Precision tube drawing for biomedical applications: Theoretical, Numerical and Experimental study
Camille Linardon

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Precision Tube Drawing for Biomedical Applications: Theoretical, Numerical and Experimental Study

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General Introduction

Minitubes is a family founded company specialized in the manufacturing of precision tubes. The principal applications include biomedical devices such as surgical implants, stents and cardiac valves or a variety of in vitro diagnostic devices such as probes. Other applications in the field of aerospace or electronics for example exist but they are less challenging in term of precision compared to biomedical applications. Indeed, the components designed to be implanted in the human body require the tightest specifications.

From its foundation, Minitubes has developed a refined know-how in the field of tube drawing. Today, this know-how enables to reach the sharpest requirements and to satisfy clients demand. In the future, Minitubes intention is to formalise the process and to build a series of tools to define the different tube manufacturing process steps. More specifically, the intention is to optimise the process in order to increase the productivity and improve the product quality.

A better understanding of the process can be achieved by conducting large series of tests. Such approach happens to be time and money consuming due to the amount of raw material needed and more especially because the tests must be performed on the industrial drawing benches. At the industrial scale, experimental studies enable to easily measure different data, such as the drawing force, the temperature, the tube surface aspect and roughness, the final tube dimensions and the tube straightness. At a laboratory scale, due to the access to more complex analysis devices, experimental tests can give information about the material structure, texture, internal residual stresses, anisotropy and the heat generated due to plastic deformation to cite some of them. The combination of the possibilities offered by both environments can deliver rich information.

In this context, finite element modelling appears to be a helpful tool to improve the process understanding. The first interest of finite element modelling is to virtually perform a large number of tests. The second interest of numerical methods is that they give access to non measurable physical values such as strains and stresses during drawing. Such informations are necessary to improve the understanding of the process and above all to link the experimentally measured data to the internal phenomena taking place during material deformation.

Evidently, the model must be physically based and a series of experimental tests must
be performed in order to build it. First, as the material deforms during the process, its mechanical behaviour must be accurately characterised by means of laboratory tests. Any mechanical test can give stress vs strain data but it is fundamental to perform the laboratory tests in representative conditions compared to the industrial tube drawing. In the case where the variety of the testing devices is limited, it is crucial to evaluate the errors that can be made when simulating the process with simpler models. Second, like in any metal forming process, the material to be formed interacts with forming tools. Most of the time the contact is lubricated. This interaction phenomenon is important to be considered as it directly influences the drawing conditions and it can influence the deformations undergone by the material surfaces. Third, as the material plastically deforms and due to the friction between the material and the forming tools, heat is generated. If this phenomenon intends to be included in a finite element model, it must be characterised with care.

In the objective of process optimisation, a major point is to identify the material formability limit. In other words it signifies to determine the maximum deformation a tube can undergo before fracture. Once the experimental formability limit is known, the goal is to be able to predict it by means of finite element method. The challenge to predict tube failure is to select the appropriate tool among all the models and criteria that were defined by different authors. Due to the industrial requirements of selecting an efficient method and because of the limited mechanical testing techniques that were available at Minitubes, the choice was oriented towards failure criteria that could be calibrated on uniaxial tensile tests only.

The different topics dealt with in this thesis fit into the above described framework. The general objective of this study is to develop the finite element modelling of the tube drawing, first in order to improve the process understanding, second to find the formability limit and to optimise the process.

The first chapter is devoted to the presentation of all the notions involved in this study and to define the vocabulary. The principle of the tube drawing process is introduced and the different physical phenomena that are involved in such forming process are detailed. The different techniques that exist to analyse the drawing process are described and the focus is put on the interest of finite element modelling compared to analytical methods. Then, as one of the main concern is the material formability during tube drawing, the different tools that were developed to study and to predict material formability are presented. This chapter ends with the description of a mechanical test called tube bulge test which is devoted to tube testing.

The second chapter presents the procedure that was used to characterise the materials mechanical and thermo-mechanical properties. The different testing techniques are introduced and the results are presented. The first objective of these experimental tests is to identify materials constitutive behaviour in order to model it. The second objective is to characterise materials failure.

The third chapter deals with an experimental tube drawing test that was designed for
the purpose of this study. The originality of this test relies on the geometry of a drawing tool that was designed to draw tubes up to failure. The principle of this test is detailed and the different measurements performed during the test are described. From these tests, the material formability limit during tube drawing is identified.

The fourth chapter is devoted to the finite element modelling of the tube drawing process. First a general description of the model is made, second, the focus is put on the development of different models considering different aspects. Three models are developed: first, a purely mechanical one considering plastic isotropy, second, a thermo-mechanical one considering also plastic isotropy and last a pure mechanical model considering plastic anisotropy. The methods used to identify contact properties by inverse analysis are detailed. Finally, once the finite element model is fully defined, the tube drawing process is analysed in term of stress and strain fields and energies.

The last chapter compiles the different methods that were developed in the previous chapters and the results that were obtained throughout this study. The goal is to try to predict the formability limit that was experimentally found by means of finite element modelling.
The objective of the introduction chapter is to present the general context of this study and to identify the different issues. As a starter, the principle of the tube drawing process is described and the different parameters involved in the process are listed. The objective is to have an insight into the issues that can be encountered during the process. In a second part, all the phenomena at the basis of the drawing process are explained. This part deals with material plasticity, friction, thermal aspects and fracture. Then, in a third part, all
the mechanical approaches that enable to analyse the process are detailed. Some analytical methods are briefly introduced and the insight is put into the Finite Element Modelling. A fourth part presents a major issue of metal forming industry, the metal formability. Indeed, the major concern of industry is to form parts safely which means without fracture occurrence. Finally, a test designed for the evaluation of mechanical properties of tubular materials is presented.

1.1 Tube drawing process

1.1.1 Introduction on tube drawing process

Cold tube drawing is a metalworking process used to produce high-quality seamless tubes with precise dimensions and good surface finish. Cold forming process compared to hot forming has three main advantages: tubes have more precise dimensions because it is not affected by thermal expansion, the surface finish is better and the mechanical properties are increased by strain hardening.

This introductory section first presents the different drawing techniques that are commonly used in the industry. Then the different operations required to manufacture the end product are explained.

1.1.1.1 Presentation of the different drawing processes

Cold tube drawing consists in reducing tube dimensions by pulling it through a die. There are four types of tube drawing process. For each of them, the tube outer diameter is calibrated by the die diameter. Their difference relies on the technique used for inner diameter calibration. The four kinds of tube drawing are tube sinking, mandrel drawing, floating plug drawing and fixed plug drawing. For illustration, the reader might refer to the figure 1.1 where the different kinds of tube drawing are shown. A brief explanation of each technique and their respective advantages and drawbacks are detailed below.

- **Tube sinking** consists in reducing the inner diameter with no tool inserted inside the tube. The inner surface is free to deform, as a consequence, the surface finish is degraded. The advantage of this technique is that it can be used in continuous drawing of coils.

- In the **mandrel drawing** technique, the tube inner diameter is calibrated by a rod, also named mandrel, which moves together with the tube. The advantage is the good surface finish which is obtained. There are two principal drawbacks associated to this technique. First the drawn length is limited by the mandrel and the drawing bench lengths. Literature reports length up to 30 m but at Minitubes the length is limited to a maximum between 5 and 6 m. Second, it requires an additional operation to remove the mandrel. This operation is called reeling. It induces dimensional changes to unstick the tube from the mandrel and introduces surface defects.
• **Floating plug** drawing is also known as **floating mandrel** drawing. It consists in inserting a specifically designed short plug inside the tube. The plug is free to move but stays located in the die vicinity due to friction forces between the mandrel and the tube. This process enables to reach a good surface roughness both inside and outside the tube. It can be used in continuous drawing of coils.

• In the **fixed plug** drawing the plug is fixed at the end of a rod. This technique is similar to the floating plug drawing and enables to reach the best surface finish.

1.1.1.2 Presentation of the drawing operations

Drawing a tube up to the wanted final dimensions requires several operations that are detailed here. The very first tube to be drawn is manufactured by successive forging, rolling and drilling. This tube is called 'ebauche'. Starting from the ebauche to end up with the final product requires different successive drawing steps called passes. At each pass the tube is drawn to a certain section and thickness reduction. Between two passes, the tube is annealed to restore the material microstructure and ductility properties. The final passes are defined according to mechanical and metallurgical characteristics that are required by the client (ultimate tensile strength, yield strength, elongation, hardness, grain size). Finally, the process is ended by a straightening step to correct the curvature the tube has developed along the process. Figure 1.2 presents in a synthetic way an example of the operations required to manufacture a classic product in Minitubes.

The first passes are generally made by means of mandrel drawing, since the drawing forces are high and a floating plug could not undergo such forces. The last passes are made on floating plug drawing to reach precise final dimensions and surface finish.
The bibliography is very rich for the study of wire drawing but less developed for tube drawing. Nevertheless, both processes have common characteristics and some of the studies concerning wire drawing can be expanded to tube drawing. In the following section, various references concerning the wire drawing are cited but one has to keep in mind that the observations transfer to the tube drawing process.

1.1.2 Process parameters

The definition of a drawing pass requires to adjust different parameters. The section below enumerates the principal process parameters.

- The section and thickness reductions: ideally they should be the highest possible to limit the number of drawing passes. The first consequence of their increases is the increase of the drawing force. But the latter must not reach the bench limit capacity. Moreover the section and thickness reductions induce variations of plastic strain imposed to the material. Thus, the additional deformation necessary for the reeling step, which is compulsory after a mandrel drawing pass, may be greater. As a consequence, in this case, the risk of deteriorating tube dimensions and aspect during reeling is increased.

- The die geometry including the die angle, the bearing length, the entry radius. The different die geometrical characteristics are detailed in figure 1.3. The die angle is the most critical parameter. It has a strong influence on the drawing force (Wistreich, 1955; Beland et al., 2011). First, it induces variations of the work of friction. Second, it influences the redundant plastic work of deformation (Aguilar et al., 2002). The redundant deformation is a radial strain heterogeneity due to localised shear and
tensile strains (Sadok et al., 1994b) which do not contribute to the section reduction. It is the cause of the loss of the cylinder shape of the tube after drawing. An example illustrating the tube deformation is shown in figure 1.4. Redundant deformation appears as a deformation with an angle $\alpha$ positive at the tube extremity. The redundant deformation can be characterised by a redundant deformation factor $\phi$ which is the ratio of the average effective strain in the cross section of the material $\epsilon_{\text{avg}}$ on the homogeneous strain imposed in the drawing process $\epsilon_h$: $\phi = \frac{\epsilon_{\text{avg}}}{\epsilon_h}$. The factor $\phi$ depends only on the die semi-angle $\alpha$ and on the section reduction of the pass $\text{RedS}$ (Chin and Steif, 1995; Aguilar et al., 2002). Thus, it is common to compute a parameter $\Delta$ to combine both parameters. It can be expressed in different manners according to different authors but the numerical results differ little. As an example, Atkins and Caddel (1968) defined the $\Delta$ parameter as:

$$\frac{1 + \sqrt{1 - \text{RedS}}}{1 - \sqrt{1 - \text{RedS}}} \sin \alpha$$  \hspace{1cm} (1.1)$$

And Backofen (1972) defined the $\Delta$ parameter as:

$$\Delta = \frac{\alpha}{\text{RedS}}(1 - \sqrt{1 - \text{RedS}})^2$$  \hspace{1cm} (1.2)$$

Beland et al. (2011) analysed the influence of the die angle on the drawing force and revealed that an optimum die angle exists leading to a minimum drawing force. An example of the evolution of the experimental drawing force as a function of the die angle is shown in figure 1.5. But the drawing force must not be the only criterion to select a die angle. Indeed, die angle has a strong effect on the level of residual stress in the tube after drawing. Residual stresses are directly linked to the inhomogeneous deformation (redundant deformation). Depending on the application of the final product, residual stresses can influence the mechanical behaviour and the durability. For example, concerning wires, tensile residual stress at the wire surface can cause stress corrosion cracking and reduce the service time of the product (Elices et al., 2004; Överstam, 2006). Die angle is not the only responsible for the presence of residual stresses, one can also mention the heat generated during the process (Lee et al., 2012). The phenomenon of heat generation is addressed in a further part. Finally, the die angle can influence the material properties: de Castro et al. (1996)
performed tensile tests on wires drawn with the same reduction and different die angles and found that the yield and tensile strength increase with die angle.

- The **drawing speed**: it can influence the friction and the material behaviour if the material behaviour is viscoplastic.

- The **lubrication**: its role is to reduce friction between the tube and the drawing tools. It enables to prevent the occurrence of surface defects like scratches or wrenching. It is also a vector for heat extraction produced by plastic deformation and friction. The lubrication is dependent on the amount of lubricant, the nature of the contacting materials, their roughness, the sliding speed, the temperature and the pressure.

All the above detailed parameters have to be analysed to understand and to optimize the process. Moreover, they have a direct influence on the different mechanical and thermal phenomena that take place during the process. Next paragraph explains in a more detailed way the mechanical and thermal phenomena encountered during tube drawing.
1.2 Phenomena to be modelled

Tube drawing like any other metal forming process involves different phenomena that must be taken into account in a modelling. First, during forming, the material deforms in an irreversible way due to plastic deformation. Second, the material interacts with tools and the respective sliding of contacting materials causes friction. Finally, when a material plastically deforms and when there is friction between two materials, heat is generated. The generated heat then transfers to the contacting parts and to the surrounding environment. This part will be devoted to the description of the three phenomena to be modelled in metal forming process:

- plasticity;
- friction;
- heat generation and thermal exchanges.

1.2.1 Plasticity

Metal forming is possible because of the material plasticity properties which is the ability of a material to undergo non-reversible deformations. The physical mechanism which is behind plastic deformation is the motion of dislocations. A dislocation is a linear defect corresponding to a discontinuity in the crystal organisation. The strain hardening is due to the accumulation of dislocations within the grains of a polycrystalline material. The dislocations can form different substructures depending on the nature of the material. Strain hardening is also due to the evolution of crystallographic texture. During plastic deformation, the tendency of the grains is to rotate towards more stable orientations and as a consequence, material hardening behaviour is modified.

Plasticity can be described by phenomenological models or by physically based ones. Phenomenological models are already implemented in FEM codes or can be easily implemented, and thus, they are convenient for industrial applications. Physical models are based on the theory of crystalline plasticity or on micromechanics. Models based on the crystal plasticity take into account the grain shape, the movement of dislocations within grains and the rotation of individual grains (Kalidindi and Schoenfeld, 2000; Van Houtte et al., 2002; Delannay et al., 2006; Li et al., 2008). All the physical phenomena at the origin of plastic deformation are modelled. Thus, the evolution of material anisotropy is naturally modelled.

Micromechanically-based models like self-consistent models also enable to take into account the material microstructure evolution on the mechanical behaviour of bulk materials. These models are coupled with the Hall-Petch relationship which models the strengthening due to the grain size and grain boundaries. Self consistent are upgradable as they can be coupled with different local constitutive behaviour associated with grains in order to analyse microstructural effects such as grain size, crystallographic texture, grain boundary, porosity and damage. Bui et al. (2013) list a number of studies dealing with the above mentioned phenomena. As an example, Segurado et al. (2012) predicted the mechanical behaviour and the development of material texture during deformation. They implemented a viscoplastic
self consistent model in an UMAT subroutine. A UMAT is a subroutine that enables to model a user-defined mechanical material behaviour in Abaqus. The originality of their work is that they considered each integration point as a crystal with a given initial texture and followed texture evolution with deformation. But the main drawback of self-consistent model combined with Hall-Petch only is that the strengthening due to dislocation density is neglected (Kapoor et al., 2010). Bui et al. (2013) developed a model to fill this gap. They modelled the strengthening due to both grain boundaries and substructures formed by dislocations and could predict the mechanical behaviour of cold drawn aluminium tubes up to various cross sectional reductions. The use of such models that enable to take into account the evolution of mechanical properties with deformation is important in the case of successive deformations analysis. For example, Karnezis and Farrugia (1998) analysed tube drawing by means of FEM and came to the conclusion that a two-pass tube drawing could be turned into a single pass. Bui et al. (2013) pointed out that Karnezis and Farrugia (1998) used the same phenomenological constitutive equation to study the two successive drawing passes and that, in this way, they did not take into account the change of mechanical properties of the tube after the first pass. They considered the mechanical properties of the tube being drawn at the second pass to be identical to the initial tube. As a comparison, Bui et al. (2011a) showed that a 36% section reduction of an aluminium tube caused the yield strength of the drawn tube to be three times higher than the initial tube and the elongation to be divided by four.

The main drawback of these models is that they require greater computational time and as a consequence they are less convenient for industrial applications.

Finally, phenomenological models are at the basis of this work due to their implementation into FEM codes and because of their ability to model material behaviour correctly.

In a general way, phenomenological models are based on the definition of different constitutive equations that are detailed in the following section.

1.2.1.1 Plastic constitutive equations

Plasticity is commonly described by three constitutive equations which are a yield condition, a flow rule and a hardening law.

- The yield condition is described following a yield function which defines a surface in the stress space corresponding to the elastic limit and the transition to the plastic deformation. Its mathematical expression describes the shape of the yield surface.

- The flow rule relates the stress and strain components and their time derivatives, it gives the plastic strain rate.

- The hardening law describes the evolution of the yield surface during deformation in terms of expansion and translation.

Each of the above mentioned constitutive equations is detailed in this section. Before presenting the constitutive equations it is necessary to define two categories of material behaviour: isotropic and anisotropic. A polycrystalline material is a solid composed of grains of different size and orientation. A material whose crystals are oriented in random directions exhibits the same mechanical properties in every loading directions.
This material is said isotropic. On the contrary a material composed of directed grains exhibits properties that depend on the testing direction. This material is said anisotropic. Thus, constitutive equations can be classified into two categories depending on whether the material is isotropic or anisotropic.

1.2.1.1 Isotropic yield functions

The oldest isotropic yield function are the Tresca (1864) and the quadratic Von Mises (1913) yield criteria. Their respective expressions in the principal basis are the following:

\[ f_{\text{Tresca}} = \frac{1}{2} \max(\sigma_i - \sigma_j) - \frac{\sigma_0}{2} \]  \hspace{1cm} (1.3)

and

\[ f_{\text{Von Mises}} = \frac{1}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} - \sigma_0 \]  \hspace{1cm} (1.4)

\( \sigma_i \) and \( \sigma_j \) are principal stresses with \( i \) and \( j \) equals to (1, 2, 3). Tresca expresses that yielding occurs when the maximum shear stress reaches a constant critical value. \( \sigma_0 \) is the yield stress in uniaxial tension.

Later Hosford (1972) extended the Von Mises yield criterion to a non-quadratic criterion based on polycrystal plasticity:

\[ f = \frac{1}{2} \left( |\sigma_2 - \sigma_3|^n + |\sigma_3 - \sigma_1|^n + |\sigma_1 - \sigma_2|^n \right)^{\frac{1}{n}} - \sigma_0 \]  \hspace{1cm} (1.5)

where \( n \) is a material parameter which is dependent upon the crystalline structure: for body-centered cubic (bcc) materials, \( n = 6 \) and for face-centered cubic (fcc) materials, \( n = 8 \). Taking \( n = 2 \) returns the Von Mises expression.

1.2.1.1.2 Anisotropic yield functions

In some metal forming processes, materials are deformed in preferred directions. The consideration of material anisotropic behaviour is crucial to study any material forming process.

As an example, in wire drawing, the wire is drawn in the axial direction several times. As a result, the final product exhibits mechanical properties variations depending on the direction of deformation. For example, Massé et al. (2011) showed anisotropy evolution throughout the cold wire drawing process. Their method to highlight material anisotropy was to study the ovalisation of small cylindrical samples cut from the wires under compression tests as shown in figure 1.6. Anisotropy appeared as an easier flow in the radial direction compared to the axial direction (the axial direction corresponding to the drawing direction).

Lopes et al. (2003) observed a decrease of 30% of uniform elongation between the rolling directions and the 45° directions during tensile tests of aluminum alloy sheet samples. As a consequence when parts are formed from metallic sheets by deep drawing or stamping, the plastic flow localization and fracture depend on the direction of deformation. Such variations are due to the fact that the initial sheet used in drawing or stamping was produced by rolling. During the rolling process, the material was deformed in preferred
Figure 1.6: scheme of the compression sampling and compression tests (Massé et al., 2011)

Figure 1.7: Example of grain elongation with reduction during sheet rolling (Park, 1999)

directions and anisotropy was induced. As a result the initial sheet showed anisotropic properties.

As a consequence, when forming materials, it is essential to know the whole deformation history in order to evaluate the material behaviour and to be able to model it with the appropriate constitutive equations. In order to explain the anisotropic behaviour, the material should be analysed at the microscopic scale.

The plastic anisotropy in metal is due to preferred orientations of grains (crystallographic texture) and dislocation microstructures (Hiwatashi et al., 1998). In the particular case of metal forming, the favoured orientation is induced by the plastic deformation that takes place in preferred directions. Park (1999) showed aluminium sheet microographies after different successive rolling passes and the grain alignment in the direction of rolling appeared clearly as it can be seen in figure 1.7. Pole figures also revealed that a texture was developing during the process. Bui et al. (2011b) also observed a grain refinement and elongation with increasing section reduction in fixed plug drawing.
Some forming processes can produce complex textures such as gradient of texture in the thickness of the formed part. As an example, Park (1999) and Cho et al. (2006) showed that a gradient of texture was developing in the part thickness during cold sheet rolling and cold wire drawing respectively. Shear strain is said to be responsible for these variations. Indeed, Cho et al. (2006) found the shear strain to increase with the distance from the center line and found a gradient of texture between the center and the wire surface. Park (1999) noticed that friction at the sheet/roll interface during sheet rolling was causing shear deformation and as a consequence, the developed texture varied between the middle and the sheet surface.

As mentioned above, the forming process can induce anisotropic properties, thus, it is important to consider material anisotropy. The following paragraph lists anisotropic yield criteria proposed throughout the years.

The first anisotropic yield criterion was introduced by Hill (1948). Hill modified the Von Mises quadratic yield criterion by introducing coefficients to describe the plastic flow direction dependency. Hill’s expression is valid for orthogonal anisotropy and writes:

\[
f = \left[ F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{23} + 2M\sigma_{31} + 2N\sigma_{12} \right] = \sigma_0^2 \tag{1.6}
\]

where \(\sigma_{ij}\) are the components of the Cauchy stress tensor and \(F, G, H, L, M, N\) are material parameters. In the case of plane stress (\(\sigma_{33} = \sigma_{13} = \sigma_{23} = 0\)) the quadratic Hill yield criterion can be expressed as a function of the Lankford coefficients \(r_0\) and \(r_90\) which are the ratio of the width to the thickness strains \(r_i = \frac{\varepsilon_{\text{width}}}{\varepsilon_{\text{thickness}}}\). The strains are measured during tensile tests in 0° and 90° with respect to the rolling or drawing direction respectively. The yield criterion expression then turns:

\[
f = \sigma_1^2 + \frac{r_0(1 + r_90)}{r_90(1 + r_0)} \sigma_2^2 - \frac{2r_0}{1 + r_0} \sigma_1 \sigma_2 = \sigma_0^2 \tag{1.7}
\]

where \(\sigma_1\) and \(\sigma_2\) are the principal stresses whose directions are aligned with the axis of anisotropy. \(\sigma_1\) is aligned with the rolling or drawing direction and \(\sigma_2\) is perpendicular.

This criterion was extensively used in different studies and led to good results (Liao et al., 1997; Zang et al., 2011).

Afterwards, Hill generalized his own criterion (Hill, 1979) by introducing an anisotropy exponent \(m\). Hill (1979) anisotropic yield criterion expresses in the space of principal stress:

\[
f = [F|\sigma_2 - \sigma_3|^m + G|\sigma_3 - \sigma_1|^m + H|\sigma_1 - \sigma_2|^m + L|2\sigma_1 - \sigma_2 - \sigma_3|^m + M|2\sigma_2 - \sigma_3 - \sigma_1|^m + N|2\sigma_3 - \sigma_1 - \sigma_2|^m] = \sigma_0^m \tag{1.8}
\]

with \(\sigma_1, \sigma_2\) and \(\sigma_3\) the principal stresses.

Hosford (1979) defined another yield criterion whose expression is similar to Hill (1979) and writes:

\[
f = F|\sigma_2 - \sigma_3|^m + G|\sigma_3 - \sigma_1|^m + H|\sigma_1 - \sigma_2|^m = 2\sigma^m \tag{1.9}
\]

where \(F, G\) and \(H\) are material constants, \(m = 6\) for bcc (body-centered cubic) materials and \(m = 8\) for fcc (face-centered cubic) materials. Even though Hill (1979) and Hosford (1979) expressions are similar, they differ in the fact that for Hill (1979) the \(m\) exponent
is dependent on the anisotropic $r$ values while it is independent in the case of Hosford (1979). Both Hill and Hosford non-quadratic anisotropic yield criteria are valid for planar/orthotropic anisotropy, when the directions of the principal stresses are superposed with the anisotropy axes. Their drawback is that they do not involve shear stresses. As a consequence, they cannot model the yield stress when the anisotropy axes do not coincide with the principal stress axes. Barlat and Lian (1989) introduced a new criterion to complete this limitation and defined a yield function which takes into account the shear stresses. Barlat and Lian (1989) yield function writes:

$$f = a|K_1 + K_2|^m + a|K_1 - K_2|^m + c|2K_2|^m = 2\sigma_s^m$$  \hspace{1cm} (1.10)

with $K_1$ and $K_2$ defined as:

$$K_1 = \frac{\sigma_{11} + h\sigma_{22}}{2} \quad \text{and} \quad K_2 = \left(\frac{\sigma_{11} - h\sigma_{22}}{2}\right)^2 + (p\sigma_{12})^2$$  \hspace{1cm} (1.11)

where $a, c, h$ and $p$ are the anisotropy coefficients and $m$ is a non quadratic exponent depending on the material crystallographic structure as for Hosford (1979). In the special case of metal forming, $11$ and $22$ refer to the rolling or drawing directions and to the perpendicular to the rolling or drawing directions respectively.

As Barlat and Lian (1989) criterion includes the shear stress component, it can be used in cases were the anisotropy axes do not coincide with the stress axes. Nevertheless, this yield criterion is limited to plane stress problem. In order to solve three dimensional stress state problems, Barlat et al. (1991) proposed another yield criterion named Yld91 and extended the isotropic Hosford (1972) yield criterion to anisotropy by introducing a modified stress tensor $\tilde{\sigma}$ obtained from a linear transformation of the Cauchy stress tensor $\sigma$:

$$\tilde{\sigma} = M\sigma$$  \hspace{1cm} (1.12)

where $M$ if a 4th-order tensor which due to the symmetry of the stress tensor can be reduced to a $6 \times 6$ matrix. In the case of an isotropic material, $M$ reduces to the unit tensor. If the plastic behaviour is pressure independent, the stress deviator $s$ can be used instead of the stress tensor. Similarly to the stress tensor, a modified stress deviator $\tilde{s}$ can be introduced by linear transformation of the stress deviator $s$:

$$\tilde{s} = Cs = CT\sigma = L\sigma$$  \hspace{1cm} (1.13)

where $C$ and $L$ are fourth order tensors containing the anisotropy coefficients. $T$ enables to transform the stress tensor $\sigma$ into the deviatoric stress tensor $s$. In the case of orthotropic materials, the matrix of linear transformation writes:

$$L = \frac{1}{3} \begin{pmatrix}
(b + c & -c & -b & 0 & 0 & 0 \\
-c & c + a & -a & 0 & 0 & 0 \\
-b & -a & a + b & 0 & 0 & 0 \\
0 & 0 & 0 & 3f & 0 & 0 \\
0 & 0 & 0 & 0 & 3g & 0 \\
0 & 0 & 0 & 0 & 0 & 3h
\end{pmatrix}$$  \hspace{1cm} (1.14)
where \(a, b, c, f, g, h\) are six independent coefficients characterising anisotropy. The Yld91 yield criterion then writes:

\[
f = |\tilde{s}_2 - \tilde{s}_3|^m + |\tilde{s}_3 - \tilde{s}_1|^m + |\tilde{s}_1 - \tilde{s}_2|^m = 2\sigma^m
\] (1.15)

Barlat et al. (1991) can be expressed in another form such as:

\[
f = \left(2\sqrt{H_1^2 + H_2^2}\right)^m \left[ |\cos \left(\frac{\theta}{3}\right) - \cos \left(\frac{\theta - 2\pi}{3}\right) |^m + |\cos \left(\frac{\theta - 2\pi}{3}\right) - \cos \left(\frac{\theta + 2\pi}{3}\right) |^m \right.
\]

\[
+ \left. |\cos \left(\frac{\theta + 2\pi}{3}\right) - \cos \left(\frac{\theta}{3}\right) |^m \right] = 2\sigma^m
\] (1.16)

with,

\[
\theta = \arccos \left(\frac{q}{p^{\sqrt{2}}}\right), 0 \leq \theta \leq \pi
\] (1.17)

\[
p = H_1^2 + H_2
\] (1.18)

\[
q = (2H_1^3 + 3H_1H_2 + 2H_3)/2
\] (1.19)

and \(H_1, H_2\) and \(H_3\) are the invariants of the transformed stress deviator:

\[
H_1 = (\tilde{s}_{11} + \tilde{s}_{22} + \tilde{s}_{33})/3
\] (1.20)

\[
H_2 = (\tilde{s}_{23}^2 + \tilde{s}_{31}^2 + \tilde{s}_{12}^2 - \tilde{s}_{22}\tilde{s}_{33} - \tilde{s}_{33}\tilde{s}_{11} - \tilde{s}_{11}\tilde{s}_{22})/3
\] (1.21)

\[
H_3 = (2\tilde{s}_{23}\tilde{s}_{31}\tilde{s}_{12} + \tilde{s}_{11}\tilde{s}_{22}\tilde{s}_{33} - \tilde{s}_{11}\tilde{s}_{23}^2 - \tilde{s}_{22}\tilde{s}_{33}^2 - \tilde{s}_{33}\tilde{s}_{12}^2)/2
\] (1.22)

Barlat et al. (1991) revealed that this criterion could predict the uniaxial tensile yield stress in different directions but the accuracy was lower in the case of the Lankford coefficient prediction. Thus to improve predictions accuracy, Barlat et al. (2003) in 2000 proposed a new criterion called Yld2000-2d. This criterion is limited to plane stress state. Its expression is based on Hosford (1972)(1.5) isotropic criterion which is expressed as a function of the principal values of the stress deviator. Hosford (1972) yield function reduces to:

\[
f = f' + f'' = 2\sigma^m \text{ where } f' = |s_1 - s_2|^m \text{ and } f'' = |2s_2 + s_1|^m + |2s_1 + s_2|^m
\] (1.23)

The expressions of \(f'\) and \(f''\) were transformed and expressed in terms of linear transformations of the stress deviator:

\[
f' = |\hat{s}_1' - \hat{s}_2'|^m \text{ and } f'' = |2\hat{s}_2'' + \hat{s}_1''|^m + |2\hat{s}_1'' + \hat{s}_2''|^m
\] (1.24)

where \(\hat{s}_1', \hat{s}_2'\) and \(\hat{s}_1'', \hat{s}_2''\) are the principal values of the transformed stress deviators \(\hat{s}'\) and \(\hat{s}''\) respectively such as:

\[
\hat{s}' = C's = C'Ts = L's
\] (1.25)

\[
\hat{s}'' = C''s = C'Ts = L''s
\] (1.26)

Finally, the linearly transformed stress deviators can be written in a matrix form as:

\[
\hat{s}' = \begin{pmatrix} \hat{s}'_{11} \\ \hat{s}'_{22} \\ \hat{s}'_{12} \end{pmatrix} = \begin{pmatrix} C'_{11} & C'_{12} & 0 \\ C'_{21} & C'_{22} & 0 \\ 0 & 0 & C'_{66} \end{pmatrix} \begin{pmatrix} \tilde{s}_{11} \\ \tilde{s}_{22} \\ \tilde{s}_{12} \end{pmatrix}, \quad \hat{s}'' = \begin{pmatrix} \hat{s}''_{11} \\ \hat{s}''_{22} \\ \hat{s}''_{12} \end{pmatrix} = \begin{pmatrix} C''_{11} & C''_{12} & 0 \\ C''_{21} & C''_{22} & 0 \\ 0 & 0 & C''_{66} \end{pmatrix} \begin{pmatrix} \tilde{s}'_{11} \\ \tilde{s}'_{22} \\ \tilde{s}'_{12} \end{pmatrix}
\] (1.27)
As a result, ten coefficients are necessary to describe plastic anisotropy. The procedure for parameters identification was given in Barlat et al. (2003). Nevertheless the Yld2000-2d criterion is limited to plane stress problems.

In order to solve three dimensions problems Barlat et al. (2005) proposed a new criterion named Yld2004-18p based on the combination of two linear transformations of the stress deviator. The expression of Yld2004-18p yield function is the following:

\[
f = f(\tilde{\mathbf{s}}', \tilde{\mathbf{s}}'') = |\tilde{s}'_1 - \tilde{s}''_1|^m + |\tilde{s}'_1 - \tilde{s}''_2|^m + |\tilde{s}'_1 - \tilde{s}''_3|^m + |\tilde{s}'_2 - \tilde{s}''_1|^m + |\tilde{s}'_2 - \tilde{s}''_2|^m + |\tilde{s}'_2 - \tilde{s}''_3|^m + |\tilde{s}'_3 - \tilde{s}''_1|^m + |\tilde{s}'_3 - \tilde{s}''_2|^m + |\tilde{s}'_3 - \tilde{s}''_3|^m\]

(1.28)

The different linear transformations applied to the stress deviators \(\tilde{\mathbf{s}}'\) and \(\tilde{\mathbf{s}}''\) are:

\[
C' = \begin{pmatrix}
0 & -c'_{12} & -c'_{13} & 0 & 0 & 0 \\
-c'_{21} & 0 & 0 & 0 & 0 & 0 \\
-c'_{31} & -c'_{32} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c'_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c'_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & c'_{66}
\end{pmatrix}
\]

and

\[
C'' = \begin{pmatrix}
0 & -c''_{12} & -c''_{13} & 0 & 0 & 0 \\
-c''_{21} & 0 & 0 & 0 & 0 & 0 \\
-c''_{31} & -c''_{32} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c''_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c''_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & c''_{66}
\end{pmatrix}
\]

(1.29)

In this case, the identification of the anisotropic coefficients requires a large number of experimental data such as uniaxial tensile test in seven directions between the rolling or drawing and the transverse directions and biaxial tests (Barlat et al., 2005). Yld2004-18p criterion was implemented by Yoon et al. (2006) in the FEM of cup drawing of a circular blank sheet and they successfully predicted the the cup heigh profile. They also showed the improved accuracy of Yld2004-18p compared to Yld96.

1.2.1.1.3 Flow rule

The flow rule gives the direction of the plastic strain rate and writes:

\[
\dot{\epsilon}^p_{ij} = d\lambda \frac{\partial g}{\partial \sigma_{ij}}
\]

(1.30)

where \(\dot{\epsilon}^p_{ij}\) are the plastic strain rate components, \(d\lambda\) a scalar coefficient and \(g\) the dissipative potential. This equation is called the non-associative flow rule as the dissipative potential \(g\) is different from the yield function. In the case where the yield function \(f\) is taken as the dissipative plastic potential, it is called the associated flow rule and writes:

\[
\dot{\epsilon}^p_{ij} = d\lambda \frac{\partial f}{\partial \sigma_{ij}}
\]

(1.31)

1.2.1.1.4 Hardening constitutive equations

The concern of the previous paragraph was to present the yield functions defining the elastic limit. The study of material mechanical properties relies also on the formulation of hardening law that defines the evolution of the yield surface during deformation. The present paragraph is devoted to the presentation of the hardening laws.
Different material models can describe the work-hardening behaviour. The Hollomon’s equation writes:

$$\bar{\sigma} = K \bar{\epsilon}_p^n$$  \(\text{(1.32)}\)

where $\bar{\sigma}$ is the equivalent flow stress, $K$ is the strength index, $\bar{\epsilon}_p$ is the equivalent plastic strain and $n$ is the strain hardening exponent. Ludwik’s equation is generally preferred since it includes the yield stress $\sigma_0$:

$$\bar{\sigma} = \sigma_0 + K \bar{\epsilon}_p^n$$  \(\text{(1.33)}\)

Voce law takes into account the variation of strain hardening exponent stating that the yield stress $\sigma_0$ approaches a saturation value $\sigma_s$. The expression of the Voce law is the following:

$$\bar{\sigma} = \sigma_s - (\sigma_s - \sigma_0) \exp(-\alpha \bar{\epsilon})$$  \(\text{(1.34)}\)

with $\alpha$ a dimensionless material parameter. Finally the Swift law can be used in the case of pre-strained materials as it is expressed as a function of a initial pre-strain $\bar{\epsilon}_0$.

$$\bar{\sigma} = C(\bar{\epsilon}_0 + \bar{\epsilon}_p)^n$$  \(\text{(1.35)}\)

The above expressions can be used in the case of both strain rate and temperature independent materials.

1.2.1.2 Viscoplastic constitutive equations

Other functions were developed to model visco-plastic materials behaviour. A comparative study of the different flow stress models was made by Banerjee (2007). In a general way, Johnson-Cook model is the most widely used. The Johnson-Cook model (Johnson and Cook, 1983) is an empirical relationship for the flow stress $\bar{\sigma}$ which is described by:

$$\bar{\sigma} = (A + B \bar{\epsilon}_p^n) \left(1 + C \ln \left(\frac{\dot{\epsilon}_p}{\dot{\epsilon}_0}\right)\right) \left(1 - T^* m\right) \text{ with } T^* = \frac{T - T_0}{T_m - T_0}$$  \(\text{(1.36)}\)

with $\bar{\epsilon}_p$ the equivalent plastic strain, $\dot{\epsilon}_p$ the plastic strain rate, $\dot{\epsilon}_0$ the reference plastic strain rate, $A$ the yield stress, $B$ the pre-exponential factor, $n$ the work-hardening coefficient, $C$ the strain rate sensitivity factor, $T$ is the temperature of the material, $T_m$ the melting temperature, $T_0$ the reference temperature and $m$ the thermal softening exponent.

1.2.1.3 Residual stresses

Tube drawing induces residual stresses in the tube. These stresses are not visible at first sight but it can be of importance to evaluate their presence for specific applications. There exist different methods to measure residual stresses in the tube: destructive and non-destructive ones. Non-destructive methods are X-ray diffraction or neutron diffraction. In destructive methods stresses are released by doing a cut and the associated deformation is measured. Figure 1.8 shows an example of drawn tube that was cut to release the residual stresses. Kuboki et al. (2008) computed the released stresses from the deformed tube geometry by measuring the circumferential expansion $S$ and the radial expansion.
\( \delta \) that are shown in figure 1.8. Figure 1.9 illustrates the process leading to the presence of residual stresses and the deformation which is induced by their release. Figure 1.9(a) shows that inner and outer surfaces are successively loaded and unloaded up to different stress levels. The outer surface is unloaded and kept in an axial tensile state while the inner surface is unloaded and remained in an axial compressive state (fig.1.9(b)). The final state shown in figure 1.9(a) is taken as the initial state shown in figure 1.9(c) and the stresses are released. Outer surface stresses release leads to a negative axial strain while inner surface stresses release leads to a positive axial strain. As a consequence, tube outer surface shortens and the inner surface extends conducting to the deformed shape observed in figure 1.8.

Drawing methods (fixed plug drawing, mandrel drawing, tube sinking) lead to different levels of residual stresses (Yoshida and Furuya, 2004). Kuboki et al. (2008) showed that floating plug drawing compared to tube sinking could lower residual stresses. Photographies of the drawn tube can be seen in figure 1.8. In this figure, the tubes were cut to release the stresses, it is clear that the tube drawn with a floating plug deforms less which is the proof that the amount of residual stresses is lower in this case. Karnezis and Farrugia (1998) showed that turning a two-passes mandrel drawing process into a single-pass one to reach the same final tube dimensions could lower the residual stresses in the tube.

The residual stresses that can be present in tubes after drawing is a complex phenomenon that was not analysed in this study.

### 1.2.2 Friction

Friction depends on several parameters such as the sliding speed, the contact normal pressure, the material roughness and the properties of contacting materials, the nature and the amount of lubricant and the temperature. These parameters are delicate to characterize in the forming process and thus they are difficult to reproduce in experimental test. Some authors studied the influence of the process and the materials parameters on the friction coefficient. Their conclusions are based on a flat-die test. This test consists in compressing a pre-lubricated sheet of material between two flat dies and drawing the sheet horizontally.
The normal load and the drawing speed are measured and controlled throughout the test. The conclusions of various studies are the following:

- the friction coefficient decreases with increasing sliding velocity (Nakamura et al., 1988; Kosanov et al., 2006; Szakaly and Lenard, 2010);

- the friction coefficient decreases with increasing normal contact pressure (Nakamura et al., 1988; Emmens, 1997; Kosanov et al., 2006; Szakaly and Lenard, 2010). Ma et al. (2010) developed a pressure dependent friction model based on the plastic deformation of surface asperities. Any surface which looks flat in appearance presents irregularities at the micro or nano scale: protrusions and depressions. When two irregular surfaces contact, and when the normal contact pressure increases, irregularities plastically deform and increase the friction surface which has a consequence on the apparent friction coefficient. For more information, the reader might refer to Szakaly and Lenard (2010) where the mechanism of lubrication is detailed;

- the friction coefficient increases with increasing material roughness (Emmens, 1997; Kosanov et al., 2006; Szakaly and Lenard, 2010);

- the friction coefficient decreases with the thickness of the lubricant film, i.e. the amount of lubricant (Nakamura et al., 1988; Emmens, 1997);

- the friction coefficient varies with the nature of the materials, harder tool materials induce lower friction coefficients (Szakaly and Lenard, 2010);

- the friction coefficient depends on the nature of the lubricant (Majzoobi et al., 2008) and its viscosity (Kosanov et al., 2006);
the friction coefficient depends on the temperature and the evolution of the friction coefficient with temperature can be described by a power law (Haddi et al., 2011):

\[
\frac{\mu}{\mu_0} = \alpha \left( \frac{T}{T_0} \right)^m
\]  

(1.37)

where \( \mu \) is the friction coefficient at the temperature \( T \), \( \mu_0 \) is the friction coefficient at the reference temperature \( T_0 \) and \( m \) and \( \alpha \) are parameters.

The different studies that were listed above reveal the complexity of the frictional behaviour. The purpose of this study is not tribology but the concern is to identify a single friction coefficient value in order to use it as an input data into the tube drawing model. Thus, it is fundamental to identify a friction coefficient corresponding to the exact friction condition during tube drawing. Next section presents different experimental methods for friction characterisation.

1.2.2.1 Tests for friction coefficient characterisation

The experimental characterisation of the friction coefficient is complex. Some experimental tests were developed in order to evaluate the friction coefficient but their validity is uncertain as the tests are more or less representative of the experimental forming process. Lazzarotto et al. (1997) developed an experimental device to identify the friction coefficient in cold wire drawing. The as-developed upsetting sliding test consists in two parts extracted from real workpieces (a wire and an indenter) that are sliding relative to each other. The wire is fixed in a specimen stand linked to a tensile test machine. The indenter penetrates into the wire with a normal force \( F_n \) and slides on the wire with a sliding velocity \( v \) equal to the drawing process one. During the test, the wire is plastically deformed by the indenter and the normal and tangential forces \( F_n \) and \( F_t \) are recorded. The friction coefficient is expressed as a function of the radial spring-back of the wire behind the indenter \( \delta \), the penetration depth \( p \), the contact length \( q \) and the measured normal and tangential forces. The expression is the following:

\[
\mu = \frac{\delta - p + qA}{q - (\delta - p)A} \quad \text{with} \quad A = \frac{F_t}{F_n}
\]  

(1.38)

Then Lazzarotto et al. (1997) introduced the as-determined friction coefficient in the Finite Element Modelling (FEM) of the wire drawing and found a relative error of 1% between the experimental and numerical drawing forces.

Vollertsen and Plancak (2002) present a push through test which is widely used for friction coefficient identification in the tube hydroforming process. A picture describing the test principle is presented in figure 1.11. In this method, a tube is expanded by an internal pressure against the tool. The tube is then pushed through the tool at a constant speed. As the tube slides inside the tool, a friction force is generated. It can be measured as a difference between the punch forces \( F_1 \) and \( F_2 \) or as a resulting force on the tool \( F_R \). The friction coefficient is then obtained from the ratio of the measured friction force on the nominal contact force which is the contact area times the internal pressure.

More recently, Vollertsen and Plancak (2002) developed a new test for friction coefficient identification considering plastic deformation. The test is called tube upsetting test and is
Figure 1.10: Design of the experimental sliding test and mechanical analysis (Lazzarotto et al., 1997)

Figure 1.11: Example of the push through test for friction coefficient identification (Vollertsen and Plancak, 2002)
Figure 1.12: Example of the tube upsetting test for friction coefficient identification (Vollertsen and Plancak, 2002)

presented in figure 1.12. It consists in upsetting a tube in a closed die while applying an internal pressure causing plastic deformation of the tube wall. The wall thickness then increases non-homogeneously due to the friction forces. The friction coefficient is then determined from the geometry of the tube wall with the help of analytical solutions or FEM.

The main purpose of the friction coefficient identification is to insert the identified values into a FEM. Next section introduces the main friction models that are currently used in contact modelling.

1.2.2.2 Friction model

There are two principal friction laws that are widely used to model the sliding behaviour between two contacting materials: the Coulomb and the Tresca models. To explain both models, it is convenient to define a contact between two rigid bodies A and B, sliding on each other at a velocity \( \vec{v}_s \). Two stresses components act at the interface: a normal contact stress \( \sigma_n \) and a shear contact stress \( \sigma_t \) which is tangent to the surface (fig.1.13).

The general expression of the Coulomb friction model is:

\[
\sigma_t \leq \mu \sigma_n
\]  

(1.39)

with \( \mu \) the friction coefficient. The model defines a condition for sticking when \( \sigma_t < \mu \sigma_n \) and a condition for sliding when \( \sigma_t = \mu \sigma_n \). The general expression for the Tresca friction model is:

\[
\sigma_t \leq g
\]  

(1.40)

with \( g \) a sliding threshold. \( g \) is a constant and expresses as \( g = m \sigma_0 \sqrt{3} \) with \( m \) the Tresca friction coefficient. This model states that when \( \sigma_t < g \) then the contact sticks and when \( \sigma_t = g \) sliding occurs.
1.2.2.3 Examples of friction coefficient

This section lists different examples of friction coefficient of a Coulomb model identified and used by different authors:

<table>
<thead>
<tr>
<th>Reference</th>
<th>Process studied</th>
<th>Identification method</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Majzoobi et al. (2008)</td>
<td>Wire drawing</td>
<td>FEM and analytical methods</td>
<td>0.035 to 0.15</td>
</tr>
<tr>
<td>Szakaly and Lenard (2010)</td>
<td>-</td>
<td>Experimental flat die tests</td>
<td>0.1 to 0.2</td>
</tr>
<tr>
<td>Karnezis and Farrugia (1998)</td>
<td>Mandrel tube drawing</td>
<td>Not mentioned</td>
<td>0.06</td>
</tr>
<tr>
<td>Yoshida and Furuya (2004)</td>
<td>Floating plug drawing</td>
<td>Not mentioned</td>
<td>0.1</td>
</tr>
<tr>
<td>Beland et al. (2011)</td>
<td>Fixed plug drawing</td>
<td>Not mentioned</td>
<td>0.035</td>
</tr>
<tr>
<td>Kuboki et al. (2008)</td>
<td>Floating plug drawing</td>
<td>Not mentioned</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 1.1: Examples of used friction coefficient in different studies

Majzoobi et al. (2008) presented a range of friction coefficients depending on the nature of the lubricant. Szakaly and Lenard (2010) carried out a study to evaluate the effect of the normal contact pressure, the sliding speed, the nature of the contacting materials and the material roughness on the friction coefficient. In a general way, the range of friction coefficient used by different authors to study wire or tube drawing goes from 0.035 to 0.2 depending on the contacting materials, sliding speed, contact pressure and lubricant. Thus any study involving friction should be fed with a friction coefficient specifically identified to ensure its validity.

1.2.3 Heat generation and transfer

Plastic deformation of metal induces heat generation. In the particular case of metal forming processes, plastic deformation is imposed by tools, and the relative friction between the part to be formed and the tools also induces heat generation. The generated heat then transfers to the part, the tools and to the environment. This section deals with the thermal aspects of metal forming. First, general basis of any thermal study is introduced. Then, in a further paragraph, the focus is put on the heat generated by plastic deformation. Next heat generated by friction is addressed and finally, the different heat exchange mechanisms are detailed.
1.2.3.1 Introduction

The basis of any thermal problem is the heat equation:

\[ \rho C_p \frac{\partial T}{\partial t} = \dot{q} + \text{div}(\overrightarrow{q}_{\text{cond}}) \]  \hspace{1cm} (1.41)

where \( \rho \) is the mass density, \( C_p \) the specific heat capacity, \( \dot{q} \) a volumetric heat source and \( \overrightarrow{q}_{\text{cond}} \) the conduction heat flux vector. The right term of the above expression is the combination of two components:

- The first component, \( \dot{q} \), corresponds to a volumetric heat source defined as the thermal power per unit volume. In metal forming process, \( \dot{q} \) have two sources, plastic deformation and friction.

- The second component, \( \overrightarrow{q}_{\text{cond}} \) represents the heat conducted from other domains that are in contact with the domain of interest. The heat flux for conduction within a body is defined according to the Fourier law which states that the local heat flux vector \( \overrightarrow{q}_{\text{cond}} \) \((W m^{-2})\) for an isotropic material is equal to the product of thermal conductivity \( k \) \((W m^{-1}K^{-1})\) and the negative local temperature gradient \((K m^{-1})\):

\[ \overrightarrow{q}_{\text{cond}} = -k \overrightarrow{\text{grad}}T \]  \hspace{1cm} (1.42)

1.2.3.2 Heat generated by plastic deformation

Mechanical energy used in cold working operations is converted both in heat and stored energy. The stored energy also known as the stored energy of cold work is in fact due to the creation or the rearrangement of crystal defects and the formation of dislocation structures.

The fraction of energy converted into heat is identified by the Taylor-Quinney coefficient \( \beta \) (Taylor and Quinney, 1933). \( \beta \) is equal to the ratio of the thermoplastic heating \( \dot{Q}^p = \beta \dot{W}^p \) on the plastic work rate \( \dot{W}^p = \text{trace}(\sigma \dot{\epsilon}^p) \). Thus \( \dot{q} \) in equation 1.41 can be replaced by:

\[ \dot{q} = \beta \text{trace}(\sigma \dot{\epsilon}^p) \]  \hspace{1cm} (1.43)

It is commonly admitted that \( \beta \) is a constant parameter bounded to the range [0.8,1] for most metals (Ravichandran et al., 2001). Mason et al. (1994) and Rosakis et al. (2000) investigated the assumption that \( \beta \) was a constant parameter and they found that the measurement of \( \beta \) varied considerably as a function of the materials, the strain and the strain rate. The risk of assuming a constant \( \beta \) is to introduce inaccuracies in the thermomechanical process analysis. Macdougall (2000) lists the \( \beta \) values measured by different authors. Some of the referenced values are presented in table 1.2. Most of the measurements of heat generation during a mechanical test are made by means of infrared radiometry. Macdougall (2000) also reported that thermocouples and calorimetry could be used but these devices were not fast enough and resulted in averaged \( \beta \) values such as the values detailed for Kapoor and Nemat-Nasser (1998).
More recently, Palengat (2009) computed an equivalent Taylor-Quinney coefficient by means of infrared measurements during tube tensile tests. Finally, Rusinek and Klepaczko (2009) estimated the fraction of plastic work converted into heat for TRansformation Induced Plasticity (TRIP) steels. This study is particular because added to the heat generated by plastic deformation, there are also heat variations due to the phase transformation induced by plastic deformation.

### 1.2.3.3 Heat generated by friction

Two surfaces sliding on each other generate heat by friction. In most forming processes the heat which is generated is just a consequence of the process and is not specifically wanted. On the other hand, some processes use the heat generated by friction to assemble materials. It is the case for friction welding or friction stir welding were friction takes place at high speed. The power of friction $P_f$ is defined as the product of the interfacial shear stress $\tau$ and the sliding speed $||\vec{v}_s||$:

$$P_f = \tau||\vec{v}_s||$$  \hspace{1cm} (1.44)

Only a part of this power is converted into heat and transmitted to the contacting materials. Heat is then distributed between the contacting surfaces according to a heat sharing coefficient $f$. The other part of the power is involved into wear phenomenon.

Heat fluxes relative to a master $q_m$ and a slave $q_s$ surfaces write:

$$q_m = f\eta P_f \quad \text{and} \quad q_s = (1 - f)\eta P_f$$  \hspace{1cm} (1.45)

with $\eta$ the fraction of power of friction converted into heat.

The heat sharing coefficient is traditionally defined as a function of the material thermal effusivity $\xi$ according to Vernotte (1956). The material effusivity expresses as function of the density $\rho$, the specific heat capacity $C_p$ and the thermal conductivity $k$:

$$\xi = \sqrt{\rho C_p k}$$  \hspace{1cm} (1.46)

Vernotte (1956) defines the heat sharing coefficient between two contacting materials as follow:

$$f = \frac{\epsilon_m}{\epsilon_m + \epsilon_s}$$  \hspace{1cm} (1.47)

$\epsilon_m$ and $\epsilon_s$ correspond to the effusivies of the materials corresponding to the master and slave surface respectively.
1.2.3.4 Surface thermal exchanges

Surface thermal exchanges can be divided into three distinct mechanisms: conduction, convection and radiation.

Concerning the interfacial heat conduction, a temperature drop is often observed at the interface between the two contacting surfaces. This phenomenon results from a thermal contact resistance existing between the surfaces in contact. Then, the thermal heat flux \( \vec{q}_{th} \) between two contacting surfaces writes:

\[
\vec{q}_{th} = k(T_m - T_s) \hat{n}
\]  
(1.48)

where \( k \) is the contact thermal conductance, \( T_m \) and \( T_s \) the temperature of the master and slave surfaces respectively and \( \hat{n} \) the normal to the body surface.

Convection is the transfer of heat by the movement of fluids: the transfer takes place between a body and its environment. The heat flux by convection (\( \vec{q}_{conv} \)) is described according to the Newton cooling law which states that the heat loss of a body is proportional to the difference in temperatures between the body and its surroundings:

\[
\vec{q}_{conv} = h(T - T_\infty) \hat{n}
\]  
(1.49)

with \( h \) the heat transfer coefficient \((Wm^{-2}K^{-1})\), \( T \) the temperature of the body and \( T_\infty \) the temperature of the environment, far from the body surface.

Radiation is the emission or absorption of electromagnetic radiation. The heat flux by radiation (\( \vec{q}_{rad} \)) is described by the Stefan-Boltzmann law:

\[
\vec{q}_{rad} = \epsilon \sigma (T^4 - T_\infty^4) \hat{n}
\]  
(1.50)

with \( \epsilon \) the material emissivity, \( \sigma \) the Stefan-Boltzmann constant equal to \( 5.67 \times 10^{-8} Wm^{-2}K^{-4} \), \( T \) the temperature of the body and \( T_\infty \) the temperature of the environment.

1.2.3.5 Intermediate conclusion concerning thermal aspects

As detailed above, conducting a thermo-mechanical analysis of a metal forming process leads to the consideration of different thermal aspects. First, there are heat sources due to plastic deformation and friction. Heat generated by plastic deformation is characterized by a material property i.e. a Taylor-Quinney coefficient \( \beta \). Heat generated by friction is characterized by a coefficient \( \eta \) which links the amount of friction work converted into heat and by a heat sharing coefficient \( f \) which defines the amount of heat transferring to the different bodies. Second, there are surface heat exchanges in the form of conduction, convection and radiation. All the heat exchange mechanisms are defined by the material thermal properties. Conduction within the material and at an interface depends on the material thermal conductivity and on the thermal contact conductance respectively. Convection is function of a heat transfer coefficient which depends on the surrounding environment. Finally, the radiation is function of the material emissivity. Thus, thermal and thermo-mechanical material properties and thermal contact properties have to be identified to carry on a thermo-mechanical analysis.
1.3 Analysis of tube drawing

At Minitubes, the drawing passes are currently defined according to an empirical know-how. In a perspective of optimizing the production time, the concern of formalizing the process has grown. There are three techniques to conduct a process optimization study. First, tests can be conducted on small scale laboratory drawing equipments and the results can be transcribed at the industrial scale. Second, tests can be performed directly on the industrial drawing bench. Compared to the first method, it requires the interruption of the production and is more costly. Finally, the process can be modelled either by analytical methods of by FEM which is the main concern of this study.

As they are less time and money consuming, the last category will be presented in this section.

1.3.1 Analytical methods

Analytical methods are limited to the approximate expression of the drawing stress which is the ratio of the drawing force on the final tube section. Deformation during tube drawing can be decomposed into three different components: the homogeneous deformation which depends only on the reduction ratio, an inhomogenous deformation also called redundant deformation linked to the geometrical parameters and finally the friction. Thus the work necessary for tube drawing can be written according to the work balance as:

\[ W = W_h + W_r + W_f \]  

with \( W_h \) the work of homogeneous deformation, \( W_r \) the work of redundant deformation and \( W_f \) the work of friction. Three analytical methods with respective advantages and drawbacks are detailed and compared in this part: the homogeneous deformation method, the slab method and the upper bound method. These methods are presented in this order as they were developed with increasing complexity. Indeed, the homogeneous deformation method considers \( W_h \) only, the slab method includes \( W_f \) and the upper bound method adds \( W_r \).

The presentation of the methods is limited to a brief introduction. The development of the methods and their application to tube drawing and in the specific case of this study will be detailed in chapter 4.

1.3.1.1 Homogeneous deformation method

This method relies on the hypothesis that all the work of external forces is converted into plastic deformation. An initial parallelepiped element transforms into a deformed parallelepiped element, no matter the intermediate deformations as shown in figure 1.14. In the first approximation the material is supposed perfectly plastic. The expression of the drawing stress is a function of the initial and final tube dimensions:

\[ \sigma_d = \frac{F}{A_f} = \sigma_0 \ln \frac{A_i}{A_f} \]  

(1.52)
with $\sigma_d$ the drawing stress, $F$ the drawing force, $\sigma_0$ the material yield stress and $A_i$ and $A_f$ the initial and final tube sections respectively.

It can be seen that the drawing stress is expressed as a function of initial and final tube dimensions and as a consequence, it is independent of the die angle. It is a purely geometric method. The method can be more realistic considering material hardening by replacing the constant yield stress by a flow stress model. Finally, the homogenous deformation method is the simplest but it considers only the homogeneous deformation and neglects the friction and the redundant shear deformation. It idealises the process.

1.3.1.2 Slab method

The first development of the slab method was made by Siebel and Von Karman in 1924 and 1925 for the rolling process. Then Sachs (1927) was the first to investigate the slab method for the drawing process. The slab method is based on three principal assumptions:

- the principal stresses do not vary on the planes perpendicular to the direction of the applied load,
- frictional effects do not cause internal distortion of the material,
- plane sections remain plane and the deformation is homogeneous.

In this method, a differential slab is considered within the deformed region. Figure 1.15 presents the different stresses acting on a slab element during mandrel drawing. The equilibrium of the stresses acting on the element is written considering both friction and homogeneous deformation. The drawing stress results from the integration of the as obtained expressions along the tube surface. The equilibrium equation of the slab in the $z$ direction writes:

$$
\sigma_1 = (\sigma_1 + d\sigma_1)(t - dt) - \sigma_2 \left( \frac{dz}{\cos \alpha} \right) - \mu_1 \sigma_2 dz - \mu_2 \sigma_2 dz
$$

(1.53)

$\mu_1$ and $\mu_2$ are the die/tube and mandrel/tube friction coefficient respectively.

Integrating this expression considering a Tresca yield criterion gives the expression of the drawing force:

$$
\sigma_1 = \frac{1 + B}{B} \left( 1 - \left( \frac{r}{r_0} \right)^B \right) \sigma^* \quad \text{with} \quad B = \frac{\mu_1 + \mu_2}{\tan \alpha}
$$

(1.54)
Figure 1.15: Stresses acting on an elemental slab during mandrel drawing (Kartik, 1995)

Figure 1.16: Evolution of the predicted drawing force as function of the die angle: comparison of different analytical methods: (a) homogeneous deformation, (b) slab method, (c) upper bound method (Luis et al., 2005)

\( \sigma^* \) is the uniaxial yield stress. More generally, the slab method can be seen as an homogeneous method completed with friction.

1.3.1.3 Upper bound method

Luis et al. (2005) showed that both the homogeneous deformation and the slab method were unable to capture the effect of the die angle on the wire drawing force and the existence of an optimum die angle. Figure 1.16 shows the evolution of the predicted drawing force as a function of the drawing angle for the different methods they compared. The plot corresponding homogeneous deformation method shows no variation of the drawing force with the die angle and the slab method plot shows a decreased drawing force with increasing die angle, but no optimum appears. Figure 1.17 presents an example of chevron like fracture that can occur in wire during drawing or extrusion. Such a fracture is a direct observation of the axial stress heterogeneity in the wire and cannot be explained by the homogeneous and slab methods since these methods neglect shear. On the contrary, the upper bound method considers the shear introduced by the changes of direction of the material flow both at the die entrance and exit.

To summarize, the upper bound method covers the three aspects, homogeneous de-
formation, friction and redundant deformation and is more complete regarding all the phenomena taking place during the process. In this method, the tube is decomposed into three different areas and two discontinuities as illustrated in figure 1.18(a and b):

- the entry zone, where the material is rigid and has a speed $v_0$;
- the ($AA'$) discontinuity corresponding to shear deformation;
- the working zone, where the material deforms. In this part, friction and homogeneous deformation are considered;
- the ($BB'$) shear discontinuity;
- the exit zone, where the material is rigid and has a speed $v_f = v_0 \frac{S_0}{S_f}$ with $S_0$ and $S_f$ are the initial and final tube sections respectively.

1.3.2 Finite Element Modelling

Finite element method is the most accurate technique to study tube drawing and any metal forming process in general. Indeed it considers the mechanical, thermal and contact aspects of the process as it includes a large number of parameters and enables to model complex geometries.

The analysis of the tube and wire drawing process has been investigated by several authors. A review of some models defined by different authors and their applications is detailed
Dixit and Dixit (1995) used an Eulerian formulation to model the plastic flow during wire drawing. In their model, the material is considered rigid-plastic, strain hardening according to a Ludwik constitutive equation and yielding according to von Mises. They neglected thermal effects and viscoplasticity. Friction was characterised by a constant friction coefficient. Through their analysis, they evaluated the effect of the process parameters (die angle, friction coefficient) on the strain rate, equivalent strain, contact pressure and drawing stress. Their main conclusions were that a die angle increase caused the contact pressure to increase and that the optimum die angle should be selected as a function of the friction conditions.

Karnezis and Farrugia (1998), compared to Dixit and Dixit (1995) added elasto-plasticity and thermal effects. The material viscoplasticity was still neglected. They modelled the tube drawing process with Abaqus/Standard considering frictional heating and heat generated by plastic deformation but considered the material mechanical behaviour to be temperature independent. The die and the mandrel were considered as deformable solids. They used FEM to validate the transformation of a two-pass drawing process into a single-pass without tube damage and evaluated damage by means of Cockcroft and Latham (1968) failure criterion. Surface temperature, drawing load and mandrel reaction force were experimentally measured to identify the friction coefficient and to validate the model. Sawamiphakdi et al. (1998) also worked with Abaqus even though they did not mention if it was an implicit or explicit solver. They considered the material being elasto-plastic, the die and the mandrel were modelled as rigid body and they considered a Tresca friction model. They did not give much details about their model. They designed a program to define a drawing pass and the corresponding process parameters to predict the final product dimensions and properties. They also estimated the drawing force. The originality of their work is that they analysed different die geometries: ellipse, square, rectangle and hexagon.

More recently, Vega et al. (2009) also conducted experimental and FEM analysis to find the optimum conditions for minimizing the drawing force. In brief, their material was modelled as viscoplastic, a Coulomb friction model was used and the die was supposed rigid. Palengat et al. (2013) with a similar model additionally included the heat generated by friction and plastic deformation.

From the above described models, it can be observed that there is a large variety of FEM studies devoted to wire or tube drawing. Through all of them, it can be deduced that whatever are the simplifications made by the authors to model the material mechanical behaviour and contact properties, the FEM of tube drawing leads to satisfactory predictive results. So far, no analysis has been made to evaluate the influence of the choice of the different hypotheses concerning the material plasticity, the friction or the thermal effects. The above mentioned studies focus on the drawing stress and the equivalent strain field while in more recent studies the focus is put on more local stress analysis.

As an example, Kuboki et al. (2008) and Yoshida and Furuya (2004) analysed the intensity of residual stresses within the tube wall thickness as a function of the drawing technique used. They focused on the local evolution of the stress field during drawing. Kuboki et al. (2008) model was not very different from the above presented models: the material was
considered elastoplastic, a von Mises yield criterion and a Coulomb friction model were used. Nevertheless, while both of their studies intended to analyse the residual stresses, they did not assess the hardening being isotropic or kinematic which can influence the state of stress after drawing. Panteghini and Genna (2010) demonstrated that the use of an isotropic hardening law led to an over-estimation of the residual stresses values while a kinematic-hardening was more accurate. However, Kuboki et al. (2008) validated the predicted residual stresses by means of experiments: they revealed the amount of residual stresses with a destructive method as presented in figure 1.8 and found good correlation with FEM. Similarly, Lee et al. (2012) analysed the effect of process parameters on the axial residual stresses in a wire but in their model they considered the heat generated by friction and plastic deformation. They showed that both the heat generated by plastic deformation and friction influenced the residual stress and that it was more accurate to consider the thermal phenomena.

A recent series of studies focuses on the drawing of aluminium tubes and in particular on the process optimization. In a first attempt, Beland et al. (2011) optimized the die geometry in order to increase the maximum area reduction achievable with a single pass. The die was designed to combine a step of hollow sinking followed by a floating plug drawing step in a single pass. The die geometry was defined according to the minimization of the axial drawing stress computed by means of FEM. Then, Bhamata et al. (2011, 2012) developed a new drawing technique to produce tube of variable thickness. This technique was further analysed by Bui et al. (2011b) who focused on the failure prediction of such tubes during drawing. Their failure criterion was a maximum drawing stress that was computed by means of FEM.

Finally, an original study made by Shinohara and Yoshida (2005) that is worth being mentioned is the analysis of surface defects evolution during wire drawing. They analysed by means of a 3D FEM the effect of successive drawing passes on the geometrical evolution of defects up to their removal from the wire surface. Their study can be a basis to analyse this kind of problematic which is also met in tube drawing.

The review of different studies concerning wire and tube drawing analysis by means of FEM reveals the wide variety of FEM that can be built to model the process. Whatever are the different hypotheses made to model the material or contact behaviour, the different analysis lead to satisfactory results. Moreover, it would be interesting to evaluate the influence of the different model hypotheses on result accuracy. Finally, this overview demonstrated the ability of the FEM tool to capture complex phenomena as FEM enables to compute stress and strain fields.

### 1.3.3 Comparison of the different methods

It is clear that analytical methods give only approximations of the drawing force and they cannot compete with FEM. Indeed the latter enables to consider more complex geometries and more realistic material and contact behaviour.

Some author’s works show good results of analytical methods. Their results are limited to the estimation of the drawing force. For example, Sawamiphakdi et al. (1998) used the slab method to compute the drawing force of tube drawing and wrote that they
found good correlation with FEM. They did not show numerical results and did not proceed to any experimental validation neither. Luis et al. (2005) have conducted a comparative study of the analytical methods applied to the wire drawing process. They have applied the homogeneous deformation method to wire drawing and have shown that the homogeneous deformation energy was independent of die angle (fig.1.16) while it is experimentally observed that the drawing force varies with die angle. The slab method and the homogeneous deformation method were dismissed since the results were inconsistent with experimental observation. These methods were unable to predict correctly the drawing stress. Finally, they have shown that FEM and the upper bound method were the more suitable for predicting the drawing stress because these methods take into account all the energies involved in the process.

1.3.4 Conclusion concerning the process analysis

Finite element modelling can compute the drawing force accurately but a special care has to be taken for the calibration of material and contact parameters. The computation time is longer than for the analytical methods but the results are accurate and enable to reach local data.

In an industrial context, it can be useful to have analytical expressions of the drawing force to be able to make quick estimates. Consequently a small part of this thesis will be dedicated to find a analytical expression of the drawing force and to provide an easy-to-use expression of the drawing force.

1.4 Formability

One of the major concern of metal forming industry is the constant improvement of productivity and product quality. The question is how far the material can be processed without appearance of defects and failure. This limit is called formability limit. It characterises the bound separating the domain where material forming is successful and the domain where damage occurs, in the form of necking, cracks or failure. Formability limit is not an intrinsic material property. It depends on the forming process and its parameters such as formed part geometry, process speed, lubrication and temperature.

Ductile fracture is the result of successive steps that are introduced in figure 1.19. Damage starts with the nucleation of micro-cavities, then voids grow and coalesce to create local cracks. The micro cracks extend and form a macroscopic crack which propagates and causes fracture. In the literature, there are four reported methods to study ductile fracture: continuous damage mechanics models (Lemaitre, 1985; Chaboche, 1988), porous solid mechanics models (Gurson, 1977; Tvergaard and Needleman, 1984), cohesive models (Barenblatt, 1962) and phenomenological models:

- continuous damage mechanics is a local approach based on the accurate description of the stress and strain fields in the vicinity of a crack tip;

- porous solid mechanics models are micro mechanical models that enable to model the void nucleation and growth during the plastic deformation Gurson (1977) model
expresses the flow stress as function of the hydrostatic stress and the void volume fraction. It was further extended by Tvergaard and Needleman (1984) who introduced the void coalescence which accelerates the fracture process. The condition under which the coalescence occurs is defined by a critical volume fraction of porosity. Maoût et al. (2009) used this method to model damage during the hemming process and found the void volume fraction to be a pertinent forming limit criterion;

- cohesive models regard fracture as a phenomenon of surface separation. The cohesive zone model is defined by laws describing the separation of the surfaces as function of the stresses acting nearby the crack tip;

- phenomenological models are based on the computation of functions of stress, strain and work of plastic deformation. They can be based on energetic expression of damage or on instantaneous variable.

The latter do not directly model physical mechanisms of ductile fracture but predict its occurrence. In industrial forming processes, the main issue is to predict failure initiation in order to avoid fracture. The point is not to understand failure mechanism but to have an effective failure indicator. As a consequence, the study of crack propagation and the development of mechanical analysis of ductile cracking is not relevant in this study. Moreover, the implementation in FEM of complex physically based models such as continuous damage mechanics models, porous solid mechanics models or cohesive models is computationally much more expensive than phenomenological models (Zadpoor et al., 2009). Valdellano et al. (2008) and Takuda et al. (1999) found failure criteria to be good competitors with physically based models. They used different failure criteria to predict fracture limits of aluminum 2024-T3 and found the same limits as Lee et al. (1997) and Tang et al. (1999) who used a continuum ductile failure criterion. For these reasons, the emphasis of this study is put on finding criteria for predicting fracture loci and deformation levels at the onset of fracture.

Processes like extrusion are best able to deform metals up to high deformation levels. These kind of processes are characterized by the development of high hydrostatic stresses that contain the growth of cavities and delay fracture. Thus, metals showing poor ductility
properties can be formed under hydrostatic stresses. Bridgman (1952) showed that fracture of mild steel subjected to tensile test could be delayed under the application of hydrostatic stress and that the section reduction of the sample could be increased. Rogers (1968) measured the density evolution of drawn aluminum 6011-T6 bands. He showed that hydrostatic pressure had a limiting effect on cavity growth, indeed, under the effect of the pressure, cavities were contained and the material density was maintained constant.

1.4.1 Formability Limit Diagram

Formability Limit Diagram is a standard tool which is widely used in the sheet metal forming industry. FLD was initially developed by Keeler (1965) and Goodwin (1968). It represents in the 2D space of major principal strain vs minor principal strain the conditions for the onset of sheet necking (fig. 1.20(a)). The diagram represents an irregular parabolic curve delimiting two areas: a material subjected to strains above the curve will fail while one subjected to strains below the curve will be formed safely.

Initially, the FLD was determined analytically or experimentally for linear strain paths. From the 1972 many researchers revealed the strain path dependency of the FLDs, Kobayashi et al. (1972); Chin-Chan (1982); Graf and Hosford (1994); Kuroda and Tvergaard (2000) and Stoughton and Zhu (2004) contributed to this work. The experimental FLDs obtained from a 2008 T4 aluminum with different strain paths are shown in figure 1.20(b). It can be seen from this figure that different strain paths lead to different FLD. The strain path dependency of the formability limit has consequences on the use of FLD in industry. For example, this tool cannot be used to predict the occurrence of failure in non linear processes such as hydroforming. Indeed during hydroforming of a part, the strain path may vary within the part itself. Such a case would require the use of different FLDs depending on the considered zone. Ideally, the number of FLDs should tend towards infinity. As a consequence, it is crucial to find other tools to characterise formability limits. Arrieux
et al. (1982); Stoughton (2000) and Zhao et al. (1996) showed that the variations of strain path induced no changes on the forming limit stresses. As a consequence, FLD could be turned into Forming Limit Stress diagram (FLSD) which represents in the 2D space of major principal stress vs minor principal stress the conditions for the onset of necking. The advantage of FLSD is that they are strain path independent. Experimentally building a FLD is a very time consuming procedure as it requires several tests up to fracture with different strain paths in order to find the Formability Limit Curve. Thus, a significant effort has been made to determine analytically the FLD with the different ductile failure models or by means of FEM. In this study, as said previously, the focus is to find a tool which is able to predict the onset of failure. Phenomenological models seem well suited for this purpose as they are easy to implement into FEM codes. Next part presents in detail the different failure criteria that are used in this study.

1.4.2 Ductile fracture criterion

1.4.2.1 Introduction on failure criteria

Historically, several failure criteria have been established. They describe the failure in terms of mechanical variables such as stress, strain or mechanical work. All the failure criteria presented in this work are based on functions which depend on these variables. If these functions reach a critical value, failure is expected. There are two simple models for failure prediction. The first one is to consider that failure occurs when a function of the current stress tensor reaches a critical value. The second model is to consider a function of current strain tensor. Both of these models are based on instantaneous damage variable \( D \).

Their general expressions are the following:

\[
D = f(\sigma) \quad \text{or} \quad D = f(\varepsilon_p)
\]  \hspace{1cm} (1.55)

where \( \sigma \) and \( \varepsilon_p \) are the current Cauchy stress and the plastic strain tensors respectively. Additionally, there are more complex failure criteria which consider mechanical work. These criteria take into account the stress and strain history. They are based on a damage accumulation variable \( D \) whose general expression is detailed below:

\[
D = \int_0^{\varepsilon_p} f(\bar{\sigma})d\bar{\varepsilon}_p
\]  \hspace{1cm} (1.56)

with \( d\bar{\varepsilon}_p \) the equivalent plastic strain increment and \( \bar{\varepsilon}_p \) the current equivalent plastic strain. Freudenthal (1950) was the first to establish a failure criterion introducing the work of plastic deformation. Cockcroft and Latham (1968) successively suggested that the largest principal stress was more likely to cause fracture and they established a failure criterion based on the highest tensile stress. Brozzo et al. (1972) introduced the level of hydrostatic stress in a new failure criterion in accordance with the experimental study of Bridgman (1952) who showed that imposing hydrostatic pressures to samples loaded in tension could contain the growth of cavities and thus improve formability. Their conclusions were reinforced recently by Wu et al. (2009). McClintock (1968b), Rice and Tracey (1969) and Oyane et al. (1980) established other failure criteria according to the void growth model and the theory of porous media.
1.4.2.2 Expressions of failure criteria

Many researchers have worked on failure criteria and they suggested different phenomenological expressions of the instantaneous damage or damage accumulation variables. Among all the failure criteria available a limited number of criteria is selected for the purpose of this study. Only criteria that can be calibrated on a single experimental test (i.e. uniaxial tensile test) are chosen. Thus, eleven failure criteria are considered. They are listed in table 1.3. In table 1.3, $D_i$ ($i = 1...11$) are the damage variables or damage accumulation variables, $\sigma_j$ ($\sigma_1 > \sigma_2 > \sigma_3$) are the three principal stresses, $\tau_{\text{max}}$ is the maximum shear stress, $\bar{\sigma}$ is the Mises equivalent stress, $\sigma_m$ is the hydrostatic stress, $\bar{\epsilon}$ is the equivalent strain and $\bar{\epsilon}_p$ is the equivalent plastic strain.

<table>
<thead>
<tr>
<th>Type</th>
<th>Abbreviation</th>
<th>Criterion</th>
<th>Damage variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>STRN</td>
<td>Equivalent strain</td>
<td>$D_1 = \bar{\epsilon}$</td>
</tr>
<tr>
<td>1</td>
<td>MSS</td>
<td>Maximum shear stress</td>
<td>$D_2 = \tau_{\text{max}} = \frac{\sigma_1-\sigma_3}{2}$</td>
</tr>
<tr>
<td>1</td>
<td>SHAB</td>
<td>Vujovic and Shabaic (1986)</td>
<td>$D_3 = \frac{3\sigma_m}{\bar{\sigma}}$</td>
</tr>
<tr>
<td>2</td>
<td>FREU</td>
<td>Freudenthal (1950)</td>
<td>$D_4 = \int_0^{\bar{\epsilon}_p} \sigma d\bar{\epsilon}_p$</td>
</tr>
<tr>
<td>2</td>
<td>COCK</td>
<td>Cockcroft and Latham (1968)</td>
<td>$D_5 = \int_0^{\bar{\epsilon}_p} \max(0, \sigma_1) d\bar{\epsilon}_p$</td>
</tr>
<tr>
<td>2</td>
<td>RICE</td>
<td>Rice and Tracey (1969)</td>
<td>$D_6 = \int_0^{\bar{\epsilon}_p} \exp(\frac{3\sigma_m}{2\bar{\sigma}}) d\bar{\epsilon}_p$</td>
</tr>
<tr>
<td>2</td>
<td>BROZ</td>
<td>Brozzo et al. (1972)</td>
<td>$D_7 = \int_0^{\bar{\epsilon}_p} \frac{2\sigma_1}{3(\sigma_1-\sigma_m)} d\bar{\epsilon}_p$</td>
</tr>
<tr>
<td>2</td>
<td>ARGO</td>
<td>Argon et al. (1975)</td>
<td>$D_8 = \int_0^{\bar{\epsilon}_p} (\sigma_m + \bar{\sigma}) d\bar{\epsilon}_p$</td>
</tr>
<tr>
<td>2</td>
<td>OH</td>
<td>Oh et al. (1976)</td>
<td>$D_9 = \int_0^{\bar{\epsilon}_p} \frac{\sigma_1}{\bar{\sigma}} d\bar{\epsilon}_p$</td>
</tr>
<tr>
<td>2</td>
<td>AYAD</td>
<td>Ayada et al. (1984)</td>
<td>$D_{10} = \int_0^{\bar{\epsilon}_p} \frac{\sigma_m}{\bar{\sigma}} d\bar{\epsilon}_p$</td>
</tr>
<tr>
<td>2</td>
<td>TREN</td>
<td>Tresca energy</td>
<td>$D_{11} = \int_0^{\bar{\epsilon}_p} (\frac{\sigma_1-\sigma_3}{2}) d\bar{\epsilon}_p$</td>
</tr>
</tbody>
</table>

Table 1.3: Details of the selected fracture criteria

As mentioned above, there are also failure criteria expressed as a function of different parameters and they require calibration with several experimental tests. Johnson and Cook (1985), Oyane et al. (1980) and Wierzbicki and Xue (2005) expressions are written in table 1.4. In table 1.4, $\eta$ and $\xi$ are the stress triaxiality and the deviatoric state parameters respectively. These criteria will not be used in our study as they require several calibration tests, but they are presented as an opening purpose.
<table>
<thead>
<tr>
<th>Type</th>
<th>Criterion</th>
<th>Damage or damage accumulation variable</th>
</tr>
</thead>
</table>
| 2    | Johnson and Cook (1985) | $D_{12} = \int_0^{\bar{\epsilon}_p} \frac{1}{\epsilon_f} d\epsilon_p$  
with $\epsilon_f = (D_1 + D_2 \exp(D_3 \frac{\bar{\sigma}_m}{\sigma})) (1 + D_4 \ln (\frac{\bar{\epsilon}_p}{\epsilon_0})) (1 - D_5 T^*)$  
and $T^* = \left( \frac{T - T_0}{T_m - T_0} \right)$ |
| 2    | Oyane et al. (1980) | $D_{13} = \int_0^{\bar{\epsilon}_p} (\eta + C) d\epsilon_p$ |
| 2    | Wierzbicki and Xue (2005) | $D_{14} = \int_0^{\bar{\epsilon}_p} \frac{1}{\frac{1}{\mathcal{F}(\eta, \xi)}} d\epsilon_p$  
with $\xi = \frac{27}{J_3^2 J_3}; J_3 = \sigma_1 \sigma_2 \sigma_3$  
and $F(\eta, \xi) = C_1 \exp(-C_1 \eta) - (C_1 \exp(-C_2 \eta) - C_3 \exp(-C_3 \eta))(1 - \xi^{1/n})^n$ |

Table 1.4: Details of fracture criteria requiring more complex calibration

1.4.2.3 Failure criteria calibration

In a general way, the onset of failure is predicted when the ratio of the damage variable (2.4) or the damage accumulation variable (1.56) to a limit value $D_{crit}$ reaches 1:

$$\frac{D}{D_{crit}} \geq 1$$  \hspace{1cm} (1.57)

The critical value $D_{crit}$ for each criterion is calibrated on mechanical tests like tensile tests or upsetting tests for example (Ko et al., 1996; Wierzbicki et al., 2005; Stoughton and Yoon, 2011). Li et al. (2011) mentioned that the quality of failure predictions was influenced by the critical value estimation. Consequently calibration tests have to be carried on accurately.

Upsetting of cylinder is the simplest and the most widespread workability test used to assess failure criteria (Venugopal-Rao et al., 2003). Ko et al. (1996) calibrated Cockcroft-Latham criterion on an uniaxial tensile test and found good predictability results both on axisymmetric extrusion and on upsetting tests.

Karnezis and Farrugia (1998) used a failure criterion based on Cockcroft-Latham damage accumulation variable and calibrated on tensile test to evaluate tube formability. They wanted to evaluate the transformation of a two-pass drawing process into a single-pass. Their study does not present experimental validation of their conclusions.

Some forming processes involve much more complex states of stress and strain than the one reached in calibration tests. Gouveia et al. (1996) stated that there was a risk to calibrate a failure criterion on a unique experimental test. They calibrated Cockcroft-Latham, Oyane, Freudenthal and Brozzo failure criteria on five upsetting tests with different local stress and strain distributions and found only Cockcroft-Latham and Oyane criteria to be accurate. They validated their accuracy with a further study (Gouveia et al., 2000) on more complex cold forging processes: radial extrusion, open die forging and blanking. Consequently, it is important to use a failure criterion whose calibrated critical value remains constant under different conditions of stress and strain.
1.4.2.4 Failure criteria predictability

The different failure criteria that were previously presented were tested and applied to different cases by numerous authors. This paragraph aims at presenting different studies that were conducted. Venugopal-Rao et al. (2003) collected experimental upsetting test data from the literature and compared criteria predictability by means of FEM. They evaluated 10 failure criteria on upsetting tests on cylindrical specimens of various aspect ratios. They classified criteria according to two specifications: the reliability of the given threshold value (statistical mean deviation) and the sensitivity (spatial variation) of the predicted fracture locci. They showed that failure criteria reliability and sensitivity depended both on the material and on the friction conditions. They found Brozzo, Oh and Kobayashi and Cockcroft-Latham failure criteria to be the most accurate and reliable failure criteria.

Wierzbicki et al. (2005) evaluated seven fracture criteria on a set of 15 tests and found the simple Tresca model also named Maximum Shear Stress model to be the most competitive. Their mechanical tests included tensile tests on unnotched and notched round bars, upsetting, shear tests and tensile tests on samples of various geometries. Zadpoor et al. (2009) calibrated the same failure criteria on a wider range of stress triaxiality to be closer to stress states that can be reached in complex metal forming processes. They concluded that the Tresca model was able to predict fracture locus qualitatively but that its quantitative performances were poor (large prediction error). They also concluded that the Tresca criterion prediction was more accurate for high stress triaxialities.

Bui et al. (2011b); Beland et al. (2011); Bihanata et al. (2012) and Yoshida and Furuya (2004) investigated the tube formability limit by considering a maximum drawing stress. Based on the large number of studies dealing with failure prediction by means of failure criteria, it is not obvious to select an appropriate failure criterion. In some cases instantaneous criteria appeared to be the most predictive (Wierzbicki et al., 2005; Yoshida and Furuya, 2004) while in some other cases cumulative damage variables were the most reliable (Oh et al., 1976; Kim et al., 2007). Thus, from this point it is not possible to select a single criterion.

Nevertheless most scientists highlight the fact that failure criteria based on cumulative damage variable are more reliable in predicting fracture. Indeed, metal working process are strain history dependent and the integration of plastic energy deformation along the deformation path is a better approach (Venugopal-Rao et al., 2003).

As a consequence, all the failure criteria that were presented above will be tested in the case of tube drawing. The failure criteria accuracy will be evaluated in term of predicted section and thickness reductions and failure initiation loci.

1.5 Tube bulge test

The principal application of tube bulge tests is linked to hydroforming industry for which the material characterization by uniaxial tensile test is not sufficient. When data obtained from a uniaxial tensile test are used for tube hydroforming analysis, results are often not acceptable (Lianfa and Cheng, 2008; Strano and Altan, 2004). Moreover, strains reached during uniaxial tensile tests are limited by localised necking. During hydroforming, effective
strains that are reached are higher and the material is submitted to a biaxial stress state. Thus tube bulge test was developed to characterise tubular materials in biaxial stress state and to reach higher effective strains. The principle of the tube bulge test is simple: a tube is clamped at two ends and a flow is introduced inside the tube to increase internal pressure. A free zone of the tube is free to expand and bulges under pressure.

Figure 1.21 presents examples of bulge tests that were developed by several researchers. Sokolowski et al. (2000), Velasco and Boudeau (2008), Bortot et al. (2008) and Ouiane et al. (2011) developed apparatus for tube bulging under pressure only. On the contrary, Lianfa and Cheng (2008) and Hwang et al. (2009) designed the tool to conduct bulge test under pressure and with the possibility of adding axial feeding. Hwang et al. (2009) apparatus is similar to Sokolowski et al. (2000) one. Bortot et al. (2008) conceived a tool to be introduced into the hydroforming equipments. They insure the fluid tightness by plastically deforming the tube ends on conical dies (similar for Velasco and Boudeau (2008)). Sokolowski et al. (2000) and Lianfa and Cheng (2008) guarantee fluid tightness by urethane rings that expand under pressure. In all the cases, the tube diameter is very large (between 40 and 50 mm) and the bulged length is short which differs from the apparatus developed in this study.

While the principle of bulge test is simple and relatively similar for all the authors working on it, the strain measurement and stress computation methods are quite different. This part first presents the different strain measurement techniques that are used by different authors. Then the methods for stress computation are detailed.

1.5.1 Strain measurement

Several authors developed different techniques to measure the deformation. Figure 1.22 presents the geometry and the geometrical characteristics of a bulged tube. The configurations can be distinguished: for a short bulged length (fig.1.22.a) the tube profile is ellipse like, for longer bulged length (fig.1.22.b), the tube keeps its cylindrical shape.
As a consequence, the parameters to be measured during a tube bulge test for further analysis differ slightly. Different parameters can be used to describe the bulge test and differ between both configurations. The common parameters between both configurations are: the internal pressure $p$, the axial force $F$, the circumferential radius of curvature $r_\theta$, and the thickness $t$. They are detailed in figure 1.22. In the case (a) the longitudinal radius of curvature $r_\phi$ is measured while it tends toward infinity in the case (b).

The strain tensor in the case of bulge test writes:

\[
\varepsilon = \begin{pmatrix}
\varepsilon_r & 0 & 0 \\
0 & \varepsilon_\theta & 0 \\
0 & 0 & \varepsilon_\phi \\
\end{pmatrix}
\]

or in case (b)

\[
\varepsilon = \begin{pmatrix}
\varepsilon_r & 0 & 0 \\
0 & \varepsilon_\theta & 0 \\
0 & 0 & \varepsilon_z \\
\end{pmatrix}
\]

The different strains can be expressed as a function of the geometrical parameters:

in the case (a),

\[
\varepsilon_r = \ln \left( \frac{t}{t_0} \right), \quad \varepsilon_\theta = \ln \left( \frac{r_\theta}{r_\theta^0} \right) \quad \text{and} \quad \varepsilon_\phi = -\varepsilon_\theta - \varepsilon_r \quad (\text{assuming incompressibility})
\]

in the case (b),

\[
\varepsilon_r = \ln \left( \frac{t}{t_0} \right), \quad \varepsilon_\theta = \ln \left( \frac{r}{r_0} \right) \quad \text{and} \quad \varepsilon_z = \ln \left( \frac{l}{l_0} \right)
\]

Geometrical characteristics necessary for strain computations can be obtained by means of different measurement techniques that can be classified into two categories: "on line" and "off line". On line measurements are made during the test while off line measurements require to remove the tube from the experimental device. Both techniques are detailed further.

In some cases, experimental measurements are not sufficient to obtain all the geometrical characteristics. As they are all necessary to compute the stresses, the missing characteristics have to be found analytically or by means of FEM.

1.5.1.1 On line measurements

A direct method for strain computation would be to obtain all the geometrical characteristics from on line measurements. This method is said to be sophisticated and costly as it requires the use of specific tools like lasers, ultrasonic sensors and non touching sensors (Koç et al., 2001). The bulge tool turns to be more complex and more efficient data acquisition systems are required.

Strano and Altan (2004) work concerns tubes that were formed from bended tubes. During the bulge tests, they measured both the pressure and the bulge height. Then, they computed the radii corresponding to the inner and outer tube profiles by assuming that the tube profile was following an analytical cosine-like function whose variable was the bulge height. The strains were then computed from the as obtained radius profile. Hwang and Lin (2006) built a model to express the bulge height as a function of the internal pressure considering the bulge shape to be an ellipsoidal surface and the tube thickness to follow a quadratic distribution.
Velasco and Boudeau (2008) elaborated a method to compute strains from the measurement of the outer bulge height only. They make the hypothesis that the tube deforms into two arcs of circumference in two perpendicular planes. From the analytical expression of the arcs they express the volume of bulged material as function of different geometrical parameters. The only unknown is the tube thickness (t) which is found by means a Newton-Raphson algorithm. For more details on the method, the reader might refer to Velasco and Boudeau (2008). This method was further used by Ouirane et al. (2011) who investigated the contributions of the different geometrical parameters (thickness and radius) on the global error made on the computed equivalent strain and stress.

Hwang et al. (2009) used a micrometer for the measurement of the bulged tube outer diameter and obtained the principal major and minor strains from the measurement of deformed grids that were electrochemically etched on the tube surface.

1.5.1.2 Off line measurements

When the on line measurement of some geometrical characteristics is impossible the off line technique is required. It consists in bulging a number a tubes up to different pressure levels and removing the tubes from the bulge tool, each pressure level corresponds to a different level of deformation. Examples of different methods are presented below.

Bortot et al. (2008) measured the tube thickness at the top of the dome with a micrometer. The tubes were cut along a plane in the radial direction. They measured the circumferential and longitudinal radii of curvature with a coordinate measurement machine. Sokolowski et al. (2000) and Lianfa and Cheng (2008) also measured the tube thickness and circumferential radius of curvature on cut tubes but obtained the missing longitudinal radius of curvature by different methods: Sokolowski et al. (2000) fitted the longitudinal radius of curvature through iterations with the tube bulge test FEM while Lianfa and Cheng (2008) obtained the longitudinal radius of curvature analytically. They measured the whole bulge
profile by measuring the bulge radii along the tube. From this operation, they obtained a list of coordinate points corresponding to the tube profile and fitted a curve using a spline function to find the equation of the bulged profile. The first and secondary derivations of this function enabled to know the longitudinal radius of curvature. This technique requires a lot of successive tests to explore the range of pressures up to tube fracture. Moreover, each set of tests must be repeated several times to ensure that the measured characteristics are accurate. Finally, off line measurement is both time and material consuming. Koç et al. (2001) compared three methods deriving from the on line and off line techniques and revealed that the on line measurements were more accurate. Finally, depending on the technique used for strain measurement, different methods can be used for stress computation.

1.5.2 Stress computation

In the case of the bulge test, the stress tensor writes:

\[
\sigma = \begin{pmatrix}
\sigma_r & 0 & 0 \\
0 & \sigma_\theta & 0 \\
0 & 0 & \sigma_\phi \\
\end{pmatrix}
\]

The most common method for stress computation is the use of the thin-walled structure approximation (Sokolowski et al., 2000; Koç et al., 2001; Lianfa and Cheng, 2008; Velasco and Boudeau, 2008). The membrane theory considers the thickness to be small enough compared to the external diameter. The stress is then assumed to distribute uniformly in the radial direction and the radial stress \(\sigma_r\) is equal to zero. It enables to express the equilibrium of an element of a bulged tube as follow:

\[
\frac{\sigma_\phi}{r_\phi} + \frac{\sigma_\theta}{r_\theta} = \frac{p}{t}
\]

Strano and Altan (2004) developed an energy method to identify the flow curve parameters. Their method relies on the minimization of a least square function which is based on the difference of the internal work of deformation and the external work. Bortot et al. (2008) identified the flow curve parameters with an iterative method. They first computed the stresses using the membrane hypothesis and obtained a first set of parameters that was inserted into a FEM. Then, they refined the flow curve parameters to obtain the best match between experimental and numerical data. The advantage of this method is that in the end, the membrane hypothesis is not made for stress computation. The drawback is that all the geometrical measurements were made off line which is time consuming. The different iterations between FEM and experimental data may be time consuming too.

1.5.3 Application of the bulge test

Strano and Altan (2004) showed that the flow curves obtained from tensile test and bulge tests differed and as a consequence, the flow stress curve parameters were also different. In their study, they mention that tubes were formed by bending sheets of stainless steel AISI
304. They do not specify the direction of folding regarding the longitudinal or transverse directions, the direction of the tube tensile test is not specified either as the material is supposed isotropic. Moreover, they do not mention if the tube was annealed after bending. Finally the flow stress curve parameters obtained from the tube bulge test gave more accurate results when inserted into the FEM of the tube hydroforming process. Hwang et al. (2009) constructed forming limit diagrams by means of bulge tests conducted with different strain path. They compared the experimental data with the FLD predicted with Hill’s localized and Swift diffused necking criteria and found good agreement.

The tube bulge test is also an experimental tool that can be used to evaluate material anisotropy (Hwang et al., 2009). More recently Boudeau and Malécot (2012) proposed an analytical model to contribute to the definition of standard tube bulge tests dedicated to industry. They introduced the need to take into account tube anisotropy. They compared Hill 1948 and Hill 1993 quadratic yield criteria and found Hill 1993 plastic criterion to fit better the experimental data.

1.6 Conclusions

In the introductory chapter, all the concepts to be used in this study were detailed. The context of the study was first presented with the description of the drawing process and the detail of the different process parameters. Then, in a second part, the basis to understand plasticity, friction and heat generation and transfer were developed. Plasticity was described in terms of constitutive equations including the yield condition, the flow rule and the hardening law. The different models of yield conditions and isotropic or anisotropic flow rules were described. A short review concerning the evolution of the friction coefficient with the contact characteristics enabled to put into relief the difficulties linked to its identification. A set of experimental tests for friction characterisation was presented and the friction models were introduced. Then, referring to the heat generation and transfer, the heat conservation equation was written, which enabled to introduce the different phenomena involved in heat generation and transfer. Heat is generated by plastic deformation and by friction, and transferred to the contacting materials and surrounding environment by conduction and convection. A third part was devoted to the presentation and the description of the methods for tube drawing analysis. Analytical methods were briefly introduced and the insight was put in the Finite Element Method. Both methods were compared with a review of various works and the necessity of using FEM was justified. Finally, in a fourth part, a mechanical test dedicated to tube testing i.e. the tube bulge test was presented.
Chapter 2

Materials experimental characterisation

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2.1 Introduction

The first step of any process modelling project is the characterisation of the material of concern in order to obtain the model input data. As seen in the previous chapter, both mechanical and thermal properties have to be identified for the purpose of this study. This chapter is devoted to the explanation of the procedure that was used for the material characterisation. In a first part, the different testing techniques are introduced. Three different mechanical testing methods used to characterise the material mechanical behaviour are presented i.e. uniaxial tube tensile test, tensile tests on samples cut from tubes and tube bulge test. In the second part, the experimental results are detailed together with the
identification of the material parameters. The first concern is the characterisation of the work hardening behaviour. The second one is the identification of the parameters of the Johnson-Cook visco-plastic hardening constitutive equation. The third one is the evaluation of the material anisotropic plasticity properties. As the properties are strongly linked to the material microstructure, it appears interesting to focus on this latter. Thus, material crystallography is analysed by means of X-ray diffraction. Finally, the measurement of the material thermo-mechanical properties is explained.

2.2 Presentation of the testing methods

Two materials were tested: a Cobalt Chromium alloy, L605 and a stainless steel, 316LVM. The composition of both materials is detailed in table 2.1 and 2.2. In all the testing methods presented below, the tubes were tested in an annealed state. Annealing temperature was 1150°C and 1050°C for the L605 and 316LVM respectively. Tubes were treated during 10 min under H₂ atmosphere.

2.2.1 Uniaxial tensile tests on tubes

2.2.1.1 Control of the test

Tube tensile tests were carried out at room temperature (21°C) on a MTS 810 hydraulic tensile testing machine with a load capacity of 100 kN. The specificity of the tensile test machine was its ability to reach high crosshead speed. The maximum reachable crosshead speed was 2 m s⁻¹ which enabled to reach high strain rates.

Two series of test were performed in order to characterise different material mechanical behaviours. First, tensile tests were conducted at a constant strain rate of 0.03 s⁻¹ in order to characterise the materials work-hardening behaviour. Second, three tensile tests were conducted at three different and constant strain rates (0.03 s⁻¹, 2.9 s⁻¹ and 8.9 s⁻¹) to characterise the visco-plastic behaviour. For low strain rate tensile tests, the crosshead velocity ($v_{cross}$) was real time controlled to impose a constant strain rate, $\dot{\varepsilon}$ as follow:

$$v_{cross} = \dot{\varepsilon}(l_0 + \Delta l) \quad \text{and} \quad \dot{\varepsilon} = \frac{i}{l}$$

with $l_0$ and $l$ the initial and current tube lengths respectively, $\Delta l$ the crosshead displacement and $\dot{\varepsilon} = \frac{dl}{dt} = v_{cross}$.

<table>
<thead>
<tr>
<th>Co</th>
<th>Cr</th>
<th>W</th>
<th>Ni</th>
<th>Fe</th>
<th>Mn</th>
<th>C</th>
<th>Si</th>
<th>P</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance</td>
<td>19-21</td>
<td>14-16</td>
<td>9-11</td>
<td>&lt;3</td>
<td>1-2</td>
<td>0.05-0.15</td>
<td>&lt;0.4</td>
<td>&lt;0.04</td>
<td>&lt;0.03</td>
</tr>
</tbody>
</table>

Table 2.1: Chemical composition of the L605 (ASTM-F90)(mass %)

<table>
<thead>
<tr>
<th>Fe</th>
<th>Cr</th>
<th>Ni</th>
<th>Mo</th>
<th>Mn</th>
<th>Si</th>
<th>Cu</th>
<th>N</th>
<th>C</th>
<th>P</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance</td>
<td>17-19</td>
<td>13-15</td>
<td>2.25-3</td>
<td>&lt;2</td>
<td>&lt;0.75</td>
<td>&lt;0.5</td>
<td>&lt;0.1</td>
<td>&lt;0.03</td>
<td>&lt;0.025</td>
<td>&lt;0.01</td>
</tr>
</tbody>
</table>

Table 2.2: Chemical composition of the 366LVM (ASTM-F138)(mass %)
However, for strain rates higher than $1 \text{s}^{-1}$, real time computation of the crosshead velocity was impossible and the tests were controlled with constant crosshead speed:

$$v_{\text{cross}} = \dot{\epsilon} l_0$$

(2.2)

In this case, once the tests were complete, real strain rates were computed in order to validate that they remained approximately constant during the test.

### 2.2.1.2 Tube positioning

The different dimensions of tested tubes are detailed in table 2.3.

<table>
<thead>
<tr>
<th>Material</th>
<th>ID (mm)</th>
<th>OD (mm)</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L605</td>
<td>9.5</td>
<td>11.5</td>
<td>1</td>
</tr>
<tr>
<td>316LVM</td>
<td>9</td>
<td>10.5</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Table 2.3: Dimensions of tubes tested by tensile tests. ID and OD are the Inner and Outer Diameters respectively.

Tested tubes were 200 mm long with a gauge length of 100 mm. They were taken from the industrial process after the 7th drawing pass (cf. fig. 1.2). Short mandrels of hard steel were inserted in both tube extremities so that the tube did not crush when closing the grips. A schematic view of the tensile test is presented in figure 2.1. A special care was taken for the shape of the mandrel extremities, more particularly for the tensile tests of L605 tubes. With cylindric mandrels having sharp edges as shown in figure 2.1.a, the tendency of the tube was to break near the mandrel close to the grips. To prevent early failure, the mandrels were machined to present a circular profile and smooth edges as illustrated in figure 2.1.b. This way, the material flow was improved near the mandrels and the failure was occurring in the center of the specimen. Concerning the tests on 316LVM tubes, fracture always occurred in the center of the specimen, even with sharp mandrels.

### 2.2.1.3 Strain measurement

Two techniques were used for strain measurement. For low strain rates, length variations were measured both with an extensometer and from the crosshead displacement. Results from the two methods were compared to validate the measure from crosshead displacement. Indeed, for higher strain rates, the use of the extensometer was not compatible and strain was measured from the crosshead displacement only. The axial strain measured from the crosshead displacement writes:

$$\epsilon_z = \ln \left( \frac{l}{l_0} \right) = \ln \left( 1 + \frac{\Delta l}{l_0} \right)$$

(2.3)

with $\epsilon_z$ the axial true strain, $l$ and $l_0$ the current and initial tube lengths respectively and $\Delta l$ the length variation.
Figure 2.1: Configuration of the tube tensile test, (a) sharp edge mandrel, (b) smooth edge mandrel

2.2.2 Tensile tests on samples cut from the tubes

The purpose of these tests is to characterise the anisotropic plastic properties of the materials.

2.2.2.1 Samples preparation

This test required a specific sample preparation as different samples had to be cut from the tubes. The different steps of sample preparation are illustrated in figure 2.2(a). First tubes of outer radius \( R \) were cut into small tubular sections of length \( L \) equal to the tube perimeter \( 2\pi R \) (1). Then the tubular sections were opened and flattened (bending operation) to form square sheets (2). Finally, specimens were cut from the flattened tubes by electrical discharge machining. Samples had a gauge length of 6 mm, a gauge width of 1.30 mm and a thickness of 0.5 mm. Stage (3) in figure 2.2(a) illustrates the arrangement of samples that were cut in the 0° direction. Other samples were cut in different orientations relative to the tube axis as shown in figure 2.2(b): 0° (Drawing Direction, DD), 22.5°, 45°, 67.5° and 90° (Transverse Direction, TD). As the flattening step induces residual stresses in the material, samples were annealed to restore the initial material properties. Annealing conditions were identical to the one conventionally used in the tube drawing process, i.e. 1150°C during 10 minutes for the L605.

2.2.2.2 Control of the test

Tensile tests on oriented samples were carried out at room temperature (21°C) on a Gabo 500N tensile testing machine presenting a maximum load of 1.5 kN. The tests were strain rate controlled. The strain rate was identical to the one imposed during tube tensile test, i.e. 0.03 s\(^{-1}\).
2.2.2.3 Strain measurement

A general view of the test can be seen in figure 2.3.a. Samples were fixed with specific grips that were designed for such small samples (Delobelle, 2012). Figure 2.3.b shows a sample inserted into the grips.

A camera was placed aligned with the sample and pictures were recorded during the test. Displacements were measured from the pictures by means of Digital Image Correlation (DIC). This technique enables to search the position of several points belonging to the sample and to follow them during deformation. The points are identified by a fine black and white painting which is projected onto the tube and forms a random pattern. An example of random pattern is presented in figure 2.3.c. Finally, strain fields are computed from the displacement fields. In this study, 7D DIC software was used (Vacher et al., 1999).

2.2.3 Tube bulge test

In chapter 1, a number of tube bulge test devices that were developed by different authors were presented. These devices were designed for large diameter tubes and only tubes of short length could be bulged. In some cases, the tubes were formed by sheet bending and welding (Yoshida and Kuwabara, 2007). The large tube diameter and small thickness enabled to bulge at low pressure (Lianfa and Cheng, 2008; Hwang et al., 2009). In our study, the tubes were not specifically designed for the test and they were extracted from the industrial process. As a consequence, the tube bulge test device was designed according to the tube dimensions that intended to be tested. The initial requirements of the apparatus were:

- to bulge the tube with pressurized water;
- to bulge the tube up to fracture;
- to use the device for tubes of different dimensions (diameter, thickness, length);
- to combine the tube bulge test with tensile test;
- to measure the pressure and the axial load;
- to see the bulged area in order to record the bulging with cameras.
2.2.3.1 Dimensions of the tubes to be tested

The apparatus was designed for the purpose of this study but also for analysis beyond the scope of this thesis. Thus as mentioned above, the objective was to make it easily usable with different tubes dimensions. Therefore, the first step was to define the tube dimensions that were planned to be tested. From the L605 and 316LVM drawing steps, the dimensions of the tube that were selected are detailed in table 2.4. Tube length is expected to range from 100 to 200 mm.

2.2.3.2 Pressure

Pressure was provided by a water pump. As mentioned previously, the tube bulge test apparatus aims at bulging the tube up to fracture. As a consequence, the maximum reachable pressure must be defined in this way. Thus, the pump capacity was defined according an estimation of the bursting pressure for every tube dimensions. The estimated bursting pressure is computed according to:

\[ P_{\text{burst}} = \frac{tR_M}{r_i} \]  

(2.4)

with \( P_{\text{burst}} \) the bursting pressure, \( t \) the tube thickness, \( R_M \) the ultimate tensile stress and \( r_i \) the tube inner radius. This expression is valid for small displacements and deformations only as it does not consider the tube radius increase and the tube wall thinning. As a consequence, the computed bursting pressure is over-estimated. The ultimate tensile stresses are equal to 1700 MPa and 1000 MPa for the L605 and the 316LVM respectively. The computed bursting pressures are detailed in table 2.4.
According to the computed bursting pressures, a pump with a capacity of 250 MPa was chosen for the purpose of this study.

<table>
<thead>
<tr>
<th>Material ID (m)</th>
<th>OD (mm)</th>
<th>Thickness (mm)</th>
<th>estimated bursting pressure (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L605 7.5</td>
<td>8.8</td>
<td>0.65</td>
<td>295</td>
</tr>
<tr>
<td>6.5</td>
<td>7.5</td>
<td>0.5</td>
<td>262</td>
</tr>
<tr>
<td>5.5</td>
<td>6.3</td>
<td>0.4</td>
<td>247</td>
</tr>
<tr>
<td>4.5</td>
<td>5.16</td>
<td>0.33</td>
<td>249</td>
</tr>
<tr>
<td>316LVM 9</td>
<td>10.5</td>
<td>0.75</td>
<td>167</td>
</tr>
<tr>
<td>7</td>
<td>8.2</td>
<td>0.6</td>
<td>164</td>
</tr>
<tr>
<td>5.8</td>
<td>6.6</td>
<td>0.4</td>
<td>145</td>
</tr>
<tr>
<td>4.5</td>
<td>5.2</td>
<td>0.35</td>
<td>156</td>
</tr>
</tbody>
</table>

Table 2.4: Dimensions of the tubes to be tested by bulge tests. ID and OD are the Inner and Outer Diameter respectively.

2.2.3.3 Tube holding system

The requirements of the tube holding system were:

- to have the possibility of bulging tubes of different dimensions easily;
- to be watertight under pressures up to 250 MPa;
- to have the largest tube outer surface visible.

The tube holding system is described in the following paragraph. A general view and a detailed view of the system are presented in figure 2.4. When reading the above paragraph, the reader might refer to this picture when coming across the numbers describing the different system parts.

The tube holding system is composed of two subsystems: a clamping (1) and a watertight (2) systems. Both systems are successively described below.

The tube clamping is ensured by adapting a classical drill holder (3) to the requirement of the bulge test. The advantage of such a tool is that tool-holder collets for different drill diameters are commercially available. Thus, a series of collets (4) covering the range of tube outer diameter is used to hold the tube (collets diameters range from 5 to 10.5 mm every 0.5 mm). Two tubes clamping systems are used, for the bottom and the top of the tube. The distance between both clamping systems can be freely chosen.

Holding a tube with such a tool requires the insertion of a mandrel inside the tube in order not to deform it when tightening it up. Such function is ensured by the watertight system (2). The latter consists in a stainless steel tool (5) whose geometry fits into the drill holders. It has the double function of locking the tube in the axial direction and ensuring watertightness. Watertightness is guaranteed by a urethane ring (6) placed between two anti extrusion rings (7). The urethane ring expands with increasing pressure and is maintained by means of the anti extrusion rings. A series of watertightness tools was designed with different diameters (9.5, 9, 8.5, 7.5, 7, 6.5, 5.8, 5.5, 4.5 mm) in order to be inserted in the different tubes to be tested. Finally, the top watertightness tool was drilled to introduce a capillary tube (8) necessary for water inlet.
The as described tube holding systems are finally inserted into 20 kN tensile testing machine and the capillary tube is connected to the hydraulic power system. From this point, preliminary tube bulge tests can be performed. The interest of such a configuration is that the tools are designed to be independently inserted in any tensile test machine. This way, the bulge test can be performed with different end conditions, either tubes ends can be fixed or the bulge can be combined to a tensile or compressive test. In this study, the tube ends were constrained in both axial and radial directions and the central region of the tube was free to expand.

### 2.2.3.4 Measurements during the test

During a bulge test, the pressure is measured by means of a pressure transducer with a capacity of 250 MPa. The pressure transducer is inserted at the pump exit, before the capillary tube. Axial force is measured by the tensile testing machine load cell. Finally, two cameras are put in front of the tube and displacements are measured by means of Stereo Digital Image Correlation (SDIC) system. For this purpose, the tube outer surface is preliminary painted with a black and white random pattern. Load, pressure and displacement measurements are synchronised by means of a central computer. A general view of the test is shown in figures 2.5 and 2.6.

Two methods can be used to control a tube bulge test. First, it can be pressure controlled.
Figure 2.5: Tube bulge test device: (a) photography of the device, (b) schematic representation of the tube bulge test in the initial configuration, (c) in the bulged configuration.

Figure 2.6: General view of the tube bulge test

A target pressure is set and the pump injects water to reach this pressure. Second, it can be injected volume controlled and the resulting pressure is measured.

2.2.3.5 User protection

Bulging a tube up to fracture leads to water projections. The pressure can be high but the volume of projected water is small. As a consequence, the water jet quickly loses intensity and the risk of injury is limited. Nevertheless adding a protection for the user is necessary. The first solution that was considered was to surround the tube with
a transparent polycarbonate shell. A photography of the protective shell is shown in figure 2.7. The shell was placed between the tube and the cameras. This configuration induced several problems: refraction phenomena could occur and induce measurement errors, cameras acquisition required lighting that caused reflection and finally, keeping a polycarbonate surface free from marks and scratches is not easy. The second solution was to place a protection pane in front of the user. The latter was chosen.

2.2.3.6 Preliminary tests

Preliminary tests were performed in order to validate the bulge test design and the measurement systems. They were performed on stainless steel tubes whose dimensions were $5.5 \times 6.5 \times 200 \text{mm}^3$. In some cases, water injection caused buckling instead of bulging. Figure 2.8(a) shows photographs of buckled and bulged tubes. The effects of initial conditions such as the alignment of the top and the bottom tools and the initial defects of the tube are not understood and were not the purpose of this study. To anticipate and prevent the appearance of such instabilities during bulge test, an extra tool was designed to reinforce the alignment of the holding tools. It is presented in figure 2.8(b). Moreover, shortening the tube lowers the risk of buckling (Koc and Altan (2002)).
Figure 2.8: (a) Example of buckled and bulged tubes, (b) Tool designed to improve the alignment of the holding tools

2.3 Mechanical characterisation

The different techniques used for material characterisation were presented in the previous section. In the following sections, the results of the experimental mechanical tests are detailed.

A first part deals with the characterisation of the material mechanical behaviour considering isotropic plasticity. The first concern is the characterisation of the work hardening behaviour and the identification of a Ludwik hardening constitutive equation. Then the second concern is the characterisation of the visco-plastic behaviour and the identification of the parameters of the Johnson-Cook constitutive equation. A second part concerns the evaluation of the material anisotropic plasticity and the identification of the Hill’s yield function parameters.

2.3.1 Work hardening characterisation

2.3.1.1 L605

Figure 2.9(a) presents the flow curve obtained for the tensile test performed at a strain rate of 0.03 \( s^{-1} \). From these data, Ludwik’s hardening model parameters were fitted. The procedure used for parameters identification is detailed below. The expression of Ludwik’s hardening constitutive equation is reminded:

\[
\bar{\sigma} = \sigma_0 + K\bar{\epsilon}_p^n
\]  

(2.5)
In order to identify \( \sigma_0 \), \( K \) and \( n \) the \( \sigma_0 \) parameter is isolated and the logarithm of the whole expression is taken:

\[
\ln(\bar{\sigma} - \sigma_0) = n \ln \dot{\epsilon} + \ln K
\]  

(2.6)

\( n \) and \( K \) parameters values are found by linear regression for a given \( \sigma_0 \) value. Thus, an interval of \( \sigma_0 \) values is explored and a single value is assigned to \( \sigma_0 \) so that the correlation coefficient of the linear regression is maximized. Figure 2.9(b) illustrates the procedure and the identification of the optimum \( \sigma_0 \). The fitted parameters are listed in table 2.5.

Figure 2.10(a) presents the flow curves obtained for tensile tests conducted at three different strain rates (0.03 \( s^{-1} \), 3 \( s^{-1} \), 9 \( s^{-1} \)). It can be seen from this figure that the yield stress increases with increasing strain rate. Such an observation is characteristic of a visco-plastic behaviour. According to this strain rate dependent behaviour, the parameters of the Johnson-Cook constitutive equation were fitted. The as mentioned constitutive equation is reminded below:

\[
\bar{\sigma} = (A + B\dot{\epsilon}_p^n)(1 + C \ln \left(\frac{\dot{\epsilon}_p}{\dot{\epsilon}_0}\right))(1 - T^*)
\text{ with } T^* = \frac{T - T_0}{T_m - T_0}
\]  

(2.7)

Starting with the thermal dependency and the thermal softening coefficient identification, Palengat et al. (2013) measured temperature increase during cold tube drawing and showed that temperatures did not exceed 100°C. In this range of temperature, largely lower than the melting temperature, the dependence of the mechanical properties on temperature can be considered as negligible. Considering this slight increase, temperature effects are neglected and the Johnson-Cook constitutive equation simplifies:

\[
\bar{\sigma} = (A + B\dot{\epsilon}_p^n)(1 + C \ln \left(\frac{\dot{\epsilon}_p}{\dot{\epsilon}_0}\right))
\]  

(2.8)

Finally, four coefficients remain to be fitted: \( A \), \( B \), \( n \) and \( C \). The methodology for parameters identification is detailed below.

Parameters \( A \), \( B \) and \( n \) can be identified with a reference test which is the test performed at a strain rate of 0.03 \( s^{-1} \). As a consequence the reference strain rate is: \( \dot{\epsilon}_0 = 0.03 \text{s}^{-1} \). Then, taking \( \dot{\epsilon}_p \) and \( \dot{\epsilon}_0 \) both equals to 0.03 \( s^{-1} \) transforms the Johnson-Cook expression into the Ludwik equation. Thus \( A \), \( B \) and \( n \) are easily identified as \( A = \sigma_0 \), \( B = K \) and
Parameter $C$ identification requires the use of tests conducted at different strain rates. $C$ is fitted by a Least Absolute Errors (LAE) minimisation procedure. $C$ value is chosen so that the sum of the absolute errors between the experimental data and the model is minimized as illustrated in figure 2.10(b). The fitted parameters are listed in table 2.5.

### 2.3.1.2 316LVM

The flow curve obtained for the tensile test performed at a strain rate of $0.03 \text{ s}^{-1}$ for the 316LVM is presented in figure 2.11(a). The parameters identification of the Ludwik constitutive equation was done with the same method used for the L605. Figure 2.11(b) shows the identified $\sigma_0$ by means of the maximisation of the correlation coefficient.

Figure 2.12(a) presents the flow curves obtained for tensile tests conducted at three different strain rates ($0.03 \text{ s}^{-1}, 3 \text{s}^{-1}, 9 \text{ s}^{-1}$). The yield stress increases with increasing strain rate and the 316LVM exhibits a visco-plastic behaviour. Thus, the parameters of the Johnson-Cook constitutive equation were fitted. The methodology is identical to the one previously used for the L605. Figure 2.12(b) shows the identification of the $C$ value by means of the LAE minimisation procedure. The fitted parameters are listed in table 2.5.
Figure 2.12: (a) Axial stress vs strain for tensile tests conducted at different strain rates, (b) minimisation of the Least Absolute Errors to identify $C$

Figure 2.13: Parameters identification for the L605: superimposition of the Johnson-Cook fit with the tensile tests conducted at different strain rates

<table>
<thead>
<tr>
<th>Material</th>
<th>$A = \sigma_0$ (MPa)</th>
<th>$B = K$ (MPa)</th>
<th>$C$</th>
<th>$n$</th>
<th>$\dot{\varepsilon}_0$ (s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L605</td>
<td>490</td>
<td>1922</td>
<td>0.023</td>
<td>0.658</td>
<td>0.03</td>
</tr>
<tr>
<td>316LVM</td>
<td>286</td>
<td>1267</td>
<td>0.021</td>
<td>0.663</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 2.5: Fitted parameters of the hardening constitutive equation

Figures 2.13 and 2.14 show the superimposition of the experimental tensile test data with the Johnson-Cook constitutive equation fed with the identified parameters. It can be observed that the Johnson-Cook constitutive equation represents the material behaviour with good correlation.
2.3.2 Anisotropy characterisation

2.3.2.1 Tube bulge test

Tube bulge tests were performed on L605 tubes only. Tested tubes dimensions were 7.5 mm and 6.5 mm for OD and ID respectively. Tubes were extracted from the process after the 10th drawing pass (cf. fig.1.2). The bulged area was 100 mm long. After a preliminary test where the bursting pressure was estimated to be 204 MPa, a series of test was conducted.

2.3.2.1.1 Strain measurement

Due to the effective tube length of 100mm, the tube bulged with a cylinder shape as shown in figure 1.22. Thus, SDIC measurements were made in the central part of the bulged tube only.

In order to validate the bulge test repeatability, several tests were performed. Figure 2.15 shows the evolution of the measured circumferential logarithmic strain ($\epsilon_\theta$) in green and axial logarithmic strain ($\epsilon_z$) in red as a function of the pressure for three different bulge tests. It enables to justify the test reproducibility as only measured data are plotted.

Figure 2.16 presents the measured circumferential and axial strains, $\epsilon_\theta$ and $\epsilon_z$ respectively, prior to failure at a pressure of 204 MPa. The radial strain $\epsilon_r$ was computed with the incompressibility hypothesis:

$$\epsilon_r = -\epsilon_\theta - \epsilon_z$$

(2.9)

From figure 2.16, it can be seen that the tube expands circumferentially as the circumferential strain $\epsilon_\theta$ is positive. $\epsilon_\theta$ ranges from 0.16 to 0.20. As the tube is constrained in the axial direction, the axial strain $\epsilon_z$ is negative and nearly zero ($\epsilon_z \approx 0$). It ranges from $-0.013$ to $-0.003$. Thus, the tube bulge test induced a plane strain state in the ($\overrightarrow{\epsilon}_\theta$, $\overrightarrow{\epsilon}_r$) plane. The standard deviation on $\epsilon_\theta$ and $\epsilon_z$ are 0.8% and 0.5% respectively.

Figure 2.14: Parameters identification for the 316LVM: superimposition of the Johnson-Cook fit with the tensile tests conducted at different strain rates
Figure 2.15: $\epsilon_\theta$ (in green) and $\epsilon_z$ (in red) as a function of pressure for three different bulge tests.

Figure 2.16: Principal strain fields of the bulged tube prior to failure ($P = 204\text{MPa}$).

2.3.2.1.2 Force measurement

Axial force measurement during tube bulge test was made with a load cell of 20kN. In a general way, considering all the bulge tests that were performed, measured forces range from 0 to -600 N. Figure 2.17 shows the evolution of the measured axial force with the pressure for a single test. All other tests showed the same tendencies. In a first period, the compressive force decreases to the minimum value of -586 N at a pressure of 121 MPa. From this point, the axial force starts to increase with irregular variations. These jumps might be due to sliding episodes of the tube inside the grips. Metallic noises were heard during the test and they may be associated with sliding. Moreover, tube sliding could explain the axial force returning to zero, since it looses contact with the load cell. According to the maximum axial strain $\epsilon_z$ that was measured, the tube axial displacement
$\Delta l$ during sliding can be computed as follow:

$$\epsilon_z = \ln \left( 1 + \frac{\Delta l}{l_0} \right) \text{ and } \Delta l = (e^{\epsilon_z} - 1)l_0 \tag{2.10}$$

The tube sliding was found to be 1.29 mm. Finally it is noticeable to mention that at the end of the test, the grips opening is done with ease and without tools. It seems that the grips open slightly during the test resulting in tube sliding.

2.3.2.1.3 Stress computation

During the bulge test, the tube deforms with a cylindrical shape up to a limit pressure. When the test is pressure controlled, once the maximum pressure is reached, the deformation localizes and the cylinder shape is lost (cf. fig.2.18). The pressure remains approximately constant until the tube fractures. Before the loss of cylindricity, the stresses expressions derive from the equilibrium of the tube elements presented in figure 2.19 considering
membrane hypothesis. Is this study, as the interest is the characterisation of the work-hardening behaviour, data are used up to the moment when the tube loses its cylindricity (phase B in figure 2.18). As a consequence, stresses are computed considering a cylindrical geometry.

In the axial direction, the equilibrium equation writes:

\[ F + P\pi r^2 - \sigma_z \pi (R^2 - r^2) = 0 \]  
\[ (2.11) \]

\( F \) is the axial force, \( P \) is the pressure, \( \sigma_z \) is the axial principal stress and \( r \) and \( R \) are the inner and outer tube radii respectively.

And in the radial direction, it writes:

\[ 2\sigma_\theta (R - r) - 2Pr = 0 \]  
\[ (2.12) \]

The axial and circumferential stresses \( \sigma_z \) and \( \sigma_\theta \) then express:

\[ \sigma_z = \frac{F + \pi Pr^2}{\pi (R^2 - r^2)} \]  
\[ (2.13) \]

\[ \sigma_\theta = \frac{Pr}{R - r} \]  
\[ (2.14) \]

Figure 2.20 presents the superimposition of axial and circumferential stresses versus strains for three different bulge tests and shows good reproducibility. All the tests were conducted in the same conditions in order to evaluate the test reproducibility.

2.3.2.1.4 Evidence of an anisotropic behaviour

In order to compare the data obtained with the tube bulge test and the tube tensile test, it is necessary to express the stresses and strains into an equivalent form. Indeed, tube bulge test is a biaxial test while the tube tensile test is an uniaxial one.

In a first step, if the material is considered as isotropic, the equivalent stress can be expressed following the isotropic von Mises yield function detailed in chapter 1. In the case of tube bulge test, the equivalent stress \( \bar{\sigma}_{VM} \) and plastic strain rate \( \dot{\epsilon}_{p,VM} \) express in the principal cylindrical basis \( (r \ \theta \ z) \) as follow:

\[ \bar{\sigma}_{VM} = \sqrt{\sigma_z^2 + \sigma_\theta^2 - \sigma_z \sigma_\theta} \quad \text{and} \quad \dot{\epsilon}_{p,VM} = \sqrt{\frac{2}{3} (\dot{\epsilon}_r^2 + \dot{\epsilon}_z^2 + \dot{\epsilon}_\theta^2)} \]  
\[ (2.15) \]

Finally, the equivalent von Mises plastic strain computes following:

\[ \dot{\epsilon}_p = \int_0^t \dot{\epsilon}_{p,VM} \, dt \]  
\[ (2.16) \]
Figure 2.20: Superimposition of $\sigma_z$ vs $\epsilon_z$ (in red) and $\sigma_\theta$ vs $\epsilon_\theta$ (in green) for different tube bulge tests

Figure 2.21: Comparison of the von Mises equivalent stress vs strain for tensile and bulge test

The von Mises equivalent stress vs strain curves were plotted both for tensile and bulge tests. Plots can be seen in figure 2.21. From this figure, it can be observed that the data do not superpose and the von Mises yield function is unable to represent the material behaviour. According to this observation, the material seems to exhibit an anisotropic behaviour. As a consequence, other yield function has to be identified in order to model the material behaviour properly.

2.3.2.1.5 Identification of anisotropic yield criteria

The evidence of plastic anisotropy was made by comparison of the experimental data of the tube bulge and tensile test. In a first attempt, the yield stress function was modelled with Hill’s quadratic yield function presented in chapter 1. The equivalent stress according to the Hill’s quadratic yield criterion (Hill, 1948) expresses...
in the cylindrical basis \((r, \theta, z)\):

\[
f(\bar{\sigma}_{\text{Hill}}) = \sqrt{[F(\sigma_{rr} - \sigma_{\theta\theta})^2 + G(\sigma_{\theta\theta} - \sigma_{zz})^2 + H(\sigma_{zz} - \sigma_{rr})^2] + 2L\sigma_{r\theta} + 2M\sigma_{\theta z} + 2N\sigma_{zr}} \tag{2.17}
\]

\(F, G, H, L, M\) and \(N\) are parameters that are obtained by uniaxial tensile tests of the material in different orientations. They are related to yield stress ratios \(R_{ij}\) as follows:

\[
F = \frac{1}{2}\left(\frac{1}{R_{rr}^2} + \frac{1}{R_{\theta\theta}^2} - \frac{1}{R_{zz}^2}\right) \tag{2.18}
\]

\[
G = \frac{1}{2}\left(\frac{1}{R_{\theta\theta}^2} + \frac{1}{R_{zz}^2} - \frac{1}{R_{rr}^2}\right) \tag{2.19}
\]

\[
H = \frac{1}{2}\left(\frac{1}{R_{zz}^2} + \frac{1}{R_{rr}^2} - \frac{1}{R_{\theta\theta}^2}\right) \tag{2.20}
\]

\[
L = \frac{3}{2R_{r\theta}} \tag{2.21}
\]

\[
M = \frac{3}{2R_{\theta z}} \tag{2.22}
\]

\[
N = \frac{3}{2R_{zr}} \tag{2.23}
\]

where \(R_{ij}\) are the anisotropic yield stress ratios defined as:

\[
R_{zz} = \frac{\sigma_{zz}}{\sigma_0} \tag{2.25}
\]

\[
R_{rr} = \frac{\sigma_{rr}}{\sigma_0} \tag{2.26}
\]

\[
R_{\theta\theta} = \frac{\sigma_{\theta\theta}}{\sigma_0} \tag{2.27}
\]

\[
R_{r\theta} = \frac{\sigma_{r\theta}}{\sigma_0} \tag{2.28}
\]

\[
R_{\theta z} = \frac{\sigma_{\theta z}}{\sigma_0} \tag{2.29}
\]

\[
R_{zr} = \frac{\sigma_{zr}}{\sigma_0} \tag{2.30}
\]

where \(\bar{\sigma}_{ij}\) is the measured yield stress value when \(\sigma_{ij}\) is applied as the only nonzero stress component. \(\sigma_0\) is the user reference yield stress which is \(\sigma_{zz}\) in the present case.

In the principal cylindrical basis, Hill’s equivalent stress expresses:

\[
\bar{\sigma}_{\text{Hill}} = \sqrt{A[F(\sigma_r - \sigma_\theta)^2 + G(\sigma_\theta - \sigma_z)^2 + H(\sigma_z - \sigma_r)^2]} \tag{2.31}
\]

with \(A = \frac{1}{2}\sqrt{\frac{3}{\Sigma}}\) and \(\Sigma = FH + FG + GH \tag{2.32}\)

\(F, G\) and \(H\) are related to yield stress ratios \(R_{j}\) as follows:

\[
R_z = \frac{1}{\sqrt{G + H}} \tag{2.33}
\]

\[
R_r = \frac{1}{\sqrt{F + H}} \tag{2.34}
\]

\[
R_\theta = \frac{1}{\sqrt{F + G}} \tag{2.35}
\]
Yield stress ratios $R_j$ correspond to the ratio of the measured yield stress in one direction $\sigma_i$ and a reference yield stress $\sigma_0$ corresponding to a reference direction.

$$R_z = \frac{\sigma_z}{\sigma_0}$$  (2.36)

$$R_r = \frac{\sigma_r}{\sigma_0}$$  (2.37)

$$R_\theta = \frac{\sigma_\theta}{\sigma_0}$$  (2.38)

$R_i$ are computed for an uniaxial test, i.e. tests where $\sigma_i$ is the only nonzero stress component.

Considering plane stress condition Hill’s equivalent stress expression simplifies:

$$\bar{\sigma}_{Hill} = \sqrt{A[F\sigma_\theta^2 + G(\sigma_\theta - \sigma_z)^2 + H\sigma_z^2]}$$  (2.39)

The equivalent strain rate expression deriving from the Hill’s equivalent stress and the flow rule writes:

$$\dot{\varepsilon}_{pHill} = \sqrt{\frac{2}{3}} \left( C_{orr} \dot{\varepsilon}_r^2 + C_{ozz} \dot{\varepsilon}_z^2 + C_{o\theta\theta} \dot{\varepsilon}_\theta^2 \right)$$  (2.40)

with

$$C_{ozz} = \sqrt{\frac{3}{\Sigma} F}, \ C_{orr} = \sqrt{\frac{3}{\Sigma} G} \text{ and } C_{o\theta\theta} = \sqrt{\frac{3}{\Sigma} H}$$  (2.41)

The above paragraph aimed at explaining the Hill parameters identification based on stresses. In some cases, it can be useful to identify the parameters from the strains directly. The flow rule according to the definition of Hill yield criterion and assuming normality rule writes:

$$d\varepsilon^p = \frac{d\lambda}{f} \begin{pmatrix} -G(\sigma_\theta - \sigma_z) + H(\sigma_z - \sigma_r) \\ F(\sigma_r - \sigma_\theta) - H(\sigma_z - \sigma_r) \\ -F(\sigma_r - \sigma_\theta) + G(\sigma_\theta - \sigma_z) \\ 2L\sigma_{zr} \\ 2M\sigma_{rz} \\ 2N\sigma_{r\theta} \end{pmatrix}$$  (2.42)

From the flow rule expression and in the case of uniaxial tests where only one stress component is non zero, the ratio of the different components of the flow rule enables to identify some parameters. As an example, in an uniaxial tensile test in the tube axial direction, $\sigma_{zz} \neq 0$ and the flow rule writes:

$$d\varepsilon^p = \frac{d\lambda}{f} \begin{pmatrix} (H + G)\sigma_{zz} \\ -H\sigma_{zz} \\ -G\sigma_{zz} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$  (2.43)

Then it turns that:

$$\frac{d\varepsilon_r^p}{d\varepsilon_z^p} = -\frac{H}{G + H}$$  (2.44)

$$\frac{d\varepsilon_\theta^p}{d\varepsilon_z^p} = -\frac{G}{G + H}$$  (2.45)

As the condition $G + H = 1$ is imposed, the ratios of the radial and circumferential plastic strain rate on the axial plastic strain rate give $H$ and $G$ respectively.
Figure 2.22: Presentation of the tube bulge model MA considering a single element

Parameters identification of Hill’s yield criterion

As presented previously, $F$, $G$ and $H$ are functions $R_i$ which are ratios of the measured yield stress in one direction $\sigma_i$ and a reference yield stress $\sigma_0$ corresponding to a reference direction. Their identifications require tests conducted in different directions and is not possible with the tube bulge test with a direct method. Thus, Hill’s anisotropic parameters ($F$, $G$ and $H$) were identified by means of an inverse analysis. The method is detailed in the next paragraph.

Tube bulge test: isotropic and anisotropic models

Tube bulge test model (MA) was made in Abaqus/Implicit. Considering the membrane hypothesis only one element of the tube wall was modelled with a 8-node linear brick with reduced integration (C3D8R). Axial and circumferential displacements $du_z$ and $du_\theta$ respectively were imposed as illustrated in figure 2.22. The imposed displacements were computed according to the experimentally measured strains as shown in figure 2.23(a). Figure 2.23(b) illustrates the strain path. Two different models were defined, one with an isotropic material $MA_{iso}$ and another with an anisotropic material $MA_{ani}$. Different series of Hill’s parameters were tested for the model $MA_{ani}$ and a set was identified due to good correlation of experimental and numerical results. The identified parameters corresponding
to Hill (1948) yield function are the following:

\[ F = 0.21, \ G = 0.41 \text{ and } H = 0.59 \]  \hspace{1cm} (2.46)

which can be expressed in terms of yield stress ratios as detailed in equations 2.33-2.35. The corresponding \( R_j \) values are the following:

\[ R_z = 1, \ R_r = 1.115 \text{ and } R_\theta = 1.265 \]  \hspace{1cm} (2.47)

Figure 2.24 shows the comparison of experimental and FEM stresses versus strains plots obtained with the identified parameters. The different observations that can be made are the following:

- When examining the \( \sigma_\theta \) vs \( \epsilon_\theta \) data corresponding to \( \text{MA}_{\text{iso}} \) and the experimental data, it is clear that a model considering isotropic plasticity is unable to model the material behaviour during tube bulge test.
- Regarding the \( \sigma_\theta \) vs \( \epsilon_\theta \) data corresponding to \( \text{MA}_{\text{ani}} \) and the experimental data, one can see that the plots superpose. The Hill’s anisotropic parameters are identified correctly.

**Validation of the anisotropic yield function**

The tube anisotropy was revealed by superposition of the von Mises equivalent stress versus strain plots for both the tube tensile and bulge tests (cf. fig. 2.21). After the identification of the Hill anisotropic parameters, the Hill equivalent stress vs strain curves were plotted both for tensile and bulge tests. Plots are presented in figure 2.25. It can be observed that both flow curves considering anisotropy superpose which validates the identified parameters.

In this part, an original method for identifying the parameters of Hill’s yield function was presented. This method relies on a tube bulge test in which the material is tested in the initial tubular form.
2.3.2.2 Tensile tests on oriented samples

In the sheet forming industry another method is widely used to characterize anisotropic behaviour. This technique consists in cutting oriented samples from the sheet and testing them in uniaxial tensile tests. This method was applied to this study in order to validate the parameters identification that was done with the tube bulge test.

Different parameters enabling the characterisation of anisotropic behaviour were previously described. These parameters are $F$, $G$, and $H$ or $R_z$, $R_r$ and $R_\theta$ and are functions of each others. Anisotropy can be described by other coefficients, namely the Lankford coefficients. The Lankford coefficients $r_k$ are defined as the ratio of the width to the thickness strain during an uniaxial tensile test (the indices $k$ indicates the direction of the tensile test). Lankford coefficients express as follow:

$$ r_k = \frac{\epsilon_{\text{width}}}{\epsilon_{\text{thickness}}} $$.  

(2.48)
Thus, a tensile test performed in the drawing direction (reference direction) or in the transverse direction enables to identify $r_0$ and $r_{90}$ respectively. The different steps for sample preparations were presented in section 1.2.2.2. As a reminder, the tubes are annealed after drawing as it is conventionally done in the process, then they are opened, flattened, cut and annealed again. The second annealing is done in the same conditions as the first one. This step is necessary to release the residual stresses that were created during the unfolding step.

Samples of different orientations were tested in uniaxial tensile tests. The samples directions were: $0^\circ$ (Drawing Direction, DD), $22.5^\circ$, $45^\circ$, $67.5^\circ$ and $90^\circ$ (Transverse Direction, TD). The results of the tests are detailed within the next paragraphs.

### 2.3.2.2.1 Strain measurement

Figure 2.26 presents an example of strain measurement on a $0^\circ$ sample. From this figure both the axial and the width strains ($\epsilon_{\text{axial}}$ and $\epsilon_{\text{width}}$ respectively) are homogeneous. Concerning this specific case, the standard deviations are 0.7% and 1.3% for $\epsilon_{\text{axial}}$ and $\epsilon_{\text{width}}$ respectively which is the evidence of good quality measurements. The tests performed on samples of different orientations exhibit strain variations in the same order of magnitude.

From figure 2.26, the Lankford coefficients $r_0$ can be computed as the ratio of the width to the thickness strain ($\epsilon_{\text{thickness}}$). For better accuracy it is necessary to plot the evolution of $\epsilon_{\text{width}}$ with $\epsilon_{\text{thickness}}$ all along the tests. Figure 2.27 presents such a plot for tensile tests conducted on different oriented samples. It can be observed that the evolution between both strains is linear.

Thus, the Lankford coefficients are computed by taking linear regression of the experimental data points. The computed Lankford coefficients are detailed in table 2.6. A more geometrical representation can be seen in figure 2.28 where the Lankford coefficients are plotted as a function of the angle relative to the Drawing Direction (DD). Coefficients listed in the table and figure 2.28 are close to 1 in every directions. As a consequence, according to the tensile tests on oriented samples, the material seems to be nearly isotropic.
Figure 2.27: Width strain versus thickness strain for tests conducted in different orientations

<table>
<thead>
<tr>
<th>$r_0$</th>
<th>$r_{22.5}$</th>
<th>$r_{45}$</th>
<th>$r_{67.5}$</th>
<th>$r_{90}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.970</td>
<td>0.927</td>
<td>0.986</td>
<td>0.910</td>
<td>0.916</td>
</tr>
</tbody>
</table>

Table 2.6: Computed Lankford coefficients

Figure 2.28: Lankford coefficients as function of the angle relative to the Drawing Direction
2.3.2.2 Flow curves

Figure 2.29 shows the stress strain response from three tensile tests (0°, 45° and 90°). The difference between the different plots is slight which reinforces the observation that the material is nearly isotropic.

2.3.2.3 Comparison of the anisotropic parameters

Two sets of Hill’s anisotropic parameters were identified by means of the tube bulge test and the tensile tests on oriented samples. In order to compare the identified parameters, it is necessary to transform the Lankford coefficients obtained by means of tensile tests into $R_r$ and $R_\theta$. The relation between both coefficients is the following:

$$R_r = \sqrt{\frac{r_90(r_0 + 1)}{r_0(r_90 + 1)}} \quad \text{and} \quad R_\theta = \sqrt{\frac{r_90(r_0 + 1)}{(r_0 + r_90)}} \quad (2.49)$$

The different $R_r$ and $R_\theta$ values are listed in table 2.7. $F$, $G$ and $H$ values are also listed.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$R_z$</th>
<th>$R_r$</th>
<th>$R_\theta$</th>
<th>$F$</th>
<th>$G$</th>
<th>$H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile tests on oriented samples</td>
<td>1</td>
<td>0.958</td>
<td>0.940</td>
<td>0.538</td>
<td>0.493</td>
<td>0.507</td>
</tr>
<tr>
<td>Tube bulge tests</td>
<td>1</td>
<td>1.115</td>
<td>1.265</td>
<td>0.26</td>
<td>0.41</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Table 2.7: Comparison of the anisotropic parameters identified by means of different tests

From table 2.7, it can be seen that the parameters identified with the different methods differ. According to the tensile tests on oriented samples, the material is nearly isotropic while it is anisotropic according to the tube bulge test.
2.3.2.4 Additional tensile test

An additional tensile test was performed on a L605 6.5×7.5 mm² tube in order to identify the most accurate set of anisotropic parameters. Moreover, this test enabled to discard the hypothesis of microstructure and properties changes induced by the tube unfolding, cutting by electrical discharge and additional annealing. In this test, the circumferential deformation was measured by means of a radial extensometer as shown in figure 2.30 and the axial deformation was computed from the cross-head displacement. The radial strain is computed by the volume conservation hypothesis. As detailed in equation 2.45 both parameters $H$ and $G$ can be identified from the ratio of the radial and circumferential plastic strain rates on the axial plastic strain rate respectively. $H$ and $G$ values are identified by linear regression of $\epsilon_r$ vs $\epsilon_z$ and $\epsilon_\theta$ vs $\epsilon_z$ respectively. The identified values are $G = 0.482$ and $H = 0.518$. These values are close to the one identified by means of the tensile tests on oriented samples (tab.2.7) and reinforce the material being nearly isotropic.

2.3.2.5 Discussion

In this section, different hypotheses to explain the discrepancies between the different identified anisotropic parameters are discussed. The first hypothesis made is that the constitutive equations selected are not compatible with the real material behaviour. The second hypothesis is that the mechanical properties are inhomogeneous in the tube thickness. A review of bibliographic references and measurements of material hardness are presented to evaluate both hypothesis. Finally, the conclusions are compared with other authors studies.

2.3.2.5.1 Incompatibility of constitutive equations

Zang et al. (2011) characterised the plastic anisotropy and hardening behaviour of steels by mean of uniaxial and biaxial tensile tests and shear tests. They evaluated different constitutive equations (yield functions and strain hardening) in order to characterise the
material anisotropy and hardening parameters. The different models were combinations of Hill 1948 and Bron-Besson yield functions (Bron and Besson, 2004) and Swift isotropic and Yoshida-Uemori (Yoshida and Uemori, 2003) kinematic hardening models. The parameters of the constitutive equations were identified by the software SiDoLo (Pilvin, 1988; Chaparro et al., 2008) which enables to search for an optimum set of parameters by minimising a cost function. The results showed that Hill (1948) used together with isotropic hardening presented the largest error in predicting the Cauchy stresses in biaxial and shear tests and in estimating the anisotropic coefficients. On the other hand, stresses were well predicted in uniaxial tensile tests. On the contrary, a model based on Bron-Besson yield function and Yoshida-Uemori kinematic hardening was able to model both the Cauchy stresses and the transverse strains (linked to anisotropic coefficients) with good accuracy for all the mechanical tests. Thus, from Zang et al. (2011) study, it can be observed that the combination of different constitutive equations induces different levels of accuracy. Moreover, as presented in chapter 1, different anisotropic yield functions were developed throughout the years and, as mentioned by Barlat et al. (2003) and Barlat et al. (2005), some yield functions are more reliable to analyse the stress state while others are more accurate to analyse strain states. From Zang et al. (2011) study, it can be deduced that the choice of the yield function together with the hardening model influences the capability of a model to represent the material behaviour. From this point and concerning this work, it can be suspected that the use of different models could lead to a better characterisation of the material anisotropy by means of tensile and tube bulge tests. Indeed, in this study, both Hill (1948) and isotropic hardening were used in FEM and Zang et al. (2011) reported their poor performances in the case of stress states other than uniaxial tension. Nevertheless, the errors made by Hill (1948) and isotropic hardening compared to the most predictive models (Bron-Besson and Yoshida-Uemori) is largely lower that the error made in this study. Thus the different anisotropic coefficients that were identified with the tensile tests and the tube bulge tests might have another origin.

2.3.2.5.2 Inhomogeneous mechanical properties

The origin of inhomogeneity

The second hypothesis to explain the inaccurate anisotropic coefficients identified with the tensile tests on oriented samples and tube bulge test is to consider that the material presents heterogeneous properties in the wall thickness. Majta and Luksza (1992b) observed that the yield stress, tensile strength and uniform elongation were non-uniformly distributed in the cross section of drawn bars and that properties heterogeneity was greater for larger die semi-angles. Aguilar et al. (2002) analysed the effective strain distribution over the cross section of a wire during wire drawing and revealed the presence of a strain gradient. The effective strain was the lowest at the wire center and increased in the wire cross section to reach a maximum at the wire surface. Moreover, the strain gradient in the wire cross section leads to a non-uniform distribution
of stored energy (Sadok et al., 1994b; Aguilar et al., 2002). As the stored energy is at the origin of grain recrystallisation during annealing, it can be expected that a gradient of stored energy in the wire cross section causes a inhomogeneous microstructure after annealing. Kazeminezhad (2007) modelled the strain, the stored energy due to deformation and the grain size distribution of as drawn and annealed wire by means of Monte Carlo models. He found the strain and microstructure heterogeneities to increase with increased redundant strain parameter $\Delta$ (Backofen, 1972):

$$\Delta = \frac{\alpha}{R} (1 + \sqrt{1 - R})^2$$

(2.50)

with $\alpha$ the semi-die angle in radian and $R$ the reduction of area. Finally, a gradient in microstructure leads to heterogeneous material properties in the wire (Sadok et al., 1994b, a, 1996).

It has been previously explained that wire drawing caused an heterogeneous microstructure in the cross section and that the stored energy of cold work was also heterogeneous. It must be noticed that when the material is annealed, a new microstructure is created by means of this stored energy. The first question that comes out is "up to which point the annealing step can reduce the microstructure heterogeneity?" and the second question is "can the annealing step totally erase the microstructure gradient and generate an homogeneous microstructure?". The answer to these questions is important in order to know whether the material properties are homogeneous in the wire or the tube after annealing and before any drawing step. The first trail analysed by Kazeminezhad (2007) is that the microstructure heterogeneity decreases with increasing annealing time.

Measurement of mechanical heterogeneity

To summarize, the heterogeneity of mechanical properties is due to non-uniform distribution of strains, microstructure gradients, texture and residual stresses (Majta and Luksza, 1992a). The means that are available to analyse the properties heterogeneity in a wire or in a tube are limited. Sadok et al. (1994a) performed tensile tests on samples cut from wires parallel to the wire axis and at various location along the wire diameter. From these experiments, they revealed an increase of yield and tensile strength and a decrease of ductility (elongation) with increasing distance from the wire axis. Such tensile tests can be envisaged on large wires only and are impossible for tubes with small thickness. The other option available in order to evaluate the heterogeneity of mechanical properties is micro-hardness measurement (Sadok et al., 1994a; Kraft et al., 1996).

Experimental measurements of tube hardness in the tube wall thickness were performed by means of a micro-indenter on L605 tubes in a annealed state with a mass of 200 g. The tubes were identical to the ones used for the tube bulge test and the tensile tests on oriented samples. The measured hardnesses are presented in figure 2.31 and were computed as an average of the three measurements made at a distance $d$ from the tube outer surface. This measurement reveals that there is a hardness heterogeneity in the tube thickness. Hardness is larger nearby the tube inner surface and decreases towards the tube outer surface. As a consequence, a gradient of mechanical properties exists in the tube wall.
Strain field during different mechanical tests

So far, the heterogeneity of mechanical properties does not fully explain the different equivalent stress vs strain behaviour characterised by means of tube bulge test and tensile tests on oriented samples. In the case of the tensile tests on oriented samples, the tube was flattened and deformed in a uniaxial way. Thus, the imposed deformation during the tensile test was homogeneous in the sample thickness. As a consequence, the measured force and the computed stresses correspond to a mean material behaviour. The heterogeneous material properties are averaged and seen as homogeneous.

In the case of the tube bulge test, the membrane hypothesis was made and the strain field was supposed homogeneous in the tube thickness. As the ratio of the tube thickness to radius is of the order of 0.15, it is clear that this hypothesis is a first approximation. This strain heterogeneity combined with the material properties heterogeneity in the tube thickness makes the analysis of the tube bulge test difficult. As a consequence, it is necessary to define a new model (MB) for the tube bulge test. In this model, the whole tube thickness requires to be considered. Only one half of the tube was modelled with an axisymmetric simplification. The tube was meshed with 4-node bilinear axisymmetric quadrilateral elements with reduced integration (CAX4R). A symmetry boundary condition was imposed on the bottom of the tube. During the experimental bulge test, tube sliding was observed in the grips due to an insufficient tightening. According to this observation, the simulation was decomposed into two steps. In the first step, a clamping boundary condition was imposed at the top tube extremity. In the second step, an axial displacement $du_z$ was imposed to the top tube to take into account tube sliding. The displacement amplitude was fitted according to the measured axial strain. Finally a uniform pressure load was applied at the inner tube surface. Figure 2.32 illustrates the axisymmetric tube bulge test model. A Ludwik constitutive equation was used combined with Hill (1948) yield function. As the material properties were identified to be non-homogeneous in the tube thickness, the ideal would have been to model this heterogeneity. However, as the local mechanical behaviour could not be characterised, the material mechanical properties were supposed homogeneous in the tube thickness.
Figure 2.32: Presentation of the tube bulge model MB considering tube axisymmetry

Figure 2.33: Comparison of the stress-strain curves for the different tube bulge test models

Figure 2.33 presents the stress vs strain curves obtained from the second model. It can be seen that the plots corresponding to both anisotropic models superpose which enables to validate the boundary conditions of the model MB.

Figure 2.34 illustrates the simulated principal plastic strains. Figure 2.34(a) presents the general view of the bulged tube at a pressure of 200 MPa and figures 2.34(b,c,d) illustrate the plastic strain field in the tube and the plastic strain profiles in the tube thickness. The positions 0 mm and 0.40 mm correspond to the tube inner and outer surface respectively.
From this figure, it can be seen that there is a plastic strain gradient in the tube thickness. $\epsilon_p$ values present a difference of 1.5% between the tube inner and outer surfaces which is negligible. The axial plastic strain field can be approximated as homogeneous in the tube thickness. The radial plastic strain component $\epsilon_p^r$ is compressive and decreases from the tube inner to outer surface: the radial strain state is more compressive close to the inner surface. The relative difference between both surfaces is 21.7%. The circumferential plastic strain component evolution is the opposite, $\epsilon_p^\theta$ decreases from the tube inner to outer surface. The relative difference between both surfaces is 16.8%. In short, the strain in the tube thickness is heterogeneous.

As a consequence, if the material properties are heterogeneous and vary in the tube thickness, then, the stress field associated with the above described strain field will vary in the tube thickness as a function of the material properties.

Finally, as the hypothesis of plastic anisotropy was made and as the heterogeneous material properties in the tube thickness was neglected, the FEM does model an average material behaviour. The accurate modelling of the tube bulge test requires to consider heterogeneous material properties in the tube wall thickness. The problematic that comes out is the characterisation of the different mechanical properties in the tube thickness. As the properties in the tube thickness were unknown, the FEM of the tube drawing considering non-homogeneous properties was not performed.

2.3.2.5.3 Comparison with other studies

Hwang and Lin (2006) and Hwang et al. (2009) who also studied the tube bulge test by means of experimental tests and FEM found better correlation between experimental tensile tests on oriented samples and tube bulge test. They considered both isotropic and anisotropic plastic behaviour and found a better agreement between flow rules that were determined from tensile tests and bulge test if anisotropy was considered. As a consequence the FEM of the bulge test gave better results when the material anisotropic behaviour was considered. Hwang and Lin (2006) developed a mathematical model using Hill’s orthogonal anisotropic theory to take into account the material anisotropy during tube bulge tests. They showed the importance of considering the material anisotropy for the evaluation of forming limit as not taking into account the anisotropy could misestimate the formability limit.

The main difference between Hwang and Lin (2006) and Hwang et al. (2009) studies and this one is the tube dimensions used for the bulge tests. Hwang and Lin (2006) and Hwang et al. (2009) used a tube with an outer radius of 25.96 mm and a thickness of 1.86 mm. The corresponding thickness vs radius ratio was equal to 0.07. In the present study the tube had an outer radius of 3.75 mm and a thickness of 0.5 mm and the thickness vs radius ratio was equal to 0.13. As a consequence, a tube presenting a lower thickness vs radius ratio is less likely to present a strain gradient in its thickness. Moreover, Hwang and Lin (2006) and Hwang et al. (2009) did not mention heterogeneous mechanical properties in the tube thickness. Such inhomogeneity can vary depending on the characteristics of
the drawing passes. The tube dimensions they used for their tests might indicate that the tubes underwent fewer drawing steps than in this study. As a consequence, it can be assumed that the tube exhibits less heterogeneity.

2.3.2.5.4 Microstructure aspects

The previous two sections aimed at characterising the material anisotropy by means of mechanical techniques. But the anisotropic behaviour can also be explained by the material crystallographic characteristics in the form of preferred crystallographic orientations. In the following section the material is analysed by means of X-ray diffraction technique.

Principle of the X-ray diffraction technique

X-ray diffraction is a technique used to determinate the atomic structure of a crystal and thus to identify the different phases of a sample (Esling and Bunge, 1997a,b). A crystal is a structure in which atoms are arranged in an ordered pattern extending in three dimensions. The elementary pattern is called elementary mesh. Elementary meshes are arranged to create parallel and equidistant planes whose characteristics are represented
Figure 2.35: (a) Example of a crystallographic structure in the (a,b,c) basis, (b) interaction of a X-ray with crystallographic planes

by the Miller indices (hkl). The distance between two planes is called the lattice spacing. Figure 2.35(a) presents three elementary meshes assembled into a crystallographic structure. (a,b,c) is the crystal basis. As an example, the plane (001) is presented in orange.

When a X-ray beam interacts with the crystallographic structure, it is diffracted into many directions. Each direction corresponds to a family of planes as illustrated in figure 2.35(b). The condition for a X-ray to be diffracted is given by the Bragg’s law:

\[ n\lambda = 2d\sin(\theta) \]  

(2.51)

where n is an integer, \( \lambda \) is the wave length of incident ray, \( d \) is the lattice spacing and \( \theta \) is the angle between the incident ray and the scattering planes. The recording of the angles and the intensities of the diffracted beams enables to produce a diagram. An X-ray diffraction diagram or diffractogram represents ray intensities as function of the angle. Every crystalline phase has a unique diffractogram characterized both by the ray position (depending on the elementary mesh dimensions) and the relative intensity of the rays.

Finally, crystal phase identification is made by superimposing the as obtained diagram with a database of powder diagrams. The interest of the powder diagram is that crystals can be statistically found in every directions and thus the material is seen as isotropic. Thus, when comparing any diagram to the corresponding powder diagram, preferred plane orientations appear automatically as the peaks are of higher intensity.

Concerning texture measurement, the drawback of X-ray diffraction is that only planes that are parallel to the sample surface diffracts. To fully characterize material texture, it is necessary to vary sample orientation to obtain information about plans with other orientations. Thus, maps presenting the different orientations of a family of planes can be drawn. These maps are called pole figures.

**Tube analysis by X-ray diffraction**

**Surface Measurements**
X-ray diffraction diagrams are plotted for three samples extracted from different stages in the process. The first sample is a piece of "ebauche" with dimensions of ID×OD = 18×24 mm². Ebauches are obtained from rough bars that are shaped by metal casting. The bars are finally drilled to end up with tubes. In the production process of the ebauche, there is no step that is likely to introduce anisotropy. The second sample is extracted after the 6th pass (cf. fig.1.2), its dimensions are ID×OD = 10.5×13 mm². The third sample is extracted after the 10th pass (cf. fig.1.2), and the tube dimensions are ID×OD = 6.5×7.5 mm². This tube has the same dimensional characteristics as the tubes used in the bulge test. Both of the tubes extracted after drawing passes were heat treated. Small curved samples were cut from the tubes in order to analyse both the inner and outer tube surfaces. The first two samples were too thick to be flattened but the curvature was correct to make an analysis. On the contrary, the third sample had a radius of curvature too important to perform the X-ray diffraction analysis and had to be flattened.

The analysis developed below refers to the X-ray diffraction diagrams shown in figures 2.37 and 2.36 for the tube inner and outer surfaces respectively. Starting with the ebauche, both inner and outer surfaces show a slight preferred orientation of type (111) but it is not significant. The material is nearly isotropic.

The second sample shows differences between inner and outer surfaces. The outer surface shows a pronounced preferred orientation of type (111) while the inner surface shows a preferred orientation of type (220). Same observations can be done for the third sample. The first remark is that there is a difference of texture on the inner and outer tube surfaces. These differences are probably due to the process but cannot be explained easily.

The comparison of the inner surface for the second and third samples reveals slight differences on the pic intensities. Indeed, (111) pic intensity is higher for the third tube. Thus, from these observations, it can be concluded that there is a texture evolution between the tubes extracted at two different passes.

The conclusions of this X-ray diffraction study are:

- the ebauche is nearly isotropic and develops texture in the primary drawing passes;
- then texture develops after successive passes. Thus, it can be expected that mechanical tests performed on different tubes that are extracted from different passes will reveal different plastic behaviour;
- there is a different preferred orientation between inner and outer surfaces. This difference is probably due to the process. As the penetration depth of the X-rays was approximately 20 µm, the transition between both orientations was not visible.

Finally, it must be mentioned that the above analysis considered the tube surfaces only. During the tube drawing process, the inner and outer tube surfaces undergo shear due to friction. As a consequence, a specific texture can develop nearby the tube surface due to friction and the above analysis is not significant for the global material texture.

**Volumetric measurements**

Volumetric measurements required to use a more energetic X-ray beam in order to analyse the whole sample volume. Thus, the analysis were performed at the ESRF in
Figure 2.36: Superimposition of the X ray diffraction of the outer tube surface for different tubes

Figure 2.37: Superimposition of the X ray diffraction of the inner tube surface for different tubes
collaboration with Caroline Curfs and Jonathan Wright, on the ID11 beamline with an energy of 80keV. A sample in the form of an arc was cut from the 6.5×7.5mm$^2$ L605 tube with a thickness of 0.5 mm. The sample was placed in a rotating device aligned with the x-ray beam and the sample orientation was identified with a coordinate system as shown in figure 2.38. The marked x,y and z directions correspond to the radial, axial and circumferential directions respectively. The sample was rotated along the z axis.

Figure 2.39 presents the pole figures obtained from the X-ray diffraction analysis. Such figures represent the intensity of the planes (111), (200) and (220) oriented in different directions.

The pole figure concerning the plane (111) presents a central spot and a ring of greater intensity. The central spot indicates that the normals to the planes (111) are oriented in the drawing direction. The ring indicates that there are also grains in which the planes (111) have a different orientation. The distance between the ring and the pole center indicates the direction of the normal to the planes. A distance corresponding to an angle of 54.7° corresponds to the direction <100>.

In the pole figure (200) the central spot indicates that the normal of this plane is oriented in the direction parallel to the drawing direction. The ring indicates that planes (200) are also oriented in the <111> direction.

The pole figure (220) indicates that the planes (220) are not oriented preferentially in the direction of drawing as there is no central spot. This measure has to be compared with the surface X-ray diffractograms. In figure 2.37 a preferred orientation of type (220) was observed for the inner tube surface. During measurements performed at the ESRF, the texture was measured for the entire tube thickness, thus the measured texture corresponds to an average. As a consequence, it signifies that the preferred orientation (220) is limited to the tube inner surface. The main remarks deduced from the pole figure are the following:

- the material exhibits a slight fibre texture <111>;
- the texture is not marked as the intensity of the spot and the ring are not important;
- the (220) preferred orientation is limited to the inner tube surface.

Kraft et al. (1996) and Chen et al. (2011) also measured <111> and <200> fibre texture in copper wire. But compared to tube drawing where the material is annealed between two successive passes, in wire drawing the wire is not annealed and the texture is more marked as the tube undergoes more deformation.

As a conclusion, as the texture in the L605 is not very marked, it indicates that the material might get close to an isotropic plastic behaviour.

2.3.2.6 Conclusion

The approach in which the material plastic behaviour is considered as isotropic appears to be founded regarding the tensile tests on oriented samples and the X-ray diffraction analysis. Nevertheless, as the material exhibits heterogeneous properties in the tube thickness, the anisotropic approach can lead to a better average of the mechanical properties in some cases. Indeed, as shown in the tube bulge tests, the anisotropic plasticity enables to model the tube bulge test while the isotropic one cannot. The stress state during tube drawing is closer to the one during tube bulge test compared to the tube tensile test.
As an example, Strano and Altan (2004) modelled the tube hydroforming process by means of FEM both with flow curves obtained from tensile tests and bulge tests. Their model showed better accuracy when the flow stress parameters were identified by bulge test compared to tensile test.

Thus, in the case of tube drawing, it can be expected that the anisotropic hypothesis identified with the tube bulge test leads to more accurate results compared to the isotropic hypothesis identified with the tensile test. In short, the anisotropic plasticity enables to model a mean mechanical behaviour which is more representative of the mechanical heterogeneity as illustrated with the FEM of the tube bulge test.

### 2.3.3 Influence of annealing temperature

The development of next part requires to refocus on the goal of this thesis work which is to evaluate tube formability. In the industrial tube drawing process, tubes can be annealed at different temperatures depending on the difficulties that can be met during the process. As a consequence, a small part of this work was devoted to the evaluation of the influence of annealing temperature on tube formability and the first step to do so is to evaluate their influence on material properties. Thus, next paragraph deals with the mechanical testing of materials annealed at different temperatures and the observation of resulting microstructures.
Tensile tests were performed on tubes with an OD of 11.5 mm and a thickness of 1 mm at constant strain rate of 0.03 s\(^{-1}\). Three series of tubes were annealed during 10 min at three different temperatures: 1100, 1150 and 1175\(^\circ\)C. The microstructures and the corresponding grain sizes of the L605 are detailed in figure 2.40.

<table>
<thead>
<tr>
<th>Annealing temperature (°C)</th>
<th>Mean grain size (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1100</td>
<td>11.2</td>
</tr>
<tr>
<td>1150</td>
<td>22.5</td>
</tr>
<tr>
<td>1175</td>
<td>37.8</td>
</tr>
</tbody>
</table>

Table 2.8: L605 mean grain size as a function of the annealing temperature

It can be observed from the micrographies in figure 2.40 that the grain size increases with increasing annealing temperature. The mean grain sizes corresponding to the different annealing temperatures are listed in table 2.8. The flow curves obtained from uniaxial tensile tests as a function of the grain size are detailed in figure 2.41. It can be seen from these curves that the materials exhibit different mechanical behaviour depending on their grain size: the ultimate tensile stress is unchanged with grain size, the maximum plastic strain or lengthening increases with increasing grain size and the yield stress decreases with increasing annealing temperature. It can be noticed that in the context of metal forming industry, it may be preferred to use a material with increased ductility, i.e. a material with larger grain size.

### 2.3.3.2 316LVM

Tensile tests were performed on tubes with an OD of 10.5 mm and a thickness of 0.75 mm at a constant strain rate of 0.03 s\(^{-1}\). Three series of tubes were annealed at three different temperatures: 950, 1000 and 1050\(^\circ\)C. The microstructures and the corresponding grain sizes of the 316LVM are detailed in figure 2.42 and table 2.9. The material behaviour is similar to the one exhibited by the L605.
2.4 Thermo-mechanical characterisation

Plastic deformation is accompanied by heat generation. This phenomenon was detailed in part 1.2.3. As the interest of this project is also to consider heat generation, it is necessary to characterise the materials thermo-mechanical properties.

2.4.1 Introduction on thermomechanics

The percentage of plastic deformation work that is converted into heat is characterised by the Taylor-Quinney coefficient $\beta$ also known as Inelastic Heat Fraction (IHF). This coefficient was identified by means of thermal measurement with an infra-red camera during the tensile tests of tubular specimens.

In a 3D thermal problem, the energy conservation equation links the temperature $T(r,\theta,z,t)$ and the heat sources $\dot{q}(r,\theta,z,t)$ for a point M of cylindrical coordinates $(r,\theta,z)$ as illustrated.
in figure 2.43. At the time \( t \) it writes:

\[
\rho C_p \frac{\partial T}{\partial t} - k \text{ lap}(T) = \dot{q}
\]

(2.52)

with

\[
\dot{q} = \beta \text{ trace}(\sigma \dot{\varepsilon}^p)
\]

(2.53)

where \( \rho \) is the mass density, \( C_p \) the specific heat capacity, \( k \) the heat conductivity.

In this study, convection is supposed to be the predominant surface heat transfer phenomenon. It is supposed that the sample thickness is small and the temperature is considered uniform in the sample thickness. The energy conservation equation then simplifies to a 2D problem (fig.2.43) and writes (Delobelle, 2012):

\[
\rho C_p \frac{\partial \tilde{T}}{\partial t} - k \text{ lap}_{2D}(\tilde{T}) + \frac{2h}{e}(\tilde{T} - T_{\infty}) = \tilde{\dot{q}}
\]

(2.54)

with \( e \) the tube thickness and \( \text{lap}_{2D} = \frac{\partial^2 T}{\partial \tau^2} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} \). The temperature and heat source reduce to \( \tilde{T}(\theta,z,t) \) and \( \tilde{\dot{q}}(\theta,z,t) \) respectively.

The temperature can be considered homogeneous in the direction \( \vec{e}_\theta \) due to the tube geometry. Then, the energy conservation equation transforms into a 1D problem (fig.2.43) and writes:

\[
\rho C_p \frac{\partial \hat{T}}{\partial t} - k \text{ lap}_{1D}(\hat{T}) + \frac{2h}{e}(\hat{T} - T_{\infty}) = \hat{\dot{q}}
\]

(2.55)

where \( \text{lap}_{1D} = \frac{\partial^2 T}{\partial z^2} \). The temperature and heat source reduce to \( \hat{T}(z,t) \) and \( \hat{\dot{q}}(z,t) \) respectively.

The temperature is inhomogeneous near the gripper due to conduction phenomenon but it can be considered homogeneous far from the gripper. Thus, considering the temperature in the middle of the tube only, the energy conservation equation turns into a 0D problem (fig.2.43) and writes:

\[
\rho C_p \frac{\partial \tilde{T}}{\partial t} + \frac{2h}{e}(\tilde{T} - T_{\infty}) = \tilde{\dot{q}}
\]

(2.56)

Finally, if the temperature field is homogeneous, it turns to be dependent on the time only: \( \tilde{T}(t) \) and \( \tilde{\dot{q}}(t) \). In the above expression, the first unknown is the heat convection coefficient \( h \). It can be identified at the end of the test, when the hot sample naturally cools down and when \( \tilde{\dot{q}} \) equals zero:

\[
h = \frac{\epsilon \rho C_p \frac{\partial \tilde{T}}{\partial t}}{2(\tilde{T} - T_{\infty})}
\]

(2.57)

Finally, combining equations 2.53 and 2.56 the expression of the Taylor-Quinney coefficient comes:

\[
\beta = \frac{\rho C_p \frac{\partial \tilde{T}}{\partial t} + \frac{2h}{e}(\tilde{T} - T_{\infty})}{\text{trace}(\sigma \dot{\varepsilon}^p)}
\]

(2.58)

### 2.4.2 Identification of the Taylor-Quinney coefficient for L605

The tubes were painted in black to reach an emissivity close to 1 (0.95) (Palengat, 2009; Delobelle, 2012) and the temperature was measured during the tensile tests. Considering
the geometry of the tube, the temperature field is supposed to vary only in the axial direction $z$. The thermal measurement was made on the whole sample but data used for further computations were acquired from a limited area $A(\theta, z)$. The selected area is limited to the center of the tube, corresponding to a small view angle from the camera (fig.2.44). The temperature was measured to be homogeneous in this area.

Figure 2.45 shows the superimposition of the evolution of the true axial stress and the temperature variation $\theta = T - T_0$ as a function of the time. During plastic deformation, the temperature increases with increasing strain. At the end of the tensile test, the tube temperature slightly decreases due to natural convection. Cooling by convection is clearly visible for the tubes annealed a temperature of $1100^\circ C$ (blue plot) and $1175^\circ C$ (green plot). During the cooling phase, there is no plastic deformation work and the sources are equal to zero. Thus, on the part of the temperature curve corresponding to this moment, the heat convection coefficient $h$ is identified and equal to $14.4 W m^{-2} K^{-1}$.

Once the heat coefficient coefficient identified, the Taylor-Quinney coefficient can be evaluated. Figure 2.46 presents the evolution of the computed $\beta$ as a function of the axial
Figure 2.45: Results of the tensile tests: Superimposition of the axial stress and temperature variation vs time for a strain rate of 0.03 s$^{-1}$. Data are plotted for three annealing temperatures.

Figure 2.46: Evolution of the Taylor-Quinney coefficient with axial strain. Data are plotted for three annealing temperatures.

strain and for materials treated at different annealing temperatures. Globally, $\beta$ increases with increasing axial strain and ranges from 0.45 to 0.7.

### 2.5 Failure characterisation

This chapter finally comes to the characterisation of the failure and the calibration of the failure criteria. The choice was made to compute the failure criteria reference values by means of uniaxial tube tensile tests. The choice of such a simple test comes from the observation that this test is one of the most widely used in the industry as it is simple to handle. The objective of this study is also to evaluate the predictability quality of fracture criteria calibrated with only tube tensile test. More complex fracture criteria requiring
Figure 2.47: Example of strain rates (s$^{-1}$) reached during tube drawing process

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Expression of the damage variable in tension</th>
<th>Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>STRN</td>
<td>$D_1 = \epsilon_1$</td>
<td>$D_1^{\text{crit}} = 0.451$</td>
</tr>
<tr>
<td>MSS</td>
<td>$D_2 = \tau_{\text{max}} = \frac{\sigma_1}{2}$</td>
<td>$D_2^{\text{crit}} = 902.6 \text{ MPa}$</td>
</tr>
<tr>
<td>SHAB</td>
<td>$D_3 = 1$</td>
<td>$D_3^{\text{crit}} = 1$</td>
</tr>
<tr>
<td>FREU</td>
<td>$D_4 = \int_0^{\epsilon_1} \sigma_1 d\epsilon_1$</td>
<td>$D_4^{\text{crit}} = 622.4 \text{ MPa}$</td>
</tr>
<tr>
<td>COCK</td>
<td>$D_5 = \int_0^{\epsilon_1} \max(0, \sigma_1) d\epsilon_1$</td>
<td>$D_5^{\text{crit}} = 622.4 \text{ MPa}$</td>
</tr>
<tr>
<td>RICE</td>
<td>$D_6 = \int_0^{\epsilon_1} \exp\left(\frac{1}{2}\right) d\epsilon_1$</td>
<td>$D_6^{\text{crit}} = 1.16$</td>
</tr>
<tr>
<td>BROZ</td>
<td>$D_7 = \int_0^{\epsilon_1} \sigma_1 d\epsilon_1$</td>
<td>$D_7^{\text{crit}} = 0.701$</td>
</tr>
<tr>
<td>ARGO</td>
<td>$D_8 = \int_0^{\epsilon_1} \frac{\sigma_1}{2} d\epsilon_1$</td>
<td>$D_8^{\text{crit}} = 829.9 \text{ MPa}$</td>
</tr>
<tr>
<td>OH</td>
<td>$D_9 = \int_0^{\epsilon_1} \sigma_1 d\epsilon_1$</td>
<td>$D_9^{\text{crit}} = 0.701$</td>
</tr>
<tr>
<td>AYAD</td>
<td>$D_{10} = \int_0^{\epsilon_1} \frac{\sigma_1}{3} d\epsilon_1$</td>
<td>$D_{10}^{\text{crit}} = 0.234$</td>
</tr>
<tr>
<td>TREN</td>
<td>$D_{11} = \int_0^{\epsilon_1} \frac{\sigma_1}{2} d\epsilon_1$</td>
<td>$D_{11}^{\text{crit}} = 311.2 \text{ MPa}$</td>
</tr>
</tbody>
</table>

Table 2.10: Details of the fracture criteria calibrated on a tube tensile test for L605

other tests were not considered.

### 2.5.1 Failure criteria calibration

#### 2.5.1.1 L605

According to the stress state taking place in the tube tensile test, the expression of the damage or damage accumulation variable can be re-written as presented in the table 2.10 where $\sigma_1$ and $\epsilon_1$ are the tensile Cauchy stress and the logarithmic strain respectively. The critical computed values determined by means of tensile tests are also detailed. Table 2.10 summarizes the calibration values of different failure criteria that are evaluated. Failure criteria were calibrated on tensile test performed at a strain rate of 9s$^{-1}$ (cf. fig. 2.10). Indeed, as presented in figure 2.47 strain rates reached during the drawing process ranges from 1 to 72s$^{-1}$ for a bench speed of 11.2 m min$^{-1}$. Predictability should be more accurate if criteria are calibrated for strain rates close to the one reached during tube drawing.

91
Strain rate \((s^{-1})\) & COCK (MPa) \\
\hline
0.03 & 557.8 \\
3 & 618.5 \\
9 & 622.4 \\
\hline

Table 2.11: Calibration values of Cockcroft-Latham failure criterion as function of strain rate

Annealing temperature (°C) & Grain size (µm) & COCK (MPa) \\
\hline
1100 & 11.2 & 526.7 \\
1150 & 22.5 & 557.8 \\
1175 & 37.8 & 633.9 \\
\hline

Table 2.12: Calibration values of Cockcroft-Latham failure criterion as function of grain size

2.5.1.1.1 Influence of visco-plasticity

The influence of strain rate on tensile test results was shown in figure 2.13. Cockcroft-Latham failure criterion was calibrated on the tests conducted at different strain rates to evaluate the effect of material visco-plasticity on failure. The computed values are listed in table 2.11. From the computed values, it can be expected that material formability improves slightly with increasing strain rate. The value of 622.4 MPa corresponding to the higher strain rate was selected for failure prediction. Indeed, the strain rate of 9 \(s^{-1}\) is closer to the strain rates reached during tube drawing.

2.5.1.1.2 Influence of grain size

The influence of grain size on tensile tests results was shown in figure 2.41. Cockcroft-Latham failure criterion was calibrated on the tensile tests performed on tubes annealed at different temperatures to evaluate the effect of grain size on failure. The computed values are listed in table 2.12. It can be seen that increasing grain size can improve metal formability.

2.5.1.1.3 Influence of anisotropy

Failure criteria were calibrated considering material anisotropy and by means of the data obtained on the tensile test with a strain rate of 9 \(s^{-1}\). The expression of failure criteria were modified to take into account the anisotropic effects. Mises equivalent stress ans strain were replaced by Hill’s equivalent stress and strain. The expression of the modified failure criteria and the corresponding calibrated values are detailed in the following table:

2.5.1.2 316LVM

The failure criteria for the 316LVM were calibrated on the tube tensile test performed at a strain rate of 9 \(s^{-1}\). The calibrated values are detailed in table 2.14.
The plastic anisotropy was further studied by means of tensile tests on oriented samples. The tube bulge test suggested the anisotropic behaviour of the material. This experimental work enabled to characterize material mechanical properties. Tensile tests on tubes enabled to identify a visco-plastic constitutive law. The influence of grain size on material properties was also addressed. This point is important when the decision of modifying the annealing temperature in the industrial process is taken. A small change in annealing temperature can have great consequence on the process and the final product. The tube bulge test suggested the anisotropic behaviour of the material.

The plastic anisotropy was further studied by means of tensile tests on oriented samples that were cut from flattened tubes. However, these tests revealed a nearly isotropic plastic behaviour. An additional tensile test on a tube was performed. In this case both the

Table 2.13: Details of the fracture criteria calibrated on a tube tensile test considering anisotropy for L605

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Expression of the damage variable in tension</th>
<th>Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>STRN$^H$</td>
<td>$D_1 = \bar{\varepsilon}^H$</td>
<td>$D_{1\text{crit}} = 0.405$</td>
</tr>
<tr>
<td>MSS$^H$</td>
<td>$D_2 = \tau_{\text{max}} = \frac{\sigma^2}{2}$</td>
<td>$D_{2\text{crit}} = 535.7$ MPa</td>
</tr>
<tr>
<td>SHAB$^H$</td>
<td>$D_3 = 1$</td>
<td>$D_{3\text{crit}} = 1$</td>
</tr>
<tr>
<td>FREU$^H$</td>
<td>$D_4 = \int_0^{\bar{\varepsilon}} \sigma^H d\varepsilon_1$</td>
<td>$D_{4\text{crit}} = 345.5$ MPa</td>
</tr>
<tr>
<td>COCK$^H$</td>
<td>$D_5 = \int_0^{\bar{\varepsilon}} \max(0, \sigma_1) d\varepsilon_1$</td>
<td>$D_{5\text{crit}} = 345.5$ MPa</td>
</tr>
<tr>
<td>RICE$^H$</td>
<td>$D_6 = \int_0^{\bar{\varepsilon}} \exp(\frac{\sigma_1}{2\sigma_T}) d\varepsilon_1$</td>
<td>$D_{6\text{crit}} = 0.709$</td>
</tr>
<tr>
<td>BROZ$^H$</td>
<td>$D_7 = \int_0^{\bar{\varepsilon}} d\varepsilon_1$</td>
<td>$D_{7\text{crit}} = 0.430$</td>
</tr>
<tr>
<td>ARGO$^H$</td>
<td>$D_8 = \int_0^{\bar{\varepsilon}} \frac{\sigma_1}{3} d\varepsilon_1$</td>
<td>$D_{8\text{crit}} = 460.6$ MPa</td>
</tr>
<tr>
<td>OH$^H$</td>
<td>$D_9 = \int_0^{\bar{\varepsilon}} d\varepsilon_1$</td>
<td>$D_{9\text{crit}} = 0.430$</td>
</tr>
<tr>
<td>AYAD$^H$</td>
<td>$D_{10} = \int_0^{\bar{\varepsilon}} \frac{\tau}{2} d\varepsilon_1$</td>
<td>$D_{10\text{crit}} = 0.143$</td>
</tr>
<tr>
<td>TREN$^H$</td>
<td>$D_{11} = \int_0^{\bar{\varepsilon}} \frac{\tau}{2} d\varepsilon_1$</td>
<td>$D_{11\text{crit}} = 172.7$ MPa</td>
</tr>
</tbody>
</table>

Table 2.14: Details of the fracture criteria calibrated on a tube tensile test for 316LVM

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Expression of the damage variable in tension</th>
<th>Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>STRN</td>
<td>$D_1 = \varepsilon_1$</td>
<td>$D_{1\text{crit}} = 0.409$</td>
</tr>
<tr>
<td>MSS</td>
<td>$D_2 = \tau_{\text{max}} = \frac{\sigma_1}{2}$</td>
<td>$D_{2\text{crit}} = 535.7$ MPa</td>
</tr>
<tr>
<td>SHAB</td>
<td>$D_3 = 1$</td>
<td>$D_{3\text{crit}} = 1$</td>
</tr>
<tr>
<td>FREU</td>
<td>$D_4 = \int_0^{\bar{\varepsilon}} \sigma_1 d\varepsilon_1$</td>
<td>$D_{4\text{crit}} = 345.5$ MPa</td>
</tr>
<tr>
<td>COCK</td>
<td>$D_5 = \int_0^{\bar{\varepsilon}} \max(0, \sigma_1) d\varepsilon_1$</td>
<td>$D_{5\text{crit}} = 345.5$ MPa</td>
</tr>
<tr>
<td>RICE</td>
<td>$D_6 = \int_0^{\bar{\varepsilon}} \exp(\frac{\sigma_1}{2\sigma_T}) d\varepsilon_1$</td>
<td>$D_{6\text{crit}} = 0.709$</td>
</tr>
<tr>
<td>BROZ</td>
<td>$D_7 = \int_0^{\bar{\varepsilon}} d\varepsilon_1$</td>
<td>$D_{7\text{crit}} = 0.430$</td>
</tr>
<tr>
<td>ARGO</td>
<td>$D_8 = \int_0^{\bar{\varepsilon}} \frac{\sigma_1}{3} d\varepsilon_1$</td>
<td>$D_{8\text{crit}} = 460.6$ MPa</td>
</tr>
<tr>
<td>OH</td>
<td>$D_9 = \int_0^{\bar{\varepsilon}} d\varepsilon_1$</td>
<td>$D_{9\text{crit}} = 0.430$</td>
</tr>
<tr>
<td>AYAD</td>
<td>$D_{10} = \int_0^{\bar{\varepsilon}} \frac{\tau}{2} d\varepsilon_1$</td>
<td>$D_{10\text{crit}} = 0.143$</td>
</tr>
<tr>
<td>TREN</td>
<td>$D_{11} = \int_0^{\bar{\varepsilon}} \frac{\tau}{2} d\varepsilon_1$</td>
<td>$D_{11\text{crit}} = 172.7$ MPa</td>
</tr>
</tbody>
</table>

2.6 Conclusion

This experimental work enabled to characterize material mechanical properties. Tensile tests on tubes enabled to identify a visco-plastic constitutive law. The influence of grain size on material properties was also addressed. This point is important when the decision of modifying the annealing temperature in the industrial process is taken. A small change in annealing temperature can have great consequence on the process and the final product. The tube bulge test suggested the anisotropic behaviour of the material.
circumferential and axial strains were measured to study anisotropy. This test enabled to confirm the results obtained from the tensile tests on oriented samples and confirmed the plasticity to be nearly isotropic.

Then, the material isotropy or anisotropy was investigated at the crystallographic scale and the tubes were analysed by means of X-ray diffraction. First, surface measurements were performed and revealed the tube inner and outer surfaces to present different preferred orientations. Second, volumetric measurements were performed at the ESRF on the ID11 beamline. These analyses enabled to draw pole figures and to characterise the material anisotropy at the crystallographic scale. A slight <111> fibre texture was observed but the intensity was not significant to ensure that the material is expected to exhibit an anisotropic behaviour. So far, the material is likely to exhibit isotropic plasticity.

In parallel, micro-hardness measurements were performed in the tube thickness and it was shown that the L605 tube exhibited heterogeneous mechanical properties in the tube thickness. As a consequence, due to the non-homogeneous mechanical properties in the tube thickness, it appears that the anisotropic approach can lead to a good average of the mechanical properties. As for the tube bulge test, it is expected that a FEM with anisotropic plasticity might model the tube drawing with better accuracy.

Finally, failure criteria were calibrated on tube tensile tests. Different failure criteria were computed depending on the strain rate, the grain size, and whether anisotropy was considered or not.

Calibrated failure criteria serve as a basis to evaluate material formability during tube drawing. However, in order to accomplish such a task, it is necessary to determine the experimental fracture limit. The latter can only be found by conducting experimental tube drawing tests which is the purpose of the following chapter.
Chapter 3

The conical mandrel tube drawing test

This chapter is dedicated to a drawing test that was developed in order to evaluate tube formability and to identify the remaining frictional and thermal properties that still have to be identified. The first method one can think to determine tube drawing limit is to perform a series of drawing tests with several mandrels of different diameters. When the use of a mandrel makes the drawing impossible the drawing limit is reached. This approach has mainly two drawbacks: it is time consuming due to the number of necessary experiments and the transition between feasible and non-feasible drawing pass is not accurately determined. The process limit can only be approached with an accuracy depending on the diameter range of the mandrels that were used. Thus, in order to fill in the disadvantages of the previously presented method, a drawing test was designed to find the drawing limit with a single drawing test. The success of this test relies on the design of a conical mandrel.
3.1 Description of the conical mandrel tube drawing test

The originality of this test relies on the design of the mandrel which was created to combine three drawing tests in a single one. Details about the tubes and the experiments are exposed in this part.

3.1.1 Mandrel geometry

The newly designed mandrel can be decomposed into three parts, each part having a distinct role during tube drawing. The different parts are presented in figure 3.1 and detailed below:

- part (1) of the mandrel is a cylindrical section of constant small diameter enabling a step of hollow sinking;
- part (2) of the mandrel is a cylindrical section of constant diameter such as outer and inner tube diameters are reduced but wall thickness is unchanged. The mandrel geometry corresponds to a classical mandrel drawing case;
- part (3) of the mandrel is the principal one. It consists in a conical part with a continuously increasing section.

The mandrel dimensions are detailed in table 3.1. The corresponding section and thickness reductions are also detailed for an initial tube dimension of 10.5×13 mm$^2$ and a die diameter of 11.5 mm. On the third mandrel part, drawing takes place with a mandrel presenting a progressively increasing diameter. As a consequence a range of thickness reduction (from 0 to 52.8%) with the corresponding section reduction (from 17.4 to 58.2%) is explored. The test starts from 0% and 17.4% thickness and section reductions respectively and ends up at failure. This way, the use of a single mandrel enables to know the thickness and section reductions at fracture with very good accuracy. Nominal mandrel dimensions with the corresponding imposed section and thickness reductions are detailed in table 3.1.

<table>
<thead>
<tr>
<th>Part (1)</th>
<th>Part (2)</th>
<th>Part (3): conical zone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ømin</td>
<td>Ømax</td>
<td></td>
</tr>
<tr>
<td>Nominal diameter (mm)</td>
<td>8.7</td>
<td>9.0</td>
</tr>
<tr>
<td>Section reduction (%)</td>
<td>16.9</td>
<td>17.4</td>
</tr>
<tr>
<td>Thickness reduction (%)</td>
<td>0</td>
<td>0.8</td>
</tr>
<tr>
<td>Length of the part (mm)</td>
<td>100</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 3.1: Nominal mandrel dimensions and expected section and thickness reductions
The conical mandrels used in the experiments were machined from straight mandrels made of hard steel. The total mandrel length was 1000 mm and the conical part was 700 mm long. Consequently the mandrel semi-cone angle was very small i.e. 0.053°. As a comparison, Bui et al. (2011b) performed the same kind of test but used a conical mandrel of shorter length (84.18 mm), a diameter ranging from 39.34 mm to 49.39 mm and a semi-cone angle of 5°. Thus, concerning the mandrel of this study, the machining was very difficult mainly due to the mandrel length and the semi-cone angle. Then, real mandrels dimensions were measured with a laser measurement system. The measured geometries of the three mandrels that were used are represented in figure 3.2. Each mandrel was used several times. Between each test, the mandrel geometry was measured by means of a laser measurement system. No significant dimensional variations were measured, the mandrel geometry remained unchanged after each drawing test.

Bui et al. (2011b) drawing test differs slightly from the mandrel drawing test described here. In their study, the test called mandrel drawing can be seen as a fixed plug test according to the vocabulary described in this thesis. The conical mandrel is a conical plug which is fixed at the end of a rod. The position of the mandrel inside the die and the imposed reductions are controlled by commanding the mandrel position with the rod.

3.1.2 Details on the tube and the dies

3.1.2.1 L605

The initial tube length was 1000 mm. The L605 tubes had an outer diameter of 13.1 mm and a wall thickness of 1.3 mm approximately. The precise initial tube dimensions and sections are detailed in the table 3.2.

The die angles have an influence on the drawing force as observed by different authors as detailed in part 1.1.2. To study the influence of die geometry on the drawing force, four series of drawing tests were conducted with dies having different semi-cone angles: 5°, 12.5°, 16° and 20°. The dies diameter was identical and equal to 11.5 mm. The die geometry is presented in figure 3.3.
3.1.2.2 316LVM

The initial tube length was 1000 mm. The 316LVM tubes had an outer diameter and wall thickness of 12.6 mm and 1.05 mm respectively. The precise initial tube dimensions and sections are detailed in the table 3.2.

In this series of tests, only one die was used. The die had a semi-cone angle of 12.5° and a diameter of 10.5 mm.

<table>
<thead>
<tr>
<th>Material</th>
<th>L605</th>
<th>316LVM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mandrel number</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Tube inner diameter (mm)</td>
<td>10.51</td>
<td>10.50</td>
</tr>
<tr>
<td>Tube outer diameter (mm)</td>
<td>13.11</td>
<td>13.11</td>
</tr>
<tr>
<td>Initial mean section (mm²)</td>
<td>48.26</td>
<td>48.39</td>
</tr>
</tbody>
</table>

Table 3.2: Initial tube dimensions used with the corresponding mandrel
3.1.3 Measurements made during the tests

The drawing tests were carried out in the same lubricant conditions and the same drawing speed as the conventional industrial process. Four different measurements were done during the conical mandrel drawing tests of L605 tubes.

- The drawing force ($F_{\text{Drawing}}$) was recorded by the use of a load cell inserted between the die and the die holding system (fig.3.1).
- The bench drawing speed was measured by means of a motion transducer that was linked to the tube gripper.
- The tube temperature was recorded by an infra-red camera Flir SC700 pointing at the die exit.
- The tube exiting the die was filmed by a high speed camera. The goal of the high speed camera was to film the tube fracture and to locate the fracture locus in the case it happened after the die.

Figure 3.4 presents the cameras positioned to observe the tube exiting the die during the conical mandrel drawing test.

Only the drawing force was measured during the conical mandrel drawing tests on 316LVM tubes.

In order to clarify the results obtained from the tests made on L605 and 316LVM tubes, both materials are presented in separate sections. A first part details the drawing tests performed on L605 tubes. Indeed, with this material, the measurements were more complete due to temperature measurements. Moreover this series of test was the very first that was done and consisted in a development test. As a consequence, the results of the drawing tests made with the 316LVM tubes are presented more briefly in a second part. This second series of tests enables to validate the testing method.
3.2 Results of the experimental drawing tests on L605 tubes

3.2.1 Drawing Force measurements

3.2.1.1 Observation of a single test

In this section, only the results of the tests performed with a die semi-cone angle of 12.5° are detailed. Figure 3.5 shows the drawing force and mandrel diameter at the die location versus mandrel displacement obtained during the three conical mandrel drawing tests. Generally speaking, the drawing force increases with increasing mandrel diameter. Looking more into details, a clear link appears between the drawing force and the mandrel diameter. For example, the mandrel 3 shows diameter irregularities while it is supposed to be straight. These irregularities are due to mandrel machining. Consequently, as shown in figure 3.5(c) the measured drawing force fluctuates following the mandrel diameter variations. Figure 3.6 superimposes the three drawing forces measured during the three different tests. In figure 3.6(a), the force is plotted as a function of the mandrel displacement. The displacement measurement related to the precise characterisation of the mandrel geometry enabled to know with very good accuracy the mandrel diameter at the die location at any time. The tube inner diameter was taken equal to the mandrel diameter and the tube outer diameter was considered equal to the die diameter. In this way, it was possible to compute tube section at any time and to compute the section and thickness reductions during the tests. Then, figure 3.6(b) presents the drawing force as a function of the section reduction. Both figures highlight the good repeatability of the tests as the three curves corresponding to the three drawing tests are very close. Figure 3.6(b) clearly shows the section reduction reached at maximum drawing force which corresponds to failure.

3.2.1.2 Influence of the die semi-cone angle

Figure 3.7 shows the evolution of the drawing force as a function of the mandrel diameter for different die angles. Three tests were performed for each die to ensure reproducibility. It can be observed that the drawing force shows little but not significant variations for the die semi-cone angles of 12.5°, 16° and 20°. In the special case of the die angle of 16° the drawing force drastically increase at the end of the test which is probably due to the die clogging. Indeed, each case the force presented such fluctuations, the die when removed from the drawing bench presented stuck material on the working part. In this case, the die were cleaned prior to any other test. The increase of die angle is expected to lower the amount of work of friction. On the contrary, it increases the amount of redundant work of deformation. It can be suspected that a balance exists between friction and redundant deformation leading to no visible influence of the die angle on the drawing force. Concerning the measured drawing force for the die with a 5° semi-cone angle, a first measure was made on a 50 kN drawing bench which stalled during the test. A second test was made on a 100 kN drawing bench but the force measurement failed. Consequently there are no force measurement for the 5° semi-cone die.
Figure 3.5: Drawing force and mandrel diameter versus displacement obtained for a conical mandrel drawing test: (a) mandrel 1, (b) mandrel 2, (c) mandrel 3.

Figure 3.6: (a) Force versus mandrel displacement and (b) force versus section reduction during the conical mandrel drawings.
3.2.2 Section and thickness reductions

The computed section reduction and measured force at fracture for a die angle of 12.5° are presented in table 3.3.

<table>
<thead>
<tr>
<th>Mandrel number</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section reduction at fracture (%)</td>
<td>54.17</td>
<td>53.37</td>
<td>53.91</td>
</tr>
<tr>
<td>Thickness reduction at fracture (%)</td>
<td>50.01</td>
<td>48.86</td>
<td>50.43</td>
</tr>
<tr>
<td>Maximum drawing Force (daN)</td>
<td>3861</td>
<td>3741</td>
<td>3710</td>
</tr>
</tbody>
</table>

Table 3.3: Maximum drawing force and corresponding section and thickness reduction at failure for a die angle of 12.5°

3.2.3 Fracture characterisation

3.2.3.1 High speed camera recording

The high speed camera recorded no failure during the tests and it can be said that failure occurred inside the die.

3.2.3.2 Fracture surface aspect

Photographies of the fractured tubes are presented in figure 3.8(a.1,b.1,c.1). It can be seen that fracture angle in the tube wall is about 45° to the drawing direction. The tube number 1 presented in fig.3.8(a.1) also shows a change in fracture surface orientation: fracture angle is unchanged but the normal to the surface alternatively points to the inside and to the outside of the tube. From these photographies, it can be observed that the fracture probably initiated at the exit of the die conical section, very close to the end of the die bearing length.

The sample observation by means of Scanning Electron Microscopy (SEM) reveals more details about fracture. Fractographies of the tubes reveal many cavities in the fracture
surfaces which is characteristic of a dimple like fracture. Some voids nucleated on inclusions of brittle carbide phase which are dispersed in the matrix (Poncin et al., 2005). Carbides particles can be observed in white in figure 3.8(b.4) which was taken with backscattered electron (BSE). After nucleation, the voids expanded and coalesced before causing full fracture.

Fractographies of the tubes also reveal the fracture propagation direction. It can be seen from figure 3.8(b.3,b.4) that there is a vertical scratch i.e. from the bottom to the top of the picture. The bottom and the top correspond to the inside and the outside of the tube respectively. More specifically in figure 3.8(b.4) it can be observed that all cavities are filled with a carbide except the cavity in the continuation of the scratch. The scratch starts from a cavity where the precipitate was initially. It indicates that the precipitate was extracted from its cavity and dragged along the surface from the bottom to the top of the picture which created a vertical scratch. So, considering SEM micrography, fracture initiated on the inner surface of the tube and propagated outward. The scratch type morphology also indicates a shear fracture. Li et al. (2011) observed such a morphology on fractographies of shear-induced tensile test.
3.2.3.3 Section and thickness reductions at fracture

3.2.3.3.1 Influence of the die angle

This part focuses on the influence of the die angle on the maximum section and thickness reductions. The measured section and thickness reductions as a function of the die angle are listed in table 3.4. It must be noticed that no value is entered for the smaller die angle as fracture did not occur in this case. Thus, section and thickness reductions are expected to be above 58.2% and 52.8% respectively. From the values detailed in table 3.4, it can be deduced that the maximum section and thickness reductions decrease with increasing die angle.

The increase of die angle is expected to lower the amount of work of friction which is expected to have little influence on the material formability. On the contrary, the amount of redundant work of deformation increases which seems to be much more detrimental for the tube. These hypotheses have to be studied with more details by means of the FEM. Finally, this result is very interesting for the industrial production. A change in the die angle can improve formability with little impact on the drawing force.

<table>
<thead>
<tr>
<th>Die semi-cone angle (°)</th>
<th>5</th>
<th>12.5</th>
<th>16</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean section reduction at fracture (%)</td>
<td>&gt; 58.2</td>
<td>53.8</td>
<td>50.7</td>
<td>49.9</td>
</tr>
<tr>
<td>Mean thickness reduction at fracture (%)</td>
<td>&gt; 52.8</td>
<td>49.8</td>
<td>47.2</td>
<td>45.4</td>
</tr>
</tbody>
</table>

Table 3.4: Section and thickness reduction at fracture as a function of die semi-cone angle

3.2.3.3.2 Influence of the grain size

It was mentioned in part 2.3.3 that the annealing temperature has an effect on the mechanical behaviour of the material. Thus, it is important to evaluate the consequences of a change of annealing temperature on material formability. To do so, conical mandrel drawing tests with a die semi-cone angle of 12.5° were performed on tubes presenting different grain sizes depending on the temperature of the thermal treatment. The different L605 materials are the same as the one tested in tensile tests. The different annealing temperatures and the corresponding grain sizes are remembered in table 3.5.

The maximum section and thickness reductions at fracture for the different materials are shown in table 3.5. The material with lower annealing temperature and smaller grain size fractures at lower section and thickness reductions compared to the material with bigger grain size. This observation is consistent with the results of the uniaxial tensile tests performed on tubes with different grain sizes: strain at fracture in uniaxial tensile test is lower for the smaller grain material.

<table>
<thead>
<tr>
<th>Annealing temperature (°C)</th>
<th>1100</th>
<th>1150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grain size (μm)</td>
<td>11.2</td>
<td>22.5</td>
</tr>
<tr>
<td>Mean section reduction at fracture (%)</td>
<td>53.3</td>
<td>53.8</td>
</tr>
<tr>
<td>Mean thickness reduction at fracture (%)</td>
<td>49</td>
<td>49.8</td>
</tr>
</tbody>
</table>

Table 3.5: Section and thickness reduction at fracture as a function of grain size
3.2.4 Thermal measurements

The temperature field induced by the plastic deformation and by the friction was measured during the conical mandrel drawing test. The infra-red camera was placed along the bench and focused on the die exit. Contrary to the tensile tests, where the tubes were painted black, the tubes were raw in the drawing tests. As a consequence, tube emissivity could not be approximated by 0.95 since the tube cannot be considered as a black body. Thus tube emissivity was calibrated.

3.2.4.1 Calibration of the emissivity

It was detailed in part 3.2.3.2 that tube surface aspect was getting shinier with increasing section reduction. Tube roughness decreases with increasing plastic deformation. As a consequence, tube emissivity, which is dependent on surface aspect (Wen and Mudawar, 2006; Fu et al., 2012), varies during the test with mandrel diameter. Thus emissivity calibration has to take into account the variation of surface aspect. Moreover at the die exit, the tube is more or less still lubricated and the presence of lubricant changes emissivity compared to dry tube. Emissivity calibration must consider two aspects:

- the variation of surface roughness during the test,
- the presence of lubricant.

A specific calibration test was made in order to calibrate the tube emissivity as a function of the above mentioned parameters.

Two samples were cut from a drawn tube produced from the conical mandrel drawing test. The tube was not removed from the mandrel in order not to modify surface aspect caused by the reeling step. The first sample was cut on the part of the tube corresponding to the beginning of the test, where the section reduction is minimum and the surface roughness the highest. It is referred as sample 1. The second sample was cut on the part of the tube the closest to tube fracture, where the section reduction is maximum, the surface aspect the shiniest and the roughness the lowest. This sample is referred as number 2. Samples were divided into five zones. Photographies of the samples in figure 3.9(a) illustrate the different zones. Tubes extremities and a middle band were painted in black. The black paint enabled to approximate tube emissivity by 0.95. The presence of two bands at the extremities and one at the middle enabled to control the temperature gradient along the tube. One of the uncovered band was left raw (R-zone) and the other was covered with lubricant (L-zone). The lubricant was applied to obtain a fine layer to get closer to drawing conditions. But the thickness of the lubricant layer was not controlled and was supposed to be homogeneous. Both samples were put in a oven at 120°C during 120 minutes. Samples were removed from the oven and temperature field was measured during cooling with an emissivity of 0.95.

The measured temperature in the black zone was taken as reference temperature. Figure 3.9(b) illustrates the acquisition made by the infra-red camera at the beginning of the test (t=0). Temperatures are computed considering an emissivity of 0.95. It is clearly visible that the measured temperatures differ depending on the considered zone. These variations are due to an incorrect estimation of the material emissivity as the temperature
field is supposed to be homogeneous. Then the emissivity was calibrated so that the temperature evolution as a function of time in the lubricated zone corresponds to the reference temperature. Same method was done for the raw zone. This method was applied for both samples.

Figure 3.9(c) shows the evolution of the as identified emissivities as function of the temperature. From this figure, there are several remarks concerning the variations of emissivity. First emissivity increases with increasing temperature. Second, the emissivity is higher for the lubricated tube than for the raw tube. Third, the emissivity increases with increasing surface roughness.

Figure 3.10 presents the validation of the identified emissivities. This figure presents the temperature evolution during cooling of the reference zone (black zone with an emissivity of 0.95). Below the reference curve is plotted the measured temperature of the lubricated zone considering an emissivity of 0.95. And superimposed to the reference curve is plotted the measured temperature of the lubricated zone considering the identified emissivity as a function of temperature. It can be seen that the temperature evolutions of the reference zone and of the lubricated zone are identical. As a consequence, the emissivity calibration is validated.

3.2.4.2 Measured temperatures

Figure 3.11 displays pictures of the conical mandrel drawing test. Figure 3.11(a) shows a general view of the tube going through the die. Figure 3.11(b) presents the corresponding thermal measurements. In a general way, a gradient of temperature is clearly visible. The farther from the die, the cooler becomes the tube. This phenomenon is explained by the fact that the heat which is generated at the die position transfers to the mandrel by conduction and to the environment by convection principally. Lee et al. (2012) performed temperature measurements during wire drawing. Parallel to the FEM of wire drawing, they found the tube surface temperature to be greater than the temperature at the center of the tube. They explained this difference stating that friction at the interface between the die and the wire caused the temperature to increase more than the overall heat generated.
by plastic deformation. Thus, the measured temperatures are surface measurement and correspond to the contribution of both friction and plastic deformation.

Looking more into details, on both pictures 3.11(a and b) and more particularly on figure (b), some helical grooves are clearly visible. These grooves are due to the mandrel removal step that is done prior to any mandrel drawing pass. Due to the grooves, local thermal variations are measured as seen in figure 3.12. The temperature variation is approximately $3.7^\circ C$. Next paragraph aims at explaining the origin of such local variations.
There are two hypotheses concerning these variations. First, grooves, which are surface defects clearly visible on the tube, are suspected to induce local emissivity variations which result in temperature measurement fluctuations. Secondly, grooves are wavelet which are formed on the tube surface during mandrel removal. Figure 3.13 shows a measure of surface profile by means of a profilometer. The tube surface shows bumps and hollows with an amplitude of $2 \mu m$. When the tube is plastically deformed inside the die, bump deforms...
more than hollows and then more heat is generated. The difference in heat generated by plastic deformation was quantified to be small. Thus, the hypothesis of local emissivity variations is selected.

In spite of the above observations, it is clear that the measured thermal fluctuations are small compared to the absolute temperatures that are measured. As the present study does not intend to analyse these temperature fluctuations, the measured temperatures are averaged. Not much information can be directly deduced from the temperature measurements as presented above. Nevertheless, these data analysed in parallel with FEM will enable to identify thermal properties such as the amount of heat generated by friction, thermal contact conductance or the heat sharing coefficient between two contacting materials. As a consequence temperature measurements are analysed with more details in chapter 4.

### 3.3 Results of the experimental drawing tests on 316LVM tubes

The results concerning the 316LVM tubes are limited to the section and thickness reduction at fracture. The values are listed in table 3.6.

<table>
<thead>
<tr>
<th>Mandrel number</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section reduction at fracture (%)</td>
<td>68.5</td>
<td>62.9</td>
</tr>
<tr>
<td>Thickness reduction at fracture (%)</td>
<td>66.4</td>
<td>60.4</td>
</tr>
</tbody>
</table>

Table 3.6: Section and thickness reductions at fracture for the 316LVM tubes

### 3.4 Conclusion

In this chapter, the handling of conical mandrel drawing tests enabled to draw L605 and 316LM tubes up to fracture. This way, the maximum section and thickness reductions a tube can undergo were identified precisely. A major conclusion directly transferable to the industrial production was revealed: the tube formability can be improved if the die angle
is decreased.
Once the experimental tube formability is known, next step is to predict its occurrence. To do so, it is necessary to build a tool enabling the computation of failure criteria. This tool is the FEM.
As a reminder, the elements necessary to build a model of tube drawing are:

- the material behaviour constitutive equation. This aspect was addressed in chapter 2;
- the friction coefficient and the different thermal properties. It was previously explained that the conical mandrel drawing test was also designed to identify these properties. But from this point, they have not been determined yet. In order to characterize them, it is necessary to first build a FEM and to identify the parameters by inverse analysis.

Finally, the interest of the conical mandrel drawing test was first to collect force measurements that enable to identify the friction coefficient by inverse analysis and second to measure thermal fields that enable to identify thermal properties by means of inverse analysis. Consequently, next step consists in building the FEM. This stage is detailed in the following chapter.
Chapter 4

Analysis of the tube drawing

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The analysis of the tube drawing process has two major objectives. The first interest is very pragmatic in an industrial context: it enables to compute the drawing force which is crucial to define the drawing passes and to select the appropriate drawing bench depending on its power. The second objective is to improve the understanding of the process in term of stress and strains undergone by the tube during the process. Such information are essential to master the process. They can explain the consequences of a change of process parameters on the tube drawing and improve the understanding of the process. Finally, in the specific context of this thesis, stress and strain data are necessary to compute the failure criteria and to predict failure during drawing.

There are two means to perform the process analysis. The first technique is FEM. The second method groups together different analytical methods. These methods were created when the FEM was not developed yet or when the computing power was not sufficient to...
run complex calculations. Both of these techniques are presented in detail in this chapter and applied to the analysis of the tube drawing process.

A first part is devoted to the presentation of the FEM and the manner different models were built. The methodology for their construction is detailed and the simulations resulting from the FEM are analysed.

A second part is dedicated to the analytical methods, from their construction to the computed drawing force. Their reliability and their limitations are evaluated and compared with the FEM.

### 4.1 FEM of the tube drawing

The FEM of the tube drawing process aims at computing the stress and strain fields with respect to process and material characteristics. Developing a FEM requires several steps including the geometry definition, the material characterisation, the contact behaviour identification and the validation by comparison with experiments. In this thesis, both tube drawings on L605 and 316LVM tubes were modelled, but in the current part, only the model concerning the L605 is presented as the construction is identical for both materials. The model devoted to the L605 was next directly extended to the 316LVM.

#### 4.1.1 General presentation of the model

There are two different solvers available in Abaqus: explicit and implicit. The explicit solver was first developed for dynamic problems but it is also well-suited for non linear quasi-static problems because of its ability to handle complex contact problems. In the case of the implicit solver, a complex contact induces more repetitive calculations and thus the computation is more time consuming. As a consequence, the choice was made to develop the model in ABAQUS/Explicit. Moreover, the choice is supported by the numerous metal forming process FEM that were performed in ABAQUS/Explicit (Abaqus, 2010).

##### 4.1.1.1 Geometry

The geometry of the system enabled to make great simplifications. Considering that the mandrel, the die and the tube were perfectly coaxial the model was reduced to an axisymmetric geometry. Figure 4.1 illustrates the different parts to be modelled and their geometries. The tube, the die and the mandrel were modelled as deformable solids. For further simplifications, both the mandrel and the die could have been modelled with rigid bodies but the contact between rigid bodies and deformable solid induces local stress variations which in turn induces greater variations of the drawing force (Affagard, 2010). In the case of an elastoplastic model, a tube length of 15 cm was modelled. This length was sufficient to reach steady state conditions. In the specific case of the conical mandrel drawing, the initial mandrel length of 1 m was also reduced to 15 cm. Thus, the modelled mandrel angle was increased compared to the real mandrel angle. Such geometrical variation was confirmed to induce no variations in the stress and strain fields. In the case
Table 4.1: Material properties of the die and the mandrel

<table>
<thead>
<tr>
<th></th>
<th>WC</th>
<th>steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (GPa)</td>
<td>650</td>
<td>210</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Density ($\text{kg m}^{-3}$)</td>
<td>$15 \times 10^3$</td>
<td>$7.9 \times 10^3$</td>
</tr>
<tr>
<td>Conductivity ($\text{W m}^{-1} \text{K}^{-1}$)</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>Specific Heat ($\text{J kg}^{-1} \text{K}^{-1}$)</td>
<td>234</td>
<td>500</td>
</tr>
</tbody>
</table>

of a thermo-mechanical model, the real mandrel geometry was modelled due to the thermal transfer aspect.

All other part dimensions were measured and drawn following their real dimensions:

- The tubes and mandrel diameters were measured with a laser measurement system;
- the die diameter was measured by means of a non contact measurement system called Micro-Vu. This system consists in measuring the projected shadow of the object to be characterised. The resolution is 1 $\mu$m.
- the die profile (fig.4.1.(b)) including the die semi-cone angle $\alpha$, the bearing length $l_b$ and the cone exit radius of curvature $r$ were measured by means of a profilometer. Apart from the die angle, the geometrical characteristics of the die remained unchanged throughout this study.

![Figure 4.1: (a) geometry of the tube and tools, (b) dimensional details of the die](image)

4.1.1.2 Material properties

4.1.1.2.1 Tools materials

The die was made of tungsten carbide (WC) and the mandrel was made of hard-steel. Both tools were modelled as elastic materials. Their mechanical and thermal properties are detailed in table 4.1.

4.1.1.2.2 Tube materials

The mechanical behaviours of both the L605 and the 316LVM were characterised by means of experimental tests as detailed in the chapter 2. Three different models were defined according to the tube material properties that were identified:
• in a first model \((M1)\) the material is considered as isotropic and visco-plastic;
• in a second model \((M2)\) the material is considered as isotropic, visco-plastic and the thermal effects are included;
• in a third model \((M3)\) the material is considered as anisotropic and the visco-plastic behaviour is neglected. Material properties corresponding to the different models are detailed further in respective parts.

### 4.1.1.3 Boundary conditions

The mechanical boundary conditions were identical for all the above detailed models \((M1, M2, M3)\). Figure 4.2 summarises the applied boundary conditions. The die was fixed and a displacement was applied to both mandrel and tube extremities. The displacement was chosen to reach a constant drawing speed. In the thermomechanical model \(M2\) the die was fully modelled: compared to the mechanical models where only the die core was considered, the thermal exchanges phenomena required the whole die volume to be modelled. Thus, as illustrated in figure 4.2(b) a part was added to the die core to increase its volume. Thermal boundary conditions were applied to the model \(M2\). All the parts were put at an initial temperature of 20°C and natural convection was applied to surfaces in contact with the air. The convection exchange coefficient was taken equal to 14.4 W m\(^{-2}\) K\(^{-1}\) as identified during the tube tensile test (part.2.4.2). The hypothesis was made that natural convection conditions were similar in the testing room and in the industrial production room.

### 4.1.1.4 Mesh

All the parts were modelled with 4-node bilinear axisymmetric quadrilateral elements with reduced integration (CAX4R). Reduced integration enables to reduce the running time. When thermal coupling was considered, the mesh elements were thermally coupled (CAX4RT). The tube was meshed with a minimum of 8 elements in the thickness (Palengat, 2009). Both the mandrel and the die were coarsely meshed with a refined zone near the contact zone. The element size of the die and mandrel surfaces contacting the tube was chosen identical to the tube mesh size. Figure 4.3 shows a mesh example.

### 4.1.1.5 Time incrementation

A mass scaling option was used to speed up the computation. The principle of mass scaling is to artificially increase the material density to lower down the number of increments required for the simulation. The following paragraph aims at explaining the mass scaling principle. The time increment \(\Delta t\) is defined by the time a dilatational wave with a speed \(c_d\) takes to cross the smallest element in the mesh. It is defined as follow:

\[
\Delta t \approx \frac{L_{\min}}{c_d}
\]  

(4.1)

with \(L_{\min}\) the smallest element length. The dilatational wave speed is computed as follow:

\[
c_d = \sqrt{\frac{\lambda + 2\mu}{\rho}}
\]  

(4.2)
Figure 4.2: Boundary conditions applied to the tube and tools

Figure 4.3: Example of mesh
with $\lambda$ and $\mu$ the Lamé's coefficients and $\rho$ the material density.
The time increment expression then transforms:

$$\Delta t \approx L_{\text{min}} \sqrt{\frac{\rho}{\lambda + 2\mu}}$$

(4.3)

If the time increment $\Delta t$ remains constant during the simulation, the number of time increments $n$ required to simulate the total time period $T$ is:

$$n = \frac{T}{\Delta t} \quad \text{and} \quad n \approx T \left( \frac{1}{L_{\text{min}}} \sqrt{\frac{\lambda + 2\mu}{\rho}} \right)$$

(4.4)

Thus, artificially increasing the mass density by a factor $f^2$ reduces the number of time increment by $\frac{1}{f}$. The total time period is unchanged and rate-dependent behaviour can still be considered in the analysis.

Nevertheless, mass scaling can cause computational errors and must be used with care. If material density is increased too much, the increased inertial forces change the predicted response. It must be ensured that the ratio of kinetic energy to internal energy stays lower than 5%. Finally, Abaqus documentation mentions that thermal solutions for coupled thermal-stress analysis are not affected by the mass scaling. Densities associated to thermal phenomenon are not scaled.

Palengat (2009) previously selected the appropriate target fixed time increment that is convenient for tube drawing modelling. In the present study and according to Palengat (2009), the target time increment for mass scaling was then set to $10^{-6}$ s. For each simulation, it was ensured that the kinetic energy named ALLKE in ABAQUS was lower than 5% of the internal energy named ALLIE.

### 4.1.1.6 Contact definition

Two contacting surfaces transmit shear and normal stresses across their interface. In this study, friction between two surfaces is modelled by a Coulomb friction model. The model assumes that no relative motion occurs between the two contacting surfaces if the equivalent frictional stress $\tau$ is less than the critical stress $\tau_{\text{crit}}$:

$$\tau_{\text{crit}} = \mu p$$

(4.5)

with $p$ the contact pressure and $\mu$ the friction coefficient that can be defined as a function of the contact pressure. When $\tau = \tau_{\text{crit}}$ slip occurs.

Surface-to-surface contact option was chosen. Contact definition relies on the selection of a master and a slave surface. The surfaces belonging to the more rigid surfaces are defined as the master surfaces i.e. the die and the mandrel surfaces. The slave surface was attributed to the tube. There are two algorithms for contact: kinematic and penalty. Kinematic contact algorithm is used when a rigid surface contacts a deformable surface and penalty contact algorithm is used when an analytical rigid surface contacts a deformable surface. Thus, the first option was chosen.

Up to this point, all the parameters shared by the models $M_1$, $M_2$ and $M_3$ were defined. The following sections present each model separately and detail their different characteristics.
4.1.2 Isotropic models

In this section, the L605 and 316LVM material properties associated to both $M_1$ and $M_2$ isotropic models are presented.

4.1.2.1 Mechanical model $M_1$

In the model $M_1$, the tube material is considered as isotropic and visco-plastic. The visco-plastic behaviour is modelled by a Johnson-Cook constitutive model whose parameters are inserted in ABAQUS/Explicit. The different material properties are recalled in table 4.2.

4.1.2.2 Thermo-mechanical model $M_2$

The second model ($M_2$) considers isotropy, viscoplasticity and thermal effects. The viscoplastic behaviour is modelled by a Johnson-Cook constitutive model identical to the model $M_1$. Concerning the thermo-mechanical properties, the amount of plastic work converted into heat is characterized by the Taylor-Quinney coefficient $\beta$ also named Inelastic Heat Fraction (IHF) in Abaqus. In chapter 2, it was shown that the Taylor-Quiney coefficient was strain dependent. In Abaqus/Implicit, a subroutine called HETVAL enables to take into account the IHF dependency with strain. HETVAL subroutine enables to compute a volumetric heat flow $r$ from the stress and strain data.

$$r = \beta(\bar{\epsilon})\sigma : \dot{\varepsilon}$$

(4.6)

where $\beta(\bar{\epsilon})$ is the strain dependent Taylor-Quinney coefficient.

Such subroutine is not available in Abaqus/Explicit. As a consequence, a mean Taylor-Quinney coefficient was calculated and inserted into the model. The mean Taylor-Quinney is computed as follow:

$$\beta_{\text{mean}} = \frac{\int (\beta\sigma : \dot{\varepsilon})\, dt}{\int (\sigma : \dot{\varepsilon})\, dt}$$

(4.7)

The different mechanical and thermal properties are detailed in table 4.2. Concerning the contact properties, the thermal contact conductance can be defined in function of the distance separating both surfaces, the contact pressure and the temperature. In this study, it was defined in function of the separation distance only and varies linearly with it. The thermal contact conductance equals zero for a separation distance greater than 0.1 mm and is maximum for a separation distance equals to zero. The maximum thermal contact conductance was identified by means of an inverse analysis. Its identification is detailed further.

The whole frictional work is supposed to convert into heat. The heat sharing coefficient which defines the amount of heat distributing between the master and slave surfaces is defined by computing the material effusivity defined in part 1.2.3.3.
Table 4.2: Material properties of L605 and 316LVM in function of the model

4.1.3 Anisotropic model

In the model M3, the plastic anisotropy is taken into account but the visco-plastic behaviour is neglected. The anisotropic yield is modelled by Hill’s potential function whose expression was detailed in part 1.2.1.1.2. The anisotropic Hill’s potential function was not used combined to Johnson-Cook constitutive equation because both models were not available simultaneously in ABAQUS/Explicit. The plastic yield stress in function of the plastic strain was defined by entering the \(\sigma_z vs \epsilon_z\) data obtained from the experimental tensile test at a strain rate of \(9 \text{s}^{-1}\). As the data extracted from the tensile tests were limited to strains up to 0.45 approximately, data for higher strains were extrapolated by considering a Ludwik constitutive equation (2.5). In the case of anisotropic plasticity, the stresses in the other directions are computed from the reference axial data by means of the anisotropic Hill’s coefficients. The different material properties are detailed in table 4.3.

Table 4.3: Material properties of L605 for the anisotropic model (M3).

4.1.4 Identification of the friction coefficient

Most of the models parameters were defined in the previous sections except the friction coefficients and the thermal contact properties. This part is devoted to the identification of the friction coefficients by inverse analysis. Such method was found to be the best adapted to identify a specific friction coefficient and to be sure that the latter is characteristic of the process. It enables to ignore the different parameters influencing friction like materials roughness, relative sliding velocity, thickness of the oil film and humidity.
4.1.4.1 Identification of constant friction coefficients

Two friction coefficients had to be calibrated independently: the friction coefficient between the mandrel and the tube $\mu_{\text{mandrel/tube}}$ and the friction coefficient between the die and the tube $\mu_{\text{die/tube}}$. The identification of both friction coefficients was done by means of the conical mandrel drawing test. The main principle of the inverse analysis is to simulate a specific drawing test with friction coefficient values chosen so that the simulated drawing force is equal to the experimentally measured one. The strategy was to first calibrate $\mu_{\text{die/tube}}$ using the hollow sinking step shown in figure 3.1 part (1). Indeed, on this part of the mandrel, friction takes place between the die and the tube only. Then $\mu_{\text{mandrel/tube}}$ was successively calibrated using the constant diameter mandrel drawing step illustrated in figure 3.1 part (2). Indeed on this mandrel part, the mandrel/tube contact is added to the die/tube contact.

According to the first identification performed on the tube sinking step and on the constant mandrel diameter drawing step, the following values were assigned to the friction coefficients: $\mu_{\text{die/tube}} = 0.065$ and $\mu_{\text{mandrel/tube}} = 0.1$.

The simulation of the full conical mandrel drawing was then performed with the previously identified parameters. The comparison of the experimental and simulated drawing force is shown in figure 4.4. The superimposition of both forces in function of time can be decomposed into three different parts. The drawing force measured and simulated in the time interval $[0,0.5]$ s corresponds to the tube hollow sinking (part (1)), the time interval $[0.5,2]$ s corresponds to the drawing on a constant diameter mandrel (part (2)) and finally, after 2 s corresponds to the drawing force measured or simulated during the conical mandrel drawing (part (3)).

It can be observed from this figure that experimental and simulated drawing forces superpose correctly up to 3 s approximately. After this point, the simulated drawing force is over-estimated. Such a result is due to a friction coefficient value which is too high. This observation highlights the fact that a constant friction coefficient is unable to model contact behaviour. The hypothesis of a pressure dependent friction coefficient can
be formulated. In order to investigate this possibility it seems interesting to analyse the normal stress in the die/tube and mandrel/tube contacts.

4.1.4.2 Analysis of the normal contact stress

Figure 4.5 shows the evolution of the normal contact stress in function of the position in the die/tube contact zone (fig.4.5(a,b)) and in the mandrel/tube contact zone (fig.4.5(c,d)). Three simulations were done for three different mandrel diameters: \( \varnothing_{\text{mandrel}}^1 = 9.0 \text{ mm} \), \( \varnothing_{\text{mandrel}}^2 = 9.32 \text{ mm} \) and \( \varnothing_{\text{mandrel}}^3 = 9.78 \text{ mm} \). Thus, the evolution of the normal contact stress can be interpreted as function of the mandrel diameter.

- Starting with the die/tube contact, it can be seen in figure 4.5(b) that the normal contact stress in the tube varies along the contact and oscillates from 0 to 2200 MPa. The contact pressure near the die shows a first peak when the tube establishes contact with the mandrel. Then, it lowers down and reaches a second peak at the exit of the die cone to finally end up at 0 MPa when the tube exits the die. The tendency is similar for all mandrel diameters but the contact pressures increase with increasing mandrel diameter. More generally, the contact pressure profile along the die/tube contact has two peaks, one near the die entry and the other at the die exit. Same observations were made by Dixit and Dixit (1995).
- Concerning the mandrel/tube contact (fig.4.5(d)), normal contact stress also varies along the contact and increases with increasing mandrel diameter.

The main conclusions resulting from this analysis are:

- contact pressures vary locally along the tool/tube contacts;
- the global contact pressure increases with increasing mandrel diameter;
- higher contact pressures are reached in the die/tube contact compared to the mandrel/tube contact.

4.1.4.3 Identification of pressure dependent friction coefficients

Putting the above observations in parallel with other studies, Petersen et al. (1997) compared the use of a constant friction coefficient and a pressure dependent one in bulk metal forming and found a better adequacy between experimental and numerical data with a pressure dependent friction coefficient. Ma et al. (2010) tried to explain the physical meaning of friction dependency with pressure. They modelled the pressure dependency of the friction coefficient taking into account the evolution of the surface contact morphology with the pressure. They found that the friction coefficient decreased with increasing pressure. Azushima and Kudo (1995) defined pressure intervals in which friction coefficient behaviour varies. They defined a low pressure interval \( p < 0.3\sigma_0 \), with \( \sigma_0 \) the yield stress where the friction coefficient was constant, and a high pressure condition \( p > 0.3\sigma_0 \) where the friction coefficient decreased with increasing pressure. Finally, the referenced studies and the observations made on the simulations reinforce the need to take into account the pressure dependency of the friction coefficient.
Finally, pressure dependent friction coefficients were identified by inverse analysis. The die/tube friction coefficient was found to vary from 0.35 to 0.002 for pressures higher than 1000 MPa and the mandrel/tube friction coefficient varied from 0.09 to 0.01. It must be noted that the mandrel/tube friction coefficient had little influence on the simulated drawing force. As the mandrel and the tube move approximately at the same speed, friction is expected to be low at the mandrel/tube contact. The set of identified pressure dependent friction coefficients are listed figure 4.6.

Figure 4.7 plots in parallel the normal and shear contact stresses and the friction coefficient for both the die/tube contact (fig.4.7(a-c)) and the mandrel/tube contact (fig.4.7(d-f)). The identified pressure dependent friction coefficients were used to compute the plotted data. The evolution of the normal and shear contact stresses and friction coefficient can be interpreted as function of the mandrel diameter:
Figure 4.6: Dependence of the friction coefficients to the contact pressure

- concerning the die/tube contact, figure 4.7(b) shows irregular variations of the shear contact stress. Shear stress values oscillate from 0 to 120 MPa which is largely lower than contact pressure. Figure 4.7(c) presents the friction coefficient computed from the ratio of the shear stress (fig.4.7(b)) to normal stress (4.7(a)). It can be seen from figure 4.7(c) that the friction coefficient varies locally along the contact and globally decreases with increasing mandrel diameter. The comparison of fig.4.7(a) and fig.4.7(c) puts in relief that the friction coefficient decreases with increasing contact pressure;
- in the case of the mandrel/tube contact, the shear contact stress (fig.4.7(e)) shows little variations and is very low. As a consequence, the computed friction coefficient in figure 4.7(f) is also low.

Finally, the superimposition of the experimental and simulated drawing forces is shown in figure 4.8. It can be seen that the sets of identified friction coefficients enables to model the evolution of the drawing force with the mandrel diameter accurately.

4.1.5 Thermal contact properties

Up to this point, the thermal contact properties remain to be characterized. The identification of the missing parameters is presented in this part.

4.1.5.1 Interfacial Heat distribution

In the present study, 100% of the frictional work was supposed to be converted into heat. The generated heat was supposed to distribute between the two surfaces proportionally to the material effusivities \( \xi = \sqrt{\rho C_p k} \). The fraction of heat \( f_{AB} \) distributing to a material \( A \) that belongs to a contact \( AB \) writes:

\[
    f_{AB} = \frac{\xi_A}{\xi_A + \xi_B} \tag{4.8}
\]

with \( \xi_A \) and \( \xi_B \) the effusivities of the A and B materials respectively.

The different material effusivities are the following detailed in table 4.4:
Figure 4.7: Evolution of the contact stresses and the friction coefficient with the position along (a-c) the die and (d-f) the mandrel and depending on the mandrel diameter.

Figure 4.8: Validation of the pressure dependent friction coefficient.
Table 4.5: Fraction of heat distributed to contacting materials

<table>
<thead>
<tr>
<th>Material</th>
<th>$\xi$ (J.K$^{-1}$.m$^{-2}$.s$^{-0.5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L605</td>
<td>6.64 x 10$^3$</td>
</tr>
<tr>
<td>Steel</td>
<td>14.1 x 10$^3$</td>
</tr>
<tr>
<td>WC</td>
<td>18.7 x 10$^3$</td>
</tr>
</tbody>
</table>

Table 4.4: Material effusivities

These materials are involved in different contacts. The die/tube contact takes place between the L605 and the WC while the mandrel/tube contact is established between the L605 and the steel. Table 4.5 lists the fractions of heat distributed to the materials involved in the die/tube and mandrel/tube contacts.

4.1.5.2 Thermal contact conductance

As high contact pressures were observed within the contact, gap conductance was supposed infinite ($1 x 10^8$ W.m$^{-2}$.K$^{-1}$) for a clearance of 0 mm and equal to zero for a clearance greater than 0.1 mm. The clearance defines the distance between two surfaces.

4.1.6 Results of FEM

4.1.6.1 Heat generation and exchanges

Figure 4.9 presents the comparison of the experimental temperature field that was measured during the conical mandrel test with the raw material emissivity (fig.4.9.a) and the temperature field computed from the simulation (fig.4.9.b). The geometry in figure 4.9.b was plotted in 3D to improve clarity but the model was axisymmetric. Both temperature fields are plotted with the same temperature scale. From this picture, it can be observed that the simulated temperature is greater close to the die and that it decreases when moving away from the die up to temperature values lower than the experimental one. The simulated temperature gradient from the die to the tube extremity is greater than the experimental one.

The experimental mean temperature nearby the die exit was measured on the one hand with the material emissivity calibrated on the lubricated tube and on the other hand with the material emissivity calibrated on the raw tube. Both measurements were compared to the simulated temperature taken at approximately the same distance from the die. Both data are plotted in function of the mandrel diameter in figures 4.10 (a) and (c) for the emissivity calibrated on the lubricated and raw tube respectively.

In these figures, the temperatures measured for the three conical mandrels are relatively
Figure 4.9: Comparison of the (a) experimental and (b) FEM temperature fields (the 2D calculus is represented in 3D for clarity).

Figure 4.10: Evolution of the mean temperature nearby the die exit in function of the mandrel diameter: (a) emissivity of the lubricated material, (c) emissivity of the raw material. Absolute error between FEM and experimental temperatures: (b) emissivity of the lubricated material, (d) emissivity of the raw material.
close to each other. Nevertheless, when the emissivity is calibrated on the lubricated tube (fig.4.10(a)), the FEM largely over-estimates the tube temperatures as seen in figure 4.10(b) where the relative errors between FEM and experimental data are plotted. Estimation error ranges from 10\% to 48\% which is not acceptable. On the contrary, if the emissivity is calibrated on the raw tube (fig.4.10(c)), the FEM prediction is in good accordance with the experimental measurements. Estimation error ranges from 0\% to 16\% as presented in figure 4.10(d).

From these observations, it can be observed that the emissivity calibration can induce large variations in the measured temperature. Thus, it is crucial to calibrate the emissivity accurately. Concerning the calibration that was performed in this study, the result showing that the measured temperatures are closer to the modelled ones when the emissivity is calibrated on the raw tube reveals that:

- the lubricant layer remaining on the tube after drawing is very thin. Intuitively, it is clear that the spacing between the tube and the die during drawing is very small, leaving little room for the lubricant layer;
- the lubricant layer that was manually applied on the tube for the emissivity calibration was too thick.

Finally, provided that the temperature is measured considering the emissivity calibrated on the raw tube, the thermo-mechanical model is satisfactory. From this point, the emissivity of the raw tube is taken for the thermal analysis. The model accuracy can be further confirmed considering the beginning of the conical mandrel drawing test. When a conical mandrel test is performed, in a first time, the tube is drawn by tube sinking and in a second time the tube is drawn on a constant diameter mandrel. As a consequence, when the temperature is measured during both of these steps a temperature drop is observed when the tube establishes contact with the mandrel. The measured temperature drop is shown in figure 4.11 and the values are reported in table 4.6. The temperature drop is supposed to be due to material contact only. Indeed, tube section and thickness reductions during tube sinking and mandrel drawing are very close and as a consequence the plastic deformations are approximately similar. The mean section and thickness reduction values are reported in table 4.6. The mandrel/tube contact establishment was simulated by means of FEM and the simulated temperature was compared with the experimental one. The simulated temperature drop was 15.2\textdegree C which is satisfactory compared to the experimental temperature drop of 18\textdegree C. As a conclusion, regarding the surface temperature, the thermo-mechanical FEM model is able to model the temperature field accurately.

Then, it appears interesting to analyse the simulated temperature inside the tube. Figure 4.12 presents the simulated temperature during the conical mandrel drawing. Data are plotted for a mandrel diameter of 10.16 mm. It can be observed that the tube which is located inside the die is at a higher temperature compared to the surface located at $T_{exit}$. Figure 4.13 presents a plot of the maximum simulated temperature ($T_{max}$) in function of the mandrel diameter during the conical mandrel FEM. The difference between the surface temperature $T_{exit}$ and the maximum internal temperature $T_{max}$ is great. Moreover $T_{max}$ can reach 220\textdegree C which is non negligible. Regarding this level of temperature, the
Figure 4.11: Temperature drop measured when the tube establishes contact with the mandrel.

Figure 4.12: Simulated temperature field during conical mandrel drawing and plotted for a mandrel diameter of 10.16 mm.

The hypothesis of neglecting the thermal softening term in the Johnson-Cook constitutive equation should be revised.

<table>
<thead>
<tr>
<th>Part</th>
<th>Mandrel 1</th>
<th>Mandrel 2</th>
<th>Mandrel 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean section reduction (%)</td>
<td>16.9</td>
<td>17.3</td>
<td>19.2</td>
</tr>
<tr>
<td>Mean thickness reduction (%)</td>
<td>0</td>
<td>0.7</td>
<td>4.0</td>
</tr>
<tr>
<td>Temperature drop (°C)</td>
<td>-</td>
<td>18</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 4.6: Mean section and thickness reductions taking place during conical mandrel drawing test on part (1) and part (2) of the mandrels, measured temperature drop
4.1.6.2 Mechanical analysis

4.1.6.2.1 General observations

This paragraph is devoted to the analysis of the Cauchy stress and strain tensors modelled for a classical mandrel drawing i.e. with a mandrel of constant diameter. Figure 4.14 presents the distribution of the different components of the Cauchy stress tensor (fig.4.14 a, c, e and g) and the strain tensor (fig.4.14 b, d, f and h) during the tube drawing. Concerning this specific analysis, the mandrel has a diameter of 9.5 mm and the die a semi-cone angle of 12.5°. This figure illustrates the tube drawing process and requires to be analysed in details as it contains much information. The components are analysed for each direction and some remarks refer to numbered zones that are pointed out in the figure.

- Starting with the axial stress and strain components as shown in figures 4.14(a) and (b), before the tube enters in contact with the die, it is submitted to bending. $\sigma_{zz} > 0$ at the outer surface (zone 1) and $\sigma_{zz} < 0$ at the inner surface (zone 2). The compressive zone corresponds to a negative axial strain $\epsilon_{zz} < 0$ (zone 5). Next, the tube is submitted to high tensile stresses (zones 3 and 4). Finally, at the die exit, a gradient of $\sigma_{zz}$ remains in the tube thickness. The resulting axial stress state is tensile nearby the tube outer surface $\sigma_{zz} > 0$ and compressive at the inner surface $\sigma_{zz} < 0$. This situation explains the tube deformation when releasing the residual stresses by cutting the tube axially as observed in figures 1.8 and 1.9 in chapter 1. The axial strain $\epsilon_{zz}$ occurs progressively throughout the conical part of the die. A small gradient of strain remains in the tube once it leaves the die. The axial strain is slightly higher on the tube inner side (zone 6).
- Concerning the circumferential stress and strain components plotted in figures 4.14(c) and (d), before the tube enters in contact with the die, it undergoes a circumferential...
compression $\sigma_{\theta \theta} < 0$ which progressively increases and reaches a peak when the tube contacts the die (zone 1). Another peak of lower intensity is visible in the zone affected by bending (zone 2). After the tube contacts the mandrel (zone 3) the circumferential stress gradually returns to tensile. Finally, when exiting the die, $\sigma_{\theta \theta} > 0$ and a gradient of circumferential stress remains in the tube thickness. $\sigma_{\theta \theta}$ is greater nearby the tube outer surface (zone 4) and decreases in the direction of the inner tube surface but remains tensile. It ranges from 300 to 1200 MPa between the inner and outer tube surfaces respectively. The radial strain $\epsilon_{\theta \theta}$ is almost homogeneous in the tube thickness and occurs in a smoother way compared to the axial component. The tube circumferentially contracts regularly when going through the die and the contraction starts before the tube contacts the die. After the die exit, a small gradient of circumferential strain remains in the tube thickness.

- In the case of the radial stress and strain components as shown in figures 4.14(e) and (f), the tube is submitted to radial tension in the bending zone (zone 1) and further to this episode, the tube thickness increases as $\epsilon_{rr} > 0$ (zone 3). The tube contact with the die leads to high compressive stresses $\sigma_{rr} < 0$. Then, when the tube is in contact both with the mandrel and the die $\sigma_{rr}$ drastically decreases and the tube undergoes most of its radial contraction ($\epsilon_{rr} < 0$). Finally, when the tube exits the die, a small gradient of radial stress and strain remains in the tube thickness. $\sigma_{rr}$ ranges from 0 MPa at the tube outer surface to 150 MPa at the tube inner surface and $\epsilon_{rr}$ ranges from -0.2 nearby the tube outer surface to -0.275 nearby the tube inner surface.

- Concerning the shear components, presented in figure 4.14(g) and (h), the tube center successively undergoes positive and negative shear stresses, alternatively going from one extremum to the other. When the tube exits the die, a gradient of shear induced strain remains in the tube thickness.

To summarize the above analysis, the main information is that the different points in the tube thickness undergo different stress and strain histories. As a consequence, the tube exiting the die exhibits stress and strain gradients in the tube thickness.

### 4.1.6.2.2 Influence of the die angle on the stress and strain fields

In this section, the die angle effect on different quantities such as maximum principal stress, shear stress, hydrostatic stress, stress triaxiality and plastic deformation is analysed. The results concern the conical mandrel drawing.

**Maximum principal stress**

Four different simulations were run with semi-cone angles of $5^\circ$, $12.5^\circ$, $16^\circ$, and $20^\circ$. Figures 4.15(a,b,c and d) show the maximum principal stress $\sigma_{\text{max}}$ field in the tube in function of the die semi-cone angle. At first sight, the maximum principal stress distribution is similar for all the simulations. Figures 4.15(e) and (f) show the comparison of the maximum principal stress profile in the tube thickness, along two different lines (l1 and l2), for different die semi-cone angles. The
Figure 4.14: Distribution of the Cauchy stress and strain tensor components in the tube during drawing. (a) $\sigma_{zz}$, (b) $\epsilon_{zz}$, (c) $\sigma_{\theta\theta}$, (d) $\epsilon_{\theta\theta}$, (e) $\sigma_{rr}$, (f) $\epsilon_{rr}$, (g) $\sigma_{rz}$, (h) $\epsilon_{rz}$
solid line \( l_1 \) corresponds to the moment when the tube exits the conical part of the die and the dashed line \( l_2 \) corresponds to the tube exiting the die. It can be observed from figure 4.15(e) that lowering the semi-cone angle causes the maximum principal stress to homogenise in the tube thickness. The effect of the die semi-cone angle is less obvious in figure 4.15(f) but \( \sigma_{\text{max}} \) decreases with decreasing \( \alpha \). Moreover, it can be seen that lowering the die semi-cone angle causes \( \sigma_{\text{max}} \) to evolve more linearly with the tube radius.

Table 4.7 summarizes the maximum values recorded along both lines in function of the die angle. From the listed values it is obvious that the maximum principal stress decreases with decreasing angles.

To summarize this brief analysis, the consequences of die semi-cone angles on the maximum principal stress are:

- the stress in the tube thickness tends to homogenise with lower angles;
- the maximum stress diminishes with lower angles.

**Hydrostatic stress**

Figures 4.16(a,b,c and d) present the hydrostatic stress field in the tube in function of the die semi-cone angle. For all the die semi-angles, at the die entry (below the dashed line), hydrostatic stress ranges from -400 MPa to 400 MPa. The zone between the two lines corresponds to the zone where major deformation occurs, In this area, the hydrostatic stress is negative. As it was previously mentioned in part 1.4, a compressive hydrostatic stress is favourable to reach high deformations. Thus, stress conditions in the tube are in favour of the forming process.

Next, the profiles of hydrostatic stress in the tube thickness and along two straight lines \( l_1 \) and \( l_3 \) are plotted in figure 4.16(e) and (f) respectively. The solid line \( l_1 \) corresponds to the moment when the tube exits the conical part of the die and the dashed line corresponds to the tube contacting the mandrel. Table 4.8 enables to compare the maximum and minimum values of the hydrostatic stress. It can be seen from both the figure and the table that lowering the die semi-angle enables to increase the compressive hydrostatic stress.

To summarize, the consequence of die semi-cone angles on the hydrostatic stress is that a lower die semi-cone angle enables to develop higher compressive hydrostatic stress which is favourable to reach higher deformations.
Figure 4.15: (a,b,c,d) Maximum principal stress field in function of the die semi-cone angle, (e,f) comparison of the maximum principal stress profiles along the solid lines l1 (e) and l2 (f) for different die semi-cone angles.
Figure 4.16: (a,b,c,d) Hydrostatic stress field in function of the die semi-cone angle, (e,f) comparison of the hydrostatic stress profiles along the solid lines $l1$ (e) and $l3$ (f) for different die semi-cone angles.
Table 4.8: Maximum and minimum values of the hydrostatic stress (in MPa) computed in the tube thickness along the solid line

<table>
<thead>
<tr>
<th>die semi-cone angle (°)</th>
<th>5</th>
<th>12.5</th>
<th>16</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>-107</td>
<td>-48.3</td>
<td>-37.8</td>
<td>-6.37</td>
</tr>
<tr>
<td>Minimum</td>
<td>-806</td>
<td>-605</td>
<td>-510</td>
<td>-472</td>
</tr>
</tbody>
</table>

Stress triaxiality

Figures 4.17(a,b,c and d) present the stress triaxiality field in the tube in function of the die semi-cone angle. As a reminder, the stress triaxiality $\eta$ is the ratio of the mean stress $\sigma_m$ to the equivalent stress $\bar{\sigma}$:

$$\eta = \frac{\sigma_m}{\bar{\sigma}} \quad (4.9)$$

$\eta < -\frac{1}{3}$ corresponds to a compression state, $\eta = 0$ corresponds to pure shear, $\eta = \frac{1}{3}$ corresponds to uniaxial traction, $\eta = \frac{2}{3}$ corresponds to equibiaxial traction and above $\frac{2}{3}$ the stress state tends toward isotropic triaxial traction. The stress triaxiality field can be divided into three principal zones. The first zone (1) is located below the dashed line and corresponds to the tube undergoing bending and when it first contacts the die. The second zone (2) corresponds to the tube which is in contact with the mandrel. The third zone (3) corresponds to the die exit. The dashed line (l3) corresponds to a first discontinuity. It is located where the inner tube surface contacts the mandrel. Finally, the solid line (l1) corresponds to the discontinuity of the conical die section end. Stress triaxialities reveal different informations depending on the zones and discontinuities studied. First in zone (1), tube is in a triaxial tensile state nearby the outer surface while it is in a compression state near the inner tube surface. In zone (2), the tube is in a compression state and in zone (3) it is in a triaxial tensile state. In a general way, there are no major differences within the stress triaxiality field for different die semi-cone angles. The profile of the stress triaxiality in the tube thickness and along two straight lines l1 and l3 are plotted in figure 4.17(e) and (f) respectively. In can be observed that a lower semi-die angle enables to lower the stress triaxiality in the tube thickness along the line l1.

Plastic deformation

Figures 4.18(a,b,c and d) present the equivalent plastic strain field in the tube in function of the die semi-cone angle. Figures 4.18(e) and (f) show the corresponding equivalent plastic strain profiles along two lines l1 and l3 crossing the tube thickness at different locations. From the $\bar{\epsilon}_p$ fields, it can be observed that the maximum $\bar{\epsilon}_p$ values increase with increasing semi-die angle. Moreover, plastic deformation takes place almost homogeneously with a semi-die angle of 5°. Indeed in figure 4.18(a) the iso-deformation bands remain almost parallel which is not the case in figures 4.18(b,c,d). This remark is more obvious in figures 4.18(e) and (f). According to the latter, a decrease of semi-die angle enables to lower the equivalent plastic strain. Moreover, it is more homogeneous in the tube thickness both along lines l1 and l3. Such an observation is consistent with Vega et al. (2009) work who simulated by FEM the wire drawing process and found the
Figure 4.17: (a,b,c,d) Triaxiality field in function of the die semi-cone angle, (e,f) comparison of the stress triaxiality profile along the solid lines l1 (e) and l3 (f) for different die semi-cone angles.
Figure 4.18: (a,b,c,d) Equivalent plastic strain field in function of the die semi-cone angle, (e,f) comparison of the equivalent plastic strain profile along the solid lines $l_1$ (e) and $l_3$ (f) for different die semi-cone angles.
4.1.6.2.3 Influence of the die angle on the drawing force and energies

Figure 4.19 presents the evolution of drawing force during the simulation of the conical mandrel drawing. Forces are plotted in function of time which is proportional to mandrel diameter as the displacement speed is constant. For the die semi-cone angle of 12.5°, 16° and 20° the drawing force evolution is identical and force ranges from 1800 daN to 5800 daN. For $\alpha = 5^\circ$ the drawing force is globally higher. From the previously presented analysis concerning the influence of the semi-die angle on various components, different remarks were made. First, increasing the die angle caused the axial stress to reach higher values and to be more heterogeneous in the tube thickness. Second, the hydrostatic stress was more compressive with lower angles. Finally the equivalent plastic strain was lowered down and tended to be more homogeneous in the tube thickness with lower die angle.

From these observations, it is complex to differentiate the contribution of each component and to explain the drawing force increase with decreasing die angle. Moreover, information about friction is missing. In this paragraph, an energetic analysis is conducted in order to explain the force evolution as function of the die angle.

The internal energy of the system is computed by Abaqus as follow:

\[
ALLIE = ALLSE + ALLAE + ALLPD + ALLFD + ALLCD + ALLDMD
\]  

(4.10)

The above mentioned variables correspond to the following energies:

- $ALLIE$: Internal energy or total strain energy
- $ALLSE$: Recoverable strain energy
- $ALLAE$: Artificial strain energy (Abaqus Analysis User’s Manual)
- $ALLPD$: Energy dissipated by plastic deformation
- $ALLFD$: Energy dissipated through frictional effects
- $ALLCD$: Energy dissipated by viscoelasticity (not included in this study)
- $ALLDMDF$: Energy dissipated by damage (not included in this study)
The order of magnitude of $ALLAE$ and $ALLSE$ is $10^7$ J while it is $10^9$ J for $ALLPD$ and $ALLFD$. $ALLCD$ and $ALLDMD$ are not available in this study. Thus, the principal contribution to internal energy ($ALLIE$) is due to the energy dissipated by plastic deformation ($ALLPD$) and the energy dissipated by friction ($ALLFD$). This observation is consistent with the analytical method presented in chapter 1 where the principal contributions to the drawing force are the plastic deformation, the friction and the redundant deformation.

The influence of the die semi-cone angle on the drawing force is analysed in terms of the balance between energies dissipated by plastic deformation and friction. Figure 4.20 shows the evolution of energy dissipated by plastic deformation ($ALLPD$) during the conical mandrel drawing simulation. Data are plotted for different die semi-cone angles. It can be seen that the greater $\alpha$, the higher the energy dissipated by plastic deformation. In parallel, figure 4.20 presents the evolution of energy dissipated by friction ($ALLFD$) during the conical mandrel drawing simulation and for different $\alpha$. In this case, the lower $\alpha$, the higher the energy dissipated by friction. To explain the influence of the die semi-cone angle on the drawing force, it is necessary to compute the sum of $ALLPD$ and $ALLFD$.

Considering the lower order of magnitude of $ALLAE$ and $ALLSE$ the expression for internal energy ($ALLIE$) can be reduced to:

$$ALLIE_{approx} = ALLPD + ALLFD$$

Figure 4.21 presents the internal energy $ALLIE_{approx}$ computed according to the approximated expression as function of time and for different die semi-cone angles. It can be observed that $ALLIE_{approx}$ is the highest for $\alpha = 5^\circ$ while it is approximately equal for $\alpha = 12.5^\circ$, $16^\circ$ and $20^\circ$ which is consistent with the observations made on the drawing force.

As a conclusion, the drawing force increases with decreasing angle because the internal energy is greater mainly due to the contribution of the energy dissipated by friction.
Figure 4.21: Sum of ALLPD and ALLFD in function of time and die semi-cone angles

4.1.6.3 Influence of the die angle on the contact stresses

Figure 4.22 exhibits the evolution of the normal and shear stresses along the tube inner surface in contact with the mandrel (fig.4.22 a and c) and the tube outer surface in contact with the die (fig. 4.22 b and d). Data are plotted for different semi-die angles. Regarding the mandrel/tube contact, data are presented in function of the position along the mandrel with the origin taken at the point facing the die exit. Concerning the die/tube contact, data are presented in function of the position along the die with the origin taken at the die exit. The corresponding friction coefficient is also plotted in figures 4.22( e and f). The objective of this figure is to compare the influence of the die angle on the contact stresses and to explain the differences observed with different die semi-cone angles in terms of friction energy. It can be observed from figure 4.22( a and b) that the range of normal contact stress values is similar for every die semi-cone angles, both for the mandrel/tube and die/tube contacts. However, the contact length between the die and the tube increases with decreasing die semi-cone angle. These observations correspond to the fact that the work of friction or the energy dissipated by friction (ALLFD) is more important for low die angles. As developed in the part devoted to FEM, it becomes evident that this method enables to deliver rich information about the process. The drawing force can be estimated with good accuracy in function of different process parameters and the FEM gives access to stress, strain, and energy data that are useful to understand the process. In the context of industrial production, a direct application of this technique is the estimation of the drawing force in function of the tube dimensions. Indeed, the drawing force has to be estimated in order to define the different drawing passes with the corresponding section and thickness reductions. It enables to dimension the drawing passes and to select the drawing benches to be used in function of their power.
Figure 4.22: Variation of the normal contact stress, shear contact stress and friction coefficient for mandrel/tube and die/tube contacts.
4.2 Analytical methods

In the past, different analytical methods were defined. They were briefly introduced in chapter 1 and are further developed in this section. As mentioned in chapter 1, the advantage of the analytical methods is that they enable to express the drawing force with an expression which is easy to use. On the other side, the different analytical methods neglect different aspects of the process and they are known to under- or over-estimate the drawing force. Finally, their efficiency is limited to the evaluation of the drawing force, and some methods cannot take into account the variations of die dimensional parameters. Nevertheless, even though the drawbacks of the analytical methods are known, their efficiency to estimate the drawing force is evaluated in this section.

4.2.1 Homogeneous deformation method

Figure 4.23(a) presents the geometry of tube drawing to be analysed by the homogeneous deformation method. Figure 4.23(b) details the dimensions necessary for this analysis.

![Diagram of tube drawing process](image)

The drawing stress $\sigma_d$ expresses according to the homogeneous deformation method as follow:

$$\sigma_d = \sigma_0 \ln \left( \frac{A_i}{A_f} \right)$$

(4.12)

with $A_i$ and $A_f$ the tube initial and final cross sectional areas respectively: $A_i = 2\pi (R_i^2 - r_i^2)$ and $A_f = 2\pi (R_f^2 - r_f^2)$. $\sigma_0$ is the material yield stress.
4.2.2 Slab method

Figure 4.24(a) introduces a slab into the general geometry of the mandrel tube drawing. The methodology consists in writing the equilibrium of the slab. The as obtained equations are next integrated along the tube length. Figures 4.24(b and c) detail the notations used to develop the method.

Figure 4.24: Slab analysis method: (a) general view of tube drawing, (b) dimensional characteristics of the working zone, (c) dimensions of a slab, (d) equilibrium of a slab in the z direction, (e and f) equilibrium of a slab in the r direction (Montmitonnet, 2006)

Figure 4.24(d) presents the different stresses acting on a slab element and the equilibrium in the z direction writes:

\[
\sigma_{zz} t - (\sigma_{zz} + d\sigma_{zz})(t + dt) - p_1 \sin \alpha \frac{dz}{\cos \alpha} - \tau_1 \cos \alpha \frac{dz}{\cos \alpha} + p_2 \sin \beta \frac{dz}{\cos \beta} - \tau_2 \cos \beta \frac{dz}{\cos \beta} = 0
\]  

(4.13)
thus:

\[ \sigma_{zz} t - (\sigma_{zz} + d\sigma_{zz})(t + dt) - p_1 \tan \alpha dz - \tau_1 dz + p_2 \tan \beta dz - \tau_2 dz = 0 \] (4.14)

where \( p_1 \sin(\alpha) \) and \( p_2 \sin(\beta) \) are the projections of \( p_1 \) and \( p_2 \) on the z axis and \( \frac{dz}{\cos(\alpha)} \) and \( \frac{dz}{\cos(\beta)} \) are the projections of the surface on which the pressure apply on the z axis.

In their study, Kartik (1995) and Rubio (2006) both developed a slab method to study tube drawing but they considered the pressures \( p_1 \) and \( p_2 \) that apply on the outer and inner tube surface respectively to be equal. Moreover, they used a Coulomb friction model which states that \( \tau_1 = \mu_1 p_1 \) and \( \tau_2 = \mu_2 p_2 \). If \( \tau_1 = \tau_2 = \tau \) and \( p_1 = p_2 = p \) then, the equilibrium equation writes:

\[ \sigma_{zz} t - (\sigma_{zz} + d\sigma_{zz})(t + dt) - p(\tan \alpha - \tan \beta) dz - p(\mu_1 + \mu_2) dz = 0 \] (4.15)

The thickness increment \( dt = t_{si} - t_{sf} \) is linked to \( dz \) following:

\[ dr_i + t_{si} = dr_f + t_{sf} \] (4.16)
\[ dt = t_{si} - t_{sf} = dr_f - dr_i = dz \tan \alpha - dz \tan \beta \] (4.17)
\[ \text{finally } dt = dz(\tan \alpha - \tan \beta) \] (4.18)

developing the equilibrium equation (4.15) and replacing \( dz \) by its expression as a function of \( dt \) (eq. 4.18) turns:

\[ \sigma_{zz} dt + d\sigma_{zz} t + pdt + p \left( \frac{\mu_1 + \mu_2}{\tan \alpha - \tan \beta} \right) dt = 0 \] (4.19)

\[ t \sigma_{zz} + (\sigma_{zz} + p(1 + B^*)) dt = 0 \text{ with } B^* = \frac{\mu_1 + \mu_2}{\tan \alpha - \tan \beta} \] (4.20)

Then a Tresca yield condition \( p = \sigma_0 - \sigma_{zz} \) is taken with \( \sigma_0 \) the yield stress:

\[ \sigma_{zz} - \sigma_{rr} = \sigma_0 \text{ and as } \sigma_{rr} = p \text{ then } p = \sigma_0 - \sigma_{zz} \] (4.21)

The pressure \( p \) which is unknown is then replaced in equation (4.20) by the expression (4.21). Finally, the as obtained equation is integrated along the strain path:

\[ \int_{0}^{\sigma_{zz}} \frac{d\sigma_{zz}}{B^* \sigma_{zz} - \sigma_0(1 + B^*)} = \int_{t_i}^{t_f} \frac{dt}{t} \] (4.22)

\[ \left[ \frac{1}{B^*} \ln \left( \sigma_{zz} - \sigma_0 \frac{1 + B^*}{B^*} \right) \right]_{t_i}^{t_f} = \left[ \ln t \right]_{t_i}^{t_f} \] (4.23)

which gives the drawing stress after transformations as follow:

\[ \sigma_d = \sigma_{zz} = \sigma_0 \left( \frac{1 + B^*}{B^*} \right) \left[ 1 - \left( \frac{t_f}{t_i} \right)^{B^*} \right] \] (4.24)

In the method presented above, a major hypothesis was to consider \( p_1 \) and \( p_2 \) to be equal.

Figure 4.25 shows the evolution of both contact pressures in the working zone during mandrel drawing. These data were computed by means of FEM for a L605 tube, for a mandrel whose diameter was 9.5 mm and a die semi-angle of 12.5°. It can be seen from figure 4.25 that \( p_1 \) ranges from approximately 500 MPa to 2100 MPa and \( p_2 \) ranges from
1300 MPa to 1400 MPa. As a consequence, \( p_1 \) and \( p_2 \) vary with \( z \) and both pressures cannot be considered as equal locally. Nevertheless, if an average pressure is computed along the contacts, \( p_{1\text{mean}} = 1142MPa \) and \( p_{2\text{mean}} = 1316MPa \), then Kartik (1995) and Rubio (2006) hypothesis can be validated.

Montmitonnet (2006) proposed a more complete version of the slab method as he considered independently \( p_1 \) and \( p_2 \) and took into account the tube diameter. Montmitonnet (2006) method is detailed below. The method starts with the expression of the equilibrium of a slab element considering the tube section:

\[
(\sigma_{zz} + d\sigma_{zz})\pi(R_{si}^2 - r_{si}^2) - \sigma_{zz}\pi(R_{sf}^2 - r_{sf}^2) + p_1\tan(\alpha)2\pi R_{sf}dz + \tau_12\pi R_{sf}dz \\
- p_2\tan(\beta)2\pi r_{sf}dz + \tau_22\pi r_{sf}dz = 0
\]  

(4.25)

Geometrically \( R_{si} = R_{sf} + dr \) and \( r_{sf} = R_{sf} - t \) and the expressions transforms:

\[
d(2R_{sf}t - t^2)\sigma_{zz} + 2R_{sf}\sigma_{zz}dr + 2R_{sf}(\tau_1 + p_1\tan \alpha)dz + 2(R_{sf} - t)(\tau_2 - p_2\tan \beta)dz = 0
\]

(4.26)

\[
\frac{d[(2R_{sf}t + t^2)\sigma_{zz}]}{dz} + 2R_{sf}(\tau_1 + p_1\tan \alpha) + 2(R_{sf} - t)(\tau_2 - p_2\tan \beta) = 0
\]

(4.27)

which can write indifferently as a function of \( r_i \) as follow:

\[
\frac{d[(2R_{sf}t + t^2)\sigma_{zz}]}{dz} + 2(R_{sf} + t)(\tau_1 + p_1\tan \alpha) + 2r_{sf}(\tau_2 - p_2\tan \beta) = 0
\]

(4.28)

The above equation is a differential equation that requires to be solved. Additional relations can be found by writing the equilibrium on an outer and inner tube slab as described in figure 4.24(e and f) (Montmitonnet, 2006):

for the outer slab: 
\[2\sigma_{\theta\theta}(r_{sf} + t - r) - 2\sigma_{rr} + (2r_{sf} + t)(p_1 - \tau_1 \tan \alpha) = 0\]  

(4.29)

which gives: 
\[2r(\sigma_{\theta\theta} - \sigma_{rr}) - 2(r_{sf} + t)(\sigma_{\theta\theta} + p_1 - \tau_1 \tan \alpha) = 0\]  

(4.30)

for the inner slab: 
\[-2r(\sigma_{\theta\theta} - \sigma_{rr}) + 2r_{sf}(\sigma_{\theta\theta} + p_2 + \tau_2 \tan \beta) = 0\]  

(4.31)
Summing equations 4.29 and 4.31 gives the expression of $\sigma_{\theta\theta}$:

$$\sigma_{\theta\theta} = \frac{r_{sf}(p_2 + \tau_2 \tan \beta) - (r_{sf} + t)(p_1 - \tau_1 \tan \alpha)}{t} \quad (4.32)$$

Then, in equation 4.29 if $r$ tends towards $r_{sf} + t$ and in equation 4.31 if $r = r_{sf}$ it gives the expression of $\sigma_{rr}$:

$$\sigma_{rr}^{ext} \approx \tau_1 \tan \alpha - p_1 \quad (4.33)$$

$$\sigma_{rr}^{int} \approx -\tau_2 \tan \beta - p_2 \quad (4.34)$$

Due to the slab hypothesis $\sigma_{rr}$ is independent of $r$. As a consequence,

$$p_1 = \tau_1 \tan \alpha - \sigma_{rr} \quad (4.35)$$

$$p_2 = -\tau_2 \tan \beta - \sigma_{rr} \quad (4.36)$$

Inserting both relations into equation 4.32 gives that $\sigma_{\theta\theta} \approx \sigma_{rr}$.

Finally, a Tresca yield criteria can be written and $\sigma_{zz} - \sigma_{rr} = \sigma_0$ and $\sigma_{rr} = \sigma_{zz} - \sigma_0$. From this point, two hypotheses can be made concerning the friction. First, friction can be modelled according to a Coulomb model, second, it can be modelled by a Tresca model.

Starting with the Coulomb friction model, $\tau_1 = \mu_1 p_1$ and $\tau_2 = \mu_2 p_2$. $p_1$ and $p_2$ are two unknowns and can be expressed as a function of $\sigma_{rr}$: from equations 4.35 and 4.36 and the Coulomb model, $p_1$ and $p_2$ express as follow,

$$p_1 \approx -\frac{\sigma_{rr}}{1 - \mu_1 \tan \alpha} \quad (4.37)$$

$$p_2 \approx -\frac{\sigma_{rr}}{1 + \mu_2 \tan \beta} \quad (4.38)$$

Inserting both expressions into the differential equation 4.28 enables to re-write it:

$$\frac{d[(2r_{sf}t + t^2)\sigma_{zz}]}{dz} + 2r_{sf}(\sigma_{zz} - \sigma_0)K + 2t(\sigma_{zz} - \sigma_0)K' = 0 \quad (4.39)$$

with

$$\begin{cases} K = -\frac{\mu_1 - \tan \alpha}{1 - \mu_1 \tan \alpha} - \frac{\mu_2 - \tan \beta}{1 + \mu_2 \tan \beta} \\ K' = -\frac{\mu_1 - \tan \alpha}{1 - \mu_1 \tan \alpha} \end{cases} \quad (4.40)$$

The first component of the differential equation 4.39 can be developed and transformed with geometrical equivalences (cf fig.4.24(c)):

$$t_{si} + dr_i = t_{sf} + dr_f \quad (4.41)$$

$$t_{si} + z \tan \beta = t_{sf} + z \tan \alpha \quad (4.42)$$

$$t_{si} = t_{sf} - z(\tan \beta - \tan \alpha) \quad (4.43)$$

Moreover,

$$dt = t_{si} - t_{sf} = -dz(\tan \beta - \tan \alpha) \quad (4.44)$$

$$\tan \beta - \tan \alpha = -\frac{dt}{dz} \quad (4.45)$$

$$t(z) = t_{sf} - \frac{dt}{dz} \quad (4.46)$$

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Similarly,
\[ r_{si} = r_{sf} + z \tan \beta \quad \text{and} \quad \tan \beta = \frac{dr}{dz} \]  \quad (4.47)

Finally, the first term of differential equation turns into:
\[ \frac{d}{dz}[(2r_{si} + t^2)\sigma_{zz}] = \frac{d}{dz}[(2r_{si} + z \frac{dr}{dz})(t_{sf} - z \frac{dt}{dz}) + (t_{sf} - z \frac{dt}{dz})^2] \sigma_{zz} \]  \quad (4.48)

which after further developments gives:
\[ (2r_{si}t + t^2) \frac{d\sigma_{zz}}{dz} + \sigma_{zz}(2t_{sf} \frac{dr}{dz} - 2t_{sf} \frac{dt}{dz} - 2r_{sf} \frac{dt}{dz}) \]  \quad (4.49)

Inserting the above term into the differential equation 4.39 and replacing \( \frac{dr}{dz} \) by \( \tan \alpha \) and \( \frac{dt}{dz} \) by \( (\tan \alpha - \tan \beta) \) gives:
\[ (2r_{si}t + t^2) \frac{d\sigma_{zz}}{dz} + \sigma_{zz}[2r_{si}(K + \tan \beta - \tan \alpha) + 2t(K' - \tan \alpha)] = \sigma_0(2r_{si}K + 2tK') \]  \quad (4.50)

which becomes:
\[ (2r_{si}t + t^2) \frac{d\sigma_{zz}}{dz} + \sigma_{zz}[2r_{si}(K + \tan \beta - \tan \alpha) + 2t(K' - \tan \alpha)] = \sigma_0(2r_{si}K + 2tK') \]  \quad (4.51)

and:
\[ (2r_{si}t + t^2) \frac{d\sigma_{zz}}{dz} + \sigma_{zz}[(2r_{si} + t)K_0(\tan \beta - \tan \alpha) + tK_1(\tan \beta + \tan \alpha)] \]
\[ = \sigma_0[(2r_{si} + t)(K_0 - 1)(\tan \beta - \tan \alpha) + t(K_1 + 1)(\tan \beta + \tan \alpha)] \]  \quad (4.52)

with
\[ K_0 = \frac{1}{\tan \beta - \tan \alpha} \left( \frac{-\mu_1 - \tan \alpha}{1 - \mu_1 \tan \alpha} - \frac{\mu_2 - \tan \beta}{1 + \mu_2 \tan \beta} \right) + 1 \approx -\frac{\mu_1 + \mu_2}{\tan \beta - \tan \alpha} \]  \quad (4.53)

and
\[ K_1 = -\frac{1}{\tan \beta - \tan \alpha} \left( \frac{\mu_1 + \tan \alpha}{1 - \mu_1 \tan \alpha} - \frac{\mu_2 - \tan \beta}{1 + \mu_2 \tan \beta} \right) - 1 \approx \frac{\mu_1 - \mu_2}{\tan \beta - \tan \alpha} \]  \quad (4.54)

The solution of this differential equation has the form:
\[ \sigma_{zz} = C(z)(2r_{si} + t)^K_1 t^{-K_0} \]  \quad (4.55)

with
\[ C(z) = C(0) + \sigma_0 \frac{(\tan \alpha - \tan \beta)K_0}{(\tan \alpha + \tan \beta)K_1} \int_0^z \left( \frac{t_0}{\tan \alpha + \tan \beta} - z \right)^{K_0 - 1} \left( \frac{2r_{sf} + t_{sf}}{\tan \alpha + \tan \beta} - z \right)^{K_1 - 1} d\]  \quad (4.56)

The exact solution has to be found numerically.

The use of the Tresca friction model leads to a simpler drawing stress expression. The Tresca friction model states that \( \tau_1 = m_1 \sigma_0 / \sqrt{3} \) and \( \tau_2 = m_2 \sigma_0 / \sqrt{3} \). In this case, \( p_1 \approx p_2 \approx -\sigma_{rr} \). After inserting these expressions into the differential equation 4.28 and after a few transformation comes the expression of the drawing stress:
\[ \frac{\sigma_z}{\sigma_0} = \left( 1 + \frac{m_1 + m_2}{(\tan \alpha - \tan \beta) \sqrt{3}} \right) \ln \left( \frac{t_i}{t_f} \right) - \frac{(\tan \alpha - \tan \beta)}{(\tan \alpha + \tan \beta)} \ln \left( \frac{2r_i + t_i}{2r_f + t_f} \right) \]
\[ - \left( 1 + \frac{m_1}{\tan \alpha \sqrt{3}} \right) \frac{2 \tan \alpha}{(\tan \alpha + \tan \beta)} \ln \left( \frac{2r_i + t_i}{2r_f + t_f} \right) \]  \quad (4.57)
4.2.3 Upper bound method

In the upper bound method, the geometry can be divided into five different zones as drawn in figure 4.26(a). Zone a corresponds to the initial tube moving forward the die. Zone b is a part of tube sinking. In this zone, the tube outer surface contacts the die and follows its profile. The tube inner surface sinks towards the mandrel but no thickness reduction occurs. Then the tube inner surface contacts the mandrel and enters in zone c. In this zone the tube thickness is decreasing. In zone d, the tube exits the die conical part. In this zone, no deformation occurs and there is only friction between the tube and the tools. Finally, the tube exits the die in zone e. Both lines FF’ and GG’ refer to discontinuities of the velocity field. Figure 4.26(b) presents a deformed mesh extracted from the FEM to justify the different zones that were defined.

The upper bound method consists in computing the rate of energy of homogeneous deformation $\dot{W}_H$, the rate of energy dissipation due to internal shear at both discontinuities $\dot{W}_{FF'}$ and $\dot{W}_{GG'}$ and the rate of energy dissipated by friction at every tool/tube contact $\dot{W}_f$. Each energy contribution is detailed in the following paragraph. The equations that are developed below refer to the notation presented in figure 4.27.
4.2.3.1 Energy dissipated by homogeneous deformation

Homogeneous deformation is linked to the initial and final tube dimensions only. In order to express the work of homogeneous deformation, the geometry can be simplified as illustrated in figure 4.23. The plastic deformation work increment per unit volume writes by analogy with the homogeneous deformation method:

\[
\hat{W}_H = \sigma_0 \epsilon_H A_f v_f = \sigma_0 \ln \frac{A_i}{A_f} A_f v_f \tag{4.58}
\]

where \(v_0\) and \(v_f\) are the initial and final velocities respectively and \(A_i\) and \(A_f\) are the initial and final tube sections respectively. Um and Lee (1997) developed another method to express the rate of energy dissipation due to homogeneous deformation. They expressed \(\hat{W}_H\) as a function of the ratio of the circumferential to axial strain increments:

\[
\hat{W}_H = \frac{2\sigma_0}{\sqrt{3}} \sqrt{1 + y + y^2} A_i v_0 \ln \frac{A_i}{A_f} \tag{4.59}
\]

with

\[
y = \frac{d\epsilon_\theta}{d\epsilon_z} = \frac{\ln \frac{R_f + r_f}{R_i + r_i}}{\ln \frac{R_f^2 + r_f^2}{R_i^2 + r_i^2}} \tag{4.60}
\]

For further details, the reader might refer to the reference Um and Lee (1997). Both expressions led to similar results.

4.2.3.2 Energy dissipation due to internal shear

Internal shear is linked to the change of direction of the velocity field that takes place at discontinuities \(FF'\) and \(GG'\). When a particle crosses the \(FF'\) discontinuity, its velocity undergoes a discontinuity \(v_{FF'}(r)\) which is proportional to its distance \(r\) from the center line. At the outer surface, the particle velocity changes from \(v_0\) to \(v_{FF'}(R_i) = v_0 \tan \alpha\) and at the inner surface it changes from \(v_0\) to \(v_{FF'}(r_i) = v_0 \tan \beta\). For a particle at the distance \(r\) from the center line, the velocity discontinuity is \(v_{FF'}(r) = \frac{r}{R_i} v_0 \tan \alpha\). The rate of energy dissipation due to internal shear at the die entry and \(FF'\) discontinuity is given by:

\[
d\hat{W}_{FF'} = 2\pi r dr v_{FF'}^*(r) \tag{4.61}
\]

Figure 4.27: Notations used to compute the different energies in (a) zone b, (b) zone c and (c) zone d. Grey parts symbolise the die and the mandrel.
with $k$ the material shear yield stress, $k = \frac{\sigma_0}{\sqrt{3}}$ with $\sigma_0$ the yield stress.

Moreover,

$$v_i^{*}\dot{F}_{F'}(r) = v_0 \frac{r}{R_i} \tan \alpha$$

(4.62)

then,

$$d\dot{W}_{F'} = 2\pi k \frac{v_0}{R_i} r^2 \tan \alpha dr$$

(4.63)

Integrating the above expression between $r_0$ and $R_0$ gives the expression of the rate of energy dissipation due to internal shear at the $F'F$ discontinuity:

$$\dot{W}_{F'} = \frac{2}{3} \pi k v_0 \frac{R_i^3 - r_i^3}{R_i} \tan \alpha = \frac{2k}{3} \frac{R_i^2 + R_i r_i + r_i^2}{R_i(R_i + r_i)} A_i v_0 \tan \alpha$$

(4.64)

where $A_i = \pi(R_i^2 - r_i^2)$.

Similarly for the $GG'$ discontinuity the rate of energy dissipation due to internal shear is equal to:

$$\dot{W}_{G'} = \frac{2k}{3} \frac{R_f^2 + R_f r_f + r_f^2}{R_f(R_f + r_f)} A_f v_f \tan \alpha$$

(4.65)

Summing both expressions for $F'F$ and $GG'$ discontinuities and using the flow conservation $v_0 A_i = v_f A_f$ leads to the following expression for the rate of energy dissipated by internal shear:

$$\dot{W}_s = \frac{2k}{3} A_i v_0 \tan \alpha \left( \frac{R_i^2 + R_i r_i + r_i^2}{R_i(R_i + r_i)} + \frac{R_f^2 + R_f r_f + r_f^2}{R_f(R_f + r_f)} \right)$$

(4.66)

with $k = \frac{\sigma_0}{\sqrt{3}}$

### 4.2.3.3 Energy dissipation due to friction

The computation of the energy dissipation due to friction is the most complex as all the different interfaces have to be considered. Figure 4.27 presents all the tool/tube interfaces that are detailed below.

- In zone b, the contact takes place between the tube and the die which is characterised by its angle $\alpha$. The inner tube surface is free. The rate of energy dissipated at the die/tube interface is $\dot{W}_{Fb}$. The lower-case letter $f$ refers to friction, $b$ refers to the zone of consideration and the upper-case letter $\alpha$ refers to the side were the contact takes place. $\alpha$ and $\beta$ are related to the die and the mandrel sides respectively. This notation is used for the other terms to be computed.
- In zone c, there are two contacts: the die/tube contact whose rate of energy dissipated by friction is $\dot{W}_{Fc}$ and the mandrel/tube contact with $\dot{W}_{Fc}$.
- In zone d there are also two contacts and the following terms can be computed: $\dot{W}_{Ff}$ and $\dot{W}_{Ff}$.

The expression of the above listed terms is detailed below. For clarity, a sketch was drawn for every section in order to detail the notations and to improve clearness.
Zone b
To start with zone b, this configuration corresponds to tube sinking as the inner tube surface is free. The geometry is detailed in figure 4.27(a). The rate of energy dissipation due to friction $\dot{W}_{fb}^\alpha$ is given by:

$$d\dot{W}_{fb}^\alpha = 2\pi \tau_\alpha v R dS$$ (4.67)

Geometrically,

$$dS = \frac{dR}{\sin \alpha} \text{ and } v = \frac{v_z}{\cos \alpha}$$ (4.68)

Thus,

$$d\dot{W}_{fb}^\alpha = 2\pi \tau_\alpha \frac{v_z}{\cos \alpha} R dR = \frac{4\pi \tau_\alpha}{\sin 2\alpha} v_z R dR$$ (4.69)

Considering flux conservation $A_z v_0 = A_z v_z$ and $v_z = \frac{A_z v_0}{A_z} = v_0 \frac{R^2 - r_f^2}{R^2 - r_i^2}$. The equation then transforms:

$$d\dot{W}_{fb}^\alpha = \frac{4\pi \tau_\alpha v_0}{\sin 2\alpha} (R_i^2 - r_i^2) \frac{R}{R^2 - r_i^2} dR$$ (4.70)

Moreover, geometrically:

$$\frac{r_i - R}{\tan \alpha} = \frac{r_i - r}{\tan \beta}$$ which transforms and gives $r = r_i - t'(R_i - R)$ with $t' = \frac{\tan \beta}{\tan \alpha}$ (4.71)

The angle $\beta$ corresponds to the angle formed by the tube inner surface and the direction of drawing. In the particular case of tube sinking, inner and outer tube surfaces are supposed to remain parallel and $\beta$ can be considered equal to $\alpha$ and $t' = 1$. Consequently, the rate of energy dissipation due to friction computes as the integral of the above expression:

$$\dot{W}_{fb}^\alpha = \int_{R_i}^{R_f} \frac{4\pi \tau_\alpha v_0}{\sin 2\alpha} (R_i^2 - r_i^2) \frac{R}{R^2 - (r_i - R_i + R)^2} dR$$ (4.72)

Such an integration requires some transformations of the term $\frac{R}{R^2 - (r_i - R_i + R)^2}$. This expression can be transformed as follow:

$$\frac{R}{R^2 - (r_i - R_i + R)^2} = \frac{a}{R - (r_i - R_i + R)} + \frac{b}{R + (r_i - R_i + R)}$$ (4.73)

where $a$ and $b$ are unknowns to be identified. After development and identification, the resulting values are $a = b = 0.5$. As a consequence:

$$\dot{W}_{fb}^\alpha = \frac{2\pi \tau_\alpha v_0}{\sin 2\alpha} (R_i^2 - r_i^2) \int_{R_i}^{R_f + R_i - r_i} \left( \frac{1}{R_i - r_i} + \frac{1}{r_i - R_i + 2R} \right) dR$$ (4.74)

The integration gives:

$$\dot{W}_{fb}^\alpha = \frac{2\pi \tau_\alpha v_0}{\sin 2\alpha} (R_i^2 - r_i^2) \left( \frac{R}{R_i - r_i} \frac{r_i + R_i - r_i}{R_i} + \frac{1}{2} \ln \left( \frac{r_i - R_i}{2} + R \right) \right)$$ (4.75)

which leads to the final expression:

$$\dot{W}_{fb}^\alpha = \frac{2\pi \tau_\alpha v_0}{\sin 2\alpha} (R_i^2 - r_i^2) \left( \frac{r_i + R_i - r_i}{R_i} + \frac{1}{2} \ln \left( \frac{2r_i + R_i - r_i}{r_i + R_i} \right) \right)$$ (4.76)
zone c

Now continuing with zone c and $dW_{fc}^\alpha$, exactly the same method can be applied as for $dW_{f0}^\alpha$. Two different configurations can be studied as shown in figure 4.27(b): first on a mandrel of constant section, $\beta = 0$, second on a mandrel presenting an angle $\beta$, in this case $t' = {\tan(\beta) \over \tan(\alpha)} \neq 1$. This second configuration aims at being used in this study in the special case of the conical mandrel drawing. As a reminder the mandrel cone angle is equal to 0.053° which can be approximated by 0. As a consequence, it is convenient to neglect the mandrel angle as it greatly simplifies the expression of rate of energy dissipation due to friction given by:

$$dW_{fc}^\alpha = {4\pi \tau_\alpha v_0 \over \sin 2\alpha} (R_i^2 - r_i^2) {R \over R^2 - r_f^2} dR$$  \hspace{1cm} (4.77)

which transforms:

$$dW_{fc}^\beta = {2\pi \tau_\alpha v_0 \over \sin 2\alpha} (R_i^2 - r_i^2) \left( {1 \over R + r_f} + {1 \over R - r_f} \right) dR$$  \hspace{1cm} (4.78)

The integration gives:

$$\dot{W}_{fc}^\alpha = {2\pi \tau_\alpha v_0 \over \sin 2\alpha} (R_i^2 - r_i^2) \int_{r_f + R_i - r_i}^{R_f} \left( {1 \over R + r_f} + {1 \over R - r_f} \right) dR$$  \hspace{1cm} (4.79)

$$\dot{W}_{fc}^\beta = {2\pi \tau_\alpha v_0 \over \sin 2\alpha} (R_i^2 - r_i^2) \left( \ln {R_f + r_f \over 2r_f + R_i - r_i} + \ln {R_f - r_f \over R_i - r_i} \right)$$  \hspace{1cm} (4.80)

Up to this point, the energy dissipated by friction was computed in the zone c at the die/tube contact. The following equations aims at developing the same quantity but for the mandrel/tube contact $dW_{f0}^\beta$.

$$dW_{fc}^\beta = 2\pi \tau_\beta v_z r_f dz$$  \hspace{1cm} (4.81)

Due to flow conservation:

$$v_0 A_i = v_z A_z \text{ and } v_0 (R_i^2 - r_i^2) = v_z (R^2 - r_f^2) \text{ then } v_z = {R_i^2 - r_i^2 \over R^2 - r_f^2} v_0$$  \hspace{1cm} (4.82)

It turns that:

$$d\dot{W}_{fc}^\beta = 2\pi \tau_\beta r_f v_0 {R_i^2 - r_i^2 \over R^2 - r_f^2} dz$$  \hspace{1cm} (4.83)

Geometrically (fig.4.27a),

$$\tan(\alpha) = {r_f + R_i - r_i \over z} \text{ and } R = r_f + R_i - r_i - z \tan(\alpha)$$  \hspace{1cm} (4.84)

Then,

$$d\dot{W}_{fc}^\beta = 2\pi \tau_\beta r_f v_0 {R_i^2 - r_i^2 \over (r_f + R_i - r_i - z \tan(\alpha))^2 - r_f^2} dz$$  \hspace{1cm} (4.85)

The above expression transforms as:

$$d\dot{W}_{fc}^\beta = \pi \tau_\beta v_0 (R_i^2 - r_i^2) \left( {1 \over R_i - r_i - z \tan(\alpha)} - {1 \over R_i - r_i + 2r_f - z \tan(\alpha)} \right) dz$$  \hspace{1cm} (4.86)

which integrates:

$$\dot{W}_{fc}^\beta = \pi \tau_\beta v_0 (R_i^2 - r_i^2) \int_0^l \left( {1 \over R_i - r_i - z \tan(\alpha)} - {1 \over R_i - r_i + 2r_f - z \tan(\alpha)} \right) dz$$  \hspace{1cm} (4.87)
and gives:

\[ W_{fc}^\beta = \pi \tau_\beta v_0 (R_i^2 - r_i^2) \left( \frac{l \tan \alpha - R_i + r_i - 2r_f}{r_i - R_i - 2r_f} - \ln \frac{l \tan \alpha - R_i + r_i}{r_i - R_i} \right) \]  (4.88)

Geometrically, \( \tan \alpha = \frac{r_f - R_f + R_i - r_i}{l} \) and \( l \tan \alpha = r_f - R_f + R_i - r_i \). Finally, replacing \( l \tan \alpha \) in the previous equation turns:

\[ W_{fc}^\beta = \pi \tau_\beta v_0 (R_i^2 - r_i^2) \ln \frac{(R_f + r_f)(R_i - r_i)}{(r_f - R_f)(r_i - R_i - 2r_f)} \]  (4.89)

zone d

In zone d, the tube is in the bearing zone of the die and the die/tube and mandrel/tube contacts are parallel. The remaining rate of energy dissipated by friction \( W_{fd}^\alpha \) and \( W_{fc}^\beta \) computes as follow:

\[ dW_{fd} = dW_{fd}^\alpha + dW_{fd}^\beta = 2\pi v (\tau_\alpha R_f + \tau_\beta r_f) dz \]  (4.90)

which integrates between 0 and \( l \) and gives:

\[ W_{fd} = W_{fd}^\alpha + W_{fd}^\beta = 2\pi v (\tau_\alpha R_f + \tau_\beta r_f) l \]  (4.91)

with \( l \) the die bearing length.

Um and Lee (1997) developed a method and combined the b and c zones into a single one. Moreover, they neglected the friction taking place in the die bearing zone (zone d)(fig.4.27)

4.2.4 Drawing force

The drawing force is computed by equating the external work rate to the sum of all the rate of dissipated energies computed previously:

\[ \sigma_d v_f A_f = \sigma_d v_0 A_i = W_{H} + W_{F} + W_{G} + W_{f} + W_{fc} + W_{fd} = \sum W_i \]  (4.92)

finally,

\[ F_d = \sigma_d A_f = \frac{\sum W_i}{v_f} \]  (4.93)

4.3 Comparison of the analytical and FEM methods

The three analytical methods that were developed in this chapter and the FEM of the tube drawing are compared in this section. The comparison is limited to the estimation of the drawing force as the analytical methods are limited to this result.

In the different expressions resulting from the analytical methods appears the yield stress \( \sigma_0 \). During tube drawing, the material strain hardens and as a consequence, the flow stress varies during a drawing pass and it is different at the die entry and exit. Generally, an approximation of the drawing force can be computed using a mean flow stress \( \sigma_{0m} \). There
are different ways to define the mean flow stress. First it can be considered as the average between the material initial yield stress \( \sigma_{0i} \) and the final yield stress \( \sigma_{0f} \):

\[
\sigma_{0m} = (\sigma_{0i} + \sigma_{0f})/2
\]

(4.94)

A better approximation of the average flow stress can be obtained by integration of the flow stress up to the final strain \( \dot{\varepsilon}_f \) (Altan et al., 1983):

\[
\sigma_{0m} = \frac{1}{\dot{\varepsilon}_f} \int_0^{\dot{\varepsilon}_f} \bar{\sigma} d\bar{\varepsilon}
\]

(4.95)

with,

\[
\dot{\varepsilon}_f = \dot{\varepsilon}_H = \ln \frac{A_f}{A_i}
\]

(4.96)

The proposition made here to set this value was to take the flow stress corresponding to the final strain \( \epsilon_f = \epsilon_H \). If a Ludwik constitutive equation was identified, then:

\[
\sigma_{0m} = \sigma_0 + K \epsilon_H^n
\]

(4.97)

Table 4.9 lists the estimated drawing forces by means of analytical methods and FEM for different drawing passes. The comparison with the experimentally measured drawing forces enables to evaluate the methods. The characteristics of each drawing test detailed in the table 4.9 are listed in table 4.10. From the drawing forces listed in table 4.9, it can be seen that the analytical methods estimate the drawing forces with great errors. The homogeneous deformation method under-estimates the force from 10% to 94%. The slab and upper bound methods both under and over-estimate the drawing force with an error ranging of 50%. Finally, the most reliable technique to estimate the drawing force is the FEM with an error ranging from up to 11.5%.

Figure 4.28 presents the predicted drawing force as a function of the die semi-angle and for the different analytical methods and the FEM. As can be seen in this figure, the drawing force determined by the homogeneous deformation method does not depend on the die angle but only on the initial and final tube dimensions. The drawing forces obtained from the slab methods decrease with increasing die angle, but no optimum die semi-angle is revealed. It is interesting to notice that the slab method proposed by Montmitonnet (2006) estimates a greater force compared to Kartik (1995); Rubio (2006). Nevertheless both methods show the same evolution of the drawing force with the die semi-angle. Finally, both the upper-bound method and the FEM reveal an optimum die semi-angle but it differs for both methods. The optimum die semi-angle estimated by the upper-bound method is between 7.5° and 10° while it is included between 15° and 20° according to the FEM.

The conclusion concerning the comparison of the analytical methods and the FEM is that the homogeneous deformation and the slab methods do not provide an accurate estimation of the drawing force. The upper-bound method gives better approximation as it consider the different energies involved in the process but it is still not satisfactory. Moreover, it must be mentioned that both the slab and homogeneous deformation methods require the friction coefficient to be known. The best and unique way which enables to identify the friction coefficient is the FEM. Thus, it is not worth running the FEM to obtain the
friction coefficient and then compute the drawing force by means of an analytical method. As a conclusion, FEM is the most reliable method to compute the drawing force. Indeed, it considers all the energies involved in the process and makes no approximation concerning the stress and strain fields. There are no geometrical approximation neither, the material strain hardening is properly taken into consideration and as a consequence, there is no need to approximate the yield stress. Finally, as a method must be chosen, the FEM is selected since it models accurately the material deformation and the contact behaviours.

Table 4.9: Drawing forces computed by means of different methods, comparison with experimental Forces. Data marked with * were extracted from Palengat (2009), the constitutive equation for the material marked with ** was taken from Affagard (2010).

<table>
<thead>
<tr>
<th>material</th>
<th>Drawing test</th>
<th>HD</th>
<th>Slab 1</th>
<th>Slab 2</th>
<th>Upper-Bound</th>
<th>FEM</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>316LVM</td>
<td>SM1900</td>
<td>F</td>
<td>59.5</td>
<td>11.7</td>
<td>103</td>
<td>67.48</td>
<td>67.35</td>
</tr>
<tr>
<td></td>
<td>SM810*</td>
<td>F</td>
<td>8.9</td>
<td>10.6</td>
<td>18.7</td>
<td>10.41</td>
<td>9.95</td>
</tr>
<tr>
<td></td>
<td>SM664*</td>
<td>F</td>
<td>2.14</td>
<td>71.4</td>
<td>6.03</td>
<td>7.08</td>
<td>7.39</td>
</tr>
<tr>
<td>L605</td>
<td>SM750*</td>
<td>F</td>
<td>5.78</td>
<td>58.4</td>
<td>13.7</td>
<td>13.0</td>
<td>13.9</td>
</tr>
<tr>
<td></td>
<td>SM630*</td>
<td>F</td>
<td>4.57</td>
<td>50.3</td>
<td>10.9</td>
<td>8.75</td>
<td>9.2</td>
</tr>
<tr>
<td></td>
<td>SM530*</td>
<td>F</td>
<td>3.59</td>
<td>30.1</td>
<td>7.69</td>
<td>5.8</td>
<td>5.2</td>
</tr>
<tr>
<td></td>
<td>SM301*</td>
<td>F</td>
<td>1.69</td>
<td>26.5</td>
<td>3.24</td>
<td>2.5</td>
<td>2.3</td>
</tr>
<tr>
<td>Platinum alloy</td>
<td>SM1848</td>
<td>F</td>
<td>41.8</td>
<td>31.0</td>
<td>91.2</td>
<td>60.7</td>
<td>60.6</td>
</tr>
</tbody>
</table>

Table 4.10: Description of the drawing tests

<table>
<thead>
<tr>
<th>Reference</th>
<th>$R_i$ (mm)</th>
<th>$r_i$ (mm)</th>
<th>$R_f$ (mm)</th>
<th>$r_f$ (mm)</th>
<th>$\alpha$ (°)</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>316LVM - SM1900</td>
<td>12</td>
<td>9</td>
<td>9.5</td>
<td>7.01</td>
<td>12.5</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>316LVM - SM810</td>
<td>5.25</td>
<td>4.5</td>
<td>4.05</td>
<td>3.5</td>
<td>16.5</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>316LVM - SM664</td>
<td>4.08</td>
<td>3.52</td>
<td>3.32</td>
<td>2.65</td>
<td>11</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>L605 - SM750</td>
<td>3.77</td>
<td>3.27</td>
<td>3.15</td>
<td>2.75</td>
<td>11.4</td>
<td>0.065</td>
<td>0.065</td>
</tr>
<tr>
<td>L605 - SM630</td>
<td>3.19</td>
<td>2.75</td>
<td>2.69</td>
<td>2.35</td>
<td>12</td>
<td>0.065</td>
<td>0.065</td>
</tr>
<tr>
<td>L605 - SM530</td>
<td>1.97</td>
<td>1.75</td>
<td>1.5</td>
<td>1.35</td>
<td>12.4</td>
<td>0.065</td>
<td>0.065</td>
</tr>
<tr>
<td>Platinum alloy - SM1848</td>
<td>10.85</td>
<td>8.15</td>
<td>9.24</td>
<td>7</td>
<td>33</td>
<td>0.07</td>
<td>0.07</td>
</tr>
</tbody>
</table>
4.4 Conclusion

In this chapter, first the FEM of tube drawing was developed and different models were presented. Three different models were detailed, a first one in which the material is considered visco-plastic and isotropic, a second where the material is anisotropic and the visco-plasticity is neglected, a third one where the material is visco-plastic and isotropic and the thermal aspects are considered. The identification of the friction coefficient and thermal contact properties was detailed and a pressure dependent friction coefficient was identified. The mechanical models enabled to analyse the stress and strain fields taking place during drawing and to evaluate the effect of the die angle on the process. It was observed that decreasing the die angle enabled to homogenise the stress and strain distribution in the tube thickness and to improve formability by increasing the compressive hydrostatic stress. The thermo-mechanical model enabled to simulate temperature fields close to the experimentally measured with a satisfactory accuracy.

Finally, analytical methods were developed and the computed drawing forces were compared with the simulated and the experimental ones. The analytical methods accuracy was not satisfactory compared to the FEM. It was shown that the homogeneous deformation and the slab method were not able to model the optimum die angle. On the contrary, the upper-bound method which considers all the energies involved in the process and the FEM enabled to estimate an optimum die angle. Nevertheless, the optimum angles computed by both methods differed greatly.

As a consequence, this chapter enabled to finalise the FEM and to prove its usefulness regarding the process analysis and the drawing force computation. Moreover, it allowed to dismiss the analytical methods whose performances are limited.
Chapter 5

Failure prediction

Sommaire

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Throughout the previous chapters, the FEM of tube drawing was built and the experimental formability limit of the material during drawing was identified. The objective of the present chapter is to predict the tube fracture by means of the failure criteria that were presented in chapter 1 and calibrated on tensile tests in chapter 2. As detailed in chapter 4, different FE models were defined and failure criteria were computed for each of them. As a reminder, in models $M_1$ and $M_2$ the material was considered as isotropic and visco-plastic. The thermal effects were included in $M_2$ only. Nevertheless, as the temperature increase was considered low enough, the material thermal softening was neglected. As a consequence, the stress/strain fields modelled by $M_1$ and $M_2$ were identical. In the $M_3$ model, the material was considered as anisotropic and the visco-plasticity was neglected. In short, both the section and thickness reductions were predicted by means of the $M_1$ and $M_3$ models and compared with the experimentally observed reductions.

Despite the discussion about the material being plastic anisotropic or not in chapter 3, both models were evaluated. It enabled to quantify the error made on failure prediction if one or the other model was used. First, both models were evaluated on the conical mandrel test performed with a die semi-cone angle of $12.5^\circ$ on the L605 material. Then, they were evaluated and validated on further conical mandrel drawing tests made with die semi-cone angles of 5, 16, 20$^\circ$. Finally, they were validated on the 316LVM tubes.
5.1 Computation of the failure criteria from FEM

The failure criteria that were presented in chapter 1 are reminded in table 5.1. They were computed independently of the FEM. A script was written in Python to extract stress and strain data from Abaqus and to compute the failure criteria. The script enables to collect the components of the Cauchy stress and strain tensor at the mesh nodes for each time increment. It also enables to collect the nodes coordinates in order to plot the 2D stress and strain maps. Criteria with an integral form are computed by finite differences.

<table>
<thead>
<tr>
<th>Type</th>
<th>Abbreviation</th>
<th>Criterion</th>
<th>Damage variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>STRN</td>
<td>Equivalent strain</td>
<td>( D_1 = \bar{\epsilon} )</td>
</tr>
<tr>
<td>1</td>
<td>MSS</td>
<td>Maximum shear stress</td>
<td>( D_2 = \tau_{\text{max}} = \frac{\sigma_1 - \sigma_3}{2} )</td>
</tr>
<tr>
<td>1</td>
<td>SHAB</td>
<td>Vujovic and Shabaic (1986)</td>
<td>( D_3 = \frac{3\sigma_m}{\sigma} )</td>
</tr>
<tr>
<td>2</td>
<td>FREU</td>
<td>Freudenthal (1950)</td>
<td>( D_4 = \int_0^{\bar{\epsilon}} \sigma d\bar{\epsilon}_p )</td>
</tr>
<tr>
<td>2</td>
<td>COCK</td>
<td>Cockcroft and Latham (1968)</td>
<td>( D_5 = \int_0^{\bar{\epsilon}} \max(0, \sigma_1) d\bar{\epsilon}_p )</td>
</tr>
<tr>
<td>2</td>
<td>RICE</td>
<td>Rice and Tracey (1969)</td>
<td>( D_6 = \int_0^{\bar{\epsilon}} \exp(3\sigma_m) d\bar{\epsilon}_p )</td>
</tr>
<tr>
<td>2</td>
<td>BROZ</td>
<td>Brozzo et al. (1972)</td>
<td>( D_7 = \int_0^{\bar{\epsilon}} \frac{2\sigma_1}{3(\sigma_1 - \sigma_m)} d\bar{\epsilon}_p )</td>
</tr>
<tr>
<td>2</td>
<td>ARGO</td>
<td>Argon et al. (1975)</td>
<td>( D_8 = \int_0^{\bar{\epsilon}} (\sigma_m + \bar{\sigma}) d\bar{\epsilon}_p )</td>
</tr>
<tr>
<td>2</td>
<td>OH</td>
<td>Oh et al. (1976)</td>
<td>( D_9 = \int_0^{\bar{\epsilon}} \frac{\sigma_1}{\sigma} d\bar{\epsilon}_p )</td>
</tr>
<tr>
<td>2</td>
<td>AYAD</td>
<td>Ayada et al. (1984)</td>
<td>( D_{10} = \int_0^{\bar{\epsilon}} \sigma_m d\bar{\epsilon}_p )</td>
</tr>
<tr>
<td>2</td>
<td>TREN</td>
<td>Tresca energy</td>
<td>( D_{11} = \int_0^{\bar{\epsilon}} \frac{(\sigma_1 - \sigma_3)}{2} d\bar{\epsilon}_p )</td>
</tr>
</tbody>
</table>

Table 5.1: Details of the selected fracture criteria

5.1.1 Mechanical model considering isotropy, M1

5.1.1.1 Evaluation of failure criteria

The approach was to follow the evolution of the different failure criteria with an increasing mandrel diameter during conical mandrel drawing. The focus was to identify the mandrel diameter at which the tube is supposed to fracture. Figure 5.1 presents the different failure criteria computed throughout the FEM of conical mandrel drawing. The damage variable are normalised by the calibrated value \( D_{\text{crit}} \) (cf. table 2.10). In this figure, the normalised damage variables are plotted as a function of the mandrel diameter and it can be observed that their values increase with increasing mandrel diameter. When a normalised damage variable crosses the horizontal black line (critical value of 1) the criterion is satisfied and the tube is supposed to fracture. Figure 5.1(b) is an enlargement of figure 5.1(a) around the critical value of 1. In this figure, the mandrel radii corresponding to the fracture limit are highlighted. The corresponding predicted section reductions are listed in table 5.2. The estimation errors are also listed. The errors were computed from the experimental section reductions that are reminded in table 5.3.

It can be observed that estimation errors range from nearly 50% to 64.6% for MSS, TREN, STRN and FREU. Estimation is improving with ARGO, RICE, OH and BROZ...
Figure 5.1: Isotropic plastic L605: evolution of failure criteria with mandrel radius, (a) global view, (b) zoom around the critical value and plot of the critical mandrel radii

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Predicted section reduction (%)</th>
<th>Mean error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>STRN</td>
<td>27.6</td>
<td>48.7</td>
</tr>
<tr>
<td>MSS</td>
<td>19.2</td>
<td>64.3</td>
</tr>
<tr>
<td>SHAB</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>FREU</td>
<td>27.6</td>
<td>48.6</td>
</tr>
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<td>COCK</td>
<td>53.9</td>
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<td>RICE</td>
<td>70.5</td>
<td>31.0</td>
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<td>BROZ</td>
<td>71.6</td>
<td>33</td>
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<td>ARGO</td>
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<td>29.1</td>
</tr>
<tr>
<td>OH</td>
<td>75.0</td>
<td>39.3</td>
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<tr>
<td>AYAD</td>
<td>&gt;75.4</td>
<td>&gt;40.1</td>
</tr>
<tr>
<td>TREN</td>
<td>25.7</td>
<td>52.2</td>
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</tbody>
</table>

Table 5.2: Predicted section reduction and section reduction prediction error (underlined criteria are expressed in MPa, others are without units).

<table>
<thead>
<tr>
<th>Experimental section reduction at fracture (%)</th>
<th>Tube 1</th>
<th>Tube 2</th>
<th>Tube 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>54.17</td>
<td>53.37</td>
<td>53.91</td>
</tr>
</tbody>
</table>

Table 5.3: Experimental section reduction at fracture obtained for the three drawing tests.
criteria with errors ranging from 28.5% to 40.5% but prediction is still not satisfactory. COCK criterion shows the lowest estimation error: 0.2%. Maximum section reduction predicted by AYAD is unknown and above 75.4% because the simulation was interrupted due to an excessive mesh distortion. Such phenomenon can be avoided by means of remeshing techniques. Finally SHAB criterion predicts failure at the very first step of tube drawing, thus, according to this criterion, drawing is not possible, which is obviously not the case. It has to be mentioned that STRN failure criterion is commonly used at Minitubes for the definition of the drawing passes. From this first analysis, it can be deduced that the use of another failure criterion could improve the productivity.

Figure 5.2 presents ten tube inner surface profiles corresponding to ten failure criteria. Figure 5.2(a) first presents the position of the tube inner and outer surfaces relative to the die and the conical mandrel for the MSS criterion only. The drawing direction is also shown. In figure 5.2(b), the mandrel was removed for clarity and several tube inner surface profiles are superimposed. Each inner tube surface is plotted for a given failure criterion and corresponds to the moment when the criterion is satisfied. The outer tube surface is common for all the failure criteria. The mean experimental tube inner radius is also plotted. Thus, one can easily compare the maximum reduction predicted by each criterion. As mentioned earlier, COCK is very close to the experimental test, RICE, OH and BROZ are overestimating the maximum reduction while ARGO, STRN, FREU, TREN and MSS underestimate it. There are no data corresponding to AYAD criterion since the simulation did not run long enough but the predicted section reduction at fracture is overestimated. The tube inner surface would be located between OH prediction and the outer surface. Figure 5.2 also shows the point where failure criteria is verified for each criterion. This point represents the locus of failure initiation. It can be seen that all the failure initiation points are located at the tube inner surface. Fracture is then expected to initiate on the inner tube surface and to propagate outward. This observation is consistent with the failure propagation direction observed on SEM in figure 3.8. The axial position of failure locus is predicted with an error linked to the Mesh deformation. Indeed, damage variables are
### Table 5.4: Experimental section reduction at fracture and Cockcroft-Latham predicted section reduction as a function of the die semi-cone angle.

<table>
<thead>
<tr>
<th>Die semi-cone angle (°)</th>
<th>5</th>
<th>13.12</th>
<th>16</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental section reduction at fracture (%)</td>
<td>&gt;56.4</td>
<td>53.8</td>
<td>50.7</td>
<td>49.9</td>
</tr>
<tr>
<td>Predicted section reduction at fracture (%)</td>
<td>59.7</td>
<td>53.9</td>
<td>51.4</td>
<td>47.1</td>
</tr>
<tr>
<td>Error (%)</td>
<td>-</td>
<td>0.2</td>
<td>1.3</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Compared computed at the mesh nodes and the initial mesh is getting more distorted with increasing tube reduction. Nevertheless, RICE, BROZ and OH predict that the fracture initiates after the die while the other criteria predict the fracture to occur inside the die. The conclusion that arises from this first analysis is that the fracture locus is well predicted by the Cockcroft-Latham failure criterion but only COCK is able to quantitatively predict fracture.

#### 5.1.1.2 Influence of the die angle on Cockcroft-Latham failure criterion

Further conical mandrel drawing tests were performed in order to validate the predictability of COCK failure criterion. Three series of complementary drawing tests were performed with dies of different semi-cone angle: $\alpha = 5^\circ$, $\alpha = 16^\circ$, and $\alpha = 20^\circ$. The drawing conditions were identical to the first series of tests. Table 5.4 reminds the experimentally observed section reductions at fracture and the COCK failure criterion prediction for the four die semi-cone angles. It can be seen from these complementary tests and simulations that Cockcroft-Latham failure criterion is able to predict failure accurately for different die semi-cone angles. During the test made with a semi-cone angle of $\alpha = 5^\circ$ the tube did not fracture. For this die semi-cone angle the experimental section reduction at fracture is unknown but above 56.4%. Furthermore it can be concluded from this series of drawing tests that the maximum section at fracture increases with decreasing die semi-cone angle. Thus, the use of smaller die angles should be considered in the tube drawing industry to improve formability.

Figures 5.3(a,b,c,d) presents the Cockcroft-Latham criterion field for different semi-die angles. Data were plotted for a mandrel diameter of 10.03 mm. The criterion profile along two straight lines for different die angles were also plotted in figures 5.3(e,f). From these figures, it can be observed that increasing the die angle increases the heterogeneity of the Cockcroft-Latham criterion in the tube thickness: Cockcroft-Latham values are lowered down nearby the outer tube surface and increased close to the inner tube surface. For example, for a semi-cone angle of 20°, Cockcroft-Latham value reaches 1.03 MPa while it reaches 0.61 MPa a semi-cone angle of 5°. As a reminder, Cockcroft-Latham is computed as the integral of the maximum principal stress (axial stress) with the increment of equivalent plastic strain. It was presented in chapter 4 that increasing die angle induced the principal axial stress nearby the inner tube surface to increase and to decrease close to the outer surface. Thus, the evolution of the axial principal stress in the tube thickness is consistent with the fact that COCK increases close to the inner tube surface with increasing die angle and that it decreases close to the outer tube surface.
Figure 5.3: (a,b,c,d) Cockcroft-Latham failure criterion field as a function of the die semi-cone angle (e,f) comparison of the Cockcroft-Latham failure criterion profiles along the solid lines l1 (e) and l3 (f) for different die semi-cone angles.
<table>
<thead>
<tr>
<th>Type</th>
<th>Abbreviation</th>
<th>Criterion</th>
<th>Damage variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>STRN</td>
<td>Equivalent strain</td>
<td>$D^H_1 = \bar{e}^H$</td>
</tr>
<tr>
<td>1</td>
<td>MSS</td>
<td>Maximum shear stress</td>
<td>$D^H_2 = \tau_{\text{max}} = \frac{\sigma_1 - \sigma_3}{2}$</td>
</tr>
<tr>
<td>1</td>
<td>SHAB</td>
<td>Vujovic and Shabaic (1986)</td>
<td>$D^H_3 = \frac{3\tau_{\text{mm}}}{2}$</td>
</tr>
<tr>
<td>2</td>
<td>FREU</td>
<td>Freudenthal (1950)</td>
<td>$D^H_4 = \int_0^{\bar{e}^H_p} \sigma_{H} d\bar{e}^H_p$</td>
</tr>
<tr>
<td>2</td>
<td>COCK</td>
<td>Cockcroft and Latham (1968)</td>
<td>$D^H_5 = \int_0^{\bar{e}^H_p} \max(0, \sigma_1) d\bar{e}^H_p$</td>
</tr>
<tr>
<td>2</td>
<td>RICE</td>
<td>Rice and Tracey (1969)</td>
<td>$D^H_6 = \int_0^{\bar{e}^H_p} \exp(\frac{2\sigma_{\text{mm}}}{3\sigma_1}) d\bar{e}^H_p$</td>
</tr>
<tr>
<td>2</td>
<td>BROZ</td>
<td>Brozzo et al. (1972)</td>
<td>$D^H_7 = \int_0^{\bar{e}^H_p} (\sigma_m + \bar{\sigma}^H_p) d\bar{e}^H_p$</td>
</tr>
<tr>
<td>2</td>
<td>ARGO</td>
<td>Argon et al. (1975)</td>
<td>$D^H_8 = \int_0^{\bar{e}^H_p} (\sigma_m + \bar{\sigma}^H_p) d\bar{e}^H_p$</td>
</tr>
<tr>
<td>2</td>
<td>OH</td>
<td>Oh et al. (1976)</td>
<td>$D^H_9 = \int_0^{\bar{e}^H_p} (\sigma_m + \bar{\sigma}^H_p) d\bar{e}^H_p$</td>
</tr>
<tr>
<td>2</td>
<td>AYAD</td>
<td>Ayada et al. (1984)</td>
<td>$D^H_{10} = \int_0^{\bar{e}^H_p} (\sigma_m + \bar{\sigma}^H_p) d\bar{e}^H_p$</td>
</tr>
<tr>
<td>2</td>
<td>TREN</td>
<td>Tresca energy</td>
<td>$D^H_{11} = \int_0^{\bar{e}^H_p} (\sigma_1 - \sigma_3)^2 d\bar{e}^H_p$</td>
</tr>
</tbody>
</table>

Table 5.5: Fracture criteria expression as function of Hill’s equivalent stress and strain. Exponent $H$ denotes the anisotropic form of the failure criteria

### 5.1.2 Mechanical model considering anisotropy

The introduction of plastic anisotropy induces the modification of the failure criteria expressions. Indeed, failure criteria are expressed in terms of Mises equivalent stress and strain. In the case of an anisotropic material, Mises equivalent stress and strain are not representative of the material behaviour. Thus failure criteria expressions were written as function of Hill’s equivalent stress and strain as reminded in table 5.5. Moreover, they were calibrated by means of the tube tensile tests considering Hill’s equivalent stress and strain. The evolution of anisotropic failure criteria as a function of the mandrel diameter during conical mandrel drawing simulation is shown in figure 5.4. From these data, and considering that the material exhibits plastic anisotropy it can be observed that Cockcroft-Latham, Brozzo and Rice failure criteria are the best able to predict section reduction at fracture with errors of 0.03%, 3.9% and 7.3% respectively. The reason why the anisotropic Cockcroft-Latham prediction is very similar to the isotropic criterion is due to the fact that the axial direction was taken as the reference direction for Hill’s parameters identification. In this way, the axial yield stress ratio $R_z = \frac{\sigma_z}{\sigma_0} = 1$ and as a consequence, the axial stress appearing in Hill 1948 is not affected by any coefficient. Finally, as $\sigma_z$ is the only stress appearing in Cockcroft-Latham expressions, the criterion value varies little if anisotropy is considered. This result is important for this study as it means that an error in characterising the material anisotropy does not induce any misestimation of the section reduction at fracture by means of Cockcroft-Latham criterion.

The predicted failure loci are illustrated in figure 5.5 and the same conclusions can be drawn for the anisotropic failure criteria compared to the isotropic ones: failure locus is predicted at the inner tube surface and failure is suspected to propagate outward.
Figure 5.4: Anisotropic plastic L605: evolution of failure criteria with mandrel radius, (a) global view, (b) zoom around the critical value and plot of the critical mandrel radii.

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Predicted section reduction (%)</th>
<th>Mean error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>STRN$^H$</td>
<td>20.8</td>
<td>61.3</td>
</tr>
<tr>
<td>MSS$^H$</td>
<td>13.5</td>
<td>74.9</td>
</tr>
<tr>
<td>SHAB$^H$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>FREU$^H$</td>
<td>20.3</td>
<td>62.3</td>
</tr>
<tr>
<td>COCK$^H$</td>
<td>53.8</td>
<td>0.03</td>
</tr>
<tr>
<td>RICE$^H$</td>
<td>49.9</td>
<td>7.3</td>
</tr>
<tr>
<td>BROZ$^H$</td>
<td>55.9</td>
<td>3.9</td>
</tr>
<tr>
<td>ARGO$^H$</td>
<td>32.4</td>
<td>39.8</td>
</tr>
<tr>
<td>OH$^H$</td>
<td>61.1</td>
<td>52.7</td>
</tr>
<tr>
<td>AYAD$^H$</td>
<td>&gt;75.4</td>
<td>40.1</td>
</tr>
<tr>
<td>TREN$^H$</td>
<td>19.8</td>
<td>53.5</td>
</tr>
</tbody>
</table>

Table 5.6: Predicted section reduction and section reduction prediction error considering plastic anisotropy (underlined criteria are expressed in MPa, others are without units).

Figure 5.5: Failure initiation loci of anisotropic plastic L605: (a) detail of the process, (b) enlargement of the zone close to the die and plot of the predicted tube inner surface and failure locus for each criterion. The outer tube surface is common for all criteria.
<table>
<thead>
<tr>
<th></th>
<th>Section reduction (%)</th>
<th>Thickness reduction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Test 1</strong></td>
<td>68.5</td>
<td>62.9</td>
</tr>
<tr>
<td><strong>Experimental</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test 2</td>
<td>66.4</td>
<td>60.4</td>
</tr>
<tr>
<td>Average</td>
<td>67.5</td>
<td>61.7</td>
</tr>
<tr>
<td><strong>FEM</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Predicted</td>
<td>60</td>
<td>52.4</td>
</tr>
<tr>
<td>Error (%)</td>
<td>11</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 5.7: 316LVM: experimental section and thickness reduction

5.1.3 Validation with 316LVM

The efficiency of Cockroft-Latham failure criterion was evaluated on the L605 with conical mandrel drawing for different semi-die angles. In the end, it is interesting to validate its predictability for another material: the 316LVM. As for the L605, the failure criteria were calibrated on tube tensile tests up to fracture, conical mandrel drawing tests were performed and simulated and the failure criteria were computed. Table 5.7 presents the experimental and predicted section and thickness reductions at fracture for the conical mandrel drawing tests on 316LVM. The estimation errors are 11% and 15% respectively. The prediction is less accurate in this case but still satisfactory.

5.2 Discussion

5.2.1 The different criteria

The different failure criteria that were presented and tested in this study were developed throughout the years. Each of them was defined by its authors according to experimental observations or based on the physical mechanism of ductile failure. Nevertheless, it appears that their ability to predict fracture during tube drawing is very disparate. The following paragraph aims at discussing the reason why some criteria might be more successful in predicting failure.

Starting with the instantaneous damage variable like constant strain (STRN), maximum shear stress (MSS) and Vujovic and Shabaic (1986)(SHAB), it was shown in chapter 1 that material formability was dependent on the deformation history. As a consequence, it is not surprising to see their inaccuracy in predicting failure.

Going on with the damage accumulation variables and the criteria based on energetics consideration different remarks can be expressed. The analysis of each criterion is detailed in the following paragraphs.

Freudenthal (1950) criterion was based on the assumption that a material can only absorb a limited amount of energy which is quantified by the work of plastic deformation. Cockcroft and Latham (1968) demonstrated that such a failure criterion was not accurate for predicting failure. Their reasoning is based as follow. During a tensile test on a ductile material, necking occurs and the strain at fracture depends on the neck geometry prior to fracture. The stresses at the minimum section in the neck zone can be calculated and considered as the sum of two components: \( \sigma = \bar{\sigma} + \sigma_m \). The first component \( \bar{\sigma} \) is the current yield stress and is constant across the specimen section. The second component \( \sigma_m \)
is an hydrostatic tension that varies from zero at the sample periphery and increases up to a peak value at the center of the specimen. If the criterion of work of plastic deformation is considered, then, only the yield stress is taken into account and integrated with respect to the plastic strain. As the current yield stress is independent of the neck geometry the consequence is that Freudenthal (1950) criterion predicts a fracture independent of the neck geometry which is contradictory with the experimental observations (Bridgman, 1952). In this way, Cockcroft and Latham (1968) set out their arguments to introduce a criterion of ductile failure based on the highest principal stress. Rice and Tracey (1969) criterion is based on the mechanism of ductile fracture by void growth and coalescence and includes the hydrostatic stress $\sigma_m$. As a hydrostatic tensile stress is known to accelerate the void growth and coalescence (Rice and Tracey, 1969), the account of $\sigma_m$ in a criterion appears to be a good indicator of its predictability due to its physical basis. Nevertheless, Rice and Tracey (1969) criterion is based on the hypothesis that the fracture is dependent on the void fraction only and that it is independent of the void size, shape and spacing. Such consideration might not reflect the real loading conditions. In the case of tube drawing, the maximum principal stress is tensile while the other two radial and circumferential components are compressive. Considering this stress state and the corresponding strain state, a cylindrical void is expected to evolve towards an ovoid geometry whose principal axis is oriented in the direction of drawing. As a consequence, the fact that Rice and Tracey (1969) criterion does not consider void geometry evolution might explain its poor accuracy to predict failure during tube drawing.

Brozzo et al. (1972) based their criterion on the fact that ductility decreases with the hydrostatic stress which is consistent with Rice and Tracey (1969) observations. The choice they made was to include this phenomenon by subtracting the hydrostatic stress to the maximum principal stress.

Argon et al. (1975) analysis is based on a local stress criterion of interfacial separation between inclusions and matrix. The decohesion between the inclusion and the matrix and thus the cavity formation is expected to occur when a critical decohesion stress is reached. The decohesion stress is a combination of the hydrostatic stress and the equivalent stress. The analysis leading to this criterion concerns large particles whose diameter is more than 100 Å. The particles are considered rigid and plastically non deformable. Thus, it can be seen that their analysis has physical basis. Moreover this criterion is expected to be more accurate than Freudenthal (1950) as the effect of hydrostatic stress is superimposed to the effective stress. Again, the hydrostatic stress was proved to have a direct effect on void growth (Bridgman, 1952). To illustrate this remark, the error of the predicted section reduction at fracture are equal to 48.6 % and 29.1 % for Freudenthal (1950) and Argon et al. (1975) respectively. Thus, as expected, Argon et al. (1975) is more accurate.

Oh et al. (1976) failure criterion derives from the the simplification of McClintock (1968a) failure criterion. McClintock (1968a) developed its criterion based on physical phenomena of void growth and coalescence. He analysed the deformation of a hole in a infinite medium considering plane strain condition. The hole has an elliptical shape and is inserted into a cylindrical cell whose dimension is of the order of the mean spacing between holes. The condition of fracture corresponds to the deformed hole reaching its cell walls. Through his study, McClintock (1968a) showed that the fracture strain was influenced by the transverse...
principal stresses $\sigma_2$ rather than on $\sigma_m$ and the maximum or axial principal stress $\sigma_1$ only ($\sigma_1 > \sigma_2 > \sigma_3$). Finally, Oh et al. (1976) criterion is obtained from McClintock (1968a) by a linear transformation of the sinh function. From this transformation, the criterion greatly simplifies and the effect of transverse principal stresses is neglected. Thus, Oh et al. (1976) criterion is expected to be less predictive than McClintock (1968a).

Vujovic and Shabaic (1986) is a confusing failure criterion as it reduces to the stress triaxiality. It was shown that stress triaxiality could influence the amount of plastic strain a material could accumulate before fracture Mirone (2007) but it cannot be a failure criterion on its own.

5.2.2 My choice

As a conclusion, the most predictive failure criterion is the phenomenological Cockcroft and Latham (1968). The reasons found to explain such accuracy are the following. All the damage variables are calibrated on uniaxial tensile tests where the major principal stress is the tensile stress and the other two are zero. While some of the damage variables are expressed in terms of hydrostatic stress or equivalent stress, such a calibration results in a great simplification of the damage variable expression. Thus, some of the failure criteria are turning equivalent when calibrated on the tensile test e.g. Cockcroft and Latham (1968) and Freudenthal (1950) or Brozzo et al. (1972) and Oh et al. (1976) (table 2.10). Moreover, when the criteria are computed during tube drawing, the stress and strain state is complex and very different from uniaxial tension. As a consequence, some failure criteria were calibrated on non representative stress and strain states and are unable to predict failure in drawing. Zadpoor (2009) observed that failure criteria calibration depended upon the range of stress triaxiality of the test they were calibrated on. Consequently, more complex failure criteria might give better results provided that they are calibrated on tests with more complex state of stress.

Concerning Cockcroft and Latham (1968) failure criterion only, it gives good results as it is expressed in terms of the largest principal stress which is tensile both on calibration test and on tube drawing. According to the large number of studies about failure criteria, scientists highlight that failure criteria based on the maximum principal stress are more reliable in predicting fracture. Venugopal-Rao et al. (2003) also showed that integration of plastic deformation energy along the deformation path was a better approach compared to instantaneous path-independent parameters because metal working process are strain history dependent.

Kim et al. (2007) and Karnezis and Farrugia (1998) can be cited to illustrate Cockcroft and Latham (1968) predictability. Kim et al. (2007) validated the geometry of a newly designed mandrel tip to prevent tube tip failure during drawing and Karnezis and Farrugia (1998) transformed a two passes drawing process into a single one.

Finally, it must be noticed that other failure criteria exist (Bao and Wierzbicki, 2004; Wierzbicki et al., 2005). As these criteria are expressed as a function of several parameters, they require to be calibrated on a series of mechanical tests. The purpose of this study was to select a criterion that could be easily calibrated on a simple uniaxial test. The goal
was to insert the methodology in the industrial process, thus, it was crucial for the method to remain easy, fast and accurate.

5.3 Conclusion

Different failure criteria were evaluated for two materials by comparison of the experimental fracture of tube during conical mandrel drawing test and the corresponding FEM. It appears from this study that Cockcroft and Latham failure criterion is the most accurate criterion. Moreover, it is able to capture the effect of die semi-angle variation on formability. Indeed, it was experimentally observed that an increase in die semi-angle caused the formability limit to be reduced. Cockcroft and Latham failure criterion was able to capture this geometrical effect and predicted an earlier failure with greater die semi-angle.

The uncertainty of material plastic anisotropy was not prejudicial for failure prediction. In the case of anisotropic model, Cockcroft and Latham again was the most accurate criterion, prediction error increased but was still satisfactory.

Finally, the prediction accuracy of Cockcroft and Latham criterion when calibrated on tensile test is satisfactory and can be used in industrial applications. It appears to be a reliable tool to define the drawing passes. Thus, compared to the criterion of constant strain which is commonly used at Minitubes and which under-estimates the formability limit, Cockcroft and Latham can be a tool to define the drawing passes in a more advanced way and to improve productivity.

To go further, it would be interesting to analyse the effect of greater section reductions on the appearance of surface defects such as cracks or cavities. Moreover, it must be ensured that the reeling step does not cause additional damage. Finally, as the material gets more deformed, the annealing conditions might get changed as the amount of stored energy is greater with increased plastic deformation.
Throughout this thesis, different aspects of the tube drawing process were presented and analysed. The different issues that were initially introduced were examined. In short:

- the material behaviour was characterised;
- experimental drawing tests up to fracture were performed;
- the finite element model was built;
- the formability limit was identified.

In the first chapter, after an introduction of the industrial context and a presentation of the tube drawing process, the different phenomena involved in the aforesaid process were detailed. Plasticity was first introduced with a review of the constitutive equations that enable to describe the plastic behaviour: isotropic and anisotropic yield functions were presented followed by the flow rule and the hardening constitutive equations. Viscoplastic constitutive equations were also introduced. Then, the friction phenomenon that is unavoidable in any forming process involving forming tools was addressed. In a third paragraph, the heat generation phenomenon due to plastic deformation and friction was presented. Once the constitutive equations, the friction model and the heat equation were presented, different methods to analyse the process were described. Both analytical methods and finite element modelling were presented and their efficiency was compared by means of a bibliographic review. Finally, as a tube bulge test was designed in this study, its principle was also presented.

The second chapter dealt with the materials experimental characterisation which was necessary to understand the material behaviour and to build the finite element model. The different mechanical testing methods that were used were first introduced: tube tensile test, tensile test on oriented samples, tube bulge test. All the tests were presented in a detailed way, in particular the tube bulge test as this specific test was designed for the purpose of this study.

After the mechanical tests presentation, the material mechanical behaviour was characterised. In a first time, tube tensile tests enabled to characterise the work hardening behaviour and to fit the parameters of the hardening constitutive equations. A Ludwik
hardening constitutive equation was first used and the material viscoplasticity was further modelled by a Johnson-Cook constitutive equation. Then, as forming processes induce material deformation in preferred directions and in a repeated way, the induced plastic anisotropy was investigated. Tube bulge test were the first indicator of an anisotropic behaviour. The parameters of Hill (1948) yield function were fitted by means of an inverse analysis between experimental results and finite element modelling. With the objective of confirming the identified parameters, tensile tests on oriented samples cut from flattened tubes in different directions relative to the drawing direction were performed. Tensile tests were performed in the drawing direction, at $45^\circ$ from the drawing direction and in the transverse direction and it was found that the material exhibited isotropic plastic properties.

Due to the inconsistency between both mechanical tests, different hypotheses were investigated to explain such different results. The first hypothesis was to consider that the selected anisotropic yield function was not suited for this study. Nevertheless a bibliographic review discarded this proposition. The second hypothesis was to investigate the material properties being heterogeneous in the tube thickness. Micro-hardness measurements were performed in the tube wall thickness and it appeared that the mechanical properties were non-homogeneous. Such characteristic could not be measured by means of the tube tensile test as it resulted in the measurement of an average behaviour. On the contrary, as the tube bulge test caused a gradient of strain in the tube thickness, a different mean mechanical behaviour was measured. As a last verification, surface and volume X-ray diffraction measurements were performed to evaluate material texture. Once the material mechanical behaviour was characterised, the thermo-mechanical properties were identified. Finally, keeping in mind that one of the principal objective of this study was to predict tube failure during drawing, different failure criteria were calibrated by means of tube tensile tests.

The third chapter was devoted to the description and the results of the conical mandrel drawing test that was designed to identify tube fracture during drawing. The specificity of this drawing test was to use a mandrel of conical geometry whose goal was to progressively increase the section and thickness reductions during drawing and to reach tube failure. The conical mandrel drawing test was described in detail. The force and temperature measurements during drawing were presented and the experimental tube section and thickness reductions at failure were identified. A parametric study involving a set of dies with different semi-cone angles revealed that the use of dies with lower angles enabled to increase the section reduction at fracture with little influence on the drawing force. Concerning the thermal measurement by means of an infra-red camera, the protocol for emissivity calibration as a function of the temperature was described.

In the fourth chapter, the finite element model was described. The geometry, the material properties, the boundary conditions, the mesh, the time incrementation and the contact definition were all presented. Three different models were defined. First, considering plastic isotropy, both a mechanical and a thermo-mechanical models were built with a Johnson-Cook hardening constitutive equation. Second, considering plastic
anisotropy, a pure mechanical model was built with Hill (1948) yield function and a Ludwik hardening constitutive equation.

As presented in the previous paragraphs, constitutive equations and thermal properties were identified by means of experimental tests. Nevertheless the friction condition remained to be characterised. This final step was performed by means of inverse analysis. The friction coefficients were identified by comparison of the drawing force that was experimentally measured and the simulated one. It resulted that a pressure dependent friction coefficient had to be identified for the experimental and simulated forces to fit.

Once the finite element model was complete, the heat generation and exchanges taking place inside the tube were analysed. Good correlation was found between the experimental and simulated temperatures. It was found that locally, the temperature of the L605 could reach 220°C.

In the case of pure mechanical analysis, the different stress and strain fields in the tube were analysed to improve the process understanding. The evolution of the distribution and levels of the maximum principal stress, the hydrostatic stress, the stress triaxiality and the plastic deformation were analysed as a function of the die angle. This analysis revealed that lowering the die angle caused the maximum principal stress to homogenise and to decrease. A decrease in die angle caused higher compressive hydrostatic stress to develop which is favourable to reach higher deformations. Finally, decreasing the die angle caused the strain to homogenise in the tube thickness which is expected to produce more homogeneous tubes. Then, the influence of the die angle on the drawing force was analysed in terms of the balance between energies dissipated by friction and plastic deformation. The energy dissipated by friction decreases with increasing die angle while the energy dissipated by plastic deformation increases with increasing die angle. Thus, the influence of the die angle on the drawing force can be explained by computing the sum of both energies.

Different analytical methods were developed in order to compare their efficiency with the finite element modelling. The homogeneous deformation method, the slab method and the upper-bound methods were applied to estimate drawing forces of different drawing passes. From this comparison, the finite element modelling appeared to be the only reliable method to estimate the drawing force.

The last chapter dealt with the estimation of tube failure by means of all the tools that were presented in the previous chapters. The experimental tube fracture that was observed during the conical mandrel drawing was compared with the FEM. The different failure criteria were evaluated and Cockcroft-Latham failure criterion was found to be the most accurate to predict tube fracture. Moreover, this criterion enabled to capture the effect of increased formability obtained with decreasing die angle.

The conclusion concerning the selected Cockcroft-Latham failure criterion was that its predictability when calibrated on tube tensile test was satisfactory. This criterion together with the use of FEM can be used to define the drawing passes and to optimise the process.

This thesis enabled to built different tools that can be used to improve the process understanding and optimisation. It was revealed that the section and thickness reductions
that are currently defined in the process can be increased. Nevertheless, it must be checked that getting closer to the failure limit during drawing does not introduce internal and non visible damage such as small cracks or cavities. Moreover, in the case of mandrel drawing, it must be ensured that the reeling step does not induce additional damage and cause the tube to fracture in this step. The reeling step could be studied by means on FEM. To do so, it is important to characterise the material mechanical behaviour during elastic unloading.

More generally, different outlooks emerged from this work:

- The thermo-mechanical modelling revealed that temperatures could locally reach 220°C for the L605. Thus, the thermal softening that was neglected in the Johnson-Cook constitutive equation should be included. Taking into account the thermal effects is relevant due to the fact that a mandrel is successively reused to draw different tubes belonging to the same pass. The first tube of any drawing pass is drawn on a cold mandrel. The mandrel warms up during drawing and is reused to draw another tube. Then, the mandrel temperature progressively increases along the process and may undergo dimensional variations due to thermal expansion. The die is also concerned by the temperature increase and thermal expansion. As a result, the final tube dimensions might differ from the first drawn tube.
- The material heterogeneous properties in the tube wall should be investigated and their effect on the finite element modelling accuracy should be evaluated. This point can be first addressed in the case of tube bulge test.
- The FEM concerned the drawing pass of rather big tubes for which the number of grains in the tube thickness is large. During the process passes, the more the tube is drawn, the more its dimensions are reduced and the less is the number of grains in the tube thickness (annealing conditions remain constant throughout the process). In some cases, only a few grains remain in the tube thickness. In the objective of modelling all the drawing passes, the effects of grain number in the tube thickness on the mechanical behaviour and on the material formability should be investigated.
- Concentricity and the evolution of surface defects such as holes or scratches could also be analysed as they are major issues of the industrial process. In this case, a 3D model should be developed.

Finally, in an industrial point of view, a tool, devoted to the definition of the different drawing passes from the ebauche to the final product, can be developed by combination of the FEM and the failure criteria computation.


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