Scheduling Handling Resources: Robotic Flowshops with Circular Layouts and a Practical Railway Problem

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Jury members

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Scheduling and transportation

Scheduling: assigning *tasks* to *resources* submitted to *constraints* to optimize a *performance criteria*

Physical handling or transportation between resources: connectivity, layout, additional specific handling resources
Scheduling and transportation

Scheduling: assigning *tasks* to *resources* submitted to *constraints* to optimize a *performance criteria*

Physical handling or transportation between resources: connectivity, layout, additional specific handling resources

Two contexts:

- **Manufacturing management**

- **Railway management**
“Trains don’t vanish”...except when they do.
EURO/Roadef Challenge 2014, SNCF
“Trains don’t vanish”...except when they do.
EURO/Roadeff Challenge 2014, SNCF

- Resource capacity
- Resource compatibility
- Conflict between trains
- ...

Railway management
“Trains don’t vanish”...except when they do.
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Optimize departure coverage and performance costs
“Trains don’t vanish”...except when they do.
EURO/Roadeff Challenge 2014, SNCF

Horizon: 14 days
200–2000 departures/arrivals
30–90 resources

- Resource capacity
- Resource compatibility
- Conflict between trains
- ...

Optimize departure coverage and performance costs
Joint work with H. Joudrier [Joudrier and T. 17]

- Simplify the station’s graph by grouping similar resources
- Sophisticated routing algorithm
- (very) simple assignment algorithm
- Up to 40% covered departures...

... And the 1st place in the junior category!
Flowshop: the parts must be processed on machine $M_1$, then $M_2$...
Manufacturing management

Flowshop: the parts must be processed on machine $M_1$, then $M_2$...

$M_1$  $M_2$  $M_3$

Sufficient model?
Manufacturing management

Flowshop: the parts must be processed on machine $M_1$, then $M_2$...

Sufficient model? Operator, tools, handling resources…
Robotic cell – Example

In/Out

M₁

M₂

M₃

In/Out
Robotic cell – Example

“Short processing times, slow robot”
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In/Out

$M_1$

$M_2$

$M_3$

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“Long processing times, fast robot”
And in-between?

That’s where the problem is

Balancing

- Total travel time
- Waiting time
Given the *processing times* and *travel time between machines*, find a robot programmation which optimizes the *throughput*:

**Maximize** \( \frac{k}{\text{Time to produce } k \text{ parts}} \)

**Minimize** \( \frac{\text{Time to produce } k \text{ parts}}{k} \)
Models: Layouts

Linear

Circular
Models: cell parameters

Waiting policy
- No-wait [Agnetis, 00; Kats et al; 09, Che et al, 12]
- Unbounded [Crama et al, 97; Rajapakshe et al., 11]
- Time-window (HSP) [Dawande et al, 09; Zhou et al, 12]

Travel metric
- General
- Additive
  - regular
- Constant

Processing times
- Identical parts
  - General
  - Balanced
Machine and robot capacity

Classical model (blocking)

Few studies for the circular layout [Rajapakshe et al., 11]

Relaxing the blocking constraint

- Dual-gripper [Sethi et al., 01 ; Jung et al., 15 ; Drobouchevitch et al., 06]
- Swapping [Jolai et al.]
- Machine buffers [Drobouchevitch et al., 06]
Describing the robot moves: Activities

Activity $A_i$

[Crama et al, 97]
Describing the robot moves: Activities

Activity $A_i$
- Go to $M_i$

[Crama et al, 97]
Describing the robot moves: Activities

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- Wait?

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Describing the robot moves: Activities

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Activity $A_i$
- Go to $M_i$
- Wait?
- Unload $M_i$
Describing the robot moves: Activities

[Crama et al, 97]

Activity $A_i$
- Go to $M_i$
- Wait?
- Unload $M_i$
- Go to $M_{i+1}$
Describing the robot moves: Activities

[Crana et al, 97]

Activity $A_i$

- Go to $M_i$
- Wait?
- Unload $M_i$
- Go to $M_{i+1}$
- Load $M_{i+1}$
Describing the robot moves: Activities

Activity $A_i$
- Go to $M_i$
- Wait?
- Unload $M_i$
- Go to $M_{i+1}$
- Load $M_{i+1}$

$A_0, A_1, \ldots, A_m$

In/Out

$M_1$, $M_2$, $M_3$
Dominance [Dawande et al, 05]

**Cycle**
- Feasible sequence of activities
- Leaves the cell in the same state
Cyclic programation

Dominance [Dawande et al., 05]

**Cycle**

- Feasible sequence of activities
- Leaves the cell in the same state
- \( k \)-cycle: produces \( k \) parts
- **1-cycle**: produces 1 part
Cyclic programmation

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1-cycles $\Leftrightarrow$ permutations of the $m$ activities $A_1 \ldots A_m$
Cyclic programmation

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1-cycles \(\iff\) permutations of the \(m\) activities \(A_1 \ldots A_m\)

Objective function
\[
\text{max throughput} \iff \min \frac{\text{cycle time}}{\text{number of parts produced}} \iff \min \frac{T(C_k)}{k}
\]
Best 1-cycle problem

1-cycles are...
- easy to describe
- easy to implement
- better known and more studied

But not necessarily optimal!
1-cycles are...
- easy to describe
- easy to implement
- better known and more studied

Finding the best 1-cycle...
- ...in linear regular cells: P (balanced [Brauner et al., 99])
- ...in circular regular cells: NP-hard ($\frac{5}{3}$ approx) [Rajapakshe et al., 11]
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Finding the best 1-cycle...
- ...in linear regular cells: P
  (balanced [Brauner et al., 99])
- ...in circular regular cells: NP-hard
  \( \left( \frac{5}{3} \approx \right. \) approx).
- ...in circular regular balanced cells: ??
Dominance?

1-cycle conjecture
1-cycles dominate all cycles

[Sethi et al., 92]
1-cycle conjecture

1-cycles dominate all cycles  
[Sethi et al., 92]

Linear layout

- Valid for regular cells, 2- to 3-machine  
[Crama et al., 97]
## 1-cycle conjecture

1-cycles dominate all cycles

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## Linear layout

- Valid for regular cells, 2- to 3-machine
  - [Crama et al., 97]

- False for regular cells, 4-machine
  - [Brauner et al., 01]
### 1-cycle conjecture

1-cycles dominate all cycles  \[\text{[Sethi et al., 92]}\]

### Linear layout

- Valid for regular cells, 2- to 3-machine  \[\text{[Crama et al., 97]}\]
- False for regular cells, 4-machine  \[\text{[Brauner et al., 01]}\]
- Valid for regular balanced cell up to 15 machines  \[\text{[Brauner, 08]}\]
### 1-cycle conjecture
1-cycles dominate all cycles

[Sethi et al., 92]

### Linear layout
- Valid for regular cells, 2- to 3-machine
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- False for regular cells, 4-machine
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- **Valid for regular balanced cell up to 15 machines**
  [Brauner, 08]

### Circular layout
???
## Dominance?

### 1-cycle conjecture

1-cycles dominate all cycles  

[Sethi et al., 92]

### Linear layout

- Valid for regular cells, 2- to 3-machine  
  [Crama et al., 97]
- False for regular cells, 4-machine  
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- Valid for regular balanced cell up to 15 machines  
  [Brauner, 08]

### Circular layout

Counter-example for regular balanced cells with 6 machines  

[T. et al. MIM 16]
Circular, identical parts, regular, balanced: \((m, \delta, p)\)

Small enough to fit in a Shadok’s head!
Representing the robot moves

![Diagram of a graph with machines and time on the x-axis, showing the movement of the robot.](image)
Representing the robot moves

IN

OUT

machines

0 2 4 6

time

$A_1$

$M_1$ $M_2$ $M_3$ $M_4$

In/Out
Representing the robot moves

machines

OUT

4

3

2

1

IN

0

2

4

6

time

$A_1$

$M_1$

$M_2$

$M_3$

$M_4$

In/Out
Representing the robot moves
Representing the robot moves

machines

IN

OUT

0 2 4 6

time

\(A_1 A_0\)
Representing the robot moves

\[ A_1 A_0 \]
Representing the robot moves

\[ \text{machines} \]

OUT

\[ \begin{array}{c}
0 & 1 & 2 & 3 & 4 \\
\hline
\end{array} \]

IN

0 2 4 6

time

\[ A_1 A_0 \]

\[ \begin{array}{c}
M_1 & M_2 & M_3 & M_4 \\
\hline
\end{array} \]

In/Out
Representing the robot moves

\[ A_1 A_0 \]
Representing the robot moves

- Machines
  - IN
  - OUT

- Time

- Graph with points: M1, M2, M3, M4
  - M1
  - M2
  - M3
  - M4

- Arrows indicating movement

- Equation: \( A_1 A_0 \)
Representing the robot moves

\[ A_1 A_0 A_4 \]
Representing the robot moves

\[ A_1 A_0 A_4 \]
Representing the robot moves

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Representing the robot moves

\[A_1A_0A_4\]
Representing the robot moves

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Representing the robot moves

A_1 A_0 A_4
Some usual 1-cycles: Identity cycle

Identity cycle $\pi_{id}$

$\pi_{id}: A_0 A_1 \ldots A_m$

Cycle time

$T(\pi_{id}) = (m + 1)\delta + mp$
Some usual 1-cycles: Downhill cycle

**Downhill cycle** $\pi_d$

$\pi_d: A_0A_mA_{m-1} \ldots A_1$

**Cycle Time**

$$T(\pi_d) = 3(m + 1)\delta + \max(0, p - (3m - 1)\delta)$$
Some usual 1-cycles: Odd-even cycle

Odd-Even cycle $\pi_{oe}$

$\pi_{oe}: A_0 A_2 \ldots A_1 A_3 \ldots$

Cycle time

$T(\pi_{oe}) = ??$
Odd-even’s cycle time

\[ T(\pi_{oe}) = 2(m + 1)\delta + \max\left(0, \frac{2\alpha - 1}{\alpha} (p - (m + 1)\delta)\right) \]

\[ \alpha = \left\lfloor \frac{m + 1}{2} \right\rfloor \leq 2p \]
Odd-even’s cycle time

\[ T(\pi_{oe}) = 2(m + 1)\delta + \max \left( 0, \frac{2\alpha - 1}{\alpha} (p - (m + 1)\delta) \right) \quad \alpha = \left\lfloor \frac{m + 1}{2} \right\rfloor \leq 2p \]
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\[ \leq 2p \]
Odd-even’s cycle time

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\]

\[
\leq 2p
\]
Odd-even’s cycle time

$$T(\pi_{oe}) = 2(m + 1)\delta + \max \left(0, \frac{2\alpha - 1}{\alpha} (p - (m + 1)\delta)\right) \quad \alpha = \lfloor \frac{m+1}{2} \rfloor \leq 2p$$

$$20 + 3 \max (0, (p - 5\delta))$$
Odd-even’s cycle time

\[ T(\pi_{oe}) = 2(m + 1)\delta + \max \left( 0, \frac{2\alpha - 1}{\alpha} (p - (m + 1)\delta) \right) \]

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\[ 20 + 3 \max \left( 0, (p - 5\delta) \right) \]

\[ \frac{20 + 3 \max \left( 0, (p - 5\delta) \right)}{2} \]
Finding the best 1-cycle

Identity

Odd-Even

Downhill

Which is the best 1-cycle?
Best 1-cycles?

Lower bounds

\[ \delta = 1, \ m = 6 \]
**Best 1-cycles?**

Lower bounds

\[ \text{LB}_1 \] \cite{Crama, 97}

\[ T(\pi) \geq p + 4\delta \]
**Best 1-cycles?**

Lower bounds

\[ LB_1 \text{ [Crama, 97]} \]
\[ T(\pi) \geq p + 4\delta \]

\[ LB_2 \text{ [Dawande et al, 02]} \]
\[ T(\pi) \geq (m + 1)\delta + m \min(\delta, p) \]
Best 1-cycles?

**Lower bounds**

- **$LB_1$ [Crama, 97]**
  \[ T(\pi) \geq p + 4\delta \]

- **$LB_2$ [Dawande et al, 02]**
  \[ T(\pi) \geq (m + 1)\delta + m \min(\delta, p) \]

- **$LB_3$ (1-cycles only)**
  \[ T(\pi) \geq 2(m + 1)\delta \]

**Graph:**
- $LB_1$, $LB_2$, $LB_3$ vs. $p$ for $\delta = 1$, $m = 6$
Best 1-cycles?

Lower bounds

- $LB_1$ [Crama, 97]: $T(\pi) \geq p + 4\delta$
- $LB_2$ [Dawande et al., 02]: $T(\pi) \geq (m + 1)\delta + m \min(\delta, p)$
- $LB_3$ (1-cycles only): $T(\pi) \geq 2(m + 1)\delta$

Diagram:

- $\delta = 1$, $m = 6$
- Graph with time vs. $p$ showing $LB_1$, $LB_2$, $LB_3$, and $\pi_{id}$.
Best 1-cycles?

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\[ T(\pi) \geq 2(m + 1)\delta \]

\( \delta = 1, \ m = 6 \)
The unknown region

\[ \delta = 1, \ m = 6 \]

Best 1-cycle

\[ p \leq \frac{m+1}{m} \delta \]

\[ \pi_{id} \]
The unknown region

\[ \delta = 1, \ m = 6 \]

Best 1-cycle

- \( p \leq \frac{m+1}{m} \delta \)
- \( \pi_{id} \)
- \( p > \frac{m+1}{m} \delta \)
- \( p \leq (m + 1) \delta \)
- \( \pi_{oe} \)
The unknown region

Best 1-cycle

\[ p \leq \frac{m+1}{m} \delta \]
\[ \pi_{id} \]

\[ p > \frac{m+1}{m} \delta \]
\[ p \leq (m + 1)\delta \]
\[ \pi_{oe} \]

\[ p \geq (3m - 1)\delta \]
\[ \pi_d \]
The unknown region

\[ \delta = 1, \quad m = 6 \]

Best 1-cycle

- \( p \leq \frac{m+1}{m} \delta \)
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- \( p \leq (m + 1)\delta \)
- \( \pi_{oe} \)
- \( p \geq (3m - 1)\delta \)
- \( \pi_d \)

between \( (m + 1)\delta \) and \( (3m - 1)\delta \)?
Exploring the unknown region

Best 1-cycles: \( \{\pi_{id}, \pi_{oe}, \pi_d\} + ?? \)

Introducing \( \pi^* \):

- \( \pi^* \) is a 1-cycle
- \( \pi^* \) crosses the yellow area
  (meaning: for some \((m + 1)\delta \leq p \leq (3m - 1)\delta\),
  \( \pi^* \) does strictly better than both \( \pi_{oe} \) and \( \pi_d \)
Best 1-cycles: \( \{\pi_{id}, \pi_{oe}, \pi_d\} + ?? \)

**Introducing \( \pi^* \)**
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  \( \pi^* \) does strictly better than both \( \pi_{oe} \) and \( \pi_d \))

**Who is \( \pi^* \)?**
What can we say about it? Does it exist?
Is there anybody in there?

Size of a minimum dominant set within 1-cycles:

<table>
<thead>
<tr>
<th>$m$</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>cardinal</td>
<td>3</td>
<td>4</td>
<td>4</td>
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Proven: $\{\pi_{id}, \pi_{oe}, \pi_d\} \cup ?$
Is there anybody in there?

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Proven computed

$\{\pi_{id}, \pi_{oe}, \pi_d\} \cup ?$

[Brauner, 99]
Is there anybody in there?

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\{\pi_{id}, \pi_{oe}, \pi_{d}\} \cup ?

Properties on the total travel time
**Notations (last ones...)**

- $\Delta(\pi)$: total travel time
- $d_i(\pi)$: travel time between the loading and unloading of machine $M_i$
- $d_{min}(\pi)$: $\min d_i(\pi)$
- $d_{min}(\pi_{oe}) = (m + 1)\delta$

**Lower bound $LB_4(\pi)$**

$$T(\pi) \geq \underbrace{\Delta(\pi)}_{\text{Total travel time}} + \underbrace{\max(0, p - d_{min}(\pi))}_{\text{Minimum waiting time}}$$

$d_{min}$ is the minimum value of $p$ for which waiting is necessary.
If $\pi^*$ exists:

- The robot travels between 2 and 3 times the size of the cell:

$$2(m + 1)\delta < \Delta(\pi^*) < 3(m + 1)\delta$$

- No waiting time if $p$ is “small” relatively to the travel time:

$$d_{min}(\pi^*) > \Delta(\pi^*) - \frac{3\alpha - 2}{2\alpha - 1}(m + 1)\delta$$
First properties

\[ \delta = 1, \ m = 6 \]
First properties

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First properties

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- \( LB_1 \)
- \( LB_3 \)
- \( \pi_d \)
- \( \pi_{oe} \)
- \( LB_4 (\pi^*) \)
First properties

\[ \delta = 1, \ m = 6 \]
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<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

proven

computed

And for $m > 8$?
Cardinal of a minimum dominant set within 1-cycles:

<table>
<thead>
<tr>
<th>$m$</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>cardinal</strong></td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
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proven
computed

And for $m > 8$?
For $m > 8$, this guy...

...always (slightly) crosses the yellow area
Two wavelets ($\pi_{2w}$)...

For $m > 8$, this guy...

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Two wavelets \((\pi_{2w})\)...

For \(m > 8\), this guy...

\[ d_{\text{min}}(\pi_{2w}) = (m + 5)\delta \]

(One turn and one wave)

...always (slightly) crosses the yellow area
Two wavelets ($\pi_{2w}$)...

\[ \delta = 1, \quad m = 10 \]
Two wavelets ($\pi_{2w}$)...

![Graph showing LB₁, LB₃, πₚ, π₀ₑ, and π₂w with δ = 1, m = 10]
For $m = 12$ and $m = 13$...
For $m = 12$ and $m = 13$...
For \( m = 12 \) and \( m = 13 \).
Three wavelets \((\pi_{3w})\)...

For \(m = 12\) and \(m = 13\)...

\[
d_{\text{min}}(\pi_{3w}) = (m + 9)\delta
\]
Three wavelets \( (\pi_{3w}) \)...

For \( m = 12 \) and \( m = 13 \)...

\[
d_{\text{min}}(\pi_{3w}) = (m + 9)\delta
\]

\( (\text{One turn and two waves}) \)
Three wavelets $\left( \pi_{3w} \right)$...

For $m = 12$ and $m = 13$...
Three wavelets \((\pi_{3w})\)...
Three wavelets ($\pi_{3w}$)...

For $m = 12$ and $m = 13$...
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Three wavelets \( (\pi_{3w}) \)...

For \( m = 12 \) and \( m = 13 \)...

\[ d_{\min}(\pi_{3w}) = (m + 9)\delta \]

(One turn and two waves)
Three wavelets ($\pi_{3w}$)...

For $m = 12$ and $m = 13$...
False for $m \geq 14$!

$$d_{\text{min}}(\pi_{3w}) = (m + 9)\delta$$
(One turn and two waves)
$d_{\min}(\pi_{nw}) = (m + 2n + 1)\delta$

(One turn and $\frac{n}{2}$ waves)

(here for $n$ even)
**Proposition:** best cycles within $\{\pi_{id}, \pi_d, \pi_{oe}\} \cup (\pi_{nw})_n$

- **Three** classical 1-cycles $\pi_{id}, \pi_d, \pi_{oe}$
- **One** $\pi_{nw}$ with the highest even value of $n$ possible
  - And/or
  - **One** $\pi_{nw}$ with the highest odd value of $n$ possible

**Conjecture:** best 1-cycle

These also dominate all 1-cycles

Proven for $m \leq 11$
Proof ideas

- 2 “turns”

\[ T(\pi^*) < 2p \]
(otherwise, dominated by \( \pi_{oe} \))

\[ T(\pi^*) < 3(m + 1)\delta \]
(otherwise, dominated by \( \pi_d \))
Proof ideas

- 2 “turns”
- \( A_i \) and \( A_{i+1} \) can’t be on the same turn in that order

\[
\begin{align*}
\Delta & \geq 2(m + 1)\delta \\
p & \geq (m + 1)\delta
\end{align*}
\]

\[
T(\pi^*) < 2p \\
\text{(otherwise, dominated by } \pi_{oe}) \\
T(\pi^*) < 3(m + 1)\delta \\
\text{(otherwise, dominated by } \pi_d)
\]
Proof ideas

- 2 “turns”
- $A_i$ and $A_{i+1}$ can’t be on the same turn in that order
- no subsequence $A_iA_{i-2}$ (rules out bigger alterations up to 11 machines)

$T(\pi^*) < 2p$
(otherwise, dominated by $\pi_{oe}$)

$T(\pi^*) < 3(m + 1)\delta$
(otherwise, dominated by $\pi_d$)
Cardinal of a minimum dominant set within 1-cycles:

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6 ≤ \( m \) ≤ 8: \( \{\pi_{id}, \pi_{oe}, \pi_d\} \)

9 ≤ \( m \) ≤ 11: \( \{\pi_{id}, \pi_{oe}, \pi_d, \pi_{2w}\} \)

Best 1 cycles for \( m \leq 11 \)...
And for any $m$?

Lower bound on the waiting time: $l, d$
(tight for $p_{nw}$ cycles)
And for any $m$?

Not proven  Regular disposition of the waves in the “turbulence” areas: seems intuitive but...

Proven  Wavelets preferable to big waves:
Not proven  Regular disposition of the waves in the “turbulence” areas: seems intuitive but...

Proven  Wavelets preferable to big waves:

And for any $m$?
If the conjecture is valid, then...

- The best 1-cycle problem in circular, regular balanced cell would be \textit{polynomial}
- Performance ratio of the usual cycles:

\[
\frac{\min(T(\pi_{oe}), T(\pi_d))}{\min_n(T(\pi_{oe}), T(\pi_d), T(\pi_{nw}))}
\]

(as a function of \(m\))
If the conjecture is valid, then...

- The best 1-cycle problem in circular, regular balanced cell would be **polynomial**
- Performance ratio of the usual cycles:

\[
\frac{\min(T(\pi_{oe}), T(\pi_d))}{\min_n(T(\pi_{oe}), T(\pi_d), T(\pi_{nw}))}
\]

(as a function of \( m \))
Throughput optimization in robotic cells with circular layout was less studied and poorly understood so far. We:

- proposed new tools and specific cycle structures...
- ... leading to a conjecture on the best 1-cycle problem.

<table>
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<tr>
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<th>Best 1-cycle</th>
<th>1-cycle conjecture</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Linear</strong></td>
<td>P</td>
<td>$m \leq 3$: valid</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$m = 4$: false</td>
</tr>
<tr>
<td><strong>Circular</strong></td>
<td>NP-hard</td>
<td>$m = 6$: false</td>
</tr>
<tr>
<td>balanced</td>
<td></td>
<td>$m \leq 15$: valid</td>
</tr>
<tr>
<td><strong>Linear</strong></td>
<td>P</td>
<td>$m \leq 11$: P</td>
</tr>
<tr>
<td>balanced</td>
<td>$m \geq 12$: also P?</td>
<td></td>
</tr>
</tbody>
</table>
(Well, aside from settling the conjecture)

Other types of production constraints:
- Non-balanced case: Improving existing approximation...
- Proportionate flow-shop

Open questions for regular balanced cells:
- Best 1-cycle for $m > 11$...
- 1-cycle conjecture for $m \leq 5$
- Cycle function for $m \geq 6$

Relationships with other layouts:
- Comparisons of layouts
- Generalization of the circular layout:

Diagram:
- In
- Out
- Circular layout with arrows indicating direction of movement.
Thank you!
This presentation features some tributes to the following works (short excerpts):

- *Wall-E* (Pixar Animation Studios, 2008), slides 1, 4, 8 and 43;
- *Up* (Pixar Animation Studios & Walt Disney Pictures, 2009), slides 25 and 43;
- *Les Shadoks* (Jacques Rouxel), slides 16 and 43.

The picture of a toy train featured on slides 1, 2, 3 and 43 is extracted from a Brio commercial.

Other clipart images (slides 4 and 10) are either public domain or released under a CC0 license.

The template and backgrounds belong to G-SCOP laboratory.