Replenishment policies for deteriorating items under uncertain conditions by considering green criteria

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Dedication

To my parents
and
To my husband, Behnam

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Politiques de réapprovisionnement pour les produits périssables dans des conditions incertaines et en considérant des critères environnementaux

Résumé

Le développement et l'application de modèles de réapprovisionnement d'articles périssables est l'une des principales préoccupations des experts en la matière, le nombre et la variété des produits périssables augmentant de façon spectaculaire. De nombreux types de produits tels que les produits pharmaceutiques, de la santé et des cosmétiques, les produits alimentaires, ainsi que les produits issus des biotechnologies, de la pétrochimie et de la matière chimique sont classés comme des produits qui se détériorent. L'une des lacunes majeures dans la littérature pour la gestion des produits périssables est que les chercheurs n'ont pas accordé suffisamment d'attention à deux aspects importants dans leurs modèles: i) les conditions stochastiques ; en particulier le délai stochastique est presque négligé car rendant les défis mathématiques plus compliqués ; *ii*) l'élaboration de politiques innovantes de réapprovisionnement prenant en compte les critères environnementaux ; en particulier la minimisation des émissions de CO_2 comme second objectif dans un contexte de modélisation multi-objectif qui est tout à fait nouvelle.

Aujourd'hui, les concepts écologiques et environnementaux ont été étendus à de nombreux domaines, y compris les chaînes logistiques. Il est évident qu'une grande partie de la législation relative à l'environnement est liée au type de produit dans les chaînes d'approvisionnement. Par conséquent, les produits périssables sont plus critiques que ceux à durée de vie infinie, ceci en raison :

- ✓ des processus de recyclage nécessaires pour les produits périmés,
- ✓ de la situation particulière qui existe pour le stockage et le transport,
- ✓ et enfin des effets toxiques de ces produits détériorés.

Dans cette thèse, nous présentons dans un premier temps une revue exhaustive de la littérature. Ensuite, les approches mathématiques pour la modélisation des processus de détérioration sont identifiées et classées en trois groupes. Par la suite, nous étudions les politiques de réapprovisionnement pour les produits périssables sous conditions stochastiques sous forme de trois problématiques différentes. Dans la première, nous développons un modèle de réapprovisionnement à révision continue (r, Q) pour un détaillant qui offre un produit périssable en prenant en compte : un horizon de planification infini, un délai d'approvisionnement stochastique, un taux de demande constante et la livraison tardive (backorder). Pour modéliser le processus de détérioration, un coût de possession de stock non linéaire est défini.

La prise en considération du délai stochastique et d'un coût de possession de stock non linéaire rend le modèle mathématique plus complexe. Nous avons donc adapté le modèle proposé pour une fonction de distribution uniforme afin de résoudre de façon optimale ce problème par une approche exacte. Pour le second problème, nous étudions la stratégie de mutualisation des risques de délai de livraison par la passation de commandes de réapprovisionnement fractionnées par lots entre plusieurs fournisseurs simultanément pour un détaillant vendant un produit périssable. Le système d'inventaire est modélisé comme un système de réapprovisionnement à révision continue (r, Q) et à délai stochastique. Nous étudions deux situations. Dans la première situation, on suppose que tous les besoins sont réapprovisionnés par une seule source, tandis que dans la seconde, deux fournisseurs sont disponibles. Etant donné que les modèles mathématiques développés sont très complexes, la programmation quadratique séquentielle (Sequential Quadratic Programming SQP) est utilisée pour résoudre ces problèmes. Ensuite, les situations dans lesquelles chaque politique d'approvisionnement est le plus économique sont déterminés.

Enfin, dans le dernier problème, nous prenons en considération les coûts de stockage et de transport, ainsi que les impacts sur l'environnement, dans une chaîne d'approvisionnement centralisée sous condition de demande incertaine et pénurie partielle (partial backordered). En raison de la caractéristique de dégradation des produits, un taux de détérioration constante et une fonction non linéaire de coût de possession de stock est pris en compte. Pour faire face à l'incertitude de la demande, est adoptée une approche de programmation stochastique en deux étapes. Par la suite, en tenant compte de la capacité de transport de véhicules, nous développons un modèle mathématique de programmation mixte en nombres entiers. De cette façon, les meilleurs véhicules de transport et les politiques de réapprovisionnement sont déterminés par la recherche d'un équilibre entre les critères financiers et environnementaux. Un exemple numérique du monde réel est également présenté pour démontrer l'applicabilité et l'efficacité du modèle proposé.

Mots-clefs: politique de réapprovisionnement, articles périssables, le coût possession de stock non linéaire, gestion de stocks, chaîne logistique verte, les conditions stochastiques.

Replenishment policies for deteriorating items under uncertain conditions by considering green criteria

Abstract

The development and application of inventory models for deteriorating items is one of the main concerns of the experts in the domain, since the number and variety of deteriorating products are dramatically increasing. Many types of products such as pharmaceuticals, health and cosmetics, foodstuffs, biotechnological, petrochemical and chemical materials are classified as deteriorating products. One of the major gaps in the deteriorating inventories literature is that researchers have not paid enough attention to two important features in their models: *i*) Considering stochastic conditions; especially stochastic lead time is almost overlooked since makes the mathematical challenges complicated, *ii*) designing innovative inventory policies by taking into account the environmental issues and particularly the CO_2 emission as a new objective in a multi-objective framework that is quite new. Today, the green principles have been expanded to many areas, including supply chains. It is obvious that much of the green legislation is relating to the type of product that is offered by supply chains. Accordingly, deteriorating products are more noticeable rather than infinite lifetime ones, because of:

- ✓ recycling process which is necessary for expired goods,
- ✓ special condition which exists for stocking and transporting,
- ✓ and even toxic effects of deteriorated products (for example in the case of radioactive substances).

In this thesis, at first a comprehensive literature review is done. Then, the mathematical approaches for modeling deterioration process are identified and classified in three groups. Subsequently, we study replenishment policy for deteriorating products under stochastic conditions in form of three different problem areas. In the first one, we develop a continuous *(r,Q)* inventory model for a retailer that offers a deteriorating product by considering infinite planning horizon, stochastic lead time, constant demand rate and backordered shortages. For modeling the deterioration process, a non-linear holding cost is defined.

Taking into consideration the stochastic lead time as well as a non-linear holding cost makes the mathematical model more complex. We therefore customize the proposed model for a uniform distribution function that could be tractable to solve optimally by an exact approach.

In second problem, we study the strategy of pooling lead time risks by splitting replenishment orders among multiple suppliers simultaneously for a retailer that sells a deteriorating product. The inventory system is modeled as a continuous

review system *(r, Q)* under stochastic lead time. We study two situations. In the first one, it is assumed that all the requirements are supplied by only one source, while in the second, two suppliers are available. Since the developed mathematical models are very complex, the Sequential Quadratic Programming (SQP) algorithm is used to solve the problems. Then, the situations in which each sourcing policy is the most economic are determined.
Finally, in the last problem, we consider inventory and transportation costs, as well as the environmental impacts in a centralized supply chain by taking into account uncertain demand and partial backordered shortages. Due to the deterioration characteristic of products, a constant deterioration rate as well as a non-linear holding cost function is considered. In order to deal with demand uncertainty, a two stage stochastic programming approach is taken. Then, by considering transportation vehicles capacity, we develop a mixed integer mathematical model. In this way, the best transportation vehicles and replenishment policy are determined by finding a balance between financial and environmental criteria. A numerical example from the real world is also presented to show the applicability and effectiveness of the proposed model.

Keywords: Replenishment policy - Deteriorating Items - Non-linear holding cost - Inventory control - Green supply chain - Stochastic conditions.

Politiques de réapprovisionnement pour les produits périssables dans des conditions incertaines en considérant des critères environnementaux

Résumé étendu:

1-Introduction

De nos jours, la complexité croissante des conditions de production et d'approvisionnement liées en particulier à la compétitivité croissante entre entreprises manufacturières, à une plus grande variété de produits, à l'évolution rapide des goûts des clients réduisant la durée de vie du produit, aux difficultés de prévision de la demande, d'une part, et au rôle clé des consommateurs et des produits périssables sur les revenus des pays d'autre part, font de la gestion des stocks et des politiques de réapprovisionnement des produits périssable une problématique intéressante tant pour les chercheurs universitaires que pour les dirigeants d'entreprises.

En règle générale, la détérioration se réfère aux dommages, dégradations, à l'évaporation, à l'assèchement, etc des produits (Goyal et Giri, 2001). Un nombre croissant de produits tels que les produits pharmaceutiques, les produits de santé et cosmétiques, les parfums, les aliments, les fruits et légumes, les substances radioactives, les produits biotechnologiques et de nombreuses matières pétrochimiques et chimiques sont classés comme des produits qui se détériorent.

Les ventes mondiales chez les détaillants ont dépassé en 2006 1000 milliards de dollars. La détérioration des produits tels que les fruits et légumes frais, les produits laitiers et la viande comptent pour plus d'un tiers de ces ventes (Broekmeulen et Donselaar, 2009). Une gestion efficace de la détérioration des stocks peut donc avoir un effet considérable sur la rentabilité d'une entreprise et constituer un avantage concurrentiel. Cependant, cette gestion est fondamentalement plus complexe que la gestion de stocks de produits à durée de vie infinie. Deux questions principales sont problématiques dans la gestion des stocks d'articles périssables : la satisfaction de la demande des clients, et la modélisation des coûts du système.

Aujourd'hui, les clients demandent une plus grande diversité de produits. En conséquence, davantage de produits se détériorent, surtout dans la catégorie des produits périssables qui ont tendance à dépasser la date de péremption, alors que la demande pour ces produits tend à être moins prévisible. Cela rend les systèmes d'inventaire difficiles à modéliser. Dans de nombreux cas réels, pour contrôler la demande, les gestionnaires utilisent souvent une politique de démarque à l'approche de la date d'expiration du produit. Ils suppriment également parfois des produits périmés. Prendre en compte ces approches dans les modèles de gestion des stocks rend ceux-ci encore plus complexes.

Le coût de stockage de produits périssables est plus compliqué à gérer que celui de produits à longue durée de vie. Dans de nombreux cas, comme les aliments surgelés, les produits pharmaceutiques et biologiques, la durée de vie des produits dépend des conditions de stockage et de transport telles que l'humidité et la température. Cette question, ainsi que les coûts associés aux produits détériorés / périmés et à leur élimination / recyclage complique la modélisation des coûts du système. Il est également évident que le type de la fonction de détérioration impacte significativement la complexité du système de coûts.

Ces défis, liés au nombre croissant d'articles périssables dans de nombreuses organisations commerciales, fait de la politique d'approvisionnement de ces produits un domaine industriel et de recherche particulièrement important. Cependant, la plupart des chercheurs ont approché ce sujet dans des conditions déterministes.

Les produits périssables sont également plus critiques que ceux qui ont une durée de vie illimitée du point de vue des critères environnementaux. Aujourd'hui, les préoccupations concernant l'appauvrissement de la couche d'ozone et le changement climatique ont été de plus en plus discutées au niveau international. Le transport, de nombreuses activités logistiques et des procédés industriels conduisent à une augmentation de l'effet de serre par le biais du dioxyde de carbone (CO_2), bien que l'effet d'autres gaz ne doive pas être sous-estimé (Harris et al. 2011).

Aux États-Unis, par exemple, le principal gaz à effet de serre émis est le CO_2, qui compte pour 85% du potentiel du changement climatique induit par toutes les émissions produites par l'homme. Les émissions des camions sont passées de 42% des émissions totales de CO_2 en 1995 à 49% en 2006 et ne montrent aucun signe de diminution (Ulku, 2012). De même, les produits périssable jouent un rôle clé en raison des émissions liées à leurs conditions particulières de stockage et de transport ainsi qu'aux processus d'élimination / recyclage. En outre, les effets toxiques ou radiations de certains types de produits périssables tels que les substances radioactives ne devraient pas été oubliés.

Dans ce contexte, les contributions de cette thèse se situent dans deux domaines principaux: *i*) Comme la plupart des modèles d'approvisionnement développés pour les produits périssable sont déterministes, nous développons deux nouveaux modèles de gestion des produits périssables en considérant des délais stochastiques (chapitres 3 et 4).

ii) Dans le chapitre 5, nous développons un nouveau modèle stochastique bi-objectif de programmation mixte en nombre entier pour le réapprovisionnement d'un produit périssable en tenant compte de deux critères économiques et environnementaux. En définissant des scénarios reposant sur la demande des consommateurs, ce modèle est également développé dans un contexte d'incertitude de la demande.

2-Types d'inventaire

Selon Goyal et Giri (2001), les biens inventoriés peuvent être classés en trois méta-classes:

✓ Les produits à durée de vie illimitée: Les produits qui n'ont pas d'obsolescence et aucune détérioration. La durée de conservation de ce type de produits peut être à durée indéterminée.

✓ Les produits de mode ou sujets à obsolescence: il s'agit de produits qui perdent leur valeur au fil du temps en raison des changements rapides de la technologie ou de l'introduction d'un nouveau produit de substitution par un concurrent. Exemple : les pièces de rechange pour avions militaires. Les produits de mode peuvent voir leurs prix fortement réduits ou éliminés lorsque la saison de vente est terminée.

✓ Les produits qui se détériorent: produits dont la détérioration résulte de dommages, de sécheresse, d'évaporation, de gaspillage, etc.

Selon Goyal et Giri (2001), les produits qui se détériorent sont classés en deux groupes principaux. Les produits qui ont une durée de vie maximale utilisable sont classés comme «produits périssables» comme les légumes verts, les produits alimentaires, les pellicules photographiques, le sang humain, etc, et ceux qui n'ont pas de durée de vie définie sont classés parmi les «produits en décomposition», tels que les substances radioactives, l'alcool, l'essence, etc. Rafaat (1991) définit les « produits périssables » comme des produits de détérioration avec une durée de vie fixe et les «produits en décomposition» comme les produits qui se dégradent de manière continue (durée de vie aléatoire).

Il a également classé les articles de détérioration par rapport à leur valeur ou leur utilité en fonction du temps comme suit :

✓ Les biens périssables à utilité constante qui ne subissent pas de perte de valeur considérable tout au long de leur durée de vie utile, comme les médicaments sur ordonnance.

✓ Les biens périssables à valeur décroissante au cours de leur vie, comme les produits frais ou les fruits.
✓ Les biens périssables dont la valeur augmente durant leur durée de vie, comme certains vins ou des antiquités.

Récemment Bakker et al. (2012) ont proposé une catégorisation pour les produits à détérioration basée sur sa durée de conservation:

- ✓ Produits à durée de vie fixe prédéterminée.
- ✓ Produits dont le taux de dégradation dépend de l'âge (Weibull etc) ce qui implique une loi de distribution de la durée de vie.
- ✓ Produits dont le taux de détérioration dépend du temps ou du stock (mais pas de l'âge). Les modèles avec un taux de détérioration constant par article stocké (dépendant donc du stock et non de l'âge) appartiennent à cette classe.

2-1-Définitions de la détérioration dans la littérature

Il existe différentes définitions de la détérioration dans la littérature. Parmi celles-ci nous retiendrons les suivantes:

- ✓ La détérioration se réfère à «des dommages, dégradations, à la sécheresse, l'évaporation, etc des produits" (Goyal et Giri, 2001).
- ✓ La détérioration est "le dommage, la détérioration, la sécheresse, l'évaporation, etc qui se traduit par (a) une diminution de l'utilité de l'article" (Wu et al. 2006).
- ✓ La détérioration est définie comme la pourriture, les dommages, les dégradations, l'évaporation, l'obsolescence, le chapardage et la perte du produit ou de sa valeur marginale en raison de son utilité décroissante (Wee, 1993).
- ✓ En général, les produits sont supposés se détériorer avec le temps entrainant une diminution de leur utilité ou de leur prix (Hsu et al. 2007).
- ✓ L'altération ou la détérioration est décrite comme un processus qui empêche d'utiliser un produit pour son usage initialement prévu par exemple: *i*) la détérioration, comme dans les denrées alimentaires périssables, les fruits et légumes, *ii*) la disparition physique, comme dans le chapardage ou l'évaporation de substances volatiles liquides (essence, alcool, etc), *iii*) la décomposition, comme dans les substances radioactives, la dégradation, comme dans les pièces électroniques, ou une perte de

puissance comme dans les produits pharmaceutiques et les films photographiques (Raafat, 1991).

- ✓ Selon Disney et al. (2012), il y a une différence entre l'inventaire de produits subissant une détérioration de celui de produits périssables. Dans le scénario de la détérioration des stocks, l'inventaire se décompose physiquement et est détruit au fil du temps, tandis que les produits périssables perdent de la valeur, mais ne sont pas détruits.

Ainsi, plusieurs définitions et classifications avec différences mineures existent dans la littérature pour les produits qui se détériorent. Notre travail dans cette thèse est basé sur la définition et la classification qui ont été fournies par Goyal et Giri (2001).

3-Les défis de la gestion des produits sujets à détérioration

Gérer et contrôler les stocks de produits sujets à détérioration devient de plus en plus important, car le nombre et la variété de ce type de produits augmentent de façon spectaculaire. Comme mentionné précédemment, selon Broekmeulen et Donselaar (2009), les ventes mondiales auprès des détaillants dépassent 1.000 milliards de dollars en 2006.

La détérioration des produits représente plus d'un tiers de ces ventes. De même, Lystad et al. (2006) a déclaré que les produits sujets à détérioration représentent environ 200 $ milliards de ventes dans le secteur de l'épicerie aux États Unis et que 30 milliards de dollars sont perdus en raison de cette détérioration.

En général, en plus des coûts de stockage habituels (de passation de commande, de détention et les coûts de pénurie), la détérioration des articles entraine également des coûts supplémentaires. Ces coûts proviennent de différentes sources, selon le type de produit. Par exemple, le coût d'achat de produits détériorés / périmés est un coût supplémentaire qui est généralement appelé un «coût de détérioration ».

Dans le cas des produits alimentaires, la qualité et en conséquence le prix de vente sont sensibles au temps, ce qui signifie que la qualité / le prix de vente diminuent considérablement à mesure que la fin de vie du produit approche. Ici, le manque à gagner résultant de la baisse des prix peut être considérée comme un coût pour le système.

Dans de nombreux cas, le coût de détention par article et par unité de temps (le taux de détention) n'est pas fixe, car le coût de chaque produit fini est variable. Par exemple, les détaillants de supermarchés sont confrontés au problème de la

diminution de la qualité des invendus comme le pain, les légumes verts et les fruits au fil du temps.
En conséquence, les coûts de détention tendent à augmenter en raison de meilleures installations de stockage acquises pour empêcher la détérioration et pour maintenir la fraîcheur des articles en stock. C'est également le cas lorsque des produits avariés sont supprimés au fil du temps. La détérioration des produits tels que les composants électroniques et les substances radioactives sont d'autres exemples qui nécessitent des stockages plus sophistiqués proportionnels à leur volume d'inventaire, afin d'assurer leur conservation et leur sécurité.
Ainsi, le taux de possession est non linéaire en fonction de l'inventaire. Par conséquent, dans de nombreuses situations, comme les exemples cités ci-dessus, le coût de chaque produit fini n'est pas fixe et dépend du temps, du niveau des stocks, etc. Ce taux non linéaire complique la gestion des coûts du système de façon importante.
En outre, de nombreux produits ayant une date d'expiration, la période de temps pendant laquelle la qualité du produit reste conforme aux normes est très limitée. Ces produits se détériorent rapidement pendant le transport et le stockage. Un équipement sophistiqué, avec des coûts supplémentaires, doit donc être utilisé pour le stockage et le transport (tels que les entrepôts et les camions frigorifiques). Récemment, certaines compagnies appliquent une nouvelle technologie appelée Intégrateur Temps Température (ITT) pour évaluer la durée de vie effective des produits par l'enregistrement de l'historique de sa température.
Enfin, l'effet de la détérioration sur l'environnement, ainsi que le coût du processus de récupération des produits détériorés / expirés, est un vrai problème, surtout dans les chaînes d'approvisionnement écologiques ou durables.
Dans la pratique, afin de réduire les coûts supplémentaires liés à la détérioration des produits, les entreprises adoptent des outils de gestion des coûts et des politiques d'approvisionnement, de production, de stockage et de transport spécifiques pour les produits sujets à détérioration. Par exemple, les gestionnaires appliquent des techniques sophistiquées pour assurer la meilleure utilisation des capacités de production, de stockage et du matériel de transport. Certains membres de la chaîne logistique amont (producteurs), appliquent une stratégie de production Make-To-Order (MTO) (ou même Make-To-Engineering (MTE)) au lieu d'un Make-To-Stock (MTS), afin de réduire le taux de détérioration et les coûts de stockage.

En résumé, les coûts supplémentaires mentionnés liés à la détérioration des produits ont un impact profond sur la rentabilité des entreprises. Ceci, compte tenu du nombre croissant et de la variété de ces produits, motive sérieusement les chercheurs universitaires pour étudier les politiques de réapprovisionnement d'articles sujets à détérioration pendant une longue période. L'intégration dans les modèles mathématiques des coûts supplémentaires mentionnées précédemment rend ceux-ci plus compliqués et soulève des challenges plus importants que ceux posés par la gestion de produits non sujets à détérioration.

4-Contributions de la thèse

Le but de cette thèse est de développer de nouveaux modèles de réapprovisionnement pour gérer les stocks de produits sujets à détérioration dans des conditions incertaines. Pour atteindre notre objectif, nous avons tenu compte à la fois du caractère stochastique des délais d'approvisionnement et de la demande, et ce dans différentes situations. Nos principales contributions sont détaillées comme suit :

Les produits sujets à détérioration ont été étudiés et classés selon différents points de vue dans la littérature existante. Dans cette thèse, nous avons fait une étude très complète sur les modèles de réapprovisionnement pour des articles sujets à détérioration. Nous présentons ensuite une nouvelle classification des modèles de gestion de stock de produits sujets à détérioration basée sur des "approches mathématiques pour la modélisation des processus de détérioration».

Notre revue de littérature a révélé que la plupart des recherches traitent de problèmes spécifiques. Par conséquent, nous étudions de nouveaux problèmes pour l'approvisionnement de produits sujets à détérioration dans des conditions incertaines. Pour chaque problème un nouveau modèle mathématique est développé sur la base de la classification proposée. Considérant les deux sources principales d'incertitude : *i*) les délais d'approvisionnement et la demande client *ii*), nous étudions deux nouveaux problèmes en se concentrant sur l'incertitude des délais de livraison et un nouveau problème en considérant l'incertitude sur la demande externe.

✓ Bien que les chercheurs et les praticiens accordent une attention croissante au développement et à l'application de modèles de gestion de stocks pour des produits sujets à détérioration, la plupart de ces travaux considèrent des temps approvisionnement nuls ou fixes, alors que dans de nombreux cas, le délai de livraison n'est pas défini et est incertain. L'imprécision du délai affecte un système de gestion de stock, et ne pas en tenir compte peut entraîner des manquants ou des coûts de possession supplémentaires. En raison du processus de détérioration, l'effet de l'incertitude sur les délais de livraison est beaucoup plus perceptible pour ces articles qui se détériorent. Dans cette thèse, nous développons des modèles de gestion des stocks pour les produits sujets à détérioration en considérant des délais d'approvisionnement stochastiques.

✓ Dans cette thèse un multiple sourcing et un fractionnement des approvisionnements avec des délais stochastiques pour des produits sujets à détérioration sont étudiés au chapitre 4. De fait, il n'a pas été accordé suffisamment d'attention dans la littérature à ces deux concepts, de fractionnement de l'ordre et de détérioration des produits.

Selon Dekker et al. (2012), la détermination de frontières éco-efficaces à l'aide de modèles multi-objectifs de programmation mixte en nombre entier est tout à fait nouvelle, en dépit de la littérature générale sur la programmation multi-objectif. Dans cette thèse, nous développons un modèle bi-objectif stochastique de programmation mixte en nombre entier pour une chaîne d'approvisionnement verte avec des produits sujets à détérioration (chapitre 5). L'objectif de ce modèle est de faire un compromis entre les coûts et les émissions de gaz à effet de serre (GES) au niveau tactique-opérationnel en déterminant une frontière éco-efficace.

En fait, nous prenons en compte des considérations économiques d'une part et des considérations écologiques, d'autre part, et étudions les interactions de ces considérations économiques et vertes dans une chaîne d'approvisionnement centralisée pour des produits sujets à détérioration et faisant l'objet d'une demande extérieure incertaine. Le modèle proposé peut donc être utilisé comme un outil d'aide à la décision systématique, car il est capable de déterminer la politique d'approvisionnement et choisir les véhicules de transport (pour éviter les sous-optimalitées résultant de décisions séparées de chaque acteur), en tenant en compte à la fois des coûts et des émissions de GES.

4-1-Les questions auxquelles entend répondre cette thèse

Notre objectif est de répondre aux questions suivantes:

- ✓ Quel est l'impact de l'incertitude sur la gestion des stocks de produits sujets à détérioration ?

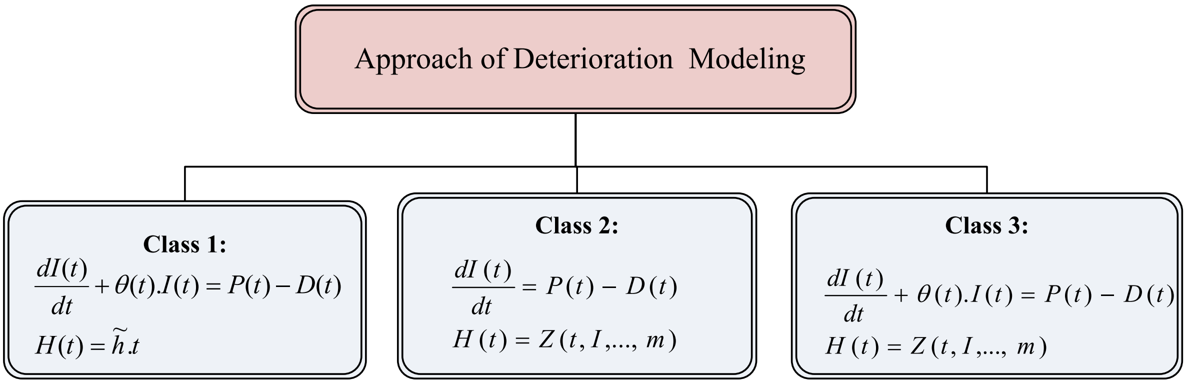
- ✓ Quel est l'impact de la détérioration des produits sur la politique d'approvisionnement ?
- ✓ Quel est l'impact des véhicules de transport sur la gestion des stocks de produits sujets à détérioration ?
- ✓ Quel est l'impact de la prise en compte des émissions de gaz à effet de serre (GES) sur la définition de la politique de gestion des stocks ?
- ✓ Quels sont les «meilleures» solutions pour concilier les préoccupations écologiques et economiques dans les chaînes d'approvisionnement pour des produits sujets à détérioration ?

5-Revue de littérature

Dans cette section, un aperçu des principales recherches sur la gestion des stocks de produits sujets à détérioration est fourni. Nous proposons une classification des travaux basée sur l'approche de modélisation de la détérioration (Figure 1). En outre, dans le mémoire (chapitre 2) différentes catégorisations sont présentées, basées sur les principaux paramètres que sont les fonctions de détérioration, de pénurie, et de la demande, etc. Les détails de ces recherches sont fournis dans le tableau (Tableau 2 - 2).
Les informations telles que l'approche de modélisation de la détérioration, le type de la demande et de la fonction délai d'approvisionnement, le type de fonction pénurie, l'approche de la solution et les composants de la fonction objective de chacun des travaux sont présentés dans ce tableau. En outre, quelques explications sur la logistique verte et l'impact des gaz à effet de serre (GES) sur l'environnement pour les produits sujets à détérioration sont fournis.
Dans ce qui suit, nous expliquons notre proposition de classification ainsi que la variété de la fonction détérioration et la fonction de demande dans les modèles existants.

5-1- Approches de modélisation du processus de détérioration

De nombreux chercheurs ont analysé la gestion des stocks d'articles sujets à détérioration selon différentes perspectives. D'une manière générale, la littérature existante dans ce domaine peut être divisée en trois classes du point de vue de la modélisation. Ces classes sont représentées schématiquement dans la Figure 1.



I(t): On-hand inventory in terms of time, *t*
Θ(t): Deterioration function in terms of time, *t*
D(t): Demand function in terms of time, *t*
P(t): Production rate in terms of time, *t*
H(t): Holding cost of one unit that is in stock for t units of time
$\tilde{h}$ **:** a positive constant
Z(t, I, …,m): a nonlinear, increasing positive function of finite number of parameters like time, *t,* on-hand inventory, *I,* etc.

Figure 1- Classification des approches de modélisation de la détérioration

5-1-1-Classe 1: Fonction non linéaire des stocks

La plupart des recherches sur la détérioration de produits stockés considère que les stocks se dégradent avec le temps, selon différents modèles. Ainsi, la fonction de stock disponible peut être déterminée par l'équation différentielle:

$$\frac{dI(t)}{dt} + \theta(t)I(t) = P(t) - D(t) \qquad ()1$$

Où *I(t)* est le niveau de stock à l'instant t, et *θ(t), P(t) et D(t)* correspondent au taux de détérioration, à la cadence de production et au taux de demande à l'instant t, respectivement.

Dans ce type de recherche, il est considéré que le coût de détention par article et par unité de temps (taux de possession) est constant. En d'autres termes, le coût de possession est linéaire en fonction de paramètres tels que le temps de stockage, *t*, le niveau des stocks, *I*, ce qui peut être énoncé comme suit : $\tilde{h}tI$, où $\tilde{h} > 0$ est constant.

Ce type de modélisation est plus approprié pour les produits sujets à décomposition et a été utilisé dans les premières recherches sur les produits qui se détériorent. Ghare et Schrader (1963) semblent avoir été les premiers à avoir mis au point un modèle de détérioration exponentiel en définissant un taux constant de décomposition. Depuis, les recherches sur la détérioration des stocks se sont poursuivies intensivement, impliquant de nombreux chercheurs.
Dans la littérature existante, diverses formes de dégradation de la fonction, $\theta(t)$, ont été prises en compte. En règle générale, on peut classer les fonctions détérioration, $\theta(t)$, en deux groupes principaux: les fonctions discrètes, et les fonctions continues.

Fonction discrète de détérioration

Les fonctions discrètes de dégradation (ou semi-continues) sont appropriées pour la modélisation de stocks de produits *«non-détériorables* instantanément " comme les légumes, les fruits, le poisson, la viande et ainsi de suite. Wu et al. (2006) introduit les *"articles à non détérioration instantanée"* correspondant aux produits qui ne se détériorent pas au cours d'une période de maintien de la qualité ou de l'état initial (temps produit frais). Ils ont supposé qu'une fraction constante du stock se détériore passé cette période. Ils ont travaillé sur la réduction des coûts d'inventaire par unité de temps en considérant un délai d'approvisionnement nul, un stock dépendant de la demande et un report partiel des commandes dépendant du temps.
Récemment, Valliathal et Uthayakumar (2011) dans le but de maximiser le profit total par unité de temps, a discuté de la tarification optimale et des politiques de réapprovisionnement basées sur un modèle de quantité économique pour des articles à non détérioration instantanée. Ils ont fait l'hypothèse d'une demande dépendante du temps et du prix et d'un report partiel des commandes dépendant du temps sur un horizon temporel infini.

Fonction de détérioration continue

S'appuyant sur Wang et al. (2011), nous pouvons classer les fonctions de détérioration continue, θ (t), en quatre catégories, comme suit:

Fonction constante: de nombreux chercheurs ont estimé que le taux de détérioration des produits est constant et indépendant du temps. Ghare et Schrader (1963) semblent avoir été les premiers à avoir développé un modèle d'inventaire à décroissance exponentielle en définissant une fonction constante

de détérioration, $\theta(t)$, dans l'équation différentielle 1. Ce modèle est approprié pour les produits comme l'huile, l'alcool et certains produits pharmaceutiques.
En 2008, Kang a travaillé sur la minimisation du coût total de wafers, incluant le coût de passation de commande et le coût d'achat. Il a déterminé la politique optimale par une programmation dynamique et une programmation linéaire mixte 0-1 (M0-1LP). Les résultats numériques ont démontré que le modèle M0-1LP est un outil efficace pour la détermination de la politique d'approvisionnement de wafers sur des périodes multiples.
En 2010, Chen et Chang, ont considéré une chaîne d'approvisionnement à deux échelons composée d'un fabricant et de plusieurs détaillants. Ils ont résolu le problème de détermination conjointe du cycle d'approvisionnement, du prix optimal de vente au détail, et du nombre de livraisons pour des produits ayant un taux de détérioration constant. Afin de discuter du processus de coordination du programme d'approvisionnement et de la politique de prix, deux modèles de maximisation du profit, selon que l'on considère une politique intégrée ou non-intégrée ont été formulées ainsi que quelques propositions structurelles. Une fonction θ (t) constante est également prise en compte dans Pakkala et Achary (1991), Chung et Wee (2008), Huang et Yao (2006), Zhou et Lau (2000), Aggarwal et Jaggi (1995), Wee et Chung (2009), Kang et Kim (1983) et Tsai (2011).

Fonction linéaire: les travaux tels que ceux de Lin et Lin (2006) font l'hypothèse d'un taux de détérioration $\theta(t) = \theta_1.t+ \theta_2$ où $0 \leq \theta(t) < 1$ et $0 \leq \theta_1,\theta_2 \leq 1$. Cette formule de détérioration linéaire est adaptée pour les produits tels que les matières radioactives.

Fonction logarithmique: cette forme de dégradation est appropriée pour les produits qui se détériorent avec un taux augmentant considérablement durant la phase initiale, pour se stabiliser rapidement. Par exemple, de nombreuses puces IC ont un taux de dégradation qui s'accroit avant leur conditionnement et se stabilise par la suite.

Fonction exponentielle: par ce type de fonction, le taux de dégradation augmente lentement dans la phase initiale, puis très rapidement, comme dans le cas de certains produits laitiers.

Fonction de Weibull de la détérioration: plusieurs chercheurs comme Giri et al. (2003) et Wu (2002), ont considéré que la forme généralisée continue de θ (t) suivait une distribution de Weibull $\theta(t)=\alpha\beta t^{\beta-1}$, $\alpha,\beta \geq 0$. En fait, les quatre fonctions continues de la détérioration peuvent être considérées comme une fonction de Weibull, en fonction de la valeur de β. Covert et Philip (1973) ont été parmi les premiers chercheurs à faire cette hypothèse. Ils ont construit un modèle EOQ avec un taux de demande constante afin de minimiser les coûts de stockage. Plus tard, d'autres chercheurs ont considéré des hypothèses plus complexes.

Par exemple, Wu et al. (1999) ont développé un modèle d'inventaire en considérant zéro délai, une demande de type rampe, un report total des commandes en cas de rupture de stock, et un taux de détérioration suivant la loi de Weibull. Deng (2005) a amélioré le modèle de Wu en omettant certaines conditions inutiles pour trouver la solution minimale, et a également étudié la forme la plus générale de leurs modèles.

5-1-2-Classe 2: coût de possession non linéaire

Le processus de détérioration affecte directement la fonction de stock disponible et ainsi la modélisation des coûts de possession des stocks. Dans cette catégorie, la forme de la fonction stock disponible est similaire à celle de produits non sujets à dégradation et peut être obtenue par l'équation différentielle:

$$\frac{dI(t)}{dt} = P(t) - D(t) \tag{2}$$

Ici, au lieu de considérer la fonction taux de détérioration, *θ(t),* dans la fonction de stock disponible, le coût de possession, *H*, est considéré comme une fonction non linéaire positive croissante de paramètres tels que le temps de stockage, *t,* ou la quantité en stock, *I*.

Un coût de possession non linéaire et dépendant du temps est plus approprié pour des produits dont la valeur et la qualité des invendus diminuent avec le temps, comme dans le cas des légumes verts. Pour les produits tels que les composants électroniques, des outils plus sophistiqués sont nécessaires pour assurer la sécurité et la sûreté des stocks, un coût de possession non linéaire et dépendant du stock peut être appropriée.

Weiss (1982) semble avoir été parmi les premiers à traiter le coût de possession unitaire comme une fonction non linéaire de la durée de détention d'un article en magasin. Il a estimé que le coût cumulé de détention d'un article ayant été conservé pendant t unités de temps est $H(t) = \tilde{h}t^{\gamma}$,, où h> 0 et γ ≥ 1 sont des constantes. Il a retenu les autres hypothèses traditionnelles d'un modèle EOQ comme le délai nul d'approvisionnement, les coûts d'achat et de passation de

commande ainsi que le prix de vente constants, et a développé deux modèles mathématiques pour une demande déterministe et stochastique.
En 1994, en considérant des stocks dépendants de la demande, Goh a utilisé une variable de coût de possession sous deux formes: une fonction continue non linéaire de la durée de détention, et une fonction continue non linéaire de la valeur du stock. En retenant les autres hypothèses du modèle EOQ des formules de forme fermées ont été développées pour déterminer la quantité optimale de commande.
Récemment Alfares (2007) a étendu le modèle de Goh à la situation dans laquelle les coûts variables sont une fonction discrète du temps de stockage avec des coûts croissants. Il a examiné deux scénarios: (1) l'augmentation des coûts de possession pour la dernière période de stockage d'un cycle d'approvisionnement est appliquée rétroactivement à toutes les périodes de stockage, de sorte que le même coût de possession est appliqué à toutes les unités de temps; et (2) l'augmentation progressive des coûts de possession qui est caractérisé comme un coût de possession proportionnel à la durée de stockage est appliquée à chaque période, de sorte que le coût de détention pour chaque période de stockage est appliquée uniquement aux produits détenus durant cette période.
En 2008, Urban (2008) a généralisé le modèle d'Alfares en utilisant un objectif de maximisation du profit et non de minimisation des coûts. Il a également assoupli la limitation de stock nul à la fin de chaque cycle de réapprovisionnement, en considérant celui-ci comme non-négatif.

5-1-3- Classe 3: Fonction non linéaire du stock et du coût de possession

Cette approche de modélisation est plus compliquée que les deux autres. Ici, à la fois le taux de détérioration, $\theta(t)$, une caractéristique de classe 1, et le coût de possession non linéaire, une caractéristique de classe 2, sont considérés pour modéliser l'évolution du stock de produits sujets à détérioration, comme dans Girl et Chaudhuri (1998). Ces auteurs ont discuté le modèle de Goh, en considérant une constante $\theta(t)$ en plus du coût de détention non-linéaire dans les deux cas : dépendance du temps, dépendance du stock.
En 2007, Teng et al. ont étudié le problème du prix et de la taille de lot pour des produits sujets à détérioration pour une production finie, en décroissance

exponentielle, avec les hypothèses de report partiel de commandes et de ventes perdues. Ils ont également examiné une détérioration dépendante de la durée de détention, avec report partiel et ventes perdues et analytiquement comparé les bénéfices nets par unité de temps en s'appuyant sur les modèles d'Abad (2003) et de Goyal et Giri (2003).

5-2- Demande Fonction-*D(t)*

Une caractéristique importante des modèles développés pour des produits sujets à dégradation est la fonction de la demande. Selon la nature de celle-ci, nous pouvons classer les recherches dans deux groupes: *i*) les études considérant une demande constante et *ii*) et les études considérant une demande variable.

5-2-1- Modèles avec fonction de demande constante

Dans de nombreuses recherches, le taux de demande est constant, indépendant du temps, du prix de vente et du niveau des stocks. Des études telles que Wang (2011), Liao (2007), Liao (2008), Rau et al. (2003), Lin et Lin (2006), Huang et Yao (2006), Lo et al. (2007), Muhlemann et Valtis-Spanopoulos (1980), et Ferguson et al. al. (2007) sont classées dans ce groupe.

5-2-2-Modèles avec une fonction de demande variable

Nous avons classé les demandes variables en trois catégories: *i*) demande stochastique, *ii*) demande dépendante et *iii*) demande floue.

Fonction de la demande stochastique

Au regard de la vraie vie, une demande stochastique est plus raisonnable, bien que moins de 20% des modèles développés dans la littérature (après 2001) la considèrent comme telle (Bakker, 2012). Avant 2001, les recherches se sont essentiellement concentrées sur le développement de modèles de base, sous certaines conditions. Nous référant à Goyal et Giri (2001), dans la littérature existante nous pouvons classer les fonctions de demande stochastiques en deux catégories :

1) Celles qui considèrent un type spécifique de fonction de distribution de probabilité (PDF), tels que Ravichandram (1995) et Weiss (1982) qui ont développé des modèles d'inventaire pour les produits sujets à détérioration en supposant que la demande suivait une loi de Poisson.
2) Celles qui considèrent une demande finale suivant une fonction de distribution de probabilité arbitraire (PDF) comme Aggoun et al. (1997) et Lian et al. (2009). Selon Bakker (2012) depuis 2001, seulement 4% des recherches développées sur les modèles de stocks pour des produits sujets à détérioration considèrent une distribution de probabilité arbitraire de la demande.

Fonction de la demande dépendante

Dans la plupart des recherches faisant l'hypothèse d'une fonction de demande dépendante, celle-ci est considéré comme dépendante du stock (Chung et Wee, 2008, Urban, 2008, Chang et al. 2006), du temps (He et al., 2010, Lin et Lin, 2006, Sana et al. 2004), du prix (Sana, 2011, Dye, 2007, Papachristo et Skouri, 2003) ou d'une combinaison de ces facteurs (Valliathal et Uthayakumar, 2011, Chen et Chang, 2010, Hsu et al., 2007). Il est intéressant de noter que dans la plupart des articles qui considèrent la demande dépendante du temps, les modèles d'inventaires sont développés sous un horizon de planification fini.

Fonction de demande floue

Un nombre limité d'études comme celles de Mahata et Goswami (2007) considèrent la demande comme un paramètre floue. Mahata et Goswami (2007), se sont intéressés au problème de la détermination de la quantité économique de commande (EOQ) d'articles sujets à détérioration dans le sens floue dans lequel des retards dans les paiements de détaillant et de client sont admissibles. Le taux de la demande, le coût de détention, les coûts d'achat et de passation de commande ont été pris comme des nombres flous. Le coût variable total dans le sens flou a été « defuzzifié » par la méthode de représentation intégrée de la moyenne graduée. Par la suite, ils ont montré que le coût variable total « defuzzifié » est convexe et qu'une solution unique existe.

6- Lacunes de la recherche

Sur la base de cette revue de littérature, nous avons constaté certaines lacunes de la recherche dans le domaine des politiques de réapprovisionnement des stocks sujets à détérioration:

✓ L'incertitude portant sur des paramètres tels que le délai d'obtention, la fraction de la demande reportée et le taux de dégradation n'a pas attiré suffisamment d'attention dans la littérature existante.

La plupart des approvisionnements nécessitent un délai, souvent difficile à déterminer avec précision, même si des améliorations importantes ont été apportées dans la communication, le transport et les systèmes de production. En outre, dans de nombreuses études où les commandes sont partiellement reportées en cas de ruptures, le taux de report partiel a été modélisé par une fraction simple. Développer des modèles pour prendre en compte ces incertitudes pourrait donc être un domaine prometteur pour d'autres recherches. En outre, étant donné qu'il existe des articles qui se détériorent au cours du transport, prendre en compte le taux de détérioration incertain et par conséquent l'incertitude sur la quantité reçue par les détaillants est une extension potentielle pour les modèles actuels. À cet égard, il est également intéressant d'utiliser l'approche floue.

✓ Les chercheurs ont également prêté peu d'attention aux approches multi objectifs dans les modèles d'inventaire des articles sujets à détérioration. Pourtant, c'est un moyen par lequel des fonctions objectives alternatives telles que le coût total (bénéfice total), la satisfaction des clients (en maximisant le niveau de service / minimiser le maximum des ruptures) et des critères environnementaux pourraient être envisagés.

✓ Le niveau des émissions de GES est important aujourd'hui, car les impacts environnementaux des processus tels que la production et la livraison sont devenus un enjeu crucial pour l'humanité et donc pour les entreprises. Intégrer les principes de la logistique verte et de la gestion de la chaîne d'approvisionnement notamment pour les produits sujets à détérioration est important en raison des effets parfois destructeurs des objets détériorés ou des systèmes de transport sur la nature. Selon les problèmes, les principes verts peuvent être pris en compte dans la fonction objectif du modèle ou sous forme de contraintes.

✓ La plupart des modèles existants d'approvisionnement pour les produits sujets à détérioration considèrent un seul fournisseur et un seul détaillant, i.e. des chaînes d'approvisionnement que l'on appelle chaînes dyadique. Toutefois, les chaînes d'approvisionnement série, convergente ou divergente ou même des formes en réseau sont tout aussi importantes dans le monde réel. Par exemple, dans des situations stochastiques (en particulier des délais stochastiques) afin de réduire les risques d'incertitudes, un détaillant peut préférer scinder une commande de réapprovisionnement entre plusieurs fournisseurs simultanément au lieu de placer une commande unique. Cette politique est appelée «fractionnement d'ordre». Alors que la politique de fractionnement est étudiée sous diverses hypothèses, les chercheurs ont accordé moins d'attention à cette politique d'approvisionnement pour les entreprises qui offrent des produits qui se détériorent.

En outre, dans les chaînes d'approvisionnement convergentes (fournisseur unique multi-détaillant comme dans Chen et Chang, 2010, Huang et Yao, 2006 et Yang et Wee, 2003, Yang et Wee, 2002) considérer la dépendance positive ou négative entre les demandes des détaillants est une question justifiant des recherches plus poussées.

- ✓ Presque toutes les études qui ont été développées pour la reconstitution des stocks l'ont été sous un horizon de planification fini, prédéterminé. La détermination de la valeur optimale de l'horizon de planification comme une variable de décision est un sujet de recherche intéressant aussi.
- ✓ Le taux de pénurie dans la plupart des recherches est constant, alors que le nombre de clients insatisfaits impacte profondément la réputation d'une entreprise, ce qui n'est pas nécessairement linéaire. Afin de persuader les décideurs à réduire les ruptures de stock (reports ou ventes perdues) autant que possible, considérer un coût de pénurie non-linéaire doit également être pris en compte.
- ✓ En général, les modèles d'inventaires multi-produits ont été moins étudiés en raison de leur complexité. La complexité des modèles d'inventaire multi-produits sont renforcées pour les produits qui se détériorent. Ainsi, des modèles multi produits sujets à dégradation ont reçu peu d'attention, alors qu'ils sont tout à fait applicables (par exemple dans les supermarchés, etc). Ainsi, le développement et l'analyse de modèles multi-produits sujets à dégradation dans certaines situations mentionnées (par exemple pour les cas de coût de détention non linéaire, les chaînes d'approvisionnement vertes, le délai stochastique, les chaînes d'approvisionnement divergentes / convergentes etc) est fortement recommandé. De cette manière, la simplification des modèles développés par des méthodes d'approximation telles que l'extension des séries de Taylor peut être utile.

- ✓ La substituabilité n'est pratiquement jamais considérée dans la gestion des stocks de produits sujets à détérioration alors qu'elle est vraiment intéressante pour les gestionnaires de stocks. La modélisation de la substituabilité dans la gestion des stocks apportera une contribution à la littérature existante. En d'autres termes, le fait qu'une décision de reconstitution dépend du stock disponible de marchandises de remplacement ne doit pas être négligé plus longtemps.
- ✓ Dans la plupart des études, l'optimisation est faite en minimisant le coût total ou en maximisant le profit total. L'application d'autres méthodes d'optimisation, comme la programmation par objectif, la programmation stochastique ou des mesures de risque ont été moins étudiées.
- ✓ La coordination et l'intégration des chaînes d'approvisionnement permet souvent de réduire les coûts (ou d'accroitre les profits) pour l'ensemble de la chaîne d'approvisionnement, mais parfois le coût total (ou le profit) de certains membres augmente (diminue). Afin de motiver ces membres à se coordonner avec les autres unités de la chaîne d'approvisionnement, nous pouvons utiliser ou définir des politiques d'incitation telles que le retard dans le paiement admissible pour les acheteurs, le prépaiement pour les vendeurs. La modélisation et l'analyse de telles incitations ont également reçu peu d'attention en particulier pour les deuxième et troisième classes de modèles de détérioration et, dans le cas des chaînes d'approvisionnement divergentes / convergentes.

- ✓ L'étude de la politique de réapprovisionnement de produits sujets à détérioration basée sur la politique de commande (1, T) est également une nouvelle zone potentielle de recherche future.
- ✓ Enfin, l'utilisation de la fonction W de Lambert pour l'analyse des modèles d'inventaires de produits sujets à détérioration est une idée nouvelle qui est appliquée par certains chercheurs comme Disney et al. (2012). En utilisant cette fonction un plus grand nombre de problèmes liés à la politique de réapprovisionnement de ces produits pourrait être résolu et analysé efficacement.

Dans les sections suivantes, nous insistons sur les orientations les plus importantes à nos yeux et les domaines connexes de recherche associés sous la forme de trois nouveaux problèmes.

7- la politique de réapprovisionnement de produits sujets à détérioration avec un coût de possession non linéaire sous délai stochastique (chapitre 3)

Dans ce chapitre le problème, presque négligé dans la littérature, de la politique de réapprovisionnement de produits périssables sous délai stochastique (RPSL) est étudié. Nous considérons une chaîne d'approvisionnement mono produit afin de minimiser les coûts totaux du commerçant. Le suivi du stock du détaillant est continu, *(r, Q),* sous un horizon de planification infini.

La seconde approche pour la modélisation du processus de détérioration est retenue. De cette façon, une fonction linéaire du stock dépendante du temps, et des coûts de possession non linéaire croissants ($H(t) = \tilde{h}t^{\gamma}$, $\gamma \geq 1$) sont considérés (comme Weiss, 1982). En 2007, Ferguson et.al ont réalisé deux études de cas réels afin d'analyser l'applicabilité de ce type de coût de possession dépendant du temps. Ils ont conclu que cette fonction de coût est appropriée pour obtenir une approximation permettant de modéliser deux principaux groupes de produits sujets à détérioration : *i*) les produits avec des dates d'expiration exigeant le retrait de ceux-ci lorsque la date limite approchait et *ii*) les produits pour lesquels les gestionnaires appliquent souvent des démarques pour stabiliser la demande lorsque la date d'expiration s'approche.

La fonction objectif de ce problème est de minimiser le coût total du détaillant composé du coût de possession (y compris les frais de détérioration), des coûts de reports et de passation de commande en déterminant les valeurs optimales du point de commande (r^*) et de la quantité de commande (Q^*). A la première étape, nous développons un modèle mathématique en considérant une fonction de probabilité de densité générale (PDF) pour le délai (l), $f_L(l)$, $l \geq 0$ (forme générale du problème RPSL désigné par GRPSL). Le problème GRPSL appartient aux problèmes d'optimisation sans contrainte multivariables avec une fonction objectif différentiable pour $Q > r > 0$, mais la preuve de convexité de cette fonction objectif est très compliquée. Par conséquent, afin de simplifier les défis mathématiques associés à cette preuve de convexité, et d'avoir une analyse plus détaillée du modèle proposé, une forme simple de $f_L(l)$ est considérée.

Dans l'étape suivante nous traitons le problème de RPSL en supposant une fonction de distribution uniforme pour le délai d'approvisionnement nommée URPSL. Ainsi, en prouvant quatre lemmes, la convexité de la fonction objectif est démontrée. Ensuite, une analyse de sensibilité est effectuée pour les paramètres influents (paramètres de coût et de possession ($\tilde{h}$ *et* γ), coût des reports par unité de temps, ainsi que les paramètres de la fonction de probabilité du délai) en nous appuyant sur un exemple tiré de la littérature (Ferguson et al. al, 2007). Les résultats montrent que pour les produits au coût de pénurie élevé et au faible taux de détérioration, il serait rentable de stocker plus de produits au point de commande, en particulier dans des conditions incertaines, pour éviter le risque de pénurie.

Pour conclure, nous formulons les suggestions suivantes correspondant à de nouvelles recherches sur ce travail :

- ✓ Considérer l'incertitude sur les autres paramètres tels que la demande, le % de commandes reportées et le taux de détérioration.
- ✓ Considérer les autres types de fonction de distribution du délai d'approvisionnement tels que la loi normale et la loi de Weibull.
- ✓ Considérer l'autre forme de coût de possession non linéaire fonction du temps et / ou du stock.
- ✓ Développer de nouveaux modèles pour examiner d'autres critères tels que la satisfaction du client, les effets environnementaux et ainsi de suite.
- ✓ Étendre la chaîne d'approvisionnement en considérant des fournisseurs multiples et / ou des détaillants multiples. De cette façon, l'étude du fractionnement de commande auprès des fournisseurs multiples avec délais de livraison stochastiques est intéressante.
- ✓ Ajouter le coût total du fournisseur au modèle proposé et optimiser la performance globale de la chaîne d'approvisionnement. A cet effet, l'étude des politiques telles que le lot-pour-lot, les tailles égales d'expédition, les livraisons retardées de tailles égales et ainsi de suite sont des pistes de nouvelles recherches fructueuses.

8- Un nouveau modèle de fractionnement des ordres avec délais stochastiques pour des produits sujets à détérioration (chapitre 4)

Dans ce chapitre, nous développons un modèle de base avec une source unique et une double source d'approvisionnement pour des articles sujets à détérioration avec des délais stochastiques.

Il existe deux grands types de politique d'approvisionnement: mono et multi sourcing. Treleven et Schweikhart (1988) définissent la source unique d'approvisionnement comme la situation dans laquelle un organisme choisit d'avoir toutes ses demandes pour un produit donné satisfaites par un seul fournisseur. Si une source unique n'est pas retenue par choix, par exemple quand un vendeur a le monopole, ces auteurs l'intitulent «fournisseur unique». Le multi sourcing se réfère à la politique dans laquelle un détaillant achète un produit identique chez deux ou plusieurs fournisseurs. Une question importante qui se pose dans les études traitant du multi sourcing est le fractionnement d'un ordre entre plusieurs fournisseurs, en lieu et place d'un ordre unique, en particulier dans des conditions stochastiques. Cette politique est appelée «fractionnement d'ordre».

En passant en revue les études de fractionnement de commande, on découvre que la plupart des chercheurs ont considéré des produits avec durée de vie infinie dans les modèles développés. Aussi sommes nous motivés pour étudier cette question dans le cas de produits qui se détériorent. La principale contribution de ce chapitre est donc de relier la littérature traitant du fractionnement et celle traitant de d'approvisionnement de produits qui se détériorent.

Nous considérons un détaillant avec un système de révision continu du stock *(r, Q)* confronté à une demande constante. Il est fait l'hypothèse que le produit se dégrade avec un taux de détérioration constante chez le détaillant. Un taux de possession constant est également considéré pour les produits stockés. Ainsi, l'approche de la classe 1 est retenue pour modéliser la détérioration.

L'objectif est de minimiser le coût total du détaillant, comprenant les coûts d'approvisionnement, de passation de commande, de possession, de rupture et de détérioration. Pour ce faire, il existe deux politiques d'approvisionnement. Le détaillant peut choisir *i*) soit un fournisseur unique auprès duquel il achète tous ses produits avec un délai exponentiellement distribué ou *ii*) une source duale d'approvisionnement avec fractionnement des commandes entre deux fournisseurs aux délais de livraison exponentiels, indépendants et non-identiques. Ainsi, deux modèles mathématiques sont développés pour deux stratégies différentes: unique source d'approvisionnement pour des produits sujets à détérioration **(SSDI)** et source duale d'approvisionnement pour des produits sujets à détérioration **(DSDI).**

Les fonctions non-linéaires de stock, la dissimilarité des fonctions de stock (avant et après rupture) et la dépendance des cycles d'approvisionnement des délais d'approvisionnement rendent la résolution exacte de ce problème insoluble. Nous utilisons donc une approche itérative de programmation quadratique séquentielle (SQP) pour résoudre ces modèles.

Puis, basé sur un exemple tiré de la littérature (Ramasesh et.al) une analyse de sensibilité est effectuée. En raison du processus de détérioration, il est montré que le temps de cycle de réapprovisionnement, dans les deux modèles SSDI et DSDI est inférieur à celui de produits sans détérioration *(Q*/D)*. En outre, chaque fois que le taux de détérioration augmente, le temps de cycle ainsi que le point de réapprovisionnement *(r)* diminue. Ceci est similaire à ce qui se passe dans la réalité. Par exemple, dans les entreprises gérant des produits qui se détériorent rapidement, la fréquence avec laquelle les ordres sont passés est plus grande que dans le cas de produits qui se détériore lentement.

Un résultat significatif dérivé de l'analyse numérique est que lorsque le taux de détérioration augmente, le montant attendu des coûts de détérioration ainsi que le coût total économisé par une stratégie de double-sourcing augmentent dans le cas de délais stochastiques, en particulier lorsque les deux fournisseurs sont plus similaires. En revanche, le fractionnement n'est pas rentable lorsque les deux fournisseurs sont très différents en termes d'incertitude sur les délais et de prix de vente. En outre, les résultats numériques indiquent que le point de commande dans le modèle de double-sourcing est inférieur à celui du modèle avec source unique. Enfin, les modèles proposés sont appliqués pour résoudre un cas réel de l'industrie pharmaceutique.

Jusqu'à présent, l'appel à de multiples fournisseurs pour des articles sujets à détérioration n'a pas reçu autant d'attention que l'utilisation d'une politique de fractionnement pour des articles non sujets à détérioration. Bien sûr, des recherches substantielles sont encore nécessaires, et il y a plusieurs domaines intéressants pouvant être étudiés:

- ✓ Il serait très utile d'étudier la politique de fractionnement des ordres pour les produits à taux de détérioration linéaire, logarithmique, exponentiel ou suivant une loi de Weibull.
- ✓ En outre, la modélisation du problème de fractionnement pour des produits sujets à détérioration et sa résolution par une méthode exacte est une question intéressante pour des recherches plus poussées. A cet effet, l'application de méthodes d'estimation (comme l'extension des séries de Taylor) pour simplifier les modèles mathématiques peuvent être utiles.
- ✓ En outre, dans le modèle développé ici nous avons supposé qu'en cas de rupture la demande sera totalement reportée. Dans un marché concurrentiel, la perte de ventes ou le report partiel des ventes sont plus réalistes et peuvent être un sujet fécond pour les recherches futures.

✓ Enfin, la demande peut dépendre de divers facteurs ou être probabiliste. Par exemple, nous savons que dans de nombreux cas, la demande dépend des prix de vente ou du cycle de vie du produit. Les dépendances ou les incertitudes sur des paramètres tels que la demande rend le modèle plus complexe, bien que plus pertinent que le modèle proposé.

9- Un modèle bi-objectif de programmation stochastique pour la chaîne d'approvisionnement verte centralisée pour des produits sujets à détérioration (chapitre 5)

Dans ce chapitre, un modèle de programmation stochastique bi-objectif en deux étapes est proposé pour une chaîne d'approvisionnement verte centralisée avec des produits qui se détériorent. L'importance des produits sujets à dégradation du point de vue des critères écologiques peut être résumée comme suit:

✓ Coût des déchets : les processus d'élimination des déchets et de recyclage sont très importants pour les produits sujets à détérioration et de nombreuses entreprises tentent de réduire ces coûts.

✓ Coûts de stockage et de transport : dans la plupart des cas, des équipements spécifiques (tels que des entrepôts et camions frigorifiques) doivent être utilisés pour le stockage et le transport des produits qui se détériorent, afin de réduire le coût de détérioration. Il devrait donc y avoir un équilibre raisonnable entre les critères tels que le coût de détérioration, la consommation d'énergie et les émissions de GES générées.

✓ Risque: dans le cas des produits pharmaceutiques sujets à détérioration tels que les substances radioactives, le risque par exemple d'un accident est susceptible d'entraîner la perte des propriétés bénéfiques des produits ainsi que des effets environnementaux. Ici, diminuer le risque d'accidents pendant le transport est indispensable.

Dans cet article, nous examinons les deux premières questions. La troisième sera traitée comme une orientation future.

Aujourd'hui, l'un des problèmes cruciaux est l'émission des gaz à effet de serre (GES). À cet égard, de nombreux règlements et contraintes ont été mis en place par les gouvernements, malgré le fait que de nombreuses entreprises craignent que ces règles vertes ne se traduisent par une baisse sensible de leurs bénéfices. En conséquence, la question essentielle est la suivante : Quels sont les

«meilleures» solutions pour concilier les préoccupations écologiques et économiques ?
Dans ce chapitre, nous avons considéré une chaîne d'approvisionnement centralisée dyadique se composant d'un fournisseur et d'un distributeur. Contrairement aux chapitres précédents, dans ce chapitre le délai de livraison est constant alors que la demande finale a été considérée comme stochastique. Les demandes non satisfaites ont été supposées être partiellement reportées. Il y a plusieurs types de véhicules de transport chez le fournisseur, chacun caractérisé par sa capacité et des frais de transport et émissions de GES associés.
En raison de la caractéristique de détérioration des produits, un taux de détérioration constante ainsi qu'une fonction non-linéaire des coûts de possession ont été pris en considération (classe 3). Afin de modéliser une demande incertaine, une approche de programmation stochastique en deux étapes a été appliquée. La prise en compte de la demande incertaine, de la détérioration des produits, de l'hypothèse de report partiel, des émissions de GES et des coûts ainsi que de la capacité de transport du véhicule rendent le problème proche de la réalité. Cependant cela rend difficile l'utilisation de modèles d'inventaires classiques (comme les modèles expliqués dans les chapitres 3 et 4). Ainsi, un modèle mixte entier mathématique est développé par l'application d'une approche de programmation stochastique en deux étapes.
L'objectif du modèle proposé est de trouver la meilleure configuration des types de véhicules et des quantités commandées, afin de répondre simultanément aux objectifs suivants: *i*) réduire le coût total de la chaîne d'approvisionnement incluant le coût de passation de commande, le coût d'achat, le coût d'exploitation, le coût de rupture, le coût du transport et le coût du recyclage après déduction des recettes induites. *ii*) réduire au minimum les émissions de GES de la chaîne d'approvisionnement incluant les quantités de GES émises par les véhicules pendant le transports des produits; par les éléments détériorés, et pendant le processus de recyclage.
Les principales contributions de ce chapitre qui la différencient des recherches existantes peuvent être résumées comme suit:

- ✓ Nous nous concentrons dans le modèle proposé sur les produits sujets à dégradation, même si celui est applicable aux autres produits. En dépit du fait que la prise en compte de la détérioration des produits rend le modèle compliqué, l'importance de la gestion des stocks pour ce type de produits dans de nombreuses sociétés, à la fois financièrement et écologiquement, justifient notre motivation.

- ✓ Un grand nombre de publications de la chaîne logistique verte a mis l'accent sur les décisions stratégiques, opérationnelles-stratégiques, et tactiques-stratégiques. Comme de nombreuses exigences aujourd'hui sont liées à des décisions tactiques-opérationnelles, notre modèle est élaboré en prenant en considération ces décisions, notamment pour les stocks et le transport.
- ✓ Malgré l'importance de la littérature sur la programmation multi-objectifs, déterminer les frontières éco-efficaces à l'aide de modèles de programmation mixte entier multi-objectifs est tout à fait nouvelle (Dekker et al., 2012). Ici, par la détermination de frontières éco-efficaces, nous déterminons la politique optimale de stockage et de transport, en tenant compte à la fois des coûts et des émissions de GES. A notre connaissance, c'est la première tentative d'aborder un nouveau modèle de programmation stochastique bi-objectif en deux étapes pour une chaîne d'approvisionnement verte centralisée avec des produits sujets à détérioration dans un contexte de demande incertaine et de report partiel des commandes en cas de rupture de stock.

Par la suite, nous appliquons le modèle proposé à un cas réel de l'industrie pharmaceutique. Puis nous proposons une analyse de sensibilité et développons des propositions managériales dans le cadre proposé. En fournissant un ensemble de solutions de Pareto il est déduit que la chaîne d'approvisionnement doit permettre une réduction minime des bénéfices totaux, afin de parvenir à une réduction considérable des émissions totales de GES produites prévu. C'est pourquoi les entreprises ne devraient pas retarder leurs actions correctives afin de réduire les gaz à effet de serre (GES) simplement en raison du risque de faillite.

Le modèle proposé n'est certainement pas définitif et le chemin est encore ouvert pour des recherches plus poussées sur les points suivants:

- ✓ Considérer le risque d'accident dans le modèle mathématique.
- ✓ Considérer une chaîne d'approvisionnement à trois échelons constituée d'un détaillant, d'un fournisseur et d'un (de multiples) fabricant(s), de sorte que le coût et les émissions de GES soient prises en compte.
- ✓ Développer le modèle en considérant des processus de logistique inverse.
- ✓ Il peut être possible d'étendre notre étude à un fournisseur de produits multiples.
- ✓ Développer un modèle pour le cas de remise en fonction des quantités.

10- Conclusion

Le nombre croissant de produits sujets à détérioration, ainsi que leurs effets notables sur les pertes et profits de l'entreprise, font des politiques de réapprovisionnement des produits sujets à détérioration une question intéressante attirant l'attention tant des chercheurs universitaires que des dirigeants d'entreprises.

Dans un premier temps, nous avons donné un aperçu sur les recherches traitant de la reconstitution des stocks de produits sujets à détérioration. Ensuite, nous avons proposé une classification des travaux basée sur des approches de modélisation de la détérioration (Figure 1) telles que: *i*) une fonction non-linéaire de stock et un taux possession constant *ii*) une fonction linéaire de stock, avec un coût de possession non linéaire croissant en fonction du temps de stockage, du niveau de stock, etc *iii*) une fonction de stock non linéaire et une fonction non linéaire croissante du coût de possession. En outre, certaines catégorisations sont présentées pour des principaux paramètres tels que les fonctions de détérioration et de demande.

Sur la base de cette revue de littérature, nous avons identifié certains manques. Celle-ci a révélé que la plupart des recherches se sont concentrées sur certaines conditions (délai d'exécution / nature de la demande). Aussi, le but de cette thèse a consisté à développer de nouveaux modèles de réapprovisionnement pour gérer les stocks de produits sujets à détérioration dans des conditions incertaines. Pour atteindre notre objectif, nous avons tenu compte à la fois d'un délai d'approvisionnement et d'une demande stochastiques dans différentes situations. La prise en compte de ces incertitudes dans nos modèles nous rend plus proche de la réalité, mais rend aussi leur formulation et leur résolution plus compliquées. Nous avons également mis l'accent sur certaines lacunes de la recherche dans nos autres modèles proposés tels que les politiques de fractionnement ou d'intégration de critères écologiques.

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- ✓ **Z.Sazvar,** M.R. Akbari Jokar, A. Baboli, 2013, "A new order splitting model with stochastic lead times for deterioration items" *International Journal of System Science (ISI).* In press.

- ✓ **Z.Sazvar**, A. Baboli, M.R Akbari Jokar, 2013, A replenishment policy for perishable products with non-linear holding cost under stochastic supply lead time, *International Journal of Advanced Manufacturing Technology (ISI)*, 64, 1087–1098.
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- ✓ **Z. Sazvar**, M.R Akbari Jokar, A. Baboli, J. P. Campagne, "Centralized replenishment policy for deteriorating items in a three echelon supply chain under stochastic lead time" INCOM 2012: *14th IFAC Symposium on Information Control Problems in Manufacturing*, Bucharest, Romania , 2012. Vol: 14, p : 493-498.

- ✓ **Z.Sazvar**, A.Baboli, M.R. Akbari Jokar, J.P. Campagne "Developing and analyzing two inventory models for deterioration items under stochastic supply lead time" *41th International Conference on Computers & Industrial Engineering (CIE41),* Los Angeles, USA, 2011. p: 780-785.

Introduction

Nowadays, increasing of difficulties in production and procurement conditions such as competitiveness among manufacturing firms, variety of products, rapidly changing of customer tastes, shortening of product lifetime and complexity of demand forecasting on one hand, and key role of consumers and deteriorating products on countries revenue on the other hand, make replenishment policies of deteriorating products challenging and also interesting for both researchers and corporate managers. Generally, products are assumed to deteriorate with time resulting in a decreasing utility or price from the original one (Hsu et al. 2007). A growing numbers of products such as pharmaceuticals, health products and cosmetics, perfumes, food, fruit and vegetables, radioactive substances, biotechnology products and many petrochemical and chemical materials are categorized as deteriorating products.

Worldwide sales at grocery retailers in 2006 exceeded $1,000 billion. Deteriorating products such as fresh produce, dairy and meat account for more than a third of these sales (Broekmeulen and Donselaar, 2009). Efficient deteriorating inventory control and management have therefore a considerable effect on a company's profitability and competitive advantage. However, it is more complicated than a simple inventory control of products with an infinite lifetime. Two main issues that make the inventory control of deteriorating items a problem are: dealing with customer demand, and modeling system costs.

Today's customers are requesting greater product diversity. As a result, more deteriorating products tend to exceed the best-before date, while demand per product tends to be less predictable. This makes the inventory systems difficult to model. In many real cases, to control the demand rate, managers frequently use a markdown policy as the product's expiration date approaches. They often also remove spoiled products. Taking these approaches into account in inventory control models make the models even more complex.

The inventory system's cost of deteriorating products is more complicated to manage than the cost of non-deteriorating products. In many cases such as frozen foods, pharmaceutical and biological products deteriorating products' lifetime is sensitive to storage and transportation conditions such as humidity and temperature. This issue as well as costs associated with deteriorated/expired products and disposal/recycling processes complicates the modeling of the system's costs. It is also obvious that the deterioration function type radically affects on the complexity of system's costs.

These kinds of challenges, along with the growing number of deteriorating items in many recent business organizations, make inventory control and replenishment policy of these products, a particularly important research and practical area. However, most researchers have focused on this subject under deterministic conditions that makes them far from real situations.

Deteriorating products are also more noticeable than ones with an unlimited lifespan from the perspective of environmental and green criteria. Nowadays, concerns about depletion of the ozone layer and climate change have been increasingly discussed at international level. Transportation, many logistics activities and industrial processes have been linked to an increase in the greenhouse effect through carbon dioxide (CO_2) emissions, although the effect of other gases should not be under-estimated (Harris et al. 2011). In the U.S., for example, the predominant greenhouse gas emitted is CO_2, which accounts for 85% of the climate change potential for all human-produced emissions. Emissions from trucks increased from 42% of total transportation CO_2 emissions in 1995 to 49% in 2006 and show no signs of decreasing (Ulku, 2012). In this way, deteriorating products play a key role because of emissions through special conditions of stocking and transporting and disposal/recycling processes. As well, toxic effects or radiations of some type of deteriorating products like radioactive substances should not been overlooked.

In this context, this dissertation has contributions in two main areas: *i*) Developing two new deteriorating inventory models by considering stochastic supply lead times (Chapters 3 and 4).

Nowadays, the complex pattern of customer demand, several substitution products, the complexity and diversity of production and transportation policies and so on force planning managers to consider uncertainties (especially in demand and delivery lead time) in their decision-making processes more and more. Considering the uncertainties in the deteriorating products planning is more significant, as the fact of sometimes not paying attention to it results in uncompensated costs.

ii) Developing a new bi-objective stochastic mixed-integer programming model for replenishment a deteriorating product in a green supply chain by considering both economical and environmental criteria (Chapter 5). By defining scenario-based end costumer's demands, this model is also developed under an uncertain condition.

Structure of the thesis

The remainder of the thesis is organized as follows.

Chapter 1: after the introduction, technical background is provided. First, some main concepts in inventory management and replenishment policy of deteriorating products are presented. Then, a general description of the thesis as well as of its contributions is given.

Chapter 2: A literature survey on deteriorating inventory replenishment is presented in this chapter. A new categorization is presented based on applied approaches for modeling deterioration process mathematically. In this way, the present deteriorating models are categorized in three classes: (*i*) Deteriorating inventory models with non-linear inventory functions and linear holding cost rates in terms of parameters such as inventory level or stocking time (*ii*) Deteriorating inventory models with linear inventory functions and non-linear holding cost rates in terms of inventory level or stocking time (*iii*) Deteriorating inventory models with non-linear inventory functions as well as non-linear holding cost functions. Finally, research gaps in existing literature are identified and presented.

Chapter 3: Based on the research gaps, an inventory problem for a main class of deteriorating items, namely perishable products, is considered, under stochastic lead time. The inventory system is modeled as a continuous review system (r, Q). Moreover, demand rate per unit time is assumed to be constant over an infinite planning horizon and the shortages could be backordered completely. Although researchers address inventory model of deteriorating products from different aspects, most researches consider zero or fixed supply lead time in models which are developed. The imprecision of lead time essentially affects an inventory system. It should therefore be considered in inventory models, since not paying attention to it may lead to huge amount of holding or shortages costs. Due to the deterioration process, the effect of uncertain lead time on inventory systems is more considerable since it may results in an uncompensated deterioration cost too. So, the main contribution of this chapter is developing an inventory model for deteriorating items under stochastic lead time.

For modeling the deterioration process, a linear inventory function as well as a non-linear holding cost is considered. Taking into account the stochastic lead time as well as a non-linear holding cost makes the mathematical model more complicated. The proposed model is customized for a uniform distribution function that could be tractable to solve optimally by means of an exact approach. Then an example taken from the literature is solved to demonstrate the effectiveness of the proposed model.

Chapter 4: having a well-organized sourcing policy is a main requirement of an efficient inventory control and management. An attractive subject appears in sourcing policy issue is the splitting of a replenishment order among a number of suppliers simultaneously instead of placing a single order, especially in stochastic conditions.

In this chapter, at first a quite comprehensive review on existing order splitting models is provided. Then, pooling lead time risks by splitting replenishment orders between two suppliers simultaneously under stochastic lead times are studied in the case of deteriorating products. For this purpose, two inventory models are developed for a retailer who offers a decaying item with constant deteriorating rate. In the first model, it is assumed that all the requirements are supplied by only one source, whereas in the second, two non-identical suppliers are available. The inventory system is modeled as a continuous review system *(r, Q)* under stochastic lead time. Demand rate per unit time is supposed to be constant over an infinite planning horizon and shortages are backordered completely. The contribution of this chapter is at first studying a new inventory model for a retailer that supplies its deteriorating inventories from a single source under exponential lead time by a new approach differed from that approach applies in Chapter 3. Then, the single-source model is extend to an order splitting case by considering two available suppliers. In this model, in order to consider ordering cost increases arising from split orders, the multiplicative approach is taken.

To develop single-source and dual-source models under a constant deterioration rate and stochastic lead times, some main challenges have to be considered (*i*) the non-linear inventory function and dissimilarity of inventory function form, before and after shortage, (*ii*) not fixed replenishment cycle times with constant ordering quantity and constant demand rate as long as products are consumed through both demand and the deterioration process until the inventory level becomes zero.

Since the developed mathematical models are very complex, the Sequential Quadratic Programming (SQP) algorithm is used to solve the problem. Then by doing sensitivity analysis, the situations in which each sourcing policy is the most economic are determined.

Chapter 5: In this chapter, a new replenishment policy in a centralized supply chain for deteriorating items is proposed by considering both environmental and

financial aspects. The supply chain composed of a single buyer and a single supplier. End customer's demand is supposed to be uncertain and unfulfilled demands are partially backordered. There are several types of transportation vehicles at the supplier for transferring goods to the retailer. Each one is characterized by its own capacity, transportation cost and greenhouse gas (GHG) emissions. Because of the deterioration characteristic of products, a constant deterioration rate in addition to a non-linear holding cost function is considered.
It is well known that in recent years, consumers and legislation have been pushing companies to design their activities in such a way as to reduce negative environmental impacts more and more. In fact, today, 'greenness' is one of the significant anxieties of industrial firms, supply chains and governmental organizations. One of the main concerns of industrial managers is that dealing with green considerations in decision-making processes may result in substantial profit reduction. The major contribution of this chapter is making a trade-off between costs and GHG emissions at tactical-operational level for a centralized supply chain with deteriorating products by determining eco-efficient frontiers. In other words, the best transportation vehicles and inventory policy is determined by finding a balance between financial and environmental criteria. In this way, a two-stage stochastic programming approach is applied to develop a mixed-integer mathematical model, in order to plan supply chain's inventory and transportation policies concurrently. The objectives include the minimization of expected total costs and expected total GHG emissions. Different types of costs such as ordering cost, purchasing, holding, transportation, shortage and recycling costs are considered taking transportation vehicles' capacities into consideration. Linearization of the mathematical model is one of the main challenges to deal with this problem.
At the end, a numerical example from the real world is presented to demonstrate the applicability and effectiveness of the proposed model. Then a sensitivity analysis is performed and some managerial insights are provided.
Chapter 6: the conclusion with a summary of the dissertation is presented. Some future work and research perspectives are also given in this chapter.

1

CONCEPTS AND DEFINITIONS

1 Concepts and definitions

1-1- Introduction

Today, control of the material flow from raw materials suppliers to final customers is a vital problem for more or less all organizations in any segment of the economy, Supply Chain Management (SCM) etc. The strategic importance of this area is completely recognized by top management. The total investment in inventories is enormous, and the control of capital tied up in raw material, work-in-progress, and finished products offers a very important potential for improvement (Axsater, 2006). Scientific techniques for inventory control and replenishment policies can therefore result in a significant competitive advantage.
As a matter of fact, inventories management cannot be separated from other areas, such as production, purchasing, sales and marketing. In fact, the goal of inventory control is often to balance conflicting objectives. One objective is to have a level of inventories as low as possible to make cash available for other purposes; it is desired by business or financial resources units. The other one is to have a high stock of finished products to offer clients a high service level which is favorable for sale departments. This is why inventory models are necessitated.
In this chapter, at first some main concepts and definitions on inventory control and management are explained in Section 1-2. The different types of inventories are then described in Section 1-3. In section 1-4, some definitions for deteriorating products existing in the literature are provided. Some challenges of deteriorating products management are described in Section 1-5. Finally, in Section 1-6, the main contributions of the thesis are described.

1-2- Some basic concepts on inventory management

One main purpose of an inventory control system is to determine *which product, when and how much (how many)* should be ordered. In reality, these decisions should be based on the product type, the anticipated demand, the inventory position etc.
In this section, some basic concepts related to inventory management used in next chapters are described based on Axsater (2006) and Silver, et al. (1998).
Let's start with some main concepts in inventory management:

- ✓ **On-hand stock:** The stock that is physically on the shelf. In other words, this quantity determines whether a particular customer demand is satisfied directly from the shelf. Subsequently, it can never be negative.

- ✓ **Net stock** that equates to *on-hand inventory* minus *backorder* level as follows:
  Net stock = (On-hand) – (Backorders)
  The concept of *Backorders* will be developed later.
- ✓ **Inventory position:** In fact, an ordering decision cannot be based only on the stock on-hand. It is necessary to include the *outstanding orders* that have not yet received, and *backorders* which are units that have been demanded but not yet delivered. Thus, the ordering process is often done based on *inventory position* characterized by the following statement and not based on *stock on-hand:*
  Inventory position = Stock on-hand + Outstanding orders – Backorders – committed.

If the customers can reserve the product for later delivery, the reserved units are usually subtracted from the inventory position as well that is stated as *committed* in the above expression.

After an ordering is placed, usually it takes time to receive. According to Axsater (2006) "*the time from the ordering decision until the ordered amount is available on shelf*" is called lead time. So, it is not only the production time in case of an internal order or the transportation time from an external supplier. It also consists of order preparation time, transit time for the order, administrative time at the supplier, time for inspection after receiving the order, and the like. The lead time, can be instantaneous, fixed or stochastic.

The interval time between placing two successive orders is called *replenishment cycle*. During each replenishment cycle/lead time, some customer's demands are filled by on-hand inventory immediately. It is also possible that the demands exceed the on-hand inventory level, and thus a part of demands is unfulfilled that is called *shortages*. It is necessary to know how the system responds to shortages or demands which cannot be filled immediately from the stock. There are three common hypotheses for shortages. All unfulfilled demand's wait to be satisfied at the future time named *backordered/backlogged/backordered demand*. It can also suppose that all unfulfilled demands are lost by satisfying from outside the system entitled *lost sale*.

Other possibility is *partial backordering/backlogging*. In partial backordering situation, a part of customer demands unfulfilled is lost (lost sale) and the rest is backordered. In fact, shortage in the form of completely lost sale or completely backordered represents special cases of the partial backordered situation.

Finally, the time interval on which the inventory planning extends into the future is *the planning horizon* that may be infinite or finite.

1-2-1- Inventory control systems

There are two basic inventory control system. Some inventory systems are designed so that the inventory position is checked continuously. Then, as soon as the inventory position is reached to a certain given point (*reorder point*), an order is triggered. This system is called *continuous review.* The triggered order will be received after a lead time.

The other type of inventory systems is *periodic review* system in which inventory position is controlled only at certain given points in time. Generally, the intervals between these reviews are constant.

1-2-2- The ordering system

In general, there are various types of ordering policy for each kind of inventory control system. A main traditional ordering policy is (r, Q). When the inventory position reaches to (or below) the *reorder point* (r) an order of size $Q>0$ is placed. If demand is continuous (or one unit at a time), the reorder point will always be hit exactly in case of continuous review. In case of periodic review (or if the triggering demand is for more than one unit in the case of continuous review) the ordering is often triggered when the inventory position is below the reorder point (r). Hence, the position ($r+Q$) after ordering will not be reached.

Another popular ordering policy is (T, S) that means each T units of time, the inventory position is checked (so it is defined in periodic review inventory control system) and if the inventory position is less than *order-up-to level* (S), a replenishment order is placed to bring the inventory position to S.

There are other types of ordering systems, most of which can be defined by the combination of the two mentioned policies. Recently, Haji and Haji (2007) have introduced a new ordering policy named $(1,T)$ which is different from classical policies. According to $(1,T)$ policy "*the time interval between any two consecutive orders is fixed (T) and the quantity of each order is one*". So, the order quantity and the time interval between two consecutive orders are both constant.

1-3- Types of Inventory

Based on Goyal and Giri (2001), inventoried goods can be broadly categorized into three meta-classes:

- ✓ Unlimited lifetime products: The products which have no obsolescence and no deterioration. The shelf-life of this type of products is indefinite.
- ✓ Style Products (Obsolescence): It refers to products that lose their value through time due to quick changes of technology or the introduction of a new substitution product by a competitor like spare parts for military aircraft. Style goods should be sharply reduced in selling price or otherwise disposed off after the season is over.
- ✓ Deteriorating products: Deterioration refers to the damage, dryness, vaporization, spoilage, etc. of the products.

According to Goyal and Giri (2001), Deteriorating products are classified in two main groups too. Products that have a maximum usable lifetime are classified as *perishable products* such as photographic films and human blood. Those products which have no shelf-life at all are classified as *decaying products* such as radioactive substances, alcohol and gasoline.

Rafaat (1991) classified *perishable products* as deteriorating products with a fixed shelf-life and *decaying products* as deteriorating items with continuous decay (random lifetime).
He also classified deteriorating items with respect to their value or utility as a function of time as follows:

- ✓ Constant-utility perishable goods undergo decay and face no considerable decrease (or increase) in value throughout their usable lifetime such as prescription drugs.
- ✓ Decreasing-utility perishable goods lose value during their lifetime like fresh produce or fruits.
- ✓ Increasing-utility perishable goods increase in value, such as some wines.

Recently Bakker et al. (2012) have provided a categorization for deteriorating products based on product's shelf-life characteristics as:

- ✓ Fixed lifetime that is predetermined deterministic lifetime.
- ✓ Age-dependent deterioration rate like Weibull or Exponential curves etc. which implies a probabilistic distributed lifetime.
- ✓ Time- or inventory-dependent (but not age-dependent) deterioration rate. Models with a constant deterioration rate per stocked item (and so inventory but not age-dependent) belong to this category.

1-4- Definitions of deterioration in the literature

There are various definitions for deterioration in the literature. Some of them are as follows:

- ✓ Decay or deterioration is described as any process which prevents an item from being used for its intended original use for example: (*i*) spoilage, as in perishable foodstuffs, fruits and vegetables; (*ii*) physical depletion, as in pilferage or evaporation of volatile liquids (gasoline and alcohol, etc.); (*iii*) decay, as in radioactive substances, degradation, as in electronic parts, or loss of potency as in pharmaceutical drugs and photographic films (Raafat, 1991).
- ✓ Deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, and loss of entity or loss of marginal value of a commodity that results in decreasing usefulness (Wee, 1993).
- ✓ Deterioration refers to "*the damage, spoilage, dryness, vaporization, etc. of the products*" (Goyal and Giri, 2001).
- ✓ Deterioration is "*the damage, spoilage, dryness, vaporization, etc. that results in (a) decrease of usefulness of the original one*" (Wu, et al. 2006).
- ✓ Generally products are assumed to deteriorate with time resulting in a decreasing utility or price from the original one (Hsu et al. 2007).
- ✓ According to Disney et al. (2012), there is a difference between deteriorating inventory and perishable inventory. In the deteriorating inventory scenario, the goods physically decay and are destroyed over time, whereas perishable goods lose value but are not destroyed.

As a result, various definitions and classifications with minor differences exist in the literature for deteriorating products. Our work in this dissertation is based on the definition and classification which have been provided by Goyal and Giri (2001).

1-5- Challenges of deteriorating products management

In recent decades, the need to efficiently deal with deteriorating products has increased in many industries since the number and variety of deteriorating products are dramatically increasing. According to Broekmeulen and Donselaar (2009), global sales at grocery retailers exceeded \$1,000 billion in 2006. Deteriorating products account for more than a third of these sales. Similarly, Lystad et al. (2006) stated that deteriorating products make up about \$200 billion sales in the United States grocery industry which about \$30 billion are lost due to deterioration.

Managing deteriorating products is also basically more challenging than products with an infinite lifetime.
In addition to usual inventory costs (procurement, ordering, holding and shortage costs), deteriorating items impose some extra costs on the system too. Those costs originate from different sources depending on the product type. For example, the procurement cost of deteriorated/expired products is an extra cost that is usually called a *deterioration cost*.
In the case of deteriorating products like foodstuffs, the quality and so the selling price is time-sensitive, which means that the quality/selling price decreases considerably as the end of the product's life approaches. Here the lost profit resulting from price reduction can be inferred as a cost for the system.
In many cases, the holding cost per unit item per unit time (the holding cost rate) for deteriorating items is not fixed, since the finished cost of each unit product is variable. For example, retailers in supermarkets face the problem of decreasing quality of unsold items like vegetables and fruits as the days go by. Therefore, the holding cost rate tends to increase, as better storage facilities are arranged to prevent spoilage and to maintain freshness of the items in stock. This case is also occurred when spoiled products are removed over time. Deteriorating products such as electronic components and radioactive substances are other examples that require more sophisticated arrangements corresponding to their inventory volume, to ensure their safety and security. Thus, their holding cost rate is non-linearly inventory-dependent. Consequently, in many situations like the above instances, the finished cost of each unit product is not fixed and depends on time, inventory level, etc. This changes the holding cost rate and complicates managing the system's costs effectively.
Furthermore, since many deteriorating products have an expiry date, the period of time for which the product quality remains up to standard is very limited. Such products rapidly deteriorate during transportation and storage. Sophisticated equipment, with extraordinary costs, should therefore be used for storage and transportation (such as refrigerated warehouses or trucks).
Additionally, the effect of deterioration on the environment, as well as the cost of deteriorated/expired products recovery process, is a real problem, especially in green or sustainable supply chains where the inventory cost modeling of deteriorating products is more complex.
In many cases, in order to reduce extra costs results from deterioration, companies take some special cost-effective policies for sourcing, production, storage and transportation of deteriorating products. For example, managers apply techniques to ensure maximum capacity of sophisticated costly production,

storage and transportation equipment. Some upstream supply chain partners (producers), apply a Make-To-Order (MTO) (or even Make-To-Engineering (MTE)) production strategy instead of a Make-To-Stock (MTS) strategy, in order to reduce high deterioration and storage costs. In recent years, some companies apply a new technology called Time Temperature Integrator (TTI) to evaluate the effective shelf-life of products by recording time temperature history ().

Bearing all that in mind, today it is more difficult to forecast demand per product for many companies since customer tastes are continuously changing and new substitution products are also released to the global market frequently (sometimes by lower price or higher quality). For deteriorating items – especially in the perishable ones – this problem is much more challenging as more products tend to exceed the best-before date.

Consequently, various mentioned challenges and costs related to deteriorating items, along with the growing number of these products, make them a principally appealing issue for researchers and corporate managers alike.

1-6- Contributions

In this dissertation, after taking the state of the art review into account, three new replenishment problems for deteriorating products are examined and analyzed. The major contributions of the dissertation are as follows:

- ✓ This dissertation will do a quite comprehensive study on deteriorating inventory management literature and will present a new classification of deteriorating inventory models based on 'mathematical approaches for modeling deterioration process' (Chapter 2). Then, some new replenishment problems for deteriorating products are studied based on new classification.
- ✓ Many existing deteriorating inventory models are developed under deterministic conditions. In this dissertation, replenishment policy of deteriorating products under uncertain conditions will be studied more. Two main sources of uncertainty are lead time and customer demands. In this way, two new problems by concentrating on uncertain lead time and a new problem by considering uncertain external demand will be analyzed.
- ✓ It is well known that the lead time affects on inventory systems and its related costs considerably. In many cases, the wrong estimation of lead time brings about a high volume of in-stock inventories or deteriorated items (in the cases of overestimation) or losing a high volume of customers (in the case of underestimation). The most existing deteriorating inventory models have been developed by assuming a certain lead time. In this dissertation, several

inventory control models for deteriorating products will be developed under stochastic lead times (Chapters 3 and 4).

- ✓ In this dissertation 'multiple sourcing and order splitting policy' for deteriorating products with constant deteriorating rate will be studied in Chapter 4. Previously, these two concepts (order splitting and deteriorating products) together have not been paid enough attention in the literature.
- ✓ According to Dekker et al. (2012), determining eco-efficient frontiers using multi-objective mixed-integer programming models is quite new, despite the broad literature on multi-objective programming. In this thesis, a bi-objective stochastic mixed-integer programming model will be developed for a green supply chain with deteriorating products (Chapter 5). The objective of this model is to make a trade-off between costs and GHG emissions at tactical-operational level by determining eco-efficient frontier. In fact we take into account economic considerations on one hand and green considerations on the other, to study interactions of economic and green considerations in a centralized supply chain for deteriorating products and under uncertain external demand. The proposed model can therefore be used as a systematic decision-aid tool, since it is able to determine inventory policy and select transportation vehicles (to avoid the sub-optimalities led from separated decisions in each part), taking into account both costs and GHG emissions simultaneously.

1-6-1- Questions of the thesis

Our purpose is to answer the following relevant questions:

- ✓ What is the impact of uncertainty on deteriorating inventory management?
- ✓ What is the impact of deterioration on sourcing policy?
- ✓ What is the impact of transportation vehicles on deteriorating inventory control?
- ✓ What is the impact of Greenhouse Gases (GHG) emissions on deteriorating inventory control?
- ✓ What are the 'best' solutions for balancing ecological and economic concerns in supply chains with deteriorating products?

2

LITERATURE REVIEW

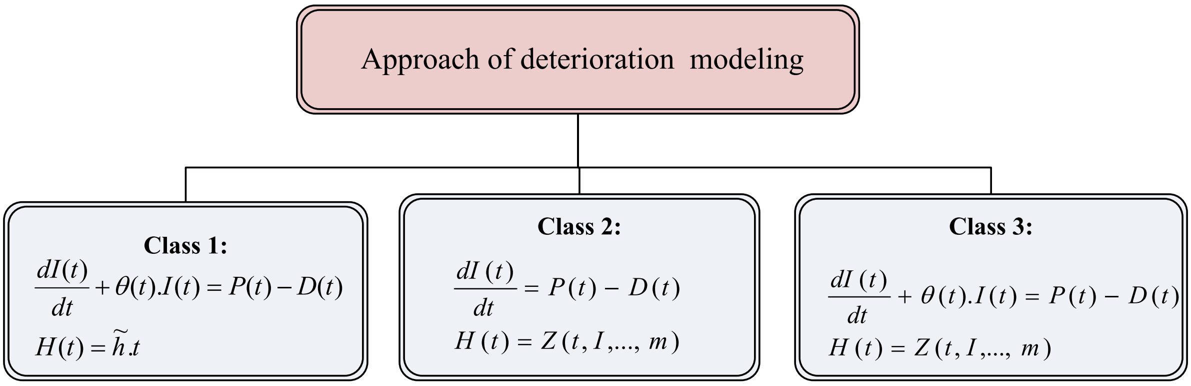
2 Literature review

2-1- Introduction

In this chapter, an overview on some main researches in deteriorating inventories replenishment is given. A new classification of works based on deterioration modeling approach is provided in Section 2-2. Some other categorizations are also presented based on objective function's components and demand functions in Sections 2-3 and 2-4 respectively. In this way, a summary of some reviewed researches are provided in a table (Table 2-2). In Section 2-5, an explanation on green logistics and the effect of greenhouse gases (GHG) on environment by considering deterioration is provided. Finally, based on the literature review, several research gaps are identified and presented in Section 2-6.

2-2- Deterioration process modeling approaches

Many researchers have analyzed inventory control of deteriorating items from different perspectives. Broadly speaking, the existing literature in this field can be divided into the following three classes from the perspective of the modeling approach. These classes are schematically illustrated in Figure 2-1.



I(t): On-hand inventory in terms of time, t
Θ(t): Deterioration function in terms of time, t
D(t): Demand function in terms of time, t
P(t): Production rate in terms of time, t
H(t): Holding cost of one unit that is in-stock for t units of time
$\tilde{h}$ **:** a positive constant
Z(t, I, …,m): a non-linear, increasing positive function of finite number of parameters like stocking time, t, on-hand inventory, I, etc.

Figure 2-1 - Categorization of deterioration modeling approaches

2-2-1- Class 1: Non-linear inventory function

Most researches on deteriorating inventory consider that inventory decays with time, in different patterns. So, the on-hand inventory function can be determined by the differential equation:

$$\frac{dI(t)}{dt} + \theta(t)I(t) = P(t) - D(t) \quad (2\text{-}1)$$

Where $I(t)$ is the inventory level at time t, and $\theta(t)$, $P(t)$ and $D(t)$ indicate the deterioration rate function, the production rate and the demand rate at time t respectively.
In this type of research it is considered that the holding cost per unit item per unit time (holding cost rate) is constant. In other words, the holding cost is linear in terms of parameters like stocking time, t, and the on-hand inventory level, I, that can be stated as $\tilde{h}tI$, where $\tilde{h} > 0$ is constant.

This kind of modeling approach is more appropriate for decaying items and was used in the earliest researches on deteriorating products. Ghare and Schrader (1963) seem to have been the first to have developed an exponentially deteriorating inventory model by defining a constant decaying rate. Since then, research on deteriorating inventory has been continued extensively by many researchers over the years. Manna and Chaudhuri (2001), Wee and Chung (2009), Wang and Chen (2001), Zhou and Lau (2000), Deng (2005), and Tsai (2011) are in this category.

More explanation on the deterioration function form, $\theta(t)$, (in Equation 2-1) is presented in Section 2-3.

2-2-2- Class 2: Non-linear holding cost

The deterioration process directly affects the on-hand inventory function and thereby inventory holding cost modeling. In this category, the on-hand inventory function form is similar to its form of non-deteriorating products and can be obtained by the differential equation:

$$\frac{dI(t)}{dt} = P(t) - D(t) \quad (2\text{-}2)$$

Here, instead of considering the deterioration rate function*, θ(t),* in the on-hand inventory function, the holding cost, H, is considered as a non-linear increasing positive function of parameters like stocking time, t, or on-hand inventory, I.
Considering a non-linear time-dependent holding cost is more suitable for deteriorating items-especially perishable ones- when the value and quality of the unsold items decrease with time, as in the case of green vegetables. For products such as electronic components, radioactive substances, volatile liquids etc., where more sophisticated tools are required for their security and safety in stock, a non-linear stock-dependent holding cost can be appropriate.
Weiss (1982) appears to have been among the first to treat holding cost per unit as a non-linear function of the length of time for which the item was held in store. He considered that the cumulative holding cost for one unit that has been kept for t units of time is $H(t) = \tilde{h}t^{\gamma}$, where $\tilde{h} > 0$ and $\gamma \geq 1$ are constant. He maintained other traditional EOQ assumptions such as zero supply lead time, constant unit cost, selling price and set-up cost, and developed two mathematical models for deterministic and stochastic demand rate.
Recently, in two case studies, Ferguson et al. (2007) have dealt with the application of the Weiss model. They indicate that this model is appropriate for getting an approximation of deteriorating items, which require price markdowns or removal of aging products such as milk, fruit, vegetables and produce, sold in small to medium-sized grocery stores.
In 1994, by considering stock-dependent demand, Goh employed a variable holding cost in two forms: a non-linear continuous function of the time the item is held in stock, and a non-linear continuous function of the amount of the on-hand inventory. By supposing other EOQ model assumptions, closed-form formulas were developed for optimum order quantity.
Recently Alfares (2007) extended Goh's model to the situation in which the variable holding costs are a discrete function as a step function of the stocking time with successively increasing costs. He considered two scenarios: (1) the retroactive holding cost increase that characterized the holding cost of the last storage period of a replenishment cycle is considered retroactively for all storage periods, so that the same holding cost is applied to all units in the cycle; and (2) the incremental holding cost increase that is characterized as a storage period holding cost is applied incrementally to each period, so that the holding cost for each storage period is applied only to the units held in that period.
In 2008, Urban (2008) generalized Alfares's model by using a profit-maximization objective instead of cost minimization. He also relaxed the

limitation that the inventory level be zero at the end of each replenishment cycle, by considering it as non-negative.

2-2-3- Class 3: Non-linear inventory function and non-linear holding cost

This modeling approach is more complicated than the other two. Here, both the deterioration rate function, *θ(t),* a feature of Class 1, and the non-linear holding cost, a feature of Class 2, are considered to model the inventory system of deteriorating products, as in Girl and Chaudhuri (1998). These authors discussed Goh's model, considering a constant *θ(t)* in addition to non-linear holding cost in two time-dependent and stock-dependent cases.

In 2007, Teng et al. studied the pricing and lot-sizing problem for a deteriorating good under finite production, exponential decay, partial backordering and lost sale assumptions. They considered time-dependent holding, backorder and lost sale costs and then analytically compared the net profits per unit time based on Abad's (2003) model and Goyal and Giri's (2003) model.

2-2-4- Deterioration function-*θ(t)*

In the existing literature, various forms of deterioration function, *θ(t),* have been considered. Generally, we can classify the deterioration functions, *θ(t),* in two main groups: discrete deterioration functions, and continuous deterioration functions.

2-2-4-1- Discrete deterioration function

Discrete (or semi-continuous) deterioration functions are appropriate for modeling the inventory system of '*non-instantaneous deteriorating items*' like fruit, fish, meat and so on. Wu et al. (2006) introduced *non-instantaneous deterioration items* as products which have no deterioration during a period of maintaining quality or the original condition (fresh product time). They assumed that a constant fraction of the on-hand inventory deteriorates after the fresh product time. They worked on minimizing inventory cost per unit time by considering zero lead time, stock-dependent demand and time-dependent partial backordering. Recently, Valliathal and Uthayakumar (2011) in order to maximize total profit per unit time discussed the optimal pricing and replenishment policies of an economic order quantity model for non-instantaneous deteriorating items. They assumed time- and price-dependent demand and time-dependent partial backorder over an infinite planning horizon.

2-2-4-2- Continuous deterioration function

Drawing on Wang et al. (2011), we can classify the continuous deterioration functions, $\theta(t)$, in four categories as follows:

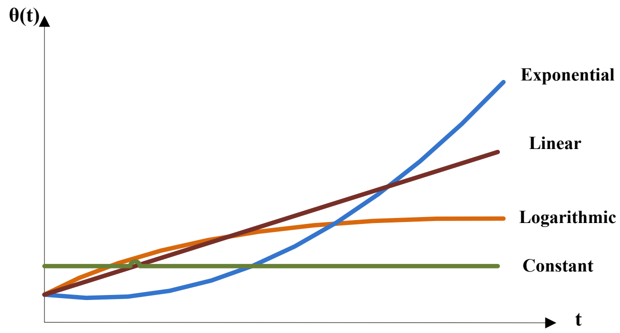


Figure 2-2 - Different types of the continuous deterioration functions

Constant function: many researchers have considered that the deterioration rate of products is constant and independent of time. This model is appropriate for products like oil, alcohol and some pharmaceutical products.

In 2008, Kang worked on minimizing the total cost of control wafers, including ordering cost, holding cost and purchase cost. He determined the optimal policy with the help of a dynamic programming approach and a mixed 0-1 linear programming (M0-1LP) model. The numerical results demonstrated that the proposed M0-1LP is an effective tool for determining the replenishment policy of control wafers for multiple periods.

In 2010, Chen and Chang, considered a two-echelon supply chain consisting of one manufacturer and multiple retailers. They solved the problem of jointly determining the replenishment cycle, the optimal retail price, and the number of shipments for products with a constant deterioration rate function. In order to discuss channel coordination, joint replenishment program, and pricing policy, two profit-maximization models including the non-integrated policy and the integrated policy were formulated and some structural insights were offered. Constant $\theta(t)$ is also considered in Pakkala and Achary (1991), Chung and Wee (2008), Huang and Yao (2006), Zhou and Lau (2000), Aggarwal and Jaggi (1995), Wee and Chung (2009), Kang and Kim (1983) and Tsai (2011).

Linear function: studies such as Lin and Lin (2006) assumed deterioration rate as $\theta(t)= \theta_1+ \theta_2.t$ where $0\leq\theta(t)<1$ and $0<\theta_1,\theta_2<1$. Linear deterioration form is suitable for modeling products like radioactive materials.

Logarithmic function: this form of deterioration function is appropriate for modeling products that deteriorate by a dramatically increasing rate in the initial phase, and then stabilize quickly. For instance, many IC chip products show that their deterioration rate increases before IC packaging and stabilizes after that.

Exponential function: by this type of function, the deterioration rate increases slowly in the initial phase, and then very rapidly, as in the case of some dairy products.

Weibull function: several researchers like Giri et al. (2003) and Wu (2002), have assumed the generalized continuous form of $\theta(t)$ as a Weibull distribution as $\theta(t)=\alpha\beta t^{\beta-1}$, $\alpha, \beta\geq 0$. In fact, all four mentioned continuous deterioration forms can be concluded from the Weibull function, depending on the value of β. In this way, $\beta =1$, $1<\beta <2$, $\beta =2$, and $\beta >2$ respectively results in constant, logarithmic, linear and exponential deterioration rates.

Covert and Philip (1973) were among the first researchers to make this assumption. They built an EOQ model with constant demand rate in order to minimize inventory system costs. Later on, other researchers applied Weibull deterioration rate under more complex assumptions. For instance, Wu et al. (1999) developed an inventory model by considering zero lead time, ramp-type demand, completely backordered demand, and Weibull deterioration rate. Deng (2005) improved Wu's model by omitting some unnecessary conditions to find the minimum solution, and also studied the more general form of their models.

2-3- Objective function

One of the distinguishing features of models in deteriorating inventory replenishment is how to define the objective function and what its constituent components are. The objective of most researches is minimizing total cost (Wang et al. 2011, He et al. 2010, Lin and Lin, 2007, Alfares, 2007, Ferguson et al., 2007, Yang and Wee, 2003, Yang and Wee, 2002, Balkhi, 1999, Girl and Chaudhuri, 1998, Weiss, 1982, Muhlemann and Spanopoulos, 1980) including usual cost elements like:

- ✓ The set up cost (or the ordering cost),
- ✓ The inventory holding cost (including the deterioration cost in the second and third class of modeling approaches),

- ✓ The shortage cost, and
- ✓ The procurement cost (including the deterioration cost in the first and third class of modeling approaches).

In this manner, some researchers have considered an infinite planning horizon and dealt with optimization of total cost/total profit per unit time while some others optimized sum of costs during a finite planning horizon.

In addition to usual cost components, other type of cost components like advertisement or transportation costs have been occasionally embedded to the objective functions which will be explained more in Section 2-3-2. The objective function's components of some main related studies are presented in Table 2-2. Some researchers such as Valliathal and Uthayakumar (2011) and Sana (2011) and Dye (2007) examined profit maximization of a deteriorating inventory system by considering selling price as a decision variable.

A number of researchers developed deteriorating inventory models to minimize the present value of total cost (or to maximize the present value of total profit) by taking into consideration time discounting concept. In 2006, Lin and Lin studied a single product deteriorating inventory system under a finite planning horizon and dealt with the minimization of the total cost's present value.

Lo et al. (2007) considered a two-echelon supply chain with a constant deterioration rate and a Weibull demand function. They dealt with minimization of the present value of total cost per unit time by taking into account a constant inflation rate. The cost components which were incorporated in the objective function were the set-up, the holding, the procurement and the deterioration costs of raw material and finished items as well as reworking cost of defective finished products at supplier's site. Since they worked on an integrated production-inventory model, the set-up, the holding, the procurement, the shortage costs and the deterioration at retailer's site were also considered.

A number of researchers coped with other criteria to optimize a deteriorating inventory system. In 1998, Andijani and A-Dajani, assumed a desired inventory and production level. Then, they minimized the system's total cost including the production and the inventory holding costs by minimizing deviations from the desired levels. In this way, the objective function was presented as follows:

$$Min\ TC = \int_0^T [h[I(t) - \acute{I}]^2 + c[p(t) - \acute{p}]^2]\mathrm{dt} \qquad (2\text{-}3)$$

where *I(t)* and $\acute{I}$ are the inventory level at time *t (unit)* and the desired inventory level (unit) respectively. Similarly, *p(t)* and $\acute{p}$ are the production rate (units/unit time) and the desired production rate (unit/unit time) respectively; *h* indicates the

inventory holding cost rate (unit cost/unit/unit time); c and T are the unit cost and the production cycle respectively.
The desired levels ($\acute{I}$ and $\acute{p}$) were assumed to be pre-determined parameters so that $\acute{I}$ could be determined by taking into consideration the available storage space and $\acute{p}$ could be determined regarding the production capability of the organization.
In the following, more detailed descriptions are provided about shortage cost as one of the main components of objective functions.

2-3-1- Shortage cost

For simplicity in modeling and solving procedures, many deteriorating inventory models existing in the literature, especially the basic ones, have been developed without considering the shortage cost (Chen and Chang, 2010, Sana, 2011, He et al., 2010, Wang, 2011, Liao, 2007, Liao, 2008, Rau et al., 2003, Weiss, 1982, Muhlemann and Valtis-Spanopoulos, 1980, Goh, 1994, Girl and Chaudhuri, 1998, and Ferguson et. al., 2007).
Some researchers have dealt with shortages as completely backordered demand. Lin and Lin (2006) by assuming a two-echelon supply chain consisting of one supplier and one retailer, allowed shortage just for retailer as completely backordered. Luo (1998) presented an integrated deteriorating inventory model in which shortage is allowed as completely backordered. He considered the influence of marketing strategies such as pricing and advertising on the profitability of the system and determined the optimal production quantity and backorder level which maximized the net profit of the system.
As mentioned before, the generalization form of completely backordered situation is partial backordered state. All studies done under partial backorder assumption can be categorized into two groups: time-independent and time-dependent models as shown in Figure 2-3. In time-independent partial backorder states, the number of unfulfilled backordered (or lost) customers is not related to the waiting time to meet the customer's requirements. Lo et al. (2007), Law and Wee (2006) and Goyal and Giri (2003) took into account that a constant percentage of demand was backordered over the length of a stock-out interval. Padmanabhan and Vrat (1995), by considering a constant deterioration rate and an inventory dependent demand function, dealt with profit maximization of a buyer under three scenarios *i*) without shortage, *ii*) completely backorder and *iii*)

partial backorder in which the rate of backordering was linearly dependent on the number of customers who were waiting (not waiting time).

In time-dependent partial backorder models, the number of backordered demands during a stock-out cycle is inversely related to the waiting time to fulfill customer's demand. In other words, more customers are willing to wait if the waiting time is short. So, it is assumed that the fraction of demand backordered during a stock-out (β) is a decreasing function of the time remaining until replenishment (τ).

Based on Pentico and Drake (2011), time-dependent partial backorder models can be classified into six main classes that are summarized in Table 2-1.

Table 2-1- Partial backorder forms

Form of $\beta(\tau)$	Equation	Range for τ
Linear 1	$\beta(\tau) = \beta_M - (\beta_M - \beta_0)(\frac{\tau}{T_1})$	$\mathbf{0 \le \tau \le T_1}$
Linear 2	$\beta(\tau) = \beta_M - \left(\frac{\beta_M}{\tau_M}\right)(\tau)$	$\mathbf{0 \le \tau \le \tau_M}$
Rational	$\beta(\tau) = \left(\frac{\beta_M}{1+a\tau}\right) \quad a > 0$	$\mathbf{0 \le \tau}$
Step	$\beta(\tau) = 1$	$\mathbf{0 \le \tau \le \tau_M}$
Exponential	$\beta(\tau) = \beta_M e^{-a\tau} \quad a > 0$	$\mathbf{0 \le \tau}$
Mixed Exponential[1]	$\beta(\tau_1, \tau_2) = \beta_1 e^{-a_1\tau_1} + \beta_2 e^{-a_2\tau_2} \quad \beta_1, \beta_2 \ge 0 \quad \beta_1 + \beta_2 \le 1 \quad a_1, a_2 > 0$	$\mathbf{0 \le \tau_1, \tau_2}$

The notations of Table 2-1 are as follows:

β_0: Initial value of $\beta(\tau)$

β_M: Maximum value of $\beta(\tau)$ over a stock-out interval

T_1: Length of time during an inventory cycle for which there is stock-out

τ_M: Maximum customer's waiting time

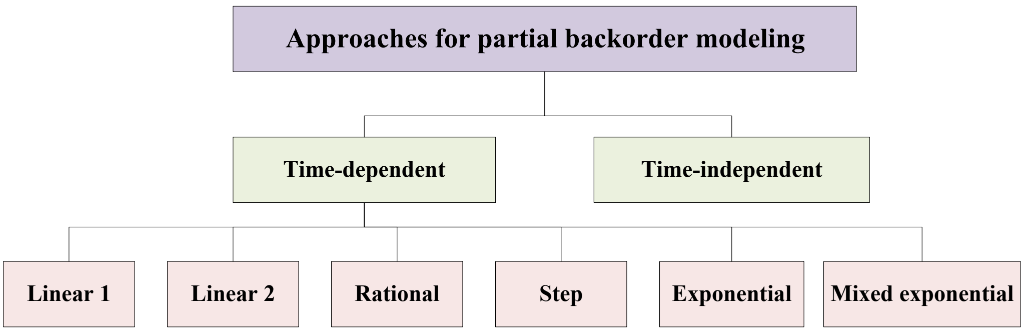


Figure 2-3 - Partial backorder forms in the literature

[1] It is assumed that there are two kinds of customers.

2-3-2- Other types of costs

As mentioned, besides usual inventory costs occasionally other types of costs are considered by researchers in deteriorating inventory models such as the inspection and rework cost (Chung and Wee, 2008), the penalty for delay in order delivery (Hsu et al., 2007), the transportation cost (Yu, 2007), the product's receiving and delivery costs (Rau et al., 2003), the pricing setting cost (Sana, 2011) and the advertisement cost (Goyal and Gunasekaran, 1995, and Luo, 1998).

Chung and Wee (2008), coped with a single-buyer single-supplier supply chain that the possibility of producing imperfect products was not negligible. So, the inspection costs as well as cost of reworking defective items were incurred by the supplier. Moreover, transportation cost, inspection cost and salvage cost of defective items were considered for the buyer as well as traditional types of costs. Note that since the two stakeholders determined the delivery quantities during the contractual period, some opportunity and flexibility to change their policy were lost because of the coordination. Thus, both the supplier and the buyer incurred the cost of less flexibility per delivery too.

Luo (1998) and Goyal and Gunasekaran (1995) studied marketing activities effects by taking into account a price-dependent and advertisement-dependent demand function as well as a constant advertising cost in the objective function.

In studies like Rau et al. (2003) and Yu (2007) a constant transportation cost was integrated into the objective function for each shipment between producer and retailer.

Urban (2008) developed the Alfares (2007) model by relaxing the assumption that inventory level was equal to zero at the end of each replenishment cycle. In this way, the inventory level can be positive at the end of each replenishment cycle and extra remaining products were sold by a selling price less than routine selling price.

Hsu et al. (2007) developed an inventory model for deteriorating products with expiration date in a two-echelon supply chain under uncertain supply lead time. They defined a managing cost for reducing lead time length (a higher managing cost resulted in shorter supply lead time) and on the contrary a penalty cost for long lead time length for the supplier in order to send retailer's order as in-time as possible.

In 2007, Yu assumed a two-echelon supply chain consisted in one retailer and one supplier under a finite planning horizon which composed of a number of equal time periods. In their research, it was assumed that the capacity of the retailer's own warehouse was limited. So, if the retailer received orders more than the own warehouse capacity, stocked extra inventory into the rented warehouse. In this way, the holding cost in rental warehouse was considered time-dependent that was added to the system's total cost.

2-4- Demand function

Another distinctive characteristic of developed deteriorating inventory models is demand function. Based on the nature of demand, researches can be classified in two groups: *i*) studies with a constant demand function and *ii*) studies developed by assuming an inconstant (variable) demand function (Figure 2-4). Let's examine all of them.

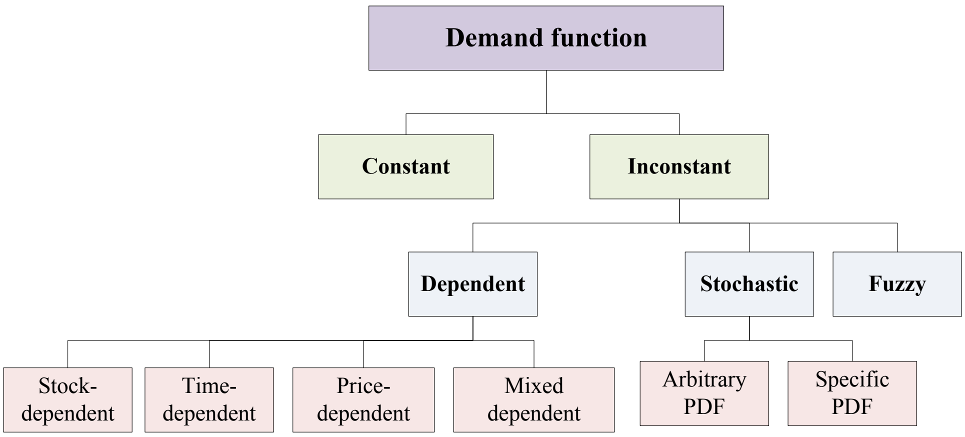


Figure 2-4 - Different modes of demand function

2-4-1- Constant demand function

The demand rate has been typically assumed as a constant parameter in terms of time, selling price and inventory level etc. Studies such as Wang (2011), Liao (2007), Liao (2008), Rau et al. (2003), Lin and Lin (2006), Huang and Yao (2006), Lo et al. (2007), Muhlemann and Valtis-Spanopoulos (1980), and Ferguson et. al. (2007) can be categorized in this group.

2-4-2- Inconstant demand function

Inconstant demand functions can be categorized in three classes: *i*) stochastic demand function, *ii*) dependent demand function and *iii*) fuzzy demand function.

2-4-2-1- Stochastic demand function

From a real life point of view, a stochastic demand distribution is more reasonable, although less than 20% of the developed models in the literature (after 2001) can be classified as such (Bakker, 2012). However, before 2001 researchers mostly concentrated on developing basic models under certain conditions. Based on Goyal and Giri (2001), stochastic demand functions in the existing literature can be seen in two ways:

1) Taking into consideration a specific type of probability distribution function (PDF) such as Ravichandram (1995) and Weiss (1982) who developed inventory models for deteriorating products assuming Poisson demand function.

2) Considering an arbitrary probability distribution function (PDF) for end customer's demand such as Aggoun et al. (1997) and Lian et al. (2009). According to Bakker (2012) since 2001, only about 4% of developed researches on deteriorating inventories provide models with an arbitrary probability distribution for demand.

2-4-2-2- Dependent demand function

In most researches which have assumed demand as a dependent function, the demand function is seen as

- ✓ Stock-dependent (Chung and Wee, 2008, Urban, 2008, Chang et al. 2006),
- ✓ Time-dependent (He et al., 2010, Lin and Lin, 2006, Sana et al. 2004). It is worth noting that in most of the articles that consider time-dependent demand function, the inventory models are developed under a finite planning horizon.
- ✓ Price-dependent (Sana, 2011, Dye, 2007, Papachristo and Skouri, 2003)

In some studies a combination of above states has been considered such as Valliathal and Uthayakumar, (2011), Chen and Chang (2010) () and Hsu et al. (2007) which assumed demand as a time- and price-dependent function.

More examples of these studies are summarized in Table 2-2.

2-4-2-3- Fuzzy demand function

A limited number of studies like Mahata and Goswami (2007) consider demand as a fuzzy parameter. Mahata and Goswami (2007), dealt with the problem of establishing the economic order quantity (EOQ) for deteriorating items in the fuzzy sense where delay in payments for retailer and customer was permissible. The demand rate, holding cost, purchasing cost and ordering cost were taken as fuzzy numbers. The total variable cost in fuzzy sense was defuzzified using the graded mean integration representation method. Afterward, they showed that the defuzzified total variable cost was convex and so a unique solution existed.

In Table 2-2 a summary of some main related research works are presented. In this way, for each study the used approaches for modeling deterioration process (according to Figure 2-1) as well as the structure of supply chain (number of echelons, number of suppliers/retailers) are presented. Moreover, type of demand, shortage, lead time and deterioration functions are reported. More details about some main assumptions such as, inventory system, solution approach, planning horizon and objective function components are presented too.

Table 2-2-Summary of selected articles on deterioration inventory models

			Num.										
Year	**Author**	**Class**	**Echelon**	**Retailer**	**Supplier**	**Solution approach**	**Demand function** CST: Constant	**Deterioration function** CST: Constant	**Lead time** CST: Constant St: Stochastic	**Inventory System**	**Shortage** **PB:** Partial backorder **CB:** Completely backorder **CL:** Completely lost sale	**Planning horizon**	**Objective Function** *R*:revenue *H*:holding cost *SH*: Shortage cost *P*:Procurement cost *S*: Setup cost D:Deterioration cost
2011	Valliath al & Uthaya kumar	1	1	1	1	Exact	$\{D(p,t)=g(p).f(t)\}$[2]	Non-instantaneous	0	EOQ[1]	*PB*: A general non-increasing time-dependent function	Infinite	*Max{R-(S+H+SH+P+the opportunity cost due to lost sales)}*
2011	Wang et al.	1	3	1	1	Heuristic	CST.	Time-dependent $\theta_i(t)=\alpha_i+\beta_i(t^{wi-1})$	0	EOQ	⊗	Infinite	$Min\{\sum_{R,D,P} S+H+D\}$[3]
2011	Wee et al.	1	2	1	1	Exact	CST.	CST.	0 (VMI)	EOQ	⊗	Single period	*Max{R-(S+D+H+ Remanufacturing cost+ Scrap processing cost)$_{supplier\ \&\ retailer}$}*

[1] EOQ refers to the Economic Order Quantity model with constant quantity order (*Q*) and also constant replenishment cycle.
[2] *g(p)* is a non-negative, continuous, convex decreasing function of the selling price(*p*) and *f(t)* is a non-negative, continuous function of time.
[3] *R*: retailer, *D*: distributor, *P*: producer.

Table 2-2-Summary of selected articles on deterioration inventory models

			Num.										
Year	**Author**	**Class**	**Echelon**	**Retailer**	**Supplier**	**Solution approach**	**Demand function** CST: Constant	**Deterioration function** CST: Constant	**Lead time** CST: Constant St: Stochastic	**Inventory System**	**Shortage** **PB:** Partial backorder **CB:** Completely backorder **CL:** Completely lost sale	**Planning horizon**	**Objective Function** ***R*:**revenue ***H*:**holding cost ***SH*:** Shortage cost ***P*:**Procurement cost ***S*:** Setup cost **D**:Deterioration cost
2011	Sana	1	1	1	0	Exact	$\{D_j(p_j)=\alpha-\beta p_j-\gamma p_j^{\wedge 2},\ \alpha, \beta, \gamma>0\}$[1] $\{D_j(p_j)=\alpha p_j^{-\varepsilon},\ \alpha, \varepsilon>0\}$[2]	CST.	0	EOQ	⊗	Finite	*Max{R-(H+P+ Pricing Setting Cost)}*
2010	Chen & Chang	1	2	n	1	Exact	$D(b_i p_j,t)=(a_i-b_i p_{j,i})exp(-\beta_i t)$ $,j=(I,NI)$[3] *i: retailer*	CST.	0	EOQ	⊗	Infinite	$Max\{\sum_n R_n-(H_n+S_n\ (Minor\ and\ Major\ set\ up)\ P_n)+(R-(H+S\ (Minor\ \&\ Major)))_{supplier}\}$
2010	He et al.	1	1	0	1	Exact	A piecewise function with time	CST.	0	EOQ	⊗	Finite	$Min\{(H+D+S)_{Finished\ products}+\{(H+D+S)_{raw\ material}\}$

[1] Quadratic price-dependent
[2] A negative power function of price
[3] (I,NI)→(Integrated, Non-Integrated)

Table 2-2-Summary of selected articles on deterioration inventory models

Year	**Author**	**Class**	**Num. Echelon**	**Num. Retailer**	**Num. Supplier**	**Solution approach**	**Demand function** CST: Constant	**Deterioration function** CST: Constant	**Lead time** CST: Constant St: Stochastic	**Inventory System**	**Shortage** **PB:** Partial backorder **CB:** Completely backorder **CL:** Completely lost sale	**Planning horizon**	**Objective Function** *R*:revenue *H*:holding cost *SH*: Shortage cost *P*:Procurement cost *S*: Setup cost D:Deterioration cost
2009	Wee & Chung	1	2	1	1	Exact	CST.	CST.	CST.	Periodic	⊗	Infinite	*Min{(H+D+S)$_{retailer}$+(H+ D+ S+ Flexibility+ Inspection+ component design-life cost)$_{supplier's\ cost\ of\ forward\ manufacturing}$ +(H+remanufacturing+repair +solvage)$_{supplier's\ cost\ of\ reverse\ manufacturing}$ }*
2008	Liao	1	1	1	0	Exact	CST.	CST.	0	Continuous	⊗	Infinite	*Min{S+H+D+ the annual capital opportunity cost}*
2008	Urban	2	1	1	_	Heuristic	Inventory-dependent $D(I)=D\times I^{\beta}$ $D>0$, $0<\beta<1$, $I>0$	Non-linearly time-dependent holding cost[1]	0	EOQ	⊗	Infinite	*Max{R$_{fresh\ and\ older\ product}$)-(H+P)}*

[1] Two cases were analyzed in this study: 1-Retroactive holding cost, 2-Incremental holding cost.

Table 2-2-Summary of selected articles on deterioration inventory models

			Num.										
Year	**Author**	**Class**	**Echelon**	**Retailer**	**Supplier**	**Solution approach**	**Demand function** CST: Constant	**Deterioration function** CST: Constant	**Lead time** CST: Constant St: Stochastic	**Inventory System**	**Shortage** PB: Partial backorder CB: Completely backorder CL: Completely lost sale	**Planning horizon**	**Objective Function** *R*:revenue *H*:holding cost *SH*: Shortage cost *P*:Procurement cost *S*: Setup cost D:Deterioration cost
2008	Chung & Wee	1	2	1	1	Exact	Linearly inventory-dependent	CST.	CST.	EOQ	⊗	Infinite	$Min\{(S+H+D+\text{Inspection cost}+\text{Rework cost}+\text{Cost of less flexibility implementing JIT})_{supplier} + (S+H+P+\text{Transportation cost}+\text{The cost of less flexibility}+\text{Inspection cost}+\text{solvage cost})_{retailer}\}$
2007	Dye	1	1	1	_	Heuristic	*D(p)*[1]	Time-dependent $\theta(t)$ $0<\theta(t)<<1$	0	Continuous	Rational *P.B*	Infinite	$Max\{R-(P+S+H+SH)\}$
2007	Ferguson et al.	2	1	1	_	Regression[2]	CST.	$h.t^r$, $r\geq 1$	0	EOQ	⊗	Infinite	$Min\{H+S\}$
2007	Liao	1	1	1	_	Exact	CST.	CST.	0	Continuous	⊗	Infinite	$Min\{S+H+D+\text{Interest payable}-\text{Interest earned}\}$

[1] *A* non-negative, continuous, convex, decreasing function of the selling price.
[2] An approximation of *h* and *r* parameters in Weiss (1982) model for products with spoilage or markdown policy through regression was done.

Table 2-2-Summary of selected articles on deterioration inventory models

			Num.										
Year	**Author**	**Class**	**Echelon**	**Retailer**	**Supplier**	**Solution approach**	**Demand function** CST: Constant	**Deterioration function** CST: Constant	**Lead time** CST: Constant St: Stochastic	**Inventory System**	**Shortage** PB: Partial backorder CB: Completely backorder CL: Completely lost sale	**Planning horizon**	**Objective Function** *R*:revenue *H*:holding cost *SH*: Shortage cost *P*:Procurement cost *S*: Setup cost D:Deterioration cost
2007	Alfares	2	1	1	_	Exact	Inventory-dependent $D(I)=D\times I^{\beta}$ D>0, 0<β<1, I>0	Non-linearly time-dependent holding cost[1]	0	EOQ	⊗	Infinite	$Min\{H+S\}$
2007	Hsu et al.	1	2	1	1	Exact[2]	Season and price dependent $D_j(t,p)=\alpha\times w(j)/p^{\beta}$ $w(j)=N-j+1/N$	Constant with expiration date	St.	EOQ	Linear *P.B* (Type 2): $B(\tau)=1-(\tau/T)$	Infinite	$Max\{R-(H+D+Managing\ cost+Delay\ cost)_{supplier}\ (P+SH+H+S+Processing\ cost)_{retailer}\}$

[1] Two cases were analyzed in this study: 1-Retroactive holding cost, 2-Incremental holding cost.

[2] They used exact methods when supplier's capital constraint was not embedded. The solution approach is not mentioned when this constraint was considered.

Table 2-2-Summary of selected articles on deterioration inventory models

			Num.										
Year	**Author**	**Class**	**Echelon**	**Retailer**	**Supplier**	**Solution approach**	**Demand function** CST: Constant	**Deterioration function** CST: Constant	**Lead time** CST: Constant St: Stochastic	**Inventory System**	**Shortage** **PB:** Partial backorder **CB:** Completely backorder **CL:** Completely lost sale	**Planning horizon**	**Objective Function** ***R:***revenue ***H:***holding cost ***SH:*** Shortage cost ***P:***Procurement cost ***S:*** Setup cost D:Deterioration cost
2007	Yu	3	2	1	1	Simulated Annealing (SA)	CST.	CST.	0	Continuous	⊗	Finite	$Min\{(S+H+D+Tranportation)_{supplier}+(S+H(OW^{1}+RW^{2}))+D+Administration\ cost)_{retailer}\}$
2007	Lin& Lin	1	2	1	1	Exact	CST.	CST.	CST.	Continuous	CB. (Only for retailer)	Finite	$Min\{(S+H+D)_{supplier\ \&\ retailer}\}$
2007	Lo et al.	1	2	1	1	Heuristic	CST.	Weibull $\theta\,(t)=\alpha\times\beta(t)_{(\beta+1)}$ $\alpha,\ \beta>0$	0	EOQ	PB. for retailer (A constant fraction of no fulfilled demands)	Infinite	$Min\{Present\ value: (S+H+D+P)_{supplier\ (raw\ material)}+(S+H+D+P+rework)_{supplier\ (finished\ goods)}+(S+H+SH+P+D)_{retailer}\}$

[1] Own Warehouse
[2] Rented Warehouse

Table 2-2-Summary of selected articles on deterioration inventory models

Year	Author	Class	Num. Echelon	Num. Retailer	Num. Supplier	Solution approach	Demand function CST: Constant	Deterioration function CST: Constant	Lead time CST: Constant St: Stochastic	Inventory System	Shortage PB: Partial backorder CB: Completely backorder CL: Completely lost sale	Planning horizon	Objective Function *R*:revenue *H*:holding cost *SH*: Shortage cost *P*:Procurement cost *S*: Setup cost D:Deterioration cost
2007	Mahata & Goswam	3	1	1	_	Exact	A fuzzy number	A constant θ+A fuzzy holding cost	0	Periodic	$\otimes$	Infinite	*Min{S+H+D+ interest payable-interest earned}*
2007	Teng et al.	3	1	1	_	Exact	A decreasing twice differentiable function of the selling price, *D(p)*	A constant θ+ *h(t)* (A general time-dependent holding cost)	0	Periodic	PB. a decreasing and differentiable time-dependent function	Infinite	*Max{R-(S+H+SH+Production cost)}*
2006	Yang & Wee	1	2	1	1	Heuristic	Linearly price-dependent *D(p)=a-b.p*	CST.	0	EOQ	$\otimes$	Infinite	*Max{R-(H+D+S+P)$_{retailer}$+(H+D+S+P)$_{supplier}$}*

Table 2-2-Summary of selected articles on deterioration inventory models

Year	**Author**	**Class**	**Num.** **Echelon**	**Retailer**	**Supplier**	**Solution approach**	**Demand function** CST: Constant	**Deterioration function** CST: Constant	**Lead time** CST: Constant St: Stochastic	**Inventory System**	**Shortage** PB: Partial backorder CB: Completely backorder CL: Completely lost sale	**Planning horizon**	**Objective Function** *R*:revenue *H*:holding cost *SH*: Shortage cost *P*:Procurement cost *S*: Setup cost D:Deterioration cost
2006	Law & Wee	1	2	1	1	Heuristic	CST.	Weibull $\theta\ (t)=\alpha\times\beta(t)_{(\beta+1)}$ $\alpha,\ \beta>0$	0	EOQ	PB. for retailer (A constant fraction of no fulfilled demands)	infinite	*Min{Present value: (S+H+ +P+ D & Ameliorating cost*[1]*)* $_{supplier\ (raw\ material)}+(S+H+D+P)_{supplier\ (finished\ goods)}+(S+H+SH+D+P)_{retailer}\}$
2006	Huang & Yao	1	2	n	1	Heuristic	CST.	CST.	0	EOQ	⊗	Infinite	$Min\{(S+H+D)_{supplier\ (raw\ material)}+(S+H+D)_{supplier\ (finished\ goods)}+\{(S+H+D)_{retailer}\}$
2006	Chang et al.	1	1	1	_	Exact	Inventory-dependent $D(t)=\alpha+\beta\times I(t)$	CST.	0	EOQ	1-Rational PB. 2-No shortage	Infinite	$Max\{R-(H+S+SH+P)\}$
2006	Lin & Lin	1	1	1	_	Heuristic	A known continuous function of time, $D(t)$	$\theta(t)=\theta_1+\theta_2 t$ $0\leq\theta(t)<1$ $0<\theta_1,\theta_2<1$	0	Periodic	Exponential PB.	Finite	*Min{Present value: (P+H+SH+D+S)}*

[1] According to this reference, 'Amelioration' occurs when the value or utility of a product increases over time like young or fast growing animals such as fish, chickens, ducks. The authors studied products with deterioration and amelioration.

Table 2-2-Summary of selected articles on deterioration inventory models

Year	Author	Class	Num. Echelon	Num. Retailer	Num. Supplier	Solution approach	Demand function CST: Constant	Deterioration function CST: Constant	Lead time CST: Constant St: Stochastic	Inventory System	Shortage PB: Partial backorder CB: Completely backorder CL: Completely lost sale	Planning horizon	Objective Function *R*:revenue *H*:holding cost *SH*: Shortage cost *P*:Procurement cost *S*: Setup cost D:Deterioration cost
2005	Dye & Ouyang	1	1	1	_	Exact	Linearly inventory-dependent $\{D(t)=\alpha+\beta\times I(t)\}$	CST.	0	EOQ	Rational PB.	Infinite	$Max\{R-(H+S+SH+P)\}$
2004	Sana et al.	1	1	1	_	Box complex algorithm (numerical)	Linearly time-dependent $\{D(t)=a+b\times t\}$ $a\geq0$, $b\neq0$	CST.	0	EOQ	CB.	Finite	*Min{S+D (Production cost of deteriorated items)+H+SH}*
2003	Rau et al.	1	3	1	1	Heuristic	CST.	CST.	0	EOQ	⊗	Infinite	$Min\{(S+H+D+\text{Delivery cost})_{supplier}+(\text{receiving cost}+H+D)_{producer\ (raw\ material)}+(S+\text{delivery cost}+H+D)_{producer\ (Finished\ cost)}+(S+\text{Recieveing cost}+H+D)_{retailer}\}$
2003	Yang & Wee	1	2	n	1	Numerical	Constant (for each retailer, D_i)	CST.	0	EOQ	⊗	Infinite	$Min\{(H+D+S)_{producer(raw\ material)}+(H+D+S)_{producer(finished\ product)}+(H+S+D)_{retailer}\}$

Table 2-2-Summary of selected articles on deterioration inventory models

Year	Author	Class	Num. Echelon	Num. Retailer	Num. Supplier	Solution approach	Demand function CST: Constant	Deterioration function CST: Constant	Lead time CST: Constant St: Stochastic	Inventory System	Shortage PB: Partial backorder CB: Completely backorder CL: Completely lost sale	Planning horizon	Objective Function *R*:revenue *H*:holding cost *SH*: Shortage cost *P*:Procurement cost *S*: Setup cost D:Deterioration cost
2003	Papachristo & Skouri	1	1	1	_	Exact	A convex decreasing function of the selling price	Weibull $\theta(t)=\alpha\times\beta(t)^{(\beta+1)}$ $\alpha, \beta>0$	0	EOQ	1-Rational PB. 2-CB.	Infinite	*Max{R-(P+S+H+SH)}*
2003	Abad	3	1	1	_	Exact	A decreasing twice differentiable function of the selling price, *D(p)*	A constant θ+ *h(t)* (A general time-dependent holding cost)	0	Periodic	PB, a decreasing and differentiable time-dependent function	Infinite	*Max{R-(S+H+SH+P(Production cost))}*
2003	Goyal & Giri	1	1	1	_	Exact and Heuristic[1]	A known functions of time, *D(t)*	A known functions of time, $\theta(t)$	0	Periodic	PB. A constant fraction of no fulfilled demands	Infinite/ finite	*Min{P+S+SH+H}*
2002	Yang & Wee	1	2	n	1	1-Exact 2-Heuristic for $n>2$	CST.	CST.	0	EOQ	⊗	Infinite	$Min\{(H+S+D)_{supplier(raw\ material\ and\ Finished\ products)}+(H+S+D)_{retailer}\}$

[1] Using exact methods in the case of infinite planning horizon and a heuristic method for finite planning horizon.

Table 2-2-Summary of selected articles on deterioration inventory models

			Num.										
Year	**Author**	**Class**	**Echelon**	**Retailer**	**Supplier**	**Solution approach**	**Demand function** CST: Constant	**Deterioration function** CST: Constant	**Lead time** CST: Constant St: Stochastic	**Inventory System**	**Shortage** PB: Partial backorder CB: Completely backorder CL: Completely lost sale	**Planning horizon**	**Objective Function** *R*:revenue *H*:holding cost *SH*: Shortage cost *P*:Procurement cost *S*: Setup cost D:Deterioration cost
1999	Balkhi	1	1	1	_	Exact	A known function of time 1-CST. 2-$D(t)=exp(\beta\times t)$	A known functions of time 1-Constant 2-a/(b-t) 3-$\gamma^{2\times t}$	CST.	EOQ	⊗	infinite	$Min\{(H+S+Item\ cost\ (P+D))_{Finished\ prodct}+\{(H+S+Item\ Cost\ (P+D))_{Raw\ Material}\}$
1998	Girl & Chaudhuri	3	1	1	_	Numerical	Inventory-dependent $D(I)=D\times I^{\beta}$ $D>0,\ 0<\beta<1,\ I>0$	A constant θ + 1- $h\times t^{r},\ r\geq 1$ 2- $h\times I^{r},\ r\geq 1$	0	EOQ	⊗	Infinite	$Min\{H+S+D\}$
1998	Luo	1	1	1	_	Numerical	$D=K(A^{ea}/P^{ep})$[1]	Weibull $\theta(t)=\alpha.\beta.t^{(\beta+1)}$ $\alpha,\ \beta>0$	0	Continuous	CL.	Infinite	$Max\{R-(S+H+SH+D+Advertisement\ cost)\}$

[1] K:Constant, A: frequency of advertisement, P: selling price per unit, ea: advertisement elasticity, e_p: price elasticity

Table 2-2-Summary of selected articles on deterioration inventory models

			Num.											
Year	**Author**	**Class**	**Echelon**	**Retailer**	**Supplier**	**Solution approach**	**Demand function** CST: Constant	**Deterioration function** CST: Constant	**Lead time** CST: Constant St: Stochastic	**Inventory System**	**Shortage** **PB:** Partial backorder **CB:** Completely backorder **CL:** Completely lost sale	**Planning horizon**	**Objective Function** ***R*:**revenue ***H*:**holding cost ***SH*:** Shortage cost ***P*:**Procurement cost ***S*:** Setup cost **D:**Deterioration cost
1998	Andijani & A-Dajani	1	1	1	_	The linear quadratic regulator (LQR) technique	*1*- CST. *2*- Linearly time-dependent *3*- Quadratic time-dependent	CST.	0	Periodic	⊗	Finite	*MIN{H+Production cost}*
1995	Padmanabhan	1	1	1	_	Exact	Inventory-dependent $D(t)=\alpha+\beta\times I(t)$	CST.	0	EOQ	1- Without shortage 2- CB. 3- PB (A constant fraction of no fulfilled demands)	Infinite	*Max{R-(H+P+SH+S)}*

Table 2-2-Summary of selected articles on deterioration inventory models

Year	Author	Class	Num. Echelon	Num. Retailer	Num. Supplier	Solution approach	Demand function CST: Constant	Deterioration function CST: Constant	Lead time CST: Constant St: Stochastic	Inventory System	Shortage PB: Partial backorder CB: Completely backorder CL: Completely lost sale	Planning horizon	Objective Function *R*: revenue *H*: holding cost *SH*: Shortage cost *P*: Procurement cost *S*: Setup cost D: Deterioration cost
1995	Goyal & Gunasekaran	1	1	1	-	Numerical (Hooke and Jeeves)	$D=K(A^{ea}/P^{ep})$[1]	CST.	0	EOQ	⊗	Infinite	$Max\{R-(S_{Raw\ material}+S_{Production}+H_{raw\ material}+H_{In\ process\ inventory}+H_{Finished\ product}+D_{raw\ material}+D_{In\ process\ inventory}+D_{Finished\ product}+Advertisement\ cost)\}$ *a multi-stage production-Inventory system
1994	Goh	2	1	1	_	Exact[2]	Inventory-dependent $D(I)=D\times I^{\beta}$ $D>0, 0<\beta<1,$ $0<I<Q$	1-$h\times t^{r}$, $r\geq 1$ 2- $h\times I^{r}$	0	EOQ	⊗	Infinite	$Min\{H+S\}$

[1] K: Constant, A: frequency of advertisement, P: selling price per unit, ea: advertisement elasticity, ep: price elasticity.
[2] He proposed a closed form formula for the optimal value of order quantity (Q) and replenishment cycle (T).

Table 2-2-Summary of selected articles on deterioration inventory models

Year	Author	Class	Num. Echelon	Num. Retailer	Num. Supplier	Solution approach	Demand function CST: Constant	Deterioration function CST: Constant	Lead time CST: Constant St: Stochastic	Inventory System	Shortage PB: Partial backorder CB: Completely backorder CL: Completely lost sale	Planning horizon	Objective Function R: revenue H: holding cost SH: Shortage cost P: Procurement cost S: Setup cost D: Deterioration cost
1982	Weiss	2	1	1	_	Exact [1]	1- CST. 2- Poisson	$h \times t^r$ $r \geq 1$	0	EOQ	⊗	Infinite	*Min{H+S}*
1980	Muhlemann & Spanopoulos	2	1	1	_	Exact	CST.	Non-linearly dependent to the average value of capital invested in stock	0	EOQ	⊗	Infinite	*Min{H+S}* *Multi-product

[1] He proposed a closed form formula for the optimal value of order quantity (Q).

2-5- Green logistics and supply chain management

The issue of supply chain sustainability has been of considerable interest for the last decade both in academia and in the practitioner's world. A key subject raised in the area of sustainable supply chains is the development of green logistics and supply chains. The definition and scope of Green Supply Chain Management (GSCM) in the literature range from green purchasing to integrated green supply chains, flowing from raw material supplier to manufacturer to customer, and reverse logistics. GSCM is defined as "*integrating environmental thinking into supply chain management, including product design, material sourcing and selection, manufacturing processes, delivery of the final product to the consumers as well as end-of-life management of the product after its useful life*" (Kumar et al., 2012). Thus, in GSCM the objectives concern not only the economic impact of logistics policies on the organizations, but also the wider effects on the planet, such as the effects on the environmental pollution, fuel consumption or waste.

Hassini et al. (2012) represented a sustainable supply chain as wheels consisting of six spokes corresponded to the major relevant functions within the chain:

- ✓ Sourcing: including using renewable resources and avoiding toxic substances etc.,
- ✓ Transformation: it includes processes like designing fair labor practices and defining sustainable practices and processes.
- ✓ Delivery: It encompasses processes such as transportation, inventory and facilities location and layout. For instance, the choice of location such as offshore vs. onshore, close to the raw material source vs. close to the customer can have a considerable influence on the GHG emissions.
- ✓ Value proposition: this is often not explicitly taken into account. Many businesses that sell environmentally friendly, sustainable or green and low carbon emission products tend to cost more. This cost is often passed on to the customers in the form of higher prices. Usually, as in the case of customers buying carbon offsets, these are tangential decisions regardless of product purchase decisions. In order to successfully market and sell sustainable or green products, many

businesses will should not only quantify the benefits but justify the value proposition to the customers.

- ✓ Customers and product use: it is related to customer education to use green energy and products however may drive up prices.
- ✓ Recycling: such as doing recycling or reusing products in an efficient way.

Accordingly, many sustainability or green functions and criteria relate directly or indirectly to the type of product that is offered by companies or supply chains. The importance of deteriorating products from the point of view of green criteria can be summarized as follows:

- ✓ Waste cost: the waste and recycling processes are very important for deteriorating products and many companies are trying to reduce these processing costs.
- ✓ Stocking and transporting: in most cases, specific equipment (such as refrigerated warehouses or trucks) should be used for storage and transportation of deteriorating products, in order to reduce the deterioration cost. So, there should be a reasonable balance between criteria like deterioration cost, energy consumption and GHGs generated.
- ✓ Risk: in the case of deteriorating products such as pharmaceutical radioactive substances, the risk for example of an accident can lead to the loss of the beneficial properties of products as well as uncompensated environmental effects. So, here decreasing the risk of accidents during transportation is essential.

In 2009, Wee and Chung developed an integrated replenishment policy for a closed loop supply chain consisting of a single buyer and single supplier with a single set-up and multiple Just In Time (JIT) delivery strategies for deteriorating products. In this way, they considered that after a period of use, the take-back product was received by the supplier, then collecting, cleaning and disassembly processes were going on. So, it was supposed that after a functional checking, a ratio of sorted product could be remanufactured. The aim of this study was to determine the optimal production lot-size and the number of the delivery interval for supporting the production and the dispatching plan when the green component value design and remanufacturing were considered. Wee et al. (2011), proposed a deteriorating inventory model by considering vendor managed inventory (VMI) strategy and conducted a life cycle cost and benefit analysis for green electronic products. The results showed that parameters such as deteriorating rate, product return rate, and remanufactured quality have a significant effect on the model. Some empirical studies have also been done to consider green criteria in deteriorating products industries such as Filho (2004) (the meat beef industry) and Matos and Hall (2007) (the oil and gas and the agricultural biotechnology industry).

A comprehensive literature review of studies on green supply chains is presented in Sbihi and W. Eglese (2010), Min and Kim (2012), Hassini et al. (2012) and Dekker et al. (2012).

2-5-1- Greenhouse Gases (GHG) emission

Concerns about depletion of the ozone layer and climate change have been increasingly motivating firms, supply chains and governments to deal with Greenhouse Gases (GHG) emissions seriously. Many companies and organizations are also switching over to green logistics and measuring their carbon footprints so that they can monitor the environmental impact of their activities. In this way, in recent decades, there has been a growing amount of research on analyzing greenhouse gas emissions (GHG) resulting from logistics activities. Some researchers have studied this issue empirically (Carlsson-Kanyama, 1998, Konyar, 2001, Gielen et al., 2002, Lehtila et al., 2005, Hirschberg, 2005, Weisser, 2007, Rizet et al., 2010, Bocken et al., 2011, Thanarak, 2012).

Besides, a number of studies have taken a mathematical approach to deal with GHG emissions in green logistics and supply chains. GHG emissions can mainly be seen in two forms in the mathematical models developed: *i*) costs of GHG gases; and *ii*) GHG emission levels.

Paksoy et al. (2010) have considered GHG emissions in a mathematical model in terms of their associated costs. They studied a closed-loop supply chain and developed a mathematical model in the form of a linear programming formulation to identify each product that had been transported and the mode of transport used, to make a trade-off between various costs, including emission costs and transportation of commodities within the chain. Zhao et al. (2012) have used the game theory to analyze the strategies selected by manufacturers to reduce life cycle environmental risk of materials and carbon emissions. In their model, the strategic choices of the manufacturers were influenced by government penalties or incentives.

Moreover, some researchers have considered GHG emission levels directly in mathematical models. Wang et al. (2011) studied a supply chain network design problem by applying a bi-objective optimization model to consider the environmental investment decision in the strategic supply network design phase. They considered total costs in the first objective function and total CO_2 emissions in all the supply chains in the second one, and determined facilities locations, material flows and environmental protection levels in each facility. Harris et al. (2011), by using a simulation method on a European case study from the automotive industry, considered strategic and operational level decisions simultaneously. They showed that the optimum design based on costs does not necessarily equate to an optimum solution for CO_2 emissions. Zhang et al. (2012) describe an empirical study of nearly one hundred questionnaires of various Chinese iron and steel companies. Then, by a regression analysis, they showed that although some CO_2 reduction practices can result in significant environmental performance, and their impacts on the improvement of economic performance are less clear.

One of the most important issues in green logistics is how to identify preferred solutions that balance environmental and financial concerns. Improving environmental quality comes at a cost, so the problem is to find the trade-off between environmental and economic concerns, and between the environmental impacts of an economic activity and its costs. The aim is to determine those solutions in which environmental damage can be decreased only if costs are increased. These solutions are called *eco-efficient*. The idea of exploring best alternatives is based on Pareto-optimality (Dekker et al., 2012). Huppes and Ishikawa (2005), with an emphasis on the necessity of eco-efficiency, presented a framework for quantifying eco-efficiency analysis at macro and micro level. Quariguasi Frota Neto et al. (2008) showed the advantages of using multi-objective programming to assess the trade-offs between the logistic network cost and its environmental impact. Then, based on a multi-objective programming approach, they introduced a technique to evaluate the efficiency of a logistics network. Quariguasi Frota Neto et al. (2009) proposed a two-phased heuristic to find eco-efficient solutions by a visual representation for multi-objective linear problems. Their study focused on: minimizing cost, cumulative energy demand, and waste in a reverse logistics network. They illustrated the approach developed by designing a complex recycling logistics network in Germany.

As regards the mathematical modeling methods, different approaches have been taken to consider green criteria in supply chains, such as goal

programming (Pati, et al., 2008), fuzzy programming (Wang and Hsu, 2010 and Pishvaee and Torabi, 2010) and robust optimization (Pishvaee et al., 2011). In this way some studies take a stochastic programming approach. El-Sayed et al. (2010) designed a multi-period multi-echelon forward-reverse logistics network under demand uncertainties. The objective was to maximize the total expected profit. A multi-stage stochastic program was applied to formulate the problem. Later, Ramezani et al. (2012) presented a stochastic multi-objective model for forward-reverse multi-product, multi-echelon supply chain design under an uncertain environment. They applied a two-stage stochastic programming approach to evaluate the systematic supply chain configuration maximizing the profit, customer service level, and quality.
Although deteriorating products have not been specifically addressed in the developed mathematical models for controlling GHG emitted by supply chains, some empirical studies have been done on greenhouse gases emission in deteriorating products industries, such as Gielen et al. (2002) (petrochemical industry) and Carlsson-Kanyama (1998) (food industry).

2-6- Research gaps

As described in the previous sections, replenishment of deteriorating products has been studied from various perspectives; however, there are several potential areas for further research. Some research gaps in this field are as follows:

- ✓ Considering uncertainty in parameters such as lead time, the fraction of demand backordered and the deterioration rate has not attracted enough attention in the existing literature.

  Most procurement systems require a replenishment lead time, which is frequently hard to determine precisely. Besides, in many studies where shortages are partially backordered, the rate of partial backorders has been modeled by a simple fraction. Developing models to consider uncertain fractions could therefore be a promising area for further research. Furthermore, since there exist items deteriorated during transportation, considering the uncertain deterioration rate and consequently the uncertain order quantity received by retailers is a potential extension for current models. In this regard, it is also interesting to use fuzzy approach.

✓ Researchers have also paid little attention to multi-objective approaches in inventory models for deteriorating items. Yet this is a way in which alternative objective functions such as total cost (total profit), customer satisfaction (maximizing service level/minimizing maximum amount of shortage) and environmental criteria could be considered.

✓ The GHG emission level is important today since the environmental impacts of processes such as producing and supplying have become a critical issue for humankind and therefore for businesses. Incorporating principles of green logistics and supply chain management especially for deteriorating products is important since sometimes destructive effects of deteriorated items or transportation system on nature is considerably dangerous. Depending on problems, the green principles can be considered in model's objective function or constraints.

✓ In existing deteriorating inventories replenishment models mostly single supplier-single retailer supply chains – called dyadic supply chains – have been studied. However serial, convergent or divergent supply chains or even network forms are equally important motivated by real world situations. For instance, in stochastic situations (especially stochastic lead times) in order to reduce risks of uncertainties, a retailer may prefer to split a replenishment order among several suppliers simultaneously instead of placing a single order. This policy is called 'order splitting' which have not been analyzed in the case of deteriorating products very well.
Moreover, in convergent supply chains (single supplier-multi retailer) (as in Chen and Chang, 2010, Huang and Yao, 2006, Yang and Wee, 2003, and Yang and Wee, 2002) considering the positive or negative dependency among retailer's demands is an appealing issue for further research.

✓ Almost in all studies which have been developed for deteriorating inventories replenishment under a finite planning horizon, the planning horizon is a pre-determined parameter. Determination of the optimal value of the planning horizon as a decision variable is an interesting research issue too.

✓ Shortage cost rate in many researches has been continuously constant. Whereas the number of unfulfilled customers has a profound relation with a company's reputation, and so the shortage cost is not necessarily linear. In order to persuade the decision-makers to reduce the shortages (backorder or lost sale) as much as possible, considering a non-linear shortage cost rate should also be noticed sufficiently.

✓ In general, multi-product inventory models have been less studied because of their complexity. The complexities of multi-products inventory models are reinforced for deteriorating products. So, multi-deteriorating products models have received less attention, while they are highly applicable (for example in supermarkets etc). So, developing and analyzing multi-deteriorating products models in some mentioned situations (for example for the cases of non linear holding cost, green supply chains, stochastic lead times, divergent/convergent supply chains) is strongly recommended for further research. In this way, simplifying the developed models by some approximation methods like Taylor Series Expansion can be useful.

✓ Substitutability is almost missed in inventory replenishment of deteriorating products while it is definitely interested by inventory managers. The modeling of substitutability in inventory control will make a contribution to existing literature. In other words, the fact that a replenishment decision is dependent on the available stock of substitute goods should not be overlooked any longer.

✓ In most studies, the optimization problem is done through minimizing total cost or maximizing total profit. Applying other approaches for optimization, such as goal programming, stochastic programming or risk measures have been less studied.

✓ Supply chains coordination and integration often leads to lower costs (higher profits) for supply chain totally; but occasionally total cost (profit) of some members increases (decreases). In order to motivate these members to be coordinated with other units in the supply chain, we can use or define some incentive policies such as permissible delay in payment for buyers, prepayment for sellers etc. Modeling and analysis of such incentives have also received little attention especially for the second and third class of deteriorating models and in the case of divergent/convergent supply chains.

✓ Studying replenishment policy of deteriorating products based on *(1,T)* ordering policy is also a new potential area for future research.

- ✓ Finally, using Lambert *W* function for analyzing deteriorating inventory models is a new idea that is applied by some researchers such as Disney et al. (2012). The Lambert function $W[z]$ is the function that satisfies $W[z]e^{W[z]}=z$ where e is the natural exponential and z is a complex number. By using this function, several problems in replenishment policy of deteriorating products might be solved and analyzed efficiently.

In the following chapters we are going to bridge some mentioned gaps such as deteriorating inventory control under stochastic lead times, order splitting policy for deteriorating products and deteriorating products replenishment in a green supply chain.

3

REPLENISHMENT POLICY FOR DETERIORATING PRODUCTS WITH NON-LINEAR HOLDING COST UNDER STOCHASTIC LEAD TIME

3 Replenishment policy for deteriorating products with non-linear holding cost under stochastic lead time

3-1- Introduction

In this chapter we are going to fill one of the mentioned gaps in the literature, "*replenishment policy for deteriorating products under stochastic lead time*". As mentioned, applying special production strategies (MTO/MTE), limited fleets of sophisticated transportation equipment, waiting time to reach acceptable production and transportation batch sizes and so on, all add uncertainties to the system. These uncertainties can be taken into account in lead time/delivery time between upstream and downstream sections of the supply chain.

This chapter is organized as follows. In the next section the problem description and motivations are presented. The modeling assumptions are then described in Sections 3-3. In Section 3-4, the general form of the proposed model is described. In Section 3-5 the proposed model with uniform supply lead time is developed. Numerical results together with related analyses are presented in Section 3-6. The chapter ends with concluding remarks in Section 3-7.

3-2- Motivation and problem description

Whereas considerable improvements have been achieved in communication, transportation and production systems, most procurement systems – except for JIT systems – necessitate a supply lead time which is often difficult to determine exactly. As mentioned, considering uncertainties in lead time (if exist) are more critical in the case of deteriorating products since neglecting it may result in some other extra costs such as deterioration, transportation or recycling costs as well as holding and shortage costs.

In this chapter we deal with the problem of Replenishment policy for Perishable products under Stochastic Lead time (**RPSL**) which is almost overlooked in the literature. For this purpose, a single-product, two-echelon supply chain for deteriorating products with a non-linear holding cost (Class 2) is considered. The objective is minimizing the retailer's total cost per unit of time by considering stochastic supply lead time. So, during lead time the retailer may be faced with shortages that are assumed to be completely backordered. Considering stochastic supply lead time together with non-linear holding cost makes the mathematical model highly complex for the general form of RPSL problem abbreviated GRPSL. So, the proposed GRPSL model is customized for a uniform distribution function –abbreviated as URPSL problem – that could be tractable to solve optimally by means of an exact approach.

Detailed explanations on modeling assumptions are presented in the next section.

3-3- Model assumptions and notations

We adjust the following notations and assumptions for the RPSL model as follows:

D:	Demand rate per unit time
$H(t)$:	The cumulative holding cost of one unit that has been stored for t units of time
π:	Backordering cost per unit of item per unit time
O_c:	Constant ordering cost per replenishment
OUT:	Ordering cost per unit time
L:	Supply lead time
$f_L(l)$:	The probability distribution function of supply lead time
a:	Lower bound of supply lead time
b:	Upper bound of supply lead time
i:	Indicator of states
j:	Indicator of areas
H_{ij}:	Expected holding cost per cycle in area j of state i
HUT_{ij}:	Expected holding cost per unit time in area j of state i
B:	Backordered cost per cycle
BUT:	Backordered cost per unit time
Q:	Order quantity (Decision variable)
r:	Reorder point (Decision variable)

Remark: a capital letter is used to show a random variable, and a small letter is used to point to the value of the random variable.

We also consider the following assumptions:

1. The inventory system is the continuous review, *(r,Q)*.
2. The demand rate per unit time, D, is constant.
3. The planning horizon is infinite.

4. The supplier's lead time is stochastic on interval $[a,b]$. So, a and b indicate the lower and upper bounds of supplier's lead time, respectively, and they are positive.
5. The order quantity, Q, is greater than the reorder point, r. The reorder point is greater than the minimum amount of demand during lead time, Da. So, $Da \leq r \leq Q$. This assumption is common in practice.
6. Shortages are allowed. So, the reorder point, r, is less than the maximum amount of demand during lead time, Db i.e. $r \leq Db$.
7. It is assumed that the replenishment cycles are renewable or regenerative. This means that the on-hand inventory level just after receiving the order exceeds the reorder level. In other words $Db \leq Q$. According to Ross (1996) a regenerative process is a stochastic process with time points at which (probabilistically) the process restarts itself. This assumption is considered in such studies as Weiss (1982).
8. In brief, according to assumptions 4, 5, 6 and 7 we have $0 \leq Da \leq r \leq Db \leq Q$.
9. Shortages are considered to be completely backordered.
10. Ordering cost, O_c, is constant.
11. The cumulative holding cost of one unit that has been stored for t units of time is assumed to be $H(t) = \tilde{h}t^{\gamma}$ where $\tilde{h}$ and $\gamma \geq 1$ are constant.

As mentioned before, this kind of holding cost function was first introduced by Weiss (1982). In 2007, Ferguson et.al surveyed two real case studies to analyze the applicability of Weiss's model. In the first case, milk was investigated as a deteriorating product to which managers frequently applied markdowns to stabilize its demand as the product's expiry date approached. In the second one, they studied blueberries as products with expiry dates requiring the removal of spoiled blueberries as the date approached. These authors concluded that Weiss's model is appropriate for obtaining an approximation to model these two main groups of deteriorating items. The values of the parameters $\tilde{h}$ and γ in Section '3-6-Numerical experiments and sensitivity analyses' are selected with the help of these two real case studies.

3-4- General form of RPSL model (GRPSL)

In this section, first the mathematical model for RPSL is described. Then, the exact method to find the global optimum replenishment policy (r^*,Q^*) is explained. Here, an arbitrary distribution function is considered for supply lead time as $f_L(l)$ for $a \leq L \leq b$.

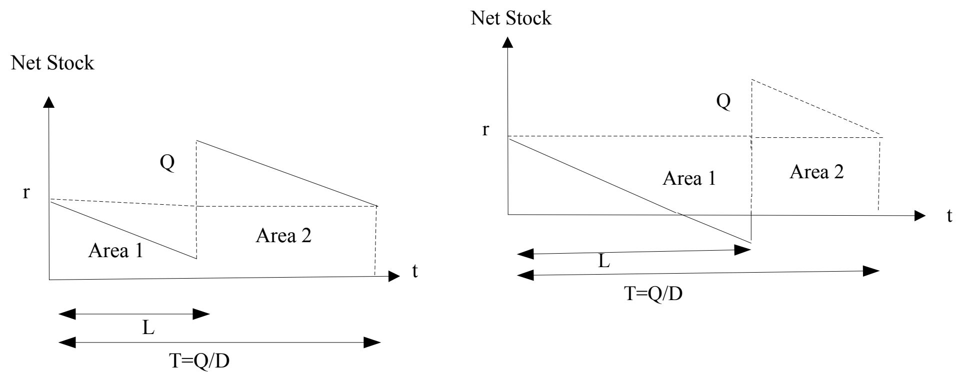
3-4-1- Mathematical model for GRPSL

We are going to formulate the retailer's expected total cost consisting of holding, shortage and ordering cost per unit time. For this purpose, its expected total cost per order (cycle) is formulated first, and then the expected total cost per unit time is derived by dividing it by the replenishment cycle length.

Given that the supplier's lead time is stochastic, and considering Assumption 8 – which implies that the reorder point, r, can cover the minimum amount of demand during lead time i.e. $Da \leq r$ but not its maximum amount i.e. $r \leq Db$ –, two separate states must be considered in order to model the retailer's inventory system. These two states are shown in Figure 3-1.

In State 1 shown in Figure 3-1(a), the reorder point, r, can cover the demand during lead time. Thus, $a \leq L \leq r/D$.

In State 2 the demand during lead time is greater than the reorder point, r, so backordered demand is occurred and $r/D < L \leq b$. This case is demonstrated in Figure 3-1(b).



a. State 1	**b. State 2**

Figure 3-1 - Retailer's net stock vs. time

In two mentioned states, the replenishment cycle – the time interval between placing orders – is equal to Q/D by considering fixed order quantity, Q, and constant demand rate, D. Because all replenishment cycles are equal over an infinite horizon, the first replenishment cycle $[0, Q/D]$ is shown in Figure 3-1 (a) and 3-1 (b). We divide the replenishment cycle by two ranges in order to simplify the formulation:

- ✓ Lead time: From the start of the replenishment cycle time up to reception of the order, $0 \leq t < l$, called Area 1 in Figure 3-1(a) and Figure 3-1(b).

- ✓ After arrival time: From receiving the order at time *l*, up to the end of replenishment cycle time, $l \leq t < Q/D$, called Area 2 in Figure 3-1(a) and Figure 3-1(b).

Now let's start to model the cost components of the retailer by formulating the holding cost per order.
According to Figure 3-1 and by considering the expectation concept, it can be concluded that the expected holding cost per order in each area of State 1 is as in Equation (3-1) and (3-2):

$$H_{11} = \int_a^{\frac{r}{D}} \int_0^l (r - Dt)\gamma \tilde{h} t^{\gamma-1} f_L(l) dt dl \tag{3-1}$$

$$H_{12} = \int_a^{\frac{r}{D}} \int_0^{\frac{Q}{D}-l} (Q + r - D(l + t))\gamma \tilde{h} t^{\gamma-1} f_L(l) dt dl \tag{3-2}$$

Note that, as in Assumption 11, $\tilde{h}t^{\gamma} = \int_0^t \gamma\, \tilde{h} t v^{\gamma-1} dv$ is the total holding cost if one unit is kept in stock during the period
[0, t]. Thus, $\int_0^t I(t)\gamma\, \tilde{h} t v^{\gamma-1} dv$ indicates the cumulative holding cost during *[0, t]* that *I*(*t*) shows its related inventory function. Equations (3-1) and (3-2) are derived in this fashion. In the same way, the expected holding cost per cycle for State 2 is as follows:

$$H_{21} = \int_{\frac{r}{D}}^{b} \int_0^{\frac{r}{D}} (r - Dt)\gamma \tilde{h} t^{\gamma-1} f_L(l) dt dl \tag{3-3}$$

$$H_{22} = \int_{\frac{r}{D}}^{b} \int_0^{\frac{Q}{D}-l} (Q + r - D(l + t))\gamma \tilde{h} t^{\gamma-1} f_L(l) dt dl \tag{3-4}$$

From Equation (3-2) and (3-4) it is inferred that H_{12} and H_{22} can easily be summarized as follows:

$$H_{T2} = \int_a^{b} \int_0^{\frac{Q}{D}-l} (Q + r - D(l + t))\gamma \tilde{h} t^{\gamma-1} f_L(l) dt dl \tag{3-5}$$

In Equation (3-5), H_{T2} indicates the total expected holding cost per cycle in Area 2.
Concerning backordered demand, it occurs only in State 2. So, the expected backordered cost per cycle, *B*, can be obtained as Equation (3-6):

$$B = \pi \int_{\frac{r}{D}}^{b} \frac{(Dl - r)^2}{2D} f_L(l) dl \tag{3-6}$$

In conclusion, by considering the ordering cost per replenishment, O_c, and the replenishment cycle time length, $T = {}^{Q}/_{D}$, the GRPSL model is as follows for

$0 \leq Da \leq r \leq Db \leq Q$:

$$Min\ ETCUT = \frac{D}{Q}(H_{11} + H_{21} + H_{T2} + B + O_c) = HUT_{11} + HUT_{21} + HUT_{T2} + BUT + OUC_c = \frac{D}{Q}\Big(\int_a^{\frac{r}{D}}\int_0^l (r - Dt)\gamma\tilde{h}t^{\gamma-1}f_L(l)dtdl + \int_{\frac{r}{D}}^{b}\int_0^{\frac{r}{D}}(r - Dt)\gamma\tilde{h}t^{\gamma-1}f_L(l)dtdl + \int_a^b\int_0^{\frac{Q}{D}-l}(Q + r - D(l + t))\gamma h t\gamma - 1 f L l dt dl + \pi r D b D l - r 2 2 D f L l dl + O c\Big) \tag{3-7}$$

The objective function of GRPSL is minimizing the retailer's expected total cost per unit time, *ETCUT,* whose components are: expected holding cost per unit time ($HUT_{11} + HUT_{21} + HUT_{T2}$), expected backordered demand per unit time (*BUT*) and expected ordering cost per unit time (OUC_c). In general, if *G(r, Q)* indicates the component of expected total cost per cycle (like H_{11}), the expected total cost per unit time, called *GUT(r,Q)*, (like HUT_{11}) is obtained as follows:

$$GUT(r, Q) = \frac{G(r,Q)}{T} = \frac{D.G(r,Q)}{Q} \tag{3-8}$$

3-4-2- Solution Approach for GRPSL

According to Equation (3-7), the GRPSL model belongs to the class of unconstrained multivariable optimization problems with a differentiable objective function at $Q>r>0$. So, in order to find the global optimum solution by an exact approach, first the convexity of the objective function, ETCUT, must be proved. The optimum values of decision variables *(r*, Q*)* could then be obtained by solving the following two equations concurrently:

$$\frac{\partial \mathrm{ETCUT}}{\partial \mathrm{Q}} = 0 \tag{3-9}$$

$$\frac{\partial \mathrm{ETCUT}}{\partial \mathrm{r}} = 0 \tag{3-10}$$

As it is comes from Equation (3-7), the convexity proof of *ETCUT* is highly complicated. Therefore, in order to simplify the mathematical challenges associated with its convexity proof, and to have more detailed analysis of the

proposed model, a simple form of $f_L(l)$ is considered. In the next section the RPSL problem is analyzed by assuming a uniform distribution function that is abbreviated to URPSL. Note that this distribution function is not too far from the real world. For example, there are several companies, especially newly established ones, which have insufficient historical data and cannot match any distribution function to their uncertain parameters. They hence rely on the lower and upper bounds of the uncertain parameters. In these cases the uniform distribution function may therefore be a good option.
This situation can also be found in a typical distribution center with multiple suppliers and multiple retailers. If the delivery time as well as the quantity of products delivered by suppliers to the distribution center are assumed to be unknown (as consignment stock), the distributor is not able to give its retailers the exact lead time. However, it can guarantee the upper and lower bounds of lead time for the retailers. This means that, if the required quantity was previously available in storage, it can immediately be sent to the retailer. In the worst case, the distributor must deliver the order to the retailers just before the upper bound of the lead time is reached. In this case, the uniform distribution function for lead time could be considered by each retailer.

3-5- RPSL model with uniform supply lead time (URPSL)

As mentioned above, solving GRPSL and finding global optimum replenishment policy, (r^*,Q^*), from Equation (3-7) is problematical. Thus, in this section the RPSL model by considering the uniform probability distribution function for supply lead time as $f_L(l) = \frac{1}{b-a}$ for $a \leq l \leq b$ is examined.

3-5-1- Mathematical model for URPSL

By substituting $f_L(l) = \frac{1}{b-a}$ in (3-1), (3-3) and (3-5), the holding cost components can be obtained as follows:

$$H_{11} = \frac{2\left(\frac{r}{D}\right)^{\gamma+1} rh}{(\gamma+2)(\gamma+1)(b-a)} + \frac{(a)^{\gamma+1}(-r\gamma-2r+Da\gamma)h}{(\gamma+2)(\gamma+1)(b-a)} \tag{3-11}$$

$$H_{21} = \frac{\left(\frac{r}{D}\right)^{\gamma} hrb}{(\gamma+1)(b-a)} - \frac{\left(\frac{r}{D}\right)^{\gamma} hr^2}{(\gamma+1)(b-a)D} \tag{3-12}$$

$$H_{T2} = -\frac{h\left(\frac{Q-Db}{D}\right)^{\gamma}(-r\gamma-Q-2r+Db)(Db-Q)}{(\gamma+2)(\gamma+1)(b-a)D} + \frac{h\left(\frac{Q-Da}{D}\right)^{\gamma}(-r\gamma-Q-2r+Da)(Da-Q)}{(\gamma+2)(\gamma+1)(b-a)D} \tag{3-13}$$

In (3-11), as H_{11} is positive, the inequality $2\left(\frac{r}{D}\right)^{\gamma+1} r \geq (a)^{\gamma+1}(r(\gamma+2) - Da\gamma)$ is satisfied. H_{21} and H_{T2} in (3-12) and (3-13) are always positive as regards the $r \leq Db$ and $Da \leq Db$ assumptions, respectively.

Besides, backordered demand per replenishment cycle is derived as Equation (3-14):

$$B = \frac{\pi D\left(b-\frac{r}{D}\right)^3}{6(b-a)} \tag{3-14}$$

So, by considering Equation (8), The URPSL model is as Equation (3-15) for $0 \leq Da \leq r \leq Db \leq Q$:

$$Min\ ETCUT = \frac{D}{Q}\{\frac{2\left(\frac{r}{D}\right)^{\gamma+1} rh}{(\gamma+2)(\gamma+1)(b-a)} + \frac{(a)^{\gamma+1}(-r\gamma-2r+Da\gamma)h}{(\gamma+2)(\gamma+1)(b-a)} + \frac{\left(\frac{r}{D}\right)^{\gamma} hrb}{(\gamma+1)(b-a)} - \frac{\left(\frac{r}{D}\right)^{\gamma} hr^2}{(\gamma+1)(b-a)D} - \frac{h\left(\frac{Q-Db}{D}\right)^{\gamma}(-r\gamma-Q-2r+Db)(Db-Q)}{(\gamma+2)(\gamma+1)(b-a)D} + \frac{h\left(\frac{Q-Da}{D}\right)^{\gamma}(-r\gamma-Q-2r+Da)(Da-Q)}{(\gamma+2)(\gamma+1)(b-a)D} + \frac{\pi D\left(b-\frac{r}{D}\right)^3}{6(b-a)} + O_c\} \tag{3-15}$$

3-5-2- Solution Approach for URPSL

Regarding Section 3-4-2, in order to find the global optimum inventory policy, first the proof of the objective function's convexity have to be done. The optimum decision variables can then be obtained by solving Equations (3-9) and (3-10) concurrently.

3-5-2-1- Proof of objective function's convexity for URPSL

The common approach for proofing the convexity of the objective function is by forming the Hessian matrix as Equation (3-16) and proving that it is positive-definite. Many researchers such as Chen and Chang (2010), Valliathal and Uthayakumar (2011) and Goh (1994) take this approach for solving the inventory model.

$$\begin{bmatrix} \frac{\partial^2 ETCUT}{\partial r^2} & \frac{\partial^2 ETCUT}{\partial r \partial Q} \\ \frac{\partial^2 ETCUT}{\partial Q \partial r} & \frac{\partial^2 ETCUT}{\partial Q^2} \end{bmatrix} \tag{3-16}$$

Because of the URPSL's objective function (*ETCUT)* form in Equation (3-15), taking the cited approach is excessively complicated and time-consuming. Moreover, *ETCUT* in Equation (3-15) cannot be broken down into a number of convex functions. Therefore, the following approach by proofing the following lemmas and prepositions is taken to confirm the convexity of the URPSL's objective function.

Lemma 1: $HUT_{11} + HUT_{21} + BUT + OUT$ is a strictly decreasing convex function in terms of Q. It is shown in Figure 3.2 (a).

Proof of Lemma 1:
$HUT_{11} + HUT_{21} + BUT + OUT$ is as follows when Equations (3-8), (3-11), (3-12) and (3-14) are applied:

$$HUT_{11} + HUT_{21} + BUT + OUT = \frac{D}{Q}\left(\frac{2\left(\frac{r}{D}\right)^{\gamma+1} rh}{(\gamma+2)(\gamma+1)(b-a)} + \frac{(a)^{\gamma+1}(-r\gamma-2r+Da\gamma)h}{(\gamma+2)(\gamma+1)(b-a)} + \frac{\left(\frac{r}{D}\right)^{\gamma} hrb}{(\gamma+1)(b-a)} - \frac{\left(\frac{r}{D}\right)^{\gamma} hr^2}{(\gamma+1)(b-a)D} + \frac{\pi D\left(b-\frac{r}{D}\right)^3}{6(b-a)} + O_c\right) = \frac{DC}{Q} \tag{3-17}$$

In Equation (3-17), it can be seen that $HUT_{11} + HUT_{21} + BUT + OUT$ is a constant in terms of Q, indicated by C, multiplied in $\frac{D}{Q}$. C is positive since H_{11}, H_{21}, B and O_c are positive.

Consequently, $\frac{\partial(HUT_{11}+HUT_{21}+BUT+OUT)}{\partial Q} = -\frac{DC}{Q^2} < 0$ and $\frac{\partial^2(HUT_{11}+HUT_{21}+BUT+OUT)}{\partial Q^2} = \frac{DC}{Q^3} > 0$ which imply that $HUT_{11} + HUT_{21} + BUT + OUT$ is strictly decreasing and convex in terms of Q respectively.

Lemma 2:
HUT_{T2} is a strictly increasing convex function in terms of Q. It is illustrated in Figure 3-2 (b).

Proof of Lemma 2:

According to Equation (3-8) and (3-13), HUT_{T2} is as follows:

$$HUT_{T2} = \frac{D}{Q}\left(-\frac{h\left(\frac{Q-Db}{D}\right)^{\gamma}(-r\gamma-Q-2r+Db)(Db-Q)}{(\gamma+2)(\gamma+1)(b-a)D}+\frac{h\left(\frac{Q-Da}{D}\right)^{\gamma}(-r\gamma-Q-2r+Da)(Da-Q)}{(\gamma+2)(\gamma+1)(b-a)D}\right) \tag{3-18}$$

So, the first differentiation of HUT_{T2} is derived as Equation (3-19):

$$\frac{\partial HUT_{T2}}{\partial Q} = -\frac{D}{Q^2}\left(\frac{h\left(\frac{Q-Da}{D}\right)^{\gamma}(Q-Da+r(\gamma+2))(Q-Da)}{(\gamma+2)(\gamma+1)(b-a)D}-\frac{h\left(\frac{Q-Db}{D}\right)^{\gamma}(Q-Db+r(\gamma+2))(Q-Db)}{(\gamma+2)(\gamma+1)(b-a)D}\right)+ \frac{D}{Q}\left(\left\{\frac{h\left(\frac{Q-Da}{D}\right)^{\gamma}\gamma(Q-Da+r(\gamma+2))}{(\gamma+2)(\gamma+1)(b-a)D}-\frac{h\left(\frac{Q-Db}{D}\right)^{\gamma}\gamma(Q-Db+r(\gamma+2))}{(\gamma+2)(\gamma+1)(b-a)D}\right\}+ \left\{\frac{h\left(\frac{Q-Da}{D}\right)^{\gamma}(Q-Da+r(\gamma+2))}{(\gamma+2)(\gamma+1)(b-a)D}-\frac{h\left(\frac{Q-Db}{D}\right)^{\gamma}(Q-Db+r(\gamma+2))}{(\gamma+2)(\gamma+1)(b-a)D}\right\}+ \left\{\frac{h\left(\frac{Q-Da}{D}\right)^{\gamma}(Q-Da)}{(\gamma+2)(\gamma+1)(b-a)D}-\frac{h\left(\frac{Q-Db}{D}\right)^{\gamma}(Q-Db)}{(\gamma+2)(\gamma+1)(b-a)D}\right\}\right) = -\frac{D}{Q^2}(M_1-M_2)+ \frac{D}{Q}(\{N_1-N_2\}+\{N_3-N_4\}+\{N_5-N_6\}) \tag{3-19}$$

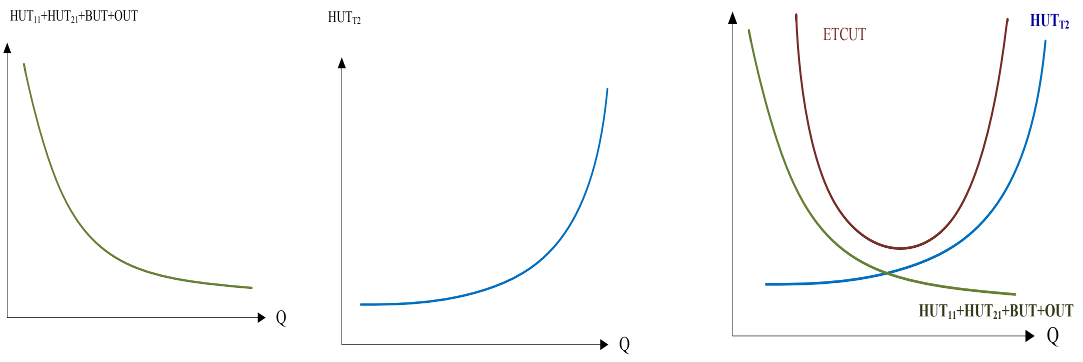
In Equation (3-19) M_1, M_2 and N_1 to N_6 is utilized for the corresponding statements in order to simplify. Because $Q \geq Db \geq Da$ then $Q\text{-}Da \geq Q\text{-}Db \geq 0$. So, we have $M_1 \geq M_2$, $N_1 \geq N_2$, $N_3 \geq N_4$ and $N_5 \geq N_6$. In other words $M_1\text{-}M_2 \geq 0$ and $\{N_1\text{-}N_2\}+\{N_3\text{-}N_4\}+\{N_5\text{-}N_6\} \geq 0$. Moreover, since $Q\text{-}Da \geq Q\text{-}Db$ then $N_1\text{-}N_2 \geq M_1\text{-}M_2$. Thus $\{N_1\text{-}N_2\}+\{N_3\text{-}N_4\}+\{N_5\text{-}N_6\} \geq M_1\text{-}M_2$. Consequently, $\frac{\partial HUT_{T2}}{\partial Q} \geq 0$ that implies HUT_{T2} is strictly increasing. In a similar way, it can be proven that $\frac{\partial^2 HUT_{T2}}{\partial Q^2} \geq 0$.

Preposition 1:

The total cost per unit of time, *ETCUT*, is a function that has only one extreme point in terms of *Q*.

Proof of Preposition 1:

Along with Lemma 1 and 2, it is wrapped up. This issue is demonstrated in Figure 3-2(c).



a. $HUT_{11} + HUT_{21} + BUT + OUT$ in terms of Q b. HUT_{T2} in terms of Q c. $ETCUT$ in terms of Q

Figure 3-2 - Behavior of *ETCUT* and its components in terms of *Q* (URPSL model)

In a similar way, the following Lemma are sustained for reorder point, *r*.

Lemma 3:

If $(\gamma + 2)r \leq \gamma bD$ (Since $Db \geq r$, this inequality is sustained approximately), the sum of expected holding cost per unit time, $HUT_{11}+HUT_{21}+HUT_{T2}$, is a strictly increasing convex function in terms of *r*. Figure 3-3(a) shows this.

Proof of Lemma 3:

In order to prove that $HUT_{11}+HUT_{21}+HUT_{T2}$ is strictly an increasing convex function in terms of *r*, it must be proved that its components are strictly increasing and convex.

According to Assumption 8 that implies $r>Da$, it is found that the first and second differentiation of HUT_{11} in terms of *r* is positive:

$$\frac{\partial HUT_{11}}{\partial r} = \frac{D}{Q}\left(\frac{2h\left(\frac{r}{D}\right)^{\gamma+1}}{(\gamma+1)(b-a)} - \frac{h(a)^{\gamma+1}}{(\gamma+1)(b-a)}\right) > 0 \tag{3-20}$$

$$\frac{\partial^2 HUT_{11}}{\partial r^2} = \frac{2Dh\left(\frac{r}{D}\right)^{\gamma+1}}{rQ(b-a)} > 0 \tag{3-21}$$

Also, HUT_{T2} is a linear (convex and concave) function in terms of *r* with a positive slope. Regarding Equation (3-18), it is derived that:

$$\frac{\partial HUT_{T2}}{\partial r} = \frac{D}{Q}\left(\frac{h\left(\frac{Q-Da}{D}\right)^{\gamma}(\gamma+2)(Q-Da)}{D(b-a)(\gamma+1)(\gamma+2)} - \frac{h\left(\frac{Q-Db}{D}\right)^{\gamma}(\gamma+2)(Q-Db)}{D(b-a)(\gamma+1)(\gamma+2)}\right) > 0 \tag{3-22}$$

Equation (3-22) is positive according to assumption $Da \leq Db \leq Q$.

Concerning HUT_{21} we have:

$$\frac{\partial HUT_{21}}{\partial r} = \frac{1}{Q}\left(\frac{hbD\left(\frac{r}{D}\right)^{\gamma}}{(b-a)} - \frac{(\gamma+2)hr\left(\frac{r}{D}\right)^{\gamma}}{(\gamma+1)(b-a)}\right) \tag{3-23}$$

$$\frac{\partial^2 HUT_{21}}{\partial r^2} = \frac{1}{Q}\left(\frac{hb\gamma\left(\frac{r}{D}\right)^{\gamma-1}}{(b-a)} - \frac{(\gamma+2)h\left(\frac{r}{D}\right)^{\gamma}}{D(b-a)}\right) \tag{3-24}$$

In Equation (3-23), if and only if $r(\gamma+2) \leq (\gamma+1)bD$ (condition I), then $\frac{\partial HUT_{21}}{\partial r}$ is positive. Moreover, (3-24) is positive if and only if $(\gamma+2)r \leq \gamma bD$ (condition II). By comparing conditions I and II, it can be concluded that condition II is dominant. As a result, if and only if $(\gamma+2)r \leq \gamma bD$, then HUT_{11}+HUT_{21}+ HUT_{T2} is a strictly increasing convex function in terms of *r*.

Lemma 4:
BUT+*OUT* is a decreasing convex function in terms of *r*. It is demonstrated in Figure 3-3(b).

Proof of Lemma 4:
Regarding Equation (3-14), it can easily be found that:

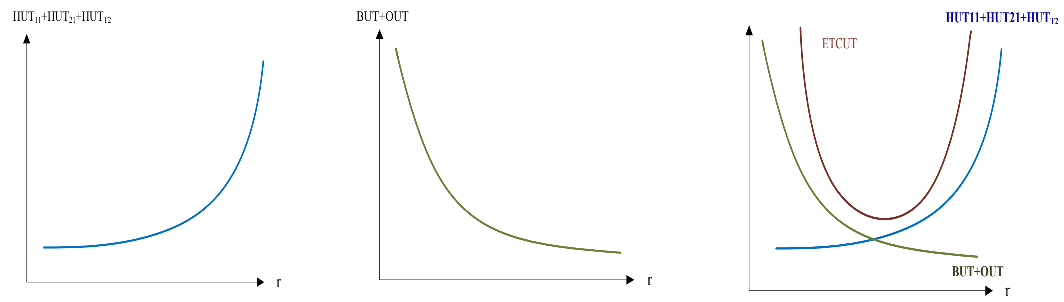
$$\frac{\partial BUT}{\partial r} = -\frac{D\pi}{2Q(b-a)}\left(b-\frac{r}{D}\right)^2 \leq 0 \tag{3-25}$$

$$\frac{\partial^2 BUT}{\partial r^2} = \frac{\pi}{Q(b-a)}\left(b-\frac{r}{D}\right) \quad \geq 0 \tag{3-26}$$

Equations (3-25) and (3-26) mean that *BUT is* a decreasing convex function in terms of *r*. Moreover, since *OUT* is constant in terms of *r*, adding it to *BUT* does not change this result.

Preposition 2: *ETCUT* has only one extreme point regarding *r*.

Proof of Preposition 2: In accordance with Lemma 3 and 4, it is directly concluded as shown in Figure 3-3(c).



4.a. $HUT_{11}+HUT_{21}+HUT_{T2}$ in terms of r 4.b. $BUT+OUT$ in terms of r 4.c. $ETCUT$ in terms of r

Figure 3-3 - Behavior of *ETCUT* and its components in terms of *r* (URPSL model)

Conclusion: According to Propositions 1 and 2, it can be concluded that the expected total cost per unit time in Equation (3-15), *ETCUT*, is a 3-dimensional function, and that its images on 2-dimentional planes are as in Figures 3-2(c) and 3-3(c). So, it can be stated that *ETCUT* is an 'Elliptic Paraboloid' function with only one extreme point regarding decision variables (Q and r). This result is explained further in the next section.

Now the optimum values of Q and r can be obtained by solving (3-9) and (3-10) concurrently. Since it is very difficult to derive closed-form formulas for r^* and Q^* from (3-9) and (3-10), the software MAPLE 11 is used to solve these equations for the URPSL model.

3-6- Numerical experiments and sensitivity analysis

Let's start the numerical analysis with an example of the URPSL problem by considering $a=0,\ b=2,\ D=1000,\ \gamma=2.2,\ O_c=1000,\ \tilde{h}=0.02$ and $\pi=0.5$. This parameter is set by the help of the existing literature Ferguson et al. (2007).

As shown in Figure 3-4, the expected total cost per unit time for URPSL, *ETCUT,* is an 'Elliptic Paraboloid' function with a minimum point in terms of r and Q, that verifies the conclusion reached in the previous section. By using Maple 11, the optimum policy is determined as $r^*=486$ unit, $Q^*=4275$ unit, with expected total cost $ETCUT^*=372$ €/unit item/unit time.

In URPSL model, to analyze the effect of holding cost parameters ($\tilde{h}$ and γ), backordered cost parameter (π) and lead time parameters ($[a,b]$) on optimum replenishment policy, their quantities are changed and sensitivity analysis is done. The results, explained below, yield some insight into the behavior of r^*, Q^* and $ETCUT^*$ as the cited parameters vary.

3-6-1- Effect of changing $\tilde{h}$ and π

In the first set of experiments, three levels for backordered demand cost per unit item per unit time are considered, i.e.: π=0.1, 0.3 and 0.5 €/unit item/unit time; and five levels for $\tilde{h}$, i.e.: $\tilde{h}$=0.006, 0.01, 0.02, 0.05 and 0.1 €/unit item/unit time. Other parameters are the same as the main example explained above. The optimum values are presented in Table 3-1.

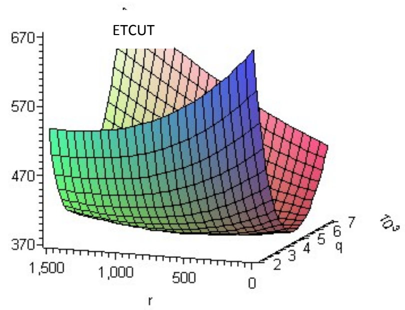


Figure 3-4 - *ETCUT* in terms of *Q* and *r* (URPSL model)

From Table 3-1, it can be seen that for each value of π, as $\tilde{h}$ increases, reorder point (r^*) decreases and expected total cost per unit time, $ETCUT^*$, increases. Since holding cost per unit increases with the time *t* that the product has been in stock, according to $H(t) = \tilde{h}t^{\gamma}$, when γ is fixed and $\tilde{h}$ increases, *H(t)* increases too. So, under fixed backorder cost parameter and lead time probability function, as *H(t)* increases the system tends to keep less inventory at reorder point, in order to avoid high inventory holding costs.

Table 3-1-Effect of $\tilde{h}$ and π on optimum inventory policy (URPSL model)

a=0	***b=2***	***D=1000***	***γ=2.2***	***O_c=1000***
$\tilde{h}$/ π		0.1	0.3	0.5
0.006	r^*	0	247	734
	Q^*	6246	6209	5861
	$ETCUT^*$	234	254	263
0.01	r^*	0	49	638
	Q^*	5412	5543	5101
	$ETCUT^*$	268	292	305
0.02	r^*	0	0	486
	Q^*	4474	4619	4275
	$ETCUT^*$	321	350	372
0.05	r^*	0	0	157
	Q^*	3508	3617	3565
	$ETCUT^*$	404	442	478
0.1	r^*	0	0	0
	Q^*	2939	3027	3109
	$ETCUT^*$	479	524	567

In addition, for each value of $\tilde{h}$, when backordered demand cost per unit item per unit time, π, increases, reorder point (r^*) and $ETCUT^*$ increase too. In this case, the inventory system tends to reduce backordered demand probability during lead time by holding more inventories to respond to the lead time demand, called reorder point.

It is obvious that *ETCUT* in Equations (3-7) and (3-15) has a direct relation with $\tilde{h}$ and π, so when they are increased, *ETCUT* increases too.

3-6-2- Effect of changing γ

It can be said that γ can be interpreted as the product deterioration speed. For products with a high deterioration rate, the holding cost of non-deteriorated items increases steeply with time. So, it can be said that the greater γ is, the higher the product's deterioration rate will be.

To study the effect of γ, for five levels of $\tilde{h}$=0.006, 0.01, 0.02, 0.05 and 0.1 €/unit item/unit time, five levels of γ are considered as γ=1, 1.6, 1.8, 2.2 and 2.8. Other parameters are the same as in the main example. The numerical results are reported in Table 3-2. Note that when γ=1, the holding cost per unit item per unit time is fixed over time, which means the product has not deteriorated.

Table 3-2- Effect of **γ** on optimum inventory policy (URPSL model)

From Table 3-2 it is seen that as γ increases, reorder point (r^*) decreases. This result is sound because by increasing γ, the holding cost per item *H(t)* increases too.

	a=0	***b=2***	***D=1000***	***π=0.5***	***O_c=1000***	
$\widetilde{h}/\gamma$		1	1.6	1.8	2.2	2.8
0.006	r^*	667	843	797	734	698
	Q^*	19148	8890	7557	5861	4476
	$ETCUT^*$	112	192	217	263	323
0.01	r^*	667	731	691	638	616
	Q^*	14837	7415	6406	5101	4002
	$ETCUT^*$	144	232	258	305	365
0.02	r^*	667	560	527	486	472
	Q^*	10501	5857	5177	4275	3483
	$ETCUT^*$	203	299	325	372	431
0.05	r^*	419	282	246	157	0
	Q^*	6873	4423	4047	3565	3193
	$ETCUT^*$	315	410	435	478	530
0.1	r^*	151	0	0	0	0
	Q^*	5131	3735	3476	3109	2753
	$ETCUT^*$	428	511	531	567	613

By analyzing the results of Tables 3-1 and 3-2, it can be concluded that: if products deteriorate at a high rate, the system tends to stock fewer inventories, unless the shortage cost is very high.

3-6-3- Effect of changing lead time uncertainty

Here lead time variance displays its uncertainty. So, four intervals for lead time is considered when $\widetilde{h}$= 0.006 and 0.01€/unit item/unit time. The numerical results are reported in Tables 3-3 and 3-4, respectively.

Table 3-3-Effect of lead time uncertainty on optimum inventory policy, $\tilde{h}$=0.006 (URPSL model)

D=1000	$\tilde{h}$*=0.006*	*γ=2.2*	*π=0.5*	O_c*=1000*
a	0	0	0	0
b	1	2	4	7
Mean	0.50	1.00	2.00	3.50
Variance	0.08	0.33	1.33	4.08
r^*	0	734	2491	4481
Q^*	5982	5861	5643	7000
$ETCUT^*$	254	263	274	311

As shown in Tables 3-3 and 3-4, when the uncertainty in lead time (lead time variance) increases, the retailer prefers to stock more inventory at reorder point, in order to respond to demand during lead time. Also, high uncertainty in supply lead time increases the retailer's inventory system costs.

Table 3-4-Effect of lead time uncertainty on optimum inventory policy, $\tilde{h}$=0.01 (URPSL model)

D=1000	$\tilde{h}$*=0.01*	*γ=2.2*	*π=0.5*	O_c*=1000*
a	0	0	0	0
b	1	2	4	7
Mean	0.50	1.00	2.00	3.50
Variance	0.08	0.33	1.33	4.08
r^*	0	638	2364	3878
Q^*	5142	5101	4927	7000
$ETCUT^*$	294	305	320	398

In brief, by increasing the parameter of the holding cost, $\tilde{h}$ and γ, the reorder point decreases. In contrast, the reorder point increases when the backordered demand parameter, π, or uncertainty in supply lead time increases in order to reduce the risk of shortage. The objective function quantity, *ETCUT*, increases in all these cases.

In all the numerical cases reported in Tables 3-1 to 3-4, Q^* and $\tilde{h}$ have an inverse relation to each other: when one of them increases, the other one decreases. This result is consistent with H. Wiess's (1982) closed form formula for Q^* that was developed under constant lead time.

3-7- Conclusion

Considering a non-linear holding cost is appropriate for modeling deteriorating products with an expiry date or limited lifetime. In the present chapter a mathematical model for an inventory system with non-linear cumulative holding cost and uncertain supply lead time was developed. The proposed model had a non-linear objective function. It is well known in optimization theory that for an unconstrained minimization problem with a convex objective function, a local optimum is also a global one. Therefore, by proving the convexity of the objective function, the local optimum solutions were determined for a uniform supply lead time that was also a global solution. A numerical example based on the literature was solved, to show the applicability of the model. The results showed that for products with a lower deterioration rate and high shortage cost it would be an economical strategy to stock more inventories at reorder point, especially under uncertain conditions, to avoid the risk of shortage.

4

A NEW ORDER SPLITTING MODEL WITH STOCHASTIC LEAD TIMES FOR DETERIORATING PRODUCTS

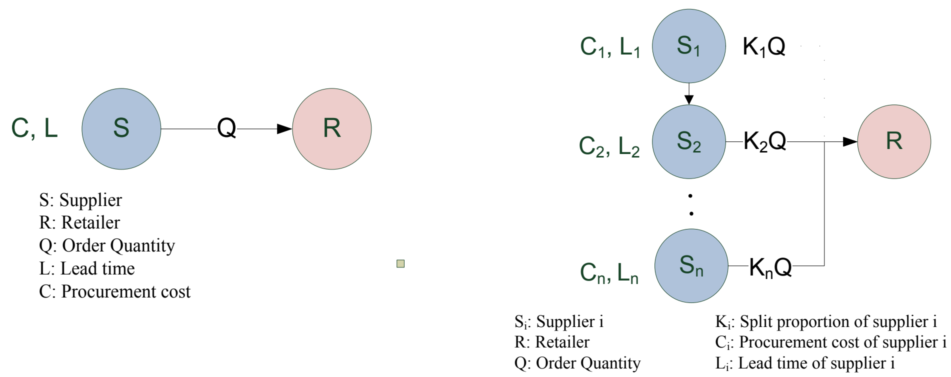
4 A new order splitting model with stochastic lead times for deteriorating products

4-1- Introduction

Establishing a well-organized inventory system to obtain the right quantity of products of the right quality, from the right source, and to have the products delivered to the right place at the right price can have a positive effect on a company's profitability and competitive advantage (Shen, et al. 2011). Determining a good sourcing policy is therefore one of the main concerns of corporations. There are two major types of sourcing policy: single and multiple sourcing.

Treleven and Schweikhart (1988) define single sourcing as the situation when an organization chooses to have all its requirements for a particular item met by a single vendor. If single sourcing is not opted for by choice, for instance when a vendor has a monopoly, these authors call it 'sole sourcing'.

Multiple sourcing refers to the policy under which a retailer buys an identical part from two or more vendors. An important issue that is raised in multiple sourcing studies is the splitting of a replenishment order among several suppliers simultaneously instead of placing a single order, especially in stochastic conditions. This policy is called *order splitting*. The concept of order splitting policy versus single sourcing is shown schematically in Figure.4-1. From the retailer's perspective, under uncertain conditions (supply lead time and/or demand) order splitting policy may be useful to reduce the uncertainty, improve quality and decrease the purchasing price. Many studies, with various assumptions, have demonstrated that by splitting orders the retailer can generate savings in inventory holding and shortage costs owing to the reduction of uncertainty, although ordering costs are increased (Lao and Zhao, 1994, Chiang and Chiang, 1996, Tyworth and Ruiz-Torres, 2000, Chiang, 2001 and Thomas and Tyworth, 2006). Hence, as the savings in holding and shortage costs outweigh the incremental costs of ordering, splitting orders among multiple suppliers concurrently is an appealing and cost-effective policy.



1(a) Single sourcing policy	1(b) Order splitting policy

Figure 4-1 - Single sourcing versus order splitting policy

Although order splitting policy is studied under various situations, researchers have paid less attention to this sourcing policy for firms that offer deteriorating products. In this chapter, we are going to start filling this gap by studying single sourcing versus the splitting of orders between two suppliers, in the case of a retailer selling deteriorating items. In this way, two inventory models are develop. In the first model it is assumed that all the requirements are supplied by only one source, whereas in the second, two suppliers are available.

The remainder of the chapter is organized as follows. In the next section some of the main researches in the field of order splitting are reviewed. The motivation of the chapter is then described in Section 4-3. The problem description and mathematical models are presented in Sections 4-4 and 4-5 respectively. The solution approach is explained in Section 4-6 and computational results together with related analyses are given in Section 4-7. In Section 4-8 we examine a real case study from the European pharmaceutical industry to demonstrate the applicability and effectiveness of the proposed models. Finally, we review the chapter and present a conclusion in Section 4-9.

4-2- A brief literature review on the order splitting policy

Before reviewing the order splitting literature, we wish to point out two key concepts in this field: *i*) the heart of the order splitting issue is the *effective lead time* concept, defined as the minimum lead time in the set of suppliers' independent, random lead times (Thomas and Tyworth, 2006); it is equivalent to $min\{L_1, L_2,\ldots,L_n\}$ in Figure 4-1(b); and *ii*) the interval between the reception of two successive orders in each replenishment cycle is called *inter-arrival time* or *inter-supply time* (Ramasesh et al., 1991).

Sculli and Wu (1981) appear to have been the first to have set up the order splitting model and introduced the concept of effective lead time. Since one of the main reasons for splitting orders is uncertain demand during supply lead time, the existing stochastic order splitting models can be categorized in three groups: *i*) models with stochastic lead time and deterministic demand, *ii*) models with deterministic lead time and stochastic demand and *iii*) models with stochastic lead time and stochastic demand. The main studies of each group are summarized in Table 4-1.

4-2-1- Order splitting models with stochastic lead time and deterministic demand

In many order splitting models, it is assumed that the supply lead time among suppliers and the retailer are stochastic, while end-customer demand is deterministic. Ramasesh et al. (1991) were among the first to adopt a cost minimization approach to order splitting policy. In their study a dual-sourcing model was considered by assuming that both suppliers had identical lead time distributions which were either uniform or exponential. Then, by considering continuous review policy and a constant demand rate, a largely inclusive comparison was done between single and dual-sourcing policies. The numerical results demonstrated that with high uncertainty in lead times and low ordering costs, dual-sourcing can be cost-effective. Soon after that, Ramasesh et al. (1993) expanded on that research which was free from the limiting assumptions that lead time distributions, split proportion and purchasing prices are the same for two suppliers, and strengthened the previous results for exponential lead times.

In 2005, Tang and Grubbstrom developed a dual-source model for a manufacturing/remanufacturing system with generic stochastic lead times and constant demand. They showed that the dual-sourcing policy has potential to reduce the average cost of the system.

Recently, Sajadieh and Eshghi (2009) considered a dual-sourcing model with constant demand and non-identical exponential lead times. They developed an all-unit discount model by considering order quantity-dependent lead times. Finally, they concluded that the relationship between lead time and order quantity increases savings from order splitting, while price-dependent order quantity decreases it. Some other studies on order splitting policy which assumed stochastic lead time and constant demand include: Tyworth and Ruiz-Torres (2000) and Ryu and Lee (2003). All these studied have been developed by assuming that the products can be stocked forever (non deteriorating products).

4-2-2- Order splitting models with deterministic lead time and stochastic demand

In some researches, deterministic lead times together with stochastic demand rate have been assumed to result in uncertain demand during lead time. Chiang and Chiang (1996), by considering normal demand and continuous review policy, demonstrated that splitting orders into multiple deliveries can significantly reduce the inventory holding cost especially when the inter-arrival times are determined optimally.

To study the effect of splitting orders on the periodic review inventory systems, Chiang (2001) developed a multi-supplier model and showed there is an optimal number of deliveries during each ordering cycle that results in the lowest total cost. Recently, Glock and Ries (2012) considered a supply chain consisting of multiple suppliers and a single retailer, and developed an integrated inventory model. They considered normal distributed demand, batch size-dependent lead times and continuous review policy, and investigated the impact of the delivery structure and the number of suppliers on the expected total costs of the supply chain and the stock-out risk. Some other order splitting studies developed under the assumption of deterministic lead time and stochastic demand are Chiang and Gutierrez (1996), Chiang (2003) and Iakovou et al. (2010).

4-2-3- Order splitting models with stochastic lead time and stochastic demand

Some researchers have studied order splitting policy under stochastic demand and stochastic lead time assumptions. Mohebbi and Posner (1998) studied the splitting of orders between two suppliers in a lost sale inventory system, by considering compound Poisson demand and non-identical exponentially distributed lead times. Regarding suppliers' reliability, Ganeshan et al. (1999)

studied a supply chain that has the choice of using two suppliers, one reliable and the other unreliable with longer lead time mean and variance.

Table 4-1-Literature review of stochastic order splitting problems and classifications

Group		#/type of Suppliers	Lead time	Demand	Inventory system	Objective	Solution approach
I.	Ramasesh et al. (1991)	2-I	Exp./Unif.	Const.	FOQ	Cost	Num[1]
	Ramasesh et al. (1993)	2-NI	Exp.	Const.	FOQ	Cost	Num.
	Lao and Zhao (1994)	2-NI	General	Const.	FOQ	Cost	Num.
	Tyworth and Ruiz-Torres (2000)	2-NI	Exp.	Const.	FOQ	Cost	Num.
	Ryu and Lee (2003)	2-NI	Exp.	Const.	FOQ	Cost	Num.
	Tang and Grubbstrom (2005)	2-NI	General	Const.	FOI	Cost	Exact
	Sajadieh and Eshghi (2009)	2-I	Exp. & BSD.	Const.	FOQ	Cost	Exact (B&B)
II.	Chiang and Chiang (1996)	2-NI	Const.	N.	FOQ	Cost	H.
	Chiang and Gutierrez (1996)	2-NI	Const.	General	FOI	Cost	Exact
	Chiang (2001)	M-NI	Const.	N.	FOI	Cost	H.
	Chiang (2003)	2-NI	Const.	N-P-Γ-Geom	FOI	Cost	Exact
	Burke et al. (2007)	M-NI	Const.	General	1-period	Profit	Exact
	Iakovou et al.(2010)	2-NI	Const.	General	1-period	Profit	Exact
	Glock and Ries (2012)	M-I	BSD.	N.	FOQ	Cost	Num. & SIM.
III.	Chiang and Benton (1994)	2- I	Shifted Exp.	N.	FOQ	Cost	Exact
	Mohebbi and Posner (1998)	2-NI	Exp.	CP.	FOQ	Cost	Num.
	Ganeshan et al. (1999)	2-NI	General	N.	FOQ	Cost	H.
	Sedarag et al. (1999)	M-NI	General	General	FOQ	Cost	GA. & SIM.
	Mohebbi and Posner (2002)	M-I	Exp.	CP.	FOQ	Cost	Num.
	Abginehchi and	M-NI	General	General	FOQ	Cost	Num.

[1] Hooke-Jeeves

Farahani (2010)						(SQP)
Abginehchi et al. (In press)	M-NI	General	Γ-Exp.	FOQ	Cost	Num. (SQP)

N: Normal variable	Γ: Gamma variable	SQP: Sequential Quadratic Programming
Exp: Exponential variable	BSD: Batch-size dependent	GA: Genetic Algorithm
CP: Compound Poisson variable	I: identical	H: Heuristic
P: Poisson variable	NI: Non-Identical	FOI: Fixed Order Interval
Geom: Geometric variable	M: Multi-supplier	FOQ: Fixed Order Quantity
Unif: Uniform variable	Num: Numerical	Const: Constant
	SIM: Simulation	B&B: Branch and Bound

They considered a broader inventory-logistics framework that included in-transit inventories and transportation costs. By assuming normally distributed demand, and lead time as a discrete random variable, it was concluded that if the unreliable supplier provides an appropriate price discount, order splitting policy can be favorable for the retailer. Dullaert et al. (2005) extended this study and analyzed a situation in which products are transported from a supplier to a retailer using diverse transport alternatives. They introduced the concept of *fastest transport alternative* in place of *effective lead time*.

Sedarag et al. (1999) considered a fairly comprehensive multi-supplier single-item continuous review inventory system, where the unit time demand and supplier lead times were general random variables and backorders were allowed. They identified the optimal number of suppliers. Moreover, they found that splitting an order up among suppliers with higher unit procurement costs and larger lead time mean/variance may be preferable to concentrating the order on a single supplier with a lower unit procurement cost and a smaller lead time mean/variance. Recently, Abginehchi and Farahani (2010) improved Sedarag's model and developed a new exact analytical one. Chiang and Benton (1994) and Hill (1996) are other examples in this group of order splitting studies.

All researches presented in Table 4-1 have studied order splitting policy for products with unlimited lifespan and deteriorating products (or products with expiration date) have not attracted enough attention in the existing literature.

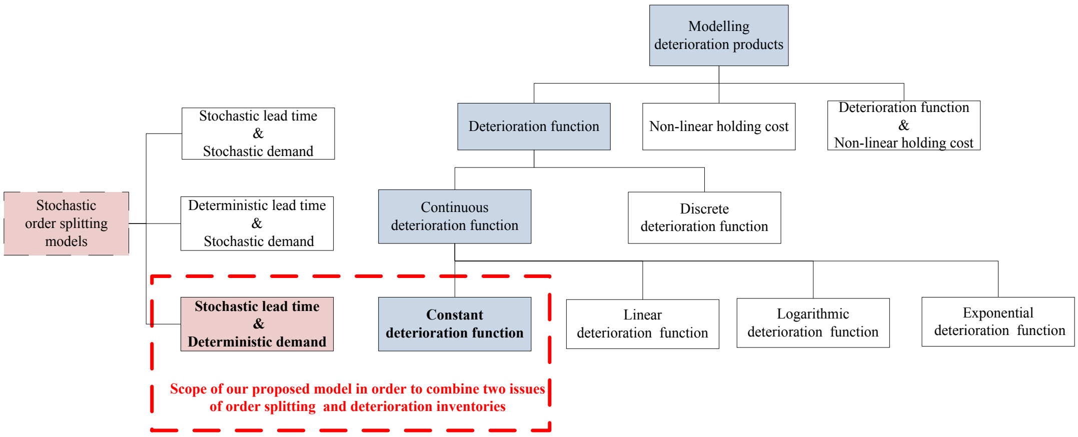


Figure 4--2 Scope of the proposed model

4-3- Motivation and Contribution

The contribution of this chapter is twofold. In the first step, a new mathematical model is proposed for a retailer that supplies its deteriorating inventories from a single source under stochastic lead time. The first class of deterioration modeling approach is opted by assuming a constant deterioration rate (non-linear inventory function) and a linear holding cost function. This assumption is appropriate for products like alcohol, gasoline, some pharmaceutical materials etc. Therefore the features distinguish the proposed single-source model in this chapter from that described in the previous chapter are the problem modeling approach as well as considering some more complex assumptions like an exponential density function for lead time in place of a uniform one.

Then, in the second step the single-source model is extended to an order splitting model by considering two available suppliers for the retailer. In fact, the concepts of order splitting and deteriorating inventory models have been studied separately from various perspectives. But these two issues together have not been paid enough attention in the literature. In other words, the main contribution of this chapter is that it combines order splitting and deteriorating inventory issues. Based on the literature classifications mentioned before, the exact scope and position of our work in the literature is shown in Figure.4-2.

In the order splitting model we assume a retailer can split its order into two non-identical suppliers (i.e. with different selling prices and lead times). A continuous inventory review policy *(r, Q)* is also considered for the retailer.

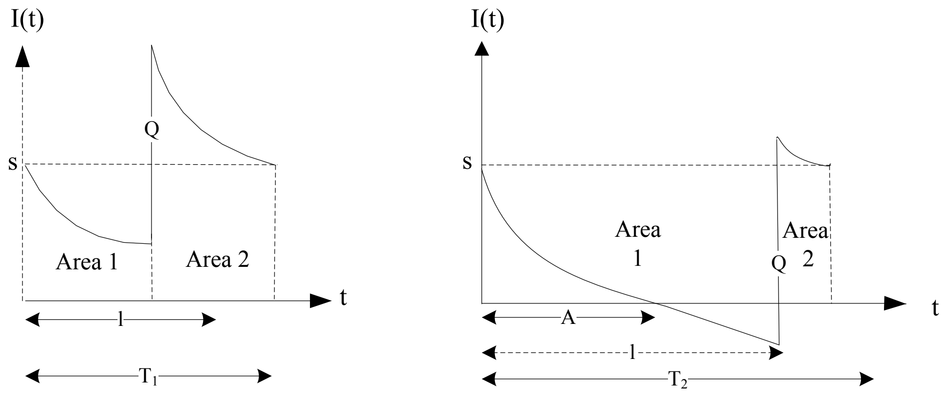
In this way, in order to formulize single-source and dual-source models under stochastic lead times and a constant deterioration rate, some important challenges have to be met. One of them is the non-linear inventory function and dissimilarity of inventory function form, before and after shortage, that makes the calculation more complicated than in usual inventory models. For example, a serious challenge is not fixed replenishment cycle times with constant ordering quantity *(Q)* and constant demand rate *(D)*. For non-deteriorated items with a fixed demand rate, stochastic lead time and backorder assumption, the replenishment cycle time is fixed and equal to *Q/D* under *(r,Q)* policy. This is explained in Ramasesh et al. (1993). But for deteriorating items under the same conditions, the replenishment cycle time is not equal to *Q/D* and its length depends on lead time length because the items are consumed through both demand and the deterioration process until the inventory level becomes zero. So, in this case, the combination of stochastic lead time and the deterioration process makes the problem very difficult to model and also to solve. These challenges become much more acute with the order splitting model, especially when the suppliers are not identical. In spite of these challenges, we deal with this fascinating and practical problem in this chapter.

4-4- Problem description

Assume a retailer that is faced with a constant end-customer demand for a product with a constant deterioration rate. In order to respond to its demand, the retailer can purchase its entire requirement from one supplier or split it between two suppliers. The lead time between the suppliers and the retailer is stochastic. In order to avoid high deterioration costs, suppliers try to deliver retailers' orders as quickly as possible. Negative exponential distributed lead times are therefore appropriate for modeling stochastic lead times. The lead time of both suppliers is identically distributed but with a different mean and variance. The longer the lead time, the lower the selling prices will be (the proposed model also covers the situation in which suppliers' lead time mean/variance are identical, as are their selling prices). Since the supply lead time is stochastic, the retailer may be faced with shortage during the lead time. The aim is to minimize the expected total cost of the retailer, consisting of procurement, ordering, holding, shortage and deterioration costs. In this way, two mathematical models are developed for two problems: Single-Source

Deteriorating Inventory problem **(SSDI)** and Dual-Source Deteriorating Inventory problem **(DSDI).**

Given that the supply lead time is stochastic, for each model the inventory curve can be in several states. For example, depending on the backorders that occurred during lead time, or not, and by considering Assumption A.8. (this assumption will be described more in Section 4.4-2), two separate states must be considered for a typical replenishment cycle of the SSDI model. These are displayed in Figure 4-3. Without loss of generality, we assume that the replenishment cycle begins at time *0*, at level $r > 0$.



(a) State 1 **(b) State 2**

Figure 4--3 Possible states of single-source (SSDI) model

In state i, the replenishment cycle time, $[0,T_i]$, is divided into a number of distinct areas to simplify the formulation. As Figure 4-3 shows, the replenishment cycle time of each state of the SSDI model is composed of two distinct areas: Area 1: before receiving the order (lead time) $(0 \leq t < L)$, and Area 2: after receiving the order $(L \leq t < T_i)$ (similar to what was done in previous chapter).

4-4-1- Notations

We use the following notations for developing the SSDI and DSDI models:

Indexes:

i: Indicator of states

j: Indicator of areas

Parameters:

D:	Demand rate per unit time
θ:	Deterioration rate per unit time
h:	Holding cost per unit item per unit time
π:	Backordering cost per unit item per unit time
O_c:	Constant ordering cost per replenishment
c_i:	Selling price of supplier *i*, *i=1*,2 per unit item
α:	Multiplier of increasing ordering cost in the DSDI model
λ_i:	Lead time parameter for supplier *i*, *i=1,2*.
L_i:	Lead time of supplier *i*, *i=1,2*.
L_e:	Effective lead time in the DSDI model
L_s	Inter arrival time in the DSDI model
UL_{ei}:	Upper bound of L_e in state *i* in the DSDI model
LL_{ei}:	Lower bound of L_e in state *i* in the DSDI model
UL_{si}:	Upper bound of L_s in state *i* in the DSDI model
LL_{si}:	Lower bound of L_s in state *i* in the DSDI model
$NS_i(t)$:	Net stock function for state *i*
$I_{ij}(t)$:	On-hand inventory function for area *j* of state *i*
$B_{ij}(t)$:	Backorder function for area *j* of state *i*
$I_i(t)$:	On-hand inventory function of state *i*
$B_i(t)$:	Backorder function of state *i*
T_i:	Replenishment cycle time of state *i*
$E(T)$:	Expected replenishment cycle time
$E_{State-i}(TCC)$	Expected total cost per cycle of state *i* of SSDI model
$E_{State-i}(TCC^k)$	Expected total cost per cycle of state *i* of DSDI model under scenario *k*
E(TCC):	Expected total cost per cycle
E(TCU):	Expected total cost per unit time
E(DCU)	Expected deterioration cost per unit time

Decision variables:

Q: Order quantity

r: Reorder point

k_i: Split proportion of supplier i, $i=1,2$

Like previous chapter, note that a capital letter is used to indicate a random variable, and a small letter is used to indicate the value of the random variable.

4-4-2- Assumptions

Problem assumptions are categorized in two clusters, inventory system and cost structure, as follows:

A. Inventory system assumptions

A.1. The planning horizon is infinite.

A.2. The inventory system is the continuous review, *(r,Q)* with constant demand per time unit.

A.3. The order quantity, Q, is greater than the reorder point, s, and the reorder point is positive. This is common in practice and in the literature, so $Q>r>0$.

A.4. The product deteriorates deterministically with a constant rate θ. As noted before, this assumption is appropriate for modeling many products such as some evaporating or pharmaceutical products.

A.5. In each cycle, Q non-deteriorated units of a product are received by the retailer.

A.6. In the dual-source model, supplier i gets the proportion k_i (i=1,2) of the order quantity Q, where $k_1+k_2=1$.

A.7. In the dual-source model, the lead times of the first and second suppliers, L_1 and L_2, are non-identical independent exponential, with parameters λ_1 (mean = $1/\lambda_1$) and λ_2 (mean = $1/\lambda_2$) respectively, that is, $\lambda_2 \leq \lambda_1$. In the single-source model, the supplier is the same as the first supplier, so its lead time, $L=L_1$, is exponentially distributed with parameter λ_1.

A.8. Also in this chapter, it is assumed that the replenishment cycles are renewable or regenerative. Here, this means that the on-hand inventory level just after the last delivery from suppliers exceeds the reorder level.

This assumption is applicable and is also considered in such order splitting studies as Ryu and Lee (2003), Sedarag et al. (1999), Abginehchi and Farahani (2010) and Abginehchi et al. (2012). With this assumption, no crossover has occurred. Later, by numerical results, we show the negligibility of the probability that the lead time becomes too long, and that the on-hand inventory falls less than the ordering point after the whole order has been received.

B. Cost structure assumptions

B.1. The selling price of supplier i is € c_i per unit, $i=1, 2$. Since the mean of the second supplier's lead time is considered to be greater than that of the first supplier ($\lambda_2 \leq \lambda_1$), it is reasonable to assume that $c_2 \leq c_1$.

B.2. The deterioration cost is assumed as the procurement cost of decayed items.

B.3. The holding cost per unit item per unit time, h, is considered only for non-deteriorated items.

B.4. Shortages are allowed and are considered to be completely backordered.

B.5. In order to model ordering cost increases arising from split orders, the multiplicative approach is taken. This means that the ordering cost per replenishment in the DSDI model is α times the ordering cost per replenishment in the SSDI model, where $\alpha \geq 1$. Studies such as Ramasesh et al. (1991), Ramasesh et al. (1993), Chiang and Benton (1994) and Mohebbi and Posner (1998) are examples of this approach.

4-5- Model Development

In two of the models developed here, we are interested in minimizing the retailer's expected total cost per unit time, that includes three constituent costs: the procurement cost; the expected inventory cost – the ordering, inventory holding, and backorder –; and the deterioration cost.

In order to formulate inventory costs, it is necessary to know the net stock function in terms of time. Because of the constant demand and deterioration rate, the net stock level at moment t, $NS(t)$, can be derived from the differential equations $NS'(t)=-\theta(t).NS(t)-D(t)$, when $NS(t) \geq 0$, and $NS'(t)=-D(t)$, otherwise. The general form of the net stock function is therefore as in Equation (4-1) where the constants β and γ can be determined by defining the right initial conditions:

$$NS(t) = \begin{cases} \beta e^{-\theta t} - \frac{D}{\theta}, \; NS(t) \geq 0 \\ -Dt + \gamma \, , \; NS(t) < 0 \end{cases} \tag{4-1}$$

According to Equation (4-1) the net stock function, *NS(t)*, can be positive or negative. If it is positive, it shows the on-hand inventory function, whereas when it is negative, it demonstrates the negative form of a backorder function.

Developed mathematical models for SSDI and DSDI problems are explained in details in next sub-sections. For this purpose, at first an arbitrary distribution function for supply lead time as $f_L(l)$ for $0 \leq L$ are considered. Then, the developed models are customized for exponential distribution function to obtain more managerial insights.

4-5-1- The Single-Source Deteriorating Inventory (SSDI) model

In this section it is assumed that the retailer uses only the supplier with purchasing cost c_1 and stochastic lead time $L=L_1$. In order to formulate the SSDI model, the following two states must be considered (Figure 4-3):

State 1: Setting a range for L that prevents backordered demand. Thus, if A indicates the time that the level of inventory falls from s to zero, then $0<L\leq A$ (Figure 4-3(a)).

State 2: Setting a range for L that results in backordered demand. So, if l_{max} is the maximum length of lead time according to assumption A.8, then $A<L\leq l_{max}$ (Figure 4-3(b)).

So, the SSDI's expected total cost in a cycle, *E(TCC)*, can be written as follows:

$$E(TCC) = \sum_{i=1}^{2} E_{State-i}(TCC) = \int_0^A (h.(\int_0^l I_{11}(t)dt + \int_l^{T_1} I_{12}(t)dt) + c_1.(Q - D.T_1)).f_L(l)dl + \int_A^{l_{max}} (h.(\int_0^A I_{21}(t)dt + \int_l^{T_2} I_{22}(t)dt) + \pi.\int_A^l B_{21}(t)dt + c_1.(Q - D.T_2)).f_L(l)dl \tag{4-2}$$

In Equation (4-2), the statement for the deterioration cost is understandable since it is defined as the procurement cost of decayed items.

Let *E(T)* be the expected replenishment cycle time that can be calculated from Equation (4-3):

$$E(T) = \int_0^A T_1 f_L(l)dl + \int_A^{l_{max}} T_2 f_L(l)dl \tag{4-3}$$

In conclusion, the final SSDI model is as Equation (4-4) for $Q \geq r \geq 0$:

$$\text{Min } E(TCU) = \frac{E(TCC)}{E(T)} \tag{4-4}$$

In Equation (4-4), the expected total cost per unit time, *E(TCU)*, is derived by dividing the expected total cost per cycle, *E(TCC)*, by the expected replenishment cycle time, *E(T).*

A closed form formula can be obtained for inventories and shortage functions in Equation (4-2). In State 1, the net stock function at time t, $NS_1(t)$, can easily be derived from general Equation (4-1) and initial conditions $I_{11}(0)=r$ and $I_{12}(l)= I_{11}(l)+Q$. So $NS_1(t)$, is determined as follows:

$$NS_1(t) = \begin{cases} I_{11}(t) = -\frac{D}{\theta} + \left(r + \frac{D}{\theta}\right) e^{-\theta t}, & 0 < t \leq l \\ I_{12}(t) = -\frac{D}{\theta} + \left(r + \frac{D}{\theta} + Qe^{\theta l}\right) e^{-\theta t}, & l < t \leq T_1 \end{cases} \tag{4-5}$$

According to Figure 4-3(a), no backorder occurred in this state. In a similar way, the net stock function of State 2, $NS_2(t)$, can be determined by initial conditions $I_{21}(0)=r$, $B_{21}(A)=0$ and $I_{22}(l)=Q-(l-A)D$ respectively as follows:

$$NS_2(t) = \begin{cases} I_{21}(t) = -\frac{D}{\theta} + \left(r + \frac{D}{\theta}\right) e^{-\theta t}, & 0 < t \leq A \\ B_{21}(t) = D(t-A), & A < t \leq l \\ I_{22}(t) = -\frac{D}{\theta} + \left(Q - D(l-A) + \frac{D}{\theta}\right) e^{-\theta(t-l)}, & l < t \leq T_2 \end{cases} \tag{4-6}$$

The point A, at which the on-hand inventory becomes zero, and the replenishment cycle time at the first and second states, can be determined by $I_{11}(A)=0$ (or $I_{21}(A)=0$), $I_{12}(T_1)=r$ and $I_{22}(T_2)=r$ respectively. These are stated in Equations (4-7), (4-8) and (4-9):

$$A = -\frac{1}{\theta} \ln\left(\frac{D}{D+r\theta}\right) \tag{4-7}$$

$$T_1 = -\frac{1}{\theta} \ln\left(\frac{D+r\theta}{D+r\theta+Q\theta e^{\theta l}}\right) \tag{4-8}$$

$$T_2 = -\frac{1}{\theta} \ln\left(\frac{D+r\theta}{(Q\theta - D\theta(l-A)+D)}\right) + l \tag{4-9}$$

An important note that must be considered is the renewal assumption (A.8) that places an upper bound on backorder quantity and consequently on lead time length. In other words, this assumption implies that the maximum number of backorders during lead time B_{max} is Q-r units, with the corresponding time interval $t_{Bmax}= (Q-r)/D$. So, the maximum length of lead time, l_{max}, is derived as Equation (4-10):

$$l_{max} = A + t_{B_{max}} = -\frac{1}{\theta}\ln\left(\frac{D}{D+r\theta}\right) + \frac{Q-r}{D} \quad (4\text{-}10)$$

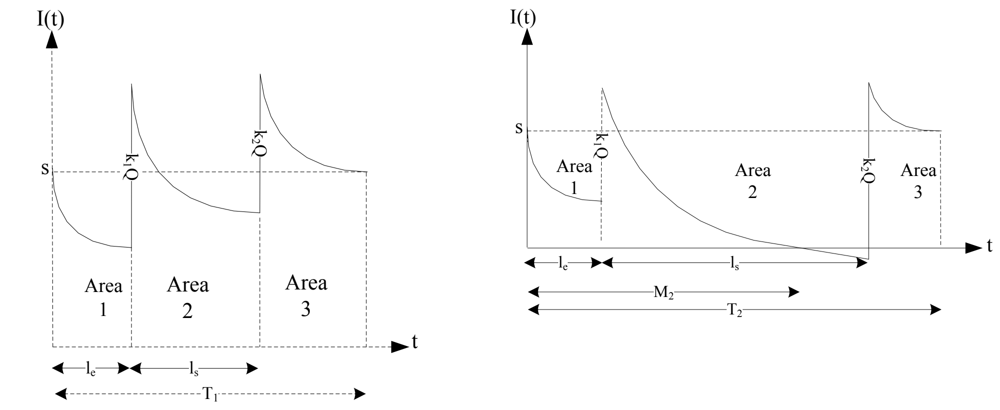
In this way, by substituting $\boldsymbol{f_L(l) = \lambda_1 e^{-\lambda_1 l}}$ for $l \geq 0$ in Equations (4-2), (4-3) and (4-4), SSDI model can be obtained for situation where supply lead time follows negative exponential distribution function (with parameter $\boldsymbol{\lambda_1}$).

4-5-2- The Dual-Source Deteriorating Inventory (DSDI) model

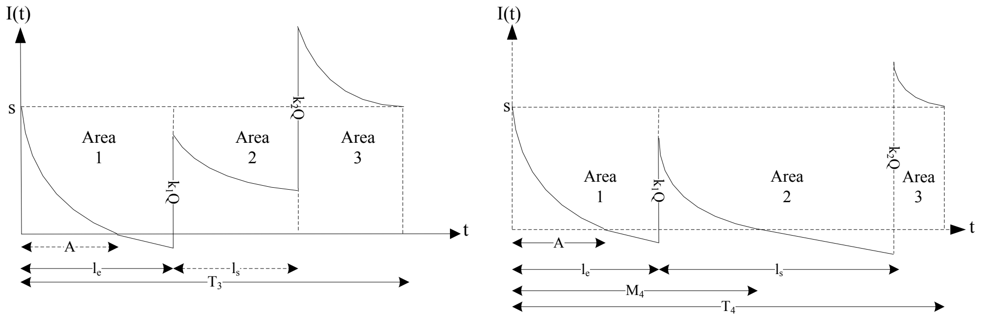
In the DSDI model, it is assumed that the retailer can order from two alternative suppliers: one is similar to the supplier of the SSDI model and the other one offers a cheaper price ($c_2 \leq c_1$) with a longer lead time mean.

For modeling the DSDI problem there will be two scenarios, depending on which supplier delivers first. In the first scenario L_1 is less than L_2 ($L_1 < L_2$). So a quantity k_1Q will be received L_e time units after the placement of orders from the first supplier. Subsequently, a quantity k_2Q will be received from the second supplier after a time lag, $L_s=L_2-L_1$. In the second scenario, inversely $L_2 \leq L_1$ and k_2Q will be received first. We develop the general formulas for the first scenario, and can then apply them to the second one.

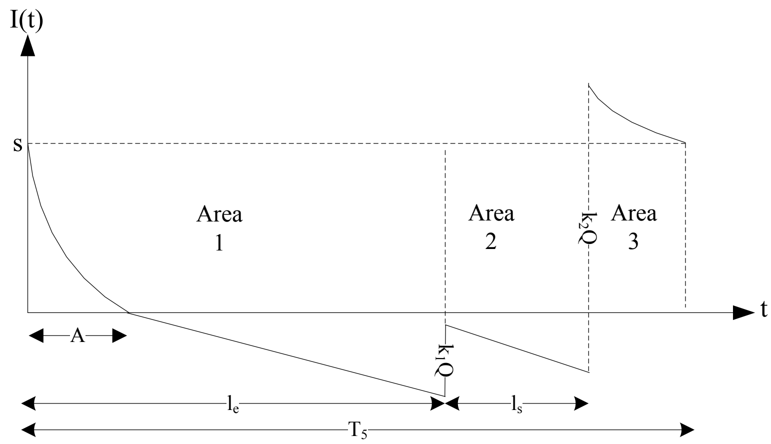
For the first scenario, as in the approach taken for the SSDI model, depending on the occurrence of backordered demand in L_e or L_s, and considering the renewal assumption (A.8), there will be five distinct combinations of L_e and L_s that would cover all possible states, as shown in Figure.4-4.



(a) State 1 (b) State 2



(c) State 3 (d) State 4



(e) State 5

Figure 4--4 Possible states of dual-source (DSDI) model (the first scenario)

According to Figure 4-4, in order to simplify mathematical modeling, the replenishment cycle time of each state i , $[0,T_i]$, $i=1,..,5$ is divided into three areas:

1. Area 1: effective lead time, $0\leq t< L_e$.
2. Area 2: Inter arrival time, $L_e \leq t< L_e+ L_s$.
3. Area 3: After arrival time, $L_e+ L_s \leq t\leq T_i$.

Generally, if $I_{ij}(t)$ and $B_{ij}(t)$ indicate on-hand inventory and backorder function respectively for state i $(i=1,\dots,5)$ and Area j $(j=1, 2, 3)$ as follows:

$$I_i(t) = \begin{cases} I_{i1}(t) & Lt_{i1} \leq t \leq Ut_{i1} \\ I_{i2}(t) & Lt_{i2} \leq t \leq Ut_{i2} \\ I_{i3}(t) & Lt_{i3} \leq t \leq Ut_{i3} \end{cases} \qquad i = 1, \dots ,5 \tag{4-11}$$

$$B_i(t) = \begin{cases} B_{i1}(t) & Ltb_{i1} \leq t \leq Utb_{i1} \\ B_{i2}(t) & Ltb_{i2} \leq t \leq Utb_{i2} \\ B_{i3}(t) & Ltb_{i3} \leq t \leq Utb_{i3} \end{cases} \qquad i = 1, \dots ,5 \tag{4-12}$$

then the first scenario's expected total cost in a cycle, $E(TCC^1)$, can be written as follows:

$$E(TCC^1) = \sum_{i=1}^{5} E_{State-i}(TCC^1) = \sum_{i=1}^{5} \int_{LL_{ei}}^{UL_{ei}} \int_{LL_{si}}^{UL_{si}} (\sum_{j=1}^{3}(h.\int_{Lt_{ij}}^{Ut_{ij}} I_{ij}(t)dt + \pi.\int_{Ltb_{ij}}^{Utb_{ij}} B_{ij}(t)dt) + c_w.(Q - D.T_i)).f_{L_s|L_e}(l_s|l_1).f_{L_e}(l_e)\, dl_s\, dl_e + \alpha.O_c + c_w.Q \tag{4-13}$$

In Equation (4-13) c_w shows the weighted procurement cost, that is, $k_1c_1+ k_2c_2$.

Let $E(T^1)$ be the expected replenishment cycle time in the first scenario. It can be evaluated from Equation (4-14) as:

$$E(T^1) = \sum_{i=1}^{5} \int_{LL_{ei}}^{UL_{ei}} \int_{LL_{si}}^{UL_{si}} T_i \, . f_{L_s|L_e}(l_s|l_1).f_{L_e}(l_e)dl_s dl_e \tag{4-14}$$

Consequently, by considering the first and second scenarios with the probability of occurrence ρ_1 and ρ_2 respectively (ρ_{1+} $\rho_{2=1}$), the DSDI's expected total cost in a cycle, $E(TCC)$, and the expected value of a replenishment cycle, $E(T)$ can be determined as follows:

$$E(TCC) = \rho_1 E(TCC^1) + \rho_2 E(TCC^2) \quad (4\text{-}15)$$
$$E(T) = \rho_1 E(T^1) + \rho_2 E(T^2) \quad (4\text{-}16)$$

where $E(TCC^2)$ and $E(T^2)$ indicate the second scenario's expected total cost in a cycle and its expected cycle time, respectively.
Finally, the DSDI model is derived as Equation (4-17) for $Q \geq r \geq 0$:

$$\text{Min } E(TCU) = \frac{E(TCC)}{E(T)} \quad (4\text{-}17)$$

s.t.
$k_1 + k_2 = 1$
$k_1,\ k_2 \geq 0$

Constraints of the DSDI model guarantee that in each replenishment cycle two non-negative order quantities whose sum is equal to Q are received.

In the following, the DSDI model's components are explained in more detail. Some formulations of the first state are explained here and the others are presented in Appendix 1.

The inventory and backorder function of State 1 are determined as follows:

$$I_1(t) = \begin{cases} I_{11}(t) = -\frac{D}{\theta} + \left(r + \frac{D}{\theta}\right) e^{-\theta t} & 0 < t \leq l_e \\ I_{12}(t) = -\frac{D}{\theta} + \left(r + \frac{D}{\theta} + k_1 Q e^{\theta l_e}\right) \mathrm{e}^{-\theta t} & l_e < t \leq l_e + l_s \\ I_{13}(t) = -\frac{D}{\theta} + \left(r + \frac{D}{\theta} + k_1 Q e^{\theta l_e} + k_2 Q e^{\theta (l_e + l_s)}\right) e^{-\theta t} & l_e + l_s < t \leq T_1 \end{cases} \quad (4\text{-}18)$$

$$B_1(t) = 0 \qquad 0 \leq t \leq T_1 \quad (4\text{-}19)$$

Equation (4-18) is determined by general Equation (4-1) and initial conditions $I_{11}(0) = r$, $I_{12}(l_e) = I_{11}(l_e) + k_1 Q$ and $I_{13}(l_e + l_s) = I_{12}(l_e + l_s) + k_2 Q$ regarding Figure4-4 (a). According to Figure 4-4(a), no backorder is incurred in this state. So, $B_{1j}(t) = 0$ for $j = 1, 2, 3$.

Thus, State 1's expected total cost in a cycle under the first scenario, $E_{State-1}(TCC^1)$, can be written as Equation (4-20):

$$E_{State-1}(TCC^1) = \int_{LL_{e1}}^{UL_{e1}} \int_{LL_{s1}}^{UL_{s1}} \left(h.\left(\int_0^{l_e} I_{11}(t)dt + \int_{l_e}^{l_e+l_s} I_{12}(t)dt + \int_{l_e+l_s}^{T_1} I_{13}(t)dt\right) + c_w(Q - D.T_1)\right).f_{L_s|L_e}(l_s|l_1).f_{L_e}(l_e)\, dl_s\, dl_e \quad (4\text{-}20)$$

In Equations (4-18) and (4-20), the cycle time length in this state, T_1, is obtained by solving $I_{13}(T_1)=r$ as follows:

$$T_1 = -\frac{1}{\theta}\ln\left(\frac{D+r\theta}{r\theta+D+k_1Q\theta e^{\theta l_e}+k_2Q\theta e^{\theta(l_e+l_s)}}\right) \quad (4\text{-}21)$$

Since there is no backorder during l_e, in Equation (4-20) $LL_{e1}=0$ and $UL_{e1}=A$. Here, the point A represents the point at which on-hand inventory becomes zero during the effective lead time (Area 1). The arithmetic formula of A obtained for all five states of the DSDI model as well as that of the SSDI model are the same. So, in the DSDI model, it can also be obtained by Equation (4-7).

With regard to l_s, we must be more accurate and consider the following two conditions:

1- $l_s+l_e \leq M_1$. As shown in Figure 4-4, Here, for each state i, $i=1,..,5$, the point at which on-hand inventory becomes zero during the inter-arrival time is denoted M_i. So, for this state, M_1 can be determined by solving $I_{12}(M_1)=0$ as Equation (4-22):

$$M_1 = -\frac{1}{\theta}\ln\left(\frac{D}{r\theta+D+k_1Q\theta e^{\theta l_e}}\right) \quad (4\text{-}22)$$

2-$I_{13}(l_e+l_s)\geq r$ to keep assumption A.8.

These conditions simultaneously result in LL_{s1} and UL_{s1} as follows:

$$LL_{s1} = 0 \quad (4\text{-}23)$$

$$UL_{s1} = \min\left\{M_1 - l_e, -\frac{1}{\theta}\ln\left(\frac{r\theta+D-k_2Q\theta}{(r\theta+D)e^{-\theta l_e}+k_1Q\theta}\right)\right\} \quad (4\text{-}24)$$

In a similar way, cost elements of other states can be obtained, which are explained in Appendix 1. For each state i, the length of replenishment cycle time, T_i, and the point at which on-hand inventory becomes zero during the inter-arrival time ,M_i , can be found by solving Equations (4-25) and (4-26) respectively:

$$I_{i3}(T_i) = s \qquad i = 1, \dots, 5 \quad (4\text{-}25)$$

$$I_{i2}(M_i) = 0 \qquad i = 1, \dots, 5 \quad (4\text{-}26)$$

DSDI model by considering negative exponential lead times: In this case, as each of the two lead times independently follow the negative exponential distribution function, the probability distribution function of effective lead time, $L_e=min\{L_1,L_2\}$, is also exponential with parameter $\lambda_1 + \lambda_2$ as follows:

$$f_{L_e}(l_e) = (\lambda_1 + \lambda_2)e^{-(\lambda_1+\lambda_2)l_e} \quad l_e \geq 0 \tag{4-27}$$

Owing to the lack-of-memory property, if $L_e=L_1$, the distribution function of inter-arrival time, $L_s=|L_1-L_2|$ is the same as that of L_2. Otherwise it is the same as the distribution function of L_1. So, by using (4-27) as the density function of the effective lead time and $f_{L_s|L_e}(l_s|l_1) = \lambda_2 . e^{\lambda_2 . l_s}, l_s \geq 0$ as the density function of the inter-arrival time in Equations (4-13) and (4-14) the formula can be derived for the first scenario.

By considering the first and second scenarios with the probability of occurrence $\rho_1 = \frac{\lambda_1}{\lambda_1+\lambda_2}$ and $\rho_2 = \frac{\lambda_2}{\lambda_1+\lambda_2}$ respectively, the DSDI's expected total cost in a cycle, *E(TCC)*, and the expected value of a replenishment cycle, *E(T)* can be determined by (4-15) and (4-16). Finally DSDI problem is modeled by considering independently exponential distributed lead times in (4-17).

4-6- Solution Approach

According to Table 4-1, most researchers have referred to the complexity of their order splitting models, which has made the calculation methods intractable, and preferred to use the numerical search methods rather than the exact ones (even for single sourcing models) . Only in limited number of researches the exact solution methods are used but in most of them a special case (e.g. single period inventory system, identical suppliers) is assumed (e.g. Iakovou, 2010 and Sajadieh and Eshghi, 2009).

Non-linear inventory functions used to consider deterioration, complicated parametric form of integral limits, and elliptic functions make the proposed model difficult either to find a closed-form statement for decision variables or to solve by an exact method. For instance showing that Hessian matrix can be positive-definite is not tractable.

The SSDI and DSDI Models are categorized as non-linear optimization models with twice continuously differentiable objective functions at $Q>r>0$. We therefore use the iterative method, Sequential Quadratic Programming (SQP) algorithm access via Maple 11 to solve the models. A general explanation on this method is provided in Appendix 2. To calculate integrals over elliptic functions, we use the trapezoidal rule method.

We solve the two models using the following parameter values that are determined with the help of the existing literature (Ramasesh et al. 1993):

D=10,000, λ_1=24, λ_2=20, c_1=5, c_2=4.95, O_c=100, π=10, h=1, α=1, θ=0.14. Using the SQP algorithm, we obtain the following:

	SSDI model	DSDI model (k_1=0.4577)
Reorder point, r^*, units	948	440
Total quantity, Q^*, units	1655	2995
Expected deterioration cost per unit time ($E(DCU)^*$), €	4978.06	3176.69
Expected total cost per unit time ($E(TCU)^*$), €	67826.66	59253.45

Thus, the dual-source model yields 12.6% saving in expected total cost per unit time and 36.2% saving in deterioration cost per unit time when $\alpha=1$. Theoretically, this is because the effective lead time will tend to have a lower mean and variance compared to those of the parent distributions of lead times. Obviously the obtained savings will decrease when $\alpha>1$. This is explained further in the next section.

According to the decision variable quantities, the probability that the assumption A.8. is contradicted in the SSDI model is:

$$P(L > E(T)) = e^{-\lambda . E(T)} = 0.0329$$

And if $Max\ \{L_1, L_2\}$ is represented by a stochastic variable L_{max}, its probability distribution function will be as follows:

$$f_{L_{max}}(l_{max}) = \lambda_1 e^{-\lambda_1 l_{max}} + \lambda_2 e^{-\lambda_2 l_{max}} - (\lambda_1 + \lambda_2) e^{-(\lambda_1+\lambda_2) l_{max}} \qquad l_{max} \geq 0 \tag{4-28}$$

So, according to Equation (4-28), the probability of contradicting the assumption A.8. in the DSDI model is equal to:

$$P(L_{max} > E(T)) = e^{-\lambda_1 . E(T)} + (1 - e^{-\lambda_1 . E(T)}) e^{-\lambda_2 . E(T)} = 0.0074$$

Clearly, these probabilities are negligible.

4-7- Sensitivity Analysis

In this section a sensitivity analysis is done for both the SSDI and DSDI models, separately. Then these two models are compared to derive managerial insights around the proposed framework.

To study the impact of deterioration rate as well as suppliers' lead time on the proposed models, first the above test problem is solved for different values of λ_I=24, 16, 14, and θ=0.04, 0.07, 0.09, 0.14, 0.24 using the SSDI model. The unit holding cost, backordering cost and ordering cost are assumed to be 1, 10 and 100, respectively. The demand rate is considered to be 10,000 and the unit procurement cost is assumed to be 5. The results are reported in Table 4-2.

Table 4-2-Effects of λ_I and θ on the SSDI model

	D=10000 h=1 $\pi = 10$ O_c=100 $c_{I=}$ 5.00					
	Deterioration rate (θ)	0.04	0.07	0.09	0.14	0.24
	Reorder point (r^*)	1612	1589	1359	948	749
	Order quantity (Q^*)	1816	1757	1721	1655	1596
	cycle time ($E(T)^*$)	0.1744	0.1634	0.1566	0.1423	0.1213
	Q^*/D	0.1816	0.1757	0.1721	0.1655	0.1596
λ_I=24	$E(DCU)^*$	4183.48	4632.00	4723.63	4978.06	6375.94
	$E(TCU)^*$	60800.35	63449.82	64568.75	67826.66	77564.20
	Reorder point (r^*)	1805	1766	1441	1169	853
	Order quantity (Q^*)	2147	2096	2030	1936	1844
	cycle time ($E(T)^*$)	0.2023	0.1914	0.1815	0.1637	0.1385
λ_I=16	Q^*/D	0.2147	0.2096	0.2030	0.1936	0.1844
	$E(DCU)^*$	4489.54	4953.05	5038.20	5396.08	6550.68
	$E(TCU)^*$	62640.22	65297.46	66217.01	69835.57	79278.46
	Reorder point (r^*)	1808	1769	1505	1206	918
	Order quantity (Q^*)	2334	2286	2273	2268	2395
λ_I=14	cycle time ($E(T)^*$)	0.2178	0.2069	0.2017	0.1892	0.1767
	Q^*/D	0.2334	0.2286	0.2273	0.2268	0.2395
	$E(DCU)^*$	4607.88	5018.93	5117.13	5450.29	6623.93
	$E(TCU)^*$	63538.32	66133.10	67193.29	69329.03	81342.41

According to Table 4-2, in all cases the cycle time is lower than Q^*/D. It is a consequence of the deterioration process. In addition, whenever the deterioration rate increases, the cycle time decreases accordingly. Therefore, it can be concluded that the cycle time has an inverse relationship with the deterioration rate.

This is similar to what happens in real cases. For example, in firms dealing with products that deteriorate fast, the frequency with which the orders are released is greater than in the case of slowly deteriorating products. Moreover,

Table 4-3-Effects of λ_2 and θ on the DSDI model (α=*1*)

D=10000 h=1 $\pi = 10$ O_c=100 $c_{1=}$ 5.00 $c_{2=}$4.95 λ_1=24						
Deterioration rate (θ)		0.04	0.07	0.09	0.14	0.24
	Reorder point (r^*)	505	456	451	440	435
λ_2=20	Order quantity (Q^*)	3289	3122	3084	2995	2922
	k_1	0.4794	0.4565	0.4569	0.4577	0.4844
	cycle time $(E(T)^*$	0.3224	0.3018	0.2950	0.2786	0.2533
	Q^*/D	0.3289	0.3122	0.3084	0.2995	0.2922
	$E(DCU)^*$	2757.64	2968.32	3022.93	3176.69	3772.98
	$E(TCU)^*$	56190.62	57414.98	57876.06	59253.45	63994.36
	Reorder point (r^*)	512	506	497	481	476
λ_2=16	Order quantity (Q^*)	3610	3545	3484	3387	3265
	k_1	0.4148	0.4272	0.4300	0.4345	0.4470
	cycle time $(E(T)^*)$	0.3548	0.3434	0.3341	0.3162	0.2785
	Q^*/D	0.3610	0.3545	0.3484	0.3387	0.3265
	$E(DCU)^*$	3262.91	3376.86	3429.15	3554.36	4312.11
	$E(TCU)^*$	57066.26	57887.90	58299.83	59726.15	65708.74
	Reorder point (r^*)	552	541	534	505	477
λ_2=12	Order quantity (Q^*)	4221	4145	4090	3858	3644
	k_1	0.4158	0.4232	0.4225	0.4267	0.4317
	cycle time $(E(T)^*)$	0.4149	0.4016	0.3925	0.3606	0.3095
	Q^*/D	0.4221	0.4145	0.4090	0.3858	0.3644
	$E(DCU)^*$	3884.79	4227.09	4294.25	4402.06	5336.70
	$E(TCU)^*$	58102.26	59075.66	59478.57	60878.78	66944.27

As seen in Table 4-2 for each $\lambda_{1,}$ the reorder point decreases when the deterioration rate increases. In fact, when the deterioration rate increases and the other parameters remain unchanged, the model tries to stay away from the deterioration by reducing the reorder point. In addition, for a specific value of θ, the reorder point value will be increased when the parameter λ_1 decreases.

Note that λ_1 is the parameter of the lead time distribution function in which the mean and variance are equal to ($1/\lambda_1$) and ($1/\lambda_1^2$) respectively.

In order to analyze the impact of λ_2 and θ on the DSDI model, a new supplier with c_2=4.95 is added by assuming $D=10000$, $h=1$, π=10, O_c=100, c_1=5 and λ_1=24. Then the parameters λ_2 and θ are changed and the problem is solved for each pair of (λ_2 and θ) by using the DSDI model. The results are reported in Tables 4-3 and 4-4.

Table 4-4-Effects of λ_2 and θ on the DSDI model ($\alpha=1.5$)

D=10000 h=1, π = 10 O_c=100 c_1= 5.00 c_2=4.95 λ_1=24						
Deterioration rate (θ)		0.04	0.07	0.09	0.14	0.24
	Reorder point (r^*)	481	478	472	459	437
λ_2=20	Order quantity (Q^*)	3308	3247	3204	3105	2939
	k_1	0.4587	0.4521	0.4625	0.4632	0.4443
	cycle time ($E(T)^*$)	0.3235	0.3119	0.3043	0.2856	0.2537
	Q^*/D	0.3308	0.3247	0.3204	0.3105	0.2939
	$E(DCU)^*$	3025.30	3261.47	3304.79	3344.39	3895.12
	$E(TCU)^*$	56724.49	57718.56	58353.27	60387.22	64456.55
	Reorder point (r^*)	535	525	519	504	477
λ_2=16	Order quantity (Q^*)	3733	3649	3596	3478	3281
	k_1	0.4217	0.4329	0.4336	0.4344	0.4467
	cycle time ($E(T)^*$)	0.3646	0.3500	0.3406	0.3186	0.2810
	Q^*/D	0.3733	0.3649	0.3596	0.3478	0.3281
	$E(DCU)^*$	3306.38	3578.77	3622.63	3778.74	4560.30
	$E(TCU)^*$	57882.88	59059.35	59649.59	61419.58	65934.93
	Reorder point (r^*)	712	701	677	570	537
λ_2=12	Order quantity (Q^*)	4563	4517	4501	4128	3874
	k_1	0.3902	0.4049	0.3883	0.4005	0.4340
	cycle time ($E(T)^*$)	0.4380	0.4200	0.4096	0.3626	0.3131
	Q^*/D	0.4563	0.4517	0.4501	0.4128	0.3874
	$E(DCU)^*$	4108.31	4360.40	4431.33	4574.63	5568.36
	$E(TCU)^*$	59946.60	61739.32	62642.62	64617.87	70509.76

As shown in Tables 4-3 and 4-4, when the deterioration rate (θ) increases, the optimal values of the reorder point (r^*) and cycle time ($E(T)^*$) in all cases will decrease. In fact, the model tries to keep the deteriorated items at the lowest level as much as possible. On the other hand, irrespective of the mean ($1/\lambda_2$) and variance ($1/\lambda_2^2$) of the second supplier lead time increase, the reorder point (r^*) will increase as well. As expected, these results are the same as those obtained with the SSDI model.

Comparing each column of Table 4-3 with its corresponding column in Table 4-4 shows that the cycle time will be longer when the ordering cost multiplier (α) increases. In other words, when the ordering cost increases the model attempts to reduce the ordering frequency. As a consequence, the cycle time increases.

4-7-1- Comparison between the SSDI and DSDI models

In order to highlight the advantages of each strategy (single and dual-source models) we compare the results of Tables 4-3 and 4-4 (the DSDI model) with the results reported in Table 4-2 for λ_1=24 (the SSDI model). In this way, the savings are calculated in both expected deterioration cost per unit time ($E(DCU)^*$) and expected total cost per unit time ($E(TCU)^*$) that are caused by dual-sourcing policy. The results obtained are reported in Tables 4-5 and 4-6.

Table 4-5-Savings due to the dual-sourcing (DSDI) model (α=*1*, λ_1=24)

Deterioration rate (θ)		0.04	0.07	0.09	0.14	0.24
λ_2=20	saving in *ETCU*	0.076	0.095	0.104	0.126	0.175
	saving in *EDCU*	0.341	0.359	0.360	0.362	0.408
λ_2=16	saving in *ETCU*	0.061	0.088	0.097	0.119	0.153
	saving in *EDCU*	0.220	0.271	0.274	0.286	0.324
λ_2=12	saving in *ETCU*	0.044	0.069	0.079	0.102	0.137
	saving in *EDCU*	0.071	0.087	0.091	0.116	0.163

One significant insight derived from Tables 4-5 and 4-6 is that once the deterioration rate, θ, increases, the amount of the expected deterioration cost ($E(DCU^*)$) as well as the total cost ($E(TCU)^*$) saved by dual-sourcing strategy increase considerably. This means that in a supply chain and under stochastic lead times, when the retailer buys deterioration items, order splitting is more attractive in comparison with the single sourcing. One can intuitively expect this result because when an order is split, the deterioration process of the second part is postponed. So, we can say that order splitting not only decreases holding and shortage costs, as shown in the literature (Thomas and Tyworth, 2006), but can also decrease the deterioration costs.

Regarding the split proportion, as the numerical results of Tables 4-3 and 4-4 show, when the deterioration rate increases, the system tends to balance the relative proportions of each supplier (which also depends on each supplier's price) to avoid getting a big order from one supplier which would consequently bear most of the deterioration cost. For example, when $\lambda_1=24$, $\lambda_2=20$ and $\alpha=1.5$, k_1 will be equal to 0.4521 at $\theta=0.07$. It then increases to 0.4632 and finally decreases to 0.4443 as θ increases.

Table 4-6-Savings due to the dual-sourcing (DSDI) model ($\alpha=1.5$, λ_1=24)

Deterioration rate (θ)		0.04	0.07	0.09	0.14	0.24
λ_2=20	saving in *ETCU*	0.067	0.090	0.096	0.110	0.169
	saving in *EDCU*	0.277	0.296	0.300	0.328	0.389
λ_2=16	saving in *ETCU*	0.048	0.069	0.076	0.094	0.150
	saving in *EDCU*	0.210	0.227	0.233	0.241	0.285
λ_2=12	saving in *ETCU*	0.014	0.027	0.030	0.047	0.091
	saving in *EDCU*	0.018	0.059	0.062	0.081	0.127

As Tables 4-3 and 4-4 indicate, the reorder quantity in the dual-source model is less than that in the single-source model. This result confirms the advantage of splitting orders to provide a more effective (at least the same) service level in a stochastic environment.

From the results presented in Tables 4-5 and 4-6, it can be seen that for a given value of λ_1, the savings decrease as λ_2 decreases. In other words, as the lead time reliability (lead time mean and variance) of the second source declines, dual-sourcing will be less appealing. This can be explained more as follows: when λ_2 decreases beyond a certain point, the savings in expected procurement, holding, backorders and deterioration costs may not be sufficient to compensate for the increase in the ordering costs. This suggests that there is a lower limit to the value of λ_2 for generating savings through split ordering. In other words, if the second supplier is much more unreliable than the first one, splitting orders might not be cost-effective. The lower limit of λ_2 depends on the other parameters of the system, such as the procurement costs of each supplier, for example.

Additionally, when the ordering cost multiplier (α) increases from 1 to 1.5, the savings from dual-sourcing decrease. So, if the cost of placing an order is too high, it would tend to make the two-source policy ineffective. Nowadays, with the improvement of e-commerce technology, it is possible to reduce the ordering cost and to use the advantages of order splitting in many situations. In the next section, a usage of proposed models in the real world is presented.

4-8- Practical example – a case study

In this section a real case is studied to assess the applicability of mathematical models in the real world. Consider a pharmaceutical network with some Hospital Pharmacies (H.P.) and care services (C.S.). In this network the care services (C.S.) meet their requirements by procurement from hospital pharmacies (H.P.). There are two types of hospital pharmacy: sellers and buyers (Figure 4-5).

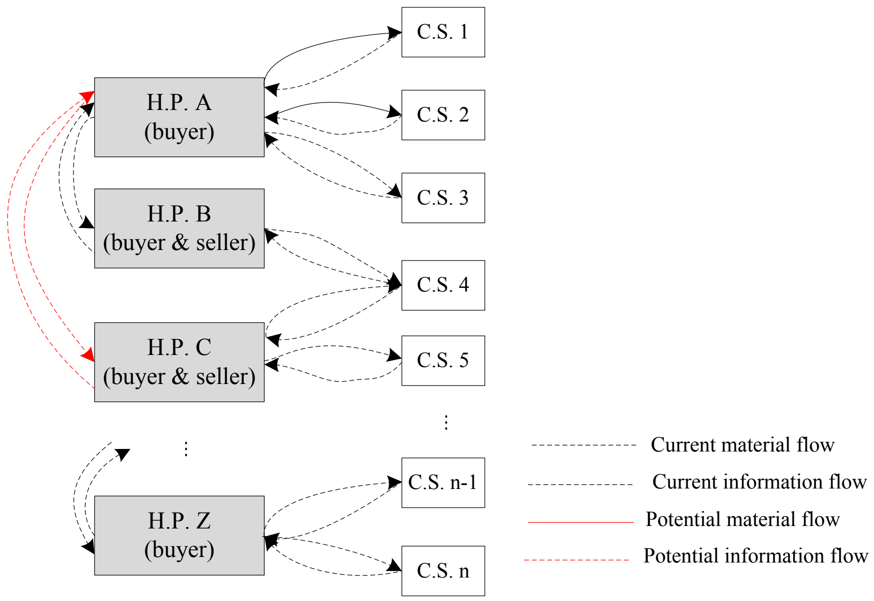


Figure 4--5 Schematic scheme of the case studies' network

Seller hospital pharmacies are those which serve their own care service(s) (external demand) as well as a number of buyer hospital pharmacies. In this way, buyer hospital pharmacies (retailers) fulfill their own requirements from seller hospital pharmacies (suppliers) in order to then meet the requirements of their care services (external demand). This structure is explained in more detail in Baboli et al. (2011).

Currently, the buyer hospital pharmacy, *A*, meets all its requirements for antibiotic *X* from the seller hospital pharmacy, *B*, based on the single-sourcing approach and a continuous review inventory *(r, Q)* policy. Annual demand from all of its care services is D=1750 units. Its holding and backorder costs are estimated as h=€70/unit/year, and π=€148/unit/year, respectively. Its ordering cost is estimated as O_c=€112. This consists of the cost of making and sending the purchase order, the cost of handling receipt transactions involving reception, incoming inspection and storage, and finally administrative and overhead costs.

The antibiotic X is not produced commercially; it is manufactured specially by hospital pharmacies. The lifespan of the product is at the most three months in the refrigerated warehouses of seller hospital's pharmacies, and it deteriorates at the rate of $\theta=0.16$ per year. Currently, hospital pharmacy B meets the requirements of hospital pharmacy A within 2 days on average ($\lambda_1=183$), with each unit selling price $c_1=150$. In order to avoid high holding and deterioration costs, the hospital pharmacy B tries to deliver the orders as soon as possible. So in this case a negative exponential distributed lead time is appropriate for modeling the supply lead time.

Our objective is to ascertain whether the hospital pharmacy A keeps its current single-sourcing policy or switches it to dual-sourcing since seller hospital pharmacy C can also produce antibiotic X.

Based on the SSDI model and using Equation (4-4), the annual expected total cost of hospital pharmacy A (retailer) is obtained as $E(TCU)^*$=€363,928. The reorder point and order quantity are also equal to $r^*=104$ and $Q^*=124$, respectively. Hence, the expected total deterioration cost incurred per year is €28,325.

If hospital pharmacy A changes its sourcing policy to dual-sourcing (sending orders to both hospital pharmacies B and C), then the ordering cost increases to O_c=€135 (α=1.2) (as a result of the increasing costs of handling receipt transactions). Since hospital pharmacy C uses a transportation system with a lower performance, it sends the orders within 4 days on average (λ_2=91). The selling price of hospital pharmacy C is c_2=€145 per unit.

Based on the DSDI model, the optimal policy of hospital pharmacy A is obtained as r^*=18, Q_B^*=51 and Q_c^*=83 with annual expected total costs $E(TCU)^*$=€306,046. In this situation, the expected total deterioration cost incurred per year is equal to €23,775. Consequently, hospital pharmacy A can take advantage of 6.13% savings in the annual expected total cost, and 16.04% savings in the annual expected deterioration cost, by switching its sourcing policy to dual-sourcing.

4-9- Conclusion

In this chapter not only a new single source inventory model was proposed for a retailer offering items with constant deterioration rate – like alcohol, gasoline and some antibiotics – under negative exponential lead time; also an order splitting model for it was developed and these two models were compared. Thus the order splitting and deteriorating inventories issues were linked up. Because of complex mathematical form of developed non-linear models, the iterative method, Sequential Quadratic Programming (SQP) algorithm was used to solve the models numerically.

The numerical results and a real case study showed that when the deterioration items are exchanged in a supply chain, the order splitting policy is more attractive, especially when the suppliers' lead times are similarly distributed and of course there is no reliable supplier.

5

A BI-OBJECTIVE STOCHASTIC PROGRAMMING MODEL FOR A CENTRALIZED GREEN SUPPLY CHAIN WITH DETERIORATING PRODUCTS

5 A bi-objective stochastic programming model for a centralized green supply chain with deteriorating products

5-1- Introduction

Today, many logistics activities and mainly transportation are recognized as main contributors to depletion of the ozone layer due to carbon dioxide (CO_2) emissions, and also other gases (Harris et al. 2011). The major Greenhouse gases (GHGs) include carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), hydro fluorocarbons (HFCs), per fluorocarbons (PFCs), and sulfur hexafluoride (SF6) (Konyar, 2001). In many cases, the predominant greenhouse gas emitted is carbon dioxide (CO_2).

In this chapter we are going to study a centralized supply chain with a single supplier and a single retailer which offers a deteriorating product. Several transportation vehicles producing various greenhouse gas (GHG) levels exist at the supplier. The end costumer demands are uncertain and unfulfilled demands will be partially backordered. The objective is minimizing total cost and GHG emissions of supply chain simultaneously by taking into account specified capacity of transportation vehicles.

Considering uncertain demand, deteriorating products, partial backorder assumption, GHG emission and costs simultaneously and transportation capacity makes the problem close to real conditions. However makes it difficult to use classical inventory models (like models explained in Chapters 3 and 4). So, a mixed integer mathematical model is developed by applying a two-stage stochastic programming approach.

The rest of the chapter is organized as follows: in the next section a brief explanation on the two-stage stochastic programming approach used in this chapter is provided. The contribution of this chapter and the problem description are then described in Sections 5-3 and 5-4 respectively. In Section 5-5 the mathematical model developed is explained. A real case study and sensitivity analysis are presented in Sections 5-6 and 5-7, respectively. The chapter ends with concluding remarks in Section 5-8.

5-2- Two stage stochastic programming

Generally, for considering uncertain demand in the decision-making process, there are four main approaches: *stochastic programming, fuzzy programming, stochastic dynamic programming and robust optimization* (Sahinidis, 2004).
In this chapter, the *two-stage stochastic programming* approach is used to deal with uncertain demand. In this approach, the decision variables of an optimization model under uncertainty are classified into two stages. The first-stage variables have to be decided before the realization of the uncertain parameters. Once a specific scenario is occurred, further design or operational policy improvements can be made by selecting the appropriate values of the second-stage variables under that scenario. The objective is to choose the first-stage variables so that the sum of the first-stage costs and the expected value of the random second-stage costs are minimized.
In the two-stage stochastic programming approach, there are two different techniques to describe uncertainty: *i*) the distribution-based approach which is applied where a continuous range of potential future outcomes can be anticipated, and *ii*) the scenario-based approach which is applicable when the uncertainty is illustrated by a set of discrete scenarios forecasting how the uncertainty might take place in the future. Each scenario is associated with a probability level signifying the decision makers' (DM's) expectation of the occurrence of a particular scenario (Mirzapour Al-e-Hashem et al., 2012). The scenario-based approach is applied in this chapter that is practicable when a continuous range of future outcomes is not available.

5-3- Motivation and Contribution

Despite pressure from the investment community, government and consumers, many companies are not implementing significant change towards greener environmental practices (Kumar et al., 2012). One of the main concerns of industrial managers is that dealing with green considerations in decision-making processes may result in substantial profit reduction. There are some recurrent questions: How much should be spent in order to improve environmental quality? And, what are ‘best’ solutions for balancing ecological and economic concerns? (Quariguasi Frota Neto et al., 2008). In many cases therefore, to avoid economic loss, they are not motivated to consider green criteria and simply comply with some obligatory requirements.

The main objective of this chapter is to develop a new bi-objective mixed-integer programming model for a two-echelon centralized supply chain under uncertain demands, to be used at tactical-operational decision levels. The objectives include the minimization of expected total costs and expected GHG emissions. The applicability and efficiency of the proposed model are examined on a real case from the pharmaceutical industry.

The major contributions of this chapter that differentiate it from existing researches can be summarized as follows:

- ✓ We concentrate on deteriorating products in the proposed model, even though it is applicable to non-deteriorating ones as well. Despite of the fact that considering deteriorating products makes the model complicated, the significant role of managing inventory for this type of products in many corporations both financially and ecologically, motivates us to cope with this problem. A constant deterioration function as well as a non-linear holding cost function is considered to model deterioration process.
- ✓ A large body of green supply chain literature has emphasized on strategic, operational-strategic, and tactical-strategic decisions. As there are many green requirements today relating to tactical-operational decisions, the model proposed in this chapter is developed by considering tactical-operational decisions, including inventory and transportation.
- ✓ With respect to Dekker et al. (2012), determining eco-efficient frontiers using multi-objective mixed-integer programming models is quite new, despite the broad literature on multi-objective programming. To the best of our knowledge, this is the first attempt to address a bi-objective stochastic mixed-integer programming model for a green supply chain with deteriorating products. Our objective is to make a trade-off between costs and GHG emissions at tactical-operational level by determining eco-efficient frontier. In fact economic considerations are taking into account on one hand and green considerations on the other, to study interactions of economic and green considerations in a centralized supply chain. The proposed model can be used as a systematic decision-aid tool.
- ✓ We offer a new two-stage stochastic programming model under uncertain demand and partially backordered demands for a green two-echelon centralized supply chain. The model is able to determine

inventory policy and select transportation vehicles (to avoid the sub-optimalities led from separated decisions in each part), taking into account both costs and GHG emissions simultaneously. Diverse costs such as ordering cost, purchasing, holding, transportation, shortage and recycling costs are considered subject to transportation vehicles' capacities.

5-4- Problem description

A centralized two-echelon supply chain is assumed with a single buyer and a single supplier providing a deteriorating product, as shown in Figure 5-1. The lead time between the retailer and the supplier is fixed. There are several types of transportation vehicles at the supplier for transferring goods to the retailer. Each one is characterized by its own capacity, transportation cost and GHG emissions.

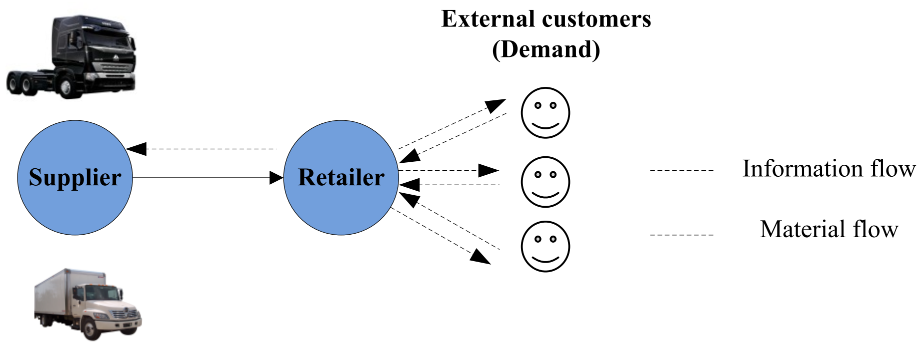


Figure 5-1 - General scheme of the considered supply chain

The retailer is faced with the customer's demand which is uncertain. Since the end-customer demands are highly variable, demand forecasting is possible only for two upcoming periods. All the planning in the supply chain is therefore based on projected demand in the two upcoming periods. This is particularly relevant to supply chains with highly variable demand, which propose highly perishable products.
The supply chain is faced with uncertain customer demand at the start of each period. Decisions have to be made about the first period's order quantity and transportation vehicles before uncertain demands (of the first and second periods) are revealed. During the first period the real demand becomes known and the inventory and/or shortage levels are specified accordingly. At the start

of the second period, according to what happened in the previous period and the uncertain nature of the demand in two next succeeding periods the order quantity must be determined and the appropriate vehicles selected accordingly. This process is applied to the third, fourth etc. periods. So the proposed model is applicable to supply chains which provide products just for two periods (like two-phase flu vaccines) or for multiple periods.

Because of the deterioration characteristic of products, it is assumed that a constant fraction of remaining products at the end of each period is deteriorated. A non-linear holding cost function is considered too. As mentioned before, for deteriorating products such as electronic components, radioactive substances, volatile liquids etc., where more sophisticated tools are required for their security and safety in stock, a non-linear stock-dependent holding cost may be appropriate. Based on the classification presented in Chapter 2 (Figure 2-1), we take the third class of deterioration modeling approach.

Moreover, a constant fraction of remaining products at each period should be recycled in order to be usable again.

The objective of the proposed two-stage stochastic programming model is to find the best configuration of vehicle types and order quantities in each period, in view of the demand uncertainty, in order to meet the following criteria simultaneously: minimizing the expected value of total costs and that of GHG emissions. The total cost of the supply chain includes the ordering cost, the purchasing cost, the holding cost, the shortage cost, the transportation cost and the recycling cost minus revenue earned. The quantity of GHG emissions is classified into three categories: *i*) the quantity of GHG produced by the vehicles during the product transportations; *ii*) the quantity of GHG produced by deteriorated items; and *iii*) the quantity of GHG produced during the recycling process.

5-4-1- Assumptions

Other assumptions of the proposed model are classified as assumptions related to the supplier, and assumptions related to the retailer, as follows:

5-4-1-1- Supplier's assumptions

- ✓ In order to respond to uncertain external demands of the supply chain, the supplier order the total quantity of each period's requirements to an external producer, then purchases the goods and finally transports them to the retailer. The ordering, purchasing and transportation costs are therefore incurred by the supplier.
- ✓ GHG produced by the supplier are due to the transportation process.
- ✓ There are several types of transportation vehicle, with a certain capacity, for carrying products from the supplier to the retailer.
- ✓ Ordering cost per replenishment, A, and purchasing cost per unit of product, C, are constant.
- ✓ It is assumed that no deterioration is taken place during transportation of the product.

5-4-1-2- Retailer's assumptions

- ✓ The retailer faces an uncertain demand in each period, which is defined under different scenarios.
- ✓ It is assumed that all the demand of each period reaches the retailer at the beginning of the period. So, the holding cost corresponds only to in-stock products remaining at the end of each period.
- ✓ Since the retailer's demand is uncertain, shortages may occur at each period, which is assumed to be partially backordered.
- ✓ Constant fraction (β) of unfulfilled demand (shortage) at each period is backordered and transferred to the next period. The rest is considered as lost sales.
- ✓ θ percent of in-stock products remaining at each period deteriorates up to the beginning of the next one.
- ✓ α percent of in-stock products remaining at each period should be recycled.
- ✓ A product unit holding cost per period, h, is inventory-dependent and considered as follows:

$$h = \begin{cases} h_1 & 0 \le I \le i_1 \\ h_2 & i_1 \le I \le i_2 \\ & \dots \\ h_m & i_{m-1} \le I \end{cases} \tag{5-1}$$

Where I is the inventory level of the period and $0<h_1< h_2<\ldots< h_m$.

- ✓ The retailer's initial inventory level, I_0, can be positive, zero or negative.
- ✓ Selling price of each unit of product, p, is independent and constant.
- ✓ Holding, shortage (backorder and lost sale) and recycling costs of the supply chain are related to the retailer. It therefore causes GHG emissions through deterioration and recycling processes.

5-4-2- Notations

In order to formulate the described problem, a bi-objective two-stage stochastic programming model is proposed. The scenario-based approach is taken to describe uncertain demand. If SCE shows the set of N scenarios, $SCE=\{(DF_1, DS_1), (DF_2, DS_2), (DF_3, DS_3),\ldots, (DF_N, DS_N)\}$. So, the pair (DF_n, DS_n) shows the nth scenario that occurred with ρ_n probability where $1\le n\le N$. In other words, DF_n and DS_n indicate the demand of the first and the second next upcoming periods respectively under scenario n $(1\le n\le N)$.

The following notations are used in the proposed model:

Indexes:

n:	Index of scenarios, $n \in \{1,\ldots,N\}$
v:	Index of transportation vehicle's type, $v \in \{1,\ldots,V\}$
m:	Index of inventory level, $m \in \{1,\ldots,M\}$

Parameters:

A:	Fixed ordering cost
C:	A product unit purchase cost
p:	A product unit selling price
h:	A product unit holding cost per period
π:	Penalty cost of a product unit lost sale (including lost profit)
γ:	A product unit backordered cost per period
SV:	A product unit recycling cost
θ:	Deterioration rate of in-stock inventories
α:	Fraction of inventory required to be recycled at the end of each period
β:	Fraction of demand backordered during a stock-out
I_0:	Retailer's initial inventory level
DF_n:	Demand at the first period under scenario n
DS_n:	Demand at the second period under scenario n

ρ_n:	Occurrence probability of scenario *n*
TA_v:	Cost of transportation by a type-*v* vehicle
CAP_v:	Capacity of a type-*v* vehicle (defined by unit of product)
G_v:	GHG emission level produced by a type-*v* vehicle
GD:	GHG emission level produced by a unit of deteriorated product
Gw:	GHG emission level produced by a unit of recycled product
M:	A large value number
FTC:	Total cost of the first-stage
STC_n:	Total cost of the second-stage under scenario *n*
FTG:	Total GHG generated of the first-stage
STG_n:	Total GHG generated of the second-stage under scenario *n*
Z_1:	Expected total cost of the supply chain
Z_2:	Expected total GHG produced in the supply chain

First-stage decision variables:

QF:	Fixed order quantity in the first period
XF_v:	Number of type-*v* used vehicles in the first period

Second-stage decision variables:

IF_n:	inventory level at the end of the first period under scenario *n*
BF_n:	shortage level at the end of the first period under scenario *n*
QS_n:	order quantity of the second period under scenario *n*
IS_n:	inventory level at the end of second period under scenario *n*
BS_n:	shortage level at the end of second period under scenario *n*
XS_{vn}:	number of type-*v* used vehicles in the second period under scenario *n*

More explanations about the first and second-stage decision variables are presented in the following section.

5-5- Mathematical model

In order to model the described problem, a two-stage stochastic optimization approach is proposed. Based on (Leung and Ng, 2007) at first, some decisions must be made before uncertainty in demand emerges. These '*here and now*' decisions are the first-stage variables (design variables) that decide what quantity should be transported with which vehicles in the first period. The *recourse decisions (control decisions),* are those made after the uncertainty of the first-stage random parameters has been revealed. In fact, these '*wait-and-see*' recourse decisions model how the decision maker conforms to the unfolding of the uncertain demand. These are inventory- and shortage-level variables (of the first and second periods), as well as the variables that decide what quantity must be transported, with which vehicles, to the retailer under each possible scenario at the second period. These two successive decisions make up the two-stage model. The proposed model is thus as follows:

$$MinZ_1 = FTC + E(STC_n) = A.V + C.QF + \sum_v (TA_v.XF_v) + E(A.U_n + C.QS_n + \sum_v (TA_v.XS_{vn}) + \sum_m h_m.(IF_{nm} + IS_{nm}) + \alpha.SV.(IF_n + IS_n) + (1-\beta).\pi.(BF_n + BS_n) + \beta.\gamma.(BF_n + BS_n) - p.(QF1_n + I_0.YF_n + DF_n.(1-YF_n)) - p.(QS1_n + (1-\theta).f_n + DS_n.(1-YS_n) + \beta.BF_n - \beta.w_n)) \tag{5-2}$$

$$MinZ_2 = FTG + E(STG_n) = \sum_v (G_v.XF_v) + E(\sum_v (G_v.XS_{vn}) + GD.\theta.(IF_n + IS_n) + Gw.\alpha.(IF_n + IS_n)) \tag{5-3}$$

s.t.

$$QF + I_0 - DF_n = IF_n - BF_n \quad \forall n \tag{5-4}$$

$$QS_n + IF_n.(1-\theta) - DS_n - \beta.BF_n = IS_n - BS_n \quad \forall n \tag{5-5}$$

$$BF_n \le M.UF_n \quad \forall n \tag{5-6}$$

$$IF_n \le M.(1-UF_n) \quad \forall n \tag{5-7}$$

$$BS_n \le M.US_n \quad \forall n \tag{5-8}$$

$$IS_n \le M.(1-US_n) \quad \forall n \tag{5-9}$$

$$\sum_v CAP_v.XF_v \ge QF \tag{5-10}$$

$$\sum_v CAP_v.XS_{vn} \ge QS_n \quad \forall n \tag{5-11}$$

$$\frac{V}{M} \le QF \le M.V \tag{5-12}$$

$$\frac{U_n}{M} \le QS_n \le M.U_n \quad \forall n \tag{5-13}$$

$$IF_n = \sum_m IF_{nm} \quad \forall n \tag{5-14}$$

$$I_m.tF_{nm} \le IF_{nm} \le I_{m+1}.tF_{nm} \quad \forall n,m \tag{5-15}$$

$$\sum_m tF_{nm} = 1 \quad \forall n \tag{5-16}$$

$$IS_n = \sum_m IS_{nm} \quad \forall n \tag{5-17}$$

$$I_m.tS_{nm} \le IS_{nm} \le I_{m+1}.tS_{nm} \quad \forall n,m \tag{5-18}$$

$$\sum_m tS_{nm} = 1 \quad \forall n \tag{5-19}$$

$$QF1_n \le (DF_n - I_0).YF_n \qquad \forall n \tag{5-20}$$

$$(1 - YF_n).(DF_n - I_0) \le QF2_n \le M.(1 - YF_n) \qquad \forall n \tag{5-21}$$

$$QF1_n + QF2_n = QF \qquad \forall n \tag{5-22}$$

$$QS1_n \le DS_n.YS_n + \beta.w_n - (1-\theta).f_n \qquad \forall n \tag{5-23}$$

$$(1 - YS_n).DS_n + \beta.BF_n - \beta.w_n - (1-\theta).IF_n + (1-\theta).f_n \le QS2_n \le M.(\tag{5-24}$$

$$-YS_n.M \le f_n \le YS_n.M \qquad \forall n \tag{5-25}$$

$$IF_n - (1 - YS_n).M \le f_n \le IF_n + (1 - YS_n).M \qquad \forall n \tag{5-26}$$

$$-YS_n.M \le w_n \le YS_n.M \qquad \forall n \tag{5-27}$$

$$BF_n - (1 - YS_n).M \le w_n \le BF_n + (1 - YS_n).M \qquad \forall n \tag{5-28}$$

$$QS1_n + QS2_n = QS_n \qquad \forall n \tag{5-29}$$

$$XF_v, XS_{vn} \ge 0, \text{integer} \qquad \forall v, n \tag{5-30}$$

$$QF, QF1_n, QF2_n, QS_n, QS1_n, QS2_n, IF_n, IF_{nm}, BF_n, IS_n, IS_{nm}, BS_n, f_n, \tag{5-31}$$

$$V, U_n, YF_n, YS_n, tF_n, tS_n, UF_n, US_n \in \{0,1\} \qquad \forall n \tag{5-32}$$

In Equation (5-2), Z_1 is the objective function related to the expected value of total cost and in Equation (5-3) Z_2 is the objective function related to the expected value of the total GHGs generated, which are inconsistent.

Based on a two-stage stochastic programming approach, each objective function has mainly two components. The first objective function, Z_1, is composed of the first component *(FTC)* which is the ordering, purchasing and transportation cost in the first period. This part is not subject to uncertainty (first-stage decision variables) and hereafter is called the '*first-stage total costs (FTC)*'. In other words, *FTC* is incurred before realization of the scenarios. The second component of Z_1 is the expected value of the second-stage costs ($E[STC_n]$), consisting of ordering cost, purchasing costs, transportation costs (related to the second replenishment period) as well as inventory holding costs, recycling costs, shortage costs (lost sale as well as backordered demand), minus revenues (related to the both first and second replenishment periods). Z_2 likewise has two main components: *FTG,* GHG emissions through transportation in the first period, and $E(STG_n)$, expected GHG emissions from transportation, deterioration and recycling processes in the second period.

Constraints (5-4) and (5-5) are the inventory balance equations at the end of the first and second periods, respectively. While the products deteriorate at rate θ, in Equation (5-5) the initial inventory of the second period is considered as $IF_n(1\text{-}\theta)$. Also, β percent of unfulfilled demand in the first period is transferred to the second period, which in Equation (5-5) is considered as a part of that period's requirements.

Constraints (5-6) and (5-7) imply that IF_n and BF_n cannot take positive values concurrently. Constraints (5-8) and (5-9) likewise means $IS_n.BS_n=0$.

Constraints (5-10) and (5-11) guarantee the necessary transportation capacity needed for carrying the quantity orders.

Since the ordering cost, A, is incurred when the order quantities (QF and QS_n) are positive, binary variables, V and U_n, in addition to the constraints (5-12) and (5-13), are defined to model ordering costs as linear expressions in the first and second periods, respectively.

As mentioned above, the holding cost rate is as Equation (5-1), which is a non-linear inventory-dependent function. In order to formulate the holding cost of the first period inventories (IF_n) as a linear expression, we define binary variables, tF_m, and non-negative variables, IF_{nm}, and add constraints (5-14) to (5-16). The procedure of linearization of the holding cost of the second period inventories (IS_n) is the same. Binary variables, tS_m, and non-negative variables IS_{nm} are defined and constraints (5-17) to (5-19) are added.

The total sale in the first period under scenario n, SF_n, is as Equation (5-33):

$$SF_n = \begin{cases} QF + I_0, & DF_n \geq QF + I_0 \\ DF_n, & DF_n < QF + I_0 \end{cases} \tag{5-33}$$

We can convert the non-linear Equation (5-33) to a linear expression, $SF_n=QF1_n+I_0.YF_n+DF_n.(1\text{-}YF_n)$, with the help of binary variables, YF_n, non-negative variables $QF1_n$ and $QF2_n$, and adding constraints (5-20) to (5-22). In this way, in Equation (5-2) the expected revenue of the first period is modeled as a linear expression as $E(p.(QF1_n+I_0.YF_n+DF_n.(1\text{-}YF_n)))$.

The total sale in the second period under scenario n, SS_n, is as Equation (5-34):

$$SS_n = \begin{cases} QS_n + (1-\theta).IF_n, & DS_n + \beta.BF_n \geq QS_n + (1-\theta).IF_n \\ DS_n + \beta.BF_n, & DS_n + \beta.BF_n < QS_n + (1-\theta).IF_n \end{cases} \tag{5-34}$$

The linearization of Equation (5-34) is implemented under two steps. In the first step, Equation (5-34) is transformed to Equation (5-35) using binary variables, YS_n, and non-negative variables $QS1_n$ and $QS2_n$:

$$SS_n = QS1_n + (1-\theta).IF_n.YS_n + (DS_n + \beta.BF_n).(1-YS_n) \tag{5-35}$$

Where the following constraints must be added to the main model:

$$QS1_n \le (DS_n + \beta.BF_n - (1-\theta).IF_n).YS_n \qquad \forall n \tag{5-36}$$

$$(DS_n + \beta.BF_n - (1-\theta).IF_n).(1-YS_n) \le QS2_n \le M.(1-YS_n) \quad \forall n \tag{5-37}$$

$$QS1_n + QS2_n = QS_n \qquad \forall n \tag{5-38}$$

Then in the second step, to transform the product terms $IF_n.YS_n$ and $BF_n.YS_n$ (in Equations (5-35) to (5-37)) into linear terms, two non-negative variables $f_n=IF_n.YS_n$ and $w_n=BF_n.YS_n$ are introduced. Thus, Equation (5-35) is changed to (5-39) as follows:

$$SS_n = QS1_n + (1-\theta).f_n + DS_n.(1-YS_n) + \beta.BF_n - \beta.w_n \tag{5-39}$$

Constraints (5-36) and (5-37) are replaced by two constraints (5-23) and (5-24). Constraints (5-25) and (5-26) are added to convert non-linear equation $f_n=IF_n.YS_n$ into a linear one. Equation $w_n=BF_n.YS_n$ is likewise replaced by Constraints (5-27) and (5-28). Constraint (5-29) is equivalent to Equation (5-38). As a result in Equation (5-2), the expected revenue of the second period is modeled as a linear expression as $E(p.(QS1_n + (1-\theta).f_n + DS_n.(1-YS_n) + \beta.BF_n - \beta.w_n))$.

Constraints (5-30) to (5-32) determine the types of variables.

5-6- Case example: a case from the downstream pharmaceutical industry

In the following the proposed model is applied to a real case in order to: *i*) show that the theoretical framework explained in the previous section can easily be implemented, *ii*) provide some sensitivity analysis and develop managerial insights around the proposed framework.

A two-echelon pharmaceutical supply chain with one retailer and one supplier is considered. A radiopharmaceutical product is provided by a central pharmacy (supplier) and transport to a radio pharmacy service (retailer) in order to treat patients. A centralized decision-making process is applied in this supply chain. Without loss of generality, we report rounded data.

Since the number of patients and the amount of medication for each is uncertain and highly variable from one period to another, only demand forecasting for the next two periods is reliable. At the start of each period, some scenarios are defined by the directors based on planned appointments, and the order quantity is set accordingly. For example, at the start of a period they define different demand scenarios as in Table 5-1. In this way, three different levels are considered for the first period demand to be optimistic (1300), most likely (1000) or pessimistic (700). Then, at each level, three different values are considered for the second period demand to be less than, more than or equal to the first period demand. Each scenario is then associated with an occurrence probability with the summation of the probabilities for all the scenarios equal to 1.

Table 5-1-Demand under different scenarios

Scenario	1	2	3	4	5	6	7	8	9
1st period demand (DF_n)	1300	1300	1300	1000	1000	1000	700	700	700
2nd period demand (DS_n)	1400	1300	1200	1100	1000	900	800	700	600
Probability(ρ_n)	0.09	0.12	0.09	0.13	0.14	0.13	0.09	0.12	0.09

The ordering and purchasing costs are estimated as *A=€500/ordering* and *C=€50/unit*. By consuming each unit of radiopharmaceutical product to treat a patient, *€150* on average is obtained. So, we can say *p=€150.* The product deterioration rate is *θ=20%*. Hence, because of the highly perishable product on one hand and highly variable demand on the other, the planning horizon is short and consists of two periods.

GHG levels produced by a unit of deteriorated and recycled product are estimated as *GD=500 gr/unit* and *Gw=900 gr/unit* respectively. It is noteworthy that these two parameters are determined by the subject-matter experts for this special case; however the applicability of the proposed model is not dependent on these quantities.

The retailer's initial inventory is $I_0=50$ units and its holding cost *(€/unit/period)* is as follows:

$$h = \begin{cases} 20 & 0 \le I < 500 \\ 40 & 500 \le I < 1000 \\ 70 & 1000 \le I \end{cases} \tag{5-40}$$

The information of cost and capacity for the transportation vehicles, as well as their GHG emissions, are summarized in Table 5-2.

Table 5-2-Transportation vehicle's data

Vehicle Type *v*	CAP_v *(unit of product)*	TA_v(€)	$G_v(gr)$
L (v=1)	1200	400	20000
M (v=2)	900	315	17000
S (v=3)	500	200	12000

The amount of each vehicle's GHG emissions is calculated by considering fuel consumption and distance between the supplier and the retailer as follows (Carbon dioxide emission calculator):

Gv(gr)=vehicle fuel consumption per 100 kilometer(Distance (Km)/100)*GHG emission produced by a unit of fuel consumed (gr)* (5-41)

In each period, based on total requirement (backordered demands as well as new uncertain demands) and supply (quantity order as well as remaining inventory from previous period), three cases may occur:
i) Requirement is equal to supply, which means no inventory or shortage exists at the end of the period;
ii) Requirement is less than supply, which means that the inventory level is positive and shortage is zero at the end of the period. In this case, α*=15%* of remaining inventory required to be recycled with cost *SV=30€/unit* and estimated GHG emission *Gw=900 gr/unit*; and
iii) Requirement is more than supply, which means that inventory and shortage levels are zero and positive, respectively, at the end of the period. In this case, β*=5%* of unfulfilled demands (shortages) is backordered and referred to the next period by cost γ=10*€/unit.* The other 95% is considered as lost sales, i.e. patients treated by emergency procurement or referred to other radio pharmacy services. In this way the lost sale cost is equal to π=100*€/unit.*
All computations were run using the *CPLEX* algorithm accessed via *IBM ILOG CPLEX 12.2* on a PC Pentium *IV-3.2GHz i5* and 4 *GB RAM* under Win

XP SP 3. Presented hereunder are the resulting solutions for which we have relied on a set of the above-mentioned data.

According to Mirzapour Al-e-Hashem et al. (2012) there are three main methods to solve multi-objective optimization problems: the *a priori* method, the *a posteriori* method, and the *interactive* method. In the *a priori* method, the decision maker (DM) states his preferences prior to the solution process and the multi-objective optimization problem is transformed into a single objective problem. Then, a classical single objective optimization algorithm is used to find the optimum. The *a priori method* can make a representative subset of the Pareto set that in most cases is adequate.

The purpose of the *a posteriori* method is to optimize all objective functions, simultaneously. In this method, first the efficient solutions of the problem, the Pareto set, are generated. Subsequently, at the end of the search process, the DM involves, with the aim of select among the Pareto set, the most preferred one.

In the interactive method the phase in which the DM becomes involved in the decision-making process, expressing his/her preferences, is interchanged with the calculation phase and the process frequently converges, after a few iterations, to the preferred solution. The DM sequentially drives the search with his/her answers towards the preferred solution.

Due to the multi-objective framework of the proposed model, a weighting technique (*a posteriori* method) is used to solve the model. In this method we ask the DM to express his/her preferences between two objectives before the solution process. Then, depending on his/her answers, the weights of each objective (w) are determined. Therefore, the proposed bi-objective optimization problem is transformed into a single objective one and the objectives could be summarized as follows:

$$MinZ = w.Z_1 + (1-w).Z_2 \qquad (5\text{-}42)$$

We first solve the model for the current situation of the above case merely with the first objective function, Z_1, (setting $w=1$) and report the optimal solution configuration and objective functions in Tables (5-3), (5-4) and (5-5). We then analyze the impact that consideration for green issues has on solutions and total costs, by changing the relative weight (w).

$w=1$ is equivalent to the single objective version of the proposed model, that is, the decision makers are not concerned with environmental challenges.

The optimal configurations of the first and second-stage decision variables are reported in Tables 5-3 and 5-4 respectively. As shown in Table 5-3, in the first period the supplier prepares 1150 units and sends it to the retailer in a type-1vehicle.

Table 5-3-Optimal value of the first-stage decision variables ($w=1, \beta=0.05$)

Variables related to the first period			
QF^*	XF_1^*	XF_2^*	XF_3^*
1150	1	0	0

The second-stage depends on the scenario. The ordering quantity and the transportation vehicle type vary accordingly. For instance, as Table 5-4 shows, in scenario 1, 1405 units are transported by a type-1 vehicle and a type-3 vehicle from the supplier to the retailer, whereas in scenario 2, 1305 units are shipped by a type-2 vehicle and a type-3 vehicle, and so on.

Table 5-4-Optimal value of the second-stage decision variables ($w=1, \beta=0.05$)

	Variables related to the first period			Variables related to the second period						
Scenario	DF_n	IF_n^*	BF_n^*	DS_n	IS_n^*	BS_n^*	QS_n^*	$XS1_n^*$	$XS2_n^*$	$XS3_n^*$
1	1300	0	100	1400	0	0	1405	1	0	1
2	1300	0	100	1300	0	0	1305	0	1	1
3	1300	0	100	1200	0	0	1205	0	1	1
4	1000	200	0	1100	0	0	940	0	0	2
5	1000	200	0	1000	0	0	840	0	1	0
6	1000	200	0	900	0	0	740	0	1	0
7	700	500	0	800	0	0	400	0	0	1
8	700	500	0	700	0	0	300	0	0	1
9	700	500	0	600	0	0	200	0	0	1
Expected values	1000	230	30	1000	0	0	817.5	-	-	-

As Table 5-5 shows, the optimum value for the first objective function (Z^*_1) is equal to -187,086 when its weight is set to 1. The worst value for GHGs is also obtained in this state (94, 330).

Table 5-5-Optimal expected values of the objective functions (*w=1, β=0.05*)

H^*	SH^*	O*	P^*	S*	T*	R*	$GHG\text{-}D^*$	$GHG\text{-}R^*$	$GHG\text{-}T^*$	Z_1^*	Z_2^*
4600	2865	*1000*	98375	1035	*759*	295720	23000	31050	40280	-187,086	94,330

H: Expected holding cost
SH: Expected shortage cost
O: Expected ordering cost
P: Expected procurement cost
S: Expected salvage cost

GHG-D: Expected GHG emission produced by deterioration
GHG-R: Expected GHG emission produced by recycling
GHG-T: Expected GHG emission produced by transportation
T: Expected transportation cost
R: Expected total revenue

To show how considering Z_2 causes positive environmental change for the solution configuration, let us resolve the above problem with $w=0.5$, that is, the DM is now concerned with GHGs as much as with costs. The results are presented in Tables 5-6, 5-7 and 5-8.

Table 5-6-Optimal value of the first-stage decision variables ($w=0.5, \beta=0.05$)

Variables related to the first period			
QF^*	XF_1^*	XF_2^*	XF_3^*
900	0	1	0

As shown in Table 5-6, here the first period ordering quantity (QF^*) is decreased to 900 that have to be transported by a type-2 vehicle. This means that consideration for environmental criteria definitely does affect the supply chain's inventory and transportation policies. This is confirmed by the second-stage's results reported in Table 5-7. A closer look at the issue, in Table 5-7, reveals that three different states can occur if green requirements ($w=0.5$) are taken into consideration: *i*) green requirements affect both inventory and transportation policies considerably (Scenario 7); *ii*) green requirements influence only inventory policy without changing transportation method (Scenario 9); *iii*) conversely they do affect the transportation method, although inventory policy is not changed to any significant degree (Scenario 3). Under Scenario 3, QS_3^* is equal to 1205 when $w=1$. It is transported by a type-2 vehicle and a type-3 vehicle. When environmental effects are considered i.e. $w=0.5$, QS_3^* has decreased slightly to 1200, transported by a type-1 vehicle. So, the transportation procedure has changed completely, although the quantity order has been modified only slightly.

Table 5-7-Optimal value of the second-stage decision variables (*w=0.5, β=0.05*)

Variables related to the first period				Variables related to the second period						
Scenario	DF_n	IF_n^*	BF_n^*	DS_n^*	IS_n^*	BS_n^*	QS_n^*	$XS1_n^*$	$XS2_n^*$	$XS3_n^*$
1	1300	0	350	1400	0	0.5	1417	1	0	1
2	1300	0	350	1300	0	0.5	1317	0	1	1
3	1300	0	350	1200	0	17.5	1200	1	0	0
4	1000	0	50	1100	0	0.5	1102	1	0	0
5	1000	0	50	1000	0	0.5	1002	1	0	0
6	1000	0	50	900	0	2.5	900	0	1	0
7	700	250	0	800	0	0	600	0	1	0
8	700	250	0	700	0	0	500	0	0	1
9	700	250	0	600	0	0	400	0	0	1
Expected values	1000	75	125	1000	0	2.1	944.1	-	-	-

By comparing Table 5-4 and 5-7, in order to have a more fuel-efficient transportation system and at the same time keeping expected costs at the lowest possible level, the optimal value of the first-stage ordering quantity (QF^*) is decreased (1150 to 900) and the optimal expected value of the second-stage ordering quantity is increased (817.5 to 944.1).

By comparing BF_n^* (or BS_n^*) values in Table 5-4 with those in Table 5-7, it can be seen that on average they increase when (*w=0.5*). In other words, the model tends to decrease GHG produced by deterioration and recycling processes, by reducing each period's inventory level and so increasing unfulfilled demands (shortages).

As Table 5-8 shows, as expected, the first objective function (expected total cost), Z_1^*, in this state (*w =0.5*) in comparison with that of *w =1* increases while the GHG emission level decreases. This reduction is a result of appropriate selection of vehicle types as well as controlling inventory level (to reduce GHG emissions of deterioration and recycling processes). As shown in Tables 5-5 and 5-8, a reduction of about 42 percent can be obtained in the supply chain's expected GHG emissions (Z_2^*), with only a 7 percent reduction in expected total profits ($-Z_1^*$).

Table 5-8-Optimal expected values of the objective functions (w=0.5, β=0.05)

H^*	SH^*	O^*	P^*	S^*	T^*	R^*	$GHG\text{-}D^*$	$GHG\text{-}R^*$	$GHG\text{-}T^*$	Z_1^*	Z_2^*
1500	12,148	1,000	92,206	337	686	281,870	*7500*	10,125	36,820	-173,993	54,445

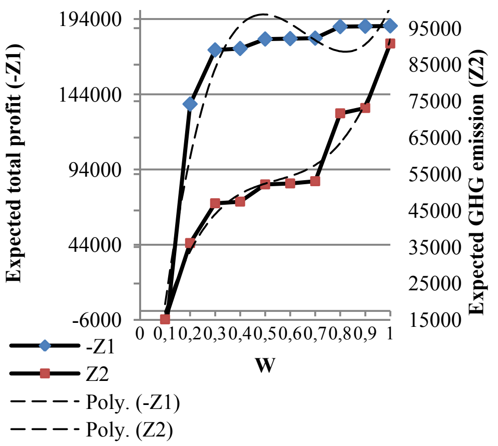
This table's notations are similar to those described in Table 5-5.

5-7- Sensitivity analysis

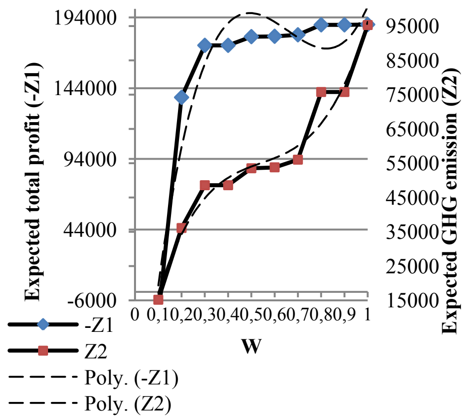
In this section a sensitivity analysis is first performed for the expected value of GHG generated, against the expected value of total profits of the supply chain ($-Z_1$). The above problem is solved by changing the relative weight of the expected profit (w) from zero to one. Figure 5-2 shows a set of Pareto solutions for deterioration rates (θ), of 0.02, 0.05, 0.12 and 0.2. In this figure the dashed lines indicate a polynomial regression of the data obtained.

As expected, for a lower value of w, the expected total profit decreases (expected total cost increases) and the GHG emission index decreases too. Conversely, for a higher value of w, the expected profit ($-Z_1^*$) of the supply chain increases and the GHG emission index increases.

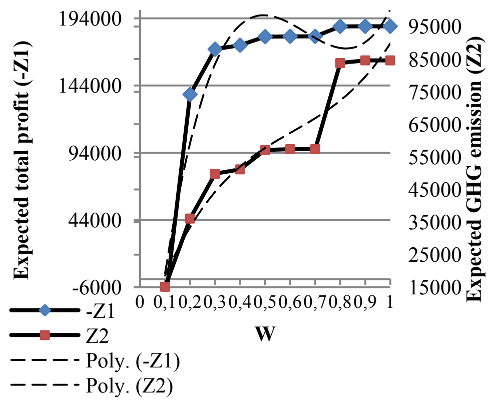
As Figure 5-2 shows, the supply chain should allow a minimal reduction in total profits, in order to achieve a considerable reduction in expected total GHG produced. This is explained more fully in the following. For example, for $\theta=0.12$ in Figure 5-2(c), when $0.8 \leq w < 1$, and even though *'green logistics'* ($w < 1$) is considered, there is not a significant reduction in the GHG emission level since the supply chain lost profit is not enough (less than 0.04%). Then, when the weight of Z_1 (w) is reduced to 0.7 (weight of Z_2 is increased to 0.3), suddenly a 32.2% reduction in expected total GHG emissions (Z_2) is obtained, with just a 3.95% reduction in expected total profits (Z_1). In other words, this supply chain needs to tolerate at least 3.95% reduction in expected total profits to obtain a satisfactory level of GHG reduction. Moreover, the slope of reduction in the GHG level is significantly steeper than the slope of the decrease in total profits. This behavior is also true for other deterioration rates. This is why companies should not postpone their corrective actions in order to reduce GHGs just because of the risk of bankruptcy. Figure 5-2 can therefore be interpreted as a tool enabling the decision maker to select the most appropriate solution.



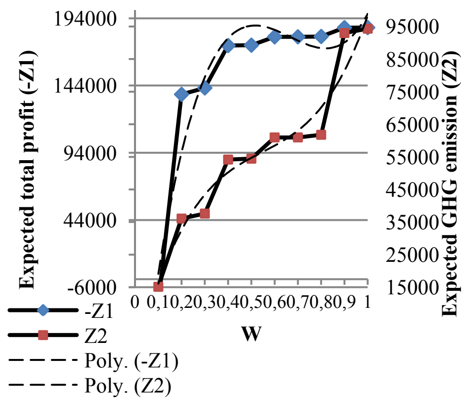
a. θ=0.02



b. θ=0.05



c. θ=0.12



d. θ=0.2

Figure 5-2 -Optimal expected profit (€) against the optimal expected GHG emission

The first and second period ordering quantity values are reported in Table 5-9. From this table the following outcomes can be deduced:

- ✓ Whenever *w* is decreased (the weight of GHG criteria (*1-w*) is increased), the optimum value of the first period order quantity (QF^*) is decreased in order to reduce the period's inventory level. In this way, GHG emissions from deterioration and recycling processes are decreased in the first period.
- ✓ At each level of *QF*, the optimum expected value of the second period order quantity ($E(QS_n^*)$) is decreased as *w* is decreased. This is because the GHG emission produced by deterioration and recycling processes is kept as low as possible.

- ✓ The optimum expected value of the second period order quantity ($E(QS_n^*)$) is increased from one level of QF^* to another when w is decreased. QF^* is decreased from one level to another, so the shortage cost is increased. Thus, we can say: $E(QS_n^*)$ is increased from one level of QF^* to another to maintain the expected shortage cost and consequently the expected total cost (Z_1) as low as possible. Moreover, when the inventory level of the supply chain is increased, GHG emissions from transportation, per product unit, are decreased; but GHG emissions through deterioration and recycling are increased. A balance between these values can thus be achieved by controlling the inventory levels in the first and second periods.
- ✓ Table 5-9 shows, for each value of w, that QF^* is decreased as the deterioration rate is increased to reduce the deterioration cost and its related GHG.

Table 5-9-Optimum order quantity of the first and second periods

w	θ=0.02		θ =0.05		θ =0.12		θ =0.2	
	QF^*	$E(QS_n^*)$	QF	$E(QS_n^*)$	QF	$E(QS_n^*)$	QF	$E(QS_n^*)$
1	1250	706	1250	715	1150	799	1150	818
0.9	1150	776	1150	783	1150	799	1150	817
0.8	1150	775	1150	783	1150	799	950	933
0.7	956	911	970	904	950	924	950	931
0.6	950	914	950	916	950	924	950	931
0.5	950	913	950	915	950	923	900	944
0.4	900	929	900	931	900	937	900	943
0.3	900	924	900	931	877	943	650	1008
0.2	650	983	650	983	650	983	650	983
0.1	0	848	0	848	0	848	0	848

By considering Table 5-9 and Figures 5-2(c), for θ=0.12, we see that for w=0.7, 0.4, 0.3 and 0.2 the slopes of GHG and profit functions are rather steep since the inventory policy is changed at these points. However, the slope of GHG emissions and profits functions is not very sensitive for other values of w. For instance, when $0.8 \leq w \leq 1$, the inventory system is almost maintainable ($QF^*=1150$) and GHG reduction is obtained by just improving the transportation system. After that, in order to reduce GHG emissions more, the transportation mode as well as the inventory policy of the deteriorating product must be changed. So, at w=0.7, QF^* is decreased from 1150 to 950. For other values of θ, a fairly considerable increase in the slope of expected total profits and GHG-generated functions can be seen when the inventory policy is changed.

The objective functions' components are presented in detail in Tables 5-10 and 5-11 for θ=0.12 and 0.2 respectively. Since similar conclusions are obtained when θ=0.02 and 0.05, the results for θ=0.12 and 0.2 are only reported.

In keeping with our comments on Table 5-9, and considering the results presented in Tables 5-10 and 5-11, It can be stated that the model tries to keep the inventory level as low as possible when w is reduced. So, the optimal value of procurement (P^*), holding(H^*) and recycling(R^*) costs, as well as GHG emissions through deterioration ($GHG\text{-}D^*$) and recycling ($GHG\text{-}R^*$) processes, decreases whenever w is decreased. Conversely, the shortage cost (SH^*) increases. The transportation cost (T^*) and its related GHG emissions ($GHG\text{-}T^*$) also decrease at each level of QF as w decreases.

Table 5-10-The objective functions' components, θ=0.12

	Expected total cost (Z_1) components						Expected total GHG (Z_2) components					
w	H^*	SH^*	O^*	P^*	S^*	T^*	R^*	$GHG\text{-}D^*$	$GHG\text{-}R^*$	$GHG\text{-}T^*$	$-Z_1^*$	Z_2^*
1	4600	2865	1000	97455	1035	759	295720	13800	31050	39760	188006	84610
0.9	4600	2865	1000	97455	1035	759	295720	13800	31050	39760	188006	84610
0.8	4600	2908	1000	97433	1035	749	295660	13800	31050	38950	187935	83800
0.7	1800	8724	1000	93698	405	771	286970	5400	12150	39820	180572	57370
0.6	1800	8724	1000	93698	405	771	286970	5400	12150	39820	180572	57370
0.5	1800	8853	1000	93630	405	763	286770	5400	12150	39550	180319	57100
0.4	1500	12288	1000	91829	338	678	281640	4500	10125	36550	174007	51175
0.3	1362	13872	1000	90999	306	678	279270	4080	9194	36550	171052	49824
0.2	0	31701	1000	81653	0	678	252460	0	0	36070	137428	36070
0.1	0	109815	395	42379	0	297	134640	0	0	15140	-18246	15140

In this table the notations are similar to those described in Table 5-5.

Table 5-11-The objective functions' components, θ=0.2

	Expected total cost (Z_1) components							Expected total GHG (Z_2) components				
W	H^*	SH^*	O^*	P^*	S^*	T^*	R^*	$GHG\text{-}D^*$	$GHG\text{-}R^*$	$GHG\text{-}T^*$	$-Z_1^*$	Z_2^*
1	4600	2865	1000	98375	1035	759	295720	23000	31050	40280	187086	94330
0.9	4600	2908	1000	98353	1035	749	295660	23000	31050	38950	187015	93000
0.8	1800	8595	1000	94125	405	781	287170	9000	12150	40630	180464	61780
0.7	1800	8724	1000	94058	405	771	286970	9000	12150	39820	180212	60970
0.6	1800	8724	1000	94058	405	771	286970	9000	12150	39820	180212	60970
0.5	1500	12142	1000	92206	338	686	281870	7500	10125	36820	173999	54445
0.4	1500	12288	1000	92129	338	678	281640	7500	10125	36550	173707	54175
0.3	0	29352	1000	82883	0	703	256150	0	0	37600	142213	37600
0.2	0	31701	1000	81653	0	678	252460	0	0	36070	137428	36070
0.1	0	109815	395	42379	0	297	134640	0	0	15140	-18246	15140

This table's notations are similar to those described in Table 5-5.

Table 5-12 shows lost profits against GHG reduction for deterioration rates (θ) of 0.02, 0.1, 0.2 and 0.3. According to this table, in all cases GHG emissions can be reduced by approximately 20 to 40 percent with less than a 5 percent increase in expected total costs. This is a valuable result for companies who postpone their corrective actions in order to reduce GHGs just because of the risk of bankruptcy.

Table 5-12-Lost profit against GHG reduction

	θ=0.02		θ=0.05		θ=0.12		θ=0.2	
w	%lost profit	%GHG reduction	%lost profit	%GHG reduction	%lost profit	%GHG reduction	%lost profit	%GHG reduction
0.9	0.13	19.47	0.11	20.51	0.00	0.00	0.04	1.41
0.8	0.26	21.09	0.11	20.51	0.04	0.96	3.54	34.51
0.7	4.31	41.59	3.85	41.22	3.95	32.19	3.67	35.37
0.6	4.48	42.26	4.40	43.57	3.95	32.19	3.67	35.37
0.5	4.61	42.56	4.54	43.86	4.09	32.51	7.00	42.28
0.4	7.93	47.76	7.79	49.05	7.45	39.52	7.15	42.57
0.3	8.44	48.26	7.79	49.05	9.02	41.11	23.99	60.14
0.2	27.44	60.27	27.27	62.15	26.90	57.37	26.54	61.76
0.1	109.63	83.32	109.66	84.11	109.71	82.11	109.75	83.95

A sensitivity analysis is done on backorder fraction (β) as well. For this purpose, the value of β is changed from zero to one for $w=0.5$ and 0.9. Note that β=0 means that all shortages are lost sales (completely lost sale system). However β=1 indicates a completely backorder system. Results are reported in Tables 5-13 and 5-14. In these tables the optimal configuration of the vehicles in the first and second periods is reported as a symbol consisting of three numbers, each one showing the number of each type of vehicle used. For example, in Table 5-13, for β=0.5, XF^*=[0 1 0] means that only a type-2 vehicle is used in the first period. Moreover, XS^*=[101, 101, 101, 100, 100, 100, 010, 010, 001] denotes that in the second period, under Scenarios 1, 2 and 3, a type-1 vehicle and a type-3 vehicle are used, under Scenario 4 a type-1 vehicle, and so on.

As Tables 5-13 and 5-14 show, optimal values of the first period order quantity (QF^*) is decreased, irrespective of which backorder fraction (β) increases. For example, when $w=0.5$, by considering completely lost sale assumption ($\beta=0$), QF^* is equal to 900 whereas it is decreased to 500 under the completely backorder assumption. In fact, when a greater percentage of customers can wait until the next period, the risk of losing customers due to inventory shortages in the first period is reduced. Thus, the first period order quantity (QF^*) is decreased.

In contrast, since the order quantity is reduced in the first period, more customers are referred to the next period. In this way the optimum value of the expected backordered demand in the first period ($E(BF_n^*)$) and the optimum value of the expected value of the second period order quantity ($E(QS_n^*)$) are increased.

5-8- Conclusion

In this chapter a new bi-objective model were proposed to optimally replenish a deteriorating product in a two-echelon supply chain, by considering both total costs of the supply chain and environmental impacts under an uncertain demand. Shortages were assumed to be partially backordered regardless waiting time of customers. The two-stage stochastic programming approach is applied to deal with demand uncertainty.

The proposed model was examined by applying it to a real case from the radiopharmaceutical industry. The numerical results show that if companies allow a minor reduction in the system's profits, they will be able to improve their GHG criteria significantly (Figure 5-2, Tables 5-10 and 5-11). Industrial managers can thus deal with the green issue without any concern about losing a considerable amount of profit. To sum up, the proposed model is especially applicable in many supply chains with highly variable demand, which propose highly perishable products.

Table 5-13- β sensitivity analysis (*w=0.5)*

β	QF^*	$E(QS_n^*)$	$E(BF^*)$	$E(BS^*)$	$E(IF^*)$	$E(IS^*)$	XF^*	XS^*	Z_1^*	Z_2^*
0	900	940	125	0	75	0	[0 1 0]	[011, 011, 100,100,100,010, 010, 001, 001]	173230	54175
0.1	900	948.7	125	3.8	75	0	[0 1 0]	[101, 011, 100, 100, 100, 010, 010, 001, 001]	174880	54445
0.2	900	963.7	125	1.3	75	0	[0 1 0]	[101, 011, 011, 100, 100, 010, 010, 001, 001]	177730	55255
0.3	900	974.9	125	2.55	75	0	[0 1 0]	[101, 011, 011, 100, 100, 010, 010, 001, 001]	179900	55255
0.4	900	990	125	0	75	0	[0 1 0]	[101, 101, 011, 100, 100, 100, 010, 001, 001]	182690	56005
0.5	770	1078.8	216	0.45	36	0	[0 1 0]	[101, 101, 101, 100, 100, 100, 010, 010, 001]	176780	47710
0.6	650	1180	300	0	0	0	[0 1 0]	[020, 101, 101, 011, 100, 100, 010, 010, 010]	174930	41050
0.7	650	1206.8	300	3.2	0	0	[0 1 0]	[020, 020, 101, 011, 100, 100, 010, 010, 010]	180190	41290
0.8	650	1234.4	300	5.6	0	0	[0 1 0]	[110, 020, 101, 011, 100, 100, 010, 010, 010]	185600	41560
0.9	650	1260.2	300	9.8	0	0	[0 1 0]	[110, 110, 020, 011, 100, 100, 010, 010, 010]	190840	42100
1	500	1450	450	0	0	0	[0 0 1]	[200, 110, 110, 101, 101, 011, 100, 010, 010]	196250	41150

Table 5-14- β sensitivity analysis (*w=0.9*)

B	QF^*	$E(QS_n)^*$	$E(BF^*)$	$E(BS^*)$	$E(IF^*)$	$E(IS^*)$	XF^*	XS^*	$Z_1{}^*$	$Z_2{}^*$
0	1150	816	30	0	230	0	[100]	[011,011,100,100,010,010, 001, 001, 001]	186820	92730
0.1	1150	819	30	0	230	0	[100]	[101,011,011,100,010,010, 001, 001, 001]	187380	93810
0.2	1150	822	30	0	230	0	[100]	[101,011,011,100,010,010, 001, 001, 001]	187950	93810
0.3	1150	825	30	0	230	0	[100]	[101,011,011,100,010,010, 001, 001, 001]	188520	93810
0.4	950	964	90	0	90	0	[100]	[101,101,011,100,100,010, 010, 001, 001]	186440	62140
0.5	950	973	90	0	90	0	[100]	[101,101,011,100,100,010, 010, 001, 001]	188150	62140
0.6	950	982	90	0	90	0	[100]	[101,101,011,100,100,010, 010, 001, 001]	189860	62140
0.7	950	991	90	0	90	0	[100]	[101,101,101,100,100,010, 010, 001, 001]	191570	62140
0.8	950	1000	90	0	90	0	[100]	[101,101,101,100,100,010, 010, 001, 001]	193280	62140
0.9	900	1052.5	125	0	75	0	[010]	[020,101,101,100,100,100, 010, 001, 001]	194560	56455
1	650	1300	300	0	0	0	[010]	[110,110,020,011,011,100, 010, 010, 010]	197620	43360

6

Conclusion and Perspectives

6 Conclusion and perspectives

Growing numbers of deteriorating products as well as their noticeable effect on the company's profit/loss have highlighted deteriorating products replenishment policies progressively for domain experts. Considering uncertainties in the deteriorating inventory replenishment models makes them closer to reality, although it also makes their formulation and solution procedure more and more complicated.

The purpose of this PhD dissertation is to develop new replenishment models to control the inventory of deteriorating products under uncertain conditions. To achieve our goal, we have considered both stochastic lead time and stochastic demand in different situations. Our main contributions were detailed in four chapters:

Chapter 2: has been an overview of researches in deteriorating inventory replenishment. We have provided a new classification of works based on deterioration modeling approach (Figure 2-1) as:

- ✓ Class 1: considering a non-linear decreasing on-hand inventory function in terms of time and a constant holding cost rate.
- ✓ Class 2: considering a linear decreasing on-hand inventory function in terms of time with a non-linear increasing holding cost in terms of parameters such as stocking time, on-hand inventory level etc.
- ✓ Class 3: taking into account both a non-linear decreasing on-hand inventory function as well as a non-linear increasing holding cost function.

Moreover, some categorizations have been presented for some main parameters in deteriorating models such as deterioration function, shortage and demand functions. In this way, details of some main related research are provided in a table (Table 2-2). Information such as deterioration modeling approach, types of demand and lead time function, types of shortage function, solution approaches and objective function components of each work have been reported in that table. Our overview has revealed that most of efforts are dedicated to covering research that specifically deals with certain (lead time/demand) conditions.

In this chapter, a review has also been done on green logistics and the effect of greenhouse gases (GHGs) on environment. The significant role of deterioration in green supply chains has been explained in more details in this chapter too.

Based on papers reviewed, several main research gaps have been recognized. Then, we emphasized on some of the most significant and related research directions in the next chapters.

Chapter 3: the problem of Replenishment policy for Perishable products under Stochastic Lead time (**RPSL**) has been studied which is almost overlooked in the literature. In this way, we have taken into account a single-product supply chain in order to minimize the retailer's total costs. The retailer's inventory system has been considered as the continuous review, *(r,Q),* under an infinite planning horizon.

A second approach for modeling deterioration process has been examined. In this way, a linear inventory function as well as a time-dependent, non-linear increasing holding cost function ($H(t) = \tilde{h}t^{\gamma}$, $\gamma \geq 1$) have been considered. In 2007, Ferguson et.al surveyed two real case studies to analyze the applicability of this type of time-dependent holding cost. They concluded that this holding cost function is appropriate for obtaining an approximation to model two main groups of deteriorating items: *i*) products with expiry dates requiring the removal of spoiled ones as the date approached and *ii*) products that managers frequently utilize markdowns to stabilize demand as the product's expiration date is coming.

The objective function of this problem has been minimizing retailer's total cost consisting of holding cost (including deterioration cost), backordered, and ordering costs. At the first step, we have developed the mathematical model by considering a general probability density function (PDF) for lead time (l) as $f_L(l)$, $l \geq 0$ (General form of RPSL problem abbreviated by GRPSL). The GRPSL problem has belonged to unconstrained multivariable optimization problems with a differentiable objective function, however the convexity proof of the objective function has been highly complicated. Therefore, in order to simplify the mathematical challenges associated with its convexity proof, and to have more detailed analysis of the proposed model, a more simple form of $f_L(l)$ has been considered.

In the next step, we have dealt with the RPSL problem by assuming a uniform distribution function for lead time that has been abbreviated to URPSL. Thus, by proving four lemmas, the convexity of objective function has been demonstrated. Then, a sensitivity analysis has been performed for influential parameters (holding cost parameters ($\tilde{h}$ and γ), backordered cost per unit time, as well as parameters of the lead time's probability function) based on an example taken from the literature. The results have showed that for products with a high shortage cost and lower deterioration rate, more savings can be obtained by stocking more inventories at reorder point, particularly under more uncertain conditions, to avoid the risk of shortage.

To conclude, the following relevant directions are suggested for further research on this work:

- ✓ Adding total costs of supplier to the proposed model and optimizing the whole supply chain performance. For this purpose, studying policies such as lot-for-lot, equal-sized shipments, delayed equal-sized shipments and so on are fruitful for further research. In fact, the proposed model can be a foundation for developing a *joint economic lot size* problem for deteriorating products under stochastic lead times.
- ✓ Extending the developed model for a convergent or divergent supply chain and in a special case for order splitting policy can be an interesting issue for further research.
- ✓ Studying and analyzing RPSL problem under situations differed from that studied in this chapter can also be a future direction. In this way, studying RPSL problem for different forms of non-linear holding cost based on time-dependent or/and inventory-dependent functions (each form is suitable for a special group of deteriorating products) can be examined. Also, analyzing other types of lead time distribution function such as Normal and Weibull ones can be fruitful for future research. Finally developing RPSL model in the case of up-to-level or *(1, T)* inventory policies might make a substantial contribution.

Chapter 4: we have developed a basic single sourcing and dual-sourcing model for deterioration items under stochastic lead times.

At the beginning of this chapter some main studies on order splitting policy have been reviewed. Since most researchers had addressed products with infinite lifetime, we have been motivated to study order splitting issue in the case of deteriorating items. The main contribution of this chapter is therefore developing an order splitting model for a retail unit that offers deteriorating products.

In this way, a retailer with a continuous review *(r,Q)* inventory system faced with constant demand has been considered. It has been supposed that the product is decayed with a constant deterioration rate at retailer's site. A constant holding cost rate has also been considered for inventories in stock. Thus, the approach of Class 1 has been taken to model deterioration.

The objective which has been minimizing the expected total cost of the retailer has consisted of procurement, ordering, holding, shortage and deterioration costs. For this purpose, there have been two different sourcing policies. The retailer could select *i*) single sourcing i.e., it purchases all its requirements from a single supplier with an exponential lead time or *ii*) dual-sourcing i.e., the retailer splits his orders between two suppliers with non-identical independent exponential lead times. Thus, two mathematical models have been developed for two different strategies: Single-Source Deteriorating Inventory problem **(SSDI)** and Dual-Source Deteriorating Inventory problem **(DSDI).**

The non-linear inventory functions, dissimilarity of inventory function form (before and after shortage) and replenishment cycle dependency on the length of lead time have made the exact problem-solving methods intractable. We have therefore used iterative approach Sequential Quadratic Programming (SQP) access via Maple to solve the problems.

Then, based on an example taken from the literature (Ramasesh et.al) a sensitivity analysis has been performed. As a consequence of the deterioration process, it has been illustrated that the cycle time – in both SSDI and DSDI model – is lower than replenishment cycle in no deterioration case (Q^*/D). Moreover, whenever the deterioration rate increases, the cycle time as well as the values of the reorder point (r) decreases accordingly. One significant insight have been derived from numerical analysis is that once the deterioration rate increases, the amount of the expected deterioration cost as well as the total cost saved by dual-sourcing strategy increase under stochastic lead times especially when two suppliers are more similar. In contrast, order splitting is not cost-effective when two available suppliers are very different from the aspects of lead time uncertainty and selling price. Furthermore, the numerical results have indicated that the reorder quantity in the dual-source model is less than that in the single-source model. Finally, the proposed models have been applied to solve a real case study from pharmaceutical industry.

Until now, the problem of using multiple suppliers under stochastic lead times for deterioration items replenishment has not received as much attention as the use of order splitting policy for non-deterioration items. Of course, substantial research is still required, and there are several interesting areas that can be investigated.

- ✓ It would be very useful to extend and analyze SSDI and DSDI for products with linear, logarithmic, exponential or Weibull deterioration rates. In addition, the models proposed in this chapter have developed by assuming a completely backordered shortage. In recent competitive markets, lost sales or partial backorder demands are more realistic and may be a fruitful subject for future research. Moreover, demand can either depend on various factors or be probabilistic. For example, it is well known that in many cases the demand rates depend on selling prices or on the product's life cycle. The dependencies or uncertainties in parameters like demand make the model more complex, although more attractive to potential researchers than the proposed model.
- ✓ Following the previous direction, it may be possible to extend SSDI and DSDI problems in situations with stochastic demand and deterministic lead times or both stochastic demand and stochastic lead times.
- ✓ Developing an integrated SSDI and DSDI models in a closed loop supply chain consisting of a single buyer and single (dual) supplier might also be attractive for further research.
- ✓ Finally, modeling order splitting problem for deteriorating products and solving it by an exact method is an interesting issue for further investigation. For this purpose, applying Lambert W function estimation methods (like Taylor Series Expansion) to simplify the mathematical models may be useful.

Chapter 5: a bi-objective two-stage stochastic programming model has been proposed for a centralized green supply chain with deteriorating products.

Today, the green principles have been expanded to many areas of work, including supply chains. Adding the '*green*' concept to the '*supply chain*' concept creates a new paradigm where the supply chain has a direct relation to the environment. Despite the fact that many companies are concerned, green rules will lead to a noticeable decrease in their profits. Accordingly, the critical question is: What are the '*best*' solutions for balancing environmental and economic concerns?

In this chapter, we have supposed a dyadic centralized supply chain consisting of one supplier and one retailer. Unlike previous chapters, in this chapter lead time has been constant whereas end-customer's demand has been considered as a stochastic parameter. The unfulfilled demands have been assumed to be partially backordered. There have been several types of transportation vehicles at the supplier side, each one being characterized by its own capacity, transportation cost and GHG emissions. Because of the deterioration characteristics of products, a constant deterioration rate as well as a non-linear inventory-dependent holding cost function has been considered (Class 3). Considering uncertain demand, deteriorating products, partial backorder assumption, GHG emission and costs simultaneously as well as transportation vehicle's capacity, all makes it difficult to use classical inventory models (such as models proposed in Chapters 3 and 4). A mixed integer mathematical model is therefore developed by applying a two-stage stochastic programming approach.

The objective of the proposed model is to find the best configuring of vehicle types and order quantities, in order to meet the following criteria simultaneously: *i*) minimizing the expected value of total costs of the supply chain includes the ordering cost, the purchasing cost, the holding cost, the shortage cost, the transportation cost and the recycling cost minus revenue earned. *ii*) minimizing expected GHG emissions by the supply chain composed of the quantity of GHG emitted by the vehicles during the product transportations; the quantity of GHG emitted by deteriorated items; and the quantity of GHG emitted during the recycling process.

We have applied the proposed model to a real case and then have developed some managerial insights around the proposed framework. By providing a set of Pareto solutions it has been derived that the supply chain should allow a minimal reduction in total profits, in order to achieve a considerable reduction in expected total GHG produced. This is why companies should not postpone their corrective actions in order to decrease greenhouse gases (GHG) just owing to the risk of bankruptcy.

The proposed model is definitely not final; it is rather a big step and the path is still open for further research in the following respects:

- ✓ Developing the model by considering more detailed reverse logistics processes may make a good contribution for future researches. In this way, studying a three-echelon supply chain consisting of one (multiple) retailer, one (multiple) supplier and one or (multiple) manufacturers is attractive, so that the cost and GHG of production have to be taken into account. As well, the ordered quantity of each retailer to each supplier must be determined as decision variables. In addition, suppliers can provide some incentives like delay in payments or quantity discounts for retailers. Analyzing these strategies is also an appropriate direction for further research. In this regard, multi-products models are also a good potential area for more investigation.

- ✓ Accident risk during transportation is a critical concern of managers for some deteriorating products like radiopharmaceutical ones. Developing the proposed mathematical model by considering the accident risk can also be attractive for researchers.

Appendix 1

The first state's formulas for the first scenario were explained in the main body of Chapter 4. Other states' formulas (of the first scenario) are explained below.

State 2: The on-hand inventory function of this state is as in Equation (A.1). This equation is obtained with the help of general Equation (4-1) and initial conditions: $I_{21}(0)=r$, $I_{22}(l_e)=I_{21}(l_e)+k_1Q$ and $I_{23}(l_e+l_s)= k_2Q-D(l_s-M_2)$:

$$I_2(t) = \begin{cases} I_{21}(t) = -\frac{D}{\theta} + \left(r + \frac{D}{\theta}\right) e^{-\theta t} & 0 < t \le l_e \\ I_{22}(t) = -\frac{D}{\theta} + \left(r + \frac{D}{\theta} + k_1 Q e^{\theta l_e}\right) e^{-\theta t} & l_e < t \le M_2 \\ I_{23}(t) = -\frac{D}{\theta} + \left(k_2 Q - D(l_e + l_s - M_2) + \frac{D}{\theta}\right) e^{-\theta(t-(l_e+l_s))} \\ \qquad l_e + l_s < t \le T_2 \end{cases} \quad (A.1)$$

In this state the points of M_2 and T_2 are determined from $I_{22}(M_2)=0$ and $I_{23}(T_2)=s$ respectively as follows:

$$M_2 = -\frac{1}{\theta} \ln \left(\frac{D}{r\theta + D + k_1 Q \theta e^{\theta l_e}}\right) \quad (A.2)$$

$$T_2 = l_e + l_s - \frac{1}{\theta} \ln \left(\frac{D + r\theta}{k_2 Q\theta - D\theta(l_e + l_s - M_2) + D}\right) \quad (A.3)$$

In this state, the backordered demand function is as follows:

$$B_2(t) = \begin{cases} B_{22}(t) = D(t - M_2) & M_2 \le t \le l_e + l_s \\ 0 & else \end{cases} \quad (A.4)$$

In this state, $L_e \in [0, A]$, so $LL_{e2}=0$ and $UL_{e2}=A$. For determining LL_{s2} and UL_{s2}, the following two conditions must be satisfied:

1- $l_s+l_e \geq M_2$, since backorder occurred in Area 2.

2-$I_{23}(l_e+l_s) \geq r$ to keep assumption A.8.

These conditions simultaneously result in the following:

$$LL_{s2} = M_2 - l_e \quad (A.5)$$

$$\mathrm{UL}_{s2} = \max\{M_2 - l_e, \frac{k_2Q-r}{D} + M_2 - l_e,\} \quad (A.6)$$

State 3: Here the initial conditions to establish the on-hand inventory function are: $I_{31}(0)=r$, $I_{32}(l_e)=k_1Q-(l_e-A)D$ and $I_{33}(l_e+l_s)=I_{32}(l_e+l_s)+k_2Q$. So, the on-hand inventory function is as follows:

$$I_3(t) = \begin{cases} I_{31}(t) = -\frac{D}{\theta} + \left(r + \frac{D}{\theta}\right)e^{-\theta t} & 0 < t \le A \\ I_{32}(t) = -\frac{D}{\theta} + \left(k_1Q - (l_e - A)D + \frac{D}{\theta}\right)e^{-\theta(t-l_e)} & l_e < t \le l_e + l_s \\ I_{33}(t) = -\frac{D}{\theta} + [\left(k_1Q - D(l_e - A) + \frac{D}{\theta}\right)e^{-\theta l_s} + k_2Q]e^{-\theta(t-(l_e+l_s))} & \\ \qquad l_e + l_s < t \le T_3 \end{cases} \quad (A.7)$$

So, for this state, the points of M_3 and T_3 are as Equations (A.8) and (A.9):

$$M_3 = l_e - \frac{1}{\theta}\ln\left(\frac{D}{k_1Q\theta - D\theta(l_e-A)+D}\right) \quad (A.8)$$

$$T_3 = l_e + l_s - \frac{1}{\theta}\ln\left(\frac{D+r\theta}{(k_1Q\theta - D\theta(l_e-A)+D)e^{-\theta l_s} + k_2Q\theta}\right) \quad (A.9)$$

In this state the backorder occurs only in Area 1, so the backorder function is as follows:

$$B_3(t) = \begin{cases} B_{31}(t) = D(t-A) & A \le t \le l_e \\ 0 & else \end{cases} \quad (A.10)$$

It is clear that $LL_{e3}=A$ and $LL_{s3}=0$ for this state (Figure 4-4(c)). For determining UL_{s3} the following two conditions should be satisfied:

1- $l_s+l_e \le M_3$ since there is no backorder in Area 2.

2-$I_{33}(l_e+l_s) \ge r$ to keep assumption A.8.

Consequently:

$$UL_{s3} = \min\{-\frac{1}{\theta}\ln\left(\frac{D}{k_1Q\theta - D\theta(l_e-A)+D}\right), -\frac{1}{\theta}\ln\left(\frac{r\theta + D - k_2Q\theta}{D-(l_e-A)D\theta + k_1Q\theta}\right)\} \quad (A.11)$$

Moreover, UL_{e3} is determined by the following two conditions:

1- $I_{32}(l_e) \ge 0$ according to Figure 4(c). So, $l_e \le A + \frac{k_1Q}{D}$

2- $L_e \le A+(Q-r)/D$ to keep assumption A.8.

So:

$$UL_{e3} = \min\{A + \frac{k_1Q}{D}, A + \frac{Q-r}{D}\} \quad (A.12)$$

State 4: The on-hand inventory function can be determined by the initial conditions $I_{41}(0)=r$, $I_{42}(l_e)=k_1Q-(l_e-A)D$ and $I_{43}(l_e+l_s)=k_2Q-(l_e+l_s-M_4)D$ as follows:

$$I_4(t) = \begin{cases} I_{41}(t) = -\frac{D}{\theta}+\left(r+\frac{D}{\theta}\right)e^{-\theta t} \quad 0<t\le A \\ I_{42}(t) = -\frac{D}{\theta}+\left(k_1Q-(l_e-A)D+\frac{D}{\theta}\right)e^{-\theta(t-l_e)} \; l_e<t\le M_4 \\ I_{43}(t) = -\frac{D}{\theta}+\left(k_2Q-(l_e+l_s-M_4)D+\frac{D}{\theta}\right)e^{-\theta(t-(l_e+l_s))} \\ \qquad l_e+l_s<t\le T_4 \end{cases} \tag{A.13}$$

In this case $M_4=M_3$ and can be obtained by Equation (A.8). Also, T_4 is as follows:

$$T_4 = l_e + l_s - \frac{1}{\theta}\ln\left(\frac{D+r\theta}{k_2Q\theta - D\theta(l_e+l_s-M_4)+D}\right) \tag{A.14}$$

Since in State 4, backorder demand exists during l_e and l_s, the backorder function is as in Equation (A.15):

$$\mathrm{B}_4(t)=\begin{cases} B_{41}(t) = D(t-A) & A\le t\le l_e \\ B_{42}(t) = D(t-M_4) & M_4\le t\le l_e+l_s \\ 0 & else \end{cases} \tag{A.15}$$

In this case, $LL_{e4}=A$. The on-hand inventory is positive after the first order arrival (it is negative in State 5) i.e. $I_{42}(l_e)\ge 0$ that results in $LL_{e4} = A + \frac{k_1Q}{D}$.

We also have $LL_{s4}=M_4-l_e$. In order to determine UL_{s4}, the following conditions must be sustained:

1- $l_e+l_s\ge M_4$ as backorder occurred during l_s.

2- $I_{43}(l_e+l_s)\ge r$ to keep assumption A.8.

The above conditions result in Equation (A.16):

$$\mathrm{UL}_{s4} = \max\left\{\frac{k_2Q-r}{D} + M_4 - l_e, M_4 - l_e\right\} \tag{A.16}$$

State 5: The on-hand inventory function in this state is as follows:

$$I_5(t) = \begin{cases} I_{51}(t) = -\frac{D}{\theta}+\left(r+\frac{D}{\theta}\right)e^{-\theta t} & 0<t\le A \\ I_{53}(t) = -\frac{D}{\theta}+[\left(Q-(l_e+l_s-A)D+\frac{D}{\theta}\right)e^{-\theta(t-(l_e+l_s))} \\ \qquad l_e+l_s<t \end{cases} \tag{A.17}$$

It is obvious that in this state $I_{52}(t)$ is equal to zero. $I_{51}(t)$ and $I_{53}(t)$ are determined by the general Equation (4-1) and initial conditions $I_{51}(0)=r$ and $I_{53}(l_e+l_s)= Q-(l_e+l_s-A)D$ respectively. The replenishment cycle time of this state is determined by $I_{53}(T_5)=r$ as follows:

$$T_5 = l_e + l_s - \frac{1}{\theta}\ln\left(\frac{D+r\theta}{Q\theta - D\theta(l_e+l_s-A)+D}\right) \tag{A.18}$$

Like previous states, the backorder function is obtained as Equation (A.19):

$$B_5(t)=\begin{cases} B_{51}(t) = D(t-A) & A \le t \le l_e \\ B_{52}(t) = D(t-A) - k_1 Q & l_e \le t \le l_e + l_s \\ 0 & else \end{cases} \tag{A.19}$$

In this state, L_e is too long, so that even after the first arrival order, the net stock is still negative. Thus, $LL_{e5} = A + \frac{k_1 Q}{D}$. The following two conditions must be satisfied to determine UL_{e5}:

1- $I_{52}(l_e) \le 0$: This is clear from Figure 4-4(e).

2- If $l_s=0$, assumption A.8 must be satisfied. In other words, in this state for $l_s=0$ the maximum amount of backorders in the effective lead time is $Q-r$, and by considering its related time interval, L_s can be lengthened up to $[A+(Q-r)/D]-l_e$. These conditions result in UL_{e5}, as follows:

$$UL_{e5} = \max\{A + \frac{k_1 Q}{D}, A + \frac{Q-r}{D}\}] \tag{A.20}$$

With regard to the lower and upper bounds of L_s, it is clear that $LL_{s5}=0$. Moreover,

$UL_{s5} = A + \frac{Q-r}{D} - l_e$, resulted from $I_{53}(L_e+ L_s) \ge r$.

APPENDIX 2

According to Bazaraa et al. (2006) Sequential Quadratic Programming (SQP), also known as successive, or recursive, quadratic programming approaches employs Newton's method to directly solve the Karush-Kuhn-Tucker (KKT) conditions for a nonlinear multi-variable problem. Consequently, the accompanying sub-problem turns out to be the minimization of a quadratic approximation to the Lagrangian function optimized over a linear approximation of the constraints. Hence, by its nature this method produces both primal and dual (Lagrange multiplier) solutions.

To present the concept of this method, consider the equality-constrained nonlinear problem P, where $x \in R^n$, and all functions are assumed to be continuously twice differentiable.

$$P: Minimize\ f(x)$$
$$Subject\ to \quad h_i(x) = 0, i = 1, \dots, l \qquad \text{(A.21)}$$

Assume that $L(x, v)$ indicates Lagrangian function related to the P problem with Lagrangian multiplier vector v. The basic SQP method can be described in the following steps.

Initialization: Set the iteration counter $k=1$ and select a (suitable) starting primal-dual solution (x_k, v_k).

Main Step: Solve the quadratic subproblem $QP(x_k, v_k)$ (A-22) to obtain a solution d_k as well as a vector of Lagrange multipliers v_{k+1}. If $d_k=0$, then (x_k, v_{k+1}) satisfies the KKT conditions for problem P and stop. Otherwise, put $x_{k+1}=x_k+d_k$, increment k by 1 and repeat the Main Step.

$$QP(x_k, v_k): Minimize\ f(x_k) + \nabla f(x_k)^t d + \frac{1}{2}.d^t \nabla^2 \mathrm{L}(x_k)\mathrm{d} \qquad \text{(A.22)}$$
$$Subject\ to\ h_i(x_k) + \nabla h_i(x_k)^t d = 0, \quad i = 1, \dots, l.$$

By defining $\nabla^2 L(x_k) = \nabla^2 f(x_k) + \sum_{i=1}^{l} v_{ki} . \nabla^2 h_i(x_k)$ to be the usual Hessian of the Lagrangian at x_k with the Lagrange multiplier vector v_k. Also, ∇ indicates gradients of functions. More details about this method have been provided in Bazaraa et al. (2006), Chapter 10.

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