Turbulent and neoclassical toroidal momentum transport in tokamak plasmas

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1. Introduction and motivations

2. Conservation of toroidal angular momentum in gyrokinetics

3. Intrinsic rotation generated by electrostatic turbulence

4. Neoclassical toroidal rotation in the presence of ripple

5. Conclusions
1 Introduction and motivations

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Toroidal momentum transport: a crucial issue for ITER

- **Goal**: obtain plasma conditions favorable for tokamak performance → importance of particle and heat transport

- **The presence of toroidal rotation can reduce heat transport through**
  - stabilization of modes which degrade confinement [Bondeson & Ward '94]
  - saturation of turbulent transport by sheared flows [Biglari et al. '90]

Perspective for ITER

- **Present experiments**: toroidal rotation dominated by external sources

- **Future experiments (e.g. ITER)**: external torque will be small
Intrinsic toroidal rotation is observed in tokamaks

- Intrinsic rotation has been observed in existing tokamaks
- Example from D-III-D using varying external sources:

\[ \Omega \text{ (rad/s)} \]

\[ T_{\text{NBI}} \approx 5 \text{ Nm (3 co + 0 ctr NB)} \]

\[ T_{\text{NBI}} \approx -2.5 \text{ Nm (1 co + 2 ctr NB)} \]

⇒ An understanding of the mechanisms for intrinsic rotation generation and transport is required to predict the toroidal rotation level and profile in ITER
What governs the evolution of toroidal rotation in tokamaks?

Basic (fluid) equation for the evolution of toroidal velocity:

$$\rho \partial_t V_\varphi = -\rho \nu_{neo} (V_\varphi - V_{\varphi neo}) - \rho \nabla \cdot \langle \tilde{V} \tilde{V} \rangle - \frac{1}{\mu} \nabla \cdot \langle \tilde{B} \tilde{B} \rangle - j_{fast} \times B$$

Key physics:

- Neoclassical friction due to collisional processes
- Turbulent generation of toroidal rotation
- Magnetohydrodynamic (MHD) effects
- Fast particles
- Boundary conditions: interaction with flows in the tokamak edge
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Models for the plasma response to electromagnetic fields

- **Fluid description (3D)**
  - Modeling of fluid density, velocity and temperature
  - Assumes weak departure from local thermodynamic equilibrium
  - Not satisfying in core tokamak plasmas, mean free path $\sim 10\, km$

- **Kinetic description (6D)**
  - Required for low collisional plasmas, includes wave-particle resonances
  - Probability distribution $F$ of particles in 6D phase-space
  - Solve Fokker-Planck equation
    $$\partial_t F(x, v) - [H, F] = C(F)$$
From kinetic (6D) to gyrokinetic (4+1D)

- 6D distribution function → prohibitive computational cost

  - Reduction of dimensionality: 6D → 5D
    - In the case of strongly magnetized plasmas
    - For frequencies < cyclotron frequency

- Resulting model: Fokker-Planck equation for \( \bar{F} \)

\[
\partial_t \bar{F} - [\bar{H}, \bar{F}] = C(\bar{F})
\]

for gyrocenter distribution function \( \bar{F}(x, v_\parallel, \mu) \)

- Magnetic moment \( \mu = \frac{mv^2}{2B} \) is an invariant of the model

\( \Rightarrow \) 4+1D model, numerically costly but accessible with modern high-performance-computing (HPC) resources
The **GYSELA** code for flux-driven gyrokinetic simulations of core plasma turbulence

- Solves gyrocenter distribution function \( \bar{F}(r, \theta, \varphi, v_\parallel, \mu) \)
- **Full-f** : no scale separation equilibrium/perturbations
- **Flux-driven** system, global geometry
- Electrostatic ITG turbulence
- Adiabatic electron response
- Gyrokinetic equation : [Brizard & Hahm, Rev.Mod.Phys. 2007]

\[
B^\star_\parallel \frac{\partial \bar{F}}{\partial t} + \nabla \cdot \left( d_{\|} B^\star_\parallel \bar{F} \right) + \frac{\partial}{\partial v_\parallel} \left( d_{\|} B^\star_\parallel \bar{F} \right) = C(\bar{F}) + S
\]

- Poisson equation : \( \nabla^2 \phi = -\frac{1}{\varepsilon_0} \sum_{\text{species}} n_s e_s \Rightarrow \delta n_e = \delta n_i \)

\[
\tau (\phi - \langle \phi \rangle) = \frac{1}{n_{eq}} \int J \cdot (\bar{F} - F_{eq}) \, d^3v + \frac{1}{n_{eq}} \nabla_\perp \cdot (n_{eq} \phi \nabla_\perp \phi)
\]
GyseLA : a massively parallel numerical code

- Semi-Lagrangian numerical scheme
- Massively parallel simulations. Results from strong scaling: 82% efficiency for 8k cores, 61% for 65k cores [G.Latu et al., 2012]
- Number of grid points \( \propto (\rho_*)^{-3} \) where \( \rho_* \equiv \rho_i/a \)

Parameters for ITER-size plasma simulation \( (\rho_* = \rho_i/a = 1/512) \)

- \((r, \theta, \varphi, v_\parallel, \mu)\) grid : \((1024, 1024, 128, 128, 16)\) for a 1/4 torus
  \(\rightarrow \sim 3.10^{11}\) grid points in 5D phase-space
- one month run on 8192 processors
  \(\rightarrow 6.10^6\) hours \(\sim 7\) centuries of computing time!
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Conservation of toroidal angular momentum in gyrokinetics

- The gyrokinetic model is a reduction of the kinetic model
- Is it accurate enough to model the transport of toroidal momentum?

Controversial issue: is toroidal angular momentum conserved in the reduced model used by GK codes?

- [Parra&Catto, PPCF'08, PoP'10] No, additional terms are required
- [Scott&Smirnov, PoP'11] Gyrokinetic field theory provides general conservation equations
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Important results obtained in the present work

1. A local conservation equation for toroidal angular momentum is derived analytically (from the equations implemented in GK codes)
2. This result is verified numerically with the GYSELA code

\[ \Rightarrow \text{Gyrokinetic codes provide an accurate description of toroidal momentum transport} \quad \text{[J. Abiteboul et al., PoP 2011]} \]
Tokamak geometry with reasonable assumptions
⇒ 3 motion invariants for particles (equilibrium motion)
  ▶ Energy (equilibrium : constant electric potential)
  ▶ Assuming slow variations of B : Adiabatic invariant $\mu$
  ▶ Axisymmetric magnetic geometry ⇒ third invariant...

...Toroidal canonical angular momentum : $P_\phi = \partial L / \partial \phi$
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...Toroidal canonical angular momentum : $P_\phi = \partial L / \partial \phi$

for gyrokinetics ⇒ gyrocenter canonical angular momentum
  ▶ $\bar{P}_\phi = \partial \bar{L} / \partial \phi$
  ▶ $\bar{P}_\phi = P_\phi +$ small terms in gyrokinetic ordering
  ▶ $\bar{P}_\phi$ is an exact invariant of unperturbed gyrocenter motion
Toroidal canonical momentum
\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) = \frac{\partial L}{\partial q_i} \]

- Tokamak geometry with reasonable assumptions
  \( \Rightarrow \) 3 motion invariants for particles (equilibrium motion)
    - Energy (equilibrium: constant electric potential)
    - Assuming slow variations of B: Adiabatic invariant \( \mu \)
    - Axisymmetric magnetic geometry \( \Rightarrow \) third invariant...

...Toroidal canonical angular momentum: \( P_\phi = \frac{\partial L}{\partial \dot{\phi}} \)

- for gyrokinetics \( \Rightarrow \) gyrocenter canonical angular momentum
  - \( \bar{P}_\phi = \frac{\partial \bar{L}}{\partial \dot{\phi}} \)
  - \( \bar{P}_\phi = P_\phi + \) small terms in gyrokinetic ordering
  - \( \bar{P}_\phi \) is an exact invariant of unperturbed gyrocenter motion

- When is \( \bar{P}_\phi \) not an invariant? Breaking of axisymmetry
  - non-axisymmetric B (e.g. due to finite number of coils)
  - turbulence (electrostatic): \( d_t \bar{P}_\phi = -e \partial_\phi \bar{\phi} \)
Local conservation equation

- Global conserved quantity: \( \int \bar{P_\varphi} d^3x d^3v \)
Global conserved quantity: \( \int \bar{P}_\varphi d^3x d^3v \)

More interesting → local (radial) conservation law?

Gyrocenter toroidal angular momentum

\[
\bar{L}_\varphi(r) = \int d\theta d\varphi d^3v \bar{P}_\varphi
\]

= particle angular momentum + small terms in gyrokinetic ordering
Local conservation equation

- **Global** conserved quantity: \( \int \bar{P}_\phi d^3x d^3v \)

- More interesting → **local** (radial) conservation law?
  Gyrocenter toroidal angular momentum

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= particle angular momentum + small terms in gyrokinetic ordering

- From the gyrokinetic equation, we derive a local equation

\[
\partial_t \bar{L}_\phi + \nabla_r \Pi^r_\phi + \nabla_r T^r_\phi = \mathcal{J}
\]

- Describes the **radial transport** of toroidal momentum

- **Exact** local conservation equation (i.e. derived from gyrokinetic model with no additional assumptions)  
  [J. Abiteboul et al., PoP 2011]
Physical interpretation of the momentum fluxes

\[
\partial_t \mathcal{L}_\varphi + \nabla_r \Pi'_\varphi + \nabla_r T'_\varphi = J
\]  

(\mathcal{L}_\varphi \sim RV_\varphi)

- \(\Pi'_\varphi \sim \langle R\tilde{V}_\varphi \tilde{V}_r \rangle\) : Reynolds stress

- \(T'_\varphi \sim \langle \frac{nm}{B^2} R E_r E_\varphi \rangle\) : Polarization stress [McDevitt et al., PRL’09]

- \(J\) : radial current of gyrocenters  
  \(J \sim 0\) with adiabatic electrons
  interpreted as exchange of momentum between field and particles using the equation for polarization \(\sigma \sim E_r\):

\[
\partial_t \sigma = -J
\]

\[
\Rightarrow \partial_t (\mathcal{L}_\varphi + \sigma) + \nabla_r \Pi'_\varphi + \nabla_r T'_\varphi = 0
\]

no source term for total toroidal momentum (field+particles)
Numerical test of the conservation law

- using gyrokinetic code \texttt{GYSELA} (conservative GK equations) with \texttt{new diagnostics} implemented for momentum transport studies
Numerical test of the conservation law

- using gyrokinetic code GYSELA (conservative GK equations) with new diagnostics implemented for momentum transport studies

![Graph showing conservation equation recovered numerically despite strong variations (both radially and in time)]

- Conservation equation recovered numerically despite strong variations (both radially and in time)

- Dominant contribution: Reynolds stress

[J. Abiteboul et al., Physics of Plasmas 2011]
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General structure of toroidal momentum transport by turbulence: \( \frac{\partial}{\partial t} V_\varphi + \nabla \Pi = 0 \)

- Radial transport governed by Reynolds stress \( \Pi^r_\varphi \sim \langle RV_\varphi \tilde{V}_r \rangle \)
Radial transport governed by Reynolds stress: $\Pi_\varphi^r \sim \langle RV_\varphi \tilde{V}_r \rangle$

Can be split into three components (e.g. [Diamond NF 2009]):

$$\Pi_\varphi^r = -\chi_\varphi \frac{\partial v_\varphi}{\partial r} + \varpi_\varphi + \Pi_\varphi^{r \, \text{res}}$$

- **Diffusive** transport with $\chi_\varphi / \chi_i = \text{Pr} \sim 1$ [Mattor PoF 1988]

- **Convective** (or “pinch”) contribution [Peeters PRL 2007; Hahm PoP 2007]

- **Residual stress** [Diamond PoP 2008; Peeters PoP 2009]
General structure of toroidal momentum transport by turbulence: \( \partial_t V_\varphi + \nabla \Pi = 0 \)

- Radial transport governed by Reynolds stress:  \( \Pi^r_\varphi \sim \left\langle R \tilde{V}_\varphi \tilde{V}_r \right\rangle \)

- Can be split into three components (e.g. [Diamond NF 2009])

\[
\Pi^r_\varphi = -\chi_\varphi \frac{\partial v_\varphi}{\partial r} + V v_\varphi + \Pi^r_{\varphi \text{res}}
\]

- Diffusive transport with \( \chi_\varphi / \chi_i = \text{Pr} \sim 1 \)  [Mattor PoF 1988]

- Convective (or “pinch”) contribution [Peeters PRL 2007; Hahm PoP 2007]

- Residual stress [Diamond PoP 2008; Peeters PoP 2009]

- In flux-driven, full-\( f \) simulations → not trivial to separate between these contributions
Initial turbulent front generates dipolar rotation

- Initialize a simulation with vanishing toroidal rotation
  \( \rightarrow \) no diffusive or convective momentum transport

Generated by the turbulent Reynolds stress (residual stress)
Dipolar structure consistent with global momentum conservation


\( \rightarrow \) no net rotation can be generated inside the simulation domain
Initial turbulent front generates dipolar rotation

- Initialize a simulation with vanishing toroidal rotation
  \(\rightarrow\) no diffusive or convective momentum transport
- Initial turbulent burst \(\Rightarrow\) generates “dipolar” rotation

- Generated by the turbulent Reynolds stress (residual stress)
- Dipolar structure consistent with global momentum conservation


\( \rightarrow \) no net rotation can be generated inside the simulation domain
The front corresponds to a cycle in Heat Flux & Reynolds stress

- Plot the turbulent heat flux and Reynolds stress for all radii
- Arrows correspond to increasing radius

- Reynolds stress front propagates earlier than the heat flux front
- Estimated delay between the fronts is \( \approx 600\omega_c^{-1} \)
  (front propagation velocity is \( \approx \rho_* v_T \approx 10 \text{ km/s} \) for ITER)
Role of edge flows in determining core toroidal rotation?

- Local conservation $\Rightarrow$ no *net* rotation generation in the core
- How is the core plasma influenced by SOL flows? [Gunn, JNM 2007]
Role of edge flows in determining core toroidal rotation?

- Local conservation $\Rightarrow$ no net rotation generation in the core
- How is the core plasma influenced by SOL flows? [Gunn, JNM 2007]

- Change position of the plasma $\rightarrow$ modify SOL flows
- Clear effect on core rotation

Other experimental results:
[LaBombard NF 2004, Hennequin EPS 2010]
Impact of boundary conditions on core toroidal rotation

Numerically : corresponds to the issue of boundary conditions

- in GYSELA replace no-slip ($V = 0$) boundary with $V_{\parallel}(r_{max}) = \pm 0.1 v_{th}$
- mimicks rotation at the top of the pedestal in H-mode
- clear effect on mean rotation in the core $\rightarrow r/a = 0.6$
- no modification for $r/a < 0.6$

- Purely diffusive – convective transport $\Rightarrow V_{\text{core}} \propto V_{\text{edge}}$
  $\rightarrow$ confirms the presence of residual Reynolds stress

[Abiteboul et al., submitted to PPCF]
Large-scale avalanches transport heat \textit{and} momentum in the steady-state regime

- Fronts are propagating in both directions (heat flux is always positive)
- Avalanches transport both heat \textit{and} momentum \((\text{propagation } v \lesssim \rho_* v_T)\)
- Strong \textbf{correlation} between heat flux and Reynolds stress \((> 0.6)\)
Similar statistics for turbulent heat flux and Reynolds stress

- Statistical distributions for: flux - $<\text{flux}>_t$
- In the steady-state regime, approx. $7 \times 10^4$ points for each distribution

Results compared with XGC1 simulations [Ku, Abiteboul, Diamond et al., Nucl. Fus. 2012]
Similar statistics for turbulent heat flux and Reynolds stress

- Statistical distributions for: flux - $<\text{flux}>_t$
- In the steady-state regime, approx. $7 \times 10^4$ points for each distribution

- Similar distributions when normalizing to standard deviation
- Strongly non-Gaussian statistics, large tails in the distributions
  - Heat flux: skewness $\simeq 0.8$, kurtosis $\simeq 1.7$
  - Reynolds stress: skewness $\simeq 0.8$, kurtosis $\simeq 1.5$

- Results compared with XGC1 simulations
Symmetry breaking mechanisms responsible for intrinsic rotation generation

- Several mechanisms proposed for symmetry breaking:
  - up-down asymmetry of magnetic configuration [Camenen PRL’09]
  - radial electric field shear $E'_r$ [Dominguez PoF’94; Gürcan PoP’07]
  - turbulence intensity gradient $I'$ [Gürcan PoP’10]

- Correlation of the mechanisms with the Reynolds stress

- Strong correlation locally for the considered symmetry breakers. Similar results with various codes in [Kwon 2011; Ku, Abiteboul et al. 2012]
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Brief overview of theoretical predictions

\[ E_r - V_\varphi B_\theta + V_\theta B_\varphi = \nabla P/Zne \]  
(radial force balance)

- **Axisymmetry** → degeneracy between \( E_r \) and \( V_\varphi B_\theta \)
- **Non-axisymmetric \( B \) → neoclassical friction on \( V_\varphi \)**

Derivation based on extremum of entropy production rate

\[ V_{\varphi}^{\text{neo}} = k_T \frac{\partial_r T}{eB_\theta} \]

⇒ **counter-current** toroidal rotation
where \( k_T \) depends on ripple amplitude and mode number, collisionality, aspect ratio etc.

[e.g. Garbet et al., Phys. Plasmas 2010]

- \( k_T \) can only be estimated analytically in a number of limit cases
Brief overview of theoretical predictions

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Gyrokinetic simulations are necessary as

- Several regimes coexist on a single flux-surface
- Competition between neoclassical friction and turbulence?
Choice of toroidal field ripple perturbation for the simulations \[ \delta B = \delta \cos(N\varphi) \]

- Tore Supra ripple experiments: \( \delta : 0.5\% \rightarrow 5\% \) with \( N = 18 \)
  - local trapping may play a role for large ripple amplitude
  - elsewhere, \( \frac{\delta}{\epsilon} \gg (Nq)^{-3/2} \) and \( \nu_* \gg Nq \left( \frac{\delta}{\epsilon} \right)^2 \)
  \[ \Rightarrow \text{“ripple-plateau collisional” regime: } k_T = 1.67, \text{ damping rate } \propto N\delta^2 \]

[Fenzi, Nuc.Fus. 2011]
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  [Fenzi, Nuc.Fus. 2011]

- **GYSELA simulations** (constant $\delta$): $\delta : 0.5\% \rightarrow 2\%$ with $N = 8$
  - for $\delta = 0.5\%$: small trapping region elsewhere ripple-plateau collisional
  $\Rightarrow$ similar to experiments
  - increase $\delta \rightarrow$ more local trapping (+ ripple-plateau weakly collisional regime?)

- **Bottom-line**: no theoretical prediction in global geometry but counter-current rotation, damping rate increases with $\delta$
Ripple implemented as perturbation to the Hamiltonian $\delta H = \mu \delta B ||$

$$||B_{eq} + \delta B|| \approx (B_{eq}^2 + 2B_{eq} \cdot \delta B)^{1/2} \approx B_{eq} + b_{eq} \cdot \delta B$$

- $\rho_* = 1/150$, $\nu_* = 0.2$
Turbulent simulations including toroidal field ripple in GYSELA

- Ripple implemented as perturbation to the Hamiltonian $\delta H = \mu \delta B_{||}$
  
  $$||B_{eq} + \delta B|| \simeq (B_{eq}^2 + 2B_{eq} \cdot \delta B)^{1/2} \simeq B_{eq} + b_{eq} \cdot \delta B$$

- $\rho_* = 1/150$, $\nu_* = 0.2$, ripple $= \delta \cos(N\varphi)$ with $\delta = 10^{-2}$, $N = 8$

> modes with low $m$ and $n = N, 2N, 3N...$ in FFT of $\delta \Phi$

(resolution in $\varphi$ limits the ripple mode number accessible in the simulations)
Competition between turbulent and neoclassical momentum transport

▶ Time evolution of the neoclassical ion heat diffusivity
  ▶ Modification of neoclassical equilibrium ⇒ rapid transient
  ▶ Neoclassical diffusivity increases with $\delta$

$\delta = 2\%$
$\delta = 0.5\%$
$\delta = 1\%$

$\delta = 5 \times 10^{-3}$: no measurable effect of TF ripple on mean $V_{\parallel}$

Higher ripple: neoclassical friction competes with turbulence

Results consistent with Tore Supra ripple experiments [Fenzi, NF 2011]
Competition between turbulent and neoclassical momentum transport

Time evolution of the neoclassical ion heat diffusivity
- Modification of neoclassical equilibrium \( \Rightarrow \) rapid transient
- Neoclassical diffusivity increases with \( \delta \)

Competition between turbulent and neoclassical momentum transport
- \( \delta = 5.10^{-3} \): no measurable effect of TF ripple on mean \( V_{||} \)
- higher ripple: neoclassical friction competes with turbulence
Competition between turbulent and neoclassical momentum transport

- Time evolution of the neoclassical ion heat diffusivity
  - Modification of neoclassical equilibrium ⇒ rapid transient
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- Competition between turbulent and neoclassical momentum transport
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- Results consistent with Tore Supra ripple experiments [Fenzi, NF 2011]
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Key mechanisms responsible for toroidal momentum transport

- Turbulence generates toroidal rotation in the absence of sources
- Neoclassical ripple-driven rotation can compete with this effect
- The resulting profile depends on boundary conditions (edge flows)
Main results of the thesis work

- Numerical developments for the GYESELA code
  [Abiteboul et al., ESAIM : Proceedings 2011]

- Analytical derivation of a local momentum conservation equation
  + Numerical test of this conservation [Abiteboul et al., Phys.Plasmas 2011]

- Statistical analysis of turbulent heat and momentum transport
  [Abiteboul et al., 2011 IAEA-TM Theory of Plasma Instabilities, oral presentation]
  + Comparisons with XGC1p code [Ku, Abiteboul et al., Nuc.Fus. 2012]

- Study on the impact of boundary conditions on core rotation
  [Abiteboul et al., submitted to Plas.Phys.Control.Fus.]

- Simulations including turbulent and neoclassical momentum transport
  [Abiteboul et al., 2012 EU-US TTF workshop, oral presentation]
Thank you for your attention!
No-slip boundary conditions lead to *net* toroidal rotation

- No-slip conditions $\Rightarrow V_\parallel = 0$ but no condition on the flux
- The *ad hoc* diffusion dissipates momentum transported to the edge

 Leads to *net rotation* $\Rightarrow$ role of boundary conditions?
Rotation profile develops on time-scale $\sim$ confinement time (usually $>\sigma$ simulation time for small $\rho_\ast$)

[Ku et al., Nuc. Fus. 2012]
Statistical analysis: Reynolds stress vs. $\partial_t \mathcal{L}_\phi$

- In terms of statistics: $\partial_t \mathcal{L}_\phi \sim \nabla \Pi^r_{\phi}$
- Very different statistics for $\partial_t \mathcal{L}_\phi$ and $\Pi^r_{\phi}$

Reynolds stress: skewness $\simeq 0.8$, kurtosis $\simeq 1.5$
$\partial_t \mathcal{L}_\phi$: skewness $\simeq 0.1$, kurtosis $\simeq 0.5$
Statistical analysis: Reynolds stress vs. $\partial_t L_\varphi$

- in terms of statistics: $\partial_t L_\varphi \sim \nabla \Pi_\varphi$
- Very different statistics for $\partial_t L_\varphi$ and $\Pi_\varphi$

Reynolds stress:
- skewness $\simeq 0.8$, kurtosis $\simeq 1.5$
- $\partial_t L_\varphi$:
  - skewness $\simeq 0.1$, kurtosis $\simeq 0.5$

Possible interpretation: large $\Pi_\varphi$ events have larger radial extent

Open issue: local vs. flux-surface averaged fluxes?
Gyro-Bohm scaling of turbulent transport is recovered

- Do the large-scale avalanches break the gyro-Bohm scaling?
- Heat transport: gyro-Bohm scaling for small values of $\rho_*$
  

Gyro-Bohm estimates for the Reynolds stress:

$$
\frac{\Pi_r}{Rv_T^2} \propto \rho_*^2
$$

$$
\frac{\partial \Pi_r}{Rv_T^2/a} \propto \rho_*
$$

- Scaling obtained from estimate of RMS fluctuations
- Slightly “worse” than gyro-Bohm scaling obtained $\propto \rho_*^{0.7}$
  
  $\rightarrow$ Needs to be confirmed with additional simulations (other codes?)

- Interpretation? Meso-scale size of the avalanches [Dif-Pradalier PRE 2010]