The Impact of the RFID Technology in Improving Performance of Inventory Systems subject to Inaccuracies

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To my family
To my wife and the little M. Yazid

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General Introduction

Problem Statement

Many companies have automated their inventory management systems and rely on an information system in critical decision making. In spite of the considerable amounts invested in information technology, a number of negative issues is not yet eliminated:

- Inventory inaccuracy: the inventory inaccuracy occurs when the Information System inventory is not in agreement with the actually available inventory. Based on a study done with a leading retailer, Raman et al. [8] reports that out of close to 370,000 SKUs investigated, more than 65% of the inventory records did not match the physical inventory at the store-SKU level. Moreover, 20% of the inventory records differed from the physical stock by six or more items.

- Out-of-stock: Based on an analysis of 52 studies that examine out-of-stocks, Gruen et al. [9] calculate an average out-of-stock level of 8.3% for the retail industry. The study does not provide any detailed data for grocery products. The out-of-stock figure varied between 7.9% in the US and 8.6% in Europe. The authors identify a number of root causes for out-of-stock. According to the study, 47% of out-of-stock situations were caused by store ordering and forecasting, 28% by upstream activities, and 25% by inadequate shelf restocking from backroom (i.e. the product was in the store, but not on the shelf).

- Unsaleable products: according to the 2000 Unsaleables Benchmarking Report[1], unsaleable products now cost the entire grocery industry more than 1% in annual sales. Damage is the biggest cause of unsaleables with 63% of all unsaleables, followed by out-of-code (16%) and discontinued items (12%).

- Inventory visibility: a key element of supply chain economics is being able to know where all the inventory is at any particular point in the supply chain, while minimizing the amount of overall product in the supply chain. With visibility into the inventory in the supply chain, a supply chain manager can make better tactical and operational decisions about redirecting it to fulfill real-time requirements at the destination. Wal-Mart reportedly estimates lost sales due to stock outages at about 4% (Spiegel [10]).

According to a number of papers published by the IBM Business Consulting Services (Alexander et al. [11], Alexander et al. [12], Alexander et al. [13] ,Alexander et al. [14]), An advanced automatic

[1] performed by the Grocery Manufacturers of America, the Food Marketing Institute and Food Distributors International

identification system based on the RFID technology is expected to address some of the root causes of the issues mentioned above.
Historically, this thesis falls under the continuation of the work completed in 2004 by Evren Sahin at the Ecole Centrale Paris (Sahin [6]). It is motivated by the development of the EPCglobal Network system developed by the Auto-ID center whose aim is to create a technology that can perform better than the bar code technology and replace it in the long term. This technology uses Electronic Product Codes and is carried by RFID tags.
The main issue considered in this thesis is the impact of the RFID technology in improving performance of inventory systems subject to inventory inaccuracy. Two major keywords can describe the context of the thesis:

1. The first keyword is RFID: Radio-Frequency IDentification (RFID) technology is the use of radio frequencies to read information on a small device known as a tag. RFID as an emerging technology has generated enormous amount of interest in the supply chain arena. With RFID technology, inventory can be tracked more accurately in real time resulting in reduced processing time and labor. More significantly, the complete visibility of accurate inventory data throughout the entire supply chain, from manufacturer to warehouses to retail stores, brings opportunities for improvement and transformation in various processes of the supply chain.

2. The second keyword is inventory inaccuracy: The inventory inaccuracy occurs when the Information System inventory is not in agreement with the actually available inventory. The inventory Information System, contrary to popular belief and assumptions in most academic papers and in spite of the considerable amounts invested in information technology, are often inaccurate.

Research question

The major aims of this thesis is to present a quantitative analysis enabling to quantify the impact of inventory inaccuracy on the performance of inventory systems and to identify what can RFID deliver to such inventory systems.
More precisely, in this PhD thesis, the following research problems and questions are to be answered:

1. After few years of the foundation of the Auto-ID Center[2], which are the main academic investigations linked to the RFID technology and the inventory inaccuracy issue?

2. How should inventory inaccuracy issue be modelled?

3. Is there any connection between models describing inventory inaccuracy issue of different sources?

4. What is the impact of inventory inaccuracy on the performance of inventory systems?

5. What can the RFID technology bring to cope with the inventory inaccuracy issue?

[2]The Auto-ID Center, founded in 1999, is sponsored by over 100 global companies, many of whom are leaders in their industries. Its aim is to create an automatic product identification system that can potentially replace bar-code technology

6. Which is the RFID cost which makes its deployment cost effective?

7. Is RFID the best and/or the unique solution against inventory inaccuracy problems?

Scope of the Thesis

This research is motivated by the ability of the RFID technology to address the potential obstacles in managing supply chains. Our first contribution, performed based on qualitative and empirical investigations, consists in a synthesis of the major impacts of the RFID technology on the supply chain performance.

As a response to the first research question mentioned above, we provide a comprehensive list of quantitative investigations (most of them are yet working papers) dealing with the inventory inaccuracy issue and the RFID technology. We also propose a classification of these investigations according to three main levels. The second research question is answered by providing a general inventory framework which permits to model the inventory inaccuracy issue. The proposed general framework enables us to answer the third research question by showing that there is a common point for all models describing the inventory inaccuracy issue. This commun point is the random yield problem: in fact we show that an inventory system subject to inventory inaccuracy is an extended version of the well known random yield problem.

Concerning the latter problem, we also contribute in extending literature related to this problem by presenting a comprehensive analysis of the Newsvendor problem under unreliable supply. The fourth and the fifth research questions are considered for all models presented in this thesis. We try in each model to compare the penalty resulting from the inventory inaccuracy and the benefice incurred by the deployment of the RFID technology. This last comparison enables us to response the sixth research question and to give insights on the other possible solutions against the inventory inaccuracy problems.

Structure of the Thesis

The thesis dissertation is composed of three parts:

1. **PART I**: the first part introduces the report by providing a review of literature on the RFID technology and the inventory inaccuracy issue. It is composed of two chapters:

   - Chapter 1: this chapter proposes a basic understanding of the RFID technology and deals with its impact on supply chain management systems in order to give an insight into the current issues and status of the technology. The presentation and discussion will help to better understand what RFID can deliver. The analysis presented in this chapter is based on qualitative studies providing business cases for RFID deployments.

   - Chapter 2: this chapter proposes a literature review in the issue of inventory inaccuracy. We first present qualitative and empirical investigations dealing with problems perturbing the information system and the physical flows. We highlight the source of inventory inaccuracy problems and ways in which inventory managers can address the issue through a combination of compensation methods such as technology, inspection policies, and process improvements. We then present quantitative investigations addressing the inventory inaccuracy issue. In particular we propose three main levels permitting to classify these investigations. This classification permits to have a visibility on topics where there is a lack of quantitative models allowing the analysis of the impact of inventory inaccuracy problems on the supply chain performance.

2. **PART II**: Models of this part of the report are built based on the Newsvendor problem where two supply chain structures are analyzed. This part is composed of four chapters:

   - Chapter 3: this chapter deals with the random yield problem where a single-period, uncertain demand inventory model is analyzed under the assumption that the quantity ordered (produced) is a random variable. We first conduct a comprehensive analysis of the well known single period production/inventory model with random yield. Then, we extend some of the results existing in literature: our main contribution is to show that earlier results are only valid for a certain range of system parameters. Under the hypothesis that demand and the error in the quantity received from supplier are uniformly distributed, closed-form analytical solutions are obtained for all values of parameters. An analysis under normally distributed demand and error is also provided. The chapter ends with an analysis of the benefit achieved by eliminating supply errors.
   - Chapter 4: this chapter considers the situation of a retail store subject to inventory inaccuracies stemming from execution problems. We assume that inventory inaccuracies are introduced by misplacement type errors that occur within the store, i.e. the whole quantity of products that is ordered and received from the supplier is not available on shelf to satisfy consumers' demand either because the replenishment process from the backroom to shelves is prone to errors (e.g. products are lost during this transfer, products are forbidden in the backroom, products are put on other shelves than where they should be...) or products are misplaced on other shelves by consumers during their visit to the store. We consider a Newsvendor model that captures this issue in a simple way: for a given quantity of products ordered from the supplier, only a random fraction is available for sales. We compare three approaches. In the first approach, the retailer is unaware of errors in the store. In the second approach, the retailer is aware of errors and optimizes its operations by taking into account this issue. The third approach deals with the case where the retailer deploys the RFID technology to eliminate errors. In particular, we provide insights on the relative benefit of implementing the RFID technology (moving from approach 2 to approach 3) compared to the benefit of optimizing the system in presence of inaccuracies (moving from approach 1 to approach 2). We also provide an analytical expression of the cost of the RFID tag which makes its deployment cost effective.
   - Chapter 5: this chapter extends the result of chapter four in the case of a decentralized supply chain. The chapter analyzes a Newsvendor type inventory model in which a manufacturer sells a single product to a retailer store whose inventory is subject to errors stemming

from execution problems. We compare two situations: in the first situation, the two supply chain actors are aware of errors and optimize their ordering decisions by taking into account this issue. The second situation deals with the case where the RFID technology is deployed in order to eliminate the errors. Each situation is developed under three scenarios: in the centralized scenario, we consider a single decision-maker who is concerned with maximizing the entire supply chain's profit; in the decentralized uncoordinated scenario, the retailer and the manufacturer act as different parties and do not cooperate. The third scenario is the decentralized coordinated scenario, where we give conditions for coordinating the channel under a buyback contract.

- Chapter 6: this chapter provides a general framework permitting to model the inventory inaccuracy issue. In particular, we show that there is a connection between inventory inaccuracy and random yield problems. This last analysis ends with deducing an elegant mathematical analysis of the optimal ordering decisions in some particular settings.

3. **PART III:** this part of the report considers the inventory inaccuracy issue and the impact of the RFID technology in a multi-period framework. It is composed of one chapter:

    - Chapter 7: this chapter considers a finite horizon, single-stage, single-product periodic-review inventory in which inventory records are inaccurate. We assume that inventory inaccuracies are introduced by theft type errors that occur within the store. Here again, we propose a comparison between three approaches, based on which the inventory system in the presence of theft can be managed: in the first approach, the inventory manager is unaware of errors in the store. In the second approach, we focus the benefits achieved through a better knowledge of errors and through taking them into account when formulating and optimizing the inventory system. In the third approach, we focus the contribution of a perfect RFID technology that prevents errors. To solve the problem, we follow two formulations: i) Optimization of underage and overage costs where dynamic programming tools are used ii) Optimization of overage cost under a service level constraint where analytical results are provided.

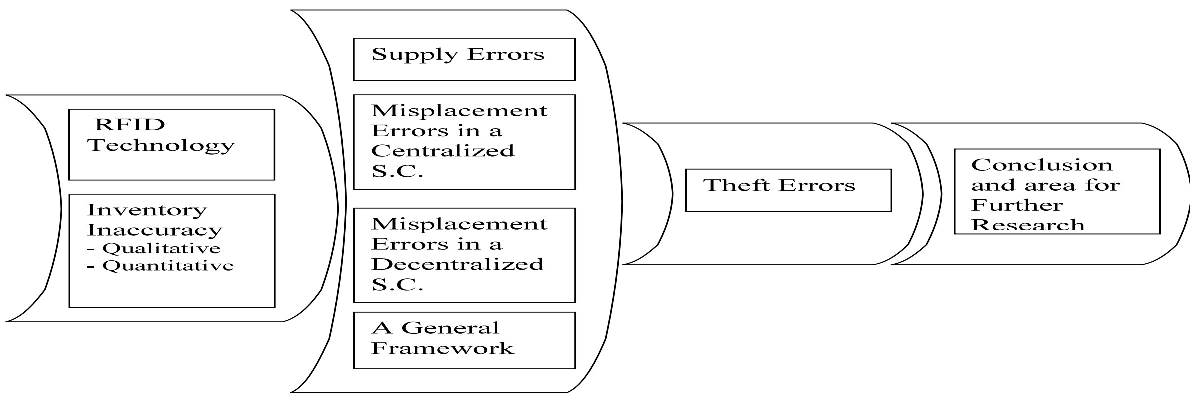


Figure 1: The structure of the Thesis

Part I

RFID and Inventory Inaccuracy: Literature Review

Preliminary Notes on Part I

This part introduces the context of our research by presenting the literature review pertaining to the RFID technology and the inventory inaccuracy issue. This part is composed of two chapters:

- Chapter 1 proposes a basic understanding of the RFID technology and focuses on its impact on supply chains in order to give an insight into the current issues and status of the technology. This chapter is based on qualitative studies providing business cases for RFID deployments.

- Chapter 2 proposes a literature review on the issue of inventory inaccuracy. We first present qualitative and empirical investigations dealing with problems perturbating the information system and the physical flows in supply chains. We highlight the source of inventory inaccuracy problems and ways in which inventory managers can address the issue through a combination of compensation methods such as technology, inspection policies, and process improvements. In the second part of this chapter, we present the quantitative investigations addressing the inventory inaccuracy issue. In particular we propose three main levels permitting to classify these investigations. This classification enables us to get more visibility on topics where there is a lack of quantitative models allowing the analysis of the impact of inventory inaccuracy problems on the supply chain performance.

Chapter 1

A Basic Understanding of the RFID Technology

Radio-Frequency IDentification (RFID) as an emerging technology has generated enormous amount of interest in the supply chain area. Concerning the inaccuracy issue, we notice that with RFID technology, inventory can be tracked more accurately in real time resulting in reduced processing time and labor. More significantly, the complete visibility of accurate inventory data throughout the entire supply chain, from manufacturer to warehouses to retail stores, brings opportunities for improvement and transformation in various processes of the supply chain. This chapter proposes a basic understanding of the RFID technology and deals with its impact on supply chains in order to give an insight into the current issues and status of the technology. The presentation and discussion will help to better understand what RFID can deliver. This chapter is based on qualitative studies providing business cases for RFID deployments.

Key words *Radio Frequency Identification (RFID), supply chain management, qualitative analysis.*

1.1 Introduction

The need to present more valuable service to customers and, at the same time, decrease the cost of logistic processes are among the main objectives of supply chain management ([15]). Ever since barcode become dominant standard in the last century there were many theorists and practitioners who realized that there are great limitations to its use and further development (Wolff [16] and Kärkkäinen and Ala-Risku [17]). In fact, the barcode technology has a number of limitations and does not meet today's needs for several reasons:

- *Damage*: Bar codes are prone to damage. Because they have to be placed on the outside of a package they can easily be physically destroyed. Additionally, in warehouses and during distribution, grease and dirt can make them difficult to read.
- *Human intervention*: Bar codes require human intervention to operate the scanning device that reads the codes. This need for close line of sight between the scanner and bar code constrains stock storage design and hence warehouse space allocation to ensure goods can be easily located.
- *Lack of information*: While bar codes have undoubtedly helped to deliver significant supply chain improvements by providing information which drives operational systems, they cannot be programmed and can only provide the most basic product number information.

Using radio waves was in many ways superior to what barcode was able to provide to its users. RFID technology is the use of radio frequencies to read information on a small device known as a tag (Das [18]). Good abilities of radio waves and there attributes were well known so they had numerous applications like: radio broadcasting, wireless telegraphy, telephone transmission, television, radar, navigational systems, and space communication. Even though it was clear very early that radio waves can find great application in supply chains, retail industry and elsewhere in business environment, the major obstacles were price and undeveloped technology. The technology, in the beginning, was not developed enough to allow feasible application.

Roots of RFID go as far as 1940's and 50's when the principle that RFID is based on, was first used in aircraft Identification Friend or Foe systems. Appendix A.1 provides a brief description of the history of the RFID technology.

This chapter proposes a basic understanding of the RFID technology. We first present the main components of this technology in Section 1.2. The impact of the RFID technology on the performance of the supply chain is analyzed in Section 1.3. Section 1.4 is concerned with the cost associated with the RFID system. Finally Section 1.5 concludes the chapter.

1.2 RFID System Components

A basic RFID system is composed of three main components: the tag, the reader and the middleware. The RFID tag is a tiny microchip or an integrated circuit with an antenna attached and embedded into labels. The antenna enables the chip to transmit the label's identification information to a reader. When the reader is prompted, the tag broadcasts the information onto its chip. The reader converts the radio waves reflected back from an RFID tag into digital information that can then be passed on to computers or computer mainframes. There the information is collected, sorted, and converted into relevant data (Cf Figure 1.1).

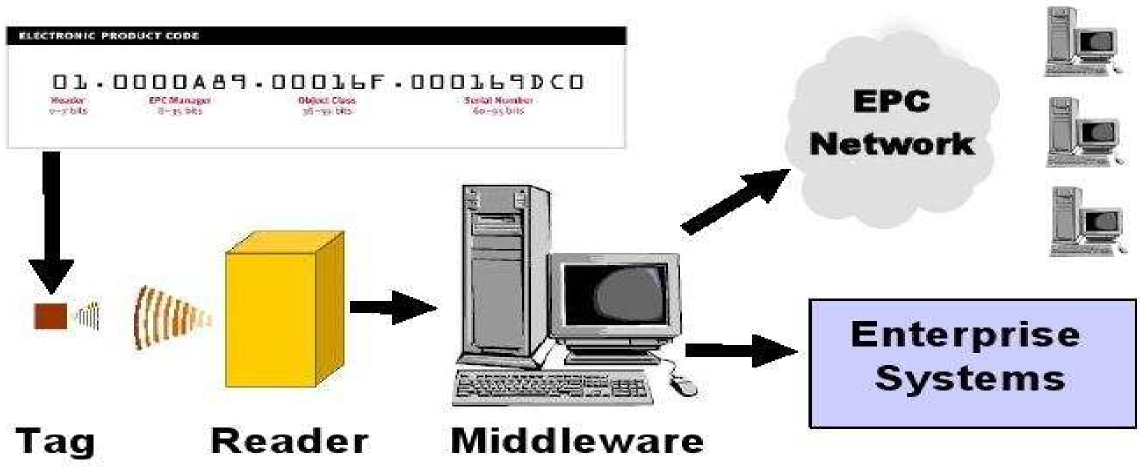


Figure 1.1: RFID System Components, source: Confino and Elmore [4]

The central data feature of RFID technology is the Electronic Product Code (EPC) (illustrated in Figure 1.2[1]), which is viewed by many in the industry as the next generation barcode or Universal Product Code (UPC)[2]. This EPC code can carry more data, than the UPC code and can be reprogrammed with new information if necessary. Like the UPC, the EPC code consists of a series of numbers that identify the manufacturer and product type. The EPC code also includes an extra set of digits to identify unique items.

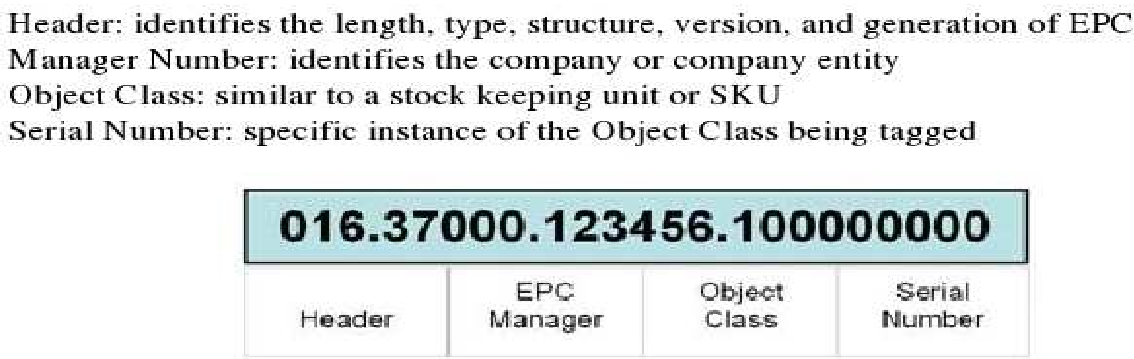


Figure 1.2: The Electronic Product Code

Information collected by RFID readers must be correctly interpreted before it is passed to an application

[1] source: http://www.EPCglobalu.s.org/Network/Electronic%20Product%20Code.html

[2] The Universal Product Code (UPC) is one of a wide variety of bar code languages called symbologies. it encodes twelve decimal digits as SLLLLLLMRRRRRRE

system. When multiple tags are within the reader's transmission range, the result is a set of responses that must be managed and processed in an orderly manner. This is the job of control software and middleware that resides on data capture devices or on specialized controllers and servers. The middleware can be in relation with an EPC network. The EPC Network is a suite of network services that enable the sharing of RFID-related data throughout the supply chain (Versign [19]). Note also that the three RFID components, i.e. the tag, the reader and the middleware, combined with the EPC network are defined as the Auto-ID technology.

1.3 What Value RFID brings into the Supply Chain

In Appendix A.2 we present some applications of the RFID technology and case studies of its deployment. The RFID system allows supply chain actors to efficiently collect, manage, distribute, and store information on inventory, business processes, and security controls. RFID allows: i) retailers to identify potential delays and shortages, ii) grocery stores to eliminate or reduce item spoilage, iii) toll systems to identify and collect auto tolls on roadways, iv) suppliers to track shipments, and in the case of critical materials, RFID allows receiving authorities to verify the security and authentication of shipped items.

The technology itself offers several improvements over its predecessor technologies the barcode and magnetic stripe cards. As mentioned in Section 1.2, the EPC can carry more data, than the UPC (Universal Product Code) code and can be reprogrammed with new information if necessary.

Among a large number of white papers and reports published in the last few years, most of them are qualitative studies providing business cases for RFID deployments. For example, IBM Business Consulting Services have published a series of papers (Alexander et al. [11], Alexander et al. [12], Alexander et al. [13] ,Alexander et al. [14]) on discussing the impact of RFID technology on supply chain performance with a focus on consumer goods and retail value chains. Topics of the white paper series range from analyzing the benefits of RFID in terms of improving product availability at the retail shelf, reducing losses associated with product obsolescence, product shrinkage, as well as the inventory inaccuracy, to articulating how RFID would affect the distributions centers and store replenishment policies to achieve better customer services and at the same time reduce the inventory cost. Other reports of a similar nature include Agarwal [20] and Kambil and Brooks [21].

According to Avhad and Ghude [7] , the basic benefits that RFID brings to the supply chain are automated real-time data capture related to product information, status information, location and environment status information. As a consequence, RFID provides a real-time view of how goods are moving through the supply chain, thereby dramatically improving the supply chain visibility, and opening up opportunities for unprecedented gains in the operational efficiency for any organization connected to the supply chain. The authors classify the benefits arising from the deployment of the RFID technology for different actors of the supply chain (the manufacturer, the retail distributor, the retail store and the freight transporter). For each supply chain actor, Table 1.1 summarizes the potential benefit of the RFID technology according to Avhad and Ghude [7].

From an other point of view focusing on the impact of the RFID technology in reducing the source of uncertainties in decision-making processes that hinder optimal supply chain performance, the anal-

Benefit	The benefit is enabled by
The manufacturer	
- Prevention of wrong production runs - Measure of actual WIP - Ensuring compliance with standards	Tagging material through the manufacturing process
- Improved demand planning - Improved availability - Reduce excess / safety inventory - Identify counterfeit products	Enhanced real-time visibility in the distribution chain
- Audit trail for key products - Improved recall management	Ability to track products from raw material to finished product in the retail store
- Increase asset utilization - Reduce production quality errors	Tracking location, condition and relevant parameters of assets
The retail distributor	
- Reduce labor costs for warehousing processes - Increase in warehouse processing accuracy and throughput - Increase in inventory accuracy - Speed up physical inventory process	Automated data capture and compare
- Reduce thefts, misplacement and misrouting - Optimize work processes to increase productivity	Tracking goods handled
- Increase on-time deliveries - Reduce inventory levels and safety stocks - Reduce inventory of obsolete products	Enhanced real-time visibility in the supply chain
- Increase asset utilization	Tracking location, condition and contents of assets
The retail store	
- Reduce labor costs, paperwork and quantity reconciliation - Increase in inventory accuracy - Speed up physical inventory process - Increased customer service levels	Automated data capture and compare
- Reduce thefts, misplacement and misrouting - Better handling of date sensitive inventory - Better returns management and warranty authentication	Tracking goods handled
- Better replenishment / re-order control - Reduce inventory levels and safety stocks - Reduce inventory of obsolete products - Higher sales	Enhanced real-time visibility in the supply chain
The freight transporter	
- Better control over shipment consolidation - Reduced labor and increased throughput - Better shipment planning - Compressed shipping times - Faster and efficient customs clearances	Automated data capture and compare
- Enhanced security during shipping - Audit trail for tracing shipments	Tracking goods handled
- Better delivery reliability and efficiency - Better route planning	Enhanced real-time visibility in the supply chain
- Increase asset utilization - Optimize asset inventories	Tracking location, condition and contents of assets

Table 1.1: Benefits of the RFID technology in the supply chain according to Avhad and Ghude [7]

ysis of Sahin [6] highlights the major benefits of using the RFID [3] technology in the supply chain. The author first characterizes the main sources of uncertainty within a supply. Those sources can be summarized in the four following points:

- Inherent factors causing fluctuations: this factors may cause fluctuations in $i)$ processes such as the production, the distribution and the reverse logistic processes, $ii)$ products and more generally components, semi finished or end products, and finally fluctuations can also concern the $iii)$ customer demand.
- The uncertainty on data captured from physical transactions: this kind of uncertainty may concern one of the four dimensions of data quality which are $i)$ the data accuracy, $ii)$ the data capture delay, $iii)$ the data granularity and $iv)$ the data availability.
- The uncertainty on the configuration of the supply chain and the deployment of resources: this uncertainty concerns structures, facilities, parties involved and the roles they perform in the supply chain.
- The uncertainty on supply chain control structure: here the uncertainty may be associated with decision process' delays or the quality of decisions.

Then, the author in Sahin [6] identifies three major properties of the RFID technology:

- The automatic identification property: as described before, tags do not need a particular positioning or physical contact with the reader. According to Sahin [6], the implication stemming from this property are its ability to $i)$ reduce identification and data capture delays and $ii)$ provide accurate information about the entities
- The item identification property: this property concerns the assignment of a unique identification number to individual logistical entities which allows a monitoring at unique item level.
- The information sharing property: the information sharing infrastructure associated with the RFID technology enables the exchange of data between the different supply chain actors.

By combining the sources of uncertainty in the supply chain and the above properties of the RFID technology, Sahin [6] deduces the major benefits associated with the deployment of this technology in the supply chain which are summarized Table 1.3.
In his PhD dissertation, Tellkamp [22] derives a conceptual framework that analyzes the impact of Auto-ID technologies on process performance. Behind the term Auto-ID technologies, the author uses the definition of McFarlane and Sheffi [23] who define Auto-ID as the 'automated extraction of the identity of an object' and considers that those technologies support two common goals: they intend to eliminate errors in the identification and data collection process and reduce the time for data capturing. The proposed framework in Tellkamp [22] distinguishes between automational, informational and transformational effects of Auto-ID technologies.

[3] The term Auto-ID is used by the author for the RFID technology and the EPC network

Using Auto ID reduces the uncertainty on		Nature of Benefit	Benefit Description	The benefit is enabled by		
				automatic identification	item level identification	an infrastructure to share information across the SC
Inherent characteristics of processes, products, demand	production/ distribution/ store/reverse logistics processes	hard	A reduction of labor cost due to the elimination of non value added control activities	+		
		hard	A reduction of the cost of delivery disputes	+		+
		hard	A reduction of losses pertaining to returns	+	+	+
		hard	A reduction of profit losses due to a faster detection of oos situations	+		
		soft	Improved reliability of production quantity and quality	+	+	
		soft	Improved efficiency of product recalls and enhanced consumer safety		+	+
		soft	A better management of after sales service		+	
		soft	Improved efficiency in recycling products		+	
	products (components, semi finished and end products)	soft	A better management of perishable items	+	+	
		soft	An enhanced control of counterfeited items		+	+
	demand	soft	An improved knowledge of consumer behavior	+		
Data (pertaining to supply chain entities) used for decision making		hard	An accurate data concerning supply chain entities (semi finished or end products, containers, pallets, tools, spare parts, other means, employees)	+		+
		hard	Reduced data capture delays	+		
		hard	An item level product data		+	
		soft	A better management of the SC (cost reduction-especially inventory holding cost- and improvement of service level)	+	+	+
Supply chain configuration		soft	An increased visibility over the supply chain network	+		+
		soft	Measurement of supply chain performance metrics	+		+
		soft	Potential re-allocation of the roles employees perform	+	+	
Supply chain control structure		soft	An enhanced coordination and opportunity to redesign policies (eliminate or reallocate processes) for improved effectiveness	+	+	+

Table 1.2: Benefits of the RFID technology in the supply chain according to Sahin [6]

- Automational effects occur if companies use the technology to reduce data capturing cost.
- Informational effects occur when the technology leads to an increase in data quality.
- Transformational effects occur when the technology acts as an enabler and allows companies to realize new processes that were not economically worthwhile before.

In the first two instances, a new Auto-ID technology such as the RFID technology merely substitutes the existing data capturing technology. It acts as an alternative means to implement the current process. The proposed framework recognizes that contextual factors and complementarities can affect the impact of an Auto-ID technology. Organizational and environmental circumstances can limit the value that companies can derive from adopting the technology. Furthermore, in order to realize the value, companies must have certain complementary technologies and practices in place or have to invest in them. Based on the results of projects with a number of companies in the FMCG (Fast-Moving Consumer Goods) industry, Tellkamp [22] describes how companies intend to apply RFID at the case and pallet level. The project work highlights that the adoption of RFID might faster changes in the FMCG industry related to RFID, but for which RFID is not a prerequisite. In these instances, RFID acts as a catalyst, i.e. it helps companies to realize benefits in the supply chain that do not rest on the capabilities of RFID, and as a consequence may lead to additional transformational effects. Then, the author identifies a number of contextual factors as well as complementary technologies and practices that can affect the value of RFID. The research points out that, due to country-, process-, strategy- and product-related circumstances, some companies may find it more difficult to benefit from RFID, while others may have to invest in complementarities before they see an improvement in operational performance.

1.4 The RFID system costs

The cost of acquiring, installing, and maintaining an RFID system is a major and determining factor in the deployment of this technology. There appears to be great diversity and little quantitative information in the overall costs of acquiring, installing, supporting and maintaining an RFID system. RFID system cost is composed of tags, readers, and processing and supporting information technology hardware and software. Higher adoption rates will cause system costs to drop and encourage more RFID users (Commerce [24]).

At present, larger retailers such as Wal-Mart, Target, Albertsons, and manufacturers like Hewlett Packard, Gillette, and Proctor and Gamble, are leading the industrial deployment of RFID.

According to Commerce [24], current tag costs range from 25 to 40 cents per tag (higher in some cases, depending upon the type of tag), making it relatively expensive for low-end consumer items. The Auto-ID Labs[4] (former Auto-ID Center) expects tag prices to drop to 10 cents in 2005 and 5 cents in 2006 for orders of 1 million units[5]. Sarma [25] describes how to reduce the costs of the tag from one dollar to five cents. He explains that the current chip design is optimized to produce better and better chips in

[4]http://www.autoidlabs.org

[5]SRI Consulting, 'RFID Technologies', 2004

the same size and for the same price. In an other article (The 5-Cent Challenge, 2004), RFID Journal[6]'s analysis indicates also that the price of simple license-plate tags will fall to 5 cents in 2007.
Finally, middleware costs include computer hardware, software, data processing, data mining, personnel salaries, and personnel training. Information technology consulting firm AMR Research estimates that a consumer products company shipping 50 million cases a year could spend upwards of $20 million for RFID implementation (Goff [26]). It is because of these associated costs that retailers currently using RFID are applying tags at the pallet or case level, rather than at the individual item level. Currently, few tags are used in the consumer marketplace, and most tags planned for introduction in the next few years are for high-value or high-cost items such as electronics, designer apparel, cosmetics, jewelry, etc. Most industry analysts predict that as RFID enters the mainstream marketplace and its volume increases, system costs will drop. Many companies report that RFID is extremely costly to use in their supply chains. However, some suggest the initial investment will generate a large return on investment from the benefits mentioned above. Companies may also be able to reduce some costs by re-using the tags at the point of sale. Some suggest that to generate real 'economic' value from RFID, companies must look beyond inventory control and asset tracking, and use it to gather intelligence that enables them to interact better with customers and streamline processes throughout the organization. However, this could lead to other problems, such as concerns about consumer and employee privacy (Commerce [24]).

1.5 Conclusion

In this chapter we proposed a basic understanding of the RFID technology, its components, its fields of application. We focussed on its impact on supply chain management systems in order to give an initial insight into the current issues and status of this technology. The analysis presented in this chapter was based on a selection of qualitative studies providing business cases for RFID deployments. We conclude that this technology would have important impacts on inventory systems. One of the aim of the rest of this dissertation is to quantify the impact of the RFID technology on inventory systems subject to inventory inaccuracies. In the following chapter, we will introduce the inventory inaccuracy issue and provide the qualitative and the quantitative literature associated with this issue.

[6] http://rfidjournal.com

Chapter 2

A literature review on the issue of inventory inaccuracy

The inventory Information Systems, contrary to popular belief and assumptions in most academic papers and in spite of the considerable amounts invested in information technology, are often inaccurate. The inventory inaccuracy occurs when the Information System inventory is not in agreement with the actually available inventory. This chapter proposes a literature review on the issue of inventory inaccuracy. We first present qualitative and empirical investigations dealing with problems perturbating the information system and the physical flows. We highlight the sources of inventory inaccuracy problems and ways in which inventory managers can address the issue through a combination of compensation methods such as technology, inspection policies, and process improvements. We then present main quantitative investigations addressing the inventory inaccuracy issue. In particular we propose three main levels permitting to classify these investigations. This classification permits us to have a visibility on topics where there is a lack of quantitative models allowing the analysis of the impact of inventory inaccuracy problems on the supply chain performance.

Key words *Inventory inaccuracy, literature review*

2.1 Introduction

Record inaccuracy has been observed in many different area such as the context of manufacturing and distribution (Wight [27]), investment banks and brokerage houses (Report [28]), government agencies (Laudon [29]), and phone and utility companies (Redman [30], Knight [31]). Thought this dissertation, the focus is steered on inaccuracy problems in inventory systems.

The standard literature on inventory models has rarely differentiated between the inventory record and the physical inventory. The two have always been considered to be the same and the main concern was on how, having observed demand and the resulting inventory levels, an inventory manager should determine when and how much to replenish. Based on recent empirical observations this implicit assumption has proven to be wrong. In fact, based on a study done with a leading retailer, Raman et al. [8] reports that out of close to 370,000 SKUs investigated, more than 65% of the inventory records did not match the physical inventory at the store-SKU level. Moreover, 20% of the inventory records differed from the physical stock by six or more items.

A general definition of accuracy includes obtaining the correct value for a measurement at the correct time (Schuster et al. [32]). According to Iglehart and Morey [33] and DeHoratius and Raman [34] , inventory inaccuracy occurs when the system inventory, i.e., what, according to the information system, is available, does not match the physical inventory, i.e. what is actually available. Various other definitions going in the same sense and measures of inventory accuracy are presented in Ernst et al. [35], Buker [36], Bernard [37], Chopra [38], Young [39] and Martin and Goodrich [40]. For example, Ernst et al. [35] proposes using a control chart to monitor the changes in the inventory accuracy. An other definition provided by Bernard [37] considers the percentage (and not the difference) error in the inventory records. Martin and Goodrich [40] define accuracy as the total dollar deviation between the actual dollar value of inventory and recorded dollar value of the inventory. As a conclusion of the last provided definitions, we say that an inventory stock is inaccurate when the record stock is not in agreement with the physical stock.

This chapter proposes a review of literature on the issue of inventory inaccuracy. We first provide a comprehensive analysis on factors generating errors in Section 2.2 where we also focus on the impact of these errors on the performance of the inventory system. The question ”how to cope with the inventory inaccuracy?” will be answered in Section 2.3. In the second part of this chapter, we consider the quantitative investigations dealing with the inventory inaccuracy issue. We begin by classifying these investigations in Section 2.4.1 and then we provide a brief description of some quantitative models in Section 2.4.2. Finally Section 2.5 concludes the chapter.

2.2 Factors generating errors and their impacts

Inventory inaccuracy can be a major obstacle to improvements in firms’ performance (Kök and Shang [41]). While companies have undertaken large investments to automate and improve their inventory management processes, inventory information system and physical inventory are rarely aligned (Raman et al. [8]). Inventory inaccuracy might result from several factors. The aim of this section is to present a comprehensive analysis of the factors generating inventory inaccuracy. Based on empirical and qualitative investigations, we try to focus on the order of magnitude of these errors.

2.2.1 Transaction errors

Transaction errors are unintentional errors occurring during inventory transactions. Some of these transactions happen when counting the inventory, receiving an order or checking out at the cash register. Errors when checking out occurs if the cash register scans one item twice, rather than each item separately, when a customer is buying two similar (but not identical) items with the same price. This innocuous action by the cash register ensures that the customer pays the correct amount and may even save the customer time as the cash register avoids handling the additional item. However, it generates a discrepancy in the inventory information system. Errors when picking impacts inventory records similarly. A warehouse employee can unintentionally ship the wrong quantity of a particular item to a store or even send the wrong item altogether. According to DeHoratius and Raman [34], in an apparel warehouse, it is quite easy for an employee to mistakenly pick a 'medium' instead of a 'large' garment. Stores typically do not scan merchandise on receipt allowing for such errors to remain invisible. In her PhD dissertation, Sahin [6] provides a comprehensive analysis of inventory systems subject to perturbations in nominal flows. According to the author, the major defects resulting in transaction errors are:

- The technical limitations of the bar code system which were discussed in Chapter 1.
- The potential failures stemming from the interaction between inventory operators and the bar code system. Those errors result in $i)$ errors made when identifying entities, $ii)$ errors made when counting and $iii)$ errors made when keypunching data.

An other interesting analysis conducted by Dehoratius [5] highlights the source of inventory record inaccuracy problems in retailing and ways in which retailers can address the problem. By understanding the steps whereby inventory arrives at retail stores, the author provides the errors that can occur in each step. The summary of this analysis is given in Figure 2.1 which provides a depiction of the retail replenishment process and the inventory accuracy pitfalls. (for further details the reader is referred to Dehoratius [5]).

Concerning the impact of transaction errors, we note that there is some empirical data available in the context of retail stores. Kang and Gershwin [42] report the results of a study conducted by a global retailer in several hundred of its stores. Inventory records were accurate for 51% of SKUs, and for 76% of SKUs, the deviation between physical inventory and inventory records was within a range of $\pm$ 5 units. This means that for close to one in four SKUs, inventory records deviated from physical inventory by six or more units. In an other empirical study conducted by DeHoratius and Raman [34] who examined inventory record accuracy at a multi-billion-dollar retailer, the authors found that the absolute difference between inventory records and physical inventory was on average close to five units. For 15% of SKUs, it was above eight items. This compares to an average target inventory of 14 units per SKU. Average inventory inaccuracy varied considerably by store, with a minimum of 2.4 units per SKU and a maximum of 7.9 units. The same investigation suggests that factors such as higher

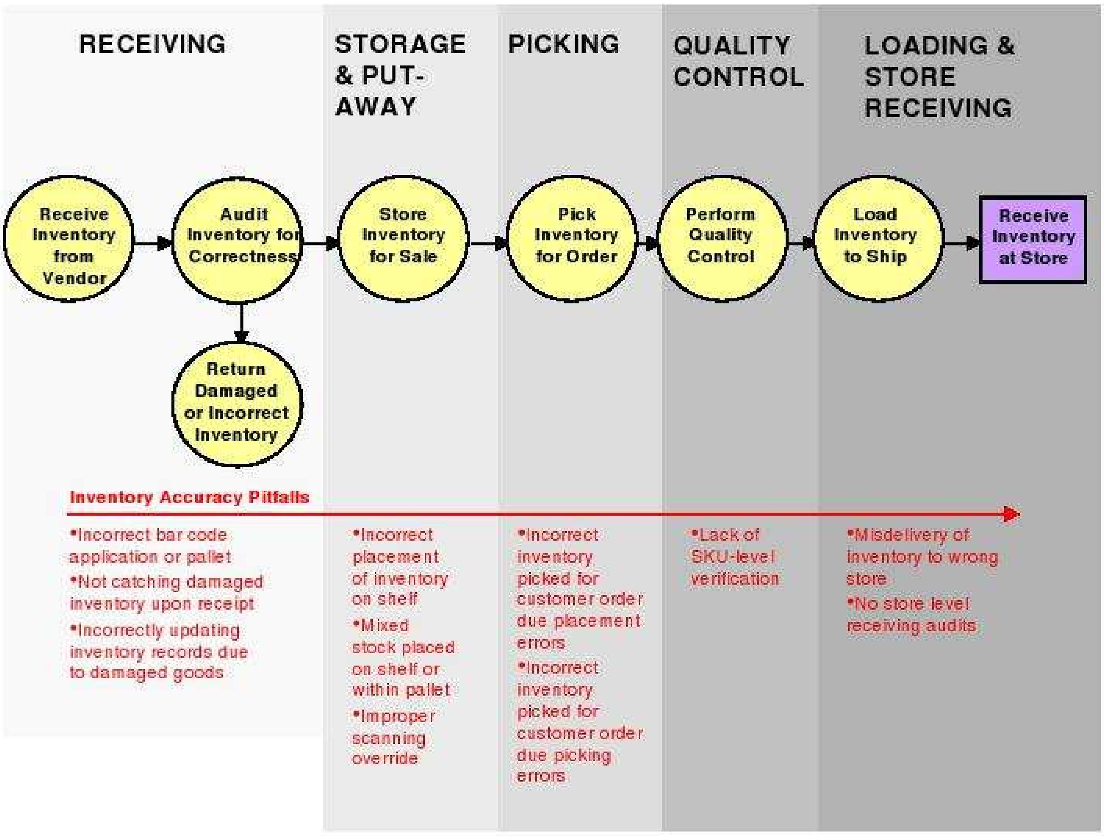


Figure 2.1: Retail replenishment process and the inventory inaccuracy pitfalls: source Dehoratius [5]

selling quantity, inventory density, and product variety are associated with higher levels of inventory record inaccuracy.

2.2.2 Misplacement errors

Misplacement errors occurs when a fraction of the inventory is misplaced, it is not available to meet a customer demand until it is found. According to Chappell et al. [43], there are several sources generating misplacement errors such as: i) Consumers picking up products and then putting them down in another location, ii) Clerks not storing products on the correct shelf at the right time and iii) Clerks losing products in the backroom. From a four-year longitudinal study of 333 stores of a large retailer, Ton and Raman [44] show that increasing product variety and inventory level per product is associated with an increase in misplaced products. The authors also show that increasing misplaced products is associated with a decrease in store sales. According to Çamdereli and Swaminathan [3], misplaced inventory is also present in warehouse operations. G.T Interactive, the creator of computer games like Doom II and Driver, suffered from low productivity due to inventory misplacement in the warehouse. Fundamentally what happens in these settings is that the product is misplaced in the supply chain and is unavailable during the sales period but can be retrieved when a cleanup is performed.

Misplaced inventory can be quite large and have a significant impact on the inventory performance. It is reported in Raman et al. [8] that customers of a 'leading retailer' cannot find 16% of the items in the store because of misplacement errors. The consequence is that misplacement errors reduce the profit by 25% at this retailer.

2.2.3 Damage and spoilage

For supermarkets, perishables are the driving force behind the industry's profitability and represent a significant opportunity for improvement, accounting for up to $200 billion in U.S. sales a year but subjecting firms to losses of up to 15 % due to damage and spoilage (Ketzenberg and Ferguson [45]). Examples with limited lifetime products are drugs and food products. In retail stores, customers can cause damages to products and as a consequence making them unavailable for sales. Some examples are tearing of a package to try on the contained cloth item, wearing down a shoes by trying it on and walking, erasing software on computers on demonstration, spilling food on clothes, and scratching a car during a test drive (Bensoussan et al. [46]). Those damage may not be detected by the inventory manager and as a consequence would cause inventory inaccuracy.
According to Sahin [6], items reaching their limit lifetime during storage is due to:

- Errors where forecasting the customer demand which may lead to an overestimation of this demand and as consequence an important quantity of products which is not sold.

- The inability to track accurately the location, condition and the age of products stored within a facility.

An industry survey performed by the Joint Industry Unsaleables Steering Committee (Lightburn [47]) provides data on the level of unsaleables in the US retail industry. According to the survey results, which are based on responses from over 60 manufacturers and retailers, the cost of unsaleable food and grocery products amount to 1% of sales in the US. Damage is the biggest cause of unsaleables with 63% of all unsaleables, followed by expired (16%) and discontinued items (12%). The rate of unsaleable products can differ by product category: frozen products, for example, reported an unsaleable rate of 0.9%, whereas the rate for refrigerated products was 1.7%. Unsaleable rates for health and beauty care and general merchandise had even higher unsaleable rates (1.9% and 2.2%, respectively), which was attributed to frequent new product introductions, shifts in fashion component, seasonality, and short shelf life.

2.2.4 Theft

Inventory theft is defined as a combination of employee theft, shoplifting, internal and external theft, vendor fraud and administrative error. The ECR[1] Europe project on shrinkage subsequently analyzed the causes of stock loss and proposed a systematic and collaborative approach to reducing the phenomenon throughout the supply chain. ECR defines 'Shrinkage' as the process errors, deceptions and internal and external thefts. according to Beck [48], some specific types of internal theft include:

[1] http://www.ecr.org/

- Staff stealing goods by either hiding them in their bags or intentionally placing them outside the building for later collection
- Collusion occurring when a staff member collaborates with a customer to steal products.During such incident, the staff member may not scan the item or the security personnel may intentionally ignores the offense as it occurs
- Grazing occurring when items stored in the warehouse are consumed by the warehouse staff

The results of the research carried by ECR Europe have shown that the scale of shrinkage in fast moving consumer goods sector is estimated to 24 mld EUR in 2003 (465 mln EUR is lost irreparably within fast moving consumer goods turnover weekly), which is 2,41% of the whole turnover value of the sector. The process errors present 27% of the whole shrinkage value, 7% deceptions, 28% internal thefts and 38 external thefts.

Based on survey data, internal and external theft, administrative errors and vendor fraud accounted for an estimated 1.8% of sales in the US retail industry in 2001, costing US retailers USD 33 billion (Hollinger and Davis [49]). For US supermarkets, the NSRG survey Supermarket [50] estimates that internal and external theft, receiving errors, damage, accounting errors and retail pricing errors amount to 2.3% of sales. These figure only take into account the item value, but not any process-related costs (e.g. for handling of damaged items).

2.2.5 Supply errors: product quality, yield and supply process

When the product quality is low or a production process has a low yield or a supply process is unreliable, the physical inventory may not be known and as a consequence may be different from the inventory in the information system (Yano and Lee [51] and Rekik et al. [52]). Products that are not conforming to quality standards can also make the inventory inaccurate. According to Bensoussan et al. [46], receipts are usually added to the inventory without a full inspection process. The consequence is that the information system may consist of both non defective products and defective products which are not available for sales.

2.3 How to cope with inventory inaccuracy

To cope with inventory inaccuracy, different compensation methods can be used. The analysis conducted by many investigations (Uckun et al. [53], Kang and Gershwin [42], Sahin [6] and DeHoratius et al. [54]) agreed that i) making decisions by considering the inventory inaccuracy may be applied to tackle with the problem of inventory inaccuracy and ii) RFID technology may help to track items through the supply chain. As well as reducing the inventory inaccuracy, the technology may also help to eliminate the reasons of inventory inaccuracy such as theft.

For the specific case of misplacement errors, inventory managers may apply other methods to tackle the problem of misplaced inventory. For example, in some apparel departments of retailers there are signs informing customers not try to return the product to their original location if the customer decides not to buy the product after trying it. Some libraries cope with the misplacement problem by putting

signs which tell the customers not to reshelve the books after use while others have designated spaces for returning books taken off the shelf (Çamdereli and Swaminathan [3])
The analysis conducted by Kang and Gershwin [42] examines various other techniques that inventory managers can use to compensate for the inventory record errors. According to the authors, the compensation techniques can be summarized in the following three points:

- Verifying manually the inventory: the inventory manager can choose the items in the facility in order to perform a manual period counting. This frequency of counting may depend on various elements, such as the availability of the labor and product characteristics, including the profit margin, sales velocity, and whether the products are highly prone to errors.
- Performing a manual reset of the inventory record: this technique is used if a direct measurement of the physical inventory is not available, inventory managers can gather and monitor the available data and search for any patterns that may be indicative of the presence of serious inventory error.
- Performing a constant decrement of the inventory record: this technique is performed if the inventory manager is aware of the presence of errors and also knows their stochastic distributions. The inventory manager may decrement the inventory record by the average of the error each period. Since the physical value of the error realization at each period is unknown, performing this constant decrement will still not eliminate the error in the inventory record. However, over time, this corrective action can be expected to perform better than leaving the inventory record unadjusted.

In her PhD dissertation, Sahin [6] proposes a set of a actions aiming to eliminate errors. These actions can be summarized in the following points:

- Re-engineering the physical organization of the facility
- Using a new product identification technology that reduced scanning errors.
- Using a technology that enables to reduce theft in the facility.
- Using a technology to accurately track products' sell by dates.
- Performing a double receiving and shipment processes.
- Improving the actual processes in the facility.
- Performing benchmarking analysis and developing personnel awareness building actions that focus on the operational weaknesses.

DeHoratius et al. [54] add two additional ways an inventory manager may respond to inventory inaccuracy problem:

- Prevention: reduce or eliminate the root causes of inventory record inaccuracy through the implementation and execution of process improvement.

- Correction: identify and correct existing inventory record discrepancies through auditing policies.

2.4 A quantitative review of literature on the inventory inaccuracy issue

There is little research in the supply chain and inventory management literature that deals with the impact of inventory inaccuracy. The inventory inaccuracy issue became apparent due to the development of RFID technology (Kang and Gershwin [42]). There has been a renewed interest in inventory inaccuracy and research so far has focused on one or a combination of the four following main issues:

1. Issue 1: determining appropriate inventory counting policies (when to conduct inventory counts, how much to count).
2. Issue 2: determining how to adjust safety stocks and replenishment policies in order to adjust for inventory inaccuracies.
3. Issue 3: examining the parameters that influence the impact of inventory record inaccuracies on product availability and other performance measures.
4. Issue 4: studying the root causes of inventory inaccuracy and their influence on supply chain performance.

The majority of investigations dealing with the fourth issue are qualitative studies providing the factors generating errors in inventory systems. Most of those investigations were cited in the first part of this chapter (especially in Sections 2.2 and 2.3). Our aim in this section is to present a comprehensive analysis and to propose a classification of the quantitative publications dealing with the first three issues above.

2.4.1 A classification of the quantitative investigations addressing the inventory inaccuracy issue

We classify the investigations dealing with the inventory inaccuracy issue based on three main levels:

- **Level 1**: is related to the *objective* of the investigation.
- **Level 2**: is related to the structure of the *supply chain* studied in the investigation.
- **Level 3**: is related to the structure of the *error(s)* causing the inventory inaccuracy issue.

We now detail each level and define sub-levels for each one:
Level 1 is composed of two sub-levels:

- **Sub-level 1.1** - *the issue considered*: we agree that a first sub-level of classification of the literature review dealing with inventory inaccuracy is made based on the issue or the issues among the ones proposed above (Issue 1,2,3) which is or are considered.

- **Sub-level 1.2** - *evaluation versus optimization*: the second sub-level deals with the way the problem of inventory inaccuracy is resolved. We distinguish two ways: 1) Some investigations try to **evaluate** the impact of inventory inaccuracy through empirical studies or through simulation analysis and 2) The other investigations try to **optimize** the inventory system subject to inaccuracy problems.

Level 2 is composed of three sub-levels:

- **Sub-level 2.1**-*Retail Store versus Warehouse context*: From inventory control point of view, the difference between the two contexts is the stemming from i) in a retail store context the customer demand is confronted to the physical available for sales inventory since the customer is physically present in the retail store ii) in a warehouse context, customer demand may be confronted to the Information System inventory at a first stage and a commitment may be done based on the level observed in the IS inventory. Then, the commitment may be confronted to the physical inventory when delivering the order. As a consequence, in a warehouse context it may exists an underage penalty that does not occur in a retail store context. This penalty is due to orders initially accepted by the warehouse inventory manager but finally not delivered because of the non agreement between the IS inventory and physical inventory in the moment of the commitment.

- **Sub-level 2.1**-*the framework considered*: the inventory framework can be considered as a sub-level in the structure of the studied supply chain. The main studied frameworks are the Newsvendor problem for the single period framework and the Economical Order Quantity model, the Period Review model and the Continuous Review model for the multi-period frameworks.

- **Sub-level 2.1**-*Centralized versus Decentralized Supply Chain*: this level concerns the number of actors considered in the supply chain. In a centralized supply chain, a unique decision maker is concerned with maximizing the entire supply chain's profit; in a decentralized supply chain two or more actors act as different parties and each one tries to maximize his own profit. In this sub-level we can also distinguish the nature of the demand: deterministic versus stochastic.

Level 3 is composed of three sub-levels:

- **Sub-level 3.1**-*The error nature*: first, let recall that the components composing an inventory system are i) The PHysical (PH) inventory, ii) The Information System (IS) inventory and iii) The decision system

  From an inventory control point of view, we think that all the factors generating the inventory inaccuracy issue (discussed in Section 2.2) can be modelled by four types of errors:

  - **Theft** type errors: those errors affect the physical inventory level and let the Information System inventory unchanged
  - **Misplacement** type errors: this type of errors affects temporally, i.e. during the selling season, the physical available for sales inventory and let the IS inventory unchanged

  - **Transaction** type errors: transaction errors affects the IS inventory and let the physical inventory unchanged

  - **Yield** (supply) type errors: those errors may affect both the IS and the physical inventory if no inspection is performed when receiving the order

  We notice that, the literature related to the last point, also known as the random yield problem, is extensive and several models that incorporate the effect of yield uncertainty or supplier unreliability on the inventory policy have been developed (Shih [55], Gerchak et al. [56], Yano and Lee [51], Inderfurth [57], and Rekik et al. [52]).

- **Sub-level 3.2**-*The error setting- additive versus multiplicative versus mixt setting*: in a general setting, if we let Q the quantity ordered from the supply process, the physical and the IS inventory can respectively be written as the following: $Q_{PH} = \gamma_{PH}Q + \epsilon_{PH}$ and $Q_{IS} = \gamma_{IS}Q + \epsilon_{IS}$ where the couple of random variables $(\gamma_{PH}, \epsilon_{PH})$ $((\gamma_{IS}, \epsilon_{IS}))$ characterizes the errors on the physical inventory level (IS inventory level). From this general setting which is called the mixt error setting, one can distinguish two particular cases:

  - The additive error setting: in this case $Q_i = Q + \epsilon_i$ where $i \in [PH, IS]$

  - The multiplicative error setting: in this case $Q_i = \gamma_i Q$ where $i \in [PH, IS]$

  The way to determine the parameters associated with errors would consist in collecting data and performing statistical analysis on Q_{PH} and Q_{IS} of different selling seasons in order to characterize the error setting and the magnitude of this error. To understand the difference between the two settings, let consider the supply errors, occurring when the received quantity from a supply system is not the same as the quantity ordered. In the additive error setting, errors in the received quantity may stem from administrative errors made by the supplier recording for instance a 7 as a 9 in the ordering process. In this case, the variability of errors does not depend on the ordered quantity. In the second case, which is also known as *stochastically proportional yield* model in the literature (Yano and Lee [51]), the variability of errors varies with the ordered quantity. Factors such as theft during the supply process can probably be modelled in this way since the higher is the ordering quantity, the higher will be the variability of the quantity stolen.

- **Sub-level 3.3**-*Deterministic versus Stochastic error*: this sub-level concerns the variables representing the error. In some investigations, the error parameter was considered as deterministic in order to emphasize the impact of the average of the error on the performance of the supply chain performance.

Based of the last sub-levels, we are able to classify the quantitative investigations dealing with the inventory inaccuracy issue. The result of this classification is summarized in Table 2.1 where The following abbreviations are used:

Ev. : Evaluate	Add. : Additive
Op. : Optimize	Mul. : Multiplicative
Mi. : Misplacement	D. : Deterministic
Tr. : Transaction	S. : Stochastic
Th. : Theft	Mo. : Mono-Period
Yi. : Yield	Mu. : Multi-Period
R. : Retail	C.S. : Centralized Supply Chain
W. : Warehouse	D.S. : Decentralized Supply Chain

Sub-level N.	1.1			1.2		2.1		2.2		2.3		3.1				3.2		3.3	
Paper	1	2	3	Ev.	Op.	R.	W.	Mo.	Mu.	C.S.	D.S.	Mi.	Tr.	Th.	Yi.	Add.	Mul.	D.	S.
Iglehart and Morey [33]	*				*	*	*	*		*			*			*			*
Shih [55]		*			*	*		*		*					*	*	*		*
Noori and Keller [58]		*			*	*		*		*					*	*	*		*
Ehrhardt and Taube [59]		*			*	*		*		*					*	*	*		*
Gerchak et al. [56]		*			*	*		*	*	*					*		*		*
Lee and Yano [60]		*			*	*		*	*	*					*	*	*		*
Henig and Gerchak [61]		*			*	*		*	*	*					*	*	*		*
Yano and Lee [51]		*			*	*			*	*					*	*	*		*
Gaukler et al. [2]		*	*		*	*		*		*	*	*					*	*	
Kang [62]		*	*	*		*			*	*				*			*		*
Dehoratius [5]		*	*	*							*	*	*						
Ton and Raman [44]		*	*	*							*	*	*						
DeHoratius and Raman [34]		*	*	*							*	*	*						
Fleisch and Tellkamp [63]		*	*	*					*										
Inderfurth [57]		*			*	*		*		*					*		*		*
Kök and Shang [41]	*	*			*	*			*	*			*			*			*
Sahin [6]		*	*		*	*	*	*		*		*	*	*	*	*	*		*
Ton and Raman [64]			*	*															
Lee et al. [65]			*	*															
Kang and Gershwin [42]			*	*		*		*		*				*			*		*

Sub-level N.	1.1			1.2		2.1		2.2		2.3		3.1				3.2		3.3	
Paper	1	2	3	Ev.	Op.	R.	W.	Mo.	Mu.	C.S.	D.S.	Mi.	Tr.	Th.	Yi.	Add.	Mul.	D.	S.
Çamdereli and Swaminathan [3]		*			*	*		*		*	*	*					*	*	
Bensoussan et al. [46]																			
Uckun et al. [53]		*	*		*						*								
DeHoratius et al. [54]		*	*		*		*			*	*								*
Atali et al. [66]		*			*	*			*	*		*	*	*		*			*
Rekik et al. [67]		*			*	*		*		*	*	*					*	*	
Rekik et al. [52]		*	*		*	*	*	*		*					*	*	*		*
Rekik et al. [68]		*	*		*	*		*		*		*					*		*
Gauckler et al. [69]	*	*		*															
Tellkamp [22]	*	*	*																

Table 2.1: Classification of the qualitative investigations addressing the inventory inaccuracy issue

During the thesis, we progressively incremented this table which helped us to get an information about topics where there is a lack of investigations. We first remarked that the literature concerning the supply type errors is sparce. So we decided to begin with such type of errors but we quickly remarked that investigations dealing with this problem under a Newsvendor framework are not complete. Chapter 3 will provide a comprehensive analysis of this problem. Motivated by the lack of investigations dealing with misplacement type errors, we decided to consider this type of errors (Cf Chapter 4). Then we remarked a tendence for the analysis of inventory inaccuracy in decentralized supply chain. This motivated us to extend our model of Chapter 4 for a decentralized supply chain. As it can be observed, recently, the tendence is the analysis of the inventory inaccuracy issue in multiperiod frameworks.

2.4.2 Quantitative literature on the inventory inaccuracy

The aim of this section is to briefly describe the main quantitative investigations addressing the inventory inaccuracy issue.

The investigations of Iglehart and Morey [33] and Kök and Shang [41] are among the rare investigations which consider transaction errors. Iglehart and Morey [33] may be the first paper to discuss the inventory inaccuracy problem in a quantitative manner. The authors consider count frequency and base-stock level in order to minimize inspection and inventory holding costs subject to the probability that showing in-stock when the item is actually out of stock is less than a pre-specified target level. The inventory inaccuracy is due only to transaction errors. They consider a single-item, periodic-review inventory system with a predefined stationary stocking policy. In other words, they do not consider establishing an optimal replenishment policy. Instead they take the control policy such as (s, S) as given. Their objective is to establish an optimal buffer stock that protects against inventory inaccuracies and to determine an optimal frequency of physical inventory counts to correct the discrepancy between inventory record and actual inventory on hand.

Kök and Shang [41] studies the inventory replenishment problem with a counting policy of an inventory system subject to transaction errors. As in Iglehart and Morey [33], the authors assume that transaction error random variables are additive and are identically and independently distributed with zero mean. In particular, they consider a periodic-review, stationary inventory system in which transaction errors accumulate until an inventory count. The manager incurs a linear ordering, holding and backorder cost and a fixed cost per count. The objective is to decide whether to count or not and how much to order to minimize the total cost of ordering and counting.

By constructing a lower bound to the original dynamic program, the authors show numerically that an Inspection-Adjusted Base-Stock policy is close to optimal for a finite horizon problem (the optimality gap is on average 0.4%). The policy is such that if the inventory Information System is below a threshold, an inventory counting is requested to correct the errors and the optimal replenishment policy is a base stock policy. Otherwise, no inspection is performed and the optimal replenishment policy is also a base-stock policy with a level depending on the number of periods since the last inspection procedure.

The investigations of Yano and Lee [51], Rekik et al. [52], Shih [55], Gerchak et al. [56], Inderfurth [57], Noori and Keller [58], Ehrhardt and Taube [59], Lee and Yano [60], Henig and Gerchak [61], Silver et al. [70] deal with the random yield problem or the unreliability of the supply system. We notice that the literature in the area of random yield in sparse and many other investigations exist (Cf Yano and Lee [51]). The references above are the more related to the inventory inaccuracy issue. We intentionally do not detail those investigations since Chapter 3 of these dissertation will focus on the random yield problem and an extension of existing results will be provided.

The investigation of Kang and Gershwin [42] is among the rare investigations which considers errors caused by the shrinkage errors and its impact on inventory management through a simulation study. The authors illustrate how shrinkage increases lost sales and results in an indirect cost of losing customers (due to unexpected out of stock) in addition to the direct cost of losing inventory. The objective is to illustrate the effect of shrinkage on lost-sales through simulation. However, the authors provide some plausible methods to compensate for inventory inaccuracy.
In their (r,Q) policy, with a random demand and a random shrinkage error[2], the authors assume that stores do not know the exact value of physical inventory at the time of ordering. And as a consequence their model distinguishes between the Information system and the physical inventory. The sequence of events in each period is assumed to be as follows:

- The on-hand record inventory is reviewed and an order is placed to the supply system
- The incoming order is received after a known and fixed lead time
- Sales and shrinkage take place: the demand for purchase is assumed to be normally distributed and the demand for shrinkage is generated from a Poisson distribution
- Demand occurring at zero actual on-hand inventory is lost

The authors also assume that when the sum of demand for purchase and the shrinkage exceeds the available physical inventory, the available physical inventory is divided proportionally to meet the two demands.
The authors simulate the inventory system and show that even small inventory inaccuracy may lead to important stockouts. In fact, according to the simulation done by the authors, even when the shrinkage is as small as 1% of the average demand, the error accumulating in the inventory record is large enough to disturb the replenishment process and make 17% of the total demand lost. They also observe a continual rise in the gap between the curve of the IS inventory and the curve of the physical inventory and they show that the system reaches a point where the inventory record stays above the reorder point r and the consequence is that no order is placed. The authors refer to this situation as the replenishment freeze. In order to confirm analytically this result, the authors analyze the model for a deterministic demand and shrinkage and provide analytical expressions of various performance measures of the system such as the time of first out-of-stock, the time of replenishment freeze and the stock out value. The authors have also provided a sensitivity analysis where they investigate in what circumstances the

[2]The authors refer to the shrinkage error as the stock-loss.

In the second part of their paper, the authors propose several compensation methods in order to reduce the effect of the inventory inaccuracy issue. Those methods were presented in the qualitative part of this chapter (Cf Section 2.3).
As a response to the lack of investigations dealing with shrinkage errors with an optimization way, we provide in Chapter 7 an inventory model where inaccuracy is caused by theft errors. We optimize the replenishment decisions with two formulations $i)$ the underage and overage costs formulation and $ii)$ the service level formulation.

The investigation of Gaukler et al. [2], Rekik et al. [67], Çamdereli and Swaminathan [3] and Uckun et al. [53] consider a decentralized supply chain. The detailed analysis of these investigations will be presented in Chapter 5 of this dissertation.

The investigation of Fleisch and Tellkamp [63] simulates a three echelon supply chain with one product in which end customer demand is exchanged between the echelons. The authors studies the relationship between inventory inaccuracy and performance in a retail supply chain. In the base model, without alignment of physical inventory and information system inventory, inventory information becomes inaccurate due to low process quality, theft, and items becoming unsaleable. In a modified model, these factors that cause inventory inaccuracy are still present, but physical inventory and information system inventory are aligned at the end of each period. They found that elimination of inventory inaccuracy can reduce supply chain costs as well as out of stock level. According to Atali et al. [66], the simulation work of Fleisch and Tellkamp [63] has some limitations. First, simulation models do not give rise to structural results. Second, the authors do not consider what decision makers can do in the presence of discrepancies. Hence, the benchmark is based on a naive inventory system, as opposed to a smarter one that would take account of the potential discrepancies to make better reorder decisions.

The investigation of DeHoratius et al. [54] considers a periodic review inventory process with unobserved lost sales, and models inventory record inaccuracy through an 'invisible' demand process that is reflected in updates of physical inventory but not recorded inventory. The decision maker is assumed to observe replenishment and sales during the day. The information state reflecting inflows and outflows that he observes by the end of the day is given.
Instead of maintaining the IS inventory, the authors propose that the retailer maintains for the purpose of inventory management the probability mass function that accounts for record uncertainty. The authors present a simple Bayesian updating procedure to solve the problem.
In order to evaluate the proposed probabilistic inventory system, the authors perform a simulation where they compare three situations: $i)$ in the first situation, the decision maker has 'Full' visibility on the physical inventory; $ii)$ in the second situation, he uses the 'Bayes' updating procedure and $iii)$ in the third situation, a 'Naïve' policy consisting in ignoring the inventory inaccuracy is performed.
The simulation shows that both the Naïve and Bayes methods require higher average inventory levels than Full to achieve the same fill rate, due to the uncertainty around the physical inventory level. However, the Bayes method is capable of achieving better fill rates than the Naïve method while holding less

inventory. To achieve a fill rate over 92%, for example, the Bayes method requires 25% less inventory than the Naïve method on average over the 90-day horizon.

The investigation of Atali et al. [66] is among the rare ones that provide an optimization procedure for an inventory system subject to misplacement, shrinkage and transaction errors. The investigation considers a finite horizon, periodic-review inventory control problem in which inventory records are inaccurate. In order to model the inventory inaccuracy issue, the total demand is grouped under four streams of demand: paying customer, misplacement, shrinkage and transaction error. These demand streams affect the system differently. Here again the context is a retail one since the paying demand is confronted against the physical available inventory.
Misplacement, shrinkage and transaction errors are undetected between consecutive inventory audits without a tracking technology such as RFID. These error terms accumulate until a physical inventory count takes place. The manager performs a physical counting of inventory. After the inventory audit, misplaced items are returned to inventory, accumulated error terms are set to zero, and the on-hand inventory record is set equal to actual on-hand inventory.
The authors first provide a model and establish an inventory control policy when the manager observes the actual inventory movement. They establish an inventory control policy that can be used when the system is RFID enabled. Second, to assess the value of prevention in addition to visibility provided by RFID, they model the special case, in which the misplaced items are returned to stock at the end of each period. Next, They provide a model and a policy that does not use RFID technology but can partially compensate for the inventory discrepancy problem. They further model the imperfect visibility case in the presence of errors due to RFID readings. At the end of their paper, they also conduct a numerical study and compare the models with and without RFID and quantify the value of RFID. This numerical study indicates that inventory errors matter and the loss due to inventory record inaccuracy can be significant, and also shows that part of this loss can be treated by intelligent strategies even without complete visibility enabled by RFID.

The investigation of Sahin [6] present a general framework, based on the Newsvendor model, allowing to evaluate the economic impact of errors perturbating the physical and/or the IS inventory levels. The author shows also the potential benefits of the deployment of the RFID technology in a such inventory system. The sequence of events in the framework provided by the author is as the following:

- Long before the beginning of the selling season, the inventory manager orders a quantity Q from a supply process. This quantity is established based on forecast information available to the inventory manager regarding the future demand
- The inventory manager receives the goods and store them to his warehouse. Because of errors, the physical inventory Q_{PH} and the IS inventory Q_{IS} may differ from the quantity ordered
- Just before the beginning of the selling season, the inventory manger receives orders from the customers. He compares the cumulative order from all the customers to the quantify observed in the Information System. If the cumulative order is less than Q_{IS}, he accepts all the order. Otherwise, he only accepts orders summing up to Q_{IS}

- Later on, the products are shipped from the warehouse and delivered to the customers. All the orders that the inventory manager has committed himself to should be satisfied, except in the case where the physical inventory is not able to satisfy the committed quantity
- Unsatisfied demand during the commitment and unsatisfied commitment are lost since there no opportunity for replenishment during the selling season

The inventory manager faces also three types of costs:

- The overage cost due to products unsold at the end of the selling season, denoted by h
- The first type of underage cost due to orders rejected by the inventory manager during the commitment, denoted by u_1
- The second type of underage cost due to orders initially accepted by the inventory manager but finally not totally delivered to the customers, denoted by u_2

The inventory manager's decision is to determine the best quantify to order from the supply system before the selling season to satisfy customers' aggregate demand. He faces three risks: $i)$ risk of having unsold products at the end of the selling season; $ii)$ risk of shortage situation and $iii)$ risk of not being able to deliver the quantity that he has made a commitment for.
We note that the second type underage cost is the parameter that characterizes what we called in level 4, the warehouse context. In a retail context, this cost does not exist since the customers are physically present at the retail store: if the product is not available, the demand is lost (or backordered) otherwise it is satisfied immediately. The second type underage cost occurs when the customer demand is confronted to the Information System inventory. To our knowledge, the investigation of Sahin [6] is the first one which make the difference between the retail and the warehouse contexts and as a consequence the first one which includes this second type underage penalty in inventory systems.
Based on the way the Information System is update, the author deduces four models associated with the proposed framework (Cf Figure 2.2):

- Model 0 corresponds to the error free case which coincides simply with the classical Newsvendor problem
- Model 3 is clearly the general one and corresponds to the case where both the physical inventory and the IS inventory are prone to errors. $Q_{PH} = Q_A$ (with Q_A a random variable function of the ordered quantity Q). The IS inventory is updated by measuring the PH inventory. However, due to errors occurring in the data collection, the IS inventory is different from the PH inventory. As a consequence $Q_{IS} = Q_B$ (with Q_B a random variable function of the ordered quantity Q)
- Model 1 is a variant of Model 3 and assumes that there are no errors in the physical flow but only errors in the information flow exist due to defects in the data capture process
- Model 2 is also a variant of Model 3 and assumes that the physical flow involves errors but the data capture processing is perfectly reliable. This model coincides with the random yield model: if a quantity Q is ordered, a quantity $Q_{PH} = Q_A$ is received and the IS inventory does not play a role here since it is aligned with physical inventory

- Model 4 corresponds to the case where the physical inventory is prone to errors and the IS inventory is updated automatically based on the ordered quantity

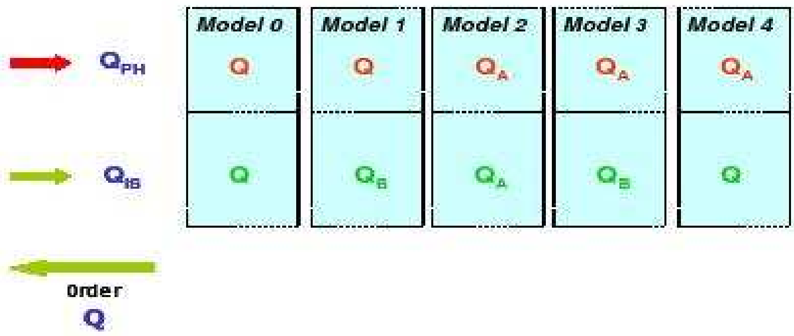


Figure 2.2: Synthesis of models deduced from the framework of Sahin [6]

According to the author Model 1 should be interpreted as a special case of the general model, i.e. Model 3, because if the inventory manager were sure that the physical quantity coincides with the order quantity, there would not be necessary to perform a scanning process to update the IS inventory and as consequence no errors would affect this latter. Model 1 was developed by the author in order to get insights on Model 3 which was not analyzed. The author provides a comprehensive analysis of Model 1 where uniform and normal distributions of the demand and the errors and where both additive, multiplicative and mixt error settings were considered. The author also assesses the impact of inventory inaccuracy by analyzing how much the performance of the inventory system is degraded where errors are ignored[3]. She also evaluates the relative cost reduction that stems from optimizing the system in presence of errors and she discusses the use of the RFID technology as a response to the inventory inaccuracy problem.

2.5 Conclusion

Two main contributions were provided in this chapter:

1. Our first contribution deals with the presentation of main qualitative and empirical investigations approaching the inventory inaccuracy issue. The focus was particularly steered on the sources of errors generating perturbations in the physical and information system flows within an inventory system. We proposed a classification of these errors, and we also focused on their impacts on supply chain performance. Concluding that the inaccuracy problems perturbate the inventory system and may be a major obstacle to improvement in entreprise's performance, we provided the main methods used to cope with the inventory inaccuracy issue.

2. Our second contribution was the classification and the description of the main quantitative investigations approaching the inventory inaccuracy issue. The classification was made based on

[3]The Naive Policy in DeHoratius et al. [54] and the Ignored Policy in Atali et al. [66]

analysis including 8 sub-levels that we proposed. This classification permitted to have a visibility on the lack of publications on certain topics and as a consequence can aid to derive future research directions in the inventory inaccuracy issue. We also briefly described the the model(s) analyzed in certain publications

Part II

RFID and Inventory Inaccuracy in a Single-Period Framework

Preliminary Notes on Part II

Models of this part of the dissertation are built based on the Newsvendor problem: the classical Newsvendor inventory problem has played an important role for many years in both the theory and applications of inventory control (Silver et al. [70] and Khouja [71]). Under a Newsvendor framework, we are concerned with seasonal type products, characterized by a short life cycle and a short selling season. Typical products that fall into this category are clothes, toys, skis, etc. These products are usually manufactured before the beginning of the season because of long production or distribution lead time constraints.

The Supply Chain under study includes three actors: the manufacturer, the wholesaler and retailers. The manufacturer produces products, the retailers are the actors selling products to the final customers. Between the manufacturer and the retailers, it may exist the wholesaler who acts as an intermediate actor that buys products from the manufacturer and resells them to the retailers. In order to differentiate between the retail store and the wholesale contexts (recall sub-level 2.1 in Section 2.4.1), we analyze two Supply Chain structures:

- Structure A: this structure focuses on the retail store context (Cf Figure 2.3). The end customers are physically present in the retail store and their demand are confronted to the physical on shelf inventory. The Information System does not play a major role in this structure. Before the beginning of the selling season, the retailer have to place a single order to the manufacturer based on the information on the end customers demand. As in the Newsvendor problem, this information is given under the form of a distribution that represents in a probabilistic way the future demand. At the beginning of the season, a quantity is received by the retailer from the manufacturer. End customers demand take place and is satisfied from the available for sales quantity. Because of errors, the available for sales quantity may be different from the quantity ordered. it also may exist a quantity which is physically present at the retail store but not available for sales. Unsatisfied demand are lost and at the end of the selling season, the unsold quantity (if any) is discounted. This structure of the Supply Chain can be modelled as illustrated in Figure 2.4)

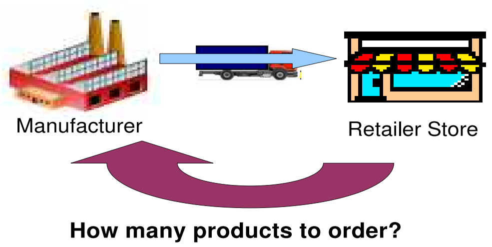


Figure 2.3: Structure A: the retailer Supply Chain

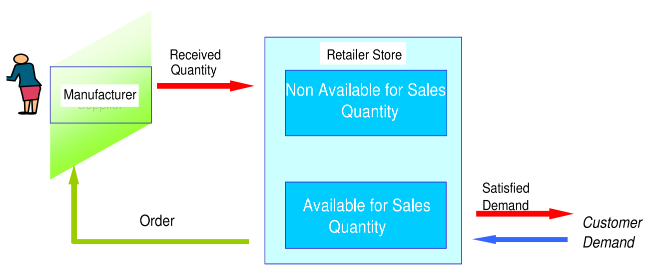


Figure 2.4: Modelling of Structure A

- Structure B: this structure focuses on the wholesale context (Cf Figure 2.5). Before the beginning of the selling season, the wholesaler orders a quantity from the manufacturer. This quantity is established based on forecast information available to the inventory manager regarding the future demand. The wholesaler receives the goods and store them to his warehouse. Just before the beginning of the selling season, the inventory manger receives orders from the customers. He compares the cumulative order from all the customers to the quantify observed in the Information System. If the cumulative order is less than the Information System quantity Q_{IS}, he accepts all the order. Otherwise, he only accepts orders summing up to Q_{IS}. Later on, the products are shipped from the warehouse and delivered to the customers. All the orders that the inventory manager has committed himself to should be satisfied, except in the case where the physical inventory is not able to satisfy the committed quantity. Unsatisfied demand during the commitment and unsatisfied commitment are lost since there no opportunity for replenishment during the selling season. The unsold quantity (if any) is discounted at the end of the selling period. The structure of this supply chain can be modeled as illustrated in Figure 2.6.

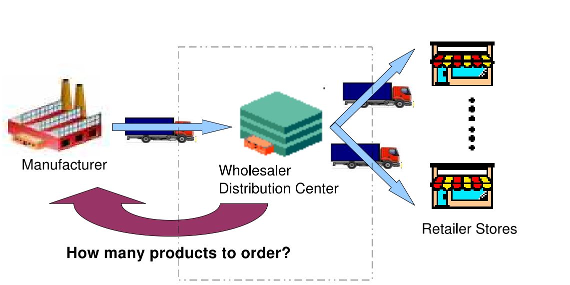


Figure 2.5: Supply Chain B: the wholesale Supply Chain

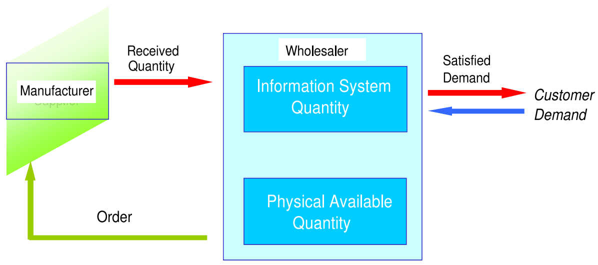


Figure 2.6: Modelling of Supply Chain B

We argue that the main difference between the two structures concerns the commitment performed by the wholesaler under Structure B. In structure A, no commitment is done by the retailer since the end customers are physically present in the store.
The part is composed of four chapters where:

- Supply (Random Yield) type errors are analyzed under Structure A in Chapter 3. Under the assumption that the total physical available inventory= the available for sales quantity=the quantity received $\neq$ the ordered quantity, the well known random yield problem is analyzed. The main contribution of this chapter is to extend the existing results pertaining to this problem.

- Misplacement type errors are analyzed under Structure A in Chapters 4 and 5. In Chapter 4, we consider a centralized supply chain with one decision maker, i.e., the retailer. In Chapter 5, we extend to the decentralized supply chain with the two actors of Structure A, i.e., the retailer and the manufacturer.

- In Chapter 6, Structure B is analyzed under general sources of errors. We provide a general framework enabling to model inventory inaccuracies.

In order to not repeat the same concepts in each chapter, we present in the following the approaches that the inventory manager may use in order to manage an inventory system subject to inventory inaccuracy. The terminology and the definitions of the proposed approaches will be used in all chapters of Parts II of this report.

Approaches for managing an inventory system subject to inaccuracy problems:

Since, among the goal of this research is to focus on the role and value of an advanced automatic identification system such as the RFID technology in better managing inventory systems with inventory inaccuracies, we should first distinguish between the case where the RFID technology is used or not.

Let us first define the role of the RFID technology in such inventory system where it exists a non agreement between the ordered quantity, the PH (PHysical) inventory, the available for sales inventory and the IS (Information System) inventory. Based on the analysis presented in the first chapter of this dissertation, we can conclude that the RFID technology has two main roles when it is confronted to the inventory inaccuracy issue:

- The visibility provided by this technology: theoretically[4], RFID enables tracking and tracing of items in stock and in the pipeline, thus, creating complete inventory visibility, leading to an accurate account of inventory discrepancy.
- The RFID may prevent or reduce the magnitude of some sources of inventory inaccuracy. In the case of theft errors, being able to distinguish customers demand and other kinds of demand (theft for example), the inventory manager can act to prevent or discourage the sources of this latter demand.

Through this dissertation, we will focus on the last role of the RFID technology, i.e., in our inventory models, deploying the RFID technology leads to the elimination of errors[5]. Through this dissertation, the situation where RFID is deployed will be referred as **Approach 3**. Under Approach 3, we assume throughout the report that the cost associated with the implementation of this technology consists in RFID tags embedded to each item individually, at a certain price t per unit. The fixed costs of investments necessary to implement the technology (such as reader systems cost, infrastructure costs, basic application integration costs, maintenance costs, support costs and overhead costs)[6] are deliberately not part of our inventory models. Estimates of these costs are provided by various studies (Cf Section 1.4 in Chapter 1) and are assumed not to vary with the model parameters. Thus, the net benefit stemming from the RFID technology will be obtained by subtracting the estimated fixed cost calculated by a net present value type analysis from the benefit provided by our models.

In the case where the RFID technology is not deployed, one should distinguish between two situations depending on whether the inventory manager is aware or not of the existence of errors. In the case where the inventory manager is unaware or simply ignores errors will be referred as **Approach 1** throughout the dissertation. In the case where the inventory manager is aware of errors occurring in the inventory system will be referred as **Approach 2**. For this last approach, one can also distinguish between two cases based on the information the inventory manager has about the error parameters. The

[4] As many electronic-based technologies, the RFID technology may not be 100% perfect

[5] The case where RFID is not 100% reliable if it only permits the reduction of errors will be briefly discussed

[6] These costs were discussed in Section 1.4 in Chapter 1

first case occurs when the inventory manager has a statistical information about the error parameter (such as a mean or the distribution of the error). The second case occurs when an exact information on the realization of the error is known. The difference between the two cases makes sense especially in a multiperiod framework. In this level of the dissertation, we do not detail more this point but we assume that Approach 2 corresponds to the first case, i.e, a statistical information on the errors is known. We will later detail this point case per case in our inventory models.
Concerning Approach 2, we notice that an estimation of the error parameters can be realized based on statistical sampling methods as reported by Pergamalis [1] who proposes a methodology for measuring stores' inventory accuracy (Cf Appendix B.1 for an example of measuring the inventory accuracy of an inventory system).
In contrast to Approach 2, Approaches 1 and 3 are easier to model and optimize. In fact, in Approach 1, the inventory manager acts as if there were no errors so, his ordering decisions or his replenishment policy coincides simply with the ordering quantity or the replenishment policy of the model without errors. For this purpose, we also define the classical Newsvendor problem as **Approach 0**, i.e., the situation without errors and without the RFID technology. As will be shown, Approach 3 is also a basic inventory problem with modified cost parameters.
To summarize, Figure 6.1 illustrates the approaches that will be used throughout this report in order to manage an inventory system subject to inaccuracy problems.

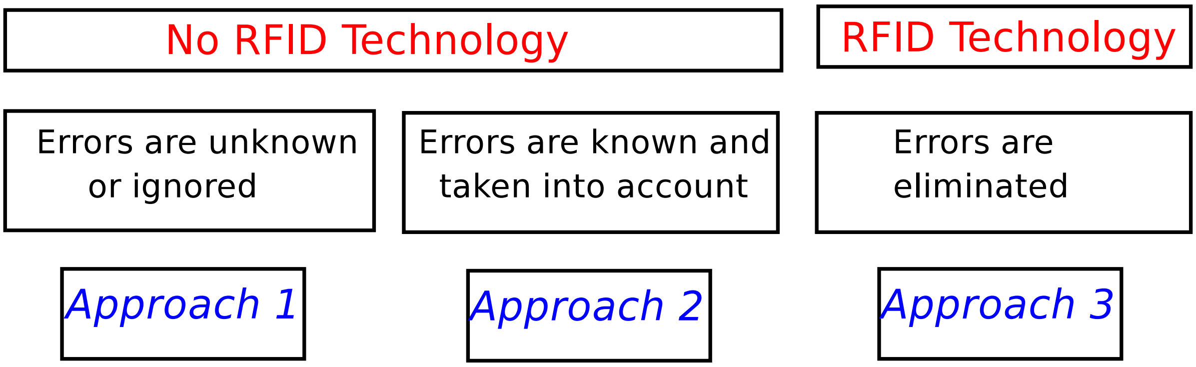


Figure 2.7: Approaches to manage an inventory system subject to inaccuracy problems

The error setting

An other issue that will be considered in the four chapters of this part is the error setting (recall sub-level 3.2 in Section 2.4.1). In a general setting, if we let Q the quantity ordered from the Manufacturer:

- Under Structure A: the available for sales inventory can be written as $Q_{available\ for\ sales} = \theta Q + \delta$ where the couple of random variables (θ, δ) characterizes the errors. From this general setting which is also called the mixt error setting, one can distinguish two particular cases:
  - The additive error setting: in this case $Q_{available\ for\ sales} = Q + \delta$
  - The multiplicative error setting: in this case $Q_{available\ for\ sales} = \theta Q$
- Under Structure B: the physical and the IS inventory can respectively be written as the following: $Q_{PH} = \gamma_{PH} Q + \epsilon_{PH}$ and $Q_{IS} = \gamma_{IS} Q + \epsilon_{IS}$ where the couple of random variables $(\gamma_{PH}, \epsilon_{PH})$

$((\gamma_{IS}, \epsilon_{IS}))$ characterizes the errors on the physical inventory level (IS inventory level). Here again, one can distinguish two particular cases:

- The additive error setting: in this case $Q_i = Q + \epsilon_i$ where $i \in [PH, IS]$
- The multiplicative error setting: in this case $Q_i = \gamma_i Q$ where $i \in [PH, IS]$

Based on this preliminary note on Part II, the following table summarizes the models that will be developed in the following four chapters:

Chapter	Error Type	Error Setting	S.C. Structure	Approaches
3	Supply (Yield)	Additive & Multiplicative	Structure A	2
4	Misplacement	Multiplicative	Structure A	1-2-3
5	Misplacement	Multiplicative	Structure A	2-3
6	All (General)	Additive & Multiplicative	Structure B	1-2

Table 2.2: Chapters of Part II

The choice to not consider the impact of the RFID technology in the model of chapter 3 is due to the fact that our main contribution in this chapter is to extend existing results pertaining to the random yield problem.

Chapter 4 deals with misplacement type errors in the retail store (Structure A). In order to derive the optimal decisions pertaining to this chapter, some concepts detailed in Chapter 3 will be used. The main contribution of this chapter is the quantification of the value of the RFID technology on such inventory system and the deduction of a critical RFID tag price which makes the deployment of this technology cost effective.

Chapter 5 extends the results obtained in Chapter 4 for the case of a decentralized supply chain where the manufacturer becomes a decision maker aiming to maximize his own profit as the retailer. In this chapter, we compare two strategies enabling for both supply chain actors the increase of their expected profits: the coordination and the deployment of the RFID technology.

Chapter 6 concludes Part II of this report by providing a general framework enabling the modelling of inventory inaccuracies. In particular, we show that a general inventory system subject to inaccuracies can be seen as an extended version of the random yield problem. This analysis ends with deducing an elegant mathematical analysis of the optimal ordering decisions under the additive error setting.

Chapter 3

Supply Errors - The Random Yield Problem

This chapter deals with the random yield problem where a single-period, uncertain demand inventory model is analyzed under the assumption that the quantity ordered (produced) is a random variable. We first conduct a comprehensive analysis of the well known single period production/inventory model with random yield. Then, we extend some of the results existing in literature: our main contribution is to show that earlier results are only valid for a certain range of system parameters. Under the hypothesis that demand and the error in the quantity received from the manufacturer are uniformly distributed, closed-form analytical solutions are obtained for all values of parameters. An analysis under normally distributed demand and error is also provided. The chapter ends with an analysis of the benefit achieved by eliminating supply errors. This chapter is based on the paper entitled "A Comprehensive Analysis of the Newsvendor Model with Unreliable Supply" by Yacine Rekik, Evren Sahin and Yves Dallery, which has been accepted for publication as a regular article in OR Spectrum Journal (Rekik et al. [52]).

Keywords: *Supply Chain Management, Random Yield, Newsvendor, Unreliable Supply*

3.1 Introduction

One of the underlying assumptions in the formulation of the Newsvendor model is that the quantity available to satisfy demand matches the quantity requisitioned from the manufacturer. In the context of inventory systems, the difference between these two quantities stems either from unreliabilities of the supply system or from internal inefficiencies such as misplaced items, perishment or internal theft. A supply system is said to be reliable when the quantity of goods effectively delivered by supplier corresponds exactly to the ordered quantity. The unreliability of the supply system may stem from: $i)$ Delivery errors or supplier frauds defined as losses happening when suppliers deliver fewer goods than ordered; and $ii)$ Theft during transportation between the vendor and the buyer. In production systems, uncertainty in production yield is also a common phenomenon observed in many processes such as electronic fabrication and assembly, and discrete parts manufacturing processes.

The literature in the area of random yield is sparse. The earliest model of a random supply inventory model with random demand was developed by Karlin [72]. This is followed by Shih [55], Noori and Keller [58], and Yano and Lee [51], among many others. Karlin [72] assumes that the only decision available is whether to order, and that if an order is placed, a random quantity is delivered. He shows that if the inventory holding and shortage cost functions are convex increasing in their respective argument, then there is a single critical initial on-hand inventory below which an order should be placed, otherwise it is optimal not to order. Shih [55] assumes that inventory holding and shortage costs are linear and that the distribution of the fraction defective is invariant with the production level. He shows that the optimal production/order quantity can be found using a variant of the Newsvendor model. For the problem considered by Shih [55], Noori and Keller [58] provide closed form solutions for the optimal order quantity for uniform and exponential demand distributions and for various distributions of the quantity received. Gerchak et al. [56] obtain the same result for the profit maximization objective. They assume continuous demand and yield and they consider a model with initial stock. They show that there is a critical level of initial stock above which no order will be placed, and this level is the same as the certain yield case. They show that when initial stock is below that critical level, the expected yield corresponding to the amount ordered will in general not be simply equal to difference. Ehrhardt and Taube [59] show that when the replenishment quantity is a random fraction of the amount ordered, an optimal single-period ordering policy can be found with a simple generalization of the traditional Newsvendor result. They also show that a simple scaling-up heuristic is an effective approximation to optimal performance. The heuristic computes an order size by starting with the order size that would be optimal with deterministic replenishment, and dividing it by the expected value of the replenishment yield fraction. They propose analytic results for the case of uniformly distributed demand. A significant theoretical contribution was made by Henig and Gerchak [61], who discuss single and multi-period models with more general assumptions about the random replenishment distribution and the cost structure. They prove that for a single-period model there exists an optimal order point that is independent of replenishment randomness. For an extensive literature on many other variants of the lot sizing problem with random yields, the reader is referred to Yano and Lee [51].

In a recent paper, Inderfurth [57] shows that in contrast to what is stated in literature, the optimal policy in the random yield model can be of a non-linear type for uniformly distributed demand and yield.

The analysis provided by the author concerns the multiplicative error setting. Demand and yield are assumed to be uniformly distributed between zero and an upper limit which also restricts the analysis. This chapter extends the work of Inderfurth [57] by examining the two error settings, i.e: the additive error setting and the multiplicative error setting. In the first setting, errors in the received quantity may stem from administrative errors made by the supplier recording for instance a 7 as a 9 in the ordering process. In this setting, the variability of errors does not depend on the ordered quantity. In the second setting, which is also known as *stochastically proportional yield* model in the literature, the variability of errors varies with the ordered quantity. Factors such as theft during transportation can probably be modelled in this way since the higher is the ordering quantity, the higher will be the variability of the quantity stolen.

We show that, depending on values that system parameters take, the optimal quantity to order may not be in the form of a Newsvendor type solution adjusted by the average error rate. We then develop a complete analysis that enables to determine the optimal order quantity in presence of errors for all values of system parameters. We also analyze the model for a normally distributed demand and received quantity and strengthen results given in the first part. We deliberately do not analyze the impact of the RFID technology on the performance of the inventory system subject to supply unreliability since the main focus of this chapter is to extend existing results pertaining to the random yield problem. Table 3.1 represents our contributions compared with the work of Inderfurth [57].

	Inderfurth 2003	Our Work
Type of errors modelled	Multiplicative errors	Additive and Multiplicative errors
Demand and error distributions	Demand and error are uniformly distributed between 0 and an upper bound	Demand and error are uniformly distributed between a lower and an upper bound Demand and error are normally distributed
Main insights	The optimal quantity to order	The optimal quantity to order and the associated expected optimal cost

Table 3.1: Main contributions

The supply chain structure considered in this chapter is Structure A. But the analysis provided is also applicable for Structure B with the assumption that $Q_{IS} = Q_{PH} = the \quad received \quad quantity$ as it is mentioned in Sahin [6]. Even if it is not implcitly stated, all investigations of the literature review mentioned above, consider the analysis of the random yield problem under Approach 2, i.e. the inventory manager is aware of the errors. Since the aim of this chapter is to extend existing results, only Approach 2 will be studied.

The chapter is organized as follows : in section 3.2, we describe both Approach 0 and Approach 2. In section 3.3, we derive the optimal order decisions when demand is uniformly distributed for both settings of errors (additive and multiplicative). The benefit of making the supplier reliable is analyzed in section 3.4. In section 3.5, we extend the model for a normally distributed demand. Finally, section 3.6 concludes the chapter.

3.2 Model Description

3.2.1 Notations

The following notations are used:

- Q_0: the order quantity under Approach 0 (the basic Newsvendor problem)
- Q_0^*: the optimal value of Q_0
- Q_2: the order quantity in the Random Yield Model under Approach 2
- Q_2^*: the optimal value of Q_2
- h: the unit overage cost
- u: the unit underage cost [1]
- x: the random variable representing demand
- μ_x: the expected demand
- σ_x: the standard deviation of x;
- if x is uniform let:
  - U_x: the upper bound of x which is given by $U_x = \mu_x + \sqrt{3}\sigma_x \geq 0$
  - L_x: the lower bound of x which is given by $L_x = \mu_x - \sqrt{3}\sigma_x \geq 0$
- $f(F)$: pdf (cdf) characterizing the demand
- $\phi(\Phi)$: the standard normal pdf (cdf)
- Q_A: the random variable representing the quantity received
- if Q_A is uniform let:
  - U_{Q_A}: the upper bound of Q_A
  - L_{Q_A}: the lower bound of Q_A
- $g(Q_A)$: pdf characterizing Q_A
- μ_{Q_A}: the expected quantity received
- σ_{Q_A}: the standard deviation of Q_A

[1] without loss of generality, in the rest of this chapter, we set $h = 1$ and use $u = k.h$ where $k \in [0.5, 10]$

3.2.2 Approach 0: The basic model without errors

The expected cost function associated with the basic one-period Newsvendor problem with zero initial inventory is given by:

$$C_0(Q_0) = kh \int_{Q_0}^{+\infty} (x - Q_0)f(x)dx + h \int_{0}^{Q_0} (Q_0 - x)f(x)dx \tag{3.1}$$

Which is minimized for Q_0^* such that:

$$F(Q_0^*) = \frac{k}{k+1}. \tag{3.2}$$

For a *uniformly distributed demand*, the optimal order quantity is given by:

$$Q_0^* = \mu_x + \sqrt{3}\sigma_x \frac{k-1}{k+1} \tag{3.3}$$

The corresponding optimal expected cost is as the following

$$C_0(Q_0^*) = \frac{\sqrt{3}kh\sigma_x}{k+1} \tag{3.4}$$

For a *normally distributed demand*, the optimal order quantity is:

$$Q_0^* = \mu_x + \sigma_x \Phi^{-1}\left(\frac{k}{k+1}\right) \tag{3.5}$$

and the corresponding optimal expected cost is as the following:

$$C_0(Q_0^*) = \sigma_x h(k+1)\phi\left\{\Phi^{-1}\left[\frac{k}{k+1}\right]\right\} \tag{3.6}$$

3.2.3 The model with errors under Approach 2

Modelling of Errors: As previously mentioned, the received quantity Q_A can be modelled by: $Q_A = \gamma Q_2 + \xi$ where

- Q_2 is the ordered quantity
- γ and ξ are random with respectively $(\mu_\gamma, \sigma_\gamma)$ and (μ_ξ, σ_ξ) as parameters μ_j being the mean of the random variable j and σ_j its standard deviation

We consider two particular settings from this general expression:

- The additive setting: the received quantity is given by $Q_A = Q_2 + \xi$, as a result we have $\mu_{Q_A} = Q_2 + \mu_\xi$. Without loss of generality we will set $\mu_\xi = 0$ for the analysis of this model since if $\mu_\xi \neq 0$, we can easily show that the optimal order quantity is simply shifted by the constant value μ_ξ. In the additive case we therefore have $\mu_{Q_A} = Q_2$ and $\sigma_{Q_A} = \sigma_\xi$ which is independent of the ordered quantity.
- The multiplicative setting: the received quantity is given by $Q_A = \gamma Q_2$, as a result we have $\mu_{Q_A} = \mu_\gamma Q_2$ and $\sigma_{Q_A} = \sigma_\gamma Q_2$ which is proportional to the ordered quantity.

Note that the analysis pertaining to the cases $Q_A = \mu_\gamma Q_2 + \xi$ and $Q_A = \gamma Q_2 + \mu_\xi$ can be deduced from our analysis on the additive and the multiplicative cases.

Expected cost function: To develop the expected total cost associated with the random yield problem, the following observation must be made: the inventory at the end of the period will be one of the two cases: (a) $x \geq Q_A$ and (b) $x \leq Q_A$.
The first case triggers an underage situation while the second generates excessive inventory. The cost incurred will be given by:

$$Cost = k \cdot h(x - Q_A)^+ + h(Q_A - x)^+ \tag{3.7}$$

The expected total cost function of the system is therefore given by:

$$\begin{aligned} C_2(Q_2) &= k \cdot h \int\limits_{Q_A=0}^{\infty} \int\limits_{x=Q_A}^{\infty} (x - Q_A) f(x) g(Q_A) dx dQ_A \\ &+ h \int\limits_{Q_A=0}^{\infty} \int\limits_{x=0}^{Q_A} (Q_A - x) f(x) g(Q_A) dx dQ_A \end{aligned} \tag{3.8}$$

The following remarks can be made:

- In contrast to Inderfurth [57], this formulation supposes that the supplier will not be paid for undelivered quantities which seems to be a realistic assumption. Note however that the analysis presented in this chapter can easily be modified to consider the case where the supplier is paid for the whole ordered quantity. In the last case, it can be shown that the expected cost function is no longer $C_2(Q_2)$ but $C_2(Q_2) + w(Q_2 - E(Q_A))$ where w is the unit product purchase cost.
- Note also that, although we assume that Q_A is known (we are under Approach 2), the cost associated with the inspection process is deliberately not part of our model. Estimates of this cost can be found by various studies and are assumed not to vary with the model parameters. Thus, the expected total cost of the model with errors can be deduced by integrating this cost component.
- Remark that we assume that there is no initial inventory. If there is an initial inventory, the optimal policy may not be of order-up-to type. However, the case with an initial inventory can be handled by following the methodology developed in this chapter. In particular, we will show that in the multiplicative errors case, the ordering quantity is not a linear function of the initial inventory level (Cf Appendix C.4).

3.3 The optimal order decision when demand and errors are uniformly distributed

3.3.1 The additive error setting

If demand is uniformly distributed, we have:

$$\begin{aligned} f(x) &= \frac{1}{U_x - L_x} \qquad for \qquad L_x \leq x \leq U_x \\ &= 0 \qquad\qquad otherwise \end{aligned}$$

We develop the analysis pertaining to this case in two steps. We first recall results existing in literature. By following a more thorough approach, we show that these results are not valid for all values of system parameters. This pushes us to conduct a more refined analysis enabling to extend results found in the earlier studies.

Previous results from literature: According to **Noori and Keller [58]**, whatever the distribution of Q_A is, the optimal quantity to order will be given by:

$$Q_2^* = Q_0^* = \mu_x + \sqrt{3}\sigma_x \frac{k-1}{k+1} \tag{3.9}$$

We note there that the optimal order quantity is independent of the standard deviation of the received quantity σ_ξ. This result is somewhat surprising, since an augmentation of σ_ξ increases the variability of the quantity received which should affect the optimal order quantity.

Extension of the results: In Noori and Keller [58], it is stated that the result above holds for all values of system parameters. We show in this section that this may not be true.
We consider the case of a uniformly distributed received quantity where

$$\begin{aligned}
g(Q_A) &= \frac{1}{U_{Q_A} - L_{Q_A}} \quad for \quad L_{Q_A} \leq Q_A \leq U_{Q_A} \\
&= 0 \qquad\qquad\quad otherwise \\
U_{Q_A} &= Q_2 + \sigma_\xi\sqrt{3} \\
L_{Q_A} &= Q_2 - \sigma_\xi\sqrt{3}
\end{aligned}$$

In order to show that the previous result is not verified for all values of system parameters, we consider a deterministic demand which is a particular case of the model above ($\mu_D = D$ and $\sigma_D = 0$). We can easily show that the optimal order quantity when demand is deterministic and the received quantity is uncertain is as follows:

$$Q_2^* = D + \sqrt{3}\,\sigma_\xi \frac{k-1}{k+1} \tag{3.10}$$

We remark that Q_2^* depends on σ_ξ, which is not compatible with the result given in (3.9). In fact, (3.9) is valid only for specific values of the standard deviation σ_ξ of Q_A. Indeed, given that x and Q_A are bounded, this result is associated with a particular positioning between the distribution of demand and Q_A which can be described by $U_{Q_A} \leq U_x$ and $L_{Q_A} \geq L_x$, i.e. Configuration 2 in Figure 3.1. If values of system parameters change, several configurations of demand and error distributions should be considered to correctly formulate the expected cost. These configurations are presented in Figure 3.1.

- Configuration 1: the variability of the received quantity is higher but the distribution of the received quantity is within the distribution of demand: $Max(Q_A) = U_{Q_A} \leq Max(x) = U_x$ and $Min(Q_A) = L_{Q_A} \geq Min(x) = L_x$
- Configuration 2: the variability of the received quantity is such that its distribution exceeds by one side the distribution of demand:

  - if $k > 1: Max(Q_A) = U_{Q_A} \geq Max(x) = U_x$
  - if $k < 1: Min(Q_A) = L_{Q_A} \leq Min(x) = L_x$
  - if $k = 1$: configuration 2 does not exist

- Configuration 3: the distribution of Q_A is no longer in the field of variation of variation of demand: $Max(Q_A) = U_{Q_A} \geq Max(x) = U_x$ and $Min(Q_A) = L_{Q_A} \leq Min(x) = L_x$

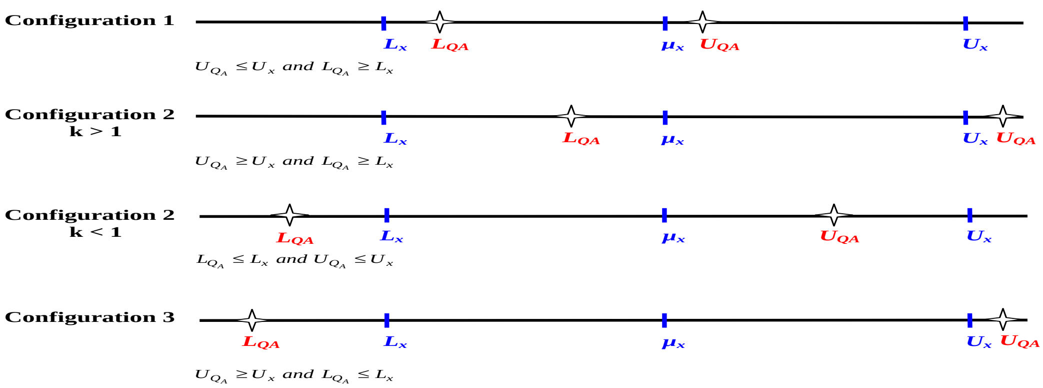


Figure 3.1: Positions between distributions of x and Q_A

In order to express the overall optimal order decision over all possible configurations, we proceed in several steps:

1. We develop the expected total cost pertaining to each configuration
2. We verify the convexity of the total cost function and derive the optimal quantity and cost for each configuration
3. For a given configuration, the expression of the optimal quantity and constraints resulting from the positions of distribution of x and Q_A enable to define an interval of σ_ξ for which results obtained are valid

Note that from a theoretical point of view, in our analysis, we consider all possible values of σ_ξ, including the ones such that $\sigma_\xi \geq \sigma_D$. This enables us to identify 3 different intervals of variation of σ_ξ, each being associated with one of our configurations.

Configuration 1

This configuration corresponds to the situation where $U_{Q_A} \leq U_x$ and $L_x \leq L_{Q_A}$. The following result states the optimal ordering quantity and the corresponding optimal expected cost pertaining to this configuration:

Result 3.1. *In configuration 1:*
For any value of k, the optimal order quantity is $Q_2^* = Q_0^* = \mu_x + \sigma_x\sqrt{3}\dfrac{k-1}{k+1}$ *with an optimal cost* $C_2(Q_2^*) = \dfrac{h(12k\sigma_x^2 + (k+1)^2\sigma_\xi^2)}{4\sqrt{3}(k+1)\sigma_x}$

Proof. For technical detail considerations cf Appendix C.1.1 □

In contrast to what is stated in Noori and Keller [58], the result above is valid only if Q_2^* verifies $Q_2^* + \sqrt{3}\sigma_\xi \leq U_x$ for the case $k \geq 1$ and $Q_2^* - \sqrt{3}\sigma_\xi \geq L_x$ for the case $k \leq 1$ (a more detailed discussion on this constraint is developed in Result 3.4). In Configuration 1, we confirm results obtained in Noori and Keller [58], and we remark that an increase of σ_ξ implies additional overage and underage costs but those costs are compensated by each other. So, the optimal ordering quantity does not depend on the standard deviation of the received quantity.

In this configuration, the overage and underage costs are compensated by each other is due to the symmetry of a uniform distribution. We analyzed configuration 1 for a triangular distribution of the received quantity and we showed that the optimal order quantity changes and depends on σ_ξ. So the result given in (3.9) is not valid for any distribution of the received quantity as stated in Noori and Keller [58].

Configuration 2

This configuration corresponds to the situation such that $U_{Q_A} \geq U_x$ and $L_{Q_A} \geq L_x$ for the case $k \geq 1$ and $U_{Q_A} \leq U_x$ and $L_x \geq L_{Q_A}$ for the case $k \leq 1$. The following result states the optimal order decisions pertaining to this configuration:

Result 3.2. *In Configuration 2:*

- ***If*** $k > 1$*:*
  The optimal order quantity is $Q_2^* = Q_0^* + \sqrt{3}\left(\sqrt{\sigma_\xi} - \sqrt{\frac{2}{k+1}\sigma_x}\right)^2$ *with an optimal expected cost* $C_2(Q_2^*) = \sqrt{3}h(\sigma_x + \sigma_\xi) - 4\frac{h\sqrt{2\sigma_x\sigma_\xi}}{\sqrt{3(k+1)}}$
- ***If*** $k < 1$*:*
  The optimal order quantity is $Q_2^* = Q_0^* - \sqrt{3}\left(\sqrt{\sigma_\xi} - \sqrt{\frac{2k}{k+1}\sigma_x}\right)^2$ *with an optimal expected cost* $C_2(Q_2^*) = \sqrt{3}hk(\sigma_x + \sigma_\xi) - 4\frac{hk^2\sqrt{2\sigma_x\sigma_\xi}}{\sqrt{3k(k+1)}}$

Proof. For technical detail considerations cf Appendix C.2.1 □

We note here that:

- In contrast to the first configuration, the optimal order quantity depends on the standard deviation of the received quantity σ_ξ and this is as expected since an increase of the variability of received quantity implies an increase of both underage and overage costs. So the decision to order more or less than the *Approach 0* depends on the value of the parameter k: for $k > 1$, the underage penalty is more important than the overage one, we have rather to order more to avoid shortage situation
- The optimal order quantity is increasing (decreasing) in σ_ξ for $k > 1$ ($k < 1$)

- Configuration 2 does not exist for the particular case of $k = 1$ since the optimal order quantity in Configuration 1 for this case is the expected demand μ_x

Configuration 3

This configuration corresponds to the situation such that $L_{Q_A} \leq L_x$ and $U_x \leq U_{Q_A}$ for the two cases ($k \leq 1$ and $k \geq 1$). The following result states the optimal order decision associated with this configuration:

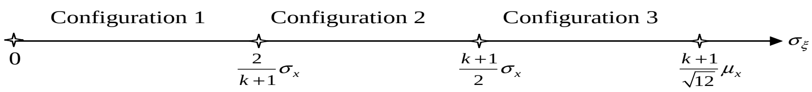
Result 3.3. *In configuration 3:*
For any value of k, the optimal order quantity is $Q_2^* = \mu_x + \sigma_\xi \sqrt{3}\dfrac{k-1}{k+1}$ *with an optimal cost*
$C_2(Q_2^*) = \dfrac{h((k+1)^2\sigma_x^2 + 12k\sigma_\xi^2)}{4\sqrt{3}(k+1)\sigma_\xi}$

Proof. For technical detail considerations cf Appendix C.3.1 □

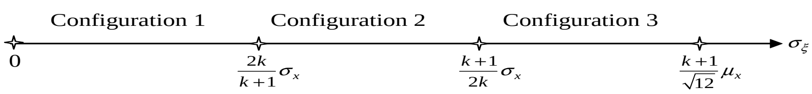
- As in the previous configuration, the optimal order quantity increases (decreases) with the standard deviation of the received quantity for $k \geq 1$ ($k \leq 1$)
- For this configuration, which corresponds to high values of σ_ξ, the optimal order quantity does not depend on the standard deviation of demand: for a given σ_ξ, an increase of σ_x will increase both underage and overage costs but those costs will be compensated by each other. Note that formulas of configuration 3 can be deduced from the ones of configuration 1 by exchanging σ_x and σ_ξ
- Note also that the result we obtained in Equation 3.10, for a deterministic demand is the same as the result obtained in Configuration 3. This is not surprising since the deterministic demand case is simply a particular case of Configuration 3.

Result 3.4. *: Summary of results:*
Based on expressions of optimal quantities obtained for each configuration and the hypothesis on the positions of distributions of demand and the received quantity, we can deduce the following intervals of variation of σ_ξ for which previous results hold:

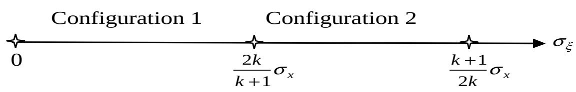
- ***Case A:*** $k \geq 1$*: The following figure shows the intervals of values of σ_ξ in which each configuration is defined:*



- ***Case B:*** $\sqrt{3}\dfrac{\sigma_x}{\mu_x} \leq k \leq 1$*: The following figure shows the intervals of values of σ_ξ in which each configuration is defined:*



- ***Case C:*** $0 \leq k \leq \sqrt{3}\dfrac{\sigma_x}{\mu_x}$*: The following figure shows the intervals of values of* σ_ξ *in which each configuration is defined:*



Proof. For technical detail considerations cf Appendix C.1.2, C.2.2 and C.3.2 □

Note that:

- We have $\dfrac{2}{k+1}\sigma_x \leq \dfrac{k+1}{2}\sigma_x \leq \dfrac{k+1}{\sqrt{12}}\mu_x$ for Case A since $k \geq 1$ and $L_x \geq 0$
- We have $\dfrac{2k}{k+1}\sigma_x \leq \dfrac{k+1}{2k}\sigma_x \leq \dfrac{k+1}{\sqrt{12}}\mu_x$ for Case B since $\sqrt{3}\dfrac{\sigma_x}{\mu_x} \leq k \leq 1$
- For the case $k \leq 1$, an assumption on parameters of demand must be made to assure the existence of Configuration 3: this assumption assures that the lower boundary of Q_A reaches zero in Configuration 3 (and not in Configuration 2). So, in the case $k \leq 1$, $CV_x = \dfrac{\sigma_x}{\mu_x}$ must satisfy $CV_x \leq \dfrac{k}{\sqrt{3}}$ to assure the existence of Configuration 3. Thus the assumption made on k ($\sqrt{3}\dfrac{\sigma_x}{\mu_x} \leq k$) for the case $k \leq 1$. For values of k such that $k \leq \sqrt{3}\dfrac{\sigma_x}{\mu_x}$ the maximal value that can take σ_ξ (to assure positive value of lower boundary of Q_A) is between $\dfrac{2k}{k+1}\sigma_x$ and $\dfrac{k+1}{2k}\sigma_x$ (Configuration 2)
- The continuity of optimal order quantities and costs at each critical value of σ_ξ is also verified. For example in configuration 2 and for $k \geq 1$, by setting $\sigma_\xi = \dfrac{2}{k+1}\sigma_x$, we retrieve the result of the second configuration $Q_2^* = Q_0^*$.

Figure 3.2 represents the variation of the optimal order quantity with σ_ξ for $\mu_x = 10$, $\sigma_x = 3$:

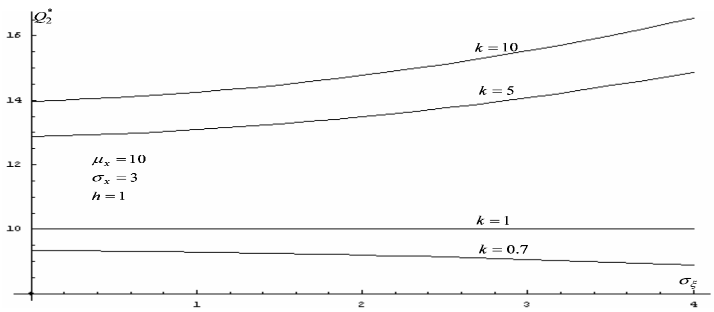


Figure 3.2: Variation of Q_2^* with σ_ξ

Remark 3.1. *If an initial inventory* I *is taken into account, we show that the results found in this section are slightly modified and the quantity ordered from the manufacturer is given by* $Max(0, Q_2^* - I)$.

3.3.2 The multiplicative error setting

We recall that the received quantity in the multiplicative error setting is $Q_A = \gamma Q_2$ with $\mu_{Q_A} = \mu_\gamma Q_2$ and $\sigma_{Q_A} = \sigma_\gamma Q_2$ (cf section 3.2.3). We also have:

$$\begin{aligned} g(Q_A) &= \frac{1}{(U_\gamma - L_\gamma)Q_2} \quad for \quad Q_2 L_\gamma \le Q_A \le Q_2 U_\gamma \\ &= 0 \quad otherwise \\ U_\gamma &= \mu_\gamma + \sigma_\gamma \sqrt{3} \\ L_\gamma &= \mu_\gamma - \sigma_\gamma \sqrt{3} \end{aligned}$$

Previous results: For such setting, Noori and Keller [58] show that:

$$Q_2^* = \frac{\mu_\gamma}{\mu_\gamma^2 + \sigma_\gamma^2} Q_0^* \tag{3.11}$$

This result is also found Gerchak et al. [56] (with the assumption that the upper bound of demand is less than the yield one and by setting the initial stock equal to zero). It is also provided in Ehrhardt and Taube [59] but also under the same condition mentioned above.

Extension of the results: In order to show that the result in (3.11) is not valid for all values of model parameters, we analyze the model for a deterministic demand ($\mu_D = D$ and $\sigma_D = 0$). We can easily show that the optimal order quantity when received quantity is uncertain is given by:

$$Q_2^* = \frac{\sqrt{(k+1)D^2}}{\sqrt{-2\sqrt{3}(k-1)\mu_\gamma\sigma_\gamma + (k+1)(\mu_\gamma^2 + 3\sigma_\gamma^2)}} \tag{3.12}$$

This result is clearly not compatible with (3.11) which is only valid for specific values of σ_γ. Again, by using the same logic as the additive error setting (cf Page 55), we have 3 configurations depending on positions of the distributions of demand and errors, and we show that (3.11) corresponds to the case where the distribution of Q_A is included in demand's one, i.e. configuration 1.
Because of the complexity of formulas that express the optimal cost, those are not provided in this dissertation but can be found in Rekik [73]. As in the additive case, based on expressions of optimal quantities and the hypothesis concerning the positions of distributions of x and Q_A, we deduce the interval of σ_γ for which each result is valid (let define σ_{ij} as the critical value of σ_γ which permits the transit from $Configuration\ i$ to $Configuration\ j$). Before presenting the optimal order decisions for the multiplicative error setting, the following result shows that we can assume that μ_γ without loss of generality:

Result 3.5. *The optimal order decision pertaining to a situation where γ has $\mu_\gamma, \sigma_\gamma$ as parameters can be obtained by determining the optimal order decision associated with the normalized case where the parameters of the distribution of γ are given by $(1, \frac{\sigma_\gamma}{\mu_\gamma})$. The optimal order quantity of the first case is equal the optimal order quantity of the second one divided by μ_γ.*

Proof. Consider a first model with error parameter setting $\mu_\gamma, \sigma_\gamma$ and an order quantity equal to $Q1$. Consider also the second model with error parameter setting $(1, \dfrac{\sigma_\gamma}{\mu_\gamma})$ and an order quantity equal to $Q2$. If we assume that $Q2 = \mu_\gamma Q1$ then the two models are equivalent since the distribution of the received quantity is the same in both models. □

In order to compare our results with those that exist in the literature, we assume that $\mu_\gamma \neq 1$ in the formulation of our model. We use the same method as the additive error setting: $i)$ determination of the total cost function; $ii)$ verification of convexity and deduction of the optimal quantity and cost; $iii)$ determination of the interval of σ_γ where the result is valid $iv)$ verification of the expressions and the sequence of critical values of σ_γ.

Result 3.6. *Expressions of the optimal order quantity for each configuration are as follows:*

Configuration	interval of σ_γ	Q_2^*
Configuration 1	$[0, \sigma_{12}]$	$\frac{\mu_\gamma}{\mu_\gamma^2+\sigma_\gamma^2} Q_0^*$
Configuration 2	$[\sigma_{12}, \sigma_{23}]$	Q_2^* is obtained by solving $aQ_2^{*^3} + bQ_2^{*^2} + c = 0$
Configuration 3	$[\sigma_{23}, \sigma_{\gamma\,\max}]$	$\frac{\sqrt{(k+1)(\mu_x^2+\sigma_x^2)}}{\sqrt{-2\sqrt{3}(k-1)\mu_\gamma\sigma_\gamma+(k+1)(\mu_\gamma^2+3\sigma_\gamma^2)}}$

Where critical values σ_{ij} are given by:

$For\ 0 \leq k \leq \frac{L_x+2U_x}{2L_x+U_x}$	$For\ k \geq \frac{L_x+2U_x}{2L_x+U_x} \geq 1$
$\sigma_{12} = \frac{\sqrt{(3Q_0^*-2L_x)(2L_x+Q_0^*)}-\sqrt{3}Q_0^*}{2L_x}\mu_\gamma$	$\sigma_{12} = \frac{\sqrt{3}Q_0^*-\sqrt{(3Q_0^*-2U_x)(2U_x+Q_0^*)}}{2U_x}\mu_\gamma$
$\sigma_{23} = \frac{(d-e)-\sqrt{(d-e)^2-e^2}}{\sqrt{3}e}\mu_\gamma$	$\sigma_{23} = \frac{(d+e)-\sqrt{(d+e)^2-e^2}}{\sqrt{3}e}\mu_\gamma$
$\sigma_{\gamma\,\max} = \frac{1}{\sqrt{3}}\mu_\gamma$	$\sigma_{\gamma\,\max} = \frac{1}{\sqrt{3}}\mu_\gamma$
$a = 2(k+1)U_\gamma^3$	$a = 2(k+1)L_\gamma^3$
$b = 3kU_x(L_\gamma^2 - U_\gamma^2) - 3L_x(U_\gamma^2 + kL_\gamma^2)$	$b = 3L_x(U_\gamma^2 - L_\gamma^2) - 3U_x(U_\gamma^2 + kL_\gamma^2)$
$c = (k+1)L_x^3$	$c = (k+1)U_x^3$
$d = kU_x^2$	$d = L_x^2$
$e = (k+1)\sigma_x(\sqrt{3}\mu_x + \sigma_x)$	$e = (k+1)\sigma_x(\sqrt{3}\mu_x - \sigma_x)$

Proof. The logic used is the same as the additive error setting, technical detail considerations are not presented in this dissertation but they can be found in **Rekik [73]** □

Note that for the case $k \leq \dfrac{L_x + 2U_x}{2L_x + U_x}$ and again like the additive setting, an assumption on k, σ_x and μ_x must be made to assure the existence of Configuration 3 and to assure that the lower boundary of Q_A reaches zero in Configuration 3 (and not in Configuration 2). So $CV_x = \dfrac{\sigma_x}{\mu_x}$ must satisfy $CV_x \leq \dfrac{\sqrt{3} - \sqrt{(3-k)(k+1)}}{k-2}$ to assure the existence of Configuration 3, otherwise the maximum value that can take σ_γ is between σ_{23} and σ_{34}.
We notice here that all results in the literature except the one of Inderfurth [57] consider only Configuration 1. The analysis of Inderfurth [57] is a particular case of our analysis with the assumption that $L_x = L_\gamma = 0$. With such an assumption, Configuration 3 does not exist and Configuration 2 is only valid for the case $k \geq 2$. For such case we can easily verify the result provided by the author $Q_2^* = \dfrac{U_x}{U_\gamma}\sqrt{\dfrac{k+1}{3}}$.
Below (see Figure 3.3) we present the variation of the optimal order quantity with σ_γ for a demand distribution with parameters $\mu_x = 10$ and $\sigma_x = 3$ and for $\mu_\gamma = 1$:

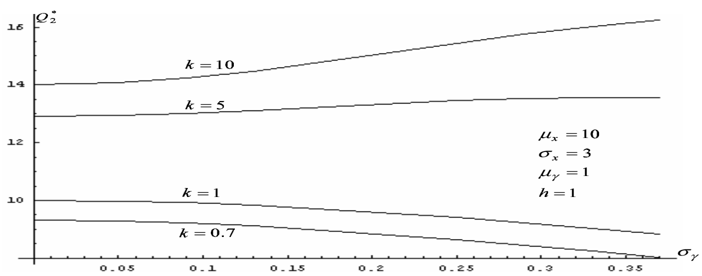


Figure 3.3: Variation of Q_2^* with σ_γ for different k

Note that:

- As the additive setting, the effect of k on the optimal order quantity is as expected intuitively
- The optimal order quantity is inversely proportional to μ_γ and this is intuitively expected: if the supplier delivers less than the inventory manager orders, he must order a larger amount
- In order to avoid negative value of received quantity, μ_γ and σ_γ must satisfy $\mu_\gamma - \sqrt{3}\sigma_\gamma \geq 0$, this is why $\sigma_{\gamma_{max}} = \dfrac{\mu_\gamma}{\sqrt{3}}$
- In some situations where k is small such as the case $k = 0.7$ presented in Figure 3.3, Configuration 3 does not exist because the lower boundary of Q_A reaches zero in Configuration 2
- The effect of σ_γ: like the additive case, the variation of optimal order quantity with σ_γ, depends on value taken by the parameter k:

  (a) If $k > \dfrac{L_x + 2U_x}{2L_x + U_x}$: underage situation is more penalizing in term of costs: an increase of σ_γ will increase the probability of falling in the underage situation, so increasing the order quantity will help the decrease of this probability. But from a certain value of σ_γ, the increase of the optimal order quantity will generate more important costs stemming from the increase of the variability of the received quantity since its boundaries are proportional to the order quantity, then, from this value of σ_γ, we have rather to order less to diminish the variability of Q_A, so Q_2^* decreases. As a consequence, in the case $k > \dfrac{L_x + 2U_x}{2L_x + U_x}$, we have two phenomena linked to the increase of σ_γ which interfere: ***i)*** if σ_γ increases, the probability to fall in a underage situation gets higher, increasing order quantity will decrease the probability of such situation. ***ii)*** If σ_γ increases, the field of variation of received quantity is higher and so its variability which is equal to $\sigma_\gamma Q_2$ is bigger and as a consequence total cost function gets bigger. Decreasing order quantity is a way to reduce costs since the boundaries of the distribution of Q_A depends on the order quantity Q_2.

(b) If $k < \dfrac{L_x + 2U_x}{2L_x + U_x}$: overage costs are less important than underage ones. An increase of σ_γ will increase both underage and overage costs, so decreasing the order quantity will be the best solution to decrease the total expected cost function by decreasing the variability of the received quantity.

Remark 3.2. *If an initial inventory I is taken into account, we observe that the ordering quantity is not a linear function of I (cf Appendix C.4).*

3.4 The benefit of making the supplier reliable

It would be worthwhile to know the benefit of making supplier 100 % reliable as a function of system parameters. This is achieved by using the ratio:

$$R = \frac{C_2(Q_2^*) - C_0(Q_0^*)}{C_2(Q_2^*)} \tag{3.13}$$

3.4.1 The additive error setting

We consider in this section, the benefit of making the supplier reliable in the additive setting. The following result states the expressions of the ration R in each configuration:

Result 3.7. *Expressions of R for the different intervals of variation of σ_ξ is given by:*

Conf.	Interval of σ_ξ		R	
	$k \le 1$	$k \ge 1$		
Conf. 1	$\left[0, \frac{2k}{k+1}\sigma_x\right]$	$\left[0, \frac{2}{k+1}\sigma_x\right]$	$1 - \frac{12k\sigma_x^2}{12k\sigma_x^2+(k+1)^2\sigma_\xi^2}$	
Conf. 2	$\left[\frac{2k}{k+1}\sigma_x, \frac{k+1}{2k}\sigma_x\right]$	$\left[\frac{2}{k+1}\sigma_x, \frac{k+1}{2}\sigma_x\right]$	$k \le 1$	$1 - \frac{3\sigma_x}{3(k+1)(\sigma_x+\sigma_\xi)-4\sqrt{2k(k+1)\sigma_x\sigma_\xi}}$
			$k \ge 1$	$1 - \frac{3k\sigma_x}{3(k+1)(\sigma_x+\sigma_\xi)-4\sqrt{2(k+1)\sigma_x\sigma_\xi}}$
Conf. 3	$\left[\frac{k+1}{2k}\sigma_x, \frac{k+1}{\sqrt{12}}\mu_x\right]$	$\left[\frac{k+1}{2}\sigma_x, \frac{k+1}{\sqrt{12}}\mu_x\right]$	$1 - \frac{12k\sigma_x\sigma_\xi}{(1+k)^2\sigma_x^2+12k\sigma_\xi^2}$	

Proof. The proof is deduced by using the expressions of $C_2(Q_2^*)$ and $C_0(Q_0^*)$ □

For our numerical example ($k = 5$, $\mu_x = 10$, $\sigma_x = 3$ and $\sigma_{Q_A} = \sigma_\xi = 4$) we have to order $Q_2^* = 15.19$ with an optimal cost $C_2(Q_2^*) = 7.50$. If the supplier were 100 % reliable, the optimal order quantity would be the Newsvendor solution $Q_0^* = 13.46$ and the associated cost $C_0(Q_0^*) = 4.33$. So, the benefit we get if the supplier is 100% reliable represents 42% of the cost of the situation with unreliable supplier.

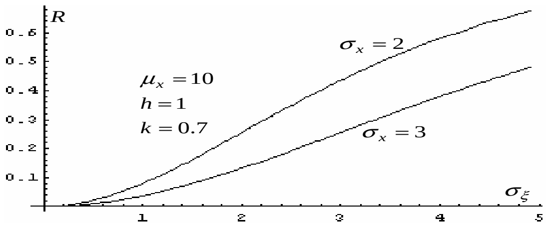


Figure 3.4: Variation of R with σ_ξ with σ_x for $k = 0.7$

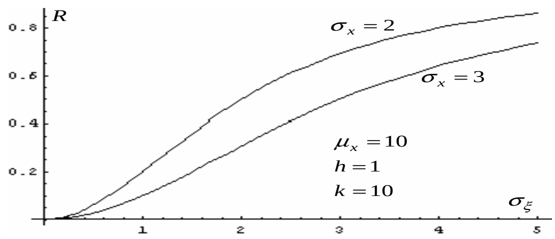


Figure 3.5: Variation of R with σ_ξ with σ_x for $k = 10$

Figures 3.4 and 3.5 present the variation of R with σ_ξ for 2 different values of σ_x for $k = 0.7$ and $k = 10$ respectively. As expected R is increasing with σ_ξ, the higher the error made by supplier is, the more important the benefits of making it 100 % reliable is. Note also, by comparing the two figures, that the benefits we get by making the supplier reliable is increasing with k and this is also expected: if k is high, the trade-off between underage and overage costs is more sensitive to supplier's errors. As we can remark, the benefits we make by making our supplier reliable is less important when the variability of demand is more important.

3.4.2 The multiplicative error setting

We consider in this section, the benefit of making the supplier reliable in the multiplicative case by analyzing the ratio R. Again, as in the additive setting, the ratio R is defined in each configuration and the continuity is checked for each critical level of σ_γ. Because of complexity, expressions of R are not provided but they can be found in Rekik [73]. We analyze graphically the variation of R with model's parameters.

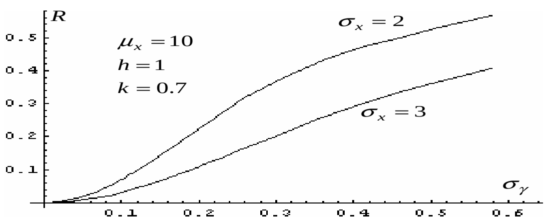


Figure 3.6: Variation of R with σ_γ with σ_x for $k = 0.7$

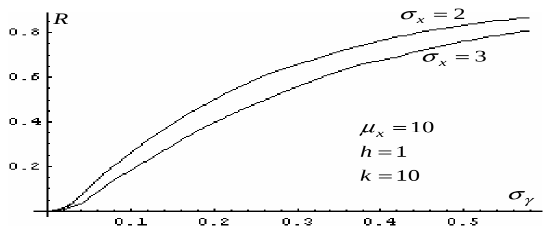


Figure 3.7: Variation of R with σ_γ with σ_x for $k = 10$

Figure 3.6 and 3.7 present the variation of R with σ_γ with σ_x for $k = 0.7$ and $k = 10$ respectively. As in the additive setting, R is increasing with σ_γ: the higher the error made by supplier is, the more important the benefit of making it 100 % reliable is. Note also that R decreases with σ_x.

3.5 The optimal order decision when demand and errors are normally distributed

In this section, we consider the case of a normally distributed x and Q_A. Our results confirm the findings we obtained in Section 3.3 for the optimal order quantity and Section 3.4 for the benefits we can get by making the supplier 100% reliable. Again, we consider the additive and the multiplicative settings.

3.5.1 The additive case

For the additive error case, a closed form analytical solution for the optimal order quantity can be determined under normally distributed demand and errors. In fact, the cost (Equation 3.7) for the additive setting is given by:

$$Cost = k \cdot hMax(x - (Q_2 + \xi), 0) + hMax((Q_2 + \xi) - x, 0) \tag{3.14}$$

which can be rewritten as the following:

$$Cost = k \cdot hMax((x - \xi) - Q_2, 0) + hMax(Q_2 - (x - \xi), 0) \tag{3.15}$$

We can define an equivalent aggregated demand $x_{eq} = x - \xi$ which is normally distributed with parameters $\mu_{x_{eq}} = \mu_x$ and $\sigma_{x_{eq}} = \sqrt{\sigma_x^2 + \sigma_\xi^2}$. It then appears that the original Newsvendor model with additive errors is simply equivalent to a classical Newsvendor with this equivalent aggregated demand. As a result, the optimal order quantity is given by:

$$Q_2^* = \mu_{x_{eq}} + \sigma_{x_{eq}} \Phi^{-1}\left(\frac{k}{k+1}\right) \tag{3.16}$$

And the optimal expected cost is:

$$C_2(Q_2^*) = \sigma_{x_{eq}} h(k+1)\phi\left\{\Phi^{-1}\left[\frac{k}{k+1}\right]\right\} \tag{3.17}$$

Based on the above expression, we obtain the following numerical results for varying values of system parameters (with $\mu_x = 10$ and $\sigma_x = 3$):

σ_ξ	k=0.7 Q_2^*	k = 1 Q_2^*	k = 5 Q_2^*	k = 10 Q_2^*
0.00	9.33	10.00	12.90	14.01
0.50	9.32	10.00	12.94	14.06
1.00	9.30	10.00	13.06	14.22
1.50	9.25	10.00	13.25	14.48
2.00	9.20	10.00	13.49	14.81
2.50	9.13	10.00	13.78	15.21
3.00	9.06	10.00	14.11	15.67
3.50	8.97	10.00	14.46	16.16
4.00	8.89	10.00	14.84	16.68

Table 3.2: The optimal order quantity with normal distributions of demand and received quantity - the additive setting

Note that results we obtained in the case with uniform distributions of x and Q_A are still valid:

- Q_2^* increases with k
- An increase (decrease) of σ_ξ produces an increase (decrease) of Q_2^* for $k \leq 1$ ($k \geq 1$)

As in the case of uniformly distributed demand and received quantity, we analyze the benefits we can get by making the supplier 100% reliable by studying the ratio R we defined in (3.13). Variation of R with σ_ξ is presented in Figure (3.8) for $k = 10$ and $k = 0.7$. Figure (3.9) presents the variation of R with σ_ξ with σ_x for $k = 3$. Note that we have the same variations as the case with uniform distributions of demand and received quantity.

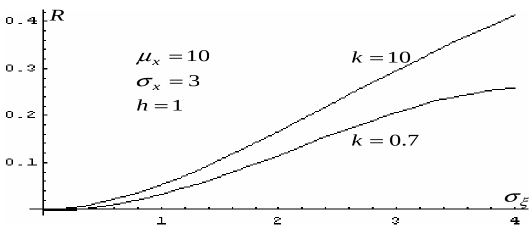


Figure 3.8: Variation of R with σ_ξ for different k

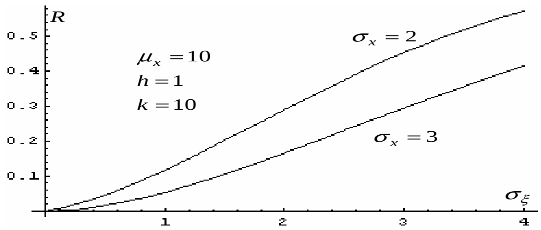


Figure 3.9: Variation of R with σ_ξ with σ_x for $k = 10$

Remark 3.3. *The equivalence between the original model and the aggregated demand model actually holds for generally distributed demand and additive errors as already noticed in Kök and Shang [41]. In particular, when demand and (additive) errors are uniformly distributed, one can also proceed to the aggregation of distributions to obtain an equivalent demand that follows a triangular distribution. We deliberately did not follow this approach and considered the individual distributions of demand and errors in Section 3.3 in order to develop a generic methodology that is valid for both additive and multiplicative errors. Note however that following the equivalent demand approach would led to a totally similar analysis as the one presented in Section 3.3.*

3.5.2 The multiplicative case

A numerical study is performed to optimize the model. Table (3.3) illustrates the impact of the different parameters on the optimal order quantity for $\mu_x = 10$ and $\sigma_x = 3$:

	k=0.7	**k = 1**	**k = 5**	**k = 10**
σ_γ	Q_2^*	Q_2^*	Q_2^*	Q_2^*
0.00	9.33	10.00	12.90	14.01
0.05	9.30	9.98	12.94	14.08
0.09	9.23	9.92	13.01	14.24
0.13	9.13	9.84	13.11	14.47
0.17	8.99	9.72	13.23	14.76
0.21	8.83	9.58	13.34	15.09
0.25	8.65	9.42	13.45	15.43
0.29	8.45	9.23	13.52	15.75
0.33	8.24	9.04	13.56	16.03
0.37	8.02	8.82	13.55	16.25

Table 3.3: The optimal order quantity for normal distributions of demand and received quantity - multiplicative case ($\mu_\gamma = 1$)

Again, all the results we obtained in Sections 3.3.2 and 3.4.2 are valid:

- The optimal order quantity increases with k
- For $k \leq 1$, an increase of σ_γ produces an increase of Q_2^*
- For $k > 1$, as in the case of uniform distributions, there are two phenomena which interfere: for small values of σ_γ the first phenomenon pushes to increase the order quantity. If σ_γ gets higher, the second phenomenon is prevailing and pushes to decrease the field of variation of the received quantity Q_A by decreasing the order quantity since the variability of Q_A is proportional to this quantity
- The variation of R with σ_γ, k and σ_x is also expected and is similar to the case with uniform distributions of demand and received quantity (cf Figures 3.10 and 3.11).

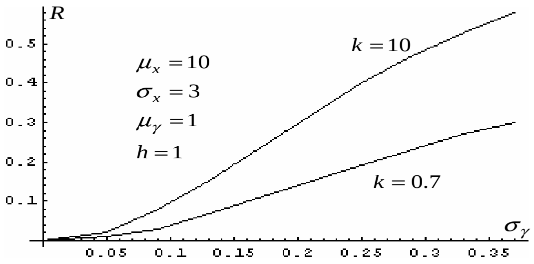


Figure 3.10: Variation of R with σ_γ for different k

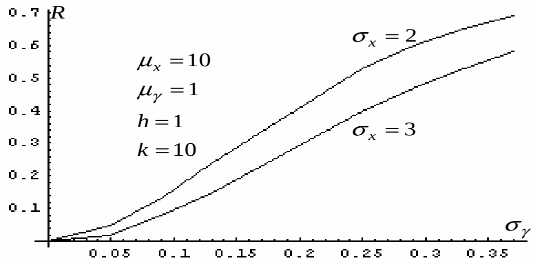


Figure 3.11: Variation of R with σ_γ with σ_x for $k = 10$

3.6 Conclusion

In this study, we consider a single stage inventory system with random yield stemming from supplier delivering incorrect ordering quantities. When demand and yield distributions are assumed to be uniform, we show that the earlier results existing in literature are not complete. In other words, we found that previous investigations developed in this area are valid for only a certain range of system parameters. This result motivates us to extend earlier found results and to propose an exhaustive inventory policy. For this purpose, we identify the different cases (or configurations) to consider, depending on values that parameters take. We express each of these cases as being an interval of variation of the standard deviation of the received quantity and determine the optimal order decision pertaining to each individual interval. The overall policy (for all possible values of system parameters) is then obtained by juxtaposing the individual optimal policies. This enables us to evaluate the penalty that would stem from using the optimal cost function that is provided in the literature, in comparison with the true optimal cost developed in our analysis.

Our results also show that the random yield issue can lead to significant losses for the inventory manager, especially when parameters pertaining to errors are important. In order to quantify this loss, we compare the optimal cost associated with the model without errors to the optimal cost associated with the model with errors. Furthermore, we strengthen our results by analyzing the case of normally distributed demand and errors.

Finally, for the transition with the following chapters let recall two important points:

- We deliberately do not analyze the impact of the RFID technology in the inventory system subject to unreliable supply process since the main contribution of this chapter was the extension of the literature pertaining to the random yield problem.

- As it will be shown is Chapter 6, an inventory system subject to inventory inaccuracy can be seen as an extended random yield problem. Some results of this chapter and particularly, the methodology used throughout this chapter will be used in analyzing the inventory inaccuracy issue, especially in the following chapter where misplacement type errors are considered in the retail supply chain.

Chapter 4

Misplacement Errors in a Centralized Supply Chain

This chapter considers the situation of a retail store subject to inventory inaccuracies stemming from execution problems. We assume that inventory inaccuracies are introduced by misplacement type errors that occur within the store, i.e. the whole quantity of products that is ordered and received from the manufacturer is not available on shelf to satisfy consumers' demand either because the replenishment process from the backroom to shelves is prone to errors (e.g. products are lost during this transfer, products are forbidden in the backroom, products are put on other shelves than where they should be...) or products are misplaced on other shelves by consumers during their visit to the store. We consider a Newsvendor model that captures this issue in a simple way: for a given quantity of products ordered from the manufacturer, only a random fraction is available for sales. We compare three approaches. In the first approach, the retailer is unaware of errors in the store. In the second approach, the retailer is aware of errors and optimizes its operations by taking into account this issue. The third approach deals with the case where the retailer deploys an advanced automatic identification technology (such as the Auto-ID system based on the Radio Frequency Identification (RFID) technology) to eliminate errors. In particular, we provide insights on the relative benefit of implementing the RFID technology (moving from approach 2 to approach 3) compared to the benefit of optimizing the system in presence of inaccuracies (moving from approach 1 to approach 2). We also provide an analytical expression of the cost of the RFID tag which makes its deployment cost effective. This chapter is based on the paper entitled "Analysis of the impact of the RFID technology on reducing product misplacement errors at retail stores" by Yacine Rekik, Evren Sahin and Yves Dallery, which has been accepted for publication in the International Journal of Production Economics (Rekik et al. [68]).

***Keywords**: retail operations, Radio Frequency Identification (RFID), random misplacement error, inventory record inaccuracies, Newsvendor model*

4.1 Introduction

Although advanced inventory control policies have been developed for almost a century, the occurrence of out-of-stock is still a significant issue in the retail supply chain. No matter how efficient the downstream supply chain operations are in shipping products to the retail store, inefficient backroom-to-shelf replenishment process and in store execution errors such as products placed on the wrong shelf, i.e. misplaced products, will lower the retailer performance.

In investigations concerning the reasons leading to the out-of-stock issue, several factors are identified (Gruen et al. [9] and Vuyk [74]): $i)$ Retail store ordering and forecasting problems, i.e. the ordered quantity is not enough to meet the actual consumer demand, $ii)$ Factor related to store shelving and replenishment practices in which products ordered are in the store but not on the right shelf. These factors may be related to shelf-space allocation, shelf-replenishment frequencies, store personnel capacity, in store execution errors etc. $iii)$ Factors related to the reliability of the supply system, i.e. the quantity received from supply process does not correspond to the ordered quantity. The literature related to the last cause, also known as the random yield problem, is extensive and several models that incorporate the effect of yield uncertainty or supplier unreliability on the inventory policy have been developed (cf Chapter 3). This chapter deals with the second cause and in particular the situation where all products ordered from the supply system are received but one part is not available on shelf due to misplacement errors arising in store.

As we have mentioned in Chapter 2, the literature addressing this cause is quite limited. We recall briefly in the following main investigations dealing with the misplacement type errors (the complete list of publications approaching this issue was presented in Chapter 2). Among empirical studies analyzing this issue, DeHoratius and Raman [34] explore the factors affecting inventory record inaccuracy, including misplacement type errors, and find that it increases with sales, with the number of stages in the supply chain, product variety, and the number of days elapsed since the last inventory audit. Ton and Raman [64] empirically study the factors affecting misplaced inventory in retail supply chains. Gaukler et al. [2] investigate the effects of the RFID technology within a retail supply chain. They build a Newsvendor model that takes into account the non efficiency of the replenishment process from the backroom to the shelf in the retail store. Then, based on this general model, they examine how the cost of the RFID implementation should be shared among supply chain actors, and determine coordinating contracts for the RFID-enabled supply chain within a Newsvendor framework.

The other issue considered in this chapter is the RFID technology which was described in Chapter 1. We will analyze the impact of this technology on the performance of an inventory system subject to misplacement errors. In particular, we will provide at the end of this chapter a critical analytical RFID tag cost which makes its deployment cost effective.

This chapter examines a store subject to inventory inaccuracies stemming from execution problems leading to products being placed in other places (another shelf, in the backroom...) than the shelf on which they normally should be. As a result, the whole products of an SKU are not available on shelf for sales to consumers. The structure of the chapter is as follows: in Section 4.2, we describe the issue of misplaced products and recall the different approaches that can be used to model the issue. The first approach is the approach where the retailer is unaware of errors occurring in the store. In the second approach, the retailer is aware of errors and optimizes its operations by taking into account errors. In Sections 4.2.3 and 4.2.4, we develop our analysis pertaining to these two approaches. In Section 4.4, we consider approach 3, i.e. the situation where errors are eliminated due to the deployment of the RFID technology. Then, in Section 4.4, we provide insights on the relative benefit of implementing the RFID technology (moving from approach 2 to approach 3) compared to the benefit of optimizing the system in presence of inventory inaccuracies (moving from approach 1 to approach 2). We also provide an analytical expression of the cost of the RFID tag which makes the deployment cost effective. Finally, Section 4.5 concludes the chapter.

4.2 Analysis of a retail store subject to misplacement errors

4.2.1 The modelling framework

This chapter deals with Supply chain Structure A previously described in the introduction of Part II. We consider a retail store who sells a single seasonal product to end consumers at a unit price r. Products are provided by the manufacturer at a unit cost w. It is assumed that, at the end of the season, products can be sold back at a discounted (salvage) price s which is strictly less than the purchase price. The stocking decision of the retailer is made within a one-period Newsvendor framework.
In order to model the impact of misplacement errors, we define θ as being the random variable which reflects the effect of misplacement errors on the real quantity which is available on shelf for consumers: θ is the ratio between the quantity on shelf which is available for sales and the total physical quantity available in the store. In other terms, for a quantity of products Q ordered and received from the manufacturer, only a random part, θQ will be available for consumers coming to the store and the remaining quantity, i.e. $(1-\theta)Q$ will not be accessible for consumers. We assume that the quantity of products lost during the selling season will be found at the end of the period, with the possibility to be salvaged at the unit salvage price s. Note that $\theta = 1$, if item level RFID technology is used in the retailer store. This last point means that the RFID technology is assumed to be able to totally eliminate the source of errors. As most of investigations within a Newsvendor framework, we assume that the distribution of demand is exogenous.

4.2.2 Approaches for managing a store inventory prone to misplacement errors

This section aims at determining the optimal quantity ordered by the retailer from the manufacturer before the season to satisfy the demand of consumers that visit a store which is perturbed by misplacement errors. As previously mentioned in the preliminary note of Part II, we consider three approaches that may be used in order to manage the inventory system:

1. Approach 1: the retailer has no information on misplacements that take place in the store so, he cannot observe the parameter θ. Thus, his decision about the ordering quantity is independent of θ.

2. Approach 2: the retailer knows that he is operating with internal misplacement errors and can estimate the distribution of θ . His decision about the ordering quantity is made by taking into

account the internal errors.

3. Approach 3: the retailer decides to remedy to the internal errors by implementing the RFID technology. There are no more errors but the retailer incurs the additional cost pertaining to the technology.

The sequence of events associated with Approaches 1 and 2 is as the following:

1. *The order:* before the beginning of the selling period, in order to satisfy consumers' demand, the retailer orders an amount of products Q_i $(i = 1, 2)$ from the manufacturer.
2. *The total physical inventory:* at the beginning of the period the retailer receives the quantity Q_i.
3. *The available for sales quantity:* due to internal errors occurring in the store, the quantity observed by consumers on the shelf, i.e. θQ_i, is different from the total quantity physically available in the store.
4. *The satisfaction of demand:* the actual demand x is observed and satisfied from the available for sales quantity.
5. The whole unsold quantity (items on shelf + misplaced items) is *discounted* at the end of the period.

The notations used throughout this chapter are as follows:

- Q_0^*: the optimal ordering quantity of the classical Newsvendor problem.
- π_0^*: the optimal expected profit of the classical Newsvendor problem.
- Q_i: the ordering quantity in Approach i $(i = 1, 2, 3)$.
- Q_i^*: the optimal value of Q_i.
- π_i: the expected profit function in Approach i.
- π_i^*: the optimal value of π_i.
- θ: the random parameter representing the error.
- g: pdf characterizing θ.
- μ_θ: the expected value of θerrors in the retail store.
- σ_θ: the standard deviation of θinternal errors in the retail store.
- L_θ: the lower bound of θ $(L_\theta \geq 0)$.
- U_θ: the upper bound of θ $(U_\theta \leq 1)$.
- x: the random variable representing demand.

- f (F): pdf (cdf) characterizing x.
- μ_x: the expected value of x.
- σ_x: the standard deviation of x.
- w: the unit product purchase cost.
- r: the unit product selling price.
- s: the unit product salvage price.
- t: the unit RFID tag cost.

In this section, we first analyze Approach 2: we begin by determining the expected profit function in presence of misplacement errors and the associated optimal quantity to order. Then, in order to illustrate the impact of error parameters (μ_θ and σ_θ) on the ordering decision, we consider two particular cases: the deterministic error case ($\sigma_\theta = 0$) where we focus on the impact of μ_θ on the ordering decision and the stochastic error case with uniformly distributed demand and error. Then, in the last part of this section we analyze Approach 1.
Note also that the analysis of the deterministic error case is also motivated by the extension which will be performed in the following chapter for the decentralized supply chain.

4.2.3 Analysis of Approach 2

In presence of errors, the retailer's profit is given by:

$$Profit = rMin(x, \theta Q_2) + s(Q_2 - Min(x, \theta Q_2)) - wQ_2 \tag{4.1}$$

Using the equality $Q_2 - Min(x, \theta Q_2) = (Q_2 - \theta Q_2) + (\theta Q_2 - Min(\theta Q_2, x) = (Q_2 - \theta Q_2) + Max(0, \theta Q_2 - x)$ leads to:

$$Profit = rMin(x, \theta Q_2) + sMax(\theta Q_2 - x, 0) - wQ_2 + s(Q_2 - \theta Q_2)$$

which can also be written as the following:

$$Profit = ux - uMax(0, x - \theta Q_2) - hMax(0, \theta Q_2 - x) - hQ_2(1 - \theta)$$

where u and h are respectively the unit underage and overage costs and are given by $u = r - w$ and $h = w - s$
The expected profit function associated with Approach 2 is also as the following:

$$\begin{aligned}
\pi_2(Q_2) &= u\mu_x - u \int_{\theta=L_\theta}^{U_\theta} \int_{x=\theta Q_2}^{+\infty} (x - \theta Q_2) f(x) g(\theta) dx d\theta \\
&- h \int_{\theta=L_\theta}^{U_\theta} \int_{x=0}^{\theta Q_2} (\theta Q_2 - x) f(x) g(\theta) dx d\theta \\
&- hQ_2 \left[1 - \mu_\theta\right]
\end{aligned} \tag{4.2}$$

We notice that the expression $\pi_2(Q_2)$ consists of two parts:

- The first part expresses the profit that the retailer would get if he orders Q_2 and receives θQ_2 from the manufacturer. This result is not surprising since the available for sales quantity is θQ_2.
- The second one expresses the overage cost that the retailer will incur because of the quantity $Q_2 [1 - \mu_\theta]$ which is misplaced.

From Equation 4.2, we notice that the first part of the expression of $\pi_2(Q_2)$ corresponds to the expected profit function of a stochastically proportional yield model which is concave in the ordering quantity (cf the analysis of Chapter 3) and the second part is a linear function of Q_2. $\pi_2(Q_2)$ is therefore concave in the order quantity Q_2. Differentiating $\pi_2(Q_2)$ with respect to Q_2 leads to the following equation from which Q_2^* can be deduced:

$$\int_{L_\theta}^{U_\theta} \theta g(\theta) \left[1 - F(\theta Q_2^*)\right] d\theta = \frac{h}{h+u} \tag{4.3}$$

In order to evaluate the effect of error parameters (μ_θ and σ_θ) on the ordering decision, we will consider two particular cases:

1. The case where the error is assumed to be deterministic ($\sigma_\theta = 0$) enabling to focus on the impact of μ_θ on the optimal ordering decision. This study is also important since its main results will be used in the following chapter where we extend the model of this chapter to the decentralized supply chain.
2. The case with a stochastic error for a uniformly distributed demand and error where the focus is on the impact of σ_θ on the optimal ordering decision.

The analysis of the first case enables to get insights on the impact of misplacement errors in environments where the information available is the mean error rate (or alternatively in conditions where σ_θ is weak). The examination of the second case is complementary to the first case and is especially interesting from an academic standpoint. Estimates of practical values taken by μ_θ and σ_θ can be found in empirical researches such as the investigation of Raman et al. [8] that states that consumers of a leading retailer cannot find in average 16% of items in the stores because those items are misplaced. To our knowledge, there is no study that provides practical values for σ_θ. Therefore, in order to be exhaustive in our analytical analysis, we will consider all feasible values that σ_θ can take. Whereas the values of σ_θ considered in our numerical examples are based on investigations that have been developed in the random yield literature which also focus on errors that perturb the physical flow of products (cf Shih [55] for example).

The case of deterministic error

Based on Equation 4.3, the following result states the optimal decisions for Approach 2 when error is deterministic:

Result 4.1. *Under a deterministic error setting:*

1. *The optimal ordering quantity for Approach 2 is such that:*

$$\begin{aligned} F(\mu_\theta Q_2^*) &= 1 - \frac{h}{h+u}\frac{1}{\mu_\theta} && for\ \mu_\theta \geq \frac{h}{h+u} \\ Q_2^* &= 0 && otherwise \end{aligned} \tag{4.4}$$

2. *The optimal expected profit for Approach 2 is such that:*

$$\begin{aligned} \pi_2(Q_2^*) &= (h+u)\int_{x=0}^{\mu_\theta Q_2^*} xf(x)dx && for\ \mu_\theta \geq \frac{h}{h+u} \\ \pi_2(Q_2^*) &= 0 && otherwise \end{aligned} \tag{4.5}$$

Proof. The Proof follows directly by the application of 4.2 and 4.3 in the deterministic error setting. □

Result 4.2. *Under a deterministic error setting, for* $\mu_\theta \geq \dfrac{h}{h+u}$*, Approach 2 is equivalent to a Newsvendor problem with a modified demand distribution* x_{eq} *with parameters* $(\mu_{x_{eq}}, \sigma_{x_{eq}})$ *such that:* $\mu_{x_{eq}} = \dfrac{\mu_x}{\mu_\theta}$*,* $\sigma_{x_{eq}} = \dfrac{\sigma_x}{\mu_\theta}$*, and modified cost parameters* h_{eq} *and* u_{eq} *such that* $h_{eq} = h$ *and* $u_{eq} = \mu_\theta u - h(1-\mu_\theta)$*.*

Proof. This follows from expressing the Newsvendor problem with the modified demand and cost parameters proposed above and comparing the results with the optimal ordering quantity and the optimal expected profit provided in result 4.1. □

Result 4.3. *In the particular case of a uniformly distributed demand, the optimal order quantity is given by:*

$$\begin{aligned} Q_2^* &= \frac{Q_0^* - (1-\mu_\theta)U_x}{\mu_\theta^2} \quad for \quad \mu_\theta \geq \frac{h}{h+u} \\ &= 0 \quad otherwise \end{aligned}$$

where

$$\begin{aligned} Q_0^* &= \frac{kU_x + L_x}{k+1} \\ L_x &= \mu_x - \sqrt{3}\sigma_x \\ U_x &= \mu_x + \sqrt{3}\sigma_x \end{aligned}$$

(L_x and U_x are respectively the lower and the upper bound of the demand distribution)

Proof. The proof follows directly by application of the result 4.1 in the case of an uniform distribution of demand □

Under the uniformly distributed demand assumption, Figure (4.1) and Figure (4.2) represent respectively the variation of Q_2^* and π_2^* with μ_θ. Note that throughout this chapter in our numerical examples, we set $\mu_x = 10$, $\sigma_x = 3$, $h = 1$ and define k such that $k = \frac{u}{h}$.

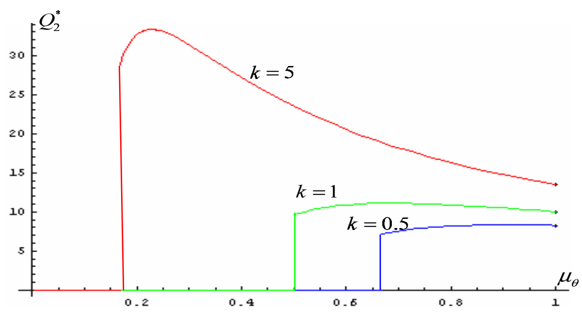


Figure 4.1: Variation of Q_2^* with μ_θ for different values of k

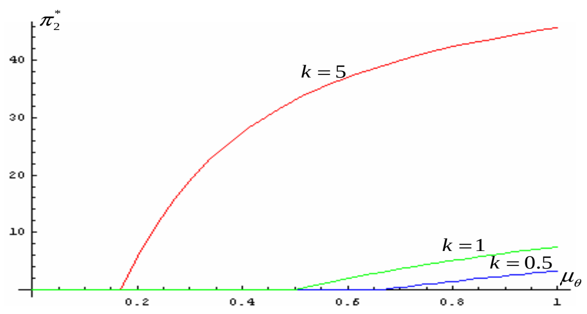


Figure 4.2: Variation of π_2^* with μ_θ for different values of k

Starting from $\mu_\theta = 1$, if one considers the evolution of Q_2^* for decreasing values of μ_θ, the following remarks can be made:

- For $\mu_\theta = 1$, there are no internal errors in the retail store and the solution of the problem is the one of the Newsvendor problem ($Q_2^* = Q_0^*$ and $\pi_2^* = \pi_0^*$)

- When μ_θ decreases, the product availability decreases since the quantity that consumers have access to is $\mu_\theta Q_2$. To compensate the reduction of product availability, the solution is to increase the quantity ordered since the available for sales quantity $\mu_\theta Q_2$ increases in the ordering quantity. As a consequence Q_2^* increases for decreasing values of μ_θ. For values of μ_θ smaller than a critical value, Q_2^* decreases as μ_θ decreases. The explanation is as follows: as μ_θ decreases, the amount of product which is not available for sales i.e. $(1-\theta)Q_2$ increases. To reduce the overage penalty that will be associated with this quantity at the end of the period, one reduces Q_2^* when μ_θ decreases.

- For values of μ_θ smaller than $\dfrac{h}{h+u}$, the available for sales quantity $\mu_\theta Q_2$ will be small. Even if one orders a large Q_2^*, the available for sales quantity would remain small. So the trade-off between underage and overage penalties is established for $Q_2^* = 0$.

Remark 4.1. *The evolution of Q_2^* can also be explained based on Result 4.2 presented earlier. Starting from $\mu_\theta = 1$, as μ_θ decreases:*

1. *On the one hand $\mu_{x_{eq}}$ increases while the coefficient of variation of x_{eq} remains equal to the coefficient of variation of x. This pushes the retailer to order more than Q_0^*, ie Q_2^* increases.*

2. *On the other hand, having $u_{eq} = u\mu_\theta - h(1-\mu_\theta)$ $(\leq u)$ pushes the retailer to reduce Q_2^* for decreasing values of μ_θ.*

The variation of Q_2^* with μ_θ results from the trade off between these two factors. The interval of μ_θ values where each factor is dominant depends on system parameters (k, μ_x, σ_x). As shown in Figure 4.1, the optimal ordering quantity is an increasing function in k which is intuitively expected.

Concerning the comparison between Approach 2 and the case without errors (Model 0), the following result states the relation between Q_0^* and Q_2^*:

Result 4.4. *Under a deterministic error setting, we have* $Q_2^* \leq \dfrac{Q_0^*}{\mu_\theta}$ *(the equality is achieved for* $\mu_\theta = 1$*).*

Proof. The proof follows from the following properties: $F(Q_0^*) = \frac{u}{u+h}$, $F(\mu_\theta Q_2^*) = 1 - \frac{h}{h+u}\frac{1}{\mu_\theta}$ and $\frac{h}{h+u} < \mu_\theta \leq 1$. □

Result 4.5. *Since* $\mu_\theta Q_2^*$ *decreases when* μ_θ *decreases, we deduce that* $\pi_2(Q_2^*)$ *decreases when* μ_θ *decreases for* $\mu_\theta > \frac{h}{h+u}$.

The case of stochastic error

In this section, in order to illustrate the effect of the variability of the misplacement errors on the optimal ordering decision, we assume that both demand and error are stochastic. Under this assumption, we examined the case where θ is uniformly distributed. Such an assumption which is explored by several authors such as Shih [55] and Inderfurth [57] enables us to derive analytical results by using the following notations:

$$f(x) = \begin{cases} \frac{1}{U_x - L_x} & \text{for } L_x \leq x \leq U_x \\ 0 & \text{otherwise} \end{cases} \qquad g(\theta) = \begin{cases} \frac{1}{U_\theta - L_\theta} & \text{for } L_\theta \leq \theta \leq U_\theta \\ 0 & \text{otherwise} \end{cases}$$

where U_θ and U_x (L_θ and L_x) are respectively the upper (lower) bounds of the random variables θ and x.

As showed in the analysis of the deterministic error case, an order is placed only if $\mu_\theta \geq \frac{h}{h+u}$. We therefore assume throughout this section that $\mu_\theta \geq \frac{h}{h+u}$. Starting from $\sigma_\theta = 0$, our aim is to analyze the evolution of Q_2^* for increasing values of σ_θ.

The same methodology used in Chapter 3 for the random yield problem is used here to derive the optimal order decisions: the expression of the expected profit depends on the positions of the distributions of x and θQ_2. In order to express it, one should distinguish the different positions that x and θQ_2 may have depending on the values of system parameters. Using the same definition of the different configurations described in the last chapter, Equation 4.3 for each configuration is also given as the following:

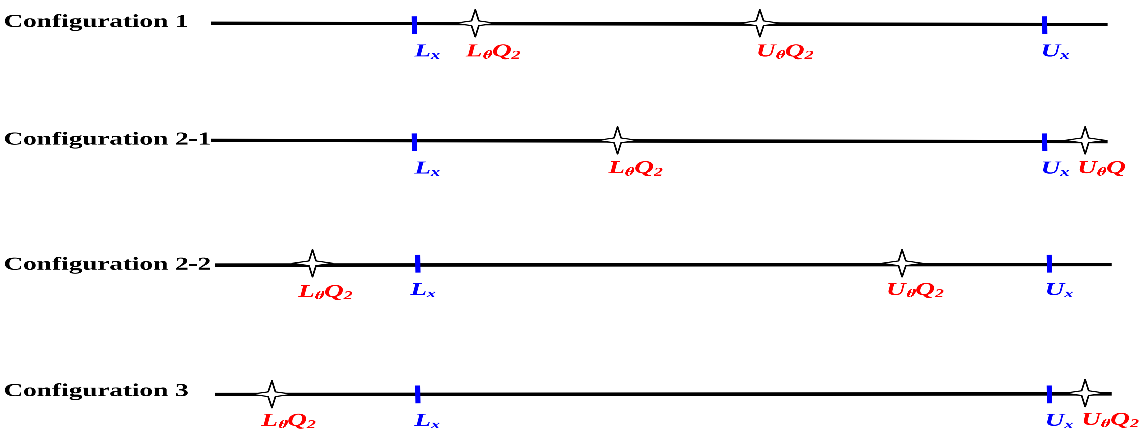


Figure 4.3: Positions between the distributions of x and θQ_2

Configuration 1	$\int_{L_\theta}^{U_\theta} \theta g(\theta)\left[1 - F(\theta Q_2^*)\right] d\theta = \frac{h}{h+u}$
Configuration 2-1	$\int_{L_\theta}^{\frac{U_x}{Q_2^*}} \theta g(\theta)\left[1 - F(\theta Q_2^*)\right] d\theta = \frac{h}{h+u}$
Configuration 2-2	$\int_{L_\theta}^{\frac{L_x}{Q_2^*}} \theta g(\theta) d\theta + \int_{\frac{L_x}{Q_2^*}}^{U_\theta} \theta g(\theta)\left[1 - F(\theta Q_2^*)\right] d\theta = \frac{h}{h+u}$
Configuration 3	$\int_{L_\theta}^{\frac{L_x}{Q_2^*}} \theta g(\theta) d\theta + \int_{\frac{L_x}{Q_2^*}}^{\frac{U_x}{Q_2^*}} \theta g(\theta)\left[1 - F(\theta Q_2^*)\right] d\theta = \frac{h}{h+u}$

For a given configuration, we can now derive the expression of the optimal quantity by solving the associated equation. Then, the constraints resulting from the positions of the distributions of x and θQ_2 enable to determine the interval of variation of σ_θ for which the expression of the optimal quantity obtained is valid.

Result 4.6. *Applying the methodology described above leads to the following expressions of the optimal ordering quantity:*

Configuration	**Interval of σ_θ**	**Expression of Q_2^***
Configuration 1	$[0, \sigma_{12}]$	$\frac{Q_0^* - (1-\mu_\theta)U_x}{\mu_\theta^2 + \sigma_\theta^2}$
Configuration 2	$[\sigma_{12}, \sigma_{23}]$	Q_2^* is obtained by solving $aQ_2^{*^3} + bQ_2^{*^2} + c = 0$
Configuration 3	$[\sigma_{23}, \sigma_{\theta\,\max}]$	$\frac{\sqrt{(k+1)(\mu_x^2+\sigma_x^2)}}{\sqrt{2U_\theta + L_\theta((k+1)L_\theta - 2)}}$

Where the critical expressions of σ_{ij}, a, b and c are given by:

For $0 \le k \le \frac{5U_x - 2L_x}{U_x + 2L_x}$	For $k \ge \frac{5U_x - 2L_x}{U_x + 2L_x} \ge 1$
$\sigma_{12} = \frac{\sqrt{de} - \sqrt{3}Q_0^*}{2L_x} + \frac{\sqrt{3}}{2}\frac{U_x}{L_x}(1 - \mu_\theta)$	$\sigma_{12} = \frac{\sqrt{3}Q_0^* - \sqrt{de}}{2U_x} - \frac{\sqrt{3}}{2}(1 - \mu_\theta)$
$\sigma_{23} = 2\frac{\sqrt{3}m + \sqrt{3m^2 - mn\mu_\theta}}{n} - \frac{\mu_\theta}{\sqrt{3}}$	$\sigma_{23} = \frac{\mu_\theta}{\sqrt{3}} - 2L_x\frac{\sqrt{3}L_x - \sqrt{3L_x^2 - \mu_\theta m}}{m}$
$\sigma_{\theta\,\max} = \frac{\mu_\theta}{\sqrt{3}}$	$\sigma_{\theta\,\max} = \frac{\mu_\theta}{\sqrt{3}}$
$a = 2(k+1)U_\theta^2$	$a = 2(k+1)L_\theta^3$
$b = 3(U_x - L_x)((k+1)L_\theta^2 - 2(U_\theta - L_\theta))$	$b = -3L_\theta^2 U_x(k+1)$
$-3(k+1)U_x U_\theta^2$	$-6(U_\theta - L_\theta)(U_x - L_x)$
$c = (k+1)L_x^3$	$c = (k+1)U_x^3$
$d = 3(Q_0^* - U_x) + \mu_\theta(3U_x - 2L_x)$	$d = 3(Q_0^* - U_x) + \mu_\theta U_x$
$e = Q_0^* - U_x + \mu_\theta(U_x + 2L_x)$	$e = Q_0^* - U_x + 3\mu_\theta U_x$
$m = (1 - (k+1)\mu_\theta)U_x^2$	$m = (k+1)(L_x - U_x)(U_x + 2L_x)$
$n = (k+1)(L_x - U_x)(2U_x + L_x)$	

Remarks:

1. We note that if $k \le \frac{L_x + 2U_x}{2L_x + U_x}$ (respectively $k \ge \frac{L_x + 2U_x}{2L_x + U_x}$), the sequence of configurations observed for increasing values of σ_θ is Configuration1, Configuration 2-1, Configuration 3 (respectively Configuration1, Configuration 2-2, Configuration 3).

2. For the case $k \le \frac{L_x + 2U_x}{2L_x + U_x}$, an additional assumption on k must be made to ensure the existence of Configuration 3 and to ensure that the lower bound of θQ_2 reaches zero in Configuration 3 (and not in Configuration 2). Therefore, if k satisfies $k \ge \frac{(\mu_x + \sqrt{3}\sigma_x)^2}{\mu_\theta(\mu_x^2 + \sigma_x^2)}$, Configuration 3 will be observed and $\sigma_{\theta_{max}} = \frac{\mu_\theta}{\sqrt{3}}$, otherwise Configuration 3 is not observed and $\sigma_{\theta_{max}} = \sigma_{23}$.

3. In order to avoid negative values of the available for sales quantity, μ_θ and σ_θ must satisfy $\mu_\theta - \sqrt{3}\sigma_\theta \ge 0$, leading to $\sigma_{\theta_{max}} = \frac{\mu_\theta}{\sqrt{3}}$. Note that although in practice σ_θ is expected to be smaller, Figures 4.4 and 4.5 consider higher values in order to enable a qualitative evaluation of the evolution of Q_2^*.

Figures 4.4 and 4.5 represent the variation of Q_2^* with σ_θ for $\mu_x = 10$, $\sigma_x = 3$: and $\mu_\theta = 0.9$. For a given value of μ_θ, the variation of Q_2^* with σ_θ depends on the value that k takes:

1. If $k < \frac{L_x + 2U_x}{2L_x + U_x}$: the overage cost is relatively more important than the underage cost. An increase of σ_θ will increase both costs, since the dominant cost is the overage cost, one should decrease Q_2^* for increasing values of σ_θ.

2. If $k > \frac{L_x + 2U_x}{2L_x + U_x}$: the underage cost is more important than the overage cost. We have two factors related to the increase of σ_θ which interfere: ***i)*** if σ_θ increases, the probability to fall in a underage situation gets higher, increasing the order quantity will decrease the probability to observe such situation. ***ii)*** If σ_θ increases, the field of variation of the available for sales quantity is higher and so, its variability $\sigma_\theta Q_2$ is higher and as a consequence, the total cost function gets higher.

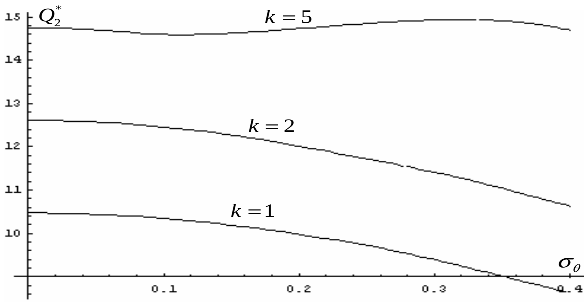


Figure 4.4: Variation of Q_2^* with σ_θ for different k and $\mu_\theta = 0.9$

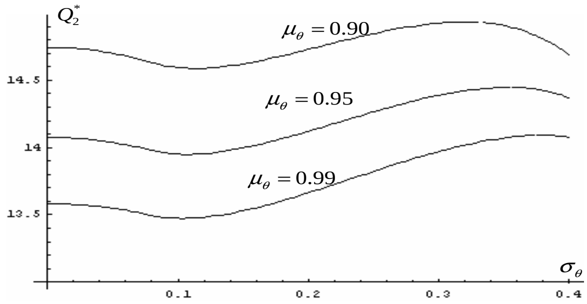


Figure 4.5: Variation of Q_2^* with σ_θ for different μ_θ and $k = 5$

Decreasing the order quantity is a way to reduce costs since the bounds of the distribution of θQ_2 depend on the order quantity Q_2. The evolution of Q_2^* for increasing values of σ_θ results from the trade of between the last two factors.

The effect of k on the optimal order quantity is as intuitively expected: the ordering quantity increases with k. The evolution of Q_2^* with σ_θ described above is verified for all possible values of μ_θ as it can be shown on Figure 4.5.

Once the expressions of the optimal ordering quantity are obtained, one can deduce the associated optimal expected profits (cf Appendix D.1). Figures 4.6 and 4.7 represent the variations of π_2^* with σ_θ:

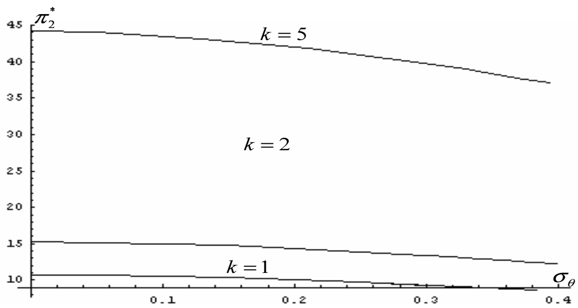


Figure 4.6: Variation of $\pi_2(Q_2^*)$ with σ_θ for different k and $\mu_\theta = 0.9$

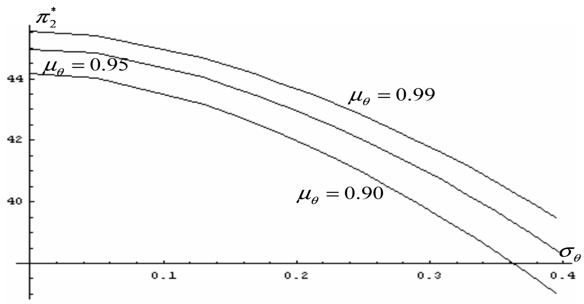


Figure 4.7: Variation of $\pi_2(Q_2^*)$ with σ_θ for different μ_θ and $k = 5$

Result 4.7. .

1. *For a given μ_θ, the optimal expected profit π_2^* decreases when σ_θ increases*

2. *For a given σ_θ, the optimal expected profit π_2^* decreases when μ_θ decreases*

Proof. For technical details cf Appendix D.1 □

To end this section, we note that results pertaining to the case where demand is normally distributed are derived numerically by using Equation 4.2 and 4.3 and the qualitative insights associated to this case are the same as the case with an uniformly distributed demand.

4.2.4 Analysis of Approach 1

Under this approach, the retailer is not aware of errors occurring in the store. He acts as if there were no errors so, his ordering quantity coincides simply with the ordering quantity of the classical Newsvendor problem's optimal ordering quantity. We therefore have $Q_1^* = Q_0^*$ where $Q_0^* = F^{-1}(\frac{u}{u+h})$.
When the retailer orders Q_1^*, the available for sales quantity will be θQ_1^* and so the real profit that the retailer will make in Approach 1 is obtained by using the profit function of Approach 2, i.e. $\pi_2(Q_1^*)$.

4.3 Analysis of a retailer store without misplacement errors

Several actions can contribute to eliminate misplacement errors occurring within the store. The re-engineering of the physical organization of the store, the definition of working procedures that improve the tracking of unreported in store product movements or the deployment of an advanced item identification technology such as RFID are among examples of such actions. Since our work is focused on getting insights on the RFID technology, we will suppose that the main action deployed to tackle in store misplacement errors is the implementation of this technology. Under Approach 3, we assume that the retailer uses this technology that enables to track the movement of goods within the store, to locate rapidly misplaced items and therefore eliminate errors. When RFID is implemented, if t represents the unit tag cost, the unit product purchasing cost is no longer w but $w + t$. The optimal ordering decision associated with Approach 3 will therefore be given by:

$$F(Q_3^*) = \frac{u-t}{u+h} \tag{4.6}$$

The associated optimal expected profit will be as follows:

$$\pi_3(Q_3^*) \quad = \quad (u+h)\int_{x=0}^{Q_3^*} xf(x)dx \tag{4.7}$$

Remarks:

1. Note that we can reinterpret the classical Newsvendor problem that can be found in literature as a particular case of either Approach 3 with $t = 0$ or Approach 2 with $\mu_\theta = 1$ and $\sigma_\theta = 0$.

2. Although we assume that RFID is $\%100$ reliable, our analysis can be extended to the case where RFID is not $\%100$ reliable. Considering the case where RFID is not 100% reliable consists in using the results obtained in the analysis of Approach 2 with an additional cost stemming from the deployment of RFID tags.

4.4 Benefits of the implementation of the RFID technology

Our aim in this section is to seek an answer to the questions: is RFID technology beneficial for the retailer? If yes, which tag cost make the implementation of this technology economically feasible?

We consider an initial situation where the retailer manages the store under Approach 1. In order to eliminate errors, the retailer chooses to implement RFID which enables to move from Approach 1 to

Approach 3. The absolute benefit achieved by this transition is therefore $\pi_3(Q_3^*) - \pi_2(Q_1^*)$. However, we argue that this difference does not enable to measure the true value of the RFID technology since:

1. One part of this benefit, i.e. $B_A = \pi_2(Q_2^*) - \pi_2(Q_1^*)$, can be achieved by getting information about the characteristics of the error (distribution, mean, variance) and integrating it in the optimization of the ordering decision. The evaluation of B_A gives further insights to the question: By how much the profit can be increased through a better replenishment policy that takes into account the probability for errors? If the distribution of θ is known, integrating this information in the ordering policy would permit to increase the profit(cf Approach 2).

2. The second part, $B_B = \pi_3(Q_3^*) - \pi_2(Q_2^*)$, is due to the elimination of errors based on the implementation of the RFID technology.

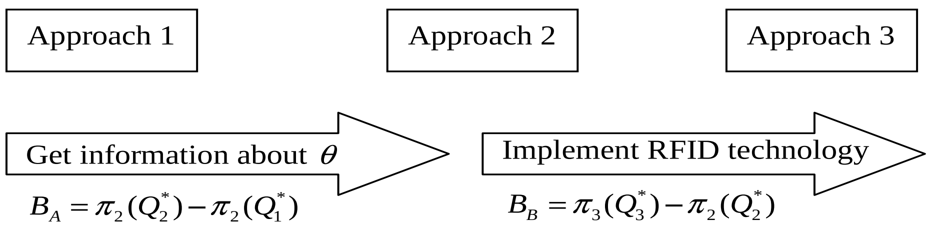


Figure 4.8: B_A versus B_B

4.4.1 Analysis of B_A

In order to evaluate the penalty resulting from ordering the inappropriate quantity in presence of errors, this section provides a comparison between the profits achieved under Approach 1 and Approach 2. From the definition of Q_2^* which maximizes π_2, it is straightforward to deduce that $\pi_2(Q_1^*) \leq \pi_2(Q_2^*)$ leading to $B_A \geq 0$.

To illustrate the impact of error parameters (μ_θ and σ_θ) on B_A, we consider in a numerical example the two cases presented previously, i.e., the deterministic case where demand is uniformly distributed and the stochastic case where both demand and error are uniformly distributed. For the deterministic case, we consider a retailer facing a uniformly distributed demand with parameters $\mu_x = 10$ and $\sigma_x = 3$. Cost parameters are such that $w = 2$, $r = 7$ and $s = 1$, the following figures represent respectively the variation of $\pi_2(Q_1^*)$, $\pi_2(Q_2^*)$ and B_A with the expected error μ_θ:

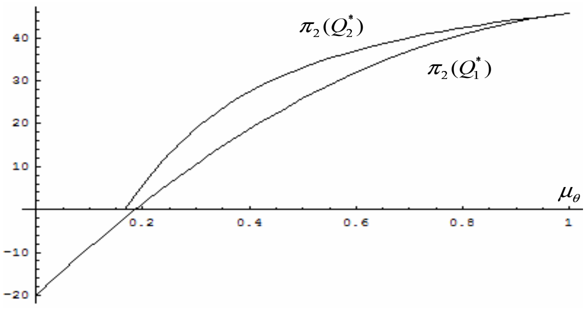


Figure 4.9: Variation of $\pi_2(Q_1^*)$ and $\pi_2(Q_2^*)$ with μ_θ

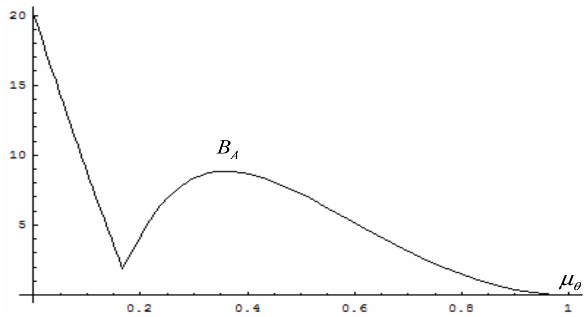


Figure 4.10: Variation of B_A with μ_θ

As explained earlier, we remember that for small values of μ_θ ($\mu_\theta < \frac{h}{h+u}$), it is optimal to not order. For such values of μ_θ, if an order is placed, $\pi_2(Q_1^*)$ would be negative and as a consequence B_A which would be equal to $-\pi_2(Q_1^*)$ could be very important for such range of μ_θ. For $\mu_\theta \geq \frac{h}{h+u}$, the evolution of B_A with μ_θ is the same as the evolution of Q_2^* with μ_θ since Q_1^* is independent of μ_θ changes.
For values of μ_θ that are most likely to be encountered in practice (i.e. $\mu_\theta \in [0.9, 1]$), the evolution of B_A for values of σ_θ different from 0 will be as follows:

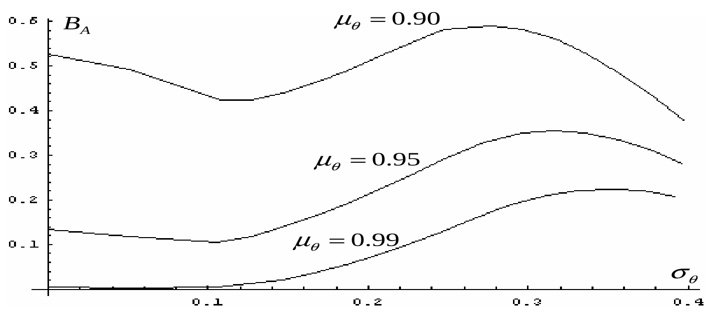


Figure 4.11: Variation of B_A with σ_θ

We notice that B_A is not an increasing function in σ_θ: this can be explained by the fact that Q_2^* is a non monotone function in σ_θ which is due to the two factors described in Section 4.2.3.

4.4.2 Analysis of B_B

Result 4.8. *If errors are deterministic, i.e. $\sigma_\theta = 0$, by comparing the expected profits of Approach 2 and Approach 3, we can identify a critical tag cost t_c such that $i)$ for $t \geq t_c$ the implementation of the RFID technology is not beneficial ($B_B \leq 0$) $ii)$ for $t \leq t_c$ the implementation of the RFID technology yields a positive benefit ($B_B \geq 0$). The expression of t_c is given by $t_c = h\dfrac{1-\mu_\theta}{\mu_\theta}$.*

Proof. The proof follows by observing that if $t \leq t_c = h\frac{1-\mu_\theta}{\mu_\theta}$, $1 - \frac{h}{h+u}\frac{1}{\mu_\theta} \leq \frac{u-t}{u+h}$. As a consequence $F(\mu_\theta Q_2^*) \leq F(Q_3^*)$. So $\mu_\theta Q_2^* \leq Q_3^*$. As a consequence
$\pi_2(Q_2^*) = (r-s)\int_{x=0}^{\mu_\theta Q_2^*} xf(x)dx \leq \pi_3(Q_3^*) = (r-s)\int_{x=0}^{Q_3^*} xf(x)dx.$ □

Starting from $\mu_\theta = 1$, we notice that the critical tag price t_c increases when μ_θ decreases: for high values of μ_θ, the tag cost should be small enough in order to be adopted by the retailer. By observing that $h = w - s$, it is important to notice also that t_c depends on the value of the unit purchase cost of the product which it will embedded to. For a small value of the purchase cost w, the tag cost should be very small to be adopted.

Result 4.9. *For the case of stochastic errors, if* $t \geq t_c = h\frac{1-\mu_\theta}{\mu_\theta}$*, the deployment of the RFID technology is not cost effective.*

Proof. The proof follows by combining the first point in Result 4.7 and Result 4.8. □

4.4.3 Comparison of B_A and B_B

This section develops a numerical analysis that compares B_A and B_B. As an example case, we consider a retailer facing a uniformly distributed demand with parameters $\mu_x = 10$ and $\sigma_x = 3$. Cost parameters are such that $w = 2$, $r = 7$ and $s = 1$. We also consider three possible values for the RFID tag i.e., $t = \alpha w$ $(\alpha = 0\%, 1\%, 5\%)$.
Let first consider the particular case where the error is deterministic $(\sigma_\theta = 0)$. The following figure represents the variation of B_A and B_B with μ_θ for different values of α:

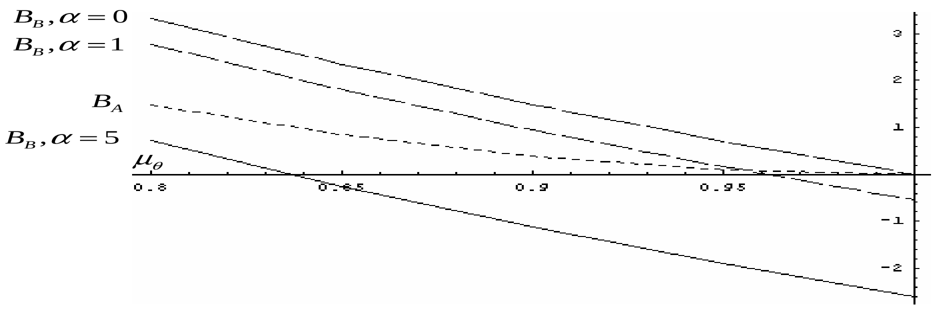


Figure 4.12: Variation of B_A and B_B with μ_θ for different values of α

For a given μ_θ, B_A can be higher or lower than B_B depending on the value of t. It appears that the benefit achieved by taking into account errors is comparable to the benefit achieved by the deployment of the RFID technology. We now assume that $\mu_\theta = 0.9$ and represent the variation of B_A and B_B with σ_θ for different values of α:

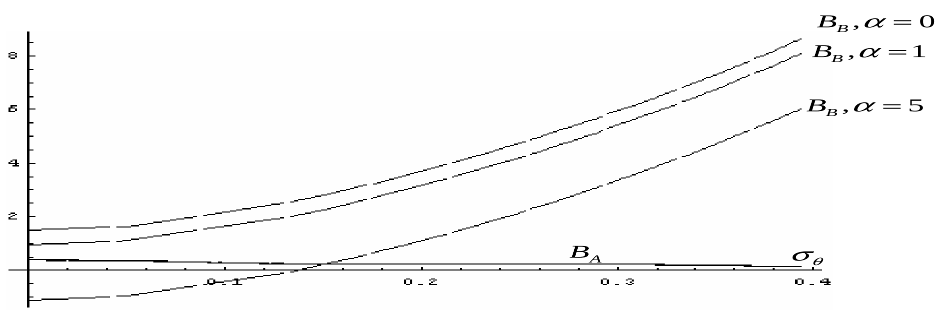


Figure 4.13: Variation of B_A and B_B with σ_θ for different values of α

According to the last analysis, the following remarks can be done:

- Getting information on misplacement errors and taking them into account when optimizing the ordering decision can lead to important savings and does not necessitate the deployment of a particular system, the retailer can benefit from this improvement by adjusting his ordering quantity.

- For the stochastic error case, we also notice that the critical tag price for which the RFID implementation yields a positive benefit depends on the value σ_θ. The less effective the current inventory management process (i.e. without RFID) is, the less important the tag cost will be for the decision whether to implement RFID or not.

Another concern is the sensitivity of potential benefits to certain parameters, especially their selling prices. A similar analysis comparing B_A to B_B can therefore be conducted with respect to the product selling price r. This type of analysis constitutes a basis for segmenting products based on values of r for a given error setting and for a given tag cost. For the same model parameters considered above, the variations of B_A and B_B with r for two values of α are as follows:

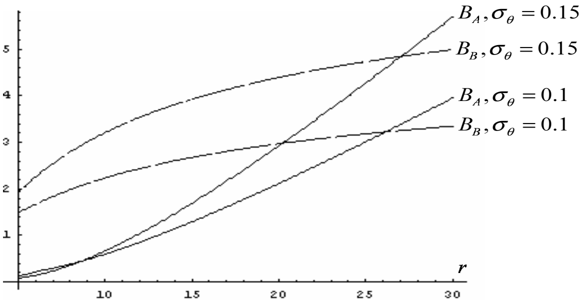


Figure 4.14: Variation of B_A and B_B with r for different values of σ_θ, $\alpha = 1$

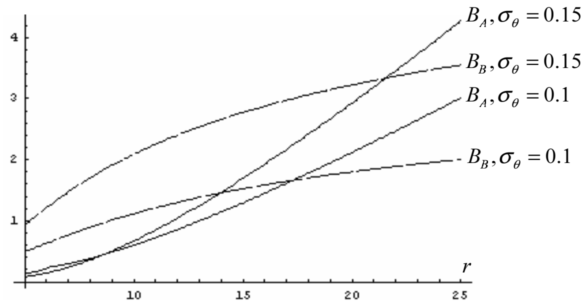


Figure 4.15: Variation of B_A and B_B with r for different values of σ_θ, $\alpha = 5$

The following remarks can be made:

- When r increases, the penalty resulting from ignoring misplacement errors increases since the unit underage cost increases. As a consequence B_A increases when r increases.

- When r increases, both $\pi_2(Q_2^*)$ and $\pi_3(Q_3^*)$ increase but it appears that $\pi_3(Q_3^*)$ increases more than $\pi_2(Q_2^*)$. B_B is an increasing function in r: RFID is more interesting for products having a high selling price. This result can be seen as a direct consequence of the fact that RFID is more interesting for products having a high w (cf the expression of t_c in Result 8).

- By comparing B_A and B_B, we deduce that taking into account errors when optimizing the inventory system is more beneficial than deploying RFID when r is high. As shown in the previous figures, it exists a critical selling price r_c which solves $B_A = B_B$ such that $B_A > B_B$ for $r > r_c$.

- As intuitively expected, r_c increases with σ_θ and decreases with α (t).

4.5 Conclusion

In this chapter, we have presented an analytical model of a single-period store inventory model subject to misplacement errors. We have compared different approaches to model this issue. In particular, we have highlighted that getting information about errors and taking them into account when optimizing the system can lead to important savings.
Concerning the difference between the quantity ordered from the manufacturer and the available for sales quantity, we have only considered misplacement as source of errors. We have also supposed that all products which are in the store (on shelves or misplaced) are sold at the salvage cost s at the end of the period. Our model can be extended to include other types of errors. To do this, we can introduce a unit cost s_c ($s_c \leq s$) that represents the additional cost that the retailer incurs in order to find a misplaced product at the end of the period. In other terms, when a misplaced product is found at the end of the period, his unit salvage price will be no more s but $(s - s_c)$. Introducing the cost parameter s_c enables to consider other types of errors. Indeed, the case of errors such as theft or perishment where the quantity which is not available to buy is not found or can not be sold at the end of the period is a particular case of the above formulation by setting $s_c = s$.
Our ongoing chapter focuses on the analysis of the inaccuracy issue in a decentralized supply chain. In Approach 2, the ordering quantity of the retailer is higher in order to remedy to the decrease of the product availability. Such an increase in the ordering quantity may make the manufacturer of the retailer less motivated to reduce errors by deploying the RFID technology and may also affect considerably the sharing of the RFID tag between supply chain partners in a decentralized context.

Chapter 5

Misplacement Errors in a Decentralized Supply Chain

This chapter extends the result of the last chapter in the case of a decentralized supply chain. The chapter analyzes a Newsvendor type inventory model in which a manufacturer sells a single product to a retailer store whose inventory is subject to errors stemming from execution problems. Within the store, all products are not available on shelf for sales either because the replenishment of the shelf from the backroom is subject to errors resulting in products forbidden in the backroom or misplaced on the other shelves of the store. We compare two approaches: in the first approach, namely Approach 2, the two supply chain actors are aware of errors and optimize their ordering decisions by taking into account this issue. The second approach, namely Approach 3, deals with the case where an advanced automatic identification system such as the Radio Frequency Identification (RFID) technology is deployed in order to eliminate the errors. Each approach is developed under three scenarios: in the centralized scenario, we consider a single decision-maker who is concerned with maximizing the entire supply chain's profit; in the decentralized uncoordinated scenario, the retailer and the manufacturer act as different parties and do not cooperate. The third scenario is the decentralized coordinated scenario, where we give conditions for coordinating the channel under a buyback contract. This chapter is inspired from the paper entitled "Evaluating the Impact of Misplacement Errors on Decentralized Retail Supply Chain" by Yacine Rekik, Zied Jemai, Evren Sahin and Yves Dallery, which has been presented in the 12^{th} IFAC Symposium on Information Control Problems in Manufacturing (INCOM'2006) where it won the price for the best paper within the track "Production Planning and Inventory Control"

Keywords: *Newsvendor model, execution errors, misplacement, RFID technology, supply chain coordination, wholesale contract, buy-back contract*

5.1 Introduction

This chapter examines the impact of store execution errors produced by misplacement type errors on the ordering decision of the retail store who faces a Newsvendor type inventory model (Supply Chain Structure A defined in the introduction of Part II). In an earlier investigation (Cf Chapter 4), we conducted this study for a centralized Supply Chain (CS). In this chapter, we consider a decentralized SC where the manufacturer and the retailer act as different parties. We propose two possible solutions to reduce the impact of errors on the performance of this supply chain.

- The first solution consists in deploying the RFID technology. RFID readers placed at different points within the store enable to detect products automatically (without human intervention) every time items flow through them and therefore contribute to the elimination of execution errors.

- The second proposed solution deals with channel coordination. Most achieve the coordination by transfer payments such that local optimization corresponds to global optimization. For example in a buy back contract, the retailer can return the excess order quantity at a partial refund, at the end of the selling season. In a revenue sharing contract, in addition to the wholesale price, the retailer gives to the manufacturer a percentage of his revenue. A further important issue to be considered in designing a contract concerns flexibility i.e. arbitrarily allocation of profit (by adjusting parameters of the contract) between the two SC actors so that each actor's profit is better off with the coordinating contact.

For this purpose, Approach 2 and Approach 3 previously defined are considered. The choice to not consider Approach 1 is essentially due to the fact that the true value of the RFID technology is calculated based on the case where the inventory manager is aware of errors by the mean of statistical tools.

Each approach is developed under three scenarios. In the centralized scenario, we consider a single decision-maker who is concerned with maximizing the entire supply chain's profit, In the decentralized uncoordinated scenario, the manufacturer and the retailer act as different parties and do not cooperate. The third scenario is the Decentralized Coordinated scenario, where SC actors cooperate in order to coordinate the channel.

Note that the idea of coordinating decentralized supply chain using contracts first originated by Pasternack [75]. Lariviere [76] and Cachon [77] present an excellent overview of decentralized supply chain control. Larivière and Porteus [78] present further results for contracting under a Newsvendor structure. Jemai et al. [79] shows that buy back contract generalizes linear transfer payment contracts. In this chapter, we build on several of results of the above papers and we use a modified buy back contract to coordinate the channel.

The main investigations dealing with RFID technology and misplacement errors in a decentralized supply chain are the works of Gaukler et al. [2] and Çamdereli and Swaminathan [3]. The two investigations are close to our framework presented in this chapter but, for the first one we point out a potential problem with the derivations of the Gaukler et al. [2] results and we suggest a way of avoiding this problem. For the second investigation, we show that results provided correspond to a particular

case of the framework presented in this chapter. Details concerning the two investigations and their relation with our research can be found in Appendices E.6 and E.7.
The main questions that this chapter addresses are:

1. What is the impact of misplacement type execution errors?

2. Which technology cost make the RFID feasible for both supply chain actors?

3. If we consider an initial situation with errors and no coordination, what is the best strategy to be adopted by supply chain actors: the deployment of the RFID technology or the coordination of the channel?

5.2 The problem setting

5.2.1 The modelling of the misplacement issue in the retail store

We consider the supply chain defined by Structure A described in the introduction of Part II: i.e. a supply chain consisting of one manufacturer and one retailer. The manufacturer produces a single seasonal product which has a unit production cost c and sells it to the retailer. The retailer sells the product in a store to end consumers at a unit price r. It is assumed that, at the end of the season, products can be sold back at a discounted (salvage) price s. The ordering decision of the retailer is made within a one-period Newsvendor framework.
Within this context, we define the parameter μ_θ which reflects the effect of errors on the physical quantity that can be sold to consumers[1]: with a quantity of products Q ordered from the manufacturer will be associated two quantities: i) an amount of products $\mu_\theta Q$ that is on shelf, thus, available to buy by consumers, and ii) an amount of products $(1-\mu_\theta)Q$ which is not available to buy since stolen or misplaced either in the backroom or on the other shelves. Concerning the portion of products which is not available to buy, one has to distinguish two cases according to the factor that generates it. Indeed, if this stems from theft or perishment, then this quantity can not be salvaged at the end of the season. If this is induced by execution type errors such as misplacement within the store, the lost quantity $(1-\mu_\theta)Q$ would be found at the end of period. In this chapter, we again consider this last case and we assume that at the end of the period, all products that are in the store (on shelves or misplaced) can be discounted. Note however that the analysis presented in this chapter could easily be modified to deal with the theft type errors or to take into account the cost associated with finding a misplaced product (cf Section 5.6). As most of investigations made within a Newsvendor framework, regarding parameters pertaining to the distribution of demand, we assume that they are provided (exogenous).

5.2.2 Models and scenarios under study

In order to examine the impact of errors and the value of the RFID technology on such supply chain, we consider two approaches:

[1]We use the same definition of the parameter μ_{μ_θ} used in Chapter 4

1. *Approach 2*: the retailer operates with internal errors and both the manufacturer and the retailer know the error parameter μ_θ. Decisions about the ordering quantity are made by taking into account μ_θ

2. *Approach 3*: the RFID technology is deployed within the store to eliminate errors. This model is a slightly modified version of the commonly known basic Newsvendor problem which includes the cost of the RFID technology

The basic Newsvendor problem, which will be called *Approach 0* appears to be a particular case of these two models: in Approach 1 if we set $\mu_\theta = 1$ we obtain Approach 0. In Approach 3 if we set the technology cost equal to zero, we also obtain Approach 0. Note also that Approach 2 and Approach 3 correspond simply with the models pertaining to Approach 2 and Approach 3, respectively, defined in Chapter 2.
For each model we examine three scenarios:

1. *The Centralized scenario* (C) where we assume that there is a single decision-maker who is concerned with maximizing the entire chain's profit.

2. *The Decentralized Uncoordinated scenario* (DU) where we consider two decision-makers, the manufacturer and the retailer, and each optimizes his own profit function.

3. *The Decentralized Coordinated scenario* (DC) where the manufacturer and the retailer cooperate in order to make the total expected profit closer to the expected profit associated with the Centralized scenario.

The following table represents the organization of the chapter:

	C scenario	DU scenario	DC scenario
Approach 2	Section 5.3.1	Section 5.3.2	Section 5.3.3
Approach 3	Section 5.4.1	Section 5.4.2	Section 5.4.3

Table 5.1: Organization of the chapter

Under the centralized scenario, the analysis of approaches described above and the comparison between them is developed in the last chapter. The analysis of Approach 0 (the basic Newsvendor problem) for each scenario is well known in the literature (cf Khouja [71] for the first scenario, Cachon [77] and Larivière and Porteus [78] for the second and the third scenarios). Appendix E.1 (E.2) summarizes the analysis of Approach 0 under the Decentralized Uncoordinated (Coordinated respectively) scenario.
In this chapter, our first contribution concerns Approach 2 which is less investigated in the literature. We examine an inventory system subject to errors and obtain analytical expressions of the optimal policy for both a centralized and a decentralized supply chain structure. Our second contribution concerns the comparison between Approach 2 and Approach 3 where we provide a sufficient condition on the values

of model parameters (especially on the cost of the RFID technology) which renders economically profitable the deployment of RFID technology for both SC actors. Our third contribution concerns the comparison between two strategies that may enable to improve the performance of a decentralized SC in presence of errors in the store. Indeed, we compare two strategies which can be adopted by supply chain actors while being in Approach 2 under the DU scenario. The first strategy consists in implementing the RFID technology while being in the DU scenario. The second strategy consists in ignoring the technology and improving the performance by coordinating the channel in presence of errors.

5.2.3 Notations

In the rest of the chapter, the following notations are used:

- Q_{ij}: the ordering quantity in Approach j $(j = 0, 2, 3)$ under scenario i $(i = C, DU, DC)$.
- Q^*_{ij}: the optimal value of Q_{ij}. item π^k_{ij}: the expected profit for entity k $(k = M, R)$ in Approach j $(j = 0, 2, 3)$ under scenario i $(i = C, DU, DC)$.
- w_{ij}: the unit product purchase price for Approach j $(j = 0, 2, 3)$ under scenario i $(i = C, DU, DC)$.
- r: the unit product selling price.
- s: the unit product salvage price.
- c: the unit production price.
- x: the random variable representing demand.
- $f(x)(F(x))$: pdf (cdf) characterizing the demand.
- μ: the expected demand.
- σ: the standard deviation of demand.
- ϕ (Φ): the standard normal pdf (cdf).

5.3 Analysis of Approach 2

Under Approach 2, we assume that both the retailer and the manufacturer are aware of errors and optimize their expected profit function by taking into account the error parameter. The sequence of events in this model is as follows:

1. The order: before the beginning of the selling period, in order to satisfy the store's demand, the retailer orders an amount of products Q_{i2} $(i = C, DU, DC)$ from the manufacturer.
2. The total physical inventory: at the beginning of the period the retailer receives the quantity Q_{i2} $(i = C, DU, DC)$ within the store.

3. The available to buy quantity: due to internal errors occurring in the store, the quantity observed by consumers in the shelf, $\mu_\theta Q_{i2}$ $(i = C, DU, DC)$, may be different from the quantity physically available to satisfy demand.

4. The satisfaction of demand: the actual demand x is observed satisfied from the available to buy quantity.

5. All the unsold quantity (on shelf + misplaced) is discounted at the end of the period.

5.3.1 Analysis of the Centralized scenario (C2)

We consider a centralized supply chain where both the retailer and the manufacturer are part of the same organization and managed by the same entity. There is a single decision-maker who is concerned with maximizing the entire chain's profit. As a consequence, we can ignore the wholesale price transaction since it is internal. The ordering decision of the centralized decision-maker is made by taking into account μ_θ. A detailed analysis of this approach (and all proofs associated with this section) was presented in Chapter 4. In this section we only present main results concerning this approach in a centralized scenario. The expected profit function of Approach 2 under the Centralized scenario is given by:

$$
\begin{aligned}
\pi_{C2}(Q_{C2}) &= (r-c)\mu - (r-c)\int_{x=\mu_\theta Q_{C2}}^{+\infty} (x - \mu_\theta Q_{C2}) f(x)dx \\
&- (c-s)\int_{x=0}^{\mu_\theta Q_{C2}} (\mu_\theta Q_{C2} - x) f(x)dx \\
&- (c-s)Q_{C2}(1-\mu_\theta)
\end{aligned}
\tag{5.1}
$$

The following proposition states the optimal ordering quantity and the optimal expected profit of Approach 2 under the Centralized scenario:

Theorem 5.1. .

a. *The expected profit function is concave in the ordering quantity Q_{C2}*

b. *The optimal ordering quantity for Approach 2 in the Centralized scenario is such that:*

$$
\begin{aligned}
F(\mu_\theta Q^*_{C2}) &= \frac{r\mu_\theta + (1-\mu_\theta)s - c}{(r-s)\mu_\theta} \quad for\ \mu_\theta \geq \frac{c-s}{r-s} \\
Q^*_{C2} &= 0 \qquad otherwise
\end{aligned}
\tag{5.2}
$$

c. *The optimal expected profit for Approach 2 in the Centralized scenario is such that:*

$$
\begin{aligned}
\pi_{C2}(Q^*_{C2}) &= (r-s)\int_{x=0}^{\mu_\theta Q^*_{C2}} xf(x)dx \ for\ \mu_\theta \geq \frac{c-s}{r-s} \\
\pi_{C2}(Q^*_{C2}) &= 0 \qquad otherwise
\end{aligned}
\tag{5.3}
$$

Further investigation leads to the following proposition:

Proposition 5.1. *For $\mu_\theta \geq \dfrac{c-s}{r-s}$, Approach 2 is equivalent to a Newsvendor problem with a modified demand distribution having parameters μ_{eq} and σ_{eq} such that $\mu_{eq} = \dfrac{\mu}{\mu_\theta}$, $\sigma_{eq} = \dfrac{\sigma}{\mu_\theta}$, and equivalent modified unit costs c_{eq}, s_{eq} and r_{eq} such that $c_{eq} = c$, $s_{eq} = s$ and $r_{eq} = r\mu_\theta + s(1-\mu_\theta)$*

The following figures, (Figures 5.1 and 5.2) represent respectively the variation of Q^*_{C2} and $\pi^*_{C2}$with μ_θ for different values of r for $c = 7$, $s = 1$, $\mu = 10$ and $\sigma = 2$:

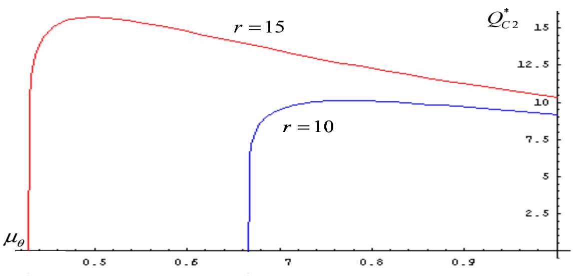


Figure 5.1: Variation of Q^*_{C2} with μ_θ for different values of r

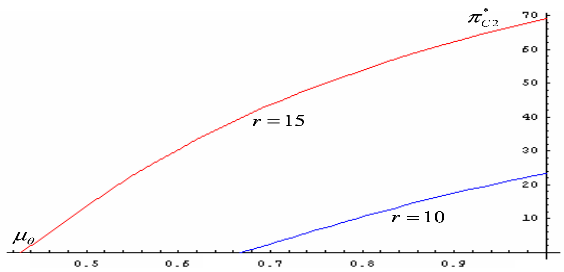


Figure 5.2: Variation of π^*_{C2} with μ_θ for different values of r

Recall that when μ_θ decreases, the product availability decreases since the quantity that the customer has access is $\mu_\theta Q_{C2}$. To remedy to the decrease of the product availability, the solution is to order more since the available to buy quantity $\mu_\theta Q_{C2}$ is increasing in the ordering quantity. As a consequence Q_{C2} increases when μ_θ decreases. But below a critical value of μ_θ, ordering more to increase the product affability increases also the quantity which is not available to buy $((1-\mu_\theta)Q_{C2})$ and which will be discounted. So, below this critical value of μ_θ, Q_{C2} decreases when μ_θ decreases. For small values of μ_θ ($\mu_\theta < \dfrac{c-s}{r-s}$), the available to buy quantity ($\mu_\theta Q_{C2}$) is small. Even if a big quantity is ordered, the available to buy quantity remains small, so the trade-off between underage and overage penalties is established for $Q^*_{C2} = 0$. Concerning the expected profit, note that, as expected, it decreases when μ_θ decreases as illustrated in Figure 5.2.

5.3.2 Analysis of the Decentralized Uncoordinated scenario (DU2)

Under this scenario we assume that the manufacturer and the retailer are two independently owned and managed firms, where each party is trying to maximize his own profit. We analyze in this section the case where the two supply chain actors do not coordinate. We consider the wholesale contract: the manufacturer chooses the unit wholesale price w_{DU2} and after observing w_{DU2}, the retailer chooses the order quantity Q_{DU2}. Recall that both the manufacturer and the retailer can observe the error parameter μ_θ and optimize their inventory systems with taking into account μ_θ. The decision action of the manufacturer depends on the decision action of the retailer and vice versa. Game theory gives precious tools to determine these actions. In this chapter, we are interested in a Stackelberg equilibrium where the manufacturer acts as a Stackelberg leader and then offers a take-it or leave-it proposition to the retailer.

The Retailer's Problem: In Approach 2, the retailer's profit function under a wholesale contract is similar to the profit function of the Centralized scenario of the same approach (Approach 2) with the exception that the retailer now pays a wholesale price w_{DU2} to the manufacturer whose unit cost is still c. The expected profit for the retailer is also as follows:

$$\begin{aligned}\pi^R_{DU2}(Q_{DU2}, w_{DU2}) \;&=\; (r - w_{DU2})\mu \\ &-\; (r - w_{DU2}) \int_{x=\mu_\theta Q_{DU2}}^{+\infty} (x - \mu_\theta Q_{DU2}) f(x)dx \\ &+\; (w_{DU2} - s) \int_{x=0}^{Q_{DU2}} (\mu_\theta Q_{DU2} - x) f(x)dx \\ &-\; (w_{DU2} - s) Q_{DU2}(1 - \mu_\theta) \qquad (5.4)\end{aligned}$$

As shown in the last section (Section 5.3.1), for $\mu_\theta \geq \frac{w_{DU2} - s}{r - s}$, the optimal ordering quantity should verify:

$$F(\mu_\theta Q^*_{DU2}) = \frac{r\mu_\theta + (1 - \mu_\theta)s - w_{DU2}}{(r - s)\mu_\theta} \qquad (5.5)$$

For the case where $\mu_\theta \leq \frac{w_{DU2} - s}{r - s}$, it is optimal for the retailer to not order because the trade off between underage and overage penalties is established for an optimal ordering quantity equal to zero. In the rest of the chapter we assume that model parameters are such that an order is placed and only results pertaining to this situation will be developed. In our numerical examples we will assume that $0.8 \leq \mu_\theta \leq 1$.

The Manufacturer's Problem: The manufacturer has the wholesale price w_{DU2} as decision variable. He is able to anticipate the retailer's order for any wholesale price. As a consequence, the function $Q_{DU2}(w_{DU2})$ is deterministic for him. The manufacturer's problem then is to choose the wholesale price w_{DU2} that maximizes his expected profit $\pi^M_{DU2}(w_{DU2})$ which is given as follows:

$$\pi^M_{DU2}(w_{DU2}) = (w_{DU2} - c) Q_{DU2}(w_{DU2}) \qquad (5.6)$$

Theorem 5.2. *For Approach 2 under an IGFR*[2] *demand distribution*

a. *The optimum is reached for Q^*_{DU2}, such that:*

$$1 - F(\mu_\theta Q^*_{DU2}) - \mu_\theta Q^*_{DU2} f(\mu_\theta Q^*_{DU2}) = \frac{c - s}{r - s} \frac{1}{\mu_\theta}$$

b. *The corresponding optimum wholesale price is:*

$$w^*_{DU2} = c + (r - s)\mu_\theta(\mu_\theta Q^*_{DU2}) f(\mu_\theta Q^*_{DU2})$$

[2] Increasing General Failure Rate. The General Failure Rate is defined by the function $g(x) = x\frac{1-F(x)}{f(x)}$ and it gives (roughly) the percentage decrease in the probability of a stock out from increasing the stocking quantity by 1%. A distribution has an increasing generalized failure rate (IGFR) if $g(x)$ is weakly increasing for all x such taht $F(x) < 1$

c. *The optimal expect profit of the manufacturer is:*

$$\pi_{DU2}^{M*} = (r - s)(\mu_\theta Q_{DU2}^*)^2 f(\mu_\theta Q_{DU2}^*)$$

d. *The optimal expect profit of the retailer is:*

$$\pi_{DU2}^{R*} = (r - s) \int_{x=0}^{\mu_\theta Q_{DU2}^*} x f(x) dx$$

Proof. cf Appendix E.3 □

Theorem 5.2 enables us to identify some interesting properties:

Property 5.1. *In Approach 2 under a wholesale price contract:*

a. *The manufacturer's optimal amount of product sold to the retailer Q_{DU2}^**

- *increases as the retail price r and the salvage price s increase*
- *decreases as the unit production cost c increases*

b. *The manufacturer's optimal wholesale price charged to the retailer w_{DU2}^**

- *decreases as the retail price r and the salvage price s increase*
- *increases as the unit production cost c increases*

For the case of a normally distributed demand, some interesting results concerning the variation of Q_{DU2}^*, w_{DU2}^* and π_{DU2}^{M*} with the error parameter μ_θ can be deduced as proposed in the following Property:

Property 5.2. *The impact of errors in the DU scenario of Approach 2 is as follows:*

a. *w_{DU2}^* decreases as μ_θ decreases*

b. *$\mu_\theta Q_{DU2}^*$, decreases as μ_θ decreases*

c. *The manufacturer's expected optimal profit, π_{DU2}^{M*}, decreases as μ_θ decreases*

d. *The retailer's expected optimal profit, π_{DU2}^{R*}, decreases as μ_θ decreases*

Proof. cf Appendix E.4 □

Note that for reasonable values of model parameters, Q_{DU2}^*, increases as μ_θ decreases. To get further insights, we consider an example where demand is normally distributed with parameters $\mu = 10$ and $\sigma = 2$, the unit production cost is $c = 7$, the unit selling price and the unit salvage price are respectively $r = 15$ and $s = 1$. Concerning the optimal ordering and wholesale price, figures bellow (Figures 5.3 and 5.4), represent respectively the variation of Q_{DU2}^* and w_{DU2}^* with μ_θ:

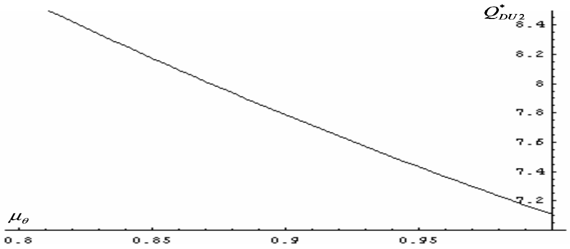


Figure 5.3: Variation of Q^*_{DU2} with μ_θ

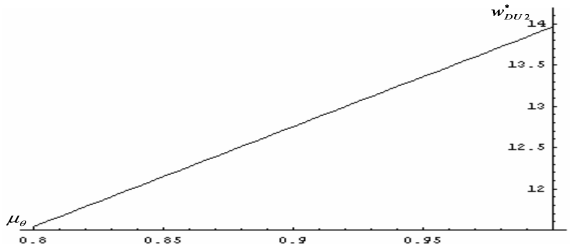


Figure 5.4: Variation of w^*_{DU2} with μ_θ

The following observations explain the variations of Q^*_{DU2} and w^*_{DU2} with μ_θ:

- When $\mu_\theta = 1$, Q^*_{DU2} corresponds to the optimal ordering quantity of Approach 0. As in the Centralized scenario, the optimal ordering quantity in Approach 2 is more important than the one of Approach 0 and increases as μ_θ decreases (for reasonable values of model parameters). Such result is not surprising since the presence of errors decreases the product's availability. Increasing the ordering quantity is the way to increase the available to buy quantity and to remedy to shelf unavailability.

- As a consequence of the last observation, the manufacturer's optimal wholesale price charged to the retailer in Approach 2 is less important than the one in Approach 0 and decreases as μ_θ decreases.

Concerning the optimal expected profit achieved by each supply chain actor, Figures 5.5 and 5.6 represent respectively the variation of π^{M*}_{DU2} and π^{R*}_{DU2} with μ_θ:

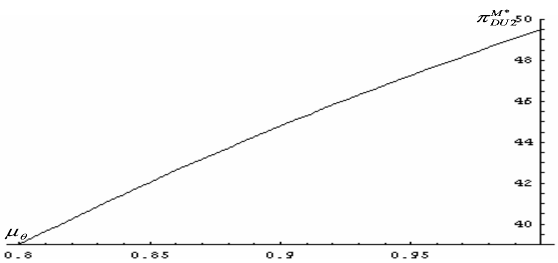


Figure 5.5: Variation of π^{M*}_{DU2} with μ_θ

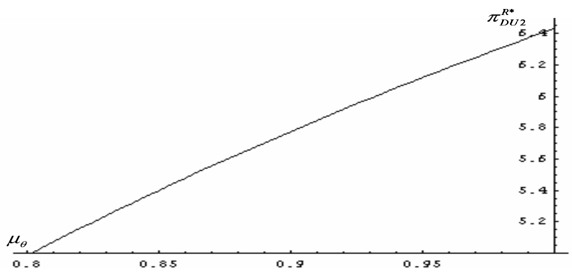


Figure 5.6: Variation of π^{R*}_{DU2} with μ_θ

As expected, the retailer suffers from the presence of errors in his store since his expected profit function decreases when μ_θ decreases. As explained later, because of errors, the manufacturer's amount of product sold to the retailer increases. So, it is not unreasonable to expect that the inventory inaccuracy might have beneficial effects on the manufacturer expected profit. This is not true because the manufacturer should decrease the wholesale price charged to the retailer. As a consequence, as illustrated in Figure 5.5, the manufacturer suffers also from inventory inaccuracy in the retailer's store.

Comparison between C2 and DU2: An important aspect to consider is supply chain efficiency which measures how efficient the Decentralized Uncoordinated scenario performs in relation to the Centralized scenario. In the Decentralized Uncoordinated scenario, the outcomes are worse for all the parties involved (manufacturer, retailer, supply chain, and consumer) compared to the Centralized scenario, because in the Decentralized scenario both the retailer and the manufacturer independently try to maximize their own profits, i.e., they each try to get a margin. This effect is called "Double Marginalization" (DM). The supply chain efficiency is defined as the ratio between the total supply chain profit in the Decentralized Uncoordinated scenario and the Centralized scenario profit.
For Approach 2, the supply chain efficiency is given by:

$$eff_2 = \frac{\pi_{DU2}^{M*} + \pi_{DU2}^{R*}}{\pi_{C2}^{*}} \tag{5.7}$$

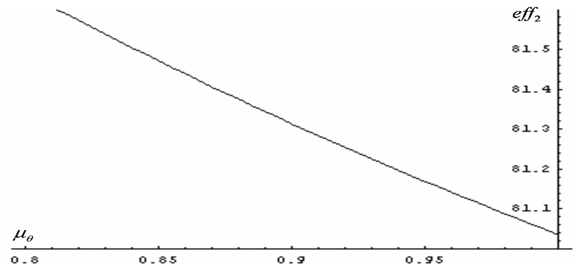


Figure 5.7: Variation of eff_2 with μ_θ

Figure 5.7 represents the variation of the supply chain efficiency with μ_θ for $\mu = 10$, $\sigma = 2$, $c = 7$, $s = 1$ and $r = 15$. We note that this efficiency increases as μ_θ decreases and this is somewhat surprising but can be explained as follows: as we have shown, Approach 2 can be considered as an equivalent Newsvendor problem with modified demand distribution such that: $\mu_{eq} = \frac{\mu}{\mu_\theta}$, $\sigma_{eq} = \frac{\sigma}{\mu_\theta}$, and equivalent modified unit costs c_{eq}, s_{eq} and r_{eq} such that $c_{eq} = c$, $s_{eq} = s$ and $r_{eq} = r\mu_\theta + s(1-\mu_\theta)$. When μ_θ decreases, both r_{eq} and w_{DU2}^* charged by the manufacturer to the retailer decrease. This induces a reduction of the double marginalization effect since r_{eq} and w_{DU2}^* are closer.

5.3.3 Analysis of the Decentralized Coordinated scenario (DC2)

In this section, we analyze the Decentralized Coordinated scenario and if and how one can design contracts such that even though each supply chain actor acts out for self interest, the decentralized solution might approach the centralized optimal solution.
We consider a modified buy-back contract: as an incentive for the retailer to order more and move toward channel coordination, the manufacturer offers to buy back the unsold quantities of the available to buy quantity (quantity which was on shelf and available to customer during the selling period). We interpret the working of our modified buy-back contract such that the manufacturer pays $(b_{DC2} - s)$ to each unsold unit of the available to buy quantity, and the retailer salvages the item for s. In other terms, the manufacturer buy back the units which were on their right place on the shelf. He also shares with the retailer the demand uncertainty risk but he will stay indifferent to inventory inaccuracy produced in

the retailer's store. With such a buy-back contract, the retailer is, essentially, getting a higher "salvage" value, b_{DC2}, for a fraction of the unsold goods, the other fraction (the non available to buy quantity) will continue to be discounted at the price s.

The Retailer's Problem: The retailer's expected profit function in Approach 2 under our modified buy-back contract is given by:

$$\begin{aligned}\pi^R_{DC2}(Q_{DC2}) &= (r - w_{DC2})\mu - (r - w_{DC2})\int_{x=\mu_\theta Q_{DC2}}^{+\infty}(x - \mu_\theta Q_{DC2})f(x)dx \\ &- (w_{DC2} - b_{DC2})\int_{x=-\infty}^{\mu_\theta Q_{DC2}}(\mu_\theta Q_{DC2} - x)f(x)dx \\ &- (w_{DC2} - s)Q_{DC2}(1 - \mu_\theta) \end{aligned} \quad (5.8)$$

By using the same optimization method as in Centralized scenario, we can show that the optimal ordering quantity Q^*_{DC2} and the optimal expected profit for the retailer, $\pi^R_{DC2}(Q^*_{DC2})$, should respectively satisfy:

$$F(\mu_\theta Q^*_{DC2}) = \frac{(r - s)\mu_\theta - (w_{DC2} - s)}{(r - b_{DC2})\mu_\theta} = \frac{r_{eq} - w_{DC2}}{r_{eq} - s_{eq}} \quad (5.9)$$
$$where\; r_{eq} = r\mu_\theta + (1 - \mu_\theta)s \;\; and \;\; s_{eq} = b\mu_\theta + (1 - \mu_\theta)s$$

$$\pi^R_{DC2}(Q^*_{DC2}) = (r - b_{DC2})\int_{x=0}^{\mu_\theta Q^*_{DC2}} xf(x)dx \quad (5.10)$$

It is straightforward to verify that the retailer's optimal ordering quantity and profit are increasing in b_{DC2} for a fixed wholesale price w_{DC2}.

The Manufacturer's Problem: With our modified buy-back contract, the expected profit of the manufacturer is given by:

$$\pi^M_{DC2}(w_{DC2}, b_{DC2}) = (w_{DC2} - c)Q_{DC2} - (b_{DC2} - s)\int_0^{\mu_\theta Q_{DC2}} F(x)dx \quad (5.11)$$

The buy-back contract is completely determined by the 2-tuple (w_{DC2}, b_{DC2}), where w_{DC2} and b_{DC2} are the wholesale price and the buy-back price, respectively. The following proposition states condition on model parameters under which channel coordination is realized:

Theorem 5.3. *There is a 2-tuple $(w_{DC2}(\varepsilon), b_{DC2}(\varepsilon))$ that is able to coordinate the decentralized scenario $w_{DC2}(\varepsilon) = r_{eq} - \varepsilon$ and $b_{DC2}(\varepsilon) = r - \varepsilon\frac{r-s}{r_{eq}-c}$ where $r_{eq} = r\mu_\theta + s(1 - \mu_\theta)$ and $\varepsilon \in (0, r_{eq} - c)$*

a. *The retailer orders the optimal solution of the Centralized scenario and system profit is also equal to the Centralized scenario profit*

b. *Retailer profit is increasing in ε. Specially $\pi^{R*}_{DC2}(w_{DC2}(\varepsilon), b_{DC2}(\varepsilon)) = \frac{\varepsilon}{r_{eq}-c}\pi^*_{C2}$*

c. *Manufacturer profit is decreasing in ε. Specially $\pi^{M*}_{DC2}(w_{DC2}(\varepsilon), b_{DC2}(\varepsilon)) = (1 - \frac{\varepsilon}{r_{eq}-c})\pi^*_{C2}$*

Proof. cf Appendix E.5 □

The parameter ε governs the distribution of market power and determines how the benefit achieved by coordination is shared between SC actors. We notice that when μ_θ decreases, the retailer gets a higher sharing of the total supply chain profit. The following property states the variation of buy back contract variables with the error parameter μ_θ:

Property 5.3. *w_{DC2} and b_{DC2} charged to the retailer decrease when the error parameter μ_θ decreases*

Proof. The proof follows directly by observing that $\frac{\partial w_{DC2}(\varepsilon)}{\partial \mu_\theta} \geq 0$ and $\frac{\partial b_{DC2}(\varepsilon)}{\partial \mu_\theta} \geq 0$. □

5.4 Analysis of Approach 3

In Approach 3, the RFID technology is deployed in order to eliminate errors in the store. As in Chapter 4, we assume that the cost associated with the implementation of this technology consists in RFID tags embedded to each item individually, at a certain tag cost t. The fixed costs of investments necessary to implement the technology (such as reader systems cost, infrastructure costs, basic application integration costs, maintenance and support costs and overhead costs) are deliberately not part of our model.

In the Centralized scenario the notion of sharing the cost t does not make sense. In the Decentralized Uncoordinated scenario, under a wholesale contract we will assume that the manufacturer will pay the whole tag cost t. We will show that the notion of sharing the tag cost will not influence the optimal solution because the manufacturer will simply adjust the wholesale price charged to the retailer so as to include on it his part pertaining to the RFID tag cost. The same remark holds also under the Decentralized Coordinated scenario. We also assume throughout our analysis that the additional cost pertaining to the RFID technology, t, will be paid by the manufacturer. We will show during the analysis that arguments presented above are justified.

When the RFID technology is deployed, the unit production price is no longer c but $c+t$: the optimization of Approach 3 under each scenario is therefore a modified Newsvendor problem with a production cost $c+t$.

5.4.1 Analysis of the Centralized scenario (C3)

Under the Centralized scenario, the general form of the expected profit as a function of model parameters is given by:

$$\begin{aligned}\pi_{C3}(Q_{C3}) &= (r-c-t)\mu - (r-c-t)\int_{x=Q_{C3}}^{+\infty}(x-Q_{C0})f(x)dx \\ &- (c+t-s)\int_{x=0}^{Q_{C3}}(Q_{C3}-x)f(x)dx \qquad (5.12)\end{aligned}$$

We can easily show that the expected profit function is concave and is maximized at the value of Q^*_{C3} such that:

$$F(Q^*_{C3}) = \frac{r-(c+t)}{r-s} \qquad (5.13)$$

The optimal expected profit for Approach 3 in the Centralized scenario is given by:

$$\pi_{C3}(Q^*_{C3}) \quad = \quad (r-s)\int_{x=0}^{Q^*_{C3}} xf(x)dx \tag{5.14}$$

5.4.2 Analysis of the Decentralized Uncoordinated scenario (DU3)

The formulation and the optimization of Approach 3 is similar to the analysis provided in Larivière and Porteus [78] with a unit production cost $c+t$ (this analysis is described in Appendix E.1).

The Retailer's Problem: The expected profit function of the retailer is given by:

$$\begin{aligned}\pi^R_{DU3}(Q_{DU3}, w_{DU3}) \quad &= \quad (r-w_{DU3})\mu - (r-w_{DU3})\int_{x=Q_{DU3}}^{+\infty}(x-Q_{DU3})f(x)dx \\ &- \quad (w_{DU3}-s)\int_{x=0}^{Q_{DU3}}(Q_{DU3}-x)f(x)dx\end{aligned} \tag{5.15}$$

The optimal ordering quantity should also verify:

$$Q^*_{DU3}(w_{DU3}) = F^{-1}\left[\frac{r-w_{DU3}}{r-s}\right] \tag{5.16}$$

The Manufacturer's Problem: The manufacturer has the wholesale price as his decision variable. The manufacturer's expected profit is given by:

$$\pi^M_{DU3}(w_{DU3}) = (w_{DU3}-(c+t))Q_{DU3}(w_{DU3}) \tag{5.17}$$

By using the inverse of $Q_{DU3}(w_{DU3})$ which is $w_{DU3}(Q_{DU3}) = (r-s)\left[1-F(Q_{DU3})\right]+s$, the expected profit function of the manufacturer can be written as follows:

$$\pi^M_{DU3}(Q_{DU3}) = \{(r-s)\left[1-F(Q_{DU3})\right]-(c-s+t)\}\, Q_{DU3} \tag{5.18}$$

The result of Larivière and Porteus [78]) can be invoked directly, as the following theorem shows:

Theorem 5.4. *For Approach 3 under an IGFR demand distribution*

a. The optimum is reached for Q^*_{DU3}, such that:

$$1-F(Q^*_{DU3}) - Q^*_{DU3}f(Q^*_{DU3}) = \frac{c-s+t}{r-s}$$

b. The corresponding optimum wholesale price is:

$$w^*_{DU3} = c+t+(r-s)Q^*_{DU3}f(Q^*_{DU3})$$

c. The optimum expect profit of the manufacturer is:

$$\pi^{M*}_{DU3} = (r-s)(Q^*_{DU3})^2 f(Q^*_{DU3})$$

d. The optimum expect profit of the retailer is:

$$\pi_{DU3}^{R*} = (r-s)\int_{x=0}^{Q_{DU3}^*} xf(x)dx$$

Proof. cf Larivière and Porteus [78] by considering $c+t$ as a unit production cost □

As expected, even if we assumed that the manufacturer pays the tag price, he adjusts his wholesale price in order to include this additional cost. This is why the notion of sharing the tag price is not relevant under a wholesale contract. To focus on this result, let consider two settings where the manufacturer pays a fraction $\alpha_1 t$ ($\alpha_2 t$) and the retailer pays the rest $(1-\alpha_1)t$ ($(1-\alpha_2)t$) in the first (second) setting. Using the same analysis as before, we can easily show that $[w_{DU3}^*]_{\alpha_2} - [w_{DU3}^*]_{\alpha_1} = (\alpha_2 - \alpha_1)t$. As a consequence $[Q_{DU3}^*]_{\alpha_2} = [Q_{DU3}^*]_{\alpha_1}$ which assures that $[\pi_{DU3}^{M*}]_{\alpha_2} = [\pi_{DU3}^{M*}]_{\alpha_1}$ and $[\pi_{DU3}^{R*}]_{\alpha_2} = [\pi_{DU3}^{R*}]_{\alpha_1}$. Some interesting properties as the ones provided in the following property can be directly deduced:

Property 5.4. *In Approach 3 under a wholesale price contract:*

a. The manufacturer's optimal amount of product sold to the retailer Q_{DU3}^*

- *increases as the retail price r and the salvage price s increases*
- *decreases as the unit production cost c and the unit tag cost t increases*

b. The manufacturer's optimal wholesale price charged to the retailer w_{DU3}^*

- *decreases as the retail price r and the salvage price s increases*
- *increases as the unit production cost c and the unit tag cost t increases*

Results concerning the tag price of the last property are expected and can be interpreted as follows: the additional cost pertaining to the RFID technology makes the unit production cost of the product higher and so the wholesale price charged to the retailer is higher. Such increase in the wholesale price obliges the retailer to decrease his ordering quantity. As a consequence, both the retailer and the manufacturer will suffer from the tag price since their expected profit function will decrease.

5.4.3 Analysis of the Decentralized Coordinated Scenario (DC3)

In this section, we examine the Decentralized Coordinated scenario for Approach 3. The buy back contract is considered: the manufacturer offers to buyback all unsold units of the retailer at the price b_{DC3}. We interpret the working of the buy-back contract such that the manufacturer pays $(b_{DC3} - s)$ to each unsold unit, and the retailer salvages the item for s. Here again, we assume that the tag price is totally paid by the manufacturer.

The Retailer's Problem: The expected profit function of the retailer is given by:

$$\begin{aligned}\pi^{R}_{DC3}(Q_{DC3}, w_{DC3}, b_{DC3}) &= (r - w_{DC3})\mu \\ &- (r - w_{DC3})\int_{x=Q_{DC3}}^{+\infty}(x - Q_{DC3})f(x)dx \\ &- (w_{DC3} - b_{DC3})\int_{x=0}^{Q_{DC3}}(Q_{DC3} - x)f(x)dx \qquad (5.19)\end{aligned}$$

By assuming $b_{DC3} < w_{DC3} < r$, the retailer's profit is strictly concave and the optimal ordering quantity Q^*_{DC3} satisfies:

$$Q^*_{DC3}(w_{DC3}, b_{DC3}) = F^{-1}\left[\frac{r - w_{DC3}}{r - b_{DC3}}\right] \qquad (5.20)$$

The Manufacturer's Problem: The expected profit function of the manufacturer is alike the model of Pasternack [75] with the exception that the unit production price is no longer c but $c + t$:

$$\begin{aligned}\pi^{M}_{DC3}(w_{DC3}, b_{DC3}) &= (w_{DC3} - (c+t))Q_{DC3}(w_{DC3}, b_{DC3}) \\ &- (b_{DC3} - s)\int_{0}^{Q_{DC3}} F(x)dx \qquad (5.21)\end{aligned}$$

The following theorem (from Pasternack [75]) outlines the coordination conditions of the buy back contract:

Theorem 5.5. *Suppose that the manufacturer offers a contract $(w_{DC3}(\varepsilon), b_{DC3}(\varepsilon))$ for $\varepsilon \in (0, r-c-t)$ where $w_{DC3}(\varepsilon) = r - \varepsilon$ and $b_{DC3}(\varepsilon) = r - \varepsilon\frac{r-s}{r-(c+t)}$:*

a. *The retailer order the optimal solution of the Centralized Scenario and system profit is also equal to the Centralized Scenario profits*

b. *Retailer profit is increasing in ε. Specially $\pi^{R*}_{DC3}(w_{DC3}(\varepsilon), b_{DC3}(\varepsilon)) = \frac{\varepsilon}{r-(c+t)}\pi^*_{C3}$*

c. *Manufacturer profit is decreasing in ε. Specially $\pi^{M*}_{DC3}(w_{DC3}(\varepsilon), b_{DC3}(\varepsilon)) = (1 - \frac{\varepsilon}{r-(c+t)})\pi^*_{C3}$*

Proof. cf Pasternack [75] □

Here again, the parameter ε governs the distribution of market power and permits the sharing of the profit of coordination between SC actors. Under the buy-back contract, as assumed the notion of sharing the tag cost is not relevant. In fact, concerning the optimal ordering quantity, recall that the aim of our contract is to assure that the ordered quantity is the same as in the Centralized scenario for which the notion of sharing the tag cost does not make sense. Concerning optimal expected profits for each SC actor, let consider two settings where the manufacturer pays a fraction $\alpha_1 t$ ($\alpha_2 t$) and the retailer pays the rest $(1 - \alpha_1)t$ ($(1 - \alpha_2)t$) in the first (second) setting. we can easily verify that it exists two values ε_1 and ε_2 that assure that both the manufacturer and the retailer achieve the same optimal expected profit in each setting. The following property states the variation of buy back contract variables with the tag price t:

Property 5.5. *The buy back price charged to the retailer is decreasing in the tag price t*

Proof. The proof of follows directly by observing that $\frac{\partial b_{DC3}(\varepsilon)}{\partial t} \geq 0$. □

5.5 Strategies reducing the impact of errors in a decentralized supply chain

Before discussing on the strategies that can be adopted in a decentralized supply chain, recall that in Chapter 4, we have shown that in a centralized supply chain it exists a critical unit tag price t_{cr} such that $i)$ For $t \geq t_{cr}$ the implementation of the RFID technology is not beneficial $ii)$ For $t \leq t_{cr}$ the implementation of the RFID technology yields a positive benefit. This critical value of t is the solution of the equation $\pi_{C3}(Q^*_{C3}) = \pi_{C2}(Q^*_{C2})$ and is given by $t_{cr} = \dfrac{1-\mu_\theta}{\mu_\theta}(c-s)$.
In this section, we consider a decentralized SC and present two strategies that may enable to both SC actors to reduce the impact of errors. We assume that they initially manage their inventory under the Decentralized Uncoordinated scenario. Our aim is to compare the performance of the following two strategies:

- *Strategy 1: introducing the RFID technology while being in a Decentralized Uncoordinated supply chain structure (i.e. the transaction from DU2 to DU3).*
- *Strategy 2: coordinating the supply chain in presence of errors (i.e. the transaction from DU2 to DC2).*

Throughout this section, we consider a normally distributed demand. Note that our analysis can be extended to deal with other demand distributions.

5.5.1 Strategy 1: introduction of the RFID technology

This section focuses on the comparison between Approach 2 and Approach 3 under a wholesale contract. Our aim being to answer the question "Under which circumstances both the retailer and the manufacturer will be interested in deploying the RFID technology?"
Concerning the comparison between Q^*_{DU2} and Q^*_{DU3}, the following proposition should be made:

Proposition 5.2. *Under a wholesale price contract, if* $t \leq (c-s)\frac{1-\mu_\theta}{\mu_\theta}$ *we have* $\mu_\theta Q^*_{DU2} \leq Q^*_{DU3}$

Proof. The proof follows by observing that $\frac{c-s}{r-s}\frac{1}{\mu_\theta} \leq \frac{c-s+t}{r-s}$ if $t \leq (c-s)\frac{1-\mu_\theta}{\mu_\theta}$. Using the fact that both $\mu_\theta Q^*_{DU2}$ and Q^*_{DU3} are less than μ and the fact that $H(y) = 1 - F(y) - yf(y)$ is decreasing in y for $y \leq \mu$ (cf Appendix E.4), the result is directly deduced. □

As a consequence of the last result we can show that:

Proposition 5.3. *Under a wholesale price contract, if* $t \leq (c-s)\frac{1-\mu_\theta}{\mu_\theta}$ *we have* $\pi^{M*}_{DU2} \leq \pi^{M*}_{DU3}$

Proof. the proof follows directly by using the result of the previous proposition and by observing that $\mu_\theta Q^*_{DU2} \leq Q^*_{DU3} \leq \mu$ *and the fact that* $f(x)$*is increasing in* x *for* $x \leq \mu$ □

As a consequence we have the result that $t \leq (c-s)\frac{1-\mu_\theta}{\mu_\theta}$ is a sufficient condition to make the manufacturer interested in deploying the RFID technology in order to remedy to inventory inaccuracy in the retailer's store. The following proposition answers the question "Is this condition interesting to the retailer also?"

Proposition 5.4. *Under a wholesale price contract, if* $t \leq (c-s)\frac{1-\mu_\theta}{\mu_\theta}$ *we have* $\pi^{R*}_{DU2} \leq \pi^{R*}_{DU3}$

Proof. The proof follows by using the fact that when $t = (c-s)\frac{1-\mu_\theta}{\mu_\theta}$ we have $\pi^{R*}_{DU2} = \pi^{R*}_{DU3}$ and the fact that the optimal expected profit of the retailer in Approach 3 is decreasing in the tag price t □

The following proposition summarizes the condition under which both the retailer and the manufacturer are interested in deploying the RFID technology:

Theorem 5.6. *Under a wholesale price contract,* $t \leq t_{cr} = \frac{1-\mu_\theta}{\mu_\theta}(c-s)$ *is a sufficient condition to make the retailer and the manufacturer choose the deployment of RFID technology*

Proof. The proof follows by using results of propositions 5.3 and 5.4. □

It is important to notice here that the critical unit tag cost provided in the last theorem (under the Decentralized Uncoordinated scenario) is the same as the one presented in the Centralized scenario. This confirms our assumption concerning the fact that the notion of sharing the tag price between the SC actors under the DU scenario does not affect optimal solutions.
To quantify the relative benefits achieved by the retailer and the manufacturer by deploying the RFID technology we introduce the following two ratios:

$$(RB_{DU2\to DU3})_M = \frac{\pi^{M*}_{DU3} - \pi^{M*}_{DU2}}{\pi^{M*}_{DU2}} * 100 \tag{5.22}$$

$$(RB_{DU2\to DU3})_R = \frac{\pi^{R*}_{DU3} - \pi^{R*}_{DU2}}{\pi^{R*}_{DU2}} * 100 \tag{5.23}$$

Which measure the Relative Benefit achieved respectively by the manufacturer and the retailer from applying Strategy 1.
For our numerical example ($\mu = 10$, $\sigma = 2$, $c = 7$, $r = 15$ and $s = 1$) and for two values of error parameter $\mu_\theta = 0.8$ and $\mu_\theta = 0.9$, the following figures (Figures 5.8 and 5.9) represent respectively the benefit that the manufacturer and the retailer achieve by applying Strategy 1 as a function of the tag price:

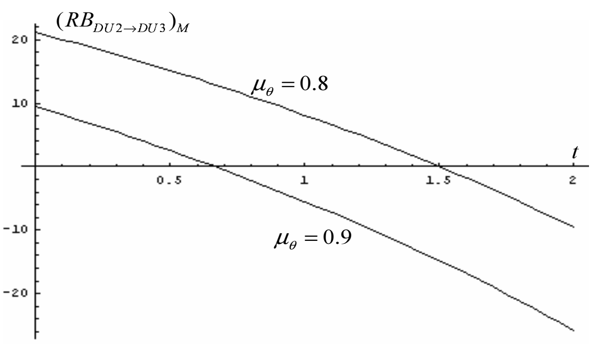


Figure 5.8: Variation of $(RB_{DU2\to DU3})_M$ with t

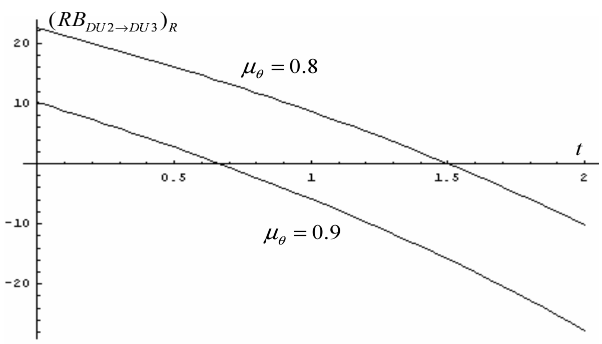


Figure 5.9: Variation of $(RB_{DU2\to DU3})_R$ with t

As shown in the previous figures: for an error parameter $\mu_\theta = 0.8$, it is beneficial for the two supply chain actors to deploy the RFID technology if the tag price is under the critical value $t_{cr} = 1.5$. It is also important to note that the critical tag price decreases when the error parameter increases. For high values of μ_θ, the tag price should be small in order to be adopted by the supply chain actors. We notice also that the critical tag price depends on the value of the product on which it will be placed. For a small value of the production cost of the product, the tag price should be very small to be adopted.

5.5.2 Strategy 2: coordination of the supply chain

In order to quantify the benefit achieved by the retailer and the manufacturer by coordinating the channel, we introduce the following two ratios:

$$(RB_{DU2 \to DC2})_M = \frac{\pi^{M*}_{DC2} - \pi^{M*}_{DU2}}{\pi^{M*}_{DU2}} * 100 \tag{5.24}$$

$$(RB_{DU2 \to DC2})_R = \frac{\pi^{R*}_{DC2} - \pi^{R*}_{DU2}}{\pi^{R*}_{DU2}} * 100 \tag{5.25}$$

An important issue to be considered in designing our modified buy back contract concerns the flexibility of the contract i.e. the fact that both the manufacturer and the retailer should obtain a profit higher than they would do without contract. Otherwise, the SC actors would not be prompted to adopt the contract. In order to assure that both supply chain actors coordinate, the following proposition states a condition on ε which assures a positive benefit for the two SC actors:

Proposition 5.5. .

a. *For a given μ_θ, both the manufacturer and the retailer achieve a positive benefit by applying Strategy 2 for ε such that $\varepsilon_{\min} \leq \varepsilon \leq \varepsilon_{\max}$ where $\varepsilon_{\min} = \frac{\pi^{R*}_{DU2}}{\pi^*_{C2}}(r_{eq} - c)$and $\varepsilon_{\max} = \frac{\pi^*_{C2} - \pi^{M*}_{DU2}}{\pi^*_{C2}}(r_{eq} - c)$*

b. *The larger of interval in which all SC actors are interested in applying Strategy 2, $\varepsilon_{\max} - \varepsilon_{\min} = (1 - eff_2)(r_{eq} - c)$, decreases when μ_θ decreases*

Proof. (a) $\varepsilon_{\min}$ and $\varepsilon_{\max}$ are derived by solving $(RB_{DU2 \to DC2})_R = 0$ and $(RB_{DU2 \to DC2})_M = 0$ respectively. (b) follows by observing that eff_2 increases and r_{eq} decreases when μ_θ decreases. □

For our numerical example ($\mu = 10$, $\sigma = 2$, $c = 5$, $r = 15$ and $s = 2$) and for two values of error parameter $\mu_\theta = 0.8$ and $\mu_\theta = 0.9$, the following figures represent respectively the benefit that the manufacturer and the retailer achieve by applying Strategy 2 as a function of the market power ε:

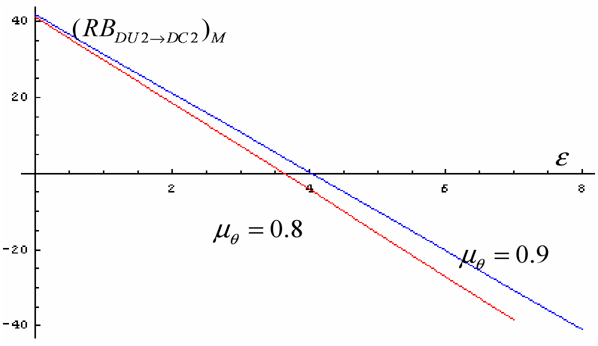


Figure 5.10: Variation of $(RB_{DU2\to DC2})_M$ with ε

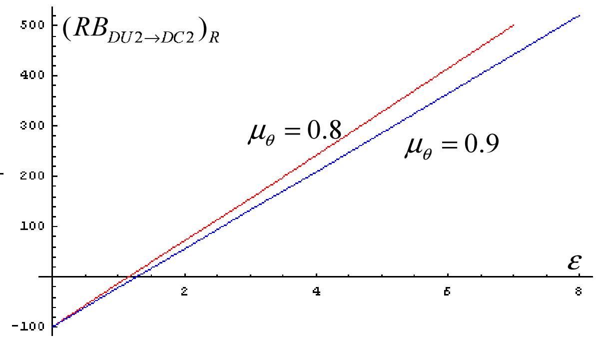


Figure 5.11: Variation of $(RB_{DU2\to DC2})_R$ with ε

5.5.3 Numerical analysis

We consider a numerical example where $\mu = 10$, $\sigma = 2$, $c = 7$, $r = 15$ and $s = 1$ with the additional hypothesis that t may take three possible values (0, 0.5, 1) (which represents respectively (0, 7, 14) % of the unit cost of production of the product). The question to be considered is now "*What is the best strategy that will be followed by a decentralized SC where the store is subject to errors?*"
In order to answer this question, we proceed in two steps. We first analyze the best strategy for the entire supply chain. We then consider the manufacturer and the retailer as being individuals. The following table represents benefits that would be achieved by the SC actors in each strategy:

	Strategy 1	Strategy 2
Manufacturer	$B_1^M = \pi_{DU3}^{M*} - \pi_{DU2}^{M*}$	$B_2^M = \pi_{DC2}^{M*} - \pi_{DU2}^{M*}$
Retailer	$B_1^R = \pi_{DU3}^{R*} - \pi_{DU2}^{R*}$	$B_2^R = \pi_{DC2}^{R*} - \pi_{DU2}^{R*}$
Supply Chain	$B_1^{SC} = B_1^M + B_1^R$	$B_2^{SC} = B_2^M + B_2^R$

Table 5.2: The benefits in each strategy

Figure 5.12 represents the variation of B_1^{SC} and B_2^{SC} with μ_θ for different value of t.

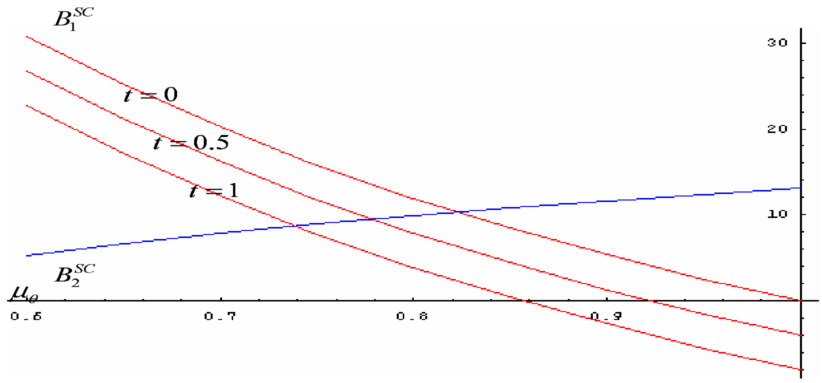


Figure 5.12: Variation of B_1^{SC} and B_2^{SC} with μ_θ for different values of t

The following observations can be made:

- Variation of B_1^{SC} with μ_θ: when $\mu_\theta = 1$, deploying the RFID technology is not necessary and may achieve a negative gain for both SC actors because of the additional tag price. When μ_θ decreases (i.e. more errors in the system) the benefit achieved by the RFID technology is more important for both the manufacturer and the retailer and as a consequence for the entire SC. For a poor shelf availability, the RFID technology is a interesting solution to remedy to errors

- Variation of B_2^{SC} with μ_θ: as explained previously, the error issue affects the efficiency of the Decentralized Uncoordinated scenario. When μ_θ decreases (i.e. more errors in the system), eff_2 increases and as consequence the total gain, B_2^{SC}, achieved by coordinating the channel decreases

- As it can be shown, for a given tag price t, it exists a critical value of μ_θ, $\mu_{\theta_{cr}}^{SC}$ which solves $B_1^{SC} = B_2^{SC}$, such that if $\mu_\theta \geq \mu_{\theta_{cr}}^{SC}$ Strategy 2 is better than Strategy 1 for the entire SC, otherwise Strategy 1 is better. As intuitively expected, note that this critical value of μ_θ decreases when t increases

Considering the retailer and the manufacturer as being individuals, we focus on the benefit that each SC actor achieves in the two strategies. In order to simplify the analysis we assume that when the supply chain is coordinated (i.e. Strategy 2 is applied), the corresponding power market ε is chosen such that the total benefit achieved by channel coordination is equitably shared among the two SC actors. In other word ε is chosen such that $\varepsilon = \dfrac{\varepsilon_{\max} + \varepsilon_{\min}}{2}$ for Strategy 2. Applying either Strategy 1 or Strategy 2 will lead to the following figures when $\mu = 10$, $\sigma = 2$, $c = 7$, $r = 15$ and $s = 1$:

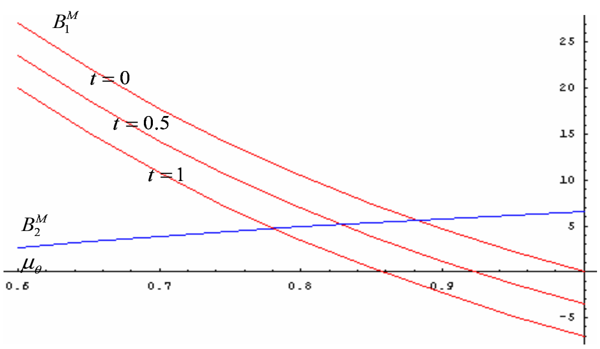


Figure 5.13: Variation of B_1^M and B_2^M with μ_θ

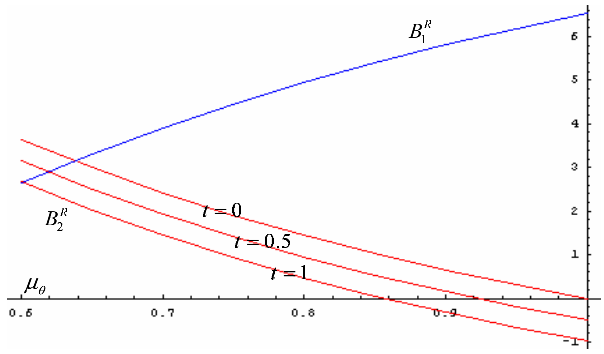


Figure 5.14: Variation of B_1^R and B_2^R with μ_θ

The following observations can be made in order to answer the main research question of this section:

- It is important to notice that the critical value $\mu_{\theta_{cr}}^{SC}$ defined above does not enable to the SC actors to choose one of the strategies presented before. Even if the gain achieved by channel coordination (Strategy 2) is supposed to be equitably shared between them, the gain achieved by the deployment of the RFID technology (Strategy 1) is not the same for the two SC actors. As illustrated, the manufacturer profits more from the RFID technology than the retailer

- It exists a critical value of μ_θ, $\mu_{\theta_{cr}}^{M}$ which solves $B_1^M = B_2^M$, which enables to the manufacturer to choose the best strategy between Strategy 1 and Strategy 2. if $\mu_\theta \geq \mu_{\theta_{cr}}^{M}$ Strategy 2 is better than Strategy 1 for the manufacturer

- It exists a critical value of μ_θ, $\mu_{\theta_{cr}}^{R}$ which solves $B_1^R = B_2^R$, which permit to the retailer to choose the best strategy between Strategy 1 and Strategy 2. If $\mu_\theta \geq \mu_{\theta_{cr}}^{R}$ Strategy 2 is better than Strategy 1 for the retailer.

- One way to make $\mu_{\theta_{cr}}^{SC} = \mu_{\theta_{cr}}^{M} = \mu_{\theta_{cr}}^{R}$ is to choose ε such that the sharing of the benefit achieved by coordination is as the same as the sharing of benefit achieved when the RFID technology is deployed

5.6 Conclusion

In this chapter, we have presented an analytical model of a single-period inventory system subject to misplacement errors. We have compared different models in different scenarios.
We have shown that coordinating the channel can lead to important savings and maybe does not necessitate the deployment of any particular system; the manufacturer and the retailer can benefit from this improvement by simply cooperating.
Concerning the quantity which is not available to buy, we have only considered errors such as misplacement. We have also supposed that all products which are in the store (on shelves or misplaced) are sold at the salvage cost s at the end of the period. Our model can be extended to include all types of errors. To do this, we can introduce the unit price s_c ($s_c \leq s$) pertaining to the additional cost that

the retailer should pay in order to find a misplaced product. This additional cost is basically the cost of scanning operations. In other terms, when a misplaced product is found at the end of the period, his unit salvage price is no more s but $(s - s_c)$. By the introduction of the cost parameter s_c, we notice that all types of errors are considered. Indeed, the case of errors such as theft or perishment where the quantity which is not available to buy is not found or is not sold at the end of the period is a particular case of the above formulation by setting $s_c = s$. For our further research on this topic, we notice that there are interesting opportunities concerning the extension of the analysis provided in this chapter for the case where the error parameter is stochastic.

Chapter 6

Inventory Inaccuracies : A General Inventory Framework

This chapter considers a wholesaler inventory system subject to inventory inaccuracies. In order to answer customers demands, the commitment of the wholesaler is made based on the Information System inventory which is not reflecting the physical available inventory. We provide a general framework permitting to model the inventory inaccuracy issue. In particular, we show that there is a connection between inventory inaccuracy and random yield problems. This last analysis ends with deducing an elegant mathematical analysis of the optimal ordering decisions in the additive error setting. The proposed framework extends the investigation of Sahin [6] by analyzing the general model (namely Model 3) defined by the author. The analysis is conducted for Approach 1 and Approach 2.

Keywords: *Newsvendor model, inventory inaccuracies, general framework*

6.1 Introduction

The aim of this chapter is to try to provide an analysis of the more general model discussed in Sahin [6], i.e. Model 3 where both the PH (PHysical) inventory and the IS (Information System) inventory are prone to errors under the wholesaler supply chain structure (Remember Structure B defined in the introduction of Part II). Our analysis is motivated by an intuition and a confirmation of this intuition in the last three chapters. We think that there is a close relationship between the inventory inaccuracy issue and the random yield problem which has been studied in Chapter 3. Remember that this connection was confirmed in our misplacement type errors model of Chapter 4[1]. We fellow our intuition and we tried to write Model 3 (in Sahin [6]) by using the cost function of a random yield problem. Our intuition was true and the result is that an inventory inaccuracy issue can be seen as an extended random yield problem. As a direct consequence of this modelling way, the case of additive error setting can now be analytically resolved and the optimal order decision is deduced. The multiplicative and the mixte error settings are mathematical more complex (the complexity is due to the difficulty in deriving simple conditions enabling the convexity of the expected cost function). We conduct our analysis for approaches 1 and 2. The chapter is composed of three sections: in section 6.2, we present the general framework and we derive the derive the expected cost. In section 6.3, we analyze the additive error setting for both approach 1 and 2. Section 6.4 concludes the chapter.

6.2 Expression of the cost function

Supply Chain Structure B is considered and the sequence of events is the one described in the introduction of Part II. We also use the same notations provided in the description of the investigation of Sahin [6] (Page 37). We also recall the unit costs define by the author:

- h: the unit overage cost which is paid by the inventory manager when a product remains in the warehouse at the end of the selling season.
- u_1: the unit type 1 underage cost which is incurred when, based on the IS system, the inventory manager is not able to satisfy a customer demand.
- u_2: the unit type 2 underage cost which is incurred when the inventory manager is not able to respect his commitment.

If D denotes the customers demand, following the sequence of events described in 2.4.2 enables us to deduce that the commitment is $C = Min(D, Q_{IS})$ for a given vector (D, Q_{PH}, Q_{IS}). The cost achieved by the inventory manager is as the following (Cf Figure 6.1):

$$\begin{aligned} Cost &= h\left[Q_{PH} - Min(D, Q_{IS})\right]^{+} + u_1\left[D - Q_{IS}\right]^{+} \\ &+ u_2\left[Min(D, Q_{IS}) - Q_{PH}\right]^{+} \end{aligned} \tag{6.1}$$

Our proposal is to write this latter cost function by using the cost function of a random yield problem:

[1] "The case of misplacement errors can be seen as an unreliable supply process plus a holding penalty resulting from the misplaced quantity"

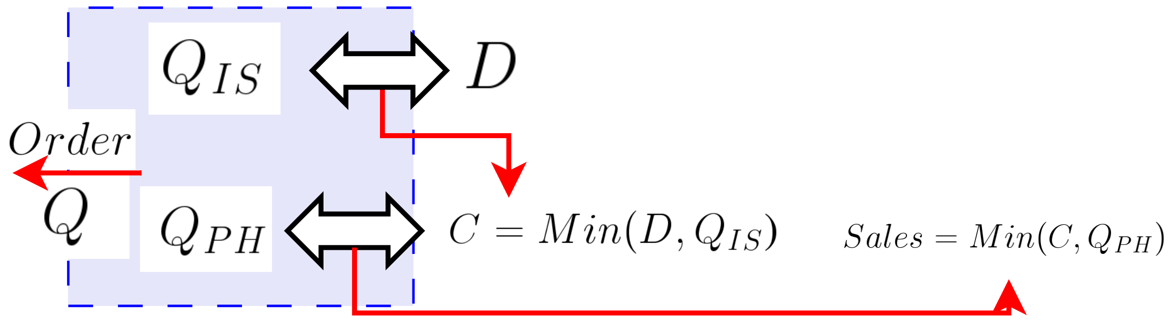


Figure 6.1: Input and output flows in the general model

Result 6.1. *For a given vector* (D, Q_{PH}, Q_{IS})*, the cost function achieved by the inventory manager is given by:*

$$\begin{aligned} Cost &= h\left[Q_{IS} - D\right]^{+} + u_1\left[D - Q_{IS}\right]^{+} \\ &+ u_2\left\{(Q_{IS} - Q_{PH}) - Min\left[(Q_{IS} - D)^{+}, (Q_{IS} - Q_{PH}\right]\right\} \\ &- hMin\left(\left[Q_{IS} - D\right]^{+}, (Q_{IS} - Q_{PH})\right) \end{aligned} \tag{6.2}$$

Proof. The proof follows directly by operating some elementary algebra and simplifying Equation 6.1. The main properties used for the purpose are $i)$ $Min(a,b) = b - (b-a)^{+}$ and $ii)$ $a = b - (b-a)^{+} + (a-b)^{+}$. We note also that the result can be verified by comparing values of equations 6.1 and 6.2 for the six possible combinations presenting the order between D, Q_{IS} and Q_{PH} ($D \leq Q_{IS} \leq Q_{PH}$; $Q_{IS} \leq D \leq Q_{PH}$; $Q_{IS} \leq D \leq Q_{PH}$; $D \leq Q_{IS} \leq Q_{PH}$; $D \leq Q_{PH} \leq Q_{IS}$ and $Q_{PH} \leq D \leq Q_{IS}$) □

Based on Result 6.1, we notice that the cost function of the inventory manager is composed of three parts:

a. The first part expresses the one-period cost function of a random yield problem where the supply system is unreliable: indeed, this part represents what will be incurred by the inventory manager if he orders a quantity Q and receives a quantity Q_{IS} from the supply system.

b. The second part expresses the penalty (type 2 underage penalty) occurring when customers orders, initially accepted by the inventory manager where the commitment is made based on the IS inventory, are not finally totally satisfied. Note that this penalty is different of zero in the case where $Q_{IS} - Q_{PH} > 0$.

c. In the first part (the random yield part), the overage cost is written based on the IS inventory. Since overage penalty should be calculated based on the physical inventory at the end of the selling season, this third part of the cost function adjusts the first one in order to write the overage penalty based on the physical inventory.

As it can be remarked, a general inventory model subject to inventory inaccuracy can be seen as extended version of the random yield problem. The aim of the following Result is to provide expressions of the unit costs h, u_1 and u_2 as a function of the unit purchase, selling and salvage costs.

Result 6.2. *By defining w as the unit purchase cost, r as the unit selling price, s as the unit salvage price and P as the unit penalty resulting from not delivering a committed product:*

a. The unit overage cost is given by $h = w - s$

b. The first type unit underage cost is given by $u_1 = r - w$

c. The second type unit underage cost is given by $u_2 = u_1 + P$

Proof. In order to demonstrate the result we need to write the profit function achieved by the inventory manager based on the last definitions of r, w, s and P. Then we should deduce the cost function written with these last parameters and we should compare it with the formulation of the cost function with the h, u_1 and u_2 parameters.
For a given vector (D, Q_{PH}, Q_{IS}), the profit achieved by the inventory manager is given by:

$$\begin{aligned} Profit &= rMin\left[Min(Q_{IS}, D), Q_{PH}\right] + s\left[Q_{PH} - Min(Q_{IS}, D)\right]^+ - wQ_{PH} \\ &- P\left[Min(Q_{IS}, D) - Q_{PH}\right]^+ \end{aligned} \tag{6.3}$$

By observing that $Min\left[Min(Q_{IS}, D), Q_{PH}\right] = Min(Q_{IS}, D) - \left[Min(Q_{IS}, D) - Q_{PH}\right]^+$, the profit can be rewritten as the following:

$$\begin{aligned} Profit &= rMin(Q_{IS}, D) + s(Q_{IS} - D)^+ - wQ_{IS} + s\left[Q_{PH} - Min(Q_{IS}, D)\right]^+ \\ &+ w(Q_{IS} - Q_{PH}) - (r + P)\left[Min(Q_{IS}, D) - Q_{PH}\right] - s\left[Q_{IS} - D\right]^+ \end{aligned}$$

Using some elementary algebra and simplifying leads to the following expression:

$$\begin{aligned} Profit &= (r - w)D - \{(w - s)(Q_{IS} - D)^+ + (r - w)(D - Q_{IS})^+ \\ &- s\left[Q_{PH} - Min(Q_{IS}, D)\right]^+ - w(Q_{IS} - Q_{PH}) + s(Q_{IS} - D)^+ \\ &+ (r + P)\left[Min(Q_{IS}, D) - Q_{PH}\right]^+\} \end{aligned}$$

The cost function can also be deduced:

$$\begin{aligned} Cost &= (w - s)(Q_{IS} - D)^+ + (r - w)(D - Q_{IS})^+ \\ &- s\left[Q_{PH} - Min(Q_{IS}, D)\right]^+ - w(Q_{IS} - Q_{PH}) \\ &+ s(Q_{IS} - D)^+ + (r + P)\left[Min(Q_{IS}, D) - Q_{PH}\right]^+ \end{aligned}$$

By using the fact that $a = b + (a - b)^+ - (b - a)^+$ and $Min(a, b) = a - (a - b)^+$, developing and simplifying the cost function leads to the following expression:

$$\begin{aligned} Cost &= (w - s)(Q_{IS} - D)^+ + (r - w)(D - Q_{IS})^+ \\ &+ (r + P - s)\left[Min(Q_{IS}, D) - Q_{PH}\right]^+ \\ &- (w - s)(Q_{IS} - Q_{PH}) \end{aligned} \tag{6.4}$$

In an another side, Equation 6.2 can be rewritten as follows:

$$\begin{aligned} Cost &= h(Q_{IS}-D)^{+}+u_{1}(D-Q_{IS})^{+} \\ &+ (u_{2}+h)\left[Min(Q_{IS},D)-Q_{PH}\right]^{+} \\ &- h(Q_{IS}-Q_{PH}) \end{aligned} \tag{6.5}$$

Comparing Equations 6.4 and 6.6 and identifying permit to show that $h = w - s$, $u_1 = r - w$ and $u_2 = u_1 + P$ □

Remark 6.1. *Based on the formulation of the previous proof, we note that we have assumed that the supply system is paid according to the physical inventory Q_{PH} (the purchase cost w is multiplied by Q_{PH} in Equation 6.3). The analysis we present in this work can easily be modified in order to consider other situations (the case where the supply system is payed based on Q_{IS} or simply based on the ordered quantity Q). Note however that our assumption is somewhere motivated by the use of the single period as an inventory framework. We can suppose that financial flows are realized at the end of the selling season based on the actual physical inventory.*

6.3 The optimal ordering decision in the additive error setting

We recall that in a general setting, if we let Q the quantity ordered from the supply process, the physical and the IS inventory can respectively be written as the following: $Q_{PH} = \gamma_{PH}Q + \epsilon_{PH}$ and $Q_{IS} = \gamma_{IS}Q + \epsilon_{IS}$ where the couple of random variables $(\gamma_{PH}, \epsilon_{PH})$ $((\gamma_{IS}, \epsilon_{IS}))$ characterizes the errors on the physical inventory level (IS inventory level). From this general setting, on can derive two particular cases (recall the additive and the multiplicative cases defined and described in Section 2.4.1).

The aim of this section is to analyze the additive error setting. For this purpose let suppose that $Q_{IS} = Q + \epsilon_{IS}$ and $Q_{PH} = Q + \epsilon_{PH}$ where Q is the ordered quantity and ϵ_{IS} and ϵ_{PH} are respectively the random variables describing the errors on the IS and the PH inventory. Let also define two additional random variables $D_m = D + \epsilon_{IS}$ and $e = \epsilon_{IS} - \epsilon_{PH}$ with f_m and F_m (g and G) the PDf[2] and the CDF[3] respectively of the random variable D_m (e respectively).

By using the last random variables and the cost function provided in Result 6.1, the cost function incurred by the inventory manager can be expressed as the following:

$$\begin{aligned} Cost &= h(Q-D_m)^{+}+u_{1}(D_m-Q)^{+} \\ &+ u_{2}\left\{e-Min\left[(Q-D_m)^{+},e\right]\right\} \\ &- hMin\left[(Q-D_m)^{+},e\right] \end{aligned} \tag{6.6}$$

The following result states the expression of the expected cost, $C(Q)$, incurred by the inventory manager for a given ordering quantity Q in the additive error setting:

[2] Probability Density Function
[3] Cumulative Distribution Function

Result 6.3. *The Expected cost is given as the following:*

$$\begin{aligned} C(Q) &= h \int_{x_m=0}^{Q} (Q - x_m) f_m(x_m) dx_m + u_1 \int_{x_m=Q}^{+\infty} (x_m - Q) f_m(x_m) dx_m \\ &+ (u_2 + h) E[A] - hE[e] \end{aligned}$$

where

$$E[A] = \int_{e=0}^{+\infty} \left[e[1 - F_m(Q)] + \int_{x_m=Q-e}^{Q} [e - (Q - x_m)] f_m(x_m) dx_m \right] g(e) de$$

Proof. Let consider the cost function defined in Equation 6.6 and let define $A = \{e - Min[(Q - x_m)^+, e]\}$. By observing that $A = 0$ if $e < 0$, the expected value of A is given by:

$$\begin{aligned} E[A] &= \int_{e=0}^{+\infty} \int_{x_m=Q-e}^{Q} [e - (Q - x_m)] f_m(x_m) g(e) dx_m de \\ &+ \int_{e=0}^{+\infty} \int_{x_m=Q}^{+\infty} e f_m(x_m) g(e) dx_m de \end{aligned} \tag{6.7}$$

The Expected cost incurred by inventory manager can also be deduced:

$$\begin{aligned} C(Q) &= h \int_{x_m=0}^{Q} (Q - x_m) f_m(x_m) dx_m + u_1 \int_{x_m=Q}^{+\infty} (x_m - Q) f_m(x_m) dx_m \\ &+ (u_2 + h) E[A] - hE[e] \end{aligned} \tag{6.8}$$

□

Analysis of the model under Approach 1 Remember that the inventory manager is not aware of errors or simply ignores them under Approach 1. His ordering decisions are also independent of the error parameters. Being in a Newsvendor framework, the inventory manager will also order the optimal Newsvendor ordering quantity independently of the presence of errors. if we let F the CDF of the demand distribution, this quantity is given by:

$$Q^*_{Newsvendor} = F^{-1} \left[\frac{u_1}{u_1 + h} \right] \tag{6.9}$$

When ordering $Q^*_{Newsvendor}$, the cost incurred by the inventory manager is not the optimal cost of a basic Newsvendor problem, but it is given by $C(Q^*_{Newsvendor})$

Analysis of the model under approach 2 Under Approach 2, the inventory manager is aware of the errors in the system. We suppose that he has an information about the distributions of ϵ_{PH} and ϵ_{IS}. Based on this information, the optimal ordering decision under Approach 2 is given in the following Result:

Result 6.4. *Under Conditions 1 and 2, there exists a unique optimal ordering quantity Q^* that minimizes the expected function $C(Q)$. Q^* solves the following equation:*

$$(u_1 + h)F_m(Q^*) - u_1 + (u_2 + h) \int_{e=0}^{+\infty} g(e) \left[F_m(Q^* - e) - F_m(Q^*)\right] de = 0 \tag{6.10}$$

where

- *Condition 1: The u_1 cost and the h cost are such that $u_1 \geq \frac{F_m(0)}{2+F_m(0)} h$*
- *Condition 2: The demand and the errors distributions are such that $\frac{\int_{e=0}^{+\infty} f_m(x-e)g(e)de}{f(x)}$ is an increasing function in x*

Proof. As the proof is quite technical and has nothing to do with inventory theory, we relegate it to Appendix F.1 □

We notice that Condition 1 ensures the existence of an optimal ordering quantity for the inventory system. Condition 2 ensures the unicity of this optimal order decision.

Remark 6.2. *From a practical point of view, both Conditions 1 and 2 are non restrictive in the inventory control context. In fact:*

- *Condition 1 which can be written in a more general manner $\frac{u_1}{h} \geq \frac{1}{3}$ (in other words: if $\frac{u_1}{h} \geq \frac{1}{3}$ is verified then Condition 1 is also verified) seems to be not in contradiction with practical values of the ratio $\frac{u_1}{h}$.*
- *Condition 2 is applicable for common distributions. Specially it holds for the case of normally distributed demand and errors.*

The following result states the expression of the optimal expected cost function $C(Q^*)$:

Result 6.5. *Under Condition 1 and 2, the optimal expected cost incurred by the inventory manager is given by:*

$$\begin{aligned} C(Q^*) &= (u_1 + h) \int_{x_m=0}^{Q^*} x_m f_m(x_m) dx_m \\ &- (u_2 + h) \int_{e=0}^{+\infty} \left[e - eF(Q^* - e) + \int_{x_m=Q^*-e}^{Q^*} x_m f(x_m) dx_m \right] g(e) de + hE\left[e\right] \end{aligned}$$

Proof. The proof is deduced directly by combining Equations 6.8 and 6.10 □

We deliberately wont present a more detailed analysis on the general framework under additive error setting. Our aim was to show that it exists a connection between the inventory inaccuracy issue and the random yield problem. We wont present a sensitivity analysis and the impact of model parameters on the performance of the inventory system since this model includes all sources of errors.

6.4 Conclusion

In this chapter, we provided a general framework enabling to model an wholesaler inventory system subject to inventory inaccuracies. The main result is that it exists a close relationship between the inventory inaccuracy issue and the random yield problem. This relationship enables us to derive an analytical analysis of the additive error setting where the optimal ordering decisions were provided. For our further research on this topic, it would be interesting:

- To analyze the multiplicative error setting. The complexity of such analysis is stemming from the deduction of simple (and practical) conditions enabling the existence and the unicity of the optimal ordering decision.
- To give further insights and details on the impact of model parameters on the optimal ordering decisions.
- To perform the analysis for a decentralized supply chain as the one conducted in the last chapter.

Part III

RFID and Inventory Inaccuracy in a Multi-Period Framework

Chapter 7

A Periodic Review Inventory Model subject to Theft Errors

We consider a finite horizon, single-stage, single-product periodic-review inventory in which inventory records are inaccurate. We assume that inventory inaccuracies are introduced by theft type errors that occur within the store. As in the last chapters, we propose a comparison between three approaches based on which the inventory system in the presence of theft errors can be managed: in the first approach, the inventory manager ignores errors occuring in the store. In the second approach, we focus on the benefit achieved through a better knowledge of errors and through taking them into account when formulating and optimizing the inventory system. In the third approach, we focus on the contribution of a perfect RFID technology that prevents errors. To solve the problem, we consider two formulations: i) The optimization of shortage and overage costs where dynamic programming tools are used ii) The optimization of the overage cost under a service level constraint where analytical results are provided. The comparison between the three approaches permits us to analyze the impact of theft errors and the value of the RFID technology on the inventory system. Here again, we propose an analytical critical tag cost which makes the deployment of the RFID technology cost effective.

Keywords: *periodic-review inventory model, RFID technology, theft errors, inventory record inaccuracy, dynamic programming, service level.*

7.1 Introduction

Let first recall that inventory theft, a combination of employee theft, shoplifting, vendor fraud and administrative error, costs United States retailers over $31 billion last year according to the latest National Retail Security Survey report on retail theft, which analyzed theft incidents from 118 of the largest U.S. retail chains. In the European side, ECR defines ”Shrinkage” as the process errors, internal and external thefts. The results of the research carried by ECR Europe have shown that the scale of shrinkage in fast moving consumer goods sector is estimated to 24 mld EUR in 2003 (465 mln EUR is lost irreparably within fast moving consumer goods turnover weekly), which is 2,41% of the whole turnover value of the sector. The process errors present 27% of the whole shrinkage value, 7% deceptions, 28% internal thefts and 38 external thefts.

This chapter focuses on the impact of theft errors on the performance of a retail inventory system and derive conditions enabling the RFID technology to be beneficial for such inventory system. Academic investigations dealing with theft errors are still rare: to our knowledge, the investigation of Kang and Gershwin [42] is among the rare ones that evaluates the impact of theft errors on inventory management through a simulation study. The authors illustrate how theft increases lost sales and results in an indirect cost of losing customers (due to unexpected out of stock) in addition to the direct cost of losing inventory. They simulate the inventory system and show that even small inventory inaccuracy may lead to important stockouts. In fact, according to their simulation, even when the theft is as small as 1% of the average demand, the error accumulating in the inventory record is large enough to disturb the replenishment process and make 17% of the total demand lost.

The lack of optimization type investigations related to the theft issue motivates the analysis provided in this chapter. For this purpose, we consider a single-stage, single-product periodic-review inventory in which inventory records can be inaccurate due to theft. Let precise that theft gnerates inventory inaccuracy since it impact the physical inventory and left the IS inventory unchanged. We assume that every N periods, the inventory manager performs an inspection operation in order to update and align the IS and the physical inventory levels. So, we consider a N-period inventory system subject to theft errors where N is fixed and given[1]. Alike our last chapters, the aim is establish and evaluate the optimal policy for our traditional three approaches enabling the management of the inventory system subject to theft. The comparison between the three approaches permits us to analyze the impact of theft errors and the value of the RFID technology on the inventory system. Here again, we propose an analytical critical tag cost which makes the deployment of the RFID technology cost effective.

The structure of the chapter is as follows: in Section 7.2, we describe the framework enabling the modeling of theft errors. In Section 7.2.1, we recall in a more detailled way (specific for the theft error type), the approaches that can be used to model the problem. The three approaches are analyzed by two different formulations. The first one is provided in Section 7.3 and the second one is performed in Section 7.4. As usual the impact of theft errors and the value of RFID technology is discussed in Section 7.5.

[1] We will give at the end of this chapters ways permitting to relax this assumption

7.2 Modeling of a retail store subject to theft type errors

First we notice that we are in a retail context, the end customers are physically present in the retail store and their demand is confronted to the physical available inventory. In order to model theft errors, we assume that the demand in each period k, is divided into two streams according to a deterministic error parameter α:

- **Demand for theft** αD_k affects only the physical inventory and leaves the IS inventory level unchanged.
- **Demand for purchase** $(1-\alpha)D_k$ affects the physical and the IS inventories.

Appendix G.1 describes the demand process proprieties that may explain the two demand streams. We also assume that the demand in each period k is independent and distributed according to a normal distribution with mean μ and standard deviation σ. The sequence of events in each period is assumed to be as follows (Cf Figure 7.1 for an graphical illustration):

1. The IS inventory x_k is reviewed and an order is placed. Because of errors the physical inventory is not x_k but is $x_k - \varepsilon_k$.
2. Lead time is zero: the incoming order is received. The IS inventory is replenished up to a level y_k.
3. Demand for purchase and demand for theft take place: theft and demand for purchase are satisfied as long as the physical inventory is available. Demand occurring at zero physical inventory is lost (no backlog). When the physical inventory is less than the total demand D_k, it is shared proportionally according to α: a fraction $(1-\alpha)$ of the physical inventory is used to satisfy demand for purchase, the other part, i.e. α, is used to satisfy the demand for theft. Sales a_k and theft b_k are also deduced.

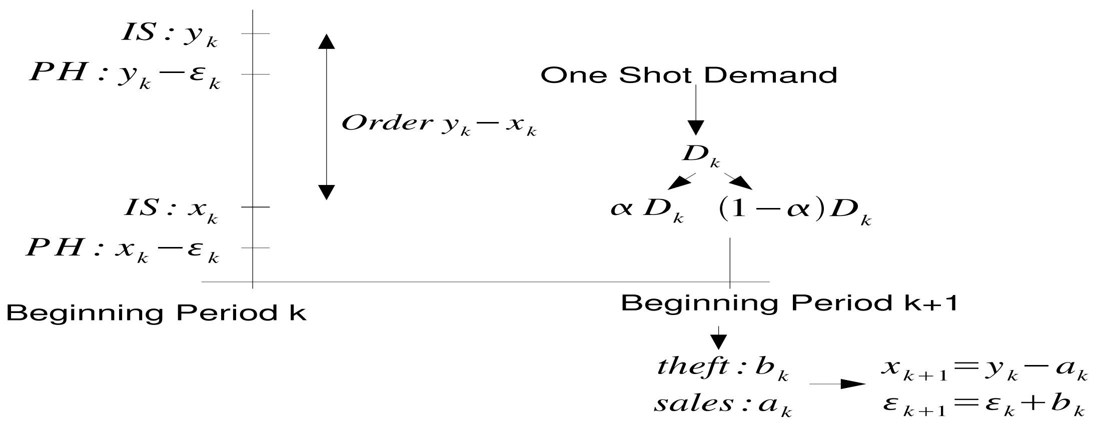


Figure 7.1: The sequence of events in Period k

Remember that the last hypothesis concerning the case where the physical inventory is less than the total demand is also assumed in the investigation of Kang and Gershwin [42]. Such an assumption will simplify the analysis since we do not care in the formulation of the expected profit about the sequence of arrival of the two demand streams. In fact, without this assumption, we have to distinguish between the case where the demand for theft arrives first or arrives last since in such case the realization of sales and theft in period k will be depending on the sequence of arrival.

7.2.1 Approaches used to model the inventory system

We recall here the approaches that the inventory manager may use in order to control the inventory system subject to theft errors. Such an inventory can be managed in two ways depending on whether an advanced automatic identification system such RFID technology is used or not. In a general setting, when RFID is no used, three possible situations can occur depending on the level of information that the inventory manager has about the theft error: i) the first situation considers the case where the inventory manager does not observe the demand for theft. Atali et al. [66] refer to this situation as the ignored one, ii) the second situation consider the case where a statistical information about the error parameter is known (such as a mean or the distribution of the error). Atali et al. [66] refer to this situation as the informed one and iii) the third situation deals with the case where an exact information about the realization of the demand for theft in each period is known. Atali et al. [66] refer to this situation as the full-visibility one.

In our problem setting, the last two situations are equivalent since a knowledge of α and sales implies an information about the realization of the demand for theft in each period.

In the other side, in the case where RFID is used, it is important to notice that such technology has two major values for the inventory manager. First, the visibility provided by this technology allows inventory records to be accurate and as a consequence eliminate the discrepancy between the physical inventory and the IS one. Second, the RFID technology will prevent or reduce the sources of errors since the inventory manager is able to monitor and to catch the demand for theft. As in the rest of this dissertation, we assume that the major role of RFID considered in this chapter is the second one, i.e., prevention and elimination of errors.

As illustrated in the Figure 7.2, in a general setting when RFID is deployed, there is a fraction β of the demand for theft that remains a demand for theft even with the deployment of RFID. The other fraction $(1-\beta)\alpha D_k$ can be divided into two sets: the first set corresponds to the fraction of the demand for theft that will be lost because the customer will abandon and leave the store without buying the product. The other fraction $(1-\gamma)(1-\beta)\alpha D_k$ is converted into a demand for purchase. For sake of simplicity we assume throughout this chapter that $\beta = \gamma = 0$. In other terms, when RFID is deployed, the demand for theft is totally converted into a demand for purchase. Moreover, the analysis presented in this chapter can easily be modified to deal with the other situations: in fact the analysis of the case where both β and γ are different from zero is simply the same analysis which will be presented in this chapter by replacing α by $\frac{\alpha(1+\beta)}{1+\alpha[1-\alpha(1-\beta)]}$.

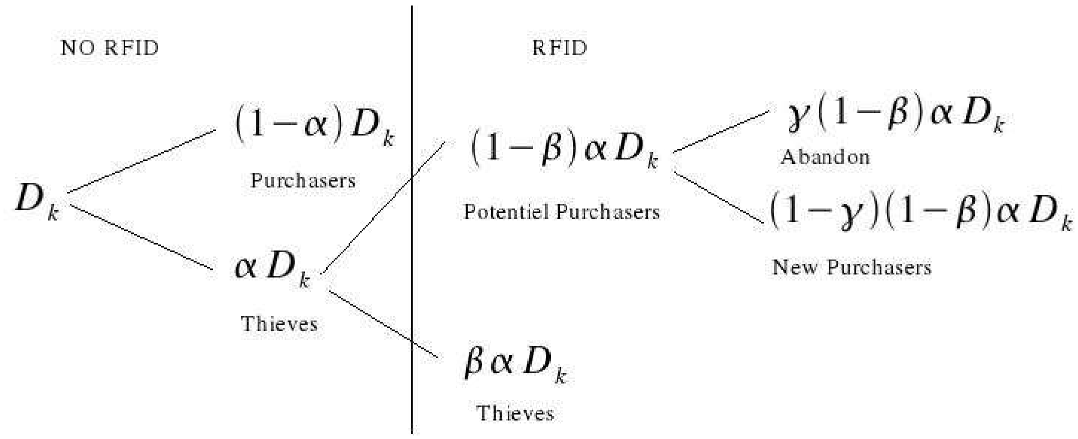


Figure 7.2: RFID vs No RFID

In order to model the non RFID and the RFID situations, we also consider our usual three approaches (which were described in Section II of Chapter 2):

- Approach 1: RFID is not deployed and the inventory manager ignores the errors occurring in his store.
- Approach 2: RFID is not deployed and the inventory manager has an information about the error parameter α. He also establishes the optimal policy by taking into account α. Again, an estimation of α can be realized based on statistical sampling methods as reported by Pergamalis [1] who proposes a methodology for measuring stores' inventory accuracy.
- Approach 3: the inventory manager decides to remedy to theft by implementing the RFID technology. As described later, we assume that there are no more errors and we suppose that the demand for theft is converted into a demand for purchase ($\beta = \gamma = 0$). An additional cost pertaining to the RFID technology is taken into account under this approach.

Under Approach 3, we assume that the cost associated with the implementation of RFID technology consists in RFID tags embedded to each item individually, at a certain tag cost t. The fixed costs of investments necessary to implement the technology (such as reader systems cost, infrastructure costs, basic application integration costs, maintenance and support costs and overhead costs) are deliberately not part of our model.
In order to solve the problem, we consider two formulations: $i)$ Formulation 1: optimization of shortage and overage costs where dynamic programming tools are used and $ii)$ Formulation 2: optimization of overage costs under a service level constraint where analytical results are provided.
Service levels are used in inventory control systems for performance evaluation and in target setting as substitutes for underage costs that are difficult to estimate. A review of standard service level measures and their relationships to underage costs and different control policies is provided by Schneider [80].
Under Formulation 2, we consider the Horizon Service Level (HSL) defined as the probability of not having a shortage over the whole horizon (N periods),i.e., the probability of not having a shortage between two successive inspection operations. We also precise that our aim in this chapter is to derive

the optimal policy for each approach for a given horizon length N (in other terms, we do not try to optimize the number of periods N[2]).

7.2.2 Notations

The notations used throughout this chapter are as the following:

- α: the parameter representing theft errors
- c: the unit product purchase cost
- r: the unit product selling price
- t: the unit RFID tag cost
- h: the unit overage cost
- D_k: the random variable representing demand in period k
- x_k: the IS inventory before ordering at the beginning of period k
- ε_k: the level of perturbation in the physical inventory at the beginning of period k
- y_k: the inventory record after ordering at the beginning of period k
- a_k: sales in period k
- b_k: theft in period k
- f (F): pdf (cdf) characterizing D_k
- μ: the expected value of D_k
- σ: the standard deviation of D_k
- $\pi_{i,j}$: the expected profit from period 1 to period N in Approach i ($i = 1, 2, 3$) under Formulation j ($j = 1, 2$)

7.3 Analysis under Formulation 1: the Underage and Overage Formulation

One way to analyze the performance of an inventory system is to use the overage (holding) and the underage costs. Remember that the unit overage cost is the cost of having one unit left over at the end of a period. On the other hand, the unit underage cost is the opportunity cost of being short by one unit. If the inventory manager shorts by one unit in meeting demand, then he loses a potential sale for the selling price r, but he also avoids the purchase cost c, so the underage cost can be expressed

[2]The case where N is not fixed is discussed in the conclusion

as $u = r - c$ per unit. This is often called the contribution margin: the difference between selling price and marginal cost. In this section underage and overage costs are used in order to evaluate the performance of the inventory system prone to theft errors.
We first begin by analyzing Approach 2 in the next subsection. Then we present Approach 1 and we end by the analysis pertaining to Approach 3.

7.3.1 Analysis of Approach 2

We recall that Approach 2 corresponds to the situation where the inventory system is prone to theft errors and the inventory manager is aware and can observe these errors. Being able to observe errors, the inventory manager also aligns the IS and the PH inventories at the end of each period. We first analyze the single-period problem, i.e., the problem in the last period and then we extend to the N-period problem.

The Single-Period Problem

We consider in this section the problem in the last period, for simplicity, we drop the subscript $k = N$ (in y_k, x_k, D_k, a_k, b_k). For a given initial situation defined by the vector (x, ε) before ordering, we argue that the IS (physical) inventory is y ($y - \varepsilon$ respectively). Under Formulation 1, the following result states the expression of the expected one-period profit:

Result 7.1. *For a given initial IS inventory level x, the expected one-period profit of Approach 2 under Formulation 1 is given by:*

$$\pi_{2,1}(y) = E_\varepsilon \left[L(x, y, \varepsilon) \right] \tag{7.1}$$

where

$$\begin{aligned} L(x, y, \varepsilon) &= u_{eq} \cdot \mu - (c + h) \int_{D=-\infty}^{y-\varepsilon} [(y - \varepsilon) - D] f(D) dD \\ &- u_{eq} \int_{D=y-\varepsilon}^{+\infty} [D - (y - \varepsilon)] f(D) dD + c(x - \varepsilon) \qquad (7.2) \\ u_{eq} &= [u(1 - \alpha) - \alpha c] \qquad (7.3) \end{aligned}$$

Proof. When the total demand exceeds the physical available inventory, the available inventory is divided proportionately to meet the two demands. As a consequence, two cases can occur:

- if $D \leq y - \varepsilon$: $a = (1 - \alpha)D$ and $b = \alpha D$
- if $D \geq y - \varepsilon$: $a = (1 - \alpha)(y - \varepsilon)$ and $b = \alpha(y - \varepsilon)$

As a consequence we can write $\begin{cases} a = (1 - \alpha) Min(D, y - \varepsilon) \\ b = \alpha Min(D, y - \varepsilon) \end{cases}$

The profit of the inventory manager is also given by:

$$\begin{aligned} Profit &= r \cdot a - h \cdot (y - \varepsilon - D)^{+} - c \cdot (y - x) \\ &= [r(1-\alpha) - c]\, D - [r \cdot (1-\alpha) - c]\, [D - (y-\varepsilon)]^{+} - (c+h)\, [(y-\varepsilon) - D]^{+} + c(x-\varepsilon) \end{aligned}$$

The expected one-period profit can be deduced by developing the last equation:

$$\begin{aligned} \pi_{2,1}(y) &= E_{\varepsilon}\{[r(1-\alpha) - c]\mu - (c+h) \int_{D=-\infty}^{y-\varepsilon} [(y-\varepsilon) - D] f(D) dD \\ &- [r(1-\alpha) - c] \int_{D=y-\varepsilon}^{+\infty} [D - (y-\varepsilon)] f(D) dD + c(x-\varepsilon)]\} \end{aligned}$$

By using the unit underage ($u = r - c$) and overage costs, the expected profit can also be written as follows:

$$\begin{aligned} \pi_{2,1}(y) &= E_{\varepsilon}\{[u(1-\alpha) - \alpha c]\mu - (c+h) \int_{D=-\infty}^{y-\varepsilon} [(y-\varepsilon) - D] f(D) dD \\ &- [u(1-\alpha) - \alpha c] \int_{D=y-\varepsilon}^{+\infty} [D - (y-\varepsilon)] f(D) dD + c(x-\varepsilon)\} \end{aligned}$$

As a consequence, the proof is deduced directly by defining an equivalent unit underage cost $u_{eq} = [u(1-\alpha) - \alpha c]$. □

Based on Result 7.1, the following remarks can be made:

- The expected profit function is built based on the physical inventory level $(y - \varepsilon)$
- During the period, we can interpret the theft error as the existence of two classes of customers where the unit selling price of the first class which represents demand for purchase (second class which represents demand for theft) is r (0 respectively). The margin for the first type of demand is $r - c$ and for the second one is $-c$.
- The difference between the physical and the IS inventory levels at the beginning of the period can be interpreted as an additive random yield problem where the supply system of the inventory manager is unreliable: when a quantity $(y - x)$ is ordered, the received quantity is $(y - x - \varepsilon)$

The inventory problem can also be seen as an additive random yield problem with an equivalent unit underage cost $u_{eq} = [u(1-\alpha) - \alpha c]$. The penalty u_{eq} can be interpreted by the fact that if a shortage situation occurs:

- There is a probability $(1 - \alpha)$ that the inventory manager losses a real sale so he occurs a penalty equal to $(1 - \alpha)u$
- There is a probability α that the inventory manger losses a demand for theft and as a consequence he gains the purchase cost of the product. The penalty of the shortage is also $-\alpha c$ in such case.

Under Approach 2, the inventory manager is aware and can observe errors occurring in the store. He also know that his inventory problem is equivalent to an additive random yield problem with the equivalent unit underage cost. As a consequence, for an initial IS inventory x, he chooses the best y that maximizes $\pi_{2,1}$. By defining $D_{eq} = D + \varepsilon$ in the expression of $\pi_{2,1}$ and letting $F_{eq}(.)$ (f_{eq}) denotes the CDF (PDF) of D_{eq}, it is clear to observe that the effect of theft errors in the one-period problem is equivalent to have an equivalent demand distribution and an equivalent unit underage cost. The optimal order-up-to level under Approach 2, $Y^*_{2,1}$, is the maximizer of $\pi_{2,1}(y)$ can easily be deduced:

$$Y^*_{2,1} = F_{eq}^{-1}\left[\frac{u_{eq}}{u_{eq} + h + c}\right] = F_{eq}^{-1}\left[\frac{u(1-\alpha) - \alpha c}{u(1-\alpha) - \alpha c + h + c}\right] \tag{7.4}$$

The optimal expected profit corresponding to the one-period problem of Approach 2 under Formulation 1 is as the following:

$$\pi^*_{2,1} = (u(1-\alpha) - \alpha c + h + c)\int_{D=-\infty}^{Y^*_{2,1}} D f_{eq}(D)dD + c(x - E[\varepsilon]) \tag{7.5}$$

Some direct proprieties on the variations of $Y^*_{2,1}$ and $\pi^*_{2,1}$ with α can be deduced:

Property 7.1. *For a given initial IS inventory level x,*

- *The optimal order-up-to level $Y^*_{2,1}$ decreases with α*
- *The optimal expected profit $\pi^*_{2,1}$ decreases with α*

Proof. The proof follows directly by observing that the equivalent underage cost u_{eq} decreases with α □

The Finite-Horizon Problem

The following figure represents the behavior of the IS and the PH inventory under Approach 2:

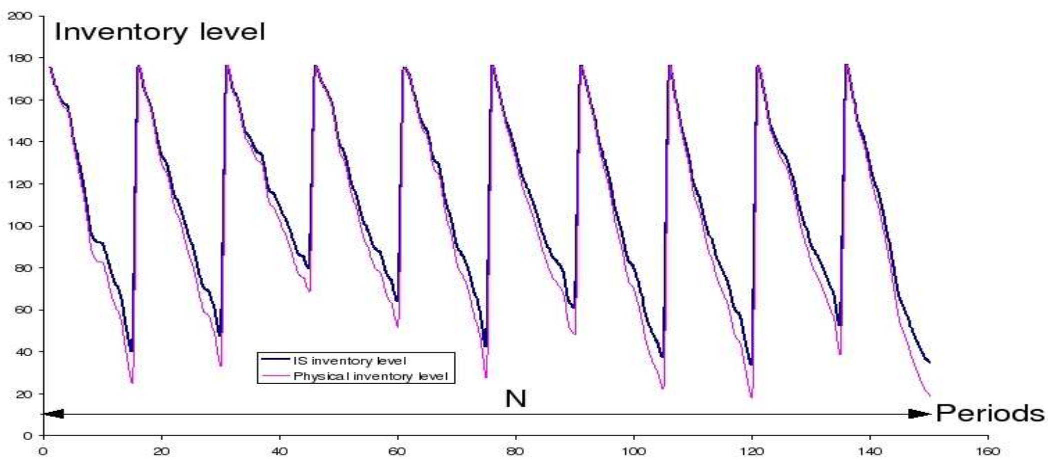


Figure 7.3: The behavior of the IS and the physical inventory- Approach 2

At the end of each period the IS and the PH inventories are aligned. Just after ordering, the PH inventory at the beginning of period k is y_k. When the total demand exceeds the physical available inventory, the available inventory is divided proportionately to meet the two demand streams. As a consequence, two situations can occur

- if $D_k \leq y_k$: $a_k = (1-\alpha)D_k$ and $b_k = \alpha D_k$
- if $D_k \geq y_k$: $a_k = (1-\alpha)y_k$ and $b_k = \alpha y_k$

As a consequence we can write $\begin{cases} a_k = (1-\alpha)Min(D_k, y_k) \\ b_k = \alpha Min(D_k, y_k) \end{cases}$
By noting that $b_k = \frac{\alpha}{1-\alpha} a_k$, we remark that a complete knowledge about α permits to deduce the realization of theft errors in each period since a_k is known and is given by point of sales data.
Managing the inventory system with an information about the error parameter can be done by assuming that the state of the inventory system is defined by the physical inventory level. Let denote x_k^{PH} the physical inventory in the beginning of period k (just before ordering), the state update is written as the following:

$$\begin{aligned} x_{k+1}^{PH} &= [y_k - (a_k + b_k)]^+ \\ &= [y_k - D_k]^+ \end{aligned} \tag{7.6}$$

Given an initial state x_k^{PH}, the optimal profit of Approach 2, J_k, from period k to N is given by the solution of the following dynamic programming problem:

$$J_k(x_k^{PH}) = \max_{y_k \geq x_k^{PH} \geq 0} \left[L(x_k^{PH}, y_k, 0) + E_{D_k} \left[J_{k+1}(x_{k+1}^{PH}) \right] \right] \tag{7.7}$$

Where[3] $J_{N+1} \equiv 0$ and $L(x, y, \varepsilon)$ is the expected single period profit function for a given vector (x, ε) defined in Equation 7.2.
The dynamic programming problem is identical to the classical inventory control with lost sales with an equivalent underage cost parameter ($u_{eq} = [u(1-\alpha) - \alpha c]$), and the key results remain valid (Zipkin [81]). In particular, an optimal policy is defined by an order-up-to level policy $y_k^* = Y_{2,1}^*$ where

$$Y_{2,1}^* = \arg\left\{ \max_{y \geq x \geq 0} \{L(y, x, 0)\} \right\} = F^{-1}\left[\frac{u_{eq}}{u_{eq} + h + c} \right] \tag{7.8}$$

$$= F^{-1}\left[\frac{u(1-\alpha) - \alpha c}{u(1-\alpha) - \alpha c + h + c} \right] \tag{7.9}$$

By using the dynamic programming of Equation 7.7, the optimal expected profit of Approach 2 under Formulation 1 is also given by:

$$\pi_{2,1}^* = J_1(x_1^{PH} \equiv 0) \tag{7.10}$$

[3] Throughout this chapter the signe "≡" means "by definition it is equal to"

7.3.2 Analysis of Approach 1

Under Approach 1, the inventory system is prone to theft errors and the inventory manager ignores them[4]. As in the analysis of Approach 2, we first perform the study of the single-period probelm and then we extend to the N-period problem.

The Single-Period Problem

The inventory manager ignores errors occurring in the store. He also follows an inventory policy established for a system that does not face theft problems. We recall our assumption concerning demand parameters which are supposed to be provided (exogenous) $D \sim N(\mu, \sigma)$ independently of the inventory system parameters. From basic inventory theory, such hypothesis is well-used especially in Newsvendor-type products[5]. In the exogenous case, the order-up-to level is independent of α and is given by:

$$Y_{1,1}^* = F^{-1}\left[\frac{u}{u+h+c}\right] \tag{7.11}$$

By setting the order-up-to level equal to $Y_{1,1}^*$, the expected profit pertaining to this policy is given by using the expected one-period profit function of Approach 2 since theft errors exist but they are ignored. For a given initial IS inventory x, the expected profit under Approach 1 is, as a consequence, as follows:

$$\pi_{1,1}^* = \pi_{2,1}(Y_{1,1}^*) \tag{7.12}$$

Remark 7.1. *We notice that ($Y_{2,1}^*$-$Y_{1,1}^*$) is decreasing with α since $Y_{2,1}^*$ decreases with α (Property 7.1) and $Y_{1,1}^*$ is independent of α. As a consequence, the relative benefit of taking into account errors when optimizing the inventory decisions, i.e., ($\frac{\pi_{2,1}^*-\pi_{1,1}^*}{\pi_{2,1}^*}$), is increasing with α.*

The following figures illustrate the last remark for $\mu = 10$, $\sigma = 3$, $c = 10$, $h = 1$ and $u = 5$:

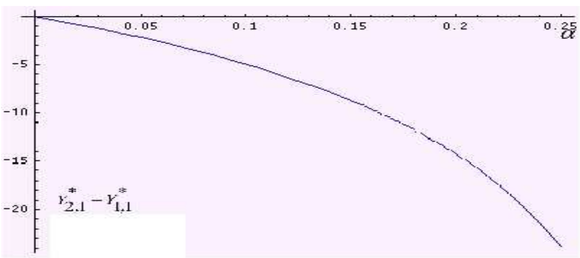


Figure 7.4: Variation of ($Y_{2,1}^*$-$Y_{1,1}^*$) with α

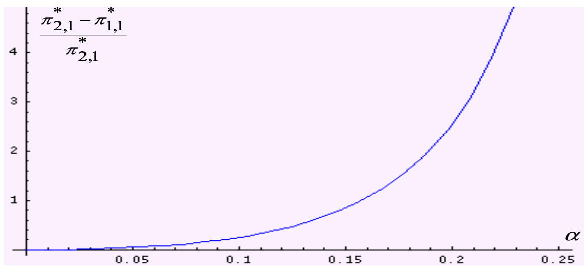


Figure 7.5: Variation of ($\frac{\pi_{2,1}^*-\pi_{1,1}^*}{\pi_{2,1}^*}$) with α

[4]The assumption "the inventory manager ignores errors" is more realistic than the assumption where he is supposed to be unaware of errors specially if the horizon length N is large since even if the inventory manager is unaware of errors, he can realize that there is an anomaly in the inventory system. This can happen if the IS inventory is positive and the sales are zero for a long interval of time.

[5]In the case where demand parameters are endogenous, the demand distribution is built based on the Point of Sales Data. In such case the inventory manager will consider the observed demand, i.e. the demand for purchase which is a function of α and is given by $D \sim N((1-\alpha)\mu, \sqrt{1-\alpha}\sigma)$. For our future research, it would be intersting to analyze the inventory system in such case.

The Finite-Horizon Problem

We now consider the N-period problem under Approach 1. By ignoring the theft occurring in the store, the inventory manger follows an inventory policy established for a system that does not face theft problem. He also follows an order-up-to policy $Y^*_{1,1} = \{y_1, y_2, ...y_N\}$. where

$$Y^*_{1,1} = F^{-1}\left[\frac{u}{u+h+c}\right] \tag{7.13}$$

Our aim is to evaluate the expected profit pertaining to this policy. The following figure illustrates the behavior of the IS and the physical inventory under Approach 1:

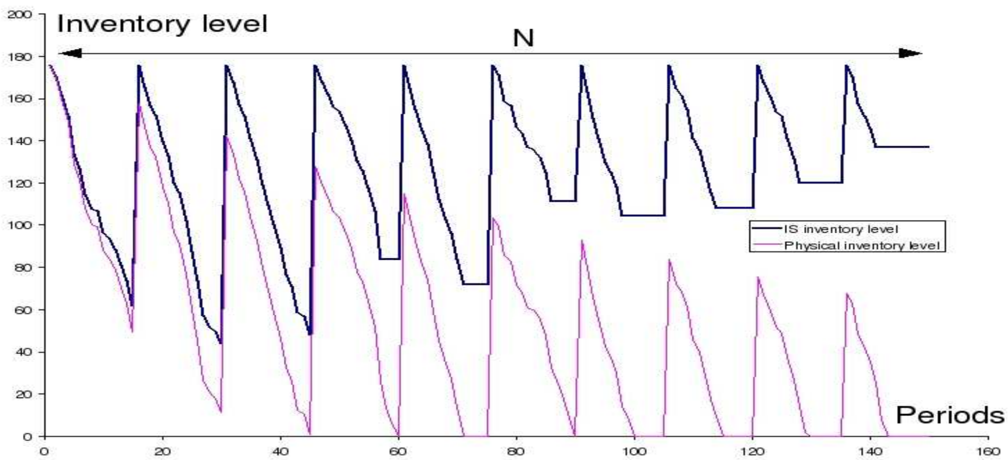


Figure 7.6: The behavior of the IS and the physical inventory- Approach 1

The physical inventory level at the beginning of period k is $Y^*_{1,1} - \varepsilon_k$ where ε_k is the perturbation in the physical level at the beginning of period k. When the total demand exceeds the physical available inventory, the available inventory is divided proportionately to meet the two demand streams. As a consequence, two situations can occur:

- if $D_k \leq Y^*_{1,1} - \varepsilon_k$: $a_k = (1-\alpha)D_k$ and $b_k = \alpha D_k$
- if $D_k \geq Y^*_{1,1} - \varepsilon_k$: $a_k = (1-\alpha)(Y^*_{1,1} - \varepsilon_k)$ and $b_k = \alpha(Y^*_{1,1} - \varepsilon_k)$

As a consequence we can write $\begin{cases} a_k = (1-\alpha)Min(D_k, Y^*_{1,1} - \varepsilon_k) \\ b_k = \alpha Min(D_k, Y^*_{1,1} - \varepsilon_k) \end{cases}$

At the beginning of period k, the inventory manager observes x_k and orders a quantity equal to $Y^*_{1,1} - x_k$. ε_k is not observed and is not taken into account when ordering. The state of the system is defined by the vector (x_k, ε_k) and evolves according to the following updates:

$$\begin{cases} x_{k+1} = Y^*_{1,1} - a_k \\ \varepsilon_{k+1} = \varepsilon_k + b_k \end{cases}$$

By using the expressions of a_k and b_k, the system updates are written as the following:

$$\begin{cases} x_{k+1} = Y_{1,1}^* - (1-\alpha)Min(D_k, Y_{1,1}^* - \varepsilon_k) \\ \varepsilon_{k+1} = \varepsilon_k + \alpha Min(D_k, Y_{1,1}^* - \varepsilon_k) \end{cases}$$

Given an initial state (x_k, ε_k) and the given order-up-to level $Y_{1,1}^*$, the expected N-period profit under Approach 1 from period k to N is given by the following dynamic programming problem:

$$I_k(x_k, Y_{1,1}^*, \varepsilon_k) = L(x_k, Y_{1,1}^*, \varepsilon_k) + E_{D_k}\left[I_{k+1}(x_{k+1}, Y_{1,1}^*, \varepsilon_{k+1})\right] \tag{7.14}$$

Which can also written as the following:

$$\begin{aligned} I_k(x_k, Y_{1,1}^*, \varepsilon_k) &= L(x_k, Y_{1,1}^*, \varepsilon_k) \\ &+ \int_{D_k=0}^{Y_{1,1}^*-\varepsilon_k} I_{k+1}(Y_{1,1}^* - (1-\alpha)D_k, Y_{1,1}^*, \varepsilon_k + \alpha D_k) f(D_k) dD_k \\ &+ \int_{D_k=Y_{1,1}^*-\varepsilon_k}^{+\infty} I_{k+1}(\alpha Y_{1,1}^* + (1-\alpha)\varepsilon_k, Y_{1,1}^*, \alpha Y_{1,1}^* + (1-\alpha)\varepsilon_k) f(D_k) dD_k \end{aligned} \tag{7.15}$$

By assuming that $I_{N+1}(.,.) \equiv 0$, the last functions can be computed recursively back in time, starting with period N and ending by period 1. Under Formulation 1, the total expected profit pertaining to approach 1 is also given by:

$$\pi_{1,1}^* = I_1(x_1 \equiv 0, Y_{1,1}^*, \varepsilon_1 \equiv 0) \tag{7.16}$$

7.3.3 Analysis of Approach 3

Under Approach 3, we assume that the inventory manager uses the RFID technology that enables to track the movement of goods within the store and to prevent theft and therefore eliminate errors. Alike other models developed in this dissertation, when RFID is implemented, if t represents the unit tag cost, the unit product purchasing cost is no longer c but $c + t$. As a consequence the unit underage cost is no longer u but $u - t$.

The Single-Period Problem

Under approach 3 and for a given initial IS inventory level x, the expected profit function is given by:

$$\begin{aligned} L_{RFID}(x, y, t) &= (u-t)\cdot\mu - (c+t+h)\int_{D=-\infty}^{y} [y-D] f(D) dD \\ &- (u-t)\int_{D=y}^{+\infty} [D-y] f(D) dD + (c+t)x \end{aligned} \tag{7.17}$$

The optimal order-up-to level associated with Approach 3 is given by:

$$Y_{3,1}^* = F^{-1}\left[\frac{u-t}{u+h+c}\right] \tag{7.18}$$

The associated optimal expected profit will be as follows:

$$\pi_{3,1}(Y_{3,1}^*) \quad = \quad (u+h+c)\int_{D=-\infty}^{Y_{3,1}^*} Df(D)dD + (c+t)x \tag{7.19}$$

Remarks:

1. Note that we can reinterpret the classical Newsvendor problem that can be found in literature as a particular case of Approach 3 with $t = 0$.

2. As noticed in section 7.2, we assume that eliminating errors consists in converting the demand for theft into a demand for purchase ($\beta = \gamma = 0$).

The Finite-Horizon Problem

Here again, the inventory system is monitored based on the physical inventory level. In the beginning of period k, given an initial physical inventory level (before ordering) x_k^{PH}, the expected profit, K_k from period k to N is given by the following dynamic programming:

$$K_k(x_k^{PH}) \quad = \quad \max_{y_k \geq x_k^{PH} \geq 0} \left[L_{RFID}(x_k^{PH}, y_k, t) + E_{D_k}\left[K_{k+1}(x_{k+1}^{PH})\right]\right] \tag{7.20}$$

The state update is expressed as the following:

$$x_{k+1}^{PH} = \left[x_k^{PH} - D_k\right]^+ \tag{7.21}$$

We also assume that $K_{N+1}(.,.) \equiv 0$ and $L_{RFID}(x, y, t)$ is the expected signle period profit function pertaining to Approach 3 (cf Equation 7.17). The dynamic programming problem is identical to the classical inventory control with lost sales and the key results remain valid. In particular, an optimizing policy is defined by an order-up-to level policy $Y_{3,1}^* = \{y_1^*, y_2^*, ...y_N^*\}$ where:

$$Y_{3,1}^* = F^{-1}\left[\frac{u-t}{u+h+c}\right] \tag{7.22}$$

The optimal profit pertaining to this policy is given by

$$\pi_{3,1}^* = K_1(x_1^{PH} \equiv 0) \tag{7.23}$$

We deliberately do not perform a numerical analysis permitting to compare the three approaches under Formulation 1. Such analysis is left for to the end of this chapter where analytical results of Formulation 2 are used. This is motivated by the fact that the managerial insights pertaining to the two formulations are going in the same sense.

7.4 Analysis under Formulation 2: the Service Level Formulation

Service levels are used in inventory control systems for performance evaluation and in target setting as substitutes for underage costs that are hard to estimate. A review of standard service level measures and their relationships to underage costs and different control policies is provided by Schneider [80].

Under Formulation 2, we consider the Horizon Service Level (HSL) defined as the probability of not having a shortage over the whole horizon (N periods)[6]. The aim is to minimize the expected overage costs in order to satisfy a target service level for the N-period horizon.
For the single-period problem, it is clear that the HSL is nothing other than the classical Cycle Service Level (CSL) (the type 2 service level defined in Silver et al. [70]). Appendix G.2 proposes the optimal control of classical newsvendor and periodic review models under the HSL constraint. As in Formulation 1, we analyze in the following subsections the inventory system in each approach.

7.4.1 Analysis of Approach 2

We recall that Approach 2 considers the situation where the inventory system is prone to theft errors and the inventory manager optimizes the system by taking them into account.

The Single-Period Problem

For the single-period problem, the Horizon Service Level corresponds simply with the classical Cycle Service Level. The aim is to minimize the expected overage costs in order to satisfy a target service level HSL_0. Since overage cost and the HSL are increasing with the order-up-to level, for each ε, the inventory manager should choose the best $Y_{2,2}$ that satisfies the service level constraint $P\left[D \leq Y_{2,2} - \varepsilon\right]$. By considering the equivalent demand distribution $D_{eq} = D + \varepsilon$, we can deduce that the optimal order-up-to level of Approach 2 under Formulation 2 is given by:

$$Y^*_{2,2} = F^{-1}_{eq}\left[HSL_0\right] \tag{7.24}$$

Using the same method as in Appendix G.2, the optimal expected profit pertaining to this policy for a given initial IS inventory level x is as the following:

$$\begin{aligned}\pi_{2,2}(Y^*_{2,2}) \quad &= \quad r(1-\alpha)\left[(1-HSL_0)Y^*_{2,2} + \int_{D_{eq}=-\infty}^{Y^*_{2,2}} D_{eq} f(D_{eq}) dD_{eq}\right] \\ &- \quad h\left[HSL_0 Y^*_{2,2} - \int_{D_{eq}=-\infty}^{Y^*_{2,2}} D_{eq} f(D_{eq}) dD_{eq}\right] \\ &- \quad c\left[Y^*_{2,2} - x - E[\varepsilon]\right]\end{aligned} \tag{7.25}$$

As in Formulation 1, we remark that the problem with an inventory system subject to theft errors is equivalent to a standard single-period problem with an equivalent demand distribution ($D_{eq} = D + \varepsilon$) and an equivalent selling price ($r(1-\alpha)$)

The Finite-Horizon Problem

For the N-period problem, the optimization of Approach 2 is the same as the classical N-period problem developed in Appendix G.2 with the exception that the selling price is no longer r but $r(1-\alpha)$. For a

[6] Remember that N periods correspond to the the time between two successive inspection operations where the physical and the IS inventory levels are updated and as a consequence aligned

given HSL_0, the optimal order-up-to level should satisfy:

$$\begin{aligned} HSL_0 &= \prod_{k=1}^{N} P[D_k \leq Y^*_{2,2}] \\ &= F[Y^*_{2,2}]^N \end{aligned} \tag{7.26}$$

As a consequence, $Y^*_{2,2}$ is as the following:

$$Y^*_{2,2} = F^{-1}\left[\sqrt[N]{HSL_0}\right] \tag{7.27}$$

The optimal expected profit for the N-Period problem is approximated by:

$$\begin{aligned} \pi^*_{2,2} &\approx r(1-\alpha)N\int_{D=-\infty}^{Y^*_{2,2}} Df(D)dD + r(1-\alpha)NY^*_{2,2}\int_{D=Y^*_{2,2}}^{+\infty} f(D)dD \\ &- hN\int_{D=-\infty}^{Y^*_{2,2}}(Y^*_{2,2}-D)f(D)dD - cN\mu \end{aligned} \tag{7.28}$$

The profit function is composed of four parts: $i)$ the first one corresponds to the revenue pertaining to sales in the case where demand is less than the physical inventory and the second part corresponds to revenue of sales in the contrary case, $ii)$ the third part corresponds to the overage cost and $iii)$ the fourth part corresponds to the purchase cost. The approximation in the profit function is stemming from the last part where we assume that purchase in each period is μ. Such an approximation is a good one if the system falls rarely in shortage, i.e. if HSL_0 is high enough. Simplifying the last equation leads to the following expression:

$$\begin{aligned} \pi^*_{2,2} &\approx r(1-\alpha)N\left[(1-\sqrt[N]{HSL_0})Y^*_{2,2} + \int_{D=-\infty}^{Y^*_{2,2}} Df(D)dD\right] \\ &- hN\left[\sqrt[N]{HSL_0}Y^*_{2,2} - \int_{D=-\infty}^{Y^*_{2,2}} Df(D)dD\right] - cN\mu \end{aligned} \tag{7.29}$$

7.4.2 Analysis of Approach 1

For the N-Period problem, in order to avoid shortage situation in period $k \in [1, N]$, the order up-to level $Y_{1,2}$ should verify $D_k \leq Y_{1,2} - \varepsilon_k$. If no shortage occurs till period k, we argue that $\varepsilon_k = \alpha \sum_{i=1}^{k-1} D_i$ since all the demands for theft (from period 1 to period $k-1$) were satisfied in this case. As a consequence for each $k \in [1, N]$, $Y_{1,2}$ verifies $D_k \leq Y_{1,2} - \alpha \sum_{i=1}^{k-1} D_i$ if no shortage happens till period k.

Let A_k denote the event "there is no shortage in period k". So the Horizon Service Level is given by:

$$\begin{aligned} HSL &= P\left[\bigcap_{k=1}^{N} A_k\right] \\ &= P\left[A_N / \bigcap_{k=1}^{N-1} A_k\right] * P\left[\bigcap_{k=1}^{N-1} A_k\right] \\ &= P\left[D_N + \alpha\sum_{i=1}^{N-1} D_i \leq Y_{1,2}\right] * P\left[\bigcap_{k=1}^{N-1} A_k\right] \end{aligned}$$

Again by using the same method we have:

$$\begin{aligned} P\left[\bigcap_{k=1}^{N-1} A_k\right] &= P\left[A_{N-1}/\bigcap_{k=1}^{N-2} A_k\right] * P\left[\bigcap_{k=1}^{N-2} A_k\right] \\ &= P\left[D_{N-1} + \alpha \sum_{i=1}^{N-2} D_i \leq Y_{1,2}\right] * P\left[\bigcap_{k=1}^{N-2} A_k\right] \end{aligned}$$

Following recursively the last method to Period 1 enables us to deduce the expression of the Horizon Service Level:

$$HSL = \prod_{k=1}^{N} P\left[D_k + \alpha \sum_{i=1}^{k-1} D_i \leq Y_{1,2}\right] \tag{7.30}$$

By letting F_k denoting the CDF of a normally distributed demand with parameters:

$$\mu_k = [1 + \alpha(k-1)]\,\mu \tag{7.31}$$

$$\sigma_k = \sqrt{1 + \alpha(k-1)}\sigma \tag{7.32}$$

permits to write the HSL as the following

$$HSL = \prod_{k=1}^{N} F_k(Y_{1,2}) \tag{7.33}$$

Remark 7.2. *Based on the last analysis and by denoting $CSL(k) = F_k(Y_{1,2})$, it appears that the Horizon Service Level of the N-Period problem corresponds to the product of N different Cycle Service Level of N single-period problems where the distribution of the demand in each period k is characterized by the parameters μ_k and σ_k provided in Equations 7.31 and 7.32. That means that in order to get the whole HSL, we need N different $CSL(k)$, $k = 1..N$ where $CSL(k) \geq HSL$ and $CSL(k+1) \leq CSL(k)$ for each $k = 1..N$. The following figures represent the evolution of $CSL(k)$ with k in order to satisfy a target HSL_0 over the N-Period horizon:*

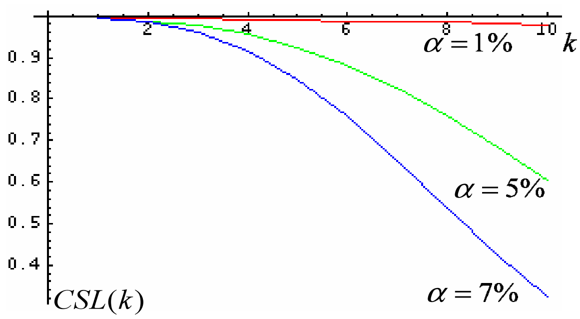


Figure 7.7: Variation of $CSL(k)$ with k for different values of α ($HSL_0 = 95\%$, $\mu = 10$, $\sigma = 2$)

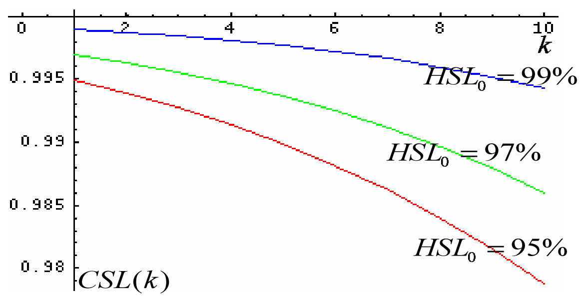


Figure 7.8: Variation of $CSL(k)$ with k for different values of HSL_0 ($\alpha = 1\%$, $\mu = 10$, $\sigma = 2$))

Two ways may exist in order to quantify the penalty resulting from ignoring theft errors:

1. For a given target HSL_0, the inventory manager chooses $Y^*_{1,2}$ without taking into account errors. So he chooses $Y^*_{1,2}$ based on the classical inventory control provided in Appendix G.2.2:

$$Y^*_{1,2} = F^{-1}\left[\sqrt[N]{HSL_0}\right] \tag{7.34}$$

In such case the effective Horizon Service Level is no more HSL_0 but is smaller and is equal to:

$$\begin{aligned} HSL_{effective} &= \prod_{k=1}^{N} F_k(Y^*_{1,2}) \\ &= \prod_{k=1}^{N} F_k(F^{-1}\left[\sqrt[N]{HSL_0}\right]) \end{aligned} \tag{7.35}$$

Where F_k is the CDF of a normal distribution defined by the parameters given in Equations 7.31 and 7.32. The following figures illustrate the variation of $HSL_{effective}$ with HSL_0 for different values of α and N. As it can remarked, $HSL_{effective}$ decreases with α and N.

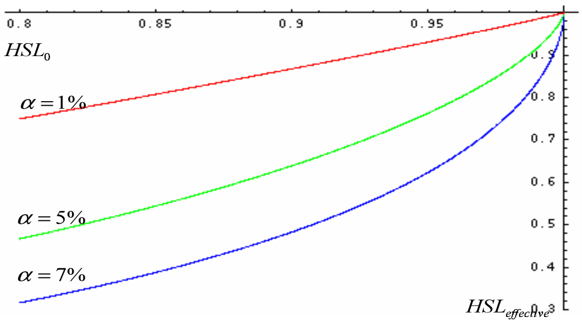


Figure 7.9: Variation of $HSL_{effective}$ with HSL_0 for different values of α ($N = 5$)

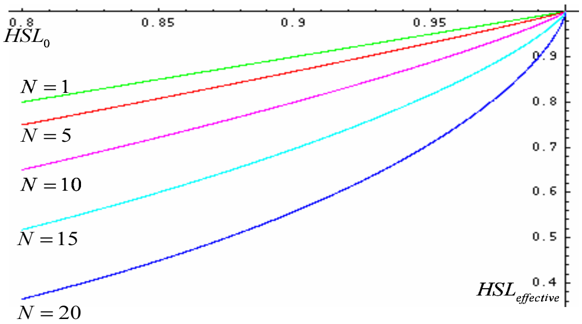


Figure 7.10: Variation of $HSL_{effective}$ with HSL_0 for different values of N ($\alpha = 1\%$)

2. The second way consists in calculating the optimal order-up-to level in order to verify a target HSL_0, $Y^*_{1,2}$ should also satisfy:

$$HSL_0 = \prod_{k=1}^{N} F_k(Y^*_{1,2}) \tag{7.36}$$

In order to quantify the penalty resulting from ignoring errors, we calculate the expected profit resulting from this policy based on the level $Y^*_{1,2}$, and we then analyze the additional overage cost which permits to satisfy the target service level HSL_0. For high values of HSL_0, the expected profit function pertaining to this policy is given by:

$$\begin{aligned}
\pi_{1,2}^* &\approx r(1-\alpha)\sum_{k=1}^{N}\left[\int_{D_k=-\infty}^{Y_{1,2}^*-(k-1)\alpha\mu} D_k f(D_k)dD_k\right] \\
&+ r(1-\alpha)\sum_{k=1}^{N}\left[\int_{D_k=Y_{1,2}^*-(k-1)\alpha\mu}^{+\infty} \left[Y_{1,2}^*-(k-1)\alpha\mu\right] f(D_k)dD_k\right] \\
&- h\sum_{k=1}^{N}\int_{D_k=-\infty}^{Y_{1,2}^*-(k-1)\alpha\mu} \left[Y_{1,2}^*-(k-1)\alpha\mu-D_k\right] f(D_k)dD_k \\
&- c(1-\alpha)\mu N
\end{aligned} \tag{7.37}$$

Here again, the last equation is an approximation since we assume that shortage situations are negligible: this is illustrated by the fact that the sales in period k are $a_k = (1-\alpha)\mu$ and theft is $b_k = \alpha\mu$. The approximation is a good one if HSL_0 is high. A high HSL_0 means that in each period k we have a higher $CSL(k)$ which means that shortage situation is negligible.

By introducing the equivalent demand $D_k^{eq} = D_k + (k-1)\alpha\mu$ and letting f_k^{eq} its PDF function, the expected profit can be written as the following:

$$\begin{aligned}
\pi_{1,2}^* &= [r(1-\alpha)+h]\sum_{k=1}^{N}\int_{D_k^{eq}=-\infty}^{Y_{1,2}^*} (D_k^{eq}-Y_{1,2}^*)f_k^{eq}(D_k^{eq}) \\
&+ N(1-\alpha)\left[rY_{1,2}^* - (c + r\alpha\frac{N-1}{2})\mu\right]
\end{aligned} \tag{7.38}$$

7.4.3 Analysis of Approach 3

Under Approach 3, we recall that errors are eliminated due to the deployment of the RFID technology. The optimization of the inventory system is similar to the classical inventory problem with a modified purchase cost which includes the cost of the RFID tag. For the single-period and the Finite-horizon problems results of Appendix G.2 are used with replacing c by $c+t$. For a given HSL_0, $Y_{3,2}^*$ is given by:

$$Y_{3,2}^* = F^{-1}\left[\sqrt[N]{HSL_0}\right] \tag{7.39}$$

If we assume that the initial physical inventory in the beginning of the horizon is zero, the optimal expected profit for the N-Period problem is approximated given by:

$$\begin{aligned}
\pi_{3,2}^* &\approx rN\left[(1-\sqrt[N]{HSL_0})Y_{3,2}^* + \int_{D=-\infty}^{Y_{3,2}^*} Df(D)dD\right] \\
&- hN\left[\sqrt[N]{HSL_0}Y_{3,2}^* - \int_{D=-\infty}^{Y_{3,2}^*} Df(D)dD\right] \\
&- (c+t)N\mu
\end{aligned} \tag{7.40}$$

7.5 Benefits of the implementation of the RFID technology

Our aim in this section is to seek an answer to the questions: is RFID technology beneficial for the inventory manager? If yes, which tag cost make the implementation of this technology economically feasible?
In order to response analytically this question, we will consider the results obtained under Formulation 2, i.e., the service level formulation. As in our analysis in the last chapters, we consider an initial situation where the inventory system is managed under Approach 1, i.e., errors are ignored. In order to eliminate errors, the inventory manager chooses to implement the RFID technology which enables to move from Approach 1 to Approach 3. The absolute benefit achieved by this transition is therefore $\pi^*_{3,2} - \pi^*_{1,2}$. However, we argue that this difference does not enable to measure the true value of the RFID technology since:

1. One part of this benefit, i.e. $B_A = \pi^*_{2,2} - \pi^*_{1,2}$, can be achieved by getting information about the error parameter and integrating it in the optimization of the ordering decision. The evaluation of B_A gives further insights to the question: By how much the profit can be increased through a better replenishment policy that takes into account the error parameter? If the value of α is known, integrating this information in the ordering policy may (but we will show that this is not the case for all values of model parameters) increase the profit

2. The second part, $B_B = \pi^*_{3,2} - \pi^*_{2,2}$, is due to the elimination of errors based on the implementation of the RFID technology.

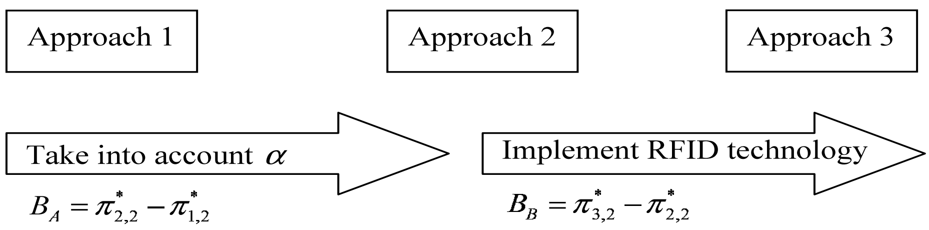


Figure 7.11: B_A versus B_B

Our criterion to compare the performance of the three approaches is that each, approach should satisfy a given service service level HSL_0. The aim is to compare the expected profits in each approach given that the service level constraint is satisfied in each one. So the inventory policy in each approach is established based on the target HSL_0.
Remember that approaches 1 and 2 can also be compared by assuming that a given HSL_0 is satisfied in Approach 2 and the same HSL_0 is supposed to be satisfied in Approach 1 but is not reached because of errors. In this last case an effective $HSL_{effective}$ would be satisfied. A comparison between approaches 1 and 2 may be not efficient in this way since:

- The comparison of expected profits is not coherent since each approach operates with a different target service level

- Even if HSL_0 is chosen to be high, the $HSL_{effective}$ under Approach 1 may be small (recall this was the case in our numerical example in Figure 7.9). In such case our approximation of the expected profit function is not of good quality

As a consequence, we choose to consider a reached HSL_0 under each approach and we compare the expected profits. For all our numerical examples in this section we let $\mu = 10$, $\sigma = 2$, $r = 10$ and $c = 2$.

7.5.1 Analysis of B_A

The following result states the relation between Approach 1 and Approach 2:

Result 7.2. *For a given target service level HSL_0, we have $Y^*_{1,2} \geq Y^*_{2,2}$*

Proof. (Proof by contradiction: Reductio ad absurdum) First, let recall that $Y^*_{1,2}$ and $Y^*_{2,2}$ satisfy respectively $HSL_0 = \prod_{k=1}^{N} F_k(Y^*_{1,2})$ (where F_k is the CDF of a normal demand distribution having as parameters $\mu_k = [1 + \alpha(k-1)]\,\mu$ and $\sigma_k = \sqrt{1+\alpha(k-1)}\sigma$ provided in Equations 7.31 and 7.32 respectively) and $Y^*_{2,2} = F^{-1}\left[\sqrt[N]{HSL_0}\right]$. Second we remark that for a given y we have $F_{k_2}(y) < F_{k_1}(y)$ if $k_2 > k_1$.
Let assume that $Y^*_{1,2} < Y^*_{2,2}$. So we have $F_k(Y^*_{1,2}) < F_k(Y^*_{2,2}) < F_1(Y^*_{2,2}) = F(Y^*_{2,2})$ for each $k \in [1, N]$. As a consequence, we have $\prod_{k=1}^{N} F_k(Y^*_{1,2}) < F(Y^*_{2,2})^N$ which is in contradiction with the fact that $\prod_{k=1}^{N} F_k(Y^*_{1,2}) = F(Y^*_{2,2})^N = HSL_0$. The last contradiction ends the proof. $\square$

Figure 7.12 represents the variation of $Y^*_{1,2} - Y^*_{2,2}$ with α for different values of N. We remark that the difference is increasing with α and with N which is intuitively expected.

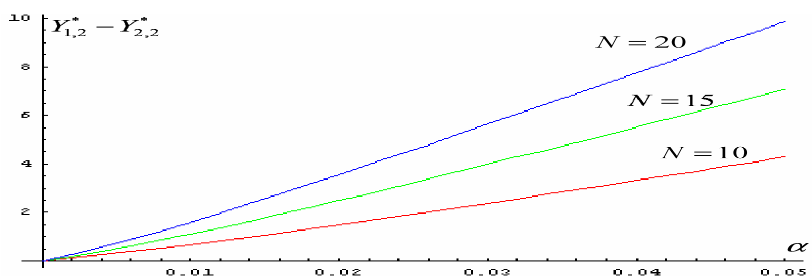


Figure 7.12: Variation of $(Y^*_{1,2} - Y^*_{2,2})$ with α for different values of N, $HSL_0 = 98\%$ and $h = 2$

Concerning the relationship between $\pi^*_{1,2}$ and $\pi^*_{2,2}$, it appears numerically (as will it be illustrated in our numerical analysis in Section 7.5.3) that B_A is positive and taking into account errors when optimizing the inventory system is intuitively beneficial. For this reason, if we want to analyze the true impact of the RFID technology on this inventory system, we should compare Approach 3 with Approach 1. The aim of the next subsection is to study this comparison through the analysis of B_B.

7.5.2 Analysis of B_B

The aim here is to compare Approaches 2 and 3 in order to derive a critical RFID tag cost which makes its deployment cost effective. First we remark that $Y_{2,2} = Y_{3,2} = F^{-1}(\sqrt[N]{HSL_0})$: this can be explained by the fact that $Y^*_{2,2}$ and $Y^*_{3,2}$ are respectively independent of α and the tag cost t. The last remark will help us to derive a simple analytical RFID tag cost which permits to compare Approaches 1 and 2. The following result states the condition under which the inventory manager would be interested in deploying the RFID technology:

Result 7.3. *For a given HSL_0:*

$$t \leq t_c = r\alpha \frac{(1 - \sqrt[N]{HSL_0})F^{-1}(\sqrt[N]{HSL_0}) + \int_{D=-\infty}^{F^{-1}(\sqrt[N]{HSL_0})} Df(D)dD}{\mu} \tag{7.41}$$

is a necessary and sufficient condition to make the deployment of the RFID technology cost effective

Proof. The proof is deduced by calculating $\pi^*_{3,2} - \pi^*_{2,2}$ and setting it positive. As mentioned, the key is the fact that $Y^*_{2,2} = Y^*_{3,2}$. □

Let now analyze the impact of model parameters on the critical RFID tag cost t_c through a numerical study (As in the previous subsection, we set $\mu = 10$, $\sigma = 2$, $r = 10$ and $c = 2$:

- We remark that t_c is increasing with the error parameter α. Such result is intuitively expected[7] since if errors are not important, the RFID tag cost should be small to be adopted by the inventory manager

- t_c is increasing with the selling price r. If r is not important, the RFID tag cost should be small to be deployed. As in our analysis of Chapter 4, such results can constitute a basis for segmenting products based on values of r for a given error setting and for a given tag cost

- t_c increases with the horizon length N: this result is also expected since the error impact is more important if N is high. In this case the RFID tag cost enabling RFID to be cost effective is also high

- As illustrated in Figure 7.13, it appears that t_c increases with HSL_0. Such result can be explained by the fact that a higher HSL_0 means that the RFID technology is easier to be adopted when the target service level is high.

[7] Recall that we have also shown the same result for our t_c of Chapters 4 and 5 for the misplacement type errors

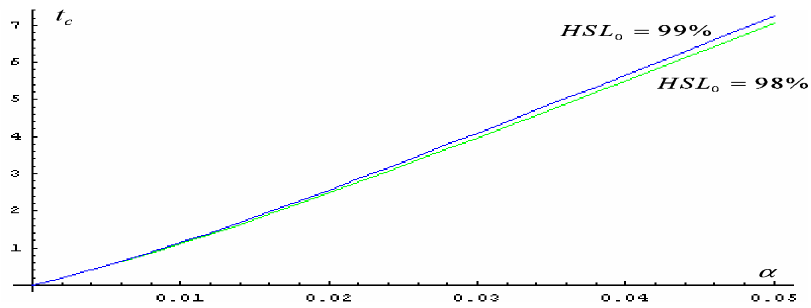


Figure 7.13: Variation of t_c with α for different values of HSL_0, $N = 15$, $h = 1$, $\mu = 10$, $\sigma = 2$, $r = 10$ and $c = 2$

7.5.3 Comparison of B_A and B_B

This section develops a numerical analysis that compares B_A and B_B. As an example case, let consider a retailer facing a normally distributed demand with parameters $\mu = 10$ and $\sigma = 2$. Cost parameters are such that $c = 2$, $r = 10$ and $h = 2$. We also consider three possible values for the RFID tag, i.e., t=m c (m=0%,1%,5%). We also consider two possible values of the target service level HSL_0 ($HSL_0 = 98\%, 99\%$). The retailer can choose between two possible values of the horizon length ($N = 10, 15$).

If N is chosen to be 15, the following figures (7.14 and 7.15) represent respectively the variation of B_A and B_B with α for the two possible values of HSL_0:

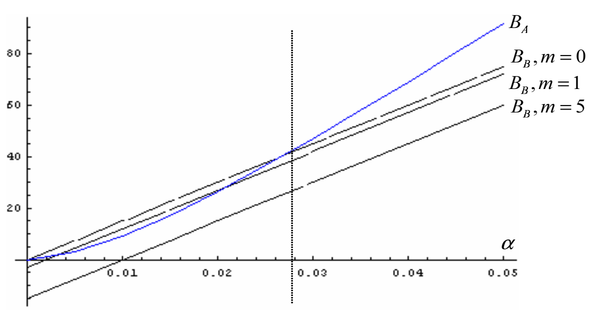


Figure 7.14: Variation of B_A and B_B with α for different values of m, $HSL_0 = 98\%$ and $N = 15$

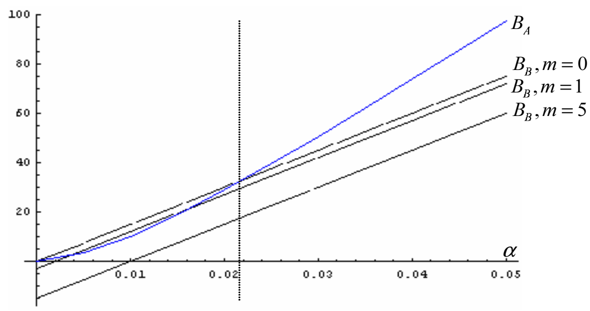


Figure 7.15: Variation of B_A and B_B with α for different values of m, $HSL_0 = 98\%$ and $N = 15$

For a given α, it appears that B_A can be higher or lower than B_B depending on the value of t. For high values of α, optimizing the inventory system by taking into account errors permits to gain more than deploying the RFID technology. This is due to the fact that penalty resulting from ignoring errors is important for high values of α. The two figures shows also that it may exist a critical value of α such that taking into account error is better (worse) than deploying RFID if α is higher (smaller) than this critical value of α. By comparing the two figures, it appears that this critical value of α decreases when HSL_0: this can be explained by the fact that the penalty of ignoring errors is more important for high value of the target service level HSL_0.

Now we set $HSL_0 = 98\%$ and let analyze the impact of the horizon length N by considering a smaller one ($N = 10$):

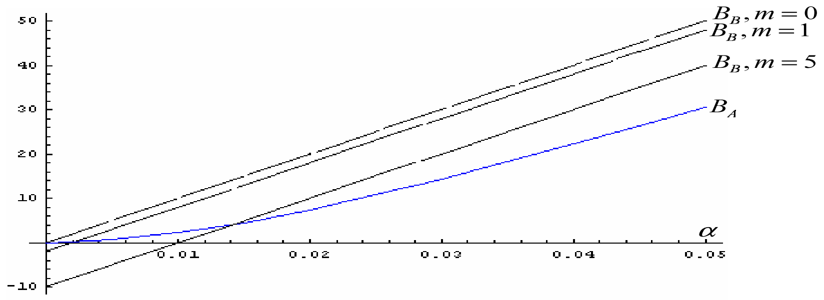


Figure 7.16: Variation of B_A and B_B with α for different values of m, $HSL_0 = 98\%$ and $N = 10$

By comparing the last figure with Figure 7.14, it appears that deploying RFID is more advantageous in the case of short horizon length: this is because the penalty of ignoring errors is more important if N is high.
The analysis conducted in this section confirm our results in the last chapters of this dissertation concerning the importance of establishing an intelligent policy that takes into account errors as a first lever against inventory inaccuracy. The RFID technology comes to complete this intelligent policy if its cost is not very high.

7.6 Conclusion

In this chapter, we considered a finite horizon, single-stage, single-product periodic-review inventory in which inventory records are inaccurate. We assumed that inventory inaccuracies are introduced by theft type errors that occur within the store. We proposed a comparison between three approaches based on which inventory systems in the presence of theft can be managed. To solve the problem, we fellow two formulations. i) Optimization of shortage and overage costs and ii) Optimization of overage costs under a service level constraint. The second formulation permits us to analytically solve the problem and to derive a critical RFID tag cost which makes RFID cost effective.
To summarize the chapter, the following points are provided:

- The originality of our error modelling is to consider that the theft quantity is a fraction of the total demand. This modelling way avoided us to define a second random variable to describe errors and as a consequence made our mathematical results cleaner.
- Our assumption concerning the sharing of the physical available stock in the case where the total demand is more than this physical quantity avoided us to consider the sequence of arrival of the two demand streams (demand for purchase and demand for theft). For our further research it would be interesting to relax this assumption in order to analyse this case.
- In the error free model, we assumed that the demand for theft is converted into a demand for purchase. The analysis provided in this chapter can be easily be modified in order to relax this

assumption.

- The horizon length is assumed to be fixed. Based on the analysis of Formulation 2, it would be interesting (and feasible) to try to optimise both the inventory policy and the horizon length. An optimal inspection based policy could be an alternative to the deployment of the RFID technology.

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Conclusion and Perspectives

The RFID technology is considered by a large number of people as a breakthrough in product identification and data capture throughout the supply chains. While the development of the different parts needed in order to implement the RFID systems (hardware, software, standardization,...) is under way, it is also important to evaluate the impact of such technology on the performance of supply chains. Benefits of the RFID technology, in terms of cost reduction, are various and among them, the elimination of inventory record errors that are currently encountered in many supply chains may be considerable. The purpose of our research project is to develop a set of models that provide qualitative and quantitative insights on the benefits of the RFID technology on the performance of supply chains in terms of cost reduction and/or improvement of service levels.

The starting point of our research is a real world observation: more and more companies are looking for other identification systems than the bar code technology enabling a more accurate tracking of products in supply chains. Most of them are also interested in evaluating the benefits of an advanced automatic identification system such as the RFID technology. They are interested in new functionalities associated with this technology in order to compare it with the performance enabled by the bar code technology. Our second observation concerns the inventory information system which is a major obstacle to achieving operational excellence. In fact, the inventory information system, in contrary to popular belief and assumptions in most academic papers and in spite of the considerable amounts invested in information technology, are often inaccurate.

Our aim is to quantify the penalty resulting from the inventory inaccuracy issue and to analyze one of the improvements stemming from the deployment of the RFID technology, namely the benefit of having accurate inventory data. We also provide other alternatives permitting to reduce the impact of the inventory inaccuracy issue. For this purpose, we organize the PhD dissertation in three parts:

Part I This part introduces the dissertation by presenting the RFID technology and the inventory inaccuracy issue.

- Based on qualitative and case study investigations, we first propose a basic understanding of the RFID technology and focuses on its impact on supply chain performance. This analysis gives a clue to the question of how companies may benefit from using the RFID technology.

- Based on qualitative and empirical investigations, we then describe the inventory inaccuracy issue by defining factors generating it and by focusing on its impact on supply chain performances. Some compensation methods permitting to cope with inventory inaccuracy is also provided.

- Based on quantitative investigations, we then provide a comprehensive review of literature on academic investigations dealing with the inventory inaccuracy issue. The focus is particularly steered on quantitative investigations where a classification is provided. Our classification is based on three main levels relative to the the objective, the supply chain structure and the errors structure of models considered in theses investigations. To be competitive, this classification helped us during the thesis in choosing the topics to be considered in priority. To our knowledge, we are the first to perform such classification and it would be a helpful mean to derive area for further research and perspectives.

Part II Our quantitative analysis starts by characterizing an inventory system subject to inventory inaccuracy where both the physical and the information system inventories are prone to perturbations. We propose two supply chain structures enabling to capture this issue, namely the retailer and the wholesaler supply chain structures.
Models of this part of the dissertation are built based on the Newsvendor framework: a single replenishment opportunity is made before the beginning of the selling period based an estimation of a demand that will be observed during the selling season.

- We first consider supply type errors or the well know random yield problem. The motivation to begin with such source of error, i.e. supply unreliability, is the sparse investigations dealing with problem. Or we quickly remark that results pertaining to this problem under a Newsvendor framework are not complete. We also propose a comprehensive analysis of this problem and we extend existing results.

- We then consider an other source of error where an analysis of a retail supply chain under misplacement type errors is provided. We first analyze this problem for a centralized supply chain where a single decision maker is aiming to maximize his own expected profit. The analysis permits to show that errors cost a lot and are particularly more penalizing if they are not known or simply ignored. We also derive a critical RFID tag cost which makes its deployment cost effective. We then analyze the same problem (with deterministic error) in a decentralized supply chain where two decision makers act as different parties and each one is aiming to maximize his own expected profit. Here, we focus on an other alternative permitting to reduce the impact of misplacement errors namely the coordination of the channel.

- We then consider a wholesaler supply chain structure subject to inventory inaccuracies stemming from different sources of errors. We provide a general framework permitting to capture this issue and we show that an inventory system subject to inventory inaccuracies can be seen as a extended version of the random yield problem. An elegant analytical analysis for the additive error setting is also performed.

Part III Motivated by the lack of investigations dealing with theft type errors in a multi-period framework, Part III of the dissertation considered a periodic review model where errors are caused by theft errors. We conduct the analysis by the mean of two formulations. In particular, the second formulation, i.e. the service level formulation, permitted us to derive an analytical critical RFID tag cost which makes the deployment of this technology cost effective. To our knowledge, we are the first to conduct such analysis especially in a multi-period framework.

Perspectives

Results obtained in this dissertation provide interesting managerial insights and stimulate the development of further research. Among research perspectives, the following ones are of special interest:

- Further analysis on the proposed general framework presented in Chapter 6 is necessary to more understand the impact of errors on the performance of the inventory system. Especially, studying more sub-models of our general framework which emphasize on the u_2 penalty (the type 2 shortage penalty) will complete the analysis of our research. For instance, it will be interesting to perform an analysis of misplacement type errors similar to the one provided in this dissertation in a wholesale context and to perform a comparison between the two contexts.

- In the short time horizon we will try to complete the analysis of the general framework for the multiplicative error setting in both centralized and decentralized supply chain structures. The sensitivity of optimal solutions to model parameters (especially the u_2 penalty) will surely give further managerial insights.

- Concerning our decentralized supply chain analysis, we are currently trying to extend to the stochastic error case. An other interesting point to be studied in this decentralized supply chain will be the case where the manufacturer gets a ”take it or leave it” choice since today most of the power lies at the retailers rather than at the manufacturer as it is described in Chapter 5.

- Evaluation of other levers enabling to reduce errors and comparing their performances with the RFID technology is also interesting to analyze. In particular, inspection policies can be an alternative to the RFID technology. A comparison taking into account the RFID tag cost and the inspection cost can lead to valuable results. Our results in Chapter 7 stimulate and help to follow this research perspective.

- Our last chapter can be considered as a first step to understand what makes multi-period framework more complex but acts as a stimulator for our further research on this topic. We will try to perform more analysis on such framework by considering other sources of errors and by relaxing some of the assumptions made in Chapter 7.

- In our research, we focus on the impact of the RFID technology in inventory system subject to inaccuracy problems. Considering other type of benefits such as the impact of the RFID in the reverse supply chain or in the planning of supply chain activities will also be of special interest.

- Even if we considered in this research “theoretical” models and results , our work is originally motivated by real world problematic expressed by many firms. A last interesting point deals with trying to make the results provided in the report useful and accessible to supply chain firms.

Appendix A

Appendix of Chapter 1

A.1 Roots of RFID

Roots of RFID go as far as 1940's and 50's when the principle that RFID is based on, was first used in aircraft Identification Friend or Foe systems. Further on, the development of integrated circuits in the 60's and the works of Richardson [82] and Vinding [83] pushed RFID prospects forward. But it was not until Charles Walton pioneered his radio frequency identification technology in the 1970's and 1980's that the real history of radio frequency identification (RFID) began (Takahashi [84]). With his patents Walton is considered by many to be the father of RFID, he created first electronic door keys that used RFID technology.

RFID had been around in various forms for years before Walton's invention of a radio-operated door lock. Earlier inventors received patents on animal control systems, a luggage handling system and a mail-sorting system. But Walton came up with a design that is popular today. His technology of the time was good enough and even better than barcode, but his 1,75$ solution was no much for 25-cent barcode. In spite of apparent problems, based on his ideas and thanks to great developments in electronics and chip making industries the future of RFID and its use looked bright.

A.2 Most common applications of RFID

RFID has the potential to improve numerous existing processes and applications. It's most common applications include (Nevshehir [85]):

- Supply chain management: RFID enhances supply chain visibility of products as they move. RFID tags allow manufacturers to see how fast a product is moving. Instead of basing manufacturing production decisions on what a company expects to sell, RFID allows the collection of data in real time to see how a product is currently selling. This optimizes profits and supply chain efficiencies.

- Work-in-progress (WIP). The most common use of RFID during WIP is to use read-write tags that allow the manufacturer to place information on the tag as the product moves through various manufacturing stages along the production line

- Improved data capture: RFID provides consumers with product information such as care instructions for clothes or technical assistance for small electronic devices.
- Security and theft prevention: RFID is more effective than short circuit televisions or security guards at controlling store theft because it does not require a line-of-sight view to detect items being stolen. RFID is also used for security (e.g., building access) and payment systems that let customers pay for items without using cash.
- Authentication of currency, documents, DVD discs and more
- Combat product counterfeiting and protect intellectual property

According to the investigation of Rochel and Joyce [86], we present in the following some case studies of the deployment of the RFID technology:

Heathcare Company A leading logistics service provider was chosen by a healthcare company and a German retail company to test RFID deployment. The task was to tag pallets for shipments bound from the healthcare company warehouse (which is operated by the logistics service provider) to certain retail distribution centres. The anticipated benefits of this RFID pilot were an enhanced supply chain visibility, more accurate and efficient scanning processes and thus cost reductions.

Fashion Company Another RFID pilot project was conducted by a global fashion and lifestyle company together with its logistics partner and a retail company. Its goal was to test current technology, examine possible applications for RFID deployment, and to identify both costs and benefits of RFID solutions in the textile trade. Logistic units as well as the items are tagged at the logistics service provider during the control of goods received. These tags are then read before as well as after the order picking process at the logistics service provider. When arriving at the retail stores the control of goods received is done by scanning the goods when they pass RFID-enabled gates.

Mailorder and Online Retailer In order to test practicability of RFID technology and to prevent theft of its high value electronic goods during shipment, a major mail-order and online retailer opted for an RFID solution. In the central warehouse RFID tags are attached to the packaging of high value goods. The passive smart label tags contain a unique article number, the shipment code, as well as a number used for returned goods. Therefore, the goods arrive at the end customers with the tag. In order to anticipate any privacy concerns, the packages comprise an informative letter about the technology and the confirmation that no personal data is stored on the tag. According to the company, the benefits of this RFID system for theft reduction alone outweigh the costs by more than 20 percent.

Supplier of Industrial Printers In cooperation with a freight forwarder and an airfreight carrier, a supplier of industrial printers tracks shipments from its Germany based warehouse alongside the route to American customers. With the intention of providing better customer service, the printer supplier needed to track shipments at shorter intervals and to gain better visibility of the shipments during the time they were handled by the freight forwarder and the airfreight carrier.

Appendix B

Appendix of Chapter 2

B.1 How to measure inventory accuracy (Pergamalis [1])

The first step to establish a system of measuring and checking the accuracy of the stock is to establish a number of samplings of circular inventory, drawn up in a specific period of time (ex. week, month). During this period, the authorized controllers undertake the checking of a certain and predefined number of random storage positions.
The number of samples chosen during the spot check of the circular inventory depends on two factors:

- On the predefined estimated stock accuracy.
- On the desired deviation from stock accuracy.

Stock accuracy a is calculated with the following formula:

$$a = 1 - (e/n) \tag{B.1}$$

Where e is the number of the errors found during the spot check and n the number of the spot checked positions. Based on the theory of statistical sampling, the number of the samples n that should be checked in every stage of the circular inventory, is calculated with the following formula:

$$n = [a(1-a)]/[(p/3)^2] \tag{B.2}$$

Where a is the estimated stock accuracy and p the maximum desired deviation from accuracy.
To understand fully the above, let's suppose that the company estimates stock accuracy in 98% and this estimation is based on a recent full or partial inventory. If we choose the desired deviation p equal to 2%, then the number of the samples is calculated using the formula B.2. Replacing the above figures in formula B.2, we have $n = [0,98(1-0,98)]/[(0,02/3)^2] = 441$.
Based on the above, 441 positions should be checked every month (or week) and according to the checking, stock accuracy should be calculated using the formula B.1. If we draw up the circular inventory in 441 random selected positions and we find 10 errors, then stock accuracy equals to 97,73%, based on the formula B.1. Considering the selected deviation from the stock at 2%, stock accuracy will range from 95,73% to 99,73%. If, after the sampling, stock accuracy differs greatly from what we

supposed before the beginning of the circular inventory, then we can use the formula B.2 and solving it regarding p to calculate the new deviation from stock accuracy:

$$p = 3[a(1-a)/n]^{1/2} \tag{B.3}$$

If, for instance, after the sampling of 441 positions, there are 35 errors, then stock accuracy is estimated at 92,06% (=35/441). Using the formula B.3 we calculate again the new deviation p from the accuracy that is equal to 3,86%, so stock accuracy ranges from 88,2% to 95,92%.

Appendix C

Appendix of Chapter 3

C.1 Technical details for configuration 1

C.1.1 Proof of Result 3.1

In configuration 1 we have three cost functions depending on the value of the received quantity compared with demand‘s one. We have:

$$C_2^1(Q_2) = h \int\limits_{Q_A=L_{QA}}^{U_{QA}} \int\limits_{x=L_x}^{L_{QA}} (Q_A - x) f(x) g(Q_A) dx dQ_A$$

$$C_2^2(Q_2) = k.h \int\limits_{Q_A=L_{QA}}^{U_{QA}} \int\limits_{x=U_{QA}}^{U_x} (x - Q_A) f(x) g(Q_A) dx dQ_A$$

$$\begin{aligned} C_2^3(Q_2) &= k.h \int\limits_{Q_A=L_{QA}}^{U_{QA}} \int\limits_{x=Q_A}^{U_{QA}} (x - Q_A) f(x) g(Q_A) dx dQ_A \\ &+ h \int\limits_{Q_A=L_{QA}}^{U_{QA}} \int\limits_{x=L_{Q_A}}^{Q_A} (Q_A - x) f(x) g(Q_A) dx dQ_A \end{aligned}$$

The total expected cost function is written as the following:

$$\begin{aligned} C_2(Q_2) &= C_2^1(Q_2) + C_2^2(Q_2) + C_2^3(Q_2) \\ &= \frac{h(-6(k-1)(Q_2 - \mu_x)\sigma_x + \sqrt{3}((-Q_2 + \mu_x)^2 + 3\sigma_x^2 + \sigma_\xi^2)))}{12\sigma_x^2} \end{aligned}$$

The convexity of $C_2(Q_2)$ is clear. Setting $\frac{\partial C_2(Q_2)}{\partial Q_2} = 0$ and solving, we get $Q_2^* = Q_0^* = \mu_x + \sigma_x \sqrt{3} \frac{k-1}{k+1}$ with an optimal cost function equal to: $C_2(Q_2^*) = \frac{h(12k\sigma_x^2 + (k+1)^2 \sigma_\xi^2)}{4\sqrt{3}(k+1)\sigma_x}$.

C.1.2 Validity of Result 3.1

The result presented above is valid till:

1. $U_{Q_A} \leq U_x$ for $k \geq 1$, so $Q_2 + \sqrt{3}\sigma_\xi \leq \mu_x + \sqrt{3}\sigma_x$.
   Replacing Q_2 by $Q_2^* = \mu_x + \sigma_x\sqrt{3}\frac{k-1}{k+1}$ and solving the last inequality, we get $\sigma_\xi \leq \frac{2}{k+1}\sigma_x$. So, configuration 1 is defined for $\sigma_\xi \in [0, \frac{2}{k+1}\sigma_x]$ if $k \geq 1$

2. $L_x \leq L_{Q_A}$ for $k \leq 1$, so $\mu_x - \sqrt{3}\sigma_x \leq Q_2 - \sqrt{3}\sigma_\xi$.
   Again replacing Q_2 by Q_2^* we get $\sigma_\xi \leq \frac{2k}{k+1}\sigma_x$. So, configuration 1 is defined for $\sigma_\xi \in [0, \frac{2k}{k+1}\sigma_x]$ if $k \leq 1$

C.2 Technical details for configuration 2

C.2.1 Proof of Result 3.2

For $k \geq 1$ we have:

$$C_2^1(Q_2) = h \int_{Q_A=U_x}^{U_{Q_A}} \int_{x=L_x}^{U_x} (Q_A - x) f(x) g(Q_A) dx dQ_A$$

$$\begin{aligned} C_2^2(Q_2) &= h \int_{Q_A=L_{QA}}^{U_X} \int_{x=L_X}^{Q_A} (Q_A - x) f(x) g(Q_A) dx dQ_A \\ &+ k.h \int_{Q_A=L_{QA}}^{U_X} \int_{x=Q_A}^{U_X} (x - Q_A) f(x) g(Q_A) dx dQ_A \end{aligned}$$

The total cost function is written as the following:
$C_2(Q_2) = C_1^1(Q_2) + C_1^2(Q_2) = \dfrac{1}{72\sigma_x\sigma_\xi}(h(-(k+1)Q_2^3C_\lambda^3 + (k+1)\mu_x^2 +$
$3\sqrt{3}(k+1)(\sigma_x\sigma_\xi)^2 + 9\mu_x((k+1)\sigma_x^2 + 2(k-3)\sigma_x\sigma_\xi + (k+1)\sigma_\xi^2) +$
$3(k+1)Q_2^2(\mu_x + \sqrt{3}(\sigma_x + \sigma_\xi)) - 3Q_2((k+1)\mu_x^2 + 3(k+1)\sigma_x^2 +$
$6(k-3)\sigma_x\sigma_\xi + 3(k+1)\sigma_\xi^2 + 2\sqrt{3}\mu_x(k+1)(\sigma_x + \sigma_\xi))))$
The second derivation of $C_2(Q_2)$ is equal to $\frac{h(k+1)[U_x - L_{Q_A}]}{12\sigma_x\sigma_\xi}$ which is all the time positive since $L_{Q_A} < U_x$, so the convexity of $C_2(Q_2)$.
Setting $\frac{\partial C_2(Q_2)}{\partial Q_2} = 0$ and solving we get $Q_2^* = Q_0^* + \sqrt{3}\left(\sqrt{\sigma_\xi} - \sqrt{\frac{2}{k+1}\sigma_x}\right)^2$ where: $Q_0^* = \mu_x + \sigma_x\sqrt{3}\frac{k-1}{k+1}$
With an optimal cost function $C_2(Q_2^*) = \sqrt{3}h(\sigma_x + \sigma_\xi) - 4\frac{h\sqrt{2\sigma_x\sigma_\xi}}{\sqrt{3(k+1)}}$.
By doing the same for the case $k \leq 1$ we get:
$Q_2^* = Q_0^* - \sqrt{3}\left(\sqrt{\sigma_\xi} - \sqrt{\frac{2k}{k+1}\sigma_x}\right)^2$ with an optimal cost function $C_2(Q_2^*) = \sqrt{3}hk(\sigma_x + \sigma_\xi) - 4\frac{\sqrt{2\sigma_x\sigma_\xi}hk^2}{\sqrt{3k(k+1)}}$

C.2.2 Validity of Result 3.2

The result obtained above is valid till:

1. If $k \geq 1$: $L_{QA} \geq L_x$ and $U_{QA} \geq U_x$ so $Q_2 - \sqrt{3}\sigma_\xi \geq \mu_x - \sqrt{3}\sigma_x$ and $Q_2 + \sqrt{3}\sigma_\xi \geq \mu_x + \sqrt{3}\sigma_x$. Again replacing Q_2 by Q_2^* we get $\frac{2}{k+1}\sigma_x \leq \sigma_\xi \leq \frac{k+1}{2}\sigma_x$ and this inequality is verified since $k \geq 1$

2. If $k \leq 1$: by doing the same as the previous case we have $\frac{2k}{k+1}\sigma_x \leq \sigma_\xi \leq \frac{k+1}{2k}\sigma_D$. Again the inequality $\frac{2k}{k+1}\sigma_x \leq \frac{k+1}{2k}\sigma_D$ is well verified since $k \leq 1$

C.3 Technical details for configuration 3

C.3.1 Proof of Result 3.3

We have:

$$C_2^1(Q_2) = k.h \int_{x=L_x}^{U_x} \int_{Q_A=L_{QA}}^{L_x} (x - Q_A) f(x) g(Q_A) dQ_A dx$$

$$C_2^2(Q_2) = h \int_{x=L_x}^{U_x} \int_{Q_A=U_x}^{U_{QA}} (Q_A - x) f(x) g(Q_A) dQ_A dx$$

$$\begin{aligned} C_2^3(Q_2) &= k.h \int_{x=L_x}^{U_x} \int_{Q_A=L_x}^{x} (x - Q_A) f(x) g(Q_A) dQ_A dx \\ &+ h \int_{x=L_x}^{U_x} \int_{Q_A=x}^{U_x} (Q_A - x) f(x) g(Q_A) dQ_A dx \end{aligned}$$

The total expected cost function is written as the following:

$$\begin{aligned} C_2(Q_2) &= C_2^1(Q_2) + C_2^2(Q_2) + C_2^3(Q_2) \\ &= \frac{h(-6(k-1)(Q_2 - \mu_x)\sigma_\xi + \sqrt{3}(k+1)((-Q_2 + \mu_x)^2 + \sigma_x^2 + 3\sigma_\xi^2))}{12\sigma_\xi} \end{aligned}$$

We can easily show that $C_2(Q_2)$ is convex, and by setting $\frac{\partial C_2(Q_2)}{\partial Q_2} = 0$ and solving, we get $Q_2^* = \left[\mu_x + \sigma_\xi \sqrt{3}\frac{k-1}{k+1}\right]$ With an optimal cost function equal to $C_2(Q_2^*) = \frac{h((k+1)^2\sigma_x^2 + 12k\sigma_\xi^2)}{4\sqrt{3}(k+1)\sigma_\xi}$.

C.3.2 Validity of Result 3.3

The result obtained above is valid till:

1. If $k \geq 1$ we have $Q_2 + \sqrt{3}\sigma_\xi \geq \mu_x + \sqrt{3}\sigma_D$ and $Q_2 - \sqrt{3}\sigma_\xi \geq 0$. So by replacing Q_2 by Q_2^* we get $\frac{k+1}{2}\sigma_x \leq \sigma_\xi \leq \frac{k+1}{\sqrt{12}}\mu_x$

2. If $k \leq 1$ we have $Q_2 - \sqrt{3}\sigma_\xi \leq \mu_x - \sqrt{3}\sigma_x$ and $Q_2 - \sqrt{3}\sigma_\xi \geq 0$. Again, by replacing Q_2 by Q_2^* we get $\frac{k+1}{2k}\sigma_x \leq \sigma_\xi \leq \frac{k+1}{\sqrt{12}}\mu_x$

For $k \geq 1$, it is easy to verify that $\frac{k+1}{2}\sigma_x \leq \frac{k+1}{\sqrt{12}}\mu_x$ since $L_x = \mu_R - \sqrt{3}\sigma_x$ is positive. $\frac{k+1}{2k}\sigma_x \leq \frac{k+1}{\sqrt{12}}\mu_x$ is verified for values of $CV_x = \frac{\sigma_x}{\mu_x}$ such that $CV_x \leq \frac{k}{\sqrt{3}}$, otherwise Configuration 4 does not exist and the maximum value that can take σ_ξ is between $\dfrac{2k}{k+1}$ and $\frac{k+1}{2k}$ (Configuration 2) is positive.

C.4 Extension to the case with initial inventory

In this Appendix, we consider the case of multiplicative errors with an initial inventory I. By following the methodology developed in Chapter 3 (cf Page 55), we extend our model and derive the optimal policy for each configuration.

If an initial inventory is taken into account, we show that that the ordering quantity in all configurations, except configuration 1, is a non linear function of the initial inventory. The following result summarizes the overall optimal policy:

Result C.1. *For a given vector (μ_x, σ_x, k, I), we distinguish two cases: Case A where $L_{Q_A} \leq L_x$ in the second configuration, i.e. $k \leq \frac{L_x+2U_x-3I}{2L_x+U_x-3I}$ and Case B where $U_{Q_A} \geq U_x$ in the second configuration i.e. $k \geq \frac{L_x+2U_x-3I}{2L_x+U_x-3I}$. Depending on system parameters, in both cases, 1 ,2 or 3 of the configurations presented in Chapter 3 may be observed. The expression of the optimal quantity to order for each configuration as well as the critical values of σ_{ij} can be determined by using the three steps approach described in Chapter 3:*

Conf.	Interval of σ_γ	Q_2^*
Conf. 1	$[0, \sigma_{12}]$	$\frac{\mu_\gamma}{\mu_\gamma^2+\sigma_\gamma^2}\,[Q_0^* - I]$
Conf. 2	$[\sigma_{12}, \sigma_{23}]$	Q_2^* is obtained by solving $aQ_2^{*^3} + bQ_2^{*^2} + c = 0$
Conf. 3	$[\sigma_{23}, \sigma_{\gamma\,\max}]$	$\frac{\sqrt{(k+1)((\mu_x-I)^2+\sigma_x^2)}}{\sqrt{-2\sqrt{3}(k-1)\mu_\gamma\sigma_\gamma+(k+1)(\mu_\gamma^2+3\sigma_\gamma^2)}}$

Case A: $k \leq \frac{L_x+2U_x-3I}{2L_x+U_x-3I}$ $\sigma_{12} = \frac{\sqrt{(3(Q_0^*+I)-2L_x)(2L_x+Q_0^*+I)-16IQ_0^*}-\sqrt{3}(Q_0^*-I)}{2(L_x-I)}\mu_\gamma$ $\sigma_{23} = \frac{(d-e)-\sqrt{(d-e)^2-e^2}}{\sqrt{3}e}\mu_\gamma$ $\sigma_{\gamma\max} = \frac{\mu_\gamma}{\sqrt{3}}$	$a = 2(k+1)U_\gamma^3$ $b = 3kU_x(L_\gamma^2 - U_\gamma^2) - 3L_x$ $(U_\gamma^2 + kL_\gamma^2) + 3(k+1)IU_\gamma^2$ $c = (k+1)(L_x - I)^3$ $d = k(U_x - I)^2$ $e = (\sqrt{3}(\mu_x - I) + \sigma_x)$ $(k+1)\sigma_x$
Case B: $k \geq \frac{L_x+2U_x-3I}{2L_x+U_x-3I}$ $\sigma_{12} = \frac{\sqrt{3}(Q_0^*-I)-\sqrt{(3(Q_0^*+I)-2U_x)(2U_x+Q_0^*+I)-16IQ_0^*}}{2(U_x-I)}\mu_\gamma$ $\sigma_{23} = \frac{(d+e)-\sqrt{(d+e)^2-e^2}}{\sqrt{3}e}\mu_\gamma$ $\sigma_{\gamma\max} = \frac{\mu_\gamma}{\sqrt{3}}$	$a = 2(k+1)L_\gamma^3$ $b = 3L_x(U_\gamma^2 - L_\gamma^2) - 3U_x$ $(U_\gamma^2 + kL_\gamma^2) - 3L_\gamma^2(k+1)I$ $c = (k+1)(U_x - I)^3$ $d = (L_x - I)^2$ $e = (\sqrt{3}(\mu_x - I) - \sigma_x)$ $(k+1)\sigma_x$

For each critical value σ_{ij} expressed in the table above, a condition on I should be satisfied to assure $\sigma_{ij} \in R^+$ and as a consequence, Configuration j exists. This condition is expressed in the form of an interval of variation of I, as represented in the table below:

Case A : $k \leq \frac{L_x+2U_x-3I}{2L_x+U_x-3I}$

Interval of I	Possible conf.
$\left[0, \mu_x - \sigma_x \frac{\sqrt{3}+\sqrt{(3-k)(k+1)}}{k}\right]$	1-2-3
$\left[0, \frac{1}{3}(2L_x + Q_0^*)\right]$	1-2
$[0, Q_0^*]$	1
$[Q_0^*, +\infty]$	Do not order

Case B : $k \geq \frac{L_x+2U_x-3I}{2L_x+U_x-3I}$

Interval of I	Possible conf.
$[0, L_x]$	1-2-3
$[0, 3Q_0^* - 2U_x]$	1-2
$[0, Q_0^*]$	1
$[Q_0^*, +\infty]$	Do not order

Remark C.1.

- *By setting $I = 0$ we find the results pertaining to the multiplicative errors case which are developed in Chapter 3*
- *For Case A, an additional condition on model parameters should be made to ensure that the lower boundary of the received quantity reaches zero in Configuration 3 and as a consequence to ensure that Configuration 3 exists. This condition is as follows:* $2 - \frac{3L_x(L_x+U_x)}{L_x^2+L_xU_x+U_x^2} \leq k \leq 3$
- *For Case B, an additional condition should be made to ensure the existence of Configuration 2 and also to assure that $3Q_0^* - 2U_x \geq 0$. This condition is as follows:* $k \geq 2 - 3\frac{L_x}{U_x}$

- *For the particular case considered in Inderfurth [57] ($L_x = 0$ and $L_\gamma = 0$), our result confirm the optimal policy provided by the author:* $Q_2^* = \begin{cases} \frac{1}{U_\gamma}\sqrt{\frac{(k+1)(U_x-I)^3}{3U_x}} \textit{ if } I \in [0, 3Q_0^* - 2U_x] \\ \frac{\mu_\gamma}{\mu_\gamma^2+\sigma_\gamma^2}\left[Q_0^* - I\right] \textit{ if } I \in [3Q_0^* - 2U_x, Q_0^*] \\ 0 \textit{ if } I \in [Q_0^*, +\infty] \end{cases}$

Appendix D

Appendix of Chapter 4

D.1 Expression of the optimal profits of Approach 2

In this appendix we provide expressions of the expected profit, $\pi_2(Q_2)$, for each configuration and we analyze the evolution of the optimal expected profit, $\pi_2(Q_2^*)$, with μ_θ and σ_θ.

Rewriting Equation 2 with respect to the position between distributions of x and θQ_2 and simplifying the expression leads to the following table:

Conf.	**Expression of** $\pi_2(Q_2)$
Conf. 1	$k\mu_x - \frac{h(-L_x^2+2Q_2(L_x-U_x)-kU_x^2-(k+1)Q_2(-2U_x\mu_\theta+Q_2\mu_\theta^2+Q_2\sigma_\theta^2))}{2(L_x-U_x)}$
Conf. 2-1	$k\mu_x - \frac{h(-Q_2(3L_\theta L_x(L_x-2Q_2)+(k+1)L_\theta^3Q_2^2-3L_x(L_x-2Q_2)U_\theta)}{6Q_2(L_\theta-U_\theta)(L_x-U_x)}$
	$-\frac{h(3Q_2^2(L_\theta(-2+L_\theta(k+1))+2U_\theta)U_x-3Q_2(kL_\theta+U_\theta)U_x^2+(k+1)U_x^3))}{6Q_2(L_\theta-U_\theta)(L_x-U_x)}$
Conf. 2-2	$k\mu_x - \frac{h(-(k+1)L_x^3+3L_x^2Q_2(U_\theta+kL_\theta)-3L_xQ_2^2(L_\theta(-2+(k+1)L_\theta)+2U_\theta)}{6Q_2(L_\theta-U_\theta)(L_x-U_x)}$
	$-\frac{h(Q_2((k+1)Q_2^2U_\theta^3+3Q_2(L_\theta-U_\theta)(-2+(k+1)(L_\theta+U_\theta)U_x+3k(U_\theta-L_\theta)U_x^2))}{6Q_2(L_\theta-U_\theta)(L_x-U_x)}$
Conf. 3	$k\mu_x + \frac{h(Q_2^2(L_\theta(-2+L_\theta(k+1))+2U_\theta)-2Q_2(kL_\theta+U_\theta)\mu_x+(k+1)(\mu_x^2+\sigma_x^2)}{2Q_2(L_\theta-U_\theta)}$

The optimal expected profit for each configuration is deduced by using the corresponding optimal ordering quantity provided in Result 6. For Configurations 1 and 3, explicit expressions of $\pi_2(Q_2^*)$ can be found. These are not provided in this appendix because of the length of formulas.

Variation of $\pi_2(Q_2^*)$ with μ_θ and σ_θ:

Result 7 is deduced from the analysis of the sign of $\frac{d\pi_2(Q_2^*)}{d\mu_\theta}$ and $\frac{d\pi_2(Q_2^*)}{d\sigma_\theta}$. For configurations 1 and 3, we have an explicit expression of $\pi_2(Q_2^*)$. As a consequence, the variation of $\pi_2(Q_2^*)$ with μ_θ and σ_θ is straightforward.

For configuration 2, we do not have an explicit expression of Q_2^*. By using the fact that Q_2^* verifies $a{Q_2^*}^3 + b{Q_2^*}^2 + c = 0$ (cf Result 6), we can explicitly deduce $\frac{dQ_2^*}{d\mu_\theta}$ and $\frac{dQ_2^*}{d\sigma_\theta}$. As a consequence $\frac{d\pi_2(Q_2^*)}{d\mu_\theta}$ and $\frac{d\pi_2(Q_2^*)}{d\sigma_\theta}$ can also be analyzed in Configuration 2 (2-1 and 2-2).

Results pertaining to the detailed analysis of $\frac{d\pi_2(Q_2^*)}{d\mu_\theta}$ and $\frac{d\pi_2(Q_2^*)}{d\sigma_\theta}$ are developed in Rekik et al. (2005-b) and can be provided on demand.

Appendix E

Appendix Chapter 5

E.1 Approach 0 under the Decentralized Uncoordinated Scenario

Recall that Approach 0 is simply the basic Newsvendor problem. The reader is referred to Larivière and Porteus [78] for a complete analysis of the wholesale contract in the context of the Newsvendor problem. In this appendix, we present main results pertaining to this issue.

In the Decentralized Uncoordinated Scenario we consider the wholesale contract. The manufacturer chooses the unit wholesale price w_{UD0} and after observing w_{DU0}, the retailer chooses the order quantity Q_{DU0}. In the game theory literature, the considered model is a dynamic game of complete information with two players, manufacturer and retailer, where the manufacture moves first and the retailer moves second. The manufacturer is called the "Stackelberg Leader" and the retailer is the "Stackelberg Follower".

The Retailer's Problem: The retailer's profit function in a wholesale contract is similar to the profit function of the Centralized Scenario with the exception that the retailer now pays a wholesale price w_{DU0} to the manufacturer whose unit cost is still c. The expected profit function of the retailer is also as follows:

$$\begin{aligned}\pi^R_{DU0}(Q_{DU0}, w_{DU0}) &= (r - w_{DU0})\mu - (r - w_{DU0})\int_{x=Q_{DU0}}^{+\infty}(x - Q_{DU0})f(x)dx \\ &- (w_{DU0} - s)\int_{x=0}^{Q_{DU0}}(Q_{DU0} - x)f(x)dx \qquad \text{(E.1)}\end{aligned}$$

As the Centralized Scenario, we can argue that the optimal ordering quantity should verify:

$$Q^*_{DU0}(w_{DU0}) = F^{-1}\left[\frac{r - w_{DU0}}{r - s}\right] \qquad \text{(E.2)}$$

The Manufacturer's Problem: The manufacturer has the wholesale price as his decision variable. Being the leader, he anticipates the retailer's order for any wholesale price. The manufacturer's expected profit is given by:

$$\pi^M_{DU0}(w_{DU0}) = (w_{DU0} - c)Q_{DU0}(w_{DU0}) \qquad \text{(E.3)}$$

Note that since the manufacturer is able to anticipate the retailer's optimal behavior, the function $Q_{DU0}(w_{DU0})$ is deterministic for him. The manufacturer's problem then is to choose the wholesale price w_{DU0} that maximizes $\pi^M_{DU0}(w_{DU0})$. By using the inverse of $Q_{DU0}(w_{DU0})$ which is $w_{DU0}(Q_{DU0}) = (r-s)\left[1 - F(Q_{DU0})\right] + s$ and by considering the transformation $\widehat{c} = c - s$ and $\widehat{r} = r - s$, the expected profit function of the manufacture can be written as follows:

$$\pi^M_{DU0}(Q_{DU0}) = \{\widehat{r}\left[1 - F(Q_{DU0})\right] - \widehat{c}\}\, Q_{DU0} \tag{E.4}$$

Several well-known wholesale price contract results from the literature can be invoked directly, as the following theorem shows.

Theorem E.1. *Larivière and Porteus [78]*
Under Approach 0, the first-order condition is sufficient and its solution is a unique global maximum for an IGFR[1] *demand distribution*

a. *The optimum is reached for Q^*_{DU0}, such that $1 - F(Q^*_{DU0}) - Q^*_{DU0} f(Q^*_{DU0}) = \frac{c-s}{r-s}$*

b. *The corresponding optimum wholesale price is $w^*_{DU0} = c + (r-s)Q^*_{DU0} f(Q^*_{DU0})$*

c. *The optimum expect profit of the manufacturer is $\pi^{M*}_{DU0} = (r-s)(Q^*_{DU0})^2 f(Q^*_{DU0})$*

Proof. Cf Larivière and Porteus [78] □

The first order condition and its solution have several interesting properties that are summarized in the following property:

Property E.1. *In Approach 0 under a wholesale price contract:*

a. *The manufacturer's optimal amount of product sold to the retailer Q^*_{DU0}*
   - *Increases as the retail price r and the salvage price s increase*
   - *Decreases as the unit production cost c increases.*

b. *The manufacturer's optimal wholesale price charged to the retailer w^*_{DU0}*
   - *decreases as the retail price r and the salvage price s increase*
   - *Decreases as the unit production cost c decreases.*

The following theorem states the relation between Q^*_{DU0} and the expected demand μ:

Theorem E.2. *In Approach 0 under a wholesale contract, we have $Q^*_{DU0} \leq \mu$*

Proof. The proof follows directly from the fact that $c \leq s$ □

[1] Increasing General Failure Rate

Finally, it is important to notice that in the Decentralized Scenario, the outcomes are worse for all the parties involved (manufacturer, retailer, supply chain, and consumer) compared to the Centralized Scenario, because in the Decentralized Scenario both the retailer and the manufacture independently try to maximize their own profits, i.e., they each try to get a margin. This effect is called "Double Marginalization" (DM). As shown in Proposition 6, it is interesting to note that for any parameter combination, the optimal amount chosen by the manufacturer for selling to the retailer is less that the expected demand value: this is due to the effect of Double Marginalization.

E.2 Approach 0 under the Decentralized Coordinated Scenario

Here, the reader is referred to Pasternack [75] and Lariviere [76] for a complete analysis of the buy-back contract in the context of the Newsvendor problem. The manufacturer stands ready to buy back any unsold stock from the retailer at a per unit rate $b < w$. We should assume that $b < w$ to ensure that the manufacturer does not create an arbitrage opportunity for the retailer, allowing him to buy stock in order to return it for a profit. Additionally, for the deal to be attractive to the retailer b must be greater than s.

The Retailer's Problem: The expected profit function of the retailer is alike Approach 0 under the Decentralized Uncoordinated Scenario with the exception that s is replaced by b:

$$\begin{aligned} \pi^R_{DC0}(Q_{DC0}, w_{DC0}, b_{DC0}) &= (r - w_{DC0})\mu - (r - w_{DC0}) \int_{x=Q_{DC0}}^{+\infty} (x - Q_{DC0}) f(x) dx \\ &- (w_{DC0} - b_{DC0}) \int_{x=0}^{Q_{DC0}} (Q_{DC0} - x) f(x) dx \end{aligned} \tag{E.5}$$

By assuming $b_{DC0} < w_{DC0} < r$, the retailer's profit is strictly concave and the optimal ordering quantity Q^*_{DC0} satisfies

$$Q^*_{DC0}(w_{DC0}, b_{DC0}) = F^{-1}\left[\frac{r - w_{DC0}}{r - b_{DC0}}\right] \tag{E.6}$$

The Manufacturer's Problem: Under the wholesale contract, the manufacturer's profit was deterministic. Now, with a buy back contract, he shares the risk of stocking out and overstocking with the retailer through the buy-back portion and he is now exposed to the possibility of a poor demand outcome. His profit is as follows:

$$\pi^M_{DC0}(w_{DC0}, b_{DC0}) = (w_{DC0} - c) Q_{DC0}(w_{DC0}, b_{DC0}) - (b_{DC0} - s) \int_0^{Q_{DC0}} F(x) dx \tag{E.7}$$

A buy-back contract is completely determined by a 2-tuple (w_{DC0}, b_{DC0}), where w_{DC0} and b_{DC0} are the wholesale price and the buy-back price, respectively. The following theorem (from Pasternack [75]) outlines the coordination conditions of the buy-back contract:

Theorem E.3. *Suppose that the manufacturer offers a contract $(w_{DC0}(\varepsilon), b_{DC0}(\varepsilon))$ for $\varepsilon \in (0, r - c)$ where $w_{DC0}(\varepsilon) = r - \varepsilon$ and $b_{DC0}(\varepsilon) = r - \varepsilon\frac{r-s}{r-c}$:*

a. *The retailer order the optimal solution of the Centralized Scenario and system profit is also equal to the Centralized Scenario profits*

b. *Retailer profit is increasing in ε. Specially $\pi^{R*}_{DC0}(w_{DC0}(\varepsilon), b_{DC0}(\varepsilon)) = \frac{\varepsilon}{r-c}\pi^*_{C0}$*

c. *Manufacturer profit is decreasing in ε. Specially $\pi^{M*}_{DC0}(w_{DC0}(\varepsilon), b_{DC0}(\varepsilon)) = (1 - \frac{\varepsilon}{r-c})\pi^*_{C0}$*

Proof. cf Pasternack [75] □

Note that the parameterε governs the distribution of market power: a high ε implies a strong retailer.

E.3 Proof of Theorem 5.2

Under the Decentralized Uncoordinated scenario, Q_{DU2} is such that:

$$F(\mu_\theta Q_{DU2}) = 1 - \frac{w_{DU2} - s}{r - s}\frac{1}{\mu_\theta}$$

By using the inverse of $Q_{DU2}(w_{DU2})$ which is given by:

$$w_{DU2} = (r_{eq} - s)(1 - F(\mu_\theta Q_{DU2})) + s$$

Where $r_{eq} = r\mu_\theta + (1 - \mu_\theta)s$. The expected profit of the manufacturer is also given by:

$$\begin{aligned}
\pi^M_{DU2}(w_{DU2}) &= (w_{DU2} - c)Q_{DU2}(w_{DU2}) \\
&= [((r_{eq} - s)(1 - F(\mu_\theta Q_{DU2})) - (c - s))(\mu_\theta Q_{DU2})]\frac{1}{\mu_\theta}
\end{aligned}$$

If we consider an IGFR distribution of the demand and the change of variable $Q^{'}_{DU2} = \mu_\theta Q_{DU2}$, the rest of the proof follows directly by using Theorem 1 in Larivière and Porteus [78] (which was described in Appendix E.1).

E.4 Proof of Property 5.2

For the case of a normally distributed demand (which is IGFR) with parameter μ and σ, we fully describe in this appendix the function $H(y) = 1 - F(y) - yf(y)$ used throughout this chapter in order to derive the optimal ordering quantity under the Decentralized Uncoordinated Scenario.
First it is important to show that $H(\mu) \leq 0$. In fact $H(\mu) = \frac{1}{2}(1 - \frac{\mu}{\sigma}\sqrt{\frac{2}{\pi}})$ is negative since demand parameters are such that the coefficient of variation $cv = \frac{\sigma}{\mu} \leq \sqrt{\frac{2}{\pi}} \approx 0.8$.
Let now analyze the sens of variation of $H(y)$. The first derivative of $H(y)$ for a normally distributed demand is given by:

$$H^{'}(y) = \frac{f(y)}{\sigma^2}(y^2 - \mu y - 2\sigma^2)$$

The first derivative is equal to zero for $y_1 = \frac{\mu - \sqrt{\mu^2 + 8\sigma^2}}{2} \leq 0$ and $y_2 = \frac{\mu + \sqrt{\mu^2 + 8\sigma^2}}{2} \geq \mu$. As a consequence we can conclude that $H(y)$ is decreasing in y for $y \in [0, y_2]$ where $y_2 \geq \mu$. In the other hand we can easily verify that $\lim_{y \to +\infty} H(y) = 0^-$. the following figure represents the variation of $H(y)$ with y for $\mu = 10$ and $\sigma = 2$.

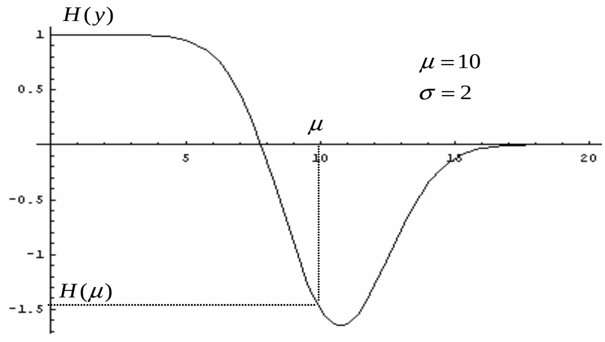

Figure E.1: Variation of $H(y)$ with y

Combining the fact that $H(y)$ is decreasing in y for $y \in [0, y_2]$ where $y_2 \geq \mu$ and the fact that $H(\mu) \leq 0$ enable us to deduce some important results used throughout the chapter. Specially the fact that Q^*_{DU0} which solves $H(Q^*_{DU0}) = \frac{c-s}{r-s}$ (with $\frac{c-s}{r-s} \geq 0$) is such that:

Result E.1. $Q^*_{DU0} \leq \mu$

Two other important results concerning Q^*_{DU2}, which solves $H(\mu_\theta Q^*_{DU2}) = \frac{1}{\mu_\theta}\frac{c-s}{r-s}$ are also deduced:

Result E.2. $\mu_\theta Q^*_{DU2} \leq Q^*_{DU0} \leq \mu$.

Result E.3. $\mu_\theta Q^*_{DU2}$ *decreases as* μ_θ *decreases*

By using the two last results and the fact the $f(x)$ is is increasing in x for $x \leq \mu$, the following results are deduced:

Result E.4. $w^*_{DU2} = c + (r-s)\mu_\theta(\mu_\theta Q^*_{DU2}) f(\mu_\theta Q^*_{DU2})$ *decreases as* μ_θ *decreases*

Result E.5. $\pi^{M*}_{DU2} = (r-s)(\mu_\theta Q^*_{DU2})^2 f(\mu_\theta Q^*_{DU2})$ *decreases as* μ_θ *decreases*

Result E.6. $\pi^{R*}_{DU2} = (r-s)\int_{x=0}^{\mu_\theta Q^*_{DU2}} x f(x) dx$ *decreases as* μ_θ *decreases*

E.5 Proof of Theorem 5.3

Observe that for all ε: $\dfrac{(r-s)\mu_\theta - (w_{DC2} - s)}{(r - b_{DC2})\mu_\theta} = 1 - \dfrac{c-s}{r-s}\dfrac{1}{\mu_\theta}$. The retailer faces the same critical fractile as the Centralized Scenario and thus orders the same amount.
To determine retailer expected profit we have:

$$
\begin{aligned}
\pi^{R*}_{DC2}(w_{DC2}(\varepsilon), b_{DC2}(\varepsilon)) &= (r - b_{DC2}) \int_{x=0}^{\mu_\theta Q^*_{DC2}} x f(x) \\
&= \varepsilon \frac{r-s}{r_{eq} - c} \int_{x=0}^{\mu_\theta Q^*_{C2}} x f(x) \\
&= \frac{\varepsilon}{r_{eq} - c} \pi^*_{C2}
\end{aligned}
$$

Total system profit is equal to π^*_{C2} so, $\pi^{M*}_{DC2}(w_{DC2}(\varepsilon), b_{DC2}(\varepsilon)) = (1 - \frac{\varepsilon}{r_{eq} - c})\pi^*_{C2}$

E.6 A note on the investigation of Gaukler et al. [2]

The purposes of this note are the following ones:

- To provide a simple presentation of the issue discussed in Gaukler et al. [2] (in a slightly different way) (Subsections 1 and 2)
- To discuss a model with perfect information that was not explicitly discussed in the Gaukler et al. [2] paper (Subsection 3)
- To discuss the model with imperfect information (the model explicitly dealt with in the Gaukler et al. [2] paper) (Subsection 4)
- To point out a potential problem with the derivations of the Gaukler et al. [2] results (Subsection 5)
- to suggest a way of avoiding this problem (Subsection 6)

E.6.1 The issue addressed by Gaukler et al. [2]

The goal is to analyze the non efficiency of the replenishment process from the backroom to the shelf in a store. This issue can be illustrated by the following chart:

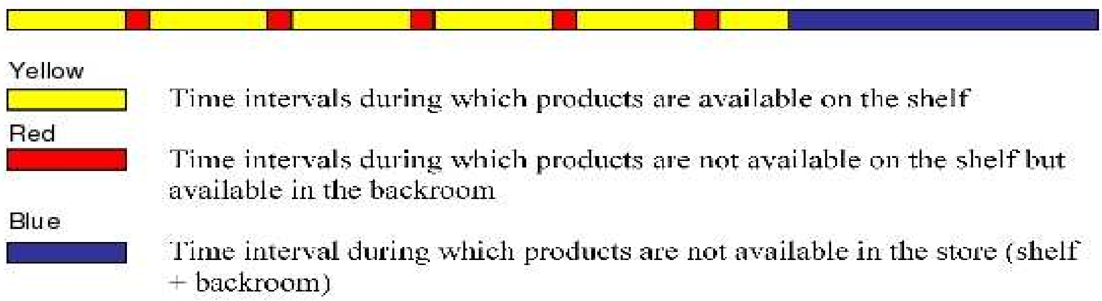

Figure E.2: The error modeling in Gaukler et al. [2]

During the last two time intervals, demands are lost.

E.6.2 Modeling of the issue addressed (done in a slightly different way compared to the Gaukler et al. [2])

In this model, it is assumed that for every unit of demand arriving to the store while there are products in the store, a fraction μ_θ will be satisfied while the remaining fraction $(1 - \mu_\theta)$ is lost. This is a simple

way of modeling the switch-over between the yellow and red time intervals (the first two time intervals) in Figure E.2. Indeed, μ_θ can be interpreted as the ratio of the length of the yellow time interval over the length of the (yellow + red) interval. μ_θ will be referred to as the replenishment efficiency parameter of the store.
The actual overall demand is a random variable given by its density function f, its cumulative probability function F, its mean μ and its standard deviation σ

E.6.3 Model A: Model with perfect information

In this model, it is assumed that the parameters of the demand μ and σ, as well as the replenishment efficiency parameter μ_θ, are known. That is, $\mu_A = \mu$ and $\sigma_A = \sigma$. Let Q_A be the quantity ordered to the supplier.
Define D_c to be the critical value of the demand such that for this value, the Q_A units are sold and there is no blue time interval. D_c is given by :

$$D_c = \frac{Q_A}{\mu_\theta} \tag{E.8}$$

Analysis of Model A: The analysis of Model A can easily be done following the classical analysis of the Newsvendor model. Indeed, two cases need to be considered depending on whether the actual demand is lower or higher than the critical value: $D \leq D_c$ and $D \geq D_c$.

Case 1: $D \leq D_c = \frac{Q_A}{\mu_\theta}$

- Number of units sold: $\mu_\theta D$
- Purchasing cost: cQ_A
- Revenue: $r\mu_\theta D$
- Salvage value: $s(Q_A - \mu_\theta D)$

$$Profit_{Case1} = r\mu_\theta D + s(Q_A - \mu_\theta D) - cQ_A = (r - s)\mu_\theta D - (c - s)Q_A \tag{E.9}$$

Case 2: $D \geq D_c = \frac{Q_A}{\mu_\theta}$

- Number of units sold: Q_A
- Purchasing cost: cQ_A
- Revenue: rQ_A
- Salvage value: 0

$$\Pr ofit_{Case2} = (r - c)Q_A \tag{E.10}$$

Remark E.1. *In both cases, there is a loss profit due to demands not being satisfied; in case 1, this loss profit is $r(1-\mu_\theta)D$; in case 2, this loss profit is $r(D-Q_A)$*

Remark E.2. *In case 2, the lack of efficiency of the replenishment has no impact on the profit. Indeed, all the Q_A units are eventually sold.*

Derivation of the optimal solution: By combining the two situations (case 1 and case 2), the profit of the retailer can be expressed as:

$$Profit_A = rMin(\mu_\theta D, Q_A) + s(Q_A - \mu_\theta D)^+ - cQ_A \tag{E.11}$$

or equivalently:

$$\begin{aligned} Profit_A &= (r-c)\mu_\theta D - (r-c)(\mu_\theta D - Q_A)^+ - (c-s)(Q_A - \mu_\theta D)^+ \\ &= \mu_\theta \left[(r-c)D - (r-c)(D - \frac{Q_A}{\mu_\theta})^+ - (c-s)(\frac{Q_A}{\mu_\theta} - D)^+ \right] \end{aligned} \tag{E.12}$$

As a result, the expected profit can be expressed as:

$$\begin{aligned} \pi_A(Q_A) &= E(Profit_A) \qquad\qquad (E.13) \\ &= \mu_\theta \left[(r-c)\mu - (r-c)\int_{x=\frac{Q_A}{\mu_\theta}}^{+\infty} (x - \frac{Q_A}{\mu_\theta}) f(x)dx - (c-s)\int_{x=0}^{\frac{Q_A}{\mu_\theta}} (\frac{Q_A}{\mu_\theta} - x) f(x)dx \right] \end{aligned}$$

Note that the term within the brackets is similar to that of the classical Newsvendor expression.
The optimal ordering quantity Q_A^* and the corresponding optimal cost $\pi_A^* = \pi_A(Q_A^*)$ are given by:

$$Q_A^* = \mu_\theta F^{-1}\left[\frac{r-c}{r-s}\right] \tag{E.14}$$

$$\pi_A^* = \pi_A(Q_A^*) = \mu_\theta (r-s)\int_0^{\frac{Q_A^*}{\mu_\theta}} xf(x) \tag{E.15}$$

Remark E.3. *Note that $Q_A^* = \mu_\theta Q_0^*$ and $\pi_A^* = \mu_\theta \pi_0^*$ where Q_0^* et π_0^* are respectively the optimal ordering quantity and the optimal expected profit of the basic Newsvendor problem (which corresponds to the case $\mu_\theta = 1$)*

Special case of a normal distribution:

$$Q_A^* = \mu_\theta \left[\mu + \sigma\Phi^{-1}(\frac{r-c}{r-s})\right] \tag{E.16}$$

$$\pi_A^* = \pi_A(Q_A^*) = \mu_\theta \left[(r-c)\mu - (r-s)\sigma\phi(\Phi^{-1}(\frac{r-c}{r-s}))\right] \tag{E.17}$$

where ϕ (resp. Φ) is the PDF (resp. CDF) of the standard normal distribution

Conclusion: The model with perfect information is equivalent to a classical Newsvendor model with an equivalent demand given by $\mu_A^{eq} = \mu_\theta \mu$ and $\sigma_A^{eq} = \mu_\theta \sigma$

E.6.4 Model B: Model with imperfect demand information (Gaukler et al. [2] model)

In this model, the retailer is unaware of the replenishment problem and in particular does not know the parameter μ_θ. What is known is an estimate of the demand where the mean μ_B and standard deviation σ_B are given by:

$$\mu_B = \mu_\theta \mu \tag{E.18}$$

$$\sigma_B = \sqrt{\mu_\theta}\sigma \tag{E.19}$$

Remark E.4. *By "unaware", we mean either that the retailer is actually unaware of the replenishment problem or that he does not have a way to measure its effect through the parameter μ_θ, or that he does want to bother including this issue in the calculation of his ordering quantity to the supplier.*

Since the retailer is unaware of the replenishment problem, he determines the optimal ordering quantity using a classical Newsvendor approach based on the parameters μ_B and σ_B.
The optimal quantity Q_B^* and the corresponding optimal profit of this model follow from the classical results of the Newsvendor model and in particular in the case of normal distribution are given by:

$$Q_B^* = \mu_\theta \mu + \sigma\sqrt{\mu_\theta}\Phi^{-1}(\frac{r-c}{r-s}) \tag{E.20}$$

$$\pi_B(Q_B^*) = (r-c)\mu_\theta\mu - (r-s)\sigma\sqrt{\mu_\theta}\phi(\Phi^{-1}(\frac{r-c}{r-s})) \tag{E.21}$$

Remark E.5. *When comparing the two models, it appears that not knowing the parameter μ_θ induces an increase of the variability of the demand in the corresponding Newsvendor model, going from $\mu_\theta\sigma$ in Model A (model with perfect information) to $\sqrt{\mu_\theta}\sigma$ in Model B (model with imperfect information). As a result, the corresponding optimal ordering quantity increases proportionally to $\sqrt{\mu_\theta}$ with respect to the average demand, i.e.:*

$$Q_B - \mu_\theta\mu = \frac{Q_A - \mu_\theta\mu}{\sqrt{\mu_\theta}} \tag{E.22}$$

E.6.5 Potential problem in Gaukler et al. [2] analysis

In their analysis, the authors use the optimal ordering quantity Q_B^* derived for Model B. This definitely makes sense. However, they also use the corresponding profit function $\pi_B(Q_B^*)$. This, however, does not seem appropriate. Indeed, in the real system, if the quantity Q_B^* is ordered, the actual corresponding expected profit will be $\pi_A(Q_B^*)$, i.e., the profit associated with the "real" model, which is the model with perfect information.
Therefore, in the comparison of the performance of the system with imperfect information (with respect to the RFID system), we think that $\pi_A(Q_B^*)$ should be used instead of $\pi_B(Q_B^*)$.
We performed some experiments. It turns out that the difference between $\pi_B(Q_B^*)$ and $\pi_A(Q_B^*)$ is (for reasonable values of the parameters) not very large. We also note that for reasonable values of the parameter μ_θ, $\pi_B(Q_B^*)$ underestimates the real profit given by $\pi_A(Q_B^*)$.
Therefore, it seems that using $\pi_B(Q_B^*)$ instead of $\pi_A(Q_B^*)$ to perform the comparison with the RFID system, both from a qualitative and from a quantitative point of view, does not cause any major problem. However, from a theoretical point of view, it becomes an issue.

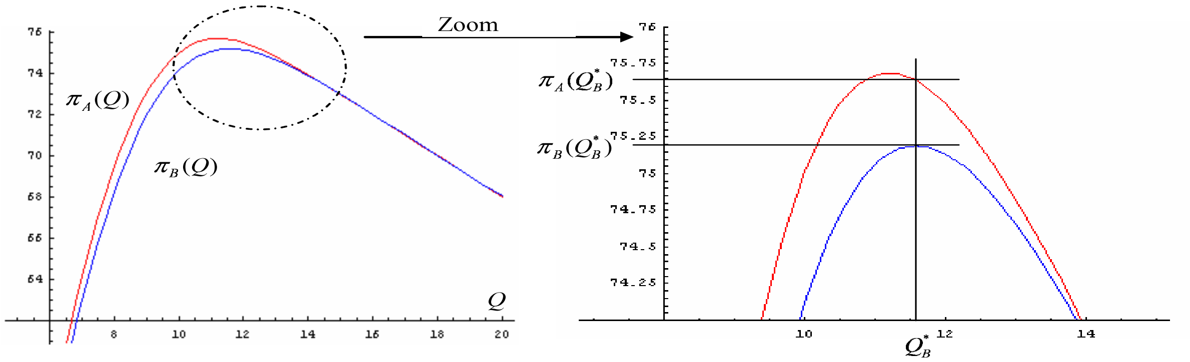

Figure E.3: $\pi_B(Q_B^*)$ vs $\pi_A(Q_B^*)$ for ($\mu = 10$, $\sigma = 3$, $c - s = 1$, $r - c = 15$ and $\mu_\theta = 0.8$)

E.6.6 Potential way of avoiding the problem

One approach to avoid this problem would be to use $\pi_A(Q_B^*)$, instead of $\pi_B(Q_B^*)$, in the comparison with the RFID system. However, this quantity does not have a closed-form expression. Therefore, all the nice theoretical results presented in Gaukler et al. [2] on competition/coordination would no longer be obtainable.

A second approach would be to slightly change the nature of the comparison by using the perfect information model (Model A) instead of the imperfect information model (Model B) in the comparison with the RFID model. Since there is an explicit Newsvendor type solution for Model A, all the analyses, including competition/coordination analysiscould be performed using this model.

E.7 Our investigation versus the investigation of Çamdereli and Swaminathan [3]

First it is important to notice that our results of Chapter 5 and the results of Çamdereli and Swaminathan [3] have been done totally independently of one another. Our results were first publicly published as a technical report dating from July 2005 (we submitted the paper on July 21, 2005 to IIE Transactions), whereas the Çamdereli and Swaminathan [3] paper is dated August 2005. So, there is obviously no anteriority of one work with respect to the other.

The two works are fairly similar in terms of the model considered. However, it appears that our work is more general. We present in this appendix the common points and the differences between the two works.

E.7.1 Common points in the two works

1. Both papers consider a supply chain with tow actors with misplacement type errors in the retail store

2. misplacement errors are modeled by the same way in both investigations

3. A Newsvendor framework is considered

4. Both papers consider 3 cases: Centralized, Decentralized without coordination and Decentralized with coordination

5. Assumptions are the same in both papers

6. Both papers analyze the impact of errors on the performance of the supply chain performance by comparing the error free model and the model with error.

E.7.2 Differences between the two works

1. In contrast to Çamdereli and Swaminathan [3], we consider the impact of an advanced identification system such as RFID technology in a decentralized supply chain subject to misplacement errors. In particular we show that:

    - As a response to the question How should the price for the tags be shared among the supply chain actors, we show that the notion of sharing the RFID tag cost between supply chain actors that has been proposed in industry circles, is a non-issue (under the assumption considered in both works)
    - As a response to the question Which technology price make its deployment cost effective we derive an analytical critical price that make both the manufacturer and the retailer motivated for the deployment of this technology. Such contribution is a major one and response an important question linked to the deployment of the RFID technology
    - As a response to the question is RFID the unique solution to the supply chain actors we show that coordinating the channel can also improve the performance and as a consequence there is no need to deploy RFID technology. The author propose a comparison between two strategies where the first one consists in deploying RFID and the second one deals with coordination and they give insights on the best strategy which should be adopted

2. All these questions and their responses are only considered in our investigation

3. Our analysis derives the optimal quantity to order (and the optimal selling price, the optimal contract prices for the decentralized case) and especially focuses on the optimal profit for each supply chain actor where analytical expressions are provided. The analysis of Çamdereli and Swaminathan [3] provides only the optimal ordering quantity and the optimal contract prices

4. In results concerning the optimal policy and optimal associated profit, we consider a general distribution of demand with the condition that it should be IGFR (has an Increasing General Failure Rate) which is the case of many demand distributions. For some specific results associated with the comparison between the two strategies discussed in Chapter 5, normal distribution was used. The analysis of Çamdereli and Swaminathan [3] considers only uniform distribution

5. Concerning the coordination of the channel, we consider a (modified) buy back contract and the authors in Çamdereli and Swaminathan [3] consider both buy back and revenue sharing contracts.

The use of the buy back contract is sufficient since Jemai et al. [79] show that buy back contract generalizes linear transfer payment contracts.

Appendix F

Appendix of Chapter 6

F.1 Proof of Result 6.4

By using Leibniz Formula[1], the first derivative of $E[A]$ with respect to Q is given by:

$$\begin{aligned}\frac{dE[A]}{dQ} &= -\int_{e=0}^{+\infty}\int_{x_m=Q-e}^{Q} f_m(x_m)g(e)dx_m de \\ &= \int_{e=0}^{+\infty}\left[F_m(Q-e)-F_m(Q)\right]g(e)de \qquad \text{(F.1)}\end{aligned}$$

The first derivative of $C(Q)$ with respect to Q can also be deduced:

$$\frac{dC(Q)}{dQ} = (u_1+h)F_m(Q)-u_1+(u_2+h)\int_{e=0}^{+\infty} g(e)\left[F_m(Q-e)-F_m(Q)\right]de$$

In order to derive the optimal ordering quantity and the corresponding optimal cost, let consider the the function H defined as:

$$H(x) = (u_1+h)F_m(x)+(u_2+h)\int_{e=0}^{+\infty} g(e)\left[F_m(x-e)-F_m(x)\right]de \qquad \text{(F.2)}$$

If the optimal ordering quantity exists, we argue that it should verify $H(Q^*) = u_1$. Our aim is to analyze the behavior of the function $H(x)$ in order to solve the equation $H(x) = u_1$. First, it is important to notice the following two properties of the function H:

- Property 1: $\lim\limits_{x\to+\infty} H(x) = u_1 + h > u_1$
- Property 2: Using Condition 1 defined in 6.4, we have $H(0) \leq u_1$

[1]Leibniz Formula: $\frac{d}{dy}\int_{a_1(y)}^{a_2(y)} h(x,y) = \int_{a_1(y)}^{a_2(y)} \frac{\partial h(x,y)}{\partial y}dx + h\left(a_2(y),y\right)a_2^{'}(y) - h\left(a_1(y),y\right)a_1^{'}(y)$

Developing $H(x)$ leads to the following expression:

$$H(x) = (nG(0) - p)F_m(x) + n \int_{e=0}^{+\infty} F_m(x-e)g(e)de \tag{F.3}$$

where $n = u_1 + p + h > 0$. The first derivative of H is given by:

$$\frac{dH(x)}{dx} = (nG(0) - p)f_m(x) + n \int_{e=0}^{+\infty} f_m(x-e)g(e)de \tag{F.4}$$

For the case where model parameters are such that $nG(0) - p < 0$, analyzing $\frac{dH(x)}{dx}$ for $x = 0$ leads to the following property:

- Property 3: $\frac{dH}{dx}(x = 0) \leq 0$

Since $\int_{e=0}^{+\infty} f_m(x-e)g(e)de \geq 0$ for each $x \geq 0$, one can distinguish two possible behaviors of $H(x)$ based on model parameters:

1. The first variation (Cf Figure F.1) corresponds to the case where $nG(0) - p > 0$. For this case, it is clear that H is monotone increasing for each $x \geq 0$. Using Properties 1 and 2, it is clear that it exists a unique solution for the equation $H(x) = u_1$. It can also be deduced that $C(Q)$ is a convex function for $Q \geq 0$ and there exists a unique optimal ordering quantity Q^* which verifies $H(Q^*) = u_1$ that optimizes $C(Q)$

2. The second variation (Cf Figure F.2) corresponds to the case where $nG(0) - p < 0$. For this case the fact that x_m and e distributions are such that Condition 2 of Result 6.4 is satisfied, one can deduce that it exists a unique critical value x_c of x such that $\frac{dH}{dx}(x_c) = 0$. In such case, by using Properties 2 and 3, we deduce that there exists a unique optimal ordering quantity $Q^* \geq x_c$ that verifies $H(Q^*) = u_1$ which minimizes $C(Q)$

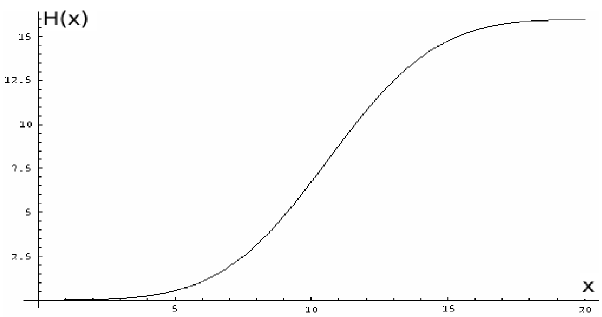

Figure F.1: Behavior 1 of $H(x)$

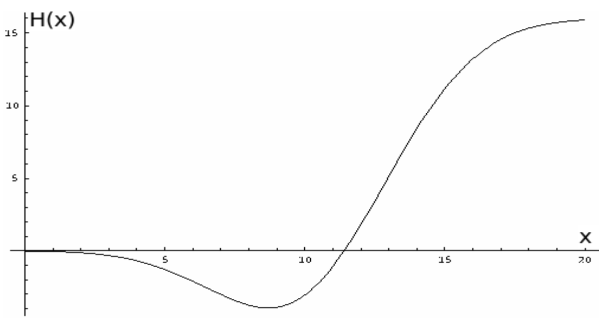

Figure F.2: Behavior 2 of $H(x)$

Appendix G

Appendix of Chapter 7

G.1 Demand Process

We describe in this section the demand process that may generate the two demand streams of the chapter. Our aim is to determine the mean and the standard deviation of each streams of end customer demand (demand for purchase and demand for theft). For this purpose, we assume that customers arrive to the store according to a Poisson process with a rate λ. For each customer there is a probability $(1-\alpha)$ that he is going to purchase and a probability α that is going to steal. As a consequence the customer flow is divided into two Poisson flows: the first one with rate $(1-\alpha)\lambda$ for customers who are going to purchase and the second one with rate $\alpha\lambda$ for the stealer's. Let Define v to be the Poisson process that counts how many customers go into the store looking for the product. Then, over the entire selling period T, the number of customers who come into the store for the product is a Poisson random variable $N(T)$with parameter λ. The total number $N(T)$ is divided into tow Poisson variables $N_p(T)$ with parameter $(1-\alpha)\lambda$ for customers who are going to purchase and $N_t(T)$ with parameter $\alpha\lambda$ for stealer's.

Let q_i be the general discrete random variable that indicates how much customer i is going to buy or to steal if there is enough inventory in the store. We call D_p (D_t) the retailer's demand for purchase (theft) distribution estimate. We assume that the retailer cannot observe lost sales and we make the following further assumptions:

- Customer i either buys or steals q_i, or nothing
- All q_i are iid to a generic random variable q

As a consequence $D_p = \sum\limits_{i=1}^{N_p(T)} q_i$ and $D_t = \sum\limits_{i=1}^{N_t(T)} q_i$ are compound Poisson random variables. D_p and D_t are defined as sums of iid distributed demand random variables. By using the central limit theorem we can deduce that the compound Poisson random variables D_p and D_t can be approximated by the random Normal random variables $N((1-\alpha)\mu, (1-\alpha)\sigma^2)$ and $N(\alpha\mu, \alpha\sigma^2)$respectively where $\mu = \lambda E\left[q\right]$ and $\sigma^2 = \lambda E\left[q^2\right]$.

G.2 The HSL in the classical inventory control

G.2.1 The newsvendor problem

Let recall the following definition:

- r: the unit selling price
- h: the unit overage cost
- c: the unit purchase cost
- F: the CDF of the demand distribution
- f: the PDf of the demand distribution

In a one-period problem, the HSL is simply the classical service level. For a target service level HSL_0, the optimal order-up-to level should verify:

$$Y^* = F^{-1}\left[HSL_0\right] \tag{G.1}$$

For a given initial physical inventory level I, the optimal expected profit pertaining to this policy is given by:

$$\pi^* = r\int_{D=0}^{Y^*} Df(D)dD + rY^*\int_{D=Y^*}^{+\infty} f(D)dD - h\int_{D=0}^{Y^*}(Y^*-D)f(D)dD - c(Y^*-I) \tag{G.2}$$

Simplifying the last equation leads to the following expression of the optimal one-period expected profit:

$$\pi^* = r\left[(1-HSL_0)Y^* + \int_{D=0}^{Y^*} Df(D)dD\right] - h\left[HSL_0Y^* - \int_{D=0}^{Y^*} Df(D)dD\right] - c\left[Y^*-I\right] \tag{G.3}$$

G.2.2 The N-Period problem

For N iid demand distributions and for a given target level HSL_0, the optimal order-up-to level should satisfy:

$$\begin{aligned} HSL_0 &= \prod_{t=1}^{N} P[D_t \leq Y^*] && \text{(G.4)} \\ &= F[Y^*]^N && \text{(G.5)} \end{aligned}$$

As a consequence, Y^* is given by:

$$Y^* = F^{-1}\left[\sqrt[N]{HSL_0}\right] \tag{G.6}$$

Under Formulation 2, it appears that the classical N-period problem is simply N single-period problems where the target service level for each period is $\sqrt[N]{HSL_0}$. The optimal expected profit for the N-Period problem can be approximated by:

$$\pi^* \approx rN \int_{D=0}^{Y^*} Df(D)dD + rNY^* \int_{D=Y^*}^{+\infty} f(D)dD - hN \int_{D=0}^{Y^*} (Y^* - D)f(D)dD - cN\mu \tag{G.7}$$

The last function is composed of four parts: $i)$ the first and the second one express the revenue pertaining to sales, $ii)$ the third one expresses the overage cost and $iii)$ the last one expresses the purchase coost.

The optimal expected profit can be written as the following:

$$\begin{aligned} \pi^* \;\; &\approx \;\; rN \left[(1 - \sqrt[N]{HSL_0})Y^* + \int_{D=0}^{Y^*} Df(D)dD\right] \\ &- \;\; hN \left[\sqrt[N]{HSL_0}Y^* - \int_{D=0}^{Y^*} Df(D)dD\right] - cN\mu \end{aligned} \tag{G.8}$$

Remark G.1. *The last expressions are approximations of the N-period expected profit since we assume that the system falls rarely in a shortage situation. A high value of the target level HSL_0 ensures that the last approximations are good ones.*

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Résumé
Contrairement à un système d’identification plus traditionnel tel que le code à barres, la nouvelle technologie RFID (Radio Frequency IDentification) utilise des ondes radio fréquence pour transmettre des données entre une étiquette et un lecteur pour pouvoir identifier, localiser ou suivre une entité dans une chaîne d’approvisionnement. Cette propriété lui procure certains avantages (facilité d’accès à l’information, suivi continu, amélioration de l’exactitude des données, détection du vol et de la contrefaçon, etc..) par rapport à d’autres systèmes d’identification et de capture de données. Nous partons du constat que l’utilisation de cette nouvelle technologie permettra aux acteurs de la chaîne logistique de pouvoir partager une information de meilleure qualité, plus exhaustive et fiable concernant le flux physique et le suivi de la localisation produits. Or, l’hypothèse implicite considérée dans la plupart des modèles classiques de gestion de stock est que l’on a une connaissance parfaite du flux entrant et sortant. L’objectif de recherche sera d’intégrer dans ces modèles des dégradations venant fausser le flux nominal et d’en analyser les conséquences (en termes de coût additionnel). Un accent fort sera mis sur le développement de solutions combinant efficacité et simplicité. L’accent sera mis aussi sur le mode de partage du coût de cette technologie entre plusieurs acteurs de la chaîne logistique : serait-il mieux de partager les bénéfices de cette technologie dans un environnement de coordination ou dans un environnement de compétitivité entre acteurs?

Les résultas de cette thèse porte sur l’élaboration de modèles théoriques -de type gestion de stock – concernant la production, la distribution et l’approvisionnement dans une chaîne logistique et faisant intervenir et le coût et les gains potentiels de cette nouvelle technologie d’identification automatique.

Abstract :
The RFID technology is considered by a large number of people as a breakthrough in product identification and data capture throughout the supply chains. While the development of the different parts needed in order to implement the RFID systems (hardware, software, standardization,...) is under way, it is also important to evaluate the impact of such technology on the performance of supply chains. Benefits of the RFID technology, in terms of cost reduction, are various and among them, the elimination of inventory record errors that are currently encountered in many supply chains may be considerable. The purpose of our research project is to develop a set of models that provide qualitative and quantitative insights on the benefits of the RFID technology on the performance of supply chains in terms of cost reduction and/or improvement of service levels. The starting point of our research is a real world observation: more and more companies are looking for other identification systems than the bar code technology enabling a more accurate tracking of products in supply chains. Most of them are also interested in evaluating the benefits of an advanced automatic identification system such as the RFID technology. They are interested in new functionalities associated with this technology in order to compare it with the performance enabled by the bar code technology. Our second observation concerns the inventory information system which is a major obstacle to achieving operational excellence. In fact, the inventory information system, in contrary to popular belief and assumptions in most academic papers and in spite of the considerable amounts invested in information technology, are often inaccurate.

Our aim is to quantify the penalty resulting from the inventory inaccuracy issue and to analyze one of the improvements stemming from the deployment of the RFID technology, namely the benefit of having accurate inventory data.