Optimal Macroeconomic Policy under Uncertainty

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Thèse de doctorat

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Introduction

Recent financial crisis has shown how scarce is our knowledge about the true structure of the economy. Uncertainty, which economic agents face when they are elaborating their strategies, can be immense. It can come in the form of stochastic shocks hitting the economy or in the form of the unexpected actions of other agents. It may prevent agents from taking the optimal decisions and can cause the considerable welfare loss. In short, uncertainty complicates the life. This is especially important, if we talk about policymakers, because the wrong policy decisions may cause considerable problems for the whole economy.

This thesis takes on board several important issues concerning the policy-making under uncertainty. The first two Chapters concentrate on the standard macroeconomic policy instruments, while the last two Chapters discuss the informational tools, which can be used by policymakers under uncertainty.

Chapter 1 is devoted to the optimal monetary policy in a currency union under model uncertainty. Model uncertainty refers to the situation when the policymaker has in its possession some model of the economy, but takes into account that this model is only a simplified representation of the real world. If the model gives wrong predictions about the policy effects, the macroeconomic policy that does not take the model uncertainty into account may provoke huge negative effects. If the policymaker accounts for possible model misspecification, it would not rely entirely on this model. Instead of this, it would elaborate robust policy, which works reasonably well across some range of possible misspecification. In Chapter 1, I study the properties of such robust monetary policy in a micro-founded model of currency union, calibrated for the euro area.

This study contributes to the existing literature on the robust monetary policy in currency areas, because it is based on a two-region model with country-specific shocks. The previous research has been based on union-wide models, which does not take into account the possible asymmetries between the countries. In Chapter 1, I show that the central bank should react differently to the asymmetric shocks in monetary union. An increase in model uncertainty leads to more aggressive reaction to the shocks is a smaller region with more flexible prices and to less aggressive reaction to the shocks in a larger region with stickier prices.

In Chapter 2, I study the issues related to the fiscal and monetary policy interaction under uncertainty about the real policy effects and uncertainty about the preferences of the government. Although Chapter 2 is based on a one-region model, which cannot be directly referred to the euro area, the questions discussed here are relevant for the European agenda, as uncertainty surrounding the fiscal policy processes in different countries seems to affect the effectiveness of the ECB policy. Similar to Chapter 1, the model in Chapter 2 assumes that the economic effects of policy are uncertain. Contrary to Chapter 1, the general structure of the economy is taken as known, and the focus of the research is shifted from the optimal policy of a sole decision-maker to a game between the central bank and the government.

The study in Chapter 2 shows that government preference uncertainty affects the equilibrium only if there is multiplicative uncertainty surrounding the possible policy effects. If the policy effects are known with certainty, the government with any preference chooses the policy which allows to reach the social optimum. This situation refers to symbiosis effect. Nevertheless, this symbiosis effect collapses if the economic effects of policy are uncertain, at least for one of the policymakers. Multiplicative uncertainty leads to the attenuation in policy action and the inefficient equilibrium, which worsens even more if it is accompanied by the uncertainty about the preferences of the government.

In Chapters 3 and 4, I switch again to a two-region framework. Contrary to the first two Chapters, I do not discuss the standard policy instruments, but concentrate on the role of public information in an uncertain world and on the optimal information structures, which could be elaborated by the social planner in such economies.

In Chapter 3, I elaborate a general two-region model, which captures three important characteristics of international financial markets: globalization of markets, segmentation of fundamentals and informational asymmetry between regions. This model allows for two types of spillovers between regions. The first spillover can be called strategic, as the strategic effects in private actions are global. The second spillover is informational. This spillover arises because the information published in one region is almost freely available to the agents in the other region. For this model, I derive the global and the regional welfare criteria and study social, regional and inter-regional value of information. The main contribution of this study to the literature is the close look on the welfare properties of information in open economies. I show that the effects of information in segmented economies differ significantly from its welfare properties in one-region models. More precisely, I explore the importance of inter-regional asymmetries for the optimal information structure in open economies and show that ignoring these asymmetries when elaborating the information policy may cause the welfare loss.

The model in Chapter 4 is closely related to the model in Chapter 3. This model studies the informational effects in open economies. Contrary to the model in Chapter 3, the attention in Chapter 4 is concentrated on the case of strategic complementarity. More precisely, an international beauty contest is studied. This beauty contest is characterized by strategic complementarity in private actions both inside and between regions and by internationally correlated fundamental shocks. This model allows for three spillover channels between the regions. These are informational and strategic channels, already studied in Chapter 3, and technological channel, which arises because of the correlation of fundamental shocks. Thus, the first contribution of Chapter 4 is the analysis of the welfare properties of information in a global economy, characterized by these three spillover effects. To the best of my knowledge, these effects have not been studied in the literature on the social value of information, although they are broadly discussed in international finance and trade studies. As it is shown in Chapter 4, the optimal informational policy is closely related to the relative strength of these spillovers. The social optimum is characterized by either full transparency or full opacity with opacity optimal only if technological spillovers between countries are weak.

The second contribution of Chapter 4 is the study of endogenous international information structure, which is defined in a non-cooperative game of two policymakers. Thus, this research is in some sense close to Chapter 2, which also discusses the policy interactions. In a model of international beauty contest, the equilibrium information strategy is never characterized by full opacity. It means that the policymakers in this open economy always disclose some part of their information. If technological spillovers are weak, the policymakers disclose all the information about the home fundamentals and hide the information about the foreign shocks. The opposite is true for strong technological spillovers. For intermediate extents of spillovers, the policymakers reveals all available information. These findings together with the social welfare properties gives some insights about the possible inefficiency of the equilibrium international information structures. According to the relative strength of international spillovers, the policymaker may publish too much or too little information.

Chapter 1

Robust Monetary Policy in a Currency Union

Abstract

A great number of recent researches reveal the importance of country-specific shocks for the optimal policy in a currency union. However, these shocks have been almost completely overlooked by the literature on optimal policy under model uncertainty. Thus, the main purpose of our paper is to fill this gap and to show that the asymmetries between regions have to be taken into account when elaborating robust monetary policy. In our research, we use a New-Keynesian model of a two-country currency union which is hit by asymmetric shocks. For this model, we derive the robust monetary policy which works reasonably well even for the worst-case model perturbations. We find the attenuation effect of uncertainty in case of shocks in a larger region with stronger price stickiness. This means that the central bank reacts to these shocks less aggressively when the extent of model uncertainty is higher. For the shocks in a smaller region with more flexible prices, we find the anti-attenuation affect of model uncertainty. The central bank reacts more aggressively to the shocks in this region, if the extent of model uncertainty is higher.

JEL Codes: E52, E58

Keywords: model uncertainty, robust monetary policy, currency union

1.1 Introduction

A lot of researches are devoted to the optimal policy in the European Monetary Union. For example, Dixit and Lambertini (2001) analyze the optimal design of fiscal and monetary policy interactions in a monetary union, whereas Gali and Monacelli (2008) and Ferrero (2009) deal with optimal macroeconomic policy in a currency union with country-specific shocks. Each of these papers is based on a precise model that is assumed to capture the main economic relationships correctly. However, nobody knows the true and extremely complex structure of the economy and nobody can be absolutely confident about the predicting power of any particular model employed for policy analysis. Thus, the problem of *model uncertainty* or uncertainty about the true structure of economy arises.

There are a number of approaches to model this uncertainty. Most research deals with more or less "parametric" uncertainty. In this case the overall structure of the economy is supposed to be known, but the values of specific parameters are uncertain. The character of this parametric uncertainty can be different. Under Bayesian uncertainty, the distributions of model parameters are known. Under Knightian uncertainty, only minimal and maximal possible values of some parameters are known. Finally, under unstructured Knightian uncertainty, neither location nor the nature of uncertainty is specified. In spite of a precise character of uncertainty, a policymaker believes that the true economy lies in the "specified neighborhood" of a baseline model (Brainard (1967)). This neighborhood includes all possible deviations from the reference framework and this approach can be interpreted as an analysis of a set of similar but not identical models (Giannoni (2002)).

One of the possible approaches to the problem of model uncertainty is searching for *robust monetary policy* that works reasonably well across a given set of model specifications. The main question in this approach concerns the comparison of robust policies and simple optimal ones, designed for the particular model. The result called *Brainard conservatism* assumes that robust policy under Bayesian uncertainty is less aggressive in the reaction to economic shocks than the policy constructed for a single model without taking model uncertainty into account (Brainard (1967)). This "attenuation effect" is usually not present if Knightian uncertainty is analyzed within minimax approach. Yet there are studies that dispute this conclusion. For example, Craine (1979) and Söderström (2002) find that an increase in uncertainty concerning the transition dynamics in a backward-looking model makes optimal policy more aggressive, although Bayesian uncertainty is assumed. This result holds for forward-looking models, as it is shown in Kimura and Kurozumi (2007) and Kurozumi (2010), who analyze Bayesian uncertainty about "deep" model parameters that influence not only structural dynamic equations but also the social loss function. On the contrary, Onatski and Stock (2000) show that Brainard principle holds for the backward-looking model despite the fact that minimax choice criterion is applied. For forward-looking models and minimax criterion, the Brainard principle has been found in Gerke and Hammermann (2016), Tillmann (2009*a*) and Tillmann (2009*b*) for uncertainty about cost-channel of monetary policy transmission and in Leitemo and Söderström (2008*a*) for open economy.

The creation of the European Monetary Union and the entrance of new member countries considerably change the economic relations between European countries. That is why the extent of uncertainty concerning the EMU models is extremely high. As a result, it is no surprise that many authors attend to the robust policy design for the euro area. For example, Adalid et al. (2005) discuss the tolerance of four models of euro area to possible misspecifications and demonstrate that the parameters of robust rules should be weighted toward the optimal policies in backward-looking models. Bihan and Sahuc (2002), Žaković, Wieland and Rustem (2007) and Kuester and Wieland (2010) find that the Brainard principle holds true for union-wide models of the euro zone. Coenen (2007) examines the properties of optimal monetary policy rules under uncertainty about inflation persistence in two small-scale estimated models of the euro area and finds that more aggressive response to inflation shocks is needed. Gerke and Hammermann (2016) investigate robust monetary policy under commitment in a calibrated union-wide model with cost-channel and imperfect interest-rate pass-through. The authors find a more aggressive response to the cost-push shocks and the shocks in loan rate under uncertainty. The response to demand shocks is less aggressive under uncertainty. Two recent papers by Afanasyeva et al. (2016) and Binder et al. (2017) discuss the robust policy issues for a wide set of estimated models of the euro area. They show that robust monetary policy implies a weaker response to inflation and output gap if financial frictions are taken into account.

Despite the huge differences in the applied methods and found results, recent studies on robust policies in the euro area generally rely on area-wide aggregated models. Nevertheless, this approach does not allow to study heterogeneity among European countries, which has been documented by a number of previous studies. For example, De Grauwe (2000) shows that the national data should be considered for the optimal policy construction because of asymmetries in the transmission of monetary policy in the EMU. More precisely, Benigno and Lopez-Salido (2006) find a huge extent of heterogeneity in inflation persistence across European countries. Different inflation persistence can provoke considerable distortions in relative prices in the case of terms of trade shocks since the speed of adjustment differs across the countries. Benigno and Lopez-Salido (2006) demonstrate that optimal monetary policy should mitigate these distortions. Account of national data is proved to be crucial if there is heterogeneity in the slopes of country-specific Phillips curves, as in De Grauwe and Senegas (2006) and Brissimis and Skotida (2008). Monteforte and Siviero (2010) and Angelini et al. (2002) also show that relying on the national variables when elaborating optimal policy rule may lead to a considerable increase in union-wide welfare.

Therefore, there is a great deal of research that shows that country-specific characteristics matter for optimal policy, but studies which take these shocks into account when constructing optimal policy under uncertainty are rare. One of the exceptions is De Grauwe and Senegas (2006) who question the necessity of national data for optimal policy elaboration in the euro area under additive and multiplicative uncertainty. For this purpose, a stylized Barro-Gordon model of a union of many countries with symmetric supply shocks and asymmetric Phillips curves slopes is applied. For this model, the use of union-wide data on inflation and output gaps are found to be sub-optimal under uncertainty. Moreover, uncertainty in policy transmission mechanism makes optimal policy less aggressive. This attenuation result holds for almost all specifications studied. Other papers which account for the possible heterogeneity between countries are Adalid et al. (2005), Orphanides and Wieland (2013), Afanasyeva et al. (2016) and Binder et al. (2017). Each of these papers includes at least one multi-country model in a model set used to study the properties of robust monetary policy in the euro area. Nevertheless, these papers do not emphasize the role of disaggregation for the robust policy and focus instead on the backward-lookingness of the model (Adalid et al. (2005), Orphanides and Wieland (2013)) and on the presence of financial frictions (Afanasyeva et al. (2016) and Binder et al. (2017)). Moreover, the policy analysis in all these studies is based on the assumption that union-wide loss is determined by the union-wide inflation and output gaps. This assumption contradicts the findings of many theoretical studies which show that the social welfare in a union of heterogeneous countries is defined by the country-specific gaps and the terms of trade between countries (for example, Benigno (2004) and Beetsma and Jensen (2005)).

The main goal of our work is to fill this remaining gap between the literature on optimal policy under uncertainty and the studies of the EMU accounting for huge heterogeneity. For this purpose we analyze a micro-founded model of a two-country currency union of Benigno (2004), which implies that the micro-founded loss function depends not only on the inflation and output gaps, but also on the terms-of-trade gap between the countries. This calibrated model allows to account for two sources of heterogeneity. The first source is the relative economic size of regions, while the second is their price stickiness. The model is used to elaborate the robust monetary policy with robust control methodology initiated by Hansen and Sargent (2001). We find that the aggressiveness of the optimal monetary policy in its reaction to shocks depends on the origin of these shocks. For the shocks in a larger region with stickier prices, the central bank should conduct less aggressive policy in case of model uncertainty. For the shocks in a smaller region with more flexible prices, the central bank should react more aggressively in case of model uncertainty. We also discuss the role of two sources of heterogeneity for the characteristics of robust monetary policy.

The remainder of the paper is organized as follows: the two-country model is presented in the next section. Then we apply robust control techniques for this model and derive the characteristics of the robust policy under commitment. After that, we demonstrate the responses of the main economic variables to different shocks. The last section concludes and outlines the possible directions for future research.

1.2 Reference model of monetary union

In this paper we assume that a unique central bank elaborates monetary policy in a two-country currency union. This bank has in its possession a single micro-founded model with sticky prices that is taken as reference, but there are some doubts concerning its quality. Thus, the monetary authority tackles a model uncertainty problem.

The reference model of the central bank is the one described in Benigno (2004). This model incorporates the main source of heterogeneity in currency union, which is heterogeneity in price stickiness. Many authors show that uncertainty about inflation dynamics is an important factor for optimal policy elaboration (e.g. Coenen (2007), Angeloni, Coenen and Smets (2003)). Studies of optimal policy in currency union emphasize that asymmetry in inflation inertia is a crucial characteristics of monetary unions and this may have a considerable impact on the optimal policy (e.g. Brissimis and Skotida (2008), De Grauwe (2000)). Thus, the model of Benigno (2004) allows to study the impact of the basic source of asymmetry of the robust policy design in a currency union. In comparison to other forward-looking disaggregated models (as in Afanasyeva et al. (2016), Binder et al. (2017)), this model is very tractable. Moreover, the use of calibration proposed in Benigno (2004) allows to get micro-founded weights in social loss function, which explicitly includes country-specific inflation rates and distortions in the terms of trade.

In the model by Benigno (2004), the currency union consists of two countries or regions (H and F). The population of this union represents a unit-continuum where the agents from $[0, n]$ interval belong to country H and the rest $(n, 1]$ are inhabitants of country F. Each country has an independent local government, which determines fiscal policy (income taxes, transfers and purchases of products produced in its own country). Here we leave the problem of fiscal policy determination out of the attention, taking fiscal variables as exogenous.

Each inhabitant is simultaneously the producer of a single differentiated good and the consumer of all goods manufactured in the union, meaning there is inter-regional trade while migration of labor force is absent. The number of goods produced in region H is equal to n, so this parameter also represents the economic size of this region or the share of the total union GDP produced in region H.

The producers in the model are monopolists in their markets. They set prices according to Calvo scheme (Calvo (1983)). Each seller faces probability $(1-\alpha)$ of adjusting his price. The parameter of price inertia α differs for two regions. The brief description of the underlying micro-foundations of the model are given in Appendix A. For the purposes of our research, we restrict our attention to the main equations, described in the next subsection.

1.2.1 Key equations

This subsection describes the law of motion of the economy. In what follows, notion $\tilde{X}_t$ goes for the deviation of the logarithm of variable X from the steady state when prices are flexible, while $\hat{X}_t$ is the deviation of logarithm of variable X from the steady state under sticky prices. Variable X^W represents the weighted average of country-specific values and X^R is the relative value in region F in comparison to region H:

$$X^W = nX^H + (1-n)\,X^F$$

$$X^R = X^F - X^H$$

The main equations, which describe the equilibrium with sticky prices in the model by Benigno (2004), are:

$$E_t\hat{C}^W_{t+1} = \hat{C}^W_t + \rho^{-1}\left(\hat{R}_t - E_t\pi^W_{t+1}\right) \tag{1.1}$$

$$\hat{Y}^W_t = \hat{C}^W_t + g^W_t \tag{1.2}$$

$$\pi^H_t = (1-n)\,k^H_T\left(\hat{T}_t - \tilde{T}_t\right) + k^H_C\left(\hat{Y}^W_t - \tilde{Y}^W_t\right) + \beta E_t\pi^H_{t+1} \tag{1.3}$$

$$\pi^F_t = -nk^F_T\left(\hat{T}_t - \tilde{T}_t\right) + k^F_C\left(\hat{Y}^W_t - \tilde{Y}^W_t\right) + \beta E_t\pi^F_{t+1} \tag{1.4}$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi^F_t - \pi^H_t \tag{1.5}$$

where C is consumption index, R is the nominal interest rate; Y is output, π^j is inflation in region $j \in \{H, F\}$, g is demand shock (e.g. government spending shock) and T stands for the terms of trade index,

$$T_t = \frac{P^F_t}{P^H_t}$$

Equation (1.1) is the log-linearization of Euler equation. Equation (1.2) represents the total demand in the currency union. As we see, the aggregate demand is equal to the sum of total consumption spending and the union-wide demand shock. This shock is a weighted combination of region-specific demand shocks g^H_t and g^F_t:

$$g^W_t = ng^H_t + (1-n)\,g^F_t.$$

Combining equations (1.1) and (1.2), we get a usual IS-curve for the whole currency area:

$$E_t\hat{Y}^W_{t+1} = \hat{Y}^W_t + \rho^{-1}\left(\hat{R}_t - E_t\pi^W_{t+1}\right) - g^W_t + E_t g^W_{t+1} \tag{1.6}$$

According to equation (1.6), the output gap depends positively on its expected future value, expected demand shocks and the expected future inflation, and negatively on the nominal interest rate.

Equations (1.3–1.4) describe the supply side of the union economy and stand for the New Keynesian Phillips curves. According to these equations, inflation rates in the regions are determined by the union-wide output gap, expectations of future inflation and the union terms of trade. Usually the inter-regional terms of trade are omitted from the analysis based on the union-wide models, so the optimal policy is constructed for the aggregate levels of inflation and output. However, equations (1.3 – 1.4) make it clear that taking trade flows between regions into account is important for policy construction.

Equation (1.5) follows explicitly from the definition of the terms of trade and represents dynamics of this variable which is determined by its past value and the current inflation rates in both countries.

As we can see in equations (1.3) and (1.4), the dynamics of inflation depends not only on dynamics of the other variables under sticky prices, but also on the dynamics of output and the terms of trade under flexible prices. These variables are moving according to the following equations:

$$\tilde{T}_t = \frac{\eta}{1+\eta}\left(g_t^R - s_t^R\right) \tag{1.7}$$

$$\tilde{Y}_t = \frac{\rho}{\rho+\eta} g_t^W + \frac{\eta}{\rho+\eta} s_t^W, \tag{1.8}$$

where g_t^R is the relative demand shock, s_t^R is the relative supply (technology) shock and s_t^W is the union-wide supply (technology) shock. Thus, we have four region-specific shocks, which compile the relative and the union-wide demand and supply composite shocks. The vector of region-specific shocks $e_t = \left[g_t^H, g_t^F, s_t^H, s_t^F\right]^T$ evolves according to the following law:

$$e_t = \rho_e e_{t-1} + \varepsilon_t,$$

where $\rho_e = \left[\rho_g^H, \rho_g^F, \rho_s^H, \rho_s^F\right] I_{4\times 4}$ is the matrix of persistence parameters and $\varepsilon_t = \left[\varepsilon_{g,t}^H, \varepsilon_{g,t}^F, \varepsilon_{s,t}^H, \varepsilon_{s,t}^F\right]^T$ is the vector of shock innovations, where each component $\varepsilon_{k,t}^j$ ($j \in \{H, F\}$ and $k \in \{g, s\}$) is i.i.d. process with zero mean and standard deviation σ_k^j. Thus, we assume that all region-specific shocks are uncorrelated.

The central bank's task is to set the nominal rate R_t that optimizes its objective function subject to equations (1.5) and (1.3 – 1.8). In what follows, we use $z_t = \left[\pi_t^H, \pi_t^F, \hat{T}_t, \hat{Y}_t^W\right]^T$ to denote the vector of policy-relevant forward-looking variables. Thus, the problem of the central bank can be rewritten in the usual state-space brief form:

$$\begin{aligned} &\min_R E_0 \left\{\sum_{t-0}^{+\infty} \beta^t L_t\right\} \\ &s.t. \begin{bmatrix} e_{t+1} \\ E_t z_{t+1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} e_t \\ z_t \end{bmatrix} + \mathbf{B} R_t + \mathbf{C}\varepsilon_{t+1} \end{aligned}, \tag{1.9}$$

where $E_t z_{t+1}$ is the expected future value of vector z computed in period t. $\mathbf{A}$ is a matrix of corresponding coefficients, $\mathbf{B}$ is 8×1 vector with all components equal to zero but the last one equal to ρ^{-1}, as only the last component in the vector of forward-looking variables ($\hat{Y}_t^W$) depends on R_t. Matrix $\mathbf{C}$ has size 8×4 with first four rows representing the identity matrix $I_{4\times4}$ and all other elements equal to zero. This means that the shock innovations in period $t+1$ influence only the values of backward-looking variables e_{t+1} and do not influence the expectations of forward-looking variables, computed in region t. These matrices are given explicitly in Appendix A.2. L_t stands for the welfare loss in period t, and is defined in the next subsection.

1.2.2 Welfare criterion

We assume that the central bank is benevolent and tries to maximize the social welfare given by $W = E_0 \left\{ \sum_{t-0}^{+\infty} \beta^t w_t \right\}$, the expected weighted sum of all future values of average utility in the union. The second-order approximation of the welfare function is based on Beetsma and Jensen (2005) and gives the following welfare criterion:
$W = -E_0 \left\{ \sum_{t-0}^{+\infty} \beta^t L_t \right\}$, where one-period loss is given by

$$L_t = \Lambda \left[\hat{Y}_t^W - \tilde{Y}_t^W \right]^2 + n\left(1-n\right)\Gamma \left[\hat{T}_t - \tilde{T}_t \right]^2 + \gamma_H \left(\pi_t^H \right)^2 + \gamma_F \left(\pi_t^F \right)^2 + t.i.p + o\left(\|\varepsilon\|^3 \right), \quad (1.10)$$

where $t.i.p.$ stands for the terms independent from policy and the last part of this relation $\|\varepsilon\|^3$ includes all parameters of more-than-second order of approximation. The weight of the inflation in region $i \in \{H; F\}$ rises with an increase in the size of the region and in the extent of price stickiness. The brief form of the objective function (1.10) of the monetary authority is the following:

$$\min_R E_0 \sum_{t=0}^{\infty} \beta^t (x_t' Q x_t),$$

where $x_t = \begin{bmatrix} e_t \\ z_t \end{bmatrix}$ represents the vector of variables that influence the social losses (1.10), Q is a 16×16 matrix of coefficients of the loss function (1.10). Appendix A.2 provides the explicit view of matrix Q.

1.2.3 Calibration

In our calibration we follow Benigno (2004). Thus, we choose the value of elasticity of producing differentiated goods η equal to 0.67. The parameter of inter-temporal substitution β is equal to 0.99. The degree of monopolistic competition σ is equal to 7.66. The risk-aversion coefficient ρ is assumed to be 1/6.

In Benigno (2004), the author allows parameters α^i to vary across a wide range of possible values. This was a necessary choice, because the empirical data on the price stickiness in euro zone were not available. In our paper, we use the estimations of price stickiness in six European countries from recent paper by Vermeulen et al. (2012). These estimations are given in Table 1.1. Six countries, listed in this study (Belgium, France, Germany, Italy, Portugal and Spain), account for around 90% of the European GDP. Thus, we can reasonably restrict our attention to the union of these countries. Nevertheless, we also discuss the optimal policy for different values of price stickiness and for different distribution of economies activities among regions in Section 1.3.3 in order to check the robustness of our results.

Table 1.1: Frequency of Price Changes and Country Weights in Euro GDP

	Frequency of price changes $(1-\alpha)$	Country weight in Euro GDP (%)
Belgium	0.24	3.1
France	0.25	20.9
Germany	0.21	31.3
Italy	0.15	21.1
Portugal	0.23	1.8
Spain	0.21	9.9
Euro area	0.21	

Source: Vermeulen et al. (2012)

We take the frequency of price changes in Table 1.1 as a proxy for the probability to change a price $(1-\alpha)$ and divide countries into two groups according to the following scheme: if the frequency of price changes is lower or equal to 0.21 (the average frequency for the union), the country belongs to region H. If this frequency is higher than 0.21, the country is a part of region F. Therefore, region H consists of Germany, Spain and Italy, while region F consists of France, Belgium and Portugal.

According to Table ***1.1***, region H produces around 70% of union output, so we take the region size as 0.7. According to the corresponding weights, we set the average frequency of price change in region H to 0.19, while this frequency for region F is equal to 0.24. These values correspond to the model parameters $\alpha^H = 0.81$ and $\alpha^F = 0.76$. According to this calibration, both price stickiness and the economic size of region H are considerably higher than those in region F. This means that inflation in the region H obtains much more weight in the objective function (1.10) of the central bank than the inflation rate in region F.

This calibration leads to the following weight coefficients in the social loss function (1.10):

$$\Lambda = 0.00942$$
$$n(1-n)\Gamma = 0.004$$
$$\gamma_H = 0.797$$
$$\gamma_F = 0.203$$

Thus, inflation rates get much more weight in the social loss function than output or the terms of trade. Moreover, the weight of inflation in region H is much higher that the weight of inflation in region F. This illustrates the idea of Benigno (2004) that the optimal policy in monetary union implies more weight of the region with stickier prices. The weights of output gap and the terms of trade under our calibration are low, although not negligible.

The auto-regressive parameters of backward-looking variables $\left[\rho_g^H, \rho_g^F, \rho_s^H, \rho_s^F\right]$ are all equal to 0.95. Each shock innovation in $\varepsilon_t = \left[\varepsilon_{g,t}^H, \varepsilon_{g,t}^F, \varepsilon_{s,t}^H, \varepsilon_{s,t}^F\right]$ evolves as i.i.d. process with zero mean and standard deviation 0.0215. This implies that the standard deviation of the terms of trade shock in (1.7) is equal to 0.0086, which is consistent with Benigno (2004).

Alternative approach, widely used in the literature, is estimation of the model instead of calibration. Nevertheless, for our research, estimation does not give considerable advantages in comparison with the use of calibrated model. First of all, calibration of the model gives micro-founded weights in social loss function (1.10). Moreover, robust-control technique explicitly deals with parameter uncertainty, and takes into account the possible gap between estimated and calibrated coefficients in the model (1.3–1.8). The results of our analysis are robust for a large set of parameters values, which also confirms the adequacy of calibration.

1.3 Optimal monetary policy under uncertainty

1.3.1 Model uncertainty specification

Now we assume that the central bank uses (1.9) as a reference model of the economy. At the same time the monetary authority fears that its reference construction does not model properly the real state of nature and there is a risk of misspecification. In other words, some perturbations of modeled economy from the real one are allowed. The possible sources of these perturbations are unknown variables or processes.

To account for this possible misspecification, the monetary authority analyses only a class of alternative models, which cannot be distinguished from the reference one with the help of statistical methods. In other words, a set of possible perturbations is limited and includes only such perturbations which will not be discovered with some fixed probability. The reason to impose this restriction on possible misspecification is quite clear – for great perturbations, when the real economy differs considerably from the reference one, there is no reason to take any decision on the base of this concrete model; adaptation of the model to reality is needed.

Thus, the task for the central bank is to construct a policy that performs reasonably well, even if there is any perturbation. In searching for such a robust policy, we implement Hansen and Sargent's approach, which is also called *robust control*. This method assumes a minimax criterion for robust policy construction; a robust policy is the one that produces the smallest loss in the case of the worst model perturbation. These perturbations from the reference model take the form of some additional shocks υ_{t+s} which are added to the standard ε_{t+s} in the model (1.9) and are induced by so-called "malevolent nature" or "evil agent", who tries to maximize the central bank loss. Clearly, there is no such an agent in reality, but this assumption helps us to design the problem of the monetary authority that minimizes the welfare losses in the worst case and insures against the model uncertainty. Thus, the robust program can be represented by simultaneous two-agent game, where the evil agent chooses a perturbation for the reference model υ_{t+s} and the central bank defines the value of the nominal interest rate. The set of possible perturbations is modeled by the restriction on the evil agent's instruments υ_{t+s} and is discussed in the next subsection.

Here and below we use the methods proposed by Giordani and Söderlind (2004) to solve the robust optimization problems.

1.3.2 Robust control problem

We assume the following inter-temporal constraint of the malevolent agent:

$$E_0 \sum_{t=0}^{\infty} \beta^t \upsilon'_{t+1} \upsilon_{t+1} \leq \Psi \tag{1.11}$$

where υ_t is a vector of disturbances initiated by the malevolent agent in the economy. In other words, (1.11) represents the allowed set of perturbations, where Ψ stands for the total possible extent of model misspecification. Moreover, the size of possible perturbations, Ψ, corresponds to the central bank's fear of misspecification. It is worth to remind that the evil agent does not exist in reality, but represents a convenient way to model the problem of the policy-making under uncertainty. If possible misspecification does not worry the monetary authority, the possible deviations of the reference model from the real world are inessential. This is modeled by assuming that the evil agent has little possibilities to interrupt the model and the value of Ψ is low. On the contrary, if there is serious fear of misspecification, we assume that the evil agent has possibilities to interfere in the model more abruptly, so the value of Ψ is high.

Taking into account (1.11), we can formulate the central bank's problem under commitment in the following way:

$$\begin{aligned} &\min_{R}\max_{\upsilon} E_0 \textstyle\sum_{t=0}^{\infty}\beta^t(x_t'Qx_t) \\ &s.t. \begin{bmatrix} e_{t+1} \\ E_t z_{t+1} \end{bmatrix} = A \begin{bmatrix} e_t \\ z_t \end{bmatrix} + BR + C\left(\varepsilon_{t+1} + \upsilon_{t+1}\right) \\ &E_0 \textstyle\sum_{t=0}^{\infty}\beta^t \upsilon_{t+1}'\upsilon_{t+1} \leq \Psi \end{aligned} \tag{1.12}$$

Using a Lagrange multiplier theorem, the problem (1.12) is converted to

$$\begin{aligned} &\min_{\{R_t\}}\max_{\{\upsilon_{t+1}\}} E_0 \textstyle\sum_{t=0}^{\infty}\beta^t(x_t'Qx_t - \theta\upsilon_{t+1}'\upsilon_{t+1}) \\ &s.t. \begin{bmatrix} e_{t+1} \\ E_t z_{t+1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} e_t \\ z_t \end{bmatrix} + \mathbf{B}R_t + \mathbf{C}\left(\varepsilon_{t+1} + \upsilon_{t+1}\right) \end{aligned} \tag{1.13}$$

where θ is a Lagrange multiplier of the constraint (1.11). A negative relation between θ and Ψ in the continuous version of the problem is derived, for example, in Hansen et al. (2006), for discrete time in Giordani and Söderlind (2004) and in Hansen and Sargent (2008). This negative relation means that when the value of Ψ is low, the corresponding Lagrange multiplier is high and vice versa. Therefore, the parameter θ can be used as an implicit characteristic of allowed model perturbations instead of Ψ. When uncertainty rises and the "budget" of malevolent nature increases, θ declines. Conversely, if $\theta \to \infty$, the size of possible perturbations is nil and Ψ is equal to zero. In this case the central bank does not account for any model misspecification and its problem corresponds to the usual optimization problem under certainty (1.9). As it is shown in Hansen and Sargent (2008), the solutions of the robust problems (1.12) and (1.13) are equivalent, but the latter is easier to solve, while the former is easier to interpret. Therefore, in this study, like in the most literature discussed earlier, we solve the problem (1.13) for the different values of θ, keeping in mind the connection between both problems.

The choice of the concrete value of θ that seems to be crucial for our analysis is based on the detection error probability approach by Hansen and Sargent (2001). According to this method, the monetary authority tries to understand whether the available data are generated by the approximating model (1.9) or by the worst case model (1.12) with perturbations created by the evil agent. We exclude from our analysis all the situations when the central bank can define the data generating model with certainty, as in these situations the probability of the wrong choice between two models is equal to zero. In this case the size of perturbations, and therefore the doubts of the quality of the reference model, are so large that the monetary authority is hardly able to use this model for the optimal policy construction. We consider only the cases with positive probability to make a wrong choice between two models and to conclude that the data are generated by the reference model while there are some perturbations or to choose the worst-case model while the data are generated by the reference model (1.9). When the extent of misspecification is high (and

θ is low), we assume that the evil agent can generate considerable distortions and the possibility of the error described earlier is low because the worst case model and the reference one differ significantly. On the contrary, when the extent of misspecification is low (high θ), there can be only slight perturbations and the probability of choosing the wrong model is high. Thus, high uncertainty corresponds to the low probability of the error in the sense described above and to the low value of θ.

The probability of error can be computed in the following way:

$$\pi(\theta) = \frac{1}{2} \Pr\left(\tilde{L}_A > \tilde{L}_W \middle| W\right) + \frac{1}{2} \Pr\left(\tilde{L}_W > \tilde{L}_A \middle| A\right), \tag{1.14}$$

where $\tilde{L}_A$ stands for the value of likelihood of the approximating model, and $\tilde{L}_W$ is the likelihood of the worst-case model. The first part of the right hand-side expression in (1.14) is the probability to treat the model as an approximating case while in reality the malevolent nature interrupts the data generating process. The second part is the probability to take the model as the worst case while there are no any actions of the evil agent.

Hansen and Sargent (2001) argue that the reasonable extent of misspecification corresponds to the detection error probability around 20%. In this case the extent of model uncertainty is neither trivial nor too high. In our analysis we suppose that the detection error probability can vary from 20% to 50% allowing the extent of model uncertainty to change considerably. It is significant to mention that the probability of 50% corresponds to the case when the central bank does not take into account model uncertainty. This means that the monetary authority always decides that the data are generated by the reference model and does not suppose that there can be any perturbation. In this case the problem of the central bank is standard (1.9), so we allow the extent of uncertainty to vary from the lowest level (where the detection error probability is equal to 50% and θ is at the highest level) to some middle magnitude (corresponding to the error probability of 20%).

Using solution techniques developed by Giordani and Söderlind (2004), we find the optimal robust policy that can be represented as a reaction of the nominal interest rate R to the shocks of the terms of trade and to the Lagrange multipliers for the constraints in the problem (1.9):

$$R_t = \tilde{R} \begin{bmatrix} e_t \\ \rho_t^z \end{bmatrix} \tag{1.15}$$

where e is a random component of the terms of trade dynamics; ρ_t^z is a (4×1) vector of the Lagrange multipliers corresponding to the constraints on the forward-looking variables in the model (1.9) and $\tilde{R}$ is a (1×8) vector of coefficients that describes the optimal policy. The presence of the Lagrange multipliers in the optimal policy ensures that today's policy measures confirm the private sector expectations formed in the past (Dennis (2007)). The brief description of the solution method, adopted from Giordani and Söderlind (2004), is given in Appendix A.3.

1.3.3 Robust policy

We compute the robust policy for several extents of model uncertainty represented by the parameter θ and by the detection error probability. The monetary policy coefficients are summarized in Table 1.2.

Table 1.2: Parameters of Robust Monetary Policy $R_t = 10^{-3}[r_1, r_2, r_3, r_4]\left[g_t^H, g_t^F, s_t^H, s_t^F\right]^\top + \tilde{R}_\rho \rho_t^z$

Error detection probability	θ	r_1	r_2	r_3	r_4
50%	400	1.02942	0.40535	-1.02942	-0.40535
40%	5.2318	1.02915	0.40561	-1.02915	-0.40561
30%	5.2273	1.02907	0.40569	-1.02907	-0.40569
20%	0.1786	1.02078	0.41398	-1.02078	-0.41398

Source: author's own calculations

Table 1.2 shows the reaction of the central bank to the shocks for different extents of possible model misspecification. Coefficient r_1 shows the reaction of the central bank to the demand shock in region H. Coefficient r_2 shows its reaction to the demand shock in region F. As we can see, for any θ, both coefficients are positive. This means that the central bank raises interest rate in response to demand shocks in the economy. Positive demand shocks lead to an increase in output and inflation. Moreover, asymmetric demand shocks lead to a disturbance in the terms of trade. To avoid the negative effect of inflation jumps on the social welfare, the central bank raises the interest rate.

As we can see, the value of coefficient r_1 is sufficiently higher than the value of coefficient r_2, meaning that the central bank reacts more actively to demand shocks in region H, than to the shocks in region F. This is consistent with the findings of Benigno (2004) and Beetsma and Jensen (2005), which show that the optimal policy in a currency union implies more weight of the region with higher price stickiness in the policy function. Under our calibration, region H is larger than region F and is characterized by the stronger price inertia. Thus, the weight of the inflation in region H in social loss function (1.10) is around 2,5 times higher than the weight of inflation in region F. This caution about the inflation in a larger region with stickier prices leads to the difference in the reaction to the demand shocks in two regions.

Coefficient r_3 represents the reaction of the policy rate to the supply shock in region H, while coefficient r_4 is its reaction to the supply shocks in region F. Both coefficients are negative, meaning that the central bank reacts to the positive supply shocks by lowering the interest rate. Positive supply shocks leads to an increase in output and a drop in inflation rate. Moreover, asymmetric shock would also disturb the terms of trade. In order to avoid these disturbances, the central bank lowers the interest rate. In this paper we do not deal with the problem of zero-lower bound and assume that the central bank can achieve the necessary drop in interest rate. Similarly to the demand shocks, the central bank reacts more actively to the supply shocks in region H, than to the shocks in regions F. Moreover, our calibration gives the same absolute values of coefficients which characterize the reaction of the central bank to supply and the demand shock inside any region. This means that a unit positive demand shock and a unit negative supply shock in region $j \in \{H, F\}$ would lead to an increase in the interest rate of the same magnitude.

The rows in Table 1.2 correspond to different extents of possible model misspecifications. The first line in Table 1.2 represents the reaction of the central bank under the lowest extent of model misspecification. As we already discussed, in this case error detection probability is equal to 50% and the problem of the central bank is equivalent to the standard rational expectation model. Under our calibration, error detection probability is equal to 50% if θ is equal to 400. Such a high value of parameter θ corresponds to a small budget of the evil agent, Ψ. In this case the evil agent does not have enough resources to disturb the underlying model. According to our computations, error detection probability is equal to 20% if θ is equal to 0.1786. In this case the evil agent has a huge budget to disturb the model and the central bank has to take possible misspecification into account.

As we can see, an increase in model uncertainty leads to different changes in the reaction of the central bank to home and foreign shocks. Higher uncertainty leads to a decrease of coefficients which correspond to region H and to an increase in coefficients which correspond to the shocks in region F. Thus, we find the asymmetric effect of model uncertainty on the robust policy in a monetary union. This finding is summarized in the following Corollary:

Corollary 1.1. *An increase in model uncertainty decreases the policy aggressiveness in the reaction to the shocks in a larger region with stickier prices (region H) and increases the policy aggressiveness in the reaction to the shocks in a smaller region with more flexible prices (region F).*

Thus, for smaller region with more flexible prices we find the "anti-attenuation" effect, meaning the more aggressive reaction to the shocks for higher extents of model uncertainty. These findings are in line with the general result of robust control techniques, while it questions the existing literature on the robust policy in the European Monetary Union, which shows that Brainard principle should hold (see Bihan and Sahuc (2002), Žaković, Wieland and Rustem (2007) and Kuester and Wieland (2010)). The main distinction between their models and ours is that we use a two-region model, while the previous studies are based on union-wide models, which do not allow to take into account the union asymmetries and the distortions in the terms of trade between the regions inside the union. This result is also different from De Grauwe and Senegas (2006), who find Brainard attenuation effect in a stylized multi-country model of a currency union. The difference in findings with this paper is based on the perfect correlation of supply shocks and the use of Bayesian uncertainty in De Grauwe and Senegas (2006), while we analyze Knightian uncertainty in a model with uncorrelated shocks.

For the larger region with stickier prices we find the Brainard attenuation effect: higher uncertainty leads to more cautions reaction to the shocks, despite the minimax approach. This results needs more explanation, as it is contrary to many studies which apply robust control method and show that the robust policy under uncertainty should be more aggressive. The rare exceptions which find that robust policy under model uncertainty may be less aggressive are Gerke and Hammermann (2016), Tillmann (2009*a*), Tillmann (2009*b*) which show that uncertainty about cost-channel of monetary policy transmission leads to the attenuation of monetary policy. As our model does not implies cost-channel, this explanation can not be applied to our results. Closer to our research stands the study by Leitemo and Söderström (2008*a*) which show that attenuation effect can be present under uncertainty about exchange rate channel. The direct effect of an increase in interest rate on inflation through aggregate demand is negative, while the indirect effect through exchange rate appreciation is positive. Thus, when uncertainty is high and central bank is concerned by the possible huge extent of indirect effect, it is more cautious in its reaction to shocks. In our model, we do not have the full exchange rate channel, as the countries share the same currency. Nevertheless, interest rate policy may strike the gap in the terms of trade between countries, which influences the social welfare, as shown in equation (1.10).

To better understand the origins of the finding stated in Corollary 1.1, we have to distinguish the effects caused by heterogeneity in economic size and price stickiness. For this purpose, we carry out two exercises. The first exercise reveals the effect of heterogeneity in price stickiness when the countries are of equal size, while the second exercise reveals the effect of heterogeneity in economic size of two regions with equal price stickiness.

For the first exercise, we consider the model of two regions of equal size ($n = 1/2$) but with different degree of price stickiness. Without loss of generality, we assume that region H demonstrates higher price stickiness than region F. We pass through a wide set of values of α^H and α^F, which give the average frequency of price change equal to 0.21. This average frequency corresponds to the estimates for the euro zone (1.1) and can be calculated as $\left(1-\alpha^H\right)^{1/2}\left(1-\alpha^F\right)^{1/2}$. For a given set of price stickiness values, we compare the coefficients of the robust rule with coefficients of the rule which does not take uncertainty into account. For simplicity, we use the same value of $\theta = 1$ for all model modifications. Thus, the results for different pairs $\left(\alpha^H, \alpha^F\right)$ should be compared with caution, as they imply different error-detection probabilities. Nevertheless, these results show the effect of model uncertainty on the coefficients of policy rules.

Table 1.3: Parameters of Optimal Monetary Policy $R_t = 10^{-3}\left[r_1, r_2, r_3, r_4\right]\left[g_t^H, g_t^F, s_t^H, s_t^F\right]^\top + \tilde{R}_\rho \rho_t^z$ **for** $n = 1/2$

$\left(\alpha^H, \alpha^F\right)$	Policy	r_1	r_2	r_3	r_4
(0.95, 0.118)	RE	1.3924	0.423	-1.3924	-0.423
	Robust	1.3931	0.417	-1.3931	-0.417
(0.9, 0.559)	RE	0.8874	0.5474	-0.8874	-0.5474
	Robust	0.8926	0.5421	-0.8926	-0.5421
(0.85, 0.706)	RE	0.8014	0.6333	-0.8014	-0.6333
	Robust	0.8018	0.6330	-0.8018	-0.6330
(0.8, 0.7795)	RE	0.7301	0.7047	-0.7301	-0.7047
	Robust	0.7302	0.7046	-0.7302	-0.7046
(0.79, 0.79)	RE	0.7174	0.7174	-0.7174	-0.7174
	Robust	0.7174	0.7174	-0.7174	-0.7174

Source: author's own calculations

RE refers to policy in rational expectations model without robustness; *Robust* refers to robust policy under model uncertainty.

Results are listed in Table 1.3. As we can see, heterogeneity in price stickiness and model uncertainty imply more aggressive response to the shocks in region with stickier prices and less aggressive response to the shocks in region with more flexible prices. The intuition is straightforward. The central bank is more cautious about inflation in region with stickier prices (in line with Benigno (2004)). Thus, uncertainty makes the monetary authority even more concerned by the shocks in this region. This implies more aggressive reaction to the shocks in this region. Reaction to the shocks in the other region with relatively flexible prices may itself provoke the undesirable volatility of inflation in region with stickier prices. Thus, central bank reacts more cautiously to the shocks in region with more flexible prices in case of model uncertainty.

For the second exercise, we assume that frequency of price change is the same for both regions and is equal to 0.21. We consider different values of the relative economic size of region H and compare the coefficients of optimal policies under model uncertainty and without it. Results are shown in Table 1.4.

Table 1.4: Parameters of Optimal Monetary Policy $R_t = 10^{-3}\left[r_1, r_2, r_3, r_4\right]\left[g_t^H, g_t^F, s_t^H, s_t^F\right]^\top + \tilde{R}_\rho \rho_t^z$ **for** $\alpha^H = \alpha^F = 0.79$

n	Policy	r_1	r_2	r_3	r_4
0.55	RE	0.7891	0.6456	-0.7891	-0.6456
	Robust	0.7891	0.6456	-0.7891	-0.6456
0.65	RE	0.9356	0.5021	-0.9356	-0.5021
	Robust	0.9356	0.5022	-0.9356	-0.5022
0.75	RE	1.0760	0.3587	-1.0760	-0.3587
	Robust	1.0759	0.3589	-1.0759	-0.3589
0.85	RE	1.2195	0.2152	-1.2195	-0.2152
	Robust	1.2194	0.2154	-1.2194	-0.2154
0.95	RE	1.3630	0.0717	-1.3630	-0.0717
	Robust	1.3630	0.0718	-1.3630	0.0718

Source: author's own calculations

RE refers to policy in rational expectations model without robustness; *Robust* refers to robust policy under model uncertainty.

As we can see, robust policy implies more aggressive reaction to the shocks in smaller region, while reaction to the shocks in larger region becomes more cautious under model uncertainty. The explanation of this finding lies in the asymmetric impact of country-specific shocks on two regions through the terms-of-trade channel. The cross-border effect of country-specific shocks in relatively large region is larger than its home effect. Model uncertainty makes the central bank even more anxious about these side-effects, thus it reacts less aggressively to the shocks in a larger region. On the contrary, the shocks in a smaller region have more pronounced home effect and less significant cross-border effect. Model uncertainty forces the central bank to pay more attention to the home effects of shocks in smaller region and its reaction to them becomes more aggressive. Nevertheless, reaction to the shocks in larger region remains much stronger than reaction to the shocks in smaller region.

Thus, the findings stated in Corollary 1.1 are defined by the opposite effects of two sources of heterogeneity. The effect of price stickiness heterogeneity is overcome by the effect of economic size. As a result, the more robust policy is characterized by less aggressive reaction to the shocks in a larger region with stickier prices and by more aggressive reaction to the shocks in a smaller region with more flexible prices. In what follows, we demonstrate dynamics of the main economic variables in the benchmark calibrated model caused by different shocks and discuss in more details the policy changes implied by model uncertainty.

1.3.4 Shocks

The response of the key variables to the demand shock in region H is given in Figure 1.1. The shock in g^H is equal to the standard deviation, i.e. 0.0215. For illustrative purposes, all the graphs are drawn for θ equal to 0.05. Such a low value of θ implies a huge budget of the evil agent and a large difference between the approximating model and the worst case. This means that the approximating model is probably sufficiently far from reality and the policymaker has to develop a new model. On the other hand, this large distance between the models allow us to show the difference in the economy dynamics under different assumptions about the model misspecification. For more reasonable and larger values of θ, the differences between the models are qualitatively the same, but differ quantitatively. For illustrative purposes we show only initial 20 periods of economy responses to the shocks.

The first 4 graphs in Figure 1.1 show the responses of the key forward-looking variables to a positive demand shock in region H: inflation in region H, inflation in region F, the terms of trade and union output under sticky prices. The next two graphs show the dynamics of the terms of trade and output under flexible prices. Part g of Figure 1.1 shows the reaction of policy rate to the demand shock. The last part shows the shocks created by the evil agent in a worst-case model.

Solid blue lines in Figure 1.1 correspond to the dynamics of rational expectation model, derived without taking model uncertainty into account. Dotted red lines represent the dynamics of the worst-case model with the additional shocks, created by the evil agent. Yellow dashed lines shows the dynamics in approximating model, with the robust policy and without any additional distortion created by the evil agent.

As we can see in Figure 1.1, a positive demand shock in period 1 leads to an increase in the union output $\hat{Y}^W$ and inflation in region H in a model with rational expectations without robustness. Nevertheless, an increase in output under flexible prices ($\tilde{Y}^W$, part f of the Figure 1.1) would be higher. Thus, the shock creates the negative output gap $\left(\hat{Y}_t^W - \tilde{Y}_t^W\right)$. According to Phillips curve (1.4), this leads to a drop in inflation in region F in period 1. An increase in π^H and a decrease in π^F leads to a drop in the terms of trade $\hat{T}$. A drop in the terms of trade under flexible prices would be larger, which is evident after comparison of Figures 1.1 c and e. Thus, a positive gap in terms of trade $\left(\hat{T}_t - \tilde{T}_t\right)$ arises. The central bank has the competing goals to lower inflation in region H, to raise inflation in region F, to close the output and the terms of trade gaps. According to the weights in its loss function (1.10), the central bank is more concerned about inflation in region H and raises its policy rate, as it is show in graph 1.g.

An increase in interest rate leads to a drop in inflation in region H which is followed by a smooth recovery to the initial level. A decrease in the terms of trade leads to an increase in inflation in region F in period 2. After that, inflation π^F smoothly decreases up to its initial

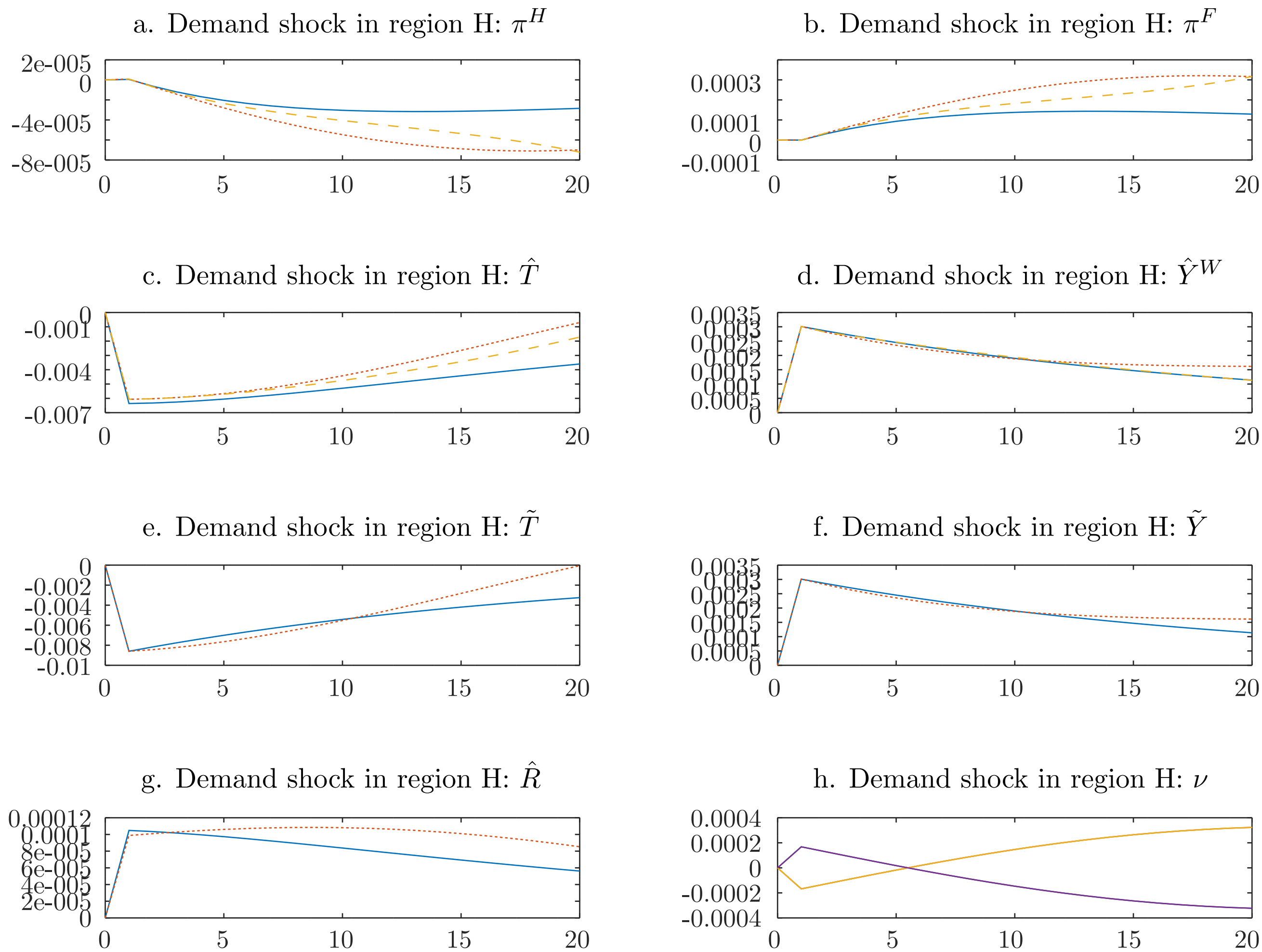

Figure 1.1: Impulse responses to the demand shock in region H. Solid lines in first seven graphs shows the dynamics in the rational expectations model; dashed lines is the dynamics in the approximating model; dotted lines shows the worst-case dynamics. The last graph shows the extra disturbances, created by the evil agent in the worst case.

value. A smooth increase in π^F and a smooth decrease in π^H ensure the recovery of the terms of trade. The gradual attenuation of the demand shock assures the recovery of all the variables to their long-run equilibrium values. The sluggishness of price reactions along with the strong shock persistence causes the slow return of the economy to the initial state. The strong reaction of the variables to the shock in period 2 is explained by a sharp reply of the central bank to the shock. This aggressive reaction is partly explained by the absence of policy smoothness component in the policy loss function. If the central bank was anxious about the policy shocks, an increase in the policy rate would be lower and it would take even more time for economy to return to the initial state.

If there is some model uncertainty, the policymaker assumes that the evil agent exists. This evil agent tries to increase the social loss by adding the extra shocks to the model. Figure 1.1 h shows the dynamics of these additional shocks. The values of shocks which are added to the demand shock in region H and the supply shock in region F (ν_g^H and ν_s^F correspondingly), coincide; their dynamics is given by the blue solid line. As we can see, the initial values of these shocks are positive. The dynamics of these shocks is similar to the dynamics of the real demand shock mentioned before. The dynamics of the demand shock in region F and supply shock in region H (ν_g^F and ν_s^H correspondingly) is the opposite; after the initial negative value there is some attenuation. Thus, all the additional shocks worsen the initial shock of the terms of trade. The asymmetries between the two regions worsen and inflation rates in both of them deviate further from the initial state, than in the model with rational expectations. This is shown by the relative position of solid and dotted lines in the first two graph. In other words, the central banker, which has some doubts about the underlying model, fears that the real asymmetries are larger than in the model. It fears that a stronger drop in home inflation and a stronger increase in foreign inflation will follow the initial shock. An increase in foreign inflation gets more concerns from the central bank, and the initial increase in policy rate is lower than in rational expectations model. The whole path of the interest rate is characterized by higher sluggishness.

The dashed lines in Figure 1.1 represent the dynamics of the economy in case of robust policy of the central bank and without additional shocks of the evil agent. As we can see, the robust policy implies the slower adjustment of inflation rates to the initial state, but quicker adjustment of the terms of trade. The dynamics of output is almost the same as it is under the policy, which is optimal for rational expectations model.

Figure 1.2 shows the responses of the key variables to the demand shock in region F. The dynamics of the inflation rates and the terms of trade are opposite to the dynamics caused by the demand shock in region H. Dynamics of the output and interest rate in a model with rational expectations is similar to the case of demand shock in region H, while the magnitude of disturbances is smaller. This can be explained by the smaller size of the region F; thus, its influence on the whole economy is smaller than the influence of region F.

The actions of the evil agent are presented in Figure 1.2 h. The shocks which increase the terms of trade (the demand shock in region F, ν_g^F, and supply shock in region H, ν_s^H) are positive in period 1, while the shocks which decrease the terms of trade (the demand shock in region H, ν_g^H, and the supply shock in region F, ν_s^F) are negative. Similar to the situation discussed above, this increases asymmetries in the union. The return of inflation rates to initial state becomes slower in comparison to the model with rational expectations. Nevertheless, output returns to initial state more rapidly than in the model with rational expectations. This can be explained by the policy response to the shock.

The reaction of the policy rate to model uncertainty is different from what we see under the demand shock in region H. According to Figure 1.2 g, the robust policy response to the demand shock in region F is more aggressive than the policy response in the model with rational expectations. Thus, we observe "anti-attenuation" effect of uncertainty, discussed above. After the initial jump in the policy rate, the following dynamics is characterized by the quicker return of the policy rate to initial state.

The response of the economy to the supply shock in region H are given by Figure 1.3. As we can see, dynamics of inflation rates, the terms of trade and output is similar to the case of the demand shock in region F. A positive technological shock in region H leads to a decrease in inflation rate in region H. The union output $\hat{Y}^W$ increases, while output under flexible prices $\tilde{Y}^W$ would increase less. The positive output gap forces price-makers in region F to raise their prices, inflation in region F increases. An increase in inflation in region F and a decrease in inflation in region H lead to a sharp increase in the terms of trade. An increase in the terms of trade causes an increase in inflation in region H and a decrease in inflation in region F. Along with attenuation of initial shock, inflation rates, the terms of trade and output return to their initial values. The central bank tries to extend the period of higher growth and pushes interest rate down. After that the interest rate smoothly returns to its initial value.

Similar to the case of demand shock in region F, the evil agent creates the shocks which strengthen the initial shock in the terms of trade. As we can see, the robust reaction of the central bank to the initial shock is less aggressive; the central bank decreases interest rate less actively, than in the model with rational expectations. In the worst case model, the inflation rates deviate further from the initial state than in the model with rational expectations.

Dynamics of the economy after the supply shock in region F is given by Figure 1.4. As we see, it is equivalent to dynamics of the economy under supply shock in region H. The main difference concerns the interest rate path. The central bank reacts to the supply shock in region F more aggressively in a worst-case model, than in the model with rational expectations. Thus, we observe "anti-attenuation" effect of model uncertainty in case of shocks in region F. In the next subsection we demonstrate in more detail the shocks created by the evil agent.

1.3.5 Worst-case shocks

In this subsection we discuss in more detail the shocks created by the evil agent. In the previous section we restricted our attention to the first 20 periods after a shock. It was made for the illustrative purposes. Nevertheless, it is worth to consider a longer period to understand better the nature of model misspecifications created by the evil agent. For this purpose, we plot the impulse

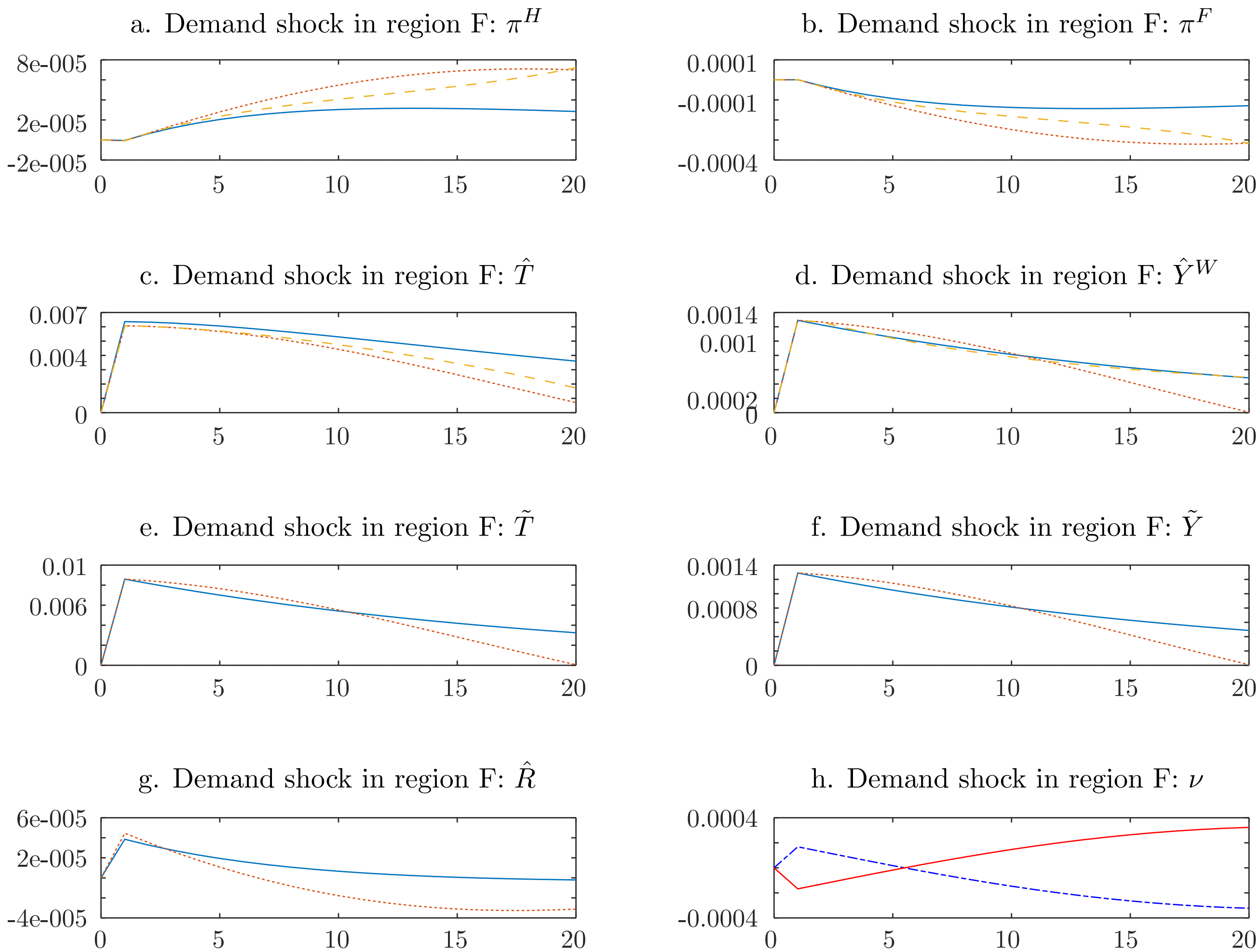

Figure 1.2: Impulse responses to a demand shock in region F. Solid lines in first seven graphs shows the dynamics in rational expectations model; dashed lines is the dynamics in approximating model; dotted lines shows the worst-case dynamics. The last graph shows the extra disturbances, created by the evil agent in the worst case; solid line is for the shocks which increase the terms of trade, while dotted line is for the shocks which decrease the terms of trade.

responses of the terms of trade and output under flexible prices along with the interest rate path and the additional shocks created by the evil agent, for 200 periods after the demand shock in region H. For the other shocks, dynamics is similar. We consider three different values of model misspecification. In the first version, θ is equal to 0.05; this value corresponds to the impulse responses in the previous section. As we discussed earlier, such a value implies unreasonably high model uncertainty. For this reason, we consider also the values of θ, equal to 0.5 and 20.

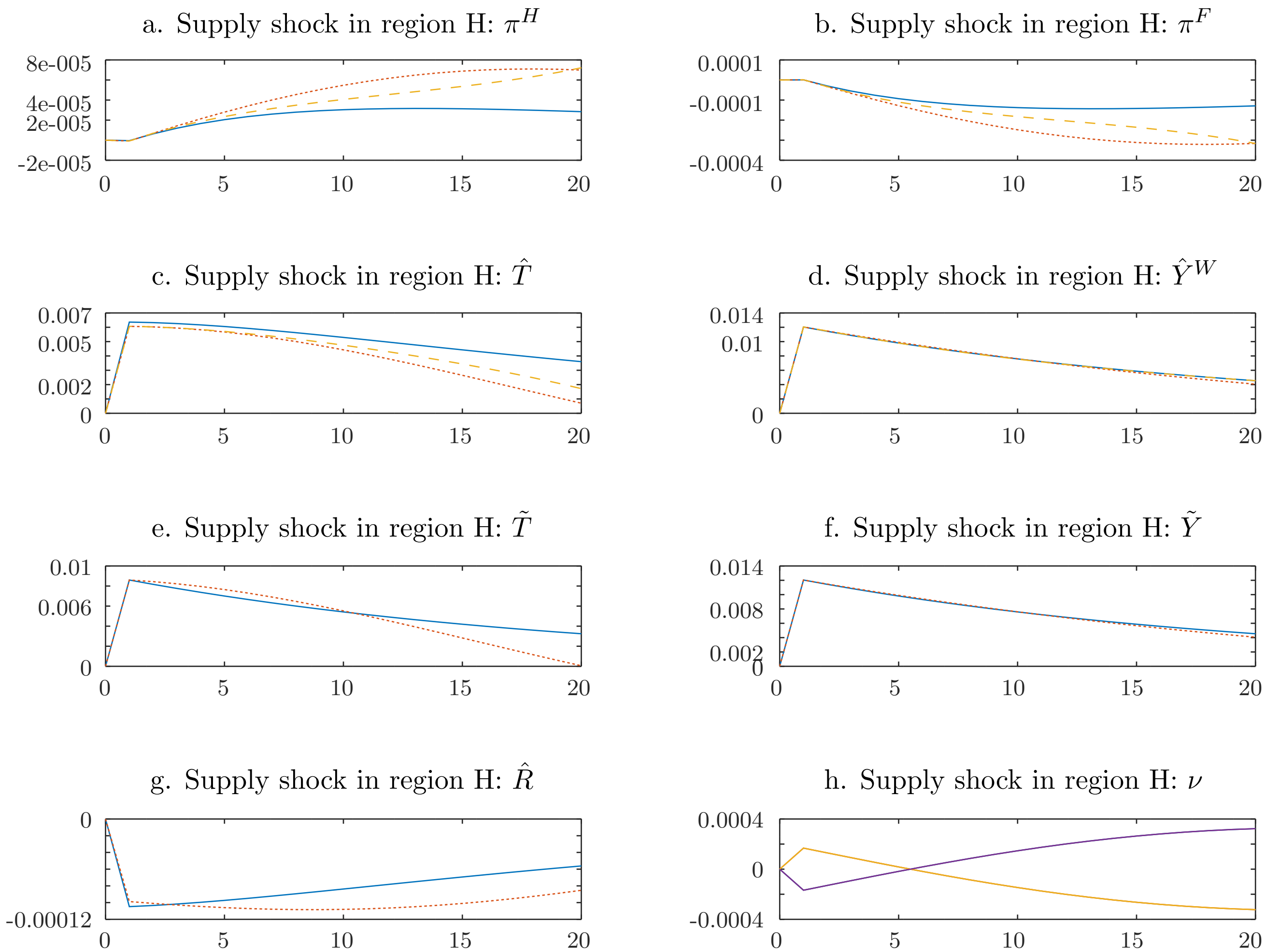

Figure 1.3: Impulse responses to the supply shock in region H. Solid lines in first seven graphs shows the dynamics in the rational expectations model; dashed lines is the dynamics in the approximating model; dotted lines shows the worst-case dynamics. The last graph shows the extra disturbances, created by the evil agent in the worst case.

Figure 1.5 shows dynamics of the mentioned variables after a positive demand shock in region H for θ equal to 0.05. As we discussed before, this shock is accompanied by a decrease in the terms of trade. The evil agent reacts by the shocks which strengthen the initial drop in the terms of trade. Graph 1.5.d sheds light on the subsequent dynamics of the additional shocks. As we can see, it creates the cycles in the terms of trade. As the evil agent is just a metaphor, this means that the central bank fears that the initial shock will not simply disappear, but will be accompanied by the cyclical volatility. Thus, the reaction of the central bank to the initial shock is not just

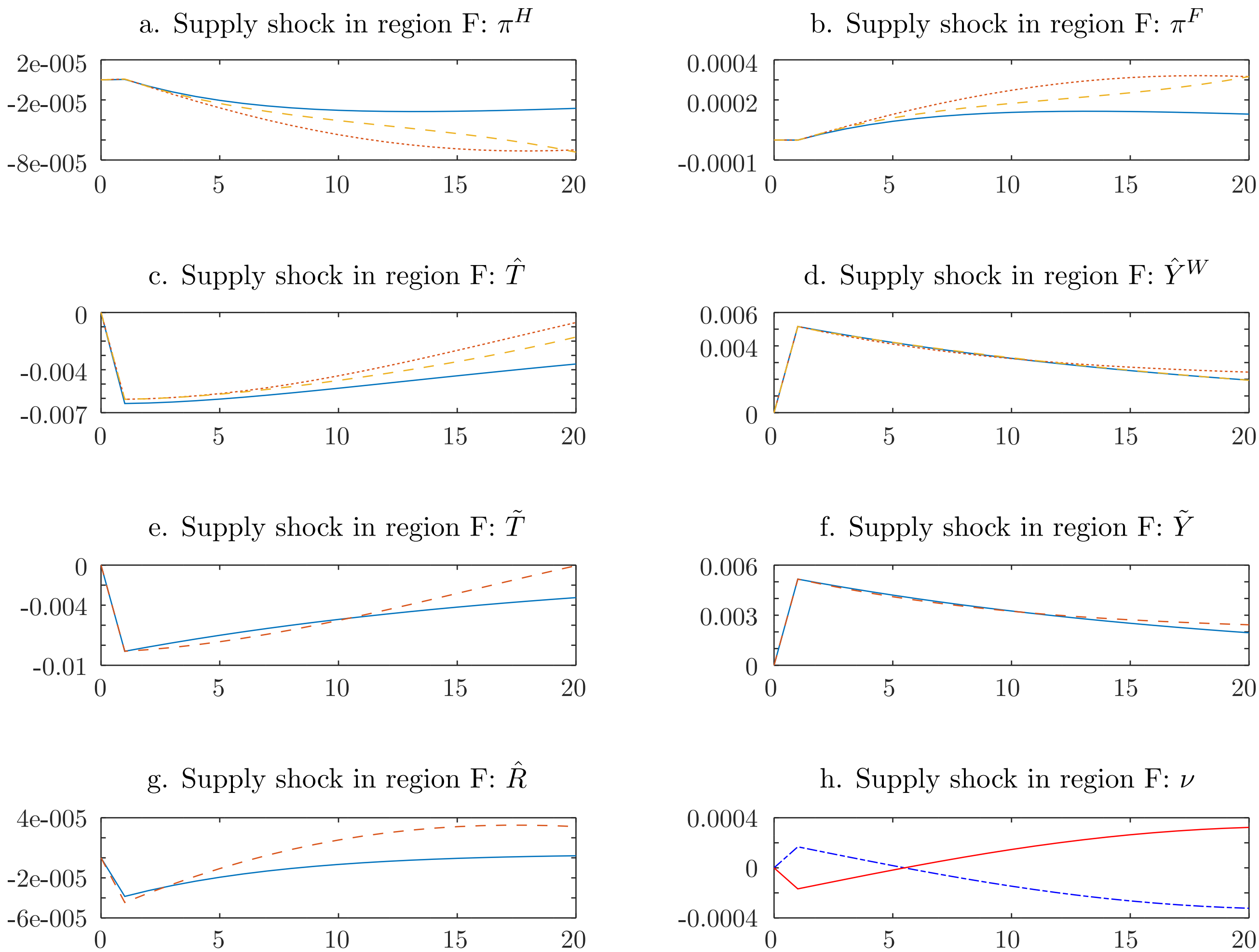

Figure 1.4: Impulse responses to the supply shock in region F. Solid lines in first seven graphs shows the dynamics in the rational expectations model; dashed lines is the dynamics in the approximating model; dotted lines shows the worst-case dynamics. The last graph shows the extra disturbances, created by the evil agent in the worst case.

an increase in interest rate, followed by the smooth return to the initial state. Graph 1.5.c shows that the central bank lets the interest rate to fluctuate around its path in the rational expectation model.

As the value of θ equal to 0.05 represents too extreme extent of model misspecification, we demonstrate the shocks created by the evil agent for $\theta = 0.5$ and $\theta = 20$. As we can see in Figure 1.6, the nature of these shocks coincides with the shocks created in the previous case, but their magnitude is lower and the speed of their attenuation is higher. Actually, the evil agent creates

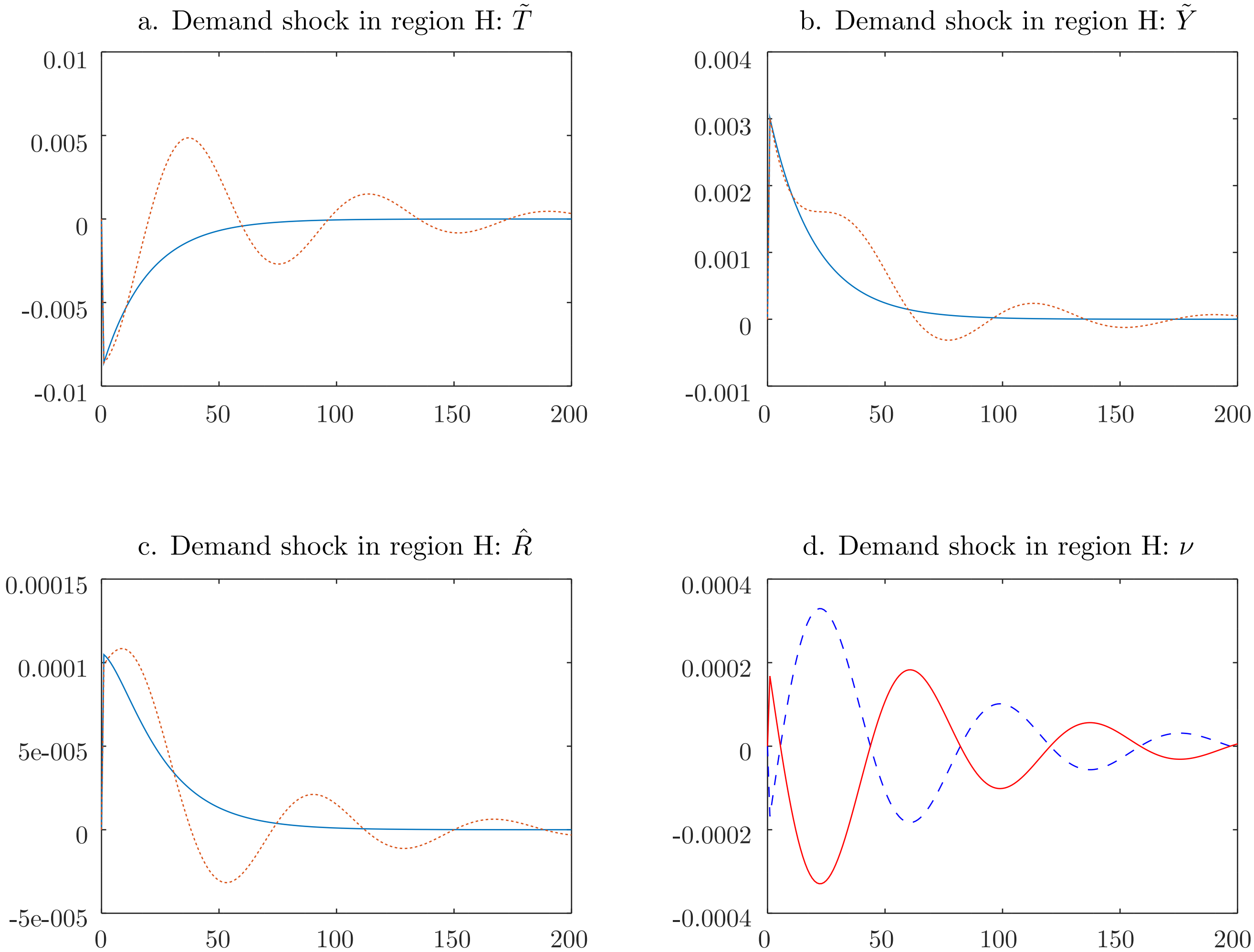

Figure 1.5: Impulse responses to the demand shock in region H. Solid lines in first three graphs shows the dynamics in the rational expectation model; dotted lines shows the worst-case dynamics. The last graph shows the extra disturbances, created by the evil agent in the worst case; solid line is for the shocks which decrease the terms of trade, while dashed line is for the shocks which increase the terms of trade.

just one considerable cycle in the terms of trade in these cases. The interest rate reaction to these shocks is qualitatively the same as in the previous case. Nevertheless, the magnitude is lower.

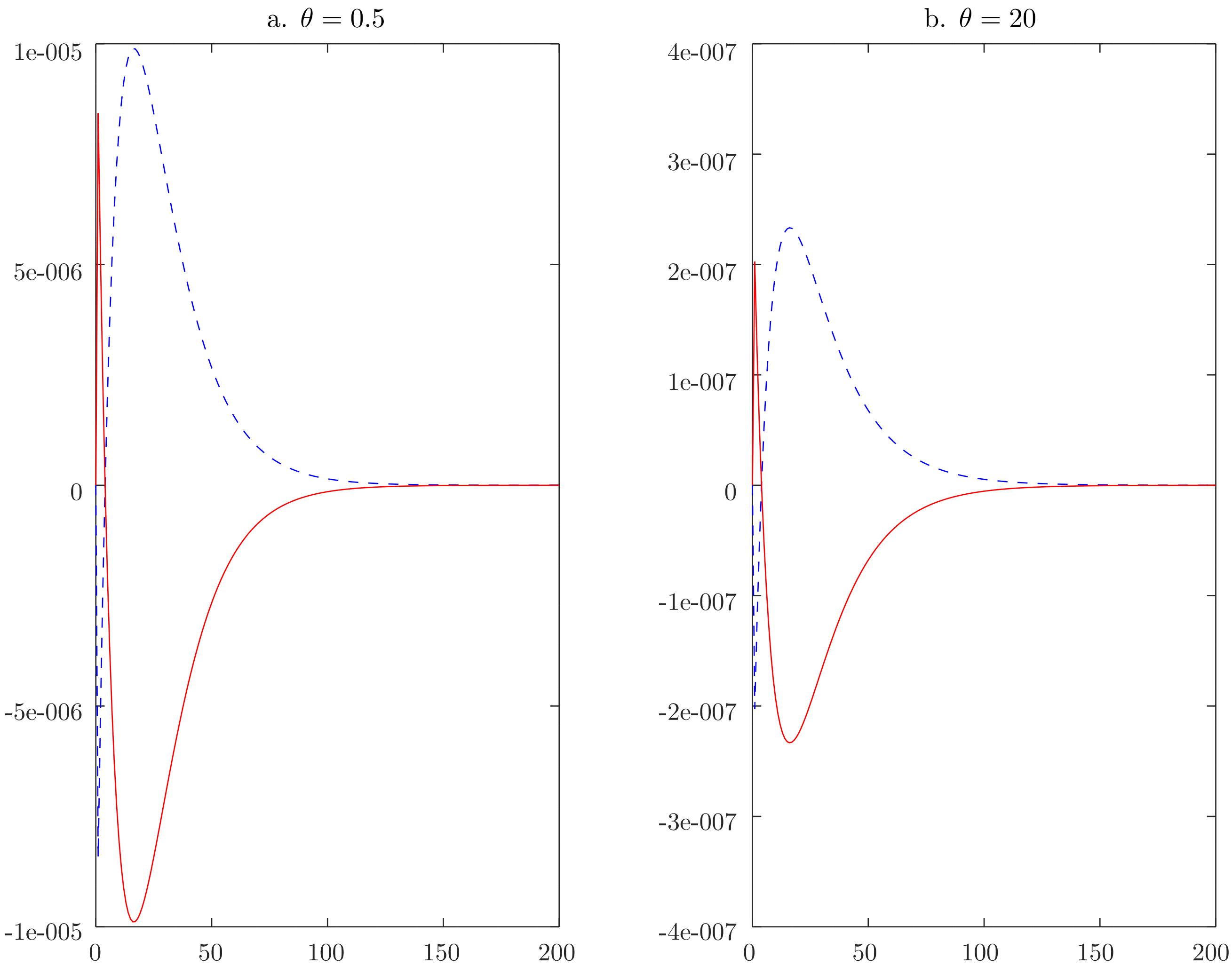

Figure 1.6: Extra disturbances, created by the evil agent in the worst case. Solid lines depict the shocks which decrease the terms of trade, while dashed line is for the shocks which increase the terms of trade.

1.4 Conclusion

For the micro-founded two-country model of a currency union by Benigno (2004), we construct robust monetary policy under commitment. We study the characteristics of this policy and find that the reaction of the central bank to an increase in model uncertainty should be different for the shocks of different origin. If the shocks happen in a larger region with stickier prices, the central bank should react to them less aggressively when the extent of possible misspecification increases. Consequently, the Brainard attenuation effect holds true for these shocks. If the shocks happen in

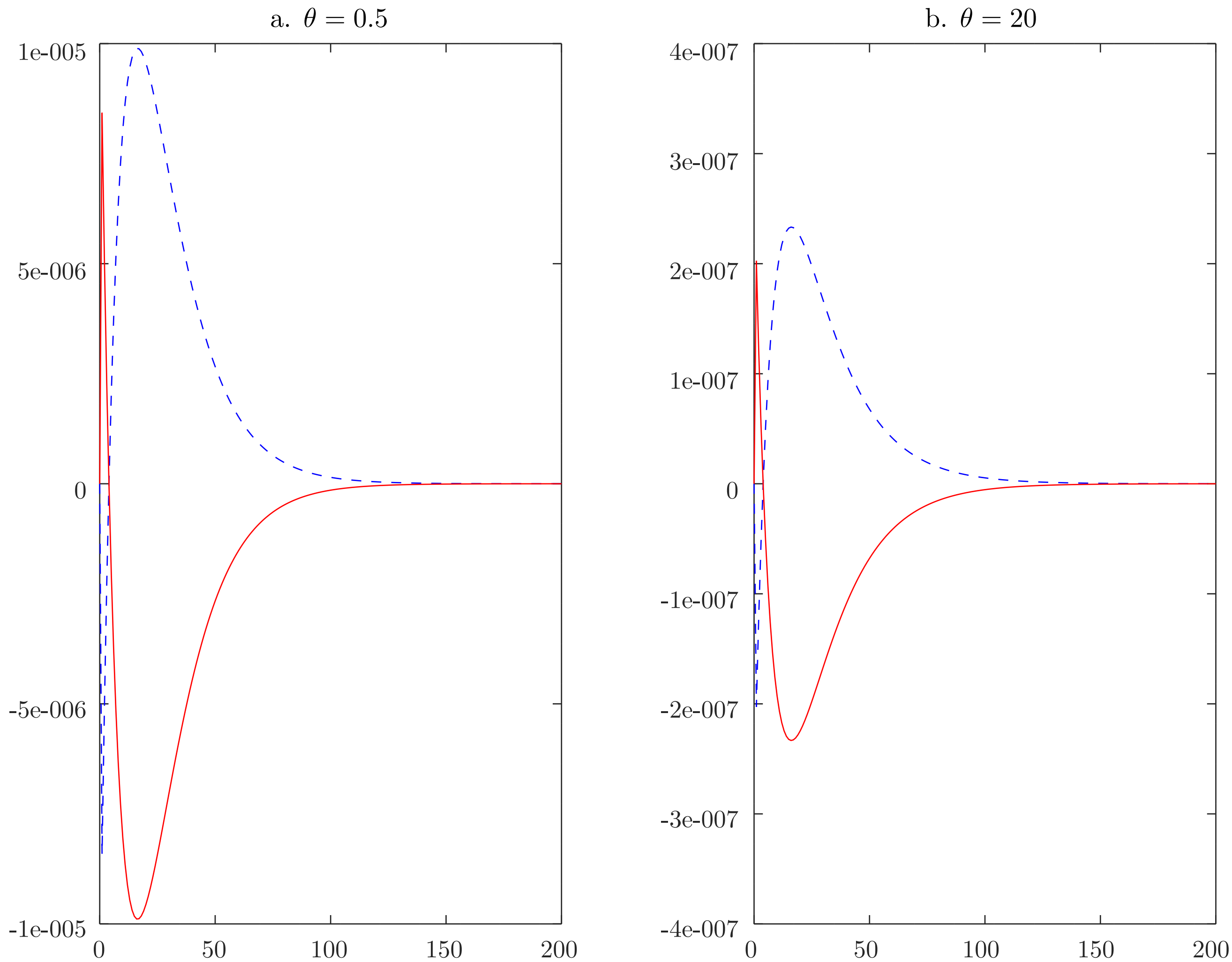

Figure 1.7: Extra disturbances, created by the evil agent in the worst case. Solid lines depict the shocks which decrease the terms of trade, while dashed line is for the shocks which increase the terms of trade.

a smaller region with more flexible prices, the central bank should be more aggressive. Thus, for these shocks the Brainard principle is violated, "anti-attenuation" effect is present.

The special discussion should concern the choice of robust policy criterion. In our paper, we rely on robust control method, which is the most widely used to elaborate optimal policy under model uncertainty. Nevertheless, this approach is sometimes criticized. For example, according to Sims (2001), this criterion assumes that the policymaker takes the decisions on a base of the least known worst cases, and this seems to be a paradoxical pattern of behavior. For this reason, some authors propose the info-gap robust satisfying approach of Ben-Haim (2006) instead of robust control by

Hansen and Sargent (2008). The info-gap approach assumes that the policymaker chooses the worst tolerable level of performance and looks for the policy which assures that the performance of the economy under all possible modifications is at least as good as this level. Thus, this approach is close but not equivalent to robust control. As info-gap approach assumes that the central bank is not willing to maximize its performance, this approach have not found substantial support in the literature. Moreover, this method requires much more computational efforts than robust control, while the concepts behind them are relatively close to each other.

One of the prominent directions for future research is the analysis of active fiscal policy in a monetary union. In our model, the shocks of government spending are described by the auto-regressive process. The inclusion of decision problem for the government would enrich the model considerably. The case of unstructured Knightian uncertainty, when the central bank has no information about the nature and the location of uncertainty, seems to be a little bit far from reality. Much more likely, the central bank should have doubts about the precise parameters of its model. This means that we should analyze structured Knightian uncertainty. The parameter of particular interest for the central bank is price stickiness in the different regions. As we found out, this parameter influences crucially the social welfare function and the objective function of the central bank, so this case is one of the most provoking and promising.

Another issue, which has become very important in the last several years, is the conduct of robust monetary policy under zero lower bound. Some researchers have already studied this issue. For example, Levine and Pearlman (2010) and Levine, McAdam and Pearlman (2012) show that ZLB constraint is crucial when elaborating robust monetary policy. Levine and Pearlman (2010) show that robust policy in a standard New-Keynesian model may imply a considerable violation of ZLB constraint and Levine, McAdam and Pearlman (2012) argue that the possible violation of ZLB should be taken into account when discussing the tolerance of different models to misspecifications. To the best of our knowledge, there has been no attempt to elaborate the robust monetary policy in a currency union with ZLB constraint. This would be a prominent direction for future research.

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Appendix

A.1 Brief description of the model by Benigno (2004)

Demand. Individual i in region j and period t maximizes his utility $U_{i,t}^{j}$ by solving the following program:

$$\max_{\left\{C_{i,k}^{j},\frac{M_{i,k}^{j}}{P_{t}^{j}},B_{i,k},B_{i,k}^{j}\right\}} U_{i,t}^{j}=E_{t}\left\{\sum_{k=t}^{\infty}\beta^{k-t}\left[U\left(C_{i,k}\right)+L\left(\frac{M_{i,k}}{P_{t}^{j}};\varepsilon_{k}^{j}\right)-V\left(y_{i,k}^{j};s_{k}^{j}\right)\right]\right\} \tag{1.16}$$

$$\begin{aligned}&\text{s.t. } E_{t}\left\{q_{t}^{j}B_{i,t}^{j}\right\}+\tfrac{B_{i,t}}{P_{t}(1+R_{t})}+\tfrac{M_{i,t}^{j}}{P_{t}}\leq\\&\leq B_{i,t-1}^{j}+\tfrac{M_{i,t-1}^{j}+B_{i,t-1}}{P_{t}}+\left(1-\tau^{j}\right)\tfrac{p_{i,t}^{j}y_{i,t}^{j}}{P_{t}(1+R_{t})}-C_{i,t}+\tfrac{Q_{i,t}^{j}}{P_{t}}\ \forall t\end{aligned} \tag{1.17}$$

where $i\in[0,1]$ is agent's index, $j\in\{H,F\}$ is region index, β is inter-temporal discount rate, $C_{i,k}$ is consumption, $\frac{M_{i,k}}{P_{k}^{j}}$ is a stock of real money balances, $y_{i,k}^{j}$ is supply of a differentiated good by the agent, $V\left(y_{i,k}^{j},s_{k}^{j}\right)$ represents labor disutility. ε_{k}^{j} is a country-specific liquidity preference shock, while s_{k}^{j} represents a productivity shock in country j. $E_{t}X_{t+k}$ stands for the expectations in the period t of the value of variable X in period $t+k$.

Every agent consumes home and foreign good bundles, which are substitutes. Consumption index $C_{i,t}$ is a combination of consumption indices for home and foreign goods: $C_{i,t}=\frac{\left(C_{i,t}^{H}\right)^{n}\left(C_{i,t}^{F}\right)^{1-n}}{n^{n}(1-n)^{1-n}}$, where $C_{i,t}^{j}$ is the amount of goods, produced in region j and consumed by agent i. Within each bundle the products are substitutes with an elasticity of substitution σ. Thus,

$$C_{i,t}^{H}=\left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}}\int_{0}^{n}c_{i,t}\left(h\right)^{\frac{\sigma-1}{\sigma}}dh\right]^{\frac{\sigma}{1-\sigma}}$$

and

$$C_{i,t}^{F}=\left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}}\int_{n}^{1}c_{i,t}\left(f\right)^{\frac{\sigma-1}{\sigma}}df\right]^{\frac{\sigma}{1-\sigma}},$$

where $c_{i,t}(h)$ is a quantity of good $h\in[0,n)$ which is produced in region H and consumed by agent i. Similarly, $c_{i,t}(f)$ is quantity of good $f\in[n,1]$, produced in region F and consumed by agent i.

Therefore, consumer price index in region j is

$$P^{j}\equiv\left(P_{H}^{j}\right)^{n}\left(P_{F}^{j}\right)^{1-n},$$

where $P_{H}^{j}\equiv\left[\left(\frac{1}{n}\right)\int_{0}^{n}p^{j}\left(h\right)^{1-\sigma}dh\right]^{\frac{1}{1-\sigma}}$ is consumer price index of the goods which are produced in region H and consumed by the agents in region j and $p^{j}(h)$ is a price of a good h sold in the region j. $P_{F}^{j}\equiv\left[\left(\frac{1}{1-n}\right)\int_{n}^{1}p^{j}\left(f\right)^{1-\sigma}df\right]^{\frac{1}{1-\sigma}}$ is consumer price index of the goods which are produced

in region F and consumed by the agents in region j and $p^j(f)$ is a price of a good f sold in the region j. With zero transaction costs, every good must be sold at equal prices in both the regions, implying that $p^H(g) = p^F(g)$ for every $g \in [0,1]$.
The terms of trade represent the relative prices in region F :

$$T_t = \frac{P_t^F}{P_t^H}.$$

Consumer's budget constraint (1.17) includes the real value of agent i portfolio of contingent securities issued in region j and denominated in units of the consumption-based price index with one-period maturity $B_{i,t}^j$; the vector of the security prices q_t^j; agent i holding of the nominal one-period non-contingent bond denominated in the union currency $B_{i,t}$; the nominal interest rate R_t; a regional proportional tax on nominal income τ^j; nominal lump-sum transfers from the fiscal authority of region j to the agent i $Q_{i,t}^j$.
Fiscal policies are determined by the local governments. Each government collects taxes τ^j, determines transfers $Q_{i,t}^j$ and purchases the goods produced in its own country G_t^j. We do not deal with the problem of fiscal policy determination, so we do not solve any programs for the transfers or taxes. We assume that the the tax rates and subsidies are chosen such that to avoid the distortions created by the monopolistic competition. Moreover, transfers and government spendings follows the autoregressive processes such that the inter-temporal budget constraint is held:

$$E\sum_{t=0}^{\infty} \frac{\tau^j Y_t^j - Q_t^j - G_t^j}{\prod_{s=0}^{t} (1 + R_s)} = 0$$

The private agents in the whole economy and the government of region j form the total demand for each good produced in this region. Thus, the total demand for goods produced in two the region are given by the following formulas:

$$Y_t^H = \frac{\left[T_t^{1-n} C_t^W + G_t^H\right]}{P_t^H}$$
$$Y_t^F = \frac{\left[T_t^{-n} C_t^W + G_t^F\right]}{P_t^F},$$

where $C_t^W = \int_0^1 C_{i,t} di$ is a union-wide consumption index.

Supply. A firm i in region j faces the probability $(1 - \alpha^j)$ to change its price. If a firm changes a price at period t, it sets a price $\tilde{p}_t(i)$ which maximizes the following function:

$$E_t \sum_{k=0}^{\infty} \left(\alpha^j \beta\right)^k \left[\lambda_{t+k} \left(1 - \tau^j\right) \tilde{p}_t(i)\, \tilde{y}_{t,t+k}(i) - V\left(\tilde{y}_{t,t+k}(i), s_{t+k}^j\right)\right]$$

Where $\lambda_{t+k} = \frac{U_c(C_{t+k})}{P_{t+k}}$represents the marginal utility of nominal income from and $\tilde{y}_{t,t+k}(i)$ is a total demand for a product of firm i in period $t+k$, if $\tilde{p}_t(j)$ is applied. This gives the following optimal price:

$$\tilde{p}_t(i) = \frac{\sigma}{(1-\tau^j)(\sigma-1)} \frac{E_t \sum_{k=0}^{\infty} V_y' \left(\tilde{y}_{t,t+k}(i), z_{t+k}^j\right) \tilde{y}_{t,t+k}(i)}{E_t \sum_{k=0}^{\infty} \lambda_{t+k} (\alpha^j \beta)^k \tilde{y}_{t,t+k}(i)}$$

Dynamics of prices in region j is as follows:

$$\left(P_t^j\right)^{1-\sigma} = \alpha^j \left(P_{t-1}^j\right)^{1-\sigma} + \left(1-\alpha^j\right)\left(\tilde{p}_t^j\right)^{1-\sigma}$$

Equilibrium with flexible prices. Linearization of equations above around the deterministic steady state if $\alpha = 0$ gives the following dynamics:

$$\tilde{C}_t^W = \frac{\eta}{\eta+\rho}\left(s_t^W - \tilde{g}_t^W\right)$$

$$\tilde{T}_t = \frac{\eta}{1+\eta}\left[g_t^R - s_t^R\right]$$

$$\tilde{Y}_t^W = \frac{\rho}{\rho+\eta} g_t^W + \frac{\eta}{\rho+\eta} s_t^W,$$

where g_t stands for the shocks of government spendings and s_t is supply (technological) shocks.

Equilibrium with sticky prices. Linearization of equilibrium conditions around the deterministic steady state if $\alpha > 0$ gives the following dynamics:

$$E_t \hat{C}_{t+1}^W = \hat{C}_t^W + \rho^{-1}\left(\hat{R}_t - E_t \pi_{t+1}^W\right)$$

$$\hat{Y}_t^H = (1-n)\hat{T}_t + \hat{C}_t^W + g_t^H$$

$$\hat{Y}_t^F = -n\hat{T}_t + \hat{C}_t^W + g_t^F$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H$$

$$\pi_t^H = (1-n) k_T^H \left(\hat{T}_t - \tilde{T}_t\right) + k_C^H y_t^W + \beta E_t \pi_{t+1}^H$$

$$\pi_t^F = -n k_T^F \left(\hat{T}_t - \tilde{T}_t\right) + k_C^F y_t^W + \beta E_t \pi_{t+1}^F$$

where $\begin{aligned} k_C^j &= \left[\frac{\left(1-\alpha^j\beta\right)\left(1-\alpha^i\right)}{\alpha^j}\right]\left[\frac{\rho+\eta}{1+\sigma\eta}\right] \\ k_T^j &= k_C^j\left[\frac{1+\eta}{\rho+\eta}\right]\end{aligned}$ and $\begin{aligned} \rho &\equiv -\frac{U_{CC}\bar{C}}{U_C} \\ \eta &\equiv \frac{V_{yy}\bar{C}}{V_y} \\ \bar{Y}_t &\equiv -\frac{V_{Y\varepsilon}\varepsilon}{V_{yy}\bar{C}}\end{aligned}$.

Combining the equilibrium expressions for output in two regions, we get

$$\hat{Y}_t^W = \hat{C}_t^W + g_t^W.$$

Welfare. Welfare criterion for the central bank is a weighted expected sum of future welfare ratios w_t:

$$W = E_0 \left\{ \sum_{t-0}^{+\infty} \beta^t w_t \right\}$$

$$w_t \equiv U(C_t) - \int_0^1 V\left(y_t(j), z_t^i\right) dj$$

Linearization of this welfare function under assumption that utility gains from liquidity services are small, gives the loss function from the main text:

$$L_t = \Lambda \left[\hat{Y}_t^W - \tilde{Y}_t^W\right]^2 + n(1-n)\Gamma\left[\hat{T}_t - \tilde{T}_t\right]^2 + \gamma_H \left(\pi_t^H\right)^2 + \gamma_F \left(\pi_t^F\right)^2,$$

where $\Lambda = \frac{1/\sigma}{n/k_C^H + (1-n)/k_C^F}$, $\Gamma = \frac{(1+\eta)/\sigma}{(n/k_C^H + (1-n)/k_C^F)(\rho+\eta)}$, $\gamma_H = \frac{n/k_C^H}{n/k_C^H + (1-n)/k_C^F}$ and $\gamma_F = \frac{(1-n)/k_C^F}{n/k_C^H + (1-n)/k_C^F}$.

A.2 Law of motion of the economy

The law of motion of the economy is described by the following system:

$$\begin{bmatrix} e_{t+1} \\ E_t z_{t+1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} e_t \\ z_t \end{bmatrix} + \mathbf{B}R + \mathbf{C}\varepsilon_{t+1},$$

where $\mathbf{A} = \begin{bmatrix} A_{ee} & A_{ez} \\ A_{\pi,He} & A_{\pi,Hz} \\ A_{\pi,Fe} & A_{\pi,Fz} \\ A_{Te} & A_{Tz} \\ A_{Ye} & \mathbf{A_{Yz}} \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} C_e \\ C_z \end{bmatrix}$.

The 4×4 matrix $\mathbf{A_{ee}}$ describes the effect of the change in backward-looking variables on their future values. Under assumptions from the main text, $\mathbf{A_{ee}} = \rho_e \equiv \begin{bmatrix} \rho_g^H & 0 & 0 & 0 \\ 0 & \rho_g^F & 0 & 0 \\ 0 & 0 & \rho_s^H & 0 \\ 0 & 0 & 0 & \rho_s^F \end{bmatrix}$. As in the model the future values of backward-looking variables do not depend on the current values of forward-looking variables, $\mathbf{A_{ez}} = \mathbf{0}_{4\times 4}$.

Equations (1.7) and (1.8) can be rewritten as:

$$\begin{bmatrix} \tilde{T}_t \\ \tilde{Y}_t \end{bmatrix} = \tilde{\mathbf{D}} \begin{bmatrix} g_t^R \\ g_t^W \\ s_t^R \\ s_t^W \end{bmatrix} \tag{1.18}$$

with $\tilde{\mathbf{D}} = \begin{bmatrix} \frac{\eta}{1+\eta} & 0 & -\frac{\eta}{1+\eta} & 0 \\ 0 & \frac{\rho}{\rho+\eta} & 0 & \frac{\eta}{\rho+\eta} \end{bmatrix}$. By definition of relative and union-wide shocks, $\begin{bmatrix} g_t^R \\ g_t^W \\ s_t^R \\ s_t^W \end{bmatrix} = \mathbf{D} \begin{bmatrix} g_t^H \\ g_t^F \\ s_t^H \\ s_t^F \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ n & (1-n) & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & n & (1-n) \end{bmatrix}$, we can rewrite (1.18) in the following way:

$$\begin{bmatrix} \tilde{T}_t \\ \tilde{Y}_t \end{bmatrix} = \tilde{\mathbf{D}}\mathbf{D} e_t, \tag{1.19}$$

where $\tilde{\mathbf{D}}\mathbf{D} = \begin{bmatrix} -\frac{\eta}{1+\eta} & \frac{\eta}{1+\eta} & \frac{\eta}{1+\eta} & -\frac{\eta}{1+\eta} \\ n\frac{\rho}{\rho+\eta} & (1-n)\frac{\rho}{\rho+\eta} & n\frac{\eta}{\rho+\eta} & (1-n)\frac{\eta}{\rho+\eta} \end{bmatrix}$.

From (1.3),

$$E_t \pi_{t+1}^H = \frac{1}{\beta} \left(\begin{bmatrix} (1-n)\, k_T^H & k_C^H \end{bmatrix} \begin{bmatrix} \tilde{T}_t \\ \tilde{Y}_t \end{bmatrix} + \begin{bmatrix} 1 & 0 & -(1-n)\, k_T^H & -k_C^H \end{bmatrix} z_t \right).$$

Using (1.19), we get $E_t \pi_{t+1}^H = \mathbf{A}_{\pi,\mathbf{He}} e_t + \mathbf{A}_{\pi,\mathbf{Hz}} z_t$,where

$$\mathbf{A}_{\pi,\mathbf{He}} = \frac{1}{\beta} \begin{bmatrix} (1-n)\, k_T^H & k_C^H \end{bmatrix} \tilde{D}D$$

$$\mathbf{A}_{\pi,\mathbf{Hz}} = \frac{1}{\beta} \begin{bmatrix} 1 & 0 & -(1-n)\, k_T^H & -k_C^H \end{bmatrix}$$

Analogically,

$$E_t \pi_{t+1}^F = \frac{1}{\beta} \left(\begin{bmatrix} -nk_T^F & k_C^F \end{bmatrix} \begin{bmatrix} \tilde{T}_t \\ \tilde{Y}_t \end{bmatrix} + \begin{bmatrix} 0 & 1 & nk_T^F & -k_C^F \end{bmatrix} z_t \right).$$

Thus,

$$\mathbf{A}_{\pi,Fe} = \frac{1}{\beta} \begin{bmatrix} -nk_T^F & k_C^F \end{bmatrix} \tilde{D}D$$

$$\mathbf{A}_{\pi,Fz} = \frac{1}{\beta} \begin{bmatrix} 0 & 1 & nk_T^F & -k_C^F \end{bmatrix}$$

As $\hat{T}_{t+1} = \hat{T}_t + \pi_{t+1}^F - \pi_{t+1}^H$, the expectations of the terms of trade are given by $E_t \hat{T}_{t+1} = \mathbf{A}_{Te} e_t + \mathbf{A}_{Tz} z_t$, where $\mathbf{A}_{Te} \equiv \mathbf{A}_{\pi,Fe} - \mathbf{A}_{\pi,He}$ and $\mathbf{A}_{Tz} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} + \mathbf{A}_{\pi,Fz} - \mathbf{A}_{\pi,Hz}$. Thus,

$$\mathbf{A}_{Te} = \frac{1}{\beta} \begin{bmatrix} -nk_T^F - (1-n)\, k_T^H & k_C^F - k_C^H \end{bmatrix} \tilde{D}D$$

$$\mathbf{A}_{Tz} = \frac{1}{\beta} \begin{bmatrix} -1 & 1 & \beta + nk_T^F + (1-n)\, k_T^H & k_C^H - k_C^F \end{bmatrix}$$

From (1.6),

$$E_t\hat{Y}^W_{t+1} = -\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}\left(\begin{bmatrix} g^R_t \\ g^W_t \\ s^R_t \\ s^W_t \end{bmatrix} - E_t\begin{bmatrix} g^R_{t+1} \\ g^W_{t+1} \\ s^R_{t+1} \\ s^W_{t+1} \end{bmatrix}\right) + \hat{Y}^W_t - \rho^{-1}\left(nE_t\pi^H_{t+1} + (1-n)\,E_t\pi^F_{t+1}\right) + \rho^{-1}\hat{R}_t \tag{1.20}$$

As $\begin{bmatrix} g^R_t \\ g^W_t \\ s^R_t \\ s^W_t \end{bmatrix} = \mathbf{D}e_t$ and $E_te_{t+1} = \mathbf{A}_{ee}e_t$, we can rewrite (1.20) in the following way:

$$E_t\hat{Y}_{t+1} = \mathbf{A}_{Ye}e_t + \mathbf{A}_{Yz}z_t + \rho^{-1}\hat{R}_t$$

where $\mathbf{A}_{Ye} = -\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}\mathbf{D}\left(\mathbf{I}_{4\times4} - \mathbf{A}_{ee}\right) - \rho^{-1}\left(n\mathbf{A}_{\pi,He} + (1-n)\,\mathbf{A}_{\pi,Fe}\right)$ and $\mathbf{A}_{Yz} = -\rho^{-1}\left(n\mathbf{A}_{\pi,Hz} + (1-n)\,\mathbf{A}_{\pi,Fz}\right) + \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$.

Matrix $\mathbf{B}$ shows the effects of policy instrument on the economy and is equal to:

$$\mathbf{B} = \begin{bmatrix} \mathbf{0}_{7\times1} \\ \rho^{-1} \end{bmatrix}$$

Matrix $\mathbf{C}$ shows the effect of shock innovations on the economy:

$$\mathbf{C} = \begin{bmatrix} \mathbf{I}_{4\times4} \\ \mathbf{0}_{4\times4} \end{bmatrix}$$

The component of loss function, which depends on the policy actions, is given by

$$\hat{L} = \Lambda\left[\hat{Y}^W_t - \tilde{Y}^W_t\right]^2 + n\,(1-n)\,\Gamma\left[\hat{T}_t - \tilde{T}_t\right]^2 + \gamma_H\left(\pi^H_t\right)^2 + \gamma_F\left(\pi^F_t\right)^2$$

Using the formulas above, we can rewrite this loss component in the following form:

$$\hat{L} = \begin{bmatrix} e_t^\top\mathbf{D}^\top\tilde{\mathbf{D}}^\top & z_t^T \end{bmatrix}\tilde{\mathbf{Q}}\begin{bmatrix} \mathbf{D}\tilde{\mathbf{D}}e_t \\ z_t \end{bmatrix},$$

where $\tilde{\mathbf{Q}} = \begin{bmatrix} \tilde{\mathbf{Q}}_\mathbf{1} & \tilde{\mathbf{Q}}_\mathbf{2} \\ \tilde{\mathbf{Q}}_\mathbf{3} & \tilde{\mathbf{Q}}_\mathbf{4} \end{bmatrix}$, $\tilde{\mathbf{Q}}_\mathbf{1} = \begin{bmatrix} n\,(1-n)\,\Gamma & 0 \\ 0 & \Lambda \end{bmatrix}$, $\tilde{\mathbf{Q}}_\mathbf{2} = \begin{bmatrix} 0 & 0 & -n\,(1-n)\,\Gamma & 0 \\ 0 & 0 & 0 & -\Lambda \end{bmatrix}$, $\tilde{\mathbf{Q}}_\mathbf{3} = \tilde{\mathbf{Q}}_\mathbf{2}^\top$, $\tilde{\mathbf{Q}}_\mathbf{4} = \begin{bmatrix} \gamma_H & 0 & 0 & 0 \\ 0 & \gamma_F & 0 & 0 \\ 0 & 0 & n\,(1-n)\,\Gamma & 0 \\ 0 & 0 & 0 & \Lambda \end{bmatrix}$.

Equivalently,

$$\hat{L} = \begin{bmatrix} e_t^\top & z_t^\top \end{bmatrix} \mathbf{Q} \begin{bmatrix} e_t \\ z_t \end{bmatrix},$$

where $\mathbf{Q} = \begin{bmatrix} \mathbf{D}^\top\tilde{\mathbf{D}}^\top\tilde{\mathbf{Q}}_1\mathbf{D}\tilde{\mathbf{D}} & \mathbf{D}^\top\tilde{\mathbf{D}}^\top\tilde{\mathbf{Q}}_2 \\ \tilde{\mathbf{Q}}_3\mathbf{D}\tilde{\mathbf{D}} & \tilde{\mathbf{Q}}_4 \end{bmatrix}$ or $\mathbf{Q} = \hat{\mathbf{D}}^\top\tilde{\mathbf{Q}}\hat{\mathbf{D}}$ and $\hat{\mathbf{D}} = \begin{bmatrix} \mathbf{D}\tilde{\mathbf{D}} & \mathbf{0}_{2\times 4} \\ \mathbf{0}_{4\times 2} & \mathbf{I}_{4\times 4} \end{bmatrix}$.

A.3 Robust policy

The choice of robust policy implies the solution of the following problem:

$$\min_{\{R_t\}} \max_{\{\upsilon_{t+1}\}} E_0 \textstyle\sum_{t=0}^{\infty} \beta^t (x_t' \mathbf{Q} x_t - \theta \upsilon_{t+1}' \upsilon_{t+1})$$
$$\text{s.t.} \begin{bmatrix} e_{t+1} \\ E_t z_{t+1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} e_t \\ z_t \end{bmatrix} + \mathbf{B} R_t + \mathbf{C}\left(\varepsilon_{t+1} + \upsilon_{t+1}\right)$$

Solution method for such a problem has been proposed by Giordani and Söderlind (2004). The equilibrium dynamics of backward-looking variables is as follows:

$$\begin{bmatrix} e_{t+1} \\ \rho_{t+1}^z \end{bmatrix} = \mathbf{M}_{\mathbf{8}\times\mathbf{8}} \begin{bmatrix} e_t \\ \rho_t^z \end{bmatrix} + \begin{bmatrix} \mathbf{I}_{4\times 4}\varepsilon_{t+1} \\ \mathbf{0}_{4\times 1} \end{bmatrix},$$

where e_t is a 4×1 vector of shocks form the main text and ρ_t^z is a 4×1 vector of shadow prices for forward-looking variables z_t.

The equilibrium dynamics of forward-looking variables is given by:

$$\begin{bmatrix} z_t \\ R_t \\ \upsilon_{t+1} \\ \rho_t^e \end{bmatrix} = \mathbf{N}_{\mathbf{13}\times\mathbf{8}} \begin{bmatrix} e_t \\ \rho_t^z \end{bmatrix},$$

where z_t is a 4×1 vector forward-looking variables from the main text, R_t is interest rate of the central bank, υ_{t+1} is 4×1 vector of additional shocks, created by the evil agent and ρ_t^e is a 4×1 vector of shadow prices for backward-looking variables e_t. Matrices $\mathbf{M}_{\mathbf{8}\times\mathbf{8}}$ and $\mathbf{N}_{\mathbf{13}\times\mathbf{8}}$ characterize the dynamics of the system and gives the impulse responses in the text. The fifth raw in matrix $\mathbf{N}_{\mathbf{13}\times\mathbf{8}}$ characterizes the optimal policy, discussed in the main text.

Chapter 2

The Role of Uncertain Government Preferences for Fiscal and Monetary Policy Interaction

Abstract

This paper explores the role of uncertain government preferences in a linear-quadratic model of fiscal and monetary policy interaction. We show that the effects of preference uncertainty are fastened on multiplicative uncertainty about the policy effectiveness. If the effects of fiscal and monetary policies on the economy are known, preference uncertainty does not affect the symbiosis result of interaction. In this case, inflation and output are equal to their targets irrespective of the central bank and the government preferences. Multiplicative uncertainty about the fiscal policy effects creates the inflation bias, and preference uncertainty deteriorates it by lowering output and rising inflation up. Multiplicative uncertainty about the monetary policy effects creates either standard inflation bias or negative inflation bias with output higher than the target and inflation lower than the target. In this case, preference uncertainty enlarges the absolute value of the output gap, while the effect on the inflation gap depends on the extent of monetary multiplicative uncertainty. Thus, under some circumstances uncertain government preferences can even reduce the negative effect of multiplicative uncertainty. If the effects of both policies are uncertain, the impact of preference uncertainty depends not only on the extent of multiplicative uncertainty, but also on the inflation and output targets. After studying the impact of uncertainty on inflation and output gaps, we proceed with the welfare properties of the equilibrium and discuss the optimal conservativeness of authorities for different types of uncertainty[1].

[1]co-written with Sergey Merzlyakov, NRU HSE, Moscow, Russia

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JEL Codes: *E52, E58, E62, E63*

2.1 Introduction

Trump's inauguration has provoked the extensive debates among economists about the future fiscal policy stance in the U.S. Many analysts worry about the macroeconomic effects of this "Trump's uncertainty". It is too early to estimate its real economic effects, but it is already obvious, that the Fed's policy may be changed in response to this uncertainty. Some hint of possible changes can be found, for example, in the speech of the Fed Governor Lael Brainard on January 17, 2017 (Brainard et al. (2017)):

"There are many sources of uncertainty affecting... the appropriate path of monetary policy. In particular, there has been speculation about significant changes to fiscal policy of late, although the magnitude, composition, and timing of any fiscal changes are as yet unknown and will depend on the incoming Administration and the new Congress as well as the vicissitudes of the budgeting process... It thus seems possible that monetary policy could be affected for some time by uncertainty surrounding fiscal policy and its effects on the economy".

Starting from the famous paper by Sargent and Wallace (1981), fiscal and monetary policy interaction has been always in the center of attention in academic literature. One of the most important issues in this literature is whether the central bank and the government can achieve the target values of output and inflation. Up to the moment, there has been no consensus in this question.

Dixit and Lambertini (2003*b*) show that fiscal and monetary policy do achieve the target values of output and inflation if the government and the central bank share their targets. This result holds for all the forms of policy interaction and for all the weights in the loss functions. This conclusion is known as the symbiosis result. However, Dixit and Lambertini (2003*a*) show that if fiscal policy creates dead-weight loss and the targets of the central bank and the government are different, the non-cooperative equilibrium is characterized by inflation bias. This inflation bias with inflation higher than the target and output lower than the target arises because of too restrictive fiscal policy and too expansionary monetary policy.

Two papers by Di Bartolomeo et al. show that the symbiosis result also does not hold in case of multiplicative uncertainty. Di Bartolomeo, Giuli and Manzo (2009) investigate central bank and government interaction under multiplicative uncertainty about the fiscal policy effectiveness. They show that even if the government and the central bank share output and inflation target levels, fiscal multiplicative uncertainty does not allow them to achieve these targets. This uncertainty forces the government to become more cautious. As a result, fiscal policy becomes less expansionary and output drops. The central bank faces time inconsistency problem and tries to raise output with too expansionary policy, which leads to an increase in inflation, and the inflation bias arises. Di Bartolomeo and Giuli (2011) analyze multiplicative uncertainty about monetary policy effectiveness and come to the same result: multiplicative uncertainty causes ineffective levels of output and inflation in equilibrium. In their model, monetary multiplicative uncertainty forces the monetary authority to lower the absolute value of its intervention. This leads to the gap between the equilibrium inflation and its target. This effect could be neutralized by the change in fiscal policy, which can be done at sake of the gap between the equilibrium output and the target level. Obviously, the government is reluctant to change considerably the policy and none of the targets is achieved.

In our paper, we examine these results in the model with uncertain government preferences. We assume that the government knows its own preferences, while for the others the government preferences are uncertain. To our knowledge, there are no other studies of fiscal and monetary policy interaction with uncertain government preferences. The role of uncertain central bank preferences has been already studied in economic literature. Ciccarone, Marchetti and Di Bartolomeo (2007), Hefeker and Zimmer (2011) show that uncertainty about the central bank preferences could reduce the macroeconomic volatility due to the fiscal disciplining effect, which is expressed in reduction of taxes, inflation and output distortions. Dai and Sidiropoulos (2011), however, note that such result can be achieved only under the Stackelberg interaction, where the government acts as a leader and the central bank acts as a follower. Dai and Sidiropoulos (2011) argue that the fiscal disciplining effect of uncertain central bank preferences could be insignificant if the government and the central bank move simultaneously. Oros and Zimmer (2015) analyze the monetary transmission mechanism in a monetary union with uncertain central bank preferences. They show that the private agents expect the central bank to be more conservative to compensate the uncertainty of the central bank preferences. This could lead to a decrease in inflation and better macroeconomic outcomes not because of a disciplinary effect, but because of the central bank's communication channel.

Thus, as we have seen, economic literature elaborates a number of applications of uncertainty about the central bank preferences for strategic interaction between fiscal and monetary policy. However, the existing research has not been dealing with uncertainty about the government preferences. Meanwhile, uncertainty about the government preferences seems to be much more significant than uncertainty about the central bank preferences, at least in developed countries. For example, the targets of the European Central Bank are clearly defined: inflation below and close to 2 percent. Moreover, Blinder et al. (2008) show that in recent years transparency of monetary policy has considerably increased all over the world. This means that the assumption of uncertain central bank preferences might be unjustified. At the same time, taking into account uncertain government preferences seems to be promising. Firstly, the government preferences are exposed to considerable changes in the election period. Moreover, fiscal authorities have not been demonstrating considerable improvements in their information policies in recent years. Almost everywhere, the governments are much less transparent than the central banks.

The goal of our paper is to study the effects of uncertain government preferences on fiscal and monetary policy interaction. We show that uncertainty about the government preferences does not change the interaction result if the policy effects are certain. However, uncertain government preferences matter in case of multiplicative policy uncertainty. Below we show how uncertainty about the government preferences affects macroeconomic equilibrium under fiscal and/or monetary multiplicative uncertainty.

The paper is organized as follows. In Section I we describe a benchmark model of fiscal and monetary policy interaction. Section II analyzes the equilibrium in the model with certain preferences. In Section III we discuss the impact of uncertain government preferences on the equilibrium. Section IV concludes.

2.2 Benchmark Model

We start our analysis with a standard benchmark model with certain preferences from Dixit and Lambertini (2000, 2003*b*). This model is described by two equations: aggregated demand (2.1) and aggregated supply (2.2):

$$\pi = \varphi m + \rho c \tau \tag{2.1}$$

$$y = \overline{y} + b(\pi - \pi^e) + a\tau \tag{2.2}$$

where π is the rate of inflation, π^e is the expected rate of inflation, y is the level of real output, $\overline{y}$ is the natural level of real output, τ is the instrument of fiscal policy (for example, transfers), m is the monetary policy instrument (for example, the growth rate of the money supply). The effect of monetary policy on inflation is prone to a multiplicative shock φ with mean 1 and variance σ_φ^2. Parameter σ_φ^2 characterizes the degree of monetary multiplicative uncertainty. The average effect of fiscal policy on inflation is given by variable c. The fiscal effect on inflation is hit by multiplicative shock ρ with mean 1 and variance σ_ρ^2. Thus, parameter σ_ρ^2 characterizes the degree of fiscal multiplicative uncertainty. Parameter $b > 0$ characterizes the indirect effect of policies on the output through inflation surprise, while a is the direct effect of fiscal policy on output.

Dixit and Lambertini (2000) and complementary appendix to Dixit and Lambertini (2003*a*) show that equations (2.1) and (2.2) represent the log-linearization of equilibrium in a micro-founded general-equilibrium model. This model describes an economy inhabited by a number of individuals each of which produces a single good, sells it in a monopolistically competitive market and consumes a bundle of goods. The central bank in this economy controls money supply. An increase in money supply leads to an increase in aggregate demand and to an increase in inflation. The government in the economy may set taxes, transfers and government spendings under constraint of balanced budget. Different fiscal policy regimes implies different signs of coefficient a and c. For example, Dixit and Lambertini (2003a) assume that government sets a proportional subsidy on sales and lump-sum taxes to balance the budget. In this case an increase in proportional subsidy leads to an increase in output and to a decrease in inflation rate, meaning that a is positive and c is negative. Dixit and Lambertini (2000) mention the case of distortionary taxes and wasting government spendings. A decrease in tax rate leads to an increase in both inflation and output. This implies that both a and c are positive, if τ is treated as the opposite to tax rate. Moreover, Dixit and Lambertini (2000) show that a is negative and c is positive, if income-tax revenues are spent on government spendings.

Thus, both a and c can be of either sign. For tractability reasons and to keep our results comparabale to Di Bartolomeo, Giuli and Manzo (2009) and Di Bartolomeo and Giuli (2011), we assume that $c > 0$ and $a > 0$. Nevertheless, all the algebra in the paper remains the same for other signs of the parameters.

Our model generalizes two papers: Di Bartolomeo, Giuli and Manzo (2009), which studies fiscal multiplicative uncertainty, and Di Bartolomeo and Giuli (2011), which studies monetary multiplicative uncertainty. The results of both papers can be easily replicated in our model by putting the corresponding variance to zero. Moreover, our model allow us to study the additional effects which arise only if both multiplicative shocks are present.

Losses of the central bank and the government are defined by the gap between inflation rate and the target inflation π^* and by the gap between output and the target output y^*:

$$L_{CB} = E\left[(\pi - \pi^*)^2 + \theta_B (y - y^*)^2\right] \tag{2.3}$$

$$L_G = E\left[(\pi - \pi^*)^2 + \theta_G (y - y^*)^2\right] \tag{2.4}$$

$$\theta_B > 0, \theta_G > 0,$$

where θ_B and θ_G characterize the preferences of the central bank and the government for output. To stay in line with the broad consensus in the literature (see, for example, Rogoff (1985)), we assume that the central bank is more conservative than the government: $\theta_G \geq \theta_B$. Moreover, the output target is higher than the natural level: $y^* > \bar{y}$. In our model, the government and the central bank choose their policies simultaneously and independently after the expectations have been formed. Minimization of losses (2.3) and (2.4) subject to constraints (2.1) and (2.2) gives the following reaction functions:

$$\tau(\theta_G) = \frac{-c\left(\overline{m}-\pi^*\right)+\theta_G\left(a+bc\right)\left(y^*-\overline{y}+b\pi^e-b\overline{m}\right)}{c^2\left(1+\sigma_\rho^2\right)+\theta_G\left(\sigma_\rho^2 b^2 c^2+\left(a+bc\right)^2\right)} \tag{2.5}$$

$$m(\theta_B) = \frac{\pi^*-c\overline{\tau}+b\theta_B\left(y^*-\overline{y}+b\pi^e-\left(a+bc\right)\overline{\tau}\right)}{\left(1+\sigma_\varphi^2\right)\left(1+\theta_B b^2\right)}, \tag{2.6}$$

where (2.5) is the reaction function of the government with preferences θ_G, (2.6) is the reaction function of the central bank with preferences θ_B, $\overline{m}$ is the expected value of monetary instrument and $\overline{\tau}$ is the expected value of fiscal instrument. As we can see from (2.5) and (2.6), the equilibrium values of both policy instruments depend positively on the inflation target π^*, expected inflation π^e and the gap between target and natural output $(y^* - \overline{y})$. The impact of the output gap on a policy instrument depends positively on the weight of output in a policymaker's loss function. According to (2.6), the absolute value of monetary instrument chosen by the central bank depends negatively on the variance of monetary multiplicative shock σ_φ^2. This phenomenon corresponds to the standard attenuation effect, explored by Brainard (1967): uncertainty about the policy instrument forces the policymaker to become more cautious and to decrease the extent of intervention. The same attenuation effect is true for the government. According to (2.5), the absolute value of fiscal instrument τ decreases with the extent of fiscal multiplicative uncertainty, measured by σ_ρ^2.

2.3 Equilibrium with certain preferences

In this Section we look for the equilibrium with certain preferences. We assume that the parameter of monetary preferences θ_B is equal to $\tilde{\theta}_B$ and the parameter of the government preferences θ_G is equal to $\tilde{\theta}_G$. As the preferences of both policymakers are known by all the agents, the expected values of their policy instruments coincide with their actual values: $\overline{m} = m(\tilde{\theta}_B)$ and $\overline{\tau} = \tau(\tilde{\theta}_G)$.

We start with the equilibrium with certain policy effects, which corresponds to the model of Dixit and Lambertini (2003*b*). Substituting $\sigma_\rho^2 = 0$, $\sigma_\varphi^2 = 0$ into reaction functions (2.5) and (2.6), we obtain the following equilibrium values of fiscal and monetary instruments:

$$\tau_0 = \frac{y^* - \overline{y}}{a} \tag{2.7}$$

$$m_0 = \pi^* - c\tau_0 \tag{2.8}$$

As the target output is higher than the natural level, in equilibrium the fiscal policy is expansionary: $\tau_0 > 0$. The value of the fiscal instrument (2.7) is chosen in a such way that the equilibrium level of output coincides with the target value: $y = y^*$. Expansionary fiscal policy would lead to an increase in the inflation rate, equal to $c\tau_0$. Nevertheless, the central bank can react to this inflationary pressure by decreasing the monetary instrument by the same value. The sign of equilibrium value of m_0 depends on the value of inflation target. If inflation target is sufficiently high, such that $\pi^* > \frac{c}{a}(y^* - \overline{y})$, monetary policy is expansionary and $m_0 > 0$. If inflation target is low, the equilibrium monetary policy is contractionary, $m_0 < 0$. As a result, the equilibrium inflation rate is equal to the target: $\pi = \pi^*$. Thus, the model with certain policy effects replicates the symbiosis result of Dixit and Lambertini (2003*b*): irrespective of their preferences, the central bank and the government achieve their inflation and output targets.

If both the policy effects are uncertain, the intersection of (2.5) and (2.6) for given $\tilde{\theta}_G$ and $\tilde{\theta}_B$ brings the following equilibrium values of fiscal and monetary instruments:

$$\tilde{\tau} = \tau_0 - \frac{\tilde{W}_\tau}{\tilde{W}}\tau_0 - \frac{\tilde{W}_\tau \Lambda_B}{\tilde{W}}\tau_0 + \frac{\tilde{W}_m - \Lambda_B \tilde{\theta}_G a\,(a+bc)}{\tilde{W}}\frac{m_0}{c} \tag{2.9}$$

$$\tilde{m} = m_0 - \frac{\tilde{W}_m}{\tilde{W}}m_0 - \frac{\tilde{W}_\tau \Lambda_B}{\tilde{W}}m_0 + \left(c + ab\tilde{\theta}_B\right)\frac{\tilde{W}_\tau}{\tilde{W}}\tau_0, \tag{2.10}$$

where $\Lambda_G = \sigma_\rho^2\left(\tilde{\theta}_G b^2 + 1\right)$, $\Lambda_B = \sigma_\varphi^2\left(\tilde{\theta}_B b^2 + 1\right)$, $\tilde{W}_\tau = c^2\Lambda_G$, $\tilde{W}_m = \Lambda_B\left(c^2 + \tilde{\theta}_G a\,(a+bc)\right)$, $\tilde{W} = W + \tilde{W}_\tau + \tilde{W}_m + \Lambda_G\Lambda_B c^2$ and $W = a\left(\tilde{\theta}_G a + \left(\tilde{\theta}_G - \tilde{\theta}_B\right)bc\right)$.

According to (2.9) and (2.10), the equilibrium values of policy instruments $\tilde{\tau}$ and $\tilde{m}$ are affected by multiplicative uncertainty. We can distinguish three effects: the direct effect of fiscal multiplicative uncertainty, the direct effect of monetary multiplicative uncertainty and the mutual effect which arises only if both uncertainties are present.

The direct effect of fiscal multiplicative uncertainty corresponds qualitatively to the process described in Di Bartolomeo, Giuli and Manzo (2009). Fiscal multiplicative uncertainty forces the government to attenuate its policy and to decrease τ. This attenuation effect is equal to $\frac{\tilde{W}_\tau}{\tilde{W}}\tau_0$ and depends positively on the uncertainty extent σ_ρ^2. Moreover, the size of the attenuation effect depends negatively on $\tilde{\theta}_G$. More the government prefers output, less is the decrease in τ in response to uncertainty. The fiscal attenuation leads to a decrease in both output and inflation, which drop lower than their desired levels. In response to a decrease in τ, the central bank starts to stimulate economy with a more expansionary policy. An increase in monetary instrument equal to $c\frac{\tilde{W}_\tau}{\tilde{W}}\tau_0$ would be enough to compensate the drop in inflation rate due to the attenuation effect of fiscal policy. Nevertheless, similarly to the famous paper Kydland and Prescott (1977), an inflation bias arises. The central bank takes inflation expectations as given and tries to push output up. With this goal, the central bank raises monetary instrument more than necessary to stabilize inflation.

As we can see from (2.10), the excess response of monetary policy to fiscal multiplicative uncertainty is equal to $ab\tilde{\theta}_B\frac{\tilde{W}_\tau}{\tilde{W}}\tau_0$. This excess increase in monetary instrument depends positively on the monetary preferences of output, $\tilde{\theta}_B$. Due to this excess increase in monetary instrument, expected inflation becomes higher than the optimal level. This, nevertheless, cannot overcome the output drop caused by the decrease in fiscal instrument, as only fiscal policy can affect the output in equilibrium.

Thus, the direct effect of fiscal multiplicative uncertainty is the inflation bias, which corresponds to the Di Bartolomeo, Giuli and Manzo (2009). Nevertheless, as the ratio $\frac{\tilde{W}_\tau}{\tilde{W}}$ depends negatively on the variance of monetary multiplicative shock, σ_φ^2, we can conclude that the presence of monetary uncertainty decreases the inflation pressure of fiscal attenuation. The intuition is straightforward: as the central bank is unsure about the monetary policy effectiveness, monetary policy also becomes more cautious. Thus, the central bank allows a lower excess increase in monetary instrument and the increase in inflation is lower.

The direct effect of monetary multiplicative uncertainty on monetary policy is equal to $-\frac{\tilde{W}_m}{\tilde{W}} m_0$ and corresponds qualitatively to the effect described in Di Bartolomeo and Giuli (2011). Uncertainty about the monetary policy effectiveness leads to the attenuation effect in monetary policy and the absolute value of monetary instrument drops. The government reacts to the attenuation effect in monetary policy by the opposite change in fiscal instrument. The change in τ equal to $\frac{\tilde{W}_m}{\tilde{W}} \frac{m_0}{c}$ would be enough to overcome the effect on inflation. Nevertheless, this would influence the output and the government varies fiscal instrument less. The change in τ is proportional to $\frac{\tilde{W}_m - \Lambda_B \tilde{\theta}_G a\,(a + bc)}{\tilde{W}}$. The stronger preferences for output $\tilde{\theta}_G$, the less change in fiscal instrument.

The influence of monetary multiplicative uncertainty on expected output and inflation depends on the sign of m_0. If $m_0 > 0$, monetary multiplicative uncertainty forces the central bank to decrease m and monetary policy becomes more contractionary. The government responds to this by an increase in fiscal instrument. This, in turn, leads to an increase in output. In order to prevent output from the excess increase, the government raises its instrument to a less extent than is necessary to overreact the influence on inflation. Moreover, the equilibrium fiscal instrument decreases with $\tilde{\theta}_G$. As a result, a negative inflation bias arises with expected inflation less than π^* and expected output greater than y^*.

On the contrary, if $m_0 < 0$, monetary multiplicative uncertainty makes monetary policy more expansionary. The government reacts by a decrease in τ. This decrease is less than necessary to overreact inflationary impact of monetary policy. As a result, expected inflation is higher than π^*, while output is lower than y^*. In other words, inflation bias arises.

As we already noted, the direct effects of fiscal and monetary uncertainties correspond qualitatively to the conclusions of Di Bartolomeo, Giuli and Manzo (2009) and Di Bartolomeo and Giuli (2011). Nevertheless, the simultaneous presence of both sources of uncertainty creates some additional effects. These effects are proportional to the product of Λ_G and Λ_B in equations (2.9) and (2.10). First of all, simultaneous uncertainty about both policies decreases the response of any policymaker to the uncertainty about the other's policy effectiveness. This follows directly from (2.9) and (2.10) if we remember that $\tilde{W}$ depends positively on the product $\Lambda_B \Lambda_G$. On the other hand, the mutual uncertainty influences the direct effects of both sources. For example, the presence of monetary uncertainty aggravates the attenuation effect which is caused by fiscal uncertainty. Fiscal instrument drops by additional amount of $\frac{c^2 \Lambda_G \Lambda_B}{\tilde{W}} \tau_0$. Moreover, this decrease is not compensated by an increase in a monetary instrument. Thus, the mutual effect strengthens the negative effect of fiscal uncertainty on the output and weakens the upward shift in inflation. The mutual effect also strengthens the attenuation in monetary policy by the amount of $\frac{c^2 \Lambda_G \Lambda_B}{\tilde{W}}$. This change in monetary instrument is not compensated by a corresponding response of fiscal authority. Thus, the mutual uncertainty weakens the effect of monetary uncertainty on inflation.

The overall effect of uncertainty on the equilibrium depends on the comparative strength of all these effects. The expected levels of output and inflation can be obtained from (2.1), (2.2) together with (2.9), (2.10) and are as follows:

$$\tilde{\pi}^e = \pi^* \left(1 - \frac{c^2 \Lambda_G \Lambda_B}{\tilde{W}}\right) + \frac{a \tilde{\theta}_B b \tilde{W}_\tau}{\tilde{W}} \tau_0 - \frac{\Lambda_B \tilde{\theta}_G a \left(a + bc\right)}{\tilde{W}} m_0 \tag{2.11}$$

$$\tilde{y}^e = y^* + \frac{a c^2 \Lambda_B}{\tilde{W}} \frac{m_0}{c} - \frac{a \tilde{W}_\tau \left(1 + \Lambda_B\right)}{\tilde{W}} \tau_0 \tag{2.12}$$

According to (2.11), the gap between expected inflation and its target depends on the direct effects of multiplicative uncertainty and the mutual effect described above. The direct effect of fiscal uncertainty is equal to $\frac{a \tilde{\theta}_B b \tilde{W}_\tau}{\tilde{W}} \tau_0$. This effect is explained by the overreaction of the central bank to the attenuation in fiscal policy. The underreaction of the government to the attenuation in monetary policy leads to the change in inflation equal to $-\frac{\Lambda_B \tilde{\theta}_G a(a + bc)}{\tilde{W}} m_0$. As we discussed earlier, this effect is positive if m_0 is negative and vice versa. The coexistence of both sources of uncertainty leads to the additional attenuation of the policies. This forces a further decrease in inflation, equal to $\frac{c^2 \Lambda_G \Lambda_B}{\tilde{W}} \pi^*$.

The attenuation effect of fiscal policy leads to a decrease in the output, equal to $\frac{a \tilde{W}_\tau}{\tilde{W}} \tau_0$. The presence of monetary multiplicative uncertainty strengthens this attenuation effect and causes a further decrease in output, equal to $\frac{a \tilde{W}_\tau \Lambda_B}{\tilde{W}} \tau_0$. The under-reaction of the government to the attenuation in monetary policy leads to the change in output equal to $\frac{a c^2 \Lambda_B}{\tilde{W}} \frac{m_0}{c}$. This amount is positive if m_0 is positive. If m_0 is negative, all the effects of uncertainty on output are negative.

The general properties of the equilibrium are summarized by Proposition 2.1:

Proposition 2.1. *For given ($\tilde{\theta}_B$, $\tilde{\theta}_G$, σ_ρ^2, σ_φ^2), there exist $\lambda_2 \geq \lambda_1$, such that in equilibrium with certain preferences:*

i) $\pi^e \geq \pi^*$ *if and only if* $\dfrac{m_0}{\tau_0} \leq \lambda_1$*;*

ii) $y^e \geq y^*$ *if and only if* $\dfrac{m_0}{\tau_0} \geq \lambda_2$*;*

where $\lambda_1 = \dfrac{c^2 \Lambda_G \left(ab\tilde{\theta}_B - c\Lambda_B\right)}{\Lambda_B \left(c^2 \Lambda_G + \tilde{\theta}_G a \left(a + bc\right)\right)}$, $\lambda_2 = \dfrac{c\Lambda_G \left(1 + \Lambda_B\right)}{\Lambda_B} \geq 0$.

Proof. See Eqs. (2.11) and (2.12). □

Proposition 2.1 indicates that there can be three different economic situations in equilibrium. If $\frac{m_0}{\tau_0} \leq \lambda_1$, there is an inflation bias problem: the expected rate of inflation exceeds its target level ($\pi^e \geq \pi^*$), while the expected rate of output is below its target level ($y^e \leq y^*$). If $\lambda_1 < \frac{m_0}{\tau_0} \leq \lambda_2$, there is the deflation bias problem: both the expected rate of inflation and output are below their target levels ($\pi^e \leq \pi^*, y^e \leq y^*$). If $\frac{m_0}{\tau_0} > \lambda_2$, there is a negative inflation bias problem: the expected rate of output exceeds its target level ($y^e \geq y^*$), while the expected level of inflation is below its target level ($\pi^e \leq \pi^*$).
We can also note that if we set $\sigma_\varphi^2 = 0$, we automatically replicate the results of Di Bartolomeo, Giuli and Manzo (2009). In this case both the thresholds λ_1 and λ_2 go to infinity and for any possible $\frac{m_0}{\tau_0}$ the economy faces the inflation bias problem. If σ_ρ^2 increases, the inflation bias problem aggravates.
If we let $\sigma_\rho^2 = 0$, we get the result of Di Bartolomeo and Giuli (2011). In this case, both the thresholds are equal to zero. This means that if $\frac{m_0}{\tau_0} < 0$, there is the inflation bias problem in the economy. If $\frac{m_0}{\tau_0} > 0$, there is negative inflation bias.
The simultaneous presence of monetary and fiscal multiplicative uncertainty makes the third type of equilibrium possible. This equilibrium is characterized by both inflation and output lower than their targets and is achieved at intermediate values of $\frac{m_0}{\tau_0} \in (\lambda_1, \lambda_2)$. It is easy to show that $\frac{\partial \lambda_1}{\partial \sigma_\rho^2} > 0$, $\frac{\partial \lambda_1}{\partial \sigma_\phi^2} < 0$, $\frac{\partial \lambda_2}{\partial \sigma_\rho^2} > 0$ and $\frac{\partial \lambda_2}{\partial \sigma_\phi^2} < 0$. Moreover, λ_1 is positive if and only if $\sigma_\phi^2 > \frac{ab\tilde{\theta}_B}{1 + b^2 \tilde{\theta}_B^2}$, while λ_2 is always positive. After characterizing the equilibrium with certain preferences, we now proceed to the search for the equilibrium with preference uncertainty.

2.4 Uncertain government preferences

In this Section, we relax the assumption of certain preferences and assume that parameter θ_G is a random variable with mean $\tilde{\theta}_G$ and cumulative distribution function $F(\theta_G)$ with support $[\underline{\theta}_G, \overline{\theta}_G]$.

Thus, we can rewrite the reaction function of the government with preferences θ_G (2.5) in the following way:

$$\tau(\theta_G) = \tau\left(\tilde{\theta}_G\right) - \Phi_G \omega(\theta_G), \tag{2.13}$$

where $\tau\left(\tilde{\theta}_G\right)$ is the value of fiscal instrument chosen by the government with preferences $\tilde{\theta}_G$, $\Phi_G = \dfrac{c^2\left(1+\sigma_\rho^2\right)(a+bc)\left(y^* - \overline{y} + b\pi^e - c\pi^*\right) + ac\left(a+bc\left(1-\sigma_\rho^2\right)\right)(m-\pi^*)}{c^2\left(1+\sigma_\rho^2\right) + \tilde{\theta}_G\left(\sigma_\rho^2 b^2 c^2 + (a+bc)^2\right)}$ and $\omega(\theta_G) = \dfrac{\tilde{\theta}_G - \theta_G}{c^2\left(1+\sigma_\rho^2\right) + \theta_G\left(\sigma_\rho^2 b^2 c^2 + (a+bc)^2\right)}$ characterizes the distance between the actual government preferences θ_G and the mean preferences $\tilde{\theta}_G$, with $\dfrac{\partial\omega}{\partial\theta_G} < 0$ and $\dfrac{\partial^2\omega}{(\partial\theta_G)^2} > 0$.

The central bank does not know the true distance between the government preferences and their mean, so the monetary policy is conducted according to equation (2.6), which is the reaction of the central bank to the expected value of fiscal instrument, $\overline{\tau}$. The expected value of fiscal instrument can be computed with the help of (2.13):

$$\overline{\tau} = \tau\left(\tilde{\theta}_G\right) - \Phi_G \Omega_G, \tag{2.14}$$

where $\Omega_G = \int\limits_{\underline{\theta_G}}^{\overline{\theta_G}} \omega(\theta_G)\, \mathrm{d}F(\theta_G)$ is the average value of $\omega(\theta_G)$. As function $\omega(\theta_G)$ is decreasing and convex, Ω_G is higher than the value $\omega\left(\tilde{\theta}_G\right)$, which is equal to zero. Obviously, the value of Ω_G depends on the extent of uncertainty about the government preferences. Due to convexity of function $\omega(\theta_G)$, the higher variance of θ_G the higher value of Ω_G.

To compute the equilibrium, we firstly find the intersection of reaction functions (2.6) and (2.14). After that, we compute expected inflation in the intersection point and substitute it into the reaction functions. The equilibrium values of policy instruments are as follows:

$$\hat{\overline{\tau}} = \tau_0 - \frac{\hat{W}_\tau(1+\Lambda_B)}{\hat{W}}\tau_0 + \frac{\hat{W}_m - \Lambda_B a(a+bc)\tilde{\theta}_G - ac^2\Omega_G\Lambda_B\left(bc\sigma_\rho^2 - (a+bc)\right)}{\hat{W}}\frac{m_0}{c} \tag{2.15}$$

$$\hat{\tau}(\theta_G) = \hat{\overline{\tau}} + \left(-\frac{\alpha_\tau}{\hat{W}}\tau_0 + \frac{\alpha_m}{\hat{W}}\frac{m_0}{c}\right)(\omega(\theta_G) - \Omega_G) \tag{2.16}$$

$$\hat{m} = m_0 - \frac{\hat{W}_m + c^2\Lambda_B\Lambda_G}{\hat{W}}m_0 + \left(c + ab\tilde{\theta}_B\right)\frac{\hat{W}_\tau}{\hat{W}}\tau_0, \tag{2.17}$$

where (2.15) is the average fiscal policy in equilibrium, (2.16) is the equilibrium policy of a government with preferences θ_G, (2.17) is the equilibrium monetary policy, $\hat{W}_\tau = \tilde{W}_\tau + \Omega_G\sigma_\rho^2 c^2 a(a+2bc)$, $\hat{W}_m = \tilde{W}_m + \Lambda_B bc^3(a+bc)\Omega_G(1+\sigma_\rho^2)$, $\hat{W} = \tilde{W} - \Omega_G c\left(a(a+bc)\left(b(a+bc)\tilde{\theta}_B + c\right) + \sigma_\rho^2 abc^2\left(b^2\tilde{\theta}_B - 1\right)\right) - bc^3(a+bc)\Lambda_B\left(1+\sigma_\rho^2\right)\Omega_G$, $\alpha_m = c^2\Lambda_B\left(a(a+bc) + \sigma_\rho^2 b^2c^2\right)$, $\alpha_\tau = \sigma_\rho^2 c^2\left[a(a+bc) + \tilde{\theta}_B ab^3 c + \Lambda_B\left(a(a+bc) - b^2c^2\right)\right]$.

If we compare (2.15) and (2.17) with the equilibrium policies with certain preferences (2.9) and (2.10), we will see that the main effects created by uncertainty are the same. These are the fiscal attenuation effect equal to $-\frac{\hat{W}_\tau (1+\Lambda_B)}{\hat{W}}\tau_0$ in (2.15) and the monetary attenuation effect equal to $-\frac{\hat{W}_m + c^2\Lambda_B\Lambda_G}{\hat{W}}m_0$ in (2.17). The reaction of the central bank to the fiscal attenuation effect is given by $-\left(c + ab\tilde{\theta}_B\right)\frac{\hat{W}_\tau}{\hat{W}}\tau_0$ in (2.17), while the average reaction of fiscal policy to the monetary attenuation effect is given by $\frac{\hat{W}_m - \Lambda_B a\,(a+bc)\,\tilde{\theta}_G - ac^2\Omega_G\Lambda_B\left(bc\sigma_\rho^2 - (a+bc)\right)}{\hat{W}}\frac{m_0}{c}$ in (2.15). These effects define the expected inflation and output in equilibrium:

$$
\begin{aligned}
\hat{\pi}^e =& \pi^* + \frac{\Lambda_B\left(\left(c^2 + a\,(a+bc)\,\tilde{\theta}_G + c^2\Lambda_G\right) + bc^3\,(a+bc)\,\Omega_G\left(1+\sigma_\rho^2\right)\right)}{\hat{W}}m_0 + \\
&+ \frac{\hat{W}_\tau\left(ab\tilde{\theta}_B - c\Lambda_B\right)}{\hat{W}}\tau_0
\end{aligned}
\tag{2.18}
$$

$$
\hat{y}^e = y^* + \frac{ac^2\Lambda_B\left(1+\Omega_G\left((a+bc)^{\,2} + b^2c^2\sigma_\rho^2\right)\right)}{\hat{W}}\frac{m_o}{c} - \frac{a\hat{W}_\tau\,(1+\Lambda_B)}{\hat{W}}\tau_0 \tag{2.19}
$$

As we can see, the equilibrium values of monetary and fiscal instruments are given by the cumbersome equations. Thus, we start the discussion of the equilibrium with the polar cases when either σ_ρ^2 or σ_φ^2 is equal to zero. After that, we describe the equilibrium in the generalized model with both σ_ρ^2 and σ_φ^2 positive.

2.4.1 Certain policy effects and uncertain fiscal preferences

We start to analyze the effects of preference uncertainty in the model with $\sigma_\rho^2 = \sigma_\varphi^2 = 0$:

Proposition 2.2. *In equilibrium with uncertain government preferences and without multiplicative uncertainty, $m = m_0$, $\tau(\theta_G) = \tau_0$ for any θ_G. Thus, for any Ω_G equilibrium output and inflation are equal to their target levels: $y = y^*$, $\pi = \pi^*$.*

Proof. Substitute $\sigma_\rho^2 = 0$ and $\sigma_\varphi^2 = 0$ into Eqs. (2.15-2.19). □

Proposition 2 indicates that in the absence of multiplicative uncertainty the government preference uncertainty does not affect the equilibrium. Irrespective of its preferences, the government with any θ_G chooses τ_0. Thus, the average fiscal policy is also equal to τ_0. The optimal reaction of the central bank to the average τ_0 is equal to m_0. As a result, in this case the uncertainty about the government preferences is not relevant and the symbiosis result of Dixit and Lambertini (2003*b*) holds: the government and the central bank are able to achieve both inflation and output targets.

2.4.2 Fiscal multiplicative uncertainty and fiscal preference uncertainty

We proceed with the model with fiscal multiplicative uncertainty. The equilibrium in this model is described in the following Proposition:

Proposition 2.3. *The equilibrium with fiscal multiplicative uncertainty and government preference uncertainty* ($\sigma_\rho^2 > 0, \Omega_G > 0, \sigma_\varphi^2 = 0$) *is such that:*

i) *For any* $\frac{m_0}{\tau_0}$*, there is the inflation bias problem: the expected rate of inflation exceeds its target level* ($\pi^e > \pi^*$)*, while the expected rate of output is below its target level* ($y^e < y^*$)*.*

ii) *Government preferences uncertainty aggravates the inflation bias problem. With higher* Ω_G*, the inflation gap and the output gap become larger:* $\frac{\partial|\pi^e - \pi^*|}{\partial \Omega_G} > 0$, $\frac{\partial|y^e - y^*|}{\partial \Omega_G} > 0$.

Proof. Substitute $\sigma_\varphi^2 = 0$ into Eqs. (2.15-2.19). □

Part i) of Proposition 3 states that the equilibrium with fiscal multiplicative and preferences uncertainty is characterized by inflation bias. The intuition is straightforward. The fiscal multiplicative uncertainty leads to the attenuation fiscal effect. The central bank does not know the true preferences of the government and has to rely on the average fiscal attenuation effect, which is given by the term $\frac{\hat{W}_\tau}{\hat{W}}\tau_0$ in (2.15). The attenuation fiscal effect leads to a decrease in both inflation and output. An increase in monetary instrument equal to $c\frac{\hat{W}_\tau}{\hat{W}}\tau_0$ would be enough to compensate the average decrease in inflation due to fiscal multiplicative uncertainty. Nevertheless, the central bank takes expectations as given and raises its instrument more in order to stimulate output. The value of the excess increase in monetary instrument is proportional to $ab\tilde{\theta}_B\frac{\hat{W}_\tau}{\hat{W}}$. This excess increase in monetary instrument pushes inflation above the target level, while expected output stays below the target.

Part ii) of Proposition 3 states that an increase in the dispersion of fiscal preferences leads to the higher inflation bias. To understand this, note that the gap between expected output and the target is proportional to the average attenuation fiscal effect. From equation (2.14), the value of the average fiscal instrument $\overline{\tau}$ is lower than $\tau(\tilde{\theta}_G)$. Thus, the average attenuation effect is higher than the attenuation of the policy by the government with preferences $\tilde{\theta}_G$. With higher preference uncertainty, measured by Ω_G, the difference between the average attenuation and the attenuation of the government with average preferences becomes larger. Consequently, the absolute value of the expected output gap also increases. Thus, the willingness of the central bank to stimulate output with the excessive increase in monetary instrument enlarges. As a result, the gap between expected inflation and the target inflation becomes larger.

The effects of fiscal multiplicative uncertainty in the model with uncertain government preferences coincide with the effects in the model with certain preferences qualitatively and are larger quantitatively. In the next subsection we analyze the effects of preference uncertainty in the model with monetary multiplicative shocks.

2.4.3 Monetary multiplicative uncertainty and fiscal preference uncertainty

Now we proceed to the model with monetary multiplicative uncertainty. The equilibrium in this model is described in the following Proposition 2.4:

Proposition 2.4. *The equilibrium with monetary multiplicative uncertainty and government preference uncertainty ($\sigma_\rho^2 = 0, \Omega_G > 0, \sigma_\varphi^2 > 0$) is such that:*

i) *If $m_0 > 0$, there is negative inflation bias problem in the economy: the expected rate of output exceeds its target level ($y^e \geq y^*$), while the expected level of inflation is below its target level ($\pi^e \leq \pi^*$). If $m_0 < 0$, there is the inflation bias problem in the economy: the expected rate of inflation exceeds its target level ($\pi^e \geq \pi^*$), while the expected rate of output is below its target level ($y^e \leq y^*$).*

ii) $\dfrac{\partial|\pi^e - \pi^*|}{\partial\Omega_G} \geq (\leq) 0$ *if and only if* $\sigma_\varphi^2 \leq (\geq) \dfrac{ab\tilde{\theta}_B}{c\left(1 + b^2\tilde{\theta}_B\right)}$. *If* $\sigma_\varphi^2 > \dfrac{ab\tilde{\theta}_B}{c\left(1 + b^2\tilde{\theta}_B\right)}$, *an increase in Ω_G lowers the inflation gap. If* $\sigma_\varphi^2 < \dfrac{ab\tilde{\theta}_B}{c\left(1 + b^2\tilde{\theta}_B\right)}$, *an increase in Ω_G enlarges the inflation gap.*

iii) *For any m_0, uncertain government preferences aggravate the gap between expected output and its target level:* $\dfrac{\partial|y^e - y^*|}{\partial\Omega_G} > 0$.

Proof. Substitute $\sigma_\rho^2 = 0$ into Eqs. (2.15-2.19). □

Part i) of Proposition 2.4 states that there is either inflation bias or negative inflation bias in the equilibrium. The logic is similar to the model with certain preferences. Monetary multiplicative uncertainty causes the attenuation monetary effect, equal to $\dfrac{\hat{W}_m}{\hat{W}}$. Similar to the case of certain preferences, to change the average fiscal instrument by $\dfrac{\hat{W}_m}{\hat{W}}\dfrac{m_0}{c}$ would be enough to compensate the influence of monetary attenuation effect on inflation. Nevertheless, the government with any preferences has a competing target of output. As the government does not want to change considerably the output level, there is the under-reaction to the monetary attenuation effect. The average size of this under-reaction is given by the term $\dfrac{-\Lambda_B a\left(a+bc\right)\tilde{\theta}_G - ac^2\Omega_G\Lambda_B\left(-\left(a+bc\right)\right)}{\hat{W}}\dfrac{m_0}{c}$ in equation (2.15). This under-reaction gives rise to the gap between expected inflation and its target, while the equilibrium average change in fiscal instrument gives rise to the gap between expected output and the target output. The signs of the inflation and output gaps depend on the sign of m_0. If m_0 is positive, negative inflation bias with low inflation and high output arises. It means that uncertain government preferences to some extent eliminate the inflation bias problem, which is caused by uncertainty about monetary multiplicative uncertainty. If m_0 is negative, uncertainty leads to a standard inflation bias.

Parts ii) and iii) of Proposition 2.4 characterize the effects of preference uncertainty on the absolute values of the output and inflation gaps. To better understand these findings, let us firstly note that the size of monetary attenuation effect, $\dfrac{\hat{W}_m}{\hat{W}}$, depends positively on Ω_G. This means that an increase in preference uncertainty aggravates the attenuation effect of monetary policy. The explanation is as follows. As we have seen in Section 2.3, if $m_0 > 0$ and preferences are certain, the equilibrium fiscal instrument is decreasing and convex function of government type. This means that under uncertain preferences the average fiscal policy is looser than the policy of the government with the average preferences. Thus, the central bank decreases m in accordance with its reaction function. This signifies an aggravation of the attenuation effect in comparison with the certain preferences model. If $m_0 < 0$, the fiscal instrument under certain preferences is an increasing concave function of the government preferences. Thus, the average fiscal policy is tighter than the policy chosen by the government with the average preferences. The central bank reacts to this by an increase in m. As the attenuation effect in this case implies the rise of m, we can conclude that uncertainty about preferences again aggravates the attenuation effect.

The gap between expected output and the target output is defined by the government reaction to this attenuation effect. The change in the fiscal instrument is proportional to the size of the attenuation effect. From here we can conclude, that the absolute value of the output gap is also proportional to the attenuation effect. Thus, an increase in preference uncertainty always aggravates the output gap which is caused by monetary multiplicative uncertainty.

The gap between expected inflation and its target is defined by the average fiscal under-reaction to the monetary attenuation effect. The under-reaction of the government with preferences θ_G is proportional to $\Lambda_B\left(\tilde{\theta}_G - c^2\omega\left(\theta_G\right)\right)a\left(a+bc\right)$. As there is no fiscal multiplicative uncertainty, the following equation holds:

$$\tilde{\theta}_G - c^2\omega\left(\theta_G\right) = \theta_G\frac{c^2+\left(a+bc\right)^2\tilde{\theta}_G}{c^2+\left(a+bc\right)^2\theta_G} \tag{2.20}$$

From (2.20) we can see that the coefficient $\tilde{\theta}_G - c^2\omega\left(\theta_G\right)$ is non-negative and depends positively on θ_G. This means that stronger the government preferences for output the less reaction to the monetary attenuation effect. Moreover, function $\tilde{\theta}_G - c^2\omega(\theta_G)$ is concave in θ_G. The average under-reaction of the government to the monetary attenuation effect, $\left.\frac{\Lambda_B\left(\tilde{\theta}_G - c^2\Omega_G\right)a(a+bc)}{\hat{W}}\right|_{\sigma_\rho^2=0}$, defines the gap between expected inflation and inflation target. The size of this gap depends on the variance of the government preferences, Ω_G. The sign of this relation is defined by the extent of monetary uncertainty. If the monetary multiplicative uncertainty is strong and $\sigma_\varphi^2 > \frac{ab\tilde{\theta}_B}{c\left(1+b^2\tilde{\theta}_B\right)}$, a decrease in Ω_G leads to an increase in the under-reaction. This means that more uncertain preferences lower the gap between expected inflation and the inflation target. On the contrary, if monetary uncertainty is weak and $\sigma_\varphi^2 < \frac{ab\tilde{\theta}_B}{c\left(1+b^2\tilde{\theta}_B\right)}$, an increase in uncertainty about the government preferences leads to an increase in the gap between the expected and target inflation rates.

2.4.4 Uncertain policy effects and uncertain fiscal preferences

After discussion of the polar cases in the previous subsections, we now proceed to the general framework. The characteristics of the equilibrium with uncertain preferences and uncertain policy effects are summarized in the following Proposition:

Proposition 2.5. *For given (σ_ρ^2, σ_φ^2, Ω_G), there exist $\lambda_2^* \geq \lambda_3^* \geq \lambda_1^*$, such that:*

i) $\pi^e \geq \pi^*$ *if and only if* $\frac{m_0}{\tau_0} \leq \lambda_1^*$;

ii) $y^e \geq y^*$ *if and only if* $\frac{m_0}{\tau_0} \geq \lambda_2^*$;

iii) $\frac{\partial(y^e - y^*)}{\partial\Omega_G} \geq 0$ *if and only if* $\frac{m_0}{\tau_0} \geq \lambda_3^*$, *and*

$$\frac{\partial(\pi^e - \pi^*)}{\partial\Omega_G} \geq 0 \text{ if and only if } \left(\frac{m_0}{\tau_0} - \lambda_3^*\right)\left(\sigma_\varphi^2 - \frac{ab\tilde{\theta}_B}{c\left(1+b^2\tilde{\theta}_B\right)}\right) > 0;$$

where $\lambda_1^* = \frac{c^2\left(\Lambda_G + a\sigma_\rho^2\Omega_G(a+2bc)\right)\left(ab\tilde{\theta}_B - c\Lambda_B\right)}{\Lambda_B\left(c^2\Lambda_G + \tilde{\theta}_G a(a+bc) + ac^2\Omega_G\left(bc\left(\sigma_\rho^2 - 1\right) - a\right)\right)}$,

$$\lambda_2^* = \frac{c(1+\Lambda_B)}{\Lambda_B}\frac{\left(\Lambda_G + a\sigma_\rho^2\Omega_G(a+2bc)\right)}{\left(1+\Omega_G\left((a+bc)^2 + b^2c^2\sigma_\rho^2\right)\right)} \geq 0,$$

$$\lambda_3^* = \frac{c\sigma_\rho^2\left(a^2 + abc\left(1+b^2\tilde{\theta}_B\right) + \Lambda_B\left(a(a+bc) - b^2c^2\right)\right)}{\Lambda_B\left(a(a+bc) + \sigma_\rho^2 b^2c^2\right)}.$$

Proof. See Eqs. (2.15-2.19). □

Parts i) and ii) of Proposition 2.5 state that if both policy effects are uncertain, there are three possible economic situations: inflation bias, deflation bias or negative inflation bias. If $\frac{m_0}{\tau_0} \leq \lambda_1^*$, there is the inflation bias problem in the economy: the expected rate of inflation exceeds its target level ($\pi^e \geq \pi^*$), while the expected rate of output is below its target level ($y^e \leq y^*$). If $\lambda_1^* < \frac{m_0}{\tau_0} \leq \lambda_2^*$, there is the deflation bias problem in the economy: the expected rate of inflation and output are below their target levels ($\pi^e \leq \pi^*, y^e \leq y^*$). If $\frac{m_0}{\tau_0} > \lambda_2^*$, the expected rate of output exceeds its target level ($y^e \geq y^*$), while the expected level of inflation is below its target level ($\pi^e \leq \pi^*$), which means that there is the negative inflation bias problem in the economy. Similar to the model with certain preferences, the deflation bias is possible only if both multiplicative shocks are present and $\frac{m_0}{\tau_0} \in (\lambda_1^*, \lambda_2^*)$.

Uncertainty about the government preferences influences the thresholds λ_1^* and λ_2^*. It is easy to show that an increase in uncertainty about the government preferences lowers λ_2^*. The effect of preference uncertainty on the value of λ_1^* depends on the sign of its value. If λ_1^* is positive, an increase in Ω_G leads to a further increase in λ_1^*. If λ_1^* is negative, an increase in Ω_G leads to a further decrease in λ_1^*.

Part iii) of Proposition 2.5 defines the effect of preference uncertainty on the equilibrium output and inflation. The effect of preference uncertainty on expected output is positive if $\frac{m_0}{\tau_0} > \lambda_3^*$ and negative if $\frac{m_0}{\tau_0} < \lambda_3^*$. This means that if $\frac{m_0}{\tau_0} < \lambda_1^*$ and the equilibrium is characterized by inflation bias with negative output gap, an increase in preference uncertainty leads to a further increase in the absolute value of this gap. If $\frac{m_0}{\tau_0} > \lambda_2^*$ and the equilibrium is characterized by the negative inflation bias with positive output gap, an increase in preference uncertainty also leads to a further increase in the absolute value of this gap. If $\frac{m_0}{\tau_0} \in (\lambda_1^*, \lambda_2^*)$, there might be non-monotonous effect of preference uncertainty on the output gap. Thus, there may be a positive effect of preference uncertainty.

The effect of preference uncertainty on expected inflation depends not only on the value of $\frac{m_0}{\tau_0}$, but also on the extent of monetary multiplicative uncertainty. For example, if $\frac{m_0}{\tau_0} > \lambda_3^*$, the equilibrium is characterized by negative gap between expected inflation and its target. The effect of Ω_G depends on the value of σ_ϕ^2. If $\sigma_\varphi^2 > \frac{ab\tilde{\theta}_B}{c(1+b^2\tilde{\theta}_B)}$, an increase in Ω_G leads to an increase in expected inflation and consequently, to a decrease in the absolute value of the inflation gap. Similarly, if $\sigma_\varphi^2 < \frac{ab\tilde{\theta}_B}{c(1+b^2\tilde{\theta}_B)}$, an increase in Ω_G leads to a decrease in expected inflation and consequently, to an increase in the absolute value of the inflation gap.

2.5 Welfare analysis

In previous Section we have analyzed the effects of uncertainty on inflation and output gaps. Now we are going to discuss the optimal design of policy decision-making under uncertainty. For this purpose, we have to define the welfare criterion. Following the consensus in the literature, we assume that this criterion is represented by the following social loss function:

$$L_S = E\left[(\pi-\pi^*)^2 + \theta_W (y-y^*)^2\right], \tag{2.21}$$

where θ_W characterizes the social preferences for output in comparison to inflation. Using equations (2.1) and (2.2) together with their expectations, we rewrite the social loss function in the following way:

$$\begin{aligned} L_S &= (\pi^e-\pi^*)^2 + \theta_W (y^e-y^*)^2 + \left(1+b^2\theta_W\right)\left(\sigma_\varphi^2 m^2 + c^2\sigma_\rho^2\overline{\tau}^2\right) + \\ &+ \left[c^2\left(1+\sigma_\rho^2\right) + \theta_W\left(b^2c^2\sigma_\rho^2 + (a+bc)^2\right)\right] E\left(\tau-\overline{\tau}\right)^2 \end{aligned} \tag{2.22}$$

As we can see, the first term in social loss represents the squared expected gap between the equilibrium inflation and its target level. The second term is the squared gap between the equilibrium output and its target level. The previous sections show that these gaps originate from sub-optimal reaction of policymakers to multiplicative uncertainty. We have also discussed the effect of preference uncertainty on these gaps. The third term in (2.22), equal to $\left(1+b^2\theta_W\right)\left(\sigma_\varphi^2 m^2 + c^2\sigma_\rho^2\overline{\tau}^2\right)$, represents the weighted volatility of inflation and output, created by multiplicative shocks. The last term represents the expected loss from uncertainty about fiscal preferences and is proportional to the variance of fiscal instrument $E\left(\tau-\overline{\tau}\right)^2$.

According to (2.16), the gap between the action of the government with preferences θ_G and the average government action is proportional to $\left(\omega\left(\theta_G\right)-\Omega_G\right)$. Thus, the variance of government actions is proportional to the variance of variable $\omega\left(\theta_G\right)$. As this function is non-linear, we cannot derive its variance explicitly without specifying the distribution of preferences. Because of that, we restrict our attention to economies with sufficiently weak uncertainty about government preferences, meaning that θ_G is fairly close to its mean $\tilde{\theta}_G$. This assumption allows us to linearize $\omega\left(\theta_G\right)$ around $\tilde{\theta}_G$ and to use a simple expression for its variance without specifying the exact distribution functions:

Assumption 2.6. *Let θ_G be fairly close to the mean $\tilde{\theta}_G$, so we can use the following approximations:*

i) $\omega\left(\theta_G\right) \approx \omega\left(\tilde{\theta}_G\right) + \omega'\left(\tilde{\theta}_G\right)\left(\theta_G-\tilde{\theta}_G\right) + \frac{1}{2}\omega''\left(\tilde{\theta}_G\right)\left(\theta_G-\tilde{\theta}_G\right)^2$

ii) $\Omega_G = E\left(\omega\left(\theta_G\right)\right) \approx \omega\left(\tilde{\theta}_G\right) + \frac{1}{2}\omega''\left(\tilde{\theta}_G\right)\sigma_\theta^2$

iii) $E\left(\omega\left(\theta_G\right) - \Omega_G\right)^2 \approx \left(\omega'\left(\tilde{\theta}_G\right)\right)^2 \sigma_\theta^2,$

where σ_θ^2 is the variance of government preferences.

Assumption 2.6 allows us to get the social loss function explicitly. Using this assumption, we substitute equilibrium policies (2.15-2.17) and equilibrium gaps from (2.18-2.19) into equation (2.22). This gives us the expression of social loss which depends on preference parameters θ_W, θ_B, $\tilde{\theta}_G$, the variances of multiplicative shocks σ_φ^2, σ_ρ^2 and the government preference uncertainty, measured by σ_θ^2. Minimization of this loss with respect to θ_B, $\tilde{\theta}_G$ would give the optimal policymakers preferences or an optimal policy design, defined as follows:

Definition 2.7. The optimal policy design is a vector of policymakers preferences $\Theta^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right) \equiv \left(\theta_B^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right), \tilde{\theta}_G^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)\right)$ such that:

$$\Theta^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right) = \arg\min_{\Theta} \tilde{L}_S\left(\theta_B, \tilde{\theta}_G, \theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right),$$

where $\Theta = \left(\theta_B, \tilde{\theta}_G\right) > 0$ and $\tilde{L}_S\left(\theta_B, \tilde{\theta}_G, \theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)$ is the expected social loss in equilibrium.

Social planner which cannot influence the extent of multiplicative uncertainty, uncertainty about the government preferences or the form of policy interaction, assigns the central bank with preferences $\theta_B^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)$ and chooses the average type of government $\tilde{\theta}_G^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)$. Unfortunately, it is impossible to find the closed-form solution of the optimal program in the general model. Thus, we use the following procedure. Firstly, we find the optimal policy preferences in the model with the only multiplicative shock (either fiscal or monetary). After that we investigate the effects of sufficiently small increase in uncertainty about the other multiplicative shock and about the government preferences on the optimal values of θ_B and $\tilde{\theta}_G$. The situation without multiplicative uncertainty is trivial. As we have seen in the previous section, in this situation the governments with any preferences choose the same value of fiscal instrument. As a result, there is no fiscal policy uncertainty and no gaps between the equilibrium values of inflation and output and their targets. Thus, for any policy preferences social loss is equal to zero. Multiplicative uncertainty of any type creates the gaps between the equilibrium levels of output and inflation and their targets, volatility of output and inflation and uncertainty about fiscal policy. This justifies the need to assign the proper policymakers which could minimize the losses created by uncertainty. Following the logic of previous sections, we start with fiscal multiplicative uncertainty (Proposition 2.8) and proceed with monetary multiplicative uncertainty (Proposition 2.9).

Proposition 2.8. *Let $\sigma_\rho^2 > 0$. Then the optimal preference parameters θ_B^* and $\tilde{\theta}_G^*$ are such that:*

i) $\theta_B^*|_{\sigma_\theta^2=0,\sigma_\varphi^2=0} = 0$ *and* $\tilde{\theta}_G^*\Big|_{\sigma_\theta^2=0,\sigma_\varphi^2=0} = \dfrac{a\theta_W}{a+bc\left(1+b^2\theta_W\right)}$;

ii) $\left.\dfrac{\partial\theta_B^*}{\partial\sigma_\theta^2}\right|_{\sigma_\theta^2=0,\sigma_\varphi^2=0} > 0$ *and* $\left.\dfrac{\partial\tilde{\theta}_G^*}{\partial\sigma_\theta^2}\right|_{\sigma_\theta^2=0,\sigma_\varphi^2=0} > 0$;

iii) $\left.\dfrac{\partial\theta_B^*}{\partial\sigma_\varphi^2}\right|_{\sigma_\theta^2=0,\sigma_\varphi^2=0} > 0$ *and* $\left.\dfrac{\partial\tilde{\theta}_G^*}{\partial\sigma_\varphi^2}\right|_{\sigma_\theta^2=0,\sigma_\varphi^2=0} > 0$, *if and only if* $\dfrac{m_0}{c\tau_0} < \dfrac{\sigma_\rho^2 c^2\left(1+b^2\theta_W\right)}{a^2\theta_W+\sigma_\rho^2 c^2\left(1+b^2\theta_W\right)}$.

Proof. See Appendix B. □

Part i) of Proposition 2.8 defines the optimal policy design without monetary multiplicative uncertainty and without preference uncertainty. As this situation is equivalent to Di Bartolomeo, Giuli and Manzo (2009), the optimal policy preferences are the same as in their model. The optimal choice of policymakers implies that both of them should be more conservative than the society. This is explained by the time-inconsistency problem. Both reaction functions (2.5) and (2.6) show that the policymaker have the incentive to push output up by inflation surprise. To avoid this, they should be sufficiently conservative. Moreover, the central bank should be more conservative than the government ($\tilde{\theta}_G^* > \theta_B^*$) and should not worry about output ($\theta_B^* = 0$). There are two reasons for this. The first reason is that the central bank cannot influence output in equilibrium. The second reason is the overreaction of the central bank to the attenuation in fiscal policy. As we have discussed earlier, fiscal multiplicative uncertainty leads to a fiscal attenuation effect which is expressed by a drop in fiscal instrument. The central bank faces the time-inconsistency problem and overreacts to this drop by too loose monetary policy. The overreaction of the central bank is proportional to its preference for output θ_B. Thus, assigning an absolutely conservative central bank without preference for output ($\theta_B^* = 0$) allows to avoid this overreaction. As a result, the expected inflation is kept at its target level.

Part ii) of Proposition 2.8 states that an increase in preference uncertainty makes the optimal conservativeness of both the central bank and the government lower. Earlier we have seen that preference uncertainty not only creates the uncertainty about fiscal policy, but also deteriorates the gaps caused by the fiscal multiplicative shock. This effect was summarized by variable Ω_G in Section 2.4. From Part ii) of Assumption 2.6, it immediately follows that Ω_G depends positively on preference uncertainty σ_θ^2 and negatively on the average government preferences $\tilde{\theta}_G$. Thus, in order to smooth the negative effect of σ_θ^2 on the output and inflation gaps, an increase in $\tilde{\theta}_G$ is needed. Moreover, from Part iii) of Assumption 2.6 along with the properties of function $\omega\left(\theta_G\right)$, we can conclude that the variances of $\omega\left(\theta_G\right)$ and $\tau\left(\theta_G\right)$ depend positively on σ_θ^2 and negatively on $\tilde{\theta}_G$. This again makes it socially desirable to assign the less conservative government. Nevertheless, higher average government preferences and more active fiscal policy lead to higher volatility of both inflation and output because of fiscal multiplicative shocks. This, however, can be compensated by a less conservative central bank. As a result, both $\tilde{\theta}_G^*$ and θ_B^* increase with an increase in σ_θ^2.

Part iii) of Proposition 2.8 explores the effect of a small increase in monetary multiplicative uncertainty on the optimal preferences. As we can see, this effect depends on the relation between policy action under certainty or, in other words, on the relation between inflation and output targets. If the inflation target is small relative to the output target, such that m_0 is small relative to τ_0, an increase in σ_φ^2 leads to a decrease in the optimal conservativeness for both policymakers. To explain this, we need to study the effect of monetary multiplicative uncertainty on the output and inflation gaps and the equilibrium policy actions. It is easy to show from (2.9- 2.12) that if $\frac{m_0}{c\tau_0} < \frac{\sigma_\rho^2 c^2 \left(1+b^2\theta_W\right)}{a^2\theta_W + \sigma_\rho^2 c^2 \left(1+b^2\theta_W\right)}$, a small increase in σ_φ^2 leads to an increase in the equilibrium monetary policy action and consequently, to an increase in the inflation gap. The fiscal policy becomes less active, output drops, the absolute value of the output gap increases. An increase in θ_G and θ_B would help to restore the output close to the target level without a large increase in inflation, as far as m_0 is sufficiently small. The opposite happens if m_0 is large and $\frac{m_0}{c\tau_0} > \frac{\sigma_\rho^2 c^2 \left(1+b^2\theta_W\right)}{a^2\theta_W + \sigma_\rho^2 c^2 \left(1+b^2\theta_W\right)}$. In this case an increase in σ_φ^2 leads to a decrease in the equilibrium monetary action and to an increase in the equilibrium fiscal policy action. As a result, the expected inflation decreases, while the expected output increases. As the initial equilibrium was characterized by inflation bias, an increase in σ_φ^2 lowers the absolute values of both gaps. Thus, more conservative government and the central bank can be assigned in order to lower τ and m and to decrease the volatility created by the corresponding multiplicative shocks.

The properties of the optimal policy design in economy with monetary multiplicative uncertainty are summarized by the following proposition:

Proposition 2.9. *Let* $\sigma_\varphi^2 > 0$

i) $\theta_B^*\big|_{\sigma_\theta^2=0,\sigma_\rho^2=0} = \dfrac{ab\theta_W^2}{c+b\theta_W\left(a+bc\right)}$ *and* $\tilde{\theta}_G^*\Big|_{\sigma_\theta^2=0,\sigma_\rho^2=0} = \dfrac{a\theta_W}{a+bc}$*;*

ii) $\dfrac{\partial\theta_B^*}{\partial\sigma_\theta^2}\bigg|_{\sigma_\theta^2=0,\sigma_\rho^2=0} > 0$ *and* $\dfrac{\partial\tilde{\theta}_G^*}{\partial\sigma_\theta^2}\bigg|_{\sigma_\theta^2=0,\sigma_\rho^2=0} > 0$ *;*

iii) $\dfrac{\partial\theta_B^*}{\partial\sigma_\rho^2}\bigg|_{\sigma_\theta^2=0,\sigma_\rho^2=0} > 0$ *and* $\dfrac{\partial\tilde{\theta}_G^*}{\partial\sigma_\rho^2}\bigg|_{\sigma_\theta^2=0,\sigma_\rho^2=0} > 0$*, if and only if* $m_0 > 0$*.*

Part i) of Proposition 2.9 describes the optimal policymakers preferences for the situation when only monetary multiplicative uncertainty is present. Similar to the situation with fiscal multiplicative uncertainty, the central bank should be more conservative than the government, and both should be more conservative than society. The reason is again the time inconsistency problem and the impossibility for the central bank to influence output in equilibrium. Contrary to the previous situation with fiscal multiplicative uncertainty, the central bank should not be absolutely conservative and have to worry about output ($\theta_B^* > 0$). The logic here is as follows. According to reaction function (2.6), the monetary multiplicative uncertainty forces the central bank to decrease its actions proportionally (monetary attenuation effect). This means that its incentives to stimulate output also weaken and time inconsistency problem becomes less pronounced. As a result, there is no need to assign the fully conservative central bank. Moreover, as $\dfrac{a\theta_W}{a+bc} > \dfrac{a\theta_W}{a+bc\left(1+b^2\theta_W\right)}$, the government under monetary multiplicative uncertainty should be also less conservative than under fiscal multiplicative uncertainty. To better understand this finding, let us remind that the reaction of the government to the attenuation effect in monetary policy depends negatively on its preferences for output θ_G. As this reaction creates the output gap, the society would be better off if the government with higher θ_G is assigned.

Part ii) of Proposition 2.9 states that the effects of preference uncertainty under monetary multiplicative uncertainty are the same as under fiscal multiplicative uncertainty. An increase in preference uncertainty lowers the optimal conservativeness of both the central bank and the government, making $\tilde{\theta}_G^*$ and θ_B^* higher. The intuition is similar. An increase in σ_θ^2 leads to an increase in Ω_G, in output gap and in the volatility of fiscal policy actions. An increase in the average government preference for output is needed to compensate for these discrepancies. An increase in θ_B^* is needed to lower the volatility of output and inflation, created by monetary multiplicative uncertainty.

Part iii) of Proposition 2.9 explores the effect of a small increase in fiscal multiplicative uncertainty on the optimal preferences. As we can see, this effect depends on sign of m_0, which, in turn, depends on the relation between the inflation and output targets. If the inflation target is sufficiently high and monetary policy under certainty is relatively loose ($m_0 > 0$), an increase in σ_ρ^2 leads to an decrease in the optimal conservativeness of both policymakers. The intuition is straightforward. If m_0 is positive, the effect of monetary multiplicative uncertainty is a decrease in m and an increase in τ, resulting in too high output and too low inflation. If fiscal multiplicative uncertainty arises in such a situation, fiscal policy becomes less expansionary. As a result, the expected inflation drops further. To avoid this drop in inflation, less conservative government and central bank should be assigned. If inflation target is sufficiently low and monetary policy under certainty is relatively tight ($m_0 < 0$), an increase in σ_ρ^2 leads to an increase in the optimal conservativeness of both policymakers. If m_0 is negative, the equilibrium with monetary multiplicative uncertainty is characterized by looser monetary policy and tighter fiscal policy, which lead to inflation bias. If we add fiscal multiplicative uncertainty, the government becomes less active, which helps to keep output closer to its target but pushes inflation up. To avoid this increase in inflation, more conservative authorities are needed and both $\tilde{\theta}_G^*$ and θ_B^* decrease.

2.6 Conclusion

This paper contributes to the existing literature on monetary and fiscal policy under uncertainty. In particular, we study the role of uncertain government preferences for policy interaction.

We show, that if the fiscal and monetary policy effects are certain, uncertainty about government preferences does not affect the equilibrium. In case of fiscal multiplicative uncertainty, uncertainty about the government preferences lowers output, increases inflation and thereby aggravates the inflation bias problem. Monetary multiplicative uncertainty can create either the inflation bias problem or negative inflation bias problem. Uncertain government preferences aggravate the problem by enlarging the absolute value of the output gap, while the effect on the inflation gap depends on the extent of uncertainty about the monetary policy effectiveness and may be beneficial. If both the policy effects are uncertain, the impact of uncertain government preference depends not only on the extent of multiplicative uncertainty, but also on the inflation and output targets. As a result, preference uncertainty may lower the absolute values of output and inflation gaps, created by multiplicative uncertainty.

Our welfare analysis is restricted to the small extents of preference uncertainty which allows us to derive the welfare function explicitly without specifying the exact distribution function. Nevertheless, higher extents of uncertainty can be also studied, probably with the use of numerical methods. Another restriction of our study is that we deal only with uncertainty about the policy effects on inflation. The direct effects of fiscal policy on output are treated as known. Nevertheless, it seems that in reality the knowledge about these policy effects is also far from completeness. Thus, incorporating uncertainty about the effects on output is a promising avenue for future research. Moreover, the problem of different forms of strategic interaction is beyond the scope of our paper: we consider that the government and the central bank conduct their policies simultaneously and independently. The analysis of the influence of uncertain government preferences on macroeconomic policy under various forms of strategic interaction is left for future studies.

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Appendix

B.1 Proof of Propositions 2.8 and 2.9

Proposition 2.8 i) and 2.9i)

The vector of optimal weights $\Theta^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)$ solves the following system of first order conditions:

$$D_\Theta \tilde{L}_S\left(\Theta, \theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right) = \underline{0}, \tag{2.23}$$

where D is the derivative operator. Substituting zeros in stead of corresponding σ_j^2, $j \in \{\varphi, \rho, \theta\}$, we get the system which can be solved for $\Theta^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)$. Normally, there are several pairs of roots but only the roots listed in i) Parts of Propositions 2.8 and 2.9 assure that the Hessian matrix of $\tilde{L}_S$ is positive semi-definite and that the found solution $\Theta^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)$ minimizes the social loss.

Proposition 2.8 ii-iii) and 2.9ii-iii)

To find the signs of corresponding derivatives, we use

$$\frac{\partial \theta_k^*}{\partial \sigma_j^2} = -\frac{|H_{kj}|}{|H|}, \tag{2.24}$$

where $k \in \{B, G\}$, $|H|$ is the determinant of the Hessian matrix and $|H_{kj}|$is the determinant of the Hessian matrix where the k-th column was replaced by the $D^2_{\Theta,\sigma_j^2}\tilde{L}_S\left(\Theta, \theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)$, computed for $\Theta^*\left(\theta_W, \sigma_\varphi^2, \sigma_\rho^2, \sigma_\theta^2\right)$. As $|H|$ is non-negative, the sign of $\dfrac{\partial \theta_k^*}{\partial \sigma_j^2}$ corresponds to the sign of $(-1)\,|H_{kj}|$. Calculations are available upon request.

Chapter 3

Value of Information in Segmented Economies

Abstract

The social value of information has been broadly discussed in economic literature. Nevertheless, almost all existing studies deal with closed economies, leaving the issues of information provision in open economies aside. Our study fills this gap and elaborate a general two-region model, which captures three important characteristics of international markets: globalization of markets, segmentation of fundamentals and informational asymmetry between regions. For this model, we derive the global and the regional welfare criteria and study social, regional and inter-regional value of information. We show that welfare properties of information in segmented economy differ significantly from its welfare properties in one-region model. For example, we show that the famous result by Angeletos and Pavan (2007) which states that the negative gap between efficient and equilibrium degree of coordination is sufficient for welfare to increase in precision of private information economies with strategic substitutability, does not hold in segmented economy. Another finding of Angeletos and Pavan (2007) states that in inefficient economies a high gap between efficient and equilibrium distributions suffices for the positive value of information, while a low gap suffices for the negative value of information about fundamental shocks. We show that this result is violated in two-region economies, if the cross-sectional dispersion in actions creates sufficiently strong externality. Moreover, we detect the conditions, under which the regional value of information differs for its social value. These findings indicate the situations in which information policy could be inefficient if conducted by the regional authorities. After discussing the general model, we illustrate our findings with a number of examples.

JEL: D82, E61

Key words: *strategic complementarity, strategic substitutability, public information, private information, value of information, segmented economy*

3.1 Introduction

The social value of information has been broadly discussed in the literature. Starting from the seminal paper by Morris and Shin (2002), most researchers which deal with this issue consider economic environments with common fundamental shocks (e.g. Angeletos and Pavan (2007), Cornand and Heinemann (2008), Ui and Yoshizawa (2015), Roca (2010), James and Lawler (2012), Walsh (2013), etc.). Some authors assume that the common shocks are complemented with agent-specific idiosyncratic shocks (e.g. Hellwig and Venkateswaran (2009), Venkateswaran (2014), Bergemann, Heumann and Morris (2015), Amador and Weill (2010)). Irrespective of the precise economic environment, all these studies investigate the role of information in closed economies, for which such shock structure may be reasonable. Nevertheless, as far as the focus is shifted to international context, these assumptions do not seem reliable any more.

In global economy, shocks are neither entirely common nor agent-specific; more likely, they are segmented or, in other words, country-specific. The segmentation or *regionalization*[1] of shocks across the international economy has been documented by a vast literature on international business cycles (e.g. Heathcote and Perri (2002), Heathcote and Perri (2004)), capital flows (Tille and van Wincoop (2014), Tille and Van Wincoop (2010)), international asset trade (Bhamra, Coeurdacier and Guibaud (2014), Devereux and Sutherland (2011)). For example, the segmentation of fundamentals can come from uncorrelated shocks to non-asset incomes across countries, country-specific productivity innovations (Tille and van Wincoop (2014)) or country-specific transaction costs (Bhamra, Coeurdacier and Guibaud (2014)).

The segmentation of shocks across the world has not found a lot of attention in the literature on the welfare properties of information. To the best of our knowledge, the only exception is the study of Arato and Nakamura (2013), who extend the beauty-contest model of Morris and Shin (2002) to a two-region version with uncorrelated country-specific fundamentals. Nevertheless, Arato and Nakamura (2013) assume that the beauty contest is not global, but region-specific, meaning that private agents have incentive to mimic the average actions only in their home region, not in the whole economy. In fact, Arato and Nakamura (2013) model two autarky economies, for which the

[1] term by Heathcote and Perri (2004)

only link is informational spillover, as the signals about region-specific fundamentals are dispersed world-wide. Thus, the model does not capture the full degree of globalization in international trade and investments, which is documented by many researchers.

Apart from segmentation of fundamental shocks, many authors confirm that there exists the informational asymmetry between countries. There is a huge literature which shows that locals have an informational advantage over foreigners (Bae, Stulz and Tan (2008), Ferreira et al. (2017), Dvořák (2005), Van Nieuwerburgh and Veldkamp (2009)). Another strand of literature shows that the informational asymmetry between countries may explain some empirical findings in international portfolio allocation (see Thapa, Paudyal and Neupane (2013) for the survey). The theoretical literature on the social value of information also discusses a specific kind of informational asymmetry. For example, Cornand and Heinemann (2008) and James and Lawler (2012) assume that the public signal reaches only a rate of population. This type of asymmetry is different from informational asymmetry in international finance literature, as all the agents have the same probability of access to this information, while in most financial studies agents have higher probability to get their home information.

The goal of this research is to fill the gap in the literature and to define the value of information in international economies. For this purpose, we explore a stylized two-country model, which captures three main characteristics of international markets: segmentation of fundamentals, informational asymmetry between countries and global strategic complementarity or substitutability in private actions. Basically, this general model is a two-country extension of the model of Angeletos and Pavan (2007), where the whole population is split between two countries with country-specific fundamentals. Informational asymmetry between countries is modeled by the different composition of private signals. We assume that private signals contain information only about the home fundamental shocks. The only source of information about the foreign shock is a public signal, which is available to all the agents in the economy. Thus, each private agent receives three signals: one public signal about the home fundamental shock, one public signal about the foreign fundamental shock and one private signal about the home fundamental shock.

For this general model, we derive social and regional loss functions, and show that social and regional welfare depends not only on the average gaps between equilibrium and optimal actions and their volatility, as in Angeletos and Pavan (2007), but also on relative gaps between regions. Our contribution is two-fold. First of all, we test the findings of Angeletos and Pavan (2007), who derive the complete classification of homogeneous economies according to their welfare properties of information. We show that the crucial parameter, which affects the value of information in two-regional economy, is the externality created by the cross-sectional dispersion. If this externality is absent, almost all the findings of Angeletos and Pavan (2007) stay relevant for segmented economy. Nevertheless, we find that some of results from Angeletos and Pavan (2007) do not hold in segmented economy in case of strategic substitutability. For example, we show that a negative gap between efficient and equilibrium degrees of coordination is not sufficient for private information to be socially needed. The reason is the fact that in this economy equilibrium coordination is inefficient not only inside the region, but also between regions. If strategic substitutability is relatively high, an increase in the precision of private information may force agents to coordinate more inside the region, but this will have a negative effect on inter-regional coordination. If the externality created by the cross-sectional dispersion is sufficiently high, all the findings of Angeletos and Pavan (2007) about the social value of information may be violated, because this externality implies the higher weight of inter-regional gaps in social loss function. For example, the negative externality of the inter-regional gap in private action implies that the social value of private information may be negative. The presence of private information, which is available only to the inhabitants of one region, automatically creates the inter-regional asymmetry in private actions. If the society values this asymmetry negatively, an increase in the precision of this information may lower social welfare, even if it would be valuable in homogeneous societies. Similarly, the positive externality of the cross-sectional dispersion may make the social value of public information negative, even if the efficient extent of coordination is positive.

The second contribution of the paper is that we characterize the regional and inter-regional value of information. When doing so, we detect the situations, in which the social value of information differs from its regional value. These differences in information structures which are optimal from the social and regional point of view, would help to detect the risks of inefficient information policy, if it is conducted by the local authorities. For example, in economies with globally efficient strategic complementarity and positive externality of cross-sector dispersion, the regional value of public information may be negative, while its social value is positive. This happens because the regional value of inter-regional gap is higher than its social value. Thus, if the social authority is the sender of public information about his home region, he or she would publish too little information.

Finally, we illustrate our findings with a number of examples which are widely used in the literature on social value of information.

The rest of the paper is organized as follows. The general two-country framework is introduced in the next Section. Sections 3.3-5 deal with the equilibrium allocation, social optimum and regional optimum, correspondingly. In Section 3.6 we discuss the welfare properties of information in several examples. The last section concludes.

3.2 Framework

In order to study the value of information in segmented economies, we extend the model of Angeletos and Pavan (2007) into a two-region version. We assume that the unit mass of private agents forms the population of an economy. This population is divided into two groups, each of which inhabits one region. Let $i \in [0,1]$ denote the index of a private agent. Agents with index $i \in [0,n] \equiv G_1$ belong to group 1 (or live in region 1) and agents with index $i \in (n,1] \equiv G_2$ belong to group 2 (or live in region 2). Thus, the size of region 1 is equal to n, while the size of region 2 is equal to $(1-n)$.

Let k_i^j denote the action taken by agent i who lives in region j. Then the average private action in this region, K_j, is given by the following expression:

$$K_j \equiv \frac{1}{n_j} \int_{i \in G_j} k_i^j \, \mathrm{d}i$$

The average private action in the economy, $K \equiv \int_{j\in\{1,2\}} \int_{i\in G_j} k_i^j \, \mathrm{d}i \, \mathrm{d}j$, is equal to the weighted average private actions in both regions:

$$K \equiv nK_1 + (1-n)\,K_2$$

The dispersion of private actions in the economy $\sigma_k^2 \equiv \int_{j\in\{1,2\}} \int_{i\in G_j} \left(k_i^j - K\right)^2 \mathrm{d}i \, \mathrm{d}j$ is defined by the dispersion of private actions in both regions and by the gap in private actions between the regions:

$$\sigma_k^2 = n\sigma_1^2 + (1-n)\,\sigma_2^2 + n\,(1-n)\,(K_1 - K_2)^2\,, \tag{3.1}$$

where $\sigma_j \equiv \left({}^1\!/_{n_j} \int_{i\in G_j} \left(k_i^j - K_j\right)^2 \mathrm{d}i\right)^{1/2}$ is the standard deviation of private actions in region j, $j \in \{1,2\}$.

The payoff of private agent i living in region j depends on his action k_i^j, average private action K, the standard deviation of private actions in the economy σ_k , fundamental parameter θ^j and is written by the following function:

$$u_i^j = U\left(k_i^j, K, \sigma_k, \theta^j\right)\,, \tag{3.2}$$

Fundamental θ^j can be interpreted as a technological parameter. This variable is normally distributed with mean $\mu^j = 0$ and variance $\sigma_{\theta,j}^2$. For simplicity, we assume that fundamentals in different region are uncorrelated. Thus, there are only local idiosyncratic technological shocks, without technological spillovers between regions. Nevertheless, the payoff function (3.2) allows for the global strategic effects, as the private payoffs depend on the global average action K.

Following the methodology of Angeletos and Pavan (2007), we assume that payoff function $U\left(k_i^j, K, \sigma_k, \theta^j\right)$ is a quadratic function with $U_{k\sigma} = U_{K\sigma} = U_{\theta\sigma} = U_\sigma\left(k_i^j, K, 0, \theta^j\right) = 0$. This means that payoff function is separable in dispersion term and the other variables. In other words, the dispersion has only non-strategic effect on private payoffs. As we will see later, this implies that the equilibrium private actions do not depend on the dispersion. Thus, the payoff function can be rewritten in the following form:

$$u_i^j = U\left(k_i^j, K, 0, \theta^j\right) + \tfrac{U_{\sigma\sigma}}{2}\sigma_k^2, \tag{3.3}$$

Moreover, we assume that the payoff function is concave in private actions ($U_{kk} < 0$). Moreover, $U_{kK} < -U_{kk}$, where U_{kK} measures the strategic effect in private actions. If $U_{kK} = 0$, the private actions are independent of the average actions in the economy. If $U_{kK} > 0$, the private payoff is higher when the private action is closer to the average action K. Thus, there is strategic complementarity in private actions and private agents have the incentive to do what others do. If $U_{kK} < 0$, the private payoff is higher when the distance between a private action and the average action in the economy is larger. As the private payoff depends on the average for the whole economy, there is a global strategic effect. The alternative version would be a local strategic effect, if the private payoff was linked to the average actions in the home region. The additional assumption is $U_{kk} + 2U_{kK} + U_{\sigma\sigma} < 0$.

The effect of dispersion in private actions can have any sign. If $U_{\sigma\sigma} > 0$, there is a positive private value of dispersion in private actions. This, for example, is the characteristic of a beauty-contest model described by Morris and Shin (2002). If $U_{\sigma\sigma} < 0$, there is a negative private value of dispersion in private actions, as in Walsh (2013). If $U_{\sigma\sigma} = 0$, private payoffs do not depend on the dispersion. Despite of the sign of this variable, we assume that $U_{kk} + U_{\sigma\sigma} < 0$. The model of Angeletos and Pavan (2007) is a special case of ours and can be obtained by choosing $n = 1$.

We assume that $U_k(0,0,0,0) = U_K(0,0,0,0) = 0$. This assumption simplifies considerably the derivations, but does not affect the conclusions about the value of public and private information. Moreover, without lack of generality, $U_{k\theta} > 0$. This assumption means that private agents have an incentive to keep their actions close to their home fundamentals.

We assume that private agents do not know the true values of the fundamentals θ^1 and θ^2. Instead of the perfect information, private agents have an excess to several imperfect signals about the fundamentals. All agents in the economy observe two public signals about the fundamentals:

$$y^j = \theta^j + \eta^j, j \in \{1,2\} \tag{3.4}$$

where $\eta^j \sim N(0, \sigma_{y,j}^2)$ is the noise of public signal y^j with variance $\sigma_{y,j}^2$. Thus, $\sigma_{y,j}^{-2}$ is the precision of a public signal about the fundamental shock in region j. If $\sigma_{y,j}^{-2} = 0$, the prediction value of this information is zero. This is equivalent to the absence of such public information. Two public signals are uncorrelated, meaning that covariance of two noises is equal to zero ($\mathrm{Cov}\,(\eta^1, \eta^2) = 0$).

Thus, the public information about fundamental θ^j consists of public signal y^j and the prior information about the fundamental μ^j. In what follows, we use the composite signal z^j to denote all public information about the fundamental shock in region j:

$$z^j \equiv \frac{\sigma_{y,j}^{-2} y^j + \sigma_{\theta,j}^{-2} \mu^j}{\sigma_{y,j}^{-2} + \sigma_{\theta,j}^{-2}}.$$

Dispersion of the noise in this composite signal is equal to $\sigma_{z,j}^2 = \left(\sigma_{y,j}^{-2} + \sigma_{\theta,j}^{-2}\right)^{-1}$ and precision of public information is equal to $\sigma_{z,j}^{-2} = \left(\sigma_{y,j}^{-2} + \sigma_{\theta,j}^{-2}\right)$. This composite signal is observed by all agents in the economy; there is no difference in the access to public information between the agents in different regions. The only difference in information available to private agents concerns their private information. Private agent i living in region j observes private signal x_i^j about the true value of θ^j:

$$x_i^j = \theta^j + \varepsilon_i^j, \tag{3.5}$$

where $\varepsilon_i^j \sim i.i.d.N(0, \sigma_{x,j}^2)$ is the noise of this private signal and $\sigma_{x,j}^2$ stands for its variance. Thus, value $\sigma_{x,j}^{-2}$ depicts the precision of private information in region j. We suppose that agents in region j do not observe any private signal about the foreign fundamental shock θ^{-j}.

Private agents use their private signals and two composite public signals to form their expectations about the fundamentals:

$$E\left[\left(\begin{array}{c} \theta^j \\ \theta^{-j} \end{array}\right) \middle| x_i^j, z^j, z^{-j}\right] = \left(\begin{array}{c} \delta^j z^j + (1-\delta^j)\, x_i^j \\ z^{-j} \end{array}\right), \tag{3.6}$$

where $\delta^j = \dfrac{\sigma_{z,j}^{-2}}{\sigma_{z,j}^{-2} + \sigma_{x,j}^{-2}}$. According to (3.6), a private agent from region j uses his own private signal x_i^j and public signal z^j to derive his expectations about θ^j. As the agent has no private information about fundamentals in the other region, his expectations about θ^{-j} are equal to public information about θ^{-j}. These expectations are used by private agents to choose their actions. The equilibrium private actions are defined in the next section.

3.3 Equilibrium

Private agents simultaneously choose their actions, which maximize their payoff (3.2). Before proceeding to the equilibrium under imperfect information, we start with the properties of the equilibrium in an economy where all the agents know the true values of fundamentals.

3.3.1 Equilibrium with complete information

The equilibrium with complete information is characterized by a pair of strategies (κ^1, κ^2): $\mathbb{R}^2 \to \mathbb{R}^2$such that

$$\kappa^j\left(\Theta^j\right) = \arg\max_{k^j} U\left(k^j, \tilde{K}\left(\theta^j, \theta^{-j}\right), \tilde{\sigma}_k\left(\theta^j, \theta^{-j}\right), \theta^j\right), \tag{3.7}$$

where $\Theta^j = (\theta^j, \theta^{-j})$ is a vector of fundamental shocks, $\tilde{K}\left(\theta^j, \theta^{-j}\right) = \int_{j\in\{1,2\}}\int_{i\in G_j} \kappa^j\left(\theta^j, \theta^{-j}\right) \mathrm{d}i\, \mathrm{d}j$ is the average private actions under complete information and $\tilde{\sigma}_k\left(\theta^j, \theta^{-j}\right) \equiv \left(\int_{j\in\{1,2\}}\int_{i\in G_j}\left(\kappa^j\left(\theta^j, \theta^{-j}\right) - K\left(\theta^j, \theta^{-j}\right)\right)^2 \mathrm{d}i\, \mathrm{d}j\right)^{1/2}$ is the standard deviation of private actions in equilibrium.

Similarly to one-region model of Angeletos and Pavan (2007), the equilibrium private strategies under complete information are linear over the fundamentals and are given by the following expression:

$$\kappa_j\left(\Theta\right) = \kappa_{j,j}\theta^j + \kappa_{j,-j}\theta^{-j}, \tag{3.8}$$

where $\kappa_{j,j}$ is the equilibrium weight of the home fundamental factor and $\kappa_{j,-j}$ is the equilibrium weight of the foreign fundamental factor in private actions in region j. These equilibrium weights are as follows:

$$\kappa_{j,j} = \kappa - \alpha\kappa(1-n_j) \tag{3.9}$$

$$\kappa_{j,-j} = \alpha\kappa(1-n_j), \tag{3.10}$$

where $\kappa = \dfrac{-U_{k\theta}}{U_{kk}+U_{kK}}$ and $\alpha = \dfrac{U_{kK}}{-U_{kk}}$.

By assumptions made before, $U_{k\theta}$ is positive while expression $U_{kk}+U_{kK}$ is negative. This implies that κ is positive. The sign of α coincides with the sign of U_{kK} and is positive, if there is strategic complementarity, and negative, if there is strategic substitutability. As $-U_{kK} > U_{kk}$, the value of α belongs to the interval $(-\infty, 1)$. This means that the weight of home fundamental shock in private actions in region j is positive. This reflects the incentive of private agents to keep their actions close to their home fundamentals. Despite the foreign fundamentals do not have the direct effect on the private payoffs, the agents also react to the foreign fundamental shock, as far as there is the strategic effect and $\alpha \neq 0$. If $\alpha > 0$ and private actions are characterized by strategic complementarity, the agents in region j also have the incentive to keep their actions close to the private actions in foreign region $-j$. As private agents in region $-j$ align their actions to foreign fundamentals θ^{-j}, strategic complementarity forces agents in region j to put a positive weight $\kappa_{j,-j}$ to this fundamental shock. The stronger strategic complementarity and the larger foreign region, the higher weight of foreign fundamentals is attached by private agents in region j. This leads to an equivalent decrease in the weight of home fundamental in private actions. If there is strategic substitutability and $\alpha < 0$, the agents want to differentiate their actions with the actions of others. This imply a negative weight of foreign fundamentals in private actions and an increase in the weight of home fundamentals.

The redistribution of the whole weight κ between the two fundamentals depends on the extent of strategic effect α and on the region size n_j. If we take the limiting case with $n_j = 1$, we get the one-region model of Angeletos and Pavan (2007) with the equilibrium private actions:

$$\kappa_j\left(\theta^j, \theta^{-j}\right) = \kappa\theta^j \tag{3.11}$$

If there is strategic complementarity ($\alpha > 0$), the weight of the local fundamentals in private actions is lower in two-regional model. The agents redistribute this weight toward the foreign shock, as there is strategic complementarity between regions. If there is strategic substitutability ($\alpha < 0$), the weight of the local fundamentals in private actions is higher in two-regional model. The agents want to keep their actions far from the foreign actions, as there is strategic substitutability between regions. Thus, the agents attach a negative weight to the foreign fundamentals and increase the weight of local fundamentals.

The average private actions in two-region economy, $\bar{\kappa}$, are proportional to the average value of fundamentals in both regions:

$$\bar{\kappa} \equiv n\kappa_1\left(\theta^1, \theta^2\right) + (1-n)\,\kappa_2\left(\theta^1, \theta^2\right) = \kappa\left(n\theta^1 + (1-n)\,\theta^2\right)$$

Nevertheless, this model in general version should not be treated as an average model with actions $\bar{\kappa}$ and fundamentals $\bar{\theta} \equiv (n\theta^1 + (1-n)\,\theta^2)$, because there is asymmetry between regions, which may affect private payoffs. This asymmetry can be illustrated by the gap between private actions in the regions:

$$\kappa_1\left(\theta^1, \theta^2\right) - \kappa_2\left(\theta^1, \theta^2\right) = \kappa\,(1-\alpha)\left(\theta^1 - \theta^2\right)$$

This gap vanishes only if the fundamental parameters are equal in two regions. As far as $\theta^1 \neq \theta^2$, private actions differ in two regions. Even if there is no dispersion in private actions inside the regions, the gap between average actions creates the dispersion between regions and the dispersion of private actions in the whole economy, according to equation (3.1). If private agents do not care about the dispersion and $U_{\sigma\sigma} = 0$, this does not affect the private payoffs. If there is the negative private value of dispersion and $U_{\sigma\sigma} < 0$, the gap between the regions creates the negative effect on private payoffs. If there is a positive private value of dispersion and $U_{\sigma\sigma} > 0$, the gap between the regions creates the positive effect on private payoffs. By assumption, this is a second-order effect which does not influence the equilibrium private actions. Nevertheless, this effect has a crucial impact on the social and regional welfare and is a crucial determinant of the social and local value of private and public information, as it will be shown later.

3.3.2 Equilibrium with incomplete information

Under incomplete information, private agents do not know the true value of fundamental shocks. Thus, they choose their actions in order to maximize their expected payoff given their information set. The information set for any agent consists of three elements. The first element is the private signal about the home fundamental. The second and the third elements are the public signal about their home fundamentals and the public signal about the foreign fundamentals. Formally, equilibrium with incomplete information is a pair of strategies (k_1, k_2): $\mathbb{R}^3 \to \mathbb{R}^2$ such that

$$k^j\left(x^j, z^j, z^{-j}\right) = \arg\max_{k'} E\left[U\left(k', K\left(\Theta, Z\right), \sigma_k\left(\Theta, Z\right), \theta^j\right)\middle| x^j, z^j, z^{-j}\right], \tag{3.12}$$

where $\Theta = (\theta^1, \theta^2)$ is a vector of fundamentals, $Z = (z^1, z^2)$ is a vector of public information, $K\left(\Theta, Z\right) = \int_{j\in\{1,2\}} \int_{x^j} k^j\left(x^j, z^j, z^{-j}\right) \mathrm{d}P\left(x^j\middle|\theta^j, z^j\right) \mathrm{d}j$ is the average private action in equilibrium and $\sigma_k\left(\Theta, Z\right) = \left(\int_{j\in\{1,2\}} \int_{x^j} \left(k^j\left(x^j, z^j, z^{-j}\right) - K\left(\theta^j, \theta^{-j}, z^j, z^{-j}\right)\right)^2 \mathrm{d}P\left(x^j\middle|\theta^j, z^j\right) \mathrm{d}j\right)^{1/2}$ is the equilibrium standard deviation of private actions.

The first-order condition, which describes the equilibrium strategies (3.12), is as follows:

$$k^j\left(x^j, z^j, z^{-j}\right) = E\left[\kappa_j\left(\Theta\right) + \alpha_{j,j}\left(K_j\left(\Theta, Z\right) - \kappa_j\left(\Theta\right)\right) + \alpha_{j,-j}\left(K_{-j}\left(\Theta, Z\right) - \kappa_j\left(\Theta\right)\right)\middle| x^j, z^j, z^{-j}\right]. \tag{3.13}$$

where $K_j\left(\Theta, Z\right)$is the average private action in region j for given fundamental shocks Θ and public information Z, $\alpha_{j,j} = \alpha - \alpha\left(1 - n_j\right)$ and $\alpha_{j,-j} = \alpha\left(1 - n_j\right)$.

According to (3.12), the optimal action of a private agent depends on his expectations about the optimal action under complete information $\kappa_j\left(\Theta\right)$, the expected gap between the average actions under incomplete and complete information in his home region, $\left(K_j\left(\Theta, Z\right) - \kappa_j\left(\Theta\right)\right)$, and the expected gap between the average actions under incomplete and complete information in the foreign region, $\left(K_{-j}\left(\Theta, Z\right) - \kappa_j\left(\Theta\right)\right)$. Value $\alpha_{j,j}$ measures the impact of the home gap in private actions on the decision of the agent. In other words, $\alpha_{j,j}$ is the regional extent of coordination. Similarly, the value $\alpha_{j,-j}$ measures the impact of the foreign gap in private actions on the decision of any agent in region j. Thus, $\alpha_{j,-j}$ is the inter-regional extent of coordination. If there is strategic complementarity ($\alpha > 0$), both regional and inter-regional extents of coordination are positive and agents are willing to mimic the average actions in both regions. The larger region, the stronger desire to mimic its average actions both inside and between regions. If there is strategic substitutability, both regional and inter-regional extents of coordination are negative.

The first-order condition (3.12) gives the linear equilibrium strategy of private agents. This strategy is described in the following proposition.

Proposition 3.1. *In a linear equilibrium, the strategy of private agents is as follows:*

$$k^j\left(x^j, z^j, z^{-j}\right) = \kappa_{j,j}\left(\gamma^j z^j + \left(1-\gamma^j\right)x^j\right) + \kappa_{j,-j}z^{-j}, \tag{3.14}$$

where γ^j is the relative weight of regional public information given by:

$$\begin{aligned}\gamma^j &= \delta^j + \frac{\delta^j\left(1-\delta^j\right)\alpha_{j,j}}{1-\left(1-\delta^j\right)\alpha_{j,j}} + \frac{\left(1-\delta^j\right)\alpha_{j,-j}}{1-\left(1-\delta^j\right)\alpha_{j,j}}\frac{\kappa_{-j,j}}{\kappa_{j,j}} = \\ &=\delta^j + \frac{\delta^j\left(1-\delta^j\right)\alpha_{j,j}}{1-\left(1-\delta^j\right)\alpha_{j,j}} + \frac{\left(1-\delta^j\right)\alpha_{j,-j}}{1-\left(1-\delta^j\right)\alpha_{j,j}}\frac{\alpha_{-j,j}}{1-\alpha_{j,-j}}\end{aligned} \tag{3.15}$$

Proposition 3.1 shows that private agents in two-region economy use the information about both regions, as far as both $\kappa_{j,j}$ and $\kappa_{j,-j}$ are non-zero. We have shown earlier that the weight of home information $\kappa_{j,j}$ is positive, while the weight of foreign information $\kappa_{j,-j}$ is positive only in case of strategic complementarity. It is negative, if there is strategic substitutability and zero, if there is no strategic effect. As the only source of information about the foreign fundamental shock is public signal z^{-j}, the weight of this signal in private action in region j coincides with the weight of foreign fundamental in private actions under complete information. As there are two sources of information about the home fundamentals, the private agent redistributes the entire weight of home information $\kappa_{j,j}$ between them. Parameter $\gamma^j \in [0,1]$ shows the relative weight of home public information, while $(1-\gamma^j)$ measures the relative weight of a private signal in the entire use of home information.

It can be easily seen that the relative weight of public home information is equal to the relative precision of public information δ^j if and only if the strategic effect is absent and $\alpha = 0$. If there is strategic complementarity, the relative weight of public signal in actions exceeds its relative precision. This can be explained by the desire of private agents to mimic the actions of others. The use of a public signal allows them to better predict the actions of others and the use of public signal increases even if this does not allow the agents to keep their actions closer to the relevant home fundamentals. If there is strategic substitutability, the agents have the desire to differentiate their actions from the actions of others. Thus, they decrease the weight of the home public signal in their actions to a level which is lower than the relative precision of public information. An increase in relative precision of public information and strategic complementarity leads to an increase in the relative weight of public home information in private actions.

The effect of region size n_j on the relative weight of public information is non-linear. The following Corollary summarizes the effect of regional size on the relative weight of its home public information:

Corollary 3.2. *The effect of region size on the relative weight of its public information is as follows:*

1. *In case of strategic complementarity,* $\frac{\partial \gamma^j}{\partial n^j} > 0$ *if and only if* $n^j < \min\left(\frac{1}{2}\left(1+\frac{\delta^j}{\alpha(1-\delta^j)}\right);1\right)$*;*

2. *In case of strategic substitutability,* $\frac{\partial \gamma^j}{\partial n^j} < 0$ *if and only if* $n^j > \max\left(\frac{1}{2}\left(1+\frac{\delta^j}{\alpha(1-\delta^j)}\right);0\right)$*.*

The first part of Corollary 3.2 describes the properties of γ^j in case of strategic complementarity. It can be easily shown that threshold $\frac{1}{2}\left(1+\frac{\delta^j}{\alpha(1-\delta^j)}\right)$ is larger than 1, if precision of local public information in region j is relatively high and $\delta^j > \frac{\alpha}{1+\alpha}$. In such situation, the relative weight is increasing in region size and the relative weight of local public information in two-region economy is lower than in one-region economy. In this case, public information is a very good predictor of home fundamentals; thus, its weight in home private actions is initially very high. When there are two regions instead of one, strategic complementarity forces the agents to switch from their home public information to foreign public information. As a result, they redistribute the use of public information as a instrument of coordination towards the foreign signal. If precision of local public information is low and $\delta^j < \frac{\alpha}{1+\alpha}$, threshold $\frac{1}{2}\left(1+\frac{\delta^j}{\alpha(1-\delta^j)}\right)$ is lower than 1 and the relative weight of public information is a hump-shaped function of n_j. Thus, the relative weight of public local information may be higher in two-region economy than in a one-region economy. In this case, the weight of home public information is not that high in a one-region model due to the relatively low precision of this information. Strategic complementarity between regions makes the inhabitants of the foreign region willing to react to the public information about region j. The population of region j knows this and may want to mimic the actions of foreigners by increasing the weight of the home public information, despite its relatively bad quality. Thus, in two-region economy agents in a large region may attach higher weight to their home public signals than they would in a one-region world. Worth to note, that this effect is present only if precision of public information is relatively low and the region is relatively large. To illustrate the reasoning, we provide the equilibrium strategies in one-region economy, which can be obtained from ours by taking $n = 1$. In this model, the equilibrium action of private agents is the function of their private signal and the public signal:

$$k^j\left(x^j, z^j\right) = \kappa\left(\hat{\gamma}^j z^j + \left(1-\hat{\gamma}^j\right)x^j\right), \tag{3.16}$$

where $\hat{\gamma}^j$ is the relative weight of regional public information in one-region economy and is given by:

$$\hat{\gamma}^j = \delta^j + \frac{\delta^j\left(1-\delta^j\right)\alpha}{1-\left(1-\delta^j\right)\alpha} \tag{3.17}$$

The second part of Corollary 3.2 summarizes the properties of γ^j in case of strategic substitutability. If precision of local public information is high and $\delta^j > \frac{-\alpha}{1-\alpha}$, threshold $\frac{1}{2}\left(1+\frac{\delta^j}{\alpha(1-\delta^j)}\right)$ is negative. Thus, an increase in region size leads to a decrease in the relative weight of local public information. Consequently, the relative weight of local public information in two-region economy is higher than in one-region economy. The reasoning is straightforward. If the quality of home public information is relatively good, the agents would use it to keep their actions close to their home fundamentals. The inter-regional strategic substitutability means that the part of the whole population is not going to use this information. Thus, the agents may increase their use of the home public information without suffering from increased coordination. If the quality of public information is relatively bad and $\delta^j < \frac{-\alpha}{1-\alpha}$, the agents may prefer to rely more on their private information to keep their actions close to the fundamentals. In this case, the relative gains of using the home public information are small and the weight of public information is a hump-shaped function of n. Thus, the relative weight of public local information may be lower in a two-region economy than in a one-region economy.

Taking into account the equilibrium strategy under incomplete information (3.14) and under complete information (3.8), we can show that the average actions in region j under incomplete information are equal to the sum of average actions in this region under complete information and the weighted errors of the public signals $(z^j - \theta^j)$ and $(z^{-j} - \theta^{-j})$:

$$K_j\left(\Theta^j, Z^j\right) = \kappa_j\left(\Theta\right) + \kappa_{j,j}\gamma^j\left(z^j - \theta^j\right) + \kappa_{j,-j}\left(z^{-j} - \theta^{-j}\right) \tag{3.18}$$

This gives the average actions in the whole economy:

$$K\left(\Theta, Z\right) = \bar{\kappa}\left(\Theta\right) + \sum_{j\in\{1,2\}}\left(n_j\kappa_{j,j}\gamma^j + \left(1-n_j\right)\kappa_{-j,j}\right)\left(z^j - \theta^j\right)\,\mathrm{d}j, \tag{3.19}$$

where $(n_j\kappa_{j,j}\gamma^j + (1-n_j)\,\kappa_{-j,j})$ is the average weight of signal z^j in private actions.

The gap between the average actions in the two regions is equal to the sum of the gap between the regions under complete information and the relative errors of the public signals:

$$K_1\left(\Theta, Z\right) - K_2\left(\Theta, Z\right) = \kappa_1\left(\Theta\right) - \kappa_2\left(\Theta\right) + \left(\kappa_{1,1}\gamma^1 - \kappa_{2,1}\right)\left(z^1 - \theta^1\right) - \left(\kappa_{2,2}\gamma^2 - \kappa_{1,2}\right)\left(z^2 - \theta^2\right), \tag{3.20}$$

where $(\kappa_{j,j}\gamma^j - \kappa_{-j,j})$ is the relative weight of signal z^j in actions in region j in comparison to its weight in region $-j$. Thus, the errors in public signals create the deviation of the average actions from their values under complete information. The use of imperfect private signals creates the dispersion of the actions inside the regions. The dispersion of private actions in region j is equal to

$$\sigma_j^2 = \kappa_{j,j}^2\left(1-\gamma^j\right)^2\sigma_{x,j}^2 \tag{3.21}$$

The welfare properties of the noise in public and private information are discussed in the next section.

3.4 Social welfare analysis

In this section, we discuss the social value of public and private information in segmented economies. We start with the description of socially efficient allocations under complete and incomplete information. After that we derive the social loss function and find the impact of precision of public and private information on this welfare criterion.

The social welfare is the sum of all private payoffs in the economy:

$$W \equiv \int_{j\in\{1,2\}} \int_{i\in S^j} U\left(k^j\left(x^j, z^j, z^{-j}\right), K\left(\Theta, Z\right), \sigma_k\left(\Theta, Z\right), \theta^j\right) \, \mathrm{d}i \, \mathrm{d}j$$

This welfare can be rewritten as a sum of two components:

$$W = \omega\left(K_1, K_2, \theta^1, \theta^2\right) + \frac{W_{\sigma\sigma}}{2}\left(n_1\sigma_1^2 + n_2\sigma_2^2\right), \tag{3.22}$$

where $\omega\left(K_1, K_2, \theta^1, \theta^2\right)$ is the component, which depends on the regional average private actions, the average private actions in the whole economy and fundamental shocks. Term $\frac{W_{\sigma\sigma}}{2}\left(n_1\sigma_1^2 + n_2\sigma_2^2\right)$ is the component which depends on the dispersion of private actions inside the regions. Coefficient $W_{\sigma\sigma} = U_{\sigma\sigma} + U_{kk}$ measures the social value of dispersion inside the regions. As it is negative, the dispersion in private actions lowers the social welfare and is undesirable from the social perspective. Worth to note, that the social value of dispersion inside regions is negative irrespective of the private value of dispersion. The component which depends on the averages is given by the following expression:

$$\omega\left(K_1, K_2, \theta^1, \theta^2\right) = n_1 U\left(K_1, K, 0, \theta^1\right) + n_2 U\left(K_2, K, 0, \theta^2\right) + \frac{U_{\sigma\sigma}}{2} n_1 n_2 \left(K_1 - K_2\right)^2 \tag{3.23}$$

The first term on the right-hand part in (3.23) is the payoff of agents in the first region, if all of them choose action K_1. The second term is the payoff of the agents in the second region, if they choose action K_2. The last term shows the global gains of private agents due to the gap in actions between the regions. If $U_{\sigma\sigma} < 0$ and there is a negative private value of dispersion, the social welfare is negatively related to the gap between regions. In other words, society values negatively the difference between regions. If $U_{\sigma\sigma} > 0$ and there is a positive private value of dispersion, society values positively the difference between regions. Thus, the social value of the gap between regions coincides with the private value of dispersion.

3.4.1 Social optimum under complete information

To find the social optimum, we assume that the social planner decides on the private actions for given values of fundamental shocks (θ^1, θ^2). As the society gets a negative value of dispersion inside the regions, the social planner chooses the same action for all agents which live in the same region. Thus, the efficient allocation with complete information is a pair of strategies (κ_1^*, κ_2^*): $\mathbb{R}^2 \to \mathbb{R}^2$ such that

$$\kappa_j^* \left(\theta_j, \theta_{-j}\right) = \arg\max_{K_j} \omega \left(K_j, \kappa_{-j}^*, \theta^j, \theta^{-j}\right), \tag{3.24}$$

where $\omega \left(K_j, \kappa_{-j}^*, \theta^j, \theta^{-j}\right)$ is welfare component (3.23). The socially efficient actions for agents in region j are linear over two fundamental shocks:

$$\kappa_j^* \left(\theta^j, \theta^{-j}\right) = \kappa_{j,j}^* \theta^j + \kappa_{j,-j}^* \theta^{-j}, \tag{3.25}$$

where the weights of fundamentals are

$$\kappa_{j,j}^* = \kappa^* - (1 - n_j) \hat{\kappa} \tag{3.26}$$

$$\kappa_{j,-j}^* = (1 - n_j) \hat{\kappa}, \tag{3.27}$$

where $\kappa^* = \frac{(U_{k\theta} + U_{K\theta})}{-W_{KK}}$, $\hat{\kappa} \equiv -\frac{(U_{k\theta} + U_{K\theta})}{W_{KK}} + \frac{U_{k\theta}}{W_{\sigma\sigma}}$ and $W_{KK} = U_{kk} + 2U_{kK} + U_{KK} < 0$.

Thus, the socially optimal private actions under complete information are the weighted sum of the two fundamentals. The optimal distribution in a one-region model can be obtained from (3.25–3.27) by choosing $n = 1$:

$$\kappa_j^* \left(\theta^j, \theta^{-j}\right) = \kappa^* \theta^j.$$

Thus, the relation between the weight of the home fundamental in a one-region model and its weight in a two-region model (3.26) is defined by the value of $\hat{\kappa}$. If $\hat{\kappa} > 0$, the optimal weight of local fundamentals is lower in two-region social optimum in comparison to one-country model. The weight of foreign fundamentals is positive. This happens if the social aversion to variance in private actions is stronger than the desire to reach the fundamentals, such that $W_{\sigma\sigma} < \frac{U_{k\theta}}{(U_{k\theta} + U_{K\theta})} W_{KK}$. This condition is equivalent to $1 - \frac{W_{KK}}{W_{\sigma\sigma}} > -\frac{U_{K\theta}}{U_{k\theta}}$. As we will see later, value $\alpha^* = 1 - \frac{W_{KK}}{W_{\sigma\sigma}}$ characterizes the socially optimal degree of coordination. If the optimal degree of coordination is high, the social planner is ready to sacrifice the closeness of private actions to the local fundamentals in order to vanish the difference between regions. As a result, the weight of fundamentals is redistributed from the local shock to the foreign one. The extent of this redistribution depends positively on the size of foreign region. Thus, the efficient distribution in a two-region model is shifted to the fundamentals in the largest region.

If $\hat{\kappa} < 0$, the optimal weight of local fundamentals is higher in two-region social optimum in comparison to one-country model. The weight of foreign fundamentals is negative. This happens if the social aversion to variance in private actions is not very high, such that $W_{\sigma\sigma} > \frac{U_{k\theta}}{(U_{k\theta} + U_{K\theta})} W_{KK}$. This is equivalent to relatively small efficient degree of coordination, $\alpha^* < -\frac{U_{K\theta}}{U_{k\theta}}$. In this case, the social planner does not care much about the variance in private actions. Thus, the planner is ready to stretch the distance between regions in order to diminish the gap between the local fundamentals and the local private actions. In this case, the private actions in two regions are shifted apart from each other and there is a substantial gap between them.

The gap between efficient actions in two regions in a model with complete information is proportional to the gap between fundamental shocks:

$$\kappa_1^*\left(\theta^1,\theta^2\right)-\kappa_2^*\left(\theta^1,\theta^2\right) \;=\; \left(\kappa^*-\hat{\kappa}\right)\left(\theta_1-\theta_2\right),$$

where coefficient $\left(\kappa^*-\hat{\kappa}\right)=-\frac{U_{k\theta}}{W_{\sigma\sigma}}$ is positive. If there is a huge social aversion to dispersion and the absolute value of $W_{\sigma\sigma}$ is high, the value of $\left(\kappa^*-\hat{\kappa}\right)$ and the gap between the regions vanish. If the social aversion to dispersion is modest, the value of $\left(\kappa^*-\hat{\kappa}\right)$ and the gap between regions are large.

The average efficient action in the economy is proportional to the average value of fundamental shock:

$$\bar{\kappa}^*\equiv n\kappa_1^*+\left(1-n\right)\kappa_2^*=\kappa^*\left(n\theta^1+\left(1-n\right)\theta^2\right).$$

This value does not depend on the private or social value of dispersion. Nevertheless, the average efficient actions in a model with incomplete information do depend on these parameters, as we will see in the next subsection.

3.4.2 Social optimum with incomplete information.

An efficient allocation with incomplete information is a pair of strategies (k_1^*,k_2^*): $\mathbb{R}^3\to\mathbb{R}^2$ such that

$$\left\{k_1^*\left(x^1,Z\right),k_2^*\left(x^2,Z\right)\right\}=\arg\max_{k'(x,Z)}E\left[W\left(k\left(x,Z\right),K\left(\Theta,Z\right),\sigma_k\left(\Theta,Z\right),\Theta\right)\right], \tag{3.28}$$

where $k\left(x,Z\right)=\left\{k_1\left(x^1,Z\right),k_2\left(x^2,Z\right)\right\}$ is a feasible set of private actions, $K\left(\Theta,Z\right)=\int_{j\in\{1,2\}}\int_{x^j}k_j\left(x^j,Z\right)\,\mathrm{d}P\left(x^j\middle|\,\Theta,Z\right)\,\mathrm{d}j$ and $\sigma_k\left(\Theta,Z\right)=\left(\int_{j\in\{1,2\}}\int_{x^j}\left(k_j\left(x^j,Z\right)-K\left(\Theta,Z\right)\right)^2\,\mathrm{d}P\left(x^j\middle|\,\Theta,Z\right)\,\mathrm{d}j\right)^{1/2}$.

This implies the following first-order condition:

$$k_j^*\left(x^j,z^j,z^{-j}\right)=E\left[\kappa_j^*\left(\Theta\right)+\alpha_{j,j}^*\left(K^j\left(\Theta,Z\right)-\kappa_j^*\left(\Theta\right)\right)+\alpha_{j,-j}\left(K^{-j}\left(\Theta,Z\right)-\kappa_{-j}^*\left(\Theta\right)\right)\middle|\,x^j,z^j,z^{-j}\right], \tag{3.29}$$

The first-order condition shows that the efficient strategy for any agent in region j is the sum of his expected efficient action under complete information $\kappa_j^*(\Theta)$ and the expected gaps between the average actions and the corresponding efficient average actions under complete information for both regions. Value $\alpha_{j,j}^* \equiv \alpha^* - (1-n_j)\dfrac{W_{KK}-U_{kk}}{-W_{\sigma\sigma}}$ is the efficient extent of coordination inside the region and $\alpha_{j,-j}^* \equiv \dfrac{W_{KK}-U_{kk}}{-W_{\sigma\sigma}}(1-n_j)$ is the efficient inter-regional extent of coordination, $\alpha^* = 1 - \frac{W_{KK}}{W_{\sigma\sigma}}$ is the efficient extent of coordination in a one-region model.

The inter-regional efficient extent of coordination is positive if the marginal social utility of average actions decreases slower than the marginal private utility of private actions, meaning that $W_{KK} > U_{kk}$. This happens if the private value of coordination is sufficiently high, such that $U_{kK} > -\frac{U_{KK}}{2}$. In this case, the social value of coordination between regions is high and the efficient inter-regional coordination is positive, $\alpha_{j,-j}^* > 0$. The regional degree of coordination diminishes by the value of the inter-regional degree of coordination and is lower than in a one-region economy. This redistribution of coordination between regions is higher for the larger size of the other region. Thus, the efficient allocation implies that the actions are shifted to the average actions in a larger region. If the private value of coordination is low, $U_{kK} < -\frac{U_{KK}}{2}$, the marginal social utility of average actions decreases faster than the marginal private utility of private actions, meaning that $W_{KK} < U_{kk}$. In this case, the efficient extent of inter-regional coordination is negative and the efficient extent of coordination inside the region is higher than in a one-region model.

The first-order condition (3.29) gives the linear efficient strategy of private agents. This strategy is described by the following proposition.

Proposition 3.3. *The linear efficient strategy of private agents is as follows:*

$$k_j^*\left(x^j, z^j, z^{-j}\right) = \kappa_{j,j}^*\left(\gamma_j^* z^j + \left(1-\gamma_j^*\right)x^j\right) + \kappa_{j,-j}^* z^{-j}, \tag{3.30}$$

where γ_j^* is the efficient relative weight of regional public information given by:

$$\gamma_j^* = \delta^j + \frac{\delta^j\left(1-\delta^j\right)\alpha_{j,j}^*}{1-\left(1-\delta^j\right)\alpha_{j,j}^*} + \frac{\left(1-\delta^j\right)\alpha_{j,-j}^*}{1-\left(1-\delta^j\right)\alpha_{j,j}^*}\frac{\kappa_{-j,j}^*}{\kappa_{j,j}^*} \tag{3.31}$$

In a one-region model the efficient action are as follows:

$$k_j^*\left(x^j, z^j\right) = \kappa^*\left(\gamma_j^* z^j + \left(1-\gamma_j^*\right)x^j\right), \tag{3.32}$$

with

$$\hat{\gamma}_j^* = \delta^j + \frac{\delta^j\left(1-\delta^j\right)\alpha^*}{1-\left(1-\delta^j\right)\alpha^*} \tag{3.33}$$

Comparison of these strategies with the equilibrium in a one-region model shows that the equilibrium is socially efficient if $\kappa = \kappa^*$ and $\alpha = \alpha^*$. In this case, the equilibrium and efficient distribution under complete information are the same and the efficient degree of coordination coincides with the equilibrium degree of coordination. In a two-region economy, these conditions are necessary but not sufficient for equilibrium to be optimal. Condition $\kappa = \kappa^*$ assures that the average actions in the equilibrium and in the optimum coincide under complete information. Nevertheless, this does not guarantee that the distribution of these averages between regions is efficient. Condition $\alpha = \alpha^*$ assures that the average degrees of coordination are efficient, but it is not sufficient for both regional and inter-regional degrees of coordination to be efficient. Comparison of equilibrium strategies (3.14) with socially efficient strategies (3.30) gives the following sufficient condition for the efficiency of equilibrium allocation:

Proposition 3.4. *Equilibrium in a two-regional model is socially efficient if and only if* $\kappa = \kappa^*$, $\alpha = \alpha^*$ *and* $U_{\sigma\sigma} = 0$.

Thus, equilibrium strategies in a two-region model is efficient if they are socially efficient in a one-region model and the private value of dispersion and the social value of the gap between two regions are equal to zero. This finding demonstrates higher importance of parameter $U_{\sigma\sigma}$ in a two-region model in comparison with a one-region model. In order to better understand this finding, we consider three possible sources of inefficiency in segmented economy: the gap between equilibrium and efficient degrees of coordination $\alpha^* - \alpha$, the gap between efficient and equilibrium average allocation under complete information $\kappa^* - \kappa$ and the externality of dispersion in private actions $U_{\sigma\sigma}$. The positive gap between α^* and α means that equilibrium coordination degrees in the model are insufficiently low, both inside and between regions. If $\kappa^* > \kappa$, the agents respond insufficiently to the shocks in both home and foreign fundamentals. Thus, the first two sources of inefficiency equally strike the agents reaction to home and foreign variables. On the contrary, the externality caused by dispersion in private actions creates an additional asymmetry. It can be easily shown that the negative externality ($U_{\sigma\sigma} < 0$) makes the regional degree of coordination inefficiently low and inter-regional degree of coordination insufficiently high. Moreover, this leads to the positive gap between the efficient and equilibrium weights of home shocks under complete information and to a negative gap between the efficient and equilibrium weights of foreign shocks in private actions. As we will see in the next section, the social welfare in a segmented economy depends on the gap in actions between two regions. Thus, the asymmetry created by this externality, can considerably change the welfare properties of information in a two-region model. In the next subsection we derive the social loss function, which is then used to study the welfare properties of public and private information.

3.4.3 Social loss function

The expected value of social welfare (3.22) can be written as:

$$EW^S = \omega^S\left(\kappa_1^*, \kappa_2^*, \theta^1, \theta^2\right) - \mathscr{L}_S^*,$$

where $\omega^S\left(\kappa_1^*, \kappa_2^*, \theta^1, \theta^2\right)$ is the first-best social welfare and $\mathscr{L}_S^*$ is the social loss which arises due to the gap between the equilibrium and social optimum. The value of this loss is as follows:

$$\begin{aligned}\mathscr{L}_S^* = {} & \frac{|W_{KK}|}{2} Var\left(K - \bar{K}^*\right) + \frac{n(1-n)}{2}|W_{\sigma\sigma}|\, Var\left(K_1 - \kappa_1^* - \left(K_2 - \kappa_2^*\right)\right) \\ & + \frac{|W_{\sigma\sigma}|}{2}\left(n\sigma_1^2 + (1-n)\,\sigma_2^2\right)\end{aligned} \tag{3.34}$$

Thus, equation (3.34) reveals the sources of inefficiency in the described economy. The first source of inefficiency is the gap between the equilibrium average actions and the socially efficient average actions. The variance of this gap is denoted by $Var\left(K - \bar{K}^*\right)$ in equation (3.34). Coefficient $\frac{|W_{KK}|}{2}$ measures the impact of this variance on the social loss. The second term in social loss comes from the possible asymmetry between the regions. The value $K_1 - \kappa_1^* - \left(K_2 - \kappa_2^*\right)$ measures the relative gap between the average regional actions and the corresponding optimal actions. If the gaps between average and optimal actions are different for the regions, the asymmetry arises and social welfare deviates further from the first-best. Coefficient $\frac{n(1-n)}{2}|W_{\sigma\sigma}|$ measures the importance of the inter-regional asymmetry for social planner. Finally, the social loss comes from the variance in private actions in both regions, which is measured by σ_1^2 and σ_2^2. For the larger region, concerns about its private actions dispersion are stronger.

The gap between the equilibrium and first-best allocations can arise because of two reasons. The first reason is the inefficient structure of the economy, such that equilibrium under complete information is not efficient. The second reason is incomplete information. These two reasons can be partially separated from each other. For example, the gap between average equilibrium and efficient actions can be represented as the sum of the gap between the average equilibrium actions under complete and incomplete information and the gap between the equilibrium actions under complete information and average efficient actions:

$$K - \bar{K}^* = (K - \bar{\kappa}) + \left(\bar{\kappa} - \bar{K}^*\right)$$

The variance of this sum is equal to the sum of variances of two gaps and their doubled covariance. As value $\left(\bar{\kappa} - \bar{K}^*\right)$ is the gap between the equilibrium and efficient action under complete information, its value does not depend on the information available to the agents. Thus, the first component in (3.34) can be represented by the sum of two terms, one of which is independent of the information quality, while the other is defined by the precision of public and private information available to agents. The same is true for the second component in social loss function. The dispersion of private actions arises only under incomplete information and thus, it is fully defined by the information precision. As a result, we can rewrite the social loss (3.34) as the sum of component L_S^0, which is independent of precision of public and private information, and component L_S, which depends on the precisions:

$$\mathscr{L}_S^* = L_S^0 + L_S,$$

where component L_S is as follows:

$$\begin{aligned} L_S^* = & \frac{|W_{\sigma\sigma}|}{2} (1-\alpha^*) \left[Var\left(K - \bar{\kappa}\right) + 2Cov\left(K - \bar{\kappa}; \bar{\kappa} - \bar{\kappa}^*\right) \right] + \\ & + n(1-n) \frac{|W_{\sigma\sigma}|}{2} \left[Var\left(K_1 - \kappa_1 - (K_2 - \kappa_2)\right) + \right] \\ & + n(1-n) |W_{\sigma\sigma}| \, Cov\left(K_1 - \kappa_1 - (K_2 - \kappa_2); \kappa_1 - \kappa_1^* - (\kappa_2 - \kappa_2^*)\right) \\ & + \frac{|W_{\sigma\sigma}|}{2} \left(n\sigma_1^2 + (1-n)\,\sigma_2^2 \right) \end{aligned} \tag{3.35}$$

The first term in (3.35) represents the variance of the gap between the average equilibrium actions K and the average actions under complete information. The gap between the average actions under incomplete and incomplete information is defined by the errors in the public signals and can be written as follows:

$$K - \bar{\kappa} = n\kappa\left(\gamma_1 + \alpha(1-n)(1-\gamma_1)\right)(z_1 - \theta_1) + (1-n)\,\kappa\left(\gamma_2 + \alpha n(1-\gamma_2)\right)(z_2 - \theta_2), \tag{3.36}$$

where $(z_j - \theta_j)$ represents the error in the public information about the fundamental θ_j. Thus, the variance of the gap is defined by the variance of two public sets of information. As two fundamentals are uncorrelated, the variance of the gap is equal to the sum of two terms, each of which is defined by the variance of one set of information:

$$Var\left(K - \bar{\kappa}\right) = Var_1\left(K - \bar{\kappa}\right) + Var_2\left(K - \bar{\kappa}\right)$$

$$Var_j\left(K - \bar{\kappa}\right) = n_j^2 \kappa^2 \left(\gamma_j + \alpha\left(1 - n_j\right)\left(1 - \gamma_j\right)\right)^2 \sigma_{z,j}^2$$

The gap between equilibrium and efficient average actions under complete information is defined as follows:

$$\bar{\kappa} - \bar{\kappa}^* = (\kappa - \kappa^*)\left(n\theta_1 + (1-n)\,\theta_2\right)$$

If $\kappa = \kappa^*$, the average equilibrium actions and efficient average actions coincide under complete information. Thus, in this case the equilibrium is efficient on average. The covariance of this gap and the gap between the average actions under complete and incomplete information can be written as the sum of two terms, each of which depends on the precision of one set of information. The term which depends on the information about fundamental θ^j is as follows:

$$Cov_j\left(K - \bar{\kappa}; \bar{\kappa} - \bar{\kappa}^*\right) = -n_j^2 \kappa \left(\kappa - \kappa^*\right) \left(\gamma_j + \alpha \left(1 - n_j\right) \left(1 - \gamma_j\right)\right) \sigma_{z,j}^2,$$

where we use $Cov(z_j - \theta_j, \theta_j) = -\sigma_{z,j}^2$

The difference between the gaps in private actions is also defined by the errors in public information:

$$\left(K_1 - \kappa_1 - \left(K_2 - \kappa_2\right)\right) = \kappa \left(\left(1 - \alpha\right) \gamma_1 - \alpha n \left(1 - \gamma_1\right)\right) \left(z_1 - \theta_1\right) - \kappa \left(\left(1 - \alpha\right) \gamma_2 - \alpha \left(1 - n\right) \left(1 - \gamma_2\right)\right) \left(z_2 - \theta_2\right)$$

Thus, the variance of this variable is also separable into two terms. For example, the term which depends on the information about region j is as follows:

$$Var_j\left(K_1 - \kappa_1 - \left(K_2 - \kappa_2\right)\right) = \kappa^2 \left(\left(1 - \alpha\right) \gamma_j - \alpha n_j \left(1 - \gamma_j\right)\right) \sigma_{z,j}^2$$

The relative gap between regions in equilibrium and social optimum under complete information is defined as follows:

$$\kappa_1 - \kappa_1^* - \left(\kappa_2 - \kappa_2^*\right) = \left(1 - \alpha\right) \kappa \frac{U_{\sigma\sigma}}{W_{\sigma\sigma}} \left(\theta_1 - \theta_2\right) \tag{3.37}$$

As we can see, this gap is present only if $U_{\sigma\sigma}$ is different from zero, meaning that the agents value the dispersion in private actions (either positively or negatively). Covariance of two measures of the gap between the regions is separable into two terms with

$$\begin{aligned} &Cov_j\left(K_1 - \kappa_1 - \left(K_2 - \kappa_2\right); \kappa_1 - \kappa_1^* - \left(\kappa_2 - \kappa_2^*\right)\right) = \\ &= -\kappa^2 \left(\left(1 - \alpha\right) \gamma_j - \alpha n_j \left(1 - \gamma_j\right)\right) \left(1 - \alpha\right) \frac{U_{\sigma\sigma}}{W_{\sigma\sigma}} \sigma_{z,j}^2 \end{aligned}$$

Finally, the variances of private actions are measured as follows:

$$\sigma_j^2 = \left(1 - \gamma_j\right)^2 \left(1 - \alpha \left(1 - n_j\right)\right)^2 \kappa^2 \sigma_{x,j}^2$$

As the variance of private actions in any region depends only on the precisions of information about this region, we conclude that social loss is separable into two arguments:

$$L_S^* = L_S^1 + L_S^2$$

where the term L_S^j is the component which depends on the information about region j:

$$L_S^j = \frac{|W_{\sigma\sigma}|}{2}\left(1-\alpha^*\right) n_j^2 \sigma_{z,j}^2 \left[\kappa^2 \left(\gamma_j + \alpha\left(1-n_j\right)\left(1-\gamma_j\right)\right)^2 - 2\kappa\left(\kappa-\kappa^*\right)\left(\gamma_j + \alpha\left(1-n_j\right)\left(1-\gamma_j\right)\right)\right] + \tag{3.38}$$

$$+ n_j\left(1-n_j\right)\frac{|W_{\sigma\sigma}|}{2}\kappa^2\sigma_{z,j}^2\left(\left(1-\alpha\right)\gamma_j - \alpha n_j\left(1-\gamma_j\right)\right)^2 +$$
$$- n_j\left(1-n_j\right)|W_{\sigma\sigma}|\kappa^2\sigma_{z,j}^2\left(\left(1-\alpha\right)\gamma_j - \alpha n_j\left(1-\gamma_j\right)\right)\left(1-\alpha\right)\rho +$$
$$+ \frac{|W_{\sigma\sigma}|}{2} n_j\left(1-\gamma_j\right)^2\left(1-\alpha\left(1-n_j\right)\right)^2\kappa^2\sigma_{x,j}^2,$$

where $\rho = \frac{U_{\sigma\sigma}}{W_{\sigma\sigma}}$. We apply this general loss function to study the social welfare properties of information in the next sub-section.

3.4.4 Social value of information

Exploring the properties of social loss function (3.38) allows to study the social value of information in a segmented economy and to compare it with its value in a homogeneous economy. The properties of information in a homogeneous economy have been described in Angeletos and Pavan (2007), who come with three main findings:

- in efficient economies with $\kappa = \kappa^*$ and $\alpha = \alpha^*$, social loss is decreasing in the precision of both public and private information;
- in economies with efficient equilibrium allocation under complete information ($\kappa = \kappa^*$) and inefficient equilibrium degree of coordination ($\alpha \neq \alpha^*$), $\alpha^* > \alpha > 0$ suffices for social loss to be decreasing in the precision of public information and $\alpha^* < \alpha < 0$ suffices for social loss to be decreasing in the precision of private information;
- in inefficient economies with $\kappa \neq \kappa^*$, there exist $\overline{\phi}$ and $\underline{\phi}$ such that social loss is decreasing in precision of both public and private information, if $\kappa^* - \kappa > \overline{\phi}$, and increasing in precision of both public and private information, if $\kappa^* - \kappa < \underline{\phi}$.

We start our testing of these results in segmented economies under assumption that the dispersion in private action does not create any externality ($U_{\sigma\sigma} = 0$). This allows us to abstract from the source of inefficiency which is present in a segmented economy, but does not affect social welfare in homogeneous economy. The findings about the social value in such economy are summarized in the following Proposition:

Proposition 3.5. ***The social value of information in economies without externality created by the dispersion in private actions.*** *In segmented economies with $U_{\sigma\sigma} = 0$ the social loss function is such that:*

1. *in economies with $\kappa = \kappa^*$and $\alpha = \alpha^*$, social loss is decreasing in precision of private and public information;*

2. *in economies with $\kappa = \kappa^*$and $\alpha \neq \alpha^*$, $\alpha^* > \alpha > 0$ is sufficient condition for social loss to decrease in precision of public information and $\alpha - \psi < \alpha^* < \alpha < 0$ with $\psi = -\frac{(1-\alpha)(1-\alpha n)}{2\alpha n(1-n)} > 0$ is sufficient condition for social loss to decrease in precision of private information;*

3. *in economies with $\kappa \neq \kappa^*$, for any (α, α^*, n) there exist $\overline{\phi}$ and $\underline{\phi}$ such that*

   a) *if $\alpha > 0$, social loss is decreasing in precision of public and private information if $\kappa^* - \kappa > \overline{\phi}(\alpha, \alpha^*, n)$ and increasing in precision of public and private information if $\kappa^* - \kappa < \underline{\phi}(\alpha, \alpha^*, n)$;*

   b) *if $\alpha < 0$, social loss is decreasing in precision of private information if $\kappa^* - \kappa > \overline{\phi}(\alpha, \alpha^*, n)$ and increasing in precision of private information if $\kappa^* - \kappa < \underline{\phi}(\alpha, \alpha^*, n)$.*

Part 1 of Proposition 3.5 shows that the social value of both private and public information is positive in segmented efficient economies. This finding corresponds to the value of information in efficient homogeneous economies. Part 2 of Proposition 3.5 implies that the sufficient condition for public information to be valuable is the same in segmented and homogeneous economies, if private actions are characterized by strategic complementarity. Similar to Angeletos and Pavan (2007), the positive gap between the efficient and the equilibrium degree of coordination ensures that the social loss is decreasing in the precision of public information.

Nevertheless, the sufficient condition for private information to be welfare-improving is now different. As we can see in Part 2 of Proposition 3.5, the social loss is necessarily increasing in the precision of public information is $\alpha^* \in (\alpha - \psi, \alpha)$. This means that for a large gap between the efficient and the equilibrium degree of substitutability, the social value of private information may be negative. The reasoning is straightforward. With the help of equation (3.35), we can show that the loss in economy with inefficient degree of coordination is equal to the loss in efficient economy plus the loss created by inefficient degree of substitutability:

$$L_S^* = L_S^*|_{\alpha=\alpha^*} + (\alpha - \alpha^*) \frac{|W_{\sigma\sigma}|}{2} Var(K - \bar{\kappa}),$$

where $Var(K - \bar{\kappa})$ is the variance of the gap between the average equilibrium actions under complete and incomplete information. As we have seen earlier in equation (3.36)), this gap is proportional to $(\gamma_j + \alpha(1 - n_j)(1 - \gamma_j))$, which stands for the normalized weight of public and private information in private actions. In homogeneous economy with $n_j = 1$, this weight is equal to the relative weight of home public information in private actions, γ_j. Thus, in a homogeneous economy this weight is positive. An increase in the precision of private information leads to a decrease in the relative weight of home public information. This leads to a lower impact of the errors in the home public information on the average actions and lower dispersion $Var\left(K - \bar{\kappa}\right)$. In a two-region economy the value $\left(\gamma_j + \alpha\left(1 - n_j\right)\left(1 - \gamma_j\right)\right)$ may be negative for sufficiently strong strategic substitutability in private actions. The negative gap between the average equilibrium actions under complete and incomplete information means that the average actions are too high in equilibrium if the value of public signal is too low in comparison with the real value of the fundamentals. This phenomenon arises because the agent tries to keep their actions apart from the actions of others not only in their home region, but also from the actions of foreigners. Because of the negative value of the gap, an increase in the precision of private information and a decrease in the relative weight of public signal lead to an increase, not decrease, in the absolute value of this gap. This implies an increase in the precision of private information together with the high value of $(\alpha - \alpha^*)$ may cause an increase in social loss.

Equivalent reasoning explains, why Part 3 of Proposition 3.5 differs from its analogue in a homogeneous economy. As we can see, the findings about the value of information in inefficient economies with strategic substitutability are different. The social loss in these economies is equal to the loss in economies with efficient equilibrium allocation under complete information plus the loss created by inefficiency in complete-information equilibrium:

$$L_S^* = L_S^*\big|_{\kappa=\kappa^*} + \left(1 - \alpha^*\right)\left|W_{\sigma\sigma}\right| Cov\left(K - \bar{\kappa}; \bar{\kappa} - \bar{\kappa}^*\right),$$

where $Cov_j\left(K - \bar{\kappa}; \bar{\kappa} - \bar{\kappa}^*\right)$ may negatively depend on the precision of public information in case of strategic substitutability and low relative precision of public information. The main differences in social value of information in segmented economies in comparison with the homogeneous economies are summarized by the following Corollary:

Corollary 3.6. *In segmented economies with strategic substitutability $\alpha < 0$ and $\rho = 0$, contrary to the corresponding homogeneous economies,*

1. *if $\kappa = \kappa^*$ and $\alpha^* < \alpha - \psi < \alpha < 0$, private information may be detrimental for social welfare;*
2. *social loss may be increasing in precision of public information, if $\kappa^* - \kappa$ is sufficiently high, and decreasing in precision of public information, if $\kappa^* - \kappa$ is sufficiently low.*

The presence of externality created by the dispersion in private actions, meaning that $U_{\sigma\sigma} \neq 0$, affects the social loss trough the gap in actions between regions:

$$L_S^* = L_S^*|_{U_{\sigma\sigma}=0} + n\,(1-n)\,|W_{\sigma\sigma}|\,Cov\,(K_1 - \kappa_1 - (K_2 - \kappa_2)\,; \kappa_1 - \kappa_1^* - (\kappa_2 - \kappa_2^*))\,,$$

where $\kappa_1 - \kappa_1^* - (\kappa_2 - \kappa_2^*) = (1-2n)\,(1-\alpha)\,\kappa\frac{U_{\sigma\sigma}}{W_{\sigma\sigma}}\,(\theta_1 - \theta_2)$ and $Cov_j\,(K_1 - \kappa_1 - (K_2 - \kappa_2)\,; \kappa_1 - \kappa_1^* - (\kappa_2 - \kappa_2^*)) = -\kappa^2\,((1-\alpha)\,\gamma_j - \alpha n_j\,(1-\gamma_j))\,(1-\alpha)\,\frac{U_{\sigma\sigma}}{W_{\sigma\sigma}}\sigma_{z,j}^2$. In case of strategic complementarity, term $((1-\alpha)\,\gamma_j - \alpha n_j\,(1-\gamma_j))\,\sigma_{z,j}^2$ is increasing in the precision of public information. Thus, the negative value of $\frac{U_{\sigma\sigma}}{W_{\sigma\sigma}}$ ensures that covariance of the two inter-regional gaps is increasing. In case of strategic substitutability, the value of this term may be decreasing in the precision of public information for high values of ζ_j. Thus, the positive value of $\frac{U_{\sigma\sigma}}{W_{\sigma\sigma}}$ is required for social loss to be increasing in the precision of public information. Value $((1-\alpha)\,\gamma_j - \alpha n_j\,(1-\gamma_j))$ is decreasing in the precision of private information for both strategic complementarity and substitutability, thus relatively high value of $\frac{U_{\sigma\sigma}}{W_{\sigma\sigma}}$ leads to a negative value of private information. As we have seen earlier, the positive gap between efficient and equilibrium allocation under complete information leads to an increase in the social value of information. Thus, more extreme values of $\frac{U_{\sigma\sigma}}{W_{\sigma\sigma}}$ are needed we retain the increasing social loss function. These findings are summarized in the following Proposition:

Proposition 3.7. ***The social value of information in economies with*** $U_{\sigma\sigma} \neq 0$***.*** *For given* $(\kappa, \kappa^*, \alpha, \alpha^*, n)$*, there exist* $0 < \overline{\rho} < 1$*,* $\underline{\rho} < 0$ *and* $\tilde{\rho} < 1$*, such that*

1. *social loss is increasing in precision of public information if* $\alpha > 0$ *and* $\rho < \underline{\rho}\,(\kappa, \kappa^*, \alpha, \alpha^*, n)$ *or if* $\alpha < 0$ *and* $\rho > \overline{\rho}\,(\kappa, \kappa^*, \alpha, \alpha^*, n)$*, at least for low values of* ζ*;*

2. *social loss is increasing in precision of private information if and only if* $\rho > \tilde{\rho}\,(\kappa, \kappa^*, \alpha, \alpha^*, n)$*;*

3. *an increase in the gap* $\kappa^* - \kappa$ *and* $\alpha^* - \alpha$ *leads to an increase in* $\overline{\rho}\,(\kappa, \kappa^*, \alpha, \alpha^*, n)$ *and* $\tilde{\rho}\,(\kappa, \kappa^*, \alpha, \alpha^*, n)$ *and to a decrease in* $\underline{\rho}\,(\kappa, \kappa^*, \alpha, \alpha^*, n)$*.*

All these results are closely related to the relative influence of strategic private motive and the externality created by the dispersion in private actions on the social loss. In case of strategic complementarity, public information is more likely to have the positive value for the social and private welfare. If it is accompanied by negative externality of the dispersion in private actions ($U_{\sigma\sigma} < 0$ and $\rho > 0$), the public information becomes even more desirable, as the social planner wants to avoid any dispersion in private actions. Sufficiently high positive value of ρ, meaning that the negative externality of the dispersion is substantial, ensures that the social loss is decreasing in the precision of public information, even if the efficient degree of coordination is lower than the equilibrium degree. On the contrary, substantial positive externality ($U_{\sigma\sigma} > 0$ and $\rho < 0$) forces the social planner to look for greater dispersion in private actions, despite of strategic complementarity. This suffices for public information to be undesirable. In case of strategic substitutability, the agents use the public signals not only to predict their home fundamental, but also to differentiate their actions from the foreign private actions. Thus, an increase in the precision of public information leads to an increase in inter-regional dispersion. If the social loss of dispersion is sufficiently high ($U_{\sigma\sigma} < 0$ and $\rho > 0$), this may lead to a decrease in the social welfare, making the social value of public information negative. Moreover, sufficiently strong negative value of dispersion assures that the social value of private information may be negative, despite the type of strategic effect in private actions.

Thus, we discussed the welfare properties of information in segmented economies. In the next section we proceed to the discussion of the regional welfare effects of public and private information.

3.5 Regional welfare analysis

In this section we discuss the regional welfare properties of information. For this purpose we describe the regionally optimal allocations under complete and incomplete information. After that we derive the regional loss function, which is then used to study the effects of public and private information on the welfare of each region.

The regional welfare is the sum of private payoffs inside the region j:

$$W^j \equiv \int_{i \in S^j} U\left(k_i^j\left(x_i^j, z^j, z^{-j}\right), K\left(\Theta, Z\right), \sigma_k\left(\Theta, Z\right), \theta^j\right) \mathrm{d}i$$

Similar to the social welfare studied in the previous section, the regional welfare consists of two terms:

$$W^j = \omega^j\left(K_j, K_{-j}, \theta^j, \theta^{-j}\right) + \frac{\left(W^j_{\sigma\sigma}\right)^T}{2}\begin{bmatrix} \sigma^2_{k,j} \\ \sigma^2_{k,j} \end{bmatrix}, \tag{3.39}$$

where $\omega^j\left(K_j, K_{-j}, \theta^j, \theta^{-j}\right)$ is the regional welfare component, which depends on the average actions and the fundamental shocks:

$$\omega^j\left(K_j, K_{-j}, \theta^j, \theta^{-j}\right) = n_j U\left(K_j, K, 0, \theta^1\right) + \frac{U_{\sigma\sigma}}{2} n_j^2\left(1 - n_j\right)\left(K_j - K_{-j}\right)^2 \tag{3.40}$$

According to (3.40), the regional welfare depends positively on the gap between regions, if the private value of dispersion $U_{\sigma\sigma}$ is positive. If private value of dispersion is negative, the regional welfare depends negatively on the gap between regions. The sign of this dependence coincides with the effect of the gap between regions on the social welfare. Nevertheless, the size of this effect is different. According to (3.23), the importance of this gap relative to the average payoff in the region is equal to $(1 - n_j)$. According to (3.40), the importance of the gap relative to the average regional payoff is equal to $n_j (1 - n_j)$. Thus, the regional society pays less attention to the gap between regions than the social planner.

The second term in regional welfare (3.39) demonstrates the regional and international values of dispersion inside the regions. Vector $W^j_{\sigma\sigma}$ is as follows:

$$W^j_{\sigma\sigma} = \begin{bmatrix} n_j (n_j U_{\sigma\sigma} + U_{kk}) \\ n_j (1 - n_j) U_{\sigma\sigma} \end{bmatrix} \tag{3.41}$$

The first element of this vector demonstrates the regional value of the dispersion in private actions in the home region. Under assumptions made at the beginning, this value is negative. Thus, the regional society does not like the variance in its home actions. Nevertheless, the absolute value of aversion to the home dispersion differs from the social aversion. In the previous section we have seen that the social aversion to dispersion in region j is equal to $n_j (U_{\sigma\sigma} + U_{kk})$. The absolute value of the aversion is lower than the local aversion if $U_{\sigma\sigma}$ is positive. If $U_{\sigma\sigma}$ is negative, the local aversion to dispersion is lower that the social aversion.

The second element of vector $W^j_{\sigma\sigma}$ demonstrates the inter-regional value of dispersion. If private value of dispersion is positive, the local society gets welfare gains from the dispersion in actions abroad. If the private value of dispersion is positive, the local society gets a welfare loss from the dispersion abroad. Thus, the inter-regional value of dispersion may differ from the social value of dispersion which is always negative. The regional planner would like to impose the infinite noise in the actions in the other region. Nevertheless, we assume that this is not possible and the regional planner cannot discriminate between private agents in the foreign region. This assumption does not change the conclusions about the regional and inter-regional value of information which are studied in the subsequent sections.

3.5.1 Regional optimum under complete information

Under assumption of impossibility to discriminate between private agents, the regionally efficient allocation is the solution of the program of the regional planner. This allocation is a pair of strategies $\left(\tilde{\kappa}^j_j, \tilde{\kappa}^j_{-j}\right)$: $\mathbb{R}^2 \to \mathbb{R}^2$ such that

$$\left\{\tilde{\kappa}^j_j \left(\Theta^j\right), \tilde{\kappa}^j_{-j} \left(\Theta^j\right)\right\} = \arg \max_{\{K_j, K_{-j}\}} \omega^j \left(K_j, K_{-j}, \theta^j, \theta^{-j}\right),$$

Thus, the regional planner chooses two strategies which maximize the component of its welfare, which does not depend on the dispersion. This representation is correct, as the planner does not value the dispersion inside its home region and chooses the same actions for all the agents in its region. Potentially, the regional planner would value the dispersion abroad, but the assumption made before does not allow the discrimination between the foreign agents. In this case the regionally optimal strategies are as follows:

$$\tilde{\kappa}_l^j\left(\theta^j,\theta^{-j}\right)=\tilde{\kappa}_{l,j}^j\theta^j,\ l\in\{j,-j\} \tag{3.42}$$

with

$$\tilde{\kappa}_{j,j}^j=\kappa^*-(1-n)\,\tilde{\kappa}_{jj} \tag{3.43}$$

$$\tilde{\kappa}_{-j,j}^j=\kappa^*-\tilde{\kappa}_{-jj}, \tag{3.44}$$

where $\tilde{\kappa}_{jj} = -\frac{(U_{kK}+U_{KK})((U_{kK}+U_{KK})U_{k\theta}+(U_{kk}+U_{kK})U_{K\theta})}{W_{KK}\left[(1-n)\left(U_{kK}^2-U_{kk}U_{kK}\right)-nW_{KK}U_{\sigma\sigma}\right]}$ and $\tilde{\kappa}_{-j,j}=\frac{(U_{kk}+U_{kK}+n(U_{kK}+U_{KK}))((U_{kK}+U_{KK})U_{k\theta}-(U_{kk}+U_{kK})U_{K\theta})}{W_{KK}\left[(1-n)\left(U_{kK}^2-U_{kk}U_{kK}\right)-nW_{KK}U_{\sigma\sigma}\right]}$

The regional optimum for region j implies that agents do not react to the shocks in region $-j$. The reason for this is that the private payoffs in region j depend only on the fundamentals in region j. Consequently, the regional planner wants all agents in the economy to base their actions on the fundamentals in region j, irrespective of the place where agents live. In equilibrium, at least the agents in region $-j$ do react to the shocks in their region. Thus, the following Corollary states the impossibility of equilibrium to be regionally optimal:

Corollary 3.8. *As far as $U_{k\theta}\neq 0$, the equilibrium is not regionally optimal.*

Moreover, the presence of externalities makes the regionally efficient allocation not optimal from the social point of view. As we have seen earlier, the regional planner do not take into account the average payoff in the other region. The relative importance of the gap between regions is lower for the regional planners than for the social planner. The absolute value of the regional aversion to the regional dispersion does not coincide with the social aversion. The sign of aversion to the foreign dispersion may be opposite to the social one. All this implies that in general, the regional optimum is not socially efficient. The direct consequence of this is the possible inefficiency of information policies if they are developed regionally.

3.5.2 Regional optimum with incomplete information

The regionally efficient allocation under incomplete information with assumption that the regional planner cannot discriminate between agents is achieved with a pair of strategies $\left(\tilde{k}_j^j,\tilde{k}_{-j}^j\right)\colon\mathbb{R}^3\to\mathbb{R}^2$ such that

$$\left\{\tilde{k}_j^j\left(x^j,Z\right),\tilde{k}_{-j}^j\left(x^{-j},Z\right)\right\}=\arg\max_{k'(x,Z)}E\left[W^j\left(k\left(x,Z\right),K\left(\Theta,Z\right),\sigma_k\left(\Theta,Z\right),\Theta\right)\right], \tag{3.45}$$

First-order condition for this problem is similar to the first-order condition of the problem of social planner:

$$\tilde{k}_l^j\left(x^l, z^j, z^{-j}\right) = E\left[\tilde{\kappa}_l^j(\Theta) + \tilde{\alpha}_{l,j}\left(K^j(\Theta, Z) - \tilde{\kappa}_j^j(\Theta)\right)\middle| x^l, z^j, z^{-j}\right] \qquad (3.46)$$
$$+ E\left[\tilde{\alpha}_{l,-j}\left(K^{-j}(\Theta, Z) - \tilde{\kappa}_{-j}^j(\Theta)\right)\middle| x^l, z^j, z^{-j}\right] \; l \in \{j, -j\},$$

where $\tilde{\alpha}_{j,j} = n\frac{n(U_{\sigma\sigma} - U_{KK}) - 2U_{kK}}{nU_{\sigma\sigma} + U_{kk}}$ is the regionally optimal coordination inside region j and $\tilde{\alpha}_{j,-j} = (1-n)\frac{n(U_{\sigma\sigma} - U_{KK}) - U_{kK}}{nU_{\sigma\sigma} + U_{kk}}$ is regionally optimal coordination between the regions. This first-order condition gives the following regionally optimal linear strategy for agents in the home region of the regional planner:

$$\tilde{k}_j^j\left(x^j, z^j, z^{-j}\right) = \tilde{\kappa}_{j,j}\left(\tilde{\gamma}_j z^j + (1 - \tilde{\gamma}_j)\, x^j\right) \qquad (3.47)$$

where

$$\tilde{\gamma}_j = \delta^j + \frac{\delta^j\left(1 - \delta^j\right)\tilde{\alpha}_{j,j}}{1 - \left(1 - \delta^j\right)\tilde{\alpha}_{j,j}} + \frac{\left(1 - \delta^j\right)\tilde{\alpha}_{j,-j}}{1 - \left(1 - \delta^j\right)\tilde{\alpha}_{j,j}}\frac{\tilde{\kappa}_{-j,j}}{\tilde{\kappa}_{j,j}} \qquad (3.48)$$

Thus, in regionally efficient distribution the agents in the home region weight their private and public information about their home fundamentals and ignore the information about foreign shocks. The actions in the foreign region $-j$, if chosen by the planner in home region j, rely only on public information about the home region z^j:

$$\tilde{k}_{-j}^j\left(x^{-j}, z^j, z^{-j}\right) = \tilde{\kappa}_{-j,j} z^j \qquad (3.49)$$

These strategies are incompatible with neither the equilibrium nor the social optimum. Thus, the information policy may be inefficient if chosen by the local authority. For example, if $U_{\sigma\sigma} > 0$, the regional planner in region j would like to impose the variance to the private actions in region $-j$ and to stretch the gap between regions. In the equilibrium, the regional authority cannot influence the private dispersion abroad, as it is defined only by the information about the foreign regional fundamental, as expression (3.21) clarifies. Nevertheless, it can impose additional noise to its signal about the regional fundamental to stretch the gap given by (3.20). This can increase the gap between the equilibrium actions in foreign region and the optimal actions, but the planner does not take this external effect into account. Thus, it would choose excessively opaque policy without publishing precise information about its region fundamentals. We will discuss the difference between the local and global welfare criteria in the next section after discussing the regional loss functions.

3.5.3 Regional loss functions

Similar to the social welfare in the previous section, the expected regional welfare can be rewritten as follows:

$$EW^j = \omega^j\left(\tilde{\kappa}_1^j, \tilde{\kappa}_2^j, \theta^1, \theta^2\right) - \mathscr{L},$$

where $\omega^j\left(\tilde{\kappa}_1^j, \tilde{\kappa}_2^j, \theta^1, \theta^2\right)$ is the value of regional welfare under the regionally optimal distribution and $\mathscr{L}_j$ is the social loss which arises due to the gap between the equilibrium and the regional optimum. The value of this loss is as follows:

$$\mathscr{L}_j = n_j \frac{|W_{KK}|}{2} Var\left(K - \tilde{K}^j\right) + n_j\left(1 - n_j\right)|U_{kk}|\left(1 - \alpha\right) Cov\left(K - \tilde{K}; K_j - \tilde{\kappa}_j^j - \left(K_{-j} - \tilde{\kappa}_{-j}^j\right)\right) \tag{3.50}$$

$$-n\left(1 - n\right)\frac{\left(1 - n\right)U_{kk} + nU_{\sigma\sigma}}{2} Var\left(K_j - \tilde{\kappa}_j^j - \left(K_{-j} - \tilde{\kappa}_{-j}^j\right)\right) + n\frac{|U_{kk} + nU_{\sigma\sigma}|}{2}\sigma_{k,j}^2 - n\left(1 - n\right)\frac{U_{\sigma\sigma}}{2}\sigma_{k,-j}^2$$

Thus, equation (3.50) reveals the sources of inefficiency from the regional perspective. The first source of inefficiency is the gap between the equilibrium average actions and the regional efficient average actions $\tilde{K}^j$. The variance of this gap is denoted by $Var\left(K - \tilde{K}^j\right)$. The second source of regional inefficiency is the covariance between the average gap $K - \tilde{K}$ and the relative gap between regions $K_j - \tilde{\kappa}_j^j - \left(K_{-j} - \tilde{\kappa}_{-j}^j\right)$. The variance of this relative gap is the third source of inefficiency from the regional perspective. The last two sources of regional loss are the dispersion of private actions in both regions. The variance in the home private actions σ_j^2 increases the regional loss. The variance in the foreign private actions σ_{-j}^2 increases the regional loss only if the private value of dispersion is negative ($U_{\sigma\sigma} < 0$). If the private value of dispersion is positive ($U_{\sigma\sigma} > 0$), the regional loss depends negatively on the dispersion in private actions in the other region.

Similar to the previous section, the regional loss can be rewritten as a sum of component L_j^0, which is independent of information structure, and component L_j, which depends on the information structure:

$$\mathscr{L}_j = L_j^0 + L_j$$

As expression L_j is rather massive, it is given in Appendix C1. Due to the absence of correlation between the regional sources of information, the loss component L_j can be represented as a sum of two components, each of which depends on the information about one of the regions:

$$L_j = L_{j,j} + L_{j,-j},$$

where $L_{j,j}$ is the regional loss in region j, caused by the incompleteness of information about region j and $L_{j,-j}$ is the loss in region j, caused by the incompleteness of information about region $-j$. Thus, $L_{j,j}$ characterizes the regional value of information about region j, while value $L_{j,-j}$ characterizes the inter-regional value of information about region $-j$.

Obviously, the social loss L_S^j, which measures the loss in social welfare because of the incompleteness of information about fundamental θ^j, is the sum of the losses in two regions:

$$L_S^j = L_{j,j} + L_{-j,j}.$$

Component $L_{-j,j}$ measures the side-effects of the information about region j suffered by region $-j$. In the next section we study the regional and inter-regional value of information in the general model.

3.5.4 Regional and inter-regional value of information

The regional loss function (3.50) reveals the importance of parameter $U_{\sigma\sigma}$ for the regional value of public and private information. Thus, we start with the efficient economy without the externality created by the dispersion in private actions ($U_{\sigma\sigma} = 0$). After that we discuss the regional and inter-regional effects of information in economies, where the only source of inefficiency is the dispersion in private actions ($\kappa = \kappa^*$, $\alpha = \alpha^*$ and $U_{\sigma\sigma} \neq 0$). We conclude with the general model, where all sources of inefficiency may be present.

The regional and inter-regional welfare effects of information in globally efficient economies are presented by the following proposition:

Proposition 3.9. ***Regional and inter-regional value of information in globally efficient segmented economies.*** *In economies with $\kappa = \kappa^*$, $\alpha = \alpha^*$ and $U_{\sigma\sigma} = 0$ for given (α, n_j), there exist $\bar{\zeta}_{j,H} \geq 0$ and $\bar{\zeta}_{j,F} \geq 0$ such that*

1. *if $\alpha > 0$, the regional loss is decreasing in precision of both public and private home information;*

2. *if $\alpha < 0$, the regional loss is decreasing in precision of public home information, but is increasing in the precision of home private information, if $\sigma_{z,j}^{-2}/\sigma_{x,j}^{-2} < \bar{\zeta}_{j,H}(\alpha, n_j)$. If $1 - \alpha n_j (2n_j - 1) > 0$, threshold $\bar{\zeta}_{j,H}(\alpha, n_j) = 0$;*

3. *the regional loss is increasing in foreign private information precision, if $\alpha > 0$, and decreasing in foreign private information precision, if $\alpha < 0$;*

4. *if $\sigma_{z,j}^{-2}/\sigma_{x,j}^{-2} < \bar{\zeta}_{j,F}(\alpha, n_j)$, the regional loss is increasing in the precision of foreign public information, if $\alpha > 0$, and decreasing in the precision of foreign public information, if $\alpha < 0$. If $\alpha\left(1 - 2n_j + \alpha n_j^2\right) > 0$, threshold $\bar{\zeta}_{j,F}(\alpha, n_j) = 0$.*

Part 1 of Proposition 3.9 states that the regional value of both home private and public information is positive, if there is strategic complementarity in private actions. If we compare this result with the social value of information in Proposition 3.5, we will see that in this case the social value of information coincides with the regional value of information. This means that if the informational policy was delegated to the regional authority, it would be socially optimal. Such a social authority would try to achieve the highest possible precision of both public and private information. This is not the case in economies with strategic substitutability.

Part 2 of Proposition 3.9 indicates, that the regional value of private information may be negative. This happens, if the regional size is relatively small ($n_j < 1/2$), strategic substitutability is sufficiently strong ($\alpha < -n_j^{-1}(1-2n_j)^{-1}$) and the relative precision of public information is sufficiently low ($\sigma_{z,j}^{-2}/\sigma_{x,j}^{-2} < \bar{\zeta}_{j,H}(\alpha, n_j)$). The intuition is as follows. An increase in the precision of home private information forces private agents to increase the weight of this information in their actions. Together with the strong strategic substitutability, this may lead to an increase in the dispersion of private actions and the dispersion of the relative gap between regions, which is detrimental for the regional welfare, according to (3.50). The effect of the dispersion in the gap between regions depends negatively on the region size. This gives Part 2 of Proposition 3.9. The negative regional value of private information means that the local authority may choose globally inefficient information structure. For strong strategic substitutability, the local authority may have the incentive to restrict the possible precision of private information.

Part 3 and 4 of Proposition 3.9 describe the inter-regional value of information. The inter-regional value of information represents the effect of information about region j on the regional welfare of region $-j$. Proposition 3.9 states that the inter-regional value of information depends on the region size, the equilibrium degree of coordination and on the relative precision of public information $\sigma_{z,j}^{-2}/\sigma_{x,j}^{-2}$.

According to Part 3 of Proposition 3.9, the inter-regional value of private information is positive, if there is strategic substitutability. In this case the agents value the coordination negatively. Thus, an increase in the precision of private information leads to an increase in its weight in the home private actions. As a result, agents in region j rely less on their home public information and the coordination between regions becomes smaller, which increases the private payoffs in region $-j$. If there is strategic complementarity, private information has negative inter-regional value. As we have already seen, an increase in the precision of private information in region j lowers the inter-regional coordination, which is undesirable for the agents in the other region.

According to Part 4 of Proposition 3.9, an increase in the precision of public information about region j leads to an increase in the regional loss in region $-j$ in case of strong strategic complementarity, such that α is positive and higher than $\frac{2n-1}{n^2}$. This happens because an increase in precision of foreign public information pushes the private actions in region $-j$ away from the relevant fundamental shock. Thus, the inter-regional effect of higher precision of public information is negative. If strategic complementarity is not so strong and $0 < \alpha < \frac{2n_j-1}{n_j^2}$, the inter-regional loss may decrease in the precision of public information up to some limit. Thus, if the value of public information precision is limited for some technological reason, there may be positive inter-regional value. Worth to note that this is possible only if the size of region j is larger than $1/2$. Otherwise, the externality created by its information is not enough to reverse its effect on the other region.

In case of strong strategic substitutability, such that α is negative and lower than $\frac{2n-1}{n^2}$, the inter-regional loss is positive, meaning that there is a negative inter-regional externality. Nevertheless, an increase in the precision of public information in region j lowers the loss in region $-j$. More precise public information about the foreign fundamentals helps private agents to better predict the foreign actions and to keep their own actions away from coordination between regions. Thus, the inter-regional value of public information is positive. If strategic substitutability is modest and $\frac{2n_j-1}{n_j^2} < \alpha < 0$, inter-regional loss is non-monotonic function of the precision of public signal. For low relative precision of public information, its increase may have negative inter-regional value. Nevertheless, this phenomenon takes place only if the size of region j is lower than $1/2$.

The presence of externality created by the dispersion in private actions may change considerably the regional and inter-regional effects of information. These effects are summarized in the following proposition:

Proposition 3.10. ***Regional and inter-regional value of information in economies, which are efficient if population is not segmented and inefficient with segmented population.*** *In economies with $\kappa = \kappa^*$, $\alpha = \alpha^*$ and $U_{\sigma\sigma} \neq 0$, for given (α, n_j) there exist $\underline{\rho}_j < 0 < \bar{\rho}_j$, $\tilde{\rho}_j$ and $\bar{\zeta}_j, \bar{\zeta}_{j,H} \geq 0$ such that:*

1. *if $\alpha > 0$ and $\rho < \underline{\rho}_j(\alpha, n_j)$ or $\alpha < 0$ and $\rho > \bar{\rho}_j(\alpha, n_j)$, the regional loss is increasing in the precision of home public information for all $\sigma_{z,j}^{-2}/\sigma_{x,j}^{-2} < \bar{\zeta}_j(\alpha, n_j)$;*

2. *the regional loss is increasing in the precision of home private information, if $\sigma_{z,j}^{-2}/\sigma_{x,j}^{-2} < \bar{\zeta}_{j,H}(\alpha, n_j)$, $\alpha < 0$ and $\rho > \tilde{\tilde{\rho}}_j(\alpha, n_j)$;*

3. *the regional loss is increasing in precision of foreign public and private information, if $\rho > \tilde{\rho}_j(\alpha, n_j)$.*

Proposition 3.10 shows that the welfare properties of the dispersion in private actions changes the regional and inter-regional value of information. Part 1 of the proposition indicates that strategic complementarity accompanied with a large positive value of dispersion ($\rho < \underline{\rho}_j(\alpha, n_j) < 0$) makes the regional value of public information negative. If $U_{\sigma\sigma}$ is positive, the regional loss depends positively on the average gap in actions between the regions. Moreover, the negative impact of the home dispersion in actions on the regional welfare is not that large. Thus, the region may be better off with the smaller precision of home public information, despite the strategic complementarity in private actions. On the contrary, sufficiently large positive value of ρ is necessary for the regional value of public information to be negative, if there is strategic substitutability.

Part 2 of Proposition 3.10 shows that the regional value of home private information is basically the same as it is in efficient economies, described in Proposition 3.9. The regional value of home private information is still negative in economies with strong strategic substitutability, if ρ is not too small. The large negative value of ρ means the large positive value of dispersion inside and between regions, that would overcome the effect of strategic substitutability on the value of private information. Thus, the regional value of private information is necessarily positive for $\rho < \tilde{\rho}_j(\alpha, n_j) < 0$.

Part 3 of Proposition 3.10 demonstrates that the inter-regional value of both public and private information is negative, if the dispersion inside and between regions has sufficiently strong negative effect on the regional welfare. Almost all these results holds in economies with socially inefficient degree of coordination. Regional and inter-regional welfare properties of information in these economies are summarized in the following Proposition:

Proposition 3.11. ***Regional and inter-regional value of information in economies with inefficient degree of coordination.*** *For given (α, α^*, n_j), there exist $\underline{\hat{\rho}} < 0 < \bar{\hat{\rho}}$, $\hat{\hat{\rho}}$, $\hat{\psi} > 0$ and $\bar{\zeta}_j \geq 0$ such that:*

1. *if $\alpha > 0$ and $\rho < \underline{\hat{\rho}}(\alpha, \alpha^*, n_j)$ or $\alpha < 0$ and $\rho > \bar{\hat{\rho}}(\alpha, \alpha^*, n_j)$, the regional loss is increasing in precision of home public information for all $\sigma_{z,j}^{-2}/\sigma_{x,j}^{-2} < \bar{\zeta}_j$. Thresholds $\underline{\hat{\rho}}(\alpha, \alpha^*, n_j)$ and $\bar{\hat{\rho}}(\alpha, \alpha^*, n_j)$ depend positively on the gap $\alpha^* - \alpha$;*
2. *the regional loss is increasing in the precision of foreign private information, if $\alpha^* - \alpha < \hat{\psi}(\alpha, \alpha^*, n_j)$and $\rho > \hat{\hat{\rho}}(\alpha, \alpha^*, n_j)$, and increasing in precision of home private information, if $\alpha^* - \alpha > \hat{\psi}(\alpha, \alpha^*, n_j)$ and $\rho > \hat{\hat{\rho}}(\alpha, \alpha^*, n_j)$;*
3. *if $\rho > \hat{\hat{\rho}}(\alpha, \alpha^*, n_j)$, the regional loss is increasing in precision of foreign public information if $\alpha > 0$, $\alpha^* - \alpha < \hat{\psi}(\alpha, \alpha^*, n_j)$ and $\rho > \hat{\hat{\rho}}(\alpha, \alpha^*, n_j)$ or if $\alpha < 0$, $\alpha^* - \alpha > \hat{\psi}(\alpha, \alpha^*, n_j)$.*

Part 1 of Proposition 3.11 shows that the regional welfare properties of public information coincide with its properties in economies with efficient degree of coordination. Nevertheless, the positive gap between the efficient and the equilibrium degree of coordination enlarges the region of values of ρ, for which the regional value of public information is negative under strategic complementarity. At the same time, it shrinks the region of values of ρ, for which the regional value of public information is negative under strategic substitutability. Higher α^* means lower impact of the volatility of the gap between the average equilibrium and regional efficient actions in the regional loss (3.50). This makes public information less valuable, if there is strategic complementarity and more valuable, if there is strategic substitutability.

Part 2 of Proposition 3.11 demonstrates that the regional value of private information may be negative, even if there is strategic complementarity in private actions. Sufficient positive gap between the efficient and equilibrium degree of coordination ($\alpha^* - \alpha > \hat{\psi}(\alpha, \alpha^*, n_j)$) and sufficiently negative private value of dispersion ($\rho > \hat{\tilde{\rho}}(\alpha, \alpha^*, n_j)$) suffices for it. Part 3 of Proposition 3.11 demonstrates that relatively large gap between the efficient and equilibrium degree of coordination ($\alpha^* - \alpha > \hat{\psi}(\alpha, \alpha^*, n_j)$) suffices for foreign public information to be regionally desirable in case of strategic complementarity. The relatively large negative gap suffices for foreign public information to be regionally desirable in case of strategic substitutability.

In the next section we illustrate the social and regional welfare properties of information by a number of examples.

3.6 Applications

In this section we apply the social and regional welfare analysis to several examples. We start with two examples of the efficient economies. The first example illustrates the efficient competitive economy with strategic complementarity, the second refers to the efficient Lucas-Phelps island economy. After that we provide two examples of beauty-contest models with inefficient degree of coordination. One of this examples assumes that the dispersion in private action does not create externalities (Hellwig and Veldkamp (2009)), while the second implies the positive externality of the dispersion (Morris and Shin (2002) beauty contest).

3.6.1 Efficient economies

Efficient competitive economy

Two regions are inhabited by a continuum of households, which consume two goods. Initially, each household has an endowment w of good 2; good 1 should be produced. Each household is a producer and a consumer at the same time. Utility of agent i living in region j is given by the following function:

$$u_i^j = \nu\left(q_{1,i}^j\right) + q_{2,i}^j, \tag{3.51}$$

where $q_{1,i}^j$ and $q_{2,i}^j$ denote the consumed quantity of two goods, $\nu\left(q_{1,i}^j, \theta^j\right) = Aq_{1,i}^j - {}^b\!/\!_2\left(q_{1,i}^j\right)^2$, $b > 0$. Goods are sold in the common market at price p, which is the same for agents in both region.

The budget constraint for the household is

$$pq_{1,i}^j + q_{2,i}^j = w + \pi_i^j, \tag{3.52}$$

where price of good 2 is normalized to unity and π_i^j is the profit of agent i:

$$\pi_i^j = pk_i^j - C\left(k_i^j\right), \tag{3.53}$$

where k_i^j is the quantity of good 1 produced by agent i in region j and

$$C\left(k_i^j\right) = \frac{\left(k_i^j\right)^2}{2} - \theta^j k_i^j \tag{3.54}$$

is the cost of producing good 1. Parameter θ^j is region-specific technology shock. An increase in θ^j means that the first good becomes cheaper to produce for agents in region j. Maximization of utility (3.51) under budget constraint (3.52) gives the demand of the agent i, living in region j, for good 1:

$$q_{1,i}^j = \frac{A-p}{b} \tag{3.55}$$

As we can see from demand function (3.55), all agents consume the same quantity of good 1. The market-clearing condition is $bK = A - p$, where K is the total quantity of good 1, produced in the economy. This gives market-clearing price $p = A - bK$. The quantity of good 1 purchased by any agent at this price, is equal to K. The quantity of good 2 consumed by agent i is equal to $w + \pi_i^j - (A - bK)\, K$. As the profit of the household is equal to $\pi_i^j = pk_i^j - C\left(k_i^j\right) = (A - bK)\, k_i^j - \frac{\left(k_i^j\right)^2}{2} - \theta^j k_i^j$, we get the following utility of the household:

$$U\left(k_i^j, K, \sigma_k, \theta^j\right) = \left(A + \theta^j - bK\right) k_i^j - \frac{\left(k_i^j\right)^2}{2} + b\frac{K^2}{2} + AK + w \tag{3.56}$$

It is easy to show that $U_{kk} = -1$, $U_{kK} = -b$, $U_{KK} = b$, $U_{k\theta} = 1$, $U_{K\theta} = U_{\sigma\sigma} = 0$. This implies that $\kappa = \kappa^* = \frac{1}{1+b}$, $\alpha = \alpha^* = -b$ and $\rho = 0$. In other words, the example describes the efficient economy with strategic substitutability in private actions, as $b > 0$ by assumption.[2]

One-region version of a similar efficient economy is derived in Angeletos and Pavan (2007), who show that the social value of both public and private information is positive. The welfare properties of information in this two-region model are listed in the following Corollary:

Corollary 3.12. *In efficient competitive economy with strategic substitutability described here,*

1. *social value of public and private information, regional value of public information and inter-regional value of private information are positive;*

2. *regional value of private information may be negative if $n_j < \frac{1}{2}$ and $b > \frac{1}{n_j(2n_j-1)}$;*

[2] Note that private actions under complete information are given by $\kappa^j(\Theta) = \frac{A}{1+b} + (1 + b(1-n_j))\,\theta^j - b(1-n_j)\theta^{-j}$. The term $\frac{A}{1+b}$ in this expression arises because $U_k(0,0,0,0) = \frac{A}{1+b} \neq 0$. This does not change the results.

3. *inter-regional value of public information about cost shock in region j may be negative if $n_j < 1/2$ and $b < \frac{1-2n_j}{n_j^2}$.*

These findings are in line with Propositions 3.9 and 3.5. The regional value of private home information may be negative in small regions with strong strategic substitutability. On the contrary, the inter-regional value of public information may be negative for weak degree of strategic substitutability.

Lucas-Phelps island economy

Myatt and Wallace (2014) show that the preference of an agent in a Lucas-Phelps island economy can be described by the following utility function:

$$U\left(k_i^j, K, \sigma_k, \theta^j\right) = \overline{u} - r\left(k_i^j - K\right)^2 - (1-r)\left(k_i^j - \theta^j\right)^2 \tag{3.57}$$

This function is also used, for example, in Baeriswyl and Cornand (2014) and Myatt and Wallace (2011).

In this economy $U_{kk} = -2$, $U_{kK} = 2r$, $U_{KK} = -2r$, $U_{k\theta} = 2(1-r)$, $U_{K\theta} = U_{\sigma\sigma} = 0$. This implies that $\kappa = \kappa^* = 1$, $\alpha = \alpha^* = r > 0$ and $\rho = 0$. Thus, this economy is efficient and is characterized by strategic complementarity. The findings about social, regional and inter-regional value of information in this economy are as follows:

Corollary 3.13. *In efficient competitive economy with strategic complementarity described here,*

1. *the social and regional value of public and private information is positive, while inter-regional value of private information is negative;*

2. *the inter-regional value of public information about cost shock in region j may be negative if $n_j > 1/2$ and $r < \frac{2n-1}{n^2}$.*

As we can see, both regional and social value of information is positive. This means that social welfare is increasing in the precisions of both private and public information and that the information policy of a local authority would be socially efficient. Nevertheless, the inter-regional value of private information is negative. Private information is available only to the agents in the home region. This informational asymmetry prevents the foreign agents from the efficient inter-region coordination. An increase in the precision of private information increases this asymmetry and creates a negative inter-regional externality. Moreover, public information about the larger region also creates a negative inter-regional effect, if the extent of strategic complementarity is relatively low.

3.6.2 Inefficient degree of coordination

Hellwig and Veldkamp (2009) study the model of price-setters with the following utility function:

$$U\left(k_i^j, K, \sigma_k, \theta^j\right) = -\left(k_i^j - rK - (1-r)\,\theta^j\right),^2 \tag{3.58}$$

where k_i^j is a price of agent i, K is the average price in the economy, θ^j is a shock to the optimal price level. In this economy $U_{kk} = -2$, $U_{kK} = 2r$, $U_{KK} = -2r^2$, $U_{k\theta} = 2(1-r)$, $U_{K\theta} = -2(1-r)$, $U_{\sigma\sigma} = 0$. This implies that $\kappa = \kappa^* = 1$, equilibrium degree of coordination is equal to $\alpha = r$ and efficient degree of coordination is equal to $\alpha^* = r(2-r)$. If $r > 0$, this model is characterized by strategic complementarity and efficient degree of coordination exceeds the equilibrium degree, meaning that $\alpha^* > \alpha > 0$. If $r < 0$, this model is characterized by strategic substitutability and efficient degree of coordination is lower than the equilibrium degree, meaning that $\alpha^* < \alpha < 0$.

The welfare properties of information are as follows:

Corollary 3.14. *In the economy described here,*

1. *if $r > 0$,*
   a) *social, regional and inter-regional value of public information is positive, while inter-regional value of private information is negative;*
   b) *regional and social values of private information in region j may be negative, if $rn_j > 1/2$;*
2. *if $r < 0$, there exists $\overline{r} < 0$ such that*
   a) *the social value of private information is positive if $r > \overline{r}$ and may be negative, if $r < \overline{r}$;*
   b) *social value of public information may be negative if $rn_j < -1$;*
   c) *the regional value of public and private information, inter-regional value of public information is positive, while inter-regional value of private information is negative.*

Part 1 of Corollary 3.14 indicates the role of information in the economy with strategic complementarity. The social value of public information is positive, which coincides with a one-region version of the model, as $\alpha^* > \alpha > 0$ suffices for that. The segmentation of the economy enlarges the set of value r, for which an increase in the precision of private information may be socially undesirable. In a one-region economy the social value of private information may be negative, if $r > 1/2$. In a two-region economy it may be negative, if $rn_j > 1/2$. In a two-region economy an increase in the precision of private information may be undesirable not only because it prevents agents inside the region from coordination, but also it disturbs the coordination between regions. Thus, in two-region economy private information is more likely to have a negative social value.

Part 2 of the Corollary 3.14 shows the value of information in the economy with strategic substitutability. In a one-region version of this economy, the social value of private information is necessarily positive, $\alpha^* < \alpha < 0$. In a two-region economy this is true only if the extent of substitutability is relatively low. This finding confirms the result listed in Part 2 of Proposition 3.5. In this economy the gap between the equilibrium and efficient degree of coordination is equal to $\alpha - \alpha^* = -r(1-r)$. This value depends negatively on r. Larger strategic substitutability means lower value of r and larger gap $\alpha - \alpha^*$. As we have seen in Proposition 3.5, the social value of private information is necessarily positive only if the gap between the equilibrium and efficient degree of coordination is not too large. Moreover, the social value of public information may also be negative for sufficiently large gap between the equilibrium and efficient degree of coordination.

3.6.3 Externality created by the dispersion in private actions

In a beauty-contest economy, described in Morris and Shin (2002), loss of a private agent is given by:

$$l_i = (1-r)(k-\theta_j)^2 + r(L_i - \bar{L}), \tag{3.59}$$

where $L_i = \int_0^1 (k_g - k_i)^2 \, \mathrm{d}g$ represents the average distance between the action of the agent and the actions of all other private agents; $\bar{L} = \int_0^1 L_g \, \mathrm{d}g$. This loss function is equivalent to the following utility function:

$$U\left(k_i^j, K, \sigma_k, \theta^j\right) = -(1-r)\left(k_i^j - \theta^j\right)^2 - r\left(k_i^j - K\right)^2 + r\sigma_k^2$$

Thus, $U_{kk} = -2$, $U_{kK} = 2r$, $U_{KK} = -2r$, $U_{\sigma\sigma} = 2r$, $U_{K\theta} = 0$, $U_{k\theta} = 2(1-r)$, $W_{KK} = -2(1-r)$, $W_{\sigma\sigma} = -2(1-r)$, $\rho = -\frac{r}{(1-r)} < 0$. This means that $\kappa = \kappa^* = 1$ and $\alpha = r > \alpha^* = 0$. This economy is characterized by two sources of inefficiency. First of all, the equilibrium degree of coordination is too large in comparison with the efficient degree of coordination. Secondly, the dispersion creates the positive externality for private agent, meaning that $U_{\sigma\sigma} > 0$ and $\rho < 0$. This source of inefficiency leads to a distortion in private actions under complete information. The equilibrium weight of the home fundamental in private actions is equal to $\kappa_{j,j} = 1 - r(1-n_j)$, while the efficient weight of the home fundamental is equal to $\kappa_{j,j}^* = 1$. Thus, the equilibrium weight of the home fundamental is lower than its efficient weight. The equilibrium weight of the foreign fundamental under complete information is equal to $\kappa_{j,-j} = r(1-n_j)$, while its efficient weight is equal to $\kappa_{j,-j}^* = 0$. Thus, the equilibrium weight of the foreign fundamental is higher than its efficient weight. Moreover, the socially optimal degree of coordination inside the region is equal to $\alpha_{j,j}^* = (1-n_j)\frac{r}{1-r}$, while the socially optimal degree of coordination between the regions is equal to $\alpha_{j,j}^* = -(1-n_j)\frac{r}{1-r}$. This means that some positive coordination inside the regions and the negative degree of coordination between regions are socially desirable. To understand this fact, let us remind that the private value of dispersion and the social value of the gap between regions is positive, as $U_{\sigma\sigma} > 0$. Thus, the negative coordination between regions and increased coordination inside them lead to a higher gap, which is socially desirable. This is different from a one-region version of the model, for which the efficient extent of coordination is equal to zero.

It is easy to show that in one-region version the social loss is decreasing in the precision of public information, if $r < 1/2$, and may be decreasing in the precision of public information, if $r > 1/2$. This means that the social value of public information may be negative only if the extent of strategic complementarity is relatively high. The social value of private information is always positive in a one-region version. The welfare properties of information in a two-region version of the model are listed in the following Corollary:

Corollary 3.15. *For the two-region beauty contest model of Morris and Shin (2002),*

1. *the social and the regional value of private information is positive for any* (r, n)*;*

2. *for any* n*, there exists* $\overline{r}_S \in (0,1)$ *such that the social value of public information is positive if* $r \geq \overline{r}_S(n)$ *and may be negative, if* $r < \overline{r}_S(n)$*. Moreover,* $\overline{r}_S(n) \geq 1/2$ *and* $\frac{\partial \overline{r}_S(n)}{\partial n} > 0$*;*

3. *if* $n \leq 1/2$*, the regional value of public information is positive. If* $n > 1/2$*, there exists* $\bar{r}_j \in (0,1)$ *such that the regional value of public information is positive if* $r \geq \bar{r}_j(n)$ *and may be negative, if* $r < \bar{r}_j(n)$*;*

4. *the inter-regional value of private information may be negative, if* $n_j > \frac{1}{4-r}$*; the inter-regional value of public information may be positive, if* $n_j < \frac{1-\alpha}{2-\alpha}$*.*

Part 1 of Corollary 3.15 indicates that the social value of private information in a two-region economy is positive, as it is in a one-region model. Part 2 shows that the social value of public information may be of negative social value, if r is sufficiently small. This contradicts to a one-region model, when the social value of public information may be negative for relatively high values of r. This distinction comes from the fact, that in two-region version there are two sources of inefficiency. An increase in r means not only an increase in the equilibrium degree of coordination, but also an increase in discrepancy created by the cross-sectional dispersion. As $\frac{\partial(\alpha^* - \alpha)}{\partial r} < 0$ and $\frac{\partial \underline{\rho}}{\partial(\alpha^* - \alpha)} > 0$, an increase in r makes it more likely that the externality becomes too low to get the negative social value of public information. On the contrary, a decrease in r leads to an increase in threshold $\underline{\rho}$ in Proposition 3.7, making the social value of information negative. Part 3 of Corollary 3.15 shows, that the regional value of public information is positive, if $n \leq 1/2$. This implies that the authority in a small region would overestimate the value of public information. Thus, the informational policy of local authorities in this economy may be too transparent from the social point of view.

3.7 Conclusion

In this paper we study social, regional and inter-regional value of information in segmented economies. We show that the externalities, which arise due to strategic and informational spillovers between regions, change considerably the welfare properties of information. For example, in economies which are efficient in a one-region model, the social value of public information may be negative in a two-region model, if the agents value dispersion in private actions.

This finding gives rise to two concerns about information policy elaboration. The first concern is about using representative-agent models. We show that the policy, elaborated on a base of such models, may be inefficient if the economy is segmented in reality. The second concern is about potential inefficiency of information policies, if they are elaborated by the local authorities. As the regional and the social values of information can differ, the regional authority may choose the policy, which is either too transparent or too opaque from the social perspective. We apply this methodology to several examples, which illustrate these issues.

The methodology can be further developed. First of all, in the current version we assume that strategic effects have global character. Distinction between local and inter-regional strategic effects may be an interesting extension of the model. Moreover, the fundamental shocks in our model are uncorrelated. Nevertheless, in reality all the economies are interconnected. The study of technological spillovers between regions would be another extension of the model.

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Appendix

C.1 Proof of Proposition 3.4

If the equilibrium is globally efficient for any information structure, it should be efficient under complete information, implying that $\kappa_{j,k} = \kappa^*_{j,k}$ with $j,k \in \{1,2\}$. Moreover, the coordination degrees in equilibrium with incomplete information should coincide with the coordination degrees in the globally optimal distribution, implying that $\alpha_{j,k} = \alpha^*_{j,k}$ with $j,k \in \{1,2\}$.

The gap between the equilibrium and the globally efficient degrees of regional coordination is as follows:

$$\alpha_{j,j} - \alpha^*_{j,j} = (\alpha - \alpha^*)\, n_j - \rho\,(1 - n_j) \tag{3.60}$$

The gap between the equilibrium and the globally efficient degrees of inter-regional coordination is as follows:

$$\alpha_{j,j} - \alpha^*_{j,j} = (\alpha - \alpha^*)\,(1 - n_j) + \rho\,(1 - n_j) \tag{3.61}$$

From (3.60) and (3.61), it is obvious, that both gaps are equal to zero if and only if $\alpha = \alpha^*$and $\rho = 0$. Analogically, the gap between the equilibrium and the globally efficient local distribution is as follows:

$$\kappa_{j,j} - \kappa^*_{j,j} = (\kappa - \kappa^*)\, n_j + \kappa\rho\,(1 - n_j)\,(1 - \alpha) \tag{3.62}$$

The gap between the equilibrium and the globally efficient inter-regional distribution is as follows:

$$\kappa_{j,j} - \kappa^*_{j,j} = (\kappa - \kappa^*)\, n_j + \kappa\rho\,(1 - n_j)\,(1 - \alpha) \tag{3.63}$$

For both (3.62) and (3.63) to be equal to zero, two conditions must held: $\kappa = \kappa^*$and $\rho = 0$. From these two findings, Proposition 3.4 comes immediately.

C.2 Proof of Proposition 3.5

Part 1 The global social loss in efficient economies with $\alpha = \alpha^*$, $\kappa = \kappa^*$ and $\rho = 0$ is equal to the following:

$\mathbf{L}_{s,1} = \frac{n_j(1-\alpha)\left(\sigma_z^{-2}(1-\alpha(1-n_j))-\sigma_x^{-2}\alpha^2 n(1-n)\right)}{\sigma_z^{-2}\left(\sigma_z^{-2}+(1-\alpha n_j)\sigma_x^{-2}\right)}(3.64)$

The function (3.64) is decreasing in both precisions: $\frac{\partial L_{s,1}}{\partial \sigma_z^{-2}} < 0$ and $\frac{\partial L_{s,1}}{\partial \sigma_x^{-2}} < 0$. Part 1 of Proposition 3.5 comes immediately.

Part 2 The global social loss in economies with inefficient degree of coordination ($\alpha \neq \alpha^*$, $\kappa = \kappa^*$ and $\rho = 0$) is as follows:

$\mathbf{L}_{s,2} = L_{s,1} + n_j^2\left(\alpha^* - \alpha\right)\hat{L}_{s,2}(3.65)$

$_{s,2} = \frac{\left(\sigma_z^{-2}+\sigma_x^{-2}\alpha(1-n)\right)}{\sigma_z^{-2}\left(\sigma_z^{-2}+(1-\alpha n_j)\sigma_x^{-2}\right)^2}$

Term $\hat{L}_{s,2}$ is decreasing in the precision of public information, if $\alpha > 0$ and my be increasing, if $\alpha < 0$. Thus, condition $\alpha^* > \alpha > 0$ suffices for the positive value of public information. The derivative of this term over the precision of private information:

$$\frac{\partial \hat{L}_{s,2}}{\partial \sigma_x^{-2}} = \frac{2\left(\sigma_z^{-2} + \sigma_x^{-2}\alpha\left(1-n\right)\right)}{\sigma_z^{-2}\left(\sigma_z^{-2} + \left(1 - \alpha n_j\right)\sigma_x^{-2}\right)^3} \tag{3.66}$$

The numerator is negative if $\alpha < \frac{\sigma_z^{-2}}{\sigma_x^{-2}(1-n)}$, thus condition $\alpha^* < \alpha < 0$ is not sufficient for the global value of private information to be positive. Taking derivative of (3.65) over the precision of private information in this case, we get that the loss is decreasing over the precision if $\alpha^* > \alpha - \frac{(1-\alpha)(1-\alpha n)}{2\alpha n(1-n)}$. This gives Part 2 of the proposition.

Part 2 The global social loss in inefficient economies ($\alpha \neq \alpha^*$, $\kappa \neq \kappa^*$ and $\rho = 0$) is as follows:

$\mathbf{L}_{s,3} = L_{s,2} + 2n_j^2\hat{L}_{s,3}\frac{\kappa^*-\kappa}{\kappa}(3.67)$

$_{s,3} = \frac{(1-\alpha^*)\left(\sigma_z^{-2}+\sigma_x^{-2}\alpha(1-n)\right)}{\sigma_z^{-2}\left(\sigma_z^{-2}+(1-\alpha n_j)\sigma_x^{-2}\right)}$

Term $\hat{L}_{s,3}$ is decreasing in the precision of private information:

$$\frac{\partial \hat{L}_{s,3}}{\partial \sigma_x^{-2}} = -\frac{\left(1-\alpha^*\right)\left(1-\alpha\right)}{\sigma_z^{-2}\left(\sigma_z^{-2} + \left(1-\alpha n_j\right)\sigma_x^{-2}\right)^2} < 0$$

Thus, for any strategic effect, sufficiently high level of gap $\frac{\kappa^*-\kappa}{\kappa}$ guarantees the positive value of private information.

The derivative of term $\hat{L}_{s,3}$ over the precision of public information:

$$\frac{\partial \hat{L}_{s,3}}{\partial \sigma_z^{-2}} = -\frac{(1-\alpha^*)\left(\zeta^2 + 2\alpha(1-n)\zeta + \alpha(1-n)(1-\alpha n)\right)}{\left(\sigma_z^{-2}\right)^2\left(\sigma_z^{-2} + (1-\alpha n_j)\,\sigma_x^{-2}\right)^2}, \tag{3.68}$$

where $\zeta = \frac{\sigma_z^{-2}}{\sigma_x^{-2}}$. As we can see, expression (3.68) is negative, if $\alpha > 0$, and may be positive, if $\alpha < 0$. The Part 3 of Proposition 3.5 comes immediately.

C.3 Proof of Proposition 3.7

Part 1 The effect of externality on the marginal loss of the public information is given by the following derivative:

$$\frac{\partial^2 L_s}{\partial \sigma_z^{-2}\partial \rho} = \frac{2n(1-\alpha)^2(1-n)\left(\zeta^2 - 2\alpha n\zeta - \alpha n(1-\alpha n)\right)}{(\zeta)^2(\zeta+1-\alpha n)^2\left(\sigma_x^{-2}\right)^2} \tag{3.69}$$

In case of strategic complementarity, expression 3.69 is negative for small values of the relative precision of public information ζ. It means that a decrease in ρ to a sufficiently large negative values would lead to a negative social value of public information. In case of strategic substitutability, expression 3.69 is positive for all values of the relative precision of public information ζ. It means that sufficiently high positive value of ρ is sufficient for the negative value of public information.

Part 2 The effect of externality on the marginal loss of the private information is given by the following derivative:

$$\frac{\partial^2 L_s}{\partial \sigma_x^{-2}\partial \rho} = \frac{2n(1-\alpha)^2(1-n)}{(\zeta)^2(\zeta+1-\alpha n)^2\left(\sigma_x^{-2}\right)^2} \tag{3.70}$$

Expression 3.70 is positive, meaning that sufficiently high value of ρ ensures the socially negative value of private information.

Part 3 The effect of the gap between efficient and equilibrium distributions under complete information of the marginal losses is as follows:

$$\frac{\partial^2 L_s}{\partial \sigma_z^{-2}\partial(\kappa^*-\kappa)} = -\frac{2n^2(1-\alpha^*)\left(\zeta^2 + 2\alpha(1-n)\zeta + \alpha(1-n)(1-\alpha n)\right)}{(\zeta)^2(\zeta+1-\alpha n)^2\left(\sigma_x^{-2}\right)^2} \tag{3.71}$$

In case of strategic complementarity, this expression is negative. Using the implicit function theorem:

$$\frac{\partial \underline{\rho}}{\partial(\kappa^*-\kappa)} = -\frac{\left.\frac{\partial^2 L_s}{\partial \sigma_z^{-2}\partial(\kappa^*-\kappa)}\right|_{\rho=\underline{\rho}}}{\left.\frac{\partial^2 L_s}{\partial \sigma_z^{-2}\partial \rho}\right|_{\rho=\underline{\rho}}}, \tag{3.72}$$

where both numerator and denominator are negative, implying that $\frac{\partial \underline{\rho}}{\partial(\kappa^*-\kappa)} < 0$. Analogically, $\frac{\partial \overline{\rho}}{\partial(\kappa^*-\kappa)} < 0$.

C.4 Proof of Proposition 3.9

The proof coincides with the proof of Propositions 3.5 and 3.7, applied for the regional loss component:

$$L_j = n_j \frac{|W_{KK}|}{2} \left[Var\left(K - \bar{\kappa}\right) + 2Cov\left(K - \bar{\kappa}; \bar{\kappa} - \bar{\kappa}^*\right) + 2Cov\left(K - \bar{\kappa}; \bar{\kappa}^* - \tilde{\kappa}^j\right)\right] + \tag{3.73}$$
$$+ n_j\left(1 - n_j\right)|U_{kk}|\left(1 - \alpha\right)\left[Cov\left(K - \bar{\kappa}; K_j - \kappa_j - \left(K_{-j} - \kappa_{-j}\right)\right) + Cov\left(K - \bar{\kappa}; \kappa_j - \kappa_j^* - \left(\kappa_{-j} - \kappa_{-j}^*\right)\right)\right]$$
$$+ n_j\left(1 - n_j\right)|U_{kk}|\left(1 - \alpha\right)\left[Cov\left(K - \bar{\kappa}; \kappa_j^* - \tilde{\kappa}_j^j - \left(\kappa_{-j}^* - \tilde{\kappa}_{-j}^j\right)\right)\right] +$$
$$+ n_j\left(1 - n_j\right)|U_{kk}|\left(1 - \alpha\right)\left[Cov\left(\bar{\kappa} - \bar{\kappa}^*; K_j - \kappa_j - \left(K_{-j} - \kappa_{-j}\right)\right) + Cov\left(\bar{\kappa}^* - \tilde{\kappa}^j; K_j - \kappa_j - \left(K_{-j} - \kappa_{-j}\right)\right)\right]$$
$$+ n\left(1 - n\right)\frac{|U_{kk} + nU_{\sigma\sigma}|}{2}\left[Var\left(K_1 - \kappa_1 - \left(K_2 - \kappa_2\right)\right) + 2Cov\left(K_1 - \kappa_1 - \left(K_2 - \kappa_2\right); \kappa_1 - \kappa_1^* - \left(\kappa_2 - \kappa_2^*\right)\right)\right]$$
$$+ n\left(1 - n\right)\frac{|U_{kk} + nU_{\sigma\sigma}|}{2}\left[2Cov\left(K_1 - \kappa_1 - \left(K_2 - \kappa_2\right); \kappa_1^* - \tilde{\kappa}_1^1 - \left(\kappa_2^* - \tilde{\kappa}_2^1\right)\right)\right]$$
$$+ n\left(1 - n\right)\frac{|U_{kk} + nU_{\sigma\sigma}|}{2}\sigma_{k,j}^2 - n\left(1 - n\right)\frac{U_{\sigma\sigma}}{2}\sigma_{k,-j}^2$$

Substituting here the corresponding variances from the main text with $\kappa = \kappa^*$, $\alpha = \alpha^*$ and taking the derivatives gives Propositions 3.9, 3.10 and 3.11.

Chapter 4

Public Communication Policies in an International Economy: What Should Policymakers Reveal?

Abstract

We study non-cooperative communication games being played by policymakers in an international economy. Each policymaker receives signals on the real idiosyncratic shocks which affect the country economies. It has the choice of revealing or not the received signals. The model is characterized by a beauty-contest argument in the utility function and cross-border real spillovers. The non-cooperative equilibrium is never characterized by no revelation. A full transparency outcome may be the equilibrium outcome and is then Pareto-optimal. From a normative point of view, no revelation may be Pareto-optimal: the social value of public information may be negative in international economies as well as in closed economies. Partial revelation schemes are possible outcomes but never Pareto-optimal[1].

JEL Codes : D82, E61

Keywords : communication policies, beauty contest, public information

4.1 Introduction

The striking result of Morris and Shin (2002) implies that public information may cause excessive volatility in a beauty-contest economy. For this reason, transparency may be detrimental in economies with high extent of strategic complementarity. This result conflicted with the existed

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consensus among the academicians and practitioners about the benefits of transparency and attracted a lot of attention.

The extensive debates about the social value of public and private information in beauty-contest economies, provoked by the Morris and Shin paper, still have not ceased. Svensson (2006) questions the main conclusion of Morris and Shin (2002) and claims that this result can only be achieved under unrealistic assumptions about the quality of public information. James and Lawler (2011) debate the criticism of Svensson (2006) and find that transparency is always detrimental in a beauty-contest model if the policymaker governs the economy with both public signals and standard policy instruments. Angeletos and Pavan (2004) agree that transparency may lower social welfare in environments with strong strategic complementarity, which may lead to multiple equilibria. Hellwig (2005) and Roca (2010) study the welfare effects of public information in models with imperfectly-informed monopolistically competitive firms and claim that public information is always welfare-improving. Nevertheless, Walsh (2013) shows that transparency may be detrimental in a New-Keynesian model with aggregate supply and demand shocks, while Myatt and Wallace (2008) argue that neither transparency nor opacity are optimal in a world without purely public signals. Angeletos and Pavan (2007) shed some light on the origins of these debates. In a general linear-quadratic framework, they explore a useful classification of economies and summarize conditions under which transparency can be detrimental.

Despite of this diversity of the views, all these papers are focused on the role of information in closed economies. In these economies, private payoffs are determined by the fundamentals and the strategic coordination inside the economy, without any recourse to the foreign sector. In reality, many markets with strategic complementarity in private actions are nowadays international, e.g. international financial markets. In these markets, investors try to guess not only their home fundamental factors and the actions of their neighbors, but also the fundamentals and the actions of foreign investors. In such circumstances, it is not surprising that the public information signals affect the actions of investors in other countries. There is the growing evidence that private actions respond to foreign signals. A number of studies reveal a significant impact of the US news on foreign financial markets (see Kim and Sheen (2000) for Australian markets, Bredin, Gavin and O'Reilly (2005) for Irish markets, Hausman and Wongswan (2011) for 49 different countries). Ehrmann and Fratzscher (2005) investigate spillovers between the European Union and the US and find that macroeconomic news affects financial markets both domestically and abroad. Büttner, Hayo and Neuenkirch (2012) and Hanousek, Kočenda and Kutan (2009) find a significant effect of European and the US macroeconomic news on financial markets in the Czech Republic, Hungary, and Poland.

The contribution of our paper is two-fold. First of all, we explore the social value of information in a two-region beauty-contest model, which captures the three important spillover channels between countries on international financial markets. The first channel is the technological spillover between countries. This spillover leads to a positive correlation between the shocks which hit the countries. The second channel is the informational spillover, caused by the publication of relevant economic information by the policymakers in both regions. As far as these signals are public, they are equally observed by private agents in both economies. The third spillover channel is the international strategic complementarity. As all the agents act on the international financial market, they have the incentive to copy the actions of other agents not only in their home region, but also abroad. Thus, our model can be seen as a model of international beauty contest. To the best of our knowledge, our study is the first attempt to derive the welfare properties of public information in a model, which captures the three spillover channels in financial markets. In some sense, the two-country model of Arato and Nakamura (2013) is close to ours, as they also analyze the informational spillover effects in a beauty-contest economy. Nevertheless, the model of Arato and Nakamura (2013) does not allow for neither technological nor strategic spillovers between regions.

The second contribution of our paper is that we study endogenous international information structure, which is defined in a non-cooperative game of two policymakers. Each of these policymakers tries to maximize the welfare of its own region. At the first stage of the game the policymakers simultaneously decide on their revelation policy. This revelation policy may be either full revelation of all the signals received, either revelation of one of them, or no revelation at all. After committing to the chosen revelation strategy, each policymaker in our model receives two signals about the two country-specific fundamentals. Thus, the policymaker chooses its revelation strategy before knowing the exact values of its signals. If the policymaker decides to reveal, it publishes all it knows about the specific fundamental shock. If it decides not to reveal, it does not publish any signal. Partial revelation refers to the situation, when the policymaker publishes only one of its signals. In some sense, our notion of partial revelation stays in between the notion of partial publicity (Cornand and Heinemann (2008), Baeriswyl and Cornand (2014), Myatt and Wallace (2014)), when only a fraction of agents receives the public signal, and partial transparency (Heinemann and Illing (2002)), which implies that all the agents receive an ambiguous public signal. In our model, partial revelation refers to the situation, when a policymaker publishes the part of its information. Thus, this signal is equally observed by all the agents in the economy and it does not contain any additional noise. Nevertheless, this signal does not contain all information available to the policymaker.

We do not discus the cheating equilibria when the policymaker publishes biased signals, which differs our paper from the literature on creative accounting (Bernoth and Wolff (2008)), strategic forecasting by central banks (Tillmann (2011); Gomez-Barrero and Parra-Polania (2014)) and the studies of regime change with information manipulation (Edmond (2013)). Moreover, we do not look for cheap-talk equilibria, which are studied, for example, in Moscarini (2007).

The endogeneity of international information structure in our model comes from the informational game between the public authorities in both regions. This source complements the existing literature, which also study the endogenous information structures. Usually, this literature links the endogeneity of informational structure to the informational acquisition of private agents (e.g. Colombo and Femminis (2008), Hellwig and Veldkamp (2009), Van Nieuwerburgh and Veldkamp (2009) and Colombo, Femminis and Pavan (2014)), learning from prices by private agents (Timmermann (1993), Banerjee (2011)) or by the central bank (e.g. Morris and Shin (2005), Bond, Goldstein and Prescott (2009), Bond and Goldstein (2015), Boleslavsky, Kelly and Taylor (2017)). Thus, we propose the new reason for endogeneity of informational structure.

The results of our study are as follows. We show that characteristics of the non-cooperative Nash equilibrium of the game played by the policymakers depend on the extent of technological spillover. If technological spillover is sufficiently weak, both policymakers are home transparent and foreign opaque, revealing their information about their home fundamentals and hiding their information about the foreign economy. When the technological spillover is weak, the efficient private actions are more linked to the home fundamental shocks, than to the foreign. Nevertheless, strategic inter-regional complementarity forces private agents to put the inefficiently high weight to the public information about the foreign shocks. Thus, the policymaker withdraws its information about the foreign shock in order to prevent the private agents from the inefficient inter-regional coordination. On the contrary, provision of the information about the home shocks is welfare-improving, as it keeps private actions closer to the relevant fundamental. Thus, the policymaker chooses home transparency. The opposite logic is true, when the technological spillover is extremely strong. In this case the equilibrium is characterized by home opacity and foreign transparency. In this equilibrium, each policymaker reveals all it knows about the foreign fundamental shock and is silent about its home fundamentals. For intermediate extents of technological spillover, the two opposing effects balance each other and there is full transparency in equilibrium. In this equilibrium policymakers reveal all their information about both economies. The equilibrium with full opacity is not possible in the studied framework.

The analysis of welfare properties of the equilibrium shows that partial revelation is never socially desirable. The social optimum is characterized by either full transparency or full opacity, meaning that the social value of public information may be negative. Full opacity is optimal only if the technological spillover is very weak. In this case both regions are close to autarky, characterized by an extensive degree of equilibrium coordination. In this economy provision of public information may be detrimental, if its quality is bad. Moreover, we show that the full revelation equilibrium is Pareto-optimal, while home opacity equilibrium is always dominated by full transparency; foreign opacity equilibrium may be dominated by full opacity. This means that there may be too much or too little public information in equilibrium, depending on the strength of technological and strategic spillovers.

The rest of the papers is organized as follows. The next Section provides the full description of the framework. Section 4.3 discusses the private game and the non-cooperative policy game, while the equilibrium is given in Section 4.4. The welfare properties of the equilibrium are studied in Section 4.5, while Section 4.6 concludes. All proofs are left for the Appendix D.

4.2 Set-up

4.2.1 The model

The economy consists of two interconnected countries, indexed by $j \in \{1, 2\}$. The economy is populated by a unit mass of private agents, which are indexed by i. Without loss of generality, we assume that agents with $i \in [0, 1/2]$ live in country $j = 1$, while agents with $i \in (1/2, 1]$ live in country $j = 2$. Thus, the countries have equal sizes n^j: $n^1 = n^2 = 1/2$.

Country j is hit by a fundamental shock Θ^j:

$$\begin{aligned} \Theta^j &= \phi\theta^j + (1-\phi)\,\theta^{-j} \\ \theta^j &\sim N\left(\mu, \sigma_\theta^2\right) \end{aligned} \tag{4.1}$$

where θ^j is a regional idiosyncratic shock of country j with mean μ and variance σ_θ^2. In what follows, we assume that μ is equal to zero. This assumption does not affect the results about the value of public information, but simplifies considerably the algebra. Parameter ϕ in equation (4.1) characterizes cross-border fundamental spillover. If $\phi = 1$, there is no cross-border real spillover and the fundamentals of country j are defined only by the country-specific shock θ^j. This case corresponds to the most of the literature on the social value of public information cited before. If $\phi = 1/2$, there is perfect correlation between the fundamentals of both countries. In this case both economies are described by the same shock, equal to the average of two country-specific shocks. If $\phi = 0$, the fundamentals of country j are defined totally by the regional shock in country $-j$.

The true values of regional fundamental shocks are not known by the agents. Nevertheless, each private agent i in country j receives a private signal x_i^j on his home regional fundamental θ^j:

$$\begin{aligned} x_i^j &= \theta^j + \varepsilon_i^j \\ \varepsilon_i &\sim i.i.d.\left(0, \sigma_x^2\right) \end{aligned} \tag{4.2}$$

where ε_i^j is the noise of the private signal x_i^j and σ_x^{-2} stands for the precision of the private signal. We assume that private agents in country j do not receive any private information about the foreign regional fundamental shock, θ^{-j}.

In each country there is a policymaker, denoted by P_j for country j. Each policymaker P_j receives dual information $\left(y_j^1, y_j^2\right)$ on the fundamentals (θ^1, θ^2), characterized by:

$$\begin{aligned} y_j^k &= \theta^k + \eta_j^k, k = 1,2 \\ \eta_j^k &\sim i.i.d.\left(0, \sigma_{y,k,j}^2\right), \end{aligned} \tag{4.3}$$

where η_j^k is the noise of a signal about regional shock θ^k, received by policymaker P_j, and $\sigma_{y,k,j}^{-2}$ stands for its precision. We call a signal about regional shock θ^j, received by policymaker P_j, the "home" information and assume that its precision is the same for both policymakers: $\sigma_{y,j,j}^{-2} = \sigma_{y,h}^{-2}$. The signal about regional shock θ^{-j}, received by policymaker P_j, is called "foreign" information. Precision of the foreign information is equal to $\sigma_{y,-j,j}^{-2} = \sigma_{y,f}^{-2}$ for $j \in \{1,2\}$. We assume that $\sigma_{y,h}^{-2} \geq \sigma_{y,f}^{-2}$. In other words, the home information cannot be less precise than the foreign information. Moreover, we assume that $\sigma_{y,f}^{-2} > \sigma_x^{-2}$. This assumption says that even the foreign policymaker information about the fundamental shock θ^j is better than the information received by private agents. This is justified by the fact that policymakers have at their disposal a professional body of statistical agencies and therefore, a superior capacity to observe shocks.

The private agent preferences are characterized by the following private loss function:

$$l_i^j = \left(\frac{1-r}{2}\right)\left(a_i^j - \Theta^j\right)^2 + \frac{r}{2}\left(L_i - \overline{L}\right) \tag{4.4}$$

with a_i^j is a private action of agent i in region j, $L_i = \int\limits_0^1 (a_k - a_i)^2 \, \mathrm{d}k$ and $\bar{L} = \int\limits_0^1 L_k \, \mathrm{d}k$. Thus, the private loss is defined by the squared distance between the private action a_i^j and the fundamentals Θ^j and by the average distance between the private action a_i^j and the actions of other private agents, or a beauty-contest argument. Parameter $r \in (0,1)$ characterizes the relative strength of the beauty-contest argument in private loss. If r is equal to zero, there is no beauty-contest effect and private actions are defined by the desire to be as close to fundamentals Θ^j as possible. If r is close to one, the beauty-contest effect is strong and private actions are defined almost entirely by the desire to be close to the actions of others. As we can see from (4.4), a private agent cares not only about the average distance between his action and the actions of other agents in his home region, but also by the distance between his actions and the actions of the agents in the other region. Thus, parameter r characterizes both the regional and the international beauty contests. The presence of the international beauty contest differentiate the loss function (4.4) from the loss function in the two-regional model by Arato and Nakamura (2013), who study only a regional beauty contest.

We can rewrite (4.4) in the following way:

$$l_i^j = \frac{1-r}{2}\left(a_i^j - \Theta^j\right)^2 + \frac{r}{4}\left[\left(\overline{a}^j - a_i^j\right)^2 + \left(\overline{a}^{-j} - a_i^j\right)^2 - \sigma_{a^j}^2 - \sigma_{a^{-j}}^2 - \left(\overline{a}^j - \overline{a}^{-j}\right)^2\right], \tag{4.5}$$

where $\overline{a}^j \equiv (n^j)^{-1}\int_{i\in S^j} a_i^j \,\mathrm{d}i$ is the average private action in country j and $\sigma_{a^j}^2 \equiv (n^j)^{-1}\int_{i\in S^j}\left(a_i^j - \overline{a}^j\right)^2 \mathrm{d}i$ is the dispersion in private actions in region j, S^j characterizes the population of country j:

$$S^j = \begin{cases} [0, 1/2], & \text{if } j = 1 \\ (1/2, 1], & \text{if } j = 2 \end{cases}$$

Equation (4.5) clarifies the factors which define the private loss. These factors are the distance to the fundamentals Θ^j, the distance to the home average actions $\overline{a}^j$, the distance to the foreign average actions $\overline{a}^{-j}$. Moreover, the private loss depends negatively on the variance of private actions in both countries and to the squared difference between the two averages, $\left(\overline{a}^j - \overline{a}^{-j}\right)^2$. The last three factors are exogenous to the private agent and are taken as given.

The policymakers are regionally benevolent, meaning that their goal is to minimize the sum of private losses in their home regions: $L_{P_j} \equiv \int_{i\in S^j} l_i^j \,\mathrm{d}i$. Taking into account (4.5), we get the loss function of the policymaker in country j:

$$L_{P_j} = \frac{1-r}{2}\int_{i\in S^j}\left(a_i^j - \Theta^j\right)^2 \mathrm{d}i + \frac{r}{8}\left[\sigma_{a^j}^2 - \sigma_{a^{-j}}^2\right] \tag{4.6}$$

As we can see, the public loss of country j is defined by the average squared distance of private actions to the corresponding fundamentals and by the variances of private actions in the home and in the foreign countries. Worth to mention that public loss depends positively on the variance of the private actions in the home country and negatively on the variance of private actions in the foreign country. For what follows, it is useful to rewrite the public loss (4.6):

$$L_{P_j} = \frac{1}{4}\left[(1-r)\left(\overline{a}^j - \Theta^j\right)^2 + \left(1 - \frac{r}{2}\right)\sigma_{a^j}^2 - \frac{r}{2}\sigma_{a^{-j}}^2\right] \tag{4.7}$$

Equation (4.7) shows that the policymaker has an incentive to keep the average private actions in its home region as close to the home fundamentals as possible. Moreover, it has the incentive to lower the home private action volatility and to raise the foreign private action volatility. The last motive comes from the positive externality, created by the dispersion in private actions. As we can see in equation (4.5), private loss depends negatively on the dispersion in private actions abroad. This term is exogenous for the private agent and does not affect his actions. Nevertheless, this term is endogenous for the policymaker and affects the equilibrium informational policy.

4.2.2 Public signals

Policymaker P_j sends two signals to private agents: a home signal, s_j^j, and a foreign signal, s_j^{-j}. Precision of signal s_j^k is denoted by $\sigma_{s,k,j}^{-2}$. We assume that policymakers cannot discriminate among private agents. Once published, signal s_j^k is equally available to all the agents in both regions. Thus, there are no informational cross-border frictions.

We assume that the policymaker chooses between revealing the true value of its own information about a fundamental shock and not revealing the true information. Thus, the signal sent by P_j about the fundamental θ_j^k is either y_j^k or empty set:

$$s_j^k \in \left\{\emptyset, y_j^k\right\} \quad , k = 1, 2.$$

If policymaker P_j chooses to reveal the information about the fundamental θ^k, the value of signal s_j^k is equal to the value of the signal y_j^k which was received by the policymaker. The precision of the sent signal $\sigma_{s,k,j}^{-2}$ is equal to $\sigma_{y,k,j}^{-2}$. In this case the policymaker is transparent about the fundamental θ^k. If policymaker P_j chooses not to reveal the information about the fundamental θ^k, the precision of signal s_j^k is equal to zero. This situation is equivalent to adding the infinite noise to signal y_j^k and is referred as opacity of policymaker P_j about the fundamental θ^k. Thus, there are four possible configurations of information policy of P_j:

1. full transparency means that a policymaker reveals all its information about the home and the foreign fundamentals;

2. home transparency and foreign opacity means that a policymaker reveals its information about the home fundamentals and does not reveal any information about the foreign fundamentals;

3. home opacity and foreign transparency means that a policymaker does not reveal its information about the home fundamentals but reveals its information about the foreign one;

4. full opacity means that a policymaker does not reveal any information.

The public signals which contain the information about fundamental θ^k constitute the composite signal s^k , which is received by all private agents:

$$s^k = \frac{\sigma_{s,k,j}^{-2} s_j^k + \sigma_{s,k,-j}^{-2} s_{-j}^k}{\sigma_{s,k,j}^{-2} + \sigma_{s,k,-j}^{-2}}, \qquad k = 1, 2; j = 1, 2.$$

Precision of composite public signal s^k on fundamental θ^k is equal to $\sigma_{s,k}^{-2} = \sigma_{s,j,j}^{-2} + \sigma_{s,j,-j}^{-2}$. If both policymakers are transparent about fundamental θ_k, we get that $\sigma_{s,k}^{-2} = \sigma_{y,h}^{-2} + \sigma_{y,f}^{-2}$. If both are opaque, we get $\sigma_{s,k}^{-2} = 0$. If there is home transparency (P_k is transparent) and foreign opacity (P_{-k} is opaque) about fundamental θ_k, $\sigma_{s,k}^{-2} = \sigma_{y,h}^{-2}$. If there is home opacity and foreign transparency about fundamental θ_k, $\sigma_{s,k}^{-2} = \sigma_{y,f}^{-2}$.

Let z^j denote a common posterior of θ^j given only public information:

$$z^j \equiv E\left(\theta^j \middle| s_j^j, s_{-j}^j\right) = \omega^j s^j + \left(1 - \omega^j\right)\mu \tag{4.8}$$

where $\omega^j = \frac{\sigma_{s,j}^{-2}}{\sigma_{s,j}^{-2} + \sigma_\theta^{-2}}$ and μ is a common prior about the fundamental shock. Precision of this common posterior is equal to $\sigma_{z,j}^{-2} = \sigma_\theta^{-2} + \sigma_{s,j}^{-2}$. As we stated before, we assume that μ is equal to zero. Thus, the common posterior z^j is given by

$$z^j = \omega^j s^j \tag{4.9}$$

In what follows we use the notion of relative precision of public information given by the following definition.

Definition 4.1. The relative precision of public information ζ^j shows the relative precision of z^j in comparison to private information about fundamental shock θ^j:

$$\zeta^j \equiv \frac{\sigma_{z,j}^{-2}}{\sigma_x^{-2}} \tag{4.10}$$

The next Section describes the game played between the policymakers.

4.3 A non-cooperative game on public information

The game played in the economy consists of several steps:

Step 1. Each policymaker decides non-cooperatively what it will reveal from what it knows, based on its expected loss function. Given that each policymaker has 4 decision possibilities, there are 16 possible outcomes at this stage of the game. Policymakers commit to their revelation strategies.

Step 2. All private and public agents receive their private signals. Public signals are emitted in accordance with decision of Step 1.

Step 3. Expectations of private agents, based on their information sets, are computed: $E\left[\dots \middle| x_i^j, s^j, s^{-j}, \sigma_{s^j}^2, \sigma_{s^{-j}}^2\right]$. Private actions $\left(a_i^j\right)$ are chosen non-cooperatively so as to minimize the expected private losses.

Step 4. Shocks are realized. Given the equilibrium of the game as well as the realized shocks, actual losses are obtained.

We proceed with the solution of the private stage of the game (step 3) and then we solve the public stage (step 1) to find the equilibrium.

4.3.1 Private actions (step 3)

Private agent i living in country j decides on his or her action a_i^j before the realization of the shocks. Thus, his or her task is to minimize the expected value of the loss (4.5) given the information set of the agent I_i^j. The optimal choice of agent i living in country j is as follows:

$$a_i^j = \arg\min_{a_i'} E\left[\frac{1-r}{2}\left(a_i' - \Theta^j\right)^2 + \frac{r}{4}\left[\left(\overline{a}^j - a_i'\right)^2 + \left(\overline{a}^{-j} - a_i'\right)^2 - \sigma_{a^j}^2 - \sigma_{a^{-j}}^2 - \left(\overline{a}^j - \overline{a}^{-j}\right)^2\right] \middle| I_i^j \right] \tag{4.11}$$

As the agent cannot influence the dispersion in private actions and the gap between average actions in two regions, the first-order condition of problem (4.11) is as follows:

$$a_i^j = E\left[(1-r)\left(\phi\theta^j + (1-\phi)\,\theta^{-j}\right) + \frac{r}{2}\left(\overline{a}^j + \overline{a}^j\right) \middle| I_i^j \right] \tag{4.12}$$

As we can see from (4.12), private actions are defined by expected fundamentals and expected average actions in both regions, according to information set I_i^j of the agent. We observe that the action of a given agent in country j is an increasing function of the average action in her country j and of the average action in the other country $-j$. The extent of this response is parameterized by r, the beauty contest parameter. If r is equal to zero, private actions do not depend on the expected average actions in the economy. In this case, the optimal private action is equal to the expected value of fundamental variable, Θ^j. If there is no technological spillover ($\phi = 1$), the action of agent i does not depend on the foreign regional shock.

The information set of agent i in region j consists of two components: the information about the home regional shock and the information about the foreign regional shock. The home information component consists of two signals, one private signal x_i^j and one public composite signal z^j. The foreign information component for agent i in region j consists of the public composite signal about the regional shock in region $-j$, z^{-j}. Thus, the whole information set I_i^j is defined as $\left(z^j, x_i^j, z^{-j}\right)$. The rational expectations of agent i in region j are given by the following expressions:

$$E\left(\theta^j \middle| z^j, x_i^j\right) = \frac{\zeta^j}{1+\zeta^j} z^j + \frac{1}{1+\zeta^j} x_i^j \tag{4.13}$$

$$E\left(\theta^{-j} \middle| z^{-j}, 0\right) = z^{-j} \tag{4.14}$$

As we can see in (4.13), the agent weights the two components of her information set according to their precisions. The weight of public signal z^j depends positively on the relative precision of the public information, ζ^j. The weight of the private signal x_i^j depends negatively on the relative precision of the public signal. The sum of the two coefficients is equal to 1. As the only source of information about the foreign regional shock is the public signal, the expectation of this shock is equal to the value of signal z^{-j}. Thus, according to equations (4.13) and (4.14), agents in the two regions use the public signals differently. The agents in region j weight the value of signal z^j with their private signal. Thus, the weight of public signal is less than 1. The agents in region $-j$ have no other information about region j but signal z^j. Thus, the weight of this signal in expectations is equal to 1.

The first-order condition (4.12) along with expectations (4.13) and (4.14) imply the following equilibrium private linear strategy:

$$a_i^j = b^j x_i^j + c^j z^j + d^j z^{-j} \tag{4.15}$$

The average private actions, computed for the linear strategies (4.15), are as follows:

$$\overline{a}^j = b^j \theta^j + c^j z^j + d^j z^{-j}, \tag{4.16}$$

where we use that $x_i^j = \theta^j + \varepsilon_i^j$ and ε_i^j are *i.i.d.* shocks.

To find the equilibrium weights b^j, $c^j$and d^j, we substitute expressions (4.13 - 4.16) into the first-order condition (4.12) and solve for the coefficients. This gives the following solution:

$$b^j = \frac{(1-r)\,\phi}{(1-{}^r\!/\!{}_2)+\zeta^j} \tag{4.17}$$

$$c^j = {}^r\!/\!{}_2 + \frac{(1-r)\,\phi\,[\zeta^j - {}^r\!/\!{}_2]}{(1-{}^r\!/\!{}_2)+\zeta^j} \tag{4.18}$$

$$d^j = (1-r)\,(1-\phi) + {}^r\!/\!{}_2 \tag{4.19}$$

First of all, the coefficients given by (4.17), (4.18) and (4.19) are positive. Moreover, it is easy to show that

$$b^j + c^j + d^j = 1$$

The weights of private signal b^j and the home public signal c^j depend on the beauty-contest parameter, r, the technological spillover ϕ and the relative precision of the public signal ζ^j. The weight of the foreign public signal depends on the beauty contest parameter, r and the technological spillover ϕ. The weight of the foreign public signal does not depend on the relative precision of public information, as this signal is the only information to predict the true value of the foreign regional shock. As each of the public composite signals consists of two signals sent by the policymakers, this gives rise to the informational spillovers. These spillovers are based on the fact that any bit of public information is available to and used by any agent in the whole economy. These information spillovers create the possibility for policymakers to influence private actions in their home region and in the foreign region.

An increase in the relative precision of the home public signal makes this signal a better predictor of both fundamental regional shock θ^j and the average private actions. The private signal x_i^j becomes a relatively worse predictor of the fundamentals and the average actions. As a result, the weight of private signal in equilibrium actions goes down, while the weight of public home signal goes up. If the relative precision of the public signal is low, this signal is a bad predictor of the home regional shock, and it is better to use private information. In this case an increase in ϕ leads to a decrease in the weight of the home public signal c^j.

An increase in ϕ leads to a decrease in d^j. The logic is straightforward. Higher ϕ means that technological spillover weakens and the agents care less about the foreign regional shocks. Thus, they do not want to rely on the foreign public information and d^j lowers. At the same time, the agents become more interested in better prediction of their home regional fundamentals, θ^j. For this reason, they increase their use of the home information, defined by the sum of coefficients b^j and c^j. It is easy to show that this sum depends positively on ϕ:

$$b^j + c^j = \frac{r}{2} + (1-r)\,\phi$$

The individual effects of an increase in ϕ on coefficients $b^j$and c^j are different. From equation (4.17), we can see that an increase in ϕ leads to an increase in the weight of private signal b^j. The effect of ϕ on the weight of the home public signal is positive if and only if the relative precision of this information is sufficiently high, such that $\zeta^j > r/2$.

The effect of the beauty contest parameter on the use of home and foreign information depends on the technological spillovers. It is easy to show that

$$\frac{\partial\left(b^j + c^j\right)}{\partial r} = -\frac{\partial d^j}{\partial r} = \frac{1-2\phi}{2}.$$

If $\phi > 1/2$, the agents are more interested in their home fundamentals and the use of the home information is already high, while the use of the foreign information, measured by d^j, is low. Thus, an increase in the beauty-contest argument cannot be satisfied by the increase in the use of the home public information, which is already close to one. Instead of this, agents become more interested in the cross-border coordination. Thus, the weight of the foreign public signal goes up, while the use of the home information, measured by $(b^j + c^j)$, goes down. On the contrary, if $\phi < 1/2$, the technological spillovers are so strong that the agents are more interested in mimicking the foreign regional shock. In this case, the weight of the foreign public signal is already so high that an increase in the beauty-contest argument cannot be satisfied by a further increase in d^j. Instead of this, the agents redistribute their use of information in favor of their home information. This helps them to better predict the average actions in their home region and to coordinate inside the region. As a result, an increase in r leads to a decrease in d^j and to an increase in $(b^j + c^j)$.

The individual effects of r on the weights of the home signals are different. It is easy to show that ${}^{\partial b^j}/_{\partial r}$ is negative. This means that an increase in the beauty-contest argument always lowers the weight of private information. Private information, which is not available to others, cannot be used to coordinate the actions with the other agents. Thus, higher strategic complementarity and stronger the desire to coordinate, lower the weight of the private signal. The effect of r on the weight of the home public signal depends on the parameter of technological spillover ϕ and on the relative precision of public information ζ^j. We can show that

$$\frac{\partial c^j}{\partial r} = {}^1/_2 - \phi \left(1 - \frac{{}^1/_2 + \zeta^j}{\left(1 - {}^r/_2 + \zeta^j\right)^2} \right). \tag{4.20}$$

Thus, for a given r and ζ^j, an increase in r may lead to a decrease in the weight of the home public signal, if ϕ is sufficiently high.

4.3.2 Public objective function (step 1)

At the first stage of the game the policymakers decide on their revelation strategies knowing that the private actions at Step 4 will be chosen according to the rule (4.15). Substituting the private strategies (4.15) into the public loss function (4.7) and taking the expectation gives the following expected public loss incurred by the policymaker P_j (for details, see Appendix D.1):

$$E\left(L_{P_j}\right) = \frac{1}{4}\left[\rho_j^j\left(\zeta^j\right) + \rho_j^{-j}\left(\zeta^{-j}\right)\right], \tag{4.21}$$

where $\rho_j^j\left(\zeta^j\right)$ is the "home" loss component, which depends on the information about fundamental θ^j, and $\rho_j^{-j}\left(\zeta^{-j}\right)$ is the "foreign" loss component, which depends on the information about fundamental θ^{-j}. The "home" loss component in region j can be expressed as follows:

$$\rho_j^j\left(\zeta^j\right) = (1-r)\left(b^j + \omega^j c^j - \phi\right)^2 \sigma_\theta^2 + \left[1 - {}^r/_2\right]\left(b^j\right)^2 \sigma_x^2 + \left(\omega^j\right)^2 (1-r)\left(c^j\right)^2 \sigma_{s,j}^2. \tag{4.22}$$

This loss components can be partially controlled by policymaker P_j through precision $\sigma_{s,j,j}^{-2}$ of its home public signal s_j^j. By definition, this precision influences the relative precision of the home public information, $\zeta^j = \frac{\sigma_\theta^{-2} + \sigma_{s,j,j}^{-2} + \sigma_{s,j,-j}^{-2}}{\sigma_x^{-2}}$. As the weights b^j, c^j and coefficient ω^j depend on the relative precision of public information about the regional shock θ^j, policymaker can influence its home loss component by deciding to reveal its home information or not. If the policymaker is home transparent, precision $\sigma_{s,j,j}^{-2}$ is equal to $\sigma_{y,h}^{-2}$ and the relative precision of public information about region j is equal to $\frac{\sigma_\theta^{-2}+\sigma_{y,h}^{-2}+\sigma_{s,j,-j}^{-2}}{\sigma_x^{-2}}$ for given $\sigma_{s,j,-j}^{-2}$. If the policymaker is home opaque, precision $\sigma_{s,j,j}^{-2}$ is equal to 0 and the relative precision of public information about region j is equal to $\frac{\sigma_\theta^{-2}+\sigma_{s,j,-j}^{-2}}{\sigma_x^{-2}}$.

The "foreign" loss component in region j, which depends on $\zeta^{-j}=\frac{\sigma_\theta^{-2}+\sigma_{s,-j,j}^{-2}+\sigma_{s,-j,-j}^{-2}}{\sigma_x^{-2}}$, can be expressed as follows:

$$\rho_j^{-j}\left(\zeta^{-j}\right)=(1-r)\left(\omega^{-j}d^j-(1-\phi)\right)^2\sigma_\theta^2-{}^{r}\!/_{2}\left(b^{-j}\right)^2\sigma_x^2+\left(\omega^{-j}\right)^2(1-r)\left(d^j\right)^2\sigma_{s,-j}^2. \tag{4.23}$$

Precision $\sigma_{s,-j,j}^{-2}$ of the signal about region $-j$, sent by policymaker P_j, influences the relative precision of public information about region $-j$: $\zeta^{-j}=\frac{\sigma_\theta^{-2}+\sigma_{s,-j,j}^{-2}+\sigma_{s,-j,-j}^{-2}}{\sigma_x^{-2}}$. As this relative precision enters into equations which describe weights b^{-j}, c^{-j} and coefficient ω^{-j}, policymaker can influence its foreign loss component by choosing his revelation action for the information about the foreign regional shock. If the policymaker is foreign transparent, precision $\sigma_{s,-j,j}^{-2}$ is equal to $\sigma_{y,f}^{-2}$. Thus, for given $\sigma_{s,-j,-j}^{-2}$, the relative precision of foreign public information is equal to $\zeta^{-j}=\frac{\sigma_\theta^{-2}+\sigma_{y,f}^{-2}+\sigma_{s,-j,-j}^{-2}}{\sigma_x^{-2}}$. If the policymaker is foreign opaque, precision $\sigma_{s,-j,j}^{-2}$ is equal to 0. Thus, for given $\sigma_{s,-j,-j}^{-2}$, the relative precision of foreign public information is equal to $\zeta^{-j}=\frac{\sigma_\theta^{-2}+\sigma_{s,-j,-j}^{-2}}{\sigma_x^{-2}}$.

Equation (4.21) shows that the function of expected public loss is separable in ζ^j and ζ^{-j}. The separability of $E\left(L_{P_j}\right)$ into two components implies that the optimal revelation strategy for information about region j is independent from the revelation strategy for information about region $-j$. In other words, the equilibrium values of precisions $\sigma_{s,j,j}^{-2}$ and $\sigma_{s,j,-j}^{-2}$ are obtained independently from the equilibrium values of precisions $\sigma_{s,-j,j}^{-2}$ and $\sigma_{s,-j,-j}^{-2}$. The definition of equilibrium at the public stage of the game is provided in the next subsection.

4.3.3 Definition of equilibrium

The equilibrium of the public game is based on mutually consistent decisions of policymakers to reveal or not their information on the fundamentals in the two countries. Formally, we define the equilibrium as follows:

Definition 4.2. The equilibrium in a policy game is the pair of strategies (P_1^*, P_2^*), where vector $P_j^*=\left(\left(\sigma_{s,j,j}^{-2}\right)^*,\left(\sigma_{s,-j,j}^{-2}\right)^*\right)$ is such that

1. $\left(\sigma_{s,j,j}^{-2}\right)^*=\arg\min_{\sigma_{s,j,j}^{-2}\in\left\{0,\sigma_{y,h}^{-2}\right\}}\rho_j^j\left(\frac{\sigma_\theta^{-2}+\sigma_{s,j,j}^{-2}+\left(\sigma_{s,-j,j}^{-2}\right)^*}{\sigma_x^{-2}}\right)$

2. $\left(\sigma_{s,-j,j}^{-2}\right)^* = \arg \min_{\sigma_{s,-j,j}^{-2} \in \left\{0, \sigma_{y,f}^{-2}\right\}} \rho_j^{-j} \left(\frac{\sigma_\theta^{-2} + \sigma_{s,-j,j}^{-2} + \left(\sigma_{s,-j,-j}^{-2}\right)^*}{\sigma_x^{-2}} \right)$

As we discussed before, separability of the loss function makes the equilibrium revelation policies on the information about region j independent from the equilibrium revelation policies in the information about region $-j$. Given 4 possible decisions of each policymakers, there are 16 types of possible equilibrium configurations in pure strategies, 4 of which are symmetric[2]. We define a symmetric equilibrium as follows:

Definition 4.3. A symmetric equilibrium is an equilibrium such that $P_1^* = P_2^*$.

Finally, we make a simple "tie-break" assumption so as to avoid the multiple solutions generating the same outcome.

Assumption 4.4. *If $\rho_j^k \left(\underline{\zeta}^k\right) = \rho_j^k \left(\overline{\zeta}^k\right)$ and $\overline{\zeta}^k > \underline{\zeta}^k \geq 0$, policymaker P_j chooses $\underline{\zeta}^k$.*

Assumption (4.4) tells that if policymaker is indifferent between two non-negative values of the relative precision of public information, it chooses higher transparency. Hence, for given $\sigma_{s,j,-j}^{-2}$, if the loss difference $\rho_j^j \left(\frac{\sigma_\theta^{-2} + \sigma_{y,h}^{-2} + \sigma_{s,j,-j}^{-2}}{\sigma_x^{-2}} \right) - \rho_j^j \left(\frac{\sigma_\theta^{-2} + \sigma_{s,j,-j}^{-2}}{\sigma_x^{-2}} \right)$ is strictly positive, policymaker P_j chooses home opacity. If this difference is either negative or equal to zero, the policymaker chooses home transparency. This assumption is used in the next section to characterize the equilibrium in the policy game.

4.4 Equilibrium

After discussion of the public loss function and the structure of the policy game, we now proceed with establishing the existence of equilibrium and characterizing its properties.

Appendix D.2 shows that the following proposition is true about the equilibrium of this game:

Proposition 4.5. *For any $\left(\sigma_\theta^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_x^{-2}, r, \phi\right)$, an equilibrium exists. This equilibrium is unique and symmetric.*

Proof. See Appendix D.2. □

According to Proposition 4.5, for any technological spillover, beauty contest parameter r and precision of information, there is a unique and symmetric equilibrium in pure strategies. Worth to mention that we did not restrict our attention to the symmetric equilibria from the beginning. This characteristic comes from the symmetry of the regions in the studied economy. The following Proposition describes the properties of this equilibrium.

[2]We do not restrict the equilibrium to be symmetric.

Proposition 4.6. *For given* $\left(\sigma_\theta^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_x^{-2}, r\right)$*, there exist* $\underline{\phi}$ *and* $\overline{\phi}$ *such that* $0 \leq \underline{\phi} < 1/2 < \overline{\phi} \leq 1$ *and*

1. *if* $\phi < \underline{\phi}$*, the equilibrium strategy for any* $j \in \{1, 2\}$ *is* $P_j^* = \left(0, \sigma_{y,f}^{-2}\right)$ *– home opacity, foreign transparency.*

2. *if* $\underline{\phi} \leq \phi \leq \overline{\phi}$*, the equilibrium strategy for any* $j \in \{1, 2\}$ *is* $P_j^* = \left(\sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}\right)$ *– home transparency, foreign transparency.*

3. *if* $\overline{\phi} < \phi$*, the equilibrium strategy for any* $j \in \{1, 2\}$ *is* $P_j^* = \left(\sigma_{y,h}^{-2}, 0\right)$ *– home transparency, foreign opacity.*

Proof. See Appendix D.2. □

As we already discussed, there are three incentives of policymaker P_j, captured by the loss functions (4.22) and (4.32). The first incentive is to help the agents in its home region to keep their actions close to the fundamental Θ^j. The second incentive is to lower the dispersion in private actions in the home region. Finally, there is the incentive to increase the dispersion in private actions in the foreign region, measured by the term $-r/2\sigma_{a^{-j}}^2$ in equation (4.7). As we can see in Proposition 4.6, the choice of the policy depends on the value of parameter ϕ.

If ϕ is low, private agents in region j are willing to keep their actions closer to the foreign regional fundamental shock θ^{-j} and not to their home regional shock θ^j. Thus, the policymaker chooses to be transparent about the foreign regional shock in order to help the agents in region j to minimize the gap between their actions and the foreign regional shock. This also helps the agents in region j to coordinate, which lowers the dispersion in private actions $\sigma_{a,j}^2$. Incentive to prevent the foreign agents from coordination forces policymaker P_j to hide the information about his home regional shock θ^j. As the agents in region $-j$ pay much attention to the shock θ^j, the lack of the information about this variable causes a sufficient increase in the private action volatility in region $-j$. At the same time, this does not lead to a large increase in the dispersion in private actions in region j. As a result, policymakers P_j chooses home opacity and foreign transparency, hiding his signal about the home regional shock and revealing his signal about the foreign regional shock. Due to symmetry, policymaker P_{-j} makes the same decision.

For high values of ϕ, situation is the opposite. The agents in region j pay almost all their attention to the information about their home regional shock θ^j, as their payoffs depend much more on the distance between their actions and the true value of the regional fundamental θ^j. Higher ϕ, closer economy to the technological autarky, where the fundamentals are defined only by the home regional shocks. As the closeness of the agents to their home regional shocks is crucial in the model with high value of ϕ, the policymaker chooses home transparency and reveal all the information about the home regional shock. This also helps the agents to coordinate and lowers the dispersion in private actions. In order to prevent the coordination of the agents in the other region, policymaker P_j chooses foreign opacity and hides his information about the foreign regional shock θ^{-j}. This, nevertheless, does not lead to a considerable increase in the volatility of private agents in region j, because they do not pay much attention to the foreign regional shock. Moreover, this also prevents the foreign agents from cross-border coordination.

For the intermediate set of ϕ, both regional shocks θ^j and θ^{-j} are relevant for private actions and payoffs. Thus, the policymakers do not want to hide any information, as they do in the previous two cases. Imagine that, similar to the case with low ϕ, policymaker decides to hide the information about the home regional shock θ^j. This prevents the agents in region $-j$ from coordination and raises the dispersion in their actions. At the same time, this does not allow the agents in the home region j to keep their actions close to the relevant shock θ^j. Moreover, as the agents in region j now pay much attention to the information about their home regional shock, the lack of information about this variable prevents them from coordination and increases the dispersion in their actions. Thus, there are too much negative consequences of non-revelation and a policymaker chooses both home and foreign transparency.

As we can see, for any value of ϕ, at least one signal is emitted by a policymaker. Thus, the equilibrium of the game is never characterized by the full opacity.

As the loss functions are highly non-monotone in their arguments, not very much can be said about the properties of functions $\underline{\phi}\left(\sigma_\theta^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_x^{-2}, r\right)$ and $\overline{\phi}\left(\sigma_\theta^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_x^{-2}, r\right)$. Nevertheless, some of the properties can be get without imposing any substantial restrictions on the model. These properties are summarized in the next Proposition.

Proposition 4.7. *Functions* $\underline{\phi}\left(\sigma_\theta^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_x^{-2}, r\right)$ *and* $\overline{\phi}\left(\sigma_\theta^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_x^{-2}, r\right)$ *are such that:*

1. *Properties of* $\underline{\phi}$*:*
   a) *Precision of prior information and policymakers information lowers* $\underline{\phi}$*:* $\frac{\partial \underline{\phi}}{\partial \sigma_\theta^{-2}} < 0$, $\frac{\partial \underline{\phi}}{\partial \sigma_{y,h}^{-2}} < 0$, $\frac{\partial \underline{\phi}}{\partial \sigma_{y,f}^{-2}} < 0$;
   b) *Precision of private information increases* $\underline{\phi}$*:* $\frac{\partial \underline{\phi}}{\partial \sigma_x^{-2}} > 0$;
   c) $\underline{\phi}$ *is monotonically increasing in* r. *If* $r = 0$, $\underline{\phi} = 0$. *If* $r = 1$, $\underline{\phi} = 1/4$.
2. *Properties of* $\overline{\phi}$*:*
   a) $\overline{\phi}$ *is monotonically decreasing in* r, *if precision of the home signal* σ_h^{-2} *is sufficiently high. If* σ_h^{-2} *is low,* $\overline{\phi}(r)$ *is a U-shaped function.*
   b) *If* $r = 0$, $\overline{\phi} = 1$. *If* $r = 1$, $\overline{\phi} = 3/4$.
3. *Properties of* $\left(\overline{\phi} - \underline{\phi}\right)$*:*

a) Precision of prior information and policymakers information enlarges the region of transparency: $\frac{\partial\left(\overline{\phi}-\underline{\phi}\right)}{\partial\sigma_{\theta}^{-2}} > 0$, $\frac{\partial\left(\overline{\phi}-\underline{\phi}\right)}{\partial\sigma_{y,h}^{-2}} > 0$, $\frac{\partial\left(\overline{\phi}-\underline{\phi}\right)}{\partial\sigma_{y,f}^{-2}} > 0$*;*

b) Precision of private information decreases the region of transparency: $\frac{\partial\left(\overline{\phi}-\underline{\phi}\right)}{\partial\sigma_{x}^{-2}} < 0$*;*

c) The beauty contest coefficient r decreases the region of transparency: $\frac{\partial\left(\overline{\phi}-\underline{\phi}\right)}{\partial r} < 0$.

Proof. See Appendix D.3. □

Part 1 of Proposition 4.7 describes the properties of threshold $\underline{\phi}$. According to our findings, an increase in the precision of public information narrows the region of home opacity in the equilibrium. The better public information, the less public gains of home opacity. The opposite is true for the quality of private information. An increase in the precision of private information leads to an increase in $\underline{\phi}$. The home opacity region widens. Finally, we show that an increase in the beauty-contest parameter also widens the region of home opacity in equilibrium. The logic is straightforward. Stronger strategic complementarity and beauty contest mean that the private agents are more prone to an excessive coordination both inside and between regions. This increases the potential gains of opacity for larger set of ϕ. Worth to remind that these findings are made under assumption $\sigma_{y,f}^{-2} > \sigma_{x}^{-2}$.

The function $\overline{\phi}\left(\sigma_{\theta}^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_{x}^{-2}, r\right)$ appears to be non-monotonic in all arguments, thus not very much can be said about this threshold without imposing further restrictions. Nevertheless, it can be shown that for sufficiently precise home signals, this functions is decreasing in the beauty-contest parameter and its value is higher than $3/4$.

Despite of the difficulties in the description of function $\overline{\phi}\left(\sigma_{\theta}^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_{x}^{-2}, r\right)$, the properties of the region of full transparency $\left(\overline{\phi}-\underline{\phi}\right)$ are defined and listed in Part 3 of Proposition 4.7. The region of full transparency in equilibrium enlarges, if the precision of public information goes up and the precision of private information goes down. An increase in beauty-contest parameter narrows the region of full transparency.

In the next section we derive the social loss function, find the socially optimal revelation policy and compare it with the equilibrium information policy.

4.5 Welfare analysis

To find the socially optimal revelation policy, we consider the problem of a social planner who minimizes the average loss of private agents in the whole economy. This social planner decides on which of the signals to reveal. As we already saw, the loss of private agents in each region are separable in the precisions of information about the two regional shocks. Consequently, the sum of losses of all agents in the economy is also separable into two components, one defined by the revelation of signals about fundamental θ^j and the other defined by the information about fundamental θ^{-j}. Thus, the decision of the social planner on the revelation of signals about one region is independent from the decision about the signals on the other region.

The social planner has 4 possibilities for the revelation of the signals about region j. It can choose full transparency and publish both signals about the regional fundamental shock θ^j: the signal with precision $\sigma_{y,h}^{-2}$ received by policymaker P_j and the signal with precision $\sigma_{y,f}^{-2}$ received by policymaker P_{-j}. This revelation policy is equivalent to publishing one composite signal on θ^j with precision $\sigma_{y,h}^{-2}+\sigma_{y,f}^{-2}$. If the social planner chooses home transparency, it publishes only the signal received by policymaker P_j. The precision of this signal is equal to $\sigma_{y,h}^{-2}$. If the social planner chooses foreign transparency, it publishes only the signal received by policymaker P_{-j}. The precision of this signal is equal to $\sigma_{y,f}^{-2}$. Finally, the social planner can choose full opacity and hide both the signals about the regional shock. This is equivalent to emitting a signal with zero precision. Formally, the problem of the social planner is defined as follows:

Definition 4.8. The social optimum is the vector $\left(\tilde{\sigma}_{s,1}^{-2}, \tilde{\sigma}_{s,2}^{-2}\right)$ such that

$$\tilde{\sigma}_{s,j}^{-2} = \arg \min_{\sigma_{s,j}^{-2} \in \left\{0, \sigma_{y,f}^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}+\sigma_{y,h}^{-2}\right\}} E\left(L_S\right),$$

where $L_S \equiv \int_{j\in\{1,2\}} \int_{i\in S^j} l_i^j \,\mathrm{d}i\,\mathrm{d}j = L_P^1 + L_P^2$ stands for the the social loss or the sum of losses of all private agents in the economy.

As the social loss is equal to the sum of public losses in the regions, we use (4.7) to obtain:

$$L_S = \frac{1}{4}(1-r)\left[\left(\overline{a}^1-\Theta^1\right)^2 + \left(\overline{a}^2-\Theta^2\right)^2 + \sigma_{a^1}^2 + \sigma_{a^2}^2\right] \tag{4.24}$$

Thus, the social loss positively depends on the squared gaps between the average actions and the fundamentals in both regions and on the dispersion in private actions. We can also rewrite the social loss as a sum of two components:

$$E\left(L_S\right) = \frac{1}{4}\left[\rho_S^j\left(\zeta^j\right) + \rho_S^{-j}\left(\zeta^{-j}\right)\right], \tag{4.25}$$

where $\rho_S^j\left(\zeta^j\right) = \rho_j^j\left(\zeta^j\right) + \rho_{-j}^j\left(\zeta^j\right)$ is the component which depends on the precision of information about fundamental θ^j and $\rho_S^{-j}\left(\zeta^{-j}\right) = \rho_j^{-j}\left(\zeta^{-j}\right) + \rho_{-j}^{-j}\left(\zeta^{-j}\right)$ is the component which depends on the precision of information about fundamental θ^j. Using equations (4.22) and (4.23), we get the following function $\rho_S^j\left(\zeta^j\right)$:

$$\rho_S^j\left(\zeta^j\right) = (1-r)\left(\left[\left(b^j+\omega^j c^j-\phi\right)^2 + \left(\omega^j d^{-j}-(1-\phi)\right)^2\right]\sigma_\theta^2 + \left(b^j\right)^2\sigma_x^2 + \left(\omega^j\right)^2\left[\left(c^j\right)^2+\left(d^{-j}\right)^2\right]\sigma_{s,j}^2\right). \tag{4.26}$$

In equation (4.26), coefficients b^j, c^j, d^j and ω^j depend on the relative precision of public information ζ^j. Thus, choosing the proper revelation policy, the social planner may lower the social loss $\rho_S^j(\zeta^j)$. Proposition 4.9 summarizes the characteristics of the social optimum:

Proposition 4.9. *For given $\left(\sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_x^{-2}, r\right)$, there exist $\hat{\sigma}$ and $\hat{\phi}$ such that:*

1. *Full transparency ($\tilde{\sigma}_{s,j}^{-2} = \sigma_{y,f}^{-2} + \sigma_{y,h}^{-2}, j = 1, 2$) is socially optimal if*

    a) if $\sigma_\theta^{-2} \geq \hat{\sigma}$, for any ϕ, or

    b) if $\sigma_\theta^{-2} < \hat{\sigma}$ and $\phi \leq \hat{\phi}$.

2. *Full opacity ($\tilde{\sigma}_{s,j}^{-2} = 0, j = 1, 2$) is socially optimal, if $\sigma_\theta^{-2} < \hat{\sigma}$ and $\phi > \hat{\phi}$.*

Proof. See Appendix D.4. □

According to Proposition 4.9.1a, there exists a threshold $\hat{\sigma}$ such that for all σ_θ^{-2} higher than this threshold, transparency is socially optimal irrespective of the technological spillover ϕ. Precision σ_θ^{-2} higher than this threshold means that the volatility of the regional fundamental shock is lower than the inverse of this threshold. In other words, this implies that economy is sufficiently stable. In a stable economy with relatively low volatility of fundamentals, the mean values of the shocks serve as good predictors of their real values and as reliable focal points for coordination both inside regions and between them. Thus, hiding some information about the regional shocks cannot prevent the excessive coordination which arise due to the beauty-contest argument in the private loss functions. Instead of this, non-revelation leads to a higher expected gap between the average actions and the corresponding fundamentals, as the information available to private agents becomes worse. Thus, the social planner does not have any incentive to hide the public information, so full transparency is a social optimum.

If σ_θ^{-2} is lower than the aforementioned threshold, the volatility of the regional shocks is sufficiently high. In this case the mean of the fundamentals is not a good predictor of the true value of the fundamentals and of the private actions in the economy. Thus, the optimal policy depends on the technological cross-border spillovers. If these spillovers are strong enough ($\phi \leq \hat{\phi}$), the private payoffs strongly depend on the gap between private actions and the foreign regional fundamentals. In such situation hiding some public information would lower an excessive coordination caused by the beauty contest, but at sake of a huge increase in the gaps between private actions and the foreign regional fundamentals, because the agents have no other information about the foreign shocks but public signals. Thus, the social planner chooses the full transparency, if the technological spillovers are strong. If the technological spillovers are weak ($\phi > \hat{\phi}$), the regions are closer to autarky and the agents are more interested in keeping their actions closer to their home regional shocks. As the agents have an additional source of the information about their home shocks in form of their private signals, hiding the public information about the fundamentals cannot damage the social loss as much as in case of strong spillovers. Consequently, the social planner may choose the full opacity, if this helps to lower the excessive coordination caused by the beauty contest.

Finally, a corollary of Proposition 4.9 is that partial transparency is never optimal. The social planner would always choose either full transparency or full opacity. From here we can conclude that the equilibria with partial transparency described in the previous section, are never socially optimal. We return to the comparison of the equilibrium with the social optimum in our model in the next Section. Now we proceed with the properties of thresholds $\hat{\sigma}$ and $\hat{\phi}$.

Proposition 4.10. *Properties of $\hat{\sigma}$:*

1. *$\hat{\sigma} > 0$, if and only if $r < 1 - \left(\sqrt{2} - 1\right)$.*
2. *There exist $\hat{r} \in (0,1)$ such that $\dfrac{\partial \hat{\sigma}}{\partial r} > 0$ for $r < \hat{r}$ and $\dfrac{\partial \hat{\sigma}}{\partial r} < 0$ for $r > \hat{r}$.*
3. *An increase in the precision of prior information and policymakers' information enlarges the region of optimal full transparency: $\dfrac{\partial \hat{\sigma}}{\partial \sigma_{y,h}^{-2}} \leq 0$, $\dfrac{\partial \hat{\sigma}}{\partial \sigma_{y,f}^{-2}} \leq 0$.*
4. *An increase in the precision of private information decreases the region of optimal full transparency: $\dfrac{\partial \hat{\sigma}}{\partial \sigma_{x}^{-2}} \geq 0$.*

Proof. See Appendix D.5. □

Part 1 of Proposition 4.10 states that full opacity may be socially optimal only if beauty-contest argument r is sufficiently small. If beauty-contest argument is large, threshold $\hat{\sigma}$ is negative, meaning that for any precision of prior information and any technological spillover, transparency is optimal. This result is opposite to Morris and Shin's result obtained for a closed economy. The paper by Morris and Shin (2002) shows that in a one-country model opacity may be optimal, if strategic complementarity is sufficiently strong. The opposite result of our paper comes from the cross-border coordination motive, which is absent in the one-country model.

Part 2 of Proposition 4.10 demonstrates the non-monotonic effect of beauty-contest argument r on threshold $\hat{\sigma}$. If beauty-contest argument is low, an increase in r enlarges the value of $\hat{\sigma}$ and widens the region for which opacity may be beneficial. In some sense, an increase in strategic complementarity makes transparency less beneficial. When beauty-contest argument is already relatively high, its further increase lowers the value of $\hat{\sigma}$ and narrows the region for which opacity may be beneficial. Thus, an increase in r makes transparency more desirable.

Parts 3 of Proposition 4.10 shows that higher precision of public information narrows the region for which opacity may be optimal and thus, makes transparency more desirable. Part 4 of Proposition 4.10 shows that higher precision of private information widens the region for which opacity may be optimal and thus, makes transparency less desirable. These two results are intuitively understandable. Better public information, higher incentives to emit it. Better private information, lower incentives to emit the imperfect public information.

The properties of the threshold on the technological spillover, $\hat{\phi}$, are summarized by the following Proposition:

Proposition 4.11. *Properties of $\hat{\phi}$:*

1. $\hat{\phi}\left(\sigma_\theta^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_x^{-2}, r\right) > \overline{\phi}\left(\sigma_\theta^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_x^{-2}, r\right)$.
2. $\hat{\phi} < 1$, *if and only if* $r < 1 - \left(\sqrt{2} - 1\right)$.
3. *Moreover, there exist* $\hat{\hat{r}}$ *such that* $\frac{\partial \hat{\phi}}{\partial r} < 0$ *for* $r < \hat{\hat{r}}$ *and* $\frac{\partial \hat{\phi}}{\partial r} > 0$ *for for* $r > \hat{\hat{r}}$.
4. *Precision of prior information and policymakers information enlarges the optimal region of full transparency:* $\frac{\partial \hat{\phi}}{\partial \sigma_\theta^{-2}} > 0$, $\frac{\partial \hat{\phi}}{\partial \sigma_{y,h}^{-2}} > 0$, $\frac{\partial \hat{\phi}}{\partial \sigma_{y,f}^{-2}} > 0$.
5. *Precision of private information decreases the optimal region of full transparency:*$\frac{\partial \hat{\phi}}{\partial \sigma_x^{-2}} < 0$.

Proof. See Appendix D.6. □

Part 1 of Proposition 4.11 shows that the threshold $\hat{\phi}$ is higher than the threshold $\overline{\phi}$, which divides the full transparency equilibrium and the equilibrium with foreign opacity (see Proposition 4.6).

Parts 2 – 5 of Proposition 4.11 correspond to Parts 1–4 of Proposition 4.10. They state that opacity may be optimal only for weak beauty-contest argument. Moreover, there is an non-monotonic effect of beauty contest on the threshold which divides the region of socially desirable transparency and the region of socially desirable opacity. We also get that an increase in the precision of public information enlarges the region of optimal full transparency, while an increase in the precision of private information narrows it and makes opacity more desirable. Finally, we show that an increase in the precision of prior information leads to an increase in $\hat{\phi}$. Thus, the region for which opacity may be optimal is smaller in more stable economies. This coincides with the findings listed in Proposition 4.10.

The described properties of thresholds $\hat{\phi}$ and $\hat{\sigma}$ allow us to compare the equilibrium with the social optimum. As we have seen in the previous section, intermediate transparency is never socially optimal. Consequently, the equilibrium is optimal neither for $\phi < \underline{\phi}$ nor for $\phi > \overline{\phi}$. If the technological spillovers are strong, such that $\phi < \underline{\phi}$, we have home opacity and foreign transparency in equilibrium. If technological spillovers are weak, such that $\phi > \overline{\phi}$, we have home transparency and foreign opacity in equilibrium. Moreover, in the previous section we show that the threshold $\hat{\phi}$ is higher than the threshold $\overline{\phi}$. As the full transparency is socially desirable for all $\phi < \hat{\phi}$ and as the equilibrium is characterized by the full transparency for $\phi \in [\underline{\phi}, \overline{\phi}]$, we can conclude that for all ϕ in $[\underline{\phi}, \overline{\phi}]$ the equilibrium coincides with the social optimum. These findings are summarized in Proposition 4.12:

Proposition 4.12. *The non-cooperative Nash Equilibrium is socially optimal if and only if* $\phi \in [\underline{\phi}, \overline{\phi}]$.

Proof. See Appendix D.7. □

To put it differently, Proposition 4.12 states that if full transparency is the equilibrium of the non-cooperative game, it is socially optimal. If partial transparency (either home transparency and foreign opacity or home opacity and foreign transparency) is the equilibrium, this is never socially optimal. Thus, for extreme values of ϕ, the non-cooperative equilibrium of the game does not produce the efficient informational structure. For small values of ϕ and strong technological spillovers, there is too little information in comparison with the social optimum. As a result, the policymaker are home opaque while the society would prefer them to be transparent. For high values of ϕ and weak technological spillovers there may be either too little or too much information in the equilibrium. For example, if $\phi \in \left[\overline{\phi}, \hat{\phi}\right]$, the policymakers are foreign opaque while the society would prefer them to be transparent. Thus, there is too little information in the equilibrium. If $\phi > \hat{\phi}$ and the fundamental shocks are sufficiently volatile, such that $\sigma_\theta^{-2} < \hat{\sigma}$, the society would prefer the full opacity, while the equilibrium policy implies home transparency. Obviously, there is too much information in this equilibrium.

The possible non-optimality of non-cooperative equilibrium gives rise to a question: is it possible to replicate the socially optimal result in such an economy? The following proposition shows, that both policymakers are better-off if they choose the socially optimal policy:

Proposition 4.13. *For given* $\left(\sigma_\theta^{-2}, \sigma_{y,h}^{-2}, \sigma_{y,f}^{-2}, \sigma_x^{-2}, r\right)$ *and* $\phi \notin [\underline{\phi}, \overline{\phi}]$, *the social optimum Pareto-dominates the non-cooperative Nash equilibrium.*

Proof. See Appendix D.8. □

When there is partial transparency, both policymakers would be better-off if the optimal information policy was enforced upon them. In other words, a commitment technology imposing full opacity when the social value of public information is negative and full transparency when the social value of public information is positive would increase the welfare in each country. Thus, suppressing "communication wars" can be beneficial for anybody in the economy and negotiations would impose a better equilibrium than the equilibrium in a non-cooperative game.

4.6 Conclusion

The famous paper of Morris and Shin (2002) shows that the social value of public information may be negative. Despite of the extensive debates about this result in the literature, it has never been questioned in the international environment. The goal of our paper is to fill this gap. The other broad issue which we address is the understanding of the process of informational policy-making in such environment.

Moving from autarky to an international environment (or more broadly, to a multi-jurisdictional environment) considerably complicates the matter. Not only multiple sources of information but also multiple policymaker deciding on their communication policy must be taken into account. This creates a strategic dimension which is absent in the simple one-region model studied by Morris and Shin (2002) and their successors.

In turn, this strategic environment generates two issues. The first issue is to find the equilibrium of the non-cooperative game played by policymakers for the sake of their own countries. The second issue is the evaluation of this equilibrium (or possibly, equilibria) with respect to a normative criterion such as the Pareto criterion or social welfare.

We address these issues by solving a communication non-cooperative game played between the country policymakers who have to decide upon which information in their possession to reveal to the public.

The multi-country model displays three types of spillovers: a real or technological spillover, a beauty-contest effect à la Morris and Shin and the informational spillovers created by the fact that the information revealed by policymakers is free and reaches the entire set of private agents in the whole economy. Policymakers can neither modify the information they reveal nor target a subset of agents benefiting from their information policy.

The results reached in this paper shed some light on the two questions mentioned above. There exists a unique linear equilibrium. This equilibrium always involves some revelation by the policymakers. In other words, full opacity is never the equilibrium. Nevertheless, this does not imply that full opacity cannot be a superior policy. Actually, we prove that for some subset of the parameter space, full opacity is Pareto-dominant to the partial transparency reached in the equilibrium. This vindicates the Morris and Shin claim: in international environment the social value of public information may be negative. On the contrary, the full transparency equilibrium which is obtained for intermediate values of the real spillover parameter is the Pareto-dominant solution. The partial communication solutions can be the equilibrium outcome but can never be optimal.

Our research leaves several interesting issues out of the discussion. For example, we study only the value of public information. Nevertheless, deriving the welfare properties of private information would give some insights about the optimal information structure in open economies. Moreover, our model is based on the private loss function from Morris and Shin (2002). Although this function is widely used with in the academic literature, its micro-foundation are still an open question. Thus, testing our finding in a more precise micro-founded example would be a reasonable direction for the future research. For example, we could consider a two-region version of a Lucas-Phelps island economy from Myatt and Wallace (2014). The results of such study could be directly linked to the literature on international monetary games. Combining the communication tools with standard policy tools appears to be a challenging but intriguing task which is also left to further research.

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Appendix

D.1 Expected public loss

From the main text, the expected public loss is given by:

$$E\left(L_{P_j}\right)=\frac{1}{4}\left[(1-r)\,E\left(\overline{a}^j-\Theta^j\right)^2+\left(1-\frac{r}{2}\right)\sigma_{a^j}^2-\frac{r}{2}\sigma_{a^{-j}}^2\right] \tag{4.27}$$

Using the expression for the average private actions (4.16), we get:

$$E\left(\overline{a}^j-\Theta^j\right)^2=E\left(b^j\theta^j+c^jz^j+d^jz^{-j}-\phi\theta^j-(1-\phi)\,\theta^{-j}\right)^2 \tag{4.28}$$

Using equation (4.9), we can rewrite the expected squared gap between the average actions and the fundamentals:

$$E\left(\overline{a}^j-\Theta^j\right)^2=E\left(\left(b^j+\omega^jc^j-\phi\right)\theta^j+\omega^jc^j\left(s^j-\theta^j\right)+\left(\omega^{-j}d^j-(1-\phi)\right)\theta^{-j}+\omega^{-j}d^j\left(s^{-j}-\theta^{-j}\right)\right)^2 \tag{4.29}$$

Taking expectations of (4.29) gives:

$$E\left(\overline{a}^j-\Theta^j\right)^2=\left(b^j+\omega^jc^j-\phi\right)^2\sigma_\theta^2+\left(\omega^{-j}d^j-(1-\phi)\right)^2\sigma_\theta^2+\left(\omega^jc^j\right)^2\sigma_{s,j}^2+\left(\omega^{-j}d^j\right)^2\sigma_{s,-j}^2 \tag{4.30}$$

The volatility of private actions is given by:

$$\sigma_{a^j}^2\equiv\left(n^j\right)^{-1}\int_{i\in S^j}\left(a_i^j-\overline{a}^j\right)^2\,\mathrm{d}i$$

Substitution of the private strategy (4.15) and the average private actions (4.16) gives the following expression:

$$\sigma_{a^j}^2=\left(b^j\right)^2\sigma_x^2 \tag{4.31}$$

Substituting (4.30) and (4.31) into expected public loss (4.27) gives the loss function components (4.22) and (4.23) in the main text.

D.2 Proof of Proposition 4.5 and Proposition 4.6

We prove Proposition 4.5 and Proposition 4.6 together. The proof consists of three steps:

- Step 1. We investigate the choice between home transparency and home opacity and show that there exists some ϕ^* such that: if $\phi<\phi^*$, policymaker chooses home opacity; if $\phi\geq\phi^*$, policymaker chooses home transparency.

- Step 2. We investigate the choice between foreign transparency and foreign opacity and show that there exists some ϕ^{**} such that: if $\phi>\phi^{**}$, policymaker chooses foreign opacity; if $\phi\leq\phi^{**}$, policymaker chooses foreign transparency.

- Step 3. We compare the values ϕ^* and ϕ^{**} and conclude about the existence, unicity and properties of equilibrium

Choice between home transparency and home opacity

The policymaker chooses either home opacity $\sigma_{s,j,j}^{-2}=0$ or home transparency$\sigma_{s,j,j}^{-2}=\sigma_{y,h}^{-2}$. The choice depends on the value of loss component ρ_j^j in the main text. Let's rewrite equation (4.22) in the following way:

$$\rho_j^j=(1-r)\left[\left(b^j+c^j-\phi\right)^2\sigma_\theta^2+\left(\omega^j-1\right)^2\left(c^j\right)^2\sigma_\theta^2-2\left(1-\omega^j\right)\left(\left(b^j+c^j-\phi\right)\right)c^j\sigma_\theta^2+\left(\omega^j\right)^2\left(c^j\right)^2\sigma_{s,j}^2\right]+ \tag{4.32}$$

$$+\left[1-{}^r\!/\!_2\right]\left(b^j\right)^2\sigma_x^2$$

As $b^j+c^j-\phi={}^r\!/\!_2\,(1-2\phi)$, we can rewrite the home loss component as follows:

$$\rho_j^j\left(\zeta^j\right)=(1-r)\left(\tilde{\rho}_j{}^j\left(\zeta^j\right)\sigma_x^2+\frac{r}{2}\left(1-2\phi\right)\sigma_\theta^2\right),$$

where

$$\tilde{\rho}_j{}^j=\left(\omega^j-1\right)^2\left(c^j\right)^2\sigma_\theta^2-2\left(1-\omega^j\right)\left(\left(b^j+c^j-\phi\right)\right)c^j\sigma_\theta^2+\left(\omega^j\right)^2\left(c^j\right)^2\sigma_{s,j}^2+\frac{\left[1-{}^r\!/\!_2\right]}{(1-r)}\left(b^j\right)^2\sigma_x^2$$

Using $\omega^j=\frac{\sigma_{s,j}^{-2}}{\sigma_{s,j}^{-2}+\sigma_\theta^{-2}}$ and $(1-\omega^j)=\frac{\sigma_\theta^{-2}}{\sigma_{s,j}^{-2}+\sigma_\theta^{-2}}$, we obtain:

$$\tilde{\rho}_j{}^j=\frac{\sigma_\theta^{-2}}{\left(\sigma_{s,j}^{-2}+\sigma_\theta^{-2}\right)^2}\left(c^j\right)^2-2\frac{c^j\left({}^r\!/\!_2\,(1-2\phi)\right)}{\sigma_{s,j}^{-2}+\sigma_\theta^{-2}}+\frac{\sigma_{s,j}^{-2}}{\left(\sigma_{s,j}^{-2}+\sigma_\theta^{-2}\right)^2}\left(c^j\right)^2+\frac{\left[1-{}^r\!/\!_2\right]}{(1-r)}\frac{\left(b^j\right)^2}{\sigma_x^{-2}}$$

$$\tilde{\rho}_j{}^j=\frac{c^j\left(c^j-r\left(1-2\phi\right)\right)}{\sigma_{z,j}^{-2}}+\frac{\left[1-{}^r\!/\!_2\right]}{(1-r)}\frac{\left(b^j\right)^2}{\sigma_x^{-2}} \tag{4.33}$$

We can rewrite further, as

$$\begin{aligned}c^j\left(c^j-r\left(1-2\phi\right)\right)&=\left[(1-r)\,\phi+{}^r\!/\!_2-b\right]\left[(1-r)\,\phi+{}^r\!/\!_2-b-r+2r\phi\right]=\\&=\left[(\phi-b)+{}^r\!/\!_2\,(1-2\phi)\right]\left[(\phi-b)-{}^r\!/\!_2\,(1-2\phi)\right]\end{aligned}$$

Thus,

$$\tilde{\rho}_j{}^j=\frac{\left(\phi-b^j\right)^2-{}^{r^2}\!/\!_4\left(1-2\phi\right)^2}{\sigma_{z,j}^{-2}}+\frac{\left[1-{}^r\!/\!_2\right]}{(1-r)}\frac{\left(b^j\right)^2}{\sigma_x^{-2}} \tag{4.34}$$

Moreover,

$$\phi - b^j = \frac{\phi}{(2-r)}\left(r + \frac{2(1-r)\,\sigma_{z,j}^{-2}}{(1-{}^{r}\!/\!{}_{2})\,\sigma_x^{-2} + \sigma_{z,j}^{-2}}\right),$$

from where

$$\tilde{\rho_j}^j\left(\zeta^j\right) = \frac{r^2\left(4\phi^2 - (2-r)^2(1-2\phi)^2\right)}{4(2-r)^2\zeta^j} + \frac{4\phi^2(1-r)}{(2-r)^2\left((1-{}^{r}\!/\!{}_{2})+\zeta^j\right)} + \frac{(1-r)\,\phi^2 r^2}{2(2-r)\left((1-{}^{r}\!/\!{}_{2})+\zeta^j\right)^2} \tag{4.35}$$

The loss component ρ_j^j depends on the relative precision of public information about fundamental $\zeta^j = \frac{\sigma_\theta^{-2}+\sigma_{s,j,j}^{-2}+\sigma_{s,j,-j}^{-2}}{\sigma_x^{-2}}$. Let Δ_j^j denote the difference between the loss under home transparency and home opacity:

$$\Delta_j^j = \tilde{\rho_j}^j\left(\sigma_{y,h}^{-2}, \sigma_{s,j,-j}^{-2}\right) - \tilde{\rho_j}^j\left(0, \sigma_{s,j,-j}^{-2}\right) \tag{4.36}$$

If $\Delta_j^j \leq 0$, the policymaker chooses home transparency (here we use tie-break assumption). If $\Delta_j^j > 0$, the policymaker chooses home opacity.

The derivative of (4.35) over $\sigma_{s,j,j}^{-2}$:

$$\frac{\partial\tilde{\rho_j}^j}{\partial\sigma_{s,j,j}^{-2}} = -\frac{r^2\left(4\phi^2-(2-r)^2(2\phi-1)^2\right)}{4(2-r)^2\left[\sigma_{z,j}^{-2}\right]^2} - \frac{4\phi^2(1-r)}{(2-r)^2\left[(1-{}^{r}\!/\!{}_{2})\,\sigma_x^{-2}+\sigma_{z,j}^{-2}\right]^2} - \frac{\phi^2 r^2(1-r)\,\sigma_x^{-2}}{(2-r)\left[(1-{}^{r}\!/\!{}_{2})\,\sigma_x^{-2}+\sigma_{z,j}^{-2}\right]^3} \tag{4.37}$$

Notice that if $\phi \in \left[\frac{2-r}{6-2r}, 1\right]$, the value $\left(4\phi^2 - (2-r)^2(2\phi-1)^2\right)$ is positive. In this case, all the terms in ((4.37)) are negative. This means that the loss ρ_j^j is decreasing in precision $\sigma_{s,j,j}^{-2}$ for all possible $\sigma_{z,j}^{-2}$. This means that for high values of ϕ loss is decreasing in home precision, $\Delta_j^j < 0$ and policymaker chooses home transparency.

To decide on the sign of Δ_j^j for $\phi < \frac{2-r}{6-2r}$, we rewrite ((4.36)) in the following way:

$$\Delta_j^j = \int_0^{\sigma_{y,h}^{-2}} \frac{\partial\tilde{\rho_j}^j\left(\sigma_{s,j,j}^{-2}, \sigma_{s,j,-j}^{-2}\right)}{\partial\sigma_{s,j,j}^{-2}}\,\mathrm{d}\sigma_{s,j,j}^{-2} \tag{4.38}$$

The derivative of (4.38) over ϕ:

$$\frac{\partial\Delta_j^j}{\partial\phi} = \int_0^{\sigma_{y,h}^{-2}} \frac{\partial^2\tilde{\rho_j}^j\left(\sigma_{s,j,j}^{-2}, \sigma_{s,j,-j}^{-2}\right)}{\partial\sigma_{s,j,j}^{-2}\partial\phi}\,\mathrm{d}\sigma_{s,j,j}^{-2} \tag{4.39}$$

From (4.37) we get:

$$\frac{\partial^2\tilde{\rho_j}^j}{\partial\sigma_{s,j,j}^{-2}\partial\phi} = \frac{r^2\left(2\phi(1-r)(3-r)-(2-r)^2\right)}{(2-r)^2\left[\sigma_{z,j}^{-2}\right]^2} - \frac{8\phi(1-r)}{(2-r)^2\left[(1-{}^{r}\!/\!{}_{2})\,\sigma_x^{-2}+\sigma_{z,j}^{-2}\right]^2} - \frac{2\phi r^2(1-r)\,\sigma_x^{-2}}{(2-r)\left[(1-{}^{r}\!/\!{}_{2})\,\sigma_x^{-2}+\sigma_{z,j}^{-2}\right]^3} \tag{4.40}$$

It is easy to show that $\left(2\phi\left(1-r\right)\left(3-r\right)-\left(2-r\right)^{2}\right)$ is negative if $\phi\in\left(0,\frac{2-r}{6-2r}\right)$. Thus, all the terms in (4.40) are negative. This means that $\frac{\partial\Delta_j^j}{\partial\phi}$ is negative and the loss difference Δ_j^j is decreasing in ϕ. We have shown earlier that $\Delta_j^j<0$ for $\phi=\frac{2-r}{6-2r}$. If ϕ is equal to 0, $\tilde{\rho}_j{}^j=-\frac{r^2(2-r)^2}{4(2-r)\sigma_{z,j}^{-2}}$ and $\Delta_j^j=-\frac{r^2(2-r)^2}{4(2-r)\left(\sigma_\theta^{-2}+\sigma_{y,h}^{-2}+\sigma_{s,j,-j}^{-2}\right)}+\frac{r^2(2-r)^2}{4(2-r)\left(\sigma_\theta^{-2}+\sigma_{s,j,-j}^{-2}\right)}>0$. Thus, for any $\sigma_{s,-j,j}^{-2}$ their exist a value $\phi^*\in\left(0,\frac{2-r}{6-2r}\right)$ such that: Δ_j^j is positive if $\phi<\phi^*$; Δ_j^j is equal to 0 if $\phi=\phi^*$;Δ_j^j is negative if $\phi>\phi^*$. Taking into account tie-break assumption, we conclude that if $\phi<\phi^*$, policymaker chooses home opacity; if $\phi\geq\phi^*$, policymaker chooses home transparency.

Choice between foreign transparency and foreign opacity

The policymaker chooses either foreign opacity $\sigma_{s,-j,j}^{-2}=0$ or foreign transparency$\sigma_{s,-j,j}^{-2}=\sigma_{y,f}^{-2}$. We rewrite the loss component ρ_j^{-j}, which depends on $\zeta^{-j}=\frac{\sigma_\theta^{-2}+\sigma_{s,-j,j}^{-2}+\sigma_{s,-j,-j}^{-2}}{\sigma_x^{-2}}$:

$$\rho_j^{-j}=\left(1-r\right)\left(d^j-\left(1-\phi\right)\right)^2\sigma_\theta^2+\left(1-r\right)\left(\omega^{-j}-1\right)^2\left(d^j\right)^2\sigma_\theta^2-2\left(1-r\right)\left(1-\omega^{-j}\right)d^j\left(d^j-\left(1-\phi\right)\right)\sigma_\theta^2+ \tag{4.41}$$

$$+\left(\omega^{-j}\right)^2\left(1-r\right)\left(d^j\right)^2\sigma_{s,-j}^2-{}^r\!/\!_2\left(b^{-j}\right)^2\sigma_x^2$$

As $\left(d^j-\left(1-\phi\right)\right)={}^r\!/\!_2\left(2\phi-1\right)$, we can rewrite the foreign loss component as follows:

$$\rho_j^{-j}\left(\zeta^{-j}\right)=\left(1-r\right)\left(\tilde{\rho}_j{}^{-j}\left(\zeta^{-j}\right)\sigma_x^2-\frac{r}{2}\left(1-2\phi\right)\sigma_\theta^2\right),$$

where

$$\tilde{\rho}_j{}^{-j}=\left(\omega^{-j}-1\right)^2\left(d^j\right)^2\sigma_\theta^2-2\left(1-\omega^{-j}\right)d^j\left(d^j-\left(1-\phi\right)\right)\sigma_\theta^2+\left(\omega^{-j}\right)^2\left(d^j\right)^2\sigma_{s,-j}^2-\frac{{}^r\!/\!_2}{\left(1-r\right)}\left(b^{-j}\right)^2\sigma_x^2 \tag{4.42}$$

Using $\omega^{-j}=\frac{\sigma_{s,-j}^{-2}}{\sigma_{s,-j}^{-2}+\sigma_\theta^{-2}}$ and $\left(1-\omega^{-j}\right)=\frac{\sigma_\theta^{-2}}{\sigma_{s,-j}^{-2}+\sigma_\theta^{-2}}$, we obtain:

$$\tilde{\rho}_j{}^{-j}=\frac{\sigma_\theta^{-2}}{\left(\sigma_{s,-j}^{-2}+\sigma_\theta^{-2}\right)^2}\left(d^j\right)^2-2\frac{1}{\sigma_{s,-j}^{-2}+\sigma_\theta^{-2}}d^j\left(d^j-\left(1-\phi\right)\right)+\frac{\sigma_{s,-j}^{-2}}{\left(\sigma_{s,-j}^{-2}+\sigma_\theta^{-2}\right)^2}\left(d^j\right)^2-\frac{{}^r\!/\!_2}{\left(1-r\right)}\left(b^{-j}\right)^2\sigma_x^2 \tag{4.43}$$

Then,

$$\tilde{\rho}_j{}^{-j}=\frac{\left(d^j\right)^2}{\sigma_{s,-j}^{-2}+\sigma_\theta^{-2}}-2\frac{d^j\,{}^r\!/\!_2\left(2\phi-1\right)}{\sigma_{s,-j}^{-2}+\sigma_\theta^{-2}}-\frac{{}^r\!/\!_2}{\left(1-r\right)}\left(b^{-j}\right)^2\sigma_x^2 \tag{4.44}$$

Finally,

$$\tilde{\rho}_j^{\,-j} = \frac{d^j\left(d^j - r\left(2\phi - 1\right)\right)}{\sigma_{z,-j}^{-2}} - \frac{r/2}{(1-r)}\frac{\left(b^{-j}\right)^2}{\sigma_x^{-2}} \tag{4.45}$$

As

$$d^j\left(d^j - r\left(2\phi-1\right)\right) = \left[\left(1-\phi\right)^2 - r^2/4\left(1-2\phi\right)^2\right], \tag{4.46}$$

we get the final expression for the foreign loss component:

$$\tilde{\rho}_j^{\,-j}\left(\sigma_{s,-j,j}^{-2}, \sigma_{s,-j,-j}^{-2}\right) = \frac{\left[\left(1-\phi\right)^2 - r^2/4\left(1-2\phi\right)^2\right]}{\zeta^{-j}} - \frac{\phi^2 r\left(1-r\right)}{2\left(\left(1-r/2\right)+\zeta^{-j}\right)^2} \tag{4.47}$$

Let Δ_j^{-j} denote the difference between the loss under foreign transparency and foreign opacity:

$$\Delta_j^{-j} = \tilde{\rho}_j^{\,-j}\left(\sigma_{y,f}^{-2}, \sigma_{s,-j,-j}^{-2}\right) - \tilde{\rho}_j^{\,-j}\left(0, \sigma_{s,-j,-j}^{-2}\right) \tag{4.48}$$

If $\Delta_j^{-j} \leq 0$, the policymaker chooses foreign transparency. If $\Delta_j^{-j} > 0$, the policymaker chooses foreign opacity.

The derivative of (4.47) over $\sigma_{s,-j,j}^{-2}$:

$$\frac{\partial\tilde{\rho}_j^{\,-j}}{\partial\sigma_{s,-j,j}^{-2}} = -\frac{\left[\left(1-\phi\right)^2 - r^2/4\left(1-2\phi\right)^2\right]}{\left(\sigma_{z,-j}^{-2}\right)^2} + r\frac{\phi^2\left(1-r\right)\sigma_x^{-2}}{\left(\left(1-r/2\right)\sigma_x^{-2} + \sigma_{z,-j}^{-2}\right)^3} \tag{4.49}$$

Notice that if $\phi \in \left[\frac{1+r/2}{1+r}, 1\right]$, the value $\left[\left(1-\phi\right)^2 - r^2/4\left(1-2\phi\right)^2\right]$ is negative. In this case, all the terms in (4.49) are positive. This means that the loss ρ_j^{-j} is increasing in precision $\sigma_{s,-j,j}^{-2}$ for all possible $\sigma_{z,-j}^{-2}$. This means that for high values of ϕ loss is increasing in foreign precision, $\Delta_j^{-j} > 0$ and policymaker chooses foreign opacity.

To decide on the sign of Δ_j^{-j} for $\phi \in \left(0, \frac{1+r/2}{1+r}\right)$, we rewrite the loss difference:

$$\Delta_j^{-j} = \int_0^{\sigma_{y,f}^{-2}} \frac{\partial\tilde{\rho}_j^{\,-j}\left(\sigma_{s,-j,j}^{-2}, \sigma_{s,-j,-j}^{-2}\right)}{\partial\sigma_{s,-j,j}^{-2}}\, \mathrm{d}\sigma_{s,-j,j}^{-2} \tag{4.50}$$

The derivative of (4.50) over ϕ:

$$\frac{\partial\Delta_j^{-j}}{\partial\phi} = \int_0^{\sigma_{y,f}^{-2}} \frac{\partial^2\tilde{\rho}_j^{\,-j}\left(\sigma_{s,-j,j}^{-2}, \sigma_{s,-j,-j}^{-2}\right)}{\partial\sigma_{s,-j,j}^{-2}\partial\phi}\, \mathrm{d}\sigma_{s,-j,j}^{-2} \tag{4.51}$$

From (4.49) we get:

$$\frac{\partial^2\tilde{\rho}_j^{-j}}{\partial\sigma_{s,-j,j}^{-2}\partial\phi} = -\frac{\left[\left(2\phi-1\right)\left(1-r^2\right)-1\right]}{\left(\sigma_{z,-j}^{-2}\right)^2} + r\frac{2\phi\left(1-r\right)\sigma_x^{-2}}{\left(\left(1-r/2\right)\sigma_x^{-2} + \sigma_{z,-j}^{-2}\right)^3} \tag{4.52}$$

The coefficient $[(2\phi - 1)(1 - r^2) - 1]$ depends positively on ϕ. If $\phi = 1$, this coefficient equals to $[1 - r^2 - 1] = -r^2 \leq 0$. From here we can conclude that coefficient $[(2\phi - 1)(1 - r^2) - 1]$ is negative for all values of ϕ. Thus, both terms in (4.52) are positive and value $\frac{\partial \tilde{\rho}_j^{-j}}{\partial \sigma_{sj-j}^{-2}}$ is increasing in ϕ. We have shown earlier that Δ_j^{-j} is positive if $\phi \in \left[\frac{1+r/2}{1+r}, 1\right]$. For ϕ equal to $1/2$, Δ_j^{-j} is negative. Consequently, for any $\sigma_{s,-j,-j}^{-2}$ their exist a value $\phi^{**} \in \left(1/2, \frac{1+r/2}{1+r}\right)$ such that: Δ_j^{-j} is positive if $\phi > \phi^{**}$; Δ_j^{-j} is equal to 0 if $\phi = \phi^{**}$; Δ_j^{-j} is negative if $\phi < \phi^{**}$. Taking into account tie-break assumption, we conclude that if $\phi \leq \phi^{**}$, policymaker chooses foreign transparency; if $\phi > \phi^{**}$, policymaker chooses foreign opacity.

Equilibrium

As we have shown, for any $\left(\sigma_x^{-2}, \sigma_\theta^{-2}, \sigma_{y,f}^{-2}, \sigma_{y,h}^{-2}\right)$, $\phi^* < \frac{2-r}{6-2r} < \frac{1}{2}$ and $\phi^{**} > 1/2$. This ensures the existence of equilibrium. The "tie-break" assumption ensures the unicity of equilibrium. Proposition 2 comes immediately with $\underline{\phi} = \phi^*\left(\sigma_{y,f}^{-2}\right)$ and $\overline{\phi} = \phi^{**}\left(\sigma_{y,h}^{-2}\right)$.

D.3 Proof of Proposition 4.7.

For this we use that $\underline{\phi} \equiv \phi^*\left(\frac{\sigma_\theta^{-2}+\sigma_{y,f}^{-2}}{\sigma_x^{-2}}, \frac{\sigma_\theta^{-2}+\sigma_{y,h}^{-2}+\sigma_{y,f}^{-2}}{\sigma_x^{-2}}\right)$ and $\overline{\phi} \equiv \phi^{**}\left(\frac{\sigma_\theta^{-2}+\sigma_{y,h}^{-2}}{\sigma_x^{-2}}, \frac{\sigma_\theta^{-2}+\sigma_{y,h}^{-2}+\sigma_{y,f}^{-2}}{\sigma_x^{-2}}\right)$. Finding the corresponding derivatives of these functions gives the results of Proposition 4.7.

D.4 Proof of Proposition 4.9.

We proceed by several steps:

- Step 1. We show that social loss is either decreasing in relative public precision $\zeta^j = \frac{\sigma_\theta^{-2}+\sigma_{s,j,j}^{-2}+\sigma_{s,j,-j}^{-2}}{\sigma_x^{-2}}$ or has an inverted-U shape. This means that either full transparency or full opacity is optimal.

- Step 2. We show that for given $\left(\sigma_{y,f}^{-2}, \sigma_{y,h}^{-2}, \sigma_x^{-2}\right)$ there exist $\hat{\sigma}$ such that: if $\sigma_\theta^{-2} \geq \hat{\sigma}$, full transparency is optimal for any ϕ; if $\sigma_\theta^{-2} < \hat{\sigma}$, full opacity may be optimal for some ϕ;

- Step 3. We describe this "some ϕ" from Step 2 and show, that there exist such $\hat{\phi}$: if $\phi > \hat{\phi}$, full opacity is optimal.

Social loss is either decreasing in public precision or has inverted-U shape.

The social loss component $\tilde{\rho}_S^j$, which depends on the relative public precision $\zeta^j = \frac{\sigma_\theta^{-2}+\sigma_{s,j,j}^{-2}+\sigma_{s,j,-j}^{-2}}{\sigma_x^{-2}}$:

$$\tilde{\rho}_S^j = \frac{\left(4r^2\phi^2 - 2r^2(2-r)^2(1-2\phi)^2 + 4(2-r)^2(1-\phi)^2\right)}{4(2-r)^2\zeta^j} + \frac{4\phi^2(1-r)}{(2-r)^2((1-r/2)+\zeta^j)} - \tag{4.53}$$
$$-\frac{(1-r)^2\phi^2 r}{(2-r)((1-r/2)+\zeta^j)^2}$$

We can derive social loss (4.53) over ζ^j:

$$\frac{\partial\rho_S^j}{\partial\zeta^j} = -\frac{\left(4r^2\phi^2 - 2r^2(2-r)^2(1-2\phi)^2 + 4(2-r)^2(1-\phi)^2\right)}{4(2-r)^2(\zeta^j)^2} - \frac{4\phi^2(1-r)}{(2-r)^2((1-r/2)+\zeta^j)^2} +$$
$$+\frac{2(1-r)^2\phi^2 r}{(2-r)((1-r/2)+\zeta^j)^3} =$$
$$= -\frac{((2-r)-2\phi(1-r))((2-r)(2-r^2)-2\phi(1-r)(2+2r-r^2))}{2(2-r)^2(\zeta^j)^2} - 2\phi^2(1-r)\frac{2\zeta^j+(2-r)(1-r+r^2)}{(2-r)^2((1-r/2)+\zeta^j)^3} \tag{4.54}$$

The second term in ((4.54)) is negative, the first term is negative if the numerator is positive. Expression $((2-r)-2\phi(1-r))$ is positive, expression $((2-r)(2-r^2)-2\phi(1-r)(2+2r-r^2))$ is positive if $\phi < \frac{(2-r)(2-r^2)}{2(1-r)(2+2r-r^2)}$. It is easy to show that $\frac{(2-r)(2-r^2)}{2(1-r)(2+2r-r^2)}$ is greater than 1, if $r > 2-\sqrt{2}$. In this case for all possible values of ϕ, expression $((2-r)(2-r^2)-2\phi(1-r)(2+2r-r^2))$ is positive and $\frac{\partial\rho_S^j}{\partial\zeta^j}$ is negative for all values of ζ^j. Thus, the social loss is decreasing in the precision of public information.

If $r < 2-\sqrt{2}$, expression $\frac{(2-r)(2-r^2)}{2(1-r)(2+2r-r^2)}$ is less than 1, thus there exist $\tilde{\phi} = \frac{(2-r)(2-r^2)}{2(1-r)(2+2r-r^2)}$, such that for all $\phi < \tilde{\phi}$, both terms in ((4.54)) are negative and the social loss is decreasing in the precision of public information for all values of ζ^j.

If $r < 2-\sqrt{2}$ and $\phi < \tilde{\phi}$, the first term in (4.54) is positive and the second term it is negative. It is easy to show that in this case there exist some positive level $\overline{\zeta}$ such that: $\zeta^j < \overline{\zeta}$, loss is increasing in the precision of public information ζ^j; if $\zeta^j > \overline{\zeta}$, loss is decreasing in the precision of public information ζ^j.

Existence of $\hat{\sigma}$.

Note that $\left.\frac{\partial\rho_S^j}{\partial\zeta^j}\right|_{\zeta^j=1-r/2} < 0$ for any ϕ. This means that public precision under full transparency is always on the decreasing part of function $\tilde{\rho}_S^j(\zeta^j)$, as $\frac{\sigma_\theta^{-2}+\sigma_{y,h}^{-2}+\sigma_{y,f}^{-2}}{\sigma_x^{-2}} > 1$ (Assumption 1). Moreover, if $r < 2-\sqrt{2}$ and $\phi < \tilde{\phi}$, the values of loss goes to minus infinity for small values of ζ^j. This means that for given $\frac{\sigma_\theta^{-2}+\sigma_{y,h}^{-2}+\sigma_{y,f}^{-2}}{\sigma_x^{-2}}$ there always exist ψ such that: $\tilde{\rho_j}^s\left(\psi+\frac{\sigma_{y,h}^{-2}+\sigma_{y,f}^{-2}}{\sigma_x^{-2}}\right)=\tilde{\rho_j}^s(\psi)$.

Let Δ_S^j denote the difference between the social loss under full transparency and full opacity:

$$\Delta_S^j=\tilde{\rho_S}^j\left(\frac{\sigma_\theta^{-2}+\sigma_{y,h}^{-2}+\sigma_{y,f}^{-2}}{\sigma_x^{-2}}\right)-\tilde{\rho_S}^j\left(\frac{\sigma_\theta^{-2}}{\sigma_x^{-2}}\right) \tag{4.55}$$

Thus, $\Delta_S^j\left(\psi;\frac{\sigma_{y,h}^{-2}+\sigma_{y,f}^{-2}}{\sigma_x^{-2}}\right)=0$. An increase in ϕ changes the value of ψ. To find this change we first rewrite Δ_S^j :

$$\Delta_S^j=\int_{\frac{\sigma_\theta^{-2}}{\sigma_x^{-2}}}^{\frac{\sigma_\theta^{-2}+\sigma_{y,h}^{-2}+\sigma_{y,f}^{-2}}{\sigma_x^{-2}}}\frac{\partial\tilde{\rho_S}^j}{\partial\zeta^j}\,\mathrm{d}\zeta^j \tag{4.56}$$

From (4.54), the derivative:

$$\begin{aligned}\frac{\partial^2\rho_S^j}{\partial\zeta^j\partial\phi}&=\frac{-8\phi(1-r)^2(2+2r-r^2)+4(1-r^2)(2-r)^2}{(2-r)^2(\zeta^j)^2}-\frac{16\phi(1-r)}{(2-r)^2((1-{}^r\!/\!_2)+\zeta^j)^2}+ &(4.57)\\&+\frac{8(1-r)^2\phi r}{(2-r)((1-{}^r\!/\!_2)+\zeta^j)^3}=\\&=\frac{-8\phi(1-r)^2(2+2r-r^2)+4(1-r^2)(2-r)^2}{(2-r)^2(\zeta^j)^2}-\frac{8\phi(1-r)((2-r)(1-(1-r)r)+2\zeta^j)}{(2-r)^2((1-{}^r\!/\!_2)+\zeta^j)^3}&(4.58)\end{aligned}$$

Note that the second term in (4.58) is negative. The first term in (4.58) is positive. This means that an increase in ϕ increases ψ. The largest possible value of ψ is reached with $\phi=1$. Denoting $\hat{\sigma}\equiv\psi|_{\phi=1}$, we comes to Proposition 5.1.

Existence of $\hat{\phi}$.

As we have shown in the previous subsection, $\left.\frac{\partial\rho_S^j}{\partial\phi}\right|_{\zeta^j=\psi}<0$ ans $\left.\frac{\partial\rho_S^j}{\partial\phi}\right|_{\zeta^j=\psi+\frac{\sigma_{y,h}^{-2}+\sigma_{y,f}^{-2}}{\sigma_x^{-2}}}>0$. If $\phi<\frac{(2-r)\left(2-r^2\right)}{2(1-r)\left(3-(1-r)^2\right)}\equiv\tilde{\phi}$, the loss is decreasing and the loss under opacity is higher than the loss under transparency: $\tilde{\rho_j}^s\left(\psi+\frac{\sigma_{y,h}^{-2}+\sigma_{y,f}^{-2}}{\sigma_x^{-2}}\right)<\tilde{\rho_j}^s(\psi)$. If $\phi=1$ and $\psi<\hat{\sigma}$, the loss under opacity is lower than the loss under transparency: $\tilde{\rho_j}^s\left(\psi+\frac{\sigma_{y,h}^{-2}+\sigma_{y,f}^{-2}}{\sigma_x^{-2}}\right)>\tilde{\rho_j}^s(\psi)$. Due to continuity, we can conclude that there exist some $\hat{\phi}\in\left(\tilde{\phi},1\right)$, such that if $\phi>\hat{\phi}$, full opacity is optimal and if $\phi\le\hat{\phi}$, full transparency is optimal.

D.5 Proof of Proposition 4.10 (Properties of $\hat{\sigma}$).

If $r \geq 1 - \left(\sqrt{2} - 1\right)$, social loss is decreasing in ζ^j, thus there is no positive $\hat{\sigma}$. The other parts of Proposition 4.10 are obtain from the derivation of implicit function $\Delta_S^j \equiv \int_{\hat{\sigma}\sigma_x^2}^{\left(\hat{\sigma}+\sigma_{y,h}^{-2}+\sigma_{y,f}^{-2}\right)\sigma_x^2} \frac{\partial \tilde{\rho_S}^j}{\partial \zeta^j}\Big|_{\phi=1} \mathrm{d}\zeta^j = 0$.

D.6 Proof of Proposition 4.11 (Properties of $\hat{\phi}$).

If $r \geq 1 - \left(\sqrt{2} - 1\right)$, social loss is decreasing in ζ^j, thus there is no feasible $\hat{\phi}$. The other parts of Proposition 4.11 are obtain from the derivation of implicit function $\Delta_S^j \equiv \int_{\frac{\sigma_\theta^{-2}}{\sigma_x^{-2}}}^{\frac{\sigma_\theta^{-2}+\sigma_{y,h}^{-2}+\sigma_{y,f}^{-2}}{\sigma_x^{-2}}} \frac{\partial \tilde{\rho_S}^j}{\partial \zeta^j}\Big|_{\phi=\hat{\phi}} \mathrm{d}\zeta^j = 0$.

D.7 Proof of Proposition 4.12.

We can show that $\tilde{\phi} \geq \frac{1+r/2}{1+r} \geq \overline{\phi}$. From that, Proposition 4.12 derives immediately.

D.8 Proof of Proposition 4.13.

As the social optimum minimizes the sum of losses,

$$\Delta_j^j \left(\left(\sigma_{s,j,j}^{-2}\right)^* + \left(\sigma_{s,-j,j}^{-2}\right)^*, \tilde{\sigma}_{s,j}^{-2}\right) + \Delta_{-j}^j \left(\left(\sigma_{s,j,j}^{-2}\right)^* + \left(\sigma_{s,j,-j}^{-2}\right)^*, \tilde{\sigma}_{s,j}^{-2}\right) < 0.$$

Due to symmetry, $\Delta_{-j}^j \left(\left(\sigma_{s,j,j}^{-2}\right)^* + \left(\sigma_{s,j,-j}^{-2}\right)^*, \tilde{\sigma}_{s,j}^{-2}\right) = \Delta_j^{-j} \left(\left(\sigma_{s,-j,j}^{-2}\right)^* + \left(\sigma_{s,-j,-j}^{-2}\right)^*, \tilde{\sigma}_{s,-j}^{-2}\right)$. Thus,

$$\Delta_j^j \left(\left(\sigma_{s,j,j}^{-2}\right)^* + \left(\sigma_{s,-j,j}^{-2}\right)^*, \tilde{\sigma}_{s,j}^{-2}\right) + \Delta_j^{-j} \left(\left(\sigma_{s,-j,j}^{-2}\right)^* + \left(\sigma_{s,-j,-j}^{-2}\right)^*, \tilde{\sigma}_{s,-j}^{-2}\right) < 0.$$

This means that each policymaker gets a negative loss difference when moving from the equilibrium to the social optimum. Thus, the social optimum is Pareto-superior.

Titre : La politique macroeconomique optimale dans le context d'incertitude

Mots clés : incertitude, politique macroeconomique optimale, valeur d'information

Résumé : La thèse se compose de quatre chapitres, qui discutent les différents aspects d'élaboration de politique macroéconomique dans le contexte d'incertitude.

Le premier chapitre est consacré à la politique monétaire robuste dans une union monétaire. Un grand nombre de recherches révèle l'importance de chocs spécifiques du pays pour la politique optimale dans une union monétaire. Cependant, ces chocs n'ont pas été étudiés par la littérature sur la politique optimale dans le contexte d'incertitude. Ainsi, le but principal de ce chapitre est de remplir cet espace et montrer que les asymétries entre les régions doivent être tenues en compte en élaborant la politique monétaire robuste. Dans cette recherche, j'utilise un modèle New-Keynesian d'une union de deux pays qui est frappée par les chocs asymétriques. Pour ce modèle, je tire la politique monétaire robuste qui est raisonnablement bonne même pour le worst-case modèle. Je trouve l'effet d'atténuation d'incertitude en cas des chocs dans une région avec la plus forte stickiness des prix. Cela signifie que la banque centrale réagit à ces chocs moins agressivement quand l'incertitude est plus haute. Pour les chocs dans une région avec les prix plus flexibles, je constate une anti-atténuation effet de l'incertitude.

Le deuxième chapitre explore le rôle de préférences gouvernementales incertaines dans un modèle d'interactions de politique monétaire et fiscale. Je montre que les effets d'incertitude de préférences sont liés à l'incertitude multiplicative de l'efficacité de politique. Si les effets de politiques monétaires et fiscales sont connus, l'incertitude de préférences n'alterne pas le résultat de symbiose d'interaction. Dans ce cas-là, l'inflation et la production sont égales à leurs cibles. L'incertitude multiplicative des effets de politique fiscale crée l'excès d'inflation. L'incertitude des effets de politique monétaire crée soit l'excès d'inflation soit l'excès d'inflation négatif avec la production plus haut que la cible et l'inflation plus bas que la cible. Dans ce cas-là, l'incertitude de préférences élargit la valeur absolue des excès. Après avoir étudié l'impact d'incertitude des excès de production et d'inflation, je poursuive les caractéristiques de bien-être dans l'équilibre et discute le design optimale d'autorités pour les types différents d'incertitude.

Le troisième chapitre étudie le rôle de l'information publique et privée dans les sociétés hétérogènes. La littérature qui étudie les impacts d'information sur le bien-être social est étendue. Néanmoins, la plupart de cette littérature est basée dans l'idée que l'économie est homogène, en signifiant que tous les agents sont frappés par les mêmes chocs fondamentaux. Dans ce chapitre je développe une économie de deux régions avec les chocs idiosyncratiques. Pour ce modèle, nous élaborons l'équilibre, l'optimum social et régional et discutons les valeurs sociales, régionales et inter-régionales d'information. Après cela, j'applique cette méthodologie à un exemple de concours de beauté.

Le dernier chapitre étudie des jeux de communication non-coopératifs étant joués par les autorités politiques dans une économie internationale. Chaque agent politique reçoit des signaux sur les chocs réels qui affectent les économies de pays. Cet agent peut révéler ou pas ces signaux reçus. Le modèle est caractérisé par un argument de concours de beauté dans l'utilité et des effets externes inter-régionales. L'équilibre non-coopératif n'est jamais caractérisé par opacité. La plaine transparence peut être le résultat d'équilibre et dans ce cas-là est Pareto-optimum. D'un point de vue normatif, opacité peut être Pareto-optimale: la valeur sociale d'information publique peut être négative dans les économies ouvertes aussi bien que dans les économies fermées. La révélation partielle est un résultat possible, mais jamais Pareto-optimum.

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ÉCOLE DOCTORALE
Sciences de l'homme et de la société (SHS)

Title : Optimal macroeconomic policy under uncertainty

Keywords : uncertainty, optimal macroeconomic policy, value of information

Abstract: The thesis consists of four chapters, which discuss the different aspects of macroeconomic policy elaboration under uncertainty.

The first chapter is devoted to the robust monetary policy in a currency union. A great number of recent researches reveal the importance of country-specific shocks for the optimal policy in a currency union. However, these shocks have been completely overlooked by the literature on optimal policy under model uncertainty. Thus, the main purpose of this chapter is to fill this gap and to show that the asymmetries between regions have to be taken into account when elaborating robust monetary policy. In my research, I use a New-Keynesian model of a two-country currency union which is hit by asymmetric shocks. For this model, I derive the robust monetary policy which works reasonably well even for the worst-case model perturbations. I find the attenuation effect of uncertainty in case of shocks in a region with stronger price stickiness. This means that the central bank reacts to these shocks less aggressively when the extent of model uncertainty is higher. For the shocks in a region with more flexible prices, we find the anti-attenuation effect of model uncertainty.

The second chapter discusses the optimal policy design in a game-theoretical framework. This chapter explores the role of uncertain government preferences in a linear-quadratic model of fiscal and monetary policy interaction. It shows that the effects of preference uncertainty are fastened on multiplicative uncertainty about the policy effectiveness. If the effects of fiscal and monetary policies on the economy are known, preference uncertainty does not alternate the symbiosis result of interaction. In this case, inflation and output are equal to their targets irrespective of the central bank and the government preferences. Multiplicative uncertainty about the fiscal policy creates the inflation bias. Multiplicative uncertainty about the monetary policy effects creates either inflation bias or negative inflation bias with output higher than the target and inflation lower than the target. In this case, preference uncertainty enlarges the absolute value of the output gap, while the effect on the inflation gap depends on the extent of monetary multiplicative uncertainty. After studying the impact of uncertainty on inflation and output gaps, I proceed with the welfare properties of the equilibrium and discuss the optimal conservativeness of authorities for different types of uncertainty.

The third chapter explores the role of public and private information in heterogeneous societies. The literature which studies the impacts of information on social welfare, is extensive. Nevertheless, most of this literature is based on the assumption of homogeneous economy, meaning that all the agents are hit by the same fundamentals shocks. In this chapter, I develop a two-region economy with idiosyncratic shocks. For this model, I derive the equilibrium, social and regional optimum and discuss the social, regional and inter-regional values of information. After that, I apply this methodology to several examples.

The last chapter studies non-cooperative communication games being played by policymakers in an international economy. Each policymaker receives signals on the real idiosyncratic shocks which affect the country economies. It has the choice of revealing or not the received signals. The model is characterized by a beauty-contest argument in the utility function and cross-border real spillovers. The non-cooperative equilibrium is never characterized by no revelation. A full transparency outcome may be the equilibrium outcome and is then Pareto-optimal. From a normative point of view, no revelation may be Pareto-optimal: the social value of public information may be negative in international economies as well as in closed economies. Partial revelation schemes are possible outcomes but never Pareto-optimal.

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