Trading strategies and endogenous asset price movement

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TRADING STRATEGIES AND ENDOGENOUS ASSET PRICE MOVEMENT

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"Ce qui est amusant avec la Bourse,
c'est qu'à chaque fois qu'un mec achète une action,
les autres ont l'impression de faire une bonne affaire."
Jean Yanne

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La théorie néoclassique postule que lorsque les marchés financiers sont parfaitement efficients, le prix de chaque actif reflète les fondamentaux économiques qui lui sont sous-jacents. L'observation des marchés financiers a cependant montré de façon assez convaincante pour faire consensus que le lien entre l'évolution des fondamentaux économiques et des marchés devient parfois plus lâche.

Cette déconnexion est particulièrement visible lors des crises financières. Durant la crise des subprimes par exemple, un choc fondamental sur le marché des CDO, qui représentait autour de 640 milliards de dollars en 2007, a occasionné pour les seuls ménages américains une perte totale de richesse financière évaluée à 8.6 trillions de dollars sur l'année 2008 (Cordell et al., 2012, IMF, 2009). Les marchés financiers eux-mêmes ont donc diffusé et amplifié le choc initial.

Ce travail de thèse étudie les stratégies des investisseurs et leurs interactions pour tenter de comprendre une partie des dynamiques non-fondamentales à l'œuvre au sein du système financier. Il s'inscrit dans l'approche "endogène" de l'évolution du prix des actifs financiers, qui postule qu'un choc peut avoir, dans un système interconnecté complexe, des effets différents selon les contraintes pesant sur les investisseurs, leur psychologie, et les règles d'investissement qu'ils suivent.

Nous considérons différentes stratégies d'investissement parmi les plus répandues

sur les divers marchés financiers et analysons théoriquement ou empiriquement comment, par le jeu des interdépendances et des dynamiques induites, la qualité intrinsèque de l'actif peut s'effacer devant les variations de prix endogènes.

Le chapitre 1 s'attache à la diversification de portefeuille, stratégie visant à réduire le risque individuel par la détention de plusieurs titres non parfaitement corrélés. Nous montrons au travers d'un modèle théorique que la diversification, par l'interdépendance qu'elle crée entre les investisseurs et entre les titres, induit des mécanismes de propagation des chocs fondamentaux qui peuvent se révéler dangereux d'un point de vue systémique. Nous mesurons les deux effets – réduction du risque individuel vs augmentation du risque systémique - afin de discuter de la désirabilité globale de la diversification.

Le deuxième chapitre considère les stratégies d'investissement basées sur le "style-investing", c'est-à-dire le groupement, en différentes classes ou "styles", de titres financiers partageant des caractéristiques communes. Nous postulons que ces stratégies créent un co-mouvement excessif entre titres d'un même style, qui vont être vendus et achetés ensemble. Appliquant cette intuition aux obligations en considérant les notes des agences en tant que styles, nous montrons empiriquement qu'une obligation qui change de note se met en effet à varier comme sa nouvelle note, même quand les fondamentaux économiques ne le justifient pas.

Dans le troisième chapitre nous étudions comment s'imbriquent les stratégies de trois types d'investisseurs opérant sur le marché des changes : les carry traders, les chartistes et les fondamentalistes[1]. Notre modèle théorique suggère que

1. La stratégie du carry trade consiste à emprunter dans des devises à taux bas pour prêter dans des devises à taux hauts. Celle des chartistes est basée sur l'achat des monnaies qui se sont récemment appréciées et la vente de celles qui se sont dépréciées. Enfin l'approche fondamentaliste consiste à acheter (ou vendre) les devises considérées comme sous-évaluées (ou sur-évaluées) par rapport à leur valeur fondamentale.

l'interaction entre ces trois règles d'investissement peut expliquer la déconnexion bien documentée entre le taux de change et sa valeur fondamentale (Meese and Rogoff,1983 ; Mark, 1995), et provoquer un effondrement endogène des taux de change.

La déconnexion entre le prix d'un actif et sa valeur fondamentale a suscité de nombreuses théories, allant des "esprits animaux" évoqués par Keynes dès 1936 au modèles de bulles rationelles. Ainsi il existe de nombreuses sources de mouvement endogène de prix. De plus, il est difficile de dissocier les composantes fondamentales et non-fondamentales dans les variations réelles du prix d'un actif, aucune des deux n'étant isolément mesurable.

Cette réalité rend difficile l'étude des "mouvements non-fondamentaux" dans une approche globale et systématique. C'est pourquoi le but de cette thèse est plus modestement de pouvoir caractériser des dynamiques endogènes émergeant dans un cadre donné.

Pour cela, nous nous appuyons dans les chapitres 1 et 3 sur des modèles théoriques. Nous choisissons de ne pas aborder le sujet vaste et complexe de la psychologie des investisseurs, en nous concentrant sur la mécanique des règles d'investissement usuelles et des contraintes de liquidité. Ces contraintes permettent de modéliser des réponses automatiques et inéluctables qui, tout en représentant une source importante de variation endogène de prix, peuvent être aisément identifiées et formalisées.

Dans le chapitre 2 qui est de nature empirique, nous élaborons une stratégie d'estimation consistant à identifier des changements de notes pour lesquels style-investing et fondamentaux économiques ont des implications différentes sur le co-mouvement. Ainsi pour ces changements, une évolution du co-mouvement conforme aux prédictions du “style investing”, dominant celles des fondamentaux, révèle la présence de co-mouvement endogène.

Le but final de cette thèse est de pouvoir identifier ex-ante certaines situations dans lesquelles le risque endogène semble s'accumuler, condition nécessaire pour élaborer des règles prudentielles. Cet effort place notre travail dans le champ d'une littérature en plein essor, dont Shin, Adrian, ou Brunnermeier sont parmi les principaux contributeurs. Nous proposons dans ce qui suit un court résumé de chaque chapitre.

Chapitre 1

Les stratégies de diversification de portefeuille, si elles sont optimales au niveau individuel, créent des liens entre titres et investisseurs qui peuvent être vecteurs de contagion sur les marchés. Dans ce chapitre nous mesurons l'ampleur de cette externalité de contagion et les implications qu'elle peut avoir sur la désirabilité de la diversification d'un point de vue systémique.

Nous proposons un modèle théorique dans lequel N investisseurs possèdent chacun n titres, et sont forcés de vendre des titres en réponse à des chocs négatifs sur leur richesse. Les investisseurs sont sujets à un biais domestique : chacun détient en priorité des titres qui lui sont proches dans le réseau financier. Considérons par exemple un marché de 5 investisseurs diversifiés à hauteur de 2 ($N = 5$ et $n = 2$). Dans notre modèle l'investisseur 1 va détenir les titres $i = 1$ et $i = 2$ tandis que $I = 2$ va détenir $i = 2$ et $i = 3$. Mathématiquement le réseau peut être défini comme ceci :

$$A = \left\{ \begin{array}{ccccc} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{array} \right\}$$

où chaque ligne représente un investisseur et chaque colonne un titre.

Le modèle étudie la propagation d'un choc exogène dans ce réseau. En pré-multipliant la matrice A par un vecteur de chocs sur les prix à la période *t-1,* noté $\triangle\mathbf{P_{t-1}}$, on obtient le vecteur des pertes totales par investisseur sur la période. Sous l'hypothèse que les ventes des investisseurs sont une fonction linéaire de la perte qu'ils subissent, la réponse en terme de vente aux chocs $\triangle\mathbf{P_{t-1}}$ peut être exprimée comme $\triangle\mathbf{Q_t^{lt}} = \frac{r}{n^2} A^\intercal A \triangle\mathbf{P_{t-1}}$, où r est un paramètre représentant la réponse en termes de ventes à une perte donnée, lié à la contrainte régulatoire. Ces ventes vont à nouveau faire baisser les prix, générant un relation de récurrence entre $\triangle\mathbf{P_{t-1}}$ et $\triangle\mathbf{P_t}$. Pour un choc donné à $t = 0$ nous obtenons le système suivant :

$$\Longrightarrow \triangle\mathbf{P_t} = [(\frac{r/h}{n^2})A^\intercal A]^t \triangle\mathbf{P_0}$$

où h traduit la facilité avec laquelle ces ventes seront absorbées par le marché. De cette dynamique des prix on déduit l'expression du vecteur des pertes totales par investisseur :

$$\mathbf{I} = \frac{1}{n} A \sum_{t=0}^{t=+\infty} [(\frac{r/h}{n^2})A^\intercal A]^t \triangle\mathbf{P_0}$$

A partir de cette expression nous approfondissons deux points.

Premièrement, nous résolvons le système pour obtenir les expressions analytiques des matrices de covariance entre investisseurs et entre titres. Elles montrent une dépendance à des variables telles que le degré de complétude des marchés et son aversion au risque. Ces covariances traduisent l'apparition de liens endogènes, alors que les titres étaient fondamentalement indépendants à l'origine. Mathématiquement la solution analytique s'appuie sur le fait que A est une matrice circulante, ce qui lui confère de puissantes propriétés.

En second lieu, nous étudions l'impact total de la diversification sur le risque systémique en observant la probabilité qu'un nombre η d'investisseurs fassent faillite, c'est-à-dire subissent une perte totale sur leur portefeuille excédant leur capital K. Cette probabilité peut être obtenue en utilisant le fait que le vecteur I est normalement distribué. On obtient le résultat suivant : quand le niveau de diversification n augmente, la probabilité qu'aucun investisseur ne fasse faillite augmente, mais la probabilité qu'un large nombre d'investisseurs chutent simultanément augmente aussi. Ainsi, en prenant en compte le coût extrêmement important des faillites massives, la désirabilité de la diversification peut être remise en question.

Cependant, cette conclusion change si l'on prend en compte la possibilité de "panique", c'est-à-dire de réponses non-linéaires des investisseurs aux chocs. Les niveaux de diversification intermédiaires s'avèrent alors l'option la moins désirable : ils créent des liens entre investisseurs qui sont vecteurs de contagion, sans toutefois aller assez loin dans la minimisation du risque au niveau individuel. Les niveaux élevés de diversification deviennent la meilleure option pour la plupart des valeurs des paramètres. Ceci suggère l'existence d'un seuil à partir duquel la diversification devient positive sans ambigüité.

Le chapitre 2 va creuser l'idée de corrélations endogènes que nous avons commencé à aborder.

Chapitre 2

Pour réduire la complexité du problème d'allocation de portefeuille et le coût de la collecte d'information sur chaque titre, les investisseurs ont tendance à adopter des stratégies de "style investing" qui les amènent à acheter et vendre ensemble les titres d'une même classe, créant ainsi un co-mouvement déconnecté de leur valeur fondamentale. Ce chapitre montre empiriquement l'existence d'un tel co-mouvement autour de ce qui constitue sans doute la plus suivie des classifications sur le marché financier : le regroupement des obligations en fonction de leur notation par les agences.

Pour mesurer ce rôle des notes, nous estimons l'impact d'un changement de note sur le co-mouvement de l'obligation avec l'index qu'elle rejoint, et celui qu'elle quitte. La régression suivante est menée 5 mois avant changement puis 5 mois après le changement :

$$R_{i,t} = \alpha_i + \beta_{i,I} R_{I,t} + \beta_{i,I'} R_{I',t}$$

où $R_{i,t}$, $R_{I,t}$ et $R_{I',t}$ représentent la différence première de l'écart de rendement par rapport au bons du trésor américains pour l'obligation i, et les deux index I et I'. Nous notons $\hat{\beta}$ et $\hat{\beta}^*$ les bêtas pré et post changement. L'évolution dans la valeur du bêta de l'index I dans la régression après un changement de note est donc donnée par $\triangle_i^\beta = \hat{\beta_{i,I}}^* - \hat{\beta_{i,I}}$, qui constitue notre indicateur de l'évolution du co-mouvement entre l'obligation i et l'index I.

Du point de vue du style-investing, l'obligation dont la note a changé devrait commencer à être échangée au sein de son nouvel index, et cesser de l'être avec l'index quitté. Elle devrait donc voir son bêta augmenter avec l'index joint, et baisser avec celui quitté : $\triangle^{\beta}_{i,in} = \hat{\beta_{i,I'}}^* - \hat{\beta_{i,I'}} > 0$, et $\triangle^{\beta}_{i,out} = \hat{\beta_{i,I}}^* - \hat{\beta_{i,I}} < 0$. La difficulté empirique est qu'il y a deux raisons qui peuvent expliquer une telle évolution : le style-investing, et le fait qu'un changement de note traduit un changement dans la caractéristique fondamentale du titre.

Pour dissocier les deux influences, nous identifions des changements de note pour lesquels ces deux sources de variation poussent en sens contraire. Deux tests sont mis en place pour distinguer les deux effets, l'un basé sur un changement quelconque de note, l'autre sur le basculement entre la catégorie "investissement" (note $\geq$ BBB-) et la catégorie "high yield" (spéculatif : note $\leq$ BB+).

La premier test s'appuie sur le fait que, dans un modèle de facteur généraliste, un titre plus mal classé est considéré comme plus risqué, et réagit donc davantage aux facteurs de risque globaux. Il doit donc avoir un bêta de marché plus fort, ce qui implique un plus fort bêta avec n'importe quel index qu'un titre mieux noté. Ainsi d'un point de vue fondamental, le bêta d'une obligation dégradée doit augmenter, avec l'ensemble qu'il quitte comme avec celui qu'il rejoint. De la même façon une obligation qui a été "upgradée" devrait voir son bêta avec les deux index baisser.

Combinant ces prédictions avec celle de l'investissement par classe, nous obtenons le tableau suivant

	downgrade	upgrade
avec index joint	$\triangle^f > 0$, $\triangle^s > 0$	$\triangle^f < 0$, $\triangle^s > 0$
avec index quitté	$\triangle^f > 0$, $\triangle^s < 0$	$\triangle^f < 0$, $\triangle^s < 0$

Dans deux cas, fondamentaux et style-investissement ont des implications opposées. Notre premier test porte donc sur le signe de $\hat{\beta_{i,I}}^* - \hat{\beta_{i,I}}$ pour ces deux cas. Si celui-ci est conforme au signe attendu de la composante $\triangle^s$ malgré l'influence contraire de $\triangle^f$, alors nous pouvons conclure à un impact significatif de l'investissent par style.

Nous concluons en effet à un rejet de l'hypothèse nulle d'absence d'effet pour le premier test.

Le second test se concentre sur un changement de note particulier : le basculement entre BBB- et BB+, frontière importante dans la psychologie de marché entre les "investment grades" et "high-yield", obligations spéculatives. Si cette partition en deux grands ensembles est utilisée dans le cadre de l'investissement par classe, alors on peut s'attendre à observer un premium de co-mouvement entre titres à ce niveau. Une obligation passant par exemple de BB+ à BBB- devrait subir un changement de bêta plus fort que les autres, puisque elle va perdre le premium de co-mouvement qu'elle avait avec les titres notés BB+, mais également le premium qu'elle avait avec l'ensemble plus large du "investment grade".

Ainsi notre deuxième test porte sur le signe de la différence entre les changements de bêtas observés pour les mouvements entre BBB- et BB+ et la moyenne. Il ne permet pas de rejeter l'hypothèse nulle. Cette absence de résultat pose question et nous conduit à explorer plus à fond les mouvements autour de cette frontière. Nous montrons ces mouvements ont en effet un statut particulier.

D'abord, les titres passant de BBB- à BB+ voient leur bêta par rapport au marché augmenter plus que la moyenne, phénomène surprenant car ces mouvements n'ont historiquement pas un impact plus important que les autres sur la probabilité de défaut de l'emprunteur. Ainsi le marché semble sanctionner (ou promouvoir) les titres passant de BBB- à BB+ (ou de BB+ a BBB-) avec une sévérité particulière, sans raison fondamentale.

Par ailleurs il semble que l'investissement par style explique une portion moindre de l'évolution des rendements pour les titres avec des notes moyennes : le R2 d'une régression du rendement d'une obligation sur celui de son index est plus haut pour les titres ayant des notes très hautes ou très basses. Dans le cas des obligations à note basse, ceci peut s'expliquer par le fait que les firmes qui émettent sont souvent plus petites et moins connues, de telle sorte que les investisseurs se fient plus aux notes en ce qui les concerne. Pour les notes hautes, ceci peut peut-être s'expliquer par des considérations régulatoires, qui incitent les investisseurs à penser plus en terme de notes que de titres individuels.

Dans le chapitre 3 nous délaissons la notion de co-mouvement ou de covariance endogènes, se concentrant sur celle de risque endogène.

Chapitre 3

On examine dans ce chapitre la possibilité que les stratégies d'investissement utilisées sur le marché des changes expliquent la déconnexion entre taux de change et fondamentaux économiques (Meese et Rogoff,1983 ; Mark, 1995) . Plus particulièrement, on s'intéresse ici à la possibilité que la structure du marché des changes puisse causer un effondrement des monnaies à fort taux d'intérêt, et ce même dans un cadre de relative stabilité des grand indicateurs macroéconomiques.

Nous présentons un modèle dans lequel s'imbriquent les stratégies de trois types d'investisseurs opérant sur le marché Forex : les "carry traders", les fondamentalistes et les chartistes. Ces investisseurs, basés dans une économie large et développée à taux d'intérêt faible, peuvent prendre des positions soit dans leur propre monnaie, soit dans celle d'un pays B plus petit ou moins développé, à fort taux d'intérêt. La demande émanant des investisseurs va impacter la valeur de la monnaie de B.

Les carry traders sont soumis à une contrainte de liquidité que nous modélisons suivant Shin (2010) par :

$$\overset{\circ}{q_t} = \frac{1 + LR_{c,t-1}}{1 + R_{c,t-1}} \overset{\circ}{q}_{t-1}$$

où $\overset{\circ}{q_t}$ représente la demande d'obligations du pays B par les carry traders. $R_{c,t-1}$ représente le différentiel de taux entre les deux pays entre *t-1* et *t*, et L représente l'effet de levier maximum autorisé par l'autorité régulatrice. Cette contrainte va être génératrice de pro-cyclicité : tout gain en capital réalisé à la période t va permettre au carry trader de financer de nouvelles positions à *t+1*, et ce tant qu'il n'a pas atteint son allocation optimale.

Pour les chartistes et les fondamentalistes nous traduisons sous une forme simplifiée les règles d'investissement observées concrètement sur le marché des changes. Néanmoins nous imposons que ces règles soient profitables. La demande des chartistes est décrite par :

$$q_{m,t} = \frac{MA(\triangle S)_x}{\beta + \alpha Var(\triangle S)_x}$$

où $MA(\triangle S)_x$ et $Var(\triangle S)_x$ représentent la moyenne et variance des évolutions du taux de change sur les x périodes précédentes, et α et β sont des constantes. Cette expression décrit la stratégie suivante : les chartistes achètent une devise si elle s'est récemment appréciée, ou la vendent si elle a baissé.

La demande des fondamentalistes est modélisée par :

$$q^*_{f,t} = \frac{\Phi(sign(F - S_t) = sign(MA(\triangle S)_x)) \times (F - S_t)}{b}$$

où $(F - S_t)$ représente la distance actuelle entre le taux de change S_t et sa valeur fondamentale F, pondérée par $\Phi(sign(F - S_t) = sign(MA(\triangle S)_x))$, la probabilité estimée par le fondamentaliste que le taux de change tende actuellement vers les fondamentaux. Les fondamentalistes vont donc acheter les devises "sous-évaluées" et vendre les "surévaluées", en prenant des positions plus larges s'ils estiment que le marché va prochainement entrer dans une phase de convergence vers les fondamentaux.

Ce cadre forme un modèle non-linéaire, que l'on simule d'abord dans sa version déterministe, puis en introduisant des chocs stochastiques. Les simulations montrent que l'interaction entre les 3 agents entraine comme attendu une déconnexion du taux de change par rapport à sa valeur fondamentale F , mais également un risque d'effondrement du taux de change des pays à taux d'intérêt fort, en accord avec les travaux de Brunnermier et al. (2009).

La dynamique est la suivante : les carry traders déplacent leur capital vers les pays à taux fort, mais ce déplacement est lent du fait de la contrainte de liquidité. Ce processus induit une tendance d'appréciation de la monnaie du pays B, tendance qui va être saisie et amplifiée par les chartistes. Mais cette appréciation va finir par s'essouffler sous l'influence des fondamentalistes, et du fait que de plus en plus de carry traders atteignent les positions visées. Cet essoufflement conduit les chartistes à se défaire de leurs positions d'achat. Les fondamentalistes intensifient alors leur action, convaincus qu'une convergence vers les fondamentaux s'opère. Les ventes combinées de ces deux agents induisent une baisse significative du taux de change. L'effondrement apparaît lorsque cette baisse est suffisante pour toucher la contrainte de liquidité des carry traders, entrainant des "appels de marge" qui les forcent à liquider leurs positions.

Dans le modèle la relation entre la contrainte de liquidité et le risque d'effondrement est non-linéaire. Lorsque les carry traders sont très contraints ils ne parviennent pas à instaurer une tendance durable d'appréciation, ce qui de fait empêche un écroulement. Lorsque la contrainte ne sature pas, en revanche, les carry traders peuvent immédiatement déplacer autant de capital qu'ils le souhaitent, en accord avec les prédictions d'un marché parfaitement efficient. Les niveaux de contrainte intermédiaires apparaissent une option risquée : ils permettent assez de mobilité du capital pour générer une tendance d'appréciation, tout en laissant les carry traders exposés aux appels de marge.

Les dynamiques générées par le modèle peuvent aussi expliquer en partie certains "puzzles" observés sur le marché des changes, tels que les profits dits excessifs des carry traders et des chartistes, ou la déconnexion entre taux des change et fondamentaux sur le court et moyen terme.

Sur le premier point, nous constatons que, dans notre cadre théorique, chartistes and carry traders génèrent tous deux d'importants profits. Dans le cas des carry traders, ces profits paraissent une rémunération légitime : ces agents sont très exposés au risque d'effondrement, notamment du fait de leur utilisation de l'effet de levier. En revanche la performance des chartistes reste bonne même en temps de crise. Ces caractéristiques sont en accord avec les travaux empirique de Darvas (2009) sur la performance des stratégies de carry trade à fort effet de levier, et de Menkhoff et al. (2012) sur la forte résistance des chartistes en temps de crise.

Pour examiner la question de la déconnexion, nous simulons 10 000 dynamiques pour la valeur fondamentale du taux de change F, dont nous tirons les 10 000 dynamiques de taux change associées via notre modèle. Le lien entre F et S_t qui s'en déduit est en accord avec la littérature empirique sur le sujet. En particulier, les déviations du taux de change par rapport à sa valeur fondamentale s'éteignent à un rythme cohérent avec les influents travaux de Mark (1995).

Ainsi chaque chapitre décrit un comportement et les mouvements de prix endogènes qu'il génère. Cette structure se refète dans les titres que nous donnons aux tois parties : diversification et risque systémique, style-investing et co-mouvement, et enfin stratégies sur le marché des changes et risque d'effondrement.

General introduction

General introduction

General motivation of the thesis

Last year I went wakeboarding, a type of "snowboarding on water". The wakeboarders were dragged by a machine similar to a ski-tow, and could go around the lake for as long as they wanted. In the morning the "wakeboard market" ran smoothly, but in the afternoon more people arrived, which led to a sizable queue. This increased the cost of a run in terms of waiting time, which triggered two responses by the participants. First, people started to stay on the wakeboard much longer to compensate for the cost. Second, beginners stopped trying because they tend to fall instantly when taking off, so that the queue became composed only of skillful wakeboarders who are able to stay longer.

Both responses thus led to a rise in the average waiting time between two people[2]. The queue then had a direct impact on the waiting time through the number of people queuing, and this indirect one, both mutually reinforcing. The initial increase in people queuing was exogenous, but its impact was potentially aggravated by the endogenous response of the participants of the wakeboard market,

2. Though the impact of beginners being driven out is ambiguous because this response also had a positive impact by reducing the size of the queue.

even though in this case those responses were perfectly rational.

Similar situations can arise in the financial markets. Investors sometimes respond to exogenous shocks in ways that may ultimately magnify their impact, sometimes much beyond the initial shock itself. Consider the subprime crisis for instance: a shock in the CDO market, which amounted to 641 billion dollars coming into the crisis, eventually lead to a total fall in financial wealth of about 8.6 trillion dollars for US households in 2008 alone (Cordell et al., 2012, IMF, 2009).

Consequently, in order to propose policies capable of enhancing financial stability, economists need to gain a deep understanding of the dynamics implied by "the actions of market participants which are hard wired in the system" (Cont and Wagalath, 2014) that may lead to such undesirable outcomes at the systemic level. The goal of this thesis is to theoretically and empirically identify such actions and their impact, by studying how non-fundamental price movements may arise from specific actions or rules followed by investors.

In particular we investigate how endogenous price movement may arise from certain investment strategies which are followed extensively by market participants. In a nutshell:

- Chapter 1 focuses on portfolio diversification, a popular investment principle based on the fact that holding many assets which are only partially correlated lowers overall portfolio risk. This chapter postulates that portfolio diversification, despite its positive impact at the individual level, may have a negative externality by raising the correlations between asset prices. The reason is that when a negative shock hits a particular asset, investors who hold it may sell other healthy assets to finance their losses, thereby lowering their prices, which in turn will impact other investors, etc.

- Chapter 2 studies style investment, an approach based on grouping assets into different classes, or "styles", and trading at the style level in order to reduce the cost of gathering information on each single asset. The second chapter posits that this strategy will also lead to non-fundamental comovement, as assets will be sold and bought together as part of a similar index. We apply this reasoning to the impact of credit rating on bond comovement.

- Chapter 3 considers the interaction of three different investment rules in the foreign exchange market: 1) Carry trade which is based on borrowing in a low interest currency and lending in a high interest one; 2) Chartism, an approach of buying/selling currencies which have recently appreciated/depreciated; and 3) fundamentalism, which is based on buying/selling currency which are below/above their fundamental values. We argue that the interaction between these rules provide an explanation for the well-documented disconnect between exchange rates and fundamentals, and may trigger "endogenous crashes". This occurs because the demand for high yield currency by carry traders will create a build-up phase for the currency, which may collapse swiftly under the influence of chartists and fundamentalists.

The thesis broad interest therefore lies with how prices may deviate from "fundamentals" through the actions of market participants, which raises wider questions: Are financial markets efficient? If not, what causes the inefficiencies? Can we prevent them? etc. In a context where the general audience is quick to condemn "greedy" or "corrupted" traders for the recent financial crises, we some take time for discussion before getting into specifics.

The academic world has long been penetrated by the issue of the efficiency of financial markets[3]. One view, led by Nobel's prize laureate Eugene Fama, holds that financial markets are organized in the most efficient manner possible. According to Fama (1970) there are 3 forms of market efficiency, and the strongest one, which he defines as a situation in which all available information is embedded into prices, has the best empirical performance. Fama (1998) also famously argues that empirical evidence challenging this strong efficiency are simply "luck results" and bound to vanish as the econometric apparatus increases.

Nevertheless, the so-called "efficient market hypothesis" has been heavily and increasingly challenged. On empirical grounds, academic research has uncovered many "puzzles" that are impossible to reconcile with perfectly efficient markets. Shiller (1981) shows that stock prices fluctuate more than warranted by their dividends. Mehra and Prescott (1985) show there exists a difference in the historical returns between stocks and government bonds, that may not be fully explained by differences in risk between both assets. Fama (1984) himself admitted a departure from his theory by showing that high interest rate currencies tend to appreciate, creating clear profit opportunities that should not exist in theory.

On theoretical grounds, the efficient market hypothesis faced similar opposition. Numerous sources of deviations from fundamentals have been put forward, which may be classified into three ensembles:

- A first group gathers theories which involve a departure from the assumption that investors always behave rationally. This ensemble notably includes the influential "prospect theory" of Kahneman and Tversky (1979), which earned Daniel Kahneman the Nobel prize in 2002. Studying the decision-making process of individuals under risk, these authors show that individuals are subject to biases such

3. see Malkiel (2003) for a survey

as "endowment effect"[4] or "loss aversion"[5]. Applied to financial markets such biases have a deep impact on the asset demands of investors, and thus on prices (Benartzi and Thaler, 1995, Barberis et al., 2005). Subsequent research identified numerous other biases, surveyed by Haselton et al. (2005).

Another departure from rationality stems from our "animal instincts", as first pointed out by Keynes (1936). This view holds that all human beings inherently react in similar fashion to similar shocks. Financial crises in particular are seen as the result of our tendency to panic in stressful situations. Recent contributors to this view include Akerlof and Shiller[6] (2009) who discuss the roles played by instincts such as "overconfidence" or "desire for fairness" during the 2008 crisis.

- A second view is based on "bounded rationality", i.e. on the idea that the investor is rational but faces a number of constraints that may prevent him from making the optimal choice. The constraint may be of an informational nature : the investor does not have access to the complete information subset to make a fully informed decision (Mishkin, 1990). It may also come from regulation (Danielsson et al., 2012) or transaction costs (Grossman and Stiglitz, 1980). Finally the investor may be limited by his own capacities. An agent faced with a complex problem and required to act quickly may not be able to efficiently treat all the information in time. In this case, he is forced to use "heuristics" rather that perfectly rational methods when taking a decision. Important contributors to this view include Simon (1972), or Rubinstein (1998).

- A last ensemble postulates that deviations from fundamentals occur because optimal actions at the micro level may not always translate into an optimal out-

4. Attaching a higher value to the goods they own
5. Preferring not making a loss to making a gain
6. Nobel's prize winners

come at the systemic level. Such situations notably arise when the interests of the market participants do not coincide with that of society. The earliest contribution of this vein again traces back to Keynes (1936), who points out that profit maximizing trading is more about guessing the actions of other investors than it is about guessing the "right" value of the asset. Episodes of "financial bubbles" (Blanchard and Watson, 1982) or "bank runs" (Diamond and Dybvig, 1983) have famously been explained through such a misalignment of interests: investors/depositors who are caught in a bubble or a bank run act rationally on an individual level, however their combined actions lead to to very costly outcomes for society. Suboptimal pricing may also emerge from perverse interactions between heterogeneous investors, as in De Grauwe and Grimaldi (2006).

These three ensembles are not mutually exclusive. Consider for instance a given observed behavior that we suspect of generating non fundamental price movement: herding. With respect to the three sources we have discussed, herding may be considered either an irrational "animal instinct", a rational response from an uniformed agent who assumes that the others have more information, or the optimal strategy of an investor who maximizes his profit by following the market trends. In practice it is likely to be a combination of all three.

This literature has had the tremendous merit of helping us understand how asset price puzzles may arise and persist. Yet it also faces a major challenge: how can we model and identify such effects? Indeed many of the mentioned sources of non-fundamental price movement are very hard to observe and quantify, notably those related to the "irrationality" of investors. What's more, their impact on prices is likely to be highly non-linear, and entangled together, as well as with more fundamental factors.

These difficulties impose important limits for researchers, notably in empirical analysis, as they make it hard to study econometrically the impact of a given "bias" or "instinct" on prices. A revealing field from this perspective is that of contagion during financial crises. The first studies in this field tested for a significant change in correlations during crises (King et al., 1990), finding what appeared to be conclusive evidence of excess comovement, or "contagion". However a paper by Forbes and Rigobon (2002) later showed that such tests were necessarily biased by the fact that correlations fundamentally increase during crises. Since then the literature has failed to reach a consensus.

Therefore a key challenge for this thesis has been to take steps towards overcoming the problems attached to mapping non-fundamental factors into prices.

In the theoretical chapters 1 and 3, we do so by choosing to focus on behaviors that are quite "mechanical". In particular in both chapters we study an investor following a given investment rule, and is subject to funding constraints. Funding constraints are an interesting vehicle because, while representing a known vector of endogenous movement, they trigger automatic and emotionless responses that are easy to capture formally. We also refrain from the larger task of modeling psychological factors. The thesis then focuses more on the two latter sources of deviations from fundamentals: bounded rationality, a wedge between the interest of investors and those of society, or a combination of both.

In the empirical chapter 2 we have tried to address these problems through a careful identification strategy. First we focus on a specific behavior, style investing, which has a precise impact a given set of assets. This investment rule is a reflection of a feature inherent to all human beings, our tendency to classify, but one that appears fairly independent of other instincts. Finally we dissociate fundamentals-driven comovement from style-driven comovement, by focusing only on rating actions for which both influences have different implications on comovement.

In this way the thesis does not aim to define an exact and unique measure of endogenous price movement, but rather provide an approximation of this movement in a particular set-up. Consequently, its goal is not to provide a global pricing model that is capable of outperforming the efficient market hypothesis, but more modestly to know enough about non-fundamental movement to spot situations in which risk is piling up, and act upon it.

This stance places this work at the heart of exciting and quickly evolving field, which has been labeled as "endogenous risk".

Notable contributors to this literature include Danielsson, Shin, and Zigrand (2011), who specify a model in which the actual price dynamics depend on the perceived risk by agents, which in turn is a function of the capital they hold. Consonantly, Adrian and Shin (2010) provide evidence that leverage for financial intermediaries is procyclical, i.e. rises during booms and falls during busts, which increases aggregate risk. Finally Brunnermeir et al. (2009) find that currency crash risk may be linked to the funding constraints faced by carry traders, while Kyle and Xiong (2001) relate asset price contagion to the wealth effects of investors.

These papers all share the same interest in how the rules and constraints faced by investors at the micro level may lead to increased risk at the system level.

Having laid out the motivation for this thesis, we now provide a more detailed outline of each chapter.

Three essays on endogenous price movements

Chapter 1

In order to study the desirability of diversification, chapter 1 provides a theoretical model in which N portfolio investors each hold an equal amount of n assets, and are forced to sell assets in response to shocks on their wealth. Investors are home-biased: they acquire assets which are close to them in the financial network. Consider for instance a five investors market with a diversification level of n=*2*. In our network investor I=*1* will hold assets i=*1* and i=*2*, while I=*2* will hold i=*2* and i=*3*, etc. Mathematically this translates into the following matrix of asset holdings:

$$A = \left\{ \begin{matrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \end{matrix} \right\}$$

where each row represents an investor and each column an asset.

Post-multiplying this matrix by the vector of the asset price shocks between two periods *t-1* and t, noted $\mathbf{P_{t-1}}$, yields the vector of total portfolio losses between *t-1* and t for each investor. Assuming the sales by investors are a linear function of their portfolio losses, the vector of total sales for each asset at time t can be expressed as $\triangle \mathbf{Q_t^{lt}} = \frac{r}{n^2} A^\intercal A \triangle \mathbf{P_{t-1}}$. Further assuming that the impact of these sales on prices is linear yields the following recurrence system for asset prices.

$$\triangle\mathbf{P_t} = (\frac{r/h}{n^2})A^{\intercal}A\triangle\mathbf{P_{t-1}}$$

which, focusing on the impact of a single stochastic shock $\triangle\mathbf{e_1^F}$ yields:

$$\implies \triangle\mathbf{P_t} = [(\frac{r/h}{n^2})A^{\intercal}A]^t\triangle\mathbf{e_1^F}$$

while the total fall in the wealth of investors following the shock $\triangle\mathbf{e_1^F}$ can be expressed as:

$$\mathbf{I} = \frac{1}{n}A\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^{\intercal}A]^t\triangle\mathbf{e_1^F}$$

. With this system the chapter moves in two directions.

First we solve the model analytically, obtaining closed-form expression for the matrix of covariance between N assets N and investors' wealth. Finding a solution is made possible by the fact that matrix A is circulant, and thus has numerous interesting properties such as the fact that its eigenvalues are given by the roots of unity. Through these expressions for covariances we show that endogenous comovement may emerge from the common asset holdings and funding constraints even when these are fundamentally uncorrelated. We then study the factors on which these endogenous covariances depend, such as market completeness or risk aversion.

Second we study systemic risk trough the likelihood that the wealth of a number η of investors goes below a given bankruptcy threshold, for different levels of diversification n. We obtain these probabilities using the fact that the vector of investors' wealth I is normally distributed. We find that while diversification maximizes the probability that no investor goes bankrupt, it also increases the odds of mass failure, which casts doubts over its desirability from a systemic perspective.

Nevertheless, extending our model to allow for "panic", i.e. for non linear responses to adverse shocks, this conclusion changes. Intermediate levels of diversification become the worst option because such levels provide links between investors through which shocks may spread, without going far enough in minimizing individual risk. Large levels of diversification become the most desirable options over the vast majority of the parameter set. This suggests the existence of a threshold: past the dangerous intermediate levels, further diversification is unambiguously positive. The cost arising from the endogenous correlations between investors becomes smaller than the benefit of having individually safer investors.

Chapter 2

Chapter 2 is an investigation of the impact of ratings on bond comovement. We start by running the following regression for bonds whose rating changed :

$$R_{i,t} = \alpha_i + \beta_{i,I} R_{I,t} + \beta_{i,I'} R_{I',t}$$

where $R_{i,t}$, $R_{I,t}$ and $R_{I',t}$ represents the first-differenced yield spreads on the bond i, the left index I, and the joined one I' respectively. The regression was ran twice, over the 5 months preceding the rating action and over the 5 months following it. The estimated betas on the post-event and pre-event regressions are noted $\hat{\beta}^*$ and $\hat{\beta}$ respectively.

As style investing should induce a comovement premium between bonds of a similar index, we expect the beta of a bond with the index it leaves to fall following the rating change, and to rise with the index joined. Mathematically $\triangle^{\beta}_{i,in} = \hat{\beta_{i,I'}}^{*} - \hat{\beta_{i,I'}} > 0$, and $\triangle^{\beta}_{i,out} = \hat{\beta_{i,I}}^{*} - \hat{\beta_{i,I}} < 0$.

Nevertheless the total change in betas may not reflect solely the influence of style investing. In particular, assets which have similar ratings are expected to comove strongly because they have similar fundamentals. Formally the change in betas following a rating event can thus be expressed as $\triangle^{\beta} = \triangle^{f} + \triangle^{s}$, where $\triangle^{f}$ and $\triangle^{s}$ are the change in beta stemming from fundamental factors and style investing respectively. In order to control for $\triangle^{f}$ we set two tests which provide a natural way of disentangling both influences.

The first test is built upon the observation that lower grade assets are by definition more fragile so their sensibility to global risk factors should be higher. In a general factor model, an asset with a high market beta should comove more than a low market beta one with any given index. Thus from a fundamental point of view the beta of a downgraded bond with both the index it joins and that it leaves should rise after the rating action. Conversely an upgraded asset should see its beta with both indexes fall.

Combining these predictions with those from style investing leads to the following table :

	downgrade	upgrade
with index joined (in)	$\triangle^{f} > 0$, $\triangle^{s} > 0$	$\triangle^{f} < 0$, $\triangle^{s} > 0$
with index left (out)	$\triangle^{f} > 0$, $\triangle^{s} < 0$	$\triangle^{f} < 0$, $\triangle^{s} < 0$

In two cases fundamentals and style investing factors should go the same way, but in the other two fundamental factors and style investing have opposite predictions. For these two cases, if the sign of $\triangle^{\beta} = \triangle^{f} + \triangle^{s}$ is in conformity with the predictions from style investing, and in contrast with those from fundamentals, then the change in comovement may be attributed to style investing alone. Our first test is thus upon the sign of $\triangle^{\beta}$ for both cases.

The second test focuses on rating movements between two particular notches: BBB- and BB+. Movements between these notches are specific because the line between speculative and investment grade lies between them. This represents an important distinction amongst investors. If there is style investing at this wider "investment versus speculative" level, then movements between BBB- and BB+ should involve a larger comovement change from a style investing perspective. From a fundamental perspective however such movement should be no different from the others. Our second test thus focuses on whether there exists a significant difference between BBB-/BB+ and average, which in theory would stem from style investing only.

We reject the null hypothesis of no style investing premium for test 1, so that style investing seems to impact comovement at the individual rating level. Test 2 on the other hand gives the expected sign, but does not achieve significance at first. Thus we cannot conclude to the presence of style driven comovement at the "investment versus speculative" level.

The second part of this chapter then focuses on investigating this failure to achieve significance for test 2. We discuss possible biases in the design of the test, which leads us to uncover interesting patterns on the bond markets. First we find that an asset moving from BBB- to BB+ will in fact experience a larger rise in its beta with the rest of the market than other movements, although moving from BBB- to BB+ does not involve a larger than average increase in default rates. In other words investors seem to sanction downgrades between BBB- and BB+ with particular severity, without fundamental reasons to do so.

A second bias arises from the fact that higher and lower grade assets seem subject to more style investing, as implied by their higher-than-average share of index driven variation in their total variance. For lower grade assets this may be explained by the fact that such bonds are usually issued by smaller, less diversified firms, for which investors rely more upon ratings. For higher grade ones, this may be explained by regulatory requirements, which give investors an extra incentive to "buy the grade rather than the asset".

Controlling for these differences across grades, the difference between BBB-/BB+ movements and average becomes significant, as expected from a style investing perspective.

Chapter 3

This last chapter examines the possibility that FX investment strategies cause the disconnect between exchange rates and fundamentals. We present a model of the FX market in which three traders interact: carry traders, chartists, and fundamentalists, where carry traders are subject to funding constraints. Our modeling of the carry trader builds on Shin (2010), and yields the following form:

$$\mathring{q}_t = \frac{1 + CR_{c,t-1}}{1 + R_{c,t-1}} \mathring{q}_{t-1}$$

where $\mathring{q}_t$ is the quantity of the foreign bond demanded by carry traders at time t, $R_{c,t-1}$ is the return on the carry position between t and $t-1$, and C is the maximum leverage allowed by regulation. This constraint will generate procyclicality in the demand by carry traders: any capital gain made at period t will loosen the VaR constraint and thus allow for a larger position at period *t+1*.

With respect to chartists and fundamentalists we take a more descriptive than normative approach, specifying exogenously simple yet profitable investment rules, as in De Grauwe and Grimaldi (2006). The demand by chartists is given by:

$$q_{m,t} = \frac{MA(\triangle S)_x}{\beta + \alpha Var(\triangle S)_x}$$

where $MA(\triangle S)_x$ and $\alpha Var(\triangle S)_x$ are the mean and variance of the exchange rate movements over the last x periods, α and β are constants. Thus the expression simply implies that momentum traders buy currencies which have recently appreciated, and sell currencies whose price has recently fallen. The demand by fundamentalists is:

$$q^*_{f,t} = \frac{\Phi(sign(F - S_t) = sign(MA(\triangle S)_x)) \times (F - S_t)}{b}$$

where $(F - S_t)$ is the current distance between the exchange rate S_t and its fundamental value F, weighted by the probability estimated by fundamentalists $\Phi(sign(F - S_t) = sign(MA(\triangle S)_x))$ that prices are currently moving according to fundamentals. This expression implies that fundamentalists buy "undervalued" currencies and sell "overvalued ones", where the size of the investment depends on their assessment of the likelihood that exchange rates are about to converge towards F in the near future.

This set-up leads to a non-linear model which we simulate with and without stochastic shocks. Our simulations show that the interactions between these three agents cause currency crash risk for high interest rate currencies, in line with the empirical findings of Brunnermier et al. (2009). Crashes occur endogenously in an otherwise stable set-up through the actions of the market participants.

The dynamics are as follows: slow moving capital from constrained carry traders induces a trend of appreciation of a high-yield currency, which is picked up by chartists. As carry traders start approaching their desired holdings and fundamentals start pushing the other way, this trend of appreciation eventually weakens. This drives chartists to exit their long positions and fundamentalists to enter the market, shorting the high yield currency. The combined selling of both agents leads to a significant drop in the price of the currency. Crashes occurs when this drop is large enough to trigger margin calls for the leveraged carry traders.

The relationship between funding constraints and crash risk is non-linear. When carry traders are very constrained fundamentalists immediately dominate so that no crash occurs. When capital moves very easily the currency immediately appreciates then adjusts slowly, in line with the prediction from a perfectly efficient market. Moderate levels of funding constraint represent the most risky option, as there is enough capital to create a build-up phase, while traders remain vulnerable to margin calls.

We also investigate how endogenous booms and busts may help to resolve the "abnormal profits" and "exchange rate disconnect" puzzles. First we examine profits. We find our set-up implies that carry trade is profitable but contains a large downside risk, while momentum performs nearly as well and appears much more resilient during crisis. This is in line with the recent work of Darvas (2009) on the possibility that downside risk may explain the seemingly abnormal returns to carry trading, and with that on Menkhoff et al. (2012) on the empirical performance of chartism during crises.

Finally we study whether the ER generated by the stochastic version of the model matches the findings of the literature on the link between exchange rates and fundamentals. We find that the actions of investors endogenously generate long-lived deviations from fundamentals, which die out at a rate consistent with the work of Mark (1995) on the exchange rate disconnect puzzle.

Each chapter required the adoption of a new skill, a deliberate choice. In chapter 1 a theoretical model is solved analytically. Chapter 2 is empirical. Chapter 3 is based on simulations. We name the chapters after the investment rule they are primarily interested in and their expected endognous impact: "portfolio diversification and endogenous risk" for chapter 1, "style investing and endogenous comovement" for chapter 2, and "Foreign exchange investment rules and endogenous currency crashes" for chapter 3.

Portfolio diversification and endogenous risk

Chapitre 1

Portfolio diversification and endogenous risk

1.1 Introduction

When thinking about financial markets and systemic risk, one can find it useful to consider a group of climbers roped together at the top of a cliff. Each climber individually favors being roped as it lowers his chances of falling off, yet one climber slipping now threatens the stability of his neighbors. The effect of being roped on the probability that many or all climbers fall is thus a priori ambiguous.

Prior to the 2008 credit crunch, both market participants and regulatory instances seemed to favor the roped equilibrium, implicitly assuming that individual soundness leads to systemic soundness. Yet the swiftness with which risk spread throughout the market, largely unanticipated, led to suspicions that "connections" may in fact carry systemic risk through the system. Basel III will apply an extra capital ratio requirement of up to 2.5% to well connected establishments. Measures of systemic importance that account for externalities, such as Covar (Adrian and Brunnermeier, 2011), or Shapley values (Tarashev et al., 2010) have recently gained in popularity.

This change in focus has implications on the desirability of portfolio diversification from society's perspective. Indeed while the individually risk reducing effect of diversification has been known since Markowitz (1952), diversification also forms connections between investors through common asset holdings, identified as a major vehicle of contagion in the presence of fire sales (Shleifer and Vishny, 2012). The goal of this chapter is to take a first step towards quantifying this contagion externality and comparing it to the individual risk-reducing effect, in order to get a primary assessment of diversification's total impact on systemic risk, as well as the factors on which this impact may depend.

We do so by setting a model in which investors react to stochastic shocks on their wealth by selling/buying assets, which further impact asset prices and wealth, and so on. This occurs in a "home biased" network, in which investors acquire assets which have low informational distance with those they already own. The model generates a normal multivariate distribution [1] of investors wealth. Systemic risk is then studied through the probability that a number η of investors fall below a given bankruptcy threshold K, for different levels of diversification. Attaching to each probability a cost that grows with η, we then discuss whether accounting for the contagion externality that arises endogenously changes the conclusion about the overall desirability of diversification.

1. Schematically, this approach allows us to map the individually risk reducing effect and the contagion externality into two distinct components of systemic risk: the former affects in the marginal distribution of each element on the vector of investors wealth, while the latter is embedded within the dependence structure between all N investors.

We find intermediate numbers of bankruptcies are less likely with a high degree of diversification, but the probability that many or all investors fail simultaneously is larger. In a context of high selling constraint and weak demand, the probability of "extreme failure" outcomes becomes non-trivial, so that no or little diversification may become optimal for society. We then introduce a touch of heuristics in the model by allowing agents on the demand side to grow more risk averse in a high variance environment. We find this strongly enhances the desirability of higher levels of diversification through a more indirect channel: spreading shocks across assets lowers the chances of triggering a panic. In this context, intermediate levels of diversification appear particularly harmful as they provide linkages through which shocks may spread without going far enough in minimizing individual risk and the likelihood that a panic occurs.

Our set-up also allows us to derive an analytical expression for the covariance between assets/investors i and j. In this case covariances depend negatively on the distance between i and j in the financial network and on the number of assets in the economy. We show that an investor may minimize the variance of his portfolio by seeking assets which are far away from his in the financial network. We discuss the systemic implications of the resulting "optimal" network in which all investors are no longer biased, and that of a "wider" network, i.e., one in which there are more assets. Moving to an optimal network is generally desirable for systemic risk, while moving to a wider one is unambiguously so. A mix of more optimality and width is particularly efficient, notably for intermediate levels of diversification which previously were the most dangerous option from a systemic perspective.

Previous work on the link between diversification and systemic has been split between three strands of literature which have rarely crossed.

The first concerns small-scale [2] models of financial contagion. Such models highlight different channels through which common asset holding may lead to fire sales, which may in turn degenerate into systemic events. Schinasi and Smith (2000) for instance show that routine portfolio rebalancing brings contagion. The scope is increased when agents are subject to wealth effects as in Kyle and Xiong (2001). Goldstein and Pauzner (2004) point out that fire sales may also result from strategic risk [3].

The second strand of literature is based on statistical analysis. One method, taken by Shaffer (1994) or Wagner (2010), is to compare the properties of a fully diversified situation to a fully undiversified one. Both authors show that the risk that all investors fail simultaneously is necessarily higher in the fully diversified situation. Our study confirms this fact but also goes more in depth by considering any level of diversification and any number of failures. A second approach deals with "fat tails", which may mitigate the strength of the variance reducing effect, as showed by Samuelson (1967). In particular, Ibragimov et al. (2011) use an indicator of the tail behavior of returns to define a diversification threshold. They find that on a given parameter range there can exist a wedge between investors interests and society ones.

2. The models usually feature only 2 assets, and diversification is defined as how evenly an investor spreads his wealth across both. A notable exception is provided by Lagunoff and Schreft (1999) who try moving to a 3 assets case. They find that the scope for contagion is decreased.

3. Others authors relate diversification to different amplification mechanisms, which deserve to be mentioned although they will not feature as such in this study. Market distress may lead to an even wider collapse if it turns into liquidity distress: Adrian and Shin (2010) shows that leverage is negatively correlated to the market value of assets. Allen et al. (2010) find that when investors need to roll debt over, being connected brings a negative reputation externality. We also leave out a potential impact of diversification on "banks not doing their homework". For instance, Jiao et al (2013) show that diversification may reduce the heterogeneity of investors beliefs, Calvo and Mendoza (2000) argue that diversification lowers the incentive for investors to acquire information about securities before selling.

The link between these first two strands is the correlation structure between asset prices. In any contagion model correlations between asset returns appear endogenously as an output, as even two securities which are "fundamentally" independent become linked through the investors who hold them. In statistical analysis high correlations are an input, as they may cause "fat tailed" portfolios.

The last strand of literature related to this work deals with network analysis of financial stability. In particular, this work may be related to previous work on the "robust yet fragile" feature of the financial system, as in Nier et al. (2007) or Amini et al (2012).

The broad method of this chapter is to bridge these three approaches, by proceeding in three steps:

1) *setting up a large scale contagion model.* N constrained "portfolio investors" who hold from 1 to N assets are forced to sell to "convergence traders" in response to negative shocks on their wealth. This lowers prices, further tightening the constraint, and so on. Mathematically this translates into a linear system of N recurrence equations of price returns, in which the amount of recurrence will depend on the weight of the constraint on investors and the discount on the sales.

2) *within a specified network.* We set two characteristics for investors: they are home biased, and all have the same pattern of asset holdings. The latter leads the network and matrix of asset holdings and to be circulant. This feature is what allows us to solve the system, obtaining analytical expressions for the covariances between assets and investors that depend on distance[4].

3) *in order to study systemic risk through statistical analysis and analytical ex-*

4. Circulant matrices are are a useful tool, and a side goal of the chapter is to contribute to widening their use and understanding beyond the fields of pure network analysis and signal theory in which they are more common.

pression of the covariances. Statistical analysis is run through the multivariate distribution of portfolio losses, which gives us the likelihood that any number of investorsη between 0 and N fail, for a given level of diversification n.

To the best of our knowledge two recent papers have used a similar approach in terms of obtaining endogenous covariances from a theoretical model. Danielsson, Shin, and Zigrand (2012) define a multivariate model which produces a matrix of covariance of unrestricted dimensions. The correlations obtained have a "fundamental" and an "endogenous risk" component, where endogenous risk is the risk resulting from "the actions of market participants which are hard wired in the system". Cont and Wagalath (2014) specify a similar but more aggregated model, which they calibrate to estimate the realized covariance matrix during well-known fire sales episodes, such as the aftermath of the collapse of Lehman brothers.

Our study places itself within this endogenous risk approach, but differs from these papers in that it puts the network of asset holding at the center of the analysis. In spirit, both papers focus on explaining the pattern of prices correlation during crisis episodes, while our interest lies primarily with the desirability of diversification from a systemic perspective.

Finally, the present work shares some of its conclusions with Caccioli et al. (2014) who find that there might be a window on the diversification spectrum for which systemic risk may become significant. Caccioli et al. (2014) study the asymptotic properties of the financial system as a function of the average number of connections of a node in an otherwise random network. Our model is micro founded, and specifies a given network based on economic evidence. This gives the model a greater granularity. In particular we show how different micro foundations lead to different conclusions on systemic risk, even within a network whose general characteristics remain unchanged.

In short, the model is somewhat "hybrid" between micro-founded and network approaches, which allows it to remain both tractable and general, and provide a wide oversight of the factors which may impact on the link between diversification and systemic risk.

Section 1.2 presents the baseline model, and how it brings recurrence in asset prices. Section 1.3 presents the distributions of investor wealth for the baseline model, and that with possible panics. In section 1.4 we go further by drawing and discussing the realized covariances between assets and investors, and studying the impact of a wider and/or a home bias free network on systemic risk. Section 1.5 concludes.

1.2 Set-up

The model presented below generates a linear system which will allow us to draw N-dimensional covariance matrices between assets. Vectors are indicated with bold characters.

1.2.1 The market

The investment period starts at t=*0* and finishes at $t=T$, where each period $t \mapsto t{+}1$ may be seen as "a day" on the markets, and T is a large but finite number. Financial markets are composed of N risky assets, which are best seen as stocks. N is finite to keep the individual benefits from diversification within bounds, so that markets are incomplete. Asset prices have three sources of movement:

$$\triangle \mathbf{P_t} = \triangle \mathbf{P^F_t} + \mathbf{MR_{t+1}} + \mathbf{e^*_{t+1}}$$

where $\triangle \mathbf{P_t}$ is the *actual price* evolution between t and *t+1*, $\triangle \mathbf{P_t^F}$ the *fundamental price* evolution. $\mathbf{MR_{t+1}}$ is the *mean reversion* vector, and $\mathbf{e_{t+1}^*}$ is the *investor-driven deviations* from fundamental value. All elements are $N \times 1$ vectors. Intuitively, these dynamics imply that the behavior of investors may induce actual prices, which are defined in equilibrium, to deviate from their fundamental value by an amount $\mathbf{e_{t+1}^*}$. Such deviations however tend to die out in the medium/long run, through $\mathbf{MR_{t+1}}$.

The *fundamental price/value* of each stock is defined as the discounted value of its future dividends. We assume dividends D follow an arithmetic Brownian motion, i.e., for stock i at time *t:* $D_{i,t+1} = D_{i,t} + u_i^D + e_{i,t+1}^D$. The Gordon growth model yields : $p_{i,t}^F = \frac{D_{i,t}}{k} + cst$, where k is an exogenous discount factor suited to the riskiness of the cash-flows, and cst a constant. Consequently $p_{i,t}^F$ also follows an arithmetic Brownian motion, which we note:

$$p_{i,t+1}^F = p_{i,t}^F + u_i + e_{i,t+1}^F$$

where u_i is the *fundamental drift*, and $e_{i,t+1}^F$ is a normally distributed shock, $N \sim (0, \sigma_F^2)$. Both are linear functions of the underlying dividend drift and shock: $u_i = \frac{u_i^D}{k}$ and $e_{i,t+1}^F = \frac{e_{i,t+1}^D}{k}$.

We assume that the fundamental drift is the same for all assets, and that fundamental shocks are independent across assets and time. Mathematically, $u_i = u$ and vector $\mathbf{e_t^F}$, the $N \times 1$ vector of general element $e_{i,t}^F$, is defined by $N \sim (0, \Sigma_F = diag(\sigma_F^2))$. These assumptions will simplify presentation and calculus, but also show how correlations between assets may arise from the actions of market participants, even starting from a situation in which they are fundamentally independent. The vector of fundamental prices is written:

$$\triangle \mathbf{P_t^F} = \mathbf{u} + \mathbf{e_{t+1}^F}$$

The *mean reversion* component ensures that prices do not go too far away from their fundamental value. In our model mean reversion will be driven by an exogenous supply of "new" investors at each period t, who will demand asset i if $p_{i,t} - p^F_{i,t} < 0$, and short sell it if $p_{i,t} - p^F_{i,t} > 0$. We set this amount of "new" investors to be quite low, so that prices only converge to their fundamental value in the medium/long run. Mathematically :

$$\mathbf{MR_{t+1}} = -\lambda(\mathbf{P_t} - \mathbf{P^F_t})$$

where λ, the speed of mean reversion, is fairly low.

Substituting for $\triangle\mathbf{P^F_t}$ and $\mathbf{MR_{t+1}}$ we may then rewrite the price vector as:

$$\triangle\mathbf{P_{t+1}} = \mathbf{u} - \lambda(\mathbf{P_t} - \mathbf{P^F_t}) + \mathbf{e^F_{t+1}} + \mathbf{e^*_{t+1}} \tag{1.1}$$

$\mathbf{e^*_{t+1}}$, the *investor driven deviations* vector, will be the focal point here. It gives the movement in prices resulting from the actions/constraints of investors that may cause prices to deviate from their fundamental value. Its exact dynamics will be derived in the following section, but it is useful to give the outline in words.

$\mathbf{e^*_{t+1}}$ is drawn from the interactions of two types of agents: "portfolio investors", whose strategy is based on fundamental drifts, and "convergence traders" who exploit the short-term deviations from fundamental value. We may also refer to them as long term and mid term investors due to their investment horizon, which we note respectively LT and MT. LT investors face a constraint which forces them to sell in response to negative shocks on their portfolio. MT investors buy those asset at a discount, which leads prices to fall below their fundamental value. This brings LT investors to sell further, and so on.

The full specialization between long-term and mid-term investors results from an ex-ante arbitrage by both agents. Convergence traders specialize on spotting and exploiting deviations of prices from their fundamental value, while portfolio investors have a more passive approach. Formally, we assume MT investors pay a fixed cost ϵ_1 to be able to successfully measure $\mathbf{P_t} - \mathbf{P_t^F}$, while LT investors pay ϵ_2 to correctly estimate the unconditional moments of vector $\triangle\mathbf{P_{t+1}}$. Such a market segmentation is close to that of Graham who differentiates an "active or enterprising approach to investing" from a "passive or defensive strategy that takes little time or effort but requires an almost ascetic detachment from the alluring hullabaloo of the market" (Graham and Zweig, 2003, p101). One may also view LT investors as mutual funds or banks, while MT investors are more speculative agents following arbitrage strategies, such as hedge funds.

Long-term investors hold a diversified portfolio, with $n \in ||[1, N]||$ the level of diversification. The level of diversification thus impacts the system in two ways. First, the total shock on the portfolio is likely to be lower when n is large, so that the wealth shocks should be lower. Second, a given shock will trigger sales on many assets when n is large, so that LT investors and assets become more correlated. Initially each long-term investor I is endowed with "his" asset i, i.e., investor 1 holds asset 1, so that they are N representative long term investors.

Note that in what follows we nearly only refer to constrained *selling* by portfolio investors, because we are primarily interested in fire sales. However the model applies equivalently to a situation in which portfolio investors are *buyers*, as their constraint gets looser following a positive shock on their wealth[5]. Convergence traders are segmented as in Merton (1987).

5. More on this assumption in section 1.2.4

We introduce some notation: for a given investor I, $q_{i,I}$ represents the actual quantity of asset i, $q^*_{i,I}$ the desired one, $q_I = \Sigma_{i=1}^{i=N} q_{i,I}$ is his total investment in all risky assets. $q_i = \sum_{I=1}^{I=N} q_{i,I}$ is the total quantity of asset i across investors.

1.2.2 Investors

Both mid-term and long-term investors have CARA utility, with risk aversion of $\frac{1}{\tau_{mt}}$ and $\frac{1}{\tau_{lt}}$ respectively. Risk aversion is similar across investors of the same type, and each agent is a price-taker.

1.2.2.1 Mid-term investors

They hold assets during $t^\star$ periods, the time it takes for investor-driven deviations shocks to return to 0. They pay a fixed cost ϵ_1 to monitor those deviations at every t. They are free of regulation and have "deep pockets", so that they may hold exactly the quantities they desire: $q^*_{i,t} = q_{i,t}$. As they are segmented each investor operates in one market only, so that we drop the vector notation for the time being. Mathematically each I solves:

$$Max\ E(-e^{-\frac{w_{I,t+t^\star}}{\tau_{mt}}})$$

$$u/c \ \ w_{I,t+t^\star} = w_{I,t} + q^*_{i,I,t}(p_{i,t+t^\star} - p_{i,t}) - t^\star \epsilon_1$$

where $w_{I,t}$ is the wealth of investor I at time t. Using the moment generating function yields the well-known solution:

$$q^{mt}_{i,I,t} = \tau_{mt} \frac{E(p_{i,t+t^\star} - p_{i,t})}{E(\sigma^2_{t+t^\star})}$$

where $\sigma^2_{t+t^\star}$ is the variance of asset price i at horizon $t + t^\star$.

When $t^\star$ is large, all trading shocks, whether they have occurred or are expected to, will vanish through mean-reversion. Setting a fairly large value for $t^\star$ we thus have:

$E(p_{i,t+t^\star} - p_{i,t}) = E(\sum_{j=t}^{j=t+t^\star-1} \triangle p_{i,j}) = E(ut^\star + \sum_{j=t+1}^{j=t+t^\star} MR_{i,j} + \sum_{j=t+1}^{j=t+t^\star} e^F_{i,j} + \sum_{j=t+1}^{j=t+t^\star} e^*_{i,j})$

$E(p_{i,t+t^\star} - p_{i,t}) = t^\star E(u) + E(p^F_{i,t} - p_{i,t})$

As mentioned, convergence traders successfully monitor deviations from fundamentals, so that $E(p^F_{i,t} - p_{i,t}) = p^F_{i,t} - p_{i,t}$.

First differencing we obtain the demand/supply shock for a given convergence trader between t and $t+1$:

$$\triangle q^{mt}_{i,I,t} = \tau_{mt} \frac{(p^F_{i,t+1} - p_{i,t+1}) - (p^F_{i,t} - p_{i,t})}{E(\sigma^2_{t+t^\star})}$$

Rearranging and setting to m the number of convergence traders operating on each market, we obtain the vector of demand/supply shifts from convergence traders between t and $t+1$:

$$\Longrightarrow \triangle \mathbf{Q^{mt}_t} = -h(\triangle \mathbf{P_t} - \triangle \mathbf{P^F_t}) \tag{1.2}$$

where $h = \frac{m\tau_{mt}}{E(\sigma^2_{t+t^\star})}$ represents the total increase in the quantity demanded in response to a rise in expected returns, and thus indicates the strength of the demand. If the distance between actual and fundamental prices remains constant between t and $t+1$, the demand by MT investors will be null. If $\triangle \mathbf{P_t} > \triangle \mathbf{P^F_t}$ they will sell, if $\triangle \mathbf{P_t} < \triangle \mathbf{P^F_t}$ they will buy.

Note also that starting from a situation in which actual prices are equal to fundamental prices, so that is no mean reversion, and portfolio investors are not constrained and thus do not supply/demand any assets, market equilibrium implies $-h(\triangle \mathbf{P_t} - \triangle \mathbf{P^F_t}) = 0 \Leftrightarrow \triangle \mathbf{P_t} = \triangle \mathbf{P^F_t}$. Therefore in a "business as usual" scenario in which the constraint of portfolio investors does not bind, actual prices should be equal to fundamental ones at any time t.

1.2.2.2 Long-term investors

Maximization problem

Their time horizon is noted $t^\diamond$. Formally:

$$Max\ E(-e^{-\frac{w_{I,t+t^\diamond}}{\tau_{lt}}})$$

$$u/c\ \ w_{I,t+t^\diamond} = w_{I,t} + \mathbf{Q_t^{*T}}(\mathbf{P_{t+t^\diamond}} - \mathbf{P_t}) - t^\diamond \epsilon_2$$

$E(\mathbf{P_{t+t^\diamond}} - \mathbf{P_t}) = E(\mathbf{P_t} + t^\diamond \mathbf{u} + \sum_{j=t+1}^{j=t+t^\diamond} \mathbf{MR_j} + \sum_{j=t+1}^{j=t+t^\diamond} \mathbf{e^F_{t+1}} + \sum_{j=t+1}^{j=t+t^\diamond} \mathbf{e^*_{t+1}} - \mathbf{P_t})$
$= E(t^\diamond \mathbf{u}) + \lambda E(\sum_{j=t+1}^{j=t+t^\diamond} (\mathbf{P^F_j} - \mathbf{P_j}))$

using equation (1.1) and the fact that $E(\mathbf{e^F_{t+1}}) = 0$. Since LT investors do not pay the cost ϵ_1 and do not observe the deviations of prices from their fundamental value, they can only estimate it with its long-term value $E(\mathbf{P^F_j} - \mathbf{P_j}) = \mathbf{0}$. However as they incur the cost ϵ_2 they successfully estimate the fundamental price drift $\mathbf{u}$ and covariance, noted $\Sigma_{t^\diamond}$. Formally this means $E(\mathbf{P_{t+t^\diamond}} - \mathbf{P_t}) = t^\diamond \mathbf{u}$ and $E(\Sigma_{t^\diamond}) = \Sigma_{t^\diamond}$. Therefore :

$$\mathbf{Q_t^{*lt}} = \frac{\tau_{lt} t^\diamond \mathbf{u}}{\Sigma_{t^\diamond}}$$

Two key elements:

- Since the unconditional moments of $\mathbf{P_{t+t^\diamond}} - \mathbf{P_t}$ are constant, the desired quantities will be constant.

- As we shall verify later in equilibrium, the matrix $\Sigma_{t^\diamond}$ is symmetric. This implies that LT investors optimally want to hold an equal share of all assets, so that:

$$\frac{q_{i,I,t}}{q_{I,t}} = \frac{1}{n} \tag{1.3}$$

Note that optimally portfolio investors want to hold as many assets as possible to maximize the benefits of diversification. The level of diversification n is thus equal to the maximum number of markets they have access to.

Constraint

We assume a linear correspondence between the total amount of risky assets that an investor may hold at *t+1* and his value-at-risk at *t*.

$$q_{I,t+1} \leq rVaR_{I,t}$$

where parameter r mitigates the strength of the selling in response to a change in the VaR. In words, when the constraint binds, a change in the VaR at time t will lead investors to sell/buy risky positions between t and *t+1* to comply with the constraint at $t+1$. Note that a more usual form would be $\mathbf{P_t Q_t} \leq rVaR_{I,t}$, i.e., the VaR leads investors to manage their monetary exposures rather than their quantities solely, we pick this form for technical reasons [6].

Each investor's VaR is given by:

$$VaR_{I,t} = E_t(\mathbf{P_{t+t^\diamond}})^\top \mathbf{G^{lt}_{I,t}} + cst\sqrt{(\mathbf{G^{lt}_{I,t}})^\top \Sigma_{t^\diamond} \mathbf{G^{lt}_{I,t}}}$$

where $\mathbf{G^{lt}_{I,t}}$ is the vector summarizing the proportions of each asset in total quantity invested by investor *I*. Using (1.3) and the fact that $E(\mathbf{P_{t+t^\diamond}}) = \mathbf{P_t} + t^\diamond \mathbf{u}$ for LT investors, we have

$E_t(\mathbf{P_{t+t^\diamond}})^\top \mathbf{G^{lt}_{I,t}} = \frac{\alpha}{n}[(\sum_i p_{i,I,t} + t^\diamond u)/n]$, where $\sum_i p_{i,I,t}$ represents the sum of the prices of assets that feature in I's portfolio. First differencing, we obtain the selling of a given asset i by investor I that operates on his constraint:

6. more on this choice in section 1.2.4

$$\implies \triangle q^{lt}_{i,I,t} = \frac{r}{n^2} \sum_i \triangle p_{i,I,t-1} \tag{1.4}$$

The message conveyed by this equation is simple: when prices increase, investors are more wealthy and thus may invest more at the following period, when prices fall they must lower their exposure to risky assets. As long as the constraint does not bind the supply/demand by portfolio investors will be null, and prices will simply track fundamental values. The rest of the chapter studies a situation in which the constraint binds for all LT investors.

Note that VaR constraints as such usually concern banks, so that one may argue the model applies primarily to markets dominated by banks, such as ABS. Yet trust funds may also be subject to an explicit constraint by their beneficiaries. What's more, in practice fire sales may also be driven by increased risk aversion, higher emphasis on the short run, etc. The constraint chosen is thus only a tractable vehicle for such sales, but captures a wider phenomenon. Therefore we avoid referring to LT investors as "banks" only, and believe our results apply to markets in general.

1.2.3 Network formation and matrix form

Network

The previous section showed how each representative investor I behaves. To study the implications of such behavior on each price we must specify a pattern of asset holdings. Figure 1.2.1 summarizes the network formation: each holding of asset i by investor J is a connection between J and the investor I who initially held the asset. Investors are nodes and asset holdings are the connections between them. The numbers on each link thus represent the assets that I and J have in common, indicating how closely related they are.

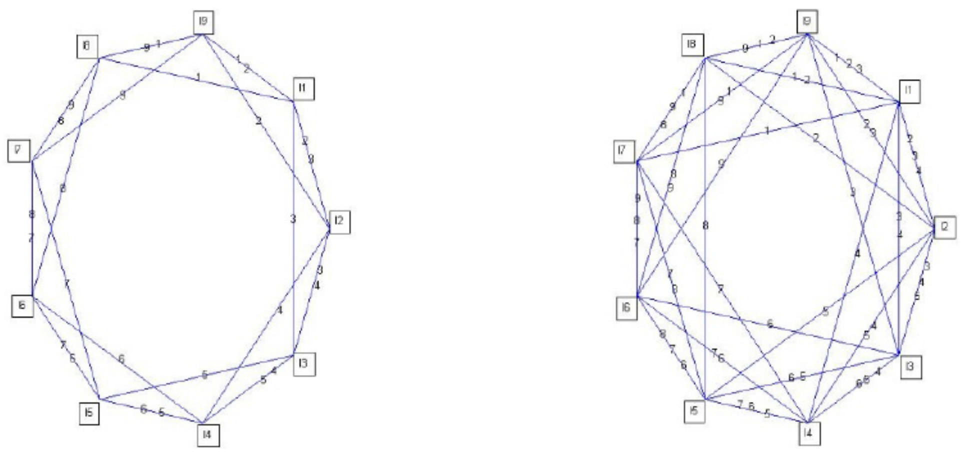

Figure 1.2.1 – Moving from a $n = 3$ to an $n = 4$ network when $N = 9$.

As the degree of diversification n increases, each I acquires the asset that is the closest to his right. Then I_k holds only asset k if $n = 1$, assets k and k+*1* if $n = 2$, etc. In general I_k holds assets k to k', where $k' \equiv k + n - 1$ [N].

The same information may be expressed in matrix form by letting each row represent an investor I and each column an asset i, and setting *1* if I holds i , 0 otherwise. For instance if N=*5* and n=*3*:

$$A = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

I_1 holds assets $1, 2, 3$ so that $a_{1,1} = a_{1,2} = a_{1,3} = 1$.

This network has two important features:

- As each investor chooses to purchase the asset of his nearest "neighbor" in the financial network, he exhibits some degree of *home bias*. Home bias refers to the tendency of investors to favor assets with the lowest distance to those they already own. It is a well documented phenomenon in the literature on international diversification. "Distance" between two assets i and j may reflect informational asymmetries between them, the transaction cost of acquiring one when possessing the other, the fact the investors who hold i are not familiar with j, etc. Here we take no stance on what factor underlie "distance", seeing it as a combination of heuristic and rational factors.

- All investors have a similar pattern of asset holdings, so that the network is *circulant,* and so is matrix A. Visually, in circulant matrices each row is equal to the previous row, shifted one spot to the right. It is important to stress that circulant networks are only the result of our assumption of symmetry across investors, not from the *home bias* feature mentioned. For instance a network in which investor 1 holds assets 1, 4, 5, is circulant if investor 2 holds 2,5,6, and so on. As we shall see in section 1.4, circulant matrices have powerful properties, which will allow us to diagonalize them easily to solve the model and provide analytical expressions for the covariance between asset i and j.

Matrix form

From section 1.2.2.2 we know that the proportion of asset i in total quantity is $1/n$ if i features in the portfolio, and 0 otherwise. The average return on investor I 's portfolio writes:

$$\sum_i a_{I,i} \triangle p_{i,t}/n$$

And the selling constraint (1.4) may be re-expressed as:

$$\triangle q_{i,I,t} = \frac{r}{n^2} \sum_i a_{I,i} \triangle p_{i,t-1}$$

i.e., the amount of each asset sold by investor I is a linear function of the product of each line I of A by the price vector.

The vector of general element $\triangle q_{i,I,t}$ summarizing the unit selling by each long-term investor I as a response to a wealth shock, may then be expressed as:

$$\triangle \mathbf{q_t^{lt}} = \tfrac{r}{n^2} A \triangle \mathbf{P_{t-1}}$$

What's more, just as each row of A tells us what assets a given investor holds, each row of $A^\intercal$ indicates what investors hold a given asset i. Thus we may express $\triangle \mathbf{Q_t^{lt}}$, the vector whose general element $\triangle q_{i,t}$ is the total selling of asset i between $t+1$ and t, as:

$$\triangle \mathbf{Q_t^{lt}} = A^\intercal \triangle \mathbf{q_t^{lt}}$$

$$\Longrightarrow \triangle \mathbf{Q_t^{lt}} = \frac{r}{n^2} A^\intercal A \triangle \mathbf{P_{t-1}} \tag{1.5}$$

Let us now take the market clearing condition $\triangle \mathbf{Q_t^{lt}} + \triangle \mathbf{Q_t^{mt}} + \triangle \mathbf{Q_t^{mr}} = 0$, where the terms on the right-hand side are the demand/supply shocks stemming from respectively portfolio investors, convergence traders, and the mean reverting exogenous supply of investors. Substituting:

$$-h(\triangle \mathbf{P_t} - \triangle \mathbf{P_t^F}) + \triangle \mathbf{Q_t^{mr}} + \tfrac{r}{n^2} A^\intercal A \triangle \mathbf{P_{t-1}} = 0$$

$$\Longrightarrow \triangle \mathbf{P_t} = \mathbf{u} + \frac{\triangle \mathbf{Q_t^{mr}}}{h} + (\frac{r/h}{n^2}) A^\intercal A \triangle \mathbf{P_{t-1}} + \mathbf{e_{t+1}^F} \tag{1.6}$$

The end product of the model is thus a stochastic recurrence system that maps how price shocks spread through time and assets. The asset price dynamics we fed into LT and MT investors maximization problems are verified in equilibrium.

Summary of the frame of the model:

- portfolio investor of type I holds assets i=I to i=I+n-1

- when faced with a negative shock I sells an equal quantity of every asset, where the amount sold depends on r*, the weight of constraint on the investor*

- convergence traders buy these assets with a discount that depends on h*, the strength of demand*

- The market clearing condition then yields $\triangle \mathbf{P_t} = \mathbf{u} + \frac{\triangle \mathbf{Q_t^{mr}}}{h} + (\frac{r/h}{n^2}) A^\intercal A \triangle \mathbf{P_{t-1}} + \mathbf{e_{t+1}^F}$

1.2.4 Vector of total change in investors wealth

As mentioned, this study aims at studying the link between systemic risk and diversification by comparing the likelihood that $\eta \in |[1, N]|$ investors go bankrupt. This requires that we get the first two moments of the the vector of total change in investors wealth during the crisis. In order to convey maximum economic intuition while minimizing quantitative heavy lifting, we make two simplifying assumptions.

First, we set the mean-reversion and fundamental drift to 0, for three reasons: a) our time unit is a "day" in the market, so that these fundamental long-term trends should be negligible anyway, b) they would overload the equations as a recurrence system with a drift is slightly more demanding and c) without changing the insights of our model, since the variance structure would not be changed by the inclusion of $\mathbf{u}$, and very marginally by $\mathbf{MR}$.

Second, we will only study how stochastic shocks on assets at *t=0* spread through time. We thus set all shocks to 0 past *t=0* for all securities, letting total losses depend on the realization of the stochastic vector $\triangle \mathbf{P_0}$. This leads to the following form for asset prices:

$$\triangle \mathbf{P_0} = \mathbf{e_1^F}$$

$$\triangle \mathbf{P_t} = (\frac{r/h}{n^2}) A^\intercal A \triangle \mathbf{P_{t-1}} \quad (1.7)$$

$$\Longrightarrow \triangle \mathbf{P_t} = [(\frac{r/h}{n^2}) A^\intercal A]^t \triangle \mathbf{e_1^F} \quad (1.8)$$

which notably implies $E(\triangle \mathbf{P_t}) = [(\frac{r/h}{n^2}) A^\intercal A]^t E(\mathbf{e_1^F}) = 0$. *Figure 1.2.2 gives a chronological description of how a vector of shock at t=0 propagates through the simplified system:*

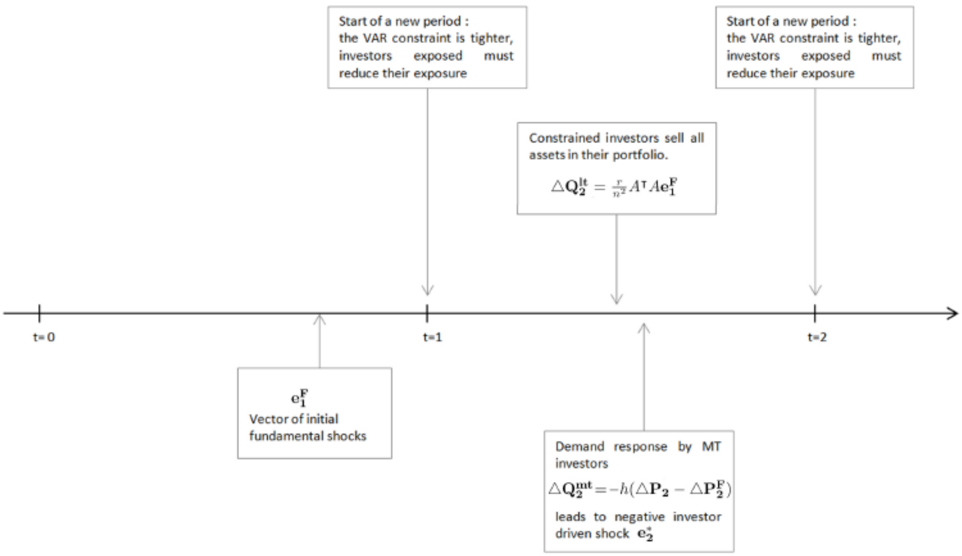

Figure 1.2.2 – Sequence of events following a shock.

We label as "change in the wealth of investor i" the evolution of the average price of the assets in i's portfolio throughout the period. Since each row i of matrix A represents the holdings of investor I, this average price shift at period

t is $\mathbf{I}_t = \frac{1}{n}A\triangle\mathbf{P_t}$, and the total price return over the period, on which we shall focus, is:

$\mathbf{I} = \frac{1}{n}A\sum_{t=0}^{t=+\infty}(\triangle\mathbf{P_t})= \frac{1}{n}A\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t\triangle\mathbf{e_1^F}$

Its expected value is null, and its variance is given by: $\Sigma_I = E([\mathbf{I} - E(\mathbf{I})][\mathbf{I} - E(\mathbf{I})]^\intercal)$, which yields:

$$\Sigma_I = \frac{\sigma_F^2}{n^2}[A\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t][A\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t]^\intercal$$

since $E(\triangle\mathbf{e_1^F}\triangle\mathbf{e_1^F}^\intercal) = diag(\sigma_F^2)$ as the fundamental shocks are independent across assets. Let us note that:

- $\mathbf{I}$ is a linear combination of the initial normally distributed shocks, so it is normally distributed.

- we have implicitly assumed that the system is not explosive, i.e., that the eigenvalues of $(\frac{r/h}{n^2})A^\intercal A$ are below one. We later prove [7] that this boils down to imposing the condition that $r < h$. The increase in demand following a drop in prices must be superior to that of constrained supply, otherwise prices are ever-falling.

1.2.5 Assumptions

At this point, it is appropriate to discuss 4 assumptions underlying the model:

1) *Stock prices are normally distributed.* The relevance of this postulate has already been discussed extensively (Andersen et al., 2001): while admittedly a poor description of reality, its merit is to be easy to manipulate, and also in our case to allow for the study of the multidimensional density function. We have

7. See appendix 1.2

chosen normality over log-normality, as the economic logic of the model dictates we work with price evolution rather than returns.

Nonetheless, one of the most common criticism attached to normal price returns is that they are incapable of capturing the actual thickness of the tails. Our model shows that under a particular set of circumstances the system endogenously generates a dependence structure between assets which make such “fat tails” appear.

2) *The model is symmetric.* We assume that negative shocks bring sales but equivalently positive shocks bring purchases. Symmetry comes partly from normality, but it also carries the implicit assumption that constrained investors may not escape the constrained region even when their wealth increases, that is the desired quantity of assets is always below that allowed by the constraint. In reality we may expect some investors not to buy assets in response to a looser constraint if they are happy with their holdings.

This assumption is purely to simplify analysis. One could include asymmetry in the model fairly easily, for instance by specifying a proportion of investors who escape the constraint binding region for each positive price shift. Nonetheless, since we are only interested in what happens in the left-tail, a choice has been made not to burden the model with parameters that affect the right-tail of the distribution.

3) *Parameters are constant through time.* So far, agents stick with their assessment of asset moments, and never question this assessment. In particular, convergence traders always provide demand with the same discount h, regardless of the current market conditions. In practice increased risk aversion, or higher emphasis on the short-term may hamper demand during crises. We account for this in section 1.3.2 by allowing risk aversion to rise in response to high selling

movements.

4) *The constraint relates linearly quantities and VaR.* As stated in section 1.2.2.2, a more common form relates monetary holdings and VaR. This form also has the advantage of being driven by theory as it is derived from the requirement that capital should exceed the VaR. However, as mentioned such a form does not allow to derive a market equilibrium, and we are forced to relate solely quantity to the VaR.

Though this choice clearly comes from necessity, it may be more justifiable on normative grounds. In reality the link between shocks on portfolios and selling decisions depends upon many factors, and its exact form is hard to know, so that a linear relationship may not be ruled out. For instance looking at the charts provided by Adrian and Shin (2010), a linear form between leverage and asset returns during crisis appears to give an acceptable fit.

1.3 Multivariate distribution of investor wealth

In this section we discuss the impact of diversification on systemic risk by drawing the likely number of bankruptcies from the distribution of the vector of total change in investors wealth. We then specify an objective function for society in which the cost is an exponential function of the number of failures. For this purpose solving the model is not required, the section consists exclusively of simulations.

1.3.1 Baseline model

1.3.1.1 Choice of parameters

Results will depend on 4 parameters: the fundamental variance σ_F^2, the maximum loss K that investors can incur before bankruptcy, the number of assets N, and finally the conditions on the markets r/h. We choose our baseline parameter set to fit the state of the financial markets coming in to the 2008 credit crunch. We normalize prices and initial quantities at $t=0$ to 1.

To estimate the fundamental variance we compute the average daily variance of an asset belonging to the S&P 500, from the first of July 1997 to the first of July 2007. This yields $\sigma_F^2 = 0.000616$. The S&P was chosen because it is designed to provide a broader description of the investment opportunities than its counterparts, and equities are the asset class which suits our model the most. With respect to the period chosen, in our model noise is only fundamental in BAU so that the window does not include the subprime crisis. Nevertheless we include the internet bubble, which we considered as an increased volatility episode rather than a systemic event.

An investor goes bankrupt when his losses exceed his capital K. We use data of the World Bank to set $K=0.08$, that is 8% of normalized assets. This value stands between the reported capital ratios of Tier 1 capital for US banks of 9,1% in 2008, and that of European ones, usually around 5%. We have somewhat arbitrarily chosen to be on the high side, to reflect the higher weight of the US markets.

These first two parameters have a similar and straightforward impact in our model: a fundamentally riskier portfolio or a lower default threshold both make each investor marginally riskier. Thus for conciseness we keep parameters σ_F^2 and

K constant throughout the study, relegating simulations with alternative values in appendix 1.1.

N represents the number of assets in our model but in essence describes more the maximum value for diversification. Anecdotal evidence from investment funds implies a number of assets between 5 and 15. Furthermore, seeing N as the maximum diversification level, we may follow a classic paper from Evans and Archer (1968), who estimate that diversification is no longer profitable past 10 securities, a belief shared amongst practitioners. We thus set *N=10*, for now. We will study in section 1.4 the impact of rising N to 20, as in Cont and Wagalath (2014).

With respect to market conditions r/h, we study 3 scenarios:

- a “mild” one in which the constrained agents are forced to sell assets in fairly small quantities when a shock hits, and the demand by active investors for such assets is strong. $r/h = 0.6$

- a “windy” scenario in which the quantities sold by constrained agents and the demand by convergence traders are both moderate. $r/h = 0.75$

- a “storm” scenario in which the pro-cyclical effect of the VaR constraint is large, and demand response is weak. $r/h = 0.9$

Studying 3 scenarios results partly from the lack of data on fire sales by investors[8], but is mostly interesting as part of our analysis.

8. There has been empirical evidence on the presence of fire sales (see for instance Coval and Stafford (2007)), but it is hard to use it for calibration as researchers can only conjecture that a given sale has been made out of necessity, and their impact on prices.

1.3.1.2 Distribution of total number of bankruptcy

Figure 1.3.1 summarizes our approach of systemic risk in the intermediate scenario $r/h = 0.75$. Each possible number of investor failures, from 0 to N=*10*, has a probability given by the multivariate cumulative normal distribution, and we study how this probability varies with the level of diversification.

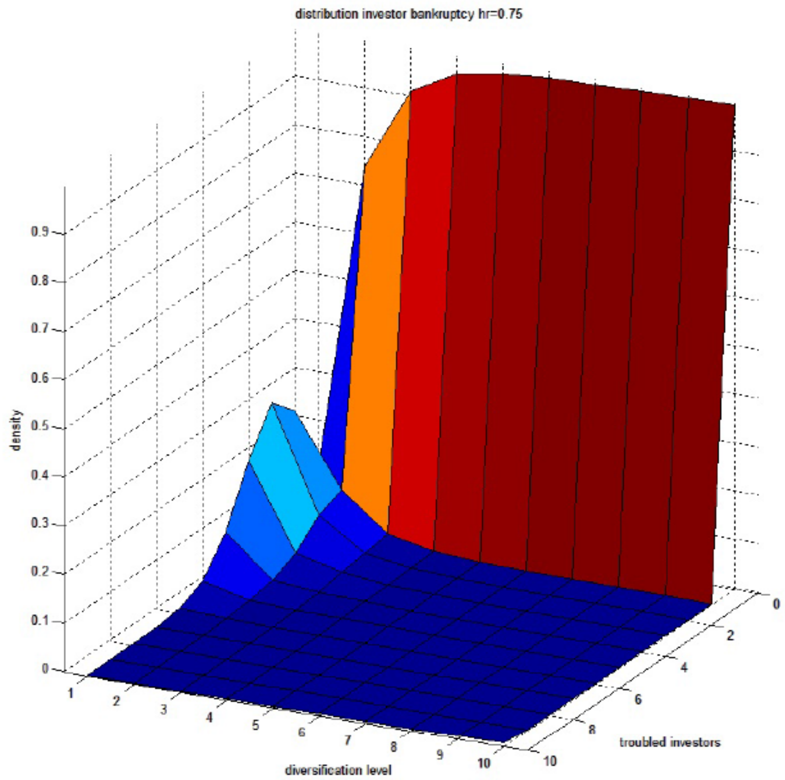

Figure 1.3.1 – Distribution of number of bankruptcies in stable regime with $r/h = 0.75$.

The dual impact of diversification appears clearly. As n rises investors become more and more dependent, outcomes in which some investors fail but other survive become less likely. In the total diversification case, only the “all survive” and the “all fail” outcomes are possible. Due to the individually risk-reducing impact of diversification, the "all survive" equilibrium increases faster in likelihood, *yet we also observe a gradual detachment from 0 of the likelihood that every investor fail,* reaching a non-trivial 0.005% for *n=10.* In this case the overall desirability is thus ambiguous, and will crucially depend on how painful mass failure is to the entire economy.

Figures 1.3.2, 1.3.3, and 1.3.4 zoom in on number of failures large enough to constitute a systemic event, for all values of r/h.

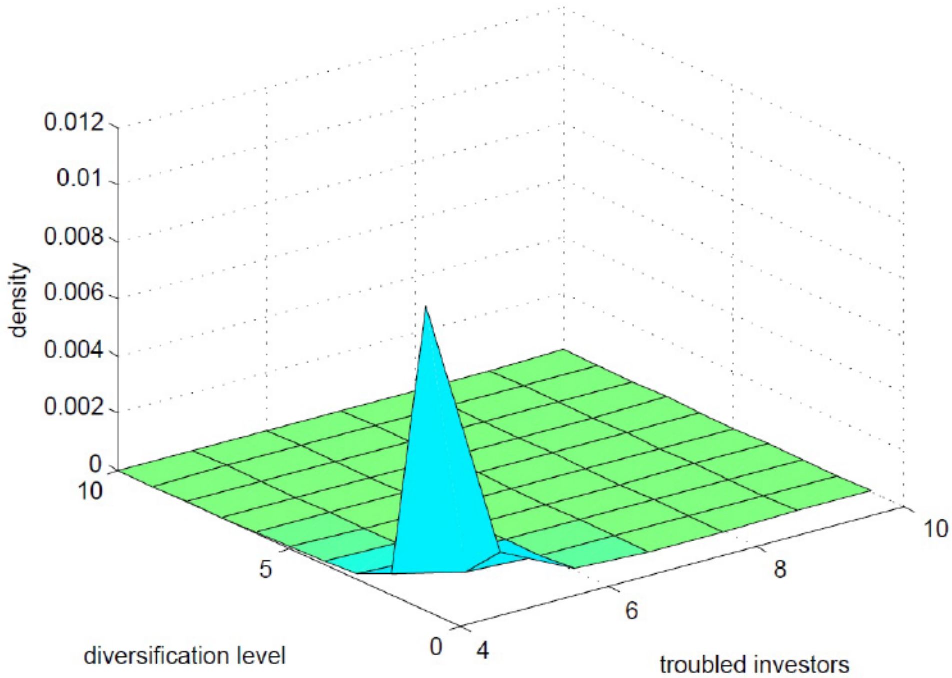

Figure 1.3.2 – Extreme bankruptcies odds, $r/h = 0.6$.

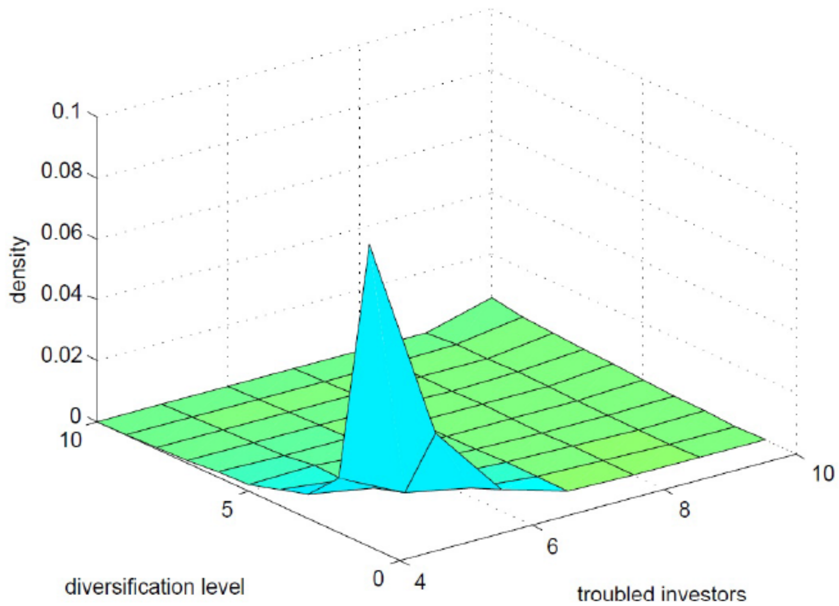

Figure 1.3.3 – Extreme bankruptcies odds, $r/h = 0.75$.

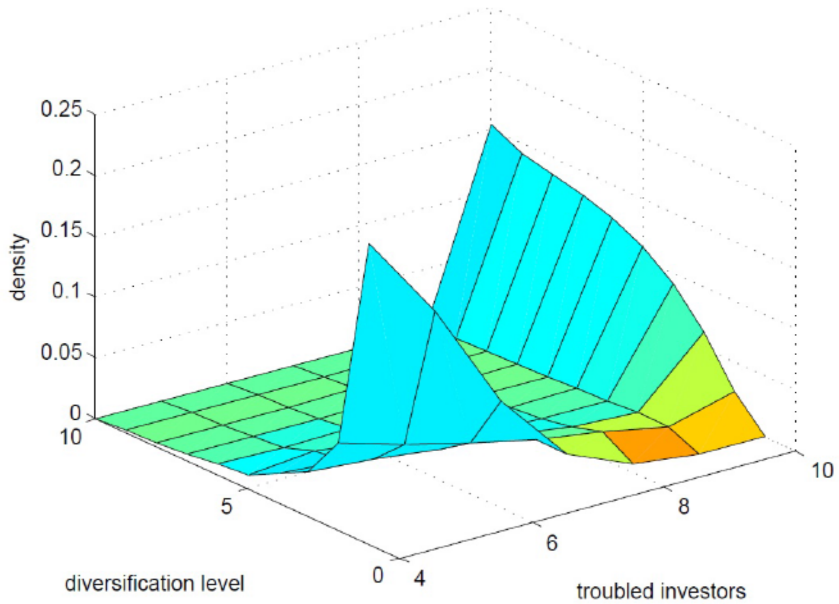

Figure 1.3.4 – Extreme bankruptcies odds, $r/h = 0.9$.

In figure 1.3.2 we see that any number of failure above 6 has a near zero likelihood. For low levels of diversification this results mainly from the higher independence between investors, for high levels of diversification it comes from the fact that with *r/h=0.6* shocks quickly die out, so that the contagion externality is limited. In figure 1.3.3 the scope for contagion is increased as we move to $r/h = 0.75$. In the extreme *r/h=0.9* case, shocks transmit almost fully across investors, so that the probability of mass failure increases drastically. In particular the odds attached to the "all-fail" outcome reaches up to 15% when n=N. In this case promoting a lower level of diversification amongst investors seems sensible.

1.3.1.3 Welfare

We weight the probability attached to a given number of failures against the cost attached to it.

If the cost to society increased in a linear fashion with the number of failures, diversification would unambiguously be desirable from society's perspective. Yet there are many theoretical reasons for which this cost may in fact grow exponentially with the numbers of defaults. In particular, the well-identified channels for contagion may be enhanced by bankruptcies: we expect surviving investors to become much more risk averse, reputation risk to sky-rock, the liquidity constraint to tighten, etc. Fiordelisi and Marqués-Ibañez(2013) for instance show empirically that bank failures have a strong impact on asset prices. Bernanke (1983) also points out that financial bankruptcies have a more than proportional impact on the real economy, through decreased money supply and increased cost of financial intermediation.

Perhaps due to this high variety of channels and non-linearity, it is hard to estimate the exact cost to society of financial bankruptcies. Authors who want to model this cost have used different mathematical artifices. For instance Ibrahimov et al. (2012) define a time to recovery for the system, which depends of the number of defaults. We simply specify the following cost function for society:

$$C(\eta) = e^{\beta\eta}$$

where η, the number of failures, is a random variable, and β mitigates the severity of the increase in the cost to society of an additional failure. We show results for values of $\beta \in]0, 0.72]$. We pick this interval because it contains the interesting cases. $\beta = 0.72$ represents a very high exponentiality of mass failures, as it implies an average cost of a single bankruptcy that is 65 times larger when $\eta = N$ than it is when $\eta = 1$.

The expected cost to society reads:

$$E(C) = \sum_{\eta=0}^{\eta=N} P(\eta)C(\eta)$$

where the probabilities $P(\eta = i)$ were obtained in the previous section. The next 3 figures show the expected cost for our three values of r/h, across β and n. The blue line and the black track the level of diversification for which the expected cost to society for a given $(r/h, \beta)$ couple is the lowest, or highest respectively.

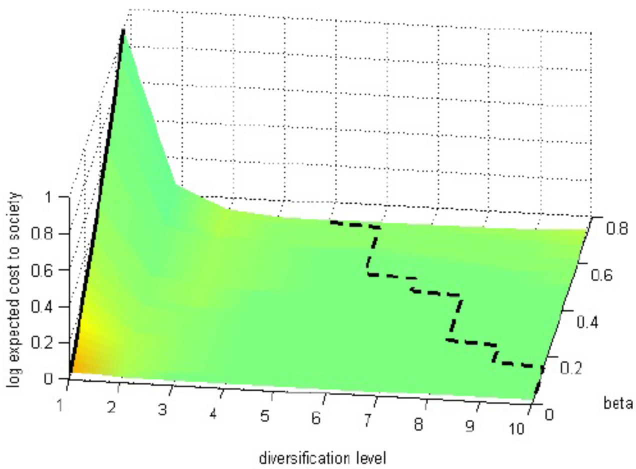

Figure 1.3.5 – Desirability of diversification, $r/h = 0.6$.

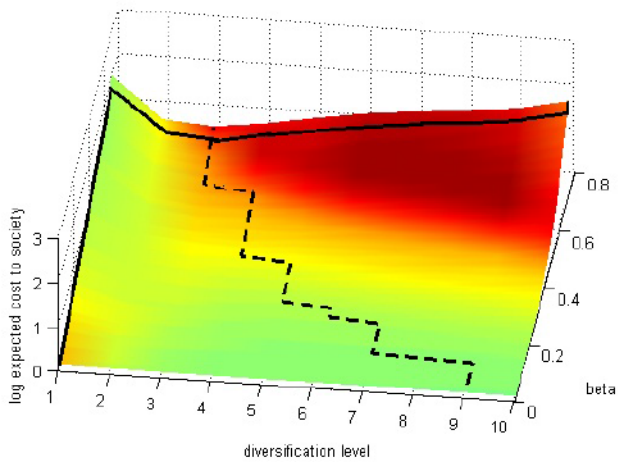

Figure 1.3.6 – Desirability of diversification, $r/h = 0.75$.

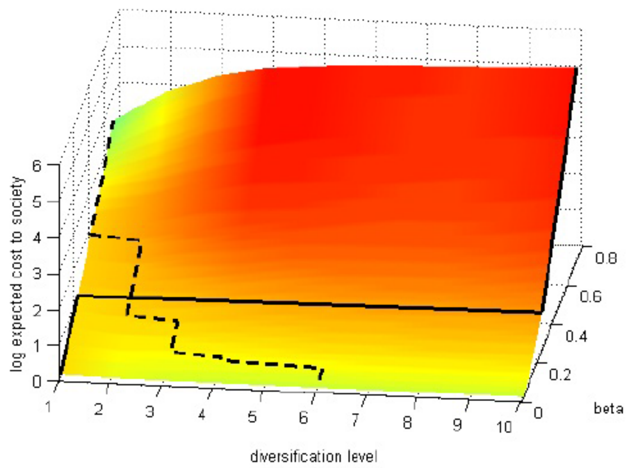

Figure 1.3.7 – Desirability of diversification, $r/h = 0.9$.

Unsurprisingly, a higher β unambiguously works against higher levels of diversification, in which mass failure is more likely. Graphically this is shown by the left turns of the blue line plotting the optimal diversification level.

The impression from the previous section remains. In the *r/h=0.6* case the contagion externality is too modest for diversification not to be desirable. However with large value of β the optimal level of diversification goes surprisingly low, reaching n=*5*. In *r/h=0.75* the same logic applies, leading the optimal level to n=*3* for a high β. In words this means the cost of an increase in the dependence between investors is still acceptable in exchange for the huge private benefits of moving from n=1 to n=2 and n=*2* to n=*3*. The contagion externality is larger, leading n=N to become the least desirable level for $\beta > 0.7$. In the last *r/h=0.9* situation, the scope for contagion is so high that no individual risk reduction justifies it past $\beta = 0.4$. The perfectly diversified situation becomes the worst possible situation rapidly, when $\beta > 0.2$.

$\beta = 0.7$ implies that the ratio of unit of failure in the "all fail" over that in the "one only" case is about 54, while $\beta = 0.4$ implies a ratio of 3.65. As mentioned this measure is based more on intuition than evidence, we leave it to the reader to assess where its true value would stand. In any case its seems fair to say that, in a situation in which financial shocks propagates linearly, *there exists a reasonable set of parameters in which any level of diversification in the economy is dominated by a situation in which investors trade only their own assets, and complete diversification is generally not the optimal level for society.*

1.3.2 With possible panic

1.3.2.1 Change in framework

Remember the demand response to a deviation from fundamentals by convergence traders is given by $h = \frac{m\tau_{mt}}{E(\sigma^2_{t+t^*})}$. We now allow the strength of this demand response to change in the face of extreme selling movement on the markets. In this set-up this could happen because MT investors review their estimation of the variance $E(\sigma^2_{t+t^*})$ in periods of higher volatility, or because their risk aversion $1/\tau_{mt}$ rises, as they begin to doubt their short-term ability to absorb escalating losses. Somewhat arbitrarily we decide to attribute the rise in r/h to risk aversion when sales go beyond a certain threshold k^*. The system now writes:

$$\triangle \mathbf{P_t} = (\tfrac{r/h^*}{n^2} A^\intercal A)^t \mathbf{e_1^F} \quad if\ \exists \triangle q_i, \mid \triangle q_i \mid > k^*$$

$$\triangle \mathbf{P_t} = (\tfrac{r/h}{n^2} A^\intercal A)^t \mathbf{e_1^F} \quad if\ \forall \triangle q_i, \mid \triangle q_i \mid \leq k^*$$

where h^* is the discount prevailing in a situation of "panic". Therefore in this framework investors "panic" when constrained sales get passed k^* on *a* given market. It may thus only take one extreme movement on one market to trigger panic.

Both experience and theory may justify this form: increased risk aversion is highly contagious so that investors operating on the troubled market may lead others to grow more risk averse, accrued counterpart uncertainty is higher when one asset falls drastically than when all fall moderately, margin calls may be triggered when losses exceed a certain level, etc.

We note α the probability that one or more market reach the sales threshold. It is given by the multivariate cumulative normal distribution Φ since sales depend linearly on the price shocks at *t=0* , which are normally distributed. We thus have $1-\alpha = prob(\forall \triangle q_i, \mid \triangle q_i \mid \leq k^*) = \Phi(k^*, ..., k^*)$. The expected value and variance of the sales vector at *t=0* are respectively 0 and $\Sigma_{\triangle Q} = \sigma_F^2 (\frac{r}{n^2} A^\intercal A)^2$.

Two things should be noted here: first this technique is consistent with our set-up since the sales are the largest at *t=0.* This means the threshold is either immediately or never reached in our framework, but in both cases r/h^* of r/h remains constant afterward. In other words the initial sales shock "sets the tone" for the rest of the crisis episode. Second, the expected value of $\triangle \mathbf{P_t}$ remains zero regardless of the regime, so that the variance with panic is given[9] by $\Sigma_t = \alpha \Sigma_t^{\mathrm{P}} + (1-\alpha)\Sigma_t^{NP}$, where subscripts P and NP refer to the "panic" and "no panic" cases respectively. Similarly the variances for investors over time is simply the weighted average of covariances matrices in the panic and no panic cases:

$$\Sigma_I = \alpha \Sigma_I^P + (1-\alpha)\Sigma_I^{NP} \tag{1.9}$$

9. Using the conditional variance decomposition formula:
$\Sigma_t = [\Sigma_t / \forall \triangle q_i :\mid \triangle q_i \mid < k*) + (E(\triangle \mathbf{P_t} / \forall \triangle q_i :\mid \triangle q_i \mid < k*))^2] P(\forall \triangle q_i :\mid \triangle q_i \mid < k*)$
$+[(\Sigma_t / \exists \triangle q_i :\mid \triangle q_i \mid > k*) + E(\triangle \mathbf{P_t} / \exists \triangle q_i :\mid \triangle q_i \mid > k*))^2] P(\forall \triangle q_i :\mid \triangle q_i \mid > k*) - [E(\triangle \mathbf{P_t})]^2$
where $E(\triangle \mathbf{P_t}) = 0$ and $E(\triangle \mathbf{P_t} / \forall \triangle q_i : \triangle q_i < k*) = E(\triangle \mathbf{P_t} / \exists \triangle q_i :\mid \triangle q_i \mid > k*))^2 = 0$, as the distribution of $\triangle Q$ is symmetric:

1.3.2.2 Distribution of investors bankruptcy

In the last section we have seen *r/h=0.6* implies a very limited scope for contagion, *r/h=0.9* a large one. These values are thus natural candidates to estimate the no panic and panic case respectively. Regarding the panic threshold, similar to last section we try three values which represent respectively low, moderate, and high tendency for short-term investors to panic, for which panic is triggered for initial shocks of respectively 0.05, 0.01 and 0.005, normalizing *r=1*, i.e., 5%, 1% and 0.5% of normalized total quantity of each asset [10]. Figure 1.3.8 to 1.3.10 summarize our findings.

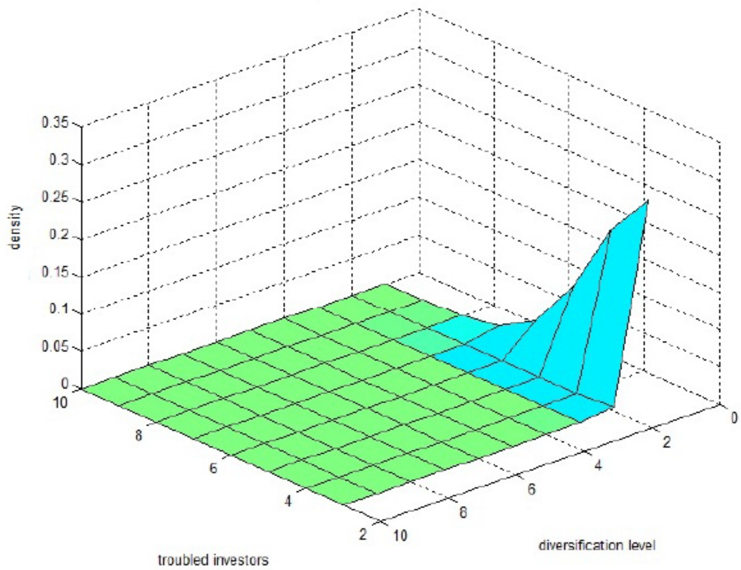

Figure 1.3.8 – Extreme bankruptcies odds, low panic.

10. These values may appear low but one should remember our time unit is a "day".

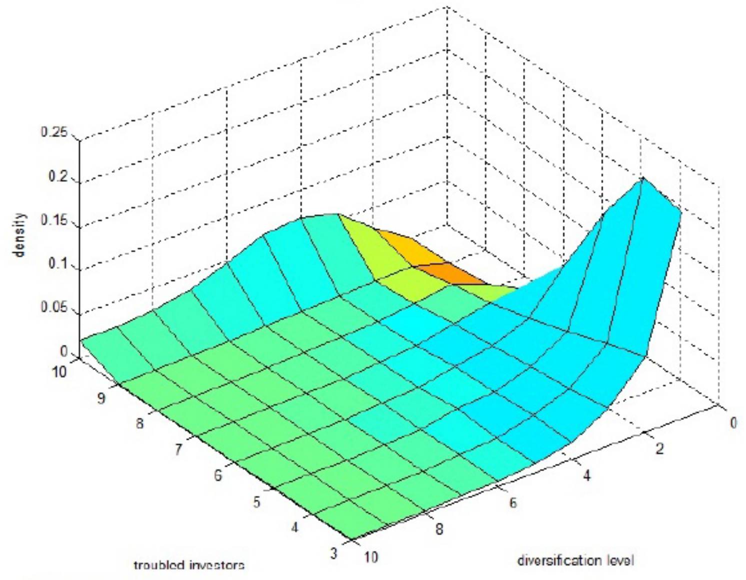

Figure 1.3.9 – Extreme bankruptcies odds, moderate panic.

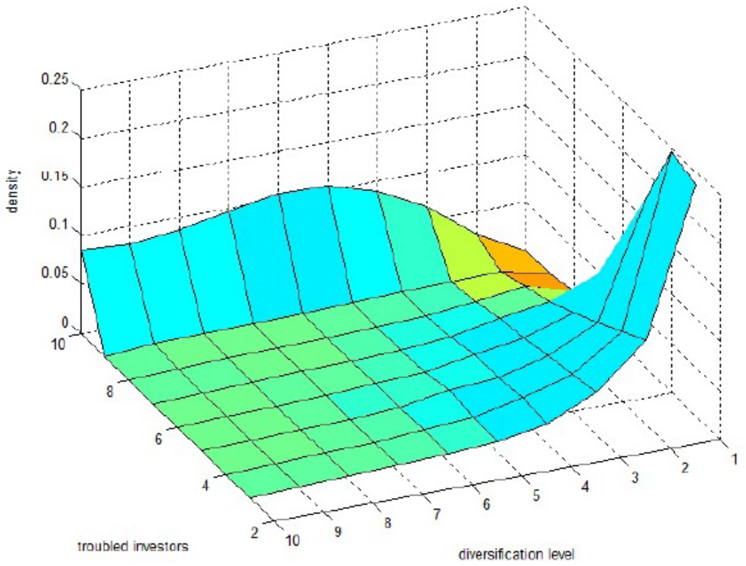

Figure 1.3.10 – Extreme bankruptcies odds, high panic.

In figure 1.3.8, the extreme selling required to trigger panic is very unlikely to happen when $n>2$, so that the figure resembles the non-panic $r/h=0.6$ case. Extreme failure, particularly the all fail outcome, is very unlikely. On the other hand, when $n=1$ the possibility of a panic exists, which may yield a considerable number of failures, with non trivial odds of as much as 80% of investors going under.

The intermediate and easy panic cases bring new light to our results. In the moderate panic case, for realistic values of β the likelihood of the all-fail outcome is maximized at $n=4$ in the intermediate case, $n=5$ in the easy panic one. In other words, *intermediate levels of diversification which were an attractive option without panic now seem particularly harmful, because such levels are not efficient enough in smoothing the wealth shocks faced by portfolio holders, but provide linkages through which shocks may spread across assets.* “Medium-low” levels also appear quite undesirable, with odds that $\eta = 9$ and $\eta = 8$ significantly above zero when $n=2$ or $n=3$.

Figure 1.3.11 on desirability confirms this intuition: in the intermediate panic case and easy panic one, $n=4$ and $n=5$ become the least desirable option for respectively $\beta = 0.42$ and $\beta = 0.3$, which implies a unit cost of failure in the all fail case over that in the one fail only of 4.38 and 1.48.

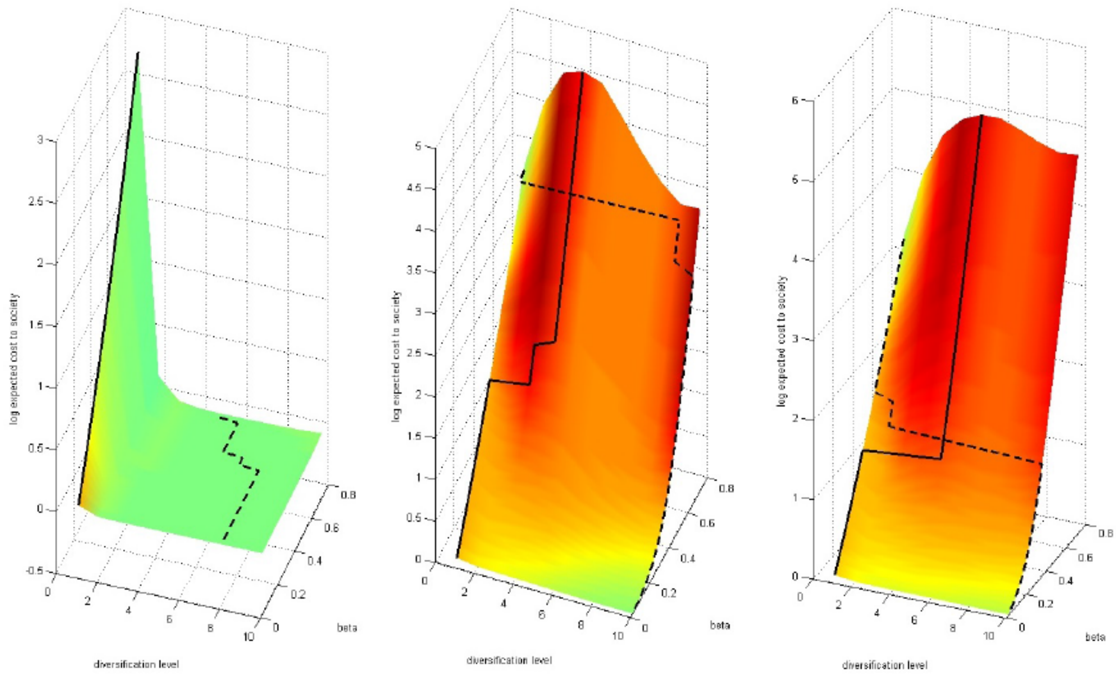

Figure 1.3.11 – Desirability of diversification, high panic.

Another interesting feature of the intermediate and easy panic cases is that the cost is a perfectly concave function of the diversification level for nearly all values of β. This leads the extreme outcomes "high diversification" and "no diversification" to become relatively more desirable, particularly the former. Indeed in the intermediate case "no diversification" becomes the preferred option for a very high β, contrary to the no panic set-up, while in both cases *very high levels of diversification are no longer the least desirable option even with a high exponentiality of mass failure, but remain the most desirable one with a low* β. A high diversification level *n=9/10* is optimal if $\beta < 0.36$ in the easy panic case and $\beta < 0.7$ in the intermediate one, implying a ratio of 2.55 and 54 respectively.

On these intervals, diversification thus provide the first-best allocation if it "goes all the way", but the worst one if it "stops halfway through". This suggests that there is a critical threshold level of diversification, past which diversification is desirable, but below which no diversification should be preferred. These conclusions complement those of Caccioli et al. (2014) who find that when leverage is above a critical value, diversification has non-monotonic impact on the odds of a cascade of default, which they define as 5% or more of investors failing.

1.4 Covariances and extensions

1.4.1 Endogenous covariances

In this section we solve the model to derive an analytical expression for covariance at a given period Σ_t, across time Σ_{tot}, and investors across time Σ_I. This allows us to relate this study to the literature on endogenous risk, and confront our form it to evidence. The discussion on the factors that impact covariances will also pave the way for the study the systemic impact of a rise in the number of assets N and a change in the pattern of asset holdings.

Analytical expression

Drawing the general element of Σ_t, Σ_{tot}, and Σ_I involves diagonalizing matrices $(A^\intercal A)^t$, $\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t$, and $\frac{1}{n}A\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t$. This is made quite straightforward by the fact that these matrices are all circulant. The demonstration is of mostly technical value, so that we relegate it to appendices 1.2 to 1.5. We obtain the following expressions for the covariance between k and j, which represent respectively asset price change per period, in total, and total investor wealth change:

$$cov(\triangle p_j, \triangle p_k)_t = \frac{\sigma_F^2}{N}(\frac{r/h}{n^2})^{2t}(2\sum_{q=1}^{q=(N-1)/2} cos(\frac{2\pi(k-j)q}{N})\left(\frac{1-cos(\frac{2\pi nq}{N})}{1-cos(\frac{2\pi q}{N})}\right)^{2t} + n^{4t}) \tag{1.10}$$

$$cov(\triangle p_j, \triangle p_k)_{tot} = \frac{\sigma_F^2}{N}((\frac{1}{1-r/h})^2 + 2\sum_{q=1}^{q=(N-1)/2} cos(\frac{2\pi(k-j)q}{N})(\frac{1}{1-(\frac{r/h}{n^2})(\frac{1-cos(\frac{2\pi kn}{N})}{1-cos(\frac{2\pi k}{N})})})^2) \tag{1.11}$$

$$cov(I_k, I_j)_{tot} = \frac{\sigma_F^2}{N}\frac{1}{n^2}[(n\frac{1}{1-r/h})^2 + 2\sum_{q=1}^{q=(N-1)/2} cos(\frac{2\pi(k-j)q}{N})\frac{1-cos(\frac{2\pi kn}{N})}{[1-cos(\frac{2\pi k}{N})][1-(\frac{r/h}{n^2})(\frac{1-cos(\frac{2\pi kn}{N})}{1-cos(\frac{2\pi k}{N})})]^2}] \tag{1.12}$$

In the case in which N is odd [11].

These covariances are defined simultaneously and depend upon the same factors: the conditions on the market [12] r/h, the fundamental risk σ_F^2, the *distance k-j* in the financial network, the *number of assets* N, and of course the *level of diversification*

11. The expressions change slightly when N is even, for instance in the case assets prices change per period: $cov(\triangle p_j, \triangle p_k)_t = \frac{\sigma_F^2}{N}(\frac{r/h}{n^2})^{2t}(2\sum_{q=1}^{q=(N/2-1)} cos(\frac{2\pi(k-j)q}{N})\left(\frac{1-cos(\frac{2\pi nq}{N})}{1-cos(\frac{2\pi q}{N})}\right)^{2t} + n^{4t} + cos(\pi q)\left(\frac{1-cos(\frac{2\pi nq}{N})}{1-cos(\frac{2\pi q}{N})}\right)^{2t}),$

12. A technical point: $h = \frac{m\tau_{mt}}{E(\sigma_{t+t^*}^2)}$, where $E(\sigma_{t+t^*}^2)$ is the assessment of the individual price variance. If this assessment were correct, i.e., $E(\sigma_{t+t^*}^2) = \sigma_{t+t^*}^2$, then deriving the covariance $\sigma_{t+t^*}^2$ would imply solving a fixed-point problem, as the expression for $\sigma_{t+t^*}^2$ would depend upon itself. However this fixed point problem is complex, and the question of whether the equilibrium is rational or not is not the main goal of the chapter. We thus let $E(\sigma_{t+t^*}^2)$ as a parameter whose value may be $\sigma_{t+t^*}^2$ or not, noting only that it enter positively in in the expression for $cov(\triangle p_j, \triangle p_k)_t$, that is when MT investor expect a high volatility asset prices covariance is higher, as expected.

n. As we will see in next section, the two expressions on which we focus, (1.11) and (1.12), will in fact move closely together.

The expressions above are in line with the literature on endogenous risk: the actions of the market participants lead to correlations between assets that would not exist otherwise. Parameters σ_F^2 and r/h in particular can be related to Danielsson et al. (2012) and Cont et al. (2014), who provide expressions of the covariance matrix that depend on the fundamental covariance structure and some economic variable that governs to the transmissibility. We add the "network" parameters *n,* N and *(k-j)*.

What if we allow for panic? Since the covariance matrix is simply the weighted average between that in the linear *h/r=0.6* case and the *h/r=0.9* one, we have $cov(\triangle p_j, \triangle p_k) = \alpha cov(\triangle p_j, \triangle p_k)^P + (1-\alpha) cov(\triangle p_j, \triangle p_k)^{NP}$ for all three forms. The impact of each factor can be split into how it impacts covariances in a given regime and how it impacts the likelihood of each regime.

Comment

We study the effect of *n,* N and *(k-j)*, impact studies of r/h and σ_F^2 are available on request [13]. For each factor we first discuss its impact in the *h/r=0.75* linear model, then that in the easy panic case which in our opinion provides a better description of the reality of crises. We keep the parameter set from section 1.3.

As could be expected from section 1.3, *diversifying amounts to transferring some variability from the neighboring assets/investors to the more distant ones.* As shown by figure 1.4.1, as n rises all assets/investors become more and more

13. Parameter σ_F^2 rises all covariances uniformly, while a high h/r increases more the covariances and correlations with more remote assets/investors, as it implies that a given shock reaches points that are further away in the network.

related and all covariances converge to a similar value[14], as shown by figure 1.4.1.

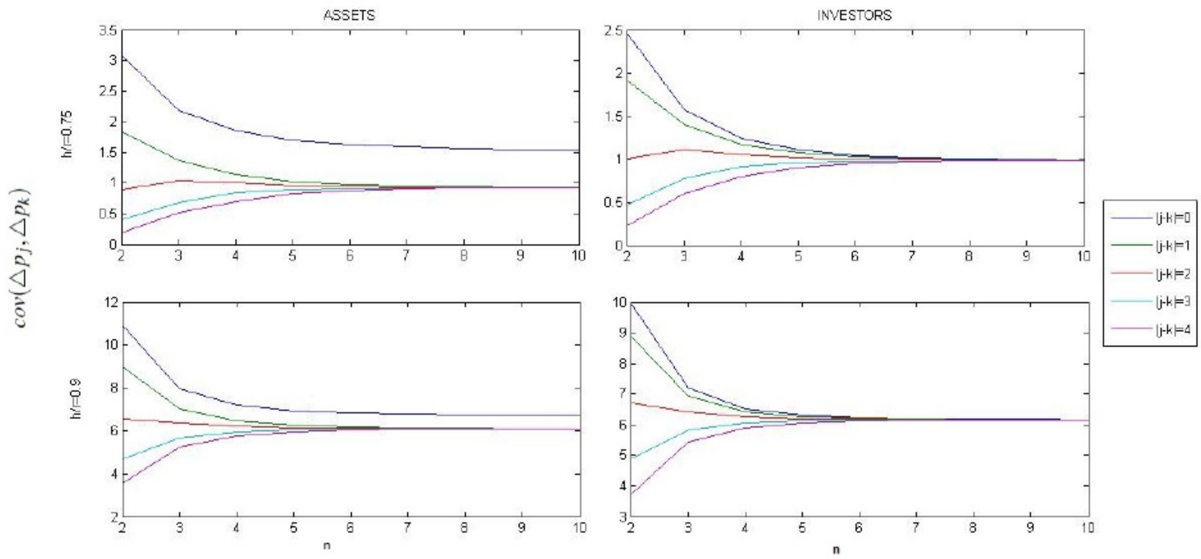

Figure 1.4.1 – Covariances as a function of n for all values of */k-j/*.

Interestingly, we see that a direct connection between two assets/investors does not necessarily enhances their covariance. For instance moving from *n=2* to *n=3* lowers the covariance between *j* and *j+2* when $h/r = 0.9$. This reflects the fact that when $h/r = 0.9$ and n=2 the losses on troubled assets snowball to the point where the gain from spreading wealth shocks through diversification exceeds the cost of a direct connection.

Figure 1.4.2 describes the panic set-up.

14. Note that compared to investors, each asset keeps a larger auto-correlation. This comes from the fact that at *t=0* its price changes are unrelated, while as n rises portfolios move in more and more similar fashion even at *t=0*.

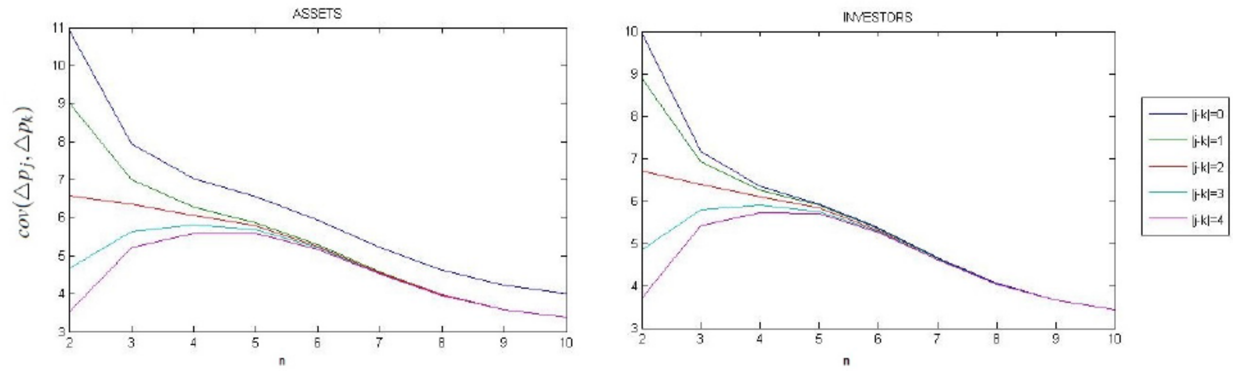

Figure 1.4.2 – Covariances as a function of n for all values of */k-j/*, panic.

As n rises wealth shocks and thus fire sales become weaker, so that the panic threshold is harder to reach. As the likelihood of the no-panic case rises, average covariances fall to their *h/r=0.6* levels, in which transmissibility is lower. However the patterns of convergence across assets remain the same. This figure is also consistent with our finding that intermediate levels of diversification are particularly harmful, as *such levels provide a unique mix of homogeneity of the covariances and significant likelihood of panic.*

The impact of *N,* which we also consider as an indicator of the completeness of the markets, is contingent on the level of diversification. To see why let us picture the financial network as a circle and a shock stemming from a particular asset as a wave running through it. When a wave has ran through all assets it returns to the asset which originated the shock, which we label as "second round" impact. In our home biased network the wave moves slowly from neighbor to neighbor, and thus such "second round" effects have little impact when n is low, even with *h/r=0.9*. However when n rises and the speed at which the shock completes the circle increases, second round effects become important and losses converge to the same value across assets/investors.

In this case *adding more assets is very desirable from a covariance minimizing perspective as it divides the total fall across more asset/investors.* When $N = +\infty$, i.e., markets are effectively complete, the covariances tend to 0. Figure 1.4.3 shows this by plotting the covariance between assets/investors j and k=j+*2* for $n \in [1, 10]$.

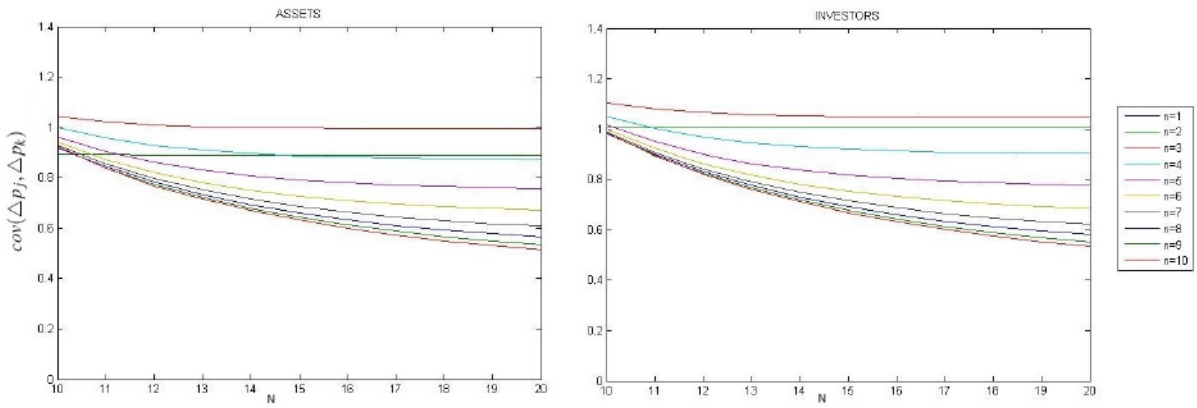

Figure 1.4.3 – Covariances between j and k=j+*2* as a function of N for all values of n.

In the panic case the impact of N becomes a priori ambiguous. On one hand fire sales are less likely as each investor is safer since the covariances between assets in his portfolio are lower. On the other more assets means that the likelihood of a single extreme movement increases. From this perspective figure 1.4.4, which plots the covariance between assets j and k=j+*2,* is interesting.

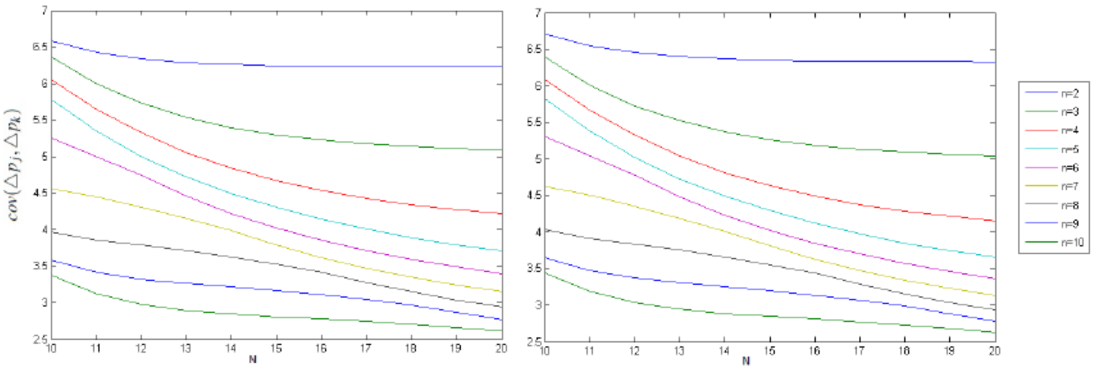

Figure 1.4.4 – Covariances between j and k=j+2 as a function of N for all values of n, panic.

First and foremost the trend is still to a decrease. This means the negative relationship between N and covariances in each regime, similar to figure 1.4.3, dominates the impact of N on the probability of panic. This relationship remains monotonic, so that *increasing N stills unambiguously lowers the covariances.*

Nevertheless, the impact of N on the probability of panic changes the patterns of falls. Looking at the highest n, the function becomes concave on some segments of $N \in [10, 20]$. On such segments the fall is then lower than its no panic-counterpart, implying that the marginal impact of N on the likelihood of panic is positive. It is logical that this occurs for high levels of diversification since for those levels the "anti-panic" impact of N is low, as investors are already quite insured against shocks through diversification. When the likelihood of panic approaches 1 however, both effects on panic die out and the function retakes its "no-panic" evolution. Interestingly, *the levels of diversification for which* N *lowers covariances the most are the intermediate ones, which appeared particularly detrimental from a systemic risk perspective.*

The role played by distance also depends on n, as when more and more investors/assets become connected the initial position in the network becomes irrelevant. Nonetheless distance is highly explicative for diversification levels below or equal to *N/2*, as Figure 1.4.5 shows. For instance in the *n=4* case asset covariance between direct neighbors is twice that with most distant ones. The relative magnitudes are very similar for *h/r=0.6* or *h/r=0.9*, so that the panic case gives the same pattern except for the fact that the average covariance falls with αsimilar to figure 1.4.2.

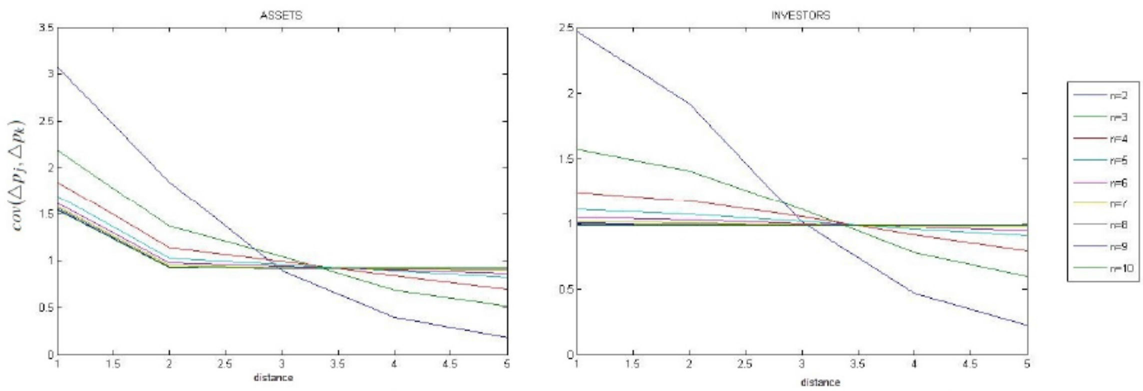

Figure 1.4.5 – Covariances j and k as a function of */j-k/*, for values of n.

This impact of distance is in line with empirical evidence. Many papers, such as Lane et al. (2004) have demonstrated empirically that informational distance is strongly negatively correlated with bilateral equity holdings. Gravity approaches to estimating correlation usually provide good fits. On the link between covariances/correlations and distance during crisis, the evidence is not as definite. This may well be due to the fact that the real impact of a financial crisis is often higher in emerging countries, so that fundamental factors dominate endogenous ones there.

Surprisingly little work has been undertaken on correlations during the Subprime crisis, but we may note that by Naoui et al. (2010) who suggest that correlations with the US increased uniformly by about 80% in developed countries during the crisis, while the rise is much variable for emerging ones. Anecdotal evidence that the crisis originated in the US also suggests that distance is important in studying contagion.

The negative relationship between covariances and distance may also be related to the so-called *home bias puzzle*: since more distant assets have lower covariances, investors apparently forgo diversification benefits by failing to include them in their portfolios. In our model investors not only potentially miss out on lower fundamental covariances, but effectively take on endogenous risk which is at least partly avoidable. Figure 1.4.6 shows the variance benefits of following a non biased strategy in our network with home bias.

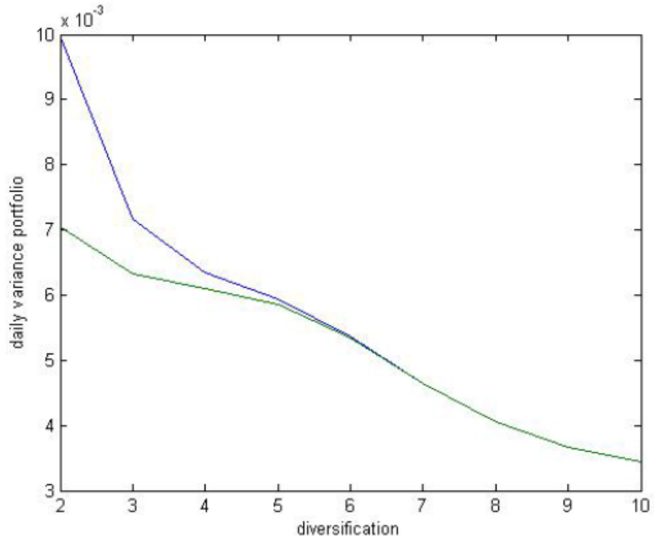

Figure 1.4.6 – Home biased versus non-biased investor.

There are substantial private benefits from deviating from home biases in the early stages of diversification. Such benefits are no longer significant around the $N/2$ level. At the investor level, accounting for endogenous risk thus increases the incentive to acquire costly information or pay higher fees.

1.4.2 Systemic risk impact of a change in the network

The impact of the completeness of markets and distance on covariances raises the question of the systemic implications of a wider network, i.e., with a larger N, and a non-biased network in which distance becomes irrelevant. For conciseness we only discuss the easy panic case, as a level of diversification desirable in the most extreme scenario must be desirable also in the other ones. We provide another extension in appendix 1.6, studying the systemic impact of introducing speculators in the market.

1.4.2.1 Optimal network

Let us imagine a network without bias in which each investor pick the portfolio that minimizes his endogenous risk. For instance if $n=3$, an investor of type I would hold the asset i he is endowed with and pick the two assets that are the least correlated with his, that is the assets that other investors of type I hold the least. Through arbitrage the average quantity by investors of type I of all assets other than their own should thus converge. The sum of these average quantities should be twice the holding of i since $n=3$ and investors ideally spread their holdings equally across assets. In general, the average holding by I should be 1 of i, and $\frac{n-1}{N}$ of other assets.

We saw in previous section that such a “home bias free” network enhances the marginal resilience of each investor by lowering the correlation between the assets he holds. However it also implies that the correlation between all investors become homogeneous more rapidly. Swapping networks thus a priori involves weighing up the same costs and benefits that those of increasing the level of diversification: higher contagion costs versus individually sounder benefits. Figure 1.4.7 plots the difference between densities in the new network and those in the previous one.

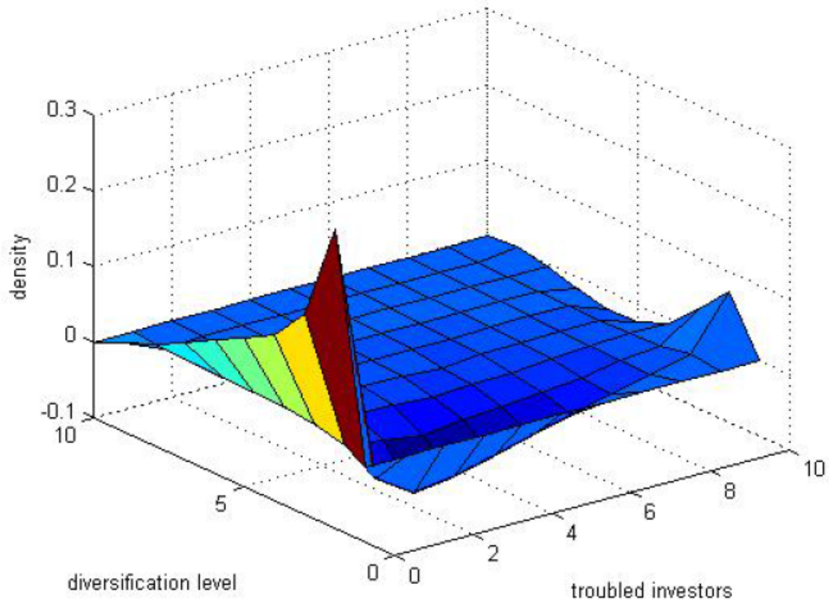

Figure 1.4.7 – Bankruptcies odds, comparison home biased/optimal networks.

The higher individual resilience is reflected in the fact that the likelihood of the “all-survive” outcome is higher in the optimal network for all levels of diversification. Yet the faster contagion cost is also visible: for low levels of diversification we have a higher probability of total failure $\eta = N$, though for such levels the increased dependence between investors also brings a strong fall in the odds of intermediate number of failures. For these levels of diversification, the desirability of moving to an optimal network should thus depend on how exponential the cost of financial failures is. For instance in the N=10 and *n=3* case the "optimal" network is preferable only when $\beta < 0.32$, which means a unit cost of failure that is 1.78 times larger when $\eta = N$ than when $\eta = 1$.

Yet the contagion cost weakens as n rises as a new effect starts kicking in: an optimal network spreads sales across assets which lowers the likelihood of panic. Therefore *past n=4, moving to an optimal network appears unambiguously desirable.* Interestingly, *the levels of diversification for which the shift appear the most desirable are the intermediate ones*, which were the most dangerous previously this easy panic case.

From a technical standpoint, note that these two networks are equivalent in Caccioli et al. (2014), but not in our set-up. Changing qualitatively the network to account for different heuristics thus has a significant impact on our results, as we shall verify in the following section.

1.4.2.2 Wider network

We move from a *N=10* network to a *N=20* one, as in Cont and Wagalath [15]. We choose [16] to compare only similar absolute levels of diversification, i.e., $n \in [1, 10]$ even for *N=20*, and normalize failures as the proportion of investors going under , i.e., $\eta = 10$ with *N=10* is equivalent to $\eta = (19, 20)$ when *N=20.* Importantly, our

15. Greenwood and Thesmar use a number of asset of 42, but in our context the computational burden rises exponentially with N, and the simulations on *N=20* case already took 10 days to complete.
16. Putting in perspective both cases requires choices. Should we compare the point ($n = 10, \eta = 10$) when *N=10* to ($n = 10, \eta = 10$) or to ($n = 20, \eta = 20$) when *N=20*?

choice of absolute n and relative η yields the situation in which the desirability of $N = 20$ over $N = 10$ is minimized. The results presented are thus a minima[17].

Figure 1.4.8 plots the difference between densities with *N=10* and those with *N=20*.

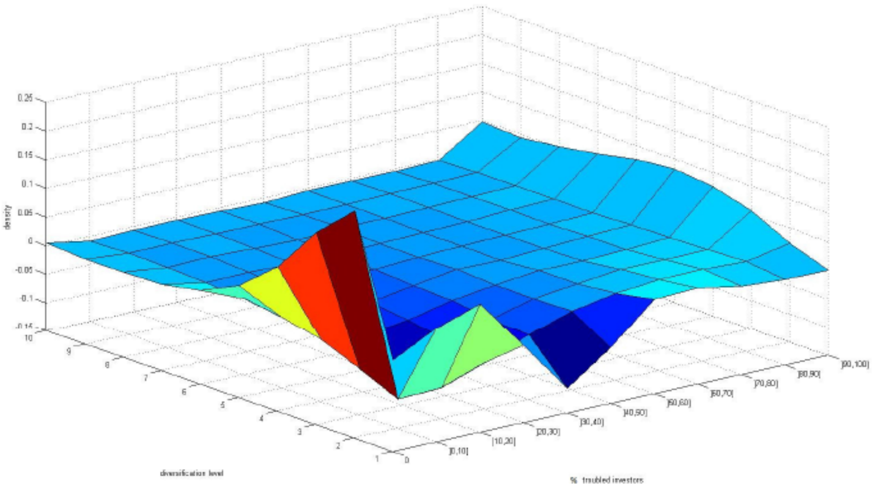

Figure 1.4.8 – Impact of increasing N on systemic risk.

The likelihood of the extreme "all fail" outcome is higher when *N=10*. Again, the difference is most pronounced for the intermediate diversification levels which were previously deemed particularly dangerous. The "all survive" outcome is also more likely. This results from the fact that increasing the number of asset induces more independence across investors, thus reducing the odds of perfectly symmetric situations. In response, the intermediate levels of failure become less likely when

17. One should also note that increasing N acts upon the covariance structure, but also has a combinatorial side which may not be relevant here. The *n=1* shows this combinatorial effect, as with no diversification the problem simply amounts to picking η out of N investors. A reassuring sight is that this value of n appears quite independent of the others, which implies the impact of N goes primarily through the covariance structure for *n>1*.

N=10, with various patterns depending on n. When n is fairly low, positive but moderate proportions of investors going bankrupt bear most of the adjustment, which probably reflect the fact that each investor is safer when *N=20* due to lower covariance across assets. As n rises and this independence naturally falls, the difference becomes more homogenous across all intermediate levels of failures.

In any case, *the fact that the "all-fail"outcome is significantly more likely in the N=10 case implies that N=20 ought to be more desirable, even for a relatively low exponentiality of the cost to society* with the number of failures. This finding is in contrast with that of Caccioli et al. (2014) who find an ambiguous impact of rising the N/n ratio.

1.4.2.3 Wider and better

Figure 1.4.9 shows the difference between an optimal and a biased network when *N=20*, to be compared with figure 1.4.7 which gives the same information when *N=10*.

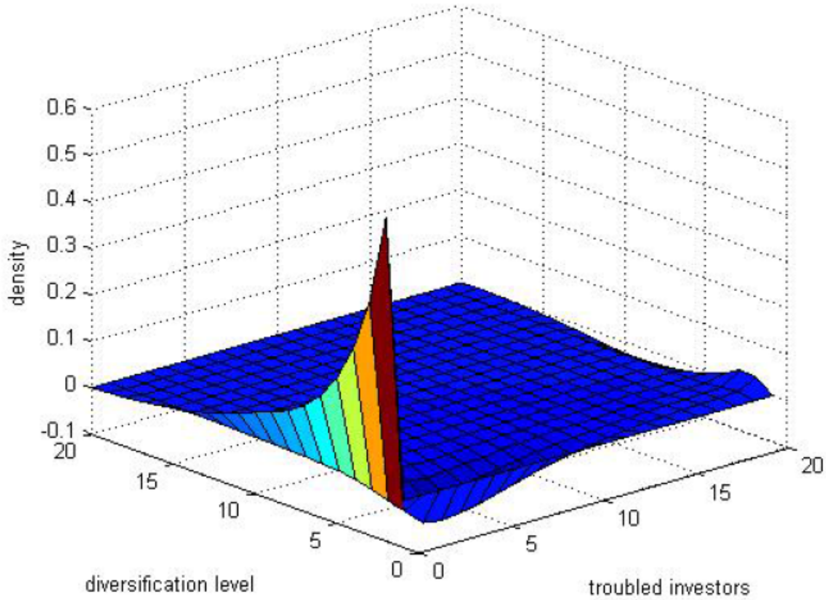

Figure 1.4.9 – Bankruptcies odds, comparison home biased/optimal networks.

Comparing this to figure 1.4.7 shows that *the systemic risk reducing impact of increasing* N *is higher without home bias.* In particular we observe that the magnitude of the increase in the likelihood of the all-fail outcome for low levels of diversification becomes relatively smaller when *N=20,* making the set of parameters for which an optimal network may be undesirable even smaller. *Therefore a "wider" and a more "optimal" network are complementary.* This is consistent with our finding in section 1.4.1 that increasing *N* is particularly desirable when the financial shocks spread quickly through the system, which is the case in an optimal network.

Figure 1.4.10 looks at the desirability of the wider and better network, to be compared with the easy panic case in figure 1.3.11..

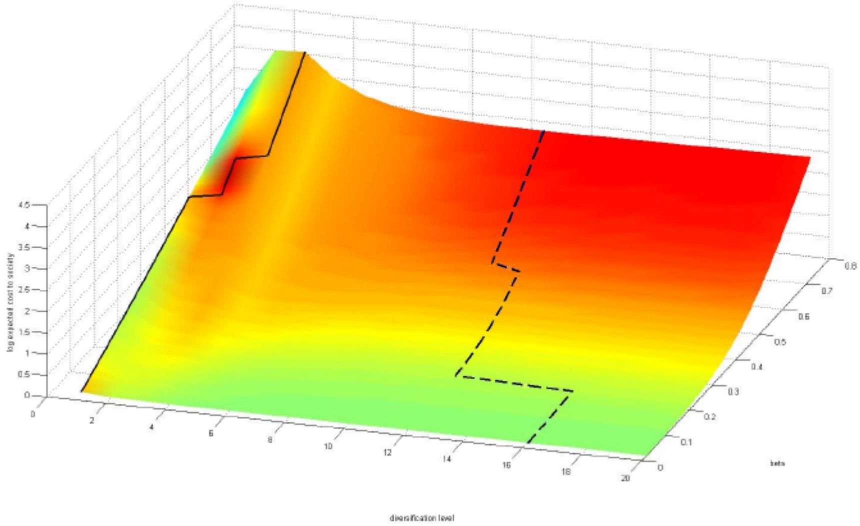

Figure 1.4.10 – Desirability, optimal and wider network.

The contrast is evident: the no diversification case remains the most undesirable option for $\beta \in]0, 0.48]$, which includes value of β for which no diversification was the preferred option in the baseline scenario of figure 1.3.11. The "worst" levels of diversification when $\beta > 0.48$ remain very low at *n=2/3*. The figure also suggests that *intermediate levels, which were quick to become the least desirable option, are now quick to become the best one.* This finding is in accordance with the significant impact that rising N and changing the pattern of asset holdings have had on the likelihood of failures.

In the "baseline" easy panic case, systemic risk was maximized around n=5, past which more diversification was desirable, but such levels were not optimal with $\beta > 0.36$. *Here the maximum is reached much earlier, and the first-best level of diversification is unambiguously beyond it. In other words when markets are complete and investors are not biased, the window for which diversification may* *be harmful is much reduced, and the first best allocation is a reachable diversified one. Therefore, the threshold past which diversification becomes preferred to no diversification falls with the completeness of markets and the "efficiency" of asset-holdings.*

1.5 Conclusion

This chapter has used a new bottom-up approach to study of systemic and asset covariances, in line with the emerging endogenous risk literature. We are able to provide a thorough analysis of the impact of diversification on systemic risk, for any possible levels and number of defaults. We find that in equilibrium, diversification increases the probability of mass failure but decreases that of all other non-zero failure outcomes. This leads to diversification to be generally desirable during business as usual periods with low transmissibility of shocks, but not when transmissibility rises and the cost of mass failure rises is high.

However in this study the link between diversification and systemic risk edges on a deeper question: is contagion driven primarily by human instincts or by hard-wired features of the financial markets? Indeed, as soon as we introduce heuristics which in our opinion provide a better description of the reality of crisis, a new desirable feature of diversification appears. Lower fluctuations to investors wealth brings lower selling movements, minimizing the possibility of panic.

In this context the relationship between diversification and systemic risk is a concave function whose maximum is reached for intermediate levels of diversification, as such levels create connections between investors without going far enough in minimizing individual risk and the scope for panic. The optimal levels of diversification then appear to be either "high" or "none", depending on the transmissibility of shocks and on how costly mass failure is, implying there exist a threshold level past which diversification becomes desirable, but below which no diversification should be preferred.

Interestingly this threshold seem to fall with the efficiency of financial markets. When markets are more complete and investors are not biased, the systemic risk maximizing level moves from intermediate to low, even in our most pessimistic scenario of "easy panics". The point at which diversification becomes worth it is reached quickly, so that intermediate levels of diversification actually become the most desirable option.

Let us look at the subprime crisis under this light. Credit backed assets were in fact much more closely fundamentally related than expected by the banks which held them. As these correlations were high, banks were in essence holding a single credit backed asset, their actual diversification level was not high. This led the wealth shocks stemming from adverse movements on ABS to be large, which in turn triggered panic through increased counterpart risk and rising risk aversion. An aggravating factor may have been that international investors tend to be biased towards US securities, a country with low informational distance to any other country. According to our set-up a lower level of diversification would have been preferable, for risk would not have spread, and a higher diversification would have the first-best allocation, as banks would have been able to digest the losses from ABS markets without triggering panic, while being safer during BAU periods.

Two policy implications may be drawn from this discussion. The first one is quite general: if one agrees with the Financial Stability Board (2009) that the most important factor to prevent a systemic crisis is maintaining confidence, then the failure of any institution constitutes a systemic event, so that favoring an approach based on micro soundness may be still be the best option from a policy perspective. Second, this chapter implies that a key to enhancing systemic stability is *diversifying both more and better*. Some of the progresses in this direction are bound to come from investors themselves, who should start accounting more for endogenous risk of their portfolio, but policy makers may also encourage this trend. In particular the promotion of international diversification though the lowering of information costs and/or taxes appears useful, as it permits both the spreading and the minimization of endogenous risk across assets.

Style investing and endogenous comovement

Chapitre 2

Style investing and endogenous comovement

2.1 Introduction

Financial crises occur when asset values grow in excess of "fundamentals". Preventing such crises thus requires separating such fundamentals from "human" factors, notably from the "instincts" of investors. Yet the task has proven very difficult, as human factors and fundamental ones are often entangled together. For instance, investors grow overconfident in good times and panic in bad ones, which makes it difficult to empirically distinguish "real" growth from changes in risk aversion. Nevertheless some human features appear quite independent of fundamentals, which makes them easier to detect.

Our natural tendency to classify is one of those features. In order to reduce the complexity of the portfolio allocation problem and the cost of gathering information on each asset, investors classify assets into different groups according to their industries, size, book-to-market ratios, etc. The resulting classes are called "styles", and trading strategies based upon these classes are called "style investing". Style investing will lead assets to be bought and sold together as part of a similar style, which will create excess comovement between assets of a similar style.

The goal of this chapter is to investigate the presence of such style-driven comovement over what is arguably the most followed classification scheme in the markets: bond credit ratings. We study the impact of downgrades/upgrades on the comovement of the bond with the rating index it leaves and that he joins. In order to be able to attribute this change in comovoment to style investing only, we focus on particular rating actions for which fundamental factors and style investing have opposite or independent impacts on the change in comovement.

We investigate style-driven comovement at two possible levels : the individual risk class one, and the wider "investment grade" versus "high-yield" one, which groups respectively bonds rated BBB- or more, and BB+ or below. Supportive evidence is found at the former individual rating level, but initially only suggestive one at the latter one. Digging deeper we find that non-significance at the wider "investment" versus "high-yield" level may be due to an interestingly peculiar status of BBB-/BB+ movements on the markets. Net of such peculiarities, style investing appears convincingly also at the investment versus high-yield level.

The study places itself within a small but conclusive literature on style investing. An important contribution is made by Boyer (2011), who finds that stocks who get reclassified between growth and value indices according to their book-to-market ratio start comoving more/less with the index they join/leave. To discard the possibility that the change in comovement simply reflects the underlying change in book-to-market ratio, the author focuses on stocks which were reclassified as growth/value even though their book-to-market ratio had risen/fallen[1], i.e. stocks for which fundamental and style investing factors do not go the same way.

Vijh (1994) finds a significant rise in comovoment for stocks that join the S&P500, using univariate regressions. Barberis and al. (2005) confirm this using bivariate regressions over S&P and non-S&P indices. Greenwood (2008) focuses on movement in and out of the Nikkei 225, and finds even stronger changes in betas. Barberis and Shleifer (2003) provide a theoretical model in which style investing leads a given style to exhibit momentum and mean reversion, besides creating co-movement. Wahal and Yavuz (2013) confirm this by showing that assets which comove more with their style usually generate higher momentum profits.

More generally, the chapter belongs to a wide and heterogeneous literature on finding movement in assets that results from "investor-driven" factors rather than fundamental ones. This includes for instance Froot and Dabora (1999) who find evidence of two stocks that refer to a similar cashflow but behave independently, or Ye (2011) who suggests that comovement patterns change with the share of active investors. We may also relate this dichotomy to some models of financial contagion that predict that endogenous factors may drive correlations above their fundamental values during crises (Raffestin, 2014).

Finally this study may also be related to the work on the impact of rating actions, though to the best of our knowledge this literature has focused on prices rather than comovement. Studies such as Norden and Weber (2004) find a strong impact of rating actions on bond or stock prices. Micu et al. (2006) uncover

1. Such cases occur because the agency that classifies the stocks wants each indice to represent 50% of total market cap

that investors tend to react more to downgrades, and are as sensitive to outlook announcements as they are to actual rating actions, if not more.

Our contribution is threefold:

1) *we focus on bonds.*

Studying another asset class is interesting per se, but also because one may suspect that style investing is particularly strong in the fixed-income market, for several reasons.

First, ratings are probably the most scrutinized classification on the markets. The recent financial crises have highlighted this importance. Downgrades of important nations have triggered angered reactions at the highest levels during the European sovereign debt episode. Practitioners and academics agree that over-reliance on credit ratings has played a large part in the subprime crisis, by fueling the building up of AAA graded CDOs.

Second, external factors may give investors an extra incentive to "buy the grade rather than the asset". From a regulatory perspective, ratings are an important input for computing capital surcharges in Basel II, and even more so in Basel III for the definition of liquid assets. From an operational perspective, bonds are usually traded more as a way to diversify portfolio risk rather than earning large returns. Thus on average investors may have less benefit in gathering idiosyncratic information on a given bond, compared to equities.

Finally casual observation strengthens our suspicions. The last 15 years have seen a large rise in the number and size of exchange traded funds (ETF) in fixed-income markets. ETFs aim at tracking the performance of a given set of assets. Since ETF are in constant need to adapt to the index they replicate so that we may expect them to buy and sell large quantities of assets, using some type of classification. Due to their size, the positions taken by these ETFs are likely to show in prices.

2) *We efficiently control for fundamental comovement.* Our identification strategy is based on two simple points driven from economic theory and evidence, which will translate into natural tests on the presence of style-driven comovement.

- The first point starts with the observation that low graded bonds are on average more risky and thus have yields that fluctuate more in response to changes in global fundamental risk factors, i.e. they have larger market betas. Mathematically, a high market beta asset must comove more with any other bond or index than a low beta one. Therefore from a fundamental perspective, following a downgrade, a bond should comove more with both the index it leaves and that it joins. Conversely an upgraded asset should have a lower comovement with both indexes.

From a style investing perspective however, following a rating action a given bond should start being bought and sold as part of the index it joins. Its comovement with the class it joins should then rise after any rating action, while that with the rating class it leaves should fall. *This naturally leads fundamental factors and style investing to have opposite predictions* on the change in comovement in two instances: between an asset and the index it joins following an upgrade, and between an asset and the index it leaves following a downgrade. We name such cases "balancers"[2].

These two balancers provide a natural way to test for the presence of style investing-driven comovement: if the sign of the total change in comovement following a rating action is consistent with style investing and in contrast to fundamental

2. A term also used by Boyer (2011)

factors, we may conclude that style investing is significantly present at the rating level despite the contrary influence of fundamentals. Our first test thus focuses on the sign of the comovement change for balancers.

- The second test is centered around the following observation: investors appear to draw an important line between "investment" and "high-yield" bonds. From our perspective, if style investing is present at this wider level, assets belonging to similar investment or high-yield index will exhibit a comovement premium. In consequence assets that move between BBB- and BB+, i.e. in and out of investment and high-yield ensembles, should experience larger absolute changes in comovement than other rating actions, as they acquire this wider ensemble comovement premium with the index they join, or lose it with the index they leave.

From a fundamental perspective however, such movements should be no different than average, as rating agencies are *in theory* designed to be an objective indicator of credit risk. Therefore a significant difference between BB+/BBB- movements and average in terms of total change in comovement should come from style investing. Test 2 focuses on finding such a difference.

We find the average change in beta is of the expected sign in both tests, but we only reject the null hypothesis for test 1, while test 2 falls short of achieving significance. Investigation of these results leads to the third contribution of this study:

3) *We uncover interesting trading patterns in the fixed-income markets.*

We examine the possibility that other forces linked to credit ratings could explain the absence of significant excess comovement at the investment versus high-yied level, which could have biased test 2 against rejection. A first bias arises if movemenst between BBB- and BB+ have a larger impact on measured fundamental comovement than average. A second bias appears if style investing is less used for assets graded BBB- and BB+, compared to the other assets.

We find that an asset moving from BBB- to BB+ indeed experiences a larger rise in its market beta than other movements, although moving from BBB- to BB+ does not involve a larger than average increase in default rates. In other words *investors seem to sanction BBB-/BB+ movements with particular severity, without fundamental reasons to.* Similarly, very high and very low grade assets are subject to more style investing, as implied by their higher-than-average share of index specific variation in total variance. *For lower grade assets this may be explained by the fact that such bonds are usually issued by smaller, less diversified firms*, for which investors rely more on ratings. For higher grade ones, this may be explained by *regulatory requirements, who give investors an extra incentive to think in terms of grade.*

Both biases are then estimated empirically to run a corrected version of test 2. We find that such corrections improve the results on the test, which now concludes to a significant impact of style investing at the high-yield versus investment level.

Section 2.2 presents the intuition behind the tests we conduct. Section 2.3 presents the data and how we implement the tests. Section 2.4 presents the results and robustness tests. Section 2.5 discusses the non significance of test 2 and runs the corrected version.

2.2 Design of the tests

This section presents the identification strategy of the chapter, which rests upon a simple economic/statistical argument. For both tests we present this reasoning in words. We also provide a concise model which formalizes the logic for test 1 and 2, in appendix 2.1 and 2.2 respectively.

2.2.1 Test 1, style driven comovement across all ratings

The fundamental impact of ratings

As ratings are an economically meaningful classification, we expect the comovement within a rating class to reflect fundamental factors. In particular, ratings are an indicator of credit risk so that we expect bonds of the same class to move together through their correlated discount factor. Different ratings may also signal different liquidity conditions, so that assets of the same risk class may have correlated liquidity premia. The dependence of bond yields upon economic activity and liquidity conditions has been established empirically, for instance by Lin et al. (2014).

The simple premise of this study is that lower grade assets are by definition more fragile so their sensibility to such risk factors should be higher. Therefore *the exposure of a given bond to any given risk factor, such as credit or liquidity, should be rising on average as its grade gets lower.* In other words we expect low graded bonds to have higher markets betas.

Other less "fundamental" factors may add to this larger yield response for lower-grade bonds. In particular a negative fundamental shock may increase the level of risk aversion of investors, leading them to turn away from risky assets. Investors may also prefer to hold liquid assets, which tend to be the safest ones in times of market turmoil. Laborda and Olmo (2014) show that investor sentiment does matter for bond pricing.

Fundamentals-driven comovement

Consistent with the above, let us assume that a rise in global credit risk drives up the yield spreads on high graded assets by 1% , and that of low graded bonds by 2% . As a 1% rise in the high-graded assets corresponds to a rise of 2% on all low-graded ones, the beta of a regression of a low-graded asset j on the high-grade index will be of 2. Conversely the beta of high-graded asset i on the low-grade index will be 0.5 (a rise of 2% in x entails a rise of 1% in y). The expected betas are summarized in the following table, with the independent variables in columns and the dependent ones in lines:

	high grade index	low grade index
high-grade asset	1	0.5
low-grade asset	2	1

The comovement of the low-grade asset with both indexes will be higher, i.e. a higher market beta implies a higher comovement. Therefore from a fundamental perspective, *when an asset is downgraded, its beta with both the index it joins and that it leaves should rise after the rating action.* Conversely an upgraded asset should see its beta with both indexes fall. Noting $\triangle^f$ the change in beta stemming from fundamental factors, these predictions may be summarized as follows:

	downgrade	upgrade
with index joined	$\triangle^f > 0$	$\triangle^f < 0$
with index left	$\triangle^f > 0$	$\triangle^f < 0$

Style-driven comovement

Let us now take a style investing perspective: since style-driven comovement results from assets being bought and sold together as part of a similar index, we expect any given asset to "style-comove" more with the index it belongs to, regardless of whether it has been upgraded or downgraded. This immediately implies that *following a rating action, an asset should start comoving more with the index it joins*, and less with that it leaves. Noting $\triangle^s$ the change in beta coming from style investing, we thus have the following predictions.

	downgrade	upgrade
with index joined	$\triangle^s > 0$	$\triangle^s > 0$
with index left	$\triangle^s < 0$	$\triangle^s < 0$

Confronting the fundamental and style investing impacts on comovement, we then have:

	downgrade	upgrade
with index joined (in)	$\triangle^f > 0$, $\triangle^s > 0$	$\triangle^f < 0$, $\triangle^s > 0$
with index left (out)	$\triangle^f > 0$, $\triangle^s < 0$	$\triangle^f < 0$, $\triangle^s < 0$

In two cases fundamentals and style investing factors should go the same way, but in the other two *fundamental factors and style investing have opposite predictions.* We name these two cases "balancers", following the terminology used by Boyer (2011) to describe stocks for which fundamentals and style investing have opposite impacts. In what follows we refer to the comovement of an asset with the index it joins as an "in" case, and that with the index it leaves as an "out" one. The balancers are thus "out-down" and "in-up".

Test

Both balancers provide a natural way to test for the presence of style-driven comovement: if the sign of the total change in comovement following a rating action $\triangle^{\beta} = \triangle^{f} + \triangle^{s}$ is consistent with style investing and in contrast to fundamentals factors, we may conclude that style investing is significantly present at the rating level. We set our null hypothesis to be consistent with no style investing. Hence our test is:

H0: $\triangle^{\beta}_{out-down} > 0$ and $\triangle^{\beta}_{in-up} < 0$

Note that we run our test on the total change in betas, not only the style investing component $\triangle^{s}$. Importantly, this means that we in fact test a stronger hypothesis than $\triangle^{s}_{out-down} > 0$ and $\triangle^{s}_{in-up} < 0$. Finding for instance $\triangle^{\beta}_{out-down} < 0$ implies that $\triangle^{s}_{out-down} < -\triangle^{f}_{out-down}$, where $\triangle^{f}_{out-down} > 0$. Thus rejecting the null hypothesis that $\triangle^{\beta}_{out-down} > 0$ means that the style-driven beta change $\triangle^{s}_{out-down}$ is not only significantly negative, but below some negative scalar $-\triangle^{f}_{out-down}$.

The same reasoning applies to the "in-up" case. In this way the "fundamentals-driven" beta change $\triangle^{f}$ acts as a bias in the test. The identification strategy of this chapter is to accept this bias as long as it goes against finding evidence of style-investing.

Comment

The suspicious reader may believe that the simple story presented above does not constitute an efficient control of fundamental factors.

Yet one should note that this theory is based upon one assumption only: that low graded bonds have a higher exposure to global risk factors than high graded ones, on average. If *this is true then the balancing cases arise naturally, in any pricing model* as long they feature some risk factors. In appendix 2.1 we show that a general factor model specification generates the same predictions with no additional assumptions.

A crucial question then becomes whether the assumption that market betas rise as grades get lower holds in practice. We provide preliminary evidence that it does in section 2.3 by looking graphically at the time-series behavior of the yield spreads for each rating index, which show that low-graded indexes react more sharply to similar shocks. We provide more direct evidence in section 2.4 as a robustness check, by running a CAPM type regression that shows that the market betas do rise as grades fall. Considering this we argue that our identification strategy involves a very low amount of modeling error.

Another less vital assumption in the argument above is that we implicitly attribute non-fundamental comovement to style investing exclusively. Indeed in theory other factors than styles could lead to non-fundamental comovement. Nevertheless as mentioned in the introduction most "investor-driven" price movements are in fact related to fundamentals, so that "panic" or any type of news-triggered behavior should be embedded in the market betas. We fail to see any investor-driven factor other than style investing that would explain the patterns of comovement uncovered in the chapter.

2.2.2 Test 2, style driven comovement across high-yield and investment grades

The BB+/BBB- premium

Let us now imagine that investors group assets at the risk class level but also at a wider "investment versus high-yield" one. As with individual risk classes, style investing at this wider level should translate into a comovement premium between high-yield and investment bonds respectively. Formally the total style-driven change in comovement following a rating action may then be decomposed into two components: $\triangle^s = \triangle^I + \triangle^w$, where $\triangle^I$ is the change in betas stemming from style investing at the individual risk class level, and $\triangle^w$ that from style investing at the wider level.

Let us note $\triangle^{\beta,w}$ the comovement change for rating actions which involve a change in ensemble between high-yield or investment, and $\triangle^{\beta,\cdot}$ the change for the other rating actions[3]. For "regular" non BBB-/BB+ actions, the comovement of the bond with other high-yield or investment assets should remain unchanged, i.e. $\triangle^w = 0$. The asset should then only lose (gain) the comovement premium from style investing at the individual level. However if a change in risk class also involves a change in wider ensemble, the asset should also lose (gain) the comovement premium from the "high-yield versus investment" level. Mathematically:

$$\begin{cases} \triangle^{\beta,w} = \triangle^{f,w} + \triangle^{I,w} + \triangle^{w} & if\ w_{out} \neq w_{in} \\ \triangle^{\beta,\cdot} = \triangle^{f,\cdot} + \triangle^{I,\cdot} & if\ w_{out} = w_{in} \end{cases}$$

3. In what follows we focus only on rating actions which do not exceed one notch, so that only movements between BBB- and BB+ qualify as changing the high yield/investment ensemble.

where $\triangle^w$ should be negative with the left index, i.e. $E(\triangle^{\beta}_{out}) < 0$, as the comovement premium is lost in this case; and positive with the joined index, i.e. $E(\triangle^{\beta}_{in}) > 0$ as the comovement premium is gained.

As a side comment, note that the comovement premium $\triangle^w$ may not have the exact same source as that of individual ratings $\triangle^I$. Indeed as pointed out by Barberis et al. (2005) style-driven comovement may result either from "category" or "habitat" trading. Category investors will use styles to manage their portfolio, and allocate funds at the class level. Habitat traders will choose to focus on certain classes only, so that the trading on these styles will reflect the situation of the investors who hold it. While we expect the category view to explain most of the comovement premium at the individual level, habitat trading may well be the main force behind the premium at the wider ensemble level[4].

Difference-in-difference approach

Taking the difference between $\triangle^{\beta,w}$ and the average beta shift across "regular" risk classes $\bar{\triangle}^{\beta}$ yields:

$$E(\triangle^{\beta,w} - \bar{\triangle}^{\beta}) = [E(\triangle^{f,w}) - \bar{\triangle}^{f}] + [E(\triangle^{I,w}) - \bar{\triangle}^{I}] + \triangle^w$$

which will be equal to $\triangle^w$ if $E(\triangle^{f,w}) - \bar{\triangle}^{f} = 0$ and $E(\triangle^{I,w}) - \bar{\triangle}^{I} = 0$. In words if BBB-/BB+ movements do not imply a different-from-average response in terms of fundamentals or style investing at the risk class level, then any difference between these rating actions and average should come from style investing at the

4. High-yield and investment bonds attract a very different audience. Investment bonds are usually seen as little risky and held as part of a portfolio, while high-yield bonds are used for more speculative purpose. Many exchange traded funds for instance chose to focus exclusively on either investment or high-yield bonds.

"high-yield versus investment" level.

Crucially, we have *a priori* no reason to expect the fundamental and risk class level style investing components $\triangle^f$ and $\triangle^I$ to be different for BBB-/BB+ movements. Rating agencies often claim they base their decisions only on underlying fundamental risk, so that we should not see a higher relative change in fundamentals $\triangle^f$ for them. Similarly there is no obvious reason why the style investing component at the risk class level should be different from average, since any difference should be captured by the wider change component.

We thus expect $E(\triangle^{\beta,w} - \triangle^{\bar{\beta}}) = \triangle^w$.

Test

As mentioned, style investing at the wider ensemble level should lead to $E(\triangle^{\beta}_{out}) < 0$ and $E(\triangle^{\beta}_{in}) > 0$, which implies $\triangle^{\beta,w}_{out} - \triangle^{\bar{\beta}}_{out} < 0$ and $\triangle^{\beta,w}_{in} - \triangle^{\bar{\beta}}_{in} > 0$. In words, the comovement with the joined (left) index should increase more (fall more) for rating actions which cross the BBB-/BB+ threshold. As in test 1 we set our null hypothesis to be consistent with no style investing, so that test 2 will be:

Test 2: H0: $\triangle^{\beta,bbb-/bb+}_{out} - \triangle^{\bar{\beta}}_{out} > 0$ and $\triangle^{\beta,bbb-/bb+}_{in} - \triangle^{\bar{\beta}}_{in} < 0$

Rejection of the null hypothesis will be interpreted as evidence of style-driven comovement at the high yield versus investment level.

Comment

This method is similar to the difference-in-difference methods of development economics. If the fundamental and risk class level style investing components $\triangle^f$ and $\triangle^s$ are constant across all risk classes, differencing should remove these components. In fact the test is also unbiased if $\triangle^f$ and $\triangle^I$ are increasing/decreasing linearly across grades, because the BB+/BBB- line lies in the middle of the grade spectrum.

On the other hand if $E(\triangle^{f,w}) - \bar{\triangle}^f = 0$ or $E(\triangle^{I,w}) - \bar{\triangle}^I = 0$ do not hold, the test will be biased. In fact we shall see later that these conditions indeed fail empirically, and will take steps towards correcting the biases. Despite this fact we have chosen to present the test in its original design, with our ex ante beliefs about $E(\triangle^{f,w}) - \bar{\triangle}^f$ and $E(\triangle^{I,w}) - \bar{\triangle}^I$. This was done to highlight the fact that the failure of these conditions constitute a puzzling finding, as there is no clear reasons for which BBB-/BB+ movements should be different from the others, notably in terms of fundamentals.

The investigation of the cause of such puzzles will be one of the main contributions of the study.

2.3 Data

This section describes the steps and choices involved in implementing our two tests empirically.

2.3.1 Gathering rating data

Data on credit ratings is obtained through the RatingXpress database provided by Datastream. RatingXpress lists the last 10 credit actions taken by the rating agency Standard and Poors for a given asset. The earliest rating action traces back to March 1991. Including other agencies such as Fitch and Moody's would be desirable but keeping one agency simplifies the analysis, and Arezki et al. (2011) find that S&P's announcements have the highest price impact of all 3 major agencies, studying sovereign bonds.

We focus on bonds issued by financial institutions and corporations only, and exchanged in the US. This choice is to keep the sample homogeneous, as one could expect bonds to behave differently according to their geographic base or their public or private nature. However we include bonds with different guarantees and seniorities, because keeping only a single category would remove too many observations, and such information is likely to be at least partly embedded in credit ratings. We also include all maturities, but will try regressions keeping only 5 to 10 year bonds to make sure differences in maturities do not drive the results.

An important first step is spotting bonds which have been issued by a similar entity, for fear they will create artificial comovement later. RatingXpress often provides the name of the underlying borrowing entity, in which case we randomly keep one bond/rating couple only. For ratings without a borrower specified, we identify bonds whose ratings actions are perfectly correlated, and keep only one occurrence.

2.3.2 Gathering bond yield data

Data on bond yields is obtained from Datastream. We use the redemption yield, i.e. the interest rate that an investor would receive if he bought the bond at market price today and held it to maturity, compounded annually. Yield spreads are obtained by taking this yield minus that on the 10 year US T-bond. The baseline regression uses the first difference of these spreads, but regressions using levels as a test of robustness are also provided. First differencing should detrend the series, while taking spreads should account at least partly for common trends from the variations in the general interest rate. These transformations are designed to reduce the amount of measured fundamental movement, and thus the bias on our test [5].

The time period for the base runs from the 31th of December 2004 to the 20th of January 2014. We could have set the price data to start at any date, but including old series may have changed the conclusions as investors attitudes towards style investing have probably changed through time. Starting in 2005 offers a good balance between this desire to stay recent and that of keeping a large number of observations, as few series had both price and rating data before 2005.

Of course the period chosen is not innocuous, as it includes the subprime and sovereign debt crises. Periods of high selling movements may make it easier to detect empirically comovement. However since investors use styles in both "good" and "bad" times, crises may reveal style investing but should not create it. We provide results on the post crisis period only as a test of robustness.

Note that most researchers in the literature on the price impact of rating actions use equities and/or CDS rather than bonds. Admittedly, working with bonds has drawbacks. They are less liquid and thus the series have a lower variability, and a given entity may issue several bonds. However in this case using bonds seems logical since we suspect style investing may be strong in this particular asset class, and it is bonds that are being rated.

5. Since we have seen that our tests may be reexpressed as $\triangle^{s}_{out-down} > -\triangle^{f}_{out-down}$ and $\triangle^{s}_{in-up} < -\triangle^{f}_{in-up}$, i.e. we test that style-driven comovement is larger in absolute value than fundamentals-driven one.

2.3.3 Constitution of the indexes

The yield spread of an index is defined as the weighted average of the yield spreads of the bonds that compose it. Each bond is weighted according to its market valuation, also provided by Datastream[6].

One must be careful that indexes are not driven by a few bonds only, particularly in this context of comovement with indexes will being the crucial point. This means avoiding that a given bond has too large a weight in the index, and dropping outliers. On the former, we specify that no bond, at any point in time, may represent more than 20% of the its index total valuation. On the latter, we use a method similar to that involved in computing the LIBOR, removing the bonds whose yield is in the top and bottom 5% at each day.

A 20% cap may appear high. This value was chosen because the valuation cap appears to have very little impact on the results[7], and a high cap allowed us to include more observations. The trimming device has a stronger impact because without it some erratic variations in the indexes appear. We picked 5% because this value is enough to remove such variations, while maximizing the number of observations.

Due to these restrictions, not all indexes feature throughout the period. Several indexes are absent at first when few series are available, and some of them, those below CCC, never make the cut. Movements across low graded assets are quite specific as they usually apply to bonds that are on the verge of default, so that their absence from the sample may not be so detrimental. In total 17 indexes appear in the sample. Figure 2.3.1 plots some of the indexes using 2-month moving averages.

6. Defined as amount in issue times the price and accrued interest for a given bond
7. Results available on request.

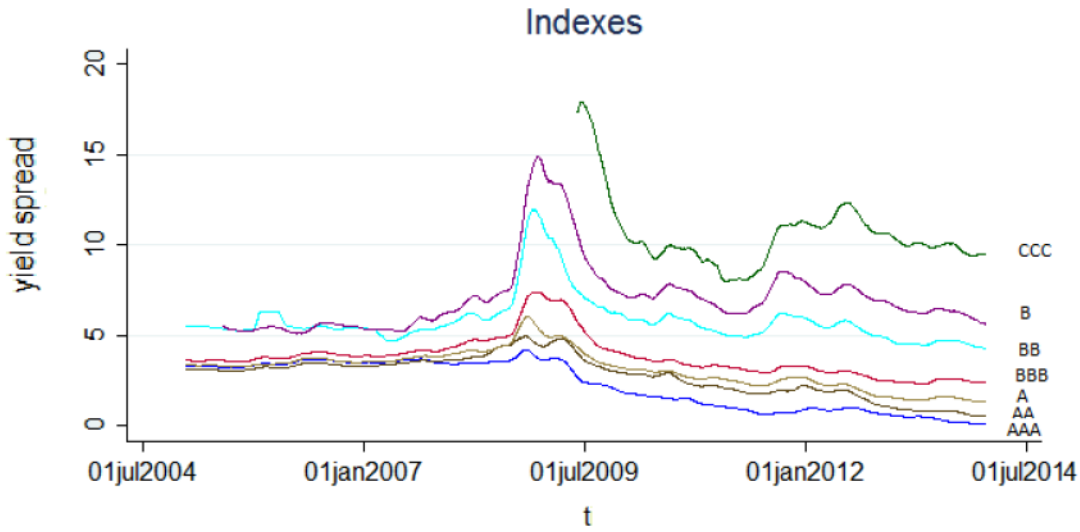

Figure 2.3.1 – Indexes yield spreads

The behavior of the indexes appears in line with expectations: as grades get worse, the yield required to hold them rises. The magnitude of the response to common shocks also grows as grades get worse, which represents preliminary evidence that market betas indeed rise as grades get lower. The relationship appears monotonic: each index seems to respond less than the next lower grade one. The same pattern remains including all indexes.

Figure 2.3.2 provides information on the composition of the indexes. The dashed and solid lines represent respectively the total number of bonds and the weight of the largest bond for each index. The values provided are averaged across periods.

Observations appear relatively well spread across and within indexes. On average an index contains 257 bonds, with the largest bond representing 7% of total

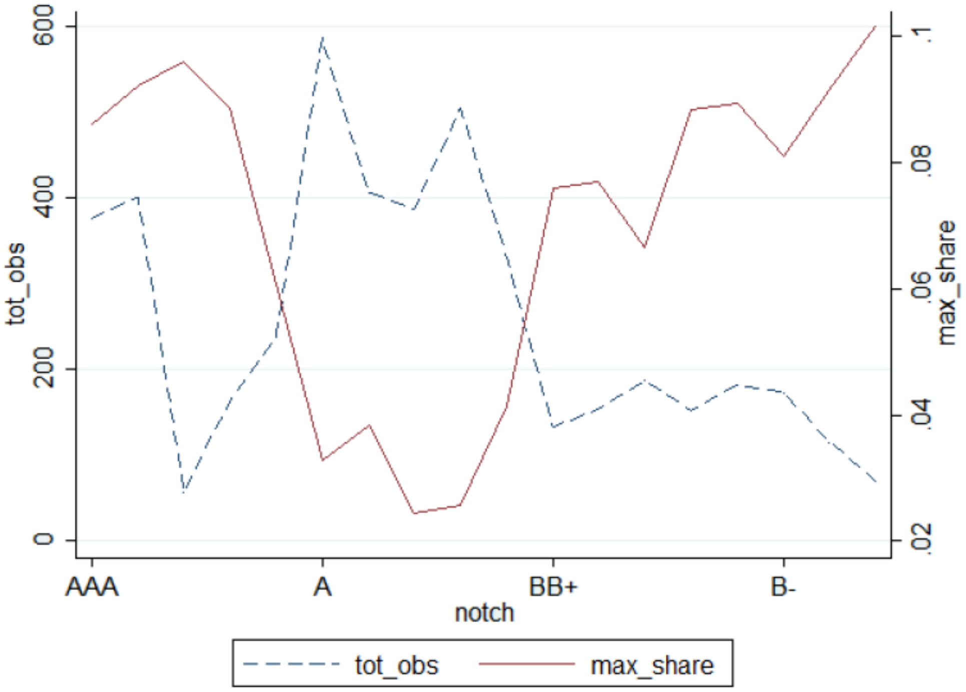

Figure 2.3.2 – Indexes yield spreads

index valuation. The lowest average amount of observations is 57, 2 for AA+. The largest weight for a single bond is 10% for CCC.

2.3.4 Regressions

We follow the standards of the style investing literature by running the following[8] :

$$R_{i,t} = \alpha_i + \beta_{i,I} R_{I,t} + \beta_{i,I'} R_{I',t} \tag{2.1}$$

8. The model presented in the appendix simplified on purpose by considering the beta of an asset with a single risk class.We also provide results for univariate regressions as a test of robustness.

where in the baseline estimation $R_{I,t}$ and $R_{I',t}$ represent the first-differenced yield spreads of the left and joined indexes respectively. The time unit is a day. We run this regression before and after the rating action, and draw the estimated betas on the post-event and pre-event regressions which are noted $\hat{\beta}^*$ and $\hat{\beta}$ respectively. Consistent with the literature, these betas are used to proxy comovement. The change in comovement following a rating action is thus $\triangle^{\beta}_{i,out} = \hat{\beta_{i,I}}^* - \hat{\beta_{i,I}}$ for the left index $\triangle^{\beta}_{i,in} = \hat{\beta_{i,I'}}^* - \hat{\beta_{i,I'}}$ for the joined one.

In the baseline regression the actual rating action is considered as the key event, but section 2.4.3 provides results considering outlooks, when present, as the defining event[9]. Furthermore, to avoid having average beta shifts driven by a few spurious individual regressions, we require that beta shifts be kept between 20 and -20[10].

Ideally this study follows Boyer (2011) in estimating the betas over the 5 months before and after the rating action. However some ratings do not stay constant for 5 months after a movement, in which case we accept regressions if we have a minimum of 3 months of data. In other words, noting t the day on which a rating changes and x the amount of months for which a rating it had stayed constant before the change, the "pre-event" regression will estimate (2.1) over the last 5 month up to *t-1* if x>*5*, and the last x months up to *t-1* if $x \in [3, 5[$. The post-event regressions follow the same rule starting from[11] *t+1*.

The initial number of bonds in the rating file, net of duplicates, was 13492.

9. Outlooks are announcements by the rating agency on the likely evolution of a given bond over the following 3 months

10. Changing this restriction has no impact on the results. The results using 10 or 5 as bounds are available on request

11. We have also tried leaving a longer window of 5 days before and after the rating actions out of the regressions. The results are very similar so were not included.

Around 30% of the bonds are not matched with price data. Removing bonds with missing values and those which were never upgraded/downgraded then leads to a total of 2720 bonds. Note that the bonds whose rating staid constant still play a role in the estimation because they are used in the constitution of the indexes. In total we have 5007 rating movements. Out of this number 2755 were exploitable. The rest involved joining/leaving an index that did not fill the requirements, a movement larger than a notch, or a rating that did not stay stable long enough before/after the movement.

2.3.5 Testing for differences in beta changes

For test 1, we take all beta shifts with left and joined index, $\triangle^{\beta}_{i,out}$ and $\triangle^{\beta}_{i,in}$, across all asset classes. Beta shifts are then split into downgrades and upgrades, which represent respectively 1370 and 1375 of our rating actions. Considering both in and out movement gives us four subsamples who correspond to the four cases identified in the model, on which we test our hypothesis. Test 1 is then in fact composed of 4 subtests for each of the four cases in-down, in-up, out-down, and out-up, where we put particular emphasis on the balancing cases in-up and out-down.

For the second test, we separate the beta shifts into two groups: those involving movements between BB+ and BBB-, which represent 145 observations, and the others. We then test for a significant difference between both groups. We do so for both in and out movements, so that test 2 has two "subtests". Note that we could have applied test 2 to the same 4 subcases as test 1. However this 1) decreases the number of observations involved in the test, 2) is not justified theoretically since the difference should be the same for all in or out movements.

For all samples the distribution of the beta shifts is moderately different from the normal distribution. It is symmetric but has a kurtosis around 5. Yet according to the central limit theorem, for fairly large sample the mean of a variable should be normally distributed even if the variable itself is not. Thus we use Student t-tests, which should be unbiased since our samples are quite large.

We conclude the section with a reminder of both tests:

- Test 1, H0: $\triangle^{\beta}_{out-down} > 0$ and $\triangle^{\beta}_{in-up} < 0$

where $\triangle^{\beta}_{out-down}$ and $\triangle^{\beta}_{in-up}$ represent the average beta shifts for the out-down and in-up cases.

- Test 2, H0: $\triangle^{\beta,bbb-/bb+}_{out} - \bar{\triangle}^{\beta}_{out} > 0$ and $\triangle^{\beta,bbb-/bb+}_{in} - \bar{\triangle}^{\beta}_{in} < 0$

where $\triangle^{\beta,bbb-/bb+}$ represents the beta shifts for rating actions between indexes BBB- and BB+ and $\bar{\triangle}^{\beta}$ the average beta shifts over all other actions.

And the null hypothesis in both tests is set to be consistent with no style investing.

2.4 Results

In this section we provide the results for test 1 and 2, provide robustness checks, and discuss how certain rating actions seem to have a particular impact on comovement.

2.4.1 Baseline results

2.4.1.1 Test 1 and 2

Table 2.4.1 summarizes our results for test 1 in our baseline regression using first-differenced yield spread.

movement	in theory	H0	value
In down	$\triangle^F > 0$, $\triangle^S > 0$	$\triangle_\beta < 0$	.117*** (.000)
In up	$\triangle^F < 0$, $\triangle^S > 0$	$\triangle_\beta < 0$	.028 (.104)
Out down	$\triangle^F > 0$, $\triangle^S < 0$	$\triangle_\beta > 0$	-.135*** (.000)
Out up	$\triangle^F < 0$, $\triangle^S < 0$	$\triangle_\beta > 0$	-.030* (.085)

Table 2.4.1 – Test 1

Balancers are highlighted in bold characters. Both are of the expected sign. It appears we strongly reject the null hypothesis of no excess comovement from style investing for the “out down” case, and fall just short of rejecting it for the “in up” one. The other two cases, even though they cannot be directly interpreted as evidence of excess comovement, are also in line with our expectations and significant at the 0.1 level.

In the down cases the magnitude of the change in betas is comparable to previous papers on style investing, who usually find beta shifts around 0.15. Overall, and considering the fact that we actually tested $\triangle^{f}_{out} > \triangle^{s}_{out}$ and not $\triangle^{s}_{out} > 0$, we view these results as indicative that there is style-driven excess comovement at the risk class level.

An immediate observation is that downgrades have a larger impact than upgrades, both in the "out" and the "in" cases. This is consistent with the existing literature on the price impact of rating actions which has showed that investor respond asymmetrically to rating actions. In our context it may also explain why the "out up" case, for which both fundamental and style component are expected to be negative, comes up higher than the out down one in which style investing only should be less than zero. We discuss possible explanations for this asymmetry in section 2.4.3.

Let us now look at the results for our second test:

movement	in theory	H0	value
In	$\triangle^{w} > 0$	$\triangle^{\beta,,bb-/bbb+} - \bar{\triangle}^{\beta} < 0$	.099 (.105)
Out	$\triangle^{w} < 0$	$\triangle^{\beta,,bb-/bbb+} - \bar{\triangle}^{\beta} > 0$	-.098 (.104)

Table 2.4.2 – Test 2

The signs are again consistent with the presence of style investing, but we fall just short of rejecting H0 at the 0.1 level for both tests. These results are thus suggestive but not definite evidence that there may be style investing at the investment versus speculative level. Robustness checks in section 2.4.2 will provide a clearer picture of the performance of Test 2.

2.4.1.2 Change in comovement across notches

Let us look at the results graphically. Figure 2.4.1 plots the average beta shifts per class in the in-down, in-up, out-down and out-up cases, with particular emphasis on BBB- and BB+[12]:

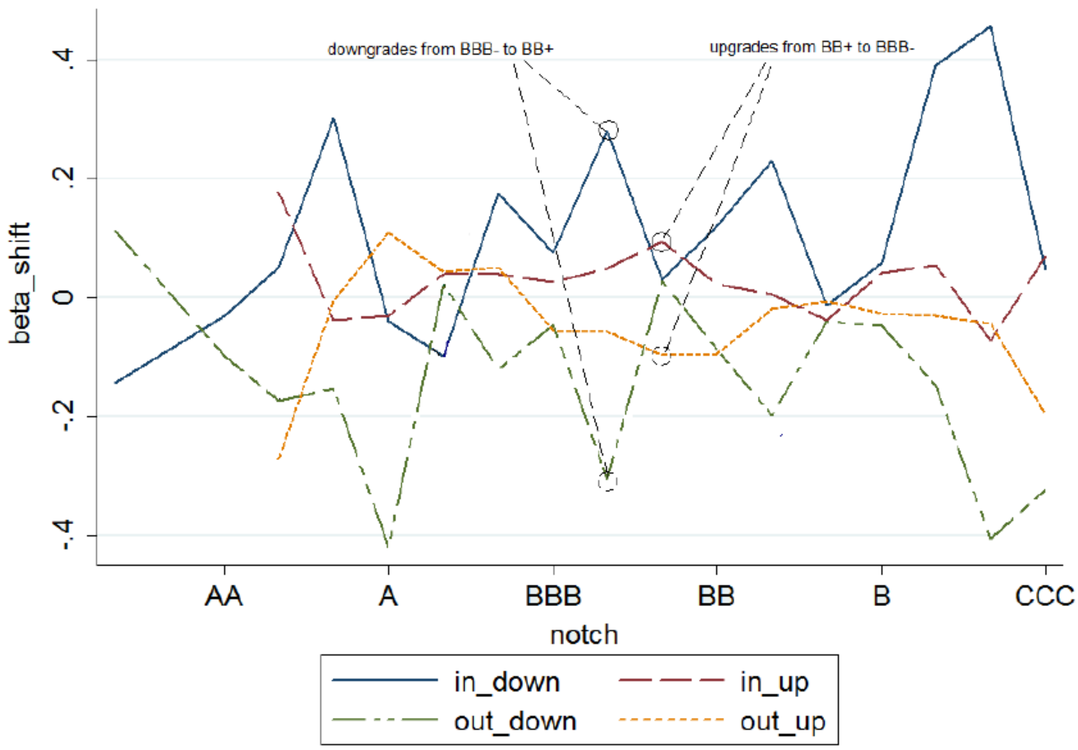

Figure 2.4.1 – Beta shifts across ratings, for all four subcases

The x-axis represents the notch of the bond before the rating action. For instance the in-up line at the point x="A" tells us how much, on average, the comovement with index A+ rises for assets that have been upgraded from index A.

12. Where we have removed indexes AA+ and CCC- which only had a few beta shifts, which in the case of CCC- gave too large an average and changed the scale of the figure.

First of all, we note that the beta shifts in all cases are fairly well-behaved across assets classes. There are no notches for which the change in beta exceeds 1 in absolute value, and the average change in beta is of the right sign in 76.6% of the cases, grouping all subcases. Besides this fairly good homogeneity, the in and out cases appear fairly symmetric. Up movements, who have a lower mean in absolute value, also have a lower variance. We view these features as encouraging from a robustness perspective.

Nevertheless two movements appear to consistently give unexpected signs: downgrades from AAA to AA+ and from A- to BBB+. This could simply result from our sample, but it may also be explained in terms of the theoretical model presented in appendix 2.1. There we show that the expected sign of $\triangle^s$ changes if the beta of the joined index on the left one is higher than 1. As mentioned this cannot be true in general, but it may be for certain indexes of peculiar status. For instance, AAA-graded bonds play a particular role in adjusting overall portfolio risk level for investors, which may lead them to buy and sell more AAA assets as part of portfolio rebalancing, driving up the measured comovement between indexes AAA and AA+.

Let us now focus on the movements between BBB- and BB+, highlighted in figure 2.4.1. Mathematically, if style investing was present at the investment versus high yield level, BBB-/BB+ movements should have a larger comovement impact in absolute value, i.e. more positive for "in" movements, and more negative for "out" ones. This is because bonds who move from BBB- to BB+ now lose not only their index level comovement with BBB-, but also the comovement at the wider investment grade level.

Looking at figure 2.4.1 this appears to hold: BBB-/BB+ movements feature larger than average betas for "in" movements, and lower ones for "out". Crucially, other notches also feature beta shifts that are relatively large in absolute values, but for BBB-/BB+ we find this in all four subcases. Only in movements between CCC+/CCC do we observe the same thing, which suggests that BBB-/BB+ movements may have a peculiar status.

To illustrate this point, we provide in figure 2.4.2 the average in and out movements, grouping upgrades and downgrades, for all possible 1-notch movements between two given indexes. This confirms that the BBB-/BB+ movements are noticeably larger than average in both the out (dashed line) and the in (solid line) which is also true, to a larger extent, only for CCC/CCC+.

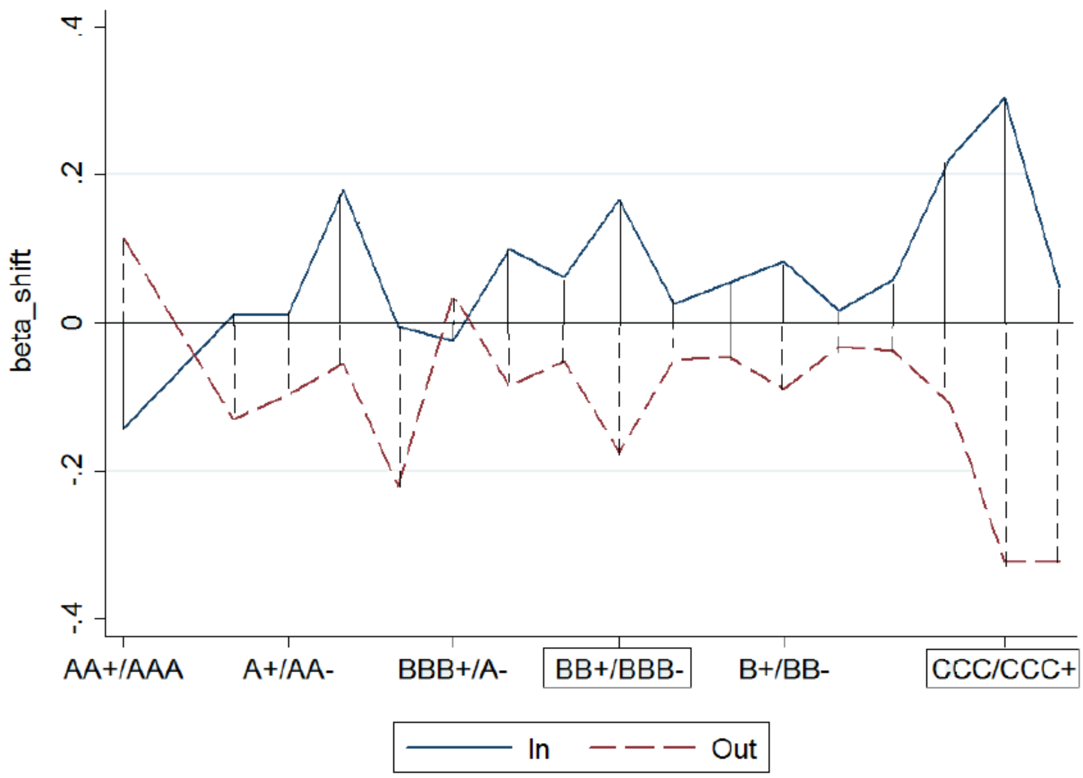

Figure 2.4.2 – Beta shifts across ratings for in and out movements

2.4.2 Robustness checks

We check in this section that the results are resilient to regressions using alternative measures, and discuss whether the assumptions involved in the design of our test hold in practice.

2.4.2.1 Alternative specifications

We provide 4 variations of our baseline regression:

1) Replacing the difference in spreads by the spreads in levels.

2) Using a single beta in line with the theoretical models in appendices 2.1 and 2.2.

3) Keeping only bonds with a lifetime of between 5 and 10 years in the sample.

4) Keeping only rating actions from 2009 onwards.

The first 2 forms should enhance the measured fundamental-driven comovement and thus have a detrimental impact on our results. Regarding form 1): first-differencing may have removed, at least partly, some of the common fundamental variation between bonds, by detrending the data. In form 2), running regressions with both joined and left indexes may have lowered the fundamental component in the beta shifts, as the second index partly controls for fundamental factors. Form 3) verifies that differences in bond maturities do not drive the results. 4) controls that the results were not driven by an insufficient amount of bonds in our indexes, as in the first part of the sample fewer series were available so that indexes had a lower atomicity. This specification will also indicate whether the results were driven by financial crises or not.

Each form usually implies a change in the number of observations available, but a number sufficient to draw conclusions remains. The lowest amount of observations is given by form 3) which features 851 downgrades and 950 upgrades, compared with 1370 and 1375 for our baseline estimation.

Table 2.4.3 summarizes the findings:

Test	Move	H0	(1)	(2)	(3)	(4)
I	In down	$\triangle_\beta < 0$	.306*** (.000)	.036* (.062)	.166*** (.000)	.113*** (.000)
I	**In up**	$\triangle_\beta < 0$	097* (.071)	-.007 (.701)	.045* (.060)	.035* (.089)
I	**Out down**	$\triangle_\beta > 0$	-.207** (.014)	-.065*** (.005)	-.151*** (.000)	-.117*** (.000)
I	Out up	$\triangle_\beta > 0$	-.122** (.020)	-.003 (.428)	-.062*** (.007)	-.027 (.144)
II	in	$\triangle^w < 0$	.108 (.330)	.012 (.423)	.072 (.225)	.093 (.147)
II	out	$\triangle^w > 0$	.019 (.5307)	-.003 (.481)	-.041 (.318)	-.071 (.203)

Table 2.4.3 – Robustness checks

The general impression is that the results appear quite resilient to alternative specifications for test 1, but not for test 2. We discuss each test separately.

Test 1

Balancers are significant at the 0.1 confidence level in 7 out of 8 cases. Globally $\triangle^{\beta}$ is of the expected sign in all but one subcases, out of 16. Focusing on downgrades the evidence is stronger, with all subcases being significant at the 0.1 level, and in 6 out of 8 at the 0.01 one.

Form (1) has little impact on the results. As expected, the single beta specification (2) is quite harmful. The sign of the in-up balancer switches, probably as a consequence of the higher influence of fundamental factors. Nevertheless strong evidence of style-driven excess comovement is found for the out-down balancer. Keeping bonds of comparable maturity (3) or focusing on past 2009 rating actions (4) appears relatively innocuous, so that differences in the nature of bonds or crooked indexes do not a priori drive the results.

Test 2

The signs are as expected in 7 of 8 subcases, but are never significant. The p-values increase in every form compared to the baseline estimation. Specifications (1) and (2) appear particularly detrimental, which comes as a surprise because in theory these forms were supposed to increase the measured fundamental comovement, and thus were expected to impact more test 1.

2.4.2.2 Are market betas increasing?

So far test 1 appears conclusive while test 2 does not. The remainder of section 2.4 will further examine the results on comovement at the risk-class level, while section 2.5 will be dedicated to the investigation of the results obtained by test 2.

The design of test 1 was based on the idea that the yield of a low-graded asset is more responsive to innovation in global risk factors than that of a high-graded one. We now verify this holds in practice by estimating the market betas of each index, using a simple CAPM-type regression:

$$R_I = \alpha_I + \beta_I R_T + \epsilon_I \tag{2.2}$$

where R_T R_I is the first-differenced yield spread on all bonds in the sample, which we use as an indicator of the systematic risk in bonds. We also run the regressions using levels instead of differences as in robustness check 1). Figure 2.4.3 plots the obtained betas across ratings.

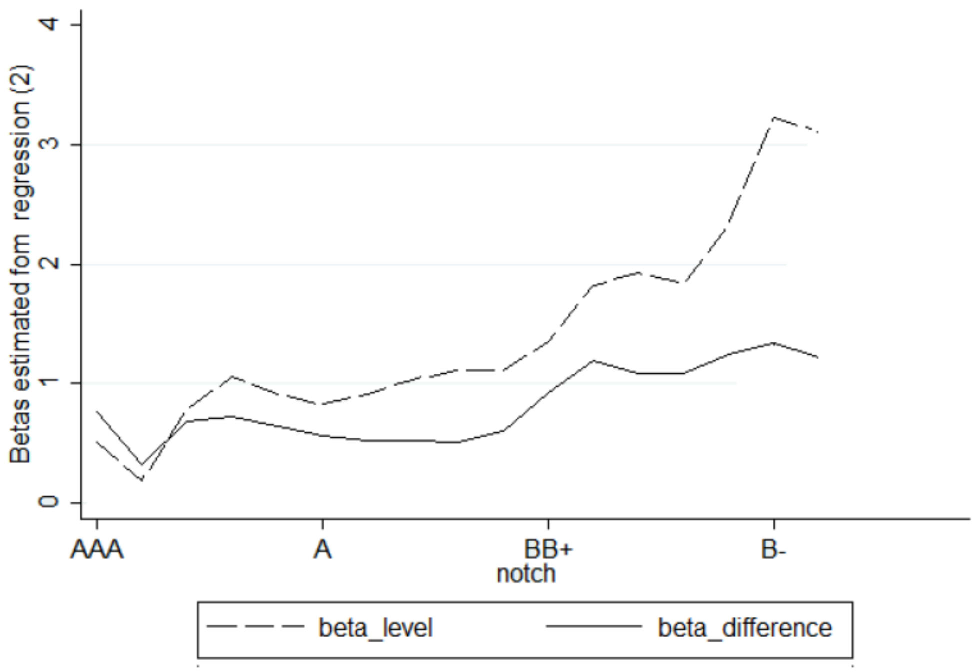

Figure 2.4.3 – Market price response compared to actual bankruptcy rate, for all ratings

The market beta indeed rises as grades get lower. In the baseline case using the first-differenced series, the rise is moderate. Using levels, it becomes more pronounced. This is not surprising since first-differences were intended to downplay the measured fundamental comovement.

In both cases the rise is not monotonic, as some indexes appear to have a larger beta than the next lower grade. Though this is a bit surprising, in our context this is not a major issue since our test only requires that the beta be rising *on average* across all rating actions, which is verified in the sample.

2.4.3 What rating actions matter?

2.4.3.1 The downgrade/ upgrade differential

A consistent finding is that downgrades have a larger comovement impact than upgrades, we now seek to explain this pattern.

A first possibility is that investors are quicker to readjust their styles following a downgrade. Loss aversion could explain such a reaction, as it implies that the cost of a downgrade is larger than the benefit of an upgrade for investors. A classic paper by Tversky and Kahneman (1991) reports that the marginal cost of losing a dollar is about 2.5 larger than the benefit of a winning one. Here the comovement reaction is about 4.5 times larger for downgrades in absolute value for both “in” and “out” cases. Even though both measures are not directly comparable, this suggests that loss aversion may not be the only factor driving the observed discrepancy between upgrades and downgrades.

Another complementary possibility lies with share of the variation that is explained by style investing following a downgrade. Güttler and Wahrenburg (2007) show that a downgrade/upgrade at a given time t is indicative of other downgrades/ upgrades at subsequent periods. This may lead some investors to pursue momentum strategies in bonds, i.e. demand more bonds which have recently been upgraded, and less those recently downgraded. In this context style investors may represent a larger share of the demand for downgraded bonds than it does for upgraded ones, which could explain their larger comovement impact.

2.4.3.2 Annoucements and issuer ratings

We now rerun test 1 with the following two modifications:

1) Using announcements rather than actual downgrades as the event day.

2) Using "underlying ratings", which are based on the credit quality of the issuer of the bond rather than the bond itself.

These two forms may be seen as robustness checks but are mostly economic interest. From the literature on the price impact of ratings, we expect that announcements should have a similar impact to the actual rating action, if not stronger. Indeed as pointed out by Micu [13] et al. (2006), announcements are likely to be followed by an actual downgrade/upgrade, so that investors should immediately account for this possibility. We also expect underlying ratings to have a starker impact, because they appear to be more followed by investors than asset specific ones.

In terms of observations, form 1) has no impact. Form 2) however leads to 244

13. In this study the authors make a difference between outlooks and reviews, the latter representing a stronger signal of future downgrades/upgrades. They find a significant impact for both, so that in this study we consider both reviews and outlooks, grouping them under the banner "announcement".

downgrades and 304 upgrades, due to the fact that changes in ratings are less common for issuers than bonds.

Test	Move	H0	(1)	(2)
I	In down	$\triangle_\beta < 0$	.141*** (.000)	.316*** (.000)
I	**In up**	$\triangle_\beta < 0$	.035* (.055)	.145*** (.004)
I	**Out down**	$\triangle_\beta > 0$	-.151*** (.000)	-.326*** (.000)
I	Out up	$\triangle_\beta > 0$	-.042** (.024)	-.112*** (.001)

Table 2.4.4 – Test results using announcements and issuer rating

Using announcements improves the results, with both balancers achieving significance. These results imply that, consistent with the literature, investors appear to use annoucements as a signal of a future rating action, and immediately act upon it.

The issuer rating form also leads to stronger results, both in terms of magnitude and significance. Taking the out-down case : a value of -0.326 means that an asset that gets downgraded sees its beta with its previous index fall from 1 to 0.674, on average. With respect to the p-values, the performance is especially strong considering the notable drop in the number of observations used for the test. Overall these results suggest that underlying rating actions indeed have particular importance for investors.

2.5 Investigating the impact of the BBB-/BB+ threshold

In this section we discuss and attempt to explain the results of section 2.4, particularly those on test 2. We investigate possible biases in the test, which will in fact lay the ground for a wider discussion of the behavior of investors around the BBB-/BB+ threshold.

2.5.1 Theoretical reasons for the weakness of the comovement premium

Section 2.4 implied that style investing appears to be present at the risk class level, but not significantly at the investment versus high-yield one. To explain this finding let us write H0 of test 2 in terms of the theoretical model provided in appendix 2.2. Taking the example of the out-down case:

$$H0: \ \frac{\alpha_I \sigma_F^2 (\alpha_{I'} - \alpha_I)}{\sigma_{R_I}^2} + (\delta_{I',I} - 1)\frac{\sigma_I^2}{\sigma_{R_I}^2} < \frac{\alpha_{bbb} \sigma_F^2 (\alpha_{bb} - \alpha_{bbb})}{\sigma_{R_{bbb}}^2} + (\delta_{bb,bbb} - 1)\frac{\sigma_{bbb}^2}{\sigma_{R_{bbb}}^2}$$

where the left hand side represents the average change in comovement over all movements but BBB-/BB+ ones. We may reexpress this expression as:

$$H0: \ (A) + (B)(C) < (A') + (B')(C')$$

where $(A) = \frac{\alpha_I \sigma_F^2(\alpha_{I'} - \alpha_I)}{\sigma_{R_I}^2}$, $(B) = (\delta_{I',I} - 1)$ and $(C) = \frac{\sigma_I^2}{\sigma_{R_I}^2}$.

Expression (A) is equal to the fundamentals-driven comovement $\triangle^f$, while $(B)(C) = \triangle^I$ represents the comovement change that is due to style investing at the risk class level.

(B) concerns the index-comovement $\delta_{I',I}$, and tells us how style investors trade the left and joined indexes together on average. If style investors see I and I' as substitutes there will be negative comovement between them, if they see them as complements, comovement will be positive[14]. This term thus depends on the "patterns" of style investing.

(C) on the other hand represents the share of the variance of a given index that is idiosyncratic, i.e. that is unrelated to global factors in the bond market. If a given risk class I is particularly subject to style investing, the covariance between the assets that compose it will be higher, and so will this share be. Therefore (C) is an indicator of the "amount" of style investing. Consequently components (B) and (C) may be seen respectively as "qualitative" and "quantitative" style investing.

Our double difference approach meant we initially expected $E(A) = E(A')$, $E(B) > E(B')$, and $E(C) = E(C')$, as the comovement premium of the "high-yield versus investment" is embedded in (B).

Thus there are three possibilities why we failed to reject H0:

- (A)<(A'), i.e. the relative change in fundamentals that accompany upgrades/downgrades is in fact higher for movements between BBB- and BB+.

- (B)≈(B'), i.e. BBB- and BB+ actually "style comoves" as much as the other

14. On average though, as we mentioned in appendix 2.1, this comovement must be below 1, so that $\delta_{I',I} - 1 < 0$ and $\triangle^I < 0$ in this out-down case.

neighboring risk classes, there is no impact of moving across wider ensembles.

- (C)>(C'), i.e the amount of style investing in BBB- and BB+ classes is lower than average.

Explanation (B) means our test was correct but there is no style investing, (A) and (C) imply the test was biased.

Different possibilities may lead to (B). The simplest one is that investors simply do not attach a particular importance to the investment versus speculative grade distinction. A second one is that portfolio investors may account for the risk that a given asset moves to a different ensemble as it approaches the BB+ to BBB- line, and act preemptively. Third, comovement could primarily be driven by passive investors who hold well diversified portfolios, so that the reaction to "speculative" versus "investment" movements could not be strong enough to show empirically. Finally we have mentioned that the BBB-/BB+ threshold may be particularly relevant from a habitat trading perspective, perhaps this type of trading has a weaker impact on comovement than category trading.

The first three options are not satisfactory. Casual observation seems to imply that the BBB-/BB+ distinction is indeed central in the market. A simple search on the internet shows that numerous ETF aim at reproducing either investment or high-yield indices. Many of such ETFs are very large, and their focus on either ensemble is complete[15]. Academic evidence seems to confirm this, notably Micu et al. (2006) who have found a higher price impact for BBB-/BB+ downgrades.

On the second possibility: if investors acted preemptively when a bond approaches the BBB-/BB+ threshold, we should see an impact for notches just above BBB- or just below BB+, which is not the case. Finally, if the reaction

15. This focus is usually made clear in the name of the ETF

to rating actions by style investors was indeed mellow, we should not obtain such good results on test 1.

The last possibility seems more likely. There may be reasons for habitat trading to generate less comovement than category one. From a qualitative standpoint habitat trading implies that investment and speculative indices behave independently i.e. $\delta_{bbb,bb} = 0$, while as mentioned in appendix 2.2 we expected $\delta_{bbb,bb} < 0$ from a category trading perspective. From a quantitative standpoint, with habitat trading, the covariance between assets of the same ensemble comes from the fact that similar assets are held by similar investors, whose selling and buying decisions are correlated. However as the subset of assets increases, the profiles of the investors who hold it becomes more heterogeneous. Thus for large subsets such as "investment" and "high-yield" bonds, the effect may be weaker.

Nevertheless before drawing any definite conclusions, we investigate the potential biases from explanations (A) and (C).

2.5.2 Was test 2 biased by heuristic factors?

In the frictionless model provided in appendix 2.2 there is no reason for the two equalities *E(A)=E(A')* and *E(C)=E(C')* not to hold. Showing a departure from these equalities thus involves proving that heuristic factors, i.e. the instincts of the investors and/or the constraints they face, leads them to react differently to movements between BBB- and BB+.

2.5.2.1 Are BBB-/BB+ movements sanctioned with more severity by the market?

The idea that $E(A) = E(A')$ relied on the fact that rating agencies claim they only judge a firm according to its fundamentals, without accounting for its peculiar status from a market sentiment perspective. Yet this sentiment, even if not based upon fundamentals, may have fundamental consequences. Indeed if investors are less willing to finance speculative bonds, the very action of moving an asset to a speculative status should rise the interests on its debt, making it de facto more risky.

More importantly, even if the “true” fundamental change required for BBB-/BB+ movement is comparable to the average, the measured one may not be. Indeed as prices are set by the market, the spread reaction to a rise in credit risk will necessarily reflect the market’s sentiment and perceptions, not necessarily fundamentals. Figure 2.3.1 seems to confirm this intuition. Neighboring indexes are usually very close one to another, but we observe a significant gap between BBB and BB.

Figure 2.4.3 which plotted the market betas from regression (2.2) also suggested a peculiar status. To investigate we zoom on the betas for the baseline first-difference case, to which we add the historical annual rate of default per class [16], for the sake of discussion.

Consistent with our suspicions the relative change between BBB- and BB+ is

16. Historical average for years 1981 to 2013 included, normalized to fit the scale. Data was obtained from S&P (2014). The data gathered all risk classes from CCC+ to C, which we plotted as CCC+. The strong increase between B- and CCC+ should then be put in perspective.

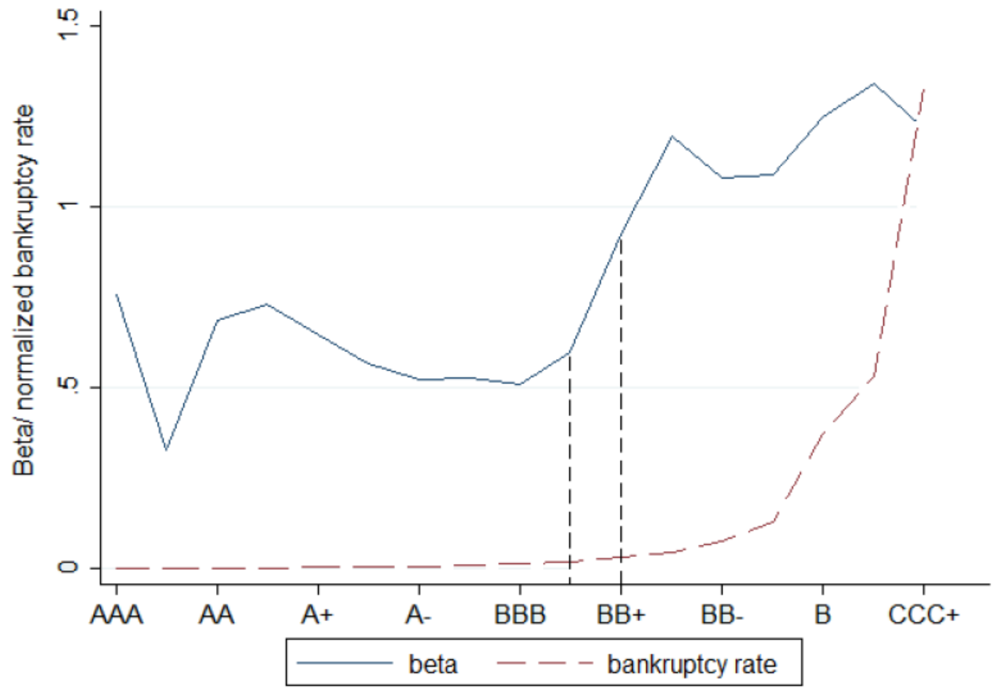

Figure 2.5.1 – Market price response compared to actual bankruptcy rate, for all ratings

one of the largest, so that our second test was indeed likely biased. This also explains why the results on test 2 were particularly harmed by robustness checks 1) and 2) which theoretically increased the magnitude of the fundamental bias. Interestingly we also see that there is a large counter-intuitive fall in betas for downgrades from AAA to AA+, which again may be due to the fact that AAA assets are particularly sensitive to changes in risk aversion, which rises and falls in response to fundamental news.

The key observation is of course the apparent disconnection between the true credit risk of assets and how the market prices them. The relative increase in bankruptcy rate between BBB- and BB+ appears no higher than average, at odds with the corresponding rise in betas. This implies first that the intuition on which test 2 was based was right in terms of “true” fundamentals, and second that as investors appear to sanction BBB-/BB+ movement with particular severity even if they have no fundamental reason to.

2.5.2.2 Is style investing stronger over assets with extreme ratings?

$E(C) = E(C')$ mathematically means that every index is expected to be equally used for style investing purposes. In other words the number of times a given asset will be bought as part of an index will be constant across indexes. Similar to last section we check that this hold in practice, by estimating $(C) = \frac{\sigma_I^2}{\sigma_{R_I}^2}$ with its empirical counterpart :$1 - R^2$ of regression (2.2), which represents the share of the variation in each index that is not explained by factors which are common to all bonds.

Of course the indicator is not perfect, as in practice there are factors other than style investing that may cause the share of idiosyncrasy to vary across notches. The results should then be seen as only partly attributable to style investing. Figure 2.5.2 plots the results.

The grades BBB- and BB+ indeed exhibit a lower share of idiosyncrasy in total variance. This suggests that test 2 was biased from this perspective also. Besides this a very interesting pattern emerges: the share of idiosyncratic variation is particularly strong for lower grade assets and for high grade ones, leading to a U-shaped curve. The evolution is remarkably consistent across notches.

Observations are well spread across indexes, so that the results do not simply reflect a higher weight of some indexes in total average. How can this results be

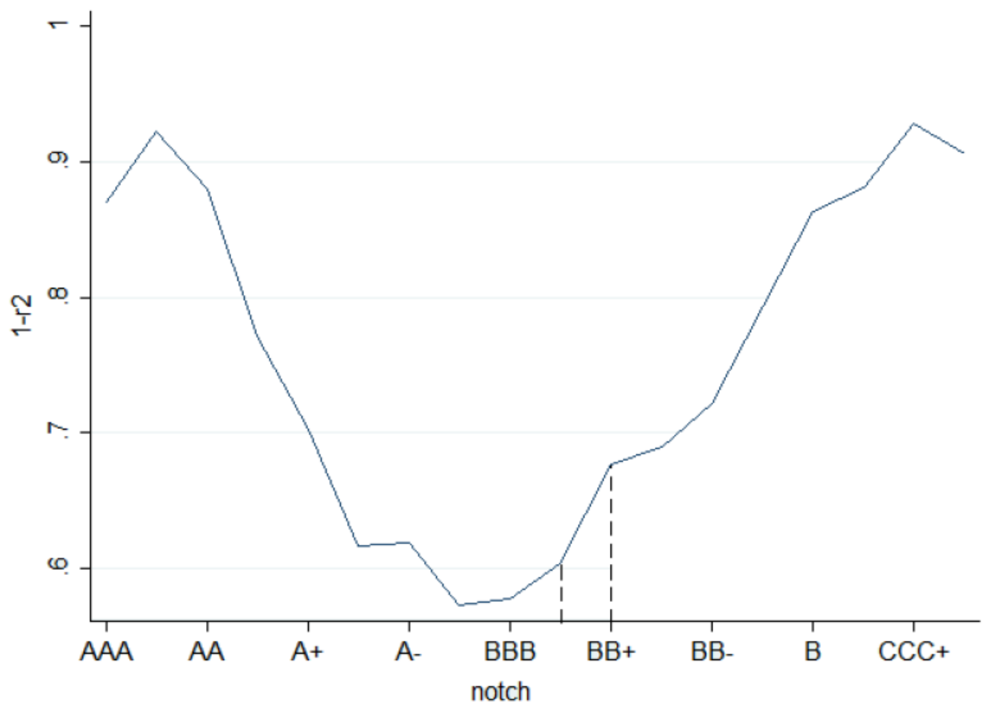

Figure 2.5.2 – Idiosyncratic share of total variation for all ratings

explained?

For high graded asset the apparent disconnection with global risk factors surely comes in part from the fact that these assets are safe enough to be immune to changes in global risk. Yet from this perspective we should have a particularly strong impact on AAA assets, which is not present compared to AA+ or AA.

A complementary explanation then lies with regulation. The sample covers a period of preparation to Basel III, which will apply a leverage ratio that particularly values AA- or more rated bonds (Basel committee, 2013). Before this Basel II had also drawn a line at AA- for capital charges (Atkinson and Blundell-Wignall, 2010). Therefore, for high graded bonds, investors may be tempted to "buy the grade" for regulatory purposes, leading to a large disconnect between these bonds and the others. This can be related to warnings that have emerged from both analysts and practitioners that Basel may shift demand away from non premium quality bonds [17].

For lower grade bonds, the disconnect with fundamentals is more surprising. A possibility is that the pricing of speculative assets classes sometimes appear to be based more on market sentiment than fundamentals. However another strong possibility, and more interesting from the perspective of this study, is that style investors rely more heavily on ratings when the cost of gathering information themselves is particularly large. Large and well diversified groups, for which information flows freely and quickly across markets, may then be less traded as part of an index compared to small less scrutinized firms. This could lead to a larger amount of style investing in lower grade indexes, who typically refer to smaller firms.

2.5.3 Comovement at the high-yield versus investment level

Section 2.5.2 has hinted that some biases may explain our results on test 2, we now use our estimates of these biases to rerun a theoretically unbiased version of this test.

Since we have $\triangle^{\beta} = (A) + (B)(C)$, removing (A) from $\triangle^{\beta}$ and dividing by (C) should give us a raw estimate of (B), the component on which we initially expected a differentiated impact for BBB-/BB+ movements. Of course correcting for (A) and (C) introduces some modeling and measurement error, so that we should be cautious notably in our interpretation of the magnitudes of the estimated (B). We focus more on the p-values of the corrected tests.

17. See for instance the speech by Dutch central banker N. Wellink in Amsterdam, 14th April 2011

(C) has been estimated in section 2.5.2.2. (A) may be expressed as the product of the relative change in fundamentals[18], which we have estimated in section 2.5.2.1, and the explained sum of squares of a regression of the index on "fundamentals", which corresponds to the R^2 of regression (2.2).

Table 2.5.1 presents the results on "corrected test 2" for our baseline estimation. We include the results of section 2.4 with no corrections, those with only the fundamental component (A) removed, and finally those with both corrections.

Correction	movement	in theory	H0	value
0	In	$\triangle^w > 0$	$\triangle^{\beta,,bb-/bbb+} - \bar{\triangle}^{\beta} < 0$	.099 (.105)
0	Out	$\triangle^w < 0$	$\triangle^{\beta,,bb-/bbb+} - \bar{\triangle}^{\beta} > 0$	-.098 (.104)
1	In	$\triangle^w > 0$	$\triangle^{\beta,,bb-/bbb+} - \bar{\triangle}^{\beta} < 0$	.184*** (.009)
1	Out	$\triangle^w < 0$	$\triangle^{\beta,,bb-/bbb+} - \bar{\triangle}^{\beta} > 0$	-.110* (.079)
2	In	$\triangle^w > 0$	$\triangle^{\beta,,bb-/bbb+} - \bar{\triangle}^{\beta} < 0$	.316*** (.004)
2	Out	$\triangle^w < 0$	$\triangle^{\beta,,bb-/bbb+} - \bar{\triangle}^{\beta} > 0$	-.203** (.040)

Table 2.5.1 – Corrected test 2

We find that accounting for (A) and (C) leads to a noteworthy change in the conclusions of test 2. Removing the "fundamental bias" only, a significant im-

18. $(A) = \frac{\alpha_I \sigma_F^2 (\alpha'_I - \alpha_I)}{\sigma_{R_I}^2} = \frac{\alpha_I^2 \sigma_F^2}{\sigma_{R_I}^2} \frac{(\alpha'_I - \alpha_I)}{\alpha_I}$

pact is present in both subcases at the 0.1 level. Accounting for both biases leads them to achieve significance at the 0.05 level. The corrected test appears to impact particularly "in" movements, who exhibit very small p-values. These results suggest that accounting for the peculiar status of BBB-/BB+ movements, assets indeed draw extra comovement from belonging to similar investment or high-yield ensembles, consistent with what we initially expected.

Figure 2.5.3 plots the average comovement change from qualitative style investment for all rating actions.

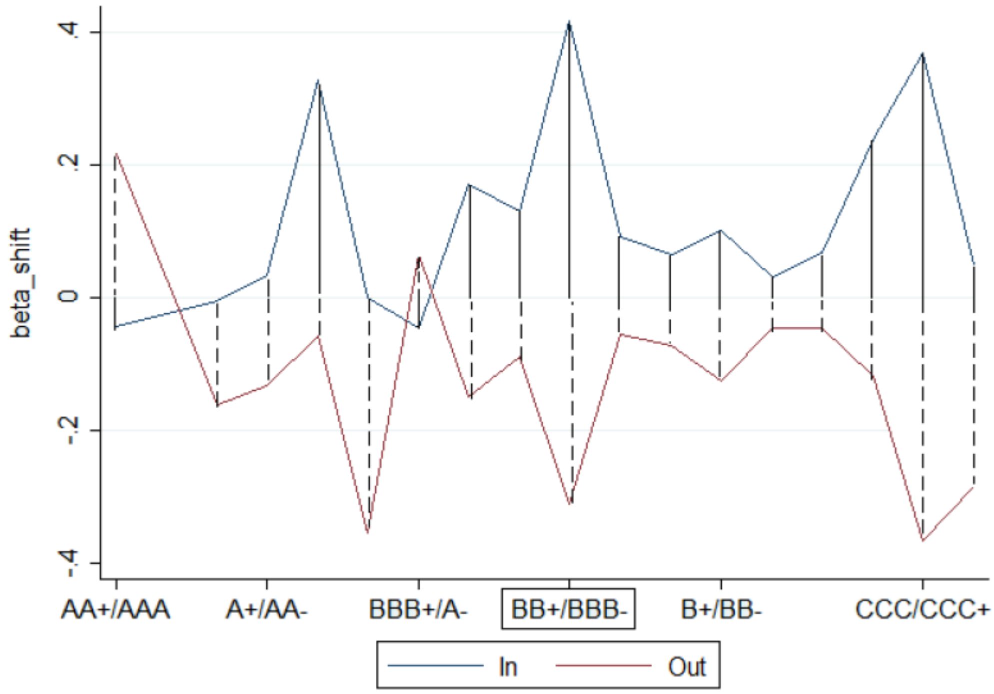

Figure 2.5.3 – Corrected beta shifts across ratings for in and out movements

The magnitude of BB+/BBB- movements has risen significantly, notably relative to movements between CCC and CCC+ and to a lesser extent between A and

A+.

Table 2.5.2 presents the results for corrected test 2 for the alternative specifications defined in section 2.4, where the first 4 forms represent our robustness checks, while (5) and (6) are the results using respectively announcements and issuer ratings.

Correction	Move	H0	(1)	(2)	(3)	(4)	(5)	(6)
0	in	$\Delta^w < 0$	.108 (.330)	.012 (.423)	.072 (.225)	.093 (.147)	.107 (.107)	.055 (.375)
0	out	$\Delta^w > 0$	.019 (.531)	-.003 (.481)	-.041 (.318)	-.071 (.203)	-.123* (.069)	-.019 (.456)
1	in	$\Delta^w < 0$	.171 (.243)	.097* (.056)	.163** (.044)	.191** (.016)	.249*** (.002)	.147 (.198)
1	out	$\Delta^w > 0$	.040 (.564)	-.015 (.407)	-.048 (.296)	-.071 (.203)	-.142** (.044)	-.025 (.443)
2	in	$\Delta^w < 0$	7.130** (.0128)	.167** (.036)	.287** (.024)	.335*** (.006)	.636*** (.000)	.267 (.164)
2	out	$\Delta^w > 0$	-2.592 (.189)	-.041 (.338)	-.109 (.185)	-.138 (.136)	-.415*** (.008)	-.055 (.418)

Table 2.5.2 – Corrected test 2 robustness checks

Applying both corrections has a strong impact of the results of test 2. “In” movements seem to perform particularly well, coming up significant at the 0.05 level in 5 out of 6 specifications. For “out” movement the evidence is not as definite, but the average beta shift is of the expected sign in every form, and its p-value is below 0.2 in 4 out of 6 cases. Overall we view this evidence as supportive of our baseline results.

Individually we see that the corrections we apply have had a strong impact on the magnitudes of the coefficients in form (1), which uses levels instead of differences. This is due to the fact that the R^2 of regression (3) approaches 1 using levels, as the regression on levels pick up more common variation which kicks up our estimates of (B). Nevertheless the performance of the t-test on this form is comparable to the other ones.

Other forms appear fairly homogeneous, except for spirit ratings who perform extremely well, and underlying ratings who perform quite poorly. Yet this result for underlying ratings is based on only 34 observations, the number of rating actions between BBB- and BB+ at the issuer level, so that further discussion would be careless. The performance of spirit ratings on the other hand strongly implies that investors do react to rating announcements more than they do to actual movements, particularly for BBB-/BB+ movements.

2.6 Conclusion

This chapter has uncovered strong evidence of style-driven excess comovement in bonds. Our first test in particular shows that assets who move from one class to another start comoving more with the class they join and less with that they leave, even when fundamental factors predict otherwise. We also find that high-yield and investment ensembles lead the assets that compose them to move together, as implied by our corrected second test.

What's more, we suggest that low and high graded assets may comove particularly strongly with their risk classes, due to informational and regulatory effects respectively. We also point out that the market pricing reaction to certain rating actions seem hard to reconcile with fundamental factors.

This chapter attempted at making a diagnosis, but many extensions may be considered from this point. From the investors perspective, certain findings in this study may have trading implications. For instance, if BB+ bonds have only a marginally higher bankruptcy rate than BBB- bonds but their price fall much more in response to negative news, a trading strategy based on selling short BBB- and buying BB+ with the proceeds may be considered following bad news.

What's more, Wahal and Yavuz (2013) have showed that a portfolio of assets that strongly covary with their indexes performs better that one with low correlation with indexes. From this perspective, passive investors may consider holding only high graded and low graded assets, who seem to be more subject to style investing. We plan on investigating such questions in a follow-up study.

The results presented may also have implications from a regulatory perspective. First further work should be undertaken to confirm that Basel III has had the unintentional effect of creating extra style investing in high graded assets. Second, "investor-driven" comovement should always be considered carefully, as it may constitute a vehicle for transmitting shocks across markets. Regulators may then be tempted to limit the amount of style investing, for instance by imposing size constraints on certain ETFs.

Foreign exchange investment rules and endogenous currency crashes

Chapitre 3

Foreign exchange investment rules and endogenous currency crashes

3.1 Introduction

Every empirical moment of the floating exchange rates seems to contain a puzzle for economists. In terms of expected values, exchange rates are related to fundamental factors only in the long run. At a shorter yet non-trivial horizon, important deviations from such "fundamentals" may arise and persist (Meese and Rogoff, 1983, Faust et al. 2003). The same disconnect has long been documented in terms of volatility (Baxter and Stockman, 1989), while the distribution of exchange rates also has fat tails (Huisman et al., 2002) and is conditionally skewed. In particular, the currencies of high interest rate countries appear subject to crashes which again do not appear linked to a clear fundamental trigger, as pointed out by Brunnermeier et al. (2009).

A gigantic literature has focused on explaining this seemingly puzzling behavior of currency prices. One broad approach has put forward macro-based explanations. Dornbusch (1976) notably argues that exchange rate volatility is a natural consequence of fundamental volatility when uncovered interest rate parity holds. Devereux and Engel (2002) postulates that the exchange rate disconnect from fundamental may be due to the fact that consumer prices are relatively insensitive to changes in the nominal rates. With respect to skewness, the "sudden stop" literature has explained crashes in emerging country through a combination of push and pull factors.

Another strand of literature has focused more on financial markets, arguing that frictions at the investor level may be the cause of non-fundamental price movement. This "microstructure" approach postulates that the rules followed by investors, the constraints they face, the structure of the markets, have a non-negligible impact on market prices. It received significant support from Evans and Lyons (2002) who showed that order flow, an indicator of the net demand by customers to foreign exchange (FX) dealers, explains most of the short-to-mid term returns in the FX market, in contrast to fundamental factors.

FX microstructure models have been successful in reproducing some empirical features of exchange rates (see King et al., 2013 for a survey). A popular approach notably explains fat tails and volatility clustering through the interactions of heterogeneous trading rules (Frankel and Froot, 1990, De Grauwe et al., 1993, De Grauwe and Grimaldi, 2006), while other models put forward information effects (Bacchetta and Van Wincoop, 2006). However these models, to our knowledge, have been silent on the issue of currency crashes.

This study fills this gap by showing that negative (positive) skewness arises naturally for high-yield (low-yield) currencies once we account for two basic features of FX investors: heterogeneous trading rules and funding constraints. We specify a model which generates dynamics that are consistent with the empirical observation that high-yield exchange rates "go up the stairs and down the elevator". Importantly, these dynamics emerge even in the absence of fundamental shocks, consistent with the suggestion by Chernov et al. (2012) that jumps in the exchange rates result from a "self exciting process".

In a nutshell, we present a model in which exchange rates move with the demand for currencies from three types of traders[1]:

- "Chartists" or "momentum traders", who aim at identifying trends in price dynamics and tag along them.

- "Fundamentalists" who try to take advantage of the tendency of the exchange rate to revert to their fundamental value in the long run.

- "Carry traders", who borrow in a low-yield currency and lend in an high-yield one, and are subject to funding constraints.

These different trading rules and their interactions cause deviations from fundamentals, by allowing the exchange rate to "feed off itself" to a certain extent. Recurrence in the exchange rate (ER) dynamics will come from two sources : 1) chartists, and to a lesser extent fundamentalists, base their expectations of future exchange rate change on the past recently observed movements; 2) past exchange rate movements impacts the capital of carry traders and thus their ability to finance new positions. Formally this translates into a complex relationship between $\triangle S_t$, the change in the exchange rate between time t and $t{+}1$, and its lagged values, which we study both in its deterministic and stochastic version.

We find that currency crashes for high-yield currencies occur endogenously as

1. See Pojarliev and Levich (2010) for evidence on the preponderance and these trading styles.

follows. Slow moving capital from constrained carry traders induces a trend of appreciation in a high-yield currency, which is picked up by chartists. When this trend weakens through the influence of fundamentals, chartists exit their long positions while fundamentalists enter the market, shorting the high yield currency. This combined selling by both agents leads to a significant drop in the price of the currency, which in turn impacts the capital of carry traders. Since carry traders are required to hold a given proportion $1/L$ of their total holdings in capital, they are then forced to sell assets to comply with their constraint. These "margin calls" lead to currency crashes. In its stochastic version, the model yields empirical moments that are comparable to those of currency couples known to attract carry traders, such as Japanese yen versus Australian or New Zealander dollar (AUD/JPY and NZD/JPY), in terms of skewness as well as variance and kurtosis.

In this way the approach of the chapter is based on combining two established findings of the exchange rate microstructure literature, which so far have been considered separately. First funding constraints have potential to turn an adverse shock into a crash, second heterogeneous trading rules may induce such adverse shocks in currency prices. This places this chapter at the crossroads of two strands of literature.

The first focuses on the link between funding issues and market prices. In a general setting, Adrian and Shin (2010) study how procyclical capital requirements foster financial cycles. Danielsson et al. (2012) argue that funding constraints may translate into higher asset volatility, leading to "endogenous risk". In the FX market in particular, Plantin and Shin (2006) argue that in the presence of high funding costs, carry trade may lead to rational speculation which causes exchange rates to depart from their fundamental value. Finally a decisive study from our perspective comes from Brunnermeier et al. (2009) who find a significant link between currency crashes and the positions of leverage constrained carry traders.

The second strand is centered around "heterogeneous agents", and studies the dynamics of the exchange rate that arise from the interactions of different types of FX investors with different demand functions for the currency. The approach pioneered by Frankel and Froot (1990) studied the interaction between 2 rules: chartist and fundamentalist (C&F). However in a recent paper Spronk et al.(2013) uses a similar set-up by adding carry traders. Our contribution again lies with the fact that these models produce exchange rates with distributions that are either symmetric, or whose skewness rises with interest rates, in contrast with what we observe empirically.

Our approach may be also compared to that of the C&F literature on technical grounds. A notable similarity is that our modeling implies a necessary departure from the rational expectation assumption. We specify investment rules for our agents which correspond more to a (simplified version of the) reality of the FX market, than to the solution to a fully informed maximization problem. As in De Grauwe and Grimaldi (2006), we simply control that such rules are valid by checking their profitability ex-post [2]. Another common feature is that the model generates dynamics that may not be solved analytically, so that this study relies mostly upon simulations.

Yet there are also important differences with the usual C&F literature. In particular models such as De Grauwe and Grimaldi (2006) generate chaotic dynamics which are defined by a strong sensitivity to initial conditions, and different possible

2. A notable difference from a technical perspective is that our dynamics are not chaotic as our set-up the exchange rate will always converge its fundamental value.

paths for a given starting point. In contrast, under general conditions our model converges asymptotically towards a single point given by the fundamental value F of the exchange rate, which represents its only steady state. Our interest lies with how realistic interactions between FX investors may lead the ER to deviate from this steady state value F, and whether this deviation will die out through a crash or a smooth adjustment.

We extend the model by studying its sensitivity to different levels of funding constraints for carry traders, and different levels of activeness for chartists and fundamentalists. We find that skewness is a non-linear function of funding constraints. When capital is very constrained no boom phase appears, and consequently no "bust". When capital is little constrained the ER immediately kicks up then gradually adjust as predicted by basic financial theory. For moderately constrained carry traders however, there is enough capital to create a build-up phase, while traders remain vulnerable to margin calls because they operate on their funding constraint. We also find that more active trading by chartists and fundamentalists typically leads to lower currency crash risk by reducing the length and stability of the build-up phase.

Finally we investigate how our funding-driven booms and busts may help resolving two well known puzzles of the FX market: the "exchange rate disconnect" from fundamentals in terms of expected values, and the seemingly "abnormal profits" to carry and momentum trading, highlighted notably by Menkhoff et al. (2012).

To investigate the "disconnect", we generate 10 000 random time paths for the fundamental value F, from which we draw the corresponding 10 000 ER dynamics implied by our model. We then regress ER on F for all realizations, in order to study what type of empirical link between fundamentals and the exchange rate arise in the model. Our results are consistent with the findings of a classic empirical study on the matter, provided by Mark (1995).

With respect to the "profit" puzzle, we uncover 1) that all strategies are profitable in equilibrium and 2) the profits generated reproduce a number of stylized facts highlighted by the literature on the matter. In particular it appears accounting for carry trader leverage reduces the puzzle of carry profits, as argued by Darvas (2009). Momentum trading on the other hand appears remarkably successful including during crashes, consistent with Menkhoff et al.(2012).

The contributions of this study may then be summarized as follows: 1) From a theoretical standpoint, the FX market may suffer from a perverse interaction between active procyclical trading rules and funding constraints; 2) A model with 3 simple trading rules and funding constraint does a good job matching the empirical features of exchange rates, notably their skewness; 3) Dynamics of booms and busts may explain other features of the FX market, notably the puzzles of the profits of the FX investors and the disconnect between the exchange rate and fundamentals.

Section 3.2 presents the model. Section 3.3 calibrates it and presents the dynamics it generates. Section 3.4 extends the discussion.

3.2 The model

This section presents the structure of the FX market, and derives the asset demands of the FX investors.

3.2.1 The market

3.2.1.1 Set-up

We study a 2-country model in which a large developed country, "Home", interacts with a smaller high-interest rate country, "Abroad". The reader may see "Home" as the US, and "Abroad" as either a smaller developed country, or a fairly open emerging one. The exchange rate between the domestic and foreign country, *expressed in terms of the value in the home currency of one foreign currency unit (FCU henceforth),* will move according to the relative demand for currencies. Currency supply is thus finitely elastic as in Evans and Lyons (2002), which allows the ER to move with demand.

We simplify the analysis by studying only the demand for FCU from the larger "Home" country, implicitly assuming that the variations in the demand for both currencies by the foreign country are not sufficiently large to impact the exchange rate. In other words the larger Home country is a currency price maker while Abroad is a price taker. Taking an example, we assume the US dollar/ New Zealand dollar exchange rate will change only marginally in response to a rise in the demand for USD by New-Zealand, but will likely be largely impacted by a rise in the US demand for NZD. We also assume both Home and Abroad have fairly advanced financial markets which allow satisfactory liquidity.

The demand for FCU by Home comes from two sources: "real" demand which represents the demand for foreign goods by Home firms and consumers, and "financial" demand which comes from the demand for FCU by Home investors. The timing is as follows : demand shocks for FCU occurs at exactly time t, and have an impact on the exchange rate at $t+\omega$, where ω may be seen as the time it takes for a dealer to execute the order and adjust his quotes if he finds evidence of excess demand. An increase in demand between *t-1* and t, $\triangle Q_{t-1} = Q_t - Q_{t-1}$, will then have an impact on the ER between t and $t+1$ $\triangle S_t = S_{t+1} - S_t$.

Formally:

$$\triangle S_t = b\triangle Q_{t-1} = b[\triangle Q_{r,t-1} + \triangle Q_{fm,t-1}] \tag{3.1}$$

where b is the marginal impact of an additional unit of FCU demanded on the exchange rate, and $\triangle Q_{r,t-1}$ and $\triangle Q_{fm,t-1}$ represent the real and financial demand shocks for the FCU between time *t-1* and time t.

3.2.1.2 Real and financial demands

We define the real demand for FCU as the current account position of the foreign country at each period t. In appendix 3.1 we derive an simple micro based expression for this current account. Plugging this expression in equation (3.1) yields:

$$\triangle S_t = \lambda(\frac{F - S_t}{S_t}) + b\triangle Q_{fm,t-1}$$

where F is the fundamental exchange rate, which in our set-up is simply defined as the exchange rate for which trade is balanced between both countries[3]. λ is a parameter which governs to the speed of convergence between S_t and its fundamental value. We set both F and λ to remain constant in order to focus on the endogenous dynamics of S_t that emerge through the behavior of investors in

3. Other definition for fundamental value may exist, notably those based on uncovered interest parity

an otherwise stable set-up.

Note that the real demand for FCU is a driver of mean reversion towards the fundamental value F. In this way our formal specification is consistent with the literature discussed in the introduction: the exchange rate will deviate from its fundamental value F through the demand from FX investors, but return to fundamentals in the long-run. We verify that this convergence holds later in the chapter.

The financial demand component $\triangle Q_{fm,t-1}$ depends on the total net demand for FCU by all traders, which includes carry traders, momentum traders, and fundamentalists, to which we add a noise trading component. The final form for the evolution of the exchange rate is thus:

$$\triangle S_t = \lambda(\frac{F - S_t}{S_t}) + b[n_c \triangle q_{c,t-1} + n_m \triangle q_{m,t-1} + n_f \triangle q_{f,t-1}] + \epsilon_{n,t} \tag{3.2}$$

where n stands for the number of traders of each profile, while the c, m, and f subscripts stand respectively for carry traders, momentum traders, and fundamentalists. $\epsilon_{n,t}$ is the noise trading component, which we set to be white noise. The evolution of the ER will then depends upon the expressions for the demand of the traders $\triangle q_{c,t-1}$, $\triangle q_{m,t-1}$, $\triangle q_{f,t-1}$, which we now derive.

Summary: the exchange rate is influenced by the real and financial demand for a currency, where the real demand is a driver of long-term convergence towards fundamentals.

3.2.2 Carry traders

3.2.2.1 Basics

Carry traders allocate capital between a riskless home bond, which may be seen as a cash position, and a risky foreign bond. The foreign bond may be seen as a generic asset whose return and risk reflect that of a diversified position in the foreign country. The balance sheet position for a carry trader in the Home currency is given by:

$$e_t = p'_t q'_t + S_t p_t q_t \tag{3.3}$$

where e_t is the capital (or equity) of the trader, expressed in the Home currency. p_t is the price of the foreign bond in the portfolio at time t, q_t the quantity held, and S_t the exchange rate, expressed as the value of one FCU in terms of the home currency. The apostrophes indicate the positions taken in the domestic currency. $p'_t q'_t$ may take a negative value, indicating that the investor is leveraged.

Normalizing the return on the domestic asset to 1, the value of the capital at the following period is given by:

$$e_{t+1} = p'_t q'_t + S_t p_t q_t (1 + R_{b,t})(1 + R_{s,t}) \tag{3.4}$$

where $R_{s,t} = \frac{S_{t+1}}{S_t} - 1$ is the realized return on the exchange rate between t and $t{+}1$, and $R_{b,t} = \frac{p_{t+1}}{p_t} - 1$ is the return of the foreign bond. Because we normalize the return on the domestic bond to 1, $R_{b,t}$ is also the spread between both countries, and we shall refer to it in this way in what follows. We set the return on the foreign risky asset to depend on the risk free rate of the foreign country, plus a premium: $1 + R_{b,t} = (1 + \delta)(1 + R_{bf}) + \epsilon_{b,t}$, where δ is the premium, R_{bf} is the risk-free rate abroad , and $\epsilon_{b,t}$ is a white noise component that captures risk. As with F, we keep δ and R_b constant for simplicity, in order to focus on endogenous price movements only.

3.2.2.2 Modeling

We consider a continuum of carry traders who wish to hold varying amount q^* of foreign bonds, depending on their risk aversion, where we simply set this desired holdings q^* to be a positive linear function of the interest rate differential. Since this differential will remain constant the desired quantity for each trader will be constant also. Carry trading demand shocks will thus not come come from changes in desired quantities, but from the evolution of the funding constraint.

Of course a richer form would allow the carry trader to have changing optimal quantities, by allowing their beliefs about future exchange rate movements, their expectations about the future interest rate differential, or their risk aversion to vary. This would notably be in line with the empirical observation that factors such as the VIX or the US interest rate expectations play a huge part in the exchange rates of high interest countries. Nevertheless, the method of the study is to show how endogenous crashes occur through the interactions of our three investors even in an otherwise perfectly stable set-up, which is why we rule out shocks in to carry demand. Note though that such shocks would likely add to both the probability and the magnitude of currency crashes, as they would involve large up and down swings in the exchange rates, where down movements may be exaggerated by margin calls.

The modeling of the VaR constraint follows closely Shin (2010).

3.2.2.3 Funding constraint

Carry traders are subject to a value-at-risk (VaR) constraint, a classic risk management tool [4] which will be our vehicle for modeling the constraint. The value-at-risk is the capital loss associated with a given realization of the portfolio return, that may occur with probability α. It is used by investors to keep enough capital to face such a capital loss, and thus keep their probability of failure to α.

We relegate to appendix 3.2 the step by step derivation of the VaR, whose final form is given by :

$$S_t p_t \mathring{q}_t = L.e_t \tag{3.5}$$

where $S_t p_t \mathring{q}_t$ is the maximum holding of the foreign asset allowed by the current equity level e_t, where maximum holding is expressed in the Home currency. L is a constant which may be seen as an indicator of maximum allowed leverage, since it represents a lower bound for the proportion of foreign investment financed by equity.

This expression implies that the positions taken by carry traders, expressed in the local currency, are a positive linear function of their capital. Any capital gain made at period t will then loosen the VaR constraint and thus allow for a larger position at period *t+1*, which will then generate procyclicality in the model [5]. If the regulatory constraint binds for the investor, i.e. if $\mathring{q}_t < q^*$, his holding of the foreign asset will simply reflect the evolution of the maximum holdings of FCU

4. Adrian and Shin (2010) provide evidence on the wide spread use of VaR across markets. This use may in fact be particularly important in the FX market, in which banks and near-bank institutions are prominent (see for instance Galati and Melvin, 2004)

5. Note that further procyclicality may arise if we allowed L itself to vary, indicating than the capital requirements are procyclical (Adrian and Shin, 2013). However for conciseness we keep only one engine of procyclicality

allowed, which plugging equation (3.4) into (3.5) may be expressed as:

$$\overset{\circ}{q}_t = \frac{1 + LR_{c,t-1}}{1 + R_{c,t-1}} \overset{\circ}{q}_{t-1} \tag{3.6}$$

where $R_{c,t-1} = (1 + R_{b,t-1})(1 + R_{s,t-1}) - 1$ is the total return on the carry position. The relative increase in the holdings of the foreign bond thus depends on the return on the portfolio at the previous period. Note the role of L here: a high leverage magnifies the gains or the losses, and thus translate into larger fluctuations in bond holdings from one period to the other.

3.2.2.4 Demand

Considering the foreign bond is issued in its own currency, the net demand for the FCU by carry traders will be given by $\triangle q_{C,t} = p_{t+1}(q_{t+1} - q_t)$, i.e. the net demand for fresh foreign bonds times the price of those newly purchased bonds.

With a constant expected return and variance, the price of the foreign bond will depend only on the distance to maturity of the bond. Accounting for changing bond prices clouds the model without adding a new dynamic, thus we assume for simplicity that carry trader will buy and sell bonds of fairly stable maturities. The price of the bonds sold and bought at each period is then assumed to be roughly constant: $\triangle q_{C,t-1} = \bar{p}(q_t - q_{t-1})$.

The net demand of FCU by carry traders is then given by:

- $\triangle q_{C,t-1} = \bar{p}(\overset{\circ}{q}_t - \overset{\circ}{q}_{t-1}) = \bar{p}(\frac{(C-1)R_{c,t-1}}{1+R_{c,t-1}})$, if the investor is constrained at both periods.

- $\triangle q_{C,t-1} = \bar{p}(q^*_t - q^*_{t-1}) = 0$, if the investor is unconstrained at both periods.

- $\triangle q_{C,t-1} = \bar{p}(\overset{\circ}{q}_t - q^*_{t-1})$ or $\triangle q_{C,t} = \bar{p}(q^*_t - \overset{\circ}{q}_{t-1})$ if the constraint stops or starts binding between *t-1* and *t*.

Summary: When the carry trader is constrained, he will demand more of the FCU at time t if the FCU has appreciated between *t-1* and *t*.

3.2.3 Chartists and fundamentalists

3.2.3.1 Basics

For simplicity we ignore interest payments on the positions of chartists and fundamentalists, so that they only benefit from the future evolution of the exchange rate S_t. This assumption is designed to keep a clear separation between agents: allowing chartists to account for interest rate differential would lead them to act partly as carry traders. Both agents are unconstrained, so that their desired holdings will always equal the actual ones, i.e. $q_t^* = q_t$.

The evolution of the wealth of the investor in his own currency is given by $W_t = q_t^{'} + q_t S_t$, and their expected change in wealth at horizon t+T is:

$$E(W_{t+T} - W_t) = q_t^* E(S_{t+T} - S_t)$$

Contrary to carry traders chartists and fundamentalists are unconstrained, thus we draw their demands from a regular unconstrained optimization. Using a classic mean-variance utility function yields the solution:

$$q_t^* = \tau \frac{E(\triangle S_T)}{Var(\triangle S_T)} \tag{3.7}$$

where $E(\triangle S_t) = E(S_{t+T} - S_t)$ is the expected change in exchange rate between t and t+T, and $Var(\triangle S_T)$ the estimated variance of ER change during the period.

Both chartists and fundamentalists face a similar optimization problem, but will differ in their investment horizons. Chartists focus on the short-run exchange rate dynamics, which are driven more by the (procyclical) demands of carry traders. Fundamentalists on the other hand will have a longer horizon, and will exploit the long term tendency for ER to return to its fundamental value.

Formally since the rate of convergence towards fundamentals λ is small, the short-run dynamics of the exchange rate may be approximated by: $E(\triangle S_{t+1}) \simeq E(b\triangle Q_{fm,t-1}) \simeq f_+(E(\triangle S_{t-1}))$. On the other hand over a sufficiently long horizon T, $E(S_T - S_t) = F - S_t$, so that price movement from order flow can be ignored. Both the short-term procyclicality and the long-run convergence to F are verified in equilibrium

3.2.3.2 Modeling

Before going further it is important to clarify our approach with respect to the modeling of both agents.

The key point is that we take a descriptive approach, as opposed to a normative one. The investment rules we specify for chartists and fundamentalists correspond to a (simplified version of the) behaviors that are known to exist on the markets, as documented by many market surveys such as Pojarliev and Levich (2010). This approach necessarily implies a departure from a rational expectation model, as naive investment strategies such as chartism in theory should not be profitable in such a set-up. Indeed a market in which all agents know the investment rule of the others, no investor should have to use past prices to make inference. Besides, as noted by De Grauwe et al. (1993) in a perfectly rational equilibrium there should be no exchange rate disconnect from its fundamental value.

This stance brings this study close to the chartist and fundamentalist (C&F) literature, and our demand functions resemble those of this literature. Another important similarity is that we control that our naive rules are realistic in an ex-post manner, by checking that they are profitable in equilibrium, similar to De Grauwe and Grimaldi (2006).

Yet the comparison with the C&F literature does not go further. In the C&F literature a single agent switches between a fundamentalist rule and a chartist one, which induces chaotic movement in the ER. Chaotic system are very sensitive to initial conditions and will generate very different dynamics for minor changes in them. In this chapter we want to discuss the interactions between three types of traders, which is more easily done within a non-chaotic system and keeping a representative agent set-up. Thus we model chartists, fundamentalists, and carry traders as separate agents. The stability of the price dynamics that results from the model is discussed in section 3.2.4.

3.2.3.3 Momentum traders

Chartists by definition use past movements to forecast future ones. We capture this by allowing them to base their estimations of the moments $E(\triangle S_t)$ and $Var(\triangle S_t)$ on recent price dynamics, as well as unconditional moments which in equilibrium are respectively 0 and $\sigma^2_{\triangle s}$. Mathematically we specify the following general form:

$E_m(\triangle S_t) = (1-\alpha) \times 0 + \alpha(\frac{1}{x}\sum_{t-x}^{t-1} \triangle S_i) = \alpha MA(\triangle S)_x$

$Var_m(\triangle S_t) = (1-\alpha)\sigma^2_{\triangle s} + \alpha Var(\triangle S)_x$

where α is the weight chartists put on recent observations compared to long-term unconditional moments. $MA(\triangle S)_x = \frac{1}{x}\sum_{t-x}^{t-1} \triangle S_i$ and $Var(\triangle S)_x$ are the two empirical moments for $\triangle S$ observed over these last x periods. Chartists thus use a moving average for their estimations, a tool widely used in practice (Lui and Mole 1998). Plugging these estimations into (3.7) yields:

$$q_{m,t} = \tau \frac{\alpha MA(\triangle S)_x}{(1-\alpha)\sigma^2_{\triangle s} + \alpha Var(\triangle S)_x} \tag{3.8}$$

where the m subscript stands for momentum trading.

The skeptical reader may see this investment rule as an over-simplification of the behavior of chartists. It is true that in practice chartists use several moving averages and within much more complex trading strategies, notably based on moving average crossovers [6]. However, this form has two advantages for our purposes.

First there are many different complex chartists rules and the investment pattern of chartists as a whole is unknown. Modeling a more complex strategy then involves choosing a precise rule, which would increase the risk of an ad-hoc model. A simple feedback strategy is more general and thus less subjective. What's more, if chartists are very heterogeneous it is not clear that this simple auto-regressive form is worse description of the investment of the entire chartist sector than a single more complex rule.

Second and most importantly, it is not apparent that this very naive rule undermines the results presented. On the contrary, the fact that simple rules give birth to complex dynamics may be seen as encouraging. More complex strategies based for instance, on moving average crossovers, would generate even larger non-linearities as such rules imply massive selling past certain endogenously de-

6. A crossover occurs when a moving average defined over a given time span x crosses another one with a time span y. For instance when the 10 days moving average gets above the 50 days one.

termined thresholds. This should increase both the possibility and the magnitude of endogenous crashes. De Grauwe and Grimaldi (2006) make a similar point.

Summary: if a currency has risen/fallen in the recent periods, chartists will be long/short q_t units of it, and the more stable the rise/fall the largest the position will be.

3.2.3.4 Fundamentalists

Fundamentalists expect deviations from fundamentals to die out, so that their expected return over a sufficiently long horizon T is given by $F - S_t$.

As we shall see, this belief that S_t will converge to F will hold in equilibrium so that the strategy can be seen as perfectly safe over a sufficiently long horizon. The risk for fundamentalists lies with the fact that this convergence to F may take a long time to occur, and may involve important losses in the short run, due to the order flow component of exchange rate dynamics which may move against fundamentals.

We allow fundamentalists to account for this risk by basing their decisions not only on the expected return $F - S_t$ they expect in a fundamentals-driven market, but also on their assessment of the likelihood that we are in such a market. This specification is in line with survey studies such as Dick and Menkhoff (2013), who show that FX traders consider of the current market dynamics before switching to a fundamentalist rule.

Mathematically we set $E_f(\triangle S_T) = Pr(regime = funda)_t \times (F - S_t)$. In short, this form implies that fundamentalists try to assess whether prices are currently driven by fundamentals or by order flow: they will not blindly try to enforce fundamentals if the market appears to be moving against them.

The probability of being in a fundamentals driven market $Pr(regime = funda)_t$ is estimated through the likelihood that the current price dynamics move in the same direction as fundamentals, which we estimate using a normal distribution whose first two moments are the empirical mean and variance of the daily exchange rate movement over the last x periods.

Of course this measure is not perfect, notably due to the fact that true distribution of the daily exchange rate will not be normal. However in practice it is accurate enough to serve its purpose, which is to allow fundamentalists to make profits in equilibrium, while capturing the stylized fact that current price dynamics have an impact on the size of fundamentalist trading. Using alternative distribution changes little the dynamics presented below [7].

The optimal quantity is then:

$$q^*_{f,t} = \tau \frac{\Phi(sign(F - S_t) = sign(MA(\triangle S)_x)) \times (F - S_t)}{\sigma^2_{\triangle s}} \tag{3.9}$$

where we have implicitly set $Var_F(\triangle S) = \sigma_{\triangle s}$, i.e. fundamentalists use the unconditional variance.

Summary: Fundamentalists will go long/short if the currency should appreciate/depreciate according to fundamentals, and the magnitude of their position will depend on how whether prices are currently moving towards fundamentals.

3.2.4 Stability analysis

We have specified a model in which the demand functions for traders and consumers generate a complex relationship between exchange rate movements and

7. Simulations using alternative distributions are available on request

their lagged values. The impact of lagged value runs through several channels: past movement have implications on the future adjustment towards fundamentals, they impact the capital of the carry trader, change the assessment of future returns for momentum traders, and the assessment by fundamentalists that we are in a fundamentals-driven regime. This recurrence will produce the dynamics described in the following section.

Before going moving on to the simulations, one issue that deserves investigation is the stability of a system with such numerous and non-linear recurrences. To investigate so we examine whether the exchange rate S_t process converges asymptotically towards a constant expected value and variance.

3.2.4.1 Stationarity of the first moment

Stability in the first moment implies $lim_{t\to\infty} E(S_t) = cst$ which must hold if the average exchange rate change tends to zero, i.e. if $lim_{t\to\infty} \frac{\sum_0^t \triangle S_t}{t} = 0$. We show this condition must hold using a reductio ad absurdum, starting with the opposite postulate $lim_{t\to\infty} \frac{\sum_0^t \triangle S_t}{t} > 0$, i.e. the expected exchange rate is ever increasing.

Let us first consider the impact of this condition on the demand by chartists. There are two possibilities for $lim_{t\to\infty} \frac{\sum_0^t \triangle S_t}{t} > 0$ to hold. Either the expected change in exchange rate tends towards a positive constant i.e. $lim_{t\to\infty} \frac{\sum_0^t \triangle S_t}{t} = cst$ where $cst > 0$, or this change is ever increasing $lim_{t\to\infty} \frac{\sum_0^t \triangle S_t}{t} = +\infty$. The first case implies $lim_{t\to\infty}\, q_{m,t} = \tau \frac{\alpha MA(\triangle S)_x}{(1-\alpha)\sigma^2_{\triangle s} + \alpha Var(\triangle S)_x}) = \frac{\alpha MA(cst)_x}{(1-\alpha)\sigma^2_{\triangle s} + \alpha cst'} = cst$, which yields $lim_{t\to\infty} \triangle q_{m,t} = 0$ and $lim_{t\to\infty} \frac{\sum_0^t \triangle q_{m,t}}{t} = 0$ i.e. the average demand shock by chartists at each period tends to *0*. The second case also implies $lim_{t\to\infty}\, q_{m,t} = 0$ because when the movements in the exchange rate are increasingly large, the variance of $\triangle S_t$ increases more rapidly than $\triangle S_t$ itself.

Thus it appears that $lim_{t\to\infty}\frac{\sum_0^t \triangle S_t}{t} > 0$ implies $lim_{t\to\infty}\frac{\sum_0^t \triangle q_{m,t}}{t} = 0$. In words there are no condition under which chartists will carry on buying FCU ad nauseam. Either the trend of appreciation is stable in which case chartists should have stable holdings, or the sequence $\triangle S_t$ is explosive in which case chartists are eventually turned off by the infinite variance that should also exist in this case.

Let us now consider the demand by carry traders. An immediate observation is that regardless of the evolution of S_t, the quantity held by carry traders $q_{c,t}$ must lie somewhere between their desired holdings q^* and 0, where the former corresponds to a situation in which the carry trader has infinite capital, and the latter to a situation in which he has none. Since $q_{c,t}$ is bounded, the sum of all changes in holdings must also be bounded, so that $lim_{t\to\infty}\frac{\sum_0^t \triangle q_{c,t}}{t} = 0$.

Now let us consider the impact of an ever increasing exchange rate on the demand by fundamentalists and the "real" demand by consumers. $lim_{t\to\infty}\frac{\sum_0^t \triangle S_t}{t} = +\infty$ yields $lim_{t\to\infty} S_t = +\infty$ which in turn implies that $lim_{t\to\infty} F - S_t = -\infty$. Since the demands by consumers and fundamentalists are a positive function of $F - S_t$, we must then have $lim_{t\to\infty}\frac{\sum_0^t \triangle q_{f,t}}{t} \leq 0$ and $lim_{t\to\infty}\frac{\sum_0^t \triangle Q_{r,t}}{t} \leq 0$. In other words when the exchange rate in increasing on average, the positions of consumers and fundamentalists are negative on average.

Equation (3.2) may be re-expressed as follows:

$$lim_{t\to\infty}\frac{\sum_0^t \triangle S_t}{t} = b[lim_{t\to\infty}\frac{\sum_0^t \triangle Q_{r,-1}}{t-1} + n_c lim_{t\to\infty}\frac{\sum_0^t \triangle Q_{c,t-1}}{t-1}$$
$$+n_m lim_{t\to\infty}\frac{\sum_0^t \triangle q_{m,t-1}}{t-1} + n_f lim_{t\to\infty}\frac{\sum_0^t \triangle q_{f,t-1}}{t-1}] + lim_{t\to\infty}\frac{\sum_0^t \epsilon_{n,t}}{t-1}$$

where $lim_{t\to\infty}\frac{\sum_0^t \epsilon_{n,t}}{t-1}$ must be zero since $\epsilon_{n,t}$ has a symmetric distribution. Since the first four limits are also below or equal to zero, it follows from the expression above that $lim_{t\to\infty}\frac{\sum_0^t \triangle S_t}{t} \leqslant 0$, which violates our initial assumption $lim_{t\to\infty}\frac{\sum_0^t \triangle S_t}{t} > 0$. Thus there exists no parameters for the exchange rate is ever increasing . There exists a threshold value for S past which all demands are either negative or constant. The exchange rate must then lie somewhere below this value.

A similar proof holds for an ever decreasing ER. Carry traders remain bounded, while the same logic works in the opposite direction for other three sources of demand. Thus we may not have $lim_{t\to\infty} \frac{\sum_0^t \triangle S_t}{t} < 0$, any drop in the exchange rate below/above F is bound to be compensated by a rise/drop in the exchange rate at subsequent periods. Thus S_t has a lower and an upper bound, so that we must have $lim_{t\to\infty} E(S_t) = cst$, S_t must be stationary in mean.

3.2.4.2 Stationarity in the second moment

Let us now move to the study of the stationarity of the variance. We use a similar method starting from the hypothesis that the variance is non-stationary, i.e. $lim_{t\to\infty} var(S_t) = \infty$. Let us consider this is true. An immediate observation is that in such a case we have $lim_{t\to\infty} q_{m,t} = \tau \frac{\alpha MA(\triangle S)_x}{(1-\alpha)\sigma^2_{\triangle s} + \alpha Var(\triangle S)_x} = 0$, the demand by chartists must converge to zero, so that $lim_{t\to\infty} var(q_{m,t}) = 0$. Similarly the demand shocks by carry traders are bounded, which implies $lim_{t\to\infty} var(q_{c,t}) = cst$

Since $var(S_t) = E(S_t^2) - E(S_t)^2$, and $E(S_T)^2$ must be a constant as the process is stationary in its first moment, we have $lim_{t\to\infty} var(S_t) = lim_{t\to\infty} E(S_T^2) = \infty$ which then implies that $lim_{t\to\infty} S_t = \pm\infty$. Re-expressing the real demand for FCU as $\triangle Q_{r,t} = \lambda(\frac{F}{S_t} - 1)$, it follows that $lim_{t\to\infty} \triangle Q_{r,t} = 0$, so that the asymptotic infinite variance $lim_{t\to\infty} var(S_t) = \infty$ may not come from $\triangle Q_{r,t}$ either.

We may rewrite equation (3.2) as follows:

$$\triangle S_t = bn_f \triangle q_{f,t-1} + \epsilon_{c,t} + \epsilon_{r,t} + \epsilon_{m,t} + \epsilon_{n,t}$$

where $\epsilon_{c,t}$, $\epsilon_{r,t}$, and $\epsilon_{m,t}$ are the demand shocks from carry traders, consumers and chartists. For our purposes these shocks may be considered simultaneously because we only require that they have finite variance. Plugging in the expression for $\triangle q_{f,t-1}$ yields

$$\triangle S_t = A_t(F - S_{t-1}) - B_t \triangle S_{t-1} + \epsilon_t$$

where $A_t = bn_f\tau\frac{\Phi_t - \Phi_{t-1}}{\sigma^2_{\triangle s}}$ and $B_t = bn_f\tau\frac{\Phi_t}{\sigma^2_{\triangle s}}$, with Φ_t the probability evaluated by fundamentalists that fundamentals dominate the market. $\epsilon_t = \epsilon_{c,t} + \epsilon_{r,t} + \epsilon_{m,t} + \epsilon_{n,t}$ is the total shock at period t with $var(\epsilon_{T,t}) = cst.$. The stochastic process thus has two components besides the shock, a mean-reversion one $A_t(F - S_{t-1})$ and a recurrence one $B_t \triangle S_{t-1}$.

A full investigation of the conditions for explosiveness of this sequence would imply studying how both components interact, which is outside the scope of this study. Here we simply note that a sufficient condition for the convergence of a simple recurrence system $\triangle S_t = B_t \triangle S_{t-1} + \epsilon_t$ is that $|B_t| < 1, \forall t$, while a sufficient condition for convergence of a mean reverting process $\triangle S_t = A_t(F - S_{t-1}) + \epsilon_t$ is $|A_t| < 1, \forall t$.

Assigning to A_t and B_t their maximum value by setting the time-varying numerator to its maximum value of *1* gives $|\frac{bn_f\tau}{\sigma^2_{\triangle s}}| < 1$ for both conditions. Therefore $|\frac{bn_f\tau}{\sigma^2_{\triangle s}}| < 1$ is sufficient to ensure that the system converges in terms of first and second moments. This condition has a limited economic meaning. It only represents a bound for the weight of fundamentalists, which are the only agent which can introduce stochastic divergence in the system.

In what follows we focus on the parameter set for which this condition is satisfied because the economic story behind non-stationary cases is limited in length and interest. Note that the exchange rate with only one steady-state is a major technical difference between this study and the C&F literature, which generates chaotic dynamics which are sensitive to initial conditions.

3.3 Baseline dynamics

3.3.1 Calibration

3.3.1.1 Exogenous parameters

We specify 10 carry traders, with a similar capital at the beginning of the period, but with varying desired holdings, i.e. $q^* \in [1, 10]$. This fairly low value of 10 was chosen because changing the number of carry traders has a trivial impact on the results as long as the total capital by carry traders stays the same, only more trader makes the simulations lengthier. Since carry traders have different desired holdings, at each period some will be constrained, i.e. have desired holdings q^* above their maximum allowed holdings q°, while others will not. The proportion of constrained carry traders at each period t is noted φ_t.

The initial exchange rate is normalized to 1, where the ER is expressed in terms of the number of units of domestic currency that one unit of FCU buys. Our time unit is a day. We chose to focus on the dynamics that emerge from a rise of the interest spread from 4% to 6% annually. A differential of 6% is in line with some of the known historical carry couples such as YEN/NZD. The previous value of 4% was chosen because it implies a rise in the desired quantities by carry traders of 50%, which seems consistent with some important historical episodes of capital flow. The daily rate implied by an annual 6% is $R_B \approx 0,00019$.

With respect to fundamentalists and chartists, we normalize the coefficient of absolute risk aversion τ to 1, and allow the variances $\sigma^2_{\triangle s}$ and σ^2_{Rs} to be defined by their empirical counterparts in the simulation [8]. We initially set x, the number of past periods they use to forge their estimation, to *30*. Thus investors base their estimations on their observations over the last month. However we later present the results with a alternative values for x to study how more active (passive) investors impact the results.

We turn to λ and α , which represent respectively the speed of convergence towards fundamentals and the weight of the recent observations in the estimations by chartists, compared to unconditional moments. As mentioned exchange rate converge only in the long run, so that a low value for λ is warranted. We set $\lambda = 0.0005$. With respect to α, the nature of chartists is to act upon recently observed dynamics, we thus settle for a high value $\alpha = 0.9$. [9]

With respect to the price impact of an additional unit of demand b, we use the study on order flow by Lyons (1995), who finds that a FX dealer would raise his quotes by 0.01 Deutschemark (DEM) for incoming orders worth \$1 billion. We thus set $b = 0.01$.

8. The definition of these parameters solves a fixed point problem, because the values we chose for $\sigma^2_{\triangle s}$ and σ^2_{Rs} impact the demand of investors and hence the realized variances. Thus the statement "we allow the variances $\sigma^2_{\triangle s}$ and σ^2_{Rs} to be defined by their empirical counterparts" should be understood as "we pick values for $\sigma^2_{\triangle s}$ and σ^2_{Rs} that provide a good approximation to the solution of the fixed point problem".

9. The reader may wonder why a value of $\alpha = 1$ was not chosen. The reason is that we set the ER to be constant and equal to F because the rise in interest occurs.$\alpha = 1$ has the undesirable feature of implying a 0 variance at the start of the period

3.3.1.2 Maximum leverage

The key parameter is L, the maximum leverage allowed in the VaR constraint, which we loosely see as an indicator of "how constrained" carry capital is. Unfortunately there is surprisingly little data on carry trader leverage. We thus resort to discussion.

On one hand the FX market appears to be hosting highly leveraged investors. Anecdotal evidence, such as press articles or discussions with traders, points to leverage being a major concern amongst practitioners. Some dealers offer leverage factors of up to 1:50. Academic research also suggests that some of the features of the FX market can only be explained through "non negligible" levels of leverage for carry traders (Darvas, 2009).

On the other hand, the FX market is dominated by bank and near-banks institution, who likely are very exposed to the capital constraints from the Basel agreements. Adrian and Shin (2010) for instance mention that the 1996 Market Risk Amendment of the Basel capital accord limits the regulatory capital is 3 times the 10 day, 99% Value-at-Risk for investment banks. We expect such a constraint at the bank level should extend to its FX trading desk. Even without considering Basel, VaR are often used internally for risk management, and it seems unlikely that a major institutional investor would allow a given trading desk to take on too much risk through leverage. Another limiting factor is that we use a wide definition of carry trading in this chapter, which includes portfolio investors who buy assets denominated in the foreign currency. These investors are likely to have a leverage factor much below 1:50.

Thus we chose to present results for a leverage factor of *1* to *10*, in order to study the impact of L across all values that we believe represent a possible value for the average carry trade position.

3.3.1.3 Model-driven parameters

3 parameters remain to be defined: the proportion of carry traders initially constrained φ, and the number of fundamentalists and momentum traders n_F and n_M.

These parameters indirectly indicate the weight of each trader in total turnover. In our set-up, if this weight is too large for a given investor, the price dynamics become driven by this trader only. If carry traders have too large a capital available they will kick up the ER instantly following a rise in the interest spread. A model in which momentum traders dominate will simply produces large price spikes and drops around F, as chartists rally and exit in response to minor price events. Finally if we allow fundamentalists to be dominant, prices immediately converge towards F, or the system will diverge as discussed in section 3.2.4.

The sensitivity of the model to the weighs attributed to each agent is an unfortunate consequence of our use of a representative agent model (Kirman, 1992). When one trader dominates 1) the intuition that the interaction between agents play a part in skewness cannot be investigated 2) the two other traders typically lose money, which makes the model unrealistic. We thus chose to focus only the interesting cases in which all agents interact, restricting the parameter set to values in which all 3 agents make profits. In the following baseline simulation we set $n_F = 0.01$,$n_M = 0.05$, and $\varphi = 0.8$, values for which the model produces representative dynamics of the parameter sets in which all 3 agents coexists.

The reader may find the list of parameter values in appendix 3.3.

3.3.2 Deterministic dynamics

To better understand the endogenous dynamics generated by the model, we start by presenting the deterministic model, i.e. the dynamics that emerge in the complete absence of exogenous shocks besides the initial interest rate rise to 6%. Formally we set $\epsilon_{b,t}$ and $\epsilon_{n,t}$, the shocks on the return of the foreign bond and the demand by noise traders, to zero.

3.3.2.1 Short-term dynamics

To ease the presentation, we chose to start with the study short-term dynamics. Figure 3.3.1 plots the dynamics that emerge in the most revealing case *L=4*, but the dynamics for the other values for *L* will be presented in the following section.

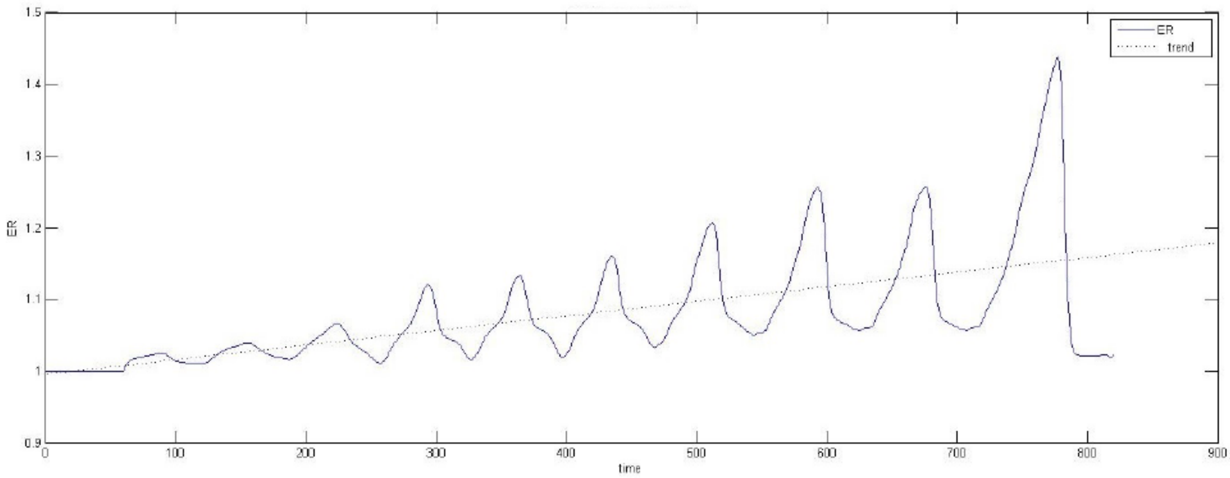

Figure 3.3.1 – Short-term endogenous cycles of the ER with C=4

We see that the exchange rate seems to be rising along a long-term trend of appreciation (dotted line) but faces increasingly large downward shifts. The importance of the currency crash, both in terms of total fall and rapidity, appears to be an increasing function of the magnitude and length of the build-up phase that precedes it.

These cycles are indicative of the dynamic at play in the model: in response to the rise in interest rate carry traders move capital towards the foreign country. However this capital is constrained, so that the ER adjusts only gradually, creating a trend of appreciation. Momentum traders spot this trend and seek to benefit from it by also going long on the currency, adding to the appreciation.

As the ER appreciates two countervailing forces start gaining strength. First a larger share of carry traders become unconstrained, second the fundamental mean reversion component becomes larger. Both influences eventually result in a weakening of the trend, which leads momentum traders to cut their exposure to the foreign currency, and fundamentalists to start taking positions against it. The combined trading of both leads the exchange rate to fall, incurring losses to the leveraged carry traders. In reaction, carry traders will be forced to reduce the foreign exposure to satisfy their funding constraint, causing the crash.

In the early stages, both chartists and carry traders have fairly small positions in the FCU. Consequently the exit by chartist leads to a moderate fall, and this fall is little magnified. However as the positions grow larger carry traders react with increasing strength to increasingly adverse shocks.

To get the full glimpse of the role of each agent in the unfolding of the crash, figure 3.3.2 plots the ER around the last and largest crash, along with the quantities demand by all agents, and the proportion of constrained carry traders in the economy.

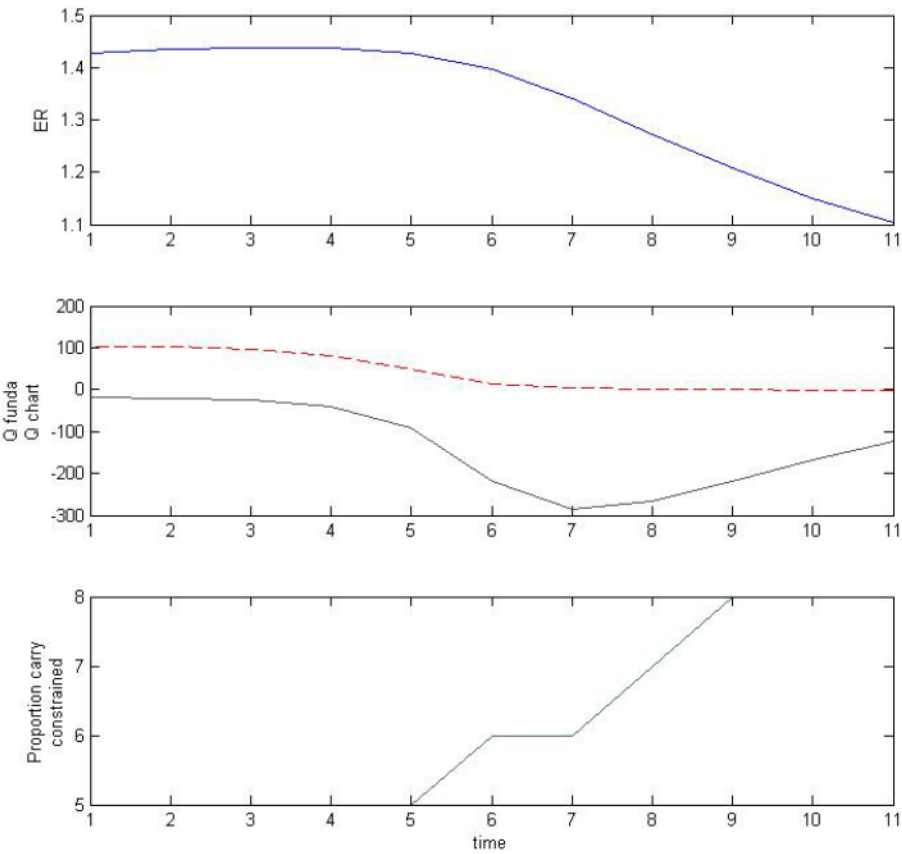

Figure 3.3.2 – Actions of market participants around the crash

The timing appears clearly: just before the crash chartists start reducing their exposures, while fundamentalists take increase their short selling. These changes are relatively smooth until *t=4*. Nevertheless it is enough to further slow the trend down, leading to a more significant selling at *t=4*, notably by fundamentalists who are starting to forge the belief that "gravity" is about to prevail. This selling leads to a notable price drop between *t=4* and *t=5*.

This is when margin calls start kicking in, which can be seen through the evolution of the share of constrained carry traders. At *t=5*, 50% of the carry traders are constrained, i.e. have holdings that are below their desired ones. At *t=5* these investors must liquidate positions to comply with their constraint, which along with further selling by chartists and fundamentalists leads to another sharp fall in the ER between *t=5* and *t=6*. The fall wipes out carry capital, so that new investors become constrained at *t=6*. A vicious cycle then begins, in which carry traders are forced to close down positions, which leads to further losses, and so on. The proportion of carry traders hit by margin calls steadily rises to 80%.

Thus crashes can occur endogenously without a fundamental trigger, through a perverse interaction between active trend chasing strategies and highly leveraged investors.

3.3.2.2 Long-term dynamics

We now present all the cases, with a longer term perspective. Figure 3.3.3 plots the different dynamics that emerge in the 10 years following our initial rise in interest rate.

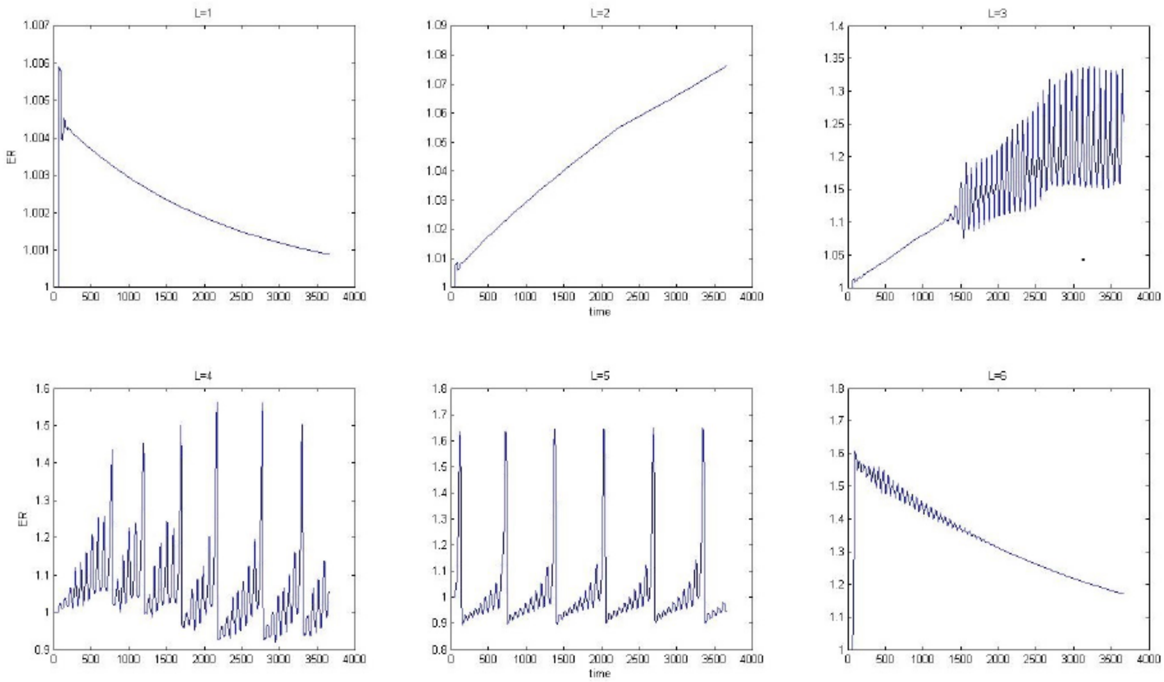

Figure 3.3.3 – Dynamics for varying levels of funding constraint

We see that funding constraints have a double impact: one lies with the emergence of shorter run cycles and the other with the overall long term behavior of the ER. We discuss each level separately to outline the non-linear impact of funding liquidity.

L=1. The ER initially kicks up before depreciating, but this initial rise is very modest. The reason is that in this case capital is very constrained, so that 1) carry traders are limited in their ability to move capital immediately and 2) in their capacity to finance new positions. Thus carry traders are immediately dominated by fundamentals and fundamentalists, no upward trend is implemented.

L=2. As carry capital is less constrained, carry traders take increasingly large positions, driving the ER upwards. Nevertheless this trend of appreciation is very gentle: carry traders remain strongly constrained so that they do not have the capacity to move capital as quickly as they wish to. The trend carries on in similar fashion until around *t=10 000,* not plotted here, at which point the ER starts converging back [10] to its fundamental value of $F = 1$.

L=3 also yields an upward trend, more pronounced because more leveraged carry traders are able to demand more FCU more quickly. The pattern of shorter-term cycles of appreciation/ depreciation start emerging. The reason is that the more vigorous appreciation has lead chartists to take larger long positions, which they then exit. Carry traders are also getting more leveraged so that price drops start translating into larger wealth effects. The skewness of $\triangle S_t$ turns negative, at -2.16. However in this case the short-term ER drops are not large enough to hamper the long-term appreciation trend.

L=4 gives the clearest "up the stairs down the elevator" pattern, discussed in the previous section. Most importantly the currency crashes are now sufficiently large to prevent carry traders from implementing a true sustainable trend of appreciation. In this way leverage has a counter counter-intuitive effect: even though capital moves more freely, it does not lead to a quicker price adjustment. Skewness falls to -5.07.

L=5 produces similar dynamics, only with even steeper rises in the price of the currency as capital flows more easily. Skewness in this case is -4.19.

Finally in L=6 the funding constraint has enough slack to allow carry traders to move to their desired holdings nearly instantly, leading to a large and immediate

10. Of course this long period of appreciation results from the fact that we keep all parameters constant and set all shocks to 0 here. In practice over such a long period there should be a change in fundamentals or in the interest rate which would change the dynamics, before the ER has adjusted.

appreciation. After this, fundamentals prevail and a slow depreciation sets in. Currency crashes disappear because carry traders are close to unconstrained, so that they have enough capital to deal with moderate drops in the exchange rate.

This case produces the dynamics that are expected in a perfectly efficient market. Indeed two conditions for efficiency coexist: 1) the demand for an asset immediately rises in response to an increase in its return 2) the subsequent evolution of the exchange rate reflects fundamentals.

Levels L=7 to L=10, not plotted, yields dynamics that are very similar to L=6 , as expected since capital is even less constrained for such levels.

Thus funding constraints have a non-linear impact on the likelihood of a currency crash. Low levels of allowed leverage constitute a guarantee against crashes as they limit the wealth effects faced by agents, but these levels have the undesirable feature of restraining the ability of capital to move quickly, which may have costly impacts on the real economy. High levels of leverage are the first best allocation in theory, as they ensure both liquid markets and a stronger resilience to shocks. Finally intermediate levels of leverage may be quite dangerous from “tail-risk” perspective, because they allow traders to take large positions while exposing them strongly to margin calls.

3.3.3 Stochastic dynamics and the empirical performance of the model

We take a look at the empirical performance of the model. To do so we release the constraint that $\epsilon_{b,t}$ and $\epsilon_{n,t}$, are zero in order to study its stochastic properties. Both shocks are modeled as idd normally distributed shocks, of variances of $1.85 \times$

10^{-5} and 2.5×10^{-5} respectively. These values were chosen because they induce a level of volatility consistent with the data, nevertheless the sensibility of this performance to the choice of the variances is very moderate. We employ a Monte Carlo method, simulating the model as specified above 10 000 times to draw the asymptotic properties of $\triangle S_t$.

Table 1 compares the model-driven moments of the daily exchange rate movement to those of 2 well known carry-trade currency couples, AUD/JPY and NZD/JPY. The mean is not included as it is zero by construction in our set-up, since in the long term the evolution of the exchange rate is given by the fundamental value F which is set constant and equal to S_0, the exchange rate at the start of the period. Out of the same desire to control for fundamentals, we report the moments for AUD/JPY and NZD/JPY over the period 2000-2007 period. During this period interest rate differential Australia/New Zealand and Japan staid fairly constant around 6% [11], and no major exogenous shock hit the FX market.

Series	variance	Skewness	Kurtosis
$L = 4$	6.3×10^{-5}	-0.32	6.06
$L = 4.5$	7.4×10^{-5}	-0.84	12.32
AUD/JPY	5.7×10^{-5}	-0.81	10.92
NZD/JPY	7.5×10^{-5}	-0.97	12.33

Our simulations match the 2 carry couples in terms of daily variance, skewness and kurtosis. *L=4*, the most interesting case in the previous section, yields estimates which are qualitatively satisfying, though a bit too small in magnitude. Specifying a slightly less binding funding constraint, *L=4.5*, gives the model a good

11. 5.38% differential with Australia and 6.53% with New Zealand, 5.96% averaged over both countries. Annualized short rates obtained from the OECD.

fit. The left-hand side diagram in figure 3.3.4 plots a representative case. Several phases of build-up emerge, with three episodes apparently materializing into currency crashes. For comparison purposes we also provide in the right-hand side a plot of the detrended AUD/JPY exchange rate, which confirms that comparable dynamics emerge.

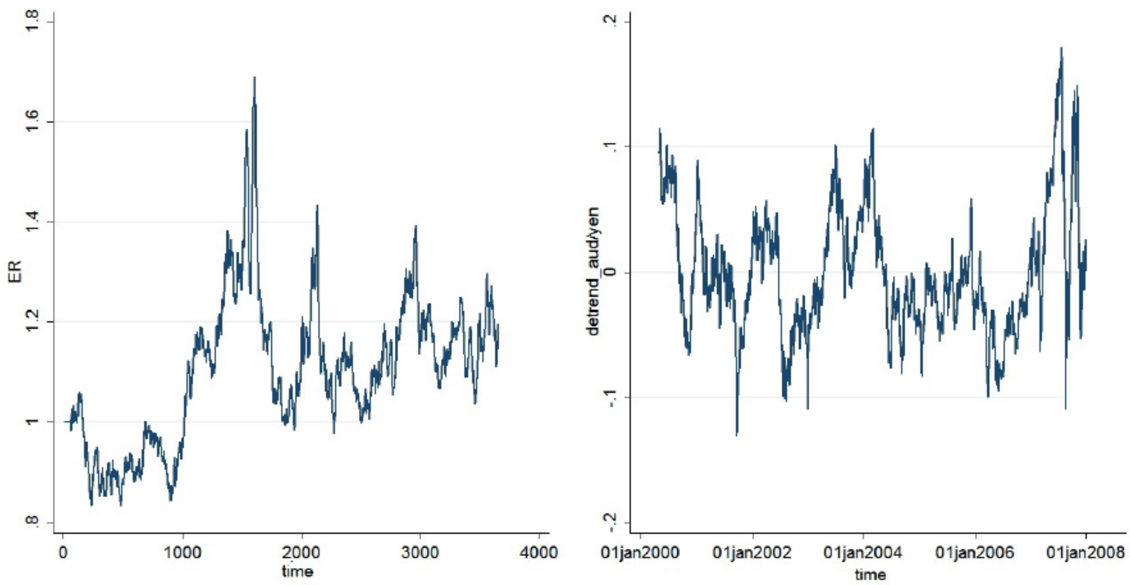

Figure 3.3.4 – Simulation of the stochastic model with C=4, compared to detrended ER AUD/YEN

We conclude that a model which endogenously generates booms and busts does a good job at matching exchange rate daily data. However two points should be noted:

- First funding issues may not be the sole explanation for "up the stairs down the elevator" pattern. Other factors may add to this dynamic, notably pro-cyclical risk aversion. Indeed carry traders may feel growingly confident about investing abroad as more and more investor do it and the benefits accumulate, but this confidence quickly evaporates as the first losses appear.

- Second, at the weekly or monthly frequency, the simulations sometimes yield deviations from fundamentals which appear slightly too large and rapid. This is likely due to our modeling of the VaR constraint and chartists, which entails a direct positive interaction between capital gains at *t-1* and quantities demanded at t during the boom phase. In reality some autocorrelation must be present in the exchange rate, otherwise momentum trading would not be profitable, but the relationship between exchange rate changes and their lagged values is likely to be much more complex and hard to gather in a single model.

3.4 Digging deeper

3.4.1 Profits

We first consider profits from a robustness check perspective, as losses would imply that we have specified unrealistic investment rules for our traders. We plot in appendix 3.4 the accumulated profits for each trading sector and all values of L. All investment rules make money over the long-run, and in the vast majority of the cases it takes little time for them to become profitable.

Zooming in on our preferred case L=*4*, we find that the profits generated by our model contain interesting patterns. We discuss how these fit with empirical findings of the existing literature on profits in the FX market. Unfortunately there is very little research on the performance of fundamentalists, likely due to the fact that designing a fundamentalist strategy requires an estimation of the fundamental exchange rate F, which will always be subject to controversy. We thus chose not to include them in the discussion. Figure 3.4.1 presents the total accumulated profits for both sectors. As an indicator of scale these accumulated earnings may be compared to the desired holdings of the carry trades $q^* \in [1, 10]$.

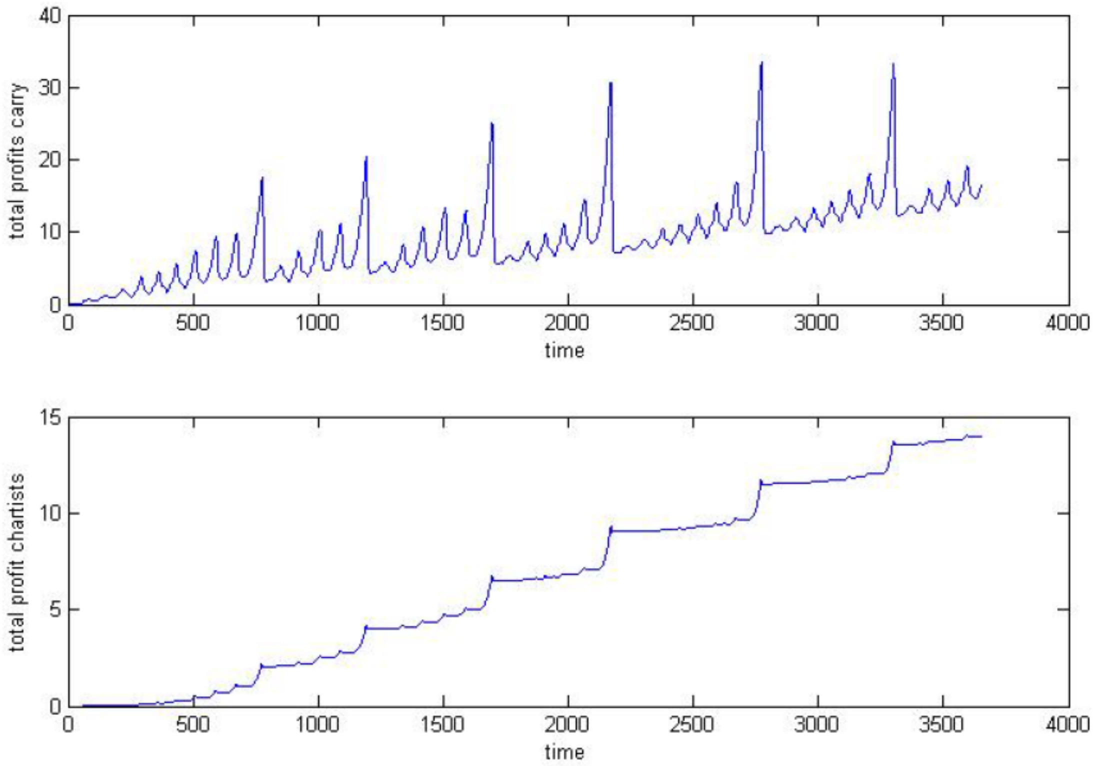

Figure 3.4.1 – C=4, Carry and momentum profits

The level of profits per se cannot directly be compared, because in the model the wealth invested to get these profits varies hugely across agents and periods. We can only say that both strategies are profitable. A more interesting discussion lies with the variance of these profits and their resilience to shocks. This variance is much higher for carry trading. In particular the total profit of carry traders appear strongly negatively skewed, reflecting the fact that a) the fact that carry trader are always long the negatively skewed high yield currency b) drops in the exchange lead to more than proportional drops in wealth because carry traders are leveraged.

This negative skewness has deep implications for the issue of the excess returns of carry trading. Indeed if carry traders face an important crash risk, the return they earn may no longer be seen as abnormal, but rather as legitimate compensation. A large literature has investigated whether crash risk is sufficient to explain carry returns, with mixed result (Burnside et al., 2008). However as noted by Darvas (2009) , these papers usually studied an unleveraged carry trade strategy, a crucial omission since leverage has a multiplicative effect on tail risk. Accounting for it, Darvas (2009) finds that skewness appears sufficient to justify carry return. Our model thus supports this view.

The profits from chartism on the other hand have a much lower variance, and no apparent crash risk. This feature appears in line with the literature on the performance of momentum trading in the FX market, both in terms of performance and strong resiliency to crashes (e.g Burnside et al., 2008). In our set-up this comes from the fact that chartists move from long to short positions before the lion's share of the adjustment occurs. Thus chartism and carry trading are negatively correlated during crashes, which is consistent with the empirical findings of Menkhoff et al. (2012). In this way our set-up may help providing an explanation to the abnormal profits from carry trading, but not to those from momentum trading.

The same patterns arise in the stochastic $L=4$ version of the model. All three rules are profitable on average. In order to compare our results to the existing literature, we study the performance of naive momentum and carry trading strategies per annum. The reader may look at appendix 3.5 for a description of the method used.

We obtain returns that are somewhat too high compared to the empirical literature for both agents, notably for momentum traders who earn a staggering *30.91* % p.a.. This is likely due to our strong auto-correlation in daily returns, which makes momentum very profitable. Carry traders also earn large returns, but these are largely offset by the important risk they face. With *L=4* we obtain a Sharpe ratio of *0.78* for momentum traders and *0.35* for carry traders. These values are consistent with the literature on momentum profits (e.g. Menkhoff et al., 2012), but noticeably lower than the *0.54* found by Burnside et al. (2008) for carry trading over single currency pairs, which adds to the impression that accounting for leverage reduces the puzzle of carry trade returns.

The ability of the model to replicate stylized fact about the profits of chartist and carry trader may also be seen as encouraging from a robustness perspective.

3.4.2 Active/passive chartists and fundamentalists

We investigate how the dynamics of the model respond to more or less active trading by chartists and fundamentalists. Activeness is measured through the number x of periods both investors go back to when forecasting future prices. A lower x means larger daily changes in holdings by both agents, as shorter-term price dynamics are more volatile. A higher x should have the opposite effect, as a strategy based on longer price dynamics should be more stable.

Figure 3.4.2 presents the ER dynamics specifying *x=10* and *x=60* for both agents. We include the baseline scenario *x=30* for comparison purposes.

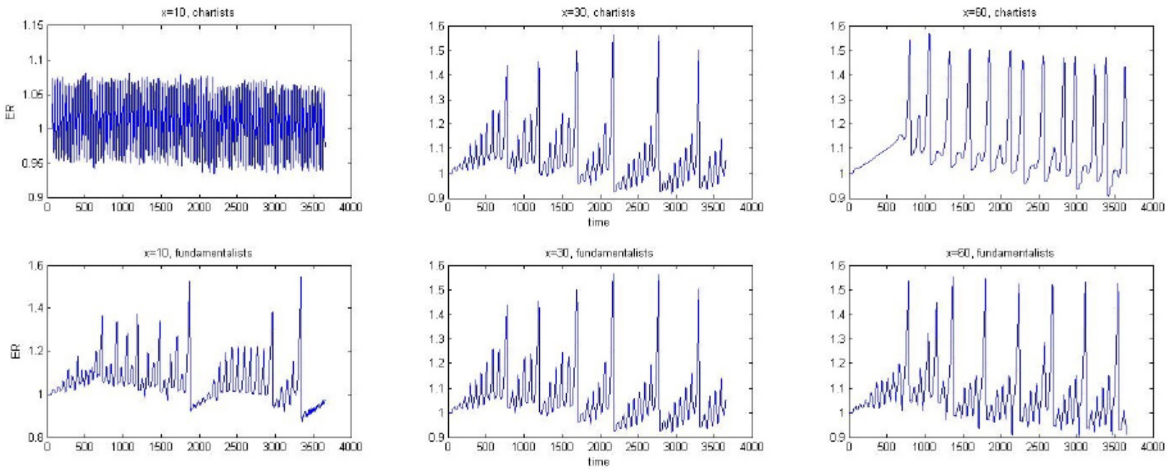

Figure 3.4.2 – C=4 dynamics with varying investment horizons by chartists and fundamentalists

First we note that only one case involves a real change in the pattern of booms and busts: more active momentum traders. The reason is that when chartists are more active, they are quicker to exit their long positions, which makes a consistent trend of appreciation difficult to implement. This option thus seems desirable from a tail risk perspective. Yet it also prevents the carry traders from kicking up the exchange rate, which should happen in an efficient market. The distribution of $\triangle S$ in this case is nearly symmetric, skewness is *0.02*.

In all the other cases the overall dynamics do not change drastically. Allowing fundamentalists to be more active in their chase for mean reversion simply reduces the magnitude of the appreciation phase, which lowers the size of the currency crashes. On the other hand more active fundamentalists also means that daily negative demand shocks can be larger. Overall both effects seem to cancel each other out, skewness is *-5.17,* comparable to the x*=30* case.

When agents have a longer perspective, i.e x*=60*, the same two effects are at play in the opposite direction. On one hand the build-up phases are larger in magnitude as the positions of chartists (fundamentalists) are consistently more long (less short). On the other hand a more passive strategy also means less drastic reactions to adverse price movements. For both cases this latter effect seems to dominate. Skewness is *-3.72* and *-3.95* for more passive chartists and fundamentalists respectively, which represents a notable improvement compared to the value of *-5.07* in the x*=30* case.

Thus the activeness of chartists has a non-linear impact on skewness, similar to funding constraints. Intermediate levels of "activeness" are sufficiently stable to create long build-up phases, and sufficiently volatile to trigger large daily drops in demand. For fundamentalists the impact of more active trading is not as clear for low values of x, but it seems past a certain point a more passive approach also reduces crash risk.

Table 2 presents the realized skewness from Monte Carlo simulations in the L*=4* case, for our different levels of x.

skewness	x=10	x=30	x=60
chartists	-0.27	-0.32	-0.29
fundamentalists	-0.58	-0.32	-0.22

This table confirms that more passive trading by both agents lowers currency risk, while the impact for more active trading differs for both agents. For chartists moving to x*=10* brings a moderate fall in realized skewness. However for fundamentalists it seems more active trading fosters crash risk in the stochastic version.

3.4.3 Can endogenous booms and busts explain the exchange rate disconnect?

We investigate whether the model is capable of reproducing some empirical features of disconnect between exchange rate and fundamentals. Long run convergence with fundamentals is a built-in feature in our set-up, so that finding a significant relationship over a sufficiently long horizon is natural. A more interesting question lies with whether the patterns of this convergence are in line with the empirical findings on the matter for our set of parameters.

To investigate we follow a classic paper by Mark (1995), who studies the predictive power of a set of fundamental variables on the exchange rate at varying horizons. In a nutshell, this author runs two regressions of the exchange rates for 4 developed countries against the US dollar (Canada, Germany, Japan and Switzerland). The first regression is based on a simple random walk model, the second is based on a set of fundamental variables. Mark then uses the regression results to draw the predicted out-of-sample exchange rate for both models, at horizons of 1,4,8,12, and 16 quarters. He then compares the performance of the random walk model and the fundamentals-based one, by comparing their root-mean-square error (RMSE), i.e. the sum of the squared differences between predicted values and actual ones.

We run a similar analysis. We first run a Monte-Carlo experiment which generates 10 000 random walks which be our vector of fundamental values, where each vector has a length of 10 years. We generate the corresponding 10 000 exchange rates vectors over 10 years. We then split the sample in half. In the first half we estimate a regression of the ER over the fundamental values. In the second we use the regression results to draw predicted values at the same horizons as Mark. Similarly we draw the predicted values from a random walk model. We then compare both RMSEs, by taking the ratio of the RMSE of the fundamentals-based regression over that of a simple random walk,

Figure 3.4.3 plots this ratio for our model, compared to that obtained by Mark (1995). We show his results for all the countries in his sample, and those excluding Canada who appears to be an outlier in his paper. Note that a high RMSE indicates a poor forecasting performance, so that a ratio above one, for instance, indicates that the fundamentals-based model is a worse predictor of future exchange rates than a random walk.

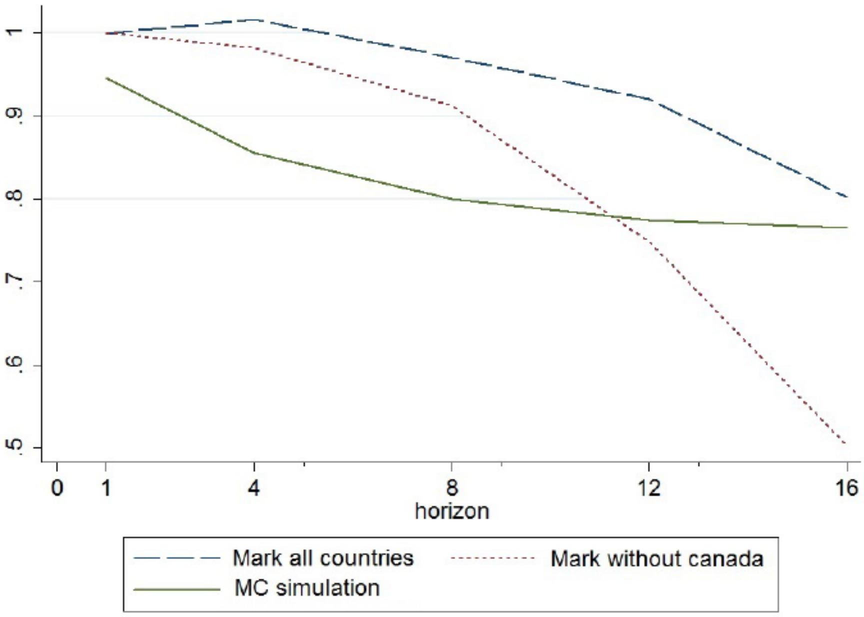

Figure 3.4.3 – C=4 dynamics with varying investment horizons by chartists and fundamentalists

We see that the model generates a forecasting power for fundamentals that is comparable to the empirical study of Mark (1995). In terms of levels first, the performance of the fundamentals-based regression dominates that of the random walk by a similar order of magnitude. In terms of evolution, we see that this dominance increases as the horizon widens in both cases, i.e. fundamentals have a better forecasting power as the horizon for prediction rises.

Nevertheless a difference is that the largest gain in terms of explanatory power occurs earlier in our set-up. In other words in Mark (1995) the main increase in the performance of fundamentals occurs when we move from a 12 to a 16 quarters horizon. For the exchange rate generated by our model on the other hand, the predictability increases more when we move from 1 to 4 quarters.

Overall in our opinion the model generates deviations from fundamentals that seem comparable, even though not perfectly so, to what we observe empirically.

With respect to the pattern of this convergence, our model is in line with the work of Taylor and Peel (2000) on non-linear adjustment towards fundamentals. These authors show empirically that the speed at which exchange rates converge to fundamentals is an increasing function of the size of the deviation. This feature is present in the ER generated by the model, since the exchange rate crashes when deviations from fundamentals become too important.

3.5 Conclusion

Previous research has highlighted that heterogeneous FX investors may be a vector of excess volatility in exchange rates, and that constrained carry traders play a part in currency crashes. The main lesson of this chapter is that both variables should be considered together, as their interaction is a potential source of currency risk.

The model provides an explanation for the appearance of seemingly inexplicable falls in exchange rates. The dynamics produced are also consistent with other empirical features of exchange rates such as excess volatility and fat tails. Finally the model helps explaining puzzles such as the profits to carry trading and the exchange rate disconnect.

The emergence of large capital inflows which are quick to evaporate has been a major policy concern, notably for emerging economies who have suffered from "sudden stops" since adopting floating currencies. Our model suggests that such stops are at least partly of endogenous nature, which may provide an extra rationale for the implementation of capital controls, as it implies even sound macroeconomic may not be sufficient to prevent crashes.

A deeper understanding of microstructure effects in large developed countries may nonetheless allow for finer forms of regulation. For instance in our model endogenous crashes occur within a certain context: high interest rate differential and/or easy access to funding in the Home country. A small interest rate differential or a tight liquidity in the developed world should thus act as natural obstacles to the arrival of speculative capital in emerging countries. These countries may then consider hardening/ loosening capital controls on all assets according to these two variables, rather than applying controls to certain assets or industries which are deemed "speculative".

Yet it appears a first and crucial step for an efficient regulation would be to increase the availability of data on the positions taken at the trader/trading desk level, so that researchers may have a more precise idea of the impact of market microstructure on currency risk.

General conclusion

General conclusion

The exponential development of global financial markets is one of the most stunning evolutions of the modern world economy. The share of the financial industry in US GDP moved from around 4% in the late 1970s to 8.3% in 2006 (Philippon, 2008). Though no global data is available on the evolution of the total amount of asset under management, all evidence suggests it has been even more dramatic. Derivative trading moved from a negligible market in 1970 to a 630 trillion dollars one in December 2014 in notional value (Bank for International Settlements, 2015). Daily turnover in the FX market more than tripled between 1998 and 2013, moving from 1.53 to 5.35 trillions of dollars (Bank for International Settlements, 2013).

Such an emergence has no precedent in history. Though it has undeniable benefits, it also brings important risks. The broad goal of this thesis has been to contribute to a better understanding of those risks.

To do so, we used a method based on splitting asset price movements into two components, which are not independent but yet different in nature. On one hand, fundamental movement can be defined as the change in an asset price that results from the evolution of underlying features of the asset. The uncertainty surrounding these underlying features brings fundamental risk, which is inherent to any investment and has always existed under its current form.

On the other hand, endogenous movements can be seen as the price changes that comes from the organization of the markets as they currently stand. The endogenous approach holds that a given fundamental shock may have very different impact on asset prices, depending on the features of the financial network, the rules market participants follow, the capital they hold, etc. The structure of the markets thus embeds different amplification mechanisms, whose impact may go well beyond that of the initial shock.

This thesis has been particularly interested in this endogenous component. In Chapter 2 we highlighted the presence of non-fundamental movement empirically, by showing the existence of label effects in the bond market. In chapter 1 and 3 we provided theoretical models in which risk carrying endogenous movement emerged through the actions of market participants. In the former risk came from the fact that portfolio diversification creates links between assets and investors which had potential to transmit adverse shocks. In the latter, it came from the possibility that the interactions between FX investors lead to currency "bubbles".

Our concern for endogenous amplification mechanisms has become shared by academics and regulators alike since the 2008 subprime crisis, in which non-fundamental factors appeared to have played a large part. Before the crisis banking regulation was based primarily on requiring banks to keep a certain proportion of their risk weighted assets in capital. The Basel III agreement marked a noteworthy turn towards a more market-wide approach, which accounts not only for risk at the entity level, but also for the risk arising from the interactions between these entities.

Key innovations from our perspective notably include:

- An extra capital charge of 2.5% of risk weighted asset for systemically important financial institutions.

- The appearance of two liquidity ratios designed to compel banks to hold enough cash to face a liquidity drought.

- The implementation of a counter cyclical capital buffer, which requires bank to put extra capital aside during boom phases.

Theoretically such measures should help managing some of the sources of endogenous risk that we have put forward in the thesis. The extra charge for systemic banks for instance should minimize the potential for propagation of risk throughout the financial network, highlighted in chapter 1. Counter-cyclical buffers should limit the likelihood of capital fueled booms such as those described in chapter 3. Liquidity buffers should help managing movements of mass selling following adverse shocks, which we have modeled in both chapters. Thus from the perspective of this thesis such rules go in the right direction.

However it seems the most difficult task for regulators is not implementing the right rule, but rather implementing it at the right time. Indeed historically each one of the Basel agreements came following a crisis [12], very few changes have been applied during quiet times. Going forward a key challenge for regulatory instances is thus to better their monitoring of the markets in order to spot situations in which endogenous risk is building up, and act preemptively.

From this perspective, it seems an important first step for regulatory instances is to pay particular attention to the impact that their own actions may have on the behavior of market participants. New rules may have a distortive effect by

12. Basel I came as response to the bankruptcy of the German bank "Herstatt" in 1974. Basel II followed the Asian financial crisis of 1998, while Basel III echoes the 2008 Subprime crisis.

making certain assets more attractive even though their fundamental values are unchanged. Securitization for instance appeared largely as a mean for banks to bypass the rules from Basel II. It allowed banks to take loans out of their balance sheet, lowering their capital requirements. This incentive likely participated to banks overlooking the fundamental characteristics of securitized assets.

Basel III could yield similar endogenous responses. Banks could notably be tempted to outsource a growing share of their activities to offshore centers. Assets defined as liquid under the new agreements will acquire a new role in the financial network. Regulators and academic research should then place these evolutions under particular scrutinity, a surveillance to which we hope to contribute in the future.

Annexes

Appendices chapter 1

Appendix 1.1

We study the impact of trying alternative values for parameter σ_F and K. Figure A.1.1 shows the impact of rising the daily variance by 25% and lowering the capital threshold for default K by 25% also, which yieds $\sigma_F = 0.0310$ and K=*6%* . We represent the intermediate linear case r/h=*0.75* and the easy panic one.

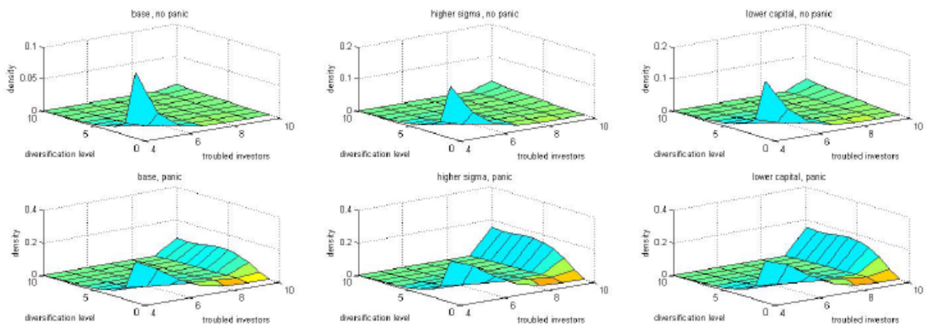

Figure A.1.1: Lower K and higher σ_F, both panic and non panic case.

First we note that these different values do not change the general picture on the link between diversification and systemic risk. Both actions have a comparable impact on systemic risk as they rise the marginal risk attached to each investor. With or without panic, all significant levels of failure are more likely, but the strongest relative increase is on mass failure. The relative rise compared to the baseline scenario falls as the transmissibility r/h rises, i.e. the rise is larger for low r/h. This is logical because both a lower K and a higher σ_F make bankruptcy more likely ceteris paribus, but this increase in risk becomes increasingly marginal as r/h rises.

Appendix 1.2

Drawing the general element of Σ_t, Σ_{tot}, and Σ_I involves diagonalizing matrices $(A^\intercal A)^t$, $\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t$, and $\frac{1}{n}A\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t$. Our method is to work our way up from a simple circulant matrix Z. This appendix gives the outline of the proof, while appendices 1.3, 1.4, and 1.5 provides details on certain steps.

From the cyclic permutation matrix Z to A

Z is the "cyclic permutation matrix", whose element $z_{i,j} = 1$ if $i \equiv j - 1$ [N], $z_{i,j} = 0$ otherwise. Taking Z to the power n shifts the one-diagonal *n-1* spots to the right. For instance if *N=5*:

$$Z = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad Z^2 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

The eigenvalues of Z are readily obtained from the observation that $Z^N = 1^N$, where 1^N is the identity matrix. Using the Cayley-Hamilton theorem, the characteristic polynomial of Z is of the form $Q[x] = x^n - 1$, and its eigenvalues are the nth-roots of unity. Thus the matrix similar to Z is $D_z = diag(\omega^0, \omega, ..., \omega^{N-1})$ where $\omega^k = e^{\frac{2i\pi k}{N}} = cos(\frac{2\pi k}{N}) + i\,sin(\frac{2\pi k}{N})$ and $\omega^{-k} = cos(\frac{2\pi k}{N}) - i\,sin(\frac{2\pi k}{N})$ according to Euler's identity. Note also that since $\omega^N = 1$, we have $\omega^{\alpha(N-k)} = \omega^{\alpha N - \alpha k} = \omega^{-\alpha k}$.

From the properties of Z, it appears that any circulant matrix C may be expressed as a polynomial in $Z : C = R(Z)$. This implies $C = R(PD_Z P^{-1}) = PR(D_Z)P^{-1}$, where P is the change of basis matrix. Therefore:

- ψ_k, the kth eigenvalue of any circulant matrix C is a polynomial in ω^k, the kth eigenvalue of Z.

- P is the change of basis matrix for all circulant matrices.

In particular for A:

$$A = R(Z) = P(\sum_{s=0}^{s=n-1} D_Z^s)P^{-1} \qquad (a.1.1)$$

where n is the level of diversification. The matrix similar to A is thus given by $D_A = \sum_{s=0}^{s=n-1} D_Z^s$. Its general element is the inverse Fourier transform $\psi_k = R(\omega_k) = \sum_{s=0}^{s=n-1} \omega^{ks}$. ψ_k is the sum a geometric series of n terms and common ratio $e^{\frac{2i\pi k}{N}}$, so that :

$$\psi_k = \frac{1-\omega^{kn}}{1-\omega^k} = \frac{1 - cos(\frac{2\pi kn}{N}) - i\,sin(\frac{2\pi kn}{N})}{1 - cos(\frac{2\pi k}{N}) - i\,sin(\frac{2\pi k}{N})}$$

except when k=*0*, in which case $\psi_0 = n$. Note that this expression also implies $\psi_{-k} = \psi_{N-k}$.

From A to $(A^\intercal A)^t$

To move from matrix A to $A^\intercal A$, we use some of the numerous useful properties of the change of basis matrix[13] P :

1) P is unitary: i.e., $P^{-1} = P^\dagger$, where $P^\dagger$ is P's conjugate transpose of general term $\frac{\omega^{-jk}}{\sqrt{N}}$

2) $P = P^\intercal$ and $P^{-1} = (P^{-1})^\intercal$, so that $A^\intercal = P^{-1} D_A P$

3) $P^2 = P^{-2} = V$ where V is a near reversal matrix[14].

We may then write $A^\intercal A = P^{-1} D_A P P D_A P^{-1} = P^{-1} D_A V D_A P^{-1}$

Pre-multiplying by PP^{-1} we get the desired result:

$$A^\intercal A = P(VD_A)^2 P^{-1} \qquad (a.1.2)$$

which implies $(A^\intercal A)^t = P(VD_A)^{2t} P^{-1}$. The matrix similar to $(A^\intercal A)^t$ is thus $(VD_A)^{2t}$. The covariance between assets price changes at a given period is then $\Sigma_t = E([\triangle\mathbf{P_t} - E(\triangle\mathbf{P_t})][\triangle\mathbf{P_t} - E(\triangle\mathbf{P_t})]^\intercal) = \sigma_F^2(\frac{r/h}{n^2})^{2t} P(VD_A)^{4t} P^{-1}$, where we used the fact that$E(\triangle\mathbf{e_1^F} \triangle\mathbf{e_1^{F\intercal}}) = diag(\sigma_F^2)$ and that $(A^\intercal A)^t$ is symmetric, so that $(P(VD_A)^{2t} P^{-1})^\intercal = P(VD_A)^{2t} P^{-1}$.

The expression of matrix V implies that $(VD_A)^2$'s element on row $k+1 > 1$ may be expressed[15] as $\phi_k = \phi_{N-k} = \psi_k \psi_{-k} = \frac{1-\omega^{kn}}{1-\omega^k} \times \frac{1-\omega^{-kn}}{1-\omega^{-k}}$, which will be positive for any k. Using (a.1.1) and rearranging:

13. We derive P in appendix 1.3
14. An example of matrix V when *N=4* is provided in appendix 1.4
15. see appendix 1.4 for a derivation through an *N=4* example

$$\phi_k = \phi_{N-k} = \frac{1-cos(\frac{2\pi kn}{N})}{1-cos(\frac{2\pi k}{N})}$$

and $\phi_0 = n^2$ if *k=0*. Note that this implies $\phi_0 = n^2$ is the maximum eigenvalue of $A^\intercal A$, we thus verify the convergence condition $r < h$ given in section 3.

Summary of the diagonalization of $A^\intercal A$:

- Matrix Z's eigenvalues are the roots of unity

- Matrix A is a polynomial of Z, so its eigenvalues are a polynomial of Z's eigenvalues

- All circulant matrices have the change of basis matrix P where $PP = V$, *V a near reversal matrix*

- This implies $A^\intercal A = P^{-1} D_A V D_A P^{-1} \implies A^\intercal A = P(VD_A)^2 P^{-1}$

From $A^\intercal A$ to $\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t$

$$\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t = P(\sum_{t=0}^{t=+\infty}(\frac{r/h}{n^2})^t (VD_A)^{2t})P^{-1} \qquad (a.1.3)$$

$\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})^t (VD_A)^{2t}]$, which we note D_{tot}, is then the matrix similar to $\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t$. The vector of total price movement after the shock is thus $\sum_{t=0}^{t=+\infty} \triangle \mathbf{P_t} = PD_{tot}P^{-1}\mathbf{e_1^F}$, and its covariance matrix is $\Sigma_{tot} = \sigma_F^2 P(D_{tot})^2 P^{-1}$, using the fact that $\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t$ is a linear function of symmetric matrices and is thus symmetric itself.

The general element of D_{tot} is thus $\xi_k = \sum_{t=0}^{t=+\infty}(\frac{r/h}{n^2}\phi_k)^t$, i.e., the sum of a geometric series of common ratio $(\frac{r/h}{n^2}\phi_k)$. As we consider total price shifts from *t=0* to $t = +\infty$, the formula for the sum of of the terms of geometric series yields:

$$\xi_k = \frac{1}{1-(\frac{r/h}{n^2}\phi_k)} = \frac{1}{1-(\frac{r/h}{n^2}\frac{1-cos(\frac{2\pi kn}{N})}{1-cos(\frac{2\pi k}{N})})}$$

with $\xi_0 = \frac{1}{1-r/h}$ since $\phi_0 = n^2$. $\xi_k = \xi_{N-k}$ since $\phi_k = \phi_{N-k}$.

From $\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t$ to $\frac{1}{n}A\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t$

Having diagonalized A and AA, we use (a.1.1) and (a.1.3) to write:

$$\frac{1}{n}A\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t = \frac{1}{n}P(D_A)P^{-1}P(D_{tot})P^{-1} = \frac{1}{n}P(D_A D_{tot})P^{-1}$$

$D_A D_{tot}$ is thus the matrix similar to $A\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t$. The covariance matrix of the vector of total wealth change can thus be re-expressed as:

$\Sigma_I = \frac{\sigma_F^2}{n^2}[PD_A D_{tot}P^{-1}][PD_A D_{tot}P^{-1}]^\intercal = PD_A D_{tot} V D_A D_{tot} P$

Post multiplying by PP^{-1} we obtain $\Sigma_I = \frac{\sigma_F^2}{n^2}P(D_A D_{tot}V)^2P^{-1}$.

The general element of $(D_A D_{tot} V)^2$is $\varrho_k = \psi_k\xi_k\psi_{-k}\xi_{-k} = \phi_k\xi_k^2$, that is:

$$\varrho_k = \frac{1 - cos(\frac{2\pi kn}{N})}{[1 - cos(\frac{2\pi k}{N})][1 - (\frac{r/h}{n^2}\frac{1-cos(\frac{2\pi kn}{N})}{1-cos(\frac{2\pi k}{N})})]^2}$$

We now have derived the eigenvectors of and matrices similar to $(A^\intercal A)^t$, $\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t$, and $\frac{1}{n}A\sum_{t=0}^{t=+\infty}[(\frac{r/h}{n^2})A^\intercal A]^t$, so that we may draw the general element of the covariance between assets price changes at a given period Σ_t, the covariance matrix of total price movement Σ_{tot}, and the covariance matrix of the vector of total wealth change Σ_I. Appendix 1.5 goes through the algebra.

Appendix 1.3

The change of basis matrix P is the same for all circulant matrices. We thus look for the change of basis matrix for Z.

$$ZP = PD_Z \iff$$

$$\begin{pmatrix} 0 & 1 & ... & 0 & 0 \\ 0 & 0 & 1 & ... & 0 \\ 0 & 0 & 0 & ... & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p_{1,1} & ... & p_{1,N} \\ & & \\ ... & ... & ... \\ & & \\ p_{N,1} & & p_{N,N} \end{pmatrix} = \begin{pmatrix} p_{1,1} & ... & p_{1,N} \\ & & \\ ... & ... & ... \\ & & \\ p_{N,1} & & p_{N,N} \end{pmatrix} \begin{pmatrix} \omega^0 & 0 & ... & & 0 \\ 0 & \omega^1 & .. & & 0 \\ 0 & 0 & ... & 0 & 0 \\ 0 & 0 & 0 & \omega^{N-2} & 0 \\ 0 & 0 & 0 & 0 & \omega^{N-1} \end{pmatrix}$$

$$\iff \begin{pmatrix} p_{2,1} & p_{2,2} & ... & p_{2,N} \\ p_{3,1} & p_{3,2} & & p_{3,N} \\ ... & & ... & ... \\ p_{N,1} & & & p_{N,N} \\ p_{1,1} & & & p_{1,N} \end{pmatrix} = \begin{pmatrix} p_{1,1}\omega^0 & p_{1,2}\omega^1 & ... & p_{1,N-1}\omega^{N-2} & p_{1,N}\omega^{N-1} \\ p_{2,1}\omega^0 & p_{2,2}\omega^1 & .. & & p_{2,N}\omega^{N-1} \\ ... & & ... & & \\ & & & & \\ p_{2N,1}\omega^0 & p_{N,2}\omega^1 & & & p_{N,N}\omega^{N-1} \end{pmatrix}$$

this yields $p_{2,1} = p_{1,1}\omega^0$, $p_{3,1} = p_{2,1}\omega^0 \implies p_{3,1} = p_{1,1}(\omega^0)^2$, $p_{4,1} = p_{3,1}\omega^0 \implies p_{4,1} = p_{1,1}(\omega^0)^3$

$p_{2,2} = p_{1,2}\omega$, $p_{3,2} = p_{2,2}\omega \implies p_{3,2} = p_{1,1}(\omega)^2$, $p_{4,2} = p_{3,2}\omega \implies p_{4,2} = p_{1,1}(\omega)^3$

$p_{2,N} = p_{1,N}\omega^{N-1}$, $p_{3,N} = p_{2,N}\omega^{N-1} \implies p_{3,N} = p_{1,N}(\omega^{N-1})^2$, $p_{4,N} = p_{3,N}\omega^{N-1} \implies p_{4,N} = p_{1,N}(\omega^{N-1})^3$

And so on, the general form is thus : $p_{i+1,j} = (\omega^{j-1})^i p_{1,j}$. Normalizing by $\frac{1}{\sqrt{N}}$ we obtain an orthonormal base for the eigenvectors of the discrete inverse Fourier transform matrix of coefficient $\frac{1}{\sqrt{N}}$, whose general term is $p_{i+1,j+1} = \frac{(\omega^{j-1})^i}{\sqrt{N}}$, with $(i,j) \in \{0, 1, .., N-1\}^2$:

$$P = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & 1 & . \ . & 1 \\ 1 & \omega & . \ . & \omega^{N-1} \\ . & . & & . \\ . & . & & . \\ 1 & \omega^{N-1} & & \omega^{(N-1)^2} \end{pmatrix}$$

Appendix 1.4

$$VD_A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_0 & 0 & 0 & 0 \\ 0 & \psi_1 & 0 & 0 \\ 0 & 0 & \psi_2 & 0 \\ 0 & 0 & 0 & \psi_3 \end{pmatrix} = \begin{pmatrix} \psi_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi_3 \\ 0 & 0 & \psi_2 & 0 \\ 0 & \psi_1 & 0 & 0 \end{pmatrix}$$

where $\psi_k = \sum_{s=0}^{s=n-1} \omega^{ks} = \frac{1-\omega^{kn}}{1-\omega^k}$

$$D_{A^\intercal A} = (VD_A)^2 = \begin{pmatrix} \psi_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi_3 \\ 0 & 0 & \psi_2 & 0 \\ 0 & \psi_1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi_3 \\ 0 & 0 & \psi_2 & 0 \\ 0 & \psi_1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} (\phi_0) & 0 & 0 & 0 \\ 0 & (\phi_1) & 0 & 0 \\ 0 & 0 & (\phi_2) & 0 \\ 0 & 0 & 0 & (\phi_1) \end{pmatrix}$$

where $\phi_0 = (\psi_0)^2$, $\phi_1 = \phi_3 = \psi_3\psi_1$, $\phi_2 = (\psi_2)^2$

Since $\psi_{-k} = \psi_{N-k}$, $\psi_3 = \psi_{-1}$, $\psi_2 = \psi_{-2}$.

Thus we verify that for $k\epsilon[0,3]$, $\phi_k = \psi_k\psi_{-k} = \frac{1-\omega^{kn}}{1-\omega^k} \times \frac{1-\omega^{-kn}}{1-\omega^{-k}}$, which yields $\phi_k = \frac{1-cos(\frac{2\pi kn}{N})}{1-cos(\frac{2\pi k}{N})}$

Appendix 1.5

Here we focus on the general element of asset per period covariance. The method is the same for total asset and investors one, only $(\phi_k)^t$ is replaced respectively by ξ_k and $\xi_k\psi_k$. From the expression of $(VD_A)^2$ in appendix 1.4, and that of P in Appendix 1.3, we may write $\Sigma_t\sigma_F^2(\frac{r/h}{n^2})^{2t}P(VD_A)^{4t}P^{-1}$ as:

$$\Sigma_t = \sigma_F^2(\frac{hr}{n^2})^{2t} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^2 & \omega^3 \\ 1 & \omega^2 & \omega^4 & \omega^6 \\ 1 & \omega^3 & \omega^6 & \omega^9 \end{pmatrix} \begin{pmatrix} (\phi_0)^{2t} & 0 & 0 & 0 \\ 0 & (\phi_1)^{2t} & 0 & 0 \\ 0 & 0 & (\phi_2)^{2t} & 0 \\ 0 & 0 & 0 & (\phi_1)^{2t} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^{-1} & \omega^{-2} & \omega^{-3} \\ 1 & \omega^{-2} & \omega^{-4} & \omega^{-6} \\ 1 & \omega^{-3} & \omega^{-6} & \omega^{-9} \end{pmatrix}$$

$$\Sigma_t = \sigma_F^2(\frac{hr}{n^2})^{2t} \begin{pmatrix} (\phi_0)^{2t} & (\phi_1)^{2t} & (\phi_2)^{2t} & (\phi_1)^{2t} \\ (\phi_0)^{2t} & \omega(\phi_1)^{2t} & \omega^2(\phi_2)^{2t} & \omega^3(\phi_1)^{2t} \\ (\phi_0)^{2t} & \omega^2(\phi_1)^{2t} & (\phi_2)^{2t} & \omega^2(\phi_1)^{2t} \\ (\phi_0)^{2t} & \omega^3(\phi_1)^{2t} & \omega^2(\phi_2)^{2t} & \omega(\phi_1)^{2t} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega^3 & \omega^2 & \omega \\ 1 & \omega^2 & 1 & \omega^2 \\ 1 & \omega & \omega^2 & \omega^3 \end{pmatrix} =$$

$$\sigma_F^2(\frac{hr}{n^2})^{2t} \begin{pmatrix} (0) & (1) & (2) & (1) \\ (1) & (0) & (1) & (2) \\ (2) & (1) & (0) & (1) \\ (1) & (2) & (1) & (0) \end{pmatrix}$$

Where we used the fact that $\omega^k = \omega^{N-k}$. Using Euler's identity $\omega^{(k-j)(N-q)}\phi_{N-k} + \omega^{(k-j)q}\phi_k = 2cos(\frac{2i\pi(k-j)q}{N})\phi_k$:

$$(0) = (\phi_0)^{2t} + 2(\phi_1)^{2t} + (\phi_2)^{2t} = (n^2)^{2t} + 2(\frac{1-cos(\frac{2\pi n}{4})}{1-cos(\frac{2\pi}{4})})^{2t} + (\frac{1-cos(\frac{2*2\pi n}{4})}{1-cos(\frac{2*2\pi}{4})})^{2t}$$

$$(1) = (\phi_0)^{2t} + (\omega^3 + \omega^1)(\phi_1)^{2t} + \omega^2(\phi_2)^{2t}$$

$= (n^2)^{2t} + [cos(\frac{2\pi}{4}) + i\ sin(\frac{2\pi}{4}) + cos(\frac{2\pi}{4}) - i\ sin(\frac{2\pi}{4})](\frac{1-cos(\frac{2\pi n}{4})}{1-cos(\frac{2\pi}{4})})^{2t} + [cos(\pi) + i\ sin(\pi)](\frac{1-cos(\pi n)}{1-cos(\pi)})^{2t}$

$= (n^2)^{2t} + 2cos(\frac{2\pi}{4})(\frac{1-cos(\frac{2\pi n}{4})}{1-cos(\frac{2\pi}{4})})^{2t} - (\frac{1-cos(\pi n)}{2})^{2t}$

$$(2) = (\phi_0)^{2t} + (\omega^2 + \omega^2)(\phi_1)^{2t} + (\phi_2)^{2t} = (\phi_0)^{2t} + (\omega^2 + \omega^{-2})(\phi_1)^{2t} + (\phi_2)^{2t}$$

$= (n^2)^{2t} + 2cos(\frac{2*2\pi}{4})(\frac{1-cos(\frac{2\pi n}{4})}{1-cos(\frac{2\pi}{4})})^{2t} + [cos(2\pi)](\frac{1-cos(\pi n)}{2})^{2t}$

The general expression for the covariance between k and j given in the body of the text (for N even) is verified:

$cov(\triangle p_j, \triangle p_k)_t = \frac{\sigma_F^2}{N}(\frac{r/h}{n^2})^{2t}[(n^{4t} + 2\sum_{q=1}^{q=N/2-1} cos(\frac{2*(k-j)q\pi}{N})(\frac{1-cos(\frac{2\pi nq}{N})}{1-cos(\frac{2\pi}{N})})^{2t} + cos(2(k-j)\pi)(\frac{1-cos(\pi n)}{2})^{2t}]$

Appendix 1.6

We introduce speculators in the model to find out whether they impact the desirability of diversification. We set speculators to follow a simple positive feedback strategies, selling the assets which have fallen steeply during the previous period. Though it is admitably stark, this rule is a way of modeling a popular speculative strategy on the markets: momentum trading. We refer the reader to De long et al. (1990) for a discussion on the existence, rationality, and behavioral finance explanations for such a strategy.

For our purposes this form simply implies that " speculators" to have an accelerating impact on losses and should on average add to the volatility of the markets. This could provide a new case for diversification: as individual asset prices snowball more quickly, the spreading of losses across assets may become more desirable.

The vector of speculator demands simply writes $\triangle\mathbf{Q}^{\mathbf{lt}}_{\mathbf{,t}} = \beta\triangle\mathbf{P}_{\mathbf{t-1}}$. Where β represents the strength of positive feedback trading in the economy. Re-expressing the market clearing condition now yields $\varepsilon^*_{t+1} + MR_{t+1} = (\frac{r/h}{n^2})T^{\intercal}T\triangle\mathbf{P}_{\mathbf{t-1}} + \frac{\triangle Q^{MR}_{i,t}}{h} + \beta\triangle\mathbf{P}_{\mathbf{t-1}}$. Plugging into the price dynamics and discarding the long-term fundamental drift and mean reversion:

$$\triangle\mathbf{P}_{\mathbf{t}} = (\frac{r/h}{n^2})[T^{\intercal}T + \beta I]\triangle\mathbf{P}_{\mathbf{t-1}}$$

Figure A.1.6.a summarizes the following difference: probabilities in the baseline model minus probabilities attributing a third of total trading to speculators, setting $r/h = 0.75$:

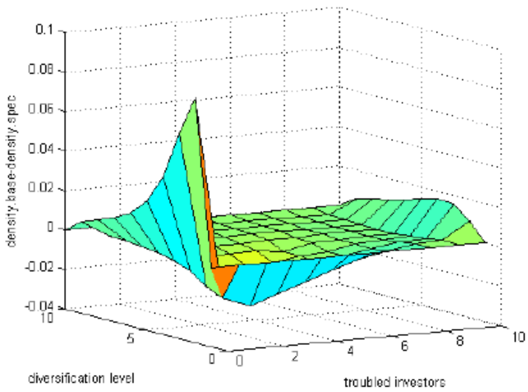

Figure A.1.6.a: Baseline vs model with speculators, without panic

We see that both extremes, no and complete failure, are more likely in the baseline model, i.e. less likely when we introduce speculators. For the no-failure case this is a natural consequence of the fact that speculators induce higher volatility at the asset level, making is less likely that no particular investor fails. In the complete failure case the finding is more surprising, the positive impact of speculator on individual risk is more than offset by the fact that assets are now more independent, which makes it less likely that they all experience large falls together. For the same reasons, the odds of all intermediary cases on the other hand increase when speculators appear.

We respect to diversification we note that this difference dies out as n increases. When the transmission between asset becomes increasingly transversal, the "asset-specific" amplification mecanisms become increasingly marginal. Looking at desirability implies lower levels of diversification are marginally more attractive when the market is subject to speculative attacks, and high levels less so.

This conclusion changes drastically when we account for possible panics, as showed by figure A.1.6.b.

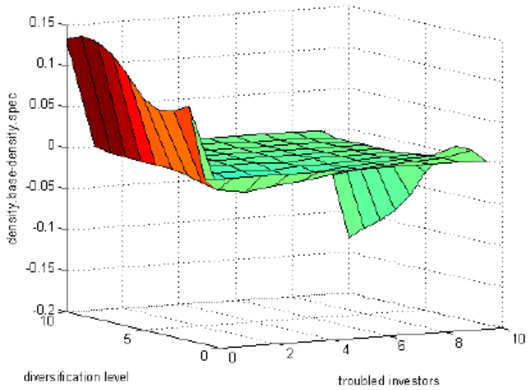

Figure A.1.6.b: Baseline vs model with speculators, with panic

As speculators increase the variance in sales for a given asset, they increase for a given asset to cross the panic threshold. The smoothing of constrained selling that diversification brings becomes very desirable in this case. Increasing the level of diversification is always unambiguously desirable for any value of β.

Appendices chapter 2

Appendix 2.1

This appendix presents a simple parsimonious model which formalizes the logic of section 1.1.

Consider a bond i whose yield spread moves according to a general factor model:

$$R_i = \alpha_i F + \varepsilon_i$$

where R_i represents the yield spread of asset i, and ε_i the idiosyncratic noise specific to asset i. F can be seen as a CAPM type risk factor that proxies the risk that is common to all bonds, and α_i the asset's exposition to it. Alternatively one could see F and α_i as vectors, with F representing the different fundamental risk factors that impact bonds. Eitherway, if asset j has a low grade, we expect $\alpha_j \geq \alpha_i$, i.e. the low graded asset should respond more starkly to increases in the global risk factor(s).

According to their sensitivity to F, assets are grouped into different exogenously defined risk classes. Index yield spread is defined by:

$$R_I = \alpha_I F + \varepsilon_I$$

where α_I is the average exposure of assets that belong to I, and ε_I the noise at the index level, also unrelated to F.

In a world in which prices only follow their fundamentals, ε_i should be i.i.d. and so should ε_I, whose variance should be much lower. Now say the number of assets is too large for investors to manage their portfolios at the asset level, so that they engage in style investing. They group assets according to their risk class, and trade at this risk class level, so that each asset i now moves with the index it belongs to. Mathematically this means $\varepsilon_i = \varepsilon_I + \varepsilon_i^*$, where ε_i^* is the "true" idiosyncratic noise on asset i, and ε_I the movement that is common to the entire index.

The comovement of an asset i with the index I it belongs to will then be:

$$\beta_{i,I} = \frac{cov(R_i, R_I)}{\sigma_{R_I}^2} = \frac{\alpha_i \alpha_I \sigma_F^2}{\sigma_{R_I}^2} + \frac{\sigma_I^2}{\sigma_{R_I}^2}$$

with $\sigma_{R_I}^2$ the total variance of index I, σ_F^2 the variance of the fundamental factor, and σ_I^2 that of the index specific movement in index I, i.e. the movement in the index that is not driven by fundamentals. Taking the average beta for all the assets i belonging to index I we have:

$$\bar{\beta_{i,I}} = \frac{\alpha_I^2 \sigma_F^2}{\sigma_{R_I}^2} + \frac{\sigma_I^2}{\sigma_{R_I}^2} = 1 \qquad (a.2.1)$$

since the exposure of index I to credit risk α_I is simply the average exposure of all the assets that belong to I, i.e. $\alpha_I = \sum_{i=1}^{i=n} \frac{\alpha_i}{n}$.

On the other hand, the average beta of an asset i' with index I, if i' does not belong to I, will be:

$$\bar{\beta_{i',I}} = \frac{\alpha_I \alpha_{I'} \sigma_F^2}{\sigma_{R_I}^2} + \frac{\delta_{I',I} \sigma_I^2}{\sigma_{R_I}^2} \qquad (a.2.2)$$

where $\delta_{I',I} = \frac{cov(\varepsilon_{I'}, \varepsilon_I)}{\sigma_I^2}$ is the beta of a regression of the index specific noise of index *I'* on that of *I* .

Consider now an asset that belonged to index I but gets upgraded/downgraded to *I'*. Before the rating action its expected comovement with I is given by (a.2.1), while after the rating action it will be given by (a.2.2). Consequently the expected change in the beta of an asset that is being upgraded/downgraded *with the index I it leaves* , which we call an "out" movement, should be :

$$E(\triangle_{out}^{\beta}) = \bar{\beta_{i',I}} - \bar{\beta_{i,I}} = \frac{\alpha_I \sigma_F^2 (\alpha_{I'} - \alpha_I)}{\sigma_{R_I}^2} + \frac{(\delta_{I',I} - 1)\sigma_I^2}{\sigma_{R_I}^2} = \triangle_{out}^{f} + \triangle_{out}^{s} \qquad (a.2.3)$$

where we have use the fact that $\delta_{I,I} = 1$, the comovement of an index with itself is one. $\triangle_{out}^{f} = \frac{\alpha_I \sigma_F^2 (\alpha_{I'} - \alpha_I)}{\sigma_{R_I}^2}$ refers to the fundamental part of total beta shift, and $\triangle_{out}^{s} = \frac{(\delta_{I',I} - 1)\sigma_I^2}{\sigma_{R_I}^2}$ the style investing one.

Conversely, the expected beta change of asset leaving I to join *I'* , or "in" movement, will be:

$$E(\triangle_{in}^{\beta}) = \bar{\beta_{i',I'}} - \bar{\beta_{i,I'}} = \frac{\alpha_{I'} \sigma_F^2 (\alpha_{I'} - \alpha_I)}{\sigma_{R_I'}^2} + \frac{(1 - \delta_{I,I'})\sigma_{I'}^2}{\sigma_{R_I'}^2} = \triangle_{in}^{f} + \triangle_{in}^{s} \qquad (a.2.4)$$

The sign that $\triangle^S$ will take will depend on the value of $\delta_{I,I'}$ or $\delta_{I',I}$, both functions of the covariance between the noise of the indexes $cov(\varepsilon_{I'}, \varepsilon_I)$. This term represents the link between two indexes arising from style investing. One cannot expect style investors to consistently buy more than 1 unit of a given index every time he buys 1 unit of another, and even less all of them to do so, for this would require that the total wealth invested varies enormously on a daily basis [16]. Therefore the average index style-driven comovement must be between 0 and 1 in absolute value, i.e. $|E(\delta_{I',I})| < 1$. In the out case, this means $\triangle^s_{out} = \frac{(\delta_{I',I}-1)\sigma^2_I}{\sigma^2_{R_I}}$ should be negative, while in the in case $\triangle^s_{in} = \frac{(1-\delta_{I,I'})\sigma^2_{I'}}{\sigma^2_{R_{I'}}}$ should be positive.

Note that for robustness we tested that this average style driven comovement is indeed below one, and confirmed at the 0.01 level.

What about $\triangle^f$? If the movement is an upgrade, asset i has become less sensitive to global risk, so that $E(\alpha_i) > E(\alpha_{i'})$, and $\triangle^f$ should be negative. Conversely for a downgrade we expect $E(\alpha_i) < E(\alpha_i')$ and thus $\triangle^f$ to be positive. Therefore we reach to same predictions as in Section 1.1:

	downgrade	upgrade
in	$\triangle^f > 0$, $\triangle^s > 0$	$\triangle^f < 0$, $\triangle^s > 0$
out	$\triangle^f > 0$, $\triangle^s < 0$	$\triangle^f < 0$, $\triangle^s < 0$

In a general factor model setting and if low graded bonds respond more to changes in the risk factor(s), *fundamental factors and style investing have opposite predictions on comovement in 2 sub-cases, which provide a natural way of testing for style-driven comovement.*

16. Besides being naturally between 0 and 1, $\delta_{I,I'}$ should also be driven towards 0 through measurement error. Indeed the trading of a given risk class may be part of a portfolio wide change, which leads to comovement between indexes, but it may also be quite independent of the other indexes, for instance as part of regular portfolio rebalancing.

Appendix 2.2

Let us now imagine that investors trade at the risk class level I but also at a wider ensemble one w, such as investment versus high-yield grade. For an asset $i \in I \subset w$, the error term should write $\varepsilon_i = \varepsilon_w + \varepsilon_I + \varepsilon_i^*$, where ε_w represents the noise of the wider ensemble, which we assume to be independent of all other terms. The return of index I will be $R_I = \alpha_I F + \varepsilon_w + \varepsilon_I$. The average beta of the regressions of all assets i belonging to $I \subset w$ on index $I' \subset w'$ will then be:

$$\bar{\beta_{i,I'}} = \frac{\alpha_I' \alpha_I \sigma_F^2}{\sigma_{R_{I'}}^2} + \frac{\delta_{w,w'} \sigma_{w'}^2}{\sigma_{R_{I'}}^2} + \frac{\delta_{I,I'} \sigma_{I'}^2}{\sigma_{R_{I'}}^2}$$

where $\delta_{w,w'}$ is the beta of the noise of wider ensemble w on that of w', and $\sigma_{w'}^2$ the idiosyncratic variance of ensemble w' . There are 3 possible cases :

- *I*=*I'* in which case we should have $\delta_{I,I'} = \delta_{w,w'} = \bar{\beta_{i,I'}} = 1$.

- $I \neq I'$ but $w = w'$ in which case only $\delta_{w,w'}$ should be 1.

- $I \neq I'$ and $w \neq w'$ in which case no coefficient is expected to be 1.

Therefore in this set-up, if a rating action concerns two risk classes that belong to the same wider ensemble, the beta shifts should have expressions similar to (a.2.3) and (a.2.4), where $\triangle^\beta = \triangle^f + \triangle^s$. However if changing of risk class also involves changing of wider ensemble, we should have, in the "out" case:

$E(\triangle_{out}^{\beta,w}) = \bar{\beta_{i',I}} - \bar{\beta_{i,I}} = \frac{\alpha_I \sigma_F^2 (\alpha_{I'} - \alpha_I)}{\sigma_{R_I}^2} + \frac{(\delta_{I',I} - 1)\sigma_I^2}{\sigma_{R_I}^2} + \frac{(\delta_{w',w} - 1)\sigma_w^2}{\sigma_{R_I}^2}$

$= \triangle_{out}^f + \triangle_{out}^s + \triangle_{out}^w$

where $\triangle_{out}^w = \frac{(\delta_{w',w} - 1)\sigma_w^2}{\sigma_{R_I}^2}$ represents the change in comovement that may be attributed to moving from a wider ensemble to another, and the notation $\triangle_{out}^{\beta,w}$ is to highlight that we focus on rating actions that provoke a change in wider ensembles w. In the "in" case:

$$E(\triangle_{in}^{\beta,w}) = \frac{\alpha_{I'}\sigma_F^2(\alpha_{I'}-\alpha_I)}{\sigma_{R'_I}^2} + \frac{(\delta_{I,I'}-1)\sigma_{I'}^2}{\sigma_{R'_I}^2} + \frac{(1-\delta_{w,w'})\sigma_{w'}^2}{\sigma_{R_I}^2}$$

Similar to previous section we expect $|\delta_{w',w}| < 1$, and $|\delta_{w,w'}| < 1$, *so that* $\triangle^w$ *should be negative in the out case, and positive in the in one.* Actually we may even expect $\delta_{w',w} \in]-1,0[$, because with only two ensembles one may be traded more against the other. For instance we may expect a category trader whose risk appetite is on the rise to buy speculative bonds and finance this by selling investment ones, which implies $cov(\varepsilon_w, \varepsilon_{w'})<0$.

Taking the difference between $\triangle^{\beta,w}$ and the average beta shift across "regular" risk classes then yields:

$$E(\triangle^{\beta,w} - \bar{\triangle^{\beta}}) = E(\triangle^f + \triangle^s + \triangle^w) - E(\triangle^f + \triangle^s) = \triangle^w$$

which again should be positive in the in case and negative in the out one.

This approach resembles the difference-in-difference methods of development economics. The idea is that the fundamental and risk class level style investing components $\triangle^f$ and $\triangle^s$ should be constant across all risk classes, so that differencing should remove these components. Of course this would be incorrect if we expected $\triangle^f$ and $\triangle^s$ to be different in $\triangle^{\beta,w}$, yet we have *a priori* no reason to expect this. Rating agencies often claim they base their decision only on underlying fundamental risk, so that we do not expect them to require a higher relative change in fundamentals $\triangle^f$ for sensitive notches. Similarly there is no obvious reason why the style investing component at the risk class level should be different from average, since any difference should be captured by the wider change component.

Which rating movements may be interpreted as changing the wider ensemble of an asset? A clear candidate lies with movements between BBB- to BB+, as the line between "investment grade" bonds and "high-yield" ones lie between these two notches. Therefore we set our base test to be:

Test 2: H0: $\triangle_{out}^{\beta,bbb-/bb+} - \triangle_{out}^{\bar{\beta}} > 0$ and $\triangle_{in}^{\beta,bbb-/bb+} - \triangle_{in}^{\bar{\beta}} < 0$

where H0 again represents the "no style investing" hypothesis.

Appendices chapter 3

Appendix 3.1

Consider that "Home" and "Abroad" each produce a single good, and both goods have a moderate degree of differentiation. Let the representative consumer[17] of the "home" country have following general utility function:

$U_t = q_t^{\alpha} q_t'^{\beta}$

where q and q' are the quantities demanded of the home and foreign good respectively, while α and β are preference parameters. The consumer faces the following budget constraint, expressed in the local currency:

$W_t = p_t' q_t' + p_t q_t$

where W_t is the consumer wealth at time t, p and p' are the prices of the foreign and domestic good respectively. Assuming perfect competition in both countries, firms are price-takers in their local currencies, we have $p_t' = MC'$ and $p_t = MC \times S_t$. The price of the domestic asset at home is the marginal cost of producing for home firms in their own currency, while the price of the foreign good at home is the marginal cost of production in the foreign currency expressed in

17. We only consider consumers in order not to overload the model. In practice this demand may also come from firms, but this demand should have a comparable form.

the local one.

For simplicity we assume all fundamentals characteristics of both economies stay constant, an assumption also made and discussed in the body of the thesis. Here this means setting *MC*, *MC'* and *W* constant.

Maximizing yields the textbook demand functions for both goods, expressed in units of the home currency.

$p'q' = \frac{\alpha}{\alpha+\beta}W$ and

$$p_t q_t = MC \times S_t \times q_t = \frac{\beta}{\alpha+\beta}W \quad (a.3.1)$$

The value of the exports for the foreign country is the home demand for foreign goods denominated in the foreign currency $X_t = \frac{p_t q_t}{S_t} = \frac{MC \times S_t \times q_t}{S_t} = MC \times q_t$. Assuming as in the body of the thesis that the foreign country is small, so that we may ignore the variations in the imports of the foreign country from "home", the current account of the foreign country at time t can then be expressed as:

$CA_t = X_t - M = MC \times q_t - M$

This expression represent the "real" net demand for FCU between t and *t+1*. Plugging in equation (a.3.1), we obtain $CA_t = \frac{A}{S_t} - M$, where $A = \frac{\beta}{\alpha+\beta}W$. Noting F the "fundamental" exchange rate for which trade is balanced i.e. $CA* = \frac{A}{F} - M = 0$, we may re-expressed this current account as:

$CA_t = \frac{A}{S_t} - M - CA* = A(\frac{1}{S_t} - \frac{1}{F}) = \frac{A}{F}(\frac{F-S_t}{S_t})$

The evolution of the exchange rate may then be expressed as $\triangle S_t = \lambda(\frac{F-S_t}{S_t}) + b\triangle Q_{fm,t-1}$, where $\lambda = b\frac{A}{F}$.

Appendix 3.2

The value at risk (VaR) gives the capital loss associated with a given realization of the portfolio return $R_{c,t}$, that may occur with probability α. Risk managers chose a threshold value for α, noted $\widetilde{\alpha}$, and require that investors hold enough capital to cover the losses associated with the corresponding return $\widetilde{R_{c,t}}$. If the realized return is below $\widetilde{R_{c,t}}$, which occurs with probability $\widetilde{\alpha}$, then the investor does have enough to face his losses and is effectively bankrupt. Through the VaR constraint risk manager then effectively set a maximum probability of bankruptcy $\widetilde{\alpha}$.

The investor is bankrupt if all of his capital, or more, has been lost. Mathematically, the bankruptcy condition at t+1 is then $e_{t+1} \leq 0$, which using expression (3.4) can be rewritten as $R_{c,t} \leq \frac{-e_t}{S_t p_t^* q_t^*}$. Therefore we have $Prob(R_{c,t} \leq \frac{-e_t}{S_t p_t^* q_t^*}) = \widetilde{\alpha}$. On the other hand the threshold return is given by $\widetilde{R_{c,t}} = \mu - x\sigma$, where x is the number of standard deviations required to have $Prob(R_{c,t} \leq \mu - x\sigma) = \widetilde{\alpha}$. We thus have:

$$Prob(R_{c,t} \leq \frac{-e_t}{S_t p_t^* q_t^*}) = \widetilde{\alpha} = Prob(R_{c,t} \leq \mu - x\sigma) \Longrightarrow \frac{e_t}{S_t p_t q_t} = x\sigma - \mu$$

thus using constant unconditional moments μ and σ, investment in the foreign bond will be linearly related to capital, and we have:

$$p_t \mathring{q}_t S_t = C.e_t$$

as put in body of the text, where $C = x\sigma - \mu$

The reader may object that specifying a constant C is a weak assumption. Indeed Shin and Adrian (2013) provide evidence that capital requirements relative to holdings do move with economic cycles, falling during booms and rising during busts. However allowing for this feature would induce further procyclicality from VaR constraint. Since in our set-up procyclicality is already present, adding a changing C would add complexity without adding a new dynamic.

Appendix 3.3

parameter	description	value
R_b	Interest spread between both countries[18]	6%
R'_b	Interest spread before the rise	4%
$\sigma^2_{\triangle s}$	Variance of daily ER change	0.0001
$\sigma^2_{\triangle R}$	Variance of daily ER return	0.0004
x	Number of lags used by chartists and fundamentalists	30
α	Weight on recent observed moments for chartists	0.9
λ	Speed of reversion toward fundamentals	0.0001
b	Marginal price impact of one unit of demand	0.01
L	Maximum allowed leverage	[1,10]
φ	Initial proportion of constrained carry traders	0.8
n_F	Number of fundamentalists	0.01
n_M	Number of momentum traders	0.05

Appendix 3.4

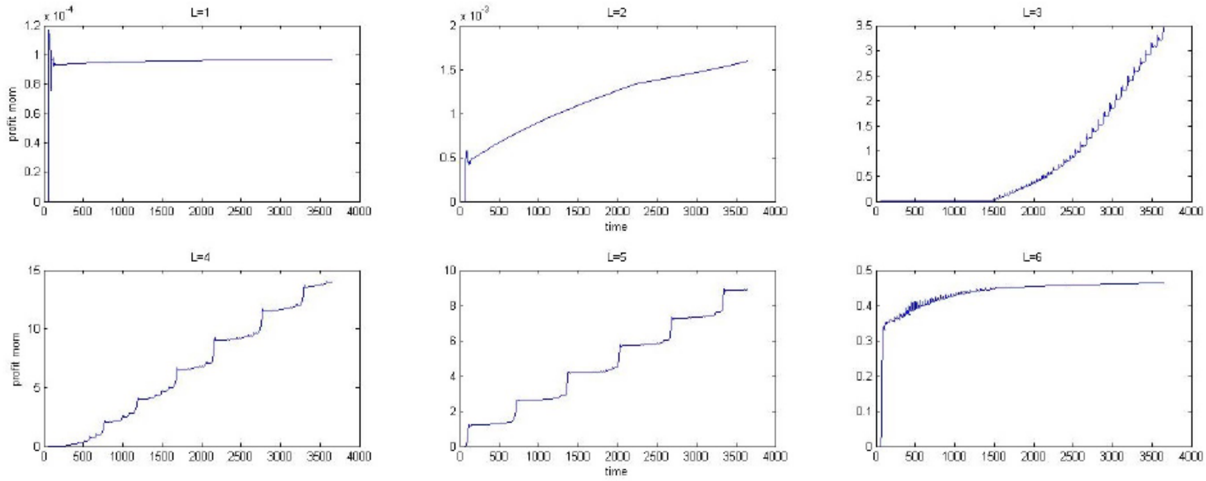

Figure 4.3.4.a: Accumulated profits for momentum trading sector

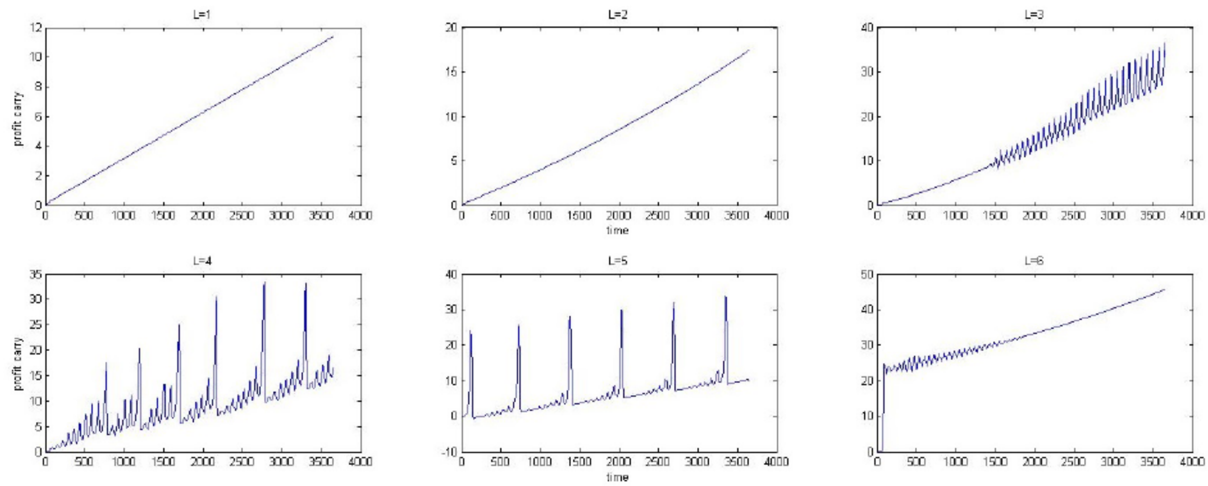

Figure 4.3.4.b: Accumulated profits for carry trading sector

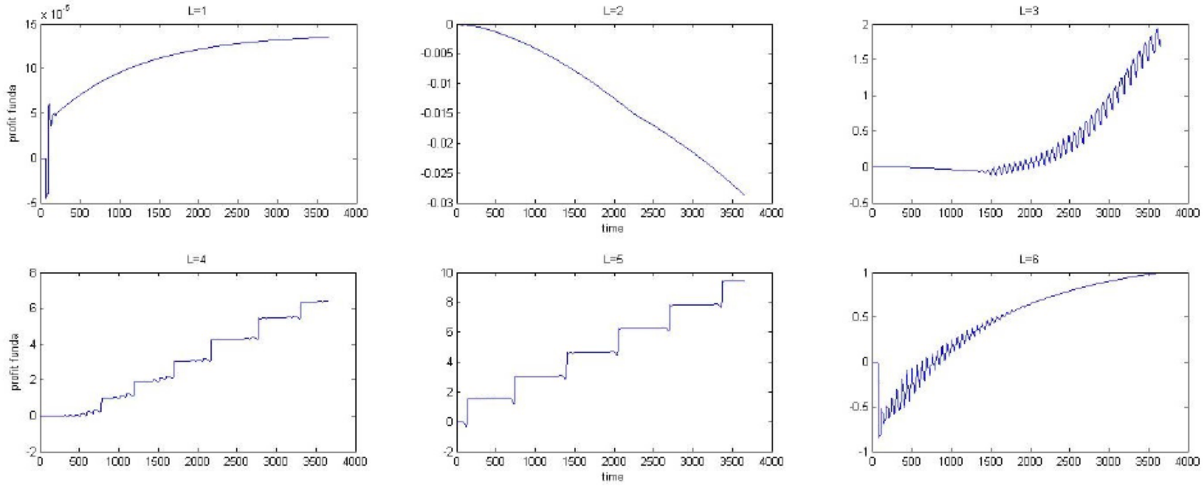

Figure A.3.4.c : Profits for fundamentalist sector

Note the $L=2$ case seems to yield losses for fundamentalists, but these vanish over a longer horizon.

Appendix 3.5

The total return for chartists and carry traders is defined as :$\frac{e_t}{e_0}$, where e_T is the wealth at the end of the year and e_o that at the beginning. To obtain yearly return we normalize e_0 to 1, and specify that investor reinvest at each period t their entire existing wealth, i.e. their wealth at *t-1* the previous period plus any capital gains realized between *t-1* and *t*. This approach is in line with the majority of the papers on FX profits, yet it involves a departure from the investment behavior we have specified in the model for both agents.

Indeed in our set-up, chartists change their exposure daily, and these exposures are unconstrained. This means that the wealth invested at time *t+1* is independent of the wealth owned at time t, so that the ratio $\frac{W_t}{W_0}$ may not be viewed as an indicator of the yearly performance of the representative chartist. Similarly the complete reinvestment of equity is not in line with carry traders when they are unconstrained, i.e. when their current holdings may not reflect their capital. Therefore these indicators can viewed as a broad estimate of the profitability of momentum trading and carry trade, comparable with that of other studies, but not as a perfect indicator of the return of the representative agents in the model.

Studying the evolution of the wealth of chartists also requires assumptions about their leverage, and how much capital is consumed by short selling. In line with the literature we assume no leverage, and that short selling is as capital intensive as buying, i.e. chartists must hold 100% of amount they short sell.

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Table des figures

Liste des tableaux

Title: Trading strategies and endogenous asset price movement

Abstract: We study how popular investment rules in financial markets may induce endogenous movements in asset prices, leading to higher market risk.

In the first chapter, we focus on portfolio diversification. We show through a theoretical model that this strategy is beneficial at the individual investor level, but also creates endogenous links between assets and investors, which can be dangerous from a systemic perspective. We measure both effects in order to discuss the overall desirability of diversification.

The second chapter considers strategies based on grouping assets that share common characteristics into different classes, or "styles". We postulate that these strategies create excess comovement between assets of a similar style, as they are traded together as part of the same class. Applying this reasoning to bond credit ratings, we show that bonds joining a new rating class indeed start comoving more with the bonds of this rating, even when fundamental factors suggest otherwise.

In the third chapter, we study three investors who operate in the foreign exchange market: carry traders, chartists and fundamentalists. We provide a theoretical model which suggests that the interaction between these trading rules may explain the well documented exchange rate disconnect from its fundamental value, and lead to endogenous currency crashes.

Keywords: Endogenous risk, comovement, crash risk, portfolio diversification, style investing, carry trade, chartism, fundamentalism

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Titre : Stratégies d'investissement et variation endogène de prix des actifs financiers

Résumé : Nous étudions des stratégies d'investissement dont l'utilisation s'est généralisée sur les marchés financiers, et leur impact sur le prix des actifs et le risque de marché.

Dans le premier chapitre nous nous intéressons aux stratégies de diversification de portefeuille. Nous montrons au travers d'un modèle théorique que si la diversification a un effet positif au niveau individuel pour l'investisseur, elle crée également des liens entre les différents investisseurs et titres, qui peuvent se révéler dangereux d'un point de vue systémique. Nous mesurons les deux effets afin de discuter de la désirabilité globale de la diversification.

Le second chapitre considère les stratégies d'investissement basées sur le groupement de titres financiers partageant certaines caractéristiques en différentes classes, ou « styles ». Nous postulons que ces stratégies créent un co-mouvement excessif entre titres d'un même style, qui seront vendus et achetés ensemble au sein d'une même classe. Appliquant cette intuition aux notes des agences sur les obligations, nous montrons qu'une obligation qui change de note se met en effet à varier comme sa nouvelle note, même quand les fondamentaux économiques ne le justifient pas.

Dans le troisième chapitre nous étudions trois types d'investisseurs opérant sur le marché des changes : les « carry traders », les chartistes et les fondamentalistes. Notre modèle théorique suggère que l'interaction entre ces trois règles d'investissement peut expliquer la déconnexion bien documentée entre le taux de change et sa valeur fondamentale, ainsi que provoquer un effondrement endogène des taux de change.

Mots-clés : Risque endogène, co-mouvement, risque d'effondrement, diversification de portefeuille, investissement par style, carry trade, chartism, fundamentalism

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