Modeling and stabilization of sociopolitical networks – application to country coalitions

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Modeling and stabilization of sociopolitical networks – application to country coalitions

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Abstract

The main subject of this thesis is a paradigm of instability and stabilization in coalition forming among countries as rational actors, presented through a Statistical Physics inspired model. This is an interdisciplinary work involving the fields of applied mathematics and sociophysics, as well as the political applications in the real cases from past and present. Applied to political, economic and social problems, the models can be used to analyze a wide variety of real cases – international alliances, economic or business alliances, coalitions of political parties, social networks, and organizational structures.

In the first part of this thesis we present and analyze the coalition forming and the instability among rational actors coupled with pairwise static historical propensity bonds that have evolved independently. Such organization leads to discordant associations into coalitions and the instability as a consequence of decentralized maximization of the individual benefits gained from joining or leaving the coalitions.

We define Natural Model of coalition forming and address the questions of instability and stabilization among actors possessing different levels of rationality. The framework presented here allows to analytically calculate the optimal and non-optimal stable configurations of actors' coalitions. We then investigate the coalition forming and the stabilization under the influence of externally-set opposing global alliances, which are represented in Global Alliance Model. The stabilization is produced through new cooperations based on the effect of polarization of several distinct interests shared by actors, which generates interest-based propensities and enables a planned coalition forming. We then investigate the effect of dissolution of a global alliance which, together with the competing alliance, has previously generated stable coalitions.

A special section of the thesis is devoted to investigation and illustration of coalition forming in real historical cases. This part presents the analysis of unstable coalitions in Europe – cycling in the England-Spain-France conflicting triangle and creation of the Italian state, as well as of the remarkable historical cases of the

Soviet global alliance collapse, of the recent internal conflict in Syria, and of the "paradoxical stability" in the Eurozone.

In the second part of this thesis, we present a simulation of the coalition forming models. The simulation allows to follow graphically the coalition forming processes. We present the methodology used in the simulation, as well as its application in the illustration of coalition forming in the prototypes of real case systems. Given exact propensity values, which is fairly considered to be the most difficult part of coalition forming modeling, the simulation tool can be used to predict optimal and non-optimal spontaneous stabilizations and globally motivated stabilities in real cases.

An independent part of the thesis is devoted to the subject of viability correction in dynamic network of actors. The model is a finite set of autonomous actors with states that evolve independently and connected into a network via their connection operators, which evolve independently as well. The network is defined to be viable if a joint evolution satisfies the centralized scarcity constraints set by the environment. In order to restore the viability of these decentralized dynamics, we apply to the method of correction by viability multipliers used in Viability Theory, where the multipliers play the role of decentralizing prices. Standing apart from the main course of the thesis, the subject of viability correction in dynamic network of actors suggests an interesting theoretic dynamical generalization of coalition stabilization in our models inspired from Statistical Physics.

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Contents

Introduction

Incipit

> I ask how far away the eye can discern a non-luminous body, as, for instance, a mountain. It will be very plainly visible if the sun is behind it; and could be seen at a greater or less distance according to the sun's place in the sky.
>
> *Leonardo da Vinci*

Study of coalition forming and instability among rational actors, which is the main subject of the present manuscript, is a product of the process of evolution and exchange between trees of knowledge. While mathematics may simulate complex mechanical dynamic processes, and the interactions between atoms may be accurately explained, the processes between individual actors, such as people, communities, nations, countries, are not yet well understood. The interest of the understanding existed since long time: ancient philosopher Empedokles observed that people interact like fluids, mathematician and meteorologist Lewis Fry Richardson developed mathematical model for the animosity dynamics between two nations, historian Imanuel Geiss described the decay of empires with Newtons law of gravitational forces. Despite the fact that peoples, countries, communities are neither dipole magnet-like atoms nor mathematical variables, the universal laws of collective behaviors expressed in the critical properties of social systems allow the application of methods from physics and mathematics in description of social aggregations.

The adjective "non-luminous" in the incipit of this manuscript refers to the degree of understanding of coalition forming among individual rational actors. Overcoming barriers between different and seeming unconnected disciplines and transferring the language and the methods from one scientific domain to another proves to be essential to shed light on those complex phenomena.

Figure 1 shows the disciplinary exchange between the cited domains as interconnected in past literature and in the present work.

The approach to the work presented in this manuscript situated in line of the methods and ideas coming from statistical physics, mathematics, history and be-

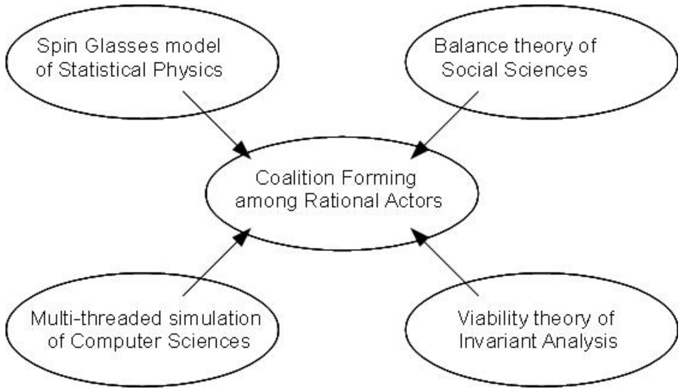


Figure 1: A pictorial representation of the topics touched in this manuscript.

yond. Namely, ideas from Spin Glass theory inspire the model of coalition forming among rational actors. From other side, the results obtained in Statistical Physics cross the Balance Theory of The Psychology of Interpersonal Relations. Further, the Spin-Glass based model of coalition forming appeals to the ubiquitous methods of multi-thread simulations, which allow the graphical visualization and testing, as well as to put forward the application in Political Sciences. Finally, The Viability Theory from the field of Invariance Analysis, which is usually applied in finances, engineering and sociological and economical sciences, helps a deeper investigation of stabilization in dynamical coalition forming.

Let us give, as a common background of the present work, a short overview of the Spin-Glass model, the coalition forming model, a brief historical overview and a general outline of the work.

Spin Glass Model

Here we present the Spin Glass model at zero temperature. In Statistical Physics a spin glass is a magnet represented by a collection of disordered interacting spins – atoms acting as a tiny dipole magnet. The interactions are competing ferromagnetic and anti-ferromagnetic bonds, randomly distributed with comparable frequency. Spins interact with each other seeking to align themselves parallel or antiparallel in order to minimize their energies. Ferromagnetic couplings make spins to align so that the configurations in which adjacent spins are of the same direction have higher probability. In anti-ferromagnetic couplings, in contrast, adjacent spins tend to have opposite directions. Thus, a spin glass forms a disordered material with a highly degenerate ground state, which at zero temperature is the system's lowest-energy state. The degeneration is brought by the competing interactions that cause high magnetic frustrations – changes of spins with no energy cost, augmented by stochastic positions of the spins.

The dynamics of a magnetic system, starting from a random distribution of spins, is determined by the pursuit of the spins to minimization of their energy. If we denote the collection of N sites by $I = \{1, 2, \ldots, N\}$ and their respective states by $S = \{S_1, S_2, \ldots, S_N\}$, then the time evolution of a magnetic system is uniquely defined by Hamilton's equation $\mathcal{H} = \mathcal{H}(S, t)$. For a closed system the Hamiltonian corresponds to the total internal energy of the system.

This is at zero temperature, that minimization of the Hamiltonian conduces to minimization of the energy. For non zero temperature, with $\mathcal{E}$ standing for the internal energy of a magnet system and $\mathcal{S}$ the entropy function, the free energy of the magnet determined by $\mathcal{F} = \mathcal{E} - \mathcal{T}\mathcal{S}$ must be minimized.

The exchange interaction between sites i, j is denoted by a constant J_{ij} which can be both ferromagnetic, $J_{ij} > 0$, and anti-ferromagnetic, $J_{ij} < 0$, or $J_{ij} = 0$ for noninteracting sites. The interactions of a given site i with all other sites results in energy contribution $\mathcal{E}_i = -S_i \sum_j J_{ij} S_j$ to the Hamiltonian $\mathcal{H} = \frac{1}{2} \sum_{i=1}^{i=N} \mathcal{E}_i$. The minus sign in the formula of the energy is conventional and impart the sense of minimization.

Ising model is known to be the simplest mathematical model to describe magnetic materials. The definition of the Ising ferromagnetic model involves discrete values for the spins with only two possible states $S_i \in \{-1, 1\}$. The model consists of N discrete variables $\{S_i\}_1^N$, called spins, that can be in one of two states *up* or *down*. The model assumes that the exchange constant of each edge i, j is a ferromagnetic coupling, $J_{ij} > 0$, and the exchange interactions are restricted to nearest neighbor spins arranged for instance in a regular cubic graph.

The model allows the representation of thermodynamic phase transitions (transformation of the system from an ordered phase or state, to a disordered one) as a simplified model of reality. The two-dimensional square-lattice Ising model is one of the simplest statistical models to exhibit a phase transition. The one-dimensional model, which has been given an analytic description, in contrast doesn't undergo phase transition.

The two-dimensional model was solved exactly. Between the dimensions 2 and 4, a solution with excellent precision was obtained by the means of renormalization group theory. In dimensions greater than four, the phase transition of the Ising model is described by mean field theory.

Figure (2) shows schematically a two-dimensional Ising model. This is the case of 8 spins located on a lattice and interacting at most with their nearest neighbors. The spins for which a shift of the state cost no energy are called "frustrated".

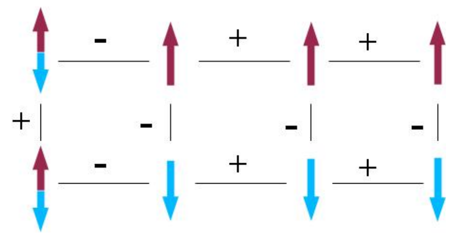


Figure 2: Ising model of 8-spins with mixture of negative and positive pair interactions. The pair propensity bonds are denoted by $+$ or $-$, and sates of the spins are denoted by the arrows. Frustrated spins, are marked by both up and down arrows. The Spin glass appears in an unstable disorder.

The mathematical model of spin glass on which we ground our models was introduced by Edward and Anderson [21] to explain some at the time new features of mixed magnetic systems. The extensions involve : 1) a mixture of both ferromagnetic and anti-ferromagnetic couplings, 2) interactions with nearest neighbors on the lattice.

The model thus consists of N discrete variables $\{S_i\}_1^N$, representing states of N spins, that can be in one of two states, *up* and *down*. The basic degrees of freedom are $S_i \in \{-1, +1\}$ placed on the vertices of a d-dimensional graph. With B standing for a magnetic external field and J_{ij} is the exchange constants on each edge ij of the graph, the spin glass Hamiltonian is defined by

$$\mathcal{H}(S, J) = -\frac{1}{2}\sum_{ij} J_{ij} S_i S_j + B \sum_i S_i.$$

The term "glass" arises from an analogy between the magnetic disorder in the collection of spins and the positional disorder of a conventional chemical glass. Spin glasses exhibit many meta-stable structures with a plenitude of phases which are difficult to explore experimentally or in simulations.

Main features of the spin-glass models are:

- *disorder*: a quenched disorder is incorporated in the Hamiltonian of the model by means of random couplings J, which are random variables that do not evolve with time – quenched. The value of a generic observable J^o depends on the realization of the disorder J. In the thermodynamic limit (the system size which goes to infinity) the distribution of the observable J^o is a delta function centered in $\bar{J}^o$, which is the average over the distribution of the disorder J.

- *frustration*: frustration in spin-glass models was firstly pointed out by G. Toulouse in [2]. The geometrical frustration is due to the fact that bonds between interacting spins can form *negative circles*: *the frustration implies the existence of a closed circle of spins connected with the bonds on which the product of total bonds is negative.* When trying to minimize the energy of a configuration, not all the spins can be "satisfied" – the no cost changes leads to frustrations in the spins.

Mattis model [37] of spin glass, which in contrast represents a random site spin glass without frustration, is defined by the Hamiltonian

$$\mathcal{H}(S,G) = -\tfrac{1}{2}\textstyle\sum_{ij} G_{ij}\epsilon_i\epsilon_j S_i S_j.$$

Here, G_{ij} are non-negative exchange parameters on each edge, and $\epsilon_i \in \{-1, 1\}$ are quenched random variables. Since all S_i all commute with each other (no negative circle is formed) the frustration nature of the problem is then irrelevant. This is readily seen by using a change of variable $\tau_i = \epsilon_i S_i$ which, due to non-negative G_{ij}s, turns $\mathcal{H}(S,G)$ into a ferromagnetic Hamiltonian $-\frac{1}{2}\sum_{ij} G_{ij}\tau_i\tau_j$.

Coalition Forming Model

The building scheme of our coalition forming model has in its first layer the extension of the Ising model – the Edward and Anderson's Spin Glass model [21] in its ground state.

In this analogy, our model involves a mixture of both ferromagnetic and anti-ferromagnetic couplings in the geometry of the lattice, and allows interactions with next neighbors.

The model is aimed to study instability and the stabilization in coalition forming among countries, which, in contrast to physical entities, possess rationality – the ability to envision a maximization of their individual benefits even through intermediate loosing states.

The model regards a system of N actors, for example countries, linked with bilateral propensity bonds, which are either positive (ferromagnetic-like) or negative (anti-ferromagnetic-like). According to the principle that "the enemy of an enemy is a friend" [57], which is manifested in practice since humanity carries out strategic cooperations, the countries are assumed to ally to one of two possible competing coalitions. To each country i is attached a discrete state variables S_i which represent the country's choice between the two possible coalitions $S_i = +1$ or $S_i = -1$. Making the same choice allies two countries to the same coalition while different choices separate them into the opposite coalitions. Thus, a configuration $S = \{S_1, S_2, S_3, \ldots, S_N\}$, a partition of the countries into coalitions, defines a particular allocation of coalitions.

The key premise is that each pair of countries, i and j, has a bilateral propensity J_{ij} to mutual interactions, which measures both the amplitudes and the directions of these exchange – cooperation or conflict. The propensity is positive if the two countries get along well together and negative if they have many sources of potential conflict. In other words, propensity is a measure of how willing the two countries are to ally to the same coalition together. Second country typically has the same source of conflict with the first, so it is assumed that propensity is symmetric $J_{ij} = J_{ji}$. Due to the symmetry, both a configuration and it's inverse define the same coalition allocation.

The product

$$J_{ij} S_i S_j \tag{1}$$

measures the countries i and j's benefit (gain) from the mutual interactions between the countries. Guided by the postulate of highest benefit (lowest energy), we expect that positive propensities encourage countries to ally to the same coalition, while negative ones encourage them to affiliate with the opposing coalition. Thus, the best coalition allocation – the one where the countries have the most comfortable and beneficial position, is achieved in an amalgam of conflict and cooperation. This defines the system's dynamics.

Country i's sum of the benefits from all its interactions makes up the its net gain:

$$H_i(S) = S_i \sum_{j \neq i} \ J_{ij} S_j. \tag{2}$$

In each particular configuration S, we can consider the total gain of the system of countries

$$\mathcal{H}(\mathcal{S}) = \tfrac{1}{2} \sum_i \ H_i(S) \tag{3}$$

.

The model have following properties and features:

- The model describes a system of countries maintaining short range interactions guided by the bilateral propensities bonds between them.
- Unlike spin glass theory, where the bilateral bonds are determined, randomly, the bonds in our model are preset and reflect the mutual relationships and interactions built by the countries through the historical experiences of the countries and their cultural traits.

  The interests and traits can be divided into ethnic, religious, territorial, ideological, economic and historical concerns, which we believe to capture the main sources of affinities and differences between countries that impact coalition forming strategies. Such typology allow operationalizations of propensities with respect to appropriate specific application of the model.

- Instability in coalition forming among countries is the key property of the model. The instability is a consequence of decentralized maximization of the individual benefits where contradictory associations into coalitions occur due to independent origination of the pairwise propensity bonds.

- In contrast to spins, the countries assumed to possess extended rationality which is a long horizon rationality that takes into account higher orders of mutual interactions. "Extended" here means the ability to envision a maximization of the individual benefits through any intermediate loosing states.

  The Spin Glass model can be compared to the coalition forming model in the system where the countries' rationality is limited to observation of an immediate gain – a spins-like system of rational actors.

- Stability reached within mixed rationality of countries is most likely not optimal. At the same time, any stability in rational actors require existence of optimal configurations.

- The conflict between countries is represented by different countries' choices. Based upon the direction of the propensities, the conflict can be beneficial to the same extent as the cooperation.

- The physical analogy to the Spin Glass model at zero temperature, and the equivalence between the Hamiltonian and the total system's gain, allow to address the bilateral propensities between countries as a guide to maximization of the countries' individual gains. This is a principal key of the coalition forming model.

- The formula for the total countries' gain in a configuration captures the idea that benefit is higher when countries that tend to cooperate are in the same coalition, while conflicting countries are in different coalitions.

Historical Overview

The first period of modeling of social phenomena succeeded the World Wars I and II, which appealed to the scientific curiosity. The classical references attempted to gives a unified view on the complex social problems, such as wars conflicts, aggregations, segregation. Among important early works can be mentioned the work of Richardson published 1919 privately and in 1935 "Mathematical psychology of war" where the author used differential equations to develop a mathematical model for how the animosity between two suspicious and well-armed nations develop overtime while the cost of war pushes for peace.

The dynamics of social aggregations were studied by psychologist Fritz Heider who in 1946 proposed a motivational theory of attitude change – Balance Theory ([27]). The theory conceptualizes the cognitive consistency motive as a drive toward psychological balance. Social network is represented as a complete graph. The balance based on the circle rule: "A network is balanced if each constituent triad is balanced".

Next period was marked by the work of Hermann and Hermann [28] in 1967 who tried to simulate the outbreak of World War I using human beings to represent the ten leaders during the war. A methodologically crucial work [23] was published by Schelling in 1971 who introduced to sociology the premises of multi-agent modeling, which later has link to Statistical Physics.

Schelling studied a model for social segregation and ghettos, and used a monte-carlo like simulation by hand. The model which gave only small ghettos, was corrected by sociologist Jones [29] in 1985 by introducing a random dynamics of the actors. Schelling model was an example where sociophysics was lacking for decades.

Using Statistical Physics in social modeling was first suggested in the work of Galam, Gefen and Shapir published in 1982 [15] who actually initiated the Sociophysics. Those models and the social application were later used by Galam and Moscovici in 1991 ([3])to create an approach on collective phenomena of consensus from the point of view of Psychology.

In a suite of inspired works of Edwards and Anderson [21], pioneering work on the model of spin-glass, and Toulouse [7], who described the energy of a system with many states as a global landscape, Axelrod and Bennett in [4] (1993) introduced the spin glass-based model a social landscape aggregation. Basing themselves on the novel results, Axelrod and Bennett defined an innovative mapping from physics to coalition forming among actors, such as nations, persons, businesses. Spin-like actors are linked by unchanging magnetic-like bonds, and the dynamics of the system correspond to decreases in energy, so that the only stable points of the system are the configurations that have locally minimum energy. Their model was criticized for theoretical inconsistencies in and a new model was proposed by Galam in [6] and [9], where the author mentioned the physical concept of frustration as introduced by Toulouse in [2] in 1977, according to which the natural condition to frustrated spins are negative cycles. In 2002 in [8], the same author, accentuating on the aspects of spontaneous coalition forming in the model of spin-like actors, attempted to extend the model with setting up extra territory coalitions – global alliances.

After the initial successful application to the study of social modeling, in recent years many systems have been described using methods and ideas borrowed from statistical physics. Let us just mention some of them:

- Group decision making ref [30]
- Evolution theory [31]
- Collective opinion formation [32]
- Social percolation [33]

Independently of this line of research, a different class of models was used in investigation of dynamics of social aggregations and interpretation of balance (stability), based on geometric graph representation. This research field has started from the Balance Theory – a motivational theory of attitude change in consistency motive as a drive toward psychological balance in a network of rational actors, proposed by psychologist Fritz Heider in 1946. The balance based on the circle rule: "a network is balanced if each constituent triad is balanced". A more general definition of a balanced network proposed by Cartwright and Harary [24] in 1956 required that each closed cycle is balanced. This definition is analogous to the above Toulouse's condition of negative cycles.

One of recent work on this research line is by Antal, Krapivsky and Redner which investigated the dynamics of social interactions – friendship and enmity (cooperation or conflict) in [17] (2006). The authors represented social network as a complete graph and considered its transition to a balanced state by the triad dynamic in which the sign of the mutual links changes. These links evolve according to natural rules that reflect a social desire to avoid imbalanced triads. The dynamics provided by actors' endeavor to arrive to a balanced state and the stabilization is based on assumption that the number of negative triads reduces – friendly links within each subnetwork can never change. An arbitrary network therefore is quickly driven to a balanced state which is a phase transition from polarity to utopia.

Both Heider and Toulouse gave the impetus to the independent development of the social aggregations modeling lying on the same geometrical base concept of instability.

The various researches have been conducted to examine and illustrate the real cases. Shelling studied the district ghettos and Warsow ghetto, Axelrod and Bennett simulated creation of the anti-Hitler coalition of World War II within European countries, Galam illustrated the Cold War, Antal, Krapivsky and Redner investigated relationships change during World War I.

Outline of the Thesis

The purpose of the present work is to present new analytical results regarding the following topics:

Coalition Forming Models

Spin glass based model of coalition forming have been the starting point for the construction of coalition forming model that represent the environment of actors possessing long horizon rationality where the dynamics arise from rational instabilities.

In chapter 2 we introduce Natural Model (NM) of coalition forming which originates from the statistical physics model and extends to a more general geometric model.

The model allows to investigate a paradigm of instability in coalition forming among countries. The instability is a consequence of decentralized maximization of the countries' individual benefits within an environment where contradictory associations into coalitions occur due to independent evolution of their pairwise propensity bonds.

The literature studying the coalition forming based on the Spin Glass model, analyzes the phenomena within a Markovian frame of spontaneous instability – a memoryless random process. In contrast to physics, where spins can only evaluate the immediate cost/benefit of a flip of orientation, countries may have a long horizon of rationality, which associates with the ability to envision a way up to a better configuration even at the cost of passing through intermediate loosing states.

The model allows a theoretical analysis for different levels of rationality and their impact of instability and the stabilization. The model with a mixture of different levels of rationality present in the same system of actors sheds a light on dynamics of many real cases. This allows to explore the complex behaviors and phenomena such as instability, infinite cycling and non-optimal stability.

As a remark for the backgrounds in the Game Theory, we note that the game-theoretic tools are not appropriate to describe the instability and stabilization in our model: the game-theoretic equilibrium represents a stable configuration only in the spins-like system, where the actor's rationality is limited to one-step horizon. Such equilibrium guarantees no stability in a model where the agents continue searching upon achieving their local maximums.

The results have been published in [18].

In parallel to coalition forming based on primary historical propensities, there exist association into two opposing alliances formed under the influence of externally-set incentives that create polarization of several distinct common interests of countries . In chapter 3, we extend the approach of the Natural Model to a so called Global Alliance Model of coalition forming (GAM). The stabilization by global alliances is achieved by either unique or multiple collective factors active over the system and proving simultaneous influence on the actors. The opposite effect, dissolution of a global alliance while its antagonist counterpart remain active is investigated within the confines of the model.

Recent economical and political crisis point out the importance of understanding of forming of coalitions under global alliances. The results shed a new light on the understanding of the complex phenomena of planned stabilization and fragmentation in coalition forming among rational actors [20], [19].

Real Cases Analysis

The models provide versatile theoretical tools for the analysis of real cases: wars and international, national conflicts or interpersonal conflicts. The conflicts and fluctuations in the England, Spain, France conflicting triangle, formation of Italian state, the Eurozone's stability and the perspectives. From the point of view of dissolution of a global alliance, two landmark historical cases – the collapse of Soviet Union and resent Syrian internal conflict are analyzed.

Those results have been published in [18], [20], [19].

Models Simulation

In order to illustrate the coalition forming in the natural and the global alliances models, we created a graphical computer simulation of the Natural Model. The simulation implements the theoretical structure of the models where the countries with different level of rationality (mastery) are represented by simultaneously running threads that make decisions in a random order based on different level of system visibility and calculative ability. The countries-threads interact through making choices motivated by maximization of their gain function. The interface of the simulation allows to screen the process of the gain maximization and the coalition forming.

The simulation's runs allows to understand the paradigms of individual selfish actors looking for their most beneficial configuration within their individual limits.

Fuzzy coalitions in dynamic networks and viability correction

Another type of model, the model of coalition forming in a dynamical network, is considered from the point of view of evolution of the system over a common centralized environment of scarce energy. The viability of dynamical fuzzy coalitions and its correction through the correction of the joint evolutions is tackled.

The fuzzy and dynamical analogy to the Natural Model of coalition forming is constructed based on the dynamical network's model. In this light, the externally motivated additional propensities in the global alliance model show themselves to play the role similar to that of the decentralized correctors in the dynamical network model. Our results have been presented in [11] and [18].

Part I

Coalition Forming and Instability

1 Comparison of Models

Here we suggest a comparison between our model of coalition forming and the initial statistical physics' models, such as Axelrod and Bennett's landscape aggregation model, Galam's extra-territory model, and the social balance model based on the Balance Theory.

Initial modeling was proposed by Axelrod and Bennett in [4] who used the physical concept of minimum energy in the spin glass to introduce a model of a social landscape aggregation. They have defined an innovative mapping from physics to coalition forming among actors, such as nations, persons, businesses. The "aggregation" means the organization of elements in patterns that tend to put highly compatible elements together and less compatible elements apart. The landscape aggregation theory and the model (LA) aimed to predict how aggregation will lead to alignments among actors which possess myopic, one-step rationality.

The authors address the problem of alignment between two competing coalitions within a group of N countries associated with sizes z_i as a reflection of the importance of the country, where the index $i = 1..N$.

The key supposition of the LA theory is that each pair of actors i and j has a propensity p_{ij} to cooperate. The propensity value is positive and large if the two get along well together and negative if they have many sources of potential conflict.

In the perspective of applications, a specific measuring of propensity is proposed by [4], which account for five factors: ethnic conflict, the similarity of the religions of the populations, the existence of a border disagreement, the similarity of the types of governments and the existence of a recent history of wars between the states. Here, due to the limitations of data, the category of economic interdependence is omitted. It is suggested that each similarity brings $+1$ and each difference brings -1 to the propensity measure, where selecting equal weights for each category is the least arbitrary way of combining the categories.

In order to describe the dynamics of the aggregations the authors introduced a concept of distance between actors. The distance d_{ij} between a pair i and j represents the choice of the actors to be either in the same coalition – 0, or in opposing coalitions – 1.

Then, the measure of "discontent" of country i in coalition S is defined by $F_i(S) = \sum_j z_j p_{ij} d_{ij}(S)$. Given a configuration S, all country "discontents" sum up to an "energy"

$$E(S) = \sum_{i<j} z_i F_i(S) = \sum_{i<j} z_i z_j p_{ij} d_{ij}. \quad (4)$$

The energy landscape is a graph which has a point in space for each possible configuration and its "energy" (see Figure 3).

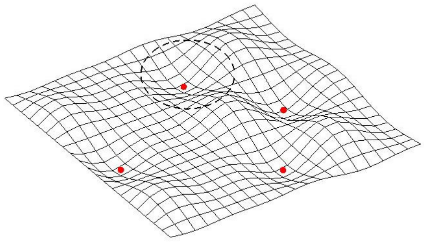


Figure 3: Axelrod and Bennet's energy landscape as a graph. The local minimums are marked by points.

A dynamics of the system is defined by single actor change which decreases the actor's local energy, its "discontent". Thus, the "energy" of the entire system is lowered and wherever the system starts on the energy landscape, it will get to an adjacent local minimum configuration.

The model try to apply the conclusions from Edward-Anderson random bond spin glass, such as minimization of each actor's energy through minimization of the hamiltonian. However, the model is actually situated in between the pure random bond spin glass and the Mattis random site spin glass without frustration.

Theoretical inconsistencies of LA model has been investigated by Galam in [6] and [9] through defining a formal position of the LA model within the field of Statistical Physics. The positioning was defined using variable s_i that associate to actor i, and which is $+1$ if the actor chooses one coalition and -1 if it chooses the opposing coalition. Then, defining $d_{ij} = \frac{1}{2}(1 - s_i s_j)$ allows to replace the "energy" formula 4 by

$$E(S) = E_0 - \tfrac{1}{2} \sum_{i<j} z_i z_j p_{ij} s_i s_j, \quad (5)$$

where $E_0 = \frac{1}{2}\sum_{i<j} z_i z_j p_{ij}$ is a constant depending on sizes. Denoting $G_{ij} = z_i z_j p_{ij}$, the dynamics are defined through $H = -\frac{1}{2}\sum_{i<j} G_{ij} s_i s_j$ which has to be minimized with respect to $\{s_i\}_i^N$, given G_{ij}. This is the Ising model Hamiltonian, and hence, the system reduces to the zero-temperature Ising ferromagnet.

To be a real random bond spin glass, containing frustrations, following lacks in LA model can be remarked:

- Absence of frustration: in spin glass frustration takes place when the total energy is 0, which results in no cost flips. In LA model, countries is defined not to be in frustrated state once a local minima is attained. This, eliminates for example the cases of negative triangles. The effect of "discontent" countries can not replace the frustration.

- Predictability: due to frustration in spin glasses, when several coalition have the same energy, it is impossible to predict the coalitions that will be the attained.

Thus, the LA model reduced to the physical framework above, where the existence of a unique minima implies that G_{ij} must be changeable, is indeed a Mattis model, i.e., a random site spin glass without frustration.

Galam hence suggested a more elaborated model inspired from the idea of spins-like short term interactions between countries exposed in the landscape theory of Axelrod and Bennet. The Extra-territory (ET) model of coalition forming among countries considers both the micro-level propensities coming from the historical frame, and the macro level interactions defining two competing bimodal Extra-territory coalitions, M and C. The parameters are the unchangeable J_{ij} featuring historical propensities between two countries i and j, variable G_{ij} representing benefit aimed interactions enabled by the extra-territory coalitions, and ϵ_i identifying the natural disposition of country i to one of the coalitions. The propensity between two countries i and j is defined as

$$p_{ij} = J_{ij} + \epsilon_i \epsilon_j G_{ij}. \tag{6}$$

In contrast to J_{ij} that can be positive, negative or zero, G_{ij}, as a benefit aimed interactions magnitude, is a non-negative variable.

The associated energy is

$$H = -\frac{1}{2}\sum_{i<j}^{N}\left[J_{ij} + \epsilon_i\epsilon_j G_{ij}\right]s_i s_j - \sum_{i<j}^{N}\nu_i b_i s_i, \tag{7}$$

where parameter b_i is local external field of country i and ν_i which can be $+1$, -1 or 0 directing the field in favor of M, C, or in neutrality.

The right part of the energy expression 7 represents the Edward-Anderson's random bond spin glass and the left part of the sum represents Mattis model. This is the superposition of unstable disorder of spin glass with a stable ferromagnetic order, which represented by itself a novel contour of physical systems.

The parameter $\sum_{i<j}^{N}\nu_i b_i s_i$ accounting form global economic or military pressures, attends to exhibit the multiple contravening tendencies in coalition forming. Thus, correspondence of countries' natural dispositions and global pressures, $\nu_i = \epsilon_i$ results in coherent tendencies in coalition forming, which, assuming $G_{ij} \geq |J_{ij}|$, provides a unique minimum of energy and an optimal configuration. Discrepancy between the values creates incoherent tendencies which leave the system within frustrated states.

Social balance model SB which address the coalition forming based on the Balance Theory, such as [17], represent the social network as a complete graph where, in contrast to the physical models, the local pair relations between actors are constantly changing thus playing the role of shifting states of actors. The model assume no static unchanging parameter such as historical primary bonds, and the actors' bonds are changing values. With this, positioning of the model within the field of Statistical Physics would rather suggest a correspondence to the Mattis model. Those models attain either intermediate stable coalitions with no negative circle inside each, or to utopia with all bonds positive.

The model also do not apply any tools for describing global tendencies, which are constantly present in the modern international interactions. Although SB models are useful to explain social processes based on actors's feeling, attitudes and believes, they cannot be used to address the complex social phenomena such as endless cycling (instabilities in the England-Spain-France triangle), the globally incited stabilizations (creation of the Eurozone), and dissolution with partial return to the initial unstable bonds configuration (the collapse of the Soviet State).

The present literature addressing the coalition forming, through the spin glass models or the social balance models, analyzes the phenomena within a Markovian frame of spontaneous instability. That is, they refer to rationality type limited to observation of an immediate gain.

Our work, in contrast, assumes the presence of long horizon rationality in the actors, which develops further the subject of instability in coalition forming and enable the mixed rationality systems. Such definition, which is more natural for simulation of countries and human actors, allows to explore and study the complex behaviors among individual actors peculiar to the real cases of forming coalitions.

In the physical sense, as it is mentioned in extra territory model of coalition forming, the effect of extensive rationality in spin-like systems appears through heating of the system. The author refers to so called "risking" actors which emerge upon $T > 0$, and adhere to long-horizon strategy simultaneously with their contestants.

It is interesting to observe that, while LA's stable configurations and those of the ET model, are corresponding to the Game Theory equilibrium, the equilibrium guarantees no stability in the extensive rationality model – a model where the agents do not stop searching upon achieving their local maximums.

In the perspective of applications of our model, we resort to a more sensitive propensity evaluation based on scaling of the values with respect to the global relative strength, from the "highly negative" to "highly positive" through mixed ones which are "negative", "neutral-negative", "neutral", "neutral-positive", "positive", with further divisions possible. The relative strengths are then attached with numbers starting from "neutral" as 0.

Another difference is that we study separately two nested models of coalition forming – Natural model (NM) and Global Alliance model (GAM), which represent respectively the random bond spin glass based disordered system and a superposition of the unstable disorder with a free mixture of two stable orders of ferromagnetic states.

In GAM, multiple collective factors which are active and equiprobable, prove simultaneous influence of independent and contravening elements in coalition forming, and affecting the countries' bonds independently from their original situation. The stabilizing effect, is produced through polarization by the of externally-set opposing global principle (the global alliances) of multiple simultaneous collective interest factors.

2 Natural Coalition Forming

The next section formally presents the natural model of coalition forming, followed by a detailed analysis of the instability rooted in the coalition forming as a decentralized maximization processes driven by each country's objective to attain its best coalition allocations. We study the terms of existence of the optimal and stable coalitions, and the ways to reach them in the ranges of extensive or limited rationality. Within these results, we investigate the interesting and little explored phenomena of information manipulation and non-optimal stability peculiar to any network of selfish actors. The theoretical framework is accompanied by analysis of real cases which justify the practical use of the model.

2.1 Natural Model of Coalition Forming

Present policies require the countries to make a choice, to agree or disagree, and thus to ally with one side or the other. Within absence of external incentives, this is based on their historical and geographical background and experience that the countries decide weather to agree or disagree on the policies, for the sake of gain which results from the actors beneficial (or non-beneficial)existence. The agreement turns to be unfavorable to countries which went through a conflict in their past, while in the same way, the disagreement is unfavorable to friendly countries.

Those primary mutual propensities between countries, which are the issues of their historical experience, can hardly be changed at a will and affect all the subsequent interactions and exchanges between the countries. The propensities do evolve with time, yet much more slowly than the coalition dynamics and therefore, as a first approximation, they are assumed to be static.

We keep the discussion with relation to countries, however it applies to any type of rational actors, either economical such as companies, political such as parties, or social such as organizations, people.

2.1.1 Natural Model Definition

Let us briefly remind the definition the *natural model* of coalition forming. Consider a group of N independent countries which had experienced geographic, cultural or economic interactions during their history. The countries are respectively denoted by characters or indicators ranging from 1 to N. Each country thus makes its choice between two competing options $+1$ and -1, corresponding to two possible coalitions. The same choice allies the countries to the same coalition, while different choices separate them into the opposite ones. According to the postulate of minimum conflict, being part of the same coalition benefits the countries with the propensities to cooperate, while countries inclined to conflict, bear loss from cooperation. A country is represented by a discrete variable that can assume one of the two state values $S = +1$ and $S = -1$. The combination $S = \{S_1, S_2, S_3, \ldots, S_N\}$ makes up the *configuration* of the choices of the countries. The configuration of choices, as well as it's inverse $S = \{-S_1, -S_2, -S_3, \ldots, -S_N\}$ by symmetry, represents a particular configuration of coalitions in the system.

Consider any two countries i and j, and denote by J_{ij} the value that measures the degree and the direction of the historical exchanges between the countries. J_{ij} represents the bond of original propensity between the countries, which is symmetric, $J_{ij} = J_{ji}$, and may vary for each pair of countries. $J_{ij} = 0$ when no mutual bond exists between the countries, which represents an absence of a direct exchange. We shall describe by $J_{ij}S_iS_j$ the measure of the interaction between the countries as a function of their choices. In case the countries agree, i.e., when $S_i = S_j$, a positive value of J_{ij} results in the positive effect for both the countries. When J_{ij} is negative, the cooperation between the countries results in a negative effect on both the countries. This describes a system of countries maintaining short range interactions guided by primary mutual propensities.

Thus, the propensities either favor the cooperation ($J_{ij} > 0$), the conflict ($J_{ij} < 0$), or signify the ignorance ($J_{ij} = 0$) where neither the agreement nor the disagreement influences the outcome of the countries.

For the sake of visualization, we represent the system of countries as a connected weighted graph with the countries in the nodes and the bilateral propensities as the weights of the respective edges, (see Figure (13)). We take red (dark) color for $+1$ choice and blue (light) color for -1 choice.

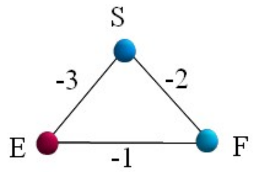


Figure 4: Triangle of three countries connected by negative bond propensities – ESF conflicting triangle.

Note that in the representation of neighbors in the model, we leave the spin glass-standard square lattices and move to the star-like $3d$ shapes.

Sum of the measures of all the interactions of a country i in the system is the net gain of the country

$$H_i(S) = S_i \sum_{j \neq i} \quad J_{ij} S_j. \tag{8}$$

The total gain (or the energy) of the system in configuration S is measured by the total of the contributions in the configuration

$$\mathcal{H}(\mathcal{S}) = \tfrac{1}{2} \sum_i \quad H_i(S). \tag{9}$$

2.1.2 Illustration Through an Historical Example

In order to illustrate a natural model, we suggest to consider historical example of the conflicting triangle of Spain, England and France. The countries were alternatively enemies and allies during a long period of time. For these countries, the period of 1521 – 1604 has been marked by a series of land and sea wars, driven by the historical background of colonization, religion, and by progress in navel technology. The commercial rivalry between England and Spain, and political and religious ambitions of France were the major forces that pushed Europe into wars. The conflicts were usually initiated between any two of the countries, with the third one joining one or other of the sides. Accordingly, the historical background of the countries during this period has defined the particular distribution of mutual negative propensities.

The natural model is shown schematically in the ESF conflicting triangle in Figure (13), where abbreviations S, E and F stand for the countries' names. The choices of the countries S, E and F are represented by the state variables S_E,S_S and S_F respectively. Different colors attached to the state nodes of the countries correspond to the different coalitions of the countries. The state nodes are linked by the following primary propensities $J_{SE} = -3$, $J_{SF} = -2$, $J_{EF} = -1$. The value of propensities are illustrative only, and these are their relative magnitudes of importance: the conflict between Spain and France is less deep than the one between Spain and England, yet it is deeper than the one between England and France.

2.2 Rationality of Countries

During the coalition forming, at any particular configuration, a country may observe another configuration where it can reach a higher gain. When it happens, the country shall take appropriate changes in order to take advantage of this opportunity. The sequence of such changes by all the countries constitutes the process of maximization of the countries' gains.

The depth of observation of a country's benefit improvements depends on its capacity of *rationality*, which associates to the ability not only evaluate the immediate benefit change, but also to envision a way up to a better configuration, which may emerge through intermediate loosing states. Rationality may be extensive, or long horizon, and limited, or short horizon rationality. The shortest horizon corresponds to the immediate improvement, which is peculiar to spin-like actors.

It may seem irrational and risky to make a change opposite to the one of an immediate best configuration, however this is the rationality in the long horizon strategy. The risk is hidden in the fact that an opponent country may possess the equivalent capacity of rationality, as well as that the changes may be performed simultaneously. Which result in causal ambiguity in the process of maximization of the countries' gains.

Extensive rationality, is a new approach to this class of models is peculiar in many type of actors in reality – countries, individuals, economical or political organizations.

The feature of long horizon rationality allows to explain the advantage a country may gain by possessing advanced technologies and knowledge in collection of system's information, such as others' strategies and possible transitions scenarios. It also provides a hint on why in some cases a country having the technological leap could gain interest in sharing its private information with a few others – knowing of existence of a common optimum benefits each involved country.

Limited rationality, on the contrary, bounds the strategic horizon of countries. In the simplest case of limited rationality, spin-like actors, this is a closest local maximum. Figure (5) shows a case of system where for the spin-like actors the process of maximization stabilizes in the local maximum $S_M = +1, +1, -1, -1, -1$. Here, the gain of the triangle's actors decreases immediately at a change which keeps the actors from fluctuating. In such system, rational actors would loop in between all the individual maximal configurations.

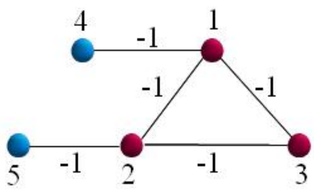


Figure 5: Configuration which is stable in Spin Glass and unstable in the system of countries.

The process of maximization of the countries' gains, which is a decentralized search for the most beneficial coalition setting, is found to produce an instability. When the countries eventually attain a common satisfactory configuration, the instability is temporary. It is a permanent one, when the maximization of the countries' gains is an endless process.

Instability of coalition forming is characterized by constant cycling of coalitions within a set of configurations. Formally, we define the instability as follows:

Definition 1 (Instability of the System of Countries) *The system of countries is said to be unstable if in any configuration of the countries' states there is a* *country which is able to foresee a latitude for individual improvement of the gain.*

2.2.1 Geometric Terms of Instability

Let us recall that in the natural model we extend the notion of instability of the physical systems where it is only caused from frustration of spins due to an immediate improvement or due to no cost of changing. In the broader sense, frustrations due to intended changes aimed to improve the current gains in further steps are also included. Such improvements are peculiar to the system of countries which are able to anticipate and adapt to the changes of others. Spins, unlike countries, are able to evaluate only the effect from the immediate changes. In order to be able to predict such an unstable character within a system of countries, we provide the necessary and sufficient conditions for the instability.

Let us take a close look at the heart of the instability. Consider separately from the rest of the system, two countries i and j connected by their propensity bond p_{ij}. When p_{ij} is the only link between the countries, we can easily valuate the contribution from cooperation or conflict between the countries to their gain: $p_{ij}S_iS_j$. What complicates the evaluation of their alternative choices is the presence of the other countries connecting between i and j indirectly.

By a circle in a graph of countries we understand a selection of nodes linked into a closed path by the propensity bonds. Imagine a situation where the countries i and j are part of a circle, and assume without loss of generality that their mutual propensity is negative while the propensities all along the rest of the circle are positive (see Figure (6)). In this case, both i and j maximize their gain in conflict with the other and in cooperation with the rest of the countries on the circle. Such an arrangement creates an everlasting competition between i and j for this maximizing arrangement. The countries continuously shift their choices, which produces the instability.

We are now ready to identify the instability formally. Denote a circle of countries by $\mathcal{C}$ and the countries composing the circle by $1, 2, \ldots, k$. Then the characteristics of the phenomena of instability in Spin Glasses [2] can be interpreted in the terms

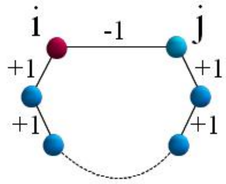


Figure 6: Circle connecting between the countries i and j by a negative and a positive links at the same time produces an everlasting competition between the countries.

of the natural model:

if there is a closed circle of countries on which the product of total propensities is negative,

$$\Pi_{i,j\in\mathcal{C}} \;\; p_{ij} < 0$$

then the system is unstable.

(10)

Terms (10) feature the instability in a system of countries as a direct consequence of geometry of the system – presence of a negative circle. We call (10) geometric terms of instability. The terms enable to estimate and predict an unstable character of system of countries. Those are the necessary and sufficient conditions of the rational instability.

The geometric instability terms, while originate from physics, provide only the necessary condition for the instability in the Spin Glass. The Spin glass with a negative circle can be stable, when, for example, an interaction with a neighbors spin, shifts the energy of the spin from zero to negative and thus keeps the spin from fluctuating. This case is illustrated in Figure (5).

In the systems of countries, terms (10) provide also a sufficient condition of the instability, that is the condition of endless competitions among the countries for the beneficial coalitions.

The geometric terms requires to examine all the possible closed paths in the graph of the system in order to identify if there is the instability in a system. At the same time the terms provide no information on the competitions between the countries.

2.2.2 Analytical Terms of Instability

Let us study the mechanisms standing behind the choices the countries make, and the nature of the competition between them.

A choice of country i realizes one of two possible system's configurations S^{+i} and S^{-i} which differ by the country i's choice S_i. Moreover, by making the choice the country keeps half of configurations that can be realizes potentially in the further steps. In order to make a choice and select among the two configurations, the country needs to classify the configurations by the order of preference based on the potential gain.

Thus, the key parameter for a country to make a choice is the order of preferences over its feasible configurations. In mathematics, a preference order over a set of elements is defined by a binary relation which determines a collection of ordered pairs of elements of the set. Each country i's function of gain H_i sets the binary relation of preference $\leq_i$ over the set of all system's configurations as follows: configuration S' is less preferred than S'', $S' \leq_i S''$ if $H_i(S') \leq H_i(S'')$. The total gain in the system $\mathcal{H}$, in turn, sets the relation $\leq_{sys}$ where $S' \leq_{sys} S''$ if $\mathcal{H}(S') \leq \mathcal{H}(S'')$. Each of the relations $\{\leq_i\}_{i=1}^{N}$ and $\leq_{sys}$ defines a partial order over the set of configurations – the order that indicates a relation between some elements, while others remain unrelated. For each particular partial order, those unrelated configurations yield the same gain to the country and therefore belong to the same equivalence classes. The equivalence classes thus are strictly ordered from minimal to maximal.

Observe that the symmetry between a configuration and its inverse leaves half of the possibilities of different coalitions (that is, 2^{N-1}), and at most half of the possibilities of different gains. The configurations are included in the same equivalence class.

Note that from the point of view of Statistical Physics, the configuration reversal, or a shift to another configuration from the same equivalence class, is instant and leaves the system's energy identical. The two configurations can not be discriminated in physical system. However, both the configurations can actually be realized in the systems of actors, and transition between them physically exists. Those systems deal with all the 2^N different configurations.

2.2.3 Optimal Configurations

The maximal configurations of a country are contained in its maximal equivalence class. Similarly, the system's maximal configurations are in the system's maximal equivalence class. Consequently, the common maximal configurations, which are those that satisfy all the countries, lie in the intersection of the maximal equivalence classes of all the countries. If we denote by $C_i^{\gamma_i}$ an equivalence class of country i corresponding to gain γ_i, and by $C_i^{\Gamma_i}$ the maximal equivalence class, then the set of common maximal configurations is determined by

$$\mathcal{S}^{\Gamma_i} = \cap_{i=1}^{N} C_i^{\Gamma_i}. \tag{11}$$

If $\mathcal{S}^{\Gamma_i} \neq \emptyset$, then the countries share the same maximal configuration in the system. The configuration is an *optimal configuration*, the one that satisfies all the countries and guarantees stability of the system.

Statement 1 *Formula (11) provides new terms of the instability reading that if*

$$\cap_{i=1}^{N} C_i^{\Gamma_i} = \emptyset \tag{12}$$

then the system is not stable in any configuration.

These are the analytical terms that, along with the indication of system's stability, provide particular optimal configurations, as well as the respective values of gains to the countries.

Within these terms, the phenomena of instability is characterized by the fact that, a priori, preference orders of different countries do not coincide, and so their maximal coalitions.

Statement 2 *Let us remark that when an optimal configuration exists, it is unique up to inverse.*

Assume, by contradiction, that there are two different optimal configurations which however are not the inverse of each other, $\mathcal{O}1$ and $\mathcal{O}2$. Then, there is at least one country whose states differ in the two configurations, and another country whose states are equal in the configurations. As far as we consider a connected system, there are necessarily two such countries which are connected by a propensity bond. Denote them by i and j. Then, $S_i^{O1} = -S_i^{O2}$ and $S_j^{O1} = S_j^{O2}$.

Assume, without loss of generality, that $J_{i,j} = J > 0$. Since $\mathcal{O}1$ and $\mathcal{O}2$ are both optimal then $\mathcal{H}_i(\mathcal{O}1) = \mathcal{H}_i(\mathcal{O}2)$. Since they differ by the states of i, there is country k such that $J_{i,k} = -J$ and the following holds: $\mathcal{H}_i(\mathcal{O}1) = \mathcal{H}_i(\mathcal{O}2) - J_{i,j} + J_{i,k}$ and $\mathcal{H}_i(\mathcal{O}2) = \mathcal{H}_i(\mathcal{O}1) + J_{i,j} - J_{i,k}$. Then, either there is a negative circle connecting i, j and k, or there is another configuration $\mathcal{O}3$ such that $\mathcal{H}_i(\mathcal{O}3) = \mathcal{H}_i(\mathcal{O}1, \mathcal{O}2) + J_{i,j} + J_{i,k}$. Which is a contradiction to the maximality of $\mathcal{O}1$ and $\mathcal{O}2$.

2.2.4 Illustration of Stable and Unstable Systems

Consider the example having no optimal configuration.

Example 1 (Unstable System)
Consider again the traditional conflicting triangle of Spain, England and France as in Figure (13). We feature the instability of the triangle from the perspective of the countries preference orders over the configurations.

Each configuration is of the form $S = (S_E, S_S, S_F)$. The 8 different configurations in total represent 4 different coalitions. The functions of the countries' gains which are $H_E = -3S_ES_S - S_ES_F$, $H_S = -3S_SS_E - 2S_SS_F$, and $H_F = -S_FS_E - 2S_FS_S$ produce the following equivalence classes of the countries:

The equivalence classes of E:

$$\begin{aligned}
\Gamma_E = 4 \quad &: C_E^4 = \{(-1, +1, +1), (+1, -1, -1)\}, \\
\Gamma_E = 2 \quad &: C_E^2 = \{(+1, -1, +1), (-1, +1, -1)\}, \\
\Gamma_E = -2 \quad &: C_E^{-2} = \{(+1, +1, -1), (-1, -1, +1)\}, \\
\Gamma_E = -4 \quad &: C_E^{-4} = \{(+1, +1, +1), (-1, -1, -1)\}.
\end{aligned}$$

The equivalence classes of S:

$$\begin{aligned}
\Gamma_S = 5 \quad &: C_S^5 = \{(-1,+1,-1),(+1,-1,+1)\},\\
\Gamma_S = 1 \quad &: C_S^1 = \{(+1,-1,-1),(-1,+1,+1)\},\\
\Gamma_S = -1 \quad &: C_S^{-1} = \{(+1,+1,-1),(-1,-1,+1)\},\\
\Gamma_S = -5 \quad &: C_S^{-5} = \{(+1,+1,+1),(-1,-1,-1)\}.
\end{aligned}$$

The equivalence classes of F:

$$\begin{aligned}
\Gamma_F = 3 \quad &: C_F^3 = \{(-1,-1,+1),(+1,+1,-1)\},\\
\Gamma_F = 1 \quad &: C_F^1 = \{(+1,-1,+1),(-1,+1,-1)\},\\
\Gamma_F = -1 \quad &: C_F^{-1} = \{(-1,+1,+1),(+1,-1,-1)\},\\
\Gamma_F = -3 \quad &: C_F^{-3} = \{(+1,+1,+1),(-1,-1,-1)\}.
\end{aligned}$$

As we can see, there is no common maximal configuration in the ESF triangle since the intersection of the maximal equivalence classes of the three countries is $C_E^4 \cap C_S^5 \cap C_F^3 = \emptyset$.

The system thus shifts between the individual maximal coalitions infinitely, driven by each country's endless search for its best position.

Figure (7) shows series of transitions in the triangle that cycles, exhibiting the endless instability.

The progression starts from $S = (+1,+1,+1)$, which is the worst configuration for all the countries. At the first step, E changes its state to -1 which yields the country an immediate benefit of $\Gamma_E = 4$. This change shifts the countries to the equivalence classes C_E^4, C_S^1 and C_F^{-1}, respectively. At the second step, country F make change and moves to C_F^1 equivalence class. Then, at the step III, E makes a change aiming to get back its best configuration in some further step. Finally, at the step IV, country S changes to -1, which brings the countries back to the original equivalence classes C_E^4, C_S^1 and C_F^{-1}.

Let us now turn to the example of a system where an optimal configuration exists.

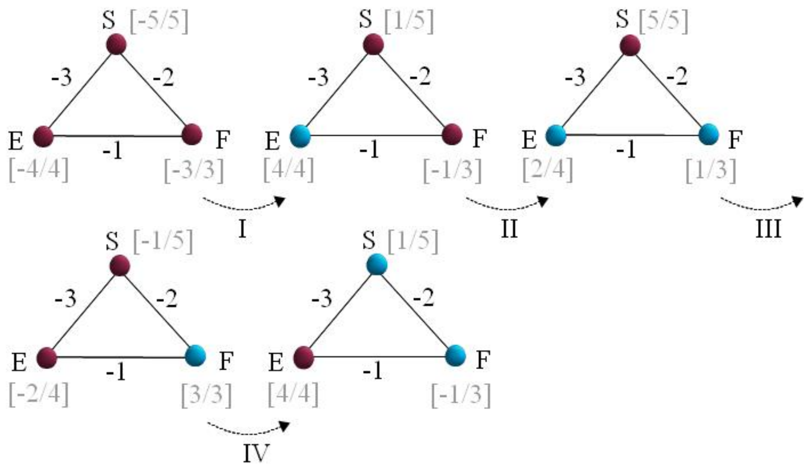


Figure 7: The diagram of successive configurations transition in the ESF conflicting triangle.

Example 2 (Stable System)

We consider a modified ESF triangle, see Figure (8). A snap shot of the short period in the midst of war against Spain, when England and France were favorable to each others, illustrates a system where an optimal configuration exists.

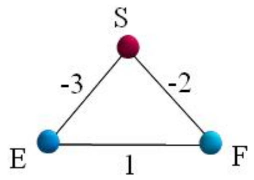


Figure 8: A stable system. Within the given distribution of the propensities, the optimal configuration of coalitions is $\{S\}$ and $\{E, F\}$ which guarantee the stability in the ESF triangle.

The maximal possible gains of E, S and F are $H_E = 4$, $H_S = 5$ and $H_F = 3$, respectively, and the maximal equivalence classes of the countries coincide: $C_E^4 = C_S^5 = C_F^3 = \{(+1, -1, +1), (-1, +1, -1)\}$. Consequently, there are two optimal configurations in the system, $C_E^4 \cap C_S^5 \cap C_3^2 \cap C_F^3 = \{(+1, -1, +1), (-1, +1, -1)\}$, each of which guarantees its stability.

2.3 Rational Maximization of Countries

The law of coalition forming among countries is determined by the character of search for the countries' maximal individual gains, the rational maximization. The rational maximum is not always an optimum, as we can see below, but a configuration satisfactory to each of the participant countries according to the strength of their capacity of rationality. It is where the rational maximum is attained the coalition is formed.

The theoretical framework for the rational maximization process is a tree-like structures that encode the configuration information and the configuration transitions for each country. The depth of each tree reflects the capacity of rationality of the respective country. Within the framework, we study the dynamics of the maximization process and formulate the law of coalition forming as a condition of termination of the process.

2.3.1 Maximization As a Sequence of Individual Choices

As we have mentioned, maximization is a sequence of the changes made by the countries that aim to improve their benefits. In our model, where the countries are individual actors possessing rationality, a change consists of rational components such as the observation and the decision making. Let us examine the mechanisms that stand behind those changes.

At any configuration S, a country i can make a choice for its state. The alternative is either to keep the current state, or to invert it and thus to move to configuration S^{-i}. Making the choice consists of two successive phases – observations and decision making. The phase of observation includes evaluation of the equivalence classes, estimation of immediate improvements, and searching for progressions to the beneficial configurations.

The phase of the decision making, in turn, includes selection of an immediate reply based on the results obtained at the observation phase. Denote by $O_i(S)$ the set of desirable configurations and by $D_i(S)$ the immediate reply, then $D_i(S) \in \{S, S^{-i}\}$. The decision can be based either on the *best reply* principle, which accepts the choices that improve the gain immediately, or on the *forecast* principle, which aims in improvement in further steps. When the alternatives are equivalent, the decision can be guided by the principle of *anticipation of feedback* which take into account the effect from the change on the other countries, or by the principle of *random choice.*

The following example illustrates the series of configurations transitions in the ESF conflicting triangle.

Example 3

Figure (9) illustrates four transitions from an initial state to the resulting configuration in the ESF conflicting triangle. Each transition is the result of particular combinations of observations and decisions made by the countries in the triangle.

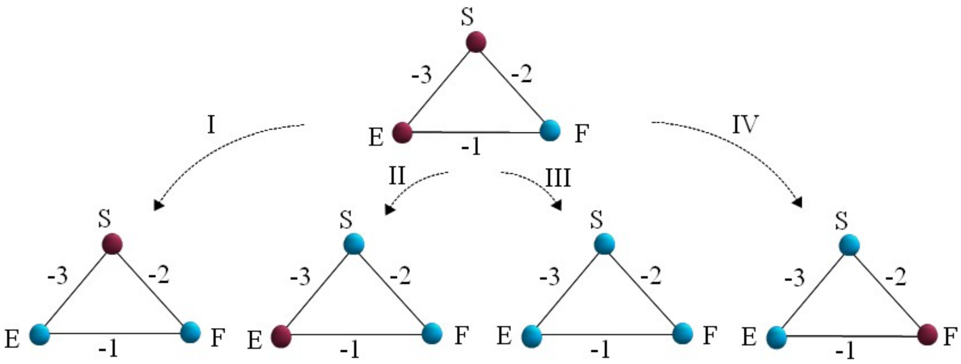


Figure 9: The diagram of transitions of configurations prompted by the decisions in the ESF conflicting triangle.

The initial configuration $S = (+1, +1, -1)$ *belongs to the equivalence classes* C_E^{-2}, C_S^{-1} *and* C_F^3 *of countries* E, S *and* F*, respectively. The transitions are marked by* $I - IV$ *in the figure. Let us analyze the transitions, their effects on the countries and on the system as a whole.*

- T_I: *Observation by* E *results in* $O_E(S) = \{(+1, +1, -1), (-1, +1, -1)\}$ *where configuration* $(-1, +1, -1)$ *is for the immediate improvement and* $(+1, +1, -1)$ *is for the improvement in a further step, which will eventually lead to the* E*'s maximal configuration* $(+1, -1, -1)$*.* E *makes the decision* $D_E(S) = (-1, +1, -1)$ *and thus moves the system to the equivalence classes* C_E^2, C_S^5 *and* C_F^1*.*

It is interesting to observe that, while the change satisfies only country S, it increases the energy of the system as a whole; the resulting configuration is the system's maximum.

T_{II}: *If the change is done by country S that makes the decision $D_S(S) = (+1,-1,-1)$, the resulting configuration belongs to the classes C_E^4, C_S^1 and C_F^{-1}.*

Though the configuration increases the energy of the system, it is not the system's maximum.

T_{III}: *If the change is performed by both the countries E and S simultaneously, with $D_E(S) = (-1,+1,-1)$ and $D_S(S) = (+1,-1,-1)$, the resulting configuration is $S = (-1,-1,-1)$ which belongs to the equivalence classes $C_E^{-4}, C_S^{-5}, C_F^{-1}$.*

This is the worst case for each of the countries, as well as for the system itself.

T_{IV}: *When changes are made by all the countries E, S and F simultaneously, the resulting configuration is $(-1,-1,+1)$. It belongs to the initial equivalence classes C_E^{-2}, C_S^{-1} and C_F^3. Transition IV thus forms a cycle.*

The results of the countries' decision making, the possible transitions of the system from a configuration to another are equiprobable: each country can decide either to make or not to make the change at any time.

2.3.2 Rationality Tree

From the point of view of a given country, the choices made by the others are the direct or indirect outcomes of the country's own decision. Such dependencies between the country's choice and the decision of the others, define a tree-like construction containing the information about configurations for each given country.

The *rationality tree* has a centralized theoretical structure that contains all the possible configurations in the system and encodes all the possible transitions between the configurations. For any country i, its tree is built as follows. At the root there is the initial configuration S of the system. The root splits into two *choice nodes* corresponding to the two possible choices of the country $+S_i$ and $-S_i$. Each of the nodes splits into 2^{N-1} *configuration nodes*, which are the possible decisions by the other countries. Starting from the configuration nodes, the tree develops the same way as it does from the root. The choice nodes of the tree continue to develop until they correspond to a configuration that has already appeared in a configuration node of the tree.

A country's rationality tree encodes transition progression of any feasible maximization processes from any initial configuration. Such informational structure enable the country to perform thorough observations, on the basis of which the country can builds a reply strategy starting from any initial configuration.

Let us look at the rationality tree of England in the ESF conflicting triangle of Figure (13).

Example 4 (Rationality Tree of England in the ESF Conflicting Triangle) *Figure (10) shows the rationality tree of E. The configuration nodes are marked by red (dark) color until they, or their inverses, reappear in the tree, at which point they are marked by a green (light) color. The values of the corresponding countries' gains are shown next to each configuration node. The choice nodes are marked in light blue.*

The lines proceeding from a configuration node correspond to the choices made by E and the dashed lines drawn from the choice nodes represent the transitions initiated by the others.

Observation includes searching for the transition pathes that bring to the best configurations. A key characteristic of such procedure is the lower bound on the complexity of the search, which is the amount of time required by the most efficient search algorithm. When the search is performed through the rationality tree, the lower bound on the complexity is determined by the size of the tree. This implies that those rational procedures have the complexity that grows exponentially as the system grows. Indeed, since only one among any two symmetric configurations is developed in the rationality tree, and each of the respective configuration nodes is followed by two choice nodes, the tree contains $\mathcal{O}(2^N)$ nodes.

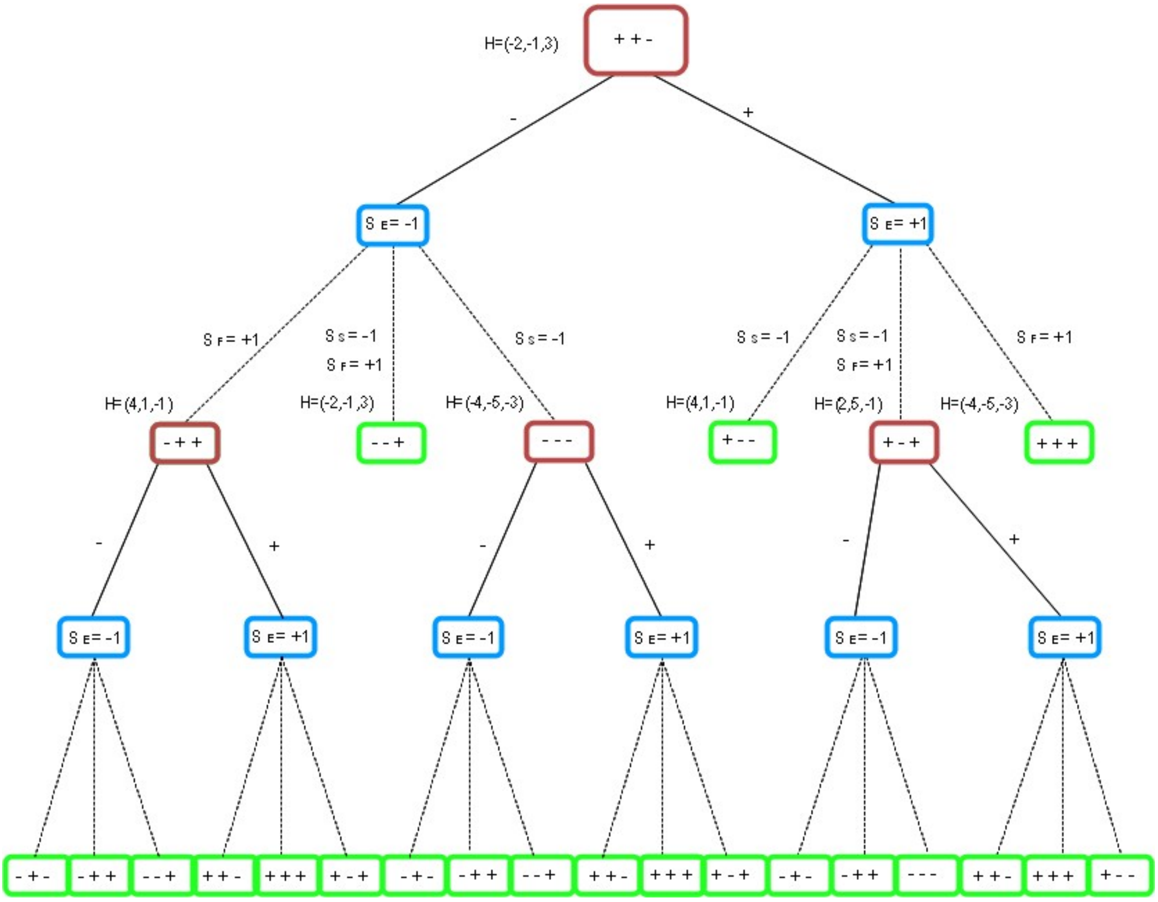


Figure 10: Rationality tree of England in the ESF conflicting triangle. In the tree, the configuration nodes are alternated with the choice nodes followed by the decision of the country. The root represents the initial configuration.

2.4 A Formal Implementation of Rational Maximization

A system of countries represents a decentralized multi-agent system of individual actors separated by geographical and cultural distances. The geographical location and the initial conditions predetermine different capacities and different degrees of impact by technological development. In such system, those distances and differences between the countries result in diverse levels of exposure to the system information, as well as in divers technical capacities of observation and decision making.

Therefore, in practice two contrast conditions of countries can be distinguished: condition where the complete system information is available to the countries and they have extensive technical capacities, and condition where the country have limited system knowledge and limited capacity. The limited knowledge of a country prevents proper observation of the other's states and decisions. The limited capacity restricts a country's ability to build the system information and to track the maximization processes. Both limitations perturb the procedure of observation by the country which introduces limitations in their rationality. The limitations become more distinct with the increase of the system's size.

Another restriction in this concern is the latency in the decision making which affects the quality of the decision output. Here, we ignore this restriction as an issue of a different subject.

In the scope of the classification of capacities, the following cases can be distinguished in practice.

I: Imagine a system which has optimal coalitions and where the countries have extensive technical capacities. The maximization process in such system converges to an optimization.

The correct configurations information available to the countries and their extensive abilities guarantee the proper observation and decision making. With the correct observation and decision making, the uniqueness of the optimum results in rapid convergence to a stable state with hight probability.

However, systems that stabilize spontaneously are rare. The probability that the system has an optimum vanish exponentially with the size.

II: The system with no optimum is more likely to happen. In this case, the complete knowledge available to the countries and the absence of the common satisfactory state produce the infinite competitions for their individual beneficial coalitions. Those competitions trap the maximization process into an infinite cycle.

III: The most real is the situation when there is no optimal state in the system and both the cases of limited and extensive knowledge present. In this case, the rationality limitations influence the maximization in the way that the process either follows infinite competitions or converge to a finite stabile state. The latter case is investigated in the next section.

2.5 Non-optimal Stability

In order to attain the maximal benefit with high probability a country must be able to forecast the behavior of the others. The forecast principle consists of screening the possible configuration transitions that can eventually lead to the maximal gain of the country. Then, it is not enough for a country to be only aware of its neighbors. In order to complete a proper forecast, the country needs to keep in view all the dependencies and bonds of the other countries linked to it indirectly.

An exact and full forecast screening by a country is necessarily based on the structure of its rationality tree. Then, the degree of completeness of the tree predefines the forecast ability of the country. Since the countries with limited system knowledge and limited capacity are unable to build the complete information trees; in a moment they may be able to foresee the maximum when if it is on their rationality horizon, however as soon as the configuration changes, they may no more see an appropriate improvement strategy. As result, they are unable to perform a proper forecast of the common behavior and make planned choices. Application of the procedures within such rationality limitations lead to wrong choices by the countries despite of their good will.

2.5.1 Rationality Limitations

Let us see an example of system which with countries having optimal configuration but have been stabilized in a local maximum in Figure (11).

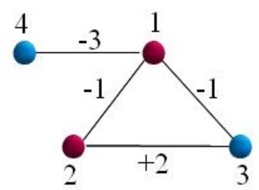


Figure 11: Countries 1, 2 and 3 are spin-like actors which are able to undertake only immediate improvement. This is the local maximum configuration where any individual change by those countries lead to immediate loose. However the change by country 1 will lead immediately to an optimal configuration with 4 switching its state.

In the figure, countries 1, 2 and 3 are spin-like which are able to undertake only immediate improvements. In the given configuration is a local maximum, where any individual change by those countries lead to immediate loose. If country 1 changes it looses -1 of its gain. However this change leads immediately to an optimal configuration with 4 shifting its state. This lost of -1 represent a rationality, or energetic, barrier between the two maximums.

The example reminds the well known game theoretic Prisoner's Dilemma. The game theory equilibrium is the local maximum configuration, and the optimal configuration represent an unattainable outline. The optimum can be attained only when the actors, at least one of the three, posses extensive rationality.

2.5.2 Formal Implementation of Non-optimal Stability

In practice, in cases of the limited rationality, the rationality itself may cease to be the criterium of choices and is substituted by inspirations of all kind, such as religious, moral or cultural codes.

Therefore, two coexisting types of countries can be distinguished in such systems: 1) countries possessing the complete system information and extensive capacities, 2) countries who have a limited system knowledge and a limited capacity.

While the knowledgeable countries forecast and make choices that choices that should benefit them in future steps, the limited countries tend to adhere to more reliable strategies. When the rationality is the criterium of choices, the limited countries, having no enough knowledge to conclude on existence of a common maximum and being unable to forecast the future steps, are interested to stay within the set of their local maximums. The countries therefore adhere to the policy where they undertake changes only upon an immediate improvement.

On this basis, we refer to the countries possessing the complete system information and extensive capacities as primary actors, and the countries who have a limited system knowledge and a limited capacity as secondary actors.

It is reasonable to assume that the configurations providing local maxima to the limited countries can be conducted by the knowledgeable ones that aims to achieve their best benefits. Such an influence is the essence of information manipulation phenomena.

The information manipulation can be described as acting to provide wrong or partial information while pursuing certain objectives. The information manipulation can be viewed as a natural consequence of the extended rationality of the primary actors.

The combination of knowledge and ignorance may bring stabilization of the system despite the negative circles. Below we illustrate the system that stabilizes regardless of lack of the optimum.

Example 5 (Information Manipulation and Non-optimal Stability) *Here, we present a system of four countries (see figure (12)) forming two triangles, a stable and an unstable, where the latest is identical to the ESF conflicting triangle.*

Assume that country 1 *is a primary actor who possesses the complete system information and the extensive capacities, while all the others are the secondary actors having limited system knowledge and limited capacities. The primary actor's node is emphasized by the additional circle around the nodes.*

The initial configuration is only profitable to country 3*. Therefore, country* 1 *initiates a maximization process by first inverting its state to* $+1$ *(step I), and then convincing* 2 *to invert it's state too asserting that* 3 *going to switch to* $S_3 = -1$ *(step II). As a result, country* 3 *changes to* $S_3 = -1$ *seeking an immediate improvement. The resulting configuration is the maximal one for* 1 *and is the stable one, as long as country* 3 *is unable to lead a profitable maximization process.*

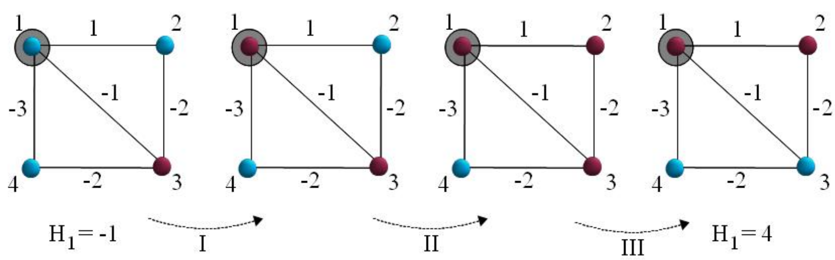


Figure 12: Stabilization of a system having a negative circle by the information manipulation.

The unlimited rationality give to the primary actor (or to group of actors having their own common maximum) an advantage over the others to govern changes leading to profitable coalitions. The widespread belief that keeping the information is secret benefits country is fully justified in this case.

However, as soon as a secondary actor supplements the lack of the information with the data gained by tracking the maximization history or from the information exchange, the non-optimal stability breaks. Therefore, a non-optimal stable state is rather a temporary one, as has been evidenced by history.

The picture changes when an optimum exists. Once the existence is known, the complete information is in the common interest of all countries. In this case, the contrary to the widespread belief about advantage of keeping information secret is valid: disclosing information helps the countries to achieve the common profitable coalitions.

The phenomena of information manipulation is intrinsic to any system of individual actors where the respective capacities vary. As we have noted, the frame of the natural model is applicable to the large spectrum of social, political or economics domains, from the dynamics of social opinion to the dynamics of interactions between public and private banks.

3 Global Alliance Model of Coalition Forming

Here we address the role of global alliances in forming of stable coalitions among the countries as rational actors. The global alliance model of coalition forming starts from a global principle which represents external incentives polarizing the whole of countries into two opposing alliances. The polarization is with regards to a particular collective factor inherent in countries, such as economical, political, religious or other key factor. The mechanism behind such polarization consists of emergence of two opposing global alliances and awakening of countries' individual dispositions to those alliances.

The dispositions, being the issue of the countries' individual tendencies and experiences, are a priori independent of given primary bilateral propensities and represent rather their pragmatic interests. The countries associate themselves to one or to the other global alliance which produces a new kind of interactions – profit based couplings. In contrast to the primary interactions that produce spontaneous coalitions, those exchanges – cooperations or conflicts, being based on the directed view of the countries's interests, are planned and intended. The new interactions supply additional, incremental or decremental, values to the countries mutual propensities allowing new allocations of the coalitions.

In real cases, coalitions in a collective of individual rational actors, when formed spontaneously are rarely stable. The probability that the system becomes stable vanishes exponentially with the size of the system. Therefore, stabilization among countries or other rational actors is more likely to happen under the external incentives of global alliances.

As often in real cases, limited and complete rationality actors are mixed in the system. The global alliances based motivations necessary to stabilization are loosened since negative circles condition is not sufficient to the instability. For the sake of simplicity and in order to respond to the most demanding case of stabilization, we assume the extensive rationality of the countries. While in this interpretation the instability is not dependent on the values of primary propensities, the stabilizing effect from the globally generated additional propensities on the stability is the value-dependent.

3.1 Global Alliance Model Formal Settings

Let us define the global alliance model (GAM) formally. One of the global alliance, denote it by M, unifies the countries that support the particular global principle, while the opposing global alliance denoted by C unifies its opponents. Given N countries $\{1,\ldots,N\}$, country i's individual disposition to the alliances is expressed through its parameter of *natural belonging* ϵ_i. The natural which is $\epsilon_i = +1$ if the country has individual disposition to alliance M, or is $\epsilon_i = -1$ for C.

Given the parameter of natural belonging of country i, its choice among the two possible state values $S_i = +1$ and $S_i = -1$ determines the country's affiliation: by $+1$ the country affiliates with alliance M and by -1 it associates with alliance C. A particular association of two countries i and j with the alliances creates new interactions between them.

The magnitude of the interaction is determined by the countries' mutual effect from the global principle. The direction of the interaction, cooperation or conflict, is determined by the parameter of natural belonging of the countries: if the countries tend to opposing alliances, this interaction will be negative as soon as the countries ally to one and the same alliance.

It is necessary to precise that in this presentation the exchange amplitudes are assumed to be unchanged. The interactions thereby continue the two main properties of the historical spontaneous interactions: they are unchanged and are of short range nature.

With G_{ij} that denotes the interaction amplitude and ϵ_i and ϵ_j that denote the parameter of natural belonging of the countries, the globally motivated interaction defines an additional propensity between the two countries $G_{ij}\epsilon_i\epsilon_j$. With respect to its value, the propensity favors either cooperation or conflict, influencing accordingly the overall propensity between the countries:

$$p_{ij} = J_{ij} + \epsilon_i\epsilon_j G_{ij}. \tag{13}$$

Thus, the net gain of country i under the influence of the global alliances is equal to

$$H_i = S_i \sum_{j \neq i} \; (J_{ij} + \epsilon_i \epsilon_j G_{ij}) S_j. \tag{14}$$

Formula (13) suggests that in the presence of external incentives of the global alliances the couplings between the countries obtain new guidance, which may allow existence of an optimal configuration – a common maximum. It also implies that in the new scheme, the countries adjust their states to the best benefit, not solely based on the spontaneous reactions, but with an additional regards to a planned profit.

3.2 Stabilization By Additional Factors

By additional factors we understands the interests inherent in countries which are different from the original primary bilateral setting. Here, we address the systems of rational actors that have no optimal configuration of actors' states and where, as result, the spontaneous stabilization can not be attained. In such systems, the global alliances based interactions allow stable coalitions among the actors while keeping the short-range nature of the couplings. These couplings, being a complex superposition of the additional interactions on several distinct collective factors, must satisfy particular stability constraints.

3.2.1 The Uni-Factor Stabilization

Consider two opposing global alliances M and C in a system of N countries. A particular collective factor of countries interests produces particular dispositions of the countries to the global alliances. With appropriate amplitudes of the produced interactions, the new scheme enables stabilization among the countries – the *uni-factor stabilization.*

With respect to a unique factor of countries interests, the necessary and sufficient condition of stability formulated upon the analytical terms of instability in the natural model (10)) reads that

$$\text{\textit{A system is stable if and only if for any circle } } \mathcal{C} \text{ \textit{in the system,}} \quad \Pi_{i,j\in\mathcal{C}} \;\; (J_{ij}+\epsilon_i\epsilon_j G_{ij}) \geq 0. \tag{15}$$

We state the existence of a stable coalition within the global alliance model.

Statement 3 (General Property of System Stabilization) *For any global alliances, which are determined by the global principle and the natural belonging parameters, there exist a collection of additional interaction amplitudes* $\{G_{ij}\}_{i<j}$ *that satisfies the system's stability condition 15.*

In other words, any pair of opposing global alliances, regardless of the global principle and of the natural belonging parameters, enables a stable coalition among countries.

In order to prove this statement, let us first observe that the product of the additional propensities $p_{ij}^{\mathcal{G}} = \epsilon_i\epsilon_j G_{ij}$ on any circle is always positive. Indeed, given circle $\mathcal{C}$,

$$\Pi_{i,j\in\mathcal{C}} \;\; G_{ij} \;\; \epsilon_i\epsilon_j = \Pi_{i,j\in\mathcal{C}} \;\; G_{ij}(\epsilon_1\epsilon_2\epsilon_3\ldots\epsilon_k)^2 = \Pi_{i,j\in\mathcal{C}} \;\; G_{ij}.$$

This implies that on any circle, the number of negative couplings produced by the global alliances is even. If the system is unstable, then there is at least one negative circle. We define the new interaction amplitude as follows. For each couple i, j whose $\epsilon_i\epsilon_j < 0$ we take $G_{ij} = 0$ if the primary propensity is negative and $G_{ij} = 2|J_{ij}|$ for the positive original coupling. When $\epsilon_i\epsilon_j > 0$, we take $G_{ij} = 2|J_{ij}|$.

Making the new propensities negative for the negative global couplings and positive for the positive ones, guarantees that there is an even number of negative couplings on the circle. This remains invariant for each circle in the system, which implies that the construction produces non-negative product on any circle in the system. The stability condition (15) holds true which concludes the proof of the statement.

3.2.2 The Case of The England-Spain-France Triangle

A typical examples of the uni-factor stabilization is stabilization of the triangle of England, Spain and France (13) during historical events of 1584 [36].

Example 6 (Stabilization in the ESF triangle with the Religious Factor) *Against the background of sequence of wars in the old Europe, the countries attained stability when in* 1584 *Catholic Spain and France formed an alliance against Protestant forces, the most notable of which were settled in England.*

In order to illustrate historical example using the GAM, we describe the propensities between the countries from "negative" to "positive" through mixed ones. Attaching to them numerical values with respect to their relative strength, taking "neutral" as 0*.*

Accounting for the historical relationship between England, Spain and France, we take the propensities as "neutral-negative", "negative" and "highly negative". There numerical interpretations, as shown in Figure (13), are arbitrary values that aim to account for a relative strength of the interactions.

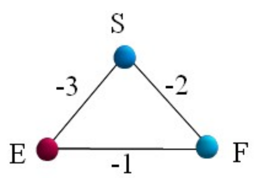


Figure 13: Triangle of England (E), Spain (S) and France (F), the ESF-conflicting triangle.

By M and C we denote the two opposing global alliances – the countries in M choose unification into a "European union" and those in C are against the unification. With respect to the religious factor, Catholic Spain and France were naturally associated to M ($\epsilon_S, \epsilon_F = 1$), while Protestant England was associated to C ($\epsilon_E = -1$). Then, $\epsilon_S\epsilon_F = 1$, $\epsilon_E\epsilon_S = \epsilon_E\epsilon_F = -1$, and the overall propensities between the three countries are:
$p_{SE} = -3 - G_{SE}$, $p_{EF} = -1 - G_{EF}$ and $p_{SF} = -2 + G_{SF}$.

Solving the inequality

$$(-3 - G_{SE})(-1 - G_{EF})(-2 + G_{SF}) \geq 0 \tag{16}$$

yields the constraint the new interaction amplitudes G_{SE}, G_{EF}, G_{SF} *must satisfy in order to stabilize the triangle. Since* G_{EF}, G_{SE} *and* $G_{SF} > 0$, *the only root of the respective equality is* $G_{SF} = 2$. *The solution space is* $G_{SF} \geq 2$, *as depicted in Figure (14), represents a three-dimensional space of the independent additional propensities.*

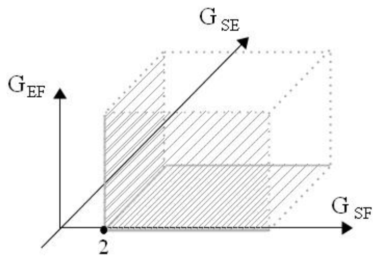


Figure 14: Three-dimensional solution space of the independent additional propensities in the uni-factor stabilization of the ESF- triangle of conflicts.

In the historical example, coalition of Spain and France against England implies that the amplitude of their new interaction belonged to the solution space. The respective stable configuration is $S = (+1, -1, -1)$, *as shown in Figure (15)) where* G_{EF}, G_{SE} *are taken to be* 0 *and* G_{SF} *to be* 3, *so that the corresponding total propensities become* -1,-3 *and* 1.

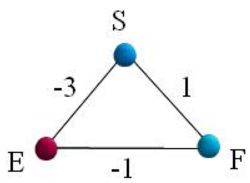


Figure 15: The global alliances model of the ESF-triangle stabilized with the religious factor in configuration $S = (+1, -1, -1)$. Here, $G_{EF} = G_{SE} = 0$, $G_{SF} = 3$, so that the respective resulting total propensities are -1,-3 and 1.

3.2.3 The Multi-Factor Stabilization

Taking into account only one collective factor would be too restricting – along with religious interests, the global principle may impact economical, ecological, moral, political or any other interest and concern. Distinct interests simultaneously influence the interactions between the countries in different ways. They modify the countries' propensities by aggregating the corresponding independent interactions – economical, political and others.

Moreover, we can observe that:

Statement 4 (Uni-factor Reduces to Natural Model) *Any system of countries in the global alliance model with a unique collective factor is reducible to a stable system represented through the Natural Model.*

Indeed, given a system in the GAM, let us define the new state variable to be $\tau_i = \epsilon_i S_i$. The variable takes a value of $\{+1, -1\}$. Then, the hamiltonian H_i of country i can be written in the terms of the new state variables as

$$H_i = \sum_{i \neq j} (J_{ij} S_i S_j + G_{ij} \epsilon_i \epsilon_j S_i S_j) = \sum_{i \neq j} (J_{ij} \epsilon_i \epsilon_j + G_{ij}) \tau_i \tau_j.$$

Here, since G_{ij} is positive, the choice of $\{G_{ij}\}_{i,j}$ such that $G_{ij} > -J_{ij} \epsilon_i \epsilon_j$ satisfies the stability condition 15 and thus produces the propensities that guarantee a stable system.

Let us define formally the multi-factor form of the GAM through two coexisting collective factors, denoted by $\mathcal{G}$ and $\mathcal{K}$ respectively. Within each factor, a country has independent natural disposition to the global alliances. Therefore, each country has two independent natural belonging parameters associated with these factors. For country i, this is $\epsilon_i = +1$ if within factor $\mathcal{G}$ the country naturally belongs to M. Similarly, $\beta_i = +1$ within factor $\mathcal{K}$. For the global alliance C, $\epsilon_i = -1$ and $\beta_i = -1$ respectively.

We denote by G_{ij} the amplitude of the exchanges between the countries i and j on factor $\mathcal{G}$, and by K_{ij} the amplitude on $\mathcal{K}$. Then, the total new propensity between the countries i and j is the superposition of those directed exchanges on the two factors: $p_{ij}^{\mathcal{G}} = \epsilon_i \epsilon_j G_{ij}$ and $p_{ij}^{\mathcal{K}} = \beta_i \beta_j K_{ij}$. The two-factor form of the GAM superposes the spontaneous interactions of the natural model with the intended interactions based on the two-dimensional choice among the global alliances: $p_{ij} = J_{ij} + \epsilon_i \epsilon_j G_{ij} + \beta_i \beta_j K_{ij}$. The net gain of country i is

$$H_i = S_i \sum_{j \neq i} \; S_j (J_{ij} + G_{ij} \epsilon_i \epsilon_j + K_{ij} \beta_i \beta_j).$$

In order to illustrate the multi-factor stabilization, we turn again to the Example (6) of stabilization of the ESF- conflicting triangle.

3.2.4 Multi-factor Stabilization of the England-Spain-France Triangle

We assume, in addition to the religious factor $\mathcal{G}$ in the ESF- conflicting triangle, that there is an economical factor $\mathcal{K}$. In this golden age Spain had a pronounced disinclination to any economical unification with its old enemies, while England and France remarked the advantages of such unification. Therefore, the respective parameters of natural belonging on the economical factor $\mathcal{K}$ are $\beta_S = -1$ and $\beta_E, \beta_F = 1$. With respect to the economical factor, the overall propensities between the countries are : $p_{SE} = -3 - G_{SE} - K_{SE}$, $p_{EF} = -1 - G_{EF} + K_{EF}$, and $p_{SF} = -2 + G_{SF} - K_{SF}$.

Solutions of inequality

$$\Pi_{i,j \in \mathcal{C}} \;\; p_{ij} = (-3 - G_{SE} - K_{SE})(-1 - G_{EF} + K_{EF})(-2 + G_{SF} - K_{SF}) \geq 0 \tag{17}$$

yield the exchange amplitudes that guarantee stability of the ESF-triangle in the multi-factor form. Since $G_{SE} + K_{SE} \geq 0$, the solution must satisfy $-G_{EF} + K_{EF} \geq 1$ and $G_{SF} - K_{SF} \leq 2$, or $-G_{EF} + K_{EF} \leq 1$ and $G_{SF} - K_{SF} \geq 2$ (see Figure (16)).

In the historical reality of this period of the countries, the economical factor $\mathcal{K}$ could not produce interactions as strong and significant as the exchanges on the religious factor. That is why $K_{EF} < 1 + G_{EF}$ and $K_{SF} < G_{SF} + 2$ which have prevented the ESF-triangle to reach the stability until religion took a secondary place conceding importance to economics. See Figure (17)), where G_{SE} is taken to

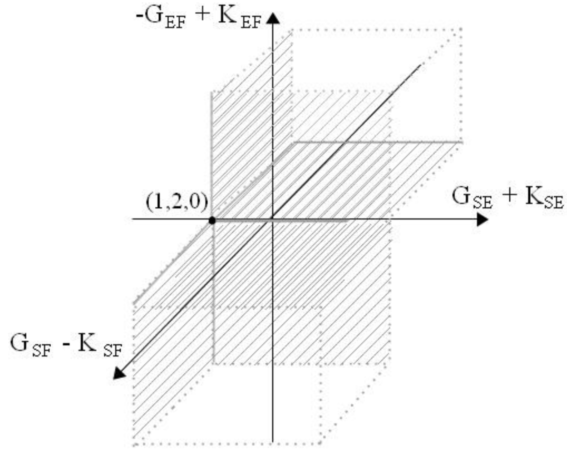


Figure 16: three-dimensional solution spaces of the independent additional propensities in the two-factor stabilization of the ESF-triangle.

be 0, G_{EF} to be 2 and G_{SF} is 3, so that the respective total propensities become 1, -3, 1.

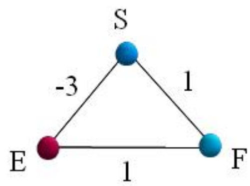


Figure 17: The global alliances model of the ESF-triangle in the multi-factor case. Here, $G_{SE} = 0$, $G_{EF} = 2$ and $G_{SF} = 3$, so that the respective total propensities become 1, -3, 1. The system remained unstable because the global exchange amplitudes did not satisfy the terms of stability – the circle remained negative.

It worth to notice that in the multi-factor form, a system in the global alliance model can be no more interpreted as a system in the natural model as soon as the choice of at least two countries differs on at least two factors. Still, the general multi-factor case can be reduced to the two-factor form of the GAM: one of the factors unifies the amplitudes of all the positive coupling and the other unifies those of all the negatives ones.

Therefore, with no restriction on the generality, the multi-factor form of GAM can be studied within the case of two coexisting collective factors. This also explains the fact that in the majority of cases, only two camps of opposing concerns play the crucial role in the coalition forming.

3.3 Physical Interpretation of the Multi-Factor Stabilization

In the context of Statistical Physics, namely the Mattis model [37] of Spin Glass, the multi-factor stabilization is equivalent to the superposition of unstable disorder of a spin glass with two stable orders (two factors) of ferromagnetic states which split the spins in two directions (two alliances). Each spin's absolute direction is the average of those ferromagnetic directions, as shown in Figure (18)). Among the two opposite directions, either one of them dominates or the two eliminate each other, thus neutralizing the ferromagnetic states on the spin. In the figure, thick arrows indicate the absolute directions of the spins, and thin arrows show their ferromagnetic directions.

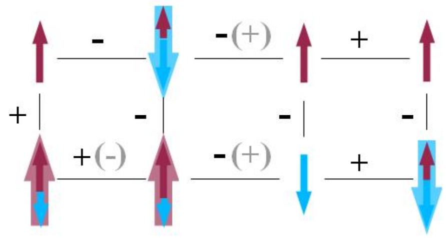


Figure 18: Ising model of 8-spins initially mixed negative and positive pair interactions (highlighted with grey color), is stabilized by mixing of two ferromagnetic states representing the Mattis model. Each spin's absolute direction (marked by the thick arrows) is the average of those ferromagnetic directions. Among the two opposite directions, either one of them dominates or the two eliminate each other, thus neutralizing the ferromagnetic states on the spin. The Spin Glass phase yields a stable disorder.

The multi-factor stabilization of coalition forming is an innovative concept in Political Sciences. In the former, it explains the multitude of elements influencing the coalition forming. In the later, it shows how in a frustrated system a stable disorder is achieved from interlocking of two ferromagnetic states of opposite directions with anti-ferromagnetic coupling among them.

The concept of multi-factor stabilization illustrates the multitude of elements influencing the coalition forming in the systems of individual actors. In physical systems, it shows how a stable disorder arises from an anti-ferromagnetic coupling achieved by the interlocking of two opposite ferromagnetic states.

4 Real Historical Cases Illustration and Analysis

This section is devoted to application of the natural and global alliance models in the illustration and analysis of real cases. It includes illustration of unstable cycling of coalitions in the Europe during the creation of Italian state, and analysis from the perspective of both the Natural and Global Alliances models of the "paradoxical stability" in the Eurozone.

4.1 Multi-factor Stabilization in Western Europe

Here we attempt to illustrate the formation of Italian state within the context of the global alliance model. It is known that, given a system from the reality, it is hard to obtain exact numerical values of the propensities in the system. Once such values are known we can explain the transitions and predict resulting configurations with arguable precision. Having no such values, we still can provide some analysis based on estimated values of the propensities extracted from the historical chronicles. Running the model with those values allows to analyze and explain the transitions and the result configuration. This cannot be done based only on the canonical representations of historical events.

Let us illustrate the Italian unification in 1856 - 1858, where four countries were involved: Italy, France, Russia and Austria [34] and [35]. The period from the end of 18th till the middle of 19th century was marked by the series of European wars including the French invasion of Italy where Austrian and Sardinian forces had to face French army in the War of the First Coalition, The War of the Fifth Coalition of Austria against French Empire.

In 1852, the new president of the Council of Ministers in an Italian region Piedmont, Camillo di Cavour, had expansionist ambitions one of which was to displace the Austrians from the Italian peninsula. An attempt to acquire British and French favor was however unsuccessful.

Then, Napoleon III, who had belonged to an Italian family originally, decided to make a significant gesture for Italy. In the summer of 1858, Cavour and Napoleon III agreed to cooperate on war against Austria. According to the agreement, Piedmont would be rewarded with the Austrian territories in Italy (Lombardy and Venice), as well as the Duchies of Parma and Modena, while France would gain Piedmont's transalpine territories of Savoy and Nice.

Despite the Russian help in crushing the Hungarian Revolt in 1849, Austria failed to support Russia in the Crimean War of the middle of 1850s. Therefore, Austria could not count on Russian help in Italy and Germany. Alexander II has agreed to support France in a fight with Austria for the liberation of the Italians, though only by showing up the army on its borders with Austria. It appeared to be enough to force the Austrians withdraw behind the borders of Venice.

However, the conquest of Venice required a long and bloody mission, which may cause revolts and threaten Napoleon III's position in France. In the private meeting with Franz Joseph, together they agreed on the principles of a settlement to the conflict according to which the Austrians have to cede Savoy and Nice to the French, yet would retain Venice. The Russian was indignant at this turn of France.

Let us reproduce the historical chronicle presented above with the help of our model. The initial states of the countries with their primary propensities are shown in Figure (19).

The value of propensities indicating the relative strength of primary interactions between the countries are taken from "negative" to "positive" through mixed ones with the respective numerical values taking "neutral" as 0 are shown in Figure 19. Thus, the historical relationships between the two absolutist monarchies Russia and Austria are estimated as "neutral" with $J_{RA} = 1$. Italy and Russia having no noticeable political relationship are "neutral" to each other. The Franco-Russian relationship built up during the French Revolutionary and the Napoleonic Wars is are rather "neutral-negative" with $J_{FR} = -1$, as well as the interactions of France with Italy and Austria who had experienced series of military conflicts. The opposition between Italy and Austria tied to the mutual territorial claims is estimated to be "significantly-negative" with $J_{IA} = -2$).

Figure (19) shows the system of the countries in its natural model. The model has two negative circles and so is unstable which appears through the historical changes before the rise of the Italian question. The instability originates from the

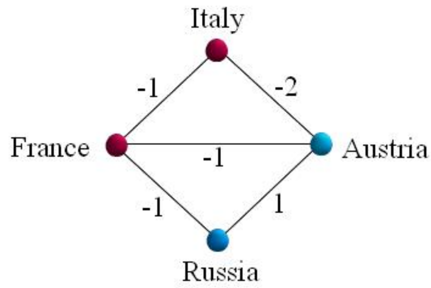


Figure 19: The unstable system of France, Russia, Italy, Austria with their relative primary propensities, 1856-1858.

fact that France gets identical benefits from the alliance with Russia and Italy as from the opposing alliance with Austria.

An external field in the model results from the principle of independent state of Italy. The respective opposing global alliances are M which associates the countries that support the independence of Italy, and C which unifies the countries opposing the independence.

Here, two respective factors influencing the historical series of events must be distinguished: external politics with the military goals, and internal politics involving the social concerns of the countries (their governing classes). Denote the two factors by $\mathcal{G}$ and $\mathcal{K}$ respectively.

With respect to their external goals, Italy and France, as well as Russia, agree to the relevance of an independent state of Italy. Yet, in the social concerns the governing classes of France, Russia and Austria agree in their rejection of socialist ideas springing over all the Italy. Therefore, the respective parameters of the countries' natural belonging to the alliances are distributed as follows. With the natural belonging parameter ϵ referring the external goals and β referring the internal social politics, for France $\epsilon_F = +1$ and $\beta_F = -1$, for Italy $\epsilon_I = +1$ and $\beta_I = +1$, for Russia $\epsilon_R = +1$ and $\beta_R = -1$, and for Austria $\epsilon_A = -1$ and $\beta_A = -1$.

The global alliance motivated propensities are given is the following chart:

Propensity	F-I	I-A	F-R	F-A	R-A
Primary	-1	-2	-1	-1	1
On $\mathcal{G}$	G_{FI}	$-G_{IA}$	G_{FR}	$-G_{FA}$	$-G_{RA}$
On $\mathcal{K}$	$-K_{FI}$	$-K_{IA}$	K_{FR}	K_{FA}	K_{RA}

The historical chronicle of the four countries is concluded in three phases: a phase of no global alliances, or the natural model phase, and two phases of global alliances rose due to the Italian question, where in the first one the external and military concerns come to picture and in the second one the internal social concerns rise over the countries.

As we have seen in Figure (19), the system in its natural model is unstable, where France fluctuates between Russia and Austria.

Let us evaluate the amplitudes of the military exchanges between the countries through numerical values providing the relative magnitudes of interactions. Russia has equally "moderate" interest in military cooperation with both France and Austria, with $G_{FR} = 2$ and $G_{RA} = 2$. Italy and France are "strongly" interested in the conflict having Italian land at stake, $G_{FI} = 4$ and $G_{IA} = 4$, while the interest between Austria and France is "moderately-strong" with $G_{FA} = 3$. A sympathy of Russian to Italian state comes up in the "basic" interest, $G_{RI} = 1$. The new propensities between the countries with respect to the external politics interests are shown in the following table:

Propensity	F-I	I-A	F-R	F-A	R-A	R-I
Primary	-1	-2	-1	-1	1	0
On $\mathcal{G}$	4	-4	2	-3	-2	1
Total	3	-6	1	-4	-1	1

As result of the interactions the system obtain a new shape shown in Figure (20). Here, absence of negative circles allow a perfectly stable coalition of France, Italy and Russia against Austria .

However, the social aspect of the internal politics of the countries dramatically interferes with the stability. The relative amplitudes of the consequent exchanges can be estimated as follows. Due to the political insularity of Russia where serfdom

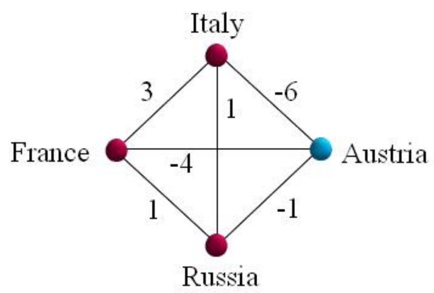


Figure 20: France, Russia, Italy and Austria, 1856-1858, with the new military propensities. It forms a stable system with the coalition of France, Italy and Russia against Austria.

still prevailed over large part of the country, the amplitudes of all its exchanges on the social aspect are "negligible", $K_{FR} = 0$ and $K_{RA} = 0$. France and Austria had a "strong" involvement in the subject, with $K_{FA} = K_{FI} = K_{IA} = 4$. The new propensities between the countries are shown in the table.

Propensity	F-I	I-A	F-R	F-A	R-A	R-I
Primary	-1	-2	-1	-1	1	0
On $\mathcal{G}$	4	-4	2	-3	-2	1
On $\mathcal{K}$	-4	-4	0	4	0	0
Total	-1	-10	1	0	-1	1

The result system with the French change in favor of cooperation with Austria is shown in Figure (21). As we can see the modified system includes three negative circles. The change of France put Russia in an unfavorable position moving it away from a most beneficial coalition configuration. At the same time, Italy and Austria found themselves in a satisfactory state.

4.2 The Stability of the Eurozone Coalition

To illustrate how the model can be useful to tackle real world challenges, we address the question of the current "paradoxical stability" of the Eurozone during the ongoing debt crisis. The Eurozone is an economic and monetary union of 17 member states of the European Union, which includes a total of 27 countries. The Eurozone is monitored by the European Central Bank (ECB), which is governed by a presi-

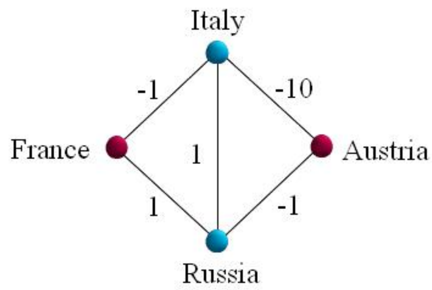


Figure 21: France, Russia, Italy, Austria, 1856-1858, with the new propensities on both the military and social factors. The result of the war for liberation of Italy is the instability in a new shape.

dent and a board of the heads of the respective national central banks. The ECB cooperates with political and financial stability institutions such as the Euro Group and the European Financial Stability Facility.

In $2008 - 2013$ the Eurozone has been experiencing a sharp financial crisis with the economic growth slow and unequally distributed across the member states. The crisis made it difficult or impossible for some countries in the Eurozone to repay or refinance their national debt, as well as to pay back cumulated interest debts. Tension have raised the temptation to reduce the number of countries that showed themselves as weaker links in the monetary union.

Due to the financial tension connected to both large state debts and inability to pay back the interest debts, centrifugal forces are active to reduce the number of countries which belong to the Eurozone. On the one hand, countries like Greece, Portugal, Spain and Cyprus are under internal pressure to leave the Eurozone and so to relieve the hard burden of austerity policies imposed as the price of crediting. On the other hand, mounting pressure is growing in wealthy countries such as Germany to expel those countries so as not to finance their unpredictable economical development at the own expense of the wealthy countries.

When Greece is apt to leave the Eurozone and Germany is inclined to expel Greece, the question arise of why both countries independently decide to adopt a policy which appears to be against their respective immediate interest. Namely, Greece is going through difficult structural reforms and privatization of government assets and Germany is accepting the emergency bailout loans, which in contrast to the interest loans shall be emanate from its own treasury ([61], [62]).

4.2.1 Eurozone Coalition within Natural Model

In order to apply the natural model to the Eurozone we introduce a system of 28 actors to represent the 27 European Union countries and the ECB. Each country have a bond with the ECB, which plays a full role of the model's actor. In addition, we assume that the connections between all pairs of countries are contingent on their historical mutual propensity and are proportional to the respective debts.

In order to define the mutual propensities formally, we characterize satisfaction of the Eurozone entry criteria through some constant $C > 0$. Bonds with ECB are subjected, in addition to C, to parameter accounting for the balance between loan interests and loan losses – the *Loan Absolute Value* L^{av}. For the debtors, a positive value guarantees the Eurozone support, whereas for the ECB (and the respective creditors) it guarantees the benefits.

Thus, propensity of a Eurozone member i with ECB is $p_{ECB,i} = C + L^{av}_i$. For a non-Eurozone member state k, the propensity is $p_{ECB,k} = -C$. The mutual propensity between any two countries i, j, given the respective part of debt D_{ij} and their historical economic propensity h_{ij}, is $p_{ij} = h_{ij} - D_{ij}$. The model is shown in Figure (22).

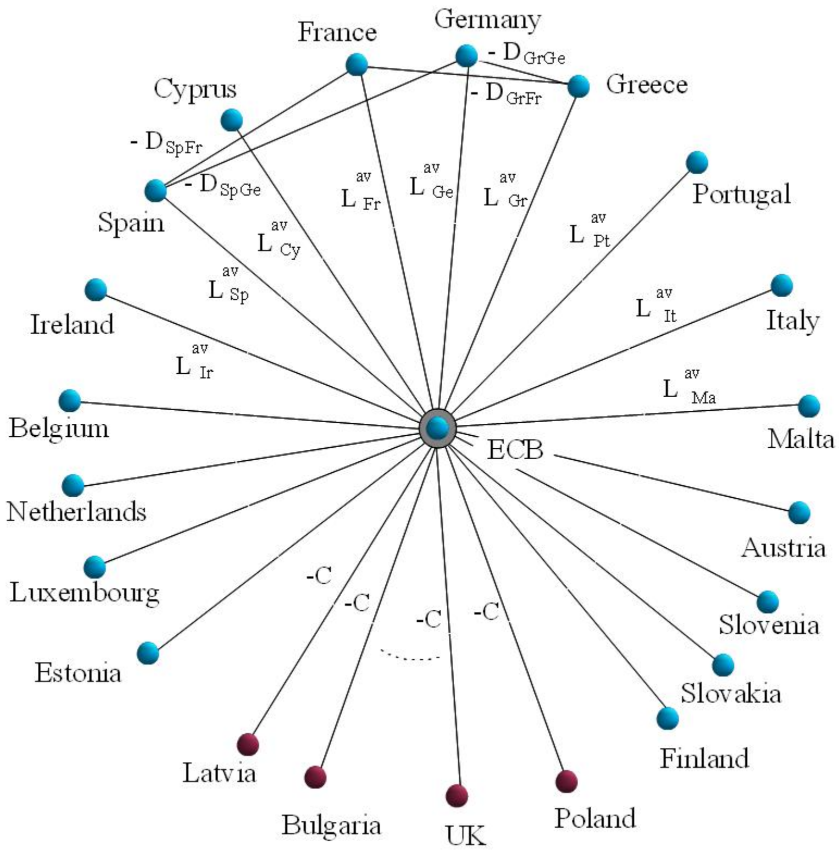


Figure 22: The natural model of the Eurozone. All the 17 Eurozone member states and only few of the non-Eurozone ones are shown in the figure. For the sake of illustration, respective part of debt D_{ij} and the L_{ij}^{av} parameter, are marked for some countries in the figure.

Let us remark that by expressing p_{ij} through the values of different dimensions we assume the existence of a common universal measure that allow to express both material and immaterial values, like goods and morals.

In order to understand the stability in the Eurozone and to make a prospect of its future, let us consider closely the sub-model involving Germany and Greece (see Figure 23).

In the present scenario, when the banks in Greece (along with that of Ireland, Italy and Spain) have received their bailout loans, Cyprus is awaiting for the bailout approval and Malta relies on its own economical strength, the debt-bond affects only the propensity between Greece and Germany.

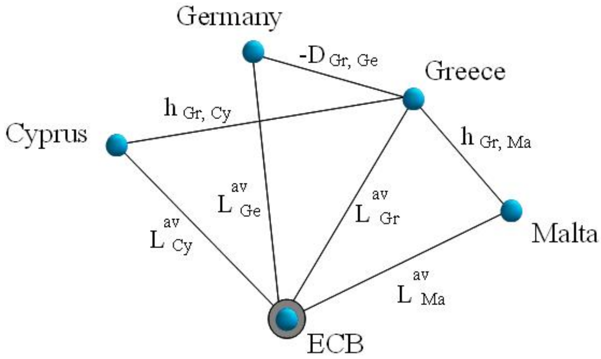


Figure 23: Eurozone model – the Greece triangle. The debt-bond affects only the propensity between Greece and Germany: the banks in Greece has received the new bailout loans, Cyprus is awaiting for the bailout approval and Malta relies on its own economical strength.

In the current situation in the Eurozone, with the ECB making efforts to supply new bailout loans, both L^{ab}_{Gr} and L^{ab}_{Ge} are still positive. This implies that the best configuration for the ECB is to keep both Greece and Germany inside the Eurozone.

With the negative mutual propensity between the two, $p_{Gr,Ge} = -D_{Gr,Ge}$, Germany, Greece and the ECB form a conflicting triangle. The triangle, yet, does not enter endless cycling: to stay in Eurozone is their local maximum wherein any shift has a cost (even a temporary) which is not affordable due to economical constraints. As result, acting as secondary actors, the countries do not change, and the triangle remains stable.

As we can see, the whole Eurozone system consists of negative triangles – whenever the individual part of debts are involved – which are in local maximum and are stable due to the attitude of the actors. As a result, despite the question of exit from the Eurozone stays open, no country leaves and no one is expelled. Economically weaker countries strive to stay in the Euro area, and the wealthy members do not leave but take measures to keep the others inside, whereas everybody assumes its sacrifices.

Within prospect on the Eurozone coalition, let us note that the model is dynamic: the propensities tend to change with time, with the Loan Absolute Value is depending on health of an economy and the debt-bonds are changing due to debt payed or owing to new bailout loans.

Being an economical function, L^{ab}'s of some countries tends to approach their critical points beyond which the propensity becomes negative. In the case of Greece, when the critical point is reached the following scenario should take place: with the historical economic propensities $h_{Gr,Cy}$ and $h_{Gr,Ma}$ of Greece with Cyprus and Malta significant enough, and L^{ab}_{Cy} and L^{ab}_{Ma} getting less important, the exit of Greece shall entail the exit of Cyprus and Malta. See Figure (24).

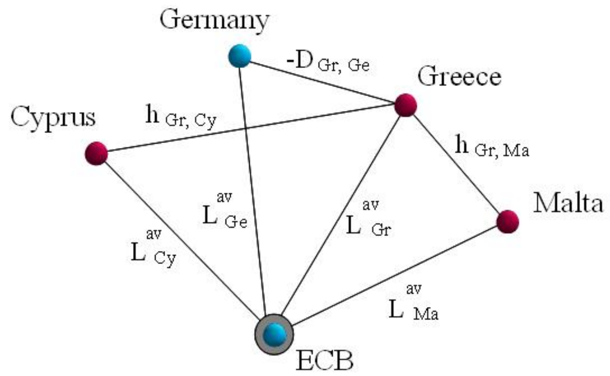


Figure 24: Eurozone model – the Greece triangle with critical value of L^{ab}_{Gr}. The historical economic propensities $h_{Gr,Cy}$ and $h_{Gr,Ma}$ are significant enough, and L^{ab}_{Cy} and L^{ab}_{Ma} are getting less important, as result the exit of Greece shall entail the exit of Cyprus and Malta.

However, until the values of L^{ab}_{Cy} and L^{ab}_{Ma} are positive and significant enough, the Eurozone coalition configuration shall stay unchanged for longer time: the ECB, being a primary actor aware of all its connections, will not be ready to let Greece to leave at the risk of the further exit of Malta and Cyprus, with which the ECB still has positive bonds. This idea sheds light on the purpose of the long term refinancing operations and low-interests bailout loans – they allow to keep the L^{ab}s positive and reduce the risk of losses.

At this point, it is important to point out that, in contrast to the dominant opinion that the decision on exit of Greece belongs to Germany, our model makes it clear that this responsibility belongs to the ECB, in assistance with the financial stability institutions, through the financial support of the Eurozone members.

As we can see, such a structure of the Eurozone model, with a completely rational and powerful primary actor in the center and the strong outgoing bonds reinforced through the entry criteria and the loans, implies that any instability in a member state propagates to the others only through the ECB and is subject to the ECB's individual stability.

This emphasizes the importance of the ECB as a centralized "controller" of a unique monetary system. In contradiction to the criticism of such a financial institution in Europe, within the economics as it is defined today, a common European financial space shall not be possible without it.

4.3 The Multi-factor Stability of the Eurozone Coalition

While the question of reducing the number weaker links in the monetary union remained open, the Eurozone financial coordinators and institutions recommended long term refinancing operations aimed to help the weaker countries to rise their economy. The prospect of a Greek exit has been viewed as leading to the Eurozone implosion; it was officially called the "domino theory" which postulates that if a country falls, one after the other the countries could be in danger of toppling ([60]). At that point, the euro will be at the risk of implosion.

In order to apply the global alliance model to the Eurozone we introduce a system of 27 actors representing European Union countries. We assume that the bonds between all pairs of countries are contingent on their historical mutual propensity and are proportional to the economical and political interests of the countries.

The natural model corresponding to the European Union is shown in Figure (25). The nodes highlighted with blue (light) color represent the countries interior to the Eurozone and the red (dark) nodes shows the countries exterior to it. The figure presents several mutual bonds between the countries and marks some of the historical propensities $h_{c_1,\, c_2}$. Naturally, the system of European countries was not stable during a long period of European history; the England-Spain-France negative triangle (Figure 16) is one of the emblematic conflicting triangles in it.

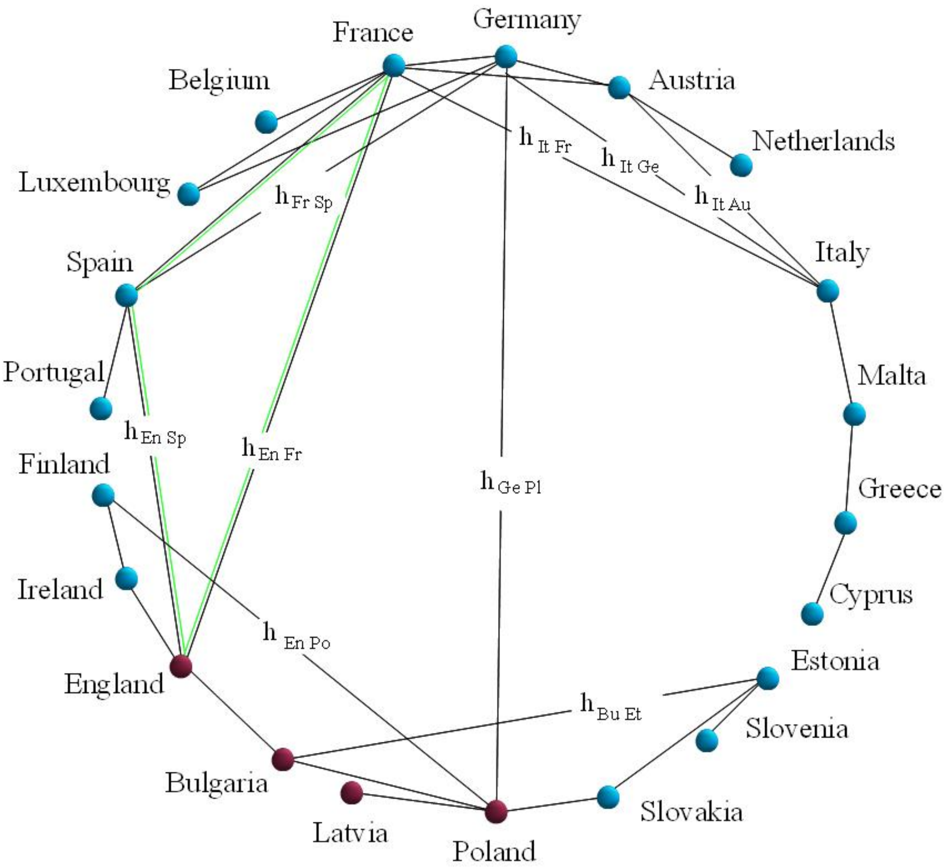


Figure 25: The natural model of the Eurozone. The 17 Eurozone member states (light blue nodes) and few of the non-Eurozone ones (dark red nodes) are shown in the figure. The figure presents several mutual bonds between the countries and marks some historical propensities $h_{c_1,\, c_2}$. The system was not stable during a long period of European history; the England-Spain-France negative triangle is one of the conflicting triangles in it.

In this presentation of the Eurozone, we only refer to the qualitative character of the propensities between the countries as to make the comparison analysis between the "natural" case and the "global-alliance" case of the Eurozone. We do not attempt to reveal the quantity alternatives of the economic propensities due to the limitations of available data.

The global alliance model corresponding to the European countries can be defined as the one that started from the global principle of common European economic and monetary space, with the Euro and non-Euro unions as respective competing alliances. The principle of European economic and monetary unification has created new motivations to the interactions among the European countries.

In order to define the effective propensities formally, we assume that the relationship between the countries rests on several collective factors attributed either to economic or political motives. The multi-factor character of the GAM starts from the Eurozone entry criteria: each of the criteria corresponds to a particular collective factor with the respective countries' natural belonging parameters, positive when the countries claims to membership in the Eurozone.

The membership requires satisfaction of all the criteria whereas the criteria was assumed to be necessary and sufficient to create a stable union of cooperating nations. The joint amplitude on the entry criteria produces a positive stimulus to cooperation between the countries of the Eurozone, while at the same time it repulses them form the rest of the countries, those which do not wish or are unable to satisfy the criteria. Thus, the total of the exchange amplitudes on the entry criteria represents a positive contribution to their primary historical propensity for the Eurozone member states $p_{i,j} = h_{i,j} + C_{i,j}$, and negative for the non-Eurozone ones $p_{i,j} = h_{i,j} - C_{i,j}$.

The criteria was supposed to be enough to create new sufficient positive bonds and to overcome the existing negative ones. That is how, for example, Germany, Greece and Italy have found themselves involved in the same coalition. The criteria later proved insufficient and the motivations were supplemented by the low interest bailout loans accompanied by political and economical austerity policies. Those new bailout or interest loans and policies define new economic and political factors which give a new turn to the countries' interactions and to the common denominator of the Eurozone stability.

In order to define the new mutual propensities formally, we offer a schematic outline of the factors as follows. With respect to the economic effect, E^I is an immediate economic effect – a loss for the creditor and a gain for the debtor, E^F is a final outcome such as payment of interests, and E^M is the mutual benefit such as the shared economic advantages. Similarly, with respect to the political effect, P^I is an immediate effect from the austerity policies – a gain for the creditor and a loss for the debtor, P^F is a final outcome, and P^M is the reciprocal benefit such as the political strength. It is worth pointing out that, with regard to those factors, the countries' natural belonging parameters to the Eurozone global alliance may happen to be negative.

Given a loan L, and the countries' natural belonging parameters ϵ^E, β^E, γ^E with respect to correspondingly the immediate effect, the final outcome and the mutual benefit, on the economic factor, and the respective ϵ^F, β^F, γ^F on the political factor, we define the loan efficiency values as follows: $E^L_{i,j} = \epsilon^E_i \epsilon^E_j E^I_{i,j} + \beta^E_i \beta^E_j E^F_{i,j} + \gamma^E_i \gamma^E_j E^M_{i,j}$ and $P^L_{i,j} = \epsilon^P_i \epsilon^P_j P^I_{i,j} + \beta^P_i \beta^P_j P^F_{i,j} + \gamma^P_i \gamma^P_j P^M_{i,j}$. The total propensity of any two European Union's countries i, j can be written as

$$p_{ij} = h_{ij} \pm C_{ij} + E^{L_1}_{ij} + P^{L_1}_{ij} + \ldots + E^{L_k}_{ij} + P^{L_k}_{ij},$$

where $\{L_1, \ldots, L_k\}$ is the progression of the bailouts.

In the present scenario of the Eurozone, Germany's belonging parameter with respect to the emergency bailout loans and Greece's belonging parameter regarding the respective austerity policies appear to be negative. The contribution to the propensity between Greece and Germany of the loan efficiency values $E^L_{Ge,Gr}$ and $P^L_{Ge,Gr}$ is thus as follows

$$-1 \cdot (+1) E^I_{Ge,Gr} + 1 \cdot (-1) E^F_{Ge,Gr} + 1 \cdot (+1) E^M_{Ge,Gr} + 1 \cdot (-1) P^I_{Ge,Gr} - 1 \cdot (+1) P^F_{Ge,Gr} + 1 \cdot (+1) P^M_{Ge,Gr}$$
$$E^M_{Ge,Gr} + P^M_{Ge,Gr} - P^I_{Ge,Gr} - E^I_{Ge,Gr} - E^F_{Ge,Gr} - P^F_{Ge,Gr}$$

It is interesting to notice that the formula suggests a scenario where, no matter how many loans are involved, the weaker the positive balance of the contributions the less the rescue effect of the supplementation.

The result GAM is shown in Figure (26).

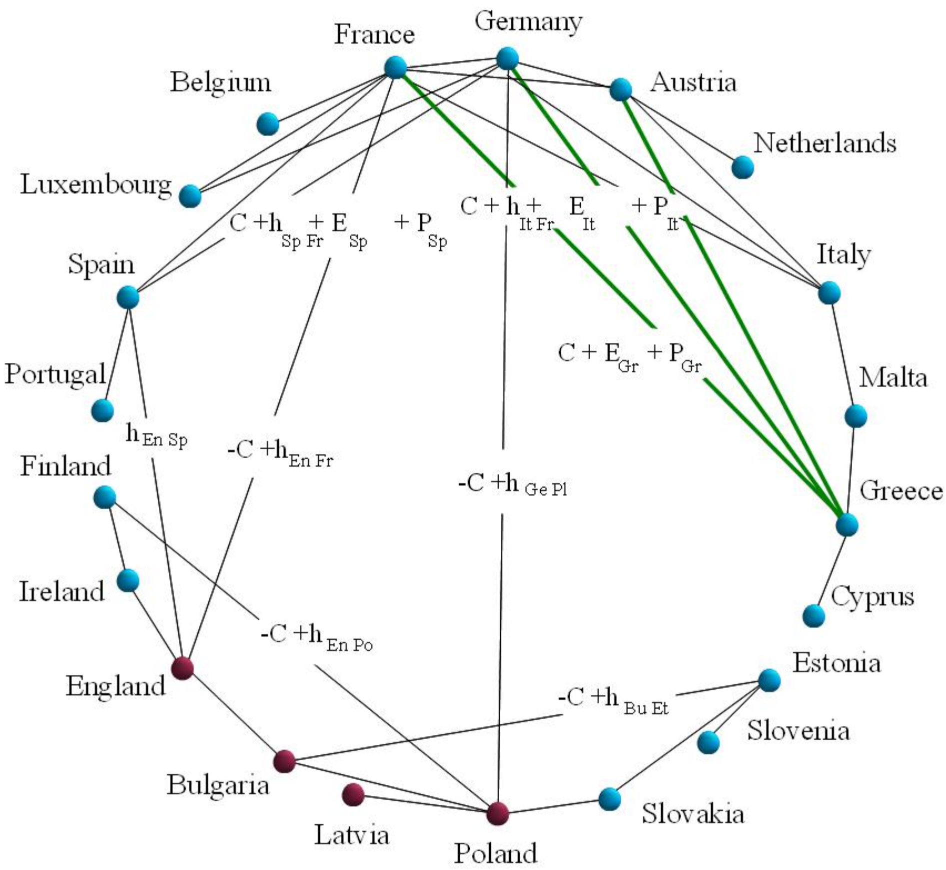


Figure 26: The GAM of the Eurozone. All the 17 Eurozone member states (blue nodes) and only few of the non-Eurozone ones (red nodes) are shown in the figure. The figure shows the global alliance motivated propensities for several countries, where $C_{i,j}$ stands for the total of the exchange amplitudes on the Eurozone convergence criteria, $P_{i,j}$ denotes the political efficiency value and $E_{i,j}$ denotes the economic efficiency value. The bonds highlighted with bold green denote the new links motivated by the global alliance. The system is stable thanks to the positive balance of the efficiency values of the loans and the austerity policies.

Within the terms of the GAM, the stability of the Eurozone coalition can be explained by the fact that the total of the globally motivated interactions between the member states is positive. While it may have approached the critical value, the exchanges between Germany and Greece – the exchanges motivated by the entry criteria $C_{Gr,Ge}$, the economical efficiency value $E_{Gr,Ge}$ and the political efficiency value $P_{Gr,Ge}$, are still producing a positive balance on their bond. So that the best configuration still includes both the countries inside the Eurozone.

Within prospect on the Eurozone coalition, let us note that the Eurozone coalition may start fragmentation once the positive balance between those values is broken. If an unpaired bond let to be negative in a given circle of the positive couplings, the fragmentation will reveal the signs of endless cycling and the instability may propagate to the whole system. This is what must happen if the balance on the bond between Germany and Greece is broken. In contrast to the initial situation where the countries had no negative historical propensities, unlike Germany and Italy, their mutual propensities this time will be negative due to all those globally motivated interactions which from cooperation would turn to antagonism. In the circle of Greece, Cyprus, Malta, Italy and Germany, eventually Cyprus, Malta and Italy will be in danger of toppling. This formally resembles the mentioned earlier "domino theory".

At this point, it is important to point out that, in contrast to the dominant opinion that the decision on exit of Greece belongs to Germany, the global alliance model makes it clear that this responsibility belongs to the Eurozone alliance. The goal involves the objectives shared by the Eurozone's board of heads, the financial and the stability institutions and the private investors: rise Germany's economy and repaying ability, and to prevent the fragmentation.

5 Dissolution of Global Alliance

This section investigates the effect of the dissolution of a global alliance in the case where two opposing alliances were coexisting producing a stable configuration in a collective of individual countries. The focus is on the effect of fragmentation and instability among the countries in the coalition that have been previously sustained by the dissolved alliance. The results shed a new light on the understanding of the complex phenomena of fragmentation of the coalition as result of dissolution of the engendering global alliance and on the prospect of historical events.

Produced by the polarization of countries' interests through their natural belongings, the global alliances lead to emergence of new propensities between the countries which generate stability for particular amplitudes. Once the stability is achieved, the system remains stable for some time – in reality, political, economical or other interests and motivations are not static, they subject to evolutionary changes.

When those propensities change, completely or partially, they may exhaust the incentive effect of a global alliance putting the respective countries, for which the stability prevailed during the existence of the alliance, back to their primary geographic-ethnic bonds. Depending on the distribution of the attraction to the global alliance and the amplitudes of the globally induced interactions, the associated coalition exhibits different effective resistance to the dissolution – the robustness of the stability that prevailed during the existence of the alliance.

5.1 Formal Definition of Dissolution

Formally, the weakening of a global alliance is a weakening of theweakening of the exchange amplitude among countries by some multiplier $\alpha \in [0, 1]$. The total dissolution takes place when $\alpha = 0$ which sets the natural belonging parameters to zero.

The weakening of global alliance, being generally a dynamic process, should be expressed in terms of dynamic parameter $\alpha(t)$ of $\alpha(t)$ of weakening, which is a continuous or discontinuous function of time. The weakening introduces a dynamical aspect into the initially unchanged model in which the changes of the primary propensities are negligible.

Definition 2 (Weakening of a Global Alliance) *Given two actors i, j and global alliance M that descends, assume without loss of generality that actor i naturally belongs to the global alliance M. Then, the weakening of the alliance is expressed through the following change of the actors' mutual propensity:*

$$p_{ij}^{total}(t) = J_{ij} + \epsilon_i \epsilon_j G_{ij} \alpha^i(t). \tag{18}$$

The robustness of the stability is naturally determined by the proximity of the new interaction amplitudes to the boundaries of the stability space. We can conclude from Formula (18) that, while the fact of stability depends on the sign of the total propensity, the robustness of the stability depends on the value of the additional, externally induced propensity G_{ij}.

It can be observed from Formula (18), taken for all the pair of countries, leaving the closed area of the stability space is always abrupt. This fact explains that in reality, dissolution tend to be followed by unexpected and brutal bursts of conflict.

In this work, we consider the system to be at a moment t of the weakening process of the alliance where the system is out of its stability space – the alliance dissolves when the competitive (negative) circles re-appear in the system. On the way, before the system reaches the dissolution, transitional stable coalitions are possible while the system is still in the stability space.

5.2 The Two Cases of Dissolution

We focus here on the dissolution of a global alliance that has previously created stability. When the dissolution has occurred, the incentive effect of the initial global concept vanishes for this particular alliance. This makes the negative circuits of the primary propensities between the respective countries to be again the instrumental in their search for optimization.

Two different effects of the dissolution can be distinguished: 1) the instability involving all the countries of both prior stable coalitions, and 2) the instability affecting only a part of the system – the countries of the respective coalition, the *semi-stability*. We illustrate below the two cases with historical examples.

5.3 Dissolution of The Global Alliances in Syria – Unstable System

Syria includes many different ethnic and religious communities unified under one government by the French mandant. Sunnis, Druzes, Alawites (a branch of Shia), Shiits and Christians are the largest religious communities of Syria, where the Alawite minority occupies key government and military positions. The politics is exclusively based on cronyism, which is characteristic of the entire East, so that the Alawite community and their allies get the political and economical benefits.

The religious composition of Syria is schematically illustrated in the diagram of Figure (27a) showing the original propensities as they appeared in the beginning of the twentieth century. Several conflicting (negative) circles are present in the system, so that the system does not have a rational stability, a stable optimal configuration.

Today's conflict in Syria exhibits a sharpest split between the ruling Alawite minority and the country's poor religious periphery – Sunni majority mostly aligned with the opposition, where the prosperous part of the Syrian people for whom religion is not of an absolute vital importance passes from side to another.

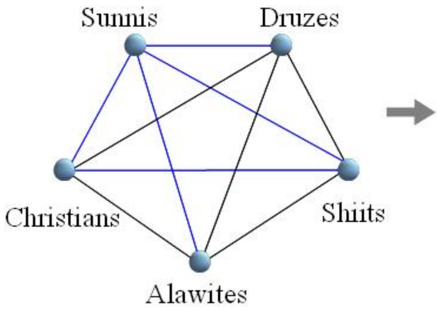
As stated in several historical sources ([58], [59]), the problem is rooted in socio-economic dimension, rather than in religious extent. Those different religious communities find themselves united under conditions of extreme poverty with neither economical nor social safety prospect, as opposed to the prosperity of the governing class. Such a discrepancy has fueled the civil uprising.

It is worth to underline that in the second half of the twentieth century, the stability have been settled by the materialization of a global alliance calling against a common enemy, the newly created Israel state. The global alliance, denoted I, has unified the frustrated and annoyed population of Syria. The alliance has neutralized all the antagonistic communities which, in contrary to the Israeli success in unification of their different ethnic and religious branches, were not able to come up with their own autonomy. The alliance I is shown in Figure (27b).

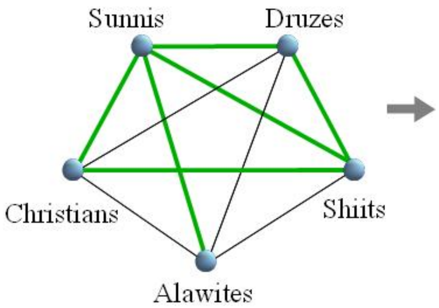
In the beginning of the twenty-first century, Egypt, Tunisia and Libya came up with public protest against their present regimes. The uprising against the government in Egypt and Tunisia was quick and decisive, in Libya the protest led to a short civil war that induced the overthrow of the government. Those examples inspired the resistance and the rebelion of unfavored Syrian population which suffered from social and economical inequalities.

The awakening leaded to the dissolution of the anti-Israel global alliance, freeing the important unstable internal conflicts. A new global alliance, denoted B, has installed immediately in opposition to the government of Bashar al-Assad with the simultaneous forming of the opposite alliance in support of the regime (see Figure (27c)). The opposition has attracted together most of Sunnis and a large part of Druze community. The global alliance B splits the population into two parts so that the system is stabilized, as shown in the figure (all the circles are positive).

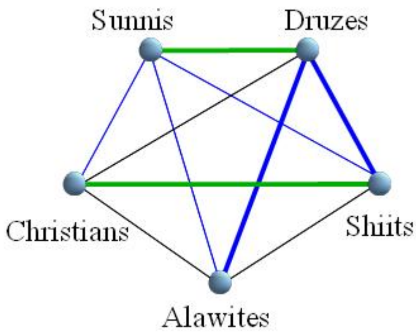
However, division of Syria into two sectors will neither produce a stable configuration. As soon as the current conflict will be resolved, the alliance B will naturally dissolve and the original ethnics and religious frustrations will be again activating the instabilities.



(a) The figure shows schematically the original system of Syrian largest religious communities in the beginning of the twentieth century. Here, the primary negative propensities are highlighted with blue while the positives propensities are marked with black. As we can see, there are several conflicting (negative) circles in the system. The system does not have a rational stability, a stable optimal configuration.



(b) A system of the Syrian religious communities in the 70s under the anti-Israel state global alliance I. The antagonistic communities, who haven't came up with their own autonomy, were unified into a stable coalition based on the ethical considerations. The new cooperative propensities are highlighted with bold green.



(c) A system of the Syrian communities from the 2010. The new conflicting propensities are highlighted with bold blue. The opposition's global alliance B has attracted, together with most of Sunnis, a part of Druze community. All the circles in the system are positive, so the global alliance splits the population into two religious stable parts.

Figure 27: Dissolution of the global alliances in Syria.

It should be noted that in reality Syrian system is larger and more complex: it accommodates many minor religious communities, and some of large communities include several different ethnics that often disfavors each other. An example can be Kurds and native Syrian Arabs that belong to Sunnis community. Nevertheless, the system of religious communities presented in the above example provides a simplified picture which already exhibits the instabilities and the complexity of Syrian conflict.

5.4 Dissolution of The Soviet Global Alliance – Semi-Stable System

The case of a semi-stable system, a system where one of the coalitions keeps stable while the other fluctuates due to the dissolution of the respective global alliance, can be illustrated with the collapse of the Soviet alliance.

In the middle of last century, the Eastern alliance represented by the Warsaw Pact and the Western alliance represented by NATO were the leading opposing global alliances in the region. On the seventh decade of its existence, the Soviet Union which held the nations together, mainly by the military-political factor, collapsed after the Warsaw Pact has dissolved. This have led to the dissolution of the entire Eastern alliance and, as a result, to the fragmentation of the Soviet coalition where any new associations among the involved countries have lacked the stability. Soviet countries, formerly unified into a strong union, turned back to the primary ethnic hostility. In contrast to the Eastern sector, the coalition of NATO remained stable.

Figure (28) schematically illustrates the system of the Soviet and the NATO countries, and the collapse of the Soviet global alliance. In both the sides negative triangles can be recognized: for example the Georgia-Armenia-Russia negative triangle on the Soviet side and the Germany-Italy-France negative triangle on the NATO side.

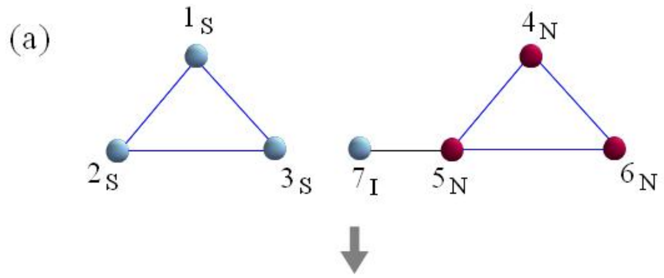
To illustrate the phenomena, we present the case of three countries on each side denoted respectively by $\{1_S, 2_S, 3_S\}$ for the Soviet part and the countries of Far East, by $\{4_N, 5_N, 6_N\}$ for the Western Europe part. The intermediary countries of Eastern Europe, such as Yugoslavia, Bosnia, are denoted by $\{7_I\}$. Primarily, before the Soviet concept has been projected and installed in the region, the system of countries $\{1_S, 2_S, 3_S, 4_N, 5_N, 6_N, 7_I\}$ formed two independent groups each having negative circuits of propensities, as shown in Figure (28a).

With the rise of the Soviet concept, Soviet global alliance S and the opposing NATO global alliance N have rose. The countries' natural belonging has distributed as follows. Countries $\{1_S, 2_S, 3_S\}$, as well as 7_I, naturally belonged to S, while $\{4_N, 5_N, 6_N\}$ belong correspondingly to N. The result externally induced interactions, shown in Figure (28b) in bold font, produce additional propensities that stabilize this originally instable system into two opposing coalitions $\{1_S, 2_S, 3_S, 7_I\}$ and $\{4_N, 5_N, 6_N\}$.

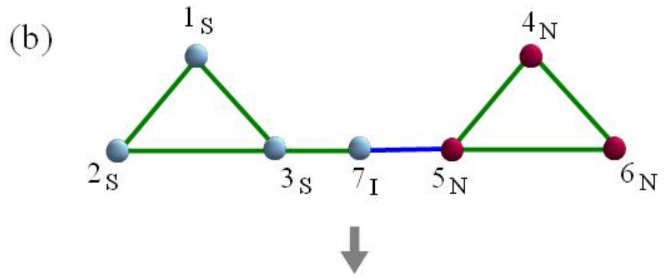
Note that country 7_I, having no significant impact on the i_S-countries initially, happens to belong naturally to the Soviet global alliance S. Due to the new interactions, 7_I is detached from the $N-$countries to which it associates originally through a positive mutual bond, and is attached to the S-coalition.

Here, the S-coalition holds the intermediary country 7_I only due to the attraction of the global alliance S. As soon as the Soviet alliance collapses, the country joins the $N-$coalition adjusting to its best configuration, see Figure (28c). The countries of the former Soviet coalition turn back to the initial negative propensities. The fluctuations of those countries do not affect the stable N-coalition: cooperative character of the interactions have persisted. The result system is semi-stable.

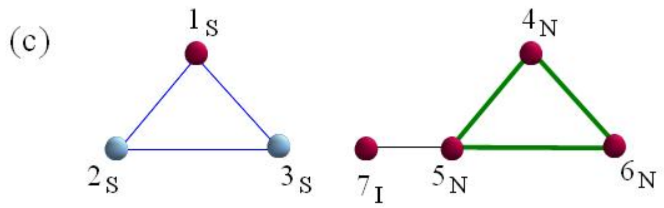
The intermediary countries are those disconnected or weakly connected to the Soviet union. Those countries served as "isolators" between the two opponent coalitions, which impeded the instability of the Eastern side to propagate to the Western one. Among those countries were Hungary, Czech Republic, Poland, Yugoslavia, Czechoslovakia, Bosnia and other countries of Northern and Eastern Europe; in 1999, the first three of the former communist countries were invited to join NATO. Membership has been expanded later to several Northern and Eastern European countries which gained them the stability. In contrast, the Caucasian region on the Eastern side till today shows up with highest instability.



(a) The system of the prior-Soviet countries and the countries of Far East $(\{1_N, 2_N, 3_N\})$, the countries Eastern and Western Europe $(\{4_N, 5_N, 6_N, 7_I\})$ with the bilateral bond propensities. Node 7_I denotes all the intermediary countries of Eastern Europe. The negative propensities are highlighted with bright blue.



(b) The Soviet S and the opposing NATO N global alliances have induced interactions that stabilized the system into two opposing coalitions $\{1_S, 2_S, 3_S, 7_I\}$ and $\{4_N, 5_N, 6_N\}$. The globally induced interactions are highlighted in green bold font.



(c) As result of the Soviet alliance's dissolution, the countries of the former Soviet coalition turn back to the initial negative propensities. intermediary countries served as "isolators" between the two opponent coalitions. Later, the country gradually joined the $N-$coalition adjusting to their best configuration. The fluctuations of the former Soviet countries do not affect the stable N-coalition: cooperative character of the interactions have persisted. The result system is semi-stable.

Figure 28: Dissolution of the Soviet global alliance.

5.5 Modeling of Dissolution

It is worth to remark that within the context rational instability, where the countries as fully rational actors can assume possible losses at maximization, the semi-stability is only possible when the system consists of two disconnected (or weakly connected – connected by negligible bond values) parts.

Let us note that modeling of the dissolution of Soviet global alliance was attempted in [8] where the descent was assumed to result from changing in the value of the countries' natural belonging parameter. The Soviet countries of Eastern Europe that joined NATO were assumed to inverse their natural disposition. Consequently, a new stability in the former Soviet region was assumed to be achievable by the inversion of the original natural belonging parameters of the respective countries and by joining the stable coalition of NATO.

While this approach is able do illustrate the dissolution of the Soviet side, it contradicts the main principle of the model – the interactions between the countries should remain of short range. The parameter of the countries' natural disposition which is the key element for the globally induced short range interaction can not be changed.

Instead, new, well designed, global alliances could bring in novel stable coalitions involving the problematic regions such as Caucasus.

Above illustrations are typical examples of dissolution in a uni-factor stability where a unique factor of interests allows each country to interact on the unique dimension of the respective global alliance.

The multi-factor stability, in contrast, implies an equiprobable influence of both the opposing global alliances on the countries. As we can see in the formula of total propensity in multi-factor form, $p_{ij}^{total} = J_{ij} + \epsilon_i \epsilon_j G_{ij} + \beta_i \beta_j K_{ij} = J_{ij} + p_{ij}^{\mathcal{G}} + p_{ij}^{\mathcal{K}}$, the both global alliance concurrently contribute to the new interactions between the countries and, thereby, to the stability of the coalitions.

The weakening of global alliance M, with any country i naturally belonging to it, is determined through the following change in the i's propensities:

$$p(t)_{ij}^{total} = J_{ij} + \alpha^i(t) p_{ij}^{\mathcal{G}} + p_{ij}^{\mathcal{K}}. \tag{19}$$

The dissolution of a global alliance in the multi-factor stability will have a weaker effect on the stable coalitions than the dissolution in the uni-factor case. This is because when the contributions from one global alliance dismiss, the coalitions may remain stable due to the contributions from the opposing, stable, alliance. Coexistence of attraction to the opposing alliances concurrently on multiple factors may thus dramatically improve the robustness of the stability.

It can be observed that the multi-factor setting in coalition forming corresponds to the democratic form of government. The stability in those settings is a priori more robust and resistant to the dissolution. In contrast, the uni-factor stability is incident to authoritarian form of government where one of the opposing ideologies solely dictates over the country's interests. For this reason, the authoritarian structures tend to collapse suddenly, bringing thereby an extensive instability followed by a burst of ongoing aggressions.

Within this context, the dissolution of the Soviet side is a dramatic example. Soviet alliance represented an authoritarian regime where the communist countries, on any factors of their interests, were focused on the Soviet ideology. The political dictatorship was reinforced by a centralized economical support. When the alliance dismissed, the coalition have collapsed all at once with the simultaneous lost of the influence that the ideology has held over all the Eastern Europe including the Caucasian region.

Comparing the stability conditions in the uni-factor stabilization case, $p_{i,j}^{total} = J_{ij} + \epsilon_i \epsilon_j G_{ij}$ and in the multi-factor stabilization case $p_{ij}^{total} = J_{ij} + \epsilon_i \epsilon_j G_{ij} + \beta_i \beta_j K_{ij}$, it appears that in the first case the condition must be satisfied for the amplitudes from a unique factor, and in the second one, it must be satisfied for several independent factors simultaneously. Therefore the stability is easier to attain within the authoritarian than within the democratic settings. At the same time, as we have seen, once the stability is reached, it is more firm within the democratic settings. This conclusion is coherent with several historical events from the past and also from the recent times.

5.6 Remarks on Applications of Dissolution

Due to the evolutionary changes in the system's environment, a global alliance which has sustained a stable coexistence with an opposing alliance sustained a stable configuration, may dissolve. Such changes produce an attenuation of the interactions between the countries previously motivated by this alliance and reveals the primary propensities between the countries. When the circuits of bonds are negative the dissolution produces the instability.

For the actors with limited rationality – the ones that are unable to foresee an improvements beyond a limited number of intermediate steps, such negative circuit may produce no changes with regards to the stability of the coalitions. However, for the countries as fully rational actors, the dissolution may result in one of two utter cases of instability. First one is when the instability, conceived in the respective coalition, propagates to the stable coalition and the entire system goes into instability. Second one is when the unstable part disconnects as a result of the dissolution from the stable one, and the system is divided into a stable and unstable parts. Various historical cases illustrate the situation, some of which is the resent conflicts in Syria and the collapse of the Soviet Union.

Within the frames of the global alliance model, re-stabilization of the resulting unstable system can be achieved by emergence of new global alliances able to bring effective interactions and motivate new stable coalitions. For the countries of the former Soviet alliance, those may be the global alliances that incite and put the focus on economical interactions. Some efforts in this direction are being made today by the former Soviet countries. For Syria, the key base of government may be shift from the traditional ethnic or religious belonging to the statehood which focuses on the concept of state with the priorities on social safety and prosperity. The exemplar can be some of Eastern countries that have successfully realized the statehood on their territories.

6 Conclusion and Remarks

The models we described in this work contain several substantial differences with the physical spin glass counterpart, where they bring new ideas and prospects.

The feature of long horizon rationality of a country is a key feature of our models. It allows the country to choose temporarily "inexpedient" coalition in order to create a dynamics of alliance shifts which may bring the initiator country to a better configuration. This possibility results from the country capacity to forecast the various patterns of rational choices other countries will make once it has done its "inexpedient" choice. Such aptitude reflexes the strategic vision which goes behind the nearest neighbor vision.

The long horizon rationality feature allows to explain the decisive advantage a country may gain by possessing advanced technologies applied in order to collect those possible scenarios. From the other side, it also provides a hint for explanation of the fact that in some cases a country having the technological leap could gain interest in sharing its exclusive information with several corresponding countries.

The long-horizon strategy carries with it the risk that a contestant rational actor also adhere to long-horizon strategy. In spin glass model, the effect of "risking" spins that appears through heating of the system reminds the effect of rationality. However, while at first sight the origin of long-horizon rationality could be comparable to the temperature effect, they have further substantial differences. The first one is that rationality is not an homogeneous feature like temperature is. The second is that the extended rationality associates to each particular country and not to all of them in uniform probability – the role of "risking" shifts randomly from spin to spin. In other words, an "inexpedient" alignment is not probabilistic, like the one caused by the temperature effect, but is a result a forecast. Moreover, the extent of the horizon may vary from one country to another.

From physical sense, the Global Alliance Model's concept of stabilization through multiple collective factors give a novel prospect on a stable disorder achieved in a frustrated system. The superimposition of a random bond spin glasses and a random site spin glasses, that is interlocking of anti-ferromagnetic couplings with two ferromagnetic states of opposite directions, is novel with respect to physics. In particular when several random site spins glasses are competing in addition to the quenched random bond distribution.

This possibility to modify the random site couplings produced by the global alliance setting also makes a difference with physics where all exchanges are given and cannot be modified using an external parameter.

Part II

Simulation of Coalition Forming, Instability and Stabilization

7 Implementation of Coalition Forming Computer Simulation

Applications of computational model is currently not common in social modeling, [38] probably because they are considered as hardline, unreliable or unsuitable for prediction compared to analytical methods. In fact, computational models and their computer simulations can deliver reliable results beyond the range of analytical tractability, and the benefit is not restricted to prediction. Reasons include explanation and illustration, revealing dynamical analogies, discovering new questions and decision support. They can enable to test mechanisms and theories, to study situations for which analytical solutions are not known, and thus go beyond the analytical approximations which, when applied to the real world problems, may result in misleading conclusions.

Here we present the implementation of our coalition forming models as a computer simulation, which aims to illustrate, to track visually, to investigate the paradigms and processes, and finally to provide a support to decision in coalition forming.

The simulation implements the theoretical structures of the Natural and Global Alliance models, where countries interact through making choices based on their bilateral propensities and natural belongings, the interactions being motivated by maximization of their gain function. It includes the computational part which provides the main model implementation and calculations, and the graphical part which accounts for screening the coalition forming process.

The simulation can be considered as a tool for tracking the system's properties and to predict dynamics convergence.

7.1 Computer Implementation of Actors and Coalition Forming

Computer implementation of a theoretical model, as well as the real case configurations, includes realization of the complex features and properties inherent to the real actors. The implementation of rationality of actors requires the presence of components appropriate to artificial intelligence simulations.

Therefore, the following functionality is built into the computer implementation of actor:

membership awareness:
: an actors belonging to a coalition depends on the actor becoming aware of its association to the union and treating fellow members differently from non-members.

knowledge:
: actor starts with a given quantity of knowledge about the system – other actors, its direct and indirect neighbors.

data collection:
: actor must posses the memory capacity necessary to collect the data from the system, to be able learn about its indirect connections within the system, and to learn at each moment about other actors' sates.

observation:
: actor's rationality property requires the ability to combine the data collection capacity and the membership awareness. Different levels of rationality capacities impose different ability of system observation.

individual interests awareness:
: an actor must be able to calculate it's gain, and be aware of its beneficial configurations at each moment of the decentralized maximization process. It must be able, as a function of its rationality capacity to calculate its maximal gain.

decision making:
: ability to react to observation of the coalitions structure and to choose an appropriate state.

action readiness:

actors must be able to make its observations and react within sufficiently small time span between the system change and the observation.

Given a set actors and propensities, the simulation creates an executable system. For simulation of large systems, the implementation enables the possibility of creation of random propensities using any required discrete distribution.

The concurrency method used in the implementation is multi-threading, where actors are represented by simultaneously running threads that make observation of system's condition and take decisions of their states.

Actors' choices are made in a random order. Implementation of the improvement of gain of an actor is reduced to the determination of the state bringing an immediate gain. Then, depending on rationality and observation capacity of the actor, if the best accessible gain does not correspond to the maximal possible benefit, the inverse choice is adopted by the actor. Thus, primary actor makes disadvantageous change putting the system out of its local maximum and as to lead to the improvement of it individual gain.

The secondary actors undertake only the changes bearing an immediate gain. They adhere to the policy of anticipation of a negative result, which guarantees a benefit without jeopardizing the whole system. This is a heuristic property that positions the simulation close to real systems of countries and rational actors. The assumption prevents no-cost fluctuations in the simulation. The rational instability is triggered by the primary actors, and never by the secondary actors.

The graphic representation of the coalition forming is depicted into a screen in the form of continuous runtime animation. It allows to track visually the gain maximization process for each actor simultaneously in the system.

The simulation allows to build, track, illustrate and study with up to 50 countries on a medium screen:

- Natural coalition forming, the phenomena of instability and the decentralized gain maximization process.

- Global alliance forming and globally motivated stabilization.

- Dissolution of a global alliance, and the partial instability.

7.2 Technical Details of the Computer Implementation

Below is the technical information on the computer implementation of the coalition forming models:

- Development kit: Java $JDK7$.
- Operation System: platform independent.
- Graphic user interface widget toolkit: Java AWT (Abstract Window Toolkit).
- Concurrency and animation method: Java Multi-threading.
- Graphic animation toolkit: Java Swing.
- Mathematical operations: Java Math.

7.3 Simulation of Coalition Forming in NM

Here, we present the simulation of coalition forming in a Natural model of a finite-size system. The system in its initial state is shown in Figure (29).

We again refer countries possessing the complete system information and extensive capacities as primary actors, and countries who have a limited system knowledge and a limited capacity as secondary actors. The system in Figure (29) include both the type of actors.

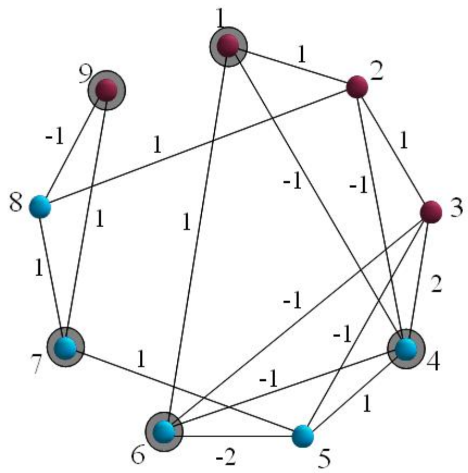


Figure 29: The initial state of a system of 9 countries. Countries 1, 4, 6, 7 and 9 are primary actors, and 2, 3, 5 and 8 are secondary actors. Two groups with disconnected primary actors are formed in the system.

Primary actors are countries 1, 4, 6 and 7, 9 emphasized by an additional circles around the nodes in the figure. The remaining countries are the secondary actors.

The primary actors countries form two groups $\{1, 4, 6\}$ and $\{7, 9\}$ which have no direct connection between them. Both groups have the local common maximums:

$$\mathcal{S}_{1,4,6} = \{(1, -1, 1), (-1, 1, -1)\} \text{ and } \mathcal{S}_{7,9} = \{(1, 1), (-1, -1)\}.$$

The negative circles of the system are $\{2, 3, 4\}$, $\{1, 2, 3, 4\}$, $\{3, 4, 5\}$, $\{3, 4, 6\}$, $\{1, 2, 3, 4, 5, 6\}$ and $\{7, 8, 9\}$. As it can be observed from the figure, the system consists of two parts, each of which formes a negative circle, the right part $\{1, 2, 3, 4, 5, 6\}$ and the left part $\{7, 8, 9\}$.

Despite the fact that the right part contains several negative circles, it is being stabilized due to particular chosen interactions between primary and secondary actors (see Figure (30)). Step (15) in the figure concludes the process of maximization in the right part of the system. At the step, the group of primary actors $1, 4, 6$ meet their common maximum, and none of the secondary actors can improve their gain at an immediate change. Thus, the right part in the last configuration exhibit the non-optimal stability.

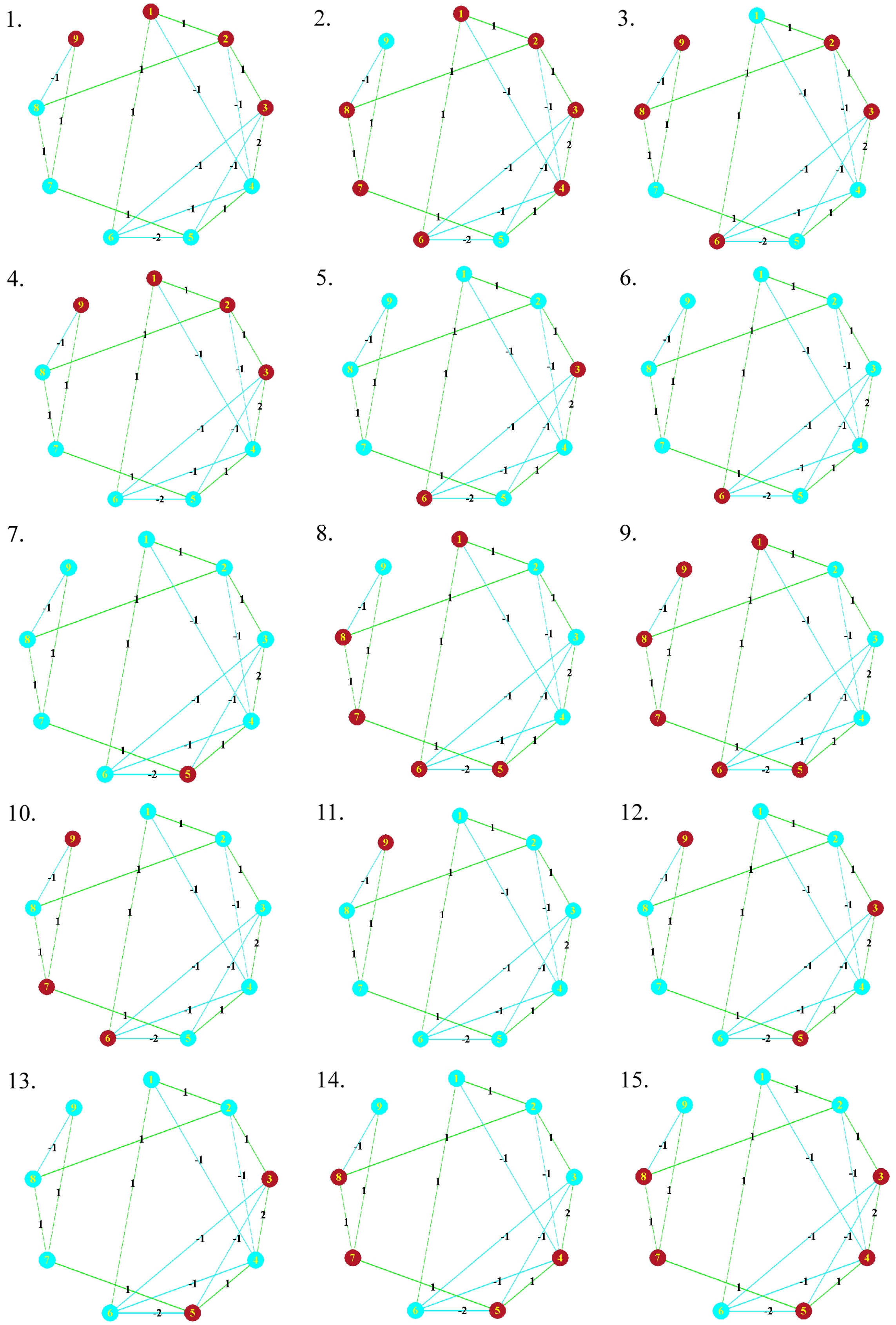


Figure 30: Simulation of the coalition forming – 15 steps of stabilization of the right part.

Let us look at the stabilization in detail:

1. The initial random state.
2. The principal actors 4 and 6 make an immediate improvement to gain simultaneously, the principal actors 7 and 9 make the change for an expected gain, and country 8, being a secondary actor, improves its gain in reply.
3. The principal actor 1 makes a change aimed at a future gain and 4 improves its gain in reply, 7 and 9 again make the change for an expected gain.
4. Country 1 improves its gain, 6 makes a change aimed at a future gain and 8 make an immediate improvement of the gain.
5. Country 6 improves its gain, at the same time 1 makes change for an expected gain, and country 2 improves its gain in reply.
6. As a result, country 3 improves its gain.
7. Country 6 makes a change for an expected gain and 5 improves its gain in reply.
8. Country 1 makes a change aimed at a future gain and 6 improves as a result, country 7 makes the change for a future gain and 8 improves in reply.
9. Country 9 makes the change for an expected gain.
10. Countries 1, 5 and 8 make an immediate improvement of their gains.
11. Countries 6 and 7 make the changes aimed at future gains.
12. As a result, countries 5 improves its gain.
13. Country 9 makes the change aimed at a future gain.
14. Countries 3 and 4 make an immediate improvements, 7 makes the change aimed at a future gain and 8 improves as a result.
15. Finally, countries 3 improves its gain.

In the given disposition, the stability of the right part is solid enough not to be broken despite the fluctuation in the left part. Figure (31) shows the instability of the triangle $\{7, 8, 9\}$, from the step 16 to the step 22.

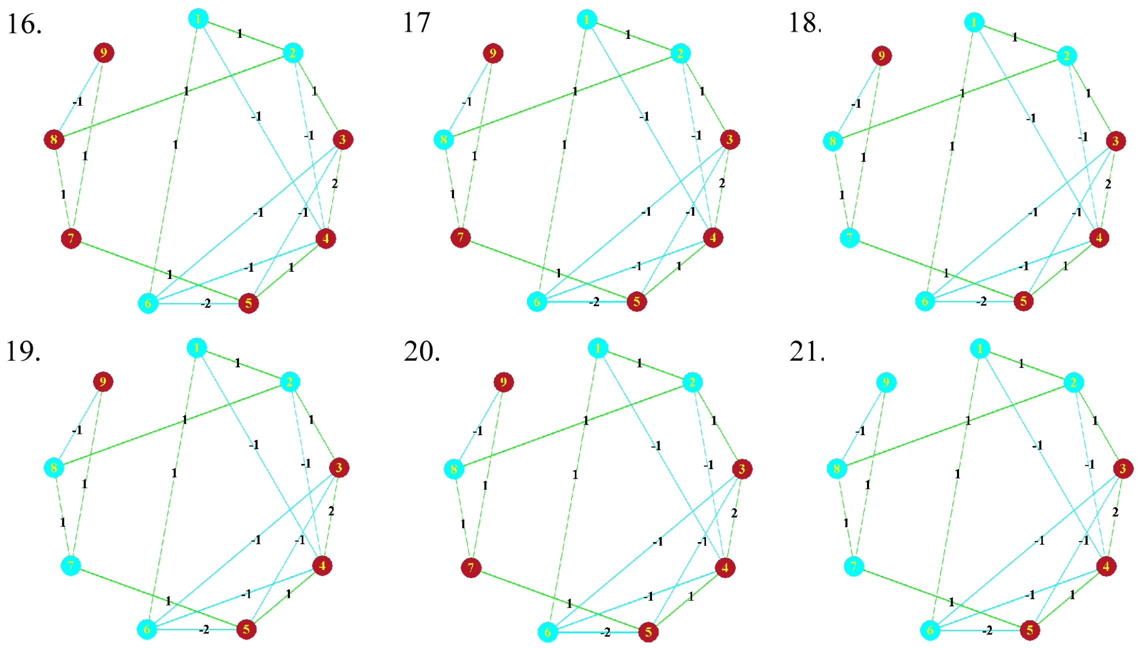


Figure 31: Partial instability – the instability of the left part versus the stability of the right part.

However, the stability of $\{1, 2, 3, 4, 5, 6\}$ is fragile since it breaks even upon a slight change of a propensity between the secondary actors. For instance, if the propensity between the countries 2 and 8 rises to 2, the instability of the left part is propagated to the stable right part. Another example is when the propensity between countries 3 and 5 decreases to -3. Then, country 3 will be constantly improving its gain immediately from -1 to 1 by inverting its choice.

7.4 Simulation of Coalition Forming in *GAM*

Let us consider an unstable system of 5 actors, all possessing extensive rationality, as depicted in Figure (32). The systems contains one negative circle– the $1, 2, 3$ negative triangle.

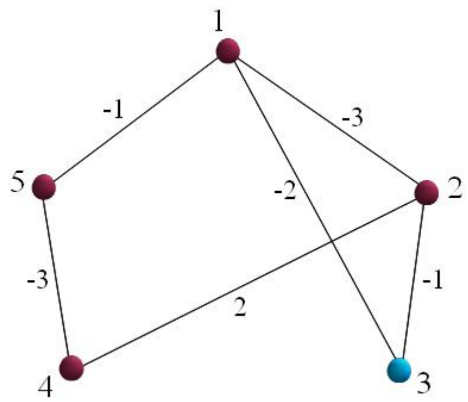


Figure 32: The initial state of the system of 5 actors with extensive rationality. The systems contains is unstable since it contains one negative circle – the $1, 2, 3$ negative triangle.

Consider now two opposing global alliances M and N set over the system, which engender the following natural belonging parameters: $\epsilon_1 = -1$, $\epsilon_2 = -1$, $\epsilon_3 = 1$, $\epsilon_4 = -1$, $\epsilon_5 = -1$.

With the additional interaction amplitudes as $G_{12} = 4$, $G_{13} = 4$, $G_{14} = 2$, $G_{15} = 0$, $G_{23} = 1$, $G_{24} = 1$, $G_{25} = 0$, $G_{34} = 2$, $G_{35} = 0$, $G_{35} = 2$, which are set at step 10 of the maximization process, as shown in Figure (33), the system become stable.

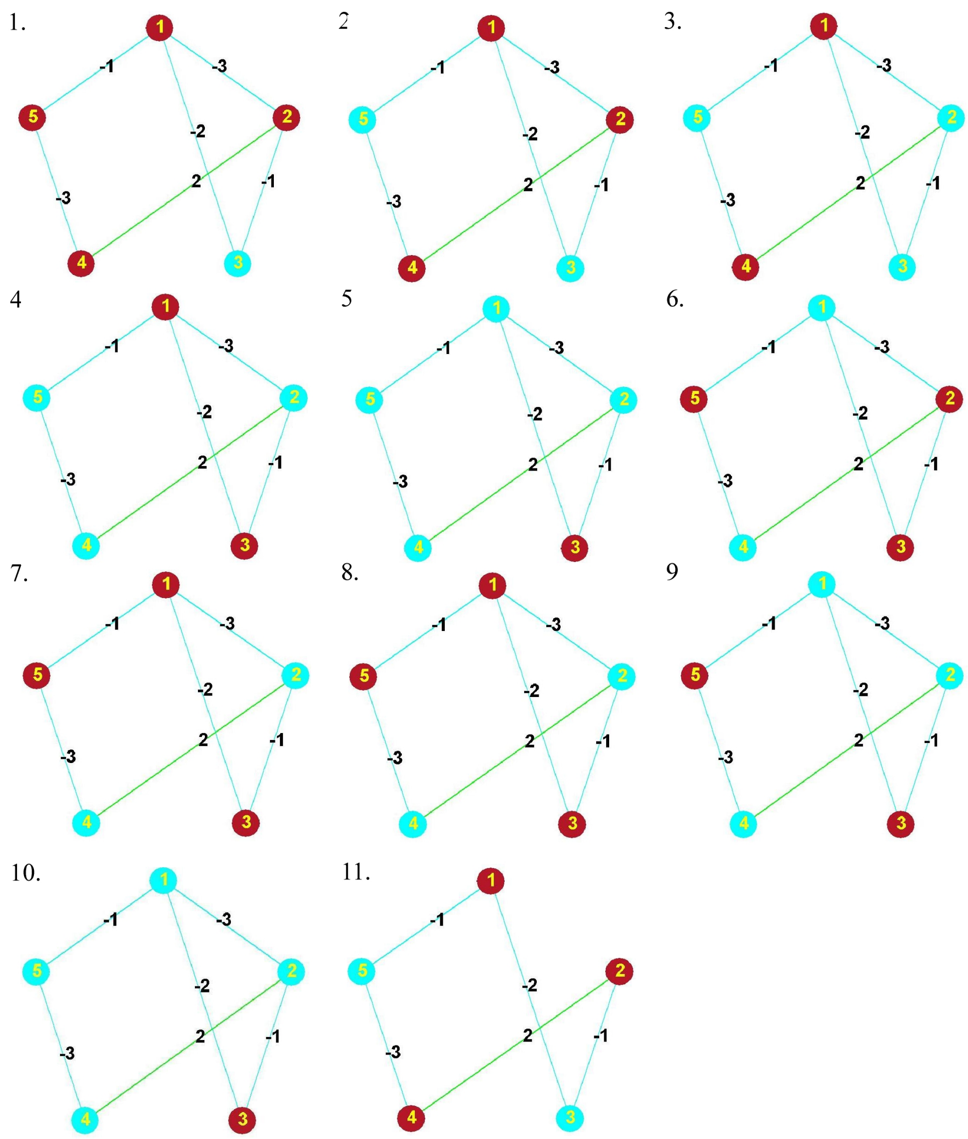


Figure 33: Show the instable cycling of the system of 5 actors in the Natural Model. The global alliances M and C set at step 10 stabilize the system immediately.

However, with additional interaction amplitude between 1 and 2 equal to $G_{12} = 2$, the system is left unstable since the interaction is insufficient to compensate the negative primary bond between those countries.

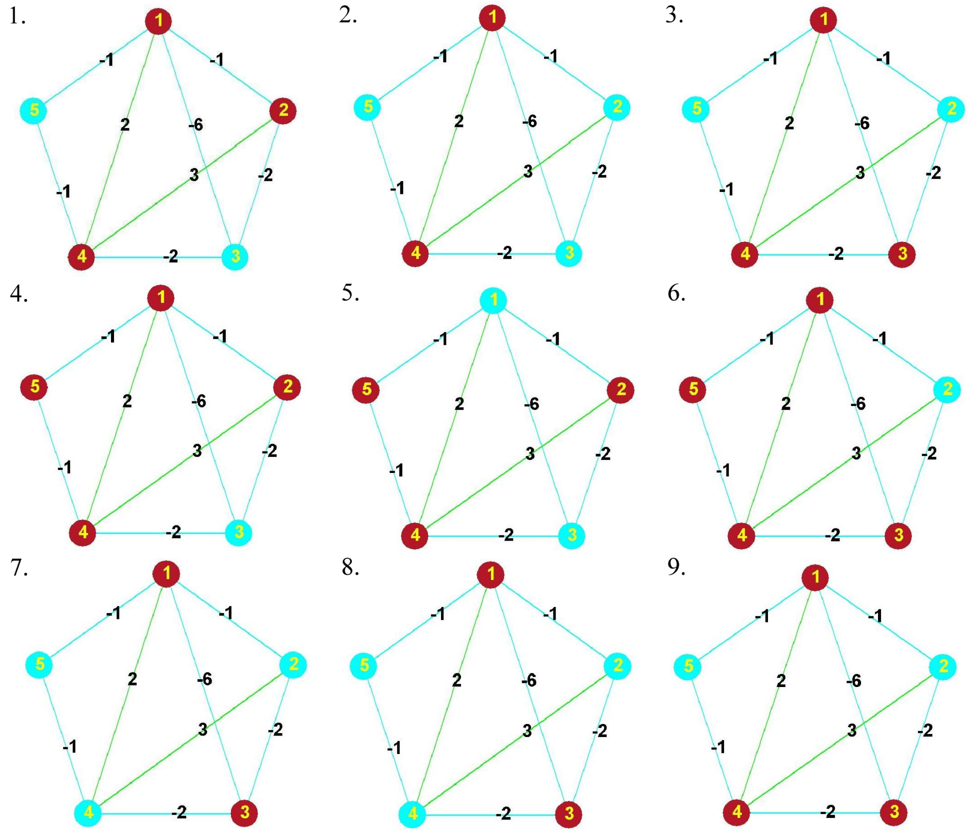


Figure 34: The global alliances M and C set at step 1 do not stabilize the system. The additional interaction amplitude of $G_{12} = 2$ is insufficient to compensate the negative primary bond between the countries 1 and 2.

The minimal additional interaction amplitudes for which the system is stable are $G_{12} = 3$, $G_{13} = G_{14} = G_{15} = G_{23} = G_{24} = G_{25} = G_{34} = G_{35} = 0$.

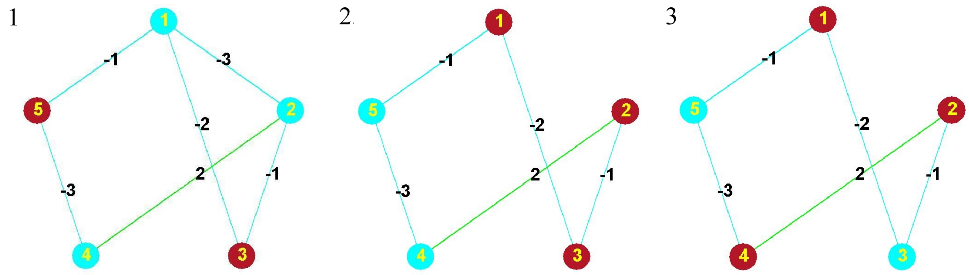


Figure 35: The global alliances M and C set at step 1 with the minimal additional interaction amplitudes that stabilize the system: $G_{12} = 3$, $G_{13} = G_{14} = G_{15} = G_{23} = G_{24} = G_{25} = G_{34} = G_{35} = 0$.

Let us remark that all the above propensity values were chosen in order to illustrate those particular system's dynamics. However, such the set of the values is not unique and many other sets can produce the same dynamics.

8 Appendix: Computer Code of Coalition Forming Simulation

```
// ConflictsAnimation.java


import javax.swing.*; import java.awt.*; import java.awt.event.*;
import java.util.Random; import java.lang.Math.*;
//import org.apache.commons.math3;

public class ConflictsAnimation extends JFrame implements Runnable {

    // member variables
    static int WIDTH = 800;
    static int HEIGHT = 720;
    static int RADIUS = 30;
    static int CFONT = 40;
    static Boolean SYS_RAND = false; // for the case the system must be
        created randomly
    static Boolean SYS_CMD = false; // system is created by user

    final static Color bkcolor = Color.WHITE;
    final static Color C_RED = new Color(180, 23, 40);
    final static Color C_BLUE = new Color(0, 255, 255);

    private Thread animator = null;
    private static Random rand = new Random();
    private Country [] system = null;
    private int count;
    private int GA = 0; //no global alliances
    private int country_propensity_waiting = -1;// name of country
        waiting coupled country for propensity setting
```

```java
private JButton GA_button = null;
private JButton DS_button = null;
private JButton ready_button = null;
private JButton [] country_buttons = null;
private int[][] usr_propensities = null;
private int[] usr_belongings = null;
private Container contentPane;

// methods:

//Constructor
public ConflictsAnimation(int c_count, int[][] c_propensity, String[]
    c_ability) {
    int[] x, y;

    if (c_count >= 3)
        count = c_count;
    else
        count = 3;

    set_screen (count);

    x = new int[count];
    y = new int[count];
    system = new Country[count];

    //prepare the canvas
    this.setTitle("Coalition Forming Simulation. By Galina Vinogradova
        for PhD degree, Ecole Polytechnique.");
    contentPane =
        this.getContentPane();//this.getContentPane().setBackground(bkcolor);
    contentPane.setBackground(bkcolor);
```

```
    //Initialize countries
    place_countries (x, y);
    fill_propensities (c_propensity);

    for ( int i = 0; i < count; i++ ) {
        if (SYS_CMD)
            system[i] = new Country (i+1, x[i], y[i], count,
                c_ability[i], c_propensity[i]);
        else //user mode, all will be set by user
            system[i] = new Country (i+1, x[i], y[i], count, null,
                null);
        System.out.println("Created country " +(i+1) +" ref " +
            system[i] + " with coordinates " + x[i] + " and "+ y[i]);
    }

    usr_propensities = new int[count][count];
    collection_system_data ();
    ready_system_listener ();

    if (!SYS_CMD){
        GA_user_dialog_new ();
        DS_user_dialog ();
    }

    setDefaultCloseOperation(JFrame.EXIT_ON_CLOSE);
    setSize(WIDTH, HEIGHT);
    setVisible(true);

    System.out.println("Initialization complete for " + count + "
        countries");

}
```

```
private void set_screen (int c){
    int factor = (int)Math.round(c/4); // till 7 countries ok, more we
        need to increase the screen and decrease the countries
    int W =
        java.awt.GraphicsEnvironment.getLocalGraphicsEnvironment().getMaximumWindowB
    int H =
        java.awt.GraphicsEnvironment.getLocalGraphicsEnvironment().getMaximumWindowB
    Boolean err = false;

    System.out.println("SIZE ..... " + W + " " + H);

    if (factor > 1 ){
        WIDTH  = 800 + 80*factor;
        HEIGHT = 720 + 70*factor;
        RADIUS = 30 - (int)Math.round(2*factor);
        CFONT = 40 - (int)Math.round(3*factor);
    }

    if (WIDTH > W | HEIGHT > H | RADIUS < 10 | CFONT < 10){
        System.err.println("Countries map can not match the screen
            size");
        System.exit(1);
    }
}

private void collection_system_data (){
    country_buttons = new JButton [count] ;
    country_propensity_waiting = -1;

    for ( int i = 0; i < count; i++ ){
        String b_name = Integer.toString(i+1);
        JButton country_button = new JButton(b_name);
        country_buttons [i] = country_button;
```

```
country_button.addActionListener(new ActionListener(){
    public void actionPerformed(ActionEvent ae){
        String user_str = null;
        String user_abl = null;
        JButton b = (JButton)ae.getSource();
        String b_name = ae.getActionCommand();
        int c_index_this = Integer.parseInt(b_name) - 1;
            //country of this chosen country_button
        int c_index_other = -1; //country of the previously
            chosen country_button

        //if GA set natural belonging
        if (GA > 0){
            Boolean blg_unset =
                (system[c_index_this].CountryGetBelongingToGA ()
                == 0);
            int user_GA_belonging = 0;
            if (blg_unset){
                user_GA_belonging = read_user_int_choice ("Enter
                    the natural belonging to global alliances M
                    and C (1 for M and -1 for C)",
                                   "Switch to Global Alliances",
                                       true, -1, 1, false, 0);
                system[c_index_this].CountrySetBelongingToGA
                    (user_GA_belonging);
                usr_belongings[c_index_this] = user_GA_belonging;
            }
        } //set the Ability
        else if (!
            (system[c_index_this].CountryIsCapacitySet()) ){
            user_abl = read_user_string_choice ("Enter the
                Rationality Capacity for country " +
```

```
                (c_index_this +1) + " among E -extensive,
                   L -low (closest neighbours)",
                "Rationality Capacity", null, 0, null);

        system[c_index_this].CountrySetCapacity (user_abl);
    }

    if (country_propensity_waiting == -1)
        country_propensity_waiting = c_index_this; //wait
            for pair country to be chosen
    else if (country_propensity_waiting == c_index_this)
        //the same country chosen twice, un-choose it
        country_propensity_waiting = -1;
    else c_index_other = country_propensity_waiting; // the
        pair was chosen already

    if (c_index_this != -1 & c_index_other != -1){
        int c_name_this = c_index_this +1;
        int c_name_other = c_index_other +1;

        //if GA read additional propensities
        if (GA > 0){
            int add_propensity = read_user_int_choice (
            "Enter the GA motivated propensity,
                non-negative, between counties " +
                c_name_this + " and " + c_name_other,
                            "GA Propensity collection",
                               false, 0, 0, true, 0);

            if (add_propensity != 0){
                usr_propensities[c_index_this][c_index_other]
                    = add_propensity;
```

```
            draw_propensity_betw
                (contentPane.getGraphics(),
                c_index_this, c_index_other,
                                  Integer.toString(add_propensity)
                                      );

            country_propensity_waiting = -1; //prepare
                for new pair of countries for propensity
                setting
        }
    }
    else{
        //two countries are chosen, we can read the
            propensity
        int propensity = read_user_int_choice (
                           "Enter the propensity between
                               counties " + c_name_this +
                               " and " + c_name_other,
                           "Propensity collection",
                               false, 0, 0, false, 0);

        if (propensity != 0){
            usr_propensities[c_index_this][c_index_other]
                = propensity;

            system[c_index_this].CountrySetPropensityWith
                (c_name_other, propensity);
            system[c_index_other].CountrySetPropensityWith
                (c_name_this, propensity);

            country_propensity_waiting = -1; //prepare
                for new pair of countries for propensity
                setting
```

```
                    }
                }
            }

        }
    });

    int c_x = system[i].CountryGetX();
    int c_y = system[i].CountryGetY();
    int leftx = c_x - RADIUS;
    int lefty = c_y - 2*RADIUS+12;
    country_button.setBounds(leftx, lefty, 50, 50); //(x, y,
        width, height)
    contentPane.setLayout(null); // set your content panel to use
        absolute layout
    contentPane.add (country_button);
  }
}

private void ready_system_listener ()
{
    ready_button = new JButton("Ready");

    ready_button.addActionListener(new ActionListener(){
        public void actionPerformed(ActionEvent ae){

            int usr_res = JOptionPane.showOptionDialog (null,
                "Finished to enter system data", "System is Ready",
                        JOptionPane.YES_NO_OPTION,
                            JOptionPane.QUESTION_MESSAGE,
                        null,
                        null, //array of option like Object[] options
                            = {"a", "b", "c"};
```

```
                null); //marked option, like options[2]
    //Ok to finish
    if (usr_res == 0) {
        //remove country buttons from the screen
        for ( int i = 0; i < count; i++ ){
            country_buttons[i].setVisible(false);
            if (GA > 0)
                system[i].CountryResume ();
            else{
                system[i].start();
                system[i].CountrySetSystemMap (system);
            }
        }

        fill_propensities (usr_propensities);

        if (GA > 0)
            ConflictsAnimationSetGA (usr_belongings,
                usr_propensities);
        else
            ConflictsAnimationSetPropensities (null,
                usr_propensities);

        clean_usr_propensities ();

        ready_button.setVisible (false);

        if (GA > 0)
            DS_button.setVisible(true);
        else
            GA_button.setVisible (true);
    }
}
```

```
    });

    //cPane.add (GA_button, BorderLayout.SOUTH);
    ready_button.setBounds(640, 30, 140, 20); //(x, y, width, height)
    contentPane.setLayout(null); // set your content panel to use
        absolute layout
    contentPane.add (ready_button);
}

//OLD using input line
private void GA_user_dialog (){
    GA_button = new JButton("Natural Model");

    GA_button.addActionListener(new ActionListener(){
        public void actionPerformed(ActionEvent ae){
            String str = null;
            Boolean create_global_alliance = false;

            if (GA == 0) create_global_alliance = true;

            if (create_global_alliance){
                //put back country buttons on the screen and suspend
                    until GA set
                for ( int i = 0; i < count; i++ ){
                    country_buttons[i].setVisible(true);
                    try{
                        system[i].wait() ;
                    }catch(InterruptedException e){
                        e.printStackTrace();}
                }
            }
```

```
if (create_global_alliance)
    //just upper half of the matrix of belongings
    str = JOptionPane.showInputDialog(null, "enter the
        natural belongings (1 for M and -1 for C) and all
        the interactions amplitudes:",
"Switch to Global Alliances", 1);
else
    str = JOptionPane.showInputDialog(null, "type 'M', 'C'
        or 'all' to remove the desired alliance:",
"Remove Global Alliances", 1);

if(!create_global_alliance & str != null){
    //dissolve one or all
    ConflictsAnimationDissolveGA (str);
}

else if(create_global_alliance & str != null){
    //data size: all the belongings + upper half of the
        matrix of additional propensities without the
        diagonal
    int ga_total_data_count = count + (count*count -
        count)/2;
    boolean user_data = true;
    int arg_indx = 0;
    int[][] ga_propensities = new int[count][count];
    int[] ga_belongings = new int[count];
    String ga_args[] = str.split(" ");

    System.out.println("Global alliance
        settings.................. data size: " +
        ga_args.length);

    if (ga_args.length < ga_total_data_count){
```

```
        JOptionPane.showMessageDialog(null, "Not enough
            data: " + str, "Switch to Global Alliances", 1);
        user_data = false;
        }

    //collect natural belongings
    for ( int i = 0; i < count & user_data; i++ ) {
        int ga_belonging_i = 0;
        try{
            ga_belonging_i =
                Integer.parseInt(ga_args[arg_indx]);
        }catch (NumberFormatException e ) {
            user_data = false;
        }catch (ArrayIndexOutOfBoundsException e){
            user_data = false;
        }

        if (user_data & (ga_belonging_i == -1 |
            ga_belonging_i == 1 | ga_belonging_i == 0))
            ga_belongings[i] = ga_belonging_i;
        else user_data = false;

        arg_indx ++;
    }

    //collect additional propensities
    for (int i = 0; i < count & user_data; i++ ){
        for (int j = i+1; j < count & user_data; j++ )
        {
            int ga_propensity_i_j = 0;
            try{
                ga_propensity_i_j =
                    Integer.parseInt(ga_args[arg_indx]);
```

```
        }catch (NumberFormatException e ) {
            user_data = false;
        }catch (ArrayIndexOutOfBoundsException e){
            user_data = false;
        }

        if (user_data) ga_propensities[i][j] =
            ga_propensity_i_j;

       arg_indx++;
    }
}

if (user_data) JOptionPane.showMessageDialog(null, "You
    entered: " + str, "Switch to Global Alliances", 1);
else JOptionPane.showMessageDialog(null, "Wrong data: "
    + str, "Switch to Global Alliances", 1);

if (user_data) {

    fill_propensities (ga_propensities);

    System.out.println("add propensities : ");
    for (int i = 0; i < count & user_data; i++ )
        for (int j = 0; j < count & user_data; j++ )
            System.out.println(ga_propensities[i][j]);

    System.out.println("belongings : ");
    for (int i = 0; i < count & user_data; i++ )
        System.out.println(ga_belongings[i]);

    ConflictsAnimationSetGA(ga_belongings,
        ga_propensities);
```

```
                }
            }

            if (str == null)
                JOptionPane.showMessageDialog(null, "You pressed cancel
                    button.", "Switch Model", 1);

            System.out.println("global_alliance_flag : " + (GA > 0));
            if(GA > 0)
                GA_button.setText("Global Alliance Model"); // change
                    to global alliance model
            else
                GA_button.setText("Natural Model"); //if global
                    alliances are removed change to natural model
        }

    });

    //contentPane.add (GA_button, BorderLayout.SOUTH);
    GA_button.setBounds(640, 30, 140, 20);//old(400, 600, 120, 20);
        //(x, y, width, height)
    contentPane.setLayout(null); // set your content panel to use
        absolute layout
    contentPane.add (GA_button);
    GA_button.setVisible(true);
}

private void GA_user_dialog_new ()
{
    GA_button = new JButton("Set GA");
    usr_belongings = new int[count];

    GA_button.addActionListener(new ActionListener(){
```

```
        public void actionPerformed(ActionEvent ae){

            GA = 2;

           //put back country buttons on the screen and suspend the
              threads until GA is set
            for ( int i = 0; i < count; i++ ){
                country_buttons[i].setVisible(true);
                //system[i].CountrySuspend (true, 600);
            }

            ready_button.setVisible(true);
            GA_button.setVisible(false);
        }
    });

    GA_button.setBounds(640, 30, 140, 20);//old(400, 600, 120, 20);
        //(x, y, width, height)
    contentPane.setLayout(null); // set your content panel to use
        absolute layout
    contentPane.add (GA_button);
    GA_button.setVisible(true);
}

private void DS_user_dialog ()
{
    DS_button = new JButton("Dissolve GA");
    usr_belongings = new int[count];

    DS_button.addActionListener(new ActionListener(){
        public void actionPerformed(ActionEvent ae){
            String usr_str = null;
            String [] GA_options = {"m", "c", "all"};
```

```
            usr_str = read_user_string_choice ("Choose a global
                alliance to dissolve, one or all: M, C, all",
            "Dissolution of GA", GA_options, 3, "Can not recognize
                your choice for the GA");

            ConflictsAnimationDissolveGA (usr_str);

            if (GA == 0)   //back to natural model
                GA_button.setVisible(true);
        }
    });

    DS_button.setBounds(640, 30, 140, 20);//old(400, 600, 120, 20);
        //(x, y, width, height)
    contentPane.setLayout(null); // set your content panel to use
        absolute layout
    contentPane.add (DS_button);
    DS_button.setVisible(false); //will set visible by Ready dialogue
}

public void ConflictsAnimationSetGA (int[] c_ga_belongings, int[][]
    c_ga_propensities)
{
    for ( int i = 0; i < count; i++ ) {
        Boolean blg_unset = (system[i].CountryGetBelongingToGA () ==
            0);
        int user_GA_belonging = 0;

        if (blg_unset){
            user_GA_belonging = read_user_int_choice (
            "Country " + (i+1) + ": Enter the natural belonging to
                global alliances M and C (1 for M and -1 for C) ",
```

```
                                "Switch to Global Alliances", true, -1,
                                   1, false,0);
            usr_belongings[i] = user_GA_belonging;
        }
        system[i].CountrySetBelongingToGA (c_ga_belongings[i]);
    }

    for ( int i = 0; i < count; i++ ) {
        system[i].CountrySetGA (GA > 0, c_ga_propensities[i]);
        System.out.println("Added global alliance to country " +(i+1));
    }
}

public void ConflictsAnimationDissolveGA (String str)
{
    String str_common = str.trim().toLowerCase();

    if (str_common.equals("all")){
        GA = 0;

        for (int i = 0; i < count; i++ ){
            system[i].RemoveGA (1, GA > 0);
            system[i].RemoveGA (-1, GA > 0);
        }
    }
    else if (str_common.equals("m")){
        GA --;
        for (int i = 0; i < count; i++ )
           system[i].RemoveGA (1, GA > 0);
    }
    else if (str_common.equals("c")){
        GA --;
        for (int i = 0; i < count; i++ )
```

```
            system[i].RemoveGA (-1, GA > 0);
        }
    }

    private int read_user_int_choice (String user_msg, String win_name,
        boolean usr_chk, int low_chk, int up_chk, boolean usr_lmt, int lmt)
    {
        String user_str = null;
        int user_int = 0;
        Boolean user_ok = false;

        while(!user_ok){
            user_str = (String)JOptionPane.showInputDialog (null,
                user_msg, win_name, 1);
            if (user_str!= null)
                user_ok = true;

            try{
                user_int = Integer.parseInt (user_str);
            }catch (NumberFormatException e ) {

                int usr_res = JOptionPane.showOptionDialog (null, "It is
                    not an integer value. Default is -1. Quit ?", win_name,
                            JOptionPane.YES_NO_OPTION,
                                JOptionPane.QUESTION_MESSAGE,
                            null,
                            null, //array of option like Object[] options
                                = {"a", "b", "c"};
                            null); //marked option, like options[2]
                //Ok to quit
                if (usr_res == 0){
                    user_int = -1;
                    user_ok = true;
```

```
            }
        else
            user_ok = false;
    }

    if (user_ok & (usr_chk || usr_lmt)){
        //check if data constraints are not satisfied
        if(usr_chk & (user_int != low_chk) & (user_int != up_chk) )
            user_ok = false;
        if(usr_lmt & !(user_int >= lmt))
            user_ok = false;

        if (!user_ok){
            int usr_res = JOptionPane.showOptionDialog (null, "The
                value does not correspond. Default is 1. Quit ?",
                win_name,
                      JOptionPane.YES_NO_OPTION,
                          JOptionPane.QUESTION_MESSAGE,
                      null,
                      null, //array of option like Object[] options
                          = {"a", "b", "c"};
                      null); //marked option, like options[2]
            //Ok to quit
            if (usr_res == 0){
                user_int = 1;  //default
                user_ok = true;
            }
            else
                user_ok = false;
        }
    }
}
```

```
    return (user_int);
}

private String read_user_string_choice (String user_msg, String
    win_name, String [] chk_options, int num, String err_msg)
{
    String user_str = null;
    Boolean user_ok = false;

    while(!user_ok){
        user_str = (String)JOptionPane.showInputDialog (null,
            user_msg, win_name, 1);
        if (user_str !=null){
            if (chk_options != null){
                String str_common = user_str.trim().toLowerCase();
                for (int i = 0; i < num & !user_ok; i++ ){
                    if (str_common.equals(chk_options[i]))
                        user_ok = true;
                }

                if (!user_ok)
                    JOptionPane.showMessageDialog (null, err_msg,
                        win_name, 1);
            }
            else user_ok = true;
        }
    }

    return (user_str);
}

public void ConflictsAnimationSetPropensities (String[] c_abilities,
    int[][] c_propensities)
```

```
{
    for ( int i = 0; i < count; i++ ) {
        String abl = "";
        if (c_abilities != null)
            abl = c_abilities[i];
        system[i].CountrySetCapacity (abl);
    }

    for ( int i = 0; i < count; i++ ) {
        system[i].SetPropensities(c_propensities[i]);
    }
}

//start and stopping the animation thread
public void start() {

  if ( animator == null ) {

    animator = new Thread( this );
    animator.start();
  }
}

//Deprecated will be called later from "close window"
/*public void stop() {

  if ( animator != null && animator.isAlive() )
     animator.stop();

  animator = null;
}  */

public void run() {
```

```java
    while (animator != null) {

        repaint(); //this calls paint()

        try {

            Thread.sleep( 1500 ); // milliseconds

        } catch ( InterruptedException e ) {

            // do nothing
        }
    }
}


//Painting the animation
public void paint(Graphics g)
{
    update(g);
}


public void update( Graphics g )
{
    super.paint(g);

    for ( int i = 0; i < count; i++ ){
        for (int j = i+1; j < count; j++){
            draw_propensity_betw (g, i, j, null);
        }
    }
```

```
    //draw the countries
    for ( int i = 0; i < count; i++ ) {
        draw_country (g, i);
    }
}

private void draw_propensity_betw (Graphics g, int c_indx_i, int
    c_indx_j, String add_prop)
{
    Font name_font = new Font ("TimesRoman", Font.BOLD, CFONT);
    int prop_i_j = get_propensity (c_indx_i, c_indx_j);

        //draw the lines connecting between countries, different
            colors for positive and negative
        //draw line where the propensity is non-zero
        if ((c_indx_i != c_indx_j) & ((prop_i_j != 0) ||
            add_prop!=null)){
            int x_i = system[c_indx_i].CountryGetX();
            int y_i = system[c_indx_i].CountryGetY();
            int x_j = system[c_indx_j].CountryGetX();
            int y_j = system[c_indx_j].CountryGetY();

            if(prop_i_j > 0)
                g.setColor(Color.green);
            if(prop_i_j < 0)
                g.setColor(Color.cyan);

            String prop_string = Integer.toString(prop_i_j);
            if (add_prop != null){
                int b_i= system[c_indx_i].CountryGetBelongingToGA ();
                int b_j= system[c_indx_j].CountryGetBelongingToGA ();
```

```
            prop_string = prop_string + " + (" + b_i*b_j + ")( " +
                add_prop + ")";
        }

        //draw line - triple line
        g.drawLine( x_i, y_i, x_j, y_j);
        g.drawLine( x_i, y_i+1, x_j, y_j+1);
        g.drawLine( x_i, y_i-1, x_j, y_j-1);

        //draw the propensities, put them inside the little circle
        int x_place, y_place;

        x_place = x_i + (x_j - x_i)*4/9;
        y_place = y_i + (y_j - y_i)*4/9;

        x_place = (int)Math.round(x_place);
        y_place = (int)Math.round(y_place);

        //range circle wherein the propensity is shown
        //g.setColor(bkcolor);
        //g.fillOval(x_place, y_place, (int)Math.round(CFONT/2),
            (int)Math.round(CFONT/2));

        g.setColor(Color.black);
        g.setFont(name_font);
        g.drawString(prop_string, x_place-9, y_place+11);
    }
}

private void draw_country (Graphics g, int c_indx_i)
{
    int c_x = system[c_indx_i].CountryGetX();
    int c_y = system[c_indx_i].CountryGetY();
```

```
int c_name = system[c_indx_i].CountryGetName();
String s_name = Integer.toString(c_name);
int leftx = c_x - RADIUS;
int lefty = c_y - RADIUS;
Font name_font = new Font ("TimesRoman", Font.BOLD, CFONT);

if(c_x < RADIUS | c_y < RADIUS)
    System.out.println(" Wrong placement of countries ");

//full capacity countries have grey circles around
if ( system[c_indx_i].CountryCapacityIsFull() ){
    int grey_circ_rad = 6;
    g.setColor(Color.lightGray);
    g.fillOval(leftx -grey_circ_rad, lefty -grey_circ_rad ,
    2*(RADIUS+grey_circ_rad), 2*(RADIUS+grey_circ_rad));
}
//belonging to global alliances is marked by a colour
if(GA > 0){
    Color ga_color = Color.green;
    int b_i = system[c_indx_i].CountryGetBelongingToGA ();
    if (b_i == 1)
        ga_color = Color.blue;//Color.orange;
    else if (b_i == -1)
        ga_color = Color.magenta;
    // if belonging is already set
    if (b_i!=0){
        int ga_circ_rad = 4;
        g.setColor(ga_color);
        g.fillOval(leftx -ga_circ_rad, lefty -ga_circ_rad ,
        2*(RADIUS+ga_circ_rad), 2*(RADIUS+ga_circ_rad));
    }
}
```

```
        if(system[c_indx_i].CountryGetDecision() == 1)
            g.setColor(C_RED);
        else
            g.setColor(C_BLUE);

        g.fillOval(leftx, lefty, 2*RADIUS, 2*RADIUS);

        g.setColor(Color.yellow);
        g.setFont(name_font);
        g.drawString(s_name, c_x-9, c_y+11);
        //graphics.drawOval(10, 10, 100, 100);
}

private void place_countries (int[] x, int[] y){
// put here the code that creat countries coordinates on the circle
    double[] d_x;
    double[] d_y;
    double segment;
    double start_angle;
    int radius_of_circle = Math.round(Math.min (WIDTH, HEIGHT)/2
        -2*RADIUS - 10); //counting for margins -> -10
    int x_center = WIDTH/2;
    int y_center = HEIGHT/2;

    segment = (2 * Math.PI)/count;

    d_x = new double[count];
    d_y = new double[count];

    // initialise coordinates of all the countries, place them on a
        uniform Polyhedron
```

```
    //first create unite circle
    if (count % 2 == 0) //if even number
        start_angle = segment/2;
    else
        start_angle = 0;

    //create points on a circle with the centre in (0,0) and radius 1
    for ( int i = 0; i < count; i++ ) {
        d_x[i] = Math.cos(-(Math.PI)/2 + start_angle);
        d_y[i] = Math.sin(-(Math.PI)/2 + start_angle);
        start_angle = start_angle + segment;
    }

    //make the circle large and move it to the real center
    for ( int i = 0; i < count; i++ ) {
        d_x[i] = radius_of_circle * d_x[i];
        d_y[i] = radius_of_circle * d_y[i];

        d_x[i] = d_x[i] + x_center;
        d_y[i] = d_y[i] + y_center;

        //round to integer coordinates
        x[i] = (int)Math.round(d_x[i]);
        y[i] = (int)Math.round(d_y[i]);
    }

}

private void fill_propensities (int [][] c_propensity){

    for ( int i = 0; i < count; i++ ){
        for (int j = 0; j < count; j++){
            if (i > j){
```

```
                //take it from where it is set already.
                if (c_propensity[i][j] != 0)
                    c_propensity[j][i] = c_propensity[i][j];
                else if (c_propensity[j][i] != 0)
                    c_propensity[i][j] = c_propensity[j][i];
            }
            else if (i == j)
                c_propensity[i][j] = 0;
        }
    }

}

private void clean_usr_propensities (){
    for ( int i = 0; i < count; i++ ){
        for (int j = 0; j < count; j++){
            usr_propensities[j][i] = 0;
        }
    }
}
private int get_propensity (int c_i, int c_j){
    int c_name_j = c_j+1;
    int ret = system[c_i].CountryGetPropensityWith (c_name_j);

    return(ret);
}

//Input format: [num of countries] [abilities letters] [right part of
    the symmetric matrix of propensities]
//ex: java ConflictsAnimation 3 f f f -1 -1 1
public static void main (String args[]) {
    int arg_count = 0;
    int arg_index = 0;
```

```java
int[][] propensity_arg = null;
String[] ability_arg = null;
boolean user_data = true;

if (args.length > 0) {
    try{
        arg_count = Integer.parseInt(args[0]);
    }catch (NumberFormatException e) {
        System.err.println("Argument must be an integer");
        System.exit(1);
    }
}
else
    user_data = false;

arg_index ++;

if (arg_count < 3){
    arg_count = 3;
    user_data = false;
    System.out.println("At least 3 countries");
}

propensity_arg = new int[arg_count][arg_count];
ability_arg = new String[arg_count];

System.out.println("number of countries = " + arg_count);

//pick up the ability levels
for (int i = 0; i < arg_count; i++ ){
    String ability_i = "";

    if(user_data){
```

```java
            try{
                ability_i = args[arg_index];
            }catch (ArrayIndexOutOfBoundsException e){
                user_data = false;
            }
        }

        ability_arg[i] = ability_i;

        arg_index ++;
    }
    //pick up propensities
    //make a function pickup_propensities (args, arg_count,
        arg_index){}
    for (int i = 0; i < arg_count & user_data; i++ ){
        for (int j = i+1; j < arg_count & user_data; j++ ){
            int propensity_i_j = 0;
            if(user_data){
                try{
                   propensity_i_j = Integer.parseInt(args[arg_index]);
                }catch (NumberFormatException e ) {
                   user_data = false;
                }catch (ArrayIndexOutOfBoundsException e){
                   user_data = false;
                }
            }

            if (user_data){
                propensity_arg[i][j] = propensity_i_j;
            }
            else{
                //System.out.println("Propensities will be created
                    randomly");
```

```
            //old: create this propensity randomly with no
               predefied distribution:
            //    propensity_arg[i][j] = generate_rand_propensity
               ();

            System.out.println("Propensities are zero by default");
        }

        arg_index ++;

        if (user_data)
            System.out.println("i and j (" + i + j + ") is "
               +propensity_arg[i][j]);
    }
}

SYS_CMD = user_data;//if false data will come from user using
   buttons

//if user data was fault or absent, and system must be created
   randomly,
//recreate the propensities matrix with the random data
if (!user_data & SYS_RAND){
    int[] propensities = generate_rand_distribution (arg_count);

    arg_index = 0;

    for (int i = 0; i < arg_count; i++ ){
        for (int j = i+1; j < arg_count; j++ ){
            propensity_arg[i][j] = propensities[arg_index];

        arg_index ++;
```

```
            System.out.println("i and j (" + i + j + ") is "
                +propensity_arg[i][j]);
            }
        }
        SYS_CMD = false;
    }


    System.err.println("count " + arg_count + " prop ref" +
        propensity_arg);
    System.err.println("count " + arg_count + " ability ref" +
        ability_arg);


    ConflictsAnimation animation = new ConflictsAnimation(arg_count,
        propensity_arg, ability_arg);
    System.out.println("Main: ");
    animation.start();

}

// UNUSED, old
private static int generate_rand_propensity (){
    int p = 0;
    if (rand.nextBoolean())
        if (rand.nextBoolean())
            p = 1;
        else p = -1;

    return (p);
}

//NOT FINISHED
```

```
private static int[] generate_rand_distribution (int count){
    int[] nums_to_generate       = new int[]   { -1,   1,    0  };
    double[] discrete_probabilities = new double[] { 0.4, 0.4, 0.2 };
    int[] samples = null;

  /*  EnumeratedIntegerDistribution distribution =
    new EnumeratedIntegerDistribution(nums_to_generate,
       discrete_probabilities);

    int[] samples = distribution.sample (count); */
    return (samples);
}
//UNUSED
public int[] GenerateRandDistribution (int count){
    return (generate_rand_distribution(count));
}

//UNUSED if to be used must be called before ConflictsAnimation
    constructor in main
public void create_random_model (int arg_count) {
    int[][] propensity_arg = null;
    //int[] args = null;
    String[] ability_arg = null;
    //boolean user_data = false;

    if (arg_count < 3)
        arg_count = 3;

    propensity_arg = new int[arg_count][arg_count];
    ability_arg = new String[arg_count];

    //create the ability levels
    for (int i = 0; i < arg_count; i++ ){
```

```
            //OLD: By default the ability is "Full"
            //take the ability from user
            ability_arg[i] = "";
        }

        //create propensities
        int arg_index = 0;
        int[] args = generate_rand_distribution (arg_count);

        for (int i = 0; i < arg_count; i++ ){
            for (int j = i+1; j < arg_count; j++ ){
                propensity_arg[i][j] = args[arg_index];
                arg_index ++;
                System.out.println("i and j (" + i + j + ") is "
                    +propensity_arg[i][j]);
            }
        }

        ConflictsAnimation animation = new ConflictsAnimation (arg_count,
            propensity_arg, ability_arg);
        animation.start();

    }
}
```

```java
// Country.java

import javax.swing.*; import java.awt.*; import java.awt.event.*;
import java.util.Random;

public class Country implements Runnable {

    //internal types
    public enum AB {E, L, U} //abilities: full, low (first neighbours),
        unset

    // member variables
    private Thread decider = null;
    private static Random rand = new Random();
    private int x, y, name, index, count;
    AB ability_enum = AB.U;
    int decision_delay;
    private int[] propensities = null; //total propensities
    private int[] orig_propensities = null; //original propensities value
        between this country and all others
    private int[] add_propensities = null; //additional propensities
    //private int[] ga_belongings = null; //belonging parameters of all
        countries except this one
    private int ga_belonging = 0; //this is default country's natural
        belonging
    private Country [] system_map = null; //relates to system visibility;
        REPLACE USAGE BY parent_system
    ConflictsAnimation parent_system = null;
    private int choice;
    private int maxgain;
    boolean GA = false; //is this a global alliances model
    boolean Suspender = false;
```

```
// methods:

public Country(int c_name, int c_x, int c_y, int c_count, String
    c_ability_string, int[] c_propensities){

    System.out.println("Initialization of country " + c_name );
    if (c_count > 3)
        count = c_count;
    else
        count = 3;

   // set the countries coordinates
    if (legal_name (c_name) & (c_x > 0)&(c_y >0)){

        name = c_name;
        x = c_x;
        y = c_y;
    }
    else{
        System.err.println( "Illegal country data" + name + c_x + c_y
            );
        System.exit(1);
    }

    index = get_index_from_name (name);

    propensities = new int[count];
    orig_propensities = new int[count];
    add_propensities = new int[count];
    //belongings = new int[count];
```

```
    set_propensities (propensities, c_propensities);
    set_propensities (orig_propensities, c_propensities); // keep
        these original propensities

    system_map = new Country[count];

    choice = make_random_decision ();//default choice is random

    if (c_ability_string != null)
        set_ability_enum (c_ability_string);

    System.out.println("Country" + name+ "s ability : " + ability_enum
        +" and decision delay : " + decision_delay );
    // TO BE USED AFTER ALL SET FROM READY FROM ANIMATION :
        CountryPrintPropensities();

}


public boolean CountrySetGA (boolean ga_flag, int[]
    c_ga_propensities){

    if (c_ga_propensities != null){
        GA = ga_flag;
        set_propensities (add_propensities, c_ga_propensities); //
            keep these additional propensities
        //set_belongings (ga_belongings, c_ga_belongings);

        add_propensities (propensities, add_propensities);
        CountryPrintPropensities();
    }

    return (GA);
```

```
}

public boolean ResetGA (boolean ga_flag)
{
    GA = ga_flag;
    set_propensities (propensities, orig_propensities); //reset
       original propensities
    add_propensities (propensities, add_propensities);

    return (GA);
}

public void RemoveGA (int rem_belonging, boolean ga_flag)
{
    for (int i = 0; i < count; i++ ) {
        if (ga_belonging == rem_belonging)
            add_propensities [i] = 0;
    }
    if (ga_flag == false)
        ga_belonging = 0;

    ResetGA (ga_flag);
}

public void SetPropensities (int[] c_propensities){
    set_propensities (propensities, c_propensities); //set original
       propensities
    set_propensities (orig_propensities, c_propensities); //set
       original propensities
    //propensities are changed, recalculate max gain
    reset_max_gain ();
}
```

```
/*  private void set_belongings(int[] to_bel, int[] from_bel) {
      //take data from the belongings arrived from user
      copy_add_array (to_bel, from_bel, ga_belonging, false);
   }*/

   private void set_propensities (int[] to_pr, int[] from_pr) {
      //take data from the propensity arrived from user
      copy_add_array (to_pr, from_pr, 0, false);
      //propensities are changed, recalculate max gain
      reset_max_gain ();
   }

   private void add_propensities (int[] to_pr, int[] from_pr) {
      copy_add_array (to_pr, from_pr, 0, true);
      //propensities are changed, recalculate max gain
      reset_max_gain ();
   }

   private void copy_add_array (int[] to_arr, int[] from_arr, int
      this_country_data, boolean add) {
      //take data arrived from user, add or reset
      for ( int i = 0; i < count; i++ ) {

         int from_val_i =0;
         if (from_arr != null)
            from_val_i = from_arr[i];

         if(i == index)
            to_arr[i] = this_country_data;
         else if (add){//add array according to belonging to global
            alliances
            to_arr[i] = to_arr[i] + from_val_i * ga_belonging *
               system_map[i].CountryGetBelongingToGA();
```

```
        }
        else //copy - reset array
            to_arr[i] = from_val_i;
    }
}

//start and stop the making decision thread
public void start() {

  if ( decider == null ) {

    decider = new Thread( this );
    decider.start();
  }
}

// Depricated will be called later from "close window"
/*   public void stop() {

  if ( decider != null && decider.isAlive() )
     decider.stop();

  decider = null;
}   */

public void run()
{
    while (decider != null) {

        try {

            Thread.sleep (decision_delay /* + 20000*/ ); // additional
                sleep for 20000 for slow tracking
```

```
            //milliseconds for tracking the simulation run slowly

        } catch ( InterruptedException e ) {

            // do nothing
        }

        make_decision();
    }
}

public void CountrySuspend (boolean suspending, int time_millisec)
{
    if (suspending)
        Suspender = true;

    while (Suspender) {
        try {

            Thread.sleep (time_millisec /* + 20000*/ ); // additional
                sleep for 20000 for slow tracking
            //milliseconds for tracking the simulation run slowly

        } catch ( InterruptedException e ) {
            // do nothing
        }

    }
}

public void CountryResume ()
{
    Suspender = false;
```

```
}

private void set_ability_enum (String c_ability_string)
{
    AB c_ability_enum = AB.E;

    if (c_ability_string != null & ability_enum == AB.U)
    {
        try
        {
            c_ability_enum = AB.valueOf
                (c_ability_string.trim().toUpperCase());
        }
        catch(IllegalArgumentException ex)
        {
        }

        ability_enum = c_ability_enum;
    }

    set_decision_delay (ability_enum);
}

public void CountrySetCapacity (String c_ability_string)
{
     set_ability_enum (c_ability_string);
}

public boolean CountryIsCapacitySet ()
{
    boolean result = true;
    if (ability_enum == AB.U)
        result = false;
```

```
    return (result);
}


public boolean CountryCapacityIsFull ()
{
    return (ability_enum == AB.E);
}

private void set_decision_delay (AB c_ability_enum){

    //fill system's map here
    for ( int i = 0; i < count; i++ ) {
        if(i == index) continue;

        switch (c_ability_enum) {
            case E: decision_delay = 1500; break;
            case L: decision_delay = 2500; break;
            default: decision_delay = 1500; break;
        }
    }
}

public void CountrySetSystemMap (Country [] c_system_map){
    //fill system's map here
    set_system_map (c_system_map);

    //only system with E abilities can find out if the system is
        basicaly stable or unstable
    //because it knows all the countries and so all the posible
        configurations
    //look for a negative circle in the system
}
```

```java
private void set_system_map (Country [] c_system_map){
    //fill system's map here
    for ( int i = 0; i < count; i++ ) {
        if(i == index) continue;

        switch (ability_enum) {
            case E: system_map[i] = c_system_map[i]; break;// full

            case L: {if (get_propensity_from_index(i) != 0)
                system_map[i] = c_system_map[i]; //only connected
                countries
                    else system_map[i] = null;} break;
            default: system_map[i] = c_system_map[i]; break;//full
        }
    }

    CountryPrintSystemMap();
}

//TODO: real calculation of visible improvement with respect to the
    capacity index, which is red from the available system map
private void reset_max_gain (){
    maxgain = 0;
    for ( int i = 0; i < count; i++ ) {
        int prop = get_propensity_from_index(i);

        if(i == index | prop == 0) continue;

        maxgain = maxgain + Math.abs(prop);
    }
    System.out.println("max gain of " + name + " is " + maxgain);
}
```

```
private int set_propensity (int c_others_index, int c_propensity){
    int ret = -1;

    if (c_others_index >= 0){
        propensities[c_others_index] = c_propensity;
        orig_propensities[c_others_index] = c_propensity;
        ret=0;
    }
    else
        System.out.println("Illegal country index " + c_others_index );

    return (ret);
}

public int CountrySetPropensityWith (int c_others_name, int
    c_propensity){
    int c_others_index = get_index_from_name (c_others_name);

    return (set_propensity (c_others_index, c_propensity));
}

public int CountryGetPropensityWith (int c_others_name){
    int ret = 0;
    int c_others_index = get_index_from_name (c_others_name);

    ret = get_propensity_from_index (c_others_index);

    return (ret);
}

private int get_propensity_from_index (int c_others_index){
```

```
    int ret = 0;

    if (c_others_index >= 0)
        ret = propensities[c_others_index];
    else
        System.out.println("Illegal country index " + c_others_index );

    return (ret);

}

public int CountryGetBelongingToGA (){
    return (ga_belonging);
}

public void CountrySetBelongingToGA (int c_ga_belonging){
    ga_belonging = c_ga_belonging;
}

private boolean legal_name (int c_name){
    if ((c_name <= count)&(c_name >=1)) //index i+1 is country "name"
        return true;
    else{
        System.out.println("Illegal name " + c_name );
        return false;
    }

}

public int get_index_from_name (int c_name){
    if (legal_name (c_name))//index i+1 is country "name"
        return (c_name - 1);
    else
```

```
        return (-1);

}

public int CountryGetName (){

    return (name);
}

//Every country will be a thread that takes its time to observe and
    to make decision
public int CountryGetDecision (){

    // return decision on request, whether it is updated or not
    return (choice);
}

private void make_decision (){
    double H = 0;
    boolean rebel = false;

    //put to normal
    if(ability_enum == AB.E)
        decision_delay = 1500;

    if (system_map == null){
        //make random choice          HERE !!!! PRIMITIVE ACTORS
            RATHER MAKE NO CHOICE ???
        choice = make_random_decision();
    }
    else{
        for (int i = 0; i < count; i++ ) {
            int prop = get_propensity_from_index (i);
```

```
        int decision_i = 0;

        if (i == index | prop == 0) continue;

        if (system_map[i] != null) {
            decision_i = system_map[i].CountryGetDecision();
        }
        H = H + prop * decision_i;
    }

    // For now the simulation supports only E and L capacities.
    // Then, only the extensive rationality countries rebel.
    // TODO: rebel if you see improvement with respect to your
        capacity index 1-step for first neighbour, 2-steps etc.
    if((ability_enum == AB.E) & (Math.abs(H) < maxgain))
        rebel = true;

    //make new choice
    if (H < 0)
        choice = -1;
    else if (H > 0)
        choice = 1;
    else{// H = 0 -> fluctuation. Only leader-E fluctuates, the
        followers-L do not change their previous choice
         // OLD: H = 0, every one fluctuates
        if (rebel) choice = make_random_decision();
    }

    //leader E rebels but only sometimes
    if(rebel & rand.nextBoolean()){
        System.out.println("Country " +name + " rebel cause its
            max gain is " + maxgain +
        " and its gain going to be " +Math.abs(H) );;
```

```
            choice = choice *(-1);
            decision_delay = 4000; //put to waiting
            System.out.println("Country " +name + " rebel ");
        }

    }

    H = choice*H; //final energy
    System.out.println("Country " +name + " making decision " + choice
        + ", his H : " + H);
}


private int make_random_decision (){
    boolean coin = rand.nextBoolean();
    int descision;

    if (coin)
        descision = 1;
    else
        descision = -1;

    return(descision);
}

public void CountryPrintPropensities (){

    System.out.println("Propensities with country " + name + " are ");

    for ( int i = 0; i < count; i++ ) {
        int prop_with_i;

        if(i == index) continue;
```

```
            prop_with_i = get_propensity_from_index (i);
            System.out.println("country " + (i+1) + " : " + prop_with_i);
        }

    }

    public void CountryPrintSystemMap (){

        System.out.println("System map of " + name + " is ");

        for ( int i = 0; i < count; i++ ) {
            Country country_i;

            if(i == index) continue;

            country_i = system_map[i] ;
            System.out.println("country " + (i+1) + country_i);
        }

    }

    public int CountryGetX (){
        return (x);
    }

    public int CountryGetY (){
        return (y);
    }
}
```

Part III

Viability Correction of Dynamic Networks

9 Dynamic Network – Viability and Decentralized Correction

A viable system is any system organized such a way as to meet the requirements of surviving in the changing environment, an adaptable system. The viable system model expresses an abstracted regulation model for a viable system capable of autonomy.

It follows the consensus that the evolution of many variables systems (organizations, networks arising in biology and human and social sciences) evolve neither in a deterministic way not in a stochastic way, as it is usually understood, but in a "Darwinian" way. Viability theory started in 1973 by writing mathematically the title of the book "Chance and Necessity" by Jacques Monod, using the differential inclusion $x'(t) \in \mathcal{F}(x(t))$ for chance and $x(t) \in \mathcal{M}$ for necessity.

The system is said to be deterministic if the set $\mathcal{F}(x(t))$ consists of only one evolution, and is contingent otherwise. Necessity is the requirement that at each moment, the evolution remains in the environment $\mathcal{M}$ described by viability constraints, which encompasses multifaceted concepts such as stability, homeostasis, adaptation, etc., expressing the idea that variables must satisfy some constraints (physical, social, biological, economic, etc.). Viability theory states itself as the confrontation of evolutionary systems and viability constraints that such evolutions must obey.

Viability theory develops mathematical and algorithmic methods for investigating the "adaptation to viability constraints" of evolutions governed by complex systems under uncertainty that are found in many domains from biological evolution to economics and social sciences, from control theory and robotics to cognitive sciences. It uses the set-valued analysis, differential inclusions and differential calculus in metric spaces.

The basic problem is to find the viability kernel of the environment – the subset of initial states in the environment such that there exists at least one evolution which is viable in the environment. The second question is then to provide the regulation map selecting such viable evolutions starting from the viability kernel. The viability kernel assumes that some controls that regulates evolutions. Otherwise, the next problem is the Tychastic kernel (tyche – chance in Greek), the subset of initial states in the environment such that all evolutions are viable in the environment, which, in contrast to uncertainty.

In our work we address the further problem of viability which is finding the regulators of the evolutions that transform any viability environment into Tychastic kernel in a finite dynamic network. We address a finite system of individual actors with states that dynamically evolve independently from each other, and which are connected into a network via the connection operators. Operators evolve independently of actors therefore representing an autonomous connecting unit. The network is defined to be *viable* if a joint evolution satisfies the centralized scarcity constraints set by the environment. The system thus forms a network with *decentralized* dynamics and *centralized* collective viability constraints.

Together with the main subject of this part, which is the correction of viability of a dynamic network by decentralization between the dynamics of individual evolving actors, we attempt to bring a bridge connecting between the two: the Spin Glass based model of coalition forming and a dynamic Network model of coalition forming among actors.

Every discipline is based and develops within a unique foundation of epistemological concepts and assumptions. This implies that even when one discipline begins to share its problems with another, there may be no sufficient contact established because of different languages. Those disciplines may still have nothing to say to each other because the questions being asked and the answers considered interesting differ greatly. Such are for example the topics around Neural and dynamical Networks, with Toulouse's work on Spin Glass model of learning by selection [7] which uses the Spin Glasses to describe Neural Networks, and Bonneuil's work on Viability in dynamic social Networks [40] which presents Networks as controls in controlled dynamic systems. To bridge between the disciplines, considerable work in translation has to be done.

9.1 Dynamic Network Definition

The network formally is described as follows. Consider finite-dimensional vector spaces $X_1, X_2, \ldots, X_n$, Y and Z, and the product $X = X_1 \times \ldots \times X_n$. Denote by $\mathcal{L}(X,Y)$ the space of all linear operators from X to Y. Introduce a linear operator $V \in \mathcal{L}(X,Y)$, n maps $f_i : X_i \mapsto X_i$, a map $\beta : \mathcal{L}(X,Y) \mapsto \mathcal{L}(X,Y)$, and a map $g : Y \mapsto Z$.

The network's actors $1, \ldots, n$ are connected by the connection operator V through the actors' states $x = (x_1, \ldots, x_n) \in X$ with the actors' collective result Vx. Actors' states evolutions are governed by the dynamics generated by the equations $x_i'(t) = f_i(x_i(t))$ and the evolution of the network's connection operator is governed by $V'(t) = \beta(V(t))$. The evolutions set by the network's data must subject to viability constraints set by the environment and requiring that at any time $t \geq 0$, the consequence of actors' actions and their connections, that is the map g applied to the actors' collective result $V(t)x(t)$, is restricted to remain in a subset $\mathcal{M}$ of Z.

The network is described by the dynamics

$$\begin{cases} \forall i = 1 \ldots n, \quad x_i'(t) = f_i(x_i(t)) \\ V'(t) = \beta(V(t)) \end{cases} \tag{20}$$

and its viability constraints are given by

$$\forall t \geq 0, \quad g(V(t)x(t)) \in \mathcal{M} \tag{21}$$

To illustrate, consider the network (see figure (36)) of four actors $1, 2, 3, 4$ with the states $x_1(t), x_2(t), x_3(t), x_4(t)$ correspondingly, connected by the connection operator

$$V(t) = \begin{pmatrix} v_{11}(t) & v_{12}(t) \\ v_{21}(t) & 0 \\ 0 & v_{32}(t) \\ v_{41}(t) & v_{42}(t) \end{pmatrix} \tag{22}$$

to their collective results $y_1(t), y_2(t)$. The results are connected with the constrained results $z_1(t), z_2(t), z_3(t)$ (marked by the gray nodes in figure (36)) by map $g(y_1(t), y_2(t)) = (y_1(t), y_1(t) + y_2(t), y_2(t))$.

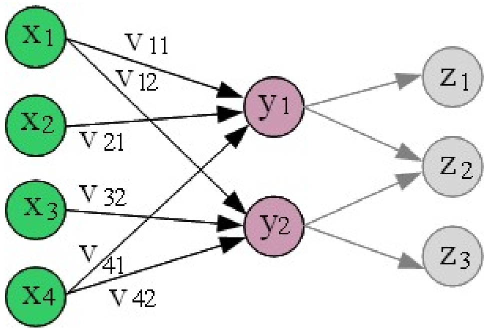


Figure 36: Scheme of the network of actors' states and their collective results.

The scheme of the network is similar to the simplified view of a neural network with the input layer consisting of actors' states, the hidden layer comprising the actors' collective results, and the output layer consisting of the constrained results ([55]).

By *environment of the network* we understand the set of all initial states $(x, V) \in X \times \mathcal{L}(X, Y)$ satisfying the viability constraints. We say that the environment of the network is *viable* under the network's dynamics if from any of the initial states starts at least one evolution governed by the dynamics that satisfies the viability constraints. The network is said to be *viable* if the environment of the network is viable under the network's dynamics.

The network most likely does not remain viable for all times when the dynamics of the actors and that of the connection operator are left to evolve by themselves. This is due to the absence of mutual dependencies between the dynamics which are required to satisfy the centralized viability constraints.

The main question we deal with in this work is that of restoring the viability of the network representing a decentralized model. We control the network's viability by modifying the decentralized settings in order to transfer to the centralized ones which satisfy the viability constraints.

9.2 Dynamic Network of a Coalition Forming Model

For creating a dynamical analogy to the natural coalition forming model we define the following dynamic model. Let us consider a system with N actors, where each actor i belongs to one of two coalitions $\mathcal{A}$ or $\mathcal{B}$, in such a way that the state of the actor is $s_i = 1$ if the actor belongs to $\mathcal{A}$ and $s_i = -1$ if it belongs to $\mathcal{B}$. Interactions between any two actors depend on their mutual propensity, which can be positive or negative. The propensity of two different actors i and j is denoted by v_{ij}. We define the gain of actor i from its interactions with other actors as follows:

$$\mathcal{H}_i = \sum_{j \neq i} \; v_{ij} s_j.$$

The actor's gain represents the system environment and the centralized scarcity constraints of the coalition forming model require that each actor i's gain is in a small space around its satisfactory value H_i^o.

To be a dynamic model, the coalitions must be fuzzy. Let $x_i \in [0, 1]$ be the value of the actor i's fuzzy belonging to the coalitions, that is i is x_i in $\mathcal{A}$ and $1 - x_i$ in $\mathcal{B}$. Then, actor i's fuzzy state s_i can be expressed in the terms of the fuzzy belonging as $s_i = x_i - (1 - x_i) = -1 + 2x_i$.

We assume the actors' states and the propensities be evolving with time according to given dynamics. Thus, the dynamic system of N actors with states $s(t) = (s_1(t), s_2(t), \dots, s_n(t))$, the connection operator $V(t) = \{v_{ij}(t)\}_{i,j}$ connecting the actors to coalitions and the actors' gains $H(t) = (\mathcal{H}_1(t), \mathcal{H}_2(t), \dots, \mathcal{H}_n(t))$ represent a dynamic network as follows:

$$\begin{cases} s'(t) = f(s(t)) \\ V'(t) = \beta(V(t)) \end{cases} \tag{23}$$

and the constraint of viability of the coalitions

$$\forall t \geq 0, \quad V(t)s(t) \in \mathcal{M}. \tag{24}$$

Here, we assume each actor's satisfactory value to be $H_i^o - \theta_i \geq \mathcal{H}_i \leq H_i^o + \theta_i$ for some

$\theta_i \in \mathbb{R}$, $\theta_i > 0$. We define $\mathcal{M} = \{(\mathcal{H}_1, \mathcal{H}_2, \ldots, \mathcal{H}_n) \in \mathbb{R}^n | \; \forall i = 1 \ldots n, \;\; H_i^o - \theta_i \geq \mathcal{H}_i \leq H_i^o + \theta_i\}$.

The constructed coalition forming model differs from the Statistical Physics based model: 1) the actors' mutual propensity v_{ij} may be non-symmetric, 2) the "self-propensity" v_{ii} may be non zero, 2) actor i's gain $\mathcal{H}_i$ does not precisely correspond to the physic's Hamiltonian. Moreover, we address the problem of viability correction of this dynamic fuzzy coalition forming model, instead of the problem of stabilization. However, the correction of the model reminds in its form the stabilization by globally motivated interactions in the Statistical Physics based model.

9.3 Viability Correction Introduction

9.3.1 Historical Overview

The recent literature on the control of network's viability, ([48], [54]) concerns the systems of individual actors whose states' evolutions are restricted by viability constraints, and handles the question of restoring the viability assuming that the actors' connection operator is not changing ([53], or providing only a general control frame for multi-linear connection operators ([54]). In practical applications, some of which are listed below, there is an interaction between the actors and their connection operator, and it is reasonable to correct the dynamics of both collectively. Therefore, in the present work, we bring into the model the connection operator that changes dynamically and treat the problem of restoring the viability as a problem of regulation of dynamics of *both* the actors' states and the connection operator. The viability is restored by correcting those dynamics using regulatory parameters, called correction prices, using the approach provided in viability theory.

Among fields where the problem of our interests is at their heart, we can mention economics, where one studies evolving economic systems faced with scarcity constraints. For example, [48] and [49] provide economists with mathematical tools necessary to handle the concepts of evolution in economics which shell replace the fundamental model of resource allocations by decentralized dynamic framework where correction prices follow the regulation law. Other recent researches which study evolving economic systems are presented in [43] and [52] which study the systems of individual agents whose behavior evolves and propose dynamic adjustment processes.

Another field which is of great interest is that of neural networks and cognitive sciences. In this case, the neural networks and cognitive systems are regarded as dynamical systems controlled by synaptic matrices ([56]).

Dynamical connectionist networks and dynamical cooperative games are also fields where the problem has a central role. Here, the authors of ([54]) provide a class of control systems able to govern the evolution of actions, coalitions and multilinear connection operators under which the architecture of a network remains viable. The controls are tensor products of the coalitions' actions and multipliers of the viability constraints space, which allow to encapsulate in the dynamical framework the concept of Hebbian learning rules (the rules that specifies how the weight of the connection between two agents should be justified in proportion to the product of their activation) in neural networks. They also use the viability and capturability approach to study the problem of characterizing the dynamic core of a dynamic cooperative game defined in a characteristic function form. Another recent research is [39] where the control of dynamics of a communication network realized using a stochastic approach.

Recently, a lot of research attention is given to sociological sciences. There, a society can be interpreted as a set of individuals which are subjected to survival or social constraints. Laws and cultural codes can be devised to provide each individual with psychological or economical means and guidelines that play the role of regulation controls ([40], [51]).

9.3.2 Correction by Viability Multipliers

In the present work, we tackle the problem of restoring the viability by correcting the network's dynamics using regulatory parameters, which introduce the missing mutual dependencies. The parameters represent control units regulating the dynamics of the network's components – the actors and their connection operator. We denote the regulatory parameters by $p(t)$ and $P(t)$, where $p(t)$ is a vector and $P(t)$

is a linear mapping. The viable corrected network has the form of

$$\begin{cases} x'(t) = f(x(t)) - p(t) \\ V'(t) = \beta(V(t)) - P(t) \end{cases} \tag{25}$$

where $p(t) = \mathbf{0}$ and $P(t) = \mathbf{O}$, the zero vector and the zero linear operator, cover the initial decentralized dynamics.

Parameter $p(t)$ is called *viability multiplier*, and parameter $P(t)$ is called *viability connection operator*.

We prove that there is a common regulatory parameter $q(t)$, called a *correction price*, such that the viability multiplier $p(t)$ and the viability connection operator $P(t)$ are derived through another linear operator applied to $q(t)$.

We show that under adequate assumptions a parameter $q_o(t)$ that minimizes the norm $||q(t)||$ of the correction price can be selected among the correction prices regulating the network's viability. In this sense $q_o(t)$ defines optimal $p_o(t)$ and $P_o(t)$.

The correction price provides the information about the changes in the network's decentralized dynamics necessary to govern evolutions satisfying the centralized constraints. This is the mean of restoring the viability of decentralized dynamics of the network by the *decentralization by price*. The corrected network (25) is said to be *decentralized by price*.

9.4 Prerequisites From Viability Theory

Consider a model that consists of N actors each of which is characterized by its state. An i'th actor's state $x_i(t)$ ranges over a vector space X_i with time t. The vector space X_i is referred as the actor i's state space, and the finite dimensional vector space $X = \Pi_{i=1}^n X_i$ of elements $\{x = (x_1, \ldots, x_n)\}$ is referred as a collective state space.

Each actor's state evolves independently from other actors. The evolution of the state of an actor i is governed by the dynamics of the state: $x_i'(t) = f_i(x_i(t))$. The map $f_i : X_i \mapsto X_i$ depends on the state x_i and not on the other actors' states, which reflects the independency of the actors' dynamics.

A subset K of the state space X is regarded as an environment of viability of the system, in which the actors' state $x(t) = (x_1(t), \dots, x_n(t))$ must remain at any time $t \geq 0$. In our framework, the environment of viability is described through the viability constraints as follows. Given a finite dimensional vector space Z and a subset $\mathcal{M}$ of Z, the viability constraints defined by a map $h : X \mapsto Z$ are

$$h(x) \in \mathcal{M}.$$

Then, the environment of viability K can be written explicitly as $K = \{x| \ \ h(x) \in \mathcal{M}\}$.

Thus, the system of the dynamics of the actors' states and the viability constraints is written as

$$\forall i = 1 \dots n, \ \ x_i'(t) = f_i(x_i(t)) \tag{26}$$

$$\forall t \geq 0, \ \ h(x(t)) \in \mathcal{M} \tag{27}$$

Definition 3 (Viable Environment) *The environment K defined by the viability constraints (27) is* viable *under the dynamics (26) if from any initial state $(x_1, \dots, x_n) \in K$ starts at least one evolution governed by the dynamics which is viable in K.*

Since there is no reason why the system, left to evolve by itself, shall always remain viable, the question of restoring the viability of the system arises. The question is resolved using the method of the viability multipliers as we describe in the next section.

9.4.1 The Viability Theorem and Viability Multipliers

Let X be a finite dimensional vector space. We denote by P a cone, by $\overline{P}$ its closure and by $\overline{co}P$ its closed convex hull. The polar cone of P is denoted by $P^- = \{p \in X^*| \ \ \forall x \in P, \ \ \langle p, x\rangle \leq 0\}$.

Definition 4 (Tangent Cone) *Consider a subset K of a finite dimensional vector space X, and a vector x in K. The* tangent cone *(or the contingent cone of Bouligand) $T_K(x)$ to set K at x is the closed cone*

$$T_K(x) = \{u \in X| \quad \lim_{h \to 0+} \inf \tfrac{d(x+hu,K)}{h} = 0\} \tag{28}$$

which coincides with the whole space X if x belongs to the interior of K.

For a convex set K, the tangent cone coincides with the tangent cone of convex analysis, which is the closed cone spanned by $K - x$: $T_K(x) = \overline{\cup_{h>0} \frac{K-x}{h}}$.

Thus, set $\overline{co}(T_K(x))$ is the closed convex hull of the tangent cone $T_K(x)$.

Definition 5 (Normal Cone) *The* normal cone *to a subset K of the vector space X at a point $x \in X$, denoted by $N_K(x)$, is defined to be*

$$N_K(x) := T_K(x)^{-} = (\overline{co}(T_K(x)))^{-} \tag{29}$$

Definition 6 (Sleekness) *A subset K of the vector space X is said to be* sleek *if the graph of the mapping $x \rightsquigarrow N_K(x)$ is closed.*

Let the vector space X be supplied with a scalar product l with the norm λ, $\lambda(x) = ||x||$, and let L be the duality map on X associated with the scalar product.

Definition 7 (Marchaud Set-valued Map) *A set-valued map $F : X \rightsquigarrow Y$ is called* Marchaud *if it has a closed graph, convex values, and a linear growth defined by*

$$\lambda(F(x)) := \sup_{v \in F(x)} \lambda(v) \leq c(\lambda(x) + 1), \textit{ for some constant } c.$$

We denote the actors' dynamics as a whole by $x'(t) = f(x(t))$, where $x(t) = (x_1(t), x_2(t) \dots, x_n(t))$ is the collective state of actors and $f(x) = (f_1(x_1), \dots, f_n(x_n))$.

Theorem 1 (Nagumo Viability Theorem) *Let K be a closed subset of vector space X, and $f : X \mapsto X$ be a continuous map with a linear growth. Then, K is viable under the differential equation $x' = f(x)$ if and only if for any $x \in K$, the* *dynamics and the constraints are linked by the following relation*

$$\forall x \in K, \quad f(x) \in \overline{co}(T_K(x)) \tag{30}$$

The general approach to restoring viability is to replace function f in the differential equation $x' = f(x)$ by a correction map $\tilde{f}$ that satisfies the requirements of the Nagumo Viability Theorem.

We define a *viability discrepancy* to be the distance between the initial and the corrected dynamics and we denote it by $c(x) = \lambda(f(x) - \tilde{f}(x))$. Obviously, the minimal viability discrepancy $c_o(x)$ is achieved for the best approximation projection of $f(x)$ on the closed convex hull of the tangent cone, $\tilde{f}(x) = \Pi_{\overline{co}(T_K(x))} f(x)$, since

$$c_o(x) = \lambda(f(x) - \Pi_{\overline{co}(T_K(x))} f(x)) = \inf_{u \in \overline{co}(T_K(x))} \lambda(f(x) - u)$$

Because $\overline{co}(T_K(x))$ is a closed convex cone and $N_K(x)$ is its polar cone, then Moreau Projection Theorem (see the Appendix) implies that $f(x)$ can be written as $f(x) = \Pi_{\overline{co}(T_K(x))} f(x) + L^{-1}\Pi_{N_K(x)} L f(x)$. If we set

$$p_o = \Pi_{N_K(x)} L f(x) \tag{31}$$

the correction function can be represented as $\tilde{f}(x) = f(x) - L^{-1} p_o(x)$, where $p_o(x)$ is considered as a regulatory parameter.

Motivated by this representation, we consider a general correction in the form of

$$x'(t) = f(x(t)) - L^{-1} p(t) \tag{32}$$

where $p(t) \in X^*$ is a regulatory parameter belonging to the set of all the regulatory parameters providing the viability corrections - the *regulation map*

$$R_K(x(t)) = \{p \in X^* | \;\; f(x(t)) - L^{-1} p \in \overline{co}(T_K(x(t)))\} \tag{33}$$

Such regulatory parameters are referred as *viability multipliers*.

Clearly, the viability multiplier $p_o(x) = \Pi_{N_K(x)} Lf(x)$ corresponds to the minimal viability discrepancy and belongs to the regulation map, $p_o(x) \in R_K(x)$. However, using only the Nagumo theorem, we cannot prove the viability of the correction since, though the tangential conditions are satisfied, the continuity properties are lost by projecting the map $f(x)$ onto the tangent cone.

In order to prove the viability of the correction (32) - (33), we are helped by the following fundamental theorem.

Theorem 2 (Fundamental Viability Theorem) *Consider differential inclusion $x' \in F(x)$, where F is a set-valued map. If $F(x)$ is Marchaud, then K is viable under F if and only if $\forall x \in K,\ \ 0 \in F(x) - \overline{co}(T_K(x))$.*

In the following theorem we present the requirements under which the correction (32) - (33) restores the viability.

Theorem 3 (Restoring Viability) *Denote by B a unit ball in X^*. Assume f be continuous with linear growth and K be sleek. Then, the environment K of the system (26) - (27) is viable under the new dynamics (32) - (33). Furthermore, it is viable under the correction with minimal viability discrepancy*

$$x'(t) = f(x(t)) - L^{-1} p_o(x(t))$$

Proof

Since the map $x \mapsto f(x)$ is continuous and $x \rightsquigarrow \overline{co}(T_K(x))$ is lower semi-continuous, we infer that the set-valued map $x \rightsquigarrow c_o(x)B$, where $c_o(x) = d(f(x), \overline{co}(T_K(x)))B$, is upper semi-continuous thanks to the Maximum Theorem. The set-valued map $G : X \rightsquigarrow X$ defined by $G(x) := f(x) - L^{-1}(c_o(x)B \cap N_K(x))$ is Marchaud because its graph is closed, its images are convex and it has linear growth since

$$\forall x \in K,\ d(f(x), \overline{co}(T_K(x))) \ \leq\ \lambda(f(x)) \ \leq\ c(\lambda(x) + 1)$$

It remains to prove that $G(x) \cap \overline{co}(T_K(x)) \neq \emptyset$.

Indeed, Moreau Theorem implies that the viability multiplier $p_o(x)$ minimizing the viability discrepancy is the projection $p_o(x) = \Pi_{N_K(x)}(Lf(x))$ onto the normal cone $N_K(x)$ of $f(x)$, and $u_o(x) = f(x) - L^{-1}p_o(x)$ is equal to $\Pi_{\overline{co}(T_K(x))}f(x)$. So the viability multiplier $p_o(x)$ satisfies $p_o(x) \in c_o(x)B \cap N_K(x)$, and hence $u_o(x) = f(x) - L^{-1}p_o(x)$ belongs to $G(x) \cap \overline{co}(T_K(x))$. Thus, the assumptions of the Fundamental Viability Theorem (Theorem 2) are satisfied, and we have proved that K is viable under the corrected differential inclusion $x'(t) \in G(x(t))$. ■

Motivated by the economic interpretation ([49]), the correction (32) - (33) is called viability correction by the *decentralization by price.*

9.4.2 Restoring Viability

The correction results described above deal with viability constrains written as $x \in K$. In case of the explicit constraints $h(x) \in \mathcal{M}$, where $h : X \mapsto Z$ and $\mathcal{M} \subset Z$, the environment K can be defined in the form $K := \{x \in X|\ \ h(x) \in \mathcal{M}\}$.

Then, under the assumption that the function $h : X \mapsto Z$ is continuously differentiable map such that its derivative $h'(x)$ is surjective, and the set $\mathcal{M}$ is sleek, the tangent and the normal cones $T_K(x)$ and $N_K(x)$ can be described in terms of the tangent and the normal cones $T_{\mathcal{M}}(h(x))$ and $N_{\mathcal{M}}(h(x))$ by the formula

$$T_K(x) = h'(x)^{-1}T_{\mathcal{M}}(h(x)) \text{ and } N_K(x) = h'(x)^* N_{\mathcal{M}}(h(x)).$$

Hence, with additional assumptions on h and $\mathcal{M}$, the correction that restores the viability is defined in the following theorem.

Theorem 4 (Restoring Viability For Explicit Constraints) *If $\mathcal{M} \subset Z$ is sleek, function $f : X \mapsto X$ is continuous with linear growth, and function $h : X \mapsto Z$ is continuously differentiable map such that its differential $h'(x)$ is surjective, then $\mathcal{M}$ is a viability domain of*

$$x' = f(x) - L^{-1}h'(x)^*q(x)$$

where $q(x)$ ranges over

$$R_{\mathcal{M}}(x) = \{q(x) \in Y^* | \ \ h'(x)f(x) - h'(x)L^{-1}h'(x)^*q(x) \quad \in \quad \overline{co}T_{\mathcal{M}}(h(x))\} \tag{34}$$

Particularly, one can take $q_o(x) \in R_{\mathcal{M}}(x)$ minimizing the viability discrepancy

$$q_o(x) = \Pi_{N_{\mathcal{M}(h(x))}} \left(\left(\ h'(x)L^{-1}h'(x)^* \ \right)^{-1} h'(x)f(x) \ \right) \in R_{\mathcal{M}}(x) \tag{35}$$

□

Here, the viability multiplier $p \in X^*$ is equal to $h'(x)^*q(x)$, where $q(x) \in Z^*$, and the particular case when the minimal viability discrepancy is achieved corresponds to $p_o(x) = h'(x)^*q_o(x)$.

9.5 Network

We define the network as follows.

Definition 8 (Network) *Given a linear operator $V \in \mathcal{L}(X,Y)$, maps $f : X \mapsto X$, $\beta : \mathcal{L}(X,Y) \mapsto \mathcal{L}(X,Y)$. Consider a system of N actors with the states $x = (x_1, \ldots, x_n) \in X$ that are governed by the decentralized dynamics $x_i'(t) = f_i(x_i(t))$ and connected by the connection operator V governed by the decentralized dynamics $V'(t) = \beta(V(t))$. The system defines a network of the actors' states connected to the actors' common results with the pattern of the connection Vx.*

We write the network as in (20)

$$\begin{cases} x'(t) = f(x(t)) \\ V'(t) = \beta(V(t)) \end{cases}$$

□

9.5.1 Network's Viability

When the actors' collective result $V(t)x(t)$ is restricted by the viability constraints (21)

$$\forall t \geq 0, \;\; g(V(t)x(t)) \in \mathcal{M},$$

the question of the network's viability raises.

By the *environment of the network* we understand the set of pairs of actors' states and connection operators (x, V) that satisfy the constraints. Evolutions governed by the network (20)'s dynamics and satisfying viability constraints (21) are called *viable evolutions*.

The environment of the network is said to be *viable* under the network's dynamics if for any initial state in the environment, there is at least one viable evolution starting from it. Then, we define a viable network as follows.

Definition 9 (Viable Network) *The network is* viable *if the environment of the network is viable under the network's dynamics.*

The network (20) with the viability constraints (21) that we study in this work represents a decentralized model which is characterized by the absence of mutual dependencies between the network's data - the actors and the connection operator. Therefore, there is nothing guaranteeing that the actors' states or the connection operator do not violate the centralized viability constraints. Hence, nothing guarantees the network's viability.

The main problem we relate in the present work is the problem of viability of the network with decentralized dynamics evolving under the given (centralized) constraints. We solve the problem by correcting the network using the method of correction by decentralization by price.

9.5.2 Restoring the Network's Viability

We assume the spaces X and Y be supplied with the scalar products which define the duality maps $L : X \mapsto X^*$ and $M : Y \mapsto Y^*$. Note that the duality map H on $\mathcal{L}(X, Y)$ is equal to $H := L^{-1} \otimes M$.

Analogously to the correction by decentralization by price displayed in the formula (32) - (33), we choose the parameters $p(t) \in X^*$ and $P(t) \in \mathcal{L}(Y^*, X^*)$ and write the correction of the network (20)with constraints (21) as the following

$$\begin{cases} x'(t) = f(x(t)) - L^{-1}p(t) \\ V'(t) = \beta(V(t)) - H^{-1}P(t) \end{cases} \tag{36}$$

Theorem 5 (Restoring the Network's Viability) *Consider the map* $J_g(x, V) \in \mathcal{L}(Z, Z^*)$,

$$J_g(x, V) = \left[\; g'(Vx) \left[\; \lambda^2(x) M^{-1} + V L^{-1} V^* \;\right] g'(Vx)^* \;\right]^{-1}$$

the regulation map $R_{\mathcal{M}} : X \times \mathcal{L}(X, Y) \mapsto Z^*$, *such that*

$$R_{\mathcal{M}}(x, V) = \{q \in Z^* |\ \ g'(Vx)(\beta(V)x + Vf(x)) - J_g(x, V)^{-1}q \quad \in \quad \overline{co} T_{\mathcal{M}}(g(Vx))\}$$

the element $q_o \in Z^*$,

$$q_o(x, V) = \Pi_{N_{\mathcal{M}}(g(Vx))} \left(\; J_g(x, V) g'(Vx) \left[\; \beta(V)x + Vf(x) \;\right] \right)$$

Assume maps f and β be continuous with linear growth, set $\mathcal{M} \subset Z$ is sleek, and map g is continuously differentiable such that derivative g' is surjective. Then, the network is viable under the corrected system

$$\begin{cases} x'(t) = f(x(t)) - L^{-1} V^* g'(Vx)^* q(t) \\ V'(t) = \beta(V(t)) - Lx \otimes M^{-1} g'(Vx)^* q(t) \end{cases}$$

where the prices $q(x, V) \in R_{\mathcal{M}}(x, V)$. Particularly, the minimal correction price $q_o(x, V)$ belongs to $R_{\mathcal{M}}(x, V)$.

□

Note that operator $J_g(x, V)$ is a duality map on Z induced by the duality map on $X \times \mathcal{L}(X,Y)$.

As one can observe from these results, the centralized constraints in the network bring into the viable correction a factor of mutual dependencies between the actors' states and the connection operator. These mutual dependencies are encapsulated in the viability multiplier $p(t) \in X^*$ and the viability connection operator $P(t) \in \mathcal{L}(X,Y)$ which are defined by

$$\begin{aligned} p(t) &= V^* g'(Vx)^* q(t) \\ P(t) &= x \otimes g'(Vx)^* q(t) \end{aligned} \tag{37}$$

9.5.3 Viability Correction in Coalition Forming Model

With $s = \{s_i(t)\}_i$ be a vector of actors' fuzzy states and $V = \{v_{ij}(t)\}_{ij}$ be an operator of mutual propensities between the actors, the dynamic fuzzy coalition forming model (23) we have defined is

$$\begin{cases} s'(t) = f(s(t)) \\ V'(t) = \beta(V(t)) \end{cases}$$

with the coalitions viability constraints

$$\forall t \geq 0, \quad V(t)s(t) \in \mathcal{M}.$$

Here, $\mathcal{M} = \{(\mathcal{H}_1, \mathcal{H}_2, \dots, \mathcal{H}_n) \in \mathbb{R}^n | \ \forall i = 1 \dots n, \quad H_i^o - \theta_i \geq \mathcal{H}_i \leq H_i^o + \theta_i\}$.

Assume maps f and β be continuous with linear growth. Since, the spaces $X, Y, Z = \mathbb{R}^n$ are euclidian (supplied with the canonical basis), the duality maps L and M are the identity maps, $V(t)^*$ is equal to the transpose matrix $V^T = V$, and map $g' = g'^* = I$. Then, according to the theorem on restoring the network's viability, since map $g = I$ is continuously differentiable and the derivative $g' = I$ is surjective, network (23) corrected by the viability multipliers $p(t)$ and viability connection operator $P(t)$ according to formula (37) is viable:

$$\begin{cases} s'(t) = f(s(t)) - V(t)q(t) \\ V'(t) = \beta(V(t)) - s(t) \otimes q(t) \end{cases} \tag{38}$$

Here, the correction price $q(t)$ belongs to

$$R_{\mathcal{M}}(s(t), V(t)) = \{q(t) \in \mathbb{R}^n | \ (\beta(V(t))s(t) + V(t)f(s(t))) - J_g(s(t), V(t))^{-1}q(t) \in \ \overline{co}T_{\mathcal{M}}(V(t)s($$

and the minimal correction price $q_o(s(t), V(t))$ is equal to

$$\Pi_{N_{\mathcal{M}}(V(t)s(t))} \left(\ J_g(s(t), V(t)) \left[\ \beta(V(t))s(t) + V(t)f(s(t)) \ \right] \right).$$

Note that in the corrected network (38), the elements of the viability connection operator $P(t) = -s(t) \otimes q(t) = -\{s_j(t)q_i(t)\}_{ij}$ play the role of the additional bilateral propensities $p_{ij} = -s_j(t)q_i(t)$ between the actors i and j, which guarantee viability of the coalitions.

In the analogy to the Statistics Physics based coalition forming model, viability correction in the dynamic fuzzy model is achieved due to the additional bilateral propensities p_{ij}, produced by supplementary exchanges between the actors. The additional propensities, as well, modify the overall propensity and the corrected gain becomes $\mathcal{H}_i = \sum_{i \neq j} s_i(v_{ij} + p_{ij})$.

Thought the result of the viability correction is not applicable directly on the natural model, it illustrates the theoretical role of the additional interactions in the natural model – correction of the viability of stability of the natural coalition forming model.

In order to give an illustration of viability correction in the dynamics fuzzy coalition forming model, consider the particular case of three actors 1, 2, and 3 whose choices for the coalitions can be described by

$$s_1(t) = -1 + 2\cos^2(t), \quad s_2(t) = -1 + 2\sin^2(t), \quad s_3(t) = -1 + 2\cos^2(t),$$

and whose bilateral propensities are

$$v_{12}(t) = -\cos(t), \quad v_{13}(t) = \cos(t), \quad v_{23}(t) = -\cos(t).$$

The system is shown in figure (37) at time $t = 0$.

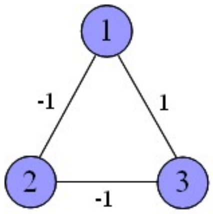


Figure 37: The model of three actors forming coalitions, at $t = 0$.

The three actors with the states $s_1(t), s_2(t), s_3(t)$ and their connection operator $V(t)$ form the network as shown in figure (38).

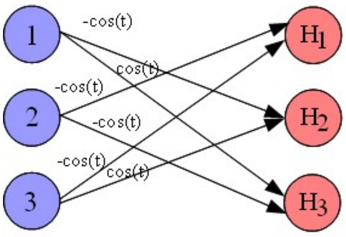


Figure 38: Scheme of the connectionist network corresponding to the model of three actors forming coalitions.

Assume $\theta_1 = \theta_2 = \theta_3 = \theta$ for some $\theta > 0$, then $\mathcal{M} = \{(\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3) \in \mathbb{R}^3 | \ \mathcal{H}_1 \geq 2 - \theta, \mathcal{H}_3 \geq 2 - \theta, \mathcal{H}_2 \leq -2 + \theta\}$. Define $z(t) = V(t)s(t)$. At the initial time $t_0 = 0$, the actors' gains $z_0 = (\mathcal{H}_1(0), \mathcal{H}_2(0), \mathcal{H}_3(0)) = (2, -2, 2)$ belong to the interior of $\mathcal{M}$. Over time, at some moment t_1 the actors' gains reach the boundary of $\mathcal{M}$ for the first time, $z_1 = (\mathcal{H}_1(t_1), \mathcal{H}_2(t_1), \mathcal{H}_3(t_1)) = (2 - \theta, -2 + \theta, 2 - \theta)$, whereupon the network runs out of viability for some time.

The correction of the networks viability in (38), at time t_1, is shown schematically in figure (39).

In this example we have seen the illustration of the practical application of the theorem, in which the regulatory parameters control the network's dynamics to keep them inside the viability domain.

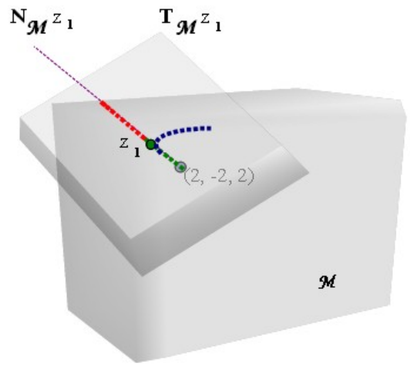


Figure 39: The collective actors' result $V(t)s(t)$ corrected at z_1 stay in the interior of set $\mathcal{M}$ (shown with the normal and tangent cones of the set at the point z_1).

9.5.4 Proof of the Theorem on Restoring the Network's Viability

In order to obtain the network's viability correction as in the form 36, we shall use the viability multipliers approach. To this end, we consider a finite dimensional vector space $X \times \mathcal{L}(X, Y)$ of pairs (x, V) representing the states. The duality map T on the state space $X \times \mathcal{L}(X, Y)$ is $T := L \times H = L \times L^{-1} \otimes M$. Then, we write the network (20) with the constraints (21) in the form of the viability system (26) - (27):

$$(x'(t), V'(t)) = \Big(\; f(x(t)), \quad \beta(V(t)) \; \Big) \tag{39}$$

$$\forall t \geq 0, \quad h(V(t), x(t)) \in \mathcal{M}, \tag{40}$$

where $h(x, V) := g(Vx)$ is restricted to remain in a subset $\mathcal{M}$ of the constrained results' space Z.

According to the Definition 9 of the viable network and the Definition (3) of the viable environment, the network (20) with the constraints (21) is viable if and only if the environment $\mathcal{M}$ is viability domain under the network's dynamics. Hence, in order to restore the network's viability we can apply the results of Theorem 4 under the theorem's assumptions.

Since, $\mathcal{M} \subset Z$ is sleek, the map $f \times \beta : X \times \mathcal{L}(X, Y) \mapsto X \times \mathcal{L}(X, Y)$ is continuous with linear growth, and $h : X \times \mathcal{L}(X, Y) \mapsto Z$ is a continuously differentiable map, such that the differentiation operator $h'(x, V)$ is surjective, then the assumptions of

Theorem 4 are satisfied. Substituting the term $[h'(x,V)\ T^{-1}\ h'(x,V)^*]^{-1}$ in the correction formulas (34) - (35) of the theorem by $J_h(x,V)$, we derive the viability correction of the network

$$\begin{cases} (x,V)' = (f(x),\beta(V)) - T^{-1}h'(x,V)^* q(x,V), \text{ where } q(x,V) \in R_{\mathcal{M}}(x,V) \\ R_{\mathcal{M}}(x,V) = \{q \in Z^* | \ h'(x,V)(f(x),\beta(V)) - J_h(x,V)^{-1}q(x,V) \ \in \ \overline{co}T_{\mathcal{M}}(h(x,V))\} \end{cases} \tag{41}$$

and particularly,

$$q_o(x,V) = \Pi_{N_{\mathcal{M}}(h(x,V))}\Big(\ J_h(x,V)h'(x,V)(f(x),\beta(V))\ \Big).$$

Note that, since $h'(x,V)$ is a surjective linear operator mapping X to Z, the operator $J_h(x,V) = [h'(x,V)\ T^{-1}\ h'(x,V)^*]^{-1}$ is a duality map on Z induced by the duality map T on $X \times \mathcal{L}(X,Y)$.

In order to continue to the next step in the proof of Theorem 5, the following lemmas are required.

Lemma 1 *Given a function of two variables $h : X \times \mathcal{L}(X,Y) \mapsto Z$ that maps a pair of a vector x and a linear operator V according to $h(x,V) = g(Vx)$, where $g : Y \mapsto Z$ is a differentiable function. Then, the differential $h'(x,V)$ in a general direction $(u,U) \in X \times \mathcal{L}(X,Y)$ complies with*

$$h'(x,V)(u,U) = g'(Vx)(Ux + Vu) \tag{42}$$

According to the definition of derivation of a function of two variables in an arbitrary direction, and taking into consideration the fact that V is a linear operator, we obtain the derivative of $h(x,V)$ in the direction of (u,U) as follows.

$$h'(x,V)(u,U) = \lim_{\alpha\mapsto 0} \frac{h(x+\alpha u, V+\alpha U) - h(x,V)}{\alpha} = \\ \lim_{\alpha\mapsto 0} \frac{g((V+\alpha U)(x+\alpha u)) - g(Vx)}{\alpha} = \lim_{\alpha\mapsto 0} \frac{g(Vx + \alpha(Ux+Vu) + \alpha^2 Uu) - g(Vx)}{\alpha}.$$

Being differentiable, g can be represented by its Taylor series expansion in a neighborhood $\alpha(Ux+Vu)$ of the point Vx of its domain. Then,

$$h'(x,V)(u,U) = \lim_{\alpha\mapsto 0}\frac{g(Vx)+g'(Vx)\alpha(Ux+Vu)+O(\alpha^2)-g(Vx)}{\alpha} = g'(Vx)(Ux+Vu),$$

as stated in the lemma.

■

We then state and prove the following property

Lemma 2 *Define a linear operator $B : X \times X^* \otimes Y \mapsto Z$ by $B(u,U) = g'(Vx)(Ux+Vu)$ for any $(u,U) \in X \times X^* \otimes Y$. Then, the transpose operator B^* maps Z^* to $X^* \times Y \otimes X^*$ such that for any arbitrary $q \in Z^*$, $B^*q = \Big(V^*g'(Vx)^*q,\ \ x \otimes g'(Vx)^*q \Big)$.*

Proof

In order to prove the statement of the lemma, we apply to properties of the transpose operation and the duality map.

Given an arbitrary $q \in Z^*$ and an instance (u,U) of $X \times \mathcal{L}(X,Y)$, consider the duality product $\Big\langle B^*q,\,(u,U)\Big\rangle$, where $B^*q \in (X \times \mathcal{L}(X,Y))^*$. Recall the property of the transpose operation stating for any $A \in \mathcal{L}(X,Y)$, $x \in X$ and $r \in Y^*$, that $\langle A^*r,\,x\rangle_X = \langle r,\,Ax\rangle_Y$ and $\langle r,\,Aa\rangle = \langle a \otimes r,\,A\rangle$, and obtain

$$\Big\langle B^*q,\,(u,U)\Big\rangle = \Big\langle q,\,B(u,U)\Big\rangle = \Big\langle q,\,g'(Vx)(Ux+Vu)\Big\rangle = \Big\langle q,\,g'(Vx)(Vu)\Big\rangle + \Big\langle q,\,g'(Vx)(Ux)\Big\rangle = \Big\langle V^*g'(Vx)^*q,\,u\Big\rangle + \Big\langle g'(Vx)^*q,\,Ux\Big\rangle = \Big\langle V^*g'(Vx)^*q,\,u\Big\rangle + \Big\langle x\otimes g'(Vx)^*q,\,U\Big\rangle.$$

As a consequence,

$$B^*q = \Big(V^*g'(Vx)^*q,\ \ x \otimes g'(Vx)^*q \Big).$$

This concludes the proof. ■

Lemma 3 *The operator $h'(x,V)$ applied to q is equal to $h'(x,V)^*q = (p,P)$, where*

$$p = V^* g'(Vx)^* q \text{ and } P = x \otimes g'(Vx)^* q.$$

Proof

By Lemma 1, the derivation operator $h'(x,V)$ for an arbitrary derivation direction (u,U) is equal to $h'(x,V)(u,U) = g'(Vx)(Ux+Vu)$. Then, according to Lemma 2,

$$h'(x,V)^*q = \Big(\ V^*g'(Vx)^*q,\quad x \otimes g'(Vx)^*q\ \Big)$$

This implies that $p = V^*g'(Vx)^*q$ and $P = x \otimes g'(Vx)^*q$. ■

Lemma 4 *Consider the linear operator $B : X \times X^* \otimes Y \mapsto Z$ defined by $B(u,U) = g'(Vx)(Ux+Vu)$ for any $(u,U) \in X \times X^* \otimes Y$. Then,*

$$BT^{-1}B^* = g'(Vx)\Big[\ \lambda^2(x)M^{-1} + VL^{-1}V^*\ \Big]g'(Vx)^*. \tag{43}$$

Proof

As it is shown in Lemma 2, B^* maps any $q \in Z^*$ to $B^*q = \Big(\ V^*g'(Vx)^*q,\quad x \otimes g'(Vx)^*q\ \Big)$.

Applying $T^{-1} = L^{-1} \times (L^{-1} \otimes M)^{-1}$ to B^*q we obtain,

$$T^{-1}B^*q = \Big(\ L^{-1}V^*g'(Vx)^*q,\quad (L^{-1} \otimes M)^{-1}x \otimes g'(Vx)^*q\ \Big)$$

or equivalently,

$$T^{-1}B^*q = \Big(\ L^{-1}V^*g'(Vx)^*q,\quad Lx \otimes M^{-1}g'(Vx)^*q\ \Big) \tag{44}$$

Then, by applying B to $T^{-1}B^*q$ we infer that

$$\begin{aligned}&B\Big(\ L^{-1}V^*g'(Vx)^*q,\quad Lx \otimes M^{-1}g'(Vx)^*q\ \Big) = \\ &g'(Vx)\Big[\ (Lx \otimes M^{-1}g'(Vx)^*q)x + V(L^{-1}V^*g'(Vx)^*q)\ \Big].\end{aligned}$$

Since $(Lx \otimes M^{-1}g'(Vx)^*q)x = \langle Lx, x\rangle M^{-1}g'(Vx)^*q = \lambda^2(x)M^{-1}g'(Vx)^*q$, we derive that

$$BT^{-1}B^*q = g'(Vx)\left[\ \lambda^2(x)M^{-1}g'(Vx)^*q + VL^{-1}V^*g'(Vx)^*q\ \right] =$$
$$g'(Vx)\left[\ \lambda^2(x)M^{-1} + VL^{-1}V^*\ \right]g'(Vx)^*q,$$

from which we deduce that

$$BT^{-1}B^* = g'(Vx)\left[\ \lambda^2(x)M^{-1} + VL^{-1}V^*\ \right]g'(Vx)^*.$$

This concludes the proof. ■

According to correction formula (41), the viability multiplier p and the viability connection operator P are subjected to $(p, P) = h'(x, V)^*q$, and therefore, the value of (p, P) follows from Lemma 3. Hence, since the correction formula follows from the fact that the duality map T on $X \times \mathcal{L}(X, Y)$ is equal to $T = X \times L^{-1} \otimes M$.

Using the notation of the correction formula, we obtain the network's regulation map

$$R_{\mathcal{M}}(x, V) = \{q \in Z^* |\ \ h'(x, V)(f(x), \beta(V)) - J_h(x, V)^{-1}q \ \ \in \ \ \overline{co}T_{\mathcal{M}}(h(x, V))\}$$

where $J_h(x, V) = [h'(x, V)\ \ (L \times L^{-1} \otimes M)^{-1}\ \ h'(x, V)^*]^{-1}$ and $h(x, V) = g(V(x))$.

Lemmas 1, 2 and 4 imply that the duality map $J_g(x, V)$ on Z is equal to

$$J_g(x, V) = \left[\ g'(Vx)\left[\ \lambda^2(x)M^{-1} + VL^{-1}V^*\ \right]g'(Vx)^*\ \right]^{-1}$$

By Lemma 1, $h'(x, V)(u, U) = g'(Vx)(Ux + Vu)$ for any (u, U), and then $h'(x, V)(f(x), \beta(V)) = g'(Vx)(\beta(V)x + Vf(x))$. Hence,

$$R_{\mathcal{M}}(x, V) = \{q \in Z^* |\ \ g'(Vx)(\beta(V)x + Vf(x)) - J_g(x, V)^{-1}q \ \ \in \ \ \overline{co}T_{\mathcal{M}}(g(Vx))\}.$$

In the similar way we calculate the viability multiplier of minimal correction price $q_o(x, V)$. This concludes the proof of Theorem 5.

9.6 Economic Interpretation of the Dynamic Network

The system represents a production company, where the actors are associated with commodities (production outputs) produced by the company. The production of commodities requires scarce resources of primary commodities (production inputs), the scarcity of which imposes constraints on the set of primary commodities needed for the production.

We assume that the commodities evolve according to predefined dynamics in independent manner, which results in the decentralized settings. The production of the commodities is represented by a technological operator V (technological matrix) reflecting the present technological capability. The operator V, applied to the commodity vector, decomposes it into intermediate commodities required for the production (energy, actions, machines, etc.). The resulting intermediate commodities are transferred by function g into corresponding primary commodities (oil, labor, investments, etc.) necessary to supply the intermediate commodities.

In the proposed economic interpretation, the common environment where the commodities evolve is the space of primary commodities where the scarcity constrains the set of available primary commodities. Thus, in order for the evolutions of commodities to be viable, the primary commodities must satisfy these (viability) constraints and remains in a specified subset $\mathcal{M}$ of the space of primary commodities.

In this system, the technological operator V connects actors' commodities to provide the collective intermediate commodities. Thus, V represents technological links between the actors. Motivated by this aspect, the technological operator V is referred as a *connection operator* and the system is viewed as a network of actors.

From the economic point of view, it is natural to think that a technological operator transferring commodities into the required intermediate commodities evolves with time, as well as the commodities themselves do. Hence, in our settings, we deal with a network of actors that evolves with time. It is natural, then, to assume that the dynamics governing the evolution of the connection operator and the dynamics of commodities are independent.

In formal terms, our network is described as follows. Let X be the space of commodities, Y be the space of intermediate commodities, and Z be the space of primary commodities, and let $x(t) = (x_1(t), \ldots, x_n(t)) \in X$ denotes the vector of commodities, and $\mathcal{M} \subset Z$ denotes the set of available primary commodities. Then, the N maps $f_i : X_i \mapsto X_i$, the linear technological operator $V : X \mapsto Y$, the function β mapping $\mathcal{L}(X, Y)$ to itself, and the function $g : Y \mapsto Z$ together define the network's dynamics (20) and the viability constraints (21).

For an intuitive demonstration of the model and the problem presented here, think of a company that produces cars. The company produces car components either from steel or plastic, achieving by thus an appropriate level of strength and lightness of their product. Following the modern achievements in the quality of plastic and its relatively low price, the company increased the tendency to use plastic details relative to the steel ones. By this tendency, however, at some moment the usage of plastic exceeds the allowable amount of the resource or violates the strength parameters which brings the company into a crisis. Hence, modifications in usage of plastic and steel details, such that the production will cost as less as possible subject to the available resources and the strength parameters, is necessary in order to provide the viability of the company.

The network correction by the regulatory parameters $p(t)$ and $P(t)$ is illustrated with the following economic interpretation. The viability multiplier $p(t)$ used in the corrected system (25) is interpreted as a price of commodities, and the viability connection operator $P(t)$ is regarded as a linear pricing operator mapping the prices of the primary commodities into the prices of the final commodities.

The price $p(t)$ and the pricing operator $P(t)$ are obtained through a pricing operator for intermediate commodities applied to the price $q(t)$ of the primary commodities.

Thus, $p(t)$ and $P(t)$ are the means of restoring the viability of the non-viable decentralized dynamics of the network by the decentralization by price. The corrected network (25), then, is said to be *decentralized by price.*

Returning to the economics interpretation of our settings, let us examine the economic meaning of p and P in the light of the results above. Observe that the viability multiplier $p = V^* g'(Vx)^* q$, being an element of the dual space X^* of the space of commodities' states, represents a pricing operator of commodities. The viability matrix-multiplier P, in its turn, is a linear operator $P = x \otimes g'(Vx)^* q$, being a mapping from Y^* to X^*, it represents a matrix of ratings of commodities. Observe that both p and P depend on $q \in Z^*$, which is a pricing operator of primary commodities, through the element $r = g'(Vx)^* q$ of the space Y^* of prices of intermediate commodities. Being a function of q, r represents an operator of pricing of intermediate commodities as a function of the prices of primary commodities. Hence, $p = V^* r$, represents an operator of pricing of commodities as a function of the prices of primary commodities, and $P = x \otimes r$ represents a matrix of ratings of commodities as a function of the rates of primary commodities. Clearly, q_o, p_o, P_o represent the minimal prices.

10 Appendix: Duality and Tensor in a Vector Space

We use mathematical techniques of the linear algebra, including tensor techniques, and the tools of viability theory as follows.

Consider the finite dimensional vector spaces X, Y and Z, and the space of real numbers $\mathbb{R}$.

- **Dual Space** X^*

  The *dual space* of a vector space X is $X^* = \mathcal{L}(X, \mathbb{R})$.

- **Duality Product** $\langle , \rangle$

  The *duality product* is a map $\langle , \rangle : X \times X^* \mapsto \mathbb{R}$ such that $\langle p, x \rangle = p(x)$.

  When X is Euclidean space $\mathbb{R}^n$ supplied with the canonical basis, then the duality product of any $x \in X$ and $p \in X^*$ can be written as $\langle p, x \rangle = p_1 \cdot x_1 + \ldots + p_n \cdot x_n$.

- **Transpose of Operator** $*$

  Consider an operator $A : X \mapsto Y$. Denote by $\langle , \rangle_X$ the duality product on X and by $\langle , \rangle_Y$ the duality product on Y. The *transpose operator* of the operator A is denoted by A^* and is defined to be a map $A^* : Y^* \mapsto X^*$ that associates with any $r \in Y^*$ the linear form $p = A^* r \in X^*$ defined by $p(x) \equiv \langle p, x \rangle_X \equiv \langle A^* r, x \rangle_X := \langle r, Ax \rangle_Y$.

  When X and Y are supplied with canonical bases, a linear operator$A$ can be written in a matrix form as $(a^i_j)_{ij}$, and then, the transpose operator of an operator coincides with the well known transpose of a matrix, i.e., $A^* = (a^j_i)_{ji}$.

- **Duality Map** L

  We denote by l a scalar product on X and by m a scalar product on Y, and by $\lambda(x) = \sqrt{l(x, x)}$ and $\mu(y) = \sqrt{m(y, y)}$ the norms corresponding to l and m respectively.

  Consider the linear form $Lx \in X^*$ defined by $u \mapsto l(x, u), \forall u \in X$. The *duality map* $L : X \mapsto X^*$ associated with the scalar product l, is defined by $x \mapsto Lx$.

The relation between scalar product l and the duality product $\langle , \rangle$ through the duality map L is following: $\langle Lx_1, x_2 \rangle = Lx_1(x_2) = l(x_1, x_2)$.

The duality map L induces a scalar product l_* on the dual space X^* as follows: $l_*(p_1, p_2) = l(L^{-1}p_1, L^{-1}p_2)$, which in the terms of duality product is equal to $\langle p_1, L^{-1}p_2 \rangle$. Observe that $L^{-1} : X^* \mapsto X$ represents the duality map on X^*.

Given a surjective (onto) linear operator $A : X \mapsto Y$, the duality map L on X induces a duality map M on Y as follows. A duality map on Y is the mapping $M : Y \mapsto Y^*$ and the corresponding duality map on Y^* is the mapping $M^{-1} : Y^* \mapsto Y$. Since A maps X to Y and A^*, correspondingly, maps Y^* to X^*, then M^{-1} can be represented as $M^{-1} = AL^{-1}A^*$ and consequently, M can be represented as

$$M = (AL^{-1}A^*)^{-1} \tag{45}$$

The scalar product l_*, in its turn, induces a scalar product m_* on Y^*. Indeed, $m_*(q_1, q_2) = \langle M^{-1}q_1, q_2 \rangle = \langle AL^{-1}A^*q_1, q_2 \rangle = \langle L^{-1}A^*q_1, A^*q_2 \rangle = l_*(A^*q_1, A^*q_2)$. Under the condition that A is a surjective linear operator, m_* is a scalar product. Then, m_* induces the scalar product m on Y.

When $X^* = X$ (for example, when $X = \mathbb{R}$) then the duality map $L = I$, yet the duality map M on Y induced by L by $A : X \mapsto Y$ is $M = (AA^*)^{-1}$, i.e., might differ from I.

- Π_K denotes projection mapping any x to its best approximation $\Pi_K x$ by elements of a closed convex set K

- **Moreau Projection Theorem**

  Moreau Projection Theorem can be found in [50].

  Let X be a vector space equipped with the scalar product l, and let P be a closed convex cone in X and v a vector in X.

  By Π_P we denote the projection mapping any v to its best approximation $\Pi_K(v)$ by the elements of the closed convex set P.

Theorem 6 (Moreau Projection Theorem) *Consider the linear form Lv and the polar cone P^- lying in the dual space X^*. We denote by u the projection $\Pi_P(v)$ of vector v to set P, and by p the projection $\Pi_{P^-}(Lv)$ of Lv to the polar cone P^-. Then, $v = u + L^{-1}p$, where $u \in P$ and $p \in P^-$, and $\langle p, u\rangle = 0$, i.e., u and p are orthogonal in the sense of the duality product.*

□

- **Tensor Product of Vectors**

  Given $p \in X^*$ and $y \in Y$, the *tensor product* of p and y, denoted by $p \otimes y$, is a linear map from X to Y, that associates with any $x \in X$ the value of $p(x)$ multiplied by the vector y. That is, $(p \otimes y)(x) = p(x)y$, $\forall x \in X$.

  Note that the set of all linear operators mapping form the vector space X to the vector space Y denoted by $\mathcal{L}(X, Y)$, is spanned by the tensor product of vectors and thus can be represented in the terms of the tensor product of vector spaces as $X^* \otimes Y$.

  The duality map H on the space $\mathcal{L}(X, Y)$ can be computed as follows. The scalar product of $\mathcal{L}(X, Y)$ corresponding to the scalar products on X, Y is equal to $l_* \otimes m$, and the respective norm is $\eta = \lambda_* \times \mu$. Consider instants $u = p \otimes x$ and $v = q \otimes y$ of $\mathcal{L}(X, Y)$. Then,

$$\begin{aligned}&\langle Hu, v\rangle = \eta(u, v) = \eta(p \otimes x, q \otimes y) = l_*(p, q)\mu(x, y) = \langle L^{-1}p, q\rangle \langle Mx, y\rangle = \\ &\langle (L^{-1} \otimes M)p \otimes x, q \otimes y\rangle = \langle (L^{-1} \otimes M)u, v\rangle .\end{aligned} \tag{46}$$

- **Tensor Product of Linear Operators**

  We assume given vector spaces X, X_1, Y, Y_1, linear operators $W : X \mapsto Y$ and $W_1 : X_1 \mapsto Y_1$, and linear operators $A : X_1 \mapsto X$ and $B : Y \mapsto Y_1$. The *tensor product* of the linear operators A^* and B denoted by $A^* \otimes B$, is a linear operator mapping from $\mathcal{L}(X, Y)$ to $\mathcal{L}(X_1, Y_1)$, that associates with any $W \in \mathcal{L}(X, Y)$ the operator $W_1 \in \mathcal{L}(X_1, Y_1)$ as per the product $BWA : X_1 \mapsto^A X \mapsto^W Y \mapsto^B Y_1$. In other words, $(A^* \otimes B)(W) = BWA$.

Remark the following property of the duality product of the tensor operator: for arbitrary $A \in \mathcal{L}(X, Y)$, $u \in X$ and $r \in Y^*$ it holds that $\langle r, Au\rangle = \langle u \otimes r, A\rangle$. Indeed, u can be viewed as an operator $u : \mathbb{R} \mapsto X$ mapping any real number α into αu, and as an operator $u : X^* \mapsto \mathbb{R}$ mapping any $p \in X^*$ into $\langle p, u\rangle$. Then $\langle r, Au\rangle = r(Au) = rAu = u \otimes r(A) = \langle u \otimes r, A\rangle$.

Observe that transpose of tensor product $A^* \otimes B$ is equal to $(A^* \otimes B)^* = B \otimes A^*$ $A \in \mathcal{L}(X, Y)$, then , and the inverse of tensor product $A^* \otimes B$ is equal to $(A^* \otimes B)^{-1} = B^{-1} \otimes (A^*)^{-1}$.

Conclusions

This thesis is devoted to the modeling and stabilization of sociopolitical networks of rational actors in the frame of the country coalitions. Thorough attention is put on both creation of theoretical and analytical framework, and to applications and analysis of real case problems.

Ideas from the extension of the Ising model to quenched disorder – the Edward and Anderson's Spin Glass model in its ground state, base our model of coalition forming. Within the physical analogy, the model involves a mixture of both ferromagnetic and anti-ferromagnetic couplings beyond the geometry of a lattice, and allows interactions with non-neighbors. While including the main features of the Spin Glass model, which are quenched disorder and frustration, in contrast to physical entities, our model's actors possess rationality – the ability to envision a maximization of their individual benefits even through intermediate loosing states.

In this thesis we investigated the problem of characterizatiing the instability which is rooted in the coalition forming as a decentralized maximization processes. We addressed the problem of the qualification of system structure in which the process converges to the stability – a configuration recognized as the most profitable by all the countries in the system. In the mixe of limited and extensive rationality of actors, such configuration may be optimal as well as non-optimal.

Our attention was also devoted to the study of the role of global alliance's external incentives in the stabilization of coalition forming, which is based on the principle of polarizing of countries' common interests and on creation of additional propensities. The introduction of multi-factor stabilization can be considered a fundamental step forward in modeling of coalition forming.

The recent political and economical crisis sharpens the necessity of a new regard and analyzes essential for understanding and identifying of rising problems. In this work, we applied our model in the investigation of several remarkable real cases, including the illustration of unstable cycling of coalitions in Europe, the stability in the Eurozone, the internal crisis in Syria, and the collapse of Soviet Union versus the stability of NATO opponent alliance. In this context, we speculated that our models may give an interesting framework to understand those problem.

The Spin-Glass based model of coalition forming appeal to the ubiquitous methods of multi-thread simulations. The simulation we build as a part of this work allows a graphical visualization and investigation of the paradigms covered by the model – instability, stabilization, dissolution. The simulation aims also to illustrate and simulate real data, which puts forward the application of the model to Political and Social Sciences.

We have finally focused on the problem of viability correction in a dynamic network of individually evolving actors from the domain of mathematics and optimal control, which widens the horizon of our research. We describe the viability correction that is attained by decentralization between the dynamics of all the network's components with the help of the regulatory parameters. Here, we attempt to create a conceptual bridge between the fuzzy dynamic version of our Spin Glass based model of coalition forming, and the dynamic network of individually evolving actors. The reduction that is built allows a new interpretation of the globally motivated interactions in the coalition forming model as viability regulatory parameters with the viability is being viewed as stability.

Undoubtedly, using Statistical Physics and the study of Spin Glass model, as well Dynamic Networks and the Viability Correction, in social modeling in 80s and 70s has led to challenging research into new methods in tackling social and political problems, and to the creation of new branches of research. However, only few social and political works adhere to this new type of modeling. Most of the research uses the classic methods and no common language has been yet founded.

Before concluding, it is appropriate to give a list of open problems and directions for future research that may stem from the present thesis:

- The model of coalition forming leans on the hypothesis that each pair of actors has a propensity to cooperate or to conflict. Being predetermined by the ongoing historical interactions as a typology of their primary interests, the propensities result from multiple coefficients such as ethnic, religious, territorial, ideological, economic, historical and others. Each of the coefficients contributes to the propensity value. In this context it would be interesting to provide a unified protocol for the numerical evaluation of the real cases. Such evaluation framework could be tested on the data by means of the simulation we provide here. This would enable more precise real case analysis using the tools we provided in this work.

- By producing new motivations to interactions among countries, global alliances introduce new energy that contributes to cooperation or conflict between actors. In the prospect of real cases, giving these energies without providing a stability may lead to the explosion of conflicts.

  For instance, Ukraine, which for last few decades has been radically divided into rich and poor sectors existing one at the expense of the others, appears now under the global alliances of Europe and Russia with the prospect (the global concept) of Ukraine joining the European Union. These global alliances with the newly motivated interactions do not have a clear tendency to contribute to the stability. They rather keep the instability active in the region while producing the new energy that fuels war and weakens the region, where US consequently attempts to access the gas production and Russia – to protect its boards from the European and US neighborhoods.

  For such cases, this would be interesting to develop a model which is able to show in a simplified and clear manner, the tendencies of globally motivated interactions toward stability or instability.

- In general, stability that bases on keeping some of actors within limited rationality and limited capacities which enables a non-optimal stability solution, will no more be effective due to the wide information accessibility growing fast. In contrast, the global alliance motivated stabilization represents a possible, programmable and responsible scheme of stabilization.

  Coming back to the Syria example, the country, together with its religious communities as shown in our illustrations, includes many different ethnic groups such as Kurds, Syrian Arab, the indigenous Arameans, Assyrians, Armenians. Some of those ethnic groups are divided in to different religious communities. Therefore, in reality, there are two systems that coexist – one is ethnic-based and the other is religion-based. Using our coalition forming model as it could allow to illustrate only a part of the pictures in the region. In this context, it could be interesting to study a global alliance motivated stabilization in the case of system represented by two or more overlapping models.

The thesis has lead to the linking of seemingly unconnected disciplines such as statistical physics, socio-political science, multi-thread simulation and optimal controls. Thus, in a certain sense, this manuscript can be viewed a "Sancho Panza thesis": while a "Don Quixote thesis" has the purpose of producing novelties and discoveries, a "Sancho Panza thesis" centers on connecting scattered innovations and theories. At the casual level, we would like to conclude this manuscript stating that *the science without connecting thesis is like Don Quixote without Sancho Panza.*

Publications

1. Correction of Dynamical Network's Viability by Decentralization by Price
   Vinogradova G.
   Journal of Complex Systems 2012

2. Rational Instability in the Natural Coalition Forming
   Vinogradova G., Galam S.
   Physica A: Statistical Mechanics and its Applications 2012

3. Dissolution of a Global Alliance: War or Peace
   Vinogradova G., Galam S.
   Policy and Complex Systems 2014

4. Global Alliances Effect in Coalition Forming
   Vinogradova G., Galam S.
   The European Physical Journal B 2014

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