Cosmologie et modifications à grandes distances de l’interaction gravitationnelle

Cosmology and modifications of gravity at large distances

Riad ZIOUR
APC, Université Paris 7 Diderot

Sous la direction de
C. Deffayet

Laboratoire APC, Mardi 19 Janvier 2010
• The acceleration of the Universe: Dark Energy or new gravity?

• Massive gravity and the Vainshtein’s mechanism

• Observations: the confrontation with reality
The acceleration of the Universe: Dark Energy or new gravity?
The expansion of the Universe

• What does “expansion” mean?
The expansion of the Universe

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The expansion of the Universe
The expansion of the Universe

- What does “expansion” mean?

Two types of units of distances:
The expansion of the Universe

- What does “expansion” mean?

Two types of units of distances:

✓ grid units
The expansion of the Universe

- What does “expansion” mean?

Two types of units of distances:

- grid units
- physical units
The expansion of the Universe

- What does “expansion” mean?

Two types of units of distances:

✓ grid units

The metric $g_{\mu\nu}$

✓ physical units
The expansion of the Universe

• What does “expansion” mean?

Two types of units of distances:

✓ grid units

The metric $g_{\mu\nu}$ encodes the space-time

✓ physical units

encodes the space-time
The acceleration of the Universe
The acceleration of the Universe

SN Ia

BAO

CMB
• Very different probes: SN Ia, BAO, CMB

The acceleration of the Universe
The acceleration of the Universe

- Very different probes: SN Ia, BAO, CMB

They all show that the Universe is accelerating!
The source of the acceleration
The source of the acceleration

- Einstein’s equations
  \[ G_{\mu\nu} = 8\pi G \, T^{(m)}_{\mu\nu} - \Lambda g_{\mu\nu} \]
The source of the acceleration

- Einstein’s equations
  \[ G_{\mu\nu} = 8\pi G \ T^{(m)}_{\mu\nu} - \Lambda g_{\mu\nu} \]

Geometry
The source of the acceleration

- Einstein’s equations

\[ G_{\mu\nu} = 8\pi G \, T^{(m)}_{\mu\nu} - \Lambda g_{\mu\nu} \]

Geometry \quad \Rightarrow \quad \text{Matter}
Einstein’s equations

\[ G_{\mu\nu} = 8\pi G \, T^{(m)}_{\mu\nu} + \Lambda g_{\mu\nu} \]

- Geometry
- Matter
- Dark Energy
The source of the acceleration

- Einstein’s equations

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Geometry

Matter

Dark Energy

Percival et al. '09
The source of the acceleration

- Einstein’s equations

\[ G_{\mu\nu} = 8\pi G \, T^{(m)}_{\mu\nu} + \Lambda g_{\mu\nu} \]

Note the coincidence problem:

- Geometry
- Matter
- Dark Energy

\[ \Omega_\Lambda, \Omega_m \]

\[ w = -1 \text{ fixed} \]

Percival et al. ‘09
The source of the acceleration

- Einstein’s equations

\[ G_{\mu\nu} = 8\pi G \, T_{\mu\nu}^{(m)} - \Lambda g_{\mu\nu} \]

- Geometry

- Matter

- Dark Energy

- Coincidence problem

- Fine tuning problem

Percival et al. ’09
The source of the acceleration

- Einstein’s equations

\[ G_{\mu\nu} = 8\pi G T^{(m)}_{\mu\nu} - \Lambda g_{\mu\nu} \]

Geometry

Matter

Dark Energy

- Coincidence problem
- Fine tuning problem

What is the nature of the Dark Energy?

Percival et al. '09
Modified gravity theories
Modified gravity theories

Aim: to generalize Einstein’s equations to explain the acceleration of the Universe without Dark Energy
Modified gravity theories

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How: by adding extra fields
Modified gravity theories

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Problem: we want to recover GR at short distances (inside the solar system) while modifying gravity at large distances
Modified gravity theories

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Suppressing the role of these extra fields at short distances:
Modified gravity theories

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Suppressing the role of these extra fields at short distances:

➡️ Chameleon mechanism
Modified gravity theories

Aim: to generalize Einstein’s equations to explain the acceleration of the Universe without Dark Energy

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Problem: we want to recover GR at short distances (inside the solar system) while modifying gravity at large distances

Suppressing the role of these extra fields at short distances:

- Chameleons
- Vainshtein’s mechanism
Modified gravity theories: a (short) review
Modified gravity theories: a (short) review

Chameleon mechanism

Khoury & Veltman ‘00
Modified gravity theories: a (short) review

**Chameleon mechanism**  
- scalar-tensor theories (and f(R) models)

*Khoury & Veltman ‘00*
Modified gravity theories: a (short) review

- **Chameleon mechanism**
  - scalar-tensor theories (and f(R) models)  
    - Khoury & Veltman '00

- **Vainshtein’s mechanism**
  - Vainshtein '72
  - Deffayet et al. '02
Modified gravity theories: a (short) review

- Chameleonic mechanism
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- Vainshtein’s mechanism
  - Massive Gravity

  *Khoury & Veltman ‘00
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[Dvali, Gabadadze and Porrati '00]

[Khouri & Veltman '00]

[Vainshtein '72]

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- 
  ✓ matter stays on the brane

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  - matter stays on the brane
  - $5D + 4D$ gravity

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- ✓ matter stays on the brane
- ✓ 5D + 4D gravity
- ✓ can provide an explanation for the cosmic acceleration

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  ✓ matter stays on the brane
  ✓ 5D + 4D gravity
  ✓ can provide an explanation for the cosmic acceleration
  ✓ 5D massless graviton = infinite tower of 4D gravitons

**References**

- Dvali, Gabadadze and Porrati ‘00
- Khoury & Veltman ‘00
- Vainshtein ‘72
- Deffayet et al. ‘02
Publications
The Vainshtein mechanism in the Decoupling Limit of massive gravity

Recovering general relativity from massive gravity

• E. Babichev, C. Deffayet, RZ, in preparation [arXiv:1001.xxxx]
Spherically symmetric solutions of massive gravity and the Vainshtein mechanism

Awarded a honorable mention at the 2009 Awards for “Essays on gravitation”
k-Mouflage gravity

Magnification bias corrections to galaxy-lensing cross-correlations
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Observations

  *Magnification bias corrections to galaxy-lensing cross-correlations*
Massive gravity
& the Vainsthein’s mechanism
• Naive idea: give a mass to the graviton with \( m \sim 1/H_0 \)

• Massive gravity = basic ingredient for models with extra-dimensions (like DGP):
  - in 5D, the gravity is massless,
  - viewed from the 4D brane, the gravity is mediated by a tower a massive Kaluza-Klein gravitons with no massless mode.

• Pathologies: non-bounded by below Hamiltonian and ghosts
  [Boulware & Deser ’72; Creminelli, Nicolis, Papucci, Trincherini ’05; Deffayet & Rombouts ‘05], singular solutions (?) [Damour et al. ‘03], but interesting toy model for more realistic theories.
What is Massive gravity?
What is Massive gravity?

- The quadratic action for the massive graviton:

\[ S = \frac{M_P^2}{2} \int d^4 x \left( "H \partial^2 H + ..." - \frac{m^2}{4} [H_{\mu\nu} H^{\mu\nu} - (H^\mu_\mu)^2] \right) + \int d^4 x \frac{1}{2} T_{\mu\nu} H^{\mu\nu} \]

- \( f \): background metric (often flat)
- \( H_{\mu\nu} \): spin 2 excitation over \( f \)
- Kinetic term
- Pauli-Fierz
- Mass term
- Matter coupling
What is Massive gravity?

- The quadratic action for the massive graviton:

\[ S = \frac{M_P^2}{2} \int d^4x \left( \frac{1}{2} (\nabla \partial H^2 + \ldots) - \frac{m^2}{4} \left[ H_{\mu\nu} H^{\mu\nu} - (H^\mu_\mu)^2 \right] \right) + \int d^4x \frac{1}{2} T_{\mu\nu} H^{\mu\nu} \]

- Kinetic term
- Pauli-Fierz Mass term
- Matter coupling
- Scalar density

- Non-linear completion: dynamical metric \( g = f + H \)

\[ S = \frac{M_P^2}{2} \int d^4x \left( \sqrt{-g} R[g] - \frac{m^2}{4} \nabla^{(a)} [g^{-1} f] \right) + S_m [g] \]
What is Massive gravity?

• The quadratic action for the massive graviton:

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- **Kinetic term**
- **Pauli-Fierz Mass term**
- **Matter coupling**

• Non-linear completion: dynamical metric \( g = f + H \)

\[ S = \frac{M_P^2}{2} \int d^4x \left( \sqrt{-g} R[g] - \frac{m^2}{4} \mathcal{V}^{(a)}[g^{-1}f] \right) + S_m[g] \]

- **Scalar density**

• Examples:

\[
\mathcal{V}^{(BD)}[g^{-1}f] = \sqrt{-f} H_{\mu\nu} H_{\sigma\tau} \left( f^{\mu\sigma} f^{\nu\tau} - f^{\mu\nu} f^{\sigma\tau} \right) \\
\mathcal{V}^{(AGS)}[g^{-1}f] = \sqrt{-g} H_{\mu\nu} H_{\sigma\tau} \left( g^{\mu\sigma} g^{\nu\tau} - g^{\mu\nu} g^{\sigma\tau} \right)
\]

Arkani-Hamed, Georgi and Schwartz ‘03

Boulware and Deser ‘72
Static Spherically Symmetric solutions
Static Spherically Symmetric solutions

- “Schwarzschild Gauge” (bi-diagonal, asymptotically flat):

\[
\begin{align*}
    g_{\mu\nu} \, dx^\mu \, dx^\nu &= -e^{\nu(R)} \, dt^2 + e^{\lambda(R)} \, dR^2 + R^2 \, d\Omega^2 : \text{"Schwarzschild" like} \\
    f_{\mu\nu} \, dx^\mu \, dx^\nu &= -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 \, e^{-\mu(R)} \, dR^2 + e^{-\mu(R)} \, R^2 \, d\Omega^2 : \text{flat}
\end{align*}
\]
Static Spherically Symmetric solutions

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\[ f_{\mu\nu}dx^\mu dx^\nu = -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)}dR^2 + e^{-\mu(R)}R^2d\Omega^2 : \text{flat} \]

- Equations of motion:

\[ e^{\nu-\lambda}\left(\frac{\lambda'}{R} + \frac{1}{R^2}(e^\lambda - 1)\right) = 8\pi G_N (T^g_{tt} + \rho e^\nu) , \]
\[ \frac{\nu'}{R} + \frac{1}{R^2}(1 - e^\lambda) = 8\pi G_N (T^g_{RR} + P e^\lambda) , \]
\[ \nabla^\mu T^g_{\mu R} = 0. \]
Static Spherically Symmetric solutions

- “Schwarzschild Gauge” (bi-diagonal, asymptotically flat):

\[
g_{\mu\nu} dx^\mu dx^\nu = -e^{\nu(R)} dt^2 + e^{\lambda(R)} dR^2 + R^2 d\Omega^2 \quad \text{"Schwarzschild" like}
\]

\[
f_{\mu\nu} dx^\mu dx^\nu = -dt^2 + \left(1 - \frac{R\mu'(R)}{2}\right)^2 e^{-\mu(R)} dR^2 + e^{-\mu(R)} R^2 d\Omega^2 \quad \text{flat}
\]

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\]

\[
\frac{\nu'}{R} + \frac{1}{R^2} \left(1 - e^\lambda\right) = 8\pi G_N \left(T_{RR}^g + P e^{\lambda}\right),
\]

\[
\nabla^\mu T_{\mu R}^g = 0.
\]

Is it possible to find a solution regular everywhere?
Solution far from the source (I)
Solution far from the source (I)

Linear massive gravity
Solution far from the source (I)

- Expansion in the Newton’s constant:
\[ \lambda = \lambda_0 + \lambda_1 + \ldots \text{ etc.}, \text{ with } \lambda_i, \nu_i, \mu_i \propto G_{N}^{i+1} \]
Solution far from the source (I)

Linear massive gravity

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  \[ \lambda = \lambda_0 + \lambda_1 + \ldots \ \text{etc., with} \ \lambda_i, \nu_i, \mu_i \propto G_N^{i+1} \]

• Linearized solution:

\[
\begin{align*}
\nu_0 &= -c \times \frac{R_s}{R} \ e^{-mR} \\
\lambda_0 &= c \times \frac{R_s}{2R} (1 + mR) e^{-mR} \\
\mu_0 &= c \times \frac{R_s}{2(mR)^2 R} \ (1 + mR + (mR)^2) \ e^{-mR}
\end{align*}
\]
Solution far from the source (I)

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  \mu_0 = C \times \frac{R_S}{2(mR)^2R} (1 + mR + (mR)^2) e^{-mR}
  \]

- for \( R \ll m^{-1} \):
  \[ \lambda_0 = \frac{C_1}{2R}, \quad \nu_0 = -\frac{C_1}{R}, \quad \mu_0 = \frac{1}{(mR)^2} \frac{C_1}{2R}. \]
Solution far from the source (I)

• Expansion in the Newton’s constant:
  \[ \lambda \ = \ \lambda_0 + \lambda_1 + \ldots \ 	ext{etc., with} \ \lambda_i, \nu_i, \mu_i \propto G_N^{i+1} \]

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\end{align*}
\]

• for \( R \ll m^{-1} \):
  \[\lambda_0 = \frac{C_1}{2R}, \quad \nu_0 = -\frac{C_1}{R}, \quad \mu_0 = \frac{1}{(mR)^2} \frac{C_1}{2R}.\]

Iwasaki ’70, van Dam, Veltman ’70, Zakharov ’70
Solution far from the source (II)

Linear massive gravity

Non GR
• Nonlinear corrections:

\[
\nu = -\frac{2}{3} \frac{R_S}{R} + \frac{R_S^2}{R^2} \frac{n_1}{(mR)^4} + O(R_S^3)
\]

\[
\lambda = \frac{1}{3} \frac{R_S}{R} + \frac{R_S^2}{R^2} \frac{l_1}{(mR)^4} + O(R_S^3)
\]

\[
\mu = \frac{1}{3(mR)^2} \frac{R_S}{R} + \frac{R_S^2}{R^2} \frac{m_1}{(mR)^6} + O(R_S^3)
\]
Solution far from the source (II)

Nonlinear corrections:

\[
\nu = -\frac{2}{3} \frac{R S}{R} + \frac{R_S^2}{R^2} \frac{n_1}{(mR)^4} + \mathcal{O}(R_S^3)
\]

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\]

Relevant at

\[R_V = \left(\frac{R_S}{m^4}\right)^{1/5}\]

New scale:
Vainshtein’s radius
Solution far from the source (II)

Linear massive gravity

Non GR

Relevant at $R_V$

New scale: Vainshtein’s radius

What happens inside the Vainshtein’s radius?

- Nonlinear corrections:

\[
\begin{align*}
\nu & = -\frac{2}{3} \frac{R_S}{R} + \frac{R_S^2}{R^2} \frac{n_1}{(mR)^4} + \mathcal{O}(R_S^3) \\
\lambda & = \frac{1}{3} \frac{R_S}{R} + \frac{R_S^2}{R^2} \frac{l_1}{(mR)^4} + \mathcal{O}(R_S^3) \\
\mu & = \frac{1}{3(mR)^2} \frac{R_S}{R} + \frac{R_S^2}{R^2} \frac{m_1}{(mR)^6} + \mathcal{O}(R_S^3)
\end{align*}
\]
Solution close to the source

Linear massive gravity

$R_V$

Non GR

$R$
Solution close to the source

- The functions $\lambda, \nu, \mu$ are expanded in $m$: $f(R) = \sum_{n=0}^{\infty} m^{2n} f_n(R)$
Solution close to the source

- The functions $\lambda, \nu, \mu$ are expanded in $m$:
  \[ f(R) = \sum_{n=0}^{\infty} m^{2n} f_n(R) \]
- Order $0 = \text{GR}$:
  \[ \lambda_0 = -\nu_0 = -\ln \left( 1 - \frac{R_S}{R} \right) \]
The functions $\lambda, \nu, \mu$ are expanded in $m$:

$$f(R) = \sum_{n=0}^{\infty} m^{2n} f_n(R)$$

- **Order 0 = GR:**
  $$\lambda_0 = -\nu_0 = -\ln \left(1 - \frac{R_S}{R}\right)$$

- Expansion:
  $$\nu = -\frac{R_S}{R} + n_1 (mR)^2 \sqrt{\frac{R_S}{R}} + \mathcal{O}(m^4)$$
  $$\lambda = \frac{R_S}{R} + l_1 (mR)^2 \sqrt{\frac{R_S}{R}} + \mathcal{O}(m^4)$$
  $$\mu = m_0 \sqrt{\frac{R_S}{R}} + m_1 (mR)^2 + \mathcal{O}(m^4)$$

Relevant at $R_V = \left(\frac{R_S}{m^4}\right)^{1/5}$
Existence of a regular solution?

Non-perturbative regime, General Relativity

\[ R_V \ll R \]

\[ R_V = \left( \frac{R_S}{m^4} \right)^{1/5} \]

linear regime, non-General Relativity

Existence of a regular solution?
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Is the Vainshtein’s mechanism valid?

Is it possible to find a solution regular everywhere?

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$R_V \ll R$

$R_V = \left( \frac{R_S}{m^4} \right)^{1/5}$

linear regime, non-General Relativity

$R \ll R_V$

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Vainshtein ’72, Boulware & Deser ’72, Jun & Kang ’86

Damour, Kogan, Papazoglou ’03
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$R_V \ll R$

$R_V = \left( \frac{R_S}{m^4} \right)^{1/5}$

$R \ll R_V$

Singularity

Vainshtein ’72, Boulware & Deser ’72, Jun & Kang ’86

Damour, Kogan, Papazoglou ’03
Our approach
Our approach

Decoupling Limit

E. Babichev, C. Deffayet, RZ,
JHEP 05 (2009) 098
Our approach

Decoupling Limit

E. Babichev, C. Deffayet, RZ, JHEP 05 (2009) 098

Analytical expansions + adapted numerical methods
Our approach

Decoupling Limit

E. Babichev, C. Deffayet, RZ,
JHEP 05 (2009) 098

General case

E. Babichev, C. Deffayet, RZ,
PRL 103 (2009) 201102

E. Babichev, C. Deffayet, RZ,
in preparation (2010)

Analytical expansions + adapted numerical methods
The Decoupling Limit: definition
The Decoupling Limit: definition

Main idea: to focus on the physics around the Vainshtein scale $R_V$
The Decoupling Limit: definition

Main idea: to focus on the physics around the Vainshtein scale $R_V$

- Remove the GR nonlinearities: $M_P \to \infty$, $M/M_P \sim \text{const}$
Main idea: to focus on the physics around the Vainshtein scale $R_V$

- Remove the GR nonlinearities: $M_P \to \infty$, $M/M_P \sim \text{const}$
- Remove the Yukawa behavior: $m \to 0$
The Decoupling Limit: definition

Main idea: to focus on the physics around the Vainshtein scale $R_V$

- Remove the GR nonlinearities: $M_P \to \infty$, $M/M_P \sim \text{const}$
- Remove the Yukawa behavior: $m \to 0$
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Decoupling Limit

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Arkani-Hamed, Georgi and Schwartz '03
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Other point of view:

\[
h_{\mu\nu} \to h_{\mu\nu} - \partial_\mu A_\nu - \partial_\nu A_\mu - 2\partial_\mu \partial_\nu \phi - \partial_\mu A_\sigma \partial_\nu A^\sigma - \partial_\mu \partial_\sigma \phi \partial_\nu \partial^\sigma \phi - \partial_\nu A^\sigma \partial_\mu \partial_\sigma \phi - \partial_\mu A^\sigma \partial_\nu \partial_\sigma \phi
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$\Lambda = \text{strong coupling scale of } \phi$
The Decoupling Limit: equations
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\[ e^{\nu-\lambda} \left( \frac{\lambda'}{R} + \frac{1}{R^2} (e^\lambda - 1) \right) = 8\pi G_N (T^g_{tt} + \rho e^\nu), \]
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Equations of motion

\[ g_{\mu\nu} dx^\mu dx^\nu = -e^\nu dt^2 + e^\lambda dR^2 + R^2 d\Omega^2, \]
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- but \( T^g_{tt} = m^2 M_P^2 \ f_t, \quad T^g_{RR} = m^2 M_P^2 \ f_R, \quad \nabla^\mu T^g_{\mu R} = -m^2 M_P^2 f_g, \)

with

\[ f_t = \frac{e^{-\lambda - 2\mu}}{4} \times \left[ (3e^{\mu + \nu} + e^\mu - 2e^\nu) \left( 1 - \frac{R\mu'}{2} \right)^2 + e^\lambda (2e^\mu - e^\nu) - 3e^{\lambda + \mu} (2e^{\mu + \nu} + e^\mu - 2e^\nu) \right], \]

\[ f_R = \frac{e^{-\nu - 2\mu}}{4} \times \left[ (3e^{\mu + \nu} - e^\mu - 2e^\nu) \left( 1 - \frac{R\mu'}{2} \right)^2 + e^\lambda (2e^\mu + e^\nu) - 3e^{\lambda + \mu} (-2e^{\mu + \nu} + e^\mu + 2e^\nu) \right], \]

\[ f_g = -\left( 1 - \frac{R\mu'}{2} \right) \frac{e^{-\lambda - 2\mu - \nu}}{8R} \times \left[ 8(e^\lambda - 1) (3e^{\mu + \nu} - e^\mu - e^\nu) + 2R \left( (3e^{\mu + \nu} - 2e^\nu) (\lambda' + 4\mu' - \nu') - e^\mu (\lambda' + 4\mu' + \nu') \right) + R^2 \left( (3e^{\mu + \nu} - 2e^\nu) (\lambda'\mu' - 2\mu'' + \mu'\nu' + (\mu')^2) - e^\mu (\lambda'\mu' - 2\mu'' + \mu'\nu' + (\mu')^2) \right) \right]. \]

For AGS potential
The Decoupling Limit: equations

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- R^2 \left( (3e^{\mu+\nu} - 2e^\nu) (\lambda' + 2\mu'' - \mu' \nu' + (\mu')^2) - e^\nu (\lambda' + 2\mu'' + \mu' \nu' + (\mu')^2) - 2e^\nu (\mu')^2 \right) \right]. \]
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\[ \frac{\lambda}{R^2} = \frac{\nu'}{2R} + Q(\mu), \quad \text{with} \quad Q = -\frac{1}{2} \left( \frac{\mu'^2}{2} + \mu \mu'' + \frac{4\mu \mu'}{R} \right). \]

E. Babichev, C. Deffayet, RZ, JHEP 05 (2009) 098
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\[ 2Q(\mu) + \frac{3}{2} m^2 \mu = \frac{R_S}{R^3} \]

Only one equation!

E. Babichev, C. Deffayet, RZ, JHEP 05 (2009) 098
The Decoupling Limit: solution
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\[ \mu \simeq \frac{2}{3} \frac{R_s}{m^2 R^3} \]
The Decoupling Limit: solution

\[ \mu \simeq \sqrt{\frac{8 R_S}{9R}} \]

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The Decoupling Limit: solution
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Linear regime: vDVZ
The Decoupling Limit: solution

Linear regime: $v_{\text{DVZ}}$

Vainshtein regime: $\sim \text{GR}$
The Decoupling Limit: solution

Linear regime:

Vainshtein regime:

\[ \text{Vainshtein regime: GR} \]

\[ \lambda, \text{DL} \quad \cdots \quad -\nu, \text{DL} \]

\[ \nu_{\text{VDVZ}} \]

- Rôle of the nonlinearities not taken into account by the DL?
The Decoupling Limit: solution

Domain of validity of the DL: between $R = 0$ and $R = m^{-1}$ for non compact sources

Rôle of the nonlinearites not taken into account by the DL?
The Decoupling Limit: solution

- Rôle of the nonlinearites not taken into account by the DL?
  - Domain of validity of the DL: between $R = 0$ and $R = m^{-1}$ for non compact sources

Is it possible to generalize this solution?
The full system solution (I)
The full system solution (I)
The full system solution (I)
The full system solution (I)
The full system solution (I)

Linear regime:

DL regime

Yukawa
The full system solution (I)

Vainshtein regime:
\( \sim \text{GR} \)

Linear regime:

\( \lambda, \text{DL} \)

\(-\nu, \text{DL}\)

\(\lambda, \text{full}\)

\(-\nu, \text{full}\)

DL regime

Yukawa

\(R/R_V\)
The full system solution (II)
The full system solution (II)
The full system solution (II)

\[ \nu \text{ inside} \]
Difference between MG and GR:

\[ \Delta \lambda \sim \frac{\sqrt{2}}{3} (mR)^2 \sqrt{\frac{R_S}{R}} \]
The full system solution (II)

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Excellent agreement with numerics
Analytical and numerical tools
Analytical and numerical tools

• Series expansions: far and inside the source
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• Cauchy problem at infinity: evidence for the existence of infinite number of solutions with the same asymptotic behavior (collaboration with J. Ecalle)
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• Simplified model (weak field)
Analytical and numerical tools

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• Simplified model (weak field)

• Numerics: relaxation method
  ✓ Decoupling Limit
  ✓ Full system (Decoupling Limit as a starting guess)
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All the methods agree: robustness of the results
Beyond Massive Gravity
Beyond Massive Gravity

- $k$-Mouflage models

Beyond Massive Gravity

- $k$-Mouflage models

\[ S = M_P^2 \int d^4x \sqrt{-g} \left( \frac{R}{2} + \frac{\gamma}{2} m^2 \phi R + m^2 H(\phi) \right) + S_m \]

Beyond Massive Gravity

• *k*-Mouflage models

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with

\[ H(\phi)_{MG} = \frac{\alpha}{2} (\Box \phi)^3 + \frac{\beta}{2} (\Box \phi_{;\mu\nu} \phi^{;\mu\nu}), \]
\[ H(\phi)_{DGP} = m^2 \Box \phi \phi_{;\mu} \phi^{;\mu}, \]
\[ H(\phi)_K = K(X), \text{ with } X = m^2 \phi_{;\mu} \phi^{;\mu} \]

E. Babichev, C. Deffayet, RZ, "*k*-Mouflage gravity", IJMP (2010)
Beyond Massive Gravity

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- **k-Mouflage models**

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Beyond Massive Gravity

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The Vainshtein mechanism is valid for a broad class of models
Observational constraints
Observational probes
Observational probes

• Laboratory and Solar system tests, and PPN parameters
Observational probes

- Laboratory and Solar system tests, and PPN parameters
- Galactic scales
Observational probes

- Laboratory and Solar system tests, and PPN parameters
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- Cosmological evolution
  - SN Ia
  - CMB
  - BAO
  - galaxy density
  - velocity field
  - weak lensing
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Combination of all these observations: best constraints
Weak lensing
Weak lensing

- The presence of matter bends the light rays trajectories.
Weak lensing

- The presence of matter bends the light rays trajectories

\[ \gamma = 0, \kappa \quad \kappa = 0, \gamma \]

magnification \quad shear

- Galaxy-lensing correlations

\[ < \delta_g(z_f) \kappa(z_b) > \quad \Rightarrow \quad \text{Informations on the foreground matter power spectrum} \]
Weak lensing

• The presence of matter bends the light rays trajectories

\[ \kappa \]

\[ \gamma = 0, \kappa \]

magnification

\[ 1 \]

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shear

• Galaxy-lensing correlations

\[ < \delta_g(z_f) \kappa(z_b) > \implies \text{Informations on the foreground matter power spectrum} \]

\[ \delta_n = \delta_g + \delta_\mu \]

Corrections that need to be taken into account 

\[ RZ, L. Hui \]

PRD '08
Conclusions
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• New results on Massive Gravity
  ✓ Decoupling Limit for Spherically Symmetric solutions
  ✓ demonstration of the existence of a solution regular everywhere
  ✓ demonstration of the validity of the Vainshtein mechanism
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• A lot of interesting open questions
  ✓ compact sources?
  ✓ stability of the found solutions
  ✓ phenomenology of the $k$-Mouflague theories
Thank you