t-1}}{dA_t}\right)^{1-\theta_w} \tag{NOLM-6}$$

$$\varepsilon^w_{F,t} = \left(\varepsilon^w_{F,t-1}\right)^{\rho_w} \left(\bar{\varepsilon}^w_F\right)^{1-\rho_w} \exp\left(\sigma_w \eta^w_{F,t} + \varsigma_w \eta^w_t\right) \qquad \eta^w_F,\ \eta^w \sim N(0,1) \tag{NOLM-7}$$

Domestic goods price setting

$$\tilde{p}^Y_{F,t} = \frac{\theta_y}{\theta_y - 1} \frac{\mathcal{M}_{1F,t}}{\mathcal{M}_{2F,t}} \tag{8}$$

$$\mathcal{M}_{1F,t} = \lambda_{F,t} \varepsilon^p_{F,t} p^Z_{F,t} Y_{F,t} \left(p^Y_{F,t}\right)^{\theta_y} + \beta \xi_y \mathrm{E}_t \left[\left(\frac{\pi_{F,t+1}}{\bar{\pi}^{1-\gamma_y} \left(\pi^Y_{F,t}\right)^{\gamma_y}} \right)^{\theta_y} \mathcal{M}_{1F,t+1} dA^{1-\sigma_c}_{t+1} \right] \tag{9}$$

$$\mathcal{M}_{2F,t} = \lambda_{F,t} Y_{F,t} \left(p^Y_{F,t}\right)^{\theta_y} + \beta \xi_y \mathrm{E}_t \left[\left(\frac{\pi_{F,t+1}}{\bar{\pi}^{1-\gamma_y} \left(\pi^Y_{F,t}\right)^{\gamma_y}} \right)^{\theta_y - 1} \mathcal{M}_{2F,t+1} dA^{1-\sigma_c}_{t+1} \right] \tag{10}$$

$$p^Y_{F,t} = \left[(1-\xi_y) \left(\tilde{p}^Y_{F,t}\right)^{1-\theta_y} + \xi_y \left(\frac{\bar{\pi}^{1-\gamma_y} \left(\pi^Y_{F,t-1}\right)^{\gamma_y}}{\pi_{F,t}} p^Y_{F,t-1} \right)^{1-\theta_y} \right]^{\frac{1}{1-\theta_y}} \tag{11}$$

$$\pi^Y_{F,t} = \frac{p^Y_{F,t}}{p^Y_{F,t-1}} \pi_{F,t} \tag{12}$$

$$\varepsilon^p_{F,t} = \left(\varepsilon^p_{F,t-1}\right)^{\rho_p} \left(\bar{\varepsilon}^p_F\right)^{1-\rho_p} \exp\left(\sigma_p \eta^p_{F,t} + \varsigma_p \eta^p_t\right) \qquad \eta^p_F,\ \eta^p \sim N(0,1) \tag{13}$$

Export price setting

$$\tilde{p}^X_{F,t} = \frac{\theta_x}{\theta_x - 1} \frac{\mathcal{N}_{1F,t}}{\mathcal{N}_{2F,t}} \tag{14}$$

$$\mathcal{N}_{1F,t} = \lambda_{F,t} \varepsilon^x_{F,t} p^Z_{F,t} \left(X_{F/E,t} + X^\star_{F,t}\right) \left(p^X_{F,t}\right)^{\theta_x} + \beta \xi_x \mathrm{E}_t \left[\left(\frac{\pi_{F,t+1}}{\bar{\pi}^{1-\gamma_x} \left(\pi^X_{F,t}\right)^{\gamma_x}} \right)^{\theta_x} \mathcal{N}_{1F,t+1} dA^{1-\sigma_c}_{t+1} \right] \tag{15}$$

$$\mathcal{N}_{2F,t} = \lambda_{F,t} \left(X_{F/E,t} + X^\star_{F,t}\right) \left(p^X_{F,t}\right)^{\theta_x} + \beta \xi_x \mathrm{E}_t \left[\left(\frac{\pi_{F,t+1}}{\bar{\pi}^{1-\gamma_x} \left(\pi^X_{F,t}\right)^{\gamma_x}} \right)^{\theta_x - 1} \mathcal{N}_{2F,t+1} dA^{1-\sigma_c}_{t+1} \right] \tag{16}$$

$$p^X_{F,t} = \left[(1-\xi_x) \left(\tilde{p}^X_{F,t}\right)^{1-\theta_x} + \xi_x \left(\frac{\bar{\pi}^{1-\gamma_x} \left(\pi^X_{F,t-1}\right)^{\gamma_x}}{\pi_{F,t}} p^X_{F,t-1} \right)^{1-\theta_x} \right]^{\frac{1}{1-\theta_x}} \tag{17}$$

$$\pi^X_{F,t} = \frac{p^X_{F,t}}{p^X_{F,t-1}} \pi_{F,t} \tag{18}$$

$$\varepsilon^x_{F,t} = \left(\varepsilon^x_{F,t-1}\right)^{\rho_x} \left(\bar{\varepsilon}^x_F\right)^{1-\rho_x} \exp\left(\sigma_x \eta^x_{F,t} + \varsigma_x \eta^x_t\right) \qquad \eta^x_F,\ \eta^x \sim N(0,1) \tag{19}$$

Imports

$$\begin{aligned} M_{F,t} = \Bigg[& (a^m)^{\frac{1}{s_m}} \left(\frac{\Sigma_E}{\Sigma_F} X_{E/F,t} \right)^{\frac{s_m - 1}{s_m}} \\ & + (1-a^m)^{\frac{1}{s_m}} \left(M^\star_{F,t} \left[1 - \frac{\chi^m}{2} \left(\frac{M^\star_{F,t} M_{F,t-1}}{M^\star_{F,t-1} M_{F,t}} - 1 \right)^2 \right] \right)^{\frac{s_m - 1}{s_m}} \Bigg]^{\frac{s_m}{s_m - 1}} \end{aligned} \tag{20}$$

$$\frac{\Sigma_E X_{E/F,t}}{\Sigma_F M^\star_{F,t}\left(1-\frac{\chi^m}{2}\left(\frac{M^\star_{F,t}M_{F,t-1}}{M^\star_{F,t-1}M_{F,t}}-1\right)^2\right)} =$$

$$\frac{a^m}{1-a^m}\left(\frac{s_{E,t}p_{M,t}}{p^X_{E,t}\left[1-\frac{\chi^m}{2}\left(\frac{M^\star_{F,t}M_{F,t-1}}{M^\star_{F,t-1}M_{F,t}}-1\right)^2-\chi^m\left(\frac{M^\star_{F,t}M_{F,t-1}}{M^\star_{F,t-1}M_{F,t}}\right)\left(\frac{M^\star_{F,t}M_{F,t-1}}{M^\star_{F,t-1}M_{F,t}}-1\right)\right]}\right)^{s_m} \tag{21}$$

$$p^M_{F,t}M_{F,t} = \frac{\Sigma_E}{\Sigma_F}p^X_{E,t}\Phi_{E/F,t}X_{E/F,t} + s_{F,t}p_{M,t}M^\star_{F,t} \tag{22}$$

$$\pi^M_{F,t} = \frac{p^M_{F,t}}{p^M_{F,t-1}}\pi_{F,t} \tag{23}$$

Production

$$\tilde{K}_{F,t} = z_{F,t}\frac{K_{F,t-1}}{dA_t} \tag{24}$$

$$V_{F,t} = \left[a_v^{\frac{1}{s_v}}\left(\tilde{K}_{F,t}\right)^{\frac{s_v-1}{s_v}} + (1-a_v)^{\frac{1}{s_v}}\left(O_{F,t}\left[1-\frac{\chi^v}{2}\left(\frac{O_{F,t}V_{F,t-1}}{O_{F,t-1}V_{F,t}}-1\right)^2\right]\right)^{\frac{s_v-1}{s_v}}\right]^{\frac{s_v}{s_v-1}} \tag{25}$$

$$Z_{F,t} = V^\alpha_{F,t}\left(\varepsilon^a_{F,t}\mathcal{L}_{F,t}\right)^{1-\alpha} \tag{26}$$

$$H_{F,t} = \left[(a^y)^{\frac{1}{s_y}}\left(Y_{F,t}\right)^{\frac{s_y-1}{s_y}} + (1-a^y)^{\frac{1}{s_y}}\left(M_{F,t}\left[1-\frac{\chi^y}{2}\left(\frac{M_{F,t}H_{F,t-1}}{M_{F,t-1}H_{F,t}}-1\right)^2\right]\right)^{\frac{s_y-1}{s_y}}\right]^{\frac{s_y}{s_y-1}} \tag{27}$$

$$C_{F,t} = \left[(a^o)^{\frac{1}{s_o}}\left(C^{ne}_{F,t}\right)^{\frac{s_o-1}{s_o}} + (1-a^o)^{\frac{1}{s_o}}\left(C^o_{F,t}\left[1-\frac{\chi^o}{2}\left(\frac{C^o_{F,t}C_{F,t-1}}{C^o_{F,t-1}C_{F,t}}-1\right)^2\right]\right)^{\frac{s_o-1}{s_o}}\right]^{\frac{s_o}{s_o-1}} \tag{28}$$

$$\frac{\tilde{K}_{F,t}}{O_{F,t}\left(1-\frac{\chi^v}{2}\left(\frac{O_{F,t}V_{F,t-1}}{O_{F,t-1}V_{F,t}}-1\right)^2\right)}$$

$$= \frac{a_v}{1-a_v}\left(\frac{s_{F,t}p^\star_{o,t}}{r^k_{F,t}\left[1-\frac{\chi^v}{2}\left(\frac{O_{F,t}V_{F,t-1}}{O_{F,t-1}V_{F,t}}-1\right)^2-\chi^v\left(\frac{O_{F,t}V_{F,t-1}}{O_{F,t-1}V_{F,t}}\right)\left(\frac{O_{F,t}V_{F,t-1}}{O_{F,t-1}V_{F,t}}-1\right)\right]}\right)^{s_v} \tag{29}$$

$$\frac{\tau^L w_{F,t}\mathcal{L}_{F,t}}{p^v_{F,t}V_{F,t}} = \frac{1-\alpha}{\alpha} \tag{30}$$

$$\frac{Y_{F,t}}{M_{F,t}\left(1-\frac{\chi^y}{2}\left(\frac{M_{F,t}H_{F,t-1}}{M_{F,t-1}H_{F,t}}-1\right)^2\right)} =$$

$$\frac{a^y}{1-a^y}\left(\frac{p^M_{F,t}}{p^Y_{F,t}\left[1-\frac{\chi^y}{2}\left(\frac{M_{F,t}H_{F,t-1}}{M_{F,t-1}H_{F,t}}-1\right)^2-\chi^y\left(\frac{M_{F,t}H_{F,t-1}}{M_{F,t-1}H_{F,t}}\right)\left(\frac{M_{F,t}H_{F,t-1}}{M_{F,t-1}H_{F,t}}-1\right)\right]}\right)^{s_y} \tag{31}$$

$$\frac{C^{ne}_{F,t}}{C^{o}_{F,t}\left(1-\frac{\chi^o}{2}\left(\frac{C^o_{F,t}C_{F,t-1}}{C^o_{F,t-1}C_{F,t}}-1\right)^2\right)}=$$

$$\frac{a^o}{1-a^o}\left(\frac{s_{F,t}p^{\star}_{o,t}}{p^{ne}_{F,t}\left[1-\frac{\chi^o}{2}\left(\frac{C^o_{F,t}C_{F,t-1}}{C^o_{F,t-1}C_{F,t}}-1\right)^2-\chi^o\left(\frac{C^o_{F,t}C_{F,t-1}}{C^o_{F,t-1}C_{F,t}}\right)\left(\frac{C^o_{F,t}C_{F,t-1}}{C^o_{F,t-1}C_{F,t}}-1\right)\right]}\right)^{s_o} \tag{32}$$

$$p^v_{F,t}V_{F,t}=r^k_{F,t}\tilde{K}_{F,t}+s_{F,t}p^{\star}_{o,t}O_{F,t} \tag{33}$$

$$p^Z_{F,t}Z_{F,t}=p^v_{F,t}V_{F,t}+\tau^L w_{F,t}\mathcal{L}_{F,t} \tag{34}$$

$$p^{ne}_{F,t}H_{F,t}=p^Y_{F,t}Y_{F,t}+p^M_{F,t}M_{F,t} \tag{35}$$

$$C_{F,t}=p^{ne}_{F,t}C^{ne}_{F,t}+s_{F,t}p^{\star}_{o,t}C^o_{F,t} \tag{NOFA-4}$$

$$C^e_{F,t}+C_{F,t}=p^{ne}_{F,t}C^{ne}_{F,t}+s_{F,t}p^{\star}_{o,t}C^o_{F,t} \tag{FA-3}$$

$$\varepsilon^a_{F,t}=\left(\varepsilon^a_{F,t-1}\right)^{\rho_a}\left(\bar{\varepsilon}^a_F\right)^{1-\rho_a}\exp\left(\sigma_a\eta^a_{F,t}+\varsigma_a\eta^a_t\right)\qquad \eta^a_F,\ \eta^a\sim N(0,1) \tag{36}$$

Markets clearing and equilibrium

$$H_{F,t}=I_{F,t}+G_{F,t}+C^{ne}_{F,t}+\bar{a}\left(\exp\left(\omega(z_{F,t}-\bar{z}_F)\right)-1\right)\frac{K_{F,t-1}}{dA_t} \tag{BASE-1}$$

$$\begin{aligned}H_{F,t}=&\,I_{F,t}+G_{F,t}+C^{ne}_{F,t}+\bar{a}\left(\exp\left(\omega(z_{F,t}-\bar{z}_F)\right)-1\right)\frac{K_{F,t-1}}{dA_t}\\&+\mu normcdf\left(\bar{x}_{F,t+1}-v_{F,t}\right)\frac{R^k_{F,t}Q_{F,t-1}K_{F,t-1}}{\pi_{F,t}dA_t}\end{aligned} \tag{FA-4}$$

$$H_{F,t}=I_{F,t}+G_{F,t}+C^{ne}_{F,t}+\bar{a}\left(\exp\left(\omega(z_{F,t}-\bar{z}_F)\right)-1\right)\frac{K_{F,t-1}}{dA_t}+cv_{F,t} \tag{LM-2}$$

$$\begin{aligned}H_{F,t}=&\,I_{F,t}+G_{F,t}+C^{ne}_{F,t}+\bar{a}\left(\exp\left(\omega(z_{F,t}-\bar{z}_F)\right)-1\right)\frac{K_{F,t-1}}{dA_t}\\&+\mu normcdf\left(\bar{x}_{F,t+1}-v_{F,t}\right)\frac{R^k_{F,t}Q_{F,t-1}K_{F,t-1}}{\pi_{F,t}dA_t}+cv_{F,t}\end{aligned} \tag{FALM-1}$$

$$\mathcal{Y}_{F,t}=C_{F,t}+I_{F,t}+G_{F,t}+X_{F/E,t}+X^{\star}_{F,t}-\frac{\Sigma_E}{\Sigma_F}X_{E/F,t}-M^{\star}_{F,t}-O_{F,t}-C^o_{F,t} \tag{NOFA-5}$$

$$\mathcal{Y}_{F,t}=C_{F,t}+I_{F,t}+G_{F,t}+X_{F/E,t}+X^{\star}_{F,t}-\frac{\Sigma_E}{\Sigma_F}X_{E/F,t}-M^{\star}_{F,t}-O_{F,t}-C^o_{F,t}+C^e_{F,t} \tag{FA-5}$$

$$Z_{F,t}=\nabla^y_{F,t}Y_{F,t}+\nabla^x_{F,t}\left(X_{F/E,t}+X^{\star}_{F,t}\right) \tag{37}$$

$$\nabla^y_{F,t}=(1-\xi_y)\left(\frac{\tilde{p}^Y_{F,t}}{p^Y_{F,t}}\right)^{-\theta_y}+\xi_y\left(\frac{\bar{\pi}^{1-\gamma_y}\left(\pi^Y_{F,t-1}\right)^{\gamma_y}}{\pi^Y_{F,t}}\right)^{-\theta_y}\nabla^y_{F,t-1} \tag{38}$$

$$\nabla^x_{F,t}=(1-\xi_x)\left(\frac{\tilde{p}^X_{F,t}}{p^X_{F,t}}\right)^{-\theta_x}+\xi_x\left(\frac{\bar{\pi}^{1-\gamma_x}\left(\pi^X_{F,t-1}\right)^{\gamma_x}}{\pi^X_{F,t}}\right)^{-\theta_x}\nabla^x_{F,t-1} \tag{39}$$

$$L_{F,t}=\nabla^w_{F,t}\mathcal{L}_{F,t} \tag{NOLM-8}$$

$$\nabla^w_{F,t}=(1-\xi_w)\left(\frac{\tilde{w}_{F,t}}{w_{F,t}}\right)^{-\theta_w}+\xi_w\left(\frac{(1+g)\bar{\pi}^{1-\gamma_w}\pi^{\gamma_w}_{F,t-1}}{\pi_{F,t}}\frac{w_{F,t-1}}{w_{F,t}dA_t}\right)^{-\theta_w}\nabla^w_{F,t-1} \tag{NOLM-9}$$

$$\frac{s_{F,t}b^{\star}_{F,t}}{R^{\star}_t}-\frac{s_{F,t}b^{\star}_{F,t-1}}{\pi^{\star}_t dA_t}+\frac{b_{E/F,t}}{R_t}-\frac{b_{E/F,t-1}}{\pi_{F,t}dA_t}=p^X_{F,t}\left(X_{F/E,t}+X^{\star}_{F,t}\right)-\Phi_{E/F,t}p^X_{E,t}\frac{\Sigma_E}{\Sigma_F}X_{E/F,t} \\ -s_{F,t}p_{M,t}M^{\star}_{F,t}-s_{F,t}p^{\star}_{o,t}\left(O_{F,t}+C^o_{F,t}\right) \tag{40}$$

Government

$$\tilde{G}_{F,t}=\frac{G_{F,t}}{\overline{\mathcal{Y}}_F} \tag{41}$$

$$\tilde{G}_{F,t}=\left(\tilde{G}_{F,t-1}\right)^{\rho_G}\left(\chi_{GY}\right)^{1-\rho_G}\exp\left(\sigma_G\eta^G_{F,t}+\varsigma_G\eta^G_t\right)\qquad \eta^G_F,\ \eta^G\sim N(0,1) \tag{42}$$

Financial accelerator

$$R^k_{F,t}=\pi_{F,t}\frac{z_{F,t}r^k_{F,t}-p^{ne}_{F,t}\bar{a}\left(\exp\left(\omega(z_{F,t}-\bar{z}_F)\right)-1\right)+Q_{F,t}\left(1-\delta\right)}{Q_{F,t-1}} \tag{FA-6}$$

$$efp_{F,t}=\frac{R^k_{F,t+1}\bar{\omega}_{F,t+1}}{1-\frac{\bar{n}_{F,t}}{Q_{F,t}K_{F,t}}}-R_t \tag{FA-7}$$

$$E_t\Bigg[\left(1-\bar{\omega}_{F,t+1}\left(1-normcdf\left(\bar{x}_{F,t+1}\right)\right)-normcdf\left(\bar{x}_{F,t+1}-v_{F,t}\right)\right)\frac{R^k_{F,t+1}}{R_t} \\ +\frac{1-normcdf\left(\bar{x}_{F,t+1}\right)}{1-normcdf\left(\bar{x}_{F,t+1}\right)-\frac{p^{ne}_{F,t+1}\mu}{v_{F,t}\sqrt{2\pi}}\exp\left[-\frac{1}{2}\bar{x}^2_{F,t+1}\right]} \\ \left(\left[\bar{\omega}_{F,t+1}\left(1-normcdf\left(\bar{x}_{F,t+1}\right)\right)+\left(1-p^{ne}_{F,t+1}\mu\right)normcdf\left(\bar{x}_{F,t+1}-v_{F,t}\right)\right]\frac{R^k_{F,t+1}}{R_t}-1\right)\Bigg]=0 \tag{FA-8}$$

$$\left[\bar{\omega}_{F,t}\left(1-normcdf\left(\bar{x}_{F,t}\right)\right)+\left(1-p^{ne}_{F,t}\mu\right)normcdf\left(\bar{x}_{F,t}-v_{F,t-1}\right)\right]\frac{R^k_{F,t}}{R_{t-1}}\frac{Q_{F,t-1}K_{F,t-1}}{\bar{n}_{F,t-1}} \\ =\frac{Q_{F,t-1}K_{F,t-1}}{\bar{n}_{F,t-1}}-1 \tag{FA-9}$$

$$\bar{n}_{F,t}=\gamma_{F,t}\left[1-\bar{\omega}_{F,t}\left(1-normcdf\left(\bar{x}_{F,t}\right)\right)-normcdf\left(\bar{x}_{F,t}-v_{F,t-1}\right)\right]\frac{Q_{F,t-1}R^k_{F,t}}{\pi_{F,t}dA_t}K_{F,t-1}+W^e_{F,t} \tag{FA-10}$$

$$C^e_{F,t}=\Theta\frac{1-\gamma_{F,t}}{\gamma_{F,t}}\left(\bar{n}_{F,t}-W^e_{F,t}\right) \tag{FA-11}$$

$$\bar{x}_{F,t}=\frac{\log\bar{\omega}_{F,t}+\frac{1}{2}v^2_{F,t-1}}{v_{F,t-1}} \tag{FA-12}$$

$$\gamma_{F,t}=\left(\gamma_{F,t-1}\right)^{\rho_\gamma}\left(\bar{\gamma}_F\right)^{1-\rho_\gamma}\exp\left(\sigma_\gamma\eta^\gamma_{F,t}+\varsigma_\gamma\eta^\gamma_t\right)\qquad \eta^\gamma_F,\ \eta^\gamma\sim N(0,1) \tag{FA-13}$$

$$\tilde{B}_{F,t}=Q_{F,t}K_{F,t}-n_{F,t} \tag{FA-14}$$

$$v_{F,t}=\left(v_{F,t-1}\right)^{\rho_v}\left(\bar{v}_F\right)^{1-\rho_v}\exp\left(\sigma_v\eta^v_{F,t}+\varsigma_v\eta^v_t\right)\qquad \eta^v_F,\ \eta^v\sim\mathcal{N}(0,1) \tag{FA-15}$$

Labor market frictions

$$m_{F,t} = \Upsilon(1-(1-s)N_{F,t-1})^{1-\varphi} v_{F,t}^{\varphi} \tag{LM-3}$$

$$\mathcal{L}_{F,t} = N_{F,t} \tag{LM-4}$$

$$N_{F,t} = ((1-s)N_{F,t-1} + m_{F,t}) \tag{LM-5}$$

$$w_{F,t}^m N_{F,t} = m_{F,t}\tilde{w}_{F,t} + (1-s)(1+g)\frac{w_{F,t-1}^m}{dA_t} N_{F,t-1} \tag{LM-6}$$

$$\Psi_{F,t} = \frac{m_{F,t}}{1-(1-s)N_{F,t-1}} \tag{LM-7}$$

$$\Phi_{F,t} = \frac{m_{F,t}}{v_{F,t}} \tag{LM-8}$$

$$\frac{p_{F,t}^{ne} c}{\Phi_{F,t}} = w_{F,t} - \tilde{w}_t + \beta E_t \frac{\lambda_{F,t+1}}{\lambda_{F,t}} dA_{t+1}^{-\sigma_c} \left[\frac{\partial \mathcal{V}_{F,t+1}}{\partial N_{F,t}} dA_{t+1} + \frac{\tilde{w}_{F,t} - w_{F,t}^m}{N_{F,t}} \frac{\partial \mathcal{V}_{F,t+1}}{\partial w_{F,t}^m} \right] \tag{LM-9}$$

$$\begin{aligned}\frac{\partial \mathcal{W}_{F,t}}{\partial N_{F,t-1}} = (1-s) \Bigg[& \lambda_{F,t}\tau^R \tau^W \left(\frac{(1+g)w_{F,t-1}^m}{dA_t} - \Psi_{F,t}\tilde{w}_{F,t} \right) - (1-\Psi_{F,t})\Gamma \\ & + \beta(1-\Psi_{F,t}) E_t \frac{\partial \mathcal{W}_{F,t+1}}{\partial N_{F,t}} dA_{t+1}^{1-\sigma_c} \\ & - \frac{\beta}{N_{F,t}} \left(\Psi_{F,t}(\tilde{w}_{F,t} - w_{F,t}^m) + w_{F,t}^m - \frac{(1+g)w_{F,t-1}^m}{dA_t} \right) E_t \frac{\partial \mathcal{W}_{F,t+1}}{\partial w_{F,t}^m} dA_{t+1}^{-\sigma_c} \Bigg]\end{aligned} \tag{LM-10}$$

$$\frac{\partial \mathcal{W}_{F,t}}{\partial w_{F,t-1}^m} = (1-s)(1+g)N_{F,t-1} \left[\lambda_{F,t}\tau^R\tau^W + \frac{\beta}{N_{F,t}} E_t \frac{\partial \mathcal{W}_{F,t+1}}{\partial w_{F,t}^m} dA_{t+1}^{-\sigma_c} \right] \tag{LM-11}$$

$$\frac{\partial \mathcal{W}_{F,t}}{\partial m_{F,t}} = \left[\lambda_{F,t}\tau^R\tau^W \tilde{w}_{F,t} - \Gamma + \beta E_t \frac{\partial \mathcal{W}_{F,t+1}}{\partial N_{F,t}} dA_{t+1}^{1-\sigma_c} + \beta E_t \frac{\tilde{w}_{F,t} - w_{F,t}^m}{N_{F,t}} \frac{\partial \mathcal{W}_{F,t+1}}{\partial w_{F,t}^m} dA_{t+1}^{-\sigma_c} \right] \tag{LM-12}$$

$$\begin{aligned}\frac{\partial \mathcal{V}_{F,t}}{\partial N_{F,t-1}} = (1-s) \Bigg[& w_{F,t} - \frac{(1+g)w_{F,t-1}^m}{dA_t} + \beta E_t \frac{\lambda_{F,t+1}}{\lambda_{F,t}} dA_{t+1}^{-\sigma_c} \Bigg(\frac{\partial \mathcal{V}_{F,t+1}}{\partial N_{F,t}} dA_{t+1} \\ & - -\frac{1}{N_{F,t}} \left(w_{F,t}^m - \frac{(1+g)w_{F,t-1}^m}{dA_t} \right) \frac{\partial \mathcal{V}_{F,t+1}}{\partial w_{F,t}^m} \Bigg) \Bigg]\end{aligned} \tag{LM-13}$$

$$\frac{\partial \mathcal{V}_{F,t}}{\partial w_{F,t-1}^m} = (1-s)(1+g) \left[-N_{F,t-1} + \beta E_t \frac{\lambda_{F,t+1}}{\lambda_{F,t}} dA_{t+1}^{-\sigma_c} \frac{N_{F,t-1}}{N_{F,t}} \frac{\partial \mathcal{V}_{F,t+1}}{\partial w_{F,t}^m} \right] \tag{LM-14}$$

$$\frac{\partial \mathcal{V}_{F,t}}{\partial m_{F,t}} = \left[w_{F,t} - \tilde{w}_{F,t} + \beta E_t \frac{\lambda_{F,t+1}}{\lambda_{F,t}} dA_{t+1}^{-\sigma_c} \left(\frac{\partial \mathcal{V}_{F,t+1}}{\partial N_{F,t}} dA_{t+1} + \frac{\tilde{w}_{F,t} - w^m_{F,t}}{N_{F,t}} \frac{\partial \mathcal{V}_{F,t+1}}{\partial w^m_{F,t}} \right) \right] \tag{LM-15}$$

$$\begin{aligned} &\xi_{F,t} \left(1 - \frac{1}{N_{F,t}} \beta E_t \frac{\lambda_{F,t+1}}{\lambda_{F,t}} dA_{t+1}^{-\sigma_c} \frac{\partial \mathcal{V}_{F,t+1}}{\partial w^m_{F,t}} \right) \frac{\partial \mathcal{W}_{F,t}}{\partial m_{F,t}} \\ &= (1 - \xi_{F,t}) \left(\lambda_{F,t} \tau^R \tau^W + \frac{\beta}{N_{F,t}} E_t \frac{\partial \mathcal{W}_{F,t+1}}{\partial w^m_{F,t}} dA_{t+1}^{-\sigma} \right) \frac{\partial \mathcal{V}_{F,t}}{\partial m_{F,t}} \end{aligned} \tag{LM-16}$$

$$U_{F,t} = 1 - N_{F,t} \tag{LM-17}$$

$$\xi_{F,t} = (\xi_{F,t-1})^{\rho_\xi} \left(\bar{\xi}_F\right)^{1-\rho_\xi} \exp\left(\sigma_\xi \eta^\xi_{F,t} + \varsigma_\xi \eta^\xi_{F,t}\right) \qquad \eta^\xi_F,\ \eta^\xi \sim N(0,1) \tag{LM-18}$$

Markups

$$mk^y_{F,t} = \frac{p^Y_{F,t}}{p^Z_{F,t}} - 1; \tag{43}$$

$$mk^x_{F,t} = \frac{p^X_{F,t}}{p^Z_{F,t}} - 1; \tag{44}$$

$$mk^w_{F,t} = \frac{w_{F,t}}{w^m_{F,t}} - 1; \tag{NOLM-10}$$

$$mk^w_{F,t} = \frac{w_{F,t}\mathcal{L}_{F,t} - cp^{ne}_{F,t} v_{F,t}}{w^m_{F,t} N_{F,t}} - 1; \tag{LM-19}$$

3.B.2 Shared equations

Import price setting

$$\tilde{p}_{M,t} = \frac{\theta_{m,t}}{\theta_{m,t}-1} \frac{p^\star_{M,t}}{1 + \frac{\chi_m \pi_{M,t}}{\bar{\pi}(\theta_{m,t}-1)} \left(\frac{\pi_{M,t}}{\bar{\pi}} - 1\right)} \tag{45}$$

$$\pi_{M,t} = \frac{p_{M,t}}{p_{M,t-1}} \frac{s_{E,t}}{s_{E,t-1}} \pi_{E,t} \tag{46}$$

$$mk^m_{F,t} = \frac{p^M_t}{p^\star_{M,t}} - 1 \tag{47}$$

$$\theta_{m,t} = (\theta_{m,t-1})^{\rho_m} \bar{\theta}_m^{1-\rho_m} \exp\left(\sigma_m \eta_{m,t}\right) \qquad \eta_m \sim N(0,1) \tag{48}$$

Exports

$$X_t = \left[(a^x)^{\frac{1}{s_x}} \left(\Sigma_E X^\star_{E,t}\right)^{\frac{s_x-1}{s_x}} + (1-a^x)^{\frac{1}{s_x}} \left(\Sigma_F X^\star_{F,t}\right)^{\frac{s_x-1}{s_x}} \right]^{\frac{s_x}{s_x-1}} \tag{49}$$

$$\frac{\Sigma_E X^\star_{E,t}}{\Sigma_F X^\star_{F,t}} = \frac{a^x}{1-a^x} \left(\frac{p^X_{F,t}}{p^X_{E,t}} \Phi_{F/E,t} \right)^{s_x} \tag{50}$$

$$p_{X,t} = \left(a^x \left(\frac{p^X_{E,t}}{s_{E,t}} \right)^{1-s_x} + (1-a^x) \left(\frac{p^X_{F,t}}{s_{F,t}} \right)^{1-s_x} \right)^{\frac{1}{1-s_x}} \tag{51}$$

$$\frac{D_t \varepsilon_{D,t}}{X_t \left(1 - \frac{a^\star}{2} \left(\frac{X_t D_{t-1}}{X_{t-1} D_t} - 1 \right)^2 \right)} = \left(\frac{p_{X,t}}{p^\star_{X,t} \left[1 - \frac{a^\star}{2} \left(\frac{X_t D_{t-1}}{X_{t-1} D_t} - 1 \right)^2 - a^\star \left(\frac{X_t D_{t-1}}{X_{t-1} D_t} \right) \left(\frac{X_t D_{t-1}}{X_{t-1} D_t} - 1 \right) \right]} \right)^{\mu^\star} \tag{52}$$

$$\pi_{X,t} = \frac{p_{X,t}}{p_{X,t-1}} \frac{s_{E,t}}{s_{E,t-1}} \pi_{E,t} \tag{53}$$

$$\varepsilon_{D,t} = \varepsilon^{\rho_S}_{D,t-1} \exp\left(\sigma_S \eta_{S,t}\right) \qquad \eta_S \sim N(0,1) \tag{54}$$

Monetary policy

$$R_t = R^{\rho_R}_{t-1} \left[\bar{R} \left(\frac{\pi_{t-1}}{\bar{\pi}} \right)^{r_\pi} \left(\frac{\pi_t}{\pi_{t-1}} \right)^{r_{\Delta\pi}} \left(\frac{\mathcal{Y}_t}{\mathcal{Y}_{t-1}} \frac{dA_t}{1+g} \right)^{r_Y} \right]^{1-\rho_R} \varepsilon_{R,t} \tag{55}$$

$$\Phi_{F/E,t} \Sigma_F b_{E/F,t} + \Sigma_E b_{F/E,t} = 0 \tag{56}$$

$$s_{F,t} = s_{E,t} \Phi_{E/F,t} \tag{57}$$

$$\Phi_{E/F,t} = \frac{\pi_{E,t}}{\pi_{F,t}} \Phi_{E/F,t-1} \tag{58}$$

$$\Phi_{F/E,t} = \frac{1}{\Phi_{E/F,t}} \tag{59}$$

$$\varepsilon^R_t = \left(\varepsilon^R_{t-1} \right)^{\rho_R} \exp\left(\sigma_R \eta_{R,t}\right) \qquad \eta_R \sim N(0,1) \tag{60}$$

Euro area aggregates

$$C_t = \Sigma_F C_{F,t} + \Sigma_E C_{E,t} \tag{NOFA-6}$$

$$C_t = \Sigma_F (C_{F,t} + C^e_{F,t}) + \Sigma_E (C_{E,t} + C^e_{E,t}) \tag{FA-16}$$

$$I_t = \Sigma_F I_{F,t} + \Sigma_E I_{E,t} \tag{61}$$

$$\mathcal{Y}_t = \Sigma_F \mathcal{Y}_{F,t} + \Sigma_E \mathcal{Y}_{E,t} \tag{62}$$

$$\mathcal{L}_t = \Sigma_F \mathcal{L}_{F,t} + \Sigma_E \mathcal{L}_{E,t} \tag{63}$$

$$w_t = \Sigma_F w_{F,t} \frac{\mathcal{L}_{F,t}}{\mathcal{L}_t} + \Sigma_E w_{E,t} \frac{\mathcal{L}_{E,t}}{\mathcal{L}_t} \tag{64}$$

$$N_t = \Sigma_F N_{F,t} + \Sigma_E N_{E,t} \tag{LM-20}$$

$$U_t = 100(1 - N_t) \tag{LM-21}$$

$$z_t = z_{F,t} \frac{\Sigma_F K_{F,t-1}}{\Sigma_F K_{F,t-1} + \Sigma_E K_{E,t-1}} + z_{E,t} \frac{\Sigma_E K_{E,t-1}}{\Sigma_F K_{F,t-1} + \Sigma_E K_{E,t-1}} \tag{65}$$

$$\pi_t = \pi^{\Sigma_F}_{F,t} \pi^{\Sigma_E}_{E,t} \tag{66}$$

$$M_t = \Sigma_F M^\star_{F,t} + \Sigma_E M^\star_{E,t} \tag{67}$$

$$dA_t = \left(dA_{t-1}\right)^{\rho_g} (1+g)^{1-\rho_g} \exp\left(\sigma_g \eta_{g,t}\right) \qquad \eta_g \sim N(0,1) \tag{68}$$

$$efp_t = \frac{\Sigma_F \tilde{B}_{F,t} efp_{F,t} + \Sigma_E \tilde{B}_{E,t} efp_{E,t}}{\tilde{B}_t} \tag{FA-17}$$

$$\tilde{B}_t = \Sigma_F \tilde{B}_{F,t} + \Sigma_E \tilde{B}_{E,t} \tag{FA-18}$$

Rest of the world

$$R_t^\star = \left(R_{t-1}^\star\right)^{\rho_{R\star}} \left(\bar{R}^\star\right)^{1-\rho_{R\star}} \exp\left(\sigma_{R\star}\eta_{R\star,t}\right) \qquad \eta_{R\star} \sim N(0,1) \tag{69}$$

$$\pi_t^\star = \left(\pi_{t-1}^\star\right)^{\rho_{\pi^\star}} \bar{\pi}^{1-\rho_{\pi^\star}} \exp\left(\sigma_{\pi^\star}\eta_{\pi^\star,t} + \varrho\eta_{O,t} + \gamma\eta_{D,t}\right), \qquad \eta_{\pi^\star} \sim N(0,1). \tag{70}$$

$$\pi_{o,t} = \frac{s_{E,t}}{s_{E,t-1}} \frac{p_{o,t}^\star}{p_{o,t-1}^\star} \pi_{E,t} \tag{71}$$

$$p_{o,t}^\star = \left(p_{o,t-1}^\star\right)^{\rho_O} \exp\left(\sigma_O\eta_{O,t} + \kappa\eta_{D,t}\right), \qquad \eta_O \sim N(0,1) \tag{72}$$

$$\pi_{M,t}^\star = \frac{s_{E,t}}{s_{E,t-1}} \frac{p_{M,t}^\star}{p_{M,t-1}^\star} \pi_{E,t} \tag{73}$$

$$p_{M,t}^\star = \left(p_{M,t-1}^\star\right)^{\rho_{M\star}} \exp\left(\sigma_{M\star}\left(\eta_{M\star,t} + \vartheta_M\eta_{M\star,t-1}\right)\right) \qquad \eta_{M\star} \sim N(0,1) \tag{74}$$

$$\pi_{X,t}^\star = \frac{p_{X,t}^\star}{p_{X,t-1}^\star} \frac{s_{E,t}}{s_{E,t-1}} \pi_{E,t} \tag{75}$$

$$p_{X,t}^\star = \left(p_{X,t-1}^\star\right)^{\rho_X^\star} \exp\left(\sigma_X^\star\eta_{X,t}^\star\right) \qquad \eta_X^\star \sim N(0,1) \tag{76}$$

$$D_t = D_{t-1}^{\rho_D} \bar{D}^{1-\rho_D} \exp\left(\sigma_D\left(\eta_{D,t} + \vartheta_D\eta_{D,t-1}\right)\right), \qquad \eta_D \sim N(0,1) \tag{77}$$

Observable variables

$$dC_{F,t} = 100\log\left(\frac{C_{F,t}}{C_{F,t-1}} dA_t\right) - 100\log(1+g) \tag{NOFA-7}$$

$$dC_{F,t} = 100\log\left(\frac{C_{F,t} + C_{F,t}^e}{C_{F,t-1} + C_{F,t-1}^e} dA_t\right) - 100\log(1+g) \tag{FA-19}$$

$$dY_{F,t} = 100\log\left(\frac{\mathcal{Y}_{F,t}}{\mathcal{Y}_{F,t-1}} dA_t\right) - 100\log(1+g) \tag{78}$$

$$dI_{F,t} = 100\log\left(\frac{I_{F,t}}{I_{F,t-1}} dA_t\right) - 100\log(1+g) \tag{79}$$

$$dW_{F,t} = 100\log\left(\frac{w_{F,t}\mathcal{L}_{F,t}}{w_{F,t-1}\mathcal{L}_{F,t-1}} dA_t\right) - 100\log(1+g) \tag{NOLM-11}$$

$$dW_{F,t} = 100\log\left(\frac{w_{F,t}^m N_{F,t}}{w_{F,t-1}^m N_{F,t-1}} dA_t\right) - 100\log(1+g) \tag{LM-22}$$

$$\tilde{\mathcal{L}}_{F,t} = 100\log\left(\frac{\mathcal{L}_{F,t}}{\bar{\mathcal{L}}_F}\right) \tag{80}$$

$$HICP_{F,t} = 100\log\left(\pi_{F,t}\pi_{F,t-1}\pi_{F,t-2}\pi_{F,t-3}\right) - 400\log\bar{\pi} \tag{81}$$

$$dM_{F,t} = 100\log\left(\frac{M_{F,t} + O_{F,t} + C_{F,t}^o}{M_{F,t-1} + O_{F,t-1} + C_{F,t-1}^o} dA_t\right) - 100\log(1+g) \tag{82}$$

$$\tilde{\pi}^X_{F,t} = 100 \log \left(\pi^X_{F,t}\right) - 100 \log \bar{\pi} \tag{83}$$

$$\tilde{\pi}_{o,t} = 100 \log \left(\pi_{o,t}\right) - 100 \log \bar{\pi} \tag{84}$$

$$dY_t = 100 \log \left(\frac{\mathcal{Y}_t}{\mathcal{Y}_{t-1}} dA_t \right) - 100 \log(1+g) \tag{85}$$

$$dC_t = 100 \log \left(\frac{C_t}{C_{t-1}} dA_t \right) - 100 \log(1+g) \tag{86}$$

$$dI_t = 100 \log \left(\frac{I_t}{I_{t-1}} dA_t \right) - 100 \log(1+g) \tag{87}$$

$$dW_t = 100 \log \left(\frac{w_t \mathcal{L}_t}{w_{t-1} \mathcal{L}_{t-1}} dA_t \right) - 100 \log(1+g) \tag{NOLM-12}$$

$$dW_t = 100 \log \left(\frac{w^m_t N_t}{w^m_{t-1} N_{t-1}} dA_t \right) - 100 \log(1+g) \tag{LM-23}$$

$$\tilde{\mathcal{L}}_t = \frac{\mathcal{L}_t + \mathcal{L}_{t-1} + \mathcal{L}_{t-2} + \mathcal{L}_{t-3}}{4\left(\Sigma_F \bar{\mathcal{L}}_F + \Sigma_E \bar{\mathcal{L}}_E\right)} \tag{88}$$

$$HICP_t = 100 \log \left(\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}\right) - 400 \log \bar{\pi} \tag{89}$$

$$dX_t = 100 \log \left(\frac{X_t}{X_{t-1}} dA_t \right) - 100 \log(1+g) \tag{90}$$

$$dM_t = 100 \log \left(\frac{M_t + \Sigma_F \left(O_{F,t} + C^o_{F,t}\right) + \Sigma_E \left(O_{E,t} + C^o_{E,t}\right)}{M_{t-1} + \Sigma_F \left(O_{F,t-1} + C^o_{F,t-1}\right) + \Sigma_E \left(O_{E,t-1} + C^o_{E,t-1}\right)} dA_t \right) - 100 \log(1+g) \tag{91}$$

$$\tilde{\pi}^\star_{X,t} = 100 \log \left(\pi^\star_{X,t}\right) - 100 \log \bar{\pi} \tag{92}$$

$$\tilde{\pi}^\star_{M,t} = 100 \log \left(\pi^\star_{M,t}\right) - 100 \log \bar{\pi} \tag{93}$$

$$\tilde{\pi}_{X,t} = 100 \log \left(\pi_{X,t}\right) - 100 \log \bar{\pi} \tag{94}$$

$$\tilde{R}_t = 100 \log R_t - 100 \log \bar{R} \tag{95}$$

$$dD_t = 100 \log \left(\frac{D_t}{D_{t-1}} dA_t \right) - 100 \log(1+g) \tag{96}$$

$$dX_{F,t} = 100 \log \left(\frac{X_{F/E,t} + X^\star_{F,t}}{X_{F/E,t-1} + X^\star_{F,t-1}} dA_t \right) - 100 \log(1+g) \tag{97}$$

$$dS_t = 100 \log \left(\frac{s_{E,t}}{s_{E,t-1}} \frac{\pi_{E,t}}{\pi^\star_t} \right) \tag{98}$$

$$\tilde{\pi}^\star_t = 100 \log \left(\pi^\star_t\right) - 100 \log \bar{\pi} \tag{99}$$

$$dn_{F,t} = 100 \log \left(\frac{n_{F,t}}{n_{F,t-1}} dA_t \right) - 100 \log(1+g) \tag{FA-20}$$

$$dn_t = 100 \log \left(\frac{\Sigma_F n_{F,t} + \Sigma_E n_{E,t}}{\Sigma_F n_{F,t-1} + \Sigma_E n_{E,t-1}} dA_t \right) - 100 \log(1+g) \tag{FA-21}$$

$$dB_{F,t} = 100 \log \left(\frac{\tilde{B}_{F,t}}{\tilde{B}_{F,t-1}} dA_t \right) - 100 \log(1+g) \tag{FA-22}$$

$$dB_t = 100 \log\left(\frac{\tilde{B}_t}{\tilde{B}_{t-1}} dA_t\right) - 100\log(1+g) \tag{FA-23}$$

$$\tilde{U}_{F,t} = U_{F,t} - 100(1 - \bar{N}_F) \tag{LM-24}$$

$$\tilde{U}_t = U_t - 100(1 - \Sigma_F \bar{N}_F - \Sigma_E \bar{N}_E) \tag{LM-25}$$

$$\tilde{z}_{F,t} = 100 z_{F,t} - 100\bar{z}_F \tag{100}$$

$$\tilde{z}_t = 100 z_t - 100\bar{z} \tag{101}$$

3.C Resolution of the steady state equilibrium

3.C.1 Baseline version (no financial or labor market frictions)

Steady state parameters and assumptions

All inflation rates are $\bar{\pi}$ and both foreign and euro area nominal interest factors are $\bar{R}$ at steady state. The Euler equations yield:

$$\Psi_F = \Psi_F^\star = \frac{\bar{\pi}}{\beta\bar{R}}(1+g)^{\sigma_c^F} \text{ and } \Psi_E = \Psi_E^\star = \frac{\bar{\pi}}{\beta\bar{R}}(1+g)^{\sigma_c^E}$$

The discount factor is set such that the weighted geometric average of financial intermediation factors in the euro area equals to one:

$$\Psi_F^{\Sigma_F}\Psi_E^{\Sigma_E} = (\Psi_F^\star)^{\Sigma_F}(\Psi_E^\star)^{\Sigma_E} = \frac{\bar{\pi}}{\beta\bar{R}}(1+g)^{\Sigma_F\sigma_c^F + \Sigma_E\sigma_c^E} = 1 \quad \Longrightarrow \quad \beta = \frac{\bar{\pi}}{\bar{R}}(1+g)^{\Sigma_F\sigma_c^F+\Sigma_E\sigma_c^E}$$

We impose that the following long run ratios can be set freely:
$\frac{G_F}{\mathcal{Y}_F} \equiv \chi_G^F$, $\frac{G_E}{\mathcal{Y}_E} \equiv \chi_G^E$, $\frac{\Sigma_F M_F^\star}{\Sigma_E X_{E/F}} \equiv \chi_M^F$, $\frac{\Sigma_E M_E^\star}{\Sigma_F X_{F/E}} \equiv \chi_M^E$, $\frac{M_F}{Y_F} \equiv \chi_Y^F$, $\frac{M_E}{Y_E} \equiv \chi_Y^E$, $\frac{X_F^\star}{\mathcal{Y}_F} \equiv \chi_X^F$, $\frac{X_E^\star}{\mathcal{Y}_E} \equiv \chi_X^E$, $\frac{C_F^{ne}}{C_F^o} = \chi_F^{ne}$, $\frac{C_E^{ne}}{C_F^o} = \chi_E^{ne}$, $\frac{I_F}{O_F} = \chi_F^{io}$ and $\frac{I_E}{O_E} = \chi_E^{io}$
which may results in constraints on some parameters of the model. In particular, the optimal choices of the domestic importer and the intermediate retailer lead to:

$$\bar{a}_m^F = \frac{1}{1+\chi_M^F\left(\frac{p_M}{p_E^X}\right)^{s_F^m}} \text{ and } \bar{a}_m^E = \frac{1}{1+\chi_M^E\left(\frac{p_M}{p_F^X}\right)^{s_E^m}}$$

$$\bar{a}_y^F = \frac{1}{1+\chi_Y^F\left(\frac{p_F^M}{p_F^Y}\right)^{s_F^y}} \text{ and } \bar{a}_y^E = \frac{1}{1+\chi_Y^E\left(\frac{p_E^M}{p_E^Y}\right)^{s_E^y}}$$

$$\bar{a}_o^F = \frac{\chi_F^{ne}}{1+\chi_F^{ne}} \text{ and } \bar{a}_o^E = \frac{\chi_E^{ne}}{1+\chi_E^{ne}}$$

Prices

The model does not determine the long run levels of several variables, among which:

- relative prices: $p_o^\star$, $p_M^\star$, $p_X^\star$;

- real exchange rates: s_E, s_F, $\Phi_{E/F}$ and $\Phi_{F/E}$.

These variables are calibrated at 1. The CES agregation technology of the final retailers and the first order conditions for investment thus yield:

$$p_F^{ne} = p_E^{ne} = Q_F = Q_E = 1$$

and the import price setting equations:

$$p_M = \tilde{p}_M = \frac{\theta_m}{\theta_m - 1}$$

From the first order conditions for capacity utilization and capital stock, we get the following constraint on the steady state value of the first derivative of the capital depreciation rate as a function of the capacity utilization rate:

$$\delta'_F = \frac{\frac{1}{\beta}(1+g)^{\sigma_c^F} - 1 + \delta_F}{z_F} \text{ and } \delta'_E = \frac{\frac{1}{\beta}(1+g)^{\sigma_c^E} - 1 + \delta_E}{z_E}$$

and the expressions of the capital return rates:

$$r_F^k = \frac{\delta'_F}{\tau_F^R} \text{ and } r_E^k = \frac{\delta'_E}{\tau_E^R}$$

The CES technology of the equipment provider yields:

$$p_F^v = \left(a_v^F \left(r_F^k\right)^{1-s_v^F} + (1-a_v^F)\right)^{\frac{1}{1-s_v^F}} \text{ and } p_E^v = \left(a_v^E \left(r_E^k\right)^{1-s_v^E} + (1-a_v^E)\right)^{\frac{1}{1-s_v^E}}$$

The first order and zero profit conditions of the producer determine the steady state production prices as functions of real hourly wages:

$$p_F^Z = \left(\frac{p_F^v}{\alpha^F}\right)^{\alpha^F} \left(\frac{\tau_F^L w_F}{1-\alpha^F}\right)^{1-\alpha^F} \text{ and } p_E^Z = \left(\frac{p_E^v}{\alpha^E}\right)^{\alpha^E} \left(\frac{\tau_E^L w_E}{1-\alpha^E}\right)^{1-\alpha^E}$$

Let us denote:

$$a_1^F \equiv \frac{\theta_x^F}{\theta_x^F - 1} \left(\frac{p_F^v}{\alpha^F}\right)^{\alpha^F} \left(\frac{\tau_F^L}{1-\alpha^F}\right)^{1-\alpha^F} \text{ and } a_1^E \equiv \frac{\theta_x^E}{\theta_x^E - 1} \left(\frac{p_E^v}{\alpha^E}\right)^{\alpha^E} \left(\frac{\tau_E^L}{1-\alpha^E}\right)^{1-\alpha^E}$$

$$a_2^F \equiv \frac{\theta_y^F}{\theta_y^F - 1} \left(\frac{p_F^v}{\alpha^F}\right)^{\alpha^F} \left(\frac{\tau_F^L}{1-\alpha^F}\right)^{1-\alpha^F} \text{ and } a_2^E \equiv \frac{\theta_y^E}{\theta_y^E - 1} \left(\frac{p_E^v}{\alpha^E}\right)^{\alpha^E} \left(\frac{\tau_E^L}{1-\alpha^E}\right)^{1-\alpha^E}$$

Then with intermediate firms price setting equations:

$$\tilde{p}_F^Y = p_F^Y = \frac{\theta_y^F}{\theta_y^F - 1} p_F^Z = a_2^F w_F^{1-\alpha^F} \text{ and } \tilde{p}_E^Y = p_E^Y = \frac{\theta_y^E}{\theta_y^E - 1} p_E^Z = a_2^E w_E^{1-\alpha^E}$$

$$\tilde{p}_F^X = p_F^X = \frac{\theta_x^F}{\theta_x^F - 1} p_F^Z = a_1^F w_F^{1-\alpha^F} \text{ and } \tilde{p}_E^X = p_E^X = \frac{\theta_x^E}{\theta_x^E - 1} p_E^Z = a_1^E w_E^{1-\alpha^E}$$

The CES technology of the domestic importers implies:

$$p_F^M = \left(\bar{a}_m^F \left(p_E^X\right)^{1-s_m^F} + (1-\bar{a}_m^F) p_M^{1-s_m^F}\right)^{\frac{1}{1-s_m^F}} \text{ and } p_E^M = \left(\bar{a}_m^E \left(p_F^X\right)^{1-s_m^E} + (1-\bar{a}_m^E) p_M^{1-s_m^E}\right)^{\frac{1}{1-s_m^E}}$$

With the expressions for a_m^F and a_m^E above, the relative price of imports can be written:

$$p_F^M = \left(\frac{p_E^X + \chi_M^F p_M}{(p_E^X)^{s_m^F} + \chi_M^F \left(p_M\right)^{s_m^F}}\right)^{\frac{1}{1-s_m^F}} \text{ and } p_E^M = \left(\frac{p_F^X + \chi_M^E p_M}{(p_F^X)^{s_m^E} + \chi_M^E \left(p_M\right)^{s_m^E}}\right)^{\frac{1}{1-s_m^E}}$$

From the CES technology of the intermediate retailer, we have:

$$p_F^{ne} = \left(\bar{a}_y^F \left(p_F^Y\right)^{1-s_y^F} + (1-\bar{a}_y^F) \left(p_F^M\right)^{1-s_y^F}\right)^{\frac{1}{1-s_y^F}} \text{ and } p_E^{ne} = \left(\bar{a}_y^E \left(p_E^Y\right)^{1-s_y^E} + (1-\bar{a}_y^E) \left(p_E^M\right)^{1-s_y^E}\right)^{\frac{1}{1-s_y^E}}$$

With the expressions for a_y^F and a_y^E above, the relative price of non energy final goods can be written:

$$p_F^{ne} = \left(\frac{p_F^Y + \chi_Y^F p_F^M}{(p_F^Y)^{s_y^F} + \chi_Y^F \left(p_F^M\right)^{s_y^F}}\right)^{\frac{1}{1-s_y^F}} \text{ and } p_E^{ne} = \left(\frac{p_E^Y + \chi_Y^E p_E^M}{(p_E^Y)^{s_y^E} + \chi_Y^E \left(p_E^M\right)^{s_y^E}}\right)^{\frac{1}{1-s_y^E}}$$

Or, using the expressions of relative prices as functions of wage rates w_F and w_E:

$$p_F^{ne} = \left(\frac{a_2^F w_F^{1-\alpha^F} + \chi_Y^F \left(\frac{a_1^E w_E^{1-\alpha^E} + \chi_M^F p_M}{\left(a_1^E w_E^{1-\alpha^E}\right)^{s_m^F} + \chi_M^F (p_M)^{s_m^F}}\right)^{\frac{1}{1-s_m^F}}}{\left(a_2^F w_F^{1-\alpha^F}\right)^{s_y^F} + \chi_Y^F \left(\frac{a_1^E w_E^{1-\alpha^E} + \chi_M^F p_M}{\left(a_1^E w_E^{1-\alpha^E}\right)^{s_m^F} + \chi_M^F (p_M)^{s_m^F}}\right)^{\frac{s_y^F}{1-s_m^F}}}\right)^{\frac{1}{1-s_y^F}}$$

$$p_E^{ne} = \left(\frac{a_2^E w_E^{1-\alpha^E} + \chi_Y^E \left(\frac{a_1^F w_F^{1-\alpha^F} + \chi_M^E p_M}{\left(a_1^F w_F^{1-\alpha^F}\right)^{s_m^E} + \chi_M^E (p_M)^{s_m^E}} \right)^{\frac{1}{1-s_m^E}}}{\left(a_2^E w_E^{1-\alpha^E}\right)^{s_y^E} + \chi_Y^E \left(\frac{a_1^F w_F^{1-\alpha^F} + \chi_M^E p_M}{\left(a_1^F w_F^{1-\alpha^F}\right)^{s_m^E} + \chi_M^E (p_M)^{s_m^E}} \right)^{\frac{s_y^E}{1-s_m^E}}} \right)^{\frac{1}{1-s_y^E}}$$

The conditions $p_F^{ne} = 1$ and $p_E^{ne} = 1$ determines a system of two non linear equations with two unknown variables w_F and w_E:

$$\begin{cases} f_1 (w_F, w_E) = 0 \\ f_2 (w_F, w_E) = 0 \end{cases}$$

with

$$\begin{aligned} f_1 (w_F, w_E) = {} & a_2^F w_F^{1-\alpha^F} + \chi_Y^F \left(\frac{a_1^E w_E^{1-\alpha^E} + \chi_M^F p_M}{\left(a_1^E w_E^{1-\alpha^E}\right)^{s_m^F} + \chi_M^F (p_M)^{s_m^F}} \right)^{\frac{1}{1-s_m^F}} \\ & - \left(a_2^F w_F^{1-\alpha^F}\right)^{s_y^F} - \chi_Y^F \left(\frac{a_1^E w_E^{1-\alpha^E} + \chi_M^F p_M}{\left(a_1^E w_E^{1-\alpha^E}\right)^{s_m^F} + \chi_M^F (p_M)^{s_m^F}} \right)^{\frac{s_y^F}{1-s_m^F}} \end{aligned}$$

$$\begin{aligned} f_2 (w_F, w_E) = {} & a_2^E w_E^{1-\alpha^E} + \chi_Y^E \left(\frac{a_1^F w_F^{1-\alpha^F} + \chi_M^E p_M}{\left(a_1^F w_F^{1-\alpha^F}\right)^{s_m^E} + \chi_M^E (p_M)^{s_m^E}} \right)^{\frac{1}{1-s_m^E}} \\ & - \left(a_2^E w_E^{1-\alpha^E}\right)^{s_y^E} - \chi_Y^E \left(\frac{a_1^F w_F^{1-\alpha^F} + \chi_M^E p_M}{\left(a_1^F w_F^{1-\alpha^F}\right)^{s_m^E} + \chi_M^E (p_M)^{s_m^E}} \right)^{\frac{s_y^E}{1-s_m^E}} \end{aligned}$$

The system is solved numerically using a Newton algorithm in order to calculate the steady state values of w_F and w_E. For that purpose, we calculate the analytical expression of the jacobian matrix of (f_1, f_2):

$$J (w_F, w_E) = \begin{pmatrix} \frac{\partial f_1}{\partial w_F} & \frac{\partial f_1}{\partial w_E} \\ \frac{\partial f_2}{\partial w_F} & \frac{\partial f_2}{\partial w_E} \end{pmatrix} \equiv \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}$$

$$\frac{\partial f_1}{\partial w_F} = \left(1-\alpha^F\right) a_2^F w_F^{-\alpha^F} - \left(1-\alpha^F\right) s_y^F \left(a_2^F\right)^{s_y^F} w_F^{(1-\alpha_F)s_y^F - 1}$$

$$\frac{\partial f_2}{\partial w_E} = \left(1-\alpha^E\right) a_2^E w_E^{-\alpha^E} - \left(1-\alpha^E\right) s_y^E \left(a_2^E\right)^{s_y^E} w_E^{(1-\alpha_E)s_y^E - 1}$$

Let

$$\Phi_F(w_E) \equiv \frac{a_1^E w_E^{1-\alpha^E} + \chi_M^F p_M}{\left(a_1^E w_E^{1-\alpha^E}\right)^{s_m^F} + \chi_M^F (p_M)^{s_m^F}} \text{ and } \Phi_E(w_F) \equiv \frac{a_1^F w_F^{1-\alpha^F} + \chi_M^E p_M}{\left(a_1^F w_F^{1-\alpha^F}\right)^{s_m^E} + \chi_M^E (p_M)^{s_m^E}}$$

With this notation, we have:

$$\Phi_F'(w_E) = \left\{\left(1-\alpha^E\right)\left(1-s_m^F\right)\left(a_1^E\right)^{1+s_m^F} w_E^{\left(1-\alpha^E\right)s_m^F-\alpha^E} + \left(1-\alpha^E\right) a_1^E w_E^{-\alpha^E} \chi_M^F p_M^{s_m^F} \right.$$
$$\left. - \left(1-\alpha^E\right) s_m^F \left(a_1^E\right)^{s_m^F} w_E^{\left(1-\alpha^E\right)s_m^F-1} \chi_M^F p_M \right\} \left[\left(a_1^E w_E^{1-\alpha^E}\right)^{s_m^F} + \chi_M^F (p_M)^{s_m^F}\right]^{-2}$$

$$\Phi_E'(w_F) = \left\{\left(1-\alpha^F\right)\left(1-s_m^E\right)\left(a_1^F\right)^{1+s_m^E} w_F^{\left(1-\alpha^F\right)s_m^E-\alpha^F} + \left(1-\alpha^F\right) a_1^F w_F^{-\alpha^F} \chi_M^E p_M^{s_m^E} \right.$$
$$\left. - \left(1-\alpha^F\right) s_m^E \left(a_1^F\right)^{s_m^E} w_F^{\left(1-\alpha^F\right)s_m^E-1} \chi_M^E p_M \right\} \left[\left(a_1^F w_F^{1-\alpha^F}\right)^{s_m^E} + \chi_M^E (p_M)^{s_m^E}\right]^{-2}$$

$$\frac{\partial f_1}{\partial w_E} = \frac{\chi_Y^F}{1-s_m^F} \Phi_F(w_E)^{\frac{s_m^F}{1-s_m^F}} \Phi_F'(w_E) \left[1 - s_y^F \Phi_F(w_E)^{\frac{s_y^F-1}{1-s_m^F}}\right]$$

$$\frac{\partial f_2}{\partial w_F} = \frac{\chi_Y^E}{1-s_m^E} \Phi_E(w_F)^{\frac{s_m^E}{1-s_m^E}} \Phi_E'(w_F) \left[1 - s_y^E \Phi_E(w_F)^{\frac{s_y^E-1}{1-s_m^E}}\right]$$

The algorithm is initialized with some value (w_F^0, w_E^0). The value of (w_F, w_E) is updated S times as explained in what follows until $f_1(w_F, w_E)$ and $f_2(w_F, w_E)$ are both close enough to zero. At each step $s \in [0, S-1]$, the initial value of the unknown vector, (w_F^s, w_E^s), is updated with the abscissa $\left(w_F^{s+1}, w_E^{s+1}\right)$ of the intercept of the tangent of (f_1, f_2) in (w_F^s, w_E^s):

$$J(w_F^s, w_E^s) \begin{pmatrix} w_F^{s+1} - w_F^s \\ w_E^{s+1} - w_E^s \end{pmatrix} = - \begin{pmatrix} f_1(w_F^s, w_E^s) \\ f_2(w_F^s, w_E^s) \end{pmatrix}$$

The solution of this equation is:

$$\begin{pmatrix} w_F^{s+1} \\ w_E^{s+1} \end{pmatrix} = \begin{pmatrix} w_F^s \\ w_E^s \end{pmatrix} + \begin{pmatrix} \frac{f_2\left(w_F^s, w_E^s\right) J_{12} - f_1\left(w_F^s, w_E^s\right) J_{22}}{J_{11}J_{22} - J_{12}J_{21}} \\ \frac{f_1\left(w_F^s, w_E^s\right) J_{21} - f_2\left(w_F^s, w_E^s\right) J_{11}}{J_{11}J_{22} - J_{12}J_{21}} \end{pmatrix}$$

Activity

On one hand, market clearing conditions imply:

$$\mathcal{Y}_F = Y_F + X_{F/E} + \left(\chi_X^F - \frac{O_F}{\mathcal{Y}_F}\right)\mathcal{Y}_F \text{ and } \mathcal{Y}_E = Y_E + X_{E/F} + \left(\chi_X^E - \frac{O_E}{\mathcal{Y}_E}\right)\mathcal{Y}_E \tag{102}$$

On the other hand, from the zero profit condition of the domestic importers, we get:

$$p_F^M \chi_Y^F Y_F = \frac{\Sigma_E}{\Sigma_F}\left(p_E^X + p_M \chi_M^F\right) X_{E/F} \text{ and } p_E^M \chi_Y^E Y_E = \frac{\Sigma_F}{\Sigma_E}\left(p_F^X + p_M \chi_M^E\right) X_{F/E} \tag{103}$$

The ratio $\frac{O}{\mathcal{Y}}$:

The first order conditions and the CES technology of the equipment providers yield[3]:

$$\frac{\tilde{K}_F}{O_F} = \frac{a_v^F}{1-a_v^F}\left(r_F^k\right)^{-s_v^F} \text{ and } \frac{\tilde{K}_E}{O_E} = \frac{a_v^E}{1-a_v^E}\left(r_E^k\right)^{-s_v^E}$$

$$\frac{V_F}{O_F} = \left(\left(a_v^F\right)^{\frac{1}{s_v^F}}\left(\frac{\tilde{K}_F}{O_F}\right)^{\frac{s_v^F-1}{s_v^F}} + \left(1-a_v^F\right)^{\frac{1}{s_v^F}}\right)^{\frac{s_v^F}{s_v^F-1}} \text{ and } \frac{V_E}{O_E} = \left(\left(a_v^E\right)^{\frac{1}{s_v^E}}\left(\frac{\tilde{K}_E}{O_E}\right)^{\frac{s_v^E-1}{s_v^E}} + \left(1-a_v^E\right)^{\frac{1}{s_v^E}}\right)^{\frac{s_v^E}{s_v^E-1}}$$

The first order conditions of the producers yield:

$$\frac{V_F}{\mathcal{L}_F} = \frac{\alpha^F}{1-\alpha^F}\frac{\bar{\tau}_F^L w_F}{p_F^v} \text{ and } \frac{V_E}{\mathcal{L}_E} = \frac{\alpha^E}{1-\alpha^E}\frac{\bar{\tau}_E^L w_E}{p_E^v}$$

From the aggregated production function:

$$\mathcal{Y}_F = V_F^{\alpha^F}\mathcal{L}_F^{1-\alpha^F} - O_F \text{ and } \mathcal{Y}_E = V_E^{\alpha^E}\mathcal{L}_E^{1-\alpha^E} - O_E$$

$$\frac{O_F}{\mathcal{Y}_F} = \left(\frac{V_F}{O_F}\left(\frac{V_F}{\mathcal{L}_F}\right)^{\alpha^F-1} - 1\right)^{-1} \text{ and } \frac{O_E}{\mathcal{Y}_E} = \left(\frac{V_E}{O_E}\left(\frac{V_E}{\mathcal{L}_E}\right)^{\alpha^E-1} - 1\right)^{-1}$$

Expression of Y as a function of $\mathcal{Y}$:

The zero profit condition of the intermediate retailers imply:

$$I_F + G_F + C_F^{ne} = p_F^Y Y_F + p_F^M M_F \text{ and } I_E + G_E + C_E^{ne} = p_E^Y Y_E + p_E^M M_E$$

$$\frac{Y_F}{\mathcal{Y}_F} = \left(p_F^Y + p_F^M \chi_Y^F\right)^{-1}\left(\frac{I_F}{K_F}\frac{K_F}{\tilde{K}_F}\frac{\tilde{K}_F}{O_F}\frac{O_F}{\mathcal{Y}_F} + \chi_G^F + \frac{C_F^{ne}}{C_F}\frac{C_F}{\mathcal{Y}_F}\right)$$

$$\frac{Y_E}{\mathcal{Y}_E} = \left(p_E^Y + p_E^M \chi_Y^E\right)^{-1}\left(\frac{I_E}{K_E}\frac{K_E}{\tilde{K}_E}\frac{\tilde{K}_E}{O_E}\frac{O_E}{\mathcal{Y}_E} + \chi_G^E + \frac{C_E^{ne}}{C_E}\frac{C_E}{\mathcal{Y}_E}\right)$$

[3] Actually, when estimating the model, we estimate the long run ratios $\frac{I_F}{O_F}$ and $\frac{I_E}{O_E}$ and deduce values for a_v^F and a_v^E from the following first order conditions. For instance, it can be shown that we have:

$$a_v^F = \frac{\frac{\chi_F^{io}\bar{z}_F}{g+\bar{\delta}_F}\left(\bar{r}_F^k\right)^{s_v^F}}{1 + \frac{\chi_F^{io}\bar{z}_F}{g+\bar{\delta}_F}\left(\bar{r}_F^k\right)^{s_v^F}}$$

where χ_F^{io} is the investment to oil long run ratio.

With the capital accumulation equation:

$$\frac{I_F}{K_F} = \frac{g+\delta_F}{1+g} \text{ and } \frac{I_E}{K_E} = \frac{g+\delta_E}{1+g}$$

By definition:

$$\frac{K_F}{\tilde{K}_F} = \frac{1+g}{z_F} \text{ and } \frac{K_E}{\tilde{K}_E} = \frac{1+g}{z_E}$$

From the first order conditions and the CES technology of the final retailers, we have:

$$\frac{C_F^{ne}}{C_F} = \bar{a}_F^o \text{ and } \frac{C_E^{ne}}{C_E} = \bar{a}_E^o$$

The steady state dispersions ∇_F^w, ∇_E^w, ∇_F^y, ∇_E^y, ∇_F^x and ∇_E^x are equal to 1. So in particular, $L_F = \mathcal{L}_F$ and $L_E = \mathcal{L}_E$. In addition, wage setting equations imply $w_F^m = \frac{\theta_s^F-1}{\theta_s^F} w_F$ and $w_E^m = \frac{\theta_s^E-1}{\theta_s^E} w_E$. The first order conditions for labor supply yield:

$$\frac{\bar{\tau}_F^R \bar{\tau}_F^W}{\bar{\tau}_F^C} w_F^m = C_F \left(1 - \frac{\eta^F}{1+g}\right) L_F^{\sigma_l^F} \text{ and } \frac{\bar{\tau}_E^R \bar{\tau}_E^W}{\bar{\tau}_E^C} w_E^m = C_E \left(1 - \frac{\eta^E}{1+g}\right) L_E^{\sigma_l^E}$$

From which we get:

$$C_F = \frac{(1+g)\,\bar{\tau}_F^R \bar{\tau}_F^W w_F^m}{\bar{\tau}_F^C \left(1+g-\eta^F\right)} \left(\frac{L_F}{V_F}\frac{V_F}{O_F}\frac{O_F}{\mathcal{Y}_F}\right)^{-\sigma_l^F} \mathcal{Y}_F^{-\sigma_l^F} \text{ and } C_E = \frac{(1+g)\,\bar{\tau}_E^R \bar{\tau}_E^W w_E^m}{\bar{\tau}_E^C \left(1+g-\eta^E\right)} \left(\frac{L_E}{V_E}\frac{V_E}{O_E}\frac{O_E}{\mathcal{Y}_E}\right)^{-\sigma_l^E} \mathcal{Y}_E^{-\sigma_l^E}$$

Let

$$a_3^F \equiv \left(p_F^Y + p_F^M \chi_Y^F\right)^{-1} \bar{a}_F^o \frac{(1+g)\,\bar{\tau}_F^R \bar{\tau}_F^W w_F^m}{\bar{\tau}_F^C \left(1+g-\eta^F\right)} \left(\frac{L_F}{V_F}\frac{V_F}{O_F}\frac{O_F}{\mathcal{Y}_F}\right)^{-\sigma_l^F}$$

$$a_3^E \equiv \left(p_E^Y + p_E^M \chi_Y^E\right)^{-1} \bar{a}_E^o \frac{(1+g)\,\bar{\tau}_E^R \bar{\tau}_E^W w_E^m}{\bar{\tau}_E^C \left(1+g-\eta^E\right)} \left(\frac{L_E}{V_E}\frac{V_E}{O_E}\frac{O_E}{\mathcal{Y}_E}\right)^{-\sigma_l^E}$$

$$a_4^F \equiv \left(p_F^Y + p_F^M \chi_Y^F\right)^{-1} \left(\frac{g+\delta_F}{z_F}\frac{\tilde{K}_F}{O_F}\frac{O_F}{\mathcal{Y}_F} + \chi_G^F\right)$$

$$a_4^E \equiv \left(p_E^Y + p_E^M \chi_Y^E\right)^{-1} \left(\frac{g+\delta_E}{z_E}\frac{\tilde{K}_E}{O_E}\frac{O_E}{\mathcal{Y}_E} + \chi_G^E\right)$$

Then:

$$Y_F = a_3^F \mathcal{Y}_F^{-\sigma_l^F} + a_4^F \mathcal{Y}_F \text{ and } Y_E = a_3^E \mathcal{Y}_E^{-\sigma_l^E} + a_4^E \mathcal{Y}_E$$

Rearranging into a two equation-system for $\mathcal{Y}_F$ and $\mathcal{Y}_E$:
With $a_5^F \equiv \chi_X^F - \frac{O_F}{\mathcal{Y}_F} - 1$ and $a_5^E \equiv \chi_X^E - \frac{O_E}{\mathcal{Y}_E} - 1$, equations (102) become:

$$a_3^F \mathcal{Y}_F^{-\sigma_l^F} + X_{F/E} + \left(a_4^F + a_5^F\right) \mathcal{Y}_F = 0 \text{ and } a_3^E \mathcal{Y}_E^{-\sigma_l^E} + X_{E/F} + \left(a_4^E + a_5^E\right) \mathcal{Y}_E = 0$$

Combining equations (102) and (103), we get:

$$p_F^M \chi_Y^F a_5^F \mathcal{Y}_F + p_F^M \chi_Y^F X_{F/E} + \frac{\Sigma_E}{\Sigma_F}\left(p_E^X + p_M \chi_M^F\right) X_{E/F} = 0$$

$$p_E^M \chi_Y^E a_5^E \mathcal{Y}_E + p_E^M \chi_Y^E X_{E/F} + \frac{\Sigma_F}{\Sigma_E}\left(p_F^X + p_M \chi_M^E\right) X_{F/E} = 0$$

Or if we substitute $X_{E/F}$ and $X_{F/E}$ by their expressions as functions of $\mathcal{Y}_F$ and $\mathcal{Y}_E$, we obtain a system of two non linear equations with two unknown variables $\mathcal{Y}_F$ and $\mathcal{Y}_E$:

$$\begin{cases} f_3\left(\mathcal{Y}_F, \mathcal{Y}_E\right) = 0 \\ f_4\left(\mathcal{Y}_F, \mathcal{Y}_E\right) = 0 \end{cases}$$

where

$$f_3\left(\mathcal{Y}_F, \mathcal{Y}_E\right) = p_F^M \chi_Y^F \left(a_4^F \mathcal{Y}_F + a_3^F \mathcal{Y}_F^{-\sigma_l^F}\right) + \frac{\Sigma_E}{\Sigma_F}\left(p_E^X + p_M \chi_M^F\right)\left[\left(a_4^E + a_5^E\right)\mathcal{Y}_E + a_3^E \mathcal{Y}_E^{-\sigma_l^E}\right]$$

$$f_4\left(\mathcal{Y}_F, \mathcal{Y}_E\right) = p_E^M \chi_Y^E \left(a_4^E \mathcal{Y}_E + a_3^E \mathcal{Y}_E^{-\sigma_l^E}\right) + \frac{\Sigma_F}{\Sigma_E}\left(p_F^X + p_M \chi_M^E\right)\left[\left(a_4^F + a_5^F\right)\mathcal{Y}_F + a_3^F \mathcal{Y}_F^{-\sigma_l^F}\right]$$

The system is solved numerically using a Newton algorithm in order to calculate the steady state values of $\mathcal{Y}_F$ and $\mathcal{Y}_E$. For that purpose, we calculate the analytical expression of the jacobian matrix of (f_3, f_4):

$$J\left(\mathcal{Y}_F, \mathcal{Y}_E\right) = \begin{pmatrix} \frac{\partial f_3}{\partial \mathcal{Y}_F} & \frac{\partial f_3}{\partial \mathcal{Y}_E} \\ \frac{\partial f_4}{\partial \mathcal{Y}_F} & \frac{\partial f_4}{\partial \mathcal{Y}_E} \end{pmatrix} \equiv \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}$$

$$\frac{\partial f_3}{\partial \mathcal{Y}_F} = p_F^M \chi_Y^F a_4^F - \sigma_l^F p_F^M \chi_Y^F a_3^F \mathcal{Y}_F^{-\sigma_l^F - 1}$$

$$\frac{\partial f_4}{\partial \mathcal{Y}_E} = p_E^M \chi_Y^E a_4^E - \sigma_l^E p_E^M \chi_Y^E a_3^E \mathcal{Y}_E^{-\sigma_l^E - 1}$$

$$\frac{\partial f_3}{\partial \mathcal{Y}_E} = \frac{\Sigma_E}{\Sigma_F}\left(p_E^X + p_M \chi_M^F\right)\left(a_4^E + a_5^E - \sigma_l^E a_3^E \mathcal{Y}_E^{-\sigma_l^E - 1}\right)$$

$$\frac{\partial f_4}{\partial \mathcal{Y}_F} = \frac{\Sigma_F}{\Sigma_E}\left(p_F^X + p_M \chi_M^E\right)\left(a_4^F + a_5^F - \sigma_l^F a_3^F \mathcal{Y}_F^{-\sigma_l^F - 1}\right)$$

The algorithm is initialized with some value $(\mathcal{Y}_F^0, \mathcal{Y}_E^0)$. The value of $(\mathcal{Y}_F, \mathcal{Y}_E)$ is updated S times until $f_3\left(\mathcal{Y}_F, \mathcal{Y}_E\right)$ and $f_4\left(\mathcal{Y}_F, \mathcal{Y}_E\right)$ are both close enough to zero. At each step $s \in [0, S-1]$, the initial value of the unknown vector, $(\mathcal{Y}_F^s, \mathcal{Y}_E^s)$, is updated with the abscissa $\left(\mathcal{Y}_F^{s+1}, \mathcal{Y}_E^{s+1}\right)$ of the intercept of the tangent of (f_3, f_4) in $(\mathcal{Y}_F^s, \mathcal{Y}_E^s)$:

$$J\left(\mathcal{Y}_F^s, \mathcal{Y}_E^s\right)\begin{pmatrix} \mathcal{Y}_F^{s+1} - \mathcal{Y}_F^s \\ \mathcal{Y}_E^{s+1} - \mathcal{Y}_E^s \end{pmatrix} = -\begin{pmatrix} f_3\left(\mathcal{Y}_F^s, \mathcal{Y}_E^s\right) \\ f_4\left(\mathcal{Y}_F^s, \mathcal{Y}_E^s\right) \end{pmatrix}$$

The solution of this equation is:

$$\begin{pmatrix} \mathcal{Y}_F^{s+1} \\ \mathcal{Y}_E^{s+1} \end{pmatrix} = \begin{pmatrix} \mathcal{Y}_F^{s} \\ \mathcal{Y}_E^{s} \end{pmatrix} + \begin{pmatrix} \frac{f_4\left(\mathcal{Y}_F^s,\mathcal{Y}_E^s\right)J_{12}-f_3\left(\mathcal{Y}_F^s,\mathcal{Y}_E^s\right)J_{22}}{J_{11}J_{22}-J_{12}J_{21}} \\ \frac{f_3\left(\mathcal{Y}_F^s,\mathcal{Y}_E^s\right)J_{21}-f_4\left(\mathcal{Y}_F^s,\mathcal{Y}_E^s\right)J_{11}}{J_{11}J_{22}-J_{12}J_{21}} \end{pmatrix}$$

Once the steady state values of $\mathcal{Y}_F$ and $\mathcal{Y}_E$ are determined, the steady state of the remaining activity variables of the model are easily obtained from the ratios calculated above.

Bonds

The equations of the model determine the long run total foreign asset position of each country F and E, but not the steady state allocation between bonds issued in the euro area and bonds issued in the rest of the world. We assume that the long run foreign asset position of each country F and E with respect to each of its trading partners (the other country of the euro area and the rest of the world) offsets its steady state trade balance with respect to that trading partner:

$$B_F = \left(\frac{1}{\bar{\bar{R}}} - \frac{1}{\bar{\pi}(1+g)}\right)^{-1}\left(p_F^X X_{F/E} - \frac{\Sigma_E}{\Sigma_F} p_E^X X_{E/F}\right)$$

$$B_E = \left(\frac{1}{\bar{\bar{R}}} - \frac{1}{\bar{\pi}(1+g)}\right)^{-1}\left(p_E^X X_{E/F} - \frac{\Sigma_F}{\Sigma_E} p_F^X X_{F/E}\right)$$

$$B_F^\star = \left(\frac{1}{\bar{\bar{R}}} - \frac{1}{\bar{\pi}(1+g)}\right)^{-1}\left(p_F^X X_F^\star - p_M M_F^\star - O_F - C_F^o\right)$$

$$B_E^\star = \left(\frac{1}{\bar{\bar{R}}} - \frac{1}{\bar{\pi}(1+g)}\right)^{-1}\left(p_E^X X_E^\star - p_M M_E^\star - O_E - C_E^o\right)$$

In order to ensure that these values correspond to a stationary equilibrium, the financial intermediation cost functions are parametrized as follows using the steady state values calculated above:

$$\bar{b}_F = \frac{B_F}{\mathcal{Y}_F}$$

$$\bar{b}_E = \frac{B_E}{\mathcal{Y}_E}$$

$$\bar{b}_F^\star = \frac{B_F^\star}{\mathcal{Y}_F}$$

$$\bar{b}_E^\star = \frac{B_E}{\mathcal{Y}_E}$$

3.C.2 Financial frictions

A major difference with the baseline case is that the rental rate of capital r_F^k cannot be computed immediately. From households first order condition with respect to investment, we have:

$$Q_F = \frac{1}{\bar{\tau}_F^R}$$

instead of $Q = 1$ in the baseline model. The steady state ratio of transfers to entrepreneurs W_F^e to capital K_F and the fraction of bankrupt entrepreneurs per period $F_F(\bar{\omega}_F)$ are considered as estimated parameters, whereas the bankrupt threshold $\bar{\omega}_F$ and the cross-sectional standard deviation of entrepreneurs productivities v_F in the steady state are obtained from long run constraints. First, we have:

$$\bar{x}_F = norminv(F_F(\bar{\omega}_F)).$$

The standard deviation of the idiosyncratic shock, v_F, is obtained from a numerical solver to satisfy both the net worth accumulation equation and the credit demand curve, that is after some transformations

$$\begin{aligned}
&\left(\frac{W_F^e}{K_F} - Q_F\right)\left(1 - \frac{\mu(1-\Gamma_F)e^{-\frac{1}{2}\bar{x}_F^2}}{\sqrt{2\pi}(1 - normcdf(\bar{x}_F))v_F} - normcdf(\bar{x}_F - v_F)\mu\right) \\
&+ \bar{\gamma}_F \frac{(1+g)^{\sigma_c - 1}}{\beta} Q_F(1-\Gamma_F) + Q_F(\Gamma_F - normcdf(\bar{x}_F - v_F)\mu) = 0,
\end{aligned}$$

with

$$\Gamma_F = (1 - normcdf(\bar{x}_F))\exp\left(v_F\bar{x}_F + \frac{1}{2}v_F^2\right) + normcdf(\bar{x}_F - v_F).$$

Then:

$$\bar{\omega}_F = \exp\left(v_F\bar{x}_F - \frac{1}{2}v_F^2\right),$$

$$\begin{aligned}
R_F^k = &\Bigg(\left[1 - \bar{\omega}_F\left(1 - normcdf\left(\bar{x}_F\right)\right) - normcdf\left(\bar{x}_F - v_F\right)\right]\frac{1 - normcdf\left(\bar{x}_F\right) - \frac{\mu}{v_F\sqrt{2\pi}}e^{-\frac{1}{2}\bar{x}_F^2}}{1 - normcdf\left(\bar{x}_F\right)} \\
&+ \bar{\omega}_F\left(1 - normcdf\left(\bar{x}_F\right)\right) + (1-\mu)\, normcdf\left(\bar{x}_F - v_F\right)\Bigg)^{-1} R,
\end{aligned}$$

and

$$r_F^k = \frac{\frac{R_F^k}{\bar{\pi}} - 1 + \bar{\delta}_F}{\bar{\tau}_F^R \bar{z}_F}.$$

With this expression of the rental rate of capital, the remaining prices and activity aggregates are computed following the same scheme as in the baseline model, except that total consumption includes entrepreneurs consumption and auditing fees consume non-oil goods, so

$$a_4^F = \left(p_F^Y + p_F^M \chi_Y^F\right)^{-1}\left(\frac{1+g}{z_F}\frac{\tilde{K}_F}{O_F}\frac{O_F}{\mathcal{Y}_F}\left(\frac{g+\delta}{1+g} + \bar{a} + \bar{a}_F^o\frac{C_F^e}{K_F} + \mu\frac{R_F^k Q_F normcdf(\bar{x}_F - v_F)}{(1+g)\bar{\pi}}\right) + \chi_G^F\right),$$

with

$$\frac{C_F^e}{K_F} = \Theta \left(1 - \bar{\omega}_F(1 - F_F(\bar{\omega}_F)) - normcdf(\bar{x}_F - v_F)\right) \frac{Q_F R_F^k}{\bar{\pi}(1+g)}.$$

Once capital K_F is computed, we get the steady state level of the transfer from government to entrepreneurs W_F^e (in real terms) from the ratio W_F^e over K_F, the same for entrepreneurs consumption, and real net worth as

$$\bar{n}_F = Q_F K_F \left(1 - \left[\bar{\omega}_F \left(1 - normcdf\left(\bar{x}_F\right)\right) + (1-\mu)\, normcdf\left(\bar{x}_F - v_F\right)\right] \frac{R_F^k}{R}\right)$$

3.C.3 Labor market frictions

The price paid by intermediate producers w_F is computed using a solver as in the baseline model. Moreover, steady state employment $\bar{N}_F$ is an estimated parameter; therefore, GDP can be computed as follows:

$$V_F = \chi_{VL} \bar{N}_F,$$

$$O_F = \frac{V_F}{\chi_{VO}},$$

where $\chi_{VO} = V_F / O_F$ and $\chi_{VL} = V_F / \mathcal{L}_F = V_F / \bar{N}_F$,

$$\mathcal{Y}_F = V_F^\alpha \left(\bar{\varepsilon}_F^a N_F\right)^{1-\alpha} - O_F.$$

The steady state share of vacancy costs in GDP and the vacancy filling rate are also estimated parameters, respectively χ_F^{vc} and $\bar{\Phi}_F$. We have immediately

$$m_F = s\bar{N}_F,$$

$$v_F = \frac{m_F}{\bar{\Phi}_F},$$

$$\Psi_F = \frac{m_F}{1 - (1-s)\bar{N}_F}.$$

Then the job creation condition together with the equation of the marginal value of existing jobs for firms implies

$$w_F^m = w_F - \breve{\varphi} \frac{\mathcal{Y}_F \chi_F^{vc}}{v_F \bar{\Phi}_F} \left(1 - \beta(1-s)(1+g)^{1-\sigma_c}\right).$$

Bargained wages are constant and equal to average wages in the long run:

$$\tilde{w}_F^m = w_F^m.$$

The matching scale parameter is updated as follows:

$$\Upsilon = \frac{m_F}{v_F^\varphi (1 - (1-s)\bar{N}_F)^{1-\varphi}},$$

and the parameter that controls the unit cost of vacancies is

$$c = \breve{\varphi}\frac{\mathcal{Y}_F \chi_F^{vc}}{m_F^{\iota\breve{\varphi}} v_F^{\breve{\varphi}-\iota\breve{\varphi}}}.$$

$$\frac{\Gamma_F}{w_F^m} = 1 - \frac{1-\xi}{\xi}\frac{\chi_{vm}}{w_F^m}\left[\frac{1}{1-F(\bar{a}_F)} + \beta(1-s)(1+g)^{1-\sigma_c}\left(1 - \frac{1/(1-F(\bar{a}_F)) - 1 + s}{1/N_F - 1 + s}\right)\right].$$

$$U_F = 1 - N_F,$$

From the zero-profit condition of domestic importers, we get:

$$Y_F = \frac{\Sigma_E}{\Sigma_F}\frac{p_E^X + p_M \chi_M^F}{p_F^M \chi_Y^F} X_{E/F}$$

which can be substituted into the market clearing condition as follows

$$\frac{\Sigma_E}{\Sigma_F}\frac{p_E^X + p_M \chi_M^F}{p_F^M \chi_Y^F} X_{E/F} + X_{F/E} = \left(1 - \chi_X^F\right)\mathcal{Y}_F + O_F$$

and similarly for country E:

$$\frac{\Sigma_F}{\Sigma_E}\frac{p_F^X + p_M \chi_M^E}{p_E^M \chi_Y^E} X_{F/E} + X_{E/F} = \left(1 - \chi_X^E\right)\mathcal{Y}_E + O_E$$

These equations give the values for intra-euro area trade flows:

$$X_{E/F} = \frac{\left(1 - \chi_X^F\right)\mathcal{Y}_F + O_F - \frac{\Sigma_E}{\Sigma_F}\frac{p_E^M \chi_Y^E}{p_F^X + p_M \chi_M^E}\left(\left(1 - \chi_X^E\right)\mathcal{Y}_E + O_E\right)}{\frac{\Sigma_E}{\Sigma_F}\frac{p_E^X + p_M \chi_M^F}{p_F^M \chi_Y^F} - \frac{\Sigma_E}{\Sigma_F}\frac{p_E^M \chi_Y^E}{p_F^X + p_M \chi_M^E}}$$

$$X_{F/E} = \frac{\left(1 - \chi_X^E\right)\mathcal{Y}_E + O_E - \frac{\Sigma_F}{\Sigma_E}\frac{p_F^M \chi_Y^F}{p_E^X + p_M \chi_M^F}\left(\left(1 - \chi_X^F\right)\mathcal{Y}_F + O_F\right)}{\frac{\Sigma_F}{\Sigma_E}\frac{p_F^X + p_M \chi_M^E}{p_E^M \chi_Y^E} - \frac{\Sigma_F}{\Sigma_E}\frac{p_F^M \chi_Y^F}{p_E^X + p_M \chi_M^F}}$$

Then $\tilde{K}_F$, K_F and I_F are calculated using the known ratios $\frac{\tilde{K}_F}{O_F}$, $\frac{K_F}{\tilde{K}_F}$ and $\frac{I_F}{K_F}$. Y_F is computed from its expression as a function of $X_{E/F}$ above and M_F is derived from the ratio χ_Y^F. The zero-profit condition of the intermediate retailer yields:

$$C_F^{ne} = p_F^Y Y_F + p_F^M M_F - I_F - G_F - cv_F$$

From which we get immediately the steady state consumption level:

$$C_F = \frac{C_F^{ne}}{\bar{a}_F^o}$$

The lagrange multiplier is also modified because the utility function does no longer include a non-separable term accounting for the desutility of working:

$$\lambda_F = \frac{1}{\bar{\tau}_F^C} C_F^{-\sigma_c} \left(1 - \frac{1}{1+g}\right)^{-\sigma_c}$$

Then, the fixed disutility of labor parameter derives from the Nash bargaining condition

$$\Gamma = \lambda_F \tau_R \tau_W \left(w_F^m - \frac{1-\xi_F}{\xi_F} \frac{(1-\Psi_F)\,\beta(1-s)(1+g)^{1-\sigma_c}}{1-\beta(1-s)(1+g)^{1-\sigma_c}} (w_F - w_F^m)\right).$$

Last, the marginal values of workers and firms are:

$$\frac{\partial \mathcal{V}_F}{\partial w_F^m} = \frac{(1-s)(1+g)\bar{N}_F}{(1-s)\beta(1+g)^{1-\sigma_c} - 1},$$

$$\frac{\partial \mathcal{W}_F}{\partial w_F^m} = -\frac{\partial \mathcal{V}_F}{\partial w_F^m}\lambda_F,$$

$$\frac{\partial \mathcal{W}_F}{\partial N_F} = \frac{(1-s)(1-\Psi_F)\,(\lambda_F w_F^m - \Gamma)}{1-(1-s)\beta(1-\Psi_F)(1+g)^{1-\sigma_c}},$$

$$\frac{\partial \mathcal{V}_F}{\partial N_F} = \frac{(1-s)\,(w_F - w_F^m)}{1-(1-s)\beta(1+g)^{1-\sigma_c}},$$

$$\frac{\partial \mathcal{W}_F}{\partial m_F} = \lambda_F w_F^m - \Gamma + \beta(1+g)^{1-\sigma_c}\frac{\partial \mathcal{W}_F}{\partial N_F},$$

$$\frac{\partial \mathcal{V}_F}{\partial m_F} = \left(w_F - w_F^m + \beta(1+g)^{1-\sigma_c}\frac{\partial \mathcal{V}_F}{\partial N_F}\right).$$

3.D Timing of decisions in the financial accelerator

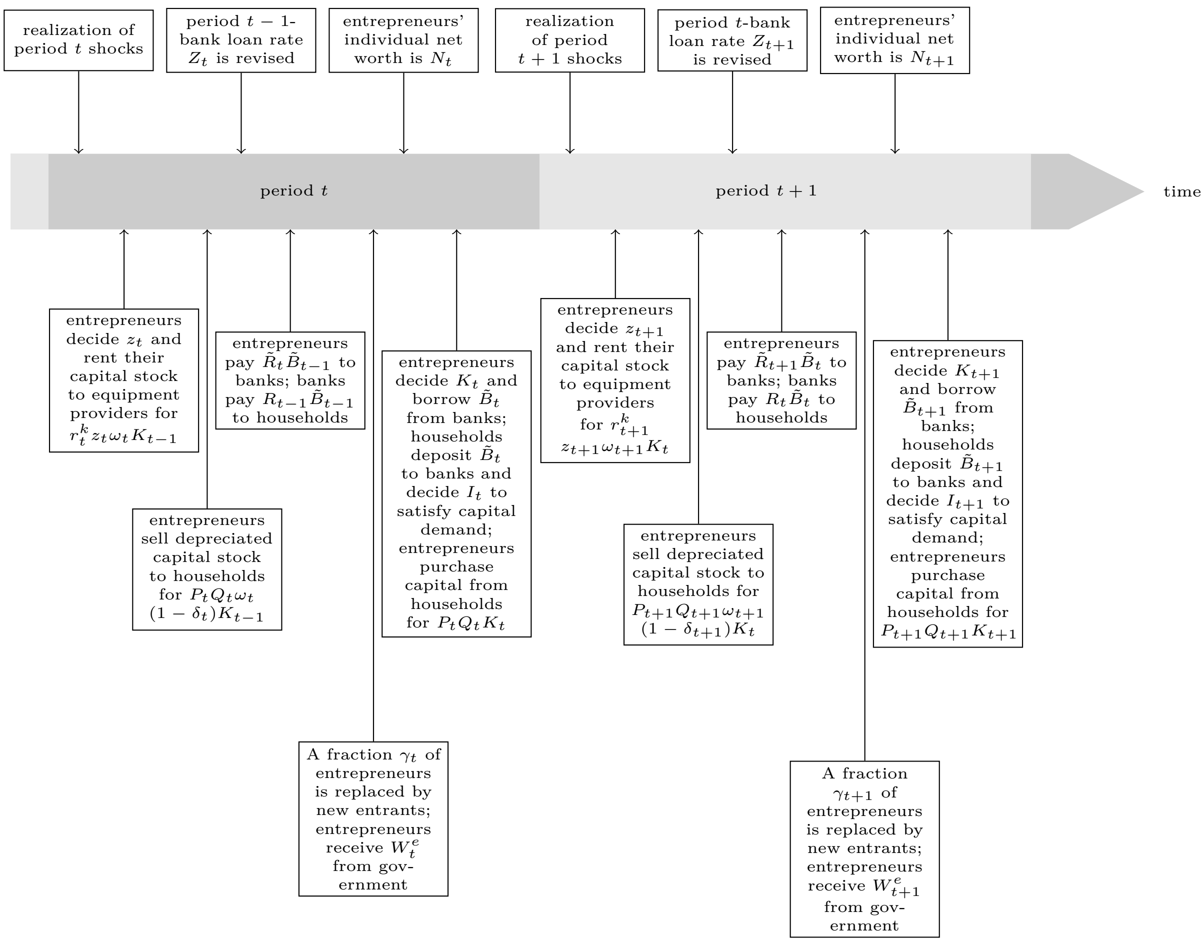


Figure 3.D.1: Timing assumptions of the financial accelerator

3.E Computable formulation of terms involving the distribution of entrepreneurs productivity

We use the log-normal distribution assumed for ω_F to provide expressions for G_F and Γ_F that can be dealt with numerically by Dynare and Matlab. As the density function of $\omega_{F,t}$ has mean 1 and standard error $\sigma_{F,t-1}$, it is:

$$f_{F,t-1}(\omega) = \frac{1}{\omega}\frac{1}{\sqrt{2\pi \log(1+\sigma_{F,t-1}^2)}} \exp\left[-\frac{1}{2\log\left(1+\sigma_{F,t-1}^2\right)}\left(\log\omega + \frac{1}{2}\log\left(1+\sigma_{F,t-1}^2\right)\right)^2\right]$$

Therefore:

$$F_{F,t-1}(\bar{\omega}_{F,t}) = \int_0^{\bar{\omega}_{F,t}} \frac{1}{\omega\sqrt{2\pi \log(1+\sigma^2_{F,t-1})}}$$
$$\exp\left[-\frac{1}{2\log\left(1+\sigma^2_{F,t-1}\right)}\left(\log\omega + \frac{1}{2}\log\left(1+\sigma^2_{F,t-1}\right)\right)^2\right] d\omega$$

We make the subsitution

$$x = \frac{\log\omega + \frac{1}{2}\log\left(1+\sigma^2_{F,t-1}\right)}{\sqrt{\log(1+\sigma^2_{F,t-1})}}$$

we have

$$\omega = \exp\left[\sqrt{\log(1+\sigma^2_{F,t-1})}x - \frac{1}{2}\log(1+\sigma^2_{F,t-1})\right] \text{ and } d\omega = \sqrt{\log(1+\sigma^2_{F,t-1})}\omega dx$$

Thus:

$$F_{F,t-1}(\bar{\omega}_{F,t}) = \int_{-\infty}^{\bar{x}_{F,t}} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}x^2\right] dx = normcdf\left(\bar{x}_{F,t}\right)$$

where

$$\bar{x}_{F,t} \equiv \frac{\log\bar{\omega}_{F,t} + \frac{1}{2}\log\left(1+\sigma^2_{F,t-1}\right)}{\sqrt{\log(1+\sigma^2_{F,t-1})}}$$

and *normcdf* is the cumulative distribution function of the standard normal distribution, available in Matlab. In the same way:

$$G_{F,t-1}(\bar{\omega}_{F,t}) = \int_0^{\bar{\omega}_{F,t}} \frac{1}{\sqrt{2\pi \log(1+\sigma^2_{F,t-1})}}$$
$$\exp\left[-\frac{1}{2\log\left(1+\sigma^2_{F,t-1}\right)}\left(\log\omega + \frac{1}{2}\log\left(1+\sigma^2_{F,t-1}\right)\right)^2\right] d\omega$$

We make the subsitution

$$x = \frac{\log\omega + \frac{1}{2}\log\left(1+\sigma^2_{F,t-1}\right)}{\sqrt{\log(1+\sigma^2_{F,t-1})}} - \sqrt{\log(1+\sigma^2_{F,t-1})}$$

we have

$$\omega = \exp\left[\sqrt{\log(1+\sigma^2_{F,t-1})}x + \frac{1}{2}\log(1+\sigma^2_{F,t-1})\right] \text{ and } d\omega = \sqrt{\log(1+\sigma^2_{F,t-1})}\omega dx$$

Thus:

$$\begin{aligned} G_{F,t-1}(\bar{\omega}_{F,t}) &= \int_{-\infty}^{\bar{x}_{F,t}-v_{F,t-1}} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}x^2\right] dx \\ &= normcdf\left(\bar{x}_{F,t} - v_{F,t-1}\right) \end{aligned}$$

where

$$v_{F,t-1} \equiv \sqrt{\log(1+\sigma_{F,t-1}^2)}$$

Then the expression for $\Gamma_{F,t-1}$ comes immediately:

$$\begin{aligned} \Gamma_{F,t-1}(\bar{\omega}_{F,t}) &= \bar{\omega}_{F,t}\left(1 - F_{\sigma_{F,t-1}}(\bar{\omega}_{F,t})\right) + G_{\sigma_{F,t-1}}(\bar{\omega}_{F,t}) \\ &= \bar{\omega}_{F,t}\left(1 - normcdf\left(\bar{x}_{F,t}\right)\right) + normcdf\left(\bar{x}_{F,t} - v_{F,t-1}\right) \end{aligned}$$

And their first derivatives are:

$$\begin{aligned} G'_{F,t-1}(\bar{\omega}_{F,t}) &= \bar{\omega}_{F,t} f_{\sigma_{F,t-1}}\left(\bar{\omega}_{F,t}\right) \\ &= \frac{1}{\sqrt{2\pi\log(1+\sigma_{F,t-1}^2)}} \exp\left[-\frac{1}{2\log\left(1+\sigma_{F,t-1}^2\right)}\left(\log\bar{\omega}_{F,t} + \frac{1}{2}\log\left(1+\sigma_{F,t-1}^2\right)\right)^2\right] \\ &= \frac{1}{v_{F,t-1}\sqrt{2\pi}} \exp\left[-\frac{1}{2}\bar{x}_{F,t}^2\right] \end{aligned}$$

and:

$$\Gamma'_{F,t-1}(\bar{\omega}_{F,t}) = 1 - F_{F,t-1}(\bar{\omega}_{F,t}) = 1 - normcdf\left(\bar{x}_{F,t}\right)$$

Chapter 4

Data and estimation results

The model is estimated using Bayesian techniques, detailed in chapter 1, using the exogenous shocks and the dataset described below.

4.1 Exogenous shocks

The model includes 28 exogenous variables (24 when financial frictions are switched off), which are driven by 28 orthogonal IID-normal random variables or structural shocks. The exogenous variables fall into three categories.

The first one includes 9 exogenous variables for each country (7 without financial frictions), namely the degree of impatience characterizing households' preferences, firms productivity, the efficiency of investment, residual demand (representing both government expenditures and changes in inventories), the relative bargaining power of firms and workers (or wage markups), domestic price markups, export price markups, and, with financial frictions, the standard deviation of entrepreneurs' idiosyncratic productivities ('risk') and the exit rate of entrepreneurs. These 18 variables are driven by 18 orthogonal IID-normal shocks. The way these orthogonal shocks affect the country-specific exogenous variables allows for cross-country correlation in the latter; the specification used is detailed in section 1.4. It implies that the orthogonal structural shocks cannot be interpreted as country-specific.

Next, there are 5 exogenous variables of the rest of the world: world demand, world inflation, oil, import and export competitors prices. They are driven by 5 orthogonal IID-normal shocks. The equations used are given in chapter 3 and imply that some shocks impact several of these variables. Specifically, world demand shocks affect world demand, world inflation and oil prices. Oil price shocks affect both oil prices and world inflation.

Last, 5 exogenous variables are common to the two countries, including the monetary policy shock, the stochastic trend in technology, import price markups, the market share of euro area firms in foreign markets, and the interest rate of the rest of the world. Each follows an AR(1) process including an IID-normal term.

In addition to these shocks, I add an IID-normal measurement error for each observable variable used in the estimation, except for those which are exogenous (foreign prices and world demand) or for the depreciation of the EUR/US nominal exchange rate. Indeed, these variables can be matched perfectly using the corresponding structural shocks already present in the model. In particular, the foreign interest rate shock plays the role of an uncovered interest rate parity shock, since the interest rate of the rest of the world is not observed for the estimation. There are hence 23 (or 19 without financial frictions) measurement errors.

4.2 Data

4.2.1 Observed cyclical time series

The model has been estimated on French and euro area quarterly data. Our dataset is made of 29 time series. Only 25 of them are used when financial frictions are switched off. It includes the growth rate in real GDP, private consumption, private investment, wage bill, exports and imports, and in loans to non-financial companies. It also includes year-on-year HICP[1] inflation, quarterly inflation of the export deflator, the unemployment rate (or hours worked in level for the versions of the model without labor market frictions), quaterly averages of the daily CAC40 and STOXX50 indices (for the entrepreneurs net worth). These variables are observed for both France and the whole euro area. Our dataset also includes the 3-month Euribor interest rate, oil price inflation expressed in euro, the growth rate in the demand of the rest of the world, the quarterly inflation rate of the US GDP deflator, of the average export price of foreign competitors to the euro area (export competitors price), and of the average export price of foreign suppliers of the euro area (import competitors price). We also observe the quarterly changes in the USD to euro nominal exchange rate.

Real variables are divided by population at working age in order to correspond to per capita averages consistently with the model's normalizing assumption stated in section 3.2. Hours worked in the euro area are only available at annual frequency between 2000 and 2012. Accordingly, we define a variable in the model that equals the sum of hours worked during the previous four quarters. This variable is only observed every four periods, at fourth quarters of each year between 2000 and 2012. The nominal exchange rate S_t in the model corresponds to the nominal effective exchange rate of the euro against foreign currencies, *i.e.* the average of foreign currencies exchange rates weighted by the size of trade flows between the euro area and the corresponding countries. Bringing the model to the data would therefore theoretically imply that we use four different time series for the nominal exchange rate: one for exports, one for non-oil imports, one for oil imports (that would correspond exactly to the euro/USD exchange rate) and one for bonds. To avoid such complication, the euro nominal exchange rate against the USD is used as a proxy for S_t in any case.

Unlike Christiano et al. (2014), the dataset does not include any measure of the slope of the interest rate term structure or of the credit spread. Defining the first variable would add a large number of forward terms in the model, which would increase the burden of numerical computation

[1]Harmonized Index of Consumer Prices.

tasks. I did not use the second one either because the adequation of time series available for France and the euro area with the concept of risk premium of the financial accelerator is questionable. Furthermore, this series is not needed to discriminate between the two financial disturbances included in the model, since wealth and risk shocks imply opposite comovements in investment and loans.

The sources of euro area and French data are respectively Eurostat and INSEE[2]. The estimation period ranges from 1995Q2 to 2013Q1, because Eurostat quarterly accounts start in 1995. In addition, we believe that a model of a monetary union would not be relevant before.

Before bringing our model(s) to the data, the consistency between the observed variables and their theoretical counterparts needs to be discussed. These observed variables, growth of real or nominal variables, are non zero mean, which is precisely what is predicted by the models. Indeed, the model predicts that all the real variables should evolve along parallel balanced growth paths. Unfortunately, this is not true in our sample: the mean growth rates of investment and output per capita are significantly different. When bringing a model to the data, two approaches are possible. First we can ignore these discrepancies between data and theory and estimate the model assuming that all the real variables share the same growth rate. We can think of this approach as a kind of structural detrending. Obviously, if the data is at odds with the model's predictions, this will eventually bias the estimates of the parameters. The second approach, which is the most common in the literature, consists in acknowledging this gap and removing the trends prior to the estimation. We follow this second approach and remove the trends prior to the estimation. In practice, all variables in the dataset are demeaned (which comes to detrending variables observed in growth rates).

However, the recent crisis being included in the sample significantly reduces the sample mean of variables (or increase it in the case of unemployment). As a result, removing the mean computed over the whole sample would make variables off-center during the pre-crisis period (above their long run levels), as if the 1995-2007 period corresponded to a long peak of the business cycle. Therefore, the mean that is removed from the gross series is computed over 1995Q2 to 2008Q3. For the same reason, the mean of unemployment rates are computed over 2000Q1-2008Q3, to exclude the high values recorded by the end of the 90s, and observe a sensible cyclical component of over the sample.

More generally, the inclusion of the crisis in the estimation sample is challenging, because the model obviously lacks many channels involved in the recession, especially a more realistic description of the banking system, of financial and housing markets. In addition, over the end of the sample period, monetary policy in the euro area has been indubitably constrained by the "zero lower bound" on policy interest rates. In the estimation of the model, this is simply captured through the identification of unexpected expansionary monetary policy shocks. Recent works suggest that this situation may have significantly altered the behavior of the economy, but taking this aspect into account in the present model is left for future research.

The following subsections describe the data and methodology used to calibrate or to determine the distribution a priori of a subset of parameters related to the steady state of the model.

[2] *Institut National de la Statistique et des Études Économiques*, the French national statistics institute.

4.2.2 Steady state of observed variables

All variables in the model are centered, whereas the timeseries used in the dataset are detrended. The detrending strategy consisted in removing the mean of the stationary observable variables computed over the pre-crisis period. In particular, the estimation sample includes 72 points which correspond to the quarters between 1995Q2 and 2013Q1. The means are computed using the first 54 points, that is the period 1995Q2-2008Q3. The steady state values of the growth rate of real aggregates g, of the inflation factor $\bar{\pi}$, of the interest rate $\bar{R}$ and of employment rates $\bar{N}$ are hence not identified in the estimation. The steady state inflation factor $\bar{\pi}$ is calibrated to $1.0199^{1/4}$ on the grounds that it corresponds to the target of the ECB, which is known to be slightly below 2% in the euro area. The discount factor β is derived from long run restrictions. For that reason, the steady state interest rate is not set to the pre-crisis mean value of the 3 month-interbank loans rate. Indeed, the latter is around 3.4%, which would imply a too high value of β. $\bar{R}$ is rather set to 5% in annual pace. The real growth rate g is set to the pre-crisis mean quarterly growth rate of the euro area real GDP per head, that is approximately 0.4%. The steady state capital utilization rates $\bar{z}$ are drawn from the pre-crisis mean of the corresponding time series, and are approximately 0.826 for both France and the rest of the euro area. Finally, the steady state employment rates $\bar{N}$ are estimated although data is detrended because these parameters impose restrictions to the level of unemployment benefits, which have strong implications for the cyclical dynamics of the model. However, the prior mode and variance of $\bar{N}$ are equal to the mean and variance of the employment rates in the data, computed over 1995Q2-2008Q3; the prior mode of $\bar{N}$ is 0.67 for France and 0.658 for the rest of the euro area, consistently with the fact that the model does not differentiate unemployment from inactivity.

Some long term ratios are introduced as parameters in the model and are estimated. It is non-oil consumption over oil consumption, investment over oil intermediate consumption in production, the share of residual demand in GDP, extra euro area exports over GDP, extra euro area imports over intra euro area imports and, last, total non oil imports over domestic production.

4.2.3 Oil shares in the economy

The ratios involving oil are computed from input-output tables for the euro area and France taken from Eurostat in 2008 and 2009. Table 4.1 is extracted from the French use tables at current prices for 2008. The share of oil in consumption at current prices is approximated by

$$\frac{P_o C_F^o}{P_c C_F} = \frac{14\,240 + 7\,902}{808\,637 + 118\,217} \approx 2.4\%.$$

Considering that the average relative price of oil was approximately 60% higher in 2008 than in 2005 or 2009, which are considered as “normal” levels reflecting the steady state, the ratio of real oil consumption over total consumption used in the calibration is

$$\frac{C_F^o}{C_F} \approx \frac{2.4\%}{1.6} = 1.5\%.$$

Table 4.1: Extract from the French input-output table for 2008

FR08		Total use of industry	Consumption of households
Use of domestic products			
Mining and quarrying		4 341	13
Coke and refined petroleum		20 326	14 240
Other		1 357 821	794 384
Total use of domestic products	(1)	1 382 489	808 637
Use of imported products			
Mining and quarrying		47 223	1
Coke and refined petroleum		10 041	7 902
Other		261 697	110 314
Total use of imported products	(2)	318 961	118 217
Taxes less subisdies on products	(3)	47 192	116 370
Total intermediate consumption	(1)+(2)+(3)	1 748 642	1 043 224
Value added	(4)	1 691 939	-
Output	(1)+(2)+(3)+(4)	3 440 581	-

The prior of the ratio of non-oil consumption over oil consumption is assumed to be beta distributed. Its mode is

$$\frac{C_F^{no}}{C_F^o} = \frac{C_F}{C_F^o} - 1 \approx \frac{1}{0.015} - 1 = 65.667,$$

and its bounds are such that the ratio $\frac{C_F^o}{C_F}$ can be 0.1 percentage point higher or lower that the value of 1.5% found. Its standard deviation is one quarter of the support. The ratio of imported oil intermediate consumption over GDP at current prices is approximated by

$$\frac{P_o O_F}{P\mathcal{Y}_F} = \frac{47\,223 + 10\,041}{1\,691\,939} \approx 3.4\%$$

Applying the same correction as for housesholds' consumption, we find

$$\frac{O_F}{\mathcal{Y}_F} \approx \frac{3.4\%}{1.6} = 2.1\%.$$

The prior distribution of the ratio of investment over oil intermediate consumption is also beta. Its mode is computed using the pre-crisis mean value of the share of private investment in GDP in France, which is 15.6%, as follows:

$$\frac{I_F}{O_F} = \frac{\text{mean}(pinv_F/gdp_F)}{0.021} \approx 7.403.$$

The bounds are computed using the minimal and maximal values of the share of private investment in GDP in the pre-crisis data, and allowing for a maximal deviation of 0.2 basis point in the ratio of oil intermediate consumption over GDP from the value of 2.1% found. Again, its standard deviation is one quarter of the support.

Table 4.2: Extract from the EA input-output table for 2008

EA08		Total use of industry	Consumption of households
Use of domestic products			
Mining and quarrying		66 356	1 064
Coke and refined petroleum		174 359	93 605
Other		7 896 849	4 137 127
Total use of domestic products	(1)	8 137 564	4 231 796
Use of imported products			
Mining and quarrying		300 305	5 512
Coke and refined petroleum		60 552	18 047
Other		862 647	281 922
Total use of imported products	(2)	1 223 505	305 482
Taxes less subisdies on products	(3)	248 951	554 307
Total intermediate consumption	(1)+(2)+(3)	9 610 020	5 091 585
Value added	(4)	8 296 055	-
Output	(1)+(2)+(3)+(4)	17 906 076	-

Tables 4.2 and 4.3 report figures extracted from the 2008 and 2009 use tables at current prices of the euro area (17 countries). Using the same method as for France, we find for 2008

$$\frac{C^o}{C} = \frac{93\,605 + 18\,047}{4\,231\,796 + 305\,482} \times \frac{1}{1.6} \approx 1.5\%,$$

and

$$\frac{O}{\mathcal{Y}} = \frac{300\,305 + 60\,552}{8\,296\,055} \times \frac{1}{1.6} \approx 2.7\%.$$

And finally, we use the same approach with 2009 figures except that no correction for high oil price is applied:

$$\frac{C^o}{C} = \frac{60\,874 + 11\,517}{4\,196\,855 + 285\,868} \approx 1.6\%,$$

and

$$\frac{O}{\mathcal{Y}} = \frac{188\,262 + 44\,279}{8\,028\,425} \approx 2.9\%.$$

On the basis of the results of 2008 and 2009, we assume that on average

$$\frac{C^o}{C} = 1.6\% \text{ and } \frac{O}{\mathcal{Y}} = 2.8\%.$$

The corresponding values for the euro area excepted France are computed by assuming that the ratios for the euro area is close to an average of the ratios for the two countries weighted by their average relative population size Σ_F and $1 - \Sigma_F$, so

$$\frac{C_E^o}{C_E} = \frac{1}{1 - \Sigma_F} \left(\frac{C^o}{C} - \Sigma_F \frac{C_F^o}{C_F} \right),$$

Table 4.3: Extract from the EA input-output table for 2009

EA09		Total use of industry	Consumption of households
Use of domestic products			
Mining and quarrying		56 782	974
Coke and refined petroleum		105 789	60 874
Other		7 220 395	4 135 007
Total use of domestic products	(1)	7 382 966	4 196 855
Use of imported products			
Mining and quarrying		188 262	3 321
Coke and refined petroleum		44 279	11 517
Other		752 237	271 030
Total use of imported products	(2)	984 778	285 868
Taxes less subisdies on products	(3)	232 273	529 818
Total intermediate consumption	(1)+(2)+(3)	8 600 017	5 012 541
Value added	(4)	8 028 425	-
Output	(1)+(2)+(3)+(4)	16 628 443	-

and

$$\frac{O_E}{\mathcal{Y}_E} = \frac{1}{1-\Sigma_F}\left(\frac{O}{\mathcal{Y}} - \Sigma_F \frac{O_F}{\mathcal{Y}_F}\right).$$

Then, we use the same methodology as for France to determine the prior distributions of the ratios C_E^{no}/C_E^{o} and I_E/O_E.

4.2.4 Population, residual demand and trade ratios

The parameter Σ_F, which represents the weight of France in the euro area is estimated. Its prior is derived from the ratio of the population aged 15 to 64 in France, over the population aged 15 to 64 in the euro area. The source is the statistics from the European Union Labor Force Survey provided by Eurostat. These times series start in 2003Q1 for France and 2005Q1 for the Euro area, so they are retropolated to 1995Q1 with working age population times series from the database of the forecasting department of the Banque de France. The prior mode and standard deviation of Σ_F are set to the pre-crisis mean and standard deviation of the ratio in the data, while its support is centered around the mode and is 4 times as large as the standard deviation.

The time series for real residual demand in France and in the rest of the euro area are computed from our dataset as

$$rdem_F = gdp_F - pinv_F - cons_F - texp_F + timp_F,$$

$$rdem_E = gdp - pinv - cons - texp + timp + rdem_F,$$

where *gdp* represents real GDP, *pinv* real private investment, *cons* real consumption of households, *texp* total real exports and *timp* total real imports. The index F refer to France whereas the absence of index indicates that the variable is for the whole euro area. In the case of the euro area, *texp* and *timp* only include extra euro area imports and exports, whereas $texp_F$ and $timp_F$ are total trade flows of France. The residual demand share series are

$$\frac{G_F}{\mathcal{Y}_F} = \frac{rdem_F}{gdp_F} \approx 0.2948$$

and

$$\frac{G_E}{\mathcal{Y}_E} = \frac{rdem_E}{gdp - gdp_F} \approx 0.2371.$$

The prior of the ratios of extra euro area exports to GDP, of extra to intra euro area imports and of imports to domestic production are built using nominal time series taken from the database of the forecasting department of the Banque de France. These series include trade flows broken down between intra and extra euro area, and are available from 1999Q1. Hence, the ratios are computed over the period 1999Q1-2008Q3. The ratios of extra euro area exports to GDP is directly available for France as

$$\frac{X_F^\star}{\mathcal{Y}_F} = \frac{XEXP_F}{GDP_F} \approx 0.1425$$

and it is

$$\frac{X_E^\star}{\mathcal{Y}_E} = \frac{XEXP - XEXP_F}{GDP - GDP_F} \approx 0.2061$$

in the rest of the euro area, where $XEXP$ is total exports toward countries outside the euro area in nominal terms and GDP is nominal GDP.

The ratios of extra to intra euro area (non oil) imports is calculated as follows: first, the time series for extra euro area non oil imports in nominal terms are

$$XIMP_F^{no} = XIMP_F - \frac{O_F}{\mathcal{Y}_F} GDP_F - \frac{C_F^o}{C_F} CONS_F,$$

and

$$XIMP_E^{no} = XIMP^{good} + XIMP^{serv} + XIMP^{ener} - \frac{O}{\mathcal{Y}} GDP - \frac{C^o}{C} CONS - XIMP_F^{no},$$

where $XIMP_F$ is total nominal imports from outside the euro area in France, $XIMP^{good}$, $XIMP^{serv}$, $XIMP^{ener}$ are respectively nominal imports of goods except energy, services and energy from outside to euro area to the euro area, GDP and $CONS$ are nominal GDP and consumption, and finally $O/\mathcal{Y}$ and C^o/C are the values of the ratios computed for real variables in section 4.2.3 and used as modes a priori for the distribution of the related parameters, which are respectively 2.1% and 1.5% for France, and 2.8% and 1.6% for the euro area. Then the ratios are

$$\frac{\Sigma_F M_F^\star}{\Sigma_E X_{E/F}} = \frac{XIMP_F^{no}}{IIMP_F} \approx 0.6665$$

for France and

$$\frac{\Sigma_E M_E^\star}{\Sigma_F X_{F/E}} = \frac{XIMP_E^{no}}{IEXP_F} \approx 4.5525$$

where $IIMP_F$ is total French imports from countries of the euro area expressed in nominal terms and $IEXP_F$ is total exports from France toward countries of the euro area in nominal terms, which also corresponds to intra euro area imports for the region made of the euro area except France.

Last, the ratios of imports to domestic production requires that we compute total non oil imports in both France and the rest of the euro area, and assume the counterpart in the data of the domestic production variable of the model. Total non oil imports in nominal terms are simply the sum of intra and extra non oil imports, and we assume that domestic production is GDP plus the oil intermediate consumption in production minus total exports. The ratios are

$$\frac{M_F}{Y_F} = \frac{IIMP_F + XIMP_F^{no}}{GDP_F\left(1 + O_F/\mathcal{Y}_F\right) - XEXP_F - IEXP_F} \approx 0.3149$$

and

$$\frac{M_E}{Y_E} = \frac{IEXP_F + XIMP_E^{no}}{GDP\left(1 + O/\mathcal{Y}\right) - GDP_F\left(1 + O_F/\mathcal{Y}_F\right) - XEXP + XEXP_F - IIMP_F} \approx 0.2383$$

The mode and standard deviation of the beta prior distributions of the parameters representing all these ratios are the pre-crisis mean and standard deviation of the corresponding time series. Their support is centered around their mode and is 4 times as large as their standard deviation.

4.2.5 Tax rates

Tax factors are calibrated parameters. Their values are computed as the mean of nominal time series taken from the database of the forecasting department of the Banque de France and starting in 1999Q1. So the period used is 1999Q1-2008Q3 as for the prior distributions of the international trade ratios described in section 4.2.4. They are

$$\tau_F^C - 1 = \frac{VAT_F}{CONS_F - VAT_F} \approx 15.15\%$$

$$1 - \tau_F^R = \frac{IT_F}{GDI_F + IT_F} \approx 11.98\%$$

$$1 - \tau_F^W = \frac{SSC_F}{WB_F} \approx 13.34\%$$

$$\tau_F^L - 1 = \frac{ESC_F}{WB_F} \approx 36.15\%$$

$$\tau_E^C - 1 = \frac{VAT - VAT_F}{CONS - CONS_F - VAT + VAT_F} \approx 12.34\%$$

$$1 - \tau_E^R = \frac{IT - IT_F}{GDI - GDI_F + IT - IT_F} \approx 12.68\%$$

$$1 - \tau_E^W = \frac{SSC - SSC_F}{WB - WB_F} \approx 11.98\%$$

$$\tau_E^L - 1 = \frac{ESC - ESC_F}{WB - WB_F} \approx 27.42\%$$

where *VAT* is total value added tax receipts, *CONS* is nominal households' consumption, *IT* is total income tax receipts, *GDI* is households' gross disposable income, *SSC* is total employee social contributions, *ESC* is total employer social contributions and *WB* is the nominal gross wage bill (including employee social contributions but net of employer social contributions).

4.2.6 Other steady state ratios

The model is expected to be consistent with the following long term ratios: the French GDP over the euro area GDP, the share of private investment in GDP in France and in the euro area and the labor share in France and in the euro area. The observed values of these ratios are computed with pre-crisis data, over 1995Q2-2008Q3, as follows. The ratio of French GDP per head over euro area GDP per head is

$$\frac{\mathcal{Y}_F}{\mathcal{Y}} = \frac{gdp_F/pop_F}{gdp/pop} \approx 1.1539$$

where *gdp* is real GDP and *pop* is total population aged 15 to 64. The share of private investment in GDP is directly available as

$$\frac{I_F}{\mathcal{Y}_F} = \frac{pinv_F}{gdp_F} \approx 0.1555$$

for France and

$$\frac{I}{\mathcal{Y}} = \frac{pinv}{gdp} \approx 0.1647$$

for the euro area, where *pinv* is real private investment. The labor shares are

$$\frac{w_F^m N_F}{\mathcal{Y}_F} = \frac{WB_F}{PC_F\, gdp_F} \approx 0.3839$$

for France and

$$\frac{w^m N}{\mathcal{Y}} = \frac{WB}{PC\, gdp} \approx 0.3790$$

for the euro area, where *WB* is the nominal gross wage bill (including employee social contributions but net of employer social contributions) and *PC* is the consumption deflator.[3]

4.3 Prior distribution

4.3.1 Model parameters

Tables 4.1 to 4.5 and 4.A.1 to 4.A.3 in Appendix 4.A summarize the assumptions used regarding the prior distributions of the parameters that are estimated. These assumptions are close to those used by previous papers related to Bayesian estimations of models à la Smets and Wouters (2007), including Christoffel et al. (2008), or Adolfson et al. (2008). The prior of financial parameters are mainly taken from Christiano et al. (2010) or Christiano et al. (2014). For the parameters characterizing labor market frictions, the priors are based on standard calibrations used in the

[3]The model labor share is $w^m N/\mathcal{Y}$ in the presence of labor market frictions, but is $w\mathcal{L}/\mathcal{Y}$ otherwise.

literature (as in Andolfatto (1996)).

The distributions a priori specified for parameters are orthogonal to each other: in Dynare, marginal densities are defined for each parameter separately, and the resulting joint density assumes that they are independant. Yet, as explained in section 4.3.2 below, I also implicitly add densities a priori to functions of parameters, so that the overall prior distribution does assume independance any more.

The shape of parameters marginal prior distributions are chosen to be consistent with their theoretical bounds: in particular, autocorrelation coefficients and consumption habit parameters follow beta distributions over [0,1], and adjustment costs and standard deviations use either gamma or inverse gamma 2 distributions to be positive.

For standard deviations of shocks, which can be numerically close to zero, I prefer inverse gamma 2 distributions with infinite variance to simple gamma distributions. The reason is that the latter feature a vertical asymptote in zero when their mode is less than $2^{0.5} - 2^{-0.5}$ times their standard deviation, whereas the former always has an horizontal asymptote.

4.3.2 Priors on moments

A number of steady state ratios are not parameters in the model but can only be computed when the static model is solved. As we view as important to match them when the model is estimated, we introduce implicite priors on their values.

We use this approach to set prior distributions for the values of the ratios of French GDP over euro area GDP, the share of private investment in GDP in France and in the euro area and the labor share in France and in the euro area. Specifically, they are beta distributions based on the numerical values computed in section 4.2.6. The support of the priors are bounded by the minimal and the maximal values of the ratios in the data. The prior mode is the sample mean and the prior standard deviation is the sample standard deviation. With financial frictions, we also include implicit prior on the value of the equity to debt ratio in France and in the euro area. For this ratio, we do not use data to determine the distribution, but the range of values reported by Christiano et al. (2010), which are 1.08-2.19 for the euro area. Accordingly, we use beta distributions over 0.5-2.0, with mode 1.0 and standard deviation 0.2.

4.4 Posterior estimates

Tables 4.1 to 4.5 report the prior distributions and the posterior estimates (mode and standard deviation) of the model's structural parameters for all versions (baseline 'base', with financial frictions only 'fa', with labor market frictions only 'lm', and with both financial and labor market frictions 'falm'). Tables 4.A.1 to 4.A.3 in Appendix 4.A provide the persistence and volatility of exogenous processes and the standard errors of measurement errors.

The inverse of the intertemporal elasticity of substitution of consumption is found slightly higher in France than in the rest of the euro area, except in the version of the model with only financial frictions. With σ_l begin significantly lower than the mode a priori of 2, the elasticity of labor supply

Table 4.1: Deep parameters 1/2

		prior shape	prior mode	prior s.d.	base mode	base s.d.	fa mode	fa s.d.	lm mode	lm s.d.	falm mode	falm s.d.
σ_c^F	cons. utility	beta	1.500	0.200	1.627	0.187	1.347	0.152	1.813	0.192	1.512	0.198
σ_c^E	cons utility	beta	1.500	0.200	1.579	0.167	1.427	0.167	1.654	0.183	1.400	0.174
σ_l^F	labor utility	gamma	2.000	1.000	1.285	0.549	1.496	0.587	–	–	–	–
σ_l^E	labor utility	gamma	2.000	1.000	0.870	0.395	0.925	0.431	–	–	–	–
ζ_w^F	calvo wages	gamma	4.000	0.500	4.328	0.451	4.172	0.442	–	–	–	–
ζ_w^E	calvo wages	gamma	4.000	0.500	4.355	0.490	4.467	0.501	–	–	–	–
θ_w^F	markup wages	gamma	12.000	1.000	11.973	0.997	11.986	0.997	–	–	–	–
θ_w^E	markup wages	gamma	12.000	1.000	11.996	0.997	11.990	0.997	–	–	–	–
ζ_y^F	calvo prices	gamma	4.000	3.000	4.031	0.747	3.409	0.717	5.813	0.763	4.806	0.695
ζ_y^E	calvo prices	gamma	4.000	3.000	6.612	1.585	3.694	0.820	6.967	0.808	5.773	0.732
ζ_x^F	calvo exports	gamma	4.000	3.000	2.878	0.687	2.724	0.672	3.529	0.412	3.000	0.399
ζ_x^E	calvo exports	gamma	4.000	3.000	3.372	0.785	2.773	0.801	4.318	0.551	3.796	0.490
α^F	capital share	beta	0.370	0.100	0.336	0.016	0.330	0.008	0.383	0.019	0.328	0.020
α^E	capital share	beta	0.400	0.100	0.379	0.007	0.384	0.006	0.390	0.021	0.371	0.014
s_v^F	subst oil/cap.	gamma	0.090	0.100	0.043	0.023	0.045	0.026	0.088	0.062	0.081	0.061
s_v^E	subst oil/cap.	gamma	0.090	0.100	0.058	0.036	0.057	0.039	0.101	0.071	0.146	0.100
s_m^F	subst imports	gamma	2.500	0.500	2.304	0.461	2.249	0.463	2.162	0.424	2.272	0.457
s_m^E	subst imports	gamma	2.500	0.500	1.637	0.304	1.616	0.311	1.918	0.437	1.561	0.331
s_y^F	subst prod/imp	gamma	2.500	0.500	3.810	0.489	3.645	0.450	3.930	0.526	3.903	0.497
s_y^E	subst prod/imp	gamma	2.500	0.500	2.472	0.284	2.325	0.237	1.768	0.214	2.265	0.269
s_o^F	subst oil/cons	gamma	0.090	0.050	0.075	0.038	0.075	0.038	0.088	0.044	0.086	0.043
s_o^E	subst oil/cons	gamma	0.090	0.050	0.082	0.041	0.086	0.044	0.092	0.046	0.092	0.046
s_x	subst exports	gamma	2.500	0.500	2.705	0.523	3.062	0.564	2.739	0.541	3.390	0.624
ψ_F	risk-prem ea	gamma	0.030	0.020	0.030	0.018	0.046	0.020	0.011	0.008	0.045	0.020
ψ_E	risk-prem ea	gamma	0.030	0.020	0.030	0.018	0.022	0.014	0.072	0.029	0.023	0.014
$\psi_F^\star$	risk-prem row	gamma	0.030	0.020	0.030	0.014	0.022	0.010	0.049	0.018	0.030	0.011
$\psi_E^\star$	risk-prem row	gamma	0.030	0.020	0.022	0.011	0.026	0.012	0.038	0.015	0.047	0.015
χ_F	rp exch rate	beta	0.500	0.200	0.246	0.062	0.193	0.047	0.223	0.080	0.204	0.053
χ_E	rp exch rate	beta	0.500	0.200	0.191	0.038	0.175	0.036	0.132	0.054	0.173	0.042

is found high, consistently with previous macro estimates based on aggregate hours (see Chetty et al. (2011)). It is even higher in the rest of the euro area than in France. For a comparison, Smets and Wouters (2003) find a value of 1.6 for σ_c, and 0.75 for σ_l.

The Calvo probabilities on wages (a Calvo probability of ξ implies an average duration of contracts of $\zeta = 1/(1-\xi)$) are found close to the value a priori, and to the value found by Smets and Wouters (2003). Regarding domestic prices, Calvo probabilities are found below their prior mode of 0.75 with financial frictions only, but above otherwise. In all cases, they are much smaller than in Smets and Wouters (2003) who find 0.91. The degree of domestic price stickiness is also found lower in France than in the rest of the euro area, although the confidence intervals sometimes overlap. Next, export prices are found much less rigid than domestic prices in both regions.

The elasticity of substitution between domestic and imported goods s_y is significantly greater in France than in the rest of the euro area. This implies that imports are more reactive to price competitivity in France.

The coefficients of the expected depreciation of the nominal exchange rate in the risk premium functions (χ_F and χ_E) are found significantly positive and around 0.2, while Adolfson et al. (2008) find 0.61 for Sweeden and Adjemian et al. (2008) find 0.13 for the euro area and the US.

The elasticities of the capital utilization rates in France and the euro area ($1/\omega^F$ and $1/\omega^E$) are found higher than in Smets and Wouters (2003) who report a value of 0.169, and in Christiano et al. (2010) with 0.04. They range between the estimates based on US data of Christiano et al. (2014), with 0.39, and of Smets and Wouters (2007), with 0.85.

Table 4.2: Deep parameters 2/2

		prior	prior	prior	base		fa		lm		falm	
		shape	mode	s.d.	mode	s.d.	mode	s.d.	mode	s.d.	mode	s.d.
Σ^F	size France	beta	0.181	0.002	0.181	0.003	0.182	0.004	0.181	0.004	0.182	0.004
θ_y^F	markup prices	gamma	12.000	0.500	11.945	0.498	11.832	0.495	11.940	0.496	11.840	0.495
θ_y^E	markup prices	gamma	12.000	0.500	11.648	0.493	11.714	0.483	11.768	0.494	11.779	0.492
θ_x^F	markup exports	gamma	12.000	0.500	11.929	0.498	11.953	0.498	11.945	0.498	11.969	0.498
θ_x^E	markup exports	gamma	12.000	0.500	12.045	0.499	12.027	0.499	12.019	0.499	12.015	0.499
θ_m	markup imports	gamma	6.000	0.500	5.914	0.480	6.021	0.480	5.660	0.455	5.551	0.432
r_π	taylor infl	gamma	1.500	0.100	1.545	0.103	1.573	0.096	1.513	0.103	1.641	0.099
$r_{\Delta\pi}$	taylor d(infl)	normal	0.610	0.500	0.715	0.127	0.752	0.140	0.742	0.121	0.874	0.136
r_Y	taylor gdp	gamma	0.400	0.150	0.409	0.101	0.726	0.155	0.399	0.100	0.709	0.153
$\mu\star$	elast exports	gamma	1.028	0.150	0.808	0.131	0.854	0.128	0.891	0.129	0.947	0.133
κ	oil to demand	gamma	5.000	2.000	6.387	1.324	6.466	1.310	6.694	1.363	6.600	1.330
ϱ	row p to oil	gamma	0.050	0.050	0.048	0.019	0.046	0.019	0.045	0.019	0.045	0.019
γ	demand to oil	gamma	0.050	0.050	0.038	0.018	0.040	0.018	0.042	0.019	0.042	0.019
ϑ_D	ma demand	beta	0.500	0.200	0.554	0.072	0.574	0.067	0.534	0.082	0.548	0.076
ϑ_m	ma import price	beta	0.500	0.200	0.499	0.055	0.493	0.055	0.495	0.064	0.505	0.059
ω^F	cap util cost	beta	2.500	0.500	2.310	0.920	1.734	0.334	1.583	0.118	1.599	0.141
ω^E	cap util cost	beta	2.500	0.500	2.933	0.675	1.970	0.660	1.572	0.102	1.549	0.070

The posterior estimates of investment adjustment costs are, in most cases, slightly higher than our prior of 10. As a comparison, Christiano et al. (2010) find 31.43 for the euro area, whereas Smets and Wouters (2007) and Christiano et al. (2014) find 5.48 and 10.78 respectively for the US. However, these estimates are not very precise since the standard deviation of their posterior densities are close to those of their prior densities.

We now turn to the parameters of the financial accelerator. Monitoring costs are found to be higher in France than in the rest of the euro area, although confidence intervals overlap. The value found by Christiano et al. (2014) for the US is 0.21, very close to our estimates and above the 0.12 suggested by Bernanke et al. (1999). These estimates are hardly changed by the inclusion of labor-market frictions into the model, consistently with the limited interaction between the two frictions emphasized in section 2.6.

Next, the proportion of exiting entrepreneurs' net worth that is consumed is not distinct from zero; this is worth emphasis since it is 1 in the calibrated original financial accelerator freamework (see Bernanke et al. (1999)). Nevertheless, it is theoretically relevant to keep a strictly positive value for this parameter since entrepreneurs' motivation to accumulate wealth over time stems from future consumption.

The posterior modes of steady state business failure rates $F(\bar{\omega})$ are found slightly below their

Table 4.3: Habits and adjustment costs

		prior shape	prior mode	prior s.d.	base mode	base s.d.	fa mode	fa s.d.	lm mode	lm s.d.	falm mode	falm s.d.
ρ_R	taylor persis	beta	0.810	0.200	0.812	0.023	0.828	0.023	0.806	0.022	0.816	0.023
η^F	cons habits	beta	0.500	0.100	0.573	0.081	0.473	0.083	0.537	0.087	0.404	0.085
η^E	cons habits	beta	0.500	0.100	0.616	0.066	0.587	0.073	0.655	0.063	0.575	0.070
ϕ^F	invest	gamma	10.000	3.000	11.323	2.503	12.503	2.399	10.445	2.368	12.416	2.588
ϕ^E	invest	gamma	10.000	3.000	10.375	2.577	11.524	2.072	9.224	2.541	11.011	2.068
χ_F^m	imports	gamma	2.500	1.000	2.459	0.981	2.677	1.082	2.393	0.939	2.658	1.006
χ_E^m	imports	gamma	2.500	1.000	2.672	1.086	3.154	1.244	4.306	1.552	3.576	1.365
χ_F^y	prod/imports	gamma	2.500	1.000	0.663	0.191	0.474	0.128	0.657	0.212	0.461	0.135
χ_E^y	prod/imports	gamma	2.500	1.000	0.540	0.151	0.379	0.110	0.833	0.266	0.590	0.183
χ_F^o	cons/oil	gamma	2.500	1.000	2.517	0.955	2.519	0.944	2.489	0.930	2.496	0.937
χ_E^o	cons/oil	gamma	2.500	1.000	2.543	0.960	2.553	0.955	2.511	0.936	2.514	0.942
χ_F^v	cap/oil	gamma	2.500	1.000	2.523	0.959	2.520	0.945	2.475	0.925	2.505	0.939
χ_E^v	cap/oil	gamma	2.500	1.000	2.582	0.973	2.594	0.978	2.555	0.952	2.618	0.977
γ_w^F	index wages	beta	0.220	0.100	0.205	0.082	0.200	0.082	–	–	–	–
γ_w^E	index wages	beta	0.220	0.100	0.163	0.079	0.162	0.080	–	–	–	–
γ_y^F	index prices	beta	0.260	0.100	0.174	0.074	0.155	0.068	0.183	0.071	0.179	0.073
γ_y^E	index prices	beta	0.260	0.100	0.312	0.101	0.297	0.097	0.301	0.080	0.314	0.086
γ_x^F	index exp p	beta	0.260	0.100	0.248	0.090	0.270	0.097	0.198	0.078	0.220	0.087
γ_x^E	index exp p	beta	0.260	0.100	0.221	0.088	0.188	0.079	0.244	0.101	0.229	0.094
$a\star$	exports	gamma	2.500	1.000	3.454	0.923	2.944	1.088	1.326	0.461	1.445	0.432
χ_m	import prices	gamma	0.500	0.200	0.075	0.031	0.058	0.023	0.374	0.169	0.462	0.158

mode a priori; they correspond roughly to 1.5% in annualized pace for the rest of the euro area, and a bit less for France. This is approximately half the value suggested by Bernanke et al. (1999), but our estimates are broadly in line with the value of 0.56% per quarter found by Christiano et al. (2014) for the US.

Table 4.4: Financial accelerator

		prior shape	prior mode	prior s.d.	base mode	base s.d.	fa mode	fa s.d.	lm mode	lm s.d.	falm mode	falm s.d.
μ_F	audit cost	beta	0.210	0.100	–	–	0.208	0.072	–	–	0.199	0.066
$\bar{\gamma}_F$	entrepr exit	beta	0.980	0.005	–	–	0.989	0.002	–	–	0.988	0.002
$100W_F^e/K_F$	transfers	beta	0.020	0.010	–	–	0.021	0.010	–	–	0.020	0.009
$F(\bar{\omega}_F)$	bankrupt rate	beta	0.007	0.004	–	–	0.002	0.001	–	–	0.003	0.002
Θ_F	entrepr cons	beta	0.500	0.250	–	–	0.008	0.011	–	–	0.009	0.012
μ_E	audit cost	beta	0.210	0.100	–	–	0.185	0.056	–	–	0.182	0.059
$\bar{\gamma}_E$	entrepr exit	beta	0.980	0.005	–	–	0.984	0.002	–	–	0.985	0.002
$100W_E^e/K_E$	transfers	beta	0.020	0.010	–	–	0.020	0.009	–	–	0.021	0.009
$F(\bar{\omega}_E)$	bankrupt rate	beta	0.007	0.004	–	–	0.005	0.002	–	–	0.004	0.002
Θ_E	entrepr cons	beta	0.500	0.250	–	–	0.033	0.020	–	–	0.031	0.019

Regarding the parameters that characterize labor market frictions, a first observation is that the elasticity of the matching technology with respect to vacancies is estimated around 0.3 in both regions instead of the standard calibration of 0.5-0.6. Firms' steady state bargaining power is somewhat lower than its prior mode in both regions, but its identification is fragile considering that the posterior standard deviations are not reduced with respect to the prior. The steady state vacancy filling rate is not identified by the estimation so its posterior value reflects a priori beliefs about this parameter; the estimation of the model would certainly be improved if statistics reflecting vacancies would have been included in the dataset, but such data is not readily available in the euro area. Finally, job separation rates in both regions are estimated around 0.07, which is below their prior mode of 0.1. This reflects the high degree of persistence in unemployment timeseries.

Table 4.5: Labor market frictions

		prior	prior	prior	base		fa		lm		falm	
		shape	mode	s.d.	mode	s.d.	mode	s.d.	mode	s.d.	mode	s.d.
φ_F	elast match	beta	0.600	0.100	–	–	–	–	0.228	0.050	0.315	0.064
s_F	job sep	beta	0.100	0.010	–	–	–	–	0.071	0.007	0.075	0.009
$\bar{\xi}_F$	barg power	beta	0.600	0.050	–	–	–	–	0.562	0.049	0.543	0.052
$vc_F/\mathcal{Y}_F$	vac cost	inv gam2	0.020	∞	–	–	–	–	0.017	0.004	0.017	0.004
$\bar{N}_F$	ss employment	beta	0.680	0.050	–	–	–	–	0.739	0.018	0.718	0.042
Φ_F	vac filling	beta	0.900	0.100	–	–	–	–	0.900	0.092	0.900	0.092
φ_E	elast match	beta	0.600	0.100	–	–	–	–	0.272	0.054	0.316	0.062
s_E	job sep	beta	0.100	0.010	–	–	–	–	0.060	0.005	0.068	0.007
$\bar{\xi}_E$	barg power	beta	0.600	0.050	–	–	–	–	0.521	0.053	0.519	0.053
$vc_E/\mathcal{Y}_E$	vac cost	inv gam2	0.020	∞	–	–	–	–	0.018	0.004	0.020	0.005
$\bar{N}_E$	ss employment	beta	0.675	0.050	–	–	–	–	0.616	0.047	0.604	0.045
Φ_E	vac filling	beta	0.900	0.100	–	–	–	–	0.900	0.092	0.900	0.092

Table 4.6: Steady state parameters

		prior	prior	prior	base		fa		lm		falm	
		shape	mode	s.d.	mode	s.d.	mode	s.d.	mode	s.d.	mode	s.d.
I_F/O_F	oil in prod	beta	7.403	0.836	8.730	0.764	8.673	0.804	7.163	1.242	7.946	1.315
I_E/O_E	oil in prod	beta	5.655	0.372	5.873	0.584	5.842	0.581	5.462	0.651	5.307	0.519
C_F^{no}/C_F^o	oil cons	beta	65.667	2.232	68.036	2.824	67.127	3.533	66.630	3.814	66.525	3.857
C_E^{no}/C_E^o	oil cons	beta	60.646	1.907	58.677	2.109	59.956	3.127	63.246	1.842	63.028	2.083
$\bar{G}_F/\bar{\mathcal{Y}}_F$	resid dem	beta	0.295	0.005	0.293	0.010	0.294	0.010	0.294	0.010	0.295	0.010
$\bar{G}_E/\bar{\mathcal{Y}}_E$	resid dem	beta	0.237	0.003	0.237	0.006	0.236	0.006	0.236	0.006	0.236	0.006
$\bar{X}_F^\star/\bar{\mathcal{Y}}_F$	extra exp	beta	0.143	0.008	0.149	0.007	0.154	0.006	0.156	0.004	0.156	0.004
$\bar{X}_E^\star/\bar{\mathcal{Y}}_E$	extra exp	beta	0.206	0.023	0.179	0.006	0.177	0.006	0.189	0.012	0.191	0.013
$\bar{M}_F^\star/\bar{X}_{E/F}$	extra imp	beta	0.667	0.071	0.797	0.016	0.797	0.016	0.793	0.021	0.794	0.020
$\bar{M}_E^\star/\bar{X}_{F/E}$	extra imp	beta	4.552	0.585	3.489	0.142	3.507	0.164	3.614	0.273	3.644	0.300
$\bar{M}_F/\bar{Y}_F$	imports	beta	0.315	0.027	0.280	0.017	0.288	0.016	0.282	0.016	0.284	0.016
$\bar{M}_E/\bar{Y}_E$	imports	beta	0.238	0.039	0.165	0.006	0.165	0.005	0.177	0.014	0.176	0.014
δ^F	cap deprec	beta	0.020	0.010	0.006	0.003	0.021	0.005	0.006	0.002	0.022	0.005
δ^E	cap deprec	beta	0.020	0.010	0.006	0.001	0.023	0.004	0.003	0.002	0.023	0.006
$\bar{\varepsilon}_a^E$	ss techno	beta	0.740	0.200	0.611	0.161	0.616	0.090	0.832	0.161	0.837	0.121

4.5 Fit and dynamic properties of the complete model

4.5.1 R^2

A measure of the fit of a model estimated with measurement errors is the R^2 statistics associated with each observed variable for which a measurement error has been included in the estimation procedure. It is computed as:

$$R^2 = 1 - \frac{\sigma^2_{\text{me}}}{\sigma^2_{\text{data}}},$$

where σ_{me} is the sample standard deviation of the series of measurement errors, and σ_{data} is the standard deviation of data. It is an indicator of the part of the dynamics of the observed timeseries which can be accounted for by the model using structural shocks only; for this part, the model provides story telling about the source of fluctuations, based on the joint observation of all the dataset. It is referred to as the 'explained' part, while the measurement errors are named 'residual' parts.

Table 4.1 provides the R^2 statistics, and sample correlation coefficients between the measurement errors and the explained parts of the observed timeseries, in the complete model (with financial and labor market frictions). This correlation should theoretically be found close to zero.

The first observation is that the fit as measured by this indicator is rather good for a majority of observed variables. However, the dynamics of real wage bills predicted by the model are rejected by the data, since the measurement errors are significantly negatively correlated with the model's predictions. This is especially true for France. Moreover, the fit of the model with respect to the growth rate of loans in France is fair-to-middling.

4.5.2 Second order moments

This picture is completed by inspecting the complete correlation structure implied by the model for the observed variables, and comparing them with the data. Appendix 4.B provides the standard errors and cross-correlations up to a 4-quarter lead and lag for these variables. The shaded area marks the 95%-confidence interval around the moments mesured in the data. These confidence areas are computed as follows. Regarding standard deviations, if the T realizations of a given variable x, denoted by $x_1, x_2, \ldots, x_T$ are considered as independant normally distributed random variables with mean 0 and variance σ^2, then

$$\frac{\sum_{j=1}^{T} x_j^2}{\sigma^2} \sim \chi^2_{T-1}.$$

Hence, with the critical values of the chi square distribution with $T-1$ degrees of freedom taken from a statistical table, the confidence interval for σ is defined by

$$p\left\{\chi^2_{T-1}(0.975) < \frac{\sum_{j=1}^{T} x_j^2}{\sigma^2} < \chi^2_{T-1}(0.025)\right\} = 0.95$$

Table 4.1: R^2 of the model with financial and labor market frictions

		explained std	residual std	data std	corr(expl,res)	R^2
$d\mathcal{Y}_F$	GDP growth France	0.50	0.01	0.51	0.24	1.00
dC_F	Consumption growth France	0.53	0.03	0.55	0.67	1.00
dI_F	Investment growth France	1.40	0.39	1.56	0.27	0.94
π_F	Inflation France	0.75	0.04	0.76	0.11	1.00
$d\mathcal{Y}$	GDP growth EA	0.59	0.14	0.62	0.17	0.95
dC	Consumption growth EA	0.39	0.05	0.42	0.50	0.99
dI	Investment growth EA	1.68	1.07	1.88	-0.13	0.68
π	Inflation EA	0.70	0.03	0.70	0.02	1.00
R	Interest rate	0.36	0.00	0.36	0.18	1.00
dW_F	Wage bill growth France	0.31	0.41	0.41	-0.39	0.01
dW	Wage bill growth EA	0.35	0.41	0.44	-0.33	0.16
dX	Exports growth EA	2.20	0.64	2.26	-0.03	0.92
dM	Imports growth EA	1.95	1.10	2.14	-0.12	0.74
dS	Exch. rate depreciation	4.18	0.00	4.18	-0.03	1.00
dX_F	Exports growth France	1.59	0.80	2.03	0.37	0.85
dM_F	Imports growth France	1.49	0.71	1.91	0.44	0.86
π^X	Export price inflation EA	0.57	0.45	0.76	0.11	0.66
π^X_F	Export price inflation France	0.54	0.34	0.75	0.40	0.79
dD	World demand growth	2.08	0.00	2.08	-0.02	1.00
π^o	Oil price inflation	14.18	0.00	14.18	-0.09	1.00
$\pi^\star$	World inflation	0.22	0.00	0.22	0.09	1.00
$\pi^\star_M$	Import price inflation	2.39	0.00	2.39	-0.22	1.00
$\pi^\star_X$	Export competitors inflation	2.76	0.00	2.76	-0.07	1.00
dn	Net worth growth EA	9.02	0.07	9.04	0.33	1.00
dn_F	Net worth growth France	8.80	0.05	8.81	0.25	1.00
$d\tilde{B}$	Loans growth EA	0.91	0.68	1.25	0.16	0.70
$d\tilde{B}_F$	Loans growth France	0.90	1.00	1.43	0.13	0.51
U	Unemployment EA	1.46	0.00	1.46	0.04	1.00
U_F	Unemployment France	1.27	0.07	1.27	0.07	1.00

so we get:

$$p\left\{\sqrt{\frac{\sum_{j=1}^{T} x_j^2}{\chi^2_{T-1}(0.025)}} < \sigma < \sqrt{\frac{\sum_{j=1}^{T} x_j^2}{\chi^2_{T-1}(0.025)}}\right\} = 0.95,$$

where $\chi^2_{T-1}(\alpha)$ is the critical value of the chi square distribution with $T-1$ degrees of freedom associated with the level α. To compute the confidence intervals around the correlation coefficients measured in the data, a standard approach consists in using a Fisher transformation, that is, for a sample correlation coefficient value r,

$$Z = \operatorname{arctanh}(r),$$

where arctanh is the inverse hyperbolic tangent function. Then Z is approximately normally distributed with mean $\operatorname{arctanh}(\rho)$ and standard deviation $1/\sqrt{T-3}$, where ρ is the true correlation coefficient. The confidence interval for r is defined by

$$p\left\{N_{0,1}(0.025) < (\operatorname{arctanh}(r) - \operatorname{arctanh}(\rho))\sqrt{T-3} < N_{0,1}(0.975)\right\} = 0.95,$$

$$p\left\{\operatorname{arctanh}(r) - \frac{1.96}{\sqrt{T-3}} < \operatorname{arctanh}(\rho) < \operatorname{arctanh}(r) + \frac{1.96}{\sqrt{T-3}}\right\} = 0.95,$$

$$p\left\{\tanh\left(\operatorname{arctanh}(r) - \frac{1.96}{\sqrt{T-3}}\right) < \rho < \tanh\left(\operatorname{arctanh}(r) + \frac{1.96}{\sqrt{T-3}}\right)\right\} = 0.95.$$

I now turn to the results. With respect to standard deviations, the model predictions are in line with the data, except for HICP inflation and unemployment rates, for which the variability is overestimated. Concerning unemployment rates, this is mainly because the model exaggerates their persistence.

Apart from unemployment, the model captures quite well the autocorrelation structure of the observed variables. Exceptions are the growth rate of investment in the euro area, which is less autocorrelated at first order in the data than in the model, and the growth rate of entrepreneurs' net worth, which is less persistent in the model than the growth rates of stock market indexes both in France and in the whole euro area.

Another observation is that the cross-correlation is almost zero for a number of couples of variables. This is explained by the fact that the model includes a large number of shocks. Hence, some variables may be primarily determined by subsets of shocks that have almost no effect on other variables of the model. The result is probably a too high degree of correlation between some of the smoothed structural shocks series obtained with a Kalman smoother over the sample.

Otherwise, if many of the model's implied cross-correlations are consistent with those measured in the data, some others are at odds. In particular, the model cannot replicate the positive correlation of foreign prices with activity. This can be expected since foreign prices are largely exogenous. Next, the model counterfactually predicts a positive correlation between HICP year-on-year inflation and real wage bills, and cannot replicate the positive correlation between consumption and investment. Consumption and exports (or consumption and world demand) are also negatively correlated in the model because the crowding out effect dominates, by contrast with the data. Finally, the model also fails to replicate the positive correlation of the wage bill with financial variables (net worth and loans).

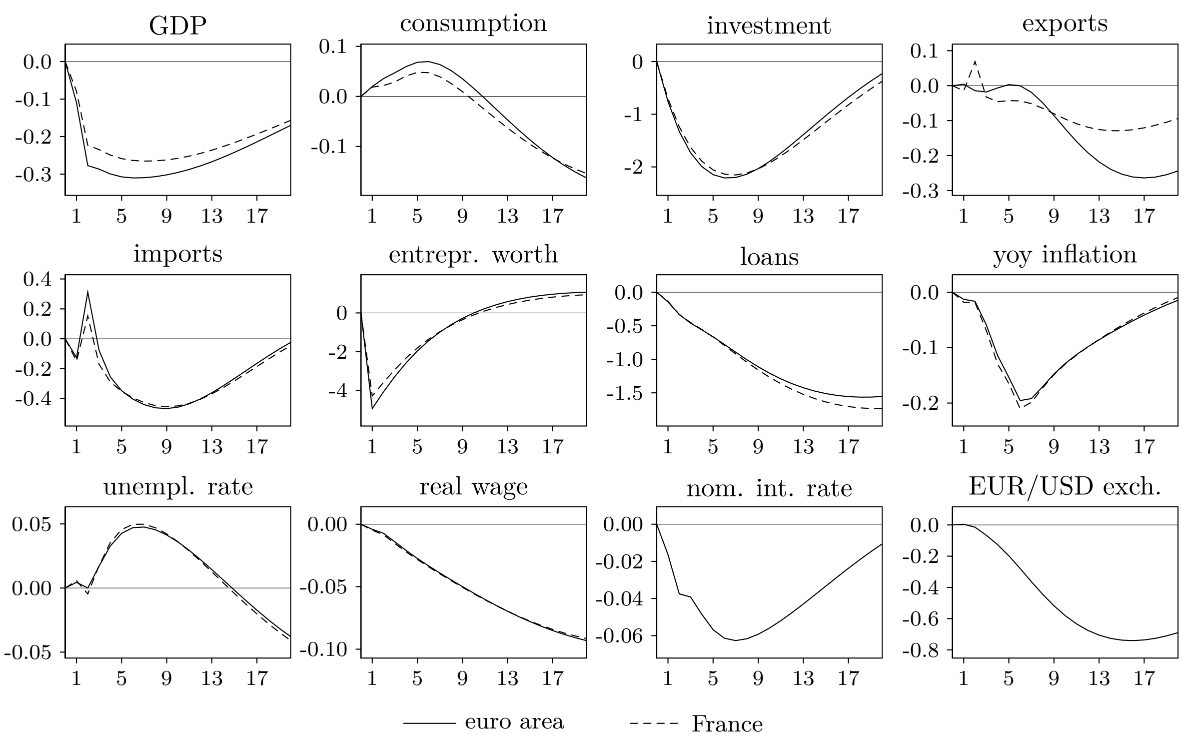
4.6 Impulse responses

This section shows the impulse responses of the complete model including both financial and labor market frictions to some shocks among those identified as the main sources of business cycle fluctuations. They are the risk shock, the permanent shock affecting the trend in world technology, the investment technology shock and the shock to the world demand addressed to the euro area. Except for the world demand shock which is obviously common to both regions, the main contributor identified in all cases is the "euro area" component of the shock. As explained in section 1.4, this component is uncorrelated with the "French" component, but does not exactly represent the disturbances affecting simultaneouly the two regions; all the most, this interpretation should be viewed as an approximation.

4.6.1 Risk shock

An unanticipated risk shock increases the interest rate charged on corporate loans ; entrepreneurs borrow less and purchase less capital. The fall in capital demand negatively affects its price and hence entrepreneurs net worth. This reduction in entrepreneurs financing capacities magnifies the contractionary effect of the shock. Initially, there is a substitution effect away from investment towards consumption, and the latter remains slightly above its steady state level for 2 years. Then it falls persistently, reflecting the decrease in produced resources. The decline in total demand for goods results in lower inflation, and higher unemployment. Monetary policy responds by reducing the nominal interest rate. With the estimated high persistence of the Taylor rule, the real interest rate of bonds in the euro area slightly increases. As a consequence, the EUR appreciates, which cancels the positive effects on exports which may have come in the short run from lower production costs. Imports remain below their long run level after 3 quarters and until the end of the simulation horizon, reflecting the slackness of domestic demand.

The short term response of consumption in the model differs from the one shown by Christiano et al. (2014). These authors find a negative response of consumption and argue that it is related to the fact that the monetary authority only smoothly reduces the interest rate. But in my framework, this mechanism does not totally cancel the substitution effect from investment towards consumption during the first quarters after the shock.

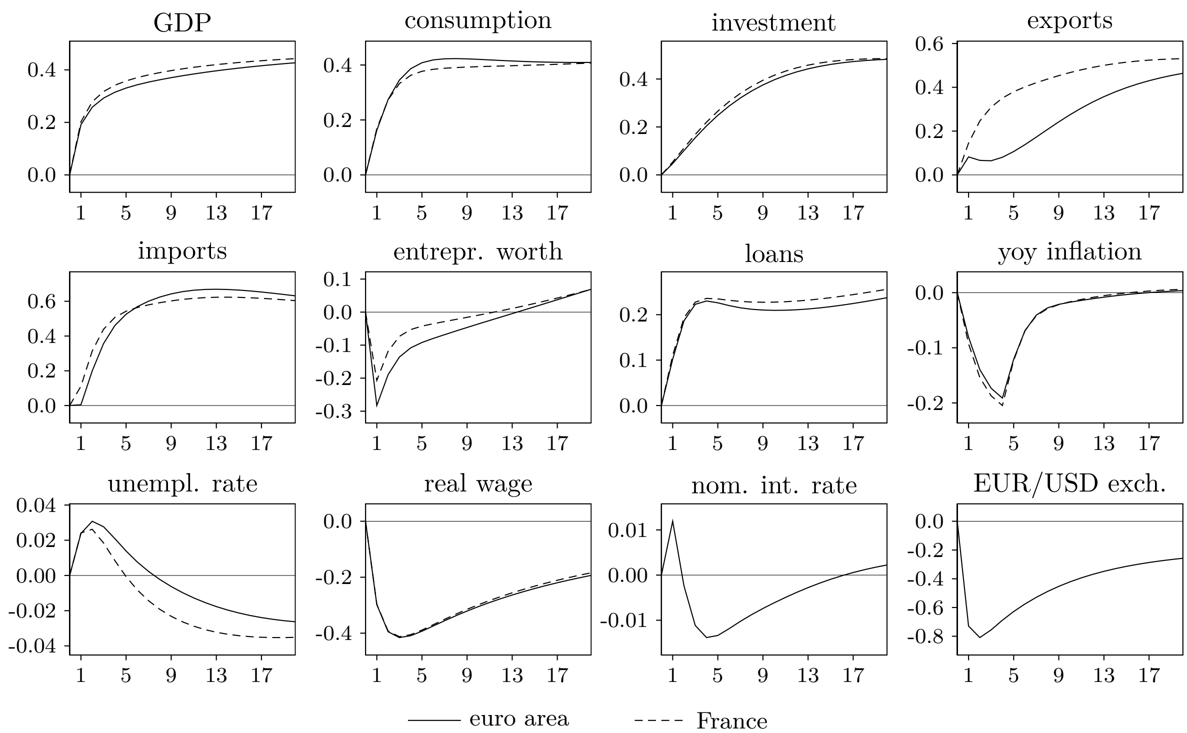


4.6.2 Technology growth

A positive shock to the trend in world technology increases permanently output. The excess supply of goods lowers inflation, which supports all the components of final demand. Yet, in the short run, employment falls because the adjustment of prices is slow, so firms are temporarily in excess production capacity.

As explained in section 2.11, the measure of output gap used in the interest rate rule is the growth rate of GDP, including the stochastic trend. Consistently, the shock implies a small positive jump in the nominal interest rate in period 1. Then, the central bank smoothly responds to the fall in inflation. As a result, the real interest rate is raised in the short run, which yields an appreciation of the EUR/USD exchange rate. Obviously, this is because the real interest rate in the rest of the world is exogenous under the small open economy assumption used for the euro area.

Last, the reduction in entrepreneurs net worth is related to the fall in the price of capital that follows from firms' excess capacity after the shock.

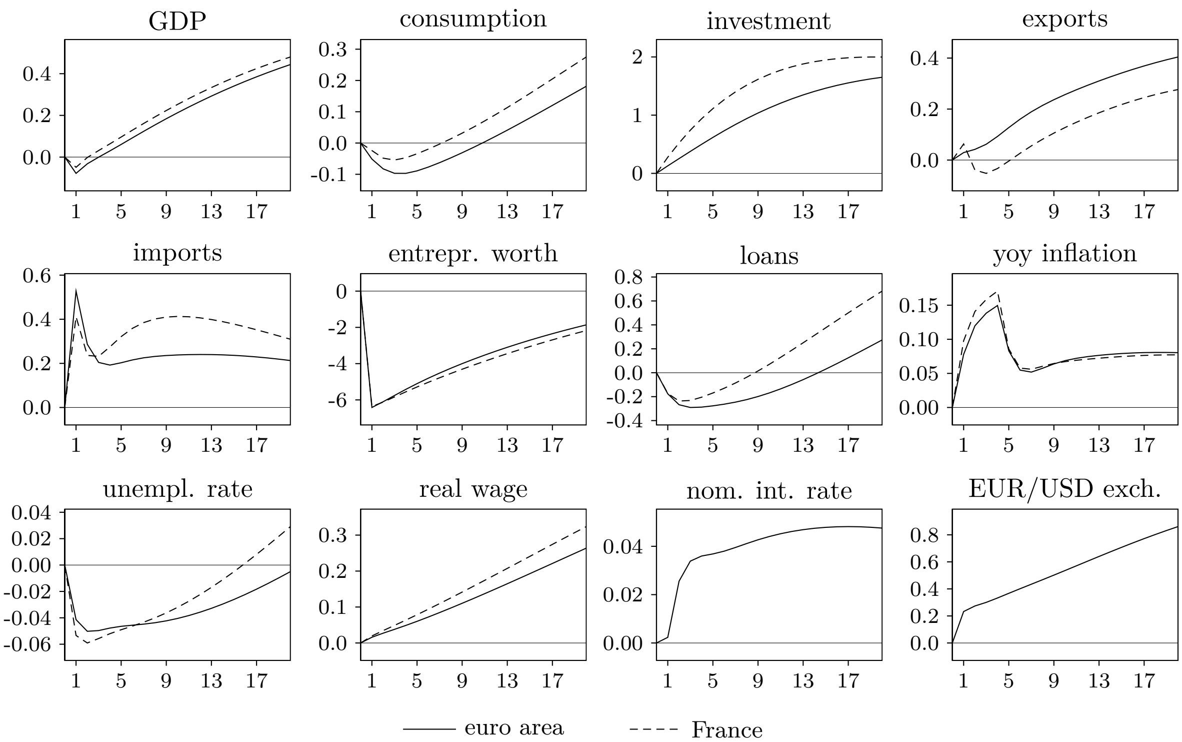


4.6.3 Investment technology

A technological shock specific to new investment goods decreases the relative price of the existing capital stock, and stimulates new investment. Although the fall in the price of capital negatively affects entrepreneurs net worth, it also leads them to purchase more capital incorporating new investment, in order to benefit from higher prospective return on capital. In addition, with the decrease in the price of capital, less external borrowing is needed.

Higher capital raises the marginal productivity of labor, so unemployment falls. Consumption declines in the short run because households preferably allocate their revenues to investment. Finally, the rise in labor demand entails an increase in the real wage rate, and hence in inflation.

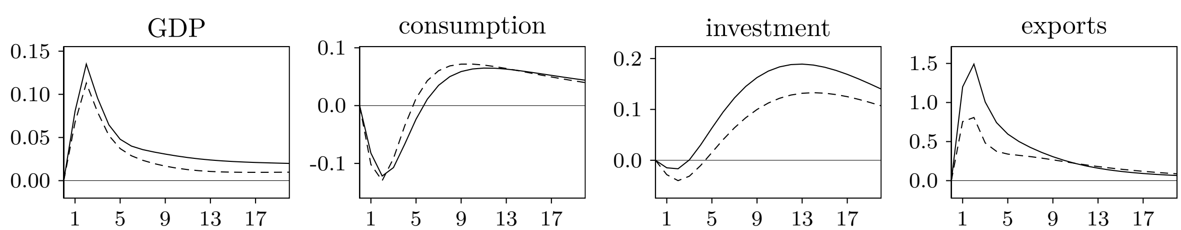
With the decrease in the real interest rate, the EUR depreciates. As a result, exports increase. On the other hand, the quick rise in domestic production prices favors substitution towards imported goods, while the exchange rate depreciation, translating into higher import prices, is gradual.

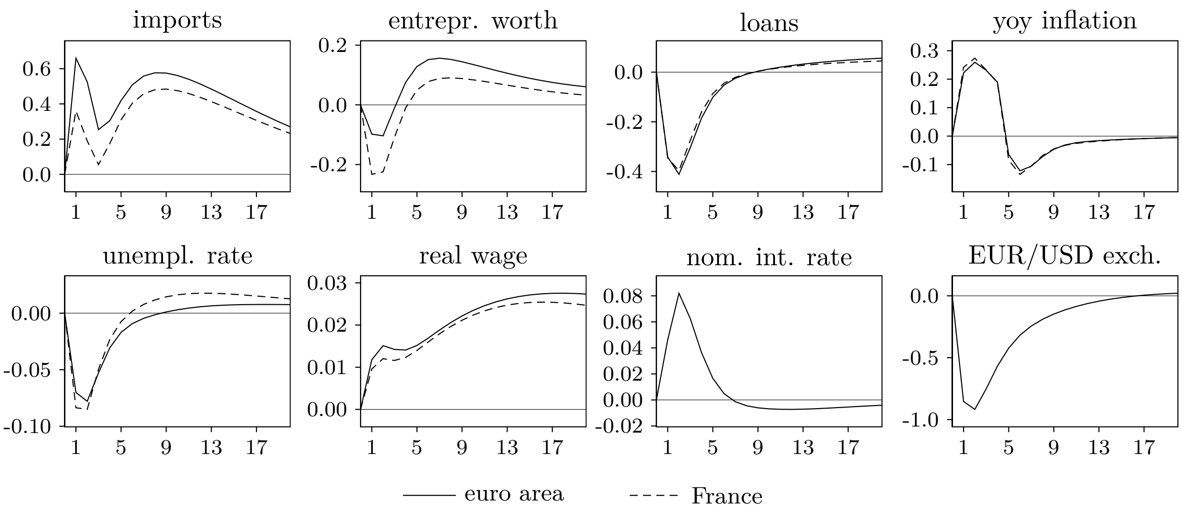


4.6.4 World demand

An unexpected increase in the world demand addressed to the euro area boosts exports. In the short term, consumption and investment are crowded out. Higher foreign demand leads firms to use capital more intensively and to raise their demand for labor. As a result, real wages increase. Added to the decrease in the supply of goods for the domestic market, this raises inflation. Despite the decrease in the real interest rate, the accrued demand for domestic goods yields an appreciation of the EUR/USD exchange rate. This appreciation makes imported goods more competitive, and reinforces the effect of the jump in production costs. As a result, imports increase.

This shock also induces a small fall in the price of capital, which translates into a reduction in both entrepreneurs net worth and loans. This is because the additional production needed to satisfy the transitory rise in foreign demand can be achieved through higher capacity utilization, without the need for additional new capital, at least in the short run.





4.7 Cross-country propagation inside the euro area

As explained in section 1.4, the model allows for a certain degree of correlation between country-specific exogenous variables. In practice, if such an exogenous variable is denoted by $\varepsilon_{F,t}$ for country F and $\varepsilon_{E,t}$ for country E, I use the following specification:

$$\varepsilon_{F,t} = \varepsilon_{F,t-1}^{\rho_F} \bar{\varepsilon}_F^{1-\rho_F} \exp\left(\sigma_F \eta_{F,t} + \varsigma_F \eta_t\right), \quad \eta_F \sim N(0,1), \quad \eta \sim N(0,1),$$

$$\varepsilon_{E,t} = \varepsilon_{E,t-1}^{\rho_E} \bar{\varepsilon}_E^{1-\rho_E} \exp\left(\sigma_E \eta_{E,t} + \varsigma_E \eta_t\right), \quad \eta_E \sim N(0,1).$$

The covariance matrix of the terms inside the exponentials is then simply

$$\Sigma = E\left[\begin{pmatrix} \sigma_F \eta_{F,t} + \varsigma_F \eta_t \\ \sigma_E \eta_{E,t} + \varsigma_E \eta_t \end{pmatrix} \begin{pmatrix} \sigma_F \eta_{F,t} + \varsigma_F \eta_t & \sigma_E \eta_{E,t} + \varsigma_E \eta_t \end{pmatrix}\right] = \begin{pmatrix} \sigma_F^2 + \varsigma_F^2 & \varsigma_F \varsigma_E \\ \varsigma_F \varsigma_E & \sigma_E^2 + \varsigma_E^2 \end{pmatrix}$$

With σ_F, ς_F and ς_E strictly positive and $\sigma_E = 0$, it actually only involves two orthogonal gaussian white noise disturbances, η_F and η, and the covariance matrix above is

$$\Sigma = \begin{pmatrix} \sigma_F^2 + \varsigma_F^2 & \varsigma_F \varsigma_E \\ \varsigma_F \varsigma_E & \varsigma_E^2 \end{pmatrix}.$$

This specification is expected to facilitate the identification by the model of the comovements between the French and the rest of the euro area economies observed in the data. It is thus necessary to question the role played by endogenous mechanisms, such as trade or financial linkages, in the cross-border propagation of shocks, and to assess the relative importance of the estimated correlations between exogenous variables in explaining the comovements between the two regions. In this section, I address this question in the complete model including both financial and labor market frictions.

For that purpose, I need a calibration which neutralizes the correlation between exogenous AR(1) variables but which leaves their variances unchanged. This is simply done by choosing the following new values, denoted with a " $'$ ", for the σ's and ς's in the specification above:

$$\sigma'_F = \sqrt{\sigma_F^2 + \varsigma_F^2}, \quad \sigma'_E = \varsigma_E, \quad \varsigma'_F = \varsigma'_E = 0,$$

and the covariance matrix of the terms inside the exponentials becomes diagonal:

$$\Sigma' = \begin{pmatrix} {\sigma'_F}^2 + {\varsigma'_F}^2 & \varsigma'_F \varsigma'_E \\ \varsigma'_F \varsigma'_E & {\sigma'_E}^2 + {\varsigma'_E}^2 \end{pmatrix} = \begin{pmatrix} {\sigma_F}^2 + {\varsigma_F}^2 & 0 \\ 0 & {\varsigma_E}^2 \end{pmatrix}.$$

First, Table 4.1 reports the estimated correlation coefficient between the country shocks of France and of the rest of the euro area (the "country shock" represents the sum of the terms in the exponential for each country). Using the notations aboved, this coefficient is calculated as

$$\varrho = \frac{\varsigma_F}{\sqrt{\sigma_F^2 + \varsigma_F^2}}.$$

Investment technology and financial shocks are strongly correlated. Markup shocks also exhibit a high degree of correlation, whereas it is moderate for the other ones.

Table 4.1: Estimated cross-correlation of exogenous shocks

pref	invest	techno	gov cons	markup	mkup export	bargain	wealth	risk
0.40	0.95	0.53	0.67	0.76	0.54	0.66	0.86	0.96

Next, I compute the standard deviations of a subset of major variables of each country when only the shocks of the other country are activated, without exogenous correlation. This provides an assessment of the contribution of pure endogenous propagation mechanisms to cyclical variations. Results expressed in percentage of unconditional volatilities are shown in Table 4.2. First, inflation rates in France and in the rest of the euro area are the most affected by endogenous cross-country propagation. Then, the endogenous propagation of French shocks has a small overall effect on the volatility of the variables of the rest of the euro area. The reverse transmission is somewhat larger, especially for inflation and investment. This is consistent with the difference in the relative sizes of the two regions.

Last, Table 4.3 shows the matrix of correlation of the same subset of variables of the two regions, conditional on the sole country-specific shocks and after neutralizing their correlation. Inside square brackets are the unconditional correlations implied by the model. Except for inflation rates in the two countries, and for French inflation with investment in the rest of the euro area, the coefficients found are virtually zero (or very low). This means either that the endogenous cross-

Table 4.2: % of volat due to the endogenous propagation of shocks specific to the other region

	GDP	cons	invest	empl	inflat
France	12.89	9.27	26.90	14.16	42.05
rest of the EA	3.12	2.39	6.75	2.49	8.53

border propagation of country shocks is weak, or that this propagation results in lagged or opposite responses.

Table 4.3: Cross-correlation implied by endogenous propagation

		France				
		GDP	cons	invest	empl	inflat
	GDP	-0.06 (0.92)	-0.05 (0.87)	-0.02 (0.78)	0.05 (-0.56)	0.01 (0.04)
rest	cons	-0.02 (0.83)	-0.07 (0.90)	0.07 (0.58)	0.09 (-0.62)	-0.18 (-0.10)
of euro	invest	-0.09 (0.82)	-0.01 (0.68)	-0.15 (0.90)	-0.02 (-0.40)	0.32 (0.16)
area	empl	0.00 (-0.40)	0.05 (-0.49)	-0.06 (-0.22)	-0.06 (0.80)	0.12 (0.09)
	inflat	0.03 (0.06)	0.06 (-0.07)	-0.01 (0.18)	-0.05 (0.10)	0.74 (0.96)

To conclude, endogenous cross-border propagation mechanisms between France and the rest of the euro area seem to play a secondary role in the dynamics implied by the estimated model. A large part of the synchronization is captured by the assumed correlation between exogenous variables – as well as by the common shocks related to the rest of the world that also hit both economies.

Yet this does not mean that international propagation channels are absent or inefficient. Indeed, the estimated values of the parameters characterizing them are affected by the presence of correlated exogenous variables. Specifically, the shock structure imposed during the estimation provides a convenient way to capture the observed comovements in the two economies, to the detriment of endogenous propagation. The latter would surely be found more important if the model only included independant exogenous variables, but the fit would not be as good as it is. This exercise should hence be considered as part of a review of the estimated model's properties, and the results do not indicate that it is rejected by the data.

Appendix

4.A Posterior estimates of exogenous processes

Table 4.A.1: Measurement errors

		prior shape	prior mode	prior s.d.	base mode	base s.d.	fa mode	fa s.d.	lm mode	lm s.d.	falm mode	falm s.d.
$d\mathcal{Y}_F$	gdp F	beta	0.252	0.126	0.159	0.032	0.113	0.041	0.169	0.037	0.056	0.056
dC_F	cons F	beta	0.276	0.138	0.354	0.050	0.246	0.066	0.226	0.083	0.105	0.101
dI_F	inv F	beta	0.767	0.384	0.403	0.130	0.537	0.093	0.532	0.117	0.547	0.097
π_F	infl F	beta	0.381	0.191	0.065	0.018	0.062	0.019	0.027	0.015	0.067	0.018
$d\mathcal{Y}$	gdp ea	beta	0.305	0.152	0.157	0.037	0.175	0.027	0.157	0.038	0.178	0.029
dC	cons ea	beta	0.204	0.102	0.165	0.040	0.096	0.060	0.181	0.035	0.102	0.057
dI	inv ea	beta	0.924	0.462	1.033	0.113	1.120	0.109	0.941	0.159	1.136	0.110
π	infl ea	beta	0.350	0.175	0.052	0.010	0.050	0.009	0.053	0.009	0.050	0.010
R	int rate	beta	0.165	0.082	0.012	0.010	0.009	0.008	0.013	0.010	0.011	0.010
dX	exports ea	beta	1.135	0.567	0.722	0.151	0.764	0.135	0.819	0.142	0.831	0.125
dM	imports ea	beta	1.061	0.530	1.474	0.141	1.480	0.144	1.329	0.145	1.264	0.148
dX_F	exports f	beta	1.020	0.510	1.056	0.108	0.988	0.105	1.037	0.115	0.917	0.113
dM_F	imports f	beta	0.953	0.476	1.115	0.124	1.159	0.130	0.851	0.144	0.915	0.158
π^X	exp infl ea	beta	0.383	0.191	0.480	0.063	0.485	0.060	0.485	0.053	0.487	0.051
π_F^X	exp infl f	beta	0.376	0.188	0.423	0.064	0.462	0.063	0.383	0.057	0.404	0.056
dW	wage bill ea	beta	0.219	0.109	0.126	0.119	0.283	0.063	0.398	0.029	0.398	0.028
dW_F	wage bill F	beta	0.203	0.101	0.091	0.061	0.073	0.065	0.388	0.019	0.387	0.020
$\mathcal{L}_F$	hours F	beta	0.826	0.413	0.023	0.023	0.023	0.023	–	–	–	–
$\mathcal{L}$	hours ea	beta	0.619	0.310	0.152	0.113	0.132	0.125	–	–	–	–
dn	net worth ea	beta	4.543	2.271	–	–	0.505	0.478	–	–	0.405	0.404
dn_F	net worth F	beta	4.429	2.215	–	–	0.400	0.405	–	–	0.353	0.355
$d\tilde{B}$	loans ea	beta	0.597	0.299	–	–	0.735	0.089	–	–	0.759	0.091
$d\tilde{B}_F$	loans F	beta	0.717	0.359	–	–	1.096	0.116	–	–	1.090	0.119
U	unempl ea	beta	0.564	0.282	–	–	–	–	0.020	0.008	0.015	0.010
U_F	unempl F	beta	0.504	0.252	–	–	–	–	0.087	0.013	0.086	0.014

Table 4.A.2: Persistence of exogenous processes

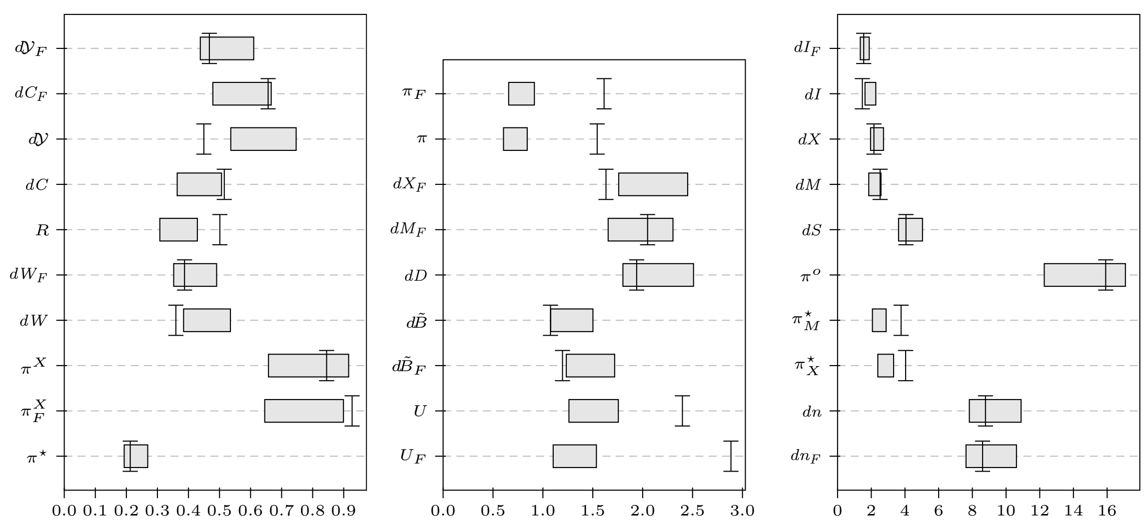
		prior shape	prior mode	prior s.d.	base mode	base s.d.	fa mode	fa s.d.	lm mode	lm s.d.	falm mode	falm s.d.
ρ_β^F	impatience	beta	0.500	0.200	0.373	0.193	0.449	0.244	0.255	0.168	0.317	0.174
ρ_β^E	impatience	beta	0.500	0.200	0.777	0.099	0.890	0.066	0.745	0.123	0.842	0.112
ρ_I^F	invest	beta	0.500	0.200	0.735	0.071	0.979	0.010	0.717	0.071	0.983	0.011
ρ_I^E	invest	beta	0.500	0.200	0.759	0.095	0.977	0.011	0.508	0.224	0.985	0.008
ρ_a^F	techno	beta	0.500	0.200	0.762	0.184	0.973	0.023	0.843	0.106	0.925	0.056
ρ_a^E	techno	beta	0.500	0.200	0.988	0.011	0.559	0.237	0.824	0.110	0.904	0.087
ρ_G^F	res demand	beta	0.500	0.200	0.638	0.271	0.582	0.285	0.404	0.245	0.355	0.210
ρ_G^E	res demand	beta	0.500	0.200	0.469	0.233	0.541	0.235	0.536	0.262	0.557	0.239
ρ_p^F	price markup	beta	0.500	0.200	0.669	0.088	0.713	0.095	0.197	0.136	0.821	0.104
ρ_p^E	price markup	beta	0.500	0.200	0.682	0.091	0.717	0.093	0.422	0.226	0.544	0.232
ρ_x^F	exp markup	beta	0.500	0.200	0.852	0.061	0.865	0.063	0.810	0.095	0.745	0.232
ρ_x^E	exp markup	beta	0.500	0.200	0.717	0.085	0.841	0.083	0.686	0.160	0.855	0.086
$\rho_{R\star}$	foreign int	beta	0.500	0.200	0.648	0.101	0.572	0.119	0.708	0.084	0.595	0.128
ρ_g	growth	beta	0.500	0.200	0.519	0.105	0.480	0.153	0.475	0.089	0.379	0.145
ρ_r	mon policy	beta	0.500	0.200	0.416	0.090	0.335	0.088	0.415	0.086	0.332	0.088
$\rho_{\pi\star}$	foreign price	beta	0.500	0.200	0.437	0.084	0.442	0.083	0.441	0.085	0.437	0.083
ρ_O	oil price	beta	0.500	0.200	0.771	0.047	0.764	0.044	0.712	0.052	0.730	0.049
$\rho_{M\star}$	imp price	beta	0.500	0.200	0.982	0.013	0.976	0.017	0.981	0.014	0.975	0.018
$\rho_{X\star}$	exp price	beta	0.500	0.200	0.929	0.031	0.938	0.032	0.931	0.029	0.903	0.031
ρ_D	world demand	beta	0.500	0.200	0.865	0.033	0.881	0.030	0.779	0.049	0.831	0.037
ρ_S	market share	beta	0.500	0.200	0.494	0.237	0.730	0.233	0.922	0.040	0.897	0.038
ρ_m	import markup	beta	0.500	0.200	0.962	0.023	0.968	0.019	0.728	0.123	0.565	0.125
ρ_w^F	wage markup	beta	0.500	0.200	0.870	0.054	0.896	0.045	–	–	–	–
ρ_w^E	wage markup	beta	0.500	0.200	0.837	0.078	0.860	0.102	–	–	–	–
ρ_ξ^F	wage bargain	beta	0.500	0.200	–	–	–	–	0.927	0.044	0.970	0.026
ρ_ξ^E	wage bargain	beta	0.500	0.200	–	–	–	–	0.887	0.049	0.966	0.035
ρ_γ^F	fin wealth	beta	0.500	0.200	–	–	0.445	0.166	–	–	0.408	0.158
ρ_γ^E	fin wealth	beta	0.500	0.200	–	–	0.448	0.251	–	–	0.428	0.224
ρ_v^F	fin risk	beta	0.800	0.200	–	–	0.932	0.016	–	–	0.938	0.017
ρ_v^E	fin risk	beta	0.800	0.200	–	–	0.949	0.014	–	–	0.936	0.017

Table 4.A.3: Standard errors of exogenous processes

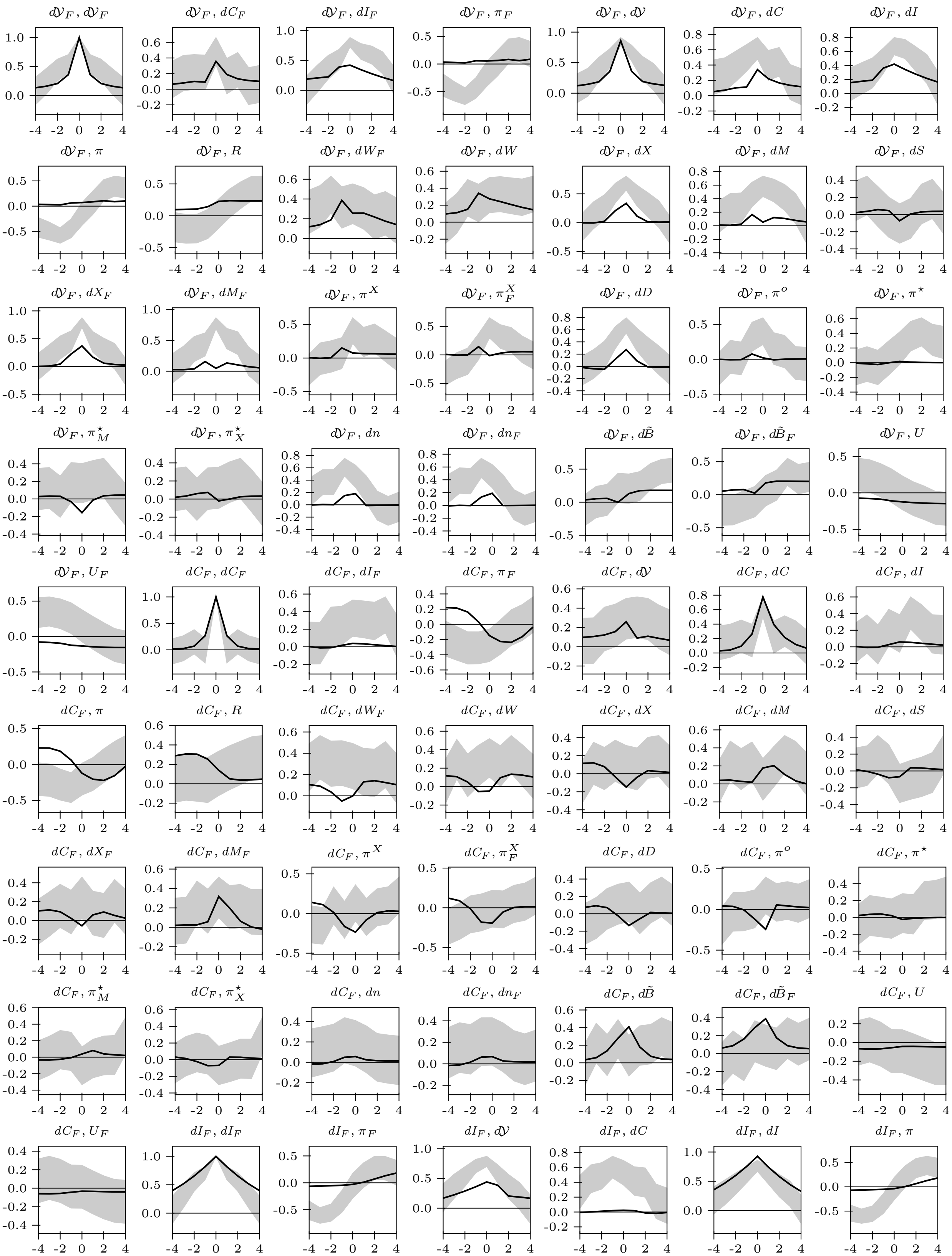
		prior	prior	prior	base		fa		lm		falm	
		shape	mode	s.d.	mode	s.d.	mode	s.d.	mode	s.d.	mode	s.d.
σ_β^F	impatience	inv gam2	0.866	∞	0.590	0.212	0.627	0.202	1.270	0.560	0.975	0.234
ς_β^F	impatience	inv gam2	0.500	∞	0.786	0.285	0.657	0.247	0.979	0.581	0.429	0.178
ς_β^E	impatience	inv gam2	1.000	∞	1.008	0.199	1.049	0.250	0.890	0.256	0.849	0.226
σ_I^F	invest	inv gam2	4.330	∞	2.051	0.659	1.226	0.217	2.046	0.654	1.260	0.227
ς_I^F	invest	inv gam2	2.500	∞	3.600	1.050	3.383	0.455	3.422	1.136	3.687	0.461
ς_I^E	invest	inv gam2	5.000	∞	3.619	1.283	2.844	0.388	5.481	3.148	2.859	0.404
σ_a^F	techno	inv gam2	0.866	∞	0.214	0.041	0.333	0.065	0.302	0.063	0.372	0.052
ς_a^F	techno	inv gam2	0.500	∞	0.220	0.051	0.298	0.077	0.251	0.068	0.235	0.062
ς_a^E	techno	inv gam2	1.000	∞	0.557	0.088	0.315	0.066	0.437	0.094	0.405	0.083
σ_G^F	res demand	inv gam2	0.866	∞	0.361	0.088	0.409	0.109	0.610	0.188	0.591	0.186
ς_G^F	res demand	inv gam2	0.500	∞	0.266	0.080	0.327	0.107	0.412	0.170	0.532	0.199
ς_G^E	res demand	inv gam2	1.000	∞	0.607	0.157	0.496	0.127	0.523	0.150	0.528	0.138
σ_p^F	price markup	inv gam2	0.866	∞	0.872	0.317	0.629	0.234	4.402	1.633	0.745	0.384
ς_p^F	p markup	inv gam2	0.500	∞	1.710	0.639	1.130	0.465	0.536	0.336	0.877	0.597
ς_p^E	p markup	inv gam2	1.000	∞	3.379	1.578	0.975	0.408	0.867	0.421	0.973	0.467
σ_x^F	exp markup	inv gam2	0.866	∞	0.465	0.160	0.490	0.156	0.704	0.290	0.769	0.371
ς_x^F	exp markup	inv gam2	0.500	∞	1.561	0.491	1.081	0.412	0.773	0.372	0.492	0.264
ς_x^E	exp markup	inv gam2	1.000	∞	2.853	1.109	1.538	0.688	1.857	0.903	1.183	0.481
$\sigma_{R\star}$	foreign int	inv gam2	1.000	∞	0.398	0.098	0.391	0.103	0.370	0.088	0.386	0.098
σ_g	growth	inv gam2	1.000	∞	0.409	0.065	0.321	0.060	0.466	0.071	0.322	0.058
σ_r	mon policy	inv gam2	0.100	∞	0.070	0.008	0.073	0.008	0.070	0.008	0.076	0.009
$\sigma_{\pi\star}$	foreign price	inv gam2	0.200	∞	0.181	0.015	0.181	0.015	0.181	0.015	0.181	0.015
σ_O	oil price	inv gam2	12.600	∞	12.753	1.057	12.659	1.043	12.763	1.065	12.696	1.053
$\sigma_{M\star}$	imp price	inv gam2	2.900	∞	2.720	0.224	2.717	0.224	2.724	0.225	2.718	0.224
$\sigma_{X\star}$	exp price	inv gam2	2.600	∞	2.650	0.219	2.645	0.218	2.652	0.219	2.673	0.223
σ_D	world dem	inv gam2	1.700	∞	1.564	0.133	1.675	0.140	1.524	0.134	1.662	0.143
σ_S	market share	inv gam2	1.000	∞	4.228	1.085	4.142	1.060	2.851	0.495	3.094	0.543
σ_m	imp markup	inv gam2	20.000	∞	20.813	3.314	20.677	3.016	51.651	19.618	66.396	22.695
σ_w^F	wage markup	inv gam2	0.866	∞	2.282	0.565	2.143	0.647	–	–	–	–
ς_w^F	wage markup	inv gam2	0.500	∞	0.637	0.370	1.196	0.743	–	–	–	–
ς_w^E	wage markup	inv gam2	1.000	∞	2.377	0.748	2.001	0.883	–	–	–	–
σ_ξ^F	wage bargain	inv gam2	8.660	∞	–	–	–	–	5.193	1.342	4.446	1.172
ς_ξ^F	wage bargain	inv gam2	5.000	∞	–	–	–	–	3.015	0.880	3.958	1.177
ς_ξ^E	wage bargain	inv gam2	10.000	∞	–	–	–	–	7.682	1.998	6.927	1.844
σ_γ^F	fin wealth	inv gam2	0.866	∞	–	–	0.335	0.089	–	–	0.331	0.086
ς_γ^F	fin wealth	inv gam2	0.500	∞	–	–	0.549	0.168	–	–	0.555	0.159
ς_γ^E	fin wealth	inv gam2	1.000	∞	–	–	0.613	0.173	–	–	0.649	0.182
σ_υ^F	fin risk	inv gam2	8.660	∞	–	–	2.610	0.622	–	–	2.542	0.598
ς_υ^F	fin risk	inv gam2	5.000	∞	–	–	11.376	2.933	–	–	9.171	2.424
ς_υ^E	fin risk	inv gam2	10.000	∞	–	–	8.194	1.440	–	–	8.833	1.821

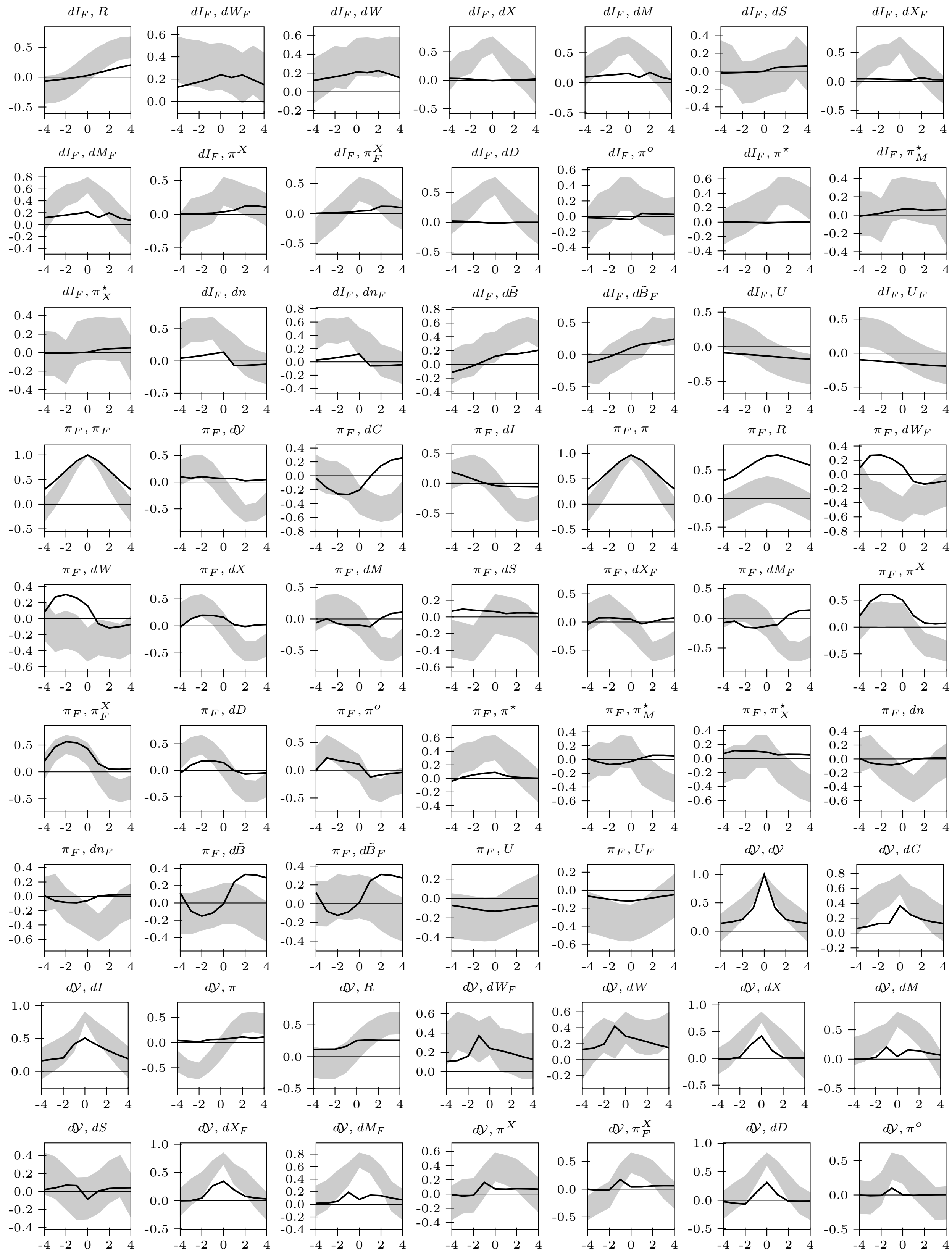
4.B Asymptotic second order moments implies by the full model

4.B.1 Inconditional standard deviations



4.B.2 Inconditional cross-correlations with lead-lag incidence

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dI_F, dW_F
dI_F, dW
dI_F, dX
dI_F, dM
dI_F, dS
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dI_F, dM_F
dI_F, π^X
dI_F, π^X_F
dI_F, dD
dI_F, π^o
$dI_F, \pi^\star$
$dI_F, \pi^\star_M$
$dI_F, \pi^\star_X$
dI_F, dn
dI_F, dn_F
$dI_F, d\tilde{B}$
$dI_F, d\tilde{B}_F$
dI_F, U
dI_F, U_F
π_F, π_F
$\pi_F, d\mathcal{Y}$
π_F, dC
π_F, dI
π_F, π
π_F, R
π_F, dW_F
π_F, dW
π_F, dX
π_F, dM
π_F, dS
π_F, dX_F
π_F, dM_F
π_F, π^X
π_F, π^X_F
π_F, dD
π_F, π^o
$\pi_F, \pi^\star$
$\pi_F, \pi^\star_M$
$\pi_F, \pi^\star_X$
π_F, dn
π_F, dn_F
$\pi_F, d\tilde{B}$
$\pi_F, d\tilde{B}_F$
π_F, U
π_F, U_F
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$d\mathcal{Y}, dC$
$d\mathcal{Y}, dI$
$d\mathcal{Y}, \pi$
$d\mathcal{Y}, R$
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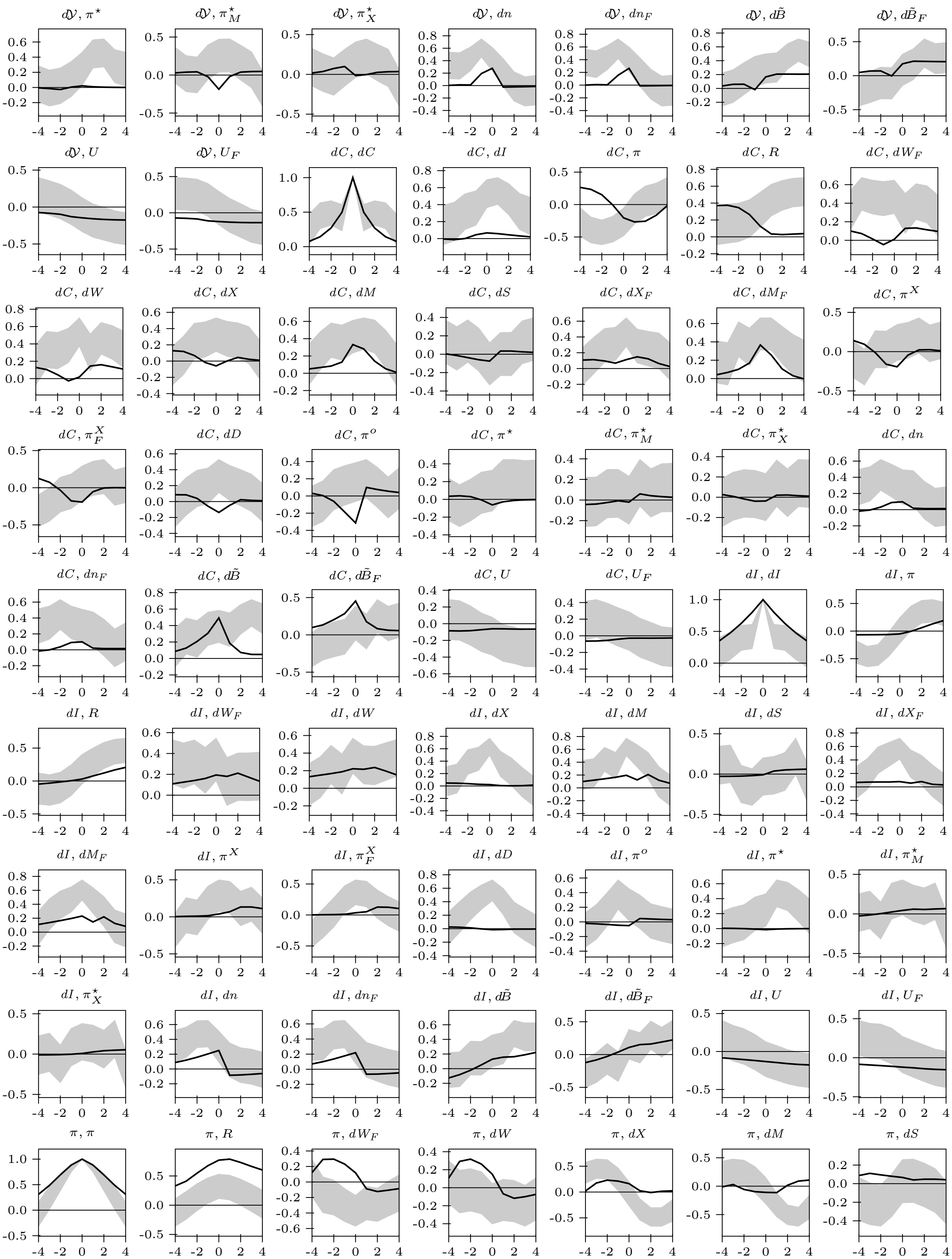
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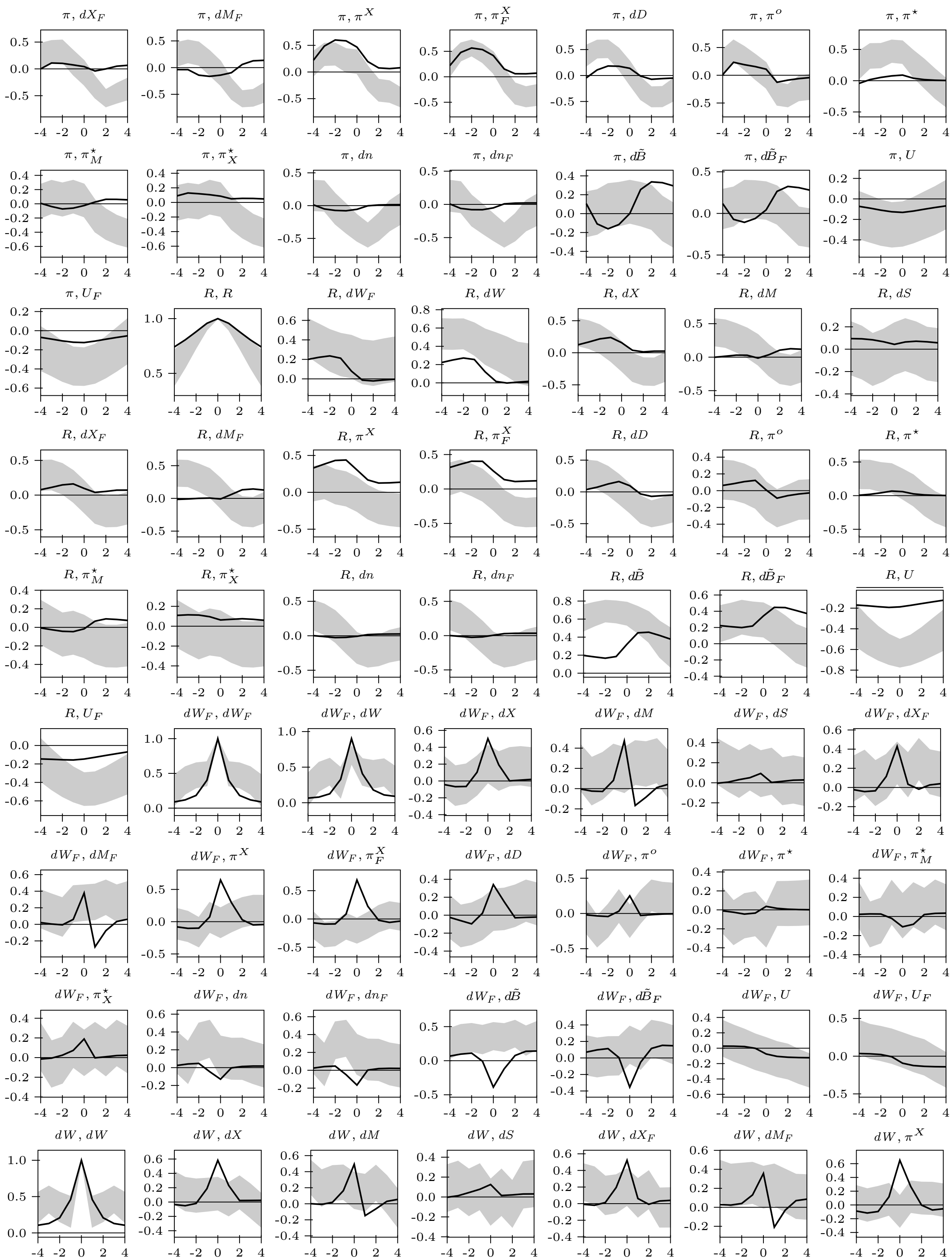
π, dW
π, dX
π, dM
π, dS



π, dX_F
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π, π^X_F
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π, U_F
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R, dW_F
R, dW
R, dX
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R, dS
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dW_F, dW
dW_F, dX
dW_F, dM
dW_F, dS
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dW_F, dM_F
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dW_F, π^X_F
dW_F, dD
dW_F, π^o
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$dW_F, \pi^\star_X$
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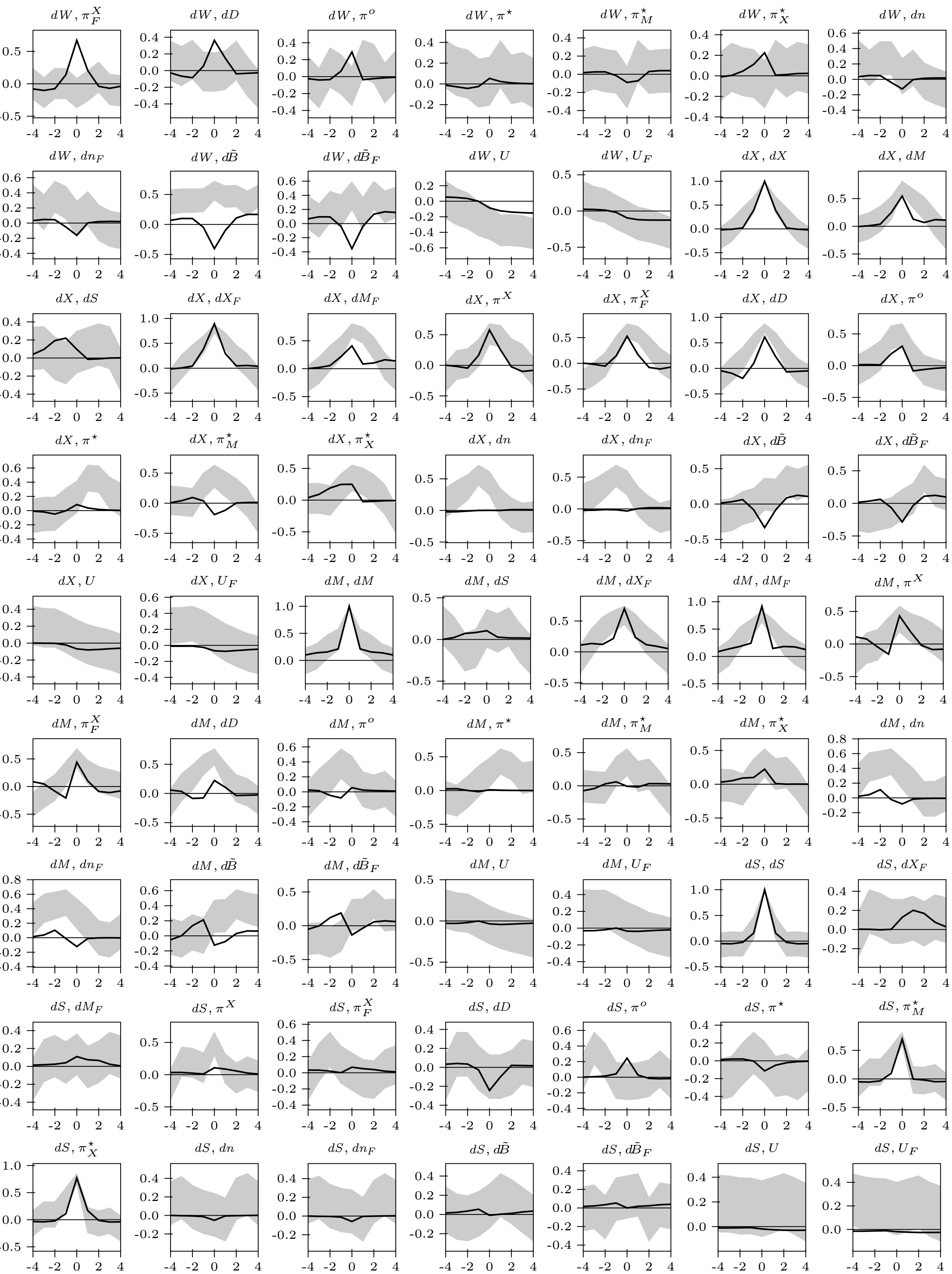
dW, dM_F
dW, π^X



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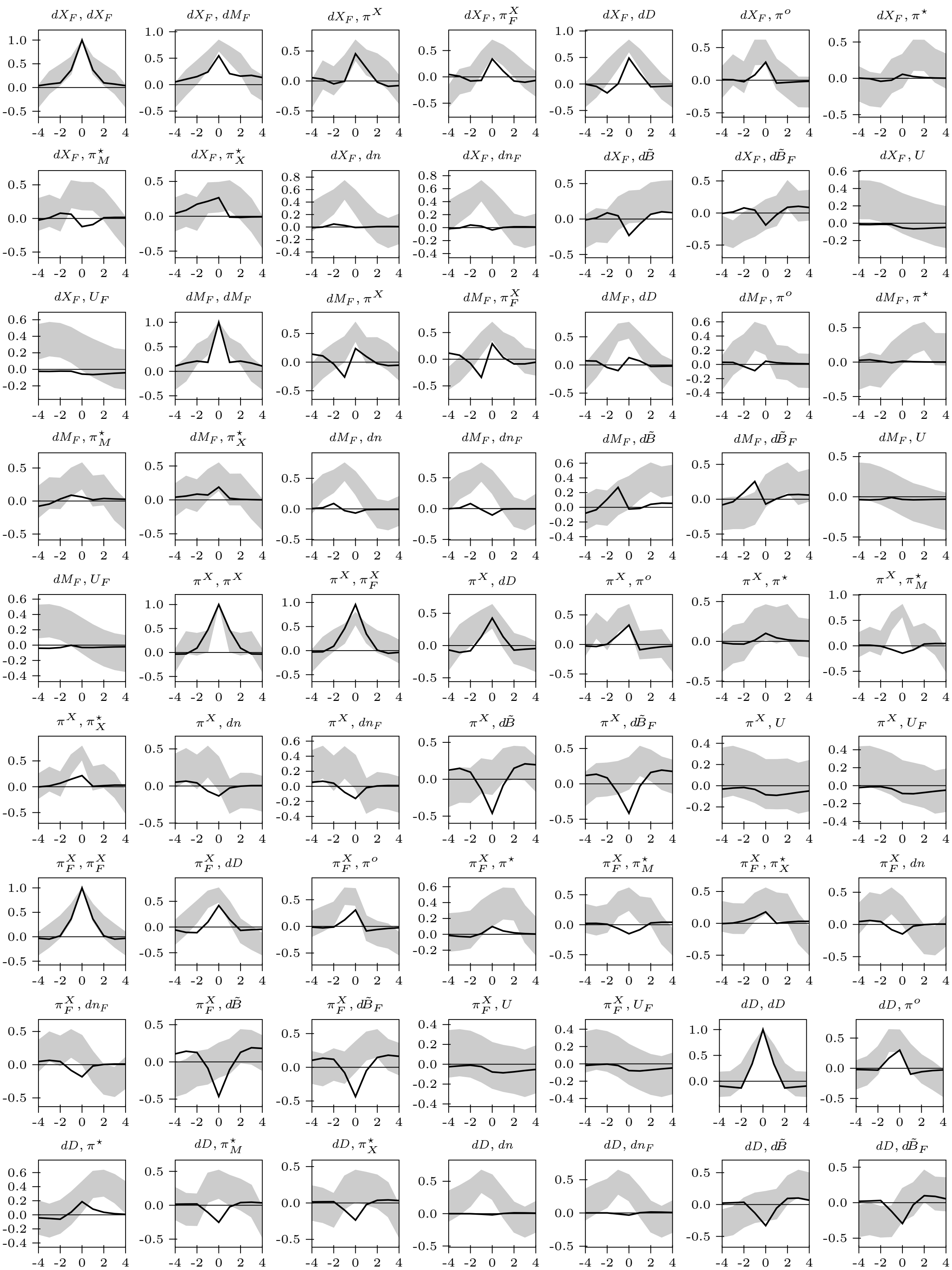
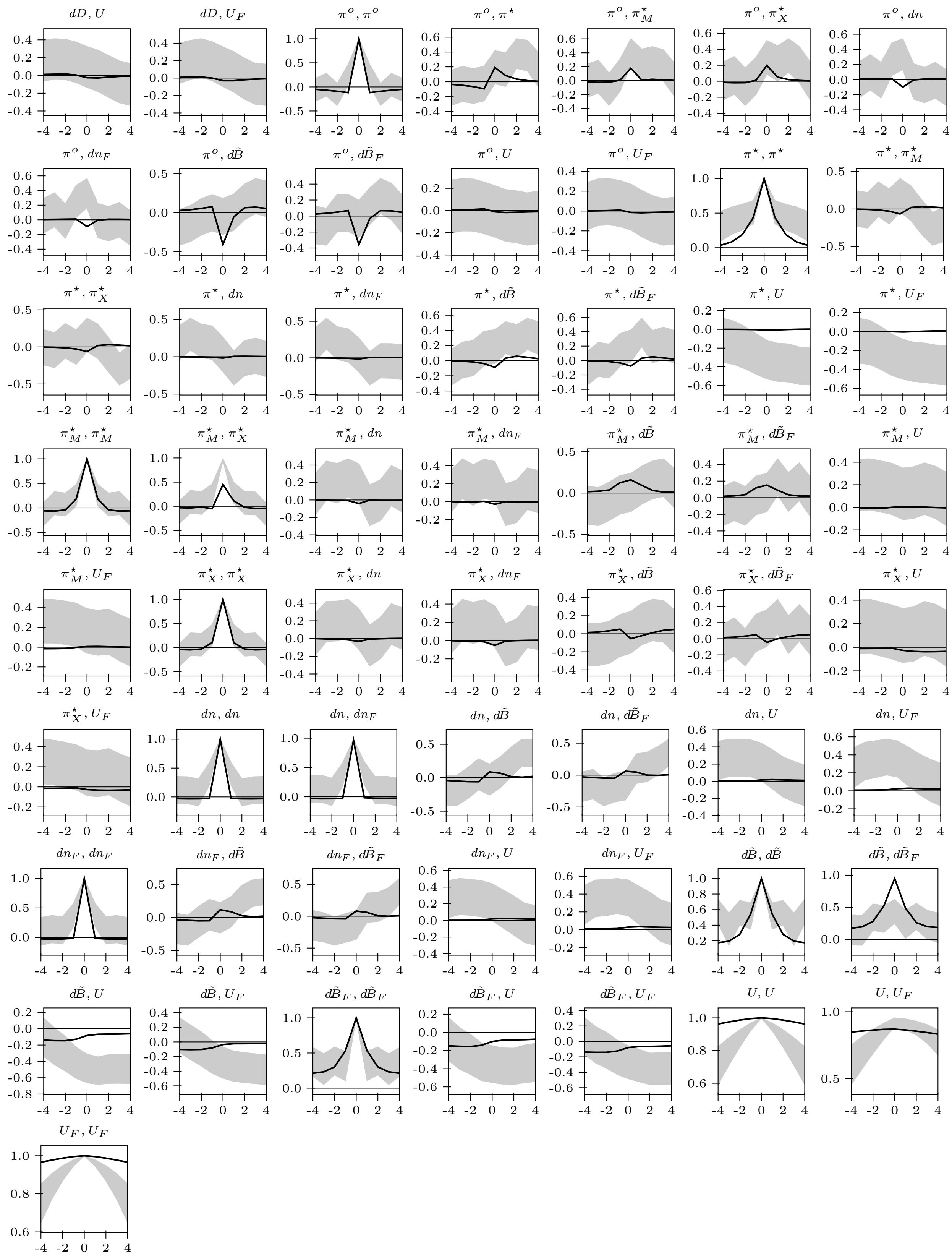
dS, U
dS, U_F



dX_F, dX_F
dX_F, dM_F
dX_F, π^X
dX_F, π^X_F
dX_F, dD
dX_F, π^o
$dX_F, \pi^\star$
$dX_F, \pi^\star_M$
$dX_F, \pi^\star_X$
dX_F, dn
dX_F, dn_F
$dX_F, d\tilde{B}$
$dX_F, d\tilde{B}_F$
dX_F, U
dX_F, U_F
dM_F, dM_F
dM_F, π^X
dM_F, π^X_F
dM_F, dD
dM_F, π^o
$dM_F, \pi^\star$
$dM_F, \pi^\star_M$
$dM_F, \pi^\star_X$
dM_F, dn
dM_F, dn_F
$dM_F, d\tilde{B}$
$dM_F, d\tilde{B}_F$
dM_F, U
dM_F, U_F
π^X, π^X
π^X, π^X_F
π^X, dD
π^X, π^o
$\pi^X, \pi^\star$
$\pi^X, \pi^\star_M$
$\pi^X, \pi^\star_X$
π^X, dn
π^X, dn_F
$\pi^X, d\tilde{B}$
$\pi^X, d\tilde{B}_F$
π^X, U
π^X, U_F
π^X_F, π^X_F
π^X_F, dD
π^X_F, π^o
$\pi^X_F, \pi^\star$
$\pi^X_F, \pi^\star_M$
$\pi^X_F, \pi^\star_X$
π^X_F, dn
π^X_F, dn_F
$\pi^X_F, d\tilde{B}$
$\pi^X_F, d\tilde{B}_F$
π^X_F, U
π^X_F, U_F
dD, dD
dD, π^o
$dD, \pi^\star$
$dD, \pi^\star_M$
$dD, \pi^\star_X$
dD, dn
dD, dn_F
$dD, d\tilde{B}$
$dD, d\tilde{B}_F$


dD, U
dD, U_F
π^o, π^o
$\pi^o, \pi^\star$
$\pi^o, \pi^\star_M$
$\pi^o, \pi^\star_X$
π^o, dn
π^o, dn_F
$\pi^o, d\tilde{B}$
$\pi^o, d\tilde{B}_F$
π^o, U
π^o, U_F
$\pi^\star, \pi^\star$
$\pi^\star, \pi^\star_M$
$\pi^\star, \pi^\star_X$
$\pi^\star, dn$
$\pi^\star, dn_F$
$\pi^\star, d\tilde{B}$
$\pi^\star, d\tilde{B}_F$
$\pi^\star, U$
$\pi^\star, U_F$
$\pi^\star_M, \pi^\star_M$
$\pi^\star_M, \pi^\star_X$
$\pi^\star_M, dn$
$\pi^\star_M, dn_F$
$\pi^\star_M, d\tilde{B}$
$\pi^\star_M, d\tilde{B}_F$
$\pi^\star_M, U$
$\pi^\star_M, U_F$
$\pi^\star_X, \pi^\star_X$
$\pi^\star_X, dn$
$\pi^\star_X, dn_F$
$\pi^\star_X, d\tilde{B}$
$\pi^\star_X, d\tilde{B}_F$
$\pi^\star_X, U$
$\pi^\star_X, U_F$
dn, dn
dn, dn_F
$dn, d\tilde{B}$
$dn, d\tilde{B}_F$
dn, U
dn, U_F
dn_F, dn_F
$dn_F, d\tilde{B}$
$dn_F, d\tilde{B}_F$
dn_F, U
dn_F, U_F
$d\tilde{B}, d\tilde{B}$
$d\tilde{B}, d\tilde{B}_F$
$d\tilde{B}, U$
$d\tilde{B}, U_F$
$d\tilde{B}_F, d\tilde{B}_F$
$d\tilde{B}_F, U$
$d\tilde{B}_F, U_F$
U, U
U, U_F
U_F, U_F

4.C Estimation results for the complete models with endogenous layoffs

An earliest version of labor market frictions used in the estimated model of France in the euro area included endogenous layoffs. As explained in section 2.5, the chosen specification of wage rigidities implied that job destructions and job creations compensated each other. Specifically, increases in employment resulted either from an increase in job destructions compensated by a larger rise in job creations, or from a decrease in job creations compensated by an even larger decrease in job destructions, according to the sign of the reaction of the bargained wage for new jobs. When the model was first estimated, both job destruction and job creation had hence excessive volatilities.

In order to cancel this undesirable effect, a second version of the estimated model included a quadratic adjustment cost on endogenous layoffs. This specification is briefly presented in what follows. Firms expenditures include

$$\frac{\psi}{2}\left(\frac{F(\bar{a}_t)}{\overline{F(\bar{a})}}-1\right)^2$$

in addition to wages and vacancy costs. In this expression, F is the cumulative density function of workers idiosyncratic productivities, $F(\bar{a}_t)$ is the number of dismissed workers in period t and ψ is the parameter that determines the size of the adjustment cost.[4] Using the notations introduced in section 2.5, the optimal layoff decision with the adjustment cost becomes

$$x_t A_t \bar{\omega}_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{\partial \mathcal{V}_{t+1}}{\partial N_t} = w_t - \psi \frac{1-F(\bar{a}_t)}{N_t}\left(\frac{F(\bar{a}_t)}{\overline{F(\bar{a})}}-1\right),$$

instead of simply

$$x_t A_t \bar{\omega}_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{\partial \mathcal{V}_{t+1}}{\partial N_t} = w_t.$$

Under the assumption that these costs are repaid evenly to households or, equivalently, paid to the government, the simple change to the model is the addition of the last term in the right hand side of the equation above describing the optimal decision for layoffs. If this specification successfully rectifies the dynamic behavior of job creations and destructions in small calibrated models, the problem is still present in the two-country model of France and the euro area once it is estimated, although it is considerably reduced. The reasons for that would need to be further investigated. In the meantime, the version put forward in chapter 4 simply ignores endogenous layoffs.

In what follows, I present some properties of the complete model of France and the euro area estimated with endogenous layoffs and quadratic adjustment costs on them. Table 4.C.1 reports the estimated values of the parameters related to the labor market in comparison to those found in the central estimation. Table 4.C.2 provides some inconditional second order moments of matches (i.e. job creations) and of endogenous job destructions, with and without ajustment costs (the ψ's are either kept at their estimated values of set to zero). I find that the presence of adjustment

[4] A formal description of a model with endogenous layoffs and wage rigidity à la Pissarides (2009) is provided in section 2.5.

costs reduces the volatility of these variables. But job creations are still positively correlated with job destructions, which shows that the costs do not totally prevent their compensation. This is confirmed by impulse responses to technology shocks, and to some extent to impatience and bargaining power shocks, as shown in Figure 4.C.1; if adjustment costs visibly improve the picture, positively correlated responses still dominate when looking at the inconditional dynamic properties of job flows.

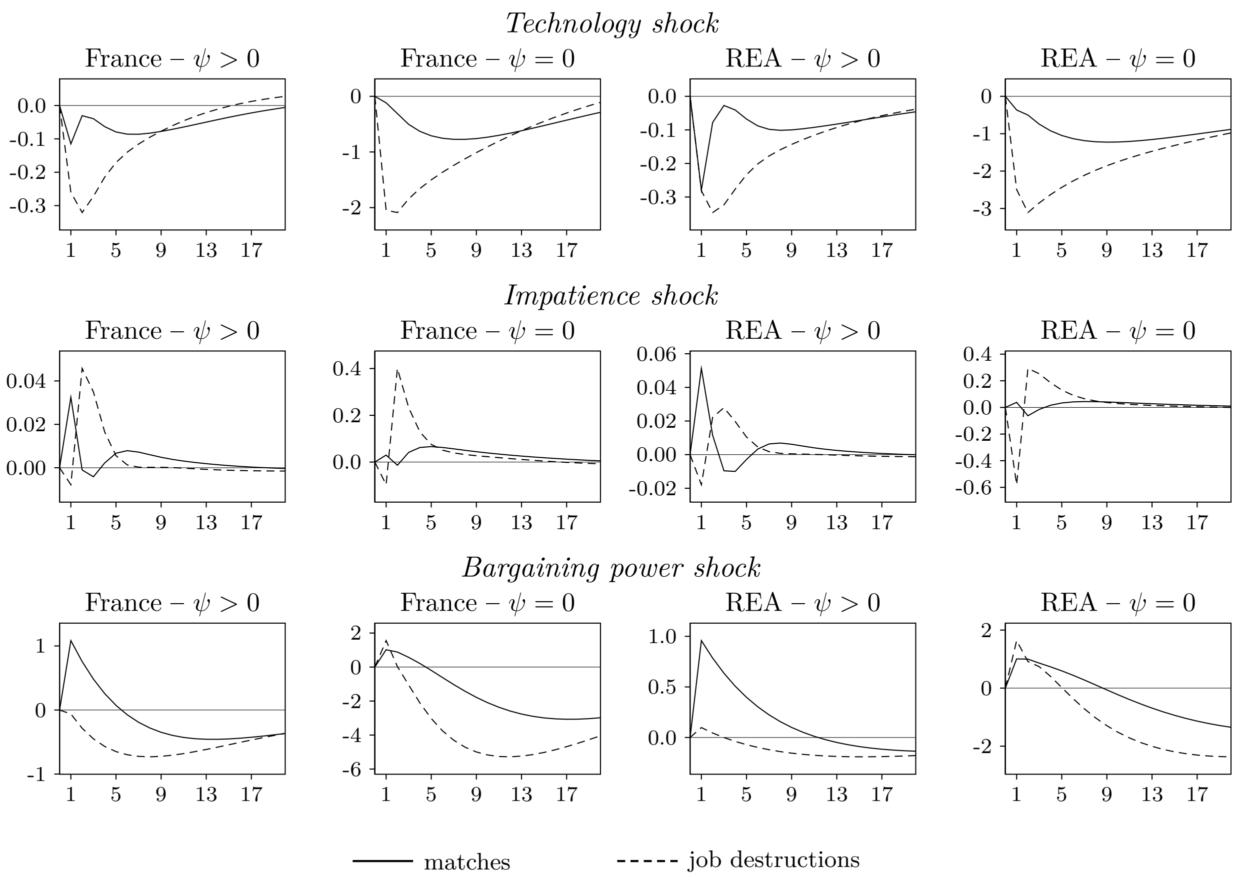
Table 4.C.1: Posterior mode estimates of labor market parameters

		France		Rest of the EA	
		baseline	endo. layoffs	baseline	endo. layoffs
φ	elast matching	0.315	0.083	0.316	0.157
s	exo job separation	0.075	0.070	0.068	0.058
$\bar{\xi}$	bargaining power	0.543	0.607	0.519	0.561
$vc/\mathcal{Y}$	vacancy cost	0.017	0.040	0.020	0.033
$\bar{N}$	steady st employment	0.718	0.579	0.604	0.568
Φ	vacancy filling rate	0.900	0.900	0.900	0.900
$F(\bar{a})$	steady st job destr	–	0.031	–	0.026
ψ	job destr adj cost	–	0.739	–	1.083

Table 4.C.2: Second order moments of job creation and job destructions with and without adjustment cost

	France		Rest of the EA	
	$\psi = 0.739$	$\psi = 0$	$\psi = 1.083$	$\psi = 0$
corr(matches,job destr.)	0.57	0.90	0.31	0.87
st. dev. matches	0.0026	0.0178	0.0020	0.0137
st. dev. job destr.	0.0036	0.0296	0.0017	0.0222

Figure 4.C.1: Impulse responses of job creations and job destructions



Chapter 5

The role of financial and labor market frictions

In this chapter, I show that including elaborate microfoundations to represent credit and labor markets can significantly alter the conclusions based on the use of the estimated model of France and the euro area developed in the previous chapters. I also illustrate the fact that accounting simultaneously for the two mechanisms is critical in spite of their apparent independence. First, from a positive point of view, historical shock decompositions during the recent crisis are found significantly different according to the version of the model used. Then, from a normative point of view, the two policy evaluation exercises considered deliver marked differences in the presence of these frictions.

These two perspectives, positive and normative, are related because the model simulations considered are affected by the structural shocks underlying fluctuations: the simulations of the effects of switching to a flexible exchange rate regime between France and the rest of the euro area are performed in the particular context of the recent crisis, which are interpreted differently by the different versions of the model. When evaluating the welfare cost of business cycles, the estimated relative importance of the structural shocks in explaining fluctuations is also of primary importance.

This work clearly supports the introduction of search and matching frictions in the labor market and of financial frictions à la Bernanke et al. (1999). Because they omit microfoundations that have significant theoretical implications, simpler tools based on Smets and Wouters (2007) can hence be viewed as hybrid forms of econometric and theoretical models, and may deliver misleading policy guidance.

5.1 Historical shock decompositions during the crisis

One interest of policy makers for estimated DSGE models lies in the fact that they can deliver an analysis of past macroeconomic developments in terms of the contributions of structural shocks. This provides story telling for business cycle analysis and hints about future economic developments, since distinct shocks propagate differently. By contrast with elementary analysis based on breakdowns into demand and supply shocks only, large scale DSGE models offer a more elaborate analysis where many possible sources of fluctuations are considered, using the information contained in a larger set of economic time series. In particular, the open economy framework allows separating the fraction of cyclical fluctuations that can be imputed to variations in specific exogenous variables, such as oil prices or world demand.

In what follows, I compare the historical shock decompositions over 2007-2013 obtained from the complete version of the model of chapter 3, with those obtained from the three other incomplete versions. This version is the most realistic representation of reality among the four considered, since it accounts for imperfections that are present in actual credit and labor markets. The analysis of the business cycle obtained with it is thus believed to be more reliable that the others. So, this exercise reveals the misinterpretations of fluctuations that result from the use of a model that is too simplistic in its representation of labor and financial markets.

First, I show that omitting labor market frictions significantly alters the analysis. This is especially true for the analyses of labor and GDP. One of the main changes is that the weights of foreign trade and of markups are overstated. Concerning inflation, the presence of labor market frictions reveals a marked positive contribution of extra euro area import prices during the 2009 recession.

Financial frictions also matter; their main interest lies in the large contribution attributed to 'risk' shocks to the business cycle fluctuations of investment, GDP, inflation, and, to a lower extent, labor. Otherwise, the analysis identifies a larger contribution of permanent technology shocks. Indeed, risk shocks are able to generate very persistent cyclical movements in activity. If absent, the unit root present in world technology captures the high degree of persistence of the recent crisis. This really makes a difference since the interpretation of the crisis would then rather be related to "supply-side" than to "demand-side" effects.

Some effects, although not predominant, of the interaction of labor market and financial frictions are also identified thanks to this exercise. For instance, the contribution of risk shocks to French employment only becomes significant in the presence of search and matching frictions. As emphasized in section 2.6, the feedback of credit frictions on job creation in the transmission of usual shocks is modest. The main interaction arises from the fact that financial shocks impact the dynamics of employment and wages. On top of that, here, the versions of the model are estimated separately. Hence, removing frictions is accompanied by a revision of the value of all parameters. This is a primary reason for which the joint inclusion of labor market and financial frictions matters more than just as the sum of the advantages of each taken separately.

A final observation is that, in spite of measurement errors, the analysis identifies, for each variable considered, shocks that are connected to it in the model's structure, as primary contributors to its cyclical fluctuations. Specifically, the fluctuations in investment are attributed to a large extent to financial shocks or, without financial frictions, to investment technology shocks, and fluctuations in labor to wage bargaining shocks or, if not, wage markup shocks.

5.1.1 France

The following histograms show the historical shock decompositions of a subset of major observed timeseries of France between 2007Q1 and 2013Q1, obtained with a Kalman smoother over the whole estimation sample. Since the model includes many shocks (24 for the baseline and the search and matching versions, 28 for the versions including a financial accelerator), only the 7 greatest contributors that are identified for each variable are plotted. Since these contributors may vary from a variable to another, and, for a given variable, from a version of the model to another, colors are not associated with the same shocks throughout this section. Instead, they are used to rank the contributors from the most important one (in red) to the 7$^{\text{th}}$ most important one (in purple).[1] The contributions of the other shocks are summed in the category 'other' (in brown). Since they are by construction smaller that those that are represented, this item does not mask large contributions compensating each other.

The complete model identifies risk, permanent technology and world demand shocks as the major explanations of the downturn of GDP in 2008. Risk shocks are also the main explanation of the persistence of the slowdown in 2012. When financial frictions are omitted in the model, permanent technology shocks become the predominant story underlying the depression and the persistence of the slowdown after the crisis. Then, if labor market frictions are omitted but not financial frictions, the findings are more markedly altered. In particular, if financial risk shocks are still critical, the crisis is attributed to a larger extent to foreign trade. Markups then also play a role: the reduction in price markups dampens the recession, while wage markups play in the opposite direction in 2009, revealing insufficient downward wage adjustments. Last, when both labor market and financial frictions are absent from the model, the story is very similar to the one obtained with the preceding version (only financial frictions) except that permanent technology shocks compensate the absence of risk shocks. To conclude, the versions of the model considered provide quite different analyses. Assuming a Walrasian labor market strongly impairs the analysis of business cycle fluctuations in French GDP that is drawn from the model, because it amplifies the role of markups and of foreign trade shocks. With financial frictions, financial shocks become major sources of output fluctuations; moreover, the interaction of both kinds of frictions makes risk shocks the largest contributor to the fall in French GDP recorded in the 2008-2009 crisis.

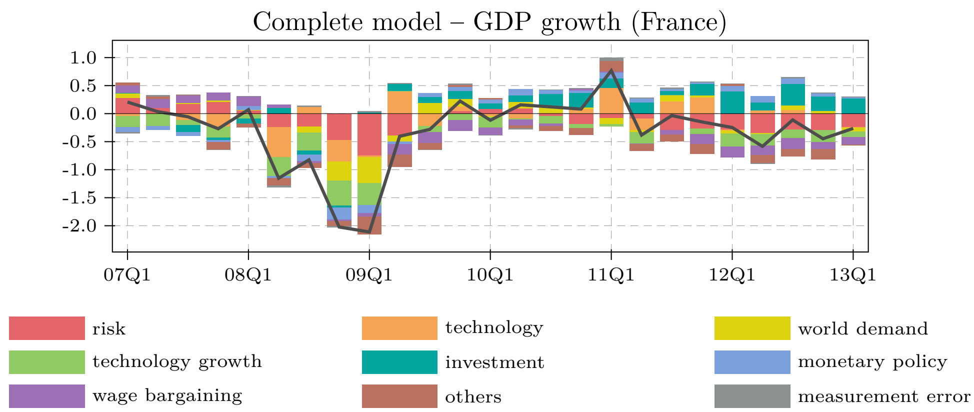
Now I turn to inflation. In the complete model, its main determinants are world demand and extra euro area import prices, and to a lower extent monetary policy and financial risk shocks. In the crisis, while relatively dynamic import prices fed inflation, the fall of world demand strongly pushed it down. In addition, high interest rates made it worse until the end of 2009 because of lags in the transmission of monetary policy decisions. The high valorization of the euro exchange rate, making imported goods cheaper, also contributed negatively to inflation in 2009 and 2010. The persistence of low inflation until the end of the sample despite the accomodative monetary policy stance is attributed almost entirely to financial risk shocks. Without financial frictions, again, the story is similar except that investment technology shocks are identified instead of risk shocks.

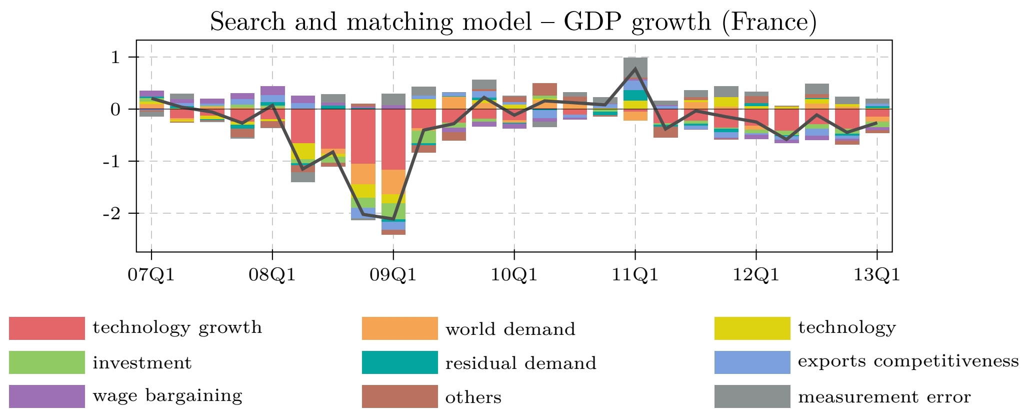
[1] The complete ordering is the following, from the most to the 7$^{\text{th}}$ most important contributor: red, orange, yellow, green, turquoise, blue, purple.

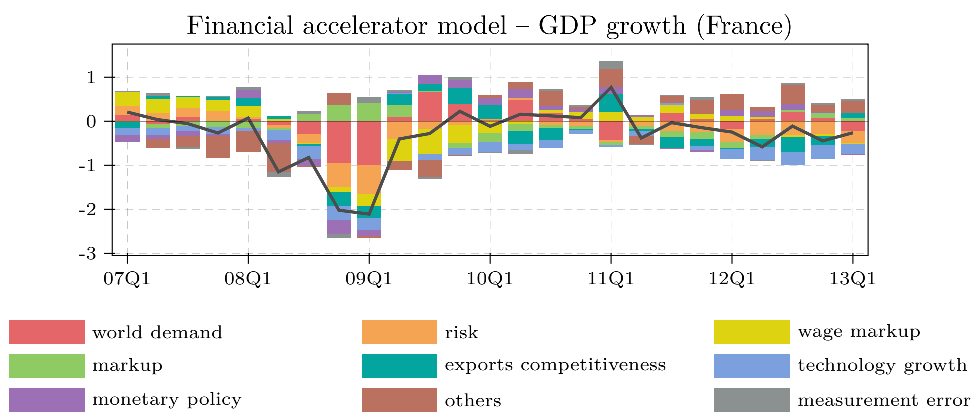
When the labor market is Walrasian but financial frictions are present, risk shocks still contribute negatively to inflation between 2009 and 2013. Yet the analysis attributes a much lower weight to world demand, preferring price and wage markup shocks to explain the 2009 reduction in inflation. The latter shock is also the main explanation underlying the crisis episode in the baseline model, followed by exchange rate effects. Moreover, in this version, the persistence of low inflation in 2011 and 2012 results from investment technology shocks, which replace risk shocks, plus a combination of many non significant (when taken individually) contributors, added together and shown in brown on the histogram. These two forces cancel the inflationary effect of monetary policy. This large contribution of the 'others' item, added to the role attributed to markup shocks, which seem like residuals in the identification, highlight the relevance of labor market frictions. Financial frictions are slightly less important in the sense that the analysis is not changed much with respect to non-financial shocks when they are omitted. Still, in the complete model, the risk shock is key to understand fluctuations in French inflation.

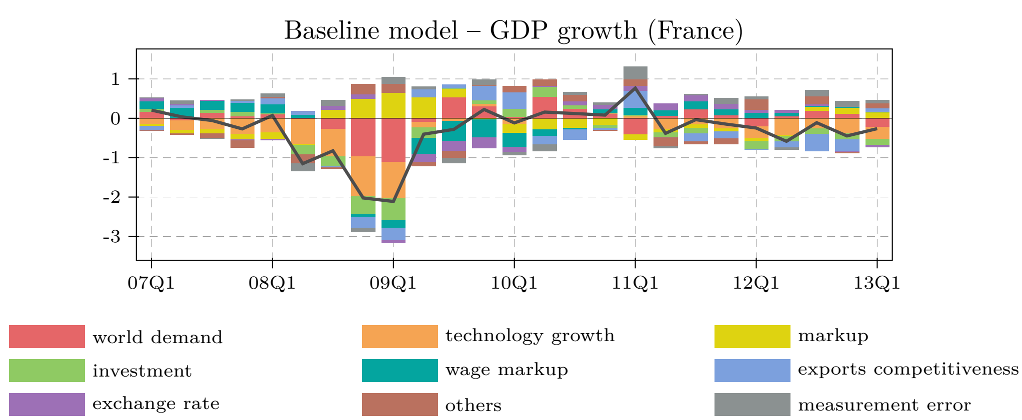
The fall in employment during the crisis is mainly attributed to the drop in world demand, to negative financial shocks and to wage bargaining shocks. The latter support employment between 2007 and 2010, but deteriorate it after 2011, pointing to an insufficient downward adjustment of wages. The benefits of the accommodative monetary policy appears progressively in 2011. A noticeable feature is the positive contribution of investment technology shocks during the whole period under review. Assuming no financial frictions implies that the contribution of financial shocks is replaced by negative investment technology shocks, while the negative contribution of wage bargaining at the end of the period is cancelled. In addition, a negative contribution of permanent technology shocks is identified. In the two models without labor market frictions, the picture is strongly changed regarding the business cycle analysis of labor. A primary reason for that is the observed time series being total hours worked instead of employment. From a general point of view, significant contributions of markup shocks are identified; this indicates that markups capture residuals resulting from the misspecification of labor markets. This conclusion supports the view expressed by Chari et al. (2009) about the important role attributed to wage markup shocks in Smets and Wouters (2007). Besides, the negative role of world demand shocks is then considerably amplified and the model identifies a visible negative contribution of exchange rate shocks between 2009 and 2011. Finally, it is worth noticing that the negative contribution of risk shocks becomes insignificant in the financial accelerator model. This highlights a benefit of the joint inclusion of financial and labor market frictions for historical shock decomposition analyses.

The picture for investment is quite simple in the complete model. The risk shock is the main contributor, but, almost systematically, it is partly compensated by investment technology shocks. The reason for this situation lies to some extent in the model's response to investment technology shocks in the presence of a financial accelerator. Indeed, a positive shock to the efficiency of new capital is plainly expansionary in a model without financial frictions, since it reduces the capital's replacement value and hence stimulates the utilization of production factors. But it propagates very differently with financial frictions. In particular, a positive shock lowers the price of the existing stock of capital, which results in a sudden devaluation of entrepreneurs assets. Hence, the latter record a loss at the date of the shock, which affects their net worth and their borrowing capacities in the subsequent periods. If investment rises because substituting old capital with new one is still profitable, capital utilization declines. All in all, negative investment technology shocks are then obviously less relevant to account for the financial crisis. Even worse, positive investment technology shocks contribute, together with recessionary risk shocks, to replicate the plunge in stock prices observed in the euro area during that period. The picture is almost unchanged if the model omits frictions in labor markets. By constrast, in the absence of financial frictions, investment technology and permanent technology shocks explain most of the deviation of investment growth from its steady state.

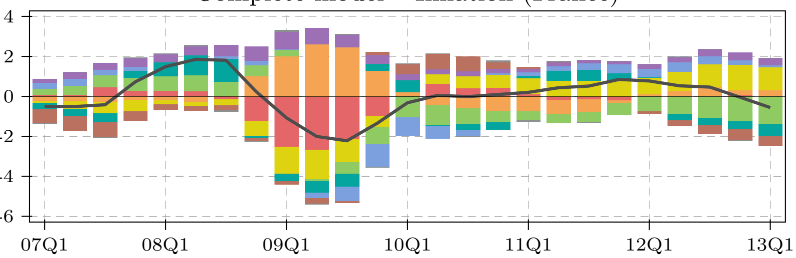

Complete model – GDP growth (France)
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
07Q1
08Q1
09Q1
10Q1
11Q1
12Q1
13Q1
risk
technology
world demand
technology growth
investment
monetary policy
wage bargaining
others
measurement error


Search and matching model – GDP growth (France)
1
0
-1
-2
07Q1
08Q1
09Q1
10Q1
11Q1
12Q1
13Q1
technology growth
world demand
technology
investment
residual demand
exports competitiveness
wage bargaining
others
measurement error


Financial accelerator model – GDP growth (France)
1
0
-1
-2
-3
07Q1
08Q1
09Q1
10Q1
11Q1
12Q1
13Q1
world demand
risk
wage markup
markup
exports competitiveness
technology growth
monetary policy
others
measurement error

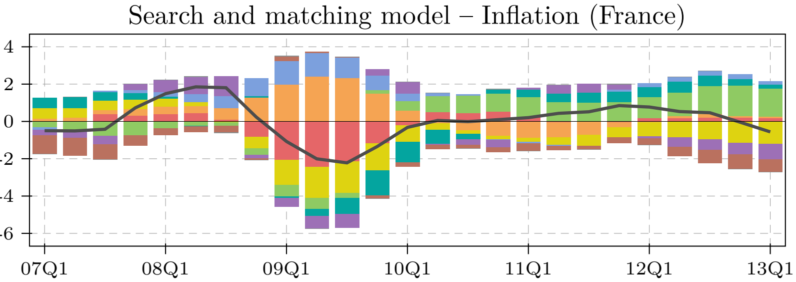

Baseline model – GDP growth (France)
1
0
-1
-2
-3
07Q1
08Q1
09Q1
10Q1
11Q1
12Q1
13Q1
world demand
technology growth
markup
investment
wage markup
exports competitiveness
exchange rate
others
measurement error

Complete model – Inflation (France)



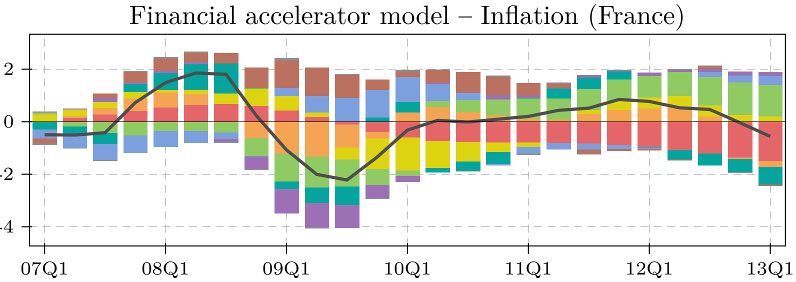
world demand | import price | monetary policy
risk | oil price | exchange rate
technology growth | others | measurement error

Search and matching model – Inflation (France)



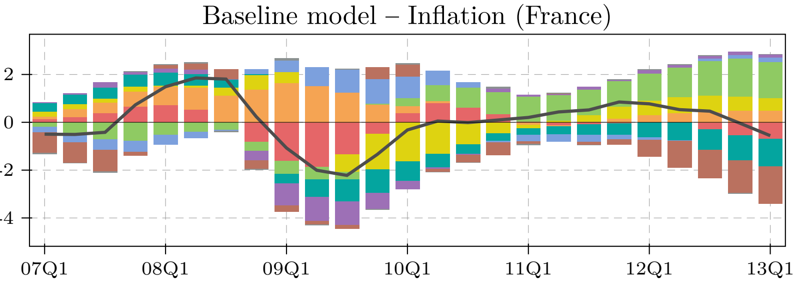
world demand | import price | investment
monetary policy | exchange rate | technology growth
oil price | others | measurement error

Financial accelerator model – Inflation (France)



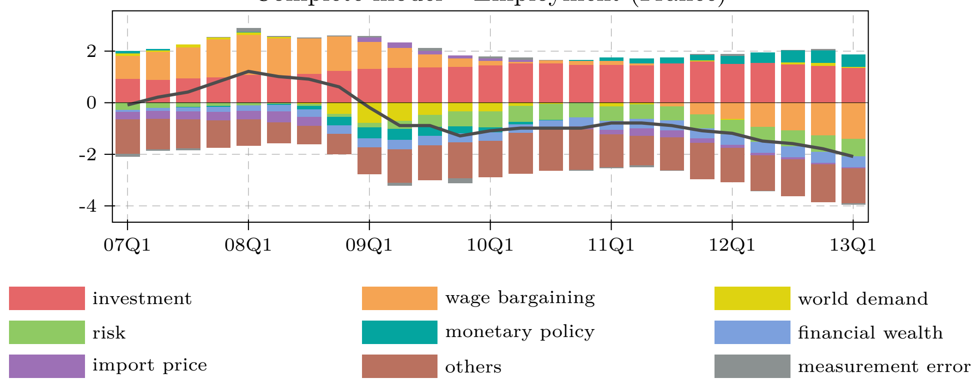
risk | markup | exchange rate
monetary policy | oil price | wage markup
world demand | others | measurement error

Baseline model – Inflation (France)

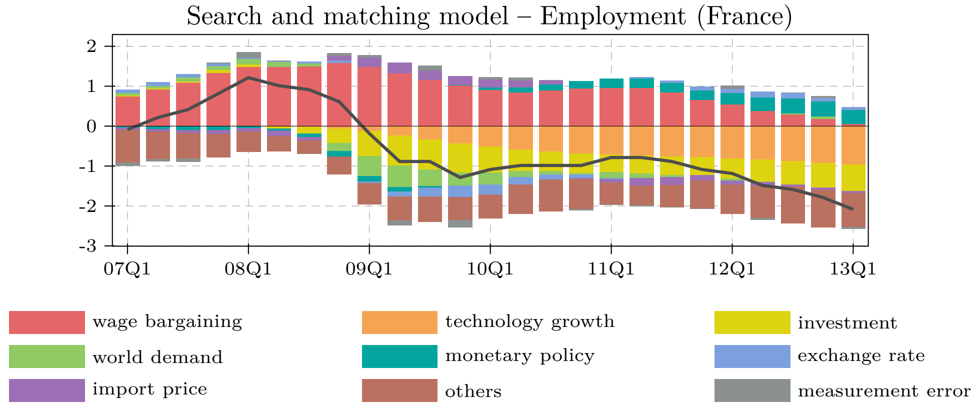


markup | technology growth | exchange rate
monetary policy | investment | wage markup
world demand | others | measurement error

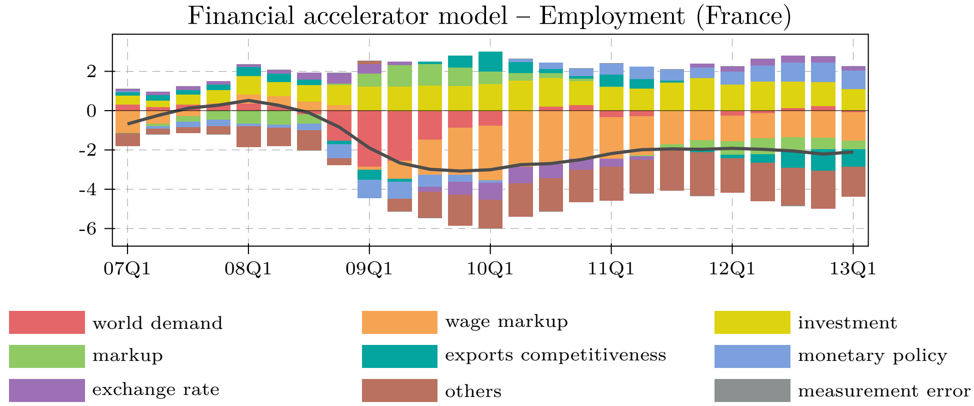
Complete model – Employment (France)



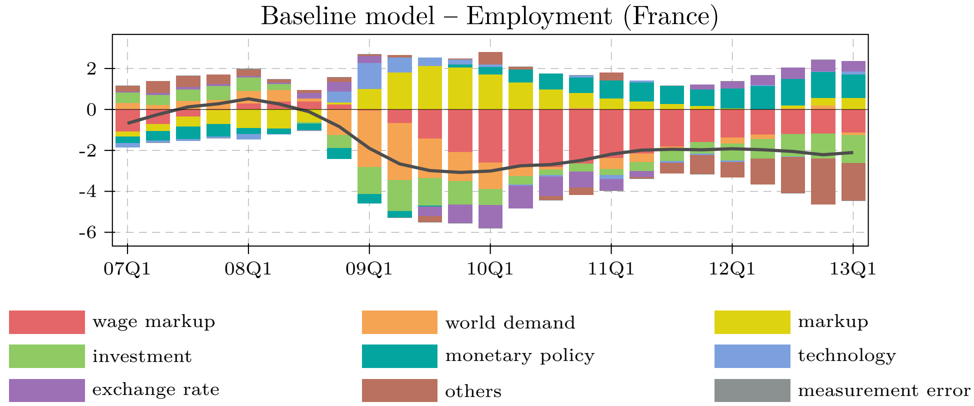
Search and matching model – Employment (France)



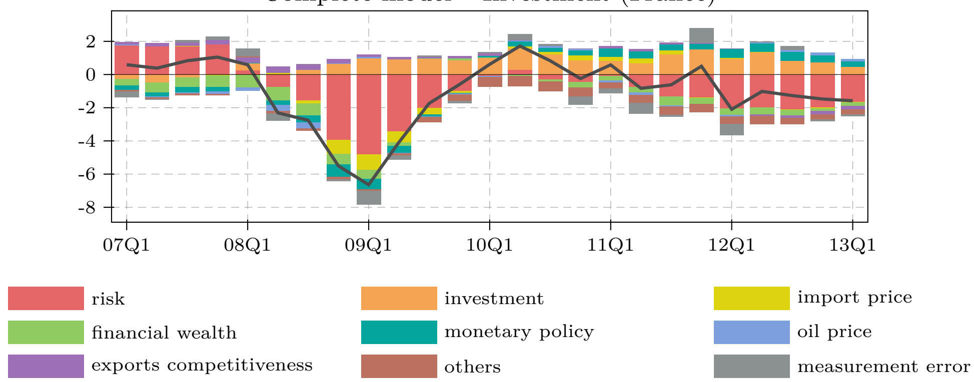
Financial accelerator model – Employment (France)



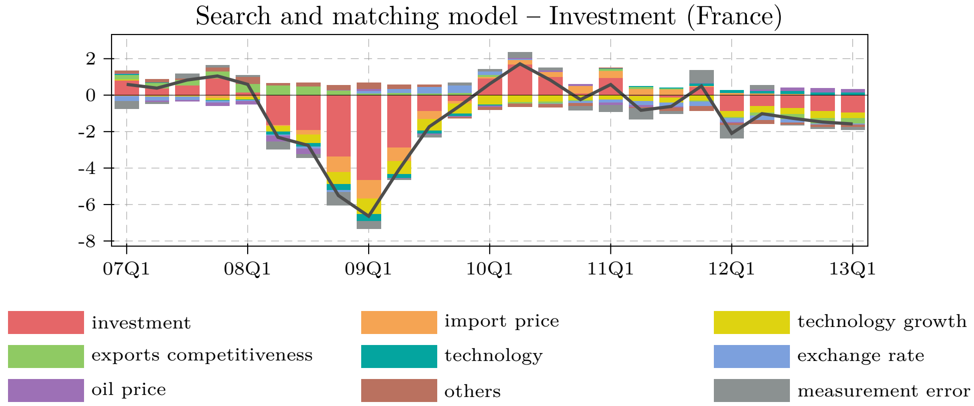
Baseline model – Employment (France)



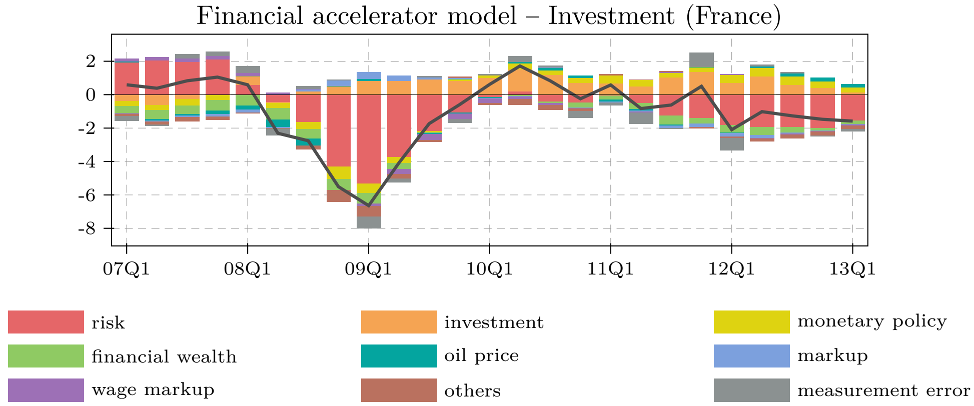
Complete model – Investment (France)



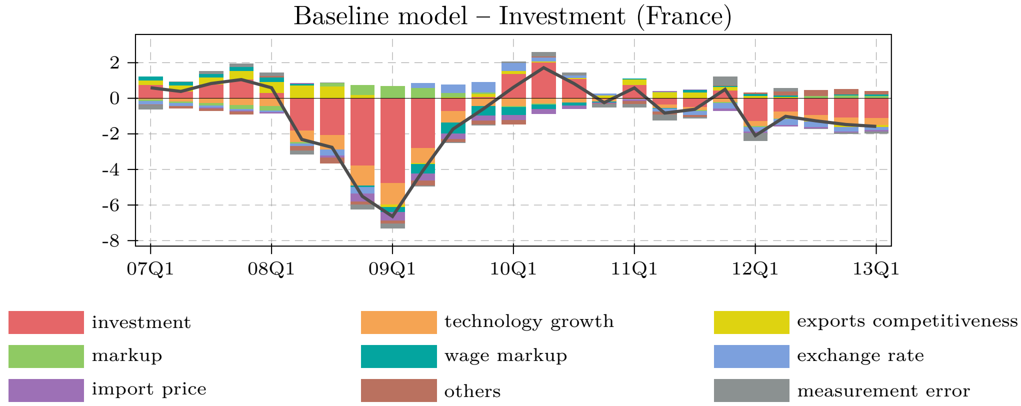
Search and matching model – Investment (France)



Financial accelerator model – Investment (France)



Baseline model – Investment (France)



5.1.2 Euro area

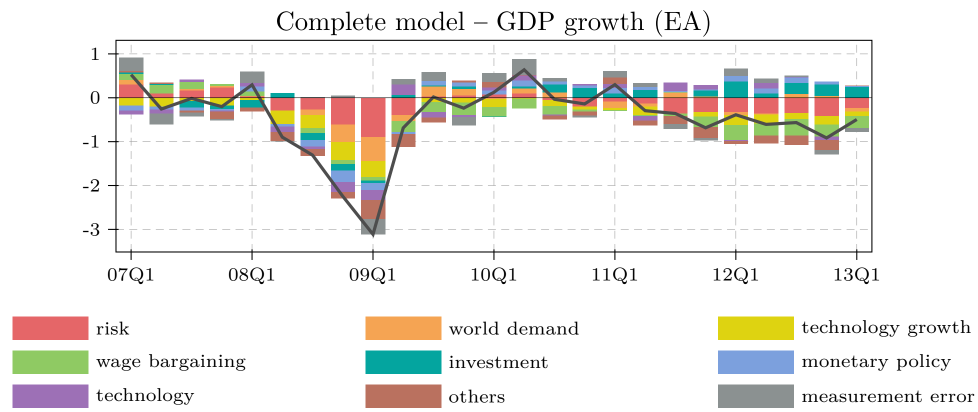
Following the same approach, the histograms below show the historical shock decomposition of GDP, inflation, labor and investment in the whole euro area between 2007Q1 and 2013Q1.

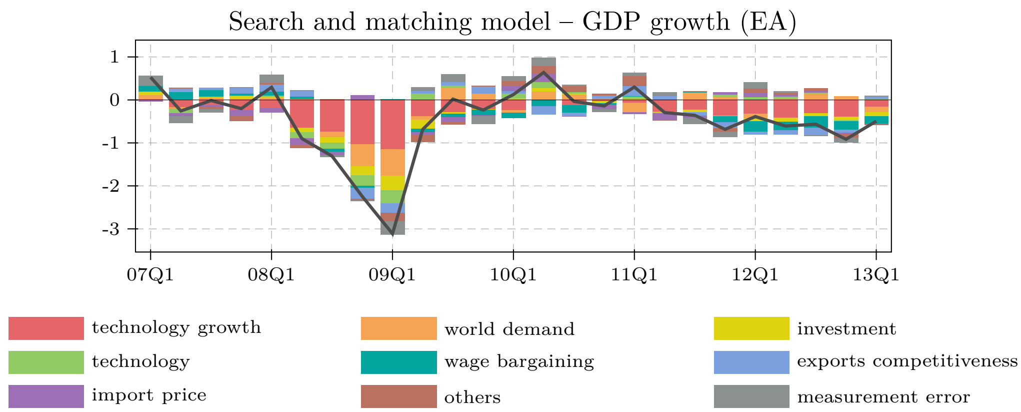
The decompositions are similar in many aspects to those obtained for France. With the complete model, the fluctuations in GDP primarily results from risk, world demand and permanent technology shocks. Wage bargaining shocks are, with risk shocks, one of the main explanations of the persistence of the economic slowdown in the euro area. Without financial frictions, risk shocks are replaced by negative permanent technology shocks and investment technology shocks. In the financial accelerator model without labor market frictions, the picture is somewhat altered. World demand plays a greater part in the 2009 recession than risk shocks. Negative export competititivity shocks apprear during the crisis and still weight on the euro area activity in 2011 and 2012. Then, wage bargaining shocks obviously disappear, whereas markups noticeably support growth during the 2008-2009 recession. Similar results are obtained in the baseline model. As in the search and matching model, risk shocks disappear in favour of permanent technology shocks explaining the persistence of weak economic conditions since the crisis, and the model exaggerates the effects of the shocks related to exports.

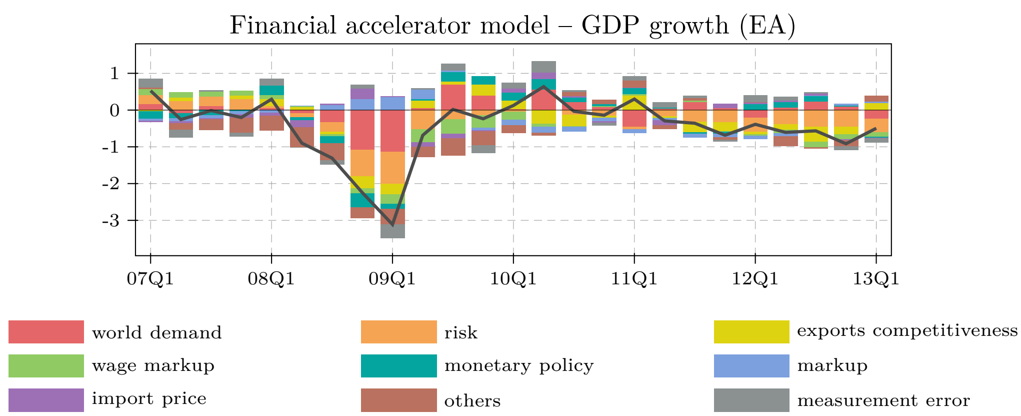
The comments which can be made about the decomposition of inflation in the euro area are the same as for France. World demand and import price shocks are crucial to account for the fluctuations in inflation recorded during the period considered. Risk and monetary policy shocks also played a significant role during and after the crisis. Omitting labor market frictions leads to overstate the role of markup shocks.

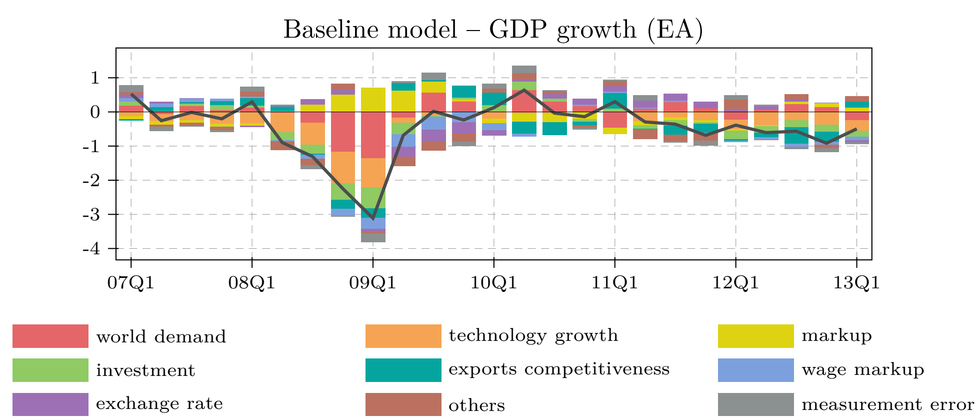
As in France, fluctuations in employment during the crisis are mainly attributed to wage bargaining shocks, meaning that the dynamics of wages were detrimental to job creation considering the weak economic conditions. Then comes a negative contribution of risk shocks since 2009Q1. The latter is counterbalanced by a persistent contribution of positive investment shocks over the whole period under analysis. The wrong interpretations resulting from assuming frictionless labor and credit markets are almost the same as for France, except that wage bargaining shocks still contribute negatively to employment from the middle of 2011 in the search and matching model, and that risk shocks still significantly contribute to lowering employment in the financial accelerator model. Assuming no financial frictions leads to negative permanent technology and investment-specific technology shocks. The model without labor market frictions attributes a significant part of the decrease in labor to foreign trade and reveals markup shocks. Last, the picture suggested by the baseline model is strongly deformed as compared to the full model. A negative effect of exchange rate shocks between 2009 and 2011 is found, while wage markup shocks appear as an important determinant of fluctuations in labor before and after the crisis.

The picture for investment growth is similar to the one obtained for France: without financial frictions, investment technology and permanent technology shocks replace the contribution of the risk shock identified otherwise.

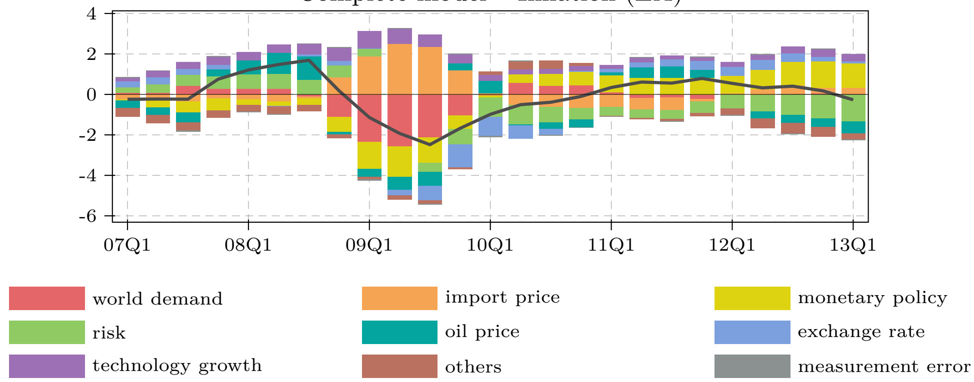

Complete model – GDP growth (EA)
1
0
-1
-2
-3
07Q1
08Q1
09Q1
10Q1
11Q1
12Q1
13Q1
risk
world demand
technology growth
wage bargaining
investment
monetary policy
technology
others
measurement error


Search and matching model – GDP growth (EA)
1
0
-1
-2
-3
07Q1
08Q1
09Q1
10Q1
11Q1
12Q1
13Q1
technology growth
world demand
investment
technology
wage bargaining
exports competitiveness
import price
others
measurement error

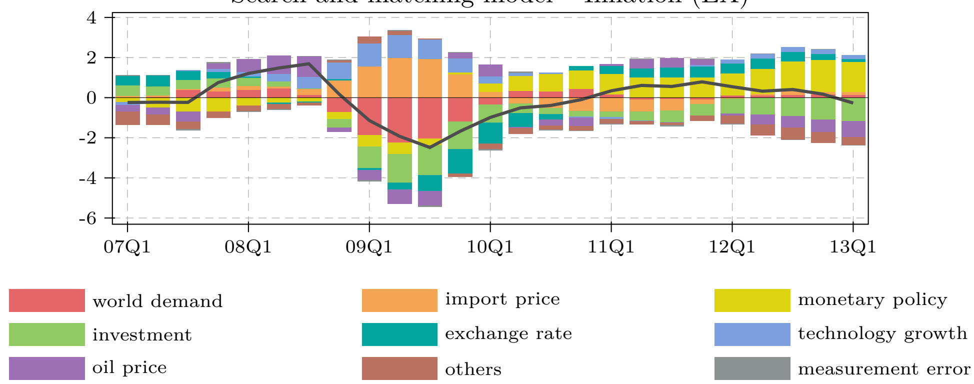

Financial accelerator model – GDP growth (EA)
1
0
-1
-2
-3
07Q1
08Q1
09Q1
10Q1
11Q1
12Q1
13Q1
world demand
risk
exports competitiveness
wage markup
monetary policy
markup
import price
others
measurement error


Baseline model – GDP growth (EA)
1
0
-1
-2
-3
-4
07Q1
08Q1
09Q1
10Q1
11Q1
12Q1
13Q1
world demand
technology growth
markup
investment
exports competitiveness
wage markup
exchange rate
others
measurement error

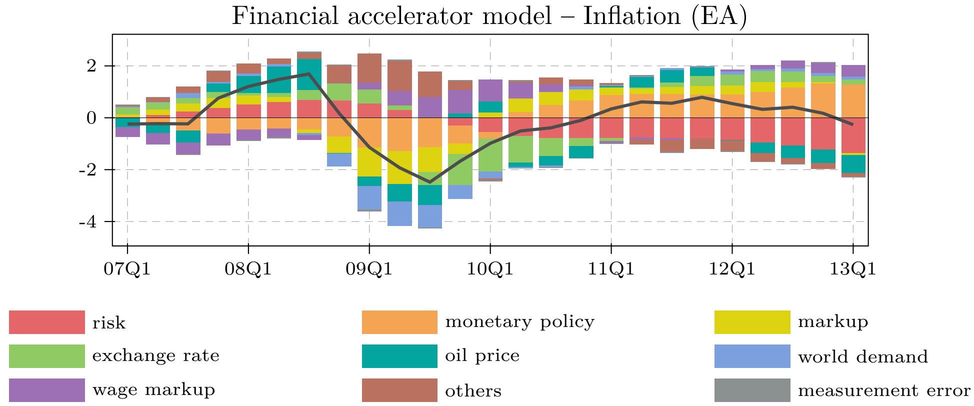
Complete model – Inflation (EA)



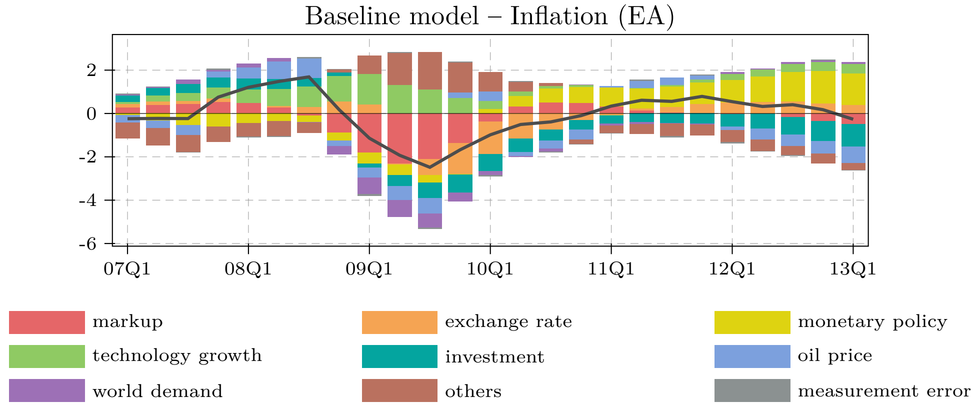
Search and matching model – Inflation (EA)

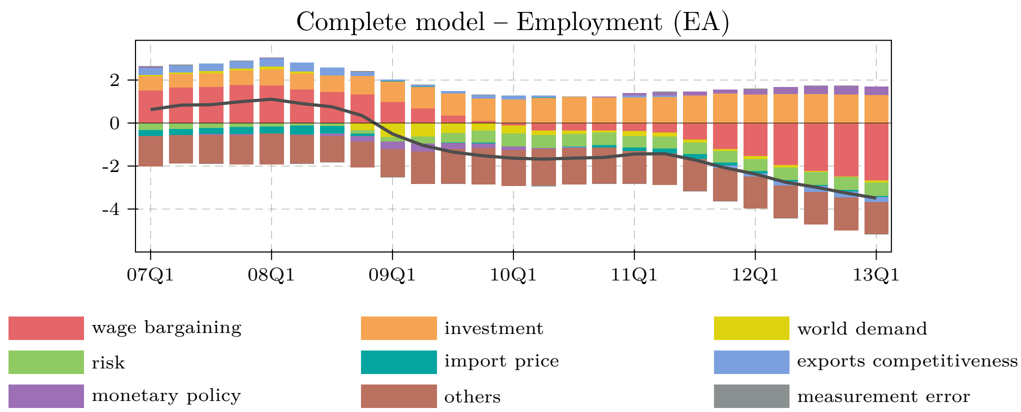


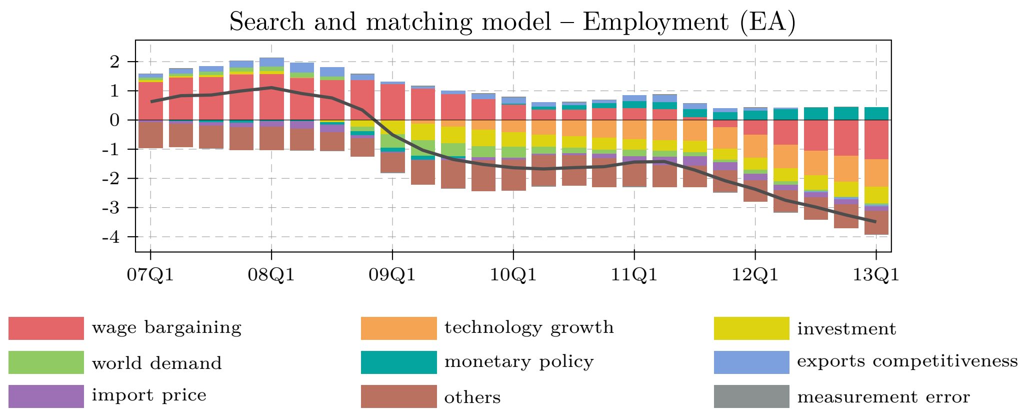
Financial accelerator model – Inflation (EA)

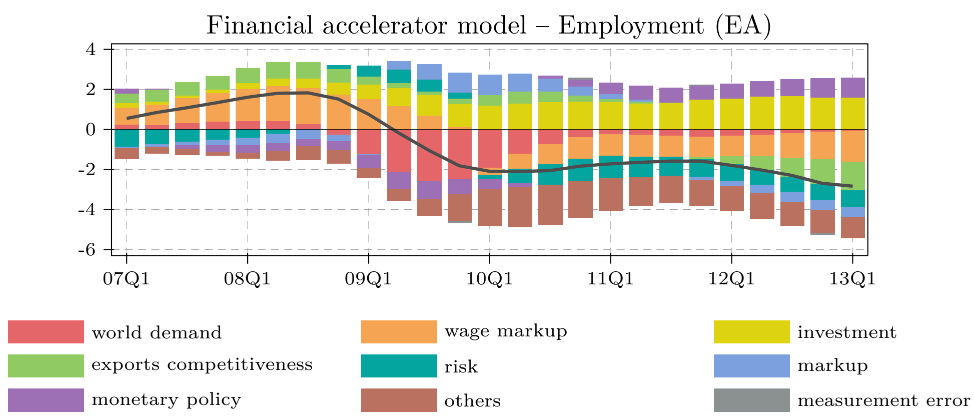


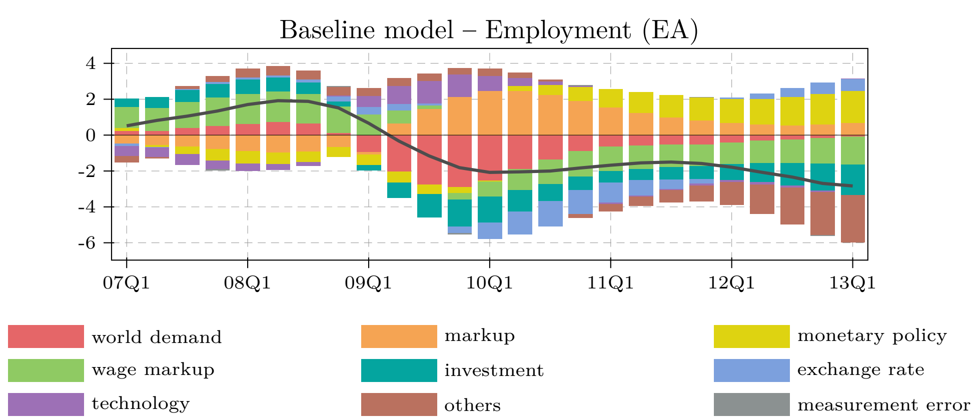
Baseline model – Inflation (EA)

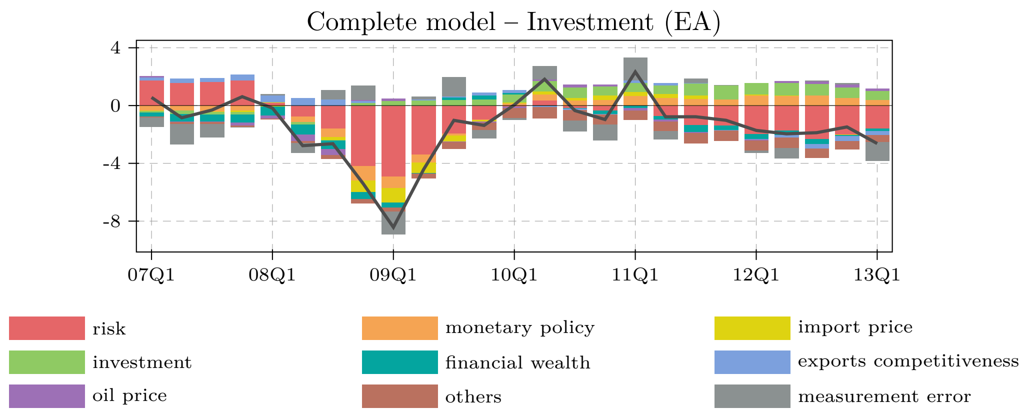


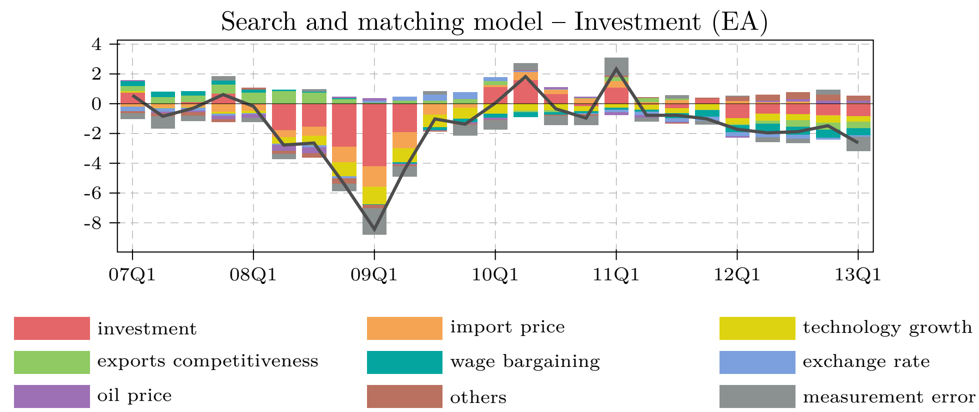

Complete model – Employment (EA)
2
0
-2
-4
07Q1
08Q1
09Q1
10Q1
11Q1
12Q1
13Q1
wage bargaining
investment
world demand
risk
import price
exports competitiveness
monetary policy
others
measurement error

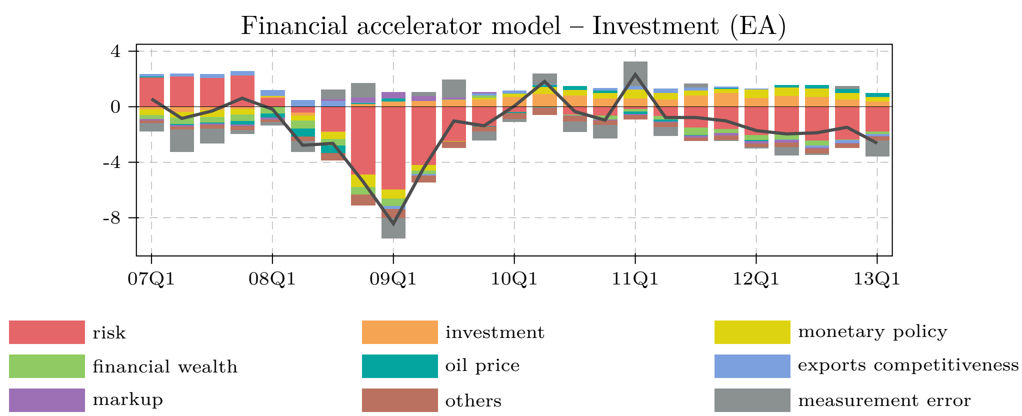

Search and matching model – Employment (EA)
2
1
0
-1
-2
-3
-4
07Q1
08Q1
09Q1
10Q1
11Q1
12Q1
13Q1
wage bargaining
technology growth
investment
world demand
monetary policy
exports competitiveness
import price
others
measurement error

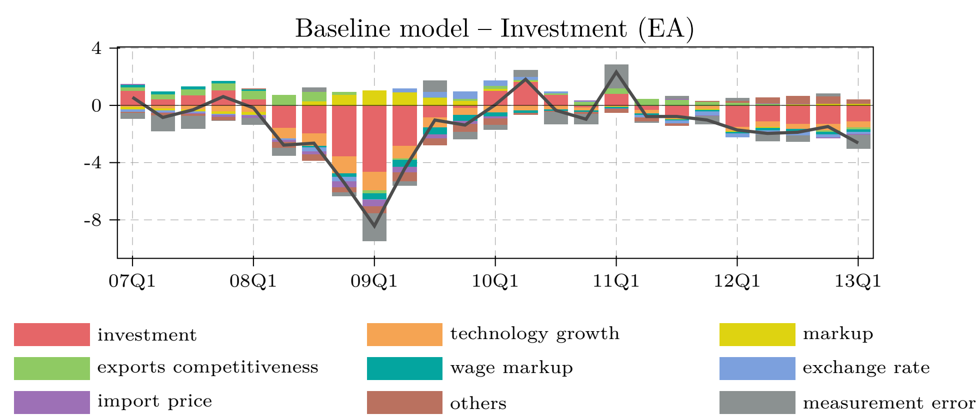

Financial accelerator model – Employment (EA)
4
2
0
-2
-4
-6
07Q1
08Q1
09Q1
10Q1
11Q1
12Q1
13Q1
world demand
wage markup
investment
exports competitiveness
risk
markup
monetary policy
others
measurement error


Baseline model – Employment (EA)
4
2
0
-2
-4
-6
07Q1
08Q1
09Q1
10Q1
11Q1
12Q1
13Q1
world demand
markup
monetary policy
wage markup
investment
exchange rate
technology
others
measurement error


Complete model – Investment (EA)
4
0
-4
-8
07Q1
08Q1
09Q1
10Q1
11Q1
12Q1
13Q1
risk
monetary policy
import price
investment
financial wealth
exports competitiveness
oil price
others
measurement error


Search and matching model – Investment (EA)
4
2
0
-2
-4
-6
-8
07Q1
08Q1
09Q1
10Q1
11Q1
12Q1
13Q1
investment
import price
technology growth
exports competitiveness
wage bargaining
exchange rate
oil price
others
measurement error


Financial accelerator model – Investment (EA)
4
0
-4
-8
07Q1
08Q1
09Q1
10Q1
11Q1
12Q1
13Q1
risk
investment
monetary policy
financial wealth
oil price
exports competitiveness
markup
others
measurement error


Baseline model – Investment (EA)
4
0
-4
-8
07Q1
08Q1
09Q1
10Q1
11Q1
12Q1
13Q1
investment
technology growth
markup
exports competitiveness
wage markup
exchange rate
import price
others
measurement error

5.1.3 Understanding the differences between France and the rest of the euro area

In reality, the recent crisis experienced by the euro area includes two crises. The first one is the financial turmoil resulting from the collapse of the subprime mortgage market in the US and the capital losses faced by the world banking system. The corresponding macroeconomic depression covers the period between 2008Q2 and 2009Q2. The second one is the sovereign debt crisis, related to the fear of insolvency of several euro area member states, and especially the critical situation of Greece leading to a partial default on its debt. In the estimation sample of the model, the macroeconomic slowdown corresponding to this second crisis spans a period from 2011Q2 to 2013Q1. The first crisis was very brutal, while the second is characterized by a smaller magnitude but a high degree of persistence. However, inflation resisted in each case, exhibiting only a moderate decline considering the size of the effects on the real economy.

During these two episodes, the French economy did significantly better than the rest of the euro area with respect to GDP growth and to the main components of demand, as illustrated by Figures 5.1 and 5.2. In particular, cumulated French GDP growth was higher than in the rest of the euro area by around 1.7pp during the first crisis and by 2pp during the second. Regarding inflation, the two economies behaved similarly, yet with slightly higher cumulated inflation in France during the first crisis.

This subsection provides an analysis of the differences recorded in the dynamics of GDP, investment, consumption and prices between France and the rest of the euro area, in terms of shock contributions. It focuses on the two consecutive crisis periods experienced recently, namely the 2008-2010 financial crisis and the 2012-2013 sovereign debt crisis. In order to provide a faithful picture of the relative importance of underlying factors, Figures 5.3 to 5.6 show the cumulated contributions of shocks. This exercise is done with the complete model (including both financial and labor market frictions). Of course, the comparison between these two episodes should take their different durations into consideration.

A general observation is that the main contributors to business cycle fluctuations during the crisis, identified in subsections 5.1.1 and 5.1.2, had in many cases comparable cumulated effects on the two regions; therefore, the analysis below puts forward other shocks, identified as secondary for the fluctuations in each country taken individually, to explain the differences. Risk shocks are an exception. Indeed, they affected more severely GDP in the rest of the euro area than in France. But the size of this difference is still small relatively to their cumulated effect on each economy.

A first finding regarding GDP growth is that all shocks significantly contributing to the difference between France and the euro area play in the same direction. In particular the better performance of France during the two crises is explained by smaller effects of investment technology and financial risk shocks. Next, the larger gap between the two economies recorded during the second crisis as compared with the first one is primarily attributed to wage bargaining shocks. If the analysis of section 5.1 points to a too small downward adjustment of real wages in both economies, this is found to be more problematic in the rest of the euro area than in France, considering the even

Figure 5.1: Cumulated variations during the first crisis

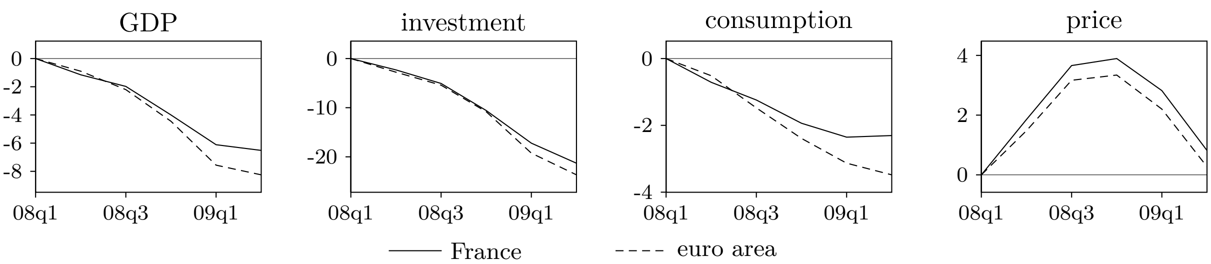
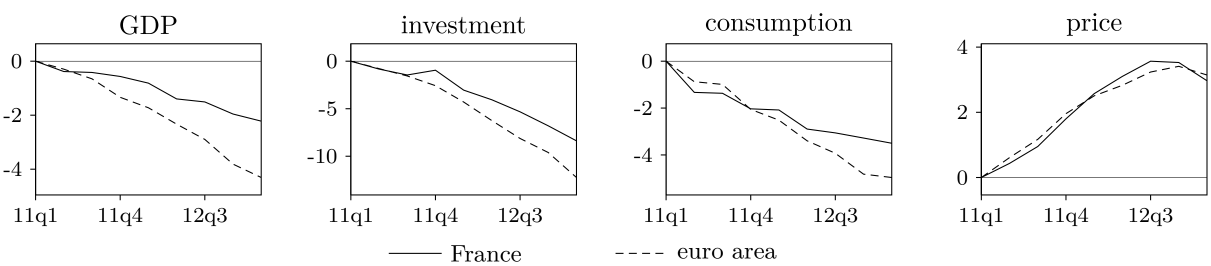
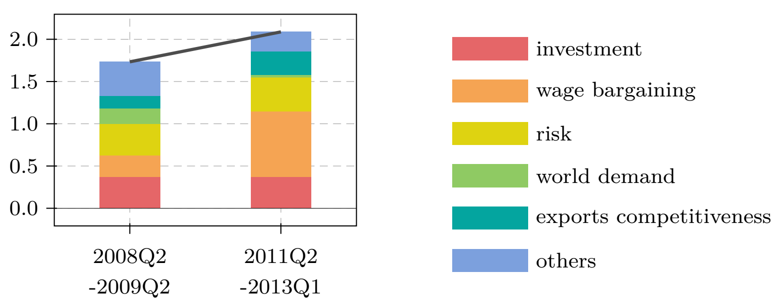


Figure 5.2: Cumulated variations during the second crisis



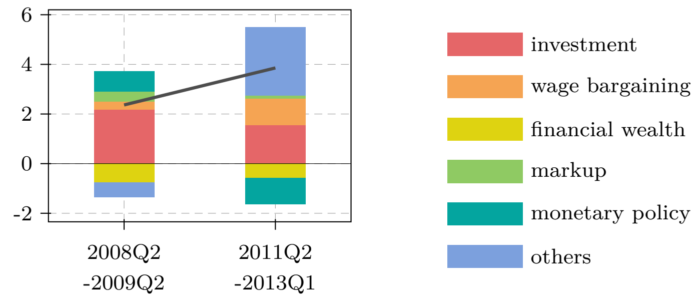
worse conditions in the labor market there.

Figure 5.3: Analysis of the cumulated difference in GDP between France and the euro area during the crises



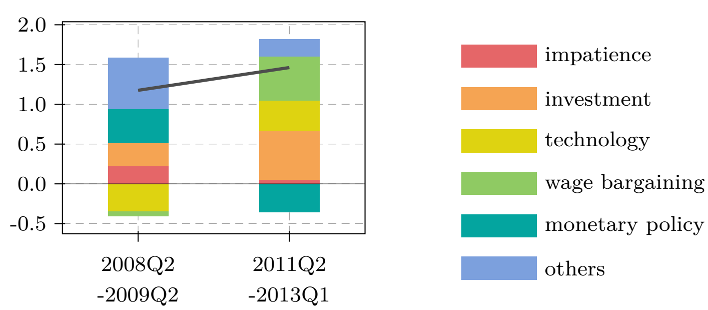
The difference of more than 2pp in the cumulated investment growth is mainly attributed to investment technology shocks during the two episodes. Monetary policy shocks contributed negatively during the first crisis but positively during the second; since the magnitude of their effects are slightly larger in the rest of the euro area than in France, they first contributed positively then negatively to the gap between the two regions. Apart from the effect of wage bargaining shocks, the fact that the investment gap was larger in the second crisis than in the first is attributed to many other minor contributors (in blue).

Figure 5.4: Analysis of the cumulated difference in investment between France and the euro area during the crises



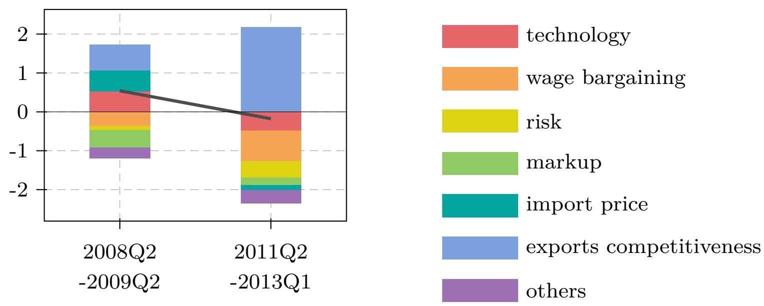
As for investment, monetary policy shocks contributed positively to the consumption gap during the first crisis and negatively during the second. Next, broadly speeking, the estimation identifies negative technology shocks during the first crisis and positive ones in the following periods. This is consistent with the particularly striking resistance of inflation recorded during the first, considering the magnitude of the recession. The present analysis reveals that technology shocks impacted more harshly France than the rest of the euro area during 2008-2009, and provided a larger support to consumption after 2011. Consistently, technology shocks' contributions to inflation (Figure 5.6) have the opposite sign. Last, as for investment and GDP, wage bargaining shocks played against consumption in the rest of the euro relatively to France during the second crisis; indeed, too low downward wage adjustments contribute to worsen unemployment, impacting aggregate consumption negatively.

Figure 5.5: Analysis of the cumulated difference in consumption between France and the euro area during the crises



Last, the model identifies extra euro area import price shocks and technology shocks as the main reasons for the slightly higher inflation recorded in France during the first crisis. Yet, this difference in inflation between the two regions is viewed as "normal" in the context of shocks moving prices and GDP in the same direction, such as risk, investment technology or world demand shocks. The other major contribution to the inflation gap during the two crisis episodes is the relatively high level of export prices in the rest of the euro area, giving rise to a positive effect on French inflation via intra euro area imports.

Figure 5.6: Analysis of the cumulated difference in price between France and the euro area during the crises



5.1.4 Concluding remarks

The different versions of the model point to different business cycle analysis during the crisis. Deeper microfoundations in modeling financial and labor markets are thus useful to identify more carefully the shocks underlying cyclical fluctuations. Implicitely, Smets and Wouters (2007)-like models assimilate the 'true' shocks that would be identified with richer models to standard macroeconomic shocks, resulting in spurious economic stories. The frictions and refinements added to the model contribute to clean up the contributions assigned to these shocks, putting them in their rightful places.

This exercise makes clear the need for using meaningful microfoundations of credit and labor markets for pure story telling in business analyses. Yet, models where these dimensions are absent fit data pretty well by identifying different combinations of shocks to explain business cycle fluctuations. This is the reason why these tools are successfully used in forecasting (see for example Adolfson et al. (2005) or Smets and Wouters (2004)). But do these frictions impact the conclusions based on simulations of the model from a normative point of view? The following sections address this issue and compare simulations obtained with the four versions of the estimated model within the context of two policy evaluation exercises.

5.2 Welfare cost of fluctuations and the Taylor rule

In this section, I compute the welfare cost of business cycle fluctuations in the estimated models, and discuss the implications of financial and labor market frictions for this measure.

5.2.1 Methodology and previous estimates

The welfare cost is computed as explained by Schmitt-Grohe and Uribe (2004) for the four versions of the estimated model. The computation is based on a second order approximation of the model, which includes a constant term. This term is a linear function of the variances and covariances of exogenous shocks, which thus disappears in a perfectly deterministic environment. It induces a permanent deviation of variables from their deterministic steady state levels.

This permanent deviation is in reality a gap between the average values of variables throughout business cycles and their deterministic steady state levels. Indeed, the non-linearities present in models result in asymmetric fluctuations around the steady state; specifically, in the presence of uncertain economics perspective, risk-averse agents react more strongly to adverse shocks than they do in response to favorable ones. As a result, this behaviour generally makes average welfare of households in business cycles lower that its steady state level. This welfare cost is generally expressed in consumption equivalent, which corresponds to the percentage of steady state consumption that households would be willing to sacrifice to eliminate business cycles.

The early estimates of the cost of business cycles by Lucas (1987) are quantitatively very low: for plausible calibrations, he finds less than 0.01% of lifetime consumption. Yet a recent literature reports much larger values (Beaudry and Pages (2001) report values between 1.4% and 4.4%, Krebs (2003) finds 3.2% in his baseline model economy, and the welfare gains of eliminating aggregate productivity shocks represent 1.44% of consumption on average in Storesletten et al. (2001); see Barlevy (2004) for a survey). A reason lies in the many frictions that are embedded in recent models. The presence of markups for instance induces strongly non-linear behaviours when agents are risk-averse. Search and matching frictions in the labor market, or financial frictions, may also add sizable welfare costs of business cycles. Hairault et al. (2010) compute the welfare costs implied by labor market frictions. They conclude that business cycles increase average unemployment, causing significant welfare costs. Regarding financial frictions, Mendoza (2002) shows that the cost is higher with credit frictions than without them, and Kolasa and Lombardo (2014) find that financial frictions significantly impact optimal monetary policy in a two-country economy. This section complements these contributions by using estimated models; the versions of the model considered do not only differ by the presence or not of frictions, but their parameter values and the respective size of shocks are updated to fit the data in each case. I compute for each of them the welfare gain of eliminating fluctuation expressed in units of lifetime consumption.

5.2.2 Effects of frictions on welfare

The welfare cost of the business cycle in each version of the model of are shown in Table 5.1. In the baseline model with perfect credit markets and no unemployment, the welfare cost of business cycles is similar in France and in the rest of the euro area, around 1.1% of consumption. The estimates obtained when the model accounts for financial frictions are much higher (+1.0pp in France and +1.1pp in the rest of the euro area). It is also somewhat amplified when the model takes into account matching imperfections in the labor market. This is especially true in France (+0.4pp)

Table 5.1: welfare cost of fluctuations in the different versions of the model

	France	Rest of EA	euro area
Basic model	-1.04	-1.10	-1.09
Financial frictions only	-2.03	-2.21	-2.18
Labor market frictions only	-1.46	-1.28	-1.31
Complete model	-1.12	-1.33	-1.29

as compared with the rest of the euro area (+0.2pp). Surprisingly, including both frictions gives estimates that are only slightly higher than in the frictionless case, with +0.1pp in France and +0.2pp in the rest of the euro area, but lower than when frictions are embedded separately.

The estimates for France are in most cases slightly below those for the rest of the euro area. The exception is the model with only labor market frictions. This suggests that the welfare costs of business cycles stemming from labor-market frictions are larger in France than in the rest of the euro area. Hairault et al. (2010) analyse the mechanisms of the search and matching model which lead to significant welfare costs. Specifically, non-linearities make average unemployment higher in the presence of fluctuations than it would be in a fully stabilized economy. This is because unemployment decreases less during economic expansions than it increases during recessions. These authors identify three sources of asymmetry. The first one originates from the equation that describes the law of motion of unemployment. It implies that low employment moderates the benefits of higher job finding rates during cycle peaks, whereas in depressions the negative effects of lower job finding rates on job creations are amplified by a higher level of unemployment. The two other non-linearities follow from the concavity of the matching technology. On the one hand, it implies that vacancies posted during expansions have decreasing chances of success. On the other hand, firms' optimal labor demand represented by job offers is a convex function of productivity. So it reacts more markedly to rises in productivity than to declines of the same magnitude. Unlike the first two mechanisms, the third one tends to reduce average unemployment and thus the welfare costs of business cycles. These authors show that lower values of the elasticity of matching with respect to vacancies augment the costs resulting from the second channel, and dampen the gains from the convexity of labor demand. Turning back to the model of France in the euro area, this leads to the conclusion that the difference in the costs of business cycles between the two regions in the model with labor-market frictions only may be partly related to the estimates of this parameter: with 0.23, it is smaller for France than for the rest of the euro area, with 0.27.

5.2.3 Alternative calibrations of the Taylor rule

An essential question for central bankers is whether the monetary authority could stabilize more efficiently aggregate fluctuations and reduce this cost. It underpins the huge literature on optimal monetary policy. The most common approach consists in solving the Ramsey problem, following in particular Woodford (2001). In a different way, Schmitt-Grohe and Uribe (2004) optimize the parameters and the specification of the Taylor rule in the model of Christiano et al. (2005), and Adjemian and Devulder (2011) undertake the same exercise in a similar model estimated to fit euro area data. Yet, computing the parameters of the Taylor rule that minimize the welfare cost of fluctuations is a tricky exercise. First, the multi-dimensional problem is strongly non-monotonic. Then, even in simple new-Keynesian models, when parameters approach their bounds, very high welfare gains of flucutations may arise, so the problem has no interior solution. For that reason, I only focus on two typical calibrations of the most simple Taylor rule possible, which is in its log-linear form

$$\hat{R}_t = \phi_\pi \hat{\pi}_t + \varepsilon_{R,t},$$

and is a particular case of the rule specified in the model. I compute the changes in welfare costs that monetary policy can achieve by strictly reacting to contemporaneous inflation, either mildly ($\phi_\pi = 1.05$), or more sharply ($\phi_\pi = 3$). This is done is all estimated versions of the model in order to see whether omitting the realistic frictions embedded in the complete model can substantially alter policy recommandations. The results are reported in Table 5.2.

Table 5.2: welfare cost of fluctuations implied by variant Taylor rules

	France			Rest of EA			euro area		
	rule 1	rule 2	rule 3	rule 1	rule 2	rule 3	rule 1	rule 2	rule 3
Basic model	-1.04	-1.30	-1.04	-1.10	-1.43	-1.09	-1.09	-1.40	-1.08
Financial frict. only	-2.03	-3.15	-2.14	-2.21	-2.90	-2.42	-2.18	-2.95	-2.36
Matching frict. only	-1.46	-1.91	-1.45	-1.28	-1.42	-1.33	-1.31	-1.52	-1.35
Complete model	-1.12	-3.36	-1.28	-1.33	-2.80	-1.63	-1.29	-2.91	-1.56

Notes: rule 1 is the estimated Taylor rule, where parameter values depend on the version of the model, rule 2 is $\hat{R}_t = 1.05\hat{\pi}_t + \varepsilon_{R,t}$, and rule 3 is $\hat{R}_t = 3\hat{\pi}_t + \varepsilon_{R,t}$. Figures of in percent of lifetime consumption.

In all versions of the model, I find that households from the euro area (both France and the rest) would be negatively affected by a loose monetary policy, stabilizing inflation less efficiently. Next, except for the complete model, the welfare cost of fluctuations would be almost unaffected under the 'sharper' monetary policy rule. This is not true in the model with both financial and labor market frictions, where housholds would be penalized under this regime, especially in the rest of the euro area. A last finding is than accounting for the presence of financial frictions augments considerably the cost of the 'mild' policy rule; in particular, in the complete model, this cost is more than doubled as compared to the benchmark case, both in France and in the rest of the euro area.

5.2.4 Concluding remarks

First, in an estimated multi-country model of France and the euro area with many imperfections, I find welfare costs of business cycles between 1.0 and 2.2% of households' lifetime consumption. It is in the range of the estimates reported in the recent literature, but much greater than in Lucas (1987).

Then, both financial and labor market frictions augment the welfare cost of fluctuations, the former having larger effects than the latter. Besides, job matching imperfections lead to higher business cycle costs in France than in the rest of the euro area. Nevertheless, the fact that the estimates are of the same order of magnitude regardless of the version of the model considered – and quantitatively large relatively to Lucas (1987) – suggests that non-linearities in goods markets is already a major source of welfare costs.

Last, these results support the view that models should account simultaneously for both labor market and financial frictions. First, because taking into account only one of the two frictions leads to overestimate the welfare cost of fluctuations. Next, because the presence of the two frictions makes a 'sharper' monetary policy rule more costly than the baseline one, whereas the simpler models may provide some support to a stronger reaction of the central bank to inflation. Even if the interaction between the two frictions is theoretically not critical with respect to the transmission of standard shocks (see section 2.6), the models may deliver contrasted monetary policy recommendations once they are estimated.[2]

5.3 Exiting from the monetary union

5.3.1 Implications for welfare

The benefits in terms of welfare of the monetary union, as compared to a flexible exchange rate regime, are unclear. On the one hand, cancelling the uncertainty surrounding fluctuations in the exchange rate undoubtably represents a gain, but on the other hand, the difficulty for a unique monetary authority to monitor several imperfectly synchronized economies may yield a cost. Kollmann (2004) computes the welfare effects of a fixed exchange rate regime using a calibrated two-country model. He finds that the monetary union may improve welfare by eliminating the uncovered interest parity shocks that are associated with the flexible exchange rate regime. This effect is significant (around +0.2% of consumption) for economies with a high degree of openness, such as those within the euro area.

By contrast with previous estimates, I use a model estimated with Bayesian techniques to address this question. In particular, the relative importance of the exogenous sources of business cycle fluctuations is computed from the data.

[2] A limitation to these evaluations comes from the fact that the estimation methodology used is based on a first order approximation of the model, while welfare cost computations use a second order approximation. Yet, the estimations follow the same methodology for all four versions, without using any particular assumption to bias the results.

The modifications of the model to switch off the peg regime are the following. First, the Taylor rule of the monetary union is replaced by two identical Taylor rules, except that each country's monetary authority only reacts to domestic inflation and output. Next, the peg condition ($S_{FR/EUR,t}/S_{FR/EUR,t-1} = 1$) is cancelled. Last, households in each country can trade bonds issued in the other country and labelled in the latter's currency. Hence, the equilibrium under flexible exchange rate includes an additional uncovered interest parity condition in each country, and the trade balance admits these two types of bonds as counterparts.

The welfare costs of the business cycle expressed in percent of lifetime consumption are reported in Table 5.1 for all the versions of the model (with/without financial and labor market frictions) under the two regimes. The result is clear: the flexible exchange rate regime does not yield any significant increase or decrease in the cost of the business cycle, for both regions and using any version of the model. The table also provides the theoretical volatility of the variation in the nominal exchange rate of FF (French franc) against EUR. It is found to be about 20 times smaller than the one of the EUR/USD exchange rate.

Table 5.1: welfare cost of fluctuations with flexible exchange rate

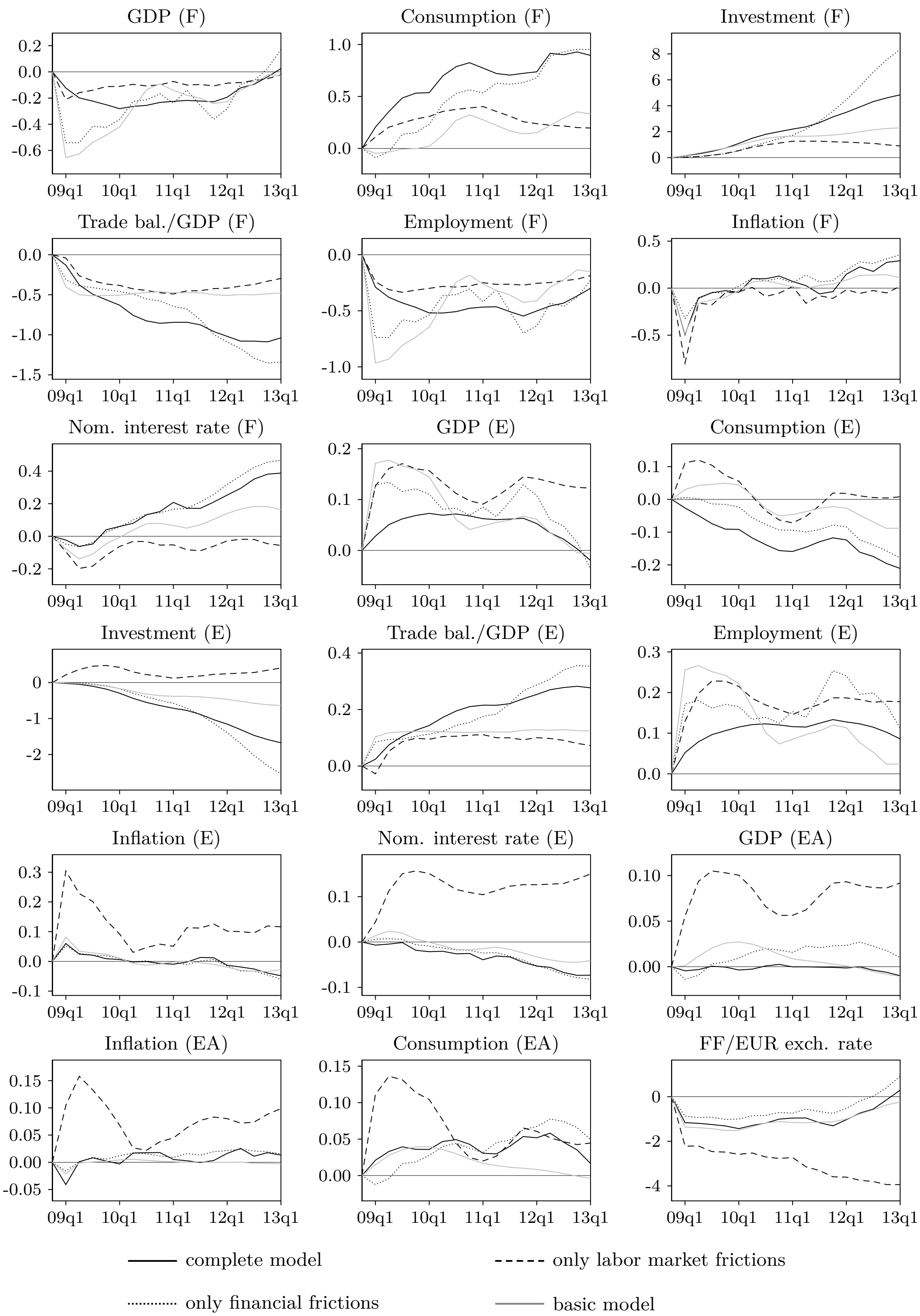
	France		Rest of EA		euro area		volatility
	MU	flex.	MU	flex.	MU	flex.	FF/EUR
Baseline model	-1.04	-1.09	-1.10	-1.08	-1.09	-1.08	0.17
Model with financial frictions	-2.03	-2.06	-2.21	-2.21	-2.18	-2.18	0.24
Model with labor market frictions	-1.46	-1.47	-1.28	-1.25	-1.31	-1.29	0.23
Complete model	-1.12	-1.11	-1.33	-1.33	-1.29	-1.28	0.26

These results are consistent with Kollmann (2004), since the sources of fluctuations, identified under the monetary union assumption, do not include uncovered interest parity shocks. So, apart from the effect of these shocks, the flexible exchange rate regime does not deteriorate welfare. Conversely, these simulations do not reveal welfare improvements either. The degree of synchronization of the two economies is high enough to be efficiently stabilized by a unique monetary policy rule. For the same reason, the model endogenously generates a very small volatility of the exchange rate.

5.3.2 Simulation of the crisis period under a flexible exchange rate regime

This section presents a counterfactual simulation of the model with flexible exchange rate between France and the rest of the euro area, starting in 2009Q1, and covering the end of the estimation sample until 2013Q1. The models are simulated using the smoothed shocks obtained under the monetary union assumption. The graphs below plot the difference between the simulated economy under the assumption that France exited the euro area in 2009Q1, and the actual economy, for each of the four versions of the estimated model.[3]

[3] Of course, these simulations do not account for a possible period of turmoil caused by the exit procedure itself.



Notes: the graphs plot the deviations between the counterfactual simulations under a flexible exchange rate regime starting in 2009Q1, and the simulations with a peg regime. The differences are in percent, except for inflation and interest rates, for which the differences are expressed in percentage points.

With the complete model, I find that the shocks which have generated the crisis would have occasioned a 1% appreciation of the FF against the EUR (which is quite a large magnitude as compare to the theoretical standard deviation of this variable, around 0.2%). This adjustment would have substantially deteriorated the French economy, with a 0.3% fall in GDP, a 0.5% decrease in employment after one year, and a deterioration of the trade balance reflecting the increase in imports and the fall in exports towards the rest of the euro area. However, private consumption and investment would have benefited from the relative deflation of imported goods, which would have pushed the inflation rate 0.5pp below its baseline level in 2009Q1. The effect on investment would have been very progressive, finally amounting to a cumulated improvement of 4% at a three-year horizon. By contrast, the economy in the rest of the euro area would have taken advantage of this minor devaluation, except for consumption and investment which would have suffered from the inflation of imports from France. From a general point of view, this asymmetry is consistent with the fact that the crisis has affected more harshly the rest of the euro area than France. Finally, the effect on the euro area economy as a whole, both in terms of GDP and inflation, would have been negligible.

A striking result is the absence of depreciation of the FF against the currency of the rest of the euro area. Indeed, political discussions about the effects of the common currency generally acknowledge that the French economy would have benefited from such a depreciation considering its competitivity gap with Germany. Nevertheless, the simulations obtained here are consistent with the representation of the rest of the euro area that is assumed for the estimation of the model: this region corresponds to an aggregate of all the other countries of the area, including Germany as well as less vigorous economies such as Italy, Spain or Greece. Since France mostly displays median characteristics within the euro area, this "averaged" economy has been affected more harshly than France by the recent crisis according to the data used. From that point of view, the counterfactual experiment presented here only considers, with the exit of France, a first step in a collapse of the euro area; in particular, a subsequent exit of Germany would have resulted in very different movements in the exchange rate of the FF. But this is outside the scope of this analysis.

The simulations obtained with the incomplete versions of the model differ somewhat from those obtained with the complete model. The responses of French output and employment in the basic model and in the one with only financial frictions are amplified in the short run but are less persistent. The model with only labor market frictions exhibits smaller reactions of French variables, but higher effects in the rest of the euro area.

Changing the exchange rate regime alters the dynamic behavior of the models through two channels: the fluctuations of the FF/EUR exchange rate and the interest being decided at country level. The propagation of a given shock is altered differently according to the presence or not of frictions and to parameter values. In addition, the various shocks of a given model are also impacted differently. Hence, the differences in the simulations above may also follow from the fact that each version of the model identifies distinct chronicles of shocks during the crisis and the post-crisis periods (see section 5.1.4).

The model with only labor market frictions predicts a larger appreciation of the nominal

FF/EUR exchange rate, ending at a 4% gap with the initial parity after four years. This yields larger effects on the rest of the euro area economy, but, surprisingly, smaller effects on the French economy, despite the 0.8pp drop in inflation at the date of the exit.

5.3.3 Concluding remarks

First, this exercise highlights the high degree of synchronization of the two economies (France and the rest of the euro area): the welfare cost of fluctuations would be unchanged under a flexible exchange rate regime, and the intra-zone nominal exchange rate would not fluctuate much. This finding does not depend on the presence of financial or labor market frictions.

Yet, there are differences in the counterfactual simulations of the crisis and post-crisis periods, depending on the version of the model that is used. The models without labor market frictions overestimate the short term effects on GDP and employment of the exit from the monetary union, whereas the model with only labor market frictions predicts a larger appreciation of the FF against the EUR, but smaller real effects in France.

Chapter 6

Social VAT in the context of the 2009 recession

In a recent paper, Farhi et al. (2014) demonstrate how real policies conducted under a fixed exchange rate regime can lead to the same real allocations than those that would prevail with a flexible exchange rate. They consider fiscal policies that manipulate a set of several tax rates; yet, the 'core' underlying policy is invariably social VAT, that is the transfer of tax bases from labor to consumption. In the last section of this paper, they illustrate the implementation of fiscal devaluations with a stylized new-Keynesian model of a small open economy in a currency union, calibrated to represent Spain. They show that, after a contractionary interest rate shock, that is supposed to mimic the 2008-2009 crisis, a permanent social VAT measure can move the economy's response close to what it would be if wages were flexible. In this exercise, they cast aside the neutralization of the indesirable effects of peg regimes, and turn to the more conventional new-Keynesian problem, that is cancelling the effects of nominal frictions. This objective is justified by the fact that the real allocations attained when prices in the broad sense are flexible are more efficient, because under a fixed exchange rate regime, nominal rigidities prevent the economy from restoring its competitiveness. By contrast with the vast literature on efficient monetary policies, the proposed approach considers real policies instead of nominal policies to reach this business cycle objective.

In this section, I confine myself to this question of the neutralization of nominal effects using real policies. Specifically, I simulate the impact of social VAT measures in the context of the 2009 recession, using the estimated model of France in the euro area including both financial and labor market frictions. By contrast to Farhi et al. (2014), the recession results from all the smoothed shocks computed over the estimation sample until 2009Q1. Social VAT measures are implemented at that date, in the form of an increase in the VAT rate and a decrease in the employer social contibution rate. These tax rates are then kept constant for a period of 3 years and then return to their initial level. This kind of one-off policy is more realistic and implementable than one with period-to-period time varying tax rates. I assume that it is only announced at the date of implementation, and perfectly anticipated from then on by all agents in the model. I keep away from the permanent fiscal devaluation considered by Fahri for two reasons.[1] First, I use a model that is estimated to match the cyclical properties of the data. Second, permanent fiscal simulations in an open economy with incomplete international bonds markets raise the issue of the determination of the long run equilibrium, as discussed in section 2.10. The measure considered is thus transitory, consistently with the business cycle objective exposed above.[2]

Fève et al. (2012) also propose an evaluation of social VAT in France, assuming permanent changes in tax rates. They find that the measure has little effects on macroeconomic aggregates and on welfare. However, their analysis ignores international trade, which is a major channel through which the measure is expected to be beneficial.

The design of the present social VAT measure is based on three considerations: (i) it is expected to prevent job relocations, as pointed up by its defenders; (ii) it is theoretically neutral for government revenues, and (iii), as already mentioned, a role assigned to policies in the new-Keynesian literature is to neutralize the inefficient allocations resulting from nominal rigidities. In this spirit, the magnitudes of tax changes are chosen to make the short run dynamics of employment in the country that implements the policy as close as possible to the one that would prevail if prices and wages were flexible in the whole euro area – or, put differently, to minimize the employment gap over the first year of the measure. At the same time, the variations in tax revenues from VAT and from employer social contributions should compensate each other over the three-year period of the measure.

In a first experiment, France implements social VAT alone. In a second one, the rest of the euro area, considered as a unique country, also applies a social VAT policy with similar objectives. The measures are assumed to be coordinated in the sense that each country takes the other one's policy into consideration to decide the size of its own tax changes. In all cases, the measures considered are entirely financed over the 3 year-period. The magnitudes of tax changes obtained are given in Table 6.1.

This exercise has been done with the four different versions of the estimated model and yields pretty similar pictures. Hence, only the simulations of the complete model are presented in what follows. The simulations obtained with the other three versions are given in Appendix 6.A.

6.1 Simulations in the complete model

The figures below plot the simulations obtained, starting in 2009Q1 and over a three-year horizon. The baseline scenario is the models' projections as of 2009Q1. The other scenarios are plotted in percentage deviation from this baseline. The plain black line corresponds to the flexible price scenario. It is obtained by simulating the model from the same initial conditions assuming that

[1]Although the policy considered by Farhi et al. (2014) in their numerical illustration also includes a decrease in the tax on capital paid by firms, I call indifferently 'fiscal devaluation' or 'social VAT' the policy described above.

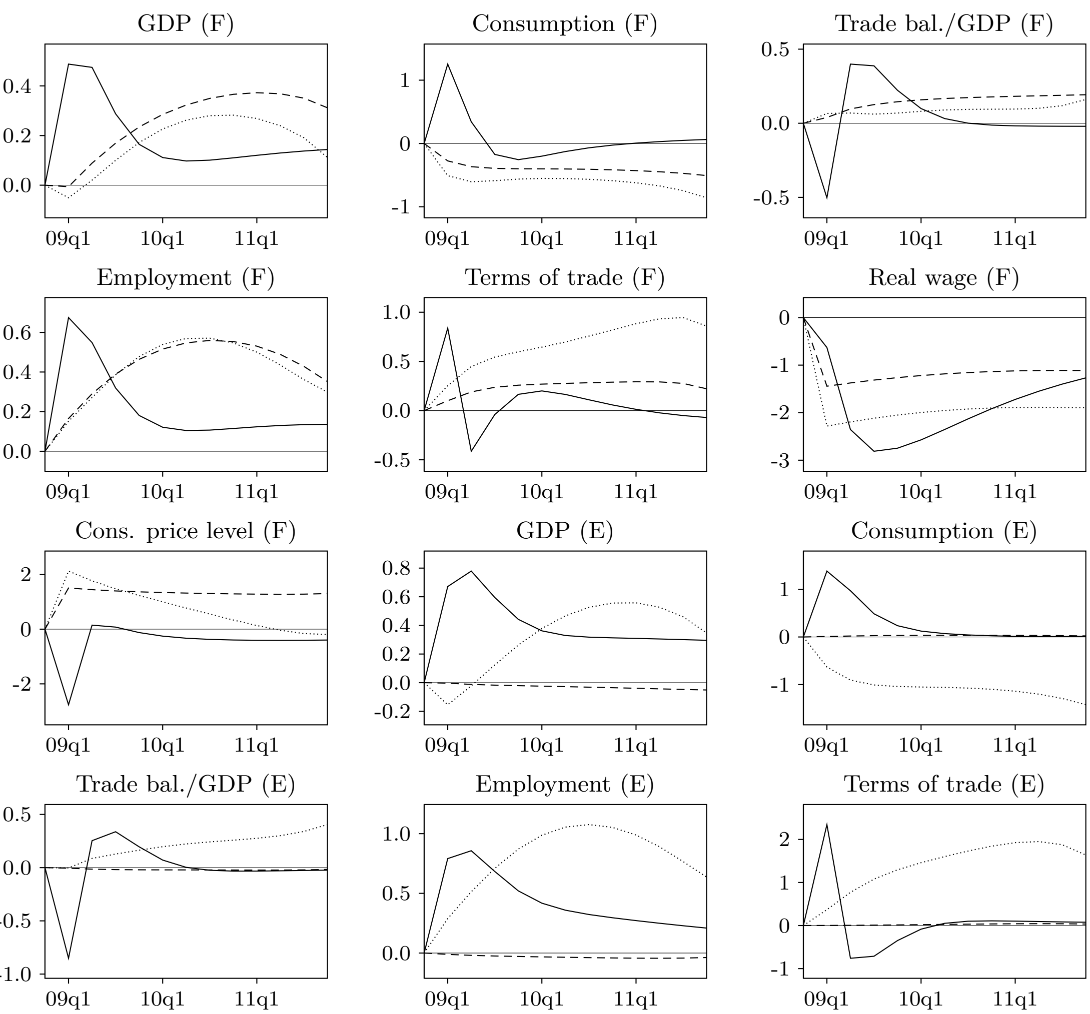
[2]Social VAT measures implemented by Germany or Denmark in the past were permanent, and political discussions generally consider this case. However, the social VAT decided in 2012 in France was abrogated the same year after the change of government. Transitory measures – or measures viewed as transitory by economic agents – may thus also result from political instability.

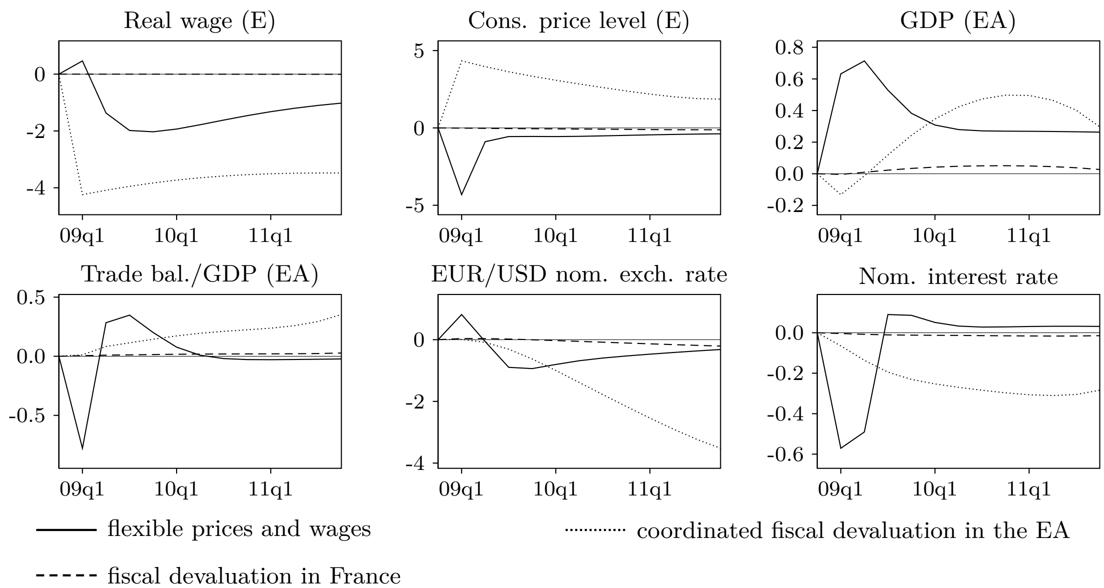
Table 6.1: Social VAT scenarios

	unilateral France	coordinated F and REA
τ_F^C	1.79	2.82
τ_F^L	-2.76	-3.98
τ_E^C	0.00	5.18
τ_E^L	0.00	-7.29

Notes: The table reports the deviation of tax rates from their steady state level in percentage points, that are put into effect between 2009Q1 and 2011Q4.

prices and wages are fully flexible from 2009Q1 onwards. For this scenario, the shocks underlying the recession really matter. The dashed line and the dotted line report the modifications of the trajectories resulting from the implementation of social VAT with sticky prices and wages, respectively in France only and in coordination with the rest of the euro area.





Notes: the graphs plot the simulated variables in deviations from the baseline scenario (presence of nominal rigidities and no social VAT measure). The differences are in percent, except for trade balance ratios and the nominal interest rate, for which the differences are expressed in percentage points.

As identified by the analysis in section 5.1, the negative shocks occuring in 2008 and 2009Q1 mainly include risk, exports and technology shocks. They translated into a sharp fall in output and employment in both countries.

In the case when prices and wages are flexible in the whole euro area, the downturn is significantly smoothed. In particular, prices in both countries decrease sharply in 2009Q1, which support consumption and exports. Yet, the trade balance ratio of the euro area is worsened because of the positive effect on GDP. The situation of employment improves markedly in 2009, then it moves back towards its level of the baseline scenario with nominal frictions. The positive reaction of consumption when nominal frictions are cancelled contrasts with the findings in Farhi et al. (2014). This may be partly explained by the fact that the recession entirely results from an interest rate shock in their simulations, and by the preferences they have specified for households.

In this economy, fiscal devaluation policies have very lagged effects on output. With the progressive pass-through of the social contribution cut to export prices, the unilateral measure leads to a 0.2% improvement in French terms of trade after one year, much later and smaller than under flexible prices and wages. In addition, the positive effects on employment are delayed. The effect on exports is also modest, but the deterioration of private domestic consumption is significant and immediate: around 0.4% in France and 1% in the rest of the euro area in the case of synchronized policies. Indeed, if the social contribution cut progressively affects nominal wages, real wages immediately plunge as the result of the mechanical rise in the price of consumption including taxes. In addition, the impact of a social VAT measure in France on the economy of the rest of the euro area is insignificant.

When both countries inside the euro area coordinate their policies to close the employment gap as much as possible, the outcome is that the magnitude of the changes in tax rates that are needed is almost doubled with respect to the unilateral case. Yet, the impact on French GDP is smaller than in the unilateral case. This is because these policies enter in competition rather than amplifying each other inside the euro area. Coordinated fiscal devaluations are thus not desirable.[3] Otherwise, the effects of these policies are qualitatively similar in both regions to the ones obtained in France in the unilateral case. Still, terms of trade and the trade balance of the euro area improve somewhat. In addition, the euro exchange rate appreciates slightly after a few quarters.

6.2 Improving the efficiency of social VAT

As can be seen in the graphs of sections 6.1, the positive effects on employment of the considered social VAT scenarios are lagged as compared to the target scenario assuming flexible prices. In addition, the benefits are rather modest considering the size of the tax adjustment. If the slow adjustment of production capacities is responsible for the lag, two mechanisms justify the magnitude of the effect. First, agents are expecting the ending of the measure, which influences current job creations. Second, search and matching inefficiencies are very important according to my estimation results. In what follows, I illustrate how these channels affect the result and suggest solutions to increase the benefits of social VAT.

6.2.1 Expectations

Frictions and especially labor market imperfections make the adjustment of employment very slow. Since firms anticipate the ending of the measure, they moderate their hires to avoid future situations of excess capacity. To highlight this mechanism, I compute the reponse of employment to social VAT measures of different durations. In order to show the particular role of labor market frictions, I simulate the four versions of the model. The changes in tax rates considered are those reported in Table 6.A.1 in Appendix 6.A, for the case of a unilateral fiscal devaluation in France (for the complete model, they are identical to those presented in Table 6.1). The results are shown in Figure 6.1.

In the models including labor market frictions, fiscal devaluations of the same magnitude have larger effects when they are continued for a longer period than 3 years.[4] This is not true in the basic model and in the model with financial frictions only. Because of the high degree of persistence implied by the search and matching framework, medium term expectations matter for the short term response of employment. Yet, in the complete model, a duration of 10 years results in less job creations in the short term than for a duration of 5 years, because firms smooth more their reaction over time.

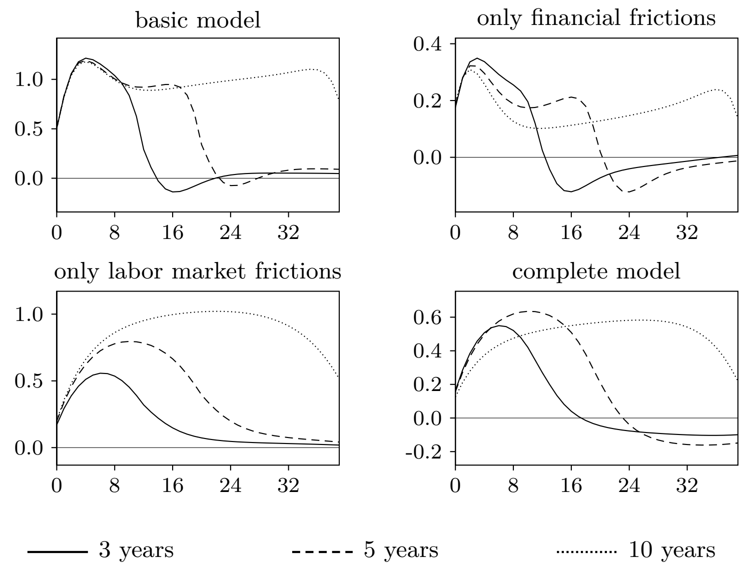
6.2.2 Inefficiency of the labor market

The second mechanism explaining the sluggish response of employment to fiscal devaluations is more specifically related to matching frictions in the labor market. These frictions yield inefficiencies:

[3] This result holds with all versions of the model.

[4] The length of 3 years is chosen because political outcomes are generally uncertain at this horizon.

Figure 6.1: Response of French employment to social VAT for different durations of the measure



Note: the graphs plot the deviation of French employment from its steady state level in percent.

labor demand is not immediately satisfied as it is in the Walrasian representation of labor markets. These market failures are reflected by the concave matching technology, and the related degree of inefficiency is measured by the value of the elasticity of matching with respect to vacancies (φ). Based on French and euro area data, the estimates of this parameter are found to be quite far from the values generally used for the US economy, as shown by Table 6.1. Specifically, the estimation reveals that firms face difficulties to hire as many workers as they would like. The labor markets in France and in the rest of the euro area can hence be viewed as strongly inefficient, as compared to the US.

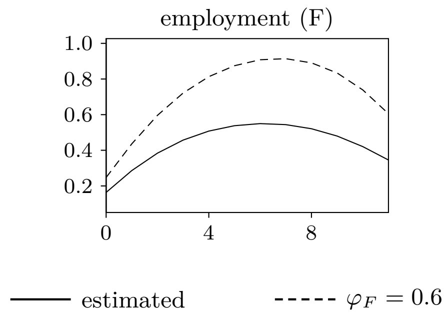
Table 6.1: Parameters characterizing the efficiency of the labor market

	usual values	model with l.m. frict.		complete model	
	US	France	rest of the ea	France	rest of the ea
φ	0.5-0.6	0.26	0.32	0.25	0.28

Note: The table reports posterior estimates and usual calibrated values for the elasticity of matching with respect to vacancies.

In order to highlight the effect of this inefficiency on the transmission of fiscal devaluations, I simulate the unilateral social VAT measure presented in Table 6.1 using the complete estimated model, but imposing different values for φ_F. Figure 6.2 shows the response of employment obtained in France. With the value of φ used for the US economy, say 0.6, the benefits of social VAT in terms of job creation would be much higher than with the estimated value. This demonstrates that real inefficiencies in the labor market are highly responsible for the sluggish response of employment to social VAT. For this reason, and considering their cost in terms of private consumption, the fiscal

Figure 6.2: Response of job creation to social VAT for different calibrations of φ_F



Note: the graph plots deviations of French employment from its steady state level in percent.

devaluations considered may be viewed as rather ineffective against unemployment in the current state of the euro area economy. Even if firms are incited by the tax cut to increase the number of job vacancies, only a small proportion of them end in actual job creations. The policy recommendation stemming from this finding is that the success of employment policies based on the reduction of labor costs, such as social VAT, crucially depends on the prior implementation of structural reforms aimed at improving the efficiency of matching between unemployed workers and firms. In the model economy, these reforms should materialize in an increase in the elasticity of matching with respect to vacancies, to a value close to the one characterizing the US labor market.

6.3 Concluding remarks

There are three main findings from these simulations. First, the estimation identifies a high degree of real inefficiencies in the labor markets of the French and euro area economies. As a consequence, in the current state of the economy, the benefits of the measures considered would have been insufficient in terms of employment. The analysis of this result leads to the following policy recommendations: on the one hand, structural reforms of the functioning of labor markets should precede the implementation of policies only based on reducing labor costs. On the other hand, governments need to commit themselves to maintaining labor tax cuts over a sufficiently long period of time.

Next, the simultaneous implementation of social VAT policies by countries inside the monetary union would have reduced their ability to attenuate the rise in unemployment during the crisis.

Last, by contrast to the findings of Farhi et al. (2014), the high level of persistence in the estimated models – particularly with labor market frictions – implies that the effects of fiscal devaluations are quite far from the allocation that would prevail in the absence of nominal frictions.

Appendix

6.A Simulation of social VAT measures using incomplete models

6.A.1 Scenarios

The same exercise is done for the three incomplete versions of the model (the models without either financial or labor market frictions and the model without both). The design of the social VAT measures considered follows the same principles. They are transitory. Each scenario consists in an increase in the VAT rate and a decrease in the employer social contribution rate to a fixed values during 3 years. At the end of the third year, the tax rates return to their initial values. The magnitudes of tax changes are chosen to make the short run dynamics of employment in the country that implements the policy as close as possible to the one that would prevail if prices and wages were flexible in the whole euro area – or, put differently, to minimize the employment gap over the first year of the measure. The values computed for each model are reported in Table 6.A.1. I consider unilateral implementation of social VAT by France, and a coordinated implementation in both countries. Finally, the measures considered are entirely financed over the 3 year-period (social contributions shortfalls are compensated by additional VAT revenues over the 3-year period in each country that implements the measure).

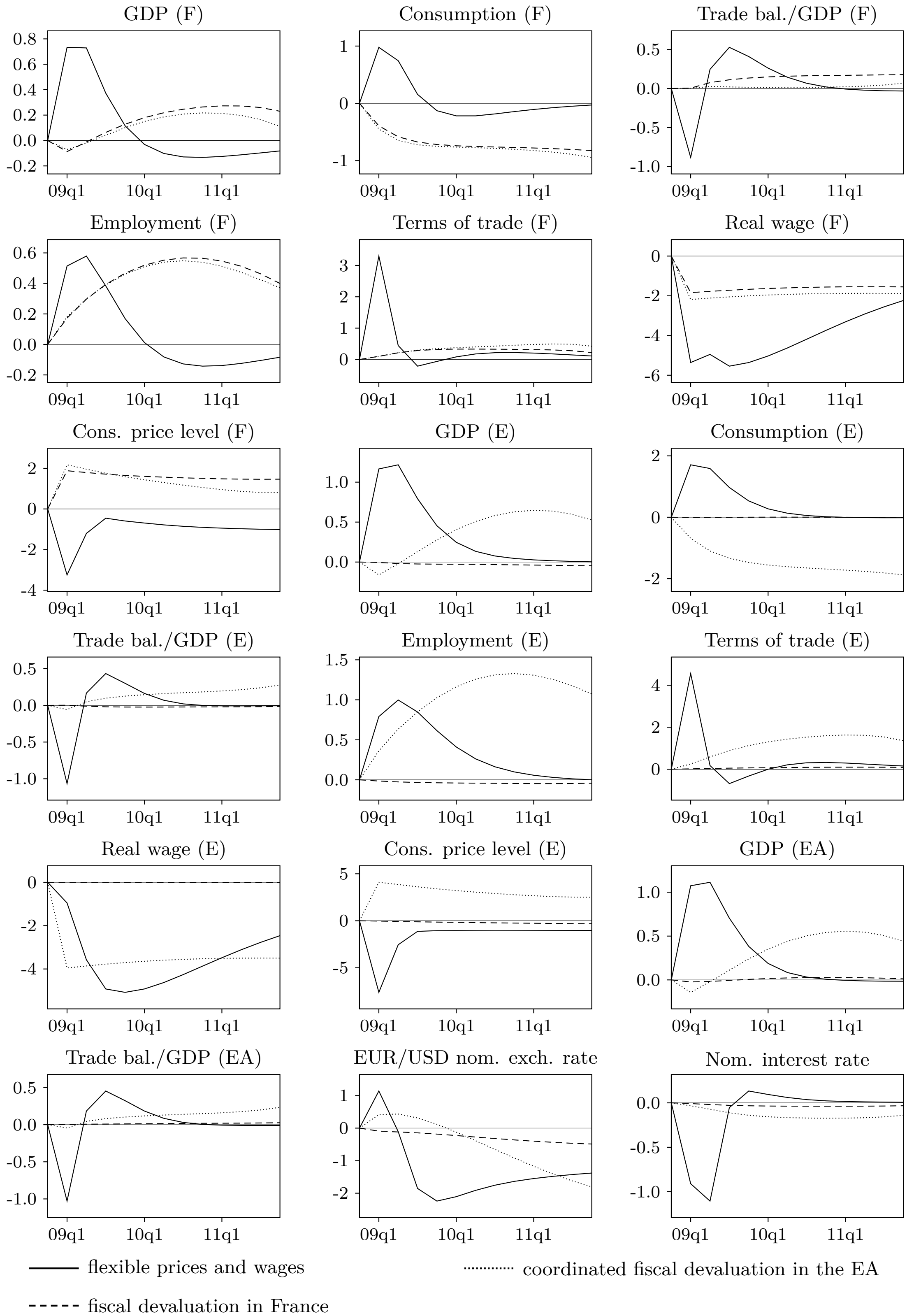
I find simulations that are pretty much similar to those obtained with the complete model.

Table 6.A.1: Social VAT scenarios

	basic model		financial frictions		labor market frict.		complete model	
	unilat.	coord.	unilat.	coord.	unilat.	coord.	unilat.	coord.
τ_F^C	2.10	4.26	0.68	1.50	2.24	2.67	1.79	2.82
τ_F^L	-3.53	-6.54	-1.15	-2.34	-3.49	-4.02	-2.76	-3.98
τ_E^C	0.00	6.70	0.00	2.11	0.00	4.76	0.00	5.18
τ_E^L	0.00	-9.78	0.00	-3.47	0.00	-7.02	0.00	-7.29

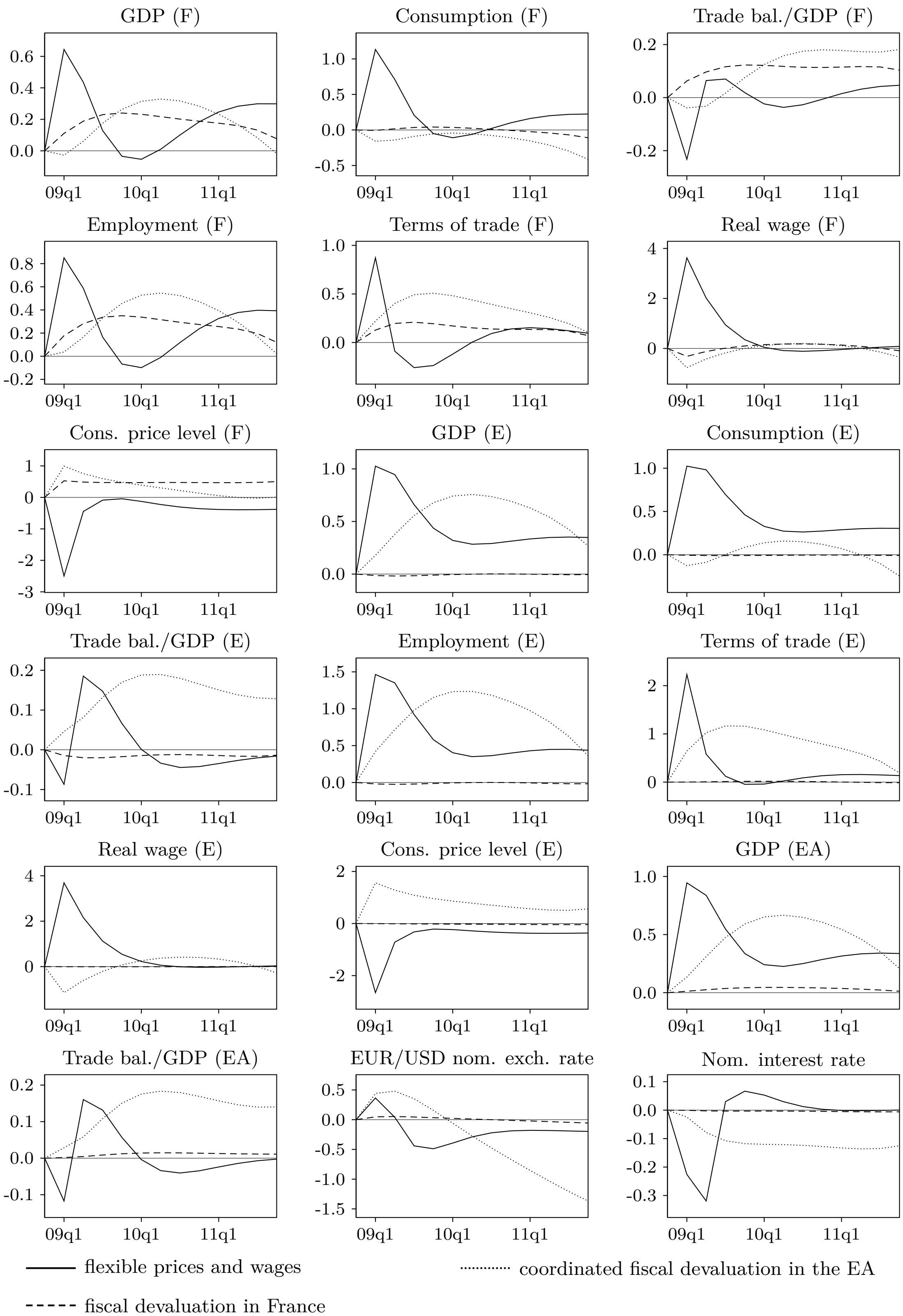
Note: The table reports the deviations of tax rates from their steady state levels in percentage points, which are put into effect between 2009Q1 and 2011Q4.

6.A.2 The model with labor market frictions

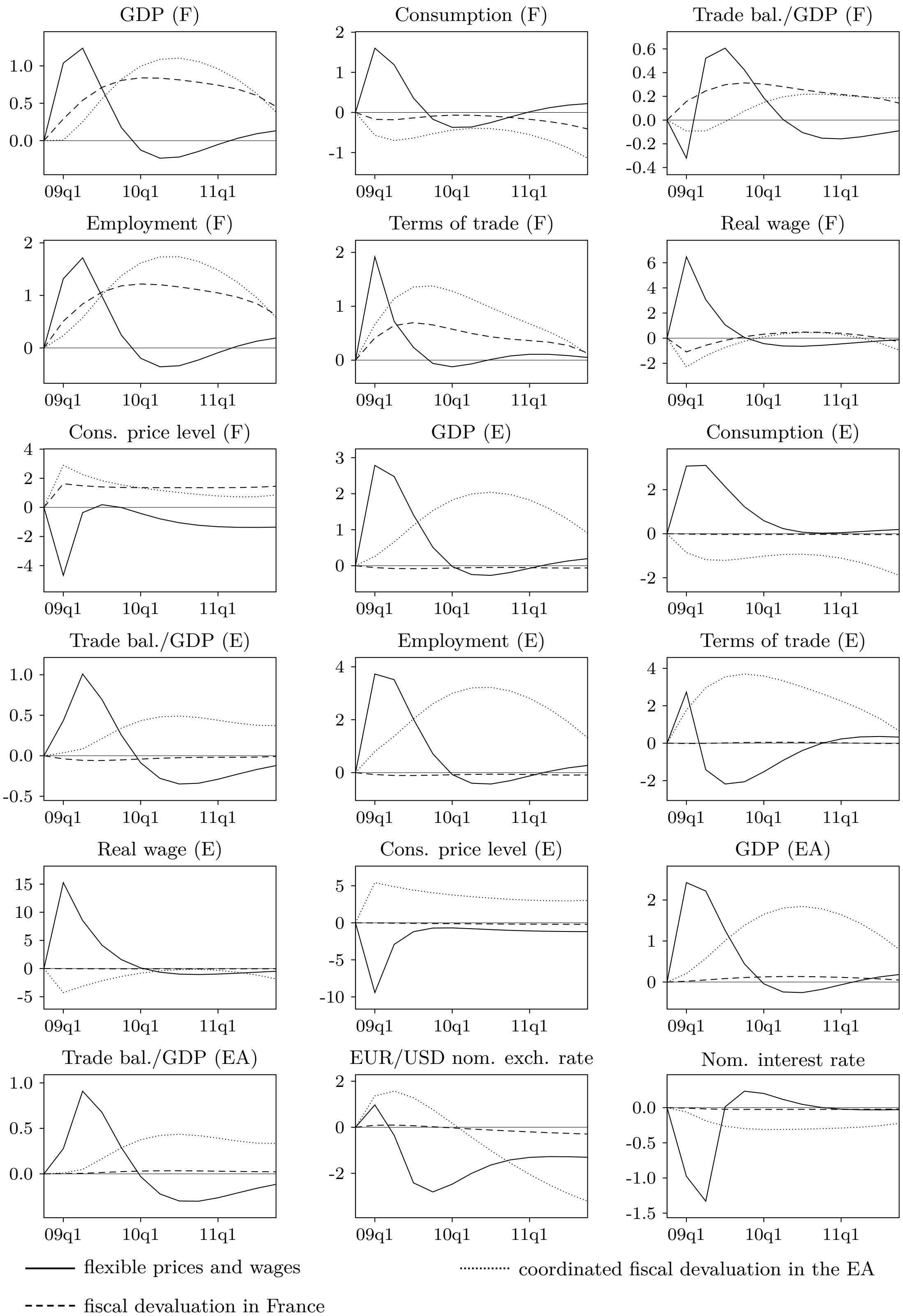


Note: The graphs show deviations from the baseline scenario in % or pp.

6.A.3 The model with financial frictions



6.A.4 The basic model



Chapter 7

Labor force participation and involuntary unemployment

Introduction

Real business cycle models including search and matching frictions in the labor market and Nash-bargained wages along the lines of Mortensen and Pissarides (1994) and Pissarides (1985) have become the standard approach to describe economies with equilibrium unemployment. In contrast to models with Walrasian labor markets, they add an endogenous propagation mechanism, provide an intuitive and more realistic description of the way of functioning of the labor market (see Andolfatto (1996)), and are consistent with many key business cycle facts. A widespread assumption is that all households always participate in the labor market so the participation rate is constant over time.[1] Recently, however, a number of authors have stressed the importance of accounting for cyclical variations in the participation rate: Elsby et al. (2013) reveal that transitions at the participation margin contribute significantly to the cyclical fluctuations in the unemployment rate, and Campolmi and Gnocchi (2011) show that allowing for endogenous fluctuations in the labor force matters for monetary policy analysis. Erceg and Levin (2013) also argue that business cycle factors account for a significant part of the post-2007 decline in the participation rate.

One major obstacle to including endogenous participation in such models has been identified by Tripier (2003), Ravn (2008) and Veracierto (2008), who extend real business cycle models incorporating search frictions:[2] their models tend to predict excessive procyclical movements in the participation rate. The result of this "participation puzzle" is counterfactual labor market dynamics: unemployment tends to be procyclical, the Beveridge curve tends to be upward sloping and the fluctuations in labor market tightness, defined as the ratio of vacancies to unemployment, are too small. Consider an economy populated by identical households, including a very large number of identical members. Households members can be affected to one out of three exclusive discrete

[1] The labor force is defined as the sum of employed and unemployed individuals; it includes people either having a job or actively searching for a job. The participation rate is the ratio of the labor force to the working age population.

[2] Tripier (2003) and Ravn (2008) use the Mortensen and Pissarides (1994) matching framework. Veracierto (2008) uses the Lucas and Prescott (1974) islands framework.

states: employment, unemployment, and non-participation, each of which being associated with a fixed utility value. With perfect risk-sharing, they all consume the same amount of market goods, while a lottery is used in order to choose those who supply labor, as in Rogerson (1988). When time is indivisible, the equilibrium is then the same as in an economy with a single agent whose utility function is linear in labor and in search effort (or in leisure time). Therefore, when the labor market becomes more attractive, it is always advantageous to increase participation until the job finding rate is deteriorated, which means that participation reacts more than employment, so that unemployment moves procyclically. Formally, Ravn (2008) shows that this situation implies a positive log-linear relationship between consumption and tightness; the model cannot replicate the high relative volatility of tightness observed in the data unless households are unrealistically risk-averse. Shimer (2013) answers by showing that rigid wages dampen fluctuations in the participation rate and therefore generate countercyclical unemployment in this framework. Another solution considered in the literature consists in assuming perfect-risk sharing with respect to home production. Individuals contribute to home production differently, depending on their status in the labor market, but then equally share the home-produced good within households. When compared with models where risk-sharing does not cover leisure time, the dynamic reponse of the participation rate is mitigated because preferences are convex with respect to home production. So Ebell (2011) proposes to calibrate the intertemporal elasticity of substitution over the home-produced good to match the relative volatility of the participation rate. This is also the strategy used by Den Haan and Kaltenbrunner (2009), while Christiano et al. (2015) also include a participation adjustment cost in the home production technology.

The microfoundations underlying these approaches are unrealistic along several dimensions. Households members do not decide voluntarily to give up leisure time to search for jobs because they expect to be better-off being employed than staying at home. Instead, households choose the desired size of the labor force and then pick up randomly participants among their members. Moreover, perfect insurance involving lotteries à la Rogerson (1988) generally implies that the employed workers are worse-off than the non-workers.[3] Besides, assuming rigid wages as put forward by Shimer (2013) contradicts empirical findings of Pissarides (2009) and Haefke et al. (2013).[4] Last, perfect risk-sharing with respect to both home production and market consumption is in itself questionable. But it is also doubtful that the response of the aggregate participation rate to cyclical changes in economic conditions is small only because individuals' marginal utility of home-production is decreasing, as suggested by the papers that use this assumption.

In actual economies, common wisdom suggests that the sluggishness of the participation rate results, at least to some extent, from heterogeneity in individual preferences: considering the variety of home tasks in which non-participants may be involved, or the differences in individual wealth, some never consider participating, others are very sensitive to the attractiveness of the labor market,

[3]Although Rogerson and Wright (1988) or Chéron and Langot (2004) use a specific form of non-separable preferences to have "involuntary" unemployment.

[4]They use worker-level data to show that the wages of newly hired workers are very sensitive to aggregate labor market conditions, when the dynamics of the aggregate wage rate reflects the sluggishness of the wages of ongoing matches.

while many workers never exit from the labor force. The dynamics of participation is also likely to be related to unemployment benefits: discouraged workers may decide to remain inside the labor force after loosing their jobs in order to benefit from insurance payments, whereas they would have left it otherwise, and thenafter decide to stay inside the labor force because economic conditions are improving. In this chapter, I propose a dynamic stochastic general equilibrium model with search and matching frictions which assumes that the low volatility and procyclicality of the participation rate result from these two explanations, and where the discrete choice to participate or not is made voluntarily by households members in order to maximize their individual welfare. For that purpose, the model adopts an imperfect insurance scheme as the one proposed by Christiano et al. (2010), where individual participation decisions are described by a principal-agent problem: the “head of a household” is unaware of the household members’ idiosyncratic utilities associated with staying outside of the labor force, but she only observes whether each one is employed or not. Information asymmetry rules out the possibility of a perfect insurance against idiosyncratic labor market outcomes that would level consumption among individuals. Instead, they enter into a contractual agreement that provides incentives to participate in the labor market by endowing the employed workers with a higher level of consumption than the non-workers. After jobs are allocated randomly to participants, the employed workers are better-off than the unemployed ones. Furthermore, the only possible source of heterogeneity in individuals’ equilibrium allocations is their consumption levels, which can take only two values at each date. Hence, the model remains tractable and can be solved with the usual methods for representative agent models.

This chapter is organized as follows. Section 7.1 uses quarterly data of the US economy to establish some stylised facts regarding the joint business cycle dynamics of output, the participation rate, unemployment and vacancies. Section 7.2 uses the partial equilibrium labor market model of Pissarides (2000) in the steady state to analyse analytically how heterogeneity in preferences and unemployment benefits help in lessening the fluctuations of the participation rate. Section 7.3 develops the DSGE model with endogenous labor force participation. It includes sticky prices consistently with the new Keynesian approach, since this class of models has been widely adopted by central banks and policy institutions, especially for monetary policy analysis. In addition, Blanchard and Galí (2010) show that the search and matching and the new Keynesian frameworks can be successfully combined. Finally, section 7.4 proposes a calibration of the model parameters and discusses its empirical properties. The behavior of the participation rate, unemployment and vacancies is found to be very close to the data. The simulations also show that, in the model, heterogeneous preferences are not sufficient to replicate the moderate procyclicality of the participation rate in the absence of unemployment benefits.

7.1 Facts

When observing the U.S. labor maket at business cycle frequency, the following facts emerge:

1. The participation rate is moderately procyclical (its correlation with output is around 0.5) and it varies five times less than output during the business cycle.

2. Unemployment and the number of vacant jobs are strongly negatively correlated, which materializes in a downward-sloping Beveridge curve.

3. Unemployment and vacancies vary approximately 10 times as much as output during the business cycle and are strongly correlated with output (negatively for unemployment and positively for vacancies); hence, the volatility of the tightness ratio is around 20 times higher than the one of output.

To compute the statistics from which these facts are drawn, I use quarterly time series of gross domestic product, total population aged 15 to 64, employment of people aged 15 to 64, the number of participants in the labor force aged 15 to 64, and vacancies in the United States. The source is the OECD database and the sample covers 1977Q1 to 2012Q4, except for the time series for vacancies which includes the number of job openings in the non-farm sector provided by the Job Openings and Labor Turnover Survey of the Bureau of Labor Statistics, and starts in 2001Q1. All series are seasonally adjusted and detrended by the population aged 15 to 64, and unemployment is computed as the difference between participation and employment of people aged 15 to 64. Their cyclical components are obtained by applying a Hodrick-Prescott filter with smoothing parameter set to 1,600 to the logarithm of the per head series. The cyclical component of the labor market tightness ratio in logarithm is computed as the difference between the cyclical components of vacancies and unemployment. The time series used are shown in Figure 7.1.

Figure 7.1: U.S. detrended quarterly data

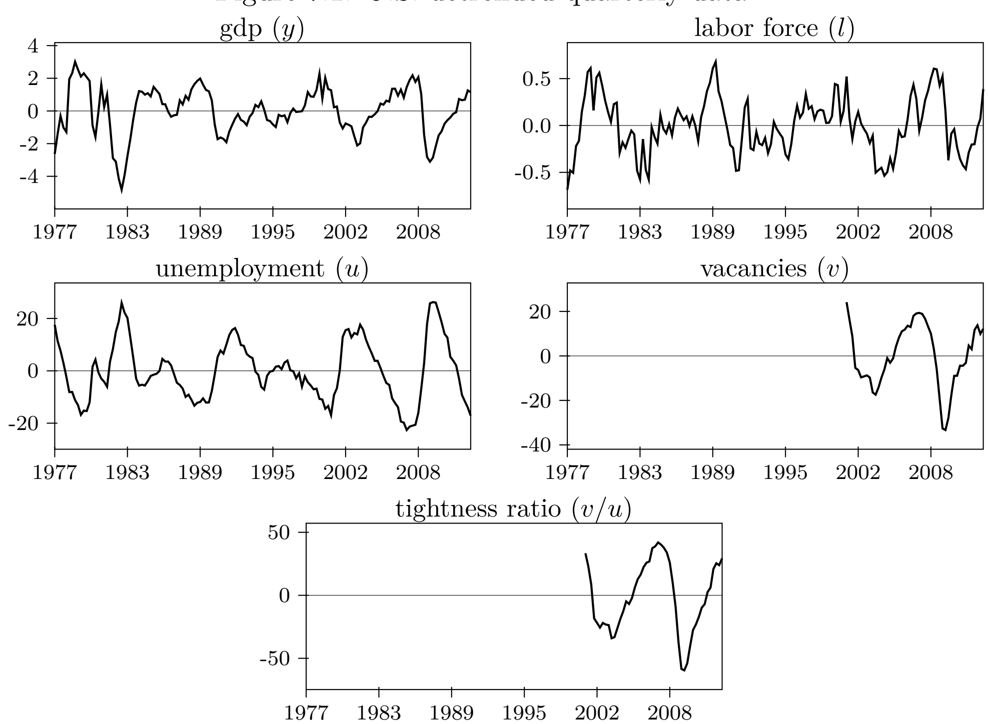


Table 7.1 reports their standard errors and correlation matrix.

Table 7.1: Statistics for quarterly U.S. data

		y	l	u	v	v/u
standard deviation		0.014	0.003	0.113	0.140	0.280
correlation matrix	y	1.000	0.471	-0.866	0.883	0.922
	l	–	1.000	-0.488	0.263	0.311
	u	–	–	1.000	-0.935	-0.984
	v	–	–	–	1.000	0.983
	v/u	–	–	–	–	1.000

Notes: The table reports the standard errors and the correlation matrix of the cyclical components of the logarithm of gross domestic product per head (y), the employment rate (n), the participation rate (l), unemployment per head (u), the number of (non-farm) job openings per head (v) and the tightness ratio (v/u), over the period from 1977Q1 to 2012Q4 (2001Q1 to 2012Q4 for v and v/u).

7.2 Analytical approach

In order to propose a parcimonious model of the participation rate that is consistent with the low volatility of participation compared to output, I start with a simple framework. In this way, the implication of each assumption can be inferred from analytical calculations. First, I find that heterogeneity across households in the value of being outside the labor force is a critical assumption. Next, I show that the difference between the non-market revenues of the unemployed and those of the non-participants gives rise to wealth effects in households' decisions to enter or exit the labor force. This second aspect points towards a calibration strategy of non-market revenues that completes the proposition of Hagedorn and Manovskii (2008): it consists in using a value of unemployed workers' revenues close to the wage rate, as they suggest, but also in setting non-participants' revenues sufficiently below those of unemployed workers.

Specifically, this section builds a partial equilibrium model based on Pissarides (2000) and derives analytically the steady state elasticity of the participation rate with respect to productivity.

7.2.1 Workers and firms' behavior

There is a continuum of individuals of working age that is assumed to be of mass 1. Let u and v denote respectively the unemployment rate and the number of vacancies as a fraction of the labor force, and $\theta \equiv v/u$. The outcome of trade in the labor market is given by a matching function $m(u, v)$, increasing in both its arguments, concave and homogeneous of degree 1. It represents the ratio of the number of job matches taking place per unit of time to the size of the labor force. The vacancy filling rate can be written as a function of θ,

$$q(\theta) = \frac{m(u, v)}{v}.$$

With this notation, the job finding rate is

$$f(\theta) = \frac{m(u,v)}{u} = \theta q(\theta).$$

Job separation occurs at the constant Poisson rate λ. Agents have access to a perfect capital market with a continuously compounded return on assets denoted by r. Let p denote the value of a job's output, c the fixed cost per unit time of holding a vacancy and w the real cost of labor for a firm. The present discounted value of expected profits from an occupied job J satisfies

$$rJ = p - w - \lambda J, \tag{7.2.1}$$

while the present discounted value of expected profits from an occupied job V is

$$rV = -c + q(\theta)(J - V), \tag{7.2.2}$$

where c is a constant vacancy cost per unit of time. In addition, free creation of vacant jobs implies that $V = 0$.

Let z and h be the real non-market incomes received respectively by unemployed workers and by non-participants, which are assumed to be the same for all individuals. z is larger than h on the grounds that $z - h$ includes unemployment benefits. In addition to h, non-participants enjoy a real return from home production which is proportional to aggregate productivity p. Moreover, this return is assumed to vary across individuals, reflecting differences in individual productivities and preferences. I also make the simplifying assumption that unemployed workers spend all their time on job search and cannot take advantage of home production. The worth of being non-participant, denoted by l_0, can hence be written

$$l_0 = ap + h, \tag{7.2.3}$$

where the proportionality coefficient $a \geq 0$ is distributed with cumulative density $H(a)$. The present discounted value of the expected income flow of an unemployed worker and an employed worker, respectively U and W, satisfy

$$rU = z + f(\theta)(W - U), \tag{7.2.4}$$

$$rW = w + \lambda(U - W). \tag{7.2.5}$$

Individuals with leisure worth l_0 participate when

$$l_0 \leq rU. \tag{7.2.6}$$

The size of the labor force denoted by L is therefore $L = H\left(a_0\right)$, where

$$a_0 = \frac{rU - h}{p}$$

is the threshold value of a for which (7.2.6) holds with equality.

Next, the wage rate satisfies a Nash sharing rule of the economic rent associated with a job match, so the wage is

$$w = (1-\beta)z + \beta(p + c\theta), \tag{7.2.7}$$

with $0 \leq \beta \leq 1$.

7.2.2 Elasticity of the participation rate

The equilibrium conditions of the model summarized above yield the elasticity of the participation rate with respect to productivity in the steady state

$$\frac{dL}{dp}\frac{p}{L} = \frac{H'(a_0)}{H(a_0)}\left[\frac{\beta f(\theta)}{r+\lambda+\beta f(\theta)}\left(\frac{(r+\lambda)\eta}{(r+\lambda)(1-\eta)+\beta f(\theta)} + \frac{z}{p}\right) - \frac{z-h}{p}\right], \tag{7.2.8}$$

where η is the elasticity of the job finding rate $f(\theta)$ with respect to θ (details are provided in Appendix 7.A).

The expression of elasticity (7.2.8) highlights the key assumptions that make small variations in participation possible, which are the degree of heterogeneity of households' preferences, and the difference between the value of unemployment and the value of non-participation.

Heterogeneity

The first key parameter is the degree of heterogeneity across individuals in the neighborhood of the marginal participant in the labor force, i.e. the individual whose leisure worth as a non-participant is equal to the threshold value rU. It is reflected by the term $H'(a_0)/H(a_0)$ in equation (7.2.8): when productivity p varies from p to $p + dp$, the threshold value of a such that $l_0 = rU$ varies from a_0 to $a_0 + da_0$. Then

$$\frac{H'(a_0)}{H(a_0)}|da_0|$$

represents the fraction of the labor force for which the difference between their leisure worth and the value of being unemployed changes sign, and thus who revise their participation decision. A small value of the density $H'(a_0)$ reflects that the number of individuals who have leisure worth close to the threshold is low, so that the response of the participation rate L to changes in productivity is weak.

In this model, heterogeneity is necessary to determine an interior dynamics of the participation rate. Indeed, assuming no heterogeneity among households, as implicitely done by Tripier (2003), Ravn (2008) and Veracierto (2008), corresponds to a constant and given for all households in the model. If free entry in the labor market is possible, the coexistence of participants and non-participants in equilibrium implies that

$$ap + h = rU, \tag{7.2.9}$$

and the model does not determine the size of the labor force. When p changes, the equilibrium condition (7.2.9) above is disrupted and all participants either enter or leave simultaneously the labor force. If the model instead assumes that the equilibrium condition (7.2.9) always holds, as it is the case in the aforementioned papers, it constrains the dynamics of tightness according to

$$\theta = \frac{1-\beta}{\beta}\frac{ap-(z-h)}{c}. \tag{7.2.10}$$

When $z = h$, this equation is a formulation of Ravn (2008)'s consumption-tightness puzzle, except that wealth is measured by a linear function of productivity p rather than by the inverse of households' marginal utility of consumption as in a standard DSGE model. It implies that $\log\theta$ and $\log p$ have the same volatility. When h is not equal to z however, the tightness ratio may vary much more than p, since its elasticity with respect to productivity resulting from equation (7.2.10) is

$$\frac{d\theta}{dp}\frac{p}{\theta} = \frac{1}{1-\frac{z-h}{ap}}. \tag{7.2.11}$$

The values of non-market acitivities

The difference between the revenues of the unemployed and the ones of the non-participants $z-h$, which includes unemployment benefits, has two contrasted effects on participation. On the one hand, it increases the steady state level of L (see equation (7.A.28) in Appendix 7.A). On the other hand, it contributes negatively to the elasticity of participation with respect to productivity, as productivity gains raise the worth enjoyed by non-participants relatively to the extra revenue $z-h$ that is immediately earned by individuals who decide to participate in the labor market. Put differently, unemployment benefits, possibly amongst other revenues, provide an additional motivation to participate in the labor market but amplify countercyclical wealth effects on labor supply.

With the same model, Hagedorn and Manovskii (2008) show that setting the revenue of the unemployed z to high values, that is using small values of $p-z$, amplifies the response of employment to productivity. Indeed, the elasticiy of tightness with respect to productivity is

$$\frac{d\theta}{dp}\frac{p}{\theta} = \frac{p}{p-z}\frac{r+\lambda+\beta\theta q(\theta)}{(r+\lambda)(1-\eta)+\beta\theta q(\theta)},$$

where $p-z$ is in the denominator. Yet, their model only consider two states: employment and non-employment. Here, I also consider the non-particiaption state, with a different revenue h, and the mechanism I put forward for non-participation to unemployment transitions is the same as the one that they highlight for unemployment to employment transitions. Whereas decreasing $p-z$ increases the volatility of employment, in a similar fashion, decreasing $z-h$ increases the volatility of the labor force. In response to Shimer (2005) who questions the lack of volatility of unemployment implied by the textbook search and matching model, these authors suggest to calibrate $p-z$ to small values. The corollary proposition stemming from the inclusion of the non-participation state in the model is to calibrate $z-h$ to a sufficiently large value, in order to replicate the sluggishness of the participation rate.

Other effects

The workers' bargaining power β has also an impact on the elasticity of participation with respect to productivity, because the expected gains from finding a job depend on the fraction of the rent associated with job creation that is taken by workers. However, it is easy to see that it is much smaller, as it is for the elasticity of tightness with respect to productivity.

The positive contribution of z/p in equation (7.2.8) can be interpreted as follows: z represents the worth of unemployment per se, i.e. for jobless workers with no job opportunity. When the job finding rate improves, the probability to remain jobless for a new participant decreases relatively to the one of finding a job. Therefore, the negative wealth effects on labor supply due to the presence of z described above are mitigated.[5]

Last, the first positive term inside the brackets reflects the fact that productivity gains improve wages and the job finding rate, making participation in the labor market more attractive.

Concluding remarks

In sum, without heterogeneity in households preferences, the dynamics of the participation rate and of the tightness ratio are strongly constrained in the model. With heterogeneity, the magnitude of the participation rate's response to changes in productivity is very sensitive to the assumed distribution of preferences across households. Moreover, unemployment benefits add a countercyclical component to the dynamics of the participation rate. These properties are used in the general equilibrium model developed in what follows to replicate the facts identified in section 7.1.

7.3 DSGE Model

7.3.1 Labor market

The number of identical jobs n_t in period t includes the jobs of the previous period minus a constant fraction λ of them that are exogeneously destroyed, plus m_t new jobs. It is

$$n_t = (1-\lambda)n_{t-1} + m_t. \tag{7.3.12}$$

As in Pissarides (1985), job creation is assumed to be well-described by a Cobb-Douglas matching function depending on unemployment and on the number of vacancies v_t posted by the representative firm. Following Blanchard and Galí (2010), the variable that enters the matching function is unemployment "before job creation", that is $L_t - (1-\lambda)n_{t-1}$, where L_t denotes the participation rate, rather than actual unemployment $L_t - n_t$, so

$$m_t = \Upsilon v_t^{\kappa}(L_t - (1-\lambda)n_{t-1})^{1-\kappa}, \tag{7.3.13}$$

[5]This mechanism involves z and not $z-h$ because the effect of loosing h for an individual who decides to participate remains unchanged.

where Υ is a scale parameter of the matching technology. New matches of period t enter immediately in the production of period t, which helps temper the large fluctuations in firms' markup that would result from rigid prices and predetermined employment.

7.3.2 Households

The economy is populated by a large number, normalized to 1, of identical housholds, which can be represented by a representative household. Each household includes a continuum of members with a mass of 1, among which is the "head of the household", referred to as "the principal" or simply "the household" in what follows. Although jobs are likely to continue over time, all workers are hired at the beginning of each period only for one period of time. Alike, all households members may enter in or exit from the labor force at the beginning of each period. The assignment of participants in the labor market to available job positions involves a standard lottery, so they all have the same probability n_t/L_t to find a job in period t.

Housholds members derive utility from consumption of an homogeneous good and from home production, in which they can engage only when they are outside of the labor force.[6] The household members' utilities of home production are different, variable and memoryless over time. For that purpose, I assume that they are subject to a privately observed idiosyncratic shock denoted by i, drawn from a uniform distribution over $[0, 1]$ independantly across individuals and across time, that determines their instantaneous utility as follows:

$$U(i, C) = \begin{cases} \log C + \zeta i^{\sigma} & \text{if out of the labor force,} \\ \log C & \text{if in the labor force,} \end{cases}$$

where C denotes individual consumption, ζ is a scale parameter and σ characterizes the curvature of the utility of home production in the cross section of the household members.[7]

The model adopts the imperfect insurance arrangement against individual labor market outcomes proposed by Christiano et al. (2010), which incites the household members with the lowest utlity of home production to voluntarily participate in the labor force. The principal and the household members agree to allocate consumption in the current period only depending on whether they have a job or not ex post. Hence, two levels of consumption are possible, the one of the employed workers and the one of the others, respectively denoted by C^e ans C^u in real terms, with $C^e > C^u$ and $nC^e + (1 - n)C^u = C$, C being aggregate real consumption of the household. Given values of C_t^e and C_t^u for a given period t, the household members decide to participate in the labor force when they draw a value of $i \in [0, 1]$ such that their expected utility as participants exceeds their

[6] As in section 7.2, the unemployed workers spend all their time searching for a job.

[7] By analogy with the partial equilibrium model of section 7.2, this corresponds to a distribution of the utility of home production across the household members with CDF $H(a) = (a/\zeta)^{1/\sigma}$ for $a \in [0, \zeta]$. Yet it does not depend on aggregate productivity in the general equilibrium model with convex preferences.

expected utility as non-participants, that is when

$$\frac{n_t}{L_t}\log C_t^e + \left(1-\frac{n_t}{L_t}\right)\log C_t^u \geq \log C_t^u + \zeta i^\sigma.$$

Any individual who has drawn i lower than the threshold value for which this condition holds with equality is a participant, so this threshold value is the size of the labor force L_t. Hence, the principal can induce any desired value of L_t by setting C_t^e and C_t^u such that

$$\frac{n_t}{L_t}\log\frac{C_t^e}{C_t^u} = \zeta L_t^\sigma. \tag{7.3.14}$$

Of course, this arrangement requires that the household members pool all their revenues, which are described in what follows. Labor is remunerated at a unique real wage w_t per period and per employee. The household also receives a constant non-market revenue z (in real terms) per unemployed worker and a constant non-market revenue h (in real terms) per out-of-the-labor-force individual, which are both introduced as benefits paid by the government. She has access to financial markets to trade nominal assets b_t, remunerated at an interest factor r_t in period $t+1$. All households also own equal shares in monopolistic firms so they are all granted dividends for div_t in real terms. Last, the government pays them evenly a real lump-sum transfer T_t. The real budget constraint of the household is hence

$$C_t + \frac{b_t}{P_t} \leq b_{t-1}\frac{r_{t-1}}{P_t} + n_t w_t + (L_t - n_t)z + (1-L_t)h + div_t + T_t,$$

where P_t is the price level of the final good in period t.

The household does not internalize the job matching technology postulated in section (7.3.1). She rather takes as given the job finding probability, defined as the ratio of the new matches of the period to unemployment just before the matching process

$$f_t \equiv \frac{m_t}{L_t-(1-\lambda)n_{t-1}},$$

so the law of motion of employment that she observes is

$$n_t = (1-\lambda)n_{t-1} + f_t(L_t-(1-\lambda)n_{t-1}).$$

The objective function of the household is the sum of her members' utilities, which is equal (up to a constant) to:

$$U_t = \log C_t^u + \frac{\zeta\sigma}{1+\sigma}L_t^{1+\sigma}.$$

Appendix 7.B provides a detailed derivation of the household's program. Her optimal behavior is described by

$$1 = \delta \mathrm{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\frac{r_t}{\pi_{t+1}}\right], \tag{7.3.15}$$

and

$$z - h + c\frac{\beta}{1-\beta}\frac{f_t}{q_t} = \left(\frac{1}{\Lambda_t} + (1+\sigma)(1-n_t)(C_t^e - C_t^u)\right)\zeta L_t^\sigma, \tag{7.3.16}$$

where δ is the discount factor that characterizes households' preference for the present, π_t is the aggregate inflation factor, $\Lambda_t = 1/C_t$ is the marginal utility of aggregate consumption and q_t is the vacancy filling rate defined in section 7.3.3. Equation (7.3.15) is a standard Euler condition, which determines the optimal intertemporal allocation of savings and consumption. Equation (7.3.16) determines the optimal size of the labor force, which is such that the marginal aggregate utility costs from higher participation compensate the marginal gains. The utility costs of higher participation result from the related decrease in home production but also from the reallocation of consumption in favor of the employed workers, aimed at increasing the incentives for the household members to enter in the labor market. This reallocation has a negative impact on aggregate utility because the non-employed have a higher marginal utility of consumption than the employed workers. Conversely, the marginal gains from increasing the participation rate are represented by the left side of equation (7.3.16). They result first from additional unemployment benefits, and, second, from the new jobs that are created thanks to a higher search effort – as measured by $L_t - (1-\lambda)n_{t-1}$ in the matching technology. At the bargained wage rate, this marginal gain is a function of the ratio of f_t to q_t.

Regarding the dynamics of the model, the presence of the positive constant $z - h$ to the left side and, when σ is high, of L_t^σ to the right side in equation (7.3.16) implies that the ratio of the job finding rate f_t to the vacancy filling rate q_t and hence the tightness ratio may vary much more than the inverse of the marginal utility of aggregate consumption.[8]

7.3.3 Firms

A representative firm uses labor input n_t to produce $p_t n_t$ of an homogeneous good, where p_t represents labor productivity and is exogenous. This good is sold at a relative price x_t to a continuum of monopolistic retailers. In order to post a vacancy, the firm has to expense a fixed cost c. As the family, she does not internalize the job matching technology, but takes as given the probability of filling a vacant position

$$q_t \equiv \frac{m_t}{v_t},$$

and observes the law of motion of employment

$$n_t = (1-\lambda)n_{t-1} + q_t v_t.$$

[8]With the "new Keynesian specification" of the matching technology (7.3.13), tightness is related to the ratio of f_t to q_t by

$$\theta_t = \frac{v_t}{L_t - n_t} = \frac{L_t - (1-\lambda)n_{t-1}}{L_t - n_t}\frac{f_t}{q_t}.$$

The firm chooses the number of vacancies v_t to post in order to maximize the expected flow of her discounted future profits subject to the law of motion of employment, as summarized by the definition of her value function

$$W_t(n_{t-1}) = \max_{v_t} \left\{ x_t p_t n_t - w_t n_t - c v_t + \delta \mathrm{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} W_{t+1}(n_t) \right] \right\}$$
$$\text{s.t.}$$
$$n_t \leq (1-\lambda) n_{t-1} + q_t v_t,$$

where the presence of the family's marginal utility of aggregate consumption Λ_t reflects the fact that the firm values her profits in terms of welfare gains for shareholders. This program yields the optimality condition

$$\frac{c}{q_t} = x_t p_t - w_t + \delta(1-\lambda)\mathrm{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{c}{q_{t+1}} \right]. \tag{7.3.17}$$

Monopolistic retailers indexed by $j \in [0,1]$ differentiate the fraction of the homogenous good that they purchase from the firm and set their prices under a lottery à la Calvo: they can only re-optimize their prices with a constant probability $1-\xi$ in every period. With probability ξ, their prices evolve automatically at a rate given by a convex combination of past aggregate inflation π_{t-1} (with weight ι) and long run inflation $\bar{\pi}$ (with weight $1-\iota$). Retailers sell their differentiated products $y_t(j)$, $j \in [0,1]$, to a competitive distributor who produces a quantity Y_t of the final good using a CES technology

$$Y_t = \left(\int_0^1 y_t(j)^{\frac{s-1}{s}} dj \right)^{\frac{s}{s-1}},$$

where the parameter s is the elasticity of substitution. The distributor chooses optimally the demand $y_t(j)$ addressed to any retailer $j \in [0,1]$. The inflation rate follows

$$1 = (1-\xi)\tilde{P}_t^{1-s} + \xi \left(\frac{\pi_{t-1}^{\iota} \bar{\pi}^{1-\iota}}{\pi_t} \right)^{1-s},$$

where $\tilde{P}_t$ is the relative price chosen by the $1-\xi$ retailers that are allowed to reoptimize in period t. Because retailers are all identical ex ante and the price setting problem is forward looking, they all choose the same optimal price. It is governed by

$$\tilde{P}_t = \frac{s}{s-1} \frac{H_{1t}}{H_{2t}} \quad \text{with} \quad \begin{aligned} H_{1t} &= x_t Y_t + \delta \xi \tfrac{\Lambda_{t+1}}{\Lambda_t} \left(\tfrac{\pi_{t+1}}{\pi_t^{\iota} \bar{\pi}^{1-\iota}} \right)^{s} H_{1t+1}, \\ H_{2t} &= Y_t + \delta \xi \tfrac{\Lambda_{t+1}}{\Lambda_t} \left(\tfrac{\pi_{t+1}}{\pi_t^{\iota} \bar{\pi}^{1-\iota}} \right)^{s-1} H_{2t+1}. \end{aligned}$$

7.3.4 Wage bargaining

The wage rate is flexible: in every period t, the representative household and the representative firm bargain over the current wage rate w_t in order to share the surplus created by successful matches. The negotiated wage is assumed to be the one that maximizes the convex combination of the marginal value of employment for the firm and for the household

$$\left(\frac{\partial W_t}{\partial n_{t-1}}\right)^{1-\beta}\left(\frac{1}{\Lambda_t}\frac{\partial V_t}{\partial n_{t-1}}\right)^{\beta},$$

where V_t denotes the value function of the household. The solution of this Nash bargaining problem is

$$(1-\beta)\frac{\partial V_t}{\partial n_{t-1}} = \beta\Lambda_t(1-f_t)\frac{\partial W_t}{\partial n_{t-1}},$$

which yields the wage equation (see Appendix 7.C for details)

$$w_t = (1-\beta)\left(z + C_t^e - C_t^u - C_t^e \log\frac{C_t^e}{C_t^u}\right) + \beta\left(x_t p_t + c\delta(1-\lambda)\mathrm{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\frac{f_{t+1}}{q_{t+1}}\right]\right). \tag{7.3.18}$$

The bargained wage is a weighted average of, on the one hand, the contribution of an additional worker to the sales of the representative firm plus the vacancy posting expenses that are expected to be saved in the next period thanks to a new hire, and, on the other hand, the household's outside options. The latter term obviously includes the non-market revenue of the unemployed z, and the effects on the household's aggregate utility of the reallocation of consumption between her employed and non-employed members that follows job creation.

7.3.5 Monetary policy and general equilibrium

The interest rate is decided by the monetary authority according to the standard Taylor rule

$$r_t = r_{t-1}^{\rho}\left(\bar{r}\left(\frac{\pi_t}{\bar{\pi}}\right)^{r_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{r_y}\right)^{1-\rho}\varepsilon_{r,t}, \tag{7.3.19}$$

including an exogenous disturbance ε_r. In this equation, $\bar{Y}$ is the steady state level of output Y_t.

The final demand of goods is assumed to include an additional term, which stands for the residual demand items of the actual economy that are not described in the model (foreign trade, investment, government spending and changes in inventories). It is modelled as government spending and its counterpart includes transfers T_t and non-employment benefits $(L_t - n_t)z + (1 - L_t)h$ paid to households, ensuring a balanced budget in every period. Its real value is assumed to be constant over time and is denoted by G. Therefore market clearing implies

$$Y_t = C_t + G + cv_t. \tag{7.3.20}$$

7.4 Quantitative analysis

7.4.1 Calibration

The steady state levels of the employment rate and of the participation rate are respectively set to 0.70 and 0.75, which correspond to their observed averages over the period 1977-2012 in the United States, computed with the data described in section 7.1. The average share of private consumption in gross domestic product is 67.8% in the data, so $G/\bar{Y}$ is set to 0.319 considering that the real gross domestic product in the model includes production Y_t less recruiting expenses cv_t, and that the latter are assumed to represent 1% of production in the steady state (as in Andolfatto (1996)). The monetary authority's inflation target is set to 2.5% a year (which is the average growth rate of the private consumption deflator over 1982-2012) and the discount factor δ is calibrated to 0.99 which implies a steady state real return on financial assets of 4.1% a year. Labor productivity is normalized to 1 in the steady state.

Usual parameters are taken from the search and matching and the new Keynesian literature. Following Andolfatto (1996), the steady state probability that a vacant position gets filled during a quarter is set to $\bar{q} = 0.90$. Regarding the job separation rate, I exploit more recent results reported by Shimer (2012). Using a model which allows for the possibility that a worker exits the labor force, he finds that the sum of monthly employment-to-unemployment and employment-to-inactivity flows averages 5% of employment from 1967 to 2010. More precisely, on the basis of the corresponding time series, the employment-to-unemployment transition rate averages 0.02 while the employment-to-inactivity transition rate averages 0.03.[9] Consistently, the quarterly job separation rate λ is set to 0.15.[10] Given the considered calibration of the steady state employment and participation rates – which yields an average unemployment rate of 6.7% that is also consistent with the BLS time series used by Shimer (2012) – and the calibration of λ, the restriction that job destruction is equal to job creation in the long run implies that the steady state quarterly job finding rate $\bar{f}$ is 0.68. When the arrival of successful matches for unemployed workers in the steady state is modelled as a Poisson process, the corresponding monthly job finding rate is $1-(1-0.68)^{1/3} = 0.32$, which matches the average monthly unemployment-to-employment transition rate computed with Shimer's time series from 1967Q2 to 2007Q2. Next, the elasticity of the matching function with respect to the number of vacancies κ and firms' bargaining power $1-\beta$ are equal, so that the Hosios (1990) conditions are satisfied and the equilibrium is socially efficient. They are set to 0.5 as in Mortensen and Pissarides (1994). The assumption that recruiting expenses represent 1% of output in the steady state implies that $c = 0.06$, and long run restrictions to the matching technology that $\Upsilon = 0.781$. The Calvo probability ξ is assumed to be 0.75, which implies that monopolist retailers can reoptimize their prices approximately once a year on average. The elasticity of substitution s is set to 6, which corresponds to a steady state markup on production prices of 20% for retailers. The degree of price indexation to past inflation ι is 0.5, and the parameters of the monetary policy rule are close to standard values: the reaction to inflation and to the output gap are $r_\pi = 1.5$ and $r_y = 0.125$ respectively. The coefficient on the lagged interest rate ρ is 0.7.

The steady state consumption replacement ratio C^u/C^e and the scale parameter of non-participants' utility of home production ζ are chosen such that the optimal participation rate equation (7.3.16) and the individual participation condition (7.3.14) are both verified in the long run. The real income of the unemployed z comes from the wage equation (7.3.18) in the steady state. In a similar way to

[9] This data is available online at https://sites.google.com/site/robertshimer/research/flows; time series are quarterly averages of monthly rates, covering the period from 1967Q2 to 2007Q2.

[10] The exact quarterly probability corresponding to a monthly job separation probability of 0.05 is $1-(1-0.05)^3 = 0.143$.

Hagedorn and Manovskii (2008), the revenues of the non-employed household members are not calibrated on the basis of actual insurance replacement rates or government benefits; here, z and h may include other non-market returns, such as family revenues, or pseudo-revenues reflecting the access to public infrastructure and services. Consistently, I do not calibrate the difference between z and h so that it represents 40% of the steady state wage rate, which is the value usually considered in the literature for unemployment benefits only. Instead, $z - h$ and the curvature of non-participants' utility of home production in the cross-section of households members σ are respectively set to 0.2038 (i.e. 24.8% of the steady state wage rate $\bar{w}$) and 7.8396 in order to match the observed volatility and correlation with output of the participation rate. The calibration of σ to replicate the dynamics of the participation rate is in line with the approach put forward by Ebell (2011) because the aggregate utility of non-participants with heterogeneity is formally similar to a representative agent's concave utility function of home goods consumption. With these values of $z - h$ and σ, I find $\zeta = 2.8301$, $z = 0.7975$ and $C^u/C^e = 0.7277$. The high value found for z (it represents 96.9% of the steady state real wage rate) directly results from vacancy costs – and therefore the firm's accounting profits – being small relatively to sales, and is consistent with the strategy used by Hagedorn and Manovskii (2008) to have large fluctuations in unemployment and vacancies. The steady state value of the consumption replacement ratio C^u/C^e is below empirical estimates of the drop in food consumption associated with becoming unemployed: according to Gruber (1997), this fall would range between 0 and a bit more than 20%, depending on the generosity of the unemployment insurance system; Chetty and Looney (2007) report a value of 10%. But as people consuming C^u in the model may also include long-term non-workers, it is sensible to use a value that is slightly lower than these estimates.

7.4.2 Exogenous processes

The model assumes two exogenous sources of fluctuations: labor productivity and monetary policy. The latter shock may include non conventional monetary policy decisions as well as all disturbances affecting the transmission of monetary policy decisions to private agents. The logarithm of labor productivity p_t is modelled as an AR(1) process with persistence 0.7 and standard deviation 0.0068. The monetary policy shock is a white noise with standard deviation 0.0062.

7.4.3 Dynamic properties

In order to assess the dynamic properties of the model, it is simulated assuming rational expectations and using a standard first-order perturbation method (see Judd (1998)). Table 7.1 reports its asymptotic properties. Since the model is very basic along several dimensions, including the absence of capital, the absence of real rigidities outside the labor market and the small number of exogenous shocks, matching quarterly autocorrelations is considered as outside the scope of this work, and these moments are not reported.[11]

[11] In RBC models with search and marching frictions as Shimer (2005), the persistence of exogenous technology is almost entirely passed to endogenous variables, whereas basic new Keynesian models generate lower persistence.

Table 7.1: Second order moments in the model

		y	l	u	v	v/u
standard deviation		0.014	0.003	0.163	0.156	0.300
correlation matrix	y	1.000	0.471	-0.823	0.626	0.772
	l	–	1.000	-0.860	0.859	0.913
	u	–	–	1.000	-0.772	-0.944
	v	–	–	–	1.000	0.938
	v/u	–	–	–	–	1.000

Notes: The table reports the theoretical standard errors and correlation matrix of the deviations of $\log(Y_t - cv_t)$, $\log L_t$, $\log(L_t - n_t)$, $\log v_t$ and $\log(\frac{v_t}{L_t - n_t})$ from their steady state levels implied by the model.

Not only the model is able to replicate exactly the volatility of the participation rate and its correlation coefficient with output, but it also performs remarkably well regarding the volatility of the tightness ratio. Consistently with the stylized facts presented in section 7.1, it also predicts volatilities of unemployment and vacancies that are around 10 times higher than output, strongly countercyclical unemployment (the correlation coefficient is -0.823 in the model versus -0.866 is the data) and a negatively sloped Beveridge curve. Yet, the model understates somewhat the correlation coefficients between unemployment and vacancies (with -0.772 versus -0.935 in the data) and between vacancies and output (with 0.626 versus 0.883 in the data). Appendix 7.D shows the impulse response functions simulated with the model.

7.4.4 Alternative calibrations

In order to investigate the dynamic properties implied by the assumptions of heterogeneous preferences and of unemployment benefits, the model is simulated first with $\sigma = 0$, and then with $z = h$. The first case ($\sigma = 0$) amounts to assuming identical preferences. The household members are then perfectly insured since the consumption arrangement developed above requires that preferences are different; their program is detailed in Appendix 7.E. With $\sigma = 0$, I consider different calibrations of $z - h$, aside from the baseline value $z - h = 0.2038$. First, it is chosen to minimize the volatility of the participation rate, and, second, to match the volatility of the tightness ratio in the data. Last, it is assumed to be the highest possible, that is $h = 0$.

The second case ($z = h$) is the absence of extra income for unemployed workers as compared to non-participants, or, in short, no unemployment benefits. I consider two alternative calibrations of σ, which are $\sigma = 7.8396$ as in the baseline and $\sigma = 34.7398$ to match the volatility of the participation rate in the data.

Finally, I set $h = z$ with $\sigma = 0$. This latter situation roughly corresponds to the models proposed by Tripier (2003), Ravn (2008) or Veracierto (2008), since both heterogeneity in preferences and unemployment benefits are cancelled. Table 7.2 reports a subset of the theoretical second order moments computed for all these cases (the full correlation matrix and standard deviations are provided in Tables 7.F.1 to 7.F.7 in Appendix 7.F).

Table 7.2: Comparison with benchmark models

	std(l)	corr(l,y)	std(v/u)	corr(u,v)
Data	0.003	0.471	0.280	-0.935
$\sigma = 7.840$ and $z - h = 0.204$	0.003	0.471	0.300	-0.772
No heterogeneity ($\sigma = 0$):				
$z - h = 0.204$	0.012	0.162	0.221	-0.313
$z - h = 0.248$ (minimizes std(l))	0.012	0.017	0.252	-0.420
$z - h = 0.290$ (matches std(v/u))	0.012	-0.113	0.280	-0.497
$z - h = 0.774$ ($h = 0$)	0.020	-0.795	0.553	-0.819
No unemployment benefits ($z - h = 0$):				
$\sigma = 7.840$	0.009	0.699	0.188	-0.436
$\sigma = 34.740$ (matches std(l))	0.003	0.678	0.294	-0.765
No heterogeneity nor u.b. ($\sigma = z - h = 0$)	0.017	0.735	0.051	0.854

Notes: The table reports the theoretical standard errors of the participation rate and the tightness ratio, and the theoretical correlation coefficients of participation and output on the one hand, unemployment and vacancies on the other hand. All variables are log-deviations from steady state levels.

With identical preferences, the model overestimates the volatility of the participation rate whatever the value of $z - h$. With h sufficiently lower than z however, it can replicate the volatility of the tightness ratio, consistently with the result found for the static model of section 7.2 when preferences are identical (see equation (7.2.11)). It can also generate a negative correlation between unemployment and vacancies. But it is at the expense of countercyclical movements in the participation rate. With heterogeneous preferences but no unemployment benefits, the model has good properties with respect to the volatility of both the participation rate and the tightness ratio when σ is increased to more than 30. However, it overestimates significantly the correlation of the participation rate with output, whatever the value of σ. When $h = z$ and $\sigma = 0$, as expected, the participation rate in the model is more procyclical than it is in the data, the volatility of the tightness ratio is strongly underestimated and the Beveridge curve is upward sloping (the correlation coefficient between u and v is 0.854). In sum, heterogeneity in preferences is needed to replicate the low volatility of the participation rate in the model. Yet, assuming that the unemployed receive a higher revenue than the non-participants is useful to match simultaneously the magnitude of the fluctuations in the participation rate and the moderate correlation between participation and output.

Concluding remarks

This chapter contributes to the literature in several ways. First, it proposes credible foundations to business cycle models with endogenous labor force participation: the sluggishness of the participation rate results from intuitively plausible explanations, that are heterogeneity in households preferences and unemployment benefits, and individuals decide to participate voluntarily in the labor market because they expect to be better-off being employed. Heterogeneity in preferences introduces concavity in the aggregate utility of non-participation. In this respect, it is formally equivalent to assuming that identical households with convex preferences equally share home production. Regarding unemployment benefits, I transpose Hagedorn and Manovskii (2008)'s response to the "Shimer puzzle", based on an adequate calibration of non-market revenues, into the "participation puzzle".[12]

Next, I successfully incorporate these mechanisms into a new Keynesian framework: the proposed model has very promising dynamic properties. Moreover, with respect to previous business cycle models achieving the objective of small fluctuations in labor force participation, I show that both a concave aggregate utility function of home production and unemployment benefits are needed to replicate simultaneously the volatility and the correlation with output of the participation rate.

Finally, the assumption of memoryless idiosyncratic shocks to preferences, based on previous works of Christiano et al. (2010), is a relatively simple modeling device to deal with heterogeneity without using the representative agent paradigm. Models such as the one proposed in this chapter can be solved and simulated numerically using a standard perturbation method while keeping a limited degree of heterogeneity. In particular, this framework could be embedded into a larger DSGE model with many frictions and a higher number of shocks, which would be able to fit the data along more dimensions, including persistence.

[12] Shimer (2005) argues that labor demand tends to be insufficiently responsive in search and matching models.

Appendix

7.A Derivation of the elasticity of the participation rate in the analytical approach

Equations (7.2.1) can be written as

$$J = \frac{p - w}{r + \lambda}, \tag{7.A.21}$$

and free creation of vacant jobs yields

$$J = \frac{c}{q(\theta)}, \tag{7.A.22}$$

so

$$p - w - \frac{(r + \lambda)c}{q(\theta)} = 0. \tag{7.A.23}$$

Equations (7.2.4), (7.2.5) and (7.2.7) imply

$$\left(r + \lambda + \theta q(\theta)\right)(W - U) = w - z = \beta(p - z) + \beta c\theta.$$

Then

$$\begin{aligned}
\left(r + \lambda + \beta\theta q(\theta)\right)(W - U) &= \left(r + \lambda + \theta q(\theta)\right)(W - U) - (1 - \beta)\theta q(\theta)(W - U) \\
&= \beta(p - z) + \beta c\theta - (1 - \beta)\theta q(\theta)(W - U).
\end{aligned}$$

Using the Nash bargaining condition and (7.A.22),

$$W - U = \frac{\beta}{1 - \beta} J = \frac{\beta}{1 - \beta} \frac{c}{q(\theta)}, \tag{7.A.24}$$

this equation becomes

$$\left(r + \lambda + \beta\theta q(\theta)\right)(W - U) = \beta(p - z) + \beta c\theta - \beta c\theta = \beta(p - z). \tag{7.A.25}$$

Substituting into equation (7.2.4)

$$rU = z + \frac{\beta\theta q(\theta)(p - z)}{r + \lambda + \beta\theta q(\theta)} = \omega(\theta)p + (1 - \omega(\theta))z, \tag{7.A.26}$$

with

$$\omega(\theta) = \frac{\beta\theta q(\theta)}{r + \lambda + \beta\theta q(\theta)}. \tag{7.A.27}$$

Hence, the participation rate is

$$L = H(a_0) = H\left(\frac{rU - h}{p}\right) = H\left(\omega(\theta)\frac{p - z}{p} + \frac{z - h}{p}\right). \tag{7.A.28}$$

Differentiating this equation yields

$$dL = \frac{1}{p}H'(a_0)\left(\omega'(\theta)(p - z)\,d\theta - \frac{(1 - \omega(\theta))z - h}{p}dp\right),$$

$$\frac{dL}{dp} = \frac{1}{p}H'(a_0)\left(\omega'(\theta)(p - z)\frac{d\theta}{dp} + \omega(\theta)\frac{z}{p} - \frac{z - h}{p}\right), \tag{7.A.29}$$

with

$$\omega'(\theta) = \frac{(r + \lambda)\beta(\theta q'(\theta) + q(\theta))}{(r + \lambda + \beta\theta q(\theta))^2}.$$

Let

$$\eta \equiv \frac{\partial \log \theta q(\theta)}{\partial \log \theta} = 1 + \theta\frac{q'(\theta)}{q(\theta)}$$

be the elasticity of the job finding rate with respect to tightness. Then

$$\omega'(\theta) = \frac{(r + \lambda)\eta\beta q(\theta)}{(r + \lambda + \beta\theta q(\theta))^2}.$$

Deriving the elasticity of participation with respect to productivity from (7.A.29) requires calculating the elasticity of tightness with respect to productivity, as in Hagedorn and Manovskii (2008): equations (7.A.23) and (7.2.7) imply that

$$(1 - \beta)(p - z) - \beta c\theta = \frac{(r + \lambda)c}{q(\theta)}. \tag{7.A.30}$$

Differentiating this equation gives

$$\begin{aligned}
(1 - \beta)\,dp - \beta c\,d\theta &= -\frac{(r + \lambda)cq'(\theta)}{q(\theta)^2}\,d\theta \\
(1 - \beta)\,dp - \beta c\,d\theta &= \frac{(r + \lambda)c(1 - \eta)}{\theta q(\theta)}\,d\theta \\
(1 - \beta)\,dp &= \left(\beta c + \frac{(r + \lambda)c(1 - \eta)}{\theta q(\theta)}\right)d\theta.
\end{aligned}$$

Equation (7.A.30) can be written as

$$1 - \beta = \frac{1}{p - z}\left(\frac{(r + \lambda)c}{q(\theta)} + \beta c\theta\right),$$

which implies

$$\frac{1}{p-z}\left(\frac{(r+\lambda)c}{q(\theta)}+\beta c\theta\right)dp=\left(\beta c+\frac{(r+\lambda)c(1-\eta)}{\theta q(\theta)}\right)d\theta$$
$$\frac{\theta}{p-z}\left(r+\lambda+\beta\theta q(\theta)\right)dp=\left(\beta\theta q(\theta)+(r+\lambda)(1-\eta)\right)d\theta$$

$$\frac{d\theta}{dp}=\frac{\theta}{p-z}\frac{r+\lambda+\beta\theta q(\theta)}{(r+\lambda)(1-\eta)+\beta\theta q(\theta)} \tag{7.A.31}$$

Substituting (7.A.31) into (7.A.29) gives

$$\begin{aligned}\frac{dL}{dp}&=\frac{1}{p}H'\left(a_0\right)\left(\omega'(\theta)\theta\frac{r+\lambda+\beta\theta q(\theta)}{(r+\lambda)(1-\eta)+\beta\theta q(\theta)}+\omega(\theta)\frac{z}{p}-\frac{z-h}{p}\right)\\&=\frac{1}{p}H'\left(a_0\right)\left(\frac{(r+\lambda)\eta\beta\theta q(\theta)}{(r+\lambda+\beta\theta q(\theta))^2}\frac{r+\lambda+\beta\theta q(\theta)}{(r+\lambda)(1-\eta)+\beta\theta q(\theta)}+\frac{\beta\theta q(\theta)}{r+\lambda+\beta\theta q(\theta)}\frac{z}{p}-\frac{z-h}{p}\right)\\&=\frac{1}{p}H'\left(a_0\right)\left[\frac{\beta\theta q(\theta)}{r+\lambda+\beta\theta q(\theta)}\left(\frac{(r+\lambda)\eta}{(r+\lambda)(1-\eta)+\beta\theta q(\theta)}+\frac{z}{p}\right)-\frac{z-h}{p}\right],\end{aligned}$$

and

$$\frac{dL}{dp}\frac{p}{L}=\frac{H'\left(a_0\right)}{H(a_0)}\left[\frac{\beta\theta q(\theta)}{r+\lambda+\beta\theta q(\theta)}\left(\frac{(r+\lambda)\eta}{(r+\lambda)(1-\eta)+\beta\theta q(\theta)}+\frac{z}{p}\right)-\frac{z-h}{p}\right].$$

7.B Households' decisions in the DSGE model

Let $\Gamma\equiv C^e/C^u>1$ denote the inverse of the consumption replacement ratio. The participation condition is:

$$n_t\log\Gamma_t=\zeta L_t^{1+\sigma}$$

Aggregate utility is:

$$\begin{aligned}U_t&=\int_0^{L_t}\left(\frac{n_t}{L_t}\log C_t^e+\left(1-\frac{n_t}{L_t}\right)\log C_t^u\right)di+\int_{L_t}^1\left(\log C_t^u+\zeta i^\sigma\right)di\\&=n_t\log C_t^e+(L_t-n_t)\log C_t^u+(1-L_t)\log C_t^u+\frac{\zeta}{1+\sigma}(1-L_t^{1+\sigma})\\&=n_t\log\Gamma_t+\log C_t^u+\frac{\zeta}{1+\sigma}-\frac{\zeta}{1+\sigma}L_t^{1+\sigma}\\&=\zeta L_t^{1+\sigma}+\log C_t^u+\frac{\zeta}{1+\sigma}-\frac{\zeta}{1+\sigma}L_t^{1+\sigma}\\&=\log C_t^u+\zeta\left(1-\frac{1}{1+\sigma}\right)L_t^{1+\sigma}+\frac{\zeta}{1+\sigma}\\&=\log C_t^u+\frac{\zeta\sigma}{1+\sigma}L_t^{1+\sigma}+\frac{\zeta}{1+\sigma}\end{aligned}$$

The sum of individual consumptions equals aggregate consumption C_t so:

$$\log C_t=\log\left(n_tC_t^e+(1-n_t)C_t^u\right)=\log\left(n_t\frac{C_t^e}{C_t^u}C_t^u+(1-n_t)C_t^u\right)=\log\left(n_t\Gamma_t+1-n_t\right)+\log C_t^u$$

Therefore aggregate utility is:

$$U_t = \log C_t - \log\left(n_t\Gamma_t + 1 - n_t\right) + \frac{\zeta\sigma}{1+\sigma}L_t^{1+\sigma} + \frac{\zeta}{1+\sigma}$$

The program of the family is (the constant in utility is omitted):

$$\begin{aligned}
V_t(b_{t-1}, n_{t-1}) = \max_{C_t, L_t, \Gamma_t, n_t, b_t} \Bigg\{ & \log C_t - \log\left(n_t\Gamma_t + 1 - n_t\right) + \frac{\zeta\sigma}{1+\sigma}L_t^{1+\sigma} + \delta \mathrm{E}_t\left[V_{t+1}(b_t, n_t)\right] \\
& + \Lambda_t\left(b_{t-1}\frac{r_{t-1}}{P_t} + n_t w_t + (L_t - n_t)z + (1 - L_t)h - C_t - \frac{b_t}{P_t} + t_t + div_t\right) \\
& + \mu_t\left((1-\lambda)(1-f_t)n_{t-1} + f_t L_t - n_t\right) \\
& + \nu_t\left(n_t \log\Gamma_t - \zeta L_t^{1+\sigma}\right) \Bigg\}
\end{aligned}$$

where the three constraints (budget constraint, law of motion of employment and participation condition) are directly written using a Lagrangian form. The first order conditions yield:

- w.r.t. C_t:

$$\frac{1}{C_t} = \Lambda_t$$

- w.r.t. Γ_t:

$$\nu_t = \frac{\Gamma_t}{n_t\Gamma_t + 1 - n_t}$$

- w.r.t. L_t:

$$\zeta\sigma L_t^{\sigma} + \Lambda_t(z - h) + \mu_t f_t = \nu_t \zeta(1+\sigma)L_t^{\sigma}$$

- w.r.t. n_t:

$$-\frac{\Gamma_t - 1}{n_t\Gamma_t + 1 - n_t} + \delta\mathrm{E}_t\left[\frac{\partial V_{t+1}}{\partial n_t}\right] + \Lambda_t(w_t - z) - \mu_t + \nu_t\log\Gamma_t = 0$$

- w.r.t. b_t:

$$\delta\mathrm{E}_t\left[\frac{\partial V_{t+1}}{\partial b_t}\right] = \frac{\Lambda_t}{P_t}$$

And the envelope conditions:

- w.r.t. b_{t-1}:

$$\frac{\partial V_t}{\partial b_{t-1}} = r_{t-1}\frac{\Lambda_t}{P_t}$$

- w.r.t. n_{t-1}:

$$\frac{\partial V_t}{\partial n_{t-1}} = \mu_t(1-\lambda)(1-f_t)$$

From the definition of aggregate consumption

$$\nu_t = \frac{\Gamma_t}{n_t\Gamma_t + 1 - n_t} = \frac{C_t^e}{C_t} = 1 + \frac{C_t^e - C_t}{C_t} = 1 + (1-n_t)\frac{C_t^e - C_t^u}{C_t} = 1 + (1-n_t)\Lambda_t(C_t^e - C_t^u)$$

and

$$\frac{\Gamma_t - 1}{n_t\Gamma_t + 1 - n_t} = \frac{C_t^e - C_t^u}{C_t} = \Lambda_t(C_t^e - C_t^u)$$

From the Nash bargaining condition:

$$(1-\beta)\frac{\partial V_t}{\partial n_{t-1}} = \beta\Lambda_t(1-f_t)\frac{\partial W_t}{\partial n_{t-1}}$$

And the optimal decision of the representative firm yields:

$$\frac{\partial W_t}{\partial n_{t-1}} = (1-\lambda)\frac{c}{q_t}$$

Therefore:

$$\frac{\partial V_t}{\partial n_{t-1}} = \frac{\beta}{1-\beta}\Lambda_t(1-f_t)(1-\lambda)\frac{c}{q_t}$$

and

$$\mu_t = \frac{\beta}{1-\beta}\Lambda_t\frac{c}{q_t}$$

Finally, the optimal participation condition becomes:

$$\Lambda_t(z-h) + c\frac{\beta}{1-\beta}\Lambda_t\frac{f_t}{q_t} = \left(1 + (1+\sigma)(1-n_t)\Lambda_t(C_t^e - C_t^u)\right)\zeta L_t^\sigma$$

The marginal utility cost of higher participation represented by the right side of this expression can be interpreted as follows: ζL_t^σ corresponds to the decrease in home production, while $(1+\sigma)(1-n_t)\Lambda_t(C_t^e - C_t^u)\zeta L_t^\sigma$ is the effect of the reallocation of consumption in favor of the employed workers.

7.C Wage bargaining in the DSGE model

The marginal value of employment for the household is:

$$\frac{\partial V_t}{\partial n_{t-1}} = (1-\lambda)(1-f_t)\left(\Lambda_t(w_t - z) - \Lambda_t(C_t^e - C_t^u) + \delta \mathrm{E}_t\left[\frac{\partial V_{t+1}}{\partial n_t}\right] + \Lambda_t C_t^e(\log C_t^e - \log C_t^u)\right)$$

The term $-\Lambda_t(C_t^e - C_t^u)$ reflects the fact that increasing the number of employed workers (with high consumption) implies that individual consumptions are diminished when aggregate consumption and the replacement ratio are unchanged. The term $\Lambda_t C_t^e(\log C_t^e - \log C_t^u)$ stands for the fact that job creation relaxes somewhat the individual participation constraint (7.3.14): since it improves the job finding rate, it makes the participant state more attractive, which allows a reallocation of consumption in favor of the non-employed household members.
The marginal value of employment for the firm is:

$$\frac{\partial W_t}{\partial n_{t-1}} = (1-\lambda)\left(x_t p_t - w_t + \delta \mathrm{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\frac{\partial W_{t+1}}{\partial n_t}\right]\right)$$

Substituting into the Nash bargaining condition yields:

$$
\begin{aligned}
&(1-\beta)\left((w_t - z) - (C_t^e - C_t^u) + \delta \mathrm{E}_t\left[\frac{1}{\Lambda_t}\frac{\partial V_{t+1}}{\partial n_t}\right] + C_t^e(\log C_t^e - \log C_t^u)\right) \\
&\quad = \beta\left(x_t p_t - w_t + \delta \mathrm{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\frac{\partial W_{t+1}}{\partial n_t}\right]\right)
\end{aligned}
$$

$$
\begin{aligned}
w_t &= \beta x_t p_t + (1-\beta)\left(z + C_t^e - C_t^u - C_t^e(\log C_t^e - \log C_t^u)\right) \\
&\quad + \beta\delta \mathrm{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\frac{\partial W_{t+1}}{\partial n_t}\right] - \beta\delta \mathrm{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}(1-f_{t+1})\frac{\partial W_{t+1}}{\partial n_t}\right]
\end{aligned}
$$

$$
\begin{aligned}
w_t &= (1-\beta)\left(z + C_t^e - C_t^u - C_t^e(\log C_t^e - \log C_t^u)\right) \\
&\quad + \beta\left(x_t p_t + \delta \mathrm{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t} f_{t+1}\frac{\partial W_{t+1}}{\partial n_t}\right]\right)
\end{aligned}
$$

$$
\begin{aligned}
w_t &= (1-\beta)\left(z + C_t^e - C_t^u - C_t^e(\log C_t^e - \log C_t^u)\right) \\
&\quad + \beta\left(x_t p_t + c\delta(1-\lambda)\mathrm{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\frac{f_{t+1}}{q_{t+1}}\right]\right)
\end{aligned}
$$

7.D Impulse response functions

Impulse responses are plotted in log-deviations from steady state levels.

Figure 7.D.1: Impulse responses to a 1% technology shock

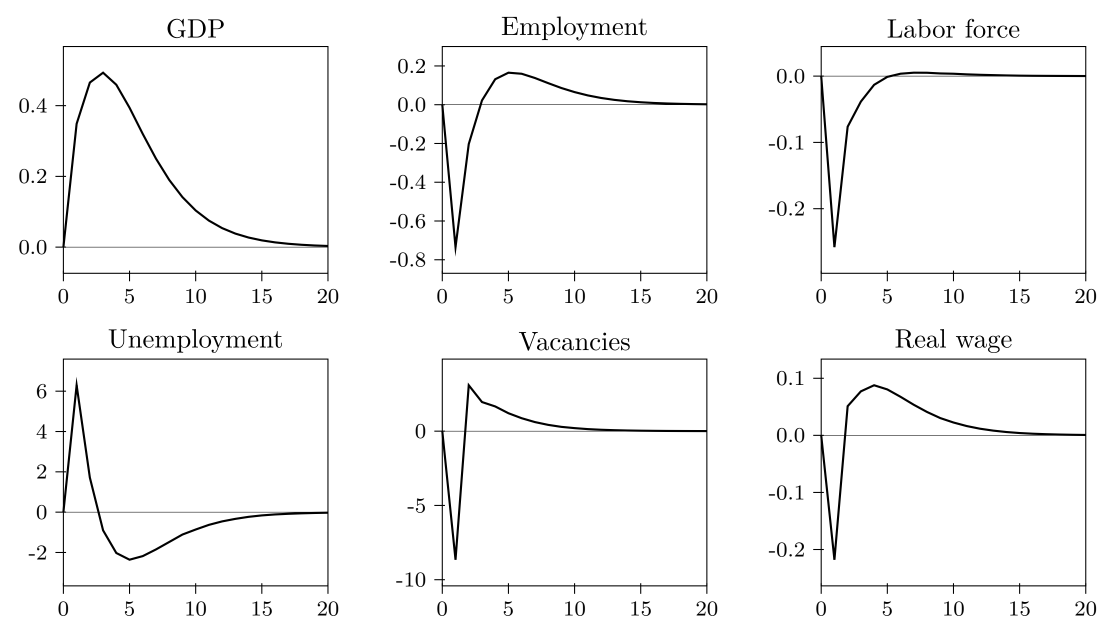
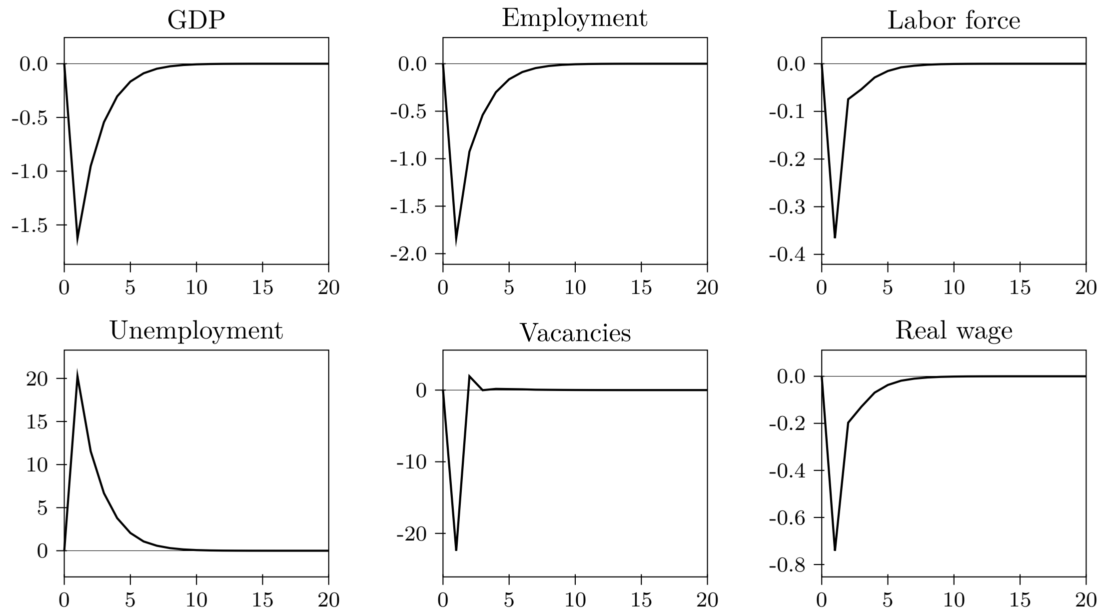


Figure 7.D.2: Impulse responses to a 1% monetary policy shock



7.E DSGE model with identical preferences

There is a large number, normalized to 1, of identical housholds. Each household includes a continuum of members with a mass of 1. The instantaneous utility of a household member who consumes C in the current period is

$$U(i, C) = \begin{cases} \log C + \zeta & \text{if out of the labor force,} \\ \log C & \text{if in the labor force,} \end{cases},$$

where ζ is the common utility of home production. The model assumes a lottery to choose among individuals those who participate in the labor market and among participants those who are actually employed. Households members insure themselves against idiosyncratic risks that arise from these lotteries: at the beginning of period t, each individual decides to buy $\tilde{b}_t$ units of participation insurance at price $\tilde{\tau}_t$ that pay $\tilde{b}_t$ in real terms whenever he is not selected to participate in the labor market during period t, and $\breve{b}_t$ units of unemployment insurance at price $\breve{\tau}_t$ that pay $\breve{b}_t$ in real terms whenever he is selected to participate but is not appointed to a job. The ex ante (i.e. before being selected or not for participation and a fortiori being hired or not) value function of an individual holding nominal assets for b_{t-1} and when employment was n_{t-1} during the previous period is:

$$V_t(b_{t-1}, n_{t-1}) = \max_{\mathcal{C}_t} \Big\{ n_t \left(\log C_t^e + \delta E_t V_{t+1}(b_t^e, n_t)\right)$$

$$+ (L_t - n_t)\left(\log C_t^u + \delta E_t V_{t+1}(b_t^u, n_t)\right) + (1 - L_t)\left(\log C_t^o + \zeta + \delta E_t V_{t+1}(b_t^o, n_t)\right) \Big\}$$

s.t.

$$C_t^e + \frac{b_t^e}{P_t} + \tilde{\tau}_t \tilde{b}_t + \breve{\tau}_t \breve{b}_t \leq b_{t-1}\frac{r_{t-1}}{P_t} + w_t + T_t + div_t$$

$$C_t^u + \frac{b_t^u}{P_t} + \tilde{\tau}_t \tilde{b}_t + \breve{\tau}_t \breve{b}_t \leq b_{t-1}\frac{r_{t-1}}{P_t} + z + \breve{b}_t + T_t + div_t$$

$$C_t^o + \frac{b_t^o}{P_t} + \tilde{\tau}_t \tilde{b}_t + \breve{\tau}_t \breve{b}_t \leq b_{t-1}\frac{r_{t-1}}{P_t} + h + \tilde{b}_t + T_t + div_t$$

$$n_t \leq (1 - \lambda)(1 - \phi_t) n_{t-1} + f_t L_t$$

Where

$$\mathcal{C}_t \equiv \left\{ C_t^e, C_t^u, C_t^o, b_t^e, b_t^u, b_t^o, \tilde{b}_t, \breve{b}_t, L_t, n_t \right\}$$

is the vector of variables that are decided by each individual in period t. Let Λ_t^e, Λ_t^u and Λ_t^o be the Lagrange multipliers associated with these budget constraints. The first order conditions yield:

$$\Lambda_t^e = \frac{1}{C_t^e}, \quad \Lambda_t^u = \frac{1}{C_t^u}, \quad \Lambda_t^o = \frac{1}{C_t^o}$$

$$\Lambda_t^e n_t \tilde{\tau}_t + \Lambda_t^u (L_t - n_t)\tilde{\tau}_t + \Lambda_t^o (1 - L_t)(\tilde{\tau}_t - 1) = 0$$

$$\Lambda_t^e n_t \breve{\tau}_t + \Lambda_t^u (L_t - n_t)(\breve{\tau}_t - 1) + \Lambda_t^o (1 - L_t)\breve{\tau}_t = 0$$

$$\delta E_t \frac{\partial V_{t+1}}{\partial b_t^e} = \frac{\Lambda_t^e}{P_t}$$

$$\delta E_t \frac{\partial V_{t+1}}{\partial b_t^u} = \frac{\Lambda_t^u}{P_t}$$

$$\delta E_t \frac{\partial V_{t+1}}{\partial b_t^o} = \frac{\Lambda_t^o}{P_t}$$

Free entry in the insurance market implies:

$$\tilde{\tau}_t = 1 - L_t$$

$$\breve{\tau}_t = L_t - n_t$$

Therefore:

$$\Lambda_t^e = \Lambda_t^u = \Lambda_t^o \quad \Leftrightarrow \quad C_t^e = C_t^u = C_t^o \equiv C_t$$

As the objective function V_{t+1} is a bijection, we get:

$$b_t^e = b_t^u = b_t^o \equiv b_t$$

From the budget constraints, we have immediately:

$$\tilde{b}_t = w_t - h$$

$$\breve{b}_t = w_t - z$$

The problem can be written with a representative household formulation. The latter has the following objective function:

$$\begin{aligned} V_t(b_{t-1}, n_{t-1}) = \max_{C_t, L_t, n_t, b_t} & \left\{ \log C_t + (1 - L_t)\zeta + \delta E_t V_{t+1}(b_t, n_t) \right\} \\ \text{s.t.} & \\ & C_t + \frac{b_t}{P_t} \leq b_{t-1}\frac{r_{t-1}}{P_t} + n_t w_t + (L_t - n_t)z + (1 - L_t)h + T_t + div_t \\ & n_t \leq (1 - \lambda)(1 - f_t)n_{t-1} + f_t L_t \end{aligned}$$

This program yields the first order condition with respect to the participation rate

$$\zeta = c\frac{\beta}{1-\beta}\Lambda_t \frac{f_t}{q_t} + \Lambda_t(z - h),$$

and the wage equation becomes

$$w_t = (1 - \beta)z + \beta \left(x_t p_t + \delta(1 - \lambda)c \mathrm{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{f_{t+1}}{q_{t+1}} \right] \right).$$

7.F Dynamic properties of the DSGE model under alternative calibrations

The dynamic properties of the model for the different calibrations considered in the paper are reported in the following tables:

Table 7.F.1: Second order moments with $\sigma = 0$ and $z - h = 0.2038$

		y	l	u	v	v/u
standard deviation		0.013	0.012	0.147	0.124	0.221
correlation matrix	y	1.000	0.162	-0.844	0.773	1.000
	l	–	1.000	0.266	0.601	0.162
	u	–	–	1.000	-0.313	-0.844
	v	–	–	–	1.000	0.773
	v/u	–	–	–	–	1.000

Notes: The table reports the theoretical standard errors and correlation matrix of the deviations of $\log(Y_t - cv_t)$, $\log n_t$, $\log L_t$, $\log(L_t - n_t)$, $\log v_t$ and $\log(\frac{v_t}{L_t - n_t})$ from their steady state levels.

Table 7.F.2: Second order moments with $\sigma = 0$ and $z - h = 0.2481$

		y	l	u	v	v/u
standard deviation		0.013	0.012	0.167	0.130	0.252
correlation matrix	y	1.000	0.017	-0.883	0.797	1.000
	l	–	1.000	0.343	0.472	0.017
	u	–	–	1.000	-0.420	-0.883
	v	–	–	–	1.000	0.797
	v/u	–	–	–	–	1.000

Table 7.F.3: Second order moments with $\sigma = 0$ and $z - h = 0.2899$

		y	l	u	v	v/u
standard deviation		0.012	0.012	0.186	0.136	0.280
correlation matrix	y	1.000	-0.113	-0.907	0.816	1.000
	l	–	1.000	0.421	0.344	-0.113
	u	–	–	1.000	-0.497	-0.907
	v	–	–	–	1.000	0.816
	v/u	–	–	–	–	1.000

Table 7.F.4: Second order moments with $\sigma = 0$ and $z - h = 0.7742$

		y	l	u	v	v/u
standard deviation		0.010	0.020	0.382	0.195	0.553
correlation matrix	y	1.000	-0.795	-0.979	0.918	1.000
	l	–	1.000	0.871	-0.548	-0.795
	u	–	–	1.000	-0.819	-0.979
	v	–	–	–	1.000	0.918
	v/u	–	–	–	–	1.000

Table 7.F.5: Second order moments with $\sigma = 7.8396$ and $z - h = 0$

		y	l	u	v	v/u
standard deviation		0.015	0.009	0.091	0.129	0.188
correlation matrix	y	1.000	0.699	-0.801	0.604	0.804
	l	–	1.000	-0.745	0.909	0.987
	u	–	–	1.000	-0.436	-0.785
	v	–	–	–	1.000	0.900
	v/u	–	–	–	–	1.000

Table 7.F.6: Second order moments with $\sigma = 34.7398$ and $z - h = 0$

		y	l	u	v	v/u
standard deviation		0.014	0.003	0.159	0.154	0.294
correlation matrix	y	1.000	0.678	-0.779	0.609	0.740
	l	–	1.000	-0.937	0.933	0.996
	u	–	–	1.000	-0.765	-0.941
	v	–	–	–	1.000	0.937
	v/u	–	–	–	–	1.000

Table 7.F.7: Second order moments with $\sigma = 0$ and $z - h = 0$

		y	l	u	v	v/u
standard deviation		0.017	0.017	0.081	0.098	0.051
correlation matrix	y	1.000	0.735	0.047	0.560	1.000
	l	–	1.000	0.547	0.837	0.735
	u	–	–	1.000	0.854	0.047
	v	–	–	–	1.000	0.560
	v/u	–	–	–	–	1.000

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Résumé détaillé

Les modèles DSGE et l'évaluation des politiques économiques

L'utilisation de modèles théoriques s'est imposée au cours des dernières décennies comme une approche incontournable de l'analyse macroéconomique. Ces outils ont le mérite de discipliner le raisonnement économique. En effet, le formalisme mathématique garantit la validité logique des développements et déductions faits à partir d'hypothèses données. La discussion des conclusions se cantonne alors à l'appréciation de la pertinence de ces hypothèses. De ce point de vue, la critique par les pairs des travaux est considérablement facilitée, à condition pour les économistes d'avoir un bagage technique suffisant. De plus, à la différence d'approches moins formelles de la macroéconomie, les prédictions des modèles sont données en termes quantitatifs, ce qui est presque toujours indispensable pour guider l'orientation des politiques économiques.

L'analyse des cycles d'affaires à partir des modèles démarre avec Kydland and Prescott (1982). Cet article, qui propose une approche dans laquelle les prix sont flexibles, est à l'origine de la théorie des cycles économiques réels (RBC en anglais). Les modèles d'équilibre général dynamiques stochastiques néo-keynésiens (DSGE en anglais), lancés par des auteurs tels que Rotemberg and Woodford (1999), s'appuient sur l'approche RBC mais font l'hypothèse que les prix ne peuvent pas s'ajuster immédiatement, ce qui suppose que les marges des entreprises varient. Cette littérature, attribuant un rôle explicite à la politique monétaire, a rencontré un succès considérable auprès des banques centrales.

Les modèles DSGE décrivent une économie fictive, censée représenter un certain nombre de caractéristiques du monde réel.[13] Ils tiennent compte des contraintes de ressources présentes dans l'économie et reflètent les décisions d'agents optimisateurs qui anticipent les effets futurs de leurs choix présents. Une approche alternative, largement utilisée par les institutions en charge de politique économique, consiste à estimer des relations statistiques entre les séries macroéconomiques observées. Cependant, l'évaluation des multiplicateurs des politiques économiques basée sur ce type de modèles sous "forme-réduite" est susceptible d'être fortement biaisée. En effet, rien n'assure que les relations statistiques mesurées soient stables dans le temps et ne dépendent pas précisément des politiques économiques qu'on chercher à étudier. Cette idée, formalisée par Lucas (1976), est appelée la "critique de Lucas". De plus, il ne fait aucun doute que les anticipations des agents économiques

[13] C'est la raison pour laquelle une part importante de la littérature est consacrée à l'étude des effets d'hypothèses de modélisation particulières sur les propriétés dynamiques des modèles et à leur comparaison avec les données macroéconomiques.

jouent un rôle dans leurs décisions présentes. Les modèles sous “forme-réduite” traitent ces anticipations de deux manières : soit elles sont simplement ignorées, soit elles sont représentées par des équations *ad hoc*, qui ne traduisent pas des choix humains rationnels. Pourtant, prendre en compte les anticipations est crucial pour l'évaluation des politiques économiques. Dans la mesure où, en réalité, les agents disposent d'informations sur le fonctionnement de l'économie et sur le processus de prise de décision des gouvernants, leurs réactions aux politiques économiques dépendent de leurs prévisions des décisions futures. Réciproquement, ces prévisions sont affectées par les décisions de politique économique présentes. Ainsi, Kydland and Prescott (1977) montrent que les politiques choisies compte tenu d'une situation existante, mais sans tenir compte de l'effet des anticipations des agents optimisateurs, peut être sous-optimale. Pour ces raisons, ces outils ne sont pas adaptés pour prévoir les effets de politiques économiques. L'utilisation de modèles cherchant à reproduire des comportements optimaux intertemporellement apparaît donc à ce jour plus crédible.

En dépit de leurs atouts, les modèles dynamiques micro-fondés peinent à reproduire les séries macroéconomiques observées de manière suffisamment satisfaisante pour s'imposer pleinement auprès des décideurs. De sévères ajustements, parfois contestables, sont nécessaires pour rapprocher les purs comportements théoriques des données. Ceci est dû pour une large part à la façon dont on suppose que les agents forment leurs anticipations sur le futur : le paradigme des anticipations rationnelles postule que les prévisions faites par les agents sont non seulement cohérentes avec le modèle – qui est évidemment mal spécifié – mais aussi avec le futur à très long terme de l'économie – qui est en réalité parfaitement inconnu. Ces hypothèses, très contraignantes, sont qualifiées “d'héroïques” par Woodford (2012). Des progrès considérables restent donc à faire pour améliorer la modélisation des anticipations. Certaines alternatives aux anticipations rationnelles, comme le *learning* (Evans and Honkapohja (1999)) sont très prometteuses.

En attendant, les modèles DSGE à anticipations rationnelles restent l'approche standard et servent de référence pour l'étude des autres approches. Ceci est dû, en premier lieu, au fait qu'ils respectent l'exigence de cohérence interne : pourquoi, en effet, les agents de l'économie fictive représentée par le modèle continueraient indéfiniment à faire de fausses prévisions sur le futur? Deuxièmement, l'hypothèse d'ancitipations rationelles, en laissant peu d'espace pour le calibrage ou les choix particuliers de spécification, contraint fortement la dynamique des modèles DSGE. Cette discipline imposée aux économistes est un argument supplémentaire en faveur de leur utilisation. Enfin, grâce aux développements des méthodes de calcul numérique et des outils informatiques, les modèles DSGE à anticipations rationnelles sont bien adaptés pour une utilisation pratique par les institutions, contrairement aux modèles s'appuyant sur des hypothèses alternatives pour décrire les anticipations des agents qui restent plus compliqués à manipuler.

Un certain nombre de travaux ont étudié les spécifications ou les calibrages qui permettent de reproduire au mieux certaines propriétés des données. D'autres ont plutôt fait le choix de s'appuyer sur des méthodes économétriques rigoureuses pour évaluer la pertinence de leurs modèles DSGE d'un point de vue quantitatif. Fève (2006) propose une revue des méthodes quantitatives utilisées dans la littérature pour estimer les modèles. Elles comprennent l'estimation par maximum de vraisemblance (Ireland (1997)), les méthodes de moments (Christiano and Eichenbaum (1992)), les approches basées sur des simulations (Jonsson and Klein (1996) ou Hairault et al. (1997) par exemple utilisent une méthode de moments simulés, tandis que Dupaigne et al. (2005) s'appuient sur l'inférence indirecte pour estimer leurs modèles), ou encore l'approche bayésienne. Récemment, cette dernière s'est popularisée sous l'impulsion d'auteurs comme Smets and Wouters (2007). Contrairement aux méthodes de moments, c'est une approche "en information complète". Les paramètres et les processus exogènes sont esimés de façon à reproduire simultanément au mieux un large éventail de propriétés dynamiques des séries observées. De la sorte, les résultats d'estimation ne sont pas influencés par la sélection de moments-cibles particuliers, laissés au choix de l'économiste. Par rapport au maximum de vraisemblance, qui est aussi en "information complète", l'approche bayésienne a deux atouts majeurs. D'abord, les données ne sont généralement pas suffisamment informatives pour identifier tous les paramètres avec un degré de précision suffisant ; l'approche bayésienne permet d'utiliser d'autres sources d'information. Ensuite, d'un point de vue pratique, elle permet d'exclure d'emblée les valeurs de paramètres non compatibles avec la théorie économique, qui peuvent éventuellement être trouvées par le maximum de vraisemblance non contraint du fait de la spécification imparfaite des modèles.

Jusqu'où enrichir les modèles DSGE pour l'analyse macroéconomique opérationnelle?

Le modèle néo-keynésien de Smets and Wouters (2003, 2007) s'est imposé comme l'outil de référence pour l'analyse du cycle, la prévision et l'évaluation de politiques par les institutions, en particulier les banques centrales. Cependant, malgré ses bonnes performances, il présente des lacunes importantes, notamment la non-prise en compte des échanges internationaux, des frictions financières et des imperfections du marché du travail. Une littérature abondante s'est attachée à proposer des extensions pour combler ces lacunes et de nombreux travaux ont démontré leur importance pour l'analyse macroéconomique. Ces extensions, incorporées à des modèles DSGE estimés, ont été confrontées aux données avec succès. Voici un bref rappel de ces avancées.

Extensions du modèle de Smets et Wouters dans la littérature

Économie ouverte

Le modèle de Smets et Wouters a d'abord été estimé sur les données de la zone euro (voir Smets and Wouters (2003)). L'économie de la zone y est représentée comme une économie fermée. Cette hypothèse peut être préjudiciable pour l'analyse économique, tant d'un point de vue positif que normatif. D'abord, les perturbations qui affectent les importations et les exportations, telles que les chocs de prix étranger ou de demande mondiale, sont confondus avec la consommation publique dans un agrégat de "demande résiduelle". Pourtant, ces chocs peuvent avoir des effets très différents sur l'économie. De même, Paoli (2009) montre que la politique monétaire optimale est différente dans une petite économie ouverte de celle obtenue en économie fermée, car le coût en bien-être du cycle d'affaires est affecté par les fluctuations du taux de change.

Le fait que la zone euro soit décrite comme une région homogène est une autre limite du modèle de Smets and Wouters (2003). D'abord, les politiques budgétaires sont décidées par pays, et la coordination de ces politiques entre les pays européens est une préoccupation importante des décideurs. Ensuite, lorsqu'on évalue des politiques économiques envisagées par un pays à l'intérieur de l'union monétaire, il semble important de distinguer les flux commerciaux intra-zone des flux extra-zone dans la mesure où seuls ces derniers sont affectés par les variations du taux de change. Quand on s'appuie sur un critère de bien-être pour ces exercices, la présence d'une union monétaire compte. Pas seulement parce que la politique monétaire est commune, mais aussi parce que l'union monétaire, comme montré par Kollmann (2004), diminue sensiblement le coût en bien-être des fluctuations par rapport à un régime de change flexible en éliminant le choc de parité non couverte des taux. Enfin, le fait de différencier les pays à l'intérieur de la zone euro permet un calibrage plus fin des paramètres de chaque région, ce qui théoriquement doit améliorer le pouvoir explicatif des modèles. Marcellino et al. (2003), par exemple, trouvent que des prévisions d'inflation pour la zone euro effectuées de manière désagrégée par pays surpassent des prévisions agrégées.

Troisièmement, Smets and Wouters (2003) ignorent les effets des variations du prix du pétrole sur l'économie. Pourtant, ceux-ci sont loin d'être négligeables, ce qui explique le suivi constant dont le prix du pétrole fait l'objet de la part des décideurs économiques. Un modèle qui en tient explicitement compte est plus pertinent pour orienter la politique monétaire lors de périodes de tension sur les prix de l'énergie. Fiore et al. (2006) et Natal (2012) mettent ainsi en évidence la supériorité d'une politique monétaire qui réagit de manière spécifique aux variations du prix du pétrole.

Récemment, un certain nombre de travaux ont intégré avec succès ces développements dans un cadre néo-keynésien. Adjemian and Darracq-Pariès (2008) estiment un modèle à deux pays (zone euro et États-unis) avec prix du pétrole, Rabanal (2009) propose un modèle à deux pays de la zone euro incluant l'Espagne, tandis que Christoffel et al. (2008) développent et estiment un modèle en petite économie ouverte de la zone euro.

Frictions financières

Smets and Wouters (2003, 2007) ignorent les frictions financières. Cependant, de nombreux auteurs, parmi lesquels Fisher (1933) ou, plus tard, Bernanke and Lown (1991), ont émis l'idée que la déterioration des conditions du marché du crédit est un facteur majeur des crises de l'économie réelle. Plus récemment, Christiano et al. (2014) identifient un choc sur le crédit, appelé "choc de risque", comme l'un des plus importants déterminants du cycle d'affaires. D'autres travaux ont proposé des modèles estimés incluant des frictions financières, notamment Christensen and Dib (2008) et Villa (2014).

Dans la ligne de ces auteurs, il y a désormais un large consensus sur le fait que les interactions entre les marchés du crédit et l'économie réelle ont joué un rôle majeur dans la grande dépression de 2008 (voir Christiano et al. (2015)). D'un point de vue normatif, la présence de frictions financières amplifie fortement la réponse de l'économie aux chocs de taux d'intérêt, ce qui susceptible d'affecter sensiblement les recommandations de politique monétaire qui peuvent être tirées d'un modèle DSGE.

De ce fait, depuis la crise, on a pu constater un regain d'intérêt des banques centrales pour les modèles de cycles pouvant rendre compte des tensions possibles sur les marchés financiers.

Frictions sur le marché du travail

Les modèles DSGE néo-keynésiens standard de type Smets and Wouters décrivent des économies dans lesquelles les mouvements d'emploi sont parfaitement volontaires, ignorant de ce fait le chômage. Ceci est paradoxal dans la mesure où c'est l'une des préoccupations majeures des gouvernements européens. Ils supposent que les individus désireux de travailler et les entreprises qui ont besoin d'employés peuvent s'accorder immédiatement et sans coût. Ce mécanisme walrasien est pourtant clairement irréaliste.

Blanchard and Galí (2010) montrent que la prise en compte de frictions réelles sur le marché du travail a des implications en termes de politique monétaire optimale. En s'appuyant sur un modèle avec des frictions de type "recherche et appariement" à la Mortensen and Pissarides (1994), ils concluent que le ciblage strict d'un taux d'inflation est inefficace dans un objectif d'amélioration du bien-être. En effet, la stabilisation parfaite de l'inflation entraine de persistantes fluctuations du chômage en réponse à des chocs technologiques, alors que l'allocation efficace qui serait décidée par un plannificateur social bienveillant prévoit au contraire un taux de chômage constant.

Gertler et al. (2008), Sala et al. (2008) ou encore Christoffel et al. (2009), parmi d'autres, estiment des modèles avec frictions sur le marché du travail. Enfin, Christiano et al. (2011) estiment un modèle en petite économie ouverte de la Suède incluant à la fois des frictions financières et des frictions sur le marché du travail.

L'importance des frictions financières et sur le marché du travail pour l'analyse macroéconomique

En dépit de l'accueil favorable reçu par ces recherches, les institutions en charge de politique économique utilisent encore largement des modèles basés sur Smets and Wouters (2003, 2007) pour l'évaluation des politiques, les prévisions ou l'analyse conjoncturelle. Si ces outils prennent désormais en compte, pour la plupart, l'ouverture de l'économie et les échanges commerciaux internationaux, ils s'appuient toujours sur une représentation rudimentaire des marchés du travail et du crédit. Ainsi, la BCE utilise le *New Area Wide Model* (Christoffel et al. (2008)) pour l'analyse conjoncturelle et la prévision et le modèle *Eagle* (Gomes et al. (2010)) pour l'évaluation des politiques structurelles. Parmi les autres exemples, on trouve le modèle *Ramses* de la Banque de Suède (Adolfson et al. (2008)), le modèle *Aino* de la Banque de Finlande (Kilponen and Verona (2014)) ou le modèle *Edo* à la Réserve Fédérale américaine (Edge et al. (2010)).[14]

Le modèle de Smets et Wouters a été adopté par les institutions principalement du fait de sa capacité à reproduire les données de manière satisfaisante. Smets and Wouters (2004) montrent en effet que ses performances prédictives sont comparables à celles d'un VAR standard. En réalité, ce

[14] La Banque de Finlande utilise actuellement une version du modèle *Aino* sans frictions financières, mais une extension incluant un secteur bancaire est en cours de développement.

succès est dû en partie au fait que les restrictions théoriques imposées par le modèle aux données sont limitées. Sept chocs exogènes, décrits par des modèles AR ou ARMA, captent une grande part de la dynamique observée. Ceci diminue quelque peu l'intérêt de cet outil pour l'évaluation de politiques économiques.

Des modèles enrichis pour tenir compte de la présence de frictions sur les marchés du travail ou du crédit sont parfois utilisés par les institutions pour traiter des questions particulières, mais ceux-ci n'ont pas encore supplanté les modèles plus simples pour l'analyse macroéconomique opérationnelle, en partie à cause de leur plus grande complexité. Il est donc légitime de s'interroger sur les gains à attendre de l'incorporation de telles frictions dans les modèles utilisés de manière récurrente pour éclairer la conduite des politiques économiques. Cette thèse traite cette question et montre que les mécanismes micro-fondés introduits dans la description des marchés du travail et du crédit peuvent modifier de manière importante les conclusions tirées de l'utilisation de modèles DSGE estimés, à la fois d'un point de vue positif que d'un point de vue normatif. Pour cela, je construis un modèle à deux pays de la France et du reste de la zone euro, avec un reste du monde exogène, et je l'estime avec et sans ces deux frictions en suivant une approche bayésienne. Par rapport au modèle de Smets et Wouters, j'ajoute un grand nombre de restrictions issues de la théorie économique. De nouvelles variables, dont la dynamique est fortement contrainte par la théorie et qui impactent fortement les principaux agrégats des deux économies, sont utilisées pour l'estimation. Ces restrictions réduisent inévitablement le pouvoir explicatif de certains chocs exogènes présents dans le modèle sans frictions, et tendent de ce fait à diminuer l'adéquation du modèle au données. En revanche, les chocs structurels et leurs canaux de propagation sont plus précisément identifiés, ce qui permet d'affiner les recommandations de politique économique tirées du modèle.

À la différence des papiers cités plus haut qui ont mis en évidence l'intérêt des frictions pour l'analyse économique, j'utilise les développements récents de la modélisation macroéconomique dans le contexte particulier de la zone euro en temps de crise. Ce contexte est caractérisé par l'existence d'une union monétaire confrontée à la question de la sortie de certains pays-membres, par des marchés du travail peu efficaces et par un intérêt croissant pour les politiques visant à réduire la charge fiscale sur l'emploi.

Je propose plusieurs exercices de simulation pour mettre en évidence le rôle des frictions sur le marché du travail et des frictions financières : (i) une décomposition des fluctuations cycliques observées pendant la crise en contributions des chocs structurels du modèle, (ii) une évaluation des effets en termes de bien-être de différentes règles de politique monétaire et (iii) une simulation contrefactuelle de la période de crise sous l'hypothèse d'une sortie de la France de la zone euro. J'utilise ensuite le modèle pour simuler les effets de la mise en œuvre de mesures de TVA sociale et formuler des recommandations de politique économique favorables à l'emploi.

Un modèle DSGE de la France dans la zone euro

Je construis et j'estime un modèle théorique dans lequel les comportements de court terme des agents dans le cycle sont cohérents avec le long terme de l'économie. Par rapport aux modèles

DSGE précédents de la littérature, il inclut un plus grand nombre de variables observables, et est enrichi dans plusieurs dimensions. Le résultat est l'un des plus gros et des plus complets des modèles DSGE estimés par méthode bayésienne.

La structure de base s'appuie sur le modèle néo-keynésien de Smets and Wouters (2003, 2007). Il inclut de nombreuses dimensions des économies ouvertes pertinentes dans le cas de la zone euro. Celle-ci est considérée comme une petite économie ouverte par rapport au reste du monde. Elle est divisée en deux pays représentant d'un côté la France et de l'autre le reste de la zone euro, qui échangent des biens et des actifs financiers. Enfin, les fluctuations du prix du pétrole impactent à la fois les secteurs productifs et les consommations finales.

Les frictions financières et sur le marché du travail sont introduites dans le modèle sous la forme de deux options indépendantes. Autrement dit, il y a en réalité quatre versions du modèle : une version de base sans ces frictions, qui est une version du modèle de Smets et Wouters en économie ouverte, une version avec seulement les frictions financières, une version avec seulement les frictions sur le marché du travail, et enfin une version complète avec les deux.

Les frictions financières prennent la forme d'un accélérateur financier à la Bernanke et al. (1999) : des entrepreneurs ont besoin d'emprunter auprès des banques pour financer leurs investissements en capital productif. Des imperfections sur le marché du crédit font apparaître une prime de risque contracyclique sur ces emprunts, qui contribue à propager et à amplifier les chocs agrégés qui frappent l'économie.

Pour les frictions sur le marché du travail, j'utilise le mécanisme de recherche et d'appariement proposé par Mortensen and Pissarides (1994) : les chômeurs sont en recherche d'emploi, tandis que les entreprises émettent des offres d'emploi. Le nombre de chômeurs et le nombre d'emplois vacants entrent dans une fonction d'appariement agrégée qui détermine le nombre d'emplois finalement créés à chaque date. Ces appariements réussis créent un surplus pour l'économie. Les employés et les entreprises négocient *à la Nash* le salaire pour partager ce surplus. La spécification utilisée s'appuie également sur le modèle de Blanchard and Galí (2010), dans la mesure où il s'agit d'un modèle néo-keynésien : les nouveaux emplois contribuent immédiatement à la production, au lieu d'attendre la période suivante comme supposé dans les modèles de cycles réels à la Mortensen and Pissarides (1994).

Cependant, cette représentation du marché du travail présente d'importantes lacunes. D'abord, elle tend à exagérer la volatilité des salaires par rapport à ce qui est observé dans les données. Deuxièmement, elle ignore les entrées et sorties sur le marché du travail qui font varier la taille de la population active et donc le chômage dans le cycle. Je propose deux extensions originales pour combler ces manques.

Améliorations de la spécification des marchés du travail

Rigidité salariale

Une originalité du modèle construit réside dans la façon dont les rigidités des salaires sont introduites. Un point de vue très répandu à ce sujet, mis en avant notamment par Shimer (2005) et Hall

(2005), consiste à considérer que les modèles de recherche et d'appariement avec négociations à la Nash sous-estiment la volatilité de l'emploi et du nombre d'emplois vacants dans le cycle parce que les salaires négociés réagissent trop fortement et de manière trop procyclique aux chocs exogènes. De ce fait, l'incitation des entreprises à émettre des offres d'emploi est atténuée par la hausse des salaires demandés en période d'expansion, et inversement lorsque l'économie se contracte. Se fondant sur l'apathie des salaires moyens dans les données, ils suggèrent que ralentir les fluctuations du salaire peut permettre à ces modèles à la fois de reproduire la dynamique des salaires et celle de l'emploi.

Dans le modèle que je propose, je m'appuie au contraire sur les conclusions de Pissarides (2009), qui met en évidence le fait que l'apathie apparente des salaires moyens est en fait le résultat de l'agrégation des salaires très dynamiques et procycliques associés aux nouveaux emplois, et des salaires peu réactifs car ne faisant pas l'objet d'une renégociation, associés aux emplois existants. Dans la mesure où les décisions d'embauche des entreprises se basent sur les perspectives de salaire des nouveaux emplois, la rigidité salariale n'est pas une solution satisfaisante pour augmenter l'amplitude des fluctuations d'emploi dans le modèle standard de recherche et d'appariement.

Ainsi, dans le modèle construit, je définis explicitement le salaire réel observé comme une moyenne entre ceux associés aux nouveaux emplois, issus d'une négociation à la Nash, et les autres, qui sont simplement indexés sur la tendance de la technologie. La dimension originale apportée dans cette thèse consiste à avoir incorporé dans un modèle DSGE estimé le mécanisme théorique développé en temps continu par Pissarides (2009). Au final, cette hypothèse rend le salaire réel moyen très inerte, tandis que la dynamique des créations d'emplois est inchangée par rapport au modèle de recherche et d'appariement standard avec salaires flexibles.

Pour permettre au modèle de reproduire l'amplitude des créations d'emplois observée dans le cycle, je mets à profit une des recommandations de calibrage de Hagedorn and Manovskii (2008) : les profits des entreprises sont quantitativement faibles, de façon à être très sensibles, en pourcentage, aux conditions économiques ; l'incitation à varier le nombre d'emplois vacants est donc amplifiée.[15]

La participation au marché du travail

Modéliser le comportement cyclique du taux de participation au marché du travail soulève un problème qui a été décrit par plusieurs auteurs, parmi lesquels Tripier (2003), Veracierto (2008) et Ravn (2008). Les modèles standard prévoient une volatilité et une corrélation avec l'activité du taux de participation bien supérieures à ce qu'elles sont dans les données.[16] Je contribue à cette littérature en proposant deux explications crédibles qui permettent de corriger le comportement cyclique du taux de participation : l'hétérogénéité des préférences et l'indemnisation du chômage. Afin de mieux comprendre les implications de ces hypothèses, je les étudie dans un petit modèle standard de recherche et d'appariement, séparément du modèle estimé de la France dans la la zone euro. Dans

[15] L'autre recommandation mise en avant par ces auteurs consiste à diminuer le pouvoir de négociation relatif des travailleurs dans la détermination du salaire. Elle a pour effet principal de réduire la volatilité des salaires négociés et n'est dont pas utile dans le présent modèle.

[16] Ces travaux s'appuient sur des données américaines.

ce modèle, l'hypothèse usuelle d'assurance parfaite destinée à agréger les comportements individuels est remplacée par un système d'allocation de la consommation tel que proposé par Christiano et al. (2010). Cette spécification rend la participation volontaire et le chômage involontaire (dans le sens où le bien-être des chômeurs est inférieur à celui des employés). Je calibre ce modèle de façon à ce qu'il reproduise la volatilité et le degré de procyclicalité du taux de participation aux États-unis, tout en respectant les principales caractéristiques dynamiques du reste de l'économie.

Á ce stade cependant, la participation au marché du travail endogène n'est pas incorporée au modèle estimé de la France dans la zone euro à ce stade. Ceci fera l'objet de travaux futurs.

En construisant le modèle, j'ai testé en détail les principales hypothèses utilisées dans de petits modèles séparés, de façon à bien comprendre les effets introduits. Ces analyses ont révélé un certain nombre d'écueils qui ont guidé mes choix de spécification du modèle final et la construction des scénarios de TVA sociale proposés dans la thèse. Ces points sont expliqués dans ce qui suit.

Difficultés techniques soulevées par la construction du modèle

Chocs "de risque" financier et utilisation du capital

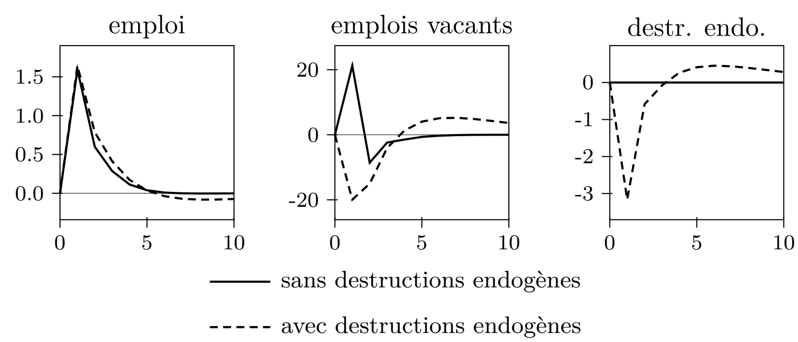
Christiano et al. (2014) identifient le choc "de risque" de leur modèle comme un contributeur majeur aux cycles d'affaires américains et européens. Pour obtenir ce résultat, leur modèle suppose, entre autres, que les ménages, propriétaires du capital, supportent des coûts de maintenance qui dépendent positivement du taux d'utilisation. Une hypothèse alternative, utilisée notamment par Greenwood et al. (1988), introduit un taux de dépréciation du capital variable en fonction du taux d'utilisation. Je trouve que cette dernière diminuerait considérablement la portée du choc "de risque" financier. En effet, il impliquerait alors une correlation négative entre investissement et taux d'utilisation, en totale contradiction avec les données.

Le mécanisme est le suivant : une hausse du risque de défaut affecte négativement l'offre de crédit, et donc la capacité d'emprunt des entrepreneurs ainsi que leur demande de capital. En conséquence, le prix du capital diminue, ce qui rend sa dépréciation moins pénalisante pour les épargnants. Le taux d'utilisation augmente alors, tandis que l'investissement chute. Le tableau ci-dessous donne les coefficients de corrélation entre investissement et taux d'utilisation du capital impliqués par le choc "de risque" dans un petit modèle calibré, en fonction de l'hypothèse retenue pour modéliser la variation du taux d'utilisation :

	corr(invest,tuc)
coût de maintenance	0.50
dépréciation variable	-0.08
données zone euro	0.86
données France	0.73

Rigidités salariales à la Pissarides (2009) et séparations endogènes

L'approche suggérée par Pissarides (2009) distingue le salaires des nouveaux embauchés et les salaires des emplois "persistants". Comme cela est expliqué ci-dessus, cette hypothèse rend les salaires moyens peu réactifs dans le cycle, sans affecter la dynamique des créations d'emplois. Cependant, les effets de cette hypothèse sont très différents dès lors que l'on inclut dans le modèle un mécanisme de séparations endogènes à la Den Haan et al. (2000). En effet, les salaires des nouveaux emplois suivent de près la dynamique du cycle d'affaires, tandis que les autres restent quasiment constants. En réponse à des chocs expansionnistes qui augmentent à la fois la demande de travail et les salaires négociés, les entreprises sont incitées à émettre moins d'offres d'emplois et, en contrepartie, à diminuer d'autant plus fortement le nombre de licenciements, de façon à accroître tout de même leur main d'œuvre. Ceci est illustré par les réponses impulsionnelles à un choc de politique monétaire obtenues pour l'emploi, le nombre d'emplois vacants et les destructions d'emplois endogènes, dans un petit modèle calibré avec frictions de type "recherche et appariement" et salaires rigides à la Pissarides, avec et sans le mécanisme de destructions d'emplois endogènes :



Bien que le remplacement de salariés chers par de nouveaux emplois moins bien payés en réponse aux fluctuations cycliques des salaires de marché soit un phénomène crédible, les prévisions quantitatives du modèle dans ce cas sont clairement contredites par les données. En outre, l'ajout d'un coût d'ajustement sur les licenciements ne semble pas suffire à régler totalement le problème. Cette solution, testée dans le modèle estimé de la France et la zone euro, a donné des résultats mitigés. Les séparations endogènes ont donc finalement été abandonnées dans sa version finale.

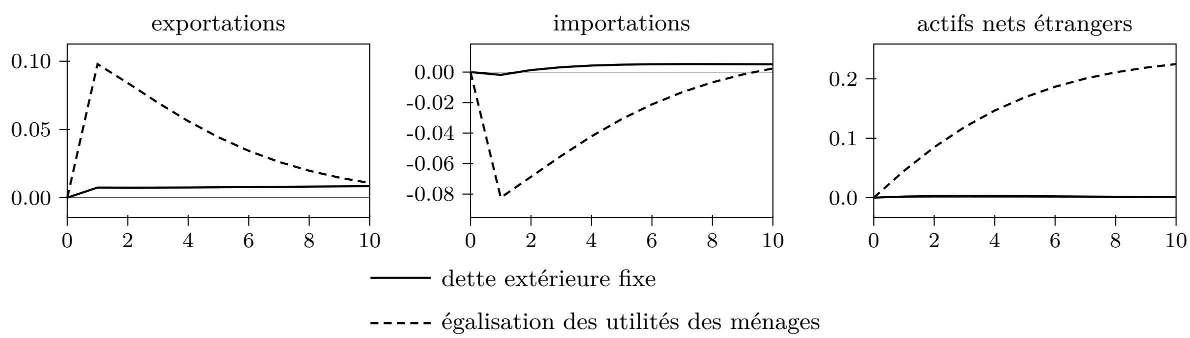
Chocs permanents dans les économies ouvertes avec marchés incomplets

Le fait que l'état stationnaire des modèles en économie ouverte avec marchés incomplets internationalement ne soit pas unique est un résultat bien connu. Une solution usuelle pour rendre ces modèles stationnaires consiste à ajouter une "prime de risque" au rendement des actifs étrangers détenus par les ménages, dépendant positivement de leur endettement net vis-à-vis de l'étranger. L'équilibre stationnaire du modèle est alors déterminé par la spécification et le calibrage de cette prime de risque.

Si les modèles DSGE servent avant tout à analyser les fluctuations cycliques de l'économie, ils sont aussi souvent utilisés pour simuler des transitions d'un état stationnaire à un autre en réponse à un stimulus permanent. Un exemple standard est la simulation de politiques fiscales prévoyant une modification permanente des taux. En présence d'une prime de risque sur les actifs étrangers, l'endettement extérieur net du pays à l'état stationnaire final est nécessairement égal à son niveau initial. En conséquence, les réponses de court terme des flux commerciaux, très influencées par les anticipations des agents à long terme, sont trouvées quantitativement faibles.

Bien que ce type d'effet soit très souvent présenté dans la littérature, l'absence d'impact à long terme des politiques sur l'endettement extérieur, impliqué par la spécification *ad hoc* de la prime de risque, est un résultat très contestable d'un point de vue théorique. Je suggère, pour simuler les effets de chocs permanents dans ce type de modèles, de choisir différemment l'état stationnaire final. En supposant que les ménages peuvent migrer d'un pays à l'autre, mais que ces migrations ne peuvent avoir lieu qu'avec un retard important, je montre qu'un état stationnaire final satisfaisant qui laisse les populations inchangées est celui qui égalise les utilités des ménages entre les pays. Je fais alors l'hypothèse que la prime de risque répond au choc pour ancrer les anticipations des agents sur cet équilibre de long terme.

Afin d'illustrer les effets de cette hypothèse alternative, je simule une mesure de TVA sociale permanente dans un petit modèle de cycles réels à deux pays. La réponse des flux commerciaux internationaux pendant la transition vers le nouvel état stationnaire est considérablement amplifiée par rapport à l'approche standard, comme illustré par les graphiques des simulations ci-dessous :



Notes : Les variables représentées sont celles du pays dans lequel la TVA sociale est mise en œuvre. Les réponses sont données en pourcentage d'écart à l'état stationnaire pour les flux commerciaux et en points de pourcentage de PIB pour les actifs nets étrangers.

Écart de production dans la règle de Taylor et croissance stochastique

Les règles de Taylor usuelles supposent que le taux d'intérêt nominal réagit à une mesure de l'écart de production. La définition de cette variable varie d'un modèle à l'autre. Certains calculent à chaque date le niveau de production qui serait observé en l'absence de frictions nominales dans l'économie, tandis que d'autres utilisent simplement la composante cyclique du PIB ou le taux de croissance du PIB. Dès lors que le modèle inclut une tendance de productivité stochastique, utiliser le niveau ou le taux de croissance du PIB "détrendé" (c'est-à-dire de la composante cyclique du PIB) à la place du taux de croissance du PIB incluant la tendance diminue l'efficacité de la règle de Taylor pour stabiliser l'inflation en réponse aux chocs technologiques permanents. L'estimation d'un tel modèle pourrait alors être délicate : en effet, soit le taux d'inflation pourrait être fortement surestimé, soit, au contraire, la volatilité du choc technologique permanent pourrait être sous-estimée, laissant une racine unitaire dans les données de cycle observées. Dans le modèle estimé de la France dans la zone euro, je fais donc l'hypothèse que le taux d'intérêt nominal répond au taux de croissance du PIB non détrendé.

Méthodologie d'estimation

Les quatre versions du modèle de la France dans la zone euro sont estimées en suivant une approche bayésienne. Contrairement à certaines estimations précédentes de modèles DSGE de grande taille, une attention particulière a été accordée aux restrictions de long terme impliquées par le modèle tout au long du processus d'estimation. J'ai aussi développé un certain nombre d'améliorations méthodologiques pour répondre aux contraintes particulières de ce projet.

Equilibre de long terme et estimation

De nombreux modèles estimés avec une approche bayésienne dans la littérature ne prennent que partiellement en compte les restrictions de long terme dans l'estimation. D'une part, les approches utilisées omettent souvent la résolution et l'examen de l'état stationnaire pour chaque jeu de paramètres structurels considéré. D'autres part, de nombreuses propriétés de long terme du modèles sont, dans de nombreux cas, seulement calibrées. Ces points sont peu discutés dans la littérature, en dehors de Del Negro and Schorfheide (2008) qui définissent des priors pour les paramètres qui conditionnent l'état stationnaire. Un certain nombre de papiers s'intéressent davantage sur un problème lié, à savoir l'identification des paramètres (see Canova and Sala (2009)).

L'approche bayésienne standard utilise en effet une approximation à l'ordre un du modèle DSGE, valide dans un voisinage de l'état stationnaire. Une fois la forme linéarisée calculée analytiquement, le modèle dynamique peut être manipulé indépendamment de son état stationnaire. Certains paramètres du modèle non-linéaire peuvent être complètement absents de la forme linéaire ; l'estimation peut alors se contenter de les calibrer ou de les ignorer. Mais il y a aussi des paramètres qui conditionnent l'état stationnaire mais qui sont aussi présents dans l'approximation linéaire. Une pratique courante pour ces derniers consiste également à les calibrer : par exemple, Smets and Wouters (2007) calibrent le taux de dépréciation du capital, le taux de markup sur les salaires ou encore le ratio des dépenses publiques (ou résiduelles) sur le PIB.

Cette procédure simplifie considérablement l'estimation mais impose des restrictions. En effet, à moins d'effectuer des tests d'identification pour ne calibrer que les paramètres non-identifiables (sans effet sur les propriétés dynamiques du modèle linéaire), autoriser l'état stationnaire à varier est susceptible d'améliorer l'ajustement aux données. Ceci est une première raison pour estimer tous les paramètres. La seconde raison, théorique, a trait au paradigme bayésien. Celui-ci suppose en effet que tous les paramètres sont des variables aléatoires caractérisées par des densités de probabilités, n'ayant donc pas de valeur numérique unique que l'estimation chercherait à révéler. De ce point de vue, calibrer les paramètres revient à choisir une densité à priori dégénérée de variance zero ou, autrement dit, à indiquer un niveau de confiance absolu dans la valeur choisie. Il est donc préférable d'utiliser des priors non-dégénérés, même si les données utilisées ne sont pas informatives pour les paramètres concernés. Dans ce cas, les distributions conditionnelles à posteriori trouvées seront identiques aux distributions a priori. En revanche, les estimations a posteriori des paramètres affectant à la fois l'état stationnaire et les propriétés cycliques du modèle seront mises à jour par l'estimation.

Estimer rigoureusement les paramètres qui affectent l'état stationnaire nécessite de résoudre numériquement le modèle statique dès que les paramètres varient, ce qui est très coûteux en temps de calcul. Si on omet cette étape en n'utilisant que la forme linéarisée du modèle, on prend le risque de considérer des jeux de paramètres pour lesquels l'état stationnaire soit n'existe pas, soit a des propriétés irréalistes.

Afin d'éviter ce problème dans l'estimation du modèle de la France dans la zone euro, l'estimation suit une approche rigoureuse en résolvant l'état stationnaire à chaque fois que la vraisemblance de la forme linéaire du modèle est calculée. Ce faisant, on peut vérifier son existence et s'assurer qu'un certain nombre de ratios de long terme restent cohérents.

Contributions méthodologiques

Les séries de données macroéconomiques incluent souvent des observations manquantes (dues soit aux délais d'établissement des derniers points, ou simplement parce que certaines variables n'étaient pas mesurées dans le passé), ou des séries de fréquence plus basse. Dans le cas présent, le problème est d'observer, pour estimer un modèle trimestriel, une série annuelle d'heures travaillées pour la zone euro. En effet, sans frictions sur le marché du travail, le modèle ne rend compte que des fluctuations du nombre total d'heures travaillées, et non pas de l'emploi à proprement parler. Si les séries trimestrielles d'emploi dans la zone euro sont disponibles avec suffisamment d'antériorité, le nombre total d'heures travaillées n'était, au moment de la constitution de ma base de données, disponible que depuis 2000 et en fréquence annuelle.[17] Je décris donc une modification du filtre de Kalman qui permet d'inclure autant de points manquants que l'on souhaite dans les séries observées.[18]

Par ailleurs, l'estimation de modèles avec de nombreuses variables observables soulève la question du choix des chocs structurels supposés à l'origine du cycle d'affaires. Quand on utilise des méthodes en information complète, le nombre de chocs doit être au minimum égal au nombre d'observables. D'après mon expérience de l'estimation, cette contrainte peut parfois contribuer à détériorer les propriétés dynamiques des modèles DSGE estimés. Ceci est lié au fait que certains chocs ont évidemment des implications contrefactuelles sur certaines variables. Pourtant, l'estimation de modèles mal spécifiés – ils le sont tous plus ou moins – est susceptible de faire ressortir ce type de chocs pour la simple raison qu'ils captent, à la façon de résidus économétriques, l'écart entre les

[17] Cette série en fréquence trimestrielle existe désormais et pourrait être utilisée pour une mise à jour ultérieure de mes estimations.

[18] Cette contribution, de même que de nombreux développements du modèle, est le fruit d'un travail joint avec Stéphane Adjemian. Elle a depuis été incorporée au logiciel Dynare.

données et les projections du modèle. J'illustre ce type d'effets au travers de deux expérimentations numériques, qui plaident en faveur de l'utilisation d'erreurs de mesure pour l'estimation bayésienne des modèles DSGE de taille moyenne à grande.

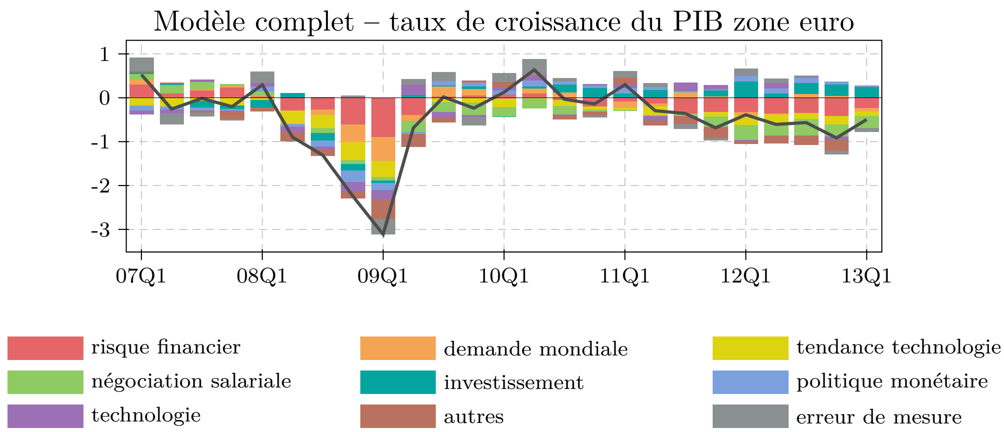
Enfin, l'estimation de “gros” modèles avec le logiciel Dynare soulève un certain nombre de problèmes computationnels. La résolution analytique complète du modèle statique n'est pas possible, mais, à l'inverse, une résolution numérique complète est source d'instabilité et, surtout, entraine des temps de calcul excessifs. Dans ce travail, je développe une approche hybride, dans laquelle la résolution numérique est limitée à des systèmes d'équations de petite taille, et est effectuée par des programmes compilés. Une autre difficulté est liée à la recherche d'un maximum d'une fonction cible dépendant de nombreux paramètres (vraisemblance ou densité postérieure). Souvent, les approches standard basées sur le calcul de gradients sont instables et nécessitent un grand nombre d'interventions manuelles. Les améliorations techniques proposées dans le cadre de ce travail ont contribué à améliorer la stabilité et la vitesse de la phase d'optimisation numérique.

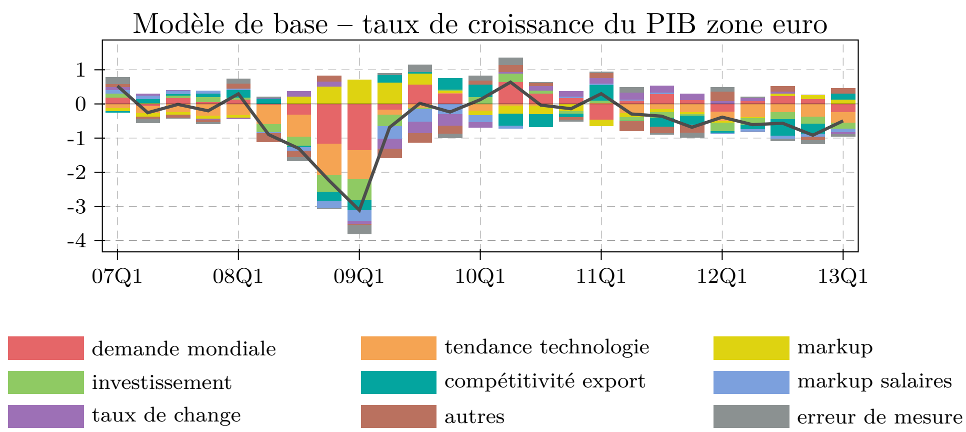
Le rôle des frictions pour l'analyse conjoncturelle et l'évaluation des politiques

Dans cette thèse, je montre qu'inclure des frictions sur le marché du travail et des frictions financières dans un modèle DSGE apporte des améliorations déterminantes pour l'utilisation en analyse conjoncturelle ou en évalution de politiques économiques. Pour cela, je soumets les quatre versions du modèle à différents exercices de simulation. La version complète est considérée comme la représentation la plus crédible de la réalité parmi les quatre, dans la mesure où elle tient compte des imperfections qui influent sur le fonctionnement des marchés du travail et du crédit réels. Ainsi, les différences avec les simulations obtenues à partir des autres modèles révèlent les biais introduits dans l'analyse par l'omission des frictions correspondantes.

Décomposition historique en contributions des chocs

Je compare les décompositions historiques en contributions des chocs sur la période 2007-2013 comprenant la crise, obtenues à partir des quatre versions du modèle. Tout d'abord, omettre les frictions sur le marché du travail conduit à surévaluer les contributions du commerce extérieur et des markups aux fluctuations du PIB et de l'emploi. Ces chocs jouent en effet le rôle de “formes-réduites” dans le modèle, captant des variations dans les données qui seraient expliquées différemment si le marché du travail était spécifié de manière plus réaliste. La présence de frictions financières révèle la contribution des chocs “de risque” aux fluctuations cycliques de l'investissement, du PIB, de l'inflation et, dans une moindre mesure, de l'emploi. Si on les omet, la persistence de la crise est captée dans une plus large proportion par les chocs technologiques permanents, mettant davantage en avant une explication de la recession basée sur des mécanismes d'offre. En guise d'illustration, les histogrammes ci-dessous représentent les décompositions obtenues pour le PIB de la zone euro avec le modèle complet et avec le modèle de base sans les frictions :





L'analyse révèle également certains gains liés à la prise en compte simultanée des deux types de frictions. Ainsi, la contribution aux fluctuations de l'emploi des chocs "de risque" financier, visible avec le modèle complet, n'est plus significative si on omet les frictions sur le marché du travail.

Politique monétaire et bien-être

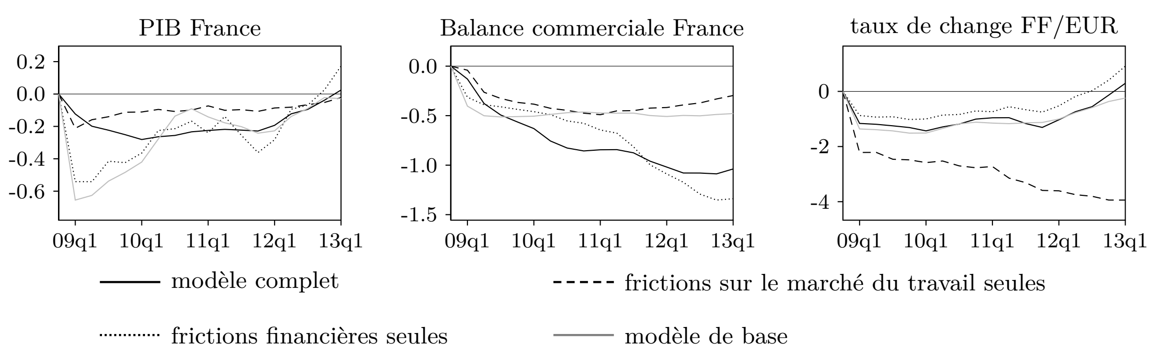
Dans un deuxième temps, je calcule le coût en bien-être du cycle d'affaires dans les deux régions (France et reste de la zone euro) à l'aide des quatre versions du modèle. Les valeurs trouvées, entre 1% et 2.2% du niveau de consommation permanent, sont en ligne avec les estimations de la littérature récente, mais bien supérieures à celles de Lucas (1987). Les frictions financières et, dans une moindre mesure, les frictions sur le marché du travail ajoutent un coût significatif. Les résultats suggèrent aussi que le coût en bien-être lié aux imperfections sur le marché du travail est légèrement plus élevé en France que dans le reste de la zone euro. Je teste ensuite les effets sur ce coût de différents calibrages du coefficient de réponse à l'inflation dans règle de Taylor. Les résultats sont donnés dans le tableau ci-dessous :

	France			reste de la zone euro		
	règle estimée	$r_\pi = 1.05$	$r_\pi = 3$	règle estimée	$r_\pi = 1.05$	$r_\pi = 3$
modèle de base	-1.04	-1.30	-1.04	-1.10	-1.43	-1.09
frict. financières	-2.03	-3.15	-2.14	-2.21	-2.90	-2.42
frict. marché travail	-1.46	-1.91	-1.45	-1.28	-1.42	-1.33
modèle complet	-1.12	-3.36	-1.28	-1.33	-2.80	-1.63

Cet exercice suggère que l'autorité monétaire n'est pas en mesure d'améliorer sensiblement le bien-être dans la zone euro en ne répondant qu'à l'inflation contemporaine.

Simulation contrefactuelle d'un scénario de sortie de la France de la zone euro

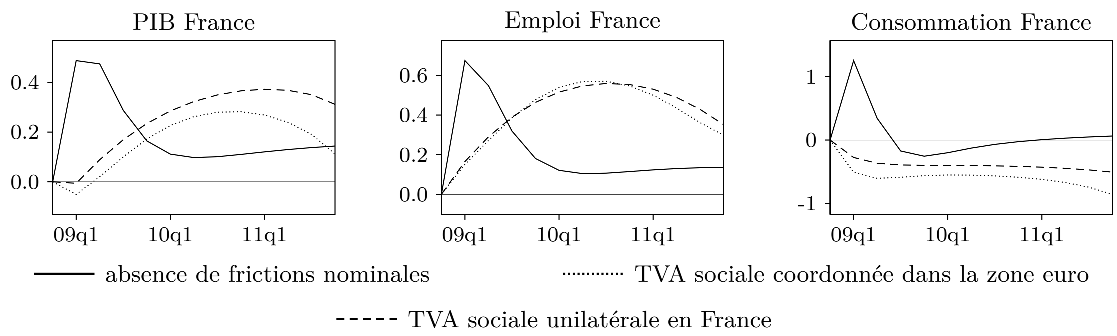
Je simule avec les quatre versions du modèle les effets d'un passage à un régime de change flexible entre la France et le reste de la zone euro. Le premier résultat est l'absence d'effet sur le coût en bien-être des fluctuations cycliques, quelle que soit la version du modèle considérée. Ensuite, une simulation contrefactuelle de la crise financière en supposant que la France a quitté la zone euro au premier trimestre de 2009 fait apparaître une légère appréciation du Franc contre l'euro. Il en découle une légère détérioration de la situation de l'économie française. L'absence de frictions financières ou sur le marché du travail déforme significativement ces simulations. L'absence de frictions sur le marché du travail amplifie les effets à court terme de la sortie de l'euro sur le PIB et l'emploi, tandis que l'absence de frictions financières, dans le modèle conservant les frictions sur le marché du travail, fait apparaître une plus forte appréciation du franc mais des effets réels plus faibles en France.



Notes : Les variables sont représentées en écart aux réalisations observées sur la période. Le PIB et le taux de change franc/euro sont en pourcentage et la balance commerciale en points de pourcentage de PIB.

TVA sociale

Les effets d'une mesure transitoire de TVA sociale sont simulés dans le contexte de la crise de 2009. On donne pour objectif à cette politique de réduire l'écart d'emploi résultant de la présence de frictions nominales dans les économies de la zone et on en fixe la durée à 3 ans. Les simulations sont données pour le PIB, l'emploi et la consommation françaises sur les graphiques suivants :

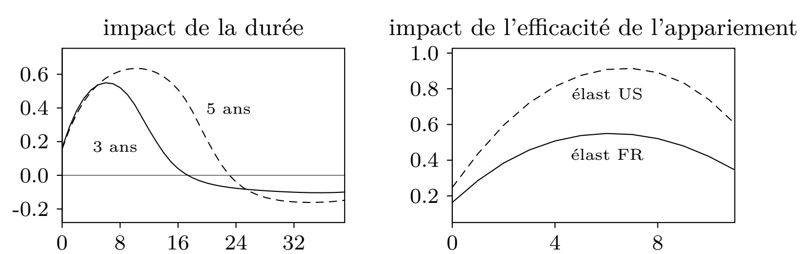


Note : Les variables sont représentées en écart en pourcentage aux réalisations observées sur la période.

Les bénéfices de la mesure sont retardés et limités du fait de la présence de nombreuses frictions, particulièrement des imperfections du marché du travail. En effet, avec le mécanisme de recherche et d'appariement, l'ajustement de l'emploi est retardé. Les entreprises suivent une statégie de recrutement à long terme, comparable à des choix d'investissement en capital productif. En conséquence, la durée de la mesure conditionne fortement l'amplitude de ses effets à court terme. Par ailleurs, l'estimation fait apparaître un degré important d'inefficacité de l'appariement en France et dans le reste de la zone euro, qui réduit fortement les effets des baisses du coût du travail en termes de créations d'emplois effectives.

Cet exercice montre aussi qu'une politique unilatérale de TVA sociale en France est préférable à une mise en œuvre coordonnée entre les pays de la zone euro.

L'efficacité de la TVA sociale pourrait être significativement améliorée. On peut faire pour cela deux recommandations en s'appuyant sur cette analyse. D'abord, les mesures devraient être maintenues pendant une période suffisamment longue, au moins 5 ans, cette durée devant être jugée crédible par les agents économiques. Ensuite, la mise en œuvre de réformes structurelles du fonctionnement du marché du travail devrait précéder les baisses de coût du travail, de façon à renforcer les effets de celles-ci. Ces réformes viseraient l'amélioration de l'efficacité des mécanismes d'appariement entre offre et demande de travail, très nettement inférieure à celle du marché du travail américain. Les graphiques ci-dessous illustrent ce point en montrant comment la réponse de l'emploi à la TVA sociale en France est augmentée lorsque la durée passe de 3 ans à 5 ans d'une part, et lorsqu'on fixe l'élasticité de l'appariement au nombre d'emplois vacants à la valeur utilisée dans les modèles de l'économie américaine d'autre part :



Note : La réponse de l'emploi est représentée en écart en pourcentage au scénario central.

Summary

Thanks to their internal consistency, DSGE models, built on microeconomic behavior, have become prevalent for business cycle and policy analysis in institutions. The recent crisis and governments' concern about persistent unemployment advocate for mechanisms capturing imperfect adjustments in credit and labor markets. However, popular models such as the one of Smets and Wouters (2003, 2007), although unsophisticated in their representation of these markets, are able to replicate the data as well as usual econometric tools. It is thus necessary to question the benefits of including these frictions in theoretical models for operational use.

In this thesis, I address this issue and show that microfounded mechanisms specific to labor and credit markets can significantly alter the conclusions based on the use of an estimated DSGE model, from both a positive and a normative perspective.

For this purpose, I build a two-country model of France and the rest of the euro area with exogenous rest of the world variables, and estimate it with and without these two frictions using Bayesian techniques. By contrast with existing models, I propose two improvements of the representation of labor markets. First, following Pissarides (2009), only wages in new jobs are negotiated by firms and workers, engendering stickiness in the average real wage. Second, I develop a set of assumptions to make labor market participation endogenous and unemployment involuntary in the sense that the unemployed workers are worse-off that the employed ones. Yet, including this setup in the estimated model is left for future research.

Using the four estimated versions of the model, I undertake a number of analyses to highlight the role of financial and labor market frictions: an historical shock decomposition of fluctuations during the crisis, the evaluation of several monetary policy rules, a counterfactual simulation of the crisis under the assumption of a flexible exchange rate regime between France and the rest of the euro area and, lastly, the simulation of social VAT scenarios.

Keywords: business cycle, Bayesian estimation, macroeconomic policy, unemployment, monetary union, labor force.

JEL classification: C11, E24, E32, E60, F45, J21.

Résumé

L'utilisation de modèles DSGE, construits à partir de comportements micro-fondés des agents économiques, s'est progressivement imposée aux institutions pour l'analyse macroéconomique du cycle d'affaires et l'évaluation de politiques, grâce à leur cohérence interne. La crise financière récente et la préoccupation que représente la persistance du chômage à un niveau élevé plaident en faveur de modèles qui tiennent compte des ajustements imparfaits de l'offre et de la demande sur les marchés du crédit et du travail. Pourtant, des modèles relativement rudimentaires dans leur représentation de ces marchés, comme celui de Smets et Wouters (2003, 2007), reproduisent aussi bien les données que des modèles économétriques usuels. On peut donc légitimement s'interroger sur l'intérêt de prendre en compte ces frictions dans la spécification des modèles théoriques destinés à l'analyse économique opérationnelle.

Dans cette thèse, je réponds à cette question en montrant que l'inclusion de mécanismes microfondés spécifiques aux marchés du crédit et du travail peut modifier très significativement les conclusions obtenues à partir d'un modèle DSGE estimé, tant d'un point de vue positif que normatif.

Pour cela, je construis un modèle à deux pays de la France et du reste de la zone euro, avec un reste du monde exogène, et l'estime avec et sans ces deux frictions, en utilisant une approche bayésienne. Par rapport aux modèles existant dans la littérature, je propose deux améliorations à la spécification du marché du travail. Premièrement, suivant Pissarides (2009), le salaire réel moyen est rendu rigide en supposant que seuls les nouveaux employés renégocient leur rémunération. Deuxièmement, le taux de participation sur le marché du travail est rendu endogène et le chômage involontaire, dans le sens où le bien-être des chômeurs est inférieur à celui des employés. L'inclusion de ce dernier mécanisme dans le modèle estimé fera cependant l'objet de travaux futurs.

Afin de mettre en évidence les effets des frictions sur les marchés du crédit et du travail, je soumets les quatre versions estimées du modèle à plusieurs exercices : une analyse en contributions des chocs structurels pendant la crise, l'évaluation de différentes règles de politique monétaire, la simulation contrefactuelle de la crise sous l'hypothèse d'un régime de change flexible entre la France et le reste de la zone euro et, enfin, la simulation de variantes de TVA sociale.

Mots clés : cycle d'affaires, estimation bayésienne, politiques économiques, chômage, union monétaire, population active.

Codes JEL : C11, E24, E32, E60, F45, J21.