Nucleon structure by lattice QCD computations with twisted mass fermions

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Outline

1. Introduction
2. Form Factors and experiments
3. Lattice QCD
4. 2 points function
5. 3 points function
6. Results
7. Conclusion
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1. Introduction
2. Form Factors and experiments
3. Lattice QCD
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5. 3 points function
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7. Conclusion
Gluon fields represented by: $A^i_\nu(x)$.
Gluonic tensor: $F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g f^{ijk} A^j_\mu A^k_\nu$.
Quarks field: $\psi^a_{\alpha f}(x)$ of flavour f, color a and spin indice $\alpha$. 

Standard Model and particles
Quantum ChromoDynamics

- $S_{QCD} = \int d^4x \left( -\frac{1}{4} (F_{\mu\nu}^i)^2(x) + \bar{\psi}(x)(i\gamma^\mu D_\mu - m_q)\psi(x) \right)$, with $D_\mu = \partial_\mu - ig A_\mu^i \lambda^i$
- only 2 parameters: $g$ and $m_q$
- Asymptotic freedom at high energy: perturbative calculation
- Non perturbative regime at low energy: need of numerical resolution, i.e. lattice QCD

To check QCD theory at low energy, comparison between LQCD and experiments

Hot topic: structure of nucleon

Figure: Running of the strong coupling constant $\alpha_s(Q) = \frac{g^2(Q)}{4\pi}$
Electric and magnetic form factors

- Elastic electron-proton diffusion
- Depends only on the photon virtuality: 
  \[ Q^2 = -q^2 = -(p' - p)^2 \]

\[ \langle N, p', s' | J_{EM}^\mu (0) | N, p, s \rangle = \bar{u}(p') \left[ \gamma_\mu F_1(Q^2) + i\sigma_{\mu\nu} q^\nu \frac{1}{2M} F_2(Q^2) \right] u(p) \]

**Electric and magnetic Sachs form factors:**

\[ G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2) \]
\[ G_M(Q^2) = F_1(Q^2) + F_2(Q^2) \]

with physical meaning in non-relativist limit of electric/magnetic charge distribution:

\[ \rho(\bar{r}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\bar{q}\bar{r}} \frac{M}{E(\bar{q})} G_E(\bar{q}^2) \]
Electric and magnetic form factors

- For proton at low $Q^2$, $G_E$ and $G_M \sim$:

$$G_D(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\Lambda^2}\right)^2}, \quad \Lambda^2 = 0.71 \text{GeV}^2$$

- $\langle p|V_\mu^3(0)|p\rangle = \langle p|J_\mu^{EM}(0)|p\rangle - \langle n|J_\mu^{EM}(0)|n\rangle$

**Figure:** Experimental values for proton $G_E$ and $G_M$ [Arrington, Melnitchouk and Tjon (2007)]

**Figure:** Experimental values for neutron $G_E$ [BLAS (2008)]

**Figure:** Experimental values for neutron $G_M$ [CLAS (2009)]
Axial and pseudoscalar form factors

- Desintegration of neutron to proton
  \[ \langle p | J_{\mu}^{CC} | n \rangle = \cos \theta_C \langle p | V_\mu^+ - A_\mu^+ | n \rangle \]
- \[ A_\mu^+ = \bar{\psi}_u \gamma_\mu \gamma_5 \psi_d \]
- Isospin symmetry \[ \rightarrow \langle N, \vec{p}', s' | A_\mu^+ | N, \vec{p}, s \rangle \]
  \[ \langle N, \vec{p}', s' | A_\mu^+ | N, \vec{p}, s \rangle = \bar{u}(p') \left[ \gamma_\mu \gamma_5 G_A(Q^2) + i \frac{q_\mu}{2M} \gamma_5 G_P(Q^2) \right] u(p) \]

Figure: Experimental values for \( G_A(Q^2) \) [Bernard, Elouadrhiri and Meissner (2002)]

Figure: Experimental values for \( G_P(Q^2) \) [Bernard, Elouadrhiri and Meissner (2002)]
Introduction

Form Factors and experiments

Lattice QCD

2 points function

3 points function

Results

Conclusion

[Wilson (1974)]
Feynmann path integral

**Goal**

To access matrix element \( \langle N, p', s' | J(0) | N, p, s \rangle \)

calculate 3 points function \( \langle P_{\alpha_f}(x_f)J(z)\bar{P}_{\alpha_i}(0) \rangle \)

Value of an operator given by **Feynmann path integral** (euclidian space-time):

\[
\langle O \rangle = \frac{1}{Z} \int DUD\psi D\bar{\psi} e^{-S_E[\psi, \bar{\psi}, U]} O[\psi, \bar{\psi}, U]
\]

Goal of lattice QCD:
Evaluate numerically this integral with a discretised QCD action and fields on a lattice. Must verify continuum limit =QCD.

Separation of gauge and fermion integration:

\[
S_E = S_G[U] + S_F[U, \psi, \bar{\psi}] \quad \text{with} \quad S_F[U, \psi, \bar{\psi}] = \bar{\psi} D[U] \psi
\]

\[
\langle O \rangle = \frac{1}{Z} \int DU \langle O(\psi, \bar{\psi}) \rangle_U \det D[U] e^{-S_G[U]}
\]
Gauge fields discretisation

4D lattice of size $L^3 \times T$ with lattice spacing $a$

- Gauge fields represented by $SU(3)$ matrix: $U_\mu(x, y) = \mathcal{P} \exp(-ig \int_x^y A_\mu(z)dz)$.
- Gauge fields lay on the link of the lattice.
- **One configuration**: ensemble of the gauge links on all the lattice

**Figure**: Representation of a gauge link
Generation of configurations

- Generate an ensemble of configurations \( U_i \), "measure" \( O \) on each configuration.

- **Average observable over \( N \) configurations:**

\[
\langle O \rangle = \frac{1}{N} \sum_{i=1}^{N} \langle O(\psi, \bar{\psi}) \rangle U_i + O\left(\frac{1}{\sqrt{N}}\right)
\]

- Give **statistical error**: evaluate by **jackknife method** of resampling. (to get rid of correlation between configurations)

- **Difficulty**: include the quark-antiquark pairs in the sea.
  Our case: light quarks in the sea, \( N_f = 2 \).
Quark propagator

- **Wick theorem**: fermionic observables $\langle O(\psi, \bar{\psi}) \rangle_U$ expressed as combinations of **quarks propagators**:
  $$\langle \psi(x_f)\bar{\psi}(x_i) \rangle_U = S(x_f, x_i)U_i$$

  ![Figure: 2 points function](image1)

  ![Figure: 3 points function](image2)

  *Figure: 2 points function*  *Figure: 3 points function*

- Quarks propagator verify:
  $$\sum_y D(x_f, y)_U S(y, x_i)_U = \delta_{x_f, x_i}$$

  where $D$ is the Dirac operator specific to the discretisation scheme.

- By fixing $x_i$, needs to solve a linear system on each configuration.
Twisted mass fermions in the continuum

[Frezzotti, Grassi, Sint and Weisz (2000)]

\[ S^{F}_{\text{QCD}}[\chi, \bar{\chi}, U] = \int d^4x \bar{\psi} [\gamma_\mu D_\mu + m_q] \psi \]

\[ S^{F}_{\text{tm}}[\chi, \bar{\chi}, U] = \int d^4x \bar{\chi} [\gamma_\mu D_\mu + m_q + i\gamma_5\tau_3\mu_q] \chi \]

- **Equivalence between QCD and twisted mass QCD** through:
  \[
  \psi = \exp(i\omega\gamma_5\tau_3/2)\chi \\
  \bar{\psi} = \bar{\chi}\exp(i\omega\gamma_5\tau_3/2)
  \]

  with \( \tan \omega = \frac{\mu_q}{m_q} \)

- **Maximal twist**: \( \omega = \frac{\pi}{2} \)
Discretised Twisted mass action

- Discretized twisted mass action:
  \[ S_{\text{tm}}^F = a^4 \sum_x \left\{ \bar{\chi}_x \left[ D_W + m_q + i \gamma_5 \tau_3 \mu_q \right] \chi_x \right\}, \]

- Parity and isospin breaking, restored in the continuum

- Advantages at maximal twist: **automatic \( O(a) \) improvement** [Frezzotti, Rossi (2004) Frezzotti, Martinelli, Papinutto and Rossi (2006)].

- After tuning at maximal twist: 2 parameters \( \beta \) (linked to \( g \)) and \( \mu_q \)
Parameters and errors

- Need 2 observables to fix the 2 parameters, for instance \((m_N, m_\pi)\). Results expressed in term of \((a, m_\pi)\): lattice spacing and pion mass.

**List of errors**

- Sample of configurations \(\rightarrow\) Statistical error
- Non zero lattice spacing \(a\) \(\rightarrow\) Discretisation effects \(O(a^2)\)
- Finite lattice size \(\rightarrow\) Finite volume effects
- Pion mass \(m_\pi\) between 260 and 460 MeV \(\rightarrow\) Chiral extrapolation

To handle with care the errors, needs several simulations with different parameters
## Ensembles

<table>
<thead>
<tr>
<th>Ensemble</th>
<th>$\beta$</th>
<th>$a$ (fm)</th>
<th>$(L/a)^3 \times T/a$</th>
<th>$L$ (fm)</th>
<th>$a\mu_q$</th>
<th>$m_\pi$ (MeV)</th>
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<td>$B_1$</td>
<td>3.9</td>
<td>0.0855</td>
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**Table:** Summary of ETMC used ensembles. Size of lattice $L^3 \times T$, inverse coupling constant $\beta$ and the corresponding lattice spacing $a$, twisted mass parameter $a\mu_q$ and the corresponding pion mass $m_\pi$. 
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Interpolating fields

2 points function: \( \langle P_{\alpha_f}(x_f) \bar{P}_{\alpha_i}(0) \rangle \)

**Proton interpolating field**

\[
P_{\alpha}(x) = \epsilon^{abc} (u^a(x)^T C \gamma_5 d^b(x)) u^c_{\alpha}(x)
\]

But excitations with the same quantum numbers.

**Smearing** : create a smeared field to improve the overlap with the ground state

- **APE smearing** of the configuration: one link replaced by an average with its neighbours
- **Gaussian smearing** of the quark propagator: average the quark propagator replacing dirac distribution by a gaussian. Applied at the sink and at the source
2 points Correlator

2 points function \( \langle P_{\alpha_f}(x_f) \bar{P}_{\alpha_i}(0) \rangle \): combination of 3 quarks propagators.

\[
C^{2pt}(t_f, \vec{p}, \Lambda^0) = \text{Tr}(\Lambda^0 \int d^3x_f e^{-i\vec{p}\vec{x}_f} \langle P_{\alpha_f}(x_f) \bar{P}_{\alpha_i}(0) \rangle)
\]

- projection on proton spin indices.
- projection on final momentum

Spectral decomposition

\[
C^{2pt}(t_f, \vec{p}, \Lambda^0) \xrightarrow{t_f, T-t_f \to \infty} |Z^N_2| \frac{(E_N + M_N)}{2E_N} e^{-E_N t_f}
\]
Energy extraction

Effective energy:

\[ E_{\text{eff}}(t_f, \vec{p}) = \ln \frac{C^{2pt}(t_f, \vec{p})}{C^{2pt}(t_f + 1, \vec{p})} \xrightarrow{t_f, T-t_f \to \infty} E_N(\vec{p}) \]

- excited states present at small \( t \)
- determination of a plateau of fit
- weak dependence on fit range
- when \( \vec{p}^2 \to \), signal to noise ratio \( \searrow \)

Figure: \( E_{\text{eff}}(t, \vec{p}^2) \) in function of \( at \).
Dispersion relation

To extract form factors need to know energy of the proton. Which one? The one of the continuum: \( E_N^2 = M_N^2 + \vec{p}^2 \).

![Figure: Difference between continuum relation and fitted energy in function of \((a\vec{p})^2\).](image)

No strong discretisation effects.
Relation valid for range of momenta used.
Effect of smearing

Spectral decomposition:

\[ C^{2pt}(t_f, \bar{p}, \Lambda^0) \xrightarrow{t_f, T-t_f \to \infty} |Z^N_2| \frac{(E_N + M_N)}{2E_N} e^{-E_N t_f} \]

**Effect of smearing on** \( Z_2 \): constant \( \to \) gaussian in \( \bar{p}^2 \), \( \equiv \) wave packet.

**Figure**: Comparison of \( \frac{Z_2(\bar{p}^2)}{Z_2(0)} \) with and without smearing.

**Figure**: Fit of \( \frac{Z_2(\bar{p}^2)}{Z_2(0)} \) by a gaussian.

Need to compensate \( Z_2(\bar{p}^2) \) factors for extraction of form factors.
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Definition

3 points function: \( \langle P_{\alpha_f}(x_f) J(z) \bar{P}_{\alpha_i}(0) \rangle \)

- Combination of 4 quark propagators.
- Restriction: isovectoriel current to avoid disconnected parts (except \( O(a^2) \) effects)

**Figure:** Illustration of 3 points function

**Figure:** Illustration of disconnected parts for 3 points function

But needs one propagator \( S(x_f, z) \) \( \forall z, x_f \).

Problems: we computed propagator \( S(x_f, x_i) \) from one fixed point source \( x_i \).
**Generalized propagator**

**Solution** [Maiani and Martinelli (1986)]: Extract the operator and the quark propagator $S(z, x_i)$. 

\[ \text{SOURCE} \quad \tilde{x}_t, t_z \]

\[ O(y', y) \quad u \]

\[ S_G(t_f, p_f, y'; \alpha_f, x_i, \alpha_i) \quad \sim S_G(z, y') \]

\[ \text{SINK} \quad p_f = 0 \]

\[ S_u(y, x_i) \quad S_u(x_f, y') \quad S_u(x_f, x_i) \quad S_d(x_f, x_i) \]
Generalized propagator

- Change the direction of the propagator \( S(x_f, z) \rightarrow S(z, x_f) \).
- **Fixed the momentum and the time at the sink.**
- **Fixed the spin indices of the proton at source and sink (projection).**
- Define the source of this generalized propagator

By "inverting" this source \( \rightarrow \) generalized propagator \( \rightarrow \) 3 points correlator.
3 points correlators

\[ C_{3pt}(t, \vec{q}, t_f, \Lambda, \mathcal{J}) = Tr(\Lambda \int d^3 x_f d^3 z e^{i\vec{q}\cdot\vec{z}} \langle P_{\alpha_f}(x_f)\mathcal{J}(z)\bar{P}_{\alpha_i}(0)\rangle) \]

Depends on:
- the operator in study
- the time of current insertion \( t \)
- momentum transferred \( \vec{q} \)
- projector on proton spin indices, \( \Lambda \) : fixed and depend on observables in study.
- time \( t_f \) : fixed, source-sink separation \( \sim 1 \text{ fm} \).
- projection at 0 momentum at the sink

Then construct the following ratio using the 2 and 3 points correlators:

\[
R(t_z, \vec{q}, t_f, \Lambda, \mathcal{J}) = \frac{C_{3pt}(t, \vec{q}, t_f, \Lambda, \mathcal{J})}{C_{2pt}(t_f, \vec{0}, \Lambda^0)} \sqrt{\frac{C_{2pt}(t_f - t_z, -\vec{q}, \Lambda^0)C_{2pt}(t, \vec{0}, \Lambda^0)C_{2pt}(t_f, \vec{0}, \Lambda^0)}{C_{2pt}(t_f - t_z, \vec{0}, \Lambda^0)C_{2pt}(t, -\vec{q}, \Lambda^0)C_{2pt}(t_f, -\vec{q}, \Lambda^0)}}
\]
Plateaux

**Ratio tends to a constant**: \( R(t_z, \vec{q}, t_f, \Lambda, \mathcal{J}) \xrightarrow{t_f - t_z, t_z \gg 1} \Pi(\vec{q}, \Lambda, \mathcal{J}) \)

In practice: fit a symmetric plateau for all \( \vec{q} \)

**Figure**: Examples of plateaux for 3 points functions

**Within our statistical errors, no dependence on plateaux range**
Extraction of form factors

- Extract the form factors from fitted plateaux
- Use all degenerate momenta and component contributing through Singular Value Decomposition (SVD).
- Consistency check of fit range
  → **plateaux safe from contaminations by excited states.**

**Figure:** Plateaux after SVD for $G_E(Q^2)$ for several momenta.

**Figure:** Plateaux after SVD for $G_A(Q^2)$ for several momenta.
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Electric form factor

- Use of exact conserved current (no renormalization).
- Charge conservation impose $G_E(0) = 1$.
- Dipole fit:
  \[ G_E(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_E^2}\right)^2} \]
- Fit range : [0-1.2] GeV.

**Figure:** Examples of $G_E^{p-n}(Q^2)$ with dipole fit. Experimental value given by black curve.
Electric mean square radius

Extract from fit with one parameter: \( \langle r_E^2 \rangle = \frac{12}{M_E^2} \)

Correspond to the slope at the origin of \( G_E(Q^2) \)

Figure: Chiral dependence of electric mean square radius.

- No evidence of discretisation effects.
- Evidence of volume effect.
Electric mean square radius

Combined fit :
- Volume effects :
  \[ M_E(m_\pi L) = A + \frac{B}{m_\pi L} e^{-m_\pi L} \]
- Chiral dependence :
  \[ \langle r_E^2 \rangle_{p-n} = c_1 + \chi_N \log \frac{m_\pi^2}{\mu^2 + m_\pi^2} \]

Need to correct finite volume effects to get chiral dependence.
Vectorial renormalization constant

Continuum vector current need to be renormalized:

\[ \Pi(\vec{q}, \Lambda^0, V^3_\mu) = \frac{G_E(Q^2)^{\nu-n}}{Z_V} \]

\(Z_V\) extracted from the ratio:

\[ \frac{\Pi(\vec{q}, \Lambda^0, V_{\mu}^{3, cons})}{\Pi(\vec{q}, \Lambda^0, V_{\mu}^{3})} = Z_V(Q^2) \]

\[ q_{\mu,a} \]

\[ \begin{array}{c|c|c|c|c}
\hline
\beta & L & Z_V & \beta & L \\
3.90 & 2.1 \text{ fm} & 0.72 & 4.05 & 2.2 \text{ fm} \\
3.90 & 2.8 \text{ fm} & 0.71 & 4.20 & 2.7 \text{ fm} \\
4.05 & 2.2 \text{ fm} & 0.70 & 4.20 & 1.8 \text{ fm} \\
4.20 & 2.7 \text{ fm} & 0.70 & 4.20 & 1.8 \text{ fm} \\
3.90 & 2.8 \text{ fm} & 0.71 & \text{Meson sector} & \text{Result of extrapolation} \\
\hline
\end{array} \]

**Figure:** Dependence of \(Z_V\) in function of twisted mass \(a\mu_q\)

Constitency between \(Z_V\) determination in meson and nucleon sector
Vectorial renormalization constant

$Q^2$ dependence of $Z_V$.

Dependence comes from discretisation effect.

Evidence of discretisation effects for $Q^2$ dependence of form factors
Magnetic form factor

- Dipole form:
  \[ G_M(Q^2) = \frac{\mu_{p-n}^2}{\left(1 + \frac{Q^2}{M_M^2}\right)^2} \]

- Simultaneous fit of \(\mu_{p-n}^2\) and \(\langle r_M^2 \rangle_{p-n}\)

**Figure:** Chiral dependence of magnetic moment.

**Figure:** Chiral dependence of magnetic mean square radius.

No evidence of finite volume neither discretisation effects.

Curvature of chiral extrapolation to the physical point for \(m_\pi < 260\) MeV.
Axial form factor

- Dipole form:
  \[ G_A(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2} \]

- \(G_A(0) = g_A\): axial coupling.
- Need to be renormalized.

**Figure:** Chiral dependence of axial coupling \(g_A\).

No evidence of finite volume nor discretisation effects.

**Curvature of chiral extrapolation to the physical point for \(m_\pi < 260\) MeV.**
Average momentum carried by quarks

- With \( \mathcal{O}_{n,f}^{\mu_1\ldots\mu_n} \), operator with derivative:
  \[
  \langle N, \vec{p}, s | \mathcal{O}_{n,f}^{\mu_1\ldots\mu_n} | N, \vec{p}, s \rangle = A_{n,f} \bar{u}(p) \gamma^\mu p^{\mu_1} \ldots p^{\mu_{n-1}} u(p)
  \]
  \[
  A_{n,f} = \langle x^{n-1} \rangle_f = \int_0^1 dx x^{n-1} (f_f(x) - (-1)^{n-1} \bar{f}_f(x))
  \]

- Average momentum carried by quarks:
  \[
  \langle x \rangle_{u-d} = \int_0^1 dx x (f_u(x) + f_{\bar{u}}(x)) - (f_d(x) + f_{\bar{d}}(x)) = A_{u-d}^2
  \]

- Needs renormalization constant.
Conclusion

- Need lattice QCD to get predictions of QCD at low energy.
- Twisted mass fermions at maximal twist: automatic improvement of lattice artefacts.
- Variety of ensembles to get under control systematical errors.
- 3 points function of nucleon challenging due to procedure and large statistical errors.
Large volume effects in electric mean square radius $\rightarrow$ combined fit needed to get chiral extrapolation.

No direct discretisation effects in mean square radius but evidence of their contribution through $Z_V(Q^2)$.

Accurate results obtained for several quantities: $\mu^{p-n}$, $\langle r_M^2 \rangle^{p-n}$, $g_A$, $\langle r_A^2 \rangle^{p-n}$, $\langle x \rangle$.

No finite volume neither discretisation effects in those observables.

Apparent discrepancy with data in pion mass range and experiments should be explain by chiral extrapolation.

Agreement with results of others collaborations having different discretisation scheme.
**Perspective**

- Need lower pion mass.
- Check source-sink time separation to control contamination by excited states.
- Improved statistical error bars: improved algorithms for quarks propagators computations.
- Widen momentum range: for large momenta with different smeared sources; for small momenta with twisted boundaries conditions.
Thanks for your attention!
Smearing for the 3 points function

APE and gaussian smearing at the source and sink
- At the source by using propagator smeared at the source
- At the sink by smearing the source of the generalized propagator : smearing fixed at the sink.

Figure: Illustration of smearing for the 3 points function
Useful tests

Comparing with the 2 points function, we test :

- Of the generalized propagator source :

\[
\begin{align*}
\text{SOURCE} & \quad u \quad \text{SINK} \\
\tilde{x}_i, \tilde{t}_i & \quad \alpha_i & \quad \beta_i \\
\cdots & \quad \cdots & \quad \cdots \\
\end{align*}
\]

\[S_u(x_f, x_i), \quad S_d(x_f, y_i), \quad S_d(x_f, x_i)\]

**Figure:** Illustration of test of the generalized propagator source

- Of the generalized propagator :

\[
\begin{align*}
\text{SOURCE} & \quad u \quad \text{SINK} \\
\tilde{x}_i, \tilde{t}_i & \quad \alpha_i & \quad \beta_i \\
\cdots & \quad \cdots & \quad \cdots \\
\end{align*}
\]

\[S_u(x_f, y'_i), \quad S_u(x_f, x_i), \quad S_d(x_f, x_i)\]

**Figure:** Illustration of test of the generalized propagator
Average of quark transversity

![Graph showing the average of quark transversity vs. \( m_\pi \) in [GeV].]
For $Q^2 \sim m^2_{\pi}$:

$$\alpha_{PPD} := \frac{G_P(Q^2)}{G_A(Q^2)} \frac{Q^2 + m^2_{\pi}}{4M^2_N} = 1$$