Specification analysis of interest rates factors : an international perspective

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Specification Analysis of Interest Rates Factors: An International Perspective

Une analyse de la spécification des facteurs des taux d'intérêts: Une perspective internationale

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Résumé

La structure par terme des taux d'intérêt (ou courbe des taux) est la fonction qui indique, à une date donnée et pour chaque maturité résiduelle, le niveau du taux d'intérêt zéro-coupon (réputé sans risque de défaut) associé. L'évolution de la courbe des taux est considérée comme informative des anticipations de marché concernant la croissance économique et l'inflation ainsi que le niveau de risque des taux à plusieurs horizons.
La mondialisation et la déréglementation des marchés ont intensifié les liens entre les marchés internationaux des taux. Elles ont également contribué à renforcer les relations économiques entre des zones géographiques différentes. Nous avons vu alors émerger un intérêt croissant pour l'analyse des phénomènes d'interdépendance, largement définis par la Banque Mondiale comme processus général de transmission des chocs à travers les marchés.
L'apport de cette thèse concerne la modélisation de la dynamique des courbes des taux (souveraines) internationales avec prise en compte de plusieurs canaux de dépendance. La spécification de ce type d'interdépendance comporte une analyse et une sélection de facteurs pertinents qui déterminent la dynamique jointe des courbes des taux associées. La recherche d'une spécification optimale est importante car elle peut améliorer la qualité de la modélisation des taux d'intérêt de chaque région économique et donc elle peut fournir une meilleure information pour ce qui concerne la politique monétaire, l'activité économique, le niveau d'inflation futur et le risque de taux.

Le premier chapître de cette thèse consiste en une revue de la littérature sur les méthodologies les plus utilisées d'interpolation de la courbe des taux. Nous considérons les méthodes d'interpolation par spline proposées par McCulloch (1971) et McCulloch (1975) et celle par bootstrap de Fama et Bliss (1987), ainsi que les méthodes d'interpolation paramétriques de Nelson et Siegel (1987) et Svensson (1994). La première et la deuxième

classe de méthodes assurent une flexible et précise réplication des données d'obligations observées tandis que la troisième méthode fournit une estimation plus lissée de la courbe des taux d'intérêt. La forme de la courbe des taux estimée dépend de la méthode d'estimation choisie.

Deuxièmement, nous comparons les différentes techniques d'interpolation en faisant attention à leurs capacités de réplication des données (analyse "in-sample") et à leur pouvoir prédictif des taux d'intérêt futurs sur plusieurs maturités et différents horizons (analyse "out-of-sample"). Grâce à l'aide des résultats empiriques de la littérature (Bliss (1997), Waggoner (1997) et Ioannidis (2003)) nous avons pu vérifier l'efficacité des différentes méthodologies sur des bases de données d'obligations qui diffèrent en termes de marché et de période considérés.

Troisièmement, nous visons à détecter une technique d'interpolation préférable par rapport aux autres. En général, les méthodes par spline et par bootstrap produisent de bons résultats "in-sample", même si, quand on considère les tests "out-of sample", l'incertitude sur la meilleure méthode à adopter domine. Sur la base de ces résultats empiriques plutôt contrastés, nous focalisons la procédure de sélection sur les besoins spécifiques de chaque agent économique (en suivant Gurkaynac, Sack et Wright (2007)). Par exemple, un trader, concerné par l'exploitation des anomalies des prix, peut être plus intéressé par une évaluation particulièrement précise des prix des obligations. Dans ce cas-là, une méthode basée sur les splines ou la procédure bootstrap peut être le bon choix. Par contre, un macro-économiste est plus intéressé par les fondamentaux macro-économiques qui déterminent les propriétés dynamiques de la courbe sur plusieurs maturités et différents horizons. Dans ce cas-là, la sélection d'un modèle d'interpolation de Nelson et Siegel (1987) pour l'estimation des taux d'intérêt est appropriée car celle-ci est à la fois suffisamment flexible, pour autoriser une variété de formes (croissante, décroissante, bombée ou inversée) de la courbe des taux, et parcimonieuse (nombre limité de paramètres) pour éviter une instabilité temporelle de ces courbes qui impliquerait un manque de fiabilité économique.

Le deuxième chapître de cette thèse est dédié d'abord à la construction d'une nouvelle base de données des courbes des taux internationales et, ensuite, à la présentation théorique et mise en oeuvre empirique des méthodes proposées dans la littérature pour l'extraction et la sélection des facteurs "locaux" et "communs" décrivant la dynamique jointe des courbes des taux de plusieurs pays.
Tout d'abord, dans la littérature sur la structure par terme pour un seul pays (voir Litterman et Scheinkman (1991), Dai and Singleton (2000), entre autres) les variations temporelles et la forme des courbes des taux sont expliquées par trois facteurs latents. Ces facteurs influencent le niveau, la pente et la courbure de la courbe des taux (Anderson, Hammond et Ramezani (2010) et Diebold et Li (2006)). Par contre, dans un cadre international, il y a une incertitude sur la quantité (combien de facteurs) et la nature (lequel d'entre ces facteurs joue le rôle de "commun" et lequel, le rôle de "local") des facteurs capables de bien décrire la dynamique jointe des courbes des taux. Par exemple, Backus, Foresi et Telmer (2001) et Ahn (2004) font des hypothèses sur le nombre des facteurs, hypothèses qui ne sont pas justifiées empiriquement. D'autres articles comme Leippold et Wu (2007), Diebold, Li et Yue (2008) et Egorov, Li et Ng (2011), parviennent à des conclusions différentes en proposant des méthodologies d'extraction et de sélection des facteurs basées sur l'analyse en composantes principales. Cette incertitude sur la meilleure combinaison des facteurs "communs" et "locaux" peut être induite par le fait que ces analyses empiriques ont été conduites dans différents pays, sur différentes périodes et/ou en utilisant différentes méthodes pour l'interpolation des courbes des taux. La première partie de ce chapître est dédiée à la construction d'une base de données unique et homogène des courbes des taux internationales (qui concerne

quatre pays, notamment Etats-Unis, Allemagne, Angleterre et Japon) qui présente les caractéristiques suiventes: une haute fréquence (quotidienne) d'observation des taux d'intérêt estimés, une même méthode de filtrage des données brutes (exclusion des obligations avec options, des obligations non activement négociées, etc.), un même degré de liquidité (on exclut toutes les obligations qui ont été émises plus récemment car elles ont un degré de liquidité supérieure par rapport aux autres obligations en circulation (Fontaine et Garcia (2009)) et de maturité résiduelle entre différents marchés et une même méthode d'interpolation utilisée (Nelson et Siegel (1987)). Clairement, ces caractéristiques facilitent une comparaison fiable entre différentes méthodes de sélection des facteurs communs et locaux décrivant la dynamique des taux d'intérêt internationaux. Dans la deuxième partie de ce chapître nous appliquons, sur la base de données construite, les méthodes proposées par Leippold et Wu (2007), Diebold, Li et Yue (2008) et Egorov, Li et Ng (2011). Nous observons que plusieurs combinaisons des facteurs semblent être équivalentes en termes de variabilité expliquée des données. De plus, d'un point de vue méthodologique, la technique par composante principale n'est pas nécessairement capable de séparer les effets communs des effets locaux associés à un pays particulier (Perignon, Villa et Smith 2007). L'incertitude sur la meilleure combinaison des facteurs et l'impossibilité de les définir précisément comme "commun" ou "local" sont, donc, les inconvénients principaux dérivant de l'utilisation de l'analyse en composantes principales. Pour ces raisons nous concluons qu'une procédure statistique alternative doit être prise en compte.

Dans le troisième chapître de thèse nous considérons une méthode nouvelle basée sur l'utilisation d'un modèle espace-état linéaire gaussien pour l'extraction et la sélection d'une combinaison optimale des facteurs explicatifs de la dynamique jointe des courbes des taux. Les avantages de cette procédure sont nombreux. Premièrement, une estimation efficace des facteurs latents est assurée par l'utilisation du Filtre de Kalman dans la procédure de maximisation de vraisemblance. Deuxièmement, ces facteurs sont identifiés comme "locaux" (affectant seulement une courbe des taux) ou "communs" (affectant toutes les courbes des taux considérées) à travers la paramétrisation de la matrice des loadings de l'équation de mesure. Troisièmement, différentes combinaisons des facteurs "communs" et "locaux" (différentes spécifications du modèle espace-état) peuvent être comparées par des critères statistiques qui se basent sur la maximisation de la fonction de vraisemblance comme le critère d'information d'Akaike (AIC). De plus, à l'aide d'une technique de bootstrap non-paramétrique (pour le modèles espace-état) de Stoffer et Wall (1991), nous calculons le bootstrap AIC ($AICb$) proposée par Cavanaugh et Shumway (1997), qui tient compte de la persistance des taux d'intérêt.

D'un point de vue méthodologique, nous utilisons l'algorithme EM (espérance-maximisation sur les facteurs lissés) pour résoudre le problème de maximisation de la fonction de vraisemblance et on traite le problème d'identification d'une façon novatrice. En particulier, l'identification du modèle espace-état est assurée, à chaque itération de l'algorithme, par une normalisation des paramètres et des facteurs qui permet de préserver les rapports de causalité entre ces derniers (communs et locaux). Par ailleurs, l'algorithme EM est particulièrement attractif dans la mesure où il permet une maximisation de la fonction de vraisemblance plus rapide que la méthode numérique classique. Sa complexité computationnelle dépend en fait de la quantité de facteurs latents à extraire (autour d'une dizaine maximum), alors que dans le cas classique cela dépend des taux observés (environ une quarantaine). De plus, cet algorithme converge assez rapidement vers le maximum de la fonction de vraisemblance.

Plusieurs combinaisons de facteurs ont été considérées et une comparaison des résultats obtenus, en estimant différents combinaisons des pays (pour des taux en niveau et en différence), conduit à la sélection d'un modèle espace-état avec deux facteurs communs

et trois facteurs locaux corrélés. Successivement, on a pu démontrer que chaque facteur commun imite la dynamique d'un facteur local extrait dans un modèle caractérisé seulement par des facteurs locaux.
La dernière partie de ce chapître est dédié à l'étude des liens entre les facteurs extraits et les variables macro-économiques de chaque pays considéré. Nous régressons le taux de croissance de la production industrielle et le taux d'inflation futurs sur la combinaison sélectionnée des deux facteurs communs et trois facteurs locaux. Nous découvrons que la présence des facteurs communs nous aide à améliorer la prévision, sur plusieurs horizons temporels, de ces variables macro-économiques par rapport à un modèle caractérisé par quatre facteurs locaux corrélés et aucun facteur commun.

Le quatrième chapître a comme objectif l'étude des facteurs déterminant les rendements (prime de risque) primes de risque dans les marchés obligataires internationaux et leurs liens avec des variables macro-économiques comme l'inflation et la croissance économique. Dans cette dernière partie de la thèse, le modèle espace-état linéaire gaussien proposé dans le chapître précédent est adapté à l'extraction des facteurs explicatifs des anticipations des rentabilités en excès d'obligations dans chaque marché obligataire. Ces facteurs seront estimés en utilisant une procédure de maximisation de vraisemblance que nous proposons comme alternative à l'analyse en composantes principales utilisée par Cochrane et Piazzesi (2005 et 2008).
D'après la littérature, plusieurs études ont été consacrées à l'analyse de la prime de risques aux États-Unis (Cochrane et Piazzesi (2005 et 2008)) et dans d'autres économies développées (Kessler et Scherer (2009) et Sekkel (2011)). Tous ces travaux concordent sur la présence d'un seul facteur explicatif de la dynamique des rentabilités en excès dans chaque économie considérée. Ce facteur est responsable de presque toute la variabilité de la prime de risque et, en même temps, contient des informations concernant

la croissance de la production industrielle future (Cochrane et Piazzesi (2005), Koijen, Lustig et van Nieuwerburgh (2012), Dahlquist et Hasseltoft (2013)).

Cependant, Duffee (2011) a démontré que la méthode d'extraction des facteurs est cruciale pour juger le pouvoir prédictif de ces derniers. Plus précisément, Duffee (2011) a estimé un modèle Gaussien affine à cinq facteurs pour la structure par terme des taux d'intérêt à travers la méthode de la maximisation de la vraisemblance (par filtre de Kalman) et a découvert un facteur utile à la prévision de variables financières comme le taux court et les rendements obligataires même si la contribution de ce facteur à expliquer la variabilité des taux est faible.

Dans le même esprit, dans l'analyse de la prime de risque international, des facteurs considérés comme négligeables par la méthode classique de maximisation de la variance expliquée des données peuvent être importants pour la prévision de variables macro-économiques et financières. On les appelle "facteurs cachés" dès qu'ils ne sont pas visibles à travers une analyse en composantes principales des rentabilités en excès des obligations. Dans ce chapître nous explorons ces facteurs en trois étapes.

Premièrement, nous sélectionnons les facteurs explicatifs de la prime de risque des différents pays. Cet objectif a abouti à la comparaison des plusieurs spécifications (nombre des facteurs) du modèle espace-état gaussien. Nous considérons un nombre de facteurs allant de un (comme suggérée par Cochrane et Piazzesi (2005) à cinq comme suggéré dans la littérature de la courbe des taux par Adrian, Crump et Moench (2013) et Duffee (2011)).

Nous observons que, contrairement à une spécification à un seul facteur conseillé par la littérature, un modèle à cinq facteurs pour les anticipations des rentabilités en excès est préférable.

Deuxièmement, nous évaluons leur pouvoir prédictif des variables financières comme les rendements obligataires ainsi que des variables macro-économiques comme le taux de croissance de la production industrielle et le taux d'inflation. Les résultats de cette analyse empirique démontrent que ces facteurs contribuent à la prévision de ces deux variables macro-économiques pour des horizons temporels de un à trois ans. Leur pouvoir prédictif est supérieur à le facteur détecté par Cochrane et Piazzesi (2005 et 2008) et les facteurs de la courbes des taux (les facteurs niveau et pente).

Troisièmement, nous avons pu vérifier que les résultats obtenus sont confirmés pour des bases de données qui diffèrent en termes de marché, de période considérées et/ou de technique d'interpolation choisie pour leur construction (Wright (2009) et Fama et Bliss (1987)).

CHAPTER 1

Comparative Study of Term Structure Estimation Techniques: *A Literature Review*

Abstract

The purpose of this chapter is to present and compare different approaches generally proposed in the economic literature to estimate the term structure of interest rates of (non-defaultable) Treasury bonds. We classify estimation methods in three main categories: bootstrapping methods, spline-based methods, and parametric/parsimonious methods. The first and second class provide a flexible and accurate fitting of the observed bond data while the third one allows, with a limited number of parameters, a smoothed representation of the term structure. The use of different methods may imply different shapes of the estimated curve and thus, different goodness-of-fit performances. According to the literature (Bliss (1997)), accurate in-sample replication of the data may lead to over-fitting problems with associated poor out-of-sample forecast performances. Taking into consideration the purpose for which the yield curve is constructed, we suggest which method is most adapted to satisfy researchers' needs. In order to do that, we consider two possible (not competitive) ways for the selection. First, we compare the empirical performances of these fitting techniques (see Bliss (1997), Waggoner (1997) and Ioannides (2003)) and then, we take into consideration the purpose for which the term structure is constructed. More specifically, in line with Gurkaynak

et al. (2007), we look at two economic agents with different preferences and we try to find the estimation methodology which best fit their needs. The economic agents are a trader and a macroeconomist. The first is more interested in a precise bond pricing, while the latter is more concerned about the macroeconomic fundamentals explaining the yield curve variations over time and maturities.

Empirical comparisons in the literature lead to contrasting results. In-sample empirical analysis are in favor of spline and bootstrapping methods even though concerns about over-fitting have to be properly considered. Conversely, out-of sample tests do not select a preferred method for the U.S. market, while, in the U.K., parametric/parsimonious methods such as the Svensson (1994) methodology seems to be the best performing ones.

Résumé

Le but de ce chapître est de présenter et comparer les méthodologies les plus utilisées d'interpolation de la structure par terme (réputé sans risque de défaut) des taux d'État. Nous classifions ces méthodes en trois catégories: les méthodes d'interpolation par bootstrap, par spline et les méthodes paramétriques.

La première et la deuxième classe des méthodes assurent une flexible et précise réplication des données d'obligations observées tandis que la troisième méthode fournit une estimation plus lissée de la courbe des taux d'intérêt. Donc, la forme de la courbe des taux estimée dépend de la méthode d'estimation choisie. Une précise réplication des données "in-sample" peut conduire à une faible capacité prédictive des taux d'intérêt sur plusieurs maturités et différents horizons (analyse "out-of-sample"). Grâce à l'aide des résultats empiriques que nous trouvons dans la littérature (Bliss (1997), Waggoner (1987) and Ioannides (2003)), nous observons que les méthodes par spline et par bootstrap produisent des bons résultats " in-sample ", même si, quand on considère les tests " out-of sample ", l'incertitude sur la meilleure méthode à adopter, domine.

Sur la base de ces résultats empiriques plutôt contrastés, on se concentre sur les besoins spécifiques de chaque agent économique (nous suivons Gurkaynak, Sack et Wright (2007)). Par exemple, un trader, concerné par l'exploitation des anomalies des prix, peut être plus intéressé par une évaluation particulièrement précise des prix des obligations. Dans ce cas-là, une méthode basée sur le spline ou bootstrap peut être le bon choix. Par contre, un macro-économiste est plus intéressé par les fondamentaux macro-économiques qui déterminent les propriétés dynamiques de la courbe des taux à différents horizons. En d'autres termes, il se concentre sur une méthode suffisamment flexible pour autoriser une variété de formes : croissante, décroissante, bombée ou inversée. Dans ce cas-là l'utilisation d'une méthode paramétrique est conseillée.

1.1 Introduction

The term structure of interest rates describes the relationship between (non-defaultable) zero-coupon bond interest rates and their time to maturity. Spot rates for different maturities are usually considered for its representation, and we refer in that case to the so called zero-coupon yield curve or spot curve. Its study is of fundamental importance for at least two main reasons. First, it provides important monetary policy information about the future economic activity, inflation and market risk premia. Second, the pricing of several (interest rate) derivative products depends on zero-coupon bond prices (e.g. forward and futures).

Zero-coupon bond prices are typically not available for all maturities of interest and they are generally issued with a time to maturity of less than one year. Useful information about the spot curve can be inferred from market prices of coupon bonds, exploiting the no-arbitrage representation of a coupon bond price as a portfolio of zero-coupon bonds and given their availability over a much wider range of (short and long) maturities. Here, numerical methods are used to extract zero-coupon rates (bootstrapping methods). Nevertheless, at any point in time the number of coupon bonds is not systematically able to cover the entire maturity spectrum and, consequently, statistical methods and interpolation techniques are used to estimate the entire term structure of interest rates (spline-based and parametric methods).

Both the bootstrapping methodology (introduced by Fama and Bliss (1987)) and the spline-based one (see McCulloch (1971), McCulloch (1975)) among others) provide a flexible and accurate fitting of the observed bond data, while the parametric technique (see Nelson and Siegel (1987) and Svensson (1994) among others) allows a smoothed representation of the term structure with a limited number of parameters.

All these interpolation techniques work on one of three equivalent forms by which the term structure of interest rate can be represented: spot curve, forward curve or discount curve. Bootstrapping methods are based on the sequential extraction of discount factors or forward rates from bond prices (see Fama and Bliss (1987)) while spline-based methods typically work on the discount function, and in some cases the forward curve (see Bliss (1997) and Waggoner (1997)), by means of a piecewise polynomial function. Parametric methods are based on a parsimonious functional specification of the forward curve over the entire maturity domain.

The use of different methods may imply different shapes of the estimated curve and thus, different goodness-of-fit performances. According to the literature (Bliss (1997)), accurate in-sample replication of the data may lead to over-fitting problems with associated poor out-of-sample forecast performances.

The purpose of this chapter is to present and compare different estimation techniques for the term structure of interest rates and to select one methodology, among the possible ones, which best fit researchers' needs. In order to do that, we consider two possible (not competitive) ways for the selection. First, we compare the empirical performance of these fitting techniques proposed in the literature (see Bliss (1997), Waggoner (1997) and Ioannides (2003)) and then, we take the purpose for which the term structure is constructed into consideration. More specifically, in line with Gurkaynak et al. (2007), we look at two economic agents with different preferences and try to find the estimation methodology which best fit their needs. The economic agents are a trader and a macroeconomist. The first is more interested in a precise bond pricing, while the latter is more concerned about the macroeconomic fundamentals explaining the yield curve variations over time and maturities.

The review will be structured as follows. In Section 1.2 we fix the notation and introduce the main concepts regarding the term structure estimation: we discuss the yield to maturity concept and the functional relationship linking spot rates, forward rates and the discount curve. In Section 1.3 we study the term structure estimation as a two dimensional problem by considering, first, the pricing function linking coupon bond and zero-coupon bond prices and, second, by specifying the functional form approximating either the discount function or the forward curve. In Section 1.4 we provide a review of bootstrapping, spline-based and parametric methods, while in Section 1.5 we compare these different methodologies by reviewing empirical results in the literature and considering the two above mentioned economic agents. Section 1.6 concludes.

1.2 Main Concepts and Notation

1.2.1 Yield to Maturity and Discount Factor

Let us consider at date t a coupon bond j, where $j \in \{1, \ldots, M\}$, with maturity date T_j, market price $CB_j(t, T_j)$ (say) and payments $C_{1,j}, \ldots, C_{n_j,j}$ at date $t_{1,j}, \ldots, t_{n_j,j}$ respectively, with $t_{n_j,j} = T_j$. The continuously compounded yield to maturity (YTM, hereafter) is the date t value $R^{CB}(t, T_j)$ (also called *gross redemption yield*) such that:

$$CB_j(t, T_j) = \sum_{i=1}^{n_j} C_{i,j} \exp[(-(t_{i,j} - t)R^{CB}(t, T_j))]. \tag{1.1}$$

The total number of cash flows is n_j and it consists of the coupon payments and the repayment of the face value at maturity. In other words, the YTM is the discount rate making the coupon bond price equal to its stream of discounted future cash flows and it is usually found by means of numerical methods (for instance, Newton-Raphson algorithm). If we consider a unitary face value zero-coupon bond maturing at date T and with price $B(t, T)$ (say) at time t, then its continuously compounded YTM is the value $R(t, T)$ such that

$$B(t, T) = \exp[-(T - t)R(t, T)], \tag{1.2}$$

and therefore we have

$$R(t, T) = -\frac{1}{T - t} \ln B(t, T). \tag{1.3}$$

In this case, the continuously compounded YTM is also called the continuously compounded zero-coupon yield, or zero-coupon rate, or spot rate for the date T. The collection of the spot rates over different maturities determines the (continuously compounded) zero-coupon yield curve or spot curve. The amount $B(t, T)$ provides the value at date t of one unit of money at date T, and for that reason it is also called discount factor.

Unfortunately, the yield curve construction faces two important problems. First, zero-coupon bonds are mostly available for short maturities (in general, less than 12 months) while for longer ones we should relay on coupon-bearing bonds. Here, the collection of YTMs on coupon bearing bonds does not immediately provide the zero-coupon yield curve given that its YTM is a complex average of spot rates[1].

Second, the number of bonds at any point in time is not sufficiently large to systematically cover the entire maturity spectrum. For instance, in the U.S. Treasury bond market, coupon bonds are commonly issued with maturities of 2, 3, 5, 7 and 10 years for Treasury Notes (T-Notes) and for 20 and 30 years for Treasury Bonds[2].

[1]Any coupon bearing bond can be viewed as a portfolio of zero-coupon bonds with maturities going from the time of the first payment to the time of the final repayment of the principal. Thus, its YTM is a complex average of spot rates.

[2]Auction frequency is monthly for coupon bonds with maturities ranging from 2 to 7 years, while for maturities ranging from 10 to 30 years, the original issue is on February, May, August and November with reopening auctions the other eight months.

For these reasons, the use of estimation techniques is required. They allow to extract the zero-coupon yield curve over the entire maturity domain from coupon bearing bonds. The estimation of term structure of interest rates is based on the functional relationship existing between spot rates, forward rates, discount function and time to maturity. In the following section we introduce these concepts, define the associated notation and the relationships among them. This is of fundamental importance in Section 1.3 and 1.4 where we present how to estimate the yield curve. We also assume that all rates are continuously compounded.

1.2.2 Spot rates, forward rates and discount functions

Let us denote with $R(t,T)$ the spot rate at time t with maturity at time T. With $h = T - t$ we denote the time-to-maturity. The spot rate at date t, considered as a function of the residual maturity h, is called the spot curve, yield curve or term structure of interest rates and it is denoted by $R(t, t+h)$. The discount factor at date t, defined as a function of h, is called discount function or discount curve and it is denoted by $B(t, t+h)$.

The relationship that links the discount function $B(t, t+h)$ and the spot rate $R(t, t+h)$ is the following:

$$B(t, t+h) = \exp[-hR(t, t+h)], \tag{1.4}$$

which implies $R(t, t+h) = -\frac{1}{h} \log B(t, t+h)$.

Finally, we define the forward rate $f(t, t+h', t+h)$ as the rate of return for an investment contracted at time t (trading date), beginning at time t' (settlement date) and ending at time T (maturity date), with $T > t' > t$, $h' = t' - t$ and $h = T - t$. The collection of all forward rates determines the forward curve and they are linked to spot rates in the following way:

$$f(t, t+h', t+h) = \frac{R(t, t+h)h - R(t, t+h')h'}{(h-h')}. \tag{1.5}$$

The instantaneous forward rate is the forward rate for an infinitesimal investment period after the settlement date:

$$f(t, t+h') = \lim_{h \to h'} f(t, t+h', t+h). \tag{1.6}$$

Intuitively, we interpret the instantaneous forward rate as the marginal increment of the rate of return if we marginally increase the length of our investment horizon.

Hence, the forward rate in (1.5) can be seen as the average of instantaneous forward rates with settlements between t' and T, as follows:

$$f(t, t+h', t+h) = \frac{1}{(h-h')} \int_{h'}^{h} f(t, t+x)dx. \tag{1.7}$$

Moreover, the average of all the instantaneous forward rates, with settlements between t and T, provides the spot rate at time t with maturity T:

$$R(t, t+h) = \frac{1}{h} \int_{0}^{h} f(t, t+x)dx. \tag{1.8}$$

The forward rate can also be expressed in terms of the spot rate by differentiating (1.8) with respect to h:

$$f(t, t+h) = R(t, t+h) + h\frac{\partial R(t, t+h)}{\partial h}, \tag{1.9}$$

and $f(t, t+h) - R(t, t+h)$ provides some information about the slope of the yield curve.

If we consider (1.4) and (1.8), it is clear that spot rate, forward rate and discount function are related in the following way:

$$B(t, t+h) = \exp[-hR(t, t+h)] = \exp\left(-\int_0^h f(t, t+x)dx\right). \tag{1.10}$$

This is clearly an important result: once we approximate, for example, the forward curve, the other two term structure representations (discount function and spot curve) are automatically obtained.

1.3 Term Structure Estimation

The term structure estimation issue can be tackled in general in two steps. First, we introduce the function that links the market price of a generic coupon bond $CB_j(t, T_j)$ (say) with its stream of future cash flows and the associate discount functions. This is the so called pricing function step. Second, we choose the function approximating $B(t, t+h)$ (or the forward curve) over the maturity spectrum of interest: the so called functional form step.

1.3.1 The Pricing Function Step

In the absence of the arbitrage opportunity principle, the relation linking the coupon bond price, coupons and zero-coupon bond prices at date t is:

$$CB_j(t, t+H_j) = \sum_{i=1}^{n_j} C_{i,j} B(t, t+h_{i,j}), \tag{1.11}$$

where $CB_j(t, t + H_j)$ is the observed price at date t of bond j, with $j \in \{1, \ldots, M\}$ and M is the number of coupon bonds traded in the market at date t. We denote with $h_{1,j}, h_{2,j}, \ldots, h_{n_j,j} = H_j$ the residual maturity between the date t and each of the payment dates of bond j. $B(t, t+h_{i,j})$ is the date t zero-coupon bond price with residual maturity $h_{i,j}$, $C_{i,j}$ is the coupon payment of bond j at date $t + h_{i,j}$ and n_j denotes its total number of cash flows.

More generally, if we consider relation (1.10), the pricing function (1.11) can be rewritten as follows:

$$CB_j(t, t + H_j) = \sum_{i=1}^{n_j} C_{i,j} B(t, t + h_{i,j}) = \sum_{i=1}^{n_j} C_{i,j} \exp[-h_{i,j} R(t, t + h_{i,j})] = \sum_{i=1}^{n_j} C_{i,j} \exp\left(- \int_0^{h_{i,j}} f(t, t + x) dx \right). \quad (1.12)$$

If this relation does not hold, the above mentioned bond market conditions are not always satisfied, and there will be arbitrage opportunities in the market. Under certain bond market conditions, that we will present in Section 1.4.2, relations (1.11) or (1.12) are exploited by bootstrapping methods to extract the term structure of interest rates from coupon bond prices. In practice, prices are never simultaneously quoted and we usually observe bid and ask spreads. Hence, we allow for error terms as follows:

$$CB_j(t, t + H_j) = \sum_{i=1}^{n_j} C_{i,j} B(t, t + h_{i,j}) + \varepsilon_t^{(j)}, \quad (1.13)$$

with $\varepsilon_t^{(j)} \sim IIN(0, \sigma^2)$ for all j, and that $Cov(\varepsilon_t^{(j)}, \varepsilon_{t+k}^{(j')}) = 0$ for all $j \neq j'$ and $k \neq 0$.

Here, the term structure estimation issue is tackled by providing and estimating a functional form for the discount function $B(t, t+h)$ or the forward curve $f(t, t+h)$. This strategy, followed by spline-based and parametric methods (see Sections 1.4.3 and 1.4.4) combines (1.13) with (1.10) and thus works with one of the following alternative regressions:

$$\begin{aligned} CB_j(t, t+H_j) = \sum_{i=1}^{n_j} C_{i,j} B(t, t+h_{i,j}) + \varepsilon_t^{(j)} &= \sum_{i=1}^{n_j} C_{i,j} \exp[-h_{i,j} R(t, t+h_{i,j})] + \varepsilon_t^{(j)} \\ &= \sum_{i=1}^{n_j} C_{i,j} \exp\left(- \int_0^{h_{i,j}} f(t, t+x) dx \right) + \varepsilon_t^{(j)}. \end{aligned} \tag{1.14}$$

1.3.2 The Approximation Function Step

This step consists of deciding the functional form approximating either the discount function or the forward curve. The relevant literature proposes two main approaches. First, a spline representation for the discount function (see McCulloch (1975) among others) or the forward curve (see Fisher, Nychka, and Zervos (1995) and Waggoner (1997)) and second, a parametric form for the forward curve (see Nelson and Siegel (1987) and Svensson (1994) among others). Therefore, polynomials and exponential functions of the time-to-maturity h (or some combinations) are typically used in practice.

Once the approximation function is selected, its parameter values have to be estimated. Two methodologies are typically followed. First, we estimate parameter values by minimizing bond pricing errors. Second, the estimation can be done by minimizing yields errors.

In the first case we have to properly consider the heteroskedasticity of pricing errors[3], while in the second case computational complexity and required estimation time make the difference. Following Bliss (1997), we adopt the first methodology. Each bond pricing error is weighted by the inverse of the bond duration. More formally, let us denote the pricing error function as

$$g = \sum_{j=1}^{M} w_j (\varepsilon_t^{(j)})^2, \tag{1.15}$$

where $(\varepsilon_t^{(j)})^2 = (\widehat{CB}_j(t, t+H_j) - CB_j(t, t+H_j))^2$. The weights are $w_j = \frac{1/D_j}{\sum_{i=1}^{M} 1/D_i}$ where is D_j is the Macaulay duration (see Appendix 1.A). $\widehat{CB}_j(t, t+H_j)$ is the theoretical (fitted) price of the coupon bond j obtained from the approximation of the discount function $\hat{B}(t, t+h)$ or the forward curve $\hat{f}(t, t+h)$. Thanks to the relationship (1.10), the spot curve $\hat{R}(t, t+h)$ is automatically obtained. Our goal consists in minimizing one of these three equivalent representations of g:

$$\begin{aligned}
& \sum_{j=1}^{M} w_j \left[\sum_{i=1}^{n_j} C_{i,j} \hat{B}(t, t+h_{i,j}) - CB_j(t, t+H_j) \right]^2 \\
= & \sum_{j=1}^{M} w_j \left[\sum_{i=1}^{n_j} C_{i,j} \exp(-h_{i,j} \hat{R}(t, t+h_{i,j})) - CB_j(t, t+H_j) \right]^2 \\
= & \sum_{j=1}^{M} w_j \left[\sum_{i=1}^{n_j} C_{i,j} \exp\left(- \int_0^{h_{i,j}} \hat{f}(t, t+x) dx \right) - CB_j(t, t+H_j) \right]^2 .
\end{aligned} \tag{1.16}$$

[3]This problem is due to non linear relationship between price and yield and it is related with the concept of duration. A given change in yields leads to a smaller change in price for bonds at shorter maturities than for bonds at longer ones. If we consider unweighted price errors we can over-fit long-end bond prices and obtain less accurate fit at the short-end.

Moreover, the choice of one estimation method implies the selection of a numerical algorithm for the estimation of model parameters. Weighted least squares are applied by McCulloch (1975), maximum likelihood by Litzenberger and Rolfo (1984) among others. Related decisions are: handling the bid/ask spread[4], the error weighing function and the specification of possible filters to rule out erroneous data from the data set.

1.4 Term Structure Estimation Techniques

We can divide the yield curve estimation techniques in three main categories: bootstrapping, spline-based and parametric methods.

The first class of methods was mostly introduced by Fama and Bliss (1987). In particular, they use the so-called "Unsmoothed Fama-Bliss" method for sequential extraction of forward rates from bond prices with successively longer maturities. The second class of methods was firstly introduced by McCulloch (1971) and McCulloch (1975) and it is based on the spline representation of the discount function. Successively, Fisher, Nychka, and Zervos (1995) and Waggoner (1997) use the spline representation for the extraction of the forward curve. The third class of methods was proposed by Nelson and Siegel (1987) and successively extended by Svensson (1994). It is based on the parsimonious representation of the forward curve with a family of exponential polynomials.

1.4.1 Criteria for choosing an estimation technique

Three main features characterize the yield curve estimation methods: goodness of fit (flexibility), smoothness of the estimated yield curve and stability of the results.

[4]A pricing error is non-zero only if the fitted price lies outside the bid-ask spread (Bliss (1997)).

Firstly, an adequate level of goodness of fit (flexibility) guarantees to capture the movements in the term structure and, at the same time, a sufficient precision of price estimates. Flexibility becomes a crucial feature when yield curve estimation is used for pricing purposes like the identification of mis-priced securities.

Instead, if the main purpose is to study the general dynamic properties of the yield curve, more smoothness can be preferred. This can be useful for monetary policy objectives such as the extraction of market expectation regarding inflation, economic activity, real and nominal interest rates as well as real and nominal term premia.

Lastly, yield curve estimation methods should be stable. Small changes in the data at one maturity should have a marginal impact at other maturities. Of course, if we remove, change or add new securities in the data set the effect on the yield curve shape has to be not excessive.

The selection of a specific technique implies to choose the adequate trade-off between flexibility, smoothness and stability according with the purpose of the research. In the following sections we will review the literature about the most common estimation methods stressing their advantages and limits.

1.4.2 Bootstrapping Methods: The Unsmoothed and Smoothed Fama and Bliss Methodologies

Bootsrapping the Yield curve

Let us consider, at time t, a zero-coupon bond with a market price of $CB_1(t, t+H_1)$ and a basket of coupon bonds with market prices of $CB_2(t, t+H_2), \ldots, CB_j(t, t+H_j), \ldots,$ $CB_M(t, t+H_M)$, that are ordered from the shortest to the longest maturity date, namely $t+H_1 < t+H_2 < \cdots < t+H_j < \cdots < t+H_M$ with $H_1 \leq 1$ year and $H_j > 1$ year for

$j \neq 1$.

Let us assume that coupon payments for any two different coupon bonds (j and j') are occurring at the same dates, namely $h_{i,j} = h_{i,j'}$ for $j \neq j'$, with $j \in \{1, \ldots, M\}$ and $h_{i,j} \in \{h_{1,j}, \ldots, h_{n_j,j}\}$. M denotes the total number of coupon bonds traded in the market at date t and n_j denotes the total number of cash flows for the bond j. The bootstrapping procedure is as follows.

First, we consider the bond with the shortest residual maturity $H_1 = h_{1,1} \leq 1$ year, namely the zero-coupon bond $CB_1(t, t + H_1)$ and we search for $B(t, t + H_1)$ from the following pricing equation:

$$CB_1(t, t + H_1) = C_{1,1}B(t, t + H_1), \tag{1.17}$$

where $C_{1,1}$ is the face value of the zero-coupon bond. From the relation (3.21) we can directly infer the discount factor $B(t, t + H_1) = \frac{CB_1(t,t+H_1)}{C_{1,1}}$.

Successively, we consider the bond $CB_2(t, t + H_2)$ paying $C_{1,2}$ at $t + H_1$ and $C_{2,2}$ at the maturity date $t + H_2$. Here we search for $B(t, t + H_2)$, given $B(t, t + H_1)$, from the relationship:

$$CB_2(t, t + H_2) = C_{1,2}B(t, t + H_1) + C_{2,2}B(t, t + H_2). \tag{1.18}$$

For a generic coupon bond $CB_j(t, t + H_j)$ paying $C_{1,j}$ at $t + H_1$, $C_{2,j}$ at $t + H_2$, ..., $C_{j,j}$ at $t + H_j$ the pricing equation will be as follows:

$$CB_j(t, t+H_j) = C_{1,j}B(t, t+H_1)+C_{2,j}B(t, t+H_2)+\cdots+C_{j-1,j}B(t, t+H_{j-1})+C_{j,j}B(t, t+H_j), \tag{1.19}$$

where $B(t, t+H_1), B(t, t+H_2), \ldots, B(t, t+H_{j-1})$ have been recursively extracted in the previous $(j-1)$ steps. We proceed in this manner until we reach the longest maturity date $t+H_M$. Here we have assumed that bonds are characterized by regularly increasing maturities with coupon payments at each period and occurring at the same dates.

However, bootstrap methods can be applied when the maturity dates of the M bonds are not all different and regularly increasing even if we still assume they have at most M different payment dates. We can organize all the cash flows $C_{i,j}$ in a square matrix $\mathbf{C} = (C_{i,j})$ with dimension $(M \times M)$. Coupon bond and zero-coupon bond prices can be stored in the vectors $\mathbf{P} = [CB(t, t+H_1), CB(t, t+H_2), \ldots, CB(t, t+H_M)]'$ and $\mathbf{B} = [B(t, t+H_1), B(t, t+H_2), \ldots, B(t, t+H_M)]'$. Using the pricing relationship in (4.1) we can write

$$\mathbf{P} = \mathbf{C} \times \mathbf{B}. \tag{1.20}$$

If the bonds payments are such that $\mathbf{C}$ is non-singular (at least one payment per date), then a unique solution for $\mathbf{B}$ exists, namely $\mathbf{B} = \mathbf{C}^{-1}\mathbf{P}$. The advantage of this procedure is that it provides an exact in-sample bond pricing. On the other hand, it has several drawbacks.

First, it does not provide smooth (forward and spot) curves. In fact, it allows the extraction of only a finite number of zero-coupon prices for a finite number of maturities and needs the use of linear interpolation for determining the discount factors at intermediate maturities. Second, we can not extract a yield $R(t, t+k)$ when $k > H_M$.

Third, we need one payment date at each period and identical payment dates for all coupon bonds considered or, more generally, the square matrix $\mathbf{C}$ has to be non-singular.

Lastly, we should take into consideration that the number of coupon bonds is typically smaller than the number of payments dates ($M < n_j$). In such a case the system (1.20) allows for multiple solutions, thus there exists a multiplicity of discount factors that, under the absence of arbitrage opportunity principle, are compatible with the observed prices.

The Unsmoothed and Smoothed Fama and Bliss Methodologies

A commonly used boostrapping technique is the one provided by Fama and Bliss (1987), the so-called Unsmoothed Fama and Bliss method.

As in the previous subsection, at time t we have a zero-coupon bond and a basket of coupon bonds that are ordered from the the shortest to the longest residual maturity.

Here we do not require to have identical payment dates for all the coupon bonds. In fact, we assume that the forward curve is constant between two successive coupon bond maturities so that the constant date t forward rate for the generic maturity interval $(t + H_{j-1}, t + H_j]$ is denoted by $F^{(j)}$. In such a way, all the cash flows falling in that interval, even if they have different payment dates, will be discounted at the same rate. The bootstrapping procedure is as follows.

First, we consider the zero-coupon bond with residual maturity $H_1 = h_{1,1} \leq 1$ year and we assume that in the time interval $(t, t + H_1]$ the forward (spot) rate is $F^{(1)}$. We search for $F^{(1)}$ from the following pricing equation:

$$CB_1(t, t + H_1) = C_{1,1} \exp\{-F^{(1)} \times H_1\}, \tag{1.21}$$

where $C_{1,1}$ is the zero-coupon bond face value. Successively, for the time interval $(t +$

$H_1, t + H_2]$, we search for $F^{(2)}$ in the following pricing equation:

$$CB_2(t, t+H_2) = \sum_{i=1}^{n_2^{(1)}} C_{i,2} \exp\{-F^{(1)} \times h_{i,2}\} + \sum_{i=n_2^{(1)}+1}^{n_2^{(2)}} C_{i,2} \exp\{-[F^{(1)} \times H_1] - [F^{(2)} \times (h_{i,2} - H_1)]\}, \tag{1.22}$$

where $n_2^{(1)}$ denotes the number of coupon payments that are in the maturity interval $(t, t + H_1]$, while $n_2^{(2)}$ denotes the number of coupon payments that are in the maturity interval $(t+H_1, t+H_2]$. Hence, $n_2^{(1)} + n_2^{(2)} = n_2$. For a generic time interval $(t+H_{j-1}, t+H_j]$ we search for $F^{(j)}$ in the following pricing equation:

$$CB_j(t, t + H_j) = \sum_{i=1}^{n_j^{(1)}} C_{i,j} \exp\{F^{(1)} \times h_{i,j}\} + \cdots + \sum_{i=n_j^{(j-1)}+1}^{n_j^{(j)}} C_{i,j} \exp\{[-F^{(1)} \times H_1] - [F^{(2)} \times H_2] - \cdots - [F^{(j-1)} \times H_{j-1}] - [F^{(j)} \times (h_{i,j} - H_{j-1})]\}, \tag{1.23}$$

with $n_j^{(1)} + n_j^{(2)} + \cdots + n_j^{(j-1)} + n_j^{(j)} = n_j$ and $F^{(1)}, F^{(2)}, \ldots, F^{(j-1)}$ have been recursively extracted in the previous stages. We proceed in this manner until we consider the last maturity interval, namely $(t + H_{M-1}, t + H_M]$.

A possible alternative is the "Smoothed Fama-Bliss" method. This is a two steps process. First, we should estimate discount factors via "unsmoothed" procedure. Second, we have to use a parametric method, such as the Nelson and Siegel or the Svensson methodologies (see Section 1.4.4), to fit a curve through the previously extracted discount factors.

1.4.3 Spline-based Methods

Cubic splines

Spline methods were firstly proposed by McCulloch (1971) and McCulloch (1975). He suggested to model the discount function by using a piecewise polynomial, the so called spline function, made up of separate polynomial sections that are joined at knot points.

Let us define first, the spline approximation of the discount function $B(t, t+h)$ with $h > 0$. Let us imagine to observe, at date t, M bonds with maturity dates $T_1 \leq T_2 \leq \cdots \leq T_M$. Furthermore, we divide the maturity interval into sub-intervals defined by the knot points $0 = h_0 < h_1 < \cdots < h_k = T_M - t$. The discount function $B(t, t+h)$ is approximated by a linear combination of basis functions $G_l(h)$[5]:

$$B(t, t+h) = \sum_{l=0}^{k-1} G_l(h) \mathbf{1}_l(h), \tag{1.24}$$

where, for each $l \in \{0, \ldots, k-1\}$, $\mathbf{1}_l(h)$ is the following step function:

$$\mathbf{1}_l(h) = \begin{cases} 1 & \text{if } \ h \geq h_l \\ 0 & \text{otherwise.} \end{cases} \tag{1.25}$$

In other words, we have:

[5]For more details see the appendix of McCulloch (1975).

$$
\begin{aligned}
B(t,t+h) &= G_0(h) && \text{for } h \in [h_0, h_1) \\
&= G_0(h) + G_1(h) && \text{for } h \in [h_1, h_2) \\
&= G_0(h) + G_1(h) + \cdots + G_l(h) && \text{for } h \in [h_l, h_{l+1}) \\
& \quad \textit{etc.}
\end{aligned}
\tag{1.26}
$$

It is required that $G_l(h)$'s functions to be continuous and twice differentiable to ensure a smooth transition in the knot points h_l. Basis functions can be represented by using polynomials. For example, McCulloch (1971) and McCulloch (1975) has introduced quadratic and cubic splines. In the first case, McCulloch uses a second-order polynomial approximation while in the latter he allowed for an extra cubic term. Here, we will show the more general case of cubic spline. The basis functions are represented as follows:

$$
G_l(h) = \alpha_l + \beta_l(h - h_l) + \gamma_l(h - h_l)^2 + \delta_l(h - h_l)^3, \tag{1.27}
$$

with $l \in \{0, \ldots, k-1\}$ and where α_l, β_l, γ_l and δ_l are constant. Combining (1.26) and (1.27) and imposing the necessary conditions on parameters to allow for smoothness and continuity around each knot point[6], we obtain a cubic-spline representation of the

[6] To guarantee a smooth transition from the first to the second subinterval, that is at the knot $h = h_1$, we impose:

- $\lim_{h \to h_1^-} B(t,t+h) = \lim_{h \to h_1^+} B(t,t+h) = B(t,t+h)$
- $\lim_{h \to h_1^-} B'(t,t+h) = \lim_{h \to h_1^+} B'(t,t+h) < \infty$
- $\lim_{h \to h_1^-} B''(t,t+h) = \lim_{h \to h_1^+} B''(t,t+h) < \infty$.

discount function as follows[7]:

$$B(t,t+h) = (1+\beta_0 h + \gamma_0 h^2 + \delta_0 h^3) + \sum_{l=1}^{k-1} \delta_l (h-h_l)^3 \mathbf{1}_l(h). \tag{1.28}$$

Considering equations (1.14) and (1.28), the bond price can be represented as follows:

$$CB_j(t,t+H_j) = \sum_{i=1}^{n_j} C_{i,j} \left[(1+\beta_0 h_{i,j} + \gamma_0 (h_{i,j})^2 + \delta_0 (h_{i,j})^3) + \sum_{l=1}^{k-1} \delta_l (h_{i,j}-h_l)^3 \mathbf{1}_l(h_{i,j}) \right] + \varepsilon_t^{(j)}. \tag{1.29}$$

Finally, the estimated market discount function is given by:

$$B(t,t+h_{i,j}) = (1+\hat{\beta}_0 h_{i,j} + \hat{\gamma_0} (h_{i,j})^2 + \hat{\delta}_0 (h_{i,j})^3) + \sum_{l=1}^{k-1} \hat{\delta}_l (h_{i,j}-h_l)^3 \mathbf{1}_l(h_{i,j}), \tag{1.30}$$

where $\hat{\theta} = (\hat{\beta}_0, \hat{\gamma_0}, \hat{\delta}_0, \ldots, \hat{\delta}_{k-1})'$ denotes the OLS estimates.

Three are the elements which determine the trade-off between smoothness and flexibility: the specification of basis functions, the number and positioning of knot points. First of all, when quadratic splines are used, the corresponding forward curve exhibits what McCulloch called "knuckles". More specifically, the discount function has a discontinuous second derivative in the knot points, thus the forward curve presents a first derivative with discontinuities in the knot points. For this reason, McCulloch (1975) replaced the quadratic splines with cubic splines. Although this specification provides a better flexibility, the discount function is not constrained to be positive and monotonously decreasing, thus forward rates may be negative. Moreover, Shea (1984) criticizes the

[7]Note that for $h \in [0, h_1)$ we have $B(t,t+h) = \alpha_0 + \beta_0 h + \gamma_0 h^2 + \delta_0 h^3$. Since $B(t,t) = 1$, it follows that $\alpha_0 = 1$.

use of quadratic and cubic basis function: estimations may be inaccurate[8] thus we can have unstable forward curves shapes. For this reasons, Shea (1984) and Steeley (1991) suggest to represent a piecewise polynomial as a linear combination of lowly correlated elements called basic splines (B-spline, hereafter).

Cubic B-spline

Widely used B-splines are the cubic B-splines. This methodology asks for an augmented numbers of knot points at the beginning and the end of the curve. In particular, in the cubic case, an additional specification of three knots below h_0 and three knots above h_k is required. The knots structure will be as follows:

$$h_{-3} < h_{-2} < h_{-1} < h_0 < h_1 < \cdots < h_k < h_{k+1} < h_{k+2} < h_{k+3}. \tag{1.31}$$

The $k+3$ cubic B-spline are:

$$\psi_l(h) = \sum_{a=l}^{l+4} \left(\prod_{b=l, b \neq a}^{l+4} \frac{1}{h_b - h_a} \right) (h - h_a)^3 \mathbf{1}_a(h), \qquad l \in \{-3, \ldots, k-1\}, \tag{1.32}$$

with $\psi_l(h) \geq 0$ in the interval $[h_l, h_{l+4}]$ and zero outside.

After having constructed the cubic B-spline, the discount factor can be approximated by three possible functional forms:

[8]The basis functions used by McCulloch can lead to a regressor matrix that is near to singularity since its columns are almost perfect collinear.

- Linear cubic B-spline

$$B(t, t+h) = \sum_{s=1}^{k+3} z_s \psi_s(h), \tag{1.33}$$

- Non-linear Exponential cubic B-spline

$$B(t, t+h) = \exp\Big(-\sum_{s=1}^{k+3} z_s \psi_s(h)\Big), \tag{1.34}$$

- Non-linear Integrated cubic B-spline

$$B(t, t+h) = \exp\Big(-\int_0^h \sum_{s=1}^{k+3} z_s \psi_s(h)\Big), \tag{1.35}$$

where z_s are coefficients that have to be estimated under the constraint: $B(t,t) = z_1\psi_1(0) + \cdots + z_{k+3}\psi_{k+3}(0) = 1$.

The use of B-splines has two main technical advantages. First, the low level of correlation between basis leads to a precise and accurate estimation. Second, constraints are easily imposed thus negative forward rates are avoided. Smoothness and goodness of fit, both for standard and B-spline piecewise polynomials, depend also on the number and positioning of knot points. Increasing the number of knot points improves the explanatory power of the estimated yield curve but at the cost of a lower smoothness and stability. Deacon and Derry (2008) show that, for a low number of knot points, the forward curve is not enough flexible to allow for typical shapes. On the other hand, for a high number of knot points, the curve tends to be too much influenced by outliers. Commonly adopted is the rule of thumb of McCulloch (1975) by which the number of knot points should be equal to the square root of the number of bonds used in the estimation process. He suggested to select the location of knot points in such a way to have the same number of bonds in the sub-intervals.

If we want to focus on a possible economic interpretations, we can use 1, 5 and the 10 years of maturity as natural knot points locations (Litzenberger and Rolfo (1984)). In such a way, the intervals broadly fit the short-term, intermediate-term and long-term segments of the bond market. Lastly, location can also be determined through optimization algorithms (De Boor (1978)). As a matter of fact, we do lose in terms of (intuitive) economic interpretation.

Smoothed Cubic Splines

A different approach is the one introduced by Fisher, Nychka, and Zervos (1995) (FNZ, hereafter) in which they propose to use a smoothed cubic spline to model the forward curve.

The relevant trade-off between smoothness and fitting is treated by a roughness parameter which controls for the degree of curvature (degree of smoothing) we are interested in. The principal aim of such a parameter is to penalize the excess curvature (measured by the square of the second derivative) of the forward curve.

The objective function (1.16) is thus modified to ensure an adequate level of smoothness and, more precisely, the forward curve is chosen to be the cubic spline minimizing[9]:

$$\sum_{j=1}^{M} w_j \left[\sum_{i=1}^{n_j} C_{i,j} \exp\left(- \int_0^{h_{i,j}} \hat{f}(t, t+x) dx \right) - CB_j(t, t+H_j) \right]^2 + \lambda \int_0^{h_k} [f''(x)]^2 dx, \tag{1.36}$$

[9]Sometimes the use of B-spline basis could be preferred for mathematical convenience. However, the choice of cubic spline or B-spline is up to the researcher. Here, we follow Waggoner (1997) and use cubic splines.

with $h_k = T_M - t$ and where $\lambda > 0$ represents the roughness/smoothness parameter that has to be set via Generalized Cross Validation (GCV)[10].

Fisher, Nychka, and Zervos (1995) assume the same degree of smoothness for the entire forward curve. In practice, given the large variability of short-term yields with respect to long ones, a flexible short end and a stiff long end of the term structure are preferred.

Waggoner (1997) allowed the roughness parameter to be maturity dependent. He introduced the so called Variable Roughness Penalty (VRP, hereafter) giving the possibility to have the desired trade-off between flexibility and smoothness for each maturity interval. As in Fisher, Nychka, and Zervos (1995), cubic splines are used to approximate the forward curve. This means that, the cubic spline forward curve is chosen to minimize the following objective function:

$$\sum_{j=1}^{M} w_j \left[\sum_{i=1}^{n_j} C_{i,j} \exp\left(- \int_0^{h_{i,j}} \hat{f}(t, t+x) dx \right) - CB_j(t, t+H_j) \right]^2 + \int_0^{h_k} \lambda(x) [f''(x)]^2 dx \quad (1.37)$$

where $\lambda(x)$ represents a maturity-dependent roughness/smoothness parameter. Waggoner (1997) defined the roughness parameters as a three steps function accordingly with the structure of the U.S. bond market. This function is constant over the interval from 0 to 1 year (for short-term U.S. treasury securities), from 1 to 10 years (for the medium U.S. bond securities -notes-) and from 10 to 30 years (for the long-term U.S. bond securities -bond-). More formally, we have:

[10] This criterion uses numerical methods for finding the optimal value of λ and requires a repeated estimation of the forward curve for a set of λ values. A high level of computational complexity may enhance the time required for the estimation. For more details, see Waggoner (1997) and Anderson and Sleath (2001).

$$\lambda(x) = \begin{cases} 0.1 & 0 \leq x \leq 1, \\ 100 & 1 \leq x \leq 10, \\ 100000 & 10 \leq x. \end{cases} \tag{1.38}$$

The roughness parameter is chosen so that the out-of-sample weighted mean absolute error is minimized. A continuous smoothing function for the roughness parameter has been proposed by Anderson and Sleath (2001) and it is defined by:

$$\log \lambda(t) = \rho_0 - (\rho_0 - \rho_1) \exp\left(\frac{-t}{\rho_2}\right), \tag{1.39}$$

with ρ_0, ρ_1 and $\rho_2 \in \mathbf{R}$ [see Anderson and Sleath (2001) for details].

Although this methodology provides a better calibration, higher computational complexity is implied, thus more time for the estimation is required.

1.4.4 Parametric Methods

A very different approach is the one proposed by Nelson and Siegel (1987) and Svensson (1994). This class of models provides a simple and parsimonious approximation of the forward curve by means of a parametric function defined over the entire maturity domain.

The Nelson and Siegel (1987) Model

The parsimonious model developed by Nelson and Siegel (1987) was originally introduced with the aim of capturing typical yield curve shapes such as monotonic, humped, or, occasionally, S shapes.

The forward rate curve $f(t, t+h)$ is modeled in the following way:

$$f(t, t+h; \Theta_{NS}) = \beta_0 + \beta_1 \exp\left(-\frac{h}{\tau_1}\right) + \beta_2 \left[\left(\frac{h}{\tau_1}\right) \exp\left(-\frac{h}{\tau_1}\right)\right], \tag{1.40}$$

where $\Theta_{NS} = (\beta_0, \beta_1, \beta_2, \tau_1)'$. This parsimonious method has several advantages. First, it directly provides a smooth and a stable forward curve representation. Second, each parameter has a precise meaning and interpretation:

- β_0 is the level parameter. It is the asymptotic value of the forward curve for $h \to \infty$, namely $\lim_{h\to\infty} f(t, t+h; \Theta_{NS}) = \beta_0 > 0$. In other words, β_0 can be identified with the long-term forward rate.
- $(\beta_0+\beta_1)$ is the starting value of the forward curve, namely $\lim_{h\to 0} f(t, t+h; \Theta_{NS}) = \beta_0 + \beta_1$. This is the risk-free rate.
- β_1 is the slope parameter. It gives the spread between the spot rate and the long-term forward rate.
- β_2 is the curvature parameter. It describes the form and the amplitude of the hump: if $\beta_2 < 0$ we have a U-shape curve while if $\beta_2 > 0$ we have a hump.
- τ_1 is the location parameter. It denotes the location of the hump (the U-shape) in the maturity domain. Besides its role of location, τ_1 can be interpreted as a scale parameter. In fact, it measures the rate at which short-term and medium-term components decay to zero.

As a result, we can observe that the term structure of forward rates is determined by three factors:

- $B_0(h) = \beta_0$, that mostly determine long-term forward rates. Therefore, it can be seen as a Level Factor;
- $B_1(h) = \beta_1 \exp\left(-\frac{h}{\tau_1}\right)$, that mostly affects the wedge between short-term and long-term forward rates and thus it can be seen as a Slope Factor;
- $B_2(h) = \beta_2 \frac{h}{\tau_1} \exp\left(-\frac{h}{\tau_1}\right)$, that mostly affects, compared to $B_1(h)$, the medium-term forward rates. Therefore, it can be seen as a Curvature Factor.

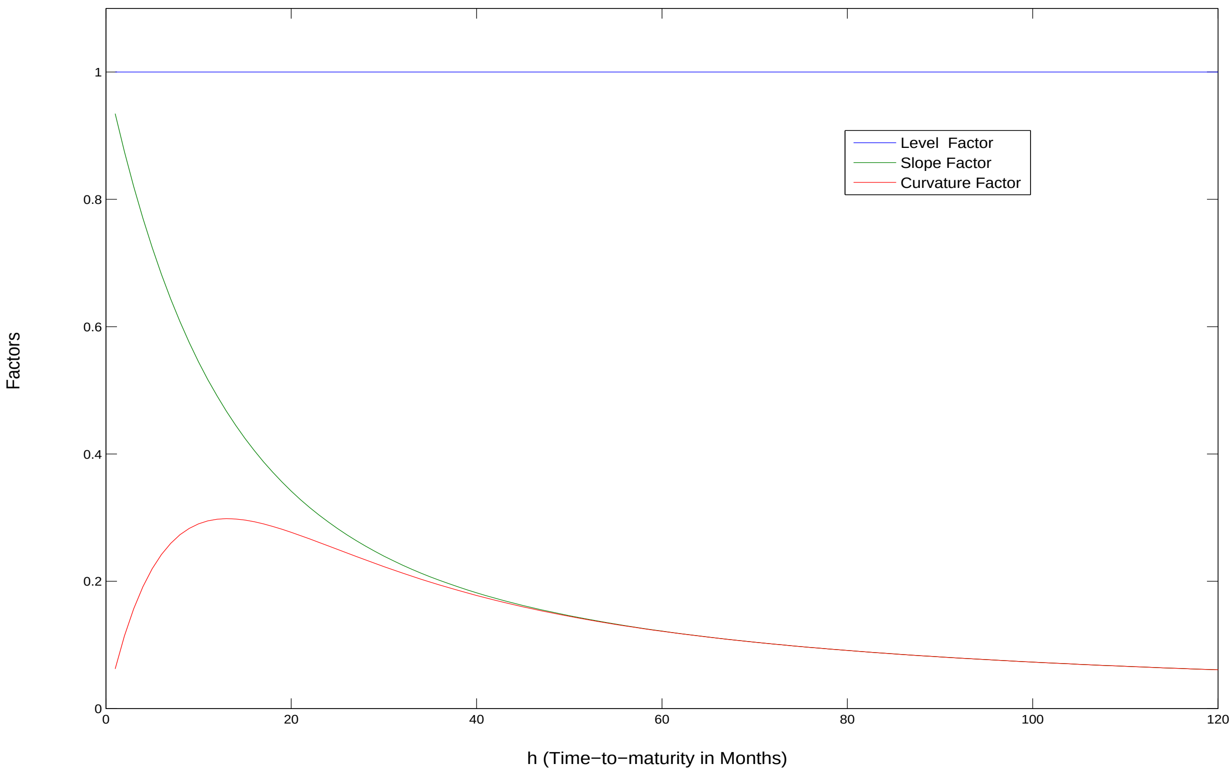

Fig. 1.1: Level, Slope and Curvature factors of Nelson and Siegel (1987) model. We assume $\beta_0 = \beta_1 = \beta_2 = 1$ and $\tau_1 = 0.0609$ as in Diebold and Li (2006).

According to (1.8), we can directly derive the spot yield curve $R(t, t+h)$ implied by (1.40):

$$R(t, t+h; \Theta_{NS}) = \beta_0 + (\beta_1 + \beta_2)\frac{1 - \exp\left(-\frac{h}{\tau_1}\right)}{\frac{h}{\tau_1}} - \beta_2 \exp\left(-\frac{h}{\tau_1}\right). \tag{1.41}$$

Considering relationship (1.10), the Nelson and Siegel (1987) discount function is:

$$B(t, t+h; \Theta_{NS}) = \exp\left[-\beta_0 h - (\beta_1 + \beta_2)\tau_1(1 - \exp\left(\frac{h}{\tau_1}\right)) - \beta_2 h \exp\left(\frac{h}{\tau_1}\right)\right]. \tag{1.42}$$

In the same spirit of (1.14), parameter estimation is based on the following regression:

$$CB_j(t, t+H_j) = \sum_{i=1}^{n_j} C_{i,j}\left\{\exp\left[-\beta_0 h_{i,j} - (\beta_1+\beta_2)\tau_1(1-\exp\left(\frac{h_{i,j}}{\tau_1}\right)) - \beta_2 h_{i,j}\exp\left(\frac{h_{i,j}}{\tau_1}\right)\right]\right\} + \varepsilon_t^j. \tag{1.43}$$

Finally, parameters have to be estimated by non-linear regression techniques, thus increasing computational complexity with respect to cubic and cubic B-splines (see Section 1.3). However, when the value of τ_1 is fixed at a pre-specified value (as in Diebold and Li (2006)), the use of ordinary least-square regressions for the parameters estimation is allowed. This notably reduces the computational effort.

The Svensson (1994) Model

Svensson (1994) extended the original model of Nelson and Siegel (1987) by allowing a more flexible description of the forward curve. The novelty is the addition of an exponential decay term that allows capturing a second hump. In this case the forward curve is modeled as follows:

$$f(t,t+h;\Theta_{SV}) = \beta_0 + \beta_1 \exp\left(-\frac{h}{\tau_1}\right) + \beta_2\left[\left(\frac{h}{\tau_1}\right)\exp\left(-\frac{h}{\tau_1}\right)\right] + \beta_3\left[\left(\frac{h}{\tau_2}\right)\exp\left(-\frac{h}{\tau_2}\right)\right], \tag{1.44}$$

where $\Theta_{SV} = (\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2)'$. Here a second "hump" is allowed by means of $B_3(h) = \beta_3 \frac{h}{\tau_2} \exp\left(-\frac{h}{\tau_2}\right)$. The Parameter β_3 describes the form and amplitude of the second hump while τ_2 defines its location and the rate at which $B_3(h)$ decays to zero.

According to (1.8), the Svensson spot yield curve is:

$$R(t,t+h;\Theta_{SV}) = \beta_0 + (\beta_1+\beta_2)\frac{1-\exp\left(-\frac{h}{\tau_1}\right)}{\frac{h}{\tau_1}} - \beta_2 \exp\left(-\frac{h}{\tau_1}\right) + \beta_3 \frac{1-\exp\left(-\frac{h}{\tau_2}\right)}{\frac{h}{\tau_2}} - \beta_3 \exp\left(-\frac{h}{\tau_2}\right). \tag{1.45}$$

Constraints are required in the estimation both for the Nelson and Siegel (1987) and Svensson (1994) methods. The discount function has to be positive and decreasing (to overcome negative forward rates). More formally, given a set of residual maturities $h_{min} < \ldots < h_k < h_{k+1} < \ldots < h_{max}$, the following constraints have to be satisfied:

$$\begin{aligned} R(t,t+h_{min}) &\geq 0 \\ R(t,t+\infty) &\geq 0 \\ \exp[-h_k R(t,t+h_k)] &\geq \exp[-(h_{k+1})R(t,t+h_{k+1})], \qquad \forall h_k < h_{max}. \end{aligned} \tag{1.46}$$

In this section we have seen that parametric methods can describe the whole yield curve by means of a reduced number of parameters and do not require a selection of knot points. Nevertheless, they provide a less flexible yield curve shape and, therefore, they do not fit the term structure as accurately as the spline interpolation techniques.

1.5 Comparison of yield curve estimation techniques

The literature of estimation techniques reviewed in the preceding section indicates that each methodology has its strengths and weakness and thus, the selection of an interpolation method is not a trivial problem to solve. Here, we will propose two possible (not competitive) ways for the discrimination.

The first one is based on the comparison between empirical results provided in the literature (see Bliss (1997), Waggoner (1997) and Ioannides (2003)). These works compare a wide range of term structure estimation techniques for the U.S. and U.K. bond markets, in two different time periods.

The second one considers the scope the analysis is intended to serve. For instance, an economic agent is interested in a high degree of flexibility of the (estimated) yield curve, with respect to another one who is more interested in a smoothed yield curve for ease of economic interpretation. The idea is to identify the estimation methodology which better fit economic agents' requirements.

We will divide this section in two parts. In the fist part, we will review the empirical results provided in the literature. In the second one, we will consider the two different economic agents perspectives.

1.5.1 Comparison of yield curve estimation techniques in the literature

Bliss (1997) shows that a high in-sample fit may not be sufficient for judging a term structure estimation method, because of possible over-fitting issues. In order to overcome this possibly distorted view of the performance, he suggests an out-of-sample test as an appropriate way for evaluating different estimation methods.

In his study, data are taken from the CRSP Government Bond files. They consist of the end-of-month prices quoted in the U.S. Treasury bond market for a period starting in January 1970 and ending in December 1995. Callable and flower bonds[11] are eliminated from the sample. Moreover, notes and bonds under one year to maturity and bills under one month to maturity are not considered. The compared estimation techniques are: Unsmoothed Fama-Bliss, Smoothed Fama-Bliss, McCulloch Cubic Spline, FNZ, Extended Nelson and Siegel (Svensson).

Another interesting comparative study is provided by Waggoner (1997). He has considered spline-based methods such as McCulloch, FNZ and VRP. The database is the same of Bliss.

Ioannides (2003), instead, analyzed a different market for a different time period but he maintains the same objectives of Bliss (1997) and Waggoner (1997). In particular, the author is interested in finding a method that, not only fits the most salient features of the historical bond data set, but also detects the most stable relationships between the parameters that are useful for out-of-sample forecasts. Here, data are closing mid-prices for U.K. treasuries and bonds (gilts) at daily frequency. They are obtained from Datastream and cover a period starting from the beginning of January 1995 and ending at the beginning of January 1999. As in Bliss (1997) and Waggoner (1997), callable and

[11] A flower bond is a bond that can be redeemed at par to pay estate taxes.

flower bonds are eliminated from the sample. Bond with less than one year to maturity are not considered. A wide range of methodologies are tested: McCulloch cubic spline, linear B-spline, non-linear exponential cubic B-spline, non-linear integrated cubic B-spline, VRP, Nelson and Siegel and Svensson.

Let us consider, first, in-sample performances[12]. According to Bliss (1997), the Unsmoothed Fama and Bliss method provides the best in-sample performance in the U.S. bond market, with cubic splines methods as second choice. The success of bootstrapping and cubic spline methods, with respect to parsimonious methods, is clearly associated to the richer parametrization of the first ones (in particular at the long end of the curve). At the short end, the FNZ is the worst performing. It attaches the same penalty across all the maturities and, therefore, it does not allow an adequate flexibility at short end which is usually required in practice. Waggoner (1997) solves this problem by means of a VRP[13]. In addition, he shows that this methodology is able to price bonds slightly better than McCulloch cubic splines over the entire maturity domain. According to Ioannides (2003) , in U.K., for maturities above 5 years, McCulloch cubic splines is the best in-sample method while FNZ and VRP are unable to fit well the long end. On the other hand, for short and medium maturities, parsimonious methods provide better in-sample performances than their splines counterparts.

Let us focus now on the out-of-sample performances. According to Bliss (1997), the clear in-sample advantage of the Unsmoothed Fama and Bliss method almost disappears. In an out-of-sample forecasts exercise, he can not identify an estimation methodology that clearly dominates the other ones. Similar conclusions are drawn by Waggoner (1997).

[12]In order to evaluate in-sample and out-of-sample performances, we can use three different measures: weighted mean absolute errors (WMAE), mean absolute errors (MAE) and hit rates. The first measures the average distance between observed and theoretical bond prices weighted by the inverse of bond duration, while the second considers the unweighted average distance between observed and theoretical bond prices. Finally, the latter indicates the percentage of bonds with a theoretical price which lie between the bid and ask quotas.

[13]However, for longer maturities (longer than 5 years) the FNZ method is preferred.

In particular, the VRP method does not provide significant out-of-sample improvements with respect to McCulloch cubic splines across all maturities. However, in these two studies, FNZ is considered the worst out-of-sample performer, especially at the short end[14].

On the other hand, for the U.K. market, Ioannides (2003) proves a better out-of-sample forecast performance for less parameterized and smoothed methods. Svensson method is found to be the best one, even if for some maturity intervals, non-linear exponential cubic B-spline and VRP perform slightly better. Overall, the Svensson methodology is the preferred choice, followed by non-linear B-spline and VRP method, while McCulloch and linear B-spline are the worst.

Another empirical analysis, relevant for model selection, is the one studying pricing residuals. Bliss (1997) found that different estimation methodologies provide common ability in fitting observed prices over time. Even though, according to (4.3), bond pricing errors should be random, the author demonstrates that they could be correlated through time (persistence of residuals) and correlated across methods (coincidence of residuals) and this seems to be linked to some economic factors. The presence of such omitted factors is confirmed by regressing bond price residuals to contemporaneous observable factors[15].

Ioannides (2003) investigates the behavior of pricing residuals with the aim of detecting misspecified techniques. The author classify the one-period holding period return for a bond in two parts: the normal and abnormal holding period return. The first one reflects general market movements while the second one is driven by pure noise. In the same

[14] However, when the sample is filtered out (he considers the same filtering rules used for the Unsmothed Fama-Bliss method) out-of-sample results improves. This means that FNZ method is very sensible to measurement errors (see Bliss (1997)).

[15] The observable factors used by the author are: time-to-maturity, premium (it is defined as $\widehat{CB}_j(t,t+H_j) - 100$ if $\widehat{CB}_j(t,t+H_j) > [CB_j^{ask}(t,t+H_j) + CB_j^{bid}(t,t+H_j)]/2$), discount (it is defined as $\widehat{CB}_j(t,t+H_j) - 100$ if $\widehat{CB}_j(t,t+H_j) < [CB_j^{ask}(t,t+H_j) + CB_j^{bid}(t,t+H_j)]/2$), age (Years since the issuing date), Time, amount outstanding, bid/ask spread.

spirit as Sercu and Wu (1997), Ioannides proposes three different alternative measures (called benchmarks) to filter out normal term structure movements (such as parallel shifts and twists) from abnormal returns [see Appendix 1.B for details]. His principal aim is to verify weather some term structure interpolation techniques may recognize abnormal returns and, more specifically, if these interpolation techniques could allow for profitable trading strategies. In practice, a trading strategy consists in opening, at date t, a long/short position on a bond when his holding period return is over/under evaluated with respect to the one it should have during "normal" market conditions, that is the model-implied return. The quantity traded is proportional to the size of the mis-pricing (contrarian trading strategy), and the position is closed at $t + k$. The optimal trading strategy is the one which provides the highest statistically significant profits under all the three benchmarks.

Ioannides (2003) shows that immediate trading strategies ($k = 0$) deriving from the implementation of parsimonious estimation methods (such as the Svensson estimation method) are the most profitable under all the three benchmarks. McCulloch spline methodology reports persistent profits under one benchmark measure only. For the other two ones, it produces (persistently) statistically significant losses. For these reasons, Ioannides (2003) suggests the Svensson estimation technique.

1.5.2 Selection

The empirical results reviewed above indicate that it is very difficult to identify one method as being superior to the other ones. For instance, the out-of-sample dominance of parsimonious functional form models is suggested by Ioannides (2003) but not found in Bliss (1997) and Waggoner (1997). Results are sample-dependent and, consequently, the selection of a method would be problematic. Therefore, we should also take into account agents' subjective preferences in order to make the choice easier. Here, in line with Gurkaynak, Sack, and Wright (2007), we consider two agents with different economic objectives: a trader and a macroeconomist.

From the point of view of a trader, a very flexible method can ensure a high precision for the estimates, thus a correct security and derivative pricing. The more flexible approaches tend to be the spline-based methods. However, spline methods also have drawbacks commonly found in Bliss, Waggoner and Ioannides. Their high level of parameterization can cause over-fitting problems thus considerable variability in the forward rates. They can not be robust to outliers, or to minor changes on the dataset and finally, they suffer of numerical instability.

However, a precise description of the database and an exact replication of outliers can be seen as strength for traders looking for small pricing anomalies. Cubic and cubic B-splines can also be suggested in that case. Moreover, given the high in-sample performance, also bootstrapping procedures, such as Unsmoothed Fama and Bliss method, are advisable.

On the contrary, when we consider a macroeconomist prospective, the purpose of the analysis is not the identification of mis-priced securities but rather the fundamental determinants of the yield curve. In such a case, are more interested to understand the relationship between macroeconomic conditions, monetary policy programs and yield curve dynamics. For these reasons, a less parameterized (i.e. more parsimonious) method could be the preferred choice. Its implementation will allow the replication of a wide range of yield curve shapes and, moreover, rules out time varying shape instability. As a result, parsimonious functional forms ensure an adequate level of smoothness and stability.

As indicated by the for International Settlements (2005), most central banks have adopted either the Nelson and Siegel or the Svensson methods to estimate the term structure of interest rate. However, some other central banks, such as the Bank of Japan and the Bank of England, use the more computational complex smoothed spline VRP method[16].

As a matter of facts, the extraordinary evolution of the computer technology has strongly reduced the time required for the estimation and thus the adoption of the VRP method becomes more feasible. Moreover, it can ensure for stability of estimates[17] and a slightly better out-of-sample forecast with respect to parsimonious methods (Anderson and Sleath (2001)). Nevertheless, the VRP methodology does not allow any macroeconomics interpretation.

Diebold and Li (2006) and Lund, Andersen, and Benzoni (2004) demonstrate that there is a close correspondence between the Nelson and Siegel model and the latent factors of Litterman and Scheinkman (1991). The three Nelson and Siegel terms can be interpreted as level, slope and curvature factors and they can explain the most of the observed variation of the yield curve.

Ang and Piazzesi (2003) and Diebold, Rudebusch, and Aruoba (2006) show that these latent factors, in particular "level" and "slope", can be related to macroeconomic variables such as inflation and real activity. Therefore, it seems that Nelson and Siegel factors have interesting macroeconomic interpretations.

Moreover, the importance of this parametric method has also been stressed by Diebold, Li, and Yue (2008) in an international bond markets setting. They adopt a dynamic version of the Nelson and Siegel model and prove the existence and the economic relevance of, not only "local" (or "country"), but also "common" (or "global") factors and their

[16]The Federal Reserve in U.S. uses the cubic spline method for its statistical releases while the Nelson and Siegel and the Svensson technique are used for research purposes.

[17]The roughness parameter is chosen so that the out-of-sample weighted mean absolute error is minimized (see Waggoner (1997)).

ability to account for a significant fraction of variation in ”local” bond yields. Additionally, they prove the linkage between such ”common” factors and ”global” macroeconomic fundamentals.

Recently, Gurkaynak, Sack, and Wright (2007) have stressed the importance, for macroeconomic purposes, to relay on a parsimonious method. In particular they estimate, from 1961 to the present[18], the U.S. treasury yield curve by using the Nelson and Siegel (1987) and Svensson (1994) techniques.

1.6 Conclusion

In this review we have considered alternative approaches typically proposed in the literature to the estimate of the term structure of interest rates. The purpose is to select an estimation technique which best fits researchers' needs. We have analyzed bootstrapping, spline-based and parametric methods, considering their goodness of fit (flexibility), smoothness and stability of the results.

Bootstrapping and spline-based methods are very flexible and provide accurate in-sample replication of the data, but may lead to over-fitting problems with consequently poor performances out-of-sample. On the contrary, parametric methods sacrifice some of the fit of the curve in favor of a higher degree of smoothness of the replicated curve, given the reduced number of parameters applied in the interpolation.

The empirical results provided in the literature indicate that it is very difficult to identify one method as being definitively superior to the others. Therefore, the selection also depends on the economic agents' objectives. A trader who is interested in exploiting profits from small pricing anomalies may be concerned with an accurate bond pricing.

[18] The updated version of yields database is available at http://www.federalreserve.gov/econresdata/researchdata.htm

In that case, bootstrapping and spline-based methods could be the choice. On the other hand, a macroeconomist is more interested in the general dynamic properties and fundamental determinants of the yield curve. As a result, he should choose a parametric method which guarantees different possible shapes and a smooth yield curve.

Moreover, in terms of macroeconomic interpretation, parametric methods are superior with respect to some spline-based counterparts, such as smoothed spline. Several studies in a single country context show the relationship between the estimated Nelson and Siegel factors, interpreted as level, slope and curvature (Diebold and Li (2006) and **?**), and macroeconomic factors (Ang and Piazzesi (2003) and Diebold, Rudebusch, and Aruoba (2006)). This is confirmed, in a multi-country environment, by Diebold, Li, and Yue (2008).

In conclusion, for macroeconomic issues, we suggest the use of parametric methods, for instance the use of the Nelson and Siegel and the Svensson methodologies (as it is suggested by Gurkaynak, Sack, and Wright (2007).

Appendix

1.A Macaulay Duration

The duration is the measure of the sensitivity of the coupon bond price to a change in interest rate. Let us consider equation (1.1) and we assume $t = 0$ as follows:

$$CB_j(0, T_j) = \sum_{i=1}^{n_j} C_{i,j} \exp[(-t_{i,j} Y)]. \tag{1.47}$$

where $R^{CB}(0, T_j) = Y$. Differentiating $CB_j(0, T_j)$ with respect to Y gives:

$$\frac{dCB_j(0, T_j)}{dY} = -\sum_{i=1}^{n_j} t_{i,j} C_{i,j} \exp[(-t_{i,j} Y)]. \tag{1.48}$$

We multiply both sides of the equation by $dY/CB_j(0, T_j)$ to get:

$$\frac{dCB_j(0, T_j)}{CB(0, T_j)} = -\frac{\sum_{i=1}^{n_j} t_{i,j} C_{i,j} \exp[(-t_{i,j} Y)]}{\sum_{i=1}^{n_j} C_{i,j} \exp[(-t_{i,j} Y)]} dY. \tag{1.49}$$

We define the duration D as:

$$D = \frac{\sum_{i=1}^{n_j} t_{i,j} C_{i,j} \exp[(-t_{i,j} Y)]}{\sum_{i=1}^{n_j} C_{i,j} \exp[(-t_{i,j} Y)]} = \sum_{i=1}^{n_j} t_{i,j} \frac{C_{i,j} \exp[(-t_{i,j} Y)]}{CB(0, T_j)}. \tag{1.50}$$

The duration D is also called Macaulay Duration and can be interpreted as weighted average of coupon dates $t_{1,j}, t_{2,j}, \ldots, t_{n_j,j}$ (expressed in years), where the weights are the present value of future cash flows divided by the bond price.

Macaulay duration measures the (first-order) sensitivity of the bond price with respect to changes in the YTM. In case of flat term structure of interest rate, YTM and term structure coincides. Therefore, Macaulay duration measures the change in the bond price due to parallel shift of the flat term structure.

1.B Benchmarks for trading Strategies

Ioannides (2003) uses three different benchmarks for evaluating abnormal returns in bonds. The first benchmark for measuring the abnormal return is the following:

$$AR_{j,t} = HP_{j,t} - E_t[HP_{j,t}|\phi_{t-1}, \phi_t], \tag{1.51}$$

where $HP_{j,t}$ is the holding period return for the bond j in the market and it is represented as follows:

$$HP_{j,t} = \frac{CP_j(t, t+H_j) - CP_j(t-1, t+H_j) + C_{i,j}}{CP_j(t-1, t+H_j)}, \tag{1.52}$$

where $C_{i,j}$ is the coupon i for the bond j that is payed for the time interval $[t-1, t)$, and

$$E_t[HP_{j,t}|\phi_{t-1}, \phi_t] = \frac{\widehat{CP}_j(t, t+H_j) - \widehat{CP}_j(t-1, t+H_j) + C_{i,j}}{\widehat{CP}_j(t-1, t+H_j)}, \tag{1.53}$$

is the expected holding period return for bond j considering a particular term structure estimation technique; $\hat{\phi}_{t-1}$ and $\hat{\phi}_t$ are the set of parameters estimated at time $t-1$ and t.

The second benchmark defines abnormal returns as follows:

$$AR_{j,t} = HP_{j,t} - [\alpha_{j,t-1} + \beta_{j,t}(HPm_{j,t} - \alpha m_{j,t})], \tag{1.54}$$

where $\alpha_{j,t-1}$ is the estimated yield for the j-th bond (calculated considering a particular term structure technique) at time $t-1$ and $\alpha m_{j,t}$ is the derived market portfolio yield at time t (both yields are annualized continuously compounded values calculated on daily basis); $\beta_{j,t}$ is the ratio between the duration of the j-th bond and the duration of the market portfolio. Finally, $HPm_{j,t}$ is the market portfolio holding period return (Ioannides (2003) chooses an appropriate Datastream benchmark index as a proxy of the market portfolio).

The last benchmark matches the return of a portfolio with the value, duration and convexity of the underlying bond. It is based on three portfolios: the first consists of one month and three months treasury bills, the second consists of bonds that mature between 1 and 8 years and the third is composed by bonds with more than 8 years to maturity.

An over-evaluation of a bond appears when $AR_{j,t} > 0$. In that case, an optimal term structure estimation technique suggests to sell that asset. On the contrary, when $AR_{j,t} < 0$, an under-evaluation is detected and thus the asset is bought.

Chapter 2

International Yield Curves and Principal Components Selection Techniques: An Empirical Assessment

Abstract

Using a common database, we provide a controlled empirical comparison of recently-proposed principal component (PC) methods for selecting a combination of common and local factors that characterize the joint dynamics of multi-country term structures. We build a database of daily Treasury yield curves for U.S., Germany, U.K. and Japan, using common criteria to filter coupon bond data, to ensure liquidity, and to interpolate the discount function. We then estimate each proposed PC method for all subgroups of these countries, using both yield levels and yield differences at weekly frequency.

We find, in general, that the proposed methods do not agree with one another on the preferred combination of common and/or local factors. We identify the explained variability decision criterion as an important source of this lack of agreement and recommend consideration of alternative statistical model selection techniques for the purpose of identifying common and local yield curve factors in international data.

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Résumé

En utilisant une nouvelle base de données de taux souverains, nous proposons une comparaison empirique des méthodes proposées récemment dans la littérature pour la sélection d'une combinaison des facteurs "locaux" et "communs" décrivant la dynamique jointe des courbes des taux de plusieurs pays. Nous avons construit cette nouvelle base de données des courbes des taux internationales à fréquence journalière pour les États-Unis, l'Allemagne, l'Angleterre et le Japon, en utilisant une même méthode de filtrage des données brutes, en garantissant un même degré de liquidité entre différents marchés et une même méthode d'interpolation des taux (à la Nelson et Siegel (1987)). Ensuite, nous avons appliqué les méthodes d'analyse en composantes principales proposées dans la littérature (Leippold et wu (2007), Diebold, Li et Yue (2008) et Egorov, Li et Ng (2011)) sur différent groupes de pays en utilisant des taux d'intérêt en niveau et en différence à une fréquence hebdomadaire.

Plusieurs combinaisons des facteurs "communs" et "locaux" semble être équivalentes en termes de variabilité expliquée de données. L'incertitude sur la meilleure combinaison des facteurs et la difficulté de les définir précisément comme "commun" et "local" semblent être les inconvénients principaux dérivant de l'utilisation de l'analyse en composantes principales.

Nous suggérons, par conséquent, une procédure statistique alternative pour l'extraction et la sélection des facteurs "communs" et "locaux" à partir d'une base de données internationale de taux d'intérêt.

2.1 Introduction

To model a single country's term structure, the empirical literature generally finds that three to five latent factors are required to match the observed time variations of yield curve shapes and to explain Treasury returns [see Adrian, Crump, and Moench (2013), Litterman and Scheinkman (1991), Dai and Singleton (2000) and Dai and Singleton (2002)), Duffee (2002) and Duffee (2011)), Cheridito, Filipovic, and Kimmel (2007), Cochrane and Piazzesi (2005) and Cochrane and Piazzesi (2008)].

In the multi-country setting, on the contrary, we observe a lack of agreement, not only about the number of latent factors that are relevant to explain the joint dynamics of two or more countries, but also about their nature, namely how many of them are *common factors* (affecting yields in all countries) and how many are *local factors* (affecting yields in one country only). On the one hand, some papers make assumptions (without empirical justification) about the number of common and local factors. For instance, Backus, Foresi, and Telmer (2001), in order to study the forward premium anomaly between pairs of countries (among U.K., Germany and Japan), consider both the case of one common factor and one local factor for each country, and the case of only two common factors. Ahn (2004) proposes a two-country (U.S. and Germany) term structure model in which the international yield curve dynamics are driven, by assumption, by one common factor and one local factor for each country, and both factors are extracted by employing a principal factor analysis of the unconditional variance-covariance matrix of the yields.

On the other hand, three papers in particular have adopted statistical methodologies for the extraction of international yield curve principal components in order to select a relevant combination of common and local factors. These methodologies, followed notably by Leippold and Wu (2007), Diebold, Li, and Yue (2008) and Egorov, Li, and

Ng (2011), have reached different conclusions.

Leippold and Wu (2007) [LW (2007), hereafter], working with U.S. and Japan swap and LIBOR rate changes observed weekly from January 1990 to December 1999, find no evidence for a common factor in these markets (in the sense that no dimensionality reduction is available by using common factors in a principal component analysis) and conclude that at most three local latent factors for each country's yield curves are needed. Egorov, Li, and Ng (2011) [ELN (2011), hereafter] focus on the joint dynamics of the U.S. and European Union (E.U.) term structures and consider daily euro interest rates (Euribor) and U.S. interest rates (LIBOR) from July 1999 to June 2006. Their analysis leads them to choose two common factors and one local factor for each country [see Table 4 in ELN (2011)], finding this combination to be more satisfactory than the case with one common factor and one local factor for each economy. Diebold, Li, and Yue (2008) [DLY (2008), hereafter], exploiting (and extending) a data base on zero-coupon government bond yields (for the U.S., U.K., Germany and Japan, from Brennan and Xia (2006)) observed monthly from September 1985 to August 2005, and generalizing to a global context the dynamic Nelson-Siegel yield curve model of Diebold and Li (2006), construct a hierarchical dynamic factor model for international yield curves in which the term structure of each country may depend on local factors, and local factors may in turn depend on common factors (but the term structures may not directly load on the common factors). They select two local factors for each country along with two common factors, where each pair of factors consists of one level factor and one slope factor.

The purpose of this chapter is to provide a controlled comparison of these three methods recently adopted by the international yield curve literature, recognizing that their different conclusions might be due to the fact that their empirical analyses have used different groups of countries, different sample periods and different data sets. To do this we build a database of daily international Treasury yield curves from the beginning of 1980 to the end of 2009, for four leading bond markets: the U.S., U.K., Germany and Japan, and then estimate using each proposed methodology with this database.

We adopt for all countries the Gurkaynak, Sack, and Wright (2007) [GSW (2007), hereafter] criteria to filter coupon bond Treasury raw data (taken from Datastream), and to guarantee a uniform level of liquidity. Indeed, like in GSW (2007), we exclude not only bonds with any option-like features and those with too short or too long maturities (i.e. not actively traded), but we also avoid the possible presence of an on-the-run/off-the-run liquidity premium by filtering out on-the-run and first off-the-run bonds. We then interpolate the discount function across residual maturities using the (parsimonious smoothed) Nelson and Siegel (1987) methodology. We thus obtain what we believe to be the first international zero-coupon Treasury yield curve data base at daily frequency with a homogeneous interpolation technique and a uniform level of bond liquidity (of an actively traded second off-the-run kind) across time, maturities and economies. These features facilitate a reliable comparison between different numbers and combinations of common and local yield factors. In addition, for completeness we have applied each of the three above-mentioned methodologies to all groups of countries using both yield levels and yield differences.

The results we have obtained are the following. If the LW (2007) approach is adopted, not only is the identification of the relevant number of local factors per country often ambiguous and completely different if we adopt interest rates in level or in difference, but also the attribution of any of them to a given country is often unclear. If we follow the ELN (2011) paper, we require (in general) one common factor, but the number of selected local factors per country changes if we consider different kind or different number of countries, or if we consider yield differences instead of yield levels. Lastly, the DLY (2008) methodology tends to always select two common (global level and slope) factors and two local (level and slope) factors per country, even if only one common factor is detected when we consider some sets of three countries and four countries with yield differences.

In summary, our empirical analysis highlights the systematic inability of the analyzed methodologies to clearly decide upon a preferred combination of common and/or local factors. We identify the explained variability decision criterion as an important source of this lack of agreement and recommend consideration of alternative statistical model selection techniques for the purpose of identifying common and local yield curve factors in international data.

This chapter is organized as follows. In Section 2.2 we present the international Treasury yield curve data set, the criteria we have adopted to build it, the associated descriptive statistics, pricing error analysis as well as a comparison with other relevant data sets used in the literature. In Section 2.3 we briefly present the three above mentioned PCs-based techniques while, in Section 2.4, exploiting our database, we detect a preferred combination of common and/or local factors for each technique across all groups of countries using both yield levels and yield differences. Section 2.5 concludes, while appendices gather all tables about the empirical assessment of the three principal component methods.

2.2 The International Treasury Yield Curves Database

One of the most important Treasury yield curve databases is the one recently proposed by GSW (2007) for the U.S. economy. In comparison with other well-known interest rates data sets [see Fama and Bliss (1987) and McCulloch (1975) and McCulloch and Kwon (1993)], three main features stand out and help determine their success: *a*) the high (daily) frequency of the (estimated) yields, *b*) the uniform level of liquidity among yields, over the entire maturity range, obtained using only second off-the-run Treasury notes and bonds (i.e., filtering out on-the-run and first off-the-run bonds) and *c*) a parsimonious parametric (Nelson-Siegel-Svensson) technique to estimate the discount function allowing, at the same time, to generate a rich family of yield curve shapes and to detect fundamental factors (such as macroeconomic variables, monetary policy interventions and announcements) behind its variation over time and in the cross-sectional dimensions.

In this section these three relevant features guide the construction of an international Treasury yield curve database for four leading bond markets (the U.S., U.K., Germany and Japan) and over a sample period from the beginning of 1980 to the end of 2009. In the international yield curve literature, the existing data do not satisfy all these important characteristics and, thus, we aim to fill this gap. For instance, the data in Ahn (2004) is given by monthly yield curves (from 1974:01 to 1988:12, for U.S. and Germany) estimated with spline techniques, while DLY (2008) propose (for U.S., Germany, U.K. and Japan) yield curve data (again) at monthly frequency (from 1985:09 to 2005:08)[1], and they use the unsmoothed Fama and Bliss (1987) technique. Wright (2011) considers a yield curves panel data of ten countries, but the frequency of observations, the data filtering criteria and the interpolation techniques are different across the economies.

[1]This database is obtained extending the one proposed by Brennan and Xia (2006) and built using cubic splines [see also Tang and Xia (2007)].

2.2.1 Raw Data, Filters and Estimation Techniques

Raw data on all outstanding sovereign coupon bonds are collected from Datastream. This database consists of bond prices, accrued interest, coupon rates, redemption dates and issue dates, in local currency for the U.S., U.K., Germany and Japan. Following GSW (2007), we apply several data filters in order to consider only securities with similar levels of liquidity and to avoid special features (like callable features). The filters we apply are the following:

- We exclude all bonds with option-like features such as bonds with warrants, floating rate bonds, callable and flower bonds and index-linked bonds.
- We consider only securities with maturity, at the issuing date, at least one year and not more than fifteen years given that bonds in the ”too short” or ”too long” segment of the maturity spectrum may be not actively traded. We also exclude Treasury bills from the U.S. market all.
- For liquidity reasons, we consider at any date all outstanding bonds with a residual maturity from three months to fifteen years. Moreover, for each available maturity, we exclude the ”on-the-run” and ”first-off-the run” issues, which have greater level of liquidity with respect to the others [see, for instance, Fontaine and Garcia (2012)].

These filters lead us to construct an ”off-the-run” international Treasury yield curve panel of data in which the liquidity level is uniform across the time series and cross-sectional dimensions. From the filtered database, the term structure of interest rates is estimated at daily frequency using the Nelson and Siegel (1987) methodology over the maturity spectrum covered by the available (filtered) data. We will post on the web a file containing the daily estimated yield curves as well as the estimated Nelson and

Siegel parameters making it possible to construct a time series of yields for any maturity. Nevertheless, we strongly suggest to generate only interest rates for maturities within the available filtered bond data[2].

2.2.2 Estimated Yield Curve Data

Using the above mentioned filters and estimation technique, daily zero-coupon yields are calculated from January 1, 1980 to December 31, 2009, with the largest available range of residual maturities being from 3 months to 10 years and available only for the U.S. market. For the German bond market, we can calculate yields for the entire sample period only for maturities from 2 to 9 years, given that short-term bonds are almost always missing at the beginning of the sample. Nevertheless, starting from 1984, we recover securities with 6, 9 and 12 months of residual maturity.

Regarding the Japanese market, bond data are extremely rare before 1982, hence we can not interpolate any term structure at that time. From 1982 to 1985, we frequently do not have securities with residual maturities from 3 to 9 months and from 8 to 10 years. It is only from 1986 on, that we have securities with residual maturity from 9 months to 10 years. For the U.K. bond market, coupon-bond data are missing at various times during the sample period, more often at short and at long maturities.

In summary, with the exception of the U.S. economy, the availability of coupon bonds is limited at the beginning of the sample period for Germany and Japan, and at various times for the U.K.. Nevertheless, when we move forward in time, coupon-bond data availability generally increases and, thus allows us to generate a panel data set of yields with a rich maturity spectrum.

[2]For instance, for Germany, Japan and U.K. (see Section 2.2 for more details), yields with residual maturity less than 12 months and more than 10 years are sometimes not available.

2.2.3 Descriptive Statistics

Descriptive statistics of daily estimated zero-coupon yields highlight important features that are common across the four different Treasury markets (see Table 2.1 and Figures 2.2.3 and 2.2):

- *Yield curves are on average upward-sloping.* More precisely, the U.S., Japanese and German term structures are upward-sloping, with German yields presenting the largest average long-term spread. The U.K. yield curve is, on average, almost flat; the difference between the sample means of the one-year and the ten-year yields is only 0.8%. The Japanese yield curve (over the 1-year/10-year maturity range) has, among the four countries, the smallest average (between 2.5% and 3.6%), while U.K. has the highest average (ranging between 7% and 8%).

- *The term structure of marginal volatility is downward-sloping.* The empirical standard deviation of yields decreases with the residual maturity both for U.S. and Germany, with a difference between the one-year and the ten-year (nine-year for Germany) maturity at around 0.60%. Volatility decreases more slightly with the maturity in Japan (the difference is about 0.50%) and in the U.K. (the difference is around 0.34%). Overall, the U.S. and U.K. markets show the highest term structure of volatilities, while the German market shows the lowest. Generally (with the exception of the shortest Japan maturity) these term structures are more volatile at shorter than at longer maturities.

- *Yields are highly persistent and with strong cross-country correlations.* Yields are highly persistent in all four markets. Indeed, they have one-day, one-week and one-month lag autocorrelation averaging 0.987 and never below 0.93. Regarding the cross-country correlations, Figure 2.2 clearly show that for any pair of countries the level of correlation is large (between 0.5 and 0.95) and generally increasing with residual maturity.

Maturity (Months)	Median	Mean	St. Dev.	Min	Max	$\rho(1)$	$\rho(7)$	$\rho(30)$	Sample
U.S.									
3	5.32	5.70	3.37	0.10	17.81	0.99	0.99	0.97	7479
12	5.57	5.97	3.34	0.26	16.52	0.99	0.99	0.98	7479
36	6.00	6.49	3.15	0.69	15.46	0.99	0.99	0.98	7479
60	6.25	6.82	2.99	1.17	15.17	0.99	0.99	0.99	7479
84	6.43	7.03	2.90	1.65	15.02	0.99	0.99	0.99	7479
108	6.53	7.18	2.83	2.14	14.98	0.99	0.99	0.99	7479
120	6.57	7.23	2.80	2.32	14.97	0.99	0.99	0.99	7479
Germany									
3	4.15	4.71	2.36	0.16	12.98	0.99	0.99	0.97	6187
12	4.38	4.97	2.37	0.65	12.21	0.99	0.99	0.98	7747
36	4.91	5.36	2.13	1.53	11.47	0.99	0.99	0.99	7828
60	5.43	5.66	1.96	2.17	10.91	0.99	0.99	0.99	7828
84	5.87	5.88	1.84	2.61	10.46	0.99	0.99	0.99	7828
108	6.15	6.03	1.77	2.78	10.16	0.99	0.99	0.99	7828
120	6.97	7.03	1.44	2.87	10.10	0.99	0.99	0.96	4579
U.K.									
3	9.26	8.76	3.50	3.05	18.89	0.99	0.98	0.93	1456
12	7.00	7.75	3.41	0.53	15.54	0.99	0.99	0.96	6573
36	6.82	7.64	3.13	1.55	15.55	0.99	0.99	0.98	7345
60	6.95	7.59	3.06	2.09	15.55	0.99	0.99	0.98	7143
84	7.04	7.55	3.06	2.48	15.74	0.99	0.99	0.98	6851
108	5.91	7.17	3.21	2.86	16.17	0.99	0.99	0.98	5217
120	5.65	6.95	3.07	3.05	16.48	0.99	0.99	0.97	4601
Japan									
3	0.42	1.85	2.47	0.02	8.23	0.99	0.99	0.98	4697
12	0.76	2.59	2.66	0.01	8.38	0.99	0.99	0.99	7305
36	1.54	2.86	2.57	0.06	8.26	0.99	0.99	0.99	7305
60	2.37	3.19	2.49	0.15	8.43	0.99	0.99	0.99	7305
84	2.93	3.47	2.42	0.25	8.66	0.99	0.99	0.99	7305
108	3.15	3.58	2.25	0.38	8.42	0.99	0.99	0.99	7112
120	3.06	3.58	2.16	0.46	8.38	0.99	0.99	0.99	6903

Table 2.1: Summary statistics for bond yields of U.S., Germany, U.K. and Japan daily yields [Source: Datastream. Estimation method: Nelson and Siegel (1987)]. 'Sample' indicates the number of observations for each maturity, while $\rho(\ell)$ denotes the sample autocorrelation for a number of lags ℓ measured in days. The sample period starts January 1, 1980 for U.S., U.K. and Germany and from January 1, 1982 for Japan. It ends for all countries in December 31, 2009. Yields are in annual basis.

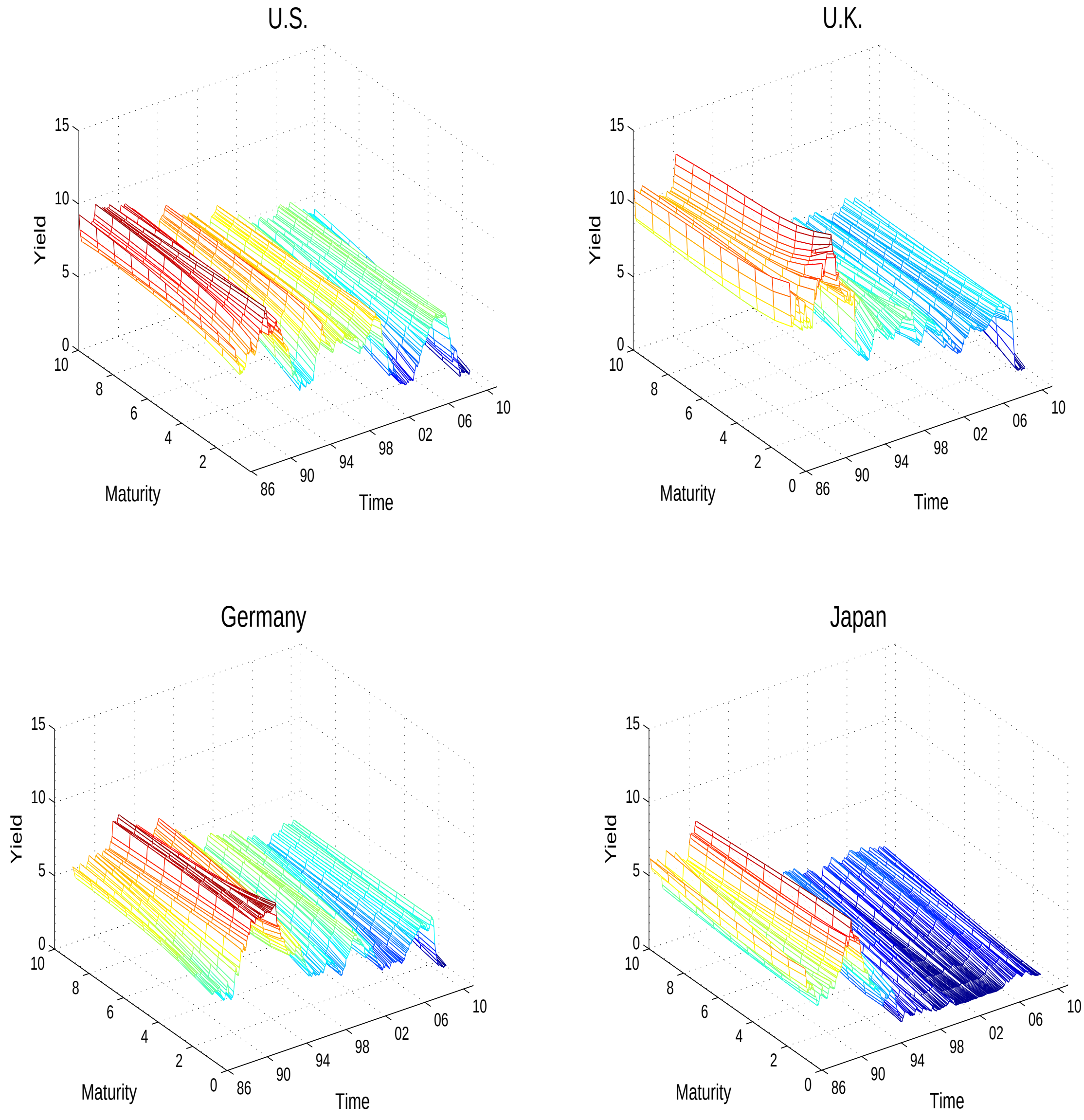

Fig. 2.1: Treasury yield curves of U.S., Germany, Japan and U.K. from January 1986 to December 2009, and for residual maturities from 1 to 9 years. The term structure of interest rates for the *U.S.*, Germany and Japan are taken from our international Treasury yield curves database while *U.K.* term structure is taken from the Bank of England data set.

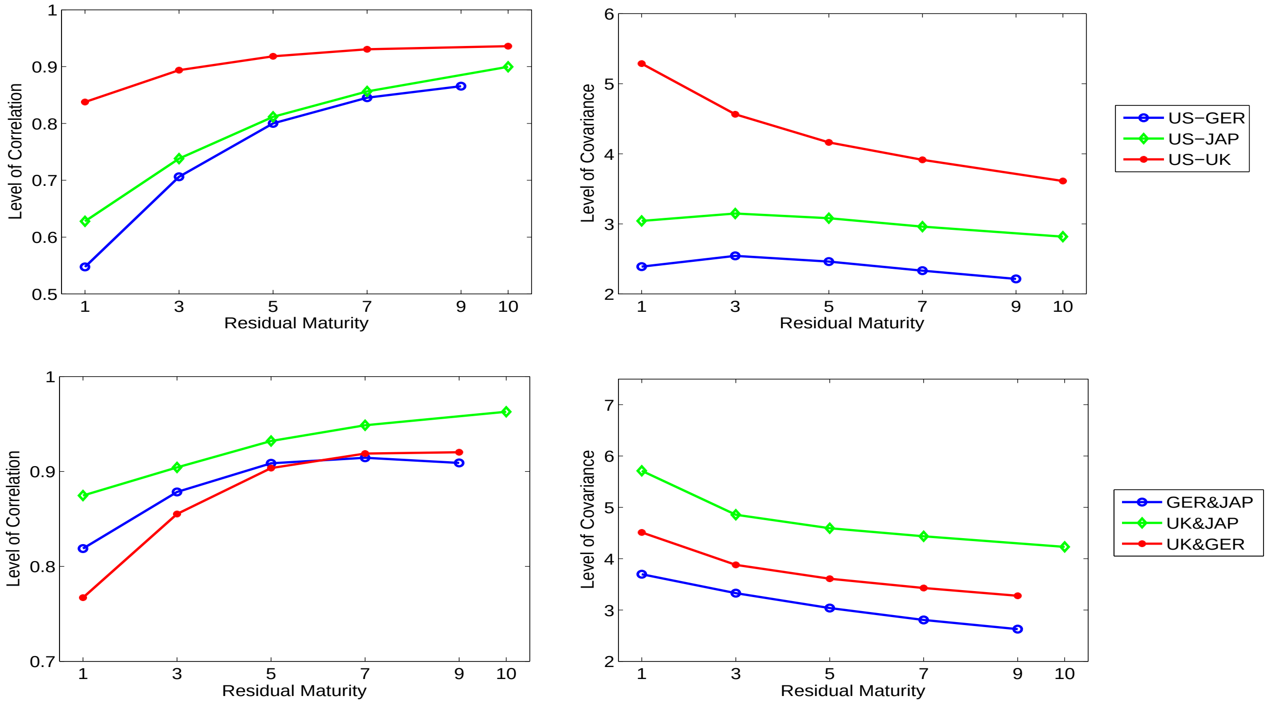

Fig. 2.2: Cross-country yield level correlations (left column) and covariances (right column) for U.S., Germany, Japan and U.K. from January 1986 to December 2009.

2.2.4 Pricing Error Analysis

Following GSW (2007), we study the fitting performances of our data set by determining, for any country, the mean absolute (continuously compounded) yield to maturity error (difference between the observed yield and the interpolated one) for three maturity buckets. The short-term maturity bucket considers securities with residual maturities from 3 months to 2 years, the medium-term maturity bucket takes bonds with time-to-maturity from 2 to 5 years, while the long-term maturity bucket takes bonds with a residual maturity between 5 and 10 years.

We can observe from Figures 2.3 and 2.4 that, for U.S., Germany and Japan, the fitting errors are quite small and similar in magnitude to those shown by GSW (2007). The largest ones are concentrated at the beginning of the sample and, in particular, for the short-term maturity bucket. This fact is mainly due, as stated by GSW (2007) for the U.S. economy, to a lack of liquidity in bond markets at the beginning of the 80's and to the idiosyncratic nature of short-term securities. In other words, it seems that bond pricing anomalies have reduced across countries and maturities and through time probably because of more active and liquid markets. As far as the U.K. bond market is concerned, we immediately see from Figure 2.4 (left column) that fitting errors are very large at the beginning of the sample and are sometimes missing entirely because filtered and even raw data are not available.

regardless the country considered, the estimated Nelson and Siegel parameters could be extremely different from the ones we have estimated the previous days. However, this typical parametric interpolation drawback[3] does not affect the estimated yields dramatically, thus the yield curve shape over the time remains quite stable.

[3]This is an identification issue well documented in Anderson and Sleath (2001) and empirically found by GSW (2007) for the U.S. market. Different combinations of parameters can provide similar yield curve shapes.

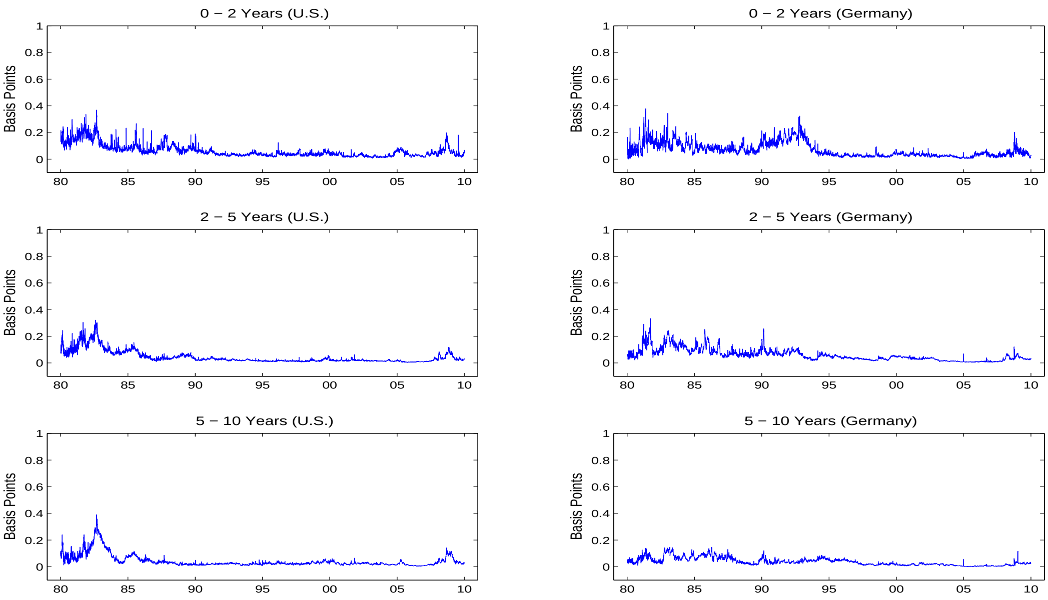

Fig. 2.3: Average absolute yield fitting errors for U.S. (left column) and Germany (right column) for three maturity buckets.

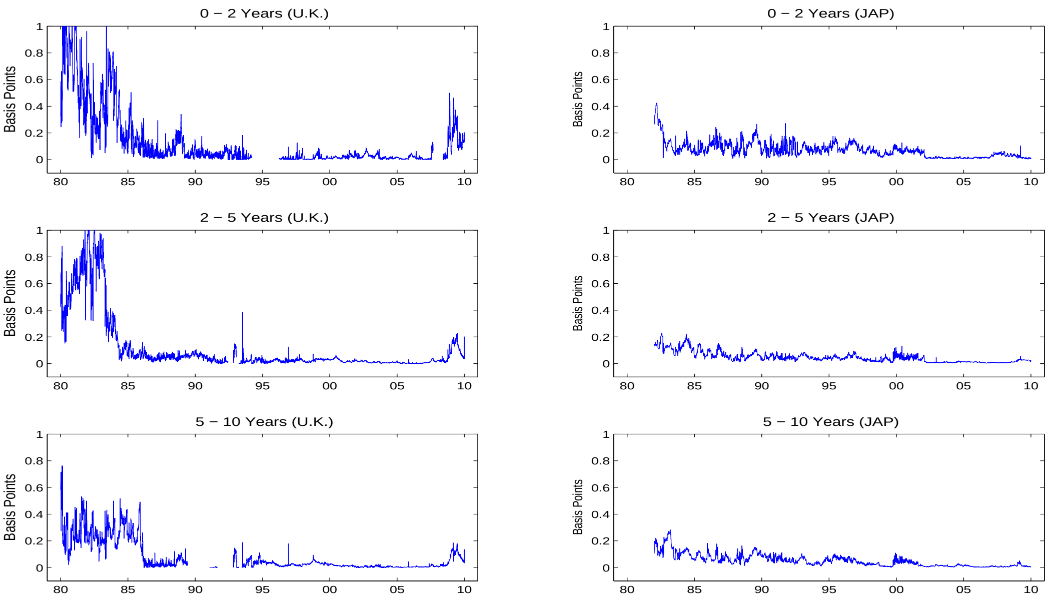

Fig. 2.4: Average absolute yield fitting errors for U.K. (left column) and Japan (right column) for three maturity buckets.

2.2.5 A Comparison with other Data Sets

In this section, in order to further evaluate the reliability of our database, we will compare our daily international Treasury yield curves with existing data from other sources. In particular, we consider three different relevant data sets: the U.S. zero-coupon yields of GSW (2007), the U.K. term structures provided by the Bank of England and the international bond yields database kindly supplied by DLY (2008).

The first database consists of daily U.S. Treasury yield curves estimates from 1961 to the present. Two parametric techniques are used to fit the term structure, namely the Nelson and Siegel (1987) one and the Svensson (1994) generalization, while Treasury quotes come from the Federal Reserve Bank of New York (FRBNY) and from CRSP. The second database consists of daily U.K. Treasury yields estimated from 1979 up to the present. A smoothed version of the cubic spline methodology, the Variable Roughness Penalty method [see Anderson and Sleath (2001) for further details] is used to interpolate bond data. The Treasury security data comes from sources including Bloomberg, Gilt Edged Market Association, Reuters and U.K. Debt Management Office. The third database consists of monthly U.S., U.K., German and Japanese Treasury yield curves estimated from 1980 through August 2005. The term structures are estimated by means of the unsmoothed Fama and Bliss (1987) methodology, while government bond data are taken from Datastream.

Given the different sample periods and frequency of observations of the above mentioned data sets, in order to provide a reliable statistical comparisons, we have selected (from our daily database) observations at both daily and monthly frequency for U.S. and U.K., and monthly frequency for Japan and Germany, covering common time intervals from beginning of 1982 to the end of 2009 in the former case while, in the latter case, the sample starts in January 1982 and ends in August 2005.

Let us consider the U.S. database first. On the basis of classical descriptive statistics, our zero-coupon daily yields estimates are very close to the ones provided by GSW (2007) (the results are available upon request from the authors). In particular, the correlations of bond yields between these two databases are always above 0.99, for all maturities and regardless of the adopted daily or monthly frequency of observations. Moreover, if we focus on the monthly frequency, a comparison with the DLY (2008) database is possible. Our estimates have slightly lower sample means and standard deviations, but correlations across yields (with the same residual maturity) are always higher than 0.98, with the only exception of the 10-year yields having correlations around 0.91[4].

We next focus on the German and Japanese monthly databases. While our German database contains fewer observations of 10-year yields, we generally provide more observations of long and of short maturities than the data of DLY (2008). Regarding the Japanese market, our database is not as rich as the DLY (2008) database for short maturities, but we provide more observations for longer maturities. The German and Japanese yield curves have mean levels and standard deviations that are slightly larger in the DLY (2008) database with respect to our estimates. Nevertheless, correlations of bond yields between these two databases are in general always above 0.97.

Finally, as far as the U.K. bond market is concerned, on a daily basis, the Bank of England database is richer than ours. However, correlations of bond yields between the two sets of term structures are very high (around 0.98 for all maturities). If we consider the monthly frequency, correlations between our estimated yields and the ones provided by DLY (2008) are above 0.94, with the only exception for the 3-month yields, perhaps because of the limited number of observations and the different methodologies applied for the interpolation.

[4]This small difference may be induced by the presence of anomalous 10-year maturity bond prices which are not smoothed out by the unsmoothed Fama and Bliss (1987) methodology.

In summary, our estimated international zero-coupon yields closely track the ones built and/or adopted by other relevant papers and institutions. Now, in order to have a richer database (over both the time series and maturity dimensions) to exploit in the second part of the chapter (focused on extracting principal components from multi-country yield curves), we will rely on the U.K. yield curve estimates provided by the Bank of England. In addition, this method is quite similar to Nelson and Siegel (1987) in terms of parameter flexibility and degree of smoothness, as also suggested by the above mentioned statistical comparison.

2.3 International Yield Curves and Principal Component Selection Techniques

The purpose of this section is to briefly present PCs-based techniques recently adopted in the literature for the purpose of selecting a preferred combination of common and local factors to characterize the joint dynamics of multi-country term structures of interest rates. In particular, the three relevant methodologies we deal with are those proposed by Leippold and Wu (2007), Egorov, Li, and Ng (2011) and Diebold, Li, and Yue (2008).

2.3.1 The Leippold and Wu (2007) Approach

LW (2007), working with U.S. and Japan swap and LIBOR rates observed weekly from January 1990 to December 1999, conclude that a common factor among these two markets does not exist and suggest three local latent factors for each yield curve. The decision strategy they follow to reach that conclusion is based on the following three steps.

First, using the correlation matrix estimated from interest rate changes, they perform a principal component analysis on the yield curve database of each country and on the pooled data set of the two countries, thus extracting single-country principal components we denote PCs and multi-country principal components denoted $PC^{(c)}s$, respectively (we focus here on their analysis of interest rates). From this analysis, they check how many principal components are required in each case to explain a large fraction (over 90%) of the interest rate variance. On the basis of this analysis they do not find a dimensionality reduction by pooling the data given that two PCs are required to explain U.S. interest rates, three PCs are required for Japan's interest rates, and five $PC^{(c)}s$ are required for the pooled data set of interest rates [see Table II in LW (2007)]. Because there is no reduction, from the factors required with local factors only, to the five factors required by the pooled analysis, they conclude that there is no dimensionality reduction achievable through the use of common factors.

Second, they regress each yield-to-maturity series on each of the first seven $PC^{(c)}s$, and report the associated percent variance explained, R^2, along with the average percent explained (over the maturity spectrum of each country) which we denote R_a^2 [see Table III in LW (2007)], and they identify the major contributing country for each $PC^{(c)}s$ according to which country has the larger R_a^2, with the sole exception of the fifth factor (which they identify as a currency movement factor largely unrelated to interest rates).

Third, they explore the nature of each factor on the basis of the pattern of correlations it has with the yields of each country [see Table IV in LW (2007), where this table represents the signed square root of the R^2 values from their Table III] finding, for example, that the third and fourth $PC^{(c)}s$ are associated with the slopes of the yield curves. They conclude that at most three local factors for each country are needed to summarize its interest rate movements, and that no dimensionality reduction is available by using common factors. We denote this strategy as the *LW*-approach.

2.3.2 The Egorov, Li, and Ng (2011) Approach

ELN (2011) focus on the joint dynamics of the U.S. and European Union (E.U.) term structures and consider daily euro interest rates (Euribor) and U.S. interest rates (LIBOR) from July 1999 to June 2006. The decision strategy they follow to select a preferred combination of local and common factors is based on the following three steps.

First, they extract local PCs and common $PC^{(c)}s$ from each economy and from the pooled term structures, respectively, and they find (in contrast with LW (2007)) that three $PC^{(c)}s$ explain more than 99% of the total variance in the pooled Euribor and LIBOR data set [see Table 2 in ELN (2011)]. Second, in order to establish how many factors are needed to explain the variation of the two curves, they regress each single-country PC on the first six $PC^{(c)}s$ and they observe that the first two $PC^{(c)}s$ are associated with the first local factor of each country, while the third and fourth $PC^{(c)}s$ contain information about the second local factors in the LIBOR and Euribor markets, respectively. They also find that the fifth (sixth, respectively) $PC^{(c)}$ is strongly linked with the third local factor in each economy. They find four factors sufficient based on the R^2 from these regressions, which show that the first four $PC^{(c)}s$ capture 99.9% of the variation in each of the first two LIBOR- and first two Euribor-PCs [see Table 3 in ELN (2011)], while using just the first three $PC^{(c)}s$ captures only 83.37% of the second local Eurobor factor, which they declare insufficient. Third, in order to choose how many of these four factors are common and how many are local, they run a principal component analysis on the residuals obtained from regressing each country's term structure on the first one and on the first two common principal components. The PCs of each country residuals (denoted $PC^{(\ell)}s$) are identified as local factors and the selected mix of common and local factors is the one such that a large percentage (around 90%, being 90.56% for LIBOR and 88.23% for Euribor after regressing on two common factors) of the variation of these residuals is explained (while 60.76% is declared insufficient for one common factor with one local LIBOR factor). We will take as their implicit criterion that at least 85% be explained. This analysis leads them to choose two common factors and one local factor for each country [see Table 4 in ELN (2011)], this combination being more satisfactory than the case with one common and one local factor for each economy.

Nevertheless, an inspection of their Table 4 also highlights that: *i*) in the LIBOR market, the (six-factor) case of two common and two local factors (for each country) dominates the (five-factor) case of one common and two locals, and that both specifications outperform the selected four-factor case (increasing the percent variance explained of residuals from 90.96% to 94.28% or 97.87%); *ii*) in the Euribor market, choosing five- or six-factors perform about equally well (98.25% or 97.63%) and both dominate the previously suggested four-factor case (88.23%). This analysis suggests that the selected specification is potentially missing relevant information, with respect to the two above mentioned larger dimensional cases, and the proposed decision strategy does not provide an objective statistical criterion. In other words, and in contrast with LW (2007), the two competitive specifications both suggest that two $PC^{(\ell)}s$ for each economy are required, while the decision's uncertainty concerns the number of $PC^{(c)}s$ (one vs two). This selection strategy is denoted *ELN*-approach.

2.3.3 The Diebold, Li, and Yue (2008) Approach

DLY (2008), exploiting (and extending) a monthly data base on zero-coupon government bond yields (for the U.S., Germany, Japan and U.K.) of Brennan and Xia (2006), and generalizing to a global context the dynamic Nelson-Siegel yield curve model of Diebold and Li (2006), construct a hierarchical dynamic factor model for international yield curves in which the term structure of each country may depend on local factors, and local factors may in turn depend on common factors (but a country's term structure does not directly depend on common factors). The sample period is from September 1985 to August 2005 and the strategy they follow to select common and local factors is based on the following steps.

First, from each yield curve they extract a level and slope factor by applying the first step of the Diebold and Li (2006) two-step procedure. Second, they observe visually that the four level factors (one for each country) exhibit commonality in their dynamics, as do the four slope factors. To assess this commonality, they conduct a PCA for each group. They infer the existence of a dominant global level factor from the fact that the first level PC explains just over 90% of the total variation of the country level factors, and infer an important global slope factor from the fact that the first slope PC explains about 50% of the total slope variation.

Their hierarchical dynamic factor model allows each country's yield curve to depend on its own level and slope factors where each of these in turn depends on the global factor (level or slope respectively), thereby constructing a model by assumption with two common factors along with two local factors for each country. Each country has direct access to only two factors (its own level and slope) because the common factors act indirectly (through the local factors) and a country is not permitted to load directly on a common factor. While the dimensionality of their model (level and slope for common and for each country, factors) exceeds the dimensionality of the data (level and slope for each country series), the inclusion of these two additional global factors allows their model to represent common, correlated movements. However, as estimated, they find that each of the two global components is closely associated with just one country's movements: Germany's level factor closely matches the global level factor, while the U.K.'s slope factor loads entirely on the global slope factor[5].

2.4 Empirical Assessment and Comparison

The purpose of this section is to use the international Treasury yield curve data set presented in Section 2.2 to provide a controlled empirical comparison of the three PCs-based techniques presented in the previous section. Using weekly observations of the U.S., U.K., German and Japanese yield curves from January 1, 1986 to December 31, 2009, results (summarized in Table 2.2) are presented for all groups of countries, and for both yield levels and yield differences in the following three sections.

[5]While interpreting the estimates from their Table 4, DLY (2008) note that each of the two global components (level and slope) is closely associated with just one country's movements. Germany's level factor closely matches the global level factor, as may be seen from the fact that its loading of 0.26 (first column of equations, second country-level-factor equation) is the largest of any country's level factor loading on the global level factor, while the apparently non-significant German-specific level factor loading of 0.07 (second column of equations in that same row) contributes little if any persistence in contrast to the other countries in this group. Similarly, the U.K.'s slope factor loads most strongly on the global slope factor with its loading of 0.77 (first column of equations, last country-slope-factor equation) which is the largest of any country's slope factor loading on the global slope factor, while the apparently non-significant U.K.-specific level factor loading of 0.03 (second column of equations in that same row) contributes little if any persistence in contrast to the other countries. In addition, neither Germany's nor Japan's slope factor appears to load significantly on the global slope factor, based on estimated loadings of 0.06 and 0.03 with posterior standard deviations 0.05 and 0.03 respectively (first column of equations, second and third country-slope-factor equation).

Countries	Techniques	Yield levels					Yield differences				
		r_c	r_1	r_2	r_3	r_4	r_c	r_1	r_2	r_3	r_4
U.S.-U.K.	*LW*	0	1 or 2	1 or 2			0	3	3		
	ELN	2	1	2			1	2	2		
	DLY	2	2	2			2	2	2		
U.S.-GER.	*LW*	0	2 or 3	1 or 2			0	3	3		
	ELN	1	1	2			1	2	2		
	DLY	2	2	2			2	2	2		
U.S.-JAP.	*LW*	0	2	2			0	3	3		
	ELN	1	1	1			1	2	2		
	DLY	2	2	2			2	2	2		
GER.-U.K.	*LW*	0	2	1			0	3	3		
	ELN	1	2	2			1	2	2		
	DLY	2	2	2			2	2	2		
GER.-JAP.	*LW*	0	1 or 2	1 or 2			0	3	3		
	ELN	1	2	2			1	2	2		
	DLY	2	2	2			2	2	2		
U.K.-JAP.	*LW*	0	1, 2 or 3	0, 1 or 2			0	3	3		
	ELN	1	2	2			1	2	2		
	DLY	2	2	2			2	2	2		
U.S.-U.K.-GER.	*LW*	0	1	2 or 3	1 or 2		0	3	3	3	
	ELN	1	1	2	2		1	1	2	2	
	DLY	2	2	2	2		2	2	2	2	
U.S.-U.K.-JAP.	*LW*	0	2 or 3	2 or 3	1		0	3	3	3	
	ELN	1	1	2	1		1	2	2	2	
	DLY	2	2	2	2		1	2	2	2	
U.S.-GER.-JAP.	*LW*	0	2	1 or 2	1 or 2		0	3	3	3	
	ELN	1	1	2	1		1	1	2	2	
	DLY	2	2	2	2		1	2	2	2	
U.K.-GER.-JAP.	*LW*	0	2	1 or 2	1 or 2		0	2	3	4	
	ELN	1	2	2	2		1	2	2	2	
	DLY	2	2	2	2		1	2	2	2	
U.S.-U.K.-GER.-JAP.	*LW*	0	2	2	2	0	0	3	2 or 3	2, 3 or 4	3 or 4
	ELN	1	1	2	2	1	1	1	2	2	2
	DLY	2	2	2	2	2	1	2	2	2	2

Table 2.2: For any given set of $n = 2, 3$ and 4 countries, and for both yields levels and yield differences, we report the number r_c of common and the number r_j of local factors (of country j in the group) suggested by each of the three PC-based techniques, namely LW, ELN and DLY. We use weekly data for a sample period starting from January 1, 1986 to December 31, 2009 (1252 observations) and residual maturities from 1 to 9 years.

2.4.1 The *LW*-approach

As far as the *LW*-approach is concerned, if we consider pairs of countries as well as yield levels, the first step suggests that three $PC^{(c)}s$ always explain more than 99% of joint interest rates variation while, if we consider yield differences, the number of factors rises to five or six [see Table 2.3] as in LW (2007)[6]. If the second step of the *LW*-approach is applied to yield levels, we have that the majority of the information (R_a^2) is concentrated in $PC_1^{(c)}$, two or three other ones [namely, ($PC_2^{(c)}, PC_3^{(c)}$) for *U.S.-U.K.*, *GER-U.K.*, *GER-JAP* and *U.K.-JAP*, and ($PC_2^{(c)}, PC_3^{(c)}, PC_4^{(c)}$) for *U.S.-GER* and *U.S.-JAP*] have a R_a^2 between 0.10 and 0.01, while the remaining ones seem to be irrelevant, having an associated R_a^2 around zero. In this case we should select, in general, the above mentioned three or four $PC^{(c)}s$ only, and the attribution to each of them as local factor of a given country would not be straightforward, given that the R_a^2 across countries are of similar magnitudes in several cases [see, for instance, $PC_3^{(c)}$ in the *U.S.-U.K.* case where we have $R_a^2 = 0.01$ for both countries, or $PC_2^{(c)}$ in the *U.S.-GER* and *GER-JAP* cases where these two groups countries show the same $R_a^2 = 0.11$ and $R_a^2 = 0.05$, respectively; see, for further details, Panel *A* of Tables from 2.4 to 2.9]. If we consider yield differences, we have that, even if the information is less concentrated in $PC_1^{(c)}$, we still need to consider as relevant a factor with $R_a^2 \approx 0.01$ if we want to have 6 $PC^{(c)}s$ to share as local factors for the two countries. In addition, while labeling any one of these six $PC^{(c)}s$ as local factor of a given country is less difficult in general, in some cases (e.g., *GER-U.K.*) again the R_a^2s across countries are quite similar to one another [see Panel *A* of Table 2.7].

In the 3-country case, the first step shows that the first 4 or 5 $PC^{(c)}s$ are required

[6] While LW (2007) also include results with relevant currency exchange rates variations, we are using criteria as they specified for choosing the number of factors based on interest rates only. When we implement the *LW*-approach including also exchange rate variation time series, we reach conclusions similar to those highlighted in Section 2.4.1. This analysis is available upon request from the authors.

to explain more than 99% of variability of interest rates levels, while the first 8 or 9 $PC^{(c)}s$ are necessary for yield differences [see Table 2.10]. Now, if the second step is applied to the levels, we have that again the identification of any of the relevant (i.e., $R_a^2 \geq 0.01$) $PC^{(c)}s$ as a local factor of a given country would not be easy for the same reasons presented in the 2-country case and the associated number of local factors for a given country may be in contradiction with the the number typically suggested by a single-country principal component analysis. For instance, in the case *U.S.-U.K.-GER* this strategy would select just one factor for the *U.S.* and would be unable to decide if $PC_4^{(c)}$ is a *U.K.* or *GER* factor being $R_a^2 = 0.02$ for both countries. Again, if we take *U.S.-U.K.-JAP*, $PC_4^{(c)}$ shares the same $R_a^2 = 0.01$ across the three countries [see Panel *A* of Tables from 2.11 to 2.14 for further details]. This problem *seems to be solved* if we work with yield differences, given that we find it easier to detect three local factors for each country among the relevant $PC^{(c)}s$. Nevertheless, in the set of three countries *U.K.-GER-JAP*, Japan receives four factors, Germany receives three factors and *U.K.* only two (namely, $PC_6^{(c)}$ and $PC_9^{(c)}$), with $PC_6^{(c)}$ providing to the latter economy an $R_a^2 = 0.04$ only slightly larger than the one provided to *GER* ($R_a^2 = 0.03$). The labeling is also not clear in the *U.S.-U.K.-GER* case, where $PC_4^{(c)}$ (respectively, $PC_5^{(c)}$) has a R_a^2 of 0.06 (respectively, 0.03) for *GER* and of 0.05 (respectively, 0.04) for *U.K.*.

In the 4-country case, we have that 5 or 6 $PC^{(c)}s$ are required for yield levels, while this number ranges between 11 and 12 for yield differences [see Table 2.15]. The second step [see Table 2.16] provides two local factors to *U.S.*, *U.K.* and *GER* and nothing to Japan if the level of interest rates is studied while, if we consider yield differences, we find difficult to associate $PC_5^{(c)}$ (respectively, $PC_7^{(c)}$) to a given country given that the R_a^2 is the same between Germany and Japan (respectively, *U.K.* and Germany). In summary, if the *LW*-approach is adopted, we find ambiguity both in the identification of the relevant number of local factors per country, and also in their attribution to specific countries.

2.4.2 The *ELN*-approach

In this section we apply the ELN (2011) approach to our term structure datab:tabase for all possible sets of countries, using both yield levels and yield differences. For the 2-country case (see Tables 2.17 to 2.19) we find for both yield levels and yield differences, and for each pair of countries, that the first four (respectively, six) $PC^{(c)}s$ always explain a large R^2 of at least 99.65% (respectively, 99.61%) of the interest rate variations of the first two (respectively, three) local PCs of both countries (second step of the analysis, presented in Tables 2.17 and 2.18). However, note that when we apply the third step by computing principal components (separately for each country) of the residuals from yields regressed on one and on two common factors, we are not always able to achieve the implicit ELN (2011) criterion of at least 85% of the variance explained by the first principal component $PC^{(\ell)}$ from this residual analysis. The average percent variance explained is just 76.78% across all cases in Table 2.19: for yield levels from one common factor to two the average percent variance explained increases from 78.64% to just 80.99%, while for yield differences the average percent explained decreases from 76.43% to 71.06%. Thus a case-by-case analysis is required.

To complete this third step for two countries, we note from Table 2.19 that just four of the 48 entries with one $PC^{(\ell)}$ exceed the 85% criterion and these are all with yield levels. For the U.S. with Japan we may choose just three factors: one common factor and one local factor for each country, because the first local U.S. factor explains 85.74% of the variance of the regression residuals from one common factor, and the first local Japan factor similarly explains 89.78%, which are both above the implicit ELN criterion of at least 85%. For the U.S. with Germany, we may choose one local U.S. factor (because it explains 85.57%) and two local German factors (which together explain 99.15%) for a total of four factors including the one common factor. The remaining entry that exceeds the 85% criterion involves the U.S. with the U.K. and does not allow us to meet the criterion with only four factors because this entry requires two common factors with one local U.S. factor (explaining 86.24%) but with two U.K. factors because the 85% criterion is not met with just one U.K. $PC^{(\ell)}$. All of the remaining nine (out of twelve) pairs of countries require five factors (one common and two local for each country will meet the criterion).

For the three-country case (see Tables from 2.20 to 2.22) we find for both yield levels and yield differences, and for each group of countries, that the first six (respectively, nine) $PC^{(c)}s$ always explain a large R^2 of at least 99.02% (respectively, 99.52%) of the interest rate variations of the first two (respectively, three) local PCs of each country (second step of the analysis, presented in Tables 2.20 and 2.21).

When we apply the third step with three countries by computing principal components (separately for each country) of the residuals from yields regressed on one and on two common factors, we are always able to achieve the implicit ELN (2011) criterion of at least 85% of the variance explained if we take the first two principal components $PC^{(\ell)}$s from this residual analysis along with one common factor, for a total of seven factors (one common plus two local for each country) using Table 2.22. In five of the eight cases we can use fewer factors. With yield levels, for the U.S. with the U.K. and Japan we can use five factors (one local U.S., two local U.K., and one local Japan, along with one common factor) because the first $PC^{(\ell)}$ for the U.S. explains more than the required 85% at 87.07%, and the U.K. explains 89.77%. Similarly, five factors suffice for the U.S. with Germany and Japan for yield levels (U.S. at 90.93% and Japan at 88.79%). Six factors suffice with both yield levels and yield differences for the U.S., the U.K., and Germany (because the first U.S. $PC^{(\ell)}$ explains 86.31% for levels and 85.26% for differences). The final case is for yield differences with the U.S., Germany, and Japan (85.50% explained by the first U.S. $PC^{(\ell)}$).

For the case of all four countries (see Tables from 2.23 to 2.25) we find for both yield levels and yield differences, that the first eight (respectively, twelve) $PC^{(c)}s$ explain a large R^2 of at least 99.81% (respectively, 99.47%) of the interest rate variations of the first two (respectively, three) local PCs of each country (second step of the analysis, presented in Tables 2.23 and 2.24). Applying the third step using Table 2.25, for yield levels we find that seven factors are required: one common factor, one local factor for the U.S. (90.35%) and for Japan (88.88%), and two local factors for each of the U.K. and Germany (for which the first $PC^{(\ell)}$ did not explain at least the 85% required). For yield differences, we find that eight factors are required: one common factor, one local factor for the U.S. (86.50%) and two local factors for each of the U.K., Germany, and Japan.

In summary, for yield levels with two countries, we need either three factors (one pair of countries), four factors (two pairs of countries) or five factors (three pairs), while for yield differences we need five factors with all six pairs of countries. For groups of three countries, we need five factors for two groups with yield levels, six factors for one group with yield levels and two groups with yield differences, and seven factors in the remaining cases (one group with levels and two groups with differences). For all four countries taken together, we need seven factors for levels and eight factors for yield differences.

2.4.3 The *DLY*-approach

Coherently with the implementation of the two previous methodologies, we have applied this third approach to each group of countries in our database, with both yield levels and yield differences (see Table 2.26).

With yield levels, we seem to have a ”global dominant level factor” for all groups of countries, although in only 9 of the 11 groups does the percent variance explained actually exceed the 90% threshold (it is close otherwise, at 88.97% for the U.S. with Germany, and 88.51% for the U.S. with Germany and Japan). We also always have one important global slope factor, with all 11 groups exceeding the 50% threshold.

With yield differences in no case do we reach the 90% threshold for a ”global dominant level factor.” The average percent variance explained by the first principal component for the level factors (averaged across all groups of countries) is just 60.44%, with a maximum of 72.24% for the U.S. with Germany, down from the average of 92.38% explained the level factors of yield levels. This suggests that, when analyzing yield differences, the global level factor might be demoted from ”dominant” to ”important” but would still be included in the hierarchical model. We have an ”important global slope factor” exceeding the 50% threshold in 7 of 11 groups of countries (the exceptions are larger groups: U.S.-U.K.-Japan at 41.95%, U.S.-Germany-Japan at 42.76%, U.K.-Germany-Japan at 42.94%, and 38.08% for all four countries). This appearance of a decrease in the percent variance explained with larger groups (with yield differences and slope factors) is confirmed by a very highly significant regression ($p = 0.000743$) of percent variance explained on the number of countries in the group. However, the DLY model does not specify how to proceed in the absence of evidence of an ”important global slope factor” for yield differences.

2.5 Conclusions and Further Developments

We have presented an extensive and detailed comparison of principal component-based techniques recently adopted for the purpose of selecting a preferred combination of common and/or local international yield curves factors. Reliability of this empirical assessment is obtained by building a database of daily Treasury yield curves for U.S., Germany, U.K. and Japan, using common criteria to filter coupon bond data, to ensure liquidity, and to interpolate the discount function.

Three relevant selection methodologies [namely, Leippold and Wu (2007), Egorov, Li, and Ng (2011) and Diebold, Li, and Yue (2008)] are applied to all subgroups of these countries, using both yield levels and yield differences at weekly frequency. Our empirical exercise shows the systematic inability of these methodologies to clearly detect a preferred combination of common and/or local factors that describe the joint dynamics of a given set of multi-country yield curves. We identify the explained variability decision criterion as a relevant source of this lack of agreement and suggest consideration of alternative statistical model selection techniques for the purpose of identifying common and local yield curve factors in international data.

Appendix

2.A The *LW*-Approach.

The 2-Country Case 1^{st} *step : Principal Component Analysis of International Bond Markets*

Table 2.3: We report principal component analysis of the term structure of interest rates of pairs of countries taken separately and jointly. The first column lists the number of principal components (four for a single country, and nine for any 2-country case), while the rest of the table is divided into three vertical blocks. Each of the first and second blocks represents an individual country, while the third block represents the two countries jointly. The first and second columns (third and fourth, respectively) of each block report marginal and cumulative percent variance explained for yield levels (yield differences, respectively). For each country we use weekly yields observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

	PCs								$PC^{(c)}s$			
	U.S.				*U.K.*				*U.S. − U.K.*			
	Levels		Differences		Levels		Differences		Levels		Differences	
	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.
1	97.22	97.22	93.55	93.55	98.10	98.10	89.96	89.96	93.48	93.48	65.16	65.16
2	2.70	99.93	5.42	98.97	1.80	99.90	8.81	98.77	4.74	98.22	26.75	91.91
3	0.07	100.00	0.88	99.85	0.09	99.99	1.09	99.87	1.36	99.57	4.63	96.54
4	0.00	100.00	0.14	99.99	0.01	100.00	0.12	99.99	0.35	99.93	2.33	98.88
5	-	-	-	-	-	-	-	-	0.05	99.98	0.56	99.44
6	-	-	-	-	-	-	-	-	0.02	100.00	0.42	99.86
7	-	-	-	-	-	-	-	-	0.00	100.00	0.07	99.93
8	-	-	-	-	-	-	-	-	0.00	100.00	0.06	99.99
9	-	-	-	-	-	-	-	-	0.00	100.00	0.01	100.00
	U.S.				*GER*				*U.S. − GER*			
	Levels		Differences		Levels		Differences		Levels		Differences	
	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.
1	97.22	97.22	93.55	93.55	96.86	96.86	87.59	87.59	86.40	86.40	65.49	65.49
2	2.70	99.93	5.42	98.97	3.00	99.86	9.33	96.91	11.28	97.68	25.14	90.62
3	0.07	100.00	0.88	99.85	0.14	99.99	2.67	99.58	1.64	99.32	4.96	95.59
4	0.00	100.00	0.14	99.99	0.01	100.00	0.40	99.98	0.59	99.90	2.36	97.95
5	-	-	-	-	-	-	-	-	0.06	99.96	1.35	99.30
6	-	-	-	-	-	-	-	-	0.03	100.00	0.42	99.72
7	-	-	-	-	-	-	-	-	0.00	100.00	0.20	99.92
8	-	-	-	-	-	-	-	-	0.00	100.00	0.07	99.99
9	-	-	-	-	-	-	-	-	0.00	100.00	0.01	100.00
	U.S.				*Japan*				*U.S. − Japan*			
	Levels		Differences		Levels		Differences		Levels		Differences	
	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.
1	97.22	97.22	93.55	93.55	98.72	98.72	84.67	84.67	88.37	88.37	52.66	52.66
2	2.70	99.93	5.42	98.97	1.18	99.91	11.34	96.01	10.18	98.55	36.46	89.12
3	0.07	100.00	0.88	99.85	0.09	99.99	3.47	99.48	0.95	99.49	5.68	94.80
4	0.00	100.00	0.14	99.99	0.01	100.00	0.48	99.97	0.44	99.94	2.69	97.49
5	-	-	-	-	-	-	-	-	0.04	99.97	1.74	99.23
6	-	-	-	-	-	-	-	-	0.02	100.00	0.44	99.67
7	-	-	-	-	-	-	-	-	0.00	100.00	0.24	99.91
8	-	-	-	-	-	-	-	-	0.00	100.00	0.07	99.98
9	-	-	-	-	-	-	-	-	0.00	100.00	0.02	100.00

Table 2.3, continued.

	PCs								*PC^{(c)}s*			
	GER				*U.K.*				*GER – U.K.*			
	Levels		Differences		Levels		Differences		Levels		Differences	
	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.
1	96.86	96.86	87.59	87.59	98.10	98.10	89.96	89.96	92.04	92.04	63.22	63.22
2	3.00	99.86	9.33	96.91	1.80	99.90	8.81	98.77	5.77	97.81	25.59	88.82
3	0.14	99.99	2.67	99.58	0.09	99.99	1.09	99.87	1.80	99.61	5.71	94.53
4	0.01	100.00	0.40	99.98	0.01	100.00	0.12	99.99	0.29	99.91	3.32	97.85
5	-	-	-	-	-	-	-	-	0.06	99.97	1.34	99.19
6	-	-	-	-	-	-	-	-	0.03	100.00	0.53	99.73
7	-	-	-	-	-	-	-	-	0.00	100.00	0.20	99.93
8	-	-	-	-	-	-	-	-	0.00	100.00	0.06	99.99
9	-	-	-	-	-	-	-	-	0.00	100.00	0.01	99.99
	GER				*Japan*				*GER – Japan*			
	Levels		Differences		Levels		Differences		Levels		Differences	
	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.
1	96.86	96.86	87.59	87.59	98.72	98.72	84.67	84.67	92.87	92.87	54.77	54.77
2	3.00	99.86	9.33	96.91	1.18	99.91	11.34	96.01	5.02	97.89	5.83	91.98
4	0.01	100.00	0.40	99.98	0.01	100.00	0.48	99.97	1.73	99.61	4.50	96.48
5	-	-	-	-	-	-	-	-	0.28	99.90	1.74	98.21
6	-	-	-	-	-	-	-	-	0.07	99.97	1.32	99.54
7	-	-	-	-	-	-	-	-	0.03	99.99	0.24	99.78
8	-	-	-	-	-	-	-	-	0.00	100.00	0.20	99.97
9	-	-	-	-	-	-	-	-	0.00	100.00	0.02	99.99
	U.K.				*Japan*				*U.K. – Japan*			
	Levels		Differences		Levels		Differences		Levels		Differences	
	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.
1	98.10	98.10	89.96	89.96	98.72	98.72	84.67	84.67	95.02	95.02	51.32	51.32
2	1.80	99.90	8.81	98.77	1.18	99.91	11.34	96.01	3.56	98.59	36.03	87.35
3	0.09	99.99	1.09	99.87	0.09	99.99	3.47	99.48	1.14	99.72	5.69	93.04
4	0.01	100.00	0.12	99.99	0.01	100.00	0.48	99.97	0.20	99.92	4.37	97.40
5	-	-	-	-	-	-	-	-	0.04	99.97	1.73	99.13
6	-	-	-	-	-	-	-	-	0.03	99.99	0.55	99.68
7	-	-	-	-	-	-	-	-	0.00	100.00	0.24	99.92
8	-	-	-	-	-	-	-	-	0.00	100.00	0.06	99.98
9	-	-	-	-	-	-	-	-	0.00	100.00	0.02	99.99

2^{nd} and 3^{rd} Steps : Regressing Yields on $PC^{(c)}s$ and Associated Correlations

		Yield levels										Yield differences									
Panel *A*		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
US	1y	0.81	0.15	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.55	0.21	0.06	0.15	0.00	0.02	0.00	0.00	0.00	0.00
	2y	0.87	0.12	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.66	0.25	0.03	0.05	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.92	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.70	0.26	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00
	4y	0.94	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.72	0.27	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	5y	0.96	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.72	0.27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.97	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.72	0.26	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.97	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.71	0.26	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.97	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.69	0.25	0.01	0.04	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.97	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.24	0.02	0.05	0.00	0.01	0.00	0.00	0.00	0.00
	R_a^2	0.93	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.68	0.25	0.02	0.04	0.00	0.01	0.00	0.00	0.00	0.00
UK	1y	0.90	0.02	0.07	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.39	0.31	0.23	0.02	0.04	0.00	0.00	0.00	0.00	0.00
	2y	0.95	0.02	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.54	0.33	0.11	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.96	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.61	0.33	0.04	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	4y	0.96	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.66	0.32	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	5y	0.96	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.69	0.30	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	6y	0.95	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.69	0.28	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.94	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.69	0.25	0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.93	0.06	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.22	0.09	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.92	0.07	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.65	0.20	0.13	0.01	0.01	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.94	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.62	0.28	0.08	0.01	0.01	0.00	0.00	0.00	0.00	0.00
Panel *B*		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
US	1y	0.90	0.39	0.17	-0.10	0.03	0.03	0.00	0.01	0.00	0.00	0.74	-0.46	-0.24	-0.39	0.06	0.16	0.04	0.00	0.00	0.00
	2y	0.93	0.34	0.08	-0.06	-0.01	0.00	0.00	-0.01	0.00	0.00	0.81	-0.50	-0.17	-0.23	-0.01	-0.04	-0.06	0.00	0.00	-0.01
	3y	0.96	0.28	0.02	-0.02	-0.02	-0.02	0.00	-0.01	0.00	0.00	0.84	-0.51	-0.10	-0.11	-0.04	-0.10	-0.03	0.00	0.00	0.01
	4y	0.97	0.23	-0.03	0.00	-0.02	-0.02	0.00	0.00	0.00	0.00	0.85	-0.52	-0.05	-0.02	-0.03	-0.10	0.01	0.00	0.00	0.01
	5y	0.98	0.19	-0.07	0.03	-0.01	-0.01	0.00	0.01	0.00	0.00	0.85	-0.52	0.00	0.06	-0.02	-0.07	0.04	0.00	0.00	0.00
	6y	0.98	0.15	-0.10	0.04	0.00	-0.01	0.00	0.01	0.00	0.00	0.85	-0.51	0.04	0.11	-0.01	-0.03	0.04	0.00	0.00	-0.01
	7y	0.98	0.11	-0.12	0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.84	-0.51	0.08	0.16	0.01	0.02	0.02	0.00	0.00	-0.01
	8y	0.98	0.08	-0.14	0.06	0.02	0.01	0.00	0.00	0.00	0.00	0.83	-0.50	0.11	0.20	0.02	0.07	-0.01	0.00	0.00	0.00
	9y	0.98	0.06	-0.16	0.07	0.02	0.02	0.00	-0.01	0.00	0.00	0.82	-0.49	0.14	0.23	0.04	0.11	-0.05	0.00	0.00	0.01
UK	1y	0.95	-0.13	0.27	0.10	0.05	-0.02	-0.01	0.00	0.00	0.00	0.62	0.56	-0.48	0.15	0.21	-0.05	0.00	-0.03	0.00	0.00
	2y	0.97	-0.14	0.17	0.06	-0.01	0.00	0.01	0.00	0.00	0.00	0.74	0.57	-0.34	0.11	-0.03	0.01	0.00	0.06	-0.02	0.00
	3y	0.98	-0.17	0.10	0.03	-0.03	0.01	0.01	0.00	0.00	0.00	0.78	0.57	-0.20	0.06	-0.11	0.03	0.00	0.03	0.01	0.00
	4y	0.98	-0.19	0.05	0.00	-0.03	0.01	0.00	0.00	0.00	0.00	0.81	0.57	-0.07	0.02	-0.11	0.03	0.00	-0.01	0.01	0.00
	5y	0.98	-0.21	0.01	-0.02	-0.02	0.01	-0.01	0.00	0.00	0.00	0.83	0.55	0.05	-0.02	-0.08	0.02	0.00	-0.04	0.00	0.00
	6y	0.97	-0.22	-0.02	-0.04	-0.01	0.01	-0.01	0.00	0.00	0.00	0.83	0.53	0.15	-0.05	-0.03	0.01	0.00	-0.04	-0.01	0.00
	7y	0.97	-0.24	-0.05	-0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.83	0.50	0.24	-0.08	0.02	-0.01	0.00	-0.02	-0.01	0.00
	8y	0.96	-0.25	-0.07	-0.07	0.02	-0.01	0.00	0.00	0.00	0.00	0.82	0.47	0.31	-0.09	0.07	-0.02	0.00	0.01	0.00	0.00
	9y	0.96	-0.26	-0.09	-0.08	0.03	-0.02	0.01	0.00	0.00	0.00	0.80	0.45	0.36	-0.11	0.11	-0.04	0.00	0.05	0.01	0.00

Table 2.4: In Panel *A* we report the R^2 obtained from the regression of each time series of U.S. and U.K. yields on the associated common principal components ($PC^{(c)}s$). In Panel *B* we report the associated correlations. We use weekly data for a sample period starting from January 1, 1986 to December 31, 2009 (1252 observations) and residual maturities from 1 to 9 years.

		Yield levels										Yield differences									
Panel *A*		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
US	1y	0.72	0.21	0.05	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.20	0.05	0.16	0.00	0.03	0.00	0.00	0.00	0.00
	2y	0.79	0.18	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.68	0.23	0.02	0.06	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.84	0.15	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.72	0.24	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00
	4y	0.88	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.74	0.24	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	5y	0.90	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.75	0.24	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	6y	0.91	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.74	0.24	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.92	0.07	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.73	0.24	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.92	0.06	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.71	0.23	0.01	0.04	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.92	0.05	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.69	0.22	0.02	0.06	0.00	0.01	0.00	0.00	0.00	0.00
	R_a^2	0.87	0.11	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.70	0.23	0.01	0.04	0.00	0.01	0.00	0.00	0.00	0.00
GER	1y	0.72	0.21	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.36	0.25	0.27	0.01	0.11	0.00	0.00	0.00	0.00	0.00
	2y	0.79	0.17	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.55	0.29	0.13	0.01	0.00	0.00	0.01	0.00	0.00	0.00
	3y	0.85	0.14	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.62	0.30	0.05	0.01	0.02	0.00	0.00	0.00	0.00	0.00
	4y	0.88	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.66	0.30	0.01	0.00	0.03	0.00	0.00	0.00	0.00	0.00
	5y	0.90	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.68	0.29	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00
	6y	0.91	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.68	0.28	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.91	0.07	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.26	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.90	0.07	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.64	0.24	0.10	0.01	0.01	0.00	0.00	0.00	0.00	0.00
	9y	0.89	0.06	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.59	0.22	0.14	0.02	0.04	0.00	0.01	0.00	0.00	0.00
	R_a^2	0.86	0.11	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.61	0.27	0.09	0.01	0.03	0.00	0.00	0.00	0.00	0.00
Panel *B*		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
US	1y	0.85	0.46	-0.22	-0.14	0.01	0.04	0.00	0.01	0.00	0.00	0.75	-0.45	-0.22	-0.40	0.02	0.16	0.01	0.04	0.00	0.00
	2y	0.89	0.43	-0.14	-0.09	0.00	-0.01	0.01	-0.01	0.00	0.00	0.82	-0.48	-0.15	-0.24	-0.03	-0.04	0.00	-0.06	0.00	0.00
	3y	0.92	0.39	-0.07	-0.04	0.00	-0.03	0.00	-0.01	0.00	0.00	0.85	-0.49	-0.09	-0.12	-0.03	-0.10	-0.01	-0.03	0.00	0.00
	4y	0.94	0.35	-0.02	0.00	0.00	-0.03	0.00	0.00	0.00	0.00	0.86	-0.49	-0.04	-0.02	-0.02	-0.10	-0.01	0.01	0.00	0.00
	5y	0.95	0.32	0.02	0.04	0.00	-0.02	0.00	0.00	0.00	0.00	0.86	-0.49	0.01	0.05	-0.01	-0.07	0.00	0.04	0.00	0.00
	6y	0.95	0.29	0.06	0.06	0.00	-0.01	0.00	0.01	0.00	0.00	0.86	-0.49	0.05	0.11	0.00	-0.03	0.00	0.04	0.00	0.00
	7y	0.96	0.26	0.09	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.85	-0.49	0.08	0.16	0.02	0.02	0.00	0.02	0.00	0.00
	8y	0.96	0.24	0.11	0.10	0.00	0.02	0.00	0.00	0.00	0.00	0.84	-0.48	0.11	0.20	0.03	0.07	0.00	-0.01	0.00	0.00
	9y	0.96	0.22	0.13	0.12	0.00	0.03	0.01	-0.01	0.00	0.00	0.83	-0.47	0.14	0.24	0.04	0.12	0.01	-0.05	0.00	0.00
GER	1y	0.85	-0.45	-0.23	0.10	0.06	0.00	-0.01	0.00	0.00	0.00	0.60	0.50	-0.52	0.10	0.33	-0.01	-0.07	0.00	0.01	0.00
	2y	0.89	-0.41	-0.18	0.06	0.00	0.00	0.01	0.00	0.00	0.00	0.74	0.54	-0.37	0.10	-0.03	-0.01	0.10	0.00	-0.02	0.00
	3y	0.92	-0.37	-0.11	0.03	-0.03	0.00	0.01	0.00	0.00	0.00	0.79	0.55	-0.21	0.07	-0.16	0.01	0.05	0.00	0.01	0.00
	4y	0.94	-0.34	-0.05	-0.01	-0.04	0.00	0.00	0.00	0.00	0.00	0.81	0.55	-0.08	0.04	-0.18	0.02	-0.02	0.00	0.02	0.00
	5y	0.95	-0.31	0.02	-0.03	-0.03	0.01	-0.01	0.00	0.00	0.00	0.83	0.54	0.05	0.00	-0.14	0.02	-0.06	0.00	0.00	0.00
	6y	0.95	-0.29	0.07	-0.06	-0.02	0.00	-0.01	0.00	0.00	0.00	0.83	0.53	0.15	-0.03	-0.07	0.01	-0.07	0.00	-0.01	0.00
	7y	0.95	-0.27	0.12	-0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.82	0.51	0.24	-0.07	0.02	0.00	-0.04	0.00	-0.02	0.00
	8y	0.95	-0.26	0.16	-0.09	0.02	0.00	0.00	0.00	0.00	0.00	0.80	0.49	0.32	-0.10	0.11	-0.01	0.01	0.00	-0.01	0.00
	9y	0.95	-0.24	0.19	-0.10	0.05	-0.01	0.01	0.00	0.00	0.00	0.77	0.46	0.37	-0.13	0.20	-0.03	0.08	0.00	0.02	0.00

Table 2.5: In Panel *A* we report the R^2 obtained from the regression of each time series of U.S. and German yields on the associated common principal components ($PC^{(c)}s$). In Panel *B* we report the associated correlations. We use weekly data for a sample period starting from January 1, 1986 to December 31, 2009 (1252 observations) and residual maturities from 1 to 9 years.

		Yield levels										Yield differences									
Panel *A*		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
US	1y	0.72	0.23	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.55	0.22	0.00	0.20	0.00	0.03	0.00	0.00	0.00	0.00
	2y	0.79	0.19	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.64	0.27	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.84	0.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.68	0.29	0.00	0.02	0.00	0.01	0.00	0.00	0.00	0.00
	4y	0.88	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.70	0.29	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	5y	0.91	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.71	0.28	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	6y	0.92	0.07	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.71	0.28	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.93	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.70	0.27	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.94	0.04	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.69	0.25	0.00	0.05	0.00	0.01	0.00	0.00	0.00	0.00
	9y	0.94	0.03	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.24	0.00	0.08	0.00	0.01	0.00	0.00	0.00	0.00
	R_a^2	0.87	0.11	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.26	0.00	0.05	0.00	0.01	0.00	0.00	0.00	0.00
JAP	1y	0.84	0.12	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.25	0.34	0.26	0.00	0.14	0.00	0.01	0.00	0.00	0.00
	2y	0.86	0.12	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.44	0.22	0.00	0.00	0.00	0.01	0.00	0.00	0.00
	3y	0.88	0.11	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.38	0.50	0.09	0.00	0.03	0.00	0.00	0.00	0.00	0.00
	4y	0.90	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.42	0.52	0.01	0.00	0.04	0.00	0.00	0.00	0.00	0.00
	5y	0.90	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.44	0.53	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00
	6y	0.91	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.44	0.51	0.03	0.00	0.01	0.00	0.01	0.00	0.00	0.00
	7y	0.91	0.08	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.49	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.91	0.08	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.40	0.45	0.14	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	9y	0.91	0.07	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.35	0.40	0.19	0.00	0.06	0.00	0.01	0.00	0.00	0.00
	R_a^2	0.89	0.10	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.38	0.46	0.11	0.00	0.03	0.00	0.00	0.00	0.00	0.00
Panel *B*		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
US	1y	0.85	-0.48	-0.23	0.06	0.03	-0.03	0.00	-0.01	0.00	0.00	0.74	-0.47	-0.05	-0.45	0.00	0.17	0.01	0.04	0.00	0.00
	2y	0.89	-0.44	-0.13	0.04	0.00	0.01	0.00	0.01	0.00	0.00	0.80	-0.52	-0.04	-0.28	-0.01	-0.04	0.00	-0.06	0.00	0.00
	3y	0.92	-0.39	-0.05	0.02	-0.01	0.02	0.00	0.00	0.00	0.00	0.83	-0.53	-0.02	-0.14	-0.01	-0.11	0.00	-0.03	0.00	0.00
	4y	0.94	-0.34	0.00	0.00	-0.02	0.02	0.00	0.00	0.00	0.00	0.84	-0.54	-0.01	-0.04	-0.01	-0.11	0.00	0.01	0.00	0.00
	5y	0.95	-0.30	0.05	-0.02	-0.01	0.02	0.00	-0.01	0.00	0.00	0.84	-0.53	0.00	0.05	-0.01	-0.07	0.00	0.04	0.00	0.00
	6y	0.96	-0.26	0.08	-0.03	-0.01	0.01	0.00	-0.01	0.00	0.00	0.84	-0.52	0.01	0.12	0.00	-0.03	0.00	0.04	0.00	0.00
	7y	0.97	-0.23	0.11	-0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.84	-0.52	0.01	0.18	0.00	0.02	0.00	0.02	0.00	0.00
	8y	0.97	-0.20	0.13	-0.05	0.01	-0.02	0.00	0.00	0.00	0.00	0.83	-0.50	0.02	0.23	0.01	0.07	0.00	-0.01	0.00	0.00
	9y	0.97	-0.18	0.15	-0.06	0.02	-0.03	0.00	0.01	0.00	0.00	0.82	-0.49	0.02	0.28	0.02	0.12	0.00	-0.05	0.00	0.00
JAP	1y	0.92	0.35	-0.11	-0.14	0.03	0.02	-0.01	0.00	0.00	0.00	0.50	0.58	-0.51	0.01	0.37	0.00	-0.09	0.00	0.01	0.00
	2y	0.93	0.34	-0.10	-0.10	0.00	0.00	0.01	0.00	0.00	0.00	0.57	0.67	-0.46	0.02	0.00	0.00	0.10	0.00	-0.03	0.00
	3y	0.94	0.33	-0.07	-0.05	-0.02	-0.01	0.01	0.00	0.00	0.00	0.62	0.71	-0.29	0.02	-0.16	0.00	0.06	0.00	0.01	0.00
	4y	0.95	0.32	-0.04	0.00	-0.03	-0.01	0.00	0.00	0.00	0.00	0.65	0.72	-0.12	0.01	-0.20	0.00	-0.01	0.00	0.02	0.00
	5y	0.95	0.31	-0.02	0.04	-0.02	-0.01	0.00	0.00	0.00	0.00	0.66	0.73	0.04	0.00	-0.17	0.01	-0.06	0.00	0.01	0.00
	6y	0.95	0.29	0.01	0.07	-0.02	-0.01	-0.01	0.00	0.00	0.00	0.66	0.72	0.17	-0.01	-0.10	0.00	-0.07	0.00	-0.01	0.00
	7y	0.95	0.28	0.03	0.08	0.00	0.00	-0.01	0.00	0.00	0.00	0.65	0.70	0.28	-0.02	0.00	0.00	-0.04	0.00	-0.02	0.00
	8y	0.96	0.27	0.06	0.09	0.02	0.01	0.00	0.00	0.00	0.00	0.63	0.67	0.37	-0.02	0.12	0.00	0.01	0.00	-0.01	0.00
	9y	0.96	0.26	0.08	0.09	0.04	0.02	0.01	0.00	0.00	0.00	0.59	0.63	0.43	-0.03	0.24	-0.01	0.10	0.00	0.02	0.00

Table 2.6: In Panel *A* we report the R^2 obtained from the regression of each time series of U.S. and Japanese yields on the associated common principal components ($PC^{(c)}s$). In Panel *B* we report the associated correlations. We use weekly data for a sample period starting from January 1, 1986 to December 31, 2009 (1252 observations) and residual maturities from 1 to 9 years.

		Yield levels										Yield differences									
Panel *A*		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
GER	1y	0.80	0.14	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.42	0.18	0.22	0.08	0.10	0.00	0.00	0.00	0.00	0.00
	2y	0.86	0.11	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.59	0.24	0.11	0.05	0.00	0.00	0.01	0.00	0.00	0.00
	3y	0.90	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.65	0.27	0.04	0.02	0.03	0.00	0.00	0.00	0.00	0.00
	4y	0.94	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.28	0.01	0.00	0.03	0.00	0.00	0.00	0.00	0.00
	5y	0.95	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.68	0.29	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00
	6y	0.95	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.68	0.29	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.95	0.03	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.65	0.29	0.04	0.02	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.94	0.02	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.61	0.27	0.07	0.04	0.01	0.00	0.00	0.00	0.00	0.00
	9y	0.93	0.02	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.56	0.25	0.09	0.05	0.04	0.00	0.01	0.00	0.00	0.00
	R^2_a	0.91	0.06	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.61	0.26	0.06	0.03	0.03	0.00	0.00	0.00	0.00	0.00
UK	1y	0.85	0.08	0.07	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.45	0.24	0.15	0.11	0.00	0.04	0.00	0.00	0.00	0.00
	2y	0.90	0.07	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.60	0.27	0.07	0.06	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.93	0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.27	0.03	0.02	0.00	0.01	0.00	0.00	0.00	0.00
	4y	0.95	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.71	0.27	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	5y	0.95	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.73	0.26	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	6y	0.95	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.72	0.25	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.95	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.70	0.24	0.04	0.03	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.94	0.05	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.22	0.06	0.04	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.93	0.05	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.63	0.21	0.08	0.06	0.00	0.01	0.00	0.00	0.00	0.00
	R^2_a	0.93	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.65	0.25	0.05	0.04	0.00	0.01	0.00	0.00	0.00	0.00
Panel *B*		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
GER	1y	0.89	0.38	0.22	-0.10	0.05	0.02	0.01	0.00	0.00	0.00	0.65	-0.42	-0.47	-0.28	0.32	0.02	-0.07	0.00	0.01	0.00
	2y	0.93	0.34	0.16	-0.04	-0.01	0.00	-0.01	0.00	0.00	0.00	0.77	-0.49	-0.32	-0.22	-0.04	0.00	0.10	0.00	-0.02	0.00
	3y	0.95	0.29	0.10	-0.01	-0.03	-0.01	-0.01	0.00	0.00	0.00	0.80	-0.52	-0.19	-0.13	-0.16	-0.01	0.05	0.00	0.01	0.00
	4y	0.97	0.25	0.03	0.02	-0.03	-0.02	0.00	0.00	0.00	0.00	0.82	-0.53	-0.08	-0.05	-0.18	-0.01	-0.02	0.00	0.02	0.00
	5y	0.98	0.21	-0.03	0.03	-0.03	-0.02	0.01	0.00	0.00	0.00	0.83	-0.54	0.03	0.02	-0.14	-0.01	-0.06	0.00	0.00	0.00
	6y	0.98	0.19	-0.09	0.04	-0.01	-0.01	0.01	0.00	0.00	0.00	0.82	-0.54	0.12	0.09	-0.07	-0.01	-0.07	0.00	-0.01	0.00
	7y	0.98	0.17	-0.14	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.81	-0.53	0.19	0.15	0.02	0.00	-0.04	0.00	-0.01	0.00
	8y	0.97	0.15	-0.18	0.05	0.03	0.01	0.00	0.00	0.00	0.00	0.78	-0.52	0.26	0.19	0.11	0.01	0.01	0.00	-0.01	0.00
	9y	0.96	0.14	-0.22	0.05	0.04	0.02	-0.01	0.00	0.00	0.00	0.75	-0.50	0.30	0.23	0.20	0.02	0.08	0.00	0.02	0.00
UK	1y	0.92	-0.27	0.26	0.08	0.04	-0.04	0.00	-0.01	0.00	0.00	0.67	0.49	-0.39	0.34	0.03	-0.21	0.00	0.03	0.00	0.00
	2y	0.95	-0.26	0.16	0.07	0.00	0.01	0.00	0.01	0.00	0.00	0.77	0.52	-0.27	0.24	-0.01	0.03	0.01	-0.06	0.00	0.02
	3y	0.97	-0.24	0.09	0.04	-0.02	0.02	0.00	0.00	0.00	0.00	0.82	0.52	-0.16	0.14	-0.03	0.11	0.00	-0.03	0.00	-0.01
	4y	0.97	-0.23	0.03	0.01	-0.02	0.02	0.00	0.00	0.00	0.00	0.84	0.52	-0.06	0.05	-0.02	0.12	0.00	0.01	0.00	-0.01
	5y	0.98	-0.22	-0.01	-0.02	-0.02	0.02	0.00	-0.01	0.00	0.00	0.85	0.51	0.04	-0.03	-0.01	0.08	-0.01	0.04	0.00	0.00
	6y	0.97	-0.21	-0.04	-0.04	-0.01	0.01	0.00	-0.01	0.00	0.00	0.85	0.50	0.12	-0.10	0.00	0.03	0.00	0.04	0.00	0.01
	7y	0.97	-0.21	-0.07	-0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.84	0.49	0.19	-0.16	0.01	-0.02	0.00	0.02	0.00	0.01
	8y	0.97	-0.21	-0.10	-0.08	0.00	-0.02	0.00	0.00	0.00	0.00	0.82	0.47	0.25	-0.21	0.02	-0.07	0.00	-0.01	0.00	0.00
	9y	0.96	-0.22	-0.12	-0.09	0.01	-0.03	0.00	0.01	0.00	0.00	0.80	0.46	0.29	-0.24	0.03	-0.11	0.01	-0.05	0.00	-0.01

Table 2.7: In Panel A we report the R^2 obtained from the regression of each time series of German and U.K. yields on the associated common principal components ($PC^{(c)}s$). In Panel B we report the associated correlations. We use weekly data for a sample period starting from January 1, 1986 to December 31, 2009 (1252 observations) and residual maturities from 1 to 9 years.

		Yield levels										Yield differences									
Panel *A*		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
GER	1y	0.84	0.07	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.42	0.18	0.04	0.26	0.00	0.10	0.00	0.00	0.00	0.00
	2y	0.89	0.07	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.56	0.28	0.03	0.12	0.00	0.00	0.00	0.01	0.00	0.00
	3y	0.93	0.06	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.60	0.31	0.01	0.04	0.00	0.03	0.00	0.00	0.00	0.00
	4y	0.95	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.63	0.33	0.00	0.01	0.00	0.03	0.00	0.00	0.00	0.00
	5y	0.95	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.64	0.34	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00
	6y	0.95	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.64	0.33	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.94	0.04	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.62	0.32	0.01	0.05	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.92	0.04	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.59	0.29	0.02	0.09	0.00	0.01	0.00	0.00	0.00	0.00
	9y	0.91	0.04	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.54	0.26	0.02	0.12	0.00	0.04	0.00	0.01	0.00	0.00
	R^2_a	0.92	0.05	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.58	0.29	0.01	0.08	0.00	0.03	0.00	0.00	0.00	0.00
JAP	1y	0.89	0.08	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.27	0.23	0.03	0.14	0.00	0.01	0.00	0.00	0.00
	2y	0.91	0.07	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.45	0.32	0.19	0.02	0.00	0.00	0.01	0.00	0.00	0.00
	3y	0.93	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.53	0.36	0.08	0.01	0.03	0.00	0.00	0.00	0.00	0.00
	4y	0.95	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.37	0.01	0.00	0.04	0.00	0.00	0.00	0.00	0.00
	5y	0.96	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.59	0.38	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00
	6y	0.96	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.59	0.37	0.03	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	7y	0.96	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.35	0.07	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.95	0.04	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.53	0.32	0.13	0.01	0.01	0.00	0.00	0.00	0.00	0.00
	9y	0.94	0.04	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.47	0.28	0.17	0.02	0.06	0.00	0.01	0.00	0.00	0.00
	R^2_a	0.94	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.51	0.33	0.10	0.01	0.03	0.00	0.00	0.00	0.00	0.00
Panel *B*		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
GER	1y	0.92	0.26	0.28	-0.07	0.05	0.02	0.00	-0.01	0.00	0.00	0.65	-0.43	-0.20	-0.51	-0.01	-0.32	0.01	0.07	0.00	-0.01
	2y	0.94	0.26	0.21	-0.03	0.00	0.00	0.00	0.01	0.00	0.00	0.75	-0.53	-0.16	-0.35	0.01	0.04	-0.02	-0.10	0.00	0.02
	3y	0.96	0.24	0.13	0.00	-0.03	-0.01	0.00	0.01	0.00	0.00	0.78	-0.56	-0.11	-0.20	0.01	0.16	-0.01	-0.05	0.00	-0.01
	4y	0.97	0.23	0.04	0.02	-0.04	-0.01	0.00	0.00	0.00	0.00	0.79	-0.58	-0.05	-0.07	0.01	0.18	0.00	0.02	0.00	-0.02
	5y	0.97	0.22	-0.03	0.03	-0.03	-0.01	0.00	-0.01	0.00	0.00	0.80	-0.58	0.00	0.04	0.01	0.14	0.01	0.06	0.00	0.00
	6y	0.97	0.21	-0.09	0.03	-0.02	0.00	0.00	-0.01	0.00	0.00	0.80	-0.58	0.05	0.14	0.01	0.06	0.01	0.07	0.00	0.01
	7y	0.97	0.20	-0.14	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.79	-0.56	0.09	0.23	0.00	-0.02	0.01	0.04	0.00	0.01
	8y	0.96	0.20	-0.19	0.03	0.02	0.01	0.00	0.00	0.00	0.00	0.77	-0.54	0.13	0.30	-0.01	-0.12	0.00	-0.01	0.00	0.01
	9y	0.95	0.20	-0.22	0.02	0.04	0.02	0.00	0.01	0.00	0.00	0.74	-0.51	0.16	0.35	-0.02	-0.20	-0.02	-0.08	0.00	-0.02
JAP	1y	0.94	-0.29	0.13	0.11	0.04	-0.02	-0.01	0.00	0.00	0.00	0.57	0.52	-0.48	0.17	0.37	-0.02	-0.09	0.01	0.01	0.00
	2y	0.95	-0.27	0.10	0.08	0.01	0.00	0.01	0.00	0.00	0.00	0.67	0.57	-0.44	0.15	0.00	-0.01	0.10	-0.01	-0.03	0.00
	3y	0.97	-0.24	0.06	0.04	-0.01	0.02	0.01	0.00	0.00	0.00	0.73	0.60	-0.28	0.09	-0.16	0.00	0.06	-0.01	0.01	0.00
	4y	0.97	-0.22	0.03	0.01	-0.02	0.02	0.00	0.00	0.00	0.00	0.76	0.61	-0.11	0.03	-0.20	0.01	-0.01	0.00	0.02	0.00
	5y	0.98	-0.20	-0.01	-0.02	-0.02	0.02	0.00	0.00	0.00	0.00	0.77	0.61	0.04	-0.02	-0.17	0.01	-0.06	0.01	0.01	0.00
	6y	0.98	-0.19	-0.04	-0.05	-0.02	0.02	-0.01	0.00	0.00	0.00	0.77	0.61	0.17	-0.06	-0.10	0.01	-0.07	0.01	-0.01	0.00
	7y	0.98	-0.19	-0.06	-0.07	-0.01	0.00	-0.01	0.00	0.00	0.00	0.75	0.59	0.27	-0.09	0.00	0.01	-0.04	0.01	-0.02	0.00
	8y	0.98	-0.19	-0.08	-0.08	0.01	-0.02	0.00	0.00	0.00	0.00	0.73	0.56	0.36	-0.11	0.12	0.00	0.01	0.00	-0.01	0.00
	9y	0.97	-0.19	-0.10	-0.08	0.02	-0.04	0.01	0.00	0.00	0.00	0.69	0.53	0.41	-0.13	0.24	-0.01	0.10	-0.02	0.02	0.00

Table 2.8: In Panel *A* we report the R^2 obtained from the regression of each time series of German and Japanese yields on the associated common principal components ($PC^{(c)}s$). In Panel *B* we report the associated correlations. We use weekly data for a sample period starting from January 1, 1986 to December 31, 2009 (1252 observations) and residual maturities from 1 to 9 years.

		Yield levels										Yield differences									
Panel *A*		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
UK	1y	0.87	0.08	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.25	0.01	0.27	0.00	0.04	0.00	0.00	0.00	0.00
	2y	0.92	0.07	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.55	0.31	0.00	0.13	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.94	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.61	0.33	0.00	0.05	0.00	0.01	0.00	0.00	0.00	0.00
	4y	0.96	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.65	0.33	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00
	5y	0.97	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.67	0.32	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	6y	0.97	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.66	0.31	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.97	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.64	0.29	0.00	0.06	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.97	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.62	0.28	0.00	0.10	0.00	0.01	0.00	0.00	0.00	0.00
	9y	0.96	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.58	0.26	0.00	0.14	0.00	0.01	0.00	0.00	0.00	0.00
	R_a^2	0.95	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.60	0.30	0.00	0.09	0.00	0.01	0.00	0.00	0.00	0.00
JAP	1y	0.92	0.04	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.28	0.26	0.00	0.14	0.00	0.01	0.00	0.00	0.00
	2y	0.94	0.04	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.38	0.39	0.22	0.00	0.00	0.00	0.01	0.00	0.00	0.00
	3y	0.95	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.46	0.09	0.00	0.02	0.00	0.00	0.00	0.00	0.00
	4y	0.96	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.46	0.49	0.01	0.00	0.04	0.00	0.00	0.00	0.00	0.00
	5y	0.96	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.47	0.50	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00
	6y	0.96	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.47	0.48	0.03	0.00	0.01	0.00	0.01	0.00	0.00	0.00
	7y	0.96	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.47	0.45	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.96	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.44	0.41	0.14	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	9y	0.96	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.40	0.35	0.18	0.00	0.06	0.00	0.01	0.00	0.00	0.00
	R_a^2	0.95	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.42	0.42	0.11	0.00	0.03	0.00	0.00	0.00	0.00	0.00
Panel *B*		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
UK	1y	0.93	0.27	0.23	-0.05	0.04	0.03	0.00	-0.01	0.00	0.00	0.65	-0.50	-0.08	0.52	0.01	0.21	0.00	-0.03	0.00	0.00
	2y	0.96	0.26	0.12	-0.04	0.00	0.00	0.00	0.01	0.00	0.00	0.74	-0.56	-0.06	0.36	-0.02	-0.03	0.00	0.06	0.00	-0.02
	3y	0.97	0.23	0.04	-0.03	-0.02	-0.02	0.00	0.01	0.00	0.00	0.78	-0.57	-0.04	0.22	-0.02	-0.11	0.00	0.03	0.00	0.01
	4y	0.98	0.20	-0.01	-0.01	-0.02	-0.02	0.00	0.00	0.00	0.00	0.81	-0.57	-0.02	0.08	-0.01	-0.12	0.00	-0.01	0.00	0.01
	5y	0.98	0.18	-0.05	0.01	-0.02	-0.01	0.00	-0.01	0.00	0.00	0.82	-0.57	0.00	-0.04	0.00	-0.08	0.00	-0.04	0.00	0.00
	6y	0.98	0.15	-0.08	0.03	-0.01	-0.01	0.00	-0.01	0.00	0.00	0.81	-0.56	0.02	-0.15	0.00	-0.03	0.00	-0.04	0.00	-0.01
	7y	0.98	0.14	-0.10	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.80	-0.54	0.03	-0.24	0.00	0.02	0.00	-0.02	0.00	-0.01
	8y	0.98	0.12	-0.12	0.05	0.01	0.01	0.00	0.00	0.00	0.00	0.79	-0.53	0.05	-0.31	0.01	0.07	0.00	0.01	0.00	0.00
	9y	0.98	0.10	-0.14	0.07	0.02	0.03	0.00	0.01	0.00	0.00	0.76	-0.51	0.06	-0.37	0.01	0.11	0.01	0.05	0.00	0.01
JAP	1y	0.96	-0.20	0.18	0.09	0.02	-0.02	-0.01	0.00	0.00	0.00	0.56	0.53	-0.51	-0.04	0.37	0.00	-0.09	0.00	0.01	0.00
	2y	0.97	-0.20	0.14	0.06	-0.01	0.00	0.01	0.00	0.00	0.00	0.62	0.62	-0.46	-0.05	0.00	0.00	0.10	0.00	-0.03	0.00
	3y	0.98	-0.20	0.09	0.02	-0.02	0.01	0.01	0.00	0.00	0.00	0.65	0.68	-0.29	-0.03	-0.16	0.00	0.06	0.00	0.01	0.00
	4y	0.98	-0.19	0.04	-0.01	-0.02	0.02	0.00	0.00	0.00	0.00	0.68	0.70	-0.12	-0.01	-0.20	0.01	-0.01	0.00	0.02	0.00
	5y	0.98	-0.19	0.00	-0.03	-0.02	0.02	0.00	0.00	0.00	0.00	0.69	0.70	0.04	0.01	-0.17	0.01	-0.06	0.00	0.01	0.00
	6y	0.98	-0.18	-0.04	-0.05	-0.01	0.01	-0.01	0.00	0.00	0.00	0.69	0.69	0.17	0.02	-0.10	0.00	-0.07	0.00	-0.01	0.00
	7y	0.98	-0.17	-0.07	-0.05	0.00	0.00	-0.01	0.00	0.00	0.00	0.68	0.67	0.28	0.04	0.00	0.00	-0.04	0.00	-0.02	0.00
	8y	0.98	-0.16	-0.09	-0.05	0.02	-0.01	0.00	0.00	0.00	0.00	0.66	0.64	0.37	0.05	0.12	0.00	0.01	0.00	-0.01	0.00
	9y	0.98	-0.15	-0.11	-0.04	0.04	-0.03	0.01	0.00	0.00	0.00	0.63	0.59	0.43	0.06	0.24	-0.01	0.10	0.00	0.02	0.00

Table 2.9: In Panel *A* we report the R^2 obtained from the regression of each time series of U.K. and Japanese yields on the associated common principal components ($PC^{(c)}s$). In Panel *B* we report the associated correlations. We use weekly data for a sample period starting from January 1, 1986 to December 31, 2009 (1252 observations) and residual maturities from 1 to 9 years.

The 3-country case. 1^{st} step : Principal Component Analysis of international bond markets

Table 2.10: We report principal component analysis of the term structure of interest rates of three countries taken separately and jointly. The first column lists the number of principal components (four for a single country, and nine for any 3-country case), while the rest of the table is divided into four vertical blocks. Each of the first, second and third blocks represents an individual country, while the fourth block represents the three countries jointly. The first and second columns (third and fourth, respectively) of each block report marginal and cumulative percent variance explained for yield levels (yield differences, respectively). For each country we use weekly yields observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

	PCs												$PC^{(c)}s$			
	U.S.				*U.K.*				*GER*				$U.S. - U.K. - GER$			
	Levels		Differences		Levels		Differences		Levels		Differences		Levels		Differences	
	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.
1	97.22	97.22	93.55	93.55	98.10	98.10	89.96	89.96	96.86	96.86	87.59	87.59	88.45	88.45	56.04	56.04
2	2.70	99.93	5.42	98.97	1.80	99.90	8.81	98.77	3.00	99.86	9.33	96.91	7.60	96.05	17.89	73.93
3	0.07	100.00	0.88	99.85	0.09	99.99	1.09	99.87	0.14	99.99	2.67	99.58	1.98	98.03	16.60	90.52
4	0.00	100.00	0.14	99.99	0.01	100.00	0.12	99.99	0.01	100.00	0.40	99.98	1.38	99.41	4.00	94.52
5	-	-	-	-	-	-	-	-	-	-	-	-	0.38	99.78	2.21	96.73
6	-	-	-	-	-	-	-	-	-	-	-	-	0.14	99.92	1.50	98.23
7	-	-	-	-	-	-	-	-	-	-	-	-	0.04	99.96	0.90	99.14
8	-	-	-	-	-	-	-	-	-	-	-	-	0.02	99.98	0.36	99.50
9	-	-	-	-	-	-	-	-	-	-	-	-	0.01	100.00	0.27	99.77
	U.S.				*U.K.*				*JAP*				$U.S. - U.K. - JAP$			
	Levels		Differences		Levels		Differences		Levels		Differences		Levels		Differences	
	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.
1	97.22	97.22	93.55	93.55	98.10	98.10	89.96	89.96	98.72	98.72	84.67	84.67	90.43	90.43	46.34	46.34
2	2.70	99.93	5.42	98.97	1.80	99.90	8.81	98.77	1.18	99.91	11.34	96.01	6.88	97.30	25.34	71.68
3	0.07	100.00	0.88	99.85	0.09	99.99	1.09	99.87	0.09	99.99	3.47	99.48	1.23	98.53	17.84	89.52
4	0.00	100.00	0.14	99.99	0.01	100.00	0.12	99.99	0.01	100.00	0.48	99.97	0.96	99.50	3.80	93.33
5	-	-	-	-	-	-	-	-	-	-	-	-	0.34	99.84	3.05	96.38
6	-	-	-	-	-	-	-	-	-	-	-	-	0.10	99.94	1.55	97.93
7	-	-	-	-	-	-	-	-	-	-	-	-	0.03	99.97	1.15	99.09
8	-	-	-	-	-	-	-	-	-	-	-	-	0.01	99.98	0.37	99.46
9	-	-	-	-	-	-	-	-	-	-	-	-	0.01	100.00	0.28	99.74

	PCs												$PC^{(c)}s$			
	U.S.				*GER*				*JAP*				$U.S. - GER - JAP$			
	Levels		Differences		Levels		Differences		Levels		Differences		Levels		Differences	
	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.
1	97.22	97.22	93.55	93.55	96.86	96.86	87.59	87.59	98.72	98.72	84.67	84.67	86.47	86.47	47.91	47.91
2	2.70	99.93	5.42	98.97	3.00	99.86	9.33	96.91	1.18	99.91	11.34	96.01	8.40	94.87	24.52	72.43
3	0.07	100.00	0.88	99.85	0.14	99.99	2.67	99.58	0.09	99.99	3.47	99.48	3.33	98.20	16.23	88.66
4	0.00	100.00	0.14	99.99	0.01	100.00	0.40	99.98	0.01	100.00	0.48	99.97	1.19	99.39	3.93	92.60
5	-	-	-	-	-	-	-	-	-	-	-	-	0.35	99.74	3.15	95.75
6	-	-	-	-	-	-	-	-	-	-	-	-	0.18	99.93	1.58	97.32
7	-	-	-	-	-	-	-	-	-	-	-	-	0.04	99.97	1.16	98.48
8	-	-	-	-	-	-	-	-	-	-	-	-	0.02	99.98	0.89	99.36
9	-	-	-	-	-	-	-	-	-	-	-	-	0.01	100.00	0.28	99.64
	U.K.				*GER*				*JAP*				$U.K. - GER - JAP$			
	Levels		Differences		Levels		Differences		Levels		Differences		Levels		Differences	
	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.
1	98.10	98.10	89.96	89.96	96.86	96.86	87.59	87.59	98.72	98.72	84.67	84.67	91.79	91.79	46.62	46.62
2	1.80	99.90	8.81	98.77	3.00	99.86	9.33	96.91	1.18	99.91	11.34	96.01	4.03	95.83	24.29	70.91
3	0.09	99.99	1.09	99.87	0.14	99.99	2.67	99.58	0.09	99.99	3.47	99.48	2.45	98.28	16.55	87.47
4	0.01	100.00	0.12	99.99	0.01	100.00	0.40	99.98	0.01	100.00	0.48	99.97	1.38	99.66	4.08	91.55
5	-	-	-	-	-	-	-	-	-	-	-	-	0.17	99.83	3.51	95.06
6	-	-	-	-	-	-	-	-	-	-	-	-	0.10	99.93	2.20	97.26
7	-	-	-	-	-	-	-	-	-	-	-	-	0.04	99.97	1.15	98.41
8	-	-	-	-	-	-	-	-	-	-	-	-	0.02	99.99	0.89	99.29
9	-	-	-	-	-	-	-	-	-	-	-	-	0.01	100.00	0.36	99.65

2^{nd} and 3^{rd} step : Regressing yields on $PC^{(c)}s$ and associated correlations

Panel *A*		Yield levels 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Yield differences 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
US	1y	0.71	0.20	0.05	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.48	0.18	0.10	0.06	0.00	0.15	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.78	0.17	0.04	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.58	0.21	0.12	0.03	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.84	0.14	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.62	0.23	0.12	0.01	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.87	0.11	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.63	0.23	0.12	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.90	0.09	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.64	0.23	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.91	0.07	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.64	0.23	0.12	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.92	0.06	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.63	0.22	0.12	0.01	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.93	0.05	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.62	0.21	0.11	0.01	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.93	0.04	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.60	0.20	0.11	0.02	0.00	0.05	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	R^2_a	0.87	0.10	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.60	0.22	0.11	0.02	0.00	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
UK	1y	0.88	0.01	0.01	0.09	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.34	0.03	0.13	0.11	0.01	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.93	0.01	0.02	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.47	0.36	0.04	0.06	0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.96	0.00	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.53	0.36	0.05	0.02	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.97	0.00	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.58	0.36	0.05	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.97	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.60	0.34	0.05	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.96	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.61	0.31	0.05	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.96	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.60	0.28	0.05	0.03	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.95	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.59	0.25	0.05	0.06	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.94	0.00	0.05	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.56	0.22	0.05	0.08	0.06	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	R^2_a	0.95	0.00	0.03	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.54	0.31	0.05	0.05	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GER	1y	0.70	0.22	0.03	0.03	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00	0.28	0.19	0.08	0.00	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.76	0.18	0.03	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00	0.34	0.09	0.05	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	3y	0.82	0.15	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.56	0.00	0.36	0.03	0.02	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.86	0.13	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.59	0.00	0.37	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.88	0.11	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.60	0.01	0.37	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.89	0.09	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.60	0.01	0.36	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.89	0.08	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.59	0.01	0.34	0.04	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.89	0.08	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.56	0.01	0.32	0.07	0.04	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.88	0.07	0.00	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.51	0.01	0.29	0.09	0.05	0.01	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	R^2_a	0.84	0.12	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.54	0.01	0.34	0.06	0.03	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 2.11: In Panel *A* we report the R^2 obtained from the regression of each time series of U.S., U.K. and German yields on the associated common principal components ($PC^{(c)}s$). In Panel *B* (following table) we report the associated correlations. We use weekly data for a sample period starting from January 1, 1986 to December 31, 2009 (1252 observations) and residual maturities from 1 to 9 years.

		Yield levels															Yield differences														
Panel *B*		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
US	1y	0.84	0.45	-0.23	-0.11	0.14	0.05	0.01	0.03	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.69	-0.42	-0.32	0.24	-0.03	-0.39	0.02	0.04	-0.16	-0.01	-0.04	0.00	0.00	0.00	0.00
	2y	0.88	0.41	-0.20	-0.04	0.08	0.02	0.00	0.00	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.76	-0.46	-0.34	0.17	-0.02	-0.23	-0.03	-0.01	0.04	0.00	0.06	0.00	0.00	0.00	0.01
	3y	0.91	0.37	-0.16	0.00	0.03	0.01	-0.01	-0.02	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.79	-0.48	-0.35	0.11	-0.01	-0.11	-0.03	-0.02	0.10	0.01	0.03	0.00	0.00	0.00	-0.01
	4y	0.93	0.33	-0.12	0.04	-0.01	0.00	-0.01	-0.02	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.80	-0.48	-0.35	0.06	-0.01	-0.01	-0.03	-0.02	0.10	0.01	-0.01	0.00	0.00	0.00	-0.01
	5y	0.95	0.30	-0.09	0.06	-0.04	-0.01	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.80	-0.48	-0.34	0.01	0.00	0.06	-0.01	-0.01	0.07	0.00	-0.04	0.00	0.00	0.00	0.00
	6y	0.96	0.27	-0.06	0.08	-0.07	-0.01	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.80	-0.48	-0.34	-0.03	0.00	0.11	0.00	0.00	0.03	0.00	-0.04	0.00	0.00	0.00	0.01
	7y	0.96	0.24	-0.04	0.10	-0.09	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.79	-0.47	-0.34	-0.07	0.01	0.16	0.02	0.00	-0.02	0.00	-0.02	0.00	0.00	0.00	0.01
	8y	0.96	0.22	-0.02	0.11	-0.11	-0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.78	-0.46	-0.34	-0.10	0.01	0.20	0.03	0.01	-0.07	-0.01	0.01	0.00	0.00	0.00	0.00
	9y	0.96	0.20	0.00	0.12	-0.12	-0.01	0.01	0.02	0.02	0.00	-0.01	0.00	0.00	0.00	0.00	0.77	-0.45	-0.33	-0.13	0.02	0.23	0.04	0.02	-0.11	-0.01	0.05	0.00	0.00	0.00	-0.01
UK	1y	0.94	0.09	0.11	-0.30	0.00	-0.08	0.04	0.03	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.59	-0.18	0.36	-0.33	0.10	0.04	0.20	0.03	0.00	0.00	-0.03	0.00	0.00	0.00
	2y	0.97	0.08	0.12	-0.20	0.01	-0.06	0.00	-0.01	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.68	0.60	-0.21	0.25	-0.24	0.07	-0.01	-0.03	0.00	-0.01	0.00	0.06	0.00	-0.02	0.00
	3y	0.98	0.05	0.14	-0.13	0.01	-0.03	-0.02	-0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.73	0.60	-0.21	0.15	-0.14	0.04	-0.02	-0.11	-0.02	0.00	0.00	0.03	0.00	0.01	0.00
	4y	0.98	0.03	0.16	-0.07	0.01	-0.01	-0.02	-0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.76	0.60	-0.22	0.05	-0.05	0.01	-0.02	-0.11	-0.02	0.00	0.00	-0.01	0.00	0.01	0.00
	5y	0.98	0.02	0.17	-0.04	0.01	0.01	-0.02	-0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.78	0.58	-0.22	-0.04	0.03	-0.02	-0.02	-0.08	-0.02	0.00	0.00	-0.04	0.00	0.00	0.00
	6y	0.98	0.01	0.19	-0.01	0.01	0.03	-0.01	-0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.78	0.56	-0.23	-0.12	0.10	-0.04	0.00	-0.03	-0.01	0.00	0.00	-0.04	0.00	-0.01	0.00
	7y	0.98	0.00	0.20	0.02	0.01	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.78	0.53	-0.23	-0.19	0.16	-0.05	0.01	0.02	0.01	0.00	0.00	-0.02	0.00	-0.01	0.00
	8y	0.97	-0.01	0.22	0.04	0.01	0.06	0.00	0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.77	0.50	-0.23	-0.24	0.21	-0.06	0.02	0.07	0.02	0.00	0.00	0.01	0.00	0.00	0.00
	9y	0.97	-0.01	0.23	0.06	0.01	0.08	0.01	0.02	-0.02	0.00	-0.01	0.00	0.00	0.00	0.00	0.75	0.47	-0.23	-0.28	0.24	-0.07	0.02	0.11	0.03	0.00	0.00	0.05	0.00	0.01	0.00
GER	1y	0.84	-0.47	-0.19	-0.18	-0.09	0.07	0.04	-0.02	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.58	0.02	0.53	0.44	0.28	0.06	0.33	-0.02	0.01	0.07	0.00	0.00	0.01	0.00	0.00
	2y	0.87	-0.43	-0.17	-0.13	-0.06	0.03	-0.01	0.00	0.00	-0.01	0.00	-0.01	0.00	0.00	0.00	0.71	-0.02	0.59	0.31	0.22	0.06	-0.03	0.00	0.01	-0.10	0.00	0.00	-0.02	0.00	0.00
	3y	0.91	-0.39	-0.14	-0.07	-0.03	0.00	-0.03	0.01	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.75	-0.04	0.60	0.18	0.14	0.05	-0.16	0.01	0.00	-0.05	0.00	0.00	0.01	0.00	0.00
	4y	0.93	-0.35	-0.11	-0.02	0.00	-0.01	-0.03	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.77	-0.06	0.61	0.06	0.05	0.03	-0.17	0.02	-0.01	0.02	0.00	0.00	0.02	0.00	0.00
	5y	0.94	-0.33	-0.08	0.04	0.03	-0.02	-0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.78	-0.07	0.61	-0.04	-0.02	0.01	-0.14	0.02	-0.02	0.06	0.00	0.00	0.00	0.00	0.00
	6y	0.94	-0.31	-0.05	0.09	0.05	-0.03	-0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.78	-0.08	0.60	-0.12	-0.09	-0.02	-0.07	0.01	-0.01	0.07	0.00	0.00	-0.01	0.00	0.00
	7y	0.95	-0.29	-0.03	0.13	0.06	-0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.77	-0.09	0.58	-0.20	-0.15	-0.04	0.02	0.00	0.00	0.04	0.00	0.00	-0.01	0.00	0.00
	8y	0.94	-0.27	-0.01	0.17	0.07	-0.03	0.03	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.75	-0.10	0.56	-0.26	-0.19	-0.06	0.11	-0.01	0.01	-0.01	0.00	0.00	-0.01	0.00	0.00
	9y	0.94	-0.26	0.00	0.20	0.08	-0.03	0.04	-0.02	0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.71	-0.10	0.53	-0.30	-0.23	-0.08	0.19	-0.02	0.03	-0.08	0.00	0.00	0.02	0.00	0.00

Panel *A*		Yield levels 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
US	1y	0.70	0.24	0.00	0.04	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.78	0.20	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.83	0.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.87	0.13	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.89	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.91	0.07	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.92	0.06	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.93	0.04	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.93	0.03	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.86	0.11	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UK	1y	0.89	0.00	0.07	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.94	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.97	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.97	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.98	0.00	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.98	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.97	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.97	0.01	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.96	0.01	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.96	0.00	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
JAP	1y	0.84	0.11	0.00	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.86	0.11	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.88	0.10	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.89	0.09	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.90	0.09	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.91	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.91	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.91	0.07	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.91	0.07	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.89	0.09	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Panel *A*		Yield differences 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
US	1y	0.50	0.05	0.21	0.01	0.05	0.15	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.60	0.06	0.25	0.00	0.02	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.64	0.06	0.26	0.00	0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.65	0.06	0.27	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.66	0.06	0.27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.66	0.06	0.26	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.66	0.05	0.26	0.00	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.64	0.05	0.25	0.00	0.01	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.63	0.05	0.23	0.00	0.02	0.05	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.63	0.06	0.25	0.00	0.01	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
UK	1y	0.35	0.04	0.31	0.01	0.22	0.02	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.49	0.05	0.33	0.00	0.11	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.56	0.05	0.33	0.00	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.61	0.05	0.32	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.63	0.05	0.30	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.64	0.05	0.28	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.64	0.05	0.25	0.00	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.62	0.05	0.22	0.00	0.09	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.60	0.04	0.20	0.01	0.12	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.57	0.05	0.28	0.00	0.07	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
JAP	1y	0.13	0.46	0.00	0.26	0.01	0.00	0.14	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	2y	0.16	0.61	0.00	0.21	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	3y	0.18	0.70	0.00	0.08	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.20	0.74	0.00	0.01	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.21	0.75	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.22	0.74	0.00	0.03	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	7y	0.22	0.70	0.00	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.21	0.64	0.00	0.13	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.19	0.56	0.00	0.18	0.01	0.00	0.06	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.19	0.66	0.00	0.11	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 2.12: In Panel *A* we report the R^2 obtained from the regression of each time series of U.S., U.K. and Japanese yields on the associated common principal components ($PC^{(c)}s$). In Panel *B* (following table) we report the associated correlations. We use weekly data for a sample period starting from January 1, 1986 to December 31, 2009 (1252 observations) and residual maturities from 1 to 9 years.

		Yield levels															Yield differences														
Panel *B*		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
US	1y	0.84	0.49	0.03	-0.20	0.12	0.04	0.02	0.01	0.03	0.00	0.00	-0.01	0.00	0.00	0.00	0.71	-0.22	-0.46	0.07	-0.23	-0.39	0.01	0.06	-0.16	-0.01	-0.04	0.00	0.00	0.00	0.00
	2y	0.88	0.45	0.06	-0.12	0.08	0.01	0.00	-0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.78	-0.25	-0.50	0.06	-0.16	-0.23	-0.01	-0.01	0.04	0.00	0.06	0.00	0.00	0.00	0.01
	3y	0.91	0.40	0.07	-0.06	0.03	0.00	-0.01	-0.01	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.80	-0.25	-0.51	0.04	-0.10	-0.11	-0.01	-0.03	0.10	0.00	0.03	0.00	0.00	0.00	-0.01
	4y	0.93	0.35	0.08	-0.01	-0.01	-0.01	-0.01	-0.01	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.81	-0.25	-0.52	0.02	-0.05	-0.02	-0.01	-0.03	0.10	0.00	-0.01	0.00	0.00	0.00	-0.01
	5y	0.95	0.31	0.08	0.03	-0.04	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.81	-0.25	-0.52	0.00	0.00	0.06	0.00	-0.02	0.07	0.00	-0.04	0.00	0.00	0.00	0.00
	6y	0.95	0.27	0.08	0.05	-0.06	-0.01	0.00	0.00	-0.01	0.00	0.00	-0.01	0.00	0.00	0.00	0.81	-0.24	-0.51	-0.01	0.04	0.11	0.00	-0.01	0.03	0.00	-0.04	0.00	0.00	0.00	0.01
	7y	0.96	0.24	0.09	0.08	-0.08	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.81	-0.23	-0.51	-0.02	0.08	0.16	0.01	0.01	-0.02	0.00	-0.02	0.00	0.00	0.00	0.01
	8y	0.96	0.21	0.09	0.09	-0.09	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.80	-0.23	-0.50	-0.03	0.11	0.20	0.01	0.02	-0.07	0.00	0.01	0.00	0.00	0.00	0.00
	9y	0.97	0.18	0.09	0.11	-0.10	0.00	0.01	0.01	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.79	-0.22	-0.48	-0.04	0.14	0.23	0.02	0.03	-0.11	0.00	0.05	0.00	0.00	0.00	-0.01
UK	1y	0.95	0.06	-0.26	-0.16	-0.05	-0.05	0.04	0.02	-0.02	0.00	-0.01	0.00	0.00	0.00	0.00	0.59	-0.19	0.56	0.09	-0.47	0.15	0.01	0.21	0.04	0.00	0.00	-0.03	0.00	0.00	0.00
	2y	0.97	0.05	-0.22	-0.07	-0.02	-0.04	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.70	-0.23	0.57	0.06	-0.33	0.11	-0.01	-0.03	-0.01	0.00	0.00	0.06	0.00	-0.02	0.00
	3y	0.98	0.03	-0.18	0.00	0.00	-0.03	-0.02	-0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.75	-0.23	0.57	0.04	-0.19	0.06	-0.01	-0.11	-0.03	0.00	0.00	0.03	0.00	0.01	0.00
	4y	0.99	0.00	-0.15	0.04	0.01	-0.01	-0.02	-0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.78	-0.23	0.57	0.02	-0.07	0.02	-0.01	-0.11	-0.03	0.00	0.00	-0.01	0.00	0.01	0.00
	5y	0.99	-0.02	-0.13	0.07	0.02	0.00	-0.02	-0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.80	-0.23	0.55	-0.01	0.05	-0.02	-0.01	-0.08	-0.02	0.00	0.00	-0.04	0.00	0.00	0.00
	6y	0.99	-0.04	-0.11	0.10	0.03	0.02	-0.01	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.80	-0.22	0.53	-0.03	0.15	-0.05	0.00	-0.03	-0.01	0.00	0.00	-0.04	0.00	-0.01	0.00
	7y	0.99	-0.06	-0.10	0.12	0.03	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.80	-0.22	0.50	-0.05	0.23	-0.08	0.00	0.02	0.01	0.00	0.00	-0.02	0.00	-0.01	0.00
	8y	0.98	-0.07	-0.08	0.14	0.04	0.05	0.01	0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.79	-0.22	0.47	-0.06	0.30	-0.09	0.00	0.07	0.02	0.00	0.00	0.01	0.00	0.00	0.00
	9y	0.98	-0.09	-0.07	0.15	0.04	0.06	0.02	0.01	-0.02	0.00	0.01	0.00	0.00	0.00	0.00	0.77	-0.21	0.45	-0.07	0.35	-0.11	0.00	0.11	0.04	-0.01	0.00	0.05	0.00	0.01	0.00
JAP	1y	0.92	-0.34	0.02	-0.16	-0.11	0.06	0.02	-0.03	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.37	0.68	0.03	0.51	0.07	0.00	0.37	0.00	0.00	0.09	0.00	0.00	0.01	0.00	0.00
	2y	0.93	-0.33	0.04	-0.14	-0.07	0.04	-0.01	-0.01	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.40	0.78	0.01	0.46	0.08	0.02	0.00	-0.01	0.00	-0.10	0.00	0.00	-0.03	0.00	0.00
	3y	0.94	-0.32	0.06	-0.11	-0.03	0.02	-0.02	0.01	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.43	0.84	0.00	0.29	0.05	0.01	-0.16	0.00	0.00	-0.06	0.00	0.00	0.01	0.00	0.00
	4y	0.95	-0.31	0.08	-0.07	0.01	0.00	-0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.45	0.86	-0.01	0.12	0.02	0.01	-0.20	0.01	0.00	0.01	0.00	0.00	0.02	0.00	0.00
	5y	0.95	-0.30	0.09	-0.04	0.03	-0.02	-0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.46	0.87	-0.02	-0.04	-0.01	0.00	-0.17	0.01	0.00	0.06	0.00	0.00	0.01	0.00	0.00
	6y	0.95	-0.28	0.10	-0.01	0.05	-0.03	-0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.47	0.86	-0.02	-0.17	-0.03	0.00	-0.10	0.01	0.00	0.07	0.00	0.00	-0.01	0.00	0.00
	7y	0.95	-0.28	0.10	0.02	0.06	-0.04	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.47	0.84	-0.01	-0.28	-0.06	-0.01	0.00	0.00	0.00	0.04	0.00	0.00	-0.02	0.00	0.00
	8y	0.95	-0.27	0.10	0.04	0.06	-0.04	0.02	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.45	0.80	0.00	-0.37	-0.07	-0.01	0.12	-0.01	0.00	-0.01	0.00	0.00	-0.01	0.00	0.00
	9y	0.96	-0.26	0.10	0.06	0.06	-0.04	0.04	-0.03	0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.43	0.75	0.01	-0.43	-0.08	-0.01	0.23	-0.01	0.01	-0.10	0.00	0.00	0.02	0.00	0.00

Panel A		Yield levels 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
US	1y	0.63	0.30	0.00	0.04	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.71	0.26	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.77	0.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.81	0.19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.84	0.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.86	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.88	0.11	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.89	0.09	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.89	0.08	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.81	0.17	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GER	1y	0.75	0.12	0.06	0.05	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.81	0.09	0.06	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.86	0.07	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.90	0.05	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.91	0.04	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.92	0.03	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.91	0.03	0.04	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.91	0.02	0.04	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.90	0.02	0.04	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.88	0.05	0.05	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
JAP	1y	0.85	0.04	0.09	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.87	0.03	0.08	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.90	0.03	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.92	0.03	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.93	0.03	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.93	0.03	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.93	0.02	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.93	0.02	0.04	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.93	0.02	0.04	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.91	0.03	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Panel A		Yield differences 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
US	1y	0.48	0.14	0.15	0.01	0.04	0.16	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.57	0.17	0.18	0.01	0.02	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.61	0.18	0.18	0.00	0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.63	0.18	0.18	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.63	0.18	0.18	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.63	0.17	0.18	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.62	0.16	0.18	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.61	0.15	0.18	0.00	0.01	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.59	0.15	0.17	0.00	0.01	0.05	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.60	0.16	0.18	0.00	0.01	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
GER	1y	0.37	0.00	0.24	0.05	0.21	0.01	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.54	0.00	0.30	0.04	0.10	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	3y	0.60	0.01	0.32	0.02	0.03	0.01	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.63	0.01	0.32	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.65	0.01	0.31	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.65	0.01	0.30	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.64	0.01	0.28	0.01	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.61	0.01	0.26	0.02	0.07	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.57	0.01	0.23	0.03	0.10	0.02	0.00	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	R_a^2	0.58	0.01	0.28	0.02	0.07	0.01	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
JAP	1y	0.16	0.41	0.03	0.21	0.05	0.00	0.14	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	2y	0.21	0.53	0.02	0.18	0.04	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	3y	0.25	0.60	0.03	0.07	0.01	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.28	0.64	0.03	0.01	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.30	0.64	0.03	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.30	0.63	0.03	0.03	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.29	0.60	0.03	0.07	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.27	0.55	0.03	0.12	0.02	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.24	0.48	0.02	0.16	0.03	0.00	0.06	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.26	0.56	0.03	0.09	0.02	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 2.13: In Panel A we report the R^2 obtained from the regression of each time series of U.S., German and Japanese yields on the associated common principal components ($PC^{(c)}s$). In Panel B (following table) we report the associated correlations. We use weekly data for a sample period starting from January 1, 1986 to December 31, 2009 (1252 observations) and residual maturities from 1 to 9 years.

		Yield levels															Yield differences														
Panel *B*		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
US	1y	0.80	0.55	0.06	0.20	0.14	0.03	0.03	-0.02	-0.02	0.00	0.00	-0.01	0.00	0.00	0.00	0.69	-0.37	0.39	0.11	-0.19	-0.40	-0.01	-0.02	-0.16	-0.01	0.01	0.04	0.00	0.00	0.00
	2y	0.84	0.51	0.06	0.12	0.08	0.01	0.00	0.00	0.01	-0.01	0.00	0.01	0.00	0.00	0.00	0.75	-0.41	0.42	0.08	-0.13	-0.24	-0.01	0.03	0.04	0.00	0.00	-0.06	0.00	0.00	0.01
	3y	0.88	0.47	0.05	0.06	0.03	0.00	-0.01	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.78	-0.42	0.43	0.05	-0.08	-0.11	-0.01	0.03	0.10	0.00	-0.01	-0.03	0.00	0.00	-0.01
	4y	0.90	0.43	0.03	0.02	-0.01	-0.01	-0.01	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.79	-0.42	0.43	0.02	-0.03	-0.02	-0.01	0.02	0.10	0.00	-0.01	0.01	0.00	0.00	-0.01
	5y	0.92	0.39	0.02	-0.02	-0.04	-0.01	-0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.80	-0.42	0.43	0.00	0.01	0.05	0.00	0.01	0.07	0.00	-0.01	0.04	0.00	0.00	0.00
	6y	0.93	0.36	0.01	-0.05	-0.07	-0.01	-0.01	0.01	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.79	-0.41	0.43	-0.02	0.04	0.11	0.00	0.00	0.03	0.00	0.00	0.04	0.00	0.00	0.01
	7y	0.94	0.33	-0.01	-0.07	-0.09	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.79	-0.40	0.42	-0.04	0.07	0.16	0.01	-0.02	-0.02	0.00	0.00	0.02	0.00	0.00	0.01
	8y	0.94	0.31	-0.02	-0.09	-0.10	-0.01	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.78	-0.39	0.42	-0.05	0.10	0.20	0.01	-0.03	-0.07	0.00	0.01	-0.01	0.00	0.00	0.00
	9y	0.95	0.28	-0.03	-0.10	-0.11	-0.01	0.01	-0.02	-0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.77	-0.38	0.41	-0.07	0.12	0.23	0.02	-0.05	-0.12	0.00	0.01	-0.05	0.00	0.00	-0.01
GER	1y	0.87	-0.35	0.24	0.23	-0.06	-0.08	0.05	0.02	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.61	-0.01	-0.49	0.23	-0.46	0.10	-0.01	-0.32	0.01	-0.01	-0.07	0.00	0.00	-0.01	0.00
	2y	0.90	-0.31	0.23	0.18	-0.05	-0.04	-0.01	0.00	0.00	-0.01	0.01	0.00	0.00	0.00	0.00	0.73	-0.07	-0.55	0.19	-0.31	0.10	0.01	0.03	0.01	0.02	0.10	0.00	0.00	0.02	0.00
	3y	0.93	-0.27	0.23	0.11	-0.02	-0.01	-0.03	-0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.77	-0.09	-0.56	0.12	-0.18	0.07	0.01	0.16	-0.01	0.01	0.05	0.00	0.00	-0.01	0.00
	4y	0.95	-0.23	0.22	0.04	0.00	0.02	-0.03	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.80	-0.10	-0.56	0.05	-0.06	0.04	0.01	0.17	-0.02	0.00	-0.02	0.00	0.00	-0.02	0.00
	5y	0.96	-0.20	0.21	-0.02	0.01	0.03	-0.03	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.81	-0.11	-0.56	-0.01	0.05	0.00	0.01	0.14	-0.02	-0.01	-0.06	0.00	0.00	0.00	0.00
	6y	0.96	-0.18	0.20	-0.08	0.03	0.04	-0.01	-0.01	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.81	-0.11	-0.55	-0.07	0.14	-0.03	0.00	0.07	-0.01	-0.01	-0.07	0.00	0.00	0.01	0.00
	7y	0.96	-0.16	0.20	-0.13	0.04	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.80	-0.10	-0.53	-0.12	0.21	-0.07	0.00	-0.02	0.00	-0.01	-0.04	0.00	0.00	0.01	0.00
	8y	0.95	-0.15	0.20	-0.17	0.04	0.04	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.78	-0.10	-0.51	-0.16	0.27	-0.10	-0.01	-0.11	0.01	0.00	0.01	0.00	0.00	0.01	0.00
	9y	0.95	-0.13	0.20	-0.21	0.05	0.03	0.04	0.02	0.00	-0.01	0.01	0.00	0.00	0.00	0.00	0.75	-0.09	-0.48	-0.19	0.32	-0.13	-0.02	-0.19	0.03	0.02	0.08	0.00	0.00	-0.02	0.00
JAP	1y	0.92	-0.19	-0.30	0.11	-0.07	0.10	0.03	-0.01	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.39	0.64	0.17	0.46	0.21	-0.03	0.37	-0.02	0.00	0.09	-0.01	0.00	0.01	0.00	0.00
	2y	0.93	-0.19	-0.28	0.09	-0.04	0.08	0.01	0.00	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.46	0.73	0.16	0.42	0.19	0.00	0.00	-0.01	0.01	-0.10	0.01	0.00	-0.03	0.00	0.00
	3y	0.95	-0.18	-0.25	0.06	0.00	0.04	-0.01	0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.50	0.78	0.16	0.27	0.12	0.01	-0.16	0.00	0.00	-0.06	0.01	0.00	0.01	0.00	0.00
	4y	0.96	-0.18	-0.23	0.03	0.02	0.01	-0.02	0.01	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.53	0.80	0.17	0.11	0.04	0.01	-0.20	0.01	0.00	0.01	0.00	0.00	0.02	0.00	0.00
	5y	0.96	-0.17	-0.21	0.00	0.04	-0.02	-0.02	0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.54	0.80	0.17	-0.04	-0.02	0.01	-0.17	0.01	-0.01	0.06	-0.01	0.00	0.01	0.00	0.00
	6y	0.96	-0.16	-0.20	-0.03	0.05	-0.04	-0.01	0.01	-0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.55	0.79	0.17	-0.16	-0.07	0.00	-0.10	0.01	-0.01	0.07	-0.01	0.00	-0.01	0.00	0.00
	7y	0.97	-0.15	-0.19	-0.05	0.05	-0.06	-0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.54	0.77	0.17	-0.26	-0.12	0.00	0.00	0.01	0.00	0.04	-0.01	0.00	-0.02	0.00	0.00
	8y	0.97	-0.14	-0.19	-0.07	0.05	-0.07	0.00	-0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.52	0.74	0.16	-0.34	-0.15	-0.01	0.12	0.00	0.00	-0.01	0.00	0.00	-0.01	0.00	0.00
	9y	0.96	-0.13	-0.19	-0.09	0.04	-0.08	0.02	-0.03	0.02	-0.01	-0.01	0.00	0.00	0.00	0.00	0.49	0.69	0.15	-0.40	-0.17	-0.02	0.24	0.00	0.01	-0.10	0.02	0.00	0.02	0.00	0.00

Panel A		Yield levels															Yield differences														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
UK	1y	0.84	0.05	0.03	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.36	0.15	0.18	0.07	0.08	0.11	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.89	0.04	0.04	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.48	0.18	0.20	0.03	0.04	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.92	0.03	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.55	0.19	0.20	0.01	0.01	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.94	0.03	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.58	0.19	0.20	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.95	0.03	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.60	0.19	0.20	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.95	0.03	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.60	0.18	0.19	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.95	0.03	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.59	0.17	0.18	0.02	0.02	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.95	0.03	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.56	0.16	0.17	0.03	0.03	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.94	0.03	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.53	0.15	0.16	0.04	0.04	0.06	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.92	0.03	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.54	0.17	0.19	0.02	0.03	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
GER	1y	0.78	0.15	0.01	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.00	0.18	0.12	0.09	0.08	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.84	0.13	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	0.01	0.26	0.07	0.04	0.05	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	3y	0.88	0.10	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.61	0.01	0.29	0.03	0.01	0.02	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.91	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.64	0.01	0.31	0.01	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.93	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.65	0.01	0.32	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.93	0.06	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.64	0.01	0.31	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.92	0.05	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.62	0.01	0.31	0.02	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.91	0.04	0.01	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.59	0.01	0.29	0.04	0.02	0.04	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.90	0.04	0.02	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.54	0.01	0.27	0.06	0.03	0.05	0.00	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	R_a^2	0.89	0.08	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.58	0.01	0.28	0.04	0.02	0.03	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
JAP	1y	0.89	0.02	0.07	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.37	0.04	0.13	0.12	0.00	0.14	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	2y	0.91	0.02	0.06	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.50	0.03	0.11	0.10	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	3y	0.93	0.01	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.27	0.58	0.03	0.05	0.04	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.95	0.01	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.62	0.03	0.01	0.01	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.95	0.01	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.63	0.03	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.96	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.62	0.03	0.02	0.02	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.96	0.01	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.58	0.03	0.04	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.95	0.01	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.29	0.53	0.03	0.07	0.07	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.95	0.01	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.46	0.03	0.10	0.09	0.00	0.06	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.94	0.01	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.54	0.03	0.06	0.05	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 2.14: In Panel A we report the R^2 obtained from the regression of each time series of U.K., German and Japanese yields on the associated common principal components ($PC^{(c)}s$). In Panel B (following table) we report the associated correlations. We use weekly data for a sample period starting from January 1, 1986 to December 31, 2009 (1252 observations) and residual maturities from 1 to 9 years.

Panel *B*		Yield levels 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Yield differences 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
UK	1y	0.92	-0.21	0.19	0.28	-0.01	0.04	0.05	0.01	0.02	0.00	0.00	-0.01	0.00	0.00	0.00	0.60	-0.39	0.43	0.27	-0.29	0.33	0.00	-0.03	-0.21	0.00	0.00	-0.03	0.00	0.00	0.00
	2y	0.94	-0.19	0.20	0.18	0.00	0.04	0.01	0.00	-0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.69	-0.43	0.45	0.19	-0.20	0.23	-0.02	0.02	0.03	0.00	0.01	0.06	0.00	0.00	0.02
	3y	0.96	-0.18	0.19	0.10	0.01	0.02	-0.02	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.74	-0.44	0.45	0.11	-0.12	0.14	-0.02	0.03	0.11	0.00	0.00	0.03	0.00	0.00	-0.01
	4y	0.97	-0.17	0.17	0.04	0.01	0.00	-0.02	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.76	-0.44	0.45	0.04	-0.04	0.05	-0.01	0.02	0.12	0.00	0.00	-0.01	0.00	0.00	-0.01
	5y	0.97	-0.16	0.16	0.00	0.01	-0.01	-0.02	-0.01	-0.01	0.00	0.00	-0.01	0.00	0.00	0.00	0.78	-0.43	0.45	-0.02	0.03	-0.03	0.00	0.01	0.08	0.00	-0.01	-0.04	0.00	0.00	0.00
	6y	0.98	-0.16	0.14	-0.04	0.01	-0.03	-0.02	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.78	-0.42	0.44	-0.08	0.09	-0.10	0.00	0.00	0.03	0.00	-0.01	-0.04	0.00	0.00	0.01
	7y	0.97	-0.17	0.13	-0.06	0.01	-0.04	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.77	-0.41	0.42	-0.13	0.14	-0.16	0.00	-0.01	-0.02	0.00	0.00	-0.02	0.00	0.00	0.01
	8y	0.97	-0.17	0.11	-0.09	0.01	-0.05	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.75	-0.40	0.41	-0.17	0.18	-0.20	0.00	-0.02	-0.07	0.00	0.00	0.01	0.00	0.00	0.00
	9y	0.97	-0.18	0.10	-0.11	0.01	-0.06	0.00	0.02	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.73	-0.39	0.40	-0.21	0.21	-0.24	0.00	-0.03	-0.11	0.00	0.01	0.05	0.00	0.00	-0.01
GER	1y	0.89	0.39	-0.08	0.21	0.08	-0.07	0.03	0.01	-0.02	0.00	0.01	0.00	0.00	0.00	0.00	0.64	-0.07	-0.43	0.35	-0.30	-0.28	-0.01	-0.32	0.02	-0.01	-0.07	0.00	0.00	-0.01	0.00
	2y	0.92	0.36	-0.04	0.17	0.04	-0.02	-0.01	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.75	-0.10	-0.51	0.26	-0.19	-0.22	0.01	0.03	0.00	0.02	0.10	0.00	0.00	0.02	0.00
	3y	0.94	0.32	0.00	0.10	0.01	0.01	-0.03	0.00	0.01	0.00	-0.01	0.00	0.00	0.00	0.00	0.78	-0.12	-0.54	0.17	-0.10	-0.13	0.01	0.16	-0.01	0.01	0.05	0.00	0.00	-0.01	0.00
	4y	0.96	0.29	0.03	0.04	-0.01	0.02	-0.03	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.80	-0.12	-0.56	0.07	-0.03	-0.05	0.01	0.17	-0.01	0.00	-0.02	0.00	0.00	-0.02	0.00
	5y	0.96	0.26	0.06	-0.02	-0.03	0.02	-0.02	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.80	-0.12	-0.56	-0.01	0.03	0.02	0.01	0.14	-0.01	-0.01	-0.06	0.00	0.00	0.00	0.00
	6y	0.96	0.24	0.09	-0.07	-0.04	0.02	-0.01	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.80	-0.12	-0.56	-0.09	0.08	0.09	0.01	0.06	-0.01	-0.01	-0.07	0.00	0.00	0.01	0.00
	7y	0.96	0.22	0.11	-0.12	-0.04	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.79	-0.11	-0.55	-0.15	0.12	0.15	0.00	-0.02	0.00	-0.01	-0.04	0.00	0.00	0.02	0.00
	8y	0.96	0.21	0.12	-0.16	-0.04	0.01	0.03	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.77	-0.10	-0.54	-0.20	0.15	0.19	-0.01	-0.11	0.01	0.00	0.01	0.00	0.00	0.01	0.00
	9y	0.95	0.20	0.13	-0.20	-0.04	0.00	0.04	0.00	-0.02	0.00	-0.01	0.00	0.00	0.00	0.00	0.73	-0.09	-0.52	-0.24	0.18	0.23	-0.02	-0.20	0.02	0.02	0.08	0.00	0.00	-0.02	0.00
JAP	1y	0.94	-0.14	-0.26	0.11	-0.09	-0.05	0.01	-0.03	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.43	0.61	0.20	0.36	0.35	0.05	0.37	-0.02	0.00	0.09	-0.01	0.00	0.01	0.00	0.00
	2y	0.95	-0.12	-0.25	0.08	-0.07	-0.02	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.49	0.71	0.18	0.33	0.32	0.03	0.00	-0.01	0.01	-0.10	0.01	0.00	-0.03	0.00	0.00
	3y	0.97	-0.10	-0.23	0.04	-0.04	0.00	-0.01	0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.52	0.76	0.17	0.21	0.20	0.01	-0.16	0.00	0.00	-0.06	0.01	0.00	0.01	0.00	0.00
	4y	0.97	-0.08	-0.21	0.00	-0.01	0.02	-0.01	0.02	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.54	0.79	0.16	0.09	0.08	0.00	-0.20	0.01	-0.01	0.01	0.00	0.00	0.02	0.00	0.00
	5y	0.98	-0.07	-0.20	-0.04	0.01	0.03	-0.01	0.02	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.56	0.79	0.16	-0.03	-0.03	-0.01	-0.17	0.01	-0.01	0.06	-0.01	0.00	0.01	0.00	0.00
	6y	0.98	-0.07	-0.18	-0.06	0.03	0.03	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.56	0.78	0.17	-0.12	-0.12	-0.01	-0.10	0.01	0.00	0.07	-0.01	0.00	-0.01	0.00	0.00
	7y	0.98	-0.07	-0.17	-0.09	0.05	0.03	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.55	0.76	0.17	-0.20	-0.20	-0.01	0.00	0.01	0.00	0.04	-0.01	0.00	-0.02	0.00	0.00
	8y	0.98	-0.08	-0.15	-0.10	0.06	0.02	0.01	-0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.54	0.73	0.16	-0.27	-0.26	-0.01	0.12	0.00	0.00	-0.01	0.00	0.00	-0.01	0.00	0.00
	9y	0.97	-0.09	-0.14	-0.12	0.07	0.01	0.02	-0.04	0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.51	0.68	0.16	-0.31	-0.30	-0.01	0.24	0.00	0.01	-0.10	0.02	0.00	0.02	0.00	0.00

The 4-country case. 1^{st} step: Principal Component Analysis of international bond markets.

Table 2.15: We report principal component analysis of the term structure of interest rates of four countries taken separately and jointly. The first column lists the number of principal components (four for a single country, and fifteen for the 4-country case), while the rest of the table is divided into five vertical blocks. Each of the first, second, third and fourth blocks represents an individual country, while the fifth block represents the four countries jointly. The first and the second column (third and fourth, respectively) of each block report marginal and cumulative percent variance explained for yield levels (yield differences, respectively). For each country we use weekly yields observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

	PCs																*$PC^{(c)}s$*			
	U.S.				*U.K.*				*JAP*				*GER*				*U.S. – U.K. – GER – JAP*			
	Levels		Differences		Levels		Differences		Levels		Differences		Levels		Differences		Levels		Differences	
	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.
1	97.22	97.22	93.55	93.55	98.10	98.10	89.96	89.96	96.86	96.86	87.59	87.59	98.72	98.72	84.67	84.67	88.26	88.26	44.62	44.62
2	2.70	99.93	5.42	98.97	1.80	99.90	8.81	98.77	3.00	99.86	9.33	96.91	1.18	99.91	11.34	96.01	6.48	94.74	19.02	63.64
3	0.07	100.00	0.88	99.85	0.09	99.99	1.09	99.87	0.14	99.99	2.67	99.58	0.09	99.99	3.47	99.48	2.63	97.37	13.42	77.06
4	0.00	100.00	0.14	99.99	0.01	100.00	0.12	99.99	0.01	100.00	0.40	99.98	0.01	100.00	0.48	99.97	1.20	98.57	12.03	89.09
5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.92	99.49	3.16	92.25
6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.27	99.76	2.68	94.92
7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.13	99.89	1.65	96.57
8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.05	99.94	1.13	97.70
9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.03	99.97	0.86	98.56
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.01	99.98	0.67	99.23
11	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.01	99.99	0.27	99.50
12	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.01	100.00	0.20	99.70
13	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.00	100.00	0.12	99.82
14	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.00	100.00	0.10	99.92
15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.00	100.00	0.04	99.95

2^{nd} and 3^{rd} step : Regressing yields on $PC^{(c)}s$ and associated correlations.

		Yield levels																			
Panel *A*		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
U.S.	1y	0.65	0.28	0.01	0.03	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.72	0.24	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.78	0.20	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.82	0.16	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.85	0.13	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.87	0.11	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.89	0.09	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.90	0.08	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.90	0.07	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.82	0.15	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
U.K.	1y	0.87	0.02	0.00	0.05	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.93	0.02	0.00	0.02	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.95	0.01	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.97	0.01	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.97	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.97	0.00	0.01	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.97	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.96	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.95	0.00	0.01	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.95	0.01	0.01	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GER	1y	0.72	0.14	0.07	0.05	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.79	0.11	0.07	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.84	0.09	0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.87	0.07	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.89	0.05	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.90	0.04	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.90	0.04	0.04	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.89	0.03	0.04	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.88	0.03	0.04	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.85	0.07	0.05	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
JAP	1y	0.85	0.04	0.08	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.87	0.04	0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.89	0.04	0.05	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.91	0.04	0.04	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.92	0.04	0.03	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.92	0.03	0.03	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.93	0.03	0.03	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.93	0.03	0.03	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.93	0.02	0.03	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.91	0.03	0.04	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

		Yield differences																			
Panel *A*		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
U.S.	1y	0.45	0.05	0.19	0.07	0.04	0.02	0.00	0.15	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.53	0.07	0.22	0.08	0.02	0.01	0.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.57	0.07	0.24	0.08	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.59	0.07	0.24	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.59	0.07	0.24	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.59	0.07	0.24	0.09	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.59	0.06	0.23	0.09	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.58	0.06	0.22	0.09	0.01	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.56	0.05	0.21	0.08	0.01	0.01	0.00	0.05	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.56	0.06	0.23	0.08	0.01	0.00	0.00	0.04	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
U.K.	1y	0.30	0.04	0.34	0.03	0.08	0.05	0.11	0.01	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.43	0.06	0.35	0.03	0.04	0.03	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.50	0.06	0.35	0.04	0.01	0.01	0.02	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.54	0.06	0.35	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.56	0.06	0.33	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.57	0.06	0.30	0.04	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.57	0.06	0.27	0.04	0.02	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.55	0.05	0.24	0.04	0.04	0.02	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.53	0.05	0.22	0.04	0.05	0.03	0.06	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.50	0.05	0.31	0.04	0.03	0.02	0.03	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GER	1y	0.34	0.00	0.00	0.26	0.14	0.06	0.08	0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.50	0.00	0.00	0.34	0.07	0.02	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.56	0.00	0.00	0.36	0.03	0.00	0.02	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.59	0.00	0.00	0.37	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.60	0.00	0.00	0.37	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.60	0.00	0.00	0.36	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.59	0.00	0.01	0.34	0.03	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.56	0.00	0.01	0.31	0.05	0.01	0.04	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.51	0.00	0.01	0.28	0.07	0.02	0.05	0.01	0.00	0.04	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.54	0.00	0.00	0.33	0.04	0.01	0.03	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
JAP	1y	0.11	0.44	0.00	0.03	0.10	0.16	0.00	0.00	0.14	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.15	0.60	0.00	0.03	0.08	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.17	0.69	0.00	0.03	0.03	0.05	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.19	0.73	0.00	0.03	0.01	0.01	0.00	0.00	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.20	0.74	0.00	0.03	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.21	0.72	0.00	0.03	0.01	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.20	0.68	0.00	0.03	0.03	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.19	0.63	0.00	0.03	0.05	0.09	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.17	0.55	0.00	0.02	0.07	0.12	0.00	0.00	0.05	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	R_a^2	0.18	0.64	0.00	0.03	0.04	0.07	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 2.16: In Panel *A* we report the R^2 obtained from the regression of each time series of U.S., U.K., Japanese and German yields on the associated common principal components ($PC^{(c)}s$). In Panel *B* (following table) we report the associated correlations. We use weekly data for a sample period starting from January 1, 1986 to December 31, 2009 (1252 observations) and residual maturities from 1 to 9 years.

Yield levels

Panel *B*		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
U.S.	1y	0.81	0.53	0.11	0.18	0.07	-0.16	-0.02	0.03	0.01	-0.03	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.85	0.49	0.10	0.11	0.08	-0.09	-0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00
	3y	0.88	0.45	0.09	0.05	0.08	-0.04	0.00	0.00	-0.01	0.02	0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.91	0.40	0.07	0.01	0.08	0.01	0.00	-0.01	-0.01	0.02	0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.92	0.37	0.06	-0.02	0.08	0.04	0.01	-0.01	0.00	0.02	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.94	0.33	0.04	-0.05	0.07	0.07	0.01	-0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	7y	0.94	0.30	0.03	-0.07	0.07	0.09	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.95	0.28	0.01	-0.08	0.07	0.11	0.01	0.00	0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.95	0.26	0.00	-0.10	0.07	0.12	0.00	0.01	0.01	-0.02	-0.01	0.01	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00
U.K.	1y	0.93	0.14	-0.07	0.22	-0.22	0.02	-0.02	-0.04	0.05	0.00	-0.01	-0.02	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	2y	0.96	0.13	-0.05	0.12	-0.20	0.01	0.00	-0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00
	3y	0.98	0.10	-0.05	0.05	-0.17	0.01	0.01	-0.03	-0.01	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.98	0.08	-0.05	0.00	-0.16	0.00	0.01	-0.01	-0.02	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.99	0.06	-0.06	-0.04	-0.14	-0.01	0.02	0.00	-0.02	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.98	0.04	-0.07	-0.07	-0.13	-0.01	0.02	0.02	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	7y	0.98	0.03	-0.09	-0.10	-0.12	-0.02	0.02	0.03	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.98	0.02	-0.10	-0.12	-0.11	-0.02	0.02	0.04	0.00	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.98	0.02	-0.11	-0.14	-0.11	-0.03	0.02	0.05	0.00	0.00	-0.01	-0.02	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00
GER	1y	0.85	-0.38	0.26	0.22	0.03	0.05	0.09	0.05	0.03	0.01	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.89	-0.34	0.26	0.18	0.02	0.03	0.04	0.01	-0.01	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.92	-0.30	0.25	0.11	0.01	0.02	0.01	-0.01	-0.03	-0.01	0.00	-0.01	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.94	-0.26	0.23	0.04	0.00	0.00	-0.02	-0.02	-0.03	-0.01	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.95	-0.23	0.22	-0.02	0.00	-0.01	-0.03	-0.02	-0.02	-0.01	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.95	-0.21	0.22	-0.08	-0.01	-0.02	-0.04	-0.01	-0.01	0.00	0.00	-0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.95	-0.19	0.21	-0.13	-0.01	-0.03	-0.04	-0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.94	-0.18	0.21	-0.17	-0.01	-0.03	-0.04	0.00	0.03	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.94	-0.17	0.20	-0.21	-0.01	-0.03	-0.04	0.01	0.04	0.01	0.00	0.01	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00
JAP	1y	0.92	-0.20	-0.28	0.14	0.04	0.06	-0.08	0.04	0.01	0.00	0.03	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2y	0.93	-0.20	-0.25	0.12	0.06	0.03	-0.07	0.02	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	3y	0.95	-0.20	-0.23	0.08	0.08	0.00	-0.04	0.01	-0.01	0.00	-0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4y	0.95	-0.20	-0.20	0.04	0.09	-0.02	-0.02	-0.01	-0.01	0.01	-0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5y	0.96	-0.19	-0.18	0.01	0.09	-0.04	0.00	-0.02	-0.01	0.01	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.96	-0.18	-0.17	-0.02	0.09	-0.04	0.02	-0.02	0.00	0.00	-0.01	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	7y	0.96	-0.17	-0.17	-0.04	0.09	-0.05	0.04	-0.03	0.01	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	8y	0.96	-0.16	-0.17	-0.06	0.09	-0.04	0.06	-0.02	0.01	-0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	9y	0.96	-0.15	-0.17	-0.08	0.09	-0.03	0.07	-0.02	0.02	-0.02	0.03	-0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Yield differences

Panel *B*		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
U.S.	1y	0.67	-0.23	-0.43	-0.27	-0.21	0.13	-0.03	-0.39	-0.01	-0.02	-0.03	0.16	0.01	-0.01	-0.04	0.00	0.00	0.00	0.00	0.00
	2y	0.73	-0.26	-0.47	-0.29	-0.15	0.09	-0.02	-0.23	-0.01	0.03	0.01	-0.04	0.00	0.00	0.06	0.00	0.00	0.00	0.00	-0.01
	3y	0.76	-0.27	-0.49	-0.29	-0.10	0.06	-0.01	-0.11	-0.01	0.03	0.02	-0.10	0.00	0.01	0.03	0.00	0.00	0.00	0.00	0.01
	4y	0.77	-0.27	-0.49	-0.29	-0.05	0.03	-0.01	-0.01	-0.01	0.03	0.02	-0.10	0.00	0.01	-0.01	0.00	0.00	0.00	0.00	0.01
	5y	0.77	-0.26	-0.49	-0.29	-0.01	0.01	0.00	0.06	0.00	0.01	0.01	-0.07	0.00	0.01	-0.04	0.00	0.00	0.00	0.00	0.00
	6y	0.77	-0.26	-0.49	-0.29	0.03	-0.02	0.00	0.12	0.00	0.00	0.00	-0.03	0.00	0.00	-0.04	0.00	0.00	0.00	0.00	-0.01
	7y	0.77	-0.25	-0.48	-0.29	0.06	-0.04	0.01	0.16	0.01	-0.02	0.00	0.02	0.00	0.00	-0.02	0.00	0.00	0.00	0.00	-0.01
	8y	0.76	-0.24	-0.47	-0.29	0.09	-0.06	0.01	0.20	0.01	-0.03	-0.01	0.07	0.00	-0.01	0.01	0.00	0.00	0.00	0.00	0.00
	9y	0.75	-0.23	-0.46	-0.29	0.11	-0.07	0.01	0.23	0.02	-0.04	-0.02	0.11	0.00	-0.01	0.05	0.00	0.00	0.00	0.00	0.01
U.K.	1y	0.55	-0.20	0.58	-0.16	-0.28	0.23	-0.33	0.10	0.01	-0.04	-0.20	-0.03	0.00	0.00	0.00	-0.03	0.00	0.00	0.00	0.00
	2y	0.66	-0.24	0.59	-0.18	-0.20	0.16	-0.23	0.07	-0.01	0.01	0.03	0.00	0.00	-0.01	0.00	0.06	0.00	0.00	0.02	0.00
	3y	0.70	-0.24	0.60	-0.19	-0.11	0.09	-0.14	0.04	-0.01	0.02	0.11	0.02	0.00	0.00	0.00	0.02	0.00	0.00	-0.01	0.00
	4y	0.73	-0.24	0.59	-0.19	-0.04	0.03	-0.05	0.01	-0.01	0.02	0.11	0.02	0.00	0.00	0.00	-0.01	0.00	0.00	-0.01	0.00
	5y	0.75	-0.24	0.57	-0.20	0.03	-0.03	0.03	-0.02	-0.01	0.02	0.08	0.01	0.00	0.01	0.00	-0.04	0.00	0.00	0.00	0.00
	6y	0.76	-0.24	0.55	-0.20	0.10	-0.07	0.10	-0.04	0.00	0.00	0.03	0.01	0.00	0.00	0.00	-0.04	0.00	0.00	0.01	0.00
	7y	0.75	-0.23	0.52	-0.21	0.15	-0.12	0.16	-0.05	0.00	-0.01	-0.02	0.00	0.00	0.00	0.00	-0.02	0.00	0.00	0.01	0.00
	8y	0.74	-0.23	0.49	-0.21	0.19	-0.15	0.20	-0.06	0.00	-0.02	-0.07	-0.02	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	9y	0.73	-0.23	0.47	-0.21	0.23	-0.17	0.24	-0.07	0.00	-0.02	-0.11	-0.03	0.01	-0.01	0.00	0.05	0.00	0.00	-0.01	0.00
GER	1y	0.59	0.06	0.03	0.51	-0.37	0.24	0.29	0.05	0.00	-0.32	0.02	-0.01	0.01	0.07	0.00	0.00	0.00	-0.01	0.00	0.00
	2y	0.71	0.03	-0.01	0.58	-0.27	0.14	0.22	0.06	0.01	0.03	0.00	-0.01	-0.02	-0.10	0.00	0.00	0.00	0.02	0.00	0.00
	3y	0.75	0.02	-0.03	0.60	-0.16	0.07	0.14	0.05	0.02	0.15	-0.01	0.00	-0.01	-0.05	0.00	0.00	0.00	-0.01	0.00	0.00
	4y	0.77	0.01	-0.05	0.61	-0.07	0.01	0.05	0.03	0.01	0.17	-0.02	0.01	0.00	0.02	0.00	0.00	0.00	-0.02	0.00	0.00
	5y	0.78	0.01	-0.06	0.61	0.02	-0.03	-0.02	0.01	0.01	0.13	-0.02	0.02	0.01	0.06	0.00	0.00	0.00	0.00	0.00	0.00
	6y	0.78	0.01	-0.07	0.60	0.10	-0.07	-0.09	-0.02	0.00	0.07	-0.01	0.01	0.01	0.07	0.00	0.00	0.00	0.01	0.00	0.00
	7y	0.77	0.02	-0.08	0.58	0.17	-0.10	-0.15	-0.04	0.00	-0.02	0.00	0.00	0.01	0.04	0.00	0.00	0.00	0.01	0.00	0.00
	8y	0.75	0.02	-0.09	0.56	0.23	-0.12	-0.19	-0.06	-0.01	-0.11	0.01	-0.01	0.00	-0.01	0.00	0.00	0.00	0.01	0.00	0.00
	9y	0.72	0.02	-0.09	0.53	0.26	-0.14	-0.23	-0.08	-0.02	-0.19	0.02	-0.03	-0.02	-0.08	0.00	0.00	0.00	-0.02	0.00	0.00
JAP	1y	0.34	0.67	0.05	-0.18	-0.31	-0.40	-0.06	-0.02	0.37	-0.02	0.00	0.00	-0.09	0.01	0.00	0.00	0.01	0.00	0.00	0.00
	2y	0.39	0.77	0.03	-0.17	-0.28	-0.37	-0.03	0.01	0.00	-0.01	0.01	-0.01	0.10	-0.01	0.00	0.00	-0.03	0.00	0.00	0.00
	3y	0.42	0.83	0.01	-0.16	-0.18	-0.23	-0.01	0.01	-0.16	0.00	0.00	0.00	0.06	-0.01	0.00	0.00	0.01	0.00	0.00	0.00
	4y	0.44	0.85	0.00	-0.16	-0.07	-0.09	0.00	0.01	-0.20	0.01	-0.01	0.00	-0.01	0.00	0.00	0.00	0.02	0.00	0.00	0.00
	5y	0.45	0.86	0.00	-0.17	0.02	0.03	0.01	0.01	-0.17	0.01	-0.01	0.01	-0.06	0.01	0.00	0.00	0.01	0.00	0.00	0.00
	6y	0.45	0.85	0.00	-0.17	0.10	0.14	0.01	0.00	-0.10	0.01	-0.01	0.01	-0.07	0.01	0.00	0.00	-0.01	0.00	0.00	0.00
	7y	0.45	0.83	0.00	-0.17	0.17	0.23	0.01	0.00	0.00	0.01	0.00	0.00	-0.04	0.01	0.00	0.00	-0.02	0.00	0.00	0.00
	8y	0.44	0.79	0.01	-0.16	0.23	0.30	0.01	-0.01	0.12	0.01	0.00	0.00	0.01	0.00	0.00	0.00	-0.01	0.00	0.00	0.00
	9y	0.42	0.74	0.02	-0.15	0.27	0.34	0.01	-0.02	0.23	0.00	0.01	-0.01	0.10	-0.02	0.00	0.00	0.02	0.00	0.00	0.00

2.B The *ELN*-Approach.

The 2-Country Case.

1^{st} *and* 2^{nd} *step : Regressing* PCs *on* $PC^{(c)}s$ *and Associated* R^2 *(Yield levels).*

Table 2.17: For any given pair of countries, we report results from regressing the single-country yield curve principal components (PCs) on the first eight common $PC^{(c)}s$, extracted by pooling the term structure data of the two countries. In both Panel A and B, the first column represents the dependent variable in the regression, naming one of the first three principal components of a country. The next columns in Panel A report the regression coefficients associated with a regression using all of the first eight $PC^{(c)}s$, while the columns of Panel B report the R^2 when the dependent variable is regressed upon subsets of the first eight $PC^{(c)}s$. We use weekly yield levels observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

Panel A	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}
$US\ 1^{st}$	0.7052	0.6566	-0.2432	0.1035	0.0398	0.0112	-0.0039	0.0004
$US\ 2^{nd}$	-0.0135	0.3613	0.6400	-0.6756	-0.0462	-0.0300	-0.0026	0.0094
$US\ 3^{rd}$	0.0014	-0.0252	0.0404	-0.0491	0.5361	0.8378	0.0739	0.0171
$UK\ 1^{st}$	0.7089	-0.6469	0.2509	-0.1190	-0.0400	-0.0144	0.0032	-0.0002
$UK\ 2^{nd}$	0.0032	0.1355	0.6826	0.7178	0.0087	0.0079	0.0018	-0.0198
$UK\ 3^{rd}$	0.0014	0.0273	-0.0257	0.0242	-0.8407	0.5363	0.0527	0.0265
Panel B	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8
$US\ 1^{st}$	0.9562	0.9983	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
$US\ 2^{nd}$	0.0126	0.4704	0.8813	0.9999	1.0000	1.0000	1.0000	1.0000
$US\ 3^{rd}$	0.0054	0.0937	0.1585	0.1833	0.6137	0.9996	1.0000	1.0000
$UK\ 1^{st}$	0.9577	0.9982	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
$UK\ 2^{nd}$	0.0011	0.0977	0.7991	1.0000	1.0000	1.0000	1.0000	1.0000
$UK\ 3^{rd}$	0.0039	0.0801	0.0994	0.1038	0.8833	0.9998	1.0000	1.0000
Panel A	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}
$US\ 1^{st}$	0.7076	0.6879	-0.0261	0.1590	0.0109	0.0016	0.0005	-0.0015
$US\ 2^{nd}$	-0.0213	0.1737	-0.6213	-0.7620	0.0504	-0.0128	-0.0002	0.0049
$US\ 3^{rd}$	0.0009	-0.0074	-0.0187	0.0062	0.1353	0.9872	0.0685	0.0443
$GER\ 1^{st}$	0.7061	-0.6882	-0.0129	-0.1664	-0.0062	-0.0042	-0.0005	0.0012
$GER\ 2^{nd}$	0.0184	0.1503	0.7819	-0.6043	-0.0040	0.0199	-0.0028	0.0112
$GER\ 3^{rd}$	-0.0024	-0.0192	0.0376	0.0327	0.9894	-0.1348	-0.0016	-0.0054
Panel B	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8
$US\ 1^{st}$	0.8898	0.9997	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000
$US\ 2^{nd}$	0.0291	0.2808	0.7484	0.9999	1.0000	1.0000	1.0000	1.0000
$US\ 3^{rd}$	0.0021	0.0203	0.0370	0.0376	0.0706	0.9995	0.9999	1.0000
$GER\ 1^{st}$	0.8893	0.9997	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000
$GER\ 2^{nd}$	0.0196	0.1897	0.8574	1.0000	1.0000	1.0000	1.0000	1.0000
$GER\ 3^{rd}$	0.0071	0.0674	0.1012	0.1104	0.9914	1.0000	1.0000	1.0000
Panel A	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}
$US\ 1^{st}$	0.7031	-0.6946	0.1016	-0.1094	-0.0130	-0.0248	-0.0047	0.0026
$US\ 2^{nd}$	-0.0237	-0.2231	-0.8672	0.4288	0.0738	0.0905	0.0091	-0.0047
$US\ 3^{rd}$	0.0022	0.0187	-0.0651	-0.0335	0.5290	-0.8407	-0.0848	0.0130
$JAP\ 1^{st}$	0.7106	0.6805	-0.1342	0.1136	0.0118	0.0298	0.0047	-0.0025
$JAP\ 2^{nd}$	-0.0074	0.0653	-0.4577	-0.8827	-0.0801	0.0216	0.0027	0.0137
$JAP\ 3^{rd}$	-0.0009	0.0062	-0.0769	0.1037	-0.8413	-0.5230	-0.0426	-0.0037
Panel B	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8
$US\ 1^{st}$	0.8987	0.9997	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
$US\ 2^{nd}$	0.0367	0.4114	0.9397	0.9997	0.9999	1.0000	1.0000	1.0000
$US\ 3^{rd}$	0.0125	0.1167	0.2345	0.2490	0.5520	0.9994	1.0000	1.0000
$JAP\ 1^{st}$	0.9041	0.9995	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
$JAP\ 2^{nd}$	0.0082	0.0816	0.4180	0.9996	1.0000	1.0000	1.0000	1.0000
$JAP\ 3^{rd}$	0.0017	0.0109	0.1416	0.2521	0.8621	0.9999	1.0000	1.0000

Table 2.17, continued.

Panel *A*	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}
GER 1^{st}	0.7044	0.6886	-0.0952	0.1373	0.0285	-0.0286	0.0036	0.0043
GER 2^{nd}	0.0124	-0.2418	-0.7629	0.5810	0.1379	-0.0493	-0.0043	-0.0142
GER 3^{rd}	-0.0014	0.0226	-0.0449	-0.2040	0.8332	0.5112	0.0184	-0.0073
UK 1^{st}	0.7097	-0.6798	0.1126	-0.1415	-0.0266	0.0288	-0.0038	-0.0031
UK 2^{nd}	-0.0054	-0.0654	0.6274	0.7438	0.2206	-0.0049	-0.0025	-0.0061
UK 3^{rd}	0.0012	0.0177	-0.0201	0.1681	-0.4853	0.8517	0.0908	-0.0433
Panel *B*	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8
GER 1^{st}	0.9430	0.9995	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
GER 2^{nd}	0.0095	0.2345	0.9333	0.9992	1.0000	1.0000	1.0000	1.0000
GER 3^{rd}	0.0026	0.0456	0.0984	0.2760	0.8884	1.0000	1.0000	1.0000
UK 1^{st}	0.9450	0.9994	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
UK 2^{nd}	0.0030	0.0303	0.8169	0.9967	1.0000	1.0000	1.0000	1.0000
UK 3^{rd}	0.0029	0.0418	0.0576	0.2357	0.5425	0.9995	0.9999	1.0000

Panel *A*	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}
GER 1^{st}	0.7033	0.7036	-0.0128	0.0990	0.0170	-0.0073	0.0017	-0.0032
GER 2^{nd}	0.0037	-0.0775	-0.9074	0.4053	0.0420	0.0664	-0.0030	0.0087
GER 3^{rd}	-0.0007	0.0135	-0.0162	-0.2022	0.8443	0.4937	0.0429	0.0135
JAP 1^{st}	0.7108	-0.6967	0.0208	-0.0930	-0.0141	0.0067	-0.0011	0.0035
JAP 2^{nd}	-0.0059	-0.1132	0.4181	0.8808	0.1608	0.1013	0.0091	0.0174
JAP 3^{rd}	0.0012	0.0232	0.0297	-0.0180	-0.5078	0.8599	0.0064	0.0314
Panel *B*	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8
GER 1^{st}	0.9486	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
GER 2^{nd}	0.0009	0.0210	0.9687	0.9998	0.9999	1.0000	1.0000	1.0000
GER 3^{rd}	0.0006	0.0140	0.0207	0.1899	0.8992	0.9999	1.0000	1.0000
JAP 1^{st}	0.9506	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
JAP 2^{nd}	0.0054	0.1142	0.6242	0.9965	0.9995	1.0000	1.0000	1.0000
JAP 3^{rd}	0.0031	0.0658	0.1012	0.1033	0.5125	0.9999	0.9999	1.0000

Panel *A*	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}
UK 1^{st}	0.7059	0.6898	-0.0840	0.1306	-0.0266	0.0329	-0.0070	-0.0065
UK 2^{nd}	-0.0067	0.2194	0.7570	-0.5998	0.1325	-0.0312	0.0211	-0.0037
UK 3^{rd}	-0.0013	0.0350	-0.0517	-0.1642	-0.6686	-0.7187	-0.0298	0.0684
JAP 1^{st}	0.7083	-0.6855	0.0936	-0.1331	0.0248	-0.0336	0.0070	0.0072
JAP 2^{nd}	-0.0031	-0.0651	0.6387	0.7399	-0.1960	-0.0331	-0.0187	0.0208
JAP 3^{rd}	-0.0009	-0.0238	0.0202	-0.1747	-0.7017	0.6897	-0.0177	-0.0110
Panel *B*	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8
UK 1^{st}	0.9652	0.9998	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
UK 2^{nd}	0.0048	0.1952	0.9195	0.9991	1.0000	1.0000	1.0000	1.0000
UK 3^{rd}	0.0034	0.0971	0.1628	0.2786	0.6940	0.9998	0.9998	1.0000
JAP 1^{st}	0.9658	0.9997	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
JAP 2^{nd}	0.0015	0.0271	0.8125	0.9971	0.9999	1.0000	1.0000	1.0000
JAP 3^{rd}	0.0020	0.0489	0.0597	0.2013	0.6958	1.0000	1.0000	1.0000

1^{st} *and* 2^{nd} *Steps : Regressing* PCs *on* $PC^{(c)}s$ *and Associated* R^2 *(Yield differences)*

Table 2.18: For any given pair of countries, we report results from regressing the single-country yield curve principal components (PCs) on the first eight common $PC^{(c)}s$, extracted by pooling the term structure data of the two countries. In both Panel A and B, the first column represents the dependent variable in the regression, naming one of the first three principal components of a country. The next columns in Panel A report the regression coefficients associated with a regression using all of the first eight $PC^{(c)}s$, while the columns of Panel B report the R^2 when the dependent variable is regressed upon subsets of the first eight $PC^{(c)}s$. We use weekly yield differences observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

Panel A	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}
US 1^{st}	0.7239	-0.6866	-0.0609	0.0286	0.0042	0.0026	-0.0014	-0.0064
US 2^{nd}	0.0035	0.0061	0.4043	0.9143	-0.0055	0.0173	-0.0032	-0.0116
US 3^{rd}	0.0012	0.0025	0.0294	-0.0290	0.2918	0.9552	-0.0015	0.0266
UK 1^{st}	0.6894	0.7237	0.0261	-0.0189	-0.0025	-0.0035	0.0021	0.0027
UK 2^{nd}	0.0270	-0.0694	0.9117	-0.4021	-0.0112	-0.0368	0.0091	0.0046
UK 3^{rd}	-0.0014	0.0033	0.0044	0.0091	0.9562	-0.2919	-0.0155	-0.0029
Panel B	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8
US 1^{st}	0.7300	0.9996	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
US 2^{nd}	0.0003	0.0007	0.2801	0.9999	1.0000	1.0000	1.0000	1.0000
US 3^{rd}	0.0002	0.0006	0.0097	0.0141	0.1220	0.9999	0.9999	1.0000
UK 1^{st}	0.6885	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
UK 2^{nd}	0.0108	0.0400	0.9142	0.9999	0.9999	1.0000	1.0000	1.0000
UK 3^{rd}	0.0002	0.0008	0.0010	0.0013	0.9339	1.0000	1.0000	1.0000

Panel A	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}
US 1^{st}	0.7329	-0.6791	-0.0372	0.0169	0.0086	-0.0001	-0.0033	0.0003
US 2^{nd}	0.0033	0.0075	0.3676	0.9266	0.0788	0.0055	0.0062	-0.0008
US 3^{rd}	-0.0004	-0.0012	0.0241	-0.0276	0.1366	0.9855	0.0941	0.0017
GER 1^{st}	0.6802	0.7327	0.0166	-0.0146	-0.0045	0.0008	0.0016	-0.0010
GER 2^{nd}	0.0159	-0.0432	0.9288	-0.3655	-0.0293	-0.0287	-0.0027	0.0021
GER 3^{rd}	-0.0030	0.0075	-0.0047	-0.0813	0.9870	-0.1379	-0.0109	-0.0071
Panel B	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8
US 1^{st}	0.7520	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
US 2^{nd}	0.0003	0.0008	0.2482	0.9969	1.0000	1.0000	1.0000	1.0000
US 3^{rd}	0.0000	0.0001	0.0066	0.0107	0.0676	0.9961	1.0000	1.0000
GER 1^{st}	0.6918	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
GER 2^{nd}	0.0035	0.0136	0.9320	0.9997	0.9999	1.0000	1.0000	1.0000
GER 3^{rd}	0.0005	0.0015	0.0016	0.0133	0.9940	1.0000	1.0000	1.0000

Panel A	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}
US 1^{st}	0.7980	-0.6024	-0.0145	0.0028	-0.0009	0.0012	0.0030	0.0002
US 2^{nd}	0.0049	0.0094	0.0684	0.9971	0.0279	0.0014	-0.0119	0.0004
US 3^{rd}	0.0033	0.0063	0.0027	-0.0025	0.0417	0.9985	0.0343	0.0003
JAP 1^{st}	0.6025	0.7980	0.0023	-0.0106	-0.0004	-0.0071	-0.0007	-0.0006
JAP 2^{nd}	0.0098	-0.0113	0.9975	-0.0683	-0.0014	-0.0028	0.0001	-0.0054
JAP 3^{rd}	0.0007	-0.0008	-0.0006	-0.0279	0.9987	-0.0417	-0.0016	0.0030
Panel B	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8
US 1^{st}	0.7171	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
US 2^{nd}	0.0005	0.0016	0.0114	0.9995	1.0000	1.0000	1.0000	1.0000
US 3^{rd}	0.0013	0.0046	0.0047	0.0048	0.0116	0.9994	1.0000	1.0000
JAP 1^{st}	0.4516	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
JAP 2^{nd}	0.0009	0.0017	0.9978	1.0000	1.0000	1.0000	1.0000	1.0000
JAP 3^{rd}	0.0000	0.0000	0.0000	0.0012	0.9996	1.0000	1.0000	1.0000

Table 2.18, continued.

Panel *A*	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}
$GER\ 1^{st}$	0.6960	-0.7171	-0.0268	0.0235	-0.0016	-0.0083	0.0007	-0.0004
$GER\ 2^{nd}$	-0.0020	-0.0082	0.7488	0.6626	-0.0125	-0.0012	0.0028	0.0056
$GER\ 3^{rd}$	-0.0014	-0.0032	-0.0144	0.0353	0.9921	0.1189	-0.0035	-0.0059
$UK\ 1^{st}$	0.7179	0.6959	0.0163	-0.0076	0.0033	0.0038	-0.0004	0.0002
$UK\ 2^{nd}$	0.0128	-0.0371	0.6619	-0.7476	0.0354	0.0051	-0.0115	-0.0021
$UK\ 3^{rd}$	-0.0031	0.0081	0.0013	-0.0010	0.1191	-0.9922	0.0346	0.0022
Panel *B*	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8
$GER\ 1^{st}$	0.6994	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$GER\ 2^{nd}$	0.0001	0.0004	0.6873	1.0000	1.0000	1.0000	1.0000	1.0000
$GER\ 3^{rd}$	0.0001	0.0003	0.0012	0.0043	0.9943	1.0000	1.0000	1.0000
$UK\ 1^{st}$	0.7244	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$UK\ 2^{nd}$	0.0023	0.0103	0.5782	0.9996	1.0000	1.0000	1.0000	1.0000
$UK\ 3^{rd}$	0.0011	0.0042	0.0042	0.0042	0.0390	0.9996	1.0000	1.0000
Panel *A*	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}
$GER\ 1^{st}$	0.7288	-0.6843	-0.0196	0.0061	0.0059	-0.0041	-0.0037	0.0004
$GER\ 2^{nd}$	-0.0016	-0.0036	0.3541	0.9350	-0.0172	-0.0036	-0.0006	0.0009
$GER\ 3^{rd}$	0.0057	0.0106	0.0332	-0.0175	-0.0574	-0.9974	-0.0192	0.0035
$JAP\ 1^{st}$	0.6846	0.7288	0.0071	0.0013	-0.0011	0.0119	0.0014	0.0010
$JAP\ 2^{nd}$	0.0106	-0.0189	0.9343	-0.3537	0.0031	0.0370	-0.0001	-0.0072
$JAP\ 3^{rd}$	-0.0033	0.0055	0.0052	0.0162	0.9981	-0.0575	-0.0022	-0.0075
Panel *B*	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8
$GER\ 1^{st}$	0.6644	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$GER\ 2^{nd}$	0.0000	0.0001	0.1568	0.9999	1.0000	1.0000	1.0000	1.0000
$GER\ 3^{rd}$	0.0013	0.0039	0.0088	0.0098	0.0141	0.9999	1.0000	1.0000
$JAP\ 1^{st}$	0.6063	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$JAP\ 2^{nd}$	0.0011	0.0031	0.9005	0.9997	0.9997	1.0000	1.0000	1.0000
$JAP\ 3^{rd}$	0.0003	0.0009	0.0010	0.0017	0.9975	1.0000	1.0000	1.0000
Panel *A*	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}
$UK\ 1^{st}$	0.7656	-0.6431	-0.0060	-0.0053	-0.0155	0.0006	-0.0012	-0.0004
$UK\ 2^{nd}$	0.0113	0.0199	0.1309	-0.9909	0.0221	-0.0004	-0.0005	0.0000
$UK\ 3^{rd}$	-0.0001	-0.0002	0.0041	0.0009	0.0368	0.9989	0.0285	-0.0021
$JAP\ 1^{st}$	0.6431	0.7654	-0.0017	0.0225	0.0024	0.0002	0.0003	0.0018
$JAP\ 2^{nd}$	0.0043	-0.0052	0.9914	0.1309	-0.0001	-0.0042	-0.0002	-0.0035
$JAP\ 3^{rd}$	0.0100	-0.0122	-0.0030	0.0217	0.9988	-0.0368	0.0003	0.0158
Panel *B*	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8
$UK\ 1^{st}$	0.6687	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$UK\ 2^{nd}$	0.0015	0.0047	0.0268	0.9998	1.0000	1.0000	1.0000	1.0000
$UK\ 3^{rd}$	0.0000	0.0000	0.0002	0.0002	0.0045	0.9996	1.0000	1.0000
$JAP\ 1^{st}$	0.5013	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
$JAP\ 2^{nd}$	0.0002	0.0003	0.9868	1.0000	1.0000	1.0000	1.0000	1.0000
$JAP\ 3^{rd}$	0.0030	0.0061	0.0061	0.0073	0.9996	1.0000	1.0000	1.0000

3rd step : Extracting $PC^{(\ell)}s$ from $PC^{(c)}s$-Based Yield Curve Residuals

Countries	$PC^{(\ell)}s$	Yield levels		Yield differences	
		after 1 $PC^{(c)}$	after 2 $PC^{(c)}s$	after 1 $PC^{(c)}$	after 2 $PC^{(c)}s$
U.S.	1	70.93	86.24	82.03	71.75
	2	28.27	12.69	15.01	25.21
	Cum.	99.20	98.93	97.04	96.96
U.K.	1	75.28	73.74	77.29	76.98
	2	23.36	22.42	19.98	21.50
	Cum.	98.64	96.16	97.27	98.48
U.S.	1	85.57	83.50	81.07	72.25
	2	13.96	15.20	15.79	25.02
	Cum.	99.52	98.70	96.86	97.27
GER	1	82.39	81.71	72.75	67.52
	2	16.76	17.21	20.61	29.26
	Cum.	99.15	98.92	93.36	96.78
U.S.	1	85.74	84.56	82.58	72.60
	2	13.81	15.01	14.57	24.86
	Cum.	99.55	99.57	97.15	97.46
JAP	1	89.78	74.10	76.69	67.16
	2	9.37	23.80	17.29	29.89
	Cum.	99.15	97.90	93.98	97.05
GER	1	71.71	82.85	71.98	67.79
	2	26.93	16.85	21.12	28.91
	Cum.	98.64	99.70	93.10	96.70
U.K.	1	80.24	81.81	75.22	77.21
	2	18.69	15.75	21.79	21.26
	Cum.	98.93	97.56	97.01	98.47
GER	1	68.55	84.34	73.46	67.81
	2	29.93	15.35	20.03	28.95
	Cum.	98.48	99.69	93.49	96.76
JAP	1	83.42	79.45	71.46	67.13
	2	15.19	19.56	21.27	29.92
	Cum.	98.61	99.01	92.73	97.05
U.K.	1	73.52	81.93	77.73	77.21
	2	25.02	17.63	19.56	21.25
	Cum.	98.54	99.56	97.29	98.46
JAP	1	76.50	77.61	74.91	67.26
	2	21.63	20.83	18.66	29.75
	Cum.	98.13	98.44	93.57	97.01

Table 2.19: For any given pair of countries we report the percent variance explained by principal components analysis of the residuals obtained from regressing each country's yields on the first one and on the first two common $PC^{(c)}s$. When yield levels are considered, the results are reported in the third and fourth columns, respectively, while when yield differences are analyzed these results are provided in the fifth and sixth columns. We use weekly data for a sample period starting from January 1, 1986 to December 31, 2009 (1252 observations) and residual maturities from 1 to 9 years.

The 3-Country Case.

1^{st} *and* 2^{nd} *Steps : Regressing PCs on* $PC^{(c)}s$ *and Associated* R^2 *(Yield levels).*

Table 2.20: For any given set of three countries, we report results from regressing the single-country yield curve principal components (PCs) on the first eleven common $PC^{(c)}s$ extracted by pooling the term structure data of the three countries. In both Panel A and B, the first column represents the dependent variable in the regression, naming one of the first three principal components of a country. The next columns in Panel A report the regression coefficients associated with a regression using all of the first eleven $PC^{(c)}s$, while the columns of Panel B report the R^2 when the dependent variable is regressed upon subsets of the first eleven $PC^{(c)}s$. We use weekly yield levels observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

Panel A	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}	11^{th}
$US\ 1^{st}$	0.5707	0.6482	-0.4133	0.1974	-0.2070	0.0196	0.0145	0.0284	0.0053	0.0041	0.0008
$US\ 2^{nd}$	-0.0190	0.1742	-0.3202	-0.3614	0.8061	0.2876	-0.0287	-0.0439	-0.0254	-0.0031	0.0058
$US\ 3^{rd}$	0.0013	-0.0066	0.0276	-0.0378	0.0206	0.1099	0.2050	0.6206	0.7438	0.0450	-0.0293
$UK\ 1^{st}$	0.5969	0.0614	0.7094	-0.3393	0.0951	0.0860	-0.0441	-0.0558	-0.0003	-0.0074	0.0033
$UK\ 2^{nd}$	-0.0012	0.0760	-0.1554	-0.5527	-0.0383	-0.7794	0.2334	0.0044	0.0274	-0.0117	-0.0110
$UK\ 3^{rd}$	0.0015	0.0012	-0.0448	0.0331	0.0208	-0.1457	-0.4954	-0.5685	0.6309	0.0888	-0.0038
$GER\ 1^{st}$	0.5634	-0.7201	-0.3521	0.1276	0.1200	-0.0882	0.0310	0.0258	-0.0087	0.0032	-0.0046
$GER\ 2^{nd}$	0.0168	0.1446	0.2688	0.6238	0.5287	-0.4650	0.1411	0.0381	0.0072	-0.0065	0.0086
$GER\ 3^{rd}$	-0.0021	-0.0195	0.0116	0.0420	-0.0416	0.2104	0.7954	-0.5299	0.1943	0.0064	-0.0130
Panel B	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8	First 9	First 10	First 11
$US\ 1^{st}$	0.8889	0.9874	0.9978	0.9995	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$US\ 2^{nd}$	0.0355	0.2913	0.5164	0.7163	0.9871	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000
$US\ 3^{rd}$	0.0066	0.0211	0.0870	0.1734	0.1804	0.2543	0.3300	0.7197	0.9998	0.9999	0.9999
$UK\ 1^{st}$	0.9637	0.9646	0.9950	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$UK\ 2^{nd}$	0.0002	0.0733	0.1529	0.8543	0.8552	0.9963	1.0000	1.0000	1.0000	1.0000	1.0000
$UK\ 3^{rd}$	0.0061	0.0065	0.1349	0.1838	0.1890	0.2846	0.6103	0.8512	0.9996	1.0000	1.0000
$GER\ 1^{st}$	0.8695	0.9915	0.9991	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$GER\ 2^{nd}$	0.0251	0.1840	0.3271	0.8639	0.9690	0.9991	1.0000	1.0000	1.0000	1.0000	1.0000
$GER\ 3^{rd}$	0.0082	0.0717	0.0775	0.1306	0.1448	0.2798	0.8487	0.9905	1.0000	1.0000	1.0000

Panel A	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}	11^{th}
$US\ 1^{st}$	0.5638	0.7107	0.3851	-0.0118	-0.1660	0.0150	0.0089	0.0264	0.0068	0.0069	-0.0024
$US\ 2^{nd}$	-0.0202	0.2259	-0.0868	-0.5707	0.7368	0.2339	-0.0162	-0.1302	-0.0182	0.0036	0.0122
$US\ 3^{rd}$	0.0020	-0.0183	-0.0231	-0.0409	0.0161	0.1809	0.3453	0.4289	0.8069	0.0722	0.0622
$UK\ 1^{st}$	0.5943	-0.0369	-0.7501	0.2514	0.1080	0.0807	-0.0339	0.0043	-0.0186	-0.0002	-0.0051
$UK\ 2^{nd}$	-0.0021	0.1077	-0.3205	-0.5764	-0.2916	-0.6641	0.1470	0.0594	0.0280	-0.0323	-0.0137
$UK\ 3^{rd}$	0.0000	0.0246	0.0063	0.0452	0.0152	-0.2117	-0.6624	-0.4388	0.5537	0.0465	0.1082
$JAP\ 1^{st}$	0.5732	-0.6530	0.3933	-0.2755	0.0708	-0.0872	0.0219	-0.0355	0.0087	-0.0062	0.0081
$JAP\ 2^{nd}$	-0.0057	-0.0599	-0.1518	-0.4426	-0.5619	0.6354	-0.2277	-0.0655	-0.0309	0.0243	0.0165
$JAP\ 3^{rd}$	-0.0010	-0.0057	0.0228	-0.0553	0.1074	-0.0820	-0.6034	0.7708	-0.1368	0.0181	-0.0266
Panel B	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8	First 9	First 10	First 11
$US\ 1^{st}$	0.8869	0.9941	0.9997	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$US\ 2^{nd}$	0.0408	0.4301	0.4404	0.7888	0.9936	0.9997	0.9997	1.0000	1.0000	1.0000	1.0000
$US\ 3^{rd}$	0.0153	0.1165	0.1451	0.2157	0.2195	0.3636	0.5249	0.6427	0.9994	0.9998	1.0000
$UK\ 1^{st}$	0.9766	0.9769	0.9980	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$UK\ 2^{nd}$	0.0007	0.1334	0.3436	0.8769	0.9250	0.9988	0.9999	1.0000	1.0000	1.0000	1.0000
$UK\ 3^{rd}$	0.0000	0.1345	0.1361	0.1998	0.2023	0.3477	0.7848	0.8757	0.9994	0.9995	1.0000
$JAP\ 1^{st}$	0.9028	0.9919	0.9977	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$JAP\ 2^{nd}$	0.0074	0.0700	0.1418	0.6206	0.8929	0.9957	0.9998	1.0000	1.0000	1.0000	1.0000
$JAP\ 3^{rd}$	0.0034	0.0111	0.0334	0.1363	0.2733	0.2968	0.6888	0.9918	0.9999	1.0000	1.0000

Panel A	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}	11^{th}
$US\ 1^{st}$	0.5582	0.8048	0.0585	0.0393	-0.1799	-0.0456	-0.0042	0.0092	-0.0338	0.0055	0.0023
$US\ 2^{nd}$	-0.0250	0.1800	0.0951	0.5098	0.8085	0.1448	0.0988	-0.0076	0.1150	-0.0158	-0.0023
$US\ 3^{rd}$	0.0018	-0.0116	-0.0250	0.0298	0.0310	0.0799	0.3216	-0.6016	-0.7183	0.0564	0.0781
$GER\ 1^{st}$	0.5807	-0.4365	0.6763	-0.0300	0.0299	0.1106	0.0199	-0.0124	0.0160	-0.0039	-0.0035
$GER\ 2^{nd}$	0.0119	0.1435	-0.0492	-0.7767	0.3645	0.4829	0.0675	0.0398	0.0338	-0.0046	0.0047
$GER\ 3^{rd}$	-0.0016	-0.0174	0.0090	-0.0457	-0.0187	-0.2063	0.8340	0.5019	-0.0761	-0.0359	-0.0047
$JAP\ 1^{st}$	0.5919	-0.3264	-0.7151	0.0338	0.1623	-0.0595	-0.0096	0.0072	0.0230	-0.0026	0.0011
$JAP\ 2^{nd}$	-0.0075	-0.0389	-0.1233	0.3586	-0.3712	0.8231	0.1336	0.1405	0.0370	-0.0187	0.0088
$JAP\ 3^{rd}$	0.0002	-0.0124	0.0221	0.0403	0.1205	0.0340	-0.4085	0.6023	-0.6699	0.0497	0.0303
Panel B	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8	First 9	First 10	First 11
$US\ 1^{st}$	0.8313	0.9992	0.9996	0.9996	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$US\ 2^{nd}$	0.0600	0.3621	0.3954	0.7379	0.9951	0.9994	0.9998	0.9998	1.0000	1.0000	1.0000
$US\ 3^{rd}$	0.0116	0.0615	0.1526	0.1988	0.2138	0.2648	0.4494	0.6947	0.9993	0.9996	1.0000
$GER\ 1^{st}$	0.9032	0.9527	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$GER\ 2^{nd}$	0.0122	0.1853	0.1933	0.9101	0.9573	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000
$GER\ 3^{rd}$	0.0050	0.0609	0.0667	0.1210	0.1237	0.2934	0.9131	0.9982	0.9999	1.0000	1.0000
$JAP\ 1^{st}$	0.9207	0.9479	0.9997	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$JAP\ 2^{nd}$	0.0124	0.0447	0.1730	0.5604	0.6844	0.9973	0.9992	1.0000	1.0000	1.0000	1.0000
$JAP\ 3^{rd}$	0.0001	0.0450	0.1017	0.1690	0.3487	0.3560	0.5933	0.7889	0.9998	0.9999	1.0000

Panel A	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}	11^{th}
$UK\ 1^{st}$	0.5795	-0.5083	0.5684	0.1576	0.1050	-0.1879	-0.0889	0.0576	0.0024	0.0061	-0.0149
$UK\ 2^{nd}$	-0.0067	-0.0406	0.1131	0.6060	-0.0940	0.6742	0.3737	-0.1054	-0.0517	-0.0242	0.0279
$UK\ 3^{rd}$	-0.0001	0.0282	0.0474	-0.0145	0.0490	0.1651	-0.4991	-0.4522	-0.7019	0.0969	0.1042
$GER\ 1^{st}$	0.5686	0.7918	0.1743	-0.0354	-0.1242	0.0158	0.0391	-0.0154	0.0216	-0.0057	0.0059
$GER\ 2^{nd}$	0.0088	-0.1922	0.2550	-0.6716	-0.5248	0.3382	0.2249	-0.0504	-0.0602	-0.0066	0.0139
$GER\ 3^{rd}$	-0.0011	0.0203	-0.0114	-0.0486	0.1845	-0.3459	0.6898	0.0724	-0.6006	-0.0087	0.0301
$JAP\ 1^{st}$	0.5837	-0.2647	-0.7379	-0.1017	0.0172	0.1697	0.0519	-0.0427	-0.0254	-0.0008	0.0088
$JAP\ 2^{nd}$	-0.0050	-0.0590	-0.1466	0.3775	-0.8070	-0.3999	-0.0689	0.0081	-0.1250	0.0146	-0.0064
$JAP\ 3^{rd}$	0.0001	0.0333	-0.0156	0.0112	-0.0321	0.2424	-0.2503	0.8704	-0.3361	0.0185	-0.0682
Panel B	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8	First 9	First 10	First 11
$UK\ 1^{st}$	0.9427	0.9746	0.9988	0.9998	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$UK\ 2^{nd}$	0.0069	0.0179	0.0701	0.9153	0.9179	0.9902	0.9996	1.0000	1.0000	1.0000	1.0000
$UK\ 3^{rd}$	0.0000	0.1032	0.2810	0.2904	0.3038	0.3880	0.7117	0.8330	0.9990	0.9995	1.0000
$GER\ 1^{st}$	0.9192	0.9976	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$GER\ 2^{nd}$	0.0071	0.1561	0.3154	0.9392	0.9869	0.9979	0.9999	1.0000	1.0000	1.0000	1.0000
$GER\ 3^{rd}$	0.0025	0.0389	0.0458	0.1173	0.2463	0.4965	0.9155	0.9176	1.0000	1.0000	1.0000
$JAP\ 1^{st}$	0.9504	0.9590	0.9995	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$JAP\ 2^{nd}$	0.0057	0.0413	0.1748	0.6743	0.9603	0.9991	0.9996	0.9996	1.0000	1.0000	1.0000
$JAP\ 3^{rd}$	0.0000	0.1558	0.1767	0.1828	0.1890	0.3850	0.4731	0.9586	0.9998	0.9998	1.0000

1^{st} and 2^{nd} Steps: Regressing PCs on $PC^{(c)}s$ and Associated R^2 (Yield differences).

Table 2.21: For any given set of three countries, we report results from regressing the single-country yield curve principal components (PCs) on the first eleven common $PC^{(c)}s$ extracted by pooling the term structure data of the three countries. In both Panel A and B, the first column represents the dependent variable in the regression, naming one of the first three principal components of a country. The next columns in Panel A report the regression coefficients associated with a regression using all of the first eleven $PC^{(c)}s$, while the columns of Panel B report the R^2 when the dependent variable is regressed upon subsets of the first eleven $PC^{(c)}s$. We use weekly yield differences observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

Panel A	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}	11^{th}
$US\ 1^{st}$	0.5990	-0.6354	-0.4802	0.0747	-0.0154	0.0305	0.0081	0.0006	-0.0029	0.0031	0.0010
$US\ 2^{nd}$	0.0043	0.0037	0.0048	-0.3507	0.0507	0.9320	0.0711	-0.0132	-0.0188	-0.0093	0.0036
$US\ 3^{rd}$	0.0005	0.0026	-0.0027	-0.0299	0.0054	-0.0400	0.1401	0.1735	-0.9681	-0.0999	0.0003
$UK\ 1^{st}$	0.5667	0.7625	-0.3093	-0.0367	0.0118	-0.0185	-0.0003	-0.0045	0.0044	0.0000	-0.0026
$UK\ 2^{nd}$	0.0249	-0.0708	-0.0113	-0.6114	0.7389	-0.2701	0.0116	-0.0086	0.0328	0.0146	-0.0092
$UK\ 3^{rd}$	-0.0028	0.0040	-0.0070	-0.0032	0.0017	0.0067	0.1236	0.9725	0.1946	-0.0256	0.0157
$GER\ 1^{st}$	0.5651	-0.0871	0.8202	-0.0015	-0.0179	-0.0057	-0.0036	0.0085	-0.0008	-0.0020	0.0018
$GER\ 2^{nd}$	0.0090	-0.0475	-0.0288	-0.7037	-0.6704	-0.2252	-0.0315	-0.0001	0.0226	-0.0001	-0.0006
$GER\ 3^{rd}$	-0.0028	0.0033	0.0070	0.0141	-0.0349	-0.0671	0.9791	-0.1468	0.1161	0.0144	0.0050
Panel B	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8	First 9	First 10	First 11
$US\ 1^{st}$	0.6449	0.8765	0.9992	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$US\ 2^{nd}$	0.0006	0.0007	0.0009	0.2729	0.2761	0.9974	0.9999	0.9999	1.0000	1.0000	1.0000
$US\ 3^{rd}$	0.0000	0.0005	0.0009	0.0130	0.0132	0.0214	0.0819	0.1186	0.9955	0.9999	0.9999
$UK\ 1^{st}$	0.6001	0.9468	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$UK\ 2^{nd}$	0.0118	0.0423	0.0431	0.5517	0.9626	0.9998	0.9999	0.9999	1.0000	1.0000	1.0000
$UK\ 3^{rd}$	0.0012	0.0020	0.0043	0.0044	0.0044	0.0046	0.0424	0.9712	0.9997	1.0000	1.0000
$GER\ 1^{st}$	0.6129	0.6175	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$GER\ 2^{nd}$	0.0015	0.0145	0.0189	0.6556	0.9752	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000
$GER\ 3^{rd}$	0.0005	0.0007	0.0016	0.0025	0.0055	0.0131	0.9871	0.9958	1.0000	1.0000	1.0000
Panel A	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}	11^{th}
$US\ 1^{st}$	0.6716	-0.2720	-0.6857	0.0249	-0.0552	0.0292	0.0064	0.0040	-0.0031	-0.0040	0.0012
$US\ 2^{nd}$	0.0053	0.0085	0.0060	-0.1112	0.3862	0.9149	0.0261	-0.0052	-0.0178	0.0118	0.0027
$US\ 3^{rd}$	0.0026	0.0064	0.0025	-0.0066	0.0273	-0.0301	0.0428	0.2748	-0.9588	-0.0327	0.0015
$UK\ 1^{st}$	0.6405	-0.2528	0.7241	-0.0052	0.0283	-0.0181	-0.0184	-0.0012	0.0021	0.0028	-0.0021
$UK\ 2^{nd}$	0.0281	0.0046	-0.0695	-0.1580	0.8996	-0.3981	0.0066	-0.0107	0.0364	0.0012	-0.0093
$UK\ 3^{rd}$	-0.0013	0.0007	0.0034	-0.0052	0.0030	0.0081	0.0379	0.9594	0.2773	-0.0270	0.0154
$JAP\ 1^{st}$	0.3711	0.9284	-0.0035	0.0006	-0.0181	-0.0030	0.0015	-0.0019	0.0063	0.0002	0.0008
$JAP\ 2^{nd}$	0.0087	-0.0067	-0.0107	-0.9808	-0.1905	-0.0388	-0.0017	-0.0047	0.0010	0.0001	0.0055
$JAP\ 3^{rd}$	0.0066	-0.0049	0.0177	0.0025	-0.0168	-0.0206	0.9977	-0.0481	0.0313	0.0004	-0.0032
Panel B	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8	First 9	First 10	First 11
$US\ 1^{st}$	0.6704	0.7305	0.9996	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$US\ 2^{nd}$	0.0007	0.0018	0.0021	0.0281	0.2802	0.9995	0.9999	0.9999	1.0000	1.0000	1.0000
$US\ 3^{rd}$	0.0010	0.0046	0.0049	0.0055	0.0133	0.0180	0.0252	0.1204	0.9994	0.9999	0.9999
$UK\ 1^{st}$	0.6339	0.6879	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$UK\ 2^{nd}$	0.0125	0.0127	0.0420	0.0744	0.9160	0.9998	0.9999	0.9999	1.0000	1.0000	1.0000
$UK\ 3^{rd}$	0.0002	0.0002	0.0008	0.0011	0.0012	0.0014	0.0060	0.9404	0.9996	1.0000	1.0000
$JAP\ 1^{st}$	0.2261	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$JAP\ 2^{nd}$	0.0009	0.0012	0.0018	0.9701	0.9994	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$JAP\ 3^{rd}$	0.0017	0.0023	0.0071	0.0071	0.0079	0.0084	0.9990	0.9998	1.0000	1.0000	1.0000

Panel *A*	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}	11^{th}
US 1^{st}	0.6439	-0.4715	0.6010	0.0233	-0.0301	0.0177	-0.0035	-0.0096	-0.0002	-0.0056	-0.0024
US 2^{nd}	0.0052	0.0087	-0.0041	-0.1719	0.3238	0.9249	0.0495	-0.0861	-0.0064	0.0129	0.0042
US 3^{rd}	0.0014	0.0070	0.0042	-0.0125	0.0206	-0.0309	0.0330	-0.1359	-0.9840	-0.0272	0.0995
GER 1^{st}	0.6378	-0.1029	-0.7628	0.0076	0.0218	-0.0127	0.0074	0.0003	-0.0022	0.0061	0.0009
GER 2^{nd}	0.0120	-0.0224	0.0383	-0.4176	0.8300	-0.3634	-0.0318	0.0276	0.0288	0.0004	-0.0037
GER 3^{rd}	0.0002	0.0144	-0.0025	-0.0298	-0.0257	-0.0847	-0.0555	-0.9844	0.1346	0.0218	-0.0148
JAP 1^{st}	0.4222	0.8753	0.2350	-0.0065	-0.0012	-0.0089	-0.0017	0.0142	0.0060	-0.0010	-0.0015
JAP 2^{nd}	0.0126	-0.0113	-0.0096	-0.8912	-0.4514	-0.0042	0.0022	0.0384	-0.0026	-0.0001	0.0074
JAP 3^{rd}	-0.0017	0.0000	0.0094	-0.0040	0.0091	-0.0612	0.9961	-0.0452	0.0421	0.0040	0.0042
Panel *B*	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8	First 9	First 10	First 11
US 1^{st}	0.6370	0.8118	0.9998	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
US 2^{nd}	0.0007	0.0018	0.0019	0.0662	0.2489	0.9948	0.9963	1.0000	1.0000	1.0000	1.0000
US 3^{rd}	0.0003	0.0044	0.0053	0.0074	0.0119	0.0171	0.0213	0.0771	0.9952	0.9956	1.0000
GER 1^{st}	0.6676	0.6765	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
GER 2^{nd}	0.0022	0.0062	0.0138	0.2345	0.9324	0.9993	0.9997	0.9999	1.0000	1.0000	1.0000
GER 3^{rd}	0.0000	0.0057	0.0058	0.0097	0.0120	0.0247	0.0287	0.9942	0.9999	1.0000	1.0000
JAP 1^{st}	0.3026	0.9682	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
JAP 2^{nd}	0.0020	0.0028	0.0032	0.8299	0.9996	0.9996	0.9997	1.0000	1.0000	1.0000	1.0000
JAP 3^{rd}	0.0001	0.0001	0.0014	0.0014	0.0016	0.0067	0.9980	0.9996	1.0000	1.0000	1.0000

Panel *A*	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}	11^{th}
UK 1^{st}	0.6211	-0.4884	0.6122	-0.0141	0.0098	-0.0094	-0.0217	-0.0032	0.0031	-0.0001	-0.0005
UK 2^{nd}	0.0162	0.0179	-0.0310	-0.4434	0.5071	-0.7363	0.0281	-0.0391	0.0058	-0.0018	-0.0112
UK 3^{rd}	-0.0027	0.0007	0.0087	-0.0046	-0.0039	-0.0011	0.0294	-0.1207	-0.9911	-0.0254	0.0374
GER 1^{st}	0.6460	-0.1233	-0.7521	0.0325	-0.0027	0.0221	0.0149	-0.0027	-0.0079	0.0036	0.0003
GER 2^{nd}	-0.0022	0.0001	-0.0092	-0.5705	0.4749	0.6693	-0.0296	0.0100	-0.0020	0.0012	0.0017
GER 3^{rd}	0.0018	0.0124	0.0014	-0.0136	-0.0421	0.0311	-0.0516	-0.9898	0.1183	0.0210	-0.0072
JAP 1^{st}	0.4433	0.8635	0.2396	0.0008	-0.0104	0.0124	-0.0004	0.0128	0.0000	-0.0014	-0.0007
JAP 2^{nd}	0.0096	-0.0067	-0.0204	-0.6902	-0.7178	-0.0798	0.0027	0.0374	0.0017	0.0003	0.0070
JAP 3^{rd}	0.0036	-0.0084	0.0251	-0.0040	0.0000	0.0420	0.9969	-0.0463	0.0355	0.0024	0.0072
Panel *B*	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8	First 9	First 10	First 11
UK 1^{st}	0.5998	0.7930	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
UK 2^{nd}	0.0042	0.0068	0.0122	0.2854	0.5927	0.9992	0.9995	1.0000	1.0000	1.0000	1.0000
UK 3^{rd}	0.0009	0.0010	0.0044	0.0046	0.0048	0.0048	0.0075	0.0429	0.9992	0.9995	1.0000
GER 1^{st}	0.6664	0.6791	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
GER 2^{nd}	0.0001	0.0001	0.0005	0.4277	0.6823	0.9996	1.0000	1.0000	1.0000	1.0000	1.0000
GER 3^{rd}	0.0002	0.0044	0.0044	0.0053	0.0122	0.0146	0.0181	0.9943	0.9999	1.0000	1.0000
JAP 1^{st}	0.3246	0.9663	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
JAP 2^{nd}	0.0011	0.0014	0.0032	0.5175	0.9960	0.9997	0.9997	1.0000	1.0000	1.0000	1.0000
JAP 3^{rd}	0.0005	0.0020	0.0110	0.0111	0.0111	0.0144	0.9980	0.9996	1.0000	1.0000	1.0000

3^{rd} *Step : Extracting* $PC^{(\ell)}s$ *from* $PC^{(c)}s$*-based Yield Curve Residuals*

Countries	$PC^{(\ell)}s$	Yield levels			Yield differences		
		after 1 $PC^{(c)}$	after 2 $PC^{(c)}s$	after 3 $PC^{(c)}s$	after 1 $PC^{(c)}$	after 2 $PC^{(c)}s$	after 3 $PC^{(c)}s$
U.S.	1	86.31	55.18	83.03	85.26	71.07	71.11
	2	13.23	43.28	15.18	12.32	24.09	25.44
	Cum.	99.54	98.46	98.21	97.58	95.16	96.55
U.K.	1	73.88	74.58	66.92	80.81	53.17	76.92
	2	24.80	24.06	26.68	16.86	40.30	21.52
	Cum.	98.68	98.64	95.60	97.67	93.47	98.44
GER	1	84.70	58.13	82.05	76.14	76.15	67.51
	2	14.56	38.70	17.70	17.99	17.94	29.25
	Cum.	99.26	96.83	99.75	94.13	94.09	96.76
U.S.	1	87.07	63.56	82.74	84.39	82.04	71.76
	2	12.53	34.19	15.86	13.05	15.01	25.18
	Cum.	99.60	97.75	98.60	97.44	97.05	96.94
U.K.	1	66.31	69.27	80.56	79.62	77.34	76.93
	2	31.72	28.87	18.52	17.91	19.93	21.54
	Cum.	98.03	98.14	99.08	97.53	97.27	98.47
JAP	1	89.77	53.96	64.32	81.48	67.18	67.25
	2	9.40	42.35	31.73	13.73	29.85	29.77
	Cum.	99.17	96.31	96.05	95.21	97.03	97.02
U.S.	1	90.93	77.38	81.45	85.50	77.41	72.24
	2	8.75	20.93	16.96	12.12	18.83	25.01
	Cum.	99.68	98.31	98.41	97.62	96.24	97.25
GER	1	80.03	69.98	83.38	73.74	73.44	67.57
	2	19.03	28.41	16.27	19.84	20.09	29.19
	Cum.	99.06	98.39	99.65	93.58	93.53	96.76
JAP	1	88.79	84.67	75.18	80.23	55.29	67.13
	2	10.18	14.00	23.13	14.65	31.92	29.92
	Cum.	98.97	98.67	98.31	94.88	87.21	97.05
U.K.	1	81.24	68.27	78.46	80.55	70.66	77.16
	2	17.65	30.31	18.93	17.08	25.78	21.29
	Cum.	98.89	98.58	97.39	97.63	96.44	98.45
GER	1	76.89	75.85	81.99	73.48	72.94	67.81
	2	22.01	21.78	17.59	19.98	20.41	28.85
	Cum.	98.90	97.63	99.58	93.46	93.35	96.66
JAP	1	83.13	80.49	78.96	79.65	54.82	67.25
	2	15.35	18.06	19.32	15.10	31.63	29.78
	Cum.	98.48	98.55	98.28	94.75	86.45	97.03

Table 2.22: For any given set of three countries we report the percent variance explained in the residuals obtained from regressing each country's yields on the first one to the first three common $PC^{(c)}s$. When yield levels are considered, the results are reported in the third to fifth columns while, when yield differences are analyzed, these results are provided in the sixth to eighth columns. We use weekly data for a sample period starting from January 1, 1986 to December 31, 2009 (1252 observations) and residual maturities from 1 to 9 years.

The 4-country Case.

1^{st} *and* 2^{nd} *Steps: Regressing PCs on* $PC^{(c)}s$ *and Associated* R^2 *(Yield levels).*

Table 2.23: In the case of four countries, we report results from regressing the single-country yield curve principal components (PCs) on the first sixteen common $PC^{(c)}s$ extracted by pooling the term structure data of all four countries. In both Panel A and B, the first column represents the dependent variable in the regression, naming one of the first three yield curve principal components of a country. The next columns in Panel A report the regression coefficients associated with a regression using all of the first sixteen $PC^{(c)}s$, while the columns of Panel B report the R^2 when the dependent variable is regressed upon subsets of the first sixteen $PC^{(c)}s$. We use weekly yield levels observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

Panel A	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}	11^{th}	12^{th}	13^{th}	14^{th}	15^{th}	16^{th}
U.S. 1^{st}	0.4821	0.7411	0.1781	0.0139	0.3885	0.1822	0.0011	0.0364	0.0019	-0.0130	-0.0260	-0.0022	-0.0071	-0.0004	-0.0035	0.0048
U.S. 2^{nd}	-0.0208	0.1800	0.1097	0.3988	0.0167	-0.8590	-0.1072	0.1615	-0.0205	0.0476	0.1310	-0.0072	-0.0006	0.0066	0.0083	-0.0174
U.S. 3^{rd}	0.0016	-0.0105	-0.0259	0.0263	-0.0174	-0.0380	-0.0471	0.2123	0.1683	-0.8480	-0.2390	0.3664	-0.0937	-0.0225	-0.0040	-0.0305
U.K. 1^{st}	0.5183	0.1347	-0.2222	-0.0340	-0.7860	-0.0454	0.1484	0.0967	-0.0970	0.0248	-0.0320	-0.0096	0.0015	-0.0156	0.0054	0.0049
U.K. 2^{nd}	-0.0034	0.0828	0.0487	0.5195	-0.2002	0.1440	-0.1624	-0.6717	0.4150	-0.0384	0.0122	0.0818	0.0354	0.0216	-0.0086	0.0078
U.K. 3^{rd}	0.0004	0.0102	0.0498	-0.0216	0.0023	0.0178	0.0376	-0.2269	-0.4822	0.0709	0.3668	0.7386	-0.1226	0.0879	-0.0295	-0.0481
GER 1^{st}	0.4918	-0.4908	0.7044	-0.0333	0.0142	-0.0202	-0.1291	0.0086	0.0391	-0.0117	0.0218	-0.0170	0.0051	0.0065	-0.0025	-0.0072
GER 2^{nd}	0.0122	0.1389	-0.0668	-0.6633	-0.0777	-0.2556	-0.5871	-0.2371	0.2340	0.0222	0.0623	0.0660	0.0086	0.0162	0.0058	-0.0021
GER 3^{rd}	-0.0016	-0.0178	0.0067	-0.0458	0.0075	0.0140	0.2280	0.3455	0.6497	0.4127	-0.0176	0.4808	0.0045	0.0342	-0.0038	-0.0260
JAP 1^{st}	0.5064	-0.3625	-0.6211	0.0936	0.4212	-0.1316	-0.0265	-0.1284	0.0564	0.0054	0.0410	0.0276	0.0005	0.0098	0.0005	-0.0040
JAP 2^{nd}	-0.0059	-0.0312	-0.1191	0.3400	-0.0744	0.3273	-0.7170	0.4470	-0.1109	0.1480	0.0533	0.0668	-0.0184	-0.0022	-0.0031	-0.0271
JAP 3^{rd}	-0.0003	-0.0133	0.0252	0.0300	0.0339	-0.1170	-0.0901	-0.1461	-0.2406	0.2702	-0.8746	0.2219	-0.0231	-0.0661	0.0333	0.0533
Panel B	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8	First 9	First 10	First 11	First 12	First 13	First 14	First 15	First 16
U.S. 1^{st}	0.8441	0.9905	0.9939	0.9939	0.9996	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
U.S. 2^{nd}	0.0565	0.3669	0.4138	0.6964	0.6968	0.9955	0.9977	0.9997	0.9997	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
U.S. 3^{rd}	0.0139	0.0560	0.1594	0.2078	0.2240	0.2471	0.2634	0.4006	0.4517	0.9098	0.9435	0.9993	0.9999	1.0000	1.0000	1.0000
U.K. 1^{st}	0.9666	0.9714	0.9767	0.9768	0.9998	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
U.K. 2^{nd}	0.0023	0.1009	0.1148	0.8344	0.9159	0.9285	0.9359	0.9881	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
U.K. 3^{rd}	0.0008	0.0299	0.3110	0.3350	0.3352	0.3390	0.3466	0.4621	0.7710	0.7733	0.8317	0.9988	0.9996	0.9999	0.9999	1.0000
GER 1^{st}	0.8814	0.9459	0.9998	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
GER 2^{nd}	0.0175	0.1843	0.1999	0.9047	0.9121	0.9360	0.9937	0.9976	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
GER 3^{rd}	0.0063	0.0661	0.0695	0.1429	0.1444	0.1460	0.3365	0.5180	0.8978	0.9519	0.9520	1.0000	1.0000	1.0000	1.0000	1.0000
JAP 1^{st}	0.9171	0.9516	0.9928	0.9932	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
JAP 2^{nd}	0.0105	0.0319	0.1582	0.6277	0.6449	0.7440	0.9625	0.9977	0.9990	0.9998	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
JAP 3^{rd}	0.0003	0.0538	0.1318	0.1821	0.2310	0.4053	0.4527	0.5045	0.5875	0.6246	0.9833	0.9996	0.9997	0.9999	0.9999	1.0000

1^{st} and 2^{nd} Steps: Regressing PCs on $PC^{(c)}s$ and Associated R^2 (Yield differences).

Table 2.24: In the case of four countries, we report results from regressing the single-country yield curve principal components (PCs) on the first sixteen common $PC^{(c)}s$ extracted by pooling the term structure data of all four countries. In both Panel A and B, the first column represents the dependent variable in the regression, naming one of the first three yield curve principal components of a country. The next columns in Panel A report the regression coefficients associated with a regression using all of the first sixteen $PC^{(c)}s$, while the columns of Panel B report the R^2 when the dependent variable is regressed upon subsets of the first sixteen $PC^{(c)}s$. We use weekly yield differences observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

Panel *A*	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}	11^{th}	12^{th}	13^{th}	14^{th}	15^{th}	16^{th}
U.S. 1^{st}	0.5608	-0.2894	-0.6495	-0.4158	-0.0656	0.0366	-0.0146	0.0306	0.0030	-0.0091	-0.0003	0.0029	0.0058	0.0021	0.0009	-0.0073
U.S. 2^{nd}	0.0053	0.0081	0.0039	0.0014	0.2912	-0.1930	0.0487	0.9306	0.0501	-0.0774	0.0135	0.0191	-0.0136	-0.0071	0.0033	-0.0106
U.S. 3^{rd}	0.0015	0.0062	0.0027	-0.0056	0.0247	-0.0159	0.0032	-0.0424	0.0336	-0.1392	-0.1589	0.9692	0.0261	-0.1052	0.0009	0.0198
U.K. 1^{st}	0.5312	-0.2684	0.7517	-0.2797	0.0336	-0.0174	0.0133	-0.0170	-0.0231	0.0004	0.0035	-0.0033	-0.0015	0.0002	-0.0025	0.0015
U.K. 2^{nd}	0.0252	0.0035	-0.0709	-0.0134	0.4710	-0.4086	0.7280	-0.2712	0.0140	-0.0137	0.0086	-0.0325	0.0009	0.0144	-0.0093	0.0056
U.K. 3^{rd}	-0.0026	0.0006	0.0039	-0.0077	0.0055	0.0034	0.0016	0.0047	0.0313	-0.1249	-0.9738	-0.1823	0.0249	-0.0285	0.0156	-0.0049
GER 1^{st}	0.5502	0.0272	-0.0692	0.8311	-0.0074	-0.0159	-0.0168	-0.0053	0.0143	-0.0007	-0.0083	0.0017	-0.0053	-0.0014	0.0017	0.0028
GER 2^{nd}	0.0076	-0.0129	-0.0485	-0.0245	0.5968	-0.3619	-0.6766	-0.2209	-0.0416	0.0300	-0.0011	-0.0228	-0.0008	0.0009	-0.0010	-0.0026
GER 3^{rd}	-0.0008	0.0139	0.0037	0.0018	0.0078	0.0438	-0.0309	-0.0703	-0.0498	-0.9770	0.1439	-0.1150	-0.0237	0.0184	0.0042	0.0032
JAP 1^{st}	0.3158	0.9181	0.0166	-0.2378	-0.0027	0.0105	-0.0122	-0.0045	-0.0010	0.0144	0.0007	-0.0054	0.0011	0.0013	0.0005	0.0020
JAP 2^{nd}	0.0103	-0.0067	-0.0127	0.0146	0.5753	0.8127	0.0790	-0.0126	0.0016	0.0389	0.0008	0.0029	-0.0001	-0.0072	0.0054	-0.0030
JAP 3^{rd}	0.0027	-0.0054	0.0190	-0.0180	0.0032	0.0013	-0.0423	-0.0548	0.9949	-0.0349	0.0429	-0.0355	-0.0033	-0.0036	-0.0027	0.0148
Panel *B*	Only 1^{st}	First 2	First 3	First 4	First 5	First 6	First 7	First 8	First 9	First 10	First 11	First 12	First 13	First 14	First 15	First 16
U.S. 1^{st}	0.6001	0.6683	0.9103	0.9992	0.9998	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
U.S. 2^{nd}	0.0009	0.0019	0.0020	0.0020	0.1995	0.2731	0.2759	0.9953	0.9969	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
U.S. 3^{rd}	0.0004	0.0038	0.0042	0.0060	0.0147	0.0178	0.0178	0.0270	0.0314	0.0905	0.1212	0.9947	0.9951	0.9999	0.9999	1.0000
U.K. 1^{st}	0.5598	0.6208	0.9579	0.9998	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
U.K. 2^{nd}	0.0129	0.0130	0.0436	0.0446	0.3625	0.5654	0.9622	0.9997	0.9998	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
U.K. 3^{rd}	0.0011	0.0011	0.0019	0.0045	0.0048	0.0049	0.0049	0.0050	0.0081	0.0463	0.9745	0.9994	0.9997	1.0000	1.0000	1.0000
GER 1^{st}	0.6170	0.6176	0.6205	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
GER 2^{nd}	0.0011	0.0025	0.0160	0.0191	0.5013	0.6517	0.9755	0.9991	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
GER 3^{rd}	0.0000	0.0056	0.0059	0.0059	0.0062	0.0139	0.0162	0.0246	0.0278	0.9875	0.9958	0.9998	0.9999	1.0000	1.0000	1.0000
JAP 1^{st}	0.2103	0.9677	0.9678	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
JAP 2^{nd}	0.0017	0.0020	0.0027	0.0036	0.3722	0.9959	0.9996	0.9996	0.9996	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
JAP 3^{rd}	0.0004	0.0010	0.0066	0.0111	0.0111	0.0111	0.0145	0.0184	0.9982	0.9991	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000

3^{rd} Step : Extracting $PC^{(\ell)}s$ from $PC^{(c)}s$-Based Yield Curve Residuals

Countries	$PC^{(\ell)}s$	Yield levels				Yield differences			
		after 1 $PC^{(c)}$	after 2 $PC^{(c)}s$	after 3 $PC^{(c)}s$	after 4 $PC^{(c)}s$	after 1 $PC^{(c)}$	after 2 $PC^{(c)}s$	after 3 $PC^{(c)}s$	after 4 $PC^{(c)}s$
U.S.	1	90.35	56.68	61.62	61.16	86.50	84.54	65.95	71.10
	2	9.32	41.55	36.29	35.69	11.29	12.92	28.34	25.42
	Cum.	99.67	98.23	97.91	96.85	97.79	97.46	94.29	96.52
U.K.	1	72.80	71.91	69.34	91.26	82.05	80.10	54.39	76.87
	2	25.64	26.39	29.20	6.70	15.76	17.49	38.72	21.56
	Cum.	98.44	98.30	98.54	97.96	97.81	97.59	93.11	98.43
GER	1	83.05	72.31	83.41	66.08	75.88	75.86	75.93	67.54
	2	16.14	26.22	16.18	31.70	18.18	18.22	18.12	29.19
	Cum.	99.19	98.53	99.59	97.78	94.06	94.08	94.05	96.73
JAP	1	88.88	83.38	54.37	64.26	81.80	55.19	55.26	67.25
	2	10.13	15.19	42.32	30.55	13.48	31.63	31.64	29.79
	Cum.	99.01	98.57	96.69	94.81	95.28	86.82	86.90	97.04

Table 2.25: In the case of four countries we report the percent variance explained in the residuals obtained from regressing each country's yields on the first one to the first four $PC^{(c)}s$. When yield levels are considered, the results are reported in the third to sixth columns while, when yield differences are analyzed, these results are provided in the seventh to tenth columns. We use weekly data for a sample period starting from January 1, 1986 to December 31, 2009 (1252 observations) and residual maturities from 1 to 9 years.

2.C The *DLY*-Approach.

Countries		Yield levels				Yield differences			
		$PC^{(L)}$s		$PC^{(S)}$s		$PC^{(L)}$s		$PC^{(S)}$s	
		Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.
U.S.-U.K.	1	96.66	96.66	79.04	79.04	73.18	73.18	62.91	62.91
	2	3.34	100.00	20.96	100.00	26.82	100.00	37.09	100.00
U.S.-GER	1	94.75	94.75	52.77	52.77	72.92	72.92	64.74	64.74
	2	5.25	100.00	47.23	100.00	27.08	100.00	35.26	100.00
U.S.-JAP	1	94.61	94.61	50.16	50.16	58.96	58.96	54.22	54.22
	2	5.39	100.00	49.84	100.00	41.04	100.00	45.78	100.00
GER-U.K.	1	97.18	97.18	74.24	74.24	71.02	71.02	63.36	63.36
	2	2.82	100.00	25.76	100.00	28.98	100.00	36.64	100.00
GER-JAP	1	95.45	95.45	74.77	74.77	62.29	62.29	54.91	54.91
	2	4.55	100.00	25.23	100.00	37.71	100.00	45.09	100.00
U.K.-JAP	1	97.86	97.89	77.92	77.92	58.47	58.47	52.37	52.37
	2	2.14	100.00	22.08	100.00	41.53	100.00	47.63	100.00
U.S.-U.K.-GER	1	94.93	94.93	59.47	59.47	63.17	63.17	51.57	51.57
	2	3.52	98.45	31.52	90.99	19.33	82.50	24.95	76.52
	3	1.55	100.00	9.01	100.00	17.50	100.00	23.48	100.00
U.S.-U.K.-JAP	1	95.18	95.18	60.24	60.24	52.34	52.34	42.95	42.95
	2	3.68	98.86	33.23	93.47	29.78	82.12	32.42	75.36
	3	1.14	100.00	6.53	100.00	17.88	100.00	24.64	100.00
U.S.-GER-JAP	1	93.25	93.25	49.96	49.96	53.59	53.59	44.78	44.78
	2	3.72	96.97	33.31	83.27	28.56	82.15	31.73	76.51
	3	3.03	100.00	16.73	100.00	17.85	100.00	23.49	100.00
U.K.-GER-JAP	1	95.78	95.78	67.55	67.55	52.40	52.40	43.41	43.41
	2	3.07	98.85	17.75	85.30	28.58	80.98	32.33	75.74
	3	1.15	100.00	14.70	100.00	19.02	100.00	24.26	100.00
U.S.-U.K.-GER-JAP	1	94.13	94.13	53.99	53.99	50.29	50.29	39.45	39.45
	2	2.82	96.95	28.54	82.52	22.36	72.65	24.35	63.80
	3	2.30	99.25	12.86	95.38	14.27	86.92	18.60	82.40
	4	0.75	100.00	4.61	100.00	13.08	100.00	17.60	100.00

Table 2.26: We conduct a *DLY*-based analysis of the term structure of interest rates of groups of two, three and four countries and report the marginal and cumulative percent variance explained. We extract, using regression, the level factor and the slope factor from each country's term structure, and then we perform a principal component analysis across countries on each set of extracted factors (level L and slope S). The first column lists the group of countries while the second column lists the number of extracted principal components. The rest of the table is divided into two vertical blocks: the first block represents yield levels and the second block represents yield differences. Each block shows the results of a principal component analysis, for levels in the first two columns and for slopes in the next two. For each group of countries, we use weekly yields observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

Chapter 3

Specification Analysis of International Treasury Yield Curve Factors

Abstract

We show how to compute patterns of variation over time, both among and within countries, that determine the international term structure of interest rates, using maximum likelihood within a linear Gaussian state-space framework. The simultaneous estimation of common factors (shared by all countries) and local factors (specific to one country) requires development of a normalization procedure beyond that of ordinary factor analysis. By jointly estimating common and local factors we avoid sequential estimation effects that may explain the lack of agreement in the multi-country term structure literature regarding not only the total number of latent factors required to explain the joint dynamics of yield curves, but also the number of common and of local factors. Using data on international yield curves of U.S., Germany, U.K. and Japan from January 1986

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to December 2009, we generally find (analyzing yields in level and in difference) that a model with two common factors and three correlated local factors is preferred to a model (of similar complexity) that includes one common factor only or a model with only correlated local factors. In addition, each common factor closely mimics (or is similar to) a local factor extracted from a pure local factor model. We also reach the conclusion that dependence across international yield curves are driven, first, by the instantaneous correlation between local factors of different countries and, then, by the (full) autoregressive matrix of latent factors and by the matrix of common loadings. A last empirical exercise highlight the importance of common factors in improving the forecast of macroeconomic variables over several forecasting horizons.

Résumé

Ce chapître est dédié à l'analyse jointe des courbes des taux de plusieurs pays. Nous avons élaboré une méthode nouvelle de selection des facteurs commun et locaux en utilisant un modèle espace-état linéaire gaussien et l'algorithme EM (espérance-maximisation) dans la procédure de maximisation de la fonction de vraisemblance. Nous avons résolu le problème d'identification associé d'une façon novatrice. En particulier, à chaque itération de l'algorithme de maximisation, nous sommes intervenus avec une normalisation des paramètres et des facteurs qui permet de préserver les rapports de causalité entre ces derniers (communs et locaux). De plus, l'estimation simultanée des facteurs " communs " (affectant toutes les courbes des taux considérées) et " locaux " (affectant seulement une courbe des taux) évite les effets d'une estimation séquentielle qui peuvent être à la base de l'absence de consensus dans la littérature sur le nombre total et la nature des facteurs décrivant la dynamique jointe des courbes des taux. En utilisant la base des données construite dans le chapître précédent, plusieurs combinaisons de facteurs et des pays ont été considérées (pour des taux en niveau et en différence) et une comparaison de résultats conduit à la sélection d'un modèle avec deux facteurs communs et trois facteurs locaux corrélés. En outre, chaque facteur commun imite la dynamique d'un facteur local extrait dans un modèle caractérisé seulement des facteurs locaux corrélés. Nous observons aussi que la structure de dépendance des taux d'intérêt internationales est entraînée, d'abord, par les corrélations instantanées entre facteurs locaux internationaux à différents pays, ensuite par la matrice autorégressive des facteurs latents et enfin par la matrice des loadings communs. Puis, nous découvrons que les facteurs "communs" contribuent à la prévision des variables macro-économiques (croissance économique et taux d'inflation) sur plusieurs horizons temporels.

3.1 Introduction

The yield curve literature, following the seminal papers of Vasicek (1977) and Cox, Ingersoll, and Ross (1985), has focused not only on the specification and estimation of models explaining the term structure of interest rates in a single economy but, more recently, has been extended to the relevant problem of specifying and estimating the joint dynamics of international yield curves[1].

In the single-country case, the estimation and implementation of dynamic yield curve models has found that three latent factors [e.g., the level, slope and curvature factors of Litterman and Scheinkman (1991)] are required to match the dynamics and the shapes of the term structure, and this holds regardless of the sample period and the data source [see Dai and Singleton (2000) and Dai and Singleton (2002) and Dai and Singleton (2003), Duffee (2002), Cheridito, Filipovic, and Kimmel (2007), Duarte (2004)]. This wide degree of robustness has made this result a fundamental building block characterizing the modeling of single-country yield curves.

In the multi-country setting, in contrast, we observe substantial lack of agreement, not only about the number of latent factors that are required to explain the joint dynamics of two or more countries' yield curves, but also about the common/local nature of the factors where each common factor affects yields in all countries while each local factor affects yields in only one country. Some researchers make *a priori* assumptions about the combination of common and local factors [e.g., Backus, Foresi, and Telmer (2001), Anderson, Hammond, and Ramezani (2010), Ahn (2004)], while others reach different

[1]See, among others, Duffie and Kan (1996), Dai and Singleton (2000), Dai and Singleton (2002) Dai and Singleton (2003), Bansal and Zhou (2002), Duffee (2002), Monfort and Pegoraro (2007), in the single-country literature, and Frachot (1995), Backus, Foresi, and Telmer (2001), Anderson, Hammond, and Ramezani (2010), Ahn (2004), Leippold and Wu (2007), Tang and Xia (2007), Diebold, Li, and Yue (2008), Egorov, Li, and Ng (2011), Gourieroux, Monfort, and Sufana (2010), Jotikasthira, Le, and Lundbland (2010), in the multi-country literature.

conclusions about the number of common and local factors based on the explained variance criterion within a principal components (PC) approach [see Leippold and Wu (2007), Diebold, Li, and Yue (2008) and Egorov, Li, and Ng (2011)].

The purpose of this chapter is to respond to this lack of agreement by directly addressing the theoretical and empirical issues that naturally characterize the selection and estimation of interest rate factors that evolve over time in a multi-country setting. We propose using maximum likelihood (ML) criteria within a linear Gaussian state-space approach, to jointly and reliably estimate the preferred combination of common and local factors required to jointly explain multi-country yield curves.

Our choice of a state-space framework is a response, in part, to two main critiques of PC-based approaches that have appeared in the literature. First, Perignon, Smith, and Villa (2007) have highlighted that the purpose of principal component analysis is to extract factors that maximize the explained variance, and do not seek to distinguish between the role of common and that of local factor in the presence of multiple groups, resulting in estimated factors that jointly capture both local and common influences without distinguishing one from the other. Second, while the factor model literature has proposed several methods for selecting the number of factors[2], the reliability of these criteria requires the presence of weak-form serial and cross-sectional dependence in the idiosyncratic component of the factor model, as well as a large N (the cross-sectional dimension) and large T (the time-series dimension) database. These conditions are clearly not all satisfied by an international yield curve panel of data, given the strong persistence and cross-correlation of interest rates, as well as the typically small dimension of the maturity spectrum; for instance, in the presence of serial dependence, the Bai and Ng (2002) criteria tend to overestimate the number of common factors, even when

[2]See, for instance, Connor and Korajczyk (1993), Forni, Hallin, Lippi, and Reichlin (2000), Bai and Ng (2002), Bai and Ng (2007), Stock and Watson (1991), Amengual and Watson (2007) and Onatski (2010).

a first-difference filter is applied to stationary data in order to mitigate the persistence [see Greenaway-McGrevy, Han, and Sul (2012), Han and Sul (2011) for details].

To be sure that the divergence in results cited in the literature was not merely due to the variety of data sets analyzed with different numbers of countries over differing sample periods, in the extensive empirical analysis in Chapter 2 we apply the same methods (as used in the literature) to a common set of data (same countries, same time period) and still finds lack of agreement among the PC-based approaches with the explained variance criterion. This controlled experiment (varying only the statistical estimation methods) involved developing an international Treasury yield curve database by applying common filtering and interpolation techniques across all countries. Not only are different combinations of common and local factors provided by the different methodologies, but even by the same methodology when it is applied to yields in level and to yields in difference. These empirical findings reinforce our choice to develop here an alternative state-space based statistical technique that can jointly estimate common and local factors within a model that reflects their distinct natures.

Our linear Gaussian state-space approach explains the joint dynamics of multi-country term structures using autoregressive stationary latent factors of two types (common and local) as specified by the measurement equation. Each common factor has loadings that are unrestricted for all countries, while each local factor is identified with just one country by restricting its loadings to zero for all other countries. We find an innovative solution to the identification problem that allows for causality across common and local factors as well as between common and local ones. We implement maximum likelihood estimation for the state-space model using the EM algorithm with the Kalman Filter and Kalman Smoother recursions [see Engle and Watson (1981), Quah and Sargent (1993), Monfort, Renne, Rüffer, and Vitale (2003), Doz, Giannone, and Reichlin (2011), Doz, Giannone, and Reichlin (2012) , Jungbacker and Koopman (2008), Bork, Dewachter, and Houssa (2009)].From among different scenarios, each specifying the numbers and combinations of common and local factors, we select the optimal combination on the basis of maximum likelihood-based model selection criteria such as the Akaike Information Criterion (AIC). In order to take into account the persistence and heteroskedasticity of interest rates we also calculate the (Nonparametric Monte Carlo) bootstrap variant of AIC ($AICb$, say) of Cavanaugh and Shumway (1997) and based on a block stationary bootstrap [see Politis and Romano (1994), Politis and White (2004), and Patton, Politis, and White (2009)]. We use the international Treasury yield curves database constructed in chapter two consisting of rates in four leading bond markets (U.S., Germany, U.K. and Japan) observed weekly from January 1, 1986 to December 31, 2009.

Our empirical analysis suggests in general, across a variety of groups of countries and for both yield levels and yield differences, that a model with two common factors and three correlated local factors is preferred to a model (of similar complexity) that includes one common factor only or a model with only correlated local factors. Careful inspection of the optimally extracted factors reveals that each estimated common factor closely mimics (or is similar to) a local factor obtained from a pure local factor model. We reach the conclusion that international Treasury yield curves dependence are driven by a preferred set of two common factors and by the strong correlation between local factors of different countries, and that the former are spanned by (pure) local factors. This conclusion exhibits one of the advantages of our proposed method as compared to PC-based approaches (for which the initial factor extraction cannot consider the distinction between a local and a common factor). In order to provide a reassessment of the link between yield curve common factors and macroeconomic variables, exploiting the multi-country perspective and the optimality of our estimation technique, we regress the future industrial production growth and inflation rate for several sets of countries on the selected mix of two common and three local (smoothed) factors. We find that presence of common factors helps to well improve the (adjusted) R^2 compared to the (competing) case where we consider 4 local smoothed factors extracted from the $MCTSM$ with $r_c = 0$.

This chapter is organized as follows. In Section 3.2 we introduce our Multi-Country Term Structure Model ($MCTSM$) that describes the joint dynamics of international yield curves (Section 3.2.1) and we specify the appropriate identification restrictions that respect the presence of common and local factors (Section 3.2.2). In Section 3.3 we describe our proposed EM-based recursive maximum likelihood estimation procedure that imposes the identification restrictions. Section 3.4 presents the empirical analysis, beginning with the database on international Treasury yield curves in Section 3.4.1, with results from various models (different groups of countries, yield levels, yield differences, various combinations of common and local factors) following in Section 3.4.2 which also includes model selection results based on the maximized likelihood. Section 3.4.3 provides parameter estimates and interpretation of factors while the focus of Section 3.4.4 is to understand what drives the dependence between international yield curves. Section 3.5 provides a macroeconomic interpretation of the extracted factors and of the behavior over time of their instantaneous covariances, while Section 3.6 concludes. Proofs, tables, and graphs are gathered from Appendix 3.A to Appendix 3.H.

3.2 The Multi-Country Term Structure Model

3.2.1 Modeling Framework

In this section we define our Multi-Country Term Structure Model ($MCTSM$) as a linear Gaussian state-space model with block structure adopted to describe the joint dynamics of international yield curves. The model is specified by the following assumptions:

Assumption 1 (Yields, Countries, Common and Local Factors:). *We denote by $Y_t^{(s)}$ the $\tau \times 1$ vector of yields observed at time t for country s, with $s \in \{1, \dots, n\}$, n being the total number of analyzed countries. $Y_t = (Y_t^{(1)'}, \dots, Y_t^{(n)'})'$ denotes the $N \times 1$ vector of the observed international yields with $N = \tau\, n$. We denote by F_t the $k \times 1$ vector of latent factors at time t that explain the international term structures of interest rates. We assume that $F_t = (F_t^{(c)'}, F_t^{(l)'})'$, where $F_t^{(c)} = (F_{1,t}^{(c)}, \dots, F_{r_c,t}^{(c)})'$ is the $r_c \times 1$ vector of factors common to all countries, and the local factors are $F_t^{(l)} = (F_{1,t}^{(l)'}, \dots, F_{n,t}^{(l)'})'$ with $F_{j,t}^{(l)} = (F_{1,j,t}^{(l)}, \dots, F_{r_j,j,t}^{(l)})'$ the $r_j \times 1$ vector of factors associated to country j only, for $j \in \{1, \dots, n\}$. The total number of local factors across all countries is denoted $r^{(l)}$, so $F_t^{(l)}$ is a $r^{(l)} \times 1$ vector and $k = r_c + r^{(l)}$.*

Assumption 2 (The Multi-Country Term Structure Model $MCTSM$:). *For a given k-dimensional latent factor F_t made up of r_c common and $r^{(l)}$ local factors, the joint dynamics of the n international yield curves $Y_t = (Y_t^{(1)'}, \dots, Y_t^{(n)'})'$ is given by:*

$$\begin{cases} Y_t = \mu + \Lambda_{\mathcal{B}}\, F_t + \varepsilon_t\,,\ \ \varepsilon_t \sim IIN(0, \Omega_{\mathcal{B}}) \\ \\ F_t = \Phi\, F_{t-1} + \eta_t\,,\ \ \eta_t \sim IIN(0, \Psi_\eta)\,, \end{cases} \tag{3.1}$$

where μ is an $N \times 1$ vector of constants and

$$\Lambda_{\mathcal{B}} = \left[\begin{array}{cc} \Lambda_c & \Lambda_l \end{array} \right] \tag{3.2}$$

is the $N \times k$ matrix of factor loadings partitioned in terms of the $N \times r_c$ matrix $\Lambda_c = [\Lambda_{c,1}, \dots, \Lambda_{c,r_c}]$ of common loadings and in the $N \times r^{(l)}$ block-diagonal matrix of local *loadings*

$$\Lambda_l = \begin{bmatrix} \Lambda_l^{(1)} & 0 & \dots & 0 \\ 0 & \Lambda_l^{(2)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Lambda_l^{(n)} \end{bmatrix}, \tag{3.3}$$

while $\Omega_{\mathcal{B}}$ is the $N \times N$ variance-covariance matrix of the Gaussian-distributed white noise ε_t. Φ is the $k \times k$ autoregressive matrix, Ψ_η is the $k \times k$ variance-covariance matrix of the k-dimensional Gaussian distributed white noise $\eta_t = \left(\eta_t^{(c)\prime}, \eta_t^{(1)\prime}, \dots, \eta_t^{(n)\prime}\right)'$ and $E(\varepsilon_t\, \eta_t') = 0$ for all t.

3.2.2 Identification Restrictions Imposed by Common and Local Factors

We now focus on the identification restrictions for this $MCTSM$ model that stem from the fact that the model's fitted values remain unchanged if we transform the model specification using any $k \times k$ non-singular matrix A because $\Lambda_{\mathcal{B}}\, F_t = (\Lambda_{\mathcal{B}}\, A)\,(A^{-1}\, F_t)$. Ordinarily, for a model that does not distinguish common from local factors (i.e., $r^{(l)} = 0$), we would impose k^2 restrictions equal to the number of free parameters in A. However, in the presence of local factors with $r^{(l)} > 0$, the matrix A has to be such that the transformed loadings matrix $\Lambda_{\mathcal{B}}^* = \Lambda_{\mathcal{B}}\, A$ maintains same required block structure (i.e., the same pattern of zeros) as we required of $\Lambda_{\mathcal{B}}$.

Proposition 1 (Identification Restrictions). *The identification restrictions for MCTSM (3.1) with loadings matrix $\Lambda_{\mathcal{B}} = [\Lambda_c\ \Lambda_l]$ from Assumption 2, requires $r^* := (r_c\, k) + \sum_{i=1}^{n} r_i^2$ restrictions that we may solve by imposing:*

R.i) $E\left(\eta_t^{(c)}\,\eta_t^{(c)\prime}\right) = I_{r_c}$, $E\left(\eta_t^{(j)}\,\eta_t^{(j)\prime}\right) = I_{r_j}$ *and* $E\left(\eta_t^{(c)}\,\eta_t^{(j)\prime}\right) = 0$ *for all* $j \in \{1,\ldots,n\}$, *that is we impose* $\Psi_\eta = \Psi_\mathcal{B}$ *where:*

$$\Psi_\mathcal{B} := \begin{bmatrix} I_{r_c} & 0 & 0 & \dots & 0 \\ 0 & I_{r_1} & \Psi_{12} & \dots & \Psi_{1n} \\ 0 & \Psi_{21} & I_{r_2} & \dots & \Psi_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \Psi_{1n} & \Psi_{2n} & \dots & I_{r_n} \end{bmatrix}, \tag{3.4}$$

and where, for any $i, j \in \{1,\ldots,n\}$ *with* $i \neq j$, *the covariance matrix* $E\left(\eta_t^{(i)}\,\eta_t^{(j)\prime}\right) = \Psi_{ij}$ *is allowed to be different from zero;*

R.ii) $(\Lambda_c'\Lambda_c)$ *and* $\Lambda_l^{(j)\prime}\Lambda_l^{(j)}$ *for all* $j \in \{1,\ldots,n\}$, *are all diagonal, that is* $\Lambda_\mathcal{B}$ *has to be such that* $\Lambda_\mathcal{B}'\Lambda_\mathcal{B} = \Pi_\mathcal{B}$, *with:*

$$\Pi_\mathcal{B} := \begin{bmatrix} \Pi_{cc}^d & \Pi_{c1} & \Pi_{c2} & \dots & \Pi_{cn} \\ \Pi_{1c} & \Pi_{11}^d & 0 & \dots & 0 \\ \Pi_{2c} & 0 & \Pi_{22}^d & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \Pi_{nc} & 0 & 0 & \dots & \Pi_{nn}^d \end{bmatrix}, \tag{3.5}$$

and where Π_{cc}^d *and* Π_{jj}^d, *for* $j \in \{1,\ldots,n\}$, *are diagonal matrices whose diagonal entries are arranged in descending order.*

(Proof: see Appendix 3.A.)

Restrictions *R.i)* and *R.ii)* allow us to estimate model parameters and to extract common and local latent factors while keeping the autoregressive matrix Φ unconstrained *and* allowing local factors to be correlated ($\Psi_{ij} \neq 0$) for distinct countries i and j, thereby allowing causality estimation. By keeping the autoregressive matrix Φ unconstrained, we avoid arbitrarily imposing a lack of causality between common and local factors, as would occur in the classical case with Φ lower triangular. Thus, we are able to estimate the impact of any factor on any other factor, regardless of whether they are both common, one common and one local, or both local either from the same country or from different countries. By allowing correlation of local factors across countries, we can estimate instantaneous causality associated with these local factors, allowing $MCTSM$ to model the presence of factors common to only a subset of the analyzed countries (*regional factors*)[3]. Regarding restrictions $R.ii$), it is important to highlight that we rank the Jordan decomposition-based eigenvalues in the main diagonal of Π^d_{cc} and Π^d_{jj}, for $j \in \{1, \ldots, n\}$, in decreasing order. Our procedure thus ranks the factors, within each group, in line with the classical level, slope and curvature order provided by the principal component approach.

In summary, the identification restrictions characterizing our $MCTSM$ model open the way to three sources of dependence across international yield curves. First, the matrix Λ_c of common loadings allowing common factors $F_t^{(c)}$ to directly impact all term structures at the same time. Second, the unconstrained autoregressive matrix Φ

[3]An alternative set of identification restrictions concerning Ψ_η might be the one imposing $\Psi_{ij} \neq 0$ for all $i \neq j$, $i, j \in \{1, \ldots, n\}$, but leaving $E\left(\eta_t^{(c)}\,\eta_t^{(j)\prime}\right) \neq 0$ and still assuming $E\left(\eta_t^{(c)}\,\eta_t^{(c)\prime}\right) = I_{r_c}$ and $E\left(\eta_t^{(j)}\,\eta_t^{(j)\prime}\right) = I_{r_j}$. Our identification method is more general than this possible alternative. First, any initially estimated Ψ_η can be transformed to $\Psi_{\mathcal{B}}$ while respecting the common-local loading structure (3.2)-(3.3), while the alternative one would potentially require nonzero loadings for each country on all of the other country local factors, which would violate the principle that a given country can load only on the common factors and its own local factors. Second, this alternative set of restrictions can not represent regional factors. Third, while restrictions $R.i$) guarantee that Ψ_η be a proper variance-covariance (thus, positive definite) matrix regardless the assumption about r_c and r_j, if in the alternative set of restrictions we assume, for instance, $r_c = r_j$ and $E\left(\eta_t^{(c)}\,\eta_t^{(j)\prime}\right) = I_{r_c}$, then the matrix cannot be a covariance matrix because the first common factor is perfectly correlated with each of the first local ones, which must in turn be perfectly correlated with one another. However, these local factors all have zero correlation with one another thus contradicting the previous statement (for instance, in the case $n = 2$ and $r_c = r_j = 1$, it is easy to verify that $|\Psi_\eta| = -1$, which is clearly not possible for a variance-covariance matrix).

allowing for causalities between all factors and, third, the instantaneous correlations between local factors of different countries ($\Psi_{ij} \neq 0$). The empirical analysis of Section 4 will thus focus on understanding not only the preferred combination of common and local factors but, also, on identifying which of the three above mentioned channels is the most important in driving international yield curves dependence.

3.3 The $MCTSM$ Recursive Maximum Likelihood Estimation Procedure

In this section we provide details of the statistical methodology to efficiently extract the optimal combination of common and local factors to represent the joint dynamics of international yield curves. This task entails specifying the estimation details to be used for each of several given combinations of common and local factors, from which the likelihood-based Akaike Information Criterion (AIC) and its bootstrap variant $AICb$ (say) will be used to select the optimal combination. In Section 3.3.1, to efficiently estimate the $MCTSM$ model given the number of common and local factors, we use the EM Algorithm (viewing the factor values as missing data) together with the recursive Kalman Filter and Kalman Smoother [see, e.g., Engle and Watson (1981), Quah and Sargent (1993) and Doz, Giannone, and Reichlin (2012)] to numerically seek the model's maximum likelihood. At each iteration of the EM algorithm the likelihood increases and (under regularity conditions) it converges to a maximum of the likelihood function. In Section 3.3.2 we present the four strategies we adopt to initialize the algorithm and the random perturbation technique of the associated estimations we use to avoid being trapped in a local maximum. The maximum likelihood estimator of any of $MCTSM$ model of interest will be the one coming from the strategy (among the four) providing the largest value of the log-likelihood function. In practice, we find that from each starting point convergence is typically obtained within 100 iterations (in a benchmark case of two common factors and two local factors for each of the four countries).

3.3.1 The Recursive *MLE* Procedure

Here is the EM-based recursive procedure to obtain maximum likelihood estimates for the set of parameters $\theta := (\mu, \Lambda_{\mathcal{B}}, \Omega_{\mathcal{B}}, \Phi, \Psi_\eta)$ of the $MCTSM$ (3.1) while imposing the identification restrictions $R.i)$ and $R.ii)$.

Proposition 2 (The $MCTSM$ Recursive Maximum Likelihood Estimation Procedure). *Each iteration of the procedure to calculate the maximum likelihood estimator denoted θ_T^{MLE}, of the parameter set θ characterizing $MCTSM$, is based on the following three steps, where step (a) defines our notation for the results of the Kalman Filter and Kalman Smoother, step (b) maximizes the expected complete data log-likelihood function, conditionally to Y^T and given the imputed factor results from step (a), and step (c) shows how the identification restrictions are satisfied:*

(a) *For a given set of $MCTSM$ input parameters denoted $\theta_{EM}^{(i)}$, and for a given data set $Y^T := (Y_1, \ldots, Y_T)$ of international yield curves, one iteration of the Kalman Filter and of the Kalman Smoother provides the log-likelihood function value denoted $\mathcal{L}\left(\widehat{\theta}_{EM}^{(i)}\right)$ and the imputed factor results denoted $\mathcal{F}_{t|T}^{(i)} := \left(F_{t|T}^{(i)}, P_{t|T}^{(i)}, P_{t-1,t|T}^{(i)}\right)$ (Expectation step i), where:*

- $F_{t|T}^{(i)} := \mathbb{E}_{\theta_{EM}^{(i)}}\left[F_t \,|\, Y^T\right]$ *is the date-t vector of smoothed factors,*
- $P_{t|T}^{(i)} := \mathbb{V}_{\theta_{EM}^{(i)}}\left[F_t \,|\, Y^T\right] = \mathbb{E}_{\theta_{EM}^{(i)}}\left[\left(F_t - F_{t|T}^{(i)}\right)\left(F_t - F_{t|T}^{(i)}\right)' \,|\, Y^T\right]$ *is the date-t smoothed variance-covariance matrix of the factors, and*
- $P_{t-1,t|T}^{(i)} = \mathbb{C}_{\theta_{EM}^{(i)}}\left[F_{t-1}, F_t \,|\, Y^T\right] = \mathbb{E}_{\theta_{EM}^{(i)}}\left[\left(F_{t-1} - F_{t-1|T}^{(i)}\right)\left(F_t - F_{t|T}^{(i)}\right)' \,|\, Y^T\right]$ *is the date-t smoothed one lag autocovariance of the factors;*

(b) *Given $\mathcal{F}^{(i)}_{t|T}$ from the previous step, the Maximization step $(i+1)$ results in the following closed form estimators:*

$$\Lambda^{(i+1)}_{\mathcal{B},T} = \mathcal{D}^{(i)}_T\,\overline{\mathcal{C}}^{(i)-1}_T + \mathcal{K}^{(i)}_{\Lambda,T}\,, \quad \mu^{(i+1)}_T = \overline{Y}_T - \Lambda^{(i+1)}_{\mathcal{B},T}\,\overline{F}^{(i)}_T$$

$$\Omega^{(i+1)}_{\mathcal{B},T} = \tfrac{1}{T}\Bigg(\mathcal{E}^{(i)}_T - \mathcal{D}^{(i)}_T\,\overline{\mathcal{C}}^{(i)-1}_T\,\mathcal{D}^{(i)\prime}_T + \mathcal{K}^{(i)}_{\Lambda,T}\,\overline{\mathcal{C}}^{(i)}_T\,\mathcal{K}^{(i)\prime}_{\Lambda,T}\Bigg) \tag{3.6}$$

$$\Phi^{(i+1)}_T = \mathcal{B}^{(i)}_T\,\mathcal{A}^{(i)-1}_T\,, \quad \Psi^{(i+1)}_{\eta,T} = \tfrac{1}{T-1}\Bigg(\mathcal{C}^{(i)}_T - \mathcal{B}^{(i)}_T\,\mathcal{A}^{(i)-1}_T\,\mathcal{B}^{(i)\prime}_T\Bigg)\,,$$

where:

$$\mathcal{A}^{(i)}_T := \textstyle\sum_{t=2}^T \left(F^{(i)}_{t-1|T}\,F^{(i)\prime}_{t-1|T} + P^{(i)}_{t-1|T}\right)\,, \quad \mathcal{B}^{(i)}_T := \sum_{t=2}^T \left(F^{(i)}_{t|T}\,F^{(i)\prime}_{t-1|T} + P^{(i)\prime}_{t-1,t|T}\right)\,,$$

$$\mathcal{C}^{(i)}_T := \textstyle\sum_{t=2}^T \left(F^{(i)}_{t|T}\,F^{(i)\prime}_{t|T} + P^{(i)}_{t|T}\right)\,, \quad \overline{\mathcal{C}}^{(i)}_T := \sum_{t=1}^T \left[\left(F^{(i)}_{t|T} - \overline{F}^{(i)}_T\right)\left(F^{(i)}_{t|T} - \overline{F}^{(i)}_T\right)' + P^{(i)}_{t|T}\right]$$

$$\mathcal{D}^{(i)}_T := \textstyle\sum_{t=1}^T \left(Y_t - \overline{Y}_T\right)\left(F^{(i)}_{t|T} - \overline{F}^{(i)}_T\right)'\,, \quad \mathcal{E}^{(i)}_T := \sum_{t=1}^T \left(Y_t - \overline{Y}_T\right)\left(Y_t - \overline{Y}_T\right)'\,,$$

$$\overline{Y}_T := \tfrac{1}{T}\textstyle\sum_{t=1}^T Y_t\,, \quad \overline{F}^{(i)}_T := \tfrac{1}{T}\sum_{t=1}^T F^{(i)}_{t|T}\,,$$

$$vec(\mathcal{K}^{(i)}_{\Lambda,T}) := (\overline{\mathcal{C}}^{(i)-1}_T \otimes \Omega^{(u,i)}_T)\,\mathcal{H}'_\Lambda\left[\mathcal{H}_\Lambda\,(\overline{\mathcal{C}}^{(i)-1}_T \otimes \Omega^{(u,i)}_T)\,\mathcal{H}'_\Lambda\right]^{-1}\left[\kappa_\Lambda - \mathcal{H}_\Lambda\,vec(\mathcal{D}^{(i)}_T\,\overline{\mathcal{C}}^{(i)-1}_T)\right]\,,$$

$$\Omega^{(u,i)}_T := \tfrac{1}{T}\Bigg(\mathcal{E}^{(i)}_T - \mathcal{D}^{(i)}_T\,\overline{\mathcal{C}}^{(i)-1}_T\,\mathcal{D}^{(i)\prime}_T\Bigg)\,, \tag{3.7}$$

and where $\mathcal{H}_\Lambda$ is a $\vartheta \times Nk$ selection matrix such that:

$$\mathcal{H}_\Lambda\,vec\,(\Lambda) = \kappa_\Lambda \tag{3.8}$$

with Λ *the unrestricted* $N \times k$ *matrix of loadings and with* κ_Λ *the* ϑ*-dimensional vector of zeros that enforces the block structure of* $\Lambda_{\mathcal{B}}$ *at each iteration of the algorithm.*

(c) *Given the updated set of estimators* $\theta_{EM}^{(i+1)} := \left(\mu_T^{(i+1)}, \Lambda_{\mathcal{B},T}^{(i+1)}, \Omega_{\mathcal{B},T}^{(i+1)}, \Phi_T^{(i+1)}, \Psi_{\eta,T}^{(i+1)}\right)$ *from the previous step, the associated normalized estimator denoted* $\theta_{EM}^{*(i+1)}$ *satisfying the indentification restrictions R.i) and R.ii), is given by:*

$$\begin{aligned} \Lambda_{\mathcal{B},T}^{*(i+1)} &:= \Lambda_{\mathcal{B},T}^{(i+1)} A^* , \;\; \mu_T^{*(i+1)} = \mu_T^{(i+1)} , \;\; \Omega_{\mathcal{B},T}^{*(i+1)} = \Omega_{\mathcal{B},T}^{(i+1)} , \\ \Phi_T^{*(i+1)} &:= (A^*)^{-1} \, \Phi_T^{(i+1)} A^* , \;\; \Psi_{\eta,T}^{*(i+1)} := (A^*)^{-1} \, \Psi_{\eta,T}^{(i+1)} \, (A^*)^{-1\prime} , \end{aligned} \tag{3.9}$$

where the normalization matrix A^* *is:*

$$A^* := \left(A_\perp \, A_{\eta,(i+1)} \, A^o_{c,l,(i+1)}\right) , \tag{3.10}$$

with

$$A_\perp^{-1} := \begin{bmatrix} I_{r_c} & 0 & \dots & 0 \\ -\left(\Psi_{10,T}^{c(i+1)}\right) \left[(\Psi_{00,T}^{c(i+1)})\right]^{-1} & I_{r_1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\left(\Psi_{n0,T}^{c(i+1)}\right) \left[(\Psi_{00,T}^{c(i+1)})\right]^{-1} & 0 & \dots & I_{r_n} \end{bmatrix} ,$$

$$\Psi_{\eta,T}^{(i+1)} := \begin{bmatrix} \Psi_{00,T}^{c(i+1)} & \Psi_{01,T}^{c(i+1)} & \dots & \Psi_{0n,T}^{c(i+1)} \\ \Psi_{10,T}^{c(i+1)} & \Psi_{11,T}^{(i+1)} & \dots & \Psi_{1n,T}^{(i+1)} \\ \vdots & \vdots & \ddots & \vdots \\ \Psi_{n0,T}^{c(i+1)} & \Psi_{n1,T}^{(i+1)} & \dots & \Psi_{nn,T}^{(i+1)} \end{bmatrix} \neq \Psi_{\eta,\mathcal{B}} ,$$

$$A^{-1}_{\eta,(i+1)} \;:=\; diag\left[A^{-1}_{\eta,c,(i+1)}, A^{-1}_{\eta,1,(i+1)}, \ldots, A^{-1}_{\eta,n,(i+1)}\right] ,$$

$$A^{-1}_{\eta,c(i+1)} \;:=\; \left(\mathcal{U}_{\eta,c(i+1)}\,\mathcal{D}^{-1/2}_{\eta,c(i+1)}\right)' , \quad A^{-1}_{\eta,j(i+1)} := \left(\mathcal{U}_{\eta,j(i+1)}\,\mathcal{D}^{-1/2}_{\eta,j(i+1)}\right)' \;\forall\; j \in \{1,\ldots,n\} ,$$

where $\mathcal{U}_{\eta,c(i+1)}$ *and* $\mathcal{D}_{\eta,c(i+1)}$ *are matrices of eigenvectors and eigenvalues of* $\Psi^{c(i+1)}_{00,T}$, $\mathcal{U}_{\eta,j(i+1)}$ *and* $\mathcal{D}_{\eta,j(i+1)}$ *are matrices of eigenvectors and eigenvalues of* $\Psi^{(i+1)}_{jj,T}$*, for* $j \in \{1,\ldots,n\}$*, and where:*

$$(A^{o}_{c,l,(i+1)})^{-1} \;:=\; diag\left[(\mathcal{U}^{o}_{c,(i+1)})^{-1}, (\mathcal{U}^{o}_{1,(i+1)})^{-1}, \ldots, (\mathcal{U}^{o}_{n,(i+1)})^{-1}\right] ,$$

with $(\mathcal{U}^{o-1}_{c,(i+1)})$ *and* $(\mathcal{U}^{o-1}_{j,(i+1)})$ *respectively denoting the eigenvector matrix of* $\Lambda^{o\prime}_{c,(i+1),T}\Lambda^{o}_{c,(i+1),T}$ *and* $\Lambda^{o(j)\prime}_{l,(i+1),T}\Lambda^{o(j)}_{l,(i+1),T}$*, for* $j \in \{1,\ldots,n\}$*, where:*

$$\Lambda^{o}_{\mathcal{B},(i+1),T} := \left[\Lambda^{o}_{c,(i+1),T} \;\; \Lambda^{o}_{l,(i+1),T}\right] = \Lambda^{(i+1)}_{\mathcal{B},T}\,\left(A_{\perp}\,A_{\eta,(i+1)}\right) .$$

(Proof: see Appendix 3.B).

3.3.2 Initialization Algorithm Descriptions

The recursive procedure presented in Proposition 2 has been initialized in four possible ways. A first method is to start with a Principal Components analysis and set $\theta^{(0)}_{EM} := \theta^{PF}_{T}$ where $\theta^{PF}_{T} := (\mu^{PF}_{T}, \Lambda^{PF}_{\mathcal{B},T}, \Omega^{PF}_{\mathcal{B},T}, \Phi^{PF}_{T}, \Psi^{PF}_{\eta,T})$ denotes the set of model parameters estimated by a three-step Principal Factor methodology (see Appendix 3.C for details). A second method is to use randomized initial values for the factors, simulating (F_t) from a normalized k-dimensional Gaussian distribution and using these values along with the observed yields to estimate all parameters by OLS regressions[4].

The last two methods, taking into account the possible presence of (one or two) common factors as suggested by the literature, aim at selecting initial parameter values by means of a strategy that favors the presence of such a common factors in the data. More precisely, the third one is based on the following steps: for any given set of common and local factors, and using the estimation methodology presented in Proposition 2, first we estimate the model with only common factors, on the residuals we estimate the model with only local factors and then we use the associated smoothed factors to select a new vector of parameter estimates, through OLS regressions, in order to provide the new starting condition to the MLE recursive procedure.

The fourth one is tailored for nested $MCTSMs$ with a fixed number of factors k, but different combination of common and locals (including $r_c = 0$), that we will analyze at the end of Section 4.2. The methodology is based on the following idea (for ease of presentation we consider here the case $n = 2$ countries): for any given number of factors k and given an estimated model with $r_c \geq 0$ commons and (r_1, r_2) locals, we move to the estimation of the nesting model with $r_c + 1$ commons and $(r_1 - 1, r_2)$ locals (say) by providing starting parameter values obtained from the linear combination of the r_1 local factors that best explains country-2 yields. More precisely, the methodology is based on the following steps. First, we regress the average yield (across maturities) of country-2, namely $\overline{Y}_t^{(2)} = \frac{1}{\tau}\sum_{i=1}^{\tau} Y_{j,t}^{(2)}$, on the r_1 local factors, the estimated linear combination of these factors is identified as the new common factor $F_{r_c+1,t}^{(c)} := \sum_{i=1}^{r_1} \beta_i \, F_{i,1,t}^{(l)}$ (say) and the associated noise is given by $\eta_{r_c+1,t}^{(c)} := \sum_{i=1}^{r_1} \beta_i \, \eta_{i,t}^{(1)}$. Second, we regress the first $(r_1 - 1)$

[4]Regarding the control for convergence of the algorithm, we adopt the following criterion:

$$\Upsilon^{(i)} = \frac{|\,\mathcal{L}(\widehat{\theta}_{EM}^{*(i+1)}) - \mathcal{L}(\widehat{\theta}_{EM}^{*(i)})\,|}{(|\,\mathcal{L}(\widehat{\theta}_{EM}^{*(i+1)})\,| + |\,\mathcal{L}(\widehat{\theta}_{EM}^{*(i)})\,|)/2}$$

and we stop the procedure after i^* iterations if $\Upsilon^{(i^*)} < 10^{-6}$.

variables $\eta_{i,t}^{(1)}$ on $\eta_{r_c+1,t}^{(c)}$ and the noise of any of these regression is denoted $\xi_{i,t}^{(1)}$. Third, we orthonormalize the $\xi_{i,t}^{(1)}$s and then, finally, we define the associated (orthonormalized) country-1 factor as the new country-1 factors of the nesting model. With the newly specified common and country-1 factors, along with the starting r_2 local factors, we select a new vector of parameter estimates, through OLS regressions, that we adopt as starting condition to estimate the nesting model following Proposition 2.

In addition, in order to overcome the possible finding of a local (instead of the global) maximum of the log-likelihood function, we randomly perturb the estimations obtained using Proposition 2 through the following procedure. First, given the smoothed factors $\widehat{F}_{t|T}^{(i^*)} := \mathbb{E}_{\theta_{EM}^{*(i^*)}}\left[F_t \,|\, Y^T\right] = \mathbb{E}_{\widehat{\theta}_T^{MLE}}\left[F_t \,|\, Y^T\right]$ and a randomly generated number $\varepsilon^{(\sigma)}$ drawn from $N(0, \sigma^2)$, we obtain a new set of smoothed factors $\widehat{F}_{t|T}^{(\sigma)} := \widehat{F}_{t|T}^{(i^*)} + \varepsilon^{(\sigma)}$. For any given $\sigma \in \{0, 1, \ldots, 5\}$, we use the associated factors $\widehat{F}_{t|T}^{(\sigma)}$ to obtain a new set parameters estimates, through OLS regressions, that are used as new starting conditions to run again the recursive MLE procedure of Proposition 2. Second, we select across the alternatives the vector of parameter estimates leading to the largest value of the log-Likelihood function, we retrieve the associated smoothed factors and we run a second set of perturbations with $\sigma \in \{0, 0.1, 0.2, \ldots, 1\}$. We then select again the parameter estimates maximizing the log-likelihood function across the alternatives and the associated factors are used for a last set of perturbation assuming now $\sigma \in \{0, 0.01, 0.02, \ldots, 0.1\}$. Finally, the vector of parameter estimates of the model of interest is thus given by the vector $\widehat{\theta}_T^{(\sigma^*)}$ (say) leading to the largest value of the log-Likelihood function across those obtained from the last perturbation stage. The associated smoothed factors are denoted by $\widehat{F}_{t|T}^{(\sigma^*)}$.

3.4 Empirical Analysis

This empirical analysis presents the optimal number of common and local factors, for each of several groups of countries and for both yield levels and yield differences, selected from $MCTSM$ models estimated using various given combinations of common and local factors. Section 3.4.1 presents the database of Treasury yield curves for the U.S., Germany, the U.K., and Japan, measured weekly from 1986 to 2009. Model estimation results, along with the optimal model for each group of countries and type of yield measurement (levels or differences) are presented in Section 3.4.2 using the estimation methods of Section 3.3 and the Akaike Information Criterion $AIC = 2\,\Xi - 2\,\mathcal{L}(\widehat{\theta}_T^{MLE})$, where $\Xi = dim(\widehat{\theta}_T^{MLE})$ denotes the number of estimated parameters of a given model. We also calculate a bootstrap variant of AIC ($AICb$) of Cavanaugh and Shumway (1997) based on the Nonparametric Monte Carlo bootstrap for state-space models of Stoffer and Wall (1991). The kind of bootstrap that is adopted is a block stationary bootstrap able to properly taking into account the persistence and the heteroskedasticity of interest rates [see Politis and Romano (1994), Politis and White (2004), and Patton, Politis, and White (2009)]. The optimal model selection is unchanged if alternative methods are used (e.g., Bayesian Information Criterion BIC or Hannan-Quinn HQ)[5]. Section 3.4.3 provides parameter estimates and interpretation of factors while Section 3.4.4 explores what drives the dependence between international yield curves.

3.4.1 The International Treasury Yield Curves Database

We use the international Treasury yield curves database constructed in chapter two consisting of four leading bond markets: the U.S., Germany, U.K. and Japan. We adopt the criteria of Gurkaynak, Sack, and Wright (2007) to filter coupon bond Treasury raw

[5]Results for BIC and HQ are available upon request from the authors.

data, to guarantee a uniform level of liquidity, and to interpolate the discount function using the (parsimonious smoothed) Nelson and Siegel (1987) methodology. We estimate with $T = 1252$ weekly observation of these four countries, for residual maturities from 1 to 9 years (for any country), covering the period from January 1, 1986 to December 31, 2009 [see Appendix 3.D, Table 2.1 and Figure 2.2.3 of Chapter 2].

3.4.2 Estimating Optimal $MCTSMs$

In this section we compare model estimation results and select the optimal combination of common and local factors, using AIC and $AICb$, for groups of 2, 3, and all 4 countries, and for both yield levels and yield differences. In each case we compare combinations of common and local factors (r_c, r_ℓ) such that the yield curve of any economy is always explained by 3 to 5 factors, following the single-country term structure literature [see Adrian, Crump, and Moench (2013) and Duffee (2011)]. Accordingly, when we assume $r_c = 0$, we fix $r_\ell = 3$, $r_\ell = 4$ and $r_\ell = 5$, while, if $r_c = 1$, we consider $r_\ell = 2$, $r_\ell = 3$ and $r_\ell = 4$ and, if $r_c = 2$, we take $r_\ell = 1$, $r_\ell = 2$ and $r_\ell = 3$.

When the number of local factors is identically r_ℓ in each of the n countries, we denote the $MCTSM$ model $\mathcal{M}_n^{r_c,r_\ell}(\Phi, \Psi_\eta)$ where r_c denotes the number of common factors. The maximum value of the log-likelihood function of each model and the associated AIC and $AICb$ values are reported in Tables 3.2 and 3.3 for yield levels, and in Tables 3.4 and 3.5 for yield differences in the Appendix . When the number of local factors is not identical in each country, we denote the $MCTSM$ model $\mathcal{M}_n^{r_c,r_j}(\Phi, \Psi_\eta)$ and specify the list for numbers of local factors by country r_j. We include the case of unequal numbers of local factors in order to compare alternative $MCTSMs$ specifications having the same factor's dimension k but different combinations of common and local factors.

Let us focus first on the case $n = 2$, that is the classical 2-country yield curve case frequently studied in the international term structure literature [see, among others, Backus, Foresi, and Telmer (2001), Ahn (2004), Bork, Dewachter, and Houssa (2009), Mosburger and Schneider (2005), Leippold and Wu (2007) and Egorov, Li, and Ng (2011)]. As can be seen from Tables 3.2 and 3.4, if we compare $MCTSMs$ providing the same number of factors to any yield curve, we always prefer the pure local factors specification $\mathcal{M}_2^{0,r_\ell}$ and this is for any pair of countries and for both for yields in level and in difference. Then, when we consider the cases $n = 3$ and $n = 4$ (Tables 3.3 and 3.5), once again we select models $\mathcal{M}_3^{0,r_\ell}$ and $\mathcal{M}_4^{0,r_\ell}$ instead of specifications where $r_c = 1$ or $r_c = 2$.

Nevertheless, as suggested by model selection literature [see Linhart and Zucchini (1986)], the above presented selection of $MCTSMs$ might be in favor of the pure local factors case $\mathcal{M}_n^{0,r_\ell}$ simply because the latter turns out to be characterized by a factor's dimension k larger than the one of the competing models $\mathcal{M}_n^{r_c,r_\ell}$. For instance, when $n = 2$ and the yield curve of any country is explained by three factors, we have that the specification $\mathcal{M}_n^{0,3}$ implies $k = 6$, while the alternative ones $\mathcal{M}_n^{1,2}$ and $\mathcal{M}_n^{2,1}$ have $k = 5$ and $k = 4$, respectively. In order to understand which specification is required by the data, we thus compare $MCTSMs$ having the same k but different combination of common and local factors including, in particular, the case $r_c = 0$. As in the previous estimations, we consider all possible combinations of countries for both yields in level and in difference. Nevertheless, for ease of presentation, the results (presented in Table 3.6 in the appendix) focus on the two pairs of countries, namely *U.S.-U.K.* and *U.S.-GER*, and then on the sets *U.S.-U.K.-GER* and *U.S.-U.K.-GER-JAP*, the remaining ones providing qualitatively the same information (and available upon request from the authors). From Table 3.6 we observe now, across alternative sets of countries and for both yield levels and differences, that the specifications with two common factors and three correlated local factors are preferred to the case $r_c = 1$ and $r_c = 0$ (the only exception being the $U.S.$-GER. case, and only for yield differences and if we consider AIC, while $AICb$ again prefers the case $r_c = 2$)[6].

3.4.3 Parameter Estimates and Interpretation of the Factors

Now, at that point of the analysis, we still do not know which is the nature of the common and local (smoothed) factors that we have extracted. We do not know, for instance, if common factors originate from a single economy or if they summarize some information over and above the one provided by local factors and if this feature depends on the number and the kind of analyzed countries. Indeed, we may have that some local factor of a given country loads also on the other economies. In other words, two questions naturally stand out: first, what the local factors extracted from the preferred $\mathcal{M}_n^{2,3}$ specification look like? Second, are the common factors in reality local factors loading on the other countries or are they common factors representing yield curves driving forces other than local ones?

Before focusing on this analysis, it is important to point out the ability of our estimated $MCTSMs$ to properly share interest rates information between common and local factors. Indeed, we may figure out the (extreme) case where, assuming (for ease of presentation) $n = 2$, $r_c = 1$, $r_\ell = 1$, the two local factors have dynamics identical to the common one, being $\Phi = \varphi\, I$ and the correlation between the two locals equal to one. In this case the two locals would look identical to the common factor and therefore distinguishing between them would be impossible. Now, if we look at the parameter estimates of $\mathcal{M}_n^{2,3}$ (yield levels) for the same set of countries analyzed in Table 3.6, we observe that this possible situation is completely and strongly overcome. Indeed, from Tables 3.10, 3.11 and 3.12 in the appendix 3.F we observe the following relevant features. First,

[6]This choice is, in addition, confirmed by likelihood ratio tests between nested models $\mathcal{M}_n^{r_c,r_j}(\Phi, \Psi_\eta)$ with a fixed k (these results are available upon request from the authors).

we have statistically significant parameters in the AR matrix over and above those in the main diagonal; in addition, the latter are rather different one each other. Second, we have estimated instantaneous correlations, between international local factors, that are statistically significant but never larger than 0.4 (in absolute value) and, in general, between 0.10 and 0.25 (in absolute value).

Let us move back now to factors' interpretations. An inspection of the estimated loadings in Tables 3.10, 3.11 and 3.12 and of the optimally extracted (smoothed) factors, provided in Appendix 3.G, leads to the following comments. First, the local factors of any given model $\mathcal{M}_n^{2,3}$ are precisely identified with some of the local factors extracted from the associated pure local factors specification $\mathcal{M}_n^{0,4}$. For instance, in the *U.S.*-*U.K.* case, the three local *U.S.* (*U.K.*, respectively) factors in $\mathcal{M}_n^{2,3}$ are the slope, curvature and 4^{th} (level, curvature and 4^{th}, respectively) factors of the specification $\mathcal{M}_n^{0,4}$ (see Table 3.10 and Figure 3.2). In the *U.S.*-*GER*. case, they are easily identified, for both countries, as level, slope and curvature factors (Table 3.10 and Figure 3.3). In the 3-country case, the *U.S.* local factors are the level, slope and 4^{th} *U.S.* factors in $\mathcal{M}_n^{0,4}$, while, for *U.K.* and *GER*., they are level, slope and curvature factors; we always find level, slope and curvature factors also in the 4-country case (see Tables 3.11 and 3.12, and Figures 3.4 and 3.5, respectively).

Second, the two common factors of any given model $\mathcal{M}_n^{2,3}$ tend in general to track quite closely two of the remaining (from the above mentioned identification) local factors obtained from the associated pure local factor model $\mathcal{M}_n^{0,4}$. For instance, in the case *U.S.*-*U.K.*, the two common factors closely track the *U.S.* level and the *U.K.* slope factors obtained from the specification $\mathcal{M}_2^{0,4}$. In we consider the joint dynamics of *U.S.* and Germany yield curves, the two common factors now look like the first *U.S.* and the 4^{th} German local factors provided by $\mathcal{M}_2^{0,4}$. If we now focus on the case *U.S.*-*U.K.*-*GER*, the two commons become similar to the fourth local *GER*. and the third local *U.S.*

factors, respectively. In the general 4-country case, the two commons are similar to the firth local German and $U.S.$ factors.

In summary, our empirical analysis highlights, first, the preference for $MCTSM$ models with $r_c = 2$ common factors that we complete with a set of $r_\ell = 3$ local factors in order to provide to each single-country yield curve five explanatory factors as suggested by the recent works of Duffee (2011) and Adrian, Crump, and Moench (2013). Second, we find that these common factors seem to be local ones (significantly) loading on the other countries.

3.4.4 What Drives the Dependence Between International Yield Curves?

The identification restrictions adopted for our $MCTSM$ model, and presented in Proposition 1, set up three possible sources of dependence between international term structures: the presence of common factors $F_t^{(c)}$ having a direct impact on all yield curves through the matrix Λ_c of common loadings, the unconstrained autoregressive matrix Φ allowing for causalities between all (common and local) latent factors and the instantaneous correlations between local factors of different countries ($\Psi_{ij} \neq 0$). The purpose of this section is to empirically assess the relative importance of these three channels in explaining term structures commonality.

In order to assess the importance of the first channel, namely the role played by Λ_c, we compare (through AIC and $AICb$) a $MCTSM$ model having $r_c = 2$ (as suggested by our empirical analysis) with another one with the same k but with $r_c = 0$. As far as the relevance of the second and third channel is concerned, we estimate $MCTSMs$ in which we first assume Φ block-diagonal but with the first r_c columns unconstrained (Φ_{bd}, say) and, then, we leave Φ unconstrained but we force $\widetilde{\Psi}_\eta = I$. In the former case, we turn off Granger-causalities between local factors of different countries, given that we maintain causalities of common towards local factors in order to guarantee a normalized estimator compatible with identification restrictions. In the latter specification, we switch off only the instantaneous causalities between international local factors. Let us denote this specifications $\mathcal{M}_n^{r_c,r_\ell}(\Phi_{bd}, \widetilde{\Psi}_\eta)$ and $\mathcal{M}_n^{r_c,r_\ell}(\Phi, I)$, respectively. It is easily seen, following the same steps as in Appendix 3.B, that the EM-based estimator of Φ_{bd} and $\widetilde{\Psi}_\eta$ are given by:

$$\Phi_{bd,T}^{(i+1)} = \mathcal{B}_T^{(i)}\, \mathcal{A}_T^{(i)-1} + \mathcal{K}_{\Phi,T}^{(i)}\,, \quad \widetilde{\Psi}_{\eta,T}^{(i+1)} = \tfrac{1}{T-1}\left(\mathcal{C}_T^{(i)} - \mathcal{B}_T^{(i)}\, \mathcal{A}_T^{(i)-1}\, \mathcal{B}_T^{(i)\prime} + \mathcal{K}_{\Phi,T}^{(i)}\, \mathcal{A}_T^{(i)}\, \mathcal{K}_{\Phi,T}^{(i)\prime}\right), \tag{3.11}$$

with:

$$vec(\mathcal{K}_{\Phi,T}^{(i)}) \;:=\; (\mathcal{A}_T^{(i)-1} \otimes \Psi_{\eta,T}^{(i+1)})\, \mathcal{H}_\Phi' \left[\mathcal{H}_\Phi\, (\mathcal{A}_T^{(i)-1} \otimes \Psi_{\eta,T}^{(i+1)})\, \mathcal{H}_\Phi{\prime}\right]^{-1} \left[\kappa_\Phi - \mathcal{H}_\Phi\, vec(\mathcal{B}_T^{(i)}\, \mathcal{A}_T^{(i)-1})\right], \tag{3.12}$$

where $\Psi_{\eta,T}^{(i+1)}$ is given in Proposition 2, and where $\mathcal{H}_\Phi$ is a $d \times k^2$ selection matrix such that:

$$\mathcal{H}_\Phi\, vec\,(\Phi) = \kappa_\Phi\,, \tag{3.13}$$

with Φ the unrestricted $k \times k$ autoregressive matrix and with κ_Φ the d-dimensional vector of zeros that guarantees to satisfy the above described structure of Φ_{bd} at each iteration of the EM algorithm[7].

Regarding the role played by matrix of common loadings, if we look at Table 3.6 (focusing on yield levels), and we compare the specification $r_c = 2$ with the one with $r_c = 0$

[7]It is straightforward to verify that the normalization matrix A^* of equation (3.10) automatically preserve the structure of Φ_{bd}.

(and $k = 8$), in the *U.S.-U.K.* case AIC rises from -359198 to -356674, and in the *U.S.-GER.* one it rises from -372030 to -371226. In the 3-country case ($k = 11$), AIC moves from -529322 to -526322 while, in the 4-country case ($k = 14$), it increases from -691060 to -688300 (we reach similar conclusions if we consider $AICb$).

As far as the role played by the autoregressive matrix Φ is concerned, the results obtained for the case $\mathcal{M}_n^{r_c,r_\ell}(\Phi_{bd}, \widetilde{\Psi}_\eta)$, presented in Table 3.7, are compared to those of $\mathcal{M}_n^{r_c,r_\ell}(\Phi, \Psi_\eta)$, in order to assess how much international local factors' dependencies are of Granger-causality kind. Let us focus again on the comparison between $r_c = 2$ and $r_c = 0$ and let us consider AIC, first. If we look at the 2-country *U.S.-U.K.* case (*U.S.-GER.* case, respectively), we observe that AIC rises of only 232 (598, respectively) while, when we close the first channel, the variation is 2524 (804, respectively). If we move to the *U.S.-U.K.-GER.* case, AIC rises of 2164 while, when we force $\Lambda_c = 0$ (and Φ unconstrained) the variation is 3000. In the 4-country case the magnitude of this positive variation is 2482 instead of 2760. A different picture stand out if we take into account interest rate persistence using $AICb$. Indeed, while we reach the same conclusion in the *U.S.-U.K* case (the first channel is more important than the second one), we end up with an opposite result in the other cases. More precisely, in the *U.S.-GER.* case, $AICb$ rises of 5864 while, when we close the first channel, the variation is 1050. In the 3-country case, $AICb$ rises of 3952 while, forcing common loadings equal to zero, induce a variation of 2232. Lastly, in the 4-country case, the size of this positive variation is 11924 instead of 4092 when we turn off common loadings. In other words, once the large interest rate dependence is taken into account through the lens of the bootstrap variant of AIC, a full AR matrix Φ seems to be (in general, but not systematically) more important than Λ_c.

Once we move to the case $\mathcal{M}_n^{r_c,r_\ell}(\Phi, I)$ (see Table 3.8), in order to assess the role played by the correlation terms Ψ_{ij}, we observe that, across the different number and set of countries, for several combination of common and local factors and regardless the fact to use *AIC* or *AICb*, the specification $\mathcal{M}_n^{r_c,r_\ell}(\Phi,\Psi_\eta)$ is strongly preferred. Indeed, in the *U.S.-U.K.* case the *AIC* (*AICb*) difference is now 11524 (7966), and in the in the *U.S.-GER.* case it is as big as 12486 (9938). If we consider the *U.S.-U.K.-GER.* and the 4-country case, the magnitude of the *AIC* (*AICb*) difference is 16802 (13678) and 25308 (19096), respectively. In addition, the fact to turn off the instantaneous causalities across locals, namely to impose $\Psi_{ij} = 0$, not only strongly induces the model to significantly reduce its ability to match the data, but it provides reductions much larger (in absolute value) than the ones we have when the two other channels of dependence are closed.

These results seem therefore to suggest that dependence between international yield curves are, first of all, driven by the instantaneous correlations between international yield curves local factors, while the second most important channel seems to be provided more by the factors' (full) autoregressive matrix Φ than by the matrix Λ_c of common loadings. In other words, common factors seem to be a scarce resource that are needed to represent truly global correlations, but they may be insufficient in their ability to represent all of the regional covariance structure.

3.5 Macroeconomic Interpretation of Common Factors

The macro-finance yield curve literature has in general identified local level and slope factors as domestic inflation and economic activity factors [see Ang and Piazzesi (2003), Ang, Piazzesi, and Wei (2006), Diebold and Li (2006)]. In the international yield curve literature, Diebold, Li, and Yue (2008) have shown that their global level and slope factors reflects the dynamics of global inflation and real activity factors, respectively.

These analysis have been drawn in general regressing the relevant principal components on the above mentioned macroeconomic variables.

The purpose of this section is to provide a reassessment of the link between yield curve common factors and macroeconomic variables exploiting the multi-country perspective and the optimality of our estimation technique allowing to *jointly* extract common and local factors while taking into account at the same time interest rates persistence *and* sources of international and domestic dependence.

The empirical exercise is run in the following way. For a given set of countries, we regress the the monthly (annualized) industrial production growth rate and inflation rate of any country j, observed at $t+h$ and denoted $g_{j,t+h}$ and $\pi_{j,t+h}$, respectively, on the preferred combination of the two common and three local smoothed factors at date t, namely $\widehat{F}_{t|T} = (\widehat{F}^{(c)\prime}_{t|T}, \widehat{F}^{(l)\prime}_{t|T})'$, and for forecasting horizons $h = 0, 6, 12, 18, 24, 30$ and 36 months. More precisely, we run the following forecasting regressions:

$$x_{j,t+h} = \beta_0 + \beta_{1,c}\,\widehat{F}^{(c)}_{1,t|T} + \beta_{2,c}\,\widehat{F}^{(c)}_{2,t|T} + \beta_{1,l}\,\widehat{F}^{(l)}_{1,j,t|T} + \beta_{2,l}\,\widehat{F}^{(l)}_{2,j,t|T} + \beta_{3,l}\,\widehat{F}^{(l)}_{3,j,t|T} + \varepsilon^{(x)}_{j,t+h}\,,$$

$$x_{j,t+h} \in \{g_{j,t+h}, \pi_{j,t+h}\}\,,\;\; h \,\in\, \{0, 6, 12, 18, 24, 30, 36\}\,, \tag{3.14}$$

where $\varepsilon^{(x)}_{j,t+h} \sim IIN(0, \sigma^2_x)$. In order to assess the relevance of our selected set of regressors and, in particular, the mix of two common and three local factors, we compare the (adjusted) R^2's with those coming from a regression in which the macro variable of any given country is explained (forecasted) by the set of 4 local smoothed factors extracted from the $MCTSM$ with $r_c = 0$. In other words, we consider:

$$x_{j,t+h} = \beta_0^* + \beta_{1,l}^* \widehat{F}_{1,j,t|T}^{(l)} + \beta_{2,l}^* \widehat{F}_{2,j,t|T}^{(l)} + \beta_{3,l}^* \widehat{F}_{3,j,t|T}^{(l)} + \beta_{4,l}^* \widehat{F}_{4,j,t|T}^{(l)} + \varepsilon_{j,t+h}^{*(x)} ,$$

$$x_{j,t+h} \in \{g_{j,t+h}, \pi_{j,t+h}\} , \quad h \in \{0, 6, 12, 18, 24, 30, 36\} , \varepsilon_{j,t+h}^{*(x)} \sim IIN(0, \sigma_{*x}^2) , \tag{3.15}$$

where $\widehat{F}_{j,t|T}^{(l)} = (\widehat{F}_{1,j,t|T}^{(l)}, \ldots, \widehat{F}_{4,j,t|T}^{(l)})'$, $j \in \{1, \ldots, n\}$, is extracted from the associated pure local factor specification $\mathcal{M}_n^{0,r_\ell}$. It is important to highlight that the benchmark we consider is quite challenging given that we do not simply consider the classical level and slope factors, but we include extra factors given their relevance in forecasting macro variables, as recently shown by Koijen, Lustig, and van Nieuwerburgh (2012) and Dahlquist and Hasseltoft (2013). In addition these factors have been optimally extracted through the EM-based Kalman filter and smoother recursions. The set of countries we consider, in line with the previous section, are *U.S.-U.K.*, *U.S.-GER.*, *U.S.-U.K.-GER.* and the 4-country case. Monthly data for industrial production and consumer price index of all countries are taken from OECD. We report in Tables 3.13 to 3.16 of Appendix 3.H parameter estimates, t-values and (adjusted) R^2 with associated 90% bootstrapped confidence intervals.

The results that stand out from these forecasting regressions are the following. From the *U.S.* perspective, the two common factors allow to well improve the (in sample) forecast of industrial production growth (inflation rate) at medium and long (short) horizons and regardless the number and kind of analyzed countries. This is probably induced by the fact that in our four sets of countries one of the common factor is always a *U.S.* factor (see Section 3.4 and Appendix 3.G). For instance, in the *U.S.-U.K.* case and focusing on economic activity, the R^2 rises from 0.22 to 0.32 if $h = 12$, from 0.19 to 0.30 when $h = 18$ and from 0.18 to 0.26 when $h = 24$. In the 3-country case, a similar improvement is obtained also for larger forecasting horizons; indeed, for $h = 30$, the R^2

rises from 0.25 to 0.42 and if $h = 36$, it increases from 0.29 to 0.43. An improvement is obtained also in the 4-country case.

If we consider now *U.K.*, the presence of common factors improve the forecast of economic activity (inflation rate, respectively) at short (short and long) horizons, and for all sets of countries we have considered. From the point of view of the German economy, we observe an improvement to forecast economic activity (inflation rate, respectively) at medium (long, respectively) horizons in the 2-country and 3-country cases. In the 4-country case, this improvement mainly concerns inflation activity at long horizons. As far as the Japanese economy is concerned, the two common factors induce a forecast improvement of the economic activity at medium and long horizons, and of the inflation rate at short horizons.

3.6 Conclusions and Further Developments

The purpose of this chapter has been to specify, exploiting a linear Gaussian state-space approach, the preferred combination of common and local factors that are required to explain international yield curves dynamics, and to efficiently estimate by Kalman Filter the factor scores when small cross-sectional and large time-series dimensions, as well as strong serial and cross-sectional dependence, characterize the database of interest.

Our extensive empirical analysis on $MCTSMs$, allowing for Granger-causalities and instantaneous causalities across factors and exploiting a fast and powerful MLE approach based on the EM algorithm and Kalman Filter-Smoother recursions, finds that the specification with $r_c = 2$ and $r_\ell = 3$ seems to be preferred to alternative ones (of similar complexity) with $r_c = 1$ and $r_c = 0$. We also find that each common factor closely mimics (or is similar to) a local factor extracted from a pure local factor model. This result comes from an inspection of the (optimally) extracted time series of common and local factors. Indeed, the common factors turns out to be almost identical, or to closely track, local factors extracted from the associated pure local factor model. We also reach the conclusion that dependence across international yield curves are driven, first, by the instantaneous correlation between local factors of different countries and, then, by the (full) autoregressive matrix of latent factors and by the matrix of common loadings. A last empirical exercise highlight the importance of common factors in improving the forecast of macroeconomic variables over several forecasting horizons.

The purpose of future research works will be to exploit what we have learned from this work in the specification and implementation of no-arbitrage international affine term structure models with latent factors and/or with macro-financial variables.

Appendix

3.A Proof of Proposition 1

Consider the identification problem induced by a non-singular matrix A for which the model fitted values remain unchanged when modifying factors and loadings because $\Lambda_{\mathcal{B}} F_t = (\Lambda_{\mathcal{B}} A)(A^{-1} F_t)$. Because the matrix of loadings $\Lambda_{\mathcal{B}} = [\Lambda_c \; \Lambda_l]$ must preserve the following block structure:

$$\Lambda_c = [\Lambda_{c,1} \; \ldots, \Lambda_{c,r_c}] = \begin{bmatrix} \Lambda_{c,1}^{(1)} & \Lambda_{c,2}^{(1)} & \ldots & \Lambda_{c,r_c}^{(1)} \\ \Lambda_{c,1}^{(2)} & \Lambda_{c,2}^{(2)} & \ldots & \Lambda_{c,r_c}^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ \Lambda_{c,1}^{(n)} & \Lambda_{c,2}^{(n)} & \ldots & \Lambda_{c,r_c}^{(n)} \end{bmatrix}, \quad \Lambda_l = \begin{bmatrix} \Lambda_l^{(1)} & 0 & \ldots & 0 \\ 0 & \Lambda_l^{(2)} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \Lambda_l^{(n)} \end{bmatrix}. \tag{3.16}$$

the matrix A has to be such that $\Lambda_{\mathcal{B}}^* = \Lambda_{\mathcal{B}} A$ has the same block structure (i.e., the same pattern of zeros) as $\Lambda_{\mathcal{B}}$. More precisely, if we partition the matrix A into blocks of size $r_c, r_1, \ldots, r_n$ as follows:

$$A = \begin{bmatrix} A_{cc} & A_{c1} & \ldots & A_{cn} \\ A_{1c} & A_{11} & \ldots & A_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{nc} & A_{n1} & \ldots & A_{nn} \end{bmatrix}.$$

where the subscript c denotes entries that impact common factors, then the condition

$\Lambda^*_{\mathcal{B}} = \Lambda_{\mathcal{B}}\, A$ forces A to be of the form:

$$A = \begin{bmatrix} A_{cc} & 0 & 0 & \dots & 0 \\ A_{1c} & A_{11} & 0 & \dots & 0 \\ A_{2c} & 0 & A_{22} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{nc} & 0 & 0 & \dots & A_{nn} \end{bmatrix}, \tag{3.17}$$

and therefore the number of free parameters is now given by $r^* := (r_c)^2 + r_c \left(\sum_{j=1}^n r_j\right) + \sum_{j=1}^n r_j^2 = (r_c\, k) + \sum_{j=1}^n r_j^2$. Accordingly, r^* is also the number of constraints we have to impose on the latent factors and/or the loadings in order to solve the identification problem of the international yield curve model (3.1)-(3.2)-(3.3) and obtain a unique model representation. The restrictions we impose are the following:

- $E\left(\eta_t^{(c)}\, \eta_t^{(c)\prime}\right) = I_{r_c}$ and $E\left(\eta_t^{(j)}\, \eta_t^{(j)\prime}\right) = I_{r_j}$ for all $j \in \{1, \dots, n\}$, and from this set of conditions we obtain $\frac{r_c(r_c+1)}{2} + \sum_{j=1}^n \frac{r_j(r_j+1)}{2}$ restrictions;
- $E\left(\eta_t^{(c)}\, \eta_t^{(j)\prime}\right) = 0$ for all $j \in \{1, \dots, n\}$, and here the number of restrictions is $r_c\, (\sum_{j=1}^n r_j)$;
- $(\Lambda_c'\Lambda_c)$ and $\Lambda_l^{(j)\prime}\Lambda_l^{(j)}$ for all $j \in \{1, \dots, n\}$, have to be all diagonal; these conditions imply $\frac{r_c(r_c-1)}{2} + \sum_{j=1}^n \frac{r_j(r_j-1)}{2}$ restrictions.

The total number of restrictions is thus exactly r^*. The first two sets of conditions force Ψ_η to satisfy relation (3.4), while the last one implies $\Lambda_{\mathcal{B}}'\Lambda_{\mathcal{B}} = \Pi_{\mathcal{B}}$.

3.B Proof of Proposition 2

(*b*)

Denoting $\widetilde{Y}_t = Y_t - \mu$, the joint log-Likelihood function of Y_t and F_t (i.e., the complete data likelihood function) can be written in the following way:

$$\begin{aligned}\ln L(Y^T, F^T) \;=\; & -\tfrac{T}{2}\ln|\Omega_{\mathcal{B}}| - \tfrac{1}{2}\textstyle\sum_{t=1}^{T}(\widetilde{Y}_t - \Lambda_{\mathcal{B}}F_t)'\Omega_{\mathcal{B}}^{-1}(\widetilde{Y}_t - \Lambda_{\mathcal{B}}F_t) \\ & -\tfrac{T-1}{2}\ln|\Psi_\eta| - \tfrac{1}{2}\textstyle\sum_{t=2}^{T}(F_t - \Phi F_{t-1})'\Psi_\eta^{-1}(F_t - \Phi F_{t-1}) \\ & -\tfrac{1}{2}\ln|V_1| - \tfrac{1}{2}(F_1 - \pi_1)'V_1^{-1}(F_1 - \pi_1) - \tfrac{T(N+k)}{2}\ln(2\,\pi)\end{aligned} \tag{3.18}$$

Using the following identities (with $F_{t|T} = \mathbb{E}_\theta\left[F_t \,|\, Y^T\right]$):

$$\begin{aligned}\widetilde{Y}_t - \Lambda_{\mathcal{B}}F_t &= \widetilde{Y}_t - \Lambda_{\mathcal{B}}F_{t|T} + \Lambda_{\mathcal{B}}(F_{t|T} - F_t) \\ F_t - \Phi F_{t-1} &= F_{t|T} - \Phi F_{t-1|T} - (F_{t|T} - F_t) + \Phi(F_{t-1|T} - F_{t-1})\end{aligned} \tag{3.19}$$

the conditional expectation $\mathbb{E}[\ln L(Y^T, F^T) \,|\, Y^T]$, namely, the criterion maximized by the EM algorithm, is given by:

$$
\begin{aligned}
&\mathbb{E}[\ln L(Y^T, F^T)|Y^T] \\
= \; & -\tfrac{1}{2}\ln|V_1| - \tfrac{1}{2}(F_1-\pi_1)'V_1^{-1}(F_1-\pi_1) - \tfrac{T(N+k)}{2}\ln(2\,\pi) - \tfrac{T}{2}\ln|\Omega_{\mathcal{B}}| - \tfrac{T-1}{2}\ln|\Psi_\eta| \\
& -\tfrac{1}{2}\mathbf{Tr}\Big\{\Omega_{\mathcal{B}}^{-1}\Big[\Big(\textstyle\sum_{t=1}^{T}\widetilde{Y}_t\,\widetilde{Y}_t'\Big) - \Big(\sum_{t=1}^{T}\widetilde{Y}_t\,F_{t|T}'\Big)\Lambda_{\mathcal{B}}' - \Lambda_{\mathcal{B}}\Big(\sum_{t=1}^{T}F_{t|T}\,\widetilde{Y}_t'\Big) \\
& +\Lambda_{\mathcal{B}}\Big(\textstyle\sum_{t=1}^{T}F_{t|T}F_{t|T}' + P_{t|T}\Big)\Lambda_{\mathcal{B}}'\Big]\Big\} \\
& -\tfrac{1}{2}\mathbf{Tr}\Big\{\Psi_\eta^{-1}\Big[\Big(\textstyle\sum_{t=2}^{T}F_{t|T}F_{t|T}' + P_{t|T}\Big) + \Phi\Big(\sum_{t=2}^{T}F_{t-1|T}\,F_{t-1|T}' + P_{t-1|T}\Big)\Phi' \\
& \qquad -\Big(\textstyle\sum_{t=2}^{T}F_{t|T}\,F_{t-1|T}' + P_{t-1,t|T}'\Big)\Phi' - \Phi\Big(\sum_{t=2}^{T}F_{t-1|T}F_{t|T}' + P_{t-1,t|T}\Big)\Big]\Big\}.
\end{aligned} \tag{3.20}
$$

If we consider, for a given $\Lambda_{\mathcal{B}}$, the first order conditions $\frac{\partial \mathbb{E}[\ln L(Y^T,F^T)|Y^T]}{\partial\,\mu} = 0$ and $\frac{\partial \mathbb{E}[\ln L(Y^T,F^T)|Y^T]}{\partial\,\Omega_{\mathcal{B}}} = 0$, we find

$$
\begin{aligned}
\mu_T &= \overline{Y}_T - \Lambda_{\mathcal{B}}\,\overline{F}_T\,, \;\text{ where } \overline{Y}_T := \tfrac{1}{T}\Big(\textstyle\sum_{t=1}^{T}Y_t\Big)\,, \overline{F}_T := \tfrac{1}{T}\Big(\sum_{t=1}^{T}F_{t|T}\Big) \\
\Omega_{\mathcal{B},T} &= \tfrac{1}{T}\Big\{\Big[\textstyle\sum_{t=1}^{T}(Y_t-\overline{Y}_T)\,(Y_t-\overline{Y}_T)'\Big] - \Lambda_{\mathcal{B},T}\Big[\sum_{t=1}^{T}(F_{t|T}-\overline{F}_T)(Y_t-\overline{Y}_T)'\Big]\Big\}.
\end{aligned} \tag{3.21}
$$

The solution of the maximization problem with respect to Φ and Ψ_η provides the Φ_T and $\Psi_{\eta,T}$ presented in equation (3.6). Then, given μ_T, if we focus on the matrix of factor loadings, the term of (3.20) that depends on $\Lambda_{\mathcal{B}}$ only, can be written in the following way:

$$-\tfrac{1}{2}\mathbf{Tr}\Bigg\{\Omega_{\mathcal{B}}^{-1}\Bigg[-\Bigg(\textstyle\sum_{t=1}^{T}\widetilde{Y}_t\,F'_{t|T}\Bigg)\Lambda'_{\mathcal{B}}-\Lambda_{\mathcal{B}}\Bigg(\textstyle\sum_{t=1}^{T}F_{t|T}\,\widetilde{Y}'_t\Bigg)$$

$$+\Lambda_{\mathcal{B}}\Bigg(\textstyle\sum_{t=1}^{T}F_{t|T}\,F'_{t|T}\Bigg)\Lambda'_{\mathcal{B}}+\textstyle\sum_{t=1}^{T}\Lambda_{\mathcal{B}}P_{t|T}\Lambda'_{\mathcal{B}}\Bigg]\Bigg\} \qquad (3.22)$$

$$= \quad -\tfrac{1}{2}\mathbf{Tr}\Bigg\{\Omega_{\mathcal{B}}^{-1}\Bigg[-\mathcal{D}\,\Lambda'_{\mathcal{B}}-\Lambda_{\mathcal{B}}\,\mathcal{D}'+\Lambda_{\mathcal{B}}\,\overline{\mathcal{C}}\,\Lambda'_{\mathcal{B}}\Bigg]\Bigg\},$$

where:

$$\overline{\mathcal{C}} \;=\; \textstyle\sum_{t=1}^{T}(F_{t|T}-\overline{F}_T)\,(F_{t|T}-\overline{F}_T)'+P_{t|T}\,,\quad \mathcal{D}=\textstyle\sum_{t=1}^{T}(Y_t-\overline{Y}_T)\,(F_{t|T}-\overline{F}_T)'\,, \qquad (3.23)$$

and the EM-based estimator of $\Lambda_{\mathcal{B}}$ is typically determined by solving the following problem:

$$\min_{\Lambda_{\mathcal{B}}}\quad \tfrac{1}{2}\mathbf{Tr}\Bigg\{\Omega_{\mathcal{B}}^{-1}\Bigg[-\mathcal{D}\,\Lambda'_{\mathcal{B}}-\Lambda_{\mathcal{B}}\,\mathcal{D}'+\Lambda_{\mathcal{B}}\,\overline{\mathcal{C}}\,\Lambda'_{\mathcal{B}}\Bigg]\Bigg\}. \qquad (3.24)$$

Nevertheless, this problem can be equivalently solved as a minimization problem with respect to the unconstrained matrix of loadings Λ, that we partition (with obvious notation) as follows:

$$\Lambda=\begin{bmatrix}\Lambda_{c1} & \Lambda_{11} & \Lambda_{12} & \ldots & \Lambda_{1n}\\ \Lambda_{c2} & \Lambda_{21} & \Lambda_{22} & \ldots & \Lambda_{2n}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ \Lambda_{cn} & \Lambda_{n1} & \ldots & \ldots & \Lambda_{nn}\end{bmatrix}, \qquad (3.25)$$

under the equality constraint:

$$\mathcal{H}_\Lambda \, vec\,(\Lambda) = \kappa_\Lambda \tag{3.26}$$

where $\mathcal{H}_\Lambda$ is a $(\vartheta \times Nk)$ selection matrix that select from $vec\,(\Lambda)$ only the matrices Λ_{ij} such that $i \neq j$, $i, j \in \{1, \ldots, n\}$ and κ_Λ is a ϑ-dimensional vector of zeros that forces Λ to be equal to $\Lambda_{\mathcal{B}}$.

The Lagrangian function is:

$$L(\Lambda) := \frac{1}{2}\,\mathbf{Tr}\Big\{\Omega^{-1}\Big[-\mathcal{D}\,\Lambda' - \Lambda\,\mathcal{D}' + \Lambda\,\overline{\mathcal{C}}\,\Lambda'\Big]\Big\} - \lambda'\Big[\mathcal{H}_\Lambda \, vec\,(\Lambda) - \kappa_\Lambda\Big], \tag{3.27}$$

and the associated first order conditions are:

$$\begin{cases} \left[vec\left\{\left[\left(\overline{\mathcal{C}}\,\Lambda' - \mathcal{D}\right)\,\Omega^{-1}\right]'\right\}\right]' - \lambda'\,\mathcal{H}_\Lambda = 0\,, \\ \\ \mathcal{H}_\Lambda \, vec\,(\Lambda) = \kappa_\Lambda\,. \end{cases} \tag{3.28}$$

If we rewrite the first equation in (3.28) as follows:

$$\left(\overline{\mathcal{C}} \otimes \Omega^{-1}\right) vec\Big(\Lambda\Big) - vec\Big(\Omega^{-1}\,\mathcal{D}\Big) = \mathcal{H}'_\Lambda\,\lambda\,, \tag{3.29}$$

we pre-multiply it by $\mathcal{H}_\Lambda\left(\overline{\mathcal{C}}^{-1} \otimes \Omega\right)$ and then we substitute from the second equation in (3.28) we find:

$$\lambda = \left[\mathcal{H}_\Lambda\left(\overline{\mathcal{C}}^{-1} \otimes \Omega\right)\mathcal{H}'_\Lambda\right]^{-1}\left\{\kappa_\Lambda - \mathcal{H}_\Lambda\left(\overline{\mathcal{C}}^{-1} \otimes \Omega\right) vec\Big(\Omega^{-1}\,\mathcal{D}\Big)\right\}. \tag{3.30}$$

We now substitute (3.30) in (3.29):

$$\left(\overline{\mathcal{C}} \otimes \Omega^{-1}\right) vec\left(\Lambda\right) - vec\left(\Omega^{-1}\mathcal{D}\right) = \mathcal{H}'_\Lambda \left[\mathcal{H}_\Lambda \left(\overline{\mathcal{C}}^{-1} \otimes \Omega\right) \mathcal{H}'_\Lambda\right]^{-1} \times$$
$$\left\{\kappa_\Lambda - \mathcal{H}_\Lambda \left(\overline{\mathcal{C}}^{-1} \otimes \Omega\right) vec\left(\Omega^{-1}\mathcal{D}\right)\right\}, \tag{3.31}$$

and let us rewrite $vec(\Omega^{-1}\mathcal{D}) = (\mathcal{D}' \otimes \Omega^{-1})\, vec(I_N)$.

Then, the term $(\overline{\mathcal{C}}^{-1} \otimes \Omega)\, vec(\Omega^{-1}\mathcal{D})$ can be written as follows:

$$\begin{aligned}\left(\overline{\mathcal{C}}^{-1} \otimes \Omega\right)(\mathcal{D}' \otimes \Omega^{-1})\, vec(I_N) &= \left(\overline{\mathcal{C}}^{-1}\mathcal{D}' \otimes \Omega\,\Omega^{-1}\right) vec(I_N) \\ &= \left(\overline{\mathcal{C}}^{-1}\mathcal{D}' \otimes I\right) vec(I_N) \\ &= vec\left(\mathcal{D}\,\overline{\mathcal{C}}^{-1}\right).\end{aligned} \tag{3.32}$$

If we substitute (3.32) in (3.31) and solve for $vec(\Lambda)$ we find that the estimator of $\Lambda_\mathcal{B}$ is:

$$vec\left(\Lambda_{\mathcal{B},T}\right) = vec\left(\mathcal{D}\,\overline{\mathcal{C}}^{-1}\right) + (\overline{\mathcal{C}}^{-1} \otimes \Omega)\, \mathcal{H}'_\Lambda \left[\mathcal{H}_\Lambda\, (\overline{\mathcal{C}}^{-1} \otimes \Omega)\, \mathcal{H}'_\Lambda\right]^{-1} \left[\kappa_\Lambda - \mathcal{H}_\Lambda\, vec(\mathcal{D}\,\overline{\mathcal{C}}^{-1})\right]. \tag{3.33}$$

Now, if we substitute (3.33) into (3.21) we find the estimator of the variance-covariance matrix of the measurement noise:

$$\Omega_{\mathcal{B},T} = \tfrac{1}{T}\left(\mathcal{E}_T - \mathcal{D}_T\,\overline{\mathcal{C}}_T^{-1}\,\mathcal{D}'_T + \mathcal{K}_{\Lambda,T}\,\overline{\mathcal{C}}_T\,\mathcal{K}'_{\Lambda,T}\right), \tag{3.34}$$

where $vec(\mathcal{K}_{\Lambda,T}) = (\overline{\mathcal{C}}^{-1} \otimes \Omega)\, \mathcal{H}'_\Lambda \left[\mathcal{H}_\Lambda\, (\overline{\mathcal{C}}^{-1} \otimes \Omega)\, \mathcal{H}'_\Lambda\right]^{-1} \left[\kappa_\Lambda - \mathcal{H}_\Lambda\, vec(\mathcal{D}\,\overline{\mathcal{C}}^{-1})\right]$.

(c)

For any given full rank $k \times k$ matrix A satisfying the structure (3.17), for a given factor F_t and associated smoothed value $F_{t|T} = \mathbb{E}_\theta \left[F_t \,|\, Y^T \right]$ we have the following re-parameterizations:

- $F_t^* = A^{-1} F_t$ and $F_{t|T}^* := \mathbb{E}_\theta \left[F_t^* \,|\, Y^T \right] = \mathbb{E}_\theta \left[A^{-1} F_t \,|\, Y^T \right] = A^{-1} F_{t|T}$;
- $P_{t|T}^* := \mathbb{E}[(F_t^* - F_{t|T}^*)(F_t^* - F_{t|T}^*)' \,|\, Y^T] = A^{-1} P_{t|T} (A^{-1})'$ and $P_{t-1,t|T}^* = A^{-1} P_{t-1,t|T} (A^{-1})'$;
- $\Lambda_{\mathcal{B},T}^* := \Lambda_{\mathcal{B},T} A$, $\Phi_T^* := A^{-1} \Phi_T A$, $\Psi_{\eta,T}^* := A^{-1} \Psi_{\eta,T} (A^{-1})'$, $\mu_T^* = \mu_T$ and $\Omega_{\mathcal{B},T}^* = \Omega_{\mathcal{B},T}$.

Let us assume now to have a given set of input parameters $\theta_{EM}^{(i)}$, satisfying the identification restrictions $R.i)$ and $R.ii)$, that we use to obtain $\mathcal{F}_{t|T}^{(i)} := (F_{t|T}^{(i)}, P_{t|T}^{(i)}, P_{t-1,t|T}^{(i)})$ from the Kalman Filter and Kalman Smoother (Expectation step i). Given $\mathcal{F}_{t|T}^{(i)}$, from the maximization step we obtain $\theta_{EM}^{(i+1)}$ but, at the same time, the updated parameter values do not satisfy the identification conditions anymore. More precisely, we have $\Psi_{\eta,T} \neq \Psi_{\mathcal{B}}$ and $\Lambda_{\mathcal{B},T}^{(i+1)\prime} \Lambda_{\mathcal{B},T}^{(i+1)} \neq \Pi_{\mathcal{B}}$. This means that we have to intervene in the EM recursions is such a way to guarantee, at each iteration, that the identification conditions be satisfied. This requirement is satisfied by means of the following steps:

- orthogonalizing common and local factor residuals: here we force common-factor autoregressive residuals to be uncorrelated with local-factor residuals in such a way to have the same patterns of zeros as $\Psi_{\mathcal{B}}$. Let us define the following matrix:

$$A_\perp^{-1} := \begin{bmatrix} I_{r_c} & 0 & \dots & 0 \\ -\left(\Psi_{10,T}^{c(i+1)}\right) \left[(\Psi_{00,T}^{c(i+1)}) \right]^{-1} & I_{r_1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\left(\Psi_{n0,T}^{c(i+1)}\right) \left[(\Psi_{00,T}^{c(i+1)}) \right]^{-1} & 0 & \dots & I_{r_n} \end{bmatrix}. \tag{3.35}$$

such that

$$\Psi^{o(i+1)}_{\eta,T} := (A^{-1}_{\perp})\,\Psi^{(i+1)}_{\eta,T}\,(A^{-1}_{\perp})' = \begin{bmatrix} \Psi^{c(i+1)}_{00,T} & 0 & \dots & 0 \\ 0 & \Psi^{o(i+1)}_{11,T} & \dots & \Psi^{o(i+1)}_{1n,T} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \Psi^{o(i+1)}_{n1,T} & \dots & \Psi^{o(i+1)}_{nn,T} \end{bmatrix}, \tag{3.36}$$

has the desired form, that is the same blocks of zeros as $\Psi_{\mathcal{B}}$.

- <u>orthogonalizing and normalizing within autoregressive residual blocks</u>: now we force the matrices in the main diagonal of (3.36) to be the identity matrix by applying a Jordan decomposition to each of them. Let us denote by $\mathcal{U}_{\eta,c(i+1)}$ and $\mathcal{D}_{\eta,c(i+1)}$ the matrix of eigenvectors and eigenvalues of $\Psi^{c(i+1)}_{00,T}$, respectively. Let us define the rotation matrix $A^{-1}_{\eta,c(i+1)} := \left(\mathcal{U}_{\eta,c(i+1)}\,\mathcal{D}^{-1/2}_{\eta,c(i+1)}\right)'$ and thus we have $A^{-1}_{\eta,c(i+1)}\,\Psi^{c(i+1)}_{00,T}\,\left(A^{-1}_{\eta,c(i+1)}\right)' = I_{r_c}$. Let us now denote by $\mathcal{U}_{\eta,i(i+1)}$ and $\mathcal{D}_{\eta,i(i+1)}$ the matrix of eigenvectors and eigenvalues of $\Psi^{o(i+1)}_{jj,T}$, respectively, for any $j \in \{1,\dots,n\}$. Let us define the rotation matrix $A^{-1}_{\eta,j(i+1)} := \left(\mathcal{U}_{\eta,j(i+1)}\,\mathcal{D}^{-1/2}_{\eta,j(i+1)}\right)'$ and thus we have $A^{-1}_{\eta,j(i+1)}\,\Psi^{o(i+1)}_{jj,T}\,\left(A^{-1}_{\eta,j(i+1)}\right)' = I_{r_j}$. We define the rotation matrix for $\Psi^{o(i+1)}_{\eta,T}$ as the block diagonal matrix
$A^{-1}_{\eta,(i+1)} := diag\left[A^{-1}_{\eta,c(i+1)}, A^{-1}_{\eta,1(i+1)}, \dots, A^{-1}_{\eta,n(i+1)}\right]$ such that:

$$\Psi^{oo(i+1)}_{T} := (A^{-1}_{\eta,(1)})\,\Psi^{o(i+1)}_{T}\,(A^{-1}_{\eta,(i+1)})' = \begin{bmatrix} I_{r_c} & 0 & 0 & \dots & 0 \\ 0 & I_{r_1} & \Psi^{oo(i+1)}_{12,T} & \dots & \Psi^{oo(i+1)}_{1n,T} \\ 0 & \Psi^{oo(i+1)}_{21,T} & I_{r_2} & \dots & \Psi^{oo(i+1)}_{2n,T} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \Psi^{oo(i+1)}_{n1,T} & \Psi^{oo(i+1)}_{n2,T} & \dots & I_{r_n} \end{bmatrix} = \Psi_{\mathcal{B}}\,. \tag{3.37}$$

In a more compact form, we define the factor's noise rotation matrix $(A^{o}_{\eta,(i+1)})^{-1} :=$

$A^{-1}_{\eta,(i+1)}\, A^{-1}_{\perp} = (A_\perp\, A_{\eta,(i+1)})^{-1}$ such that $(A^o_{\eta,(i+1)})^{-1}\, \Psi^{(i+1)}_{\eta,T}\, [(A^o_{\eta,(i+1)})^{-1}]' = \Psi_{\mathcal{B}}$.

- forcing orthogonality within blocks of loadings: here we intervene in the matrix of factor loadings, given that $\Lambda^{(i+1)\prime}_{\mathcal{B},T}\Lambda^{(i+1)}_{\mathcal{B},T} \neq \Pi_{\mathcal{B}}$. We know that, given the previously defined matrix $(A^o_{\eta,(i+1)})^{-1}$ rotating factor's noise, the associated rotation of the loadings is given by $\Lambda^{o(i+1)}_{\mathcal{B},T} := \Lambda^{(i+1)}_{\mathcal{B},T}\, A^o_{\eta,(i+1)}$, where $A^o_{\eta,(i+1)} = A_\perp\, A_{\eta,(i+1)}$.

  The matrix $\Lambda^{o(i+1)\prime}_{\mathcal{B},T}\Lambda^{o(i+1)}_{\mathcal{B},T}$ has the same blocks of zeros as $\Pi_{\mathcal{B}}$ but the matrices in the main diagonal are not diagonal matrices. Let us perform a Jordan decomposition of each of them. Let us diagonalize first the positive definite symmetric matrix $\Lambda^{o(i+1)\prime}_{c,\mathcal{B},T}\Lambda^{o(i+1)}_{c,\mathcal{B},T}$ associated to common factors:

$$\begin{cases} \mathcal{U}^{o\prime}_c \left(\Lambda^{o(i+1)\prime}_{c,\mathcal{B},T}\Lambda^{o(i+1)}_{c,\mathcal{B},T}\right) \mathcal{U}^o_c = \mathcal{D}^o_c \\ \\ \mathcal{U}^{o\prime}_c \mathcal{U}^o_c = \mathcal{U}^o_c \mathcal{U}^{o\prime}_c = I_{r_c}\,,\ \ \mathcal{U}^{o\prime}_c = (\mathcal{U}^o_c)^{-1}\,, \end{cases}$$

  and any positive definite symmetric matrix $\Lambda^{o(i+1)\prime}_{j,l,\mathcal{B},T}\Lambda^{o(i+1)}_{j,l,\mathcal{B},T}$ associated to the local factors of any country $j \in \{1,\ldots,n\}$:

$$\begin{cases} \mathcal{U}^{o\prime}_j \left(\Lambda^{o(i+1)\prime}_{j,l,\mathcal{B},T}\Lambda^{o(i+1)}_{j,l,\mathcal{B},T}\right) \mathcal{U}^o_j = \mathcal{D}^o_j \\ \\ \mathcal{U}^{o\prime}_j \mathcal{U}^o_j = \mathcal{U}^o_j \mathcal{U}^{o\prime}_j = I_{r_j}\,,\ \ \mathcal{U}^{o\prime}_j = (\mathcal{U}^o_j)^{-1}\,, \end{cases}$$

  and let us define the matrix $(A^o_{c,l,(i+1)})^{-1} := diag\left[(\mathcal{U}^o_{c,(i+1)})^{-1}, (\mathcal{U}^o_{1,(i+1)})^{-1}, \ldots, (\mathcal{U}^o_{n,(i+1)})^{-1}\right]$ such that $(A^o_{c,l,(i+1)})^{-1} = A^{o\prime}_{c,l,(i+1)}$. We define

$$\Lambda^{*(i+1)}_{\mathcal{B},T} := \Lambda^{o(i+1)}_{\mathcal{B},T}\, A^o_{c,l,(i+1)} = \Lambda^{(i+1)}_{\mathcal{B},T}\, A^o_{\eta,(i+1)}\, A^o_{c,l,(i+1)} = \Lambda^{(i+1)}_{\mathcal{B},T}\, \left(A_\perp\, A_{\eta,(i+1)}\, A^o_{c,l,(i+1)}\right) \tag{3.38}$$

and we have

$$\Lambda^{*(i+1)\prime}_{\mathcal{B},T}\Lambda^{*(i+1)}_{\mathcal{B},T} = A^{o\prime}_{c,l,(i+1)} \left(\Lambda^{o(i+1)\prime}_{j,l,\mathcal{B},T}\Lambda^{o(i+1)}_{j,l,\mathcal{B},T}\right) A^{o}_{c,l,(i+1)} = \Pi_{\mathcal{B}} \,. \tag{3.39}$$

This matrix $A^{o}_{c,l,(i+1)}$ does not perturb the structure already imposed on the factor's noise variance-covariance matrix given that, by block orthogonality, we have

$$\Psi^{*(i+1)}_{\eta,T} := (A^{o}_{c,l,(i+1)})^{-1}\, \Psi^{oo(1)}_{\eta,T}\, [(A^{o}_{c,l,(i+1)})^{-1}]' = A^{o\prime}_{c,l,(i+1)}\, \Psi^{oo(i+1)}_{\eta,T}\, A^{o}_{c,l,(i+1)} = \Psi_{\mathcal{B}} \,. \tag{3.40}$$

Thus, the normalization matrix $A^{*} := \left(A_{\perp}\, A_{\eta,(i+1)}\, A^{o}_{c,l,(i+1)}\right)$ is such that $\Lambda^{*(i+1)}_{\mathcal{B},T}$ and $\Psi^{*(i+1)}_{\eta,T}$ satisfy the indentification restrictions $R.i)$ and $R.ii)$, respectively. Moreover, it implies the factor's rotation $F^{*}_{t|T} := (A^{*})^{-1}\, F_{t|T}$ and the rotated AR matrix $\Phi^{*(i+1)}_{T} := (A^{*})^{-1}\, \Phi^{(i+1)}_{T}\, A^{*}$.

3.C A 3-Step Principal Factor Estimation Procedure.

The purpose of this appendix is to briefly present a Principal Factor (PF) estimation procedure adapted to a linear Gaussian state-space model with a block structure characterizing the matrix of factor loadings (i.e., in presence of VAR distributed common and local factors). This estimation methodology is based on the following three steps:

First Step: we estimate Λ_c and $F_t^{(c)}$ by PF assuming $\Lambda_l = 0$. More precisely, denoting $\widetilde{Y}_t = Y_t - \mu$ and for any $t \in \{1, \ldots, T\}$, we have $F_t^{*(c)} := D_c^{-1/2}\, P_c' \widetilde{Y}_t$ and $\Lambda_{c,T}^{PF} := P_c\, D_c^{1/2}$ where $D_c = \text{diag}(\lambda_1^{(c)}, \ldots, \lambda_{r_c}^{(c)})$ is the diagonal matrix of eigenvalues (in decreasing order of magnitude) of the variance-covariance matrix denoted $\mathcal{S}$ of the centered data $\widetilde{Y}_t$, and where $P_c = (p_1^{(c)}, \ldots, p_{r_c}^{(c)})$ is the $N \times r_c$ orthogonal matrix of associated unitary eigenvectors. Given $F_t^{*(c)}$ and $\Lambda_{c,T}$, we calculate the errors $\widetilde{Y}_t^e := Y_t - \mu_T^{PF} - \Lambda_{c,T}^{PF}\, F_t^{*(c)}$, with $\mu_T^{PF} := \frac{1}{T} \sum_{t=1}^{T} Y_t$, and the associated variance-covariance matrix denoted $\mathcal{S}_j^e$ for any country $j \in \{1, \ldots, n\}$.

Second Step: we estimate $\Lambda_l^{(j)}$ and $F_{j,t}^{(l)}$ by PF on $\mathcal{S}_j^e$ and for any $j \in \{1, \ldots, n\}$. We obtain $F_{j,t}^{*(l)} := (D_l^{(j)})^{-1/2}\, P_l^{(j)\prime} \widetilde{Y}_t^{(j)}$ and $\Lambda_{j,l,T}^{PF} := P_l^{(j)}\, (D_l^{(j)})^{1/2}$ where $D_l^{(j)} := \text{diag}(\lambda_{1,l}^{(j)}, \ldots, \lambda_{r_j,l}^{(j)})$ denotes the diagonal matrix of eigenvalues (in decreasing order of magnitude), and $P_l^{(j)} = (p_{1,l}^{(j)}, \ldots, p_{r_j,l}^{(j)})$ the $\tau \times r_j$ orthogonal matrix of associated unitary eigenvectors of $\mathcal{S}_j^e$.

Third Step: given $F_t^* = \left(F_t^{*(c)\prime}, F_t^{*(l)\prime}\right)'$, where $F_t^{*(l)} = \left(F_{1,t}^{*(l)\prime}, \ldots, F_{n,t}^{*(l)\prime}\right)'$, μ_T^{PF} and $\Lambda_{\mathcal{B},T}^{PF} := [\Lambda_{c,T}\ \Lambda_{l,T}]$, we obtain $\Omega_{\mathcal{B},T}^{PF}$ from yield errors $Y_t - \mu_T^{PF} - \Lambda_{\mathcal{B},T}^{PF}\, F_t^*$ while, from the regression of F_t^* on F_{t-1}^*, we estimate Φ_T^{PF} and then $\Psi_{\eta,T}^{PF}$ from associated model residuals.

Observe that, with this estimation procedure, the identification restrictions may be taken to be:

- $\mathbb{E}\left(F_t^{(c)}\,F_t^{(c)\prime}\right) = I_{r_c}$ and $\mathbb{E}\left(F_{j,t}^{(l)}\,F_{j,t}^{(l)\prime}\right) = I_{r_j}$ for all $j \in \{1, \ldots, n\}$;
- $\mathbb{E}\left(F_t^{(c)}\,F_{j,t}^{(l)\prime}\right) = 0$ for all $j \in \{1, \ldots, n\}$;
- $(\Lambda_c'\Lambda_c)$ and $\Lambda_l^{(j)\prime}\Lambda_l^{(j)}$, for all $j \in \{1, \ldots, n\}$, have to be diagonal.

That is, we naturally require to the marginal variance-covariance matrix of the latent factors, denoted Ψ_f, to be equal to $\Psi_{\mathcal{B}}$.

3.D International Treasury Yields Summary Statistics and Graphs.

Maturity (Months)	Median	Mean	St. Dev.	Min	Max	$\rho(1)$	$\rho(4)$	$\rho(12)$	$\rho(36)$
U.S.									
12	5.00	4.71	2.17	0.26	9.74	0.99	0.98	0.94	0.80
24	5.18	5.00	2.10	0.48	9.70	0.99	0.98	0.93	0.80
36	5.40	5.25	2.01	0.69	9.65	0.99	0.98	0.93	0.81
48	5.56	5.45	1.94	0.94	9.60	0.99	0.98	0.93	0.82
60	5.64	5.61	1.87	1.21	9.54	0.99	0.98	0.93	0.83
72	5.74	5.75	1.81	1.51	9.56	0.99	0.98	0.93	0.83
84	5.82	5.85	1.76	1.75	9.64	0.99	0.98	0.93	0.84
96	5.88	5.94	1.72	1.95	9.70	0.99	0.98	0.93	0.84
108	5.93	6.01	1.69	2.14	9.74	0.99	0.98	0.94	0.85
Germany									
12	3.89	4.63	2.02	0.67	9.11	0.99	0.99	0.96	0.85
24	4.09	4.53	1.90	1.16	8.80	0.99	0.99	0.95	0.84
36	4.41	4.73	1.79	1.59	8.81	0.99	0.99	0.95	0.84
48	4.65	4.91	1.71	1.94	8.80	0.99	0.99	0.95	0.85
60	4.91	5.07	1.65	2.24	8.79	0.99	0.99	0.95	0.86
72	5.08	5.20	1.60	2.46	8.81	0.99	0.99	0.96	0.86
84	5.20	5.32	1.57	2.64	8.84	0.99	0.99	0.96	0.87
96	5.36	5.41	1.54	2.78	8.86	0.99	0.99	0.96	0.87
108	5.43	5.50	1.52	2.85	8.88	0.99	0.99	0.96	0.88
U.K.									
12	5.86	6.57	2.92	0.56	14.36	0.99	0.98	0.93	0.82
24	6.19	6.62	2.67	1.18	13.74	0.99	0.98	0.93	0.83
36	6.23	6.69	2.54	1.72	13.34	0.99	0.98	0.93	0.84
48	6.28	6.74	2.47	2.07	13.09	0.99	0.98	0.94	0.85
60	6.22	6.78	2.43	2.28	12.93	0.99	0.98	0.94	0.86
72	6.14	6.81	2.40	2.45	12.81	0.99	0.98	0.95	0.87
84	6.09	6.83	2.39	2.63	12.69	0.99	0.98	0.95	0.88
96	6.09	6.84	2.37	2.82	12.57	0.99	0.98	0.95	0.89
108	6.07	6.84	2.35	3.00	12.43	0.99	0.98	0.96	0.89
Japan									
12	0.59	1.91	2.24	0.01	8.35	0.99	0.99	0.97	0.91
24	0.78	2.01	2.17	0.01	8.28	0.99	0.99	0.97	0.90
36	0.99	2.18	2.12	0.07	8.21	0.99	0.99	0.96	0.90
48	1.25	2.36	2.07	0.10	8.13	0.99	0.99	0.96	0.90
60	1.47	2.52	2.03	0.15	8.06	0.99	0.99	0.96	0.90
72	1.62	2.68	1.99	0.20	7.98	0.99	0.99	0.96	0.90
84	1.74	2.83	1.96	0.26	7.90	0.99	0.99	0.96	0.90
96	1.88	2.96	1.93	0.33	7.82	0.99	0.99	0.96	0.91
108	1.99	3.08	1.90	0.40	7.74	0.99	0.99	0.96	0.91

Table 3.1: Summary Statistics for bond yields of U.S., Germany, U.K. and Japan daily yields. $\rho(\ell)$ denotes the sample autocorrelation for a number of lags ℓ measured in days. The sample period is from January 1, 1986 to December 31, 2009. Yields are in annual basis.

Treasury yield curves across countries and time

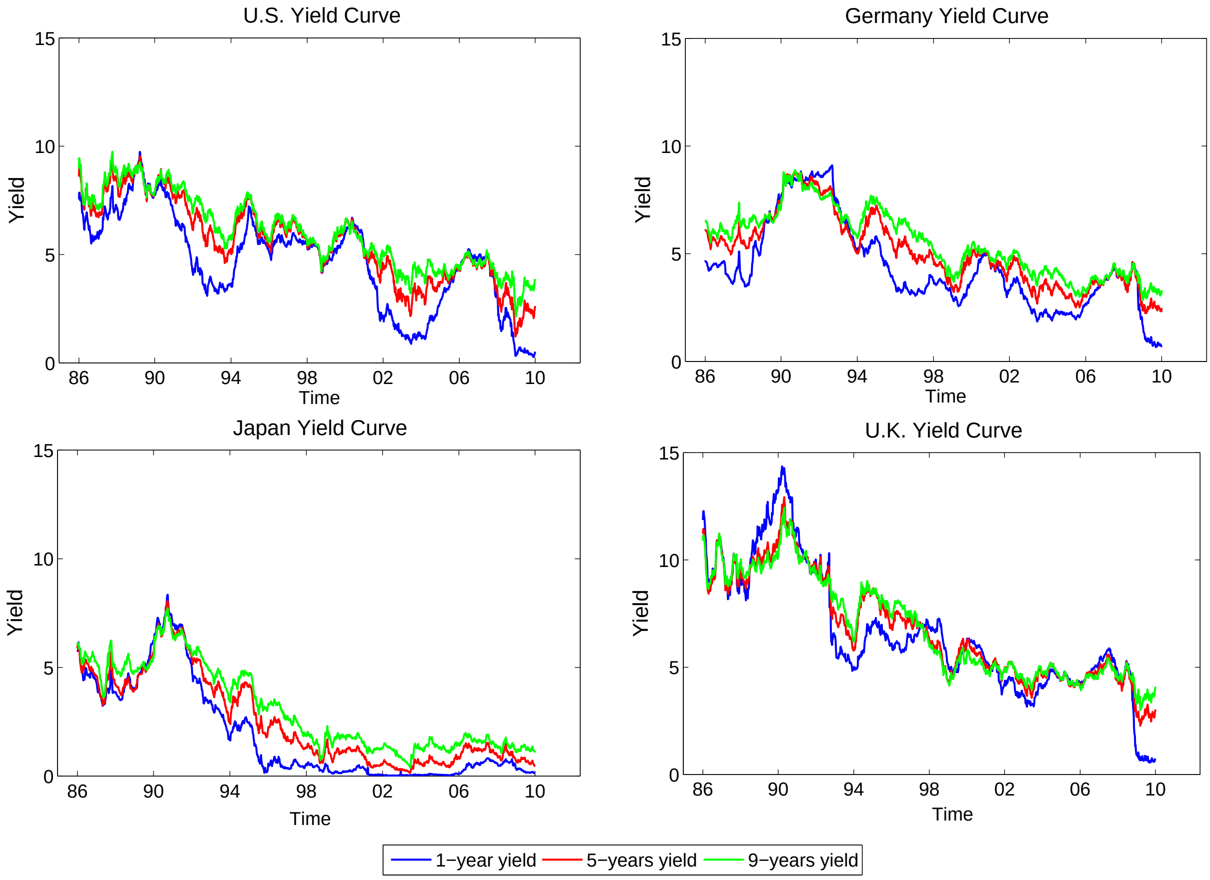


Fig. 3.1: Treasury yield curves of U.S., Germany, Japan and U.K. and for residual maturities 1, 5 and 9 years. The term structures of interest rates of *U.S.*, Germany and Japan are the one constructed in Chapter 2 while *U.K.* term structures are taken from the Bank of England data set.

3.E Maximum Log-Likelihood of $MCTSMs$ and Model Selection.

$\mathcal{M}_n^{r_c,r_\ell}(\Phi,\Psi_\eta)$ *for yield levels;*

The 2-Country Case

U.S. – U.K.								*U.S. – GER.*							
r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$	r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	135	159778	-319286	-322112	0	3	3	6	135	164955	-329640	-333404
0	4	4	8	188	178525	-356674	-360092	0	4	4	8	188	185801	-371226	-375164
0	5	5	10	251	191826	-383150	-386914	0	5	5	10	251	198543	-396584	-401122
1	2	2	5	119	149652	-299066	-301856	1	2	2	5	119	153828	-307418	-310800
1	3	3	7	166	170927	-341522	-344418	1	3	3	7	166	176150	-351968	-356308
1	4	4	9	223	186159	-371872	-375132	1	4	4	9	223	192456	-384466	-388910
2	1	1	4	107	138919	-277624	-280736	2	1	1	4	107	140508	-280802	-283416
2	2	2	6	148	160764	-321232	-324218	2	2	2	6	148	165476	-330656	-334980
2	3	3	8	199	179798	-359198	-361954	2	3	3	8	199	186214	-372030	-376214
U.S. – JAP.								*U.K. – GER.*							
r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$	r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	135	162186	-324102	-325156	0	3	3	6	135	160199	-320128	-322470
0	4	4	8	188	184013	-367650	-369030	0	4	4	8	188	177953	-355530	-358312
0	5	5	10	251	197486	-394470	-397430	0	5	5	10	251	190553	-394470	-384090
1	2	2	5	119	151547	-302856	-302432	1	2	2	5	119	151689	-303140	-305824
1	3	3	7	166	173371	-346410	-348280	1	3	3	7	166	170884	-341436	-344394
1	4	4	9	223	190845	-381244	-383178	1	4	4	9	223	184642	-368838	-371960
2	1	1	4	107	140173	-280132	-280060	2	1	1	4	107	138761	-277308	-280490
2	2	2	6	148	162576	-324856	-326524	2	2	2	6	148	161144	-321992	-325470
2	3	3	8	199	184357	-368316	-369678	2	3	3	8	199	179642	-358886	-361306
GER. – JAP.								*U.K. – JAP.*							
r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$	r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	135	162666	-325062	-328264	0	3	3	6	135	157484	-314698	-315300
0	4	4	8	188	183493	-366610	-369434	0	4	4	8	188	176205	-352034	-350272
0	5	5	10	251	195508	-390514	-394096	0	5	5	10	251	188779	-377056	-378488
1	2	2	5	119	151143	-302048	-304832	1	2	2	5	119	147837	-295436	-297934
1	3	3	7	166	173732	-347132	-350520	1	3	3	7	166	169090	-337848	-339070
1	4	4	9	223	189533	-378620	-382858	1	4	4	9	223	182946	-365446	-364368
2	1	1	4	107	140653	-281092	-281184	2	1	1	4	107	136423	-272632	-274402
2	2	2	6	148	162903	-325510	-328752	2	2	2	6	148	158907	-317518	-319896
2	3	3	8	199	183775	-367152	-370498	2	3	3	8	199	177459	-354520	-352702

Table 3.2: For any given set of $n = 2$ countries and for any given number of latent factors k, shared between r_c common factors and r_ℓ local factors, we provide the number of parameters (Ξ), the maximum value of the log-likelihood function ($\mathcal{L}(\widehat{\theta}_T^{MLE})$), the associated Akaike Information Criterion (AIC), and its bootstrap variant ($AICb$) of $MCTSMs$ $\mathcal{M}_n^{r_c,r_\ell}(\Phi,\Psi_\eta)$. We use for any country weekly yields (in level) observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

The 3-Country and 4-Country Case

U.S. – U.K. – GER.									*U.S. – U.K. – JAP.*								
r_c	r_1	r_2	r_3	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$	r_c	r_1	r_2	r_3	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	3	9	243	242733	-484980	-491432	0	3	3	3	9	243	239937	-479388	-483898
0	4	4	4	12	354	271533	-542358	-549436	0	4	4	4	12	354	269563	-538418	-541068
0	5	5	5	15	489	290810	-580642	-587562	0	5	5	5	15	489	288959	-576940	-583108
1	2	2	2	7	196	223047	-445702	-452496	1	2	2	2	7	196	219448	-438504	-442416
1	3	3	3	10	289	254223	-507868	-514970	1	3	3	3	10	289	251755	-502932	-507008
1	4	4	4	13	406	278610	-556408	-564154	1	4	4	4	13	406	276865	-552918	-556256
2	1	1	1	5	163	197180	-394034	-397888	2	1	1	1	5	163	195551	-390776	-393626
2	2	2	2	8	238	234589	-468702	-476422	2	2	2	2	8	238	231298	-462120	-468380
2	3	3	3	11	337	264998	-529322	-528836	2	3	3	3	11	337	262946	-525218	-541276
U.S. – GER. – JAP.									*U.K. – GER. – JAP.*								
r_c	r_1	r_2	r_3	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$	r_c	r_1	r_2	r_3	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	3	9	243	245072	-489658	-496280	0	3	3	3	9	243	240351	-480216	-486776
0	4	4	4	12	354	276840	-552972	-559126	0	4	4	4	12	354	269122	-537536	-541712
0	5	5	5	15	489	295646	-590314	-597880	0	5	5	5	15	489	287695	-574412	-580700
1	2	2	2	7	196	222296	-444200	-449490	1	2	2	2	7	196	219371	-438350	-445270
1	3	3	3	10	289	256369	-512160	-519226	1	3	3	3	10	289	252061	-503544	-509000
1	4	4	4	13	406	283476	-566140	-572512	1	4	4	4	13	406	275945	-551078	-556400
2	1	1	1	5	163	198341	-396350	-399524	2	1	1	1	5	163	196220	-392114	-397642
2	2	2	2	8	238	234520	-468564	-475800	2	2	2	2	8	238	232278	-464080	-471688
2	3	3	3	11	337	267502	-534330	-530336	2	3	3	3	11	337	262646	-524618	-528064

U.S. – U.K. – GER. – JAP.									
r_c	r_1	r_2	r_3	r_4	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	3	3	12	378	323397	-646038	-655300
0	4	4	4	4	16	568	362485	-723834	-733018
0	5	5	5	5	20	802	388039	-774474	-783988
1	2	2	2	2	9	285	290509	-580448	-590858
1	3	3	3	3	13	439	334773	-668668	-678682
1	4	4	4	4	17	637	369489	-737704	-747770
2	1	1	1	1	6	222	255090	-509736	-516604
2	2	2	2	2	10	340	303748	-606816	-618450
2	3	3	3	3	14	502	346032	-691060	-700900

Table 3.3: For any given set of $n = 3$ and $n = 4$ countries and for any given number of latent factors k, shared between r_c common factors and r_ℓ local factors, we provide the number of parameters (Ξ), the maximum value of the log-likelihood function ($\mathcal{L}(\widehat{\theta}_T^{MLE})$), the associated Akaike Information Criterion (AIC), and its bootstrap variant ($AICb$), of $MCTSMs$ $\mathcal{M}_n^{r_c,r_\ell}(\Phi, \Psi_\eta)$. We use for any country weekly yields (in level) observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

$\mathcal{M}_n^{r_c,r_\ell}(\Phi, \Psi_\eta)$ *for yield differences*

The 2-Country Case

U.S. – U.K.								U.S. – GER.							
r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$	r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	135	178797	-357324	-363034	0	3	3	6	135	180839	-361408	-367448
0	4	4	8	188	192625	-384874	-388406	0	4	4	8	188	194919	-389462	-392958
0	5	5	10	251	201177	-401852	-405164	0	5	5	10	251	205097	-409692	-413870
1	2	2	5	119	172329	-344420	-350652	1	2	2	5	119	172591	-344944	-349458
1	3	3	7	166	186301	-372270	-376756	1	3	3	7	166	188330	-376328	-381174
1	4	4	9	223	197519	-394592	-398148	1	4	4	9	223	199857	-399268	-403090
2	1	1	4	107	164085	-327956	-332634	2	1	1	4	107	164296	-328378	-331552
2	2	2	6	148	179358	-358420	-361948	2	2	2	6	148	180856	-361416	-367574
2	3	3	8	199	192673	-384948	-388464	2	3	3	8	199	194948	-389498	-393066
U.S. – JAP.								U.K. – GER.							
r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$	r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	135	180717	-361164	-366994	0	3	3	6	135	178714	-357158	-362968
0	4	4	8	188	194520	-388664	-391706	0	4	4	8	188	191642	-382908	-386264
0	5	5	10	251	204786	-409070	-412776	0	5	5	10	251	200250	-399998	-403584
1	2	2	5	119	172463	-344688	-349080	1	2	2	5	119	172174	-344110	-350110
1	3	3	7	166	188187	-376042	-380574	1	3	3	7	166	185104	-369876	-374526
1	4	4	9	223	199491	-398536	-401942	1	4	4	9	223	196562	-392678	-396470
2	1	1	4	107	164319	-328424	-331868	2	1	1	4	107	163212	-326210	-332166
2	2	2	6	148	180722	-361148	-367048	2	2	2	6	148	178676	-357056	-362906
2	3	3	8	199	194558	-388718	-391732	2	3	3	8	199	191685	-382972	-386440
GER. – JAP.								U.K. – JAP.							
r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$	r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	135	180588	-360906	-366592	0	3	3	6	135	178545	-356820	-362294
0	4	4	8	188	193529	-386682	-389722	0	4	4	8	188	191247	-382118	-385060
0	5	5	10	251	203834	-407166	-411104	0	5	5	10	251	199937	-399372	-402502
1	2	2	5	119	171453	-342668	-347142	1	2	2	5	119	172003	-343768	-347496
1	3	3	7	166	187193	-374054	-378512	1	3	3	7	166	184927	-369522	-373960
1	4	4	9	223	198503	-396560	-399956	1	4	4	9	223	196218	-391990	-395446
2	1	1	4	107	164216	-328218	-331524	2	1	1	4	107	163342	-326470	-332160
2	2	2	6	148	180611	-360926	-366600	2	2	2	6	148	178629	-356962	-362546
2	3	3	8	199	193539	-386680	-389714	2	3	3	8	199	191271	-382144	-385144

Table 3.4: For any given set of $n = 2$ countries and for any given number of latent factors k, shared between r_c common factors and r_ℓ local factors, we provide the number of parameters (Ξ), the maximum value of the log-likelihood function ($\mathcal{L}(\widehat{\theta}_T^{MLE})$), the associated Akaike Information Criterion (AIC), and its bootstrap variant ($AICb$), of $MCTSMs$ $\mathcal{M}_n^{r_c,r_\ell}(\Phi, \Psi_\eta)$. We use for any country weekly yields (in difference) observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

The 3-Country and 4-Country Case

U.S. – U.K. – GER.									*U.S. – U.K. – JAP.*								
r_c	r_1	r_2	r_3	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	*AIC*	*AICb*	r_c	r_1	r_2	r_3	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	*AIC*	*AICb*
0	3	3	3	9	243	269345	-538204	-547028	0	3	3	3	9	243	269141	-537796	-546420
0	4	4	4	12	354	289903	-579098	-584214	0	4	4	4	12	354	289437	-578166	-582962
0	5	5	5	15	489	303370	-605762	-611596	0	5	5	5	15	489	302932	-604886	-610362
1	2	2	2	7	196	254623	-508854	-516378	1	2	2	2	7	196	254330	-508268	-515792
1	3	3	3	10	289	276859	-553140	-560646	1	3	3	3	10	289	276627	-552676	-558732
1	4	4	4	13	406	295056	-589300	-594850	1	4	4	4	13	406	294216	-587620	-592986
2	1	1	1	5	163	238002	-475678	-480106	2	1	1	1	5	163	238033	-475740	-480960
2	2	2	2	8	238	262827	-525178	-534116	2	2	2	2	8	238	262595	-524714	-533586
2	3	3	3	11	337	283347	-566020	-572388	2	3	3	3	11	337	283067	-565460	-571926
U.S. – GER. – JAP.									*U.K. – GER. – JAP.*								
r_c	r_1	r_2	r_3	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	*AIC*	*AICb*	r_c	r_1	r_2	r_3	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	*AIC*	*AICb*
0	3	3	3	9	243	271224	-541962	-550812	0	3	3	3	9	243	269102	-537718	-546316
0	4	4	4	12	354	291724	-582740	-587990	0	4	4	4	12	354	288506	-576304	-581190
0	5	5	5	15	489	306868	-612758	-618998	0	5	5	5	15	489	301962	-602946	-608724
1	2	2	2	7	196	253829	-507266	-513396	1	2	2	2	7	196	253412	-506432	-513882
1	3	3	3	10	289	278711	-556844	-558732	1	3	3	3	10	289	275679	-550780	-557998
1	4	4	4	13	406	296510	-592208	-597602	1	4	4	4	13	406	293252	-585692	-591232
2	1	1	1	5	163	238028	-475730	-479152	2	1	1	1	5	163	237111	-473896	-478184
2	2	2	2	8	238	262981	-525486	-533022	2	2	2	2	8	238	262469	-524462	-533278
2	3	3	3	11	337	285356	-570038	-576552	2	3	3	3	11	337	282110	-563546	-569884

U.S. – U.K. – GER. – JAP.									
r_c	r_1	r_2	r_3	r_4	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	*AIC*	*AICb*
0	3	3	3	3	12	378	359853	-718950	-730844
0	4	4	4	4	16	568	386765	-772394	-779328
0	5	5	5	5	20	802	403614	-805624	
1	2	2	2	2	9	285	336575	-672580	-680120
1	3	3	3	3	13	439	367232	-733586	-744314
1	4	4	4	4	17	637	391400	-781526	-788674
2	1	1	1	1	6	222	311688	-622932	-628596
2	2	2	2	2	10	340	344911	-689142	-699856
2	3	3	3	3	14	502	373710	-746416	-755034

Table 3.5: For any given set of $n = 3$ and $n = 4$ countries and for any given number of latent factors k, shared between r_c common factors and r_ℓ local factors, we provide the number of parameters (Ξ), the maximum value of the log-likelihood function ($\mathcal{L}(\widehat{\theta}_T^{MLE})$), the associated Akaike Information Criterion (AIC), and its bootstrap variant ($AICb$), of $MCTSMs$ $\mathcal{M}_n^{r_c, r_\ell}(\Phi, \Psi_\eta)$. We use for any country weekly yields (in difference) observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

$\mathcal{M}_n^{r_c,r_j}(\Phi,\Psi_\eta)$ *for both yield levels and differences*

yield levels

U.S. – U.K.

r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	135	159778	-319286	-322112
0	4	4	8	188	178525	-356674	-360092
1	3	2	6	141	160650	-321018	-323982
1	3	4	8	193	179576	-358766	-361692
2	2	2	6	148	160764	-321232	-324218
2	3	3	8	199	179798	-359198	-361954

U.S. – GER.

r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	135	164955	-329640	-333404
0	4	4	8	188	185801	-371226	-375164
1	2	3	6	141	165279	-330276	-334518
1	4	3	8	193	186180	-371974	-375812
2	2	2	6	148	165476	-330656	-334980
2	3	3	8	199	186214	-372030	-376214

U.S. – U.K. – GER.

r_c	r_1	r_2	r_3	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	2	3	8	211	232124	-463826	-469930
0	4	3	4	11	314	263475	-526322	-533518
1	3	2	2	8	224	234184	-467920	-475558
1	4	3	3	11	325	264395	-528140	-535298
2	2	2	2	8	238	234589	-468702	-476422
2	3	3	3	11	337	264998	-529322	-535750

U.S. – U.K. – GER. – JAP.

r_c	r_1	r_2	r_3	r_4	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	2	2	3	3	10	299	301081	-601564	-611526
0	4	3	3	4	14	467	344617	-688300	-696808
1	2	2	2	3	10	319	302975	-605312	-616618
1	4	3	3	3	14	484	345486	-690004	-697722
2	2	2	2	2	10	340	303748	-606816	-618450
2	3	3	3	3	14	502	346032	-691060	-700900

yield differences

U.S. – U.K.

r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	135	178797	-357324	-363034
0	4	4	8	188	192625	-384874	-388406
1	3	2	6	141	179331	-358380	-361856
1	3	4	8	193	192656	-384926	-388290
2	2	2	6	148	179358	-358420	-361948
2	3	3	8	199	192673	-384948	-388464

U.S. – GER.

r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	135	180839	-361408	-367448
0	4	4	8	188	194919	-389462	-392958
1	2	3	6	141	180849	-361416	-367618
1	4	3	8	193	194944	-389502	-393040
2	2	2	6	148	180856	-361416	-367574
2	3	3	8	199	194948	-389498	-393066

U.S. – U.K. – GER.

r_c	r_1	r_2	r_3	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	2	3	8	211	262322	-524222	-532094
0	4	4	3	11	314	283103	-565578	-572062
1	2	2	3	8	224	262349	-524250	-532146
1	3	3	4	11	325	283327	-566004	-571762
2	2	2	2	8	238	262827	-525178	-534116
2	3	3	3	11	337	283347	-566020	-572388

U.S. – U.K. – GER. – JAP.

r_c	r_1	r_2	r_3	r_4	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	2	2	3	3	10	299	343934	-687270	-695888
0	4	4	3	3	14	467	372925	-744916	-753436
1	2	2	2	3	10	319	343969	-687300	-696162
1	4	3	3	3	14	484	373007	-745046	-753830
2	2	2	2	2	10	340	344911	-689142	-699856
2	3	3	3	3	14	502	373710	-746416	-755034

Table 3.6: For any given set of n countries and for any given number of latent factors k, shared between r_c common factors and r_j local factors, we provide the number of parameters (Ξ), the maximum value of the log-likelihood function ($\mathcal{L}(\widehat{\theta}_T^{MLE})$), the associated Akaike Information Criterion (AIC), and its bootstrap variant ($AICb$), of $MCTSMs$ $\mathcal{M}_n^{r_c,r_j}(\Phi,\Psi_\eta)$. We use for any country weekly yields observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

$\mathcal{M}_n^{r_c,r_j}(\Phi_{bd}, \widetilde{\Psi}_\eta)$ *for both yield levels and differences*

yield levels

U.S. – U.K.								U.S. – GER.							
r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$	r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	117	159679	-319124	-319660	0	3	3	6	117	164845	-329456	-328330
0	4	4	8	156	178508	-356704	-357864	0	4	4	8	156	182945	-365578	-363390
1	3	2	6	124	160300	-320352	-320978	1	3	2	6	124	165101	-329954	-330694
1	3	4	8	162	179378	-358432	-360822	1	4	3	8	162	185875	-371426	-370624
2	2	2	6	132	160734	-321204	-323554	2	2	2	6	132	165264	-330264	-332096
2	3	3	8	169	179652	-358966	-361694	2	3	3	8	169	185885	-371432	-370350

U.S. – U.K. – GER.									U.S. – U.K. – GER. – JAP.										
r_c	r_1	r_2	r_3	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$	r_c	r_1	r_2	r_3	r_4	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	2	3	8	169	232058	-463778	-464598	0	2	2	3	3	10	225	300958	-601466	-602048
0	4	3	4	11	234	263179	-525890	-526390	0	4	3	3	4	14	321	342878	-684358	-686292
1	3	2	2	8	185	233904	-467438	-469188	1	2	2	2	3	10	250	302700	-604900	-606586
1	4	3	3	11	249	263198	-525898	-527638	1	4	3	3	3	14	345	342904	-685118	-686426
2	2	2	2	8	202	234361	-468318	-471578	2	2	2	2	2	10	276	303387	-606222	-609128
2	3	3	3	11	265	263844	-527158	-531798	2	3	3	3	3	14	370	343561	-686382	-688976

yield differences

U.S. – U.K.								U.S. – GER.							
r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$	r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	117	178030	-355826	-361132	0	3	3	6	117	180602	-360970	-366920
0	4	4	8	156	192515	-384718	-388068	0	4	4	8	156	194788	-389264	-392662
1	3	2	6	124	178088	-355928	-361286	1	2	3	6	124	180615	-360982	-366858
1	4	3	8	162	192548	-384772	-387962	1	3	4	8	162	194820	-389316	-392664
2	2	2	6	132	178165	-356066	-360794	2	2	2	6	132	180736	-361208	-367344
2	3	3	8	169	192557	-384776	-388092	2	3	3	8	169	194833	-389328	-392806

U.S. – U.K. – GER.									U.S. – U.K. – GER. – JAP.										
r_c	r_1	r_2	r_3	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$	r_c	r_1	r_2	r_3	r_4	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	2	3	8	169	262213	-524088	-531752	0	2	2	3	3	10	225	344013	-687576	-696454
0	4	4	3	11	234	282981	-565494	-571710	0	4	3	4	3	14	321	373285	-745928	-753908
1	2	2	3	8	185	262258	-524146	-531976	1	2	2	2	3	10	250	344029	-687558	-696668
1	4	3	3	11	249	283066	-565634	-571934	1	4	3	3	3	14	345	373389	-746088	-755074
2	2	2	2	8	202	262260	-524108	-531940	2	2	2	2	2	10	276	344310	-688068	-697324
2	3	3	3	11	265	283093	-565656	-572090	2	3	3	3	3	14	370	373448	-746156	-755336

Table 3.7: For any given set of n countries and for any given number of latent factors k, shared between r_c common factors and r_j local factors, we provide the number of parameters (Ξ), the maximum value of the log-likelihood function ($\mathcal{L}(\widehat{\theta}_T^{MLE})$), the associated Akaike Information Criterion (AIC), and its bootstrap variant ($AICb$), of $MCTSMs$ $\mathcal{M}_n^{r_c,r_j}\left(\Phi_{bd}, \widetilde{\Psi}_\eta\right)$. We use for any country weekly yields observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

$\mathcal{M}_n^{r_c,r_j}(\Phi, I)$ *for both yield levels and differences*

yield levels

U.S. – U.K.

r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	126	153344	-306436	-311544
0	4	4	8	172	171278	-342212	-348900
1	3	2	6	135	154842	-309414	-314440
1	3	4	8	181	173022	-345682	-352214
2	2	2	6	144	155261	-310234	-315958
2	3	3	8	190	174027	-347674	-353988

U.S. – GER.

r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	126	158113	-315974	-322558
0	4	4	8	172	178253	-356702	-363412
1	2	3	6	135	159761	-319252	-325756
1	4	3	8	181	179397	-358432	-365276
2	2	2	6	144	159847	-319406	-325208
2	3	3	8	190	179962	-359544	-366276

U.S. – U.K. – GER.

r_c	r_1	r_2	r_3	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	2	3	8	190	222523	-444666	-452714
0	4	3	4	11	274	252780	-505012	-513968
1	3	2	2	8	208	226558	-452700	-463024
1	3	3	4	11	292	255021	-509458	-520720
2	2	2	2	8	226	227664	-454876	-465198
2	3	3	3	11	310	256370	-512120	-522072

U.S. – U.K. – GER. – JAP.

r_c	r_1	r_2	r_3	r_4	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	2	2	3	3	10	262	288629	-576734	-588374
0	4	3	3	4	14	394	330392	-659996	-673336
1	2	2	3	2	10	289	292316	-584054	-596682
1	4	3	3	3	14	421	333407	-665972	-681088
2	2	2	2	2	10	316	295071	-589510	-603812
2	3	3	3	3	14	448	333732	-666568	-681804

yield differences

U.S. – U.K.

r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	126	176873	-353494	-357296
0	4	4	8	172	190640	-380936	-384030
1	3	2	6	135	176903	-353536	-357594
1	4	3	8	181	190743	-381124	-384290
2	2	2	6	144	176969	-353650	-357594
2	3	3	8	190	190812	-381244	-384260

U.S. – GER.

r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	126	179222	-358192	-362544
0	4	4	8	172	193020	-385696	-388548
1	2	3	6	135	179240	-358210	-362712
1	4	3	8	181	193064	-385766	-388718
2	2	2	6	144	179339	-358390	-362722
2	3	3	8	190	193149	-385918	-389134

U.S. – U.K. – GER.

r_c	r_1	r_2	r_3	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	2	3	8	190	259813	-519246	-525386
0	4	4	3	11	274	280156	-559764	-565166
1	2	2	3	8	208	259908	-519400	-525790
1	3	3	4	11	292	280233	-559882	-565368
2	2	2	2	8	226	260168	-519884	-526274
2	3	3	3	11	310	280387	-560154	-565728

U.S. – U.K. – GER. – JAP.

r_c	r_1	r_2	r_3	r_4	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	2	2	3	3	10	262	341160	-681796	-689238
0	4	4	3	3	14	394	369721	-738654	-746008
1	2	2	2	3	10	289	341173	-681768	-689366
1	4	3	3	3	14	421	370294	-739746	-747654
2	2	2	2	2	10	316	341592	-682552	-689568
2	3	3	3	3	14	448	370300	-739704	-747754

Table 3.8: For any given set of n countries and for any given number of latent factors k, shared between r_c common factors and r_j local factors, we provide the number of parameters (Ξ), the maximum value of the log-likelihood function ($\mathcal{L}(\widehat{\theta}_T^{MLE})$), the associated Akaike Information Criterion (AIC), and its bootstrap variant ($AICb$), of $MCTSMs$ $\mathcal{M}_n^{r_c,r_j}(\Phi, I)$. We use for any country weekly yields observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

$\mathcal{M}_n^{r_c,r_j}(\Phi_{bd}, I)$ *for both yield levels and differences*

yield levels

U.S. – U.K.

r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	108	153180	-306144	-307770
0	4	4	8	140	171073	-341866	-346398
1	3	2	6	118	154329	-308422	-310210
1	4	3	8	150	173132	-345964	-347736
2	2	2	6	128	155585	-310914	-311542
2	3	3	8	160	174076	-347832	-351980

U.S. – GER.

r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	108	158086	-315956	-317854
0	4	4	8	140	177248	-354216	-357754
1	3	2	6	118	159600	-318964	-319624
1	3	4	8	150	179799	-359298	-362646
2	2	2	6	128	160279	-320302	-319996
2	3	3	8	160	180114	-359908	-359762

U.S. – U.K. – GER.

r_c	r_1	r_2	r_3	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	2	3	8	148	222160	-444024	-445862
0	4	3	4	11	194	252354	-504320	-506624
1	2	2	3	8	169	225683	-451028	-453308
1	3	3	4	11	216	255333	-510234	-512588
2	2	2	2	8	190	228490	-456600	-460682
2	3	3	3	11	238	256089	-511702	-517102

U.S. – U.K. – GER. – JAP.

r_c	r_1	r_2	r_3	r_4	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	2	2	3	3	10	188	287904	-575432	-575928
0	4	3	3	4	14	248	329806	-659116	-661400
1	3	2	2	2	10	220	292286	-584132	-585020
1	4	3	3	3	14	282	333440	-666316	-688284
2	2	2	2	2	10	252	295940	-591376	-595346
2	3	3	3	3	14	316	333843	-667054	-671976

yield differences

U.S. – U.K.

r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	108	176441	-352666	-356844
0	4	4	8	140	190637	-380994	-383978
1	3	2	6	118	176481	-352726	-356956
1	4	3	8	150	190715	-381130	-384290
2	2	2	6	128	177060	-353864	-358022
2	3	3	8	160	190927	-381534	-384816

U.S. – GER.

r_c	r_1	r_2	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	3	6	108	179148	-358080	-362512
0	4	4	8	140	192944	-385608	-388492
1	2	3	6	118	179210	-358184	-362492
1	4	3	8	150	192973	-385646	-388658
2	2	2	6	128	179440	-358624	-363000
2	3	3	8	160	193298	-386276	-389320

U.S. – U.K. – GER.

r_c	r_1	r_2	r_3	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	3	2	3	8	148	259654	-519012	-524880
0	4	4	3	11	194	280166	-559944	-564860
1	2	2	3	8	169	259675	-519012	-525056
1	3	3	4	11	216	280232	-560032	-565346
2	2	2	2	8	190	259777	-519174	-525200
2	3	3	3	11	238	280800	-561124	-566678

U.S. – U.K. – GER. – JAP.

r_c	r_1	r_2	r_3	r_4	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	AIC	$AICb$
0	2	2	3	3	10	188	340965	-681554	-688634
0	4	4	3	3	14	248	369494	-738492	-744938
1	2	2	2	3	10	220	340968	-681496	-688788
1	4	3	3	3	14	282	369536	-738508	-745832
2	2	2	2	2	10	252	341371	-682238	-690458
2	3	3	3	3	14	316	370744	-740856	-748418

Table 3.9: For any given set of n countries and for any given number of latent factors k, shared between r_c common factors and r_j local factors, we provide the number of parameters (Ξ), the maximum value of the log-likelihood function ($\mathcal{L}(\widehat{\theta}_T^{MLE})$), the associated Akaike Information Criterion (AIC), and its bootstrap variant ($AICb$), of $MCTSMs$ $\mathcal{M}_n^{r_c,r_j}(\Phi_{bd}, I)$. We use for any country weekly yields observed from January 1, 1986 to December 31, 2009 (1252 observations) and with residual maturities from 1 to 9 years.

3.F Parameters Estimates

U.S. – U.K.

$\Lambda_{\mathcal{B}} \times 10^{-3}$

$\Lambda_{c,1}^{(1)}$	$\Lambda_{c,1}^{(2)}$	$\Lambda_{c,2}^{(1)}$	$\Lambda_{c,2}^{(2)}$	$\Lambda_{l}^{(1)}$			$\Lambda_{l}^{(2)}$		
0.9057**	0.4906**	-0.0543*	-1.0331**	-0.8897**	-0.3545**	-0.0471**	1.3687**	0.5412**	0.0426**
(15.24)	(4.23)	(-1.94)	(-7.15)	(-20.29)	(-20.02)	(-11.48)	(16.93)	(16.20)	(9.06)
1.1490**	0.6400**	0.0145	-0.8467**	-0.8104**	-0.0110**	0.0800**	1.3282**	0.0800**	0.0307**
(19.64)	(6.91)	(0.62)	(-7.59)	(-24.56)	(-3.17)	(12.94)	(20.47)	(12.60)	(8.80)
1.2921**	0.6708**	0.0545	-0.6077**	-0.6830**	0.1471**	0.0566**	1.3312**	-0.1219**	0.0433**
(20.17)	(8.16)	(0.12)	(-6.03)	(-22.67)	(15.54)	(12.73)	(20.62)	(-18.11)	(15.89)
1.3674**	0.6661**	0.0887	-0.3760**	-0.5494**	0.1890**	-0.0042	1.3321**	-0.1787**	0.0546**
(19.98)	(8.89)	(0.75)	(-3.75)	(-19.71)	(22.18)	(-0.97)	(20.42)	(-18.40)	(14.25)
1.4026**	0.6530**	0.1208	-0.1701	-0.4252**	0.1695**	-0.0524**	1.3198**	-0.1633**	0.0495**
(19.55)	(9.40)	(1.36)	(-1.39)	(-17.35)	(20.65)	(-14.68)	(20.48)	(-18.42)	(14.45)
1.4152**	0.6397**	0.1503*	0.0030	-0.3146**	0.1211**	-0.0717**	1.2937**	-0.1169**	0.0227**
(19.01)	(9.78)	(1.93)	(0.79)	(-15.98)	(16.59)	(-15.51)	(20.69)	(-18.97)	(13.55)
1.4155**	0.6269**	0.1770**	0.1412**	-0.2177**	0.0615**	-0.0608**	1.2579**	-0.0638**	-0.0247**
(18.34)	(10.03)	(2.46)	(2.66)	(-16.06)	(12.03)	(-15.37)	(20.89)	(-15.26)	(-16.07)
1.4094**	0.6130**	0.2006**	0.2460**	-0.1331**	-0.0006	-0.0239**	1.2177**	-0.0169**	-0.0875**
(17.52)	(10.14)	(2.93)	(4.15)	(-19.95)	(-0.63)	(-13.45)	(20.94)	(-2.55)	(-16.82)
1.4001**	0.5967**	0.2214**	0.3221**	-0.0589**	-0.0609**	0.0337**	1.1774**	0.0177*	-0.1585**
(16.52)	(10.13)	(3.34)	(5.26)	(-7.86)	(-16.48)	(13.62)	(20.75)	(1.67)	(-16.91)

Φ								Ψ_{12}		
0.9838**	-0.0167*	-0.0124**	0.0207**	-0.0027	-0.0092	-0.0143*	0.0046	0.0809**	0.0222	-0.0387
(162.69)	(-1.90)	(-2.30)	(2.55)	(0.99)	(-0.56)	(-1.70)	(1.31)	(2.16)	(0.21)	(-0.71)
0.0075*	0.9551**	-0.0095	0.0099	-0.0110	0.0043	-0.0202	0.0236	0.0297	0.0619**	-0.0137
(1.73)	(165.20)	(-0.53)	(0.28)	(-1.09)	(1.16)	(-0.18)	(1.43)	(1.09)	(2.20)	(-0.65)
-0.0223**	0.0031	0.9862**	0.0139	0.0554**	-0.0165**	-0.0032	0.0039	0.0145	-0.0884**	-0.0310
(-2.29)	(1.49)	(154.90)	(-0.0320)	(3.67)	(-2.79)	(0.63)	(0.03)	(0.88)	(-2.45)	(-1.14)
-0.0059	0.0239**	0.0249**	0.9374**	-0.0201	0.0039	0.0244**	-0.0159**			
(-1.56)	(2.69)	(2.50)	(144.80)	(-0.78)	(-0.25)	(2.10)	(-2.73)			
0.0113	0.0056	0.0066	0.0111	0.8702**	0.0068	-0.0031	-0.0073			
(-0.08)	(0.70)	(-0.32)	(0.91)	(118.12)	(0.24)	(-0.20)	(-0.69)			
-0.0205	0.0049	0.0049	-0.0125*	-0.0132	0.9789**	0.0110	0.0026			
(-0.82)	(1.04)	(1.59)	(-1.66)	(-0.98)	(181.93)	(0.82)	(-1.62)			
-0.0035	-0.0288	0.0001	0.0095	-0.0116	0.0084	0.9606**	0.0021			
(-0.39)	(0.24)	(1.54)	(-1.18)	(-0.91)	(0.54)	(147.41)	(-1.58)			
0.0065	0.0132	-0.0058	0.0090	0.0026	0.0116	0.0048	0.9742**			
(0.56)	(-0.57)	(-1.32)	(1.11)	(0.02)	(0.66)	(-0.19)	(237.28)			

U.S. – GER.

$\Lambda_{\mathcal{B}} \times 10^{-3}$

$\Lambda_{c,1}^{(1)}$	$\Lambda_{c,1}^{(2)}$	$\Lambda_{c,2}^{(1)}$	$\Lambda_{c,2}^{(2)}$	$\Lambda_{l}^{(1)}$			$\Lambda_{l}^{(2)}$		
-0.5830**	-0.2665**	0.1469**	-0.0173	1.0964**	-0.3662**	0.2188**	-0.7448**	0.5043**	-0.3220**
(-12.07)	(-7.52)	(3.52)	(-0.0721)	(23.77)	(-14.09)	(14.55)	(-14.14)	(13.25)	(-14.25)
-0.9648**	-0.3580**	0.0632	-0.2441**	0.9561**	-0.3349**	0.0110**	-0.8369**	0.4106**	-0.0073*
(-18.74)	(-9.42)	(1.63)	(-5.32)	(23.43)	(-17.23)	(3.32)	(-18.96)	(21.86)	(-1.91)
-1.1296**	-0.4110**	0.0629*	-0.2820**	0.9013**	-0.1975**	-0.1108**	-0.8913**	0.2556**	0.1486**
(-20.12)	(-10.18)	(1.67)	(-5.55)	(21.05)	(-13.37)	(-19.79)	(-21.32)	(12.88)	(13.29)
-1.1867**	-0.4338**	0.0726**	-0.2520**	0.8737**	-0.0414**	-0.1495**	-0.9139**	0.0955**	0.1885**
(-20.22)	(-10.57)	(2.08)	(-4.47)	(18.21)	(-5.45)	(-21.27)	(-22.19)	(5.94)	(19.09)
-1.1986**	-0.4422**	0.0802**	-0.2183**	0.8462**	0.0963**	-0.1314**	-0.9127**	-0.0475	0.1568**
(-19.86)	(-10.73)	(2.53)	(-3.49)	(15.85)	(6.21)	(-20.09)	(-21.96)	(-0.37)	(19.83)
-1.1959**	-0.4459**	0.0864**	-0.2058**	0.8097**	0.2045**	-0.0787**	-0.8952**	-0.1676**	0.0842**
(-19.15)	(-10.69)	(2.99)	(-2.97)	(14.09)	(18.81)	(-16.49)	(-20.89)	(-9.14)	(13.73)
-1.1923**	-0.4497**	0.0933**	-0.2207**	0.7624 **	0.2830**	-0.0062**	-0.8673**	-0.2655**	-0.0098
(-18.12)	(-10.47)	(3.46)	(-2.88)	(12.73)	(23.74)	(-2.33)	(-19.28)	(-17.32)	(-0.37)
-1.1932**	-0.4555**	0.1022**	-0.2608**	0.7057**	0.3358**	0.0768**	-0.8329**	-0.3443**	-0.1133**
(-16.85)	(-10.07)	(3.89)	(-3.04)	(11.59)	(22.32)	(13.77)	(-17.46)	(-16.87)	(-9.26)
-1.1999**	-0.4636**	0.1135**	-0.3215**	0.6419**	0.3677**	0.1644**	-0.7951**	-0.4076**	-0.2193**
(-15.50)	(-9.54)	(4.21)	(-3.29)	(10.54)	(19.53)	(18.01)	(-15.63)	(-14.34)	(-13.16)

Φ								Ψ_{12}		
0.9610**	-0.0068	-0.0283*	-0.0239	0.0104**	-0.0008	0.0088	0.0146	-0.2381**	0.0346	-0.0449
(128.49)	(-0.47)	(-1.95)	(-1.42)	(1.96)	(-0.41)	(1.45)	(1.04)	(-5.48)	(-0.96)	(-1.06)
-0.0084	0.8113**	0.0059	0.0082	0.0063	0.0045	0.0168	-0.0110	-0.0448	-0.2279**	-0.0017
(0.16)	(83.69)	(1.01)	(1.06)	(-0.14)	(1.15)	(1.38)	(-1.25)	(-0.85)	(-6.13)	(0.23)
-0.0662**	-0.0051	0.9443**	-0.0502**	0.0258	0.0129**	-0.0009	-0.0128**	0.0343	-0.1104**	-0.0656*
(-3.57)	(-0.37)	(114.86)	(-2.88)	(-0.27)	(3.18)	(-1.35)	(-1.99)	(0.52)	(-2.67)	(-1.78)
-0.0074**	0.0210	-0.0213**	0.9725**	-0.0091**	-0.0145**	0.0034	0.0194**			
(3.66)	(1.20)	(2.43)	(149.37)	(-2.87)	(-4.13)	(1.10)	(3.07)			
0.0199	0.0192	0.0449**	0.0246	0.9329**	-0.0017	-0.0030	0.0066			
(0.34)	(0.29)	(2.44)	(1.49)	(118.10)	(0.92)	(-0.74)	(-0.08)			
0.0450 **	0.0224	0.0113	0.0200*	-0.0057	0.9777**	-0.0142	0.0112			
(2.17)	(-0.84)	(0.27)	(1.66)	(0.80)	(217.36)	(-1.24)	(-0.74)			
0.0035	0.0278	0.0104	-0.0008	-0.0056	-0.0005	0.9924**	0.0018			
(-1.30)	(-0.11)	(0.47)	(-0.90)	(-0.45)	(1.43)	(223.13)	(0.26)			
-0.0004	-0.0240	-0.0067	-0.0102	0.0001	0.0120	-0.0095	0.9521**			
(-0.07)	(-1.30)	(-1.04)	(-0.83)	(0.26)	(-0.08)	(-1.08)	(182.31)			

Table 3.10: We report the maximum likelihood estimates of $\Lambda_{\mathcal{B}}$, Φ and Ψ_{η} parameters and the associated bootstrap t-values (in parenthesis). We use Nonparametric Monte Carlo block stationary bootstrap [see Stoffer and Wall (1991) and Politis and Romano (1994); the optimal block sizes are chosen following Politis and White (2004) and Patton, Politis, and White (2009)]. One and two asterisks denote statistical significance at 10% and 5% levels, respectively. The (statistically significant) parameter estimates of μ and Ω are not reported for ease of presentation.

U.S. – U.K. – GER.														
$\Lambda_{\mathcal{B}} \times 10^{-3}$														
$\Lambda_{c,1}^{(1)}$	$\Lambda_{c,1}^{(2)}$	$\Lambda_{c,1}^{(3)}$	$\Lambda_{c,2}^{(1)}$	$\Lambda_{c,2}^{(2)}$	$\Lambda_{c,2}^{(3)}$		$\Lambda_{l}^{(1)}$			$\Lambda_{l}^{(2)}$			$\Lambda_{l}^{(3)}$	
-0.5913**	-0.1354**	-0.2318**	-0.1468**	-0.3759**	0.0496	1.0749**	-0.4659**	-0.0860**	1.7600**	-0.7916**	-0.2316**	-0.8004**	0.4190**	0.3181**
(-10.96)	(-2.12)	(-6.42)	(-4.64)	(-3.99)	(1.67)	(22.11)	(-20.73)	(-11.96)	(14.90)	(-16.57)	(-14.48)	(-15.46)	(11.10)	(13.91)
-0.8316**	-0.3804**	-0.3505**	-0.0797**	-0.0225	0.3074**	1.1051**	-0.1503**	0.1105**	1.6198**	-0.4908**	0.0250**	-0.8315**	0.3738**	0.0180**
(-15.31)	(-6.20)	(-8.83)	(-3.15)	(0.75)	(8.77)	(25.81)	(-14.55)	(15.07)	(18.18)	(-17.02)	(3.67)	(-19.89)	(22.29)	(2.57)
-0.9812**	-0.4874**	-0.4248**	-0.0933**	0.0638**	0.3769**	1.0698**	0.0475**	0.1117**	1.5216**	-0.2059**	0.1428**	-0.8507**	0.2353**	-0.1395**
(-16.36)	(-8.02)	(-9.94)	(-3.69)	(2.64)	(9.86)	(25.18)	(3.23)	(16.02)	(20.54)	(-9.17)	(17.29)	(-22.32)	(13.74)	(-12.07)
-1.0781**	-0.5401**	-0.4651**	-0.1232**	0.0754**	0.3714**	0.9933**	0.1450**	0.0471**	1.4383**	0.0189	0.1662**	-0.8525**	0.0778**	-0.1861**
(-16.73)	(-8.96)	(-10.68)	(-4.74)	(2.90)	(9.61)	(23.83)	(19.36)	(12.46)	(21.53)	(1.20)	(18.99)	(-22.37)	(5.60)	(-19.39)
-1.1449**	-0.5720**	-0.4890**	-0.1539**	0.0681**	0.3523**	0.8976**	0.1769**	-0.0200**	1.3571**	0.1945**	0.1315**	-0.8363**	-0.0651	-0.1605**
(-16.93)	(-9.65)	(-11.18)	(-5.78)	(2.66)	(9.04)	(22.58)	(24.54)	(-7.63)	(21.52)	(9.17)	(18.89)	(-20.89)	(-1.40)	(-19.94)
-1.1945**	-0.5960**	-0.5077**	-0.1824**	0.0602**	0.3454**	0.7958**	0.1693**	-0.0654**	1.2740**	0.3342**	0.0632**	-0.8050**	-0.1826**	-0.0919**
(-17.08)	(-10.22)	(-11.38)	(-6.67)	(2.36)	(8.13)	(21.74)	(21.53)	(-16.65)	(20.79)	(14.30)	(14.22)	(-18.90)	(-10.35)	(-13.34)
-1.2339**	-0.6155**	-0.5261**	-0.2088**	0.0578**	0.3586**	0.6951**	0.1391**	-0.0831**	1.1900**	0.4483**	-0.0217**	-0.7626**	-0.2740**	0.0006
(-17.12)	(-10.73)	(-11.27)	(-7.29)	(2.23)	(7.16)	(21.44)	(18.20)	(-17.37)	(19.58)	(16.70)	(-6.54)	(-16.87)	(-18.79)	(-0.72)
-1.2669**	-0.6300**	-0.5462**	-0.2338**	0.0615**	0.3916**	0.5988**	0.0965**	-0.0751**	1.1084**	0.5444**	-0.1104**	-0.7128**	-0.3425**	0.1049**
(-16.92)	(-11.12)	(-10.89)	(-7.58)	(2.27)	(6.46)	(21.82)	(15.60)	(-17.46)	(18.01)	(17.22)	(-15.36)	(-14.94)	(-18.25)	(8.71)
-1.2959**	-0.6385**	-0.5686**	-0.2578**	0.0698**	0.4414**	0.5083**	0.0476**	-0.0457**	1.0322**	0.6275**	-0.1939**	-0.6584**	-0.3921**	0.2138**
(-16.43)	(-11.34)	(-10.36)	(-7.56)	(2.41)	(6.02)	(23.07)	(12.90)	(-17.12)	(16.26)	(16.88)	(-17.36)	(-13.14)	(-14.99)	(13.53)

Φ											$\Psi_{1,2}$			$\Psi_{1,3}$			$\Psi_{2,3}$		
0.9612**	0.0319	-0.0192**	-0.0248**	0.0206*	-0.0184*	-0.0207**	-0.0024	-0.0022	0.0061	-0.0008	0.2547**	-0.0879**	-0.0225	-0.2461**	0.1393**	0.0150	-0.3222**	0.0699**	-0.0080
(119.15)	(1.45)	(-3.06)	(-3.11)	(1.71)	(-1.80)	(-2.73)	(-0.75)	(-0.26)	(-0.23)	(0.96)	(6.40)	(-2.06)	(-0.85)	(-5.31)	(4.29)	(0.48)	(-8.50)	(2.46)	(-0.30)
0.0217	0.8170**	0.0108	-0.0010	0.0152*	0.0030	0.0131*	0.0278**	-0.0244**	-0.0425**	-0.0202*	-0.0088	-0.0325	0.0033	-0.0564	-0.0063	-0.0755*	-0.0003	-0.1558**	-0.0060
(0.66)	(64.03)	(0.07)	(-0.78)	(1.80)	(0.37)	(1.83)	(2.39)	(-2.06)	(-2.03)	(-1.77)	(-0.80)	(-0.23)	(0.32)	(-1.05)	(-0.52)	(-1.90)	(-1.39)	(-4.63)	(-0.11)
-0.0324**	0.0287	0.9724**	-0.0293	0.0598**	0.0121*	-0.0126	0.0096*	0.0279**	0.0003	0.0375**	0.0046	-0.0300	0.1112**	-0.0085	0.0643*	-0.0261	-0.0713	0.0172	-0.0765**
(-2.35)	(0.85)	(119.53)	(-1.19)	(3.41)	(1.75)	(-0.47)	(1.65)	(3.90)	(-0.66)	(3.32)	(0.73)	(-0.64)	(3.63)	(-0.71)	(1.87)	(-0.82)	(-1.62)	(1.00)	(-2.15)
-0.0092	0.0282	-0.0368*	0.9549**	0.0217	-0.0108**	-0.0176	-0.0092**	-0.0072**	0.0010	0.0059									
(0.74)	(0.71)	(-1.78)	(138.45)	(0.06)	(-2.64)	(-1.43)	(-2.66)	(-2.84)	(0.34)	(-1.29)									
0.0173	0.0093	0.0036	-0.0022	0.8845**	0.0122	0.0001	0.0147	0.0096*	-0.0032	0.0167**									
(-0.47)	(0.74)	(-1.47)	(-1.47)	(122.11)	(0.99)	(-0.77)	(0.92)	(1.68)	(-0.44)	(2.15)									
-0.0217	0.0117	0.0159	0.0081	0.0050	0.9725**	-0.0041	-0.0133*	0.0022*	0.0030	0.0074									
(-0.36)	(-0.19)	(1.51)	(1.38)	(-0.12)	(129.07)	(0.48)	(1.74)	(1.75)	(-0.01)	(0.62)									
-0.0104	0.0498	-0.0120	-0.0007	-0.0027	-0.0058	0.9703**	-0.0156	0.0041	0.0053	0.0182									
(0.60)	(0.58)	(-0.12)	(0.33)	(-0.47)	(-0.02)	(129.27)	(-1.07)	(0.64)	(1.47)	(0.15)									
-0.0045	0.0183	0.0081	0.0081	0.0116	-0.0326*	0.0055	0.9577**	-0.0036	0.0073	0.0062									
(-0.58)	(0.24)	(-0.50)	(-0.63)	(1.04)	(-1.65)	(0.08)	(140.83)	(0.09)	(0.30)	(0.77)									
0.0285	-0.0483	-0.0010	0.0059	-0.0251	-0.0181	0.0059	-0.0126**	0.9620**	-0.0251	-0.0283									
(1.41)	(0.57)	(-0.66)	(0.14)	(-1.60)	(-1.41)	(0.33)	(-2.05)	(114.94)	(-0.38)	(-0.10)									
0.0058	-0.0588	0.0040	-0.0015	-0.0054	-0.0053	-0.0292**	0.0083	-0.0213	0.9602**	-0.0009									
(-1.12)	(-0.05)	(0.54)	(0.02)	(0.12)	(0.17)	(-2.92)	(0.88)	(-0.18)	(143.61)	(0.68)									
-0.0042	-0.0390	0.0049	0.0017	-0.0073	-0.0156	0.0024	-0.0064	-0.0261	0.0031	0.9416**									
(-0.45)	(-1.49)	(0.35)	(-0.05)	(-0.44)	(-0.23)	(-1.15)	(0.33)	(-1.46)	(-0.64)	(128.77)									

Table 3.11: We report the maximum likelihood estimates of $\Lambda_{\mathcal{B}}$, Φ and Ψ_η parameters and the associated bootstrap t-values (in parenthesis). We use Nonparametric Monte Carlo block stationary bootstrap [see Stoffer and Wall (1991) and Politis and Romano (1994); the optimal block sizes are chosen following Politis and White (2004) and Patton, Politis, and White (2009)]. One and two asterisks denote statistical significance at 10% and 5% levels, respectively. The (statistically significant) parameter estimates of μ and Ω are not reported for ease of presentation.

U.S. – U.K. – GER. – JAP.

$\Lambda_{\mathcal{B}} \times 10^{-3}$

$\Lambda_{c,1}^{(1)}$	$\Lambda_{c,1}^{(2)}$	$\Lambda_{c,1}^{(3)}$	$\Lambda_{c,1}^{(4)}$	$\Lambda_{c,2}^{(1)}$	$\Lambda_{c,2}^{(2)}$	$\Lambda_{c,2}^{(3)}$	$\Lambda_{c,2}^{(4)}$	$\Lambda_{l}^{(1)}$			$\Lambda_{l}^{(2)}$			$\Lambda_{l}^{(3)}$			$\Lambda_{l}^{(4)}$		
0.0691**	-0.3359**	0.0633**	-0.0340	0.0030	-0.0469	0.0173	0.0337	1.1204**	0.6323**	-0.2475**	1.6742**	-0.9017**	-0.2467**	-0.8160**	-0.4650**	-0.3035**	-0.6280**	0.4677**	-0.2535**
(2.32)	(-2.94)	(1.72)	(-0.59)	(0.86)	(-0.30)	(0.34)	(1.00)	(21.23)	(24.04)	(-18.41)	(14.28)	(-20.29)	(-16.04)	(-16.27)	(-11.65)	(-12.28)	(-11.77)	(14.65)	(-15.49)
0.2445**	0.0765**	0.3295**	0.0497**	-0.0551	0.0005	0.0756	-0.0079	1.3004**	0.4143**	0.0588**	1.6333**	-0.5350**	0.0378**	-0.8706**	-0.4042**	-0.0217**	-0.6837**	0.4163**	-0.0580**
(5.45)	(2.49)	(6.98)	(2.49)	(-0.13)	(0.06)	(0.99)	(-0.37)	(25.21)	(26.12)	(8.16)	(18.24)	(-20.40)	(5.86)	(-19.95)	(-22.30)	(-2.69)	(-14.69)	(19.54)	(-5.82)
0.2878**	0.2027**	0.4135**	0.0843**	-0.0279	-0.0118	0.0855	-0.0180	1.4007**	0.2122**	0.1621**	1.5757**	-0.2281**	0.1554**	-0.9148**	-0.2602**	0.1325**	-0.7569**	0.2865**	0.0839**
(5.91)	(4.50)	(7.70)	(3.54)	(0.32)	(-0.20)	(0.94)	(-0.69)	(25.18)	(15.19)	(20.94)	(20.98)	(-10.25)	(19.62)	(-21.93)	(-13.24)	(11.77)	(-17.47)	(16.09)	(12.34)
0.2927**	0.2360**	0.4140**	0.0937**	0.0065	-0.0306	0.0743	-0.0158	1.4357**	0.0474**	0.1530**	1.5126**	0.0053	0.1729**	-0.9379**	-0.1019**	0.1823**	-0.8134**	0.1404**	0.1527**
(5.85)	(4.95)	(7.15)	(3.73)	(0.69)	(-0.60)	(0.84)	(-0.55)	(24.20)	(6.37)	(22.22)	(22.30)	(0.87)	(21.12)	(-22.36)	(-6.01)	(18.45)	(-18.78)	(6.89)	(20.70)
0.2903**	0.2432**	0.3951**	0.0919**	0.0184	-0.0472	0.0594	-0.0100	1.4340**	-0.0813**	0.0954**	1.4452**	0.1861**	0.1321**	-0.9414**	0.0423	0.1611**	-0.8472**	0.0019	0.1583**
(5.69)	(4.99)	(6.33)	(3.63)	(0.68)	(-1.01)	(0.72)	(-0.27)	(22.87)	(-5.72)	(21.22)	(22.64)	(9.07)	(20.93)	(-21.79)	(0.03)	(17.89)	(-19.08)	(-0.66)	(21.37)
0.2900**	0.2473**	0.3859**	0.0868**	0.0010	-0.0587	0.0483	-0.0064	1.4123**	-0.1800**	0.0236**	1.3744**	0.3309**	0.0588**	-0.9294**	0.1637**	0.0955**	-0.8603**	-0.1220**	0.1141**
(5.53)	(5.01)	(5.59)	(3.47)	(0.38)	(-1.34)	(0.59)	(-0.08)	(21.47)	(-22.72)	(9.76)	(22.24)	(14.74)	(15.08)	(-20.70)	(8.45)	(12.28)	(-18.80)	(-7.72)	(17.45)
0.2937**	0.2540**	0.3961**	0.0826**	-0.0433	-0.0622	0.0431	-0.0096	1.3806**	-0.2556**	-0.0462**	1.3024**	0.4505**	-0.0296**	-0.9066**	0.2619**	0.0038	-0.8564**	-0.2310**	0.0312**
(5.25)	(5.18)	(5.06)	(3.39)	(-0.15)	(-1.50)	(0.50)	(-0.14)	(20.17)	(-26.06)	(-10.54)	(21.25)	(14.75)	(-9.20)	(-19.38)	(17.46)	(1.36)	(-18.10)	(-14.98)	(3.95)
0.3010**	0.2617**	0.4264**	0.0815**	-0.1092	-0.0565	0.0437	-0.0225	1.3446**	-0.3136**	-0.1077**	1.2315**	0.5520**	-0.1207**	-0.8767**	0.3398**	-0.1019**	-0.8386**	-0.3269**	-0.0823**
(4.78)	(5.45)	(4.75)	(3.38)	(-0.79)	(-1.49)	(0.47)	(-0.60)	(19.06)	(-23.63)	(-16.65)	(19.79)	(18.65)	(-17.55)	(-17.96)	(16.19)	(-8.59)	(-16.87)	(-18.04)	(-11.21)
0.3110**	0.2668**	0.4742**	0.0845**	-0.1911	-0.0432	0.0494	-0.0475	1.3073**	-0.3583**	-0.1589**	1.1641**	0.6397**	-0.2059**	-0.8422**	0.4008**	-0.2138**	-0.8099**	-0.4125**	-0.2210**
(4.18)	(5.70)	(4.59)	(3.27)	(-1.34)	(-1.26)	(0.48)	(-1.37)	(18.13)	(-21.69)	(-18.57)	(18.07)	(18.55)	(-19.27)	(-16.52)	(13.03)	(-13.80)	(-15.01)	(-15.47)	(-17.29)

Φ

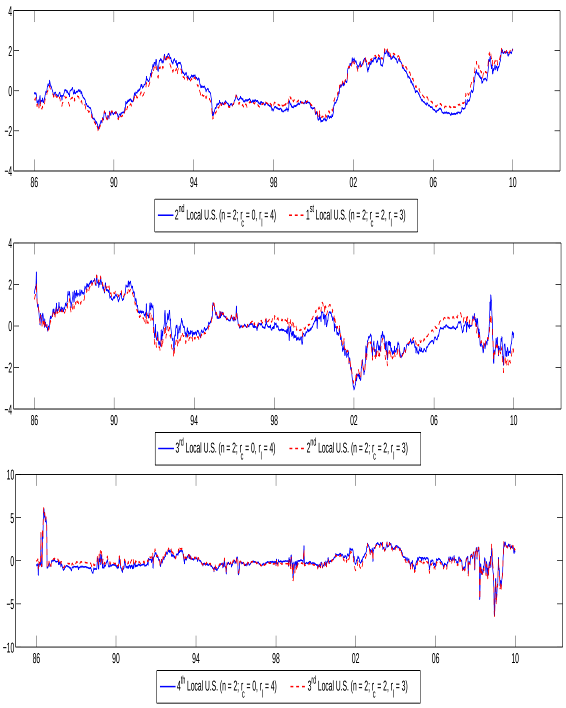
0.7992**	-0.0220	0.0304**	-0.0095	-0.0017	0.0033	-0.0022	0.0334*	0.0046	0.0369	0.0258	-0.0198	-0.0068	0.0064
(56.67)	(-0.65)	(2.21)	(-1.03)	(0.41)	(1.22)	(1.33)	(1.74)	(1.01)	(0.43)	(1.38)	(-1.61)	(-0.02)	(1.42)
-0.0069	0.8632**	0.0046	0.0006	0.0310	-0.0223	-0.0216	-0.0017	-0.0115	0.0112	0.0127	-0.0079	-0.0094	-0.0066
(-0.06)	(94.06)	(0.03)	(0.95)	(1.58)	(-1.50)	(-0.92)	(-0.40)	(-1.37)	(0.55)	(0.61)	(-0.46)	(-0.57)	(-0.76)
0.0444	-0.0108	0.9739**	0.0127	0.0174*	0.0083*	-0.0171	0.0050	0.0175**	0.0099	-0.0388*	-0.0146	-0.0059	-0.0049
(0.44)	(0.23)	(110.49)	(0.60)	(1.81)	(1.70)	(0.25)	(0.81)	(2.87)	(0.68)	(-1.78)	(-1.24)	(0.13)	(0.86)
-0.0021	-0.0378*	0.0252	0.9803**	-0.0105	0.0107	-0.0066	0.0141*	0.0220**	-0.0012	-0.0102*	-0.0030	-0.0114**	0.0061
(0.90)	(-1.67)	(1.50)	(158.41)	(0.21)	(1.48)	(-1.00)	(1.78)	(3.38)	(0.51)	(-1.84)	(-1.07)	(-1.96)	(-0.15)
0.0238	-0.0023	-0.0183*	-0.0068	0.9373**	-0.0043	-0.0238	0.0092	-0.0008	0.0106	-0.0270	-0.0039	0.0011	0.0046
(1.24)	(-0.68)	(-1.73)	(0.11)	(157.85)	(-1.29)	(-1.54)	(-0.20)	(-1.32)	(1.35)	(-1.42)	(-0.21)	(1.06)	(-0.10)
0.0174	0.0167	0.0182*	0.0048	0.0211	0.9672**	-0.0142	-0.0048	0.0005**	0.0160	-0.0262	-0.0101	0.0099*	-0.0017
(-0.13)	(0.78)	(1.66)	(0.15)	(1.51)	(93.13)	(0.78)	(1.52)	(2.24)	(0.60)	(-1.30)	(-0.25)	(1.83)	(1.31)
0.0332	0.0105	-0.0243	0.0005	0.0045	-0.0241	0.9365**	-0.0061	-0.0050	0.0189	-0.0279	-0.0303	-0.0172	-0.0074
(0.07)	(0.38)	(-0.79)	(-0.17)	(0.20)	(-0.19)	(79.12)	(-1.13)	(0.73)	(-0.27)	(-0.52)	(-1.37)	(-0.34)	(-0.93)
0.0201	-0.0080	0.0087	0.0021	0.0160	-0.0354	0.0054	0.9581**	-0.0084	-0.0037	-0.0080	0.0015	0.0025	-0.0006
(0.50)	(-0.74)	(-0.27)	(0.79)	(0.59)	(-1.63)	(-0.31)	(121.01)	(-0.65)	(0.43)	(-0.63)	(-0.33)	(0.23)	(-0.85)
-0.0482	0.0033	-0.0154**	-0.0006	-0.0171	-0.0304*	-0.0031	-0.0191**	0.9542**	0.0077	0.0511	0.0018	-0.0264*	-0.0300*
(0.37)	(0.41)	(-2.32)	(-0.02)	(-1.44)	(-1.66)	(-0.32)	(-2.14)	(100.32)	(-0.50)	(0.94)	(0.70)	(-1.86)	(-1.67)
0.0509	0.0067	-0.0101	-0.0006	0.0048	0.0331	0.0620**	-0.0235*	0.0245	0.9458**	0.0093	0.0185	0.0052	0.0231
(-0.14)	(0.26)	(-0.96)	(-0.38)	(-0.33)	(1.38)	(3.35)	(-1.83)	(1.41)	(89.70)	(1.25)	(1.24)	(-0.15)	(1.47)
0.0488*	0.0153	-0.0149	0.0107	0.0143	0.0166	-0.0103	0.0193	0.0284	0.0258	0.9038**	-0.0056	0.0325*	0.0245
(1.80)	(0.37)	(-1.52)	(1.52)	(0.95)	(0.60)	(0.87)	(0.02)	(1.12)	(0.67)	(84.86)	(-0.01)	(1.90)	(1.05)
-0.0247	-0.0028	-0.0158	-0.0004	-0.0201	-0.0161	-0.0133	0.0132	-0.0186**	0.0015	0.0396**	0.9793**	-0.0341**	-0.0234**
(-0.39)	(-0.11)	(-1.45)	(0.44)	(-1.13)	(-0.99)	(0.78)	(0.06)	(-2.48)	(-1.35)	(2.96)	(109.35)	(-2.44)	(-2.81)
-0.0296	-0.0026	0.0098	-0.0007	-0.0131	-0.0212	-0.0126	-0.0109	-0.0057	0.00004	0.0121	-0.0066	0.9714**	-0.0213
(-0.29)	(0.03)	(1.28)	(0.17)	(-0.47)	(-1.27)	(-0.01)	(-0.94)	(-0.19)	(0.36)	(-0.48)	(0.79)	(125.97)	(0.12)
0.0049	0.0146	-0.0287*	0.0226	0.0070	-0.0059	0.0043	0.0091	-0.0212	-0.0027	0.0165	0.0055	-0.0266	0.9316**
(0.18)	(-0.004)	(-1.69)	(1.40)	(0.09)	(0.35)	(0.34)	(1.03)	(-0.83)	(-0.49)	(0.62)	(0.82)	(-0.94)	(108.99)

$\Psi_{1,2}$			$\Psi_{1,3}$			$\Psi_{1,4}$			$\Psi_{2,3}$			$\Psi_{2,4}$			$\Psi_{3,4}$		
0.4001**	0.1324**	-0.0884**	-0.4144**	0.0671	-0.0117	-0.1619**	-0.0487*	0.0054	-0.4007**	-0.0234	0.0034	-0.1504**	-0.0261	-0.0824**	0.2500**	0.0287	0.0123
(13.12)	(3.79)	(-3.94)	(-12.66)	(1.30)	(-0.15)	(-4.68)	(-1.84)	(-1.05)	(-11.45)	(-1.20)	(0.10)	(-3.79)	(-1.08)	(-3.11)	(6.28)	(1.18)	(0.50)
0.0048	-0.2008**	0.0618**	0.0434	-0.2296**	-0.0368	0.0499	0.0373	0.0203	-0.0879**	0.2301**	-0.0009	-0.0826**	-0.0403**	-0.0019	0.0033	-0.0893**	-0.0132
(1.01)	(-5.21)	(2.11)	(0.77)	(-6.35)	(-1.10)	(0.84)	(0.81)	(0.94)	(-3.54)	(6.40)	(0.02)	(-3.48)	(-2.01)	(-0.66)	(0.69)	(-2.30)	(-0.52)
-0.0138	-0.1319**	0.0861**	-0.0303	-0.1254**	0.0642*	0.0291	0.0293	0.0254	-0.0466	-0.0514**	0.0775**	-0.0128	0.0086	-0.0098	0.0829	0.0440*	0.0060
(-0.14)	(-2.88)	(2.61)	(-0.76)	(-3.14)	(1.82)	(0.50)	(0.35)	(0.81)	(-0.41)	(-2.47)	(2.64)	(0.56)	(0.36)	(0.10)	(1.51)	(1.68)	(1.07)

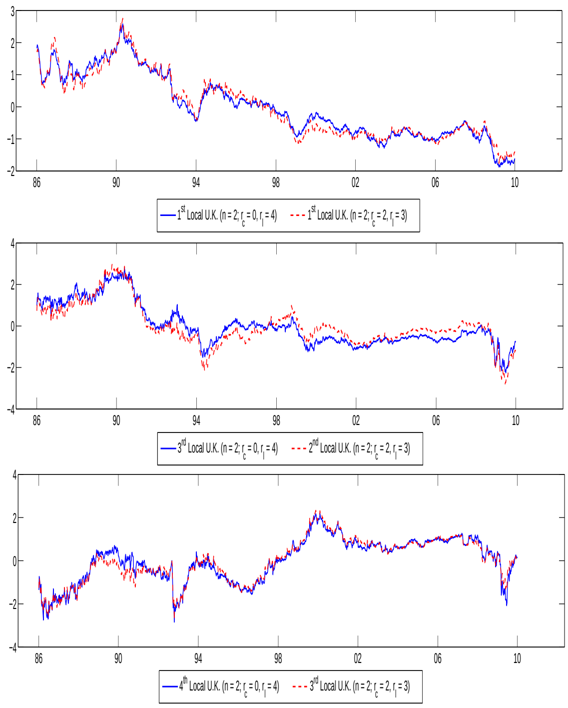
Table 3.12: We report the maximum likelihood estimates of $\Lambda_{\mathcal{B}}$, Φ and Ψ_η parameters and the associated bootstrap t-values (in parenthesis). We use Nonparametric Monte Carlo block stationary bootstrap [see Stoffer and Wall (1991) and Politis and Romano (1994); the optimal block sizes are chosen following Politis and White (2004) and Patton, Politis, and White (2009)]. One and two asterisks denote statistical significance at 10% and 5% levels, respectively. The (statistically significant) parameter estimates of μ and Ω are not reported for ease of presentation.

3.G Smoothed Common and Local Factors

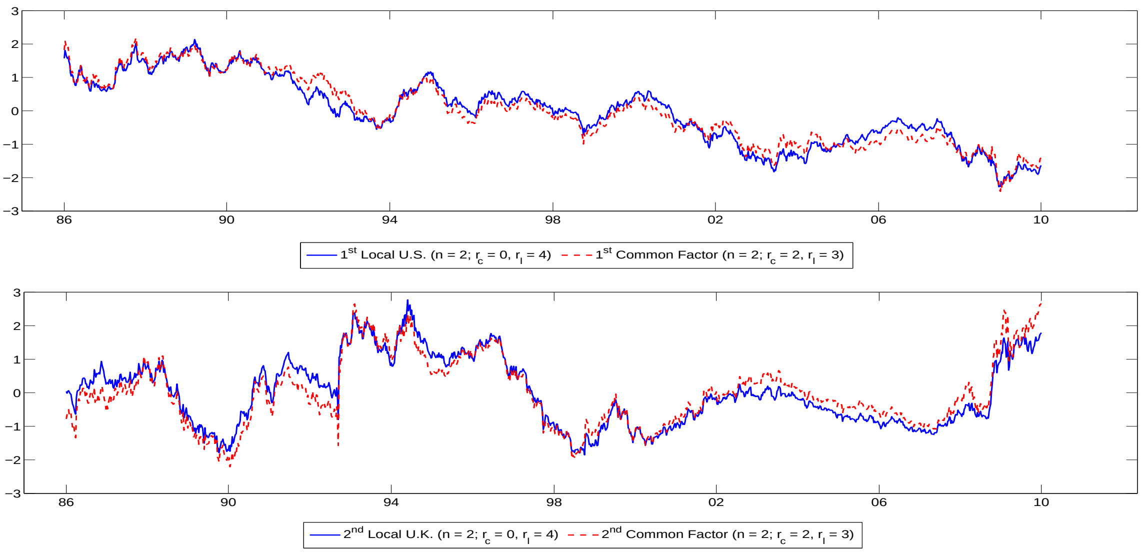
2-country case: U.S.-U.K.



(a) Local *U.S.* factors of $\mathcal{M}_2^{0,4}$ and $\mathcal{M}_2^{2,3}$



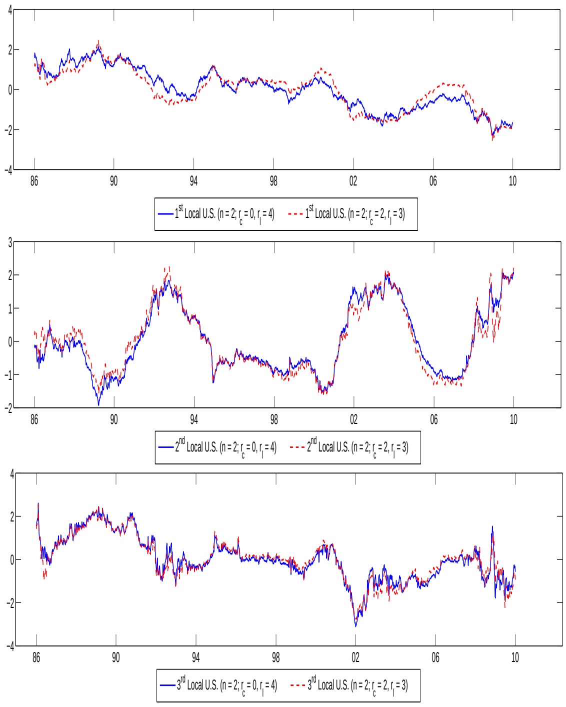
(b) Local *U.K.* factors of $\mathcal{M}_2^{0,4}$ and $\mathcal{M}_2^{2,3}$



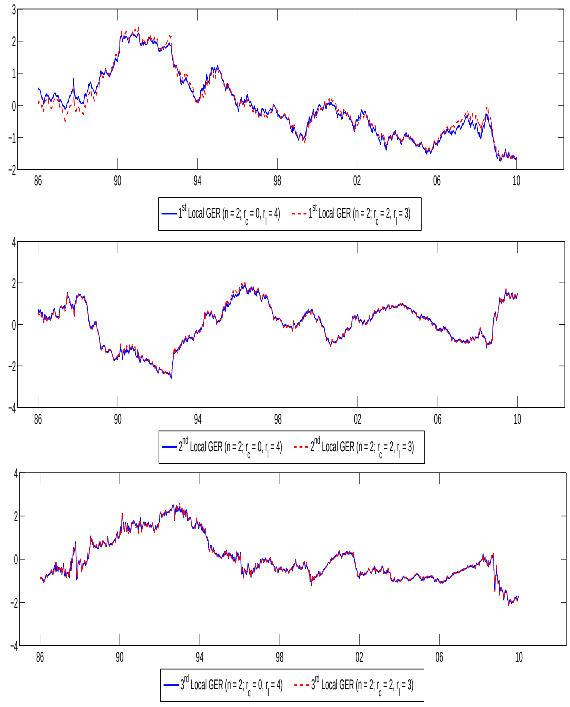
(c) 1^{st} and 2^{nd} common factor of $\mathcal{M}_2^{2,3}$

Fig. 3.2: Smoothed factors in the 2-country *U.S.-U.K.* case when $(r_c = 0, r_\ell = 4)$ and $(r_c = 2, r_\ell = 3)$.

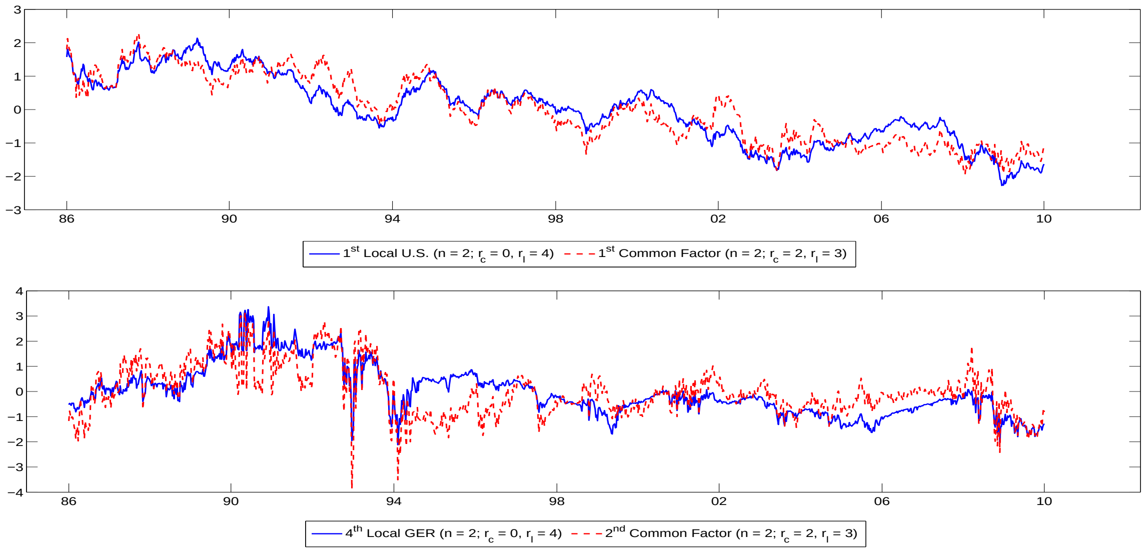
2-country case: U.S.-GER.



(a) Local *U.S.* factors of $\mathcal{M}_2^{0,4}$ and $\mathcal{M}_2^{2,3}$



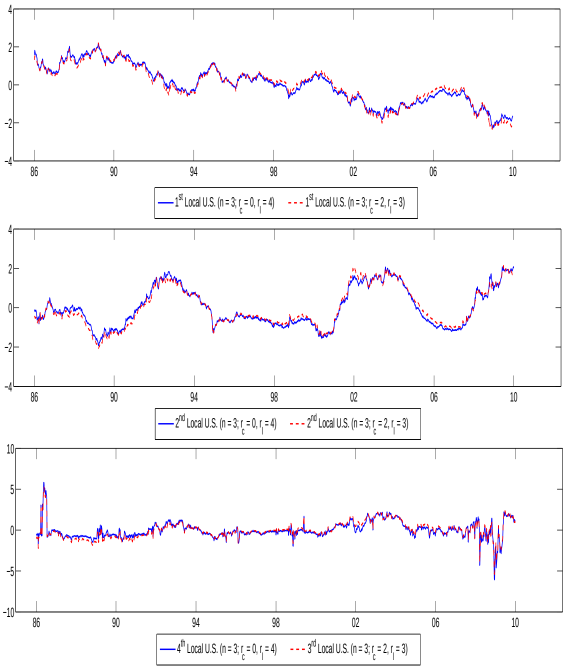
(b) Local *GER* factors of $\mathcal{M}_2^{0,4}$ and $\mathcal{M}_2^{2,3}$



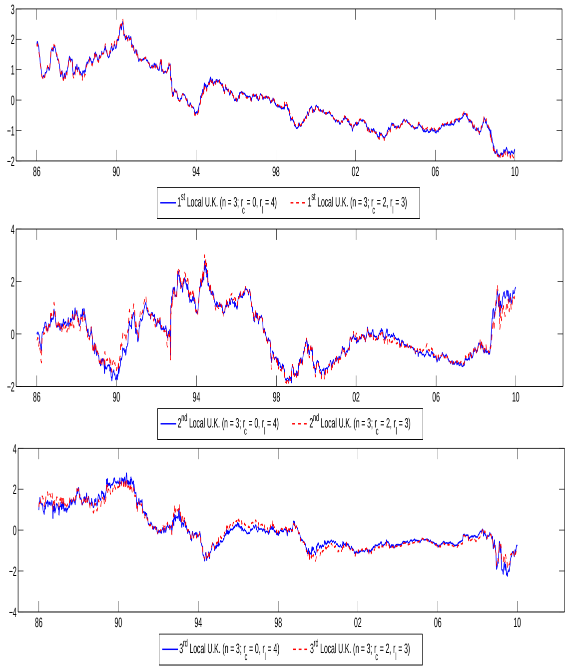
(c) 1^{st} and 2^{nd} common factor of $\mathcal{M}_2^{2,3}$

Fig. 3.3: Smoothed factors in the 2-country *U.S.-GER* case when $(r_c = 0, r_\ell = 4)$ and $(r_c = 2, r_\ell = 3)$.

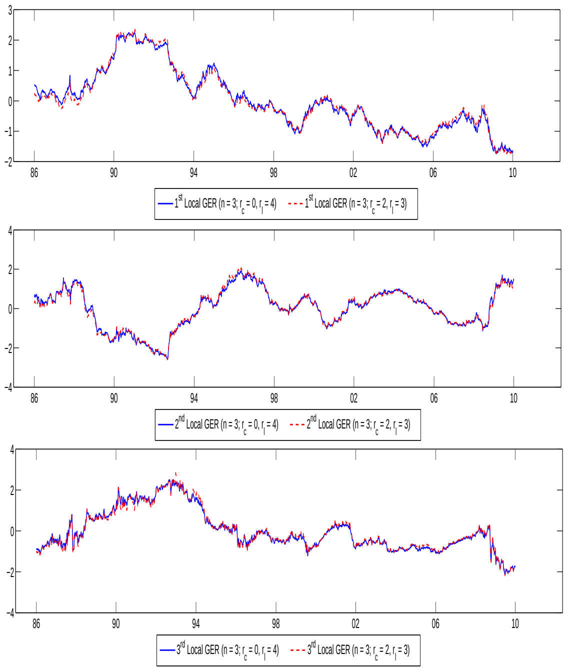
3-country case: U.S.-U.K.-GER.



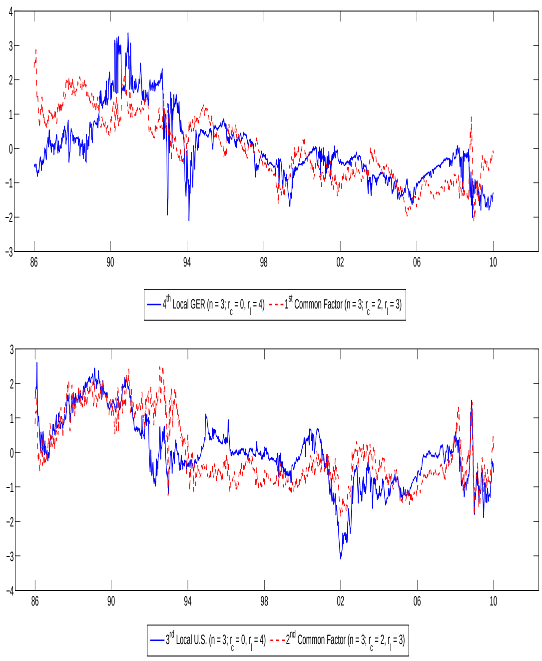
(a) Local *U.S.* factors of $\mathcal{M}_3^{0,4}$ and $\mathcal{M}_3^{2,3}$



(b) Local *U.K.* factors of $\mathcal{M}_3^{0,4}$ and $\mathcal{M}_3^{2,3}$



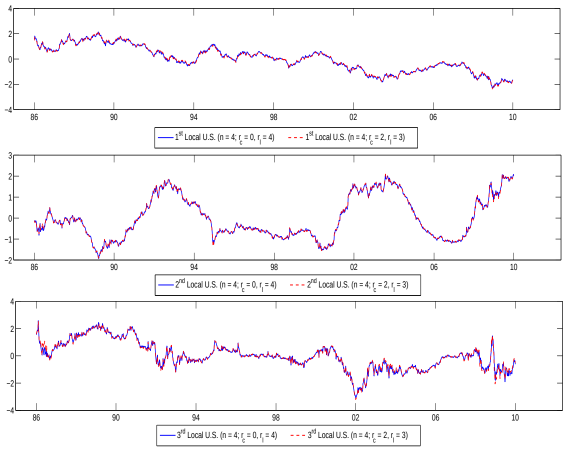
(c) Local *GER* factors of $\mathcal{M}_3^{0,4}$ and $\mathcal{M}_3^{2,3}$



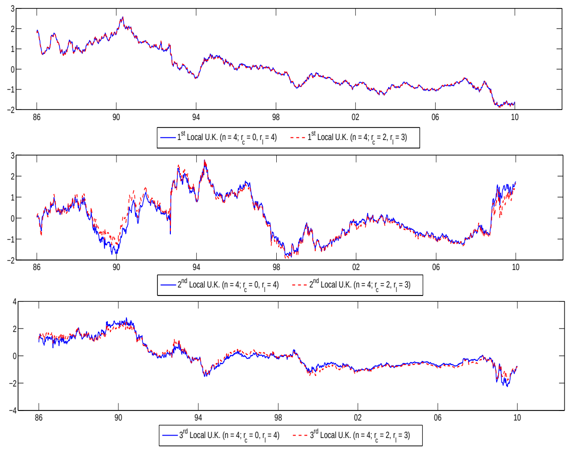
(d) 1^{st} and 2^{nd} common factor of $\mathcal{M}_3^{2,3}$

Fig. 3.4: Smoothed factors in the 3-country case *U.S.* – *U.K.* – *GER* when $(r_c = 0, r_\ell = 4)$ and $(r_c = 2, r_\ell = 3)$.

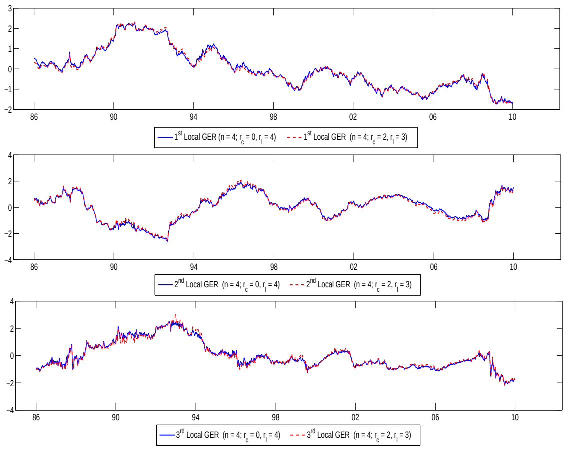
4-country case: U.S.-U.K.-GER-JAP.



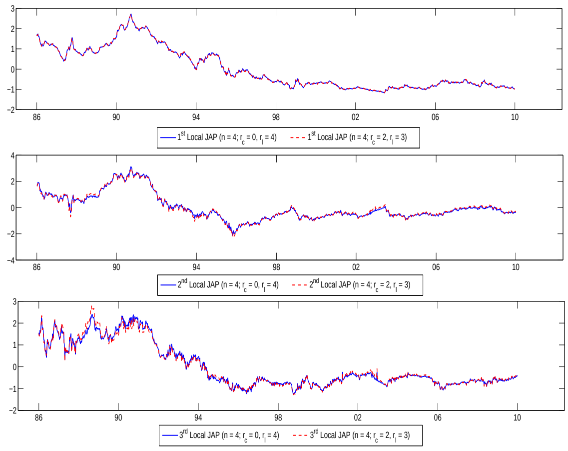
(a) Local *U.S.* factors of $\mathcal{M}_4^{0,4}$ and $\mathcal{M}_4^{2,3}$



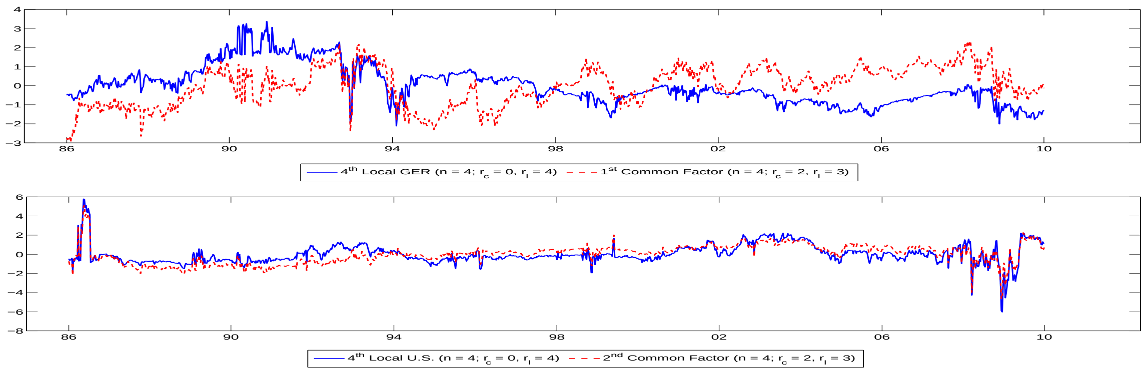
(b) Local *U.K.* factors of $\mathcal{M}_4^{0,4}$ and $\mathcal{M}_4^{2,3}$



(c) Local *GER* factors of $\mathcal{M}_4^{0,4}$ and $\mathcal{M}_4^{2,3}$



(d) Local *JAP* factors of $\mathcal{M}_4^{0,4}$ and $\mathcal{M}_4^{2,3}$



(e) 1^{st} and 2^{nd} common factor of $\mathcal{M}_4^{2,3}$

Fig. 3.5: Smoothed factors in the 4-country case when $(r_c = 0, r_\ell = 4)$ and $(r_c = 2, r_\ell = 3)$.

3.H Macroeconomic Interpretation of Factors

	h	$\beta_{1,c}$	$\beta_{2,c}$	$\beta_{1,l}$	$\beta_{2,l}$	$\beta_{3,l}$	R^2	$\beta^*_{1,l}$	$\beta^*_{2,l}$	$\beta^*_{3,l}$	$\beta^*_{4,l}$	R^2
$g_{U.S.,t+h}$	0	0.79	0.08	-2.21**	1.57	2.31	0.28	-1.48	-1.31**	1.58	-2.36	0.28
		(1.04)	(0.06)	(-2.63)	(0.95)	(0.74)	{0.16; 0.60}	(-1.63)	(-2.21)	(0.93)	(-0.73)	{0.15; 0.58}
	6	1.73**	-0.61	-2.76**	4.41**	4.09	0.30	-2.66**	-0.58	4.11**	-3.95	0.28
		(2.42)	(-0.75)	(-2.59)	(2.56)	(1.06)	{0.20; 0.59}	(-2.79)	(-1.12)	(2.41)	(-1.07)	{0.18; 0.56}
	12	1.77**	-1.96**	-2.61**	4.93**	4.08	0.32	-2.82**	0.13	4.54**	-3.84	0.22
		(2.27)	(-2.83)	(-2.45)	(2.96)	(1.21)	{0.22; 0.59}	(-2.58)	(0.25)	(2.61)	(-1.21)	{0.13; 0.53}
	18	1.72	-2.52**	-1.86**	4.07*	4.08**	0.30	-2.60*	0.76	3.61	-3.75**	0.19
		(1.47)	(-2.75)	(-2.03)	(1.72)	(2.37)	{0.20; 0.60}	(-1.79)	(1.40)	(1.38)	(-2.41)	{0.10; 0.47}
	24	1.52	-2.59**	-0.29	1.98	2.12	0.26	-1.79	1.58**	1.44	-1.41	0.18
		(1.38)	(-2.62)	(-0.53)	(1.27)	(1.60)	{0.17; 0.56}	(-1.54)	(2.19)	(0.84)	(-0.87)	{0.07; 0.48}
	30	1.92*	-2.03**	0.65	1.36	-0.25	0.28	-1.78*	2.10**	0.93	0.77	0.25
		(1.82)	(-2.13)	(1.07)	(1.13)	(-0.25)	{0.17; 0.58}	(-1.85)	(2.61)	(0.71)	(0.56)	{0.11; 0.56}
	36	2.36**	-1.88**	0.71	1.46	0.18	0.32	-2.17**	2.24**	0.93	0.29	0.29
		(2.48)	(-2.08)	(0.97)	(1.52)	(0.11)	{0.19; 0.64}	(-2.66)	(2.76)	(0.95)	(0.16)	{0.15; 0.62}
$g_{U.K.,t+h}$	0	2.10**	0.51	0.36	0.38	0.40	0.29	1.34*	-0.57	0.95	0.20	0.23
		(3.77)	(0.52)	(0.42)	(0.41)	(0.39)	{0.18; 0.58}	(1.74)	(-0.61)	(0.80)	(0.18)	{0.13; 0.53}
	6	2.82**	0.00	1.38**	0.74	-0.66	0.27	0.99	-0.19	1.15	-0.87	0.13
		(3.34)	(0.00)	(2.19)	(1.27)	(-0.71)	{0.19; 0.54}	(1.34)	(-0.18)	(1.02)	(-0.76)	{0.07; 0.42}
	12	2.52**	0.27	1.80**	1.30*	-2.33**	0.34	0.41	-0.12	1.21	-2.59**	0.23
		(2.51)	(0.26)	(3.55)	(1.66)	(-4.02)	{0.23; 0.59}	(0.57)	(-0.14)	(1.02)	(-3.08)	{0.11; 0.53}
	18	1.68*	0.97	1.17**	1.95**	-3.15**	0.41	0.33	-0.30	1.54*	-3.60**	0.38
		(1.91)	(1.15)	(2.62)	(2.72)	(-6.39)	{0.28; 0.66}	(0.63)	(-0.45)	(1.77)	(-5.57)	{0.22; 0.66}
	24	0.67	1.38	0.06	2.25**	-2.95**	0.39	0.55*	-0.35	1.72**	-3.48**	0.39
		(0.94)	(1.59)	(0.13)	(3.62)	(-5.22)	{0.25; 0.67}	(1.67)	(-0.52)	(2.64)	(-5.00)	{0.23; 0.67}
	30	-0.20	1.22	-1.22**	2.22**	-1.72**	0.32	0.97**	-0.05	1.78**	-2.24**	0.31
		(-0.30)	(1.43)	(-2.49)	(3.36)	(-3.08)	{0.21; 0.62}	(2.35)	(-0.08)	(2.27)	(-3.15)	{0.17; 0.60}
	36	-0.63	0.41	-2.12**	1.80**	-0.12	0.30	1.28**	0.64	1.48*	-0.47	0.26
		(-0.99)	(0.55)	(-3.17)	(2.74)	(-0.23)	{0.18; 0.59}	(2.88)	(1.10)	(1.72)	(-0.70)	{0.15; 0.55}

	h	$\beta_{1,c}$	$\beta_{2,c}$	$\beta_{1,l}$	$\beta_{2,l}$	$\beta_{3,l}$	R^2	$\beta^*_{1,l}$	$\beta^*_{2,l}$	$\beta^*_{3,l}$	$\beta^*_{4,l}$	R^2
$\pi_{U.S.,t+h}$	0	0.29*	0.24	0.31*	-1.03**	-0.56	0.40	-0.15	-0.01	-0.96**	0.51	0.37
		(1.83)	(0.89)	(1.73)	(-3.72)	(-1.35)	{0.30; 0.61}	(-0.80)	(-0.06)	(-3.03)	(0.93)	{0.21; 0.62}
	6	0.35*	0.29	0.14	-0.73	0.33	0.37	-0.26	-0.06	-0.76*	-0.43	0.34
		(1.68)	(1.31)	(0.53)	(-1.52)	(0.35)	{0.24; 0.60}	(-0.94)	(-0.40)	(-1.65)	(-0.40)	{0.20; 0.62}
	12	0.47**	0.31	-0.07	-0.18	0.56	0.29	-0.45**	-0.08	-0.25	-0.64	0.28
		(2.77)	(1.44)	(-0.28)	(-0.52)	(0.89)	{0.17; 0.56}	(-1.97)	(-0.50)	(-0.68)	(-0.98)	{0.14; 0.54}
	18	0.58*	0.43**	0.06	-0.10	0.97*	0.27	-0.50	0.06	-0.25	-1.06*	0.23
		(1.86)	(2.13)	(0.17)	(-0.16)	(1.71)	{0.15; 0.55}	(-1.35)	(0.33)	(-0.40)	(-1.73)	{0.12; 0.53}
	24	0.52*	0.22	0.33	-0.36	0.80**	0.21	-0.37	0.29	-0.53	-0.86**	0.19
		(1.80)	(0.98)	(1.43)	(-0.87)	(2.09)	{0.11; 0.52}	(-1.37)	(1.36)	(-1.32)	(-1.98)	{0.09; 0.50}
	30	0.59**	0.18	0.36	-0.06	0.42	0.18	-0.44*	0.40	-0.22	-0.49	0.17
		(1.97)	(0.74)	(1.41)	(-0.16)	(0.87)	{0.09; 0.48}	(-1.71)	(1.63)	(-0.53)	(-1.05)	{0.07; 0.47}
	36	0.63**	0.21	0.34	0.10	0.82	0.19	-0.48*	0.44*	-0.10	-0.92	0.18
		(2.09)	(1.04)	(1.41)	(0.30)	(1.19)	{0.10; 0.46}	(-1.76)	(1.93)	(-0.29)	(-1.37)	{0.08; 0.46}
$\pi_{U.K.,t+h}$	0	-0.42*	-0.55	-1.15**	-0.65	0.32	0.62	0.46	0.57*	-0.87*	0.56	0.58
		(-1.68)	(-1.47)	(-3.00)	(-1.53)	(0.85)	{0.51; 0.84}	(1.39)	(1.83)	(-1.70)	(1.37)	{0.46; 0.82}
	6	-0.54**	-0.77**	-1.09**	-1.25**	0.45	0.73	0.29	0.59**	-1.46**	0.85**	0.69
		(-2.77)	(-2.76)	(-4.49)	(-6.10)	(1.39)	{0.66; 0.88}	(1.41)	(2.30)	(-4.74)	(2.20)	{0.60; 0.86}
	12	-0.28	-0.97**	-0.85**	-1.75**	0.72**	0.82	0.28*	0.52**	-1.81**	1.20**	0.81
		(-1.51)	(-3.52)	(-5.05)	(-7.27)	(2.90)	{0.77; 0.90}	(1.88)	(2.17)	(-6.27)	(4.12)	{0.76; 0.89}
	18	0.14	-0.76**	-0.44*	-1.72**	0.61*	0.75	0.31*	0.24	-1.65**	1.05**	0.75
		(0.60)	(-2.10)	(-1.83)	(-6.19)	(1.75)	{0.70; 0.87}	(1.94)	(0.73)	(-5.64)	(2.73)	{0.70; 0.87}
	24	0.52	-0.94	0.13	-1.76**	0.32	0.58	0.08	0.30	-1.72**	0.79	0.57
		(1.55)	(-1.59)	(0.39)	(-4.01)	(0.60)	{0.48; 0.81}	(0.54)	(0.64)	(-3.59)	(1.15)	{0.44; 0.79}
	30	0.99**	-1.10*	0.87**	-1.81**	-0.25	0.52	-0.22	0.33	-1.86**	0.27	0.44
		(2.22)	(-1.67)	(1.99)	(-3.51)	(-0.44)	{0.42; 0.77}	(-1.15)	(0.67)	(-2.95)	(0.35)	{0.31; 0.76}
	36	1.20**	-0.78	1.18**	-1.44**	-0.79	0.47	-0.28	0.11	-1.53**	-0.38	0.35
		(2.28)	(-1.18)	(2.52)	(-2.91)	(-1.44)	{0.36; 0.76}	(-1.25)	(0.23)	(-2.30)	(-0.52)	{0.22; 0.71}

Table 3.13: We report the results of forecasting regressions of *U.S.* and *U.K.* economic activity (g_t) inflation rates (π_t) using as regressors the smoothed factors extracted from $\mathcal{M}_2^{2,3}$ and $\mathcal{M}_2^{0,4}$. We run regressions for annualized growth in industrial production and consumer price index over forecasting horizons of $h = 0, 12, 18, 24, 30$ and 36 months. We report estimates for parameters (multiplied by 10^3; constant terms, included in the regressions, are omitted from the table) along with t-values (in parenthesis) based on Newey and West (1987) standard errors, considering conditional heteroscedasticity and serial correlation up to 12 lags. One and two asterisks denote statistically significance at the 10% and 5% levels, respectively. We also report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use monthly data for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations).

	h	$\beta_{1,c}$	$\beta_{2,c}$	$\beta_{1,l}$	$\beta_{2,l}$	$\beta_{3,l}$	R^2	$\beta^*_{1,l}$	$\beta^*_{2,l}$	$\beta^*_{3,l}$	$\beta^*_{4,l}$	R^2
$g_{U.S.,t+h}$	0	-0.17	-1.79	2.35	-0.31	-2.28	0.28	1.45	1.32**	1.59	-2.37	0.28
		(-0.12)	(-0.71)	(1.08)	(-0.21)	(-0.90)	{0.16; 0.62}	(1.62)	(2.20)	(0.93)	(-0.74)	{0.14; 0.61}
	6	-1.31	-3.69*	2.86	0.35	-5.09*	0.30	2.62**	0.60	4.14**	-3.96	0.28
		(-0.96)	(-1.83)	(1.14)	(0.22)	(-1.74)	{0.19; 0.58}	(2.79)	(1.14)	(2.40)	(-1.07)	{0.17; 0.57}
	12	-1.89	-4.20**	2.33	0.71	-5.30**	0.26	2.78**	-0.11	4.58**	-3.85	0.22
		(-1.42)	(-2.27)	(1.06)	(0.49)	(-2.11)	{0.17; 0.54}	(2.58)	(-0.20)	(2.61)	(-1.21)	{0.12; 0.53}
	18	-1.79	-4.86**	1.98*	1.53	-4.13*	0.23	2.58*	-0.74	3.64	-3.76**	0.19
		(-1.09)	(-2.44)	(1.84)	(1.34)	(-1.75)	{0.15; 0.51}	(1.80)	(-1.36)	(1.39)	(-2.41)	{0.09; 0.47}
	24	-1.98	-4.70**	0.48	1.76*	-1.21	0.22	1.80	-1.57**	1.45	-1.42	0.18
		(-1.49)	(-2.20)	(0.58)	(1.77)	(-0.76)	{0.12; 0.52}	(1.55)	(-2.19)	(0.84)	(-0.87)	{0.08; 0.49}
	30	-2.96**	-3.70*	-0.80	1.22	0.13	0.28	1.80*	-2.09**	0.93	0.76	0.25
		(-2.16)	(-1.65)	(-1.22)	(1.64)	(0.10)	{0.18; 0.57}	(1.87)	(-2.62)	(0.71)	(0.56)	{0.12; 0.56}
	36	-3.07**	-3.00	-0.34	1.56	-0.27	0.31	2.18**	-2.24**	0.93	0.28	0.29
		(-2.85)	(-1.31)	(-0.30)	(1.34)	(-0.17)	{0.19; 0.64}	(2.66)	(-2.76)	(0.94)	(0.16)	{0.13; 0.63}
$g_{GER,t+h}$	0	-1.48	-0.21	-0.66	2.21**	3.36	0.12	-1.64*	-1.86**	-3.58	0.89	0.12
		(-1.18)	(-0.11)	(-0.92)	(2.49)	(1.34)	{0.07; 0.46}	(-1.76)	(-2.51)	(-1.44)	(0.46)	{0.05; 0.43}
	6	-2.62*	0.22	0.06	0.79	2.97	0.07	-1.82*	-0.12	-3.38*	1.50	0.05
		(-1.65)	(0.10)	(0.08)	(0.96)	(1.48)	{0.04; 0.39}	(-1.84)	(-0.13)	(-1.86)	(0.62)	{0.03; 0.37}
	12	-2.77*	1.81	1.23	-0.90	0.49	0.14	-0.75	1.64	-0.92	0.90	0.10
		(-1.66)	(0.70)	(1.60)	(-1.03)	(0.28)	{0.09; 0.47}	(-0.54)	(1.49)	(-0.55)	(0.32)	{0.03; 0.43}
	18	-2.93*	3.46	2.38**	-1.61**	-2.11*	0.22	0.27	2.41**	1.66	0.56	0.15
		(-1.64)	(1.37)	(3.35)	(-2.04)	(-1.66)	{0.12; 0.51}	(0.20)	(2.07)	(1.34)	(0.18)	{0.06; 0.48}
	24	-2.96**	1.47	2.33**	-1.75	-2.70	0.22	-0.07	2.65*	2.14	2.10	0.17
		(-2.22)	(0.66)	(3.05)	(-1.50)	(-1.42)	{0.12; 0.48}	(-0.08)	(1.84)	(1.15)	(0.87)	{0.08; 0.40}
	30	-2.81**	-1.79	1.65**	-1.88	-2.55	0.20	-0.95	2.81*	1.88	4.02*	0.18
		(-2.42)	(-0.97)	(2.03)	(-1.37)	(-1.36)	{0.13; 0.33}	(-1.17)	(1.84)	(1.05)	(1.82)	{0.08; 0.40}
	36	-1.64	-3.00	0.54	-2.63*	-2.69	0.19	-1.15	3.18**	2.28	3.45	0.19
		(-1.47)	(-1.55)	(0.54)	(-1.69)	(-1.27)	{0.11; 0.52}	(-1.37)	(2.10)	(1.12)	(1.41)	{0.08; 0.48}

	h	$\beta_{1,c}$	$\beta_{2,c}$	$\beta_{1,l}$	$\beta_{2,l}$	$\beta_{3,l}$	R^2	$\beta^*_{1,l}$	$\beta^*_{2,l}$	$\beta^*_{3,l}$	$\beta^*_{4,l}$	R^2
$\pi_{U.S.,t+h}$	0	0.04	2.08**	0.13	0.02	0.83**	0.48	0.16	0.01	-0.97**	0.51	0.37
		(0.22)	(4.10)	(0.43)	(0.07)	(2.22)	{0.40; 0.57}	(0.85)	(0.04)	(-3.03)	(0.93)	{0.23; 0.63}
	6	0.27	1.52**	0.66	0.36	0.25	0.40	0.27	0.06	-0.77*	-0.44	0.34
		(0.89)	(2.56)	(1.06)	(0.90)	(0.34)	{0.28; 0.61}	(0.97)	(0.40)	(-1.64)	(-0.40)	{0.20; 0.59}
	12	0.05	0.59	0.74*	0.33	-0.20	0.28	0.45**	0.08	-0.24	-0.64	0.28
		(0.18)	(1.08)	(1.73)	(1.06)	(-0.40)	{0.17; 0.54}	(2.00)	(0.51)	(-0.68)	(-0.98)	{0.14; 0.55}
	18	0.10	-0.08	0.93**	0.68**	-0.32	0.23	0.50	-0.05	-0.25	-1.06*	0.23
		(0.24)	(-0.15)	(2.18)	(2.32)	(-0.47)	{0.12; 0.53}	(1.37)	(-0.30)	(-0.39)	(-1.73)	{0.12; 0.51}
	24	0.08	-0.29	0.70**	0.86**	0.05	0.19	0.38	-0.28	-0.53	-0.86**	0.19
		(0.27)	(-0.61)	(2.65)	(2.58)	(0.12)	{0.09; 0.50}	(1.41)	(-1.34)	(-1.31)	(-1.98)	{0.09; 0.49}
	30	-0.25	-0.32	0.39	0.67*	-0.03	0.17	0.44*	-0.40	-0.22	-0.49	0.17
		(-0.62)	(-0.57)	(1.57)	(1.89)	(-0.07)	{0.08; 0.49}	(1.74)	(-1.62)	(-0.52)	(-1.05)	{0.07; 0.47}
	36	-0.13	-0.24	0.60	0.86**	-0.37	0.17	0.49*	-0.43*	-0.09	-0.92	0.18
		(-0.34)	(-0.39)	(1.50)	(2.13)	(-0.84)	{0.08; 0.47}	(1.78)	(-1.91)	(-0.28)	(-1.37)	{0.07; 0.46}
$\pi_{GER,t+h}$	0	0.53**	0.00	-0.14	0.18	-1.22**	0.70	0.26*	-0.33**	1.29**	-0.27	0.68
		(3.28)	(0.00)	(-1.05)	(1.40)	(-6.27)	{0.64; 0.80}	(1.74)	(-2.68)	(6.02)	(-0.72)	{0.59; 0.80}
	6	0.34*	0.46	-0.17	0.16	-1.01**	0.65	0.17	-0.29**	1.09**	-0.52	0.65
		(1.85)	(1.06)	(-1.10)	(1.09)	(-3.81)	{0.56; 0.78}	(0.88)	(-2.18)	(3.82)	(-1.25)	{0.56; 0.76}
	12	0.08	1.03**	-0.26*	0.17	-0.44	0.56	0.01	-0.29*	0.52*	-0.90*	0.56
		(0.37)	(2.12)	(-1.66)	(1.13)	(-1.63)	{0.48; 0.69}	(0.07)	(-1.94)	(1.88)	(-1.86)	{0.47; 0.69}
	18	-0.10	1.37**	-0.41**	0.21	0.26	0.54	-0.18	-0.32**	-0.17	-1.19**	0.53
		(-0.41)	(2.96)	(-2.23)	(1.35)	(1.15)	{0.43; 0.73}	(-0.89)	(-2.08)	(-0.74)	(-2.45)	{0.41; 0.72}
	24	-0.36	1.50**	-0.29	0.33	0.52**	0.47	-0.25	-0.36*	-0.48*	-1.05*	0.44
		(-1.57)	(2.54)	(-1.53)	(1.62)	(1.98)	{0.38; 0.69}	(-1.32)	(-1.78)	(-1.78)	(-1.76)	{0.32; 0.66}
	30	-0.69**	1.74**	-0.03	0.43**	0.54*	0.44	-0.24	-0.38	-0.57*	-0.88*	0.32
		(-3.41)	(2.99)	(-0.21)	(1.98)	(1.93)	{0.33; 0.67}	(-1.39)	(-1.47)	(-1.90)	(-1.73)	{0.20; 0.62}
	36	-1.05**	1.39**	0.24	0.40*	0.27	0.38	-0.33*	-0.23	-0.43	-0.21	0.18
		(-3.21)	(2.85)	(1.02)	(1.78)	(0.84)	{0.28; 0.65}	(-1.86)	(-0.84)	(-1.24)	(-0.40)	{0.08; 0.57}

Table 3.14: We report the results of forecasting regressions of $U.S.$ and $GER.$ economic activity (g_t) inflation rates (π_t) using as regressors the smoothed factors extracted from $\mathcal{M}_2^{2,3}$ and $\mathcal{M}_2^{0,4}$. We run regressions for annualized growth in industrial production and consumer price index over forecasting horizons of $h = 0, 12, 18, 24, 30$ and 36 months. We report estimates for parameters (multiplied by 10^3; constant terms, included in the regressions, are omitted from the table) along with t-values (in parenthesis) based on Newey and West (1987) standard errors, considering conditional heteroscedasticity and serial correlation up to 12 lags. One and two asterisks denote statistically significance at the 10% and 5% levels, respectively. We also report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use monthly data for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations).

	h	$\beta_{1,c}$	$\beta_{2,c}$	$\beta_{1,l}$	$\beta_{2,l}$	$\beta_{3,l}$	R^2	$\beta^*_{1,l}$	$\beta^*_{2,l}$	$\beta^*_{3,l}$	$\beta^*_{4,l}$	R^2
$g_{U.S.,t+h}$	0	0.14	1.84	2.55	1.20	-1.60	0.29	1.46	1.32**	-1.57	-2.36	0.28
		(0.22)	(1.15)	(1.43)	(0.65)	(-0.56)	{0.16; 0.60}	(1.61)	(2.20)	(-0.93)	(-0.74)	{0.15; 0.59}
	6	-0.44	3.78**	3.60*	3.86*	-2.44	0.31	2.65**	0.60	-4.08**	-3.95	0.28
		(-0.60)	(2.97)	(1.87)	(1.88)	(-0.74)	{0.22; 0.61}	(2.78)	(1.13)	(-2.40)	(-1.07)	{0.16; 0.57}
	12	-0.95	4.59**	3.09*	4.31**	-2.25	0.29	2.82**	-0.11	-4.52**	-3.84	0.22
		(-1.38)	(3.13)	(1.88)	(2.41)	(-0.79)	{0.20; 0.57}	(2.58)	(-0.21)	(-2.61)	(-1.21)	{0.13; 0.51}
	18	-1.49*	5.44**	1.86	3.45*	-2.71*	0.29	2.61*	-0.74	-3.59	-3.75**	0.19
		(-1.93)	(3.33)	(1.23)	(1.72)	(-1.66)	{0.18; 0.53}	(1.80)	(-1.36)	(-1.38)	(-2.41)	{0.10; 0.48}
	24	-2.29**	5.87**	-0.38	1.30	-1.86	0.33	1.80	-1.56**	-1.44	-1.41	0.18
		(-2.51)	(3.57)	(-0.45)	(0.99)	(-1.46)	{0.23; 0.56}	(1.55)	(-2.18)	(-0.84)	(-0.87)	{0.08; 0.47}
	30	-3.08**	5.79**	-1.32**	0.48	-0.16	0.42	1.79*	-2.09**	-0.92	0.76	0.25
		(-3.08)	(3.35)	(-2.05)	(0.53)	(-0.16)	{0.31; 0.64}	(1.86)	(-2.62)	(-0.71)	(0.56)	{0.12; 0.55}
	36	-3.35**	5.21**	-0.95	0.89	-0.54	0.43	2.18**	-2.23**	-0.93	0.28	0.29
		(-3.52)	(3.08)	(-1.08)	(0.87)	(-0.30)	{0.31; 0.69}	(2.65)	(-2.77)	(-0.94)	(0.16)	{0.13; 0.61}
$g_{U.K.,t+h}$	0	-1.32*	2.30*	0.67	-1.39	0.40	0.27	-1.34*	-0.57	-0.95	-0.21	0.23
		(-1.79)	(1.77)	(0.70)	(-1.40)	(0.37)	{0.18; 0.57}	(-1.74)	(-0.61)	(-0.80)	(-0.18)	{0.12; 0.54}
	6	-2.31**	1.86	-0.49	-0.69	0.02	0.19	-0.99	-0.17	-1.16	0.87	0.13
		(-2.93)	(1.62)	(-0.68)	(-0.66)	(0.02)	{0.13; 0.50}	(-1.34)	(-0.17)	(-1.02)	(0.76)	{0.07; 0.43}
	12	-2.34**	1.26	-1.29**	0.44	-0.65	0.19	-0.40	-0.08	-1.25	2.61**	0.23
		(-2.49)	(0.83)	(-2.32)	(0.48)	(-0.96)	{0.12; 0.48}	(-0.57)	(-0.10)	(-1.05)	(3.07)	{0.12; 0.53}
	18	-2.26**	1.17	-1.40**	0.93	-0.81	0.24	-0.33	-0.26	-1.60*	3.64**	0.38
		(-2.00)	(0.73)	(-2.36)	(1.15)	(-1.24)	{0.15; 0.53}	(-0.63)	(-0.39)	(-1.83)	(5.60)	{0.22; 0.65}
	24	-2.12*	1.51	-1.04	0.90	-0.60	0.25	-0.55*	-0.31	-1.78**	3.52**	0.39
		(-1.65)	(0.92)	(-1.32)	(1.38)	(-0.71)	{0.18; 0.54}	(-1.68)	(-0.46)	(-2.71)	(5.02)	{0.23; 0.67}
	30	-1.32	1.34	0.04	0.98*	0.19	0.23	-0.97**	-0.02	-1.83**	2.26**	0.31
		(-1.01)	(0.81)	(0.05)	(1.73)	(0.21)	{0.14; 0.52}	(-2.35)	(-0.03)	(-2.30)	(3.15)	{0.19; 0.60}
	36	-0.64	0.98	0.99	1.09**	0.98	0.25	-1.27**	0.65	-1.50*	0.47	0.26
		(-0.50)	(0.90)	(1.24)	(2.50)	(1.23)	{0.16; 0.55}	(-2.88)	(1.12)	(-1.71)	(0.70)	{0.15; 0.56}
$g_{GER,t+h}$	0	-0.21	2.05	-1.34**	2.09**	-3.30	0.12	1.62*	1.84**	3.58	-0.89	0.12
		(-0.21)	(1.07)	(-1.96)	(2.07)	(-1.34)	{0.07; 0.47}	(1.76)	(2.51)	(1.44)	(-0.46)	{0.04; 0.43}
	6	-1.40	1.77	-0.42	0.86	-3.08	0.05	1.80*	0.12	3.37*	-1.49	0.05
		(-1.19)	(0.88)	(-0.44)	(0.86)	(-1.51)	{0.05; 0.37}	(1.84)	(0.13)	(1.86)	(-0.62)	{0.02; 0.34}
	12	-1.65	0.02	0.73	-0.93	-0.86	0.11	0.74	-1.62	0.92	-0.89	0.10
		(-1.14)	(0.01)	(0.78)	(-0.93)	(-0.50)	{0.06; 0.45}	(0.54)	(-1.49)	(0.55)	(-0.32)	{0.03; 0.42}
	18	-1.91	-2.03	1.83**	-1.71**	1.52	0.19	-0.28	-2.39**	-1.65	-0.55	0.15
		(-1.25)	(-0.73)	(2.15)	(-2.16)	(1.13)	{0.10; 0.51}	(-0.20)	(-2.08)	(-1.34)	(-0.18)	{0.06; 0.48}
	24	-2.63**	-1.23	2.39**	-1.44	2.25	0.22	0.07	-2.62*	-2.14	-2.08	0.17
		(-2.07)	(-0.65)	(2.65)	(-1.23)	(1.22)	{0.12; 0.50}	(0.08)	(-1.84)	(-1.15)	(-0.87)	{0.08; 0.40}
	30	-3.18**	1.23	2.42**	-1.06	2.38	0.21	0.94	-2.78*	-1.88	-3.98*	0.18
		(-2.81)	(0.74)	(2.75)	(-0.80)	(1.33)	{0.13; 0.49}	(1.17)	(-1.84)	(-1.05)	(-1.82)	{0.09; 0.40}
	36	-2.87**	1.62	1.83*	-1.74	2.72	0.21	1.15	-3.15**	-2.28	-3.41	0.19
		(-2.58)	(0.89)	(1.71)	(-1.13)	(1.31)	{0.13; 0.51}	(1.36)	(-2.10)	(-1.12)	(-1.41)	{0.09; 0.48}

	h	$\beta_{1,c}$	$\beta_{2,c}$	$\beta_{1,l}$	$\beta_{2,l}$	$\beta_{3,l}$	R^2	$\beta^*_{1,l}$	$\beta^*_{2,l}$	$\beta^*_{3,l}$	$\beta^*_{4,l}$	R^2
$\pi_{U.S.,t+h}$	0	-0.28	-1.60**	-0.09	-0.72**	0.28	0.46	0.15	0.01	0.96**	0.51	0.37
		(-1.59)	(-4.59)	(-0.28)	(-2.46)	(0.77)	{0.36; 0.60}	(0.79)	(0.04)	(3.03)	(0.93)	{0.24; 0.63}
	6	-0.17	-1.22**	0.23	-0.32	-0.50	0.40	0.26	0.06	0.76*	-0.44	0.34
		(-0.98)	(-3.40)	(0.46)	(-0.59)	(-0.61)	{0.34; 0.48}	(0.94)	(0.41)	(1.66)	(-0.40)	{0.21; 0.61}
	12	-0.17	-0.84**	0.53	0.12	-0.56	0.30	0.45**	0.08	0.25	-0.64	0.28
		(-1.07)	(-2.14)	(1.48)	(0.36)	(-1.00)	{0.19; 0.56}	(1.97)	(0.52)	(0.69)	(-0.98)	{0.15; 0.54}
	18	-0.21	-0.81*	0.57	0.35	-0.96*	0.26	0.50	-0.05	0.25	-1.06*	0.23
		(-0.95)	(-1.68)	(1.12)	(0.60)	(-1.95)	{0.16; 0.53}	(1.36)	(-0.29)	(0.40)	(-1.73)	{0.11; 0.52}
	24	-0.33	-0.86*	0.18	0.13	-0.89**	0.21	0.37	-0.28	0.53	-0.86**	0.19
		(-1.28)	(-1.67)	(0.70)	(0.34)	(-2.33)	{0.11; 0.54}	(1.40)	(-1.33)	(1.33)	(-1.98)	{0.10; 0.48}
	30	-0.45	-0.67	0.13	0.29	-0.48	0.18	0.44*	-0.40	0.22	-0.49	0.17
		(-1.30)	(-1.20)	(0.47)	(0.83)	(-0.97)	{0.09; 0.49}	(1.73)	(-1.61)	(0.53)	(-1.05)	{0.07; 0.48}
	36	-0.42	-0.41	0.21	0.49	-0.86	0.18	0.49*	-0.43*	0.10	-0.92	0.18
		(-1.36)	(-0.71)	(0.72)	(1.40)	(-1.31)	{0.09; 0.48}	(1.78)	(-1.90)	(0.30)	(-1.37)	{0.08; 0.47}
$\pi_{U.K.,t+h}$	0	0.34*	-1.89**	0.87**	0.71*	-0.08	0.64	-0.46	0.56*	0.88*	-0.57	0.58
		(1.66)	(-2.94)	(2.90)	(1.95)	(-0.22)	{0.55; 0.83}	(-1.39)	(1.81)	(1.71)	(-1.36)	{0.46; 0.82}
	6	0.67**	-1.50**	0.95**	0.56**	-0.48*	0.71	-0.29	0.58**	1.48**	-0.85**	0.69
		(3.41)	(-2.36)	(4.08)	(1.98)	(-1.95)	{0.61; 0.86}	(-1.42)	(2.26)	(4.76)	(-2.18)	{0.60; 0.87}
	12	0.50*	-1.05*	0.84**	0.09	-0.73**	0.73	-0.28*	0.50**	1.84**	-1.21**	0.80
		(1.86)	(-1.72)	(3.53)	(0.32)	(-2.33)	{0.68; 0.86}	(-1.89)	(2.10)	(6.32)	(-4.09)	{0.76; 0.89}
	18	0.15	-0.96	0.62**	-0.32	-0.75*	0.67	-0.31*	0.22	1.67**	-1.06**	0.75
		(0.44)	(-1.55)	(2.12)	(-1.00)	(-1.94)	{0.61; 0.83}	(-1.95)	(0.69)	(5.70)	(-2.72)	{0.69; 0.87}
	24	-0.24	-1.44*	0.10	-0.29	-0.92**	0.55	-0.09	0.29	1.74**	-0.80	0.57
		(-0.60)	(-1.94)	(0.30)	(-0.81)	(-2.30)	{0.45; 0.79}	(-0.55)	(0.62)	(3.56)	(-1.15)	{0.44; 0.80}
	30	-0.91*	-1.46**	-0.70*	-0.32	-1.36**	0.52	0.22	0.32	1.87**	-0.27	0.44
		(-1.87)	(-2.03)	(-1.89)	(-0.92)	(-3.47)	{0.43; 0.78}	(1.14)	(0.66)	(2.91)	(-0.34)	{0.30; 0.76}
	36	-1.39**	-1.12*	-1.12**	-0.40	-1.50**	0.51	0.28	0.11	1.53**	0.39	0.35
		(-2.25)	(-1.91)	(-2.74)	(-1.24)	(-4.00)	{0.40; 0.78}	(1.24)	(0.22)	(2.26)	(0.53)	{0.20; 0.72}
$\pi_{GER,t+h}$	0	0.47**	-0.18	-0.21	0.06	1.25**	0.69	-0.26*	0.32**	-1.29**	0.26	0.68
		(2.98)	(-0.57)	(-1.42)	(0.40)	(6.25)	{0.63; 0.81}	(-1.72)	(2.68)	(-6.02)	(0.72)	{0.58; 0.80}
	6	0.33	-0.57*	-0.28*	0.00	0.99**	0.65	-0.17	0.29**	-1.08**	0.52	0.65
		(1.61)	(-1.64)	(-1.86)	(0.01)	(3.88)	{0.57; 0.78}	(-0.86)	(2.18)	(-3.82)	(1.25)	{0.56; 0.76}
	12	0.18	-0.98**	-0.45**	-0.05	0.35	0.56	-0.01	0.28*	-0.52*	0.89*	0.56
		(0.73)	(-2.21)	(-2.50)	(-0.21)	(1.35)	{0.50; 0.67}	(-0.06)	(1.93)	(-1.88)	(1.86)	{0.47; 0.69}
	18	0.04	-1.23**	-0.63**	-0.03	-0.39*	0.55	0.18	0.32**	0.17	1.18**	0.53
		(0.16)	(-2.68)	(-3.30)	(-0.14)	(-1.74)	{0.46; 0.73}	(0.89)	(2.07)	(0.74)	(2.45)	{0.42; 0.71}
	24	-0.14	-1.30**	-0.54**	0.10	-0.68**	0.48	0.25	0.36*	0.48*	1.04*	0.44
		(-0.55)	(-2.54)	(-2.25)	(0.38)	(-2.58)	{0.37; 0.69}	(1.32)	(1.78)	(1.78)	(1.76)	{0.31; 0.67}
	30	-0.36*	-1.51**	-0.33*	0.23	-0.76**	0.45	0.24	0.37	0.57*	0.87*	0.32
		(-1.69)	(-2.78)	(-1.70)	(0.96)	(-2.59)	{0.35; 0.69}	(1.39)	(1.46)	(1.90)	(1.73)	{0.20; 0.64}
	36	-0.75**	-1.28**	0.03	0.32	-0.49	0.42	0.33*	0.22	0.43	0.21	0.18
		(-3.12)	(-2.83)	(0.15)	(1.45)	(-1.54)	{0.30; 0.68}	(1.86)	(0.84)	(1.24)	(0.40)	{0.09; 0.56}

Table 3.15: We report the results of forecasting regressions of $U.S.$, $U.K.$ and $GER.$ economic activity (g_t) inflation rates (π_t) using as regressors the smoothed factors extracted from $\mathcal{M}_3^{2,3}$ and $\mathcal{M}_3^{0,4}$. We run regressions for annualized growth in industrial production and consumer price index over forecasting horizons of $h = 0, 12, 18, 24, 30$ and 36 months. We report estimates for parameters (multiplied by 10^3; constant terms, included in the regressions, are omitted from the table) along with t-values (in parenthesis) based on Newey and West (1987) standard errors, considering conditional heteroscedasticity and serial correlation up to 12 lags. One and two asterisks denote statistically significance at the 10% and 5% levels, respectively. We also report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use monthly data for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations).

	h	$\beta_{1,c}$	$\beta_{2,c}$	$\beta_{1,l}$	$\beta_{2,l}$	$\beta_{3,l}$	R^2	$\beta^*_{1,l}$	$\beta^*_{2,l}$	$\beta^*_{3,l}$	$\beta^*_{4,l}$	R^2
$g_{U.S..t+h}$	0	0.47	2.83	1.48	1.01*	0.90	0.28	-1.46	-1.33**	1.56	2.35	0.28
		(0.22)	(0.90)	(1.17)	(1.90)	(0.52)	{0.17; 0.61}	(-1.61)	(-2.19)	(0.93)	(0.73)	{0.15; 0.59}
	6	2.57	5.37	2.32*	0.09	2.49	0.32	-2.65**	-0.63	4.07**	3.94	0.28
		(1.61)	(1.34)	(1.88)	(0.19)	(1.64)	{0.22; 0.60}	(-2.78)	(-1.17)	(2.41)	(1.07)	{0.17; 0.57}
	12	3.50**	5.25	2.23**	-0.60	2.73*	0.29	-2.82**	0.08	4.51**	3.83	0.22
		(2.45)	(1.51)	(2.05)	(-1.21)	(1.91)	{0.20; 0.55}	(-2.58)	(0.14)	(2.61)	(1.21)	{0.13; 0.51}
	18	3.61**	4.75**	1.88	-1.22**	1.65	0.25	-2.61*	0.71	3.57	3.74**	0.19
		(2.00)	(2.74)	(1.48)	(-2.02)	(0.73)	{0.16; 0.52}	(-1.80)	(1.32)	(1.38)	(2.41)	{0.09; 0.47}
	24	3.74*	2.13	1.02	-1.73**	0.11	0.23	-1.80	1.55**	1.43	1.41	0.18
		(1.89)	(1.55)	(1.06)	(-2.25)	(0.07)	{0.14; 0.52}	(-1.55)	(2.18)	(0.84)	(0.87)	{0.08; 0.49}
	30	3.65*	-0.19	1.01	-1.97**	0.26	0.28	-1.79*	2.07**	0.93	-0.76	0.25
		(1.75)	(-0.19)	(1.28)	(-2.63)	(0.22)	{0.17; 0.58}	(-1.86)	(2.62)	(0.71)	(-0.56)	{0.12; 0.55}
	36	2.76	0.07	1.63**	-2.19**	0.43	0.31	-2.18**	2.22**	0.93	-0.29	0.29
		(1.34)	(0.04)	(2.22)	(-2.70)	(0.44)	{0.20; 0.64}	(-2.66)	(2.76)	(0.95)	(-0.16)	{0.15; 0.63}
$g_{U.K..t+h}$	0	2.51*	0.90	1.23	-0.99	0.70	0.27	-1.35*	-0.57	0.95	0.20	0.23
		(1.67)	(0.45)	(1.37)	(-1.11)	(0.68)	{0.16; 0.58}	(-1.75)	(-0.62)	(0.81)	(0.17)	{0.13; 0.53}
	6	2.41*	4.68**	0.96	0.31	0.18	0.25	-0.99	-0.20	1.13	-0.87	0.13
		(1.74)	(2.02)	(1.23)	(0.39)	(0.23)	{0.17; 0.53}	(-1.34)	(-0.20)	(1.02)	(-0.77)	{0.07; 0.42}
	12	2.16	3.77*	0.20	1.33*	-0.51	0.22	-0.42	-0.14	1.18	-2.57**	0.23
		(1.34)	(1.66)	(0.26)	(1.74)	(-0.67)	{0.14; 0.49}	(-0.58)	(-0.17)	(1.00)	(-3.09)	{0.12; 0.51}
	18	1.81	3.10*	0.04	1.77**	-0.65	0.25	-0.35	-0.33	1.49*	-3.56**	0.38
		(1.08)	(1.81)	(0.07)	(2.38)	(-0.97)	{0.14; 0.52}	(-0.65)	(-0.50)	(1.73)	(-5.56)	{0.23; 0.66}
	24	1.33	3.45**	0.42	1.74**	-0.31	0.26	-0.56*	-0.38	1.67**	-3.44**	0.39
		(0.87)	(3.42)	(0.79)	(2.70)	(-0.45)	{0.15; 0.52}	(-1.71)	(-0.56)	(2.59)	(-4.98)	{0.24; 0.68}
	30	0.02	4.46**	1.22**	1.55**	0.51	0.27	-0.98**	-0.07	1.74**	-2.21**	0.31
		(0.02)	(3.57)	(2.42)	(2.94)	(0.75)	{0.17; 0.55}	(-2.36)	(-0.12)	(2.24)	(-3.14)	{0.19; 0.59}
	36	-0.87	4.07**	1.76**	1.37**	1.24**	0.31	-1.26**	0.62	1.47*	-0.46	0.26
		(-0.96)	(2.15)	(3.32)	(3.41)	(2.15)	{0.20; 0.60}	(-2.87)	(1.06)	(1.72)	(-0.69)	{0.16; 0.55}
$g_{GER.t+h}$	0	1.66	1.01	-1.00	-2.15**	3.17	0.11	1.56*	1.77**	3.54	-0.88	0.12
		(0.89)	(0.25)	(-1.42)	(-2.32)	(1.32)	{0.05; 0.45}	(1.72)	(2.49)	(1.44)	(-0.46)	{0.05; 0.43}
	6	2.12	3.62	-1.09	-0.45	2.93	0.06	1.74*	0.06	3.33*	-1.47	0.05
		(1.09)	(0.65)	(-0.92)	(-0.44)	(1.37)	{0.05; 0.36}	(1.79)	(0.07)	(1.87)	(-0.61)	{0.03; 0.35}
	12	1.39	1.34	-0.23	1.40	0.70	0.10	0.73	-1.63	0.90	-0.87	0.10
		(0.53)	(0.28)	(-0.21)	(1.29)	(0.41)	{0.05; 0.43}	(0.53)	(-1.51)	(0.54)	(-0.31)	{0.03; 0.41}
	18	0.61	-0.81	0.59	2.26**	-1.70	0.15	-0.25	-2.35**	-1.64	-0.54	0.15
		(0.21)	(-0.23)	(0.73)	(2.32)	(-1.13)	{0.08; 0.47}	(-0.18)	(-2.06)	(-1.35)	(-0.18)	{0.06; 0.49}
	24	2.02	-0.96	0.85	2.18	-2.56	0.17	0.11	-2.58*	-2.11	-2.06	0.17
		(0.95)	(-0.41)	(1.03)	(1.50)	(-1.22)	{0.08; 0.41}	(0.13)	(-1.84)	(-1.15)	(-0.87)	{0.07; 0.42}
	30	3.75*	1.97	0.47	2.08	-2.74	0.17	0.98	-2.75*	-1.86	-3.96*	0.18
		(1.94)	(0.90)	(0.50)	(1.34)	(-1.39)	{0.09; 0.39}	(1.20)	(-1.86)	(-1.05)	(-1.82)	{0.09; 0.38}
	36	3.00	4.72*	-0.31	2.77	-2.91	0.20	1.18	-3.11**	-2.25	-3.40	0.19
		(1.41)	(1.76)	(-0.29)	(1.60)	(-1.37)	{0.11; 0.50}	(1.40)	(-2.12)	(-1.12)	(-1.41)	{0.09; 0.48}
$g_{JAP.t+h}$	0	1.24	-0.46	-0.47	-0.88	-1.31	0.05	-0.79	0.85	-0.99	1.80	0.05
		(0.33)	(-0.09)	(-0.38)	(-0.41)	(-0.70)	{0.04; 0.32}	(-0.99)	(0.49)	(-0.56)	(0.76)	{0.03; 0.32}
		4.94**	6.24	-0.23	-1.82	-3.48*	0.15	-0.40	2.98	-3.85*	0.59	0.09
		(2.03)	(0.87)	(-0.25)	(-0.95)	(-1.76)	{0.09; 0.45}	(-0.53)	(1.38)	(-1.66)	(0.29)	{0.04; 0.36}
	12	5.58**	6.84	0.32	-2.21	-5.13**	0.19	0.17	3.59	-5.66**	0.53	0.12
		(2.22)	(1.15)	(0.54)	(-1.36)	(-2.89)	{0.10; 0.44}	(0.28)	(1.58)	(-2.37)	(0.28)	{0.06; 0.34}
	18	2.73	10.05**	0.12	-3.00*	-6.19**	0.19	0.46	3.62	-6.20**	0.19	0.12
		(1.05)	(2.69)	(0.20)	(-1.71)	(-3.26)	{0.11; 0.46}	(0.77)	(1.57)	(-2.51)	(0.09)	{0.06; 0.34}
	24	2.63	8.31**	0.37	-3.07*	-6.69**	0.17	0.65	3.53	-6.61**	0.48	0.12
		(0.87)	(3.21)	(0.64)	(-1.65)	(-2.74)	{0.10; 0.43}	(0.87)	(1.56)	(-2.19)	(0.22)	{0.06; 0.36}
	30	4.45	4.62**	0.51	-2.55	-5.66**	0.15	0.36	3.34	-5.80*	0.99	0.11
		(1.30)	(2.53)	(0.71)	(-1.39)	(-2.54)	{0.09; 0.44}	(0.45)	(1.52)	(-1.92)	(0.43)	{0.05; 0.36}
	36	4.33	7.01**	0.00	-2.45	-4.74**	0.13	0.23	3.51*	-5.37*	-1.36	0.08
		(1.58)	(2.67)	(-0.01)	(-1.47)	(-1.97)	{0.08; 0.43}	(0.35)	(1.70)	(-1.93)	(-0.68)	{0.04; 0.36}

	h	$\beta_{1,c}$	$\beta_{2,c}$	$\beta_{1,l}$	$\beta_{2,l}$	$\beta_{3,l}$	R^2	$\beta^*_{1,l}$	$\beta^*_{2,l}$	$\beta^*_{3,l}$	$\beta^*_{4,l}$	R^2
$\pi_{U.S..t+h}$	0	-1.46**	-1.06**	0.40**	0.03	-0.51**	0.51	-0.15	0.00	-0.95**	-0.51	0.37
		(-3.27)	(-2.65)	(2.06)	(0.24)	(-2.20)	{0.45; 0.60}	(-0.78)	(0.00)	(-3.03)	(-0.93)	{0.22; 0.62}
	6	-1.10**	-0.04	0.49	0.00	-0.57	0.40	-0.26	-0.06	-0.76*	0.43	0.34
		(-2.64)	(-0.04)	(1.58)	(0.00)	(-1.62)	{0.28; 0.62}	(-0.93)	(-0.37)	(-1.66)	(0.40)	{0.19; 0.61}
	12	-0.38	0.48	0.55**	0.00	-0.28	0.28	-0.44**	-0.08	-0.25	0.64	0.28
		(-0.83)	(0.72)	(2.27)	(-0.02)	(-0.92)	{0.17; 0.56}	(-1.97)	(-0.51)	(-0.69)	(0.98)	{0.15; 0.55}
	18	0.00	0.95	0.53	-0.18	-0.49	0.23	-0.50	0.05	-0.25	1.06*	0.23
		(0.00)	(1.50)	(1.44)	(-1.14)	(-0.94)	{0.13; 0.54}	(-1.36)	(0.29)	(-0.41)	(1.73)	{0.11; 0.51}
	24	0.29	0.74	0.33	-0.38*	-0.80**	0.19	-0.37	0.28	-0.53	0.85**	0.19
		(0.58)	(1.61)	(1.32)	(-1.69)	(-2.16)	{0.11; 0.53}	(-1.39)	(1.35)	(-1.33)	(1.98)	{0.09; 0.49}
	30	0.52	0.47	0.35*	-0.45*	-0.45	0.18	-0.44*	0.40	-0.22	0.49	0.17
		(0.85)	(1.23)	(1.68)	(-1.93)	(-1.05)	{0.09; 0.48}	(-1.73)	(1.61)	(-0.53)	(1.05)	{0.07; 0.45}
	36	0.48	0.91	0.42*	-0.54**	-0.42	0.19	-0.49*	0.43*	-0.10	0.92	0.18
		(0.82)	(1.58)	(1.72)	(-2.43)	(-1.18)	{0.08; 0.47}	(-1.78)	(1.90)	(-0.30)	(1.37)	{0.07; 0.46}
$\pi_{U.K..t+h}$	0	-1.35**	-1.67**	0.50*	0.38	-0.31	0.66	-0.45	0.58*	-0.86*	0.56	0.58
		(-2.11)	(-3.23)	(1.78)	(1.17)	(-0.83)	{0.56; 0.83}	(-1.36)	(1.86)	(-1.69)	(1.37)	{0.43; 0.82}
	6	-1.34**	-1.29**	0.45**	0.12	-0.66**	0.71	-0.28	0.61**	-1.44**	0.84**	0.69
		(-2.08)	(-2.89)	(2.26)	(0.44)	(-2.84)	{0.62; 0.86}	(-1.35)	(2.36)	(-4.74)	(2.20)	{0.60; 0.86}
	12	-0.76	-0.93**	0.44**	-0.25	-0.87**	0.72	-0.27*	0.54**	-1.79**	1.19**	0.81
		(-1.35)	(-2.40)	(2.18)	(-0.98)	(-2.91)	{0.67; 0.85}	(-1.78)	(2.26)	(-6.23)	(4.13)	{0.76; 0.89}
	18	-0.29	-0.46	0.45**	-0.52	-0.88**	0.66	-0.30*	0.26	-1.63**	1.03**	0.75
		(-0.58)	(-1.04)	(2.03)	(-1.63)	(-2.60)	{0.58; 0.83}	(-1.90)	(0.78)	(-5.60)	(2.72)	{0.69; 0.86}
	24	0.03	-1.06	0.07	-0.36	-1.12**	0.51	-0.08	0.32	-1.70**	0.78	0.56
		(0.05)	(-1.31)	(0.29)	(-1.02)	(-3.37)	{0.39; 0.77}	(-0.49)	(0.67)	(-3.60)	(1.15)	{0.44; 0.79}
	30	0.65	-1.54	-0.42	-0.12	-1.56**	0.45	0.23	0.34	-1.84**	0.26	0.44
		(1.10)	(-1.25)	(-1.41)	(-0.38)	(-4.37)	{0.32; 0.75}	(1.20)	(0.70)	(-2.97)	(0.35)	{0.30; 0.76}
	36	1.15	-1.11	-0.55	-0.01	-1.64**	0.40	0.28	0.12	-1.53**	-0.38	0.35
		(1.53)	(-0.80)	(-1.51)	(-0.04)	(-3.72)	{0.28; 0.73}	(1.26)	(0.25)	(-2.33)	(-0.53)	{0.22; 0.72}
$\pi_{GER.t+h}$	0	-0.69*	0.00	-0.05	-0.17	-1.17**	0.68	-0.24	0.34**	-1.27**	0.26	0.68
		(-1.91)	(0.00)	(-0.42)	(-1.23)	(-5.23)	{0.60; 0.80}	(-1.62)	(2.84)	(-6.01)	(0.72)	{0.60; 0.80}
	6	-0.79**	-0.20	-0.18	-0.10	-0.91**	0.64	-0.16	0.30**	-1.07**	0.51	0.65
		(-2.02)	(-0.38)	(-1.34)	(-0.60)	(-3.63)	{0.57; 0.77}	(-0.81)	(2.35)	(-3.81)	(1.25)	{0.56; 0.76}
	12	-0.88*	-0.57	-0.36**	-0.08	-0.28	0.56	-0.01	0.29**	-0.51*	0.89*	0.56
		(-1.90)	(-1.14)	(-2.53)	(-0.43)	(-1.11)	{0.47; 0.70}	(-0.04)	(2.04)	(-1.88)	(1.86)	{0.46; 0.70}
	18	-0.86*	-0.83	-0.52**	-0.11	0.45**	0.54	0.17	0.32**	0.17	1.17**	0.53
		(-1.83)	(-1.57)	(-3.59)	(-0.65)	(2.23)	{0.43; 0.72}	(0.86)	(2.11)	(0.73)	(2.45)	{0.42; 0.72}
	24	-0.61	-1.33**	-0.43**	-0.23	0.69**	0.47	0.24	0.35*	0.47*	1.03*	0.44
		(-1.12)	(-2.92)	(-2.46)	(-1.00)	(2.73)	{0.37; 0.68}	(1.26)	(1.79)	(1.76)	(1.77)	{0.32; 0.69}
	30	-0.38	-1.93**	-0.27	-0.32	0.72**	0.40	0.23	0.36	0.56*	0.87*	0.32
		(-0.88)	(-2.55)	(-1.60)	(-1.25)	(2.52)	{0.28; 0.66}	(1.32)	(1.46)	(1.90)	(1.73)	{0.18; 0.64}
	36	0.24	-2.09	-0.08	-0.33	0.41	0.28	0.32*	0.22	0.42	0.21	0.18
		(0.50)	(-1.61)	(-0.33)	(-1.13)	(1.24)	{0.17; 0.62}	(1.82)	(0.83)	(1.24)	(0.41)	{0.07; 0.55}
$\pi_{JAP.t+h}$	0	-0.79**	-0.88**	-0.41**	0.34**	0.63**	0.64	-0.35**	-0.48**	0.60**	-0.43	0.60
		(-2.76)	(-3.23)	(-8.21)	(2.86)	(3.18)	{0.54; 0.78}	(-4.67)	(-2.99)	(2.34)	(-1.59)	{0.50; 0.75}
	6	-0.94**	-0.85	-0.32**	0.03	0.03	0.53	-0.26**	-0.23	0.07	-0.20	0.48
		(-2.48)	(-1.53)	(-3.36)	(0.17)	(0.18)	{0.43; 0.74}	(-3.07)	(-1.09)	(0.26)	(-0.59)	{0.35; 0.71}
	12	-0.55	-0.65	-0.21**	-0.15	-0.48*	0.48	-0.21*	-0.02	-0.38	0.25	0.46
		(-1.23)	(-1.27)	(-1.99)	(-0.62)	(-1.66)	{0.36; 0.70}	(-1.65)	(-0.07)	(-1.15)	(0.72)	{0.32; 0.68}
	18	0.01	-0.71	-0.14	-0.08	-0.50	0.45	-0.26*	-0.04	-0.29	0.83**	0.48
		(0.02)	(-1.41)	(-1.20)	(-0.29)	(-1.47)	{0.33; 0.69}	(-1.86)	(-0.15)	(-0.85)	(2.73)	{0.35; 0.69}
	24	0.42	-1.41**	0.00	-0.13	-0.71**	0.46	-0.20**	0.11	-0.59**	0.93**	0.46
		(1.04)	(-2.79)	(0.00)	(-0.40)	(-2.75)	{0.34; 0.68}	(-2.28)	(0.34)	(-2.28)	(3.67)	{0.35; 0.69}
	30	1.01**	-0.74*	0.08	-0.18	-1.07**	0.54	-0.15	0.24	-0.91**	1.16**	0.55
		(2.29)	(-1.71)	(0.83)	(-0.98)	(-4.42)	{0.43; 0.72}	(-1.51)	(1.09)	(-3.73)	(5.18)	{0.44; 0.73}
	36	1.39**	0.26	0.07	-0.14	-1.18**	0.57	-0.13	0.23	-0.96**	1.20**	0.55
		(3.19)	(0.51)	(0.74)	(-1.00)	(-4.71)	{0.49; 0.74}	(-0.99)	(1.29)	(-2.95)	(3.86)	{0.45; 0.73}

Table 3.16: We report the results of forecasting regressions of *U.S.*, *U.K.*, *GER.* and *JAP.* economic activity (g_t) inflation rates (π_t) using as regressors the smoothed factors extracted from $\mathcal{M}_4^{2,3}$ and $\mathcal{M}_4^{0,4}$. We run regressions for annualized growth in industrial production and consumer price index over forecasting horizons of $h = 0, 12, 18, 24, 30$ and 36 months. We report estimates for parameters (multiplied by 10^3; constant terms, included in the regressions, are omitted from the table) along with t-values (in parenthesis) based on Newey and West (1987) standard errors, considering conditional heteroscedasticity and serial correlation up to 12 lags. One and two asterisks denote statistically significance at the 10% and 5% levels, respectively. We also report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use monthly data for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations).

Chapter 4

Economic Evidence of Hidden Factors in the International Bond Risk Premia

Abstract

This chapter investigates the relevance of hidden factors in international bond risk premia to forecast future excess bond returns and macroeconomic variables such as economic growth and inflation rate. Using maximum likelihood estimation of a linear Gaussian state-space model, adopted to explain the dynamics of expected excess bond returns of a given country, associated selection criteria detect as relevant, factors otherwise judged negligible by the classical explained variance approach adopted by Cochrane and Piazzesi (2005) and Cochrane and Piazzesi (2008). We call these factors hidden, meaning that they are not visible through the lens of a principal component analysis of expected excess bond returns. We find that these hidden factors are useful predictors of both future economic growth and inflation rate given that they add forecasting power over and above the information contained both in the Cochrane and Piazzesi (2008) and in yield curve factors. These empirical findings are robust across different sample periods and countries as well as with respect to the interpolation technique used in the construction of the international bond yield data sets.

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Résumé

Ce chapître examine l'importance des facteurs cachés, dans les rendements obbligataires des taux d'intérêts, pour prévoir des variables macro-économiques comme la croissance économique et l'inflation. Le modèle espace-état linéaire gaussien proposé dans le chapître trois est adapté à l'extraction et à la sélection d'une combinaison optimale de facteurs explicatifs des anticipations des rendements d'obligations sur les marchés internationaux. A l'aide d'une procédure de maximisation de la vraisemblance nous sélectionnons des facteurs qui seraient autrement considérés comme négligeables par la méthode classique de maximisation de la variance expliquée des données. On les appelle "facteurs cachés" dès qu'ils ne sont pas visibles à travers une analyse en composantes principales des rendements d'obligations. Nous découvrons que ces facteurs contribuent à la prévision de l'inflation et du taux da croissance de la production industrielle future en apportant un pouvoir prévisionnel au-delà de l'information fournie par le facteur détecté par Cochrane et Piazzesi (2008) et Cochrane et Piazzesi (2005) et par les facteurs qui déterminent la courbe des taux (Ang et Piazzesi (2003), Ang, Piazzesi et Wei (2006)). Pour finir, nous avons pu vérifier que les résultats obtenus sont confirmés dans des bases de données qui diffèrent en termes de nombre de type des pays, de périodes considérées et/ou de la technique d'interpolation choisie pour leur construction.

4.1 Introduction

The bond risk premia literature has focused not only on the problem of detecting factors to forecast future excess bond returns but it has also considered the problem of predicting important macroeconomic variables such as the economic growth [see Koijen, Lustig, and van Nieuwerburgh (2012) and Dahlquist and Hasseltoft (2013), among others].

In the first case, Cochrane and Piazzesi (2005) and Cochrane and Piazzesi (2008) have found that the tent shaped factor accounting for almost all the variability in (expected) excess bond returns (in the U.S. economy) is also a relevant predictor for them. Recent papers by Kessler and Scherer (2009) and Sekkel (2011), extending the analysis of Cochrane and Piazzesi (2005) and Cochrane and Piazzesi (2008) outside the U.S. economy, confirm the presence of one single return-forecasting factor in international bond markets. However, we conjecture a possible weakness behind their analysis given that, according to Duffee (2011), the methodology used in the extraction of factors is crucial to judge their importance from a time-series (forecast) perspective. More precisely, Duffee (2011) uses a Kalman filter-based maximum likelihood approach to estimate a five-dimensional Gaussian affine term structure model and he detects a factor which is found to be a good predictor of both the short rate and bond returns, even if it is hidden in the cross section of yields, being the associated explained variance negligible. In other words, we may have factors with a small explained variance but showing, at the same time, an important forecasting power about excess bond returns.

In the second case, several studies [see Cochrane and Piazzesi (2005), Koijen, Lustig, and van Nieuwerburgh (2012), Dahlquist and Hasseltoft (2013), among others] have investigated the link between the return-forecasting factor and economic growth for the U.S. and for several developed countries. In general, they find that this factor is countercyclical and contains information about future economic growth while the relationship with inflation is ignored, given that it is the yield curve level, and not excess bond returns, seems as related to inflation [see Ang and Piazzesi (2003), Diebold, Li, and Yue (2008), Cochrane and Piazzesi (2005) and Cochrane and Piazzesi (2008)]. However, as mentioned above, information in hidden factors could help in predicting economic growth and could disclose some forecasting power for inflation as well.

The purpose of this chapter is to fill this gap in the literature by investigating the relevance of factors hidden in international bond risk premia to forecast future excess bond returns as well as macroeconomic variables such as economic growth and inflation rates. We propose, using maximum likelihood (ML) criteria and parameters significance, within a linear Gaussian state-space approach, to estimate and select a preferred number of factors to explain expected excess bonds returns in four leading bond markets (namely, the U.S., U.K., German and Japanese bond markets). Then, these factors are used to predict excess bond returns, economic growth and inflation rates.

Our choice of a linear Gaussian state-space approach is important at least for two reasons. First, the Kalman filter-based maximum likelihood estimation of the state-space model might judge as important factors otherwise seen as negligible when the standard explained variance methodology is used. Second, the large time-series and cross-sectional dependence of bond returns and the typical small dimension of its maturity spectrum prevent us from successfully applying selection criteria of factor model analysis in order to detect the number of factors required to explain bond risk premia [see, among the others, Bai and Ng (2002), Greenaway-McGrevy, Han, and Sul (2012)].

This chapter is organized in two parts. In the first part, autoregressive stationary latent factors are used within our linear Gaussian state-space approach in order to explain expected excess bond returns for each of the four bond markets. We consider alternative model specifications characterized by a number of factors going from one, as in Cochrane and Piazzesi (2005) and Cochrane and Piazzesi (2008), to five, as suggested by the recent term structure literature [Adrian, Crump, and Moench (2013) and Duffee (2011)]. Maximum likelihood estimation of the state-space model is performed by using the EM algorithm with Kalman Filter and Kalman Smoother recursions [Engle and Watson (1981), Quah and Sargent (1993) and Doz, Giannone, and Reichlin (2011), Doz, Giannone, and Reichlin (2012)]]. We derive forward rates and construct expected excess bond returns by using the international Treasury yield curves database constructed in the Chapter 2, observed monthly from January 1, 1986 to December 31, 2009. We select the preferred number of factors by means of maximum likelihood criteria, namely the bootstrap variant of Akaike Information Criteria ($AICb$) of Cavanaugh and Shumway (1997) and the significance of parameters by performing a Nonparametric Monte Carlo bootstrap for state-space models of Stoffer and Wall (1991). The kind of bootstrap we adopt is a block stationary bootstrap able to properly taking into account the persistence of bond returns [Politis and Romano (1994) and Politis and White (2004), Patton, Politis, and White (2009)].

Given our extracted factors, in the second part of the chapter we investigate their relationships with future excess bond returns and macroeconomic variables. More precisely, we first evaluate the prediction of future excess bond returns by running regressions of one-year excess bond returns on the extracted factors and then, we check the contribution of the latters in forecasting both industrial production growth and inflation rates. Finally, we compare our results with competitors in the literature.

Our empirical analysis suggests that a state-space model with five factor is generally preferred. However, only one factor is statistically important to predict future excess bond returns and it closely tracks the return-forecasting factor. As a matter of fact, it seems that factors that marginally contribute to explain the variance of expected bond returns, are not useful in predicting them. Conversely, when we evaluate their contribution in predicting future macroeconomic variables in each economic area, we uncover an appealing result. More precisely, factors usually seen as capturing only idiosyncratic movements in bond returns, reveal to be important predictors for both economic growth and inflation rates. They add an incremental forecasting power over and above the information contained in both the return-forecasting factor and yield curve factors [see Ang and Piazzesi (2003), Diebold, Li, and Yue (2008)]. In other words, after having controlled for the forecasting power contained in the yield curve slope (level, respectively), we calculate the difference in percentage points (DPP) between the (adjusted) R^2 of predictive regressions of future industrial production growth (inflation rates, respectively) using our extracted factors and regressions which use the return-forecasting factor and yield curve factors instead. We evaluate these differences for forecasting horizons up to three years. Finally, we calculate the average difference across forecasting horizons ($ADDP$) and we construct a measure that we call, for the industrial production growth, $ADPP_g$ and, for inflation rates, $ADPP_\pi$. We find always positive values for the $ADPP_g$ across countries. They range from 14.6% in the U.K. to 3.2% in the U.S.. In Germany and Japan, we have an $ADPP_g$ of 6.6% and 4%, respectively. Our extracted factors play an important role in forecasting inflation as well. The $ADPP_\pi$ is always positive and ranges from 11.2% in Japan to 2.2% in U.S.. In Germany we obtain a value of 7% while in U.K. it is as big as 3.6%.

In order to check that these results are not due to the adopted sample period, to the market analyzed or to the interpolation technique used in the construction of forwards rates we perform a battery of robustness checks. We estimate state-space models by using international data from different sources [namely, Wright (2011), Fama and Bliss

(1987)] and for different sample periods by adopting both an increasing and a rolling window of observations in the estimations. Our main conclusions do not change: hidden factors are important predictors of economic growth and inflation rates across markets and sample periods and regardless the interpolation technique used in the construction of international data.

The chapter is organized as follows. In Section 4.2 we introduce the bond notation, the typical approaches used in the literature in order to predict excess bond returns and macroeconomic variables and we describe the state-space model used in the extraction of factors across different economies. Section 4.3 presents the empirical analysis, starting with the data description in Section 4.3.1 and state-space models estimations in Section 4.3.2 (for different countries and different number of factors). In Section 4.3.3 we present the results of in-sample forecasting regressions for excess bond returns and in Section 4.3.4 the in-sample forecasting regressions for the macroeconomic variables. Section 4.3.5 focus on robustness checks, while Section 4.4 concludes.

4.2 Literature

4.2.1 Bond Notation

Let us define by $p_{j,t}^{(n)}$ the log-price at time t in a country j of a discount bond with a residual maturity n. The relationship linking the log-price and the log-yield is:

$$y_{j,t}^{(n)} = -\frac{1}{n} p_{j,t}^{(n)}. \tag{4.1}$$

The (short) log-forward rate $f_{j,t}^{(n)}$ at time t for loans beginning at time $t+n-1$ and ending in $t+n$ is:

$$f_{j,t}^{(n)} = p_{j,t}^{(n-1)} - p_{j,t}^{(n)}. \tag{4.2}$$

The log-return from buying at time t, a bond with residual maturity n and selling it, in $t+1$, when its residual maturity is $n-1$ is:

$$r_{j,t+1}^{(n)} = p_{j,t+1}^{(n-1)} - p_{j,t}^{(n)}. \tag{4.3}$$

The log-return in excess over the one-year yield is:

$$rx_{j,t+1}^{(n)} = r_{j,t+1}^{(n)} - y_{j,t}^{(1)}. \tag{4.4}$$

4.2.2 Bond Returns Predictability and the link with the Macroeconomy

Several empirical studies have shown that forward rates contain important information about future excess bond returns. Fama and Bliss (1987), pioneers in this area of finance, use the spread between the n-year forward rate and the one-year yield in order to predict the one-year excess bond return at maturity n. They run the following regressions:

$$rx_{j,t+1}^{(n)} = \alpha_j^{(n)} + \beta_j^{(n)}(f_{j,t}^{(n)} - y_{j,t}^{(1)}) + \varepsilon_{j,t+1}^{(n)}, \tag{4.5}$$

where j is the U.S. and obtain an R^2 up to 14%. More recently, Cochrane and Piazzesi (2005) regress one-year excess bonds returns onto a constant and a set of forward rates:

$$\begin{aligned} rx_{j,t+1}^{(n)} &= \beta_{j,0}^{(n)} + \beta_{j,1}^{(n)} y_{j,t}^{(1)} + \beta_{j,2}^{(n)} f_{j,t}^{(2)} + \cdots + \beta_{j,k}^{(n)} f_{j,t}^{(k)} + \varepsilon_{j,t+1}^{(n)}, \\ &= \beta_{\mathbf{j}} \mathbf{f_{j,t}} + \varepsilon_{j,t+1}^{(n)}. \end{aligned} \tag{4.6}$$

where $\varepsilon_{j,t}^{(n)} \sim IIN(0, \sigma_j^{2(n)})$, $\beta_{\mathbf{j}} = [\beta_{j,0}^{(n)}, \beta_{j,1}^{(n)}, \beta_{j,2}^{(n)}, \ldots, \beta_{j,k}^{(n)}]$ and $\mathbf{f_{j,t}} = [1, y_{j,t}^{(1)}, f_{j,t}^{(2)}, \ldots, f_{j,t}^{(k)}]'$ and k is the biggest forward rates maturity available[1] and country j is the U.S.. They find a suggestive tent-shape pattern characterizing the regression coefficients[2] and detect a single-factor structure characterizing the bond risk premia at all maturities. In light of this result, Cochrane and Piazzesi (2005) describe the one-year excess bond returns as follows:

$$\begin{aligned} rx_{j,t+1}^{(n)} &= b_{j,n}(\gamma_{j,0} + \gamma_{j,1} y_{j,t}^{(1)} + \gamma_{j,2} f_{j,t}^{(2)} + \cdots + \gamma_{j,k} f_{j,t}^{(k)}) + \varepsilon_{j,t+1}^{(n)} \\ &= b_{j,n}(\gamma_{\mathbf{j}} \mathbf{f_{j,t}}) + \varepsilon_{j,t+1}^{(n)}, \end{aligned} \tag{4.7}$$

where $\gamma_{\mathbf{j}} = [\gamma_{j,0}^{(n)}, \gamma_{j,1}^{(n)}, \gamma_{j,2}^{(n)}, \ldots, \gamma_{j,k}^{(n)}]$ and $\frac{1}{k-1} \sum_{n=2}^{k} b_{j,n} = 1$ [see Cochrane and Piazzesi (2005) for details]. They estimate the above regression in two steps. First, they regress the average (across maturities) excess bond return on the available forwards rates:

$$\begin{aligned} \overline{rx}_{j,t+1} &= \gamma_{j,0} + \gamma_{j,1} y_{j,t}^{(1)} + \gamma_{j,2} f_{j,t}^{(2)} + \cdots + \gamma_{j,k} f_{j,t}^{(k)} + \overline{\varepsilon}_{j,t+1}, \\ &= \gamma_{\mathbf{j}} \mathbf{f_{j,t}} + \overline{\varepsilon}_{j,t+1}, \end{aligned} \tag{4.8}$$

[1] In their case k is equal to five since they use Fama and Bliss (1987) forward rates from one to five years of maturity. In our empirical analysis, we consider the case where k is equal to five and nine.

[2] The pattern of coefficients seems to depend on the interpolation technique used in the construction of the forward rates [Dai, Singleton, and Wei (2004)]. Contrary to the unsmoothed techniques, parametric methods, such as the ones proposed by Nelson and Siegel (1987) and Svensson (1994), smooth forwards rates across maturities. As a result, regression coefficients in (4.6) have a W-shape reflecting strong multicollinearity between regressors. In order to overcome this problem, Wright and Zhou (2009), Singleton (1978) and Sekkel (2011) use only three forward rates to explain the one-year excess bond returns. Others, like Cochrane and Piazzesi (2008) regress the one-year excess bond returns calculated using the Gurkaynak, Sack, and Wright (2007) database onto a constant and the Fama and Bliss forwards rates.

where $\overline{rx}_{j,t+1} = \frac{1}{k-1}\sum_{n=2}^{k} rx_{j,t+1}^{(n)}$. They then take the fitted values and construct the so called return-forecasting factor or Cochrane and Piazzesi factor (CP factor, hereafter):

$$x_{j,t} \quad := \quad \widehat{\gamma}_{\mathbf{j}}\mathbf{f_{j,t}} \tag{4.9}$$

Second, they estimate $b_{j,n}$ by regressing the one-year excess bond returns on the CP factor, as follows:

$$rx_{j,t+1}^{(n)} = b_{j,n}x_{j,t} + \varepsilon_{j,t+1}^{(n)}, \tag{4.10}$$

and they find an explanatory power for these regressions of 35% in the U.S[3].

In some recent studies, this analysis has been extended outside the U.S.. For instance, Dahlquist and Hasseltoft (2013) construct the CP factor and study excess bond predictability also in U.K., Germany and Switzerland while Sekkel (2011) enlarge the analysis to nine developed countries in the data sets of Wright (2011). Although they have found also that the CP factor is responsible of bond return predictability also at an international level, some evidence of heterogeneity appears as well, since its performance depends upon the country and the adopted sample period.

More recently, Cochrane and Piazzesi (2008) have proposed another way to construct the CP factor and show that it dominates the variance of expected returns. In practice, they calculate first expected excess bond returns:

$$E_t(rx_{j,t+1}^{(n)}) \quad = \quad \widehat{\beta}_{\mathbf{j}}\mathbf{f_{j,t}}. \tag{4.11}$$

[3]This value rises to 44% if three-month moving averages of forward rates are adopted instead of the current one alone. They argue that measurement errors is the main cause for this increase in the performance.

Afterwards, they conduct an eigenvalue-eigenvector decomposition of the covariance matrix of expected excess bond returns so that $Q\Lambda Q' = cov[E_t(rx_{j,t+1}^{(n)})]$, where Q are the eigenvectors and Λ is the diagonal matrix of eigenvalues. They define the CP factor as follows:

$$x_{j,t} = q'_\tau[E_t(rx_{j,t+1}^{(n)})], \tag{4.12}$$

where q_τ is the eigenvector in the matrix Q corresponding to the largest eigenvalue in the matrix Λ. This factor accounts for almost all (more than 99%) the variability in the expected excess bond returns, while the remaining ones are only responsible of idiosyncratic movements in each bond maturity and thus judged not economically meaningful.

They also use yield curve factors to predict excess bond returns:

$$rx_{j,t+1}^{(n)} = \alpha_j^{(n)} + \beta_j^{(n)} L_{j,t} + \gamma_j^{(n)} S_{j,t} + \delta_j^{(n)} C_{j,t} + \varepsilon_{j,t+1}^{(n)}, \tag{4.13}$$

where $\varepsilon_{j,t}^{(n)} \sim IIN(0, \sigma_j^{2(n)})$ and $L_{j,t}$, $S_{j,t}$ and $C_{j,t}$ are the level, slope and curvature factors, respectively. They find that these three factors do not perform as well as the CP factor in predicting excess bond returns and therefore the CP factor is judged not fully spanned by the yield curve. This finding is confirmed outside the U.S. economy by Dahlquist and Hasseltoft (2013).

From a macroeconomic perspective, the CP factor seems to contain important information about future economic growth. In particular, Koijen, Lustig, and van Nieuwerburgh

(2012) runs the following regressions:

$$g_{j,t+h} = \alpha_{j,h} + \beta_{j,h} x_{j,t} + \varepsilon_{j,t+h}, \tag{4.14}$$

where $\varepsilon_{j,t} \sim IIN(0, \sigma_j^2)$, h = 12, 18, 24, 30 and 36 months indicates the forecasting horizon, $g_{j,t}$ is the date t economic growth measured by the Chicago Fed National Activity Index and country j is the U.S.. They find that the CP factor is a strong predictor of future economic growth at 12 and 24 months ahead.

In an international context, Dahlquist and Hasseltoft (2013) run regressions where the yield curve slope, $S_{j,t}$, is the only predicting variable for industrial production growth:

$$g_{j,t+h} = \alpha_{j,h} + \gamma_{j,h} S_{j,t} + \varepsilon_{j,t+h}, \tag{4.15}$$

and regressions where the CP factor and the slope of the yield curve are both used as regressors,

$$g_{j,t+h} = \alpha_{j,h} + \beta_{j,h} x_{j,t} + \gamma_{j,h} S_{j,t} + \varepsilon_{j,t+h}. \tag{4.16}$$

They find that, at some forecasting horizons, both factors are statistically important in predicting future industrial production growth.

4.2.3 The state-space Approach

In this section we define the linear Gaussian state-space model that we adopt to describe the dynamics of expected excess bond returns of a given country. The model is specified by the following assumptions:

ASSUMPTION 1 (EXPECTED EXCESS BONDS RETURNS AND LATENT FACTORS): *We denote by $E_t(rx^{(n)}_{j,t+1})$ the $n \times 1$ vector of expected excess bond returns observed at time t for country j. We denote by $F_{j,t}$ the $k \times 1$ vector of latent factors at time t that explain expected excess bond returns, at all maturities, for country j.*

ASSUMPTION 2 (THE SINGLE-COUNTRY BOND RISK PREMIA MODEL): *For a given k-dimensional latent factor $F_{j,t}$, the dynamics of expected excess bonds returns $E_t(rx^{(n)}_{c,t+1})$ is given by:*

$$\begin{cases} E_t(rx^{(n)}_{j,t+1}) & = & \mu + \Lambda\, F_{j,t} + \varepsilon_t\,, \;\; \varepsilon_t \sim IIN(0,\Omega) \\ \\ F_{j,t} & = & \Phi\, F_{j,t-1} + \eta_t\,, \;\; \eta_t \sim IIN(0,\Psi_\eta)\,, \end{cases} \tag{4.17}$$

where μ is an $n \times 1$ vector of constants, Λ is $n \times k$ matrix of factors loadings and Ω is the $n \times n$ variance-covariance matrix of the Gaussian-distributed white noise ε_t. Φ is the $k \times k$ autoregressive matrix and Ψ_η is a $k \times k$ identity matrix .

To efficiently estimate the model for expected excess bond return, for any given country, we follow the EM-based recursive maximum likelihood estimation procedure adopted in section 3.2.1. While their methodology is adopted in a multi-country setting, here we want to explain the dynamics of expected excess bond returns in a single-country context. Following their notation, our model has $r^{(l)} = 0$, $r^{(c)} = k$, $\Lambda_l = 0$ and $\Lambda_{\mathcal{B}} = \Lambda_c = \Lambda$.

4.3 Empirical Analysis

The purpose of our empirical analysis is to select the number of factors required to explain expected excess bond returns in four leading bond markets and then to evaluate their ability to predict industrial production growth and inflation rates. The international data we use are described in Section 4.3.1. Model estimation results, adopting the estimation method of Section 4.2.3, along with the selection of preferred number of factors in each country, using the parameters'statistical significance and the bootstrap variant of the Akaike Information Criteria of Cavanaugh and Shumway (1997) ($AICb$, hereafter), are presented in Section 4.3.2. In Section 4.3.3 we assess excess bond return predictability while in Section 4.3.4 we focus on industrial production growth and inflation rates. In all these cases, we compare predictions obtained using our extracted factors with the ones we get by competitors in the literature (CP and yield curve factors). Finally, in Section 4.3.5 we run a battery of robustness checks in order to test whether our findings depend on the adopted sample period, on the analyzed market or on the interpolation technique used in the construction of the international data.

4.3.1 Data

We use the international Treasury yield curve database constructed in Chapter 2, [PST hereafter], consisting of four leading bond markets: the U.S., U.K., Germany, and Japan[4]. Yields are observed monthly, from January 1986 to December 2009.

We also consider data from other sources for robustness checks. In particular, for the U.S., we use yield data of Wright (2011)[5] and Fama and Bliss (1987). For Germany and

[4]The database is characterized by uniform level of liquidity and a (parsimonious smoothed) interpolation technique for the discount function. More precisely, Treasury coupon bonds data are filtered using the Gurkaynak, Sack, and Wright (2007) criteria and the discount function is interpolated using the Nelson and Siegel (1987) methodology.

[5]Yield curves for the *U.S.* economy are taken from Gurkaynak, Sack, and Wright (2007).

Japan, we consider the term structures in the international database of Wright (2011)[6]. Finally, we use monthly data for industrial production and consumer price index taken from OECD and calculate the annualized growth in industrial production growth and inflation rates in each economic area.

4.3.2 Model Selection: State-Space and Principal Component Factors Selection

In this section we estimate model (4.17) for several k and select the preferred number of factors required to explain expected excess bond returns in international bond markets. For any given country, we use the international yield curve database of PST, with residual maturity from one to nine years, to derive forward rates and construct expected excess bond returns. The latters are used to estimate the linear Gaussian state-space model described in Section 4.2.3. Different model specifications characterized by a number of factors going from 1, as in Cochrane and Piazzesi (2005) and Cochrane and Piazzesi (2008), to 5 factors, as suggested by the recent term structure literature [Adrian, Crump, and Moench (2013) and Duffee (2011)] are considered. We also perform principal components analysis of expected excess bond returns as in Cochrane and Piazzesi (2005) and Cochrane and Piazzesi (2008).

Table 4.1 provides the marginal and cumulative percentage of variance explained by factors extracted from the covariance matrix of expected excess bond returns. Table 4.2 reports the maximum value of the log-likelihood function for each estimated linear Gaussian state-space model along with the value of the $AICb$, while Table 4.3 shows the maximum likelihood estimates of the matrix loadings in Λ and of the autoregressive

[6] In the Appendix 4.D we also consider the international bond yields database kindly supplied by Diebold, Li, and Yue (2008). In this data set, term structures are estimated by means of the unsmoothed Fama and Bliss (1987) methodology.

coefficients in Φ as well as the associated bootstrap standard errors.

As can be seen from Table 4.1, the first principal component accounts for more than 95% of expected returns variance across bond markets suggesting a single-factor structure behind the international bond risk premia. However, when we consider maximum likelihood estimation results, additional factors seem to help explaining expected returns. From Table 4.2 we notice a substantial rise in the log-likelihood values when we increase the number of factors and, according to $AICb$, we systematically select the 5-factors model. As suggested by the model selection literature [Linhart and Zucchini (1986)], we may make this choice simply because this model has a larger factor's dimension with respect to the competing ones. In order to understand if this specification is truly required by the data, we evaluate whether these factors are statistically important to explain the dynamics of expected returns across countries. Form a careful inspection of Table 4.3, we note that loadings in Λ are all statistically different from zero at 5% significance level. We also find statistically significant coefficients outside the main diagonal of the matrix Φ thus suggesting causality between the estimated factors.

Clearly, the statistical approach adopted is relevant in the choice of the number of factors. Factors that we are tempted to consider responsible of only idiosyncratic movements on bond maturities, since they marginally contribute to explain the variance of expected returns, turn out to be important when we take into account the dynamics (and persistence) in bond risk premia[7].

At this point of the analysis, we have shown that hidden factors, i.e. not detected by the classical principal components methodology, are clearly helpful in describing expected excess bond returns. In the next sections, we will study their contribution in predicting

[7] Table 4.19 in the Appendix 4.E provides residual analysis of regressions of expected excess bond returns at different maturities on the CP factor. Residuals are conditionally heteroskedastic, strong serially correlated and, at some maturities, far from normality.

future excess bond returns and macroeconomic variables such as industrial production growth and inflation rates.

4.3.3 Predictability of Excess Bond Returns

In this section we present results from forecasting regressions concerning one-year excess bond returns. First, we consider classical predictors in the literature like the forward-spot spread, the CP factor and yield curve factors. In practice, we run regressions (4.5), (4.10) and (4.13), for each analyzed country. Results are reported in Table 4.4[8]. Then, we consider the following regressions:

$$rx^{(n)}_{j,t+1} \quad = \quad \mathbf{b_j}\widehat{F}_{j,t|T} + \varepsilon^{(n)}_{j,t+1}, \tag{4.18}$$

where $\mathbf{b_j} = [b^{(n)}_{j,0}, b^{(n)}_{j,1}, b^{(n)}_{j,2}, \ldots, b^{(n)}_{j,5}]$ and $\hat{F}_{j,t|T}$ is our 5-dimensional vector of extracted smoothed factors for country j. Table 4.5 shows the associated results.

Let us consider classical predictors first. Forward-spot spread provides the worst performances, while regressions involving the CP factor supply the best ones across markets, and regression coefficients associated to this factor are statistically significant across countries. The three (level, slope and curvature) yield curve factors do not over-perform the CP factor but they do better than the forward-spot spread. Looking at the statistical significance of coefficients, we notice that the level does not help in predicting

[8] We do not show results concerning the CP factor calculated as in (4.12). The reason is that the correlation between factors calculated either following (4.9) or (4.12) is always above 0.99. As a matter of fact, they provide similar forecasting results.

excess bond returns while the slope is important in Japan and, at long bond maturities, in Germany. Curvature is a statistically significant predictor both in U.K. and Japan.

As far as the five extracted smoothed factors are considered, only the first one reveals to be statistically important in describing future bond returns as highlighted in Table 4.5. The only exception is Japan, where the second factor presents statistically significant regressions coefficients at short maturities. The R^2 of these forecasting regressions are close to the ones using the CP factor[9] unlike a slight improvement for short bond maturities. However, 90 % confidence intervals for the R^2 have slightly higher lower and upper bounds than other competitors in the literature[10].

From this analysis we conclude that the first one of the five extracted factors contributes to forecast one-year excess bond returns. It produces R^2 similar to the CP ones (the best predictor among competitors in the literature) with a slight improvement for short bond maturities and in confidence intervals.

4.3.4 Predictability of Macroeconomic Variables

Predictability of Economic Growth

In this section we present forecasting regressions results of industrial production growth, and we consider forecasting horizons up to three years. We first run the classical regressions (4.14) and (4.15) to assess the predictive power contained in the CP factor and the yield curve slope, and then we use both predictors jointly by running regressions

[9] The first extracted factor closely tracks the CP one being the associated correlation above 0.98 across countries.

[10] Confidence intervals are calculated adopting the block stationary bootstrap method of Politis and Romano (1994) and Politis and White (2004). The blocks size is chosen following Patton, Politis, and White (2009). This procedure permits to properly taking into account the persistence and heteroskedasiticy of bond returns. Similar results are obtained using alternative bootstrap methods.

(4.16). Afterwards, we consider the following regressions:

$$g_{j,t+h} \quad = \quad \mathbf{b_{j,h}}\hat{F}_{j,t|T} + \varepsilon_{j,t+h}. \tag{4.19}$$

where $h = 12,\ 18,\ 24,\ 30$ and 36 months indicates the forecasting horizon, $\mathbf{b_{j,h}} = [b_{j,h,0}, b_{j,h,1}, b_{j,h,2}, \dots, b_{j,h,5}]$, and $\hat{F}_{j,t|T}$ is our 5-dimensional vector of extracted factors for country j. In this case we also take into account the predictability contained in the yield curve slope by running the following regressions:

$$g_{j,t+h} \quad = \quad \mathbf{b_{j,h}}\hat{F}_{j,t|T} + c_{j,h}S_{j,t} + \varepsilon_{j,t+h}. \tag{4.20}$$

where $S_{j,t}$ is the yield curve slope.

Results from classical methods are presented in Table 4.6, while the ones concerning our extracted factors are provided in Table 4.7.

Let us focus again on classical predictors first. When we adopt the CP factor and the yield curve slope as regressors, the former seems to provide additional information with respect to the latter at all forecasting horizons in U.K. (except for the 12 months) and at 12 and 18 months horizons in Germany. In Japan, it improves forecast precision till 24 months ahead. In fact, in all these cases, we observe statistically significant coefficients associated to the CP factors yielding higher R^2 with respect to the case where the yield curve slope is the only predicting variable. In U.S. only coefficients associated to the yield curve slope are statistically significant. Confidence intervals for the R^2, in regressions where the slope of the yield curve and the CP factor are considered jointly, have higher lower and upper bounds than regressions considering these two factors separately.

Next, we consider our extracted factors as predictors of future industrial production growth. An inspection of Table 4.7 highlights that, in all the bond markets considered, many predictability coefficients associated to these factors are statistically significant even when we control for the forecasting power provided by the yield curve slope. In this latter case, we notice higher R^2 with respect to the classical case where the CP and the yield curve slope are considered jointly. For example, in U.S. we observe an increase in the R^2 from 6% and 11% to 11% and 15%, for predictions 12 and 18 months ahead, respectively. In U.K., we notice a rise in the R^2 of 19% at 36 months horizon of 17% at 12 months horizon. For the other horizons, the average increment is around 12%. Also in Germany we observe, at all forecasting horizons, a general improvement in the R^2 but with a lower magnitude with respect to the case of U.K.. Indeed, the best results are obtained at 12 and at 36 months horizons: the R^2 rises from 13% and 16% to 18% and 24%, respectively. Finally, also in Japan we gain in terms of explanatory power by using our extracted factors. The peak is for forecasts 30 months ahead, where we move from a R^2 of 1% using the CP factor to 10% when the five extracted smoothed factors are adopted. Generally, lower and upper bound on the 90% confidence intervals for the R^2 are higher in these regressions than the one using CP factor and the yield curve slope as predictive variables.

To sum up results presented above, it is interesting to observe that our EM-based smoothed factors strongly help predicting industrial production growth and it outperforms the forecasting power of classical CP tent-shaped factor even when we control for the yield curve slope predictive power.

Predictability of Inflation

In this section, we present the results of forecasting regressions about inflation rates. Also in this case we consider forecasting horizons up to three years. We run five type of predicting regressions. First, we use the CP factor to predict future inflation rate as follows:

$$\pi_{j,t+h} \quad = \quad \alpha_{j,h} + \beta_{j,h} x_{j,t} + \varepsilon_{j,t+h}, \tag{4.21}$$

where $\varepsilon_{j,t} \sim IIN(0, \sigma^2_{j,t})$, h is the forecasting horizon and π_j is the inflation rate of country j. Second, we evaluate the predictive power of the yield curve level of country j, $L_{j,t}$ as follows:

$$\pi_{j,t+h} \quad = \quad \alpha_{j,h} + \beta_{j,h} L_{j,t} + \varepsilon_{j,t+h}. \tag{4.22}$$

Third, we consider the CP factor and yield curve level jointly as follows:

$$\pi_{j,t+h} = \alpha_{j,h} + \beta_{j,h} x_{j,t} + \gamma_{j,h} L_{j,t} + \varepsilon_{j,t+h}. \tag{4.23}$$

Fourth, we use our selected five smoothed factors:

$$\pi_{j,t+h} = \mathbf{b_{j,h}} \hat{F}_{j,t|T} + \varepsilon_{j,t+h}, \tag{4.24}$$

where $\mathbf{b_{j,h}} = [b_{j,h,0}, b_{j,h,1}, b_{j,h,2}, \ldots, b_{j,h,5}]$ and, finally, we consider these five factors and the yield curve level jointly:

$$\pi_{j,t+h} = \mathbf{b_{j,h}} \hat{F}_{j,t|T} + \gamma_{j,h} L_{j,t} + \varepsilon_{j,t+h}. \tag{4.25}$$

Given that excess bond returns are not related to inflation, considering the CP factor in the forecasting regressions (4.21) and (4.23) may appear useless. However, this allow us to precisely assess the forecasting power added by our extracted factors over and above the information contained in the return-forecasting factor.

Table 4.8 shows that the variable which mostly matter in predicting future inflation rates is the yield curve level. However, in regressions considering yield curve level and CP factor jointly, the latter appears to be an important predictor in U.K. even if, in the other economies, it has a modest impact on forecasts. For instance, it is only statistically significant at 36 months horizons in U.S. and for forecasts of 12, 18 and 24 months ahead in Germany, and only at 12 and 24 months horizons in Japan.

As far as our five extracted factors are concerned, Table 4.9 shows associated statistically significant predictive coefficients leading to higher R^2 with respect to regressions using the CP factor as predictive variable. As for industrial production growth, also for inflation rates the statistical significance of our factors is robust to the introduction of the level of the yield curve in the regressions. In addition, explanatory power of these regressions is higher than the ones considering the CP and the yield curve level jointly. For example in Germany, we find improvements at any forecasting horizons, in particular at the 12 months ahead horizon. In this case, predictive regressions considering our factors, along with the yield curve level, reach a R^2 of 59% while the one considering the CP factor and level jointly obtains a R^2 of 46%. In U.K. the R^2 rises from 76% to 82% and from 38% to 44% for the 12 and 36 months horizons, respectively. In Japan, the highest improvements are for 30 and 36 months ahead horizons, where our factors allow us to improve to 18% the R^2 compared to the case where the CP and level factors are used jointly. Finally, in the U.S., improvements seem to concern more long forecasting horizons, with R^2 rising by 5% for 36 months ahead forecasts. In all the countries analyzed, predictive regressions including our extracted factors and the yield curve level jointly produce higher lower and upper bound of 90% confidence intervals, for the R^2, than classical competitors.
This analysis seems to suggest that, our factors are not only important in forecasting future industrial production growth but they play an important rule also in predicting future inflation rates as well.

In summary, from our empirical analysis two important findings arise. First, factors not detected by the classical explained variance approach of Cochrane and Piazzesi (2005) and Cochrane and Piazzesi (2008), turn out to be important in describing the dynamic of expected excess bond returns, when a maximum likelihood estimation approach is applied to select them. Second, these factors are economically meaningful since they provide an incremental forecasting power over and above the information contained in classical regressors proposed in the literature[11].

4.3.5 Robustness Checks

The objective of this section is to evaluate whether the usefulness in using our factors to forecast industrial production growth and inflation rates is robust to a battery of robustness checks. We divide the analysis in two parts.

In the first part, we check the impact of different yield curve interpolation techniques on the predictability of future excess bond returns and macroeconomic variables. Forecasting results exploiting three data sets are compared. The first database is the one of PST already presented in section 4.3.1. The second one consists of monthly yield curves of U.S., Germany and Japan in the database of Wright (2011) which are estimated according to the Svensson methodology. The third one is composed by un-smoothed Fama and Bliss (1987) data for the U.S..

Given the different maturity spectrum and sample periods of the above mentioned data sets, we decide to provide reliable statistical comparisons of forecasting results by selecting common sample periods and maturity spectrum across databases. In particular, we select monthly observations from January 1986 to December 2009, while we take care

[11] In Table 4.18 in the Appendix 4.E, we verify that these findings are robust to the presence of measurement errors in the data. In particular, as in Cochrane and Piazzesi (2005) and Cochrane and Piazzesi (2008), we use a 3-months moving average forward rates in the construction of both the CP factor and expected excess bond returns used to estimate our state-space model. From this analysis we conclude that our EM-based factors outperform other competitors in the literature even after the moving average correction.

of different maturity spectrum in two ways. First, we select residual maturity from one to nine years for U.S., Germany and Japan in the database of Wright (2011). Second, we reduce the maturity spectrum of PST and Wright data sets to five years in order to include the Fama and Bliss (1987) data set in the comparison. As in section 4.3.2, we adopt a five factor model specification when a nine maturity spectrum is applied. However, for numerical reasons, we reduce the number of factors to three when we consider yield curves with residual maturity up to five years ($n = 4$).

In the second part, we focus on the impact of different sample periods on the predictability of future excess bond returns and macroeconomic variables. We select common time intervals for U.S., U.K. and Germany in the data set of Wright (2011) covering the sample period from 1973:01 to 2009:12. Coherently, we consider the same time interval in the data set of Fama and Bliss (1987). We decide to remove Japan from this analysis since data for this country starts only from January 1985. Here, in order to check for the predictive power over different time periods, we adopt both an increasing and a rolling window of observations in the regression estimations. In the former case, we start estimations with a sample of 180 monthly observations (15 years) and we add 1 observations (1 month) at each new estimation. In the latter case, we fix the width of the window to 288 observations and we roll it through the above mentioned period. Since Fama and Bliss (1987) data are included in the analysis, we decide to adopt a common maturity spectrum form one to five years across countries and a three factor linear Gaussian state-space model for estimations.

The extra forecasting power of our extracted factors, with respect to the CP one, in predicting the one-year excess bond return, industrial production growth (after having controlled for the predictive power of the yield curve slope) and inflation rates (after having controlled for the predictive power of the yield curve level) are evaluated. In particular, we calculate the difference between the R^2 of predictive regressions of future excess bond returns using our extracted factors and the ones which use the CP factor (denoted $DPP_{rx^{(n)}_{t+1}}$ hereafter). We also calculate the average $DPP_{rx^{(n)}_{t+1}}$ across maturity, namely $ADPP_{rx_{t+1}}$. Then, we calculate the difference between the R^2 of predictive regressions of future industrial production growth (inflation rates, respectively) using our extracted factors and the ones using CP factor, and we denote it $DPP_{g_{t+h}}$ ($DPP_{\pi_{t+h}}$, respectively). We also calculate their average across forecasting horizons, namely $ADPP_g$ ($ADPP_\pi$, respectively).

Empirical Results

As far as predictions of future excess bond returns are concerned, Table 4.10 shows that, in the sample period starting in January 1986 and ending in December 2009, the $DPP_{rx^{(n)}_{t+1}}$ is positive at short maturities while it tends to be slightly negative at longer ones[12]. If we look at the $ADPP_{rx_{t+1}}$, we notice values around zero across countries and databases with the only exception of Germany when we consider five years maturity spectrum data sets. We reach the same conclusions when different sample period are adopted[13]. On the basis of these results we conclude that the extracted factors do not outperform the CP one with the only exception of short maturities.

We focus next on the forecast of macroeconomic variables, and start with industrial production growth. Table 4.11 shows that the $ADPP_g$ are always positive regardless the yield curve estimation technique and the maturity spectrum considered in the analysis[14]. The highest $ADPP_g$ is detected in U.K. with a value of 14.6%, while the lowest is in

[12] We observe a general decrease in the $DPP_{rx^{(n)}_{t+1}}$ when a five maturity spectrum is considered with Germany being the only exception.

[13] Results are available upon request from the author.

[14] However, we observe a general reduction in the $ADPP_g$ when a five years maturity spectrum is considered.

Germany with 1.2% in the PST data set. When we extend the analysis to different sample periods we notice from Figure 4.2 that, regardless the fact to use an increasing or a rolling window of observations in the estimations, we always detect positive values for the $ADPP_g$.

Our factors contain useful information to forecast future inflation rates as well. The $ADPP_\pi$ is always positive but with different amplitude across countries. We note a general reduction in the $ADPP_\pi$ when we consider a five years maturity spectrum. However, this is not the case for U.S. and Japan yields when we use the PST data set. Moreover, we notice that $DPP_{\pi_{t+h}}$ is equal to zero for 12 and 18 months ahead forecasts in U.S., when a maturity spectrum from 1 to 9 years is considered. More precisely, in Japan we find the best performances, both using a nine and five years maturity spectrum, with $ADPP_\pi$ of 11.2% and 12.8% respectively, when the PST database is considered. Conversely, in U.K. the $ADPP_\pi$ is of 3.6% for a nine years maturity spectrum and only 1% when yields from one to five years are used. The $ADPP_\pi$ is also (always) positive across different sample periods in all the countries considered, as highlighted by Figure 4.1[15].

Tables 4.12 to 4.14 display that, regardless the database considered and for both the industrial production growth and inflation rates, lower and upper bounds of the 90% confidence intervals for the R^2 are higher in predictive regressions using our factors than competing regressors in the literature.

To sum up the results presented above, we have empirically verified that our findings in Section 4.3.3 and 4.3.4 are robust to a battery of robustness checks. In other words, considering different methods to interpolate international yield curves or different sample periods do not prevent our extracted factors outperform classical competitors in

[15] Although the $ADPP_\pi$ is always positive in U.S., the amplitude depends upon the interpolation technique used in the construction of the data.

forecasting macroeconomic variables[16].

4.4 Conclusions

This chapter documents the presence of relevant economic information hidden in the cross-section of expected excess bond returns of four leading markets (U.S., U.K., Germany and Japan).

We estimate and select the preferred number of factors driving excess bond returns both using a classical principal component approach and maximum likelihood criteria ($AICb$) and parameters significance within a linear Gaussian state-space approach.

We show that factors explaining only a small fraction of the variance of expected bond returns are judged important when a Kalman-filter based maximum likelihood estimation is applied. More importantly, these factors add a forecasting power over and above the information contained in classical predictors proposed by the literature, such as the return-forecasting factor and the yield curve factor [see Ang and Piazzesi (2003), Diebold, Li, and Yue (2008), Cochrane and Piazzesi (2005) and Cochrane and Piazzesi (2008)].

Our results are robust to battery of robustness checks making sure that our findings are not sample or country-dependent and do not rely on the interpolation technique used in the construction of international yield curves data.

[16] In Table 4.17 we show that we reach similar conclusions when we use the data set kindly supplied by Diebold, Li, and Yue (2008).

Appendix

4.A Model Estimation

	U.S.		*U.K.*		*GER*		*Japan*	
	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.	Marg.	Cum.
1	95.40	95.40	97.48	97.48	97.20	97.20	97.59	97.59
2	4.41	99.81	2.35	99.82	2.64	99.84	2.35	99.94
3	0.18	99.99	0.17	100.00	0.16	99.99	0.05	99.99
4	0.01	100.00	0.00	100.00	0.01	100.00	0.01	100.00
5	0.00	100.00	0.00	100.00	0.00	100.00	0.00	100.00
6	0.00	100.00	0.00	100.00	0.00	100.00	0.00	100.00
7	0.00	100.00	0.00	100.00	0.00	100.00	0.00	100.00
8	0.00	100.00	0.00	100.00	0.00	100.00	0.00	100.00
9	0.00	100.00	0.00	100.00	0.00	100.00	0.00	100.00

Table 4.1: We report principal component analysis of the expected excess returns of each given country. The first column lists the number of principal components, while the rest of the table is divided into four vertical blocks. Each of these blocks represents an individual country. The first and second columns of each block report marginal and cumulative percent variance explained for expected excess bond returns. We derive forward rates and calculate expected excess bond returns by using the international Treasury yield curve database constructed in Chapter 2. For any country, yields are observed monthly from January 1, 1986 to December 31, 2009 (288 observations) and with residual maturities from 1 to 9 years.

U.S.				U.K.			
k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	$AICb$	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	$AICb$
1	25	-928	1222	1	25	-1481	2234
2	36	1769	-4604	2	36	698	-2350
3	49	3462	-7514	3	49	2613	-5518
4	64	6135	-12658	4	64	2979	-6090
5	81	7756	-14800	5	81	3148	-6434
GER				**JAP**			
k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	$AICb$	k	Ξ	$\mathcal{L}(\widehat{\theta}_T^{MLE})$	$AICb$
1	25	-1243	1730	1	25	-1531	2076
2	36	1033	-2858	2	36	1128	-3098
3	49	3431	-7382	3	49	2615	-5918
4	64	4643	-9670	4	64	5145	-10798
5	81	6799	-10214	5	81	5783	-11932

Table 4.2: For any given country and for any given number of latent factors k we provide the number of parameters Ξ, the maximum value of the log-likelihood function ($\mathcal{L}(\widehat{\theta}_T^{MLE})$) and the associated bootstrap variant of the Akaike Information Criteria of Cavanaugh and Shumway (1997). We derive forward rates and calculate expected excess bond returns by using the international Treasury yield curve database constructed in Chapter 2. For any country, yields are observed monthly from January 1, 1986 to December 31, 2009 (288 observations) and with residual maturities from 1 to 9 years.

Country	$\Lambda_{\mathcal{B}}$					Φ				
	0,23**	0,12**	-0,04**	0,02**	0,00**	0,78**	-0,07	0,06**	0,04	-0,08
U.S.	(13,12)	(13,05)	(-10,58)	(11,67)	(14,15)	(17,13)	(-1,58)	(2,25)	(1,21)	(-1,56)
	0,46**	0,20**	-0,04**	0,00*	0,00**	-0,05	0,79**	-0,09**	-0,02	0,04
	(14,68)	(13,85)	(-16,37)	(-1,78)	(-12,55)	(-0,21)	(16,59)	(-2,66)	(-0,59)	(0,54)
	0,64**	0,22**	-0,01**	-0,02**	0,00**	0,05	0,00	0,85**	-0,09	0,13**
	(15,79)	(13,14)	(-2,90)	(-14,05)	(-15,18)	(0,92)	(-0,71)	(20,99)	(-1,55)	(2,03)
	0,78**	0,18**	0,02**	-0,01**	0,00**	0,03	-0,04	-0,12**	0,65**	0,09**
	(16,35)	(11,34)	(8,53)	(-11,76)	(15,38)	(1,47)	(-0,89)	(-2,17)	(11,90)	(2,07)
	0,88**	0,10**	0,03**	0,00**	0,00**	0,07	0,01	0,06*	-0,09	0,58**
	(15,84)	(7,81)	(15,66)	(4,37)	(16,10)	(1,59)	(0,26)	(1,72)	(-1,09)	(9,43)
	0,96**	-0,01	0,03**	0,01**	0,00**					
	(14,19)	(-0,74)	(10,15)	(11,31)	(-14,26)					
	1,04**	-0,15**	0,01**	0,01**	0,00**					
	(12,07)	(-14,52)	(2,92)	(14,17)	(-16,12)					
	1,12**	-0,30**	-0,03**	-0,01**	0,00**					
	(10,13)	(-15,31)	(-12,39)	(-10,03)	(16,33)					
	-0,48**	0,08**	0,09**	-0,01**	0,00**	0,51**	-0,15**	-0,20**	0,17**	0,18**
U.K.	(-22,03)	(10,09)	(20,87)	(-17,43)	(2,01)	(8,15)	(-3,34)	(-3,56)	(4,13)	(2,57)
	-0,83**	0,16**	0,06**	0,00	0,00**	-0,23**	0,79**	-0,18**	0,03	0,18**
	(-22,42)	(18,00)	(20,11)	(1,63)	(3,46)	(-4,35)	(17,84)	(-3,54)	(0,55)	(2,55)
	-1,10**	0,20**	0,02**	0,01**	0,00	-0,30**	-0,11*	0,60**	0,09	0,24**
	(-22,59)	(21,47)	(11,27)	(17,18)	(-1,56)	(-5,06)	(-1,95)	(9,64)	(1,50)	(2,67)
	-1,32**	0,18**	-0,02**	0,00**	0,00**	0,13*	0,11**	0,10*	0,89**	-0,09*
	(-22,83)	(21,08)	(-12,08)	(11,42)	(-4,41)	(1,64)	(2,88)	(1,70)	(23,88)	(-1,81)
	-1,51**	0,12**	-0,05**	0,00**	0,00**	0,36**	0,13	0,36**	-0,07	0,73**
	(-23,12)	(17,72)	(-20,89)	(-2,07)	(7,95)	(2,55)	(1,54)	(2,69)	(-0,94)	(14,27)
	-1,65**	0,02**	-0,04**	-0,01**	0,00**					
	(-23,23)	(5,77)	(-19,61)	(-11,19)	(-10,49)					
	-1,81**	-0,13**	-0,02**	0,00**	0,01**					
	(-23,08)	(-21,18)	(-15,26)	(-4,16)	(9,59)					
	-1,94**	-0,31**	0,04**	0,01**	0,00**					
	(-22,50)	(-21,35)	(15,86)	(13,57)	(-6,52)					
	-0,27**	-0,12**	0,07**	-0,02**	0,00**	0,51**	-0,16**	0,15**	-0,08	0,07**
GER	(-15,49)	(-11,90)	(13,94)	(-17,43)	(14,49)	(8,52)	(-2,27)	(4,59)	(0,14)	(3,10)
	-0,60**	-0,22**	0,07**	0,00*	0,00**	-0,07	0,81**	0,02	-0,01*	0,03
	(-16,25)	(-14,19)	(14,66)	(-1,85)	(-17,00)	(-1,11)	(18,71)	(0,72)	(-1,69)	(-0,84)
	-0,92**	-0,26**	0,03**	0,01**	0,00**	0,13**	0,01	0,76**	-0,14**	-0,03**
	(-16,52)	(-14,25)	(9,85)	(17,01)	(-13,73)	(3,72)	(0,26)	(18,92)	(-2,52)	(-2,13)
	-1,20**	-0,23**	-0,02**	0,01**	0,00**	-0,02	-0,04	-0,05	0,56**	-0,21**
	(-16,48)	(-12,89)	(-7,71)	(15,55)	(16,73)	(0,78)	(-1,13)	(-1,32)	(8,48)	(-4,76)
	-1,45**	-0,15**	-0,05**	0,00**	0,00**	0,07	-0,08**	0,00	-0,18*	0,67**
	(-16,09)	(-10,32)	(-14,24)	(-4,18)	(17,06)	(1,25)	(-2,00)	(-0,51)	(-1,88)	(20,65)
	-1,68**	-0,02**	-0,05**	-0,01**	0,00**					
	(-15,28)	(-3,71)	(-14,26)	(-16,66)	(-13,38)					
	-1,89**	0,14**	-0,01**	-0,01**	0,00**					
	(-14,15)	(14,49)	(-8,27)	(-15,83)	(-16,70)					
	-2,10**	0,32**	0,06**	0,01**	0,00**					
	(-12,88)	(13,69)	(13,46)	(16,19)	(15,58)					
	0,37**	0,12**	-0,03**	0,03**	0,00**	0,70**	0,09**	0,19**	-0,16**	0,08**
JAP	(13,58)	(12,23)	(-7,55)	(13,84)	(-14,44)	(13,02)	(2,50)	(4,10)	(-3,03)	(3,63)
	0,76**	0,22**	-0,04**	0,01**	0,00**	-0,02	0,86**	0,00	0,03	0,00
	(13,48)	(13,16)	(-12,45)	(10,86)	(14,28)	(0,41)	(23,85)	(-1,28)	(0,58)	(-1,01)
	1,09**	0,27**	-0,03**	-0,01**	0,00**	0,13**	0,04	0,78**	0,01	-0,07**
	(13,58)	(13,30)	(-12,15)	(-8,83)	(13,66)	(3,54)	(0,38)	(14,58)	(0,71)	(-2,60)
	1,36**	0,25**	0,00	-0,01**	0,00**	-0,24**	0,10**	0,13**	0,74**	0,07**
	(13,78)	(12,69)	(-0,50)	(-14,15)	(-13,92)	(-3,94)	(2,43)	(3,00)	(16,27)	(2,58)
	1,58**	0,17**	0,03**	0,00**	0,00**	0,00	0,02	-0,08	0,03	0,94**
	(13,98)	(10,61)	(12,43)	(-7,57)	(-14,12)	(0,24)	(0,12)	(-1,49)	(0,36)	(35,39)
	1,76**	0,02**	0,04**	0,01**	0,00**					
	(13,99)	(2,53)	(13,83)	(7,21)	(12,06)					
	1,90**	-0,17**	0,02**	0,01**	0,00**					
	(13,60)	(-13,47)	(11,03)	(11,09)	(13,39)					
	2,00**	-0,41**	-0,05**	-0,01**	0,00**					
	(12,63)	(-13,36)	(-13,53)	(-7,27)	(-13,18)					

Table 4.3: We report the maximum likelihood estimates of $\Lambda_{\mathcal{B}}$ and Φ parameters and the associated bootstrap t-students (in parenthesis) for state-space models in (4.17) for U.S., U.K., GER and JAP. We use Nonparametric Monte Carlo bootstrap [see Stoffer and Wall (1991) and Politis and Romano (1994); the optimal block sizes are chosen following Politis and White (2004) and Patton, Politis, and White (2009)]. One and two asterisks denote statistically significance at 10 and 5 percent levels, respectively.

4.B Forecasting Excess bond Returns

	n	$x_{U.S.,t}$	R^2	$f^{(n)}_{U.S.,t}-y^{(1)}_{U.S.,t}$	R^2	$L_{U.S.,t}$	$S_{U.S.,t}$	$C_{U.S.,t}$	R^2		n	$x_{GER,t}$	R^2	$f^{(n)}_{GER,t}-y^{(1)}_{GER,t}$	R^2	$L_{GER,t}$	$S_{GER,t}$	$C_{GER,t}$	R^2
$rx^{(n)}_{U.S.,t+1}$	2	0,23	0,05	0,15	0,00	4,43	-8,91	-32,91	0,02	$rx^{(n)}_{GER,t+1}$	2	0,15*	0,06	0,19	0,00	0,72	5,94	-52,80	0,00
		(1,43)	{0,00; 0,20}	(0,34)	{0,00; 0,16}	(1,06)	(-0,35)	(-0,25)	{0,01; 0,32}			(1,68)	{0,00; 0,21}	(0,41)	{0,00; 0,14}	(0,17)	(0,24)	(-0,74)	{0,00; 0,27}
	3	0,50*	0,06	0,30	0,01	6,71	-29,55	-135,69	0,03		3	0,41**	0,11	0,37	0,01	2,42	30,46	-179,00	0,03
		(1,66)	{0,00; 0,23}	(0,60)	{0,00; 0,17}	(0,91)	(-0,62)	(-0,55)	{0,01; 0,28}			(2,46)	{0,01; 0,28}	(0,73)	{0,00; 0,17}	(0,29)	(0,67)	(-1,16)	{0,01; 0,31}
	4	0,75*	0,08	0,39	0,01	7,94	-54,35	-240,01	0,04		4	0,68**	0,15	0,48	0,02	4,08	62,27	-295,87	0,05
		(1,79)	{0,00; 0,23}	(0,74)	{0,00; 0,15}	(0,82)	(-0,84)	(-0,70)	{0,01; 0,29}			(3,06)	{0,03; 0,33}	(0,87)	{0,00; 0,19}	(0,34)	(1,01)	(-1,25)	{0,02; 0,35}
	5	0,96*	0,08	0,47	0,01	8,85	-79,94	-323,14	0,04		5	0,93**	0,18	0,58	0,03	5,21	97,11	-377,17	0,07
		(1,88)	{0,00; 0,23}	(0,84)	{0,00; 0,14}	(0,78)	(-1,03)	(-0,77)	{0,01; 0,28}			(3,55)	{0,06; 0,36}	(0,98)	{0,00; 0,21}	(0,34)	(1,32)	(-1,20)	{0,02; 0,38}
	6	1,14**	0,08	0,54	0,02	9,78	-105,33	-384,61	0,05		6	1,16**	0,21	0,69	0,03	5,64	133,48	-418,95	0,09
		(1,96)	{0,00; 0,24}	(0,94)	{0,00; 0,11}	(0,77)	(-1,19)	(-0,80)	{0,01; 0,27}			(3,95)	{0,07; 0,38}	(1,08)	{0,00; 0,21}	(0,32)	(1,59)	(-1,09)	{0,03; 0,37}
	7	1,31**	0,09	0,62	0,02	10,81	-130,44	-429,91	0,06		7	1,37**	0,22	0,82	0,04	5,34	170,83*	-424,51	0,10
		(2,05)	{0,02; 0,18}	(1,03)	{0,00; 0,15}	(0,78)	(-1,34)	(-0,81)	{0,01; 0,24}			(4,29)	{0,07; 0,40}	(1,21)	{0,00; 0,22}	(0,27)	(1,84)	(-0,96)	{0,03; 0,39}
	8	1,48**	0,09	0,71	0,02	11,95	-155,35	-464,44	0,06		8	1,56**	0,23	0,98	0,05	4,38	208,98**	-398,90	0,11
		(2,14)	{0,01; 0,23}	(1,12)	{0,00; 0,15}	(0,79)	(-1,47)	(-0,80)	{0,01; 0,22}			(4,57)	{0,09; 0,42}	(1,36)	{0,00; 0,23}	(0,20)	(2,08)	(-0,80)	{0,03; 0,39}
	9	1,64**	0,09	0,80	0,02	13,17	-180,18	-492,12	0,07		9	1,74**	0,24	1,17	0,07	2,81	247,79**	-346,93	0,12
		(2,25)	{0,01; 0,22}	(1,20)	{0,00; 0,16}	(0,81)	(-1,59)	(-0,78)	{0,02; 0,27}			(4,82)	{0,10; 0,42}	(1,53)	{0,00; 0,25}	(0,12)	(2,29)	(-0,64)	{0,03; 0,39}
	n	$x_{U.K.,t}$	R^2	$f^{n}_{U.K.,t}-y^{1}_{U.K.,t}$	R^2	$L_{U.K.,t}$	$S_{U.K.,t}$	$C_{U.K.,t}$	R^2		n	$x_{JAP,t}$	R^2	$f^{n}_{JAP,t}-y^{1}_{JAP,t}$	R^2	$L_{JAP,t}$	$S_{JAP,t}$	$C_{JAP,t}$	R^2
$rx^{(n)}_{U.K.,t+1}$	2	0,35**	0,29	0,50*	0,06	-0,80	-22,58	-206,48**	0,11	$rx^{(n)}_{JAP,t+1}$	2	0,26**	0,52	1,50**	0,19	3,31	-53,78**	302,21**	0,43
		(4,00)	{0,14; 0,49}	(1,77)	{0,00; 0,27}	(-0,17)	(-1,20)	(-2,09)	{0,04; 0,38}			(7,11)	{0,38; 0,72}	(2,14)	{0,04; 0,49}	(1,15)	(-2,32)	(5,65)	{0,27; 0,72}
	3	0,62**	0,29	0,58	0,05	1,30	-37,90	-387,88**	0,12		3	0,55**	0,57	1,78**	0,25	6,59	-129,03**	616,61**	0,48
		(4,29)	{0,16; 0,44}	(1,64)	{0,00; 0,23}	(0,17)	(-1,09)	(-2,29)	{0,05; 0,37}			(8,13)	{0,42; 0,75}	(2,57)	{0,07; 0,54}	(1,19)	(-2,90)	(5,90)	{0,32; 0,74}
	4	0,84**	0,29	0,61	0,04	3,95	-55,06	-524,99**	0,13		4	0,81**	0,59	1,84**	0,24	8,49	-207,24**	859,35**	0,48
		(4,60)	{0,15; 0,45}	(1,36)	{0,00; 0,17}	(0,39)	(-1,11)	(-2,41)	{0,04; 0,35}			(8,62)	{0,45; 0,75}	(2,51)	{0,06; 0,51}	(1,09)	(-3,29)	(5,82)	{0,33; 0,73}
	5	1,01**	0,28	0,60	0,03	6,28	-76,45	-608,37**	0,13		5	1,02**	0,58	1,88**	0,21	9,41	-281,74**	1011,25**	0,47
		(4,78)	{0,14; 0,45}	(1,13)	{0,00; 0,13}	(0,52)	(-1,19)	(-2,39)	{0,04; 0,36}			(8,61)	{0,44; 0,74}	(2,29)	{0,05; 0,46}	(0,97)	(-3,54)	(5,52)	{0,32; 0,71}
	6	1,15**	0,27	0,63	0,02	8,17	-101,75	-644,38**	0,12		6	1,19**	0,57	1,89**	0,17	9,86	-350,95**	1073,48**	0,45
		(4,83)	{0,13; 0,44}	(1,03)	{0,00; 0,13}	(0,59)	(-1,30)	(-2,28)	{0,04; 0,37}			(8,26)	{0,49; 0,65}	(2,03)	{0,03; 0,41}	(0,87)	(-3,68)	(5,06)	{0,29; 0,69}
	7	1,26**	0,25	0,68	0,02	9,47	-129,29	-636,41**	0,12		7	1,32**	0,54	1,83*	0,13	10,18	-414,74**	1051,85**	0,42
		(4,72)	{0,11; 0,43}	(1,00)	{0,00; 0,12}	(0,61)	(-1,40)	(-2,10)	{0,03; 0,35}			(7,69)	{0,39; 0,70}	(1,77)	{0,01; 0,35}	(0,80)	(-3,78)	(4,46)	{0,28; 0,65}
	8	1,35**	0,23	0,80	0,03	10,14	-158,77	-595,26*	0,11		8	1,41**	0,50	1,71	0,10	10,58	-473,05**	951,41**	0,40
		(4,52)	{0,09; 0,41}	(1,10)	{0,00; 0,14}	(0,60)	(-1,50)	(-1,86)	{0,03; 0,34}			(6,94)	{0,33; 0,69}	(1,54)	{0,00; 0,31}	(0,75)	(-3,84)	(3,71)	{0,24; 0,61}
	9	1,42**	0,21	0,95	0,04	10,17	-189,48	-527,15	0,10		9	1,45**	0,45	1,53	0,08	11,17	-525,58**	775,14**	0,37
		(4,24)	{0,09; 0,39}	(1,19)	{0,00; 0,16}	(0,55)	(-1,58)	(-1,58)	{0,03; 0,34}			(5,99)	{0,29; 0,64}	(1,39)	{0,00; 0,26}	(0,74)	(-3,90)	(2,74)	{0,23; 0,56}

Table 4.4: We report the results of forecasting regressions of *U.S.*, *U.K.*, *GER* and *JAP* excess bond returns (rx_{t+1}) using as regressors the CP factor, the forward-spot spread and the yield curve factors (Level, Slope and Curvature).We report estimates for parameters (multiplied by 10^2; constant terms, included in the regressions, are omitted from the table) along with t-students (in parenthesis) based on Newey and West (1987) standard errors considering conditional heteroscedasticity and serial correlation up to 12 lags. One and two asterisks denote statistically significance at 10% and 5% levels, respectively. We also report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use monthly data (*PST* database) for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations) and with residual maturities from 1 to 9 years.

	n	$b^{(n)}_{U.S.,1}$	$b^{(n)}_{U.S.,2}$	$b^{(n)}_{U.S.,3}$	$b^{(n)}_{U.S.,4}$	$b^{(n)}_{U.S.,5}$	R^2		n	$b^{(n)}_{GER,1}$	$b^{(n)}_{GER,2}$	$b^{(n)}_{GER,3}$	$b^{(n)}_{GER,4}$	$b^{(n)}_{GER,5}$	R^2
$rx^{(n)}_{U.S.,t+1}$	2	-0,23**	-0,12	0,04	0,02	0,00	0,07	$rx^{(n)}_{GER,t+1}$	2	-0,27**	0,12	0,07	0,02	0,00	0,07
		(-2,07)	(-1,23)	(0,55)	(0,26)	(-0,01)	{0,04; 0,31}			(-2,82)	(1,42)	(1,50)	(0,38)	(0,06)	{0,06; 0,31}
	3	-0,46**	-0,20	0,04	0,00	0,00	0,07		3	-0,60**	0,22	0,07	0,00	0,00	0,12
		(-2,20)	(-1,09)	(0,25)	(-0,01)	(0,01)	{0,04; 0,31}			(-3,34)	(1,46)	(0,68)	(0,01)	(-0,03)	{0,08; 0,36}
	4	-0,64**	-0,22	0,01	-0,02	0,00	0,07		4	-0,92**	0,26	0,03	-0,01	0,00	0,15
		(-2,21)	(-0,87)	(0,05)	(-0,07)	(0,00)	{0,04; 0,30}			(-3,69)	(1,29)	(0,17)	(-0,08)	(-0,01)	{0,10; 0,39}
	5	-0,78**	-0,18	-0,02	-0,01	0,00	0,07		5	-1,20**	0,23	-0,02	-0,01	0,00	0,18
		(-2,16)	(-0,59)	(-0,06)	(-0,04)	(0,00)	{0,04; 0,29}			(-3,91)	(0,97)	(-0,09)	(-0,05)	(0,01)	{0,12; 0,42}
	6	-0,88**	-0,10	-0,03	0,00	0,00	0,07		6	-1,45**	0,15	-0,05	0,00	0,00	0,20
		(-2,11)	(-0,29)	(-0,10)	(0,01)	(0,00)	{0,03; 0,28}			(-4,04)	(0,55)	(-0,17)	(0,01)	(0,01)	{0,12; 0,44}
	7	-0,96**	0,01	-0,03	0,01	0,00	0,07		7	-1,68**	0,02	-0,05	0,01	0,00	0,21
		(-2,07)	(0,02)	(-0,08)	(0,04)	(0,00)	{0,03; 0,24}			(-4,12)	(0,08)	(-0,14)	(0,04)	(0,00)	{0,12; 0,45}
	8	-1,04**	0,15	-0,01	0,01	0,00	0,08		8	-1,89**	-0,14	-0,01	0,01	0,00	0,23
		(-2,04)	(0,32)	(-0,02)	(0,03)	(0,00)	{0,03; 0,27}			(-4,17)	(-0,41)	(-0,04)	(0,03)	(-0,01)	{0,13; 0,47}
	9	-1,12**	0,30	0,03	-0,01	0,00	0,08		9	-2,10**	-0,32	0,06	-0,01	0,00	0,24
		(-2,03)	(0,61)	(0,08)	(-0,03)	(0,00)	{0,03; 0,29}			(-4,23)	(-0,89)	(0,13)	(-0,03)	(0,01)	{0,14; 0,48}
	n	$b^{(n)}_{U.K.,1}$	$b^{(n)}_{U.K.,2}$	$b^{(n)}_{U.K.,3}$	$b^{(n)}_{U.K.,4}$	$b^{(n)}_{U.K.,5}$	R^2		n	$b^{(n)}_{JAP,1}$	$b^{(n)}_{JAP,2}$	$b^{(n)}_{JAP,3}$	$b^{(n)}_{JAP,4}$	$b^{(n)}_{JAP,5}$	R^2
$rx^{(n)}_{U.K.,t+1}$	2	-0,48**	-0,08	-0,09	0,01	0,00	0,32	$rx^{(n)}_{JAP,t+1}$	2	-0,37**	0,12**	0,03	0,03	0,00	0,58
		(-4,22)	(-0,87)	(-0,84)	(0,14)	(0,02)	{0,22; 0,48}			(-5,87)	(2,58)	(0,65)	(1,00)	(-0,07)	{0,49; 0,72}
	3	-0,83**	-0,16	-0,06	0,00	0,00	0,30		3	-0,76**	0,22**	0,04	0,01	0,00	0,61
		(-4,55)	(-1,08)	(-0,35)	(-0,03)	(0,01)	{0,18; 0,49}			(-6,08)	(2,48)	(0,44)	(0,17)	(0,01)	{0,49; 0,81}
	4	-1,10**	-0,20	-0,02	-0,01	-0,01	0,29		4	-1,09**	0,27**	0,03	-0,01	0,00	0,61
		(-4,75)	(-1,11)	(-0,09)	(-0,07)	(-0,04)	{0,18; 0,50}			(-6,12)	(2,03)	(0,22)	(-0,09)	(-0,01)	{0,49; 0,79}
	5	-1,32**	-0,19	0,01	-0,02	-0,01	0,28		5	-1,36**	0,25	0,00	-0,01	0,00	0,59
		(-4,76)	(-0,90)	(0,03)	(-0,06)	(-0,07)	{0,18; 0,49}			(-6,02)	(1,42)	(0,02)	(-0,10)	(-0,02)	{0,46; 0,77}
	6	-1,50**	-0,12	0,03	-0,01	-0,01	0,26		6	-1,59**	0,17	-0,03	0,00	0,00	0,56
		(-4,64)	(-0,53)	(0,08)	(-0,04)	(-0,07)	{0,16; 0,49}			(-5,86)	(0,76)	(-0,11)	(-0,02)	(-0,01)	{0,45; 0,74}
	7	-1,65**	-0,02	0,01	-0,01	-0,03	0,24		7	-1,76**	0,02	-0,04	0,01	0,00	0,53
		(-4,39)	(-0,10)	(0,02)	(-0,02)	(-0,11)	{0,15; 0,46}			(-5,66)	(0,09)	(-0,15)	(0,05)	(0,00)	{0,42; 0,71}
	8	-1,80**	0,13	-0,01	-0,02	-0,02	0,23		8	-1,90**	-0,17	-0,02	0,01	0,00	0,50
		(-4,23)	(0,45)	(-0,01)	(-0,04)	(-0,07)	{0,13; 0,45}			(-5,46)	(-0,57)	(-0,07)	(0,06)	(0,00)	{0,38; 0,70}
	9	-1,94**	0,30	-0,07	-0,03	-0,03	0,22		9	-2,00**	-0,41	0,05	-0,01	0,00	0,47
		(-4,05)	(0,98)	(-0,14)	(-0,06)	(-0,11)	{0,13; 0,42}			(-5,26)	(-1,22)	(0,13)	(-0,03)	(-0,01)	{0,35; 0,67}

Table 4.5: We report the results of forecasting regressions of $U.S.$, $U.K.$, GER and JAP excess bond returns (rx_{t+1}) using as regressors the smoothed factors extracted from the five factors state-space model.We report estimates for parameters (constant terms, included in the regressions, are omitted from the table) along with t-students (in parenthesis) based on Newey and West (1987) standard errors considering conditional heteroscedasticity and serial correlation up to 12 lags. One and two asterisks denote statistically significance at 10% and 5% levels, respectively. We also report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use monthly data (PST database) for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations) and with residual maturities from 1 to 9 years.

4.C Forecasting Macro Variables

	h	$x_{U.S.,t}$	R^2	$S_{U.S.,t}$	R^2	$S_{U.S.,t}$	$x_{U.S.,t}$	R^2
$g_{U.S.,t+h}$	12	0,84**	0,06	-86,00*	0,03	-8,60	0,80	0,06
		(2,01)	{0,00; 0,26}	(-1,83)	{0,00; 0,26}	(-0,13)	(1,36)	{0,00; 0,30}
	18	1,02*	0,10	-140,75**	0,09	-81,14	0,60	0,11
		(1,67)	{0,00; 0,31}	(-2,03)	{0,00; 0,35}	(-1,18)	(0,85)	{0,01; 0,38}
	24	1,05*	0,10	-187,52**	0,18	-176,00**	0,12	0,17
		(1,73)	{0,00; 0,32}	(-2,12)	{0,02; 0,45}	(-2,35)	(0,36)	{0,04; 0,43}
	30	1,34**	0,16	-221,32**	0,25	-192,05**	0,30	0,25
		(2,09)	{0,02; 0,39}	(-2,50)	{0,06; 0,52}	(-2,58)	(1,04)	{0,09; 0,52}
	36	1,44**	0,18	-240,53**	0,27	-205,40**	0,37*	0,28
		(2,52)	{0,04; 0,45}	(-2,73)	{0,07; 0,56}	(-2,39)	(1,75)	{0,08; 0,57}

	h	$x_{GER,t}$	R^2	$S_{GER,t}$	R^2	$S_{GER,t}$	$x_{GER,t}$	R^2
$g_{GER,t+h}$	12	0,02	0,00	195,54	0,09	289,99**	-0,79**	0,13
		(0,06)	{0,00; 0,08}	(1,59)	{0,00; 0,34}	(2,29)	(-3,14)	{0,03; 0,42}
	18	0,02	0,00	208,64*	0,10	306,91**	-0,83**	0,14
		(0,04)	{0,00; 0,07}	(1,78)	{0,00; 0,35}	(2,59)	(-3,35)	{0,05; 0,39}
	24	0,31	0,01	227,94*	0,12	287,90*	-0,50	0,13
		(1,01)	{0,00; 0,11}	(1,83)	{0,01; 0,34}	(1,93)	(-1,45)	{0,02; 0,36}
	30	0,71**	0,05	255,59*	0,14	252,91	0,02	0,14
		(2,36)	{0,00; 0,22}	(1,93)	{0,01; 0,36}	(1,62)	(0,07)	{0,02; 0,38}
	36	0,91**	0,08	280,15**	0,16	249,73*	0,25	0,16
		(2,98)	{0,00; 0,29}	(2,29)	{0,02; 0,43}	(1,68)	(0,64)	{0,02; 0,43}

	h	$x_{U.K.,t}$	R^2	$S_{U.K.,t}$	R^2	$S_{U.K.,t}$	$x_{U.K.,t}$	R^2
$g_{U.K.,t+h}$	12	0,29	0,05	-121,41**	0,14	-113,25**	0,07	0,14
		(1,59)	{0,00; 0,22}	(-3,40)	{0,02; 0,40}	(-2,99)	(0,43)	{0,03; 0,39}
	18	0,48**	0,13	-143,76**	0,20	-115,16**	0,25*	0,23
		(2,66)	{0,01; 0,33}	(-3,07)	{0,05; 0,45}	(-2,56)	(1,71)	{0,07; 0,49}
	24	0,59**	0,20	-134,17**	0,18	-87,55*	0,42**	0,26
		(3,88)	{0,05; 0,44}	(-2,91)	{0,03; 0,42}	(-1,87)	(3,01)	{0,11; 0,51}
	30	0,58**	0,20	-110,33**	0,13	-59,69	0,46**	0,23
		(4,28)	{0,05; 0,46}	(-2,67)	{0,01; 0,37}	(-1,31)	(3,08)	{0,08; 0,49}
	36	0,54**	0,18	-90,65**	0,09	-39,90	0,47**	0,19
		(4,04)	{0,03; 0,43}	(-2,28)	{0,00; 0,32}	(-1,09)	(4,00)	{0,06; 0,46}

	h	$x_{JAP,t}$	R^2	$S_{JAP,t}$	R^2	$S_{JAP,t}$	$x_{JAP,t}$	R^2
$g_{JAP,t+h}$	12	-0,11	0,00	-161,96	0,02	-381,71**	-0,73**	0,05
		(-0,35)	{0,00; 0,09}	(-0,97)	{0,00; 0,18}	(-2,04)	(-4,17)	{0,01; 0,27}
	18	-0,18	0,00	-112,96	0,01	-322,79*	-0,69**	0,04
		(-0,69)	{0,00; 0,11}	(-0,67)	{0,00; 0,14}	(-1,68)	(-3,84)	{0,00; 0,23}
	24	-0,22	0,00	-77,52	0,00	-275,71	-0,66**	0,03
		(-0,85)	{0,00; 0,13}	(-0,48)	{0,00; 0,11}	(-1,36)	(-2,22)	{0,00; 0,25}
	30	-0,04	0,00	-96,99	0,00	-210,09	-0,37	0,01
		(-0,16)	{0,00; 0,10}	(-0,63)	{0,00; 0,12}	(-1,01)	(-1,10)	{0,00; 0,18}
	36	0,16	0,00	-104,31	0,00	-107,72	-0,01	0,00
		(0,76)	{0,00; 0,11}	(-0,77)	{0,00; 0,13}	(-0,59)	(-0,04)	{-0,01; 0,17}

Table 4.6: We report the results of forecasting regressions of *U.S.*, *U.K.*, *GER* and *JAP* economic activity (g_{t+h}) using as regressors the CP factor, the yield curve slope and the CP factor and the yield curve slope jointly. We run regressions for annualized growth in industrial production over forecasting horizons of $h = 12$, 18, 24, 30 and 36 months. We report estimates for parameters (multiplied by 10^2; constant terms, included in the regressions, are omitted from the table) along with t-students (in parenthesis) based on Newey and West (1987) standard errors considering conditional heteroscedasticity and serial correlation up to 12 lags. One and two asterisks denote statistically significance at 10% and 5% levels, respectively. We also report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use monthly data (*PST* database) for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations) and with residual maturities from 1 to 9 years.

	h	$b_{U.S.,h,1}$	$b_{U.S.,h,2}$	$b_{U.S.,h,3}$	$b_{U.S.,h,4}$	$b_{U.S.,h,5}$	R^2	$b_{U.S.,h,1}$	$b_{U.S.,h,2}$	$b_{U.S.,h,3}$	$b_{U.S.,h,4}$	$b_{U.S.,h,5}$	$S_{U.S.,t}$	R^2
$g_{U.S.,t+h}$	12	-0,50	0,00	-0,15	0,31	-0,52	0,10	-0,85*	0,20	-0,29**	0,49	-0,65	133,68	0,11
		(-1,50)	(-0,02)	(-0,98)	(1,06)	(-0,89)	{0,04; 0,42}	(-1,73)	(0,78)	(-2,68)	(1,28)	(-1,04)	(1,22)	{0,09; 0,42}
	18	-0,56	0,09	-0,23	0,50**	-0,52	0,15	-0,70	0,16	-0,28**	0,57*	-0,58	51,85	0,15
		(-1,33)	(0,37)	(-1,49)	(2,05)	(-1,34)	{0,07; 0,46}	(-1,08)	(0,44)	(-2,00)	(1,69)	(-1,17)	(0,35)	{0,09; 0,46}
	24	-0,49	0,35	-0,26**	0,49**	-0,32	0,18	-0,25	0,24	-0,19	0,38	-0,21	-82,14	0,18
		(-1,57)	(1,34)	(-2,30)	(2,02)	(-1,39)	{0,09; 0,47}	(-0,72)	(0,76)	(-1,24)	(1,53)	(-0,81)	(-0,85)	{0,10; 0,49}
	30	-0,68**	0,56**	-0,17	0,37**	-0,18	0,25	-0,37	0,41*	-0,07	0,23*	-0,03	-106,72	0,26
		(-2,15)	(2,13)	(-1,35)	(2,17)	(-0,89)	{0,13; 0,53}	(-1,46)	(1,66)	(-0,43)	(1,83)	(-0,13)	(-1,52)	{0,16; 0,57}
	36	-0,80**	0,60**	-0,14	0,49	-0,07	0,28	-0,37**	0,40	-0,01	0,29	0,14	-146,34	0,30
		(-2,79)	(2,11)	(-0,81)	(1,53)	(-0,29)	{0,15; 0,59}	(-2,53)	(1,46)	(-0,06)	(1,42)	(0,52)	(-1,58)	{0,18; 0,63}

	h	$b_{GER,h,1}$	$b_{GER,h,2}$	$b_{GER,h,3}$	$b_{GER,h,4}$	$b_{GER,h,5}$	R^2	$b_{GER,h,1}$	$b_{GER,h,2}$	$b_{GER,h,3}$	$b_{GER,h,4}$	$b_{GER,h,5}$	$S_{GER,t}$	R^2
$g_{GER,t+h}$	12	0,97**	-1,24**	-0,31	-0,81*	-0,45	0,18	0,98**	-1,21**	-0,30	-0,80	-0,44*	9,81	0,18
		(2,69)	(-3,08)	(-1,59)	(-1,82)	(-1,39)	{0,11; 0,46}	(2,54)	(-2,66)	(-1,47)	(-1,44)	(-1,66)	(0,08)	{0,12; 0,46}
	18	0,58*	-1,26**	0,16	-0,57	-0,33	0,14	0,83**	-0,67	0,26	-0,24	-0,05	205,68*	0,16
		(1,80)	(-2,77)	(0,63)	(-1,32)	(-1,10)	{0,07; 0,42}	(3,16)	(-1,48)	(1,04)	(-0,61)	(-0,21)	(1,83)	{0,09; 0,45}
	24	-0,03	-1,06*	0,42	-0,41	-0,31	0,11	0,31	-0,20	0,58*	0,08	0,10	298,20*	0,15
		(-0,11)	(-1,86)	(1,37)	(-0,90)	(-0,94)	{0,04; 0,35}	(0,82)	(-0,45)	(1,77)	(0,21)	(0,49)	(1,74)	{0,07; 0,38}
	30	-0,72**	-0,85	0,56**	-0,25	-0,36	0,13	-0,38	0,00	0,72**	0,24	0,04	296,08	0,17
		(-2,56)	(-1,55)	(2,15)	(-0,58)	(-0,96)	{0,06; 0,43}	(-1,01)	(0,01)	(2,54)	(0,61)	(0,15)	(1,60)	{0,12; 0,27}
	36	-1,15**	-0,60	0,63**	0,12	-0,51	0,15	-0,66*	0,58	0,86**	0,80	0,05	421,42**	0,24
		(-3,45)	(-1,29)	(2,28)	(0,25)	(-1,40)	{0,07; 0,46}	(-1,84)	(1,35)	(3,24)	(1,63)	(0,17)	(2,06)	{0,15; 0,52}

	h	$b_{U.K.,h,1}$	$b_{U.K.,h,2}$	$b_{U.K.,h,3}$	$b_{U.K.,h,4}$	$b_{U.K.,h,5}$	R^2	$b_{U.K.,h,1}$	$b_{U.K.,h,2}$	$b_{U.K.,h,3}$	$b_{U.K.,h,4}$	$b_{U.K.,h,5}$	$S_{U.K.,t}$	R^2
$g_{U.K.,t+h}$	12	0,00	0,09	-0,21	-0,03	-0,42**	0,22	0,25**	-0,41	-0,12	-0,61**	-0,35**	-247,90**	0,31
		(-0,03)	(0,49)	(-1,06)	(-0,18)	(-2,87)	{0,11; 0,48}	(2,33)	(-1,55)	(-0,74)	(-2,28)	(-2,62)	(-3,22)	{0,19; 0,56}
	18	0,00**	0,00	0,00**	0,00	0,00**	0,58	-0,20	-0,14	-0,18	-0,55**	-0,29**	-200,81**	0,37
		(4,70)	(-0,86)	(4,09)	(-1,02)	(-2,26)	{0,18; 0,56}	(-1,54)	(-0,66)	(-1,25)	(-2,43)	(-3,04)	(-2,50)	{0,24; 0,64}
	24	0,01**	0,00	0,00**	0,00	0,00**	0,55	-0,47**	0,13	-0,34**	-0,16	-0,36**	-50,47	0,37
		(4,46)	(-0,48)	(3,65)	(-0,67)	(-2,12)	{0,23; 0,64}	(-3,61)	(0,92)	(-2,35)	(-1,03)	(-3,09)	(-1,03)	{0,25; 0,64}
	30	0,01**	0,00	0,00**	0,00	0,00*	0,53	-0,43**	0,21*	-0,40**	0,28**	-0,45**	80,93**	0,35
		(4,00)	(0,39)	(2,61)	(-0,65)	(-1,77)	{0,22; 0,62}	(-2,39)	(1,88)	(-3,61)	(2,43)	(-4,07)	(2,44)	{0,23; 0,63}
	36	0,00**	0,00	0,00**	0,00	0,00	0,44	-0,44**	0,36**	-0,31**	0,72**	-0,42**	211,97**	0,38
		(3,28)	(1,03)	(2,08)	(-1,09)	(-1,18)	{0,17; 0,60}	(-4,07)	(3,24)	(-1,97)	(6,14)	(-4,45)	(6,00)	{0,28; 0,67}

	h	$b_{JAP,h,1}$	$b_{JAP,h,2}$	$b_{JAP,h,3}$	$b_{JAP,h,4}$	$b_{JAP,h,5}$	R^2	$b_{JAP,h,1}$	$b_{JAP,h,2}$	$b_{JAP,h,3}$	$b_{JAP,h,4}$	$b_{JAP,h,5}$	$S_{JAP,t}$	R^2
$g_{JAP,t+h}$	12	0,05	-1,19*	-0,12	0,10	0,22	0,08	1,21**	0,02	0,93*	1,46**	-0,04	-1056,89**	0,11
		(0,13)	(-1,83)	(-0,28)	(0,36)	(0,68)	{0,04; 0,32}	(2,44)	(0,03)	(1,75)	(2,27)	(-0,16)	(-2,54)	{0,07; 0,39}
	18	0,03	-1,15*	0,02	0,16	0,15	0,07	0,81**	-0,34	0,73	1,08*	-0,03	-714,13**	0,08
		(0,10)	(-1,70)	(0,04)	(0,49)	(0,45)	{0,03; 0,20}	(2,29)	(-0,59)	(1,45)	(1,71)	(-0,10)	(-2,06)	{0,05; 0,26}
	24	-0,04	-1,33**	0,12	0,23	0,16	0,10	0,07	-1,22**	0,21	0,36	0,13	-97,94	0,09
		(-0,10)	(-2,08)	(0,29)	(0,73)	(0,49)	{0,05; 0,20}	(0,15)	(-2,21)	(0,47)	(0,67)	(0,49)	(-0,25)	{0,05; 0,19}
	30	-0,39	-1,34**	0,22	0,30	0,16	0,10	-0,57	-1,53**	0,05	0,08	0,20	170,49	0,10
		(-1,08)	(-2,70)	(0,54)	(0,87)	(0,46)	{0,06; 0,37}	(-1,03)	(-2,77)	(0,10)	(0,16)	(0,65)	(0,37)	{0,07; 0,39}
	36	-0,70**	-1,22**	0,02	0,61*	0,19	0,08	-0,63	-1,15**	0,08	0,69	0,17	-62,17	0,08
		(-2,35)	(-2,76)	(0,05)	(1,84)	(0,61)	{0,04; 0,38}	(-0,92)	(-2,33)	(0,16)	(1,01)	(0,69)	(-0,11)	{0,05; 0,38}

Table 4.7: We report the results of forecasting regressions of $U.S.$, $U.K.$, GER and JAP economic activity (g_{t+h}) using as regressors the the smoothed factors extracted from the five factors state-space model. We run regressions for annualized growth in industrial production over forecasting horizons of $h = 12$, 18, 24, 30 and 36 months. We report estimates for parameters (constant terms, included in the regressions, are omitted from the table) along with t-students (in parenthesis) based on Newey and West (1987) standard errors considering conditional heteroscedasticity and serial correlation up to 12 lags. One and two asterisks denote statistically significance at 10% and 5% levels, respectively. We also report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use monthly data (PST database) for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations) and with residual maturities from 1 to 9 years.

	h	$x_{U.S.,t}$	R^2	$L_{U.S.,t}$	R^2	$L_{U.S.,t}$	$x_{U.S.,t}$	R^2		h	$x_{GER,t}$	R^2	$L_{GER,t}$	R^2	$L_{GER,t}$	$x_{GER,t}$	R^2
$\pi_{U.S.,t+h}$	12	0,12	0,01	12,80**	0,28	13,64**	-0,09	0,28	$\pi_{GER,t+h}$	12	-0,04	0,00	17,35**	0,44	17,90**	-0,10**	0,46
		(1,19)	{0,00; 0,14}	(3,39)	{0,12; 0,51}	(3,53)	(-1,28)	{0,13; 0,52}			(-0,83)	{0,00; 0,10}	(4,53)	{0,24; 0,65}	(4,84)	(-2,05)	{0,28; 0,67}
	18	0,17	0,03	11,63**	0,21	11,62**	0,00	0,21		18	-0,04	0,00	17,37**	0,46	17,84**	-0,10*	0,48
		(1,06)	{0,00; 0,17}	(2,60)	{0,06; 0,46}	(2,69)	(0,00)	{0,06; 0,47}			(-0,59)	{0,00; 0,10}	(4,25)	{0,28; 0,67}	(4,70)	(-1,66)	{0,31; 0,68}
	24	0,16	0,03	8,91**	0,12	8,45*	0,05	0,12		24	-0,09	0,02	15,39**	0,36	16,02**	-0,14*	0,41
		(1,06)	{0,00; 0,17}	(1,98)	{0,00; 0,36}	(1,93)	(0,37)	{0,01; 0,38}			(-0,96)	{0,00; 0,15}	(3,45)	{0,14; 0,62}	(3,91)	(-1,66)	{0,23; 0,64}
	30	0,24	0,06	6,85	0,07	5,26	0,17	0,09		30	-0,11	0,02	12,84**	0,25	13,43**	-0,14	0,30
		(1,34)	{0,00; 0,24}	(1,51)	{0,00; 0,31}	(1,31)	(1,12)	{0,01; 0,34}			(-1,17)	{0,00; 0,17}	(2,85)	{0,04; 0,56}	(3,25)	(-1,57)	{0,11; 0,59}
	36	0,27	0,07	5,22	0,04	3,23	0,23*	0,08		36	-0,10	0,02	10,56**	0,17	11,06**	-0,13	0,20
		(1,61)	{0,00; 0,25}	(1,18)	{0,00; 0,24}	(0,87)	(1,66)	{0,01; 0,31}			(-1,13)	{0,00; 0,16}	(2,37)	{0,00; 0,51}	(2,67)	(-1,45)	{0,04; 0,54}
	h	$x_{U.K.,t}$	R^2	$L_{U.K.,t}$	R^2	$L_{U.K.,t}$	$x_{U.K.,t}$	R^2		h	$x_{JAP,t}$	R^2	$L_{JAP,t}$	R^2	$L_{JAP,t}$	$x_{JAP,t}$	R^2
$\pi_{U.K.,t+h}$	12	-0,18	0,06	19,35**	0,60	21,23**	-0,29**	0,76	$\pi_{JAP,t+h}$	12	-0,05	0,01	13,95**	0,44	14,69**	-0,09*	0,48
		(-1,64)	{0,00; 0,34}	(5,52)	{0,44; 0,80}	(9,59)	(-6,79)	{0,68; 0,86}			(-0,57)	{0,00; 0,18}	(5,30)	{0,27; 0,63}	(6,06)	(-1,75)	{0,31; 0,69}
	18	-0,22*	0,10	18,56**	0,54	20,61**	-0,33**	0,74		18	-0,05	0,01	13,55**	0,41	14,23**	-0,09	0,45
		(-1,84)	{0,00; 0,40}	(4,73)	{0,31; 0,77}	(9,35)	(-6,85)	{0,67; 0,84}			(-0,56)	{0,00; 0,19}	(5,09)	{0,22; 0,64}	(5,63)	(-1,51)	{0,30; 0,67}
	24	-0,27**	0,14	15,89**	0,39	17,85**	-0,34**	0,62		24	-0,07	0,02	12,47**	0,34	13,21**	-0,11*	0,40
		(-2,13)	{0,00; 0,44}	(3,53)	{0,14; 0,71}	(5,16)	(-5,40)	{0,49; 0,78}			(-0,84)	{0,00; 0,24}	(4,48)	{0,14; 0,61}	(4,90)	(-1,94)	{0,24; 0,63}
	30	-0,30**	0,18	13,34**	0,27	15,20**	-0,36**	0,52		30	-0,07	0,02	12,16**	0,32	12,86**	-0,11	0,38
		(-2,38)	{0,01; 0,50}	(2,71)	{0,03; 0,62}	(3,23)	(-4,09)	{0,35; 0,76}			(-0,76)	{0,00; 0,21}	(4,02)	{0,12; 0,57}	(4,32)	(-1,53)	{0,20; 0,63}
	36	-0,26**	0,13	11,51**	0,20	12,99**	-0,31**	0,38		36	-0,06	0,01	11,77**	0,30	12,34**	-0,09	0,34
		(-2,10)	{0,00; 0,46}	(2,31)	{0,01; 0,59}	(2,45)	(-2,88)	{0,18; 0,71}			(-0,64)	{0,00; 0,21}	(3,48)	{0,08; 0,59}	(3,72)	(-1,31)	{0,15; 0,61}

Table 4.8: We report the results of forecasting regressions of $U.S.$, $U.K.$, GER and JAP inflation rates (π_{t+h}) using as regressors the CP factor, the yield curve slope and the CP factor and the yield curve slope jointly. We run regressions for annualized growth in industrial production over forecasting horizons of $h = 12$, 18, 24, 30 and 36 months. We report estimates for parameters (multiplied by 10^2; constant terms, included in the regressions, are omitted from the table) along with t-students (in parenthesis) based on Newey and West (1987) standard errors considering conditional heteroscedasticity and serial correlation up to 12 lags. One and two asterisks denote statistically significance at 10% and 5% levels, respectively. We also report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use monthly data (PST database) for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations) and with residual maturities from 1 to 9 years.

	h	$b_{U.S.,h,1}$	$b_{U.S.,h,2}$	$b_{U.S.,h,3}$	$b_{U.S.,h,4}$	$b_{U.S.,h,5}$	R^2	$b_{U.S.,h,1}$	$b_{U.S.,h,2}$	$b_{U.S.,h,3}$	$b_{U.S.,h,4}$	$b_{U.S.,h,5}$	$L_{U.S.,t}$	R^2
$\pi_{U.S.,t+h}$	12	-0,17**	-0,06	0,15**	0,18**	0,05	0,20	0,05	-0,06	0,01	0,00	-0,05	13,59**	0,28
		(-2,21)	(-0,63)	(2,44)	(2,44)	(0,34)	{0,09; 0,46}	(0,68)	(-0,70)	(0,18)	(-0,05)	(-0,38)	(2,33)	{0,16; 0,55}
	18	-0,17	-0,02	0,10	0,23**	0,04	0,16	0,02	-0,04	-0,03	0,09	-0,09	12,24	0,21
		(-1,50)	(-0,31)	(1,50)	(3,61)	(0,30)	{0,08; 0,46}	(0,18)	(-0,48)	(-0,41)	(1,14)	(-0,52)	(1,51)	{0,12; 0,50}
	24	-0,11	0,08	0,07	0,23**	-0,03	0,12	0,07	0,05	-0,05	0,10	-0,19	12,18*	0,15
		(-1,16)	(1,09)	(1,07)	(2,70)	(-0,28)	{0,05; 0,41}	(0,77)	(0,80)	(-0,59)	(1,41)	(-1,42)	(1,88)	{0,07; 0,46}
	30	-0,16	0,07	0,02	0,16**	-0,01	0,09	0,00	0,05	-0,09	0,05	-0,16	11,12**	0,12
		(-1,35)	(0,77)	(0,32)	(2,04)	(-0,07)	{0,04; 0,37}	(0,01)	(0,60)	(-1,45)	(0,57)	(-1,42)	(2,17)	{0,06; 0,42}
	36	-0,17	0,04	-0,08*	0,24**	0,09	0,13	-0,10	0,03	-0,13*	0,19**	0,02	4,66	0,13
		(-1,51)	(0,43)	(-1,80)	(3,62)	(1,00)	{0,06; 0,41}	(-1,08)	(0,37)	(-1,84)	(2,72)	(0,16)	(0,69)	{0,07; 0,41}

	h	$b_{GER,h,1}$	$b_{GER,h,2}$	$b_{GER,h,3}$	$b_{GER,h,4}$	$b_{GER,h,5}$	R^2	$b_{GER,h,1}$	$b_{GER,h,2}$	$b_{GER,h,3}$	$b_{GER,h,4}$	$b_{GER,h,5}$	$L_{GER,t}$	R^2
$\pi_{GER,t+h}$	12	0,01	0,12	-0,20*	0,13	0,16*	0,18	0,02	0,02	-0,04	0,10	0,20**	18,18**	0,59
		(0,09)	(1,14)	(-1,69)	(1,32)	(1,95)	{0,10; 0,47}	(0,48)	(0,26)	(-1,14)	(1,63)	(5,59)	(5,70)	{0,50; 0,76}
	18	0,14	0,10	-0,29**	0,10	0,03	0,18	0,15**	0,00	-0,16**	0,07	0,06	16,40**	0,54
		(1,37)	(1,06)	(-2,63)	(1,12)	(0,36)	{0,11; 0,48}	(2,84)	(0,07)	(-4,54)	(1,17)	(1,53)	(4,73)	{0,43; 0,74}
	24	0,26**	0,10	-0,31**	0,04	-0,02	0,23	0,28**	0,01	-0,21**	0,01	0,00	13,67**	0,47
		(2,29)	(0,95)	(-2,84)	(0,49)	(-0,24)	{0,13; 0,51}	(3,93)	(0,12)	(-4,10)	(0,16)	(0,08)	(4,10)	{0,37; 0,69}
	30	0,25**	0,13	-0,25**	0,04	-0,05	0,19	0,26**	0,05	-0,16**	0,01	-0,04	11,03**	0,35
		(3,12)	(1,28)	(-2,95)	(0,44)	(-0,78)	{0,10; 0,49}	(2,85)	(0,59)	(-2,42)	(0,14)	(-0,72)	(3,06)	{0,22; 0,65}
	36	0,25**	0,08	-0,16**	0,05	-0,16**	0,12	0,26**	0,00	-0,08	0,02	-0,15**	9,93**	0,25
		(3,10)	(0,94)	(-2,11)	(0,58)	(-2,58)	{0,07; 0,46}	(3,26)	(-0,01)	(-1,17)	(0,32)	(-3,21)	(2,57)	{0,16; 0,63}

	h	$b_{U.K.,h,1}$	$b_{U.K.,h,2}$	$b_{U.K.,h,3}$	$b_{U.K.,h,4}$	$b_{U.K.,h,5}$	R^2	$b_{U.K.,h,1}$	$b_{U.K.,h,2}$	$b_{U.K.,h,3}$	$b_{U.K.,h,4}$	$b_{U.K.,h,5}$	$L_{U.K.,t}$	R^2
$\pi_{U.K.,t+h}$	12	0,47**	-0,05	0,32**	0,01	-0,13**	0,50	0,23**	0,12**	-0,11	0,05	0,21**	33,61**	0,82
		(4,96)	(-0,53)	(3,00)	(0,15)	(-3,02)	{0,37; 0,76}	(5,05)	(2,19)	(-1,26)	(1,18)	(4,22)	(7,96)	{0,79; 0,89}
	18	0,00**	0,00	0,00**	0,00	0,00**	0,58	0,29**	0,05	0,07	-0,03	0,15**	25,41**	0,76
		(4,70)	(-0,86)	(4,09)	(-1,02)	(-2,26)	{0,47; 0,77}	(5,38)	(0,88)	(0,77)	(-0,51)	(3,08)	(5,13)	{0,72; 0,87}
	24	0,01**	0,00	0,00**	0,00	0,00**	0,55	0,38**	0,05	0,16	-0,03	0,08	16,57**	0,63
		(4,46)	(-0,48)	(3,65)	(-0,67)	(-2,12)	{0,43; 0,75}	(5,05)	(0,58)	(1,56)	(-0,37)	(0,93)	(2,28)	{0,55; 0,81}
	30	0,01**	0,00	0,00**	0,00	0,00*	0,53	0,45**	0,08	0,21*	-0,03	0,00	8,45	0,55
		(4,00)	(0,39)	(2,61)	(-0,65)	(-1,77)	{0,41; 0,76}	(4,36)	(0,92)	(1,83)	(-0,45)	(0,01)	(1,02)	{0,45; 0,79}
	36	0,00**	0,00	0,00**	0,00	0,00	0,44	0,37**	0,11	0,24*	-0,08	-0,04	3,20	0,44
		(3,28)	(1,03)	(2,08)	(-1,09)	(-1,18)	{0,31; 0,72}	(3,30)	(1,26)	(1,82)	(-1,02)	(-0,37)	(0,39)	{0,34; 0,74}

	h	$b_{JAP,h,1}$	$b_{JAP,h,2}$	$b_{JAP,h,3}$	$b_{JAP,h,4}$	$b_{JAP,h,5}$	R^2	$b_{JAP,h,1}$	$b_{JAP,h,2}$	$b_{JAP,h,3}$	$b_{JAP,h,4}$	$b_{JAP,h,5}$	$L_{JAP,t}$	R^2
$\pi_{JAP,t+h}$	12	0,08	-0,06	-0,02	-0,11**	0,20**	0,42	0,18**	-0,02	0,01	-0,02	-0,08	20,38**	0,49
		(1,05)	(-0,93)	(-0,32)	(-2,12)	(4,66)	{0,29; 0,66}	(3,32)	(-0,41)	(0,20)	(-0,46)	(-1,22)	(3,99)	{0,37; 0,72}
	18	0,05	-0,05	0,09	-0,16**	0,16**	0,43	0,17**	-0,01	0,13**	-0,04	-0,18*	24,84**	0,53
		(0,60)	(-0,75)	(1,61)	(-2,48)	(3,81)	{0,30; 0,66}	(2,70)	(-0,19)	(2,94)	(-0,82)	(-1,79)	(3,30)	{0,43; 0,74}
	24	0,05	-0,11*	0,12*	-0,14*	0,14**	0,42	0,16**	-0,08	0,15**	-0,04	-0,17**	22,78**	0,51
		(0,79)	(-1,76)	(1,85)	(-1,92)	(3,69)	{0,32; 0,65}	(2,37)	(-1,42)	(2,85)	(-0,71)	(-2,11)	(3,41)	{0,41; 0,70}
	30	-0,02	-0,22**	0,17**	-0,07	0,13**	0,50	0,07	-0,19**	0,19**	0,02	-0,13**	18,99**	0,56
		(-0,30)	(-3,78)	(3,12)	(-1,15)	(3,66)	{0,40; 0,71}	(1,12)	(-4,17)	(3,64)	(0,31)	(-2,06)	(3,52)	{0,47; 0,74}
	36	-0,09	-0,23**	0,20**	-0,01	0,11**	0,48	-0,01	-0,21**	0,22**	0,07	-0,11	16,35**	0,52
		(-0,95)	(-4,12)	(2,89)	(-0,16)	(2,89)	{0,37; 0,69}	(-0,11)	(-4,51)	(2,92)	(0,84)	(-1,59)	(3,29)	{0,42; 0,72}

Table 4.9: We report the results of forecasting regressions of $U.S.$, $U.K.$, GER and JAP inflation rates (π_{t+h}) using as regressors the the smoothed factors extracted from the five factors state-space model. We run regressions for annualized growth in industrial production over forecasting horizons of $h = 12$, 18, 24, 30 and 36 months. We report estimates for parameters (constant terms, included in the regressions, are omitted from the table) along with t-students (in parenthesis) based on Newey and West (1987) standard errors considering conditional heteroscedasticity and serial correlation up to 12 lags. One and two asterisks denote statistically significance at 10% and 5% levels, respectively. We also report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use monthly data (PST database) for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations) and with residual maturities from 1 to 9 years.

4.D Other Databases

	n	9 Years Maturity SPT	9 Years Maturity W	5 Years Maturity SPT	5 Years Maturity W	5 Years Maturity FB
$DPP_{rx^{(n)}_{U.S.,t+1}}$	2	2,00	6,00	1,00	1,00	0,00
	3	1,00	2,00	0,00	0,00	0,00
	4	-1,00	1,00	-1,00	-1,00	-1,00
	5	-1,00	0,00	-1,00	-1,00	0,00
	6	-1,00	-1,00			
	7	-2,00	-1,00			
	8	-1,00	0,00			
	9	-1,00	1,00			
	$ADPP_{rx_{U.S.,t+1}}$	***-0,50***	***1,00***	***-0,25***	***-0,25***	***-0,25***
$DPP_{rx^{(n)}_{U.K.,t+1}}$	2	3,00		1,00		
	3	1,00		0,00		
	4	0,00		0,00		
	5	0,00		0,00		
	6	-1,00				
	7	-1,00				
	8	0,00				
	9	1,00				
	$ADPP_{rx_{U.K.,t+1}}$	***0,38***		***0,25***		
$DPP_{rx^{(n)}_{GER,t+1}}$	2	1,00	2,00	0,00	1,00	
	3	1,00	1,00	2,00	5,00	
	4	0,00	0,00	4,00	6,00	
	5	0,00	-1,00	5,00	7,00	
	6	-1,00	-1,00			
	7	-1,00	-1,00			
	8	0,00	-1,00			
	9	0,00	0,00			
	$ADPP_{rx_{GER,t+1}}$	***0,00***	***-0,13***	***2,75***	***4,75***	
$DPP_{rx^{(n)}_{JAP,t+1}}$	2	6,00	1,00	1,00	0,00	
	3	4,00	1,00	0,00	-1,00	
	4	2,00	0,00	0,00	-1,00	
	5	1,00	0,00	0,00	-1,00	
	6	-1,00	0,00			
	7	-1,00	-1,00			
	8	0,00	0,00			
	9	2,00	1,00			
	$ADPP_{rx_{JAP,t+1}}$	***1,63***	***0,25***	***0,25***	***-0,75***	

Table 4.10: We report the $DPP_{rx^{(n)}_{t+1}}$ of U.S., U.K., GER and JAP using as regressors the the smoothed factors extracted from five factors state-space models and the ones using the CP factor. We calculate the $DPP_{rx^{(n)}_{t+1}}$ for $n = 2, 3, \ldots, 9$ and we average them across maturity ($ADPP_{rx_{t+1}}$). The first column lists bond maturities for each country, while the rest of the table is formed by one main block. The first and the second columns report results considering residual maturities from 1 to 9 years while the third, the fourth and the fifth columns consider residual maturities from 1 to 5 years, respectively. We consider the SPT and Wright databases for a maturity spectrum of nine years while we include also the Fama and Bliss data set when we consider a five maturity spectrum instead. We use monthly data for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations).

		9 Years		5 Years					9 Years		5 Years		
	h	SPT	W	SPT	W	FB		h	SPT	W	SPT	W	FB
$DPP_{g_{U.S.,t+h}}$	12	8,00	9,00	5,00	7,00	8,00	$DPP_{\pi_{U.S.,t+h}}$	12	0,00	0,00	2,00	-1,00	4,00
	18	4,00	13,00	1,00	5,00	8,00		18	0,00	0,00	3,00	0,00	7,00
	24	1,00	5,00	0,00	1,00	1,00		24	3,00	6,00	8,00	8,00	7,00
	30	1,00	1,00	1,00	1,00	0,00		30	3,00	10,00	7,00	10,00	6,00
	36	2,00	0,00	3,00	-1,00	1,00		36	5,00	13,00	8,00	10,00	8,00
$ADPP_{g_{U.S.}}$		***3,20***	***5,60***	***2,00***	***2,60***	***3,60***	$ADPP_{\pi_{U.S.}}$		***2,20***	***5,80***	***5,60***	***5,40***	***6,40***
$DPP_{g_{U.K.,t+h}}$	12	17,00		6,00			$DPP_{\pi_{U.S.,t+h}}$	12	6,00		3,00		
	18	14,00		2,00				18	2,00		1,00		
	24	11,00		5,00				24	1,00		-1,00		
	30	12,00		10,00				30	3,00		0,00		
	36	19,00		17,00				36	6,00		2,00		
$ADPP_{g_{U.K.}}$		***14,60***		***8,00***			$ADPP_{\pi_{U.K.}}$		***3,60***		***1,00***		
$DPP_{g_{GER,t+h}}$	12	5,00	11,00	1,00	2,00		$DPP_{\pi_{GER,t+h}}$	12	13,00	8,00	3,00	1,00	
	18	2,00	1,00	0,00	2,00			18	6,00	6,00	3,00	4,00	
	24	2,00	4,00	1,00	0,00			24	6,00	2,00	3,00	1,00	
	30	3,00	6,00	2,00	0,00			30	5,00	1,00	2,00	2,00	
	36	8,00	3,00	2,00	3,00			36	5,00	3,00	0,00	-1,00	
$ADPP_{g_{GER}}$		***4,00***	***5,00***	***1,20***	***1,40***		$ADPP_{\pi_{GER}}$		***7,00***	***4,00***	***2,20***	***1,40***	
$DPP_{g_{JAP,t+h}}$	12	6,00	2,00	6,00	4,00		$DPP_{\pi_{JAP,t+h}}$	12	1,00	5,00	5,00	5,00	
	18	4,00	5,00	6,00	2,00			18	8,00	7,00	7,00	3,00	
	24	6,00	9,00	9,00	3,00			24	11,00	10,00	10,00	0,00	
	30	9,00	15,00	11,00	5,00			30	18,00	12,00	20,00	1,00	
	36	8,00	11,00	7,00	3,00			36	18,00	21,00	22,00	8,00	
$ADPP_{g_{JAP}}$		***6,60***	***8,40***	***7,80***	***3,40***		$ADPP_{\pi_{JAP}}$		***11,20***	***11,00***	***12,80***	***3,40***	

Table 4.11: We report the difference in percentage points between the adjusted R^2 of predictive regressions of $U.S.$, $U.K.$, GER and JAP economic activity (g_{t+h}) and inflation rates (π_{t+h}) using the smoothed factors extracted from five factors state-space models and the ones which use the CP factor ($DPP_{g_{t+h}}$ and $DPP_{\pi_{t+h}}$). We run regressions for industrial production growth and inflation rates over forecasting horizons of h = 12, 18, 24, 30 and 36 months. We also display the their average difference in percentage points across forecasting horizons ($ADPP_g$) and ($ADPP_\pi$). We consider the SPT and Wright databases for a maturity spectrum of nine years while when we include the Fama and Bliss data set when we consider a five maturity spectrum instead. We use monthly data for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations) .

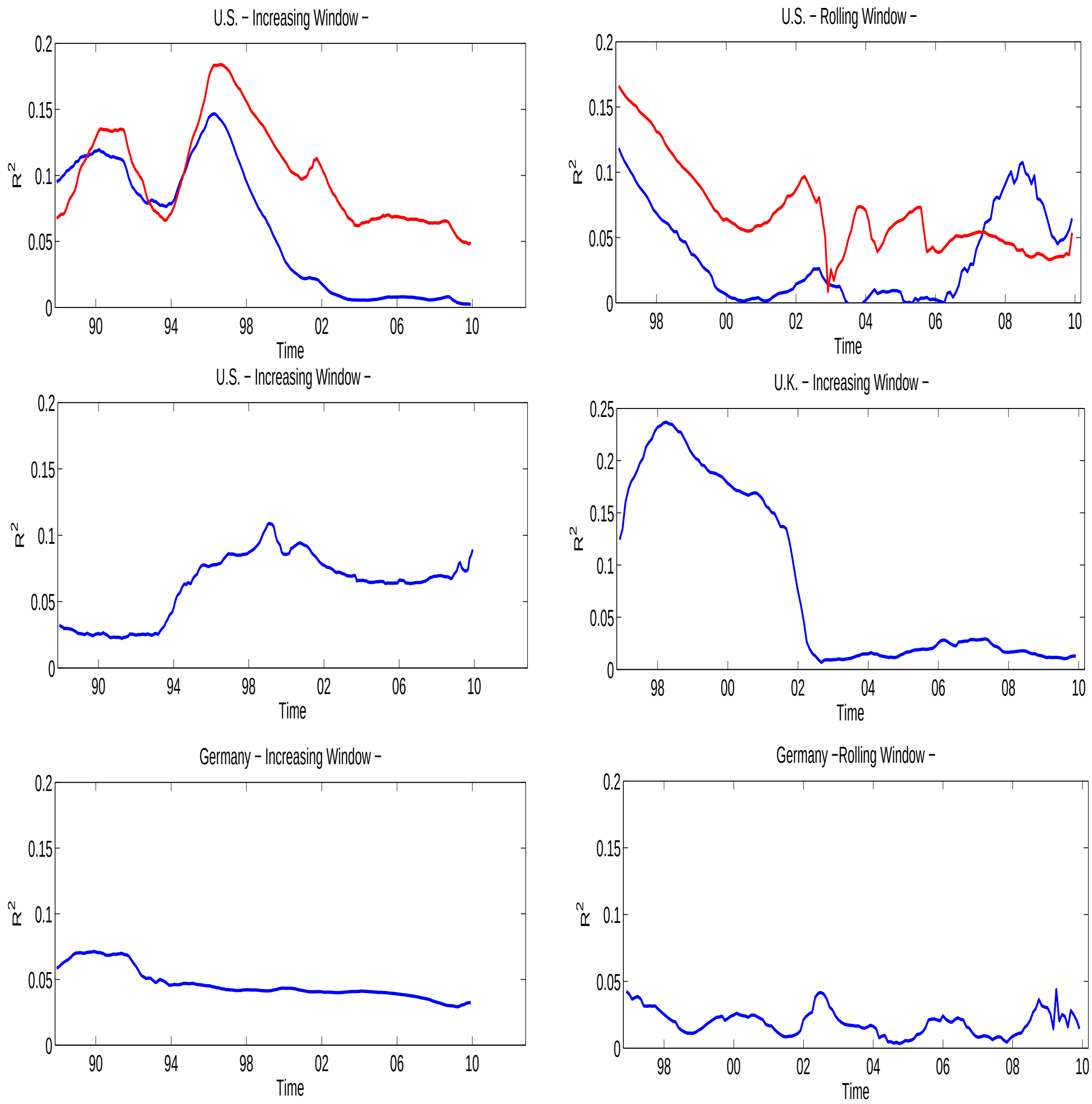

Fig. 4.1: $ADPP_\pi$ for $U.S.$, $U.K.$ and GER. The left column shows results adopting rolling window of observations in the estimation while the right column display results adopting rolling window of observations in the estimation. The top two figures show the $ADPP_{\pi_{U.S.}}$ using either Wright (red line) or Fama and Bliss (blue line) data set. The medium two figures show the $ADPP_{\pi_{U.K.}}$ using the Bank of England data set. The bottom two figures show the evolution through time of $ADPP_{\pi_{GER}}$ using the Wright data set. We use monthly yields observed from November 1973 till December 2009 and with residual maturities from 1 to 5 years.

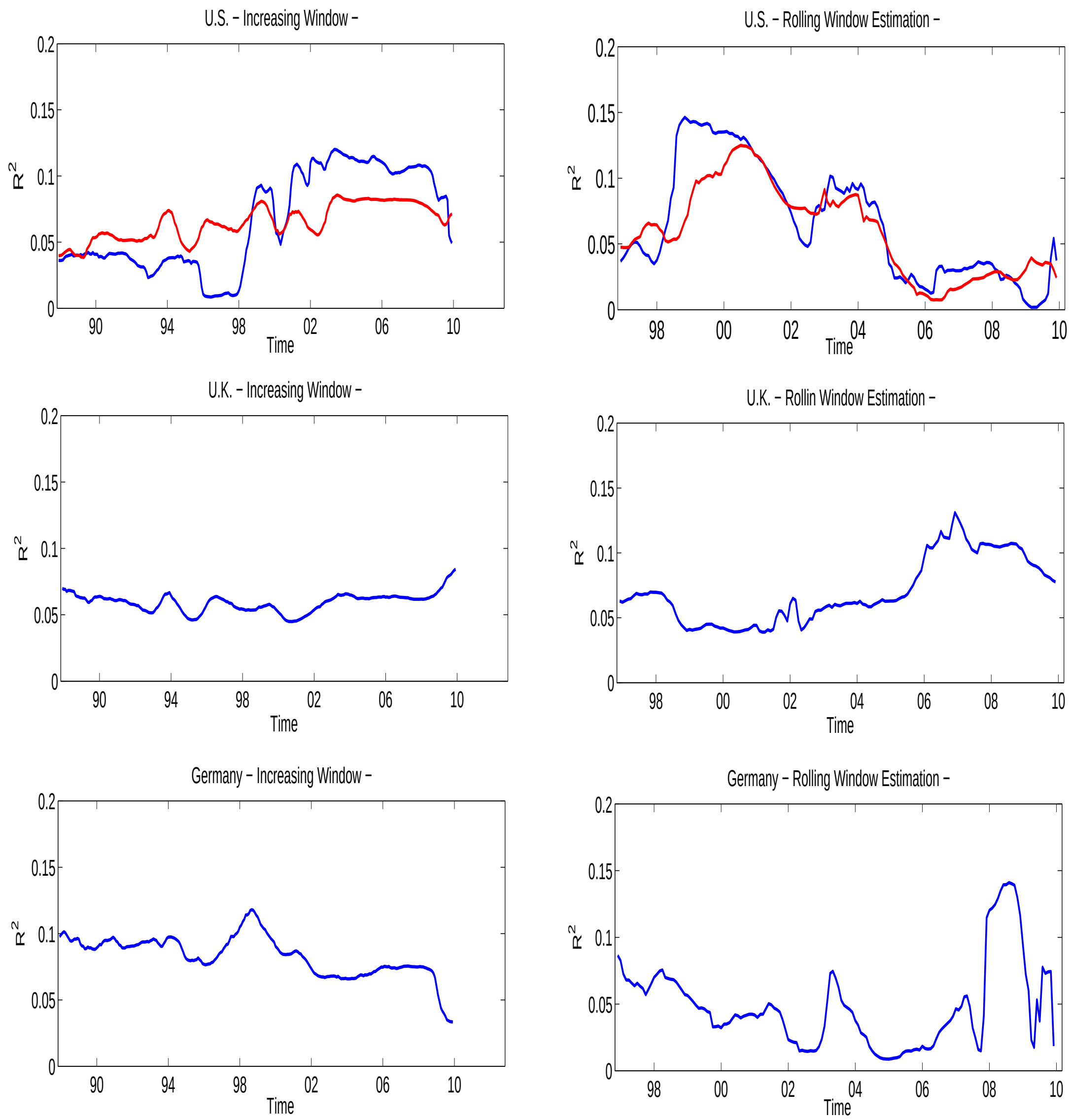

Fig. 4.2: $ADPP_g$ for $U.S.$, $U.K.$ and GER. The left column shows results adopting rolling window of observations in the estimation while the right column display results adopting rolling window of observations in the estimation. The top two figures show the $ADPP_{g_{U.S.}}$ using either Wright (red line) or Fama and Bliss (blue line) data set. The medium two figures show the $ADPP_{g_{U.K.}}$ using the Bank of England data set. The bottom two figures show the evolution through time of $ADPP_{g_{GER}}$ using the Wright data set. We use monthly yields observed from November 1973 till December 2009 and with residual maturities from 1 to 5 years.

	$g_{U.S.,t+h}$								$\pi_{U.S.,t+h}$							
	PST - 1 to 9 years maturity -				*PST* - 1 to 5 years maturity -				*PST* - 1 to 9 years maturity -				*PST*- 1 to 5 years maturity -			
h	$x_{U.S.,t}$	$x_{U.S.,t}+S_{U.S.,t}$	$\hat{F}_{U.S.,t\mid T}$	$\hat{F}_{U.S.,t\mid T}+S_{U.S.,t}$	$x_{U.S.,t}$	$x_{U.S.,t}+S_{U.S.,t}$	$\hat{F}_{U.S.,t\mid T}$	$\hat{F}_{U.S.,t\mid T}+S_{U.S.,t}$	$x_{U.S.,t}$	$x_{U.S.,t}+L_{U.S.,t}$	$\hat{F}_{U.S.,t\mid T}$	$\hat{F}_{U.S.,t\mid T}+L_{U.S.,t}$	$x_{U.S.,t}$	$x_{U.S.,t}+\hat{F}_{U.S.,t\mid T}$	$\hat{F}_{U.S.,t\mid T}$	$\hat{F}_{U.S.,t\mid T}+L_{U.S.,t}$
12	0,06	0,06	0,10	0,11	0,13	0,13	0,14	0,18	0,01	0,28	0,20	0,28	0,07	0,27	0,29	0,29
	{0,00 ; 0,26}	{0,00; 0,31}	{0,04; 0,40}	{0,08; 0,41}	{0,01; 0,37}	{0,03; 0,39}	{0,04; 0,44}	{0,10; 0,48}	{0,00; 0,16}	{0,12; 0,54}	{0,09; 0,47}	{0,16; 0,56}	{0,00; 0,25}	{0,12; 0,54}	{0,15; 0,53}	{0,17; 0,56}
18	0,10	0,11	0,15	0,15	0,11	0,14	0,15	0,15	0,03	0,21	0,16	0,21	0,07	0,20	0,23	0,23
	{0,00 ; 0,33}	{0,01; 0,39}	{0,07; 0,46}	{0,10; 0,46}	{0,00; 0,32}	{0,02; 0,41}	{0,05; 0,44}	{0,08; 0,44}	{0,00; 0,18}	{0,06; 0,45}	{0,08; 0,44}	{0,11; 0,49}	{0,00; 0,27}	{0,07; 0,46}	{0,11; 0,50}	{0,11; 0,53}
24	0,10	0,17	0,18	0,18	0,08	0,19	0,19	0,19	0,03	0,12	0,12	0,15	0,05	0,11	0,18	0,19
	{0,00 ; 0,30}	{0,03; 0,45}	{0,08; 0,47}	{0,10; 0,50}	{0,00; 0,30}	{0,05; 0,46}	{0,07; 0,47}	{0,08; 0,49}	{0,00; 0,17}	{0,02; 0,39}	{0,06; 0,40}	{0,08; 0,45}	{0,00; 0,22}	{0,02; 0,37}	{0,07; 0,49}	{0,09; 0,50}
30	0,16	0,25	0,25	0,26	0,10	0,26	0,27	0,27	0,06	0,09	0,09	0,12	0,08	0,10	0,16	0,17
	{0,02 ; 0,40}	{0,08; 0,53}	{0,12; 0,53}	{0,15; 0,57}	{0,00; 0,34}	{0,09; 0,54}	{0,13; 0,55}	{0,15; 0,58}	{0,00; 0,23}	{0,01; 0,35}	{0,04; 0,37}	{0,05; 0,44}	{0,00; 0,27}	{0,01; 0,35}	{0,05; 0,43}	{0,07; 0,46}
36	0,18	0,28	0,28	0,30	0,13	0,29	0,32	0,32	0,07	0,08	0,13	0,13	0,09	0,09	0,15	0,17
	{0,03 ; 0,42}	{0,09; 0,56}	{0,15; 0,58}	{0,17; 0,63}	{0,00; 0,39}	{0,10; 0,58}	{0,17; 0,61}	{0,18; 0,65}	{0,00; 0,24}	{0,01; 0,31}	{0,07; 0,42}	{0,07; 0,41}	{0,00; 0,29}	{0,01; 0,31}	{0,05; 0,42}	{0,08; 0,48}
	W - 1 to 9 years maturity -				*W* - 1 to 5 years maturity -				*W* - 1 to 9 years maturity -				*W*- 1 to 5 years maturity -			
h	$x_{U.S.,t}$	$x_{U.S.,t}+S_{U.S.,t}$	$\hat{F}_{U.S.,t\mid T}$	$\hat{F}_{U.S.,t\mid T}+S_{U.S.,t}$	$x_{U.S.,t}$	$x_{U.S.,t}+S_{U.S.,t}$	$\hat{F}_{U.S.,t\mid T}$	$\hat{F}_{U.S.,t\mid T}+S_{U.S.,t}$	$x_{U.S.,t}$	$x_{U.S.,t}+L_{U.S.,t}$	$\hat{F}_{U.S.,t\mid T}$	$\hat{F}_{U.S.,t\mid T}+L_{U.S.,t}$	$x_{U.S.,t}$	$x_{U.S.,t}+\hat{F}_{U.S.,t\mid T}$	$\hat{F}_{U.S.,t\mid T}$	$\hat{F}_{U.S.,t\mid T}+L_{U.S.,t}$
12	0,02	0,08	0,11	0,17	0,06	0,07	0,14	0,14	0,00	0,28	0,13	0,28	0,02	0,28	0,12	0,27
	{0,00 ; 0,19}	{0,01; 0,34}	{0,06; 0,40}	{0,10; 0,49}	{0,00; 0,26}	{0,00; 0,35}	{0,05; 0,44}	{0,06; 0,46}	{0,00; 0,09}	{0,11; 0,52}	{0,06; 0,39}	{0,16; 0,55}	{0,00; 0,19}	{0,12; 0,53}	{0,04; 0,37}	{0,15; 0,56}
18	0,02	0,18	0,17	0,31	0,09	0,13	0,17	0,18	0,00	0,23	0,15	0,23	0,04	0,20	0,12	0,20
	{0,00 ; 0,19}	{0,06; 0,49}	{0,11; 0,48}	{0,20; 0,60}	{0,00; 0,33}	{0,01; 0,41}	{0,06; 0,46}	{0,07; 0,49}	{0,00; 0,09}	{0,09; 0,49}	{0,07; 0,40}	{0,12; 0,53}	{0,00; 0,22}	{0,05; 0,47}	{0,04; 0,39}	{0,10; 0,49}
24	0,00	0,24	0,15	0,29	0,11	0,19	0,18	0,20	0,00	0,13	0,15	0,19	0,02	0,09	0,16	0,17
	{0,00 ; 0,14}	{0,09; 0,52}	{0,08; 0,46}	{0,19; 0,57}	{0,00; 0,35}	{0,05; 0,47}	{0,06; 0,44}	{0,09; 0,49}	{0,00; 0,08}	{0,02; 0,43}	{0,08; 0,40}	{0,11; 0,50}	{0,00; 0,19}	{0,01; 0,36}	{0,06; 0,40}	{0,08; 0,44}
30	0,01	0,25	0,14	0,26	0,18	0,28	0,26	0,28	0,00	0,07	0,14	0,17	0,02	0,05	0,15	0,15
	{0,00 ; 0,14}	{0,09; 0,54}	{0,06; 0,43}	{0,16; 0,56}	{0,03; 0,44}	{0,11; 0,56}	{0,11; 0,53}	{0,14; 0,57}	{0,00; 0,08}	{0,00; 0,32}	{0,06; 0,39}	{0,08; 0,45}	{0,00; 0,20}	{0,00; 0,28}	{0,06; 0,40}	{0,07; 0,44}
36	0,04	0,28	0,15	0,28	0,23	0,33	0,29	0,32	0,00	0,03	0,14	0,16	0,04	0,05	0,15	0,15
	{0,00 ; 0,24}	{0,09; 0,57}	{0,07; 0,47}	{0,16; 0,62}	{0,06; 0,49}	{0,16; 0,61}	{0,11; 0,57}	{0,16; 0,63}	{0,00; 0,08}	{0,00; 0,26}	{0,07; 0,38}	{0,08; 0,44}	{0,00; 0,21}	{0,00; 0,26}	{0,05; 0,41}	{0,06; 0,42}
					FB - 1 to 5 years maturity -								*FB* - 1 to 5 years maturity -			
h					$x_{U.S.,t}$	$x_{U.S.,t}+S_{U.S.,t}$	$\hat{F}_{U.S.,t\mid T}$	$\hat{F}_{U.S.,t\mid T}+S_{U.S.,t}$					$x_{U.S.,t}$	$x_{U.S.,t}+L_{U.S.,t}$	$\hat{F}_{U.S.,t\mid T}$	$\hat{F}_{U.S.,t\mid T}+L_{U.S.,t}$
12					0,05	0,07	0,05	0,15					0,01	0,29	0,31	0,33
					{0,00; 0,26}	{0,01; 0,36}	{0,02; 0,36}	{0,08; 0,46}					{0,00; 0,17}	{0,14; 0,54}	{0,16; 0,53}	{0,19; 0,58}
18					0,10	0,14	0,11	0,22					0,03	0,20	0,27	0,27
					{0,00; 0,35}	{0,02; 0,43}	{0,04; 0,43}	{0,12; 0,49}					{0,00; 0,20}	{0,06; 0,47}	{0,13; 0,52}	{0,15; 0,53}
24					0,15	0,20	0,20	0,21					0,05	0,11	0,18	0,18
					{0,01; 0,40}	{0,05; 0,46}	{0,08; 0,49}	{0,10; 0,50}					{0,00; 0,24}	{0,01; 0,40}	{0,07; 0,46}	{0,09; 0,46}
30					0,18	0,27	0,26	0,27					0,07	0,09	0,15	0,15
					{0,02; 0,44}	{0,10; 0,55}	{0,12; 0,55}	{0,14; 0,56}					{0,00; 0,28}	{0,01; 0,35}	{0,05; 0,44}	{0,06; 0,45}
36					0,19	0,30	0,30	0,31					0,08	0,08	0,16	0,16
					{0,02; 0,47}	{0,12; 0,58}	{0,12; 0,61}	{0,16; 0,62}					{0,00; 0,27}	{0,01; 0,32}	{0,05; 0,42}	{0,07; 0,46}

Table 4.12: We report the results of forecasting regressions of *U.S.* economic activity (g_{t+h}) and inflation rates (π_{t+h}) using as regressors the CP factor, the CP factor and the yield curve factor (the slope for forecasting economic activity while the level for forecasting inflation rates) jointly, the five smoothed factors extracted from the State-Space model and the five smoothed factors and the yield curve factor jointly. We run regressions for annualized growth in industrial production and consumer price index over forecasting horizons of h = 12, 18, 24, 30 and 36 months. We report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use monthly data from the *PST*, Wright and Fama and Bliss databases for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations) with a maturity spectrum of five and nine years

	$g_{U.K.,t+h}$								$\pi_{U.K.,t+h}$							
	PST - 1 to 9 years maturity -				*PST* - 1 to 5 years maturity -				*PST* - 1 to 9 years maturity -				*PST*- 1 to 5 years maturity -			
h	$x_{U.K.,t}$	$x_{U.K.,t}+S_{U.K.,t}$	$\hat{F}_{U.K.,t\|T}$	$\hat{F}_{U.K.,t\|T}+S_{U.K.,t}$	$x_{U.K.,t}$	$x_{U.K.,t}+S_{U.K.,t}$	$\hat{F}_{U.K.,t\|T}$	$\hat{F}_{U.K.,t\|T}+S_{U.K.,t}$	$x_{U.K.,t}$	$x_{U.K.,t}+L_{U.K.,t}$	$\hat{F}_{U.K.,t\|T}$	$\hat{F}_{U.K.,t\|T}+L_{U.K.,t}$	$x_{U.K.,t}$	$x_{U.K.,t}+\hat{F}_{U.K.,t\|T}$	$\hat{F}_{U.K.,t\|T}$	$\hat{F}_{U.K.,t\|T}+L_{U.K.,t}$
12	0,05	0,14	0,22	0,31	0,03	0,16	0,18	0,22	0,06	0,76	0,50	0,82	0,07	0,74	0,54	0,77
	{0,00 ; 0,23}	{0,03; 0,41}	{0,12; 0,50}	{0,20; 0,58}	{0,00; 0,16}	{0,04; 0,43}	{0,07; 0,44}	{0,11; 0,46}	{0,00; 0,33}	{0,69; 0,86}	{0,36; 0,76}	{0,79; 0,89}	{0,00; 0,32}	{0,66; 0,85}	{0,39; 0,76}	{0,72; 0,87}
18	0,13	0,23	0,31	0,37	0,04	0,21	0,22	0,23	0,10	0,74	0,58	0,76	0,08	0,68	0,57	0,69
	{0,01 ; 0,33}	{0,07; 0,50}	{0,18; 0,57}	{0,25; 0,63}	{0,00; 0,20}	{0,07; 0,48}	{0,09; 0,49}	{0,11; 0,49}	{0,00; 0,36}	{0,67; 0,85}	{0,48; 0,76}	{0,72; 0,87}	{0,00; 0,33}	{0,59; 0,81}	{0,42; 0,78}	{0,62; 0,84}
24	0,20	0,26	0,37	0,37	0,05	0,17	0,21	0,22	0,14	0,62	0,55	0,63	0,09	0,52	0,45	0,51
	{0,05 ; 0,46}	{0,10; 0,53}	{0,23; 0,65}	{0,25; 0,64}	{0,00; 0,22}	{0,05; 0,44}	{0,10; 0,48}	{0,11; 0,50}	{0,00; 0,44}	{0,49; 0,78}	{0,45; 0,74}	{0,52; 0,81}	{0,00; 0,34}	{0,35; 0,75}	{0,29; 0,71}	{0,38; 0,76}
30	0,20	0,23	0,34	0,35	0,06	0,13	0,20	0,23	0,18	0,52	0,53	0,55	0,11	0,40	0,38	0,40
	{0,05 ; 0,46}	{0,09; 0,50}	{0,20; 0,62}	{0,22; 0,64}	{0,00; 0,28}	{0,03; 0,40}	{0,08; 0,46}	{0,14; 0,50}	{0,01; 0,49}	{0,35; 0,76}	{0,40; 0,75}	{0,42; 0,79}	{0,00; 0,39}	{0,20; 0,70}	{0,21; 0,70}	{0,25; 0,74}
36	0,18	0,19	0,31	0,38	0,07	0,11	0,20	0,28	0,13	0,38	0,44	0,44	0,10	0,31	0,32	0,33
	{0,04 ; 0,45}	{0,05; 0,47}	{0,17; 0,60}	{0,26; 0,68}	{0,00; 0,26}	{0,01; 0,35}	{0,08; 0,49}	{0,16; 0,57}	{0,00; 0,45}	{0,20; 0,70}	{0,32; 0,73}	{0,34; 0,74}	{0,00; 0,36}	{0,12; 0,65}	{0,16; 0,66}	{0,21; 0,71}

	$g_{GER,t+h}$								$\pi_{GER,t+h}$							
	PST - 1 to 9 years maturity -				*PST* - 1 to 5 years maturity -				*PST* - 1 to 9 years maturity -				*PST*- 1 to 5 years maturity -			
h	$x_{GER,t}$	$x_{GER,t}+S_{GER,t}$	$\hat{F}_{GER,t\|T}$	$\hat{F}_{GER,t\|T}+S_{GER,t}$	$x_{GER,t}$	$x_{GER,t}+S_{GER,t}$	$\hat{F}_{GER,t\|T}$	$\hat{F}_{GER,t\|T}+S_{GER,t}$	$x_{GER,t}$	$x_{GER,t}+L_{GER,t}$	$\hat{F}_{GER,t\|T}$	$\hat{F}_{GER,t\|T}+L_{GER,t}$	$x_{GER,t}$	$x_{GER,t}+\hat{F}_{GER,t\|T}$	$\hat{F}_{GER,t\|T}$	$\hat{F}_{GER,t\|T}+L_{GER,t}$
12	0,00	0,13	0,18	0,18	0,05	0,08	0,09	0,09	0,00	0,46	0,18	0,59	0,00	0,53	0,05	0,56
	{0,00 ; 0,08}	{0,02; 0,39}	{0,10; 0,45}	{0,12; 0,49}	{0,00; 0,25}	{0,01; 0,33}	{0,02; 0,34}	{0,04; 0,38}	{0,00; 0,11}	{0,28; 0,67}	{0,09; 0,47}	{0,50; 0,75}	{0,00; 0,15}	{0,39; 0,70}	{0,02; 0,31}	{0,46; 0,75}
18	0,00	0,14	0,14	0,16	0,00	0,12	0,08	0,12	0,00	0,48	0,18	0,54	0,01	0,48	0,07	0,51
	{0,00 ; 0,08}	{0,03; 0,41}	{0,07; 0,40}	{0,08; 0,46}	{0,00; 0,12}	{0,02; 0,38}	{0,01; 0,31}	{0,05; 0,43}	{0,00; 0,10}	{0,32; 0,68}	{0,10; 0,47}	{0,43; 0,73}	{0,00; 0,16}	{0,33; 0,70}	{0,02; 0,35}	{0,38; 0,71}
24	0,01	0,13	0,11	0,15	0,00	0,14	0,08	0,15	0,02	0,41	0,23	0,47	0,02	0,38	0,11	0,41
	{0,00 ; 0,11}	{0,03; 0,37}	{0,04; 0,35}	{0,08; 0,38}	{0,00; 0,13}	{0,03; 0,40}	{0,02; 0,32}	{0,07; 0,39}	{0,00; 0,16}	{0,22; 0,65}	{0,13; 0,51}	{0,37; 0,70}	{0,00; 0,19}	{0,20; 0,65}	{0,03; 0,41}	{0,28; 0,68}
30	0,05	0,14	0,13	0,17	0,02	0,15	0,11	0,17	0,02	0,30	0,19	0,35	0,01	0,27	0,10	0,29
	{0,00 ; 0,22}	{0,02; 0,38}	{0,06; 0,40}	{0,12; 0,27}	{0,00; 0,18}	{0,04; 0,33}	{0,03; 0,36}	{0,09; 0,33}	{0,00; 0,17}	{0,10; 0,60}	{0,08; 0,48}	{0,23; 0,65}	{0,00; 0,19}	{0,07; 0,61}	{0,02; 0,40}	{0,16; 0,61}
36	0,08	0,16	0,15	0,24	0,02	0,17	0,11	0,19	0,02	0,20	0,12	0,25	0,01	0,18	0,06	0,18
	{0,00 ; 0,29}	{0,02; 0,44}	{0,07; 0,47}	{0,14; 0,52}	{0,00; 0,19}	{0,04; 0,45}	{0,03; 0,38}	{0,08; 0,48}	{0,00; 0,16}	{0,05; 0,56}	{0,07; 0,47}	{0,15; 0,63}	{0,00; 0,18}	{0,02; 0,54}	{0,02; 0,35}	{0,08; 0,59}

	W - 1 to 9 years maturity -				*W* - 1 to 5 years maturity -				*W* - 1 to 9 years maturity -				*W* - 1 to 5 years maturity -			
h	$x_{GER,t}$	$x_{GER,t}+S_{GER,t}$	$\hat{F}_{GER,t\|T}$	$\hat{F}_{GER,t\|T}+S_{GER,t}$	$x_{GER,t}$	$x_{GER,t}+S_{GER,t}$	$\hat{F}_{GER,t\|T}$	$\hat{F}_{GER,t\|T}+S_{GER,t}$	$x_{GER,t}$	$x_{GER,t}+L_{GER,t}$	$\hat{F}_{GER,t\|T}$	$\hat{F}_{GER,t\|T}+L_{GER,t}$	$x_{GER,t}$	$x_{GER,t}+\hat{F}_{GER,t\|T}$	$\hat{F}_{GER,t\|T}$	$\hat{F}_{GER,t\|T}+L_{GER,t}$
12	0,00	0,09	0,20	0,20	0,02	0,09	0,10	0,11	0,00	0,46	0,06	0,54	0,00	0,55	0,14	0,56
	{0,00 ; 0,11}	{0,00; 0,34}	{0,11; 0,46}	{0,12; 0,49}	{0,00; 0,16}	{0,00; 0,35}	{0,02; 0,37}	{0,04; 0,41}	{0,00; 0,08}	{0,29; 0,67}	{0,03; 0,40}	{0,43; 0,71}	{0,00; 0,13}	{0,39; 0,71}	{0,05; 0,45}	{0,47; 0,73}
18	0,00	0,11	0,12	0,12	0,00	0,10	0,12	0,12	0,02	0,46	0,09	0,52	0,01	0,49	0,11	0,53
	{0,00 ; 0,09}	{0,01; 0,35}	{0,05; 0,39}	{0,05; 0,41}	{0,00; 0,11}	{0,01; 0,35}	{0,03; 0,39}	{0,04; 0,40}	{0,00; 0,16}	{0,30; 0,68}	{0,04; 0,41}	{0,41; 0,72}	{0,00; 0,15}	{0,33; 0,70}	{0,03; 0,41}	{0,41; 0,71}
24	0,01	0,11	0,15	0,15	0,01	0,09	0,10	0,09	0,00	0,37	0,09	0,39	0,02	0,39	0,14	0,40
	{0,00 ; 0,12}	{0,01; 0,35}	{0,07; 0,40}	{0,06; 0,42}	{0,00; 0,17}	{0,01; 0,31}	{0,02; 0,35}	{0,03; 0,36}	{0,00; 0,11}	{0,19; 0,63}	{0,05; 0,42}	{0,26; 0,68}	{0,00; 0,16}	{0,21; 0,65}	{0,05; 0,48}	{0,26; 0,67}
30	0,01	0,14	0,19	0,20	0,03	0,11	0,10	0,11	0,00	0,27	0,10	0,28	0,02	0,27	0,15	0,29
	{0,00 ; 0,14}	{0,03; 0,38}	{0,09; 0,46}	{0,11; 0,48}	{0,00; 0,20}	{0,01; 0,34}	{0,02; 0,37}	{0,04; 0,39}	{0,00; 0,09}	{0,06; 0,60}	{0,04; 0,43}	{0,14; 0,65}	{0,00; 0,17}	{0,07; 0,60}	{0,05; 0,48}	{0,15; 0,62}
36	0,00	0,19	0,17	0,22	0,02	0,13	0,15	0,16	0,00	0,18	0,08	0,21	0,02	0,18	0,07	0,17
	{0,00 ; 0,10}	{0,04; 0,44}	{0,08; 0,45}	{0,13; 0,53}	{0,00; 0,19}	{0,02; 0,39}	{0,04; 0,44}	{0,06; 0,47}	{0,00; 0,10}	{0,01; 0,54}	{0,04; 0,38}	{0,11; 0,62}	{0,00; 0,18}	{0,02; 0,52}	{0,02; 0,41}	{0,07; 0,55}

Table 4.13: We report the results of forecasting regressions of *U.K.* and *GER* economic activity (g_{t+h}) and inflation rates (π_{t+h}) using as regressors the CP factor, the CP factor and the yield curve factor (the slope for forecasting economic activity while the level for forecasting inflation rates) jointly, the five smoothed factors extracted from the State-Space model and the five smoothed factors and the yield curve factor jointly. We run regressions for annualized growth in industrial production and consumer price index over forecasting horizons of h = 12, 18, 24, 30 and 36 months. We report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use monthly data from the *PST* and Wright databases for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations) with a maturity spectrum of five and nine years

	$g_{JAP,t+h}$								$\pi_{JAP,t+h}$							
	PST - 1 to 9 years maturity -				*PST* - 1 to 5 years maturity -				*PST* - 1 to 9 years maturity -				*PST*- 1 to 5 years maturity -			
h	$x_{JAP,t}$	$x_{JAP,t}+S_{JAP,t}$	$\hat{F}_{JAP,t\|T}$	$\hat{F}_{JAP,t\|T}+S_{JAP,t}$	$x_{JAP,t}$	$x_{JAP,t}+S_{JAP,t}$	$\hat{F}_{JAP,t\|T}$	$\hat{F}_{JAP,t\|T}+S_{JAP,t}$	$x_{JAP,t}$	$x_{JAP,t}+L_{JAP,t}$	$\hat{F}_{JAP,t\|T}$	$\hat{F}_{JAP,t\|T}+L_{JAP,t}$	$x_{JAP,t}$	$x_{JAP,t}+\hat{F}_{JAP,t\|T}$	$\hat{F}_{JAP,t\|T}$	$\hat{F}_{JAP,t\|T}+L_{JAP,t}$
12	0,00	0,05	0,08	0,11	0,00	0,06	0,12	0,12	0,00	0,45	0,49	0,50	0,00	0,45	0,49	0,50
	{0,00 ; 0,10}	{0,01; 0,26}	{0,04; 0,32}	{0,07; 0,39}	{0,00; 0,14}	{0,01; 0,31}	{0,04; 0,36}	{0,05; 0,36}	{0,00; 0,17}	{0,29; 0,68}	{0,37; 0,70}	{0,37; 0,69}	{0,00; 0,17}	{0,27; 0,67}	{0,36; 0,71}	{0,38; 0,71}
18	0,00	0,04	0,07	0,08	0,01	0,05	0,09	0,11	0,00	0,42	0,48	0,49	0,00	0,42	0,48	0,49
	{0,00 ; 0,12}	{0,00; 0,24}	{0,03; 0,21}	{0,05; 0,27}	{0,00; 0,15}	{0,00; 0,26}	{0,03; 0,24}	{0,05; 0,31}	{0,00; 0,18}	{0,26; 0,64}	{0,34; 0,68}	{0,37; 0,66}	{0,00; 0,18}	{0,26; 0,63}	{0,34; 0,68}	{0,37; 0,66}
24	0,00	0,03	0,10	0,09	0,02	0,04	0,09	0,13	0,00	0,36	0,44	0,46	0,00	0,36	0,44	0,46
	{0,00 ; 0,12}	{0,00; 0,24}	{0,05; 0,19}	{0,06; 0,20}	{0,00; 0,18}	{0,00; 0,25}	{0,03; 0,23}	{0,06; 0,37}	{0,00; 0,16}	{0,20; 0,62}	{0,31; 0,65}	{0,34; 0,63}	{0,00; 0,17}	{0,18; 0,62}	{0,29; 0,65}	{0,34; 0,64}
30	0,00	0,01	0,10	0,10	0,01	0,02	0,08	0,13	0,01	0,36	0,51	0,56	0,01	0,36	0,51	0,56
	{0,00 ; 0,08}	{0,00; 0,18}	{0,06; 0,39}	{0,07; 0,41}	{0,00; 0,14}	{0,00; 0,22}	{0,01; 0,31}	{0,06; 0,38}	{0,00; 0,20}	{0,18; 0,62}	{0,39; 0,71}	{0,46; 0,71}	{0,00; 0,18}	{0,19; 0,61}	{0,39; 0,71}	{0,46; 0,72}
36	0,00	0,00	0,08	0,08	0,00	0,01	0,05	0,08	0,01	0,34	0,50	0,56	0,01	0,34	0,50	0,56
	{0,00 ; 0,11}	{0,00; 0,17}	{0,05; 0,40}	{0,06; 0,39}	{0,00; 0,11}	{0,00; 0,19}	{0,02; 0,30}	{0,03; 0,35}	{0,00; 0,19}	{0,16; 0,62}	{0,38; 0,69}	{0,46; 0,74}	{0,00; 0,19}	{0,16; 0,63}	{0,38; 0,70}	{0,45; 0,73}

	W - 1 to 9 years maturity -				W - 1 to 5 years maturity -				W - 1 to 9 years maturity -				W- 1 to 5 years maturity -			
h	$x_{JAP,t}$	$x_{JAP,t}+S_{JAP,t}$	$\hat{F}_{JAP,t\|T}$	$\hat{F}_{JAP,t\|T}+S_{JAP,t}$	$x_{JAP,t}$	$x_{JAP,t}+S_{JAP,t}$	$\hat{F}_{JAP,t\|T}$	$\hat{F}_{JAP,t\|T}+S_{JAP,t}$	$x_{JAP,t}$	$x_{JAP,t}+L_{JAP,t}$	$\hat{F}_{JAP,t\|T}$	$\hat{F}_{JAP,t\|T}+L_{JAP,t}$	$x_{JAP,t}$	$x_{JAP,t}+\hat{F}_{JAP,t\|T}$	$\hat{F}_{JAP,t\|T}$	$\hat{F}_{JAP,t\|T}+L_{JAP,t}$
12	0,00	0,02	0,04	0,04	0,00	0,02	0,00	0,06	0,00	0,46	0,34	0,51	0,00	0,45	0,32	0,49
	{0,00 ; 0,09}	{0,00; 0,23}	{0,03; 0,31}	{0,04; 0,32}	{0,00; 0,10}	{0,00; 0,22}	{0,00; 0,21}	{0,03; 0,33}	{0,00; 0,17}	{0,31; 0,67}	{0,24; 0,57}	{0,42; 0,71}	{0,00; 0,16}	{0,29; 0,66}	{0,19; 0,57}	{0,37; 0,70}
18	0,00	0,02	0,07	0,07	0,00	0,00	0,00	0,02	0,00	0,43	0,33	0,50	0,01	0,41	0,33	0,45
	{0,00 ; 0,07}	{0,00; 0,22}	{0,03; 0,28}	{0,04; 0,28}	{0,00; 0,09}	{-0,01; 0,18}	{0,00; 0,22}	{0,01; 0,27}	{0,00; 0,19}	{0,25; 0,64}	{0,22; 0,59}	{0,40; 0,70}	{0,00; 0,25}	{0,26; 0,64}	{0,18; 0,61}	{0,33; 0,69}
24	0,00	0,02	0,10	0,11	0,00	-0,01	0,02	0,02	0,00	0,35	0,31	0,45	0,02	0,34	0,20	0,34
	{0,00 ; 0,08}	{0,00; 0,20}	{0,05; 0,30}	{0,06; 0,35}	{0,00; 0,08}	{0,00; 0,14}	{0,00; 0,26}	{0,00; 0,29}	{0,00; 0,17}	{0,19; 0,61}	{0,20; 0,57}	{0,35; 0,68}	{0,00; 0,25}	{0,17; 0,60}	{0,06; 0,55}	{0,20; 0,62}
30	0,00	0,01	0,14	0,16	0,00	-0,01	0,04	0,04	0,00	0,33	0,33	0,45	0,02	0,32	0,15	0,33
	{0,00 ; 0,07}	{0,00; 0,19}	{0,07; 0,39}	{0,08; 0,44}	{0,00; 0,09}	{0,00; 0,13}	{0,01; 0,29}	{0,01; 0,31}	{0,00; 0,16}	{0,14; 0,59}	{0,22; 0,60}	{0,34; 0,69}	{0,00; 0,28}	{0,14; 0,62}	{0,04; 0,47}	{0,18; 0,64}
36	0,00	0,00	0,10	0,11	0,00	0,00	0,03	0,03	0,00	0,31	0,38	0,52	0,01	0,29	0,12	0,38
	{0,00 ; 0,08}	{-0,01; 0,18}	{0,03; 0,40}	{0,06; 0,43}	{0,00; 0,10}	{0,00; 0,15}	{0,01; 0,27}	{0,01; 0,30}	{0,00; 0,17}	{0,13; 0,58}	{0,28; 0,62}	{0,43; 0,72}	{0,00; 0,26}	{0,10; 0,60}	{0,04; 0,46}	{0,24; 0,66}

Table 4.14: We report the results of forecasting regressions of *JAP* economic activity (g_{t+h}) and inflation rates (π_{t+h}) using as regressors the CP factor, the CP factor and the yield curve factor (the slope for forecasting economic activity while the level for forecasting inflation rates) jointly, the five smoothed factors extracted from the State-Space model and the five smoothed factors and the yield curve factor jointly. We run regressions for annualized growth in industrial production and consumer price index over forecasting horizons of $h = 12$, 18, 24, 30 and 36 months. We report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use monthly data from the *PST* and Wright databases for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations) with a maturity spectrum of five and nine years

	$rx^{(n)}_{U.S.,t+1}$ - n from 2 to 9 years							
	PST				*W*			
n	$x_{U.S.,t}$	$f^{(n)}_{U.S.,t} - y^{(1)}_{U.S.,t}$	$YF_{U.S.}$	$\hat{F}_{U.S.,t\|T}$	$x_{U.S.,t}$	$f^{(n)}_{U.S.,t} - y^{(1)}_{U.S.,t}$	$YF_{U.S.}$	$\hat{F}_{U.S.,t\|T}$
2	0,05 {0,00; 0,22}	0,00 {0,00; 0,15}	0,02 {0,01; 0,28}	0,07 {0,03; 0,32}	0,29 {0,15; 0,45}	0,00 {0,00; 0,14}	0,02 {0,01; 0,30}	0,35 {0,25; 0,53}
3	0,06 {0,00; 0,21}	0,01 {0,00; 0,18}	0,03 {0,01; 0,29}	0,07 {0,03; 0,31}	0,32 {0,18; 0,47}	0,01 {0,00; 0,16}	0,02 {0,01; 0,27}	0,34 {0,25; 0,51}
4	0,08 {0,00; 0,23}	0,01 {0,00; 0,18}	0,04 {0,01; 0,28}	0,07 {0,04; 0,30}	0,33 {0,20; 0,48}	0,01 {0,00; 0,17}	0,03 {0,01; 0,27}	0,34 {0,24; 0,51}
5	0,08 {0,00; 0,24}	0,01 {0,00; 0,15}	0,04 {0,01; 0,27}	0,07 {0,03; 0,29}	0,33 {0,20; 0,48}	0,02 {0,00; 0,17}	0,04 {0,01; 0,27}	0,33 {0,24; 0,50}
6	0,08 {0,01; 0,24}	0,02 {0,00; 0,12}	0,05 {0,01; 0,27}	0,07 {0,03; 0,29}	0,32 {0,19; 0,47}	0,02 {0,00; 0,13}	0,06 {0,01; 0,27}	0,31 {0,22; 0,49}
7	0,09 {0,02; 0,19}	0,02 {0,00; 0,15}	0,06 {0,01; 0,24}	0,07 {0,03; 0,24}	0,31 {0,18; 0,47}	0,03 {0,00; 0,15}	0,07 {0,01; 0,27}	0,30 {0,21; 0,49}
8	0,09 {0,00; 0,24}	0,02 {0,00; 0,15}	0,06 {0,01; 0,22}	0,08 {0,03; 0,26}	0,29 {0,16; 0,44}	0,03 {0,00; 0,17}	0,08 {0,03; 0,25}	0,29 {0,18; 0,48}
9	0,09 {0,01; 0,23}	0,02 {0,00; 0,17}	0,07 {0,02; 0,26}	0,08 {0,04; 0,30}	0,27 {0,15; 0,43}	0,03 {0,00; 0,18}	0,09 {0,02; 0,31}	0,28 {0,18; 0,46}

	$rx^{(n)}_{U.S.,t+1}$ - n from 2 to 5 years							
	PST				*W*			
n	$x_{U.S.,t}$	$f^{(n)}_{U.S.,t} - y^{(1)}_{U.S.,t}$	$YF_{U.S.}$	$\hat{F}_{U.S.,t\|T}$	$x_{U.S.,t}$	$f^{(n)}_{U.S.,t} - y^{(1)}_{U.S.,t}$	$YF_{U.S.}$	$\hat{F}_{U.S.,t\|T}$
2	0,04 {0,00; 0,20}	0,00 {0,00; 0,16}	0,02 {0,01; 0,29}	0,05 {0,01; 0,29}	0,08 {0,00; 0,26}	0,00 {0,00; 0,12}	0,04 {0,01; 0,29}	0,09 {0,03; 0,32}
3	0,05 {0,00; 0,23}	0,01 {0,00; 0,17}	0,03 {0,00; 0,28}	0,05 {0,01; 0,28}	0,07 {0,00; 0,25}	0,01 {0,00; 0,16}	0,03 {0,00; 0,27}	0,07 {0,02; 0,29}
4	0,06 {0,00; 0,21}	0,01 {0,00; 0,15}	0,03 {0,01; 0,27}	0,05 {0,01; 0,26}	0,07 {0,00; 0,23}	0,01 {0,00; 0,15}	0,03 {0,01; 0,26}	0,06 {0,01; 0,27}
5	0,06 {0,00; 0,21}	0,01 {0,00; 0,15}	0,04 {0,01; 0,29}	0,05 {0,01; 0,26}	0,07 {0,00; 0,23}	0,02 {0,00; 0,16}	0,04 {0,01; 0,28}	0,06 {0,02; 0,26}
					FB			
					$x_{U.S.,t}$	$f^{(n)}_{U.S.,t} - y^{(1)}_{U.S.,t}$	$YF_{U.S.}$	$\hat{F}_{U.S.,t\|T}$
					0,08 {0,00; 0,25}	0,00 {0,00; 0,13}	0,04 {0,01; 0,32}	0,08 {0,02; 0,30}
					0,07 {0,00; 0,24}	0,01 {0,00; 0,17}	0,03 {0,00; 0,27}	0,07 {0,02; 0,27}
					0,08 {0,00; 0,23}	0,02 {0,00; 0,18}	0,04 {0,01; 0,28}	0,07 {0,03; 0,27}
					0,05 {0,00; 0,20}	0,01 {0,00; 0,16}	0,04 {0,01; 0,26}	0,05 {0,01; 0,25}

	$rx^{(n)}_{U.K.,t+1}$ - n from 2 to 9 years				n - from 1 to 5 years			
n	$x_{U.K.,t}$	$f^{(n)}_{U.K.,t} - y^{(1)}_{U.K.,t}$	$YF_{U.K.}$	$\hat{F}_{U.K.,t\|T}$	$x_{U.K.,t}$	$f^{(n)}_{U.K.,t} - y^{(1)}_{U.S.,t}$	$YF_{U.K.}$	$\hat{F}_{U.K.,t\|T}$
2	0,29 {0,13; 0,48}	0,06 {0,00; 0,27}	0,11 {0,04; 0,38}	0,32 {0,22; 0,47}	0,19 {0,04; 0,42}	0,06 {0,00; 0,26}	0,07 {0,02; 0,35}	0,20 {0,10; 0,42}
3	0,29 {0,15; 0,44}	0,05 {0,00; 0,23}	0,12 {0,04; 0,37}	0,30 {0,19; 0,51}	0,20 {0,07; 0,36}	0,05 {0,00; 0,22}	0,08 {0,02; 0,31}	0,20 {0,10; 0,39}
4	0,29 {0,15; 0,45}	0,04 {0,00; 0,17}	0,13 {0,04; 0,37}	0,29 {0,19; 0,49}	0,20 {0,11; 0,32}	0,04 {0,00; 0,19}	0,09 {0,03; 0,33}	0,20 {0,10; 0,37}
5	0,28 {0,14; 0,44}	0,03 {0,00; 0,13}	0,13 {0,05; 0,37}	0,28 {0,18; 0,49}	0,19 {0,12; 0,26}	0,03 {0,00; 0,12}	0,10 {0,03; 0,33}	0,19 {0,09; 0,36}
6	0,27 {0,13; 0,44}	0,02 {0,00; 0,11}	0,12 {0,04; 0,36}	0,26 {0,16; 0,47}				
7	0,25 {0,12; 0,42}	0,02 {0,00; 0,12}	0,12 {0,04; 0,35}	0,24 {0,15; 0,45}				
8	0,23 {0,11; 0,40}	0,03 {0,00; 0,14}	0,11 {0,04; 0,32}	0,23 {0,13; 0,45}				
9	0,21 {0,08; 0,38}	0,04 {0,00; 0,17}	0,10 {0,03; 0,33}	0,22 {0,13; 0,43}				

Table 4.15: We report the results of forecasting regressions of *U.S.* and *U.K.* excess bond returns (rx_{t+1}) using as regressors the CP factor, the forward-spot spread, the yield curve factors (Level, Slope and Curvature) and the five smoothed factors extracted from the state-space model. We report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use monthly data from *PST*, Wright and Fama and Bliss databases for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations) and with residual maturities from 1 to 9 and from 1 to 5 years.

$rx^{(n)}_{GER,t+1}$ - n from 2 to 9 years

n	PST: $x_{GER,t}$	PST: $f^{(n)}_{GER,t} - y^{(1)}_{GER,t}$	PST: YF_{GER}	PST: $\hat{F}_{GER,t\|T}$	W: $x_{GER,t}$	W: $f^{(n)}_{GER,t} - y^{(1)}_{GER,t}$	W: YF_{GER}	W: $\hat{F}_{GER,t\|T}$
2	0,06 {0,00; 0,20}	0,00 {0,00; 0,15}	0,00 {0,01; 0,28}	0,07 {0,05; 0,30}	0,06 {0,00; 0,23}	0,01 {0,00; 0,15}	0,02 {0,01; 0,28}	0,08 {0,06; 0,31}
3	0,11 {0,01; 0,28}	0,01 {0,00; 0,18}	0,03 {0,01; 0,32}	0,12 {0,08; 0,35}	0,11 {0,02; 0,27}	0,01 {0,00; 0,19}	0,05 {0,02; 0,33}	0,12 {0,08; 0,36}
4	0,15 {0,03; 0,33}	0,02 {0,00; 0,20}	0,05 {0,02; 0,35}	0,15 {0,11; 0,39}	0,15 {0,04; 0,33}	0,02 {0,00; 0,20}	0,08 {0,03; 0,36}	0,15 {0,09; 0,40}
5	0,18 {0,05; 0,37}	0,03 {0,00; 0,20}	0,07 {0,02; 0,36}	0,18 {0,10; 0,42}	0,19 {0,05; 0,36}	0,03 {0,00; 0,21}	0,10 {0,04; 0,38}	0,18 {0,11; 0,42}
6	0,21 {0,07; 0,40}	0,03 {0,00; 0,21}	0,09 {0,02; 0,37}	0,20 {0,12; 0,44}	0,21 {0,08; 0,38}	0,03 {0,00; 0,20}	0,11 {0,04; 0,39}	0,20 {0,13; 0,44}
7	0,22 {0,09; 0,41}	0,04 {0,00; 0,20}	0,10 {0,03; 0,37}	0,21 {0,12; 0,45}	0,23 {0,10; 0,41}	0,04 {0,00; 0,21}	0,12 {0,04; 0,39}	0,22 {0,14; 0,46}
8	0,23 {0,10; 0,41}	0,05 {0,00; 0,24}	0,11 {0,03; 0,38}	0,23 {0,12; 0,46}	0,25 {0,12; 0,43}	0,05 {0,00; 0,23}	0,13 {0,04; 0,39}	0,24 {0,16; 0,46}
9	0,24 {0,10; 0,43}	0,07 {0,00; 0,25}	0,12 {0,03; 0,40}	0,24 {0,14; 0,49}	0,26 {0,12; 0,42}	0,05 {0,00; 0,24}	0,14 {0,05; 0,42}	0,26 {0,16; 0,48}

$rx^{(n)}_{GER,t+1}$ - n from 2 to 5 years

n	PST: $x_{U.S.,t}$	PST: $f^{(n)}_{GER,t} - y^{(1)}_{GER,t}$	PST: YF_{GER}	PST: $\hat{F}_{GER,t\|T}$	W: $x_{U.S.,t}$	W: $f^{(n)}_{GER,t} - y^{(1)}_{GER,t}$	W: YF_{GER}	W: $\hat{F}_{GER,t\|T}$
2	0,01 {0,00; 0,15}	0,00 {0,00; 0,15}	0,00 {0,00; 0,28}	0,00 {0,00; 0,23}	0,02 {0,00; 0,18}	0,01 {0,00; 0,14}	0,01 {0,01; 0,28}	0,02 {0,01; 0,25}
3	0,04 {0,00; 0,22}	0,01 {0,00; 0,17}	0,03 {0,01; 0,31}	0,03 {0,01; 0,26}	0,06 {0,00; 0,24}	0,01 {0,00; 0,17}	0,05 {0,02; 0,31}	0,06 {0,02; 0,30}
4	0,06 {0,00; 0,26}	0,02 {0,00; 0,19}	0,06 {0,02; 0,33}	0,06 {0,01; 0,30}	0,09 {0,00; 0,28}	0,02 {0,00; 0,20}	0,07 {0,03; 0,34}	0,08 {0,03; 0,32}
5	0,08 {0,00; 0,28}	0,03 {0,00; 0,20}	0,08 {0,03; 0,33}	0,08 {0,01; 0,34}	0,10 {0,01; 0,29}	0,03 {0,00; 0,19}	0,09 {0,04; 0,36}	0,10 {0,03; 0,35}

$rx^{(n)}_{JAP,t+1}$ - n from 2 to 9 years

n	PST: $x_{JAP,t}$	PST: $f^{(n)}_{JAP,t} - y^{(1)}_{JAP,t}$	PST: YF_{JAP}	PST: $\hat{F}_{JAP,t\|T}$	W: $x_{JAP,t}$	W: $f^{n}_{JAP,t} - y^{1}_{JAP,t}$	W: YF_{JAP}	W: $\hat{F}_{JAP,t\|T}$
2	0,52 {0,34; 0,72}	0,19 {0,03; 0,50}	0,43 {0,27; 0,73}	0,58 {0,49; 0,72}	0,31 {0,10; 0,59}	0,07 {0,00; 0,31}	0,18 {0,05; 0,53}	0,32 {0,22; 0,65}
3	0,57 {0,42; 0,74}	0,25 {0,06; 0,55}	0,48 {0,32; 0,73}	0,61 {0,49; 0,80}	0,35 {0,18; 0,61}	0,13 {0,01; 0,39}	0,23 {0,08; 0,58}	0,36 {0,25; 0,65}
4	0,59 {0,45; 0,75}	0,24 {0,06; 0,50}	0,48 {0,32; 0,73}	0,61 {0,49; 0,79}	0,38 {0,19; 0,62}	0,16 {0,02; 0,41}	0,26 {0,08; 0,58}	0,38 {0,25; 0,65}
5	0,58 {0,43; 0,74}	0,21 {0,04; 0,46}	0,47 {0,32; 0,72}	0,59 {0,47; 0,78}	0,38 {0,20; 0,63}	0,16 {0,01; 0,41}	0,27 {0,10; 0,60}	0,38 {0,26; 0,64}
6	0,57 {0,50; 0,64}	0,17 {0,03; 0,42}	0,45 {0,30; 0,69}	0,56 {0,43; 0,75}	0,38 {0,20; 0,61}	0,16 {0,02; 0,40}	0,28 {0,12; 0,57}	0,38 {0,25; 0,63}
7	0,54 {0,39; 0,71}	0,13 {0,01; 0,35}	0,42 {0,25; 0,65}	0,53 {0,41; 0,72}	0,38 {0,21; 0,59}	0,16 {0,02; 0,37}	0,29 {0,11; 0,57}	0,37 {0,24; 0,62}
8	0,50 {0,33; 0,69}	0,10 {0,00; 0,30}	0,40 {0,24; 0,63}	0,50 {0,38; 0,69}	0,37 {0,19; 0,59}	0,16 {0,02; 0,38}	0,29 {0,12; 0,57}	0,37 {0,24; 0,61}
9	0,45 {0,30; 0,64}	0,08 {0,00; 0,26}	0,37 {0,25; 0,56}	0,47 {0,36; 0,67}	0,36 {0,19; 0,56}	0,16 {0,02; 0,37}	0,30 {0,14; 0,55}	0,37 {0,24; 0,59}

$rx^{(n)}_{JAP,t+1}$ - n from 2 to 5 years

n	PST: $x_{JAP,t}$	PST: $f^{(n)}_{JAP,t} - y^{(1)}_{JAP,t}$	PST: YF_{JAP}	PST: $\hat{F}_{JAP,t\|T}$	W: $x_{JAP,t}$	W: $f^{n}_{JAP,t} - y^{1}_{JAP,t}$	W: YF_{JAP}	W: $\hat{F}_{JAP,t\|T}$
2	0,43 {0,22; 0,67}	0,19 {0,04; 0,49}	0,35 {0,18; 0,69}	0,44 {0,28; 0,71}	0,27 {0,08; 0,59}	0,07 {0,00; 0,30}	0,18 {0,06; 0,53}	0,27 {0,12; 0,61}
3	0,48 {0,28; 0,72}	0,25 {0,08; 0,54}	0,40 {0,23; 0,69}	0,48 {0,33; 0,72}	0,33 {0,12; 0,63}	0,13 {0,01; 0,43}	0,23 {0,07; 0,56}	0,32 {0,16; 0,64}
4	0,49 {0,29; 0,71}	0,24 {0,07; 0,52}	0,42 {0,24; 0,70}	0,49 {0,33; 0,73}	0,35 {0,14; 0,61}	0,16 {0,01; 0,40}	0,26 {0,10; 0,60}	0,34 {0,17; 0,63}
5	0,48 {0,28; 0,68}	0,21 {0,05; 0,46}	0,42 {0,25; 0,69}	0,48 {0,33; 0,68}	0,35 {0,15; 0,60}	0,16 {0,02; 0,40}	0,27 {0,11; 0,58}	0,34 {0,18; 0,62}

Table 4.16: We report the results of forecasting regressions of GER and JAP excess bond returns (rx_{t+1}) using as regressors the CP factor, the forward-spot spread, the yield curve factors (Level, Slope and Curvature) and the five smoothed factors extracted from the state-space model. We report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use monthly data from PST and Wright databases for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations) and with residual maturities from 1 to 9 and from 1 to 5 years.

Diebold Database

h	$g_{U.S.,t+h}$				$g_{U.K.,t+h}$				$\pi_{U.S.,t+h}$				$\pi_{U.K.,t+h}$			
h	$x_{U.S.,t}$	$x_{U.S.,t}+S_{U.S.,t}$	$\hat{F}_{U.S.,t\|T}$	$\hat{F}_{U.S.,t\|T}+S_{U.S.,t}$	$x_{U.K.,t}$	$x_{U.K.,t}+S_{U.S.,t}$	$\hat{F}_{U.K.,t\|T}$	$\hat{F}_{U.K.,t\|T}+S_{U.K.,t}$	$x_{U.S.,t}$	$x_{U.S.,t}+L_{U.S.,t}$	$\hat{F}_{U.S.,t\|T}$	$\hat{F}_{U.S.,t\|T}+L_{U.S.,t}$	$x_{U.K.,t}$	$x_{U.K.,t}+L_{U.S.,t}$	$\hat{F}_{U.K.,t\|T}$	$\hat{F}_{U.K.,t\|T}+L_{U.K.,t}$
12	0,00	0,08	0,10	0,12	0,01	0,03	0,05	0,05	0,07	0,24	0,23	0,24	0,10	0,71	0,34	0,75
	{0,00 ; 0,13}	{0,01; 0,33}	{0,06; 0,41}	{0,07; 0,41}	{0,00; 0,11}	{0,00; 0,20}	{0,03; 0,31}	{0,03; 0,31}	{0,00; 0,33}	{0,08; 0,51}	{0,12; 0,54}	{0,14; 0,55}	{0,00; 0,34}	{0,62; 0,82}	{0,23; 0,65}	{0,69; 0,85}
18	0,01	0,06	0,09	0,09	0,02	0,05	0,06	0,06	0,05	0,12	0,16	0,16	0,06	0,62	0,34	0,67
	{0,00 ; 0,18}	{0,00; 0,29}	{0,05; 0,39}	{0,06; 0,40}	{0,00; 0,14}	{0,00; 0,24}	{0,03; 0,32}	{0,04; 0,33}	{0,00; 0,28}	{0,01; 0,39}	{0,07; 0,47}	{0,08; 0,47}	{0,00; 0,30}	{0,48; 0,77}	{0,21; 0,65}	{0,59; 0,82}
24	0,03	0,05	0,06	0,06	0,02	0,03	0,05	0,10	0,07	0,09	0,14	0,14	0,03	0,54	0,41	0,61
	{0,00 ; 0,22}	{0,00; 0,29}	{0,03; 0,37}	{0,04; 0,39}	{0,00; 0,13}	{0,00; 0,20}	{0,03; 0,33}	{0,07; 0,36}	{0,00; 0,34}	{0,00; 0,39}	{0,07; 0,49}	{0,08; 0,49}	{0,00; 0,22}	{0,35; 0,74}	{0,27; 0,68}	{0,51; 0,80}
30	0,02	0,08	0,11	0,12	0,01	0,01	0,02	0,04	0,06	0,07	0,09	0,09	0,02	0,42	0,34	0,48
	{0,00 ; 0,19}	{0,01; 0,35}	{0,04; 0,43}	{0,07; 0,44}	{0,00; 0,12}	{0,00; 0,17}	{0,02; 0,29}	{0,03; 0,32}	{0,00; 0,33}	{0,00; 0,39}	{0,04; 0,47}	{0,05; 0,47}	{0,00; 0,21}	{0,20; 0,69}	{0,20; 0,67}	{0,36; 0,74}
36	0,02	0,13	0,15	0,15	0,10	0,10	0,16	0,16	0,07	0,08	0,13	0,13	0,04	0,35	0,24	0,40
	{0,00 ; 0,23}	{0,02; 0,42}	{0,06; 0,46}	{0,07; 0,48}	{0,01; 0,25}	{0,02; 0,29}	{0,09; 0,43}	{0,09; 0,43}	{0,00; 0,36}	{0,01; 0,40}	{0,07; 0,51}	{0,07; 0,52}	{0,00; 0,23}	{0,11; 0,67}	{0,10; 0,61}	{0,26; 0,72}

h	$g_{GER,t+h}$				$g_{JAP,t+h}$				$\pi_{GER,t+h}$				$\pi_{JAP,t+h}$			
h	$x_{GER,t}$	$x_{GER,t}+S_{GER,t}$	$\hat{F}_{GER,t\|T}$	$\hat{F}_{GER,t\|T}+S_{GER,t}$	$x_{JAP,t}$	$x_{JAP,t}+S_{GER,t}$	$\hat{F}_{JAP,t\|T}$	$\hat{F}_{JAP,t\|T}+S_{JAP,t}$	$x_{GER,t}$	$x_{GER,t}+L_{GER,t}$	$\hat{F}_{GER,t\|T}$	$\hat{F}_{GER,t\|T}+L_{GER,t}$	$x_{JAP,t}$	$x_{JAP,t}+L_{GER,t}$	$\hat{F}_{JAP,t\|T}$	$\hat{F}_{JAP,t\|T}+L_{JAP,t}$
12	0,00	0,10	0,10	0,11	0,00	-0,01	0,09	0,09	0,04	0,48	0,15	0,50	0,01	0,51	0,17	0,54
	{0,00 ; 0,09}	{0,02; 0,30}	{0,04; 0,34}	{0,05; 0,36}	{0,00; 0,11}	{0,00; 0,15}	{0,05; 0,35}	{0,05; 0,37}	{0,00; 0,22}	{0,27; 0,71}	{0,07; 0,46}	{0,35; 0,74}	{0,00; 0,23}	{0,34; 0,73}	{0,09; 0,47}	{0,42; 0,77}
18	0,00	0,03	0,09	0,10	0,00	-0,01	0,04	0,03	0,01	0,42	0,11	0,46	0,01	0,41	0,12	0,42
	{0,00 ; 0,09}	{0,00; 0,20}	{0,05; 0,34}	{0,05; 0,33}	{0,00; 0,12}	{0,00; 0,16}	{0,03; 0,29}	{0,04; 0,29}	{0,00; 0,19}	{0,20; 0,67}	{0,06; 0,42}	{0,30; 0,73}	{0,00; 0,22}	{0,23; 0,66}	{0,07; 0,43}	{0,26; 0,72}
24	0,00	0,00	0,01	0,01	0,01	0,02	0,05	0,04	0,00	0,33	0,12	0,34	0,01	0,33	0,13	0,34
	{0,00 ; 0,07}	{-0,01; 0,14}	{0,01; 0,24}	{0,02; 0,25}	{0,00; 0,14}	{0,00; 0,20}	{0,02; 0,31}	{0,04; 0,32}	{0,00; 0,13}	{0,11; 0,63}	{0,08; 0,45}	{0,19; 0,67}	{0,00; 0,25}	{0,13; 0,62}	{0,07; 0,44}	{0,20; 0,67}
30	0,02	0,03	0,05	0,06	0,04	0,07	0,12	0,11	0,00	0,19	0,11	0,20	0,00	0,28	0,16	0,31
	{0,00 ; 0,15}	{0,00; 0,22}	{0,03; 0,28}	{0,04; 0,30}	{0,00; 0,21}	{0,01; 0,26}	{0,06; 0,39}	{0,06; 0,39}	{0,00; 0,15}	{0,04; 0,54}	{0,06; 0,46}	{0,11; 0,60}	{0,00; 0,20}	{0,09; 0,59}	{0,08; 0,50}	{0,17; 0,66}
36	0,05	0,08	0,15	0,16	0,05	0,06	0,08	0,08	0,00	0,07	0,07	0,09	0,00	0,24	0,20	0,31
	{0,00 ; 0,21}	{0,01; 0,31}	{0,08; 0,40}	{0,09; 0,42}	{0,00; 0,25}	{0,01; 0,26}	{0,04; 0,35}	{0,06; 0,36}	{0,00; 0,15}	{0,01; 0,42}	{0,05; 0,42}	{0,06; 0,51}	{0,00; 0,18}	{0,06; 0,57}	{0,12; 0,53}	{0,18; 0,67}

n	$rx^{(n)}_{U.S.,t+1}$				$rx^{(n)}_{U.K.,t+1}$				$rx^{(n)}_{GER,t+1}$				$rx^{(n)}_{JAP,t+1}$			
n	$x_{U.S.,t}$	$f^{(n)}_{U.S.,t}-y^{(1)}_{U.S.,t}$	$YF_{U.S.}$	$\hat{F}_{U.S.,t\|T}$	$x_{U.K.,t}$	$f^{(n)}_{U.K.,t}-y^{(1)}_{U.K.,t}$	$YF_{U.K.}$	$\hat{F}_{U.K.,t\|T}$	$x_{GER,t}$	$f^{(n)}_{GER,t}-y^{(1)}_{GER,t}$	YF_{GER}	$\hat{F}_{GER,t\|T}$	$x_{JAP,t}$	$f^{(n)}_{JAP,t}-y^{(1)}_{JAP,t}$	YF_{JAP}	$\hat{F}_{JAP,t\|T}$
2	0,19	0,00	0,20	0,20	0,11	0,07	0,15	0,35	0,03	0,04	0,10	0,16	0,17	0,04	0,12	0,20
	{0,05 ; 0,41}	{0,00; 0,05}	{0,08; 0,46}	{0,11; 0,47}	{0,01; 0,27}	{0,01; 0,18}	{0,05; 0,40}	{0,25; 0,56}	{0,00; 0,19}	{0,00; 0,14}	{0,04; 0,38}	{0,07; 0,44}	{0,03; 0,40}	{0,00; 0,12}	{0,04; 0,42}	{0,11; 0,48}
3	0,19	0,00	0,19	0,19	0,22	0,02	0,10	0,34	0,10	0,06	0,14	0,19	0,20	0,05	0,14	0,23
	{0,06 ; 0,40}	{0,00; 0,06}	{0,08; 0,43}	{0,11; 0,45}	{0,09; 0,38}	{0,00; 0,13}	{0,02; 0,33}	{0,25; 0,54}	{0,01; 0,28}	{0,00; 0,20}	{0,06; 0,42}	{0,11; 0,47}	{0,05; 0,43}	{0,00; 0,20}	{0,04; 0,42}	{0,14; 0,51}
4	0,23	0,01	0,19	0,24	0,22	0,00	0,13	0,27	0,22	0,08	0,22	0,26	0,20	0,01	0,15	0,20
	{0,08 ; 0,43}	{0,00; 0,11}	{0,08; 0,44}	{0,14; 0,50}	{0,10; 0,37}	{0,00; 0,06}	{0,04; 0,38}	{0,18; 0,49}	{0,07; 0,43}	{0,00; 0,24}	{0,12; 0,49}	{0,17; 0,52}	{0,05; 0,42}	{0,00; 0,15}	{0,05; 0,40}	{0,10; 0,48}
5	0,22	0,01	0,23	0,22	0,25	0,08	0,15	0,29	0,19	0,03	0,21	0,23	0,21	0,05	0,17	0,21
	{0,07 ; 0,41}	{0,00; 0,10}	{0,10; 0,47}	{0,13; 0,47}	{0,12; 0,40}	{0,00; 0,23}	{0,06; 0,40}	{0,20; 0,48}	{0,05; 0,38}	{0,00; 0,18}	{0,10; 0,47}	{0,13; 0,50}	{0,05; 0,44}	{0,00; 0,21}	{0,07; 0,45}	{0,11; 0,48}
6	0,25	0,04	0,21	0,25	0,23	0,05	0,11	0,22	0,16	0,03	0,11	0,20	0,31	0,17	0,26	0,30
	{0,10 ; 0,44}	{0,01; 0,08}	{0,10; 0,44}	{0,15; 0,48}	{0,12; 0,37}	{0,00; 0,16}	{0,04; 0,33}	{0,15; 0,41}	{0,04; 0,38}	{0,00; 0,17}	{0,03; 0,38}	{0,13; 0,48}	{0,12; 0,54}	{0,03; 0,38}	{0,12; 0,52}	{0,17; 0,55}
7	0,18	0,00	0,20	0,20	0,20	0,01	0,13	0,21	0,16	0,03	0,17	0,21	0,30	0,07	0,28	0,30
	{0,05 ; 0,37}	{0,00; 0,06}	{0,08; 0,42}	{0,11; 0,44}	{0,09; 0,36}	{0,00; 0,09}	{0,04; 0,36}	{0,14; 0,41}	{0,03; 0,35}	{0,00; 0,15}	{0,07; 0,45}	{0,13; 0,51}	{0,12; 0,49}	{0,00; 0,24}	{0,14; 0,52}	{0,18; 0,55}

Table 4.17: We report the results of forecasting regressions of $U.S.$, $U.K.$, GER and JAP economic activity (g_{t+h}), inflation rates (π_{t+h}) using the CP factor, the CP factor and the yield curve factor (the slope for forecasting economic activity while the level for forecasting inflation rates) jointly, the five smoothed factors extracted from the State-Space model and the five smoothed factors and the yield curve factor jointly. We run regressions for annualized growth in industrial production over forecasting horizons of $h = 12$, 18, 24, 30 and 36 months. We also provide the results of forecasting regressions of $U.S.$, $U.K.$, GER and JAP excess bond returns (rx_{t+1}) using as regressors the CP factor, the forward-spot spread, the yield curve factors (Level, Slope and Curvature) and the five smoothed factors extracted from the state-space model. We report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use monthly data (DLY database) for a sample period starting from January 1, 1981 till December 31, 2005 (286 observations) and with residual maturities from 1 to 7 years.

4.E Additional checks PST database

PST Database (3-months MA Forwards Rates)

h	$g_{U.S.,t+h}$				$g_{U.K.,t+h}$				$\pi_{U.S.,t+h}$				$\pi_{U.K.,t+h}$			
	$x_{U.S.,t}$	$x_{U.S.,t}+S_{U.S.,t}$	$\hat{F}_{U.S.,t\|T}$	$\hat{F}_{U.S.,t\|T}+S_{U.S.,t}$	$x_{U.K.,t}$	$x_{U.K.,t}+S_{U.K.,t}$	$\hat{F}_{U.K.,t\|T}$	$\hat{F}_{U.K.,t\|T}+S_{U.K.,t}$	$x_{U.S.,t}$	$x_{U.S.,t}+L_{U.S.,t}$	$\hat{F}_{U.S.,t\|T}$	$\hat{F}_{U.S.,t\|T}+L_{U.S.,t}$	$x_{U.K.,t}$	$x_{U.K.,t}+L_{U.K.,t}$	$\hat{F}_{U.K.,t\|T}$	$\hat{F}_{U.K.,t\|T}+L_{U.K.,t}$
12	0,03	0,04	0,10	0,12	0,00	0,14	0,24	0,28	0,00	0,28	0,15	0,29	0,06	0,73	0,40	0,78
	{0,00 ; 0,20}	{0,00; 0,31}	{0,05; 0,43}	{0,09; 0,43}	{0,00; 0,10}	{0,03; 0,41}	{0,15; 0,49}	{0,18; 0,54}	{0,00; 0,10}	{0,13; 0,54}	{0,07; 0,44}	{0,18; 0,57}	{0,00; 0,32}	{0,65; 0,86}	{0,27; 0,70}	{0,74; 0,88}
18	0,05	0,10	0,16	0,16	0,09	0,23	0,25	0,30	0,00	0,21	0,13	0,21	0,08	0,70	0,47	0,74
	{0,00 ; 0,22}	{0,01; 0,35}	{0,09; 0,48}	{0,11; 0,49}	{0,00; 0,28}	{0,09; 0,47}	{0,14; 0,51}	{0,20; 0,58}	{0,00; 0,12}	{0,06; 0,45}	{0,07; 0,42}	{0,11; 0,52}	{0,00; 0,34}	{0,60; 0,83}	{0,34; 0,74}	{0,69; 0,86}
24	0,06	0,17	0,22	0,23	0,17	0,26	0,28	0,29	0,00	0,12	0,09	0,19	0,16	0,64	0,52	0,67
	{0,00 ; 0,24}	{0,04; 0,46}	{0,13; 0,52}	{0,13; 0,56}	{0,04; 0,44}	{0,10; 0,52}	{0,16; 0,59}	{0,19; 0,60}	{0,00; 0,10}	{0,02; 0,39}	{0,05; 0,40}	{0,12; 0,49}	{0,01; 0,45}	{0,52; 0,79}	{0,41; 0,74}	{0,60; 0,83}
30	0,11	0,25	0,28	0,30	0,17	0,23	0,28	0,29	0,01	0,07	0,08	0,17	0,27	0,61	0,60	0,65
	{0,00 ; 0,34}	{0,08; 0,52}	{0,16; 0,59}	{0,20; 0,62}	{0,03; 0,43}	{0,09; 0,49}	{0,17; 0,57}	{0,20; 0,58}	{0,00; 0,16}	{0,00; 0,33}	{0,04; 0,40}	{0,11; 0,48}	{0,08; 0,56}	{0,48; 0,79}	{0,52; 0,78}	{0,59; 0,83}
36	0,09	0,27	0,24	0,33	0,17	0,20	0,30	0,35	0,01	0,04	0,12	0,13	0,17	0,42	0,48	0,51
	{0,00 ; 0,31}	{0,10; 0,57}	{0,13; 0,59}	{0,21; 0,67}	{0,04; 0,41}	{0,07; 0,46}	{0,19; 0,59}	{0,26; 0,63}	{0,00; 0,15}	{0,00; 0,26}	{0,07; 0,41}	{0,09; 0,45}	{0,01; 0,48}	{0,25; 0,71}	{0,37; 0,74}	{0,43; 0,75}

h	$g_{GER,t+h}$				$g_{JAP,t+h}$				$\pi_{GER,t+h}$				$\pi_{JAP,t+h}$			
	$x_{GER,t}$	$x_{GER,t}+S_{GER,t}$	$\hat{F}_{GER,t\|T}$	$\hat{F}_{GER,t\|T}+S_{GER,t}$	$x_{JAP,t}$	$x_{JAP,t}+S_{JAP,t}$	$\hat{F}_{JAP,t\|T}$	$\hat{F}_{JAP,t\|T}+S_{JAP,t}$	$x_{GER,t}$	$x_{GER,t}+L_{GER,t}$	$\hat{F}_{GER,t\|T}$	$\hat{F}_{GER,t\|T}+L_{GER,t}$	$x_{JAP,t}$	$x_{JAP,t}+L_{JAP,t}$	$\hat{F}_{JAP,t\|T}$	$\hat{F}_{JAP,t\|T}+L_{JAP,t}$
12	0,00	0,16	0,26	0,26	0,00	0,07	0,09	0,12	0,00	0,45	0,21	0,57	0,01	0,48	0,42	0,49
	{0,00 ; 0,12}	{0,04; 0,45}	{0,16; 0,56}	{0,17; 0,56}	{0,00; 0,12}	{0,01; 0,31}	{0,05; 0,33}	{0,09; 0,37}	{0,00; 0,11}	{0,27; 0,66}	{0,14; 0,54}	{0,49; 0,76}	{0,00; 0,21}	{0,31; 0,70}	{0,29; 0,67}	{0,39; 0,74}
18	0,00	0,15	0,17	0,18	0,01	0,05	0,08	0,11	0,01	0,48	0,27	0,57	0,02	0,46	0,45	0,51
	{0,00 ; 0,11}	{0,05; 0,42}	{0,10; 0,48}	{0,11; 0,52}	{0,00; 0,15}	{0,01; 0,28}	{0,04; 0,16}	{0,07; 0,20}	{0,00; 0,13}	{0,33; 0,69}	{0,17; 0,58}	{0,49; 0,75}	{0,00; 0,24}	{0,30; 0,67}	{0,35; 0,67}	{0,41; 0,72}
24	0,01	0,12	0,12	0,16	0,01	0,04	0,11	0,11	0,05	0,44	0,33	0,55	0,04	0,41	0,45	0,52
	{0,00 ; 0,15}	{0,02; 0,34}	{0,05; 0,43}	{0,09; 0,38}	{0,00; 0,13}	{0,00; 0,27}	{0,06; 0,29}	{0,06; 0,23}	{0,00; 0,24}	{0,28; 0,66}	{0,20; 0,62}	{0,47; 0,75}	{0,00; 0,27}	{0,25; 0,63}	{0,35; 0,68}	{0,42; 0,72}
30	0,07	0,15	0,16	0,21	0,00	0,01	0,10	0,10	0,06	0,34	0,26	0,40	0,03	0,39	0,55	0,57
	{0,00 ; 0,28}	{0,03; 0,42}	{0,08; 0,48}	{0,13; 0,44}	{0,00; 0,11}	{0,00; 0,21}	{0,06; 0,41}	{0,06; 0,43}	{0,00; 0,26}	{0,16; 0,62}	{0,16; 0,60}	{0,29; 0,71}	{0,00; 0,27}	{0,21; 0,65}	{0,46; 0,75}	{0,47; 0,76}
36	0,08	0,17	0,16	0,30	0,00	0,00	0,07	0,07	0,04	0,23	0,16	0,28	0,02	0,34	0,49	0,50
	{0,00 ; 0,31}	{0,04; 0,44}	{0,08; 0,51}	{0,19; 0,59}	{0,00; 0,12}	{0,00; 0,17}	{0,05; 0,40}	{0,05; 0,43}	{0,00; 0,23}	{0,07; 0,57}	{0,10; 0,57}	{0,18; 0,68}	{0,00; 0,23}	{0,17; 0,63}	{0,38; 0,71}	{0,40; 0,71}

Forecasting HPRX

n	$rx^{(n)}_{U.S.,t+1}$				$rx^{(n)}_{U.K.,t+1}$				$rx^{(n)}_{GER,t+1}$				$rx^{(n)}_{JAP,t+1}$			
	$x_{U.S.,t}$	$f^{(n)}_{U.S.,t}-y^{(1)}_{U.S.,t}$	$YF_{U.S.}$	$\hat{F}_{U.S.,t\|T}$	$x_{U.K.,t}$	$f^{(n)}_{U.K.,t}-y^{(1)}_{U.K.,t}$	$YF_{U.K.}$	$\hat{F}_{U.K.,t\|T}$	$x_{GER,t}$	$f^{(n)}_{GER,t}-y^{(1)}_{GER,t}$	YF_{GER}	$\hat{F}_{GER,t\|T}$	$x_{JAP,t}$	$f^{(n)}_{JAP,t}-y^{(1)}_{JAP,t}$	YF_{JAP}	$\hat{F}_{JAP,t\|T}$
2	0,05	0,02	0,02	0,10	0,39	0,00	0,11	0,44	0,09	0,00	0,00	0,15	0,57	0,01	0,43	0,64
	{0,00 ; 0,21}	{-0,01; 0,20}	{0,00; 0,29}	{0,06; 0,38}	{0,22; 0,58}	{0,00; 0,14}	{0,04; 0,38}	{0,35; 0,64}	{0,00; 0,28}	{-0,01; 0,15}	{0,00; 0,27}	{0,10; 0,43}	{0,42; 0,74}	{0,00; 0,21}	{0,27; 0,71}	{0,51; 0,82}
3	0,08	0,01	0,03	0,10	0,41	0,00	0,12	0,44	0,16	0,00	0,03	0,20	0,61	0,02	0,48	0,66
	{0,00 ; 0,25}	{-0,01; 0,18}	{0,01; 0,27}	{0,05; 0,36}	{0,27; 0,58}	{-0,01; 0,11}	{0,04; 0,37}	{0,34; 0,59}	{0,03; 0,37}	{-0,01; 0,16}	{0,01; 0,31}	{0,13; 0,47}	{0,47; 0,75}	{0,00; 0,25}	{0,31; 0,73}	{0,55; 0,81}
4	0,09	0,01	0,04	0,09	0,42	0,01	0,13	0,44	0,22	0,01	0,05	0,23	0,62	0,04	0,48	0,65
	{0,00 ; 0,24}	{-0,01; 0,16}	{0,01; 0,28}	{0,05; 0,34}	{0,33; 0,52}	{-0,01; 0,13}	{0,05; 0,37}	{0,33; 0,62}	{0,07; 0,42}	{0,00; 0,17}	{0,02; 0,34}	{0,16; 0,50}	{0,49; 0,74}	{0,00; 0,29}	{0,34; 0,73}	{0,54; 0,81}
5	0,10	0,01	0,04	0,09	0,42	0,01	0,13	0,42	0,25	0,02	0,07	0,26	0,61	0,05	0,47	0,62
	{0,01 ; 0,26}	{-0,01; 0,15}	{0,01; 0,27}	{0,05; 0,32}	{0,29; 0,57}	{0,00; 0,15}	{0,04; 0,36}	{0,33; 0,60}	{0,09; 0,46}	{0,00; 0,18}	{0,02; 0,36}	{0,17; 0,53}	{0,49; 0,74}	{0,00; 0,30}	{0,30; 0,71}	{0,50; 0,79}
6	0,10	0,01	0,05	0,09	0,41	0,01	0,12	0,41	0,28	0,03	0,09	0,27	0,59	0,05	0,45	0,59
	{0,01 ; 0,27}	{-0,01; 0,14}	{0,01; 0,26}	{0,06; 0,31}	{0,28; 0,56}	{0,00; 0,14}	{0,04; 0,36}	{0,31; 0,59}	{0,11; 0,49}	{0,00; 0,20}	{0,03; 0,39}	{0,17; 0,53}	{0,47; 0,73}	{0,00; 0,29}	{0,29; 0,69}	{0,49; 0,76}
7	0,10	0,01	0,06	0,09	0,40	0,01	0,12	0,39	0,30	0,03	0,10	0,29	0,57	0,05	0,42	0,56
	{0,05 ; 0,19}	{0,00; 0,10}	{0,01; 0,25}	{0,05; 0,20}	{0,26; 0,54}	{0,00; 0,15}	{0,04; 0,35}	{0,30; 0,58}	{0,13; 0,50}	{0,00; 0,21}	{0,03; 0,38}	{0,18; 0,54}	{0,43; 0,71}	{0,00; 0,26}	{0,27; 0,65}	{0,45; 0,73}
8	0,11	0,01	0,06	0,10	0,38	0,01	0,11	0,37	0,31	0,03	0,11	0,30	0,53	0,04	0,40	0,53
	{0,01 ; 0,26}	{0,00; 0,14}	{0,02; 0,22}	{0,04; 0,30}	{0,24; 0,53}	{0,00; 0,15}	{0,03; 0,35}	{0,28; 0,55}	{0,15; 0,50}	{0,00; 0,21}	{0,03; 0,38}	{0,17; 0,57}	{0,38; 0,70}	{0,00; 0,24}	{0,24; 0,62}	{0,41; 0,70}
9	0,11	0,02	0,07	0,10	0,36	0,02	0,10	0,36	0,31	0,03	0,12	0,31	0,48	0,03	0,37	0,49
	{0,01 ; 0,26}	{-0,01; 0,14}	{0,02; 0,28}	{0,05; 0,32}	{0,22; 0,52}	{0,00; 0,16}	{0,03; 0,33}	{0,26; 0,56}	{0,14; 0,52}	{0,00; 0,20}	{0,03; 0,39}	{0,19; 0,56}	{0,33; 0,67}	{0,00; 0,19}	{0,22; 0,56}	{0,38; 0,70}

Table 4.18: We report the results of forecasting regressions of *U.S.*, *U.K.*, *GER* and *JAP* economic activity (g_{t+h}), inflation rates (π_{t+h}) using the CP factor, the CP factor and the yield curve factor (the slope for forecasting economic activity while the level for forecasting inflation rates) jointly, the five smoothed factors extracted from the State-Space model and the five smoothed factors and the yield curve factor jointly. We run regressions for annualized growth in industrial production over forecasting horizons of h = 12, 18, 24, 30 and 36 months. We also provide the results of forecasting regressions of *U.S.*, *U.K.*, *GER* and *JAP* excess bond returns (rx_{t+1}) using as regressors the CP factor, the forward-spot spread, the yield curve factors (Level, Slope and Curvature) and the five smoothed factors extracted from the state-space model. We report adjusted R^2 with 90% bootstrapped confidence intervals in curly brackets. We use 3-months moving average forward rates, monthly data (*PST* database) for a sample period starting from January 1, 1986 till December 31, 2009 (288 observations) and with residual maturities from 1 to 9 years.

U.S.				U.K.			
n	$J-B$	$B-G$	$ARCH$	n	$J-B$	$B-G$	$ARCH$
1	0.8717	193.2	111.0	1	0.0000	230.3	159.3
2	0.9237	180.2	91.6	2	0.0001	228.5	155.7
3	0.3288	172.8	80.4	3	0.0059	224.2	157.5
4	0.0170	168.6	80.0	4	0.0981	216.2	151.1
5	0.0000	164.6	90.2	5	0.8896	210.8	152.4
6	0.0000	172.4	115.7	6	0.0000	219.6	119.8
7	0.2805	172.4	80.6	7	0.0055	218.4	149.1
8	0.0037	167.3	81.9	8	0.1286	216.8	153.5
GER				JAP			
n	$J-B$	$B-G$	$ARCH$	n	$J-B$	$B-G$	$ARCH$
1	0.0000	128.1	19.8	1	0.2756	221.1	166.4
2	0.0000	149.3	36.5	2	0.9129	218.0	168.9
3	0.0000	165.8	40.7	3	0.3188	213.7	161.8
4	0.0002	178.5	40.7	4	0.0035	211.5	165.4
5	0.4106	189.0	54.1	5	0.0000	211.4	182.1
6	0.0000	185.2	71.0	6	0.0000	205.9	194.1
7	0.0000	165.9	38.1	7	0.2762	211.5	154.5
8	0.0356	183.2	41.8	8	0.0000	215.1	182.1

Table 4.19: We report analysis of residuals obtained from regressions of expected excess bond returns at different maturities on the CP factor. For any given country and for any given bond maturity n we display the p-value associated to the Jarque-Bera statistic, the chi-squared values for both the Breusch-Godfrey test for serial correlation and the ARCH test of Engle (1982) for conditional heteroskedasticity. We derive forward rates and calculate expected excess bond returns by using the international Treasury yield curve database constructed in Chapter 2. For any country, yields are observed monthly from January 1, 1986 to December 31, 2009 (288 observations) and with residual maturities from 1 to 9 years.

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