Joint Source-Network Coding & Decoding

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Codage/Décodage Conjoint Source-Réseau

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Abstract

While network data transmission was traditionally accomplished via routing, network coding (NC) broke this rule by allowing network nodes to perform linear combinations of the upcoming data packets. Network operations are performed in a specific Galois field of fixed size q. Decoding only involves a Gaussian elimination with the received network-coded packets. However, in practical wireless environments, NC might be susceptible to transmission errors caused by noise, fading, or interference. This drawback is quite problematic for real-time applications, such as multimedia content delivery, where timing constraints may lead to the reception of an insufficient number of packets and consequently to difficulties in decoding the transmitted sources. At best, some packets can be recovered, while in the worst case, the receiver is unable to recover any of the transmitted packets.

In this thesis, we propose joint source-network coding and decoding schemes in the purpose of providing an approximate reconstruction of the source in situations where perfect decoding is not possible. The main motivation comes from the fact that source redundancy can be exploited at the decoder in order to estimate the transmitted packets, even when some of them are missing. The redundancy can be either *natural*, *i.e*, already existing, or *artificial*, *i.e*, externally introduced.

Regarding artificial redundancy, we choose multiple description coding (MDC) as a way of introducing structured correlation among uncorrelated packets. By combining MDC and NC, we aim to ensure a reconstruction quality that improves gradually with the number of received network-coded packets. We consider two different approaches for generating descriptions. The first technique consists in generating multiple descriptions via a real-valued frame expansion applied at the source before quantization. Data recovery is then achieved via the solution of a *mixed integer linear problem.* The second technique uses a correlating transform in some Galois field in order to generate descriptions, and decoding involves a simple Gaussian elimination. Such schemes are particularly interesting for multimedia contents delivery, such as video streaming, where quality increases with the number of received descriptions.

Another application of such schemes would be multicasting or broadcasting data towards mobile terminals experiencing different channel conditions. The channel is modeled as a binary symmetric channel (BSC), which transition probability ε is described by a probability distribution $f(\varepsilon)$, and for which we study the effect on the decoding quality for both proposed schemes. Performance comparision with a traditional NC scheme is also provided.

Concerning natural redundancy, a typical scenario would be a *wireless sensor network*, where geographically distributed sources capture spatially correlated measures. We propose a scheme that aims at exploiting this spatial redundancy, and provide an estimation of the transmitted measurement samples via the solution of an *integer quadratic problem*. The obtained reconstruction quality is compared with the one provided by a classical NC scheme.

Résumé

Dans les réseaux traditionnels, la transmission de flux de données s'effectuaient par routage des paquets de la source vers le ou les destinataires. Le codage réseau (NC) permet aux noeuds intermédiaires du réseau d'effectuer des combinaisons linéaires des paquets de données qui arrivent à leurs liens entrants. Les opérations de codage ont lieu dans un corps de Galois de taille finie q. Aux destinataires, le décodage se fait par une élimination de Gauss des paquets codés-réseau reçus. Cependant, dans les réseaux sans fils, le codage réseau doit souvent faire face à des erreurs de transmission causées par le bruit, les effacements, et les interférences. Ceci est particulièrement problématique pour les applications temps réel, telle la transmission de contenus multimédia, où les contraintes en termes de délais d'acheminement peuvent aboutir à la réception d'un nombre insuffisant de paquets, et par conséquent à des difficultés à décoder les paquets transmis. Dans le meilleurs des cas, certains paquets arrivent à être décodés. Dans le pire des cas, aucun paquet ne peut être décodé.

Dans cette thèse, nous proposons des schémas de codage conjoint source-réseau dont l'objectif est de fournir une reconstruction approximative de la source, dans des situations où un décodage parfait est impossible. L'idée consiste à exploiter la redondance de la source au niveau du décodeur afin d'estimer les paquets émis, même quand certains de ces paquets sont perdus après avoir subi un codage réseau. La redondance peut être soit naturelle, c'est-à-dire déjà existante, ou introduite de manière artificielle.

Concernant la redondance artificielle, le codage à descriptions multiples (MDC) est choisi comme moyen d'introduire de la redondance structurée entre les paquets non corrélés. En combinant le codage à descriptions multiples et le codage réseau, nous cherchons à obtenir une qualité de reconstruction qui s'améliore progressivement avec le nombre de paquets codés-réseau reçus. Nous considérons deux approches différentes pour générer les descriptions. La première approche consiste à générer les descriptions par une expansion sur trame appliquée à la source avant la quantification. La reconstruction de données se fait par la résolution d'un problème d'*optimisation quadratique mixte*. La seconde technique utilise une matrice de transformée dans un corps de

Galois donné, afin de générer les descriptions, et le décodage se fait par une simple élimination de Gauss. Ces schémas sont particulièrement intéressants dans un contexte de transmission de contenus multimédia, comme le streaming vidéo, où la qualité s'améliore avec le nombre de descriptions reçues.

Une seconde application de tels schémas consiste en la diffusion de données vers des terminaux mobiles à travers des canaux de transmission dont les conditions sont variables. Le canal est modélisé par un canal binaire symétrique (BSC), dont la probabilité de transition aléatoire ε est décrite par une densité de probabilité $f(\varepsilon)$. Dans ce contexte, nous étudions la qualité de décodage obtenue pour chacun des deux schémas de codage proposés, et nous comparons les résultats obtenus avec ceux fournis par un schéma de codage réseau classique.

En ce qui concerne la redondance naturelle, un scénario typique est celui d'un réseau de capteurs, où des sources géographiquement distribuées prélèvent des mesures spatiallement corrélées. Nous proposons un schéma dont l'objectif est d'exploiter cette redondance spatiale afin de fournir une estimation des échantillons de mesures transmises par la résolution d'un problème d'*optimisation quadratique à variables entières*. La qualité de reconstruction est comparée à celle obtenue à travers un décodage réseau classique.

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Acronyms

AMPL	A Mathematical Programming Language
BB	Branch and Bound
BSC	Binary Symmetric Channel
CRC	Cyclic Redundancy Check
DCT	Discrete Cosine Transform
DF	Decode and Forward
DSC	Distributed Source Coding
IQP	Integer Quadratic Problem
MAC	Media Access Control
MAP	Maximum a Posteriori
MDC	Multiple Description Coding
MDS	Maximum Distance Separable
MIQP	Mixed Integer quadratic Program
NC	Network Coding
Pe	Error Probability
RLNC	Random Linear Network Coding

SDC Single Description Coding

SEQ Sequence Number

SNR Signal to Noise Ratio

UEP Priority Encoding Protection

UEP Unequal Error Protection

WSN Wireless Sensor Network

Première partie

Résumé Français

Codage-Décodage Conjoint Source-Réseau

0.1 Contexte de la thèse

Le codage réseau [ACLY00a, LYC03a] et plus particulièrement le codage réseau aléatoire [KM03, CWJ03a, HKM+03, HMK+06] est un outil puissant de diffusion d'information dans un réseau. Les techniques de codage réseau aléatoire permettent une implantation du codage réseau complètement distribuée et relativement indépendante de la structure du réseau considéré. Ainsi, [HMK+06] montre que la capacité d'un réseau dans lequel le codage aléatoire est effectué peut être atteinte avec une probabilité qui s'approche exponentiellement de un avec la taille du corps de Galois dans lequel les opérations de codage ont lieu. Ces travaux ont conduit à de nombreux schémas pratiques tels que COPE, ANC, MIXIT, MORE... [KKH+05, KK07, KGK07].

En contre partie, le codage réseau est très sensible à l'égard d'erreurs de transmission, de paquets corrompus intentionnellement par des agents malveillants, et de pertes de paquets. En effet, les recombinaisons effectuées par chaque nœud entrainent une contamination progressive de l'ensemble des paquets sains par les paquets erronés, rendant le décodage impossible au récepteur. Par ailleurs, même en l'absence d'erreurs, les pertes de paquets, conduisent à un nombre insuffisant de paquets au récepteur, et rendent l'exploitation des paquets déjà reçus impossible.

Dans ce contexte, cette thèse se focalise sur le problème de reconstruction de données émises par la source, dans des situations où le décodage parfait est impossible au niveau du récepteur en raison des pertes ou des variations de capacité sur certains liens du réseau. En particulier, nous cherchons à évaluer l'effet de l'exploitation de la redondance de la source sur la qualité de décodage par le biais d'estimateurs permettant une recontruction approximative des paquets transmis. Cette redondance peut être naturellement présente, comme les données prélevées dans un réseau de capteurs, ou introduite artificiellement par l'intermédiaire de techniques de codage réseau correcteur d'erreurs.

Les techniques de *codage réseau correcteur d'erreurs* ont pour objectif de protéger les paquets transmis vis-à-vis de paquets erronés et/ou de pertes. Ces techniques introduisent un certain niveau de *redondance* et sont similaires dans leur principe aux codes correcteurs d'erreurs classiques. Deux familles de codes peuvent être distinguées. Les codes introduits dans [CY02, Zha08] considèrent conjointement le codage réseau et l'introduction de redondance. Ces codes nécessitent une connaissance *a priori* de l'architecture du réseau et de la manière dont le codage réseau est effectué. Ces résultats sont étendus dans le cadre d'un codage réseau aléatoire dans [HMK+06, BYZ09]. Les techniques introduites dans [JLK+07, KK08a, SKK08, AA09] exploitent le fait qu'en l'absence d'erreurs, le codage réseau aléatoire conserve l'espace vectoriel engendré par les paquets transmis. Dans ce cas, le code est constitué d'un ensemble d'espaces vectoriels. La source transmet des paquets formant la base de l'un de ces espaces, et la destination cherche à retrouver l'espace envoyé par la source. Les codes réseaux robustes obtenus ont des propriétés indépendantes de la structure du réseau.

Les techniques de *décodage conjoint* exploitent la redondance présente au sein du réseau de communication [DK09]. Dans le cas du *décodage conjoint réseau-canal* [HH06, Tho08], la diversité temporelle, spatiale, ou la présence de codes de canal [KDH07, GHW+09] sont mises à profit pour combattre le bruit introduit par les canaux de communication, en particulier sans fils. Les techniques de *décodage conjoint source-réseau* permettent quant à elles de restaurer tout ou une partie des paquets initiaux, même en présence d'un nombre insuffisant de paquets reçus, en exploitant la corrélation existant entre paquets de données transmis. Ces techniques apportent donc une certaine robustesse à l'égard de pertes de paquets.

Dans cette thèse, au paragraphe 0.2, nous proposons un schéma de codage source-réseau conjoint, basé sur le principe du codage à descriptions multiples (MDC), et nous étudions l'effet de l'introduction de la redondance, par expansion sur trame ou par transformée, sur la qualité de décodage au récepteur.

Nous évaluons ensuite l'effet de la taille du corps de Galois ainsi que les conditions du canal sur la qualité moyenne du signal reconstruit, dans un contexte de diffusion de données vers des utilisateurs disposant de canaux sans fils à caractéristiques variées. Ceci est effectué dans le cas où

1. seul le codage réseau est effectué
2. une combinaison de codage réseau et de codage à descriptions multiples est mis en oeuvre au paragraphe 0.3.

Enfin, au paragraphe 0.4, nous considérons un réseau de capteurs effectuant des mesures spatialement corrélées, et nous proposons un estimateur permettant l'exploitation de cette corrélation, naturellement existante, afin de fournir une reconstruction approximative des mesures transmises.

Le paragraphe 0.5 présente la conclusion de cette thèse et introduit quelques perspectives.

0.2 Décodage réseau de données redondantes

Concernant la robustesse à l'égard de pertes ou à des variations de capacité de certains liens du réseau, nous proposons une solution consistant à combiner des techniques de codage à descriptions multiples [Goy98], [GKK01] et de codage réseau. L'objectif est d'exploiter la redondance introduite par ces techniques de codage afin de permettre une amélioration progressive de la qualité des données reconstruites en fonction du nombre de paquets indépendants reçus au niveau du décodeur [IKLAA11]. Cette redondance artificielle est introduite soit par expansion sur trame, voir le paragraphe 0.2.1, soit par transformée redondante dans un corps fini, voir le paragraphe 0.2.2.

0.2.1 Corrélation introduite par expansion sur trame

On considère une source qui génère des échantillons $\mathbf{x} \in \mathbb{R}^k$, supposés iid, gaussiens, de moyenne nulle et de variance σ^2.

0.2.1.1 Schéma de codage

On note cette technique NC-MDC-F. La redondance est introduite à l'aide d'une expansion sur trame [GKK01] des données générées par la source, voir la figure 1.

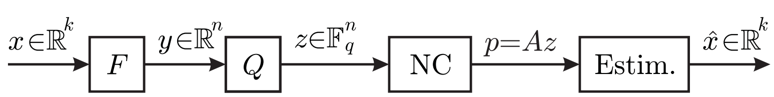

FIGURE 1 – Codage source-réseau conjoint avec redondance introduite via une expansion sur trame

Les échantillons $\mathbf{x} \in \mathbb{R}^k$ générés par la source sont transformés à l'aide d'une expansion sur une trame $F \in \mathbb{R}^{n\times k}$ de manière à obtenir $\mathbf{y} = F\mathbf{x} \in \mathbb{R}^n$ avec $n > k$. Les échantillons corrélées $\mathbf{y}$ sont ensuite quantifiés à l'aide d'un quantificateur uniforme de pas Δ à q niveaux pour obtenir un vecteur $\mathbf{z} \in \mathbb{F}_q^n$. Chaque composante z_i de $\mathbf{z}$ est placée dans un paquet différent. Ces paquets sont ensuite transmis dans le réseau où ils subissent un codage réseau représenté par une matrice A.

Une estimée $\widehat{\mathbf{x}}_F$ de $\mathbf{x}$ est finalement évaluée à l'aide de l'ensemble des paquets reçus groupés dans un vecteur $\mathbf{p}$, ainsi qu'en exploitant les différentes contraintes imposées par le système.

Dans ce qui suit, chaque étape de la figure (1) est décrite afin de mettre en évidence les contraintes liant les différentes variables du système, ce qui permettra d'évaluer l'estimée $\widehat{\mathbf{x}}_F$ de $\mathbf{x}$ au niveau du décodeur.

0.2.1.2 Codage à descriptions multiples via une expansion sur trame

Une trame sur $\mathbb{R}^k$ est un ensemble de $n > k$ vecteurs $\{\varphi_i\}_{i=1\ldots n}$ tel qu'il existe $B > 0$ et $C < \infty$ satisfaisant pour tout $\mathbf{x} \in \mathbb{R}^k$,

$$B \left\|\mathbf{x}\right\|^2 \leq \sum_{i=1}^{n} \left\langle \mathbf{x}, \varphi_i \right\rangle^2 \leq C \left\|\mathbf{x}\right\|^2, \tag{1}$$

où $\langle \cdot, \cdot \rangle$ est le produit scalaire de $\mathbb{R}^k$. L'opérateur de trame F associé aux vecteurs $\{\varphi_i\}_{i=1\ldots n}$ est un opérateur linéaire de $\mathbb{R}^k$ à $\mathbb{R}^n$, défini par

$$\left(F\mathbf{x}\right)_i = \left\langle \mathbf{x}, \varphi_i \right\rangle, \, i = 1 \ldots n. \tag{2}$$

Pour tout $\mathbf{x} \in \mathbb{R}^k$, l'opérateur de trame F produit un vecteur

$$\mathbf{y} = F\mathbf{x} \in \mathbb{R}^n. \tag{3}$$

Le taux de redondance introduit par l'expansion sur trame est $r = n/k$.

0.2.1.3 Quantification

Les échantillons corrélées $\mathbf{y}$ sont quantifiés à l'aide d'un quantificateur uniforme Q de pas Δ à q niveaux pour obtenir un vecteur $\mathbf{z}$, dont les échantillons $z_i = Q(z_i)$ appartiennent à $\mathbb{F}_q$. Les intervalles de quantification sont $\left[\left(i - q/2\right)\Delta, \left(i - q/2 + 1\right)\Delta\right]$, $i = 0 \ldots q - 1$. Les niveaux de reconstruction sont choisis au milieu des intervalles de quantification, $r_i = \left(i - q/2 + 1/2\right)\Delta$, $i = 0 \ldots q - 1$. Δ est choisi de manière à garantir que toutes les entrées de $\mathbf{y}$ se retrouvent à l'intérieur de l'un des intervalles de quantification. L'opérateur de quantification inverse Q^{-1}, associe à chaque indice de quantification z_i une reconstruction

$$Q^{-1}(z_i) = \alpha z_i + \beta, \tag{4}$$

avec $\alpha = \Delta$ et $\beta = (-q+1)\,\Delta/2$. Les niveaux de quantification étant tous pris au milieu des intervalles de quantification, $Q^{-1}(z_i)$ satisfait

$$y_i - Q^{-1}(z_i) \leq \Delta/2, \tag{5}$$

$$-y_i + Q^{-1}(z_i) \leq \Delta/2. \tag{6}$$

0.2.1.4 Codage réseau

Le vecteur d'échantillons quantifiés $\mathbf{z}$ est transmis dans le réseau. Chaque composante z_i de $\mathbf{z}$ est transmise dans un paquet separé. Un codage réseau linéaire et aléatoire est effectué au niveau des noeuds intermédiaires du réseau. On suppose que le ℓ ème récepteur dispose de m paquets indépendants $p_\mu \in \mathbb{F}_q$, $\mu = 1 \ldots m$, avec $m \leq n$. Comme ces paquets sont codés, la relation entre $\mathbf{p} = (p_1, ..., p_m)^T$ et $\mathbf{z}$ peut être exprimée par

$$\mathbf{p} = A\mathbf{z}, \tag{7}$$

où $A \in \mathbb{F}_q^{m \times n}$ est la matrice du réseau contenant les coefficients de codage globaux, disponible au niveau du récepteur. Les coefficients de A peuvent être obtenus à partir des entêtes des paquets reçus [CWJ03a]. En général, $m = n$ paquets doivent être reçus pour pouvoir reconstruire les paquets non codés. Cependant, l'obtention de n paquets au niveau du récepteur n'implique pas forcément que la matrice de codage A est de rang plein n. D'autre part, si le nombre de paquets reçus n'est pas suffisant, c'est-à-dire, $m < n$, la matrice de codage A n'est pas de rang plein, et les paquets non codés ne peuvent pas être reconstruits directement.

0.2.1.5 Estimation du vecteur source

Une estimée $\widehat{\mathbf{x}}_F$ du vecteur source $\mathbf{x}$ est évaluée à partir de l'ensemble des paquets reçus $\mathbf{p}$, du fait qu'une expansion sur trame a été effectuée sur $\mathbf{x}$, et en respectant l'ensemble des égalités et inégalités derivées de (3), (5), (6), et (7). L'ensemble des contraintes sont rassemblées dans le système suivant

$$\begin{cases} y_i = \sum_{j=1}^{k} f_{i,j} x_j, & i = 1 \dots n \\ y_i - (\alpha z_i + \beta) \leq \Delta/2, & i = 1 \dots n \\ -y_i + (\alpha z_i + \beta) \leq \Delta/2, & i = 1 \dots n \\ z_i \in \{0, \dots, q-1\} & i = 1 \dots n \\ p_\mu = \sum_{j=1}^{n} a_{\mu j} z_j, & \mu = 1 \dots m \end{cases} \tag{8}$$

où $F = (f_{ij})_{i=1\dots n, j=1\dots k}$ et $A = (a_{\mu j})_{\mu=1\dots m, j=1\dots n}$. Dans la dernière égalité du système (8), les opérations sont toutes effectuées dans $\mathbb{F}_q$. Ce système contient $n+m$ équations, $2n$ inéquations, et $2n+k$ inconnues, notamment $\mathbf{x} \in \mathbb{R}^k$, $\mathbf{y} \in \mathbb{R}^n$, et $\mathbf{z} \in \mathbb{F}_q^n$. Comme $\mathbf{x}$ est quantifié, il ne peut être reconstruit de manière exacte, même si la matrice de codage A est de rang plein n.

0.2.1.6 Un nombre suffisant de paquets est reçu

Lorsque A est de rang plein n, une estimée $\widehat{\mathbf{x}}_F$ de $\mathbf{x}$ peut être obtenue à partir des paquets reçus $\mathbf{p}$ par inversion de la matrice de codage réseau A, ou d'une sous-matrice de A de rang plein n, ce qui donne une estimée

$$\widehat{\mathbf{z}} = Q^{-1}\left(A^{-1}\mathbf{p}\right), \tag{9}$$

de $\mathbf{z}$, avec Q^{-1} la fonction de desindexation du quantificateur. Cette estimée est ensuite utilisée pour obtenir l'estimée au sens des moindres carrés de $\mathbf{x}$ à partir de $\widehat{\mathbf{z}}$

$$\widehat{\mathbf{x}}_F = \left(F^T F\right)^{-1} F^T \widehat{\mathbf{z}}. \tag{10}$$

0.2.1.7 Un nombre insuffisant de paquets est reçu

Lorsqu'un nombre insuffisant de paquets est reçu, la matrice de codage A ne peut pas être inversée. Il n'existe dans ce cas pas d'estimée unique de $\mathbf{z}$ à partir des paquets reçus $\mathbf{p} \in \mathbb{F}^m$, et par conséquent, $\mathbf{x}$ est encore plus difficile à estimer. On choisit par conséquent l'estimée de norme minimale

$$\widehat{\mathbf{x}}_F = \arg\min \mathbf{x}^T\mathbf{x} \tag{11}$$

En combinant (11), et (8), on obtient un problème d'optimisation sous contraintes délicat car il combine les variables réelles $\mathbf{x}$ et $\mathbf{y}$, ainsi que des variables appartenant à un corps de Galois $\mathbf{z}$. Lorsque la taille du corps de Galois q est première, les opérations de codage réseau peuvent être exprimées dans l'anneau des entiers $\mathbb{Z}$ en introduisant un vecteur de variables additionnelles $\mathbf{s} \in \mathbb{Z}^m$ de manière à exprimer la contrainte (7) sous la forme

$$\mathbf{p} = A\mathbf{z} + q\mathbf{s}. \tag{12}$$

La solution de (11) sous les contraintes du Système (8) où la dernière contrainte est remplacée par (12) nécessite la résolution d'un problème d'optimisation quadratique mixte. Nous avons modélisé ce problème avec AMPL et l'avons résolu avec CPLEX.

La complexité d'estimation est bien supérieure qu'avec une technique de codage réseau classique. Cependant, une partie du bruit de quantification peut être supprimée lorsqu'un nombre élevé de paquets est reçu.

0.2.2 Corrélation introduite par transformée redondante

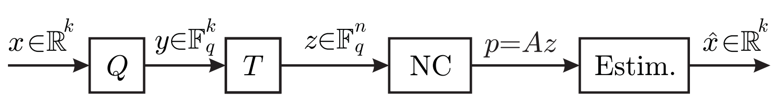

FIGURE 2 – Codage source-réseau conjoint avec redondance introduite par transformée

Dans cette version, notée NC-MDC-T, les échantillons $\mathbf{x} \in \mathbb{R}^k$ générés par la source sont quantifiés sur q niveaux, puis transformés à l'aide d'une transformée redondante $T \in \mathbb{F}^{n\times k}$ de rang plein k pour obtenir $\mathbf{z} = T\mathbf{y}$. Par conséquent, il existe une matrice $D \in \mathbb{F}^{(n-k)\times n}$ de rang plein $n-k$ telle que

$$D\mathbf{z} = \mathbf{0}.$$

Les éléments de $\mathbf{z}$ sont ensuite placés chacun dans un paquet et ces derniers sont ensuite transmis dans le réseau où ils subissent un codage réseau représenté par la matrice A au niveau des nœuds intermédiaires. Au décodeur, la matrice

$$B = \begin{pmatrix} A \\ D \end{pmatrix}$$

est construite à partir des entêtes des paquets reçus. S'il existe une sous-matrice B' de B telle que B' est de rang plein n, alors les éléments de $\mathbf{z}$ peuvent être reconstruits par simple élimination de Gauss. Dans le cas contraire, le décodage est impossible.

Cette approche fournit une bonne robustesse à l'égard de pertes avec une complexité de décodage du même ordre de grandeur qu'avec un codage réseau classique.

0.2.2.1 Résultats de simulations

Le scénario NC-MDC-F décrit au paragraphe 0.2.1.1, et le scénario NC-MDC-T décrit au paragraphe 0.2.2 sont simulés avec la même source. On choisit les paramètres $k = 4$ et $n = 7$. Dans les deux cas, la source génère k échantillons iid, gaussiens, de moyenne nulle et de variance $\sigma^2 = 1$, limité à $\pm 3\sigma$.

Pour NC-MDC-F, F est construite à l'aide des lignes 2 à 5 d'une matrice de transformée DCT de dimension $n \times n$. Un quantificateur uniforme avec des cellules de quantification partionnant l'intervalle $[-3\sigma, +3\sigma]$ est choisi. On considère q niveaux de quantification avec $q \in \{7, 17, 31, 61\}$. Par conséquent, les indices de quantification appartiennent à $\mathbb{F}_q$. La matrice de codage A de dimension $m \times n$ est générée aléatoirement avec $m \leq n$, afin de simuler l'effet du codage réseau.

Pour NC-MDC-T, les mêmes niveaux de quantification avec $q \in \{7, 17, 31, 61\}$ sont considérés, aboutissant à des indices de quantification appartiennent à $\mathbb{F}_q$. La redondance est introduite par

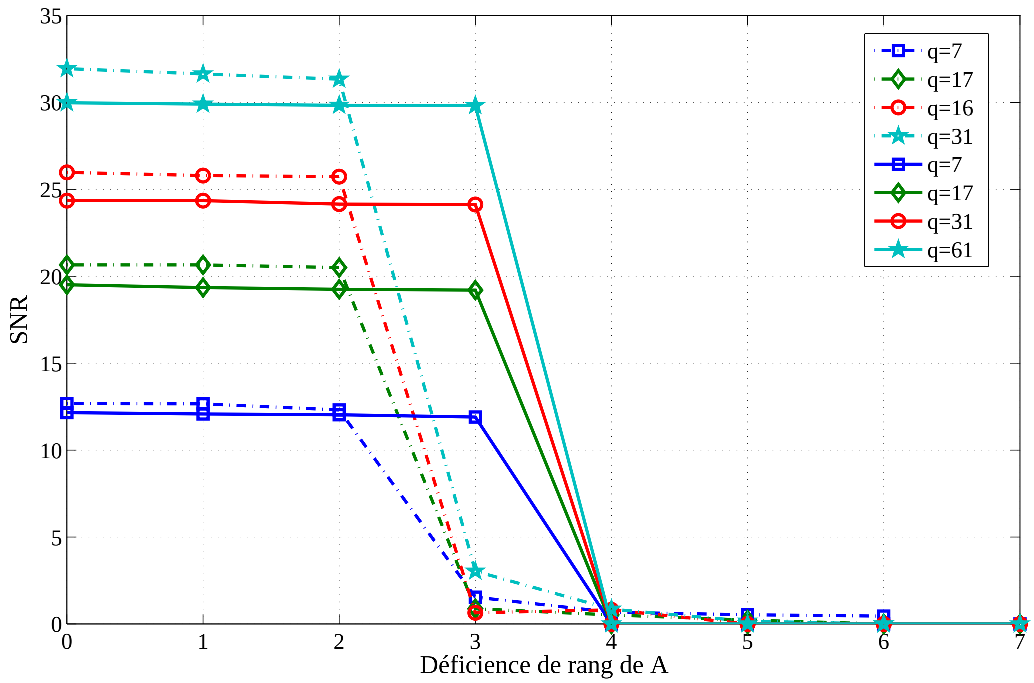

Figure 3: Rapport signal à bruit (SNR) en fonction de la déficience de rang de A et de la taille du corps de Galois considéré. NC-MDC-F en traits discontinus, NC-MDC-T en traits continus.

une matrice fixe $T \in \mathbb{F}_q$, de dimension $n \times k$, et de rang plein k. La matrice D, de dimension $(n-k) \times n$ et de rang plein $n-k$ est choisie orthogonale à T.

Dans les deux scénarios, les résultats de simulations sont moyennés sur 1000 réalisations de la source et de la matrice de codage A.

La figure 3 représente le rapport signal à bruit (SNR) moyen defini par

$$SNR_{dB} = 10 \log_{10} \frac{\|\mathbf{x}\|^2}{\|\mathbf{x} - \widehat{\mathbf{x}}\|^2} \tag{13}$$

et résultant de la reconstruction des échantillons transmis en fonction de la déficience de rang de A, et pour différentes tailles de corps de Galois. NC-MDC-F est moins robuste aux pertes que NC-MDC-T, puisqu'avec NC-MDC-T, une perte de plus est tolérée. Par contre, quand suffisamment de paquets indépendants sont reçus, la qualité de reconstruction est meilleure avec NC-MDC-F. En effet, la redondance introduite par l'expansion sur trame permet de réduire une partie du bruit

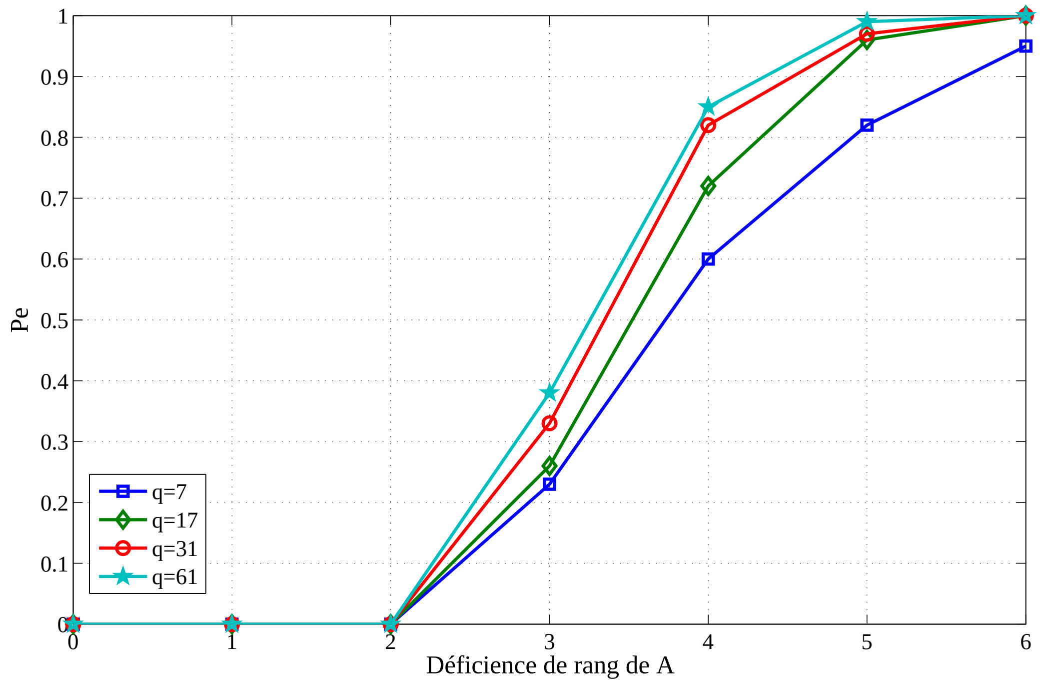

Figure 4: NC-MDC-F: Proportion d'erreurs de reconstruction en fonction de la déficience de rang de A, pour différents corps de Galois dans lequel se fait le codage réseau

de quantification, ce qui n'est pas le cas avec NC-MDC-T.

Les figures 4 et 5 représentent l'évolution des erreurs de décodage, lorsque $\widehat{\mathbf{z}} \neq \mathbf{z}$, en fonction de la déficience de rang de A dans les deux scénarios.

Avec le scénario NC-MDC-T, si le nombre de paquets indépendants perdus est supérieur au nombre de paquets redondants, c'est-à-dire, supérieur à $n-k$ dans le cas considéré, la probabilité d'erreur de décodage est de 1. La reconstruction est par conséquent impossible. Par contre, dans le cas de NC-MDC-F, une fraction non nulle des paquets transmis peut être décodée correctement, même si le nombre de pertes est supérieur au nombre de paquets introduits lors de l'expansion sur trame. D'autre part, la probabilité d'erreur de décodage augmente de manière progressive.

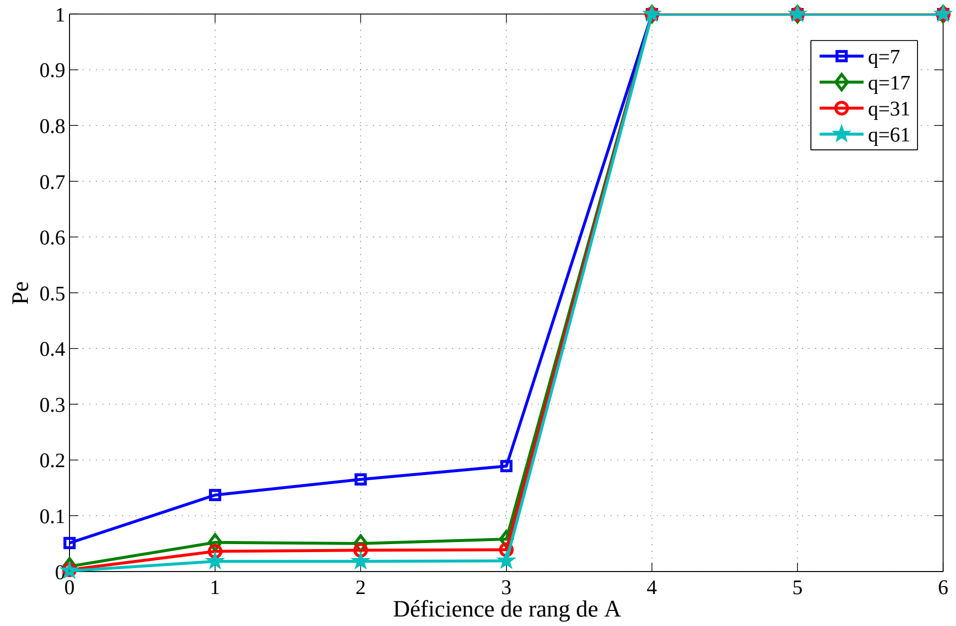

Figure 5: NC-MDC-T: Proportion d'erreurs de reconstruction en fonction de la déficience de rang de A, pour différents corps de Galois dans lequel se fait le codage réseau

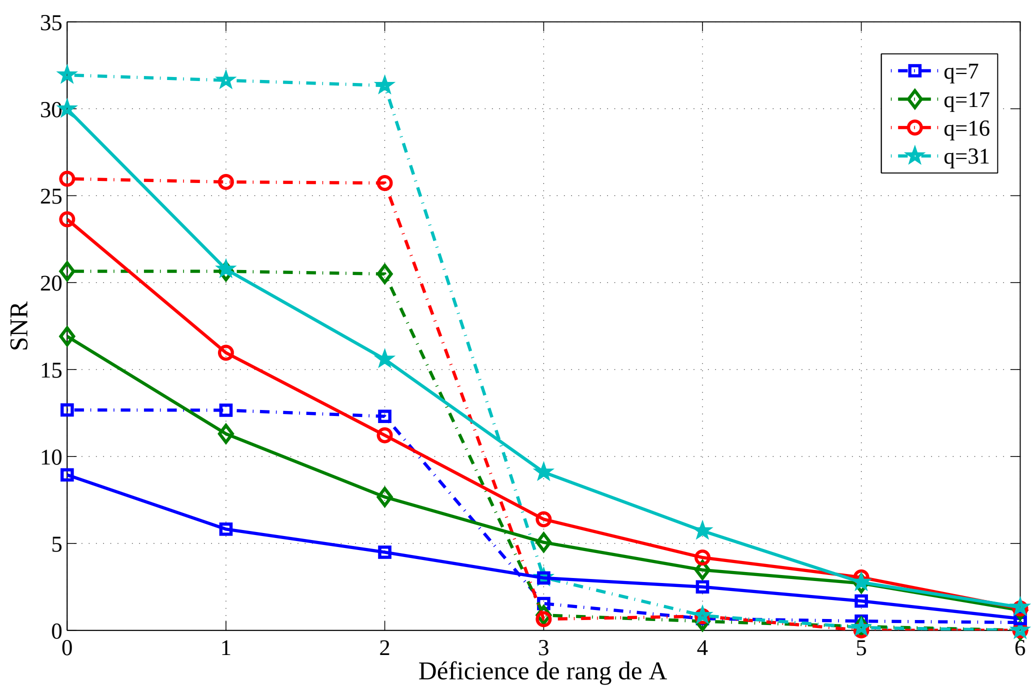

Figure 6: NC-MDC-F (discontinu) comparé à NC-SDC (continu) où le codage réseau est effectué uniquement entre les paquets d'une même description

La figure 6 permet d'évaluer l'amélioration apportée par la reconstruction MIQP, en comparant le scénario NC-MDC-F avec un scénario MDC-F où le codage réseau est réalisé uniquement sur des paquets correspondant à la même description. Dans le dernier cas, si un paquet indépendant est perdu, la description entière est perdue. La reconstruction MIQP n'est pas possible. Les résultats montrent que la reconstruction MIQP permet d'avoir une reconstruction avec la meilleure qualité possible, même lorsqu'un certain nombre de paquets indépendants sont perdus. D'un autre coté, quand le nombre de pertes est assez important, le schéma NC-SDC présente une diminution plus progressive du SNR que celle avec NC-MDC-F.

0.3 Evaluation de performance des schémas MDC

Nous considérons maintenant un scénario de multicast entre une source unique S, et des utilisateurs desservis chacun par un canal de transmission dont les caractéristiques sont variables. Le canal entre la source et l'utilisateur i est modélisé par un canal binaire symétrique (CBS), avec une probabilité de transition aléatoire ε, décrite selon une densité de probabilité $f(\varepsilon)$. La qualité du reconstruction moyenne de données dépend largement de $f(\varepsilon)$.

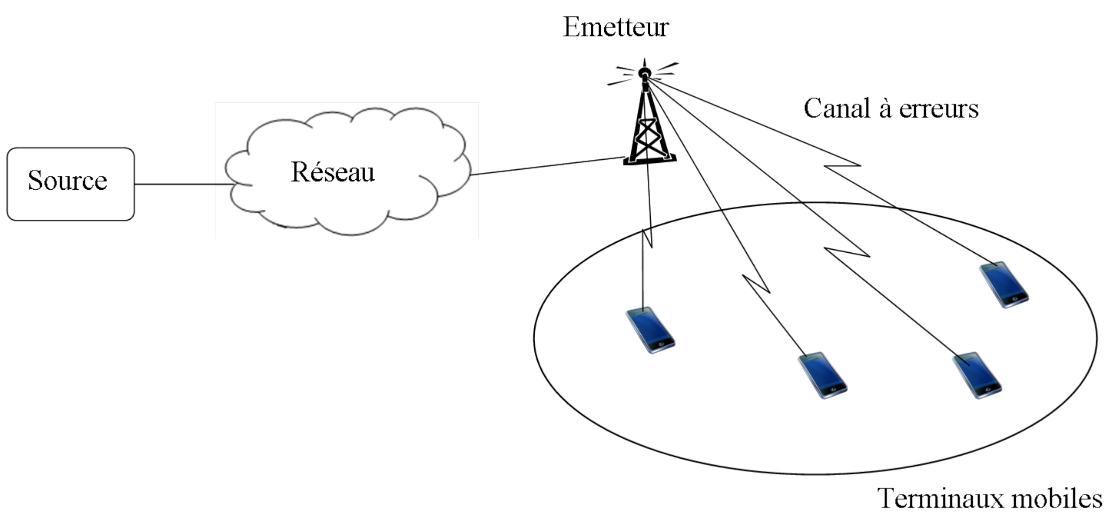

FIGURE 7 – Schéma de transmission multicast vers des terminaux mobiles

Dans ce contexte, l'objectif de cette partie consiste à évaluer les performances des deux scénarios proposés NC-MDC-F et NC-MDC-T, et de les comparer à un scénario classique NC-SDC, où seul un codage réseau linéaire et aléatoire (RLNC) est effectué au niveau du réseau.

Ceci permet de caractériser pour les trois schémas, le SNR moyen au niveau des récepteurs, et ceci pour différentes $f(\varepsilon)$. Le SNR est évalué en fonction de

1. la taille du corps de Galois q dans lequel les opérations de codage ont lieu

2. la probabilité d'erreur ε pour une taille fixe q

0.3.1 Calcul du SNR moyen

On définit la taille d'une génération, notée g, comme étant le nombre de paquets codés-réseau envoyés au cours de chaque transmission de la source. Pour NC-SDC, $g = k$, alors que pour NC-MDC-T et NC-MDC-F, $g = n$. On suppose que chacun des N récepteurs collecte n paquets erronnés à partir desquels une estimée de $\mathbf{x}$ doit être évaluée. Pour chacun des récepteurs et quel que soit le scénario considéré, on détermine la distribution de probabilité $P_\Gamma(\gamma \mid g, n, q)$ du nombre γ de paquets indépendants reçus, c'est-à-dire, la distribution de probabilité du rang de la matrice de codage A, sachant que g paquets ont été combinés, que n paquets erronnés ont été reçus, et que la taille du corps de Galois est q.

Si i est le nombre de paquets corrects reçus, alors

$$P_\Gamma(\gamma \mid g, n, q) = \sum_{i=\gamma}^{n} P_R(\gamma \mid i, g, q)\, P_C(i \mid g, n, q)\,, \tag{14}$$

où $P_C(i \mid g, n, q)$est la probabilité de réception de i paquets non-erronnés (elle dépend de $f(\varepsilon)$) et $P_R(\gamma \mid i, g, q)$ est la probabilité d'avoir γ paquets indépendants parmi les i paquets corrects reçus (ceci étant la probabilité que la matrice A ait un rang γ). Tout calcul fait, $P_C(i \mid g, n, q)$ peut être exprimé comme suit

$$P_C(i \mid g, n, q) = \int P(i \mid g, n, q, \varepsilon)\, f(\varepsilon)\, d\varepsilon \tag{15}$$

avec

$$P(i \mid g, n, q, \varepsilon) = \binom{n}{i} (1-\varepsilon)^{iL} \left(1 - (1-\varepsilon)^L\right)^{n-i} \tag{16}$$

où $P(i \mid g, n, q, \varepsilon)$ est la probabilité d'avoir i paquets non-erronnés parmi n reçus, et ceci pour une réalisation ε de la probabilité de transition du CBS. $L = (g + \ell + \ell_{\text{seq}} + \ell_{\text{crc}}) \lceil \log_2 q \rceil$ représente la longueur totale d'un paquet en bits, où ℓ_{seq} et ℓ_{crc} représentent respectivement la longueur du numéro de séquence (SEQ) et celle du CRC, et où ℓ représente la longueur de la charge utile du paquet (payload).

D'autre part, on a

$$P_R(\gamma \mid i, g, q) = \frac{\mu(i, g, \gamma, q)}{q^{ig}} \tag{17}$$

où $\mu(n_1, k_1, r_1, q)$ [vLW92] est le nombre de matrices appartenant à $\mathbb{F}_q^{n_1 \times k_1}$ et dont le rang est égal à r_1

$$\mu(n_1, k_1, r_1, q) = \begin{bmatrix} k_1 \\ r_1 \end{bmatrix}_q \sum_{i=0}^{r_1} (-1)^{(r_1-1)} \begin{bmatrix} r_1 \\ i \end{bmatrix}_q q^{n_1 i + \binom{r_1 - i}{2}} \tag{18}$$

et $\begin{bmatrix} n_1 \\ k_1 \end{bmatrix}_q$ est le coefficient gaussien [vLW92] donné par

$$\begin{bmatrix} n_1 \\ k_1 \end{bmatrix}_q = \begin{cases} 1 & k_1 = 0 \\ \frac{(q^{n_1}-1)(q^{n_1-1}-1)\dots(q^{n_1-k_1+1}-1)}{(q^{k_1}-1)(q^{k_1-1}-1)\dots(q-1)} & k_1 > 0. \end{cases} \tag{19}$$

On note $\text{SNR}_s(\gamma, q)$, le SNR moyen relatif à chaque scénario $s = 1, \dots, 3$, et qui correspond à la réception de γ paquets indépendants et codés dans $\mathbb{F}_q$. Par conséquent, en tenant compte de la distribution $f(\varepsilon)$ de la probabilité de transition du CBS, le SNR moyen peut être exprimé comme

$$\text{SNR}_s(q) = \sum_{\gamma=0}^{n} \text{SNR}_s(q, \gamma) P_\Gamma(\gamma \mid g, n, q). \tag{20}$$

0.3.2 Résultats expérimentaux

Les simulations sont réalisées pour $k = 6$ et $n = 9$. La source génère k échantillons distribués selon $\mathcal{N}(0,1)$. q niveaux de quantification avec $q \in \{7, 17, 31, 61\}$ sont choisis et la matrice de codage A est générée aléatoirement dans le corps de Galois $\mathbb{F}_q$ correspondant. Les résultats de simulations sont moyennés sur 1000 réalisations de la source et de la matrice de codage A.

0.3.2.1 SNR moyen en fonction de ε

Dans ce cas, nous cherchons à évaluer le SNR moyen en fonction de la probabilité de transition du canal ε. La figure 8 représente le SNR obtenu quand ε varie de 10^{-6} à 10^{-1}, et ceci pour un corps de Galois de taille $q = 31$ et pour une longueur de charge utile des paquets $\ell = 100$ symboles.

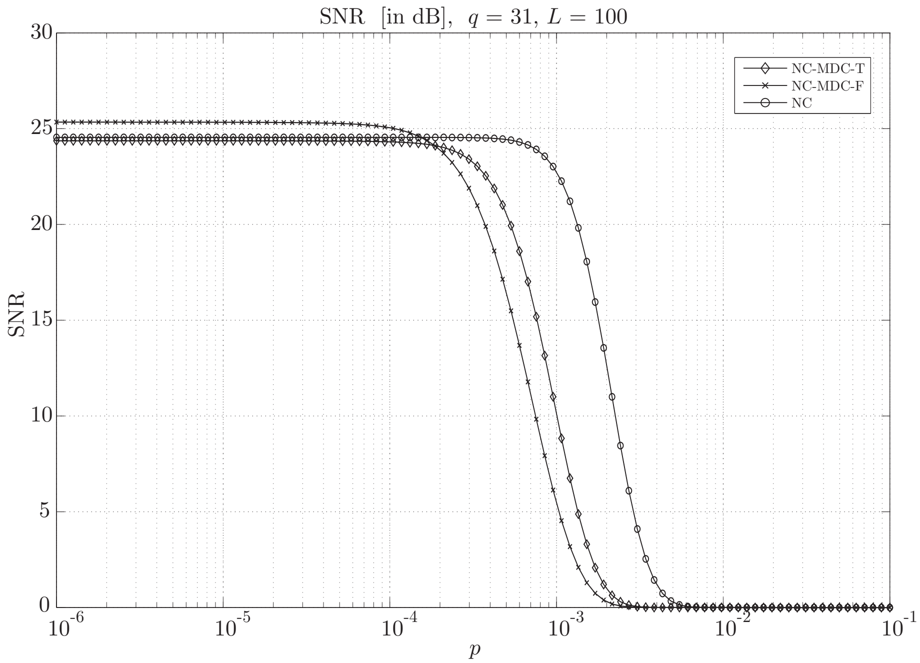

Figure 8: SNR moyen en fonction de la probabilité de transition ε

On observe que NC-MDC-F présente un meilleur SNR que NC-SDC pour des valeurs de $\varepsilon \in [10^{-6}, 1.5 \times 10^{-4}]$, alors que pour des valeurs plus élevées de ε, c'est NC-SDC qui présente les valeurs de SNR les plus élevées.

0.3.2.2 SNR en fonction de $f(\varepsilon)$

On cherche maintenant à évaluer le SNR moyen obtenu en fonction de la distribution $f(\varepsilon)$ de la probabilité de transition du CBS. Pour cela, on considère que $f(\varepsilon)$ suit, d'abord, une loi de distribution uniforme

$$f_{\mathrm{U}}(\varepsilon) = \begin{cases} \frac{1}{a} & 0 \leq \varepsilon \leq a, \\ 0, & \end{cases} \tag{21}$$

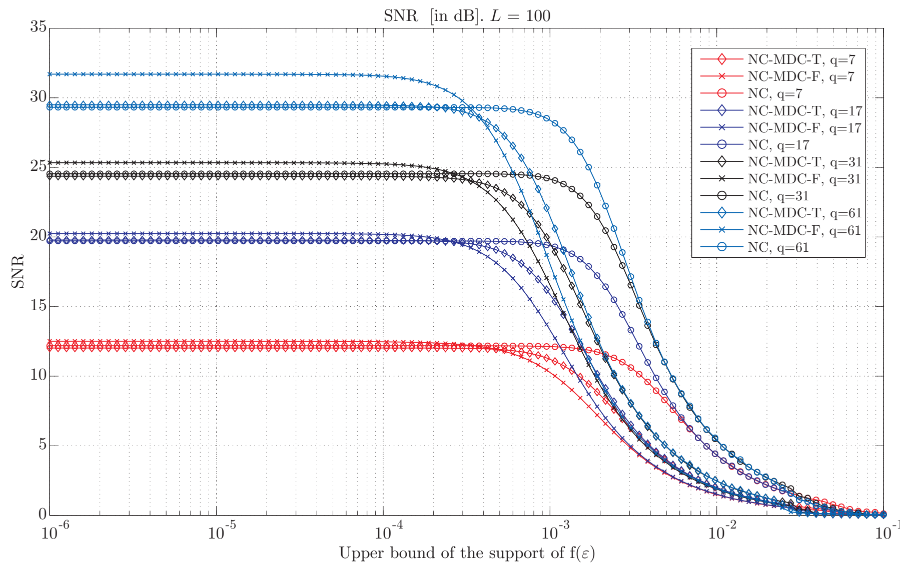

Figure 9: SNR moyen pour une distribution uniforme en fonction de la borne supérieure du support de $f_{\mathrm{U}}(\varepsilon)$

et ensuite, une distribution exponentielle

$$f_{\mathrm{E}}(\varepsilon) = \begin{cases} \frac{\ln(10)10^{-\varepsilon}}{10^{-a}-10^{-b}} & a \leq \varepsilon \leq b, \\ 0. \end{cases} \tag{22}$$

L'expression de $P_C(i \mid g, n, q)$ devient

$$P_C(i \mid g, n, q) = \frac{1}{a}\binom{n}{i}\int_0^a (1-\varepsilon)^{iL}\left(1-(1-\varepsilon)^L\right)^{n-i} d\varepsilon. \tag{23}$$

pour la distribution uniforme, et pour la distribution exponentielle elle devient

$$P_C(i \mid g, n, q) = \frac{\ln(10)}{10^{-a}-10^{-b}}\binom{n}{i}\int_a^b 10^{-\varepsilon}(1-\varepsilon)^{iL}\left(1-(1-\varepsilon)^L\right)^{n-i} d\varepsilon. \tag{24}$$

Le SNR moyen est calculé à l'aide (20) dans les deux cas. Le nombre de paquets à combiner est $g = n$ pour NC-MDC-T et NC-MDC-F, alors que pour NC-SDC, $g = k$.

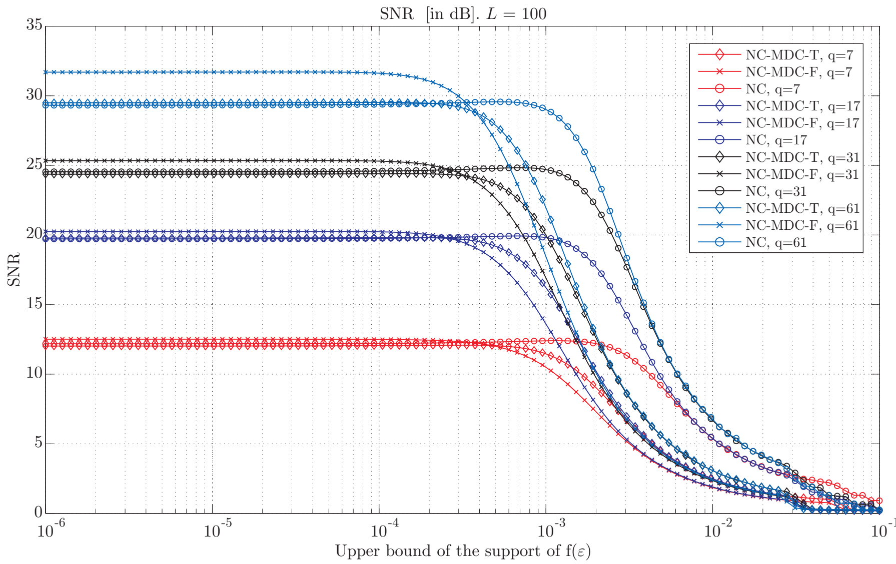

Figure 10: SNR moyen pour une distribution exponentielle en fonction de la borne supérieure du support de $f_{\mathrm{E}}(\varepsilon)$

La figure 9 représente le SNR obtenu pour les trois scénarios dans le cas d'une distribution uniforme, et ceci pour différents tailles de corps de Galois, $q \in \{7, 17, 31, 61\}$. Le SNR est tracé en fonction de a, qui correspond à la borne supérieure du support de ε.

La figure 10 représente le SNR obtenu dans le cas de la distribution exponentielle, avec $a = 10^{-10}$, et $b \in [10^{-6}, 10^{-1}]$.

Dans les deux cas, pour de bonnes conditions de canal, NC-MDC-F présente de meilleures performances que NC-MDC-T, qui lui même est meilleur que NC-SDC. Par contre, quand les conditions de canal sont mauvaises ($a > 3.10^{-4}$ pour la distribution uniforme et $b > 3.10^{-4}$ pour la distribution exponentielle), NC-SDC devient meilleur.

0.4 Exploitation de la corrélation existante : réseaux de capteurs

L'objectif de ce paragraphe consiste à exploiter la redondance présente au niveau des données générées par la source, lorsque cette dernière est présente naturellement. Un scénario typique est celui d' un réseau de capteurs effectuant des mesures corrélées spatialement comme l'indique la figure 11.

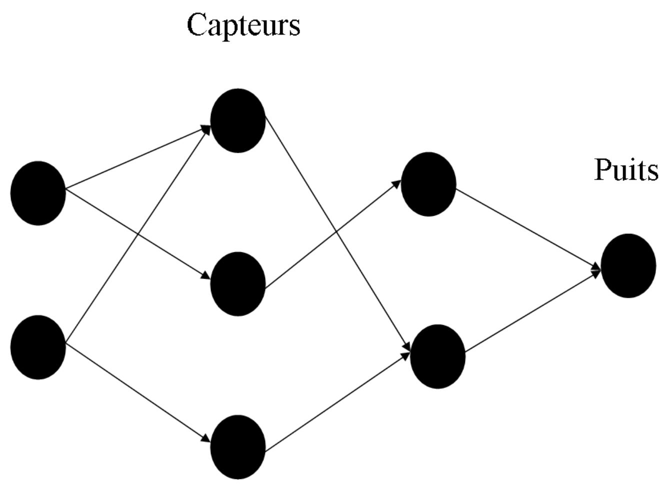

Figure 11: Schéma d'un réseau de capteurs destiné à collecter des données par un destinataire

On considère un ensemble de k capteurs dispersés dans un espace bi-dimensionnel. On note $\boldsymbol{\theta}_i \in \mathbb{R}^2$ la position du i-ème capteur. Ce dernier mesure une quantité physique x_i, supposée être la réalisation d'une variable aléatoire X_i. Le vecteur de mesures $\mathbf{x} = (x_1, ..., x_k) \in \mathbb{R}^k$, est supposé être la réalisation d'un vecteur gaussien de moyenne nulle et de matrice de covariance

$$\Sigma = \begin{bmatrix} \sigma^2 & \sigma^2 e^{-\lambda d_{1,2}^2} & \cdots & \cdots & \sigma^2 e^{-\lambda d_{1,k}^2} \\ \sigma^2 e^{-\lambda d_{1,2}^2} & \sigma^2 & & & \sigma^2 e^{-\lambda d_{2,k}^2} \\ \vdots & \ddots & \sigma^2 & & \vdots \\ \vdots & & & \sigma^2 & \sigma^2 e^{-\lambda d_{k-1,k}^2} \\ \sigma^2 e^{-\lambda d_{1,k}^2} & \cdots & \cdots & \sigma^2 e^{-\lambda d_{k-1,k}^2} & \sigma^2 \end{bmatrix} \tag{25}$$

où σ^2 est la variance de chaque source, λ est une constante, et $d_{i,j} = \sqrt{(\theta_{i,1} - \theta_{j,1})^2 + (\theta_{i,2} - \theta_{j,2})^2}$ est la distance entre les capteurs i et j. La corrélation entre les mesures prélevées par deux capteurs i, et j, diminue lorsque $d_{i,j}$ augmente.

Chaque capteur i effectue sa propre mesure x_i. Chaque mesure x_i est ensuite quantifiée à l'aide d'un quantificateur uniforme à q niveaux et de pas de quantification Δ. Les mesures quantifiées, qu'on note z_i, sont alors plaçées chacune dans un paquet différent. Chaque capteur effectue ensuite une combinaison linéaire du paquet contenant sa mesure quantifiée $z_i \in \mathbb{F}_q$ avec les autres paquets provenant de ses voisins. Le nouveau paquet codé est retransmis aux noeuds voisins. Nous supposons que chaque capteur dispose d'un intervalle de temps propre pendant lequel il transmet alors que les autres capteurs écoutent. Ce processus se répète jusqu'à ce que le récepteur estime qu'il dispose d'un nombre suffisant de paquets pour garantir la qualité d'estimation de $\mathbf{x}$ désirée.

On suppose que $m \leq k$ paquets indépendants sont disponibles au décodeur. Le codage réseau est représenté par l'équation

$$\mathbf{p} = A\mathbf{z}. \tag{26}$$

où $A \in \mathbb{F}_q^{m \times k}$ est la matrice de codage connue par le décodeur, puisque les coefficients de codage globaux sont inclus dans les entêtes des paquets reçus, et $\mathbf{p}$ est le vecteur des paquets reçus. Le schéma global de codage et de transmission est représenté par la figure 12.

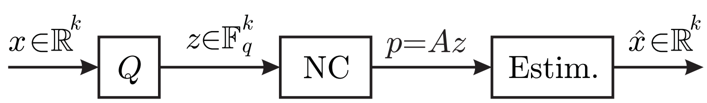

Figure 12: Schéma de codage et de transmission proposé

Le récepteur cherche à évaluer une estimée $\widehat{\mathbf{x}}$ de $\mathbf{x}$ à partir de $\mathbf{p}$ et en exploitant le fait que les mesures $\mathbf{x}_i$ sont corrélées. Σ est supposée connue par le récepteur.

0.4.1 Estimation des mesures de la source

Plutôt que d'estimer directement $\mathbf{x}$, on choisit d'abord d'estimer le vecteur des mesures quantifiées $\mathbf{z}$. Ensuite, $\mathbf{x}$ peut être facilement déduit par une quantification inverse. On choisit le quantificateur de manière à ce qu'il existe deux constantes α et β, tel que $x_i = \alpha z_i + \beta$.

0.4.1.1 Un nombre suffisant de paquets est disponible au récepteur

Le vecteur $\mathbf{z}$ à estimer comprend k inconnues. Si le récepteur reçoit un nombre suffisant de paquets, c'est-à-dire, si la matrice de codage A est de rang plein k, cette dernière, ou une sous-matrice de cette dernière, peut être inversée afin d'obtenir une estimée $\widehat{\mathbf{z}}$ de $\mathbf{z}$

$$\widehat{\mathbf{z}} = A^{-1}\mathbf{p}. \tag{27}$$

Ensuite, $\widehat{\mathbf{x}}$ s'écrit en fonction de $\widehat{\mathbf{z}}$ de la manière suivante

$$\widehat{\mathbf{x}} = \alpha\widehat{\mathbf{z}} + \beta. \tag{28}$$

0.4.1.2 Le nombre de paquets reçus n'est pas suffisant

Si le récepteur ne reçoit pas un nombre suffisant de paquets, c'est-à-dire, si le rang de A est inférieur à k, l'inversion (27) n'est pas possible. On propose dans ce cas un estimateur au sens du maximum a postériori (MAP), qui permet en se basant sur les paquets reçus $\mathbf{p}$ ainsi que la matrice de covariance Σ, de fournir une estimée $\widehat{\mathbf{z}}$ de la source. Soit $m < k$ le nombre de paquets indépendants disponibles au niveau du décodeur. Le rang de la matrice de codage A est donc égal à m. Ceci nous permet de trouver une partition $\mathbf{z}_0 = (z_{i_1}, ..., z_{i_m})^T$ et $\mathbf{z}_1 = (z_{i_{m+1}}, ..., z_{i_k})^T$ des éléments de $\mathbf{z}$, dont la partition correspondante de la matrice de codage s'écrit

$$A = \Pi\left[A_0\, A_1\right]. \tag{29}$$

avec Π, une matrice de permutation de colonnes. Cette partition permet d'écrire

$$\mathbf{p} = \Pi A_0 \mathbf{z}_0 + \Pi A_1 \mathbf{z}_1, \tag{30}$$

avec ΠA_0 de rang plein m. Le rang de la matrice A étant égal à m, une telle partition existe toujours après une permutation convenable des colonnes de A et des entrées *inconnues* de $\mathbf{z}$. L'expression de l'estimateur MAP s'écrit

$$\begin{aligned}
\widehat{\mathbf{z}}_1 &= \arg\max_{\mathbf{z}_1} P(z_{i_{m+1}}, \ldots, z_{i_k} | \mathbf{p}) && (31)\\
&= \arg\max_{\mathbf{z}_1} \sum_{\mathbf{z}_0} P(\mathbf{p} | z_{i_1}, \ldots, z_{i_k}) P(z_{i_1}, \ldots, z_{i_k}). && (32)
\end{aligned}$$

Tous calculs faits, l'expression finale de l'estimateur s'écrit

$$\widehat{\mathbf{z}}_1 = \arg\min_{\mathbf{z}_1} \begin{bmatrix} \alpha\mathbf{z}_0 + \beta \\ \alpha\mathbf{z}_1 + \beta \end{bmatrix}^T \Sigma^{-1} \begin{bmatrix} \alpha\mathbf{z}_0 + \beta \\ \alpha\mathbf{z}_1 + \beta \end{bmatrix}. \tag{33}$$

Les opérations à effectuer ont toutes lieu dans $\mathbb{R}$. Par contre, l'évaluation de

$$\mathbf{z}_0 = (\Pi A_0)^{-1} (\mathbf{p} - \Pi A_1 \mathbf{z}_1) \,. \tag{34}$$

a lieu dans $\mathbb{F}_q$.

En supposant q premier, (34) peut être réécrite en introduisant un vecteur de variables additionnelles $\mathbf{s} \in \mathbb{Z}^m$

$$\mathbf{z}'_0 = (\Pi A_0)^{-1}(\mathbf{p} - \Pi A_1 \mathbf{z}_1) + q\mathbf{s} \tag{35}$$

avec $\mathbf{z}'_0 \in \{0, ..., q-1\}^m$.

Les opérations dans (35) sont ainsi exprimées dans $\mathbb{Z}$. L'espression de l'estimateur devient donc

$$\widehat{\mathbf{z}}_1 = \arg\min_{\mathbf{z}_1} \begin{bmatrix} \alpha\mathbf{z}'_0 + \beta \\ \alpha\mathbf{z}_1 + \beta \end{bmatrix}^T \Sigma^{-1} \begin{bmatrix} \alpha\mathbf{z}'_0 + \beta \\ \alpha\mathbf{z}_1 + \beta \end{bmatrix} \tag{36}$$

La solution de (36) implique la résolution d'un problème d'optimisation quadratique à variables entières (IQP). Ce type de problème de minimisation peut être modélisé avec AMPL et résolu avec CPLEX.

0.4.2 Résultats de simulations

On considère un réseau de $k = 10$ capteurs, dispersés de manière uniforme sur un carré de 1 km de coté. Les capteurs localisés dans un cercle de rayon $d_0 = 0.4$ km autour d' un capteur s_i, constituent l'ensemble des voisins de s_i, et peuvent communiquer directement avec s_i. La communication est supposée parfaite et sans erreurs. L'ensemble des voisins de s_i est noté $\mathcal{N}(s_i)$. Le modèle de corrélation entre les mesures des différents capteurs est supposé être représenté par (25), avec $\sigma^2 = 0.9$ and $\lambda = 0.4$ km^{-2}.

Un protocole de transmission assez simple est considéré. Le temps est divisé en intervalles et les capteurs sont synchronisés. Dans l'intervalle associé au i-ème capteur, seul le capteur s_i peut transmettre, alors que les autres capteurs écoutent. Le i-ème capteur combine le paquet contenant sa propre mesure, avec les paquets déjà reçus de ses voisins. Le nouveau paquet obtenu est ensuite

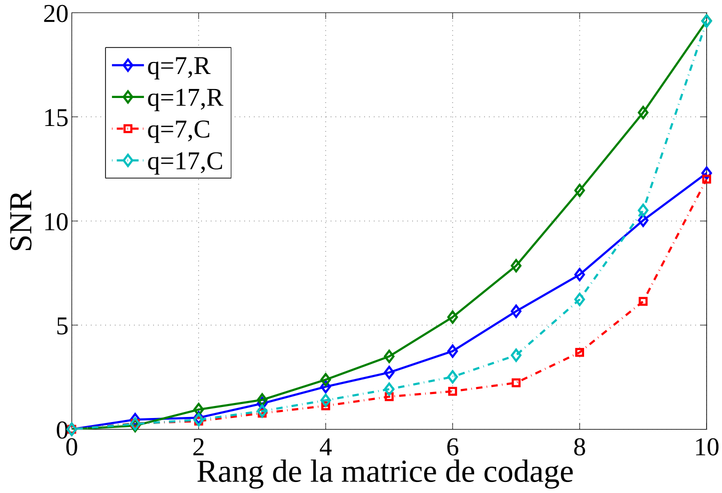

Figure 13: SNR en fonction du nombre de paquets indépendants disponible au récepteur pour $q = 7$ et $q = 17$, avec l'approche proposée (R) et une élimination de Gauss classique (C)

transmis par s_i à tous les capteurs appartenant à $\mathcal{N}(s_i)$. Parmi les paquets disponibles au niveau du décodeur, seuls les paquets linéairement indépendants sont gardés.

Nous comparons la technique proposée avec un scénario de codage réseau classique où la corrélation entre les mesures n'est pas prise en considération. Dans cette approche, k mesures sont émises par les capteurs et nous supposons que le décodeur dispose de $n \geq k$ combinaisons linéaires de ces k mesures. La matrice de codage A est donc de dimension $n \times k$. Le décodage complet n'est possible que si A est de rang plein k, ou s'il existe une sous-matrice A' de A qui soit de rang k.

Les simulations sont moyennées sur 1000 réalisations de la source et de la matrice de codage A.

Les figures 13 et 14 représentent le SNR moyen obtenu en fonction du rang de la matrice de codage A, c'est-à-dire du nombre de paquets indépendants disponibles au décodeur, et ceci pour des tailles de corps de Galois $q \in \{7, 17, 31, 61\}$. La méthode d'estimation proposée notée (R), est comparée à un schéma de codage réseau classique, noté (C), où le décodage se fait par une élimination de Gauss classique.

L'élimination de Gauss est possible puisqu'un certain nombre de paquets non-codés atteint le décodeur. Les mesures non estimés sont alors remplaçées par leur moyenne. On observe que le SNR augmente de manière progressive, avec une meilleure performance pour (R) par rapport à (C). En effet, pour les mêmes valeurs de SNR, un nombre inférieur de mesures est nécessaire.

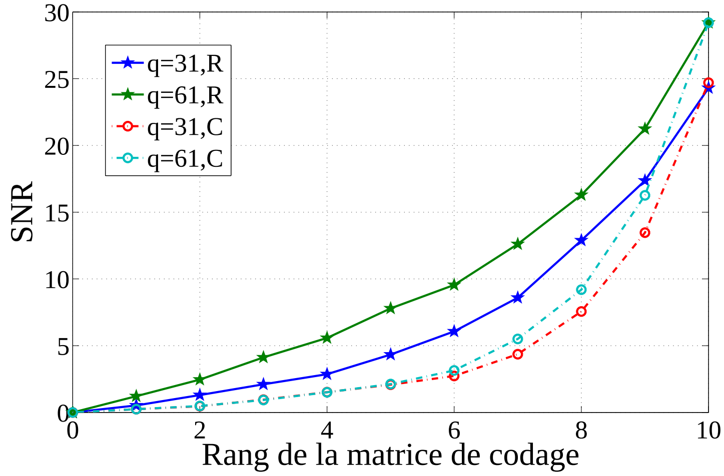

Figure 14: SNR en fonction du nombre de paquets indépendants disponible au récepteur pour $q = 31$ et $q = 61$, avec l'approche proposée (R) et une élimination de Gauss classique (C)

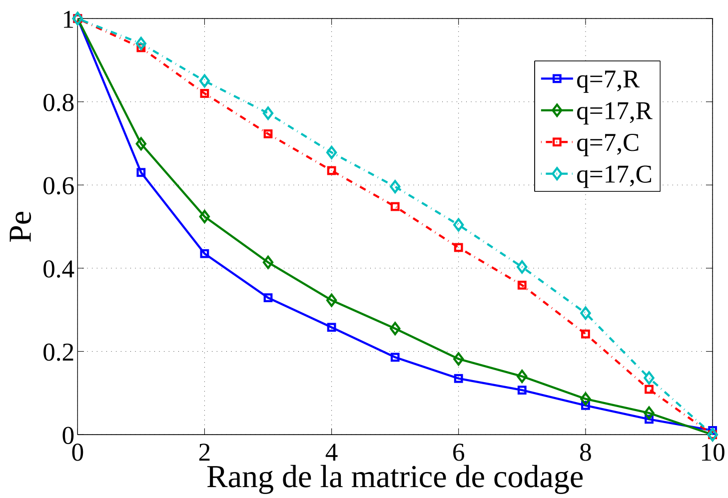

Figure 15: Proportion d'échantillons quantifiés mal reconstruits en fonction du rang de la matrice A au décodeur, pour $q = 7$ et $q = 17$, avec l'approche proposée (R) et une élimination de Gauss classique (C)

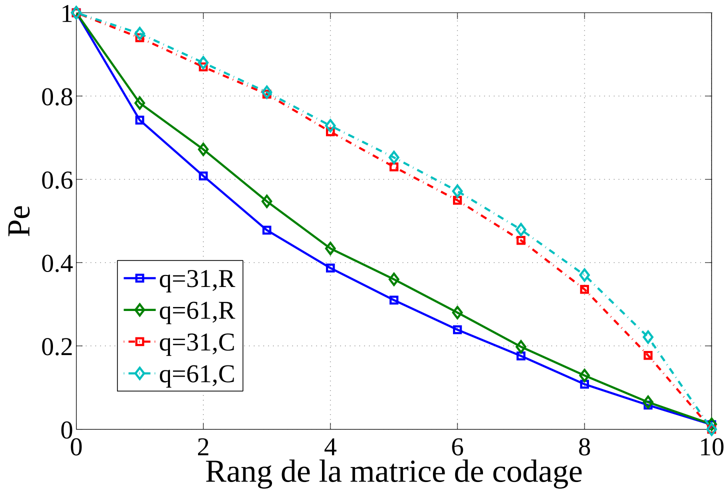

Figure 16: Proportion d'échantillons quantifiés mal reconstruits en fonction du rang de la matrice A au décodeur, pour $q = 31$ et $q = 61$, avec l'approche proposée (R) et une élimination de Gauss classique (C)

La proportion d'échantillons quantifiés z_i reconstruits de manière erronnée en fonction du rang de la matrice A, et pour $q \in \{7, 17, 31, 61\}$, est représentée dans les figures 15 et 16. On observe que le rang de A nécessaire pour obtenir une certaine probabilité d'erreur, est plus petit avec (R) qu'avec (C). En effet, suivant la probabilité d'erreur tolérée, 2 ou 3 mesures de moins sont nécessaires.

0.5 Conclusion & Perspectives

Cette thèse présente plusieurs techniques de codage source-réseau robuste à l'égard de pertes introduites sur les liens ou au niveau des noeuds du réseau.

Ces techniques permettent également d'avoir une amélioration plus progressive de la qualité des messages décodés à mesure que le nombre de paquets reçus par un destinataire augmente. Ceci peut être intéressant pour la transmission de contenu multimédia, par exemple dans des réseaux de type pair à pair, voir [CYC+07, MS09, MF09] pour plus de détails.

Enfin, le codage réseau peut constituer un outil très intéressant, en vue de la collecte efficace de données au sein de réseaux de capteurs. En effet, moins de paquets sont nécessaires pour garantir une qualité de reconstruction donnée.

Plusieurs pistes peuvent être envisagées pour des travaux futurs. Une première perspective à envisager consiste à généraliser l'étude des schémas de codage conjoint proposés dans des corps d'extension $\mathbb{F}_q^r$. Les opérations de codage sont donc effectués dans $\mathbb{F}_q^r$ et non dans $\mathbb{F}_q$. La clé pour résoudre ce problème consiste dans le fait que les élements appartenant à $\mathbb{F}_q^r$ peuvent être vus comme des polynômes dans $\mathbb{F}_q[D]$.

Une deuxième perspective consiste à élargir l'étude de l'influence de la variabilité des canaux de transmission à des situations où le nombre de paquets erronés reçus au niveau de chaque récepteur n'est pas la même. Ceci constitue un modèle plus réaliste de la réception de paquets dans les réseaux ad-hoc. L'influence de la localisation de l'utilisateur sur la rapport signal à bruit dans le contexte d'un réseau cellulaire peut être aussi évaluée.

Concernant l'étude de la corrélation au sein des réseaux de capteurs, une des perspectives consiste à considérer des réseaux plus denses, donc avec un nombre plus important de capteurs, ainsi que des portées différentes entre les différents capteurs. Une seconde perspective consiste à fournir une estimation des mesures transmises par les capteurs via un estimateur MAP classique et de comparer la qualité de reconstruction avec celle obtenue avec l' approche proposée.

Au delà des aspects traités au cours de cette thèse, une situation d'intérêt pour les réseaux ad-hoc et la transmission coopérative est la situation de relayage, dans laquelle quelques utilisateurs peuvent aussi aider dans la transmission d'une autre communication. Ce problème a de multiples facettes, parmi lesquelles la stratégie de coopération à la couche physique, qui est actuellement de grand intérêt. Cette proposition est fortement liée à la stratégie nommée « decode and forward » (DF). En utilisant DF, un relais décode d'abord le signal entrant et le retransmet ensuite au récepteur. Des études classiques sur ce sujet ont négligé l'interaction de cette stratégie avec le codage canal. Cependant, un codage réseau peut être réalisé au niveau du relais, permettant ainsi la transmission de plusieurs mots de codes à partir de celui déjà reçu. Cette proposition permet ainsi de considérer le lien existant entre le codage de réseau, les codes fontaines ainsi que le relayage dans les communications sans fil, étant donné que ces derniers partagent le même but et presque les mêmes outils. Cette approche pourrait être considérée comme une extension assez intéressante du travail déjà effectué.

Deuxième partie

Robust Source-Network Coding

Chapter 1

Introduction

1.1 Introduction

In traditional communication networks, information delivery is accomplished through routing: intermediate nodes simply store and forward data according to a defined metric, and processing is performed only at the destination nodes. This approach has been mostly applied in the context of unicast transmissions, but has also been extended to multicast transmissions.

The concept of Network Coding [ACLY00b] was introduced to allow intermediate nodes to mix packets by performing basic operations such as linear combinations over finite fields of the content of packets reaching their incoming edges [LYC03a]. The linear coefficients are then stored into the headers of the outgoing generated packet, and are consequently available at the receiver side. Consequently, when a source transmits n packets to one or several destinations, the decoders need to collect n linearly independent combinations of the original transmitted packets in order to perform perfect decoding. Original packets are then recovered via a simple Gaussian elimination.

In theory, NC permits to achieve the multicast capacity of a network. Significant througput gains, in wired and wireless networks, are observed in practice [KKH+08, CJKK07]. In wireless contexts, this is mainly due to the ability of NC to exploit wireless broadcast and to take advantage of opportunistic reception.

However, wireless networks are often subject to dynamic changes caused by noise, fading, or interference, and consequently to packet losses. By construction, random linear network coding

(RLNC) is quite robust against losses of coded packets. A receiver has only to wait until enough linearly independent packets have been received to perform decoding. However, bad channels often require retransmissions of lost packets, which can rapidly increase the waiting time. This is particularly critical for real-time multimedia communications, where most applications are delay sensitive, since it leads to many situations where the network may time out [LRM+06], leaving receivers without enough packets to perform network decoding.

A possible solution is the use of *error-correcting network coding* techniques, which basically aim at protecting packets from transmission errors, erroneous packets and/or losses. The basic idea of these techniques consists in introducing a certain level of *redundancy*, and are similar in principle to classical error correcting codes. Coherent network codes [CY02, Zha08] are one family of error correcting network codes that focus on combining NC with some introduced redundancy. However, these codes require an *a priori* knowledge of the network and the way in which network coding is carried out.

Joint source-network coding approaches represent an alternative way of coping with transmissions in lossy networks. These techniques allow the recovery of all or part of the transmitted data, in cases where the number of linearly independent packets collected at the decoder is not enough, by exploiting the source redundancy whether it is naturally present or artificially introduced. These techniques also enable the distributed compression of correlated messages generated by geographically distributed sources.

Multiple description coding [Goy01] is one way of introducing redundancy which has always been associated with robust network communications since multiple description codes are designed to exploit the path and server diversities of a network. The basic idea of multiple description coding is the following: a source signal is encoded into several equally important bitstreams. Each bitstream, called a description, is independently decodable, and can mutually enhance the quality of each other. In fact, each description is supposed to be good enough to meet some decoder requirements if it gets through by itself. The more descriptions are available at the receiver, the higher the quality of the reconstructed data.

This feature enables multiple description coding to be widely applied to various channels. Applications of multiple description coding may include robust transmission of audio, image and video over unreliable networks, distributed storage, etc. However, multiple description coding should only be used in applications involving packet loss, because only in this case the overhead in the communication volume can be justified.

In this thesis, we aim to further investigate joint source-network coding and decoding approaches by deploying them in wireless transmission scenarios, such as multimedia content delivery for multicast, data broadcasting towards mobile terminals, and efficient data collection in wireless sensor networks. We focus on the problem of data reconstruction when perfect decoding is impossible due to the reception of an insufficient number of linearly independent packets, and we aim to investigate the effect of exploiting source redundancy on the quality of the achieved decoding.

Naturally existing redundancy originates, e.g., from the spatial correlation between measures collected in a wireless sensor network, and which can be exploited to provide an approximate estimation of the transmitted samples. Regarding artificial redundancy, we choose multiple description coding as a way of introducing structured redundancy, and we propose two different schemes combining network coding and multiple description coding, where descriptions are generated either via frame expansion [GKV98], or via a correlating transform [Goy98]. The main goal is to take advantage of the capability of multiple description codes to provide gradual improvement of the decoding quality with the number of received descriptions. Comparision with classical scenarios is performed in both cases.

1.2 Contributions

This thesis has two goals:

1. take advantage of the benefits of network coding in terms of use of the network resources

2. exploit the source correlation, whether natural or artifical, in order to ensure an approximate reconstruction when not enough innovative network-coded packets are collected at the destination side

We divide our contributions into two parts:

Artificial redundancy using MDC schemes. First, we propose two joint NC-MDC schemes, in which multiple description coding allows to introduce structured redundancy among uncorrelated source packets. We consider two different approaches for generating descriptions, one via frame expansion (NC-MDC-F), and the other via a correlating transform (NC-MDC-T). In the first approach, quantization of the transmitted source samples is performed *after* the source is being expanded, while in the second approach, quantization is done *before* applying a correlating transform belonging to some Galois field of finite size q on the quantized samples. In both cases, linear combinations of packets is performed *only* at the intermediate nodes of the network.

We implement a decoding scheme requiring the solution of a mixed integer quadratic problem for NC-MDC-F, while a simple Gaussian elimination is sufficient for NC-MDC-T. The performance of both approaches is compared in terms of signal-to-noise ratio, and reconstruction error probability.

Second, we apply both schemes to a multicat scenario where a source transmits data to several receivers experiencing different channel conditions, and we study the effect of the channel variability on the average performance of both approaches. Finally, we compare the performance of the two proposed methods to a classical network coding scenario.

Spatial redundancy in Wireless Sensor Networks. In this part, we propose a *maximum a posteriori* estimator that exploits existing correlation in order to provide an estimate of the original measurements collected in a wireless sensor network.

We consider a scenario where a certain number of sensors is spread over some 2-dimensional area, and where each sensor can communicate with a set of neighboring nodes. The transmission protocol is defined such that each sensor is allowed to transmit only during its own time slot, while the other sensors are listening. The sensor combines its own quantized measurements with packets already received, and transmits the resulting combination to its neighbors. This process continues until the coded measurements reach the sink.

The original data is reconstructed based on the solution of an integer quadratic problem, and we study the performance of the proposed approach in terms of signal to noise ratio and reconstruction error probability. Finally, we compare the proposed technique to a traditional network decoding scenario, where redundancy is not taken into account in the reconstruction process.

1.3 Outline

This thesis is organized as follows:

Chapter 2 recalls the basics of network coding, and presents a brief overview of the main existing work linked to network error correction.

In Chapter 3, we propose two joint MDC and NC scenarios and we implement a decoding scheme that exploits the artificially introduced redundancy in order to provide an approximate reconstruction of the source. In addition, we study the effect of the channel conditions on the performance of both schemes in the context of a multicast scenario from one source towards several mobile terminals, and we compare it with the performance of a traditional NC scenario.

In Chapter 4, we provide an approximate estimation of spatially correlated data collected in a wireless sensor network when potentially not enough network-coded packets are received at the decoder side. We develop a MAP estimator that exploits the spatial correlation, and we evaluate the decoding performance. Finally, we compare the performance of the proposed scenario to the one obtained using a classical NC decoding scheme.

Chapter 5 concludes this thesis and present some possible future research directions.

Chapter 2

Reliable Network Coding

2.1 Introduction

Research on NC was initiated by the seminal paper [ACLY00b] and has since then attracted significant interest from the research community. Many initial contributions on NC focused on establishing multicast connections. It was shown in [ACLY00b] that the capacity of multicast networks (*i.e.*, the maximum number of packets that can be sent from a source to a set of destinations per time unit) can be achieved by coding over the network, *i.e.*, by allowing the mixing of data at the intermediate nodes of the network. A few years later, in [LYC03a] it was shown that, for multicast networks, linear coding at the intermediate nodes suffices to achieve the capacity limit, which is the max–flow from the source to each receiving node. In [KM03], the authors extended the results in [LYC03a] to arbitrary networks and introduced a very powerful algebraic framework for NC. The approach establishes a useful connection between a NC problem and the solution of certain systems of polynomial equations. In [CWJ03a], the authors conceived a practical NC scheme with centralized knowledge of neither the network topology nor the encoding/decoding functions. The fundamental idea of [CWJ03a] consists in including within each transmitted packet the global encoding vector along the edge. This way, these encoding vectors, which are needed to decode the network-coded packets at any receiver, can be found in the packets themselves. With the cost of a reasonable overhead, the approach can offer a totally decentralized solution to NC over dynamic networks. In [HMK+06], the authors capitalized on the analytical formulation of [KM03] and the practical scheme in [CWJ03a] by proposing a distributed and fully randomized method to design the network codes. Moreover, it was shown in [HMK$^+$06] that the network capacity can be achieved with probability exponentially approaching one with the code length. Finally, on a more practical side, Katti *et al.* conceived several solutions, *i.e.*, COPE, ANC, MIXIT, MORE, to efficiently exploit the NC paradigm over wireless networks [KGK07]–[KKBM08].

A short overview of network coding basics is recalled in Section 2.2. Previous work in the litterature addressing the problem of reliability and robustness of network coding is presented in Section 2.3. Readers well experienced in the domain, may skip these sections and move immediately to Subsection 2.3.5.

2.2 Network Coding Overview

2.2.1 Introductory Example : The Butterfly Network

Figure 2.1 depicts what is commonly known as the *butterfly network* in the network coding litterature. A communication network is referred to by a directed acyclic graph where nodes represent terminals and edges represent channels. The time is slotted, and only one bit per time slot is supposed to be able to pass through the channels. The source node S produces two bits b_1 and b_2 and multicasts them to both receiver nodes Y and Z.

Traditional routing In Figure 2.1 (a), every channel can carry either the bit b_1 or the bit b_2. The intermediate nodes simply replicate and send out the bit(s) received from upstream. In this scenario, a multicast rate of 1 bit per unit time is achieved.

The same network but with one less channel between intermediate nodes W and X, is represented in Figures 2.1 (b) and 2.1 (c), and show a way of multicasting 3 bits b_1, b_2 and b_3 from S to Y and Z in 2 time units. In this scenario, a multicast rate of 1.5 bits per unit time is achieved, which is actually the maximum rate possible when the intermediate nodes perform only bit replication.

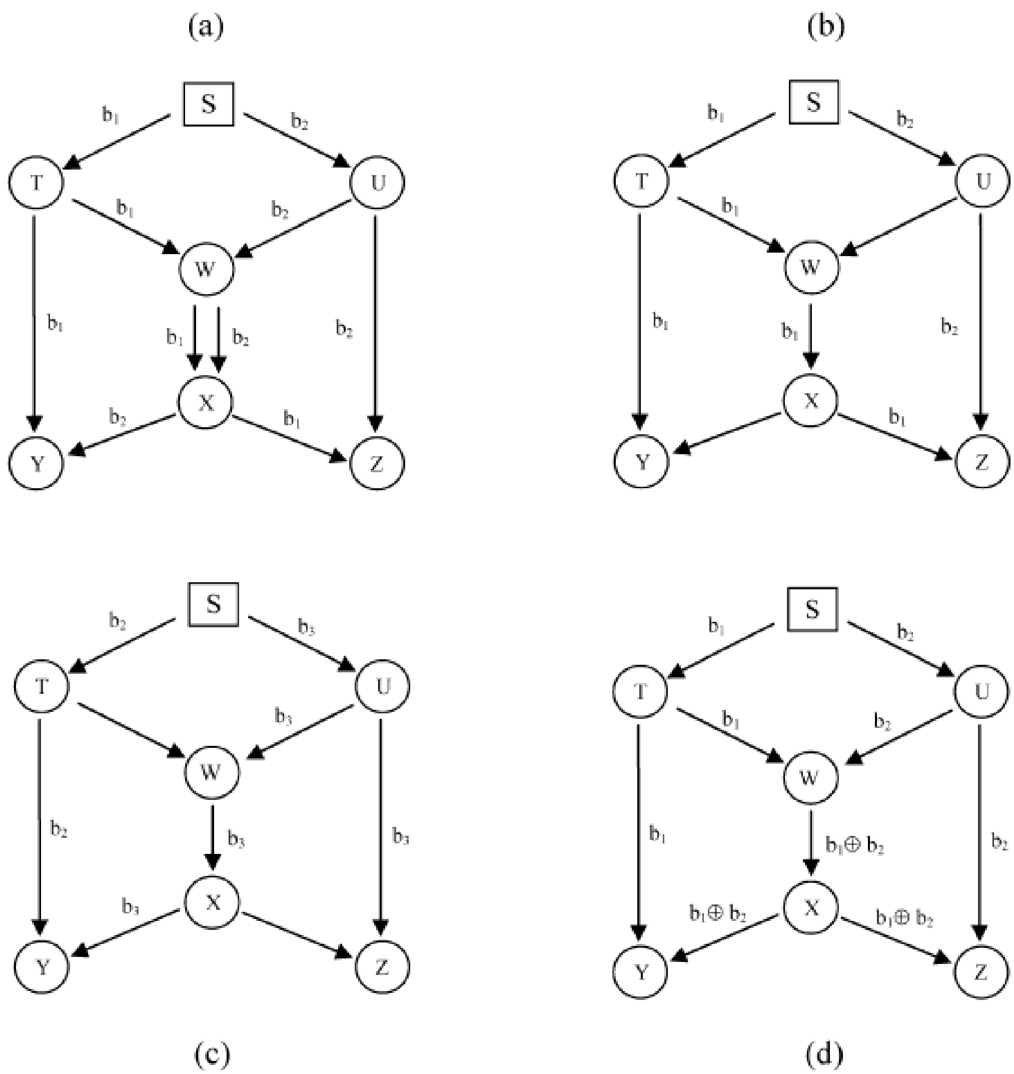

Figure 2.1: Multicasting over a communication network

Network coding on the butterfly network Figure 2.1 (d) depicts a different way of multicasting the two bits $b_1$and b_2 from the source node S to the receivers nodes Y and Z . In this scenario, the intermediate node W receives the bits b_1 and b_2 and derives the exclusive-OR bit $b_1 \oplus b_2$. The channel between W and X transmits $b_1 \oplus b_2$, which is then replicated at X in order to pass on to Y and Z. The node Y receives b_1 and $b_1 \oplus b_2$, from which the bit b_2 can be decoded. Similarly, the node Z decodes the bit b_2 from the received bits b_2 and $b_1 \oplus b_2$. In this way, all the 9 channels in the network are used exactly once.

Deriving the exclusive-OR bit is the simplest form of network coding. If the same goal is to be achieved by a simple routing, at least one channel in the network must be used twice so that the total number of channel usage would be at least 10. Thus, coding offers the potential advantage of minimizing both latency and energy consumption, since it reduces the number of transmissions, and at the same time maximizing the bit rate.

2.2.2 Main Network Coding Theorems

Network multicast refers to simultaneously transmitting the same information to multiple receivers in the network. Research is mainly concerned with necessary and sufficient conditions that the network has to satisfy to be able to support the multicast at a certain rate. For the case of unicast, when only one receiver at the time uses the network, such conditions are known for the past fifty years, and, clearly, it is required that they hold for each receiver participating in the multicast. The important fact that the main network coding theorem brings is that the conditions necessary and sufficient for unicast at a certain rate to each receiver are also necessary and sufficient for multicast at the same rate, provided that the intermediate network nodes are allowed to combine and process different information streams.

2.2.2.1 The Min-Cut Max-Flow Theorem

Consider a directed acyclical graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ with unit capacity edges, a source node S, and a receiver node R, with S and R belonging to $\mathcal{V}$. $\mathcal{V}$ is the set of nodes, while $\mathcal{E}$ is the set of edges.

Definition 1 *A cut between S and R is a set of graph edges whose removal disconnects S from*

R. A min-cut is a cut with the smallest (minimal) value. The value of the cut is the sum of the capacities of the edges in the cut.

For unit capacity edges, the value of a cut equals the number of edges in the cut and is sometimes referred to as the size of the cut. There exists only one min-cut value, but possibly several min-cuts as seen in the example below. A min-cut can actually be seen as the bottleneck for information transmission between the source S and the receiver R.

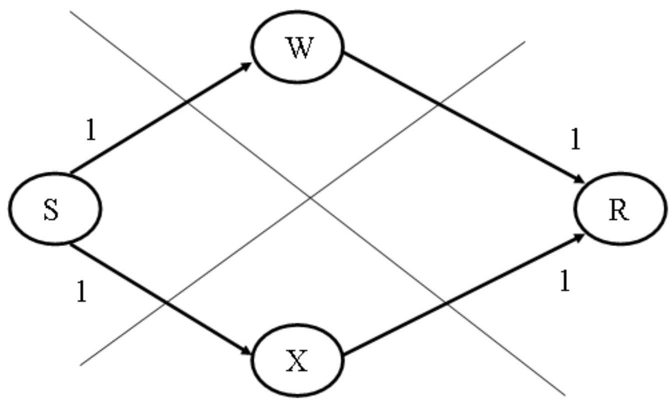

Figure 2.2: The min-cut between S and R equals 2. There exists two edge disjoint paths between S and R that brings the transmitted data to the receiver R.

Theorem 1 *If the min-cut between S and R equals h, then the information can be send from S to R at a maximum rate of h. Equivalently, there exist exactly h edge-disjoint paths between S and R.*

2.2.2.2 The Main Network Coding Theorem

Consider now a multicast scenario over a network $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ where h unit rate sources $S_1, ..., S_h$ located on the same network node S (source) simultaneously transmit information to N receivers $R_1, ..., R_N$. Assume that $\mathcal{G}$ is an acyclic directed graph with unit capacity edges, and that the value of the min-cut between the source node and each of the receivers is h. Assume also zero delay, meaning that during each time slot all nodes simultaneously receive all their inputs and send their outputs.

Theorem 2 *Consider a directed acyclic graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ with unit capacity edges, h unit rate* *sources located on the same vertex of the graph and N receivers. Assume that the value of the min-cut to each receiver is h. Then there exists a multicast transmission scheme over a large enough finite field $\mathbb{F}_q$, in which intermediate network nodes linearly combine their incoming information symbols over $\mathbb{F}_q$, that delivers the information from the sources simultaneously to each receiver at a rate equal to h.*

Based on the *min-cut max-flow theorem*, there exist exactly h edge-disjoint paths between the sources and each of the receivers. Thus, if any of the receivers R_j, is using the network by itself, the information from the h sources can be routed to R_j through a set of h edge disjoint paths. When multiple receivers are using the network simultaneously, their sets of paths may overlap. The conventional wisdom says that the receivers will then have to share the network resources, which leads to reduced rates. However, the main NC theorem shows that by allowing intermediate network nodes to not only forward but also combine their incoming information flows, each one of the receivers will be getting the information at the same rate as if it had sole access to network ressources.

2.2.3 Random Linear Network Codes

It has been shown in [LYC03b], that when dealing with multicast scenarios, linear network codes are sufficient to reach the max-flow bound. On the other hand, using linear codes makes both coding and decoding easier and faster to implement in practice [YLCZ05]. A linear combination is a sum of packets weighted by coefficients belonging to a finite field $\mathbb{F}_q$. The assignment of network coding coefficients can be performed in several ways. For instance, a greedy algorithm for code construction has been proposed in [LYC03a].

A simple yet powerful encoding scheme is referred to as *random linear network coding.* The network nodes transmit linear combinations of the received packets, with the encoding coefficients randomly chosen over the finite field $\mathbb{F}_q$. When the field size q is sufficiently large, the probability that the receiver(s) obtain linearly independent combinations (and thus, innovative information) approaches one. It should however be noted that, although random network coding has an excellent throughput performance, if a receiver obtains an insufficient number of packets, the recovery of all of the original packets becomes impossible.

2.2.4 Benefits of Network Coding

Network coding promises to offer benefits along very diverse dimensions of communication networks, such as throughput, wireless resources, security, complexity, and resilience to link failures.

2.2.4.1 Throughput

The main idea behind network coding was to design networks being able to achieve the maxflow bound on the information transmission rate in a multicast scenario. The major finding in [RAY00] was that it is in general not optimal to consider information to be multicast in a network as a "fluid" which can simply be routed or replicated at the intermediate nodes. On the contrary, network coding had to be employed to achieve optimality [RAY00]. In network coding based multicast, not only the shortest path is used but also other paths are used to transmit the data. So the traffic in the network is distributed to multiple links and network coding can have an effect of load balancing. Hence, by using network coding, traffic load can be distributed to the entire network [TN03].

2.2.4.2 Ressource Savings

Another advantage of network coding is the saving in bandwidth when network coding is allowed. Compression of the information will result from using network coding because bits (or packets) from the input links are incorporated (encoded) into one bit (or packet) and sent to the output links. Instead, in traditional multicast schemes they are just replicated and sent to the output links. So, network coding is able to save in bandwidth. Another benefit is offered in terms of battery life, energy efficiency, delay and interference.

2.2.4.3 Network Management & Robustness

Network coding is not only applicable to networks in order to achieve capacity, but can also be used to recover from link failures in networks. The failures considered here are long term failures due to a link cut, or the permanent removal of an edge, or other disconnection. Currently, such failures are dealt with through the use of rerouting, such as link or path protection. Previous work [RK02] shows that network codes that operate under certain failure scenarios can be designed. Also, no network management overhead is required for multicast connections, but sometimes a change of codes needs to be initiated by network management in more general cases. So, proper network coding with minimum changing of codes may lead to a much optimal way for network management to respond to a failure in the network.

2.2.4.4 Security:

Sending linear combinations of packets instead of uncoded data offers a natural way to take advantage of multipath diversity for security against wiretapping attacks. Thus systems that only require protection against such simple attacks, can get it "for free" without additional security mechanisms.

2.3 Network Coding in lossy networks

Besides the many potential advantages and applications of NC over classical routing, (see, *e.g.*, [FS07b], [FS07a]), the NC principle is not without drawbacks. A fundamental problem that NC needs to face over lossy networks is the so–called *error control problem*: corrupted packets injected by some intermediate nodes might propagate through the network until the destination, and might make it impossible to decode the original information. In contrast to routing, this problem is crucial in NC due to the algebraic operations performed by the internal nodes of the network. The mixing of packets within the network makes every packet flowing through it statistically dependent on other packets: even a single erroneous packet might affect the correct detection of all other packets. On the contrary, the same error in networks using just routing would affect only a single source–destination path. Broadly speaking, possible errors in NC might arise for three main reasons [Zha08]: i) *erasures*, which lead to an insufficient number of received packets at the destination to solve the NC problem and retrieve the transmitted messages, ii) *errors*, which are due to using, for complexity and practical reasons, not powerful enough link-to-link error-correcting codes or are caused by the need to avoid a retransmission of all corrupted packets, and iii) the presence of intentional *jammers*, who might introduce erroneous packets at the application layer, whose effects might not be recovered at the physical layer by the destination node.

Error-correcting network coding techniques aim at protecting packets from transmission errors, erreneous packets, and/or losses. Error correcting network coding techniques introduce a certain level of *redundancy* and are similar in principle to classical error correcting codes. We can distinguish two families of codes. The codes introduced in [CY02, Zha08] both focus on network coding and the introduction of redundancy. These codes require an *a priori* knowledge of the architecture of the network and of the way in which network coding is carried out, see Section 2.3.1 for further details. These results are extended to the framework of random network coding in [HMK+06, BYZ09], see Section 2.3.2. The techniques introduced in [JLK+07, KK08a, SKK08, AA09] exploit the fact that, in the absence of errors, random network coding preserves the vector space spanned by the transmitted packets. The proposed robust network codes have properties that are relatively independent from the way the network coding is carried out, see Section 2.3.3.

Joint decoding techniques exploit the existing redundancy in the transmitted packets flow [DK09]. In the case of *joint channel-network decoding* [HH06, Tho08], temporal or spatial diversity or the presence of channel codes [GHW+09, KDH07] is used to combat the noise introduced by the communication channels, in particular wireless channels, see Section 2.3.4. *Joint source-network decoding* allows the recovery of all or part of the initial packets, even in the presence of an insufficient number of received packets, by exploiting the correlation between transmitted data packets, see Section 2.3.5. Therefore, these techniques provide a certain robustness against packet loss.

2.3.1 Coherent network error correction codes

The notations and the content of this section are largely inspired from [Zha11, YYN11, Zha08]. For this type of network error correcting codes, the topology of the network as well as the considered network code are assumed to be known by each destination node [YC06, CY06].

A communication network is described by a directed acyclical graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. A link $e = (i, j) \in \mathcal{E}$ represents a channel linking the nodes $i \in \mathcal{V}$ and $j \in \mathcal{V}$. The set of links emerging from a node $i \in \mathcal{V}$ is written as $O(i)$, the set of links converging at i is written as $I(i)$. A multicast network is a triple $(\mathcal{G}, s, \mathcal{T})$, where $\mathcal{G}$ is a network, $s \in \mathcal{V}$ is the source and $\mathcal{T}$, the set of destination nodes. We assume that $I(s) = \emptyset$, $O(t) = \emptyset$ for every $t \in T$. Let $n_s = |O(s)|$. Subsequently, $\mathbb{F}$ represents the Galois field with q elements.

The source node s encodes the message to transmit as a row vector $\mathbf{x} = [x_1, \ldots, x_{n_x}] \in \mathbb{F}^{n_x}$ called a *codeword*. The set of codewords is written as $\mathcal{C}$. Each component of $\mathbf{x}$ is therefore sent on one of the links of $O(s)$. An error vector $\mathbf{z} \in \mathbb{F}^{|\mathcal{E}|}$ allows us to describe the errors introduced by the links in the network. If we denote $\bar{f}_e$ and f_e as the input and output of the link e and if an error z_e is introduced on the link $e \in \mathcal{E}$, then $f_e = \bar{f}_e + z_e$. For every subset of links $\rho \in \mathcal{E}$, we introduce the two vectors $\mathbf{f}_\rho = [f_e, e \in \rho]$ and $\bar{\mathbf{f}}_\rho = \left[\bar{f}_e, e \in \rho\right]$. A code for the network $\mathcal{G}$ is therefore defined by a set of codewords $\mathcal{C} \subset \mathbb{F}^{n_s}$ and a family of local coding functions $\left\{\bar{\beta}_e, e \in \mathcal{E}\backslash O(s)\right\}$, with $\bar{\beta}_e : \mathbb{F}^{|I(\text{source}(e))|} \to \mathbb{F}$ such that

$$\bar{f}_e = \bar{\beta}_e \left(F_{I(\text{source}(e))}\right) \tag{2.1}$$

and where $\text{source}(e)$ indicates the node from which e emerges. Assume that the destination node t receives the vector $\mathbf{u}_t = (u_e,\ e \in I(t))$. An iterative application of (2.1) allows to express $\mathbf{u}_t$ as a function of $\mathbf{x}$ and of the error vector $\mathbf{z}$

$$\mathbf{u}_t = F_{st}(\mathbf{x}, \mathbf{z}). \tag{2.2}$$

where $F_{st}(\mathbf{x}, \mathbf{z})$ represents the set of network coding operations taking place between the source s and destination t. In the case of coherent network codes, the structure of $F_{st}(\mathbf{x}, \mathbf{z})$ is assumed to be known at the decoder and is used to perform the estimation of $\mathbf{x}$ from $\mathbf{u}_t$. In order to characterize the error correction capacity of a network code, it is necessary to introduce the notion of distance between codewords [Zha08]. For that purpose, consider the set of vectors that can be received by the node t when the source transmits a codeword $\mathbf{x}$ and the network introduces an error vector $\mathbf{z}$

with a Hamming weight $w_H(\mathbf{z})$ less than c

$$\Phi_t(\mathbf{x}, c) = \{F(\mathbf{x}, \mathbf{z}) \text{ st } w_H(\mathbf{z}) \leqslant c\}. \tag{2.3}$$

It is possible to deduce from $\Phi_t(\mathbf{x}, c)$ a pseudo-distance between two codewords $\mathbf{x}$ and $\mathbf{y}$ emitted by the source

$$D_t(\mathbf{x}, \mathbf{y}) = \min\{c_1 + c_2 \text{ st } |c_1 - c_2| \leqslant 1 \text{ and } \Phi_t(\mathbf{x}, c_1) \cap \Phi(\mathbf{x}, c_2) \neq \emptyset\} \tag{2.4}$$

and a minimal distance for the network code at the node t

$$d_{\min,t} = \min\{D_t(\mathbf{x}, \mathbf{y}), \mathbf{x} \neq \mathbf{y}\}. \tag{2.5}$$

The decoder seeking the minimum weight error vector (maximum likelihood decoder if all code words have same probability) can therefore be constructed in the following way. First, we search for the set $\mathcal{P}$ of pairs $(\mathbf{x}, \mathbf{z})$ satisfying (2.2). In the sub-set $\mathcal{P}_w \subset \mathcal{P}$ of the pairs $(\mathbf{x}, \mathbf{z})$ whose H amming weight $\mathbf{z}$ is minimal, if all the pairs have same $\mathbf{x}$, then the error is said to be *correctable* and $\mathbf{x}$ is the estimation of the transmitted message. If this is not the case, the error is not *correctable*. It has been shown in [YYZ08] that the correction capacity of a network code (with a decoder that searches for the minimum weight error vector) is $\lfloor (d_{\min} - 1)/2 \rfloor$, where $\lfloor \cdot \rfloor$ indicates the rounding towards $-\infty$. In the case of linear network codes, the functions $\bar{\beta}_e$ are linear and for every $e \in \mathcal{E} \backslash O(s)$, we have

$$\bar{f}_e = \sum_{e' \in \mathcal{E}} \beta_{e',e} F_{e'} \tag{2.6}$$

where $\beta_{e',e}$ is the local coding coefficient of the node e' towards the node e. Using (2.6), [KM03] has shown that (2.2) can be written as

$$\mathbf{u}_t = \mathbf{x}\mathbf{F}_{s,t} + \mathbf{z}\mathbf{F}_t, \tag{2.7}$$

where $\mathbf{F}_{s,t}$ and $\mathbf{F}_t$ can be deduced from (2.2) and are perfectly known. In the case of linear network codes, (2.4) becomes

$$D_t(\mathbf{x},\mathbf{y}) = \min\{c \text{ st } (\mathbf{x}-\mathbf{y})\mathbf{F}_{s,t} \in \Phi_t(c)\}, \tag{2.8}$$

with

$$\Phi_t(c) = \left\{\mathbf{z}\mathbf{F}_t,\, \mathbf{z} \in \mathbb{F}^{|\mathcal{E}|},\ w_H(\mathbf{z}) \leqslant c\right\}, \tag{2.9}$$

the set of messages received when the zero code word is sent. The main bounds in terms of error correction codes have been extended to network codes in [YC06, CY06, YYN11] such as the Hamming, the Singleton, and the Gilbert-Varshamov bound, as well as in [Byr08] for the Plotkin and Elias bounds. For the Hamming and Singleton bounds, let

$$d_{\min} = \min_{t\in\mathcal{T}} d_{\min,t} \tag{2.10}$$

and

$$n = \min_{t\in\mathcal{T}} \operatorname{maxflow}(s,t). \tag{2.11}$$

In the case of a network code for which $\operatorname{rank}(\mathbf{F}_{s,t}) = r_t$ and $d_{\min,t} > 0$, the Hamming bound may be written as

$$|\mathcal{C}| \leqslant \min_{t\in\mathcal{T}} \frac{q^{r_t}}{\sum_{i=0}^{\tau_t} \begin{pmatrix} r_t \\ i \end{pmatrix} (q-1)^i}, \tag{2.12}$$

with $\tau_t = \lfloor (d_{\min,t}-1)/2 \rfloor$. The Singleton bound becomes

$$|\mathcal{C}| \leqslant q^{r_t - d_{\min,t}+1} \tag{2.13}$$

for every node t, see [YYN11]. The Singleton bound (2.13) allows to extend the notion of *maximum distance separable* (MDS) codes to network codes [YLCZ05]. A network code where the Singleton bound is reached is said to be MDS. It is optimal in the sense that it exploits all the redundancy in the network error correcting code. A code construction method enabling the Singleton bound (2.13) to be reached has been proposed in [YYN11]. The technique consists of first constructing the local coding coefficients which ensure that the rank of matrices $\mathbf{F}_{s,t}$ is always sufficient. This can be done using the Jaggi-Sanders algorithm [JSC+05]. The codewords are then generated so that there is sufficient distance between them regardless of which destination node t is considered. The associated decoding algorithms are presented in [YC06, CY06]. See also [Mat07, Zha08] as well as [BYZ09] for further details on this type of codes.

2.3.2 Codes for non-coherent networks, random codes

The random network codes proposed in [HKM+03, HMK+06] can be seen as a practical solution to network coding which can easily adapt to variations in the network topology since they are decentralized. In the case of random coding, the matrices $\mathbf{F}_{s,t}$ and $\mathbf{F}_t$ introduced in (2.7) are random. While it is possible to deduce $\mathbf{F}_{s,t}$ from the received packet headers (assuming that they have not been corrupted), $\mathbf{F}_t$, on the other hand, cannot be easily deduced. In the absence of transmission errors, the probability that a destination node $t \in \mathcal{T}$ is not capable of decoding the message received can be expressed as a function of the rank of $\mathbf{F}_{s,t}$

$$P_e^{(t)} = \Pr\left(\text{rank}(\mathbf{F}_{st}) < n_x\right). \tag{2.14}$$

The probability that at least one of the destination nodes is incapable of decoding the received message is deduced from (2.14)

$$P_e = \Pr\left(\exists t \in \mathcal{T} \text{ st } \text{rank}(F_{st}) < n_x\right), \tag{2.15}$$

see [HMK+06]. If c_t denotes the min-cut capacity between s and t, then $\delta_t = c_t - n_x$ corresponds to the redundancy at t. The error probability at the receiver t is therefore bounded as follows

$$P_e^{(t)} \leqslant 1 - \sum_{i=n_x}^{n_x+\delta_t} \binom{n_x+\delta_t}{i} \left(1-p-\frac{1-p}{q}\right)^{Li} \left(1-\left(1-p-\frac{1-p}{q}\right)^{L}\right)^{n_x+\delta_t-i} \tag{2.16}$$

where L indicates the length of the longest path between s and t and p is the link erasure probability. When the links are perfectly reliable ($p = 0$), (2.16) becomes

$$P_e^{(t)} \leqslant 1 - \sum_{i=0}^{\delta_t} \binom{C_t}{i} \left(1 - \frac{1}{q}\right)^{L(C_t - i)} \left(1 - \left(1 - \frac{1}{q}\right)^L\right)^i . \tag{2.17}$$

In the presence of errors, the results of [Zha08] briefly presented in Section 2.3.1 can be extended. However, for a given code, the minimum distance $d_{\min,t}$ introduced in (2.5) becomes a random variable $D_{\min,t}$. Once the code $\mathcal{C}$ is fixed, the distance $d_{\min,t}$ will depend on the (random) elements of $\mathbf{F}_{st}$. A partial characterization of $D_{\min,t}$ has been proposed in [BYZ09]

$$\Pr\left(D_{\min,t} < \delta_t + 1 - d\right) \leqslant \frac{\binom{|\mathcal{E}|}{\delta_t - d}\binom{d+|\mathcal{J}|+1}{|\mathcal{J}|}}{(q-1)^{d+1}}, \tag{2.18}$$

where $\mathcal{J} \subset \mathcal{E}$ is the set of internal nodes in the network. This result allows to deduce the probability of existence of an MDS code according to the size q of the Galois field in which the coding operations take place, see [BYZ09] for further details.

2.3.3 Codes for non-coherent networks, subspace codes

The network coding error correcting techniques proposed in [KK08a, SKK08] are very different from the ones previously introduced. A non-coherent network model is considered, where neither the coder nor the decoder need to know the topology of the network nor the way in which combinations of packets are carried out. This work is motivated by the fact that, in the absence of errors, network coding preserves the vector space spanned by the transmitted packets. The coding operation is carried out via the transmission of a vector space inside a set of possible vector spaces (which represents the set of codewords). A destination node must identify the vector subspace belonging to the code found to be the closest (in a sense to be defined) to the vector space spanned by the received packets. The received vector space can be different from the one that has been transmitted, depending on the packet losses, transmission errors or erroneous packets deliberately injected by malicious nodes.

2.3.3.1 Principle of subspace codes

In this approach, the transmission of information from the source s to a destination node t is performed by the injection into the network of a vector subspace $V \subset \mathbb{F}^n$ and by the reception of a subspace $U \subset \mathbb{F}^n$. Let $\mathbf{x} = \{\mathbf{x}_1, \ldots, \mathbf{x}_{n_s}\}$, with $\mathbf{x}_i \in \mathbb{F}^n$, be the set of vectors (data packets) injected by the source s and forming a base of V. In the absence of errors, $t \in T$ receives a set of packets $\mathbf{u} = \{\mathbf{u}_1, \ldots, \mathbf{u}_{n_t}\}$ formed by linear combinations of $\{\mathbf{x}_1, \ldots, \mathbf{x}_{n_s}\}$, such that $\mathbf{u}_j = \sum_{i=1}^{n_s} h_{ji}\mathbf{x}_i$, where the h_{ji} are random coefficients of $\mathbb{F}$. The effect of potential transmission errors is modeled by the introduction of *packets* of errors $\mathbf{z} = \{\mathbf{z}_1, \ldots, \mathbf{z}_{n_z}\}$ throughout the network. Since these packets can be injected into any link or node in the network, at receiver side, one gets

$$\mathbf{u}_j = \sum_{i=1}^{n_s} h_{ji}\mathbf{x}_i + \sum_{k=1}^{n_z} g_{jk}\mathbf{z}_k, \tag{2.19}$$

where the $g_{jk} \in \mathbb{F}$ are again random. In matrix form, one obtains

$$\mathbf{u} = H\mathbf{x} + G\mathbf{z}. \tag{2.20}$$

The model (2.19) is close to (2.7), but in (2.7), symbols belonging to $\mathbb{F}$ are transmitted while in (2.19) packets are sent through the network. In (2.7), $\mathbf{F}_{s,t}$ and $\mathbf{F}_t$ are perfectly known when the network structure and the network coding operations are known, which is not the case with the coefficients h_{ji} and g_{jk} (this is why we consider here non-coherent network codes). With this type of model, the aim of the receiver cannot be to precisely identify $\mathbf{x}$, but rather to identify the vector subspace V spanned by the vectors of $\mathbf{x}$, based on the knowledge of the vector subspace U created by the elements of $\mathbf{u}$. To introduce the notion of subspace codes, we consider a vector space W of dimension n on $\mathbb{F}$, for example $\mathbb{F}^n$. $\mathcal{P}(W)$ is the set of all the vector subspaces of W. The dimension of a subspace $V \in \mathcal{P}(W)$ is written as $\dim(V)$. In [KK08a], it has been shown that for every $A \in \mathcal{P}(W)$ and $B \in \mathcal{P}(W)$

$$d(A, B) = \dim(A + B) - \dim(A \cap B) \tag{2.21}$$

is a distance between vector subspaces. A subspace code is therefore a subset of $\mathcal{C} \subset \mathcal{P}(W)$. A *codeword* of $\mathcal{C}$ is a vector subspace of $\mathcal{C}$. The minimum distance of $\mathcal{C}$ is the minimum distance between two distinct codewords while using the distance (2.21)

$$d_{\min}(\mathcal{C}) = \min_{X,Y \in \mathcal{C},\ X \neq Y} d(X,Y). \tag{2.22}$$

The maximum dimension of the code words of $\mathcal{C}$ is $\ell(\mathcal{C}) = \max_{X \in \mathcal{C}} \dim(X)$. When the dimension of all the codewords of $\mathcal{C}$ is the same, then the code is of constant dimension. Assume that a codeword $V \in \mathcal{C}$ is sent by the source, that U is received by a destination $t \in \mathcal{T}$, it is possible to describe the behavior of the network in terms of modifications of the vector subspaces as

$$U = H_k(V) \oplus Z \tag{2.23}$$

with $k = \dim(U \cap V)$, $H_k(V)$ is a subspace of V with dimension k such that $H_k(V) \cap Z = 0$. This type of model illustrates the impact of network coding and the introduction of errors in terms of operations on vector subspaces. With this model, the network introduces $\rho = \dim(V) - k$ cancellations and $n_z = \dim(Z)$ errors. In this case, [KK08a] shows that if $2(n_z + \rho) < d_{\min}(\mathcal{C})$, then a decoder with a minimum distance allows getting V from U. A generalization of the Singleton bound is proposed for these codes [KK08a]. A construction of codes on subspaces similar to Reed-Solomon codes allowing the Singleton bound to be reached as well as a decoding algorithm with minimum distance for this family of codes is detailed in [KK08a], emphasizing constant dimension codes.

2.3.3.2 Recent developments

These results have lead to a number of recent developments. Constant dimension codes are studied in [XF08] and applied to network coding. Johnson-type bounds are also calculated. In [GB08], several new codes and bounds exploiting the distance between sub-spaces (2.21) are explored. In [SKK08], a wide class of constant dimension codes is studied, a new distance considering the rank metrics is introduced. Codes associated with this metric are introduced and an effective decoding algorithm for this family is proposed. Several constant dimension codes are introduced in [KK08b] with a larger number of codewords than in the case of the previously examined codes. Performance bounds as well as construction methods for the code family introduced in [KK08a] are proposed in [AA09]. An analysis of the geometric properties of the codes using rank metric is carried out in [GY09]. The lower and upper bounds of the cardinality of codes of given rank are evaluated which enables an analysis of the performance of these codes. In [ES09], a new multi-level approach examining the construction of subspace codes is presented. The authors show that the codes proposed in [KK08a] represent a specific case of the proposed family of codes. A Gilbert-Varshamov bound relative to the codes constructed in [SK09] is introduced in [KK09]. Finally, [CGY09] studies the practical implantation of the codes introduced in [KK08a]. The construction of these codes for small Galois fields and limited error correction capacity is feasible, and improves the network performance in terms of throughput.

2.3.4 Joint network-channel coding/ decoding

This section aims to show how, in a wireless context, the redundancy existing at the network level can help to improve channel decoding performance by performing joint network-channel decoding. This joint approach allows to reduce the number of packets lost due to transmission errors on wireless networks. This is achieved by using, on one hand, the network spatial diversity and on the other hand, the redundancy introduced by channel codes on the low layers of communication protocols. This research is motivated by [EMH$^+$03], which highlights the limits of the coding approaches in which the network and the channel or the source and the network are considered separately. Studies carried out on canonical networks demonstrate that source-channel separation remains valid for some networks although this is not the case for network-channel separation. In [LMH$^+$07], it is also shown that despite the fact that separation remains valid in some cases, a separate processing, for example source-network, results in higher costs, for example in terms of bandwidth or energy, than a joint treatment.

Wireless networks are a privileged area of application for joint network-channel decoding techniques. In contrast to wired networks where lower layers of the protocol stack are supposed to

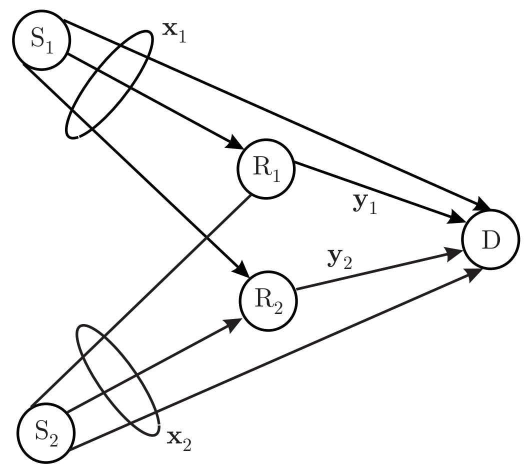

Figure 2.3: Wireless networks with two sources, two relays and one destination

provide error-free links, wireless networks provide packets which may be erroneous. Joint network-channel decoding techniques exploit the redundancy introduced by the network coding operations in order to improve the capacity of the channel code to correct transmission errors. Instead of focusing on guaranteeing an error-free transmission on each link, we are more interested in guaranteeing error-free decoding at the destination nodes. The latter uses the data received from incoming links for decoding. In the presence of links providing a certain level of redundancy, error-free decoding is possible even if decoding at the level of each individual link is not possible. Joint network-channel decoding is therefore only useful when network coding introduces redundancy. The first practical application of this concept to networks with relays has been proposed in [HH06]. Iterative network-channel decoding methods for relay networks as well as for multiple access relay channels have been proposed in [HD06] and [HH06].

2.3.4.1 Principle

Consider a wireless network topology consisting of two sources S_1 and S_2, two intermediate relay nodes R_1 and R_2, and a destination node D, see Figure 2.3. The sources generate two information messages $\mathbf{x}_1$ and $\mathbf{x}_2$ of k symbols each, and protect them using channel codes in order to obtain two independent packets with n symbols each, $\mathbf{p}_1$ and $\mathbf{p}_2$, which are then transmitted towards D. The relays receive the two packets, process them and retransmit them to D. To simplify, the links are assumed to be without errors and the communications are carried out on two orthogonal channels where mutual interference is negligible. As a result, D receives four packets from which it attempts to recover the information messages $\mathbf{x}_1$ and $\mathbf{x}_2$ sent by the sources. Assume that the two packets $\mathbf{p}_1$ and $\mathbf{p}_2$ can be expressed as a function of $\mathbf{x}_1$ and $\mathbf{x}_2$ as follows

$$\mathbf{p}_1 = \mathbf{x}_1 G_1 \text{ et } \mathbf{p}_2 = \mathbf{x}_2 G_2, \tag{2.24}$$

where G_1 and G_2 are two channel coding matrices of dimension $k \times n$ with elements belonging to $\mathbb{F}$, a Galois field with q elements. The relay nodes R_1 and R_2 directly receive $\mathbf{p}_1$ and $\mathbf{p}_2$ and are therefore capable of decoding them to obtain $\mathbf{x}_1$ and $\mathbf{x}_2$, which are then re-encoded using a channel code and a network code to obtain

$$\mathbf{y}_1 = a_{11}\mathbf{x}_1 G_{11} + a_{12}\mathbf{x}_2 G_{12} \tag{2.25}$$

and

$$\mathbf{y}_2 = a_{21}\mathbf{x}_1 G_{21} + a_{22}\mathbf{x}_2 G_{22} \tag{2.26}$$

where the a_{ij} are network coding coefficients and where the matrices G_{ij} are channel code generator matrices used at the relay. As a result, D receives four packets $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{y}_1$, and $\mathbf{y}_2$ from which it has to estimate $\mathbf{x}_1$ and $\mathbf{x}_2$ transmitted by the source. By adopting a matrix notation, one obtains the following equations

$$\begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{y}_1 \\ \mathbf{y}_1 \end{bmatrix} = \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \\ a_{11}G_{11} & a_{12}G_{12} \\ a_{21}G_{21} & a_{22}G_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = G_{\text{joint}} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} \tag{2.27}$$

where G_{joint} represents the generator matrix for the joint network-channel code. From (2.27), one sees that channel and network codes can be considered as a unique code from the point of view of the network extremities and that the latter can be represented by a unique generator matrix G_{joint}. As a result, in the presence of transmission errors, the messages $\mathbf{x}_1$ and $\mathbf{x}_2$ can be decoded at the destination by directly exploiting G_{joint} or by the use of an iterative decoding method, see for example [GHW+09].

2.3.4.2 Recent developments

Moving from the basic idea in [HH06] and [HD06], various studies about the performance improvement of joint network and channel-decoding are available in the literature. Most of these studies have the main objective to analyze the effectiveness of such a joint decoding design for the robust and reliable operation of network-coded wireless architectures over lossy networks and to overcome some initial assumptions retained in, e.g., [HH06]. For example, in [HH06] ideal error-correcting codes are assumed for the source-to-relay channels, which results in having error-free communication over these links, as well as in introduc ing a diversity loss since the local channel code blocks the whole frame if just a single bit is erroneous (see, e.g., [XFKC07], [AHGH09]). Some examples of recent research results addressing the exploitation and the benefits of a joint network-channel code design and decoding can be found in [BL06, YSW09].

2.3.5 Joint source-network coding/ decoding

Joint source-network coding and decoding enable all or some of the packets transmitted by the sources to be recovered in the presence of an insufficient number of received network-coded packets, by exploiting the existing or artificially introduced correlation between the transmitted data packets. These techniques also enable the distributed compression of correlated messages generated by geographically distributed sources.

Regarding distributed compression, distributed source coding [SW73, WZ76, CBLV05] can perform separate compression of correlated sources and may be as effective (when there are no losses) as joint compression. This technique is interesting in the case of sensor networks where it is possible to perform efficient compression even in the absence of coordination between sensors [TPCO09, HF08]. This solution does not, however, allow to completely exploit the capacity of the network and assumes that the sensors have a precise estimate of the level of correlation between the data they produce. In this context, network coding is a natural solution for correlated data transmission on a network with diversity. The application of network coding for the compression of correlated sources has been proposed in [BS06, HMEK04, WSXK05] in the case of lossless coding. The proposed techniques provide efficient distributed algorithms which are capable of exploiting diversity whether at the source or the channel level. In the case of coding with losses, *compressed sensing* [Don06, CT06] allows an approximate reconstruction of the source by exploiting its properties of compressibility using random combinations of its samples. Network coding techniques inspired by compressed sensing have been proposed in [SKJ08] using network codes on the real fields. However, the data taken from wireless sensor networks are, in general, quantized and network coding in the case of real fields is therefore questionable.

Regarding the robustness against losses, or capacity variations on some of the links of the network, an alternative solution to the network coding techniques presented in Sections 2.3.1 to 2.3.3 consists of combining multiple description coding techniques [Goy98], [GKK01] and network coding.

2.3.5.1 Using multiple description coding (MDC) to combat loss

Multiple Description Coding (MDC) techniques, see [Goy01] and the references therein, represent an alternative approach to cope with lossy networks. Structured redundancy is introduced during compression to generate packets which may be partitioned in equally important descriptions of the source data. Reconstruction with a given quality is possible with the reception of one description and gradually improves with the number of received descriptions. Multiple description may be performed using adapted quantization [LP03], correlating transforms [Goy98], frame expansions [GKV98], *etc.* Optimized routing techniques have been proposed in [SW05] with the concept of Rainbow Networks, where descriptions are assimilated as colors, the goal being to determine which combination of colors has to be associated to each link of the network to maximize reconstruction quality at the various receivers. Nevertheless, the proposed optimization approach is centralized and requires some knowledge of the network topology.

Several alternative joint source-network coding approaches have been proposed recently, trying to get the best of MDC techniques and NC. A first attempt to combine NC with rainbow network flow is presented in [SWS08] where higher network throughput is achieved compared to the original rainbow network flow solution, however, only intra-layer NC is performed. In [DSW09] and [KLS+10], techniques to perform intra- and inter-layer NC are proposed. Concatenated MDC and RLNC have been introduced in [WW08], where a concatenation of an *inner* network code and an *outer* PET code is considered to allow receivers with low resources to network decode the most important packets without requiring all packets to be received. In [VS10], unequal error protection (UEP) is combined with NC. Source packets are again grouped into priority classes. NC is performed in such a way that a small amount of network-coded packets allow to decode the highest priority class. In [PTF10], the correlation existing between packets transmitted, *e.g.*, by neighboring sensors of a sensor network is exploited to perform an *approximate decoding* when not enough network-coded packets have been received to perfectly perform network decoding. The influence of the size of the Galois field in which quantization and NC are performed, is studied and an optimal size in terms of reconstruction noise is evaluated.

2.4 Conclusion

In this chapter, we provided a short overview of the network coding basics. We illustrated the principle of network coding through the so called *butterfly network*. We recalled that the maximum capacity in a network can be achieved by using linear network codes, and that random linear network codes can ensure increasing decoding probability with the size of the finite field in which the coding coefficients are picked from. We also presented the many advantages of network coding in terms of throughput, wireless resources, security, complexity, and resilience to link failures.

Finally, we have provided an overview of the main research fields concerning the design of robust network-coded wireless architectures over lossy networks: error-correcting code design in projective spaces, joint network-channel iterative decoding, and joint source-network coding. It is in this latter context that the work presented in this thesis is situated.

Chapter 3

Artificially Introduced Correlation

Network codes are *all or nothing* codes. The sink cannot recover the complete information sent by the source unless it receives a number of linearly independent network-coded packets at least equal to the number of the original transmitted packets. We propose two code designs combining multiple description coding and network coding (NC-MDC) in order to overcome this limitation by keeping the good properties of network coding, *i.e*, a better use of network resources, while adding the multiple description codes capability of being decoded with a quality that improves gracefully with the number of received network-coded packets.

We consider two ways of introducing redundancy using MDC and we aim to investigate the effect of exploiting this redundancy on the average quality received at the destination, in the context of a multicast scenario where network coding of packets is performed at the intermediate nodes of the network.

MDC is performed either via frame expansion (NC-MDC-F) [GKK01] or via correlating transform (NC-MDC-T) [Goy98]. With NC-MDC-F, redundancy is introduced before quantization, whereas with NC-MDC-T, redundancy is introduced after quantization.

We first show that the redundancy introduced *before* quantization using NC-MDC-F may be efficiently used at receiver side to decode network-coded packets and to mitigate part of the quantization noise. Focusing on the case of missing network-coded packets, we show that the estimation of the original packets using the received combinations and taking into account the redundancy introduced during frame expansion can be performed via the solution of a mixed integer quadratic program (MIQP) . This provides some robustness to packet losses, with a quality of the reconstructed packets increasing with the number of received packets.

Then, we consider the case of NC-MDC-T. In this scheme, redundancy is introduced *after* quantization, and the reconstruction in presence of missing combinations may be done via a simple Gaussian elimination as in [PTF10]. However, contrary to MDC via frame expansion, as soon as enough packets are received to be able to perform decoding, there is no advantage in receiving more packets.

In the second part of this chapter, we consider a scenario where a single source multicasts to several receivers experiencing different wireless channel conditions, and we evaluate the effect of these conditions on the average signal quality received. The channels are modeled as binary symmetric channels (BSC) with random transition probability ε, distributed according to some probability density function (pdf) $f(\varepsilon)$. This allows to characterize the average SNR among receivers for various $f(\varepsilon)$ for the two proposed joint coding schemes, and to compare it to a traditional multicast scenario denoted as NC-SDC (single description coding), and where only RLNC is performed within the network.

The chapter is organized as follows. The conventional NC scenario is presented in Section 3.1. The two proposed joint NC-MDC schemes are presented in Sections 3.2 and 3.3. Performance comparision is done in Section 3.4. MDC schemes evaluation is presented in Section 3.5, and simulation results are described in Section 3.6 . Conclusions are drawn in Section 3.7.

3.1 Conventional Network Coding Scenario (NC-SDC)

We start by introducing a conventional multicast scenario where only network coding is performed at the intermediate nodes of the network. This scenario is denoted as NC-SDC (single description coding), since each transmitted packet is assumed to be a description by itself.

Consider a network with a source S, and several intermediate and receiver nodes. Assume that S has to transmit some realization $\mathbf{x} \in \mathbb{R}^k$ of a random vector to N receiver nodes. The source vector entries x_i, $i = 1, ..., k$ are supposed to be uncorrelated. Each entry is then quantized using a q-level uniform scalar quantizer to get a vector of quantized indexes $\mathbf{z} \in \mathbb{F}_q^k$, where $\mathbb{F}_q$ is the Galois field with q elements. Each quantized index z_i, $i = 1, ..., k$ is put in a separate packet. Then $n \geqslant k$ random linear combinations of packets over $\mathbb{F}_q$ are transmitted. Further packet combinations are performed within the network. A receiver then gets $m \leqslant n$ packets $p_j \in \mathbb{F}_q$, $j = 1, ..., m$, see Figure 3.1. The effect of the network and of the various network coding operations is modeled by the network coding matrix A linking $\mathbf{z} = (z_1, ...z_k)^{\mathrm{T}}$ and $\mathbf{p} = (p_1, ...p_m)^{\mathrm{T}}$ as follows:

$$\mathbf{p} = A\mathbf{z}. \tag{3.1}$$

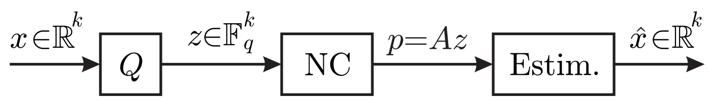

Figure 3.1: Conventional random linear network coding scenario

An estimate $\widehat{\mathbf{x}}$ of $\mathbf{x}$ at the receiver side is easily obtained when A is full rank k. In this case, there exists a $k \times k$ submatrix $\widetilde{A}$ of A of rank k. Let $\tilde{\mathbf{p}}$ be the subvector of $\mathbf{p}$ obtained by selecting all components of $\mathbf{p}$ corresponding to the rows of A selected in $\widetilde{A}$. $\mathbf{z}$ can then be obtained via a simple matrix inversion

$$\widehat{\mathbf{z}} = \widetilde{A}^{-1}\tilde{\mathbf{p}} \tag{3.2}$$

and $\widehat{\mathbf{x}}$ via inverse quantization of $\widehat{\mathbf{z}}$

$$\widehat{\mathbf{x}} = \alpha\widetilde{A}^{-1}\tilde{\mathbf{p}} + \beta \tag{3.3}$$

If A is of rank less than k, (3.2) is not possible and no decoding can be performed. In this case, no estimate better than the mean value of $\mathbf{x}$ may be obtained. However, a partial Gaussian elimination might be sometimes useful in recovering some of the transmitted samples. In fact, if with some permutations of the rows and columns, A can be rewritten in the form

$$A = \begin{bmatrix} A' & 0 \\ B' & C' \end{bmatrix} \tag{3.4}$$

with A' being of full rank k', then k' transmitted samples can be reconstructed.

Note that the reconstruction process would be the same if we choose to put several successive quantized entries z_i in each sent packet, see Figure 3.2. This also applies to all the scenarios proposed later.

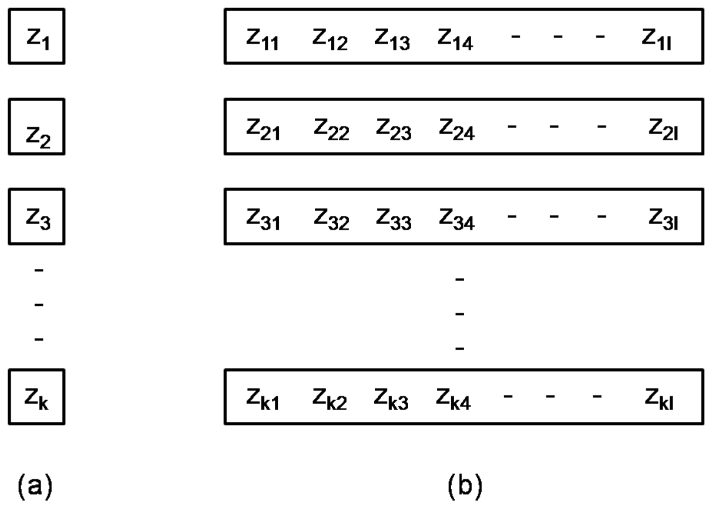

Figure 3.2: Two possible approaches for sending packets: packets containing one quantized sample each (a), or packets containing several quantized samples (b)

3.2 Introducing redundancy via frame expansion

This section introduces the first proposed NC-MDC scheme, denoted as NC-MDC-F. MDC is performed via a frame expansion [GKV98] applied at the source.

3.2.1 NC-MDC-F Coding scheme

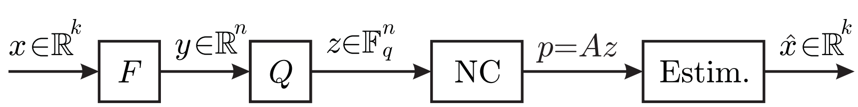

Figure 3.3: Block diagram of the proposed system

The source vector $\mathbf{x} \in \mathbb{R}^k$ is expanded using a frame expansion matrix F, producing the vector $\mathbf{y} = F\mathbf{x} \in \mathbb{R}^n$, with $n > k$. Each entry of $\mathbf{y}$ is quantized with a q-level uniform scalar quantizer to get a vector of indexes $\mathbf{z} \in \mathbb{F}_q^n$. Each index is then put in a separate packet and the n packets are transmitted over the network. Random linear network coding (RLNC) is performed at the intermediate nodes of the network. At receiver side, a set of m network-coded packets is obtained.

An estimate $\widehat{\mathbf{x}}_F$ of $\mathbf{x}$ from the received packets grouped in a vector $\mathbf{p}$ using the various constraints imposed by the system is finally evaluated. In what follows, each step in Figure 3.3 is described to evidence the constraints linking the variables of the system, which will help obtaining $\widehat{\mathbf{x}}_F$ at the receiver nodes.

3.2.1.1 Multiple description using frame expansion

MDC is performed via a real-valued frame expansion. A real-valued frame of $\mathbb{R}^k$ [GVT98] is a set of $n > k$ vectors $\{\varphi_i\}_{i=1\dots n}$ such that there exists $B > 0$ and $C < \infty$ (the frame bounds) satisfying for all $\mathbf{x} \in \mathbb{R}^k$,

$$B \left\|\mathbf{x}\right\|^2 \leq \sum_{i=1}^{n} \langle \mathbf{x}, \varphi_i \rangle^2 \leq C \left\|\mathbf{x}\right\|^2, \tag{3.5}$$

where $\langle \cdot, \cdot \rangle$ is the inner product of $\mathbb{R}^k$. The frame operator F associated to $\{\varphi_i\}_{i=1\dots n}$ is the linear operator from $\mathbb{R}^k$ to $\mathbb{R}^n$ defined as

$$\left(F\mathbf{x}\right)_i = \langle \mathbf{x}, \varphi_i \rangle, \ i = 1 \dots n. \tag{3.6}$$

For any $\mathbf{x} \in \mathbb{R}^k$, the frame operator F produces a vector

$$\mathbf{y} = F\mathbf{x} \in \mathbb{R}^n. \tag{3.7}$$

The redundancy rate introduced by the frame expansion is $r = n/k$.

3.2.1.2 Quantization

A q-level uniform scalar quantization Q with step size Δ is performed on each entry of $\mathbf{y}$, resulting in a vector of quantization indexes $\mathbf{z}$, whose entries $z_i = Q(z_i) \in \mathbb{F}_q$. The quantization intervals are $[(i - q/2)\,\Delta, (i - q/2 + 1)\,\Delta]$, $i = 0 \dots q - 1$. The reconstruction levels are chosen at the middle of the quantization intervals, $r_i = (i - q/2 + 1/2)\,\Delta$, $i = 0 \dots q - 1$. The inverse quantizer Q^{-1} takes a quantization index z_i and associates a reconstruction

$$Q^{-1}(z_i) = (z_i - q/2 + 1/2)\,\Delta, \tag{3.8}$$

$$= \alpha z_i + \beta \tag{3.9}$$

with $\alpha = \Delta$ and $\beta = (-q+1)\,\Delta/2$. Since all reconstruction levels have been taken at the middle of the quantization intervals, $Q^{-1}(z_i')$ satisfies

$$y_i - Q^{-1}(z_i) \leq \Delta/2, \tag{3.10}$$

$$-y_i + Q^{-1}(z_i) \leq \Delta/2. \tag{3.11}$$

3.2.1.3 Network Coding

The vector of quantized indexes $\mathbf{z}$ is then transmitted over the network. Each entry of $\mathbf{z}$ is transmitted in a separate packet. No combinaison is done at the source. RLNC is performed at the intermediate nodes of the network. Assume that the ℓ-th receiver has access to m independent packets $p_\mu \in \mathbb{F}_q$, $\mu = 1 \ldots m$, with $m \leq n$. Since these packets have been network-coded, the relation between $\mathbf{p} = (p_1, ...p_m)^{\mathrm{T}}$ and $\mathbf{z} = (z_1, ...z_n)^{\mathrm{T}}$ may be written as follows

$$\mathbf{p} = A\mathbf{z} \tag{3.12}$$

where the network matrix $A \in \mathbb{F}_q^{m \times n}$ is the matrix of global NC coefficients. The coefficients of A may be recovered from the headers of each received packet [CWJ03b]. Usually, $m = n$ packets have to be received in order to recover the uncoded packets. However, even if $m = n$, A is not necessarily full rank n. Moreover, when not enough linearly independent packets have been received, A is not full rank n and all the uncoded packets cannot be recovered directly.

3.2.2 Estimation of the source vector

An estimate $\widehat{\mathbf{x}}_F$ of the source vector $\mathbf{x}$ based on the received network-coded packets $\mathbf{p}$ and using the fact that $\mathbf{x}$ has been expanded into $\mathbf{y}$ before quantization and transmission has to satisfy a system of equations and inequalities derived from (3.7), (3.10), (3.11), and (3.12). Gathering all constraints, one gets

$$\begin{cases} y_i = \sum_{j=1}^{k} f_{i,j} x_j, & i = 1 \dots n \\ y_i - (\alpha z_i + \beta) \leq \Delta/2, & i = 1 \dots n \\ -y_i + (\alpha z_i + \beta) \leq \Delta/2, & i = 1 \dots n \\ z_i \in \{0, \dots, q-1\} & i = 1 \dots n \\ p_\mu = \sum_{j=1}^{n} a_{\mu j} z_j, & \mu = 1 \dots m \end{cases} \tag{3.13}$$

where $F = (f_{ij})_{i=1\dots n, j=1\dots k}$ and $A = (a_{\mu j})_{\mu=1\dots m, j=1\dots n}$ have been defined in Section 3.2.1. In the last line of (3.13), all operations are done in $\mathbb{F}_q$. This system contains $n+m$ equations, $2n$ inequalities, and $2n+k$ unknows, namely $\mathbf{x} \in \mathbb{R}^k$, $\mathbf{y} \in \mathbb{R}^n$, and $\mathbf{z} \in \mathbb{F}_q^n$. Due to the quantization, $\mathbf{x}$ cannot be recovered exactly, even if A is full rank.

3.2.2.1 Enough network-coded packets have been received

When A is full rank n, $i.e$, enough linearly independent network-coded packets have been received, a full rank n submatrix $A' \in \mathbb{F}_q^{n \times n}$ of A may be inverted to get an estimate $\widehat{\mathbf{y}}$ for $\mathbf{y}$

$$\widehat{\mathbf{y}} = \alpha (A')^{-1} \mathbf{p}' + \beta. \tag{3.14}$$

where $\mathbf{p}'$is the subvector of $\mathbf{p}$ that corresponds to the rows of A'. Then, a least-squares estimate $\widehat{\mathbf{x}}_F$ of $\mathbf{x}$ is easily obtained from $\widehat{\mathbf{y}}$ as

$$\widehat{\mathbf{x}}_F = \left(F^T F\right)^{-1} F^T \widehat{\mathbf{y}}. \tag{3.15}$$

3.2.2.2 Network-coded packets are missing

Now, when A is not full rank n, A cannot be inverted. Since there is not a unique $\widehat{\mathbf{x}}_F$ satisfying (3.13), one may search for an estimate of minimum norm

$$\widehat{\mathbf{x}}_F = \arg \min_{\mathbf{x} \text{ satisfying } (3.13)} \mathbf{x}^T \mathbf{x} \tag{3.16}$$

Nevertheless, this optimization problem is quite hard to solve, since it involves real variables and variables belonging to $\mathbb{F}_q$. Moreover, equality and inequality constraints have to be considered. Assuming that the Galois field is of prime size (q is prime), one may transform the last line in (3.13) by introducing m *slack* variables $s_\mu \in \mathbb{Z}$ as follows

$$p_\mu = \sum_{j=1}^{n} a_{\mu j} z_j + q s_\mu,\ \mu = 1 \dots m \tag{3.17}$$

Now, standard additions and multiplications may be performed in (3.17).

Solving (3.16) with the modified system where (3.17) has been put in (3.13) consists in solving a mixed integer quadratic problem (MIQP). This kind of minimization problem may be modeled using AMPL [FG02]. Since this MIQP only involves convex quadratic forms, it can be solved using CPLEX [CPL]. CPLEX implements a Branch-and-Bound (BB) search whose node bounds are computed by solving a continuous relaxation of the MIQP. Since this relaxation is convex, it can be solved using either a modified simplex method or a barrier method (in case the quadratic terms are in the constraints). The whole BB solution algorithm runs in exponential time in the worst case.

Note that (3.17) holds only when working in $\mathbb{F}_q$. A generalization of the decoding process when operations take place in the extension field $\mathbb{F}_q^r$ is presented in Appendix A.

3.3 Alternative MDC scheme: MDC via correlating transform

This section introduces the second proposed NC-MDC scheme, denoted as NC-MDC-T. MDC is performed via a correlating transform. This technique is inspired by [PTF10].

In [PTF10], an approximate decoding scheme based on existing correlation between *quantized* packets is proposed. Several correlated sources are considered and no frame expansion is performed on the source signals before quantization. The correlation between source symbols is still present on the quantized indexes $\mathbf{z}$ and translates into the fact that there exists some *known* matrix H of size $(n-k)\times n$ such that

$$H\mathbf{z} = \mathbf{0}. \tag{3.18}$$

H plays a role similar to a parity-check matrix of a traditionnal error-correcting code. The relation (3.18) may then be exploited at receivers which do not have access to enough network-coded packets.

3.3.1 Probability of existence of a full rank coding matrix

Consider a network matrix A of dimension $m\times n$, with $m \leq n$. If there exists a submatrix B' of size $n\times n$ of

$$B = \begin{pmatrix} A \\ H \end{pmatrix} \tag{3.19}$$

that is full rank n, then B' may be inverted to get an estimate $\widehat{\mathbf{z}}$ of the quantized source samples.

The probability that such matrix B' exists may be shown to be as follows.

$$\Pr(\text{rank}\begin{pmatrix} A \\ H \end{pmatrix} = n) =$$

$$\begin{cases} 0 & if\, m < k \\ \prod_{i=1}^{k}(1-q^{-i}) & if\, m = k \\ \sum_{\substack{i_1,\ldots i_m \\ i_1<i_2<\ldots<i_{m-k}}}^{m} q^{-k+i_1-1}\ldots q^{-k+i_{m-k}-(m-k)}\prod_{i=1}^{k}(1-q^{-i}) & if\, m > k \end{cases} \tag{3.20}$$

The proof of (3.20) is presented in Appendix C.

3.3.2 Proposed coding scheme

We consider the scenario represented in Figure 3.4. The source vector $\mathbf{x}$ is quantized to get quantization indexes $\mathbf{y} \in \mathbb{F}_q^k$. Then, a full-rank k correlating transform, randomly generated in the Galois field in which the operations take place, $T \in \mathbb{F}_q^{n\times k}$, is applied to $\mathbf{y}$ to get a vector $\mathbf{z} = T\mathbf{y} \in \mathbb{F}_q^n$. Entries of $\mathbf{z}$ are again put in separate packets which are then network coded, and $\mathbf{p} \in \mathbb{F}_q^m$ represents the received packets, see (3.12) and Figure 3.4. The coding matrix A is of dimension $m \times n$, with $m \leq n$.

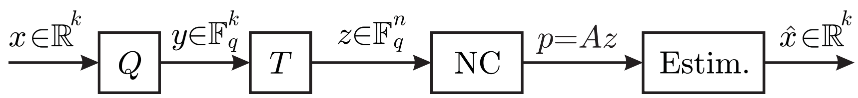

Figure 3.4: Block diagram of the MDC scheme via a correlating transform (NC-MDC-T)

Since $T \in \mathbb{F}_q^{n\times k}$ is full rank k, there exists a *parity-check* matrix $H \in \mathbb{F}_q^{(n-k)\times n}$ of rank $n-k$ such that $H\mathbf{z} = 0$ for all $\mathbf{z}$ such that there exists $\mathbf{y} \in \mathbb{F}_q^k$ with $\mathbf{z} = T\mathbf{y}$. This property may be used at receiver side to estimate $\widehat{\mathbf{x}}_T$ from $\mathbf{p}$. If there exists an $n \times n$ submatrix A' of

$$B = \begin{pmatrix} A \\ H \end{pmatrix} \tag{3.21}$$

that is full rank n, then $\mathbf{z}$ may be obtained via a simple matrix inversion

$$\widehat{\mathbf{z}} = (A')^{-1}\mathbf{p}' \tag{3.22}$$

where

$$\mathbf{p}' = \begin{pmatrix} \mathbf{p} \\ \mathbf{0} \end{pmatrix} \tag{3.23}$$

such that $\mathbf{p}' \in \mathbb{F}_q^n$, and where the components of $\mathbf{p}'$correspond to the rows of A'. Let $\widetilde{\mathbf{z}}$ be the subvector of $\widehat{\mathbf{z}}$ of dimension $k \times 1$, and $\widetilde{T}$, the submatrix of T of dimension $k \times k$ and full rank k. In this case, $\widehat{\mathbf{y}}$ can be calculated as

$$\widehat{\mathbf{y}} = \widetilde{T}^{-1}\widetilde{\mathbf{z}} \tag{3.24}$$

and $\widehat{\mathbf{x}}_T$ can be deduced from $\widehat{\mathbf{y}}$ via inverse quantization

$$\widehat{\mathbf{x}}_T = \alpha \widetilde{T}^{-1} \widetilde{\mathbf{z}} + \beta \tag{3.25}$$

When A' is of rank less than n, again, in general, no better estimate than the mean value of $\mathbf{x}$ may be obtained. However, a partial Gaussian elimination can be performed, as described in (3.4).

Compared to the approach introduced in Section 3.2, here, redundancy is introduced *after* quantization. The decoding process is quite simple, since it involves only Gaussian elimination.

3.4 Simulation results

The NC-MDC-F transmission scheme described in Section 3.2.1 is simulated with $k = 4$, and $n = 7$. The source generates vectors of k independent and identically distributed Gaussian samples with zero-mean and variance $\sigma^2 = 1$, cropped to $\pm 3\sigma$. F is built with lines 2 to 5 of an $n \times n$ DCT transform matrix. The uniform quantizer with quantization cells partitioning the interval $[-3\sigma, 3\sigma]$ is chosen[1]. Quantization with $q \in \{7, 17, 31, 61\}$ quantization intervals is considered, leading to quantization indexes in $\mathbb{F}_q$. The $m \times n$ network matrix A is chosen at random with $m \leq n$ to simulate the effect of network coding, and lost packets.

The NC-MDC-T transmission scheme described in Section 3.3 is also simulated with the same source, k, and n. Like NC-MDC-F, quantization with $q \in \{7, 17, 31, 61\}$ is considered, leading to quantization indexes in $\mathbb{F}_q$. A fixed $n \times k$ random matrix $T_c \in \mathbb{F}_q$ with full rank k is chosen to introduce redundancy. An $(n-k) \times n$ matrix H with full rank $n-k$ is chosen to be orthogonal to T_c.

In both cases, simulations results are averaged over 1000 realizations of the source and of the network matrix A.

[1]Depending on the rate, better rate-distortion performance may be obtained by adjusting more carefully the boundaries of the quantization domain

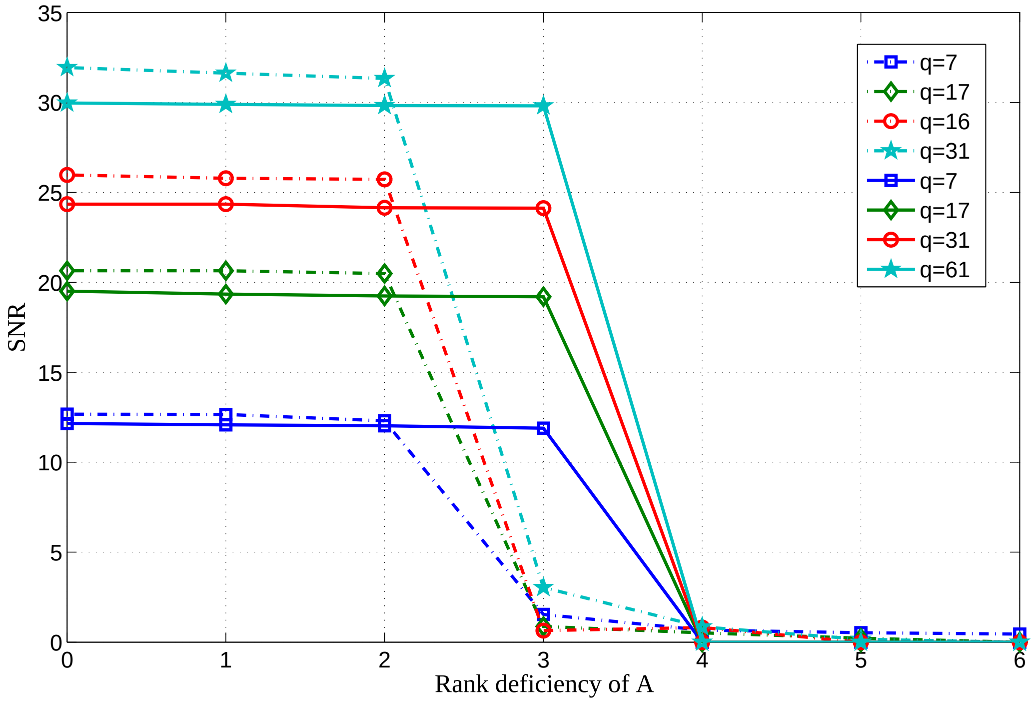

Figure 3.5: SNR as a function of the rank deficiency of A and of the size of the considered Galois field; NC-MDC-F is in dashed lines and NC-MDC-T is in plain lines

Figure 3.5 represents the average Signal-to-Noise Ratio (SNR)

$$SNR_{dB} = 10 \log_{10} \frac{\|\mathbf{x}\|^2}{\|\mathbf{x} - \widehat{\mathbf{x}}\|^2} \tag{3.26}$$

obtained from the reconstruction of the transmitted message as a function of the rank deficiency of A for various sizes of the Galois field used in quantization and NC operations. NC-MDC-F is less robust to erasures than NC-MDC-T, since one more erasure is tolerated. However, when enough linearly independent packets have been received, the reconstruction quality is better using NC-MDC-F. The frame expansion allows to reduce the effect of quantization noise. This effect is not obtained with the correlating transform, since its applied after the source is being quantized.

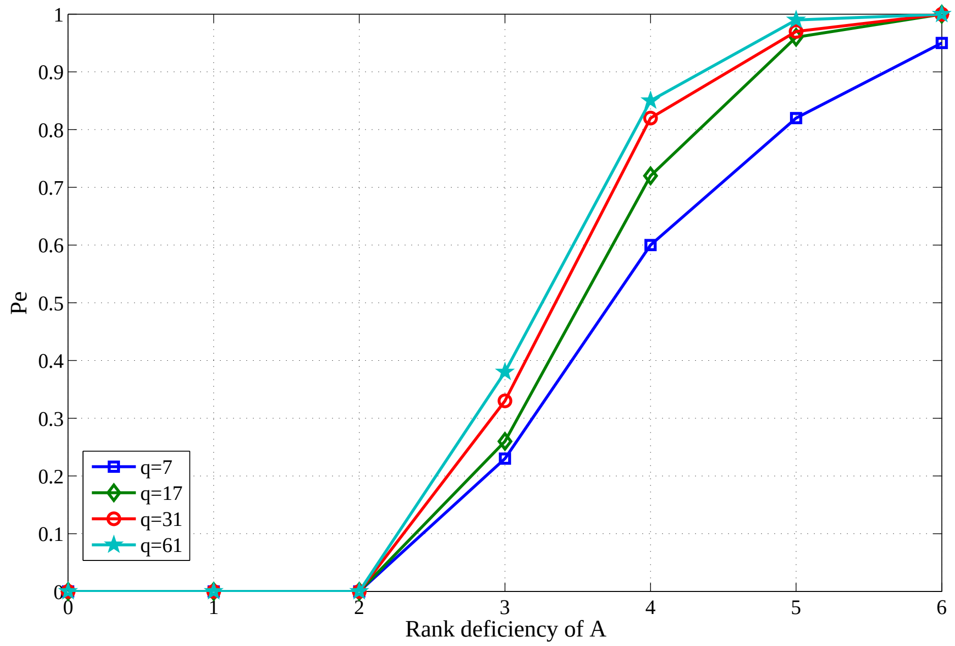

Figure 3.6: NC-MDC-F: Proportion of reconstruction errors as a function of the rank deficiency of A and of the size of the considered Galois field

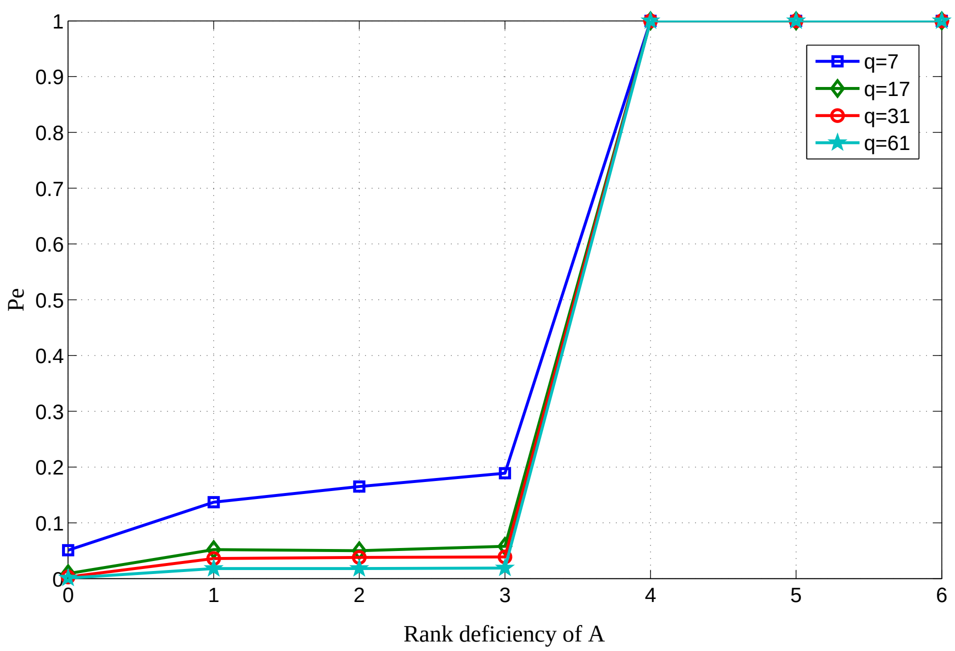

Figure 3.7: NC-MDC-T: Proportion of reconstruction errors as a function of the rank deficiency of A and the size of the considered Galois field

Figure 3.6 and Figure 3.7 provide the evolution of the decoding errors (when $\widehat{\mathbf{z}} \neq \mathbf{z}$) as a function of the rank deficiency of A for both schemes.

With the NC-MDC-T scheme, when the number of losses is larger than the number of redundant packets, $i.e$, larger than $n - k = 3$ in our case, the decoding error probability Pe is one. The reconstruction therefore is impossible. Whereas in the case of NC-MDC-F, a non-zero fraction of the transmitted packets can still be correctly decoded, even if the number of missing independent packets is larger than the one being introduced during the frame expansion process. The decoding error probability increases smoothly. When 3 packets have not been received, about 70% of the packets are still correctly decoded with NC-MDC-F and the poor SNR observed in Figure 3.5 is mainly due to erroneously recovered source samples. Being able to detect when samples were not reconstructed correctly may significantly improve the performance when many NC packets are lost.

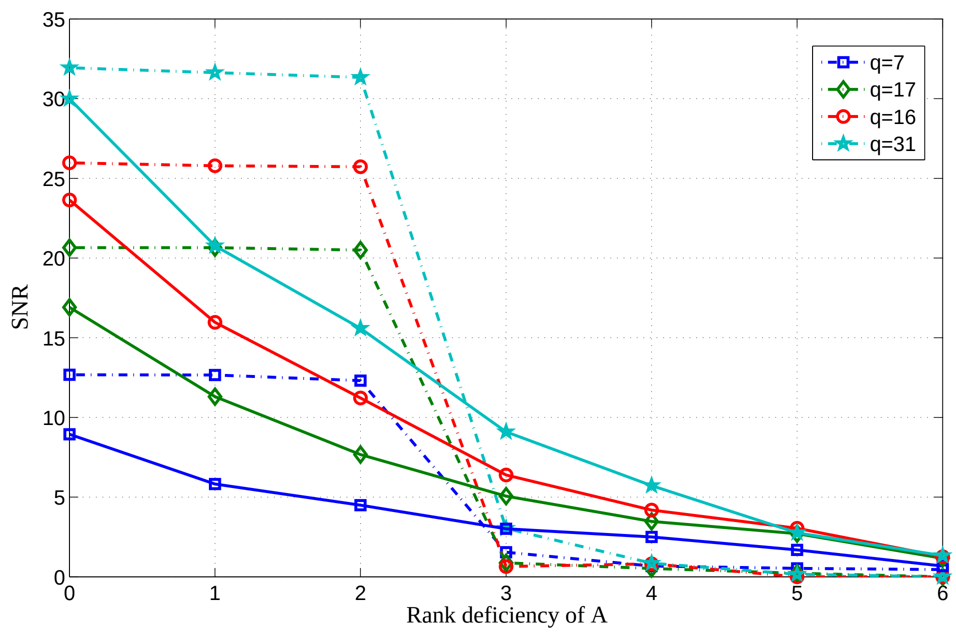

Figure 3.8: NC-MDC-F (dashed) compared to a MDC-F scheme (plain) where NC is only performed between packets of the same description

Figure 3.8 evaluates the improvement provided by the MIQP reconstruction, by comparing a NC-MDC-F scheme and a scheme where MDC-F is performed and NC is done on packets corresponding to the same description, see Figure 3.9. In the latter case, it is assumed that when a packet is lost, the whole description is lost. No MIQP reconstruction is possible. One sees that MIQP allows to get almost the maximal reconstruction quality even if some packets are lost. However, when too much packets are lost, the MDC-F scheme provides a much smoother SNR decrease than the NC-MDC-F.

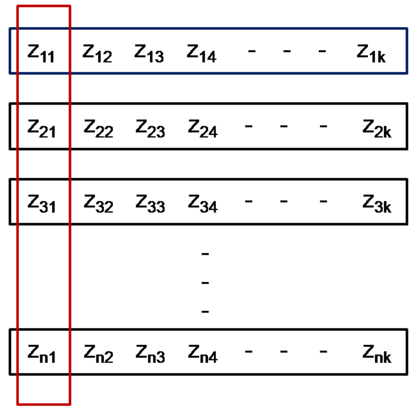

Figure 3.9: MDC-F : combines samples belonging to the same packet (horizontal blue line), NC-MDC-F: combines packets corresponding to different descriptions (red vertical line)

3.5 MDC schemes evaluation in a multicast scenario

Consider the wireless multicast scenario illustrated by Figure 3.10. A source S multicasts data to a set of mobile terminals experiencing different channel conditions. Each channel is modeled as a binary symmetric channel (BSC) with a random transition probability ε, distributed according to some probability density function (pdf) $f(\varepsilon)$. The BSC represents the whole transmission chain, including modulation, channel encoding, decoding, and demodulation.

In this context, we aim to evaluate the performance of both proposed NC-MDC approaches and to compare it with a traditional multicast scenario that we denote as NC-SDC (single description coding), where linear combinations are performed both at the source and at the intermediate nodes of the network, see Section 3.1. Our goal is to characterize for the three coding schemes the average SNR among receivers for various $f(\varepsilon)$.

The SNR is evaluated as a function of

1. the size of the Galois field q in which the network coding operations take place

2. the distribution of the transition probability ε for a fixed field size q

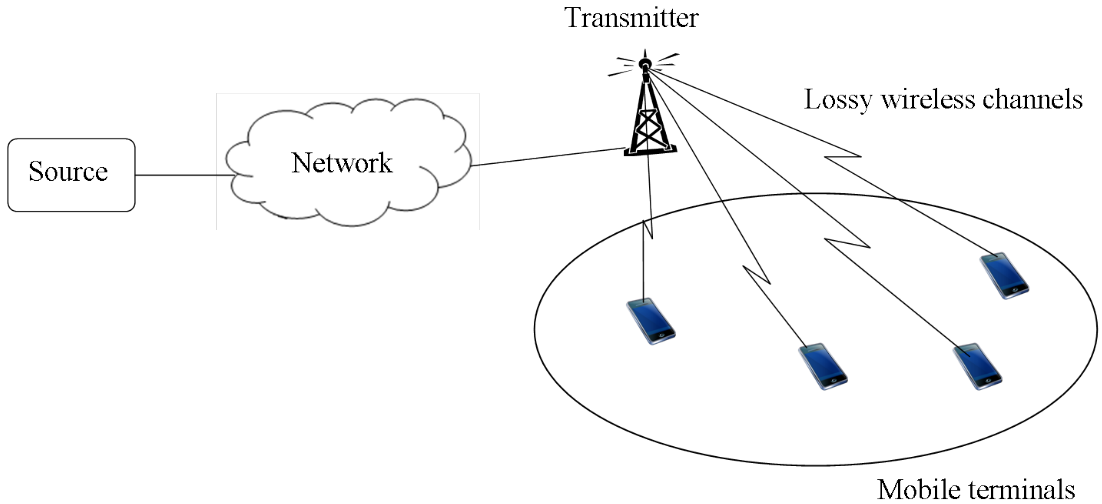

Figure 3.10: Considered wireless transmission scheme

3.5.1 Hypotheses

The size of a generation, *i.e*, the number of network-coded packets during each transmission, is denoted as g. For NC-SDC one has $g = k$, since no redundancy is introduced, whereas for NC-MDC-T and NC-MDC-F, $g = n$. It is assumed that each of the N receivers obtains n noisy packets from which an estimate of $\mathbf{x}$ has to be evaluated. The packets are assumed to have passed through a binary symmetric channel with transition probability ε distributed according to some probability density function (pdf) $f(\varepsilon)$. The pdf is chosen to be identical for all receivers.

3.5.2 Average signal-to-noise ratio

The average SNR observed by a given receiver depends of both the number of correctly received packets at the destination, and of the size of the Galois field q in which the NC operations take place. For a given user, whatever the considered scenario, one has to determine the probability mass function (pmf) $P_\Gamma\left(\gamma \mid g, n, q\right)$ of the number γ of linearly independent packets received, *i.e.*, the pmf of the rank of A, given that g packets were combined, that n noisy combinations were received, and that the size of the Galois field is q.

If i is the number of error-free packets received, one gets

$$
\begin{aligned}
P_{\Gamma}(\gamma \mid g, n, q) &= \sum_{i=\gamma}^{n} P(\gamma, i \mid g, n, q) \\
&= \sum_{i=\gamma}^{n} P(\gamma \mid i, g, n, q)\, P(i \mid g, n, q) \\
&= \sum_{i=\gamma}^{n} P_R(\gamma \mid i, g, q)\, P_C(i \mid g, n, q)\,. \qquad (3.27)
\end{aligned}
$$

In (3.27), $P_C(i \mid g, n, q)$ is the probability of getting i noise-free (Clean) packets (this probability depends on $f(\varepsilon)$) and $P_R(\gamma \mid i, g, q)$ is the probability of having γ informative packets among i received packets without error (this is the probability that the rank of A is γ).

To evaluate $P_C(i \mid g, n, q)$, and knowing that $\varepsilon \in [0, 1]$, one has to introduce $f(\varepsilon)$ as follows

$$
\begin{aligned}
P_C(i \mid g, n, q) &= \int_0^1 P(i, \varepsilon \mid g, n, q)\, d\varepsilon \\
&= \int_0^1 P(i \mid g, n, q, \varepsilon)\, f(\varepsilon \mid g, n, q)\, d\varepsilon \\
&= \int_0^1 P(i \mid g, n, q, \varepsilon)\, f(\varepsilon)\, d\varepsilon, \qquad (3.28)
\end{aligned}
$$

since $f(\varepsilon)$ depends neither on n, nor on g, or q.

Then

$$
P(i \mid g, n, q, \varepsilon) = \binom{n}{i} (1-\varepsilon)^{iL} \left(1 - (1-\varepsilon)^{L}\right)^{n-i} \qquad (3.29)
$$

indicates the probability of receiving i packets without errors among n received packets sent for a realization ε of the BSC transition probability. The format of the transmitted packets is shown in Figure 3.11. The MAC wireless header is simplified and reduced to the SEQ field, to which are appended the NC coefficients.

Figure 3.11: Format of the considered network-coded packets

In (3.29), $L = (g + \ell + \ell_{\text{seq}} + \ell_{\text{crc}}) \lceil \log_2 q \rceil$ represents the total length in bits of a packet. ℓ_{seq} and ℓ_{crc} stand for the number of symbols in $\mathbb{F}_q$ used to represent the sequence number (SEQ) and the CRC , respectively, and ℓ is the payload. SEQ and CRC form a part of the header of network-coded packets [CWJ03a]. The CRC is used to protect the data, the coefficients involved in the linear combination and the SEQ.

Now, $P_R(\gamma \mid i, g, q)$ represents the pmf of the number γ of independent linear combinations among the i received packets, which is given by

$$P_R(\gamma \mid i, g, q) = \frac{\mu(i, g, \gamma, q)}{q^{ig}} \tag{3.30}$$

where $\mu(n_1, k_1, r_1, q)$ [vLW92] is the number of matrices in $\mathbb{F}_q^{n_1 \times k_1}$ with rank equal to r_1

$$\mu(n_1, k_1, r_1, q) = \begin{bmatrix} k_1 \\ r_1 \end{bmatrix}_q \sum_{i=0}^{r_1} (-1)^{(r_1-1)} \begin{bmatrix} r_1 \\ i \end{bmatrix}_q q^{n_1 i + \binom{r_1 - i}{2}} \tag{3.31}$$

and $\begin{bmatrix} n_1 \\ k_1 \end{bmatrix}_q$ is the Gaussian coefficient [vLW92] given by

$$\begin{bmatrix} n_1 \\ k_1 \end{bmatrix}_q = \begin{cases} 1 & k_1 = 0 \\ \frac{(q^{n_1}-1)(q^{n_1-1}-1)\dots(q^{n_1-k_1+1}-1)}{(q^{k_1}-1)(q^{k_1-1}-1)\dots(q-1)} & k_1 > 0 \end{cases} \tag{3.32}$$

Assume now that the average SNR for Scenario s, with $s \in \{1, 2, 3\}$, when receiving γ independent packets coded in $\mathbb{F}_q$ is given by $\text{SNR}_s(\gamma, q)$. The value of s refers to one of the three considered scenarios: NC-SDC, NC-MDC-T and NC-MDC-F. The average SNR, taking into account the transition probability distribution $f(\varepsilon)$, can then be deduced from (3.28) and (3.30) and is expressed as

$$\mathrm{SNR}_s(q) = \sum_{\gamma=0}^{n} \mathrm{SNR}_s(q,\gamma) P_\Gamma\left(\gamma \mid g, n, q\right). \tag{3.33}$$

The values of $\mathrm{SNR}_s(q,\gamma)$ are obtained experimentally, see Section 3.4. The following section characterizes the average SNR for various $f(\varepsilon)$ and sizes of the Galois field using (3.33).

3.6 Experimental results

Simulations are performed with $k = 6$ and $n = 9$. The source generates vectors of k independent and identically distributed Gaussian samples with zero-mean and variance $\sigma^2 = 1$. Quantization with $q \in \{7, 17, 31, 61\}$ outputs is considered, leading to quantization indexes in $\mathbb{F}_q$. The network matrix A is chosen at random in the corresponding Galois field to simulate the effect of network coding. Simulation results are averaged over 1000 realizations of the source and of the network matrix A.

3.6.1 Performance as a function of the coding matrix rank deficiency

First, the average SNR resulting from the reconstruction of the transmitted message is drawn as a function of the number of the rank deficiency of the coding matrix A for various sizes of the Galois field used in the quantization and NC operations. These curves correspond to the representation of $\sum_\gamma \mathrm{SNR}_s(q,\gamma) P_R\left(\gamma \mid i, g, q\right)$ with the number of missing independent packets being $n - i$. The potential rank deficiency is thus taken into account, but not the effect of transmission errors.

Figures 3.12 and 3.13 represent the SNR for NC-MDC-F, NC-MDC-T, and NC-SDC as a function of the field size. For NC-SDC, n network-coded packets containing k independent packets are transmitted (the redundancy is almost the same as in the two first scenarii, only the size of the header is somewhat reduced due to the reduced number of independent packets).

Figures 3.12 and 3.13 show that NC-MDC-F can mitigate part of the quantization noise, providing better results than the NC-MDC-T when the BSC transition probability vanishes. NC-MDC-T provides an increased robustness to packet losses compared to NC-MDC-F.

Figures 3.12 and 3.13 also show a degradation of the SNR due to the rank deficiency, even if

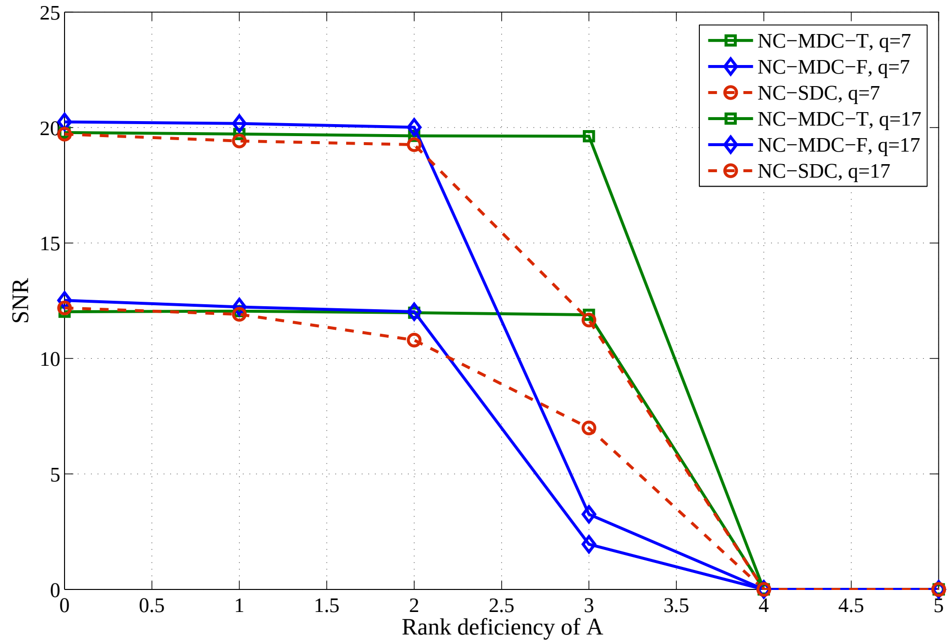

Figure 3.12: SNR as a function of the rank deficiency of A and the Galois field size q, for $q = 7$ and $q = 17$. NC-MDC-F (diamond), NC-MDC-T (square), and NC-SDC (circle)

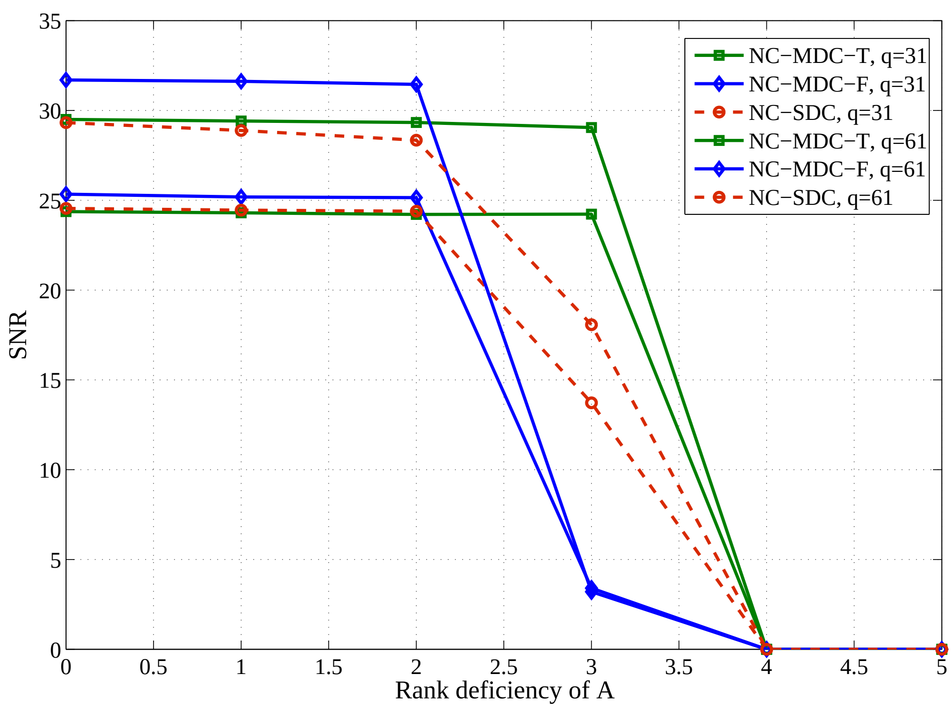

Figure 3.13: SNR as a function of the rank deficiency of A and the Galois field size q, for $q = 31$ and $q = 61$. NC-MDC-F (diamond), NC-MDC-T (square), and NC-SDC (circle)

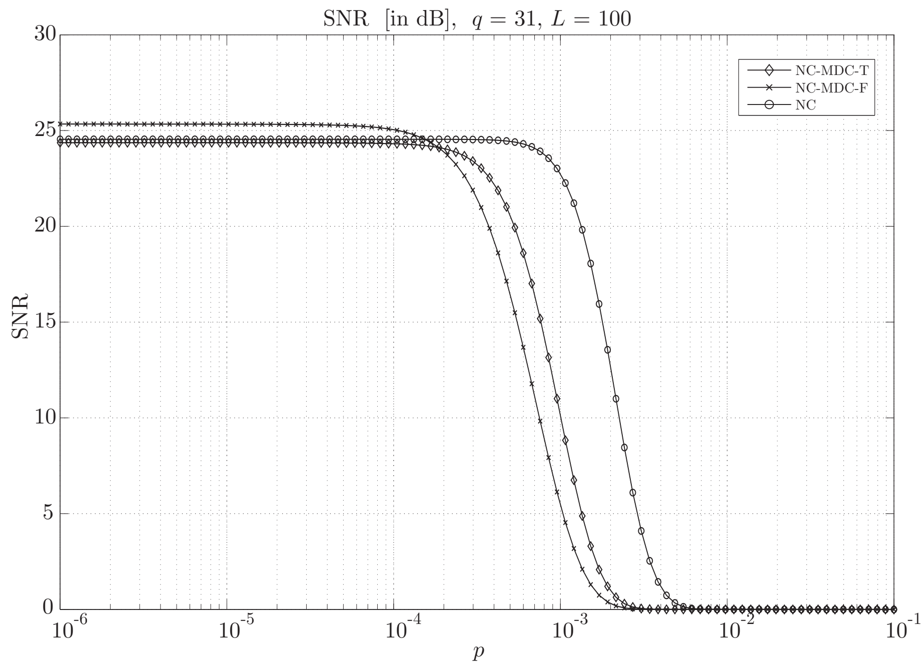

Figure 3.14: Average SNR as a function of the transition probability ε

enough packets have been received. When not enough independent packets are received, the SNR is null.

3.6.2 Average SNR as a function of ε

In this case, the average SNR for a single user is evaluated as a function of the channel transition probability ε, with $q = 31$. Figure 3.14 represents the SNR as a function of ε when ε varies between 10^{-6} and 10^{-1} for packets with payloads of length $\ell = 100$ symbols. NC-MDC-F outperforms NC-SDC for values of ε belonging to $[10^{-6}, 1.5 \times 10^{-4}]$, while for larger values of ε the average SNR obtained with NC-SDC is larger.

3.6.3 Average SNR for various distributions of ε

In this case, we study the average received signal quality of many users, when their channel random transition probability ε is described by two different pdfs. The uniform distribution

$$f_{\mathrm{U}}(\varepsilon) = \begin{cases} \frac{1}{a} & 0 \leq \varepsilon \leq a \\ 0 & \text{elsewhere} \end{cases} \tag{3.34}$$

and the exponential distribution

$$f_{\mathrm{E}}(\varepsilon) = \begin{cases} \frac{\ln(10)10^{-\varepsilon}}{10^{-a}-10^{-b}} & a \leq \varepsilon \leq b \\ 0 & \text{elsewhere} \end{cases} \tag{3.35}$$

are considered.

Using (3.34), (3.28) becomes

$$P_C(i \mid g, n, q) = \frac{1}{a}\binom{n}{i}\int_0^a (1-\varepsilon)^{iL}\left(1-(1-\varepsilon)^L\right)^{n-i} d\varepsilon. \tag{3.36}$$

Using (3.35), (3.28) becomes

$$P_C(i \mid g, n, q) = \frac{\ln(10)}{10^{-a}-10^{-b}}\binom{n}{i}\int_a^b 10^{-\varepsilon}(1-\varepsilon)^{iL}\left(1-(1-\varepsilon)^L\right)^{n-i} d\varepsilon. \tag{3.37}$$

The average SNR is computed using (3.33) after substituting (3.36) or (3.37) in (3.27). With both NC-MDC-F and NC-MDC-T, the number of mixed packets is $g = n$, whereas for NC-SDC, $g = k$.

Figure 3.15 represents the expected SNR for the studied techniques for various field sizes $q \in \{7, 17, 31, 61\}$ using the uniform pdf for the transition probability as a function of a, the upper bound of the support of the uniform pdf.

Similarly, Figure 3.16 represents the expected SNR using the exponential pdf for the transition probability. We choose $a = 10^{-10}$, and b varying from 10^{-6} to 10^{-1}.

In both cases, when there are not too many users with bad channel characteristics, NC-MDC-F outperforms NC-MDC-T, which itself outperforms NC. When the proportion of users with bad channel gets larger ($a > 3.10^{-4}$ for the uniform pdf and $b > 3.10^{-4}$ for the exponential pdf), NC-SDC becomes better.

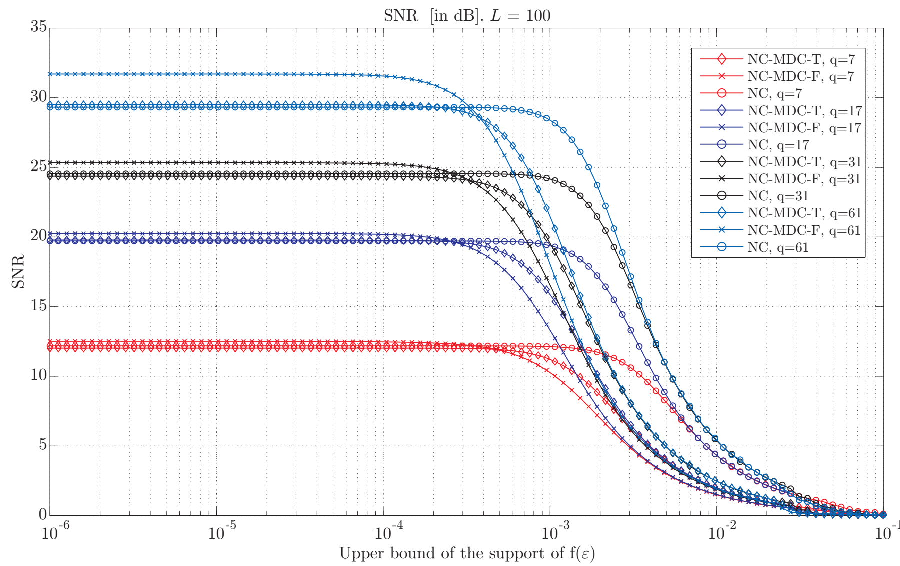

Figure 3.15: Average SNR for the Uniform Distribution as a function of the upper bound of the support of $f_{\mathrm{U}}(\varepsilon)$

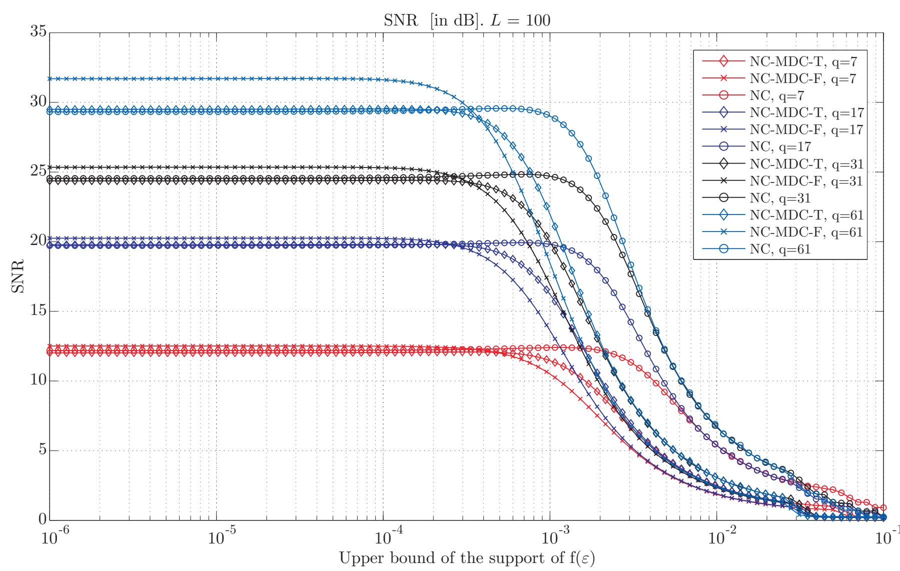

Figure 3.16: Average SNR for the Exponentiel Distribution as a function of the upper bound of the support of $f_{\mathrm{E}}(\varepsilon)$

3.7 Conclusion

This chapter introduces a transmission scheme combining MDC and NC. Multiple descriptions have been obtained either using a frame expansion (NC-MDC-F) or using a correlating transform (NC-MDC-T). In the first case, the reconstruction is performed via the resolution of a mixed integer quadratic problem (MIQP). In the second case, a reconstruction algorithm using a simple Gaussian elimination and derived from [PTF10] has been employed. In both cases, a very good robustness to erasures has been observed. When the number of lost packets is small, the NC-MDC-F provides better SNRs thanks to a reduction of a part of the quantization noise. The price to be paid is a decreased robustness to losses. When the number of lost packets increases, a reconstruction is still possible for some packets, even if the number of losses is larger than $n-k$, the number of redundant packets.

Packets containing only a single expanded sample have been considered. Packets carrying more samples could be introduced in both methods. The NC-MDC-T would not be affected by the number of introduced samples, since the reconstruction performance is only determined by the network matrix A and the parity-check matrix D. In the case of NC-MDC-F, additional constraints and inequalities could be introduced in (3.13) to help obtaining the correct solution when the number of lost packets is close to or above $n-k$.

This chapter also considers the effects of the field size and channel conditions on the average quality of the received signal, in a scenario where a single source multicasts or broadcasts data to a set of users experiencing various channel conditions, using NC or a combination of MDC and NC. Packets are assumed to be received at the output of a BSC whose transition probability ε is taken as random according to some pdf $f(\varepsilon)$. The performance of the three schemes largely depends on $f(\varepsilon)$. When a user is likely to have good channel conditions, NC-MDC-F provides the best results. On the other hand, when the channel conditions are quite bad, plain NC-SDC becomes better.

Chapter 4

Exploiting Existing Correlation

The upsurge of sensor networks in the recent years directed research towards the design and implementation of low-complexity sensing techniques, along with efficient solutions for information collection. The transmission of information between sensors and to a data collection sink is typically performed in a distributed manner, mainly on ad-hoc network topologies.

Distributed source coding (DSC) is one enabling technology for sensor networks, as it allows to perform distributed compression without coordination between sensors [CBLV05]. When sensors provide spatially correlated data, these data may be compressed, *e.g.*, by channel codes [SW73, WZ76] and eventually jointly decoded at the data collection sink [TPCO09, HF08]. However, this solution does not allow to fully exploit the network capacity, moreover, it requires the sensors to be aware of the correlation level between the data they produce and the data produced by neighboring sensors.

In this context, DSC based on NC schemes [BS06, HMEK04, WSXK05] is known to be an efficient method for building distributed data gathering algorithms in networks with channel and source diversity. However, wireless networks are often subject to packet losses.

Compressed sensing [Don06, CT06] and its distributed variant [SKJ08] is an alternative solution that allows to perform an approximate reconstruction of the source by exploiting some sparsity properties. Nevertheless, this approach requires network codes over real fields [SKJ08], and does not apply to finite field network codes, better suited to transmission by wireless sensors of quantized data.

This chapter introduces a scheme to better collect data in a wireless sensor network by allowing nodes to perform and transmit linear combinations of collected measures. The goal is to reduce the number of measurements required for decoding, by exploiting the spatial correlation existing among transmitted data.

A MAP estimator is provided to recontruct the source samples using the correlation between data contained in the network-coded packets collected at the sink. The MAP estimator is not straightforward, since it has to cope with packets containing linear combinations in a Galois field of the data to be estimated and with a correlation between this data which is expressed in the real field. The temporal correlation between successive samples collected by the sensors is not considered here. If such correlation exists, it may be easily removed via a decorrelating transform such as a DCT.

The proposed coding scheme is introduced in Section 4.1. The MAP estimator is derived in Section 4.2, and simulation results are presented in Section 4.3. Some conclusions are drawn in Section 4.5.

4.1 Coding and transmission scheme

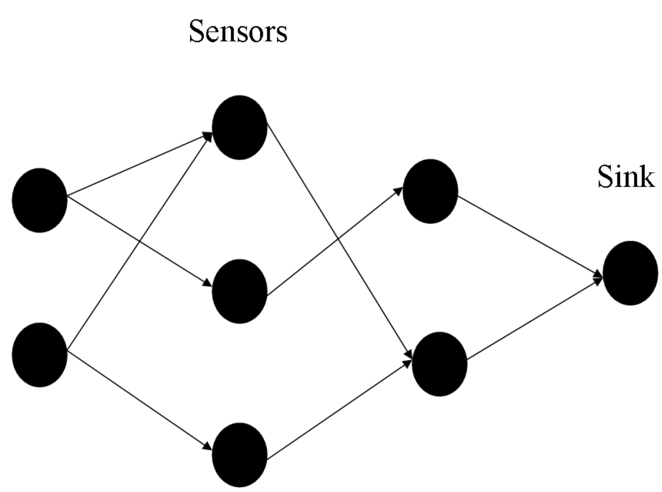

Figure 4.1: Wireless Sensor Network

Consider a set of k wireless sensors spatially spread over some two-dimentionnal area. The location of the i-th sensor is denoted by $\boldsymbol{\theta}_i \in \mathbb{R}^2$. It measures some physical quantity x_i, assumed to be the realization of some random variable X_i. The vector $\mathbf{x} = (x_1, ..., x_k)^{\mathrm{T}} \in \mathbb{R}^k$ gathering all measurements is assumed to be the realization of a zero-mean Gaussian vector $\mathbf{X} = (X_1, ...X_k)^{\mathrm{T}}$ with covariance matrix

$$\Sigma = \begin{bmatrix} \sigma^2 & \sigma^2 e^{-\lambda d_{1,2}^2} & \cdots & \cdots & \sigma^2 e^{-\lambda d_{1,k}^2} \\ \sigma^2 e^{-\lambda d_{1,2}^2} & \sigma^2 & & & \sigma^2 e^{-\lambda d_{2,k}^2} \\ \vdots & \ddots & \sigma^2 & & \vdots \\ \vdots & & & \sigma^2 & \sigma^2 e^{-\lambda d_{k-1,k}^2} \\ \sigma^2 e^{-\lambda d_{1,k}^2} & \cdots & \cdots & \sigma^2 e^{-\lambda d_{k-1,k}^2} & \sigma^2 \end{bmatrix} \tag{4.1}$$

where σ^2 is the variance of each source, λ is some constant, and $d_{i,j} = \sqrt{(\theta_{i,1} - \theta_{j,1})^2 + (\theta_{i,2} - \theta_{j,2})^2}$ is the distance between sensors i and j. The chosen model allows us to have a correlation that decreases with the distance separating two sensors.

Prior to transmission, each x_i is quantized with a q-level uniform scalar quantizer $Q: \mathbb{R} \to \mathbb{F}_q$ with stepsize Δ to get $z_i \in \mathbb{F}_q$. Each sensor then performs a linear combination of the packet containing its own quantized measurement with the packets it receives from its neighbors using RLNC. The new coded packet is transmitted to the neighboring nodes. This process is repeated until the sink considers that it has received enough packets to be able to recover the data with the desired quality.

Assume that the data processing sink has received $m \leqslant k$ *linearly independent* network-coded packets. It is very likely that it has received more packets, but it keeps only linearly independent packets. The content of the received packets is grouped in a vector $\mathbf{p} \in \mathbb{F}_q^m$. The effect of network coding is represented by the network coding matrix $A \in \mathbb{F}_q^{m \times k}$ linking $\mathbf{p}$ and $\mathbf{z}$ as follows

$$\mathbf{p} = A\mathbf{z}. \tag{4.2}$$

Each network-coded packet contains the result of the linear combinations of the quantized source samples z_i, in addition to the global coding coefficients which are stored in an extra header added to each packet [CWJ03a]. Therefore, the coding matrix A is known by the sink. The measurement, quantization, network coding, and estimation scheme is represented in Figure 4.2. Note that even if $\mathbf{x} \in \mathbb{R}^k$ is a vector, each of its entries is spatially spread.

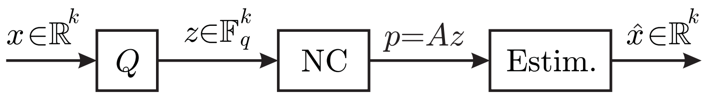

Figure 4.2: Block diagram of the proposed system

The receiver has to evaluate an estimate $\widehat{\mathbf{x}}$ of $\mathbf{x}$ from the received packets $\mathbf{p}$ using the fact that the entries of $\mathbf{x}$ are correlated. For that purpose, Σ is assumed to be known by the sink.

4.2 Estimation of the source samples

Instead of estimating directly $\mathbf{x}$, we choose to perform first an estimate $\widehat{\mathbf{z}}$ of the vector $\mathbf{z} = (z_1, ..., z_k)^{\mathrm{T}}$ of quantization indexes based on the received network-coded packets $\mathbf{p} = (p_1, ..., p_m)^{\mathrm{T}}$. Then, an estimate $\widehat{\mathbf{x}}$ for $\mathbf{x}$ is easily obtained from $\widehat{\mathbf{z}}$, either via inverse quantization or via a classical MAP estimator.

Provided that the scalar quantizer Q is well designed, there exist two constants α and β such that the reconstructed sample after inverse quantization may be expressed as $\alpha z_i + \beta$ and such that

$$\begin{cases} x_i - (\alpha z_i + \beta) \leq \Delta/2, & i = 1 \dots k \\ -x_i + (\alpha z_i + \beta) \leq \Delta/2, & i = 1 \dots k \end{cases} \tag{4.3}$$

Moreover, exploiting the network coding matrix, one has

$$p_\mu = \sum_{j=1}^{k} a_{\mu j} z_j, \; \mu = 1...m. \tag{4.4}$$

where $a_{\mu j}$ are the global encoding coefficients of the received network-coded packets corresponding to the entries of A. Two possible scenarios can be considered:

1. The sink collects enough *linearly independent* network-coded packets to perform decoding using a simple Gaussian elimination

2. An insufficient number of *linearly independent* network-coded packets is available at the sink, a MAP estimator of the source is proposed in this case

4.2.1 Scenario 1: enough network-coded packets are received

The vector $\mathbf{z}$ to estimate contains k unknowns. When enough network-coded packets are received, *i.e.*, when A is full rank k, or when there exists a submatrix A' of A such that A' is full rank k, A' can be inverted in order to obtain an estimate $\widehat{\mathbf{z}}$ of $\mathbf{z}$

$$\widehat{\mathbf{z}} = (A')^{-1}\mathbf{p}', \tag{4.5}$$

$\mathbf{p}'$ being the correpondant subvector of $\mathbf{p}$. $\widehat{\mathbf{x}}$ is then estimated as

$$\widehat{\mathbf{x}} = \alpha\widehat{\mathbf{z}} + \beta. \tag{4.6}$$

Note that the correlation among source entries could have been taken into consideration by estimating $\mathbf{x}$ using a classical MAP estimator.

4.2.2 Scenario 2: network-coded packets are missing

When not enough network-coded packets are received, *i.e.*, when $\text{rank}(A) < k$, the inversion (4.5) is not possible. Let $\mathbf{z}_0 = (z_1, ..., z_m)^{\mathrm{T}}$ and $\mathbf{z}_1 = (z_{m+1}, ..., z_k)^{\mathrm{T}}$ be a partition of the entries of $\mathbf{z}$ such that the corresponding partition of the coding matrix

$$A = \Pi\,[A_0\,A_1] \tag{4.7}$$

Π being a column permutation matrix, leads to

$$\mathbf{p} = \Pi A_0\mathbf{z}_0 + \Pi A_1\mathbf{z}_1, \tag{4.8}$$

with ΠA_0 of full rank m. Since the rank of A is m, such partition always exist up to a suitable permutation of the columns of A and of the *unknown* entries of $\mathbf{z}$.

Our aim is now to provide a MAP estimate of $\mathbf{z}_1$ using $\mathbf{p}$ and Σ

$$\begin{aligned}\widehat{\mathbf{z}}_1 &= \arg\max_{\mathbf{z}_1} P(z_{m+1}, \ldots, z_k|\mathbf{p}) && (4.9)\\ &= \arg\max_{\mathbf{z}_1} \sum_{\mathbf{z}_0} P(z_1, \ldots, z_k|\mathbf{p}) && (4.10)\\ &= \arg\max_{\mathbf{z}_1} \sum_{\mathbf{z}_0} P(\mathbf{p}|z_1, \ldots, z_k) P(z_1, \ldots, z_k). && (4.11)\end{aligned}$$

Assuming that the quantization step Δ is small, one can write

$$P(z_1, ..., z_k) = f(Q^{-1}(z_1), ..., Q^{-1}(z_k))\Delta^k \tag{4.12}$$

where f is the *a prior* pdf of $\mathbf{x}$

$$f(\mathbf{x}) = \frac{1}{(2\pi)^{k/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}\mathbf{x}^T\Sigma^{-1}\mathbf{x}\right). \tag{4.13}$$

Using (4.8) and the fact that A_0 is of full rank m, one gets

$$\mathbf{z}_0 = (\Pi A_0)^{-1} (\mathbf{p} - \Pi A_1 \mathbf{z}_1). \tag{4.14}$$

Combining (4.12) and (4.14) in (4.11), and since Δ^k is constant, one gets

$$\begin{aligned}\widehat{\mathbf{z}}_1 = \; & \arg\max_{\mathbf{z}_1} P(\mathbf{p} \mid (\Pi A_0)^{-1}(\mathbf{p} - \Pi A_1 \mathbf{z}_1), \mathbf{z}_1)\\ & f(Q^{-1}(z_1), ..., Q^{-1}(z_k)). && (4.15)\end{aligned}$$

For the first term in (4.15), one has

$$P(\mathbf{p} \mid (\Pi A_0)^{-1}(\mathbf{p} - \Pi A_1 \mathbf{z}_1), \mathbf{z}_1) = \begin{cases} 1 & \text{if } \mathbf{p} = A \begin{pmatrix} (\Pi A_0)^{-1}(\mathbf{p} - \Pi A_1 \mathbf{z}_1) \\ \mathbf{z}_1 \end{pmatrix} \\ 0 & \text{else.} \end{cases}$$

Consequently, using the fact that $Q^{-1}(z_i) = \alpha z_i + \beta$, one can write

$$\widehat{\mathbf{z}}_1 = \arg\max_{\mathbf{z}_1} f(\alpha((\Pi A_0)^{-1}(\mathbf{p} - \Pi A_1 \mathbf{z}_1)) + \beta, \alpha \mathbf{z}_1 + \beta) \tag{4.16}$$

Using (4.13), (4.16) may be written as

$$\widehat{\mathbf{z}}_1 = \arg\min_{\mathbf{z}_1} \begin{bmatrix} \alpha \mathbf{z}_0 + \beta \\ \alpha \mathbf{z}_1 + \beta \end{bmatrix}^T \Sigma^{-1} \begin{bmatrix} \alpha \mathbf{z}_0 + \beta \\ \alpha \mathbf{z}_1 + \beta \end{bmatrix}. \tag{4.17}$$

4.2.3 Integer quadratic problem formulation

Obtaining $\widehat{\mathbf{z}}_1$ requires the solution of a quadratic optimization problem. The main difficulty comes from the fact that the evaluation of $(\Pi A_0)^{-1}(\mathbf{p} - \Pi A_1 \mathbf{z}_1)$ involves operations in $\mathbb{F}_q$, whereas all other operations have to be done in $\mathbb{R}$. To address this issue, we procede in the same way as in Chapter 3. Assuming that q is prime, (4.17) is rewritten by introducing a vector of slack variables $\mathbf{s} \in \mathbb{Z}^m$ as follows

$$\widehat{\mathbf{z}}_1 = \arg\min_{\mathbf{z}_1} \begin{bmatrix} \alpha \mathbf{z}'_0 + \beta \\ \alpha \mathbf{z}_1 + \beta \end{bmatrix}^{\mathrm{T}} \Sigma^{-1} \begin{bmatrix} \alpha \mathbf{z}'_0 + \beta \\ \alpha \mathbf{z}_1 + \beta \end{bmatrix} \tag{4.18}$$

with

$$\mathbf{z}'_0 = (\Pi A_0)^{-1}(\mathbf{p} - \Pi A_1 \mathbf{z}_1) + q\mathbf{s} \tag{4.19}$$

and with the constraints

$$\mathbf{z}'_0 \in \{0, ..., q-1\}^m. \tag{4.20}$$

The inverse $(\Pi A_0)^{-1}$ of ΠA_0 is still computed in $\mathbb{F}_q$. Now all operations in (4.19) may be done in $\mathbb{Z}$.

The solution of (4.18) with the constraints (4.19) and (4.20) requires now the solution of an Integer Quadratic Problem (IQP) . This type of minimization problem can be modeled using AMPL [FG02]. Since the IQP problem involves only convex quadratic forms, CPLEX [CPL] may be used to solve it.

4.3 Simulations Results

We consider a WSN consisting of $k = 10$ sensor nodes. The nodes are uniformly spread over a square of 1 km width. All sensors located within a circle of radius $d_0 = 0.4$ km centered in the sensor s_i are neighbors of s_i and can directly communicate with s_i. The set of neighbours of s_i is denoted as $\mathcal{N}(s_i)$. The correlation between measurements is assumed to be represented by (4.1), with $\sigma^2 = 0.9$ and $\lambda = 0.4$ km^{-2}.

A very simple transmission protocol is considered. Time is slotted, all sensors are synchronized. In the time slots devoted to the i-th sensor, only s_i is allowed to transmit, the other sensors are listening. The i-th sensor performs a linear combination over $\mathbb{F}_q$ of the packet containing its own quantized measurement and the packets it has already received from its neighbours. The resulting network-coded packet is then transmitted and received by all sensors in $\mathcal{N}(s_i)$. Among packets reaching the sink, only linearly independent packets are kept.

We compare the performance of the proposed approach with a classical NC scenario, where correlation among the source samples is not taken into consideration at the sink. In this approach, a partial Gaussian elimination is employed when some uncoded packets reach the data processing sink, see (3.4), and measurements not estimated in this case are replaced by their mean.

Simulations are averaged over 1000 realizations of the network and of the data samples.

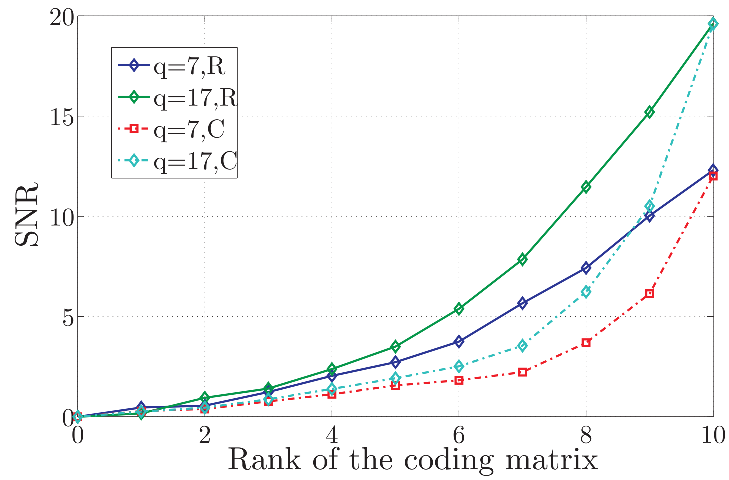

Figure 4.3: SNR as a function of the number of linearly independent packets available at the sink for $q = 7$ and $q = 17$, with the proposed approach (R) and with a conventional Gaussian elimination (C)

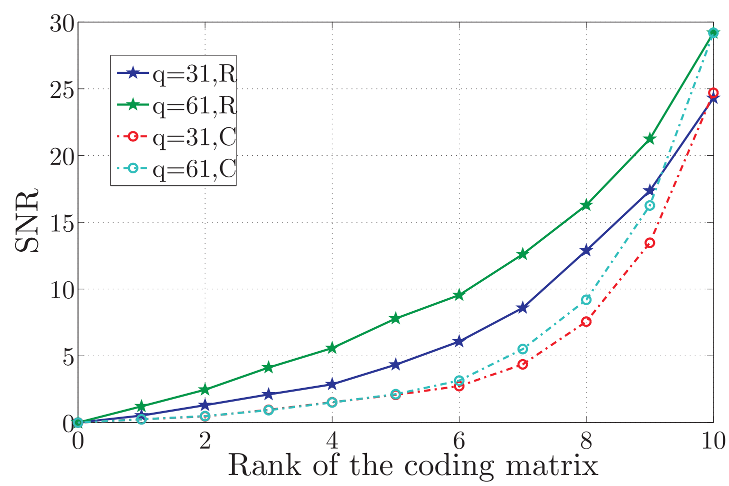

Figure 4.4: SNR as a function of the number of linearly independent packets available at the sink for $q = 31$ and $q = 61$, with the proposed approach (R) and with a conventional Gaussian elimination (C)

Figures 4.3 and 4.4 compare the average Signal-to-Noise Ratio (SNR) obtained at the sink as a function of the number of available linearly independent packets, *i.e.*, of the rank of A, and for $q \in \{7, 17, 31, 61\}$. The estimation method presented in Section 4.2.2 (denoted R) is compared to a classical Gaussian elimination performed on the received packets (denoted C). The SNR gracefully increases with the number of received packets, R outperforming C, *i.e.*, for the same SNR, fewer measurements are required.

The proportion of quantized data samples z_i erroneously estimated as a function of the rank of A for $q \in \{7, 17, 31, 61\}$ is represented in Figures 4.5 and (4.6). Consistently with the results in Figures 4.3 and 4.4, the rank of A required to obtain a given probability of error is smaller with R than with C. About 2 to 3 fewer measurements are necessary in average.

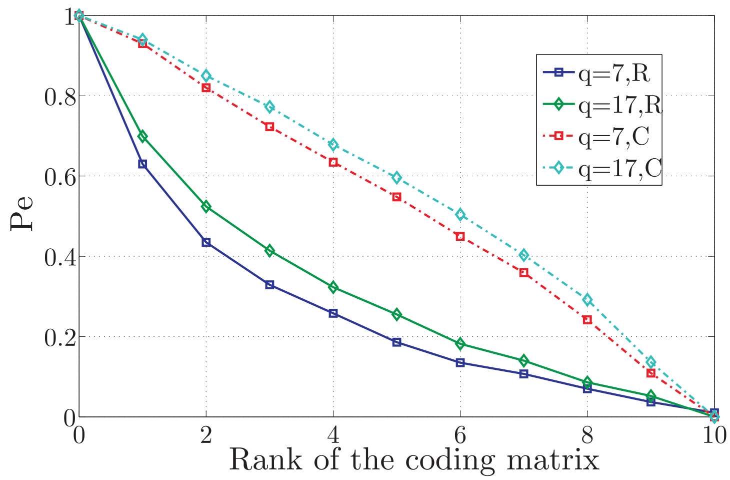

Figure 4.5: Proportion of erroneously reconstructed quantized samples as a function of the rank of A sink for $q = 7$ and $q = 17$, with the proposed approach (R) and with a conventional Gaussian elimination (C)

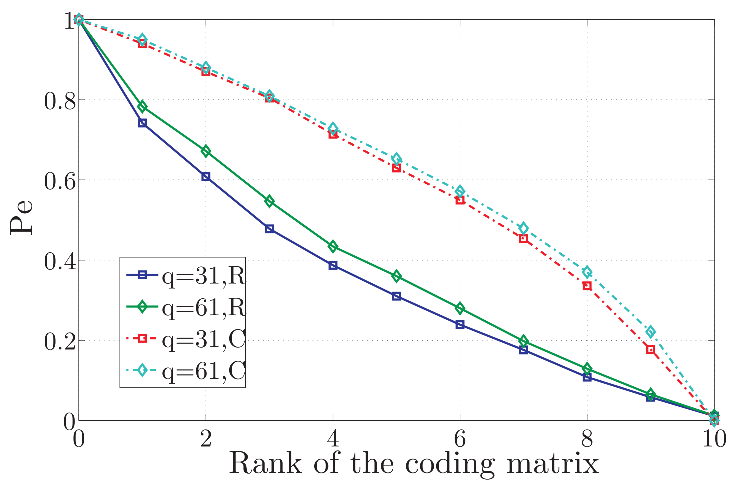

Figure 4.6: Proportion of erroneously reconstructed quantized samples as a function of the rank of A sink for $q = 31$ and $q = 61$, with the proposed approach (R) and with a conventional Gaussian elimination (C)

Figure 4.7 illustrates the effect of the level of correlation on the proportion of correctly estimated packets.

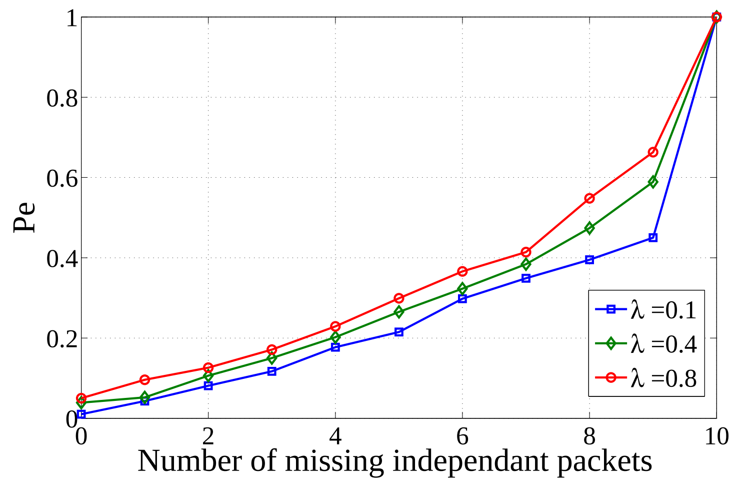

Figure 4.7: Proportion of erroneously reconstructed quantized samples as a function of the number of missing packets and the level of correlation at the source, for $q = 17$, with the proposed approach (R)

As shown by (4.1), the chosen correlation model is represented as

$$\Sigma_x = (\sigma^2 e^{-\lambda d_{i,j}^2})_{i,j}$$

The level of correlation among transmitted source samples, is thus highly dependant on the choice of λ. A low value of λ implies that the existing correlation between samples is high, and consequently, reconstruction can be achieved with a lower error decoding probability. In Figure 4.7, three values of λ is chosen, *i.e.*, three different levels of correlation. As expected, a better reconstruction quality is observed for the lowest λ value, *i.e.*, $\lambda = 0.1$, while the highest value $\lambda = 0.8$ corresponds to the worst reconstruction quality.

4.4 Exploiting redundancy using a classical MAP estimator

An alternative approach to reconstruct the transmitted measurements is via the use of a classical MAP estimator. The problem formulation and the expression of the MAP estimator is presented in Appendix B. Future work consists in evaluating the performance of such estimator, and comparing it with the performance obtained using the approach proposed in this chapter.

4.5 Conclusion

In this chapter, we investigate the problem of transmission of spatially correlated sources when random linear network coding is performed through the transmission network. We provide a MAP estimator in order to reconstruct the transmitted packets using the spatial correlation between quantized measurements stored in the network-coded packets. The reconstruction is done via the solution of an integer quadratic problem. The quality gracefully increases with the number of received packets. For comparable probability of reconstruction error or SNR, fewer packets are required using the proposed estimator than with a conventional Gaussian elimination.

Chapter 5

Conclusion & Future work

This thesis addresses the problem of source reconstruction, when not enough network-coded packets are available at the destination node to perform perfect decoding due to losses and/or variable link capacities. We focus on joint source-network coding/decoding as one possible solution to address this issue. In particular, we aim at investigating the effect of exploiting source redundancy in order to estimate the missing packets. This redundancy can be either, artificial, *i.e,.* introduced by some external correlating techniques, or natural, *i.e*, already existing.

Regarding *artificial* redundancy, multiple description coding is used to introduce structured redundancy among source samples, while aiming to achieve a progressive improvement of the decoding quality with the number of received descriptions. Regarding *natural* redundancy, we consider the case of spatially correlated measurements collected in a wireless sensor network. In both cases, random linear network coding is performed at the intermediate nodes of the network to provide a better use of the network resources.

Three main contributions are presented in the thesis. First, we propose joint NC-MDC schemes and we evaluate the ability of two proposed decoding approaches to provide an approximate estimation of the source, even in the absence of a sufficient number of linearly independent packets at the decoder. Second, we study the effect of the variations of the channel conditions in the performance of the proposed schemes when applied in a wireless multicast scenario. Finally, we consider the case of a wireless sensor network, and we propose an estimator that exploits the spatial correlation that exists among the original captured measures in order to provide an approximate reconstruction of the source at some collection point. Comparisions with classical schemes are provided in each case.

5.1 Joint NC-MDC schemes

The first contribution consists in introducing a new joint source-network scheme that allows to cope with lossy networks. For that purpose, we proposed two schemes combining multiple description coding and network coding. Network coding permits a better use of network ressources. Multiple description coding allows to introduce structured redundancy that can be exploited at the decoder. An approximate reconstruction of the source is then possible even when not enough network-coded packets are available at the receiver side.

Multiple descriptions have been obtained either using a frame expansion (NC-MDC-F) or using a correlating transform (NC-MDC-T). In the first case, the reconstruction is performed via the solution of a mixed integer quadratic problem. In the second case, a reconstruction algorithm derived from [24], and involving a simple Gaussian elimination, is employed.

The performance of both proposed schemes is evaluated, first, as a function of the rank deficiency of the coding matrix A, *i.e.*, the nomber of linearly independent packets that the decoder still needs to be able to perform decoding, and second, as a function of the size of the Galois field in which the coding operations take place.

In both cases, a good robustness to missing NC packets has been observed. When the number of lost packets is small, the NC-MDC-F provides better SNRs thanks to a reduction of a part of the quantization noise. The price to be paid is a decreased robustness to losses. When the number of lost packets increases, a reconstruction is still possible for some packets, even if the number of losses is larger than the number of redundant packets.

When combining packets containing only samples from the same description, classical least-squares reconstruction techniques may be employed when some descriptions are missing. This allows to get smooth performance degradation with increased number of lost descriptions. This property is somewhat lost with NC-MDC-F. An optimization of the way NC is performed has to be found to get smoother performance degradation.

5.2 Performance evaluation of NC-MDC in a multicasting scheme

The second contribution consists in evaluating the efficiency of the proposed joint NC-MDC schemes, in a scenario where a single source S multicasts data to a set of mobile terminals, experiencing different channel conditions.

The channels are modeled as binary symmetric channels, with transition probability ε distributed according to some probability density function $f(\varepsilon)$. The average SNR observed by a given receiver depends on the number of linearly independent packets among the ones correctly received at the decoder. Thus, the average SNR depends highly on $f(\varepsilon)$.

We evaluate the performance of both NC-MDC-F and NC-MDC-T as a function of the channel transition probability ε first, and then for various distributions $f(\varepsilon)$. When a user is likely to have good channel conditions, NC-MDC-F provides the best results. When his channel conditions may be quite bad, then plain NC-SDC becomes better.

5.3 Exploiting redundancy in wireless sensor networks

The third contribution addresses the problem of efficient data collection in a wireless sensor network. Network coding is used to fully exploit the network resources. The correlation between data measured by neighboring sensors is exploited to reduce the amount of network-coded packets that have to be collected at the receiver side in order to perform decoding.

We considered a scenario where sensors measure spatially correlated data and transmit them to some data processing sink. A very simple transmission protocol is considered, in which each sensor is allowed to transmit only in its own time slot, and where linear combinations are done such that each sensor combines its own measure with the measures already received from the neighboring nodes and then sends the coded measure to its neighbors.

A MAP estimator is considered at the sink to exploit the spatial correlation between data samples and provide a reconstruction of the original transmitted measures. The SNR is then evaluated as a function of the rank of the coding matrix available at the sink for different sizes of the Galois field in which the coding operations take place.

We observe that the reconstruction quality gracefully increases with the number of received packets. For comparable probability of reconstruction error or SNR, less packets are required using the proposed estimator than with a conventional Gaussian-elimination based estimator.

5.4 Future work perspectives

Several improvements can be considered in each one of the three contributions presented in this thesis.

5.4.1 Generalization of the NC-MDC schemes

One interesting aspect is to generalize the two proposed joint NC-MDC schemes presented in Chapter 3 to extended Galois fields. Coding operations are performed in $\mathbb{F}_q^r$ instead of $\mathbb{F}_q$. The key to solving this problem is to realize that elements belonging to $\mathbb{F}_q^r$ can be seen as polynomials over $\mathbb{F}_q[D]$.

The problem formulation is already done, and presented in Appendix A. Future work includes performing simulations in order evaluate the performance of these schemes over extended Galois fields.

5.4.2 More realistic MDC schemes evaluation

In the second part of Chapter 3, we studied the effect of the channel variations on the performance of the two proposed NC-MDC techniques. We considered a scheme that suppose that the same amount of noisy packets is available at each receiver side.

In future work, a distribution of the number of noisy packets obtained by the various receivers may be introduced to have a more realistic description of packet reception in wireless ad-hoc networks. One may also evaluate the distribution of the channel SNR as a function of the user location when considering a cellular network. The effect of relays can also be better taken into account.

5.4.3 Reconstruction in a wireless sensor network

Two aspects can be considered within chapter 4 as short term perspectives.

5.4.3.1 Experiments withing larger sensor networks

One of the limiting aspects of this chapter is the fact that we considered relatively small sensor networks (10 sensors), with a fixed range between the sensors.

Future work would include conducting further experiments with more network nodes and various communication range between sensors.

5.4.3.2 Exploiting redundancy using a classical MAP estimator

In chapter 4, we proposed a MAP estimator that allows us to reconstruct the transmitted source samples via the solution of an integer quadratic problem. An alternative method that would be interesting to consider, is to evaluate the reconstruction quality obtained while estimating the source using a classical MAP estimator.

The problem formulation as long as the expression of the MAP estimator are already done and presented in Appendix B. Future work includes performing simulations in order to compare the performance of such scheme with the one proposed in this chapter.

5.4.4 Network coding in cooperative relay networks

Beyond the aspects of this thesis, a second situation of interest for ad-hoc networks and cooperative transmission is the relaying situation, in which some users may also help in the transmission of another communication.

This problem has many facets, among which the cooperation strategy at the physical layer is currently of great interest. This proposal has a strong connection with the so-called decode and forward (DF) strategy. When using DF, a relay first decodes the incoming signal, and then retransmits it to the receiver. Classical studies on this topic overlooked the interaction of the strategy with channel coding. However, the relay could also use some kind of network coding strategy to retransmit other codewords than the one he received. This proposal will thus consider the hidden, yet deep connection existing between network coding, fountain codes, and relaying in wireless communications, as all of these share the same goal and almost the same tool.

This approach could be considered as an interesting extension of the work already done in this thesis.

Appendix A

Reconstruction in $\mathbb{F}_q^r$

Consider again the following scenario

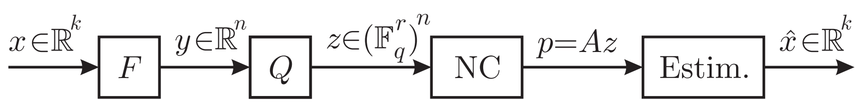

Figure A.1: Block diagram of the proposed system

We aim to generalize the decoding approach NC-MDC-F presented in Chapter 3 to extended Galois fields $\mathbb{F}_q^r$. Since the received packets have been network-coded, the relation between $\mathbf{p} = (p_1, ..., p_m)^{\mathrm{T}}$ and the quantized indexes $\mathbf{z} = (z_1, ..., z_n)^{\mathrm{T}}$ can still be expressed as

$$\mathbf{p} = A\mathbf{z} \tag{A.1}$$

or equivalently

$$p_j = \sum_{k=1}^{n} a_{j,k} z_k \tag{A.2}$$

with $p_j \in \mathbb{F}_q^r$, $a_{j,k} \in \mathbb{F}_q^r$, $z_k \in \mathbb{F}_q^r$, $1 \leq k \leq n$ and $1 \leq j \leq m$.

Equation (A.1) holds in $\mathbb{F}_q^r$. To express it in $\mathbb{Z}$, one uses the fact that elements of $\mathbb{F}_q^r$ may be viewed as polynomials of $\mathbb{F}_q[D]$. Let $A(D)$ be the network coding matrix of polynomials, each coefficient $a_{j,k} \in \mathbb{F}_q^r$ of A in (A.1) being represented by a polynomial $a_{j,k}(D)$. Since $\mathbb{F}_q^r$ is isomorphic to $\mathbb{F}_q[D]/g(D)\mathbb{F}_q[D]$, where $g(D)$ is a generator polynomial of $\mathbb{F}_q^r$, (A.1) can be expressed in $\mathbb{F}_q[D]$ as

$$p(D) = A(D)z(D) + s(D)g(D), \tag{A.3}$$

where $p(D) = (\pi_1(D), ..., \pi_m(D))^T$, $z(D) = (z_1(D), ..., z_n(D))^T$. Here, $s(D) = (s_1(D), ..., s_m(D))^T$ is a vector of slack polynomials. All operations in (A.3) are now in $\mathbb{F}_q$.

To express (A.3) in $\mathbb{Z}[D]$, an additional vector of slack polynomials $\lambda(D) = (\lambda_1(D), ..., \lambda_m(D))^T$ needs to be introduced. Then, (A.3) can be expressed as

$$p(D) = A(D)z(D) + s(D)g(D) + \lambda(D) \tag{A.4}$$

where all operations are in $\mathbb{Z}$ and with $\lambda_{j,i} \in \mathbb{Z}$, $0 \leq s_{j,i} \leq q-1$, $0 \leq z_{k,i} \leq q-1$, $0 \leq a_{j,k,i} \leq q-1$, $0 \leq g_i \leq q-1$, $0 \leq p_{j,i} \leq q-1$. The subscript i, for example in $s_{j,i}$, indicates the coefficient of degree i of the polynomial $s_j(D)$. Consequently, (A.4) can be expressed as

$$p_j(D) = \sum_{k=1}^{n} a_{j,k}(D)z_k(D) + s_j(D)g(D) + \sum \lambda_{j,i}D^i, \tag{A.5}$$

with $1 \leq j \leq m$.

Consider the term of degree ℓ in (A.5)

$$p_{j,\ell} = \sum_{k=1}^{n}\sum_{i=0}^{\ell} a_{j,k,i}z_{k,\ell-i} + \sum_{i=0}^{\ell} s_{j,i}g_{\ell-i} + \lambda_{j,\ell} \tag{A.6}$$

In particular, when $\ell > \deg p_j(D)$, one gets

$$0 = \sum_{k=1}^{n}\sum_{i=0}^{\ell} a_{j,k,i}z_{k,\ell-i} + \sum_{i=0}^{\ell} s_{j,i}g_{\ell-i} + \lambda_{j,\ell}. \tag{A.7}$$

An estimate $\widehat{\mathbf{x}}$ of the source vector $\mathbf{x}$ based on the received network-coded packets $\mathbf{p}$ and using the the fact that $\mathbf{x}$ has been expanded into $\mathbf{z}$ before quantization and transmission may be obtained by minimizing $\mathbf{x}^{\mathbf{T}}\mathbf{x}$ under the following set of constraints

$$\begin{cases} y_u = \sum_{v=1}^{k} f_{u,v} x_v, & u = 1 \dots n \\ y_u - (\alpha z_u + \beta) \leq \Delta/2, & u = 1 \dots n \\ -y_u + (\alpha z_u + \beta) \leq \Delta/2, & u = 1 \dots n \\ p_{j,\ell} = \sum_{k=1}^{n} \sum_{i=0}^{\ell} a_{j,k,i} z_{k,\ell-i} + \sum_{i=0}^{\ell} s_{j,i} g_{\ell-i} + \lambda_{j,\ell}, & \\ \qquad j = 1 \dots m,\ \ell = 1, ..., 2(r-1) & \\ 0 \leq z_u \leq q-1 & u = 1 \dots n \\ 0 \leq s_{j,i} \leq q-1 & \\ (q-1)[1 - \ell(q-1)(n+1)] \leq \lambda_{j,\ell} \leq 0 & \end{cases} \tag{A.8}$$

ℓ is the index of the coefficient of $p_j(D)$ to estimate, with $0 \leq \ell \leq 2(r-1)$. In fact $\deg(s_j(D)G(D)) = \deg(A(D)z_j(D)) = 2(r-1)$, where $\deg()$ is the degree of a polynomial. Since $\deg g(D) = r$, then $\deg s_j(D) \leq r-2$. In the worst case, $(r-1)m$ slack variables $s_{j,i}$ need to be introduced, in addition to $(2r-1)m$ slack variables $\lambda_{j,\ell}$. The total number of slack variables to introduce is then equal to $(r-1)m + (2r-1)m = (3r-2)m$, whereas in the particular case of $r = 1$ only m slack variables needed to be introduced.

Appendix B

Classical MAP Estimation in a WSN

B.0.5 Problem Formulation

Consider again the transmission scheme introduced in Section 4.1

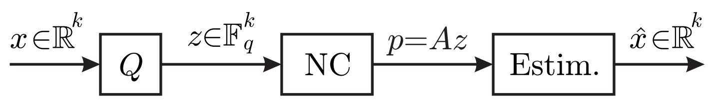

Figure B.1: Block diagram of the proposed system

The measurement vector $\mathbf{x} \in \mathbb{R}^k$ is modeled as $\mathbf{x} \sim \mathcal{N}(0, \Sigma_{\mathbf{x}})$, where $\Sigma_{\mathbf{x}}$ is the non-diagonal covariance matrix. After quantization, the transmitted samples $z_i \in \mathbb{F}_q$ are network-coded, and m linearly independent packets are collected at the sink. We consider the case where not enough network-coded packets are available at the decoder, i.e, $m < k$. Again, we aim to provide an estimate $\widehat{\mathbf{z}}$ of the vector of quantized samples $\mathbf{z}$. Let $\widetilde{\mathbf{z}}$ be the vector of the known uncoded quantized samples available at the decoder. $\widetilde{\mathbf{z}}$ can be expressed as

$$\widetilde{\mathbf{z}} = \phi \mathbf{z} \tag{B.1}$$

where $\mathbf{z}$ of dimension $k \times 1$ is the vector of quantized samples, and ϕ of dimension $\ell \times k$ is the matrix indicating the index of the received coding coefficients.

An estimate of the entries of $\mathbf{x}$ corresponding to $\widetilde{\mathbf{z}}$ can then be expressed as

$$\widetilde{\mathbf{x}} = \phi \mathrm{Q}^{-1}(\widetilde{\mathbf{z}}) \tag{B.2}$$

$$\widetilde{\mathbf{x}} = \phi \mathbf{x} + \mathbf{b} \tag{B.3}$$

where Q is a scalar uniform quantizer. $\mathbf{b} \sim \mathcal{N}(0, \Delta^2 \mathtt{I}/12)$ with $\mathtt{I}$ the identity matrix and Δ the quantization step size.

Our goal is to provide an estimate $\widehat{\mathbf{z}}$ based on the knowledge of $\widetilde{\mathbf{z}}$.

B.0.6 Expression of the MAP Estimator of x

The MAP estimator of $\mathbf{x}$ can be expressed as

$$\widehat{\mathbf{x}} = \arg\max \mathrm{P}(\mathbf{x} \mid \widetilde{\mathbf{x}})$$

$$= \arg\max_{\mathbf{x}} \mathrm{P}(\widetilde{\mathbf{x}} \mid \mathbf{x}) \mathrm{P}(\mathbf{x})$$

$$= \arg\min_{\mathbf{x}} [-\log \mathrm{P}(\widetilde{\mathbf{x}} \mid \mathbf{x}) - \log \mathrm{P}(\mathbf{x})]$$

$$= \arg\min_{\mathbf{x}} [(\widetilde{\mathbf{x}} - \phi\mathbf{x})^{\mathrm{T}} 12/\Delta^2 (\widetilde{\mathbf{x}} - \phi\mathbf{x}) + \mathbf{x}^{\mathrm{T}} \mathbf{\Sigma^{-1}} \mathbf{x}]$$

$$= \arg\min_{\mathbf{x}} [12/\Delta^2 (\widetilde{\mathbf{x}}^{\mathrm{T}} \widetilde{\mathbf{x}} - \widetilde{\mathbf{x}}^{\mathrm{T}} \phi \mathbf{x} - \mathbf{x}^{\mathrm{T}} \phi^{\mathrm{T}} \widetilde{\mathbf{x}} + \mathbf{x}^{\mathrm{T}} \phi^{\mathrm{T}} \phi \mathbf{x}) + \mathbf{x}^{\mathrm{T}} \mathbf{\Sigma^{-1}} \mathbf{x}] \tag{B.4}$$

Deriving B.4 over $\mathbf{x}$ we get

$$12/\Delta^2 (-\phi^{\mathrm{T}} \widetilde{\mathbf{x}} - \phi^{\mathrm{T}} \widetilde{\mathbf{x}} + 2\phi^{\mathrm{T}} \phi \mathbf{x}) + 2\Sigma^{-1} \mathbf{x} = 0 \tag{B.5}$$

$$(12/\Delta^2 \, \phi^{\mathrm{T}} \phi + \Sigma^{-1}) \, \mathbf{x} = 12/\Delta^2 \, \phi^{\mathrm{T}} \widetilde{\mathbf{x}} \tag{B.6}$$

$$\mathbf{x} = 12/\Delta^2 \, (12/\Delta^2 \, \phi^{\mathrm{T}} \phi + \Sigma^{-1})^{-1} \phi^{\mathrm{T}} \widetilde{\mathbf{x}}. \tag{B.7}$$

Appendix C

Proof of (3.20)

Theorem 3 *Consider* $\mathbf{H} \in (\mathbb{F}_q)^{(n-k)\times n}$ *of full row rank* $n-k$*. Consider a random network matrix* $\mathbf{A} \in (\mathbb{F}_q)^{m\times n}$, *where the entries are realizations of iid uniform random variables over* $\mathbb{F}_q$*. The probability that the rank of* $\begin{pmatrix} \mathbf{A} \\ \mathbf{H} \end{pmatrix}$ *is* n *is*

$$\Pr\left(rank \begin{pmatrix} \mathbf{A} \\ \mathbf{H} \end{pmatrix} = n \right) = \begin{cases} 0 \text{ if } m < k, \\ \prod_{i=1}^{k} \left(1 - q^{-i}\right) \text{ if } m = k, \\ \sum_{\substack{i_1,\ldots,i_m \\ i_1 < i_2 < \ldots < i_{m-k}}}^{m} q^{-k+i_1-1} \ldots q^{-k+i_{m-k}-(m-k)} \prod_{i=1}^{k} \left(1 - q^{-i}\right) \text{ if } m > k. \end{cases}$$

To prove Theorem 3, consider the following lemma.

Lemma 1 *Consider* $\mathbf{B} \in (\mathbb{F}_q)^{m\times n}$ *a matrix of rank* r*. Consider a row vector* $\mathbf{a} \in (\mathbb{F}_q)^{n}$ *which entries are realizations of iid uniform random variables over* $\mathbb{F}_q$*. Then*

$$\Pr\left(rank \begin{pmatrix} \mathbf{a} \\ \mathbf{B} \end{pmatrix} = \nu \right) = q^{r-n} \tag{C.1}$$

and thus

$$\Pr\left(rank \begin{pmatrix} \mathbf{a} \\ \mathbf{B} \end{pmatrix} = \nu + 1 \right) = 1 - q^{r-n}. \tag{C.2}$$

If $\mathbf{B}$ is of row rank r, then there exist r linearly independent rows of $\mathbf{B}$ denoted as $\mathbf{b}_1, \ldots, \mathbf{b}_r$. When the rank of $\begin{pmatrix} \mathbf{a} \\ \mathbf{B} \end{pmatrix}$ is still r, $\mathbf{a}$ is a linear combination of $\mathbf{b}_1, \ldots, \mathbf{b}_r$. There are q^r such combinations in $(\mathbb{F}_q)^n$ among q^n possible vectors. Thus $(C.1)$ is proved and $(C.2)$ is immediately deduced.

Now, one may prove Theorem 3. The case $m < k$ is trivial, since the number of rows of $\begin{pmatrix} \mathbf{A} \\ \mathbf{H} \end{pmatrix}$ is strictly less than n. When $m = k$, since the rank of $\mathbf{H}$ is $n-k$ and $\mathbf{A}$ has k rows, each row of $\mathbf{A}$ supplementing $\mathbf{H}$ should increase the rank, *i.e.*, be linearly independent of the rows of $\mathbf{H}$ and of the rows of $\mathbf{A}$ already added to $\mathbf{H}$. The result is thus proved by induction on the rows of $\mathbf{A}$ applying $(C.2)$. For the case $m > k$, we assume that the m rows of $\mathbf{A}$ are added one after the other. The i-th row $\mathbf{a}_i$ of $\mathbf{A}$ stacked over

$$\mathbf{B}_{i-1} = \begin{pmatrix} \mathbf{a}_{i-1} \\ \vdots \\ \mathbf{a}_1 \\ \mathbf{H} \end{pmatrix}$$

may increase the rank of $\mathbf{B}_{i-1}$, or may not increase its rank. In total, $m-k$ rows of $\mathbf{A}$ will not increase the rank. Let $i_1, \ldots, i_{m-k}$ the indexes of the rows of $\mathbf{A}$ which do not increase the rank. Assume that $i_1 < \cdots < i_{n-k}$. Using Lemma 1, one may thus show by induction that

$$\begin{aligned} \Pr\left(rank\left(\mathbf{B}_{i_1-1}\right) = n-k+i_1-1\right) &= \left(1-q^{-k}\right)\left(1-q^{-k+1}\right)\ldots\left(1-q^{-k+i_1-2}\right) \\ &\prod_{i=k-i_1+2}^{k}\left(1-q^{-i}\right) \end{aligned}$$

and that

$$\Pr\left(rank\left(\mathbf{B}_{i_1}\right) = n-k+i_1-1 | rank\left(\mathbf{B}_{i_1-1}\right) = n-k+i_1-1\right) = q^{-k+i_1-1}$$

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