Thermodynamique des gaz de fermions ultrafroids

Thèse de doctorat de l'Université Pierre et Marie Curie - Paris VI

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sous la direction de Christophe Salomon & Frédéric Chevy

11 juin 2010

Outline

1. Introduction: Universal Thermodynamics of a Fermi Gas
2. Methods
3. Equation of State of a Strongly-Interacting Fermi Gas
4. Superfluid Equation of State for Arbitrary Interaction Strength
5. Spin-Imbalanced Fermi Gases

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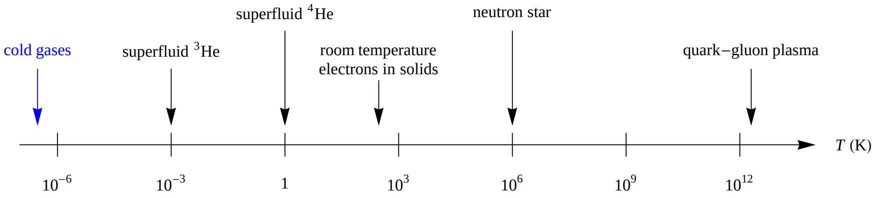
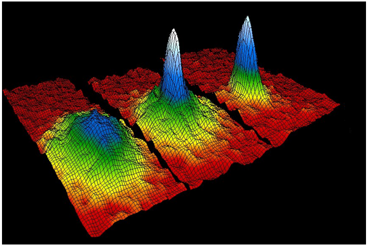
Several quantum degenerate systems





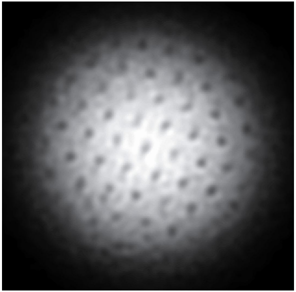
Unconventional parameter scales:

- density $n \sim 10^{13}$ to 10^{15} atoms/cm^3
- temperature $T \sim 1\ \mu$K

Strong connections between these systems despite the difference in orders of magnitude

Superfluidity in Ultracold Fermi Gases

Different kinds of superfluidity are observed as a function of the strength of interactions.



M. Zwierlein *et al*, Nature **435**, 1047 (2005)

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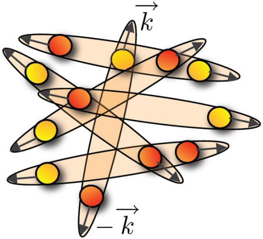
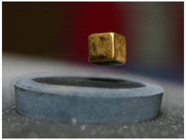
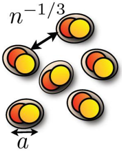
weakly attractive interactions
BCS superfluidity



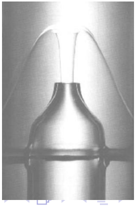
weakly-bound Cooper pairs



strongly attractive interactions
molecular BEC



deeply-bound molecules



Universal Thermodynamics of a Fermi Gas

Equation of state in the grand-canonical ensemble:

$$\Omega(V, \mu, T) = -P(\mu, T)V = E - TS - \mu N$$

Here we directly measure $P(\mu, T)$.

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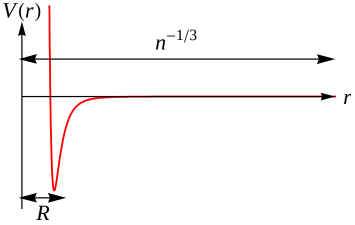
Simple description of short-range interactions

- interparticle distance $n^{-1/3} \gtrsim 100$ nm
- size of the inter-atomic potential $R \sim 1$ nm



Universal Thermodynamics of a Fermi Gas (2)

The inter-atomic potential can be replaced by

$$V(\mathbf{r}) = \frac{4\pi\hbar^2 a}{m}\delta(\mathbf{r}),$$

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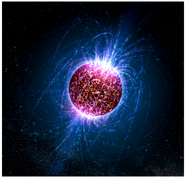
Other ('natural') universal Fermi gas: superfluid neutron matter

$$a = -18.6 \text{ fm}, \quad R = 2.7 \text{ fm} \ll a$$

Universal regime at low density $n \ll R^{-3}$
Simulation of neutron matter in the laboratory!



BEC-BCS Crossover in Ultracold Fermi Gases

Our system: a mixture of ^{6}Li atoms in the two lowest internal states

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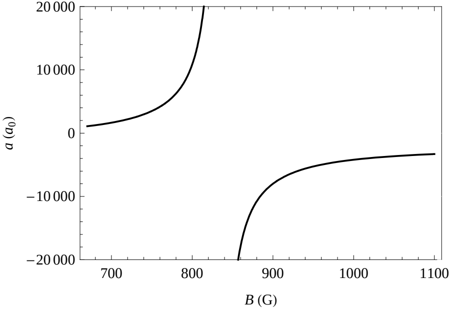
Feshbach resonance for ^{6}Li



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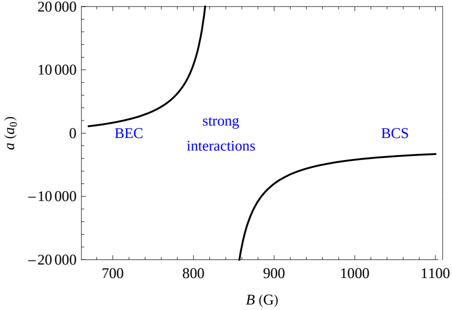

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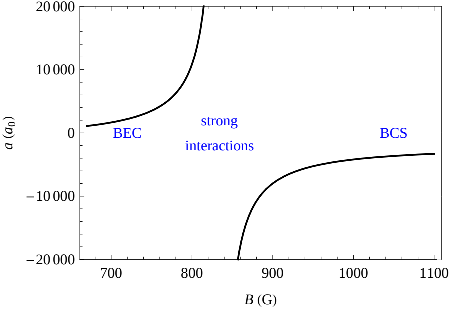

Motivations:

- Understanding a model superfluid system in all aspects

  A. J. Leggett, J. Phys. **C7**, 19 (1980)

  P. Nozieres, S. Schmitt-Rink, J. Low Temp. Phys. **59**, 195 (1985)

  $>$ 300 theoretical papers since the 80's

- Benchmark for many-body theories
- Quantum simulation of neutron matter

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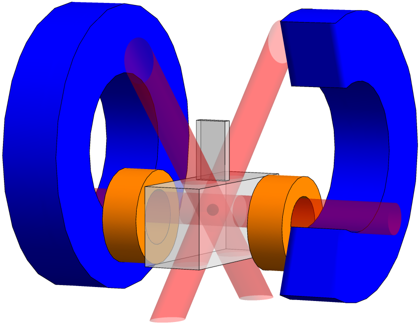
Experimental Setup

^{6}Li-^{7}Li experiment

- ^{7}Li-^{6}Li magneto-optical trap



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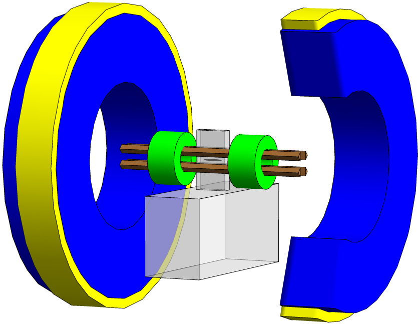

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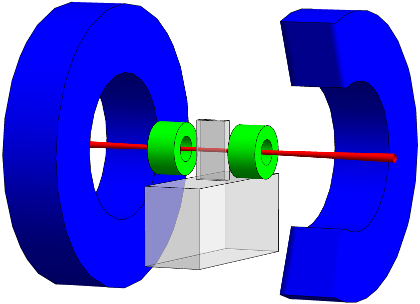

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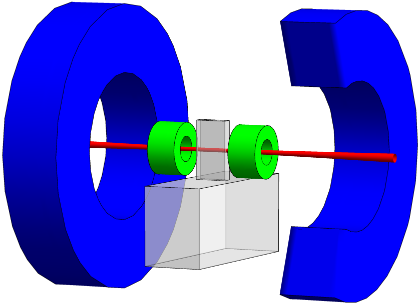

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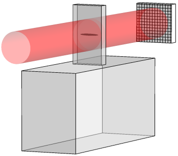

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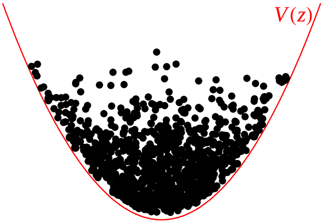
Performances of our setup:

- 10^5 atoms in each spin state at $T \simeq 0.03\, T_F$
- repetition rate: 1 per minute

Measuring the Equation of State of an Ultracold Gas

How to compare many-body theories for homogeneous gases to trapped gases in the laboratory?



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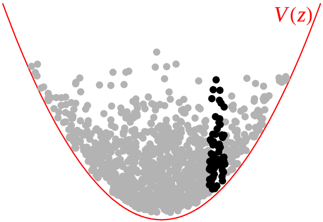

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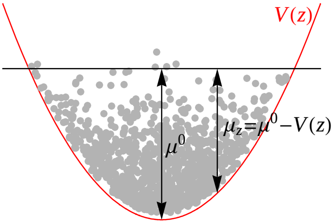

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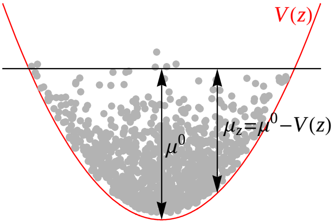
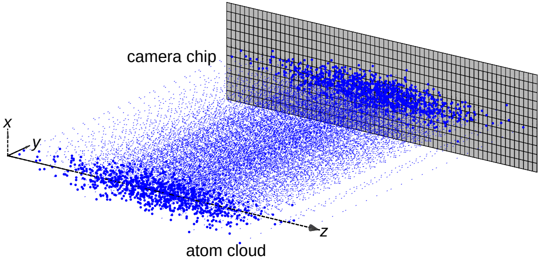


Measuring $P(\mu, T)$ requires to

- **locally** probe the pressure,
- measure the temperature.

Experimental Challenge (1): Probing the Local Pressure

Issue: *in situ* absorption imaging integrates the density over y.



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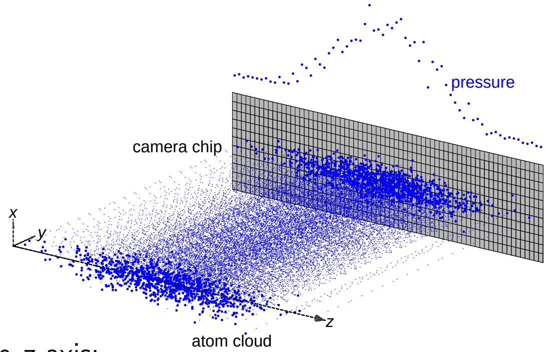


Pressure on the z axis:

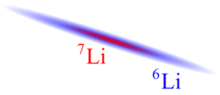
$$P(\mu_z, T) = \frac{m\omega_r^2}{2\pi}\overline{n}(z), \quad \text{where} \quad \overline{n}(z) = \int \mathrm{d}x\,\mathrm{d}y\, n(x,y,z)$$

J. Fuchs *et al*, Phys. Rev. A **68**, 043610 (2003), T. Ho & Q. Zhou, Nature Phys. **6**, 131 (2009)

S. N., N. Navon, K.-J. Jiang, F. Chevy & C. Salomon, Nature **463**, 1057 (2010)

Experimental Challenge (2): Thermometry

Our thermometer: ^{7}Li atoms at thermal equilibrium with ^{6}Li



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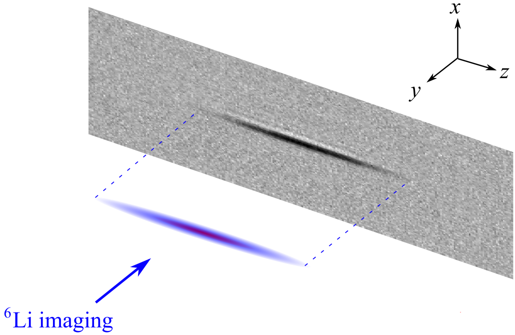

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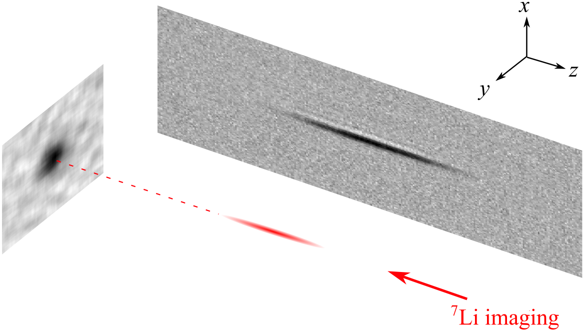

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Fermi Gases with Resonant Interactions: State of the Art

$a = \infty$: 'unitary limit'

universal scaling: E_F and $k_B T$ only energy scales in the problem

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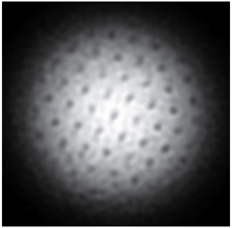

Low-temperature physics:

- Equation of state $\mu = \xi_s E_F$, $\xi_s = 0.41$ (Fixed Node Monte Carlo simulations)

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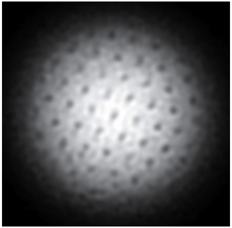

Finite temperature:

- Analytic treatment extremely hard
- 'Sign problem' for Monte Carlo simulation
- Equation of state averaged over the trap

J. Stewart *et al*, Phys. Rev. Lett. **97**, 220406 (2006)

L. Luo *et al*, Phys. Rev. Lett. **98**, 080402 (2007)

Equation of state of a Fermi Gas at Unitarity

We express it as

$$P(\mu, T, a = \infty) = P_0(\mu, T) h_T \left(-\mu / k_B T\right),$$

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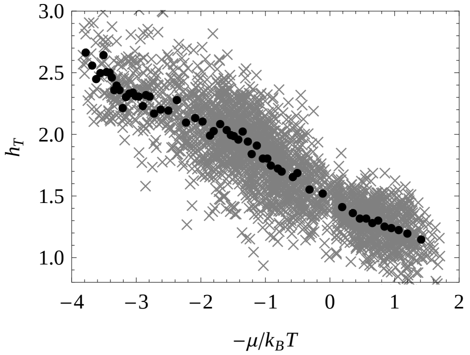

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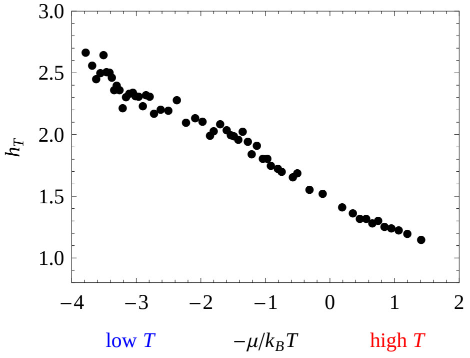

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statistical noise $< 5\%$

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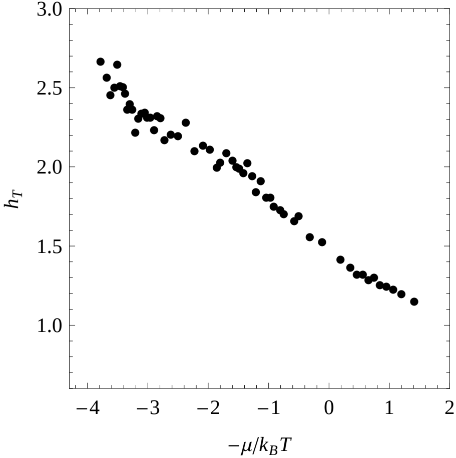
Direct Comparison with Many-Body Theories



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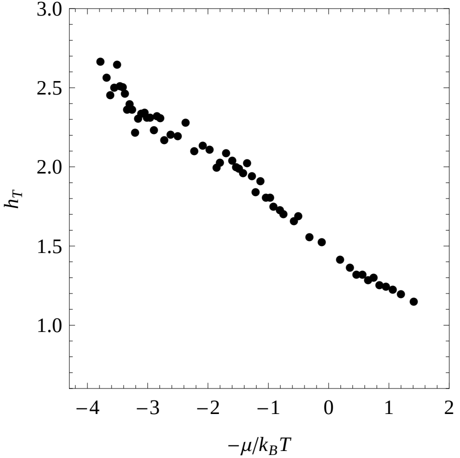

Mean-field theory

Numerical simulations

Semi-analytical theories

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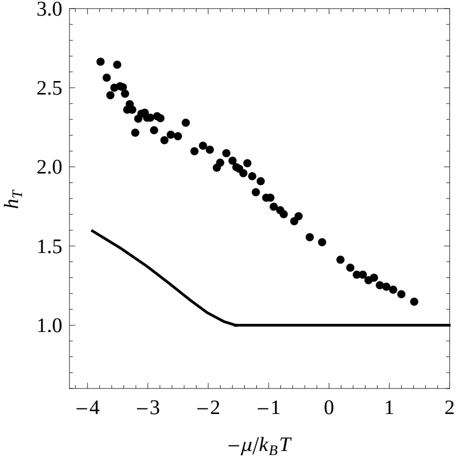

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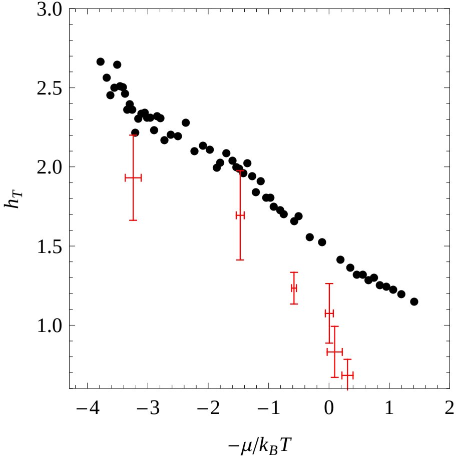

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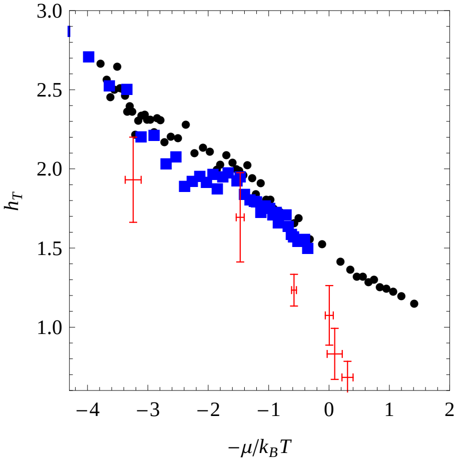

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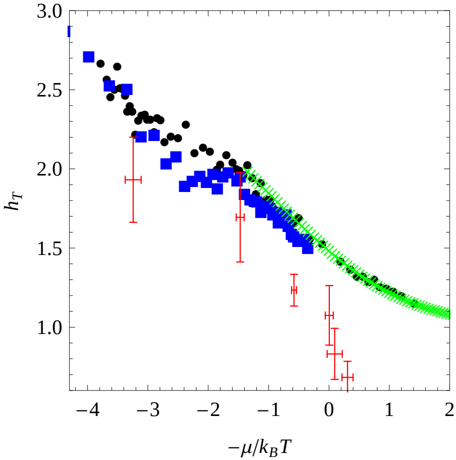

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K. Van Houcke *et al*, to be published

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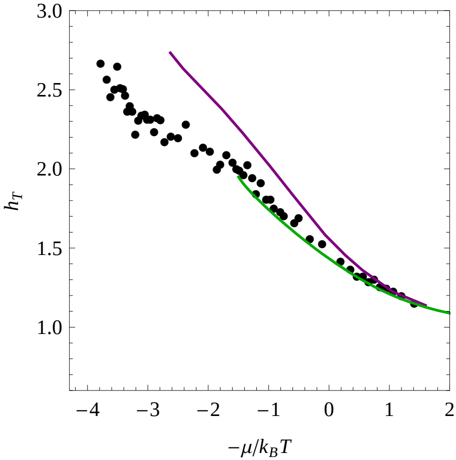

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Semi-analytical theories

—: *GG* perturbation theory

R. Haussmann *et al*, Phys. Rev. A **75**, 023610 (2007)

—: Ladder diagrams approx.

R. Combescot *et al*, Phys. Rev. A **79**, 053640 (2009)

Virial Expansion

High temperature virial expansion: fugacity $z = e^{\mu/k_B T} \to 0$

$$P(\mu, T) = \frac{2k_B T}{\lambda_{dB}^3(T)} \left(z + b_2 z^2 + b_3 z^3 + b_4 z^4 + \ldots\right),$$

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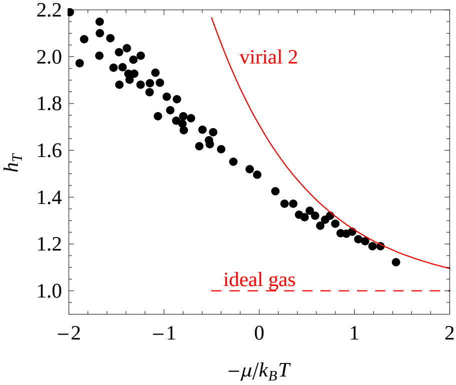

$b_2 = 3/4\sqrt{2}$: used for calibration

T. Ho *et al*, Phys. Rev. Lett. **92**, 160404 (2004)

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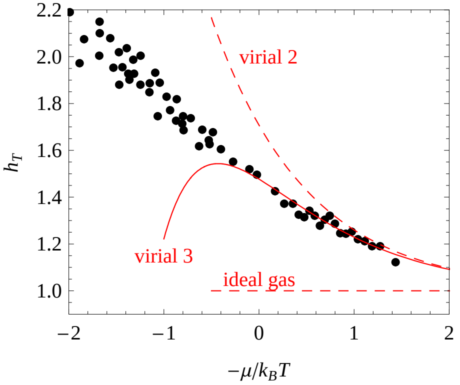

Two predictions for b_3:

- $b_3 = -0.291$

  F. Werner *et al*, Phys. Rev. Lett. **97**, 150401 (2006)

  X. Liu *et al*, Phys. Rev. Lett. **102**, 160401 (2009)

- $b_3 = 1.11$

  G. Rupak, Phys. Rev. Lett. **98**, 090403 (2007)

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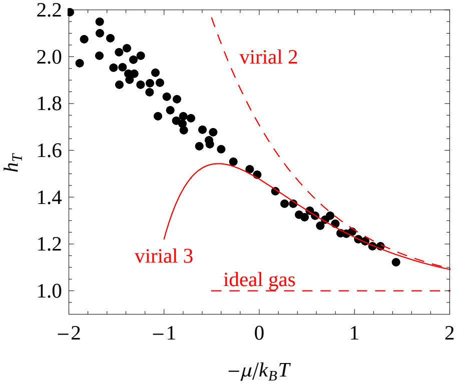


Two predictions for b_3:

- $b_3 = -0.291$

  F. Werner *et al*, Phys. Rev. Lett. **97**, 150401 (2006)

  X. Liu *et al*, Phys. Rev. Lett. **102**, 160401 (2009)

- $b_3 = 1.11$

  G. Rupak, Phys. Rev. Lett. **98**, 090403 (2007)

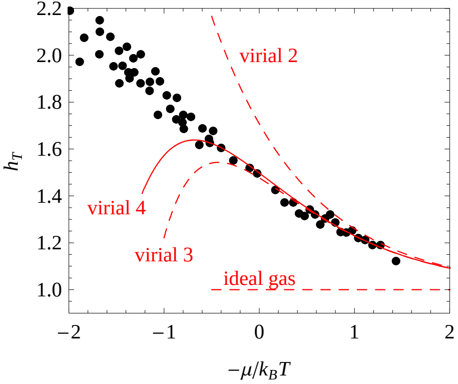
We obtain $b_3 = -0.29(2)$

Virial Expansion

High temperature virial expansion: fugacity $z = e^{\mu/k_B T} \to 0$

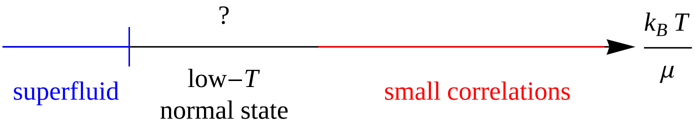
$$P(\mu, T) = \frac{2k_B T}{\lambda_{dB}^3(T)} \left(z + b_2 z^2 + b_3 z^3 + b_4 z^4 + \ldots\right),$$

b_k is calculated from spectrum of the k-body problem.

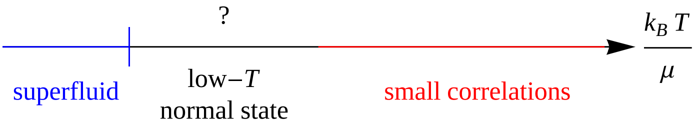


$b_4 = 0.05(1)$: benchmark for the resolution of the 4-body problem

Low-temperature regime

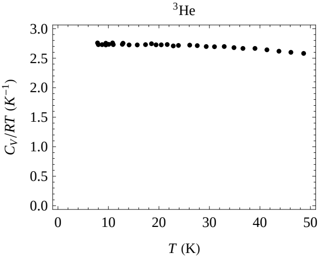


Low-temperature regime



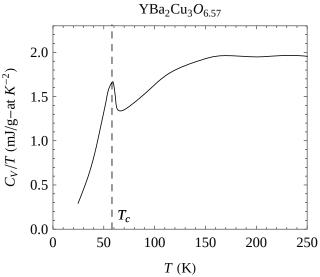
Fermi liquids (most metals, ^{3}He)

Pseudogap phase (high-T_c materials)



D. Greywall, Phys. Rev. B **27**, 2747 (1983)

$$C_v \underset{T\to 0}{\propto} T$$



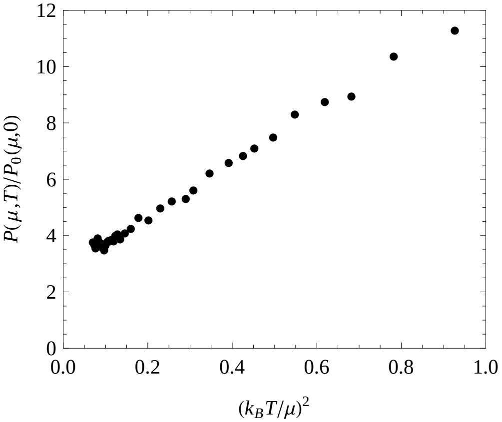
J. Loram *et al*, Phys. Rev. Lett. **71**, 1740 (1993)

$$C_v \underset{T\to 0}{\not\propto} T$$

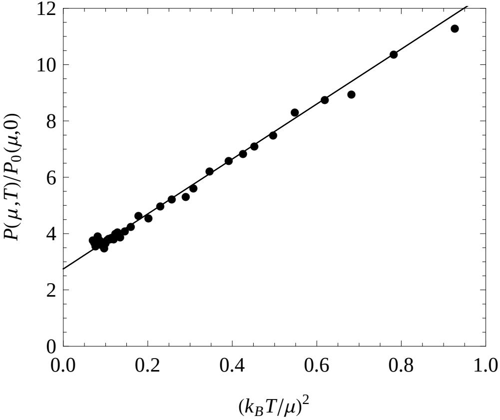
We plot the data as

$$\frac{P(\mu, T)}{P_0(\mu, 0)} \quad \text{vs} \quad \left(\frac{k_B T}{\mu}\right)^2$$

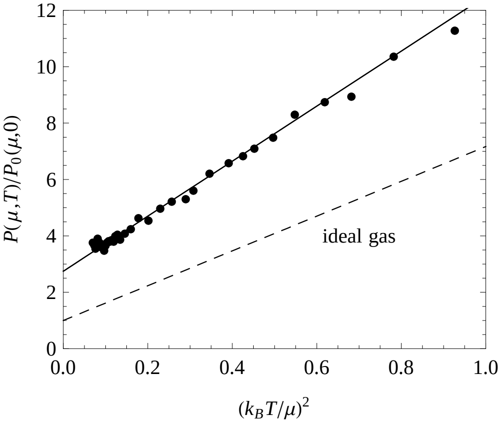
Fermi Liquid Behavior



Fermi Liquid Behavior



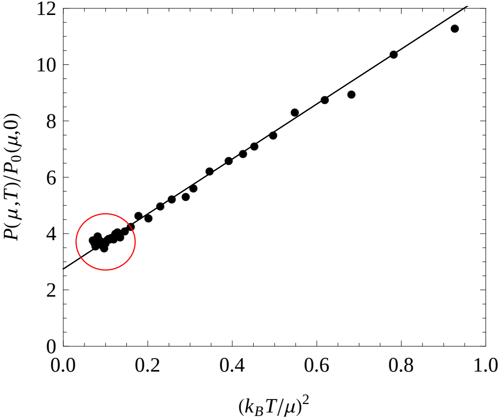
equivalent to $C_v \underset{T\to 0}{\propto} T$



Fermi liquid parameters:

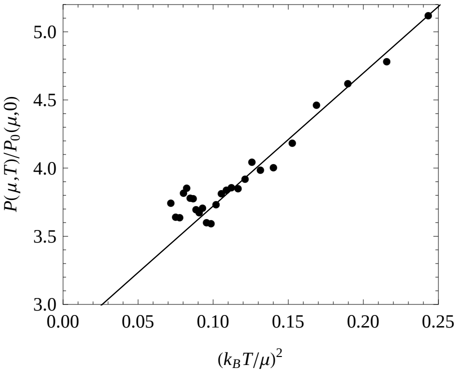
- Low-T equation of state: $\mu = \xi_n E_F$, $\xi_n = 0.51(2)$
  Monte Carlo simulation: $\xi_n = 0.56$ C. Lobo *et al*, Phys. Rev. Lett. **97**, 200403 (2006)
- Effective mass of quasi-particles: $m^* = 1.13(3)m$

Superfluid Transition



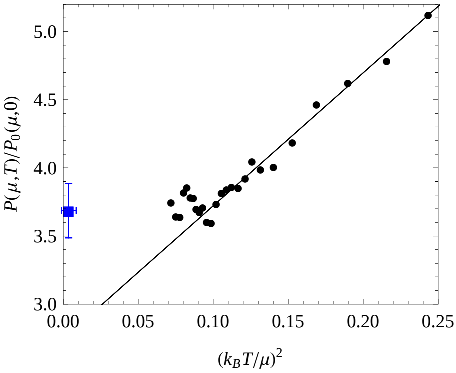
Superfluid transition?

Superfluid Transition



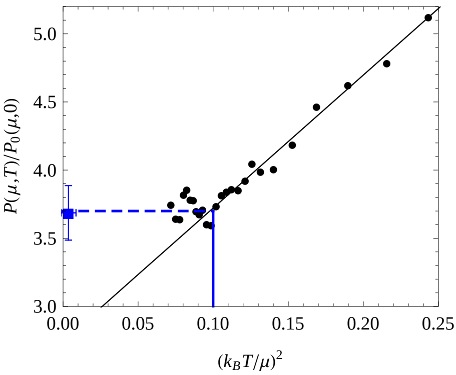
Small deviation for $(k_B T/\mu)^2 < 0.1$

Superfluid Transition



■: independent measurement at the lowest temperature

Superfluid Transition

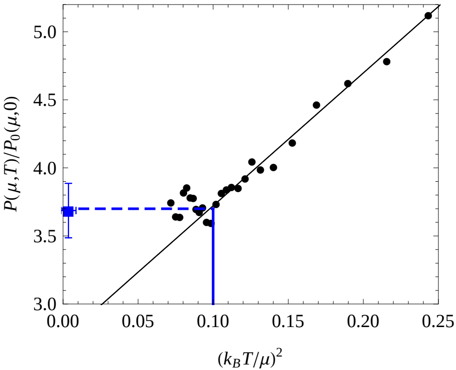


We obtain the superfluid transition temperature:

$$\left(\frac{k_B T}{\mu}\right)_c = 0.32(3)$$

'High-T_c' system

Superfluid Transition



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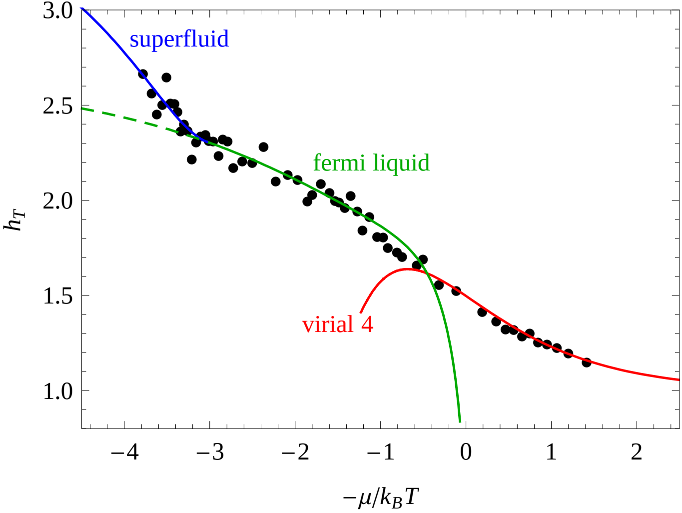
'High-T_c' system

Comparison with many-body theories:

- diagrammatic Monte Carlo, E. Burovski *et al*, Phys. Rev. Lett. **96**, 160402 (2006): 0.32(2)
- quantum Monte Carlo, A. Bulgac *et al*, Phys. Rev. A **78**, 023625 (2008): 0.35(3)
- renormalization group, K. Gubbels *et al*, Phys. Rev. Lett. **100**, 140407 (2008): 0.41

Summary



superfluid
fermi liquid
virial 4
h_T
$-\mu/k_BT$
3.0
2.5
2.0
1.5
1.0
−4
−3
−2
−1
0
1
2

Outline

Superfluid Equation of State

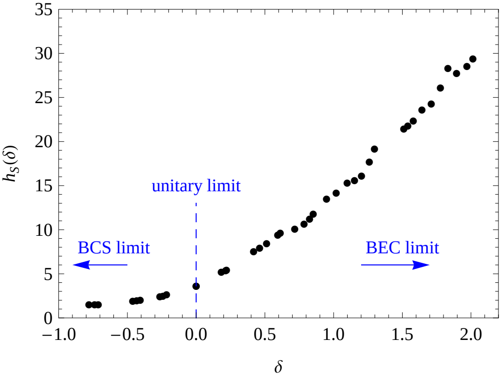
Using the same method we measure the pressure

- at the lowest temperatures $k_B T \sim 0.03\,\mu$
- for different scattering length values a

Superfluid Equation of State

We obtain the superfluid equation of state in the BEC-BCS crossover

$$P(\mu, T = 0, a) = P_0(\mu) h_S \left(\delta = \frac{\hbar}{\sqrt{2m\mu a}} \right)$$



N. Navon, S. N., F. Chevy & C. Salomon, Science **328**, 729 (2010)

Physical Content of our Data

Unitary limit

BCS limit

Physical Content of our Data

Unitary limit

$$E = \frac{3}{5} N E_F \left(\xi_s - \zeta \frac{1}{k_F a} + \dots \right)$$

$\xi_s = 0.41(1)$ determines the $T = 0$ thermodynamics
$\zeta = 0.93(5)$ related to short-distance pair correlations S. Tan, Ann. Phys **323**, 2971 (2008)

- from dynamic structure factor: $\zeta = 0.92(3)$ H. Hu *et al*, arXiv:1001.3200
- experiments in JILA: $\zeta = 0.91(5),\ 0.67(7),\ 0.63(7)$ J. Stewart *et al*, arXiv:1002.1987
- Monte Carlo simulations: $\zeta = 0.95$ C. Lobo *et al*, Phys. Rev. Lett. **97**, 100405 (2006)

BCS limit

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BCS limit

$$E = \underbrace{\frac{3}{5} N E_F}_{\text{ideal gas}} \left(1 + \underbrace{\frac{10}{9\pi} k_F a}_{\text{mean field}} + \underbrace{c_{\mathrm{LY}} (k_F a)^2}_{\text{Lee}-\text{Yang correction}} + \dots \right)$$

theory: $c_{\mathrm{LY}} = \frac{4(11 - 2\log 2)}{21\pi^2} \simeq 0.186$ T. Lee & C. Yang, Phys. Rev. **105**, 1119 (1957)
our value: $c_{\mathrm{LY}} = 0.18(2)$

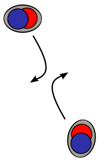
Beyond-Mean Field Interactions in a Bosonic Superfluid

Equation of state of a molecular Bose-Einstein condensate:

$$E = \underbrace{\frac{N}{2}E_b}_{\text{binding energy}} + \underbrace{\frac{N\pi\hbar^2 a_{dd}}{2m}n}_{\text{mean field}}\left(1 + \underbrace{c_{\text{LHY}}\sqrt{n a_{dd}^3}}_{\text{LHY correction}} + \dots\right)$$

dimer-dimer interactions: $a_{dd} = 0.60a$

D. Petrov *et al*, Phys. Rev. Lett. **93**, 090404 (2004), I.V. Brodsky *et al*, Phys. Rev. A **73**, 032724 (2006)



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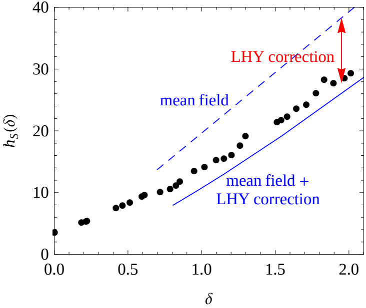
theory: $c_{\text{LHY}} = \frac{128}{15\sqrt{\pi}} \simeq 4.81$

T. Lee, K. Huang & C. Yang, Phys. Rev. **106**, 1135 (1957), X. Leyronas & R. Combescot, Phys. Rev. Lett. **99**, 170402 (2007)



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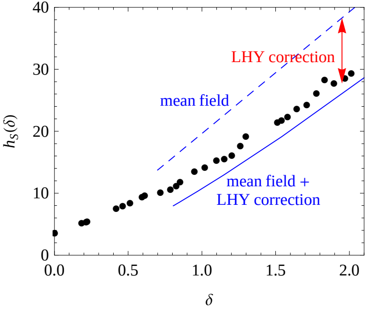
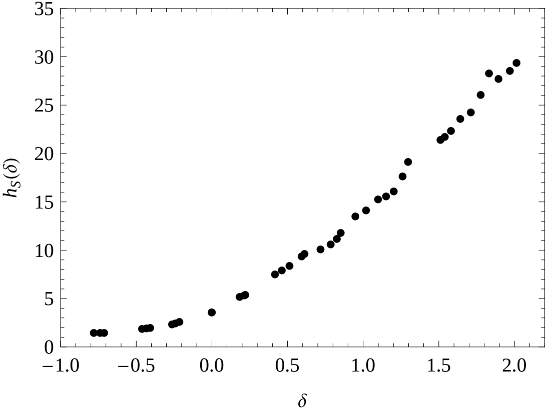


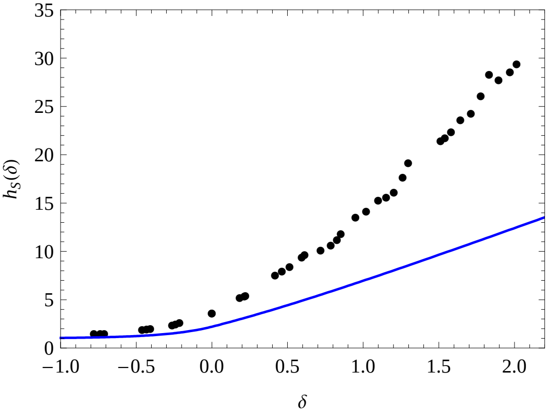
We obtain

$$c_{\mathrm{LHY}} = 4.4(5)$$

Validating Fixed-Node Monte Carlo Simulations

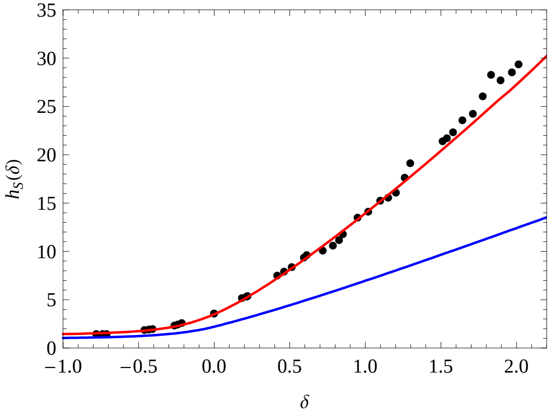


Validating Fixed-Node Monte Carlo Simulations



—: BCS mean-field theory

Validating Fixed-Node Monte Carlo Simulations



—: BCS mean-field theory

—: Fixed-node Monte Carlo simulations

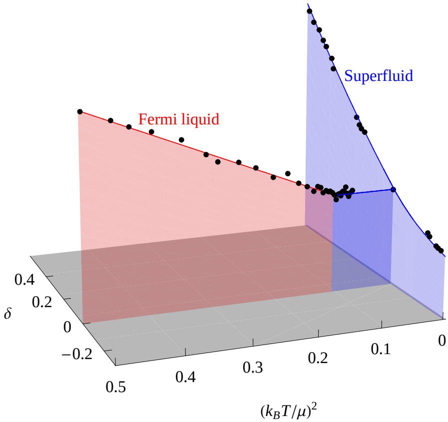
S. Chang *et al*, Phys. Rev. A **70**, 043602 (2004)

G. Astrakharchik *et al*, Phys. Rev. Lett. **93**, 200404 (2004)

S. Pilati *et al*, Phys. Rev. Lett. **100**, 030401 (2008)

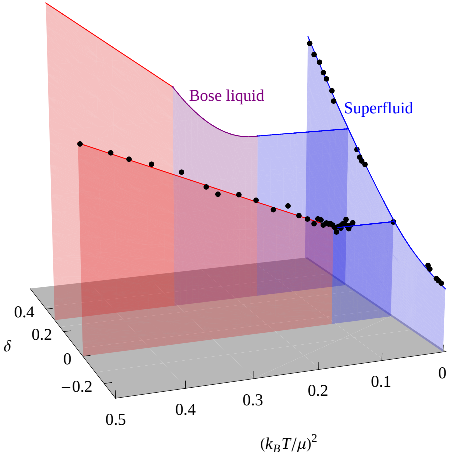
Conclusion (1)

Low-temperature behaviors observed in this thesis: superfluid & Fermi liquid



Conclusion (1)

Low-temperature behaviors observed in this thesis: superfluid & Fermi liquid
On the BEC side of the resonance: Bose liquid ($T_c < T < E_b/k_B$)

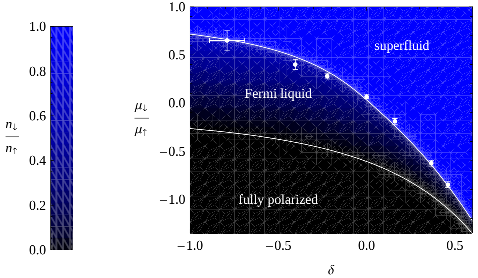


Conclusion (2): Other Applications of the Method

- Generalization to multi-component gases

Spin-imbalanced Fermi gases

$$P(\mu_\uparrow, \mu_\downarrow, T=0, a)$$



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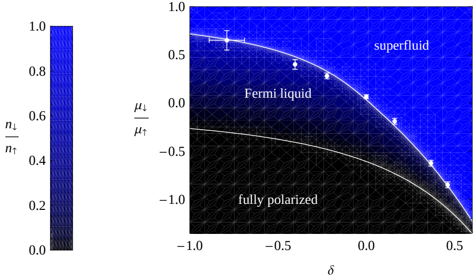
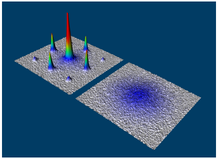


- Application to ultracold gases in optical lattices: simple thermodynamic signature of a Mott insulator behavior

Bosonic Mott insulator

M. Greiner *et al*, Nature **415**, 39 (2002)



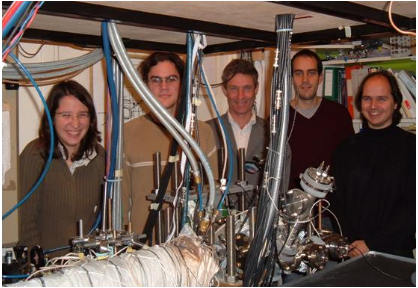
Fermionic Mott insulator

U. Schneider *et al*, Science **322**, 1520 (2008)

R. Jördens *et al*, Nature **455**, 204 (2008)

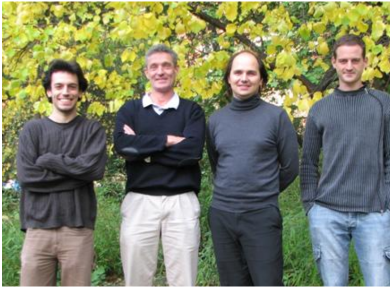
Remerciements

Merci à toute l'équipe Lithium,



Grainne

Kai-Jun



Leticia Martin Jason Nir Christophe Frédéric

aux autres équipes du groupe atomes froids,

aux théoriciens du département et théoriciens étrangers,

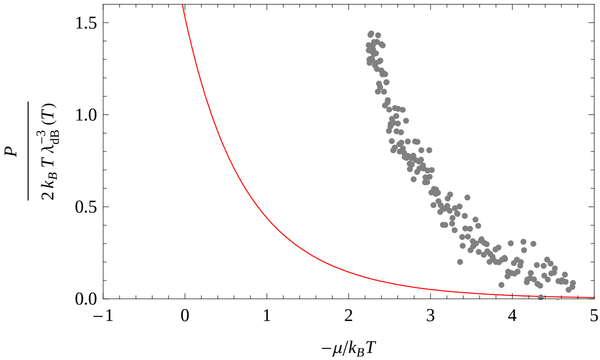
et aux membres du secrétariat, de la bibliothèque, des ateliers, des services généraux. . .

Determination of μ^0

- Determination of μ^0 for high-temperature clouds

Determination of μ^0

- Determination of μ^0 for high-temperature clouds

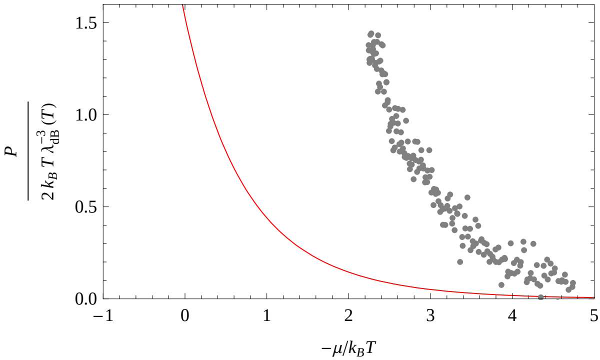


Reference (——): classical gas

$$P(\mu, T) = \frac{2k_B T}{\lambda_{dB}^3(T)} e^{\mu/k_B T}$$

Determination of μ^0

- Determination of μ^0 for high-temperature clouds



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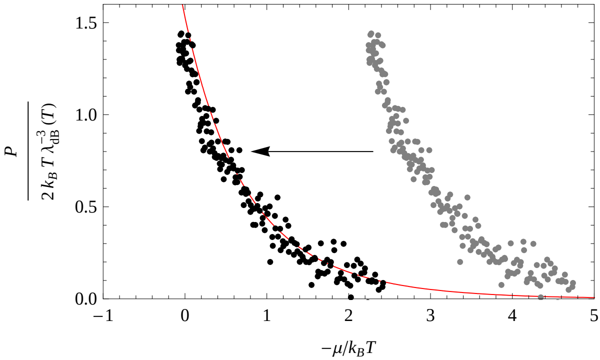

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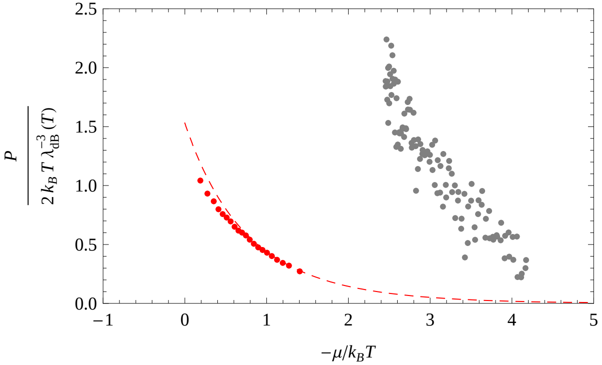

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Determination of μ^0

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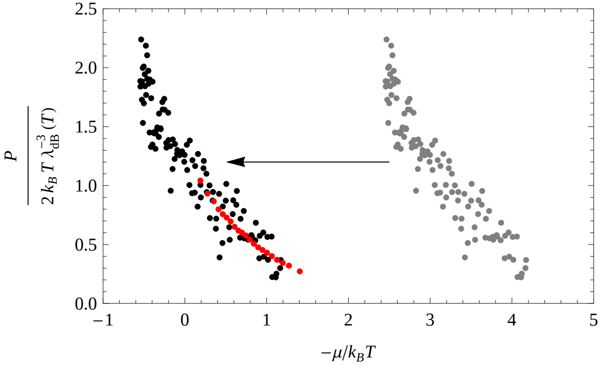
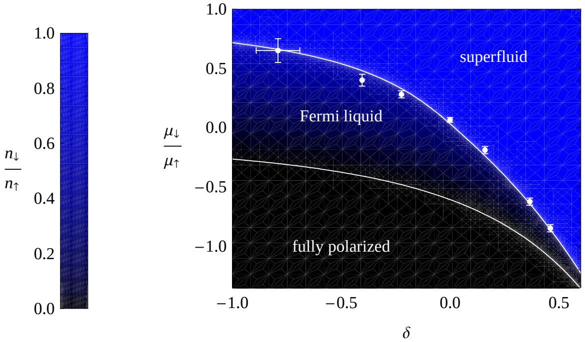


Reference (•): EoS determined from hotter clouds

Spin-Imbalanced Fermi Gases

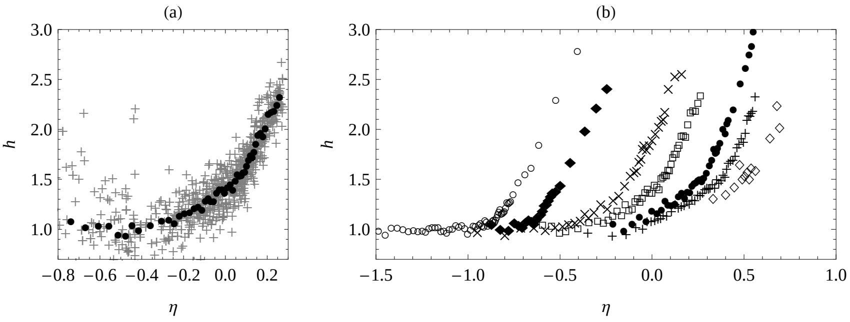
We addressed the physics of spin-imbalanced Fermi gases: $N_\uparrow - N_\downarrow$.



- Superfluid phase unpolarizable up to a 'critical field' $(\mu_\uparrow - \mu_\downarrow)_c$
- Partially polarized phase: Fermi liquid
- Impurity problem $n_\downarrow \ll n_\uparrow$: static and dynamic properties

S. N., N. Navon, K.-J. Jiang, L. Tarruell, M. Teichmann, J. McKeever, F. Chevy & C. Salomon, PRL **103**, 170402 (2009)

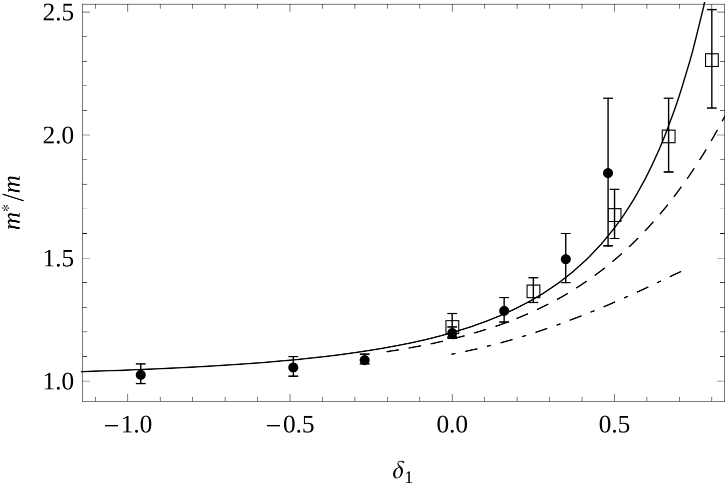
Equation of State of a Spin-Imbalanced Fermi Gas



(a): unitary limit

(b): in the BEC-BCS crossover

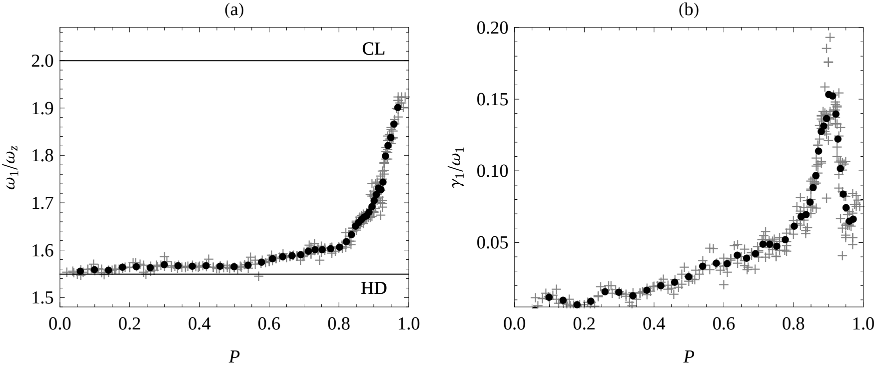
Polaron Effective Mass



In-Phase Axial Breathing Mode

ω_z: axial trapping frequency
$P = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$: spin polarization

$$R(t) = R^0 \left(1 + A_1 e^{-\gamma_1 t} \cos(\omega_1 t + \phi_1)\right)$$



Out-of-Phase Axial Breathing Mode

ω_z: axial trapping frequency
$P = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$: spin polarization