Supply Chain optimization with sustainability criteria: A focus on inventory models
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Supply chain optimization with sustainability criteria: 
A focus on inventory models

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Summary

Sustainability concerns are increasingly shaping customers’ behavior as well as companies’ strategy. In this context, optimizing the supply chain with sustainability considerations is becoming a critical issue. However, work with quantitative models is still scarce. Our research contributes by revisiting classical inventory models taking sustainability concerns into account. We believe that reducing all aspects of sustainable development to a single objective is not desirable. We thus reformulate single and multi-echelon economic order quantity models as multiobjective problems. These models are then used to study several options such as buyer-supplier coordination or green technology investment. We also consider that firms are becoming increasingly proactive with respect to sustainability. We thus propose to apply multiple criteria decision aid techniques instead of considering sustainability as a constraint. In this sense, the firm may provide preference information about economic, environmental and social tradeoffs and quickly identify a satisfactory solution.

**Keywords:** sustainable supply chain, multiobjective optimization, multiple criteria decision aid, inventory control, buyer-supplier coordination, green technology investment.
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Acronyms

**CSR**: Corporate Social Responsibility

**DM**: Decision Maker

**EOQ**: Economic Order Quantity

**KPI**: Key Performance Indicator

**LCA**: Life-Cycle Assessment

**MCDA**: Multiple Criteria Decision Aid

**OM**: Operations Management

**OR**: Operational Research

**SOQ**: Sustainable Order Quantity
Research questions and contributions

1 Observations

In a book entitled “J’accuse l’économie triomphante”, Albert Jacquard criticizes the ascendancy of the economic theories in today’s world and uses the following metaphor:

“Sur le Titanic en train de sombrer, est-il raisonnable de consacrer beaucoup d’efforts et d’intelligence à obtenir une meilleure cabine?”  

Even if Jacquard does not refer to sustainable development in this quotation, the above metaphor can be reinterpreted with a sustainability perspective as follows: Is it reasonable to constantly try to increase the worldwide wealth while globally damaging ecological and social welfare? Indeed, the Titanic metaphor is often used in the debates about sustainable development issues. As an example, the title of Schellnhuber (2007) is: “Kyoto: no time to rearrange deckchairs on the Titanic”. The concept of strong sustainable development is in accordance with Jacquard’s quotation as this concept states that substitutability between the economic, environmental and social dimensions of the sustainable development is not desirable (Gasparatos et al., 2008; Neumayer, 2004). A certain level of ecological and social welfare is indeed required and thus, the aspects of sustainable development cannot be reduced to a single objective.

The companies all over the world are increasingly recognizing the concept of strong sustainability. Generally starting from a denial position in the seventies, the companies are increasingly proactive with sustainability issues. Nowadays, the first motivation for most of the companies when implementing sustainability programs is the increase in stakeholder’s awareness (customers, public opinion, shareholders, employees…) about sustainable development. The companies have indeed started to understand that sustainable development is not a transient trend but a long term movement that may deeply modifies individual and collective behaviors all over the world. Some companies are thus starting sustainable

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1 This can be translated as: Is it reasonable to devote a lot of effort and intelligence to get a better cabin while the Titanic is sinking?
development projects in spite of their low short-term profitability by assessing that these projects may be profitable in the long-term.

Supply chains have a major role to play in implementing sustainable development strategies. Indeed, both academics and practitioners recognize that operations management practices strongly impact the environment, the society and the economy. The sustainable supply chain literature is thus continuously growing. We believe that sustainability is an emerging issue that will shape the research in supply chain management for years.

2 Research questions

Among the numerous questions related to sustainable supply chain management, we believe that supply chain optimization with sustainability criteria do deserve further attention. Some authors indeed mention the lack of model based research that deals with sustainable supply chains (Benjaafar et al., 2010). We thus aim at contributing to the model based research on sustainable supply chains by taking the above observations into account.

Before aiming at optimizing the supply chain with sustainability concerns, the notion of sustainable supply chain performance has to be clearly defined. The following first research question is thus considered:

| Research question 1: How to define and evaluate sustainable supply chain performance? |

Defining and evaluating sustainable supply chain performance is a prerequisite in our research. Contrarily to traditional supply chain performance that is evaluated on criteria such as cost, service level and leadtime, sustainability issues require taking a broader view of supply chain performance. There is an emerging field of literature on sustainable supply chain performance. This literature mainly applies product life-cycle assessment (LCA) tools. The proposed performance frameworks for sustainable supply chains are indeed product-oriented and set up sustainable development indicators for all the product life-cycle stages. We believe that the existing product LCA frameworks do not behave satisfactorily when focusing on sustainable supply chains. We thus aim at proposing a new set of key performance indicators (KPIs) to assess sustainable supply chain performance. To do so, an appropriate methodology is also required.
As a second step in our research, we aim at optimizing sustainable supply chain performance. Although sustainable supply chains have to be considered globally while performing optimization, this global optimization is very difficult even if focusing only on cost. To tackle this issue, hierarchical optimization is traditionally applied. This method consists in isolating the supply chain decisions to perform optimization. Applying hierarchical optimization in the context of sustainable supply chain, we decide to focus on inventory control decisions. Indeed, the few published papers teach us that sustainable inventory optimization is effective to improve the sustainability of supply chains. Moreover, this operational decision can be easily adjusted in connection with the other decisions if required. The second research question may thus be formulated as follows:

**Research question 2:** How to optimize inventory models with sustainability criteria?

The few papers that include sustainability criteria into inventory optimization models mainly focus on regulatory policies. By doing so, the authors consider that companies include sustainability considerations in their supply chain management practices mainly due to regulatory pressures. We believe that regulation is no longer the predominant sustainability pressures for companies. We thus aim at proposing new methods that reflect the proactive role of companies with respect to sustainability. Moreover, we acknowledge the concept of strong sustainability that states that reducing all aspects of sustainable development to a single objective is not desirable. However, the most commonly used optimization technique consisting in aggregating the different sustainable development criteria into a single metric (for instance by setting a price for carbon emissions) is in opposition with this principle. We thus advocate finding new ways to include sustainability criteria into inventory optimization, being in accordance with the strong vision of sustainable development.

Once the appropriate ways of including sustainability criteria into inventory models are setup, an underlying question is also discussed. Indeed, several management options are often available to optimize inventory models with sustainability criteria. For instance, firms may develop buyer-supplier coordination practices. The companies may also invest in green technologies or adjust their operations. Such options may have different impacts in terms of sustainability performance. We thus aim at comparing the efficiency of such options to provide effective managerial solutions for sustainable inventory models optimization.
3 Contributions

Our first contribution consists in assessing the performance of supply chains in terms of sustainability. We start by drawing insights from a classification of the existing key performance indicators sets for sustainability. We then propose a new methodology for KPIs set building in the context of sustainable procurement and distribution supply chains. Finally, this methodology is applied to propose a new set of KPIs for such supply chains. This KPIs set was validated by sustainable development managers and applied in an industrial context.

Secondly, our research contributes by revisiting classical inventory models taking sustainability criteria into account. We believe that reducing all aspects of sustainable development to a single objective is not desirable. Indeed, we propose to use multiobjective optimization techniques to avoid substitutability between the economic, environmental and social dimensions of the sustainable development. We thus reformulate single and multi-echelon economic order quantity models as multiobjective problems. The models are compatible with the proposed KPIs set; however, we use a broader formulation by considering general sustainability objectives. The multiobjective version of the economic order quantity model is called the sustainable order quantity (SOQ) model. For the single and multi-echelon SOQ models, the set of efficient solutions is analytically characterized. We finally propose to apply multiple criteria decision aid (MCDA) techniques to reflect the proactive positioning of companies in terms of sustainability. By doing so, we recognize that companies face several types of opportunities and threats that imply their positioning about sustainability issues. In this sense, the firm may provide preference information about economic, environmental and social tradeoffs and quickly identify a satisfactory solution.

These proposed multiobjective models are then adapted to compare several managerial options in terms of sustainability performance. First, we compare operational adjustment and technology investment by modeling both options in the SOQ model. The results show that operational adjustment may be a valuable alternative in comparison to technology investments. We also provide analytical conditions under which one of both options is the most interesting for two classical regulatory policies, i.e. the carbon cap and the carbon tax policies. The second type of managerial option under study concerns buyer-supplier coordination practices. Different outcomes of buyer-supplier coordination are indeed illustrated. Among them, a new model of a supplier leader supply chain is introduced and
discussed. The impact of buyer-supplier coordination on the supply chain economic and environmental performances is then challenged.

4 Thesis structure

The remaining part of this PhD dissertation is structured as follows:

Chapter 1: Introduction

This introductory chapter begins by defining some concepts related to supply chain and to supply chain performance. The second section is devoted to the presentation of sustainable development issues. The emergence of the concept from the economic theories to the international scene is discussed. We also define the concept of strong sustainability as a building block of our research. The companies’ positioning with respect to sustainable development is then presented. We argue that firms are becoming increasingly proactive with respect to sustainable development. In a last section, we present a literature review on sustainable supply chain management where we focus more particularly on sustainable inventory models optimization. We conclude that the few published papers in this category adopt a regulatory perspective about sustainable development issues. This regulatory perspective does not appropriately reflect the new companies’ positioning with respect to sustainable development.

Chapter 2: Measuring sustainable supply chain performance

Defining and measuring sustainable supply chain performance is a prerequisite when aiming at optimizing the supply chain with sustainability concerns. Our research focuses on inventory models that mainly deal with the procurement and the distribution stages of the supply chain. In this chapter, we show that the existing performance frameworks for sustainable supply chain do not behave satisfactorily when focusing on the procurement and distribution stages. We thus contribute by proposing a new set of KPIs for assessing sustainable procurement and distribution supply chain performance. We also propose a new methodology for KPIs set building in this context.

Chapter 3: Including sustainability criteria into inventory models

In this chapter, we reformulate the classical economic order quantity model as a multiobjective problem. By doing so, we aim at including sustainability criteria into inventory
models by adopting the strong vision of sustainability. We refer to this model as the sustainable order quantity model. We also study a multi-echelon extension of the sustainable order quantity model. For both models, the set of efficient solutions is analytically characterized. We also propose a new interactive procedure that allows the decision maker to quickly identify the best option among these solutions. This interactive procedure acknowledges the proactive role of companies with respect to sustainability issues.

Chapter 4: Adjust or invest: Assessing two management principles in a low-carbon inventory model

In this chapter, the sustainable order quantity model is adapted to support green technology investment decisions. This option is compared to operational adjustment. The results show that operational adjustment may be a valuable alternative in comparison to investments in carbon-reducing technologies. We also provide analytical conditions under which an option outperforms the other one for two classical regulatory policies, i.e. the carbon cap and the carbon tax policies. The results can also be directly extended to the case where several technologies are available. Finally, the results are used to illustrate the effectiveness of different regulatory policies to control carbon emissions. Some potentially impacting practical insights on this topic are thus drawn.

Chapter 5: Economic and environmental performance of buyer-supplier coordination

In the multi-echelon extension of the sustainable order quantity model proposed in chapter 3, the supply chain is assumed to be centrally optimized. This situation may be encountered either when the supply chain is controlled by a single entity or when independent entities decide to coordinate their operations in order to improve the system performance. In practice, the buyer-supplier negotiation may lead to several outcomes. In this chapter, the different outcomes of buyer-supplier coordination are illustrated by several models. Among them, a new model of a supplier leader supply chain is introduced and discussed. The impact of buyer-supplier coordination on the supply chain environmental performance is challenged in this chapter. We show that the total supply chain carbon emissions may be greater when buyer and supplier ordering policies are fully coordinated. Moreover, the setting of a carbon price may also lead to a similar outcome.
Conclusions and future research directions

The conclusions highlight the main findings of this PhD thesis. Moreover, several future research directions are discussed.

Appendix A: Multiobjective optimization and MCDA

This appendix presents some basic features on multiobjective optimization and MCDA methods. First, we define some concepts of multiobjective optimization and we highlight some of underlying issues. Second, we introduce MCDA methods and we focus on the main methods linking multiobjective optimization with MCDA. The reader is thus encouraged to refer to appendix A that may help providing the required theoretical background of this PhD dissertation.
Chapter 1: Introduction

This introductory chapter begins by defining some concepts related to supply chain and to supply chain performance. The second section is devoted to the presentation of sustainable development issues. The emergence of the concept from the economic theories to the international scene is discussed. We also define the concept of strong sustainability as a building block of our research. The companies’ positioning with respect to sustainable development is then presented. We argue that firms are becoming increasingly proactive with respect to sustainable development. In a last section, we present a literature review on sustainable supply chain management where we focus more particularly on sustainable inventory models optimization. We conclude that the few published papers in this category adopt a regulatory perspective about sustainable development issues. This regulatory perspective does not appropriately reflect the new companies’ positioning with respect to sustainable development.
1 Supply chain management

This section is an introduction to supply chain management. We first define the concepts of supply chain and supply chain management. Then we focus on supply chain performance notions.

1.1 The supply chain concept

According to Chopra and Meindl (2001), “a supply chain consists of all parties involved, directly or indirectly, in fulfilling a customer request. The supply chain includes not only the manufacturers and suppliers, but also transporters, warehouses, retailers, and even customers themselves.” The concept of supply chain refer to both products and services organizations. The structure of a supply chain may be very complex and may widely differ from industry to industry and from firm to firm. Figure 1.1 is an example of a simple supply chain.

The management of supply chain may be defined as follows: “Supply chain management aims at designing, managing and coordinating material/product, information and financial flows to fulfill customer requirements at low costs and thereby increasing supply chain profitability” (Rosic, 2011). According to Simchi-Levi et al. (2003), supply chain management is “a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system wide costs while satisfying service level requirements.” These two definitions highlight the existing trade-off between costs and service level.
Supply chain management decisions can be classified into three main categories i.e. strategic, tactical and operational. Strategic decisions are typically made over a longer time horizon. These decisions are related to the corporate strategy and mainly deal with design problems. Tactical decisions are taken with a mid-term horizon and focus on the planning of operations. Finally, the operational decision level refers to the short-term and very short-term horizon and deals with flow management and scheduling problems. Decisions on the three levels occur at the procurement, the production and the distribution stages of the supply chain (figure 1.2).

Figure 1.2: Supply chain decision levels

Source: Dallery (2000)

Our research focuses on inventory control optimization which is one of the main issues of operational decisions. Inventory control may be defined as the management of inventory at all stages of the supply chain, i.e. raw material, work-in-progress and finished goods. The objective of inventory control is often to balance conflicting parameters. On one hand, the stock levels at all stages may be kept down to make cash available for other purposes. On the other hand, having a high stock level enables economies of scale and prevents operations problems due to uncertainties in supply, production and sales (Axsäter, 2006). The balance is seldom trivial, that is why inventory models are required. Modern inventory control is based on quite advanced and complex decision models.
1.2 Supply chain performance

The notion of performance has totally evolved during the last fifty years, mainly due to a huge increase in competition between firms. Three main phases may be distinguished. From 1945 to 1975, the demand exceeded the supply. The performance was evaluated only with a cost perspective. From 1975 to 1990, the supply has balanced then exceeded the demand. The notion of performance has started to include other criteria such as reliability, quality and leadtime. From 1990 up to the present, a much wider vision of performance has emerged. From now on, the notion of performance also includes environmental and social aspects. Nowadays, the concept of corporate social responsibility (CSR) is strongly linked to the concept of performance. CSR is based on the stakeholder theory (Freeman, 2010). The stakeholder theory is an alternative to the traditional view and to the input-output model of the company. In the traditional view, the shareholders own the company thus their needs are considered at first. In the input-output model of the corporation, the firm transforms the inputs of investors, employees, and suppliers into outputs bought by customers. Firms thus only address the needs and wishes of these four parties. In the stakeholder theory, the company has to satisfy all the stakeholders who have cooperative as well as rival interests. The stakeholders include suppliers, customers, shareholders, employees, investors, communities, government, creditors, media and the society. As the interest of the various stakeholders may often diverge, performance is tightly dependent on which stakeholder has to be satisfied. Performance has thus become a relative notion. Moreover, performance measurement always implies strategic orientations (Lebas, 1995). This feature is clearly presented in the following extract from a report from the United Nations Commission on Sustainable Development: “We measure what we value, and value what we measure” (UNCSD, 2001). The notion of performance has thus become a relative notion that should include several dimensions.

Supply chain performance is traditionally evaluated based on costs and customer service level. These two performance measures are generally conflicting i.e. there is a trade-off between financial efficiency and responsiveness (Nahmias, 2001). Nowadays, supply chain performance is becoming a major issue for companies due to the globalization phenomenon (Botta-Genoulaz, 2005). The supply chain performance should be evaluated on each supply chain process. Chardine-Baumann (2011) analyses five major supply chain performance frameworks to identify the main supply chain processes (table 1.1).
The frameworks under consideration are the SCOR model (SCC, 2008), the Cooper et al. model (Cooper et al., 1997), the Porter model (Porter, 1990), the ASLOG model (ASLOG, 2006) and the EVALOG model (EVALOG, 2007). These frameworks are based on similar processes but with different names. Chardine-Baumann (2011) identifies seven common processes i.e. design, purchasing, procurement, production, selling, distribution and return.
The same kind of analysis is also proposed in Gruat La Forme et al. (2007) where the supply chain processes are grouped into four categories i.e. downstream part, internal part, upstream part and cross-supply part.

Each framework identifies several KPIs for each supply chain process. These supply chain performance frameworks traditionally measure performance on criteria such as cost, flexibility, reliability, quality and leadtime. Nowadays, CSR issues have drastically broadened the notion of supply chain performance. “Single optimization of an economic criterion is insufficient as the business is dominated by customers with strong social and environmental commitments” (Benyoucef, 2008). These frameworks are continuously evolving and integrate now CSR perspectives. For instance, the SCOR model includes the GREENSCOR that proposes environmental best practices as well as environmental indicators. However, these supply chain performance frameworks are based on benchmark measures and do not adopt a holistic view of sustainability issues. The inclusion of environmental and social issues is indeed partial. These frameworks were first designed by industrials based on best practices. They are not designed to evaluate sustainable supply chain performance and may be hardly modified to this extent.

2 Sustainable development

This section introduces the concept of sustainable development from its origin in the economic theories to its emergence on the international scene. We also discuss the concept of strong sustainability. Then, we identify the main sustainable development pressures for companies. Finally, we discuss the evolution of the companies’ positioning with respect to sustainable development.

2.1 The concept of sustainable development

As mentioned in Simpson et al. (2005) the concept of sustainable development can be traced back into the economical debate about scarcity and growth. Traditional economists (e.g. Malthus) indeed predicted that the scarcity of natural resource may lead to retardation or eventual cessation of economic growth (Barnett and Morse, 2010). Stiglitz (1974) synthesizes this idea as follows: “The proposition that limited natural resources provide a limit to growth and to the sustainable size of population is an old one.” As an answer, the author proposes “an
Chapter 1: Introduction

attempt to determine more precisely under what conditions a sustainable level of per capita consumption is feasible”. The problem of natural resource depletion was popularized 40 years ago by a report entitled “the limits to growth” (Meadows et al., 1972). This report from an MIT research team was commissioned by a worldwide think tank created in 1968 called the Club of Rome. A computer model was created to assess the consequences of interaction between earth’s and human systems. The simulation showed a decline of the global system by the mid to latter part of the 21st century. Even if several controversies still remain about the hypotheses and the method used, “the limits to growth” generated an electroshock. In 1972, Stockholm hosted the United Nations Conference on the Human Environment. This first United Nations' conference on environmental issues marked a turning point in the development of international environmental politics. Even if the meeting only ends up with a declaration, this paves the way for an international recognition of the concept of sustainable development. Nevertheless, the success of this concept was not as quick as expected, mainly due to financial downturn caused by the oil crises of the 70ies.

The worldwide recognition of the concept of sustainable development may be linked with the publication of the Bruntland’s report entitled “our common future” (WCED, 1987). In this document, sustainable development is defined as a “development that meets the needs of the present without compromising the ability of future generations to meet their own needs”. This definition is in correlation with the conclusion of “the limits to growth” as it states that current development may affect future generations’ welfare. The Bruntland’s definition is undeniably helpful by acting as a universal catalyst. However, this definition does not directly enable the concept of sustainable development to be converted into actions. This happened during the Rio Earth Summit in 1992. This major international conference brought together more than 170 governments. Several international agreements were signed out. Among them, the well-known Agenda 21 may be seen as an action plan in favor of sustainable development. One of the most famous initiatives resulting from the Rio conference is the adoption of the Kyoto Protocol in 1997. The protocol requires each country to publish their inventories in terms of greenhouse gases emissions. In addition, they must implement national programs to mitigate climate change. The slogan “People, Planet, Prosperity” was adopted at the 2002 World Summit on Sustainable Development in Johannesburg. It allows making operational the concept of sustainable development so that a balance is required between economic growth, environmental protection and social development. These three pillars of sustainable
development are often referred as the triple bottom line (Elkington, 1998). This clearly states that several dimensions should be taken into account when dealing with sustainable development. As the Bruntland’s definition is quite vague, the proposed definitions of sustainable development are numerous. For instance, Pearce (1996) identifies more than 50 definitions of sustainable development. In our work, we acknowledge the definition proposed by Mihelcic et al. (2003): “design and operation of human and industrial systems to ensure that humankind’s use of natural resources and cycles do not lead to diminished quality of life due either to losses in future economic opportunities or to adverse impacts on social conditions, human health and the environment”. This definition indeed combines the triple bottom line idea with the definition of the Bruntland’s report. Moreover, this definition may be directly used in an industrial engineering context.

2.2 Strong sustainable development

Sustainability is a political concept. However, this one is rooted into the economical theories. According to Pezzey (1997), the economic definition of sustainability means that the current actual utility must not exceed the current maximum sustainable utility. A decline in future utility may occur if this maximum sustainable utility is exceeded. Note that in economic terms, utility is a synonym of well-being and is a mix of human-made and natural capital. The economic definition of sustainable development may thus be summarized as a forever non-declining utility. A critical debate about this definition comes from the substitutability of human-made and natural inputs in producing this non-decreasing utility. The two options are often presented in terms of weak versus strong sustainability. Weak sustainability assumes significant possibilities for substitution between natural capital and other inputs to sustain their well-being (Simpson et al., 2005). Strong sustainability states that some ecological services are critical to life support, i.e. that substitutability between the different sustainable development dimensions is not desirable (Gasparatos et al., 2008; Neumayer, 2004). With this terminology, every technique that translates sustainability impacts into monetary units may be considered to follow a weak sustainability principle.

The concept of strong sustainability seems very promising. Daly (1990) made this concept operational and stated that strong sustainability requires:

- ecological services critical to life support to be maintained, and pollution stocks to be prevented from increasing beyond certain critical levels,
renewable resource stocks to be used no faster than they are renewed,
- depletion of non-renewable resources to be offset by investment in the production of comparable services from renewable resources.

2.3 Companies’ five sustainable pressures

The role of companies in implementing sustainable development actions is indisputably recognized. “It has become increasingly clear that business must play a central role in achieving the goals of sustainable development strategies” (Elkington, 1994). Indeed, companies may strongly leverage sustainability actions. For instance, companies may design and produce environmentally and socially responsible products. However, the companies need motivations to pursue sustainable development goals. In this section, a synthesis of some sustainable pressures is presented. We refer to sustainable pressures to encompass both opportunities and threats that the companies may take into account to act in a more sustainable way.

We propose to classify the sustainable pressures into five categories:

Natural resources depletion:
The first type is linked with non-renewable and renewable resource depletion. The concept of sustainable development was initially proposed as an answer to resource depletion issues. Resource depletion may indeed affect companies in several ways. A dependency on a scarce resource may affect companies due to speculation, price manipulation and political intentions. Resource depletion may also cause civil and transnational conflicts influencing both supply and demand. To tackle resource depletion problems, companies may diversify their supply sources, design products requiring low resource consumption or invest in cleaner technologies. Natural resource depletion problems thus require companies to implement sustainable development actions.

Regulations:
The second type of sustainable pressure is generated by governments and international bodies. The United Nations and national governments have indeed been the driving force behind sustainable development. Once the central role of companies in achieving the goals of sustainable development strategies has been recognized, governments and national bodies started setting up political tools requiring companies to operate in a sustainable manner. The
first way to do so consists in favoring sustainable innovation and sustainable business by providing financial supports for these activities. The second way consists in enacting regulations. Several types of regulatory policies may be found such as taxes on supply or waste (e.g. ecotax), targets on collection and recycling of used products (e.g. the waste electrical and electronic equipment directive) or cap and trade systems (carbon emissions trading system). The third and last way consists in requiring companies to communicate about their sustainability-related performance. For instance, the companies may be required to publish a sustainability report. Carbon labeling also enters into this category. In response, the companies may find ways to operate in a more sustainable way. The risk of more stringent regulations in the future may also motivate companies. “To start with, corporations get involved with sustainability programs forced by legislation. Some companies anticipate such legislative changes, in order to gain some competitive advantage from acting as first movers” (de Brito et al., 2008).

**Customer awareness:**
The third reason for companies to focus more on sustainable development actions is the increasing customer awareness on sustainability issues (Blengini and Shields, 2010; Jaffry et al., 2004; Vlosky et al., 1999). Customer awareness is indeed a strong pressure for companies as it may positively affect the business for two reasons. First, selling sustainable products may be a way to attract more customers. Second, sustainable products may deserve a price premium. The DHL green trends survey illustrates these facts (DHL, 2010). Half of the interviewed consumers indeed expressed the view that they would favour a company with green solutions over a cheaper one in the next ten years. The customer willingness to pay for a price premium is however hardly predictable as a gap often exists between intention and action (Vermeir and Verbeke, 2006).

**Company image:**
Enhancing the company image is often argued to be a company motivation for establishing sustainability programs. Continuous improvements in information technologies indeed lead to the advent of the global information society. Whatever happens wherever in the world may thus affect the company’s business. In this context, companies are under pressure to disclose more and more about their environmental goals and performance (Elkington, 1994). This public opinion, non-governmental organization and other stakeholders’ pressure is very intense for companies. This phenomenon is amplified by government regulations requiring companies to communicate about their sustainability performances.
Employees’ motivation:
Finally, the fifth identified sustainable pressure is self-motivation. Companies may indeed have “the desire to do the right thing” as reported by Lieb and Lieb (2010) survey. In this sense, sustainability may be viewed as an entire part of company’s values. This integration of sustainable development in the strategic vision of companies may also be beneficial for employees’ motivation. Modern employees are more and more focused on a positive and responsible company culture. This may also be a valuable argument to attract skilled employees as more and more employees argue that they would choose working for a company with strong sustainability commitments.

This classification is not purposed to be exhaustive. However, we have tried to give an overview of the main sustainable pressures on companies.

2.4 Companies’ positioning with respect to sustainability
Due to the emergence of these pressures, the companies’ positioning with respect to sustainable development has drastically evolved during the last thirty years. Three periods can be distinguished.

In the seventies, a lot of companies were at first reluctant to include sustainable development concerns into their business model. Firms were mainly convinced that sustainable development issues would erode their competitiveness. Regulatory policies were only seen as constraints on business activities. This first period may be seen as the denial phase.

The situation has evolved in the nineties and the link between sustainability and profitability became a true debate (Porter and van der Linde, 1995; Hart and Ahuja, 1996). This debate is still open in the literature. To help in clarifying this question, we propose to adopt the concept of eco-socio-efficiency as a balance of economic, environmental and social performance (Quariguasi Frota Neto et al., 2008; Huppes and Ishikawa, 2005). Eco-socio-efficiency is based on a multiobjective optimization analysis. The current situation is assumed to be generally eco-socio inefficient (i.e. that some win-win situations still exist). It does not mean that it is always profitable to follow sustainability principles. Precisely, trade-offs will become inevitable while progressing into sustainable development practices. In this second period, the companies carried on adopting sustainable principle mainly due to regulatory pressures.
However, sustainability programs were more easily adopted as companies expected financial payoff. This second period may be seen as the bargaining phase.

Nowadays, firms are becoming increasingly proactive with respect to sustainable development. A recent survey of 582 European companies highlights that regulation is no longer considered as the most important reason to establish sustainability programs as pictured in figure 1.3 (Bearing Point, 2010).

We can conclude that a shift has occurred. Nowadays, the first motivation for companies to implement sustainability programs is the increase in stakeholders’ awareness (customers, public opinion, shareholders, employees…). This trend is also reflected in a 2008 survey of 40 chief executive officers from many of the largest third-party logistics industries worldwide (Lieb and Lieb, 2010). In order of importance, the top three reasons to establish sustainability programs were “The corporate desire to do the right thing”, “The pressure from customers” and “The corporate desire to enhance company image”. This third period may be seen as the
integration phase. In the future, companies may go further. The Bearing Point 2010 survey indeed highlights that “more than one third of the 582 interviewed companies declare being ready to start up environmental actions in spite of their low present profitability provided they create value in the medium term” (Bearing Point 2010).

3 Literature review on sustainable supply chain optimization

Our research focuses on linking sustainability issues and operations management (OM) problems. The concept of sustainable supply chain management may be defined as follows: “management of material, information and capital flows as well as cooperation among companies along the supply chain while taking goals from all three dimensions of sustainability, i.e., economic, environmental and social, into account which are derived from customer and stakeholder requirements” (Seuring and Müller, 2008). According to Seuring and Müller (2008), environmental issues are dominating social ones in the sustainable supply chain management literature. Srivastava (2007) thus introduces the concept of green supply chain management that is defined as “integrating environmental thinking into supply-chain management, including product design, material sourcing and selection, manufacturing processes, delivery of the final product to the customer as well as end-of-life management of the product after its useful life.” In this section, we start by giving an overview of the literature on sustainable and green supply chain management. Then, a focus on sustainable inventory optimization is taken, as this is a special interest of our research. Some observations on this review are finally proposed.

3.1 Sustainable supply chain management

The literature dealing with sustainability and supply chains is very extensive. Several reviews dealing with sustainable supply chain management are firstly mentioned. Bloemhof-Ruwaard et al. (1995) is one of the first papers establishing a strong link between operational research (OR) and sustainability issues. This paper reviews the early literature on this field. About 50 papers are already mentioned. In Corbett and Kleindorfer (2001a, 2001b), two special issues of Production and Operations Management are introduced. The first one deals with manufacturing and eco-logistics and the second one is about integrating operations and environmental management systems. According to the author, an on-going integration of environmental management and operations is occurring. Midgley and Reynolds (2004)
reinforce the link between OR and environmental planning for sustainable development. The author found that the two fields share three generic issues i.e. the complexity and uncertainty, multiple and often conflicting values and political effects. Corbett and Klassen (2006) focus on total quality management and supply chain management to analyze how adopting environmental perspectives may produce unexpected side benefits. The number of literature review papers on sustainable supply chain management has drastically increased these last years. We refer to Linton et al. (2007), Srivastava (2007), Carter and Rogers (2008), Seuring and Müller (2008), Kleindorfer et al. (2009), Ilgin and Gupta (2010), Mollenkopf et al. (2010), Halldorsson and Kovacs (2010), Sarkis et al. (2011) and Dekker et al. (2012) for reviews. These papers cite up to 450 articles, illustrating the extent to which the sustainable supply chain management literature has grown.

Comparing to this extensive literature on sustainable supply chain management, the model-based literature is less developed. Moreover, model-based literature has mainly focused on reverse logistics and waste management (Sbihi and Eglese, 2007). Several authors mention a lack of model based research on sustainable supply chain management (Benjaafar et al., 2010). Several traditional OM problems have nevertheless been revisited with sustainability considerations. Among other, several papers dealing with sustainable supply chain design problems may be found. These papers use multiobjective optimization techniques to explicitly include LCA criteria into the design of a supply chain (see e.g. Nagurney et al., 2006; Quariguasi Frota Neto et al., 2008; Chaabane et al., 2011; Cachon, 2011; Chaabane et al., 2012). The transportation mode selection problem has also attracted some research (see e.g. Cholette and Venkat, 2009; Corbett et al., 2009; Pan et al., 2010; Hoen et al., 2011). Indeed, finding more sustainable ways of transportation seem to be a challenging issue. The relationship between the supply chain’s actors may deeply influence the sustainable performances of the supply chain. Several types of relationship as coordination, cooperation or competition are studied in the sustainable supply chain literature (see e.g. Corbett and DeCroix, 2001; Ni et al., 2010; Caro et al., 2011; Liu et al., 2012). These papers give examples of OM problems that can be addressed with a sustainability perspective.
3.2 Literature review on sustainable inventory optimization

Inventory management is a very strong field of research. However, including sustainability concerns into inventory models has not attracted a lot of research yet. Apart from our own contribution (Bouchery et al., 2012), only three published papers were found in this category. Hua et al. (2011) extend the economic order quantity (EOQ) model to take carbon emissions into account under the cap and trade system. Analytical and numerical results are presented and managerial insights are derived. Bonney and Jaber (2011) briefly present an illustrative model that includes vehicle emissions cost into the EOQ model. The authors refer to this model as the environmental economic order quantity. Finally, Jaber et al. (2012) include emissions from manufacturing processes into a two-echelon supply chain model. Different emissions trading schemes are studied. Analytical and numerical results are used to provide managerial insights. The efficiency of the different emissions trading schemes under study is also discussed.

Several working papers may also be found. Avci et al. (2012) use a repairable inventory model to study the adoption of a battery-switching station for electric vehicles. Customers’ adoption and usage as well as environmental impact are studied. Several insights are derived. Among them, the authors show that a well-intended policy intervention may actually be harmful to the environment. Absi et al. (2011) include carbon emissions constraints on a lot-sizing model. Four types of constraints are proposed and analyzed. One case is shown to be solvable in polynomial time, while the three others are NP-hard. Benjaafar et al. (2010) include carbon emission constraints on single and multi-stage lot-sizing models with a cost minimization objective. Four regulatory policy settings are considered. Insights are derived from an extensive numerical study. Velázquez-Martínez et al. (2011) examine the limit of aggregate carbon emissions models by studying different aggregate approaches for transportation carbon emissions in a lot-sizing model. Their numerical experimentation shows that the magnitude of errors can be substantial. Chen et al. (2011) investigate how operational adjustment can be used to reduce carbon emissions under a constraint on carbon emissions in the EOQ model. Finally, Saadany et al. (2011) study a two-echelon supply chain model where the demand is assumed to be a function of the price and product’s environmental quality.
3.3 What can we deduce from the review?

First of all, we can observe a growing body of literature aiming at optimizing inventory problems with sustainability criteria. This field of research is still in its infancy and there is a huge potential for future research. The situation is indeed very unbalanced comparing to the literature on sustainable supply chain management. Several working papers intend to fill the gap.

Moreover, several trends are pointed out in the proposed literature review. It may be noticed that carbon emissions play a major role in nowadays’ vision of sustainability. The majority of the published papers focus on both cost and carbon emissions. This is certainly a narrow vision of sustainable development. However, global warming is a major problem that catches all the considerations. Moreover, the most common way to include carbon emissions in supply chain optimization models is to focus on carbon emissions regulatory policies. Two main observations may be deduced from this trend.

First, this regulation based integration of sustainable development issues into inventory models implies to understand sustainable development in its weak sense. The most often used regulatory policies are indeed the cap-and-trade and the carbon tax regulatory policies. For these two policies, a price is given to carbon emissions. This amount to aggregate the different sustainability criteria into a single metric, thus that substitutability between the different sustainable development dimensions is possible. Other optimization techniques may be used to favor the strong vision of sustainable development.

Second, the regulation is not the only green pressure for companies. Indeed, firms are becoming increasingly proactive with respect to sustainable development. One possible way to reflect this new trend is the one followed by Saadany et al. (2011) where the demand is assumed to be a function of the price and product’s environmental quality. Several other ways of including sustainability criteria into inventory models are available. The model-based research literature may also consider this new trend in company positioning in order to develop new models. In our research, we assume that the firm will decide on economic, environmental and social tradeoffs by taking into account all the sustainable pressures that are faced. To do so, multiple criteria decision aid (MCDA) techniques seem to be a valuable and promising tool.
4 Conclusion

Sustainable development is now integrated into the organizations’ strategy. Moreover, we believe that the concept of strong sustainable development may deeply impact the research on sustainable supply chain. On the other hand, the presented literature review on sustainable inventory optimization models highlights the lack of research in this field. Thus, this PhD dissertation aims at contributing to this literature by acknowledging the concept of strong sustainability and by taking the new companies’ positioning on sustainability issues into account.
Chapter 2: Measuring sustainable supply chain performance

Defining and measuring sustainable supply chain performance is a prerequisite when aiming at optimizing the supply chain with sustainability concerns. Our research focuses on inventory models that mainly deal with the procurement and the distribution stages of the supply chain. In this chapter, we show that the existing performance frameworks for sustainable supply chain do not behave satisfactorily when focusing on the procurement and distribution stages. We thus contribute by proposing a new set of KPIs for assessing sustainable procurement and distribution supply chain performance. We also propose a new methodology for KPIs set building in this context.
1 How to measure sustainable supply chain performance?

In this section, a new classification of the literature on KPIs for sustainability is proposed. This classification allows drawing several insights. First, the emerging literature on sustainable supply chain performance mainly applies product LCA tools. Second, we argue that these frameworks do not behave satisfactorily when focusing on procurement and distribution supply chain.

1.1 Introduction

Measuring sustainable supply chain performance is a prerequisite when aiming at optimizing the supply chain with sustainability concerns. Our research focuses on inventory models that mainly deal with the procurement and the distribution stages of the supply chain. As the procurement of a link in supply chain may generally be seen as the distribution of another link, we propose to use the term of distribution supply chain in the rest of the document. This kind of system is often constituted by several locations (central warehouses, consolidation centers, distribution centers, retailers) linked by several transportation modes. Distribution supply chain is a traditional focus in operations management as this stage of the product life-cycle traditionally accounts for a substantive part in the supply chain performance. Distribution supply chain is also very innovative while dealing with sustainable development as the impacts of this stage of the product life-cycle are generally visible to the final consumer.

These observations legitimate considering distribution supply chain when reporting on sustainable development. Nevertheless, is it relevant to adopt this perspective to measure sustainable supply chain performance? We first would like to stress that the implementation of this kind of analysis in practice should not be considered solely. This would indeed result in adopting a myopic view of sustainability issues as distribution is only one stage of the product life-cycle. A KPIs set for sustainable distribution supply chain performance should rather be implemented as a part of an overall sustainability portfolio. However, establishing KPIs for sustainable distribution supply chain is justifiable because the impacts of the distribution are rarely negligible and are generally different from the other life-cycle stages.
Chapter 2: Measuring sustainable supply chain performance

We aim at finding KPIs set applicable for sustainable distribution supply chain in the existing literature.

### 1.2 KPIs for sustainability, a classification

The literature dealing with KPIs sets for sustainability is very vast. In order to draw insights from this literature, we present a new classification by focusing on the system under consideration. We indeed classify the existing literature into four main categories, i.e. the geographic level, the corporate level, the product level and the sector level.

Since the United Nations and national governments have been the driving force behind sustainable development, the first initiatives for KPIs set building have been focused on national, regional and community level (Labuschagne et al., 2005). The main international framework at geographic level is the United Nations Commission on Sustainable Development KPIs set (UNCSD, 2001), but a lot of other initiatives exist (see e.g. Daly and Cobb, 1989; Palme et al., 2005).

In the second half of the 1990s, new KPIs sets were created at company level. The main initiative in this category is the Global Reporting Initiative (GRI, 2002). This international guideline is one of the most prevalent standards for sustainability reporting. Nevertheless, many other institutional and academic frameworks exist (see e.g. Labuschagne et al., 2005; Azapagic and Perdan, 2000; Veleva and Ellenbecker, 2001; IChEn, 2002; Krajnc and Glavic, 2005).

Even if KPIs for sustainability at corporate level is an important tool to report on sustainable development performance, it has two important drawbacks. First, large efforts may result in small overall improvements as sustainable development performance is related to the supply chain as a whole. Moreover, companies are mainly attracted to overcome local impacts for which they are directly held responsible. Global impacts for which all companies in the supply chain are responsible can be left behind (Tsoulfas and Pappis, 2008). The concept of sustainable supply chain performance has thus emerged, mainly based on LCA tools. LCA is the most common technique to assess environmental impacts associated with all the stages of a product's life from cradle to grave. The main life-cycle stages are raw material extraction, materials processing, manufacturing, distribution, use, repair and maintenance, and disposal.
or recycling. The life-cycle stages are thus very similar to the supply chain processes, the main difference lies into the inclusion of the use phase into the LCA framework. LCA is traditionally used to assess environmental impacts of products. However, several authors have tried to fine-tune the method in order to include economic and social analyses (Kumaran et al., 2001; Norris, 2001; Benoît et al., 2010; Gauthier, 2005). As they are based on LCA tools, all the frameworks for sustainable supply chain are product-oriented and set up sustainable development indicators for all the product life stages (see e.g. Tsoulfas and Pappis, 2008; Clift, 2003; Quariguasi Frota Neto et al., 2008).

In our literature review, we distinguished a fourth level of reporting based on a sector analysis. KPIs for sustainability at a sector level are neither product- nor company-oriented but activity-oriented. An example of this kind of framework in the transport sector is used in Joumard and Nicolas (2010).

1.3 Insights

It can firstly be noticed that the literature on sustainable supply chain performance is rapidly growing as a part of the sustainable supply chain management literature. This literature mainly applies LCA tools to measure the sustainable supply chain performance. The proposed KPIs sets for sustainable supply chain performance measurement include economic, environmental and social criteria. However, it is often claimed that the social impacts of supply chain are harder to evaluate. The instinctive way of reporting about sustainable development performance for distribution supply chain is to focus on this literature by using an extended product LCA analysis (that includes environmental, economic and social aspects) and by focusing on the appropriate stage of the product life-cycle. Distribution supply chain generally concerns several actors; this is thus very hard to rely on KPIs for sustainability at corporate level.

However, we believe that the existing product LCA frameworks do not behave satisfactorily when focusing on distribution supply chain. For instance, Tsoulfas and Pappis (2008) frame their first principle for transportation as “minimizing distance covered”. Nevertheless, several factors in outbound supply chain are known to potentially dominate distances (Cholette and Venkat, 2009). Reporting only on distance will certainly favor road transportation, as shipping and rail transportation are known to be inappropriate for short distances. This example proves
that existing product LCA tools for sustainability are not precise enough for our purpose. Moreover, LCA analyses require a huge amount of data that are often unavailable. This type of analysis would thus be difficult to perform for companies. These considerations lead us to propose in the next section a new set of KPIs to assess sustainable distribution supply chain.

2 KPIs for sustainable distribution supply chain: methodology and application

In this section, we first present our methodology for KPIs set building in the context of sustainable distribution supply chains. The proposed methodology is then applied to propose a set of KPIs to assess the sustainable performance of distribution supply chain.

2.1 KPIs set building methodology

KPIs set building deserves a reliable underlying methodology. The proposed methodology consists into six steps:

Step 1: Definition of sustainability and derivation of the underlying dimensions.
Step 2: Delimitation of the system under study and characterization of its sub-processes.
Step 3: Setting of the strategic orientations.
Step 4: Derivation of KPIs goals and KPIs main characteristics.
Step 5: Definition of impact oriented criteria for all sub-processes of the defined system.
Step 6: Definition of the indicator to be used for each criterion.

Step 1 enables defining what is aimed to be measured. Sustainable development should indeed be clearly defined in order to precise the dimensions that have to be taken into account. Step 2 enables defining the system under consideration and its sub-processes. These two steps are essential to define sustainability criteria in Step 5. These criteria are indeed impact oriented, i.e. they should be defined for all the sustainability dimensions taken into consideration. Moreover, these criteria have to be setup for all sub-processes of the considered system. Most of the methodologies for sustainability KPIs set building found in the literature disregard that performance measurement is subjective by essence. Applying such methodologies may thus lead to a misleading objectivity belief. In the above methodology, we propose to explicitly state the chosen strategic orientations implied by performance measurement in Step 3. KPIs
goals and characteristics can then be derived in step 4. Knowing the kind of information (step 5) and the characteristics (step 4) needed, KPIs can be set up in step 6. We propose to apply this methodology to build a KPIs set in the context of sustainable distribution supply chain in the next section.

2.2 Applying the proposed methodology to build a KPIs set for sustainable distribution supply chain

Step 1: Definition of sustainability and derivation of the underlying dimensions:
The proposed definition of sustainable development is the “design and operation of human and industrial systems to ensure that humankind’s use of natural resources and cycles do not lead to diminished quality of life due either to losses in future economic opportunities or to adverse impacts on social conditions, human health and the environment” (Mihelcic et al., 2003). In this definition, the human health is separated from the three traditional sustainable development dimensions. However, we decide to include it into the social pillar as human health is related to social aspects.

Step 2: Delimitation of the system under study and characterization of its sub-processes:
We focus on the SCOR model (SCC, 2008) classification as this guideline is an international reference. Our work deals with distribution supply chain. In the SCOR model, distribution supply chain corresponds to the deliver process. This process consists of four sub-processes: order, warehouse, transportation and delivery. Out of these four sub-processes, we believe that transportation and warehousing are the most impacting ones. We focus on these two sub-processes in what follows.

Step 3: Setting of the strategic orientations:
We distinguish three major KPIs related debates in the literature. The first one deals with the possibility to create a standard KPIs set for sustainability. This debate remains open in the literature, some authors arguing that indicators need to be established on a case-by-case basis (e.g. Clift, 2003), some others arguing that it is possible to have a standard set of indicators (e.g. Veleva and Ellenbecker, 2001). The second question is about the possibility to aggregate the criteria into a composite indicator\(^2\). The recurring appeal for “keeping it simple” explains

\(^2\) A composite indicator is formed when individual indicators are compiled into a single index on the basis of an underlying model (Nardo et al., 2005).
the intensive use of composite indicators in the literature (Gasparatos et al., 2008). The third traditional debate held with some passion in the literature can be formulated as: Is it profitable to be green?

We thus recommend adopting the three following strategic orientations. First, a standard set of KPIs should be created. The Global Reporting Initiative experience (GRI, 2002) indeed allows expecting standardization, as this international guideline is followed by more and more companies from different size and different sectors all over the world. Second, we advocate not aggregating the criteria into a composite indicator by separating criteria for each potential impact and for each sub-process. Data aggregation into a composite indicator indeed implies compensability and substitutability between criteria (Munda and Nardo, 2008). These drawbacks seem to be hardly compatible with the strong vision of sustainability. Finally, we propose a quantified set of KPIs in order to be able to measure the eco-socio-efficiency (i.e. the economic, ecological and social efficiency) of the distribution supply chain. By doing so, it is also possible to track improvement and to benchmark the current situation with other supply chains.

**Step 4: Derivation of KPIs goals and KPIs main characteristics:**

The KPIs goals should be derived from the recommended strategic orientations, i.e. creating a standard set of non-aggregated quantified KPIs. Creating a standard set of KPIs allows comparison over time and evaluation among different supply chains which are two important KPIs goals. This is also a prerequisite for effective communication both internally (information and motivation of the workforce) and externally (sustainability reports). The second recommended strategic orientation requires the separation of criteria for each sustainability impact. This feature is necessary to get an effective technical support for sustainable development management systems. Moreover, these criteria can be used as a basis to derive and to pursue sustainability targets. The last recommended strategic orientation is to propose a quantified set of KPIs. Doing so, optimization potentials can be highlighted and the potential win-win situations can be identified. These KPIs goals derived from the recommended strategic orientations are in accordance with Jasch (2000).
Step 5: Definition of impact oriented criteria for all sub-processes of the defined system:
KPIs characteristics are well discussed in the literature (see e.g. Veleva and Ellenbecker, 2001; Schaltegger et al., 1996). From our point of view, it is possible to classify KPIs characteristics into two categories. The first one includes very common and essential features for KPIs such as being simple yet meaningful, based on available and reliable data, relevant to the information needs of stakeholders, and including a manageable number of indicators. The second type of KPIs characteristics have to be derived from the chosen strategic orientations. In our case, KPIs should be standard thus comparable over time and against relevant benchmark. Standardization also implies that KPIs should not be technology oriented (as sustainable development indicators should not be based on the assumption that only one path of development is valid as state in Anderson, 1991). The second recommended strategic orientation requires a set of impact oriented indicators rather than a single composite index. Finally, the indicators should be expressed with metrics in order to obtain a quantified set of KPIs.

Step 6: Definition of the indicator to be used for each criterion:
Table 2.1 and table 2.2 thus present the criteria and indicators for the two sub-processes under consideration.

Table 2.1: Proposed KPIs for transportation

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Indicator</th>
<th>Metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economic Pillar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial Performance</td>
<td>Transportation Cost</td>
<td>€ / ton</td>
</tr>
<tr>
<td>Service Level</td>
<td>% of Product Deliver in Time</td>
<td>%</td>
</tr>
<tr>
<td>Environmental Pillar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy Consumption</td>
<td>Energy Use</td>
<td>kJ / ton</td>
</tr>
<tr>
<td>Resource Consumption</td>
<td>Material Use</td>
<td>kg / ton</td>
</tr>
<tr>
<td>Global Warming</td>
<td>GHG Emissions</td>
<td>kg (CO₂eq) / ton</td>
</tr>
<tr>
<td>Social Pillar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Human Toxicity</td>
<td>Human Toxicity Potential</td>
<td>DALY³ / ton</td>
</tr>
<tr>
<td>Congestion</td>
<td>% of Time Lost due to Congestion</td>
<td>%</td>
</tr>
<tr>
<td>Work Conditions</td>
<td>Absenteeism Rate</td>
<td>number / ton</td>
</tr>
<tr>
<td>Safety</td>
<td>Injury Rate</td>
<td>number / ton</td>
</tr>
</tbody>
</table>

³ DALY (Disability Adjusted Life Year): Assesses emissions and noise (see Hofstetter and Müller-Wenk, 2005)
Table 2.2: Proposed KPIs for warehousing

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Indicator</th>
<th>Metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economic Pillar</td>
<td>Warehouse Cost</td>
<td>€ / ton</td>
</tr>
<tr>
<td>Service Level</td>
<td>Fill Rate</td>
<td>%</td>
</tr>
<tr>
<td>Environmental Pillar</td>
<td>Energy Use</td>
<td>kJ / ton</td>
</tr>
<tr>
<td>Resource Consumption</td>
<td>Material Use</td>
<td>kg / ton</td>
</tr>
<tr>
<td>Global Warming</td>
<td>GHG Emissions</td>
<td>kg(CO₂eq) / ton</td>
</tr>
<tr>
<td>Space Utilization</td>
<td>Space Use</td>
<td>m² / t</td>
</tr>
<tr>
<td>Social Pillar</td>
<td>Absenteeism Rate</td>
<td>number / ton</td>
</tr>
<tr>
<td></td>
<td>Injury Rate</td>
<td>number / ton</td>
</tr>
</tbody>
</table>

It may be noticed that the majority of the proposed KPIs are applicable for both transportation and warehousing. Moreover, all the indicators are technology independent and expressed as metrics to allow comparison between supply chains. To keep a manageable number of KPIs, we focus on the major impacts of transportation and warehousing found in the literature. The work conditions criterion is one of the most difficult to evaluate. According to several discussions with academics and practitioners, we decide to use the absenteeism rate as an indicator of the work conditions.

To validate our methodology, we contacted a French retail chain and a third party logistics to discuss our results with sustainable development managers. The proposed KPIs set was judged useable in an industrial context and representative of the major distribution supply chains impacts. The proposed methodology was seen valuable as managers face a real lack of methodological concerns while dealing with sustainable development. We have also validated the proposed KPIs set by applying this one to several distribution supply chains of a French retail chain. We then compare the proposed KPIs to the existing set used by the company. By discussing with our industrial partner, we conclude that our proposition was easily implementable and more general than the existing KPIs set. This feature allows simple comparison over time and among different supply chains. This also simplifies internal and external communication and provides an effective technical support for sustainable development management systems. This KPIs set can be used as a basis to derive and to
pursue sustainability targets. The KPIs set proposed by the present work thus achieves our main goals.

3 Conclusion

The aim of this chapter is to provide an evaluation framework to assess the performance of distribution supply chain in terms of sustainability. To perform this assessment, we advocate the use of a standard set of KPIs. The study of the existing performance frameworks for sustainable supply chain allows concluding that these frameworks do not behave satisfactorily when focusing on distribution supply chain. We thus propose a methodology for KPIs set building in the context of sustainable distribution supply chain. An illustration of this methodology is given and a set of KPIs is proposed. The proposed methodology and the related KPIs are validated by sustainable development managers and applied in an industrial context. In the next chapter, sustainability criteria are included into inventory models by using multiobjective optimization techniques. The models are compatible with the KPIs set proposed in this chapter. However, a broader formulation of sustainability objectives is used in what follows. This allows academics and practitioners applying the following models both with their own fine-tuned KPIs sets or with the proposed KPIs set.
In this chapter, we reformulate the classical economic order quantity model as a multiobjective problem. By doing so, we aim at including sustainability criteria into inventory models by adopting the strong vision of sustainability. We refer to this model as the sustainable order quantity model. We also study a multi-echelon extension of the sustainable order quantity model. For both models, the set of efficient solutions is analytically characterized. We also propose a new interactive procedure that allows the decision maker to quickly identify the best option among these solutions. This interactive procedure acknowledges the proactive role of companies with respect to sustainability issues.
1 Introduction

The literature review on sustainable supply chain optimization shows a lack of operational models addressing this issue. We thus aim at including sustainability criteria into inventory models as inventory decisions aim at finding a good balance between transportation and warehousing impacts. The concept of strong sustainability states that reducing all aspects of sustainability to a single objective is not desirable. We thus study a multiobjective formulation of the EOQ model. We refer to this model as the sustainable order quantity (SOQ) model. A multi-echelon extension of the SOQ model is also studied. For both models, the set of efficient solutions is analytically identified. We also consider that firms are becoming increasingly proactive with respect to sustainability. MCDA techniques may be applied in this context. We thus propose an interactive multiobjective optimization procedure that enables the firm to provide preference information about economic, environmental and social tradeoffs in order to quickly identify a satisfactory solution. The contribution of this chapter is thus threefold. First, innovative inventory models including sustainable development criteria are presented. Second, multiobjective optimization results are provided for the two proposed models. Third, the proposed interactive procedure enables users to identify quickly a satisfactory solution and to implement the model in practice.

The chapter is organized as follows. The proposed procedure is presented in section 2 after a review of the related background. In section 3, the multiobjective formulation of the EOQ model is presented. An extension of the SOQ model to the multi-echelon case is studied in sections 4 and 5. Section 4 is devoted to the study of stationary ordering policies while section 5 focuses on non-stationary policies. Finally, some conclusions are drawn in section 6.

2 A new interactive procedure helping the decision maker to identify a satisfactory solution

2.1 Theoretical background

Methods developed for multiobjective optimization problems can be classified into four classes i.e. no-preference methods, a priori methods, a posteriori methods and interactive methods, depending on the role of the decision maker (DM) in the solving process.
The method proposed in this chapter belongs to the latter class. In interactive methods, the preference information obtained from the DM is used to direct the process and only a subset of solutions is generated and evaluated. Interactive multiobjective optimization problem solving is a constructive process consisting of several iterations where the DM builds a conviction of what is possible. Moreover the DM confronts this knowledge with his/her preferences that may also evolve through the process. In this setting, the most important stopping criterion is the DM’s conviction that a satisfactory solution has been reached (Branke et al., 2008).

In this chapter, a non-empty set of alternatives (operational decisions) \( A \) is evaluated on a family of \( n \) criteria \( Z_1;Z_2;...;Z_n \) with \( Z_i : A \rightarrow \mathbb{R} \) (the symbol \( \forall \) corresponds to “for all”). We assume that the criteria represent sustainable development impacts that should be minimized. An alternative \( a \in A \) is said to be dominated if \( \exists b \in A \) so that \( \forall i \in [1,n] \), \( Z_i(b) \leq Z_i(a) \) with at least one strict inequality (the symbol \( \exists \) corresponds to “there exists”). The non-dominated solutions are called efficient solutions and the set of efficient solutions is called the efficient frontier.

To rank the different alternatives of \( A \), an aggregation model is constructed on the basis of preference information provided by the DM. This aggregation model is called a preference model. The preference model considered in this work is in the form of an additive value function \( V : A \rightarrow \mathbb{R} \), such that \( \forall a \in A \),

\[
V(a) = \sum_{i=1}^{n} v_i(Z_i(a)),
\]

where \( v_i \) are monotonic decreasing marginal value functions, \( v_i : \mathbb{R} \rightarrow \mathbb{R}, \forall i \in [1,n] \) (Keeney and Raiffa, 1976). The bigger is \( V(a) \), the better is alternative \( a \) for the DM. One possible way to elicit such a preference model is to directly ask the DM for some parameters of the targeted value function. Another approach consists of deducing value functions that are compatible with preference information given by the DM. In this second approach known as the preference disaggregation paradigm (Jacquet-Lagreze and Siskos, 1982), a finite subset of \( A \), called the learning set \( A_L \), is proposed to the DM who is required to compare some of these alternatives. This approach allows the DM to gain more insights about his/her own
preferences and a better knowledge of the problem. Furthermore, judgments on alternatives are acknowledged as less demanding in terms of cognitive effort. The main difficulty encountered when using preference disaggregation is that several value functions are often compatible with the information obtained from the DM. The available methods can then be classified into two classes, depending on how they handle the multiplicity of compatible value functions. The first one includes UTA-GMS (Greco et al., 2008) and GRIP (Figueira et al., 2009). These methods deal with all the value functions compatible with the preference information obtained from the DM and seek robust conclusions. For the second class of methods known as meta-UTA techniques, a particular value function is selected by using some predefined rules (Jacquet-Lagrèze and Siskos, 2001). There are four main meta-UTA techniques, i.e. UTA* (Siskos and Yanacopoulos, 1985), UTAMP I (Beuthe and Scannella, 1996), UTAMP II (Beuthe and Scannella, 2001) and ACUTA (Bous et al., 2010). Moreover, Kadziński et al. (2012) propose a method for selecting a representative value function in the GRIP framework.

Combining preference disaggregation and interactive methods is not a new idea. Jacquet-Lagrèze et al. (1987) propose a method that optimizes an additive value function, which has been interactively assessed, to focus on a particular alternative of \( A \). However, this method does not allow the DM to learn about the problem as the value function assessment is the unique interactive phase. Stewart (1987) proposes an interactive method for the progressive elimination of elements from a finite set of alternatives. In this method, the set of utility functions compatible with the preference information given by the DM is used to eliminate elements of \( A \). Siskos and Despotis (1989) use UTA to select a value function that is optimized within a feasible region defined at each iteration on the basis of satisfaction levels. Figueira et al. (2008) present an interactive procedure where GRIP is used to build a set of additive value functions compatible with the preference information obtained from the DM. This set is applied to \( A \) to deduce necessary and possible rankings that will help the DM to either select a solution or give new preference information.

The proposed interactive procedure combines the idea of Jacquet-Lagrèze et al. (1987) consisting in optimizing a particular additive value function to focus on a new solution with the interactive methodology proposed Figueira et al. (2008).
2.2 The proposed interactive procedure

The study of all efficient solutions can become too time-consuming in practice, especially in an operational context where decisions may be taken several times a day. In this context, it can be useful to start with a rather small but representative learning set and to present a new interesting solution to the DM. Our interactive procedure is based on this idea and consists of a number of iterations. At each iteration, a value function reflecting the preference information given by the DM is obtained by using the preference disaggregation approach. This value function is then optimized on $A$ to highlight a new solution that is proposed to the DM. The procedure stops when a satisfactory solution is found. The proposed interactive procedure is described in figure 3.1.

Figure 3.1: The proposed interactive procedure

This interactive procedure allows the DM to learn about the problem and identify what is possible as a new solution $a^*$ is presented at each iteration. It also enables the DM to have evolving preferences as he / she can come back to the preference information given in Step 2. Moreover, the generated value function is not required to represent perfectly DM’s preferences. Indeed, this value function is used only to point out a possibly interesting solution $a^*$. If $a^*$ is judged unsatisfactory, new preference information can be given and a new value function can be generated.
The proposed procedure is compatible with any meta-UTA techniques. In what follows, we decide to use the ACUTA method (Bous et al., 2010) as an example. In ACUTA, the chosen piecewise linear decreasing value function is generated by computing the analytic center of the feasible value functions polyhedron. This definition is implicit and ensures uniqueness. Being situated as far as possible from the boundaries of the feasible value functions’ polyhedron, the solution may also be considered as representative. There is however no guarantee that the selected value function perfectly represents DM’s preferences. As already explained, the procedure enables the DM to either validate or reject the result. Note that the computation of the analytic center is not a linear problem. However, computations were performed using the Diviz software platform (Consortium Decision Deck, 2006) and computation time remains reasonable in all of our experiments.

### 2.3 Discussion

As already mentioned, several value functions are generally compatible with the preference information obtained from the DM. In the proposed interactive procedure, a specific one is chosen without any validation by the DM. We have indeed argued that this value function is used only to point out a possibly interesting solution. Instead of validating the preference model, the DM can either validate or reject the solution found by optimizing a specific value function. Another method is proposed by Stewart (1987) where the optimality of every alternative in $A$ is checked for every utility function compatible with the preference information obtained from the DM. If the optimality of an alternative is inconsistent in every case, this one is eliminated. In this method, a non-eliminated alternative is randomly added to $A_L$ and the DM is asked to indicate some preference information taking this new element into account. However, the work of Stewart (1987) is limited to the case where $A$ is finite. As it is shown in the following models, operational decision problems are often characterized by an infinite decision space. Moreover, the interactive method of Stewart (1987) does not allow the DM to have evolving preferences. By contrast, our procedure enables the DM to come back to the preference information given in Step 2.

It may also happen that the preference information obtained from the DM in Step 2 leads to an empty set of compatible value functions. In this case, two options can be considered. Either the DM can reduce the number of pairwise comparisons made by focusing on the ones he /
she is more comfortable with. Doing so, the problem of finding a compatible value function is less constrained. Or it can be concluded that the DM’s preferences are not compatible with an additive value function model. The proposed algorithm is also compatible with non-additive value function models. For instance, Angilella et al. (2004) propose a preference disaggregation method for non-additive value functions.

In our procedure, the appropriateness of the result is deeply influenced by the selection of the learning set in step 1. The learning set should not contain too many alternatives, yet it should be representative enough of the problem. The problem of selecting the most appropriate learning set may deserve future research. However, the proposed procedure can be easily modified to make the learning set denser in the region of the proposed solution \( a^* \). Instead of presenting only one solution to the DM at each iteration, some solutions in the neighborhood of \( a^* \) could also be proposed. We nevertheless focus on the procedure proposed in figure 3.1 in what follows.

The procedure can also take strict limits (e.g. a carbon cap) on some criteria into account. In this case, it can be assumed that the additive value function generated in step 3 represents the DM’s preferences under reasonable limits. The learning set can be restricted to alternatives that respect the caps and the limitations can be added to the optimization problem in step 4 by using constraints.

3 The sustainable order quantity model

3.1 Including sustainable development criteria into the EOQ model

The EOQ model was first derived by Harris (1913). Assuming a constant and continuous demand, a fixed leadtime and no shortage allowed, the average total cost per time unit has the following expression:

\[
Z(Q) = PD + \frac{Q}{2}h + \frac{D}{Q}c,
\]  

(3.2)
Chapter 3: Including sustainability criteria into inventory models

with:

\( Q \) = batch quantity (decision variable),

\( P \) = fixed purchasing cost per product unit,

\( D \) = demand per time unit,

\( h \) = constant inventory holding cost per product unit and time unit,

\( O \) = fixed ordering or setup cost.

As the cost function \( Z \) is strictly convex for \( Q \in \mathbb{R}_+^* \), the optimal batch quantity has the following expression:

\[
Q^* = \sqrt{\frac{2OD}{h}}. \tag{3.3}
\]

It can be noticed that the value \( P \) does not affect the optimal order quantity. This parameter is thus omitted in what follows.

Considering that minimizing the cost may not be the unique company objective, environmental and social objectives are included into the model. We refer to this multiobjective extension of the EOQ model as the SOQ model. Note that we propose a methodology to build sustainable KPIs for distribution supply chains in chapter 2. A set of such KPIs for delivery and warehousing processes is also suggested. The SOQ model is compatible with the proposed KPIs set; however, we use a broader formulation by considering general sustainability objectives.

From a general point of view, environmental and social impacts may be associated with any process of the product life-cycle. In our work, we aim at including sustainable development criteria into inventory models. In the EOQ model, decision on the order quantity affects both ordering and warehousing operations. A structure similar to formula 3.2 is thus used to quantify sustainable development impacts. This assumption is also used in other papers (Arslan and Turkay, 2010; Benjaafar et al., 2010; Hua et al., 2011). We can also notice that the KPIs proposed in chapter 2 are by a majority applicable for both ordering and warehousing processes.
Let \( n \) be the number of criteria (\( n \in \mathbb{N}^\ast \)). Each economic, environmental or social impact \( Z_i \) is thus evaluated by using the following formula:

\[
Z_i(Q) = Q h_i + \frac{D}{Q}, \quad \forall i \in [1,n].
\] (3.4)

with:
- \( h_i, i \in [1,n] = \) constant inventory holding impact per product unit and time unit pertaining to criteria \( i \),
- \( O_i, i \in [1,n] = \) fixed ordering impact pertaining to criteria \( i \).

In the decision space, the set of possible values for \( Q \) is \( A = \mathbb{R}^\ast_+ \). Let \( Z : A \to \mathbb{R}^a, Z(a) = \{Z_1(a),...;Z_n(a)\} \), \( \forall a \in A \), with \( Z_i \) defined by formula 3.4, \( \forall i \in [1,n] \). \( A^Z = Z(A) = \{(Z_1(Q),...,Z_n(Q))|Q \in A\} \) is the image of \( A \) in the criterion space (evaluation space). From a practical point of view, some alternatives of \( A \) are not of interest to the DM as there exists other alternatives that have lower impacts in every criteria. We analytically determine the efficient frontier \( E \) of the SOQ model and derive some properties of its image \( E^Z = Z(E) \) in the criterion space.

We also introduce the following notations:
- \( \mathbb{R}^n_+ = \{(x_1,...,x_n)|x_i \in \mathbb{R}^+, \forall i \in [1,n]\} \) is the nonnegative subset of \( \mathbb{R}^n \),
- Let \( S_1 \) and \( S_2 \) two subsets of \( \mathbb{R}^n \): \((S_1 + S_2) = \{s_1 + s_2|s_1 \in S_1, s_2 \in S_2\} \) is the Minkowski sum, \( E^Z_+ = (E^Z + \mathbb{R}^n_+) \). For \( n = 2 \), \( E^Z_+ \) thus includes all the elements of \( E^Z \) as well as all the elements situated at the top right of \( E^Z \) (see figure 3.3 for a graphical example).

As \( Z_i(Q) \) is strictly convex for \( Q \in \mathbb{R}^n_+, \forall i \in [1,n] \), the single objective minimum is expressed as follows:

\[
Q_i^* = \sqrt{\frac{2O_iD}{h_i}}.
\] (3.5)

We can assume without loss of generality that the criteria are arranged so that \( Q_1^* \leq ... \leq Q_n^* \).
Theorem 3.1. Let $E$ be the efficient frontier of the SOQ problem and $E^Z$ its image in the criterion space, then:

$$E = [Q_1^*, Q_2^*],$$

$E^Z$ is convex.

Proofs of chapter 3 are provided in appendix 3A. Note that theorem 3.1 is valid as soon as $Z$ is a general strictly convex function. We illustrate the results with two criteria ($n = 2$), for instance the cost and the carbon footprint in example 3.1. Let $D = 20$ product units per time unit, $O_{\text{cost}} = O_1 = 50$, $h_{\text{cost}} = h_1 = 1.5$, $O_{\text{emissions}} = O_2 = 200$ and $h_{\text{emissions}} = h_2 = 0.4$. It can be noticed that the parameters’ units are omitted. Indeed, they are not useful as only the ratios $O_i / h_i$ matter. The parameters must nevertheless be expressed with the same unit within a criterion. Applying formula 3.5, we obtain that $Q_1^* \approx 37$ and $Q_2^* \approx 141$. Figure 3.2 illustrate the results.

Figure 3.2: Cost and carbon emissions in function of the batch size

By applying theorem 3.1, we obtain that $E = [37;141]$. The image of the efficient frontier is illustrated in figure 3.3.
Chapter 3: Including sustainability criteria into inventory models

Figure 3.3: The image of the efficient frontier in the criterion space

It can be noticed that a significant carbon emissions reduction can be achieved by an operational adjustment that requires only a small financial effort. In example 3.1, the carbon emissions can be reduced by 22% (from 116 to 90) for a 5% cost increase (from 55 to 58) starting from the minimal cost (see figure 3.3). This highlights that operational adjustments are effective to improve the sustainability of supply chains. On the contrary, the financial effort will increase when getting closer to the minimum amount of emissions. In this case, the firms will tend to invest in carbon-reducing technologies in addition to operational adjustments (see chapter 4).

In the next section, a numerical example is used to illustrate the type of interaction and the type of result that the interactive procedure proposed in section 2.2 can produce for the SOQ model.

3.2 Applying the proposed interactive procedure to the SOQ model

In example 3.2, three criteria are taken into account for the SOQ model. We do not advocate that the proposed criteria are the most relevant ones but they are proposed as an example. As greenhouse gases reduction is nowadays a key issue, we decide to choose the carbon footprint as an environmental criterion. The fixed amount of carbon emissions per order represents the emissions related to order processing and transportation. An amount of carbon emissions can also be associated with the storage of each unit per time unit. The social dimension of sustainable development has received less attention in the literature (White and Lee, 2009).
There is a lack of consensus on how to assess the social performance of operations. In example 3.2, the injury rate is used as a social criterion. Injuries are indeed a major social impact of operations and are caused both by ordering and warehousing operations. These two KPIs are included in the KPIs set for sustainable distribution supply chain proposed in chapter 2. We focus on a didactic example and we imagine an interaction with a fictitious DM so as to illustrate the type of interaction and the type of result that the proposed method can produce. For the numerical calculation, the chosen values are presented below.

Table 3.1: Example 3.2 parameter’s values

<table>
<thead>
<tr>
<th></th>
<th>demand rate (D)</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>ordering cost (O₁)</td>
<td>100</td>
<td>320</td>
</tr>
<tr>
<td>ordering carbon footprint (O₂)</td>
<td></td>
<td>119</td>
</tr>
<tr>
<td>holding cost (h₁)</td>
<td>1</td>
<td>0.45</td>
</tr>
<tr>
<td>holding carbon footprint (h₂)</td>
<td></td>
<td>0.27</td>
</tr>
</tbody>
</table>

Applying formula 3.5, the three single objective optima can be calculated (see table 3.2).

Table 3.2: Single objective optima

<table>
<thead>
<tr>
<th></th>
<th>Q₁</th>
<th>Cost Z₁</th>
<th>Carbon Emissions Z₂</th>
<th>Injuries Z₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>economic order quantity</td>
<td>71</td>
<td>70.7</td>
<td>128.7</td>
<td>51.5</td>
</tr>
<tr>
<td>environmental order quantity</td>
<td>189</td>
<td>107.7</td>
<td>84.9</td>
<td>41.3</td>
</tr>
<tr>
<td>social order quantity</td>
<td>148</td>
<td>90.9</td>
<td>87.4</td>
<td>40.1</td>
</tr>
</tbody>
</table>

Applying theorem 3.1, the efficient frontier consists of any batch sizes between [71; 189]. The range on each criterion also appears in table 3.2. The final solution will depend on the relative importance the DM gives to each of the three criteria.

Iteration 1:

Step 1: We decide to include the economic order quantity a₁, the environmental order quantity a₅ and the social order quantity a₇ into the learning set. The corresponding batch sizes are 71, 189 and 148 respectively (see table 3.2). Only the images of the alternatives in the criterion space are presented to the DM (see table 3.3). We also include two other solutions a₂ and a₄ into the learning set with corresponding batch sizes of 110 (in the middle of [71; 148]) and 169 (in the middle of [148; 189]).
Table 3.3: The initial learning set

<table>
<thead>
<tr>
<th></th>
<th>Cost $Z_1$</th>
<th>Carbon emissions $Z_2$</th>
<th>Injuries $Z_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>70.7</td>
<td>128.7</td>
<td>51.5</td>
</tr>
<tr>
<td>$a_2$</td>
<td>77.7</td>
<td>97.5</td>
<td>41.9</td>
</tr>
<tr>
<td>$a_3$</td>
<td>90.9</td>
<td>87.4</td>
<td>40.1</td>
</tr>
<tr>
<td>$a_4$</td>
<td>99.3</td>
<td>85.4</td>
<td>40.4</td>
</tr>
<tr>
<td>$a_5$</td>
<td>107.7</td>
<td>84.9</td>
<td>41.3</td>
</tr>
</tbody>
</table>

*Step 2:* Assume that the DM provides the following preference information: $a_2 \succ a_4 \succ a_1$ ($\succ$ corresponds to strict preference).

*Step 3:* ACUTA is used with the provided preference information to compute a compatible value function:

$$V(Q) = \sum_{i=1}^{3} v_i(Z_i(Q)).$$

*(3.6)*

*Step 4:* $V(Q)$ can then be maximized. The optimum is found for $Q = 120$, the corresponding alternative is $a_6$ (80.8; 93.7; 41.0).

*Step 5:* The DM considers that $a_6$ is not satisfactory, this one is added to $A_L$.

**Iteration 2:**

*Step 2:* The DM provides the following additional information: $a_2 \succ a_6 \succ a_4 \succ a_1$.

*Steps 3 and 4:* With this new information, a new value function can be generated and optimized. The optimum is found for $Q = 102$, the corresponding alternative is $a_7$ (75.5; 101.4; 42.9).

*Step 5:* The DM considers that $a_7$ is not satisfactory, this one is added to $A_L$.

**Iteration 3:**

*Step 2:* The following preference information is given by the DM: $a_2 \succ a_7 \succ a_6 \succ a_4 \succ a_1$.

*Step 3 and 4:* The optimum of the new value function is found for $Q = 109$, the corresponding alternative is $a_8$ (77.4; 98.0; 42.0).

*Step 5:* Assume that the solution $a_8$ is satisfactory for the DM, the procedure stops.
It can be noticed that the resulting solution is relatively close to alternative $a_2$ which was randomly generated in Step 1. However, the DM feels more confident with alternative $a_8$ as he/she has learnt about the problem and about his/her own preferences. The proposed procedure enables an effective interaction with the DM as a satisfactory solution is quickly identified.

### 3.3 Sensitivity analysis

The previous section has shown that the proposed interactive procedure allows the DM to quickly find a satisfactory solution for the SOQ model. However, this procedure will be used in practice only if it ensures a certain kind of robustness. The following result proves that the procedure is quite insensitive to a slight change or an estimation error for any parameter of the model.

Recall that in the SOQ model, $n$ criteria ($n \in \mathbb{N}^*$) are evaluated by using formula 3.4,

$$Z_i(Q) = \frac{Q}{2} h_i + \frac{D}{Q} Q_i, \forall i \in [1, n].$$

Assume that the value function generated in the last iteration of the example 3.2 represents DM’s preferences. This value function is noted $V^*(Q) = \sum_{i=1}^{n} v_i^*(Z_i(Q))$ and is maximal for $Q = Q^*$. By using ACUTA, $\forall i \in [1, n]$, $v_i^*$ is piecewise linear decreasing. The following theorem proves that $V^*$ behaves as a cost function $Z_{eq}(Q) = \frac{Q}{2} h_{eq} + \frac{D}{Q} O_{eq}$ in a neighborhood of $Q^*$, with $h_{eq} = \sum_{i=1}^{n} \alpha_i h_i$ and $O_{eq} = \sum_{i=1}^{n} \alpha_i O_i$. It implies that $V^*$ has the same robustness as the cost function in the EOQ model.

**Theorem 3.2.** There exists $Q_{\min} < Q_{\max} \in \mathbb{R}^*$, such that:

$$Q^* \in [Q_{\min}, Q_{\max}].$$

$$\forall Q \in [Q_{\min}, Q_{\max}], V^*(Q^*) - V^*(Q) = Z_{eq}(Q) - Z_{eq}(Q^*).$$
The coefficients $\alpha_i$ can be obtained by using the following formula for $Q \in [Q_{\min}, Q_{\max}]$ such that $Q \neq Q^*$:

$$\alpha_i = \frac{v_i^*(Z_i(Q) - Z_i(Q^*))}{Z_i(Q) - Z_i(Q^*)}.$$  \hspace{1cm} (3.7)

For $Q \notin [Q_{\min}, Q_{\max}]$, a deviation appears between $V^*(Q^*) - V^*(Q)$ and $Z_{eq}(Q) - Z_{eq}(Q^*)$.

Figure 3.4 illustrates theorem 3.2, the chosen value function is the one obtained in iteration 3 of section 3.2. For this example, recall that $Q^* = 109$. $V^*$ behaves like a cost function in the EOQ model for a wide range of values as the segment $[Q_{\min}, Q_{\max}]$ is equal to [95,140].

This result strengthens the proposed interactive procedure for two reasons. First, this ensures robust results even if an error occurs when estimating a parameter of the model. This is a crucial point when dealing with sustainability criteria as companies often face difficulties to get reliable sustainability measures. Second, this implies valid results for a longer period of time. Slight changes in parameter values often occur in operational situations. As the procedure is quite insensitive to these changes, performing the interactive procedure is not required every time. Note that theorem 3.2 also implies that $V^*$ can be considered as a weighted sum of the criteria in the neighborhood of $Q^*$.

Figure 3.4: Illustration of theorem 3.2
4 The two-echelon serial sustainable order quantity model

4.1 Problem presentation and preliminary results

This section presents an extension of the EOQ model in a multi-echelon case. The considered model is a serial system with 2 echelons, where one warehouse supplies a single retailer (see figure 3.5). The model was first studied by Schwarz (1973).

Figure 3.5: The two-echelon serial system

The retailer faces a constant continuous demand. Leadtimes are assumed to be zero for clarity (fixed leadtimes can be easily handled) and no shortage is allowed. Moreover, initial inventories are assumed to be zero. Fixed ordering costs and linear holding costs are supported at each location. Let $Q_r$ and $Q_w$ be the batch quantities ordered respectively by the retailer and by the warehouse. An entire batch is delivered at the same time. The following result is taken from Schwarz (1973).

<table>
<thead>
<tr>
<th>Preliminary Result. An optimal policy is stationary-nested and respects the zero-inventory condition i.e.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_r$ and $Q_w$ are time invariant,</td>
</tr>
<tr>
<td>$Q_w = k Q_r$, with $k \in \mathbb{N}^*$,</td>
</tr>
<tr>
<td>The retailer orders only if its inventory level is null,</td>
</tr>
<tr>
<td>The warehouse orders when both the retailer and the warehouse have no inventory.</td>
</tr>
</tbody>
</table>

To simplify the notations, let $Q_r = Q$. The total cost can then be expressed as a function of $Q$ and $k$ :

$$Z(k,Q) = (h_r + (k-1)h_w) \frac{Q}{2} + (O_r + \frac{Q_w}{k}) \frac{D}{Q},$$  \hspace{1cm} (3.8)
with:

- $Q$ = batch quantity at the retailer (first decision variable),
- $k$ = strictly positive integer such that $Q_w = kQ$ (second decision variable),
- $D$ = demand per time unit,
- $h_r$ = constant inventory holding cost per product unit and time unit at the retailer,
- $h_w$ = constant inventory holding cost per product unit and time unit at the warehouse,
- $O_r$ = fixed ordering cost at the retailer,
- $O_w$ = fixed ordering cost at the warehouse.

If $h_r < h_w$, the minimum of formula 3.8 is found for $k^* = 1$. Else, let $k^\text{inf} = \sqrt{\frac{O_w(h_r-h_w)}{O_r h_w}}$.

$k^*$ is a strictly positive integer that can be found by using the following rule. If $k^\text{inf} < 1$, it is optimal to choose $k^* = 1$. Else, let $k^i \leq k^\text{inf} \leq k^i + 1$ with $k^i \in \mathbb{N}$.* If $\frac{k^\text{inf}}{k^i} \leq \frac{k^i + 1}{k^\text{inf}}$ then it is optimal to choose $k^* = k^i$. Otherwise, $k^* = k^i + 1$ (Axsäter, 2006). It follows that,

$$Q^* = \frac{2D(O_r + \frac{O_w}{k^*})}{h_r + (k^* - 1)h_w}. \quad (3.9)$$

We now consider the case where several criteria ($n \geq 2$) have to be taken into account and we refer to this problem as the two-echelon serial SOQ problem. Theorem 3.3 proves that each efficient ordering policy (efficient solution) can be found in the set of basic policies.

**Theorem 3.3.** For the two-echelon serial SOQ problem, an ordering policy leading to an efficient solution is basic i.e.:

- The retailer orders only if its inventory level is null,
- The warehouse orders when both the retailer and the warehouse have no inventory,
- All deliveries made to the retailer between successive deliveries to the warehouse are of equal size.

This section focuses on stationary policies then non-stationary policies are studied in section 5. When dealing with stationary policies, theorem 3.3 can be strengthened. An ordering policy
leading to an efficient solution is then stationary nested and respects the zero inventory condition as in preliminary result. The simplified notations $Q$ and $k$ are kept. Each sustainable development criterion is thus evaluated by using the following formula:

$$Z_i(k, Q) = (h_{ir} + (k - 1)h_{iw}) \frac{Q}{2} + (O_{ir} + \frac{O_{iw}}{k}) \frac{D}{Q}, \forall i \in [1, n],$$  \hspace{1cm} (3.10)$$

with:

- $h_{ir}, i \in [1, n]$: constant inventory holding impact $i$ per product unit and time unit at the retailer,
- $h_{iw}, i \in [1, n]$: constant inventory holding impact $i$ per product unit and time unit at the warehouse,
- $O_{ir}, i \in [1, n]$: ordering impact $i$ per order at the retailer,
- $O_{iw}, i \in [1, n]$: ordering impact $i$ per order at the warehouse.

$\forall i \in [1, n], \text{ if } h_{ir} < h_{iw}, k_i^* = 1$. Else, $k_i^*$ is a strictly positive integer that can be found by using the rule described earlier with $k_i^{\text{inf}} = \sqrt[\frac{O_{ir}(h_{ir} - h_{iw})}{O_{ir}h_{iw}}}, \forall i \in [1, n].$

The minimum of formula (3.10) is found for:

$$Q_i^* = \sqrt{\frac{2D(O_{ir} + O_{iw})}{h_{ir} + (k_i^* - 1)h_{iw}}}, \text{ and } k_i^* \text{ defined above, } \forall i \in [1, n].$$ \hspace{1cm} (3.11)$$

### 4.2 Multiobjective optimization of the two-echelon serial SOQ model

In this section, some theorems that characterize the efficient frontier of the two-echelon serial SOQ problem are presented. Compared with the single-echelon SOQ model, a strictly positive integer $k$ that represents the warehouse-retailer batch size multiplier is added as decision variable. Let $n$ be the number of criteria ($n \in \mathbb{N}^*$). In the decision space, the set of possible alternatives $A$ is $\{k, Q|k \in \mathbb{N}^*, Q \in \mathbb{R}_+^*\}$. Let $Z : A \rightarrow \mathbb{R}^n$, $Z(a) = \{Z_1(a), \ldots, Z_n(a)\}, \forall a \in A$, with $Z_i$ defined by formula (3.10), $\forall i \in [1, n]$. The image of $A$ in the criterion space is
Chapter 3: Including sustainability criteria into inventory models

\[ A^Z = \{(Z_1(k,Q),...,Z_n(k,Q)) | (k,Q) \in A \} \]. Let \( E \) be the efficient frontier of the problem and \( E^Z = Z(E) \) its image in the criterion space. Finally, let \( E^Z_k = (E^Z + \mathbb{R}^+_{\infty}) \).

We first consider the case with \( k \) fixed. \( A^Z_k = \{(Z_1(k,Q),...,Z_n(k,Q)) | Q \in \mathbb{R}^+ \}, \forall k \in \mathbb{N}^* \). The efficient frontier of this sub-problem is noted \( E_k \) and \( E^Z_k \) is its image in the criterion space. Let \( E^Z_k = (E^Z_k + \mathbb{R}^+_{\infty}) \). As formula 3.11 is strictly convex in \( Q \), assume that \( Q^*_i \) minimizes \( Z_i(k,Q) \).

**Theorem 3.4.** Let \( E_k \) be the efficient frontier of the two-echelon serial SOQ with \( k \) fixed and \( E^Z_k \) its image in the criterion space, then:

\[ E_k = [\min_i(Q^*_i), \max_i(Q^*_i)] \],

\( E^Z_k \) is convex.

It can be noticed that \( E^Z \subset \bigcup_{k=1}^{\infty} E^Z_k \). We could intuitively expect that \( E^Z \subset \bigcup_{k=\min(k^*_i)}^{\max(k^*_i)} E^Z_k \).

However, a counterexample can be found even for \( n = 2 \) as shown by example 3.3 (table 3.4).

**Table 3.4: Example 3.3 data set**

<table>
<thead>
<tr>
<th>demand rate (D)</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>holding impact 1 at the retailer (h_{11})</td>
<td>10</td>
</tr>
<tr>
<td>holding impact 1 at the warehouse (h_{1w})</td>
<td>6</td>
</tr>
<tr>
<td>holding impact 2 at the retailer (h_{2r})</td>
<td>4</td>
</tr>
<tr>
<td>holding impact 2 at the warehouse (h_{2w})</td>
<td>0.5</td>
</tr>
<tr>
<td>ordering impact 1 at the retailer (O_{1r})</td>
<td>50</td>
</tr>
<tr>
<td>ordering impact 1 at the warehouse (O_{1w})</td>
<td>500</td>
</tr>
<tr>
<td>ordering impact 2 at the retailer (O_{2r})</td>
<td>10</td>
</tr>
<tr>
<td>ordering impact 2 at the warehouse (O_{2w})</td>
<td>10</td>
</tr>
</tbody>
</table>

Applying formula 3.11 to example 3.3, we obtain that \( k^*_1 = 3 \) and \( k^*_2 = 3 \). It could then be tempting to conclude that \( E^Z = E^Z_3 \). However, some elements of \( E^Z_4 \) are also efficient. This can be seen in figure 3.6. In this example, \( E^Z \subset \{E^Z_3 \cup E^Z_4 \} \).
Theorem 3.5 states that a lower bound $k_{\min}$ and an upper bound $k_{\max}$ exist such that
\[ E^Z \subseteq \bigcup_{k=k_{\min}}^{k_{\max}} E_k^Z. \]

**Theorem 3.5.** There exists $(k_{\min}, k_{\max}) \in \mathbb{N}^2$ such that:

\[
\begin{align*}
1 & \leq k_{\min} \leq \min_i (k_i^*) , \\
\max_i (k_i^*) & \leq k_{\max}, \\
E^Z & \subseteq \bigcup_{k=k_{\min}}^{k_{\max}} E_k^Z.
\end{align*}
\]

It can also be noticed in the above example that $E_i^Z$ is non convex. This result can be generalized as soon as $E^Z$ is not included into a single set $E_k^Z$. This condition holds when $\min_i (k_i^*) \neq \max_i (k_i^*)$. However, the example shows that the converse is not true. This result is stated in theorem 3.6.

**Theorem 3.6.** If $\min_i (k_i^*) < \max_i (k_i^*)$, then $E_i^Z$ is non convex.

An illustration of the two-echelon serial SOQ problem is given with two criteria (the cost and the carbon footprint) in example 3.4. Parameter values can be found in table 3.5.
Table 3.5: Example 3.4 data set

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>demand rate (D)</td>
<td>20</td>
<td>holding impact 1 at the retailer (h_{1r})</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>holding impact 2 at the retailer (h_{2r})</td>
<td>2</td>
</tr>
<tr>
<td>holding impact 1 at the warehouse (h_{1w})</td>
<td>4</td>
<td>holding impact 2 at the warehouse (h_{2w})</td>
<td>0.15</td>
</tr>
<tr>
<td>ordering impact 1 at the retailer (O_{1r})</td>
<td>80</td>
<td>ordering impact 2 at the retailer (O_{2r})</td>
<td>45</td>
</tr>
<tr>
<td>ordering impact 1 at the warehouse (O_{1w})</td>
<td>350</td>
<td>ordering impact 2 at the warehouse (O_{2w})</td>
<td>70</td>
</tr>
</tbody>
</table>

Figure 3.7: The image of the efficient frontier in the criterion space

It can be noticed that $E^Z_+$ is non convex in example 5.4 (See figure 3.7). In this case, some efficient solutions cannot be generated by using a linear combination of the objectives. For instance, $E^Z_2 \cap E^Z_5$ is an efficient solution that cannot be found by optimizing a linear combination of the two objectives. However, this solution can represent a desirable trade-off for the company. The interactive procedure described in section 2.2 enables such solutions to be proposed by optimizing an additive value function instead of a simple weighted sum. This strengthens the proposed interactive procedure.
5 The two-echelon serial SOQ with non-stationary policies

5.1 Introduction to non-stationary policies

A policy is called stationary if each facility orders at equally-spaced points in time and in equal amount. Stationary policies are known to be optimal for the classical two-echelon serial EOQ model (Schwarz, 1973). Intuitively, it may be expected that the two-echelon serial SOQ model behaves as the two-echelon serial EOQ model. However, the former model is a multiobjective version of the latter. The complexity induced by moving from an EOQ model to an SOQ one may thus be seen as similar to moving from a single retailer model to a multi-retailer one where stationary policies are proven to be non-optimal (Roundy, 1985).

In the two-echelon serial SOQ model, \( n > 1 \) objectives \( Z_i(k, Q) \) defined by formula 3.10 should be minimized. As non-stationary policies are allowed, both \( k \) and \( Q \) may vary over time. In multiobjective optimization, the concept of optimality is replaced by the concept of efficiency. In section 4, we have proven that efficient ordering policies for the two-echelon serial SOQ model may be found in the set of basic policies (theorem 3.3).

The interest of non-stationary policies is illustrated with example 3.5. The related data can be found in table 3.6.

Applying formula 3.11, we obtain that \( k_1^* = 2 \) and \( k_2^* = 1 \). The stationary ordering policies are illustrated in figure 3.8. We recall that \( E_i^Z \) represents the image in the criterion space of the efficient frontier (set of the efficient solutions) for the restricted sub-problem with \( k = 1 \).

<table>
<thead>
<tr>
<th>Table 3.6: Example 3.5 data set</th>
</tr>
</thead>
<tbody>
<tr>
<td>demand rate (D)</td>
</tr>
<tr>
<td>holding impact 1 at the retailer (( h_{1r} ))</td>
</tr>
<tr>
<td>holding impact 2 at the retailer (( h_{2r} ))</td>
</tr>
<tr>
<td>holding impact 1 at the warehouse (( h_{1w} ))</td>
</tr>
<tr>
<td>holding impact 2 at the warehouse (( h_{2w} ))</td>
</tr>
<tr>
<td>ordering impact 1 at the retailer (( O_{1r} ))</td>
</tr>
<tr>
<td>ordering impact 2 at the retailer (( O_{2r} ))</td>
</tr>
<tr>
<td>ordering impact 1 at the warehouse (( O_{1w} ))</td>
</tr>
<tr>
<td>ordering impact 2 at the warehouse (( O_{2w} ))</td>
</tr>
</tbody>
</table>
For example 3.5, figure 3.8 shows that the set of efficient ordering policies is $E = E_1 \cap E_2$ if the problem is restricted to stationary policies. Choosing $k = 1$ will favor objective 2 while choosing $k = 2$ will favor objective 1. As efficient ordering policies are basic, $k$ should necessarily be an integer. It is thus impossible to balance the two objectives by choosing $k = 1.5$. However, switching from $k = 1$ to $k = 2$ when the warehouse places an order may be of interest if non-stationary ordering policies are allowed. This situation is illustrated in figure 3.9. Note that $E_{(1,2)}^Z$ represents the image in the criterion space of the non-stationary policy consisting in alternately choosing $k = 1$ and $k = 2$.

Example 3.5 illustrates the effectiveness of non-stationary ordering policies in generating efficient solutions. The next section is devoted to the analytical exploration of non-stationary policies in the two-echelon serial SOQ model.
5.2 The exploration of non-stationary policies

Non-stationary policies in a two-echelon serial SOQ context can be divided into three classes. In the first one, we consider that only \( Q \) may vary over time. In the second one that will deserve most of our attention, we consider that only \( k \) may vary over time. Finally, the third class is the most general case where both \( k \) and \( Q \) are taken as non-stationary.

Let us consider that only \( Q \) may vary over time. It may first be noticed that the average inventory level at the warehouse \( \overline{IL}_W \) is directly related to the average inventory level at the retailer \( \overline{IL}_R \) as \( \overline{IL}_W = (k - 1) \overline{IL}_R \). In this case, ordering with different values of \( Q \) is sub-optimal as the policy consisting in ordering equal batch size with the same average frequency reduces the inventory holding costs without increasing the ordering costs. The situation is exactly similar in the EOQ model where non-stationary ordering quantities are sub-optimal for the same reason. Efficient ordering policies may thus not be found in this first class.

Assume now that only \( k \) may vary over time. Example 5.5 shows that this type of ordering policies may be efficient. Let \( K \) be the list (tuple) of successive values taken by \( k \). \((K, Q)\) is the ordering policy with \( Q \) being stationary and \( k \) successively taking the values included in \( K \). It can first be noticed that each element of \( K \) is in \( \mathbb{N}^* \). Moreover, the ordering policy with
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\( K = (1,2,1,2) \) is the same as the ordering policy with \( K = (1,2) \). These two policies have also the same evaluation as the ordering policies with \( K = (2,1) \) or \( K = (2,1,1,2) \) and may thus be seen as equivalent. To simplify the presentation of the results, we focus on the list \( K \) that contains the minimum number of elements sorted in ascending order. Let \( \overline{K} \) be the arithmetic mean of the elements of \( K \). The following theorem states that an efficient ordering policy \((K,Q)\) has a quite simple structure as \( K \) contains in maximum two types of elements.

**Theorem 3.7.** If \((K,Q)\) is an efficient ordering policy, then \( K \) contains in maximum two types of elements i.e. \([\overline{K}]\) and \([\overline{K}]\).

It may also be noticed that \( \overline{K} \geq 1 \) and that there exists \( a \in \mathbb{N} \) and \( b \in \mathbb{N}^* \) such that

\[ \overline{K} = \left\lfloor \overline{K} \right\rfloor + \frac{a}{b} \]

with \( a < b \) and \( \gcd(a,b) = 1 \) as \( \overline{K} \) is the arithmetic mean of elements from \( \mathbb{N}^* \). Moreover, \( a \) and \( b \) are unique if \( a \neq 0 \).

By applying theorem 5.7, we can conclude that if \((K,Q)\) is an efficient ordering policy, then:

- \( K = (\overline{K}) \) if \( K \in \mathbb{N}^* \),
- \( K = ([\overline{K}], \ldots, [\overline{K}], [\overline{K}], \ldots, [\overline{K}]) \) else.

Non-stationary efficient ordering policies of type \((K,Q)\) are thus very simple to identify as the only required information is the value \( \overline{K} \). For instance, the efficient ordering policy \((K,Q)\) with \( \overline{K} = 1.4 = 1 + \frac{2}{5} \) is \( K = (1,1,1,2,2) \). This class of non-stationary ordering policies may also be easily evaluated on each objective \( Z_i \), as for all \( i \in [1,n] \):

\[
Z_i(K,Q) = \left( h_w + \left\lfloor \overline{K} \right\rfloor \left( 1 - \frac{b-a}{b\left\lfloor \overline{K} \right\rfloor + a} \right) h_{lw} \right) \frac{Q}{2} + \left( O_w + \frac{O_{lw}}{K} \right) \frac{D}{Q}.
\]

(3.12)

In example 3.6, we focus on \((K,Q)\) policies with \( \overline{K} \) limited to one decimal place. The related data can be found in Table 3.7.
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Table 3.7: Example 3.6 data set

<table>
<thead>
<tr>
<th>demand rate (D)</th>
<th>10 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>holding impact 1 at the retailer (h₁r)</td>
<td>3.0</td>
</tr>
<tr>
<td>holding impact 2 at the retailer (h₂r)</td>
<td>2.5</td>
</tr>
<tr>
<td>holding impact 1 at the warehouse (h₁w)</td>
<td>1.2</td>
</tr>
<tr>
<td>holding impact 2 at the warehouse (h₂w)</td>
<td>0.5</td>
</tr>
<tr>
<td>ordering impact 1 at the retailer (O₁r)</td>
<td>250</td>
</tr>
<tr>
<td>ordering impact 2 at the retailer (O₂r)</td>
<td>300</td>
</tr>
<tr>
<td>ordering impact 1 at the warehouse (O₁w)</td>
<td>1 250</td>
</tr>
<tr>
<td>ordering impact 2 at the warehouse (O₂w)</td>
<td>80</td>
</tr>
</tbody>
</table>

Applying formula 3.11, we obtain that $k_1^* = 3$ and $k_2^* = 1$. Note that $E_{2.5}$ corresponds to the efficient frontier of the non-stationary policy $(K, Q)$ with $\overline{K} = 2.5$ thus $K = (2,3)$. For this example, it can be proven that any ordering policies of type $(K, Q)$ with $\overline{K} > 3$ is dominated by the stationary policy with $k = 3$. Figure 3.10 shows the image in the criterion space of the stationary ordering policies with $k \in \{1;2;3;4;5\}$ as well as the non-stationary ordering policies with $\overline{K}$ limited to one decimal place and $1 < \overline{K} < 3$.

Figure 3.10: An example of non-stationary ordering policy in the criterion space
As for example 3.5, non-stationary policies of type \((K,Q)\) are effective in generating efficient solutions. These ordering policies may easily be implemented in practice as the change in ordering quantity only occurs at the warehouse and is limited to two different order quantities. In practice, \(\overline{K}\) may not be limited to one decimal place. However, this may be interesting to limit the inventory cycle time at the warehouse. In this example, choosing \(\overline{K} = 1.9\) and \(Q = 1575\) leads to an efficient ordering policy. As \(D = 10000\) product units per year, the cycle time at the warehouse is almost 3 years. \((1 + 2 \times 9)Q \approx 30000\) units will indeed be ordered by the warehouse before coming back to \(k = 1\). It may also be worth to notice that having \(1 < \overline{K} < 3\) is not a sufficient condition to obtain an efficient ordering policy. For instance, choosing \(\overline{K} = 2.1\) does not lead to any efficient solution. The results presented above only give necessary conditions to obtain non-stationary efficient ordering policies.

The analysis of the third class of non-stationary ordering policies where both \(k\) and \(Q\) are taken as non-stationary is left for future research. We did not find any example of efficient ordering policies in this class; however, a deepest analysis is required.

## 6 Conclusion

In this chapter, we use multiobjective optimization to include sustainability criteria into inventory models. Both single and multi-echelon formulations of the economic order quantity model are studied. For both models, the efficient frontier is analytically characterized. We also propose a new interactive method that enables the companies to quickly identify their most preferred solution. By doing so, some efficient methods enabling sustainable supply chains optimization are proposed.

One of the main sustainable supply chain challenges consists in reducing the carbon emissions issued from operations. The remainder of this PhD dissertation thus focuses on finding desirable balances between cost and carbon emissions in inventory models by considering several options. In chapter 4, the sustainable order quantity model is used to study the effectiveness of green technology investment to reduce the carbon footprint of the supply chain. Then, the two-echelon SOQ model is used in chapter 5 to study the impacts of buyer-supplier coordination in terms of cost and carbon emissions.
Appendix 3A

Proof of theorem 3.1:
Identification of the efficient frontier:
If $Q^*_i = Q^*_n$, $E = Q^*_i$ as $Z_i(Q^*_i)$ is the unique minimum on each criteria $i$.

Assume that $Q^*_i < Q^*_n$:

$Z_i(Q)$ is strictly increasing on $[Q^*_i, Q^*_n]$,

$Z_*(Q)$ is strictly decreasing on $[Q^*_i, Q^*_n]$,

$\forall i \in [1,n]$, $Z_i(Q)$ is strictly increasing on $[Q^*_i, \infty)$ and strictly decreasing on $(0, Q^*_i]$ then the solution is dominated if $Q \not\in [Q^*_i, Q^*_n]$,

then $E = [Q^*_i, Q^*_n]$.

Convexity:
As $\mathbb{R}^*_+$ is convex, we only have to prove that $\forall(a,b) \in E^\mathbb{Z} \times E^\mathbb{Z}$, the segment $[a,b]$ is included into $E^\mathbb{C}$.

Let $(a,b) \in E^\mathbb{Z} \times E^\mathbb{Z}$, if $a = b \in E^\mathbb{Z}$ by definition.

Else, let $a = Z(Q_a)$ and $b = Z(Q_b)$ with $(Q_a, Q_b) \in [Q^*_i, Q^*_n] \times [Q^*_i, Q^*_n]$.

$a \in E^\mathbb{Z}$ and $b \in E^\mathbb{Z}$.

$\forall \lambda \in [0,1], \text{ let } x_\lambda = \lambda a + (1-\lambda)b$.

As $Z$ is strictly convex, $x_\lambda$ is dominated by $Z(\lambda Q_a + (1-\lambda)Q_b)$.

So, $x_\lambda \in E^\mathbb{Z}$.

Proof of theorem 3.2:

$v_i$ are piecewise linear decreasing then there exits $(Q_{\min}, Q_{\max}) \in \mathbb{R}^*_+ \times \mathbb{R}^*_+$ such that:

$Q^* \in [Q_{\min}, Q_{\max}]$,

$\forall i \in [1,n]$, there exists $\alpha_i \in \mathbb{R}^*_+$ \ $\forall Q \in [Q_{\min}, Q_{\max}]$,

$v_i^*(Z_i(Q)) = v_i(Z_i(Q^*)) - \alpha_i (Z_i(Q) - Z_i(Q^*))$.

By applying formula 3.6, we obtain that $\forall Q \in [Q_{\min}, Q_{\max}]$:
\[ V^*(Q) = \sum_{i=1}^{n} v_i^*(Z_i(Q)) = V^*(Q^*) + \left( \frac{h_{eq} Q^*}{2} + \frac{O_{eq} D}{Q^*} \right) - \left( \frac{h_{qy} Q}{2} + \frac{O_{qy} D}{Q} \right), \] 
with \( h_{qy} = \sum_{i=1}^{n} \alpha_i h_i \) and 
\[ O_{eq} = \sum_{i=1}^{n} \alpha_i O_i. \]

It follows that:
\[ \forall Q \in [Q_{\min}, Q_{\max}], \quad V^*(Q^*) - V^*(Q) = Z_{eq}(Q) - Z_{qy}(Q^*). \]

**Proof of theorem 3.3: Similar to that of Schwarz (1973)**

(1) The retailer orders only if its inventory level is null:
Consider any feasible policy that does not satisfy (1) at some time \( t \). Every holding impacts in the interval \([0, t]\) will be reduced by reducing the amount of the preceding delivery by the inventory on hand at time \( t \) (or to zero) and increasing the amount of the delivery at time \( t \) by the same amount. This adjustment does not increase the number of deliveries and ordering impacts are thus reduced or kept equal. By repeating this adjustment for every retailer delivery time, a policy satisfying (1) will result.

(2) The warehouse orders when both the retailer and the warehouse have no inventory:
The fact that the warehouse orders when its inventory level is null is proven in the same manner as (1). To prove that the warehouse orders when the retailer has no inventory, we remark that on the other case, the warehouse order can be postponed until the retailer orders. This will decrease every holding impact at the warehouse without modifying the ordering impacts. By applying (1), this condition happens when the inventory at the retailer is null.

(3) All deliveries made to the retailer between successive deliveries to the warehouse are of equal size:
Assume that there are \( n \) deliveries to the retailer of lot sizes \( Q_k, k \in [1, n] \) such that \( \sum_{i=1}^{n} Q_i = Q \) between any two successive deliveries to the warehouse. The only impacts affected by these lot sizes are the holding impacts at the retailer. As \( D \) is constant, the minimum of all holding impacts at the retailer is reached when \( \forall k \in [1, n], \ Q_k = \frac{Q}{n} \).

**Proof of theorem 3.4:**
Similar to that of theorem 5.1.
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Proof of theorem 3.5:
The existence of $k_{\text{min}}$ is trivial.

Moreover, the mono-objective optima defined in formula 3.11 are included in $E$ by definition, then $1 \leq k_{\text{min}} \leq \min_{i} (k'_{i})$.

It also implies that if $k_{\text{max}}$ exists, $\max_{i} (k'_{i}) \leq k_{\text{max}}$.

$\forall i \in [1, n]$, $Z_{i}(k, Q)$ tends to infinity as $k$ tends to infinity. Let $e(k, Q) \in E$.

There exists $t \in \mathbb{N}^*$ such that $\forall i \in [1, n]$, $\forall Q \in \mathcal{R}^{+}_{s}$, $\forall n \in \mathbb{N}$, $Z_{i}(k, Q) < Z_{i}(t + n, Q)$.

Then $e$ dominates all elements of $\bigcup_{k=1}^{k_{\text{max}}} E_{k}$. That proves the existence of $k_{\text{max}}$.

Proof of theorem 3.6:

By using theorem 3.5, $E^{Z} \subset \bigcup_{k=1}^{k_{\text{max}}} E_{k}^{Z}$.

As $\min_{i} (k'_{i}) < \max_{i} (k'_{i})$, there exists $e_{k_{\text{min}}} \in E_{k_{\text{min}}}^{Z} | e_{k_{\text{min}}} \in E^{Z}$ and $e_{k_{\text{max}}} \in E_{k_{\text{max}}}^{Z} | e_{k_{\text{max}}} \in E^{Z}$.

$E_{\min (k'_{i})}^{Z} \neq E_{\max (k'_{i})}^{Z}$ and both are convex by using theorem 3.4 thus $E^{Z}$ is non convex.

Proof of theorem 3.7:

Assume that there exists an efficient ordering policy $(K, Q)$ such that $K = (\ldots, k_{1}, \ldots, k_{2}, \ldots)$ with $k_{2} - k_{1} > 1$. Consider now the ordering policy $(K^{*}, Q)$ with $K^{*} = (\ldots, k_{1} + 1, \ldots, k_{2} - 1, \ldots)$. $\overline{K}^{*} = \overline{K}$ thus the ordering impacts of $(K^{*}, Q)$ are similar to the ones of $(K, Q)$. The inventory holding impacts at the retailer are also similar for the two considered ordering policies. The average inventory at the warehouse is lower for the ordering policy $(K^{*}, Q)$, $(K, Q)$ may thus not be an efficient ordering policy.

As the maximum difference for the element of $K$ leading to an efficient ordering policy is equal to 1, $K$ contains in maximum two types of elements i.e. $[\overline{K}]$ and $[\mathcal{K}]$. 

66
In this chapter, the SOQ model is adapted to support green technology investment decisions. This option is compared to operational adjustment. The results show that operational adjustment may be a valuable alternative in comparison to investments in carbon-reducing technologies. This gives additional flexibility to supply chain managers who are likely to focus solely on carbon-reducing technologies investments. We also provide analytical conditions under which one of both options is the most interesting for two classical regulatory policies, i.e. the carbon cap and the carbon tax policies. The results can also be directly extended to the case where several technologies are available. Finally, the results are used to illustrate the effectiveness of different regulatory policies to control carbon emissions. Some potentially impacting practical insights on this topic are thus drawn.
Chapter 4: Adjust or invest: Assessing two management principles in a low-carbon inventory model

1 Introduction

Environmental awareness has considerably increased since the Brundtland’s report publication (World Commission on Environment and Development, 1987). Nowadays, customers, investors, employees and other stakeholders consider that greening the supply chain is a key issue for companies. In response, two thirds of the European companies have for instance intensified their green actions over the past three years (Bearing Point, 2010). One of the main challenges when greening a supply chain consists in reducing carbon emissions. The logistics industry is indeed responsible for around 5.5% of global greenhouse gas emissions worldwide. These emissions are mainly generated by transportation. Nevertheless, warehousing contributes to 13% of the sector’s carbon footprint mainly due to indirect emissions from electricity consumption (World Economic Forum, 2009).

When intending to reduce the carbon footprint of a supply chain, companies first focus on investments that quickly lead to win-win situations, i.e. projects that contribute to reduce both costs and carbon emissions in the short term. These projects may be found below the x-axis of the McKinsey’s carbon abatement cost curve (McKinsey, 2009) as shown in figure 4.1. However, less than 30% of the total carbon abatement potential identified in McKinsey’s report corresponds to win-win investments. Companies have thus begun to exhaust these low-hanging fruits leading to short term win-win situations and start thinking that “sustainability can only be attained by optimizing seemingly conflicting targets” (DHL, 2010). This chapter thus focuses on situations where investments leading to win-win situations are not available anymore. In this case, carbon footprint reduction may only be achieved by increasing the operational costs. These situations may still be interesting for companies as the Bearing Point 2010 survey highlights that “more than one third of the 582 interviewed companies declare being ready to start up environmental actions in spite of their low present profitability, provided they create value in the medium term” (Bearing Point, 2010).

Going beyond the win-win situations does not seem so trivial. For instance, a third-party logistics company can invest in greener trucks. In the short term, this investment will increase the operational costs while reducing the carbon footprint of the supply chain (this investment is indeed above the x-axis of the McKinsey’s carbon abatement cost curve). However, it may be profitable for the company in the long term. Several technology investments of this type
may be applied to transportation and warehousing in order to reduce the carbon footprint of the supply chain. Another option is proposed by Benjaafar et al. (2010) who “study the extent to which carbon reduction requirements can be addressed by operational adjustments, as an alternative (or a supplement) to costly investments in carbon-reducing technologies”. Chen et al. (2011) have indeed demonstrated that significant reductions in carbon emissions can be obtained without significantly increasing costs by making only adjustments in the ordering quantities for the EOQ model.

In this chapter, we thus intend to assess operational adjustment and technology investment options in terms of costs and carbon emissions. To do so, the SOQ model proposed in chapter 3 is extended to allow modeling both options. The results show that operational adjustment may be an effective alternative to investments in carbon-reducing technologies. This gives additional flexibility to supply chain managers who are likely to be focused solely on investing in carbon-reducing technologies. We also provide analytical conditions under which an option outperforms the other one for two classical regulatory policies. The results can also be directly extended to the case where several technologies are available.
Chapter 4: Adjust or invest: Assessing two management principles in a low-carbon inventory model

The chapter is organized as follows. Section 2 is devoted to the presentation of the model and to the multiobjective optimization results. Operational adjustment and technology investment options are first modeled in the SOQ framework. Then we show that operational adjustment may be a valuable alternative comparing to investments in carbon-reducing technologies when intending to lower the carbon footprint of the supply chain. Section 3 is devoted to the study of two common regulatory policies. The first one consists of choosing an upper limit on carbon emissions and the second one is based on carbon pricing. For both of them, we provide analytical conditions under which an option outperforms the other one. Finally, Section 4 is devoted to insights discussion and to the conclusion.

2 Model formulation

2.1 Modeling carbon emissions in the EOQ framework

As shown in chapter 3, the average total cost per time unit has the following expression in the EOQ model:

\[ Z_c(Q) = \frac{Q}{2} h_c + \frac{D}{Q} O_c, \]  

(4.1)

with:

- \( Q \) = batch quantity (decision variable),
- \( D \) = demand per time unit,
- \( h_c \) = constant inventory holding cost per product unit and time unit,
- \( O_c \) = fixed ordering or setup cost.

Moreover, the optimal batch quantity can then be expressed as follows:

\[ Q_c^* = \sqrt{\frac{2O_cD}{h_c}}. \]  

(4.2)

The amount of carbon emissions is a sustainability impact that should be minimized. We thus adopt the same expression as in chapter 3 to estimate the average carbon footprint per time unit:

\[ Z_e(Q) = \frac{Q}{2} h_e + \frac{D}{Q} O_e, \]  

(4.3)
with:

\( Q \) = batch quantity (decision variable),
\( D \) = demand per time unit,
\( h_E \) = constant inventory holding emissions per product unit and time unit,
\( O_E \) = fixed ordering or setup emissions.

The fixed amount of carbon emissions per order \( O_E \) represents the emissions related to order processing and transportation. An amount of carbon emissions \( h_E \) is also associated with the storage of each unit per time unit. This amount can become important in case of refrigeration. These emissions parameters correspond to both direct emissions from fuel consumption and indirect emissions from electricity consumption.

The batch quantity that minimizes the emissions function \( Z_E \) has the following expression:

\[
Q_E^* = \frac{2 O_E D}{h_E}.
\]  
(4.4)

### 2.2 Operational adjustment

By adopting the strong vision of sustainability, we consider that minimizing carbon emissions is, in itself, an objective for the company like the economic cost of operations. In this case, two conflicting objectives (the cost and the carbon footprint) have to be minimized. The set of possible values for \( Q \) is \( A = \mathbb{R}^*_+ \). Let \( Z : A \to \mathbb{R}^2 \), \( Z(a) = \{Z_C(a); Z_E(a)\} \), for all \( a \in A \), with \( Z_C \) defined by formula 4.1 representing the total cost of operations and \( Z_E \) defined by formula 4.3 representing the total carbon emissions. \( Z(A) = \{(Z_C(Q); Z_E(Q)) | Q \in A\} \) is the image of \( A \) in the criterion space. The efficient frontier is a subset of \( A \) noted \( E \). Its image in the criterion space is \( Z(E) \). By applying theorem 3.1, we obtain that \( E = [\min(Q_C^*, Q_E^*); \max(Q_C^*, Q_E^*)] \).

It shows that it is possible to reduce the carbon emissions of a supply chain by modifying the batch size (from the economic order quantity) if \( Q_E^* \neq Q_C^* \).
Chapter 4: Adjust or invest: Assessing two management principles in a low-carbon inventory model

This condition is equivalent to:

\[ \frac{O_E}{h_E} \neq \frac{O_C}{h_C}. \]  

(4.5)

In what follows, this batch size modification is called an operational adjustment.

Let us consider example 4.1. Let \( D = 25 \) product units per time unit, \( O_C = 200 \), \( h_C = 1 \), \( O_E = 250 \) and \( h_E = 0.3 \). Applying formula 4.2 and 4.4 implies that \( Z_C(Q) \) is minimum for \( Q_C^* \approx 100 \) and \( Z_E(Q) \) for \( Q_E^* \approx 204 \). Figure 4.2 illustrates the results.

Figure 4.2: Cost and carbon emissions in function of the ordering quantity

![Graph showing cost and carbon emissions in function of the ordering quantity](image)

By applying theorem 3.1, we obtain that \( E = [\min(Q_C^*, Q_E^*), \max(Q_C^*, Q_E^*)] = [100; 204] \). Figure 4.3 displays the results in the criterion space. The x-axis represents the costs and the y-axis represents the carbon emissions of the available alternatives. Both the image of feasible solutions \( Z(A) \) and the image of the efficient frontier \( Z(E) \) are displayed.

Assume that the current situation is cost optimized. Figure 4.3 shows that a significant carbon emissions reduction can be achieved by increasing the batch size starting from \( Q_C^* \). Moreover, the required financial effort remains reasonable for a significant carbon emissions reduction.
For instance, the carbon emissions can be reduced by almost 15% for a 5% cost increase in the presented example. This feature is due to the fact that the flat region of the cost function coincides with a steeper region of the emissions function (see figure 4.2). Chen et al. (2011) provide conditions under which the relative reduction in emissions is greater than the relative increase in cost for the EOQ model. On the opposite, the financial effort will increase as $Q$ is getting closer to $Q^*_E$, the ordering quantity that minimizes the amount of carbon emissions.

Figure 4.3: The images of the feasible solutions and the efficient frontier in the criterion space

2.3 Technology investment

In the previous section, the operational adjustment option is defined and illustrated through an example. However, companies can also invest in carbon-reducing technologies to curb emissions. In this section, we show how to model a green technology investment option in the SOQ framework.

In the SOQ framework, carbon emissions result from both ordering and warehousing. An investment in a carbon-reducing technology can then modify the ordering and / or the holding parameters of the model. We recall that we focus only on situations where investments leading to decrease both the costs and carbon emissions are not available. In this case, a carbon-reducing technology investment will increase the operational costs while decreasing the supply chain carbon emissions. For instance, investing in hybrid or electric vehicles will
decrease the emissions related to transportation while increasing the ordering costs. This investment can be done directly by the company but it can also be made by a supplier. A third party logistics provider may for instance be asked to use greener trucks. The logistics provider may thus charge the customers with a fixed cost per delivery to support this investment.

In summary, the carbon-reducing technologies investments considered in this chapter enable reducing a carbon emissions parameter (either $O_E$ or $h_E$) by requiring an increase in a cost parameter (either $O_C$ or $h_C$). In what follows, we focus on ordering parameters as transportation is recognized as a major source of carbon emissions in supply chains. Moreover, the McKinsey’s report (McKinsey & Company, 2009) shows that carbon-reducing technologies investments for heavy-duty trucks are generally above the x-axis, i.e. that these projects generally increase the operational costs.

An investment in a carbon-reducing technology may thus be modeled as follows:
- The new fixed ordering carbon emissions parameter is $O_{E}^{Tech}$ with $O_{E}^{Tech} < O_{E}$,
- the new fixed ordering costs parameter is $O_{C}^{Tech}$ with $O_{C}^{Tech} > O_{C}$.

The new average cost function is:

$$Z_{C}^{Tech}(Q) = \frac{Q}{2} h_{C} + \frac{D}{Q} O_{C}^{Tech}.$$  \hfill (4.6)

The new average carbon emissions function has the following expression:

$$Z_{E}^{Tech}(Q) = \frac{Q}{2} h_{E} + \frac{D}{Q} O_{E}^{Tech}.$$  \hfill (4.7)

By directly applying the results of sections 2.1 and 2.2, we obtain that:

$$Q_{C}^{Tech*} = \sqrt{\frac{2O_{C}^{Tech} D}{h_{C}}} > Q_{C}^{*},$$  \hfill (4.8)

$$Q_{E}^{Tech*} = \sqrt{\frac{2O_{E}^{Tech} D}{h_{E}}} < Q_{E}^{*},$$  \hfill (4.9)

and $E_{Tech}^{*} = \min(Q_{C}^{Tech*}, Q_{E}^{Tech*}); \max(Q_{C}^{Tech*}, Q_{E}^{Tech*})$}.  \hfill (4.10)
with $E^{Tech}$ being the efficient frontier of the SOQ problem while investing in the technology $Tech$.

As $O^{Tech}_{E} < O_{E}$, the following expression holds:

$$Z_{E}^{Tech}(Q_{E}^{Tech^{*}}) = \sqrt{2O_{E}^{Tech}Dh_{E}} < Z_{E}(Q_{E}^{*}) = \sqrt{2O_{E}Dh_{E}}.$$  \hspace{1cm} (4.11)

Finally, as $O^{Tech}_{C} > O_{C}$, we obtain that:

$$Z_{C}^{Tech}(Q_{C}^{Tech^{*}}) = \sqrt{2O_{C}^{Tech}Dh_{C}} > Z_{C}(Q_{C}^{*}) = \sqrt{2O_{C}Dh_{C}}.$$  \hspace{1cm} (4.12)

### 2.4 Operational adjustment option versus technology investment option

Let us assume that a company is considering both operational adjustment and technology investment options to green its supply chain. To illustrate the situation, the example 4.1 is adapted by assuming that the company has also the possibility to invest in a technology with the following parameters: $O^{Tech}_{C} = 220 (> O_{C} = 200)$ and $O^{Tech}_{E} = 180 (< O_{E} = 250)$. Figure 4.4 represents the image of the feasible solutions in the criterion space for both the operational adjustment option and the technology investment one.

Note that $Z^{Tech}(A) = \{(Z_{C}^{Tech}(Q); Z_{E}^{Tech}(Q)) | Q \in A\}$ corresponds to the image of the feasible solutions for the technology investment option in the criterion space. It can be noticed in figure 4.4 that there is a single intersection point between $Z(A)$ and $Z^{Tech}(A)$.

More generally, the following result holds:

**Theorem 4.1.** Let $Z(A)$ and $Z^{Tech}(A)$ be the images of the feasible solutions for the operational adjustment option and for the technology investment option then:

$Z(A) \cap Z^{Tech}(A)$ contains at most a single element.

This result is proven in Appendix 4A. Figure 4.5 illustrates the trade-offs that a company can face when deciding on technology investment and on the ordering quantity. In general, the
image of the global problem efficient frontier is included into $Z(E) \cup Z^{Tech}(E^{Tech})$. However, we cannot assert that all elements of $Z(E)$ and $Z^{Tech}(E^{Tech})$ are efficient.

Figure 4.4: operational adjustment case and technology investment case in the criterion space

Figure 4.5: Images of the efficient frontiers in the criterion space

In this example, there exists an intersection point $\{C_{\cap}, E_{\cap}\} \approx \{117,62\}$ between $Z(E)$ and $Z^{Tech}(E^{Tech})$. The image of the efficient frontier for the global problem is thus composed by
the elements of \( Z(E) \) with \( Z_C \leq 117 \) and by the elements of \( Z^{Tech}(E^{Tech}) \) with \( Z^{Tech}_C \geq 117 \). By using formula 4.11 and 4.12, we can assert that in the general case, the image of the global problem efficient frontier contains at least one element of \( Z(E) \). Operational adjustment may thus be a valuable alternative comparing to investments in carbon-reducing technologies in certain situations. In the proposed example, we can notice that operational adjustment is more effective than technology investment for \( Z_E \in (62,78) \). The best option to green a supply chain will depend on the chosen trade-off. Two common regulatory policies are studied in the following section. The first one consists of choosing an upper limit on carbon emissions and the second one is based on carbon pricing.

3 The best option to green a supply chain

3.1 The carbon cap case

In this chapter, we aim at evaluating operational adjustment and technology investment options with respect to both costs and carbon emissions. Results of section 2 show that operational adjustment may be an effective alternative to investments in carbon-reducing technologies. However, identifying the best option to green a supply chain clearly required to set a trade-off between costs and carbon emissions.

In this section, we consider that the regulatory policy consists of choosing an upper limit on carbon emissions. This decision can be imposed by government regulations; however, it can also come from a voluntary effort of the company. This upper limit is noted \( CAP \) and is expressed in the same unit as \( h_E, O_E \) and \( O^T_E \). We further assume that \( CAP \geq Z^T_E(Q^*_E) \), otherwise, no feasible solution exists for the given technology investment option. In this context, operational adjustment will perform better if the carbon cap is high enough and technology investment is the best option for low values of \( CAP \). This result is stated in theorem 4.2.
Theorem 4.2. Assume that the company faces an upper limit on carbon emissions noted \( CAP \), then there exists a threshold \( L_E \) on carbon emissions such that:

- If \( CAP > L_E \), operational adjustment performs better than technology investment,
- if \( CAP < L_E \), technology investment is the best option.

Theorem 4.2 is proven in appendix 4A. It may also be noticed that the value of \( L_E \) is not necessarily unique. We refer to appendix 4B for the analytical derivations of feasible values for \( L_E \). Moreover, when \( CAP = L_E \), the best option has to be determined in a case by case basis. For instance, we can notice that \( L_E = 62 \) for the example of figure 4.5. For \( CAP = L_E \) in this case, operational adjustment and technology investment options are equivalent. Nevertheless, operational adjustment may be preferred in this case as this operational decision can be quickly reassessed relatively to technology investment option.

3.2 The carbon tax case

In this section, we prove that the best option among operational adjustment and technology investment is obtained by verifying a simple condition on the company’s parameters through a carbon tax policy. So let us consider that a cost is associated to carbon emissions. This cost can be imposed to the company in the case of a carbon tax. However, it can also come from an internal evaluation from the company, by considering the cost of the energy used or the cost issued from an environmental accounting analysis. This cost per amount of carbon emissions is noted \( \alpha \in [0; \infty) \). The decision problem can then be formulated as determining:

\[
\min_{Q \in \mathbb{R}^*} \left( Z_c(Q) + \alpha Z_E(Q) ; Z_{E}^{tech}(Q) + \alpha Z_{Tech}^{tech}(Q) \right). \tag{4.13}
\]

In this context, there exists a value \( L_c \in (0; \infty) \) such that if \( \alpha < L_c \), the operational adjustment option performs better than the technology investment one. On the opposite, the technology
investment option is the best option if \( \alpha > \lambda_c \). Moreover, \( \lambda_c = \frac{O_c^{Tech} - O_c}{O_E - O_E^{Tech}} \). This result is stated in theorem 4.3.

**Theorem 4.3.** Assume that a carbon cost noted \( \alpha \in \{0; \infty\} \) is given, then:

- If \( \alpha < \lambda_c = \frac{O_c^{Tech} - O_c}{O_E - O_E^{Tech}} \), then the operational adjustment option outperforms the technology investment one,

- if \( \alpha > \lambda_c = \frac{O_c^{Tech} - O_c}{O_E - O_E^{Tech}} \), then technology investment is the best option.

Theorem 4.3 is proven in appendix 4A. This result is illustrated with the example provided in section 2. For this example, \( \lambda_c = \frac{O_c^{Tech} - O_c}{O_E - O_E^{Tech}} \approx 2.33 \). The situation is illustrated in figure 4.6.

In the criterion space, for \( \alpha \in \{0; \infty\} \), the problem stated in formula 4.13 is equivalent to find the tangent points between \( Z(E) \cup Z^{Tech}(E^{Tech}) \) and a straight line of slope \( \frac{1}{\alpha} \). It is thus
equivalent to minimize \( y_0 \in \mathbb{R} \) such that \( \left\{ x \in \mathbb{R}; y = y_0 - \frac{x}{\alpha} \right\} \cap \left\{ Z(E) \cup Z^T (E^T) \right\} \) is not empty.

If \( \alpha < L_c, \quad \frac{1}{\alpha} < -\frac{1}{L_c} \), then the problem stated in formula 4.13 is solved with an operational adjustment. On the other hand, if \( \alpha > L_c, \quad \frac{1}{\alpha} > -\frac{1}{L_c} \), then the problem stated in formula 4.13 is solved with a technology investment.

4 Discussion and conclusion

4.1 Discussion

Two classical regulatory policies were studied in the previous sections. For both the carbon cap and the carbon tax policies, we have proven that there exists a limit value that allows deciding between the operational adjustment option and the technology investment one. Two types of questions must be answered when emissions have to be reduced in response to regulatory policies. First, policy makers should determine and implement the most effective regulatory policy. Then companies answer by identifying the best option to comply with the regulation. The results presented in the previous sections answer to the second question. However, they can also be used to discuss the first question. Our results indeed show that controlling emissions via a carbon price has some technical drawbacks. Carbon emissions are controlled by a carbon price for the carbon tax policy as well as for the cap and trade system. Hua et al. (2011) have indeed proven that emissions levels depend only on the carbon price in the EOQ model with a fixed carbon price under the cap and trade system. In this case, the minimum amount of emissions cannot be achieved as it would imply an infinite carbon price. Moreover, the financial effort will considerably increase as getting closer to the minimum amount of emissions as both operational costs and emissions costs will significantly increase.

The case where the carbon cost is \( \alpha = \frac{O_{Tech}^C - O_C}{O_E - O_{Tech}^E} \) reveals another drawback of the carbon tax policy and the cap and trade system. Operational adjustment and technology investment...
indeed give the same overall result (operational costs + tax) and the optimal ordering quantity is also the same:

\[
Q^* = \sqrt{\frac{2(O_{Tech}^T O_E - O_C O_{Tech}^T)D}{h_{Tech}(O_E - O_{Tech}^T) + h_E(O_{Tech}^T - O_C)}}. \tag{4.14}
\]

In the proposed example, we obtain that \( Q^* \approx 152 \), \( Z_{Tech}(Q^*) + L_{Tech}Z_{Tech}(Q^*) \approx 109 + 2.33 \times 64 \approx 258 \) and \( Z_{E}(Q^*) + L_{E}Z_{E}(Q^*) \approx 120 + 2.33 \times 59 \approx 258 \). For this given carbon price, operational adjustment and technology investment give the same overall result with different costs and carbon emissions levels. At a macroeconomic level, this operational flexibility implies that the total amount of carbon emissions is hardly controllable by setting a carbon price. Whatever the chosen value of \( \alpha \), some companies may face \( \alpha = \frac{O_{Tech}^T - O_C}{O_E - O_{Tech}^T} \). These companies may thus be able to choose among several carbon emissions levels. However, governments are interested in designing regulatory policies that enable to predict and manage the global amount of carbon emissions as many countries have ratified the Kyoto protocol mainly based on a negotiated carbon cap for each country (UNFCC, 1997).

A regulatory policy based on a carbon price gives unexpected flexibility to companies but, on the other hand, it limits the possibilities. Some interesting operational solutions are indeed ruled out whatever the chosen carbon price is. In figure 4.6, each efficient solution with an emissions level between \((59, 64)\) is unreachable for any given value of \( \alpha \in [0; \infty) \). This can be seen as a limitation induced by setting a carbon price.

As a result, using an upper limit on carbon emissions seems to be more effective to green supply chains as the previous drawbacks are avoided. Moreover, using a carbon cap is in accordance with the concept of strong sustainability. However, this kind of regulatory policy may be harder to implement as there is need to setup a different cap for each company.

### 4.2 Conclusion

In this chapter, we use a multiobjective formulation of the EOQ model called the SOQ model to evaluate how operational adjustment and technology investment can be used to green the supply chain. In Section 2, we prove that operational adjustment may be an effective alternative to investments in carbon-reducing technologies. Both options may thus be
considered when intending to green a supply chain. Two classical regulatory policies are then studied in Section 3. In the carbon cap case, we prove that the best option among operational adjustment and technology investment is obtained by verifying a simple condition on the company parameters. The same kind of result is also demonstrated in the carbon tax case. These results give additional flexibility to supply chain managers who are likely to be focused on investing in carbon reducing technology. Some practical insights are then discussed. We prove that controlling the carbon emissions by setting a carbon price may have several limitations.
Appendix 4A

Proof of theorem 4.1:

Assume that there exists \((Q, Q^{Tech}) \in A \times A\) such that \(Z(Q) = Z^{Tech}(Q^{Tech})\) i.e.:

\[
\begin{align*}
Z_C(Q) &= Z^{Tech}_C(Q^{Tech}) \\
Z_E(Q) &= Z^{Tech}_E(Q^{Tech})
\end{align*}
\]

\[
\begin{align*}
\left\{ \frac{h_C}{2D}(Q - Q^{Tech}) = \frac{O^{Tech}_C}{Q^{Tech}} - \frac{O_C}{Q} \right. \\
\left. \frac{h_E}{2D}(Q - Q^{Tech}) = \frac{O^{Tech}_E}{Q^{Tech}} - \frac{O_E}{Q} \right.
\end{align*}
\]

\[
\Rightarrow \frac{h_E}{Q^{Tech}} \left( \frac{O^{Tech}_C}{Q^{Tech}} - \frac{O_C}{Q} \right) = \frac{h_C}{Q^{Tech}} \left( \frac{O^{Tech}_E}{Q^{Tech}} - \frac{O_E}{Q} \right) \Rightarrow Q^{Tech} = \frac{h_E O^{Tech}_C - h_C O^{Tech}_E}{h_E O^{Tech}_C - h_C O^{Tech}_E} Q = KQ
\]

As both \(Q\) and \(Q^{Tech}\) belongs to \(\mathbb{R}_+^*\), \(Z(A) \cap Z^{Tech}(A)\) is empty if \(K \leq 0\).

Else, \(Z_C(Q) = Z^{Tech}_C(Q^{Tech}) \Leftrightarrow \frac{h_C}{2D}Q + \frac{O_C}{Q} = \frac{h_C}{2} KQ + \frac{O^{Tech}_C D}{KQ} \Leftrightarrow Q^2 = \left( \frac{D \left( \frac{O^{Tech}_C}{K} - O_C \right)}{1 - K} \right) \frac{h_C}{2} = L
\]

If \(L \leq 0\), then \(Z(A) \cap Z^{Tech}(A)\) is empty, else \(Q = \sqrt{L}\) and \(Q^{Tech} = KQ\), thus there is at most a single intersection point between \(Z(A)\) and \(Z^{Tech}(A)\).

Proof of Theorem 4.2:

The following notations are introduced:

\(\mathbb{R}_+^2 = \{ (x_1, x_2) | x_1 \in \mathbb{R}_+, \forall i \in [1,2] \}\) is the nonnegative subset of \(\mathbb{R}_+^2\),

Let \(S_1\) and \(S_2\) two subsets of \(\mathbb{R}^n\): \((S_1 + S_2) = \{ s_1 + s_2 | s_1 \in S_1, s_2 \in S_2 \}\) is the Minkowski sum, \(Z(E)_+ = (Z(E) + \mathbb{R}_+^n)\). \(Z(E)_+\) thus includes all the elements of \(Z(E)\) as well as all the elements situated at the top right of \(Z(E)\).

By using the results of the theorem 3.1, we obtain that \(Z(E)_+\) and \(Z^{Tech}(E^{Tech})_+\) are convex.

As \(Z(E) \cap Z^{Tech}(E^{Tech})\) contains at most a single element by using theorem 4.1, we obtain that:

- If there exist a value \(L_E^+\) such that the operational adjustment option is the best one for \(CAP = L_E^+\), then the operational adjustment option is the best one for all values of \(CAP \geq L_E^+\),
- If there exist a value \( L_E^- \) such that the technology investment option is the best one for \( CAP = L_E^- \), then the technology investment option is the best one for all values of \( CAP \leq L_E^- \).

If \( CAP = Z_E(Q^*_C) \), the operational adjustment option is the best option then we can choose \( L_E^+ = Z_E(Q^*_C) \). If \( CAP = Z_E^{Tech}(Q^{Tech^*}_E) \), the technology investment option is the best option then we can choose \( L_E^- = Z_E^{Tech}(Q^{Tech^*}_E) \). It can then be concluded that there exists \( L_E \) with \( L_E^- \leq L_E \leq L_E^+ \) that allow deciding among the two options.

**Proof of Theorem 4.3:**

By using the same argumentation as in theorem 4.3, we obtain that:

- If there exists \( L_C \in \mathbb{R}^+ \) such that \( \min_{Q \in \mathbb{R}^+}(Z_C(Q) + L_C \cdot Z_E(Q)) \leq \min_{Q \in \mathbb{R}^+}(Z_C^{Tech}(Q) + L_C \cdot Z_E^{Tech}(Q)) \), then for all \( \alpha \in \mathbb{R}^+ \), \( \alpha < L_C \), \( \min_{Q \in \mathbb{R}^+}(Z_C(Q) + \alpha \cdot Z_E(Q)) < \min_{Q \in \mathbb{R}^+}(Z_C^{Tech}(Q) + \alpha \cdot Z_E^{Tech}(Q)) \).

- If there exists \( L_C^+ \) such that \( \min_{Q \in \mathbb{R}^+}(Z_C^{Tech}(Q) + L_C^+ \cdot Z_E^{Tech}(Q)) \leq \min_{Q \in \mathbb{R}^+}(Z_C(Q) + L_C \cdot Z_E(Q)) \), then for all \( \alpha > L_C^+ \), \( \min_{Q \in \mathbb{R}^+}(Z_C^{Tech}(Q) + \alpha \cdot Z_E^{Tech}(Q)) < \min_{Q \in \mathbb{R}^+}(Z_C(Q) + \alpha \cdot Z_E(Q)) \).

Let \( L_C = \frac{O_C^{Tech} - O_C}{O_E - O_E^{Tech}} \), then:

- For all \( \alpha \in \mathbb{R}^+ \), such that \( \alpha < L_C \), \( \min_{Q \in \mathbb{R}^+}(Z_C(Q) + \alpha \cdot Z_E(Q)) < \min_{Q \in \mathbb{R}^+}(Z_C^{Tech}(Q) + \alpha \cdot Z_E^{Tech}(Q)) \).

- For all \( \alpha > L_C \), \( \min_{Q \in \mathbb{R}^+}(Z_C(Q) + \alpha \cdot Z_E(Q)) < \min_{Q \in \mathbb{R}^+}(Z_C^{Tech}(Q) + \alpha \cdot Z_E^{Tech}(Q)) \).

**Appendix 4B**

**Analytical derivations of \( L_E \):**

Two cases must be considered depending on the efficiency of \( Z(Q_E^*) \) for the global problem.

**Case 1:**

If \( Z(Q_E^*) \) is an efficient solution for the global problem, then \( L_E = Z_E(Q_E^*) = \sqrt{2O_E D h_E} \).
As \( Z(Q^*_E) \) is included into \( Z(E) \), it can only be dominated by an element of \( Z^{Tech}(E^{Tech}) \). Moreover, due to the properties of \( Z \) and \( Z^{Tech} \) demonstrated in appendix 4A, \( Z(Q^*_E) \) is dominated if and only if there exists \( Q_D \in \Re^*_+ \) such that

\[
\begin{cases}
\left( \frac{Z_c^{Tech}(Q_D)}{Z_c^{Tech}(Q^*_E)} = Z_c(Q^*_E) \right) \\
\left( Z_E^{Tech}(Q_D) < Z_E(Q^*_E) \right)
\end{cases}
\]

(4B.1)

The condition “\( Z(Q^*_E) \) is an efficient solution for the global problem” can thus be expressed as follows:

\[ Z_c^{Tech}(Q) = Z_c(Q^*_E) \implies Z_E^{Tech}(Q) > Z_E(Q^*_E) \text{ for all } Q \in \Re^*_+ . \]

(4B.2)

In expression 4B.2, the equation \( Z_c^{Tech}(Q) = Z_c(Q^*_E) \) is equivalent to:

\[
\frac{h_c}{2} Q^2 - \sqrt{\frac{D}{2O_E h_E}} (O_c h_E + O_E h_c) Q + O_c^{Tech} D = 0 .
\]

(4B.3)

If equation 4B.3 does not have any feasible solution then expression 4B.2 is verified. Else, assume that \( Q_1 \) and \( Q_2 \) are the roots of equation 4B.3 (not necessarily distinct). By calculating \( Z_E^{Tech}(Q_1) \), \( Z_E^{Tech}(Q_2) \) and \( Z_E(Q^*_E) \), condition 4B.2 can be easily verified.

**Case 2:**

If \( Z(Q^*_E) \) is not an efficient solution for the global problem, two subcases should be considered.

**Case 2.1:**

If \( Z(E) \cap Z^{Tech}(E^{Tech}) \) is non empty, then the single intersection point is noted \( \{C_\cap; E_\cap\} \) and \( L_E = E_\cap \).

By applying theorem 4.1, we know that there exists at most a single solution \( (Q; Q^{Tech}) \) such that:

\[
\begin{cases}
Z_c(Q) = Z_c^{Tech}(Q^{Tech}) \\
Z_E(Q) = Z_E^{Tech}(Q^{Tech})
\end{cases}
\]

(4B.4)
If \( Q \in E \) and \( Q^{Tech} \in E^{Tech} \) then, \( L_E = Z_E(Q) = Z^{Tech}_E(Q^{Tech}) \). Else \( Z(E) \cap Z^{Tech}(E^{Tech}) \) is empty.

**Case 2.2:**

If \( Z(Q^*_E) \) is not an efficient solution for the global problem and if \( Z(E) \cap Z^{Tech}(E^{Tech}) \) is empty, then there exists \( Q_{L_E} \) such that \( Z_C(Q_{L_E}) = Z^{Tech}_C(Q^{Tech}_E) = \sqrt{O^{Tech}_C} \frac{Dh_c}{O_C} \) and \( L_E = Z_E(Q_{L_E}) \).

Moreover, \( Q_{L_E} = \arg \min \left( Z_E \left( \sqrt{\frac{2D}{h_c}(O^{Tech}_C - O^{Tech}_C - O_C)} \right) ; Z_E \left( \sqrt{\frac{2D}{h_c}(O^{Tech}_C + O^{Tech}_C - O_C)} \right) \right) \).
Chapter 5: Economic and environmental performance of buyer-supplier coordination

In the two-echelon serial SOQ model proposed in chapter 3, the supply chain is assumed to be centrally optimized. This situation may be encountered either when the supply chain is controlled by a single entity or when independent entities decide to coordinate their operations in order to improve the system performance. In practice, the buyer-supplier negotiation may lead to several outcomes. In this chapter, the different outcomes of buyer-supplier coordination are illustrated by several models. Among them, a new model of a supplier leader supply chain is introduced and discussed. The impact of buyer-supplier coordination on the supply chain environmental performance is challenged in this chapter. We show that the total supply chain carbon emissions may be greater when buyer and supplier ordering policies are fully coordinated. Moreover, the setting of a carbon price may also lead to a similar outcome.
1 Introduction

Supply chains are generally composed of several independent entities aiming at optimizing their individual performance. In this situation, the companies should try to coordinate their operations in order to optimize the system performance instead of their individual one (Li and Wang, 2007). In practice, the buyer-supplier negotiation may lead to several outcomes depending on the respective bargaining power and willingness to collaborate of the different entities. In this chapter, we aim at exploring the economic and environmental performance of buyer-supplier supply chain according to different coordination relationships. An emerging idea presented in the sustainable supply chain management literature states that sustainability concerns may foster coordination. “In this context, ecological sustainability becomes one of the driving forces for a more cooperative business environment in terms of vertical cooperation between customers, suppliers and service providers, as well as horizontal cooperation between industrial companies” (DHL, 2010). We thus aim at analyzing this new trend in buyer-supplier relationships.

The literature dealing with buyer-supplier relationship and sustainability has rapidly grown. In this review, we restrict our attention to papers including sustainability concerns into single-buyer single-supplier models. To the best of our knowledge, the first paper analyzing buyer-supplier relationships by taking sustainability concerns into account is Corbett and DeCroix (2001). In this paper, the authors assess indirect material consumption in a single-buyer single-supplier supply chain. They prove that a well designed “shared-savings” contract can allow both parties to benefit from a consumption reduction. Vachon and Klassen (2008) examine the impact of environmental collaboration on manufacturing performance based on a survey of North American manufacturers. They highlight that green collaboration with suppliers generally leads to superior delivery and flexibility performance. On the other hand, they found that green collaboration with customers generally leads to better quality performance. Ni et al. (2010) include CSR into a single-buyer single-supplier model. They study how CSR should be allocated by using game-theoretical analysis on six different games. They prove that economic performance is not aligned with CSR performance and propose an optimal allocation scheme. Benjaafar et al. (2010) include carbon emission constraints on a multi-stage lot-sizing model with a cost minimization objective. The impact of collaboration is numerically studied under several carbon regulatory policies. Among others, they observe that the presence of carbon
constraints may increase the value of supply chain collaboration. Saadany et al. (2011) focus on a Joint Economic Lot Size (JELS) problem where the demand is assumed to be a function of product’s price and environmental quality. Analytical results and numerical examples are provided. In Ghosh and Shah (2012), the buyer-supplier relationship is analyzed by including green investment in a game-theoretical framework. They find that collaboration leads to higher greening level and higher retail price. Finally, Jaber et al. (2012) include carbon emissions into a JELS problem by considering different emissions trading schemes. Carbon emissions are assumed to be a function of the production rate. Their numerical study proves that coordination minimizes the total system cost without automatically reducing carbon emissions.

Combining the numerical observations of Benjaafar et al. (2010) and Jaber et al. (2012) may apparently provide contradictory results. The presence of carbon constraints may thus foster collaboration that would minimize the total system cost without automatically reducing carbon emissions. In this chapter, the link between buyer-supplier coordination and carbon emissions is formally analyzed by focusing on simple inventory models. We prove that the total supply chain carbon emissions may be increased when companies coordinate their operations. We also prove that a higher carbon price can lead to higher total carbon emissions in non-coordinated situations. We thus demonstrate that even if sustainability seems to be an incentive to increase collaborative behaviors, collaboration may have a negative impact on sustainability.

This chapter is organized as follows. Several models illustrating different outcomes of the buyer-supplier negotiation are presented in section 2. First, the centralized case is analyzed. In a second model, the buyer is placed in the position of the supply chain leader. Finally, a new model that enables the supplier to act as the supply chain leader is presented. In section 3, the models presented in section 2 are compared both in terms of cost and carbon emissions. Several insights are given. Finally, section 4 is devoted to the conclusion.
2 Different outcomes in buyer-supplier relationships

In this section, a brief overview of the buyer-supplier literature is given before stating the assumptions. Then, different outcomes of the buyer-supplier negotiation are illustrated by three models.

2.1 Literature review

The operations management literature dealing with buyer-supplier relationships is very vast. In this paper, we restrict our attention to single-buyer single-supplier situations with deterministic constant demand. Schwarz (1973) derives the centralized problem optimal solution. In this case, a single decision maker controls the entire supply chain. This centralized solution may be seen as a benchmark for independent firms aiming at coordinating their operations. The first paper dealing with this problem known as the Joint Economic Lot Size (JELS) problem is Goyal (1977). Several papers refine Goyal’s model by taking more realistic assumptions into account. The optimal solution for a general shipment policy with finite production rate and lot streaming is derived in Hill (1999). We refer to Goyal and Gupta (1989) for a review on early works on the JELS problem and to Ben-Daya et al. (2008) for a review on recent extensions of the JELS problem.

Even if coordination’s benefits are extensively recognized, non-coordinated supply chains are still very common in practice. In non-coordinated supply chains, an entity often acts independently so as to minimize its individual cost. In what follows, this entity is called the supply chain leader. Goyal (1977) presents a model where the buyer is the supply chain leader. Contrarily, in Lu (1995), the supplier seeks to minimize his total cost subject to the maximum cost the buyer is willing to incur. In these two situations, a side-payment contract can be designed so as to entice the leader to modify his behavior to achieve coordination. We refer to Cachon (2003), Sarmah et al. (2006) and Leng and Zhu (2009) for reviews on coordination under side-payment contracts. Note that game theory is often used in such situations to find an equilibrium solution.
2.2 Assumptions and preliminary results

In this chapter, the considered supply chain is composed of a single supplier (vendor) delivering a single product to a single buyer (customer). Figure 5.1 describes the supply chain under consideration.

Figure 5.1: The supply chain structure

The supplier produces the item with an infinite production rate. The product is then sent in batch to the buyer who faces a constant continuous demand. We assume that the entire batch is delivered to the buyer at the same time. Leadtimes are assumed to be zero for clarity (fixed leadtimes can be easily handled) and no shortage is allowed. Moreover, initial inventories are assumed to be zero. Fixed ordering costs and linear holding costs are supported by both the supplier and the buyer. Finally, we consider an infinite time horizon.

Let $Q_b$ be the batch quantity ordered by the buyer and $Q_s$ be the production lot size at the supplier. The following preliminary results were first derived by Schwarz (1973).

<table>
<thead>
<tr>
<th>Preliminary Results</th>
<th>An optimal policy is stationary-nested and respects the zero-inventory condition i.e.:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_B$ and $Q_S$ are time invariant,</td>
</tr>
<tr>
<td></td>
<td>$Q_S = k Q_B$, with $k \in \mathbb{N}^*$,</td>
</tr>
<tr>
<td></td>
<td>The buyer orders only if its inventory level is null,</td>
</tr>
<tr>
<td></td>
<td>The supplier orders when both the buyer and the supplier have no inventory.</td>
</tr>
</tbody>
</table>
In the following notations, B and S represent the buyer and the supplier respectively. C is used to identify the cost parameters (in opposition to E that identifies carbon emissions parameters):

\[ Q_B \] = ordering quantity at the buyer (first decision variable),
\[ Q_S \] = production lot size at the supplier,
\[ k \] = strictly positive integer such that \( Q_S = k Q_B \) (second decision variable),
\[ D \] = demand per time unit at the buyer,
\[ h_{CB} \] = constant inventory holding cost per product unit and time unit at the buyer,
\[ h_{CS} \] = constant inventory holding cost per product unit and time unit at the supplier,
\[ O_{CB} \] = fixed ordering cost at the buyer,
\[ O_{CS} \] = fixed production cost at the supplier.

Even if sustainable development is a vast concept that embraces economic, environmental and social aspects, global warming problem seems to overwhelm other concerns. Carbon footprint is now extensively adopted as an indicator of environmentally friendly supply chains activities. We thus focus on carbon emissions and we model carbon emissions in accordance to chapter 3.

In the following notations, E identifies carbon emissions parameters:

\[ h_{EB} \] = constant inventory holding emissions per product unit and time unit at the buyer,
\[ h_{ES} \] = constant inventory holding emissions per product unit and time unit at the supplier,
\[ O_{EB} \] = fixed ordering emissions at the buyer,
\[ O_{ES} \] = fixed production emissions at the supplier.

2.3 The centralized model: Model (c)

In the centralized model, the buyer and the supplier coordinate their operations in order to improve the system performance. Buyer’s and supplier’s operations performance is then jointly optimized. The cooperation mechanism that enables the distribution of coordination’s benefits among both parts is not made explicit. We refer to this model as model (c).
With the assumptions presented in section 2.2, the total supply chain cost $Z_C$ can be expressed as a function of $Q_B$ and $k$:

$$Z_C(k, Q_B) = (h_{CB} + (k - 1)h_{CS}) \frac{Q_B}{2} + \left( O_{CB} + \frac{O_{CS}}{k} \right) \frac{D}{Q_B}. \quad (5.1)$$

The total supply chain carbon emission function $Z_E$ has the following expression:

$$Z_E(k, Q_B) = (h_{EB} + (k - 1)h_{ES}) \frac{Q_B}{2} + \left( O_{EB} + \frac{O_{ES}}{k} \right) \frac{D}{Q_B}. \quad (5.2)$$

When coordinating their operations, the buyer and the supplier may aim at optimizing their economic and/or their environmental performance. In the present framework, optimizing the economic (respectively the environmental) performance of the system corresponds to minimizing the total cost function $Z_C$ (respectively the total carbon emission function $Z_E$).

In the present chapter, only single objective optimization is considered. The aim of the model is thus to minimize $Z_i, i \in \{C; E\}$.

The optimal values of $Q_B$ and $k$ noted respectively $Q_i^{*(c)}$ and $k_i^{*(c)}$, can be calculated as follows:

If $h_{ib} < h_{is}$, the minimum of $Z_i$ is found for $k_i^{*(c)} = 1$. Else, let $k_i^{inf} = \frac{O_{is}(h_{ib} - h_{is})}{O_{ib}h_{is}}$.

$k_i^{*(c)}$ is a strictly positive integer that can be found by using the following rule:

If $k_i^{inf} < 1$, it is optimal to choose $k_i^{*(c)} = 1$. Else, let $k' = k_i^{inf} \leq k_i^{*(c)} + 1$ with $k' \in \mathbb{N}$.

If $\frac{k_i^{inf}}{k'} \leq \frac{k_i^{*(c)} + 1}{k_i^{inf}}$ then it is optimal to choose $k_i^{*(c)} = k'$. Otherwise, $k_i^{*(c)} = k_i^{*(c)} + 1$ (Axsäter, 2006).

It follows that:

$$Q_i^{*(c)} = \sqrt{\frac{2(O_{ib} + O_{is}/k_i^{*(c)})D}{h_{ib} + (k_i^{*(c)} - 1)h_{is}}}. \quad (5.3)$$

Model (c) can be interpreted as a perfect buyer-supplier coordination situation (see figure 5.2).
2.4 Some decentralized models

In this section, two different non-coordinated situations are considered. First, we assume that the buyer is the supply chain leader. This situation is referred as model (b). Second, the supplier is assumed to be the supply chain leader. A new model referred as model (s) is proposed.

2.4.1 The buyer is the supply chain leader: Model (b)

In this model, we consider that the buyer has the strongest bargaining power and so is acting as the supply chain leader. The buyer thus optimizes its operations without taking the whole supply chain performance into account. The supplier then reacts by optimizing its operations. We refer to this model as model (b).

In this case, the buyer would be better ordering the quantity that minimizes the following function:

\[
Z_{ib}(Q_B) = h_{ib} \frac{Q_B}{2} + O_{ib} \frac{D}{Q_B},
\]  

(5.4)

with \( i \in \{C; E\} \).

The minimum of formula 5.4 is the economic (respectively environmental) order quantity:

\[
Q_{ib}^{*(b)} = \sqrt{\frac{2O_{ib}D}{h_{ib}}},
\]  

(5.5)
The supplier then chooses the optimal value $k_{i}^{*(b)}$ minimizing the following function:

$$Z_{is}(k) = h_{is} (k - 1) \frac{Q_{ib}^{*(b)}}{2} + O_{is} \frac{D}{k Q_{ib}^{*(b)}}.$$  \hfill (5.6)

Let $k_{inf} = \sqrt{\frac{O_{is} h_{is}}{O_{ib} h_{is}}}$. $k_{i}^{*(b)}$ is a strictly positive integer that can be found by using the rounding rule described in section 2.3.

Figure 5.3 illustrates the decision process of Model (b).

Figure 5.3: Illustration of model (b)

2.4.2 The supplier is the supply chain leader: Model (s)

In this section, we consider that the supplier has an advantage over the buyer in the purchasing negotiation. As stated in Lu (1995), this situation can be encountered when the supplier is the sole vendor of an item and the buyer lacks of bargaining power to ask for a price discount. As shown in formula 5.7, the supplier objective function $Z_{is} , i \in \{C; E\}$ depends on both $Q_{B}$ and $k$:

$$Z_{is}(Q_{B}, k) = h_{is} (k - 1) \frac{Q_{B}}{2} + O_{is} \frac{D}{k Q_{B}}.$$  \hfill (5.7)

Formally, this objective function may be reduced to zero if the supplier requires a very large order quantity $Q_{B} \to \infty$ and chooses $k = 1$. However, this may not be possible in practice as the buyer may not accept such situation ($Z_{is}$ as defined in formula 5.4 tends to infinity). Lu (1995) thus proposes to minimize $Z_{is}$ subject to the maximum increase in the objective function that the buyer is prepared to incur. To our knowledge, this is the only single-buyer
single-supplier deterministic model that assumes that the supplier is the supply chain leader. In what follows, we propose a new model that addresses such situation. This model has several advantages over that studied in Lu (1995) as shown hereafter.

Based on the preliminary results stated in section 2.2, it is interesting for the supplier to meet up orders that are synchronized with its production pattern. This synchronization may reduce supplier’s inventory as some items can be sent to the buyer as soon as produced avoiding the warehousing operations (Wang, 2004). To achieve such synchronization, the supplier may require that the buyer orders with a minimal frequency $N$. More frequent orders may also be accepted given that the buyer’s ordering frequency is a multiple of $N$. Based on the chosen frequency $N$, the supplier decides on the production lot size $Q_s = \frac{D}{N}$. The buyer then decides on its ordering quantity $Q_b = \frac{kD}{N}$ by choosing $k \in \mathbb{N}^*$. This negotiation process leads to stationary-nested ordering policies and is thus consistent with the preliminary results stated in section 2.2. We refer to this model as model (s).

Mathematical derivations of model (s) can be found in appendix 5A. Theorem 5.1 states that the supplier may decide on the production lot size $Q_{is}$ that will minimize $Z_{is}$ (as defined by formula 5.7) by using the following rule:

\begin{align*}
\text{Theorem 5.1.} & \quad \text{There exists } (k_{i1}; k_{i2}) \in \mathbb{N}^* \times \mathbb{N}^* \text{ such that:} \\
& \quad 1 < k_{i1} \leq k_{i2}, \\
& \quad Q_{is}(k) = \sqrt{k(k+1)}Q_{ib}^{*(b)} \text{ for all } k < k_{i1}, \\
& \quad Q_{is}(k) = \sqrt{\frac{k}{k-1}} \sqrt{\frac{2Q_{is}D}{h_{is}}} \text{ for all } k \text{ such that } k_{i1} \leq k < k_{i2}, \\
& \quad Q_{is}(k) = \sqrt{(k-1)k}Q_{ib}^{*(b)} + \varepsilon \text{ for all } k \geq k_{i2}, \text{ with } \varepsilon \text{ being a small positive number.}
\end{align*}

Then we prove that:

\begin{align*}
k_{i1}^{*(a)} \leq k_{i1} \leq k_{i1}^{*(b)} + 2, \quad (5.8)
\end{align*}
and that:

\[ Q_{ib}^{*(s)} = \begin{cases} \frac{k_i^{*(s)} + 1}{k_i^{*(s)}} Q_i^{*(b)} & \text{if } k_i^{*(s)} < k_i^{*(b)} , \\ \frac{1}{k_i^{*(s)} (k_i^{*(s)} - 1)} \sqrt{\frac{2O_{ib} D}{h_S}} & \text{else}. \end{cases} \]  

(5.9)

Figure 5.4 illustrates the decision process of model (s).

Figure 5.4: Illustration of model (s)

Model (s) has several advantages. First, the supplier would share a part of his savings with the buyer even if he perfectly dominates the buyer. This feature is common with the model developed in Lu (1995). Second, we prove that the maximal buyer’s increase in objective function is limited to 6.1% in comparison to model (b). Indeed, the buyer order quantity cannot exceed \( \sqrt{2Q_{ib}^{*(b)}} \) due to the negotiation process described above. Finally, this negotiation process may favor horizontal cooperation between several buyers as they are required to pass their orders at given time intervals. In this setting, it may be possible to consolidate shipments (Minner, 2007).

In order to implement model (s), an important practical issue should be considered. The supplier indeed need numerical estimates of \( k_i^{*(b)}, Q_{ib}^{*(b)} \) and \( D \) to determine his optimal inventory policy. To estimate these parameters, the supplier only needs to know buyer’s demand and previous order frequency as in Lu (1995). These parameters may be inferred from buyer’s past ordering behavior. Other assumptions are proposed in the literature. For instance, Li et al. (2012) propose a single-supplier single-buyer inventory model where the buyer’s cost information is private. The same assumption is taken in Ha (2001).
Chapter 5: Economic and environmental performance of buyer-supplier coordination

3 Buyer-supplier relationships and carbon emissions

In this section, the effect of supply chain coordination on costs and carbon emissions is analyzed. We prove that supply chain coordination can have a negative impact on the total amount of carbon emissions. Finally, we focus on situations where a tax is associated to carbon emissions.

3.1 Economic performance of coordinated versus non-coordinated models

We first focus on the economic performance of the buyer-supplier coordination. Some typical situations are illustrated by the following numerical examples. The data related to example 5.1 taken from Goyal (1977) are presented in table 5.1.

Table 5.1: Example 5.1 data set

<table>
<thead>
<tr>
<th>demand rate (D)</th>
<th>12 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>buyer inventory holding cost (h_{CB})</td>
<td>0.30</td>
</tr>
<tr>
<td>Supplier inventory holding cost (h_{CS})</td>
<td>0.24</td>
</tr>
<tr>
<td>buyer ordering cost (O_{CB})</td>
<td>10</td>
</tr>
<tr>
<td>supplier ordering cost (O_{CS})</td>
<td>100</td>
</tr>
</tbody>
</table>

The related optimal values and resulting costs are presented in table 5.2.

Table 5.2: Example 5.1 results

<table>
<thead>
<tr>
<th>Q_{CB}^*</th>
<th>k_C^*</th>
<th>Z_{CB}</th>
<th>Z_{CS}</th>
<th>Z_C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model (b)</td>
<td>894</td>
<td>4</td>
<td>268.33</td>
<td>657.40</td>
</tr>
<tr>
<td>Model (s)</td>
<td>1 033</td>
<td>3</td>
<td>271.11</td>
<td>635.17</td>
</tr>
<tr>
<td>Model (c)</td>
<td>1 633</td>
<td>2</td>
<td>318.43</td>
<td>563.38</td>
</tr>
</tbody>
</table>

As shown in table 5.2, we obtain that $Q_{CB}^*(b) < Q_{CB}^*(c)$. Moreover, $k_C^*(b) > k_C^*(c)$. In general, the results of formula 5.10 hold:

\[
\begin{cases}
Q_{CB}^*(b) < Q_{CB}^*(c) \\
k_C^*(b) \geq k_C^*(c)
\end{cases} \quad (5.10)
\]
Formula 5.10 is proven in appendix 5B. By comparing model (b) to model (c), we notice that the buyer has to increase its ordering quantity in order to achieve coordination. Hence, the buyer may allow the supplier to produce lots with larger size while reducing its inventory. This trend is often observed in multi-echelon inventory systems. In coordinated supply chains, the buyers often increase their average inventory levels in order to reduce the inventory level at the supplier, but the increase in buyer’s supply chain cost is less than the decrease in supplier cost. Quantity discounts are thus often proposed by the supplier to foster independent buyers to increase their ordering quantities (Li and Wang, 2007). This type of side payment is extensively studied in the literature (Sarmah et al., 2006).

When considering model (s), it may be noticed that the negotiation process entices the buyer to reasonably increase its order quantity in order to reduce the supplier cost. In the above example, the buyer is not willing to accept a cost increase leading to the results of model (c). However, an increase of 15.6% in order quantity is possible in exchange of an increase of 1.04% in buyer’s cost. This result is due to the relative insensitivity of the economic order quantity model to a variation of the ordering quantity in the neighborhood of the optimal value.

A particular case is presented in example 5.2. The related data are presented in table 5.3. The related optimal values and resulting costs are presented in table 5.4.

Table 5.3: Example 5.2 data set

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>demand rate (D)</td>
<td>12 000</td>
</tr>
<tr>
<td>buyer inventory holding cost (h_{CB})</td>
<td>1.50</td>
</tr>
<tr>
<td>Supplier inventory holding cost (h_{CS})</td>
<td>0.3975</td>
</tr>
<tr>
<td>buyer ordering cost (O_{CB})</td>
<td>25</td>
</tr>
<tr>
<td>supplier ordering cost (O_{CS})</td>
<td>78</td>
</tr>
</tbody>
</table>

Table 5.4: Example 5.2 results

<table>
<thead>
<tr>
<th></th>
<th>Q^*</th>
<th>k^*</th>
<th>Z_{CB}</th>
<th>Z_{CS}</th>
<th>Z_C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model (b)</td>
<td>632</td>
<td>3</td>
<td>948.68</td>
<td>744.72</td>
<td>1 693.40</td>
</tr>
<tr>
<td>Model (s)</td>
<td>730</td>
<td>3</td>
<td>958.51</td>
<td>717.52</td>
<td>1 676.03</td>
</tr>
<tr>
<td>Model (c)</td>
<td>730</td>
<td>3</td>
<td>958.51</td>
<td>717.52</td>
<td>1 676.03</td>
</tr>
</tbody>
</table>
In example 5.2, Model (s) leads to the same results as model (c). The negotiation process of model (s) may thus imply to reach perfect buyer-supplier coordination without any side payment agreement while considering independent entities. This result strengthens model (s) as it may be seen as a balanced buyer-supplier relationship without any side-payment contract.

3.2 The effect of buyer-supplier coordination on environmental performance

Even if coordination’s financial benefits are extensively recognized, non coordinated supply chains are still very common in practice. Several barriers such as communication, mutual trust or benefit sharing issues may indeed discourage the companies from collaborating. The sustainable supply chain literature often argues that sustainability issues may encourage the firms to coordinate their operations. However, is the buyer-supplier coordination always environmentally friendly? To answer this question, we aim at evaluating the environmental performance of the models defined in section 2. An illustration is presented in example 5.3 with related data provided in table 5.5. The related optimal values and resulting costs and carbon emissions are presented in table 5.6.

<table>
<thead>
<tr>
<th>Table 5.5: Example 5.3 data set</th>
</tr>
</thead>
<tbody>
<tr>
<td>demand rate (D)</td>
</tr>
<tr>
<td>buyer inventory holding cost (h_{CB})</td>
</tr>
<tr>
<td>Supplier inventory holding cost (h_{CS})</td>
</tr>
<tr>
<td>buyer ordering cost (O_{CB})</td>
</tr>
<tr>
<td>supplier ordering cost (O_{CS})</td>
</tr>
<tr>
<td>buyer inventory holding emissions (h_{EB})</td>
</tr>
<tr>
<td>Supplier inventory holding emissions (h_{ES})</td>
</tr>
<tr>
<td>buyer ordering emissions (O_{EB})</td>
</tr>
<tr>
<td>supplier ordering emissions (O_{ES})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5.6: Example 5.3 results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q_{CB}^*</td>
</tr>
<tr>
<td>Model (b)</td>
</tr>
<tr>
<td>Model (s)</td>
</tr>
<tr>
<td>Model (c)</td>
</tr>
</tbody>
</table>

In example 5.3, coordinating operations with a cost minimization objective leads to an increase in the total supply chain carbon emissions comparing to decentralized models.
Model (c) leads to an increase in total supply chain emissions in the following conditions stated in theorem 5.2 and theorem 5.3.

**Theorem 5.2.** Model (c) leads to an increase in the total supply chain carbon emissions comparing to model (b) if the following conditions are verified:

\[
k^*_C = k^*_C = k^*_E,
\]

\[
Q^*_{CB} \geq Q^*_{EB}.
\]

**Theorem 5.3.** Model (c) leads to an increase in the total supply chain carbon emissions comparing to model (s) if the following conditions are verified:

\[
k^*_C = k^*_C = k^*_E,
\]

\[
Q^*_{CB} \geq Q^*_{EB}.
\]

These results are proven in appendix 5C. Note that the conditions stated in theorems 5.2 and 5.3 are only sufficient ones. In the previous example, these conditions are verified by model (s). On the other hand, we can observe that \( Q^*_{CB} \leq Q^*_{EB} \) ( \( Q^*_{CB} = 2191 \) and \( Q^*_{EB} = 2627 \)). Nevertheless, model (b) performs better than model (c) in terms of carbon emissions.

### 3.3 The impacts of a carbon tax regulatory policy

The carbon tax is a commonly used regulatory policy to foster companies to reduce their carbon emissions. In this section, we consider that a price is associated to carbon emissions. The notion of carbon price is indeed more general than a carbon tax. For instance, this price can be setup by the company through an internal evaluation, by considering the cost of the energy used or the cost issued from an environmental accounting analysis. Hua et al. (2011) have also proven that emissions levels depend only on the carbon price in the economic order quantity model under a cap and trade regulation. In what follows, we assume that both the buyer and the supplier are charged with the same carbon price \( \alpha \in [0; \infty) \) per unit of carbon emissions.
In this context, the companies aim at minimizing their total cost resulting from both carbon emission cost and supply chain cost. It can be noticed that the results of section 2 can be directly applied in this context by replacing $h_{ib}$ by $h_{cb} + \alpha h_{eb}$, $O_{ib}$ by $O_{cb} + \alpha O_{eb}$ and by introducing the same modification for the supplier’s parameters. In what follows, the implications of setting up or increasing a carbon price are studied. It is proven that setting up a carbon tax may have a negative impact on total supply chain emissions in certain situations.

*The carbon emissions are non-linear in function of the carbon price:* Consider model (c). An increase in $\alpha$ necessarily implies a decrease in total carbon emissions. However, this decrease is non-linear in $\alpha$ and may also be discontinuous. Such situation is illustrated in example 5.4 with related data presented in table 5.7.

Table 5.7: Example 5.4 data set

<table>
<thead>
<tr>
<th>Demand rate (D)</th>
<th>12 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buyer inventory holding cost ($h_{cb}$)</td>
<td>2.50</td>
</tr>
<tr>
<td>Supplier inventory holding cost ($h_{cs}$)</td>
<td>0.50</td>
</tr>
<tr>
<td>Buyer ordering cost ($O_{cb}$)</td>
<td>25</td>
</tr>
<tr>
<td>Supplier ordering cost ($O_{cs}$)</td>
<td>150</td>
</tr>
<tr>
<td>Buyer inventory holding emissions ($h_{eb}$)</td>
<td>1.00</td>
</tr>
<tr>
<td>Supplier inventory holding emissions ($h_{es}$)</td>
<td>0.30</td>
</tr>
<tr>
<td>Buyer ordering emissions ($O_{eb}$)</td>
<td>150</td>
</tr>
<tr>
<td>Supplier ordering emissions ($O_{es}$)</td>
<td>75</td>
</tr>
</tbody>
</table>

Figure 5.5: Carbon emissions in function of the carbon price

$Z_{E}$
Figure 5.5 illustrates the variation of the total supply chain carbon emissions $Z_E$ in function of the carbon price $\alpha \in [0;25]$. We can observe some discontinuities in $Z_E$. For instance, a slight variation of the carbon price from $\alpha = 1.0215$ to $\alpha = 1.0216$ would imply carbon emissions to decrease from more than 6% (from $Z_E = 2320$ to $Z_E = 2179$). This feature has several implications. First, this would imply that if the carbon price is setup by the company thanks to an internal evaluation, then the precision of this evaluation is of crucial importance. Second, if the company faces a cap and trade regulation, a tiny variation of the carbon price is likely to have major impacts on company’s optimal carbon emissions.

The total supply chain carbon emissions may be increasing in the carbon price:
Consider then model (b). In this case, the buyer’s emissions are decreasing in $\alpha$. On the other hand, it may happen that the supplier’s emissions increase in $\alpha$. The total supply chain carbon emissions may thus be increasing in $\alpha$. Example 5.5 illustrates this situation. The related data are presented in table 5.8. The related optimal values and resulting costs and carbon emissions are presented in table 5.9.

Table 5.8: Example 5.5 data set

<table>
<thead>
<tr>
<th>Demand rate (D)</th>
<th>10 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buyer inventory holding cost ($h_{CB}$)</td>
<td>0.50</td>
</tr>
<tr>
<td>Buyer inventory holding emissions ($h_{EB}$)</td>
<td>2.00</td>
</tr>
<tr>
<td>Supplier inventory holding cost ($h_{CS}$)</td>
<td>15.0</td>
</tr>
<tr>
<td>Supplier inventory holding emissions ($h_{ES}$)</td>
<td>5.00</td>
</tr>
<tr>
<td>Buyer ordering cost ($O_{CB}$)</td>
<td>15</td>
</tr>
<tr>
<td>Buyer ordering emissions ($O_{EB}$)</td>
<td>25</td>
</tr>
<tr>
<td>Supplier ordering cost ($O_{CS}$)</td>
<td>150</td>
</tr>
<tr>
<td>Supplier ordering emissions ($O_{ES}$)</td>
<td>120</td>
</tr>
</tbody>
</table>

Table 5.9: Example 5.5 results

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$Q_{opt}^{(b)}$</th>
<th>$k_{opt}^{(b)}$</th>
<th>$Z_{CB}$</th>
<th>$Z_{CS}$</th>
<th>$Z_C$</th>
<th>$Z_{EB}$</th>
<th>$Z_{ES}$</th>
<th>$Z_E$</th>
<th>$Z_{C}+\alpha.Z_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>775</td>
<td>1</td>
<td>387.30</td>
<td>1 936.49</td>
<td>2 323.79</td>
<td>1 097.35</td>
<td>1 549.19</td>
<td>2 646.54</td>
<td>2 323.79</td>
</tr>
<tr>
<td>0.5</td>
<td>606</td>
<td>1</td>
<td>399.10</td>
<td>2 477.17</td>
<td>2 876.27</td>
<td>1 018.39</td>
<td>1 981.73</td>
<td>3 000.13</td>
<td>4 376.33</td>
</tr>
<tr>
<td>1</td>
<td>566</td>
<td>1</td>
<td>406.59</td>
<td>2 651.65</td>
<td>3 058.24</td>
<td>1 007.63</td>
<td>2 121.32</td>
<td>3 128.95</td>
<td>6 187.18</td>
</tr>
<tr>
<td>10</td>
<td>508</td>
<td>1</td>
<td>422.12</td>
<td>2 950.06</td>
<td>3 372.18</td>
<td>1 000.14</td>
<td>2 360.04</td>
<td>3 360.19</td>
<td>36 974.03</td>
</tr>
</tbody>
</table>

In this example, the total supply chain emissions are increased by 27% (from $Z_E = 2647$ to $Z_E = 3360$) by setting up a carbon price $\alpha = 10$. This surprising result implies that a carbon tax regulatory policy may be ineffective in reducing carbon emissions in certain situations.
The same conclusion may be drawn for a cap and trade regulatory policy. Setting up a carbon tax or a carbon price in model (b) would indeed entice the buyer to reduce its emissions by modifying its ordering quantity. $Q^{*}(\alpha)$ is indeed monotonous in $\alpha$. This change in buyer’s ordering quantity may negatively affect the supplier performances both in terms of cost and carbon emissions. The same analysis can be performed with model (s).

An increase in the carbon tax may favor coordination without decreasing carbon emissions:
The modification of buyer’s ordering quantity induced by the setup or the increase in carbon price may favor the supplier in some cases. The supplier total cost (operations cost + carbon cost) may indeed be lower than the operations cost before the change in buyer’s ordering quantity. This situation is illustrated for model (b) in example 5.6 with related data presented in table 5.10. The related optimal values and resulting costs and carbon emissions are presented in table 5.11.

Table 5.10: Example 5.6 data set

<table>
<thead>
<tr>
<th>demand rate (D)</th>
<th>2 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>buyer inventory holding cost ($h_{CB}$)</td>
<td>5.00</td>
</tr>
<tr>
<td>buyer inventory holding emissions ($h_{EB}$)</td>
<td>2.00</td>
</tr>
<tr>
<td>Supplier inventory holding cost ($h_{CS}$)</td>
<td>15.0</td>
</tr>
<tr>
<td>Supplier inventory holding emissions ($h_{ES}$)</td>
<td>0.10</td>
</tr>
<tr>
<td>buyer ordering cost ($O_{CB}$)</td>
<td>50</td>
</tr>
<tr>
<td>buyer ordering emissions ($O_{EB}$)</td>
<td>25</td>
</tr>
<tr>
<td>supplier ordering cost ($O_{CS}$)</td>
<td>800</td>
</tr>
<tr>
<td>supplier ordering emissions ($O_{ES}$)</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 5.11: Example 5.6 results for Model (b)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$Q_{all}^{*}(\alpha)$</th>
<th>$k_{\alpha}^{*}(\alpha)$</th>
<th>$Z_{CB}$</th>
<th>$Z_{EB}$</th>
<th>$Z_{CB}+\alpha.Z_{EB}$</th>
<th>$Z_{CS}$</th>
<th>$Z_{ES}$</th>
<th>$Z_{CS}+\alpha.Z_{ES}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>200</td>
<td>2</td>
<td>1 000.00</td>
<td>450.00</td>
<td>1 000.00</td>
<td>5 500.00</td>
<td>17.50</td>
<td>5 500.00</td>
</tr>
<tr>
<td>1</td>
<td>207</td>
<td>2</td>
<td>1 000.60</td>
<td>448.54</td>
<td>1 449.14</td>
<td>5 417.01</td>
<td>17.60</td>
<td>5 434.61</td>
</tr>
<tr>
<td>5</td>
<td>216</td>
<td>2</td>
<td>1 002.97</td>
<td>447.48</td>
<td>3 240.37</td>
<td>5 323.47</td>
<td>17.74</td>
<td>5 412.19</td>
</tr>
</tbody>
</table>

In this example, the supplier total cost is decreasing for $\alpha = 1$ and $\alpha = 5$. On the other hand, the buyer faces a huge increase in his own total cost. Assume that coordination was not feasible before setting up a carbon price. For instance, the buyer who is the supply chain leader may not be willing to share the benefit of coordinating operations with the supplier. The setup of the carbon price may change this situation. The buyer who faces a huge increase
in his total cost may be more prone to share coordination’s benefits. Table 5.12 presents the results obtained with model (c) for the same parameters.

Table 5.12: Example 5.6 results for Model (c)

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( Q^{\alpha(c)} )</th>
<th>( k^\alpha(c) )</th>
<th>( Z_{CB} )</th>
<th>( Z_{EB} )</th>
<th>( Z_{EB} + \alpha.Z_{EB} )</th>
<th>( Z_{CS} )</th>
<th>( Z_{ES} )</th>
<th>( Z_{CS} + \alpha.Z_{ES} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>825</td>
<td>1</td>
<td>2 182.82</td>
<td>882.26</td>
<td>2 182.82</td>
<td>1 940.29</td>
<td>3.64</td>
<td>1 940.29</td>
</tr>
<tr>
<td>1</td>
<td>708</td>
<td>1</td>
<td>1 910.58</td>
<td>778.36</td>
<td>2 688.94</td>
<td>2 260.80</td>
<td>4.24</td>
<td>2 265.04</td>
</tr>
<tr>
<td>5</td>
<td>512</td>
<td>1</td>
<td>1 475.01</td>
<td>609.54</td>
<td>4 522.73</td>
<td>3 125.86</td>
<td>5.86</td>
<td>3 155.16</td>
</tr>
</tbody>
</table>

In this case, setting up a carbon price \( \alpha = 5 \) would favor collaboration, however, the total supply chain carbon emissions increases:

Before the carbon price setup, \( Z_E = Z_{EB} + Z_{ES} = 450 + 17.50 = 467.50 \) for model (b) (see table 5.11).

After setting up \( \alpha = 5 \), \( Z_E = Z_{EB} + Z_{ES} = 609.54 + 5.86 = 615.40 \) for model (c) (see table 5.12).

### 4 Conclusion

In this chapter, we investigate the economic and environmental performance of buyer-supplier coordination. The study is based on a single-buyer single-supplier supply chain. Several situations illustrating different outcomes of the buyer-supplier negotiation are presented. We propose a new model that enables the supplier to act as the supply chain leader. This model has several advantages comparing to the existing models. We prove that the maximal cost increase for the buyer is limited to 6.1% comparing to the buyer’s economic order quantity. This model may also be easily implemented in practice.

Sustainability is becoming an essential feature in supply chain management. The sustainable supply chain management literature often argues that sustainability issues may encourage the firms to coordinate their operations. However, we show that coordination may increase the total supply chain carbon emissions. The same result is also established in case of a carbon price setting up. Finally, we show that an increase in carbon price may favor collaborative behaviors without necessarily having a positive effect on carbon emissions. These counterintuitive results may warn both practitioners and policy makers.
Appendix 5A: Analytical derivations of model (s)

The buyer-supplier negotiation process may be described as follows:

**Negotiation process:** The supplier first decides on the production lot size $Q_S \in (0; \infty)$. This value is then transmitted to the buyer that decides on its order quantity $Q_B = \frac{Q_S}{k}$ by choosing $k \in \mathbb{N}^*$.

The buyer still aims at minimizing its own objective function $Z_{ib}$ given by formula 5.4. The buyer’s decision is made as follows. If $Q_S \leq Q_{ib}^{*(b)}$, then it is optimal for the buyer to choose $k = 1$, else there exists $k' \in \mathbb{N}^*$ such that $\frac{Q_S}{k'+1} < \frac{Q_S}{k'} \leq \frac{Q_S}{k'}$. If $Z_{ib}\left(\frac{Q_S}{k'+1}\right) < Z_{ib}\left(\frac{Q_S}{k'}\right)$ then it is optimal to choose $k = k'+1$, else it is optimal to choose $k = k'$.

Due to the structure of formula 5.4, the interval $(0; \infty)$ can thus be divided into subintervals $(Q_{iS}^{\min}(k); Q_{iS}^{\max}(k))$ such that the buyer decides to choose the given integer $k$ for any proposed value of $Q_S \in (Q_{iS}^{\min}(k); Q_{iS}^{\max}(k))$.

**Proposition 7.1:** For all $k \in \mathbb{N}^*$:

$Q_{iS}^{\max}(k) = \sqrt{k(k+1)}Q_{ib}^{*(b)}$,

$Q_{iS}^{\min}(k) = \begin{cases} 0 & \text{if } k = 1 \\ \sqrt{(k-1)k}Q_{ib}^{*(b)} & \text{else.} \end{cases}$

**Proof:** The buyer decides to choose the given integer $k$ for any proposed value of $Q_S \in (Q_{iS}^{\min}(k); Q_{iS}^{\max}(k))$ if and only if:

$Z_{ib}\left(\frac{Q_S}{k}\right) \leq \frac{Z_{ib}(Q_S)}{Z_{ib}(Q_{ib}^{*(b)})} \iff 1\left(\frac{Q_S}{k} + \frac{Q_{ib}^{*(b)}k}{Q_S}\right) \leq 1\left(\frac{Q_S}{k+1} + \frac{Q_{ib}^{*(b)}k+1}{Q_S}\right) \iff Q_S \leq \sqrt{k(k+1)Q_{ib}^{*(b)}}$. It follows that $Q_{iS}^{\max}(k) = \sqrt{k(k+1)Q_{ib}^{*(b)}}$ for all $k \in \mathbb{N}^*$.  

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By using the calculation above, it follows that for all \( k > 1 \), the buyer decides to choose the given integer \( k \) for any proposed value of \( Q_S \in (Q_{IS_{min}}(k); Q_{IS_{max}}(k)) \) if and only if:

\[
Z_{ib} \left( \frac{Q_S}{k-1} \right) > Z_{ib} \left( \frac{Q_S}{k} \right) \iff Q_S > \sqrt{k(k-1)Q_{ib}^{*(b)}}.
\]

It follows that \( Q_{IS_{min}}(k) = \sqrt{k(k-1)Q_{ib}^{*(b)}} \) for all \( k > 1 \). Moreover, \( Q_{IS_{min}}(1) = 0 \).

On the other hand, the supplier aims at minimizing its total objective function \( Z_{is} \) as given by formula 5.7. For any given value of \( k \in \mathbb{N}^* \), the minimum of \( Z_{is} \) is obtained in \( Q_{is}^{*}(k) \) as given in proposition 5.2:

\[
\begin{align*}
\text{Proposition 5.2:} & \quad \text{For all } k \in \mathbb{N}^*: \\
Q_{is}^{*}(k) = & \begin{cases} 
\infty & \text{if } k = 1 \\
\sqrt{\frac{k}{k-1} \frac{2O_{is}D}{h_{is}}} & \text{else.}
\end{cases}
\end{align*}
\]

\[
\text{Proof:} \quad \text{The supplier objective function is expressed as follows:}
\]

\[
Z_{is}(Q_S, k) = h_{is} \left( \frac{(k-1)}{k} \frac{Q_S}{2} + O_{is} \frac{D}{Q_S} \right).
\]

For \( k = 1 \), \( Z_{is} \) tends to zero as \( Q_S \) tends to infinity.

For any given value of \( k > 1 \), the minimum of \( Z_{is} \) can be obtained by setting the first derivative of \( Z_{is} \) with respect to \( Q_S \) equal to zero.

\[
\frac{\partial Z_{is}(Q_S, k)}{\partial Q_S} = h_{is} \left( \frac{k}{k-1} \right) \frac{Q_S}{k} + O_{is} \frac{D}{Q_S^2} = 0 \iff Q_{is}^{*}(k) = \sqrt{\frac{k}{k-1} \frac{2O_{is}D}{h_{is}}} \implies Q_S = \sqrt{\frac{k}{k-1} \frac{2O_{is}D}{h_{is}}}.
\]

Then, for all \( k \in \mathbb{N}^* \),

\[
Q_{is}^{*}(k) = \begin{cases} 
\infty & \text{if } k = 1 \\
\sqrt{\frac{k}{k-1} \frac{2O_{is}D}{h_{is}}} & \text{else.}
\end{cases}
\]
In other words, the supplier would like to choose \( Q_{s}^{*}(k) \) but is required to choose \( Q_{s}(k) \in (Q_{s_{\min}}(k); Q_{s_{\max}}(k)) \) due to the proposed negotiation process. The supplier may thus choose \( Q_{s}(k) = \max\left(Q_{s_{\min}}(k) + \varepsilon; \min\left(Q_{s}^{*}(k); Q_{s_{\max}}(k)\right)\right) \), with \( \varepsilon \) being a small positive number.

**Theorem 5.1.** There exists \( (k_{i_{1}}; k_{i_{2}}) \in \mathbb{N}^{2} \) such that:

\[
1 < k_{i_{1}} \leq k_{i_{2}},
\]

\[
Q_{s}(k) = \sqrt{k(k+1)}Q_{iB}^{*}(b) \quad \text{for all } k < k_{i_{1}},
\]

\[
Q_{s}(k) = \frac{2O_{s}D}{h_{s}} \quad \text{for all } k \text{ such that } k_{i_{1}} \leq k < k_{i_{2}},
\]

\[
Q_{s}(k) = \sqrt{(k-1)kQ_{iB}^{*}(b)} + \varepsilon \quad \text{for all } k \geq k_{i_{2}}, \text{ with } \varepsilon \text{ being a small positive number.}
\]

**Proof:** \( Q_{s}(k) = \max\left(Q_{s_{\min}}(k) + \varepsilon; \min\left(Q_{s}^{*}(k); Q_{s_{\max}}(k)\right)\right). \)

\( Q_{s_{\min}}(k) \) and \( Q_{s_{\max}}(k) \) are strictly increasing in \( k \). On the opposite, \( Q_{s}^{*}(k) \) is strictly decreasing in \( k \). Moreover, \( Q_{s}(1) = Q_{s_{\max}}(1) = \sqrt{2}Q_{iB}^{*}(b) \).

**Proposition 5.3 gives additional information on \( k_{i_{1}} \) and \( k_{i_{2}} \):**

**Proposition 5.3:****

\[ k_{i}^{*}(b) \leq k_{i_{1}} \leq k_{i_{2}} \leq k_{i}^{*}(b) + 2. \]

**Proof:** By definition of \( k_{i_{1}} \), we obtain that:

\[
\begin{align*}
\{ &Q_{s}^{*}(k_{i_{1}} - 1) > Q_{s_{\max}}(k_{1} - 1) \quad (A) \\
&Q_{s}^{*}(k_{i_{1}}) \leq Q_{s_{\max}}(k_{1}) \quad (B) \\
\}
\end{align*}
\]

\[
(A) \iff \frac{k_{i_{1}} - 1}{k_{i_{1}} - 2} > \frac{2O_{s}D}{h_{s}} \iff \frac{2O_{s}D}{h_{s}} \iff \frac{k_{i_{1}}(k_{i_{1}} - 2)}{h_{s}} \leq \frac{O_{s}h_{iB}}{2O_{iB}h_{s}} \leq k_{i}^{*}(b) + 1
\]

\[
\Rightarrow k_{i_{1}} - 2 < \frac{k_{i_{1}}(k_{i_{1}} - 2)}{h_{s}} \leq k_{i}^{*}(b) + 1 \iff \frac{k_{i_{1}}(k_{i_{1}} - 2)}{h_{s}} \leq k_{i}^{*}(b) + 2.
\]

\[
(B) \iff \frac{k_{i_{1}}}{k_{i_{1}} - 1} \leq \frac{2O_{s}D}{h_{s}} \iff \frac{2O_{s}D}{h_{s}} \iff \frac{(k_{1} + 1)(k_{1} - 1)}{h_{s}} \geq \frac{O_{s}h_{iB}}{2O_{iB}h_{s}} \geq k_{i}^{*}(b) - 1
\]
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\[ \iff k_{i1} > \sqrt{(k_{i1}+1)(k_{i1}-1)} \geq k_{i}^{*(b)} - 1 \iff k_{i1} \geq k_{i}^{*(b)}. \]

We thus obtain that \( k_{i}^{*(b)} \leq k_{i1} \leq k_{i}^{*(b)} + 2. \)

By definition of \( k_{i2} \), we obtain that:

\[
\begin{align*}
A & : Q_{iS}^*(k_{i2} - 1) > Q_{iSmin}(k_{i2} - 1) \\
B & : Q_{iS}^*(k_{i2}) \leq Q_{iSmin}(k_{i2})
\end{align*}
\]

\( A \iff k_{i2} - 1 \sqrt{2O_{iS}D h_{iS}} > (k_{i2}+1)(k_{i2}^2 - 2) \sqrt{2O_{iB}D h_{iB}} \iff k_{i2} - 2 < \frac{O_{iS}h_{iB}}{O_{iB}h_{iS}} \leq k_{i}^{*(b)} + 1 \)

\( \iff k_{i2} \leq k_{i}^{*(b)} + 2. \)

\( B \iff k_{i2} \sqrt{2O_{iS}D h_{iS}} \leq (k_{i2}+1)k_{i2} \sqrt{2O_{iB}D h_{iB}} \iff k_{i2} - 1 \geq \frac{O_{iS}h_{iB}}{O_{iB}h_{iS}} \geq k_{i}^{*(b)} - 1 \iff k_{i2} \geq k_{i}^{*(b)}. \)

We thus obtain that \( k_{i}^{*(b)} \leq k_{i2} \leq k_{i}^{*(b)} + 2. \)

Moreover, \( 1 < k_{i1} \leq k_{i2} \), thus:

\( k_{i}^{*(b)} \leq k_{i1} \leq k_{i2} \leq k_{i}^{*(b)} + 2. \)

The supplier may thus aim at finding \( k_{i}^{*(s)} \) that minimizes \( Z_{iS}(Q_{iS}(k),k) \). Proposition 5.4 enables restricting the search space for \( k_{i}^{*(s)} \):

**Proposition 5.4:**

\[ k_{i}^{*(s)} \leq k_{i1} \leq k_{i}^{*(b)} + 2. \]

**Proof:** For all \( k > 1 \),

\[ Z_{iS}(Q_{iS}^*(k)) = \frac{h_{iS}}{2} \frac{(k-1)}{k} \sqrt{\frac{k}{k-1}} \sqrt{\frac{2O_{iS}D h_{iS}}{h_{iS}}} + O_{iS} D \sqrt{\frac{k-1}{k}} \frac{h_{iS}}{2O_{iS}D h_{iS}} = \frac{k-1}{k} \sqrt{2h_{iS}O_{iS}D}. \]

In addition, \( Z_{iS}(Q_{iS}^*(1)) = 0 \). \( Z_{iS}^*(Q_{iS}^*(k)) \) is thus strictly increasing in \( k \).

Moreover, for all \( k \in \mathbb{N}^* \), \( Z_{iS}(Q_{iSmin}(k+1)+\epsilon) > Z_{iS}(Q_{iSmax}(k)) \).

If \( k_{i1} < k_{i2} \), then \( Q_{iS}(k_{i1}) = Q_{iS}^*(k_{i1}) \).

For all \( k > k_{i1} \):

- either \( Q_{iS}(k) = Q_{iS}^*(k) \), then \( Z_{iS}(Q_{iS}(k)) > Z_{iS}(Q_{iS}(k_{i1})) \) as \( Z_{iS}(Q_{iS}^*(k)) \) is thus strictly increasing in \( k \),

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or \( Q_{is} (k) = Q_{is\min} (k) \), then \( Z_{is} (Q_{is} (k)) > Z_{is} (Q^{*}_{is} (k)) > Z_{is} (Q_{is\max} (k)) \) as \( Z_{is} (Q_{is} (k)) \) is thus strictly increasing in \( k \).

Else \( Q_{is} (k_{i1}) = Q_{is\min} (k_{i1}) + \varepsilon \).

In this case, \( Q_{is} (k_{i1} - 1) = Q_{is\max} (k_{i1} - 1) \). It follows that \( Z_{is} (Q_{is} (k_{i1})) > Z_{is} (Q_{is} (k_{i1} - 1)) \) as \( Z_{is} (Q_{is\min} (k + 1) + \varepsilon) > Z_{is} (Q_{is\max} (k)) \) for all \( k \in \mathbb{N}^* \).

Moreover, for all \( k \geq k_{i1} \), \( Z_{is} (Q_{is\max} (k)) > Z_{is} (Q_{is\min} (k)) \) as \( Z_{is} \) is convex in \( Q_{is} \) and \( Q^{*}_{is} (k) < Q_{is\min} (k) < Q_{is\max} (k) \). Then \( Z_{is} (Q_{is} (k + 1)) > Z_{is} (Q_{is\max} (k)) > Z_{is} (Q_{is} (k)) \). By induction, we obtain that \( k^{*(s)}_{i} \leq k_{i1} - 1 \).

It follows that \( k^{*(s)}_{i} \leq k_{i1} \leq k^{*(b)}_{i} + 2 \). \( \square \)

It is thus possible to assess \( Z_{is} (Q_{is} (k)) \) for all \( k \in [1; k^{*(b)}_{i} + 2] \) and to deduce \( k^{*(s)}_{i} \).

The following algorithm can be used to determine the optimal ordering policy for model (s):

### Step 1: Estimate \( k^{*(b)}_{i} \).

### Step 2: For all \( k \in [1; k^{*(b)}_{i} + 2] \), compute \( Q_{is\max} (k) \), \( Q_{is\min} (k) \) and \( Q^{*}_{is} (k) \) by using Propositions 1 and 2.

### Step 3: Obtain \( k_{i1} \) and \( k_{i2} \) by using Theorem 1.

### Step 4: For all \( k < k_{i1} \), compute \( Z_{is} (Q_{is\max} (k), k) \).

### Step 5: If \( k_{i1} \neq k_{i2} \), compute \( Z_{is} (Q^{*}_{is} (k_{i1}), k_{i1}) \).

### Step 6: Obtain \( k^{*(s)}_{i} \) and \( Q_{is} (k^{*(s)}_{i}) \).

Then we have thus proven that:

\[
q^{*(s)}_{is} = \begin{cases} 
\frac{k^{*(s)}_{i} + 1}{k^{*(s)}_{i}} Q^{*}_{i} & \text{if } k^{*(s)}_{i} < k_{i1} \\
\frac{1}{k^{*(s)}_{i}} (k^{*(s)}_{i} - 1) \sqrt{\frac{2O_{is} D}{h_{is}}} & \text{else.}
\end{cases}
\]
Appendix 5B: Proof of formula 5.10

Comparison of model (b) and model (c):

\[ Q^{(b)}_{CB} = \sqrt{\frac{2O_{CB}D}{h_{CB}}} \] is independent of \( k \).

\[ Q^{(c)}_{CB}(k) = \sqrt{\frac{2(O_{CB} + \frac{Q_{CS}^k}{k})D}{h_{CB} + (k-1)h_{CS}}} \] is strictly decreasing in \( k \) and tends to \( Q^{(b)}_{CB} \) as \( k \) tends to infinity, thus \( Q^{(c)}_{CB}(k) > Q^{(b)}_{CB} \) for all \( k \in \mathbb{N}^* \).

It follows that \( Q^{(c)}_{CB} > Q^{(b)}_{CB} \).

For model (c), \( k^c_{inf} = \sqrt{\frac{O_{CS}(h_{CB} - h_{CS})}{O_{CB}h_{CS}}} \) is rounded by using the rule presented in section 2.3 to obtain \( k^c \). For Model (b), \( k^b_{inf} = \sqrt{\frac{O_{CS}h_{CB}}{O_{CB}h_{CS}}} > \sqrt{\frac{O_{CS}(h_{CB} - h_{CS})}{O_{CB}h_{CS}}} \) is rounded by using the same rule to obtain \( k^b \) thus \( k^b \geq k^c \).

Appendix 5C: Proofs of theorem 5.2 and 5.3

Proof of theorem 5.2:

Assume that \( k^c = k^b = k^e \) and \( Q^{(b)}_{CB} \geq Q^{(c)}_{EB} \).

By applying theorem 3.4, we obtain that the efficient frontier of the two-echelon serial SOQ problem restricted to \( k = k^c \) is \( E_{k^c} = [\min(Q^{(b)}_{CB}, Q^{(c)}_{EB}), \max(Q^{(b)}_{CB}, Q^{(c)}_{EB})] = [Q^{(b)}_{EB}, Q^{(c)}_{CB}] \) as \( Q^{(c)}_{CB} > Q^{(b)} \geq Q^{(c)}_{EB} \) by applying formula 5.10.

It follows that \( Q^{(b)}_{CB} \in E_{k^c} \). By definition of an efficient solution, we finally obtain that:

\[ Z_E(k^b_{CB}, Q^{(b)}_{CB}) < Z_E(k^c_{CB}, Q^{(c)}_{CB}) \quad \text{as} \quad Z_C(k^b_{CB}, Q^{(b)}_{CB}) > Z_C(k^c_{CB}, Q^{(c)}_{CB}) \]

\[ \Box \]
Proof of theorem 5.3:

Idem as theorem 5.2.
Conclusions and future research directions

1 Conclusions

Among the numerous questions related to sustainable supply chain management, this PhD thesis mainly aims at contributing to the model-based research on sustainable supply chain optimization. Two key observations structure the research. First, we believe that the concept of strong sustainable development is going to deeply impact the practices and the research on sustainable supply chain. This concept indeed states that reducing all sustainability aspects to a single objective is not desirable. Second, we acknowledge the proactive positioning of companies with respect to sustainable development issues. This actual trend is not properly reflected in the existing literature. Indeed, the firm is usually assumed to face a single sustainable pressure source (e.g. a regulatory policy) and tries to minimize its cost under the considered pressure constraint.

These two key observations lead us to combine multiobjective optimization and MCDA techniques to propose new sustainable supply chain optimization methods. Multiobjective optimization appropriately reflects the strong sustainable development concept by considering that several objectives (i.e. sustainable development impacts) have to be considered in order to optimize the sustainable performance of the supply chain. By applying MCDA techniques, we assume that the firm can decide on economic, environmental and social tradeoffs by taking into account the different sustainable pressures that are faced. This positioning is complementary to the existing literature. This also enables firms to go beyond strict regulatory requirements in terms of sustainability performances. Multiobjective optimization and MCDA are two layers of analysis that contribute in finding the most preferred solution. Even if these tools are connected, both enable providing interesting insights that may be left behind when directly providing the final solution.

We decide to focus on inventory models in our research for two main reasons. First, the few published papers teach us that sustainable inventory optimization is effective to improve the sustainable performance of supply chains. Moreover, this operational decision can be easily adjusted in connection with the other decisions if required. The provided optimization tools
should thus be implemented as a part of an overall sustainable supply chain optimization tools portfolio. Even if the proposed methods were designed for inventory decisions, they may also be efficiently applied in other operations management contexts such as sustainable supply chain design, facility location and distribution optimization for example. Applying these methods to other operations management problems may deserve future research.

Defining and measuring sustainable supply chain performance is a prerequisite when aiming at optimizing sustainable supply chain. Our first contribution thus consists in assessing the performance of supply chains in terms of sustainability. We start by drawing insights from a classification of the existing key performance indicators sets for sustainability. We then propose a new methodology for KPIs set building in the context of sustainable procurement and distribution supply chains. Finally, this methodology is applied to propose a new set of KPIs for such supply chains. This KPIs set was validated by sustainable development managers and applied in an industrial context.

Secondly, our research contributes by revisiting classical inventory models taking sustainability criteria into account. We reformulate single and multi-echelon economic order quantity models as multiobjective problems. The multiobjective version of the EOQ model is called the SOQ model. For the two proposed models, the set of efficient solutions is analytically characterized. Two main findings can be highlighted when focusing on multiobjective optimization results. First, operational adjustment is proven to be an effective way to reduce sustainable development impacts. In the SOQ model, the flat region of the cost function indeed corresponds to a steeper region of the other criteria functions. It enables reducing any sustainable development impact by requiring a small increase in cost. Second, we have identified problems with non-convex efficient frontiers. In this case, some efficient solutions cannot be generated by using a weighted sum of objectives. We also propose an interactive procedure that enables to quickly take advantage of operational adjustment. This procedure is proven to be robust and allows focusing on all efficient solutions even if the efficient frontier is non-convex.

Finally, the proposed multiobjective models are adapted to compare several managerial options in terms of sustainability performance. In chapter 4, we compare operational adjustment and technology investment by modeling both options in the SOQ model. The
results show that operational adjustment may be a valuable alternative in comparison to technology investments. We also provide analytical conditions under which an option outperforms the other one for two classical regulatory policies, i.e. the carbon cap and the carbon tax policies. Some practical insights are also discussed. We prove that controlling the carbon emissions by setting a carbon price may have several limitations. In chapter 5, different outcomes of buyer-supplier coordination are illustrated. Among them, a new model of a supplier leader supply chain is introduced and discussed. This model has several advantages comparing to the existing models. We prove that the maximal cost increase for the buyer is limited to 6.1% comparing to the buyer’s economic order quantity. This model may also be easily implemented in practice. The impact of buyer-supplier coordination on the supply chain economic and environmental performances is then challenged. Several counterintuitive results may warn both practitioners and policy makers.

2 Future research directions

Several research directions can then be considered. First of all, other inventory models could be revisited. For instance, Benjaafar et al. (2010) as well as Absi et al. (2011) include carbon emissions constraints on single and multi-stage lot-sizing models with a cost minimization objective. However, both papers highlight the difficulty that appears when focusing on more sophisticated inventory models. Absi et al. (2011) indeed prove that the single-stage lot-sizing problem with a carbon constraint is NP-hard for several types of constraints. In this case, close to optimal solutions could be used. A parallel may indeed be done with more sophisticated inventory problems where finding closed to optimal solutions with guaranteed performance has attracted a lot of attention in the past (see e.g. Crowston et al., 1973; Roundy, 1985; or Roundy, 1986).

Moreover, introducing stochastic variables into the presented models may also be considered for future research. The proposed models consider that both the demand and the leadtimes are deterministic. These simple assumptions enable providing useful insights but may be relaxed to focus on more realistic situations. The amount of carbon emissions may also be considered as a stochastic variable due to variations in production setup and waste or to imperfect operations that could lead to possible defective items that require rework, recycling or scrap.
Considering non deterministic demand, leadtime and production processes could indeed affect the sustainable development performance of the supply chain.

In chapter 3, we show that optimal solutions may be very complex even for the two-echelon serial SOQ model. We indeed prove that non-stationary ordering policies may lead to efficient solutions. Non-stationary ordering policies for the two-echelon serial SOQ problem may be very complex. Instead of trying to identify the optimal policies, we thus focus on a class of simple ordering policies with good performance. The complexity induced by moving from an EOQ model to an SOQ one may be seen as similar to moving from a single retailer model to a multi-retailer one. The logic of our approach may be seen as similar to that of Roundy (1985). The main difference lies in the fact that Roundy (1985) manage to compare the proposed power of two ordering policies to a lower bound to obtain guaranteed performance. The same idea may certainly be of interest for the two-echelon serial SOQ model and may deserve future research.

Moreover, the sustainable development criteria could be modeled with more precision. In this PhD dissertation, a structure similar to the classical cost function of the EOQ model is used as a first attempt. Alternative structures could be used in future work. As an example, a more accurate evaluation of the carbon footprint including vehicle capacity could be of practical interest. Note that the presented multiobjective optimization results for the SOQ and the two-echelon serial SOQ models are valid as soon as the criteria are modeled by using general strictly convex functions.

Finally, chapter 5 focuses on a simple supply chain structure with a single buyer. The effect of coordination may perhaps be different in a multi-buyer context. Studying the effect of sustainability considerations in a single-supplier multi-buyer context thus deserves future research. Model (s) may also be extended to a multi-buyer context. This negotiation process may indeed favor horizontal cooperation between buyers as they would be required to pass their orders at given time intervals. In this setting, it may be possible to consolidate shipments in order to improve the sustainable performances of the supply chain.

Our research has focused on very simple inventory models and may thus be seen as a stepping stone for future research. On the other hand, this focus on simple situations has allowed us
providing analytical results as well as managerial insights. Simple inventory models behave quite unexpectedly while including sustainability criteria. These counterintuitive behaviors may warn academics, practitioners as well as policy makers in their analysis of close to real life situations.

3 Epilogue

Based on a distinguished fellows presentation given at the University of Michigan, Cachon (2012) proposes his personal view of the essential characteristics of interesting research in operations management:

“Interesting research raises more questions than it answers. It is controversial. It invokes responses like “that can’t be true” or “this is obviously incomplete.” Interesting research should initially leave the reader a little discontent, unnerved, or motivated to prove it wrong or at least incomplete” (Cachon, 2012).
Appendix A: Multiobjective optimization and Multiple Criteria Decision Aid

This appendix presents some basic features on multiobjective optimization and MCDA methods. First, we define some concepts of multiobjective optimization and we highlight some of underlying issues. Second, we introduce MCDA methods and we focus on the main methods linking multiobjective optimization with MCDA.
Appendix A: Multiobjective optimization and MCDA

1 Multiobjective optimization

In this section, some basic concepts of multiobjective optimization are presented. We define what we call a multiobjective optimization problem and we summarize its main characteristics. The concepts of decision space, criterion space and non-supported solutions are then presented.

1.1 Introduction, example and characterization

Multiobjective optimization is the process of simultaneously optimizing several objectives. However, in most of the case, objectives are conflicting and one cannot identify a single solution that simultaneously optimizes each objectives. Thus, the aim of multiobjective optimization is to identify particular solutions such that, when attempting to improve an objective further, other objectives suffer as a result. Historically, multiobjective optimization can be traced back in the work of Pareto (1896). Multiobjective optimization has been efficiently used in various fields such as product and process design, finance or operations management.

The following example taken from Ehrgott (2005) will be used as an illustration. We consider a problem with two objective functions that should be minimized and one decision variable. The objective functions $f_1$ and $f_2$ are defined for all $x \in \mathbb{R}$ as follows:

$$f_1(x) = \sqrt{x+1} \quad \text{and} \quad f_2(x) = x^2 - 4x + 5.$$  \hspace{1cm} (A.1)

The multiobjective problem that has to be solved is stated as follows:

$$\min_{x \geq 0} (f_1(x), f_2(x)).$$  \hspace{1cm} (A.2)

The two objective functions are plotted in figure A.1. It can be noticed that $f_1$ is strictly increasing on $\mathbb{R}$ and that $f_2$ is strictly decreasing on $[0,2]$ and then strictly increasing on $[2,\infty)$. The question is, what are the “minima” in this problem? Note that the corresponding optimization problem is easy for each function taken individually.
Appendix A: Multiobjective optimization and MCDA

Figure A.1: The considered objective functions

In multiobjective optimization, the concept of optimality is replaced by the concept of efficiency. A solution called efficient (or Pareto optimal) is a solution such that, when attempting to improve an objective further, other objectives suffer as a result. Applying this concept in our example, all \( x \in [0,2] \), where one function is increasing and the other one is decreasing, are efficient solutions.

It may be noticed that both \( f_1 \) and \( f_2 \) are strictly increasing on \((2, \infty)\). Thus, for all \( x \in (2, \infty) \), it is possible to improve both objective functions by choosing \( x = 2 \). These solutions are called dominated solutions. “The fundamental importance of efficiency is based on the observation that any \( x \) which is not efficient cannot represent a most preferred alternative for a decision maker” (Ehrgott, 2005). Identifying the set of efficient solutions is thus important for implementing a multiple criteria decision aid (MCDA) method.

In our research, we assume that the objective functions (or criteria) represent sustainability impacts that should be minimized. In a multiobjective optimization problem with \( n > 1 \) objectives, a solution (or alternative) \( a \) is said to be dominated if there exists another alternative \( b \) such that that for all \( i \in [1,n] \), \( f_i(b) \leq f_i(a) \) with at least one strict inequality. The set of efficient solutions is called the efficient frontier and is noted \( E \).

Several distinctions may be made in characterizing multiobjective optimization problems. Some problems which have a countable number of alternatives are called discrete. The other
Appendix A: Multiobjective optimization and MCDA

problems are called continuous. The proposed example is an illustration of a continuous problem. This class of problem will be of particular importance in our research as the decision variables of inventory models are often continuous variables as in the EOQ model. In this case, non-trivial multiobjective problems are characterized by an infinite efficient frontier, i.e. an efficient frontier composed by an infinite number of alternatives.

As proposed by Ehrgott (2005), the elements of a multiobjective optimization problem are summarized as follows:

- the set of feasible solutions $\mathcal{A}$ (e.g. $\mathbb{R}$ in the proposed example),
- the objective function vector $f = (f_1, \ldots, f_n) : \mathcal{A} \to \mathbb{R}^n$,
- the objective space $\mathbb{R}^n$,
- the ordered set $(\mathbb{R}^n, \preceq)$.

The choice of an order $\preceq$ on $\mathbb{R}^n$ enables defining the meaning of “min” in formula A.2. The classical definition of the min is in relation with the componentwise order $\leq$ i.e. “less or equal to”. This order will be the only one considered in our work. We refer to Ehrgott (2005) for a formal definition of an order and for other examples.

1.2 Decision space, criterion space and non-supported solutions

The set of feasible solutions noted $\mathcal{A}$ is called the feasible set. The space of which the feasible set is a subset is called the decision space. In the proposed example, the feasible set is $\mathcal{A} = \{ x \in \mathbb{R}; x \geq 0 \} = \mathbb{R}_+$. Then, the decision space is $\mathbb{R}$ as $\mathcal{A} \subset \mathbb{R}$. Figure A.1 is plotted in the decision space.

The criterion space represents the space where the feasible solutions are evaluated. In the proposed example, for all $i \in \{1,2\}$, $f_i : \mathbb{R} \to \mathbb{R}$ thus the criterion space is $\mathbb{R}^2$. To obtain the image of the feasible set $f(\mathcal{A}) = \{ f_1(x); f_2(x) | x \in \mathcal{A} \} = A^f$ in the criterion space, we substitute $y_1$ for $f_1(x)$ and $y_2$ for $f_2(x)$. Figure A.2 represents the image of the feasible set in the criterion space for the proposed example. Note that the condition $x \geq 0$ translates into $y_1 \geq 1$. 

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Appendix A: Multiobjective optimization and MCDA

Figure A.2: The image of the feasible set in the criterion space

The criterion space is particularly interesting in multiobjective optimization problems with two objectives as the image of the efficient solution may be easily plotted. Note that the criterion space is very complex to use with more than three objectives. In the criterion space, the image of the efficient frontier may be easily determined. Figure A.3 represents the image of the efficient frontier \( f(E) = \{f_1(x), f_2(x)|x \in E\} = E^f \) in the criterion space for the considered problem. The right angle associated to the efficient point \((y_1, y_2)\) shows that there does not exist any solution that dominates \((y_1, y_2)\). Note that the monoobjective minima of both \(f_1\) and \(f_2\) for \(x \in A\) are among the efficient solutions.

The definition of the feasible set may deeply influence the results of a multiobjective optimization problem. In the previous example, it is possible to enlarge the feasible set by now considering that \(A = [-1, \infty)\). In this case, we obtain that \(E = [-1, 2]\). Figure A.4 represents the image of \(E = [-1, 2]\) in the criterion space.
We also would like to introduce the following notations:

\[ \mathbb{R}_+^n = \{ (x_1, \ldots, x_n) | x_i \in \mathbb{R}_+, \forall i \in [1, n] \} \] is the nonnegative subset of \( \mathbb{R}^n \).

Let \( S_1 \) and \( S_2 \) two subsets of \( \mathbb{R}^n \):

\[ (S_1 + S_2) = \{ s_1 + s_2 | s_1 \in S_1, s_2 \in S_2 \} \]

is the Minkowski sum, \( E^f_+ = (E^f + \mathbb{R}_+^n) \).

For \( n = 2 \), \( E^f_+ \) thus includes all the elements of \( E^f \) as well as all the elements situated at the top right of \( E^f \).

For \( A = \mathbb{R}_+ \), it may be noticed that the \( E^f_+ \) is convex. That is not the case for \( A = [-1, \infty) \).

This allows us distinguishing between two types of efficient solutions, i.e. supported solutions and non-supported ones (Geoffrion, 1968). Supported solutions are situated in the convex hull of the \( E^f \). This type of solution can be generated by using a linear combination of the objectives. This is not the case for non-supported solutions. Non-supported solutions are of
interest in our research as we prove that some of the considered multiobjective problems are non-convex. The existence of non-supported solutions (i.e. the existence of non-convex problem) is in contradiction with the use of methods based on a weighting sum of the objectives to generate the efficient frontier. Using this type of method for non-convex problems may provide a completely misleading impression to the decision maker about the feasible solutions available as non-supported solutions would be left behind.

### 2 Multiple Criteria Decision Aiding (MCDA)

In this section, the basic principles of MCDA are first stated. Then, we focus on the methods linking multiobjective optimization and MCDA.

#### 2.1 Introduction to MCDA

As the efficient frontier may contain a lot of solutions, extra information is often required so as to determine a final solution. This most preferred solution is selected by a DM based on some preference information. The process of guiding the DM to obtain the final solution is called decision aiding and may be defined as follows: “Decision aiding is the activity of the person who, through the use of explicit but not necessarily completely formalized models, helps obtain elements of responses to the questions posed by a stakeholder in a decision process” (Roy, 1996). MCDA is the process of solving a multiobjective optimization problem by helping a DM in considering the multiple objectives simultaneously and in finding the efficient solution that please him/her the most (Branke et al., 2008).

The concept of alternative and the concept of criterion are central notions in MCDA. An action in MCDA is the synonym of a solution in multiobjective optimization. The concept of action does not necessarily include the notion of feasibility. An action is qualified as potential when this one is feasible. The concept of alternative is more widely used than the concept of potential action in the literature. The only difference is that several alternatives may not be conjointly chosen due to mutual exclusion. As an alternative has to be of interest for the DM, this corresponds to an efficient solution of the associated multiobjective optimization problem. A criterion is constructed for evaluating alternatives according to a well-defined point of view. The evaluation of an alternative according to a certain criterion is called the performance. This one is often evaluated by using real numbers. A criterion in MCDA may thus be seen as the synonym
of an objective in multiobjective optimization. However, other type of scale may be used in MCDA. Note that we consider only quantitative scales in our research. An MCDA problem is thus often synthesized by its performance table, i.e. the set of alternative evaluated on every criterion.

The MCDA problems are often divided into three classes. First, the choice problematic consists in selecting a small number of alternatives that may be defined as good options. It may sometimes be possible to select a single alternative that outperforms the other ones from the DM point of view and may thus be seen as the most preferred solution. Second, the sorting problematic deals with the assignment of alternatives into predefined categories. Third, the ranking problematic aims at ranking the alternative so as to build a complete or partial preorder on the set of possible alternatives. It may also be noticed that a fourth class of problematic is sometimes proposed (Roy, 2005). This fourth class called problem setting only consists in building the performance table of the MCDA problem. Defining the available alternatives, building the family of suitable criteria and providing the evaluation of each alternative on every criterion may indeed be seen as an entire problematic.

As mentioned by Roy (2005), the most frequently used aiding methods are based on mathematically explicit multiple criteria aggregation model called the preference model. The preference model is based on inter-criteria parameters such as weights, scaling constant, veto, aspiration levels, rejection levels… Moreover, this one is also required to specify the possible dependence between criteria as well as the conditions under which compensation is accepted or refused between good and bad performance. Two main classes of MCDA models may be found in the literature. The first one is based on a synthesizing criterion. A formal rule that takes into account all the performance of any alternative allows assigning each alternative a well defined position on an appropriate scale. This leads to the definitions of a total preorder that allows classifying the alternatives. The most preferred alternative is the one that obtains the best score. The second class of MCDA model is based on a synthesizing preference relational system and is often labelled as outranking models. This type of model does not aim at constructing a global scale enabling classifying each alternative. In this case, the preference model is based on pairwise comparisons so as to design a synthesizing preference relational system. This type of approach may cause some intransitivity or some incomparability to appear. They may thus be
harder to handle. However, these models are very popular as they allow reflecting a lot of DM behaviour.

To conclude this section, we would like to highlight the difference between MCDA models and MCDA methods. As an example, the analytical hierarchy process method (Saaty, 2005) allows determining the weights of a weighted sum model. Other methods may be used to determine these weights. MCDA model thus refers to preference model whereas MCDA method refers to the procedure implemented to infer DM’s preference information in order to determine the parameters of preference model. These two terms are often confused as a model generally needs a method to be implemented.

### 2.2 Non-interactive methods

MCDA methods developed for multiobjective optimization problems can be classified into four classes i.e. no-preference methods, a priori methods, a posteriori methods and interactive methods, depending on the role of the DM in the solution process (Miettinen, 1999). This sections focuses on non-interactive methods, the three first types of methods are thus presented.

In no-preference methods, the DM is assumed to be unavailable for expressing his / her preference information. It may also happen that the DM has no special expectations of the solution. In these cases, the task is to find a compromise solution that lies somehow in the “middle” of the efficient frontier. Some assumptions are then made about what a reasonable compromise could be. Two main classes of no-preference methods have been developed. In the first one called the method of global criterion, the aim is to find a solution that minimizes the distance between a desirable reference point and the efficient frontier (Cochrane and Zeleny, 1973; Yu, 1973). The ideal point, i.e. a fictitious alternative having the best performance on each criterion, is often used as a reference point. The distance is generally measured by using the Chebyshev metric. The second class of no-preference methods is referred to as the neutral compromise solution method (Wierzbicki, 1999). The aim of this method is to find a compromise solution in the “middle” of the efficient frontier by averaging the best and worst possible performance on each criterion.

In a priori methods, the DM first provides preference information. The solution procedure then tries to find an efficient solution that satisfies as much as possible the aspirations of the
DM. In general, only a part of the efficient frontier is generated in a priori methods. This requires the DM to have a clear idea of what may be possible in the problem and how realistic his/her own expectations are. Three main classes of a priori methods are generally used. In, the value function method (Keeney and Raiffa, 1976), the preference model based on a synthesizing criterion is in the form of a value function. This method may be of particular interest if the DM is able to specify an explicit mathematical formulation for the value function and if that function can capture and represent all his/her preferences. Note that the value function theory will be used in the interactive procedure proposed in chapter 5. The second class of method is called the lexicographic ordering method (Fishburn, 1974). In this method, the DM must arrange the objective functions according to their absolute importance. This means that a more important objective is infinitely more important than a less important one. The third class of a priori method is referred to as the goal programming method (Charnes et al., 1955). In this method, the DM is required to specify aspirations levels on each criterion. Then, deviations from these aspiration levels are minimized. The aspiration levels are assumed to be selected so that they are not achievable simultaneously.

The last type of non-interactive method is referred to as a posteriori methods. In this case, the efficient frontier is firstly generated by using multiobjective optimization. The DM is then supposed to select the most preferred solution among the set of efficient ones. As mentioned in Branke et al. (2008), this approach allows giving the DM an overview of the different solutions available. On the other hand, it may be difficult for the DM to analyze this large amount of information. Moreover, generating the set of efficient solutions may be computationally expensive. Two main classes of a posteriori methods are generally used. The first one is called the method of weighted metrics. In this case, the idea of the method of global criterion is generalized by letting the DM proposing the search direction from the ideal point. This is made possible by weighting the metrics. The second class of methods is referred to as the achievement scalarizing functions method. The idea of this method is to ask the DM to provide desirable aspiration levels on each criterion and to project this reference point on the efficient frontier by using achievement scalarizing function.

### 2.3 Interactive methods

In interactive methods, an interactive algorithm is repeated several times. At each iteration, some information is given to the DM who is asked to provide some preference information (as
in a posteriori methods). This preference information is then used in the next iteration to explore the solution space (as in a priori methods). Solving a multiobjective optimization problem interactively is a constructive process where the DM builds a conviction of what is possible and confronts this knowledge with his / her preferences that may also evolve. In this setting, the most important stopping criterion is the DM’s conviction that a satisfactory solution has been reached (Branke et al., 2008).

An interactive algorithm generally consists of the following steps:

- **Step 1:** Generate some efficient solutions as a starting point.
- **Step 2:** Ask for preference information to the DM.
- **Step 3:** Generate new efficient solutions in accordance to the provided preference information.
- **Step 4:** Ask the DM if a satisfactory solution has been found, if true then stop, else go to step 2.

Three main classes of interactive methods have been developed depending on the type of information asked to the DM. The classes are methods based on tradeoff information, reference point approaches and classification based methods.

In methods based on tradeoff information, tradeoffs are used to direct the search of the most preferred solution. The main methods of this category includes the Zionts-Wallenius method (Zionts and Wallenius, 1976), the Geoffrion-Dyer-Feinberg method (Geoffrion et al., 1972), the SPOT method (Sakawa, 1982) and the GRIST method (Yang, 1999).

In the reference point approaches, the DM is required to specify his / her aspiration by proposing a reference point. The “closest” efficient solution is then proposed. The DM is free to modify the reference point during the interactive process. The notion of distance may also evolve as this one may be based on some preference information. The main methods of this type includes the Chebyshev method (Steuer, 1986), the Pareto race method (Korhonen and Laakso, 1986) and the REF-LEX method (Miettinen and Kirilov, 2005).

The classification-based methods use the trading off principle. The DM indeed indicates his / her preference by classifying objective functions. By doing so, the DM indicates which
objective functions should improve and which ones could impair from their current values. In addition, desirable amount of improvement may be asked to the DM. The main methods of this type includes the STEM method (Benayoun et al., 1971), the STOM method (Nakayama and Sawaragi, 1984) and the NIMBUS method (Miettinen, 1999).

Interactive methods are very effective and the number of iterations is often limited. These methods are of practical interest when the DM has limited availability to obtain his / her most preferred solution. The number of interaction depends on the preciseness that the DM wants to obtain on the result as well as on the idea that the DM has about what may be possible when starting the procedure.

3 Conclusion

Multiobjective optimization and MCDA are two layers of analysis that participate in finding the most preferred solution. Even if these tools are connected, both enable providing interesting insights that may be left behind when directly providing the final solution.
References


