Preference Elicitation for Aggregation Models based on Reference Points : Algorithms and Procedures

Jun Zheng

ÉCOLE CENTRALE DES ARTS
ET MANUFACTURES
« ÉCOLE CENTRALE PARIS »

THÈSE

présentée par

Jun ZHENG

pour l'obtention du

GRADE DE DOCTEUR

Spécialité : Génie Industriel

Laboratoire d'accueil : Laboratoire Génie Industriel

SUJET : Elicitation des Préférences pour des Modèles d'Agrégation basés sur des Points de Référence: Algorithmes et Procédures

soutenue le 14 juin, 2012

devant un jury composé de :

Ahti Salo	Professeur, Aalto University	**Rapporteur**
Roman Słowiński	Professeur, Poznań University of Technology	**Rapporteur**
Alexis Touskiàs	Professeur, Université Paris-Dauphine	**Rapporteur**
Chengbin Chu	Professeur, Ecole Centrale Paris	**Examinateur**
Marc Pirlot	Professeur, UMONS, Belgique	**Examinateur**
Vincent Mousseau	Professeur, Ecole Centrale Paris	**Directeur de thèse**

Preference Elicitation for Aggregation Models based on Reference Points: Algorithms and Procedures

Jun ZHENG

September 30, 2012

Acknowledgement

I would like to show my appreciation to all the people who made this thesis possible and an unforgettable experience for me. First of all, I would like to express my sincere gratitude to my advisor Prof.Vincent Mousseau for accepting me to carry on this research and guiding me to the field of MCDA. I benefited a lot from his continuous support during my three years Ph.D study. Without his encouragement when I was facing difficulties, I wouldn't have completed the thesis. Besides the scientific knowledge, I also learnt his positive and optimistic attitude in concentrating on the bright side of things, which I believe is not only important for the complete of the present thesis, but also for my future life.

Besides my advisor, I would like to thank the rest of my thesis committee, with whom I feel so lucky to have the chance to interact. My deeply thanks goes to Prof. Alexis Tsoukiàs. Without him, I wouldn't have the chance to pursue my Phd in France, which changed my entire life. I am grateful to Prof. Alexis Tsoukiàs also for providing financial support to attend the 2nd Algorithmic Decison Theory Conference. I worked with Prof. Marc Pirlot for the elicitation problems of ELECTRE TRI with optimistic rule. His insightful perspectives are always valuable and helpful, and it was very pleasant to work with him because of his sense of humor and kindness. I warmly thank Prof. Ahti Salo for giving me many precious suggestions to improve my work concerning portfolio selection problems with remarkable patient. I also appreciate the time that Prof. Roman Slowiński and Prof. Chengbin Chu spent in reviewing my thesis, providing insightful comments, and asking constructive questions.

I wish to express my warm and sincere thanks to Dr. Antoine Rolland, Dr. Stéphane Aimé Metchebon Takougang and Prof. Julie Stal-Le Cardinal for their excellent collaborations with me during my PhD.

I thank my fellow officemate Olivier Cailloux for the stimulating discussions and his friendliness. I also wish to thank Dr. Wassila Ouerdane for revising my manuscript and giving me detailed advices. My sincere thanks are due to the labmates in Laboratoire Génie Industriel for their sympathetic help when a Chinese was struggling with her French. Also I thank my Chinese friends in Paris: Yan Zhang, Erbao Li, Xin Liu, Yanpei Chen, Shuangqing Liao, Jiliang Wang and Ya Liu for their support at the difficult moments of the thesis and for sharing the happy time with me in Paris.

I would like to thank my family: my parents Shengli Zheng and Congzhi Lei for doing their best to support me so that I can pursue my dreams. I am indebted to my sister Ying Zheng to support my family when I was abroad.

Last but not the least, I owe my special thanks to my boyfriend Yunhui Wang. His spiritual support gives me tremendous courage to overcome all the difficulties I have met.

Abstract

Multiple Criteria Decision Aid (MCDA) aims at supporting decision makers (DM) facing decisions involving several conflicting objectives. DM's preferences play a key role in the decision aiding process, since the recommendations are meaningful and acceptable only if the DM's values are taken into consideration. A preference elicitation tool is therefore necessary to help the analyst to incorporate appropriately the DM's preferences in the decision models. We are interested in developing preference elicitation tools for two aggregation models based on reference points, namely ELECTRE TRI and a new Ranking method based on Multiple reference Points (RMP).

Firstly, we consider ELECTRE TRI using the optimistic assignment rule. We propose a preference elicitation tool which elicits the preference parameters of the model from assignment examples provided by the DM, and also analyzes the robustness of the assignments related to the imprecise nature of the preference information. Secondly, a preference elicitation tool is developed for portfolio selection problems formulated as constrained sorting problems using ELECTRE TRI. The DM's preferences both at the individual and portfolio level are considered to elicit the ELECTRE TRI model. The elicited model evaluates intrinsically the individuals and simultaneously selects a satisfactory portfolio as a group. Thirdly, we are interested in preference elicitation for RMP model, which determines a weak order by comparing alternatives with reference points. A preference elicitation tool is provided which infers a parsimonious RMP model from the DM's pairwise comparisons. Lastly, three web services implementing the preference elicitation tools for ELECTRE TRI have been developed and integrated to Decision Deck software. The proposed preference elicitation tools consist of algorithms solving mixed integer programs. Extensive numerical experiments have been performed to study the performance and behavior of the proposed elicitation tools to give insights into their applicability in practice. Moreover, the tools have been successfully applied to three real-world cases.

Key words: Multiple Criteria Decision Aid, Preference elicitation, Reference points, Portfolio selection, ELECTRE TRI , Ranking method

Résumé

L'Aide Multicritère à la Décision (AMCD) vise à aider un décideur (DM) confronté à un problème de décision impliquant plusieurs objectifs contradictoires. Les préférences du DM jouent un rôle important au sein du processus d'aide à la décision, puisque les recommandations ne sont pertinentes et acceptables que si le système de valeurs du DM est pris en considération. Un outil d'élicitation des préférences est donc nécessaire pour aider l'analyste à intégrer les préférences du DM de façon appropriée dans les modèles de décision. Nous sommes intéressés par le développement d'outils d'élicitation des préférences pour deux modèles d'agrégation basés sur des points de référence, à savoir ELECTRE TRI et une méthode de Rangement basé sur des Points de référence Multiples (RPM).

Tout d'abord, nous considérons ELECTRE TRI en utilisant la règle d'affectation optimiste. Nous proposons un outil d'élicitation des préférences, qui infère les paramètres de préférence de ce modèle à partir d'exemples d'affectation du DM, et analyse également la robustesse des affectations résultant de la nature imprécise de l'information préférentiel. En second lieu, un outil d'élicitation des préférences est développé pour le problème de sélection de portefeuille formulée comme des problèmes de tri contraint en utilisant ELECTRE TRI. Les préférences du DM à la fois au niveau individuel et au niveau du portefeuille sont considérés pour infère le modèle Electre Tri. Le modèle élicité évalue intrinsèquement les individus et sélectionne simultanément un portefeuille satisfaisant comme un groupe. Troisièmement, nous nous intéressons à l'élicitation des préférences pour le modèle RPM, qui détermine un pré-ordre comparant des alternatives avec des points de référence. Nous proposons un outil qui infère un modèle RPM parcimonieux à partir de comparaisons par paires du DM. Enfin, trois web services implémentent des outils d'élicitation des préférences pour ELECTRE TRI et ont été intégrées au logiciel de Decision Deck. Les outils d'élicitation des préférences proposés consistent en des algorithmes qui résolvent des programmes linéaires en nombres mixtes. Des expériences numériques approfondies ont été réalisées pour étudier la performance et le comportement des outils d'élicitation proposées. Ces expériences éclairent sur l'applicabilité pratique de ces outils. De plus, les outils ont été appliqués avec succès à trois cas.

Mots clés: L'Aide Multicritère à la Décision, Elicitation des préférences, Points de références, Sélection de portefeuille, ELECTRE TRI , Méthode de rangement

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Motivation

1.1.1 MCDA

Decision making has attracted researchers from various domains including phycology, business, engineering, economy, systems engineering, social choice, and management science (Plous, 1993; Clemen, 1997; Dwarakanath and Wallace, 1995; Simon, 1959, 1977; Sen, 1986; Horvitz et al., 1988). Real-world decisions are frequently too complex to be investigated with a single point of view. In fact, the decision makers (DM) often confront multiple and conflicting objectives. Therefore, it is more reasonable to consider simultaneously all points of view that are pertinent to the problem. To address a multiple criteria decision problem, one needs to aggregate the various dimensions at stake, and therefore an aggregation model. Such aggregation methods have been studied in variety of disciplines, such as statistical approaches, artificial intelligence techniques, Multiple Criteria Decision Aid (MCDA) methodologies (Hamburg, 1970; Doumpos and Zopounidis, 2011; Figueira et al., 2005a; Beliakov et al., 2008).

In this thesis we are concerned with MCDA, which constitutes an important field of operations research. It concentrates on developing decision aiding tools and methodologies to deal with complex decision problems involving multiple criteria. In fact, it is a discipline aimed at supporting decision makers who are faced with decision involving several conflicting objectives. In the framework of MCDA, the DM plays a key role in the decision aiding process. Firstly, the DM's preference is vitally important to the decision result. Obviously, it is unlikely for the DM to accept and implement the outcome of the decision aiding process when it doesn't conform to his preferences and judgements. Only when his own preferences, experiences, and decision-making policy are considered in such a process, the result can make sense to him. This is a significant issue that should be kept in mind during the development of MCDA models. Secondly, it should be emphasized that the DM has to be deeply involved in the decision aiding process. The MCDA models are not built by an analyst through a sequential process following a standard routine. Instead, the analyst and the DM interacts extensively along such process. The analyst guides the DM to clarify and express his preferences. The DM himself can also get more insights into the problem though their communication. The preferences of the DM are analyzed and then represented as consistently as possible in an appropriate MCDA model. This interactive procedure to investigate the DM's preferences is one of the most fundamental features of MCDA, which distinguish MCDA from other methodologies such as statistical and optimization approaches (Figueira et al., 2005a; Tsoukiàs, 2008; Roy and Vanderpooten, 1996).

Among the MCDA methods and tools, the proposed approaches (see Chapter 2 for a review) in the literature are classfied into three kinds: (1) multi-attribute value theory (Keeney and Raiffa, 1993; Wallenius et al., 2008), (2) outranking relations approach (Roy, 1985), (3) rule based models (Greco et al., 2001a). In the large literature on management science and operational research there is an increasing number of real-world applications of MCDA, including Environmental Management and Energy planning, Finance and economics, Marketing, Transportation, etc (see Wallenius et al., 2008; Hämäläinen, 2004; Keefer et al., 2004; Bana e Costa et al., 1999, for example).

1.1.2 Preference Elicitation

As discussed previously, the incorporation of the DM's preferences is an inevitable and vitally important issue in a decision aiding process. The parameters of aggregation models, such as weights of criteria, marginal value functions, thresholds allow to elaborate the models taking into account the DM's preferences. With such preferential parameters, we can expect the outcomes of the aggregation models to make sense to the DM (Bouyssou et al., 2006).

From a theoretical point of view, the aggregation models are characterized by groups of properties that we call axioms, which are the conditions of preference relations that fit the models (see Greco et al., 2004; Bouyssou and Pirlot, 2005, for example). In a particular decision context, the analyst has to check if the DM's preferences fulfil such conditions. Moreover, the signification of the parameters are strongly related to the specific model. For example, weights of value based methods represent tradeoffs between criteria. However, for outranking methods, weights stand for voting powers of criteria, and the compensation of performances among criteria is not allowed. In addition, the aggregation models are becoming more and more complex in order to better characterize the DM's preference, such as Choquet Integral which takes into account the interaction between criteria (Grabisch et al., 2008).

From a practical point of view, the DM with little expertise in MCDA methodology is unaware of the axiomatic foundations of the aggregation methods. It is not always easy to understand such axioms in order to choose a suitable method for his specific problem and preference. The meanings of such parameters are not well understood by the DM as well, which may comes from the fact that he has little time to understand it or it requires too much effort. Furthermore, the DM usually can only express his values, believes, and preferences in an intuitive and even ambiguous way and his preference can be controlled by factors that may appear irrelevant to the problem (Kahneman and Tversky, 1982).

Thus, how to represent the DM's preferences by the values of the parameters is unclear for most aggregation models. Therefore, preference elicitation tools are necessary in order to build a bridge between the analyst and the DM. The goal of such tools is to help the analyst to represent the DM' preferences meaningfully in MCDA models.

There are two paradigms of preference elicitation techniques: direct and indirect ones. For direct elicitation techniques, the DM is asked directly the parameters of the aggregation models which are then used to obtain global preferences (see Figueira and Roy, 2002; Edwards, 1977, for example). However, such elicitation methods are generally acknowledged to be too difficult for the DM due to the reasons previously discussed. In the indirect elicitation technique framework (disaggregation method), partial information on comprehensive preferences is supposed to be known a priori (provided by the DM) and a consistent criteria aggregation model is inferred from this information (see Jacquet-Lagrèze and Siskos, 2001, for a review).

We are interested in the disaggregation method for preference elicitation. Within such a framework, the elicitation method is mostly designed based on a specific model, since different aggregation models imply different logics of aggregation and different signification of parameters. Although many aggregation approaches of MCDA have been proposed in the literature, for many of these methods, there does not exist well defined preference elicitation tools. Hence difficulties arise when it comes to implementing such models in order to support actual DMs involved in real world decision problems. We shall concentrate on the development of such elicitation tools for two specific models (see the following section).

1.2 Objectives of the Thesis

We aim at developing preference elicitation tools particularly for two aggregation methods based on reference points, namely ELECTRE TRI and a Ranking method based on Multiple reference Profiles (RMP). More precisely, three types of algorithmic tools are needed to address the following issues arising from the decision aiding process. Firstly, aggregation models should be inferred from the DM's indirect preference information. Secondly, we are concerned with the robustness analysis related to the incomplete nature of preference information. Thirdly, when the DM's information is inconsistent, we are interested in how to detect and resolve such inconsistencies.

The preference elicitation tools often consist of complex optimization algorithms. Our additional concern is that the disaggregation algorithm should be able to scale well in terms of computation time when large scale problems are considered. Moreover, we have sought opportunities to apply our elicitation tools to real world decision problems in order to examine their usability. The four main objectives of the present thesis are given as follows.

1.2.1 Preference Elicitation for a Sorting Problem: ELECTRE TRI

Multiple criteria sorting problems aim at assigning alternatives to predefined ordered categories considering multiple criteria (see Zopounidis and Doumpos, 2002, for a review). Various methods have been proposed to address such kind of problems, such as ELECTRE TRI (Yu, 1992), UTADIS (Doumpos and Zopounidis, 2004), rough set based sorting (Greco et al., 2002b), cased-based distance sorting (Chen et al., 2008), etc. We consider the widely studied and used sorting method ELECTRE TRI , which compares alternatives to several profiles separating consecutive categories and uses either the so-called pessimistic rule or optimistic rule to assign these alternatives to one of the categories. To define an ELECTRE TRI model involving several parameters, the approach to learn the model from decision examples of that alternatives should be assigned to specific categories has been investigated carefully as far as pessimistic rule is considered (Mousseau et al., 2000; Dias et al., 2002; Dias and Mousseau, 2006; Mousseau and Słowiński, 1998). However, the learning approach has been rarely investigated for ELECTRE TRI using the optimistic rule. Though there exists an evolutionary approach to construct ELECTRE TRI model considering both the two rules (Doumpos et al., 2009), we are more interested in using linear programming to tackle the difficulties of preference elicitation for ELECTRE TRI with the optimistic rule. We aim at developing algorithms to elicit parameter values and compute corresponding robust assignment from assignment examples through linear optimization.

Part of this contribution has been published in Metchebon T. et al. (2010b) and a paper version will be submitted to a journal soon.

1.2.2 Preference Elicitation for Portfolio Selection Problems

This part of the present thesis is motivated by a student selection problem at Ecole Centrale Paris, France. The main concern of the DM is to find students who not only best fulfill the requirements of a specific major (Industrial Engineering) but also form a group who has a good gender balance, a good distribution among professional tracks, etc (Le Cardinal et al., 2011). Such student selection problem is generalized as a portfolio selection problem which aims at selecting a subset of alternatives considering not only the performance of the alternatives evaluated on multiple criteria, but also the performance of the portfolio as a whole, on which balance over alternatives on specific attributes is required by the DMs. Thus, the DMs' preference information both at individual and portfolio level should be taken into consideration during the portfolio selection process.

There exists a large number of methods for evaluating and selecting portfolios, such as multicriteria decision analysis (Philips and C.Bana e Costa, 2007; Duarte and Reis, 2006), weighted scoring (Coldrick et al., 2005), etc. However, expressing the DM's sophisticated preferences on portfolio selection remains a challenge. Some researchers combine preference programming with portfolio selection considering incomplete preference information (Liesiö et al., 2007, 2008). A balance model (Farquhar and Rao, 1976) is developed which measures the distribution of specific attributes by dispersion and uses such measurement to select subsets of multiattribute items. Golabi et al. (1981) uses constraints to eliminate the ones which do not fit in the requirement on whole portfolio. However, to our knowledge, MCDA outranking methods have rarely been applied to portfolio selection problem, despite the fact that they have been widely applied to many other domains (for example, see Parsaei et al., 1993; Boer et al., 1998).

Our aim is to apply outranking methods to the selection of portfolio. More precisely, we formulate such problem to the evaluation of individuals to predefined ordered categories using constrained ELECTRE TRI. Such formulation necessitates the incorporation of the DM's preference both at individual and portfolio level to the evaluation ELECTRE TRI model. Therefore, we investigate the preference elicitation issue of ELECTRE TRI particularly for portfolio selection.

Our work in this chapter has been published in Zheng et al. (2011) and Le Cardinal et al. (2011).

1.2.3 Preference Elicitation for a Ranking Problem: S-RMP model

Suppose we want to aggregate profiles of weak orders and there are at least three alternatives. Due to Arrow's impossibility theorem in social choice theory (Arrow, 1953), it is impossible to develop an ideal aggregation method in MCDA satisfying simultaneously Weak Order, Independence of Irrelevant Alternatives, Non-dictatorship, Universality and Unanimity (see Section

3.3.1). For example, Condorcet method satisfies Independence of Irrelevant Alternatives, Non-dictatorship, Universality and Unanimity, but it doesn't always yield a transitive global preference relation (Bouyssou et al., 2006). Some methods relaxing one of these properties have been studied (Campbell and Kelly, 2002). For instance, Fishburn (1975) proposed a lexicographic aggregation method which weakens the non-dictatorship property.

We try here to present a simple outranking-based method which focuses on weakening the independence condition that the preference of two alternatives depends on other third alternatives, i.e, the preference is based on some reference alternatives, which we refer to as reference points. Psychological evidence reports people make their decisions based on some references, which can be the current status or their expectations (Knetsch, 1989; Tversky and Kahneman, 1991; Samuelson and Zeckhauser, 1988; Köszegi and Rabin, 2006). The concept of reference point was firstly introduced in the domain of psychology, sociology (Tversky and Kahneman, 1991), social choice theory (Sen, 1986), and multicriteria sorting (Yu, 1992). Rolland (2008) adopted the idea to an outranking method for ranking problem, and developed a new method Ranking method based on Multiple reference Points (RMP) whose potentials and properties have been investigated. Such a method compares two alternatives based on the way they compare with some reference points. The preference relation of the two alternatives is lexicographically determined by their relations with each reference point.

We are interested in the preference elicitation of RMP method. As an outranking method, RMP seems to be appealing, since it obtains a weak order satisfying invariance with respect to a third irrelevant alternative, while other outranking methods dealing with ranking problems (e.g, ELECTRE III Figueira et al., 2005b) violate such properties. However, the application of such model is restrained as a result of the lack of tools to construct a meaningful RMP model considering the DM's preference. We aim at designing such elicitation tool to answer the three following questions: (1) how many reference points should we use? (2) what should we set the values of these reference points to? (3) which lexicographic order the reference points should be used?

The third concern of the thesis is to elicit preference for S-RMP model indirectly from pairwise comparisons.

1.2.4 Developing Preference Elicitation Web-services for ELECTRE TRI

The literature of MCDA has proposed numerous aggregation methods (Roy, 1985; Figueira et al., 2005a; Keeney and Raiffa, 1993; Wallenius et al., 2008, see also Section 2.2) which have been applied to real-world decision problems (see Hämäläinen, 2004; Keefer et al., 2004, for example). These applications have been independently implemented in an uncoordinated way using different tools and programming language (see Korhonen et al. (1992); Doumpos and Zopounidis (2010) and the MakeItRational software for AHP for example). For people who are interested in applying MCDA methods in their own domains, they face difficulties in implementing such methods since they are not experts in MCDA methodology.

The Decision Deck project aims at collaboratively developing open Source software tools, whose components implement the common functionalities of a large range of MCDA methods (see Decision Deck Consortium, 2012a). The Diviz software is one of the initiatives of the project which is an open source Java client and server for designing, executing and sharing MCDA methods, via the composition of XMCDA web services (Decision Deck Consortium, 2012b). The developers implement their interested MCDA methods using the programming language, operation system they know best while respecting the data model of Diviz: XMCDA. These implementations are integrated to Diviz software with a friendly user interface. The users all over the world can download such software for free for their own use.

We are interested here in implementing web services for ELECTRE TRI method. More precisely, three issues need to be considered: (1) inferring ELECTRE TRI model from preference information; (2) robustness analysis when the preference information is not precisely known; (3) inconsistency resolution when the preference information is conflicting.

1.3 Structure of the Thesis

Chapter 2 provides a background and literature review of MCDA and preference elicitation. Chapter 3 is devoted to the introduction of aggregation models involved in this thesis: two reference-based aggregation models. More precisely, we are interested in the widely used sorting model ELECTRE TRI and a newly developed ranking model Ranking based on Multiple reference Points (RMP) both based on the decision rule of comparing alternatives to some reference profiles. Chapter 4 tackles the difficulties of preference elicitation for ELECTRE TRI using optimistic rule. In Chapter 5 we propose a preference elicitation method to handle decision situation where a portfolio has to be selected considering not only the preference on the evaluation of individuals but also the preference on the formation of overall portfolio. Chapter 6 aims at designing preference elicitation tool for the RMP method. In Chapter 7, we concern the developement of software tool for ELECTRE TRI method as web services which are to be integrated to Diviz software. Chapter 8 contains a summary of the main contributions of the research and suggestions for future research.

Chapter 2

MCDA and Preference Elicitation

In this chapter, a literature review of Multiple Criteria Decision Aid (MCDA) is presented to provide a basis of the thesis. The key concepts and notations of MCDA are firstly introduced briefly. Then the concept of MCDA process is explained to show its main activities. The Decision Maker's (DM) preference information is crucial in the process, so the way to represent such information is discussed afterwards. Three types of multicriteria aggregation procedures in the literature are reviewed: multiattribute value theory, outranking methods and rule-based methods. Preference elicitation methods which aim at constructing the aggregation models reflecting the DM's value system are distinguished to two types: direct and indirect elicitation. The implementation of the two types of elicitation techniques are presented based on the three different kinds of aggregation procedures.

2.1 An Introduction to MCDA

People are facing decisions every time and everywhere. Some decisions are too complex, involving multiple stakeholders or decision makers (decision maker is referred to as DM along the thesis), uncertainties, conflicting objectives, to be handled only based on intuitive judgements. Decision aiding aims at supporting the DMs to better understand the problem and give some responses to them.

The DMs usually assess potential actions based on multiple points of view, which can seldom be measured on a common unit (for example, we cannot assign a definite "value" to any human being). As a result, it is difficult to define a unique criterion to take into account all the considerations of the DMs. In the framework of MCDA (Belton and Stewart, 2002; Roy, 1985; Figueira et al., 2005a), the pros and the cons of different points of view are explicitly considered. The methodologies of MCDA provides a variety of tools to help the DM make justified decisions based on multiple criteria. Note that the term "MCDA" is commonly used in the Europe, while in US, "Multiple Criteria Decision Making (MCDM)" is more widely accepted.

2.1.1 MCDA Concepts and Notations

Actors

The actors are the people who are involved in the decision problem. Normally, there are at least two actors: a DM and an analyst. The DM is usually the person who is responsible for the decision. The DM can also be an expert in a specific domain who is capable of providing knowledge of the problem. There may exist multiple DMs who have common or conflicting objectives. The DM should not be assumed to have the background of MCDA methodology. On the other hand, the analyst, specialized in MCDA methodology, is supposed to support the DM. The analyst interacts with the DM during the decision aiding process in order to help him (for convenience, we use masculine form to refer to the DM along the thesis) to better structure the problem, to better understand his desires, to choose a MCDA model suitable for his needs, to explain the logic of the model to him, give a final recommendation, etc. The distinction of the DM and the analyst has been made by Roy (1993); Tsoukiàs (2007) in the framework of decision aiding, and the objective of the decision aiding process is to achieve a shared representation of the problem between the two actors and make a positive influence on it.

Alternatives

Alternatives refer to potential actions which are to be evaluated in the decision aiding process. We denote A the set of potential actions. When all possible alternatives can be defined, we say A is finite: $A = \{a_1, \ldots, a_n\}$, where n is the number of alternatives in the set. A can also be infinite, for example in some design problems, it's impossible to enumerate all feasible design parameters. In this thesis, we consider the case where A is finite.

Criteria

To evaluate alternatives, the DM considers multiple points of view. Each alternative is characterized by a set of criteria based on these points of view. The family of criteria is defined as $F = \{g_1, g_2, \ldots, g_m\}$, where m is the number of criteria. The evaluation of an alternative a_i on a criterion g_j is usually called its performance on this criterion, denoted as $g_j(a_i)$, which measures a_i directly on g_j. The set of all possible performance on g_j is denoted as X_j, which is a complete ordered set enabling the comparison between potential alternatives, and is called the scale associated with criterion g_j. There exists different kinds of scales (Stevens, 1946; S. and Roberts, 1994):

- Ordinal scale: the alternatives are measured in an ordinal way. It usually occurs when the DM accesses alternatives using linguistic grades. This type of scale permits the measurement of degrees of difference, but not the specific amount of difference. All strictly increasing transformations are admissible transformations for such scales.

- Quantitative (or ratio) scale: real-valued numbers are associated with this type of scales. The numbers give meaning to a unit allowing us to interpret each degree as the addition of a given number (integer or fractional) of such unit. The ratio of two levels in the scale is meaningful with the associated numbers. All positive homothetic transformations (the form $\phi(x) = \alpha x$, $\alpha > 0$) of the numbers are admissible to preserve the information of such scale. For example, the cost of a project in euro is a quantitative scale.

- Interval scale: real-valued numbers are associated with the criteria, and the ratio of differences in two levels on the scale is meaningful. All positive affine transformations of the numbers (the form $\phi(x) = \alpha x + \beta$, $\alpha > 0$) are admissible to preserve the information of such scale. A typical example of interval scale is temperature in Celsius degree.

In MCDA, knowing which type of scale we are working with is critical in order to be sure that its degrees are used in a meaningful way.

2.1.2 Multiple Criteria Decision Aiding Process

The decision aiding process is viewed as the activities of the participating actors, namely the DM and the analyst who both are involved in the decision process (Bana e Costa et al., 1999; Simon, 1997). It has been emphasized that decision aiding is not only solve a well-defined decision problem, but a complex process during which the actors gain better understanding of an ill-structured decision situation (Tsoukiàs, 2007; Bana e Costa et al., 1999). The importance of problem structuring and formulation has been recognized. For example, von Winterfeldt and Edwards (1986) claimed that problem structuring is the most difficult part in decision aiding process. With such a perspective, Tsoukiàs (2007) models the decision aiding process through its main results to four main phases: the representation of problem situation, problem formulation, evaluation model and final recommendation. The four phases are not a linear process, but rather a recursive one. During the decision aiding process, new information and new insights of the problem would invalid the previous outcomes, therefore updates are often necessary.

The representation of problem situation

During this phase, the DM and the analyst should work together to clarify the decision situation which is usually a "mess" at the starting point. They need to identify the persons who face the problem, who are involved or influential in the problem, and who are responsible for the consequence. The stakes and concerns of the DM should be analyzed as well as his goals and objectives. The roles of different actors and their different concerns are to be discussed and clarified. In this phase they use the human natural language to communicate and the task is just to describe the problem. The analyst is supposed to guide the DM to express his knowledge of the situation in an explicit way, which is rather important to shed light on the problem.

Ostanello and Tsoukiàs (1993) developed a " public" interaction space, i.e. an interorganizational informal structure to facilitate the communication. Using such a method, the interactions can be regulated and uncertainties can be reduced.

Problem formulation

Formulating the problem refers to modeling it as a formal and abstract model (Tsoukiàs, 2007). The first question needs to be answered is that: "what are the potential actions?". It is not trivial as the DM may not be able to provide them directly or what they provide doesn't include all interesting alternatives that should be considered. Keeney (1996) has presented a value-focused thinking method to create better alternatives, to identify decision opportunities more appealing and to use the DM's fundamental values to guide the decision aiding activities. Secondly, the points of view under which the alternatives are analyzed and evaluated should be established, and concerns of different stakeholders need to be taken into account. Lastly, the problem statement should be determined, because different problem statements may lead to different recommendations. For example, in a student selection problem, admitting the top best students or selecting the students who are suitable for a special program are totally different, and the two different problem statements would result in divergent selection results (for example, see French et al., 1998). According Roy (1985), decision problems can be classified into three different types, which means that the objective of most real world problems can be structured as one of the following three problematics.

- **Sorting problems**: assigning each alternative to one of the predefined ordered categories.
- **Ranking problems**: the alternatives are to be ranked to an order, while ties and incompatibilities may occur.
- **Choice problems**: the objective is to select a smallest number of the best possible alternatives.

Remark 1 *For sorting problems, the assignment of an alternative to a category depends on the intrinsic evaluations of the alternative while other alternatives are irrelevant to the assignment. Therefore the sorting of an alternative is considered as absolute evaluation of such alternative. On the contrary, the results of ranking or choice problems depend on all alternatives in A. Thus the result should be seen as relative evaluation of the alternatives.*

Evaluation model

With the result of problem formulation, the analyst who has methodological expertise in MCDA should be able to choose an evaluation model as a consequence of the understanding of the problem. At this stage of the decision aiding process, the aim of an evaluation model is to synthesize the available information into a global relation on the set of alternatives A. Such set A needs to be established and be evaluated on different dimensions D. To do so, a scale X is associated to each element of D. Moreover, the family of criteria F should be built based on D to take into account the DM's preferences. An evaluation model is to be chosen based on its theoretical and operational meaningfulness (Tsoukiàs, 2007). In other words, the chosen model has to use the information correctly and be understood by the DM without too many difficulties. Moreover, the model usually involves many parameters (such as importance weights of criteria) which reflect the DM's preferences and should be set with numerical values. In this thesis we shall investigate the techniques of determining these parameters based on the DM's preferences (see Chapter 4-6).

Recommendation

It is necessary to investigate some issues before concluding a final recommendation. Sensitivity analysis and robustness analysis are performed to examine the evaluation result when the values of parameters in the evaluation models are changed or different scenarios are considered. Furthermore, we need to pay attention to the legitimation of the recommendation, which is related to the organizational context of the decision aiding process. Thus the cultural, ethnical aspects should be included in order to convince all people involved to accept and implement the recommendation in practice.

2.1.3 Preference Representation

To incorporate the DM's preferences in the decision aiding process, the preferences should be represented in a formal way. Let us consider two alternatives a and b, which are presented to the DM to ask how he compares them. Generally, he may respond: "I prefer the first to the second (or vice versa)", which can be formally expressed as $a \succ b$ ($b \succ a$ respectively), where $\succ$ means strict preference. If the DM says that " I am indifferent between the two alternatives", we can represent such preference by $a \sim b$ where $\sim$ means no preference. The union of strict preference and indifference is defined as $\succsim$, which is called weak preference.

Generally, preference relations satisfy asymmetry and negative transitivity, which are the basic requirements of rationality the DM should have. Preference asymmetry is defined such that if $a \succ b$ then not $b \succ a$. Negative transitivity means that if not $a \succ b$ and not $b \succ c$, then not

$a \succ c$. If the preference relation $\succ$ is asymmetric and negatively transitive, it is a weak order.

Preference relations may satisfy other properties. The transitivity of indifference is defined such that if $a \sim b$ and $b \sim c$, then $a \sim c$. The intransitivity of indifference has been studied, for example, by a cup of sweetened tea (Luce, 1956; Tversky, 1969). By introducing thresholds (see Section 2.2.2 for the use of thresholds in outranking methods) to the preference relation, such intransitivity can be modeled (Böckenholt, 2001).

Transitivity of preference is defined such that if $a \succ b$ and $b \succ c$, then $a \succ c$. Although it seems unnatural to violate such property, there are empirical evidences which show the violation can occur in decision experience (May, 1954).

Incomparability may occur when the DM states: "I can't compare the two alternatives". Such preference is represented by $a?b$. Certain situations, such as lack of information, uncertainty, ambiguity, multi-dimensional and conflicting preferences, can create incomparability between alternatives (Tsoukiàs et al., 2002). Section 3.2.2 provides the modeling of incomparability in ELECTRE TRI.

A preference structure is a collection of binary relations defined on the set A. In other terms a preference structure defines a partition of the set $A \times A$ (Öztürk et al., 2005). An order of the alternatives is an important preference structure as it allows to operate such structure. Different forms of orders are defined with different properties. The total order structure consists of an arrangement of alternatives from the best to the worst without any ex aequo. The weak order is defined by adding indifference relation to the total order. Relaxing the property of transitivity of indifference results in two well-known structures: semi-order and interval order. When incomparability is added to the preceding orders (total order, weak order, semi-order, interval order), partial order, partial preorder (quasi-order), partial semi-order and partial interval order are respectively obtained. The readers are referred to Öztürk et al. (2005) for the details of different orders.

In the decision aiding process, given a set A and a set of preference relations between the elements of A, it is important to know whether such preferences fit a specific preference structure. If so, it is possible to replace the elements of A with their numerical values. Moreover, different preference structures correspond to different ways of numerical representations (Öztürk et al., 2005).

2.2 Multicriteria Aggregation Procedure

To compare alternatives in a comprehensive way, an evaluation model based on a mathematical procedure is used. Such procedure is called Multiple Criteria Aggregation Procedure (MCAP). MCAP, which is the logic of aggregating criteria, is a central element in decision aiding process. We introduce here the three main types of MCAPs, among which we focus on outranking methods in this thesis.

2.2.1 Multiattribute Value Theory (MAVT)

Taking into account the performance of an alternative on m dimensions, MAVT uses some functions to assign a well-defined degree to each $a \in A$ on an appropriate scale (Fishburn, 1970; Keeney and Raiffa, 1993). It is generally acknowledged that value functions represent the order of preference; on the other hand, utility functions refer to preference under risk. Here we concentrate on the cases where no risk is involved.

If strict preference $\succ$ on A is a weak order and A is finite and denumberable, then there exists a real-valued function $v(\cdot)$ on A such that

$$a \succ b \Leftrightarrow v(a) > v(b) \tag{2.1}$$

$$a \sim b \Leftrightarrow v(a) = v(b) \tag{2.2}$$

The value function captures the order of the preference, but the value of $v(\cdot)$ should not be over interpreted. That is to say, the difference of values between two alternatives is meaningless.

The key problem is to decompose the value function $v(\cdot)$ into simple forms. Based on the properties of the DM's preferences, a value function may be expressed in different forms such as Analytic Hierarchy Process (AHP) (Saaty, 1990), weighted sum (Fishburn, 1967), Measuring Atrractiveness by a Categorical Based Evaluation TecHnique (MACBETH) (Bana e Costa and Vansnick, 1994). The most widely used is the additive value function, where $v_j(g_j(a))$ is the value of a on criterion j.

$$v(a) = \sum_{j=1}^{m} v_j(g_j(a)) \tag{2.3}$$

The value function $v(\cdot)$ can be normalized in the sense that

$$v_N(a) = \sum_{j=1}^{m} w_j v_j^N(g_j(a)) \tag{2.4}$$

where $w_1, \ldots, w_m$ (with $\sum_{j=1}^{m} wj = 1$) are non-negative constants representing the criteria trade-offs. They define the respective roles of different criteria and are interpreted as substitution rates (or exchange rates), which describe how a loss on one criterion may be compensated by a gain on another. $v_j^N(g_j(a))$ ($j \in M$) are the marginal value functions of criterion g_j, which is scaled such that $v_j^N(g_j(a_{j*})) = 0$ and $v_j^N(g_j(a_j^*)) = 1$ (a_{j*} and a_j^* are respectively the least and the most preferred level of criterion j). It should be emphasized that the weight w_j reflects the increase of overall value $v_N(a)$ when the performance on corresponding criterion g_j is improved from the worst $g_j(a_{j*})$ to the best level $g_j(a_j^*)$ (Salo and Hämäläinen, 1997).

MAVT associates a real-valued number to each alternative. It offers the following features. (1) The existence of incomparability is excluded and transitivity of preference and indifference is ensured (see Section 2.1.3 for the definitions of incomparability and transitivity); (2) The performances of m criteria are synthesized into a common scale (e.g., monetary scale, utility scale); (3) A poor performance on one criterion can be compensated by a good performance on another one; (4) The methods provide sound axiomatic foundations which allow to fit different types of preference in diverse contexts with different forms of value functions (Deutsch and Malmborg, 1985).

2.2.2 Outranking Methods

The value based methods usually result in a complete and transitive preference order, and the existence of value functions have to be enforced by some axioms (Deutsch and Malmborg, 1985). However, the DM's behavior can violate the axioms, as shown in many psychological experiments (see Section 2.1.3 for references). Outranking methods allow to model more complicated preferences, namely intransitivity of indifference and incomparability relation (Roy, 1991). This family of methods have been developed in France and have achieved a wide application in Europe (for example, see Parsaei et al., 1993; Boer et al., 1998). There are two well-known outranking methods: ELimination Et Choix Traduisant la REalité (ELECTRE) and Preference Ranking Organisation MeTHod for Enrichment Evaluations (PROMETHEE), which we shall present as follows.

To model the preference relations, namely indifference, preference and incomparability, the methods of ELECTRE family introduces two thresholds to the criterion on which the numerical values of performances are subject to imprecision, uncertainty, and indetermination. A criterion with thresholds is called pseudo-criterion. Formally, a pseudo-criterion is a function g_j associated with two threshold functions $q_j(\cdot)$ and $p_j(\cdot)$, satisfying the following condition. For all ordered pairs of actions $(a,b) \in A \times A$ such that $g_j(a) \geq g_j(b)$, $g_j(a) + p_j(g_j(b))$ and $g_j(a) + q_j(g_j(b))$ are non-decreasing monotone functions of $g_j(b)$, where $p_j(g_j(b)) \geq q_j(g_j(b))$ (Figueira et al., 2010). $p_j(g_j(b))$ is called preference threshold, which is the smallest performance difference of two alternatives on criterion g_j for a preference relation between them. On the other hand, $q_j(g_j(b))$ is called indifference threshold, which is the largest performance difference of two alternatives on criterion g_j for an indifferent relation between them. The two thresholds can be constants.

With the two thresholds, the binary relation of two alternatives on a criterion can be defined, and three situations are possible:

1. $g_j(a) - g_j(b) > p_j(g_j(b)) \Leftrightarrow a$ is preferred to b on criterion g_j, i.e, aP_jb
2. $q_j(g_j(b)) < g_j(a) - g_j(b) \leq p_j(g_j(b)) \Leftrightarrow$ There is a hesitation between the assertion "*a* is preferred to *b*" and the assertion "*a* is indifferent with *b*" on criterion g_j, i.e, aQ_jb
3. $-q_j(g_j(b)) < g_j(a) - g_j(b) \leq q_j(g_j(b)) \Leftrightarrow a$ is indifferent with b on criterion g_j, i.e, aI_jb

A partial outranking relation $\succsim_j$ is defined on criterion g_j such that $a \succsim_j b$ means "*a* is at least as good as *b*". We can see that $\succsim_j = P_j \cup Q_j \cup I_j$.

To define comprehensive preference relation of two alternatives, the concepts of concordance and discordance are used (Figueira et al., 2010). The concordance condition means that to validate the assertion $a \succsim b$ there should be a sufficient majority of criteria in favor of such an assertion. The discordance condition guarantees that there should not be a minority of criteria which are strongly against the assertion $a \succsim b$. The details of how to verify such two conditions are given in Chapter 3 for an outranking sorting method ELECTRE TRI .

Once the binary preference relations on all criteria have been aggregated to a comprehensive preference relation, an exploitation procedure is necessary to draw recommendations. This procedure has led to various methods. ELECTRE I, ELECTRE IV, and ELECTRE IS are proposed for choice problems, ELECTRE II, ELECTRE III, and ELECTRE IV are developed for ranking problems and ELECTRE TRI-B, ELECTRE TRI-V, and ELECTRE TRI-NC deal with sorting problems (see Figueira et al., 2005b, for a review of these methods).

We show the main features of outranking methods (Figueira et al., 2010), which can be viewed as the reasons for which we have chosen this family of methods for the thesis.

- **Heterogeneity of scales** The criteria can be evaluated on heterogeneous scales. It is not necessary to recode the original performance data as what is done in utility based methods.

- **Qualitative scales** Very often, the criteria are evaluated on qualitative scales by the DM. Outranking methods deal with such evaluations directly without the need of recoding them. Even the quantitative data are treated in a qualitative way.

- **Non-compensatory nature** Many MCDA methods are based on tradeoffs between criteria. However, Outranking methods don't allow the compensation of performances among criteria. In other words, the degradation of performances on some criteria cannot be compensated by improvements of performances on other criteria. This point will be further explained in the introduction of ELECTRE TRI in Section 3.2.2.

- **Imprecise preference information** With preference and indifference thresholds, outranking methods model imprecise preference information of the evaluations, which means that the small variation of some performance will not influence the preference relation in a significant way.

There are certain limitations of this family of approaches. (1) When the scale of a criterion is quantitative, the quantitative character of the performance is lost, since the methods merely use its order. (2) It is not possible to assign a value to each alternative, but in some cases it may be desirable. Consequently, the approaches are incapable of giving meaning to the strength of preference. In other words, we can only say a is better than b, but we can't say how much a is better than b.

2.2.3 Rule based Approaches

In the field of machine learning (Alpaydin, 2010), rule-based models have attracted much attention, since decision rules are simple to understand and interpret. Among these rule-based methods, we present here the rough set theory which has been successfully adopted to the domain of MCDA.

Rough set theory is a very powerful tool to analyze and represent ambiguous information (Pawlak, 1982; Pawlak and Slowiński, 1994). Rough sets can be considered as sets with fuzzy boundaries, sets that can't be precisely characterized using the available set of attributes. The key concept of "indiscernibility " representing the relation of two objects which are indiscernible by the set of attributes. Thus the concept induces a partition of the universe into blocks of indiscernible objects. Rough set can handle quantitative and qualitative data, while inconsistent information can be represented as well. Moreover, the insight of data can be gained by obtaining the dependencies between attributes and the importance of attributes. The output is in the form of "if...then" which is rather comprehensible.

Rough set methodology has been adopted to MCDA during the last decades as a complete and well-axiomatized system known as Dominance-based Rough Set Approach (DRSA). Greco et al. (2001a) present a good review of DRSA. The classical rough set only considers the indiscernibility relation of two objects, but it is insufficient to model preference relation which is a central concept in MCDA. So it is modified by taking into account the preference order of criteria and categories to be suitable for MCDA. The dominance relation is defined which permits to deal with multicriteria sorting problems (Greco et al., 1998). Each induced "if ... then ..." type decision rule is composed of a condition part specifying a profile to which an alternative is compared using the dominance relation, and a decision part assigning an alternative to "at least" or "at most" a class. To capture the preference relation of pairwise comparisons, the graded dominance relation is proposed to handle pairwise comparison table with which preference in choice and ranking problems can be modeled (Greco et al., 1999). The induced decision rules consist of a condition part which compares two alternatives to some reference alternative using the graded dominance relation, and a decision part which gives the binary preference relation of the two alternatives (outranking or uncertain relation). An exploitation procedure is necessary to obtain a final recommendation.

It is worth highlighting that DRSA is able to deal with heterogeneous information, including qualitative and quantitative, criteria and attributes, crisp and fuzzy evaluation, ordinal and missing values. With the modification described previously, the rough set theory is strengthened as an interesting tool for MCDA.

Some extension of DRSA has been proposed. The strict dominance relation is relaxed to avoid the decrease of cardinality of lower approximations to an unacceptable extent when large data are considered (Greco et al., 2001b). Stochastic DRSA has been introduced to deal with noise data (Kotlowski et al., 2008). DRSA has been applied to many other contexts (Slowiński et al., 2007; Greco et al., 2006, 2008a, 2007, 2010a), and also real decision problems (Witlox and Tindemans, 2004; Liou and Tzeng, 2010).

Other decision rule based methods are proposed in the literature as well. For example, decision tree is used as a visual and analytical tool which calculates the expected values of alternatives (Quinlan, 1986). In other words, it maps observations of an item to conclusions about the item's target value. Decision tree models can be viewed as rule based since they produce "if...then" format decision rules.

2.2.4 Remarks

We have discussed respectively the features that each kind of methods offer. However, it is still worth emphasizing that different methods have their own strengths and weakness. As is known, the value based methods assign a degree to each alternative, therefore a complete order can be obtained, with a sound axiomatic foundation. That is why they are more widely used. But the restrictive axioms to be satisfied require much cognitive effort from the DM, which is often too difficult. Meanwhile, outranking methods are relatively more flexible, but the results they get with incomparabilities are poorer compared to the complete order value functions obtain. Some of the methods are not axiomatically well founded, which has been discussed in Tsoukiàs (2008). Other discussion on the comparisons of these different kinds of methods can be found in Roy (1993).

Although we have presented the above three kinds of methods in a separate way, many researchers have proved that there are links between such methods using conjoint measurement (see for example Greco et al., 2004, 2002a; Bouyssou et al., 1997).

To use the various aggregation methods we have just presented, we have to set many parameter values, which reflect the DM's value system. The parameters which construct the aggregation models are very important, as they give sense to the recommendation of the decision aiding process. A preference elicitation tool is therefore needed which aims at meaningfully setting these parameters. We shall present the techniques of preference elicitation in what follows.

2.3 Preference Elicitation

2.3.1 Objective

Preference elicitation aims at helping the analyst to appropriately elicit the DM's preferences to be represented in the decision models. On one hand, the aggregation models become more and more complex to better model the DM's values in complicated decision situations. On the other hand, the DM who has limited knowledge of the aggregation models, can only express his preferences in a rather intuitive and ambiguous way. A preference elicitation tool is designed to facilitate the communication between two actors involved in a decision aiding process.

Generally speaking, there are two paradigms of preference elicitation approaches, namely direct and indirect paradigms. In the direct aggregation paradigm, the parameter values are supposed to be directly provided by the DM through an interactive communication with the analyst. The aggregation model is firstly constructed with these parameters and then applied to the alternative set to obtain the DM's comprehensive preferences. Within such a paradigm, the DM should make enough effort to understand the meaning and the roles of these parameters and to associate appropriate values to them, which may be beyond his cognitive capacities. On the contrary, in the disaggregation-aggregation paradigm (see Jacquet-Lagrèze and Siskos, 2001, for a review), the partial comprehensive preference is known a priori and a consistent criteria aggregation model is inferred from this information. The given comprehensive preference information is usually represented by constraints. Disaggregation-aggregation methods infer an aggregation model as compatible as possible with given preferential structures using the regression technique. We call this type of elicitation methods indirect ones.

2.3.2 Direct and Indirect Elicitation Methods: an Overview

Direct elicitation methods

In the framework of direct aggregation, the DM is required to specify the parameter values of the models directly within an interaction process with the analyst. We introduce some techniques as follows.

In many situations, the DM is just asked to give the precise values of the weights which are then normalized to one. Sometimes the DM is required to give information on the comparisons of criteria and even the intensities of such comparisons. Simos proposed a technique, namely

SRF method, which allows the DM to express his preference on weights, even if he has little knowledge of MCDA (see Figueira and Roy, 2002). We shall introduce the method in the following paragraph because it is used in one of the case studies in this thesis (see chapter 4).

For SRF method, the DM is asked to play cards which represent the criteria and the weights are derived from the way the DM places the cards. More precisely, the DM is given a set of cards, on each of which the corresponding criterion is written. The DM is asked to order the card from the least important to the most important. If the DM feels that some criteria are equally important, he should put the related cards in the same position. Consequently, a complete pre-order of the criteria is obtained by the ranking of the cards. The DM is also demanded to insert white cards between the successive criteria (or two successive subsets of *ex aequo* criteria). The white cards reflect the intensity of the difference in the successive cards. More white cards means bigger difference in the importance of the two successive criteria (or two successive subsets of *ex aequo* criteria). Figueira and Roy (2002) proposed an algorithm to compute the values of weights based on such a procedure and a software has been developed. This method is very intuitive and understandable, and it has been applied in many real-world applications (e.g, see Fontana et al., 2011; Merad et al., 2004).

However, many critics have been raised to show that it is problematic to use the directly elicited values in the aggregation models. The weights are often interpreted as importance of criteria, but their definition is not yet clear. Bouyssou et al. (2006) provide an example which explains clearly the problem that the numerical values provided by the DM don't imply the logic of the chosen aggregation method. Suppose the DM is asked to assign values to weights for absolute majority method involving 3 criteria. Feeling that the criterion two is more important than criterion three, but less important than criterion one, the DM set respectively the weights as 0.45, 0.40 and 0.15 for the criteria. The weights construct an absolute majority model that no criterion plays a major role, but any two criteria is strong enough to pass the threshold (0.5). Therefore, the weights are in fact equally important in the model, but it doesn't conform to the DM' preferences.

Furthermore, the weights in different aggregation method don't play the same role. Let us consider two alternatives evaluated on three criteria as follows: $a = \{15, 10, 10\}$ and $b = \{10, 12, 12\}$. If the DM just feel in an very ambiguous way that there is no significant difference in the importance of each criterion, he can assign each criterion with a weight $\frac{1}{3}$. Using a weighted sum method, a should be judged as preferred to b as the score of a (11.67) is greater than the one of b (11.33). When the absolute majority method is considered, b is preferred to a as on two criteria b is better than a and the coalition passes the threshold 0.5. If we then assign the same weights to qualified majority method with a threshold 0.7, we find that not $a \succ b$ and not $b \succ a$ so that the two alternatives are incomparable. This example shows that different aggregation methods with the the same weights lead to totally different preference relations. The reason lies in the fact that the weights of different aggregation methods don't imply the same meanings, as pointed out by Roy and Mousseau (1996). A criterion is defined by specifying how to take into account the consequences attached to a given point of view, and encoding a criterion means to choose a real value function on the basis of which the preferences, relative to the attached viewpoint, may be argued (Roy and Mousseau, 1996). For weighted sum method, the values of weights are dependent on the units that we choose for the criteria, so it is inconsistent to set the values of the weights without taking into account the encoding. Moreover, the weights of this method allow to compensate a disadvantage on some criteria by a sufficient advantage on other criteria, and therefore represent substitution rates between criteria. For both absolute majority method and qualified majority method, the weights represent the intrinsic importance of criteria and are independent on the criteria's encodings. However, the weights of the two methods are dependent on the majority threshold (the threshold is 0.5 for absolute majority method). Therefore, giving numerical values to the weights without considering the threshold results in the fact that the values don't actually reflect the opinion of the DM.

With the arguments above, we believe that it's meaningless to elicit the preference parameters as long as the MCAP is not specified. The parameter should only be defined in relation to the MCAP in which they are used.

Some methods are proposed for specific MCAPs. In this case, the analyst should make the effort to explain the logic of the aggregation procedures and the meaning of the parameters to the DM. Meanwhile, the DM has to gain some understanding on how his preference is represented in the model. As discussed previously, the weights in MAVT reflect substitution rates between criteria, so elicitation of weights without this interpretation is not really meaningful. Many rating techniques have been proposed to access the weights, such as Simple MultiAttribute Rating Technique (SMART) (Edwards, 1977; von Winterfeldt and Edwards, 1986), swing weighting (von Winterfeldt and Edwards, 1986). These methods ask the DM to give directly the numerical estimates of weight ratios although different elicitation questions are used (see, for example Montibeller et al., 2006). Because of their intuitive features, they have won popularity in many applications. However, behavioral experiments have shown that the DM's response may be influenced by scale effect (Pöyhönen and Hämäläinen, 2001), range effect (von Nitzsch and Weber, 1993) and splitting bias (Pöyhönen et al., 2001), and therefore is not reliable enough. Eigenvalue method has been developed for AHP to specify the weights (see Saaty and Hu, 1998; Saaty, 2005). The DM should express his preferences by comparing alternatives and giving either a number or a verbal judgement of this preference's intensity. Then the weights are directly derived from these comparisons. However, the method has been criticized extensively on many aspects (see Salo and Hämäläinen, 1997; Bana e Costa and Vansnick, 2008). From our point of view, AHP method is a variant of MAVT, so the weights from direct elicitation are dependent on the encoding of criteria. The weights which stand for intrinsic importance of criteria may not fit into the method.

Indirect elicitation methods (disaggregation methods)

With the criticism on direct elicitation approaches discussed above, it is acknowledged that the elicitation process should be based on specific aggregation models. Thus, the disaggregation of MCDA models are often considered more appropriate to elicit the preferential parameters. Instead of asking precise values of the parameters from the DM, the disaggregation methods just require him to give some global preference judgements.

The objective of disaggregation method is to estimate values of the parameters defining an aggregation model as close as possible with the "real" preference of the DM. But the "real" preference is unknown a priori, and doesn't even pre-exist in the DM's mind. Thus we use holistic preference judgements, which can come from different resources. The DM may provide some previous decision examples, or evaluate some representative alternatives according to his knowledge of the problem. In some cases, the analyst presents the DM some fictitious alternatives and asks him to make judgements. Disaggregation methods infer an aggregation model from these decision examples in a way that the inferred model is able to reproduce them as much as possible. Then the inferred model is used to evaluate the alternative set to get global preference.

Keeney and Raiffa (1993) use a tradeoff method to elicit value functions by indifference judgements. The DM assigns a performance level of criteria to make an alternative indifferent to a given alternative. The method can determine the value functions with high efficiency, but it needs the criterion scales to be continuous. So fictitious alternatives are introduced, though the answers to the tradeoff questions are not considered reliable in this case.

As a result, the task turns into an optimization problem (see Function (2.5) below) to elicit the aggregation models from more diverse forms of preference representation (preference relation, assignment examples). Based on different problems, the input of the disaggregation methods have various forms. For example, the DM can express his preference by ranking some reference alternatives to an order, giving several pairwise comparisons or assigning a set of reference alternatives to specific categories. The disaggregation methods produce divergent results depending on the aggregation methods employed. For value function based models, the disaggregation methods infer the weights and the marginal value functions. For outranking methods, the parameters (weights, thresholds, etc.) involved are determined by the disaggregation methods. For rule-based aggregation methods, the output of disaggregation methods takes the form of some decision rules, which include a condition and a conclusion of the rules.

Let us denote the reference alternatives in the input information A^*, $E(A^*)$ the evaluation of alternatives in A^* according to the DM while $M(A^*)$ representing the evaluation of these alternatives based on the inferred model. f is a function which measures the distance of the two evaluations. Pa^* stands for the inferred parameters by minimizing such distance function of the two evaluations.

$$Pa^* = \min f(M(A^*), E(A^*)) \tag{2.5}$$

Through the solution of the optimization problem (2.5), the inferred parameters are supposed to establish an aggregation model which expresses the DM's preference. However, it is not always true in reality due to a number of reasons.

Firstly of all, unlike direct elicitation techniques, the disaggregation methods only constraint the parameters to some feasible value space. The decision examples may be too few, the knowledge of reality is usually uncertain or imprecise, so the space is not sufficiently constrained. Then the optimal solution of (2.5) can only be an arbitrarily chosen set of parameters in this space. Although the solution is good enough which means the inferred model is able to reproduce the DM's decision examples without any error, the model is rather unlikely to conform to the DM's preference. This problem is due to the multiple solutions of the optimization problem. Roy (2010) uses the term "robust" referring to a capacity for withstanding "vague approximations" and/or "zones of ignorance " in order to prevent undesirable impacts, notably the degradation of the properties to be maintained.

Secondly, the optimization program can result in an empty feasible space. This may be due to the fact that the DM's preference involves some noisy statements, or that the well-defined aggregation model doesn't fit into the real-life context. We refer to this situation as "inconsistency". The first reason of inconsistency requires the DM to modify his statements to make them compatible with each other. In the second case, if the DM is not willing to change his preference judgements, the preference model should probably be re-considered.

In the disaggregation framework, three kinds of algorithmic tools are then needed to deal with the above issues for supporting the decision aiding process.

- Disaggregation/inference procedure: inferring a single set of parameters from the possible value space based on a specific criterion, which is the loss function in Equation (2.5). The function used is not a universally accepted criterion. Take ranking problems as an example, measurements such as spearman's footrule (Spearman, 1987) which measures the sum of absolute differences between ranks, kendall's distance (Kendall, 1938) can be employed.

- Elicitation and computation of robust results: recommendations should be given while taking into account the ambiguity of preference information. For example, for a sorting problem, it is possible to assign one alternative to several categories considering all preference parameters compatible with the information provided by the DM rather than only to one single category using a determined sorting model.

- Inconsistency detection and resolution: to detect the confliction of the preference information when no preference model is compatible with the DM's preference information. Suggestions should be provided either to modify his statements so that the preference can be represented by a model, or to use another more suitable model to express his preferences.

2.4 Indirect Preference Elicitation Methods: Implementations

We introduce the implementations of indirect preference elicitation methods (disaggregation methods) and corresponding tools for the three different MACP approaches (see Section 2.2).

2.4.1 Value based Models

Value based models consist in defining explicitly an unique criterion which associates a value synthesizing m criteria to each alternative. Formally, for an alternative a, we have $v(a) = v[g_1(a), \ldots, g_m(a)]$.

The pairwise comparisons $a \succ b$ and $a \sim b$ are conveniently expressed by

$$a \succ b \Leftrightarrow v(a) > v(b)$$

$$a \sim b \Leftrightarrow v(a) = v(b)$$

For a sorting problem, the DM could express his preference as: "I think this alternative should be assigned to the best category". Let us define the aim of the sorting problem is to assign alternatives to ordered categories $C_1, C_2, \ldots, C_h, \ldots, C_k$, and l_{h-1} (l_h resp.) the maximum (minimum resp.) value of an alternative in C_h. Then this kind of preference statements can be modeled by the following condition:

$$a \rightarrow C_h \Leftrightarrow l_{h-1} \leq v(a) \leq l_h$$

Jacquet-Lagreze and Siskos (1982) proposed the UTA method which infers additive value functions from a given ranking of a reference set A^*. Without loss of generality, the alternatives are assumed to be rearranged from the best alternative to the worst one, so the alternative set becomes $A^* = \{a_1, a_2, \ldots, a_{na}\}$. We present the optimization model here as it can be seen as the general philosophy of the disaggregation method.

$$\begin{aligned}
\min \quad & \sum_{e=1}^{na} \sigma(a_e) \\
\text{s.t.} \quad & v(a_e) - v(a_{e+1}) + \sigma(a_e) - \sigma(a_{e+1}) \geq \varepsilon \quad \forall a_e \succ a_{e+1} \\
& v(a_e) - v(a_{e+1}) + \sigma(a_e) - \sigma(a_{e+1}) = 0 \quad \forall a_e \sim a_{e+1} \\
& v(a_*) = 0 \quad v(a^*) = 1 \\
& \sigma(a_e) \geq 0 \quad \forall a_e \in A^*
\end{aligned} \tag{2.6}$$

The error variables $\sigma(a_e)$ and $\sigma(a_{e+1})$ stand for the overestimation and underestimation errors. a_* (a^* resp.) is the ideal (anti-ideal resp.) alternative, which is defined as $a_* =$

$(g_1(a)_*, g_2(a)_*, \ldots, g_m(a)_*)$ ($a^* = (g_1(a)^*, g_2(a)^*, \ldots, g_m(a)^*)$ resp.). ε is an arbitrary small positive number. The program minimizes the sum of error variables to identify a value function which restores the DM's preference order as much as possible.

Many extensions have been proposed based on UTA method (see Siskos et al., 2005, for a review). Figueira et al. (2009) and Oral and Kettani (1989) consider the intensity of the DM's preference statements. UTADIS (UTilités Additives DIScriminantes) (see Doumpos and Zopounidis, 2002, 2004) and MHDIS method (Zopounidis and Doumpos, 2000) are proposed to deal with sorting problems. The UTA method is also incorporated to the solution of multiobjective programming problems (e.g., Siskos and Despotis, 1989).

For Choquet integral considering the interactions between criteria, Marichal and Roubens (2000) develop a disaggregation method to implement the method on the basis of the knowledge of a partial ranking over a reference set of alternatives (prototypes), a partial ranking over the set of criteria, and a partial ranking over the set of interactions between pairs of criteria (see Grabisch et al., 2008, for a review).

Other formulation of optimization criterion can be used in the optimization problem (2.6), such as Kendall's τ Kendall (1938). Interested readers can see different formulations in (Siskos et al., 2005). Apparently, finding solutions with different criteria will result in different solutions of the optimization program, as there may exist multiple optimal solutions. Jacquet-Lagrèze and Siskos (2001) proposed a post-optimality analysis based on heuristic method for near optimal solutions search. An alternative way to address this issue is to use fuzzy relations based on the results of the UTA models (Siskos, 1982). The fuzzy outranking relation is characterized by a membership function which represents the degree of credibility of the fuzzy relation (S.A. and Orlovsky, 1978).

Greco et al. (2008b) propose UTA$^{\text{GMS}}$ to consider the whole set of additive value functions compatible with pairwise comparisons as input preference information. The output of the model are the necessary weak preference relation which holds for any two alternatives a, b from set A if and only if for all compatible value functions a is preferred to b, and the possible weak preference relation which holds for this pair if and only if for at least one compatible value function a is preferred to b. These relations establish a necessary and a possible ranking of alternatives from A, being, respectively, a partial preorder and a strongly complete relation. Later, the method was extended in Figueira et al. (2009) which takes into account the intensities of preference among alternatives. Greco et al. (2010b) proposed to select a most representative value function from all compatible ones to deal with the robustness issue.

2.4.2 Outranking Methods

To implement outranking methods, disaggregation approaches are considered as suitable to set the values of involved parameters (for example, weights, thresholds, etc). Compared with value based models, outranking models have more parameters, which make the disaggregation rather complex.

The parameters to be elicited in ELECTRE TRI (Yu, 1992) include category limits, discrimination and veto thresholds, weights and majority level λ (see Chapter 3 for the definitions of these parameters). Mousseau and Slowiński (1998) firstly propose to infer all parameters of ELECTRE TRI from decision examples. The method uses a set of mathematical constraints to model the holistic preference judgements. A non-linear programming problem is then solved to infer simultaneously all parameters of ELECTRE TRI.

Later, some simplifications make it possible to infer partially the parameters by linear programming technique. Assuming the category limits, discrimination and veto thresholds are known, Mousseau et al. (2001a) infer the weights and majority level λ through linear programming. Another work pursues the idea of partial inference by considering the complementary subproblem which determines the category limits (the weights being fixed) by solving a linear programming problem. Dias and Mousseau (2006) complement the above work by inferring veto-related parameters given fixed values of the remaining parameters. The proposed method is also possible to be used for the inference of ELECTRE III. In Mousseau and Dias (2004), a slight adaptation of the valued outranking relation is proposed to preserve the original discordance concept and the modified outranking relation makes it easier to solve inference programs.

Recently, researchers consider evolutionary algorithms to elicit the parameters of outranking methods. Doumpos et al. (2009) propose a differential evolutionary algorithm to infer ELECTRE TRI. Leyva López et al. (2008) build a fuzzy outranking relation using ELECTRE III and then uses a genetic algorithm or a multiobjective evolutionary algorithm to exploit the relations to obtain a recommendation.

As far as robustness is concerned, Dias et al. (2002) combines the inference and robustness algorithm together. Besides inference of a set of parameters which best match the preference information, the proposed interactive approach considers all feasible value space defined by the constraints stemming from decision examples and computes the best and worst categories of each alternative compatible with these constraints. The robust assignment increases the insight of the DM into the model during the elicitation process. Tervonen et al. (2009) carry on a stability analysis for the inference of ELECTRE TRI method using SMAA-TRI (Stochastic Multicriteria Acceptability Analysis) based on Monte Carlo simulation method. The finite space of arbitrarily distributed parameter values is analyzed to compute the share of parameter values that have a given alternative assigned to a given category. The analysis result can be seen as robust conclusions of the assignments. Recently, Greco et al. (2011) present a new method ELECTRE$^{\mathrm{GKMS}}$ which employs robust ordinal regression to construct a set of outranking models compatible with preference information (pairwise comparisons). The compatible models allow to define two kinds of relations, the necessary and possible outranking relation. The method is supposed to be used with the DM interactively. During the elicitation process, the DM should be able to provide more and more pairwise comparisons with the inspirations of robustness computation. The increase of pairwise comparisons enriches the necessary relations and impoverishes the possible relations.

Roy (2010) proposes three measurements of robustness in order to find a solution of the optimization problem in a more reasonable way. The DM is supposed to give some boundaries of robustness level. Three ways of achieving robust conclusions are then suggested: perfectly robust conclusions, approximately robust conclusions and pseudo robust conclusions. By considering different robustness measurements, the responses of robustness concern can be more diverse.

The inconsistency issue has been investigated by Mousseau et al. (2003b). The inconsistency resolution algorithm suggests to delete a subset of constraints which are drawn from preference information, and the system becomes consistent after such deletion. These suggestions help the DM and the analyst to identify the pieces of conflicting information. The method is not restricted to outranking models but can be used in a general context where constraints representing preference information are conflicting. Later, Mousseau et al. (2006) extend the above method specifically for sorting problems. The extension concerns the possibility to relax (rather than to delete) assignment examples. The confidence attached to each assignment example is also taken into account.

2.4.3 Rule based Models

In the framework of disaggregation approaches, preference information is represented as value functions in value based models or as parameters in relational models, by minimizing some loss functions which measure the differences in the evaluations of the alternatives produced by the model and perceived by the DM. Using the similar philosophy, decision rules are induced from decision example to construct rule based models, which however don't involve explicit parameters as in the other two types of models. Instead, DRSA produces decision rules which transparently describe the preference information. The weights of importance and interaction of criteria are calculated from data.

One important advantage of the approach is its ability to handle inconsistent information, as the rough set approach is more general than other aggregation approaches (Greco et al., 2004). The inconsistent information is represented in the decision rules. For example, during the exploitation process for ranking problems, four kinds of situations may happen: *"true"*, *"false"*, *"contradictory"*, or *"unknown"* outranking relation. These relations are based on four-valued logic proposed in Tsoukiàs and Vincke (1995). Another advantage is that we can get more insight into the problem as the importance and dependencies of criteria are computed based on the characterizations of performance table.

Taking into account fuzzy evaluations and even missing values as input preference information, DRSA infers decision rules in a robust way in the sense that each rule is matched by at least an object.

2.5 Our Concerns and Conclusions

Although we have presented various preference elicitation techniques in the literature, no elicitation tool has been developed for many aggregation methods. The lack of elicitation tools restrains the application of the methods to real life decision problems, as incorporating the DM's preference in the aggregation models is an inevitable step in the decision aiding process. We are interested in the indirect elicitation methods (disaggregation methods) which are considered as more appropriate than direct ones. However, the disaggregation of preference information is not easy because it consists of representing the DM's preference statements meaningfully in the aggregation models, and often requires to use complex optimization algorithms. Moreover, the preference information of the DM can be rather limited, ambiguous, unstable and conflicting, which increases the difficulty. Another concern is that the disaggregation algorithm should be able to scale well in terms of computation time when large scale problems are considered.

The present thesis concentrates on the development of indirect preference elicitation tools for two specific aggregation methods. More precisely, two aggregation methods based on reference points are considered, namely ELECTRE TRI and RMP, which will be introduced in Chapter 3. The rest of the thesis presents our work on developing usable tools for such two methods.

Chapter 3

Aggregation Models based on Reference Points

This chapter is devoted to the introduction of aggregation models involved in this thesis: two aggregation models based on reference points. More precisely, we are interested in the widely used sorting model ELECTRE TRI and a newly developed ranking method both based on the decision rule of comparing alternatives to some reference points. Firstly, theoretical foundations of a simplified version of ELECTRE TRI is given. The standard version of ELECTRE TRI method which considers the imprecise nature of the preference information is then presented. Secondly, we introduce a Ranking method based on Multiple reference Points (RMP). Moreover, the characteristics and axiomatic foundations of this method are discussed to help understand the method. The main work of this thesis consists in investigating algorithmic and procedural aspects of preference elicitation concerning the two methods.

3.1 Introduction

3.1.1 Reference Dependent Preference

The behavior of people preferring to remain the current situation against some achievable improvements is described as "status quo bias". Knetsch (1989) carried out an experiment in which two groups of undergraduate students were asked to make decisions in two different scenarios. The first group were given a decorated mug while the other group received a large bar of Swiss chocolate. The costs of the two gifts (mug and chocolate) were only slightly different. The first group were offered the opportunity to change their gifts to chocolate, while the second were offered the possibility to change to mug. It turned out that approximately 90% of the participants kept their previous gifts no matter what they already owned were mugs or chocolates. There are also other experiments which show evidence of "status quo bias " (see Samuelson and Zeckhauser, 1988).

Loss aversion effect means that the negative impact on people of losing a good thing he already owns is greater than the positive impact of obtaining the same thing (see Tversky and Kahneman, 1991, for rigorous experiments). As Samuelson and Zeckhauser (1988) point out, loss aversion implies the status quo bias in the sense that people prefer to remain the current state because of the dislike of loss. However, status quo bias can be explained by costs of thinking, transaction costs and psychological commitment to previous choices without involving loss aversion.

These psychological evidences reveal the phenomenon that people make their decisions depending on some reference points, which can be explained by the willingness to remain in status quo status or the expectation of the outcome. With reference points, the preference of two alternatives can be reversed when the reference point is changed. The reference point can be referred to as the initial state of the DM, his expectation, or comes from some social comparison (Tversky and Kahneman, 1991).

3.1.2 MCDA Methods based on Reference Points

The concept of reference points is firstly introduced in the domain of psychology, sociology (Tversky and Kahneman, 1991) and social choice theory (Sen, 1986). Some MCDA methods have been proposed using the idea of reference points. For instance, TOPSIS method evaluates an alternative by measuring its Euclid distance to an ideal and anti-ideal point (Chen, 2000). If an alternative is closer to the ideal point and more far from the anti-ideal point, the alternative is more preferred. Another example is ELECTRE TRI (Yu, 1992), an outranking sorting method, compares alternatives with ordered reference points which represent the lower and upper bound of the categories. For rule-based methods, Dominance-based Rough Set Approach (DRSA) assigns alternatives to categories based on decision rules with condition parts which are partial reference profiles (Greco et al., 2001a). Recently, Rolland (2008) proposes a new ranking method, in which the preference relation of two alternatives is based on the way they compare with some reference points. Such a method is called Ranking method with Multiple reference Points (RMP). The output of the method is a relatively precise ranking of the alternatives (ties may exist) rather than a classification.

We are interested in two particular reference based aggregation methods: ELECTRE TRI and RMP. More precisely, this thesis concentrates on preference elicitation issues for such two methods. This chapter is devoted to the basis of our work: the introduction of the two methods and their axiomatic foundations. In section 3.2, we introduce ELECTRE TRI in detail and give its theoretical foundations which have been studied by Bouyssou and Marchant (2007a,b). Section 3.3 introduces the RMP model and its characterization. With this chapter, the readers should be able to understand the logic of the two aggregation methods and the definitions of their parameters to be elicited.

3.2 ELECTRE TRI Sorting Model

A simplified version of ELECTRE TRI method has been fully characterized in Bouyssou and Marchant (2007a,b). We recall the main results of these two papers to show the theoretical foundations of ELECTRE TRI . The detailed presentation of the more complicated version is given afterwards.

3.2.1 Axiomatic Foundations

Notations

We consider a finite set of alternatives A evaluated on a set of criteria $g_1, g_2, ..., g_m$, X_j being the set of evaluation levels of the associated scales of the criteria g_j. Let M denote the set of the indices of the criteria. We define a p-fold partition (C_1, C_2, …, C_p, $P = \{1, 2, \ldots, p\}$), without loss of generality, we assume C_1 contains the least desirable alternatives, and C_p contains the most preferred ones. The nonempty subset of criteria M is denoted by J. The union of categories which are more preferred by C_h is represented by $C_{\geqslant h}$, while the union of categories which are less preferred by C_h is represented by $C_{\leqslant_h}$. The set of criteria in (out of resp.) subset J is defined as $\prod_{j\in J} X_j$ ($\prod_{j\notin J} X_j$ resp.), denoted by X_J (X_{-J} resp.). (x_J, y_{-J}) represents the element $w \in A$ such that $w_j = x_j$ if $j \in J$ and $w_j = y_j$ otherwise. When $J = \{j\}$, X_J is written as X_j while (x_J, y_{-J}) is simplified as (x_j, y_{-j}).

Conjoint Measurement to Deal with Partitions

The theory of conjoint measurement was independently discovered by the French economist Debreu (1959) and by the American mathematical psychologist and statistician Luce and Tukey (1964). Its aim is to provide measurement techniques that would be adapted to the needs of Social Science in which, most often, multiple dimensions have to be taken into account (Bouyssou and Pirlot, 2005). The theory examines the conditions under which a relation on a set of objects described by a vector of evaluations can be determined by a sort of synthetic measurement that takes the relevant attributes of the objects into account in an appropriate manner. MCDA adopts the theory to study the aggregation of preferences. For a decision problem, the measurements are not intrinsic properties of the objects, but reflect the DM's subjective preferences (Bouyssou et al., 2006).

Conjoint measurement is used here to deal with partitions. $X = X_1 \times X_2 \times \ldots X_j \ldots \times X_m$ is a set of objects, where X_j is the scale associated with criterion g_j, $j \in M$. Let us define $< C_h >_{h\in R}$ a partition of X. A real-valued function v_j is defined on X_j and f is a real-valued function on $\prod_{j=1}^{m} v_j(X_j)$ that is increasing in all its arguments. Equation (3.1) is a conjoint model (we call M1) in which $\sigma_1, \sigma_2, \ldots, \sigma_{p+1}$ are real numbers such that $\sigma_1 < \sigma_2 < \ldots < \sigma_{p+1}$. The weakening of M1 in which f is only supposed to be nondecreasing in all its arguments will be called M2. For all $x \in X$,

$$x \in C_h \Leftrightarrow \sigma_h < f(v_1(x_1), v_2(x_2), \ldots, v_m(x_m)) < \sigma_{h+1} \tag{3.1}$$

On every criteria g_j, a preference relation $\succsim_j$ is defined such that,

$$x_j \succsim_j y_j \Leftrightarrow [\text{for all } a_{-j} \in X_{-j} \text{ and all } h \in P, (y_j, a_{-j}) \in C_h \Rightarrow (x_j, a_{-j}) \in C_{\geqslant h}] \tag{3.2}$$

The partition $\{C_1, C_2, \ldots, C_p\}$ is considered to be P-linear on criteria $j \in M$ if, for all $x_j, y_j \in$

X_j, all h, and all $a_{-j}, b_{-j} \in X_{-j}$,

$$\left.\begin{array}{c}(x_j, a_{-j}) \in C_h \\ \text{and} \\ (y_j, b_{-j}) \in C_l\end{array}\right\} \Rightarrow \left\{\begin{array}{c}(y_j, a_{-j}) \in C_{\geqslant h} \\ \text{or} \\ (x_j, b_{-j}) \in C_{\geqslant l}\end{array}\right. \tag{3.3}$$

If for all criteria $j \in M$ the partition is linear, the partition $\{C_1, C_2, \ldots, C_p\}$ is said to be P-linear.

Bouyssou and Marchant (2007b) give the necessary and sufficient conditions of P-linearity as follows.

Lemma 3.1 *A partition $\{C_1, C_2, \ldots, C_p\}$ is P-linear iff $\succsim_j$ is complete, for all $j \in M$.*

They also connect the function u_j with the binary relation $\succsim_j$ by Proposition 3.1

Proposition 3.1 *A partition $\{C_1, C_2, \ldots, C_p\}$ has a representation in M1 (Equation 3.1) iff it is P-linear and for all $j \in M$, there is a finite or countable set $X'_j \subseteq X_j$ that is dense in X_j for $\succsim_j$. Furthermore,*

- *if $\{C_1, C_2, \ldots, C_p\}$ has a representation in M1 (Equation 3.1), it has a representation in which, for all $j \in M$, v_j is a numerical representation of $\succsim_j$,*
- *models (M1) and (M2) are equivalent.*

Model M1 (Equation 3.1) is the general form of many sorting models. For example, when the measurements on the criteria conform to an additive model, for all $x \in A$,

$$f(v_1(x_1), v_2(x_2), \ldots, v_m(x_m)) = \sum_{j=1}^{m} v_j(x_j) \tag{3.4}$$

When the value function f takes the form of Equation (3.4), it becomes the sorting model UTADIS (Zopounidis and Doumpos, 1999a; Chen, 2000). There are also other sorting models contained in Equation (3.1), such as the pessimistic version of ELECTRE TRI, decision rule based sorting models, etc.

A noncompensatory sorting model is defined if the following conditions are satisfied:

- for each criterion $j \in M$, there are sets $\mathscr{A}_j^h, \mathscr{V}_j^h \subseteq X_j$ such that:

  1. for all $j \in M$, $\mathscr{A}_j^p \subseteq \mathscr{A}_j^{p-1} \subseteq \ldots \subseteq \mathscr{A}_j^2$,

2. for all $j \in M$, $\mathscr{V}_j^p \subseteq \mathscr{V}_j^{p-1} \subseteq \ldots \subseteq \mathscr{V}_j^2$,
3. for all $h_1, h_2 \in P$, if $h_1 < h_2$, $x_j \in \mathscr{A}_j^{h_1}$, $y_j \in \mathscr{U}_j^{h_1}$, and $x_j \in \mathscr{V}_j^{h_2}$, then $y_j \in \mathscr{V}_j^{h_2}$, where $\mathscr{U}_j^h = X_j \backslash [\mathscr{A}_j^h \cup \mathscr{V}_j^h]$

- there is a subset $\mathscr{F}$ of 2^M ($\mathscr{F}^p \subseteq \mathscr{F}^{p-1} \subseteq \ldots \subseteq \mathscr{F}^2$) such that, for all $h \in P$, and all $I, J \in 2^M, [I \in \mathscr{F}^h$ and $I \subseteq J] \Rightarrow J \in \mathscr{F}^h$,

$$x \in C_{\geqslant h} \Leftrightarrow [\{j \in J : x_j \in \mathscr{A}_j^h\} \in \mathscr{F}^h \text{ and } \{j \in M : x_j \in \mathscr{V}_j^h\} = \varnothing] \tag{3.5}$$

The interpretation of the partition of X_j is as follows. The set X_j is partitioned by several subsets $\mathscr{A}_j^h$ (for all $j \in J, h \in P$) which contains the elements of X_j that are considered desirable for an alternatives to be assigned at least to C_h. To assign an alternative to $C_{\geqslant h}$, x_j the evaluation of x should belong to the subset $\mathscr{A}_j^h$ on a sufficiently important coalition of criteria, which is $\mathscr{F}^h$. $\mathscr{A}_j^h \subseteq \mathscr{A}_j^{h-1}$ means that an evaluation that is considered satisfactory for C_h should be judged as satisfactory at any category below it. Veto effect is considered also in this model. For each $h \in P$, there is a set $\mathscr{V}_j^h$ which is repulsive for $C_{\geqslant h}$. Condition 2 means that an evaluation which is repulsive for a specific category should be repulsive for all higher categories. Condition 3 can be explained as follows. If $x_j \in \mathscr{A}_j^{h_1}$, $y_j \in \mathscr{U}_j^{h_1}$, we can say x_j is preferred to y_j. So for $h_1 < h_2$, when x_j belongs to the repulsive set $\mathscr{V}_j^{h_2}$, y_j should also belong to it. Thus, (3.5) means that to assign x to $C_{\geqslant h}$, x_j ($j \in M$) need to belong to $\mathscr{A}_j^h$ on a sufficiently important coalition of criteria, and none of x_j should belong to the repulsive set $\mathscr{V}_j^h$.

The noncompensatory sorting model is characterized in detail in Bouyssou and Marchant (2007a,b) and the simplified version of ELECTRE TRI , which we will present in what follows, can be seen as a particular case of the model.

3.2.2 ELECTRE TRI

A simplified version of ELECTRE TRI : ELECTRE TRI$_{\mathbf{BM}}$

We denote the set of alternatives in ELECTRE TRI $A = \{a_1, a_2, \ldots, a_n\}$. ELECTRE TRI assigns alternatives to predefined ordered categories based on the comparisons of the alternatives with several reference limits (Roy, 1985; Yu, 1992) . The limit between two consecutive categories is formalized by what is called a profile (which is equivalent to a reference point), denoted by b_h being the upper limit of category C_h and the lower limit of category C_{h+1}, $h = \{1, 2, ..., p-1\}$.

$g_j(a)$ represent the evaluation of a with respect to criterion g_j ; hence, $g_j(a)$ is a real-valued function $g_j(a): A \to \mathbb{R}$.

We associate with every criterion g_j a degree of importance of represented by weight w_j and a veto threshold $v_j(b_h)$. The weights are assumed to be normalized, i.e, $\sum_{j=1}^{m} w_j = 1$. For all $a_i \in A$, and all $p \in P$, as a particular case of the general form of noncompensatory sorting model (3.5), ELECTRE TRI defines $\mathscr{A}^h$, the subset $\mathscr{F}$ and $\mathscr{V}^h$ as follows. $\mathscr{A}^h$ is defined as the coalition of criteria on which a is at least as good as b_h: $\mathscr{A}^h = \{j \in M : g_j(a) \geq g_j(b_h)\}$. Let us denote λ a threshold that the coalition of criteria should exceed in order to verify $\mathscr{A}^h \in \mathscr{F}$, then $\mathscr{F}$ is defined as the union of sets J which has a coalition of criteria whose importance of weights exceed λ: $J \in \mathscr{F}$ whenever $\sum_{j \in J} w_j \geq \lambda$. Now we define that any criterion g_j on which b_h is far better than a is in the set $\mathscr{V}^h, h \in P$: $\mathscr{V}^h = \{j \in M : b_h \gg_j a\}$. To define the discordance relation $\precsim_j$, $v_j(b_h)$ is used as the smallest difference which is compatible with the assertion $a \succcurlyeq b_h$ on criterion g_j with respect to limiting profile b_h. Whenever a is "bad" enough on any criterion, i.e, $g_j(b_h) - g_j(a) > v_j(b_h)$, we say $b_h \gg_j a$. Hence $\forall a \in X$ and $h \in P$,

$$a \in C_{\geqslant h} \Longleftrightarrow \sum_{j \in M : g_j(a) \geq b_{h-1}} w_j \geq \lambda \text{ and [Not } [b_{h-1} \gg_j a], \text{ for all } j \in M \text{]} \tag{3.6}$$

Defining $\mathscr{F}$, $\mathscr{A}^h$ and $\mathscr{V}^h$ amounts to determining the preference relation of alternatives and limiting profiles. The assertion $a \succcurlyeq b_h$ means "a is at least as good as b_h". Asymmetric part of $\succcurlyeq$ is denoted by $\succ$ which corresponds to strict preference ("better than" relation) and the symmetric part $\sim$ represents the indifference relation. Since $\succcurlyeq$ is usually not complete, we denote by ? the incomparability relation, i.e. $a?b_h$ if not $a \succcurlyeq b_h$ and not $b_h \succcurlyeq a$.

The version of ELECTRE TRI in Equation (3.6) implies pessimistic rule. The original presentation of ELECTRE TRI$_{\text{BM}}$ (Bouyssou and Marchant, 2007a,b) uses the pessimistic rule in the sense that such rule assigns alternative a to the highest category C_h for which $a \succcurlyeq b_{h-1}$ and not($a \succcurlyeq b_h$). It is easy to verify it is equivalent to Equation (3.6). We shall introduce how the optimistic rule works later on.

Other ELECTRE TRI versions

We have presented the simplified version of ELECTRE TRI method (called ELECTRE TRI$_{\text{BM}}$) which ignores preference and indifference thresholds involved in the standard concordance condition (Yu, 1992). To better model the DM's preference on the criteria, we associate two thresholds $p_j(b_h)$ and $q_j(b_h)$ on each criterion g_j to take into account the imprecise evaluations. $p_j(b_h)$, the preference threshold, is the smallest difference to verify the assertion a is preferred to b_h on criterion g_j, i.e, aP_jb_h. $q_j(b_h)$, the indifference threshold, is the largest difference to verify the assertion a is indifferent to b_h on criterion g_j, i.e, aI_jb_h. The verification of $a \succcurlyeq b_h$ (the definitions of $\mathscr{F}$, $\mathscr{A}^h$ and $\mathscr{V}^h$) is realized in a more complicated way.

On each criterion g_j, ELECTRE TRI builds a partial concordance index $c_j(a,b_h) \in [0,1]$ to evaluate the degree of credibility of the assertion $a \succcurlyeq_j b_h$. Such index can be computed by:

$$c_j(a,b_h) = \begin{cases} 1 \ \text{ if } \ g_j(a) + p_j(b_h) \geq g_j(b_h) \\ 0 \ \text{ if } \ g_j(a) + q_j(b_h) \leq g_j(b_h) \\ \frac{p_j(b_h)+g_j(a)-g_j(b_h)}{p_j(b_h)-q_j(b_h)} \ \text{ otherwise} \end{cases} \tag{3.7}$$

The partial concordance indices $c_j(a,b_h)$ ($j \in M$) are aggregated to a comprehensive concordance index $c(a,b_h)$ which represents the degree of credibility of the assertion $a \succcurlyeq b_h$.

$$c(a,b_h) = \sum_{j \in M} w_j c_j(a,b_h) \tag{3.8}$$

The partial discordance index $d_j(a,b_h)$ of the discordance relation $\gg$ is computed as:

$$d_j(a,b_h) = \begin{cases} 1 \ \text{ if } \ g_j(b_h) - p_j(a) > v_j(b_h) \\ 0 \ \text{ if } \ g_j(b_h) - g_j(a) \leq p(b_h) \\ \frac{g_j(b_h)-g_j(a)-p_j(b_h)}{v_j(b_h)-p_j(b_h)} \ \text{ otherwise} \end{cases} \tag{3.9}$$

The credibility index $\sigma(a,b_h)$ of the outranking relation $a \succcurlyeq b_h$ combines the concordance index $c(a,b_h)$ and the discordance index $d_j(a,b_h)$:

$$\sigma(a,b_h) = c(a,b_h) \prod_{j \in \mathscr{M}(a,b_h)} \frac{1-d_j(a,b_h)}{1-c(a,b_h)} \tag{3.10}$$

where $\mathscr{M}(a,b_h) = \{j \in M : d_j(a,b_h) \geq c(a,b_h)\}$.
The values of $c(a,b_h)$, $c(b_h,a)$ and λ determine the preference situation between a and b$_h$:

- $c(a,b_h) \geq \lambda$ and $c(b_h,a) \geq \lambda \Leftrightarrow a \succcurlyeq b_h$ and $b_h \succcurlyeq a \Leftrightarrow a \sim b_h$, i.e., a is indifferent to b_h,
- $c(a,b_h) \geq \lambda$ and $c(b_h,a) < \lambda \Leftrightarrow a \succcurlyeq b_h$ and not $b_h \succcurlyeq a \Leftrightarrow a \succ b_h$, i.e., a is preferred to b_h (weakly or strongly),

- $c(a, b_h) < \lambda$ and $c(b_h, a) \geq \lambda \Leftrightarrow$not $a \succcurlyeq b_h$ and $b_h S a \Leftrightarrow b_h \succ a$, i.e., b_h is preferred to a (weakly or strongly),

- $c(a, b_h) < \lambda$ and $c(b_h, a) < \lambda \Leftrightarrow$not $a \succcurlyeq b_h$ and not $b_h \succcurlyeq a \Leftrightarrow a?b_h$, i.e., a is incomparable to b_h.

Besides the pessimistic rule which has been discussed in Section 3.2.2 (the pessimistic rule assigns alternative a to the highest category C_h for which $a \succcurlyeq b_{h-1}$ and not $(a \succcurlyeq b_h)$), the optimistic rule is also proposed in the literature which assigns a to the lowest category C_h for which $b_h \succ a$ and not $(b_{h-1} \succ a)$. The characterization of ELECTRE TRI in Bouyssou and Marchant (2007a,b) restricts to pessimistic rule only.

Remark 2 *Bouyssou and Marchant (2007a,b) show that only the pessimistic version of* ELECTRE TRI *fits into the framework of noncompensatory sorting model. This may be linked with the fact that the optimistic version of* ELECTRE TRI *is based on the strict preference $\succ$ rather than the outranking relation $\succcurlyeq$.*

Remark 3 *The optimistic rule always assigns alternatives to higher categories than the pessimistic rule. If no incomparability relation between the alternatives and the profiles exist, the two rules yield the same result.*

It is worth pointing out that there are two ways of defining categories in a sorting problem statement. In the case of ELECTRE TRI, the categories are defined by limiting profiles indicating the limit of each category. Alternatively, the definition of categories can be norms modeled as prototypes of alternatives belonging to a category. Such definition leads to ELECTRE TRI-C and ELECTRE TRI-NC (Figueira et al., 2011; Almeida-Dias et al., 2012), which takes into account one or several reference alternatives for characterizing each category. Bouyssou et al. (2006) illustrate these two ways of defining categories in a sorting problem statement by considering the case of the evaluation of students in an academic programme. A "good" student may be defined using examples of past students in the programme. This would define the prototypes or reference of the category of "good students". Alternatively, we could define, as is done in the French bacalauréat, an average grade above which, students are considered to be "good". E.g, in the French bacalauréat on average grade above 16 on a scale going from 0 to 20 implies that the exam is passed magna cum laude (see Chapter 5 for a real-world case study where the students are evaluated by comparing them to some limiting profiles).

Now let us explain the non-compensatory nature of ELECTRE TRI to better understand such feature (which has already been mentioned in Section 2.2.2). The feature comes from two facts:

1. When computing the partial concordance index, only the fact that alternative a outranks alternative b on one criterion matters. However, the fact that a is far better than b doesn't contribute to the partial concordance index.

2. As a result of considering veto effect in outranking methods, when alternative a is far worse than alternative b, a can't outrank b no matter how good the performance of a is on other criteria compared with b.

In this thesis, Chapter 4 deals with preference elicitation for a simplified ELECTRE TRI (more precisely the optimistic rule is used). Chapter 5 applies ELECTRE TRI to portfolio selection problem.

3.3 Ranking Method based on Reference Points

3.3.1 Motivation

We can induce a ranking using the assignment of ELECTRE TRI, saying that alternative a is ranked higher than alternative b if a is classified in an upper category than the one b is assigned to. But the ranking is very rough, so here we are interested in obtaining a more precise ranking using reference points directly to determine the preference relation between two alternatives. Roughly speaking, it consists in comparing two alternatives by respectively comparing them with some reference points.

In decision theory, numerous axiomatic results show the theoretical and practical difficulties due to the aggregation of preference relations which are partially in conflict. One of the main problems of this type of methods is that transitivity of the obtained preference relation is generally not compatible with the independence of irrelevant alternatives property. This problem has been investigated extensively in social choice theory which aims at aggregating the opinions of voters on candidates to be ranked or chosen. Arrow's impossibility theorem (Arrow, 1953) highlights that it is impossible to find an aggregation procedure which satisfies several desirable properties.

Theorem 3.1 *When voters have three or more distinct alternatives (options), no voting system can aggregate the ranked preferences of individual voters into a (complete and transitive)* *ranking while satisfying no dictatorship, universality, independence, unanimity.*

- Complete order: the ranking should be a complete order of the alternatives, with ties allowed.
- Independence: the preference of two alternatives depends only on the individual preferences between them. The change of preference on other candidates should not reverse the preference relation of the two.
- No dictatorship: no voter should have the privilege to the voting result regardless of other voters' preferences.
- Universality: any preference order of the candidates is acceptable.
- Unanimity: if every individual prefers a particular candidate to another, then so must the resulting overall preference order.

Some results of social choice theory have been adapted to MCDA field because of the strong link between them (Bouyssou and Perny, 1992; Marchant, 1996). The result in MCDA corresponding to Arrow's impossibility theorem means that there exist no ideal aggregation method which can find a transitive ranking while respecting independence, no dictatorship, universality and unanimity simultaneously. Some methods relaxing one of the above properties have been studied. For example, Fishburn (1975) proposes a lexicographic aggregation method which weakens the non-dictatorship property. The collective rationality requirement in Arrow's theorem is weakened to get a social quasi-ordering (a reflexive and transitive but not necessarily complete binary relation) (Weymark, 1984) . In the ELECTRE-type outranking methods, the preference relation between the alternative is often not transitive. We try here to present a simple outranking-based method which focuses on weakening the independence condition that the preference of two alternatives is dependent on other third alternatives, i.e, the preference is based on some reference alternatives, which we refer to as reference points. The method is developed by Rolland (2008), and its potentials and properties have been studied.

3.3.2 S-RMP: a Simplified Ranking Model based on Reference Points

We concentrate here on a multi-criteria preference model which uses several reference points to rank alternatives. Formally, the model involves k reference points $p^1, p^2, \ldots, p^h, \ldots, p^k$, where

$p^h \in X$, $h \in P$, P being the set of indices of the reference points ($P = \{1, 2, ..., h, ..., k\}$). The evaluation of p^h on criterion g_j is denoted as p^h_j, $h \in P, j \in M$. The aggregation method consists in a three-step procedure as follows.

1. On each criterion g_j, comparing each alternative a_i ($i \in N$) to every reference point p^h ($h \in P$).

2. Aggregating the preference relations on m criteria to obtain a preference relation on $X \times X$ depending on the reference point p^h ($h \in P$).

3. Aggregating these k preference relations with respect to k reference points into a global preference relation.

The first step of the procedure gives preference relations between alternative a_i ($i \in N$) and reference point p^h ($h \in P$) on m criteria. In step two, we are only interested in the criteria on which a_i is at least as good as p^h. $C(a_i, p^h)$ represents the set of criteria for which the evaluation of a_i is considered as least as good as the evaluation of p^h: $C(a_i, p^h) = \{j \in M$ such that $g_j(a_i) \geq p^h_j\}$. We can then define an importance relation $\trianglerighteq$ on sets of criteria 2^M by decomposing $C(a_i, p^h)$ additively.

$$C(a_i, p^h) \trianglerighteq C(a_{i'}, p^h) \iff \sum_{j \in C(a_i, p^h)} w_j \geq \sum_{j \in C(a_{i'}, p^h)} w_j \tag{3.11}$$

As shown in Rolland (2008), an important result derived from social choice theory (Fishburn, 1975) indicates that the only importance relation which aggregates the k preference relation (with respect to k reference points) and leads to transitive relation on each possible set of alternatives is obtained by a lexicographic order on the reference points. Therefore, a permutation σ on P is used lexicographically in step three to aggregate the k preference relations. The first used reference point is denoted by $p^{\sigma(1)}$, the second one by $p^{\sigma(2)}$ and so on. To compare a_i and $a_{i'}$, we look at the first reference point $p^{\sigma(1)}$. If a_i is strictly better than $a_{i'}$ according to $p^{\sigma(1)}$, then a_i is claimed globally preferred to $a_{i'}$ without even considering the other reference points. Similarly, if $a_{i'}$ is strictly better than a_i according to $p^{\sigma(1)}$, then $a_{i'}$ is claimed globally preferred to a_i ignoring the other reference points. But if a_i and $a_{i'}$ are indifferent with respect to $p^{\sigma(1)}$, we shall look at the second reference point $p^{\sigma(2)}$. If a_i is strictly better than $a_{i'}$ according to $p^{\sigma(2)}$, then a_i is claimed globally preferred to $a_{i'}$. If we can't make a difference between a_i and $a_{i'}$ using $p^{\sigma(2)}$, we proceed with reference point $p^{\sigma(3)}$, then $p^{\sigma(4)}$ and so on until we can make a difference or until all reference points have been used. In that case, a_i and $a_{i'}$ are globally tied. Formally,

$$\begin{array}{rcl} a_i \succ a_{i'} & \Longleftrightarrow & a_i \succ_{p^{\sigma(1)}} a_{i'} \\ & \text{or} & a_i \sim_{p^{\sigma(1)}} a_{i'} \text{ and } a_i \succ_{p^{\sigma(2)}} a_{i'} \\ & \ldots & \ldots \\ & \text{or} & a_i \sim_{p^{\sigma(1)}} a_{i'} \text{ and } \ldots \text{ and } a_i \sim_{p^{\sigma(k-1)}} a_{i'} \text{ and } a_i \succ_{p^{\sigma(k)}} a_{i'} \\ a_i \sim a_{i'} & \Longleftrightarrow & a_i \sim_{p^h} a_{i'} \ \ \forall h \in P \end{array} \tag{3.12}$$

Decomposing the importance relation of subsets of criteria leads to the Simplified Ranking method with Multiple reference Points (S-RMP), in which the preference relation between two alternatives can be computed by:

$$\begin{array}{rcl} a_i \succ a_{i'} & \Longleftrightarrow & \sum_{j \in C(a_i, p^{\sigma(1)})} w_j \geq \sum_{j \in C(a_{i'}, p^{\sigma(1)})} w_j \\ & \text{or} & \sum_{j \in C(a_i, p^{\sigma(1)})} w_j = \sum_{j \in C(a_{i'}, p^{\sigma(1)})} w_j \\ & & \text{and } \sum_{j \in C(a_i, p^{\sigma(2)})} w_j \geq \sum_{j \in C(a_{i'}, p^{\sigma(2)})} w_j \\ & \ldots & \ldots \\ a_i \sim a_{i'} & \Longleftrightarrow & \sum_{j \in C(a_i, p^{\sigma(h)})} w_j = \sum_{j \in C(a_{i'}, p^{\sigma(h)})} w_j \ \ \forall h \in P \end{array} \tag{3.13}$$

There is no lack of generality to impose a dominance relation among reference points, as shown in Rolland (2008), and this relation means that $\forall i \in N, \forall h, h' \in P$, $p_j^{\sigma(h)} \geq p_j^{\sigma(h')}$ or $p_j^{\sigma(h)} \geq p_j^{\sigma(h')}, \forall j \in M$.

3.3.3 RMP: a General Ranking Model based on Reference Points

S-RMP model presented above is in fact a very specific model of the general Ranking model based on Multiple reference Points (RMP). The three-step procedure of S-RMP can be generalized by generalizing the aggregation procedures of step 2 and 3. Different ways of generalization can lead to numerous variants of the general model. We introduce here Rolland's work on the general RMP model, particularly the one based on the concept of outranking (Rolland, 2008).

To aggregate the m preference relations between an alternative and a reference point on m criteria, let us consider the importance relation on the set of criteria with respect to the reference point p^h denoted by $\trianglerighteq_{p^h}$ on subsets of 2^M. We can then induce a preference relation $\succsim_{p^h}$ on $X \times X$ such that:

$$a_i \succsim_{p^h} a_{i'} \iff C(a_i, p^h) \trianglerighteq_{p^h} C(a_{i'}, p^h) \tag{3.14}$$

This preference relation $\succsim_{p^h}$ expresses how pairs of alternatives compare with respect to the reference point p^h. This importance relation $\trianglerighteq_{p^h}$ on the sets of criteria can be rather general and can take different forms. In fact, all set functions are formally acceptable to be used as importance relation $\trianglerighteq_{p^h}$. For example, it can be based on a weighted sum of weights as in outranking relations like ELECTRE (Roy, 1996), a capacity function as in a Choquet integral (Grabisch and Roubens, 2000), or a general importance relation as in a general concordance relation (Dubois et al., 2003). The S-RMP considers $\trianglerighteq_{p^h}$ $(h \in P)$ as identical for all reference points, and the unique importance relation $\trianglerighteq$ is defined by a concordance rule (see Section 3.3.2).

On the basis of the relations $\succsim_{p^h}$ $(h \in P)$, when comparing two alternatives $a_i, a_{i'} \in A$, we consider $P(a_i, a_{i'}) = \{h : a_i \succsim_{p^h} a_{i'}\}$ the set of indices of reference points with respect to which a_i is at least as good as $a_{i'}$. Finally, we can define a preference relation $\succsim$ on alternatives of A as follows :

$$a_i \succsim a_{i'} \iff P(a_i, a_{i'}) \succsim_P P(a_{i'}, a_i) \tag{3.15}$$

We denote $\succsim_P$ an importance relation on coalitions of reference points. In other words, a_i is at least as good as $a_{i'}$ if the set of reference points with respect to which a_i is at least as good as $a_{i'}$ is more important than the set of reference points with respect to which $a_{i'}$ is at least as good as a_i. Similarly, the importance relation $\succsim_P$ on the set of reference points can also be build based on different models: majority method, weighted sum, capacity function, etc. Particulary, S-RMP model uses a lexicographic order of dictatorial reference points as importance relation $\succsim_P$ on subsets of reference points 2^P.

3.3.4 Characterization of the Method

Rolland (2008) provided the axiomatic foundations of RMP model. We recall the main results here to help understand the method and its properties. To facilitate the characterization, let us denote A^h the set $C(a, p^h) = \{j \in M, g_j(a) \geq p^h_j\}$ which represents the set of criteria on which alternative a is at least as good as p^h.

Axiom 1 *Conditional Independence with respect to reference points (CIP)*

$$\begin{bmatrix} (A^1,\ldots,A^k) = (C^1,\ldots,C^k) \\ (B^1,\ldots,B^k) = (D^1,\ldots,D^k) \end{bmatrix} \Rightarrow a \succsim b \Leftrightarrow c \succsim d \tag{3.16}$$

The axiom defines the CIP property that the preference relation of two alternatives depends only on how they compare to all reference points, and is independent on any other alternatives.

The necessary and sufficient conditions for the existence of an importance relation $\succsim_P$ defining the preference relation $\succsim$ are given as follows.

Theorem 3.2 *If the preference relation $\succsim$ satisfies CIP, then there is an importance relation $\succsim_P$ on $(2^M)^p$ such that*

$$a \succsim b \Leftrightarrow (A^1,\ldots,A^k) \succsim_P (B^1,\ldots,B^k) \tag{3.17}$$

There are two main approaches to obtain the global preference information. The first one is to aggregate the values on all criteria to an unique utility and then to compare the utilities to obtain global preference relation. The second approach compares alternatives on each criterion to get partial preference relations, which are then aggregated into global preference relation. Here, we only consider the second approach which uses the concordance rule.

The standard concordance rule consists in comparing the set of criteria where alternative a is at least as good as alternative b and the set of criteria where alternative b is at least as good as alternative a by an importance relation on the sets of criteria. Following a generalized concordance rule on the subset of criteria and the subsets of P, the preference relation $\succsim$ of two alternatives in Equation 3.17 is defined as:

$$a \succsim b \Leftrightarrow \{h \in P \mid A^h \succsim'_h B^h\} \succsim_P \{h \in P \mid B^h \succsim'_h A^h\} \tag{3.18}$$

Where $\succsim'_h$ are importance relations on the subsets of M and $\succsim_P$ is an importance relation on the subsets of P.

To obtain the preference relation (denoted by $\succsim_h$) of alternatives with respect to a specific reference point p^h using only the importance relations ($\succsim'_h$), the preference $\succsim$ should satisfy SEP condition defined as follows.

Axiom 2 *Separability with respect to reference points (SEP)*

$$\begin{bmatrix} A^h = C^h & A^{h'} = B^{h'} & \forall h' \neq h \\ B^h = D^h & C^{h'} = D^{h'} & \forall h' \neq h \end{bmatrix} \Rightarrow a \succsim b \Leftrightarrow c \succsim d \tag{3.19}$$

With SEP condition, we can compare subsets of criteria to obtain the preference relation $\succsim_h$:

$$a \succsim_h b \Leftrightarrow A^h \succsim'_h B^h \tag{3.20}$$

Then it is necessary to aggregate the k preference relations $\succsim_h$ ($h = \{1, 2, \ldots, k\}$) to get the global preference relation $\succsim$. To do so, the condition CIIR should be satisfied.

Axiom 3 *Conditional Independence with respect to the Induced Relations (CIIP): let $\succsim$ be a preference relation satisfying axiom SEP and the induced relations $\succsim_h$ ($h = \{1, 2, \ldots, k\}$.*

$$\begin{bmatrix} \forall h \in P, & \begin{matrix} a \succsim_h b \Leftrightarrow c \succsim_h d \\ b \succsim_h a \Leftrightarrow d \succsim_h a \end{matrix} \end{bmatrix} \Rightarrow a \succsim b \Leftrightarrow c \succsim d \tag{3.21}$$

Now the preference relation in Equation 3.18 can be characterized.

Theorem 3.3 *If the preference relation $\succsim$ satisfies SEP and CIIR, then there exist k importance relation $\succsim'_h$ on the subsets of M and an importance relation $\succsim_P$ on the subsets of P such that*

$$a \succsim b \Leftrightarrow \{h \in P \mid A^h \succsim'_h B^h\} \succsim_P \{h \in P \mid B^h \succsim'_h A^h\}$$

We define several properties preference relation $\succsim$ is supposed to satisfy, including unanimity (UNA), Richness of the framework (RICH).

Axiom 4 *Unanimity*

$$\forall a, b \in A, [\forall j \in M, a \succsim_j b] \Rightarrow a \succsim b \tag{3.22}$$

Axiom 5 *Richness of the framework*

1. *$\exists a, b \in A$ such that $a \succ b$*

2. *$\forall h \in P$, there exists $a, b \in A$, $a \succ_{p^h} b$*

Arrow proves that the aggregation method which leads to a transitive global preference is a dictatorship of one criterion. Similarly, Rolland (2008) proves the following theorem in the framework of multiple reference points.

Theorem 3.4 *If the aggregation procedure considers at least 3 reference points, and the preference relation $\succsim$ satisfies*

- *SEP and CIIR*
- *RICH and UNA*

then there exist a reference point $h \in P$ such that

$$\forall a, b \in A, a \succ_{p^h} b \Leftrightarrow a \succ b$$

Theorem 3.4 shows that the only aggregation procedure which produces a transitive global preference relation is the dictatorship of a single reference point. When a reference point fails to differentiate two alternatives, i.e, the two alternatives are indifferent with respect to the reference point, other reference points should be considered. A lexicographic principle (Fishburn, 1975) is applied to the aggregation procedure, and we denote the first reference point $p^{\sigma(1)}$, the second one $p^{\sigma(2)}$ and so on. Formally,

$$a \succ b \iff a \sim_{p^{\sigma(h')}} b \text{ and } a \succ_{p^{\sigma(h)}} b \quad \forall h' < h \tag{3.23}$$

The above formula is interpreted as follows. The lexicographic principle only considers the reference point $p^{\sigma(h)}$ when all reference points lexicographically before it ($p^{\sigma(h')}, h' < h$) judges the two alternatives as indifferent. If the two alternatives are indifferent with respect to all reference points, they are said to be indifferent.

Theorem 3.5 gives the necessary and sufficient conditions of the existence of the lexicographic order on reference points.

Theorem 3.5 *If the aggregation procedure considers at least 3 reference points. The preference relation $\succsim$ satisfies*

- *SEP and CIIR*

- *is a complete pre-order satisfying a strict unanimity axiom on the reference points:* $\forall a, b \in A$,

$$\left[\begin{array}{l} \forall h \in P,\ \ a \succsim_h b \\ \exists h \in P,\ \ a \succ_h b \end{array}\right] \Rightarrow a \succ b$$

Then there exists an order $\{\sigma(1), \ldots, \sigma(k)\}$ *on reference points* $p^1, \ldots, p^k$ *and k preference relations* $\succsim'_{p\sigma(h)}$ *on the subsets of criteria such that*

$$\begin{array}{rcl} a \succ b & \Longleftrightarrow & A^{\sigma(1)} \succ'_{p^{\sigma(1)}} B^{\sigma(1)} \\ & or & A^{\sigma(1)} \sim'_{p^{\sigma(1)}} B^{\sigma(1)} \ \ and\ \ A^{\sigma(2)} \succ'_{p^{\sigma(2)}} B^{\sigma(2)} \\ & \ldots & \\ & or & \forall h' < h, A^{\sigma(h')} \sim'_{p^{\sigma(h')}} B^{\sigma(h')} \ \ and\ \ A^{\sigma(h)} \succ'_{p^{\sigma(h)}} B^{\sigma(h)} \\ a \sim b & \Longleftrightarrow & \forall h \in P,\ \ A^{\sigma(h)} \sim'_{p^{\sigma(h)}} B^{\sigma(h)} \end{array}$$

As discussed in Section 3.2, there are two assignment rules of ELECTRE TRI, according to the order of comparing alternatives to reference profiles in the assignment procedure (from the lowest point to the highest or the other way around). The lexicographic order of RMP can take many possible orders, such as top-down, bottom-up, etc.

3.4 Conclusion

We have introduced two outranking methods, ELECTRE TRI and RMP, based on a specific type of preference, namely preference based on reference points. ELECTRE TRI deals with sorting problems, while RMP handles ranking problems. Many parameters, whose definitions and meanings have been provided in this chapter, are involved in the two models. To implement ELECTRE TRI, we need to set values of weights, profiles, majority level and thresholds (depending on whether thresholds are considered or not). To construct a S-RMP model, the reference points, their lexicographic order and importance weights of criteria have to be determined. All these parameters should be set meaningfully based on the DM's preferences, therefore preference elicitation tools are required. In the succeeding chapters, we attempt at developing such tools for the two aggregation methods based on reference points.

Chapter 4

Preference Elicitation for ELECTRE TRI using the Optimistic Assignment Rule

Multiple criteria sorting problems aim at assigning alternatives to predefined ordered categories considering multiple criteria. We consider a widely studied and used sorting method ELECTRE TRI. As is presented in chapter 3, two assignment rules have been proposed in the method (the so-called pessimistic rule and optimistic rule). To define an ELECTRE TRI model involving several parameters, the approach to learn the model from decision examples of alternatives that should be assigned to specific categories has been investigated carefully as far as the pessimistic rule is considered. However, no corresponding tool exists for ELECTRE TRI using the optimistic rule. We tackle the difficulties of preference elicitation for ELECTRE TRI using the optimistic rule. Algorithms are proposed to elicit parameter values and compute corresponding robust assignment from assignment examples through solving Mixed Integer Program (MIP). Moreover, several numerical experiments are conducted to test the performance of the algorithms with respect to the issues including the ability of elicited models to reproduce the DM's preference, robustness computation and the ability to identify conflicting preference information in case of inconsistency. The empirical study gives insights into the practical application of ELECTRE TRI using the optimistic rule.

4.1 Introduction

Multiple criteria sorting problems, which aim at assigning each alternative of a set to pre-defined ordered classes taking into account several criteria, have gained extensive research interests during the past few decades. The literature considers machine learning, rough sets, fuzzy sets and MCDA methodologies (see Zopounidis and Doumpos (2002) for a review).

In the framework of MCDA methodologies, many different aggregation models have been proposed based on utility function (Zopounidis and Doumpos, 1999b), outranking relation (Roy, 1985), or decision rules(Greco et al., 2001a). We are interested in the preference elicitation approaches for these models, more precisely, the indirect elicitation methods (see Section 2.4). For additive utility based models, Doumpos and Zopounidis (2004) present a disaggregation method UTADIS as a variant of UTA model for sorting purpose. For rule-based models, the disaggregation method infers a set of "if...then..." decision rules derived from rough approximations of decision examples (Greco et al., 2002b).

In this chapter, we focus on the disaggregation methodology for ELECTRE TRI method (see Section 3.2.2). ELECTRE TRI is one of the most widely used methods based on outranking relation and many successful applications have been reported (eg. Mousseau et al. (2001b), Siskos et al. (2007),Mavrotas et al. (2003)). The model involves several parameters including a set of profiles that define the limits between categories, weights, discrimination thresholds. Several authors have proposed disaggregation methodologies to establish an ELECTRE TRI model from decision examples provided by the DM (Mousseau and Słowiński, 1998; Dias and Climaco, 1999; Mousseau et al., 2000). These methodologies propose to infer the preference parameters that best match the DM's preference information and to compute robust categories to which an alternative can be assigned, considering all combinations of parameter values compatible with the DM's preference statements. These disaggregation procedures consider the pessimistic rule only. Recently, an evolutionary approach has been presented considering both the optimistic and the pessimistic rule to infer parameters of ELECTRE TRI model (Doumpos et al., 2009). To the best of our knowledge, all papers in the literature don't provide a mathematical formulation concerning elicitation of ELECTRE TRI model for optimistic rule, even though in practice there is a need to apply the method using optimistic rule (Metchebon T. et al., 2010a; Metchebon T., 2010). For pessimistic rule, Mousseau et al. (2001a) considers partial elicitation, i.e, assuming the profiles are known and determining the weights and majority level only, so the inference of weights implies solving linear programs, which is relatively simple. For optimistic rule, it is not the case, that's the reason why such problem has been neglected.

Our first contribution of this thesis is a preference elicitation tool for ELECTRE TRI method using optimistic rule. The decision examples are represented by linearized constraints, which make it possible to infer preferential parameters and analyze robustness. The chapter is organized as follows. Section 4.2 presents the parameter elicitation algorithm and robustness analysis algorithm. In Section 4.3, extensive numerical experiments are designed to test their computational behaviors. Section 4.4 conducts a real-world case study to illustrate the proposed algorithms. This chapter details conclusions that can be derived from our work.

4.2 Algorithms to Elicit Preferential Parameters

We consider a simplified version of ELECTRE TRI method which ignores discrimination thresholds (preference and indifference threshold) and veto threshold involved in the standard non-discordance condition. The use of an indifference threshold can be avoided by an appropriate translation of the limit profiles. Such simplification is in line with the axiomatic study of Bouyssou and Marchant (2007a,b) avoiding indifference and preference thresholds on criteria.

4.2.1 Decision Variables and Constraints

In order to reduce the complexity of the problem, we consider here a partial elicitation in which the limits of the categories are known. Hence, the only parameters to be set are the weights w_j and the majority level λ, which are thereby the decision variables.

We aim at eliciting the preferential parameters from the comprehensive preference statements of the DM. The preference information that we use consists of some assignment examples, which refer to some alternatives the DMs are able to assign holistically to a category. Let us define the set of assignment examples as $A^* = \{a_1, a_2, \ldots, a_{na}\}$ ($A^* \subset A$, $E = \{1, 2, \ldots, na\}$). The assignment examples are used as a training set to establish an ELECTRE TRI model which can reproduce these training assignments. The elicited model reflects the DM' implicit preference. Then the obtained model is applied to assigning alternatives in A to categories.

We use a regression-like technique to identify such model (Jacquet-Lagrèze and Siskos, 2001). The preference information (in the form of assignment examples) is represented by linear constraints. A set of parameters of such model are to be determined to the greatest satisfaction of these constraints. The elicitation algorithm for ELECTRE TRI using pessimistic rule has already been studied with regression-like technique (eg. Mousseau et al. (2001a) Dias et al. (2002)). However, when considering the optimistic rule, it appears that the necessary conditions to verify the assertion that a is assigned to C_h involve a disjunction (we will explain later the conditions in detail), which induce a difficulty in using disaggregation method for the optimistic rule and might explain why the literature so far concerns the pessimistic rule only.

In the optimistic assignment, to be assigned at a specific category C_h, an alternative a must satisfy the following conditions:

$$b_h \succ a \tag{4.1}$$

$$\text{and } \ not(b_{h-1} \succ a) \tag{4.2}$$

The condition (4.1) means

$$b_h \succsim a \ \ i.e. \ \ c(b_h, a) \geq \lambda \tag{4.3}$$

$$\text{and } \ not(a \succsim b_h) \ \ i.e. \ \ c(a, b_h) < \lambda \tag{4.4}$$

The condition (4.2) means that one of situations below occurs

$$a \succ b_{h-1} \ \ i.e. \begin{cases} c(a, b_{h-1}) \geq \lambda \\ c(b_{h-1}, a) < \lambda \end{cases}$$

$$\text{or } \ a ? b_{h-1} \ \ i.e. \begin{cases} c(a, b_{h-1}) < \lambda \\ c(b_{h-1}, a) < \lambda \end{cases}$$

$$\text{or } \ a \sim b_{h-1} \ \ i.e. \begin{cases} c(a, b_{h-1}) \geq \lambda \\ c(b_{h-1}, a) \geq \lambda \end{cases}$$

Hence we can simply rewrite the condition (4.2) as equivalent to the following proposition:

$$c(b_{h-1}, a) < \lambda \text{ or } c(a, b_{h-1}) \geq \lambda \tag{4.5}$$

Finally, (4.3), (4.4) and (4.5) are necessary and sufficient conditions for an alternative a to be assigned to a specific category C_h in the optimistic assignment.

The condition (4.5) is not linear. A standard way to linearize this condition can be used (see, e.g. Williams, 1999), by introducing two binary variables δ_1, δ_2. The two binary variables are the logical values to indicate whether or not each of the two propositions $\mathscr{P}_1$ and $\mathscr{P}_2$ in (4.5) is satisfied, where $\mathscr{P}_1$ represents $c(b_{h-1}, a) < \lambda$ and $\mathscr{P}_2$ represents $c(a, b_{h-1}) \geq \lambda$. when δ_1

(δ_2 resp.) is 1, the constraint $\mathscr{P}_1$ ($\mathscr{P}_2$ resp.) always holds as if it is deleted. The constraint $\delta_1+\delta_2 \leq 1$ ensures that at least one of $\mathscr{P}_1$ and $\mathscr{P}_2$ holds.

$$\begin{cases} c(b_{h-1},a)-\lambda-\delta_1+\varepsilon \leq 0 \\ c(a,b_{h-1})-\lambda+\delta_2 \geq 0 \\ \delta_1,\delta_2 \in \{0,1\} \\ \delta_1+\delta_2 \leq 1 \end{cases} \tag{4.6}$$

Hence, (4.6) is equivalent to (4.5), where ε is an arbitrary small positive value to transform strict inequality to non-strict.

Other constraints such as bounds of weights w_j and λ are considered as well.

$$\begin{cases} 0.5 \leq \lambda \leq 1 \\ 0 \leq w_j \leq 0.5, \forall j, \ \sum_j w_j = 1 \end{cases}$$

4.2.2 Eliciting Algorithm

We aim at establishing an ELECTRE TRI model which can restore the DM's assignment examples. The assignment examples impose constraints on the parameters to be determined as discussed in Section 4.2.1. We maximize α the minimum value of slack variable in the constraints representing these assignment examples, as α is considered as the robustness of the elicited model to reproduce the assignment examples. When α can be non-negative, all assignment examples are satisfied. Let us suppose that the DM assigns $a_e \in A^*$ to category C_{eh} ($a_e \rightarrow C_{eh}$). b_{eh} is the upper limit of C_{eh-1} and lower limit of C_{eh}. The MIP to be solved is as follows.

$$\max \quad \alpha \tag{4.7}$$

$$\text{s.t.} \quad c(b_{eh},a_e)-\beta_e=\lambda \qquad \forall a_e \in A^* \ \ \forall e \in E \tag{4.8}$$

$$c(a_e,b_{eh})+\gamma_e+\varepsilon=\lambda \qquad \forall a_e \in A^* \ \ \forall e \in E \tag{4.9}$$

$$c(b_{eh-1},a_e)-\lambda-\delta_{i1}+\eta_e+\varepsilon=0 \qquad \forall a_e \in A^* \ \ \forall e \in E \tag{4.10}$$

$$c(a_e,b_{eh-1})-\lambda+\delta_{i2}-\mu_e=0 \qquad \forall a_e \in A^* \ \ \forall e \in E \tag{4.11}$$

$$\delta_{e1}+\delta_{e2} \leq 1 \qquad \forall a_e \in A^* \ \ \forall e \in E \tag{4.12}$$

$$\delta_{e1},\delta_{e2} \in \{0,1\} \qquad \forall a_e \in A^* \ \ \forall e \in E \tag{4.13}$$

$$\alpha \leq \beta_e, \;\; \forall e \in E \tag{4.14}$$

$$\alpha \leq \gamma_e, \;\; \forall e \in E \tag{4.15}$$

$$\alpha \leq \eta_e, \;\; \forall e \in E \tag{4.16}$$

$$\alpha \leq \mu_e, \;\; \forall e \in E \tag{4.17}$$

$$0 \leq w_j \leq 0.5, \;\; \forall j \in M \tag{4.18}$$

$$\textstyle\sum_{j=1}^{m} w_j = 1 \tag{4.19}$$

$$0.5 \leq \lambda \leq 1 \tag{4.20}$$

The constraints (4.8)-(4.13) correspond to all the assignment examples in A^* that the DM could provide. β_e, γ_e, η_e and μ_e are the slack variables of such constraints, and ε is a small positive value to ensure the inequality. Constraints (4.14)-(4.17) define α as the minimum value of slack variables. The natural constraints on weights and cutting level are expressed by (4.18)-(4.20). In this program the constraints in which $c(a_e, b_{eh})$ intervenes are linear (as the category limits are known, $c(a_e, b_{eh})$ can be computed, see Section 3.2.2). The situation is also true for $c(b_{eh}, a_e)$ in the MIP.

If the previous program is feasible and its optimal value α^* is non-negative, then there exists a combination of parameter values that satisfy all the constraints in (4.8)-(4.20) simultaneously. Hence the preference information provided by the DM matches ELECTRE TRI model. The corresponding inferred combination of parameters are identified to construct an ELECTRE TRI model, otherwise the DM should reconsider his/her statements. In that case an inconsistency resolution algorithm (see (Mousseau et al., 2003b)) should be performed to help the DM identify the inconsistencies.

4.2.3 Robustness Analysis

As the preference information in the form of assignment examples is represented by linear constraints, the set of acceptable values for the weights satisfying the constraints can be considered as a polyhedron. Therefore, in such polyhedron there might exist multiple combinations of the parameters which satisfy the preference information. In this context, we are interested in the following question: "Does there exist a combination of parameters which would lead alternative a_i to be assigned to category C_h?". In order to answer this question, we should compute the robust assignment of a_i.

The objective function considered for robustness analysis algorithm is to maximize ε the parameter that intervenes in the transformation of strict inequalities into large inequalities. The mathematical program to be solved to compute whether a_i is possible to be assigned to category C_h is shown as follows.

$$\max \quad \varepsilon \tag{4.21}$$

$$\text{s.t.} \quad c(b_h, a_i) \geq \lambda \qquad \forall a_i \in A,\ \forall i \in N,\ \forall h \in P \tag{4.22}$$

$$c(a_i, b_h) \leq \lambda - \varepsilon \qquad \forall a_i \in A,\ \forall i \in N,\ \forall h \in P \tag{4.23}$$

$$c(b_{h-1}, a_i) \leq \lambda + \delta_{r1} - \varepsilon \qquad \forall a_i \in A,\ \forall i \in N,\ \forall h \in P \tag{4.24}$$

$$c(a_i, b_{h-1}) \geq \lambda - \delta_{r2} \qquad \forall a_i \in A,\ \forall i \in N,\ \forall h \in P \tag{4.25}$$

$$\delta_{i1} + \delta_{i2} \leq 1 \qquad \forall i \in N \tag{4.26}$$

$$\delta_{i1}, \delta_{i2} \in \{0, 1\} \qquad \forall i \in N \tag{4.27}$$

$$c(b_{eh}, a_e) \geq \lambda \qquad \forall a_e \in A^* \tag{4.28}$$

$$c(a_e, b_{eh}) \leq \lambda - \varepsilon \qquad \forall a_e \in A^* \ \ \forall e \in E \tag{4.29}$$

$$c(b_{eh-1}, a_e) \leq \lambda + \delta_{i1} - \varepsilon \qquad \forall a_e \in A^* \ \ \forall e \in E \tag{4.30}$$

$$c(a_e, b_{eh-1}) \geq \lambda - \delta_{i2} \qquad \forall a_e \in A^* \ \ \forall e \in E \tag{4.31}$$

$$\delta_{e1} + \delta_{e2} \leq 1 \qquad \forall e \in E \tag{4.32}$$

$$\delta_{e1}, \delta_{e2} \in \{0, 1\} \qquad \forall e \in E \tag{4.33}$$

$$0 \leq w_j \leq 0.5, \qquad \forall j \in M \tag{4.34}$$

$$\textstyle\sum_{j=1}^{m} w_j = 1 \tag{4.35}$$

$$0.5 \leq \lambda \leq 1 \tag{4.36}$$

If the preference information provided by the DM is consistent, in other words, the above program is feasible and its optimal value ε^* is strictly positive, we can say there exist a combination of parameter values with which the model can assign a_i to category C_h.

We now let the algorithm find the robust assignment ranges of each alternative a_i in A. The computation of robust assignment for a_i consists in checking whether a_i can be assigned to each category C_h, $h = 1, 2, \ldots, p+1$. It is performed by using constraints as in (4.22)-(4.27) of the above MIP to represent this particular assignment. The alternative a_i can be assigned to the category C_h whenever the program is feasible and ε^* is strictly positive. To describe this algorithm, we introduce b_{p+1} and b_0, the upper limit of the best category and the lower limit of the worst category respectively: $\forall a \in A, b_{p+1} \succ a$ and $a \succ b_0$. The two fictitious limits can be determined by satisfying $b_{p+1} : \forall j \in M, g_j(b_{p+1}) > \max_{a\in A}\{g_j(a)\}$ and $b_0 : \forall j \in M, g_j(b_0) < \min_{a\in A}\{g_j(a)\}$. For each alternative a_i, we define the set $\mathscr{R}_{a_i}$ to be the set of categories a_i can be assigned to. To compute the set $\mathscr{R}_{a_i}$, Algorithm 1 is conducted.

Algorithm 1 Procedure to compute robust assignment.

Input:

The set of assignment examples A^*;
The performance of alternatives a: $g_j(a)$;
The limits of profiles b_h, $h = 1, 2, \ldots, p$

Output:

Robust assignment for each alternative a_i: $\mathscr{R}_{a_i}$;

$\mathscr{R}_{a_i} \leftarrow \emptyset$
for $h = 1$ to $p+1$ **do**
  represent constraints corresponding to $a_i \rightarrow C_h$ in MIP
  solve the program
  if $\varepsilon^* > 0$ **then**
    $\mathscr{R}_{a_i} \leftarrow \mathscr{R}_{a_i} \cup \{h\}$
  end if
end for

4.3 Experiment Design and Results

4.3.1 General Modeling Scheme

The experiments are designed to address three issues: (1) the learning ability of the elicitation algorithm; (2) the behavior and performance of the robustness analysis algorithm; (3) the ability to deal with inconsistent information. Moreover, the possible influencing factors on the algorithms are also investigated, including the number of assignment examples, the number of criteria, the number of categories. Similar experiments concerning the learning ability and inconsistency identification for ELECTRE TRI with pessimistic rule can be found in Leroy et al. (2011)

We consider the following strategy to answer the questions. We assume that the DM has the "true" preference in his/her mind, which is represented by an ELECTRE TRI model (the so-called original model). The model is characterized by several profiles b_h ($h = 1, 2, \ldots, p-1$), a set of weights w_j ($j = 1, 2, \ldots, m$) and λ. The randomly generated original model assigns some alternatives to categories, and then the alternatives are used to infer an ELECTRE TRI model (so-called inferred model). We are interested in how close the original model and the inferred model are.

Firstly, a set of *na* alternatives is generated as assignment examples. The evaluations of the alternatives are drawn from [0,99] with a uniform distribution. Then the original model is generated randomly. To generate the weights, we take a set of numbers between 1 and $m-1$ (m being the number of criteria), and then normalize them to sum to one. This method ensures that no criterion represents a majority of weights alone. The majority threshold λ is randomly taken from [0.5,1] interval. We generate p profiles by partitioning [0,99] interval into $p+1$ equal intervals. During the elicitation process, the performances of the profiles are considered known and are the same with the original model.

A set of experiments are designed by varying the complexity of the original model, namely the number of criteria and the number of categories. We also test the algorithms with different amounts of preference information, i.e, the number of assignment examples. The parameter settings of the experiments are shown in Table 4.1.

Parameters	Values considered
Number of alternatives	100
Number of assignment examples	2, 5, 7, ..., 98 (Experiments 1 and 2) 20, 30, ..., 100 (Experiments 3)
Number of criteria	3, 6, 9, 12
Number of categories	2, 4, 6, 8

Table 4.1: Parameter settings in the experiments

4.3.2 Experiments and Results

Eliciting ability

Experiments The experiments study the ability of the elicitation algorithm to retrieve the original ELECTRE TRI model. It is expected that with little input information. In other words, if the number of assignment examples is rather limited, there should exist many ELECTRE TRI models compatible with the assignment examples, which means the inferred model is very arbitrary. It is expected that it requires more cognitive effort from the DM to give more assignment examples which lead to closer inferred model with respect to the original model. This investigation studies the tradeoff between the effort from the DM and the “closeness” of the two models. We are interested in how many assignment examples are necessary to obtain an ELECTRE TRI model which is “close” enough to the original one. To compare the two models, we generate randomly 100 alternatives (which are called test alternatives) and use both the original and the elicited models to assign them to categories. Afterwards, the assignment results are compared to check if they yield to the same assignment. The proportion of the same assignment by the original and elicited model is calculated. Moreover, the computation time is also collected to show the tractability of the elicitation algorithm. The experiments are run 500 times to cancel out the arbitrariness of each run.

Results Figure.4.1 presents the average proportion of correct assignments as a function of the number of assignment examples when the original models involving 4 categories and a varied number of criteria. Figure 4.2 gives the corresponding average computation time.

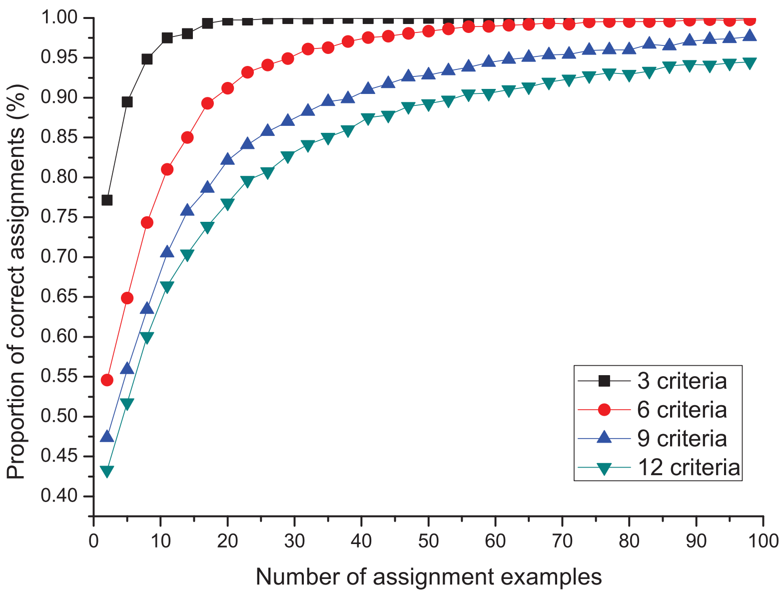


Figure 4.1: Proportion of correct assignments versus the number of assignment examples: experiments to elicit ELECTRE TRI models involving varied number of criteria, 4 categories.

In Figure 4.1 we find that the proportion of right assignments increases when more assignment examples are provided, which means the inferred model is getting closer to the original model with more preference information. For instance, in the case of ELECTRE TRI models with 4 categories and 5 criteria, providing 23 assignment examples, the algorithm can elicit

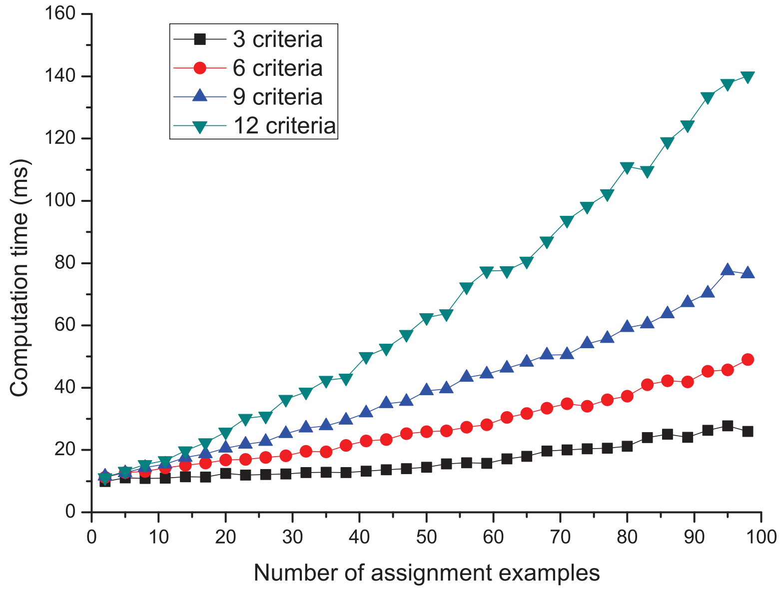


Figure 4.2: Computation time versus the number of assignment examples: experiments to elicit ELECTRE TRI models involving varied number of criteria, 4 categories.

models which are close enough to the DM's "true preference " so that they are able to assign 95% alternatives correctly. The phenomenon is obvious because the more the assignment examples, the smaller the feasible polyhedron of the constraints and thus the more determined the original model. Figure.4.1 also indicates that with more criteria considered, more assignment examples are required by the algorithm to obtain models with a certain level of closeness to the original models. This can be explained as follows. The increase of criteria number implies more variables in the linear program, thus more input information is necessary to determine their values. For experiments considering different number of criteria, we collect the average number of assignment examples with which the algorithm could infer ELECTRE TRI models assigning 95 % of assignments correctly, as given in Table 4.2.

m	3	4	5	6	7	8	9	10	11
$n_{0.95}$	9	15	23	30	41	53	65	80	92

Table 4.2: The number of assignment examples ($n_{0.95}$) used to infer ELECTRE TRI models which lead to 95% correct assignments for different number of criteria (m)

From a linear regression analysis of the table, we observe that the number of assignment examples required has a strong positive correlation with the number of criteria. In fact, the relation of $n_{0.95}$ and m in table 4.2 can be expressed as $n_{0.95} = 10.57m - 28.6$ with a correlation coefficient $r = 0.991$. Such relation might be interpreted as the following: if one more criterion is added to an ELECTRE TRI model, approximately 10 more assignment examples are necessary to get the same level of reliability of the elicited model.

In Figure 4.2, it can be observed that the maximum average computation time of all the experiments with different settings is only 144 ms, which is quite acceptable for a decision aiding process. Furthermore, the computation time grows rapidly with the number of assignment examples and the number of criteria. It is reasonable as the number of criteria implying the number of variables in the MIP, and each assignment example is transformed to constraints and introduces two binary variables as well.

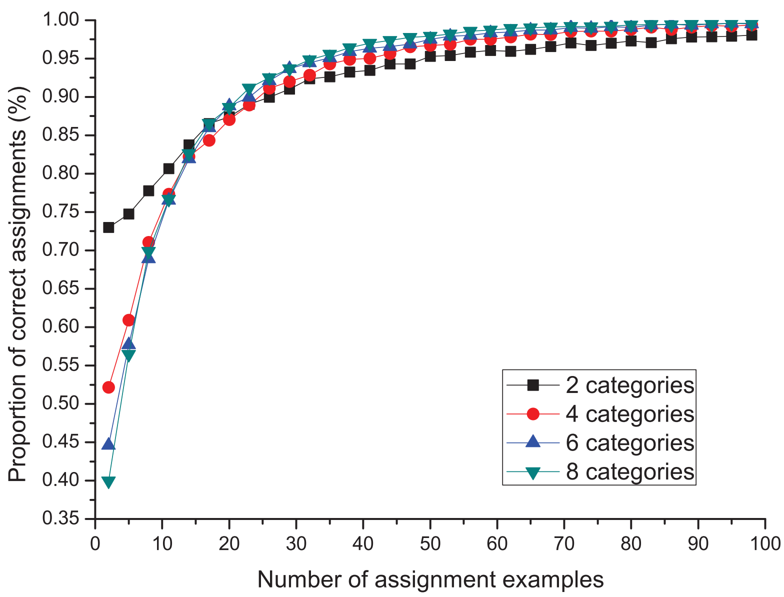


Figure 4.3: Proportion of correct assignments versus the number of assignment examples: experiments to elicit ELECTRE TRI models involving 7 criteria, varied number of categories.

Figure 4.3 summarizes the results of experiments where ELECTRE TRI models involve a varied number of categories and 7 criteria. It can be found that when the number of assignment examples is insufficient (less than 15), the proportions of correct assignments corresponding to experiments with different number of categories are significantly different. With the lack of input information, the inferred ELECTRE TRI models are rather arbitrary. A larger number of categories means more possibility to make mistakes. When the number of assignment examples is not too small (greater than 15 in our experiments), there is no significant difference in the proportions of right assignments for experiments with different number of categories. The curves representing the correct assignment proportions for different number of categories converge to 100% almost at the same speed. We explain the phenomenon as follows. More categories make the constraints stemming from assignment examples tighter, and thus a smaller feasible polyhedron. At the same time, it becomes easier to assign an alternative to a wrong category with more categories. The two effects cancel out so that the number of categories doesn't influence the number of assignment examples required to infer a "close" model to the DM's preference.

Robustness analysis

Experiments

We consider robust assignments as a property of the algorithm's output. The experiment studies the relation between the amount of input preference information (in the form of assignment examples) and the robustness level quantitatively. The average cardinality of $\mathscr{R}_{a_i}$ ($\forall a_i \in A$) is computed as the indicator of the robustness level. We vary the amount of input preference information (the number of assignment examples) and the complexity of the original model (the number of categories and criteria considered) to study their influence on $\mathscr{R}_{a_i}$. For each parameter set, the experiments are run 50 times based on preliminary results. The computation time is not presented because the check of each possible assignment amounts to performing the inference algorithm once.

Results The results of experiments which elicit ELECTRE TRI models with 4 categories and varied number of criteria are shown in Figure 4.4. A significant decrease in the average $\mathscr{R}_{a_i}$ can be seen with an increase in assignment examples. This phenomenon is consistent with the results in the previous experiment to test the learning ability of the inference algorithm, as more assignment examples exert more constraint on the original ELECTRE TRI models, hence less flexibility for the models and more robustness for the assignments. When assignment examples are relatively limited, the average $\mathscr{R}_{a_i}$ is relatively large. This is because many ELECTRE TRI models are compatible with the preference information, so the alternatives are possible to be assigned to several categories using different "versions" of the models. When the number of assignment examples is relatively large, the average $\mathscr{R}_{a_i}$ falls to the bottom, (i.e, 1), which means almost every alternative can only be assigned to a single category. In this case, only a few models conform to the preference information and the original model is nearly determined. The number of criteria has a strong effect on the average $\mathscr{R}_{a_i}$. When the number of assignment examples remains the same, the average $\mathscr{R}_{a_i}$ in the experiments which elicit ELECTRE TRI models considering more criteria is smaller than the ones considering less criteria. It is because more criteria give more flexibility for the ELECTRE TRI models, and the flexibility leaves more possibility for an alternative to be assigned to a certain category.

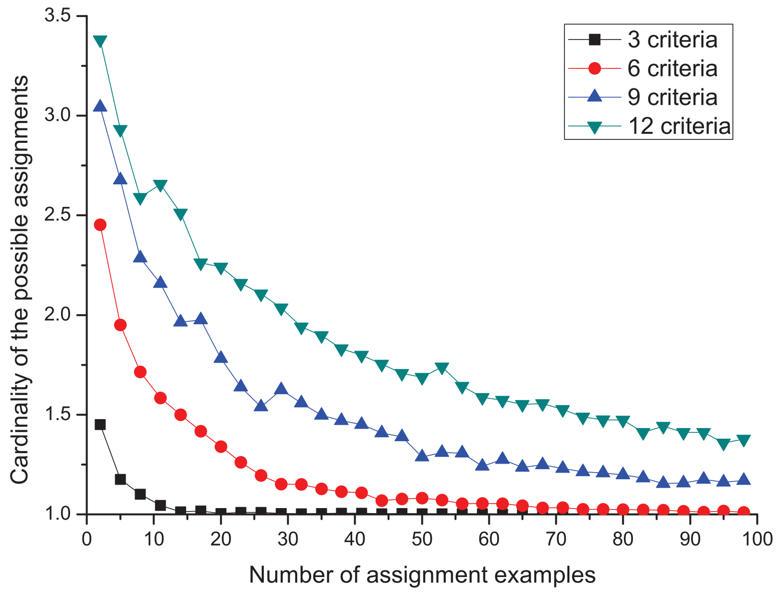


Figure 4.4: Cardinality of possible assignments versus the number of assignment examples: experiments to compute robust assignments of ELECTRE TRI models involving varied number of criteria, 4 categories.

Figure 4.5 can be deduced from the experiments which elicit ELECTRE TRI models with 7 criteria and a varied number of categories. We find that with insufficient assignment examples (less than 15), the average $\mathscr{R}_{a_r}$ corresponding to different number of categories is markedly different. When the number of assignment examples is not too small (greater than 15 in our experiments), there is no significant difference of the average $\mathscr{R}_{a_i}$ for different number of categories. The curves corresponding to different category number reach the lowest point (i.e, 1) almost at the same speed, which means the robustness levels are similar. The trend in Figure 4.5 can be explained similarly as Figure 4.3, and they both illustrate that the number of categories doesn't have any impact on the robustness level when relatively sufficient input preference information is provided.

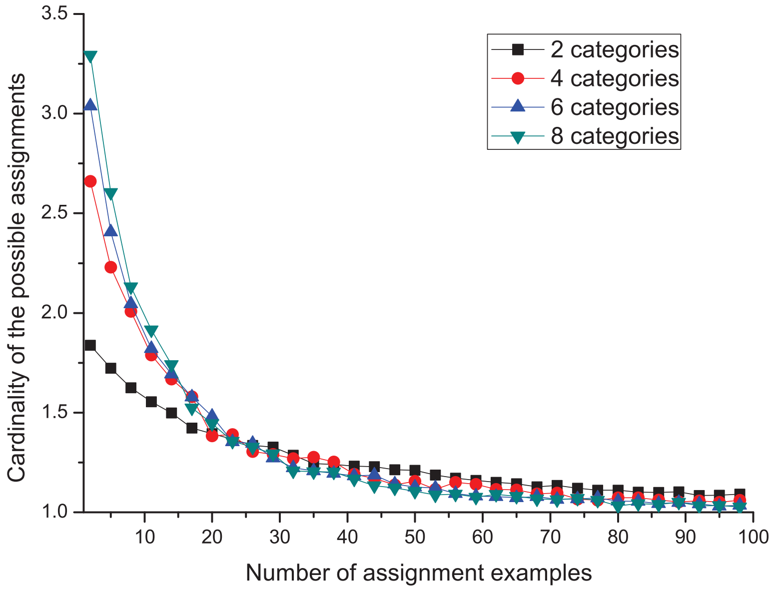


Figure 4.5: Cardinality of possible assignments versus the number of assignment examples: experiments to compute robust assignments of ELECTRE TRI models involving 7 criteria, varied number of categories.

Inconsistency identification

Experiments During the decision aiding process, inconsistency often occurs because there does not exist an ELECTRE TRI model which matches all the DM's preference information. The algorithms from Mousseau et al. (2003b) could help the DM to identify the conflicting pieces of information which are a set of statements he/she has asserted. The algorithm suggests a subset of assignment examples which should be removed to make the rest of his statements consistent. However, the algorithm in Mousseau et al. (2003b) is limited to the pessimistic rule. The situation for the optimistic rule is different as additional binary variables are introduced in the constraints stemming from assignment examples. We design here the numerical experiments to study the ability of a similar inconsistency resolution algorithm to identify inconsistent preference information for optimistic rule.

The experiments are grounded on the following idea. Firstly, the DM's inconsistent preference is simulated by introducing a certain proportion of assignment errors to the assignment example set. The errors introduced consist in assigning several alternatives to their neighboring categories (for example, assigning an alternative to C_1 instead of C_2). Secondly, the inference algorithm is used to elicit ELECTRE TRI models which may be compatible with only part of the assignment examples, that is to say, the optimal objective value might be negative. We are only interested in the cases of inconsistency. Finally, the inconsistency resolution algorithm identifies a maximum subset of assignment examples that can be represented in an ELECTRE TRI model. We focus on the proportion of assignment examples that can be represented.

Based on a preliminary test, we run each experiment in specific parameter setting 100 times. Three error levels in the assignment examples are considered: 10%, 20%, or 30% assignment errors are introduced intentionally. The number of assignment examples is raised from 20 to 100 assignment examples with an increment of 10 assignment examples each time. The parameters are chosen to ensure the number of wrong assignments to be integer. The numbers of criteria and categories are varied to test their influences.

Results Figure 4.6 shows the results of experiments in which different levels of errors are introduced (10%, 20%, 30% respectively) to the set of assignment examples, and the original ELECTRE TRI models involve 7 criteria and 4 categories.

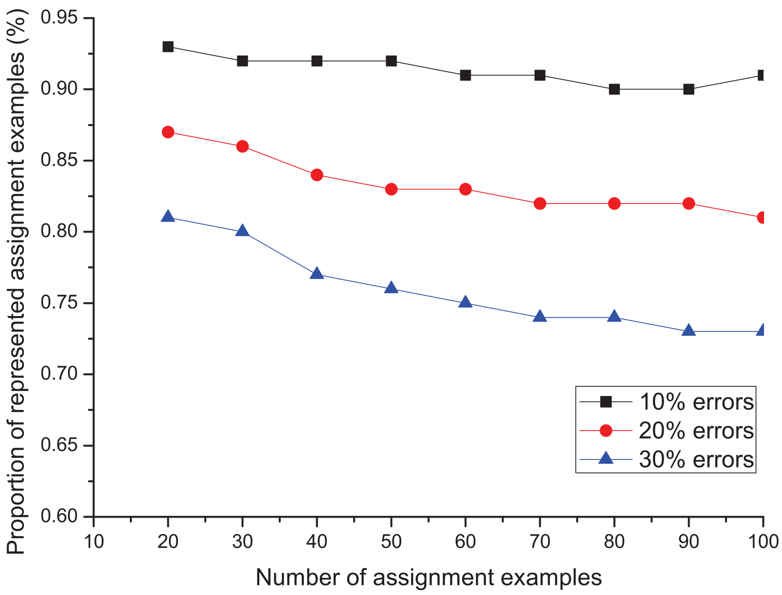


Figure 4.6: Proportions of represented assignment examples by ELECTRE TRI models involving 7 criteria, 4 categories. Different levels of errors introduced in the assignment example set.

Figure 4.6 indicates that when more assignment examples are provided, the average proportions of represented examples go down and reach the bottom line. More precisely, such proportions approximate to 90%, 80%,70%, corresponding to the error levels 10%, 20%,30%.

The phenomenon is consistent with our knowledge that more assignment examples produce a more determined model, thus it becomes much harder for an ELECTRE TRI model to "tolerate" the assignment errors. When the size of assignment examples are large, the proportions of assignment examples which can't be represented by the models almost equal to the proportions of assignment errors introduced, although these assignments unrepresentable don't necessarily correspond to the assignment errors in assignment examples. The computation time presented in Table 4.3 (AEs means assignment examples) grows with the increase of error levels and the number of assignment examples. The computation time is still acceptable even for the extreme cases of experiments which elicit ELECTRE TRI models with 7 criteria, 4 categories, using 100 assignment examples with 30% errors introduced (25.5s at average for 100 runs).

Error levels	20 AEs	30 AEs	40 AEs	50 AEs	60 AEs	70 AEs	80 AEs	90 AEs	100 AEs
10%	0.04	0.07	0.15	0.22	0.37	0.44	0.64	0.69	0.86
20%	0.04	0.10	0.18	0.43	0.65	1.04	1.31	1.64	2.04
30%	0.08	0.22	0.34	0.90	2.77	2.93	7.87	7.61	25.51

Table 4.3: Computation time (s) to identify inconsistency for different error levels introduced in assignment examples: to elicit ELECTRE TRI models involving 7 criteria, 4 categories.

We also designed the experiments to test the inconsistency resolution algorithm when criteria in different numbers are considered in the original ELECTRE TRI models, which take into account 4 categories, and 20% errors are introduced in the assignment example set A^*. The results in Figure 4.7 reveal that the proportions of representable assignment examples in experiments which consider ELECTRE TRI models with more criteria are greater than the ones with fewer criteria. It can be explained as follows. More criteria lead to more flexible ELECTRE TRI models, so that more "tolerant " of the errors. The computation time corresponding to these experiments are shown in Table 4.4, and we find the computation time explodes when ELECTRE TRI models involve 12 criteria and 20% errors are introduced to assignment example set (more than 80). However, we think such a situation won't occur very often in practice.

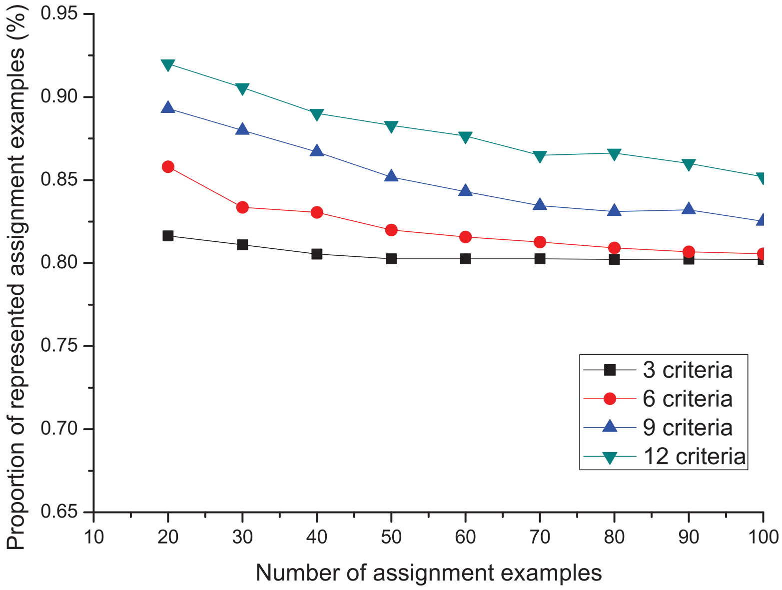


Figure 4.7: Proportions of represented assignment examples by ELECTRE TRI models involving a varied number of criteria, 4 categories, 20% errors introduced in assignment example set

	20 AEs	30 AEs	40 AEs	50 AEs	60 AEs	70 AEs	80 AEs	90 AEs	100 AEs
3 Cri.	0.03	0.04	0.06	0.07	0.10	0.13	0.19	0.30	0.39
6 Cri.	0.04	0.06	0.12	0.27	0.39	0.47	0.64	0.71	0.80
9 Cri.	0.05	0.14	0.53	1.66	2.41	6.63	51.45	75.41	168.16
12 Cri.	0.05	0.13	0.75	3.46	10.14	48.18	82.72	91.65	1756.91

Table 4.4: Computation time (s) to identify inconsistency when 20% errors introduced in assignment examples: ELECTRE TRI models involving a varied number of criteria, 4 categories.

When the number of categories is varied, no significant difference is discovered, so the result is not presented here.

4.4 Case Study

We consider a case study which was studied extensively in Metchebon T. (2010), and some elements in the following case description is drawn from the thesis. Our focus is to design a robust analysis tool for the case. We will describe the case briefly and give a whole picture of the decision aiding process, which show how the algorithm can be used in real-world applications. The robustness analysis tool we developed for ELECTRE TRI model using optimistic rule is also presented.

4.4.1 Case Description

The decision problem aimed at the assessment of degraded landscape of a region located in the center-north of Burkina Faso (West Africa). The assessment should consider multidimensional information (physical, biological, technical, economic, social, cultural). Based on this assessment, the DMs had to prioritize the actions to be undertaken against degradation for sustainable development. In the case, the analyst didn't have opportunity to interact with the real DMs, but an environmentalist expert played the role of DM and provided his expertise in the way the SUs should be evaluated (for more detail on this application, see Metchebon T. et al. (2010a)).

4.4.2 Problem Structuring

The region had been partitioned into 229 spatial units (SUs), each of which had been labeled with a number. Four fundamental objectives had been defined by the DM as the principles of degradation of landscapes.

- Principle 1 (ERO): Soil erosion is limited
- Principle 2 (BIO): The loss of biodiversity is limited
- Principle 3 (FER): Soil fertility is maintained
- Principle 4 (PRO): Good agricultural productivity is enhanced

Then the human activities which had negative influence on the principles were considered as major factors. 11 criteria of limitation of the degradation were derived from these factors as indicators, as shown in Table 4.5. The criteria were all evaluated on a [0,5] ordinal scale.

The environmentalist agreed to use ELECTRE TRI method to evaluate the 229 SUs, which amounted to assigning SUs to 4 categories of response to the risk of degradation: "*Adequate*", "*Moderately Adequate*", "*Weakly Adequate*" and "*Not Adequate*". SUs assigned to the first or the second category corresponded to degraded SUs, while SUs assigned to two other categories

Table 4.5: Principles and criteria for limiting the damage (CES: Conservation Water and Soils; PS: Soil preparation)

Principles	Criteria
ERO	Choosing appropriate morpho pedological (CR11) Proper application techniques CES (Cr12) Appropriate application of technical PS (Cr13) Limitation of soil compaction (CR14)
BIO	Limiting the expansion of cultivated areas (CR25) Maintaining the integrity of ecosystems (CR26) Limitation of bush fires (Cr27)
FER	Adequate application of culture techniques (CR38) Adequate practice of fertilization of soil(Cr39)
PRO	Technical support for farmers (Cr410) Improved production (Cr411)

corresponded to not degraded zones. Moreover, the optimistic assignment rule was considered suitable according to the DM's expertise. The evaluation results were forwarded to administrative offices to make the decisions which actions would be undertaken.

4.4.3 Constructing an Evaluation Model

Direct preference elicitation

During the elicitation of the ELECTRE TRI model, the DM was able to provide category limits without veto (as shown in Table 4.6). Moreover, all indifference thresholds were 0 and all preference thresholds were 1 except for g_{10} the preference threshold was 2. However, the DM found difficulties in providing information about the importance of criteria by providing directly a single weight vector.

Table 4.6: Parameter values

Category limits \ Criterion	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
b1	1	2	1	1	1	1	2	4	1	1	1
b2	2	2	2	2	3	1	3	4	2	3	4
b3	2	3	3	2	3	1	3	5	3	3	5

Indirect preference elicitation

Instead of asking the DM the exact values of weights and the majority level, we considered two kinds of indirect preference elicitation techniques. The first kind refereed to a set of constraints on the parameter values reflecting the imprecise information that the DMs were able to provide, while the second kind originated from assignment examples.

(1) Expressing preference on weights by SRF Method

SRF method (presented in detail in chapter 2) is considered to elicit the DM's preference on the relative importance of the criteria. We don't use SRF method (Figueira and Roy, 2002) to set the values of weights directly, since there are some drawbacks of doing so, as already discussed in chapter 2. Instead, the SRF method is just used as an inspiration tool for the DM to think and express his preference. The preference information is treated as imprecise information, which is represented by linear constraints in the elicitation program specifically for ELECTRE TRI method. Thus, the preference information on the ranking of importance related to each criterion should be reflected meaningfully in the preference model.

Without loss of generality, we suppose that the set of criteria $\{g_1, ..., g_m\}$ is ordered w.r.t. the importance: g_1 is the least important criterion and g_m the most important one. The difference in the importance of criterion g_{j+1} and g_j (or subsets of *ex aequo* criteria) is expressed in the SRF method by the number of white cards n_j that the DM has put between them, as illustrated by Figure 4.8.

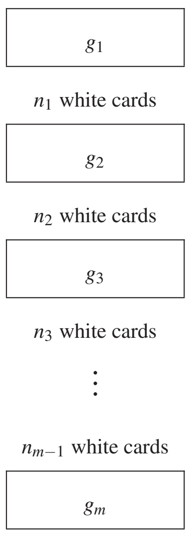


Figure 4.8: Input information for SRF

The way the DM ranges the cards and inserts white cards leads to the following constraints:

$$\begin{cases} w_1 < w_2 < ... < w_m \\ \forall i, j \in \{1, 2, ..., m\} \;\; n_i > n_j \Rightarrow w_{i+1} - w_i > w_{j+1} - w_j \end{cases} \tag{4.37}$$

To avoid the arbitrariness of the ratio between the most important criterion and the least important criterion, the SRF method involves the elicitation of the ratio. This ratio (denoted z) is usually considered in many application cases to be difficult for the DMs to determine its value (see, e.g. Metchebon T. et al., 2010a). Generally, in the most favorable cases, z can be set in an interval ($z \in [\alpha,\ \beta]$, $\alpha, \beta > 0$, $\beta > \alpha$). This leads to the following inequalities: $\alpha w_1 \leq w_m \leq \beta w_1$. The constraints referring to the bounds of decision variables can be stated as follows:

$$0 \leq w_j \leq 0.5, \forall j,\ \sum_j w_j = 1 \tag{4.38}$$

Assignment examples

The DM is asked to provide some assignment examples to express his expertise on evaluating the spatial units. Let us recall that when alternative a is assigned to category C_h, the corresponding constraints should be (4.39), adding 2 binary variables per assignment example.

$$\begin{cases} \sigma(b_h, a) \geq \lambda \\ \sigma(a, b_h) < \lambda \\ \sigma(b_{h-1}, a) - \lambda - \delta_1 + \varepsilon \leq 0 \\ \sigma(a, b_{h-1}) - \lambda + \delta_2 \geq 0 \\ \delta_1, \delta_2 \in \{0, 1\} \\ \delta_1 + \delta_2 \leq\ 1 \end{cases} \tag{4.39}$$

4.4.4 Robust Analysis

We have performed the robustness analysis algorithm (Algorithm 1) to provide robust assessment results of the 229 SUs. More precisely, the preference information of the DM was provided at one time. Instead, the preference was expressed as the decision aiding process was proceeded, which stimulated the DM to a better understanding of the problem. Therefore, the pieces of preference information was added step by step to show the DM the evolutionary process of the robust assignments.

First phase

Firstly, when no preference information was available according to the DM, robust assignments were computed, which was based on the dominant outranking relation between alternatives and b_h as well as the natural constraints on the parameters. We proceeded by using the proposed algorithm to compute robust assignments. Thus a relevant result for each SU concerned its robust assignment with ELECTRE TRI optimistic assignment rule. It was not surprising to find the wide range of the robust assignments. The result showed that the average cardinality of the set $\mathscr{R}_{a_i},\ i = \{1,2,\ldots,229\}$ for the 299 alternatives was 3.59. For instance, concerning the spatial unit number 37 ($a_{37} = (1,2,2,1,3,1,2,4,1,3,4)$), the computation procedure was the following. For category C_1, the program to be solved involved 2 binary variables and 13 continuous variables, and 8 constraints. We solved this program using CPLEX v11 on a Intel Core Duo CPU 3Ghz with 2 GBytes RAM in less than 0.01 seconds. Solving this program lead to a solution $\varepsilon^* < 0$, hence no acceptable weights would let a_{37} be assigned to C_1. Finally, computations proved that $\mathscr{R}_{a_{37}} = \{C_2, C_3, C_4\}$.

Second phase

In this phase, the preference information from SRF method was added to the input of the algorithm. Indeed we have used SRF method to elicit more information about the weights. The DM gave us a rank for those weights and the importance difference between them, which was shown in Figure 4.9. But concerning the ratio z considered in the SRF method, it was difficult for the DM to determine this value and he could just give the interval [8, 10] which he thought that the value of this parameter z could belong to.

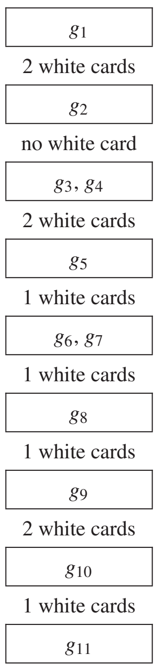


Figure 4.9: Presentation of the information given by the set of cards

We found the polyhedron of all acceptable parameters was not empty and then robust assignment was computed for each alternative. Given an imprecise definition of the importance of criteria to the algorithm, the average cardinality of the set $\mathscr{R}_{a_j}$ for the 299 alternatives declined to 2.34. And a_{37} can not be assigned to C_4 any more, i.e. $\mathscr{R}_{a_{37}} = \{C_2, C_3\}$.

Third phase

The DM also provided indirect preference information in the form of assignment examples. In fact, the DM provided an assignment example ($a_{27} = (1,1,1,1,1,1,1,1,1,1,1,1)$) and claimed this alternative should be assigned to the lowest category C_1. The assignment example ($a_{27} \rightarrow C_1$) was taken into account in this phase. Therefore, 5 more constraints were added to the program. Computations showed all the preference information was still consistent, thus robust assignment was computed. We found that the polyhedron was smaller after such an assignment example was introduced. The average cardinality of the set $\mathscr{R}_{a_j}$ decreased to 2.13 and $\mathscr{R}_{a_{37}} = \{C_2\}$ at this time.

The process continued until the decision maker was satisfied with the robust assignments and the corresponding values for the parameters. We would like to mention that the algorithm proposed was able to solve the program in an acceptable time. When 6 assignment examples were provided as input to the algorithm, i.e.12 binary variables were introduced into the program, the program was solved in 15 seconds using the simulation environment we have described.

4.5 Concluding Remarks

We present an algorithm to infer parameters of the model and compute robust assignments of the ELECTRE TRI using the optimistic rule, when the importance of criteria is not precisely known but rather expressed by a polyhedron of acceptable values. This polyhedron is expressed by the DM through some assignment examples. Different from pessimistic rule, ELECTRE TRI model with the optimistic rule induces a disjunction which causes difficulties when formulating the conditions to assign an alternative to a specific category. We propose to linearize the conditions by introducing binary variables. The linearization permits to infer weights and compute robustness by linear programming.

Firstly, several experiments are conducted to study the eliciting ability of the algorithms. The results reveal that the elicitation is relatively demanding in terms of input preference information. The DM has to make enough cognitive effort and provide a certain number of assignment examples in order to obtain an ELECTRE TRI model “close” enough to his/her preference and to get “robust” sorting results. It is also found that more criteria involved in ELECTRE TRI model demands more information to determine the model while the number of categories does not have obvious effect on the elicitation. The quantitative study of the amount of input preference information and the quality of output models gives important guidelines to the DM and the analyst during the decision aiding process, when the criteria and the categories should be formulated and the DM should provide some assignment examples. Secondly, we investigate the inconsistency issue by intentionally introducing some errors to the assignment example set. The algorithm appears to be less “tolerant” of the errors with more preference information available. Taking into account more criteria again makes ELECTRE TRI models more flexible, and results in more “tolerance” of the inconsistent information. Lastly, the algorithm proved to be efficient in terms of computing time. To sum up, we believe the experiments give rich information to the application the proposed eliciting algorithms in practice, when the analyst and the DM structure the problem, construct the model, and make final recommendations.

In Chapter 4, we have developed preference elicitation tools for ELECTRE TRI using the optimistic rule, with the assumption that the profiles of the categories are known as a priority, and no threshold is taken into consideration. Further research could be carried out to relax these assumptions.

The extension of eliciting profiles as unknown parameters is rather straightforward as it can be formulated similarly as in Chapter 5 and 6 where binary variables are introduced to indicate the way two objects are compared. However, such a formulation results in more binary variables in the optimization problem for the elicitation of ELECTRE TRI using the optimistic rule compared with the one using the pessimistic rule, since two additional binary variables are introduced to represent each assignment example as linear constraints using the optimistic rule. Thus the elicitation can be computational costly.

The elicitation can also be extended by taking into account thresholds. The veto threshold and other parameters (profiles, weights and majority level) can be elicited simultaneously using additional binary variables, which can undoubtedly increase computation complexity. But eliciting the preference and indifference thresholds with other parameters together leads to non-linear program (see Mousseau and Slowiński, 1998, for complete inference of ELECTRE TRI with the pessimistic rule using non-linear optimization).

We also present a case study which assesses the land degradation in Burkina Faso. We show the decision aiding process of the assessment. ELECTRE TRI model with the optimistic rule is considered as the evaluation model, and robustness analysis is conducted using the proposed algorithm.

Chapter 5

Preference Elicitation for Portfolio Selection Problems

The chapter focuses on portfolio selection problems which aim at selecting a subset of alternatives considering not only the performance of the alternatives evaluated on multiple criteria, but also the performance of portfolio as a whole, on which balance over alternatives on specific attributes is required by the DMs. The DMs' preference information both at individual and portfolio level is considered.

We propose a two-level method to handle such decision situation. First, at the individual level, the alternatives are evaluated by the sorting model ELECTRE TRI_{BM} . The DMs' preferences on alternatives are expressed by some assignment examples they can provide, which reduces the DMs' cognitive efforts. Second, at the portfolio level, the DMs' preferences express requirements on the composition of portfolio and are modeled as constraints on category size. The method proceeds through the resolution of a Mixed Integer Program (MIP) and selects a satisfactory portfolio as close as possible to the DMs' preference. The method can be used widely in portfolio selection situations where the decision should be made taking into account the individual alternative and portfolio performance simultaneously.

We apply this method to a real-world student selection problems in which the group criteria conflict with the constraints relative to the student portfolio. The decision aiding process of the application is described in detail.

5.1 Introduction

Let us consider the student enrollment in universities every year. Universities want to select students with good performances on several criteria (such as GPAs, motivation, maturity, ...). At the same time, the selected students should satisfy some specific requirements at a collective level. For instance, the number of students in each department should be more or less balanced. Each department tries to achieve a gender (nationality, etc.) diversity. Moreover, the positions available are limited. Therefore, the universities face a decision which consists of selecting a certain number of students, designing a waiting list and rejecting the other students (see similar example in universities Le Cardinal et al. (2011)). Another example of such portfolio selection problems concerns allocating grants to research proposals. The committee evaluates the merit of the proposal, including originality, novelty, rigor and the ability of the researchers to carry out the research individually. On a whole level, they try to balance the funding among disciplines, institutions and even regions. Therefore, a decision is to be made to select certain research proposals within limited budget.

The two problems above share some characteristics. Firstly, they involve evaluating individual alternatives according to their performances on multiple criteria. Secondly, a portfolio is to be selected based not only on individual alternative's performance, but also on the performance of the whole portfolio. Such situation typically corresponds to a portfolio selection problem.

There is a large number of methods in literature for evaluating and selecting portfolios (Golabi et al., 1981; Rao et al., 1991; Hall et al., 1992; Archer and Ghasemzadeh, 1999; Chien, 2002). Cost-benefit analysis (Philips and C.Bana e Costa, 2007), multiattribute utility theory (Duarte and Reis, 2006), weighted scoring (Coldrick et al., 2005) are widely used. In terms of solution methods, there is an extensive literature on portfolio decision analysis or mathematical programming formulations (e.g. Miyaji et al. (1988), Polyashuk (2006); Montibeller et al. (2009); Liesiö et al. (2008); Ghasemzadeh and Archer (2000); Archer and Ghasemzadeh (1999). In the aggregation of criteria, methodologies either use a synthesis criterion (e.g. multiple attribute value theory, Philips and C.Bana e Costa (2007)) or outranking based methods (e.g. Leyva López (2005), see also Figueira et al. (2005b)). Depending on the aggregation procedure used, the criteria can be quantitative or ordinal. Moreover, many models require a complete specification of preferences to induce recommendations, while a limited number of papers allow imprecise specification of preferences. Some researchers combine preference programming with portfolio selection considering incomplete preference information (Liesiö et al., 2007, 2008). However, to our knowledge, MCDA outranking methods have rarely been applied to portfolio selection problem. Furthermore, the ability of the methods to express sophisticated preference on portfolios has little been explored. A balance model (Farquhar and Rao, 1976) is developed which measures the distribution of specific attributes by dispersion and uses such measurement to select subsets of multiattribute items. Golabi et al. (1981) uses constraints to eliminate the ones which do not fit in the requirement on whole portfolio.

We propose a two-level method for such portfolio selection problems. At individual level, the paper uses ELECTRE TRI method (Roy, 1991, 1996) to evaluate the alternatives on multiple criteria, which assigns alternatives to predefined ordered categories by comparing an alternative with several profiles. The DMs' preference on individual evaluation can be taken into account by some assignment examples. At portfolio level, a wide class of preferences on portfolios (resource limitation, balance of the selected items over an attribute…) are represented using general category size constraints. An optimization procedure is performed by solving a MIP to infer the values of preference parameters and to identify a satisfactory portfolio.

The present chapter is organized as follows. Section 5.2 formulates portfolio selection problem as a constrained multicriteria sorting problem. Section 5.3 presents a mathematical program which computes the portfolio that best matches the DMs' preferences. Section **??** illustrates the proposed method with an example. We apply the method to a real-world case of student selection problem in Section 5.5. The last section groups conclusions.

5.2 Problem Formulation

5.2.1 Evaluating Alternatives with ELECTRE TRI Method

Alternatives to be included on a portfolio are evaluated by ELECTRE TRI (Roy, 1991, 1996). For example, for the enrollment problem described in section 5.1, the DMs want to sort the students into three categories: *accepted, waiting list* or *rejected* according to students' performances on multiple criteria. Thus the two profiles could be two frontiers which separate these three categories. ELECTRE TRI_{BM} is used in this chapter.

In order to implement ELECTRE TRI_{BM} , an elicitation process is necessary to determine the values of preference parameters (profiles b_h, weights w_j and majority level λ). From a portfolio selection perspective, we consider DMs' preferences at two levels. At alternative level, the DMs express preferences on alternatives individually. At a portfolio level, they express preferences on portfolios as a whole (resource limitation, balance of the selected items over an attribute, ...). These two preference levels are distinguished, as they are elicited in different ways, and could be provided by different DMs who have expertise and understanding of the portfolio selection at different levels.

5.2.2 DMs' Preference on Alternatives

The DMs have little understanding of the precise semantics of the preference parameters involved in ELECTRE TRI$_{\mathrm{BM}}$. On the contrary, they can easily express their expertise on which category an alternative should be assigned to. Therefore, we propose to elicit the DMs' preference in an indirect way, in accordance with the disaggregation-aggregation paradigm. Instead of providing precise values for the parameters, the DMs provide assignment examples, i.e. alternatives which they are able to assign confidently to a category. For instance, in a student selection problem, the DMs may state that one particular student should be assigned to the best category (the set of accepted students). Inference procedure can thus be used to compute values for the preference parameters that best match the assignment examples. In this chapter, we assume all the preference parameters (profiles b_h, weights w_j and majority level λ) are variables and infer them by solving a MIP.

5.2.3 DMs' Preference Information on Portfolios

The DMs' preferences can also be expressed at the portfolio level (resource limitation, balance on the composition of categories w.r.t. an attribute, ...). We formalize such preferences as general constraints on category size. For example, in the student enrollment case, let us denote the category of rejected students C_1, the category of waiting list C_2 and the category of admitted students C_3. Suppose the university only have 100 positions available, and such constraint can be modeled as the number of students in C_3 cannot exceed 100. Moreover, balancing gender in the selected students (100 students in total) can also be modeled as a constraint that the number of female students in C_3 should not be lower than 30. Adding such constraints to the selection process may result in rejecting some male students whose performances are better than the accepted female ones. However, such portfolio is more satisfactory for the DMs in terms of gender balance. Modeling the DMs' preference as constraints eliminates some portfolios which don't satisfy their requirements on the whole portfolio.

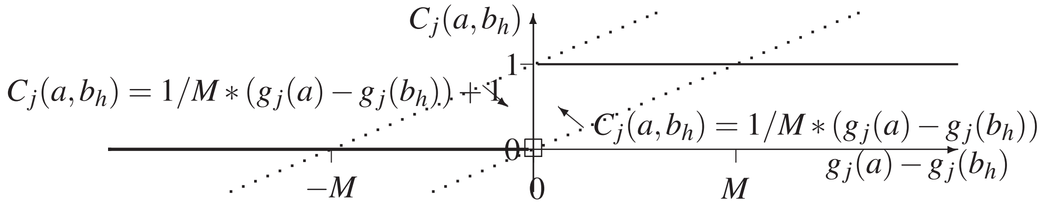


Figure 5.1: constraining $C_j(a_e, b_h)$ to the appropriate value

5.3 Mathematical Program Formulation

5.3.1 Stating the Problem and Decision Variables

The goal of the program is to determine the performances of profiles $g_j(b_h), \forall j \in M, 1 \leq h \leq p-1$, weights w_j and majority threshold λ, satisfying all the constraints given by the DMs in the form of assignment examples and portfolio constraints.

The MIP also defines additional variables involved in the way ELECTRE TRI$_{\text{BM}}$ assigns alternatives to categories. The binary variables $C_j(a,b_h), \forall a \in A, j \in M, 1 \leq h \leq p-1$ represent the partial concordance indices such that $C_j(a,b_h) = 1$ if and only if the performance of the alternative a on the criterion j is at least as good as the performance of the profile b_h. The sum of support in favor of the outranking of an alternative a over a profile b_h, $\sum_{j \in M: g_j(a_i) \geq g_j(b_h)} w_j$, can also be written $\sum_{j \in M} C_j(a_i, b_h) w_j$ with $C_j(a_i, b_h)$ equals to one iff $g_j(a_i) \geq g_j(b_h)$. Constraints (5.1) define the binary variables $C_j(a_i, b_h), \forall j \in M, a_i \in A, 1 \leq h \leq p-1$, where ε is an arbitrary small positive value, and L is an arbitrary large value. See also Fig. 5.1.

$$\frac{1}{L}((g_j(a_e) - g_j(b_h)) + \varepsilon) \leq C_j(a_i, b_h) \leq \frac{1}{L}(g_j(a_i) - g_j(b_h)) + 1 \ . \tag{5.1}$$

The continuous variables $c_j(a,b_h)$ represent the weighted partial concordance indices, they are such that $c_j(a,b_h) = w_j$ if and only if $C_j(a,b_h) = 1$. Therefore, $c(a_i,b_h) = \sum_{j \in M} c_j(a_i,b_h)$. The following constraints define the variables $c_j(a,b_h)$ while avoiding the non-linear expression $c_j(a_i,b_h) = C_j(a_i,b_h) w_j$ (Meyer et al., 2008). See also Fig. 5.2.

$$\forall j \in M, a_i \in A, 1 \leq h \leq p-1 : \begin{cases} c_j(a_i,b_h) \leq w_j \\ c_j(a_i,b_h) \geq 0 \\ c_j(a_i,b_h) \leq C_j(a_i,b_h) \\ c_j(a_i,b_h) \geq C_j(a_i,b_h) + w_j - 1 \ . \end{cases} \tag{5.2}$$

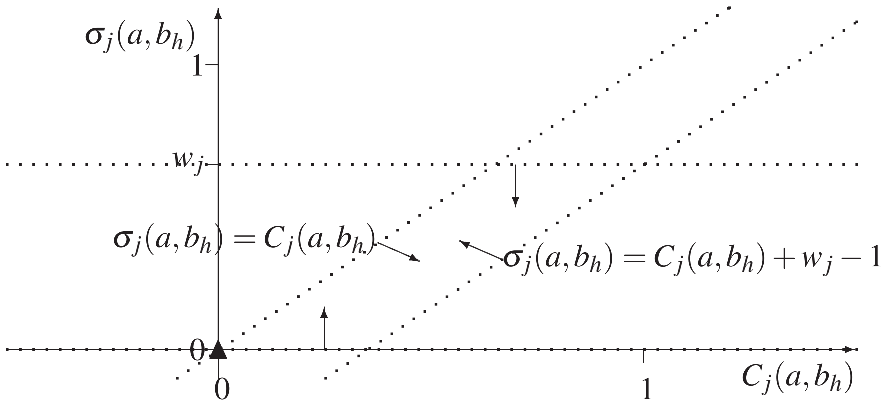


Figure 5.2: constraining $c_j(a_i,b_h)$ to the appropriate value

We also define, for simplicity of use in the next constraints, $\forall j \in M, a_i \in A$: $c_j(a_i,b_0) = w_j$ and $c_j(a_i,b_p) = 0$.

Finally, binary variables $n(a_i,h), \forall a_i \in A, h \in K$ are defined so that $n(a_i,h) = 1$ if and only if alternative a is assigned to category h. Recall that satisfying an assignment example (a_i,h) $(a_i \to C_h)$ amounts to satisfy both $\sum_{j\in M:g_j(a_i)\geq g_j(b_{h-1})} w_j \geq \lambda$ and $\sum_{j\in M:g_j(a_i)\geq g_j(b_h)} w_j < \lambda$, for all $a_i \in A$. The following constraints define the binary variables $n(a_i,h)$, $\forall a_i \in A, 1 \leq h \leq p$, so that $n(a_i,h)$ equals one iff a is assigned to category C_h, that is, $\sum_{j\in M} c_j(a_i,b_{h-1}) \geq \lambda$ and $\sum_{j\in M} c_j(a_i,b_h) < \lambda$. The first constraints force that $n(a_i,h) = 1$ requires that a goes to category h, and the last ones force that exactly one $n(a_i,h)$ among all h equals one. A slack variable s is used in the objective function which appreciates the ability of the ELECTRE TRI$_{\text{BM}}$ model to reproduce the assignment examples in a robust way.

$$\forall a_i \in A, 1 \leq h \leq p : \begin{cases} n(a_i,h) \leq 1 + \sum_{j\in M} c_j(a_i,b_{h-1}) - \lambda - s \ , \\ n(a_i,h) \leq 1 + \lambda - \sum_{j\in M} c_j(a_i,b_h) - \varepsilon - s \ . \end{cases} \tag{5.3}$$

$$\forall a_i \in A : \sum_{1\leq h\leq p} n(a_i,h) = 1. \tag{5.4}$$

The constraint $\sum_{j\in M} w_j = 1$ is posed, and the following constraints are used to ensure a correct ordering of the profiles defining the categories: $\forall j \in M, 2 \leq h \leq p-1 : g_j(b_{h-1}) \leq g_j(b_h)$.

5.3.2 Constraints Stemming from Preferences at Individual Level

We need to ensure that each assignment example $a_e \in A^* \to C_{eh}$ is satisfied. The necessary and sufficient conditions for this statements are:

$$\forall a_e \in A^* : n(a_e, eh) = 1 \quad . \tag{5.5}$$

5.3.3 Constraints Stemming from Preferences at Portfolio Level

Suppose the DMs want to impose, in a student selection problem, that at least 30 students in the best category (i.e. C_p) are females. To model this, we define a function *Gender* on the set of alternatives that equals one if the student a_i is a female student and zero otherwise, and set as a constraint that the sum of $Gender(a_i)$ on each alternative a_i assigned to C_p should be at least 30 ($\sum_{a_i \to C_p} Gender(a_i) \geq 30$). In a project selection problem, suppose the DMs want to make sure that the sum of the costs of the selected projects (say, the projects in the best category) do not exceed the available budget x. A function *Cost* would be defined on the set of alternatives representing their cost attribute, and a constraint is added to ensure that the sum of $Cost(a_i)$ on alternatives a_i assigned to the best category should be no greater than the budget ($\sum_{a_i \to C_p} Cost(a_i) \leq x$).

More generally, portfolio preferences are represented as a set T of tuples $\left\langle h, \underline{n_h}, \overline{n_h}, F \right\rangle, 1 \leq h \leq p, \underline{n_h}, \overline{n_h} \in \mathbb{R}, F$ a function from A to $\mathbb{R}$, representing the constraint that the preference model inferred by the program should be such that the number of alternatives from A assigned to C_h weighted by their attribute F should be at least $\underline{n_h}$ and at most $\overline{n_h}$: $\underline{n_h} \leq \sum_{a_i \to \mathrm{Cat}_h} F(a_i) \leq \overline{n_h}$.

These variables permit to ensure correctness of the group sizes.

$$\forall \left\langle h, \underline{n_h}, \overline{n_h}, F \right\rangle \in N : \underline{n_h} \leq \sum_{a_i \in A} n(a_i, h) F(a_i) \leq \overline{n_h} \quad . \tag{5.6}$$

5.3.4 Objective Function and Resolution Issues

In order to maximize the separation between the sum of support and the majority threshold, the objective of the MIP is set to maximize the slack variable s as defined in Constraints (5.5). The slack variable evaluates the ability of the ELECTRE TRI$_{\text{BM}}$ model to “reproduce” the assignment examples in a robust way.

However the preference information of the DMs does not lead univocally to a single compatible portfolio. The optimization procedure finds out one of the compatible portfolios. In an interactive perspective, the DMs can provide further preference information considering the results of the MIP, and the information can be added to the optimization procedure to get a more satisfactory portfolio. The decision aiding process can proceed with several interactions until the DMs are content with the selected portfolio. In case an infeasible problem had been reached at some point during the process, some constraints would have had to be relaxed or deleted. The reader will find in Mousseau et al. (2006) algorithms on how to proceed for constraints relaxation.

5.4 Illustrative Example

Let us illustrate the method with the following hypothetical decision situation. A government board has the responsibility to choose which research projects to finance among a list of 100 research proposals. The selection process involves sorting these proposals into three categories: projects that are considered very good and should be funded (category *Good*); projects that are good and should be funded if supplementary budget can be found (category *Average*); projects that are of insufficient quality and should not be funded (category *Bad*). To sort these projects in these three categories, the board agrees to use the following six criteria.

sq The project's scientific quality, evaluated on a 5 points ordinal scale.

rq The proposal's writing quality, evaluated on a 5 points ordinal scale.

a The proposal's adequacy with respect to the government priorities, evaluated on a 3 points ordinal scale.

te The experience of the researcher teams submitting the project, evaluated on a 5 points ordinal scale.

ic Whether the proposal includes international collaboration, a binary assessment.

ps The researchers' publication score evaluated by an aggregate measure of the total quality of publications of the researchers involved in the proposal (evaluated on a [0,100] scale).

The scales on all criteria are defined such that a greater value corresponds to a better evaluation.

Table 5.1: Some of the research projects to be evaluated. The budget is in tens of K€.

	evaluations criteria						descriptive attributes		
Project	rq	ps	a	sq	te	ic	budget	domain	country
Pr001	2	47	2	3	1	0	27	Stat.	Germany
Pr002	2	3	2	4	4	0	29	Stat.	France
Pr003	5	63	1	5	1	0	20	Stat.	Italy
Pr004	1	92	3	5	5	1	34	AI	Germany
Pr005	4	13	2	4	2	0	32	Stat.	Germany
Pr006	5	5	3	5	1	0	22	Stat.	Netherlands
Pr007	1	27	3	2	5	1	34	OR	Germany
⋮									

Table 5.2: Some research project examples and their respective assignments.

Project	rq	ps	a	sq	te	ic	Cat
Ex01	4	50	2	3	3	0	Average
Ex02	4	85	3	1	5	1	Good
Ex03	3	95	1	2	5	1	Average
Ex04	5	91	2	2	5	1	Good
Ex05	5	89	1	5	3	0	Good
Ex06	3	5	3	2	2	1	Average
⋮							

Supplementary to these six criteria, the 100 projects to be evaluated are described by three attributes: the research domain to which the project belongs (Operational Research (OR), Artificial Intelligence (AI) or Statistics); the budget the project asks funding for; the originating country. Table 5.1 shows the data about the first 7 projects in the list (complete data lists for the whole example are available at (`http://www.lgi.ecp.fr/~mousseau/ADT2011/`). In order to determine an appropriate preference model, the board gives as a first stage 30 examples of past research proposals whose performances on the six criteria and final quality evaluation are known. A part of this data is shown in Table 5.2.

The inference program is ran with these assignment examples, and without supplementary portfolio constraints. Table 5.3 lists the resulting profiles and weights. Note that the profiles performances values in all our tables have been rounded up. Because each alternative used in this example has integer performance values on all criteria, doing so does not impact the way each alternative compares to these profiles. The resulting preference model is used to evaluate the 100 research projects, which leads to 22 projects being evaluated as good projects. The

Table 5.3: Profiles, weights and majority threshold inferred during the first stage.

	rq	ps	a	sq	te	ic	λ
b1	2	73	4	1	2	1	
b2	4	96	4	5	3	1	
w	0.2	0.2	0	0.2	0.2	0.2	0.5

Table 5.4: Profiles, weights and majority threshold inferred with supplementary budget constraint.

	rq	ps	a	sq	te	ic	λ
b1	2	2	2	1	2	1	
b2	3	84	2	4	3	2	
w	0.143	0.143	0.143	0.143	0.286	0.143	0.643

board is not satisfied with this set of projects because accepting these projects induces a total funding cost of 718 which exceeds the available budget (400). The program is thus ran again with a supplementary constraint on the sum of the budget of the projects being assigned to the Good category to ensure that it stays below the available budget.

This second stage inference yields other profiles and weights, given in Table 5.4, and a new list of assignments of which a part is displayed in Table 5.5. At this stage 11 projects are assigned to category Good and therefore are to be financed, leading to a total cost below 400. However the board is not fully satisfied yet because one domain is largely favored by this result, as the AI domain has 7 projects selected whereas only 1 project in the OR domain is to be financed. In a third stage, the inference program is thus ran again with a new constraint requiring that the domain OR has at least 2 projects in the category Good. The final assignment results, shown partly in Table 5.6, are considered satisfactory.

The process could have continued had the board wished a better balance among the originating countries, or had they wished to consider more closely also the Average category.

5.5 A case study

5.5.1 Introduction

We address a real world example dealing with students selection at Ecole Centrale Paris, France, and use this case study to discuss the usability of the proposed methodology in this chapter for

Table 5.5: A part of the assignment of the research projects with the preference model inferred during the second stage.

Project	rq	ps	a	sq	te	ic	budget	domain	country	Cat
Pr001	2	47	2	3	1	0	27	Stat.	Germany	Bad
Pr002	2	3	2	4	4	0	29	Stat.	France	Average
Pr003	5	63	1	5	1	0	20	Stat.	Italy	Bad
Pr004	1	92	3	5	5	1	34	AI	Germany	Good
Pr005	4	13	2	4	2	0	32	Stat.	Germany	Average
Pr006	5	5	3	5	1	0	22	Stat.	Netherlands	Bad
Pr007	1	27	3	2	5	1	34	OR	Germany	Average
⋮										

Table 5.6: A part of the assignment of the research projects with the preference model inferred during the third stage.

Project	rq	ps	a	sq	te	ic	budget	domain	country	Cat
Pr001	2	47	2	3	1	0	27	Stat.	Germany	Average
Pr002	2	3	2	4	4	0	29	Stat.	France	Average
Pr003	5	63	1	5	1	0	20	Stat.	Italy	Bad
Pr004	1	92	3	5	5	1	34	AI	Germany	Good
Pr005	4	13	2	4	2	0	32	Stat.	Germany	Average
Pr006	5	5	3	5	1	0	22	Stat.	Netherlands	Average
Pr007	1	27	3	2	5	1	34	OR	Germany	Average
⋮										

more general recruitment/staffing problem. The goal of this selection is to choose candidates among students having similar competencies (trained in the same school). The main concern for the decision maker involved in this selection is to find students who best fulfill the requirements of the courses, based on their behavior competencies and abilities.

Our contribution is an innovative two stage methodology to support such staffing decision. In a first step, the ELECTRE TRI_{BM} method sorts students to ordered predefined categories according to how each student individually fulfills the selection requirements. On the basis of the individual evaluations, a second step of the analysis combines the results of the first step with the requirements to select a group of students which are individually good and satisfactory as a group. This second step involves a mathematical programming formulation and identifies students portfolios which are good compromise between group constraints and the ELECTRE TRI_{BM} students classification.

5.5.2 Literature on Multiple Criteria Student Selection

The management of students in universities has long been an issue for academic institutions: deciding which student should enter an academic program? how to compose groups of students to form teams for projects? How to assign students to academic majors?

Student selection problems can be schematically divided into two different types of decision situation. The first type of problem concerns the partition of the set of students. Depending on the situation, the partition of students can refer to groups required for team projects, or to the distribution of students among majors according to their preferences, see Reeves and Hickman (1992), Weitz and Jelassi (1992), Bafail and Moreb (1993), Saber and Ghosh (2001), Miyaji et al. (1988). The second problem refers to the selection of students for entering an academic program or a particular major, in which case the issue is not to partition a set but, rather, to identify the students with the highest merit to enter the program; such questions are related to recruitment issues, see Kuncel et al. (2001), Yeh (2003), Leyva López (2005).

According to the problem type, the information used as input to solve the question can vary. The type of information considered can be either:

- Students' preferences, for the assignment to majors, or the composition of groups (e.g. Miyaji et al. (1988), Reeves and Hickman (1992), Saber and Ghosh (2001)); the preferences in such case correspond to an order on majors, or wishes on the group formation; or
- Criteria or constraints on the group formation such as gender issues, similarity of performance among groups, or diversity within groups (e.g. groupMiyaji et al. (1988), Reeves and Hickman (1992), Weitz and Jelassi (1992), Saber and Ghosh (2001)); or
- Student individual performance, GPA, predictive success attributes (e.g. Kuncel et al. (2001), Yeh (2003), Leyva López (2005)).

Earlier approaches give recommendations based on different multiple criteria decision models and diverse methodologies. Montibeller et al. (2009) and Philips and C.Bana e Costa (2007) make use of problem structuring methodologies. Some papers involve a ranking of students/alternatives to select the *k* best ones (e.g. LeyvaLopez2005), yet no approaches exist, to our knowledge, which consider these problems as multicriteria sorting problems with constraints on categories (which is the case of the methodology proposed in this work, see Mousseau et al. (2003a)).

Our approach of using a two stage methodology has been applied to a student admission problem in a academic program (Industrial Engineering major). In a first stage, each student is evaluated individually to determine wether (s)he fulfils the admission requirements. This first stage is modeled as a multicriteria sorting model (in which criteria evaluate students' individual performance), and involves the ELECTRE TRI_{BM} outranking method. The second stage, which involves a mathematical programming formulation, identifies subsets of students, which best fit the selection goals of the decision maker, and sets priorities among incompatible requirements on the group formation. We do note provide in this theis an extensive presentation of ELECTRE TRI_{BM} (see Section 3.2.2), but the interested reader will find detailed presentation of outranking methods in Roy (1991) and Figueira et al. (2005b).

5.5.3 Case Study Description

Context

In France, to enter most engineering schools (so-called "*grandes écoles*"), two years of preparatory studies are required before a competitive examination. During these two years, students acquire high level knowledge mainly in mathematics, physics and chemistry. The competitive examination is a selection process in which each engineering school selects its own admitted students. As one of the engineering schools, Ecole Centrale Paris (ECP) selects its student based on a nationwide competitive examination after the two years preparatory studies: 350 places (for more than 10 000 candidates) are available through a specific entrance examination. The other places are filled through different methods of selection: 150 places are available to university transfer students and foreign students that meet specific selection requirements.

ECP is an institution of higher education whose principal mission is to prepare highly qualified, general engineers for professions in industry and research. The educational program of ECP is based upon an integrated multidisciplinary approach that combines basic scientific and technical education, and has a solid orientation to economic, social and human realities in industry.

The students study three years at ECP. With respect to international standards, the first year (ECP1) at ECP corresponds to the final year of their undergraduate studies; the two remaining years (ECP2 and ECP3) corresponds to graduate studies. At the end of ECP2, students choose a major for year ECP3. We are concerned with the selection of students for the admission to one of the majors proposed in ECP3 (see Figure 5.3).

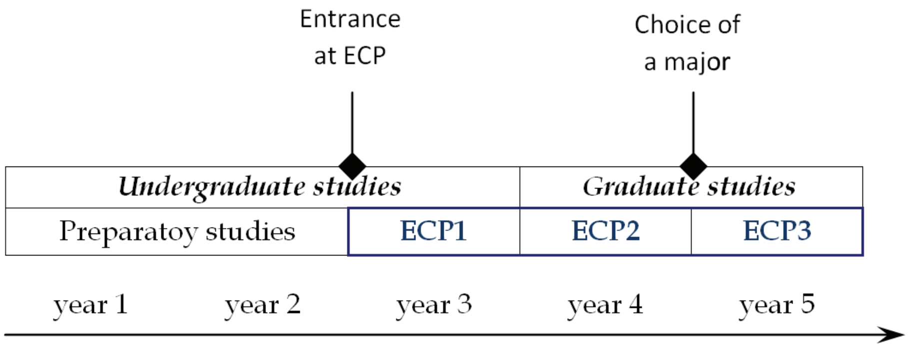


Figure 5.3: Organisation of the studies at ECP

For their final year (ECP3), the students have to choose a major among the nine ones listed; in addition to their major, students have to choose a "*professional track*" among 6 professionally oriented set of courses (entrepreneurship (E), design of innovative systems (DIS), operations management (OM), international project management (IPM), research (R), strategy and finance (SF)). During ECP3, students follow courses proposed by their major and other courses corresponding to their professional tracks (ECP3 is composed as an alternance of periods devoted to the courses of majors and the courses of professional tracks). Thus, students acquire both knowledge and competency in engineering, to become adapted to an ever-changing job market, fluctuating growth sectors, and the emergence of new activity fields.
The nine majors proposed at ECP are:

- Industrial Engineering major (IE),
- Applied mathematics major,
- Sustainable civil engineering major,
- Energy major,
- Environment and Biotechnologies major,
- Mechanical and aerospace engineering major,
- Physics and applications major,
- Information sciences major,

- Advanced systems major.

Students applying for the IE major should make an additional choice among four possible streams of courses. These streams correspond to "sub-specialization" within the IE major (product/service design, production/industrialization, supply chain, management). The choice of a stream defines a choice of specific elective courses. This happens to be important for the selection process because the minimum number of students required to open a course is 10, which imposes to select at least 10 students per stream.

Case description

Each year, a decision is to be made concerning which students should be admitted to the Industrial Engineering (IE) major. The limit established by the Dean of Studies is 50 students per major, and the number of students who apply for the IE major always exceeds the available places. Two persons are in charge of this selection process: the head and vice-head of the IE major. They work collaboratively on this problem and decide which students will enter the IE major (these two stakeholders have very similar views and agree on which type of students should be selected). Therefore, in the following the Decision Maker (DM) refers to this group of two persons.

The time line of the selection process is the same each year. Each student is required to specify a preference order on three majors. Each major receives the applications of students who have selected this major as a first choice. Applications of the students who are not admitted to their first choice of major are forwarded to the head of their second choice major, and eventually to their third choice. Concerning the IE major, the DM receives the files of all students applying as a first choice to the major at the end of April. This application includes a curriculum vitae, the GPAs obtained during the two first years at ECP (ECP1 and ECP2) and a personal motivation letter. In addition, this file includes also a choice of professional track, and the second and the third choice of the majors. For the students who apply to the IE major, an addition choice is required from the students: the choice of a stream (one of the four sub-specializations).

Before interviewing students, the DM examines all applications and, individual interviews are planned in the beginning of May. After three or four days of interviews, DM makes a first decision, which consists in a list of 50 accepted students, complemented with a waiting list of 10 students (the remaining students are redirected to their second choice major). In May, a few accepted students might resign from IE major and make it possible for students on the waiting list to be admitted. At the end of May, this progressive withdrawal process stops.

Until 2009, the admission of students to the IE major was based on an ad hoc selection process which was not thoroughly formalized and did not incorporate any formal decision support, but proceeded through an intensive interaction with the students and multiple exchanges and discussion between the head and vice-head of the IE major. The DM analyzed the students results and letters of motivation before having an individual interview with each of them. So as to assess whether a given student fulfills or not the different requirements, the DM considered the following aspects:

- motivation to follow courses in the IE major,
- professional project in relation to his/her formal industrial experiences,
- maturity and personality,
- knowledge about Industrial Engineering,
- GPAs during the two first year at ECP.

After each interview, each student was classified into one of the three following categories:

- the student should be accepted to the IE major,
- the student should be placed on a waiting list,
- the student should not be accepted to the IE major.

Apart from these five mentioned requirements, the DM also considered additional issues: to balance the gender on the set of selected students, an additional advantage was implicitly assigned to female students (less than 20% of candidates are girls). Similarly, the DM sought to build a balanced student group with respect to the choice of professional track is an important issue for the DMs; this was done by giving a penalty to students who chose a frequently demanded professional track. At the end of the interview process, the DM decided the 50 selected students and designed a waiting list. Very often, rejected students requested a second interview or wanted to know why they had been rejected.

Stakes involved in the students selection process

In order to reach a balanced distribution of students among the nine majors, the Dean of Studies imposes a maximum of 50 students per major. For some of the majors, this constraint is not an issue, as, each year, there are no more than 20 students that apply. As the IE major is the most popular major, each year, more than 70 students apply to this major. For the DM, this constraint is crucial on many respects:

- The decision is to be explained to the rejected students. The DM must be able to make the reasons explicit why a student is not selected.
- The DM considers that rejected students are not "bad" students but rather students who do not fit the IE major orientation,
- The DM needs to justify to the Dean of Studies and heads of other majors the decision process and the result of selection. Namely, the DM wants to avoid the selection to be viewed as "IE major admits all students with high GPA".
- The DM makes the selection decision on the basis of how well each student individually fits the IE major, but also with respect to the coherence of the group of students as a whole (gender issue, balance among professional tracks...).
- The minimum number of students required to open a course is 10. Because some courses are restricted to students that are assigned to one of the four streams, an important aspect of the selection process is to try to admit at least 10 students per stream.

Considering these stakes, the DM needs decision support tools to be introduced in the students selection process; The introduction of such tools should lead to gain in efficiency by reducing the time spent by the DM, to improve transparency of the selection process (namely for the students and dean of studies), and, to perpetuate a systematic yearly selection process.

5.5.4 A Methodology for Admitting Students to the IE Major at ECP

The objective of the DM is to have a systematic selection process that can also help him justifying the final decision concerning the selection of a students in the IE major. In addition to this main purpose, one of the aims of this paper is also to investigate the interest of the introduction of such tool in students selection processes, and in a broader sense into selection processes.

The analyst, who supported the DM in designing a decision support methodology, was a specialist in multicriteria decision analysis. The proposed methodology had been developed through a strong interaction between the DM and the analyst. The structuring phase consisted in several working sessions. In a first stage, the DM and analyst worked on a common understanding of the existing process (decision to be made, criteria to consider, stakeholders involved in the decision process, constraints of the process,...). Simultaneously, key data from former years were gathered. At this point, application files from year 2009 were analyzed. Then, the process consisted in discussions between the DM and the analyst to identify, clarify, and justify the criteria on which the selection of students was done. This led to distinguish among the criteria which characterize each student:

- individual criteria: relative to the quality of the application (compliance to the IE major requirements); these criteria are used in the model which evaluates students individually, see section 5.5.4.

- group criteria: factors to consider at a group level (balance in gender, equilibrate distribution among professional tracks, ...),see section 5.5.4

This distinction justifies the elaboration of a two-levels methodology. In a first step, each student application is evaluated individually, i.e., without considering the factors that are relevant at the group level (gender, professional track, ...). This results in a list of students who individually deserve to be admitted to the IE major based on their individual merits. However, this group of students may not be satisfactory for the DM (unbalanced in gender, or among professional track,). The second step establishes which students should be admitted in order to select students who are individually good, on one hand, and to form a group that is collectively satisfactory, on the other hand. Moreover, this second step should provide insights to the DM for how to integrate and compromise these two aspects.

Evaluating students individually

The available information has been structured to evaluate students on 6 criteria: the two first criteria correspond to the student's GPA in first and second year study (in France, grades are given on a [0..20] scale, 20 being the best possible grade); the four last criteria (motivation, professional project, maturity/personality, knowledge about IE) are qualitatively evaluated on a

5 level scale (5 being the best evaluation). They are defined as follows (the precise description of criteria and evaluation scales is provided in Appendix A.1):

- **Motivation**: Perceived motivation of the student in the choice of the IE major as judged by the DM through the interview and by reading the cover letter.
- **Professional Project**: Ability of the student to articulate his/her future professional project with his/her previous achievements (courses, ...). (S)he takes into account the logic, consistency and variety of what (s)he has done previously, the reasons for his/her choice to come to IE projects and employability. The coherence of the choice of major with the Professional Track is considered here.
- **Maturity / Personality**: Maturity and the openness of the student that brought her/him to focus on the Industrial Engineering in a large sense and beyond to general society concerns.
- **General knowledge of Industrial Engineering and its career opportunities**: Ability to define what Industrial Engineering is, in particular knowledge of the contents of the IE Major at ECP and the various outcomes.

On the basis of these 6 criteria, the DM first assigns each student to one of the four categories defined hereafter, according to whether or not they fulfil the requirements for entering the IE major.

- C_1: Students who do not meet at all the requirements to enter the IE major, and consequently who can really not be accepted.
- C_2: Students who do not really meet the requirements to enter the IE major. They should not normally be accepted unless the categories C_3 and C_4 contain a limited number of students.
- C_3: Students who fairly well correspond to the requirements to enter the IE major. These students could be admitted or appear in the waiting list.
- C_4: Students who fully correspond to the requirements to enter the IE major, and therefore should be admitted.

This first analysis aims at appreciating each application individually, and check how well each student fulfils the requirements for admission to the IE major. The choice of four categories was made by the analyst and the DM as a good compromise between the discriminating power and the over-complexity of this model.

Considerations relative to the group of students to be admitted

However, the classification stemming from step 1 (individual evaluation) is made independently for each student, without any consideration dealing with the number of available positions in the IE major. Hence, it remains insufficient for the DM for defining which students to select. It should be emphasized that when the number of "sufficiently good" students ($C_3 \cup C_4$) is less than the number of positions (50), the methodology makes it possible to decide either to admit less than 50 students, or to consider admitting some students assigned to C_2. On the contrary, if the number of "sufficiently good" students exceeds 50, the DM has some flexibility to choose among them a subset of 50 students which will be balanced in terms of gender, and other considerations relative to the group of admitted students. A way to integrate this limitation in the number of admitted students is to impose that the number of alternatives (students) assigned to these categories is lower or equal to 50.

Moreover, the DM takes other considerations into account, which gives additional constraints on the group of students to be selected:

- the DM wishes to have a good balance in terms of gender,
- the students in the IE major choose one of the 4 existing "streams" (product/service design, production/industrialization, supply chain, management) which corresponds to specific courses. Therefore, the number of students in each stream should not be too low (a course is opened if it contains at least 10 students).
- During their last year study, in addition to their major, students are assigned to a "professional track" among 6 (professionally oriented set of courses). The DM wants to have a group of students well distributed among "professional track", so that the group of students can be representative of the various jobs available for industrial engineers on the labor market.

The above considerations can be formulated in the model through the addition of constraints on the number of students assigned to C_3 or C_4 who exhibit a specific property. For instance, a good balance among the professional tracks , can be formulated by a constraint stating that the number of students assigned to C_3 or C_4 and who chose a specific professional track should not exceed 20.

Obviously, constraints concerning the balance (in gender, streams and professional tracks) might not match the group of students assigned to C_3 or C_4, and not be compatible with the students evaluation model. In order to appreciate how these constraints conflict, we identify the various ways to relax these constraints. These relaxations characterize sets of incompatible constraints from which the DM can get insights on the conflicting aspects of the selection problem, and be supported in finding a reasonable compromise between arguments relative to the quality of the selected students and the desired quality of the group of selected students as a whole.

Interest of MCDA methodology

The introduction of the proposed methodology in the actual selection process improves this decision process as perceived by the DM and other stakeholders. Specifying a structured process based on a formal method of selection is a real advantage for many reasons:

- It is easier for the DM to justify the decisions: criteria are explicit, the student evaluation model can be explained and is calibrated based on past examples.
- All stakeholders involved in the selection process (the DM, students, dean of studies, heads of other majors) have a better understanding of the decision process, with less perceived arbitrariness.
- For non-selected students who want to have explicit explanations why they are refused, it is easier for the DM to show them the result of the decision through this formal process.

These elements contribute to the acceptability of the selection process and its result by the stakeholders. Another important argument of the proposed methodology is related to the fact that it is decomposed into two “phases”, and fits the process used by the DM to select students. In a first phase, application files are studied, students are interviewed, and the DM evaluates each student individually . In a second phase, once each student is evaluated (classified in one of the four categories), arguments relative to the group of selected students are considered. Then the DM conducts an analysis of how considerations about each student individually conflict with arguments related to the set of students as a whole. Such analysis makes it possible to define a final set of selected students. Hence, the students evaluation tool is intended to be used during the interview, while the tool which analyzes how to balance group issues with the students' individual qualities should be used after the end of the interview process.

5.5.5 Models Involved in the Selection Process

In this section, we present how the methodology proposed in the previous section can be implemented. The description of the implementation is applied to the dataset corresponding to the students selection process which took place in 2009. The full dataset (76 applicants in 2009) is provided in Appendix A.2.

Sorting students using ELECTRE TRI$_{\mathrm{BM}}$

Problem Statement

So as to evaluate the quality of student applications individually, we assign each student to one of the four categories ($C_4 \lhd C_3 \lhd C_2 \lhd C_1$) using the ELECTRE TRI$_{\mathrm{BM}}$ (introduced in detail in chapter 3). We choose the ELECTRE TRI$_{\mathrm{BM}}$ method to assign the students to categories, because this method is very well adapted to problems in which ordinal criteria (i.e., "qualitative" criteria for which differences of evaluations have no meaning; evaluations can be interpreted in terms of order only) are involved. In our case study, 6 ordinal criteria $g_1, g_2, g_3, g_4, g_5, g_6$ are considered.

During the interaction with the DM, we interpreted ELECTRE TRI$_{\mathrm{BM}}$ assignment procedure in our student evaluation context as follows.

- In order to be assigned to C_4 (best category), a student evaluations must be at least as good as b_3 (limit between C_3 and C_4) on a "*majority*" of criteria, and should not have, on any criterion, a "*veto evaluation*" which precludes the assignment of this student to C_4, otherwise

- In order to be assigned to C_3 (second best category), a student evaluations must be at least as good as b_2 on a"*majority*" of criteria, and should not have, on any criterion, a "*veto evaluation*" which precludes the assignment of this student to C_3, otherwise

- In order to be assigned to C_2, a student evaluations must be at least as good as b_1 on a"*majority*" of criteria, and should not have, on any criterion, a "*veto evaluation*" which precludes the assignment of this student to C_2, otherwise

- The student is assigned to C_1

In the above process, we also clarified to the DM that the weights w_j do not represent tradeoffs but are used here to determine whether a subset of criteria constitutes (or not) a majority. Hence, the information conveyed by the weights can be interpreted as a "voting power" associated with each criterion. A subset of criteria is considered as a majority winning coalition of criteria when the sum of the weights of these criteria exceeds the majority threshold λ. Moreover, the notion of *veto evaluation* corresponds to an evaluation on a criterion g_j which is sufficiently bad to forbid a student to be assigned to a category C_h. To specify these *veto evaluations*, ELECTRE TRI$_{\text{BM}}$ uses a set of veto thresholds $(v_1(b_h),...,v_j(b_h),...,v_6(b_h))$, $h \in \{1,2,3\}$ such that the *veto evaluation* is equal to $g_j(b_h) - v_j(b_h)$.

Elicitation of the ELECTRE TRI$_{\text{BM}}$ student evaluation model

In order to define an ELECTRE TRI$_{\text{BM}}$ model for our students selection problem, the preference related parameters should be elicited:

- category limits ($g_j(b)$, $i = 1..6, b \in B$)
- the relative importance of criteria:
  - the weight vector w_j and the majority level λ used in the concordance condition,
  - the veto evaluations v_{jh}, $j = 1..6$, $h = 2,3,4$ used in the non-discordance condition.

In order to set the values for these parameters, interviews were conducted with the DM. The category limits ($g_j(b)$, $j = 1..6, b \in \{b_1, b_2, b_3\}$) were induced directly from such discussions. For instance, the DM was able to state that if a student a has evaluation on criterion g_1 greater or equal to 13 (g_1: GPA in first year study at ECP, ranging in France in [0,20]) then its GPA in first year will "vote" (contribute to the assignment) to the category C_4. The values of $g_j(b_h)$ are provided in Table 5.7. Furthermore, during the interviews, the DM made the statements given below. These statements are relevant to define "*veto evaluations*" v_{jh}:

1. a student having a score less than 3 on the criterion "Motivation" (g_3) cannot be assigned to category C_4,
2. a score 1 on one of the four last criteria forbids a student to be admitted (i.e., cannot be assigned to category C_3 nor C_4),

Frontiers \ Criteria	g_1	g_2	g_3	g_4	g_5	g_6
$g_i(b_1)$	10	10	2	2	2	2
$g_i(b_2)$	11	11	3	3	3	3
$g_i(b_3)$	13	13	4	4	4	4

Table 5.7: Values of limiting profiles b_h

Veto evaluations \ Criteria	g_1	g_2	g_3	g_4	g_5	g_6
v_{i2}	-	-	-	-	-	-
v_{i3}	-	-	1	1	1	1
v_{i4}	-	-	2	1	1	2

Table 5.8: Veto evaluations

3. a student evaluated 1 or 2 on criterion "general knowledge on Industrial engineering" (g_6) cannot be assigned to category C_4.

At this stage of the elicitation process, the frontiers separating consecutive categories and veto evaluations were determined. The parameters which remains to be elicited are the weights w_j and the majority level λ. The DM was not comfortable with providing precise values about such parameters, but was confident about his judgments on the assignment of some students, statements which indirectly (the limits of categories and veto thresholds being fixed) provided information about the weights w and majority level λ.

From his experience and expertise, the DM was confident in his judgment concerning 5 given students. In other words, he managed to provide 5 assignment examples given below (which constitute the subset $A^* \subset A$).

1. a_{10} and a_{61} should be assigned to category C_3 ($a_{10} \rightarrow C_3$, $a_{61} \rightarrow C_3$)

2. a_{22} should be assigned to category C_2 ($a_{22} \rightarrow C_2$)

3. a_{59} and a_{68} should be assigned to category C_4 ($a_{59} \rightarrow C_4$, $a_{68} \rightarrow C_4$)

These assignment examples induce linear constraints to infer weights and majority level. For instance, $a_{68} \rightarrow C_4$ implies that $\sum_{j:g_j(a_{68}) \geq g_j(b_3)} w_j \geq \lambda$.

Moreover, the DM believed that the second criterion (second-year grade) should not be more important than the third (motivation), fourth (professional project) or fifth (maturity/personality) criterion, which lead to the following constraints: $w_2 \leq w_3$, $w_2 \leq w_4$, and $w_2 \leq w_5$.

From this indirect information concerning the weights and majority level (assignment examples and additional constraints on weights), we inferred a weight vector and cutting level using the inference algorithm proposed by (see Mousseau et al. (2001a)). The inference program (see Appendix A.3) minimizes an error function subject to constraints which represent the assignment examples. The inferred values were $w = (0.16, 0.17, 0.21, 0.21, 0.21, 0.05)$ and $\lambda = 0.54$. This completed the elicitation of the model and allowed to assign each student to a category (C_4, C_3, C_2, or C_1).

From this model, it results that the number of students assigned to C_4, C_3, C_2, C_1, respectively, is 29, 27, 4, 16, respectively (all results are provided in Appendix A.2). From this analysis, it appears that 56 students meet the requirements to be admitted to the IE major (C_4 and C_3). Among these 56 students, 18 are girls (38 are boys). The distribution among the four streams (product/service design, production/industrialization, supply chain, management) is 19, 14, 12, and 11, and the distribution among the six professional tracks is (0, 13, 26, 0, 8, 9).

Although, this first model provides interesting insights for the DM (which student deserves to be selected, based on his evaluations), the results do not fully match the particularities of the decision situation. Namely, one important characteristic of the decision to be made deals with the fact that the maximum number of students to be selected is 50. Obviously, the proposed model does not account for the size of category C_3 and C_4 (it only assesses each student individually and assigns each of them to a category). With this model, it is possible that the number of students assigned to categories C_4 and C_3 exceeds 50 (which is the case in our dataset), in which case, the DM should be supported in choosing the 50 students to be selected out of the ones assigned to C_4 and C_3 (56 in our dataset).

Moreover, the DM wants to evaluate the group of students to be selected as a whole on various aspects. For instance, gender issue is involved and a reasonable balance concerning "streams" is desired by the DM. A good balance among the "professional tracks" is also viewed by the DM as highly desirable. The second phase of the methodology aims at integrating such constraints in the selection of the students.

Remark 4 *The mathematical formulation proposed in Section 5.3 doesn't take into account the veto effect. However, we consider for this case study the* ELECTRE TRI$_{\text{BM}}$ *model involving veto threshold because of the veto statements of the DM.*

Remark 5 *In this case, we elicit only partly the parameters of* ELECTRE TRI$_{\text{BM}}$ *model (category limits are given by the DM) while the mathematical formulation of Section 5.3 elicits* *all the parameters of* ELECTRE TRI$_{\mathrm{BM}}$ *model. However, both optimization programs yield to linear programming.*

Remark 6 *The portfolio selection method proposed in the chapter considers preference information at two levels (individual and group level). Mathematical formulation of Section 5.3 infers an* ELECTRE TRI$_{\mathrm{BM}}$ *model from the information at two levels, and the individuals to be selected are identified simultaneously in the preference elicitation process. For this case study, preference information of the two levels is used separately in the two-step method. That is to say, during the evaluation process, only the preference of evaluating the students is incorporated; preference at group level is used in the selection step to draw the final recommendation. The two step method corresponds to the DM's previous decision practice and is easier for him to perceive, and that's why we treat separately the two kinds of preference information.*

Remark 7 *For this case, only one fixed* ELECTRE TRI$_{\mathrm{BM}}$ *model is used as evaluation model to assign students to a single category, although multiple models may conform to his preference. It is also because evaluating students to a single category fits into the DM's previous two-step experience. Assigning students to a range of categories considering all compatible models was found to be not easy for the DM to understand.*

Modeling constraints to design the group of selected students

We will propose in the following a methodology to select a group of students which are individually as good as possible and collectively conform as much as possible to the constraints specified by the DM on the group to be admitted. In order to do so we will pursue the analysis of the data of the year 2009. In this case, the question amounts at deciding which students to select out of the 56 who were judged as fulfilling the IE major requirements.

Defining the group constraints

From the first step of the analysis, it follows that 56 students reasonably deserve to be admitted to the IE major. Among these 56 students, 29 are assigned to category C_4 (students who fully correspond to the requirements to enter the IE major, and therefore should be admitted), and 27 to category C_3 (students who fairly well correspond to the requirements to enter the IE major). These students could be admitted or appear in the waiting list). In the following, all students assigned to category C_4 are unconditionally admitted, while the ones assigned to C_3 will be considered for admission depending on the constraints on the group.

In order to specify constraints relative to the group of selected students, we will proceed as follows. Let us consider one binary variable Θ_i, i=1..27, for each student assigned to C_3. The semantic of such variables is defined by: $\Theta_i = 1$ when the student i is admitted (the i^{th} student in the list of 27 students assigned to C_3), $\Theta_i = 0$ otherwise.

The first type of constraints to be considered refers to the number of actually admitted students. The objective of the DM is to limit the number of admitted student to 50; as the 29 students assigned to C_4 are all admitted, this can be formulated by the following constraint: $\sum_{i=1}^{27} \Theta_i \leq 21$. If the DM considers relaxing this constraints, the successive relaxations take the form of $\sum_{i=1}^{27} \Theta_i \leq 21$, $\sum_{i=1}^{27} \Theta_i \leq 22$, $\sum_{i=1}^{27} \Theta_i \leq 23$, ...$\sum_{i=1}^{27} \Theta_i \leq 27$. Note that the last relaxation amounts to not considering size constraints.

The second type of constraints is related to the distribution of admitted students among the four "streams". Let us define S the 4×27 matrix of choice of streams by student assigned to C_3 ($s_{ij} = 1$ if the i^{th} student assigned to C_3 chooses the stream j, $s_{ij} = 0$ otherwise). As each course should have at least 10 students, the number of students in each stream should be at least 10. Such constraint can be formulated as: $\sum_{i=1}^{27} s_{ij}\Theta_i + n_j(C_4) \geq 10,\ j = 1..4$, where $n_j(C_4)$ denotes the number of students assigned to C_4 (and therefore admitted) who chose the stream j. If the DM considers relaxing these constraints concerning the streams, the successive relaxations take the form of $\sum_{i=1}^{27} s_{ij}\Theta_i + n_j(C_4) \geq 10,\ j = 1..4$, $\sum_{i=1}^{27} s_{ij}\Theta_i + n_j(C_4) \geq 9,\ j = 1..4$, $\sum_{i=1}^{27} s_{ij}\Theta_i + n_j(C_4) \geq 8,\ j = 1..4$, etc.

The third type of constraints concerns the distribution of admitted students among the six "professional tracks". Let us define PT the 6×27 matrix of choice of professional tracks by student assigned to C_3 ($pt_{ij} = 1$ if the i^{th} student assigned to C_3 chooses the professional track j, $pt_{ij} = 0$ otherwise). The goal of the DM is to select students well spread among professional tracks and therefore to limit the number of admitted students in a given professional track to 20 at most. Such constraint can be formulated as: $\sum_{i=1}^{27} pt_{ij}\Theta_i + m_j(C_4) \leq 20,\ j = 1..6$, where $m_j(C_4)$ denotes the number of students assigned to C_4 (and therefore admitted) who chose the professional track j. If the DM considers relaxing these constraints concerning the professional tracks, the successive relaxations take the form of $\sum_{i=1}^{27} pt_{ij}\Theta_i + m_j(C_4) \leq 20,\ j = 1..6$, $\sum_{i=1}^{27} pt_{ij}\Theta_i + m_j(C_4) \leq 21,\ j = 1..6$, $\sum_{i=1}^{27} pt_{ij}\Theta_i + m_j(C_4) \leq 22,\ j = 1..6$, etc.

The fourth type of constraints is related to gender issue. The DM considers that a good balance between girls and boys is when the number of accepted girls is in the interval [20, 30]. Let us define $g_i = 1$ if the i^{th} student assigned to C_3 is a girl, $g_i = 0$ otherwise). This constraint can be formulated as: $\sum_{i=1}^{27} g_i\Theta_i \geq 20$ and $\sum_{i=1}^{27}(1-g_i)\Theta_i \geq 20$. The successive relaxations take the form of $\sum_{i=1}^{27} g_i\Theta_i \geq 19$ and $\sum_{i=1}^{27}(1-g_i)\Theta_i \geq 19$, $\sum_{i=1}^{27} g_i\Theta_i \geq 18$ and $\sum_{i=1}^{27}(1-g_i)\Theta_i \geq 17$, etc.

It is obvious that all the above constraints are not compatible. The issue for the DM is then to identify the alternative ways to relax the constraint so as to make the problem feasible, and to choose the best solution among these.

Identifying "best compromises" among the list of constraints In order to help the DM select a "best compromise" among incompatible requirements, all possible minimal sets of constraints relaxations yielding to a feasible set of constraints have been computed.

First of all, note that all constraints considered are linear and can be written as $\sum_{i=1}^{27} \alpha_i\Theta_i \geq \beta$. Please note also that the number of relaxations of a constraint (which are also linear) is finite (and even a small number). In the following, we will consider the (unfeasible) set of constraints defined in the previous section and all their respective relaxations. It is obvious that the relaxed constraints are redundant initial constraints. Let us suppose that the number of relaxed constraints is equal to p. The k^{th} constraint can be written as:

$$\sum_{i=1}^{27} \alpha_{ik}\Theta_i \geq \beta_k \tag{5.7}$$

Let us now define y_k ($k = 1..p$), p new binary variables (one for each relaxed constraint) and rewrite the k^{th} constraint as:

$$\sum_{i=1}^{27} \alpha_{ik}\Theta_i + L.y_{ik} \geq \beta_k, \text{ where } L \text{ is an arbitrary large positive value} \tag{5.8}$$

It is clear that when $y_{ik} = 0$, (5.8) is equivalent to (5.7), but when $y_{ik} = 1$, (5.8) is always verified, and it is as if (5.7) is "deleted". Moreover, it should be noted that when the k^{th} constraint is deleted ($y_{ik} = 1$), then one of its relaxed constraint which was initially redundant, becomes active. We consider as objective function $z = \sum_{i=1}^{27}\sum_{k=1}^{p} y_{ik}$ which should be minimized subject to the set of p constraints defined as in (5.8) and defined the following mathematical program:

$$\begin{cases} Min\ z = \sum_{i=1}^{27}\sum_{k=1}^{p} y_{ik} \\ \quad s.t. \sum_{i=1}^{27} \alpha_{ik}\Theta_i + M.y_{ik} \geq \beta_k, k : 1..p, i : 1...27 \end{cases} \tag{5.9}$$

The optimal solution of this mathematical program identifies the smallest set of constraints and relaxed constraint whose deletion leads to a feasible set of constraints. Let us denote by S the set of indices of these constraints ($ik \in S$ such that $y^*_{ik} = 1$). So as to find alternative solution to reach feasibility, we add to the above mathematical program a constraint which forbids to find this optimal solution: $\sum_{ik \in S} y_{ik} \leq card(S)$, where $card(S)$ denotes the cardinality of the set S. Solving this second mathematical program leads to find a new way to relax the constraints. This iterative process continues until the resulting mathematical program to be solved is unfeasible, which means that there does not exist any other way to relax constraints. The algorithm described here, is very similar to the one used in Mousseau et al. (2003b) and Mousseau et al. (2006) to solve inconsistencies among a set of inconsistent preference statements.

Results on the 2009 dataset

So as to identify the possible "compromises" among the list of constraints on the set of 2009 applicants, we applied the algorithm described in the previous subsection. The mathematical program to be solved at each iteration (to identify one minimal set of constraint relaxation) involved 95 binary variables (27 Θ_i variables for each student assigned to C_3, and 68 y_{ik} variables for all constraint relaxations). The algorithm has been implemented using CPLEX v.11, and solved on a Intel Core Duo CPU 3Ghz with 2 GB RAM; the CPU time was 0.76 second. The limited computing time (for 95 binary variables) makes it conceivable to consider even larger datasets.

On the given data, the algorithm did run four iterations, that is to say four alternative minimal sets of constraint relaxations were identified. These four sets correspond to the only four ways to account for the infeasibility of the constraints on the group of selected students (admit a higher number of selected students, worse balance in each stream, professional track and gender). These solutions are described in Table 5.10.

Solutions	♯ students (girls/boys)	♯ stream (1/2/3/4)	♯ prof. track (E/SF/IPM/OM)
Sol_1	50 (15/35)	(16/14/10/10)	(13/20/8/9)
Sol_2	50 (16/34)	(17/13/10/10)	(12/21/8/9)
Sol_3	50 (17/33)	(16/13/11/10)	(11/22/8/9)
Sol_4	50 (18/32)	(16/13/11/10)	(12/23/7/8)

Table 5.10: Minimal sets of constraint relaxations

So as to interpret the solutions stemming from the algorithm, one should consider the four degrees of liberty to relax the constraints concerning the group of selected students:

- Accept more than 50 students,
- Accept to worsen the gender balance,
- Accept to worsen the balance among streams, or
- Accept to worsen the balance among professional tracks.

In the case of this dataset, the interpretation of these four solutions is straightforward. It appears clearly in the four solutions that the total number of admitted students (50) and the minimum number of students per stream (10) are fully conform to the DM's wishes. Moreover, deteriorating the situation on these two aspects does not make possible to improve the situation on the constraints concerning gender nor professional track. The only way to solve infeasibility in the set of constraints involves gender balance and balance among professional tracks.

If the DM wishes to admit a maximum number of girls (18), he will have to accept to have 23 students in the second professional track (Sol_4); conversely, if he wants to limit to 20 the number of admitted students in the second professional track, this will limit the number of admitted girls to 15 (Sol_1). There also exists two intermediate solutions (Sol_2 and Sol_3) that provide reasonable compromise among these two issues. Finally, considering these four solutions, the DM chose to admit 18 girls (Sol_4). The list of selected students is provided in Appendix A.2.

5.5.6 Insights from the Model Implementation

The first tangible result concerns the set of selected students stemming from the use of the methodology. The methodology was successfully adopted by the DM in 2010, and is now considered as a tool for the students selection process. For this case study, the development of the methodology has also established criteria for the evaluation of students together with precise evaluation scales, and their uses in the decision process. A key success factor of the adoption of the methodology has been a strong involvement of the DM in the elaboration of the methodology. We have shown how to proceed on the 2009 dataset, but the decision process makes it possible to use it each year repeatedly, in a consistent manner. Moreover, the annual use of this methodology guarantees that the selection policy remains consistent over time.

Second, the development process of the methodology are interesting in its own right. The four categories in ELECTRE TRI$_{\text{BM}}$, along with their semantics and multicriteria frontiers, help the DM clarify requirements to enter the IE major. Setting values to weights of criteria and veto thresholds, forced the DM to consider the relative importance he attaches to the criteria. The result of this preference elicitation process makes explicit how he compares criteria in terms of importance.

Another insight for the DM is linked to the data and decision process structure. Indeed, the implementation of the model shows that there are some conflicts between group and individual preference. For instance, it is not always possible to admit 50 students who satisfy all the constraints concerning gender balance, balance among streams and balance among professional tracks. This recognition highlights that the DM may have to relax the constraints to make his decision, and the methodology forces him to explicit these choices.

The use of a decision model makes the decision process more transparent, because it is based on a well-known and sound methodology. Results support the DM in providing explanations about the selection process to the various stakeholders (students, heads of other majors and dean of studies).

Yet, there are still some controversial aspects with respect to the constraint relaxation in the second phase of our methodology. Indeed, why should one constraint rather than others be relaxed ? Are there any priorities among constraints? Here, the methodology forces the DM to make explicit choices about which constraints to relax, whereas an informal process would make it possible for him to select students without acknowledging the underlying compromises among constraints. Another aspect of the decision process which could need a decision support is the definition of the waiting list (a ranking procedure of the students assigned to C_3 who are not admitted could be a reasonable solution).

Methodologically, the proposed selection process pays much attention to the individual evaluation of students (ELECTRE TRI$_{\text{BM}}$ method at step 1), and considers group constraints only in a second step. An advantage of this formulation is that if the number of students assigned to C_3 and C_4 does not reach the quota (50), the DM is proposed to admit less than 50 students (only the ones that fulfil the admission requirements). This is one of the characteristics of our approach whereas standard portfolio decision analysis methodologies would rather select the best group of 50 students.

5.6 Concluding Remarks

We apply constrained ELECTRE TRI model to portfolio selection problems in order to select a satisfactory portfolio considering DMs' preferences both at individual and portfolio level. Using a sorting model, the alternatives are evaluated by their intrinsic performances on criteria. Unsatisfactory portfolios which do not meet the DMs' requirements on portfolios as a whole are screened out by adding category size constraints to ELECTRE TRI_{BM} model. Because of such category size constraints, the assignment of an alternative is dependent on its evaluation but also on other alternatives.

Our formalization permits to tackle the challenges the DMs may face during the decision of portfolio selection. (1) At individual level, an alternatives is evaluated on multiple criteria which can be qualitative or quantitative criteria. Moreover, the DMs express their preferences on alternatives by assignment examples easily. (2) At portfolio level, the best alternatives do not necessarily compose the best portfolio. Our method takes into account the overall portfolio performance by modeling the DMs' preference on portfolio as constraints. (3) The preference information at the two levels (individual classification of alternatives and preference at the portfolio level) can be elicited from different stakeholders. (4) The proposed method involves the DMs deeply by asking them preference in an intuitive way.

The case studied is related to the selection of students from the same academic institution (ECP in Paris). Selected students are not assigned to different tasks but are altogether in the same activity field (IE major). Moreover, selected students are not directly compared one to each other, but rather to norms which define admissibility; hence the methodology proceeds through an overall evaluation, without any ranking. This individual evaluation is complemented, in a second phase, by the introduction of constraints on the set of selected students. A direct extension of this student selection work is supporting the elaboration of the waiting list. This could be performed by the introduction of a ranking model applied to the students who fit the admission requirements, but who were not admitted.

Although the case is not a direct application of the proposed method, as illustrated in Remark 4-7, the two parts of the chapter (a general portfolio selection method and the case study) can be viewed as complementary possible results of a decision aiding process. Section 5.4 illustrates how the method is used if the DM's preference at individual and group level is consistent and a satisfactory portfolio is selected (not necessarily an optimal nor unique one). The case in Section 5.5 shows how to deal with inconsistent preference concerning individual and group evaluation, and a compromise that some constraints are relaxed is to be made.

The proposed method can be used widely in portfolio selection situations where the decision should be made taking into account the individual alternative and portfolio performance simultaneously. The proposed syntax of category size constraints has a broad descriptive ability for portfolio decision modeling.

Chapter 6

Preference Elicitation Algorithm for Ranking Models with Reference Points

The chapter concentrates on the preference elicitation of a newly proposed Ranking model based on Multiple reference Points (RMP), which is presented in detail in Chapter 3. More precisely, we are interested in the simplified version of RMP (S-RMP). As an outranking method, it obtains a weak order satisfying invariance with respect to an irrelevant alternative. However, the application of such model is restrained as the result of the lack of tools to construct a meaningful S-RMP model incorporating the DM's preference. We aim at designing such preference elicitation tool to answer the following three questions: (1) how many reference points should we use? (2) what should we set the values of these reference points to? (3) which lexicographic order the reference points should be used? These questions are answered by eliciting preference information in the form of pairwise comparisons. Based on these pairwise comparisons, which are represented by linear constraints, the proposed elicitation algorithm infers a parsimonious S-RMP model with minimum number of reference points. Numerical experiments are conducted to investigate the usability and behavior of the algorithm. In addition, an application is studied which assesses the priority of treating pollutant substances. S-RMP evaluation models were elicited with a compromise between the simplicity and expressibility of the elicited models. In other words, we obtain models with one reference point which represent as much as possible the DMs' preference. The decision aiding experience shows how the elicitation algorithm can be applied in practice.

6.1 Introduction

The Ranking method based on Multiple reference Points (RMP) has some appealing properties (see Section 3.3 for the detailed presentation of the method). Firstly, it is able to handle criteria evaluated on qualitative scales where only poor information is available. For instance, the customer satisfaction with a service can only be assessed by qualitative judgement or verbal terms. Secondly, the method is able to rank the alternatives to a weak order, which means transitivity of preference is satisfied. Such feature is a significant advantage, since most outranking methods for ranking problems don't satisfy transitivity, due to Arrow's impossibility theorem (see Section 3.3). For example, ELECTRE III violates transitivity with the presence of preference cycles ($a \succ b$, $b \succ c$ and $c \succ a$), but the cycles are intuitively difficult to be understood in most cases. Last but not least, the preference relation of two alternatives is only determined by the reference points, and is invariant with the presence or absence of other alternatives. This property is important to preference elicitation because adding new alternatives doesn't reverse the preference relation of two alternatives when the property holds. Otherwise, it is difficult to understand the reverse of preference on two alternatives if new alternative is taken into account.

However, up to now, only few real-word applications have been reported using RMP model (Botreau and Rolland, 2008; Rolland and Zighed, 2011). The main reason is that it is difficult to parameterize the model to be implemented in practice. The aim of the chapter is then to propose a tool to elicit the parameters of the simplified version of RMP (S-RMP), in a similar way as parameter learning method used by ELECTRE TRI (e.g, Mousseau et al., 2001a).

In Section 6.2 we develop a preference elicitation algorithm for S-RMP which infers the model's parameters from pairwise comparisons. An illustrative example is given in Section 6.3. We conduct a numerical analysis in Section 6.4 to test the usability and behavior of the elicitation algorithm. Section 6.5 describes a case study which uses S-RMP model to represent the DMs' preferences on the priority of treating pollutant substances.

6.2 Learning a S-RMP Model

6.2.1 Parameters to be Elicited and General Presentation of the Algorithm

We are interested in eliciting the parameters of S-RMP in an indirect way (see Section 2.4 for more information on indirect elicitation methods). The DM is supposed to provide input preference information through some pairwise comparisons. Eliciting a S-RMP model amounts to setting values for the following parameters:

- k, the number of reference points involved in the S-RMP model;
- the k reference points $p^h = (p^h_1, \ldots, p^h_j, p^h_m)$, $h \in P, j \in M$. Without loss of generality, we assume that the reference points are numbered such that $p^h \succ p^{h+1}$, $h = 1, \ldots, k-1$;
- the criteria weights w_j, $j \in M$;
- the lexicographic order on reference points, defined as a permutation σ on the set of indices of reference points P, i.e., $p^{\sigma(1)}$ is the first reference point to which alternatives are compared to, $p^{\sigma(2)}$ is the second one, etc.

A general presentation of the elicitation algorithm is presented in Algorithm 2. The number of reference points k required to restore the pairwise comparisons provided by the DM is not known beforehand. Our strategy of the elicitation procedure consists in searing a parsimonious S-RMP model which can restore the preference statements with a minimum number of reference points. The algorithm proceeds iteratively, checking first of all whether there exists a S-RMP model with one reference point compatible with all DM's pairwise comparisons. If no model exists, the algorithm considers S-RMP models with two reference points. The number of reference points is increased gradually until a parsimonious S-RMP model is found which restores all DM's pairwise comparisons. To check whether there exists a S-RMP model with a given number of points k, we examine all the $k!$ possibilities of lexicographic orders. For a particular permutation σ, the examination of the existence of S-RMP model amounts to testing the "if" condition of line 6 in Algorithm 2, which is done by solving a Mixed Integer Program (MIP). The formulation of the MIP to be solved is provided in the next section.

6.2.2 Representing Pairwise Comparisons in the Elicitation Program

We define the set of pairwise comparisons as B, and the alternatives in B as $A^* = \{a_1, a_2, ..., a_e, ..., a_{na}\}$, $A^* \subset A$. The set of indices of alternatives in A^* is denoted as $E = \{1, 2, \ldots, na\}$.

Algorithm 2 Procedure to elicit a S-RMP model.

Input:

The set of pairwise comparisons B;
The performance of alternatives $g_j(a_e), j \in M, a_e \in A^*$;

Output:

k the number of reference points;
A set of reference points $p^1, p^2, \ldots, p^k$;
Criteria weights $w_j, j \in M$;

```
k ← 1
problem solved ← false
while Problem solved = false do
   while an unchecked lexicographic order on reference points exists or Problem solved do
      Select a permutation σ corresponding to an unchecked lexicographic order
      if all preference statements can be restore by a S-RMP model with the lexicography σ
      then
         Problem solved = true
         Break
      end if
   end while
   k ← k + 1
end while
```

To facilitate the elicitation process, we introduce several variables here. δ_{ej}^h ($e \in E$, $j \in M$ and $h \in P$) are binary variables which represent the binary relation between a_e and p^h on criterion g_j such that $\delta_{ej}^h = 1$ iff $g_j(a_e) \geq p_j^h$ and 0 otherwise. That is to say, δ_{ej}^h indicates whether the assertion "a_e is at least as good as p^h" is true or not. Constraints in (6.1) define δ_{ej}^h, where ε is an arbitrary small positive value, and L is an arbitrary large positive value.

$$\begin{cases} L(\delta_{ej}^h - 1) \leq g_j(a_e) - p_j^h \\ g_j(a_e) - p_j^h + \varepsilon \leq L \cdot \delta_{ej}^h \end{cases} \tag{6.1}$$

On criterion g_j, the importance degree of the support in favor of the assertion "a_e is at least as good as p^h" can be computed by $c_{ej}^h = \delta_{ej}^h w_j$ ($e \in E, j \in M, h \in P$), which is a non-linear expression because both δ_{ej}^h and w_j are variables. The following linear constraints define c_{ej}^h while avoiding the difficulty of non linear problem (Meyer et al., 2008).

$$\forall h \in P, j \in M, a_e \in A^* : \begin{cases} c_{ej}^h \leq w_j \\ c_{ej}^h \geq 0 \\ c_{ej}^h \leq \delta_{ej}^h \\ c_{ej}^h \geq \delta_{ej}^h + w_j - 1 \quad . \end{cases} \tag{6.2}$$

We recall that $C(a_e, p^h)$ represents the set of criteria for which the evaluation of a_e is considered as least as good as the evaluation of p^h: $C(a_e, p^h) = \{j \in M \text{ such that } g_j(a_e) \geq p_j^h\}$ (see Section 3.3). $w(a_e, p^h)$ $(e \in E, h \in P)$ is defined as the importance of the coalition of criteria in $C(a_e, p^h)$. For S-RMP model, the corresponding importance relation $\trianglerighteq$(see Section 3.3) on the subsets of criteria has an additive decomposition, which computes $w(a_e, p^h)$ as follows:

$$w(a_e, p^h) = \sum_{j \in C(a_e, p^h)} w_j = \sum_{j \in M} c_{ej}^h \tag{6.3}$$

With these variables, it is possible to state the necessary and sufficient conditions to guarantee the statement "$a_e \succ a_{e'}$" according to the specific S-RMP model, which involves k reference points with a given lexicographic order σ of these reference points. The two alternatives a_e and $a_{e'}$ are compared with the k reference points following a lexicographic order such that:

$$\begin{aligned}
a_e \succ a_{e'} \quad \Longleftrightarrow \quad & a_e \succ_{p^{\sigma(1)}} a_{e'} && (6.4)\\
\text{or} \quad & a_e \sim_{p^{p^{\sigma(1)}}} a_{e'} \text{ and } a_e \succ_{p^{p^{\sigma(2)}}} a_{e'} && (6.5)\\
\ldots \quad & \\
\text{or} \quad & a_e \sim_{p^{\sigma(1)}} a_{e'} \text{ and } \ldots \text{ and } a_e \sim_{p^{\sigma(h-1)}} a_{e'} \text{ and } a_e \succ_{p^{\sigma(h)}} a_{e'} && (6.6)\\
\ldots \quad & \\
\text{or} \quad & a_e \sim_{p^{\sigma(1)}} a_{e'} \text{ and } \ldots \text{ and } a_e \sim_{p^{\sigma(k-1)}} a_{e'} \text{ and } a_e \succ_{p^{\sigma(k)}} a_{e'} && (6.7)
\end{aligned}$$

To determine the preference relation between a_e and $a_{e'}$ with respect to reference point p^h, $w(a_e, p^h)$ and $w(a_{e'}, p^h)$ are compared as below:

$$a_e \succ_{p^h} a_{e'} \iff w(a_e, p^h) > w(a_{e'}, p^h) \tag{6.8}$$

A slack variable $s_{ee'}^h$ is introduced to determine the comparison of a_e and $a_{e'}$ with respect to p^h:

$$w(a_e, p^h) - w(a_{e'}, p^h) - s_{ee'}^h = 0 \tag{6.9}$$

$$s_{ee'}^h > 0 \iff a_e \succ_{p^h} a_{e'} \tag{6.10}$$

$$s_{ee'}^h = 0 \iff a_e \sim_{p^h} a_{e'} \tag{6.11}$$

Consequently, the conditions in (6.4)-(6.7) to ensure can be described using these slack variables.

$$
\begin{aligned}
a_e \succ a_{e'} \quad \Longleftrightarrow \quad & s_{ee'}^{\sigma(1)} > 0 && (6.12)\\
\text{or} \quad & s_{ee'}^{\sigma(1)} = 0 \text{ and } s_{ee'}^{\sigma(2)} > 0 && (6.13)\\
\dots \quad & \dots && \\
\text{or} \quad & s_{ee'}^{\sigma(1)} = 0 \text{ and } \dots \text{ and } s_{ee'}^{\sigma(h-1)} = 0 \text{ and } s_{ee'}^{\sigma(h)} > 0 && (6.14)
\end{aligned}
$$

The above conditions in (6.12)-(6.14) can be rewritten as:

$$s_{ee'}^{\sigma(1)} \geq 0 \tag{6.15}$$

$$s_{ee'}^{\sigma(1)} = 0 \Rightarrow s_{ee'}^{\sigma(2)} \geq 0 \tag{6.16}$$

$$s_{ee'}^{\sigma(1)} = s_{ee'}^{\sigma(2)} = 0 \Rightarrow s_{ee'}^{\sigma(3)} \geq 0 \tag{6.17}$$

$$\vdots$$

$$s_{ee'}^{\sigma(1)} = \dots = s_{ee'}^{\sigma(k-1)} = 0 \Rightarrow s_{ee'}^{\sigma(k)} \geq 0 \tag{6.18}$$

$$\text{at least one of the variables } s_{ee'}^{\sigma(1)}, s_{ee'}^{\sigma(2)}, \dots, s_{ee'}^{\sigma(k)} \text{ is strictly positive} \tag{6.19}$$

Condition in (6.15)-(6.18) express the lexicographic order on P, which means that if a_e is indifferent with $a_{e'}$ with respect to reference points $p^{\sigma(1)}$, $p^{\sigma(2)}, \dots, p^{\sigma(h-1)}$, then a_e should be preferred to or indifferent with $a_{e'}$ with respect to reference point $p^{\sigma(h)}$. Moreover, constraints (6.19) ensure that the preference relation between a_e and $a_{e'}$ is differentiated by one reference point.

Constraint (6.19) can be formulated using additional binary variables $\mu_{ee'}^h \in \{0,1\}, h \in P$ which indicates whether $s_{ee'}^h$ is strictly positive:

$$\mu_{ee'}^h = 1 \quad \Leftrightarrow \quad s_{ee'}^h > 0 \tag{6.20}$$

$$\mu_{ee'}^h = 0 \qquad \text{otherwise} \tag{6.21}$$

These $\mu_{ee'}^h$ variables can be defined by the following constraints:

$$s_{ee'}^h - \mu_{ee'}^h \leq 0 \tag{6.22}$$

$$\mu_{ee'}^{h}(1+\varepsilon)-s_{ee'}^{h} \quad \leq \quad 1 \tag{6.23}$$

where ε is an arbitrary small positive value. Hence, (6.19) can be expressed as $\sum_{h=1}^{k}\mu_{ee'}^{h} \geq 1$.

The constraints (6.15)-(6.18) should also be linearized. To do so, we introduce the following additional binary variables:

- $\alpha_{ee'}^{h}=1$ if $s_{ee'}^{h} \geq 0$, $\alpha_{ee'}^{h}=0$ otherwise,
- $\beta_{ee'}^{h}=1$ if $s_{ee'}^{h}=0$, $\beta_{ee'}^{h}=0$ otherwise,

Constraints (6.15)-(6.18) can be rewritten using these variables:

$$\alpha_{ee'}^{1}=1 \tag{6.24}$$
$$\alpha_{ee'}^{2} \geq \beta_{ee'}^{1} \tag{6.25}$$
$$\alpha_{ee'}^{3} \geq \sum_{h=1}^{2}\beta_{ee'}^{h} \tag{6.26}$$
$$\vdots$$
$$\alpha_{ee'}^{k} \geq \sum_{h=1}^{k-1}\beta_{ee'}^{h} \tag{6.27}$$

The binary variables $\alpha_{ee'}^{h}, \beta_{ee'}^{h}, h=1..k$ are defined by linear constraints similarly to the definitions of $\mu_{jj'}^{h}$ variables by (6.22)-(6.23) (see e.g, Williams, 1999).

As shown in Rolland (2008), for a S-RMP model, it is always possible to find an equivalent RMP model whose reference points are dominated with each other. Such dominance order imposes corresponding constraints. With the previous assumption on the numbering of reference points, the dominance relations can be expressed as:

$$p_j^h \geq p_j^{h+1} \qquad 1 \leq h \leq k-1, \quad j \in M \tag{6.28}$$

Now for a S-RMP model with k reference points and a given permutation σ on P, the constraints to be satisfied to guarantee the statement "$a_e \succ a_{e'}$" are all linearized.

To check whether there exists a S-RMP model which can represent all pairwise comparisons in B, a variable s_{min}^{1} subjects to $s_{min}^{1} \leq s_{ij}^{1}$ is maximized by solving a Mixed Integer Program (see Appendix B.1 for the MIP). If $s_{min}^{1*} \geq 0$ then we find a model which is able to reproduce the comparisons with one preference model. This is how we check the "if"condition in line 6 of Algorithm 2.

It can be seen that the MIP involves many binary variables. We summarize the number of binary variables for a given MIP in Table 6.1.

Binary variables	Number
δ_{ej}^{h}	$na \cdot m \cdot k$
$\alpha_{ee'}^{h}$	$nc \cdot k$
$\beta_{ee'}^{h}$	$nc \cdot (k-1)$
$\mu_{ee'}^{h}$	$nc \cdot k$

Table 6.1: Number of binary variables in the MIP: *nc* pairwise comparisons (*na* alternatives in these comparisons), *m* the number of criteria, *k* the number of reference points

6.3 Illustrative Example

We illustrate the proposed preference elicitation algorithm in the following example. Holiday proposals are to be ranked based on four criteria, namely price, comfort, distance and attractiveness, which are evaluated on scales described as follows. Obviously the lower the price (in euro) of a holiday proposal is, the more it is preferred. The comfort of the holiday is evaluated on a one to five ordinal scale. The distance to the destination is sorted to four levels: A, B, C and D (A being the most desirable while D being the least desirable). The attractiveness of places to visit is evaluated into three levels: "+“,"=“,"-“ from the best to the worst.

To rank the holiday proposals in an order, S-RMP is considered as evaluation model. We use the DM's preference information which is an order of six alternatives: $a_3 \succ a_2 \succ a_4 \succ a_6 \succ a_5 \succ a_1$. The evaluations of such six alternatives are presented in Table 6.2. The elicitation algorithm aims at inferring a S-RMP model which is able to restore such order. Then the inferred S-RMP model can be used to rank all the holiday proposals. The elicitation algorithm (Algorithm 2) is able to find a S-RMP model with two reference points compatible with the preference order. The parameters of the inferred S-RMP model (the reference points and weights of criteria) are given in Table 6.3.

We now verify that the inferred model can restore the order the DM provide. To do so, we compute $w(a_e, p^h), e = \{1,2,\ldots,6\}$ the set of criteria on which the evaluation of a_e exceeds p^h, $h = \{1,2\}$. The computation result is given in Table 6.4, which is then used to determine the preference relation of each pairwise comparisons in the order. Comparing all alternatives

Table 6.2: Evaluation table of holiday proposals

	1	2	3	4
a_1	60	***	C	+
a_2	60	**	B	+
a_3	80	****	A	=
a_4	80	***	B	=
a_5	70	****	C	=
a_6	70	**	A	=

Table 6.3: The inferred S-RMP model with two reference points

	g_1	g_2	g_3	g_4
$p^{\sigma(1)}$	60	****	B	+
$p^{\sigma(2)}$	70	***	C	=
w	0.01	0.26	0.49	0.24

to $p^{\sigma(1)}$, we find that $a_3 \succ_{p^1} a_2 \succ_{p^1} a_4 \sim_{p^1} a_6 \succ_{p^1} a_5 \succ_{p^1} a_1$. Because of the lexicographic dictatorship of $p^{\sigma(1)}$, the preference order is partly reproduced so far, except that the relation between a_4 and a_6 is not determined yet. Then the alternatives a_4 and a_6 are compared to $p^{\sigma(2)}$. It is clear that $a_4 \succ_{p^2} a_6$, thus $a_4 \succ a_6$. At this point the complete order is all reproduced.

It is worth mentioning that finding a solution for such example costs 0.03s using CPLEX v11 on a Intel Core Duo CPU 3Ghz with 2 GBytes RAM, although the MIP with two reference points involves 68 binary variables totally.

Table 6.4: Coalition of weights $w(a_e, p^h)$ in favor of $a_e \succ p^{\sigma(h)}$

	$p^{\sigma(1)}$	$p^{\sigma(2)}$
a_1	0.25	1
a_2	0.74	0.74
a_3	0.75	0.99
a_4	0.49	0.99
a_5	0.26	1
a_6	0.49	0.74

6.4 Tractability Issues and Numerical Analysis

6.4.1 Experiment Design

Several issues emerge when the preference elicitation algorithm is applied to real-world decision problems. Firstly, the elicitation ability of the algorithm is one of our concerns, more precisely, we try to answer the question that how many pairwise comparisons approximately are needed to elicit S-RMP models "close" enough to the "true" preference of the DM? We shall define how the degree of "closeness" is measured afterwards. Secondly, we are interested in the question: " is the algorithm able to represent the pairwise comparisons with S-RMP models that are not too complex"? As the number of reference points is not known beforehand, the algorithm adds an additional reference point when it cannot find a model compatible with the comparisons using a given number of reference points. In some cases, the algorithm may identify a model with a large number of reference points as a result of overfitting. However, we find such situation unacceptable, because it is difficult to interpret such models. Hence, we investigate the required number of reference points of the elicited models. Thirdly, we concern the algorithm's computation time as it is crucial during a decision aiding process with the DM in an interactive way.

The three issues are investigated through a series of numerical experiments based on the idea in Figure 6.1. The "true" preference of the DM is simulated by a S-RMP model (which is called original model M_o). A set of reference alternatives A^* are generated with random evaluations. First of all, the original model M_o is used to rank the alternatives A^*. Secondly, some pairwise comparisons are chosen from the order of the alternatives in A^* as preference information, which is used as input of the elicitation algorithm. Thirdly, a S-RMP model is inferred based on such comparisons. Finally, we use respectively the original model M_o and the elicited model (M_e) to rank a set of test alternatives A which are also generated randomly.

The preference relations of each pair calculated respectively by M_o and M_e are compared. If the two preference relations are identical, M_e is considered to compare correctly the pair. The proportion of correct comparisons is collected as indicator of the "closeness " of the two models. A high proportion indicates that the elicited model is able to reproduce a large percentage of pairwise comparisons ranked by the original model, therefore the elicited model is viewed as relatively close to the original model. The average number of reference points of M_e corresponding to a given number of pairwise comparisons is collected when a M_o with certain

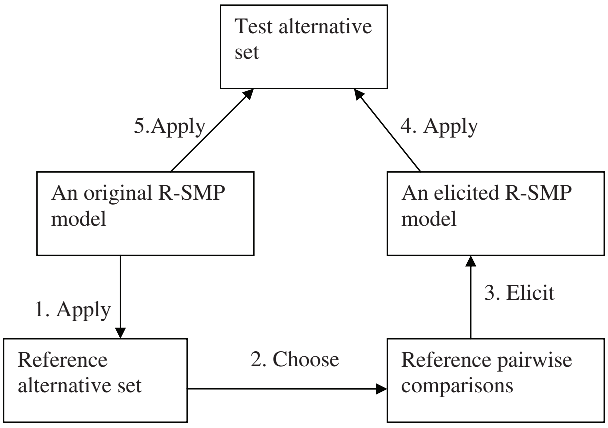


Figure 6.1: Work flow of the experiments

complexity is to be elicited. We also collect the average computation time of the optimization program.

Obviously, to address the three issues, it is necessary to take into account the complexity of M_o, which is related to the number of criteria m, the number of reference points k of M_o and the nature of scales (discrete or continuous). We consider different levels of complexity of M_o to study their influence in the elicitation algorithm. We conducted three sets of experiments:

- Experiment 1: M_o with 2 reference points, 3/5/7 criteria evaluated on continuous scale
- Experiment 2: M_o with 3 reference points, 5 criteria evaluated on continuous scale
- Experiment 3: M_o with 2 reference points, 5 criteria evaluated on 5-level scale

6.4.2 Generation of M_o

To generate the reference alternatives in A^*, for criteria of continuous scales, the evaluation of alternatives are generated randomly on [0,1] scale. For criteria with 5-level scale, the evaluations of alternatives are randomly chosen from $\{1,2,3,4,5\}$. In our experiments, 20 alternatives of A^* are generated.

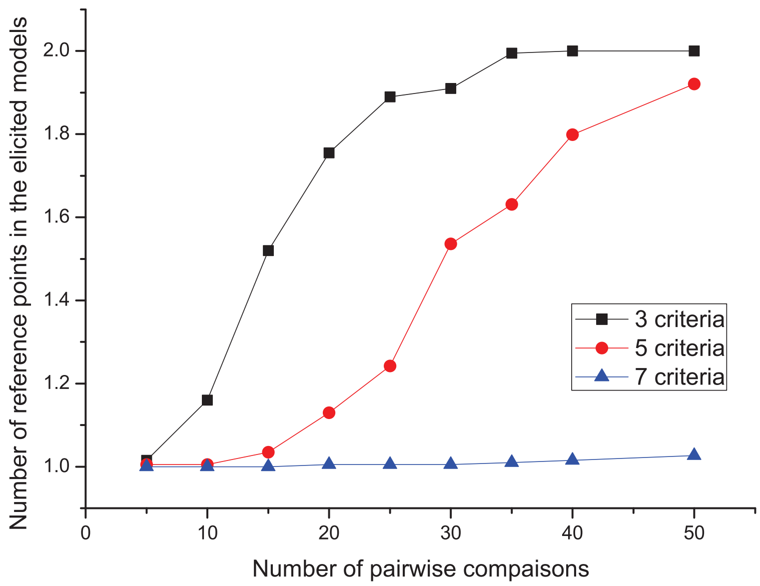
The reference points are generated as to guarantee their dominance relations. For criteria of continuous scales, the scale [0,1] is divided into k equal pieces, k being the number of reference points. Then p^k is generated randomly on $[0, \frac{1}{k}]$, p^{k-1} is generated randomly on $[\frac{1}{k}, \frac{2}{k}]$, etc. Similarly, the reference points with evaluations on 5-level scale are generated. These reference points are used with a lexicographic order σ which is a random permutation of P. The weights are generated randomly on [0, 0.5] and are normalized to one.

With the generated reference points, their lexicographic order and weights, a specific M_o can be constructed. Using such model, the preference relation of each pair in A^* is able to be determined. We randomly choose a number of pairwise comparisons (the number increases from 5 to 50) out of the 400 comparisons. For any size of pairwise comparisons, 50 runs are performed.

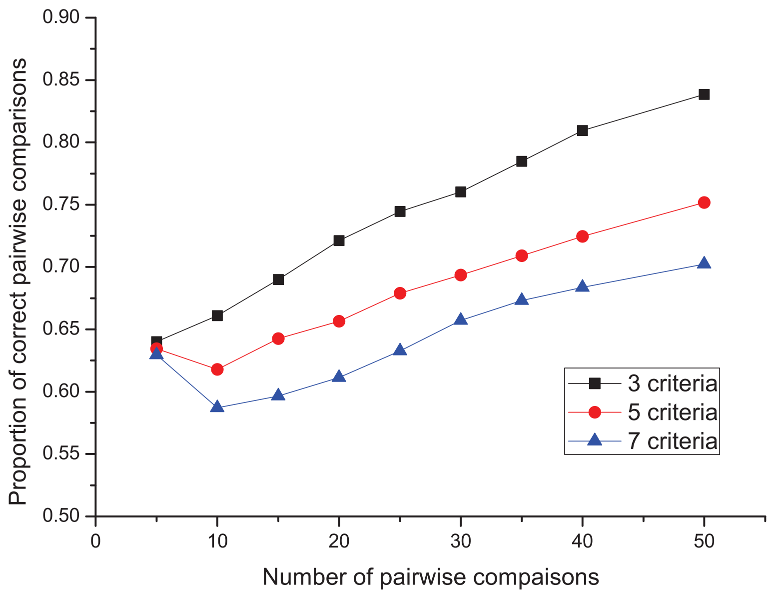
6.4.3 Results and Discussions

The results of Experiment 1 are shown in Figure 6.2. We observe that with the increase of preference information (increasing the number of pairwise comparisons nc), the number of reference points of M_e grows, as well as the proportion of correct pairwise comparisons and the computation time. We explain such trends in what follows. More pairwise comparisons lead to more constraints. On the other hand, adding one reference point means adding more variables ($p_j^h, \delta_{ej}^h, \alpha_{ee'}^h, \beta_{ee'}^h$ and $\mu_{ee'}^h$, $h \in P, j \in M, a_e, a_{e'} \in A^*$) to the optimization program, thus leaving M_e more flexibility. Therefore, a higher number of referencee points is required to satisfy such constraints. Moreover, the phenomenon that the proportion of correct pairwise comparisons goes up with the increase of nc indicates that M_e is getting closer to M_o. Such phenomenon is in line with the fact that M_o is more determined by more constraints stemming from preference information.

From Figure 6.2(a), we can easily find that the number of reference points of M_e is bigger when M_o has less criteria. For example, to represent 50 pairwise comparisons, all M_e with 3 criteria has to use 2 reference points, while 97.3% of M_e with 7 criteria needs only 1 reference point. The reason is that with less criteria, M_e is less flexible with less variables of weights, which requires more reference points to represent all these pairwise comparisons. Another result in line with the previous one is presented in Figure 6.2(b), which illustrates that M_e is closer to M_o when less criteria is considered. Figure 6.2(c) provides an interesting result that it is more computationally costly to elicit M_o with 5 criteria than 3 or 7 criteria, which can be explained as follows. On one hand, more criteria imply more weight variables, which makes the optimization program more difficult to solve. On the other hand, more criteria results in a lower number of reference points in M_e, thus many of instances of the experiments with more criteria can find a M_e involving only 1 reference point without the need of solving the optimization program with 2 reference points. Figure 6.2(c) presents the two opposite effects of considering more criteria.

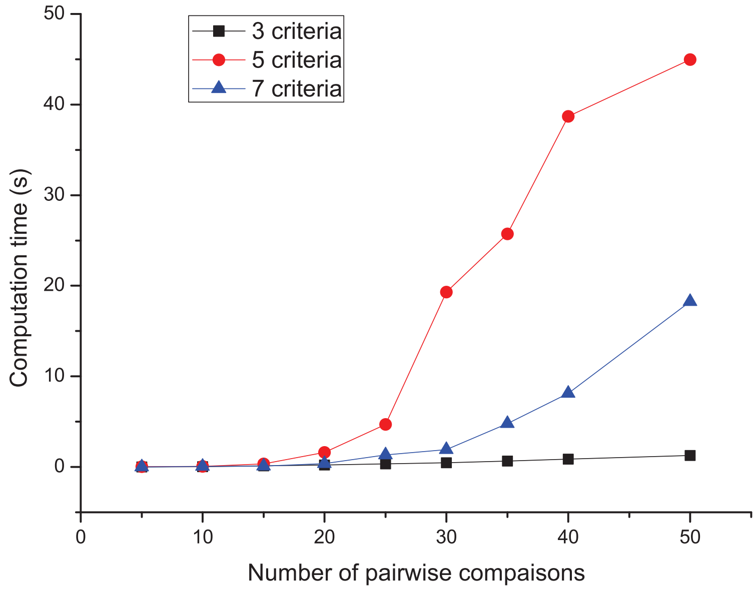


(a)



(b)

Figure 6.2: Eliciting M_o with 2 reference points, 3/5/7 criteria evaluated on continuous scale.



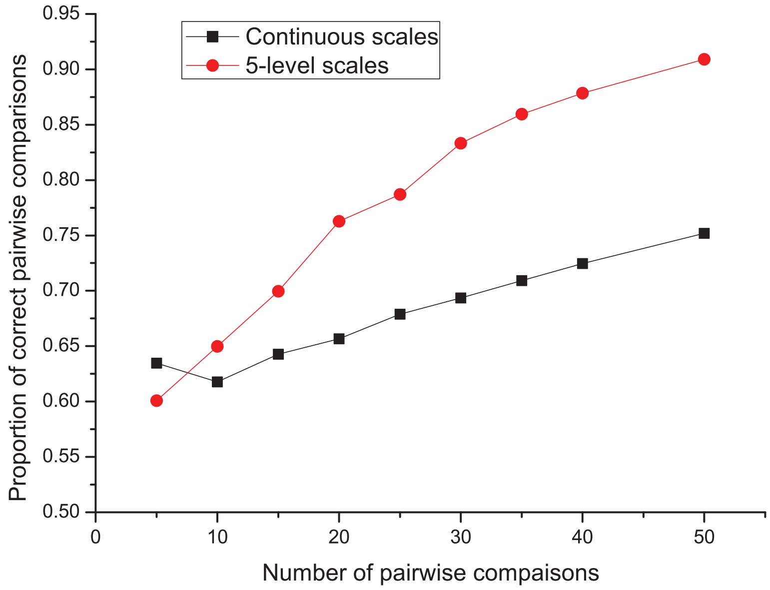
(c)

Figure 6.2: Eliciting M_o with 2 reference points, 3/5/7 criteria evaluated on continuous scale (cont.)

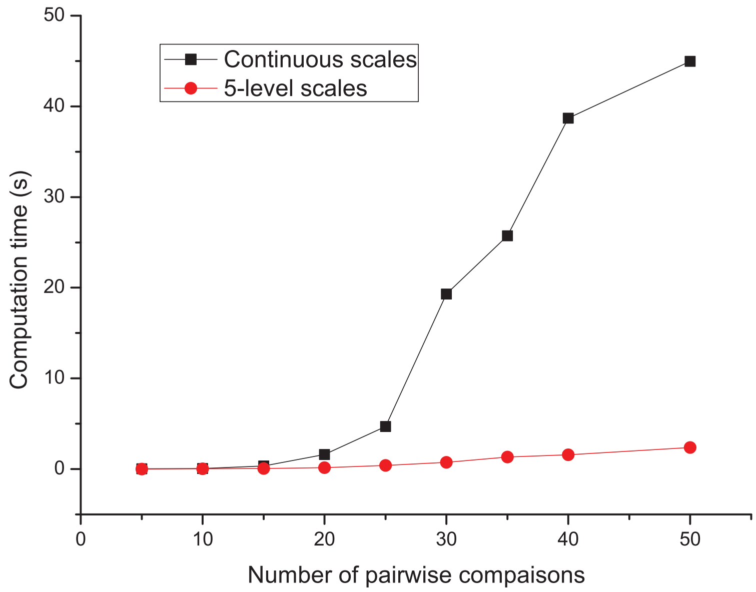
The results of Experiment 2 are presented in Appendix B.2, where we can find there are indeed some instances that S-RMP models are elicited with 3 reference points. Figure B.2(a)-B.2(c) give the distribution of the elicited S-RMP models with 1, 2 or 3 reference points. Nevertheless, there is no significant difference in the results of Experiment 1 and 2. However, when we tried to elicit M_o with 4 or more reference points from more than 50 pairwise comparisons, there were some instances which couldn't solve the optimization program within 30 minutes. This is due to the presence of more binary variable and more constraints. Based on these preliminary experiments, we limit the number of reference points of M_o to 2 and 3, and nc to 50, because it becomes difficult for extensive numerical analysis as a result of the increase of computation time.

Monte Carlo simulation is used to analyze whether it is a serious problem that the elicitation algorithm should be used with the number of reference points limited to 3. We randomly generated M_o with 8 reference points (see Section 6.4.2 for how M_o is generated) and used such model to rank 10000 randomly generated alternatives. The result shows that on 99.83% pairwise comparisons are able to be differentiated by the first 3 reference points. Therefore, limiting the number of reference points to 3 doesn't restrict the application of the elicitation algorithm in most cases.

We compare here the results of Experiments 3 which elicits M_o involving 5-level scales with the one of Experiment 1 dealing with 5 criteria with continuous scales. No remarkable difference has been found in the number of reference points of M_e for Experiments 1 and 3. However, the results concerning the proportion of correct comparisons show noticeable differences in the two experiments, as illustrated in Figure 6.3(a). It can be seen that the proportion corresponding to 5-level scales is much higher than the one corresponding to continuous scales. Elicited from 50 pairwise comparisons, M_e with 5-level scales determine 91% of the preference relation of pairwise comparisons in A correctly, while M_e with continuous scales only have 75% correctness. Such result reveals that M_e with 5-level scales is "closer" to M_o than M_e with continuous scales. Indeed, it is more difficult to differentiate two alternatives evaluated on 5-level scales than those on continuous scales because they are more similar. Thus the pairwise comparisons with 5-level criteria are more informative than those with continuous criteria. As a result, the optimization programs for 5-level scales are more constrained, while lead to M_e "closer" to M_o. Figure 6.3(b) gives the comparisons of computation time of the two experiments. Experiment 1 with continuous scales is more costly in terms of computation time than Experiment 3 with 5-level scale. As the two experiments have the same numbers of variables and constraints, the difference in computation time comes from the fact that there are less feasible solutions to the optimization programs for 5-level scales.



(a)



(b)

Figure 6.3: Comparing Experiments 1 and 3 which elicit M_o with 2 reference points, 5 criteria evaluated on continuous or 5-level scales.

6.5 A Case Study

6.5.1 Context

We consider a case in which a team in Eureval (Centre Européen d'Expertise et d'Evaluation) was supported to analyze the preferences of a group of participants who studied the risks of using polluted products. The team of Eureval itself was mandated by INERIS (Institut National de l'EnviRonnement Industriel et des riSques), which aimed at establishing a ranking model to evaluate the priority of treating the pollutants. A treatment referred to an action which can reduce the pollutants' harmful impact on human being or the environment, or a proposal to the legislation, etc. Corresponding treatments to be undertaken should be determined based on the ranking. The objective of our work was to propose MCDA models which were able to represent the preference statements of the group as much as possible.

Three sessions were held to interview the group and help them to express their preferences. In the first two sessions with a lot of discussions and interactions, eight criteria were defined to assess the priority of the pollutants' harm, and then evaluations of twenty pollutants were provided according to the eight criteria. We used the twenty pollutants as the basis for the participants to express their preferences. They were asked to classify the twenty pollutants into four categories *"very dangerous", "modestly dangerous","a little dangerous","not dangerous"*. ELECTRE TRI method was used to represent these assignment examples. To best restore the decision examples, the nine participants were divided into two groups in a way that the participants in each group shared similar preference (see Cailloux et al., 2012, for more details on how the participants were divided into two groups).

In the third session, each group of participants provided a complete order of the twenty pollutants. At this time, the ranking is not individual but collective preference. In other terms, it reflects the consensus of the group. Our work presented in this chapter concerns the analysis of preference information resulting from the third session.

6.5.2 The Structured Decision Problem

The participants agreed to access the priority of treatment for the pollutants on eight criteria.

- g_1: the proportion of anthropogenic origin of the pollutant. The origin of the pollutant can be natural or anthropogenic, and a higher proportion of anthropogenic origin is considered more dangerous.

- g_2: The extent of dispersion in the environment the substance can lead to decided by some of its characteristics.
- g_3: The remaining time of the substance in the environment (water, air, soil, living things).
- g_4: The degree of danger the substance may give to the health of human being.
- g_5: The grade of specific risk the substance involves to particular populations, in particular pregnant women and children.
- g_6: The grade of risk the substance give to the health or survival of wildlife and plants.
- g_7: The important risk of the substance to human health according to French and European public authorities.
- g_8: The availability of treatment (in terms of economy and technology) to reduce emissions of the substance in the environment.

Criterion g_1 is evaluated on a [0, 100] continuous scale, where 0 means the pollutant is totally caused by nature, while 100 represents that human activities cause the pollutant. Criteria g_2, g_3, g_4, g_5 and g_8 are qualitatively evaluated on a five-level ordinal scale (0 being the worst evaluation, and 4 being the best evaluation). Similarly, we use a four-level ([0,3]) ordinal scale to evaluate criteria g_6 and g_7 qualitatively .

The evaluations of the twenty pollutants are given in appendix B.3. Group one was able to put the twenty alternatives in the following order of treating priority: $K \succ F \succ I \succ C \succ Q \succ H \succ E \succ O \succ B \succ S \succ R \succ D \succ A \succ N \succ G \succ P \succ T \succ J \succ M \succ L$. Group two also provided its priority order: $F \succ C \succ Q \succ N \succ P \succ O \succ E \succ K \succ H \succ B \succ S \succ R \succ I \succ J \succ A \succ D \succ T \succ G \succ M \succ L$.

6.5.3 Choosing a MCAP Model

The clients from Eureval preferred to use outranking method because the substances were evaluated on pseudo criteria, which allowed to take into account imprecise data and preferential uncertainties. The S-RMP model was considered as appropriate for this case. Another option can be ELECTRE III, but the method doesn't respect the property of "invariance against a third irrelevant alternative". If an ELECTRE III model is elicited from some preference information and then used to a larger set of alternatives, the preference information can't be reproduced any more. In our case an evaluation model should be elicited based on the twenty substances, and then the model would be used to evaluate more substances. The behavior of ELECTRE III is difficult to interpret and doesn't satisfy our requirement. Therefore, it is not appropriate for this case. As already discussed in Chapter 3, for the S-RMP model, the preference relation of two alternatives only depends on the comparisons of alternatives with respect to reference points, and is irrelevant to a third alternative. Therefore, the S-RMP model is more suitable for this case.

6.5.4 Inferring S-RMP Models

Based on preliminary analysis, we found that it is impossible to represent both the two orders from the two groups with S-RMP models which used less than three reference points. However, the team of Eureval thought that the models with more than two reference points were too complex and difficult to understand. In fact, they hoped to have S-RMP models with only one reference point. Accepting a tradeoff between the simplicity of the inferred models and the ability to restore the order, we considered the following strategy to fulfill their requirements. First of all, we try to obtain S-RMP models with only one reference point which represent all of the order except that one alternative was mistakenly placed. If failed, similar attempts are made by ignoring two alternatives in the order. Finally, S-RMP models involving only one reference point are identified when a minimum number of alternatives are wrongly ordered.

To represent the preference order of group one, a S-RMP model (M1) is inferred when three alternatives C, Q and K are not correctly ordered. The parameters of M1 are shown in Table 6.5.

	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8
Reference point	80	3	2	4	1	3	1	3
Weights	0.3207	0.1628	0.0070	0.1708	0.1618	0.0040	0.0040	0.1688

Table 6.5: S-RMP model for group one: M1

Appendix B.3 presents the score associated with each substance computed by M1, which indicates the credibility that a substance outranks the reference point. The score is calculated by adding the weights of criteria on which the substance is at least as good as the reference point. Taking substance A (which is evaluated as $<100, 0, 0, 4, 0, 0, 3, 0>$) as an example, it gets a score of 0.3207 + 0.1708 + 0.0040 = 0.4955 as A is at least as good as the reference point on g_1, g_4 and g_7. Using the result from Appendix B.3, the priority order computed by M1 is $Q \succ F \sim K \succ I \succ H \succ E \succ O \succ B \succ S \succ R \succ D \succ C \succ A \succ N \succ G \succ P \succ T \succ J \succ M \succ L$. We can discover from that the priority order of group one ($K \succ F \succ I \succ C \succ Q \succ H \succ E \succ O \succ B \succ S \succ R \succ D \succ A \succ N \succ G \succ P \succ T \succ J \succ M \succ L$) is reproduced except C, Q and K, as Q is the most risky pollutant, while K and F are indifferent according to M1. In fact, Q has a higher or equal evaluation compared with K and I on all criteria except on criterion g_5, and that it has a higher or equal evaluation compared with C on all criteria except on criterion g_3. Thus it is unsurprising that the position of Q in the order provided by group one can't be respected. A second reference point can be added to the model to differentiate K and F so that only two substances (C and Q) are wrongly ordered. However, it is impossible to find a model with two reference points which can reproduce the order with only one incorrectly positioned substance.

The elicitation has been performed in a similar way with the preference order of the second group, which leads to M2 (see Table 6.6). With such model, the priority order can be derived from appendix B.3: $F \succ Q \succ P \succ O \succ C \succ E \succ K \succ B \succ S \succ H \succ R \succ I \succ J \succ A \succ D \succ N \succ T \succ G \succ M \succ L$. We can see that three substances (C, N and H) are not correctly ordered as the way the participants of group two ranked them ($F \succ C \succ Q \succ N \succ P \succ O \succ E \succ K \succ H \succ B \succ S \succ R \succ I \succ J \succ A \succ D \succ T \succ G \succ M \succ L$). Indeed, the feedback from the leader of the group was that the three substances were exactly the pollutants on which the participants changed their opinions most often when they expressed their preferences. In other words, the ranking they provided concerning the three substances were very unstable.

	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8
Reference point	80	2	1	2	2	1	2	3
Weights	0.2247	0.1798	0.0674	0.1910	0.0112	0.1011	0.1685	0.0562

Table 6.6: S-RMP model for group two: M2

There exist several models which involve only one reference point and can restore the order expect the three substances mistakenly positioned. M1 and M2 are chosen, because they lead to least errors in terms of the number of wrong pairwise comparisons in the orders.

We are aware of the fact that deleting the substances in the order the participants provided us is not the only approach to establish S-RMP models which can reproduce as much as possible the preference. For example, some loss functions can be minimized to find models which can represent the preference order with least errors based on different measurements of the errors, such as Spearman's footrule (Spearman, 1987), Kendall's distance (Kendall, 1938), etc.

6.5.5 Insights of the Application

The Eureval team was supported to use the two models (M1 and M2) to evaluate all the pollutants (200 in total). They did find the models easy to understand, because they only needed to compare the substances with the reference point to compute a score to each substance and then rank the substances using such scores. In this way, the elicited two models appeared attractive in terms of implementation. The team was able to use the models even by implementing them in spreadsheet, which also helped them to understand the procedure of the evaluation.

From the point view of analysts, we find that explaining the idea of the reference-based ranking to the participants doesn't require much effort. Moreover, the compromise between the models' simplicity and the respect to the preference information has to be made in this case, although it was easily accepted by the participants.

6.6 Concluding Remarks

In this chapter, we develop a preference elicitation tool for S-RMP model. From a given set of pairwise comparisons provided by the DMs, the algorithm infers the parameters of the model including the reference points (whose number is unknown), their lexicographic order and the weights of criteria. The algorithm involves many binary variables and linear constraints which cause difficulties in terms of computation time, nevertheless the algorithm proves to be usable based on extensive experiments. We suggest that the algorithm should limit the search of S-RMP model to up to 3 reference points, which is found to be sufficient in most cases. With such limitation, the algorithm appears to be tractable and applicable. Furthermore, the experiments reveal that considering more criteria requires more preference information to determine the models. Additional comparative experiments show that it is easier to elicit S-RMP models with discrete scales than continuous ones.

The elicitation algorithm was applied to a real-world decision problem aiming at evaluating the priority of treating some pollutants. S-RMP model was considered not only because it is rather simple to implement but also because it produces a ranking respecting transitivity and invariance with respect to a third irrelevant alternative. According to the clients' requirement, we tried to elicit a S-RMP with one reference point to represent their preferences. However, it turned out that no such model existed. The compromise between the simplicity of the model and the respect of the preference information was made after the communication with the clients. Consequently, two S-RMP models with one reference point were inferred to represent a maximum number of alternatives in the preference order. The application provides interesting experience of using the proposed elicitation tool when the preference information of the DM can't be represented by a simple S-RMP model.

Chapter 7

Software Development for ELECTRE TRI Method

The literature of MCDA has proposed numerous aggregation methods which have been applied to real-world decision problems. Such ad-hoc applications are independently implemented in an uncoordinated way. The Decision Deck project aims at collaboratively developing open source software tools implementing MCDA. Within the project, the Diviz software is developed as an open source Java client and server for designing, executing and sharing MCDA methods, via the composition of web services. Such web services implement the common functionalities of a large range of MCDA methods using XMCDA standard as data model.

We are interested in implementing web services for ELECTRE TRI method. More precisely, three issues are considered: (1) inferring ELECTRE TRI model from preference information; (2) robustness analysis taking into account the imprecision nature of the preference information; (3) inconsistency resolution when the preference information is conflicting. Three interoperable web services are accordingly developed communicating in the XMCDA standard. The developed web services are integrated to Diviz software with a friendly user interface. An illustrative example is given to show the usage of such services.

7.1 Context

7.1.1 Decision Deck Project

A multitude of MCDA methods have been proposed in the literature (Roy, 1985; Figueira et al., 2005a; Keeney and Raiffa, 1993; Wallenius et al., 2008, see also Section 2.2). Growing number of real-world applications have been reported (Hämäläinen, 2004; Keefer et al., 2004). These applications have been independently implemented in an uncoordinated way using different tools and programming language. Either the implementations are ad-hoc, that is to say, they are only designed for a specific case (Korhonen et al., 1992; Doumpos and Zopounidis, 2010; Bana e Costa et al., 1999; Doumpos and Zopounidis, 2010), or are commercial softwares for a particular method (for example, the MakeItRational software for AHP). For people who are interested in applying MCDA methods in their own domains, they face difficulties in implementing such methods since they are not experts on MCDA methodology.

Decision Deck project aims at collaboratively developing open source software tools implementing a platform composed of modular and interconnected software components of MCDA methods (see Decision Deck Consortium, 2012a). These software components implement the common functionalities of a large range of MCDA methods (Ros, 2011).

Consequently, several complementary initiatives focusing on different aspects contribute to Decision Deck project:

- XMCDA : a standardized XML recommendation to represent objects and data structures issued from the field of MCDA. Its main objective is to allow different MCDA algorithms to interact and be easily callable;
- Diviz : an open source Java client and server for designing, executing and sharing MCDA methods, via the composition of XMCDA web services;
- d2 : a rich open source Java software containing several MCDA methods;
- d3 : an open source rich internet application for XMCDA web services management.

7.1.2 XMCDA

(Bisdorff et al., 2008)

Now let us introduce XMCDA standard which is a key concept of the Decision Deck project. XML schema defines a common language for concepts used in a specific domain of knowledge.XMCDA is such schema, i.e, a standardized XML (eXtensible Markup Language) proposal to represent objects and data of MCDA. Its main objective is to allow different MCDA algorithms to interact and to be easily callable from a software like, e.g., the Diviz platform of the Decision Deck project. Using XMCDA, the MCDA data elements can be represented in XML according to a clearly defined grammar. If a decision problem is represented in XMCDA standard, the unified representation can be used in various algorithms. Moreover, the visual representation of MCDA concepts and data structures via standard tools like web browsers (Ros, 2011).

A XMCDA file may contain several tags under the root element. These tags allow to describe various MCDA related data from a few general categories:

- Project description;
- Input information for methods (parameters) and output messages from methods (log or error messages);
- Description of major MCDA concepts such as attributes, criteria, alternatives, categories;
- The performance table;
- Further preference information related to criteria, alternatives, attributes or categories.

We introduce here the definition of some basic MCDA concepts by representing the decision problem of Section 6.3 with XMCDA standard. The following code describes the six holiday proposals of Section 6.3 defined under the *"alternatives tag"*. The id of an alternative is mandatory. The alternatives can be either active or not and be either real or fictive. In addition, the alternatives can also be flagged as reference alternatives. In the case of Section 6.3, p representing the reference profile is a fictive and reference alternative.

```
<alternatives>
        <alternative id="a01" name="Paris">
            <active>true</active>
        </alternative>
        <alternative id="a02" name="Rome">
            <active>true</active>
        </alternative>
```

```
        <alternative id="a03" name="Venice">
            <active>true</active>
        </alternative>
        <alternative id="a04">
            <active>true</active>
        </alternative>
        <alternative id="a05">
            <active>true</active>
        </alternative>
        <alternative id="a06">
            <active>true</active>
        </alternative>
        <alternative id="p">
            <active>true</active>
            <type>fictive</type>
            <reference>true</reference>
        </alternative>
    </alternatives>
```

Criteria are defined under the *"criteria"* tag. For each criterion one has to define its id. In the following code, we define the first two criteria to evaluate the holiday proposals. The code illustrates that the first criterion g_1 standing for the price of the proposal is evaluated on quantitative scale. The preference on such criterion defined by *"preferenceDirection"* tag is to minimize the price. The second criterion g_2 representing the comfort of the proposal is evaluated on a five level quantitative scale, which is to be maximized.

```
<criteria>
        <criterion id="g1" name="Price">
            <scale>
                <quantitative>
                    <preferenceDirection>min</
                       preferenceDirection>
                </quantitative>
            </scale>
        </criterion>
        <criterion id="g2" name="Comfort">
            <scale>
                <qualitative>
                    <preferenceDirection>max</
                       preferenceDirection>
                    <rankedLabel>
                        <label>*</label>
                        <rank>1</rank>
```

```
                    </rankedLabel>
                    <rankedLabel>
                        <label>**</label>
                        <rank>2</rank>
                    </rankedLabel>
                    <rankedLabel>
                        <label>***</label>
                        <rank>3</rank>
                    </rankedLabel>
                    <rankedLabel>
                        <label>****</label>
                        <rank>4</rank>
                    </rankedLabel>
                    <rankedLabel>
                        <label>*****</label>
                        <rank>5</rank>
                    </rankedLabel>
                </qualitative>
            </scale>
        </criterion>
    </criteria>
```

The performance table is defined with the tag *“performanceTable”*. It contains, for each alternative (given by its id), a list of performances, given by a criterion id (or attribute id) and a corresponding performance value. The following code gives the performance of a_1 in Section 6.3.

```
<performanceTable>
        <alternativePerformances>
            <alternativeID>a1</alternativeID>
            <performance>
                <criterionID>g1</criterionID>
                <value>
                    <real>60</real>
                </value>
            </performance>
            <performance>
                <criterionID>g2</criterionID>
                <value>
                    <label>***</label>
                </value>
            </performance>
            <performance>
                <criterionID>g3</criterionID>
```

```
                <value>
                    <label>C</label>
                </value>
            </performance>
            <performance>
                <criterionID>g4</criterionID>
                <value>
                    <label>+</label>
                </value>
            </performance>
        </alternativePerformances>
    </performanceTable>
```

XMCDA is able to define additional information of a decision problem such as attributes, categories, and preference information, see Bisdorff et al. (2008).

7.1.3 Diviz Software

Within the initiatives of Decision Deck project, we are interested in the Diviz software, which is an open source Java client and server available at the website (Decision Deck Consortium, 2012b) for free download.

Algorithmic components

Algorithmic components have been collaboratively developed for Diviz with the efforts of various developers. These components are some useful functionalities for the implementation of MCDA methods. Generally, such components are classified into computation or tool components. The computation components implement MCDA methods including some popular value based methods (e.g, weighted sum) and outranking methods (e.g, ELECTRE TRI , PROMETEE). The tool components are helpful to the computation components. For example, *"plotAlternativesComparisons"* component generates a graph representing a partial preorder on the alternatives.

Figure 7.1 shows a simple component represented by a box: the weighted sum method. The entries of the component are connected with input files, which include the definition of alternatives, criteria, performance table and weights. These files are all conformed to XMCDA standard. The output, also respecting XMCDA standard, contains a *"messages"* file which states whether the component has been successfully executed and a *"alternativesVales"* file which is the computation result of the value for each alternative.

The execution of the algorithm is performed via web service, which allows the user to access the developed components without having to install them on their computers.

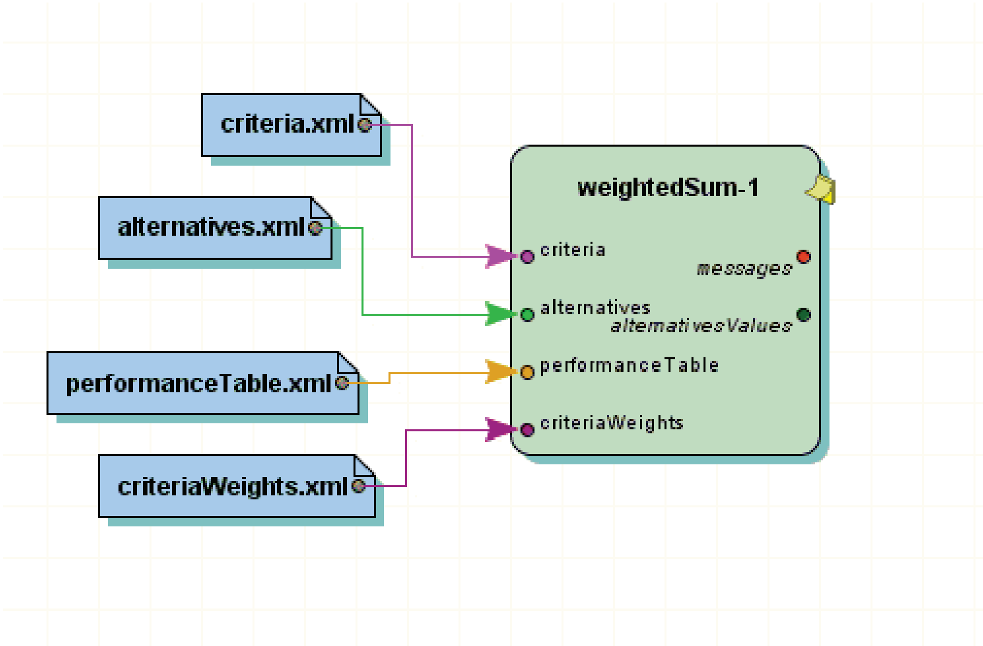


Figure 7.1: Weighted sum web service

Workflow

With the various algorithmic components, it's convenient for the user to create a workflow using Diviz. The workflow refers to a combination of components to fulfil some purpose, which is usually the implementation of an algorithm being decomposed into several components. By splitting an algorithm, the users (maybe students learning MCDA methods with Diviz) can get intermediate results of the algorithm. Moreover, some algorithms which share common logics can reuse the same component without developing in redundant way (Cailloux, 2010). The components of a workflow interact with each other using the language of XMCDA, which means that the input of a component can be the output of another one. Such workflow is managed by Diviz with a nice graphic interface.

For the end users to create a new workflow in Diviz, they just need to drag and drop components to the workspace and then connect them with each other. Figure 7.2 shows a workflow which connects three components *"weightedSum"*, *"plotNumericPerformanceTable"* and *"plotAlternativesValues"*. The workflow is available on line (Meyer, 2012) and readers are re-

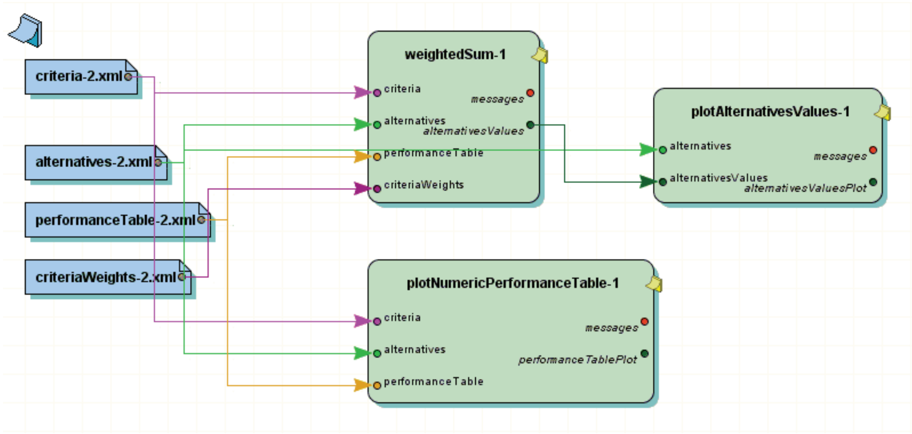


Figure 7.2: Weighted sum workflow

ferred to download such workflow and import it to their own Diviz software to access the input files. The *"weightedSum"* component computes the overall value for each alternative using the weighted sum method, and such values are provided by the output file also validated by XMCDA grammer. Then the *"plotAlternativesValues"* component generates a barplot (Figure 7.3) to represent such value. A barplot (Figure 7.4) is also generated representing the performance table by *"plotNumericPerformanceTable"*.

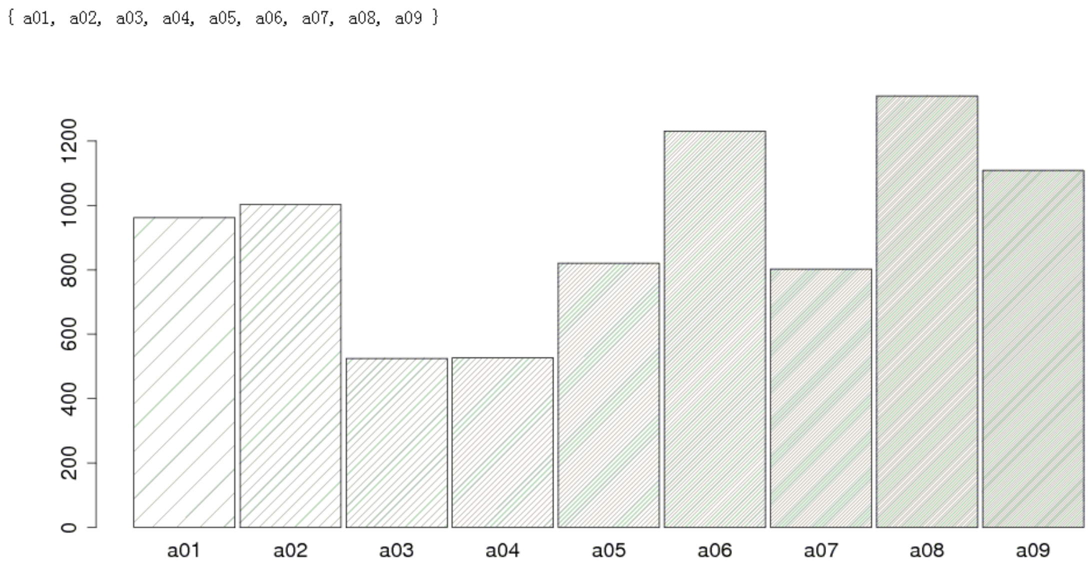


Figure 7.3: Output of *"plotAlternativesValues"* component

Features

Researchers can construct algorithmic MCDA workflows (which are MCDA methods) from elementary components as open source software bricks. All these elementary MCDA components

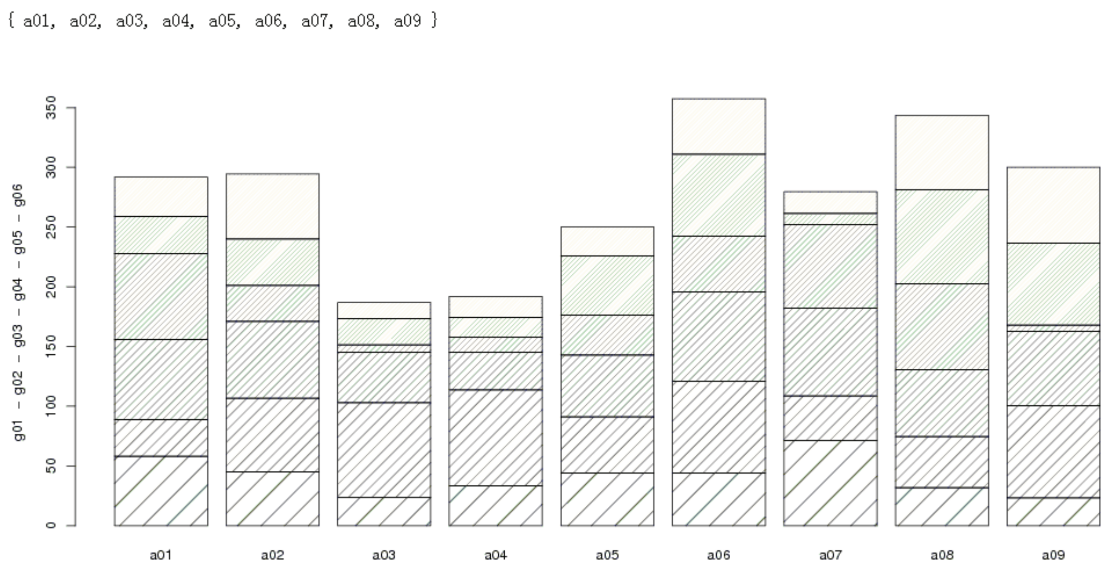


Figure 7.4: Output of *"plotNumericPerformanceTable"* component

are currently (open source) web services, which are interoperable with the use of XMCDA. Moreover, the input and output data can be visualized by standard visualization tools of XML. Additionally, the Diviz client software is written in Java and is therefore independent of the operating system.

7.2 Developing Web Services for ELECTRE TRI Method

Our fourth contribution of the present thesis is the development of three web services concerning the application of ELECTRE TRI for Diviz software. The standard ELECTRE TRI model using pessimistic rule is considered (see Section 3.2.2).

7.2.1 Objectives and General Description

The first task of our development is to support the user to infer an ELECTRE TRI model from his preference information, which can be some assignment examples and some linear constraints on the parameters of the model (see Section 3.2.2). It is assumed that the profiles are given, so the only parameters to be elicited are weights and the majority level. Moreover, the robust assignment is to be computed as the result of the imprecise nature of preference information. Furthermore, the consistency of preference information should be tested to inform the user whether the information he provide can be represented by an ELECTRE TRI model. If not, insights should be given to the users how the inconsistency occurs and how to resolve it.

To address the issues described previously, three web services are developed, namely *"IRIS-testInconsistency"*, *"IRIS"* and *"inconsistencyResolution"*, where IRIS stands for Interactive Robustness analysis and parameters' Inference for multicriteria Sorting problems. The web services are based on the algorithms proposed in literature which correspondingly deal with the inference of ELECTRE TRI model(Mousseau et al., 2001a), robust analysis (Dias et al., 2002) and inconsistency resolution (Mousseau et al., 2006). Figure 7.5 shows the connections of the three web services. The *"IRIS-testInconsistency"* service is used firstly to test whether there exist a combination of parameters which are compatible with the preferences the user provide. If yes, the *"IRIS"* service is then executed to infer an ELECTRE TRI model which assigns alternatives to one of the categories and compute robust assignments. When the preference can not be represented by an ELECTRE TRI model, the *"IRIS-testInconsistency"* outputs a set of linear constraints which stem from the preference of the user. In that case, the *"inconsistencyResolution"* service should be performed which suggests several subsets of such linear constraints that are consistent. Based on the suggestions, the user is supposed to modify his preference statements. Our three web services are coded in c++ with CPLEX solver and has been integrated to Diviz software.

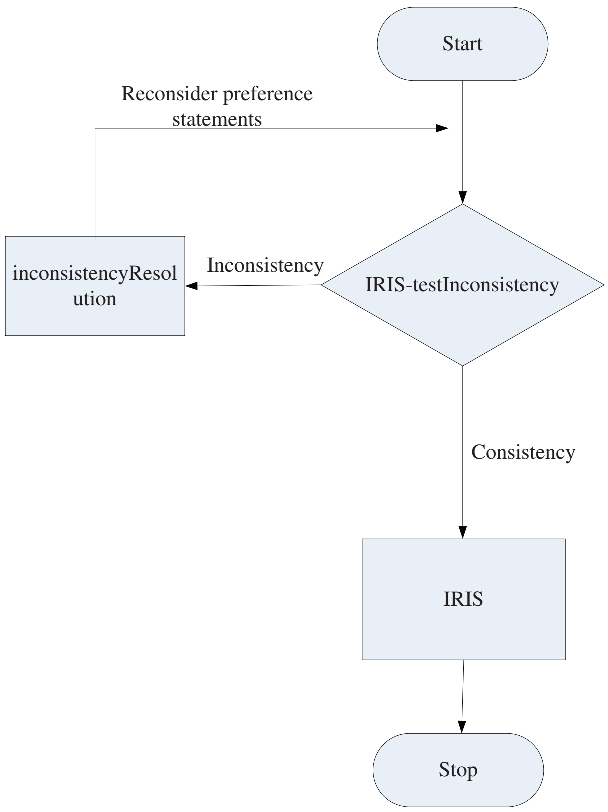


Figure 7.5: Three web services for ELECTRE TRI method

A workflow which combines the three web services can be built as Figure 7.6. We detail the functionalities of each service in the following sections.

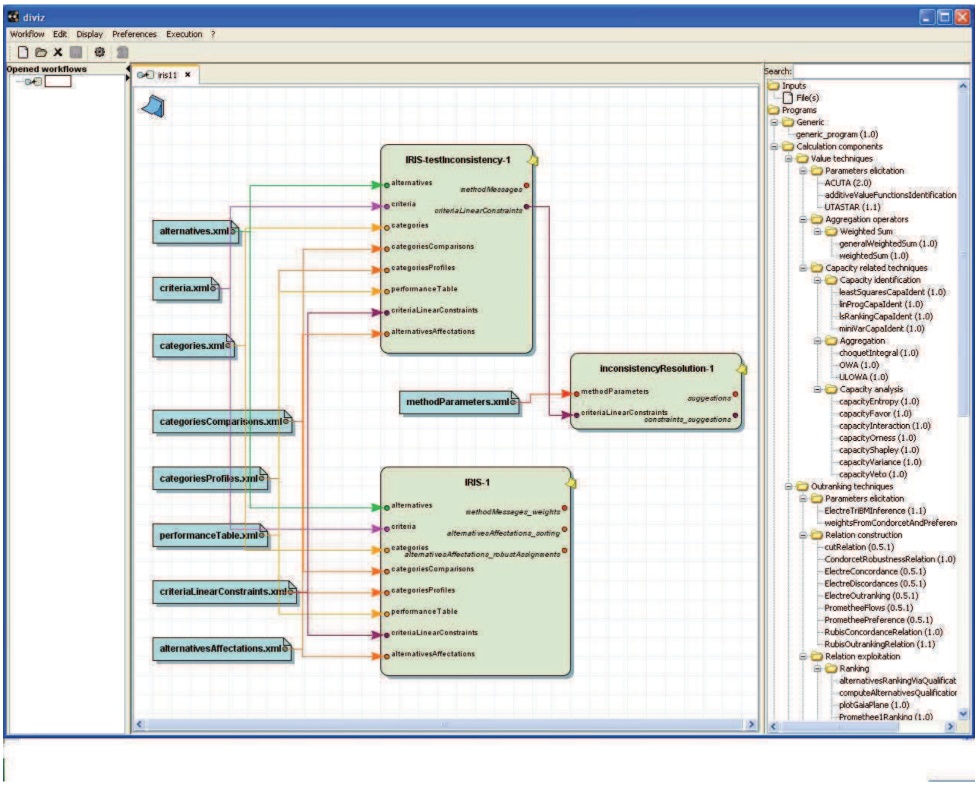


Figure 7.6: Web service workflow

7.2.2 *IRIS-testInconsistency*

The web service tests whether the preference information of the user is able to be represented by an ELECTRE TRI model. As illustrated in Figure **??**, the input files of this web service define the elements of the sorting problem, including alternatives, criteria, categories, the order of the categories, the profiles separating the categories and performance table of the alternatives. The preference information related to criteria and majority level is expressed in *"criteriaLinearConstraints"* file which is composed of some linear constraints. The user can also formulate his preference by some assignment examples restored in the *"alternativesAffectations"* file. A confidence level can be attached (not obligational) to each piece of preference information as the strength of such preference for resolving inconsistency (if it happens). All the files conform to XMCDA standard. The consistency of preference information is tested using the algorithm of Mousseau et al. (2001a). As one of the output files, the *"methodMessages"* file notifies whether the information is consistent. If not, *"criteriaLinearConstraints"* file is given which contains a set of linear constraints generated from the preference statements for further analysis .

7.2.3 *inconsistencyResolution*

The web service *"inconsistencyResolution"* identifies subsets of constraints which should be removed to obtain a consistent system, using the algorithm in Mousseau et al. (2006). For each constraint, the algorithm introduces a binary variable to indicate whether such constraint holds. We maximize the number of constraints which are satisfied to get a maximum subset of constraints which are compatible with each other. Through successive optimizations, a list of such subsets are identified. Two rule are proposed for the optimization. The first rule identifies subsets of constraints with maximum cardinality, that is to say, a minimum number of constraints are violated. In the second rule, confidential levels can be considered, which means it's only possible to delete the constraints of low confidence level.

The first input file of *"inconsistencyResolution"* web service is the output file of *"IRIS-testInconsistency"* web service, which contains a set of incompatible constraints. Several parameters are also required as input information. The first parameter is the resolution criterion, which can either be *"confidencelevel"* or *"cardinality"*. If the resolution criterion is based on confidence level, an additional parameter should be specified to define at which confidence level to resolve inconsistency. In other words, the constraints whose confidence level is higher than the specified level are not considered to be given up. The second parameter is the maximum number of constraints to be deleted to make the rest of the constraint set consistent.

The web service outputs a *"suggestions"* file which gives a list of suggestions for the resolution of inconsistency. Another output is a "*constraints_suggestions*" file with the details of such suggestions, which are a list of subsets of constraints. After one of such subset is deleted, the system becomes consistent.

We would like to mention that all the input and output files are XML files conforming to XMCDA schema, which enables them to be used by other web services. Furthermore, the web service *"inconsistencyResolution"* can be used in a general way, in the sense that any set of inconsistent linear constraints can apply this web service to get suggestions to revise their problems in order to get a consistent subset.

7.2.4 *IRIS*

If the preference information is consistent according to *"IRIS-testInconsistency"* web service, the *"IRIS"* web service is used to infer an ELECTRE TRI model which best matches such preference information (Mousseau et al., 2001a). The alternatives are assigned to one of the categories according to the inferred model and the robust assignment of each alternatives are computed as well.

The web service takes the same input files as *"IRIS-testInconsistency"*. The outputs consist of a "*methodMessages_weights*" file which gives weights and majority level of the inferred model. Another output file "*alternativesAffectations_sorting*" is the assignments of alternatives by the inferred model. A "*alternativesAffectations_robustAssignments*" file is also an output which provides robust assignments for each alternative and the combination of values of weights and majority level corresponding to such assignment.

7.3 An Illustrative Example

We illustrate the use of the three web services by an example whose data are from Dias (2003). The aim of such example is to assign 20 alternatives to 4 categories.

7.3.1 Representing the Problem with XMCDA Files

To use the web services, the first step is to represent the problem by XML files using XMCDA standard (see Section 7.1.2 and Bisdorff et al. (2008)). We only present here how to express the user's preference information.

Two kinds of preference statements are available. Firstly, the user claims that criterion g_2 is no less important than any other criterion, which leads to:

$$w_2 \geq w_j \quad j = \{1,3,4,5\}$$

Moreover, there are other constraints related to weights and the majority level, including their bounds and normalization. Such preference information is described by using the tag *"criteriaLinearConstraints"*. Secondly, three assignment examples are provided as follows.

$$a_{00} \rightarrow [C_2, C_4]; \quad a_{01} \rightarrow [C_3, C_4]; \quad a_{02} \rightarrow C_1$$

This information is restored under the tag *"alternativesAffectations"*, and is provided in Appendix C.1.

7.3.2 First Interaction: *"IRIS-testInconsistency"*

The second step is to create a workflow using only the web service *"IRIS-testInconsistency"*. The XML files built in the previous step should be connected to the entries of such web service. The workflow is executed and gives the output message below, which indicates that the preference information is inconsistent.

Method messages

The preference information is inconsistent, you can get suggestions by InconsistencyResolution!

The inconsistent preference information results in linear constraints which are described with XMCDA standard and can be visualized by web browser (see Appendix C.2).

7.3.3 Second Interaction: *inconsistencyResolution*

The *inconsistencyResolution* web service is used to resolve the inconsistency of these linear constraints. Suppose that the user prefer to obtain a maximum cardinality of subset of these inconsistent linear constraints which are consistent, he can specify the resolution criterion to *"cardinality"*, see Appendix C.3. Moreover, he also limits the number of deleted constraints to 5. Two suggestions are given, as illustrated below. Appendix C.4 details the linear constraints that each suggestion asks to remove, where the first set of constraints corresponds to Suggestion 1 and the second set corresponds to Suggestion 2.

Method messages

```
Suggestion 1 is to delete the constraints: C[a02]<=1
Suggestion 2 is to delete the constraints: C[a00]>=2 C[a01]>=3 lamda<=0.99
```

7.3.4 Refining Preference Information

After seeing the suggestions, the user decides to follow suggestion 1 which deletes the assignment $a_{02} \rightarrow C_1$. The file *"alternativesAffectations"* is changed accordingly.

7.3.5 Third Interaction

After changing the preference information, an Electre Tri model should exist compatible with such information. The *"IRIS"* web service is executed and an Electre Tri model is identified.

```
Method messages
weights [0.168333 0.326667 0.168333 0.168333 0.168333 ] lamda 0.758333
```

The model sorts the 20 alternatives to one of the 4 categories (see Appendix C.5). Moreover, the robust assignment of each alternative is given in Appendix C.6.

7.4 Concluding Remarks

We have developed three web services for the usage of Electre Tri method in practice. With *"IRIS-testInconsistency"*, the user can test if there exists an Electre Tri model compatible with his preference. The inconsistent information can be identified and suggestions to resolve such inconsistency are given by *"inconsistencyResolution"* web service. If the information is consistent, *"IRIS"* web service infers an Electre Tri model which assigns each alternative to a category. Moreover, it computes the robust assignment of each alternative considering the imprecision of preference information.

These services have been integrated to Diviz software, which allows them to interact with each other. and also with other services provided by other developers of Diviz. Interested readers are referred to the website of Diviz to download the software in order to use these web services.

Chapter 8

Conclusions and Future Research

8.1 Conclusions

Multiple Criteria Decision Aid (MCDA) aims at supporting decision makers (DM) facing with decisions involving multiple and conflicting criteria. During the decision aiding process, the analyst with adequate methodological expertise in MCDA interacts with the DM who has his knowledge, values and judgements in the decision problem to elicit the DM's preference. Such preference elicitation is a crucial element as it enables the two actors to communicate in a meaningful way so that the DM's preference can be represented appropriately in the decision models.

Two types of elicitation techniques are used in practice, direct and indirect ones. Direct elicitation methods ask the DM to specify the values of the parameters of the aggregation models, or provide a range of such values. However, they are criticized that the parameters elicited in this way don't appropriately reflect the DM's preferences, since the parameters of a specific model have their own interpretations that the DM is unaware of. Moreover, the direct elicitation methods require too much cognitive effort from the DM, which may be beyond his limitations. Instead, we are more interested in the indirect elicitation methods, which elicits an aggregation model from the DM's holistic judgements on the alternatives. The indirect elicitation tools are dependent on the aggregation models, for most of which there doesn't exist well-defined such tools.

This thesis concentrates on the development of preference elicitation tools for two aggregation models based on reference points, namely ELECTRE TRI and a Ranking method based on Multiple reference Points (RMP). We presented in the introduction of the thesis the four objectives of our work concerning the development of such tools. Now let us summarize our main contributions to achieve these objectives.

Preference elicitation tool for a sorting problem: ELECTRE TRI

The first contribution is the development of preference elicitation tools for ELECTRE TRI with the optimistic rule, which elicit parameter values and compute corresponding robust assignment from assignment examples through solving Mixed Integer Programs (MIP).

Numerical experiments have been conducted to investigate the performance of the algorithms with respect to the issues including learning ability, robustness computation and the ability to identify conflicting preference. The experiments of learning ability provides insights on the amount of input preference information needed to infer an ELECTRE TRI model in a reliable way. The experiments also show that taking into account more criteria in the model requires more preference information to determine such a model, while the number of categories considered doesn't appear to affect the elicitation algorithm. The experiments of robustness computation get the results in line with the one investigating learning ability. Another result concerning inconsistency issue indicates that more preference information increases the ability of the algorithm to identify the conflict while considering a larger number of criteria decreases such ability.

A case study has been presented which assesses degraded landscape of a region located in the center-north of Burkina Faso. The robustness computation algorithm is used to analyze the robust evaluation of the spatial units using ELECTRE TRI with the optimistic rule.

Preference elicitation tool for portfolio selection problems

We propose a two-level preference elicitation method to handle the DM's sophisticated preferences at two levels on portfolio selection. First, at the individual level, the DM's preferences on alternatives are expressed by some assignment examples, which are modeled by linear constraints on the parameter of evaluation model ELECTRE TRI. Second, at the portfolio level, the DM's preferences on the overall portfolio are modeled as the category size constraints of ELECTRE TRI. The elicitation algorithm is able to elaborate an ELECTRE TRI model as close as possible to the DMs' preference by the resolution of MIP. A satisfactory portfolio is then selected using the elaborated ELECTRE TRI model. The method can be used widely in portfolio selection situation where the decision should be made taking into account the individual and portfolio performance simultaneously.

The proposed method has been applied to a real-world student selection problem at Ecole Centrale Paris, France. This application shows how the method can be used to make a compromise when the preference at portfolio level is not compatible with preference at individual level.

Preference elicitation tool for a ranking problem: S-RMP model

We propose a preference elicitation tool for S-RMP model, which elicits a parsimonious S-RMP model with as fewest as possible reference points from the DM's pairwise comparisons.

Numerical experiments have been performed to investigate the useability of the elicitation tool. The results shed light on the required number of pairwise comparisons to elicit the S-RMP model close to the DM's "real" preferences. Moreover, more criteria in the model increases the difficulty in eliciting such a model. The experiments also finds that it's more difficult to elicit S-RMP model with continuous scales than with discrete ones. An important result obtained is that the elicitation algorithm is suggested to be used limiting the search of S-RMP models with no more than 3 reference points, due to computational difficulties. This limitation is proved to be not a serious problem in most cases.

In addition, an application in Eureval (Centre Européen d'Expertise et d'Evaluation, France) has been studied which evaluates the priority of treating pollutant substances. The elicitation tool is used to build S-RMP evaluation models considering both the simplicity and expressibility of such models. The case shows how the elicitation tool can be applied when there doesn't exist a simple S-RMP model which can restore the DM's preference. Indeed, we proposed to represent a maximum subset of the preference order to elicit two S-RMP models with only one reference point.

We expect a wide use of S-RMP model, because (1) it is easy to be understood by the DM who has no strong knowledge in MCDA, as the concept of reference point is rather natural in a decision experience; (2) it avoids the problem of preference circles obtained by most outranking aggregation methods; (3) it is easy to implement, even with a spread sheet; (4) the proposed elicitation tool facilitates the method's application as it constructs a S-RMP model from the DM' indirect preference statements.

Development of three web-services concerning preference elicitation for ELECTRE TRI

We have developed three web services for ELECTRE TRI , namely *"IRIS-testInconsistency"*, *"IRIS"* and *"inconsistencyResolution"*. Such three web services are integrated to Diviz software with a friendly user interface. The users can benefit from the implementation by downloading them as open source software and using them for their own problems.

It is worth pointing out that our research gives insights into the applicability of the two specific models. The numerical experiments not only investigate the behavior of the proposed elicitation algorithms, but also provide useful information on how to interact with the DM during the decision aiding process. Firstly, we study the amount of preference information required to obtain a reliable model. Secondly, robustness analysis is concerned to deal with the DM's incomplete preference information. Thirdly, we investigate the situation when the DM's preferences are inconsistent and give possible suggestions to resolve such a problem. Moreover, the case studies we present illustrate clearly our concerns about the three issues and how we handle them using the proposed preference elicitation tools.

8.2 Future Research

The research in this dissertation can of course be extended and expanded. We address specific extensions of each contribution for potential future research in what follows.

In Chapter 4, we have developed preference elicitation tools for ELECTRE TRI using the optimistic rule, with the assumption that the profiles of the categories are known as a priority, and no threshold is taken into consideration. Further research could be carried out to relax these assumptions.

The extension of eliciting profiles as unknown parameters is rather straightforward as it can be formulated similarly as in Chapter 5 and 6 where binary variables are introduced to indicate the way two objects are compared. However, such a formulation results in more binary variables in the optimization problem for the elicitation of ELECTRE TRI using the optimistic rule compared with the one using the pessimistic rule, since two additional binary variables are introduced to represent each assignment example as linear constraints using the optimistic rule. Thus the elicitation can be computationally costly.

The elicitation can also be extended by taking into account thresholds. The veto threshold and other parameters (profiles, weights and majority level) can be elicited simultaneously using additional binary variables, which can undoubtedly increase computation complexity. But eliciting the preference and indifference thresholds with other parameters together leads to non-linear program (see Mousseau and Slowiński, 1998, for complete inference of ELECTRE TRI with the pessimistic rule using non-linear optimization).

In Chapter 5, we have proposed elicitation method which elicits a portfolio selection model (ELECTRE TRI in our case) from the DM's preferences both at individual and portfolio level. The DM's preferences at individual level are expressed as assignment examples while at portfolio level they are described as constraints. Obviously such preference information is imprecise, which results in the fact there exists multiple ELECTRE TRI models compatible with such preferences. We have proposed to maximize a slack variable which represents the ability of the elicited model to reproduce the assignment examples to elicit an ELECTRE TRI model.

The first extension of the elicitation method consists of selecting robust individual considering the imprecise preference information. The individuals are evaluated by the whole set of compatible ELECTRE TRI models instead of one somehow arbitrarily chosen one. The non-dominated portfolios have to be computed based on the robust evaluations. Thus the selection of a satisfactory portfolio leads to the question of how to evaluate a portfolio, which remains a challenge.

The second extension of the elicitation method is to formulate the DM's preference at portfolio level as objectives rather than constraints which only eliminate the unsatisfactory portfolios. This formulation would lead to the selection of portfolio as multiobjective problem, which seems to be not trivial, also because the non dominated portfolios should be computed and the evaluation of a portfolio needs further analysis.

In Chapter 6, we have provided a preference elicitation tool for S-RMP which elicits a parsimonious model compatible with the DM's pairwise comparisons. The elicitation tool is suggested to be used limiting the search to S-RMP models with no more than 3 reference points. Many interesting problems may be explored in the future.

For the first direction, the computational efficiency of the elicitation tool should be improved, which will allow the tool to be used with larger data set. Possible way of the improvement could be designing a more effective search strategy in the elicitation algorithm.

For the second direction, we should consider the pairwise comparisons as imprecise preference information and elicit all S-RMP models which are compatible with such preference. The ranking outcome should be derived from all these models. We can investigate such an issue by applying robust ordinal regression, which proposes taking into account all the sets of parameters compatible with the preference information, in order to give a recommendation in terms of necessary and possible consequences of applying all the compatible preference models on the considered set of alternatives (Greco et al., 2010b).

For the third direction, the interpretation of the elicited models should be studied. For example, what are the interpretations of the reference points for the DM? What are the meanings of the different lexicography orders? Probably more decision aiding experiences using the elicitation tool would give more insights to the questions.

For the fourth direction, we concern the detection of the inconsistency in terms of the DM's preference information during the elicitation. As shown in the case study, this can be tackled by finding a maximum subset of preference information which can be represented by a

S-RMP model with a limited number of reference points. However, other approaches to deal with inconsistency in S-RMP model should be investigated.

Further developments of web services can be considered. Firstly, elicitation tools implementing ELECTRE TRI using the optimistic assignment rule can be developed as web services for Diviz. Secondly, such an implementation can also be carried out for the elicitation tool of S-RMP, which will surely enhance the application of the method.

Appendix A

Appendix of Chapter 5

A.1 Definition of the qualitative evaluation criteria

- **Motivation** : Perceived motivation of the student in the choice of the IE major as judged by the DM through the interview and by reading the cover letter.

  1. “I come to IE because it is the only non-technical option at ECP, I don’t know how to find my way”, sloppy letter graphically and in terms of content,
  2. She/He doesn’t know exactly why she/he wants the IE Major, cover letter correctly written but not revealing a particularly strong motivation,
  3. Student motivation and looks inspired by the offer of the Major, however she/he could consider other options,
  4. Motivated Student, able to project her/him into the future (his future employability and academic year) and that clearly expresses how the IE Major corresponds to expectations,
  5. Highly motivated student, saying that she/he is willing to invest in the "life of the Major" (students’ delegate or other responsibility) and showing in her/his letter and her/his interview that the choice of the IE Major is the natural continuation of its courses and enables her/him to make a clearly formulated career plan.

- **Professional Project**: Ability of the student to articulate his/her future professional project with his/her previous achievements (courses, ...). She/he takes into account the logic, consistency and variety of what she/he has done previously, the reasons for his choice to come to IE projects and employability. The coherence of the choice of major with the Professional Track is considered here.

1. Student unable to express or that has no career plans, has never visited a factory, unaware of what constitutes a factory and what it means to work in one,
2. Professional project still unclear, despite the training and professional experiences,
3. Professional project starting to be worked on whilst not being specific enough, but is able to make some elements of where she/he wants move to,
4. The student is clear in expressing her/his projects proved by internships, but still hesitating between different career paths (which are clearly specified),
5. The professional project is clear and well defined; she/he has done a series of courses of various sorts and other experiences that are part of this logic.

- **Maturity / Personality**: Maturity and the openness of the student that brought her/him to focus on the Industrial Engineering in a large sense and beyond to general society concerns

1. Student whose maturity is not asserted, that justifies her/his answer by default, or very vague or even evasive,
2. Student still fairly young at heart, "I'm hesitating about what I want to do but I have some ideas on certain types of jobs, so I want to test the idea by this Major" she/he didn't made internships, wants to be hired just to get a clearer picture of IE,
3. Student who remains a bit unsure about his/her career choice but realizes (s)he must move forward on this issue, although (s)he has already forged a few elements of views in his/her professional experiences,
4. Student mature, dynamic, and the year IE Major should still allow her/him to mature and arrive at the job market very positively,
5. Mature student, shows dynamism, able to express clearly her/his choices for her/his projects, shows a cultural openness beyond the strict academic requirements at ECP.

- **General knowledge of Industrial Engineering and its career opportunities**: Ability to define what industrial engineering is, in particular knowledge of the contents of the Industrial Engineering Major of ECP and the various outcomes

1. Student can not describe what Industrial Engineering is, knowing nothing of the contents of the Major, and not knowing the jobs of Industrial Engineering,
2. Student able to express a veneer of knowledge about Industrial Engineering, but being unable to go beyond the general speech,
3. Student with some knowledge in Industrial Engineering, jobs in general, but without a detailed vision of these elements; she/he doesn't necessarily know the contents of the major,
4. Student well aware of what may represent the Industrial Engineering, but whose vision may still remain vague about the contents of the option and/or outcomes,
5. Student very aware of what constitutes the field of Industrial Engineering, having ascertained precisely the content of the option, which can even specify the choice of electives, knows in detail the various opportunities in terms of jobs.

A.2 Evaluation of 2009 students

	$g_1(a_i)$	$g_2(a_i)$	$g_3(a_i)$	$g_4(a_i)$	$g_5(a_i)$	$g_6(a_i)$	gender	prof. track	stream	Category	Accepted
a_1	11.87	12.66	4	4	4	4	M	E	1	C_4	1
a_2	13.82	13.82	4	4	4	3	M	SF	2	C_4	1
a_3	15.66	15.66	5	4	4	5	F	SF	1	C_4	1
a_4	13.77	13.77	4	4	4	4	M	SF	4	C_4	1
a_5	13.72	13.72	1	1	2	1	M	SF	3	C_1	0
a_6	12.8	12.8	5	4	5	4	F	SF	3	C_4	1
a_7	14.15	14.15	4	4	4	3	M	E	2	C_4	1
a_8	13.96	13.96	3	2	2	2	M	SF	2	C_2	0
a_9	12.87	12.87	1	0	1	1	M	E	4	C_1	0
a_{10}	12.44	12.65	3	2	3	2	M	E	1	C_3	0
a_{11}	13.43	13.43	3	3	2	1	M	IPM	4	C_1	0
a_{12}	12.63	12.63	3	3	4	3	F	SF	3	C_3	1
a_{13}	11.5	12.31	3	3	3	3	M	IPM	2	C_3	0
a_{14}	12.64	12.64	2	3	2	2	M	SF	4	C_2	0
a_{15}	13.63	13.63	4	5	5	4	M	OM	1	C_4	1
a_{16}	13.38	13.87	3	4	4	3	M	OM	1	C_3	0
a_{17}	13.12	13.05	5	4	5	3	M	SF	1	C_4	1
a_{18}	13.68	13.68	5	5	5	5	M	OM	3	C_4	1
a_{19}	12.87	12.87	3	3	2	2	F	SF	4	C_3	1
a_{20}	13.37	13.37	3	3	4	4	M	IPM	1	C_3	1
a_{21}	13.45	13.45	2	1	1	2	M	SF	3	C_1	0
a_{22}	12.42	12.42	2	2	2	2	M	SF	2	C_2	0
a_{23}	13.33	13.33	4	3	4	3	M	SF	1	C_4	1

following next page

	$g_1(a_i)$	$g_2(a_i)$	$g_3(a_i)$	$g_4(a_i)$	$g_5(a_i)$	$g_6(a_i)$	gender	prof. track	stream	Category	Accepted
a_{24}	13.18	13.18	5	4	5	5	M	OM	2	C_4	1
a_{25}	12.68	12.68	3	2	3	3	M	SF	3	C_3	0
a_{26}	14.08	14.08	4	5	4	3	M	SF	4	C_4	1
a_{27}	12.96	12.96	3	3	3	2	M	SF	4	C_3	0
a_{28}	12.73	12.73	4	4	5	2	F	SF	1	C_3	1
a_{29}	12.59	12.59	3	3	4	3	M	E	2	C_3	1
a_{30}	12.68	13.39	5	4	4	4	M	SF	4	C_4	1
a_{31}	14.32	14.29	4	5	3	3	M	E	3	C_4	1
a_{32}	13.52	14.32	4	4	5	4	M	E	3	C_4	1
a_{33}	13.08	13.08	2	2	2	1	M	OM	4	C_1	0
a_{34}	13.54	13.54	3	3	4	3	F	SF	2	C_3	1
a_{35}	13.36	13.36	4	4	4	3	M	OM	3	C_4	1
a_{36}	13.78	13.78	4	4	4	3	F	SF	3	C_4	1
a_{37}	12.17	12.17	3	3	3	3	M	SF	1	C_3	0
a_{38}	11.55	11.55	1	1	1	1	M	E	4	C_1	0
a_{39}	12.81	12.81	1	1	1	1	F	SF	2	C_1	0
a_{40}	12.78	12.87	4	3	4	4	F	SF	3	C_3	1
a_{41}	13.24	13.24	1	2	1	1	M	OM	2	C_1	0
a_{42}	12.42	12.67	2	3	3	4	M	IPM	1	C_3	1
a_{43}	14.16	14.16	4	4	4	4	F	SF	1	C_4	1
a_{44}	12.57	12.57	4	4	5	3	M	OM	2	C_4	1
a_{45}	12.86	12.86	2	1	1	1	M	IPM	4	C_1	0
a_{46}	12.73	12.73	3	3	4	3	M	E	3	C_3	1
a_{47}	12.75	12.75	3	3	4	4	F	SF	1	C_3	1
a_{48}	12.93	12.87	3	4	5	3	M	IPM	2	C_3	1
a_{49}	13.54	13.54	3	3	4	3	F	SF	4	C_3	1
a_{50}	12.65	12.65	3	4	4	4	M	E	2	C_3	1
a_{51}	12.61	12.31	1	1	2	1	M	E	3	C_1	0
a_{52}	11.01	12.03	3	3	3	3	M	E	3	C_3	1
a_{53}	12.21	14.11	4	4	4	3	M	IPM	1	C_4	1
a_{54}	11.43	12.41	2	3	2	3	F	SF	4	C_3	1
a_{55}	13.02	14.12	4	4	4	5	F	SF	4	C_4	1
a_{56}	10.94	12.24	3	2	1	4	M	SF	3	C_1	0
a_{57}	11.87	12.77	3	3	4	3	M	E	2	C_3	1
a_{58}	12.87	14.24	3	4	4	3	M	IPM	1	C_3	1
a_{59}	12.40	13.84	5	5	5	4	M	OM	1	C_4	1
a_{60}	12.43	13.43	4	3	3	3	F	SF	4	C_3	1
a_{61}	11.51	13.28	4	4	3	3	M	E	4	C_4	1
a_{62}	11.93	13.43	4	4	4	3	M	SF	3	C_4	1
a_{63}	12.54	13.77	1	2	2	1	M	IPM	1	C_1	0
a_{64}	11.84	13.64	4	4	4	4	F	SF	2	C_4	1
a_{65}	13.54	13.01	1	1	1	2	M	IPM	4	C_1	0
a_{66}	11.45	13.03	3	4	4	3	F	OM	2	C_3	1
a_{67}	11.71	12.11	2	2	2	1	M	IPM	2	C_1	0
a_{68}	12.18	13.51	2	2	3	3	F	SF	1	C_3	1
a_{69}	12.53	14.77	4	4	5	4	F	IPM	1	C_4	1
a_{70}	11.43	12.53	2	2	1	2	M	SF	2	C_1	0

following next page

	$g_1(a_i)$	$g_2(a_i)$	$g_3(a_i)$	$g_4(a_i)$	$g_5(a_i)$	$g_6(a_i)$	gender	prof. track	stream	Category	Accepted
a_{71}	12.33	13.15	2	2	2	2	M	E	4	C_2	0
a_{72}	13.72	13.72	2	1	1	1	M	IPM	3	C_1	0
a_{73}	13.82	13.91	2	3	4	5	M	E	1	C_4	1
a_{74}	12.52	12.56	3	4	2	3	M	E	4	C_3	1
a_{75}	12.96	13.54	5	4	4	4	M	OM	2	C_4	1
a_{76}	10.71	12.37	3	2	1	4	M	IPM	2	C_1	0

A.3 Mathematical program to infer ELECTRE TRI$_{\mathbf{BM}}$ weights

$$
\begin{aligned}
\max \quad & \alpha \\
s.t. \quad & \alpha \leq x_k \\
& \alpha \leq y_k \\
& \textstyle\sum_{j:g_j(a_i)\geq g_j(b_3)} w_j - x_k = \lambda, * \quad \text{for } \ i = \{59, 68\} \\
& \textstyle\sum_{j:g_j(a_i)\geq g_j(b_2)} w_j - x_k = \lambda, \quad \text{for } \ i = \{10, 61\} \\
& \textstyle\sum_{j:g_j(a_i)\geq g_j(b_3)} w_j + y_k = \lambda, \quad \text{for } \ i = \{10, 61\} \\
& \textstyle\sum_{j:g_j(a_{22})\geq g_j(b_1)} w_j - x_k = \lambda \\
& \textstyle\sum_{j:g_j(a_{22})\geq g_j(b_2)} w_j + y_k = \lambda \\
& \textstyle\sum_{j=1}^{m} w_j = 1 \\
& w_j \in [0, 0.5], \quad \forall j = \{1, 2, ..., 6\} \\
& w_2 \leq w_j, \quad \forall j = \{3, 4, 5\} \\
& \lambda \in [0.5, 1]
\end{aligned}
$$

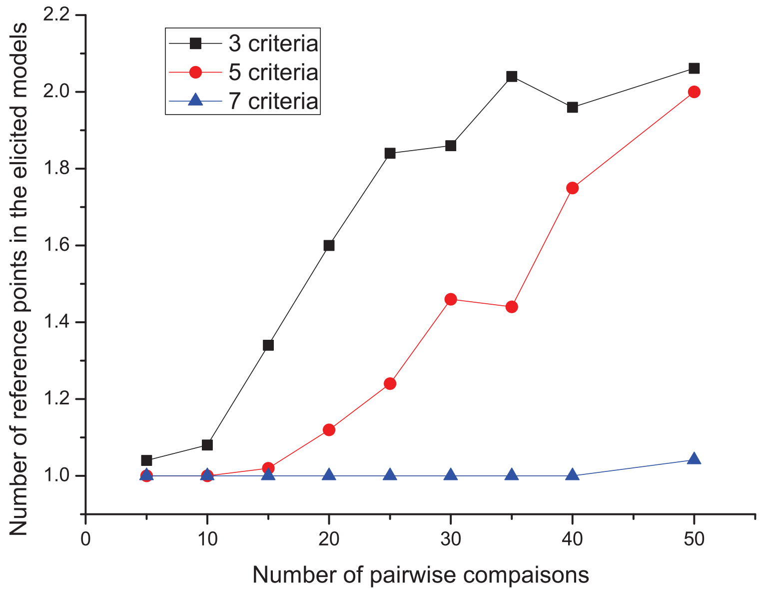
Appendix B

Appendix of Chapter 6

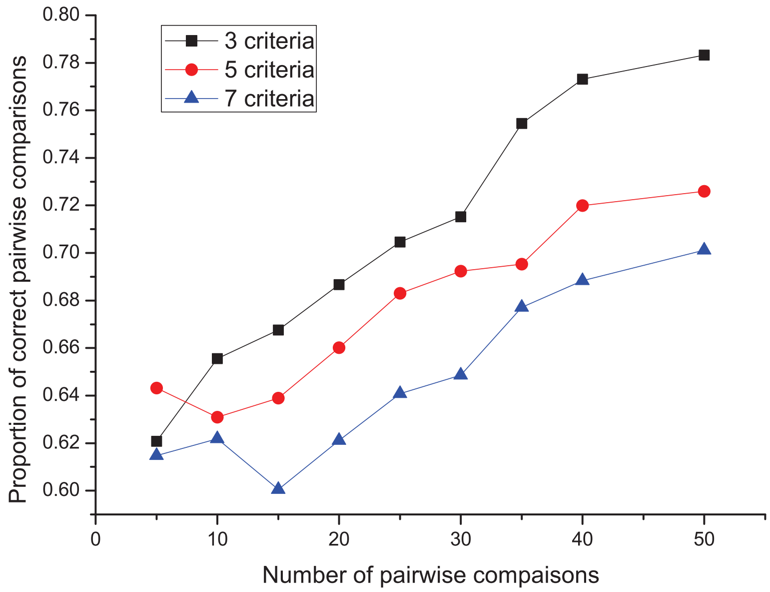
B.1 Mathematical program to infer reference points

$$
\begin{aligned}
\max \quad & s^1_{min} \\
\text{s.t.} \quad & \sum_{j=1}^{m} c^h_{ej} - \sum_{j=1}^{m} c^h_{e'j} - s^h_{ee'} = 0 \qquad \forall a_e, a_{e'} \in A^* \;\; h \in P \\
& \alpha^h_{ee'} = 1 \Leftrightarrow s^h_{ee'} \geq 0 \qquad \forall a_e, a_{e'} \in A^* \;\; h \in P \\
& \beta^h_{ee'} = 1 \Leftrightarrow s^h_{ee'} = 0 \qquad \forall a_e, a_{e'} \in A^* \;\; h \in P \\
& \alpha^h_{ee'} \geq \sum_{z=1}^{h-1} \beta^z_{ee'} \qquad \forall a_e, a_{e'} \in A^* \;\; h \in P \\
& \mu^h_{ee'} = 1 \Leftrightarrow s^h_{ee'} > 0 \qquad \forall a_e, a_{e'} \in A^* \;\; h \in P \\
& \sum_{h=1}^{k} \mu^h_{ee'} \geq 1 \qquad \forall a_e, a_{e'} \in A^* \;\; h \in P \\
& \alpha^h_{ee'}, \beta^h_{ee'}, \mu^h_{ee'} \in \{0,1\} \qquad \forall a_e, a_{e'} \in A^* \;\; h \in P \\
& p^h_j \geq p^{h+1}_j \qquad 1 \leq h \leq k-1 \\
& 0 \leq w_j \leq 0.5, \;\; j = 1,2,...,m \\
& \textstyle\sum_{j=1}^{m} w_j = 1 \\
& s^1_{min} \leq s^1_{ee'} \qquad \forall a_e, a_{e'} \in A^*
\end{aligned}
$$

B.2 Results of Experiment 2

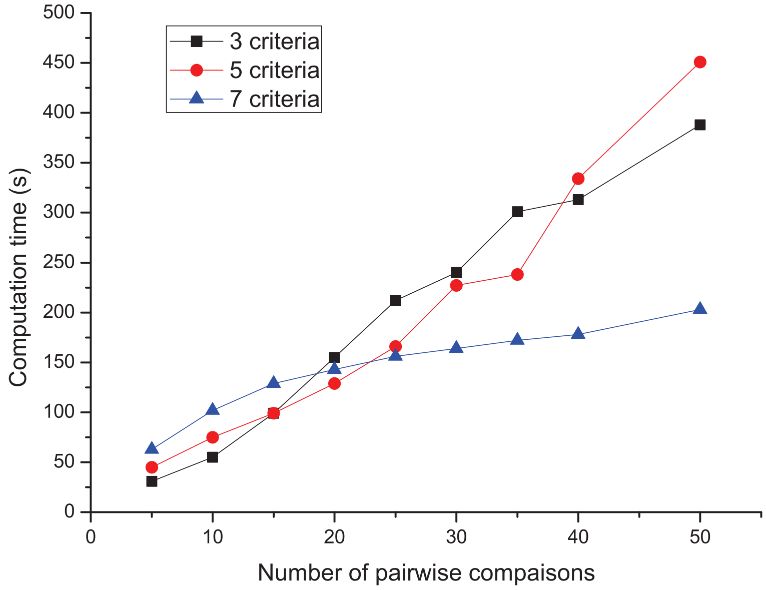


(a)



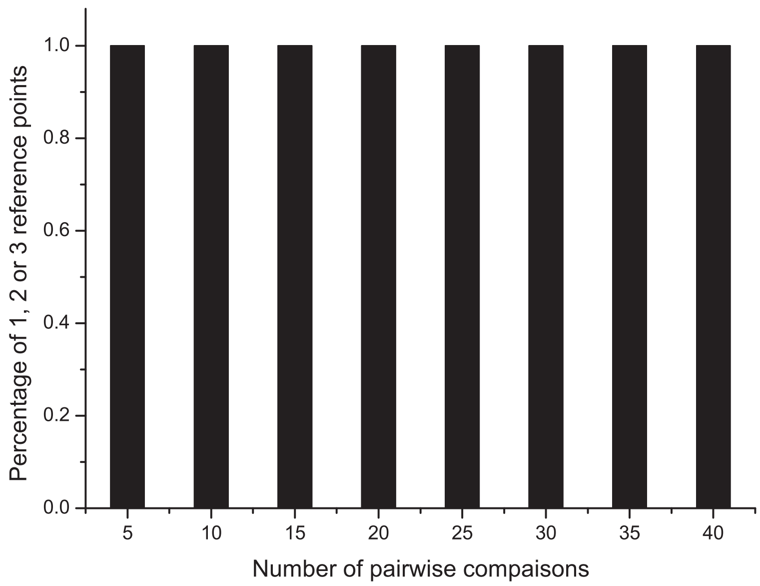
(b)

Figure B.1: Eliciting M_o with 3 reference points, 3/5/7 criteria evaluated on continuous scale.



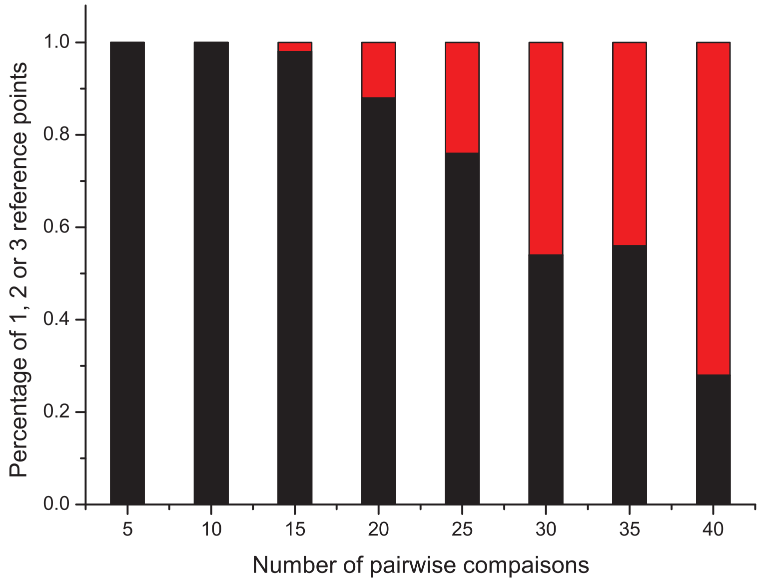
(c)

Figure B.1: Eliciting M_o with 3 reference points, 3/5/7 criteria evaluated on continuous scale. (cont.)

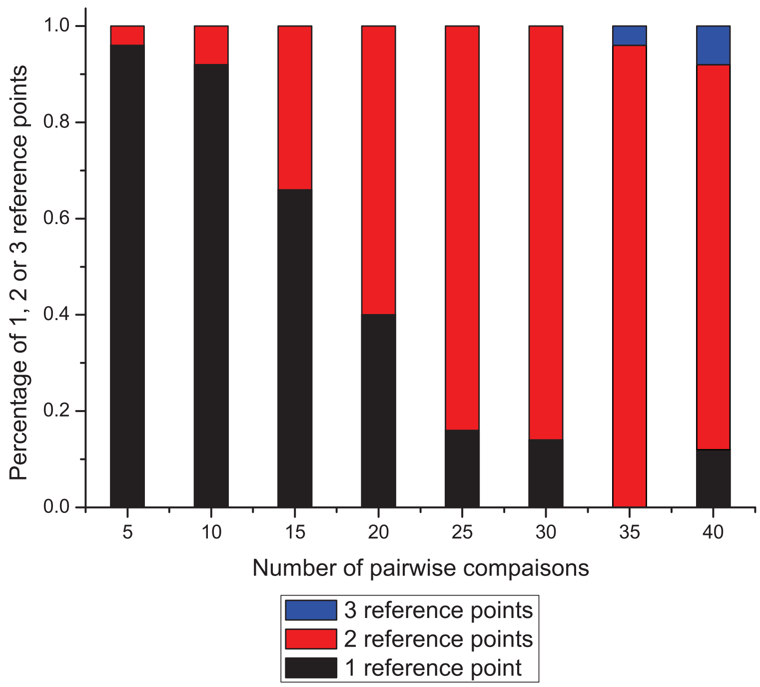


(a) 7 criteria

Figure B.2: Eliciting M_o with 3 reference points, criteria evaluated on continuous scale



(b) 5 criteria



(c) 3 criteria

Figure B.2: Eliciting M_o with 3 reference points, criteria evaluated on continuous scale(cont.)

B.3 Evaluation of twenty pollutants

	$g_1(a_i)$	$g_2(a_i)$	$g_3(a_i)$	$g_4(a_i)$	$g_5(a_i)$	$g_6(a_i)$	$g_7(a_i)$	$g_8(a_i)$	Score (M1)	Score (M2)
A	100	0	0	4	0	0	3	0	0.4955	0.5843
B	100	1	0	4	2	1	3	0	0.6573	0.6966
C	100	2	3	3	0	0	0	3	0.4965	0.7191
D	90	3	4	0	0	3	1	2	0.4985	0.5730
E	100	4	3	3	0	0	0	3	0.6593	0.7191
F	90	4	1	4	4	2	3	1	0.8202	0.9438
G	100	1	2	3	4	0	0	0	0.4895	0.4944
H	90	2	3	4	2	0	0	2	0.6603	0.6742
I	100	3	0	3	4	0	1	3	0.8182	0.6629
J	10	4	0	3	2	1	3	1	0.3287	0.6517
K	100	3	0	4	4	1	1	2	0.8202	0.7079
L	100	2	0	0	0	0	0	0	0.3207	0.4045
M	90	2	3	0	0	0	0	2	0.3277	0.4719
N	100	4	4	0	0	2	1	2	0.4945	0.5730
O	80	2	3	3	4	0	0	3	0.6583	0.7303
P	80	4	0	3	0	0	3	0	0.4875	0.7640
Q	100	3	0	4	2	1	2	4	0.9890	0.9326
R	100	3	3	2	4	0	0	2	0.6523	0.6742
S	90	4	0	1	4	3	3	0	0.6533	0.6854
T	80	2	0	0	2	3	0	0	0.4865	0.5169

Appendix C

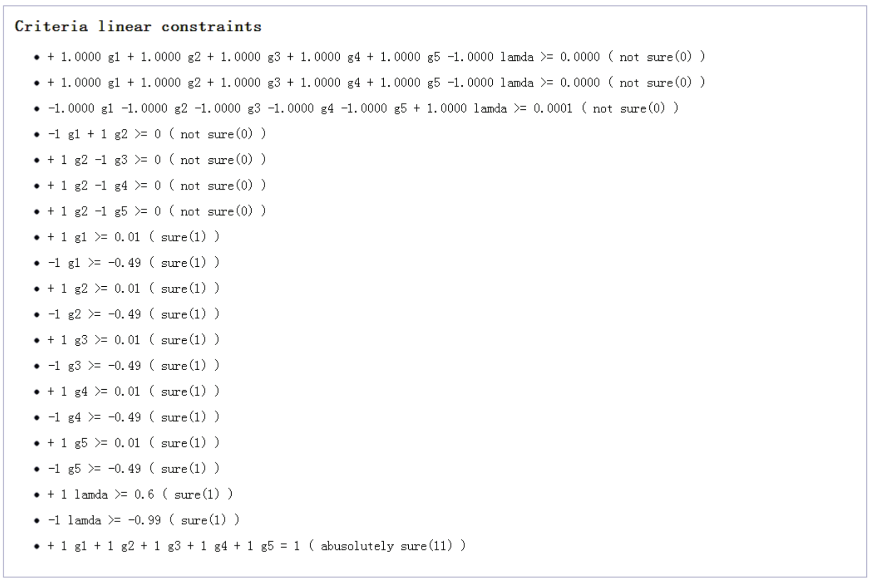
Appendix of Chapter 7

C.1 Assignment examples

```
<alternativesAffectations>
        <alternativeAffectation>
            <alternativeID>a00</alternativeID>
            <categoriesInterval>
                <lowerBound>
                    <categoryID>c2</categoryID>
                </lowerBound>
                <upperBound>
                    <categoryID>c4</categoryID>
                </upperBound>
            </categoriesInterval>
            <value>
                <rankedLabel>
                    <label>not sure</label>
                    <rank>0</rank>
                </rankedLabel>
            </value>
        </alternativeAffectation>
        <alternativeAffectation>
            <alternativeID>a01</alternativeID>
            <categoriesInterval>
                <lowerBound>
                    <categoryID>c3</categoryID>
                </lowerBound>
                <upperBound>
                    <categoryID>c4</categoryID>
                </upperBound>
            </categoriesInterval>
            <value>
                <rankedLabel>
```

```
                    <label>not sure</label>
                    <rank>0</rank>
                </rankedLabel>
            </value>
        </alternativeAffectation>
        <alternativeAffectation>
            <alternativeID>a02</alternativeID>
            <categoriesInterval>
                <lowerBound>
                    <categoryID>c1</categoryID>
                </lowerBound>
                <upperBound>
                    <categoryID>c1</categoryID>
                </upperBound>
            </categoriesInterval>
            <value>
                <rankedLabel>
                    <label>not sure</label>
                    <rank>0</rank>
                </rankedLabel>
            </value>
        </alternativeAffectation>
    </alternativesAffectations>
```

C.2 Inconsistent linear constraints



Criteria linear constraints

- + 1.0000 g1 + 1.0000 g2 + 1.0000 g3 + 1.0000 g4 + 1.0000 g5 -1.0000 lamda >= 0.0000 (not sure(0))
- + 1.0000 g1 + 1.0000 g2 + 1.0000 g3 + 1.0000 g4 + 1.0000 g5 -1.0000 lamda >= 0.0000 (not sure(0))
- -1.0000 g1 -1.0000 g2 -1.0000 g3 -1.0000 g4 -1.0000 g5 + 1.0000 lamda >= 0.0001 (not sure(0))
- -1 g1 + 1 g2 >= 0 (not sure(0))
- + 1 g2 -1 g3 >= 0 (not sure(0))
- + 1 g2 -1 g4 >= 0 (not sure(0))
- + 1 g2 -1 g5 >= 0 (not sure(0))
- + 1 g1 >= 0.01 (sure(1))
- -1 g1 >= -0.49 (sure(1))
- + 1 g2 >= 0.01 (sure(1))
- -1 g2 >= -0.49 (sure(1))
- + 1 g3 >= 0.01 (sure(1))
- -1 g3 >= -0.49 (sure(1))
- + 1 g4 >= 0.01 (sure(1))
- -1 g4 >= -0.49 (sure(1))
- + 1 g5 >= 0.01 (sure(1))
- -1 g5 >= -0.49 (sure(1))
- + 1 lamda >= 0.6 (sure(1))
- -1 lamda >= -0.99 (sure(1))
- + 1 g1 + 1 g2 + 1 g3 + 1 g4 + 1 g5 = 1 (abusolutely sure(11))

C.3 Parameters for resolving inconsistency

```xml
<methodParameters>
    <parameters>
        <parameter mcdaConcept="confidence level at which
           to resolve inconsistency" id="confidencelevel">
            <value>
                <rankedLabel>
                    <rank>0</rank>
                    <label>not sure</label>
                </rankedLabel>
            </value>
        </parameter>
        <parameter mcdaConcept="maximum constraints to be
           deleted" id="maxcount">
            <value>
                <integer>5</integer>
            </value>
        </parameter>
        <parameter mcdaConcept="resolution criterion(
           confidencelevel or cardinality)" id="
           resolutioncriterion">
            <value>
                <label>cardinality</label>
            </value>
        </parameter>
    </parameters>
</methodParameters>
```

C.4 Suggestions to remove constraints

Criteria linear constraints

- -1.0000 g1 -1.0000 g2 -1.0000 g3 -1.0000 g4 -1.0000 g5 + 1.0000 lamda >= 0.0001 (not sure(0))

Criteria linear constraints

- + 1.0000 g1 + 1.0000 g2 + 1.0000 g3 + 1.0000 g4 + 1.0000 g5 -1.0000 lamda >= 0.0000 (not sure(0))
- + 1.0000 g1 + 1.0000 g2 + 1.0000 g3 + 1.0000 g4 + 1.0000 g5 -1.0000 lamda >= 0.0000 (not sure(0))
- -1 lamda >= -0.99 (sure(1))

Sorting result based on parameter values that best match the provided information

- a00 -> c4
- a01 -> c3
- a02 -> c4
- a03 -> c3
- a04 -> c3
- a05 -> c3
- a06 -> c4
- a07 -> c4
- a08 -> c3
- a09 -> c3
- a10 -> c3
- a11 -> c3
- a12 -> c3
- a13 -> c3
- a14 -> c2
- a15 -> c3
- a16 -> c2
- a17 -> c3
- a18 -> c3
- a19 -> c1

C.5 Sorting results

C.6 Robustness assignments

Robust assignments

- a00 -> categories set { c4 }
- a01 -> categories set { c3 }
- a02 -> categories set { c2, c4 }
- a03 -> categories set { c2, c3, c4 }
- a04 -> categories set { c3 }
- a05 -> categories set { c3, c4 }
- a06 -> categories set { c3, c4 }
- a07 -> categories set { c3, c4 }
- a08 -> categories set { c3 }
- a09 -> categories set { c2, c3, c4 }
- a10 -> categories set { c3 }
- a11 -> categories set { c2, c3 }
- a12 -> categories set { c3 }
- a13 -> categories set { c3 }
- a14 -> categories set { c2, c3 }
- a15 -> categories set { c3 }
- a16 -> categories set { c1, c2, c3 }
- a17 -> categories set { c3, c4 }
- a18 -> categories set { c3 }
- a19 -> categories set { c1, c2 }

Bibliography

Almeida-Dias, J., Figueira, J., and Roy, B. (2012). A multiple criteria sorting method where each category is characterized by several reference actions: The Electre Tri-NC method. *European Journal of Operational Research*, 217(3):567 – 579.

Alpaydin, E. (2010). *Introduction to Machine Learning*. The MIT Press, 2nd edition.

Archer, N. P. and Ghasemzadeh, F. (1999). An integrated framework for project portfolio selection. *International Journal of Project Management*, 17(4):207–216.

Arrow, K. J. (1953). *Social Choice and Individual Values*. Cowles Foundation Monographs; New York: Wiley 1964.

Bafail, A. and Moreb, A. (1993). Optimal allocation of students to different departments in an engineering college. *Computers & Industrial Engineering*, 25(1-4):295–298.

Bana e Costa, C. A., Ensslin, L., Cornêa, E. C., and Vansnick, J.-C. (1999). Decision support systems in action: Integrated application in a multicriteria decision aid process. *European Journal of Operational Research*, 113(2):315 – 335.

Bana e Costa, C. A. and Vansnick, J.-C. (1994). MACBETH: An interactive path towards the construction of cardinal value functions. *International Transactions in Operational Research*, 1(4):489 – 500.

Bana e Costa, C. A. and Vansnick, J.-C. (2008). A critical analysis of the eigenvalue method used to derive priorities in AHP. *European Journal of Operational Research*, 187(3):1422 – 1428.

Beliakov, G., Pradera, A., and Calvo, T. (2008). *Aggregation Functions: A Guide for Practitioners*. Springer Publishing Company, Incorporated, 1st edition.

Belton, V. and Stewart, T. (2002). *Multiple criteria decision analysis: an integrated approach*. Kluwer.

Bisdorff, R., Meyer, P., and Veneziano, T. (2008). Quick dive into XMCDA 2.0. technical report, Université du Luxembourg and Institut Télécom-Télécom Bretagne.

Böckenholt, U. (2001). Thresholds and intransitivities in pairwise judgments: A multilevel analysis. *Journal of Educational and Behavioral Statistics*, 26(3):pp. 269–282.

Boer, L., van der Wegen, L., and Telgen, J. (1998). Outranking methods in support of supplier selection. *European Journal of Purchasing & Supply Management*, 4:109 – 118.

Botreau, R. and Rolland, A. (2008). Evaluation multicritère du bien-être animal en ferme : une application de méthodes développé es en aide à la décision multicritère. In *ROADEF 08*.

Bouyssou, D. and Marchant, T. (2007a). An axiomatic approach to noncompensatory sorting methods in MCDM, i: The case of two categories. *European Journal of Operational Research*, 178(1):217–245.

Bouyssou, D. and Marchant, T. (2007b). An axiomatic approach to noncompensatory sorting methods in MCDM, II: more than two categories. *European Journal of Operational Research*, 178(1):246–276.

Bouyssou, D., Marchant, T., Pirlot, M., Tsoukiàs, A., and Vincke, P. (2006). Evaluation and decision models with multiple criteria : stepping stones for the analyst.

Bouyssou, D. and Perny, P. (1992). Ranking methods for valued preference relations: A characterization of a method based on leaving and entering flows. *European Journal of Operational Research*, 61:186 – 194.

Bouyssou, D. and Pirlot, M. (2005). Conjoint measurement tools for MCDM. In *Multiple Criteria Decision Analysis: State of the Art Surveys*, volume 78 of *International Series in Operations Research & Management Science*, pages 73–112. Springer New York.

Bouyssou, D., Pirlot, M., and Vincke, P. (1997). A general model of preference aggregation. In M.H. Karwan, J. S. and Wallenius, J., editors, *Essays in Decision Making*, pages 120–134. Springer-Verlag.

Cailloux, O. (2010). ELECTRE and PROMETHEE MCDA methods as reusable software components. In *Proceedings of the 25th Mini-EURO Conference on Uncertainty and Robustness in Planning and Decision Making (URPDM 2010)*. University of Coimbra, Portugal.

Cailloux, O., Zheng, J., and Mousseau, V. (2012). Rapport étude substances. technicial report, Ecole Centrale Paris.

Campbell, D. E. and Kelly, J. S. (2002). Impossibility theorems in the arrovian framework. In Arrow, K. J., Sen, A. K., and Suzumura, K., editors, *Handbook of Social Choice and Welfare*, volume 1 of *Handbook of Social Choice and Welfare*, chapter 1, pages 35–94. Elsevier.

Chen, C.-T. (2000). Extensions of the topsis for group decision-making under fuzzy environment. *Fuzzy Sets and Systems*, 114(1):1 – 9.

Chen, Y., Li, K. W., Kilgour, D. M., and Hipel, K. W. (2008). A case-based distance model for multiple criteria abc analysis. *Computers & Operations Research*, 35(3):776 – 796.

Chien, C. (2002). A portfolio-evaluation framework for selecting R&D projects. *R & D Management*, 32(4):359–368.

Clemen, R. T. (1997). *Making Hard Decisions: An Introduction to Decision Analysis (Business Statistics)*. South-Western College Pub, 2 edition.

Coldrick, S., Longhurst, P., Ivey, P., and Hannis, J. (2005). An R&D options selection model for investment decisions. *Technovation*, 25(3):185–193.

Debreu, G. (1959). Topological methods in cardinal utility theory. *Cowles Foundation Discussion Papers*.

Decision Deck Consortium (2012a). Decision Deck Project. `http://www.decision-deck.org/`.

Decision Deck Consortium (2012b). Diviz software. `http://www.decision-deck.org/diviz/index.html`.

Deutsch, S. J. and Malmborg, C. J. (1985). Measurable multiattribute value functions defined on binary attributes. *European Journal of Operational Research*, 19(2):186 – 190.

Dias, L. and Climaco, J. (1999). On computing ELECTRE's credibility indices under partial information. *Journal of Multi-Criteria Decision Analysis*, 8(2):74–92.

Dias, L., Mousseau, V., Figueira, J., and Clímaco, J. (2002). An aggregation/disaggregation approach to obtain robust conclusions with ELECTRE TRI. *European Journal of Operational Research*, 138(2):332–348.

Dias, L. C. and Mousseau, V. (2006). Inferring electre's veto-related parameters from outranking examples. *European Journal of Operational Research*, 170(1):172 – 191.

Dias, L., V. M. (2003). IRIS-Interactive Robustness analysis and parameters' Inference for multicriteria Sorting problems (Version 2.0) - User Manual. Technical report, Documents of INESC Coimbra.

Doumpos, M., Marinakis, Y., Marinaki, M., and Zopounidis, C. (2009). An evolutionary approach to construction of outranking models for multicriteria classification: The case of the electre tri method. *European Journal of Operational Research*, 199:496 – 505.

Doumpos, M. and Zopounidis, C. (2002). *Multicriteria Decision Aid Classification Methods*. Springer, New York.

Doumpos, M. and Zopounidis, C. (2004). Developing sorting models using preference disaggregation analysis: An experimental investigation. *European Journal of Operational Research*, 154(3):585 – 598.

Doumpos, M. and Zopounidis, C. (2010). A multicriteria decision support system for bank rating. *Decision Support Systems*, 50(1):55 – 63.

Doumpos, M. and Zopounidis, C. (2011). Preference disaggregation and statistical learning for multicriteria decision support: A review. *European Journal of Operational Research*, 209(3):203 – 214.

Duarte, B. P. and Reis, A. (2006). Developing a projects evaluation system based on multiple attribute value theory. *Computers & Operations Research*, 33(5):1488–1504.

Dubois, D., Fargier, H., Perny, P., and Prade, H. (2003). A characterization of generalized concordance rules in multicriteria decision making. *International Journal of Intelligent Systems, special issue on preference modeling application*, (18):751–774.

Dwarakanath, S. and Wallace, K. (1995). Decision-making in engineering design: Observations from design experiments. *Journal of Engineering Design*, 6(3):191–206.

Edwards, W. (1977). How to use multiattribute utility measurement for social decision making. *Systems, Man and Cybernetics, IEEE Transactions on*, 7(5):326 –340.

Farquhar, P. H. and Rao, V. R. (1976). A balance model for evaluating subsets of multiattributed items. *MANAGEMENT SCIENCE*, 22(5):528–539.

Figueira, J., Almeida-Dias, J., Matias, S., Roy, B., Carvalho, M., and Plancha, C. (2011). Electre Tri-C, a multiple criteria decision aiding sorting model applied to assisted reproduction. *International Journal of Medical Informatics*, 80(4):262 – 273.

Figueira, J., Greco, S. G., and Ehrogott, M. (2005a). *Multiple Criteria Decision Analysis:State of the Art Surveys*, volume 78 of *International Series in Operations Research & Management Science*.

Figueira, J., Mousseau, V., and Roy, B. (2005b). ELECTRE methods. In Figueira, J., Greco, S., and Ehrgott, M., editors, *Multiple Criteria Decision Analysis: State of the Art Surveys*, pages 133–162. Springer Verlag, Boston, Dordrecht, London.

Figueira, J. and Roy, B. (2002). Determining the weights of criteria in the ELECTRE type methods with a revised Simos' procedure. *European Journal of Operational Research*, 139:317–326.

Figueira, J. R., Greco, S., Roy, B., and Slowiński, R. (2010). ELECTRE methods: Main features and recent developments. In Zopounidis, C., Pardalos, P. M., Pardalos, P. M., and Hearn, D., editors, *Handbook of Multicriteria Analysis*, volume 103 of *Applied Optimization*, pages 51–89. Springer Berlin Heidelberg.

Figueira, J. R., Greco, S., and Slowiński, R. (2009). Building a set of additive value functions representing a reference preorder and intensities of preference: GRIP method. *European Journal of Operational Research*, 195(2):460 – 486.

Fishburn, P. C. (1967). Additive utilities with incomplete product sets: Application to priorities and assignments. *Operations Research*, 15(3):pp. 537–542.

Fishburn, P. C. (1970). *Utility theory for decision making*. Wiley.

Fishburn, P. C. (1975). Axioms for lexicographic preferences. *Review of Economic Studies*, 42(3):415–19.

Fontana, M., Morais, D., and de Almeida, A. (2011). A MCDM model for urban water conservation strategies adapting simos procedure for evaluating alternatives intra-criteria. In Takahashi, R., Deb, K., Wanner, E., and Greco, S., editors, *Evolutionary Multi-Criterion Optimization*, volume 6576 of *Lecture Notes in Computer Science*, pages 564–578. Springer Berlin / Heidelberg.

French, S., Simpson, L., Atherton, E., Belton, V., Dawes, R., Edwards, W., Hämäläinen, R. P., Larichev, O., Lootsma, F., Pearman, A., and Vlek, C. (1998). Problem formulation for multicriteria decision analysis: report of a workshop. *Journal of Multi-Criteria Decision Analysis*, 7(5):242–262.

Ghasemzadeh, F. and Archer, N. P. (2000). Project portfolio selection through decision support. *Decision Support Systems*, 29(1):73–88.

Golabi, K., Kirkwood, C. W., and Sicherman, A. (1981). Selecting a portfolio of solar energy projects using multiattribute preference theory. *MANAGEMENT SCIENCE*, 27(2):174–189.

Grabisch, M., Kojadinovic, I., and Meyer, P. (2008). A review of methods for capacity identification in choquet integral based multi-attribute utility theory: Applications of the kappalab r package. *European Journal of Operational Research*, 186(2):766 – 785.

Grabisch, M. and Roubens, M. (2000). Application of the Choquet integral in multicriteria decision making. In Grabisch, M., Murofushi, T., and Sugeno, M., editors, *Fuzzy Measures and Integrals - Theory and Applications*, pages 348–374. Physica Verlag.

Greco, S., Kadzinski, M., Mousseau, V., and Słowiński, R. (2011). ELECTRE-GKMS: Robust ordinal regression for outranking methods. *European Journal of Operational Research*, 214(1):118–135.

Greco, S., Matarazzo, B., and Słowiński, R. (1998). A new rough set approach to multicriteria and multiattribute classification. In Polkowski, L. and Skowron, A., editors, *Rough Sets and Current Trends in Computing*, volume 1424 of *Lecture Notes in Computer Science*, pages 60–67. Springer Berlin Heidelberg.

Greco, S., Matarazzo, B., and Słowiński, R. (1999). Rough approximation of a preference relation by dominance relations. *European Journal of Operational Research*, 117(1):63 – 83.

Greco, S., Matarazzo, B., and Słowiński, R. (2001a). Rough sets theory for multicriteria decision analysis. *European Journal of Operational Research*, 129(1):1 – 47.

Greco, S., Matarazzo, B., and Słowiński, R. (2002a). Preference representation by means of conjoint measurement and decision rule model. In Bouyssou, D., Jacquet-Lagrèze, E., Perny, P., Slwinski, R., Vanderpooten, D., and Vincke, P., editors, *Aiding Decisions with Multiple Criteria*, volume 44 of *International Series in Operations Research & Management Science*, pages 263–313. Springer US.

Greco, S., Matarazzo, B., and Słowiński, R. (2002b). Rough sets methodology for sorting problems in presence of multiple attributes and criteria. *European Journal of Operational Research*, 138(2):247 – 259.

Greco, S., Matarazzo, B., and Słowiński, R. (2004). Axiomatic characterization of a general utility function and its particular cases in terms of conjoint measurement and rough-set decision rules. *European Journal of Operational Research*, 158(2):271 – 292.

Greco, S., Matarazzo, B., and Słowiński, R. (2006). Dominance-based rough set approach to case-based reasoning. In Torra, V., Narukawa, Y., Valls, A., and Domingo-Ferrer, J., editors, *Modeling Decisions for Artificial Intelligence*, volume 3885 of *Lecture Notes in Computer Science*, pages 7–18. Springer Berlin Heidelberg.

Greco, S., Matarazzo, B., and Słowiński, R. (2008a). Dominance-based rough set approach to interactive multiobjective optimization. In Branke, J., Deb, K., Miettinen, K., and Słowiński, R., editors, *Multiobjective Optimization*, volume 5252 of *Lecture Notes in Computer Science*, pages 121–155. Springer Berlin Heidelberg.

Greco, S., Matarazzo, B., and Słowiński, R. (2010a). Dominance-based rough set approach to decision under uncertainty and time preference. *Annals of Operations Research*, 176:41–75. 10.1007/s10479-009-0566-8.

Greco, S., Matarazzo, B., Słowiński, R., and Stefanowski, J. (2001b). Variable consistency model of dominance-based rough sets approach. In Ziarko, W. and Yao, Y., editors, *Rough*

Sets and Current Trends in Computing, volume 2005 of *Lecture Notes in Computer Science*, pages 170–181. Springer Berlin / Heidelberg.

Greco, S., Mousseau, V., and Słowiński, R. (2008b). Ordinal regression revisited: Multiple criteria ranking using a set of additive value functions. *European Journal of Operational Research*, 191(2):416 – 436.

Greco, S., Słowiński, R., Figueira, J. R., and Mousseau, V. (2010b). Robust ordinal regression. In Ehrgott, M., Figueira, J. R., and Greco, S., editors, *Trends in Multiple Criteria Decision Analysis*, volume 142 of *International Series in Operations Research & Management Science*, pages 241–283. Springer US.

Greco, S., Słowiński, R., and Yao, Y. (2007). Bayesian decision theory for dominance-based rough set approach. In Yao, J., Lingras, P., Wu, W.-Z., Szczuka, M., Cercone, N., and Zak, D. S., editors, *Rough Sets and Knowledge Technology*, volume 4481 of *Lecture Notes in Computer Science*, pages 134–141. Springer Berlin Heidelberg.

Hall, N. G., Hershey, J. C., Kessler, L. G., and Stotts, R. C. (1992). A model for making project funding decisions at the national cancer institute. *Operations Research*, 40(6):1040–1052.

Hämäläinen, R. P. (2004). Reversing the perspective on the applications of decision analysis. *Decision Analysis*, 1(1):26–31.

Hamburg, M. (1970). *Statistical analysis for decision making*. New York : Harcourt, Brace & World. Inc.

Horvitz, E. J., Breese, J. S., and Henrion, M. (1988). Decision theory in expert systems and artificial intelligence. *International Journal of Approximate Reasoning*, 2(3):247 – 302.

Jacquet-Lagreze, E. and Siskos, J. (1982). Assessing a set of additive utility functions for multicriteria decision-making, the uta method. *European Journal of Operational Research*, 10(2):151 – 164.

Jacquet-Lagrèze, E. and Siskos, Y. (2001). Preference disaggregation: 20 years of MCDA experience. *European Journal of Operational Research*, 130(2):233 – 245.

Kahneman, D. and Tversky, A. (1982). The psychology of preferences. *Scientific American*, 246(1):160–173.

Keefer, D. L., Kirkwood, C. W., and Corner, J. L. (2004). Perspective on decision analysis applications, 1990-2001. *Decision Analysis*, 1(1):4–22.

Keeney, R. L. (1996). Value-focused thinking: Identifying decision opportunities and creating alternatives. *European Journal of Operational Research*, 92(3):537 – 549.

Keeney, R. L. and Raiffa, H. (1993). *Decisions with Multiple Objectives*. Cambridge University Press.

Kendall, M. G. (1938). A new measure of rank correlation. *Biometrika*, 30(1/2):pp. 81–93.

Knetsch, J. L. (1989). The endowment effect and evidence of nonreversible indifference curves. *The American Economic Review*, 79(5):pp. 1277–1284.

Korhonen, P., Moskowitz, H., and Wallenius, J. (1992). Multiple criteria decision support - a review. *European Journal of Operational Research*, 63(3):361 – 375.

Köszegi, B. and Rabin, M. (2006). A model of reference-dependent preferences. *The Quarterly Journal of Economics*, 121(4):1133–1165.

Kotlowski, W., Dembczyński, K., Greco, S., and Slowiński, R. (2008). Stochastic dominance-based rough set model for ordinal classification. *Information Sciences*, 178(21):4019 – 4037.

Kuncel, N., Hezlett, L., and Ones, D. (2001). Assigning students to academic majors. *Psychological Bulletin*, 127(1):162–181.

Le Cardinal, J., Mousseau, V., and Zheng, J. (2011). An application of constrained multicriteria sorting to student selection. In Salo, A., Keisler, J., and Morton, A., editors, *Portfolio Decision Analysis*, volume 162 of *International Series in Operations Research & Management Science*, pages 213–240. Springer New York.

Leroy, A., Mousseau, V., and Pirlot, M. (2011). Learning the parameters of a multiple criteria sorting method. In Brafman, R., Roberts, F., and Tsoukiàs, A., editors, *Algorithmic Decision Theory*, volume 6992 of *Lecture Notes in Computer Science*, pages 219–233. Springer Berlin / Heidelberg.

Leyva López, J., Dautt Sánchez, L., and Aguilera Contreras, M. (2008). A multicriteria decision support system with an evolutionary algorithm for deriving final ranking from a fuzzy outranking relation. *Operational Research*, 8:47–62. 10.1007/s12351-008-0006-6.

Leyva López, J. C. (2005). Multicriteria decision aid application to a student selection problem. *Pesquisa Operacional*, 25(1).

Liesiö, J., Mild, P., and Salo, A. (2007). Preference programming for robust portfolio modeling and project selection. *European Journal of Operational Research*, 181(3):1488 – 1505.

Liesiö, J., Mild, P., and Salo, A. (2008). Robust portfolio modeling with incomplete cost information and project interdependencies. *European Journal of Operational Research*, 190(3):679–695.

Liou, J. J. and Tzeng, G.-H. (2010). A dominance-based rough set approach to customer behavior in the airline market. *Information Sciences*, 180(11):2230 – 2238.

Luce, R. and Tukey, J. W. (1964). Simultaneous conjoint measurement: A new type of fundamental measurement. *Journal of Mathematical Psychology*, 1(1):1 – 27.

Luce, R. D. (1956). Semiorders and a theory of utility discrimination. *Econometrica*, 24(2):pp. 178–191.

Marchant, T. (1996). Valued relations aggregation with the borda method. *JOURNAL OF MULTI-CRITERIA DECISION ANALYSIS*, 5:127–132.

Marichal, J.-L. and Roubens, M. (2000). Determination of weights of interacting criteria from a reference set. *European Journal of Operational Research*, 124(3):641 – 650.

Mavrotas, G., Diakoulaki, D., and Capros, P. (2003). Combined MCDA-IP approach for project selection in the electricity market. *Annals of Operations Research*, 120:159–170.

May, K. O. (1954). Intransitivity, utility, and the aggregation of preference patterns. *Econometrica*, 22(1):pp. 1–13.

Merad, M., Verdel, T., Roy, B., and Kouniali, S. (2004). Use of multi-criteria decision-aids for risk zoning and management of large area subjected to mining-induced hazards. *Tunnelling and Underground Space Technology*, 19(2):125 – 138.

Metchebon T., S. A. (2010). *Contribution à l'aide à la déision en matière de gestion spatialisée. Etude de cas en management environnemental et développement de nouveaux outils*. PhD thesis, Faculté polytechnique, Université de Mons, Belgium.

Metchebon T., S. A., Pirlot, M., Some, B., and Yonkeu, S. (2010a). Multicriteria assesment aid for degraded landscape: a case study. In Bisdorft, R., Dias, L., Mousseau, V., and Pirlot, M., editors, *Evaluation and Decison models: applications*. Springer, Berlin.

Metchebon T., S. A., Vincent, M., and Zheng.Jun (2010b). Computing robust assignments for the electre tri optimistic rule. In et al., C. H. A., editor, *Uncertainty and Robustness in Planning and Decision Making (URPDM 2010)*. Springer, Coimbra, Portugal.

Meyer, P. (2012). Sample workflow: weighted sum. `http://www.decision-deck.org/diviz/workflow.methodWeightedSum.html`.

Meyer, P., Marichal, J., and Bisdorff, R. (2008). Disaggregation of bipolar-valued outranking relations. In Le Thi, H. A., Bouvry, P., and Pham Dinh, T., editors, *Proc. of MCO'08 conference*, pages 204–213, Metz, France. Springer.

Miyaji, I., Ohno, K., and Mine, H. (1988). Solution method for partitioning students into groups. *European Journal of Operational Research*, 33(1):82–90.

Montibeller, G., Franco, L. A., Lord, E., and Iglesias, A. (2009). Structuring resource allocation decisions: A framework for building multi-criteria portfolio models with area-grouped options. *European Journal of Operational Research*, 199(3):846 – 856.

Montibeller, G., Gummer, H., and Tumidei, D. (2006). Combining scenario planning and multi-criteria decision analysis in practice. *Journal of Multi-Criteria Decision Analysis*, 14(1-3):5–20.

Mousseau, V. and Dias, L. (2004). Valued outranking relations in ELECTRE providing manageable disaggregation procedures. *European Journal of Operational Research*, 156(2):467–482.

Mousseau, V., Dias, L., and Figueira, J. (2003a). On the notion of category size in multiple criteria sorting models. Dimacs research report 2003-02, Rutgers Univerity.

Mousseau, V., Dias, L., and Figueira, J. (2006). Dealing with inconsistent judgments in multiple criteria sorting models. *4OR: A Quarterly Journal of Operations Research*, 4:145–158.

Mousseau, V., Dias, L., Figueira, J., Gomes, C., and Clímaco, J. (2003b). Resolving inconsistencies among constraints on the parameters of an MCDA model. *European Journal of Operational Research*, 147(1):72–93.

Mousseau, V., Figueira, J., and Naux, J. (2001a). Using assignment examples to infer weights for ELECTRE TRI method: Some experimental results. *European Journal of Operational Research*, 130(2):263–275.

Mousseau, V., Roy, B., and Sommerlatt, I. (2001b). Development of a decision aiding tool for the evolution of public transport ticket pricing in the Paris region. In A. Colorni, M. P. and Roy, B., editors, *A-MCD-A Aide Multicritère à la Décision - Multiple Criteria Decision Aiding*, pages 213–230. Joint Research Center, European Commision, Luxembourg.

Mousseau, V. and Slowiński, R. (1998). Inferring an ELECTRE TRI model from assignment examples. *Journal of Global Optimization*, 12(2):157–174.

Mousseau, V., Slowiński, R., and Zielniewicz, P. (2000). A user-oriented implementation of the ELECTRE TRI method integrating preference elicitation support. *Computers & Operations Research*, 27(7-8):757–777.

Oral, M. and Kettani, O. (1989). Modelling the process of multiattribute choice. *The Journal of the Operational Research Society*, 40(3):pp. 281–291.

Ostanello, A. and Tsoukiàs, A. (1993). An explicative model of 'public' interorganizational interactions. *European Journal of Operational Research*, 70(1):67 – 82.

Öztürk, M., Tsoukiàs, A., and Vincke, P. (2005). Preference modelling. In *Multiple Criteria Decision Analysis: State of the Art Surveys*, volume 78 of *International Series in Operations Research & Management Science*, pages 27–59. Springer New York.

Parsaei, H. R., Wilhelm, M. R., and Kolli, S. S. (1993). Application of outranking methods to economic and financial justificaton of CIM systems. *Computers & Industrial Engineering*, 25:357 – 360.

Pawlak, Z. (1982). Rough sets. *International Journal of Parallel Programming*, 11:341–356.

Pawlak, Z. and Slowiński, R. (1994). Rough set approach to multi-attribute decision analysis. *European Journal of Operational Research*, 72(3):443 – 459.

Philips, L. and C.Bana e Costa (2007). Transparent prioritisation, budgeting and resource allocation with multi-criteria decision analysis and decision conferencing. *Annals of OR*, 154:51–68.

Plous, S. (1993). *The psychology of judgment and decision making*. England: Mcgraw-Hill Book Company, New York, NY.

Polyashuk, A. (2006). Formulation of portfolio selection problem with multiple criteria. *JM-CDA*, 13(2-3):135–145.

Pöyhönen, M. and Hämäläinen, R. P. (2001). On the convergence of multiattribute weighting methods. *European Journal of Operational Research*, 129(3):569 – 585.

Pöyhönen, M., Vrolijk, H., and Hämäläinen, R. P. (2001). Behavioral and procedural consequences of structural variation in value trees. *European Journal of Operational Research*, 134(1):216 – 227.

Quinlan, J. R. (1986). Induction of decision trees. *Machine Learning*, 1:81–106. 10.1007/BF00116251.

Rao, V. R., Mahajan, V., and Varaiya, N. P. (1991). A balance model for evaluating firms for acquisition. *Management Science*, 37(3):331–349.

Reeves, G. and Hickman, E. (1992). Assigning MBA students to field study project teams: A multicriteria approach. *Interfaces*, 22(5):52–58.

Rolland, A. (2008). *Procédures d'agrégation ordinale de préférences avec points de référence pour l'aide a la décision*. PhD thesis, Université Paris VI.

Rolland, A. and Zighed, D. (2011). Evaluation multicritère pour le classement et la sélection des étudiants. In *ROADEF 11*.

Ros, J. C. (2011). Introduction to Decision Deck-Diviz: Examples and user guide. Technical report, Universitat Rovira i Virgili.

Roy, B. (1985). *Méhodologie Multicritère d'Aide à la Déision*. Economica, Paris.

Roy, B. (1991). The outranking approach and the foundations of ELECTRE methods. *Theory and Decision*, 31:49–73.

Roy, B. (1993). Decision science or decision-aid science? *European Journal of Operational Research*, 66(2):184 – 203.

Roy, B. (1996). *Multicriteria Methodology for Decision Aiding*. Kluwer Academic, Dordrecht.

Roy, B. (2010). Robustness in operational research and decision aiding: A multi-faceted issue. *European Journal of Operational Research*, 200(3):629 – 638.

Roy, B. and Mousseau, V. (1996). A theoretical framework for analysing the notion of relative importance of criteria. *Journal of Multi-Criteria Decision Analysis*, 5:145–159.

Roy, B. and Vanderpooten, D. (1996). The European school of MCDA: Emergence, basic features and current works. *Journal of Multi-Criteria Decision Analysis*, 5(1):22–38.

S., F. and Roberts (1994). Limitations on conclusions using scales of measurement. In S.M. Pollock, M. R. and Barnett, A., editors, *Operations Research and The Public Sector*, volume 6 of *Handbooks in Operations Research and Management Science*, pages 621 – 671. Elsevier.

S.A. and Orlovsky (1978). Decision-making with a fuzzy preference relation. *Fuzzy Sets and Systems*, 1(3):155 – 167.

Saaty, T. (2005). The analytic hierarchy and analytic network processes for the measurement of intangible criteria and for decision making. In *Multiple Criteria Decision Analysis: State of the Art Surveys*, volume 78 of *International Series in Operations Research and Management Science*, pages 345–405. Springer New York.

Saaty, T. and Hu, G. (1998). Ranking by eigenvector versus other methods in the analytic hierarchy process. *Applied Mathematics Letters*, 11(4):121 – 125.

Saaty, T. L. (1990). How to make a decision: The analytic hierarchy process. *European Journal of Operational Research*, 48(1):9 – 26.

Saber, H. and Ghosh, J. (2001). Assigning students to academic majors. *Omega*, 29(6):513–523.

Salo, A. A. and Hämäläinen, R. P. (1997). On the measurement of preferences in the analytic hierarchy process. *Journal of Multi-Criteria Decision Analysis*, 6(6):309–319.

Samuelson, W. and Zeckhauser, R. (1988). Status quo bias in decision making. *Journal of Risk and Uncertainty*, 1:7–59.

Sen, A. (1986). Social choice theory. volume 3 of *Handbook of Mathematical Economics*, pages 1073 – 1181. Elsevier.

Simon, H. A. (1959). Theories of decision-making in economics and behavioral science. *The American Economic Review*, 49(3):253 – 283.

Simon, H. A. (1977). *The New Science of Management Decision*. Prentice Hall PTR, Upper Saddle River, NJ, USA.

Simon, H. A. (1997). *Administrative Behavior, 4th Edition*. Free Press, 4 sub edition.

Siskos, J. (1982). A way to deal with fuzzy preferences in multi-criteria decision problems. *European Journal of Operational Research*, 10(3):314 – 324.

Siskos, J. and Despotis, D. (1989). A DSS oriented method for multiobjective linear programming problems. *Decision Support Systems*, 5(1):47 – 55.

Siskos, Y., Grigoroudis, E., Krassadaki, E., and Matsatsinis, N. (2007). A multicriteria accreditation system for information technology skills and qualifications. *European Journal of Operational Research*, 182(2):867 – 885.

Siskos, Y., Grigoroudis, E., and Matsatsinis, N. (2005). UTA methods. In *Multiple Criteria Decision Analysis: State of the Art Surveys*, volume 78 of *International Series in Operations Research and Management Science*, pages 297–334. Springer New York.

Slowiński, R., Greco, S., and Matarazzo, B. (2007). Dominance-based rough set approach to reasoning about ordinal data. In Kryszkiewicz, M., Peters, J., Rybinski, H., and Skowron, A., editors, *Rough Sets and Intelligent Systems Paradigms*, volume 4585 of *Lecture Notes in Computer Science*, pages 5–11. Springer Berlin Heidelberg.

Spearman, C. (1987). The proof and measurement of association between two things. *The American Journal of Psychology*, 42:441–471.

Stevens, S. S. (1946). On the theory of scales of measurement. *Science*, 103(2684):677–680.

Tervonen, T., Figueira, J. R., Lahdelma, R., Dias, J. A., and Salminen, P. (2009). A stochastic method for robustness analysis in sorting problems. *European Journal of Operational Research*, 192(1):236 – 242.

Tsoukiàs, A. (2007). On the concept of decision aiding process: an operational perspective. *Annals of Operations Research*, 154:3–27.

Tsoukiàs, A. (2008). From decision theory to decision aiding methodology. *European Journal of Operational Research*, 187(1):138 – 161.

Tsoukiàs, A., Perny, P., and Vincke, P. (2002). From concordance/discordance to the modelling of positive and negative reasons in decision aiding. In Bouyssou, D., Jacquet-Lagrèze, E., Perny, P., Slwinski, R., Vanderpooten, D., and Vincke, P., editors, *Aiding Decisions with Multiple Criteria*, volume 44 of *International Series in Operations Research & Management Science*, pages 147–174. Springer US.

Tsoukiàs, A. and Vincke, P. (1995). A new axiomatic foundation of partial comparability. *Theory and Decision*, 39:79–114.

Tversky, A. (1969). Intransitivity of preferences. *Psychological Review*, 76:31–48.

Tversky, A. and Kahneman, D. (1991). Loss aversion in riskless choice: A reference-dependent model. *Quarterly Journal of Economics*, 106(4):1039–1061.

von Nitzsch, R. and Weber, M. (1993). The effect of attribute ranges on weights in multiattribute utility measurements. *Management Science*, 39(8):pp. 937–943.

von Winterfeldt, D. and Edwards, W. (1986). *Decision Analysis and Behavioral Research*. Cambridge University Press.

Wallenius, J., Dyer, J. S., Fishburn, P. C., Steuer, R. E., Zionts, S., and Deb, K. (2008). Multiple criteria decision making, multiattribute utility theory: Recent accomplishments and what lies ahead. *Manage. Sci.*, 54(7):1336–1349.

Weitz, R. and Jelassi, M. (1992). Assigning students to groups: A multi-criteria decision support system approach. *Decision Sciences*, 23(3).

Weymark, J. (1984). Arrow's theorem with social quasi-orderings. *Public Choice*, 42:235–246.

Williams, J. (1999). *Model building in mathematical programming*. John Wiley and sons, Chichester.

Witlox, F. and Tindemans, H. (2004). The application of rough sets analysis in activity-based modelling. opportunities and constraints. *Expert Systems with Applications*, 27(4):585 – 592.

Yeh, H. (2003). The selection of multiattribute decision making methods for scholarship student selection. *International Journal of Selection and Assessment*, 11(4):289–296.

Yu, W. (1992). *Aide multicritère à la décision dans le cadre de la problématique du tri: méthodes et applications*. PhD thesis, LAMSADE, Université Paris Dauphine, Paris.

Zheng, J., Cailloux, O., and Mousseau, V. (2011). Constrained multicriteria sorting method applied to portfolio selection. In *Proceedings of the Second international conference on Algorithmic decision theory*, ADT'11, pages 331–343, Berlin, Heidelberg. Springer-Verlag.

Zopounidis, C. and Doumpos, M. (1999a). Business failure prediction using the utadis multicriteria analysis method. *Journal of the Operational Research Society*, 50(11):1138–1148.

Zopounidis, C. and Doumpos, M. (1999b). A multicriteria decision aid methodology for sorting decision problems: The case of financial distress. *Computational Economics*, 14:197–218. 10.1023/A:1008713823812.

Zopounidis, C. and Doumpos, M. (2000). Building additive utilities for multi-group hierarchical discrimination: the mhdis method. *Optimization Methods and Software*, 14(3):219–240.

Zopounidis, C. and Doumpos, M. (2002). Multicriteria classification and sorting methods: A literature review. *European Journal of Operational Research*, 138(2):229 – 246.