Observations bruitées d’une diffusion
Estimation, filtrage, applications

Benjamin Favetto

MAP5 - Université Paris Descartes

30 septembre 2010

Thèse préparée sous la direction de Valentine Genon-Catalot
Stochastic Differential Equations

**Figure:** Diffusion models (neuronal data)

- Continuous-time stochastic models
- Mean behaviour ruled by an ordinary differential equation
- **Aim**: estimation of a parameter $\theta$ of interest
Model

Stochastic Differential Equation

\[ dX_t = b(X_t, \theta)dt + \sigma(X_t, \theta)dB_t, \quad X_0 = \eta \] (1)

N+1 observations at discrete times

\[ 0 = t_0 < t_1 < \cdots < t_N = T \quad \text{with} \quad t_{i+1} - t_i = \Delta_N \]

\((X_t)\) ergodic, with stationary probability \(\pi_0\).

Literature review

- [Bibby and Sørensen, 1995], [Kessler and Sørensen, 1999], [Kessler, 2000], [Sørensen, 2009] for the estimating-equations method
- [Ditlevsen and Lansky, 2005], [Höpfner and Brodda, 2006], [Ditlevsen and Ditlevsen, 2007] for examples of applications
Examples of diffusions

- **Ornstein-Uhlenbeck (on $\mathbb{R}$)**
  \[ dX_t = \theta(\mu - X_t)dt + \sigma dB_t \]  
  (2)

- **Cox-Ingersoll-Ross (on $(0, \infty)$)**
  \[ dX_t = \theta(\mu - X_t)dt + \sigma \sqrt{X_t} dB_t \]  
  (3)

- **Hyperbolic diffusion (on $\mathbb{R}$)**
  \[ dX_t = \theta X_t dt + \sigma \sqrt{1 + X_t^2} dB_t \]  
  (4)
Hidden diffusions and statistical problems

Observations

\[ Y_{t_k} = F(X_{t_k}, \varepsilon_{t_k}) \]  \hspace{1cm} (5)

F (known) function, \((\varepsilon_{t_k})\) iid random variables independent of \((X_t)\):
observation noise

→ to model a measurement error (physical or biological measure), a microstructure noise (finance)...

→ Particular case of hidden Markov model, two questions:
1. estimation of \(\theta \in \Theta\) when \((X_t)\) is ergodic
2. exact computation and approximation of the filter

\[ \pi_{k|k:0}(f) = \mathbb{E}(f(X_{t_k})|Y_{t_0}, \ldots, Y_{t_k}) \]

Example of additive noise

\[ Y_{t_k} = X_{t_k} + \varepsilon_{t_k} \]  \hspace{1cm} (6)

often with Gaussian distribution.
\begin{equation}
\begin{cases}
    dX_t = b(X_t, \theta)dt + \sigma(X_t, \theta)dB_t, & X_0 = \eta \\
    Y_{t_k} = F(X_{t_k}, \varepsilon_{t_k})
\end{cases}
\end{equation}

1. Parameter estimation for noisy observations of a diffusion: [Gloter and Jacod, 2001], [Zhang et al., 2005], [Jacod et al., 2009] on [0, 1]

Survey

1. Parameter estimation for a bidimensional partially observed Ornstein-Uhlenbeck process with biological application (chapters 2 and 3)
   - Application to medical data [Favetto et al., 2009], submitted

2. Parameter estimation by contrast minimization for noisy observations of a diffusion process (chapters 4 and 5)
   - Contrasts, consistency of the estimators and simulations [Favetto, 2010], submitted
   - Estimating equations for noisy observations of ergodic diffusions, preprint

3. On the asymptotic variance in the Central Limit Theorem for particle filters (chapter 6, [Favetto, 2009] accepted in ESAIM P&S)
Parameter estimation for a bidimensional partially observed Ornstein-Uhlenbeck process

[Favetto and Samson, 2010] for the theoretical results
Medical data : joint work with Adeline Samson (MAP5), Daniel Balvay (HEGP), Isabelle Thomassin (HEGP), Valentine Genon-Catalot (MAP5), Charles-André Cuénod (HEGP) and Yves Rozenholc (MAP5)

Medical problem

- New treatments in anti-cancer therapy : anti-angiogenesis
- Evaluation *in vivo*
- Based on several sequences of medical images
- Comprehension of microvascularization phenomena
- With a pharmacokinetic model for the contrast agent
MRI data

**Figure**: Female pelvis
Biological model

Deterministic pharmacokinetic model

1. \( Q_P(t) \) quantity of contrast agent at time \( t \) in the plasma
2. \( Q_I(t) \) quantity of contrast agent at time \( t \) in the interstitium

Driven by two coupled ODE.

Only access to the sum \( S(t) = Q_P(t) + Q_I(t) \) (quantity of contrast agent in a voxel.) [Brochot et al., 2006], [Fournier et al., 2007], [Brix et al., 2004]
Stochastic version of the model

Hidden diffusion (bidimensional Ornstein-Uhlenbeck model)

\[
\begin{align*}
    dU(t) &= (F_\theta(t) + G_\theta U(t))dt + \Sigma_\theta dB(t) \\
    U(t_0) &= U_0
\end{align*}
\]

with \( U(t) = \begin{pmatrix} S(t) \\ \ast \end{pmatrix} \) and \( \Sigma_\theta = \begin{pmatrix} \sigma_1 & \sigma_2 \\ 0 & \sigma_2 \end{pmatrix} \)

Discrete-time noisy observations (unidimensional)

\[
y_i = J U(t_i) + \sigma \varepsilon_i, \quad J = (1, 0), \quad \varepsilon_i \sim \mathcal{N}(0, 1)
\]

with \( \Delta = t_{i+1} - t_i \).

→ Aim : estimation of \( \theta \) (microvascularization parameters and diffusion coefficients).

Stochastic models in biology : [Picchini et al., 2006], [Picchini et al., 2008]
Computation of the Maximum Likelihood Estimator

Log-likelihood

\[ \ell_{0:n}(\theta) = \ell(\theta, y_0, \ldots, y_n) = \sum_{i=0}^{n} \log p(y_i | y_{i-1}, \ldots, y_0; \theta). \]

Maximum Likelihood Estimator

Aim: compute \( \hat{\theta} = \arg\max_{\theta \in \Theta} \ell(\theta, y_0, \ldots, y_n). \)

Kalman algorithm for the log-likelihood

- iterative computation of the log-likelihood using Kalman recursions
- Gradient and Hessian of the log-likelihood also exactly computed by similar recursions
- \( \rightarrow \hat{\theta} \) obtained by a gradient method

Advantages of an iterative computation of the likelihood: no need to invert large matrix, faster than direct method
Theoretical properties of $\hat{\theta}$

(in stationary regime)

Link with ARMA processes

- $(y_i)$ is an ARMA (2,2) process (can be extended to $d$-dimensional hidden diffusions)
- Hessian and information matrix $\lim_{n \to \infty} \left( -\frac{1}{n} \frac{\partial^2}{\partial \theta^2} \ell_0: n(\theta) \right) = I(\theta)$
- Central Limit Theorem $\sqrt{n}(\hat{\theta}_n - \theta_0) \overset{n \to \infty}{\longrightarrow} \mathcal{N}(0, I^{-1}(\theta_0))$

Identifiability

- difficulty: number of identifiable parameters on the spectral density
- when $\Delta$ is small, five parameters (out of six) are identifiable
Simulation results

- x-axis: time
- y-axis: quantity of contrast agent
- blue: $S(t)$
- purple: $Q_P(t)$
- green: $Q_I(t)$
- constant AIF

- Comparison with the EM algorithm
- Validation of the method
- Influence of $\Delta$
- Curves obtained with the estimated parameters
Medical data

- comparison with the ODE model (no stochastic part in the hidden process)
- different behaviours for the ODE and the SDE-based methods
  1. ODE similar to SDE
  2. slightly different
  3. important differences

→ the SDE-based method performs at least as well as the ODE-based, and sometimes much better

- stability of the SDE-based method
Conclusion on this part

- strongly based on Gaussian framework
- stability of the SDE method
- importance of numerical results (simulations and medical data)
Parameter estimation by contrast minimization for noisy observations of a diffusion process

Model

\[ dX_t = b(X_t, \kappa)dt + \sigma(X_t, \lambda)dB_t, \quad X_0 = \eta \quad \text{(hidden)} \]
\[ Y_{i\delta_N} = X_{i\delta_N} + \rho_N \varepsilon_{i\delta_N} \quad \text{(observed)} \]

Hidden unidimensional diffusion, with general drift and diffusion coefficient

- Aim: estimate \( \theta = (\kappa, \lambda) \)
- Discretization step \( \delta_N \to 0 \), number of observations \( N \to \infty \) over \([0, T]\) with \( T = N\delta_N \to \infty \) (high frequency data)
- \((X_t)\) ergodic, with stationary distribution \( \nu_0(dx) = \nu_0(x)dx \)
- \(\rho_N\) is known here
- \((\varepsilon_{i\delta_N})\) i.i.d. centered random variables, \( \mathbb{E}( (\varepsilon_{i\delta_N})^2 ) = 1 \)
Assumptions on the noise

**Figure**: Ornstein-Uhlenbeck process with high-frequency noisy observations. $N = 1000$, $\delta = 0.1$, $\rho^2 = 1$

Two possible cases for the observations:

(B1) $\rho_N^2 = \rho^2 > 0$

(B2) $\rho_N^2 \to 0$ when $N \to \infty$

Assumption (B2) corresponds to the case $\sqrt{\delta_N} \epsilon_i \delta_N = V_{(i+1)}\delta_N - V_i\delta_N$ with $(V_t)$ Brownian motion independent of $(B_t)$. 
Minimum contrast estimation for a discretely observed diffusion

The Euler Scheme

For $\delta_N \rightarrow 0$,

$$X_{(j+1)\delta_N} - X_{j\delta_N} \approx \mathcal{N}(b(X_{j\delta_N}, \kappa)\delta_N, \sigma(X_{j\delta_N}, \lambda)^2\delta_N).$$

- [Kessler, 1997] $\hat{\theta} = (\hat{\kappa}, \hat{\lambda})$ consistent and asymptotically Gaussian estimator built as minimum of a contrast based on the loglikelihood of Gaussian observations
- [Gloter, 2000] and [Gloter, 2006] : $\int_{j\delta_N}^{(j+1)\delta_N} X_s ds$ observed $\rightarrow$ minimum contrast estimators, for observations on $[0, 1]$ and for $T = N\delta_N \rightarrow \infty$

**Aim** : build a contrast based on noisy data, then obtain an estimator $\hat{\theta} = (\hat{\kappa}, \hat{\lambda})$ and derive consistency and asymptotic normality
Local means of the observations

Local means and noise reduction

\( p_N, k_N \) such that \( p_N = \delta_N^{-\frac{1}{\alpha}} \) for \( 1 < \alpha \leq 2 \), \( N = p_N k_N \). Let \( \Delta_N = p_N \delta_N = \delta_N^{1-\frac{1}{\alpha}} \). Hence \( N \delta_N = k_N \Delta_N \). Define

\[
Y^j_\bullet = \frac{1}{p_N} \sum_{i=0}^{p_N-1} Y^j_{i \Delta_N + i \delta_N} = X^j_\bullet + \rho_N \varepsilon^j_\bullet
\]

Idea : \( Y^j_\bullet \approx X^j_\bullet \approx \Delta_N^{-1} \int_{j \Delta_N}^{(j+1) \Delta_N} X_s ds \approx X^j_{j \Delta_N} \rightarrow \) Build a contrast function based on \( (Y^j_\bullet) \) in the ergodic case.
Minimum contrasts estimation for an hidden diffusion

Contrast for $(Y_j^\bullet)$

$$\mathcal{E}_N(\theta) = \sum_{j=1}^{k_N-2} \left\{ \frac{3}{2} \frac{(Y_{j+1}^\bullet - Y_j^\bullet - \Delta_N b(Y_{j-1}^\bullet, \kappa))^2}{\Delta_N c(Y_{j-1}^\bullet, \lambda)} + \log(c(Y_{j-1}^\bullet, \lambda)) \right\}$$

where $c(., \lambda) = \sigma(., \lambda)^2$.

Modified contrast for $(Y_j^\bullet)$

$$\mathcal{E}_{N}^{\rho_N}(\theta) = \sum_{j=1}^{k_N-2} \left\{ \frac{3}{2} \frac{(Y_{j+1}^\bullet - Y_j^\bullet - \Delta_N b(Y_{j-1}^\bullet, \kappa))^2}{\Delta_N c_{N,\rho_N}(Y_{j-1}^\bullet, \lambda)} + \log(c_{N,\rho_N}(Y_{j-1}^\bullet, \lambda)) \right\}$$

where $c_{N,\rho_N}(x, \lambda) = \sigma(x, \lambda)^2 + 3\Delta_N^{\frac{2-\alpha}{\alpha-1}} \rho_N^2$, for $1 < \alpha \leq 2$. 
Minimum contrast estimators (I)

Let $\hat{\theta}_N = \arg\inf_{\theta \in \Theta} \mathcal{E}_N(\theta)$ and $\hat{\theta}^{\rho_N} = \arg\inf_{\theta \in \Theta} \mathcal{E}^{\rho_N}(\theta)$.

Consistency

1. If (B1/2) holds, with $p_N = \delta_N^{-\frac{1}{\alpha}}$ with $\alpha \in (1, 2)$, the estimator $\hat{\theta}_N$ is consistent.

2. If (B1/2) holds, with $p_N = \delta_N^{-\frac{1}{\alpha}}$, $\alpha \in (1, 2]$, the estimator $\hat{\theta}^{\rho_N}$ is consistent.

→ Results based on Taylor expansions for $Y^{\check{\bullet}}_j$, and the asymptotic behaviour of the variation and the quadratic variation of $Y^{\check{\bullet}}_j$

→ Special case $\alpha = 2$
Minimum contrast estimators (II)

Asymptotic normality

Assume that \( N\delta_N^{2-\frac{1}{\alpha}} \to 0 \), when \( N \to \infty, \delta_N \to 0, N\delta_N \to \infty \),
\[
\begin{align*}
k_N &= N\delta_N^{\frac{1}{\alpha}} \to \infty, \\
\Delta_N &= \delta_N^{1-\frac{1}{\alpha}} \to 0.
\end{align*}
\]
If \( \rho_N = \rho \) (B1) and \( \alpha \in (1, 2) \) or \( \rho_N \to 0 \) (B2) with \( \alpha \in (1, 2] \),

\[
\frac{\sqrt{N\delta_N}(\hat{\kappa}_N^{\rho_N} - \kappa_0)}{\sqrt{N\delta_N^{\frac{1}{\alpha}}}(\hat{\lambda}_N^{\rho_N} - \lambda_0)} \xrightarrow{d} \mathcal{N}(0, V(\theta_0))
\]

where

\[
V(\theta_0) = \begin{pmatrix}
\nu_0 \left( \frac{(\partial_{\kappa}b(.\kappa_0))^2}{c(.,\lambda_0)} \right) & 0 \\
0 & \frac{9}{4} \nu_0 \left( \frac{(\partial_{\lambda}c(.,\lambda_0))^2}{c(.,\lambda_0)^2} \right)
\end{pmatrix}^{-1}
\]

In the case \( \alpha = 2 \) with (B1) \( (\rho_N = \rho > 0) \), the asymptotic variance \( V(\theta_0) \) is increased by the noise variance \( \rho^2 \).

\rightarrow Estimation rate and asymptotic variance for \( \hat{\lambda}_N^{\rho_N} \) ([Gloter, 2006])
Estimation with unknown $\rho^2$

Quadratic variation of the observations

- $\hat{\rho}_N^2 = \frac{1}{2N} \sum_{i=0}^{N-1} (Y_{i+1}\delta_N - Y_i\delta_N)^2$
- If $N\delta_N^2 \rightarrow 0$, then $\sqrt{N}(\hat{\rho}_N^2 - \rho^2) \xrightarrow{\mathcal{L}} N(0, 3\rho^4)$

Associated estimator

The estimator $\hat{\theta}\hat{\rho}_N$ is consistent.

→ Essential difference with direct observations, local means to deal with the parameters of the hidden diffusion.
Numerical results

Ornstein-Uhlenbeck model with additive noise

- explicit estimators
- simulations with different \( \delta \) and \( N \): large number of data needed, with high frequency sampling
- with \( \alpha = 2 \), \( \alpha = 1.5 \) and \( \alpha \) close to 1: \( \alpha = 1.5 \) is a pretty good choice
- no influence of the distribution of \( \varepsilon_i \delta_N \)
- \( \hat{\lambda}_N \) is poorly estimated when \( \rho \) is very large
- comparison with direct observations

Other models

- Cox-Ingersoll-Ross model with multiplicative noise
- Hyperbolic diffusion with additive noise
Conclusion on this part

Other results

- Parameter estimation for neuronal data
- Case of an integrated diffusion process observed with small noise
- General framework of estimating functions
On the asymptotic variance in the Central Limit Theorem for particle filters

General hidden Markov model \((X_k, Y_k)\) (not only an hidden diffusion model):

**Quantities of interest**

**Aim:** Compute

\[
\pi_{k|k:0}(f) = \mathbb{E}(f(X_k)|Y_k, \ldots, Y_0) \quad \text{(filter)}
\]

\[
\eta_{k|k-1:0}(f) = \mathbb{E}(f(X_k)|Y_{k-1}, \ldots, Y_0) \quad \text{(prediction)}
\]
Filtering : problems and goals

- Except for a few models ([Kalman, 1960], [Genon-Catalot and Kessler, 2004], [Chaleyat-Maurel and Genon-Catalot, 2006]), exact algorithmic computation of $\pi_{k|k:0}(f)$ and $\eta_{k|k-1:0}(f)$ untractable
- Numerical approximation needed: particle filter ([Künsch, 2001], [Doucet et al., 2001])
- (Asymptotic) confidence interval for the method? Accuracy of the method? ([Del Moral and Jacod, 2001b], [Van Handel, 2009])
Assumptions and notations

On the hidden Markov chain

- Kernel $Q(x, dx')$ with stationary probability $\pi(dx)$
- There exists a probability measure $\mu$ and $0 < \epsilon_- \leq \epsilon_+$ such that

$$\forall x \in \mathcal{X}, \forall B \in \mathcal{B}(\mathcal{X}) \quad \epsilon_- \mu(B) \leq Q(x, B) \leq \epsilon_+ \mu(B). \quad (10)$$

On the observations

- $f(y|x)$ conditional density of $L(Y_k|X_k)$

With $g_k(x) = f(Y_k|x)$ and $L_k(x, dx') = g_k(x)Q(x, dx')$,

$$\mathbb{E}(f(X_k)|Y_{k-1}, \ldots Y_0) = \eta_{k|k-1:0}(f) = \frac{\eta_0 L_{0,k-1} f}{\eta_0 L_{0,k-1} \mathbf{1}} \quad (11)$$
Particle Monte-Carlo method

- Algorithm which provides recursively empirical measures $\pi^N_{k|k:0}$ and $\eta^N_{k|k-1:0}$ which are good approximations of $\pi_{k|k:0}$ and $\eta_{k|k-1:0}$, for a fixed set of observations $Y_{0:k}$

- Based on the simulation of the evolution of $N$ particles (need to know how to simulate under $Q$), which give the accuracy of the method

See e.g. [Künsch, 2001], [Doucet et al., 2001], [Del Moral et al., 2001]
Central Limit Theorem

[ Pitt and Shephard, 1999], [Del Moral and Jacod, 2001a], [Chopin, 2004]

Central Limit Theorem for the prediction

$$\sqrt{N}(η^N_k|k−1:0(f) − η_k|k−1:0(f)) \xrightarrow{N \to \infty} \mathcal{N}(0, \Delta_k|k−1:0(f))$$

Central Limit Theorem for the filter

$$\sqrt{N}(π^N_k|k:0(f) − π_k|k:0(f)) \xrightarrow{N \to \infty} \mathcal{N}(0, \Gamma_k|k:0(f))$$

As $N \to \infty$, $\Gamma_k|k:0(f)$ and $\Delta_k|k−1:0(f)$ give the accuracy of the approximations (confidence interval) for a fixed sequence of observations. $\Gamma_k|k:0(f)$ and $\Delta_k|k−1:0(f)$ can be seen as functions of $Y_{0:k}$, i.e. random variables depending on the observations.

Question: behaviour when $k \to \infty$? Aim: prove tightness of the sequences, to have uniform (in time) confidence intervals.
Litterature review

Numerous references about particle filter(s). For the asymptotic behaviour:

- [Chopin, 2004] CLT and tightness under stronger assumptions, Bayesian approach of particle algorithms
- [Del Moral and Jacod, 2001b] Tightness in the particular case of Kalman filter, with analytic method
- [Van Handel, 2009] Other criterion of time-stability, under assumptions close to us
The asymptotic variances

Asymptotic variance of the prediction

\[ \Delta_{k|k-1:0}(f) = \sum_{i=0}^{k} \eta_{i|i-1:0} \left( \left( \frac{L_{i,k-1} \mathbf{1}(\cdot)}{\eta_{i|i-1:0} L_{i,k-1}} \right)^2 \left( \eta_{k|k-1:i} f(\cdot) - \eta_{k|k-1:0} f \right)^2 \right) \]

Asymptotic variance of the filter

\[ \Gamma_{k|k:0}(f) = \sum_{i=0}^{k} \eta_{i|i-1:0} \left( \left( \frac{L_{i,k-1} \mathbf{1}(\cdot)}{\eta_{i|i-1:0} L_{i,k-1}} \right)^2 \left( \eta_{k|k-1:i} (g_k f)(\cdot) - \pi_{k|k:0} f \right)^2 \right) \cdot \left( \eta_{k|k-1:0} (g_k) \right)^2 \]
Tightness of \((\Delta_{k|k-1:0}(f))_k\) and \((\Gamma_{k|k:0}(f))\)

Additional assumption

\((\textbf{B})\) For some \(\delta > 0\)

\[
\sup_{k \geq 0} \mathbb{E} \left| \log \left( \eta_{k|k-1:0}(g_k) \right) \right|^{1+\delta} < \infty,
\]

where \(\mathbb{E}\) denotes the expectation with respect to the distribution of \((Y_k)_{k \geq 0}\).

Remark: Technical assumption, but easy to check on common examples.

Main result:

Assume \((\textbf{B})\). Then, for any bounded function \(f\), the sequences of variances \((\Delta_{k|k-1:0}(f))\) and \((\Gamma_{k|k:0}(f))\) are tight.
Exponential stability of the prediction

Tool for the proof: forget the initial distribution at exponential rate ([Douc et al., 2009])

Proposition: forgetting the initial distribution

Setting $\rho = 1 - \frac{\epsilon^2}{\epsilon^2_+}$, then for all $k, \nu, \nu'$ and all set $y_0:k-1$ of values:

$$\|\eta_{\nu,k}[y_0:k-1] - \eta_{\nu',k}[y_0:k-1]\|_{TV} \leq \rho^k,$$

where $\|\cdot\|_{TV}$ denotes the total variation distance.

Consequence:

For $f$ bounded,

$$\Delta_{k|k-1:0}(f) \leq \|f\|_\infty^2 \frac{\epsilon^2_+}{\epsilon^2_-} \sum_{i=0}^{k} \eta_i|i-1:0 \left( \frac{g_i}{\eta_i|i-1:0 g_i} \right)^2 \rho^{2(k-i)}.$$

→ conclusion with the tightness lemma [Del Moral and Jacod, 2001b].
A simple Markov chain

\[ X_0 \sim \pi, \quad X_{k+1} = 1_{X_k < \alpha} U_{k+1} + 1_{X_k \geq \alpha} V_{k+1} \]

\((U_k)\) and \((V_k)\) independent of \(X_k\) with distributions \(u(x)dx\) and \(v(x)dx\).

Then \(\mu(dx) = 4(x \wedge 1 - x)dx\), \(\epsilon_- = \frac{1}{4}\) and \(\epsilon_+ = \frac{3}{2}\).
Simulations and numerical results

**Figure**: Simulation of the example, with additive noise (observations on [0, 50])

Observations: $Y_i = X_i + \varepsilon_i$, $\varepsilon_i \sim iid \mathcal{N}(0, 0.2)$.

Concluding remarks and perspectives

- Generalization of the contrast-based method for high-frequency noisy observations of a diffusion to the multidimensional case, with general noise.
- Assumption on the transition kernel of the hidden chain used to prove the tightness could be weakened.
- Other implementations of the contrast-based method on real data.
Concluding remarks and perspectives

- Generalization of the contrast-based method for high-frequency noisy observations of a diffusion to the multidimensional case, with general noise.
- Assumption on the transition kernel of the hidden chain used to prove the tightness could be weakened.
- Other implementations of the contrast-based method on real data.

Thanks!
References I


References II


References III


References IV


On the asymptotic variance in the Central Limit Theorem for particle filters.
Preprint MAP5 2009-14, to appear in ESAIM P & S.

Favetto, B. (2010). 
Consistent parameter estimation by contrast minimization for noisy discrete observations of a hidden diffusion process.
MAP5 2010-13.

Parameter estimation for a bidimensional partially observed Ornstein-Uhlenbeck process with biological application. 
*Scand. J. Statist.*, 37(2) :200–220.
References VI

Blood micro-circulation parameters extraction from Dynamic Contrast Enhanced MRI data using stochastic differential equations.
Submitted.

Approximate discrete-time schemes for statistics of diffusion processes.
References VII

Fournier, L., Thiam, R., Cuénod, C.-A., Medioni, J., Trinquart, L.,
Balvay, D., Banu, E., Balcaceres, J., Frija, G., and Oudard, S.
(2007).
Dynamic contrast-enhanced CT (DCE-CT) as an early biomarker of
response in metastatic renal cell carcinoma (mRCC) under
anti-angiogenic treatment.
*J. of Clinical Oncology - ASCO Annual Meeting Proceedings
(Post-Meeting Edition)*, 25.

Maximum contrast estimation for diffusion processes from discrete
observations.

On the estimation of the diffusion coefficient for multi-dimensional
diffusion processes.
References VIII

Random scale perturbation of an AR(1) process and its properties as a nonlinear explicit filter.
*Bernoulli*, 10(4) :701–720.

Discrete sampling of an integrated diffusion process and parameter estimation of the diffusion coefficient.

Parameter estimation for a discretely observed integrated diffusion process.

Diffusions with measurement errors. II. Optimal estimators.
References IX

A stochastic model and a functional central limit theorem for information processing in large systems of neurons.

Microstructure noise in the continuous case : the pre-averaging approach.

A new approach to linear filtering and prediction problems.

Estimation of an ergodic diffusion from discrete observations.
*Scand. J. Statist.*, 24(2) :211–229.
Simple and explicit estimating functions for a discretely observed diffusion process.

Estimating equations based on eigenfunctions for a discretely observed diffusion process.

State space and hidden Markov models.
References XI

Modeling the euglycemic hyperinsulinemic clamp by stochastic differential equations.  

Maximum likelihood estimation of a time-inhomogeneous stochastic differential model of glucose dynamics.  

Filtering via simulation : auxiliary particle filters.  

Efficient estimation for ergodic diffusions sampled at high frequency.  
Preprint.
References XII

Uniform time average consistency of Monte Carlo particle filters.

Estimation for diffusion processes from discrete observation.

A tale of two time scales: determining integrated volatility with noisy high-frequency data.