Piezoelectric power transducers and its interfacing circuitry on energy harvesting and structural damping applications
Yu-Yin Chen

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THESE DE DOCTORAT
DE L'ECOLE NORMALE SUPERIEURE DE CACHAN

Présentée par

Monsieur Yu-Yin Chen

pour obtenir le grade de

DOCTEUR DE L'ECOLE NORMALE SUPERIEURE DE CACHAN

Domaine :

ELECTRONIQUE –ELECTROTECHNIQUE-AUTOMATIQUE

Sujet de la thèse :

Piezoelectric power transducers and its' interfacing circuitry on energy harvesting and structural damping applications

Thèse présentée et soutenue à Taipei le 28 janvier 2013 devant le jury composé de :

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Piezoelectric power transducers and its' interfacing circuitry on energy harvesting and structural damping applications

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January, 2013
中文摘要

現在，能源成為了相當重要的議題，從環境當中獲取能源更是受到高度重視。此論文主要圍繞在透過各種設計來改進壓電能量擷取裝置，希望可與低耗電裝置與無線監測網路結合，來延長裝置電池壽命與直接提供能量為最終目標。機械結構具有高品質因子，當壓電能量擷取裝置操作在遠離共振頻時，輸出功率會快速下降，本論文提出可調變共振頻率的技術，成功的將共振頻率的頻寬延展，獲取更多的能量，此技術也成功的與無線監測網路結合，可在累積足夠能量後將量測資料無線傳出。為了將可用頻寬延展，本論文提出結合非線性雙穩態懸臂樑結構與切換式介面電路的架構，透過永久磁鐵的設計，使懸臂樑成為非線性系統，成功提升在非共振頻時的輸出功率，透過零速度偵測的技術，使切換式電路成功的使用在非線性振動系統當中，由工作週期的討論顯示出兩種技術結合的成果。在低耦合系統中，同步切換為相當成功之介面電路，不同於以往峰值偵測的方式，本論文提出零速度偵測與三片壓電片分離的架構，成功完成自供電同步切換壓電能量擷取系統。當系統為非低耦合時，同步切換技術可應用在系統減震上，最大優點為犧牲少部分減震能力完成自供電減震系統，自供電減震系統的限制與成果透過理論分析、時域與頻率域的結果被成功驗證，整體系統如同回授控制般，當結構振動高於限制時，自供電減震系統將會啟動，並成功的抑制結構振動。

關鍵字：壓電能量擷取，自供電，零切換偵測，同步切換，結構減震
Abstract

Nowadays with the world oil price soaring, the energy issue is becoming a significant topic and the possibility of harvesting ambient energy receiving much attention. In this dissertation, the main topic surrounds improving the piezoelectric energy harvesting device in several aspects and the final objective is to integrate it with low power consumption device, for example a wireless sensor network node to extend the battery lifetime and further supply the energy to device directly. Based on the high mechanical quality factor of the structure, the output power of the piezoelectric energy harvesting device will decrease rapidly when the exciting frequency is out of the resonant frequency range. The tunable resonant frequency technique is proposed to broaden the resonant frequency range and to increase the output power effectively. Then this technique is successfully combined with a wireless sensor module to transmit the radio frequency signal. To broaden resonant frequency another method is proposed, based on a bistable vibrating cantilever beam and a switching-type interfacing circuit. It is a new and interesting concept to combine these two techniques. The magnets are used to make mechanical behavior non-linear and increase the output power at non-resonance. The synchronized switching technique through zero-velocity detection can work well when system is driven in non-linear system. The experimental and simulation results through work-cycles discussion show good performance of combining these two techniques. Synchronized switching harvesting on an inductor have been verified to be a
successful technique to increase output power in low-coupling system. In order to make use of the synchronized switching technique in the real application, the velocity control self-powered system is proposed. Unlike the conventional peak detector technique, the zero-velocity detection is used to make the switching time more accurate. The energy flow is separated into three paths to construct the above-mentioned velocity control self-powered synchronized switching system and the experimental results show good performance.

When the system is not low-coupled, the synchronized switching harvesting on an inductor technique will damp vibration. This technique is synchronized switching damping on an inductor. Based on the self-powered technique and zero-velocity detection used in energy harvesting, these techniques are further applied in structural damping to construct a self-powered synchronized switching damping system. The major advantage is that it is only necessary to sacrifice a small amount of damping performance to make the system fully self-powered. The theoretical analysis and experimental results of time domain comparison and frequency response testing show the limit and performance of this technique. The self-powered damping system is like a feedback loop system and when the displacement is over the limit the system will effectively damp the vibration.

**Keywords:** piezoelectric energy harvesting, self-powered, zero-velocity detection, synchronized switching, structural damping.
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Chapter 1. Introduction

1.1 Backgrounds and Motivations

In recent decades, world oil price soars and the energy issue becomes the most important issue all over the world. Many researches focus on finding alternative energy source. The alternative energy source includes large scale power sources such as solar and wind energy [1-8] used to replace the conventional energy source and small scale power sources such as energy harvested from vibration, acoustic noise [9] and temperature gradient [10] used to extend the battery lifetime of electronic devices. Methods adopted to make portable devices or sensors retrieve energy from the environment are so called “Power harvesting” or “Energy harvesting.” In recent years, the size of portable devices such as mobile phone, mp3 players, flashlight and sensor nodes become smaller and the latest semiconductor fabrication technologies advancement significant lowered the power consumption of portable devices [11, 12]. It makes more sense to harvest energy from ambient directly and to power the portable devices or to elongate the battery lifetime which the energy harvested is limited.

Thermal gradient [1, 2], solar [3, 4], wind [5], humans activities [6-8], barometric fluctuations, ocean wave [13], etc. are good ambient energy sources to be harvested and active materials are required to convert the ambient energy sources into useful electrical
Most common materials used to retrieve the ambient energy are photovoltaic materials, piezoelectric material and electromagnetic materials. Photovoltaic cells can convert ambient light energy [14] such as sunlight to electrical energy. To get enough intensity of solar light, the photovoltaic cells are typically located at the place with direct sunlight such as roof, windows of buildings, roadway signs, sailboats, and other marine locations. Energy from mechanical vibration in some situations may also be taken effectively by using two kinds of mechanisms. They are 1) piezoelectricity that converts mechanical vibrations to electric energies [11, 15-17] and 2) electromagnetism that generates electricity by moving coils crossing magnetic fields [18-20].

The efforts of most researches are working on harvesting energy from ambient mechanical vibrations with piezoelectric materials due to its high energy density per unit volume, high electromechanical coupling, and no external power source required. Researchers also tried to combine power harvesting devices with emerging wireless sensor network applications [21, 22, 26]. Roundy [22] presented that the energy density of the piezoelectric material based energy harvester is around 35.4 (mJ/cm$^3$) and is higher than electromagnetic material (24.8 mJ/cm$^3$) and electrostatic (4 mJ/cm$^3$) based energy harvesters. Comparing with different energy sources, the power density of the piezoelectric material is around 250 (µW/cm$^3$), which is higher than other materials.
when excited by external vibrations [26]. In 2001, Paradiso and Feldmeier [27] designed a self-powered wireless RF identification device. A digital ID code will be transmitted when a button on the device is pushed.

Since 2002, papers published with energy harvesting as keywords increased significantly. Figure 1-1 shows the WTI (West Texas Intermediate) crude oil price (US dollars per barrel) from 1997 to 2012 and the crude oil price soars around from 2000 [30]. Figure 1-2 shows the paper record count of energy harvesting including all kinds of energy source and using different materials. Figure 1-3 shows only the paper record count of piezoelectric energy harvesting. The data of the paper record count are obtained from the SCIE database by using keywords "energy harvesting" and "piezoelectric energy harvesting". The paper record count in Figure 1-2 and Figure 1-3 compared with Figure 1-1, the time of the crude oil pricing soaring agrees quite well with the time of energy harvesting paper count growing vigorously. Before 2000, the number of papers published with energy harvesting keywords per year keep almost the same number. After 2000, the papers published with energy harvesting keywords have boosted for almost 5 times. In 2000, there was only one energy harvesting paper using piezoelectric material published. However, in 2011, there are total of 253 papers published. As shown in Figure 1-3, the number of papers published using piezoelectric
material in energy harvesting significant boosted over the past 10 years. From Figure 1-4, we can see papers published using piezoelectric materials is around 8.74% of the total energy harvesting papers.

With the growing interests of on using piezoelectric materials in energy harvesting researches, piezoelectric materials are chosen to be the active material of energy harvesters to transfer the ambient vibration energy into electrical energy in this dissertation. Several techniques to boost the output power and useful bandwidth will be presented in this dissertation.

![Figure 1-1. WTI crude oil price chart](image1.png)

![Figure 1-2. All energy harvesting paper statistics](image2.png)
1.2 Literature review

Typical piezoelectric energy harvesting devices can be divided into two parts as shown in Figure 1-5. The first part is the mechanical part composed of piezoelectric
material and host structures and the second part is the electrical part composed of the interfacing circuit and loading stage with electrical load or storage devices. The piezoelectric material and host structures determine the efficiency of electrical energy transformed from the vibration mechanical energy. The interfacing circuit and the loading stage with electrical loads or storage devices determine the efficiency of the energy transformed from the piezoelectric material into storage components such as capacitors and charging batteries. According to the different loading stages used, choosing the proper interfacing circuit can effectively increase the efficiency. The simplest loading stage is composed of a regular capacitor and a pure resistive load, which can be equivalent to charging a Ni-Cd (Nickal-Cadmium) battery. Based on the different parts design of piezoelectric energy harvesting devices and applications, the literatures review is divided into several sub-sections followed.

Figure 1-5. Schematic of the typical piezoelectric energy harvesting device.
1.2.1 Mechanical part: Design of the piezoelectric material and host structures

Many researchers made efforts in developing energy harvesting devices from vibrations using cantilever beam based energy harvesters due to its simplicity and high efficient on generating large strain and power output [30-38]. Tang et al. [28] and Khaligh et al. [29] demonstrated the state of the art vibration piezoelectric energy harvesting setup based on simple cantilever beam design. Typical cantilever beam structure is shown in Figure 1-6. Instead of using the conventional one-layer piezoelectric cantilever beam type energy harvester, Roundy and Wriht in 2004 [22] developed, validated, and optimized the basic analytical model for a two-layers bending element type (bimorph type) piezoelectric vibration based energy harvester as shown in Figure 1-7. They also designed a power generation circuit to demonstrate the application worked with wireless sensor networks.

![Figure 1-6. (a) Typical cantilever beam (b) Cantilever beam deflection at first mode](image)

Figure 1-6. (a) Typical cantilever beam (b) Cantilever beam deflection at first mode
Instead of using the conventional 1-D cantilever beam design [40, 41], Ericka et al. [42] presented a 2-D piezoelectric membrane based device to harvest energy from pulsing vibration sources and established the 2-D piezoelectric plate model as shown in Figure 1-8. The PZT (lead zirconate titanate) plate was bounded onto the aluminum plate to form a 2-D circular piezoelectric energy harvesting device. From their results, the energy harvested can be enhanced by patterned polarization of the piezoelectric material.
Goldfarb, Jones [43] and Umeda et al. [30, 44] reported the key parameters of the piezoelectric materials which affect the power efficiency of the power harvesting devices. The results showed that high mechanical quality factor ($Q_m$), high electromechanical coupling coefficient ($k^2$), and low dielectric loss ($\tan \delta$) will increase the efficiency of the piezoelectric energy harvester. Richards et al. [45] developed an exact formula to predict the power conversion efficiency of the piezoelectric energy harvester and established the relation between the electromechanical coupling coefficients, quality factor and power generation efficiency for piezoelectric oscillators. From their results, the magnitudes of $Q$ and $k^2$ are coupled together; it cannot be optimized and designed separately. The energy conversion efficiency is basically a trade-off between $Q$ and $k^2$.

The power output from energy harvesters is not a normalized quantity, and thus it is hard to fairly compare the performance of different power harvesting devices. Roundy [33] in 2005 provided a general theory that could be applied to compare different power harvesting vibration-based generators and he presented a general theory that could also be applied to electromagnetic, piezoelectric, magnetostrictive, and electrostatic transducer technologies. In addition to the input parameters of the vibrations, the general form "effectiveness" is composed of system coupling coefficient, the quality
factor of the device, the mass density of the generator and the degree to which the
electrical load maximizes power transmission.

1.2.2 Electrical part: Design of the interfacing circuit and loading stage

Besides the material and host structure design issues of the energy harvesting
devices, there were also lots of researches focusing on electrical interface to improve the
efficiency and power output of the energy harvesting devices. Lesieutre et al. [46] indicated that when the piezoelectric patches connect to the electrical load through a
bridge rectifier, the electrical load would absorb the energy from the piezoelectric patch
and it would increase the damping of the vibration structure. The electric signal
generated from a piezoelectric energy harvesting device is in alternating current (AC)
form a crossing its electrodes, and thus the simplest interfacing circuit would be a
standard interface with simple rectifier circuit (full bridge diode rectifier). The the
efficiency of the standard interface is of course not an optimal design for maximum
efficiency and power output [36], and thus there are lots of studies devoted the efforts to
design and to optimize the interfacing circuits improving the efficiency and maximizing
the output power. Ottman et al. [21, 31] tried to adopt the concept of impedance
matching and using adaptive control technique to design an interfacing circuit
composed of an AC-DC rectifier and a DC-DC step-down converter. The interfacing circuit increased the harvested power by around 325%. According to Ottman's results, maximum energy harvesting can be obtained accurately by determined the optimal duty of the dc-dc converter.

Figure 1-9. Experimental circuit setup of energy harvesting device with a DC-DC converter [21].

Lefeuvre et al. [34] proposed a switching circuit with switches operated synchronously with the vibration of the host structure and showed significant boosting on the output power of the energy harvesting devices. Several synchronized switching circuit topologies and corresponding switching control mechanism were then proposed and studied. The switching techniques can be classified into two groups according to the placement between the full-wave bridge rectifier and the switches. The first group of the switching circuits places the switches before the full-wave bridge rectifier, such as parallel-SSHII (Synchronized Switching Harvesting on an Inductor) shown in Figure
1-10(a) and series-SSHI [34, 35] shown in Figure 1-10(b); the second group places the switches after the full-wave bridge rectifier, such as SECE technique shown in Figure 1-10(c) [34]. The optimal load of the series and parallel SSHI techniques are different. [34-38, 48-50]. The optimal load value of parallel-SSHI (around mega ohms) is higher than that of series-SSHI (around hundred ohms) [36]. In SECE technique, the power output is not as high as the SSHI technique, but the power output is independent of the load. In these techniques, the switching circuit only turns “ON” at the extreme value of the displacement or at the zero crossing of velocity to shift the phase of the voltage across the piezoelectric element. These techniques are used because the piezoelectric-generator is weakly coupled to the host structure, i.e. only a small amount of mechanical energy is taken from the structure and converted in electricity. The electrical behavior of the piezoelectric-generator with the SSHI circuit is equivalent to an operation under strong coupling conditions by increasing the output voltage [51].
1.2.3 Self-powered energy harvesting system

Proper interfacing circuit can effectively increase the harvested energy but most of early researches of the interfacing circuits using external power sources to build up the system and test the effectiveness of the interfacing circuit. Therefore, after the effectiveness of the interfacing circuits being proved, the next stage is to have the interfacing circuit being self-powered, which means the interfacing circuit is powered by the energy harvester itself. In 2001, Shenck and Paradiso [52] designed an integrated circuit to harvest the primary signal peak with minimal loss and also with the bootstrapped “cold” start-up feature. Elvin et.al. [53, 54] and Ng and Liao [39] reported some new applications and innovative design ideas. They use piezoelectric materials not...
only being a power harvesting device but also a sensor. The self-powered sensor system with a power harvesting device and a RF transmitter is shown in Figure 1-11 and the voltage on the charging capacitor is shown in Figure 1-12 [53]. When the voltage on the capacitor is over 1.1V, the electronic switch (s) are switched on and deliver power to the RF transmitter to send out the sensor data.

![Figure 1-11. Sensor system with a power harvester and an RF transmitter [53].](image1)

![Figure 1-12. The terminal voltage of the storage capacitor being charge by the energy harvesting device [53].](image2)
The self-powered system proposed by Elvin in 2001 [53] used a simple half-bridge rectifier to regulate the energy. In order to make synchronized switching technique more efficient, Lallart et al. in 2008 [55] proposed a self-powered SSHI switching circuit shown in Figure 1-13. They presented a basic theoretical analysis and the experiment shows the circuit can be fully self-powered and may work even the voltage output from the harvester is low. Thus, the design could be potentially integrated with MEMS generators. In 2012, Liang et al [56] proposed a modified self-powered SSHI interfacing circuit shown in Figure 1-14. This circuit is the improved version of Lallart's circuit and the entire system is accurately analyzed, including the switching time lag and inversion factor. The conclusion also shows the self-powered SSHI is better than the standard interfacing circuit only if the excitation level is high enough.

Figure 1-13. Unipolar electronic switch on maxima (a) block diagram (b) implementation [55].
1.2.4 Nonlinear energy harvesting technique

Although the piezoelectric materials exhibit high power density, the linear piezoelectric energy harvester are efficient only when the mechanical system is excited at the resonance frequency. When the harvester is excited on resonance, the maximum output power can be obtained and which is significant larger than the power output when excited at off-resonant frequencies because of high quality factor of the mechanical structure. However, in practice, the frequency of ambient vibration is not a constant and it varies within a frequency range around the resonant frequencies of the host structure [26]. It is impossible to excite the energy harvester at specific resonance frequency and to keep the system operating on the maximum power point. In order to
increase the power at off-resonant frequency, designing a mechanical system to work in a wider frequency range is important for real world applications. Leland et al. in 2006 [57] enlarged the frequency bandwidth by applying an axial force as the preload force. The resonant frequency of a piezoelectric cantilever beam is successfully tuned as Figure 1-15. These methods are active techniques and the mechanical system is still operated within the linear regime. Lin et al. in 2010 [59] made a non-linear or bistable vibration of a cantilever beam to enlarge the workable bandwidth shown in Figure 1-16. By using simple fixed magnets, this passive technique make the mechanical system improve the harvesting efficiency within off- resonant regime without any external power.

Figure 1-15. The schematic of a simply supported piezoelectric bimorph vibration energy harvester [57].
1.2.5 Piezoelectric energy harvesting device used in real application

MIT Media Laboratory studied the possibility of adopting power harvesting by embedding piezoelectric devices to insole of walking shoes, which extract electricity from the foot pressure as shown in Figure 1-17 [52, 63]. Both piezoelectric polymer such as PVDF (polyvinylidene fluoride) and piezoelectric ceramics such as PZT (lead, zirconate, titanate) were used as the active material in the energy harvesting devices. One of the main challenges lie on how to charge the battery efficiently by using the electricity retrieved. To gain maximum power output, the PVDF and PZT based devices were designed to fit both the shoe shape and the way of walking. Flexible piezoelectric devices based on PVDF were adopted in the front of the shoes as both compressed and bending forces.
A few companies are working on commercializing power harvesting devices in past years worldwide. For example, Ferro Solutions Energy Harvesters (FSEH) developed by Ferro is an independent power source that generates electricity from environment vibrations to power wireless transceivers, sensors, micro-motors and actuators. FSEH successfully demonstrated the potential to replace batteries in many applications by providing devices with a continuous, nearly endless supply of electricity.

The prototype developed by Ferro looks like a clear spool just under two inches in height and in diameter. With power output in the range of 0.4 mW when the external vibrations is on the order of 20 milli-Gs in strength, which is barely enough to be felt on the surface by using a bare hand. Stronger vibrations of 100 milli-Gs were found to generate 9.3 mW, which further demonstrate the future potential of power harvesting
technology.

Wireless sensor networks (WSN) can be used to monitor the health of the structures, environment, wild animals, tire pressure of running cars, etc. In most of the WSN applications, the device is far from the power line reach. In some situations, the device needs to be embedded into the structure to monitor. All of which makes it hard to use power line to supply energy to the sensor devices and battery becomes the major solution to supply power. However, there are lots of disadvantages associated with using batteries as the only power source for these applications. The major problem is limited battery life time, a WSN node can only be operated using 3V battery for 1~2 years and the cost to replace the battery is usually very high. For embedded applications, it is even impossible to replace the batteries. Harvesting the ambient energy closed to the sensor nodes of a WSN node is thought to be the most likely and suitable solution to extend the battery life time of WSN [21-25].

1.2.6 Piezoelectric material used in structural damping

The basic theory of using piezoelectric material in energy harvesting is the same as in structural damping. Piezoelectric material are used to transform the mechanical energy into electrical energy. For energy harvesting devices, the electrical energy is delivered to the electrical load or stored in storage devices. For the structural damping
applications, the electrical energy transformed from mechanical vibration can be simply dissipated in resistors by Joule effect, the vibration of the structure will then be significantly reduced and this is called structural passive damping [64]. Nevertheless, to maximize the dissipated energy, most techniques and interfacing circuits used in the energy harvesting can be used in the structural damping application. There are many shunt techniques based on the design of piezoelectric materials used for damping applications. The simplest one is the passive technique that uses a matched inductor and resistor network connected to the piezoelectric patch as shown in Figure 1-18 [65, 66]. However, this technique has a major disadvantage. In low frequency applications, the optimal shunt inductor is too large for feasibly implementation. In most cases, the inductance required would be approximately one hundred Henry, which can only be implemented with active circuitry. To implement a shunt inductance with active circuitry, an external power source is needed. The most effective active damping technique is to use active controllers, power amplifiers, and analog or digital processors that generate an out-of-phase signal to control the structural vibration. This technique usually provides better damping performance than passive ones [67, 68]. The advantages of the active damping techniques are good performance and a wider working frequency range, whereas the disadvantage is that the active circuits require external
power. The implementation and algorithms for active techniques can be much more complex and the cost is much higher than passive damping techniques.

Figure 1-18. Piezoelectric transducer with an RL shunt circuit [66].

Considering the trade-offs in the cost, difficulty of implementation, power consumption required and damping performance, some switching shunting or semi-passive techniques [69-71] have proved to be the most effective methods. Several switching shunt circuit topologies and corresponding switching mechanisms were proposed. The two most popular switching shunt-damping techniques are SSDI (Synchronized Switch Damping on an Inductor) as Figure 1-19(a) [72] and SSDV (Synchronized Switch Damping on a Voltage source) as Figure 1-19(b) [72-75]. Reihard et al. first proposed the SSD (Synchronized Switch Damping) technique in 1999 as shown in Figure 1-20 and this technique was further designed and enhanced to be the
SSDI and SSDV technique. SSDI technique has attracted more attention because its offers several advantages. It does not require a very large inductor for low frequencies and it’s robustness to the environmental changes. Moreover, this technique only needs little power to operate the switches and can be design to be self-powered easily. The fundamental concept of SSDI technique is the same as the SSHI technique to utilize the resonance of the piezoelectric clamped capacitance and a shunt inductance when closing the switch during a short time interval to inverse the piezoelectric voltage.

Figure 1-19. (a) SSDI electric circuit (b) SSDV electric circuit [75].

Figure 1-20. SSDS electric circuit [69].
1.3 Organization of the dissertation

There are 7 chapters in this dissertation. The framework and summary of each chapter is as follows

**Chapter 1** presents the background introduction and literature review.

**Chapter 2** presents the fundamental mechanical modal, equivalent model and the analysis of the piezoelectric cantilever beam based energy harvesting devices. Several famous and effective synchronized switching techniques are presented, analyzed, discussed, compared and summarized.

**Chapter 3** proposed a tunable resonant frequency cantilever beam type energy harvesting to increase the power output of the piezoelectric energy harvesting device. The tunable resonant frequency technique is based on the characteristic of the piezoelectric material to shift the resonant frequency of the cantilever beam and make the available bandwidth broaden. The average harvested power output increases almost 30% under chirping and random frequency from 72Hz to 76Hz (resonant frequency is 73.5Hz). From the experimental results, this tunable frequency system can be successfully combined with a wireless sensor node to transmit the RF signal.

**Chapter 4** proposed a self-powered piezoelectric energy harvesting device using the velocity control synchronized switching technique. In this chapter, the self-powered
technique and velocity sensing technique are used to make the popular and effective synchronized switching technique work without any external instruments. The experimental results show better performance and lead to a gain of around 200% compared to the standard DC approach.

Chapter 5 proposed a broad bandwidth and efficient piezoelectric energy harvesting device by using the magnetic force combined with synchronized switching technique. The magnetic force is used to broaden the available bandwidth to make piezoelectric energy harvesting device can obtain more energy on the off-resonant frequency. Combining the magnetic force with the traditional synchronized switching technique, a high efficiency and wide-band piezoelectric energy harvesting device design can be achieved. The frequency response and analysis of the work-cycle show the performance of the results of combing these two techniques to build the piezoelectric energy harvesting device.

Chapter 6 proposed a self-powered semi-passive piezoelectric structural damping technique based on zero-voltage crossing detection. The drawback of the traditional semi-passive damping technique is that the system needs external instrument. In this chapter, the self-powered technique is used to make the semi-passive technique damping work like the passive technique without any external instruments and by using
zero-voltage crossing detection to reduce the switching time lag of the synchronized switching technique. Compared to the case in which all of the piezoelectric patches are used for structural damping and driven by an external function generator and a power supply, the efficiency of the proposed self-powered damping system is approximately 86%. Compared to the ideal switching case in which the same size of piezoelectric patch is used for SSDI damping and is driven by an external function generator and power supply, the efficiency of the proposed self-powered system is approximately 95%.

The major advantage of the proposed technique is that it is only necessary to sacrifice a small amount of damping performance to make the system fully self-powered.

Chapter 7 summarized the entire dissertation and conclude with a conclusion session. The conclusion will point out the main innovation and contribution of this dissertation. Also, the future work will be presented in the end of this chapter.
Chapter 2 Review of the electric interfaces for energy harvesting and damping

This chapter presents a review of the literature about electric interface called SSHI (Synchronized Switching Harvesting on an Inductor) for the energy harvesting and structural damping applications. The basic governing equations, equivalent circuit models, waveforms, optimal loads and maxima output power are analyzed and discussed. A comparison of different interfaces trough work-cycle will be presented and discussed.

Since the efficiency of the electromechanical conversion of piezoelectric transducer depends of the electrical load, an electrical circuit must be introduced to optimize the conversion and to adapt the piezoelectric voltage to the storage device. The interfacing circuit plays a very important role to regulate the alternating current into direct current and decides the efficiency of the energy harvester [36]. In order to easily analyze and combine with the interfacing circuit, the piezoelectric beam will be studied with the equivalent circuit representation. Starting from the equations of motion and the constitutive equations of the piezoelectric material. The model of the cantilever beam type piezoelectric energy harvesting device will be established firstly. Several interfacing circuits and corresponding waveforms of piezoelectric terminal voltage and
output power will be studied based on the model. Moreover, a work-cycle representation to compare the performances of the interfacing circuits will be investigated.

2.1 Basic theory of piezoelectric materials

According to IEEE standard 176-1987 [76], constitutive law of piezoelectric materials can be expressed as equation (0.0). The constitutive law presents the relations between strain $S$, stress $T$, electric displacement $D$ and electric field $E$ of the materials. Table 2-1 shows the representations. According to the constitutive law, the behavior of the piezoelectric materials can be obtained and through the further analyzing the mechanical modal can be easily established.

\[
\begin{bmatrix}
T_p \\
D_i
\end{bmatrix} =
\begin{bmatrix}
e^{e_{pq}} & -e^{q_p} \\
e^{p_q} & \varepsilon_{ik}
\end{bmatrix}
\begin{bmatrix}
S_q \\
E_k
\end{bmatrix}
\]  

(0.0)

Where

<table>
<thead>
<tr>
<th>$T_p$</th>
<th>stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_q$</td>
<td>strain</td>
</tr>
<tr>
<td>$D_i$</td>
<td>electric displacement</td>
</tr>
<tr>
<td>$E_k$</td>
<td>electric field respectively</td>
</tr>
</tbody>
</table>

Table 2-1. Representations of constitutive law of piezoelectric materials
<table>
<thead>
<tr>
<th>c</th>
<th>elastic constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon )</td>
<td>permittivity constant</td>
</tr>
<tr>
<td>e</td>
<td>piezoelectric constant</td>
</tr>
</tbody>
</table>

Superscripts E and S represent constant.

Superscripts i,k=1~3, p,q=1~6 represent the coordinates index which is shown in Figure 2-1.

Considering the piezoelectric patches bound on a cantilever beam can be regarded as a simple energy harvesting device. The dimensions of piezoelectric patches, schematic of the beam and coordinate directions are shown in Figure 2-1. When the cantilever beam vibrates, for example as Figure 2-2 shows the first mode of the cantilever beam, the force acts on the piezoelectric patches can be simplified to 1-D model and regarded as a force \( F_P \) acts on the lateral surface as Figure 2-1 shows. In the cantilever beam type piezoelectric energy harvesting application, the first mode vibration is discussed because there is the largest strain. The larger strain means that more energy can be generated. On the assumption that the strain distribution is homogeneous and the 3-1 type piezoelectric patch is used and constitute equation (0.0) can be rearranged in Force, displacement, charge and voltage (electric potential) as
shown in equation (0.0).

\[
\begin{bmatrix}
F_p \\
Q
\end{bmatrix} = \begin{bmatrix}
\frac{E}{12} & \frac{E}{12} & & & & \\
\frac{E}{12} & \frac{E}{12} & & & & \\
\frac{E}{12} & \frac{E}{12} & & & & \\
\frac{E}{12} & \frac{E}{12} & & & & \\
\end{bmatrix} \begin{bmatrix}
x \\
V_p
\end{bmatrix}
\]

(0.0)

where \( w \) is the width, \( l \) is the length and \( t \) is the thickness of the piezoelectric patch.

**Table 2-2. Representations of the re-arranged constitutive law.**

<table>
<thead>
<tr>
<th>Force</th>
<th>( F_p = T_l \cdot w \cdot t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>( x = S_l \cdot l )</td>
</tr>
</tbody>
</table>

Figure 2-1. Schematic of piezoelectric harvesting cantilever beam

Figure 2-2. First mode vibration of the cantilever beam.
In equation (0.0), the relation between force \( F_P \) and displacement \( x \) means short circuit stiffness \( K_p^E \), the relation between force \( F_P \) and piezoelectric terminal voltage \( V_P \) can be induced a new parameter, force-voltage coupling factor \( \alpha \), and the relation between \( Q \) can \( V \) is clamped capacitance \( C_0 \). The equation (0.0) can be rewritten as equation (0.0). The Table 2-2 & Table 2-3 give the new quantities and parameters.

Table 2-3. Representations of the governing equations.

<table>
<thead>
<tr>
<th>Short circuit stiffness</th>
<th>( K_p^E = e_{11}^E \cdot \frac{w \cdot t}{l} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force-voltage coupling factor</td>
<td>( \alpha = e_{31} \cdot w )</td>
</tr>
<tr>
<td>Clamped capacitance</td>
<td>( C_0 = \varepsilon_{33}^S \cdot \frac{w \cdot l}{t} )</td>
</tr>
</tbody>
</table>

\[
\begin{bmatrix}
F_P \\
Q
\end{bmatrix} = \begin{bmatrix}
K_p^E & \alpha \\
\alpha & -C_0
\end{bmatrix} \begin{bmatrix}
x \\
V_P
\end{bmatrix}
\]

(0.0)

To study the dynamic behavior of the piezoelectric patches, we need the relations between displacement \( x \) and velocity \( \dot{x} \) in mechanical part and the
relations between voltage ($V_p$) and current ($I$) in electrical part. So take Laplace transform of displacement ($x$) and charge ($Q$) into frequency domain. The governing equation of piezoelectric patches can be shown as equation (0.0).

$$\begin{align*}
F_p &= K_p^E x + \alpha V_p \\
I &= \alpha \dot{x} - C_0 \dot{V}_p
\end{align*}$$

(0.0)

According to equation (0.0), the schematic model of the piezoelectric patch can be plotted as shown in Figure 2-3. The force-voltage factor ($\alpha$) is the conversion parameter between mechanical part and electrical part. Under same forcing condition, when the force-voltage factor ($\alpha$) is large, the piezoelectric patch can generate more energy. However, the above relation only exists when piezoelectric patch is under open-circuit condition. When the piezoelectric patch is connected to the load, the output voltage will be influenced and then there will be a force generated in the mechanical part to induce the damping effect [46]. The damping effect will decrease the displacement of the cantilever beam and then decrease the output voltage.
2.2 Model of piezoelectric energy harvesters

The schematic of general cantilever beam type piezoelectric energy harvesting device is shown in Figure 2-1. The mechanical structure can be simply modeled as an equivalent mechanical model with a mass, a damper, a spring and a piezoelectric element as shown in Figure 2-4.
$F_E$ : External driving force on the structure
$F_D$ : Damping force from damper
$F_S$ : Spring force from structure stiffness
$F_P$ : Force from piezoelectric structure
$x$ : Displacement
$V_p$ : Voltage across the piezoelectric patch
$I$ : Current flow out from piezoelectric patch

According to dynamic equation, the force equation of whole structure can be represented as equation (0.0).

![equation](image)

$$F_E = m\ddot{x} + D\dot{x} + K^E x + \alpha V_p$$  \hspace{1cm} (0.0)

<table>
<thead>
<tr>
<th>Damping ratio of structure</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffness of structure</td>
<td>$K_s$</td>
</tr>
<tr>
<td>Equivalent stiffness of the circuit</td>
<td>$K^e = K_s + K_p^e$</td>
</tr>
</tbody>
</table>

The governing equations of the piezoelectric patch bounded on structure are shown in equation (0.0). The energy equations can be obtained by multiplying velocity ($\dot{x}$) into equation (0.0) and integrating over time as shown in equation (0.0). The definition of energy term is shown in Table 2-4. The input energy is divided into four terms: kinetic energy, elastic energy, mechanical losses and converted energy. The converted energy
represents the sum of the energy stored in the piezoelectric capacitance and the energy delivered to the electrical load.

\[
\begin{align*}
\begin{cases}
    m\ddot{x} + D\dot{x} + K^E x &= F_E - \alpha V_p \\
    I &= \alpha \dot{x} - C_0 \dot{V}_p
\end{cases}
\end{align*}
\] (0.0)

\[
\begin{align*}
\int F_\epsilon \dot{x} dt &= \frac{1}{2} M\dddot{x} + \frac{1}{2} K^E \dot{x}^2 + \int D\dddot{x}^2 dt + \int \alpha V_p \dot{x} dt \\
\int \alpha V_p \dot{x} dt &= \frac{1}{2} C_0 V_p^2 + \int V_i dt
\end{align*}
\] (0.0)

<table>
<thead>
<tr>
<th>Table 2-4. Definitions of energy terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input energy</td>
</tr>
<tr>
<td>Kinetic energy</td>
</tr>
<tr>
<td>Elastic energy</td>
</tr>
<tr>
<td>Mechanical losses</td>
</tr>
<tr>
<td>Converted energy</td>
</tr>
</tbody>
</table>
2.3 Standard interfacing circuit

2.3.1 Standard AC approach

The schematic diagram of piezoelectric energy harvesting device with simple resistor load is shown in Figure 2-5(a) and Figure 2-5(b) shows the waveforms including the terminal voltage of the piezoelectric patch ($V_p$), equivalent current generated from piezoelectric patch ($I_{eq}$) and displacement ($x$). This diagram is called standard AC approach. The equivalent circuit model of the piezoelectric energy harvester is shown in Appendix A.1. Figure 2-6 is the equivalent circuit modal of the standard AC approach and this model can be used to calculate the optimal load $R$ and maximum power $P_{max}$.

![Diagram](image1)

(a) Schematic diagram of piezoelectric energy harvesting device with resistor load. (Standard AC approach). (b) Waveform of the standard approach.
The current $I_R$ equals to the equivalent current ($I_{eq}$) minus the current flowing in clamped capacitor ($I_C$). The voltage $V_C$ can then be expressed in frequency domain as equation (0.0) shown. The relation between the external force ($F_E$) and displacement ($\ddot{x}$) also can be expressed in frequency domain as shown in equation (0.0). Considering the piezoelectric energy harvesting device is driven at the resonant frequency ($\omega = \omega_0$), the force and the velocity ($\dot{x} = s\ddot{x}$) are in phase, equation (0.0) can be further simplified as shown in equation (0.0).

$$\ddot{V}_C = \frac{\alpha R}{1 + sC_0R} s\ddot{x} \quad (0.0)$$

, where $s$ is the Laplace operator.

$$\ddot{F}_E = \left[ sM + \frac{K^E}{s} + D + \frac{\alpha^2 R}{1 + (\omega_0C_0R)^2} \right] s\ddot{x} \quad (0.0)$$

37
The power output from piezoelectric energy harvesting devices with a resistor load can be calculated using simple equation \( \frac{V_c^2}{R} \). Because the waveform across the resistor is sinusoidal, when we calculate the output power, RMS value is taken here. Also because the \( V_c \) is complex, the complex conjugate should be used and the power output can be expressed as equation (0.0).

\[
\hat{F}_E = \left[ D + \frac{\alpha^2 R}{(\omega_n C_0 R) + 1} \right] s\hat{x}
\]

(0.0)

According to the relationship between external force and displacement as equation (0.0), the output power can be further expressed using external force amplitude as equation (0.0). When the piezoelectric patch is low coupled to the cantilever beam, the electromechanical coefficient \( (k^2) \) is small and the force-voltage factor is close to zero \( (\alpha \rightarrow 0) \) and equation (0.0) can be simplified as equation (0.0).
\[
P = \left( \frac{\alpha^2 R}{1 + (\omega_n C_0 R)^2} \right) \frac{1}{D + \left( \frac{\alpha^2 R}{1 + (\omega_n C_0 R)^2} \right)^2} \frac{F_E^2}{2D^2} \tag{0.0}
\]

In order to calculate the optimal resistance value \( R_{\text{opt}} \), take a partial differential with respect to \( R \) and equals it to zero, the optimal resistor load can be obtained as equation (0.0). Substitute equation (0.0) into equation (0.0), the maxima power output of the standard AC approach can be obtained as equation (0.0) shown.

\[
R_{\text{opt}} = \frac{1}{\omega_n C_0} \tag{0.0}
\]

\[
P_{\text{max}} = \frac{\alpha^2}{4\omega_n C_0 D^2} \cdot F_E^2 \tag{0.0}
\]

Because the power output of the piezoelectric energy harvesting device depends on the electromechanical coefficient and mechanical quality factor of the structure, according to the definition of the electromechanical coefficient and the mechanical quality factor, a new parameter is defined in equation (0.0). The electromechanical
coefficient is shown in Appendix A.2. Through equation (0.0), the output power of the standard AC approach can be plotted as a function of the load R and the parameter $k^2 Q_m$ at resonance as Figure 2-7 shown. $Q_m$ is the mechanical quality factor and $k^2$ the electromechanical coupling factor. For weakly coupled structure, $k^2 Q_m$ is lower than 2 [77]. When $k^2 Q_m$ is lower than 2, the SSHI technique can effectively increase the power than the standard interfacing circuit.

$$k^2 Q_m = \frac{\alpha^2}{\sqrt{\frac{MK}{DC_0}K_p^5}}$$

, where mechanical quality factor $Q_m = \frac{\sqrt{MK}}{D}$.

Figure 2-7. Normalized power as a function of the normalized load resistance and the electromechanical parameters.
Using work-cycle to compare the generating energy from the piezoelectric energy harvesting device is a good method. It's easy to show the generating energy from the plot. Figure 2-8 shows the work-cycle of the standard AC approach. The y-axis is the equivalent force ($\alpha V$) and the x-axis is the displacement ($x$). The work-cycle area means the energy which the piezoelectric energy harvesting device can generate, so if the area of work-cycle is larger, more energy can be obtained per cycle.

Figure 2-8. Work cycle of the standard AC approach (optimal load).
2.3.2 Standard DC approach

The schematic diagram of piezoelectric energy harvesting device with full-bridge rectifier and a resistor load is shown in Figure 2-9. This is called standard DC approach. Assuming the structure is excited at resonant frequency and from the governing equation, the exciting source can be modeled as a current source shown in Figure 2-10.

Figure 2-11 shows the waveform of standard DC approach including terminal voltage across piezoelectric \( V_p \), current source \( I_{eq} \) and displacement \( x \).

![Figure 2-9. Schematic diagram of piezoelectric energy harvesting transducer with a simple resistor load. (Standard DC approach).](image1)

![Figure 2-10. The equivalent circuit diagram of the Standard DC approach.](image2)
The detailed time interval discussion is shown in Appendix A.3 and the output voltage $\hat{V}_c$ can be obtain as the equation (0.0) shown.

$$\hat{V}_c = \frac{2\alpha R\omega_0}{2C_0 R\omega_0 + \pi} \hat{x}$$ (0.0)

The output power from piezoelectric energy harvesting transducer with standard DC approach can be calculated using simple equation $\frac{\hat{V}_c^2}{R}$ as shown in equation (0.0).

$$P = \frac{\hat{V}_c^2}{R} = \frac{4\alpha^2 R\omega_0^2}{(2C_0 R\omega_0 + \pi)^2} \hat{x}^2$$ (0.0)
In order to calculate the optimal load $R_{opt}$, take a partial differential with respect to $R$ of equation (0.0) and equals it to zero. $R_{opt}$ can then be obtain as equation (0.0).

$$R_{opt} = \frac{\pi}{2C_0 \omega_0} \tag{0.0}$$

Substituting the $R_{opt}$ into equation (0.0), the maxima power output ($P_{max}$) can be obtained as equation (0.0) shown.

$$P_{max} \bigg|_{R=R_{opt}} = \frac{\alpha^2 \omega_0}{2\pi C_0} \hat{x}_c^2 \hat{2} \tag{0.0}$$

Substituting the optimal load ($R_{opt}$) into equation (0.0) and equation (0.0), the voltage $\hat{\hat{V}_C}$ and time $T_2$ during optimal load condition can be obtained as equation (0.0) and equation (0.0).

$$\hat{\hat{V}_C} \bigg|_{R=R_{opt}} = \frac{2\alpha \left( \frac{\pi}{2C_0 \omega_0} \right) \omega_0}{2C_0 \omega_0 \left( \frac{\pi}{2C_0 \omega_0} \right) + \pi} \hat{x} = \frac{\alpha}{2C_0} \hat{x} \tag{0.0}$$

$$(T_2 - T_1) \bigg|_{R=R_{opt}} = \frac{T_2}{4} \tag{0.0}$$

The work cycle of the standard DC approach with optimal load can be plotted as
Figure 2-12. Because the full-bridge rectifier, the $\alpha V$ is constrained between $\alpha \hat{V}_c$ and $-\alpha \hat{V}_c$. The shape is a parallelogram.

2.4 Analysis of the synchronized switching technique

In energy harvesting applications, the piezoelectric elements convert the vibration energy of the host structure into the electrical energy, and then the generated electrical energy is stored in a storage buffer. Since the piezoelectric element has large clamped capacitance, an impedance matching circuit is required to maximize the generated power. It is known that an inductor can be added to compensate the contribution of the
piezoelectric clamped capacitor [26], but it cannot be adaptive to the environmental variations and the value of the inductance is too large in a low frequency range. To overcome this drawback, switching-type interfaces were proposed and popularly used in recent years [78]. In the switching circuits, the switches are operated synchronously with the vibration of the host structure in order to optimize the power flow.

Several synchronized switching circuit topologies and corresponding switching mechanisms were proposed. They can be classified into two groups according to the placement of the rectifier and the active switches. The first group of the switching circuits places the switches between piezoelectric element and the rectifier, such as parallel-SSHI (Synchronized Switching Harvesting on an Inductor) and series-SSHI [35, 79]. This group of techniques is used to modify the waveform of the piezoelectric voltage, i.e. the voltage across the piezoelectric element, in order to increase the collected power in the weakly coupled structure. The second group places the switches between the rectifier and the storage buffer, such as SECE technique [34]. This second group of techniques is used to modify the charging current flowing into the storage buffer in order to fasten the charging speed and to make a load adaptation.

In this dissertation, the parallel-SSHI technique and series-SSHI technique will be used to increase the efficiency. Thus, in the following two sub-suctions these two
technique will be discussed in detail.

2.4.1 Synchronized Switch Harvesting on Inductor in parallel (parallel-SSHI)

The schematic diagram of Synchronized switching harvesting on inductor in parallel (parallel-SSHI) piezoelectric energy harvesting device with full-bridge rectifier to a simple resistive load is shown in Figure 2-13. Assuming the structure is excited at resonant frequency and from the governing equation, the piezoelectric energy harvesting device can be modeled as a current source parallel with a clamped capacitor and the equivalent circuit of entire system is shown as Figure 2-14. Figure 2-15 shows the waveform of SSHI-parallel including voltage across piezoelectric $V_p$, current source $I_{eq}$ and displacement $x$.

In this technique, a bi-directional switch and an inductor $L$ are added in parallel with the piezoelectric patch. The switch is conducted at each maximum and minimum of the displacement or at the zero crossing of the vibration velocity, in order to reverse the voltage across the piezoelectric element and put it in phase with velocity. The result is that the energy stored in the structural clamped capacitor ($C_0$) is extracted by the LC resonance and achieve to a minimum value, and thus the piezoelectric voltage can be increased [80]. The harvested energy of the system with the SSHI technique is similar to
that using the standard interface under the strongly coupled condition [51]. When the vibration velocity crossing zero, the switch is conducted, the inductor L and the clamped capacitor \( (C_0) \) begin to oscillate. This resonant circuit increases the magnitude and changes the polarity of the voltage across the piezoelectric capacitance sinusoidal, and thus put voltage \( (\nu_p) \) and velocity \( (\dot{x}) \) in phase, which indicates that more energy is extracted from the vibration source.

![Figure 2-13. Schematic diagram of parallel-SSH energy harvesting device with full bridge rectifier to a simple resistor load.](image)

![Figure 2-14. The equivalent circuit diagram of the parallel-SSH piezoelectric energy harvesting device.](image)
Figure 2-15. Waveform of the parallel-SSHI piezoelectric energy harvesting device.

The detailed analysis of each time interval (From $T_1$ to $T_4$) is shown in Appendix A.4. $\hat{V}_C$ can be obtained and the result is shown in equation (0.0).

\[
\hat{V}_C = \frac{2\alpha R\omega_0}{\pi + (1-q_{lc})C_0R\omega_0} \hat{x} 
\]

(0.0)

The power output from piezoelectric energy harvesting device with parallel-SSHI can be calculated using simple equation $V^2/R$ as equation (0.0) shown.

\[
P = \frac{\hat{V}_C^2}{R} = \frac{4\alpha^2 R\omega_0^2}{\left[\pi + (1-q_{lc})C_0R\omega_0\right]^2} \hat{x}^2 
\]

(0.0)
In order to calculate the optimal resistor $R_{\text{opt}}$, take a partial differential with respect to $R$ on equation (0.0) and equals it to zero as equation (0.0). The optimal resistor ($R_{\text{opt}}$) can be obtained as equation (0.0) shown.

\[
\frac{\partial P}{\partial R} = \frac{4\alpha^2 \cdot \left[\pi + (1-q_{\text{LC}})C_0R\omega_0\right]^2 - (4\alpha^2 R) \cdot 2 \cdot (1-q_{\text{LC}})C_0\omega_0 \cdot \left[\pi + (1-q_{\text{LC}})C_0R\omega_0\right]}{\left[\pi + (1-q_{\text{LC}})C_0R\omega_0\right]^4} \omega_0^2 \hat{x}^2 = 0
\]  

(0.0)

\[
R_{\text{opt}} = \frac{\pi}{(1-q_{\text{LC}})C_0\omega_0}
\]  

(0.0)

Substituting the optimal resistor ($R_{\text{opt}}$) into the equation (0.0), the maxima power output $P_{\text{max}}$ can be obtained as equation (0.0) shown.

\[
P_{\text{max}} \bigg|_{R=R_{\text{opt}}} = \frac{\alpha^2 \omega_0^2}{\pi \left(1-q_{\text{LC}}\right)C_0\omega_0} \hat{x}^2
\]  

(0.0)

The voltage $\hat{V}_C$ and time $T_3$ under optimal load condition can be obtained as equation (0.0) and equation (0.0) shown.
\[ \dot{V}_c \bigg|_{R=R_{opt}} = \frac{2\alpha\omega_0}{\pi + \pi (1-eq_{LC})C_0\omega_0} \dot{x} = \frac{\alpha}{C_0(1-q_{LC})} \dot{x} \quad (0.0) \]

\[ T_2\big|_{R=R_{opt}} = \frac{1}{\omega_0} \cos^{-1} \left[ \frac{C_0}{\lambda\alpha} (1-q_{LC}) \dot{V}_c \bigg|_{R=R_{opt}} - 1 \right] = \frac{1}{\omega_0} \cos^{-1} (0) \quad (0.0) \]

\[ \Rightarrow T_2\big|_{R=R_{opt}} = \frac{T_0}{4} \]

The work-cycle of the parallel-SSI with optimal load can be plotted as Figure 2-16.

![Work-cycle of parallel-SSI with optimal load](image)

Figure 2-16. Work-cycle of the parallel-SSI.
2.4.2 Synchronized Switch Harvesting on Inductor in Series (Series-SSHI)

The schematic diagram of synchronized switching harvesting on inductor in series (Series-SSHI) piezoelectric energy harvesting device with full-bridge rectifier to a simple resistor load is shown in Figure 2-17. The piezoelectric energy harvesting device can be modeled as a current source parallel with a clamped capacitor and the equivalent circuit of entire system is shown as Figure 2-18. Figure 2-19 shows the waveforms of series-SSHI including the voltage across piezoelectric terminals $V_p$, voltage across the load $V_C$, current source $I_{eq}$ and displacement $x$.

Figure 2-17. Schematic diagram of series-SSHI piezoelectric energy harvesting device with a full bridge rectifier and a resistor load

Figure 2-18. The equivalent circuit diagram of the series-SSHI piezoelectric energy harvesting device.
The detailed analysis of each time interval (From $T_1$ to $T_4$) is shown in Appendix A.5. The maxima $V_C$ ($\hat{V}_C$) can be obtained and the result is shown as (0.0).

$$\hat{V}_C = \frac{2\alpha R (1+q_{LC})}{2R\omega_0 C_0 (1+q_{LC}) + \pi (1-q_{LC})} \omega_0 \hat{x}$$  \hspace{1cm} (0.0)$$

The power output from piezoelectric energy harvesting devices with series-SSHI can be calculated using simple equation $V^2/R$ as equation (0.0) shown.

$$P = \frac{\hat{V}_C^2}{R} = \frac{4\alpha^2 R (1+q_{LC})^2}{2R\omega_0 C_0 (1+q_{LC}) + \pi (1-q_{LC})} \omega_0^2 \hat{x}^2$$ \hspace{1cm} (0.0)$$
In order to calculate the optimal resistor $R_{\text{opt}}$, take a partial differential with respect to $R$ on equation (0.0) and equals it to zero. $R_{\text{opt}}$ can be obtain as equation (0.0).

$$ R_{\text{opt}} = \frac{\pi (1-q_{LC})}{2\omega_0 C_0 (1+q_{LC})} $$

Substituting $R_{\text{opt}}$ into the equation (0.0), maxima power output ($P_{\text{max}}$) of series-SSI can be obtained as equation (0.0) shown.

$$ P_{\text{max}} \bigg|_{R=R_{\text{opt}}} = \frac{\alpha^2 (1+q_{LC})}{2\pi C_0 (1-q_{LC})} \alpha_0 \hat{x}^2 $$

Substituting $R_{\text{opt}}$ into equation (0.0), the voltage $\hat{V}_{C}$ under optimal load condition can be obtained as equation (0.0).

$$ \hat{V}_{C} \bigg|_{R=R_{\text{opt}}} = \frac{2\alpha R_{\text{opt}} (1+q_{LC})}{2R_{\text{opt}} \alpha_0 C_0} \alpha_0 \hat{x} + \pi \left(1-q_{LC}\right) $$

$$ \Rightarrow \hat{V}_{C} \bigg|_{R=R_{\text{opt}}} = \frac{\alpha}{2C_0} \hat{x} $$

According to the waveform in the Figure 2-20, the work cycle can be plotted as Figure 2-20.
2.5 Discussion of the energy harvesting interfacing circuits

2.5.1 Power output discussion

The above sub-sections show the theoretical analysis of the standard interfacing circuits and the synchronized switching technique circuits and in this sub-section the power output of the interfacing circuits are compared and discussed. From equation (0.0), equation (0.0) and equation (0.0), the power output of the standard DC approach, parallel-SSH, and series-SSH can be obtained by using the same displacement to make system be driven under the same excitation. The output power and the load resistor can be normalized by the maximal output power and the optimal resistor load of the standard DC approach and the results are shown in Figure 2-21. The device is weakly...
coupled and $k^2 Q_m$ is $1.6 \times 10^{-4}$ (much lower than 2) on plotting Figure 2-21. The standard AC approach was not put into comparison here because in real applications, the AC output energy needs to be further rectified before powered most electronic devices nowadays. The parallel and series SSHI is plotted using three different quality factors: $Q_i = 1.5, 2.5$ and 3.5. The optimal resistor load and maxima output power of different techniques are marked in star in Figure 2-21. The comparison of the power output and resistor load is followed.

1. Resistor load comparison.

From the results, it's obviously that the optimal load of parallel-SSHI are always higher than standard DC approach. From $Q_i = 1.5$ to 3.5, the optimal load of parallel-SSHI are 3.08, 4.28 and 5.52 times of the standard case. For series-SSHI, the optimal resistor load is opposite to parallel-SSHI. From $Q_i = 1.5$ to 3.5, the optimal load of series-SSHI are 0.48, 0.32 and 0.24 times of the standard case. In the parallel-SSHI case, the higher quality factor $Q_i$ needs a higher optimal resistor load. On the contrary, in series-SSHI, the higher quality needs a lower optimal resistor load.

According to the different load, the proper synchronized switching technique can be chosen to achieve the maxima output power.

2. Output power comparison
Comparing the synchronized switching technique with standard DC approach whatever parallel-SSHI or series-SSHI, the power output is always higher and it shows the switching technique is an effective technique to increase the output power of the piezoelectric energy harvesting device. The higher quality factor $Q_I$ can lead to higher output power. From $Q_I = 1.5$ to 3.5, the maxima power output of parallel-SSHI are 3.08, 4.29 and 5.53 times of the standard case. For series-SSHI the optimal load are 2.08, 3.29 and 4.52 times of the standard case. Comparing the parallel-SSHI to series-SSHI, under the same quality factor $Q_I$, the power output of parallel-SSHI is little higher than series-SSHI. However, in the real application the quality factor $Q_I$ is constrained and the range from 1.5 to 3.5 is the reasonable value. If the system wants to achieve very high $Q_I$ value such as 10, it needs very expensive elements in the circuit and the cost will be very high. But it's meaningless to establish a expensive energy harvesting device.
2.5.2 Work-cycle discussion

According to energy equation (0.0), the energy transferred from the external energy can be easily obtained by measuring the terminal voltage of the piezoelectric patch and the tip displacement \((x)\) of the cantilever beam and the energy transferred can be calculated from the terminal voltage times the force-voltage factor \((\alpha)\) to get exerted force and then times the displacement. Work-cycle plot can then be plotted with exerted force in y-axis and displacement in the x-axis, and the enclosed area on the plot represent the transferred energy. Work-cycle is a simple way to show and compare the increased efficiency of the interfacing circuits for the energy harvesting devices. When
the enclosed area is larger, more energy are transferred from the vibration energy. If the external input energy is the same, larger enclosed area means higher efficiency. According to equations (0.0), (0.0), and (0.0), we can plot the work-cycles of standard DC approach, parallel-SSHI and series-SSHI techniques in the same plot at the optimal load as Figure 2-22 by giving the same displacement ($x$), force-voltage factor ($\alpha$), inverting quality factor ($q_{LC}$) and clamped capacitor ($C_0$), assumed the same piezoelectric patch connected to different interfacing circuits.

![Figure 2-22. Work-cycles comparison of different interfacing circuits.](image)

As the enclosed area is the transferred energy from the piezoelectric patch, the
following are the discussions of each technique.

1. Standard DC approach:

   The work-cycle of the standard DC approach is a parallelogram, so the area can be calculated by equation (0.0). And substitute equation (0.0) into the equation (0.0), the transferred energy of standard DC approach at optimal load can be obtain with equation (0.0).

   \[
   E_{DC} = 2\alpha\hat{V}_{c-DC} \cdot 2\hat{x} = 4\alpha\hat{V}_{c-DC} \hat{x} \\
   E_{DC} = 2\frac{\alpha^2}{C_0} \hat{x}^2
   \]

2. Parallel SSHI technique:

   From Figure 2-22, the enclosed area of the parallel-SSHI can be calculated as equation (0.0). And by substituting \( \hat{v}_{c-p} \) into equation (0.0), the transferred energy of parallel-SSHI at the optimal load can be obtained as equation (0.0).

   \[
   E_{Parallel-SSHI} = 2\alpha\hat{v}_{c-p} \cdot 2\hat{x} - (\alpha\hat{v}_{c-p} - \alpha\hat{v}_{c-p} \cdot q_{LC}) \hat{x} \\
   = 4\alpha\hat{v}_{c-p} \hat{x} - (1 - q_{LC})\alpha\hat{v}_{c-p} \hat{x}
   \]
\[ E_{\text{Parallel--SSHI}} = 4 \frac{\alpha^2}{C_0 (1-q_{LC})} \hat{x}^2 - \frac{\alpha^2}{C_0} \hat{x}^2 \]  

(0.0)

3. Series-SSHI:

The enclosed area of the series-SSHI is also a parallelogram like standard DC approach, so the area can be calculated as equation (0.0). By substitute the equation of \( \hat{V}_c \) into equation (0.0), the transferred energy of series-SSHI at optimal load can be obtained as (0.0).

\[
E_{\text{Series--SSHI}} = \left[ \hat{V}_{p-s} + \left( \hat{V}_{c-s} + \left( \hat{V}_{p-s} - \hat{V}_{c-s} \right) q_{LC} \right) \right] \cdot 2\alpha \hat{x}
\]

(0.0)

\[
E_{\text{Series--SSHI}} = 4 \left[ \frac{\alpha^2 \hat{x}^2}{C_0} - \alpha \hat{x} \hat{V}_c \right] \frac{(1+q_{LC})}{(1-q_{LC})}
\]

\[
E_{\text{Series--SSHI}} = 2 \frac{\alpha^2 (1+q_{LC})}{C_0 (1-q_{LC})} \hat{x}^2
\]

(0.0)

Using the transformed energy of the standard DC approach as the standard, the ratio of the parallel-SSHI to standard technique is \( E_{\text{Parallel--SSHI}} \) as equation (0.0) and the ratio of the series-SSHI to standard technique is \( E_{\text{Series--SSHI}} \) as equation (0.0). Using different
inverting quality factor \( q_{LC} \) the energy ratio curves can be plotted as Figure 2-23. From the plot, the energy ratio of the parallel-SSHI is always little higher than series-SSHI and when the inverting quality factor is higher, the series-SSHI is much closer to the parallel-SSHI. For low inverting quality factor, the energy ratio is higher than 1, which means even high electrical losses in the synchronized switching technique the efficiency is still better than standard technique.

\[
E_{\Delta 1} = \frac{E_{\text{parallel-SSHI}}}{E_{\text{DC}}} = \frac{2}{(1-q_{LC})} - \frac{1}{2} \tag{0.0}
\]

\[
E_{\Delta 2} = \frac{E_{\text{series-SSHI}}}{E_{\text{DC}}} = \frac{(1+q_{LC})}{(1-q_{LC})} \tag{0.0}
\]
2.6 Theoretical analysis of interfacing circuits of structural damping

2.6.1 Synchronized Switching Damping on a Short circuit (SSDS)

Another application of the previous SSHI interfaces is in semi-passive structural damping. In this application, the synchronized switching technique is used to extract energy from the structure in order to damp the vibration. One of these techniques is called SSDS (Synchronized Switching Damping on Short circuit), a semi-passive technique first presented by Richard et al. in 1999 [69]. The schematic diagram of SSDS technique using piezoelectric patch is shown in Figure 2-25. The SSDS technique is
composed of a piezoelectric patch and two-way switches. The two-way switches usually are composed of two MOSFET (metal oxide semiconductor field effect transistor). The equivalent circuit of SSDS technique is shown in Figure 2-25. The waveforms of the SSDS technique are shown in Figure 2-26. The switches turn on when the displacement $x$ or piezoelectric terminal voltage $V_p$ reaches to the maxima and minima value.

Figure 2-24 Schematic diagram of synchronized switching damping on short circuit (SSDS).

Figure 2-25. The equivalent circuit diagram of the synchronized switching damping on a short circuit (SSDS).
According to the energy equation (0.0) and integrating over a period (T), the energy equation can be expressed as equation (0.0). The first and second terms in the right-hand side are kinetic and elastic energies. When integrating over a period, these two terms will vanish as they are periodic function and the energy equation will be simplified as equation (0.0). It means that the external energy will turn into viscous energy (mechanical losses) and electrical energy dissipated in the interfacing circuit. Thus the switching damping energy is shown in equation (0.0). The Vp waveform of SSDS can be decomposed into two waveforms $V_1(t)$ and $V_2(t)$ shown in Figure 2-27(b) and Figure 2-27(c). The piezoelectric patch is short-circuited during the LC
resonance and most time in a period is open-circuited ($I_p = 0$). The magnitude of $V_p(t)$ can be obtained from integrating the current flow out from piezoelectric patch ($I_p$) over half LC resonant period and is shown in equation (0.0).

$$\int_0^T F \dot{x} \, dt = \frac{1}{2} M \dot{x} + \frac{1}{2} K \dot{x}^2 + \int_0^T D \ddot{x} \, dt + \int_0^T \alpha V_p \dot{x} \, dt$$  \hspace{1cm} (0.0)

$$\int_0^T F \dot{x} \, dt = \int_0^T D \ddot{x} \, dt + \int_0^T \alpha V_p \dot{x} \, dt$$  \hspace{1cm} (0.0)

$$E_s = \int_0^T \alpha V_p \dot{x} \, dt$$  \hspace{1cm} (0.0)

$$\int_0^{\frac{T}{2}} I_p \, dt = \int_0^{\frac{T}{2}} (\alpha \ddot{x} - C_0 V_p) \, dt = 0$$  \hspace{1cm} (0.0)

$$\Rightarrow V_{p,SSDS} = 2 \frac{\alpha}{C_0} \ddot{x}$$

Figure 2-27. Waveforms of (a) SSDS can be decomposed of (b) $V_1(t)$ and (c) $V_2(t)$.
As \( V_p(t) \) can be decomposed into two waveforms, the switching damping energy can be rewritten as equation (0.0). \( V_1(t) \) integrate with velocity \( (\dot{x}) \) will be zero because they have 90 degree phase difference. \( V_2(t) \) is a 50% duty cycle square wave, thus the integration for a period will equal to integration for half active period. Finally, the SSDS switching damping energy can be expressed as equation (0.0).

\[
E_s = \alpha \int_0^T V_1(t) \dot{x} dt + \alpha \int_0^T V_2(t) \dot{x} dt
\]

\[
E_{s-SSDS} = \alpha \int_0^T V_2(t) \dot{x} dt = \alpha \int_0^{T/2} \left( \frac{\alpha}{C_0} \dot{x} \right) \dot{x} dt
\]

\[
\Rightarrow E_{s-SSDS} = 4 \frac{\alpha^2}{C_0} \dot{x}^2
\]

### 2.6.2 Synchronized switching damping on an inductor (SSDI)

The synchronized switching damping on an inductor (SSDI) technique is detailed in this sub-section. The SSDI technique is further improved from the SSDS technique and presented by Richard et Al. in 2000 [70]. The damping ability of SSDI technique is more powerful than the SSDS technique. The schematic diagram of SSDI technique using piezoelectric patch is shown in Figure 2-28. The SSDI technique is composed of a
piezoelectric patch, two-way switches and an inductor. The inductor used here is like in SSHI technique and through the resonance between the inductor and the clamped capacitor of piezoelectric, more energy can be extracted from the piezoelectric patch to damp the structural vibration. The equivalent circuit of SSDI technique is shown in Figure 2-29 and the waveforms of the SSDI technique is shown in Figure 2-30. The switches turn on when displacement $x$ or piezoelectric terminal voltage $V_p$ reaches to the maxima and minima value. From Figure 2-30, there is a transient period, and in this transient period, the SSDI turns on and the terminal voltage of the piezoelectric patch will increase first and then decrease to the stable state. In SSHI technique, the piezoelectric patch is low coupled to the structure, so the displacement is assumed to keep the same. When SSHI technique turns on, the displacement is keeping the same and the terminal voltage will also keep the same. However, in SSDI technique, when SSDI turns on the displacement will start to decrease and make the terminal voltage decreases. Finally the system will reach to the stable state.

![Diagram of Synchronized Switching Damping with Inductor](image)

Figure 2-28. Schematic diagram of synchronized switching damping with and Inductor
Figure 2-29. The equivalent circuit diagram of SSDI technique.

Figure 2-30. Waveforms of the SSDI technique.

The SSDI waveform of the steady state (Figure 2-31(a)) is also can be decomposed of two waveforms as Figure 2-31(b) and Figure 2-31(c) like SSDS. As the equations derived from the SSDS sub-section, the SSDI switching damping energy can also expressed as (0.0). From the waveform, the behavior of SSDI technique is almost the same as the series-SSHI and the only difference is the voltage of regulated capacitor.
(\dot{V}_C) \) equals to zero. So according to the equation (0.0) and equation (0.0) and setting the \( \dot{V}_c \) equals to zero, the voltage \( \dot{V}_p \) and voltage \( \ddot{V}_p \) can be obtained as equation (0.0) and equation (0.0). The \( V_1(t) \) integrate with the velocity \( \dot{x}(t) \) will turn into zero because they are 90 degree phase lag and the SSDI switching damping energy will only be composed of \( V_2(t) \). Finally, the SSDI switching damping energy can be expressed as equation (0.0).

Figure 2-31. Waveforms of (a) SSDI can be decomposed of (b) \( V_1(t) \) and (c) \( V_2(t) \).

\[
\dot{V}_{p-SSDI} = \frac{2\alpha}{C_0 (1 - q_{LC})} \dot{x}
\]  

(0.0)
\[
\dot{V}_{p-\text{SSDI}} = \dot{V}_{p-\text{SSDI}} \cdot q_{LC} = \frac{2\alpha}{C_0} \frac{q_{LC}}{1-q_{LC}} \dot{x}
\] (0.0)

\[
E_{s-\text{SSDI}} = \alpha \int_0^T V_2(t) \dot{x} dt = \alpha \int_0^T \left( \frac{2\alpha}{C_0} \frac{q_{LC}}{1-q_{LC}} \right) \dot{x} dt 
\Rightarrow E_{s-\text{SSDI}} = \frac{8\alpha^2}{C_0} \frac{q_{LC}}{1-q_{LC}} \dot{x}^2
\] (0.0)

### 2.6.3 Discussion of the structural damping circuits

Comparing the switching damping energy of SSDI with the SSDS, the switching damping energy ratio \( E_{\Delta D} \) can be expressed as equation (0.0). Using different inverting factor, the switching damping energy ratio can be plotted as Figure 2-32. Higher inverting quality factor can make SSDI technique more efficient than SSDS technique but there is a limit of SSDI. If the SSDI technique is efficient than SSDS, the inverting quality factor needs to be higher than 1/3 as equation shown. In normal system, the inverting quality factor will be higher than 0.6 and the efficiency of SSDI is usually higher than the SSDS around 4 times.

\[
E_{\Delta D} = \frac{E_{s-\text{SSDI}}}{E_{s-\text{SSDS}}} = 2 \frac{q_{LC}}{(1-q_{LC})}
\] (0.0)
2.7 Summary of the interfacing circuits

In this chapter, a review of several interfacing circuits in energy harvesting and damping applications is presented. The basic governing equation, equivalent circuit model, waveform, optimal load and maxima output power are analyzed and discussed. The output power and transferred energy are compared of each interfacing circuit. The
synchronized switching circuits in energy harvesting are theoretically always better than the standard circuit composed of full-bridge rectifier. For normal inverting quality factor, the parallel-SSHI and series-SSHI can theoretically increase around 400% power output at optimal load. From the discussion of work-cycle, the transferred energy of parallel-SSHI and series-SSHI are theoretically increased around 4 times. When the synchronized switching technique is used in the structural damping, the performance of SSDI is better than SSDS but there is a limit of the inverting quality factor. For normal inverting quality factor, the switching damping energy of SSDI is around 4 times of SSDS. No matter in energy harvesting application or structural damping application, higher inverting quality factor can lead to better performance. However, there is a trade-off between inverting quality factor and the cost. The interfacing circuits proposed will be used in the following chapters to establish more useful piezoelectric energy harvesting device and piezoelectric structural damping system.
Chapter 3 Tunable Resonant Frequency Power Harvesting Devices

Methodologies of using piezoelectricity to convert mechanical power to electrical power with a cantilever beam excited by external environmental vibration were widely discussed and examined. Operating in resonant mode of the cantilever beam was found to be the most efficient power harvesting method, but in most cases that the resonant frequencies of the cantilever beam is hardly matching with the frequency of external vibration sources, such as mounting on a real world bridge. A cantilever beam based tunable resonant frequency power harvesting device which will shift its resonant frequency to match the external vibrations was developed and verified and will be presented in this chapter. From the networks analysis results, the useful bandwidth can be successfully extended. This system utilizes a variable capacitive load to shift the gain curve of the cantilever beam and a low power microcontroller sampling the external frequency and adjust the capacitive load to match external vibration frequency in real-time. The underlying design thoughts, methods developed, and preliminary experimental results will be presented. Potential applications of this newly developed power harvesting to wireless sensor network will also be detailed.
3.1 Introduction

From the research activities mentioned [11, 15-17, 30-38], researchers are improving the power harvesting devices from all aspects, including mechanics design, electrical signals, best materials, magnetic fields and adaptive power circuit design. In comparison with these technologies, the newly developed technology presented in this chapter derives its innovation from the interaction with the external excitation sources. It is known that the mechanical behavior of the structure is hard to be controlled, and most control technique will consume lots of the energy. In the energy harvesting application, if the energy harvesting device needs to be used in the real application, it is meaningless to control the mechanical behavior by using the external instruments and external energy.

In the view of basic mechanics, when the external force excites one of the natural frequencies of the system, resonant condition are meet and leads to large structure vibrations. If the resonant frequencies of the power harvesting devices coincide with the natural frequencies of the structure it mounted, much higher power output could be expected due to higher gain on the resonant frequencies. However, the natural frequency of the structure which power harvesting devices mounted on, such as bridges, scaffolds etc., may change with time when the structure is under different loading conditions. For
a power harvesting device based on a cantilever-beam structure has only constant resonant frequencies, it will not always be working at the best condition. A high mechanical quality factor cantilever beam can bring larger displacement at resonant frequency and generator more energy, but when the exciting frequency is away from the resonant frequency high mechanical quality factor will cause larger attenuation. Thus, the best solution for this trade-off problem is to design a high mechanical quality factor cantilever beam and the resonant bandwidth of this cantilever beam can be tuned. In a short region bandwidth, the voltage gain of the cantilever beam will not attenuate and the harvesting efficiency can be increased. An innovative real-time resonant frequency tuning system which can tune the resonant frequency to match the natural frequencies drifting of the mounting structure are proposed. The device can always work at best condition during a short frequency range by tuning its resonant frequency to match the external excitation frequency.

The technology proposed in this chapter utilized an ultra-low-power microcontroller on real-time sampling the external excitation frequency which can be combined with the wireless sensor on wireless sensor network (WSN) nodes, so as a more efficient self-powered wireless sensor could be built. Sensor network typically finds its applications in remote or difficulty to access areas, long-lasting batteries and
wireless technologies are the two subsystems mostly used. Power harvesting techniques provide the user with an opportunity to eliminate or at least reduce the frequency of battery replacement, which is particularly attractive for long-term applications related to highly dangerous or remote areas. Adopting such techniques to improve the usage time of portable electronics without increasing the pack weight is another area worth noting considering the mobile computing trend in today’s information application.

3.2 Theoretical Analysis

In this section, theoretical analysis of a piezoelectric cantilever beam will be conducted and through the simple analysis and the concept of the tunable frequency system would be derived. The piezoelectric cantilever beam bend at 1st mode can be shown as Figure 3-1, when boundary condition is one end fixed and the other end free.

![Figure 3-1. Cantilever beam bends at 1st Mode.](image)

According to the theoretical analysis in the chapter 2, if the external electric load is a capacitive load $C_L$ shunt with a resistive load $R_L$, the equivalent circuit model can further be simplified by reflecting the static capacitor $C_0$, capacitive load $C_L$ and the load $R_L$ to the primary side of the transformer by times the square of the transformer
turns ratio (force-voltage factor, $\alpha$) and the simplified equivalent circuit model is shown in Figure 3-2. The $C'_0$ equals to $\alpha^2C_0$, $C'_L$ equals to $\alpha^2C_L$, and $R'_L$ equals to $\alpha^2R_L$ after reflection.

![Simplified equivalent circuit model](image)

Figure 3-2. Simplified equivalent circuit model.

The equivalent circuit model in Figure 3-2 is a simple serial-parallel loaded resonant network (SPLR), the voltage gain $A_v\left(\frac{\alpha V_o}{F_i}\right)$ represents the force to voltage output gain, and it can be written as equation (0.0).

$$A_v = \frac{\alpha V_o}{F_i} = \frac{R'_L \parallel 1/j\alpha C_p}{(R_m + j\omega L_m + 1/j\omega C_m) + R'_L \parallel 1/j\omega C_p}$$ \hspace{1cm} (0.0)

,where $C_p = (C'_0 + C'_L)$

Equation (0.0) can be further simplified and non-dimensionized with $k = \frac{C_p}{C_m}$,

$$\omega_0 = \frac{1}{\sqrt{L_mC_m}}, \quad Q_L = \frac{\omega_0 L_m}{R_L} \quad \text{and} \quad Q_S = \frac{\omega'_0 L_m}{R_m} \quad \text{as equation (0.0).}$$
\[ A_v = \frac{1}{\sqrt{(1+k)^2 - k\left(\frac{\omega}{\omega_0}\right)^2 + \frac{Q_L}{Q_s}\left[Q_s\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) + k\left(\frac{\omega}{\omega_0}\right)^2\right]^2}} \]  

From equation (0.0), the two resonant frequencies of the RLC oscillator can be obtained. One is the series resonant frequency and the other is the shunting resonant frequency as shown in equation (0.0) and equation (0.0).

\[ \omega_s = \frac{1}{\sqrt{L_mC_m'}} = \omega_0' \]  

\[ \omega_p = \frac{1}{\sqrt{L_m\left(\frac{C_mC_p'}{C_m + C_p}\right)}} = \omega_0'\sqrt{1 + \frac{1}{k}} \]

For short circuit condition, \( C_p = 0 \) (\( k = 0 \)), the circuit becomes a serial loaded resonant network and \( \omega_s \) is the resonant frequency. When \( C_p \) is much larger than \( C_m \) (large \( k \)), \( \omega_p \approx \omega_s \) and the resonant would be closed to \( \omega_s \) again. For other finite smaller capacitive loading values, the resonant frequency will be shifted between \( \omega_s \) and \( \omega_p \) with \( C_p = C_0' \) (the open circuit condition). The value of capacitive load \( C_L' \)
can then be varied to tune the resonant frequency within this frequency range. The force to voltage output gain versus non-dimensionalized $\omega/\omega_0'$ is plotted in Figure 3-3 by using $Q_e = 1$, $Q_s = 1000$ and $k = 0, 0.5, 1, 1.5$ and 2. It's obviously to see that the in different k value, the resonant frequency is successfully be shifted and when k value is larger, the voltage gain increases.

From the view of mechanics, the stiffness of the cantilever beam is varied when the electrical loading condition changed. The frequency tuning can be achieved by a simple analog circuit or by sampling the external excitation frequency with a microprocessor, and switch in an adequate capacitive value to tune the resonant frequency matching to the external excitation frequency in a small range. The circuit can be put on the sensor node and control the frequency tuning with the low power microprocessor on the sensor node.
3.3 Experimental validation and discussion

3.3.1 Real bridge frequency measurement

In this sub-section, we want to know the behavior of the real bridge when the time passes or the loading of the bridge changes. It is obviously that the amounts of the cars go through a bridge will not be the same all day long and the cars may have traffic jams on the bridge, so it means that the loading of the bridge is varying over time. From the structural dynamics perspective, when the loading of the bridge changes, the natural frequency of the bridge will be changed. When the time passes or the bridge encounters to the natural disaster, the bridge will be fatigued and be destroyed and these reason will also cause the natural frequency of the bridge be changed. If a cantilever-beam typed power harvesting device combined with wireless sensor networks is placed on a real bridge to monitor the bridge, the resonant frequency of the cantilever beam will be within narrow band width. When the natural frequency of the bridge changes, the cantilever-beam typed power harvesting device cannot work at the best condition. This result cause the power harvesting device cannot always work matching the resonant frequency of the bridge.

Figure 3-4(a) shows the experimental setup for measuring the vibration signal of
the bridge. The accelerometer is placed on the middle of the two bridge piers, because the middle point is the maximum displacement point and it is the best point to harvesting the vibration energy. Figure 3-4(b) shows how to record the accelerometer data. Through the conditioning, DAQ card and LabVIEW program of the notebook, the measuring data of the accelerometer can be recorded in the notebook and then be analyzed.

Figure 3-4. (a) Bridge Vibration Measurement (b) Measuring data record setup

Figure 3-5 shows the results of the real bridge vibration measurement. The bridge is Jhonghsing bridge (中興橋) located in Taipei, Taiwan. There are three data sets measured in different time in the Figure 3-5. Each measuring time interval here is 800 sec. During each measuring time interval, the loading of the bridge must be different as different amount of the cars pass the bridge. Because the natural frequency of the bridge
must be very low and the frequency range we concern is under 100Hz, the measuring sampling rate is set to be 5000Hz. From the measuring result, we can obviously see that the three vibration signal are different from each other when the loading is different.

Then three measuring data sets are taken into Fast Fourier Transformation (FFT), and the FFT results are shown in Figure 3-6. The results are sorted from the frequency range from 65Hz to 85Hz and three FFT results show the resonant frequencies are a little shifting from each other under different loading. The resonant frequency range is around 2.5Hz. If the resonant frequency of the tunable energy harvesting device is
designed to match in this 2.5Hz range, it can work better and harvest more energy.

![Vibration Signal FFT of the bridge](image)

Figure 3-6. FFT of the vibration signals.

### 3.3.2 Piezoelectric energy harvesting cantilever beam testing

The experimental setup of the piezoelectric energy harvesting cantilever beam is shown in Figure 3-7. The experimental setup is composed of a bimorph piezoelectric clamped at fixed end, a function generator generating the vibration signal, a vibrating shaker generating exciting source and a photonic sensor measuring the displacement. The instruments and model are shown in Table 3-1. The piezoelectric patch used here is bimorph type made by Mide Corporation and the model is QP25W. The dimension and the parameters are shown in Table 3-2. The bimorph piezoelectric cantilever beam is composed two piezoelectric patches as Figure 3-8 and the two patches can be used
separately. This QP25W bimorph piezoelectric is suitable for power harvesting as its good performance for charging the battery [81].

Figure 3-7. Experimental setup of the tunable energy harvesting device.

Table 3-1. Instrument list

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Company</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function generator</td>
<td>Tektronix</td>
<td>AFG320</td>
</tr>
<tr>
<td>Power Amplifier</td>
<td>Brüel &amp; Kjær</td>
<td>4809</td>
</tr>
<tr>
<td>Vibration Shaker</td>
<td>NF Corporation</td>
<td>HSA4052</td>
</tr>
<tr>
<td>Fotonic Sensor</td>
<td>MTI Instruments Inc.</td>
<td>MTI2000</td>
</tr>
</tbody>
</table>

Figure 3-8. Bimorph piezoelectric cantilever (QP25W) from Mide corporation.
Table 3-2. Dimension and parameters of the piezoelectric cantilever beam.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value (unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>Length×Width×Thickness</td>
<td>2×1.5×0.02 (in³)</td>
</tr>
<tr>
<td>f_{op}</td>
<td>Open circuit resonant frequency</td>
<td>76 Hz</td>
</tr>
<tr>
<td>f_{sh}</td>
<td>Short circuit resonant frequency</td>
<td>73.55Hz</td>
</tr>
<tr>
<td>k²</td>
<td>Electromechanical coupling coefficient</td>
<td>0.0677</td>
</tr>
<tr>
<td>ζ</td>
<td>Damping ratio</td>
<td>0.054</td>
</tr>
<tr>
<td>Q_M</td>
<td>Mechanical quality factor</td>
<td>9.19</td>
</tr>
<tr>
<td>M</td>
<td>Mass</td>
<td>5.1 g</td>
</tr>
<tr>
<td>K^E</td>
<td>Equivalent stiffness when all piezoelectric element is in short circuit</td>
<td>1163.2 N/m</td>
</tr>
<tr>
<td>K^D</td>
<td>Equivalent stiffness when all piezoelectric element is in open circuit</td>
<td>1089.4 N/m</td>
</tr>
<tr>
<td>D</td>
<td>Damping coefficient</td>
<td>0.265 N/m/s</td>
</tr>
<tr>
<td>α</td>
<td>Force-voltage factor</td>
<td>0.35 N/V</td>
</tr>
<tr>
<td>C_0</td>
<td>Clamped capacitance of</td>
<td>330nF</td>
</tr>
</tbody>
</table>

The piezoelectric energy harvesting device is tested under different excitations at short-circuit resonance and the results are shown in Figure 3-9. Figure 3-9(a) shows the capacitor's voltage versus charging time curves when the device charges to a 0.047F super capacitor under different excitations. When the exciting source increases, the charging time decreases. The total charging energy can be calculated by using $(1/2)CV^2$, where $C$ is the capacitance value and $V$ is the terminal voltage of the capacitor. And the average harvesting power can be calculated by dividing the total charging energy by the charging time when the voltage reaches the target value. The target
voltage here is set at 4V for calculating the average power. Figure 3-9(b) shows the results of the power output of piezoelectric patches under different excitations. When the displacement is under 0.08mm, the relation between power output and displacement is almost a linear line. However, when excitation goes too large the power output won't increase linearly. This result shows that the excitation should be limited in the elastic region or the device may be broken. In order to keep the piezoelectric cantilever beam working in the linear region, the acceleration of the following testing is set to 0.5m/s² (displacement = 0.017mm).

![Figure 3-9. Piezoelectric energy harvesting cantilever beam testing results. (a) Charging time curve (b) Output power under different excitation.](image)

### 3.3.3 Network Analysis

In order to demonstrate the function of the tunable frequency system, the shifting resonant frequency effect will first be verified through SRS Network Signal Analyzer
SR780, and the experimental setup is shown in Figure 3-10. The microprocessor is used to choose different capacitor load according to the exciting frequency. The upper patch of the bimorph piezoelectric configuration is used for the frequency tuning purpose through the microprocessor to sample the exciting frequency and connected to different capacitors loadings. According to the theory when the capacitor loadings are changed, the resonant frequency of the piezoelectric cantilever beam can be tuned to match the exciting frequency. The lower piezoelectric patch is used to harvest energy and regulated to a DC voltage by a full-bridge rectifier to charge a 0.047F super capacitor. The super capacitor can then provide the extra energy for wireless sensor network nodes and extend the battery life time.

The testing results are shown in Figure 3-11. The short-circuit condition (star points) and the open-circuit condition (triangle points) are two extreme conditions and the resonant frequencies are 73.5Hz and 76Hz respectively. Around 2.5Hz frequency range is the tunable bandwidth on this system. The resonant frequency of the system can be changing within this 2.5Hz range by switching in different capacitive loads as Figure 3-11 shown. This tunable bandwidth and resonant frequency almost fit the measuring results from the real bridge and can be used in the real application. When the tuning patch of the bimorph piezoelectric is shunted to the 0.16uF and 0.078uF capacitor, the
gain curve can be tuned between the two extreme conditions and through proper
switching control the gain curve of the tunable frequency system can be extended and
smoothly changed between short-circuit and open-circuit condition as the experimental
results (round points). Comparing the tunable system curve with short-circuit, open-circuit and other single capacitive loads, the resonant bandwidth is obviously
wider and the harvesting efficiency can be effectively increased.

Figure 3-10. Network analysis of the tunable energy harvesting device.

Figure 3-11. Experimental results of the network analysis.
3.3.4 Charging the Capacitor with Chirping and Random Frequency Excitations

3.3.4.1 Frequencies slightly away from the resonance test

In order to evaluate the difference of harvesting efficiency when the piezoelectric energy harvesting cantilever beam excited under slightly different frequencies around the resonance, a 0.047F super capacitor is charged by the power harvesting device under short-circuit condition (resonant frequency = 73.5Hz) at acceleration=0.5m/s$^2$. The testing results are shown in Figure 3-12.

We can see that the super capacitor reaches the target voltage 4V in shortest time when excited at the resonant frequency (73.5Hz). When the system is excited at 72.5Hz, 1Hz away from the resonance, the charging time spends more than 50 seconds to reach the target voltage. And when the system is excited at 71.5Hz, 2Hz away from the resonance, the charging time spent are longer than 200 seconds.

When piezoelectric energy harvesting cantilever beam is excited at resonant frequency (73.5Hz), its average harvesting power output is around 0.859mW. When the beam is excited 1Hz away from the resonant frequency (72.5Hz), its average harvesting power is around 0.778mW and 2Hz away from the resonant frequency (73.5Hz), its average harvesting power is around 0.578mW. From average power results, if the resonant frequency can be tuned for 1Hz, the average harvesting power will increase
10.4%, and if the system can be tuned for 2Hz, the average harvesting power will increases 48.6%. When 3Hz or more away from the resonant frequency, the voltage of the super capacitor is hardly to reach to 4V, even cannot reach the target voltage. This results show that in a quality factor system, to match excitation frequencies with the resonance is very important.

![Comparing different frequency charging time](image)

**Figure 3-12.** Charging time of external excitations at different frequency.

### 3.3.4.2 Chirping and random frequencies excitation testing

The testing of the above sub-section excites the energy harvesting system at the single frequency under short-circuit condition to show how the tunable technique use to increase the energy harvesting efficiency. The real-time frequency tuning energy
harvesting device is then tested in chirping frequency and random frequency excitation under different frequency range. The exciting signal source is provided by the LabVIEW program to generate the chirping frequency and random frequency with a DAQ card (USB 6259) and drive the vibration shaker through a power amplifier. The testing chirping and random frequency ranges are both from the wider frequency range to the narrower frequency range. There are four testing ranges: 1. Bandwidth=40Hz (55~95Hz), 2. Bandwidth=20Hz (65~85Hz), 3. Bandwidth=10Hz (70~80Hz) and 4. Bandwidth=4Hz (72~76Hz). The tunable energy harvesting device still charges to a 0.047F super capacitor and the charging voltage curve versus time of the four testing ranges are shown in Figure 3-13 to Figure 3-16, the testing curves (a) are all chirping testing and (b) are all random testing.

![Figure 3-13. Chirping (a) & Random frequency (b) from 55Hz to 95Hz.](image-url)
Figure 3-14. Chirping (a) & Random frequency (b) from 65Hz to 85Hz.

Figure 3-15. Chirping (a) & Random frequency (b) from 70Hz to 80Hz.

Figure 3-16. Chirping (a) & Random frequency (b) from 72Hz to 76Hz.
These experimental results can verify our tunable energy harvesting system. Table 3-3 and Table 3-4 summarize the power output of the chirping and random frequency testing results. The increased power is calculated by using the short-circuit condition to be the reference and the increased power is calculated using equation (0.0).

### Table 3-3. Chirping frequency testing results.

<table>
<thead>
<tr>
<th>Frequency range</th>
<th>Short-Circuit (mW)</th>
<th>Open-circuit (mW)</th>
<th>Tunable System (mW)</th>
<th>Increased power (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>55 to 95 Hz</td>
<td>0.276</td>
<td>0.298</td>
<td>0.33</td>
<td>19.57</td>
</tr>
<tr>
<td>65 to 85 Hz</td>
<td>0.345</td>
<td>0.372</td>
<td>0.409</td>
<td>18.55</td>
</tr>
<tr>
<td>70 to 80 Hz</td>
<td>0.561</td>
<td>0.66</td>
<td>0.723</td>
<td>28.88</td>
</tr>
<tr>
<td>72 to 76Hz</td>
<td>0.606</td>
<td>0.671</td>
<td>0.737</td>
<td>21.62</td>
</tr>
</tbody>
</table>

### Table 3-4. Random frequency testing results.

<table>
<thead>
<tr>
<th>Frequency range</th>
<th>Short-Circuit (mW)</th>
<th>Open-circuit (mW)</th>
<th>Tunable System (mW)</th>
<th>Increased power (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>55 to 95 Hz</td>
<td>0.165</td>
<td>0.184</td>
<td>0.219</td>
<td>32.73</td>
</tr>
<tr>
<td>65 to 85 Hz</td>
<td>0.261</td>
<td>0.284</td>
<td>0.313</td>
<td>19.92</td>
</tr>
<tr>
<td>70 to 80 Hz</td>
<td>0.384</td>
<td>0.427</td>
<td>0.495</td>
<td>28.91</td>
</tr>
<tr>
<td>72 to 76Hz</td>
<td>0.495</td>
<td>0.57</td>
<td>0.66</td>
<td>33.33</td>
</tr>
</tbody>
</table>

Increased power\(=\frac{\text{Tunable system power output}}{\text{Short-circuit power output}}\) (0.0)

In the chirping frequency testing results, four testing frequency ranges can all reach
mW level power output and the average power output is around 0.3mW, 0.38mW, 0.65mW and 0.67mW for each range. The tunable resonant frequency energy harvesting device can increase the power output around 19.57%, 18.55%, 28.88% and 21.62% for each frequency range. The maxima increased power is 28.88% and occurs when the chirping testing frequency range is 70 to 80Hz. The charging time can be shorted around 170 seconds.

In the random frequency testing results, the average power is around 0.19mW, 0.29mW, 0.44mW and 0.58mW for each range. The tunable resonant frequency energy harvesting device can increase the power output around 32.73%, 19.92%, 28.91% and 33.33% for each frequency range. The maxima increased power is 33.33% and occurs when the chirping testing frequency range is 72 to 76Hz. The charging time can be shorted around 160 seconds.

In the two testing conditions, the both average power increases when the testing range narrows. Comparing the chirping frequency testing results with the random frequency testing results, the output power of the chirping testing is higher than random testing. However, the tunable energy harvesting system used in random frequency testing can increase power output more than in chirping testing case. That's because when exciting signal is random frequency and if the exciting frequency is changed
instantaneously from the resonance to non-resonance, the piezoelectric voltage will
decrease immediately to induce the full-bridge rectifier turning into open-circuit
condition and the charging current discontinued. Because the charging current
discontinued, the charge time will increase. The tunable frequency energy harvesting
system is very suitable to be used in the random exciting source and the random
exciting source is more closed to the vibration sources in real world. The tunable
technique make the resonance of the system changed with the exciting frequency and
this wide resonant bandwidth keeps the charging current continuous and effectively
increase the output power. This real-time resonant frequency tuning system shows
significant improvement on average harvesting power output.

3.3.5 Implement the tunable frequency power harvesting function on a Wireless
sensor network transceiver module

The real-time frequency tuning capability can be achieved by integrating with a
wireless sensor with the low-power microcontroller on a wireless sensor to sample the
external excitation frequency and changing the loading capacitor to tune the resonant
frequency of the cantilever beam. Figure 3-17 show a wireless sensor transceiver
module which uses an integrated Chipcon CC1010 microcontroller with built-in
wireless transceiver circuit and analog-to-digital converters for sensor signal interfacing.
The microcontroller was programmed to control the frequency tuning harvesting device. It has to be noted that general microcontroller will consume several tens mW power which is much higher than the energy harvesting devices can generate. However, modern low-power microprocessor can operate in μW level. It would be worthy to pay the price of μW power consumption in operating microcontroller to tune the frequency and gain much higher harvesting power generation in mW range.

Figure 3-17. The wireless sensor transceiver module using Chipcon CC1010 integrated microprocessor.

Figure 3-18 shows the circuit schematic of the wireless sensor network (WSN) transceiver module implemented the tunable frequency function for piezoelectric energy harvester and Figure 3-19 shows the photos. The energy harvested from the piezoelectric cantilever beam is stored in the super capacitor C1 through a full-bridge rectifier. Here, we still use a battery to supply the energy to the WSN module and the
piezoelectric energy harvesting device provide the extra energy to extend the lifetime of the battery by proper switching control.

Figure 3-20 shows the schematic diagram of the switching control for supplying energy to the WSN transceiver module. The voltage of the capacitor C1 and the battery's voltage of the WSN transceiver module are compared by a Schmitt trigger circuit which is composed of a TLV3494 voltage comparator. When the C1's voltage is charged higher than the battery's voltage, the analog switch (TS5A4596) will switch the C1’ terminal to connect to the WSN module and supply power to the WSN transceiver module. The switching signal will also be sent to the WSN transceiver module and the triggers the transmitting procedure. When the WSN transceiver module started transmission, the C1's voltage will drop immediately and the switching control will switch the power supply back to the battery. And then the C1 will be charged by piezoelectric energy harvesting device until the voltage is higher the target level. The regulator (TLV70230) regulates the voltage of the super capacitor to provide a stable voltage. In this circuit, Schmitt trigger plays an important role to generator the hysteresis. The hysteresis can make the system more stable, because when the voltage of the capacitor decreases, the power switching control will not switch the circuit to the battery supply mode immediately.
Figure 3-18. Electric circuit for the wireless sensor network combined with piezoelectric energy harvesting system.

Figure 3-19. Electric circuit photos.

Figure 3-20. Schematic diagram of the switching control for supplying to the WSN node.
Figure 3-21 shows experimental result of the battery, energy harvester switching and the transceiver transmission scheme. In Figure 3-21, left half part shows the module tries to register itself in the registering interval and right half part shows the communication behaviors after it is successfully registered. If in this registering time interval, switching control switches the system immediately to the piezoelectric energy harvester to supply power, the system will go back to the battery supply mode just as shown in left half part. That's because the piezoelectric energy harvester cannot provide the enough energy for the module working in the registering interval. The switching control will switch the power supply mode back and forth between battery supply and energy harvester. After all the sensor nodes have already registered, the sensor node can be arranged by the local control center to sleep in idle mode or to transmit the RF signal in active mode. When sensor node works in idle mode, the battery provides the system energy and in this time interval the piezoelectric energy harvester will harvest the ambient vibration energy. When the voltage is charged over around 2.3V (set by comparator), the switching control will switch the system to the piezoelectric supply mode to provide energy and sensor node will be in the active mode to transmits the RF signal. After sensor node transmitting the RF signal, the voltage drops down and the system will go back to the battery supply mode. Under this switching scheme, the
sensor network node can work longer through combining with piezoelectric energy harvesters and batteries.

![Graph showing battery switching and the transceiver module transmission scheme.]

**Figure 3-21.** Experimental result of battery switching and the transceiver module transmission scheme.

### 3.4. Conclusion

In this chapter, we present the theoretical analysis and experimental results of the tunable resonant frequency system on a piezoelectric energy harvesting cantilever beam device. The real-time resonant frequency tuning system is further demonstrated by using a microcontroller on a wireless sensor in sensor networks. The tunable frequency technique can extend the resonant frequency range around 2.5Hz and increase the
average harvesting power output almost 30% when under chirping and random frequency excitation testing. This significant power improvement can be expected due to the nature of high mechanical quality factor. From the final experiments, this tunable frequency system can be successfully combined with the wireless sensor network to transmit the RF signal. By integrating the tunable resonant frequency harvesting device with wireless sensor network system, a more powerful self-powered wireless sensor could be built and the battery lifetime can be effectively extended.
Chapter 4 A self-powered switching circuit for piezoelectric energy harvesting with velocity control

In this chapter, a self-powered piezoelectric energy harvesting device is proposed based on the velocity control synchronized switching harvesting on inductor technique (V-SSHI). In chapter 3, in order to use the tunable technique in real applications, a WSN module is necessary to be used to get the exciting frequency and tuning the electrical load. To further improve the power output, synchronized switching techniques is proved to be effective on enhance the overall power output. To simplify the overall system design, the main focus in this chapter is to realize the synchronized switching technique to be a self-powered system.

Comparing to the standard DC approach using a full-bridge rectifier, synchronized switching harvesting on inductor (SSHI) technique can significantly improve the harvesting efficiency. However, in real applications, when the energy harvesting device is associated with wireless sensor network (WSN) nodes, the SSHI technique needs to be implemented and requires to be self-powered for a reasonable and neat design. The conventional technique to implement self-powered SSHI uses bipolar transistors and diode as voltage peak detector. In this chapter, a new self-powered design is proposed, using velocity control to switch the MOSFETs more accurately than in the conventional
technique. The concept of the design and the theoretical analysis are presented in detail and experimental results are used to examine to concept of the design.

4.1 Introduction

Nowadays with improvement and rapid growth of low-power electronics, it is possible to supply portable devices such as mobile phone, MP3 player, wireless sensors and human or animal detecting devices, with harvest energy from ambient. Among these low-power devices, wire-less sensor network (WSN) is one of the most important and valuable applications which is highly investigated. Wireless sensor networks can be used to monitor the health of structures, environment, wild animals, tire pressure of running cars, etc. In most of WSN applications, the devices are far from the power line or the devices need to be embedded into the structure to monitor. So, it is hard to use power line to transmit energy to device; battery is the only conventional solution. However, there are lots of disadvantages with using batteries. The major problem is the lifetime: using a 3 V battery a WSN module can only be operated for 1 or 2 years. The batteries cannot be a permanent energy supply for a WSN module. A WSN module with self-powered system can be operated for a longer time without replacing the battery. Harvesting the ambient energy close to the sensor nodes of the WSN is the most likely
and suitable solution to extend its lifetime [21-25, 82-86].

As mentioned in chapter 1 and chapter 2, the Synchronized Switch Harvesting on Inductor (SSHI) technique is a very successful and efficient technique to boost the output power from piezoelectric [34, 35, 48, 87]. This approach is derived from a semi-passive damping technique: Synchronized Switch Damping on Inductor (SSDI) [69, 70]. The SSHI technique consists in adding up a nonlinear switching. This nonlinear process increases the output voltage of the piezoelectric elements that increase the output power. The switching device is triggered at the zero crossing of velocity. In order to realize the synchronized switching technique in real applications without external power source to supply the system, many researches present self-powered supply system for piezoelectric energy harvesting devices [23, 39, 55, 56]. The design concept of self-powered system proposed by Lallart and Guyomar [55] is shown in Figure 4-1. This conventional self-powered system works by using a peak voltage detector to control the switching time for SSHI technique. However, the energy supplying to the peak detector and the switching control is drawn from the piezoelectric device. The energy losses in the circuit can be accurately controlled by circuit design; the larger excitation levels leading to relatively smaller losses. As the conventional self-powered system uses peak detector, there is always a phase lag between the peak
voltage and the actual switching time. Moreover, the phase lag for large excitation level is less than for a small one [56].

In this chapter, we present a new self-powered piezoelectric energy harvesting system using velocity control SSHI technique, called "V-SSHI". The schematic design concept of self-powered velocity control SSHI is shown in Figure 4-2. The SSHI used here is series type (inductor and switch are in series with piezoelectric patch). Comparing to the conventional design concept, the piezoelectric material is separated into three parts. The main part is dedicated to harvest ambient vibration energy. The second small part is designed to supply energy for switching MOSFET and the last small part is designed for velocity control and for switching on the optimal time. There are two major advantages of this new technique: (1) theoretically, there is no phase lag by using velocity control signal to determine the switching time; (2) the supply energy for the switching driver can be designed and optimized by the size of the piezoelectric material.

The energy flow chart of the conventional self-powered technique and the self-powered V-SSHI technique is shown in Figure 4-3. There is a common path for the main stream of energy and for the supply of the self-switching system. In the V-SSHI technique, they are three energy paths. The energy supplying the self-switching system
and the velocity control patches can be designed optimally. The theoretical analysis and modeling of the self-powered V-SSHI is presented in detail in section 4.2. The experimental results comparing standard DC approach, conventional self-switched technique and V-SSHI technique are presented in section 4.3. The experimental results show higher output power of the V-SSHI technique over conventional technique.

Figure 4-1. Schematic design concept of conventional self-switched system.

Figure 4-2. Schematic design concept of velocity control SSHI self-switched system.
4.2 Theoretical Analysis of the self-powered V-SSHII technique

4.2.1 Standard DC technique

Before talking about the models of the V-SSHII technique, the standard DC technique is proposed to be a reference. The schematic diagram of piezoelectric energy harvesting transducer with full bridge rectifier connected to a resistor is shown in Figure 4-4(a). Figure 4-4(b) shows also the key waveforms of the standard DC approach. When the absolute voltage value of the piezoelectric patch $V_P$ is less than voltage $V_C$, the diode bridge is in open-circuit. The diodes conduct and piezo-patches charge the load only when $V_P$ reaches load voltage $V_C$. The detail theoretical analysis is already derived and discussed in the chapter 2. Because in the next sub-suction, the theoretical and experimental results of the standard DC approach will be shown to compared with V-SSHII, here the voltage crossing the resistor ($V_C$) and output power ($P$) can be
expressed as equation (0.0) and equation (0.0)

\[
V_c = \frac{2\alpha R}{(2RC_0\alpha_0 + \pi)} \frac{F_B}{D} 
\]

\[
P = \frac{V_c^2}{R} = \frac{4\alpha^2 R}{(2RC_0\alpha_0 + \pi)^2} \frac{F_B^2}{D^2}
\]

4.2.2 Self-powered V-SSHİ technique

According to the self-powered V-SSHİ concept presented in Figure 4-2, the model can be easily separated into three parts. The details are presented hereunder.

4.2.2.1 Main patch for SSHI

The main patch concerned with our new concept is designed to act like a classic
SSHII technique. The schematic diagram of a SSHII technique is shown in Figure 4-5. The fundamental concept of SSHII is to use an inductor \( L \) and achieving a LC\(_0\) resonance between piezo-patch and \( L \). Through LC\(_0\) resonance and switches to confine the current flow, more power can be harvested from the piezo-patch. Assuming the structure is excited at the mechanical resonance frequency, the excitation source can be modeled as a current source \( I_{eq} \). Figure 4-6 shows the waveform of series-SSHII including voltage across piezoelectric \( V_P \), current source \( I_{eq} \) and displacement \( x \). The detail equations are already derived and discussed in the chapter 2. The voltage crossing the load resistor \( (V_C) \) and power output \( (P) \) can be expressed as equation (0.0) and equation (0.0). The theoretical results and experimental results will be shown, calculated and compared in the next sub-section.

![Schematic diagram of SSHII piezoelectric energy harvesting device with full bridge rectifier to a resistor load.](image)

Figure 4-5. Schematic diagram of SSHII piezoelectric energy harvesting device with full bridge rectifier to a resistor load.
Figure 4-6. Waveform of the SSHI piezoelectric energy harvesting device.

\[
\dot{V}_c = \frac{2\alpha R \left( 1 + e^{-\frac{\pi}{\omega_0}} \right)}{2R\omega_0 C_0 \left( 1 + e^{-\frac{\pi}{\omega_0}} \right) + \pi \left( 1 - e^{-\frac{\pi}{\omega_0}} \right)} \omega_0 \dot{x}
\]

\[
P = \frac{\dot{V}_c^2}{R} = \frac{4\alpha^2 R \left( 1 + e^{-\frac{\pi}{\omega_0}} \right)^2}{2R\omega_0 C_0 \left( 1 + e^{-\frac{\pi}{\omega_0}} \right) + \pi \left( 1 - e^{-\frac{\pi}{\omega_0}} \right)} \omega_0^2 \dot{x}^2
\]

4.2.2.2 Auxiliary patch for supplying comparator

The second piezoelectric patch is designed to create two stable supply voltages $+V_{cc}$ and $-V_{cc}$ to supply energy to a comparator and make V-SSHI self-powered and self-switched. The velocity control input signal of the comparator is discussed in next part and the output signal of the comparator drives the two switches (NMOS and PMOS)
pair). The equivalent circuit of the supplying circuit is depicted Figure 4-7. The two diodes $D_A$ and $D_B$ rectify the positive and negative current flow, $C_r$ regulates the voltage between $+V_{CC}$ and $-V_{CC}$, $C_P$ regulates the voltage between $+V_{CC}$ to GND and $C_N$ regulates the voltage between $-V_{CC}$ to GND. The voltage $V_{CC}$ can be obtained by integrating from the interval $T_1$ to $T_3$ as equation (0.0) and the output power can be represented by $V_{CC}^2 / R_{eq}$ as equation (0.0). Resistor $R_{eq}$ is the equivalent load between $+V_{CC}$ and $-V_{CC}$.

![Equivalent Circuit](image1.png)

Figure 4-7. (a) The equivalent circuit diagram of the supplying circuit. (b) Waveform of the supplying circuit.

$$V_{CC} = \frac{\alpha R_{eq}}{(C_0 R_{eq} \alpha_b + \pi) D} \hat{F}_k$$

(0.0)
4.2.2.3 Sensor patch for velocity control

The third patch is designed for generating the velocity control signal. The equivalent circuit is shown in Figure 4-8(a). A low value resistor $R_C$ is connected in parallel with the patch to sense the mechanical current $I_{eq}$. A passive low-pass filter is used to reduce the high frequency noise. When SSHI works, the high frequency noise of the velocity signal is very large, so it is impossible to apply directly voltage $V_P$ to comparator and the high frequency noise will make the comparator output unstable during the switching interval. The current sensing resistor used herein must be small enough to avoid the effect of the piezoelectric capacitance. The low-pass filter should be carefully designed to guarantee there is no phase lag for the considered frequency. The key waveforms are also shown in Figure 4-8(b). The blue line $V_P$ is the open-circuit waveform of the piezoelectric patch and the red line $V_S$ is the velocity control signal which is in phase with $I_{eq}$. There is 90 degree phase lag inherently between $V_S$ and $V_P$. When the circuit switches by velocity control, the switching time can be accurate; the current is always in phase with voltage when SSHI works. The power output from
piezoelectric can be always positive.

![Equivalent Circuit Diagram](image1)

![Waveforms](image2)

**Figure 4-8.** (a) The equivalent circuit diagram of the sensor patch (b) Waveforms.

### 4.3. Experimental results and discussion

#### 4.3.1 Experimental setup

The experimental structure under testing is a cantilever steel beam. Three 31-type PZT-QA piezoelectric ceramic patches provide by the company ELECERAM were bonded on the beam. Table 4-1 gives the dimensions of the beam and the patches. Figure 4-9 shows the experimental setup and the self-powered V-SSH1 circuit diagram. Figure 4-10 shows a picture of the experimental setup. In the experimental setup, the SSH1 circuit part is a little bit different from the one of Figure 4-6, but it works identically. The four diodes act like a full bridge rectifier to confine the current flow and the inductor is in series with P1 patch.
The cantilever beam is excited by a vibration shaker (LDS-V406). Three piezoelectric patches (P₁ to P₃) are bounded close to the fixed end. An accelerometer (PCB-353B03) is situated at the fixed end to measure acceleration; a laser vibrometer (LK-G32) measures the displacement at the free end. P₁ is the main patch for harvesting power. The circuit connected to P₁ for SSHI is composed of several parts: an inductor L for LC₀ resonance to enhance the power; four Schottky diodes (D₁ to D₄) for confining the current flow; the load composed of a resistor and a capacitor; NMOS (2N7002) and PMOS (NDS0610) pair for positive and negative switching. There are two stages for switching:

- when velocity crosses zero from negative to positive, voltage P₁ is at the maximal positive value, the NMOS is switched at this time and the SSHI process will occur through the path L-D₁-LOAD-D₃-NMOS.

- the negative stage works with the same logic through the path L-D₂-LOAD-D₄-PMOS.

Patch P₂ is connected to circuit composed of two Schottky diodes and three capacitors to generate the positive voltages +V_CC and −V_CC for supplying comparator (TLV3701). Voltage V_CC should be larger than 2.5V to make sure that the comparator fully works to drive MOSFET. The comparator chosen here is a nano-power comparator from TI and the supplying current is only 560nA/per channel. This nano-power
comparator is very easy to drive and suitable for low power circuit design.

Patch P₃ is designed for velocity control. It is connected to a current sensing resistor followed by a passive low-pass filter. The resistor used herein is small enough to make sure there is no phase lag. The velocity signal noise (sine wave ideally) is attenuated by the low-pass filter. A comparator is used in order to generate the switching signal (square wave ideally) to drive NMOS and PMOS. The low-pass filter is designed to reduce high-frequency noise without phase lag. Figure 4-11 shows the experimental waveforms of the self-powered V-SSHI device.

<table>
<thead>
<tr>
<th>Table 4-1. Dimension of the electromechanical transducer.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel beam</td>
</tr>
<tr>
<td>Length x Width x Thickness</td>
</tr>
<tr>
<td>168.5mm x 94.3mm x 15mm</td>
</tr>
<tr>
<td>First bending mode</td>
</tr>
<tr>
<td>41.4Hz</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Piezoelectric paths (PZT-QA)</td>
</tr>
<tr>
<td>P1</td>
</tr>
<tr>
<td>38.1mm x 16.5mm x 0.5mm</td>
</tr>
<tr>
<td>P2</td>
</tr>
<tr>
<td>15mm x 5mm x 0.5mm</td>
</tr>
<tr>
<td>P3</td>
</tr>
<tr>
<td>15mm x 5mm x 0.5mm</td>
</tr>
</tbody>
</table>
4.3.2 Experimental results

Figure 4-11 shows three waveforms:
- black line $V_P$ is the waveform of the piezoelectric patch $P_1$,

- blue line $V_{Cout}$ is the output waveform of the comparator,

- red line $V_S$ is the velocity control signal after the low-pass filter.

Although there is still some high frequency noise in the velocity control signal $V_S$, the comparator work still well; it is a trade-off between reducing noise and phase lag. Observing waveform $V_P$, we can note that the switching time occurs almost at the peak value of the voltage. The model parameters, identified by measurements, are given in Table 4-2. The experimental and theoretical results of output power are shown in Figure 4-12. All experimental data are acquired for the same acceleration ($a = 2.5 \text{m/s}^2$). The theoretical curves for standard DC and standard SSHI are drawn from equations (0.0), equation (0.0) and parameters are in Table 4-2. The standard DC experiment (measured using Figure 4-4(a) circuit) and "SSHI-Experiment" (measured by power supply and function generator using Figure 4-5 circuit) are the reference lines compared to theoretical lines; results show good agreement with predictions. Piezoelectric patch $P_3$ can be replaced by a smaller one. So, in the experiments of this study, the effect of patch $P_3$ is neglected. The experimental results (blue point) called "SSHI-Experiment" are measured on the conventional SSHI technique powered by external switching signal. The self-powered V-SSHI technique (red point) is measured by only one patch $P_1$.
output power of V-SSHI circuit is lower than the one of “SSHI-Experiment” circuit, because the energy is split to supply the auxiliary self-powered circuit. In order to establish the self-powered system, there has to spend parts of energy to supply the electrical circuit. The conventional self-powered technique proposed by [56] is the line with green points. Experimental results show that the maximum output power of self-powered V-SSHI is higher than the conventional technique, essentially due to the efficient phase control.

Figure 4-11. Experimental waveform of the self-powered V-SSHI.
Table 4-2. Measurements and model parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0$</td>
<td>Short circuit resonance frequency</td>
<td>41.41 Hz</td>
</tr>
<tr>
<td>$f_1$</td>
<td>Open circuit resonance frequency</td>
<td>41.45 Hz</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Open circuit damping coefficient</td>
<td>0.00105</td>
</tr>
<tr>
<td>$Q_1$</td>
<td>Quality Factor</td>
<td>2.6</td>
</tr>
<tr>
<td>$V_D$</td>
<td>Diode drop voltage</td>
<td>0.3 V</td>
</tr>
<tr>
<td>$C_0$</td>
<td>Clamped capacitance of the piezoelectric element</td>
<td>$P_1$ = 25nF, $P_2$ = 3.5nF, $P_3$ = 3.5nF</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Force-voltage coupling factor</td>
<td>0.00069 N/V</td>
</tr>
<tr>
<td>$k^2$</td>
<td>Electromechanical coupling coefficient</td>
<td>0.00193</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass</td>
<td>182 g</td>
</tr>
<tr>
<td>$K^E$</td>
<td>Equivalent stiffness of the structure</td>
<td>12320 Nm$^{-1}$</td>
</tr>
<tr>
<td>$D$</td>
<td>Damping ratio of the structure</td>
<td>0.1 Nm$^{-1}$ s$^{-1}$</td>
</tr>
</tbody>
</table>

Figure 4-12. Experimental results.
4.4 Conclusion

In this chapter, a self-powered V-SSHI piezoelectric energy harvesting is proposed and this is a new design concept which is different from the traditional design. Based on the outstanding performance of SSHI technique, the self-powered V-SSHI circuit is fully self-powered, requiring no external power supply and though the velocity control, the switching time can be more accurate than with state-of-the-art techniques. The performance of the conventional self-powered circuit is close to the theoretical values of the SSHI; however, it requires an excitation level high enough to work properly. In the self-powered V-SSHI technique, the excitation level doesn't influence the performance and when the supply voltage of the comparator is larger than 2.5V, the whole circuit fully works. The experimental results show better performance and lead to a gain of around 200% compared to the standard DC approach. Of course, the V-SSHI output power is lower than the theoretical SSHI, because the energy is split to supply the auxiliary self-powered circuit. The architecture proposed in this chapter is more beneficial and represents a new step of the design concept. This circuit is easily used in real applications and may be combined with wireless sensor networks.
Chapter 5 Study of a Piezoelectric Switching Circuit for Energy Harvesting with Bistable Broadband Technique by Work-cycle Analysis

In order to increase the output power of the piezoelectric energy harvesting in all aspect including mechanical part design and electrical part design, in this chapter, a piezoelectric energy harvesting device comprised of a bistable vibrating cantilever beam and a switching-type interfacing circuit (SSHI) is proposed, and the resulting performance are compared to the traditional linear technique. The main contribution focuses on combining two non-linear techniques to achieve an efficient broad band piezoelectric energy harvesting device. It was known that the synchronized switching techniques increase efficiency and the output power of the piezoelectric energy harvester for low-coupled structures. However, the traditional piezoelectric energy harvester based on a cantilever beam is only efficient at resonance. To broaden the available bandwidth, a bistable non-linear technique was proposed. In this paper, the bistable technique and SSHI interface are combined together to accomplish a more efficient broadband piezoelectric energy harvester. The power flow and work-cycles are adopted to simplify the analysis of the switching techniques and then summarize the increasing performance of the non-linear piezoelectric harvester. Finally, simulation results and experimental validations show that the proposed integrated device owns
larger bandwidth and collects more harvested energy

5.1 Introduction

Although the piezoelectric materials exhibit high power density, the linear piezoelectric energy harvester are efficient only when the mechanical system is excited at the resonance frequency; there is largest strain, largest vibration displacement and maximum output power compared to work at non-resonant frequency. However, in practice, the exciting frequency of the ambient vibration source is random and it varies within a frequency range [26]. It is impossible to excite the energy harvester at specific resonance frequency and to keep the system operating on the maximum power point. In the cantilever beam system, the mechanical quality factor is commonly very high. It causes that the device has high harvesting power only at single resonant frequency. In order to increase the power at non-resonant frequency, designing a mechanical system to work in a wide frequency range is necessary. This design concept to enlarge the frequency bandwidth is based on applying external forces. By applying an axial force, the resonant frequency of a piezoelectric cantilever beam can be successfully tuned [57, 58], but these methods are active techniques and the mechanical system is still operated within the linear regime. Another method consists to make a non-linear or bistable vibration of a cantilever beam to enlarge the workable bandwidth [59-62]. By using
simple fixed magnets, this passive technique makes the mechanical system improve the harvesting efficiency within non-resonant regime without any external power.

In this chapter, the performances, drawbacks and system requirements of magnetic non-linear piezoelectric generators combined with the SSHI technique, shown in Figure 5-1(c), will be discussed. According to comparisons with others linear standard generator (Figure 5-1(a)) and linear SSHI generator (Figure 5-1(b)), the voltage waveform across the piezoelectric element and displacement are used to show the efficiency of the bistable piezoelectric energy harvester through the work-cycle representation. The simulation and experimental results show that the SSHI technique is advantageous over the standard interface for both linear and non-linear cases and non-linear case is advantageous over linear for both the standard interface and SSHI techniques. The theoretical analysis, equivalent circuit model, simulation and experimental results will be presented in following sections.
5.2 Electromechanical Linear Model

As detailed theoretical analysis in the chapter 2, a mechanical model based on a spring–mass system gives a good description of the vibration behavior near the resonance of the host structure. Therefore, for simplicity, this system can be modeled as a one degree-of-freedom system of a mass $M$, a spring $K^E$ and a damper $D$. According to dynamics equation, the differential governing equation of this electromechanical system can be expressed as equations (0.0).

$$M\ddot{x} + D\dot{x} + K^E x + \alpha V_p = F_E$$  \hspace{1cm} (0.0)

where $F_E$ is external force and $x$ is displacement exerted on the host structure. The
equation (0.0) is linear equations. However, the bistable energy harvesting technique is a non-linear method essentially, and it is not easy to analyze. To make the analysis more intuitive, the work-cycle (or energy cycle) is adopted here to analyze the non-linear circuit and vibration. The work-cycle is the trace in piezoelectric force-displacement plane. The observing point of interest is the power generated from the mechanical part. At this point, the average power converted into the electric part in a period can be expressed as equation (0.0).

\[ P = \frac{1}{T} \int_{0}^{T} \alpha xV_p \, dt \] (0.0)

Where \( T \) represents the period of the vibration, i.e. \( T = \frac{2\pi}{\omega} \). Accordingly, the energy flowing out of the piezoelectric in one vibration cycle can be expressed as equation (0.0).

\[ E = \alpha \int_{0}^{u(T)} V_p \, dt \] (0.0)

The integration in equation (0.0) stands for the area in the force-displacement plane, representing the energy flowing out of the piezoelectric element. The real energy, which
flows out of the piezoelectric element, is the key issue in energy harvesting design. The energy that flows out of the piezoelectric element is larger when the vibrating energy harvested by the electronic circuit in each cycle is larger.

5.3 Switching Control Strategy

The equivalent circuit of the single-mode piezoelectric harvester including the switching circuit is represented in Figure 5-2. In this figure, \( \dot{x} \) represents the velocity of the host structure at a particular location, which also can be viewed as the current in the equivalent circuit. The voltage \( V_P \) is the voltage across the piezoelectric element. In this following, \( V_P \) is directly named piezoelectric voltage for simplicity.

![Figure 5-2. Equivalent electric circuit of the single-mode piezoelectric harvester.](image)

Usually in energy harvesting applications, the piezoelectric patches and structure
are weakly coupled. This means that the energy extraction from piezoelectric patches
doesn’t disturb the vibration behavior of the structure and the magnitude of velocity \( \dot{x} \) can be assumed unchanged. As we mentioned in the chapter 2, for weakly coupled structure \( k^2 Q_m \) is lower than 2 [77]. When \( k^2 Q_m \) is lower than 2, the SSHI technique can effectively increase the power than the standard interfacing circuit.

According to equation (0.0) for the weakly coupled structure, the purpose of the switching circuit is to change the waveforms of piezoelectric voltage \( V_p \) to enlarge the extracted energy and to keep similar magnitude of velocity. According to equation (0.0), in order to have the best performance, the piezoelectric voltage \( V_p \) should be in phase with velocity \( \dot{x} \) and the voltage amplitude should be large to harvest the larger energy as well. The circuits studied here are the standard DC rectifier (Figure 5-1(a)) and the series-SSHI technique (Figure 5-1(b)) operated at resonance. Then these two initial techniques are applied to a non-linear bistable structure (Figure 5-1(c)). The key waveforms are given in Figure 5-3.
5.4 Series-SSHI Technique

The SSHI technique used here is like the classical series-SSHI technique and the detailed theoretical analysis is studied and discussed in the chapter 2. In this sub-section, we just talk and show the significant equations, waveforms and behavior. Figure 5-3(c) shows the theoretical waveforms of series-SSHI. When the vibration velocity crosses zero, the switch is conducted, the inductor L and the piezoelectric capacitor C₀ begin to oscillate. This resonant circuit increases the magnitude and changes the polarity of the voltage across the piezoelectric capacitance sinusoidally, and thus put voltage \( V_P \) and
velocity \dot{x} in phase, which indicates that more energy is extracted from the vibration source.

To quantify the performances of energy harvesting devices, the force-displacement diagram is employed to illustrate the energy conversion cycle. In the case of a purely capacitive load on the piezoelectric element, the displacement and voltage are in phase; the area of the cycle is null, so the harvesting energy is equal to zero. When a resistive load is added, a phase shift appears between displacement and voltage (Figure 5-3(a)).

Figure 5-4 shows the force-displacement locus under three conditions. The first condition corresponds to the simple resistive load. The area enclosed by the locus represents the vibratory energy converted into electrical energy. The second condition corresponds to the full-wave bridge rectifier. The extracted energy by the full-wave bridge is lower than in the case of simple resistance because the maximum piezoelectric voltage is lower, but the maximal value of the displacement remains the same. The third condition corresponds to the series-SSHI technique. The energy harvested by the series SSHI technique is much higher than the previous cases because the \( LC_0 \) resonance increases the magnitude of \( V_p \) in the low coupling condition and the magnitude of displacement does not change.
From the energy conversion cycle shown in Figure 5-4, and based on the geometric relations among the area in different colors, we can calculate the extracted energy. For the series-SSHI, the transferred energy $E_{\text{SSHI}}$ can be expressed as follows:

$$E_{\text{Series–SSHI}} = 2 \frac{\alpha^2 (1 + q_{LC})}{C_0 (1 - q_{LC})} \hat{x}^2$$ \hspace{1cm} (0.0)

where $q_{LC} = e^{-\frac{\pi}{2Q_{0}\phi}}$ is a function of the quality factor $Q_{LC}$ of the resonant $LC_0$ circuit. The usual value of the $q_{LC}$ is around 0.7 in the normal experiment because if the
system have very high quality factor it will lead bulky and expensive inductor [36]. For the standard DC rectifier, the transferred energy $E_{DC}$ can be expressed as follows:

$$E_{DC} = 2 \frac{\alpha^2}{C_0} x^2$$ \hspace{1cm} (0.0)

The conclusion that we can get from the work-cycle observations is we can evaluate the performances of the energy harvesting circuits by the size of the area. This maximum corresponds actually to the rectangular shape in the force-displacement plane for the same external voltage and displacement.

5.5 Bistable Energy Harvester

The most piezoelectric energy harvesting system is a linear electromechanical device excited at resonance. Considering that most realistic vibration environments are more accurately described as multi-frequency and time varying, narrowband linear systems are inefficient under these conditions. Non-linear systems, on the other hand, are capable of responding over a broad frequency range. The solution is to use a bistable inertial oscillator comprised of permanent magnets and a piezoelectric cantilever beam.
The bistable behavior is obtained with two magnets. One is mounted on the tip of the beam and the other one is fixed on a stage. Because the two magnets repulse to each other, the system will be a bistable system and there will be two possible stable positions as shown in Figure 5-5. When the distance between these two magnets is designed properly, the non-linear behavior can broaden the available bandwidth [57, 88].

Figure 5-5. Principle of the broadband energy harvesting device with a destabilized zero equilibrium position.

The non-linear magnetic repulsion force $F_M$ given by the interaction of the magnets can be simplified to one-dimensional model and it is acting only in vertical direction [88]. The magnetic force $F_M(x)$ is a variable value and depends on the displacement of the cantilever beam $x$ and the distance between the moving and the fixed magnets. By using the curve fitting method, the magnetic force $F_M$ for a specific distance can be
expressed as equation (0.0) [59].

\[ F_M(x) = \frac{ax}{1 + bx^3} \]  

where a and b are the fitted parameters.

In order to analyze the non-linear energy harvester with an electric interface, in this chapter we adopt an electric equivalent impedance representation. According to equations (0.0) and (0.0), the equivalent circuit model can be represented as shown in Figure 5-6. The host structure with piezoelectric elements mechanical is modeled by the classical equivalent circuit. The magnetic force \( F_M \) is taken into account by adding a non-linear magnetic feedback loop. The main advantage of this equivalent circuit is that it can be easily simulated and does not need to use numerical methods. This method has some limitations. First of all, it considers the equivalent spring as a linear one in which the stiffness is independent of the position of the mass. In the present case, this is true only when the displacement is small with respect to the distance between the moving and the fixed magnets. When this hypothesis is not verified, significant errors can take place.
5.6 Simulation, experimental results and discussion

5.6.1 Experimental setup

In order to demonstrate the performances of the energy harvesting devices, a simple experimental test was performed on a clamped cantilever steel beam with 31-type PZT-QA elements provided by the Eleceram Technology Co., Ltd. There were two piezoelectric elements. The first one is the main element to harvest the energy, and it was connected to the series-SSHI interface. The second one is smaller size, and it was used only to sense the velocity and to generate the driving signal for the switches of the SSHI interface [38]. The electronic components in this experiment were supplied by an external DC source. A picture of the tested beam and SSHI circuit is shown in Figure 5-7 and the detailed experimental setup is presented in Figure 5-8. The distance between
the two magnets was 3.5 mm. The dimensions of the beam and the piezoelectric elements are shown in Table 5-1.

Figure 5-7. (a) Experimental beam structure (b) SSHI circuit.
The beam was excited at the fixed end by an electromagnetic shaker (Brüel & Kjær 4809). The shaker is driven by a data acquisition card (NI-DAQ USB-6259). To realize the SSHI circuit, two diodes and two MOSFET switches (Metal Oxide Semiconductor Field Effect Transistor) were used. When the velocity signal goes zero crossing from negative to positive, the NMOS switch (IRFU210) is switched-on and when the signal...
goes zero crossing from positive to negative, the PMOS switch (IRF9640) is switched-on. The two diodes confine the current flow, and the inductor L resonates with the clamped capacitor $C_0$ of the piezoelectric-element. Parameters of the model were identified from the experimental measurements. The tip displacement of the beam $x$ was measured when the piezoelectric element is in open circuit and in short circuit. $D, M, K^E, \alpha$ and $C_P$ were calculated with Equations (0.0) to (0.0).

\[
\alpha = \frac{V_{op}}{x_{op}} C_0 \quad (0.0)
\]

\[
K^E = \alpha \frac{V_{op}^2}{x_{op}} \frac{f_{sh}^2}{f_{op}^2 - f_{sh}^2} \quad (0.0)
\]

\[
M = \frac{K^E}{\omega_{op}^2} \quad (0.0)
\]

\[
D = 2\zeta M\omega_{op} \quad (0.0)
\]

where $V_{op}$ is the open-circuit measured piezoelectric voltage for a given tip displacement $x_{op}$ of the beam. The parameter model values are given in Table 5-2.

The parameter $b$ of the magnetic force, in equation (0.0), sets the static beam tip displacement and parameter $a$ sets the maximum value of the magnetic force as shown.
in Figure 5-9. First, the parameter $b$ was calculated according to the experimental measure of displacement $x_{\text{max}}=1.2\text{mm}$. Then, the parameter $a$ was obtained by fitting the experimental voltage curve in Figure 5-11 & Figure 5-12. According to the experiment results, the proper parameters could be chosen.

Table 5-2. Measured values and model parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value (unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c$</td>
<td>Open-circuit resonant frequency when all piezoelectric elements are in open circuit</td>
<td>10.4 Hz</td>
</tr>
<tr>
<td>$f_p$</td>
<td>Short-circuit resonant frequency when all piezoelectric elements are in short circuit</td>
<td>10.307 Hz</td>
</tr>
<tr>
<td>$k_e$</td>
<td>Electromechanical coupling coefficient</td>
<td>0.0018</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Damping ratio</td>
<td>0.02</td>
</tr>
<tr>
<td>$Q_m$</td>
<td>Mechanical quality factor</td>
<td>2.09</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass</td>
<td>49 g</td>
</tr>
<tr>
<td>$k_a$</td>
<td>Equivalent stiffness when all piezoelectric elements are in short circuit</td>
<td>209.22 N/m</td>
</tr>
<tr>
<td>$k_c$</td>
<td>Equivalent stiffness when all piezoelectric elements are in open circuit</td>
<td>211 N/m</td>
</tr>
<tr>
<td>$D$</td>
<td>Damping coefficient</td>
<td>0.15 N/m/s</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Force–voltage factor</td>
<td>0.00007716 N/V</td>
</tr>
<tr>
<td>$\theta_c$</td>
<td>Inversion factor</td>
<td>0.7</td>
</tr>
<tr>
<td>$\zeta_c$</td>
<td>Clamped capacitance</td>
<td>15.57 nF</td>
</tr>
<tr>
<td>$L$</td>
<td>Resonance inductor in SSHI</td>
<td>10 mH</td>
</tr>
<tr>
<td>$a$</td>
<td>Fitting magnetic force parameter</td>
<td>280</td>
</tr>
<tr>
<td>$b$</td>
<td>Fitting magnetic force parameter</td>
<td>$1.5 \times 10^{11}$</td>
</tr>
</tbody>
</table>
Figure 5-9. The magnetic force $F_M$ as a function of the beam tip displacement $x$.

5.6.2 Frequency sweeping

The main interest of this work is broadening the frequency range from which energy can be extracted. To show the interest of bistable non-linear technique to broaden the available bandwidth the excitation frequency was linearly increasing. This was accomplished using an excitation of the form $\gamma = A_0 \cos \left( (\omega_0 + \omega_s t) t \right)$ where $\omega_0$ is the initial pulsation and $\omega_s$ is the frequency sweep rate. $A_0$ is the amplitude of acceleration: $2 \text{m/s}^2$.

The experimental testing was performed on the linear and non-linear clamped cantilever beam shown in Figure 5-8. The simulation was carried out with Matlab and PSIM software packages as shown in Figure 5-10. The module Simcoupler in PSIM software is used to make a link between Simulink in Matlab and PSIM. The driving chirp frequency of the input force is sent from Matlab to the electric circuit implemented in PSIM. The simulated results will be sent back to Matlab and organized. Figure 5-11 and Figure 5-12 show the experimental and simulation results of increasing frequency sweeping for the case of standard DC rectifier interface and SSHI interface, respectively. The experimental driving signal was chirping with frequency range from 5 Hz to 30Hz in 250 seconds. The simulation driving signal was ranging from 1Hz to 30
Hz in 300 seconds and its sweeping rate was the same as the experimental conditions ($\omega_s=0.1$ Hz/s). The sweeping rate was kept sufficiently small in order to reflect the non-linear response [89]. The output voltage for the linear system without magnetic force and the output voltage for the case with the magnetic force are plotted in Figure 5-11. The load resistance was chosen 2M$\Omega$ to show the piezoelectric terminal voltage. Comparing to the experimental results, the simulation shows good agreement with the experimental data. The results of Figure 5-11 & Figure 5-12 show that the piezoelectric voltage $V_P$ at 10.4 Hz (resonant frequency in linear system) is almost the same for the both systems, but at non-resonance frequencies the bistable system can improve the output power obviously. The non-linear effect at the resonance is limited unlike in the non-resonant region. At the resonance, the driving force from resonant effect is larger than the magnetic force in our experiment, so the non-linear magnetic coupling technique cannot work effectively. The results also show considerable chaotic motion when f<5 Hz and between 10Hz and 17Hz. For other frequencies the motion is would be periodic response. This result is in agreement with study in Stanton et al. [88] and Thompson [90]. Over a wide frequency range, there is enough energy imparted into the bistable system to enable drive the beam from one stable position to the other.

In the non-linear system there is a critical frequency when the potential energy is
not enough to drive the system from one stable position to another [62, 91]. In our experimental results shown in Figure 5-11 & Figure 5-12, the critical frequency is around at 23Hz. When the driving frequency is higher than 23Hz, the piezoelectric terminal voltage in non-linear system is the same as in linear system.

Figure 5-10. Non-linear simulation setup (a) Matlab Simulink and (b) PSIM
Figure 5-11. (a) Experimental results (b) simulation results of nonlinear piezoelectric energy harvester combined with standard DC rectifier interface: increasing frequency sweeps.

Figure 5-12. (a) Experimental results (b) simulation results of nonlinear piezoelectric energy harvester combined with SSHI interface: increasing frequency sweeps.
5.6.3 Work cycles study

Two specific frequencies (at-resonance: \( f = 10.4 \) Hz and off-resonance: \( f = 5 \) Hz) were chosen to be the examples to analyze the work cycles. Figure 5-13 and Figure 5-14 show simulation results of the voltage \( V_p \) across the piezoelectric element, the velocity and the voltage-displacement diagrams at resonance frequency of the structure (10.4 Hz) for standard DC rectifier and SSHI technique. The maximum displacement is \( x_{\text{max}} = 2.5 \) mm. According to the Equation (9), the energy by period for the SSHI technique is \( E_{\text{SSHI}} = 13.5 \mu J \) and thus the power is \( P_{\text{SSHI}} = 140.4 \mu W \). According to the Equation (10), the \( E_{\text{DC}} = 2.39 \mu J \) and power for standard DC rectifier is \( P_{\text{DC}} = 24.86 \mu W \). Figure 5-15 and Figure 5-16 show voltage \( V_p \) across the piezoelectric element, the velocity and the voltage-displacement diagram at non-resonance frequency of the structure (5Hz) for standard DC rectifier and SSHI technique.

Figure 5-13 to Figure 5-16 clearly show that the SSHI interface enlarges the work-cycle area by increasing the piezoelectric voltage and non-linear bistable technique increase the work-cycle area by increasing beam displacement at non-resonance. Therefore, if we compare the energy harvested at the non-resonance frequency (Figure 5-15 and Figure 5-16) the work-cycle area of bistable device is much wider. That means that for both cases, standard DC rectifier and series SSHI, the
bistable device keeps good performances at the would-be resonance but increases them at the non-resonance.

Figure 5-17 shows the output power for the SSHI technique in the case of bistable and linear devices and for the two frequencies: at-resonance 10.4Hz and non-resonance frequency 5Hz. The maximum output power at resonance frequency (10.4 Hz) is 0.14 mW. This power at non-resonance frequency (5 Hz) with linear device is only 0.3μW, but with bistable vibration the output power is 8μW. Comparing bistable system to linear system results, the output power close to the resonance frequency is almost the same, but more energy can be harvested in bistable system at non-resonant frequency when the displacement is large enough to drive the beam from one stable position to the other stable position.

Figure 5-13. Standard DC rectifier @ f = 10.4 Hz, (a) Piezoelectric voltage and velocity (b) Work cycle.
Figure 5-14. Series SSHI @ f = 10.4 Hz, (a) Piezoelectric voltage and velocity (b) Work cycle.

Figure 5-15. Standard DC rectifier @ f = 5 Hz, (a) Piezoelectric voltage and velocity (b) Work cycle.
Figure 5-16. Series SSHI @ f = 5 Hz, (a) Piezoelectric voltage and velocity (b) Work cycle.

Figure 5-17. Experimental results of the output power for SSHI technique.

5.7 Conclusion

This chapter studies the performances of magnetic non-linear piezoelectric generator combined with a series-SSHI interface in the weak coupling case. The
equations of motion for a one-degree-of-freedom piezoelectric cantilever beam with magnetic non-linear force were derived and an equivalent electric circuit is proposed. Then, this equivalent electric circuit is used to simulate a bistable piezoelectric generator with the series-SSHI technique. Finally, the non-linear generator was tested experimentally and compared with work-cycle to standard interface and linear technique. The SSHI technique has proved that it is an effective technique to improve output power over the standard interface in both linear and bistable cases. Moreover, the non-linear coupling technique has proved that it is not only advantageous over linear technique for standard interface but also for SSHI interface. It is interesting to combine these two remarkable techniques and the results show that these two techniques can work well together. The SSHI interface enlarges the work-cycle area by increasing the piezoelectric voltage in the weak coupling case and non-linear bistable technique increase the work-cycle area by increasing beam displacement inducing voltage to increase. According to the analysis of work-cycles, the synchronized switching interface and non-linear bistable technique are two major factors for designing a broad bandwidth and efficient energy harvester. Through these two non-linear techniques, the piezoelectric harvester can work more efficiently and more output power at a broadened frequency range can be gained.
Chapter 6 Self-Powered Semi-Passive Piezoelectric Structural Damping Based on Zero-Velocity Crossing Detection.

In recent years, semi-passive vibration damping using non-linear synchronized switching methods has been intensively investigated and discussed. In this chapter, a self-powered synchronized switch damping on inductor (SSDI) technique based on zero-velocity crossing detection is proposed and investigated. The control signal used to drive the switches is obtained by sensing velocity as we used in the self-powered V-SSHI technique in energy harvesting. A totally self-powered damping system powered by harvested energy using SSDI technique with velocity sensing and without external power is established. Compared with the conventional technique based on voltage peak detector, this technique do not generate lag in detection of switching time. The theoretical model, the experimental evaluation and the drawback of the self-powered zero-velocity crossing detection switching technique are discussed in this study. The system performance is also compared with the externally powered system.

6.1 Introduction

Many successful applications of piezoelectric materials for structure vibration suppression have been developed in recent decades. In these applications, piezoelectric
materials convert the vibration energy of the host structure into electrical energy, and then the generated electrical energy is dissipated in a shunt circuit. The piezoelectric shunt techniques were widely used due to their simple configurations and compact size, but these techniques are better to be self-powered to reduce the system complexity. In some applications, like automotive and aeronautics, the external power is limited [90, 93], the self-powered design can eliminate the requirements of external power supply.

Several versions of self-powered SSDI technique have been proposed [94, 95]. The conventional method is based on peak voltage detection [95, 96] using a small energy storage capacitor. The peak detection is made using an envelope detector: a comparison between envelope and piezoelectric voltage is made with a bipolar transistor. The drawback of this method is the lag in detection of switching time due to the use of the transistor, which degrades the damping performance. The principle of the peak detector method is shown in Figure 6-1(a).

In this chapter, a self-powered SSDI technique based on zero-velocity crossing detection is proposed and investigated. Hereafter this technique in this chapter is called SP-SSDI. Based on the self-powered system used in piezoelectric energy-harvesting devices (V-SSHII) [38], SP-SSDI does not require external instruments. The control signal used to drive the switches is obtained by sensing velocity, and then compare to
zero. A totally self-powered damping system powered by harvested energy using SSDI technique with velocity sensing and without external power is established. The concept of the proposed system is shown in Figure 6-1(b).

The chapter is organized as follows: the second section 6.2 summarize the SSDI technique and present the theoretical analysis. The next section 6.3 presents the detailed analysis of the self powered SSDI based on zero-velocity crossing detection technique. Section 6.4 presents the experimental results that include the time domain evaluation for different excitation levels, the measurement of the efficiency and the system frequency response results. Finally, the last section 6.5 concludes this chapter.

Figure 6-1. (a) Principle of voltage peak detector method (b) Principle of zero velocity crossing detector method.
6.2 SSDI Technique

The electrical circuit of the semi-passive damping technique called SSDI (Synchronized Switching Damping on an Inductance) is represented in Figure 6-2(a). An inductor $L$, a resistance $R$ and a switch $K$ are connected in series with the piezoelectric patch. The piezoelectric patch voltage $V_P$ is switched across the LR shunt circuit. The dissipated energy of the structure depends on the voltage amplitude across the piezoelectric patches. The role of these additional patches is to increase the amplitude of voltage $V_P$ and thus to increase the damping effect. The switch $K$ is turned ON when a maximum of displacement $x$ occurs and the voltage $V_P$ starts to oscillate, until $K$ is turned OFF. The switching ON period is equal to a half of the resonant period of the $LC_0$ circuit. Assuming that the electrical resonant period is very small compared to the mechanical vibration period, voltage $V_P$ can make the inversion in this short period. The same function can be obtained by turning switch $K$ ON at the minimum displacement. Theoretical waveforms of the displacement $x$, the velocity $dx/dt$ and the voltage $V_P$ of the SSDI technique are shown in Figure 6-2(b). The amplitude of voltage $V_P$ is limited by the loss of energy during the inversion process.
The absolute value of the voltage of the piezoelectric patch after inversion is less than the initial voltage. This difference occurs because of the energy losses that occur due to the energy flow between the capacitance and the inductor during the switching interval. The absolute value of the inverted voltage is \( q_{LC} \hat{V}_P \), where \( q_{LC} = \frac{\pi}{2Q} \) is a function of the quality factor \( Q_{LC} \) of the resonant LC\(_0\) circuit.

Analytic calculation of the displacement amplitude at resonance \( \omega_0 \) can be made from previously expressed equations. The expression of the mechanical displacement \( \hat{x}_{SSD} \) as a function of external force amplitude \( F_E \) and the resonant circuit factor \( q_{LC} \) is given in equation (0.0) [75],

\[
\hat{x}_{SSD} = \frac{F_E}{D\omega_b + \frac{4\alpha^2(1+q_{LC})}{\pi C_0(1-q_{LC})}} \quad (0.0)
\]

Figure 6-2. (a) Electric circuit of the SSDI technique (b) Key waveforms of the SSDI technique.
6.3 Self-powered zero-velocity crossing detection for SSDI Technique

Due to the use of switches in the SSDI technique, the shunt circuit requires an external power source. To design a totally self-powered device, a part of extract energy for damping can be used to power the electronics. Moreover, because it is a synchronous technique, the displacement or the velocity must be measured accurately to obtain the driving signal for the switches. The technique proposed here is to divide the piezoelectric patches into three parts. The largest part, called P₁, behaves like the conventional piezoelectric patch used in the SSDI technique to dissipate the vibration energy. The second piezoelectric patch, called P₂, is a smaller patch that works like energy harvesting device to provide power supply to other electronic circuit. The third piezoelectric patch, called P₃, is a smaller patch designed to sense the velocity and to generate driving signal to control the switches at the optimal time. All three patches contribute to damp the structure.

A schematic of the complete electronic circuit is shown in Figure 6-3 with the functions of different sub-circuits labeled in the same figure. The system will be analyzed in detail in the following subsections. To obtain a precise velocity signal, P₃ is cut from the electrode of the lower piezoelectric patch, and the patch is positioned on the centerline of the cantilever, as shown in Figure 6-3.
6.3.1 Zero-velocity crossing detector (piezoelectric-patch P₃)

Figure 6-4 shows the electric circuit and the theoretical waveforms of the zero-velocity crossing detector. Since the mechanical current in the electric equivalent circuit of a piezoelectric patch can be assumed to be velocity; if the output terminal is short-circuited, the current through this short-circuit is proportional to velocity, i.e. \( i_{sh} = \alpha \dot{x} \). To convert this current into voltage, this short-circuit can be made by a small resistance. Thus, voltage across the shunt resistance represents the velocity of the structure. The value of the current-sensing resistance must be much smaller than the output impedance of the piezoelectric patch. As shown in Figure 6-4(b), the velocity...
signal $V_S$ (black curve) is in phase with the output current $dx/dt$ (green curve). To avoid the noise problems when SSDI is active, a low-pass filter is necessary, because the voltage inversion introduces high frequency vibrations into the system. A third-order low-pass filter is used; it is composed of two capacitors and an inductor. The design goal of the filter is to ensure that the high frequency noise is reduced sufficiently to generate an accurate control signal with minimum phase lag to keep the efficiency of the SSDI system. Finally, the filter output voltage $V_S$ is connected to a comparator, and the output of the comparator $V_{C_{out}}$ is used to drive the switches, as shown the red curve in Figure 6-4(b).

![Figure 6-4](image)

**Figure 6-4.** The velocity zero crossing detector: (a) electric circuit (b) theoretical waveforms.

Compared to the conventional switching method using a voltage peak detector, the velocity-synchronized signal should theoretically cause the switches to switch more precisely at the optimal time. Because the conventional peak detector uses the diode drop to detect the peak voltage, it produces a time lag between the peak time and the
switching time.

Figure 6-5 shows the Bode diagram of the filter of zero-velocity crossing detector, as measured by an SR780 dynamic signal analyzer. The Bode diagram shows that the corner frequency is approximately 340Hz. The first natural frequency of the cantilever beam is approximately 34Hz; approximately 10 times lower than the corner frequency of the low-pass filter. The low-pass filter design reduces the high frequency noise of the velocity signal effectively without altering the phase.

![Bode diagram](image)

Figure 6-5. Bode diagram of the filter of the zero velocity crossing detector.

6.3.2 Power supply (piezoelectric-patch P₂)

Figure 6-6 shows the electric circuit and the theoretical waveforms of the power supply part. This circuit provides two DC voltages sources, \( V_{CC} \) and \( -V_{CC} \), to supply power to the comparator and the switches of the SSDI circuit. The power supply circuit
is composed of two diodes (DA and DB) and three capacitors (CP, CN and Cr). The two diodes maintain the correct positive and negative current flows to charge the capacitors. CP regulates the positive voltage between VCC and ground, and CN regulates the negative voltage between –VCC and ground. Cr regulates the voltage between VCC and –VCC and acted as an energy storage buffer. Two Zener diodes Dz are used to limit excursion and to regulate the DC voltage. The two regulated voltages, VCC and –VCC, are connected to the comparator, and the output of the comparator is used to drive the switches. The comparator used in this application is a Nano-power comparator TLV3701 from Texas Instrument (Dallas USA) that sinks a small and constant current (Icomp=560nA) for a voltage VCC higher than 2.5V. Assuming that the equivalent load of the comparator between the two regulated voltages is \( R_{eq} = \frac{2V_{cc}}{I_{comp}} \), the voltage VCC can be expressed as in equation(0.0) [79].

\[
V_{CC} = \frac{1}{2} \frac{\alpha R_{eq} a_0}{C_0 R_{eq} a_0 + \pi} \hat{x} \tag{0.0}
\]

Using equation(0.0) given the relation between \( \alpha \), \( C_0 \) and the parameters of the piezoelectric patches, the voltage VCC can be rewritten as equation(0.0). The parameters
of the piezoelectric patches described in Table 6-1, $C_0$ and $\alpha$ are calculated as equation (0.0).

\[
\alpha = e_{31} w, \quad C_0 = \frac{\varepsilon_{33}^s wt}{t}
\]

\[
V_{CC} = \frac{1}{2} \frac{e_{31}^E 2V_{CC}}{\varepsilon_{13}^s \frac{w}{t} \frac{2V_{CC}}{I_{Comp}} \omega_0} \hat{x} + \pi
\]

Table 6-1. piezoelectric physical parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>Width of the piezoelectric patches</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness of the piezoelectric patches</td>
</tr>
<tr>
<td>$l$</td>
<td>Length of the piezoelectric patches</td>
</tr>
<tr>
<td>$\varepsilon_{11}^E$</td>
<td>Elastic rigidity of equivalent patches in short-circuit</td>
</tr>
<tr>
<td>$\varepsilon_{31}$</td>
<td>Permittivity of piezoelectric patches</td>
</tr>
<tr>
<td>$\varepsilon_{33}^s$</td>
<td>Piezoelectric coefficient of equivalent patches</td>
</tr>
</tbody>
</table>

Since voltage $V_{CC}$ must be higher than 2.5V, we can find a relation between the amplitude of displacement and the size of piezoelectric patch $P_2$, as shown in equation (0.0).
\[ V_{CC} = \frac{e_{31} \omega_0 \dot{x} - \pi I_{comp}}{2 \varepsilon_{33}^s \frac{w_1}{l} \omega_0} \geq 2.5V \] (0.0)

6.4 Experimental results and discussion

6.4.1 Experimental setup

Figure 6-7 shows the experimental setup and pictures of the zero-velocity crossing based SSDI damping system. The experimental structure is a cantilever steel beam with three 31-type PZT-QA patches provided by the Eleceram Technology Co., Ltd. (Taoyuan Taiwan). The fixed end of the beam is excited by a shaker (Brul & Kjaer 4809), and the shaker-driving signal is generated by a DAQ card (NI USB-6259 from National Instrument, Austin USA) on a notebook computer. A vibrometer (LK-G3001P+LK-G32
from Keyence, Osaka Japan) is used to measure the beam tip displacement, and an accelerometer (Brüel & Kjaer 4381 from Brüel & Kjær Sound & Vibration Measurement A/S, Nærum Denmark) is used to measure the acceleration at the fixed end. The piezoelectric voltage, the displacement and the acceleration are recorded by the DAQ card. The dimensions of the cantilever beam and the piezoelectric patches are shown in Table 6-2. The component measured values and model parameters are shown in Table 6-3. The first natural frequency of the cantilever beam is 34Hz. The dimension and the clamped capacitance of the main piezoelectric patch are much greater than those of the two small piezoelectric patches.

Figure 6-7. Experimental setup and pictures.
Table 6-2. Dimensions of the piezoelectric patches.

<table>
<thead>
<tr>
<th>Steel Beam</th>
<th>Length×Width×Thickness</th>
<th>140mm×35mm×0.5mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piezoelectric Patchches</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P1</td>
<td>50mm×65.5mm×0.6mm</td>
<td></td>
</tr>
<tr>
<td>P2</td>
<td>30mm×5mm×0.5mm</td>
<td></td>
</tr>
<tr>
<td>P3</td>
<td>50mm×4.5mm×0.6mm</td>
<td></td>
</tr>
</tbody>
</table>

Table 6-3. Component values and model parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value (unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{op}$</td>
<td>Open circuit resonant frequency when all piezoelectric patch is in open circuit</td>
<td>34 Hz</td>
</tr>
<tr>
<td>$f_{sh}$</td>
<td>Short circuit resonant frequency when all piezoelectric patch is in short circuit</td>
<td>33.97 Hz</td>
</tr>
<tr>
<td>$k^2$</td>
<td>Electromechanical coupling coefficient</td>
<td>0.0018</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Damping ratio</td>
<td>0.02</td>
</tr>
<tr>
<td>$Q_M$</td>
<td>Mechanical quality factor</td>
<td>2.09</td>
</tr>
<tr>
<td>M</td>
<td>Mass</td>
<td>28 g</td>
</tr>
<tr>
<td>$K^E$</td>
<td>Equivalent stiffness when all piezoelectric patch is in short circuit</td>
<td>1276 N/m</td>
</tr>
<tr>
<td>$K^D$</td>
<td>Equivalent stiffness when all piezoelectric patch is in open circuit</td>
<td>1278 N/m</td>
</tr>
<tr>
<td>D</td>
<td>Damping coefficient</td>
<td>0.24 N/m/s</td>
</tr>
<tr>
<td>$\alpha^{P1}$</td>
<td>Force-voltage factor of $P_1$</td>
<td>0.000368 N/V</td>
</tr>
<tr>
<td>$\alpha^{P2}$</td>
<td>Force-voltage factor of $P_2$</td>
<td>0.0000242 N/V</td>
</tr>
<tr>
<td>$\alpha^{P3}$</td>
<td>Force-voltage factor of $P_3$</td>
<td>0.0000227 N/V</td>
</tr>
<tr>
<td>$Q_{LC}$</td>
<td>Quality factor of resonant L-CP</td>
<td>4.4</td>
</tr>
<tr>
<td>R</td>
<td>Equivalent resistor of resonant L-CP</td>
<td>0.48 Ω</td>
</tr>
<tr>
<td>$q_{LC}$</td>
<td>Inversion factor</td>
<td>0.7</td>
</tr>
<tr>
<td>$C_{P1}$</td>
<td>Clamped capacitance of $P_1$</td>
<td>67.6 nF</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Value</td>
</tr>
<tr>
<td>--------</td>
<td>--------------------------------------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>$C_{P2}$</td>
<td>Clamped capacitance of $P_2$</td>
<td>4.44 nF</td>
</tr>
<tr>
<td>$C_{P3}$</td>
<td>Clamped capacitance of $P_3$</td>
<td>4.16 nF</td>
</tr>
<tr>
<td>$L$</td>
<td>Resonant inductor in SSDI</td>
<td>10 mH</td>
</tr>
<tr>
<td>$C_r$</td>
<td>Regular capacitor in supply circuit</td>
<td>4.7 uF</td>
</tr>
<tr>
<td>$C_P$</td>
<td>Regular capacitor in supply circuit</td>
<td>2.2 uF</td>
</tr>
<tr>
<td>$C_N$</td>
<td>Regular capacitor in supply circuit</td>
<td>2.2 uF</td>
</tr>
<tr>
<td>$D_Z$</td>
<td>Zener diode in supply circuit</td>
<td>15 V</td>
</tr>
<tr>
<td>$L_f$</td>
<td>Low pass filter inductor in zero velocity crossing detector</td>
<td>700 mH</td>
</tr>
<tr>
<td>$C_f$</td>
<td>Low pass filter capacitor in zero velocity crossing detector</td>
<td>470 nF</td>
</tr>
</tbody>
</table>

### 6.4.2 Experimental results

Experimental data were taken to validate the self-powered velocity-synchronized semi-passive system presented in this sub-section and to demonstrate the operation of the circuit.

Figure 6-8 shows the displacement, the sensed velocity, the switching signal and the piezoelectric voltage of uncontrolled and self-powered SSDI systems for acceleration of 0.16 m/s$^2$. Comparing the results with and without the SSDI system active, the tip displacement is reduced from approximately 1.12 mm to 0.72 mm by the SSDI damping effect. Because the current-sensing resistance of 1 kΩ is much lower than
the output impedance of the piezoelectric patch (around 1.05MΩ), the signal is very small, with a peak value of approximately 0.01V. When SSDI is active, the high frequency noise is easily introduced into the velocity control signal, but it does not influence the power supply effectiveness because the capacitors in the rectifier regulate the supplied voltage.

Figure 6-8. Experimental results of the zero-velocity crossing detection circuit (a) without SSDI active and (b) with SSDI active (green trace: velocity $V_S$, black trace: $V_{\text{Cout}}$, blue curve: piezoelectric voltage $V_P$, and red trace: beam tip displacement $x$).

The operating limit of the self-powered technique is obtained when the voltage $V_{CC}$ is lower than 2.5V. Once the SSDI circuit works, the decrease of the vibration
magnitude leads to a decrease in terms of harvested power and thus leads to a decrease of voltage $V_{CC}$. Therefore there is a minimum value of the displacement magnitude. Figure 6-8 we can see that the DC voltage $V_{CC}$ is equal to 3.26 V when the displacement magnitude is 1.12 mm, and 2.51 V for a displacement of 0.72 mm. The experimental limit of the system is 0.7 mm to have $V_{CC}$ greater than 2.5V to power the comparator in the circuit. Figure 6-9 shows the theoretical and experimental value of $V_{CC}$ as a function of displacement.

![Image of graph showing Vcc as a function of displacement x]

Figure 6-9. Voltage VCC as a function of displacement x.

This minimum value of the displacement magnitude can be set by the size of the
piezoelectric patch $P_2$. If we consider the thickness is fixed and vary the width, equation(0.0) can be used to predict the minimal displacement magnitude required for the corresponding width. The theoretical minimum value of magnitude of displacement as a function of the width is therefore plotted in Figure 6-10 for thickness, $t=0.5\text{mm}$, and length $l=30\text{mm}$. Figure 6-10 shows that for a width $w=5\text{ mm}$, the theoretical value of the displacement magnitude is $0.7\text{ mm}$.

Figure 6-10. Minimum value of displacement magnitude as a function of width of piezoelectric-patch $P_2$.

### 6.4.3 Comparison

For comparison, experiments were carried out in four different cases; these cases
are listed in Table 6-4.

Table 6-4. Four experimental cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Piezoelectric patches</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Uncontrolled</td>
<td></td>
</tr>
</tbody>
</table>
| 2    | \(P_1 + P_2 + P_3\)    | External energy: Power supply  
Optimal switching point: Function generator |
| 3    | \(P_1\)                | External energy: Power supply  
Optimal switching point: Function generator |
| 4    | \(P_1\)                | No External energy. Using self-powered supplying circuit.  
Switching point: Velocity control circuit. |

- Case 1 is the reference case, used to show the undamped situation. The piezoelectric voltage is in phase with the tip displacement and has a phase lag of approximately 90 degrees with respect to the velocity signal.

- Case 2 is the maximal damping condition. All three piezoelectric patches \((P_1 + P_2 + P_3)\) are controlled using the SSDI damping technique, and the SSDI is operated by external instruments. The power source is an external DC power supply, and the optimal switching signal is provided by a function generator.

- Case 3 provides an experimental control for comparison with case 4. In case 3, only one piezoelectric patch \((P_1)\) is controlled using the SSDI damping technique. The power source is an external DC power supply, and a function generator provides the switching signal.
- Case 4 is the experimental condition and uses the self-powered technique (SP-SSDI) presented in this chapter without any external instruments.

The following two subsections will compare the damping performance of each of the cases when the beam is driven at different excitation levels. The results will be compared in both the time domain and the frequency domain.

**6.4.3.1 Time domain comparison**

The cantilever beam is driven at its first natural frequency (34Hz) at different accelerations. Figure 6-11 and Figure 6-12 show the experimental results of the displacement and the work cycles calculated for the 4 cases for acceleration of 0.13m/s² and 0.16m/s², respectively. To quantify the performances of damping technique, the force-displacement diagram (work-cycle) is employed to illustrate the energy conversion cycle. In the case of a purely capacitive load on the piezoelectric patch (case 1), the displacement and voltage are in phase; the area of the cycle is null, so the extracted energy is equal to zero. When the SSDI technique is active, the LC₀ resonance circuit increases the magnitude of voltage \( V_p \) and decreases the displacement \( x \). The area of the cycle is the extracted energy. Case 2 is the most effective one because all piezoelectric patches are used for the damping control with SSDI technique. This is the
ideal experimental case because the SSDI circuit is powered by external power supply.

If we compare case 3 (external drive) and case 4 (self-powered), we obtain a good evaluation of the zero velocity crossing detection technique (same area of piezoelectric patches with SSDI). For an acceleration of $0.13m/s^2$; the self-powered SSDI circuit damps the structure only up to 0.7mm (0.5mm for the case with external supply) due to the limit of operation of the comparator. For the acceleration of $0.16m/s^2$ and higher, the results show that the amplitude of voltage and displacement are in phase, which means that with the proposed technique the switching occurs at nearly the optimal time. The little difference is due to the high frequency noise in the velocity sense signal.

Figure 6-11. Experimental results (acceleration=$0.13m/s^2$) (a) displacement (b) work-cycle.
Figure 6-12. Experimental results (acceleration=0.16m/s$^2$) (a) displacement (b) work-cycle.

Figure 6-13 shows the experimental results and theoretical value of the displacement magnitude as a function of acceleration. The self-powered technique operates successfully for acceleration higher than 0.16m/s$^2$. For a lower acceleration, the damped displacement is kept at 0.7mm until the uncontrolled displacement is higher than 0.7mm. The system behaves like a feedback control loop; SSDI decrease displacement but when the displacement is lower than 0.7mm, the comparator stop to work and the displacement increases again.
Table 6-5 summarizes the experimental results for 4 accelerations: 0.16m/s², 0.19m/s², 0.21m/s² and 0.24m/s². The tip displacement increases as the imposed acceleration increases. Two measures of efficiency are proposed and defined in equations(0.0) and equation(0.0); there are presented in Table 6-5. The first efficiency compares the self-powered technique with velocity control, called SP-SSDI here to the use of all of the piezoelectric patches (P₁+P₂+P₃) for SSDI damping. From Table 6-5, the average efficiency is approximately 86%. This result means that if the size of the
piezoelectric patches used in the system is not increased, the proposed SP-VSSDI technique provides approximately 14% less damping. The second efficiency compares the SP-VSSDI technique to the case in which only the $P_1$ patch is used for SSDI, and the patch is controlled with an external function generator with perfect timing and powered by external power sources. This measure provides a fairer comparison because the area of the piezoelectric patch used for the SSDI is the same as that used for the self-powered system. From Table 6-5, the average of this efficiency measure is approximately 95%. This means that the phase lag generated by the inherent structure and the passive low-pass filter degrades the damping performance by approximately 5%. Based on the two comparisons, the SP-SSDI technique demonstrates high efficiency and good damping ability while maintaining a fully self-powered system.

Table 6-5. Experimental results for different excitation levels.

<table>
<thead>
<tr>
<th>Acceleration (m/s²)</th>
<th>Tip Displacement (mm)</th>
<th>Efficiency¹ (%)</th>
<th>Efficiency² (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1: Uncontrolled</td>
<td>Case 2: $P_1+P_2+P_3$</td>
<td>Case 3: $P_1$</td>
<td>Case 4: SP-SSDI</td>
</tr>
<tr>
<td>0.16</td>
<td>1.12</td>
<td>0.65</td>
<td>0.7</td>
</tr>
<tr>
<td>0.19</td>
<td>1.23</td>
<td>0.75</td>
<td>0.81</td>
</tr>
<tr>
<td>0.21</td>
<td>1.42</td>
<td>0.86</td>
<td>0.917</td>
</tr>
<tr>
<td>0.24</td>
<td>1.52</td>
<td>1.013</td>
<td>1.039</td>
</tr>
<tr>
<td>Average</td>
<td>85.74</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Efficiency _1 = \frac{\text{Uncontrolled} - \left[\text{Self - powered}\right]}{\text{Uncontrolled} - \left[\text{P}_1 + \text{P}_2 + \text{P}_3\right]} \quad (0.0)
Figure 14 shows the experimental results in the time domain at acceleration of $0.19\text{m/s}^2$. The results show that when the SP-SSDI system starts to take effect, the tip displacement decreases rapidly, and the piezoelectric terminal voltage increases rapidly. The results also demonstrate that the SP-SSDI system provides good, stable damping during the period when it is active. The SSDI damping effect is not influenced when the self-powered system is active.
6.4.3.2 Frequency response

Figure 6-15 shows, the frequency responses of the system for the 4 cases and for acceleration of 0.13m/s², 0.16m/s², 0.21m/s², and 0.24m/s². The testing frequency ranges from 28Hz to 40Hz. For an acceleration of 0.13m/s², we can see clearly the limit of the system; the SSDI circuit dumps the structure only up to 0.7mm, it is not plenty effective. As we say, the system works like a feedback control loop and regulates displacement at 0.7mm. When the uncontrolled displacement is lower than 0.7 mm the SSDI is not active. The acceleration of 0.16m/s² is the limit case; the damped displacement is 0.7mm with self-power technique (case 4) and with external supply (case 3), but the bandwidth is smaller with self-powered technique (2Hz). For the acceleration of 0.21m/s², and 0.24m/s², the self-powered technique provides almost the same damping ability as the case in which the P₁ patch is controlled with external instruments. The working bandwidth of the self-powered system is approximately 3.5 Hz at 0.21m/s² acceleration, and 3.5Hz at 0.24m/s² acceleration. Compared to the cases in which the P₁+P₂+P₃ patches and the P₁ patch are driven by external instruments (with working bandwidths of approximately 5Hz and 6Hz, respectively), the self-powered technique does not provide as much damping. Consequently, when the exciting acceleration is low, the working bandwidth is small. However, when the
exciting acceleration is sufficiently high, the working bandwidth is almost the same as when external instruments are used.

Figure 6-15. Experimental frequency response results: (a) acceleration=0.13 m/s², (b) acceleration=0.16 m/s², (c) acceleration=0.21 m/s², and (d) acceleration=0.24 m/s².
6.5 Conclusion

In this chapter, a self-powered SSDI technique based on zero-velocity crossing detection is proposed. The control signal used to drive the switches is obtained by sensing the velocity signal. This technique makes the semi-passive damping technique SSDI become the passive damping technique. The system concept is to divide the piezoelectric patch into three parts. The largest part behaves like the conventional piezoelectric patch used in the SSDI technique to dissipate the vibration energy. The second piezoelectric patch is a smaller patch that works like energy harvesting device to provide the power supply circuit. The third piezoelectric patch is a smaller patch designed to sense the velocity and to generate driving signal to control the switches. Because the three components are designed individually, each can be analyzed and optimized separately. Compared to the case in which all of the piezoelectric patches \((P_1+P_2+P_3)\) are used for structural damping and driven by an external function generator and a power supply, the efficiency of the proposed self-powered system is approximately 86%. Compared to the ideal switching case in which only the main piezoelectric patch is used for SSDI damping and is driven by an external function generator and power supply, the efficiency of the proposed self-powered system is approximately 95%. The major advantage of the proposed technique is that it is only
necessary to sacrifice a small amount of damping performance to make the system fully self-powered. The circuit design and the implementation of the system are quite simple, and the study shows the effectiveness of this new design. The drawback of this technique is the narrow bandwidth in the frequency response for low excitation level due to the decrease of harvested energy when the SSDI circuit works. When the exciting acceleration is sufficiently high, the working bandwidth is nearly equal to the bandwidth of the system driven by external instruments. To improve the system performance, the high frequency noise generated by the inversion could be further processed; the damping performance and efficiency of the self-powered system could also be improved.
Chapter 7 Summary and Discussion

The main topic of this dissertation is about improving the output power of the piezoelectric energy harvesting device. The objective was to build a totally self-powered energy harvester and to broaden the bandwidth of the frequency response. As the interfaces used in energy harvesting application are similar to the ones used in damping applications, the self-powered technique was also applied in the damping system. According to different techniques proposed in this dissertation, Figure 7-1 shows a schematic diagram of our contribution to enhance the performances of the piezoelectric energy harvester.

Figure 7-1. Schematic diagram of different techniques improving the power output of the piezoelectric energy harvesting device.
7.1 Summary and conclusion of the major results

The techniques proposed in this dissertation can be summarized as follows.

1. Tunable resonant frequency piezoelectric energy harvesting system

The tunable resonant frequency technique is based on the characteristic of the piezoelectric material and shifts the resonant frequency in a short region by connecting the piezo-patch to different capacitors in the electrical part and then influences the behavior in the mechanical part. Finally the resonant region can be broadened around 2.5 Hz in the experimental results.

Since this technique is performed through changing the loads in the electrical part to influence the mechanical behavior, the electromechanical parameter $k^2Q_m$ must be at least close to the medium coupling region. When the electromechanical parameter $k^2Q_m$ is much lower than 2 is in the weak-coupling region and in our experiment in chapter 3 the $k^2Q_m = 1.25$ is in the medium-coupling region, that is why the tunable resonant frequency technique can shift the resonant frequency.

In the results of the tunable resonant frequency energy harvesting device, the maximal output power can be increased by around 30% and the charging time can be shortened to around 200s. The tunable resonant frequency system is successfully combined with the a WSN node to transmit the RF signal. The tunable system can make
a WSN node transmit more RF data during the same time period.

2. Self-powered velocity detection SSHI energy harvesting system:

Due to the success of the SSHI technique improving the output power for the piezoelectric energy harvesting device, the main contribution in this dissertation is to make the SSHI technique into a fully self-powered system through the velocity detection to switch more accurately than classical peak detector technique. The energy flows are separated into three parts so it can be designed respectively. The velocity signal is detected from the characteristic of the piezoelectric patch and it can theoretically make switches work at the optimal time.

The SSHI techniques achieve good performance over standard techniques when the harvester is weakly coupled. In this case the electromechanical parameter $k^2Q_m$ is much lower than 2. The self-powered V-SSHI increases the output power by enlarging the work-cycle area. From the results, the self-powered V-SSHI can lead to a gain of around 200% compared to the standard DC approach without any external energy and have better performance than using peak detector technique.

3. Bistable broadband technique combined synchronized switching technique:

In order to harvest the vibration energy over a broader frequency range than that of the traditional linear beam harvester we proposed a bistable harvester. This bistable
harvester was combined with synchronized switching technique. The bistable broadband technique enhances the bandwidth and makes mechanical behavior nonlinear through proper magnets design in the mechanical part. The classical SSHI technique enlarges the work-cycle area in the electrical part and is combined with bistable broadband technique to construct a complete system.

From the results of our example, at resonance (f = 10.4 Hz) the output power of the bistable broadband technique combined with SSHI is 140.4 μW and the output power of the bistable broadband combined with standard DC technique is 24.86 μW. The output power is increased around 5.64 times at resonance. At non-resonance (f = 5Hz) the output power of the bistable broadband technique combined with SSHI is 8μW and the output power of the bistable broadband combined with standard DC technique is 0.3 μW. The output power is increased around 26.67 times at non-resonance.

The greatest advantage of combining these two non-linear techniques is that they can be designed individually and will not influence each other. The performance of the bistable broadband technique and the SSHI technique are integrated together.

4. Self-Powered Semi-Passive Piezoelectric Structural Damping Based on Zero-Velocity Crossing Detection

When the piezoelectric patch is not weakly coupled to the host structure, the
piezoelectric energy harvester will produce damping effects. Based on this characteristic, the self-powered synchronized switching technique used in the energy harvesting application can be also used in the structural damping application. The advantage of the self-powered technique is that it can fully perform in damping application but the limit of the self-powered damping technique is that this method needs a minimum structure displacement to harvest energy to supply electronic devices. As the structure displacement is the key parameter for the self-powered technique, in the damping application the system behaves like a feedback loop when the displacement is over the limit level, the self-powered semi-passive damping system will start to damp the structural vibration effectively. Compared to the case when the electronics are supplied with an external source, the efficiency of the proposed self-powered semi-passive damping system is approximately 86 %. Compared to the ideal switching case in which only the main piezoelectric patch used for SSDI damping with external source, the efficiency of the proposed self-powered semi-passive damping system is approximately 95 %. The major advantage of the this technique is that it is only necessary to sacrifice a small amount of damping performance to make the system fully self-powered and also make the system also have good damping performance.
7.2 Future work

To further increase the efficiency and the output power of energy harvesting devices so as to facilitate real applications such as WSN nodes, some possible directions of future work are examined below.

1. Piezoelectric material is the active material to convert the mechanical energy into electrical energy. As the piezoelectric properties of the single crystal piezoelectric material is much higher than PZT, substituting the single crystal piezoelectric material for PZT is a potential method to increase energy directly.

2. Design proper interfacing circuit for capacitive load. The interfacing circuit designed in this dissertation is suitable for the resistor load. However, if in the application, the electrical energy does not supply to the load directly, the capacitor is needed to be the buffer to store the energy temporarily. The steady-state analysis presented in this dissertation will not apply to interfacing circuit with capacitive loads and thus the interfacing circuit design consideration will be different.

3. Design a better and proper WSN communication framework for energy harvesting devices. There are already some research works attempt to improve the communication mechanism for the network design by powering the nodes using energy harvesting devices. Through combing these specially designed communication
mechanism, a self-sustained wireless network without the need to use batteries can hopefully be realized in the future.
Appendix A.

A.1 Equivalent circuit of the piezoelectric energy harvester

In order to analyze and discuss the piezoelectric energy harvesting device with the interfacing circuit, the equivalent circuit of the piezoelectric energy harvesting is presented. From the governing equation of piezoelectric (equation (0.0)), the equivalent circuit of mechanical part and electrical part can be modeled and separated by an ideal transformer and the ratio of the transformer is force-voltage coupling factor ($\alpha$) as shown in Figure A-1. The equivalent inductor $L_m$ is given by equivalent mass, the equivalent capacitor $C_m$ is given by equivalent stiffness $\frac{1}{K_E}$ and the equivalent resistor $R_m$ is given by damping ratio $D$.

Figure A-1. Equivalent circuit model of piezoelectric and structure.

The mechanical part of the equivalent circuit can be transformed into electrical part as Figure A-2 shown. $v_{eq}$ and $i_{eq}$ is the equivalent voltage and current which transformed from the mechanical part. Figure A-2 is the equivalent circuit of the
piezoelectric energy harvesting device and can be used to analyze with the interfacing circuit. The impedance of the mechanical part is shown in equation (0.0).

\[ Z_{\text{mech}} = j \omega \frac{M}{\alpha^2} + \frac{K^E}{j \omega \alpha^2} + \frac{D}{\alpha^2} \]  

(0.0)

**A.2 Electromechanical coupling coefficient**

Electromechanical coupling coefficient (EMCC) is another important parameter and it indicates the effectiveness of piezoelectric materials to convert the mechanical energy into electrical energy. EMCC can be presented as equation (0.0) [97]. This equation is general formula and suitable for both dynamic and static condition. In the real application, it's hard and complex to measure energy and put it into equation (0.0) to calculate the EMCC. So when the structure driving at the resonance, the equation (0.0) can be extended as equation (0.0) shown [97, 98] and this EMCC is called effective electromechanical coupling coefficient \( k^2_{\text{eff}} \). Effective electromechanical coupling
coefficient \( k_{\text{eff}}^2 \) can be given by open-circuit resonant frequency and short-circuit resonant frequency as following equations (0.0) shows.

\[
k_s = \frac{U_m}{\sqrt{U_d U_d}}
\]  

(0.0)

Where

Table A-1. Definitions of the EMCC energy terms.

<table>
<thead>
<tr>
<th>Elastic energy</th>
<th>( U_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric energy</td>
<td>( U_m )</td>
</tr>
<tr>
<td>Mutual energy</td>
<td>( U_e )</td>
</tr>
</tbody>
</table>

\[
k_{\text{eff}}^2 = \frac{\omega_D^2 - \omega_E^2}{\omega_D^2}
\]  

(0.0)

The effective electromechanical coefficient shown in equation (0.0) is dynamic definition. According to the static definition of open-circuit resonant frequency and short-circuit resonant as equation (0.0) shown, the static electromechanical coupling coefficient \( k^2 \) can be represented in open-circuit stiffness and short-circuit stiffness as equation (0.0) shown and it's also called global electromechanical coupling coefficient.
Effective mass

| Effective open-circuit stiffness | $K_p^D$ |
| Effective short-circuit stiffness | $K_p^E$ |

The effective open-circuit stiffness ($K_p^E$) can be calculated by the piezoelectric equation when the piezoelectric patch is in short-circuit condition as equation (0.0).

When the piezoelectric patch is in short-circuit condition, there is no piezoelectric effect and the piezoelectric material like only a normal ceramic. In order to calculate the effective open-circuit stiffness ($K_p^D$), let the piezoelectric patch in open-circuit condition and it means there is no current flow out from piezoelectric patch. The output current (I) in piezoelectric equation is zero and substitute the relation between velocity and voltage into the governing equation (0.0). Open-circuit stiffness ($K_p^D$) can be obtained as equation (0.0). When the piezoelectric patch is driving under low-coupled condition and small displacement, the displacement can be assumed constant ($x = x'$). $x$ represents the
displacement when the system is driven under open-circuit condition and $x'$ represents the displacement when the system is driven under short-circuit condition. The relation between effective open-circuit stiffness ($K^E_p$) and effective open-circuit stiffness ($K^D_p$) can be expressed as equation (0.0).

$$F_p = K^E_p \cdot x$$  \hspace{1cm} (0.0)

$$F_p = \left( K^D_p + \frac{\alpha^2}{C_0} \right) x'$$  \hspace{1cm} (0.0)

$$K^D_p = K^E_p + \frac{\alpha^2}{C_0}$$  \hspace{1cm} (0.0)

**A.3 Time interval discussion of Standard DC approach**

In this sub-section, the time interval behavior of the standard DC approach is discussed. It is assumed that the displacement is sinusoidal and the displacement $x$, velocity $\dot{x}$, equivalent current $I_{eq}$ can be represented as equation (0.0), (0.0) and (0.0).

$$x(t) = -\dot{x} \cos(\omega_d t)$$  \hspace{1cm} (0.0)

$$\dot{x}(t) = \omega_d \dot{x} \sin(\omega_d t) = \ddot{x} \sin(\omega_d t)$$  \hspace{1cm} (0.0)

$$I_{eq}(t) = \alpha \dot{x} = \alpha \omega_d \ddot{x} \sin(\omega_d t) = \dot{I}_{eq} \sin(\omega_d t)$$  \hspace{1cm} (0.0)

where
\[
\begin{align*}
\dot{x} &= \alpha_0 \dot{x} \\
\dot{I}_e &= \alpha \omega_0 \dot{x}
\end{align*}
\]

According to the Figure 2-11, the time when \( V_p \) equals \(-\dot{V}_c\), \( x \) equals \(-\dot{x}\) and \( I_{eq} \) equals zero is \( T_1 \), the time when \( V_p \) reaches to the \( \dot{V}_c \) is \( T_2 \) and the time when \( x \) equals \( \dot{x} \) is \( T_3 \). The behavior of these three time intervals is discussed in detail as following.

(i) \( t = T_1 \sim T_2 \)

As \( V_c < \parallel \dot{V}_c \parallel \), the full bridge rectifier is disconnected and \( I_R, I_P \) flow into \( R \) and \( C_0 \) respectively as equation (0.0) shown. Integrating the second equation in equation (0.0) from \( T_1 \) to \( T_2 \), the time \( T_2 \) can be obtained as equation (27) shown.

\[
\begin{align*}
\begin{cases}
I_R &= -I_C \\
I_P &= \alpha \dot{x} - C_0 \dot{V}_C = 0
\end{cases} \\
\int_{T_1}^{T_2} \dot{V}_C \, dt &= \frac{\alpha}{C_0} \int_{T_1}^{T_2} \dot{x} \, dt \\
\Rightarrow \quad T_2 &= \frac{1}{\alpha_0} \cos^{-1} \left( \frac{2C_0 \dot{V}_C}{\dot{x} \alpha} - 1 \right)
\end{align*}
\]

(ii) \( t = T_2 \sim T_3 \)
The voltage $V_p$ reaches to the $\dot{V}_C$, so the full bridge rectifier is connected. $I_p$ equals $I_C + I_R$ and flow through full bridge to into the rectifier capacitor $C_r$ and load resistor $R$ as equation (0.0) shown.

$$I_p = I_C + I_R$$  \hspace{1cm} (0.0)

(iii) $t = T_1 \sim T_3$

Considering half cycle period (the time interval from $T_1$ to $T_3$) and assuming the rectified capacitor $C_r$ of load is large enough, so the output voltage $V_C$ during the time interval form $T_1$ to $T_3$ can be regarded as a constant value and the net current through $C_r$ equals zero. As this assumption, the sum of the current output $I_p$ from piezoelectric patch equals to the sum of the current $I_R$ flow through the resistor load $R$ as equation (0.0) shown. Integrating over the half cycle from time $T_1$ to $T_3$, the output voltage $\dot{V}_C$ can be obtain as the equation (0.0) shown.

$$\int_{T_1}^{T_3} I_R \, dt = \int_{T_1}^{T_3} I_p \, dt$$  \hspace{1cm} (0.0)

$$\int_{T_1}^{T_3} \frac{\dot{V}_C}{R} \, dt = \int_{T_1}^{T_3} \left( \alpha \dot{x} - C_0 \ddot{V}_C \right) \, dt$$

$$\Rightarrow \dot{V}_C = \frac{2\alpha R\omega_b}{2C_0 R\omega_b + \pi} \ddot{x}$$  \hspace{1cm} (0.0)
A.4 Time interval discussion of Parallel-SSHI

In this sub-section, the time interval behavior of the parallel-SSHI is presented. According to the Figure 2-15, the time when $V_p$ equals $-\hat{V}_c$, $x$ equals $-\hat{x}$ and $I_{eq}$ equals zero is $T_1$. Let $T_i$ is initial time and equals zero. The time when the clamped capacitor $C_0$ and inductor $L$ is resonant during half resonant cycle and $V_p$ reaches to the $\hat{V}_c \cdot q_{LC}$ is $T_2$. $T_2$ equals $\frac{1}{2} T_{LC}$. $q_{LC} = e^{\frac{-\pi}{\omega_0}}$ is the inverting quality factor of the LC resonance and the $Q_i$ is quality factor of whole energy harvesting device and equals $\frac{Q_L L}{R_{LC}}$. The $R_{LC}$ in the $Q_i$ can be regarded as whole electrical losses in the system. The time when $V_p$ reaches to $\hat{V}_c$ is $T_3$ and the time when $x$ equals $\hat{x}$ is $T_4$. The behavior of the these time interval is discussed in detail as following.

(i) $t \in [T_1, T_2]$

The time interval from $T_1$ to $T_2$ equals to the half LC resonant period ($\frac{1}{2} T_{LC}$). In this interval the inductor will resonate with the clamped capacitor of the piezoelectric patch and the terminal voltage of piezoelectric patch reverses from the negative voltage to positive voltage. The terminal voltage of the piezoelectric patch can be expressed as equation (0.0) shown during oscillating period. Substitute the $t = \frac{T_{LC}}{2}$ into the equation (0.0) and the terminal voltage at time $T_2$ can be obtained as equation (0.0).
\[
V_p(t) = \hat{V}_p \cdot e^{-\frac{\alpha LC}{2Q_L} t} \left[ \frac{1}{2Q_L} \sin(\omega_{I_C} t) + \cos(\omega_{I_C} t) \right] 
\]

(ii) \( t \in [T_1, T_3] \)

As \( V_c < \|\hat{V}_c\| \), the full-bridge rectifier is disconnected and \( I_R \) and \( I_T \) can be represented as equation (0.0) shown. Integrating the piezoelectric equation from \( T_1 \) to \( T_3 \), the time \( T_3 \) can be obtained as equation (0.0) shown.

\[
\begin{align*}
I_R &= -I_C \\
I_T &= I_p + I_L = \alpha \hat{x} - C_v \hat{V}_c + I_L = 0
\end{align*}
\]

\[
\int_{T_1}^{T_3} \left( \alpha \hat{x} - C_v \hat{V}_c + I_L \right) dt \\
\Rightarrow T_3 = \frac{1}{\omega_L} \cos^{-1} \left[ \frac{C_v}{\frac{\omega_L}{\omega_{I_C}}} (1 - q_{I_C}) \hat{V}_c - 1 \right]
\]

(iii) \( t \in [T_3, T_4] \)

The voltage \( V_p \) reaches to the \( \hat{V}_c \), so the full-bridge rectifier is connected. \( I_T \) equals \( I_C + I_R \) and flow through full-bridge to into the rectifier capacitor \( C_c \) and load
resistor $R$ as equation (39) shown.

$$I_R = I_T - I_c = I_p + I_L - I_C$$  \hspace{1cm} (0.0)

(iv) $t \in [T_1, T_4]$  

Assuming the rectified capacitor $C_r$ of the load is large enough, so the output voltage $V_C$ during the time interval form $T_1$ to $T_3$ can be regarded as a constant value and the net current through $C_r$ equals zero. As this assumption, the sum of the current output $I_p$ from piezoelectric patch equals to the sum of the current $I_R$ flow through the resistor load $R$ as equation (0.0) shown. When integrate the current $I_R$ from the time $T_1$ to $T_4$, the $\hat{V}_C$ can be obtained and the result is shown in equation (0.0).

$$I_R = I_p + I_L$$  \hspace{1cm} (0.0)

$$\int_{T_1}^{T_4} \frac{\hat{V}_C}{R} \, dt = \int_{T_1}^{T_4} \left( \alpha \hat{x} - C_0 \hat{V}_C + I_L \right) \, dt$$

$$\Rightarrow \hat{V}_C = \frac{2 \alpha R \omega_0}{\pi + \left(1 - q_{lc}ight) C_0 R \omega_0} \hat{x}$$  \hspace{1cm} (0.0)

**A.5 Time interval discussion of Series-SSH**

In this sub-section, the time interval behavior of the series-SSH is presented.
According to Figure 2-19, the time when $V_p$ equals $-\hat{V}_p$, $x$ equals $-\hat{x}$ and $I_q$ equals zero is $T_1$. Let $T_1$ is initial time and equals zero. The time when the clamped capacitor $C_0$ and inductor $L$ is resonant during half resonant cycle and $V_p$ reaches to the $-\hat{V}_c + (\hat{V}_p - \hat{V}_c) \cdot q_{LC}$ is $T_2$. $T_2$ equals $\frac{T_{LC}}{2}$ and $x(T_2) \approx x(T_1) = -\hat{x}$. $q_{LC} = e^{-\frac{\pi}{2Q_0}}$ is also the inverting quality factor of the LC resonance. The time when $V_p$ reaches to $\hat{V}_p$, $x$ equals $\hat{x}$ is $T_3$. The behavior of these time interval is discussed in detail as following.

(i) $t \in [T_1, T_2]$

During this time interval, the inductor resonate with the clamped capacitor of the piezoelectric patch and comparing with the one oscillating period of the SSHI, the interval of LC resonance is much shorter. As LC resonance, the full-bridge rectifier is connected and $I_R$ and $I_L$ can be represented as equation (0.0) shown. During the oscillating period, the relation between voltage $V_p$ and voltage $V_C$ can be expressed as equation (0.0). Substitute the initial condition into equation (0.0), and the voltage $V_p$ at time $T_1$ and $T_2$ can be obtained as equation (50) shown.

\[
\begin{align*}
\begin{cases}
I_R = I_L - I_C \\
I_L = -C_0 \frac{dV_p}{dt}
\end{cases}
\end{align*}
\]
\[ (V_p - V_c) \left( \frac{T_{LC}}{2} \right) = (-\hat{V}_p + \hat{V}_c) \cdot e^{\frac{-\pi}{\omega_{LC}}} \left[ \frac{1}{2Q_t} \sin \left( \frac{\omega_{LC} \cdot \pi}{\omega_{LC}} \right) + \cos \left( \frac{\pi}{\omega_{LC}} \right) \right] \]

\[ \Rightarrow (V_p - V_c) \left( \frac{T_{LC}}{2} \right) = (\hat{V}_p - \hat{V}_c) \cdot e^{\frac{-\pi}{2Q_t}} = (\hat{V}_p - \hat{V}_c) \cdot q_{LC} \quad \text{(0.0)} \]

Considering integrating \( I_L \) during half resonant period, the result is shown in equation (0.0). 

\[ \int_{T_1}^{T_2} I_L \cdot dt = \int_{0}^{\frac{T_{LC}}{2}} C_0 \frac{dV_p}{dt} \cdot dt \]

\[ \Rightarrow \int_{0}^{\frac{T_{LC}}{2}} I_L \cdot dt = C_0 \hat{V}_p \left( 1 + q_{LC} \right) \quad \text{(0.0)} \]

(ii) \( t \in [T_2, T_3] \)

During this period LC resonance is off, so the full bridge rectifier is disconnected.

The \( I_R \) and \( I_p \) can be represented as equation (0.0). By integrating \( I_p \) from \( T_2 \) to \( T_3 \), the relation between \( \hat{V}_p \) and \( \hat{V}_c \) can be obtained as equation (0.0) shown.

\[ \begin{cases} I_R = -I_c \\ I_p = I_L = 0 \end{cases} \quad \text{(0.0)} \]
\[
\int_{T_1}^{T_3} I_p \cdot dt = \int_{T_1}^{T_3} (\alpha \dot{x} - C_p V_p) dt = 0 \\
\Rightarrow \dot{V}_p = \frac{2\alpha \dot{x}}{C_o (1-q_{LC})} - \frac{(1+q_{LC})}{(1-q_{LC})} \dot{V}_C 
\]

(iii) \( t \in [T_1, T_3] \)

Considering the time period from \( T_1 \) to \( T_3 \) and integrating \( I_R \), the result is shown in equation (0.0). The result shows that the current flow through \( I_R \) from \( T_1 \) to \( T_3 \) equals to the current during resonant period.

\[
\int_{T_1}^{T_3} I_R = \int_{T_1}^{T_3} I_R + \int_{T_1}^{T_3} I_R = \int_{T_1}^{T_3} (I_L - I_C) + \int_{T_1}^{T_3} (-I_C) \\
\Rightarrow \int_{T_1}^{T_3} I_R = \int_{T_1}^{T_3} I_L 
\]

According to the equation (0.0) and substituting the equation (0.0) into equation (0.0), the \( \dot{V}_C \) can be obtained as equation (0.0) shown.

\[
\dot{V}_C \pi \frac{p}{R} \omega_b = C_o \left( \dot{V}_p - \dot{V}_C \right) (1+q_{LC}) \\
\dot{V}_C \pi \frac{p}{R} \omega_b = C_o \left[ \frac{2\alpha \dot{x}}{C_o (1-q_{LC})} - \frac{(1+q_{LC})}{(1-q_{LC})} \dot{V}_C - \dot{V}_C \right] (1+q_{LC}) \\
\Rightarrow \dot{V}_C = \frac{2\alpha R (1+q_{LC})}{2R\omega_b C_o (1+q_{LC}) + \pi (1-q_{LC})} \omega_b \dot{x} 
\]
[Reference]

1998.


33. S. Roundy, "On the effectiveness of vibration-based energy harvesting," Journal of


91. H. B. Stewart, et al., "Optimal escape from potential wells-patterns of regular and


