Optimization of Resource Allocation in Small Cells Networks: A Green Networking Approach

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OPTIMIZATION OF RESOURCE ALLOCATION IN SMALL CELLS NETWORKS

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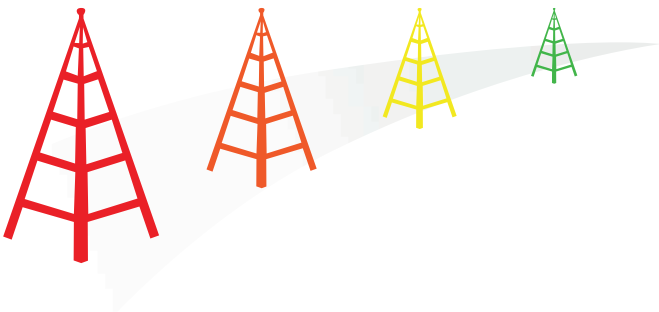

A Green Networking Approach

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to my lovely wife and daughter
as well as my family in Macedonia...

ABSTRACT

The term “green networking” refers to energy-efficient networking technologies and products, while minimizing resource usage as possible. This thesis targets the problem of resource allocation in small cells networks in a green networking context. We develop algorithms for different paradigms. We exploit the framework of coalitional games theory and some stochastic geometric tools as well as the crowding game model.

We first study the mobile assignment problem in broadcast transmission where minimal total power consumption is sought. A green-aware approach is followed in our algorithms. We examine the coalitional game aspects of the mobile assignment problem. This game has an incentive to form grand coalition where all players join to the game. By using Bondareva-Shapley theorem, we prove that this coalitional game has a non-empty core which means that the grand coalition is stable. Then, we examine the cost allocation policy for different methods.

In a second part, we analyze a significant problem in green networking called switching off base stations in case of cooperating service providers by means of stochastic geometric and coalitional game tools. The coalitional game herein considered is played by service providers who cooperate in switching off base stations.

We observed the Nash stability which is a concept in hedonic coalition formation games. We ask the following question: Is there any utility allocation method which could result in a Nash-stable partition? We address this issue in the thesis. We propose the definition of the Nash-stable core which is the set of all possible utility allocation methods resulting in stable partitions obtained according to Nash stability.

We finally consider games related to the association of mobiles to an access point. The player is the mobile which has to decide to which access point to connect. We consider the choice between two access points or more, where the access decisions may depend on the number of mobiles connected to each access points. We obtained new results using elementary tools from congestion and crowding games.

Last but not least, we extend our work to cooperative transmissions. We formulate the partner selection problem in cooperative relaying based on a matching theoretic approach. Partner selection is described as a special stable roommate problem where each player ranks its partners by some criterion. We adapted Irving’s algorithm for determining the partner of each player. We introduced a decentralized version of the Irving’s algorithm.

RÉSUMÉ

Le terme “réseau vert” ou pour éviter une traduction directe, “réseau propre” repose sur la sélection de technologies et de produits réseaux économes en énergie, et grantissant un usage minimal des ressources (radio, bande passante,...) quand cela est possible. Cette thèse vise à étudier les problèmes d’allocation des ressources dans les petits réseaux de cellules dans un contexte de réseau propre. Nous développons des algorithmes pour différents paradigmes. Nos travaux reposent principalement sur le contexte de la théorie des jeux de coalition, mais également sur des outils de géométrie stochastique ainsi que d’un modèle de jeu de surpeuplement.

Nous étudions tout d’abord le problème d’association de mobiles à des stations de base dans les applications de diffusion d’un flux commun, sous contrainte de minimisation de la consommation d’énergie totale: nos algorithmes suivent une approche préservant l’énergie. Nous examinons le problème d’association des mobiles sous le prisme des jeux de coalition. Ce jeu tend à former la grande coalition, qui se caractérise par le fait que tous les joueurs forment une coalition unique. En utilisant le théorème de Bondareva-Shapley, nous prouvons que ce jeu de coalition a un noyau non vide ce qui signifie que la grande coalition est stable. Ensuite, nous examinons la politique de répartition des coûts pour différentes méthodes.

Dans une deuxième partie, nous analysons un problème important dans les réseaux propres qui consiste à étteindre les stations de base qui ne sont pas indispensables. Nous abordons ce problème de facon statistique, dans le cas de fournisseurs de services coopérant au moyen d’outils de jeux de coalition vus sous un angle de la géométrie stochastique. Le jeu coalitionnel considéré est joué par les fournisseurs de services qui collaborent à éteindre leurs stations de base.

Nous avons analysé la stabilité de Nash qui est un concept utilisé pour les jeux de coalition hédoniques. Nous posons la question suivante: Existe-t-il une méthode de répartition de la fonction d’utilitè qui se traduit par un partitionnement Nash-stable? Nous répondons à cette question dans la thèse. Nous démontrons que le noyau Nash-stable, défini comme l’ensemble des méthodes de répartition des couts conduisant à un partitionnement stable au sens de la stabilité de Nash.

Nous considérons finalement les jeux liés à l’association des mobiles à un point d’accès non plus dans le cas d’un broadcast, mais dans le cas général. Le jeu consiste à décider à quel point d’accès un mobile doit se connecter. Nous considérons le choix entre deux points d’accès ou plus. Les décisions d’association dépendent du nombre de mobiles connectés à chacun des points d’accès. Nous obtenons de nouveaux résultats en utilisant des outils élémentaires de jeux de congestion et déviction.

Enfin, nous nous intéressons aux transmissions coopératives. Nous étudions le problème de la sélection de partenaires dans le cas de constitution de binomes gagnant-gagnant, ou chacun des partenaire s’appuie sur l’autre pour sa propre transmission. Nous proposons d’assimiler la sélection des partenaires au problème classique en théorie des jeux de recherche stable de colocataire

où chaque joueur établi une liste de préférence parmi les partenaires possibles; Nous adaptons l'algorithme de Irving pour déterminer le partenaire de chaque joueur et nous introduisons une version décentralisée de l'algorithme de Irving.

PUBLICATIONS

During the thesis study, the following publications have been yielded:

Journal Papers

1. C. Hasan, E. Altman, J.-M. Gorce, "The Nash-stable core," *to be submitted to IEEE Trans. on Automatic Control*.
2. C. Hasan, J.-M. Gorce, E. Altman, "The mobile assignment problem: Green-aware broadcast transmission in cellular networks," *to be submitted to IEEE Trans. on Networking*.

Conference Papers

1. C. Hasan, E. Altman, J.-M. Gorce, "The coalitional switch-off game of service providers," *GROWN 2013 jointly held with IEEE WiMob 2013*, Lyon, 2013.
2. C. Hasan, E. Altman, J.-M. Gorce, "Partner selection for decode-and-forward relaying: A matching theoretic approach," *IEEE PIMRC 2013*, London, 2013.
3. C. Hasan, E. Altman, J.-M. Gorce, M. Haddad, "Non-cooperative association of mobiles to access points revisited," *IEEE WiOpt 2012*, Padeborn, May 2012.
4. E. Altman, C. Hasan, J.-M. Gorce, L. Roullet, "Green networking: Downlink considerations," *NetGCoop 2011*, Paris, Oct. 2011.
5. C. Hasan, E. Altman, J.-M. Gorce, "A coalition game approach to the association problem of mobiles in broadcast transmission," *IEEE WiOpt 2011*, pp.236-240, May 2011.

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Cengis Hasan
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INTRODUCTION

1.1 GREEN NETWORKING

Cellular networks market has been growing tremendously last two decades. The demand for cellular communication as well as the number of subscribers imposes the mobile operators for better link quality. The introduction of *smart devices*, such as tablets, smartphones, hybrid tablet-laptops, and the success of social networking actors, boost the demand for cellular data traffic. The following key indicators present some striking information where the cellular trends go [1]:

- **Global mobile data traffic grew 70 percent in 2012**. Global mobile data traffic reached 885 petabytes per month at the end of 2012, up from 520 petabytes per month at the end of 2011.
- **Last year's mobile data traffic was nearly twelve times the size of the entire global Internet in 2000**. Global mobile data traffic in 2012 (885 petabytes per month) was nearly twelve times greater than the total global Internet traffic in 2000 (75 petabytes per month).
- **Mobile video traffic exceeded 50 percent for the first time in 2012**. Mobile video traffic was 51 percent of traffic by the end of 2012.

Predictions about 2017 are interesting, as well

- Monthly global mobile data traffic will surpass 10 exabytes in 2017.
- The number of mobile-connected devices will exceed the world's population in 2013.
- The average mobile connection speed will surpass 1 Mbps in 2014.
- Due to increased usage on smartphones, handsets will exceed 50 percent of mobile data traffic in 2013.
- Monthly mobile tablet traffic will surpass 1 exabyte per month in 2017.
- Tablets will exceed 10 percent of global mobile data traffic in 2015.

All these observations imply that the energy expenditure due to the wireless devices is also expected to grow significantly. There are currently more than 4 million base stations (BSs) serving mobile users, each consuming an average of 25 MWh per year [3].

The term "green networking" is described in [2] as

> *the practice of selecting energy-efficient networking technologies and products, and minimizing resource use whenever possible.*

It covers all dimensions of the network such as personal computers, peripherals, switches, routers, etc. Having a green network may allow to reduce CO_2 emissions and thus will help mitigating the global warming [4]. However, having a significant impact on the overall energy consumption call for improving the energy efficiency of all network components.

With a growing awareness to the dangers related to large scale energy consumption and drafting of many international agreements as well as legislation have reduced energy consumption in several sectors [5]. There is also a growing willingness to reduce energy consumption in wireless networks. On the one hand, wireless communication infrastructures, like the ones managed by mobile network operators, are a major contributor to the ever-increasing energy consumption of the ICT industry, which calls for the adoption of energy-efficient solutions in their design and operation. Moreover, the recent explosive growth of smartphones market adoption and the consequent mobile internet traffic requirements have prompted waves of research and standard development activities to meet the expected future demands in an energy-efficient manner. On the other hand, wireless networks will also be a major component of the communication infrastructure required by other "green" solutions for the efficient management of energy, since they enable practices like telecommuting (e.g., traffic reduction) and remote administration (e.g., the Smart Energy Grid), which are expected to significantly help reduce the environmental footprint of many human activities.

1.2 SMALL CELLS NETWORKS

A *heterogeneous cellular network* includes a hierarchy of the following base station types [7]:

- **Tower-mounted traditional macro base stations:** Expensive (over $100K, plus high OpEx), 40W Tx Power plus high gain antenna, medium to long-range (1-10 km), fast dedicated backhaul, crucial for universal basic coverage and mobility support.
- **Picocells:** Small, short-range (nearly 100m), 1-2W, low-cost ($5-40K CapEx, small OpEx), deployed, maintained and backhauled (X2 interface, usually fiber or wireless) by service provider; typically targeting traffic "hotspots" or dense areas.
- **Femtocells:** Wi-Fi-esque range, power (100-200mW), cost ($100), and backhaul (IP, e.g. DSL, coax). Licensed spectrum, cellular protocols, must interoperate with cellular network with minimal coordination.

That is the reality in today's deployment of cellular networks. In dense urban centers, the rise of bandwidth demand per surface unit yields a shrink of the cell's coverage area and therefore increases the number of cells sites, with straight consequences on the complexity of distributed collaboration and on network scalability issues.

Energy consumption and electromagnetic pollution are main societal and economical challenges that developed countries have to tackle. The evolution of future communication infrastructures will be very concerned about these aspects. The energetic crisis has modified the trade-off between economical variables of the networks for telecommunication operators, public institutions or corporate companies, which include the energetic consumption in the recurrent exploitation cost (weighted by the local cost of electricity KWh). The importance of the concept of *small cells networks* (in short SCN) arise in that context.

A typical *small cell*'s (femto, pico, or micro cell environment) coverage area is possible to range from a few meters to hundreds of meters. Small cells have an important role in orchestrating a cellular network that can overcome the mobile traffic. The cell size reduction offers theoretically higher capacity and energy efficiency, but this reduction increases the complexity of all the operator tasks: cell planning, site acquisition, parameters configuration and tuning.

1.2.1 *Benefits of Small Cell Networks*

As density of mini bases stations increases, classical off-line planning techniques based on frequency, space reuse, power control and antenna tilting are not able to cope with interference due the increasing number of equipments. The SCN concept gets rid of this 3D propagation model planning. Instead of this planning, SCN takes benefit of the high number of interacting devices to increase the spectral efficiency frontier. In fact, a higher number of devices offer more opportunities to schedule information transmission efficiently with a consequent enhancement of the global throughput or equivalently, a reduction of the necessary resources (power, frequency band, etc.).

Existing cellular networks like GSM and WiMAX are designed to cater to large coverage areas, which do not achieve the expected throughput to ensure seamless mobile broadband in the up-link, as one moves farther away from the BS. This is due to an increase in the Inter-cell interference (ICI) as well as constraints on the transmit power of the mobiles. Another limitation of the macro cell approach is the poor indoor penetration. But, for a given radio architecture, efficient frequency reuse by dividing a large (macro) cell into number of small (pico) cells is one of the most effective ways to increase system capacity [8]. While improving the overall throughput achievable in the macro cell, this also brings down deployment costs. In addition, the capability of macro cell is intrinsically limited in an urban environment:

- Path loss in distance attenuation in cities will not change: this will cost energy and lead to high power RF emission (for long range);
- Each 3dB saving means to double the number of antenna;
- Real estate new BTS sites renting and building will be more and more difficult. The today cellular operators assets is mainly made of these sites themselves (and not the installed equipment).

The conventional frequency reuse schemes are not efficient in terms of power and frequency band consumption to deal with the increased level of interference from adjacent cells when their size and density increase. In addition, SCN are characterized by an advanced level of distributed control functions which enhance their self-organizing and self-healing capabilities. In this context, a paradigm shift is required: the new cellular networks need to allocate dynamically resources from a common pool while maintaining decentralized control functions, high level of efficiency in the use of the resources, and an acceptable level of signalling (refer [8, 9, 10, 11] for more details).

1.3 THESIS OVERVIEW

Chapter 4: We study the mobile assignment problem in broadcast transmission where minimal total power consumption is sought. A green-aware approach is followed in our algorithms. A centralized recursive algorithm called the hold minimum is suggested to find optimal assignments. Knowing the NP-hard complexity of the mobile assignment problem, we propose a centralized polynomial-time heuristic algorithm called the column control which is very efficient, and produces near-optimal solutions when the operational power costs of base stations are taken into account. We also develop the distributed column control algorithm. For the fast-moving users, we propose the nearest base station algorithm which is distributed, gives near-optimal solutions, and has polynomial-time complexity. Based on our achievements in Section 2.3.4 and as a novel group formation game, we develop hedonic decision algorithm. Simulation results were used to verify the performance of the algorithms. Furthermore, we simulate the proposed algorithms in the Poisson point process deployment model of nodes over an area.

Chapter 5: We examine the coalitional game aspects of the mobile assignment problem. This game has an incentive to form grand coalition where all players join to the game. By using Bondareva-Shapley theorem, we prove that this coalitional game has a non-empty core which means that grand coalition is stable. Then, we examine the cost allocation policy for different methods such as egalitarian allocation, proportional repartition of total cost, the Shapley value and the nucleolus. We also conclude that if the nucleolus is used as the cost allocation algorithm, the players maintain the grand coalition satisfying the minimization of total cost for broadcast transmission.

Chapter 6 and 7: We analyze a significant problem in green networking called switching off base stations in case of cooperating service providers by means of stochastic geometric and coalitional game tools. The coalitional game herein considered is played by service providers who cooperate in switching off base stations. When they cooperate, any mobile is associated to the nearest BS of any service provider. Given a Poisson point process deployment model of nodes over an area and switching off base stations with some probability, it is proved that the distribution of signal to interference plus noise ratio remains unchanged while the transmission power is increased up to preserving the quality of service. The coalitional game behavior of a typical player is called to be hedonic if the gain of any player depends solely on the members of the coalition to which the player belongs, thus, the coalitions form as a result of the preferences of the players over their possible coalitions' set.

Chapter 8: We consider games related to the association of mobiles to an access point. It consists of deciding to which access point to connect. We consider the choice between two access points or more, where the access decisions may depend on the number of mobiles connected to each one of the access points. We obtain new results using elementary tools in congestion and in crowding games.

Chapter 9: A matching theoretic approach to study the partner selection in cooperative relaying is followed. Partner selection is considered as a special stable roommate problem where each player ranks its partners by some criterion. Each agent aims here at finding a "good" partner in order to exploit efficiently the spatial diversity achieved with cooperation. We adapt Irving's algorithm for determining the partners of each player. The ranking criterion here is chosen to be outage probability such that each player comprises its own preference list according to outage probability from the lowest to the highest. The first player in the preference list provides the lowest outage probability. We introduce a decentralized version of Irving's algorithm. Then, we compare the results obtained by stable-matching with the global optimum and random selection results. From the computational results, we observe that stable-matching results are near to global optimum as well as superior than random selection in terms of average outage probability.

Part I

MATHEMATICAL TOOLS

2

GAME THEORETIC TOOLS

2.1 COALITIONAL GAMES

We represent a coalitional game in utility function form as

Definition 2.1.1 *$\langle N, u\rangle$ where $N = (1, 2, \dots n)$ is a non-empty finite set of players who consider different cooperation possibilities, and $u : 2^N \rightarrow \Re$ is the utility function. Each subset $S \subset N$ is referred to as a* crisp coalition. *The set N is called the* grand coalition *and $\emptyset$ is called the* empty coalition *where $u(\emptyset) = 0$. We denote the collection of coalitions, i.e. the set of all subsets of N by 2^N.*

These games are usually called coalitional games with transferable utility (TU games, for short) where its members can jointly guarantee themselves and which can be transferred without loss between them [22].

In TU games, it is supposed that the utility is freely transferable from one player to another. This is, in particular, possible in the presence of "ideal money", i.e. commodity whose utility is directly proportional to quantity, and independent of any other assets, which a player may have. In general, unfortunately, the situation is not so simple–players' utility for money may be not linear, it may depend on other assets of players, or, in some cases, side payments may even be forbidden. In such situations it is better to represent each coalition's possibilities not by a single number, but rather by a set of all payoff vectors, which the coalition can obtain for its members. We then speak about coalition games with nontransferable utility (NTU games, for short) which is also called as *games without side payments*. [22]. An NTU game is defined as

Definition 2.1.2 *Given the set of players N and V which is a mapping of a set of feasible utility vectors, a subset $V(S)$ of $\Re^S$ to each coalition of players, $S \subseteq N$, such that $V(\emptyset) = \emptyset$, and $\forall S \subseteq N$, $S \neq \emptyset$, the following must hold in order that (N, V) to be an NTU-game:*

1. *$V(S)$ is a closed subset of $\Re^S$*
2. *$V(S)$ is comprehensive, i.e. if $u^S \in V(S)$ and $\tilde{u}^S \leq u^S$ then $\tilde{u}^S \in V(S)$*
3. *The set of vectors in $V(S)$ in which each player in S receives no less than the maximum that he can obtain by himself is a nonempty, bounded set.*

Further discussion about NTU games can be found in [23, 98].

In the following, we concentrate on TU games.

2.1.1 *Properties of Utility Function*

Let us now explain the properties of utility function.

Definition 2.1.3 ***Monotonicity***: *A utility function is said to be monotonic if it satisfies*

$$u(S) \geq u(T), \quad \text{if } T \subseteq S. \tag{2.1}$$

It means that the utility increases or remains unchanged while a coalition expands.

Definition 2.1.4 ***Superadditivity***: *A utility function is said to be superadditive if it satisfies*

$$u(S \cup T) \geq u(S) + u(T), \quad \text{if } T \cap S = \emptyset, \quad \forall S, T \subseteq N. \tag{2.2}$$

From the superadditivity, we infer that whenever two separate coalitions combine, the utility is higher or equal compared to the sum while coalitions are separated. The superadditivity property determines if the grand coalition can be reached. In other words, all players have incentive to join the game provided that the utility increases by combining separate coalitions in a unique coalition.

2.1.2 *Utility Allocation*

In this section, we mention about the sharing the total gain u to the players.

The utility of player $i \in S$ is denoted by ϕ_i^S. The meaning is that what player i gains being in coalition S. The sum of utilities in a coalition S must be equal to the total utility which is called *efficiency*: $\sum_{i \in S} \phi_i^S = u(S)$. The gain vector of player i for all possible coalitions is denoted by $\phi_i \in \Re^{2^{n-1}}$. For example, let $N = (1, 2)$ then $\phi_1 = \left\{\phi_1^{(1)}, \phi_1^{(1,2)}\right\}$. Moreover, we call as *allocation method* $\phi \in \Re^{n2^{n-1}}$ the gains of all possible coalitions of all players, i.e., $\phi = \{\phi_1, \phi_2, \dots, \phi_n\}$.

2.1.3 *The Core*

The core is the set of allocation methods that guarantee the grand coalition. Therefore, no coalition has incentive to leave the grand coalition and receive a larger payoff. Formally, the core is defined as following:

$$\text{core} = \left\{ \phi \in \Re^n \,\middle|\, \sum_{i \in N} \phi_i = u(N), \quad \sum_{i \in S} \phi_i \geq u(S), \forall S \subset N \right\}. \tag{2.3}$$

In general, the core can be found by the following linear program:

$$\min_{\phi} \sum_{i \in N} \phi_i, \quad \text{subject to} \quad \sum_{i \in N} \phi_i = u(N) \text{ and } \sum_{i \in S} \phi_i \geq u(S), \forall S \subset N. \tag{2.4}$$

Provided that the above linear program is feasible, then the core is said to be non-empty [20].

2.1.4 *Coalition Formation Games*

In some cases acting together may be difficult, costly or illegal, or the players may for various personal reasons not wish to do so [21]. Then, the question arises: how the coalitions does have to be formed in order that the players do not deviate from?

A coalition formation game is given by a pair $\langle N, \succ \rangle$, where $\succ = (\succeq_1, \succeq_2, \ldots, \succeq_n)$ denotes the preference profile, specifying for each player $i \in N$ his preference relation $\succeq_i$, i.e. a reflexive, complete and transitive binary relation.

Definition 2.1.5 *A coalition structure or a* coalition partition *is defined as the set* $\Pi = \{S_1, \ldots, S_l\}$ *which partitions the players set N, i.e.,* $\forall k$, $S_k \in N$ *are disjoint coalitions such that* $\bigcup_{k=1}^{l} S_k = N$. *Given* Π *and i, let* $S_\Pi(i)$ *denote the set* $S_k \in \Pi$ *such that* $i \in S_k$ *[18].*

We may think of situations where a player evaluates a coalition partition as a whole, but generally it is more realistic when a player only formulates its preferences according to its own utilities. Such coalition formation games are called *hedonic*. In [21], Drèze and Greenberg introduced the hedonic aspect in players' preferences in a context concerning local public goods. Moreover, purely hedonic games and stability of hedonic coalition partitions were studied by Bogomolnaia and Jackson in [18].

2.1.5 *Hedonic Coalition Formation*

A coalition formation game is classified as hedonic if [18]

1. *The gain of any player depends solely on the members of the coalition to which the player belongs.*
2. *The coalitions form as a result of the preferences of the players over their possible coalitions' set.*

Definition 2.1.6 ***Nash Stability****: A partition* Π *is said to be Nash stable if no player can benefit from moving from his coalition* $S_\Pi(i)$ *to another existing coalition* S_k*, i.e.,* $\forall i, S_\Pi(i) \succeq_i S_k \cup \{i\}$ *for all* $S_k \in \Pi \cup \{\emptyset\}$ *[18].*

Nash-stable partitions are immune even to those movements of individuals when a player who wants to change does not need permission to join an existing coalition [18].

Remark 2.1.1 *In the literature ([18, 22]), the stability concepts being immune to individual deviation are* Nash stability, individual stability, contractual individual stability. *Nash stability is the strongest within above. We concentrate our analysis on the partitions that are Nash-stable. Also,* core stability *is used in models where immunity to coalition deviation is required [22]. Accordingly, the stability concepts aiming* hedonic conditions *can be summarized as following [25]: a partition could be* individually stable, Nash stable, core stable, strict core stable, Pareto optimal, strong Nash stable, strict strong Nash stable. *We refer to [25] for further discussions concerning the stability concepts in the context of hedonic coalition formation games.*

2.1.6 *Properties of Preferences*

The preference relation of a player can be defined over a *preference function*. Let us denote by $\pi_i : 2^N \rightarrow \Re$ the preference function of player i. Thus, player i prefers the coalition S to T iff,

$$\pi_i(S) \geq \pi_i(T) \Longleftrightarrow S \succeq_i T. \tag{2.5}$$

If the preference relation is chosen to be the utility allocated to the player in a coalition, then $\pi_i(S) = \phi_i^S$. Furthermore, we are able to define the preference relation by means of a function which characterizes how a player prefers another player when they share the same coalition. In the following, we define this function. The preferences of player i is said to be *additively separable* if there exists a function $v_i : N \to \mathfrak{R}$ such that $\forall S, T \subseteq N$

$$\sum_{j\in S} v_i(j) \geq \sum_{j\in T} v_i(j) \Leftrightarrow S \succeq_i T, \tag{2.6}$$

where we normalize by setting $v_i(i) = 0$ [18].

A profile of additively separable preferences, represented by $(v_i, \ldots, v_n)$, satisfies *symmetry* if $v_i(j) = v_j(i), \forall i, j$.

2.1.7 *The Existence of Nash-stable Preferences*

In [18], it is proved that if players' preferences are additively separable and symmetric, then a Nash stable coalition partition exists. For further discussion on additively separable and symmetric preferences, we refer to [19].

2.2 HEDONIC COALITION FORMATION AS A NON-COOPERATIVE GAME

In this section, we develop a decentralized algorithm to find a Nash-stable partition whenever one exists in a hedonic coalition formation game.

We model the problem of finding Nash stability as a non-cooperative game. Let us denote as Σ the set of strategies. We assume that the number of strategies is equal to the number of players. We choose this way because the players that select the same strategy is interpreted as a coalition. Therefore, the total gain of the players that share the same coalition S is the utility $u(S)$ of the corresponding coalition. If the game possesses a *Nash equilibrium*, it is an indicator of the *Nash stability*, as well.

We consider a *random round-robin* fashion where each player determines its strategy in its turn according to a *scheduler* which is randomly generated for each round. A scheduler in round ℓ is denoted as $\mathbf{t}^{(\ell)} = \left(t_1^{(\ell)}, t_2^{(\ell)}, \ldots, t_n^{(\ell)}\right)$ where $t_k^{(\ell)}$ is the player in kth turn. Clearly, first player in $\mathbf{t}^{(\ell)}$ takes first turn and the remaining players follow the preceding player according to the scheduler. Note that a scheduler is a random permutation of the set of players N.

A *strategy tuple* in step s is denoted as $\sigma^{(s)} = \left\{\sigma_1^{(s)}, \sigma_2^{(s)}, \ldots, \sigma_n^{(s)}\right\}$, where $\sigma_i^{(s)}$ is the strategy of player i in step s. The relation between a step and a round can be given by $s = n(\ell-1)+k$ which means that in step s, player $t_k^{(\ell)}$ takes its turn. In each step, only one dimension is changed in $\sigma^{(s)}$. We further denote as $\Pi_s^{(\ell)}$ the partition in step s and round ℓ. Define as $S_i^{(s)} = \left\{j : \sigma_i^{(s)} = \sigma_j^{(s)}, \forall j \in N\right\}$ the set of players that share the same strategy with player i. Thus, note that $\cup_{i\in N} S_i^{(s)} = N$ for each step. The preference function of player i can be shown as $\pi_i\left(\sigma^{(s)}\right)$ which implies the following:

$$\pi_i\left(\sigma^{(s)}\right) \geq \pi_i\left(\sigma^{(s-1)}\right) \Leftrightarrow S_i^{(s)} \succeq_i S_i^{(s-1)}, \tag{2.7}$$

where note that player i takes its turn in step s. Any sequence of strategy-tuple in which each strategy-tuple differs from the preceding one in only one coordinate is called a *path* [80], and a unique deviator in each step strictly increases the utility he receives is an *improvement path*. Obviously, any *maximal* *improvement path* which is an improvement path that can not be extended is terminated by stability. Therefore,

> *if a hedonic coalition formation game admits a maximal improvement path, it always possesses a Nash-stable partition.*

If there exists an *infinite improvement path*, then a *cycle* is found, which leads to an infinite loop. Such an improvement path indicates that the game considered does not possess a Nash-stable partition.

An algorithm for hedonic coalition formation can be given as in Algorithm 1.

Algorithm 1 Nash stability establisher

```
set stability flag to zero
while stability flag is zero do
  generate a scheduler
  according to the scheduler, each player chooses its strategy
  check improvement path
  if there exists an infinite imporevement path then
    there is no stability
    break while
  else
    check stability
  end if
end while
```

Lemma 2.2.1 *The proposed Algorithm 1 (Nash stability establisher) always converges to a stable partition whenever there exists one.*

Proof 2.2.1 *A non-cooperative game is classified as* weakly acyclic *if every strategy-tuple is connected to some pure-strategy Nash equilibrium by a best-reply path. Weakly acyclic games have the property that if the order of deviators is decided more-or-less randomly, and if players do not deviate simultaneously, then a best-reply path almost surely reaches an equilibrium [80]. Therefore, a hedonic coalition formation game can be directly considered as a weakly acyclic game whenever a random round-robin scheduler is used.*

Let us denote as Π_0 the initial partition where each player is alone. It corresponds to the case where each player chooses different strategy; thus each player is alone in its strategy: $S_i^{(0)} = i, \forall i \in N$. Since in each round the algorithm generates different schedulers, there must be a scheduler in which a finite improvement path exists whenever the game possesses a Nash-stable partition. The transformation of strategy tuple and partition in each step can be denoted as following, respectively:

$$\Pi_0 \to \Pi_1^{(1)} \to \Pi_2^{(1)} \to \ldots \to \Pi_n^{(1)} \to \Pi_{n+1}^{(2)} \to \ldots \to \Pi_{s^*}^{(\ell^*)}, \tag{2.8}$$

$$\sigma^{(0)} \to \sigma^{(1)} \to \ldots \to \sigma^{(s^*)}, \tag{2.9}$$

where ℓ^* and s^* represent the round and the step, respectively in which the stable partition occurs. Note that during the steps $s^*+1, s^*+2, \ldots, s^*+n-1$ the strategy tuple and the partition do not change.

2.3 THE NASH-STABLE CORE

This section presents a new approach to obtain preference profiles that give Nash-stable partitions. We develop a rule ensuring that such utility allocation method always produces a Nash-stable partition.

We denote as $\langle N, u, \succ \rangle$ the hedonic TU game (since u is transferable to the players, we consider hedonic coalition formation games based on transferable utility). Let us assume that the preference function of player i is the gain obtained in the corresponding coalition, i.e., $\pi_i(S) = \phi_i^S$. Define the operator $\mathbb{F} : \mathscr{S}_\succ \mapsto \mathscr{P} \cup \emptyset$, where $\mathscr{S}_\succ$ and $\mathscr{P}$ are the set of all possible preference profiles and partitions, respectively. Clearly, for any preference profile $\succ$, the operator $\mathbb{F}$ finds the Nash-stable partition Π, i.e. $\mathbb{F}(\succ) := \Pi$. If a Nash-stable partition can not be found, the operator maps to empty set. Moreover, the inverse of the operator is denoted as $\mathbb{F}^{-1}(\Pi \in \mathscr{P})$ which finds the set of all possible preferences $\mathscr{S}_\succ^\Pi$ that gives the Nash-stable partition Π. Thus, the Nash-stable core includes all those efficient allocation methods that build the following set:

$$\mathscr{N}\text{-core} = \left\{ \phi \in \Re^{|\phi|} \,\middle|\, \mathbb{F}^{-1}\left(\Pi_\phi \in \mathscr{P}\right) := \mathscr{S}_\succ^{\Pi_\phi} \right\}, \tag{2.10}$$

where Π_ϕ is the preference profile occurring due to ϕ.

2.3.1 *Non-emptiness of the Nash-stable Core*

We consider the preference relation which evaluates the possible coalitions of a player according to its utility gained in the corresponding coalition. We ask the question: *Is there any utility allocation method which results in a Nash-stable partition?*

Theorem 2.3.1 *The Nash-stable core can be non-empty.*

Proof 2.3.1 *Algorithmically, for any partition $\Pi \in \mathscr{P}$, if the following linear program is feasible, then the Nash-stable core is non-empty.*

$$\min_{\phi \in \Re^{|\phi|}} \left\{ \sum_{\forall S \in 2^N} \sum_{\forall i \in S} \phi_i^S \,\middle|\, \phi_i^{S_\Pi(i)} \geq \phi_i^T, \forall T \in \Pi \cup \emptyset, \forall i \in N \textit{ and } \sum_{j \in S} \phi_j^S = u(S), \forall S \in 2^N \right\}, \tag{2.11}$$

where $|\phi| = n(2^{n-1} - 1)$ as well as we denote as $\mathscr{C}_\Pi := \{\phi_i^{S_\Pi(i)} \geq \phi_i^T, \forall T \in \Pi \cup \emptyset, \forall i \in N\}$ the set of constraints arising due to partition Π. Combining all possible constraint sets provides the sufficient condition of the non-emptiness of Nash-stable core:

$$\mathscr{N}\text{-core} = \min_{\phi \in \Re^{|\phi|}} \left\{ \sum_{\forall S \in 2^N} \sum_{\forall i \in S} \phi_i^S \,\middle|\, \bigcup_{\Pi \in \mathscr{P}} \mathscr{C}_\Pi \textit{ and } \sum_{j \in S} \phi_j^S = u(S), \forall S \in 2^N \right\}, \tag{2.12}$$

where $\bigcup_{\Pi \in \mathscr{P}} \mathscr{C}_\Pi$ is the union of all possible constraint sets. Note that it is a non-trivial problem as well as the union could result in a non-convex set.

2.3.2 *Special Case: Grand Coalition as a Nash-stable Partition*

We ask the following question: what is the condition of existence of the grand coalition as a Nash-stable partition? Let $\Pi = \{N\}$, then the following must hold

$$\phi_i^N \leq u(i), \forall i \in N, \tag{2.13}$$
$$\sum_{i \in N} \phi_i^N = u(N). \tag{2.14}$$

These two conditions result in the following:

$$u(N) \geq \sum_{i \in N} u(i). \tag{2.15}$$

Those cooperative TU games that satisfy this condition is said to be *essential*.

2.3.3 *Symmetric Relative Gain*

Let us consider the case where the gain of player i in coalition S is denoted as $\phi_i^S = u(i) + \delta_i^S$ in which δ_i^S is called as *the relative gain*. Note that when player i is alone, then $\delta_i^i = 0$. Thus, the preference relation can be determined over the relative gain:

$$\delta_i^S \geq \delta_i^T \Leftrightarrow S \succeq_i T. \tag{2.16}$$

The total allocated utilities in coalition S is $\sum_{i \in S} \phi_i^S = \sum_{i \in S} u(i) + \sum_{i \in S} \delta_i^S = u(S)$. Therefore, $\sum_{i \in S} \delta_i^S = u(S) - \sum_{i \in S} u(i) = \Delta(S)$, where $\Delta(S)$ is the *clustering profit* due to coalition S. Let us assume that the relative gain is equal to the equally divided clustering profit in a coalition, i.e.

$$\delta_i^S = \frac{\Delta(S)}{|S|}, \quad \forall i \in S. \tag{2.17}$$

This choice means that each player in coalition S has the same gain; thus the effect of coalition S is identical to the players within it.

Corollary 2.3.1 ***Equivalent Evaluation****: Assume that $S \cap T \neq \emptyset$. Due to eq. (2.17), the following must hold*

$$\frac{\Delta(S)}{|S|} \geq \frac{\Delta(T)}{|T|} \Leftrightarrow S \succeq_i T \quad \forall i \in S \cap T. \tag{2.18}$$

It means that all players in $S \cap T$ prefer coalition S to T whenever the relative gain in S is higher than T.

Lemma 2.3.1 *There is always a Nash-stable partition when $N = \{1,2\}$ in case of symmetric relative gain.*

Proof 2.3.2 *Let us enumerate all possible partitions and corresponding conditions of Nash-stability:*

1. $\Pi = \{(1),(2)\}$:

$$0 \geq \delta_1^{12}$$
$$0 \geq \delta_2^{12}$$
$$\delta_1^{12} + \delta_2^{12} = \Delta(1,2) \tag{2.19}$$

2. $\Pi = \{1,2\}$:

$$0 \leq \delta_1^{12}$$
$$0 \leq \delta_2^{12}$$
$$\delta_1^{12} + \delta_2^{12} = \Delta(1,2) \tag{2.20}$$

According to Corollary 2.3.1, $\delta_1^{12} = \delta_2^{12} = \delta = \frac{\Delta(1,2)}{2}$. *Thus, combining all constraint sets of all possible partitions, we have the following result constraint set:* $\mathscr{C}_\Pi := \{0 \leq \delta\} \cup \{0 \geq \delta\} \Leftrightarrow \delta \in [-\infty, \infty]$. *It means that for any value of* $\Delta(1,2)$*, symmetric relative gain always results in a Nash-stable partition for two players case.*

Lemma 2.3.2 *There is always a Nash-stable partition when* $N = \{1,2,3\}$ *in case of symmetric relative gain.*

Proof 2.3.3 *Note that there are* 5 *possible partitions in case of* $N = \{1,2,3\}$. *Thus, according to equally divided clustering profit, the following variables occur:* $\delta_1^{12} = \delta_2^{12} = \delta_1$, $\delta_1^{13} = \delta_3^{13} = \delta_2$, $\delta_2^{23} = \delta_3^{23} = \delta_3$, $\delta_1^{123} = \delta_2^{123} = \delta_3^{123} = \delta_4$. *Enumerating all possible partitions results in the following conditions:*

1. $\Pi = \{(1),(2),(3)\}$:

$$\delta_1 \leq 0, \delta_2 \leq 0, \delta_3 \leq 0, \tag{2.21}$$

2. $\Pi = \{(1,2),(3)\}$:

$$\delta_1 \geq 0, \delta_1 \geq \delta_2, \delta_1 \geq \delta_3, \delta_4 \leq 0, \tag{2.22}$$

3. $\Pi = \{(1,3),(2)\}$:

$$\delta_2 \geq 0, \delta_2 \geq \delta_1, \delta_2 \geq \delta_3, \delta_4 \leq 0, \tag{2.23}$$

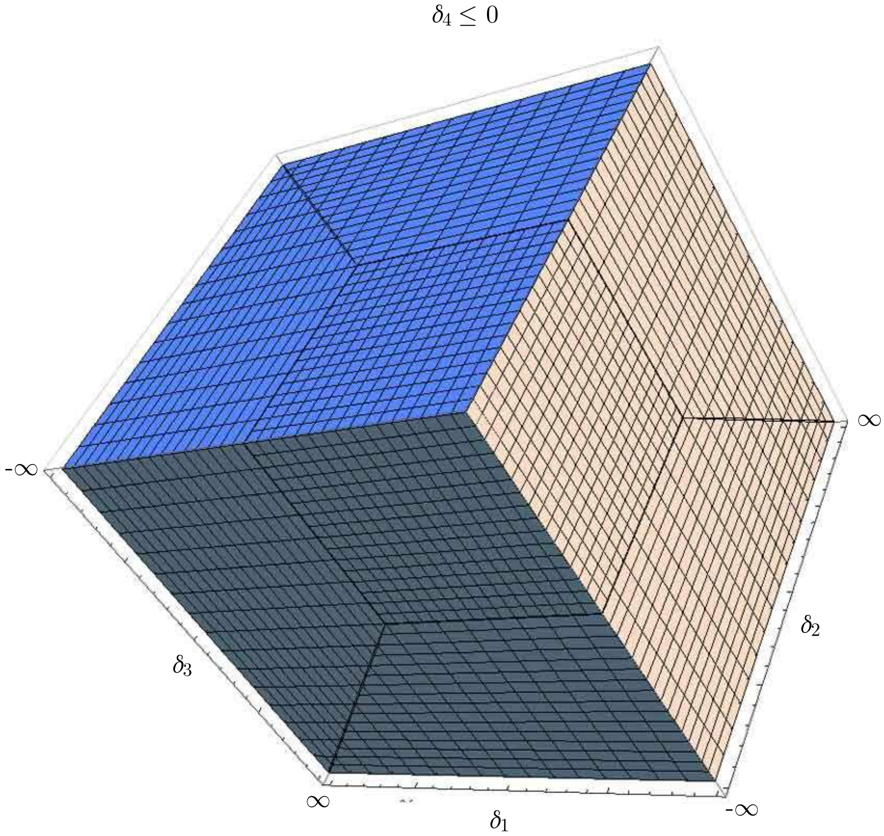
4. $\Pi = \{(2,3),(1)\}$:

$$\delta_3 \geq 0, \delta_3 \geq \delta_1, \delta_3 \geq \delta_2, \delta_4 \leq 0, \tag{2.24}$$

5. $\Pi = \{1,2,3\}$:

$$\delta_4 \geq 0. \tag{2.25}$$

Note that the constraint set $\mathscr{C}_\Pi$ covers all values in $\delta_1, \delta_2, \delta_3$ in case of $\delta_4 \geq 0$. Further, it also covers all values when $\delta_4 \leq 0$. We are able to draw it since there are three dimensions:



Thus, we can conclude that symmetric relative gain always results in a Nash-stable partition when $n \leq 3$. However, there could not be a Nash-stable partition when $n > 3$. We can find many counter examples that justify it.

2.3.4 *Additively Separable and Symmetric Utility Case*

Consider eq. (2.6) meaning that player i gains $v_i(j)$ from player j in any coalition. In case of symmetry, $v_i(j) = v_j(i) = v(i,j)$ such that $v(i,i) = 0$. Further, we denote as $\phi_i^S = u(i) + \sum_{j \in S} v(i,j)$ the utility that player i gains in coalition S. Then, the sum of allocated utilities in coalition S is given by

$$\sum_{i \in S} \phi_i^S = \sum_{i,j \in S} v(i,j) + \sum_{i \in S} u(i) = u(S). \tag{2.26}$$

Observe that $\sum_{i,j \in S} v(i,j) = 2 \sum_{i,j \in S : j > i} v(i,j)$ (for example, $S = (1,2,3)$, $\sum_{i,j \in S} v(i,j) = 2[v(1,2) + v(1,3) + v(2,3)]$). Therefore, the following determines the existence of additively separable and symmetric preferences when the utility function u is allocated to the players:

$$\sum_{i,j \in S : j > i} v(i,j) = \frac{1}{2}\left(u(S) - \sum_{i \in S} u(i)\right), \tag{2.27}$$

where $\Delta(S) = \frac{1}{2}\left(u(S) - \sum_{i\in S} u(i)\right)$ is the *clustering profit* due to coalition S. Actually, the existence of v is too limited in such a setting. Even for example $N = (1,2,3)$, the following is very restrictive

$$\begin{aligned} v(1,2) &= \frac{1}{2}[u(1,2) - u(1) - u(2)], \\ v(1,3) &= \frac{1}{2}[u(1,3) - u(1) - u(3)], \\ v(2,3) &= \frac{1}{2}[u(2,3) - u(2) - u(3)], \\ v(1,2) + v(1,3) + v(2,3) &= \frac{1}{2}[u(1,2,3) - u(1) - u(2) - u(3)]. \end{aligned} \tag{2.28}$$

2.3.5 *Balancedness of Utility Function*

In this section, we find the Nash-stable core based on additively separable and symmetric preferences. Further, we analyze the balancedness of utility function according to Bondareva-Shapley theorem.

In the following, we derive the balancedness condition of utility function u utilizing Bondareva-Shapley theorem's proof [63, 64].

Let us denote as

- $\mathscr{V}$ the all possible bipartite coalitions such that $\mathscr{V} := \{(i,j) \in 2^N : j > i\}$. Note that $|\mathscr{V}| = n(n-1)/2$.
- $\mathscr{I}$ the index set of all possible bipartite coalitions. So, $\mathscr{V}(k \in \mathscr{I})$ is the kth bipartite coalition.
- $\mathbf{v} = (v(i,j))_{(i,j)\in\mathscr{V}} \in \Re^{|\mathscr{V}|}$ which is the vector demonstration of all $v(i,j)$.
- $\mathbf{c} = (1,\dots,1) \in \Re^{|\mathscr{V}|}$.
- $\mathbf{b} = (b_S)_{S\in 2^N} \in \Re^{2^n}$ such that $b_S = \frac{1}{2}\Delta(S)$ where $\Delta(S) = u(S) - \sum_{i\in S} u(i)$.
- $\mathbf{A} = (a_{S,k})_{S\in 2^N, k\in\mathscr{I}} \in \Re^{2^n \times |\mathscr{V}|}$ is a matrix such that $a_{S,k} = \mathbf{1}\{S : \mathscr{V}(k) \in S\}$.

By using these definitions, the Nash-stable core is non-empty whenever the following linear program is feasible

$$\begin{aligned} (L) \quad & \min \mathbf{cv} \quad \text{subject to} \\ & \mathbf{Av} = \mathbf{b}, \mathbf{b} \in [-\infty, \infty]. \end{aligned} \tag{2.29}$$

The linear program that is dual to (L) is given by

$$\begin{aligned} (\hat{L}) \quad & \max \mathbf{wb} \quad \text{subject to} \\ & \mathbf{wA} = \mathbf{c}, \mathbf{w} \in [-\infty, \infty], \end{aligned} \tag{2.30}$$

where $\mathbf{w} = (w_S)_{S\in 2^N} \in \Re^{2^n}$ denote the vector of dual variables. Let $\mathbf{A}^k$ denote the kth column of $\mathbf{A}$. Then $\mathbf{wA}$ implies

$$\mathbf{wA}^k = \sum_{S\in 2^N} w_S a_{S,k} = \sum_{S\in 2^N : \mathscr{V}(k)\in S} w_S = 1, \quad \forall k \in \mathscr{I}. \tag{2.31}$$

This result means that the feasible solutions of $(\hat{L})$ exactly correspond to the vectors containing balancing weights for balanced families. More precisely, when $\mathbf{w}$ is feasible in $(\hat{L})$, $\mathscr{B}_{\mathbf{w}} := \{S \in 2^N | w_S > 0\}$ is a balanced family with balancing weights $(w_S)_{S\in\mathscr{B}_{\mathbf{w}}}$.

According to the *weak duality theorem* the objective function value of the primal (L) at any feasible solution is always greater than or equal to the objective function value of the dual $(\hat{L})$ at any feasible solution, i.e. $\mathbf{cv} \geq \mathbf{wb}$ which implies

$$\mathbf{cv} = \sum_{k\in\mathscr{I}} v_k = \sum_{\forall S\in 2^N} \sum_{i,j\in S:j>i} v(i,j) = \frac{1}{2}\Delta(N), \tag{2.32}$$

and

$$\mathbf{wb} = \frac{1}{2}\sum_{S\in 2^N} w_S \Delta(S) \leq \frac{1}{2}\Delta(N). \tag{2.33}$$

Combining these results, we have the following balancedness conditions of u:

$$\begin{aligned} &\sum_{S\in 2^N} w_S \Delta(S) \leq \Delta(N), \\ &\sum_{S\in 2^N:\mathscr{V}(k)\in S} w_S = 1, \quad \forall k \in \mathscr{I}, \\ &w_S \in [-\infty, \infty] \quad \forall S \in 2^N. \end{aligned} \tag{2.34}$$

2.3.6 *Relaxed Efficiency*

By *relaxed efficiency*, we mean that the sum of allocated utilities in a coalition is not strictly equal to the utility of the coalition, i.e. $\sum_{i\in S} \phi_i^S \leq u(S)$ which results in

$$\sum_{i,j\in S:j>i} v(i,j) \leq \frac{1}{2}\Delta(S). \tag{2.35}$$

The motivation behind relaxed efficiency is the following: in case of the individual deviations, the efficiency principle is not important since there is no group interest; therefore, we can relax this condition (thus, we call it relaxed efficiency).

Lemma 2.3.3 *The Nash-stable core is always non-empty in case of relaxed efficiency.*

Proof 2.3.4 *A feasible solution of the following linear program guarantees the non-emptiness of the Nash-stable core:*

$$\begin{aligned} &\max \sum_{\forall S\in 2^N} \sum_{i,j\in S:j>i} v(i,j) \textit{ subject to} \\ &\sum_{i,j\in S:j>i} v(i,j) \leq \frac{1}{2}\Delta(S), \forall S \in 2^N, \end{aligned} \tag{2.36}$$

which is equivalent to

$$\begin{aligned} (LRE) \quad &\max \mathbf{cv} \quad \textit{subject to} \\ &\mathbf{Av} \leq \mathbf{b}, \mathbf{b} \in [-\infty, \infty]. \end{aligned} \tag{2.37}$$

Note that (LRE) is always feasible since

- *there are no any inconsistent constraints, i.e. there are no at least two rows in* **A** *that are equivalent,*
- *the polytope is bounded in the direction of the gradient of the objective function* **cv**.

2.4 STABLE MATCHING GAMES

A *matching game* can be considered as a special NTU-game where the cardinality of each basic coalition is at most 2 so that $S = (i, j)$, $i, j \in N$. Note that i can be equal to j which means a player is possible to be alone in coalition S. The *stable marriage problem* is primal problem formalised in the context of two-sided market [85].

Specifically, a *matching* $\mathscr{M}$ is a partition of the set of players N, which we denote as $\mathscr{M}(N)$ such that

$$\bigcup_{S \in \mathscr{M}(N)} = N. \tag{2.38}$$

The utility of player $i \in S$ is represented as u_i^S. Player i is said to *prefer to or is indifferent* between coalition S and T showed by $S \succeq_i T$ whenever $u_i^S \geq u_i^T$. The preference relation $\succeq$ is a reflexive, complete and transitive binary relation. If $S \succ_i T \Leftrightarrow u_i^S > u_i^T$ which means that player i *strictly prefers* coalition S to T.

Definition 2.4.1 *A coalition B is said to block a matching $\mathscr{M}$ whenever both $i \in S$ and $j \in T$ prefers coalition B to S and T, respectively, i.e. $B \succ_i S$ and $B \succ_j T$.*

Definition 2.4.2 *A matching $\mathscr{M}$ is said to be stable whenever there does not exist a blocking coalition.*

Definition 2.4.3 *A set of players X are matched to another set of players Y whenever a stable matching $\mathscr{M}$ exists such that $X \cup Y = N$.*

2.4.1 *Stable Roommates Problem*

The stable roommates problem (SRP) corresponds to a one-sided market. Each person aims to find his best roommate. Therefore, the preference list of a specific person is composed of a descending order all possible partners. Note that a player can also rank himself in the list. In this case, the player remains alone. *Incomplete lists* in a SRP mean that a person does not include all the roommates in his preference list.

The problem of finding stable bipartite coalitions of mobiles where a mobile is let to order his possible partners according to some preference relation can be seen a SRP as described in Section 9.2.

2.4.2 *An Efficient Algorithm for Solving SRP*

The stable roommates problem had been a nontrivial open problem, until Irving [86] constructed the first polynomial time algorithm which determines whether a given instance of the stable roommates problem admits a stable matching, and if so, finds one [98]. Irving, in his paper [86], proves that the proposed algorithm has $O(n^2)$ complexity.

We name the algorithm here as *alg*-IRVING consisting of two phases:

First Phase
Each player in his turn *do a bid* to his partners. This sequence of bids proceeds with each individual pursuing the following strategies:

1. If i receives a bid from j, then
   a) he rejects it if he already holds[1] a better bid from someone higher than j in his preference list;
   b) he holds it for consideration simultaneously rejecting his current bid being poorer than j.
2. If i is rejected by someone in his preference list, he continues proposing until accepted by a partner.

This phase of algorithm will terminate

(i) with every person holding a bid, or

(ii) with one person rejected by everyone

In case of (ii), the algorithm will terminate with no matching meaning that the problem is not stable.

If the first phase of the algorithm terminates with every person holding a bid, then the preference list of possible partners for j, who holds a bid from i, can be "reduced" by deleting from it

Reductions

- all those to whom j prefers i;
- all those who hold a bid from a person whom they prefer to j.

We denote as **Reductions** the procedure that performs these operations. If the reduced preference list of player i and j contains only j and i, respectively, then it is said that they are *matched*; therefore, in the second phase of algorithm, they are out of consideration.

We denote by $\mathscr{S}$ the ordered pairs of the form (x, y), where y holds a bid from x. It is said that y is x's current *favorite* which is the first in his reduced preference list.

Second Phase
The second phase of *alg*-IRVING deals with finding a *rotation* ρ in $\mathscr{S}$. In case of a rotation, the set $\mathscr{S}$ is repeatedly changed by the application of rotations. After applying rotation, if two players are matched, then they are removed from $\mathscr{S}$.

A rotation relative to $\mathscr{S}$ is a sequence

$$\rho(\mathscr{S}) = \{(x_1, y_1), (x_2, y_2), \ldots, (x_k, y_k), (x_1, y_1)\}$$

such that $\forall(x_l, y_l) \in \rho(\mathscr{S})$, y_{l+1} is x_l's current second favorite in his reduced preference list.

If an even length rotation is found such that $x_{l+1} = y_l$ forall l, this is the case referred to *even party* which is also an indicator of no stable matching.

In case of no an even party, the application of rotation involves replacing the pairs (x_l, y_l) in $\mathscr{S}$ by the pairs (x_l, y_{l+1}) and performing again the procedure **Reductions** on the preference lists of corresponding players. The second phase continues until not finding a rotation which indicates that a stable matching is found.

In the following we give an example to show how the algorithm works.

Example 2.4.1 *Consider the following preference lists of players:*

$$\begin{aligned} 1 &: \quad (3,4,2,5,6) \\ 2 &: \quad (1,6,5,3,4) \\ 3 &: \quad (6,4,1,5,2) \\ 4 &: \quad (1,2,6,3,5) \\ 5 &: \quad (2,1,3,4,6) \\ 6 &: \quad (5,2,4,3,1) \end{aligned} \tag{2.39}$$

We would like to find a stable matching if there exists one. The procedure concerning first phase is performed as following:

1 proposes to 3; 3 holds 1

2 proposes to 1; 1 holds 2

3 proposes to 6; 6 holds 3

4 proposes to 1; 1 holds 4 and rejects 2

Since 1 rejects 2, they remove each other in their preference lists:

$$\begin{aligned} 1 &: \quad (3,4,5,6) \\ 2 &: \quad (6,5,3,4) \\ 3 &: \quad (6,4,1,5,2) \\ 4 &: \quad (1,2,6,3,5) \\ 5 &: \quad (2,1,3,4,6) \\ 6 &: \quad (5,2,4,3,1) \end{aligned} \tag{2.40}$$

Then,

2 proposes to 6; 6 holds 2 and rejects 3

which results in the following preference lists:

$$\begin{aligned} 1 &: \quad (3,4,5,6) \\ 2 &: \quad (6,5,3,4) \\ 3 &: \quad (4,1,5,2) \\ 4 &: \quad (1,2,6,3,5) \\ 5 &: \quad (2,1,3,4,6) \\ 6 &: \quad (5,2,4,1) \end{aligned} \tag{2.41}$$

The algorithm continues as following:

3 proposes to 4; 4 holds 3
5 proposes to 2; 2 holds 5
6 proposes to 5; 5 holds 6

Then, the `reductions()` *is run, which produces the following preference lists:*

$$\begin{aligned} 1: &\quad (3,4) \\ 2: &\quad (6,5) \\ 3: &\quad (4,1) \\ 4: &\quad (1,3) \\ 5: &\quad (2,6) \\ 6: &\quad (5,2) \end{aligned} \tag{2.42}$$

Thus, the first phase ends with the set

$$\mathscr{S}[0] = \{(6,5)(5,2)(3,4)(2,6)(4,1)(1,3)\}. \tag{2.43}$$

In second phase, the algorithm seeks a rotation in $\mathscr{S}[0]$ *which is found to be*

$$\rho(\mathscr{S}[0]) = (1,3),(3,4),(4,1),(1,3).$$

This is a rotation because the second player in the preference list of player 1 is 4, the second player in the preference list of player 3 is 1, and the second player in the preference list of player 4 is 3. Moreover, note that this is also an even party where $x_{l+1} = y_l$ *forall* $x_l \in \rho(\mathscr{S}[0])$.

Therefore, we conclude that there is no a stable matching in the considered example.

2.5 CROWDING GAMES

A crowding game is represented by triple $\Gamma = \langle M, \Sigma^m, (u_i)_{i\in M}\rangle$ where $M = \{1,2,\ldots,m\}$ is the set of *players*, Σ is the set of *strategies* shared by all the players and $u_i : \sigma \to \mathfrak{R}$ is the *utility function* of player $i \in M$. Each player $i \in M$ chooses exactly one element from the r alternatives in Σ. The choices of players are represented by $\sigma = \{\sigma_1, \sigma_2 \ldots, \sigma_m\} \subseteq \Sigma^m$ which is called the *strategy-tuple* (σ_i shows the strategy chosen by player i).

The utility that player i receives for playing the jth strategy is monotonically non-increasing function u_i of the total number of n_j of players playing the jth strategy [80]. The number of players playing each strategy corresponding to σ can be presented by a *congestion vector* $n = (n_1, n_2, \ldots, n_r)$, where $n_j \geq 0$ is the number of players who have chosen a $j \in \Sigma$. The strategy-tuple σ is a Nash equilibrium iff each σ_i is a best-reply strategy [80]:

$$u_i(n_{\sigma_i}) \geq u_i(n_j + 1) \quad \forall i \in M \text{ and } \forall j \in \Sigma. \tag{2.44}$$

A crowding game becomes a congestion game (symmetric crowding game) if all players share the same set of utility functions. Clearly, the crowding games arise if there exist *player-specific* utility functions. Nonsymmetric crowding games, however, generally do not admit a potential function (for further information about potential function, refer to [79]).

Table 2.1: Utility Matrix

		Mobile 2		
		BS 1	BS 2	BS 3
	BS 1	$(2, 1.3)$	$(4^{*}, 8^{*})$	$(4, 6.3)$
Mobile 1	BS 2	$(5.2, 2.6)$	$(2.7, 4)$	$(5.2^{*}, 6.3^{*})$
	BS 3	$(2, 2.6)$	$(2, 8)$	$(1, 3.15)$

2.5.1 *An Algorithm for Finding Nash Equilibrium*

Milchteich establishes the following [80]:

Theorem 2.5.1 *Consider a crowding game. Assume that the utility of player i for choosing resource j is*

- *a function of i, j and the number of players that choose resource j,*
- *decreasing in this number*

Then

- *(i) There exists a pure Nash equilibrium,*
- *(ii) There exists a sequence of best responses of players that converges to an equilibrium within finitely many steps.*
- *(iii) Assume that the number of resources is 2. Consider any sequence of best responses in which each player has infinitely many opportunities to change its decision. Then already after a finite number of steps, the sequence reaches an equilibrium.*

In view of this Theorem, we can use a best response algorithm to compute an equilibrium. We are guaranteed that it will converge within a finite time if the number of resources is two, or if there is a unique best response decision at every step. Under these conditions it can be used as an algorithm that yields convergence to an equilibrium within a finite number of steps. The Algorithm is summarized below (see Algorithm 2).

Proof 2.5.1 *The proof of Theorem 2.5.1 is given in the proof of Theorem 2 of [80].*

Example 2.5.1 *In this section, we show by an example scenario how the introduced algorithm converges to an equilibrium in the context of throughput competition.*

In Figure 2.1, it is depicted the utilities for each BS-mobile pair when one mobile uses one BS. For example, the utility is $u_1(1) = 4$ if mobile 1 is served by BS_1. In case of multiple usage, the utility decreases, for example: $u_1(2) = 2$, $u_2(2) = 1.3$ if both mobile 1 and mobile 2 use BS_1 which results in the strategy tuple $\sigma = \{\sigma_1, \sigma_2\} = \{BS_1, BS_1\}$.

Let us then find the Nash equilibrium of this scenario. We have two players and three strategies. First, we show the utility matrix of this game (Table 1). From the utility matrix, we find easily the equilibria $(4, 8)$ and $(5.2, 6.3)$.

Algorithm 2 Utility and Strategy-tuple in Nash Equilibrium

```
σ(0) ← {0,0,...0} Set the initial strategy-tuple
u_i(σ(0)) ← 0, ∀i ∈ M
c ← 0 Set the convergence variable to zero
p ← 1 Set the player variable to 1
l ← 1 Set the step variable of strategy-tuple to 1
while c == 0 do
  Find the best-reply strategy of player p: σ*_p(l)
  Calculate u_i(σ(l)), ∀i ∈ M
  if u_p(σ*_p(l)) ≥ u_p(σ*_p(l−1)) then
    l ← l+1
    if p < m then
      p ← p+1
    else
      p ← 1
    end if
  else
    u_p(σ*_p(l)) ← u_p(σ*_p(l−1))
    σ_p(l) ← σ_p(l−1)
    l ← l+1
    if p < m then
      p ← p+1
    else
      p ← 1
    end if
  end if
  if l > m+1 then
    if σ_p(l−1) == σ_p(l−2) ∀p ∈ M then
      c ← 1
    end if
  end if
end while
end
```

Example scenario demonstrating how the algorithm converges to a Nash equilibrium

The utilities are shown for each BS-mobile pair when one mobile uses one BS

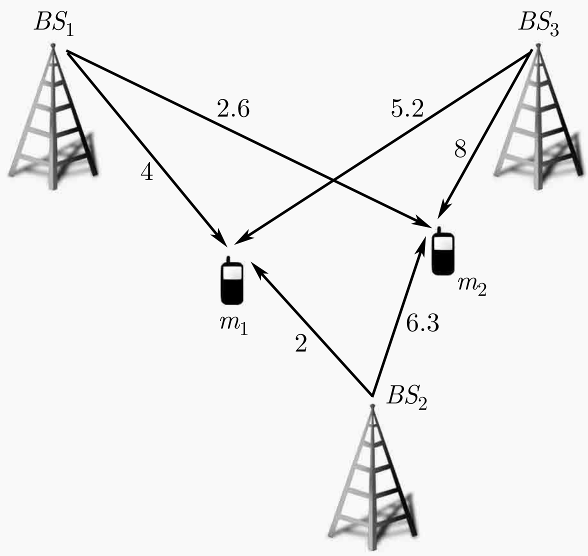


Figure 2.1: Example scenario.

Secondly, we run the algorithm for this example scenario that is introduced in Algorithm 1. Let us assume that in the step $l = 0$*, the initial strategy-tuple be as* $\sigma(0) = \{BS_1, BS_1\}$*. Then the utilities become*

$$u_1(\sigma(0)) = 2, u_2(\sigma(0)) = 1.3, \tag{2.45}$$

in which $u_i(\sigma(l))$ *represents the utility of player i in case of strategy-tuple* $\sigma(l)$.

We set player 1 as the first player which looks for the best-reply strategy. Player 1 finds out that the best-reply strategy $\sigma_1(1) = BS_2$ *in the step* $l = 1$*. The utilities are calculated as*

$$u_1(\sigma(1)) = 5.2, u_2(\sigma(1)) = 2.6, \tag{2.46}$$

where $\sigma(1) = \{BS_2, BS_1\}$*. In the next step,* $l = 2$*, player 2 searches the best-reply strategy which turns out to be* $\sigma_2(2) = BS_3$*. The strategy-tuple then is as* $\sigma(2) = \{BS_2, BS_3\}$ *which results in the following utilities*

$$u_1(\sigma(2)) = 5.2, u_2(\sigma(2)) = 6.3. \tag{2.47}$$

In the next step, player 1 can not find a best-reply strategy. Consequently, the algorithm converges to the Nash equilibrium which coheres with the one of utility matrix that we found as $(5.2, 6.3)$.

3

POINT PROCESSES

Stochastic geometry is a rich branch of applied probability which allows the study of random phenomena on the plane or in higher dimensions. It is intrinsically related to the theory of point processes [27]. A point process (p.p.) Φ can be depicted as a *random collection of points* in space. More formally, Φ is a random, finite or countably-infinite collection of points in the space $\mathbb{R}_d$, without accumulation points [26]. A point measure is a measure which is locally finite and which takes only integer values on some space E. Each such measure can be represented as a discrete sum of Dirac measures on E

$$\Phi = \sum_i \delta_{X_i}. \tag{3.1}$$

The random variables $\{X_i\}$ taking values in E are the points of Φ. The *intensity measure* Λ of Φ on B is defined as $\Lambda(B) = \mathbb{E}\Phi(B)$ denoting the expected number of points in $\Phi \cap B$ where B is assumed to be a Borel set.

3.1 POISSON POINT PROCESSES

A p.p. on some metric space E with intensity measure Λ is Poisson if for all disjoint subsets $A_1, \dots, A_n$ on E, the random variables $\Phi(A_i)$ are independent and Poisson. For some $\mathrm{d}x$, if $\Lambda(\mathrm{d}x) = \lambda \mathrm{d}x$ is multiple of Lebesgue measure, we call Φ a *homogeneous Poisson p.p.* and λ is its intensity parameter [26].

Definition 3.1.1 ***Superposition****: The superposition of Poisson point processes Φ_k with intensities Λ_k is the sum $\Phi = \sum_k \Phi_k$ with intensity measure $\sum_k \Lambda_k$ [26].*

Definition 3.1.2 ***Thinning****: The thinning of Poisson p.p. Φ with a* retention function $q : \mathbb{R}^d \to [0, 1]$ *is a p.p. given by*

$$\Phi^q = \sum_i \varepsilon_i \delta_{X_i} \tag{3.2}$$

where the random variables $\{\varepsilon_i\}_i$ are independent given Φ, and $\mathbf{P}\{\varepsilon_i = 1 | \Phi\} = 1 - \mathbf{P}\{\varepsilon_i = 0 | \Phi\} = q(X_i)$. Thus, the retention probability q yields a Poisson p.p. of intensity measure $q\Lambda$ [26].

3.2 MARKED POINT PROCESSES

In a marked point process (m.p.p.), a mark belonging to some measurable space and carrying some information is attached to each point. In our context, the points are the BSs and the marks are considered to be the transmitted power by each BS.

Consider a $d \geq 1$ dimensional Euclidean space $\mathbb{R}^d$ as the *state space* and a $\ell \geq 1$ dimensional *space of marks* $\mathbb{R}^\ell$. A marked p.p. $\tilde{\Phi}$ can be represented as a discrete sum of Dirac measures

$$\tilde{\Phi} = \sum_i \delta_{(X_i, M_i)} \tag{3.3}$$

where $\delta_{(X,M)}$ is the Dirac measure on the Cartesian product $\mathbb{R}^d \times \mathbb{R}^\ell$ with an atom at (X, M).

Definition 3.2.1 (from [26]) ***Independent marking****: A marked p.p. is said to be independently marked (i.m.) if, given the locations of the points $\Phi = \{X_i\}$, the marks are mutually independent random vectors in $\mathbb{R}^\ell$, and if the conditional distribution of the mark M of a point $X \in \Phi$ depends only on the location of this point X it is attached to; i.e.,* $\mathbf{P}\{M \in \cdot|\Phi\} = \mathbf{P}\{M \in \cdot|X\} = F_X(\mathrm{d}M)$ *for some* probability kernel or marks $F_\cdot(\cdot)$ *from* $\mathbb{R}^d$ *to* $\mathbb{R}^\ell$.

Definition 3.2.2 (from [26]) *An independently marked Poisson p.p. $\tilde{\Phi}$ with intensity measure Λ on $\mathbb{R}^d$ and marks with distributions $F_X(\mathrm{d}M)$ on $\mathbb{R}^\ell$ is a Poisson p.p. on $\mathbb{R}^d \times \mathbb{R}^\ell$ with intensity measure*

$$\tilde{\Lambda}(A \times K) = \int_A \tilde{p}(X, K)\Lambda(\mathrm{d}x), \quad A \subset \mathbb{R}^d, K \subset \mathbb{R}^\ell. \tag{3.4}$$

where $\tilde{p}(X, K) = \int_K F_X(\mathrm{d}M)$. The independently marked Poisson p.p. can be seen as a transformation of the (non-marked) Poisson p.p. of intensity measure Λ on $\mathbb{R}^d$ by a probability kernel $p(X, A \times K)$ [26].

Part II

NETWORK APPLICATIONS

4

THE MOBILE ASSIGNMENT PROBLEM IN BROADCAST TRANSMISSION: OPTIMIZATION ASPECTS

4.1 INTRODUCTION

It is a requirement of the Long-term Evolution (LTE) specifications to support the delivery of broadcast/multicast data under the name of the Multimedia Broadcast/Multicast Service (MBMS) [30]. There is no difference between broadcast and multicast downlink data transmissions at the physical layer. While broadcast services are available to all users without the need for subscriptions to particular services, multicasting can thus be seen as "broadcast via subscription", with the possibility of charging for the subscription [30].

Broadcast is in particular well adapted to wireless channels, where one may use resources (in frequency and/or time) that are common to all destinations. We assume that the cost for a base station (BS) to transmit to a multicast group is proportional to the power needed to reach the most remote mobile among the group, and that the latter is a function of the distance to that mobile and the effect of shadowing. A BS may broadcast the same information to several multicast groups. In that case each multicast group is charged the cost to reach the most remote mobile in that group. In our setting, the most remote mobile should be understood as the one for which is needed the highest transmission power, rather than a remoteness due to a geographical measure.

We consider the situation where there is one common information that every one of m mobiles is interested to receive, and which can be obtained from any one of n BSs. The broadcast information could be some content, such as streaming transmission of a sport or cultural event, or it could be some signalling such as a beacon for time synchronization or for power control purposes. We seek such assignments of mobiles to BSs so that *the total power is minimized*. Further, we take into account the *operational power costs* (e.g. power amplifiers, cooler, etc.) of a typical BS. Indeed, the mobile assignment problem (MAP) in the context of broadcast transmission that we study in this work is built upon *min-size k-clustering problem* suggested and examined in [31]. In min-size k-clustering problem, the objective is to assign the points to at most k clusters so that the sum of all distances between points in the same cluster (k-clustering) is minimized. In [31], the typical cost for a BS-mobile pair is assumed to be only a function of distance between the BS and the mobile, which is formulated as $\sum_{N_j \in \mathscr{C}} \max_{i \in N_j} d_{ij}^{\alpha} + P_0^j$. Here, N_j is a *cluster* of mobiles assigned to BS j, $\mathscr{C}$ is the set of clusters called as *clustering*, d_{ij} is the distance between mobile i and BS j, α is the path loss exponent, P_0^j is the operational power cost loaded to BS j. However, here, we add the effect of shadowing to such a cost producing the following total cost $\sum_{N_j \in \mathscr{C}} \max_{i \in N_j} d_{ij}^{\alpha}/\Psi_{ij} + P_0^j$ where Ψ_{ij} denotes the shadowing effect between mobile i and BS j.

4.1.1 *Related Work*

The computational geometric approaches to the MAP can be found in [31, 32, 33, 34]. In [32], the authors examine the 1-dimensional version of the MAP, where the effects of shadowing and operational power cost are not taken into account. Polynomial time solutions via dynamic programming are proposed. The paper [33] suggests approximation algorithms (and an algebraic intractability result) for selecting an optimal line on which to place BSs to cover mobiles, and a proof of NP-hardness for any path loss exponent $\alpha > 1$.

The papers [36, 37, 38, 39] focus on source-initiated broadcasting of data in static all-wireless networks. Data are distributed from a source node to each node in a network. The main objective is to construct a minimum-energy broadcast tree rooted at the source node.

In [35], we study the combined problem of (i) deciding what subset of the mobiles would be assigned to each BS, and then (ii) sharing the BSs' cost of multicast among the mobiles. The subset that we wish to assign to a given BS is said to be its target set of mobiles. This problem can be conceived as a coalition game played by mobiles which we call as *association game of mobiles*. This game has an incentive to form grand coalition where all players join to the game. We also conclude that if *the nucleolus* is used as the cost allocation algorithm, the players maintain the grand coalition satisfying the minimization of the total cost for broadcast transmission. However, we do not study how to reach to a minimal cost in [35].

4.1.2 *Our Contribution*

The referred papers concentrate on the geometry aspect of the MAP where basically, the coverage area of a BS is assumed to be a disc which issues from omnidirectional antenna pattern. However, *the effect of shadowing*, special designed *antenna patterns* as well as *the operational power costs* could change the BS-mobile assignments. Here, we take into account these effects. To this end, we represent by *power cost matrix* each BS-mobile pairing power cost. Then, we propose dynamic programming based algorithms performing operations on power cost matrix.

Besides, "green-aware" approaches [44] which aim to reduce the energy consumption in wireless environments has to be taken into account in designing the relevant algorithms. In this context, switching off some fraction of BSs is considered to be a way of decreasing dramatically the total energy consumption. Heterogeneous networks include macro and small cells with coordination. In this work, we assume that small cells are subject to switching off operation while macro cells are always turned on, which serve moving mobiles in order to decrease the number of hand-offs. The small cells are deployed intensively, therefore the transmission power is lower than macro cells'.

Comparing the transmission power being in milliwatt levels with the operational power costs which is nearly tens of Watts, it is very efficient to turn off some fraction of BSs for reducing the total energy consumption. This method could be abundant when the users do not move. However, the fast-moving users are considered to be served by macro BSs which are not switched off generally [11].

Subsequently, we propose a recursive algorithm called *the hold minimum algorithm* which solves the considered problem optimally. The hold minimum

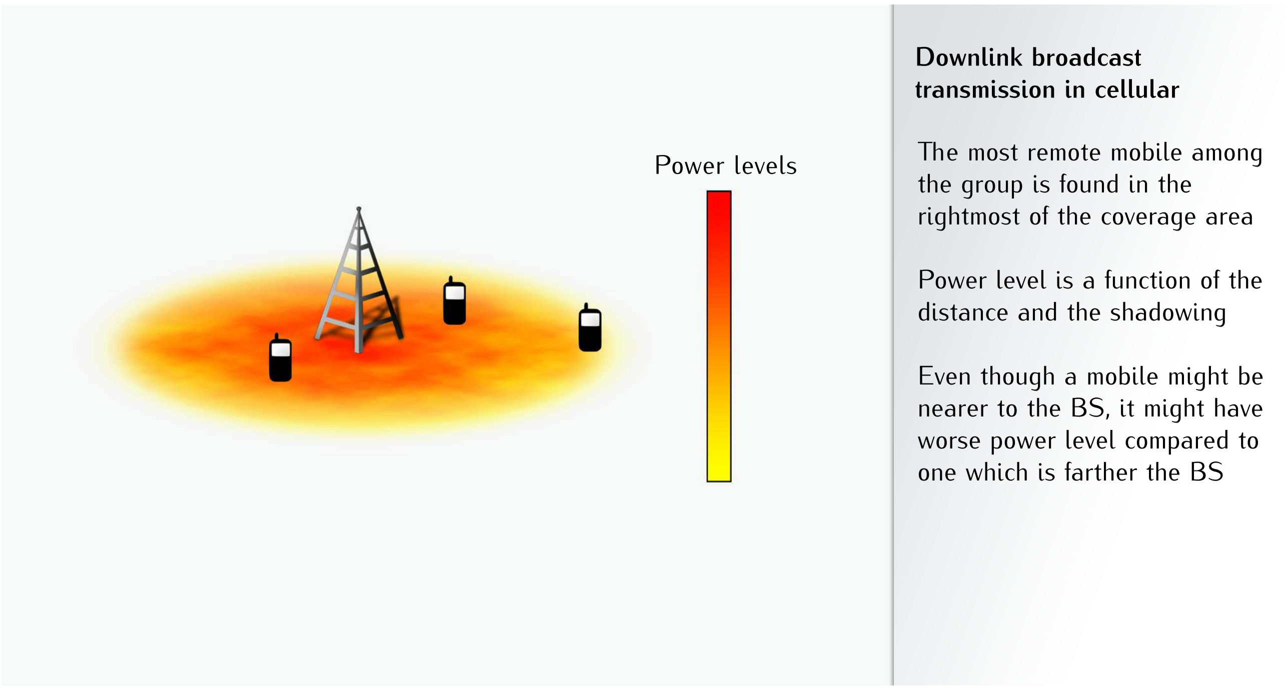


Figure 4.1: Broadcast transmission in cellular networks.

algorithm operates in a centralized way, which requires the whole knowledge for each BS-mobile pairing power cost. We also suggest another centralized polynomial-time heuristic algorithm called the *the column control* which produces optimal assignments when taking into account the operational power cost. Moreover, we develop a distributed approach to the column control, where each mobile gathers the local information from the BSs that can transmit to it. We On the other hand, *the nearest base station algorithm*, a distributed heuristic algorithm which runs in polynomial-time is offered. This algorithm is efficient for the fast-moving users served by macro BSs. We also introduce a new algorithm based on group formation games, which we call as the *hedonic decision algorithm*. This formalism is constructive: a new class of group formation games is introduced where the utility of players within a group is separable and symmetric being a generalized version of parity affiliation games. Furthermore, the hedonic decision algorithm can be suitable for any set covering problem.

4.2 THE PROBLEM

We consider the coverage problem in case of broadcast transmission in cellular networks. We assume each BS transmits simultaneously to the mobiles. The distance between the mobile i and BS j is represented as d_{ij}. The power needed to receive the transmission is given by P_r. We consider basic signal propagation model capturing path loss as well as shadowing effect formulated as

$$P_{ij} = P_r \frac{d_{ij}^{\alpha}}{\Psi_{ij}}, \tag{4.1}$$

where P_{ij} and α denote transmitted power from BS j to mobile i and path loss exponent, respectively. The random variable Ψ is used to model slow fading effects and commonly follows a log-normal distribution, i.e., the variable $10\log_{10}\Psi$ follows a normal distribution.

The required transmission power is related to the mobile having the worst signal level from the BS (Figure 4.1). At this power level, we are guaranteed that all mobiles receive at a sufficient power. We also consider the operational power cost denoted as P_0^j which captures the energy expenditure of a typical BS j for operational costs (power amplifiers, cooler, etc.). So, the total power cost (transmission power + operational power cost) of a typical transmission between BS j and mobile i is denoted as

$$p_{ij} = P_{ij} + P_0^j. \tag{4.2}$$

Let $M = (1, \ldots, m)$ and $N = (1, \ldots, n)$ be the sets of mobiles and BSs, respectively. Representing the *power cost matrix* $\mathbf{P} = (p_{ij}) \in \Re^{m\times n}$, we assume $p_{ij} \in [0, \infty]$ where if $P_{ij} > P_{max}$, then $p_{ij} = \infty$ (P_{max} denotes a maximal power, for instance, in WiFi, it is 100 mW).

4.2.1 *The MAP as a Clustering Problem*

The clustering is a rich branch of combinatorial problems which have been extensively studied in many fields including database systems, image processing, data mining, molecular biology, etc. [31]. Consider the set of mobiles M; a *cluster* is any non-empty subset of M. A *clustering* is a partition of M. Many different clustering problems can be defined. The mostly studied problems are as following and their objectives are to assign the points to at most k clusters so that

- *k-centre*: the maximum distance from any point to its cluster centre is minimized,
- *k-median*: the sum of distances from each point to its closest cluster centre is minimized,
- *k-clustering*: the sum of all distances between points in the same cluster is minimized.

The problem of clustering a set of points into a specific number of clusters so as to minimize the sum of cluster sizes is called as min-size k-clustering problem. In [31], the typical cost for a BS-mobile pair is assumed to be only a function of distance between the BS and the mobile, which is formulated as $\sum_{N_j\in\mathscr{C}} \max_{i\in N_j} d_{ij}^{\alpha} + P_0^j$. Here, N_j is a *cluster* of mobiles assigned to BS j, $\mathscr{C}$ is the set of clusters called as *clustering*, d_{ij} is the distance between mobile i and BS j, α is the path loss exponent, P_0^j is the operational power cost loaded to BS j. In this work, we add the effect of shadowing to such a cost producing the following total cost $\sum_{N_j\in\mathscr{C}} \max_{i\in N_j} d_{ij}^{\alpha}/\Psi_{ij} + P_0^j$. Thus, note that the shadowing effect breaks the monotonicity due to the distance (see Figure 4.1). Moreover, in the MAP, the objective is not to place BSs in order to minimize the total power.

4.2.2 *Brute-force Search Solution*

The enumerating all possible solutions and choosing the one which produces the lowest cost is known as brute-force search or generate and test.

We represent by $\mathbf{A} = (a_{ij}) \in \Re^{m\times n}$ the *assignment matrix* where $a_{ij} \in (0,1)$. If mobile i is assigned to BS j, then $a_{ij} = 1$, otherwise $a_{ij} = 0$. Notice that each row of assignment matrix includes only unique "1" which means that a mobile is served by only one BS, i.e. $\sum_j a_{ij} = 1$. Denoting the collection of assignment matrices $\mathscr{A}$, actually, we would formalize our problem as following:

$$p = \min_{\mathbf{A}\in\mathscr{A}} \left(\sum_{i\in M} \max_{j\in N} \mathbf{A} \otimes \mathbf{P} \right), \tag{4.3}$$

where $\otimes$ is the element-wise product. Note that the total number of possibilities of assignment matrices can be calculated as $|\mathscr{A}| = n^m$.

4.2.3 *The MAP as a Set-partitioning Problem*

In this section, we formalize the MAP as a binary integer program.

Let the power set of M be $\wp(M)$. Then, the power set of M contains $|\wp(M)| = 2^m$ elements. Thus, the total number of elements corresponding to the sum of combinations of each power set of n BSs is given by $\kappa = n(2^m - 1)$. We represent by $\mathscr{S}$ the collection of total possibilities. The index set of $\mathscr{S}$ is denoted by $L = (1, \ldots, \kappa)$. Let $\mathbf{U} = (u_{il}) \in \Re^{m\times\kappa}$ be a 0-1 matrix, and $q = (q_l) \in \Re^{\kappa}$ be a κ-dimensional vector. The value q_l represents the optimal power of $(S_l; j) \in \mathscr{S}$ by which we denote a pair which consists of a set of mobiles S_l assigned to BS j. Clearly

$$q_l = \max_{i\in(S_l;j)} p_{ij}. \tag{4.4}$$

We let $u_{il} = 1$ if mobile i belongs to the set S_l. The formulation of this problem is given by

$$\begin{aligned} (P) \qquad & p = \min \sum_{l\in L} q_l x_l \\ & \text{s.t.} \quad \sum_{l\in L} u_{il} x_l = 1, \quad i \in M, \\ & x_l \in \{0,1\}, \quad l \in L \end{aligned} \tag{4.5}$$

where the term $\sum_{l\in L} u_{il} x_l = 1$ follows only one BS is associated with a mobile. By this way, we do not let a mobile to be assigned to several BSs. As result, the optimal clustering is denoted as $\mathscr{C}^* \subset \mathscr{S}$ such that $\mathscr{C}^* = \{(S^*; j^*) \in \mathscr{S}, \forall j \in N\}$ where $(S^*; j^*)$ is the pairing in case of optimal solution.

We can consider the MAP as a version of the set partitioning problem. In the MAP the set M is associated with another set N. Therefore, the collection $\mathscr{S}$ contains those subsets of M each of which is associated with every element of the set N. Consider the following example.

Example 4.2.1 *Let us have a power cost matrix given by*

$$\mathbf{P} = \begin{bmatrix} 3 & 6 \\ 5 & 1 \end{bmatrix}. \tag{4.6}$$

The collection of total possibilities:

$$\mathscr{S} = \{(1;1),(2;1),(1,2;1),(1;2),(2;2),(1,2;2)\}. \tag{4.7}$$

Recall that $(S_l; j)$ *denotes the cluster of mobiles* S_l *assigned to BS* j*. The optimal values for each possibility is given by* $q = (q_l) = (3,5,5,6,1,6)$*. Then, we define the following matrix:*

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}. \tag{4.8}$$

The optimal total power thus is calculated by the following binary integer program:

$$\begin{aligned} p = &\min(3x_1 + 5x_2 + 5x_3 + 6x_4 + x_5 + 6x_6) \\ &\text{s.t.} \quad x_1 + x_3 + x_4 + x_6 = 1, x_2 + x_3 + x_5 + x_6 = 1, \\ &x_l \in \{0,1\}, \quad l \in (1,\dots,6). \end{aligned} \tag{4.9}$$

The values $x_1 = 1$ *and* $x_5 = 1$ *result in the optimal total power of the example scenario, i.e.,* $p = 3 + 1 = 4$ *with the optimal clustering* $\mathscr{C}^* = \{(1;1),(2;2)\}$*.*

Set partitioning problem is well known to be NP-hard [40]. Consequently, the MAP being a special set partitioning problem is also NP-hard. The collection set of the MAP possesses a large scale nature. For example, even for $m = 30$, $n = 10$, the size of collection set is $\kappa = 10(2^{30} - 1) \approx 1.074 \times 10^{10}$.

4.2.4 *Set Cover Relaxation: A Solution of Binary Integer Program*

We relax the condition of associating only one BS to a cluster of mobiles in (P) such that it is possible a cluster of mobiles to be served by more than one BS: $\sum_{l \in L} u_{il} x_l \geq 1$. Thus, this arrangement turns the MAP into so called *set covering* problem.

Moreover, consider a set of mobiles S assigned to BS j which we have been denoting as $(S; j)$. When a group of mobiles $T \subset S$ deviates to another BS k then the cost due to S becomes additively. Let the cost of $(S; j)$ and $(T; k)$ be $q_S = \max_{i \in (S;j)} p_{ij}$ and $q_T = \max_{i \in (T;k)} p_{ik}$, respectively. We denote the total cost before deviation of T as p and after deviation of T as p', respectively [1], which can be given by

$$p = p_r + q_S, \tag{4.10}$$

$$p' = p'_r + q_{S \setminus T} + q_T, \tag{4.11}$$

where p_r and p'_r are the remaining costs before and after deviation, respectively. There is always a potential (probability) increasing the total cost when a deviation occurs, i.e. $p' > p$. For better observation, let us consider the following power cost matrix:

$$\mathbf{P} = \begin{bmatrix} 10 & 15 & 25 \\ 27 & 20 & 33 \\ 32 & 31 & 30 \end{bmatrix}. \tag{4.12}$$

1 Note that these costs are not optimal. We only would like to show what is the effect of deviation of a group of mobiles from theirs current BS.

Let $(S;j) = (1,2,3;1)$ and $(T;k) = (1,2;3)$, respectively. Then, $q_S = \max(10,27,32) = 32$, $q_T = \max(25,33) = 33$, and $q_{S\backslash T} = \max(32) = 32$ resulting in the following total costs:

$$p = 32 \quad (4.13)$$

$$p' = 32 + 33 = 65, \quad (4.14)$$

where $p_r = p'_r = 0$.

Utilizing this property,

> *we delete from the collection $\mathscr{S}$ all those assignments $(S\backslash T;j)$ whenever the cost of $(S;j)$ is equal to the cost of $(S\backslash T;j)$ such that $T \subset S$.*

For example, let us consider the last example where $M = (1,2,3)$ and $N = (1,2,3)$. For $j = 1$, all possible assignments are

$$(1;1),(2;1),(3;1),(1,2;1),(1,3;1),(2,3;1),(1,2,3;1)$$

corresponding to the cost vector $q = (10,27,32,27,32,32,32)$. Note that the cost of $(3;1)$, $(1,3;1)$, $(2,3;1)$, $(1,2,3;1)$ are equal each other. Therefore, we remove $(3;1)$, $(1,3;1)$, $(2,3;1)$ from the collection. The reduced collection of assignments becomes as following:

$$\mathscr{S}' = \{(1;1),(1,2;1),(1,2,3;1),(1;2),(1,2;2),$$
$$(1,2,3;2),(1;3),(1,3;3),(1,2,3;3)\}. \quad (4.15)$$

Thus, the binary integer program of finding the solution of the problem is given by

$$\begin{aligned} p = &\min(10x_1 + 27x_2 + 32x_3 + 15x_4 + 20x_5 + 31x_6 + 25x_7 + 30x_8 + 33x_9) \\ \text{s.t.} \quad & x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 \geq 1, \\ & x_2 + x_3 + x_5 + x_6 + x_9 \geq 1, \\ & x_3 + x_6 + x_8 + x_9 \geq 1, \\ & x_l \in \{0,1\}, \quad l \in (1,\ldots,9). \end{aligned} \quad (4.16)$$

The solution of this problem is found to be $x_6 = 1$ and $x_l = 0, \forall l \in (1,\ldots,9)$ which fits to the optimal one.

By such an elimination, the size of collection of assignments reduces from $n(2^m - 1)$ to nm.

4.3 THE ALGORITHMS

In this section, we propose our algorithms used for calculation the MAP. We first introduce dynamic programming based two new algorithms which are centralized: *the hold minimum algorithm* and *the column control algorithm*. Also, a distributed approach to the column control as well as another distributed algorithm which we call as *the hedonic decision* derived from hedonic coalition formation are proposed. A greedy solution of the problem is introduced as *the nearest base station* approach.

Because of the large scale nature of the collection set $\mathscr{S}$, we develop the algorithms by making operations on the power cost matrix. This approach facilitates to remove elements of the collection set, and thus, converge quickly to a solution.

4.3.1 *Optimal Solution: The Hold Minimum (HM) Algorithm*

The HM algorithm solves the problem *optimally*. We explain the algorithm by an example. Consider the power cost matrix which is given by

$$\mathbf{P} = \begin{bmatrix} 9 & 3 \\ 1 & 4 \\ 2 & 8 \end{bmatrix}. \tag{4.17}$$

The power cost matrix can also be shown as $\mathbf{P} = (p_1, p_2, \ldots, p_n)$, where $p_j = (p_{1j}, p_{2j}, \ldots, p_{mj})^T$. In each step, the algorithm removes a group of values p_{ij} of the power cost matrix. Removing p_{ij} means that we eliminate those clusters that include the mobile i and BS j from the collection set. The algorithm compares maximum n clusters and holds only the cluster minimizing the total cost. Thus, it terminates in a step Q where each mobile is assigned to only one BS. In step s, the power cost matrix and collection set is denoted as $\mathbf{P}[s] = (p_1[s], p_2[s], \ldots, p_n[s])$ and $\mathscr{S}[s]$, respectively.

Let us now turn to the example. In the initial step $s = 0$, we assume that $\mathbf{P}[0] = \mathbf{P}$ and $\mathscr{S}[0] = \mathscr{S}$ given by

$$\begin{aligned} &\mathscr{S}[0] = \{(1;1),(2;1),(3;1),(1,2;1),(1,3;1),(2,3;1), \\ &(1,2,3;1),(1;2),(2;2),(3;2),(1,2;2),(1,3;2), \\ &(2,3;2),(1,2,3;2)\}. \end{aligned} \tag{4.18}$$

Recall that assigning a cluster of mobiles S_l to BS j has a cost $\max_{i \in S_l} p_{ij}$. Therefore, if we find the maximum value of p_j, we obtain the total cost in case of all mobiles in column j are assigned to BS j. For example, $\max p_1 = \max(9, 1, 2) = 9$. This means that if all mobiles are assigned to BS 1, then the total cost is 9.

The algorithm runs as following: in step $s = 1$, we find the maximum value of each column of power cost matrix, then eliminate all values in power cost matrix except minimum of the calculated maximum values. Namely, $\max(9, 1, 2) = 9$ and $\max(3, 4, 8) = 8$, then 9 is eliminated by putting an ∞

$$\mathbf{P}[1] = \begin{bmatrix} \infty & 3 \\ 1 & 4 \\ 2 & 8 \end{bmatrix}. \tag{4.19}$$

Thus, the collection set reduces to the following

$$\begin{aligned} &\mathscr{S}[1] = \{(2;1),(3;1),(2,3;1),(1;2),(2;2),(3;2),(1,2;2), \\ &(1,3;2),(2,3;2),(1,2,3;2)\}. \end{aligned} \tag{4.20}$$

First column contains an ∞ which means that mobile 1 must be assigned to another BS (here, there is only one BS, it is BS 2). In fact, this represents the recursiveness of the algorithm where we run the algorithm for a sub power cost matrix. In this simple example, the sub power cost matrix is 3. In general case, the algorithm does the following

$$P_j[s] = \begin{cases} \max p_j[s] + \mathfrak{h}(\mathbf{P}_j^{sub}[s]), & \text{if sub power cost matrix;} \\ \max p_j[s], & \text{otherwise.} \end{cases} \tag{4.21}$$

where $P_j[s]$ represents the total cost if we assign all mobiles to BS j except those that can not be assigned to, and the optimal cost occurring due to sub power cost matrix $\mathbf{P}_j^{sub}[s]$ in step s. Here, $\mathfrak{h} : \Re^{m\times n} \rightarrow \{\Re, \mathscr{A}\}$ is the function which gives the optimal value and assignments obtained by running HM algorithm.

For $s = 2$, we calculate $P_1[2] = \max(1,2) + \mathfrak{h}(3) = 2 + 3 = 5$, where $\mathbf{P}_1^{sub}[2] = (3)$. On the other hand, we do not need to calculate $P_2[2]$ since it is kept in the memory. Therefore, $P_2[2] = P_2[1]$. Then, the algorithm holds minimum value of $\min(P_1[2], P_2[2]) = P_1[2] = 5$ meaning that we remove 8 resulting in the following

$$\mathbf{P}[2] = \begin{bmatrix} \infty & 3 \\ 1 & 4 \\ 2 & \infty \end{bmatrix}, \tag{4.22}$$

and

$$\mathscr{S}[2] = \{(2;1),(3;1),(2,3;1),(1;2),(2;2),(1,2;2)\}. \tag{4.23}$$

Then, for $s = 3$, $P_2[3] = \max(3,4) + \mathfrak{h}(2) = 4 + 2 = 6$, where $\mathbf{P}_2^{sub}[3] = (2)$, and $P_1[3] = P_1[2]$. We remove 4, since $\min(P_1[3], P_2[3]) = P_1[3]$. This gives the following matrix, collection set, assignments, and optimal total power,

$$\mathbf{P}[3] = \begin{bmatrix} \infty & 3 \\ 1 & \infty \\ 2 & \infty \end{bmatrix}, \tag{4.24}$$

$$\mathscr{S}[3] = \{(2;1),(3;1),(2,3;1),(1;2)\} = \{(2,3;1),(1;2)\}, \tag{4.25}$$

$$\mathfrak{h}(\mathbf{P}) := \left\{ p = 5, \mathbf{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \right\}, \tag{4.26}$$

respectively. The pseudo-code of this algorithm is given in Algorithm 3.

Theorem 4.3.1 The HM algorithm terminates in finite step. At this step, the total power cost is minimum.

Proof 4.3.1 *The power cost matrix is transformed in each step by*

$$\mathbf{P}[0] \rightarrow \mathbf{P}[1] \rightarrow \ldots \rightarrow \mathbf{P}[Q]. \tag{4.27}$$

In each step, at least $n-1$ values are removed from the power cost matrix. Removed values are those that increase the total cost. The value that is held is the minimum one in the corresponding step. So, in the terminal step $s = Q$, it arrives to such a cost that is the lowest. Each mobile is assigned to exactly one BS in the terminal step.

Algorithm 3 The Hold Minimum

function $(p, \mathbf{A}) = \mathfrak{h}(\mathbf{P})$
  while each row includes several "1"s in **A** **do**
    $k \leftarrow 1$
    for j = indices of columns of **P** not including all ∞ **do**
      $V_k \leftarrow$ maximum of column j except ∞
      if any row in column j includes ∞ **then**
        $\mathbf{P}^{sub} \leftarrow$ rows including ∞ of **P**
        $p_{sub} \leftarrow \mathfrak{h}(\mathbf{P}^{sub})$
        $V_k \leftarrow V_k + p_{sub}$
      end if
      $k \leftarrow k + 1$
    end for
    $(i_{min}, j_{min}, V_{min}) \leftarrow \min V$
    Hold V_{min} using (i_{min}, j_{min}) and put ∞ and 0 in all indices causing $P_{ij} \geq V_{min}$ in **P** and **A**
    Put 0 in all indices causing $P_{ij} \geq V_{min}$ in **A**
  end while
  $p \leftarrow \sum \max \mathbf{A} \bigotimes \mathbf{P}$
end

4.3.2 *Greedy Solution: The Column Control (CC) Algorithm*

Let us denote the CC algorithm by $\mathfrak{c} : \mathfrak{R}^{m \times n} \to \{\mathfrak{R}, \mathscr{A}\}$. Take into account the operational power cost P_0 which is fairly higher than the transmitted power, i.e. $P_0 \gg P_{ij}$, which dominates the energy consumption of a typical BS. We could use this advantage of the broadcast transmission. The aim here is to assign many mobiles to only one BS. Recall that cluster N_j (set of mobiles that can be assigned to BS j) has the transmission cost $\max_{i \in N_j} P_{ij}$ and the operational cost P_0^j. For better understanding, consider the following power cost matrix in which the operational power cost is assumed to be 12 W:

$$\mathbf{P} = \begin{bmatrix} 12.50 & 12.40 & 12.32 & \infty \\ 12.30 & 12.30 & 12.43 & \infty \\ 12.20 & 12.45 & 12.15 & 12.23 \\ \infty & 12.43 & 12.25 & 12.35 \\ \infty & \infty & \infty & 12.29 \end{bmatrix}. \tag{4.28}$$

In such a scenario, we can assign $|N_1| = 3$ mobiles to BS 1 with cost 12.50, $|N_2| = 4$ mobiles to BS 2 with cost 12.45, $|N_3| = 4$ mobiles to BS 3 with cost 12.43, and $|N_4| = 3$ mobiles to BS 3 with cost 12.35.

The logic behind the CC algorithm is the following:

1. *Find how many mobiles can be assigned to each BS*
2. *Choose the BS to which can be assigned the most mobiles*
3. *If there are multiple BSs in the state of 2), then choose the BS which can serve the mobiles with minimal cost*

Applying these rules to the last example, it turns out that BS 3 can cover the most mobiles $|N_3| = 4$ which are $N_3 = (1, 2, 3, 4)$ with the lowest cost 12.43.

Then, the algorithm assigns only a cluster of mobiles. In the following step, the CC algorithm performs the same rules to the remained mobiles which produces the sub power cost matrix $\mathbf{P}^{sub}$. In the last example, it is given by

$$\mathbf{P}^{sub} = \begin{bmatrix} \infty & \infty & \infty & 12.29 \end{bmatrix}. \quad (4.29)$$

This results in the assignment of mobile 5 to BS 4, because mobile 5 can not be assigned to other BSs. This shows the recursiveness of the CC algorithm. A pseudo-code is given in Algorithm 4.

Formally, in step s, we denote as following the set of mobiles assigned to BS j:

$$R[s] = \left\{\text{Assigned mobiles in step } s\right\}. \quad (4.30)$$

Thus, the collection set is reduced as following in step s:

$$\mathscr{S}[s] = \left\{\mathscr{S}[s-1] \setminus (S;k) : i \in S, \forall i \in R[s] \text{ and } \forall k \in N \setminus j\right\}. \quad (4.31)$$

Considering the last example, in step $s = 1$, $R[1] = (1,2,3,4)$. By assigning these mobiles to BS 3, the CC algorithm removes those assignments which include mobiles $(1,2,3,4)$ except $(S;k) = \{(1,2,3,4),3\}$. Thus, the total power and assignments are given by

$$\mathfrak{c}(\mathbf{P}) := \left\{ p = 24.72, \mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right\}. \quad (4.32)$$

Moreover, for any power cost matrix, we conclude that in a final step Q, the CC algorithm converges to the case where each mobile is assigned to merely one BS.

4.3.3 *Distributed Column Control (DCC) Algorithm*

Assume that each BS broadcasts its own power vector and identities of mobiles that it can serve. Recall that we denote the power vector of BS j as p_j. The power vector of BS 1 given in the power cost matrix of eq. (4.28) is $p_1 = (12.50, 12.30, 12.20, \infty, \infty)^T$. In fact, the infinite power costs in p_1 are not produced by BS 1, rather these values are used in a central unit being aware of all mobiles and BSs. Therefore, we have to show the power vector as $p_1 = (12.50, 12.30, 12.20)^T$. Also, the identity vector of mobiles that can be served by BS j is represented as $h_j = (h_{jk}) \in \mathbb{N}^{|p_j|}$. For example, $h_1 = (1,2,3)^T$.

Moreover, each mobile receives power vectors from all BSs that can transmit to it. Then, each mobile generates the power cost matrix from received power vectors. For example, mobile 1 receives from BS 1, BS 2, and BS 3 the power vectors

$$p_1 = (12.50, 12.30, 12.20)^T,$$

$$p_2 = (12.40, 12.30, 12.45, 12.43)^T,$$

and

$$p_3 = (12.32, 12.43, 12.15, 12.25)^T$$

Algorithm 4 The Column Control

function $(p, \mathbf{A}) = \mathfrak{c}(\mathbf{P})$
 $v' =$ Find how many ∞ has each column of $\mathbf{P}$
 $v_{min} \leftarrow \min v'$
 $v'' =$ Find which row of v' is equal to v_{min}
 for $l =$ columns of $\mathbf{P}$ determined by $|v''|$ **do**
  $V_l \leftarrow$ maximum value of column l of $\mathbf{P}$
 end for
 Find $V_{min} = \min V_l$ and the corresponding column l_{min}
 if $|v'| == 0$ **then**
  Put 1s to the column l_{min} in $\mathbf{A}$
 else
  $\mathbf{A}' \leftarrow$ Put 1s to the column l_{min} in $\mathbf{A}$
  Find sub power cost matrix $\mathbf{P}^{sub}$ which is composed by those mobiles that are not assigned
  $\mathbf{A}^{sub} \leftarrow$ Find assignments by running $\mathfrak{c}(\mathbf{P}^{sub})$
 end if
 $\mathbf{A} \leftarrow$ Combine $\mathbf{A}^{sub}$ and $\mathbf{A}'$
 $p \leftarrow \sum \max \mathbf{A} \bigotimes \mathbf{P}$
end

with identity vectors $h_1 = (1,2,3)^T$, $h_2 = (1,2,3,4)^T$, and $h_3 = (1,2,3,4)^T$, respectively. Mobile 1 decides that the power cost matrix is as following:

$$\begin{bmatrix} 12.50 & 12.40 & 12.32 \\ 12.30 & 12.30 & 12.43 \\ 12.20 & 12.45 & 12.15 \\ \infty & 12.43 & 12.25 \end{bmatrix}. \tag{4.33}$$

Note that mobile 1 realizes from h_2 and h_3 that BS 1 can not transmit to mobile 4. Therefore, it puts an infinite cost corresponding to mobile 4 and BS 1 in the power cost matrix.

By this rule each mobile determines its own power cost matrix. Thus, each mobile finds the assignments according to CC algorithm, and selects the BS from which it will receive data. For example, mobile 1 obtains the following assignment matrix by running CC algorithm

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}. \tag{4.34}$$

It turns out that mobile 1 chooses BS 3 for reception the broadcast data.

Remark 4.3.1 *Through the advantage of the decentralization, the DCC algorithm makes possible the following:* if the mobiles do not send any assignment information to a BS, then the corresponding BS is considered to be switched off. *On the other hand, the CC algorithm is centralized, therefore,* the network determines according to the assignments which BS is switched off.

4.3.4 *Greedy Solution: The Nearest BS (NBS) Algorithm*

To solve the problem heuristically, the easiest way is to assign each mobile to the nearest BS. By "nearness", we do not mean a geographical measure, instead, it is the lowest power cost that the corresponding mobile needs from the corresponding BS. So, the mobile selects the BS transmitting with the lowest power. We assume that a mobile is capable to know the power costs corresponding to those BSs that can transmit to it.

Clearly, mobile i knows the vector $p_i = (p_{i1}, p_{i2}, \ldots, p_{in_i})$ where n_i denotes the number of BSs that mobile i can be served. Then, mobile i only calculates the minimal value of p_i, and chooses the corresponding BS,

$$a_{i,j} = 1: \quad j = \arg\min_j p_i, \tag{4.35}$$

where $a_{i,j} \in \mathbf{A}$ is the BS that mobile i selects. Actually, this corresponds to remove from collection set $\mathscr{S}$ all assignments related to mobile i and the BSs being out BS j.

The NBS algorithm is very efficient and quick, and in most cases, it gives optimal assignments when the operational power cost $P_0 = 0$ (See Section 4.6).

4.3.5 *Group Formation Game Solution: The Hedonic Decision (HD) Algorithm*

Recall that the required power for serving the group of mobiles S_j by BS j is denoted as $\max_{x \in S} p_{xj}$. Consider this as a *transferable virtual cost* among the mobiles sharing the same BS. Here, we assume that a BS is *passive* which does not make any strategic decision. Thus, let us represent as $u(S_j; j) = -\max_{x \in S} p_{ij} \leq 0$, the *transferable virtual utility*. Note that $u(S_j; j) \leq 0, \forall S_j \subseteq M, \forall j \in N$ is a monotonically decreasing function [45]. The marginal utility is given by $\Delta(S; j) = u(S; j) - \sum_{i \in S} u(i; j)$. Note that $u(i; j) = -p_{ij}$. Therefore, $\Delta(S; j) = \sum_{i \in S} p_{ij} - \max_{i \in S} p_{ij} \geq 0$. Moreover, since $u(S; j)$ is always superadditive [45], the following is obtained:

$$\begin{aligned} &\Delta(S; j) + \sum_{i \in S} u(i; j) + \Delta(T; j) + \sum_{i \in T} u(i; j) \leq \Delta(S \cup T; j) + \sum_{i \in S \cup T} u(i; j), \\ &\Delta(S; j) + \Delta(T; j) \leq \Delta(S \cup T; j), \quad \forall S, T \subseteq N, S \cap T = \emptyset. \end{aligned} \tag{4.36}$$

Assume that the mobiles are *strategic decision makers*. The strategic decision is performed in the following way:

mobile x prefers BS j to BS k whenever $\phi_x^{(S_j;j)} > \phi_x^{(S_k;k)}$.

In the following, we prove that the last condition is sufficient to formalize the MAP as a *group formation game* (see [48], [47], [49]). We also adopt the additively separable and symmetric utility allocation within a group where for any couple of mobiles $x, y \in M$ receive the symmetric utility $v(x, y; j)$ whenever they share the same BS j. Note that $v(x, x; j) = 0$ which means that a mobile does not gain from itself. Therefore, the allocated utility to player x in coalition S_j can be given by

$$\phi_x^{(S_j;j)} = \sum_{y \in S_j} v(x, y; j) + u(x; j). \tag{4.37}$$

Let us represent as $\sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_m\}$ the strategy tuple of mobiles where σ_x denotes the BS that mobile x chooses. The partition of mobiles that is a result of strategy tuple σ is denoted as $\Pi_\sigma = \{S_j\}_{j\in N}$. Thus, we denote as $\mathcal{G}_{\mathrm{HD}} = \langle M, N, \Pi_\sigma \rangle$ the group formation game of mobiles.

Theorem 4.3.2 *$\mathcal{G}_{\mathrm{HD}}$ is a potential game.*

Proof 4.3.2 *A non-cooperative game is a potential game [78] whenever there exist a function Ψ*

$$\Psi(j, \sigma_{-x}) - \Psi(k, \sigma_{-x}) = \phi_x^{(S_j;j)} - \phi_x^{(S_k;k)}, \tag{4.38}$$

meaning that when a mobile switches from BS j to k the difference of its utility can be given by the difference of a function Ψ. This function is called as potential function. Let us choose as following the potential function Ψ:

$$\begin{aligned}\Psi(\sigma) &= \sum_{j\in N}\left[\sum_{a\in S_j(\sigma)} u(x;j) + \sum_{a\in S_j(\sigma)}\sum_{b\in S_j(\sigma)} v(a,b;j)\right]\\ &= \sum_{j\in N}\sum_{a\in S_j(\sigma)} u(a;j) + \sum_{j\in N}\sum_{a\in S_j(\sigma)}\sum_{b\in S_j(\sigma)} v(a,b;j),\end{aligned} \tag{4.39}$$

where $S_j(\sigma)$ is the group of mobiles served by BS j in case of strategy tuple σ. Let us rewrite the potential function as following:

$$\begin{aligned}\Psi(\sigma_x, \sigma_{-x}) =& u(x;\sigma_x) + \sum_{y\in S_{\sigma_x}} v(x,y;\sigma_x)\\ &+ \underbrace{\sum_{j\in N}\sum_{a\backslash x\in S_j(\sigma)} u(a;j) + \sum_{j\in N}\sum_{a\backslash x\in S_j(\sigma)}\sum_{b\backslash x\in S_j(\sigma)} v(a,b;j)}_{I}.\end{aligned} \tag{4.40}$$

When mobile x switches from σ_x to σ'_x (the other mobiles do not change their strategies), then the strategy tuple is transformed from σ to σ', and the potential becomes

$$\begin{aligned}\Psi(\sigma'_x, \sigma_{-x}) =& u(x;\sigma'_x) + \sum_{y\in S_{\sigma'_x}} v(x,y;\sigma'_x)\\ &+ \underbrace{\sum_{j\in N}\sum_{a\backslash x\in S_j(\sigma')} u(a;j) + \sum_{j\in N}\sum_{a\backslash x\in S_j(\sigma')}\sum_{b\backslash x\in S_j(\sigma')} v(a,b;j)}_{I'}.\end{aligned} \tag{4.41}$$

Note that $I = I'$ since the total utility due to the other mobiles is equal in σ and σ'. Thus, the difference of potentials is given by

$$\begin{aligned}\Psi(\sigma_x, \sigma_{-x}) - \Psi(\sigma'_x, \sigma_{-x}) =& u(x;\sigma_x) + \sum_{y\in S_{\sigma_x}} v(x,y;\sigma_x)\\ &- u(x;\sigma'_x) - \sum_{y\in S_{\sigma'_x}} v(x,y;\sigma'_x).\end{aligned} \tag{4.42}$$

On the other hand, the difference of the utility of player x is calculated as

$$\phi_x^{(S_{\sigma_x};\sigma_x)} - \phi_x^{(S_{\sigma'_x};\sigma'_x)} = u(x;\sigma_x) + \sum_{y \in S_{\sigma_x}} v(x,y;\sigma_x) - u(x;\sigma'_x) - \sum_{y \in S_{\sigma'_x}} v(x,y;\sigma'_x). \quad (4.43)$$

By this result we conclude that

$$\Psi(\sigma_x, \sigma_{-x}) - \Psi(\sigma'_x, \sigma_{-x}) = \phi_x^{(S_{\sigma_x};\sigma_x)} - \phi_x^{(S_{\sigma'_x};\sigma'_x)}$$

which proves that $\mathcal{G}_{\mathrm{HD}}$ is a potential game.

Corollary 4.3.1 The proof 4.3.2 is constructive: *any group formation game possessing additively separable and symmetric utility gain of players within a group always converges to a pure Nash equilibrium. Actually, the game $\mathcal{G}_{\mathrm{HD}}$ is a straightforward generalization of party affiliation game [46].*

4.3.5.1 *What should be the additively separable and symmetric gain allocation?*

To guarantee the stability, the allocation of utilities among in a group S served by BS j must be additively separable and symmetric which is studied in Section 2.3.4. To this end, we would choose as following the condition in order that the allocated utilities become additively separability and symmetric:

$$v(x,y;j) = \theta \Delta(x,y;j) = \theta(p_{xj} + p_{yj} - \max(p_{xj}, p_{yj})), \text{ if } x \neq y$$
$$= 0, \text{ if } x = y. \quad (4.44)$$

where $v(x,y;j)$ is the symmetric utility gain of mobile x and y when served by BS j. θ is a parameter that must be adjusted according to the environment. It is affected by the intensity of mobiles and BSs as well as the area over which the algorithm is run. Note that we are able to represent as $v(x,y;j) = \theta \min(p_{xj}, p_{yj})$ if $x \neq y$. Thus, the preference function of any player $x \in M$ can be given by

$$\phi_x^{(S;j)} = \theta \sum_{y \in S} \Delta(x,y;j) + u(x;j) \text{ then,}$$
$$\phi_x^{(S;j)} = \theta \sum_{y \in S} \min(p_{xj}, p_{yj}) - p_{xj}, \text{ if } S \neq \emptyset$$
$$- p_{xj} \text{ if } S = \emptyset. \quad (4.45)$$

Remark 4.3.2 ***The interpretation of*** θ*: Consider the term* $\min(p_{xj}, p_{yj})$ *of the last preference function in eq. 4.45. Note that a mobile will prefer a BS where its neighbour mobiles are intensified near to itself since the sum becomes higher. Observe that this property is advantageous for decreasing the total power cost. The importance of θ reveals here. Because, calibrating θ impacts the total power cost in the following way:*

> *When the operational power costs dominate the transmission power cost,[2] the value of θ is relatively small compared to the case where*

2 It can be imagined as small cells deployment since we add the switching on/off or sleep mode attributes; thus, when the small cell is switched off or in sleep mode, then the operational power cost is zero.

there is no operational power cost (it corresponds to the macro cell deployment model since macro cells are always switched on). In other words, to increase the impact of $\sum_{y \in S} \min(p_{xj}, p_{yj})$*, we have to choose a higher* θ *when there is no operational power cost.*

In the computational results section, we find the optimal θ^* *which minimizes the total power cost for different scenarios.*

Assuming that each mobile is capable to discover those BSs that can transmit to it, we can adapt the Algorithm 1 (Nash stability establisher) to determine the assignments. Recall the scheduler that we discuss in Section 2.1.7. We can produce a scheduler in the following way: each BS generates a random clock-time for all those mobiles that it can transmit; then each mobile selects randomly a clock-time from those BSs that it can discover. We need to produce the clock-times by such a way that the collision of the turns of mobiles is minimal. In case of a collision, the clock-times of the corresponding mobiles are regenerated by corresponding BSs.

In Algorithm 5, the pseudo-code of the HD is given. Note that this is an algorithm preformed in both BS and mobile side by an exchange of the information in a separated channel.

Algorithm 5 The Hedonic Decision

```
Base Station:
Check stability
if there is no stability then
    Send information to each mobile about the current partition
    Generate clock-times for each mobile
else
    Stop the procedure and inform each mobile that the stability is found
end if
Mobile:
Select randomly a clock-time and inform each BS
Determine the preferred BS according to eq. 4.45
Send the preferred BS to each BS
```

Corollary 4.3.2 *In the literature, the use of game models for set covering problems is called as* set covering games *[50], [51], [52]. The HD algorithm is a novel approach for set covering games. It is suitable for any set covering and also facility location problem where the agents are allowed to make strategic decisions.*

4.4 TIME COMPLEXITY ANALYSIS

In this section, we calculate time complexities of the proposed algorithms.

Let us assume that $m = kn$. The input size is supposed to be the total number of elements of the power cost matrix, denoted as $x = nm$. This choice provides us to calculate the time complexity $T(x)$ in terms of x. It is straightforward to obtain that $n = \sqrt{\frac{x}{k}}$ and $m = \sqrt{kx}$.

Theorem 4.4.1 The time complexity of the HM algorithm is $O\left(x^{\frac{1}{2}(1-\log_q x)}\right)$.

Proof 4.4.1 *Note that there are at most* $n - 1 = \sqrt{\frac{x}{k}} - 1$ *operations finding the maximum value of a column of the power cost matrix in a step, and a single* *operation for finding minimum of a vector which could have n values, at most $m = \sqrt{kx}$ steps until a convergence to the optimal case. The selection algorithms (finding maximum or minimum) have the time complexity $O(x)$ [43]. Moreover, in each step a sub power cost matrix occurs with time complexity $T(qx)$ where $0 < q < 1$. qx is the number of values of sub power cost matrix. q is a random variable which is affected by the shadowing and the position of mobiles and BSs. Then, we are able to express the time complexity as*

$$\begin{aligned} T(x) &= \sqrt{kx}\left[\left(\sqrt{\frac{x}{k}}-1\right)\left(O\left(\sqrt{kx}\right)+T(qx)\right)+O\left(\sqrt{\frac{x}{k}}\right)\right] \\ &\leq \sqrt{kx}\left[\sqrt{\frac{x}{k}}\left(c\sqrt{kx}+T(qx)\right)+c\sqrt{\frac{x}{k}}\right] \\ &= cx\sqrt{kx}+xT(qx)+cx. \end{aligned} \tag{4.46}$$

Solving this difference equation, we obtain the following complexity:

$$T(x) = O\left(x^{\frac{1}{2}\left(1-\log_q x\right)}\right). \tag{4.47}$$

Note that q determines significantly the complexity of the HM algorithm. Increasing values of q means that the algorithm has to be run for further sub power matrices.

Theorem 4.4.2 The time complexity of the CC algorithm is $O\left(\frac{x}{1-q}\right)$.

Proof 4.4.2 *The CC algorithm performs $m = \sqrt{kx}$ operations for finding how many mobiles can be assigned to each BS. We assume that this operation has $O(\sqrt{kx})$ complexity. Choosing the BS to which can be assigned the most mobiles has the same complexity as finding maximum given by $O(\sqrt{\frac{x}{k}})$ operations, and there is also a minimization operation. Then, the time complexity can be calculated by*

$$\begin{aligned} T(x) &= \sqrt{\frac{x}{k}}O\left(\sqrt{kx}\right)+2O\left(\sqrt{\frac{x}{k}}\right)+T(qx) \\ &\leq c_1x+2c_2\sqrt{\frac{x}{k}}+T(qx). \end{aligned} \tag{4.48}$$

Thus, the time complexity is found to be

$$T(x) = O\left(\frac{x}{1-q}\right)+\frac{2c_2\left(\sqrt{x}-1\right)}{\sqrt{k}\left(1-\sqrt{q}\right)}-\frac{c_1}{1-q}+1. \tag{4.49}$$

Theorem 4.4.3 The time complexity of the NBS algorithm is $O\left(\sqrt{\frac{x}{k}}\right)$.

Proof 4.4.3 *It is straightforward because the only operation that is performed in the NBS algorithm is to find the minimum value of a vector of power cost matrix having a dimension at most $n = \sqrt{\frac{x}{k}}$. Therefore, the time complexity is $O\left(\sqrt{\frac{x}{k}}\right)$.*

4.5 DOWNLINK SYSTEM MODEL

The cellular network model consists of BSs arranged according to some homogeneous Poisson point process Φ of intensity λ_b $(points/m^2)$ in the Euclidean plane [26]. Also, we consider an independent collection of mobile users, located according to some independent homogeneous Poisson point process with intensity λ_m $(points/m^2)$. The main weakness of the Poisson model is that because of the independence of the Poisson point process, BSs will in some cases be located very close together but with a significant coverage area. This weakness is balanced by two strengths: the natural inclusion of different cell sizes and shapes and the lack of edge effects, i.e. the network extends indefinitely in all directions [13]. The expected value of a homogeneous Poisson point process is $\mathbf{E}[\Phi] = \lambda A$, where $A \subset \Re^2$ denotes some area. The snapshot depicted in Figure 4.2 shows the distribution of BSs and mobiles with intensity $\lambda_b = 44.4 \times 10^{-5} \frac{points}{m^2}$ and $\lambda_m = 2.5 \times 10^{-5} \frac{points}{m^2}$ where $A = 9\ km^2$.

Moreover, the deployment scenario in the figures 4.3, 4.4, 4.5, 4.6, 4.8 is considered to be in the small cell network context. Clearly, the general term "small cell networks" covers a range of radio network design concepts which are all based on the idea of deploying BSs much smaller than typical macro cell devices to offer public or open access to mobile terminals [10]. We assume $P_r = -10\ dBm$, being the typical received signal power of a wireless network as well as we set the transmission power $P_{ij} = \infty$ if $P_{ij} \geq 20\ dBm$ which is the upper bound in WiFi.

We assume to be the equal operational power cost for each BS, $P_0 = 12\ W$. Furthermore, we enumerate the BSs and mobiles according to the distance between the corresponding node (BS, mobile) and the origin assumed to be $(0,0)$ (Figure 4.2).

4.6 COMPUTATIONAL RESULTS

We compare the proposed algorithms for different values of λ_m and λ_b. We also aimed to obtain the results in a range milliseconds of running time in order to calculate the average total power when comparing the algorithms with the HM algorithm. Moreover, we set the path loss exponent $\alpha = 3$.

The average total power was calculated by Monte Carlo simulations by running the algorithms for different generated power cost matrices for some iteration number t and taking the mean of the result, which can be given by

$$\begin{aligned} &P(0) = 0, \\ &P(i+1) = P(i) + p, \quad i = 0, \dots, t \\ &\bar{p} = \frac{P(t+1)}{t}. \end{aligned} \tag{4.50}$$

In Table 4.1, the comparison of the SC and the developed algorithms is given for different example power cost matrices in case of the operational power cost $P_0 = 0$ for all BSs. We consider the hexagonal-type deployment of macro cells The transmission power $P_{ij} = \infty$ if $P_{ij} \geq 48\ dBm$. It turns out that the HD algorithm is very efficient to converging to optimal assignments when we do not take into account P_0. This result is meaningful since we do not lose switching on any BS in order to decrease the total power. The NBS algorithm also produces near optimal results in many examples. Generally, the fast-moving users are considered to be served by macro BSs [11]. Basically, in the green networking approach, the switching off operation is not performed for macro cells In this context, *it is reasonable, the fast-moving users assumed to be served by macro BSs to use the NBS algorithm* for choosing the BS to receive the broadcast data.

Table 4.3 compares the SC algorithm with all developed algorithms introduced in this work in case of the operational power cost $P_0 = 12\ W$. The deployment is considered to be in the small-cell context. The CC algorithm produces optimal assignments nearly all examples as well as the DCC algorithm is very efficient for the considered examples. However, the DCC algorithm naturally performs worse when the number of BSs increases (we mention detailed in the sequel about that result). Moreover, the NBS algorithm does not give any optimal result in the comparison. This is due to the fact that the operational power cost which is relatively higher than the transmission power privilege large cells. On the other hand, the HD algorithm is also efficient when the parameter θ is calibrated properly. In many scenarios, it gives optimal or near-optimal results. In the sequel, we explain the advantages of the HD algorithm compared to the others.

Figure 4.3 plots the average total power $\bar{p}$ with respect to intensity of BSs λ_b when increasing the intensity of mobiles

$$\lambda_m = (0.016, 0.025, 0.044, 0.1) \times 10^{-3} \frac{points}{m^2}$$

in an area $A = 4\ km^2$. The figure implies that $\bar{p}$ increases exponentially when the intensity of mobiles goes up in case of $P_0 = 12\ W$. Although the average total power decreases while the intensity of BSs increases in case of $P_0 = 0$, the effect of operational power costs arises dramatically. The reason is that the cell size diminishes when increasing the intensity of BSs, resulting in more switched on BSs, and consequently, too high average total costs emerge.

Looking into Figure 4.4, the advantage of the CC algorithm is obvious. It copes very efficiently with increasing intensity of BSs, where the average total power decreases significantly. We have this performance since the CC algorithm privileges larger cells which provide less switched on BSs.

Furthermore, in Figure 4.5, we depict the performance of the DCC algorithm under the same conditions. It is an expected result that the average total power increases when the intensity of BSs is getting high. Because, while each mobile develops its own power cost matrix, the probability of activation of a cell increases when intensifying the number of BSs. Thus, the mobiles might choose different BSs resulting in smaller cells. On the other hand, comparing the DCC algorithm with NBS algorithm, we observe from Table 4.4, the advantage of the DCC algorithm. The ratio of the average total power of the NBS and the DCC is given in Table 4.4. When the intensity of BSs increases to $\lambda_b = 10 \times 10^{-5}\ \frac{points}{m^2}$, the average total power of the NBS is 7.83 times higher than the average total power of the DCC.

4.6.1 *Comparison of Proposed Algorithms with respect to the Intensity of Mobiles*

We adopt the greedy set cover algorithm (greedy-SC) which is developed in [42] for comparison with proposed algorithms.

Figure 4.6 plots the change of the average total power with respect to the intensity of mobiles for small cells scenario. The assumptions are as following: $\lambda_b = 1.11 \times 10^{-5} \frac{points}{m^2}$, $\theta = 0.002$ (in Figure 4.8, the optimal θ is found), and area $A = 6.25 km^2$. Note that the HD algorithm performs efficiently even though it is decentralized. For example, in case of $\lambda_m = 8 \times 10^{-8}$, the average number of mobiles is given by $8 \times 10^{-8} \cdot 6.25 \times 10^6 = 50$; thus, the average power used per mobile is calculated as following: a) the HD algorithm: $63/50 = 1.26W$, b) the CC algorithm: $55/50 = 1.1W$, c) the greedy-SC algorithm: $50/50 = 1W$, d) the SC algorithm: $43.83/50 = 0.88W$.

Figure 4.9 depicts the change of the average total power with respect to the intensity of mobiles for macro cell deployment. $\lambda_b = \frac{80points}{3600km^2}$, $A = 3600km^2$ ($60km \times 60km$ area), and $\theta = 0.21$ (in Figure 4.9, we plot the change of average total power with respect to θ, and choose the optimal value). Here, we observe that the HD algorithm produces remarkable results. Calibrating θ properly is significant, otherwise the HD algorithm may not converge to the near optimal results. On the other hand, the NBS algorithm is also efficient in the macro cell deployment. The drawback of greedy-SC algorithm reveals here since it works with a mechanism where the larger cells are privileged.

In Figures 4.8 and 4.9, the normalized average total power is plotted with respect to θ. From the figures and our observations in experiments performed in MATLAB, it might be considered that θ is mainly affected by the area over which the algorithm is run. For example, in Figure 4.9, the normalized average total power has a minimum in the same value of intensity of BSs, but it moves to a higher value when the area is enlarged from $2500km^2$ to $3600km^2$.

Figure 4.10 shows the change of the average number of rounds (see Section 2.2) of the HD algorithm for converging to a Nash equilibrium with respect to the area. The figure implies that the average number of rounds has a logarithmic characteristic. Moreover, when the operational power costs are zero, the average number of rounds increases since smaller cells are formed; therefore, the HD algorithm needs more rounds to converge to a Nash equilibrium.

4.7 CONCLUSION

We considered the MAP in broadcast transmission in the “green” context. We proposed a centralized optimal recursive algorithm (the HM) as well as a centralized polynomial-time heuristic algorithm (the CC). Further, we developed a distributed approach to the CC algorithm (the DCC), and another distributed one called the NBS algorithm. We also introduced a new algorithm based on group formation games, which we call as the hedonic decision (HD) algorithm. This formalism is constructive: a new class of group formation games is introduced where the utility of players within a group is additively separable and symmetric being a concept in hedonic coalition formation games. Simulation results were used to verify the performance of the algorithms. We realized that the HD algorithm produces near-optimal solutions.

Table 4.1: $A = 2500km^2$, $\lambda_b = \frac{6points}{2500km^2}$, $\lambda_m = \frac{1point}{25km^2}$, $P_0 = 0\ W$, $\theta = 0.11$.

Example i	m	n	HM	SC	CC	DCC	NBS	HD
1	54	6	294.0360	294.0360	294.0360	295.5705	294.0360	294.0360
2	43	6	299.8159	299.8159	323.6194	352.1657	299.8159	299.8159
3	46	6	250.4830	250.4830	271.4828	271.4828	250.4830	250.4830
4	41	6	270.4417	270.4417	302.8145	287.9684	284.5738	283.0740
5	51	6	307.7226	307.7226	361.7673	361.7673	320.2662	307.7226
6	45	6	278.5206	278.5206	317.5086	317.5086	278.5206	278.5206
7	41	6	305.1243	305.1243	345.3096	345.3096	306.8924	312.2107
8	34	6	221.5681	221.5681	236.7168	236.7168	256.0736	221.5681
9	58	6	360.3562	360.3562	363.1885	363.1885	360.3562	360.3562
10	52	6	310.0721	310.0721	310.0721	310.0721	332.0735	332.0735
11	44	6	283.9244	283.9244	313.8116	339.6551	283.9244	283.9244
12	49	6	312.9718	312.9718	312.9718	312.9718	325.5796	337.9936
13	53	6	229.9336	229.9336	255.3125	255.3125	229.9336	232.6530
14	60	6	308.6002	308.6002	308.6002	308.6002	320.2059	320.2059
15	57	6	329.2667	329.2667	342.8460	354.0650	329.2667	335.9528

Table 4.2: $A = 4\ km^2$, $\lambda_b = \frac{6points}{4km^2}$, $\lambda_m = \frac{18points}{4km^2}$, $P_0 = 12\ W$, $\theta = 0.003$.

Example i	m	n	HM	SC	CC	DCC	NBS	HD
1	14	4	24.153	24.153	24.153	24.153	48.183	24.153
2	14	7	24.140	24.140	36.226	36.226	72.139	24.152
3	23	7	36.138	36.138	48.177	48.275	84.042	48.185
4	16	7	36.195	36.195	36.195	36.257	72.208	48.204
5	19	4	24.109	24.109	24.109	36.174	48.126	36.184
6	7	3	12.070	12.070	12.070	12.070	24.073	12.070
7	15	8	36.178	36.178	36.178	36.222	84.241	36.222
8	15	7	24.110	24.110	24.110	24.144	72.101	24.154
9	18	9	36.099	36.099	36.148	36.253	84.042	36.148
10	24	6	36.111	36.111	36.191	36.191	60.134	36.191
11	21	3	24.149	24.149	24.149	24.149	36.162	24.149
12	18	10	36.111	36.111	36.111	48.141	84.053	48.152
13	23	5	36.105	36.105	36.105	48.143	60.218	36.105
14	13	4	24.082	24.082	24.104	24.104	48.123	24.110
15	17	8	36.190	36.190	36.190	36.217	60.217	36.190

Table 4.3: $\lambda_b = 0.10 \times 10^{-3} \frac{points}{m^2}$, $\lambda_m = 1.11 \times 10^{-3} \frac{points}{m^2}$, $P_0 = 12\ W$, $\theta = 0.008$.

	Number of rounds			
Example	$A = 0.98km^2$	$A = 1.28km^2$	$A = 1.62km^2$	$A = 2.00km^2$
1	3	2	3	3
2	3	3	3	3
3	3	3	2	3
4	2	3	3	3
5	3	2	3	3
6	3	3	3	3
7	3	3	3	4
8	3	3	3	4
9	3	3	3	3
10	3	3	3	4

Table 4.4: $A = 4\ km^2$, $\lambda_m = 0.16 \times 10^{-4} \frac{points}{m^2}$, $P_0 = 12\ W$.

λ_b	1.6×10^{-5}	2.5×10^{-5}	4.4×10^{-5}	10×10^{-5}
$\frac{\bar{P}_{NBS}}{\bar{P}_{DCC}}$	5.91	6.54	7.15	7.83

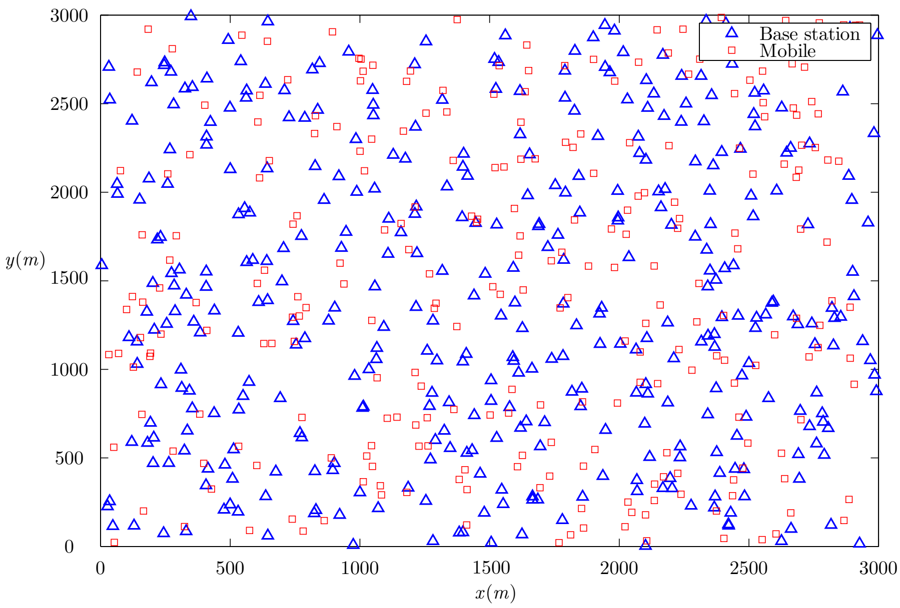


Figure 4.2: Distribution of BSs and mobiles in Euclidean plane. $\lambda_b = 44.4 \times 10^{-5} \frac{points}{m^2}$, $\lambda_m = 2.5 \times 10^{-5} \frac{points}{m^2}$.

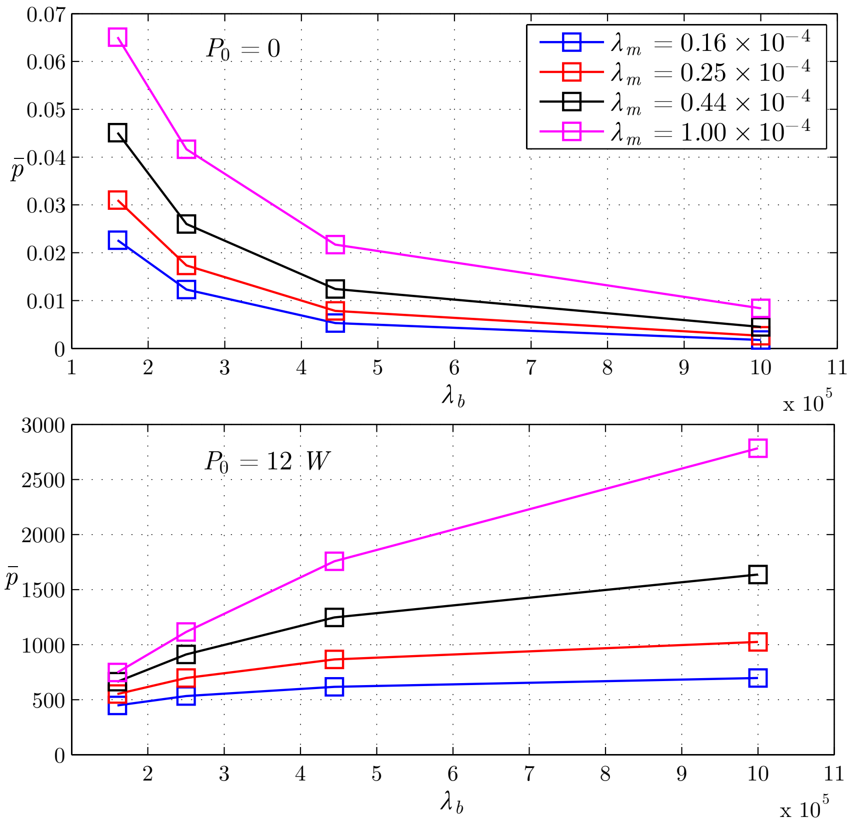


Figure 4.3: The NBS Algorithm: Change of the average total power $\bar{p}$ with respect to intensity of BSs λ_b for increasing values of intensity of mobiles $\lambda_m = (0.16, 0.25, 0.44, 1.00) \times 10^{-4}\ \frac{points}{m^2}$, $A = 4\ km^2$.

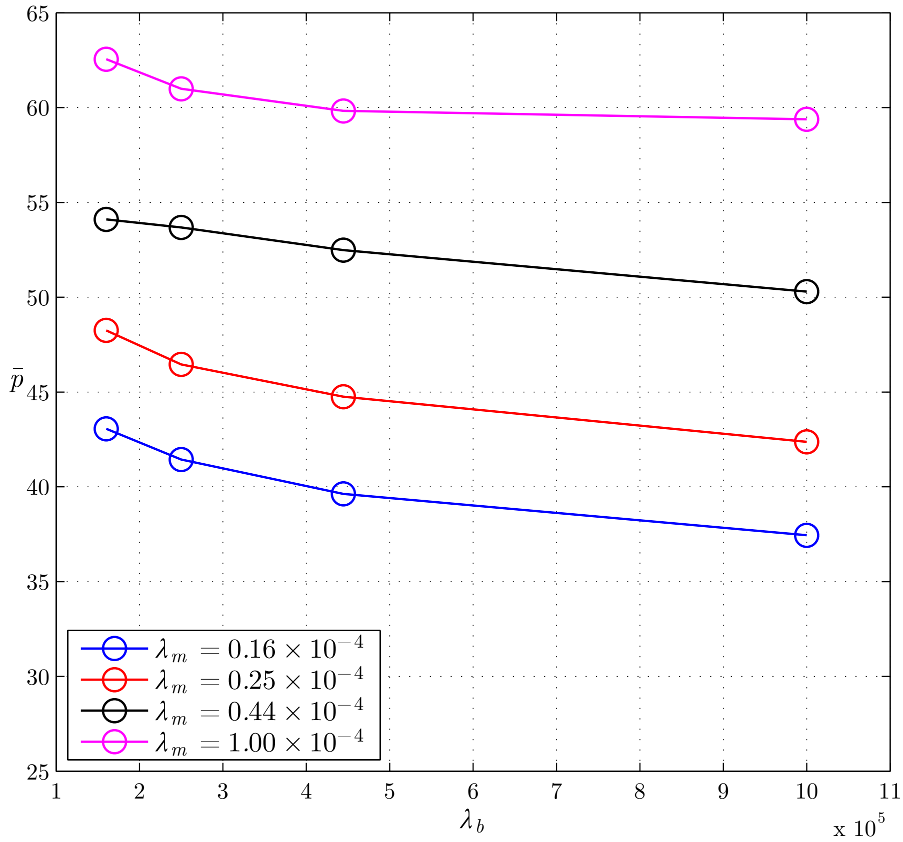


Figure 4.4: The CC Algorithm: Change of the average total power $\bar{p}$ with respect to intensity of BSs λ_b for increasing values of intensity of mobiles $\lambda_m = (0.16, 0.25, 0.44, 1.00) \times 10^{-4}\ \frac{points}{m^2}$, $A = 4\ km^2$, $P_0 = 12\ W$.

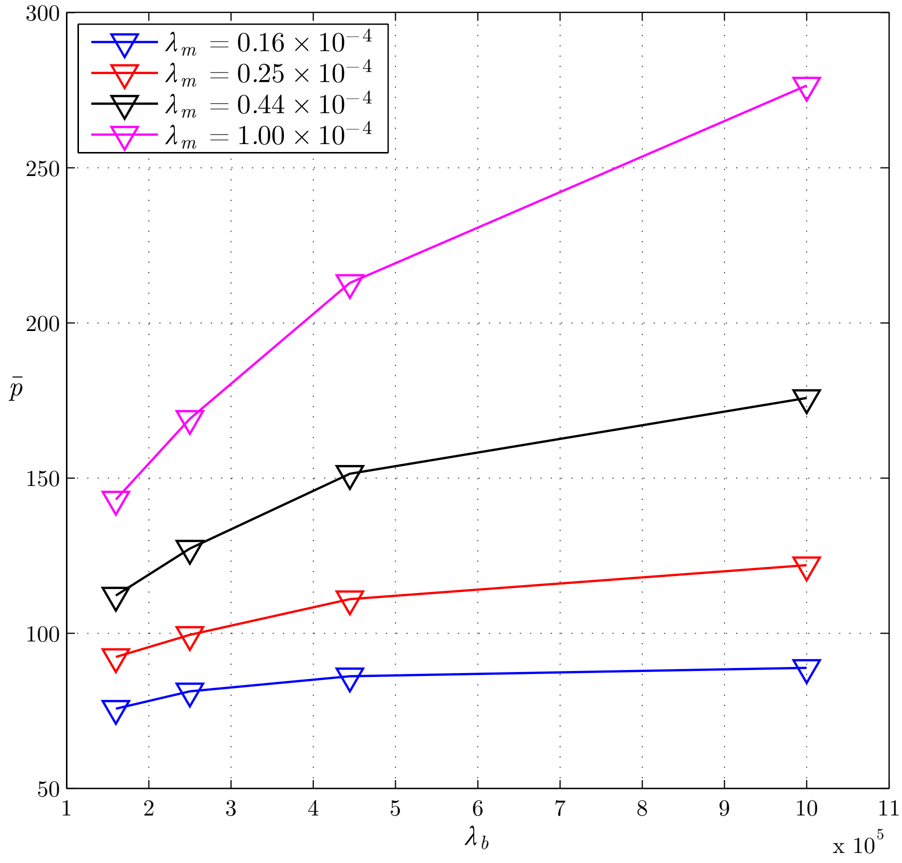


Figure 4.5: The DCC Algorithm: Change of the average total power $\bar{p}$ with respect to intensity of BSs λ_b for increasing values of intensity of mobiles $\lambda_m = (0.16, 0.25, 0.44, 1.00) \times 10^{-4}\ \frac{points}{m^2}$, $A = 4\ km^2$, $P_0 = 12\ W$.

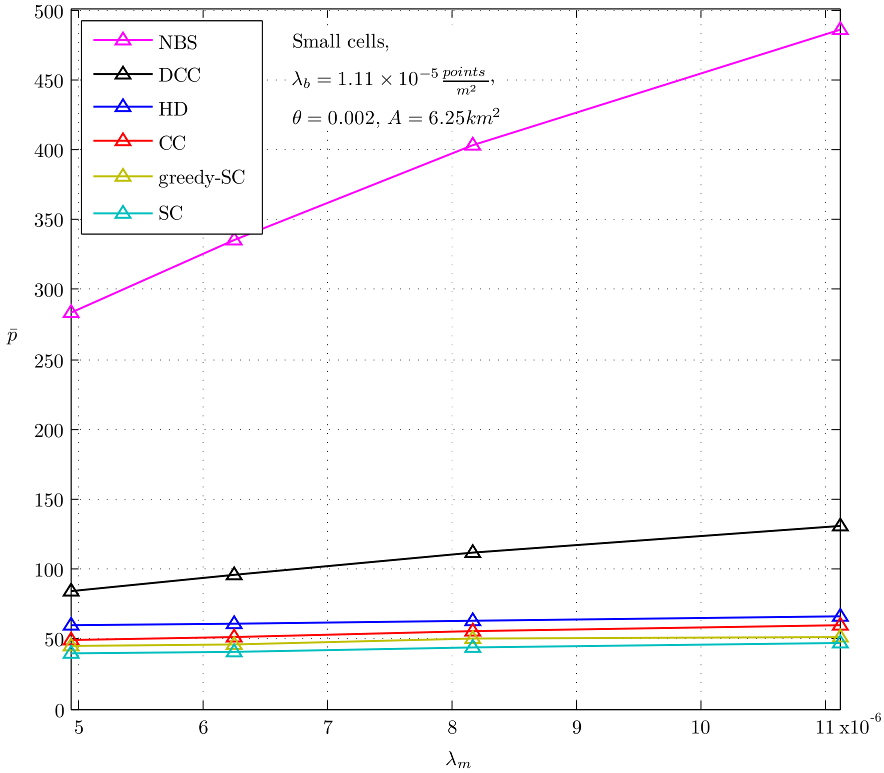


Figure 4.6: Small cells: Change of the average total power $\bar{p}$ with respect to the intensity of mobiles.

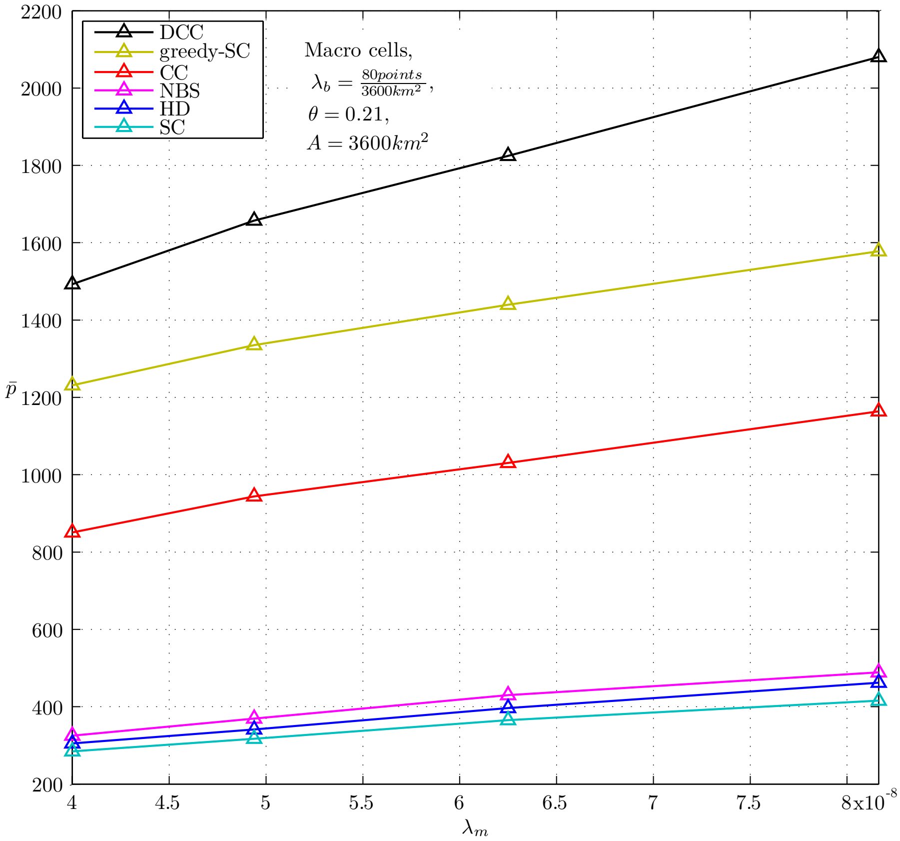


Figure 4.7: Macro cells: Change of the average total power $\bar{p}$ with respect to the intensity of mobiles.

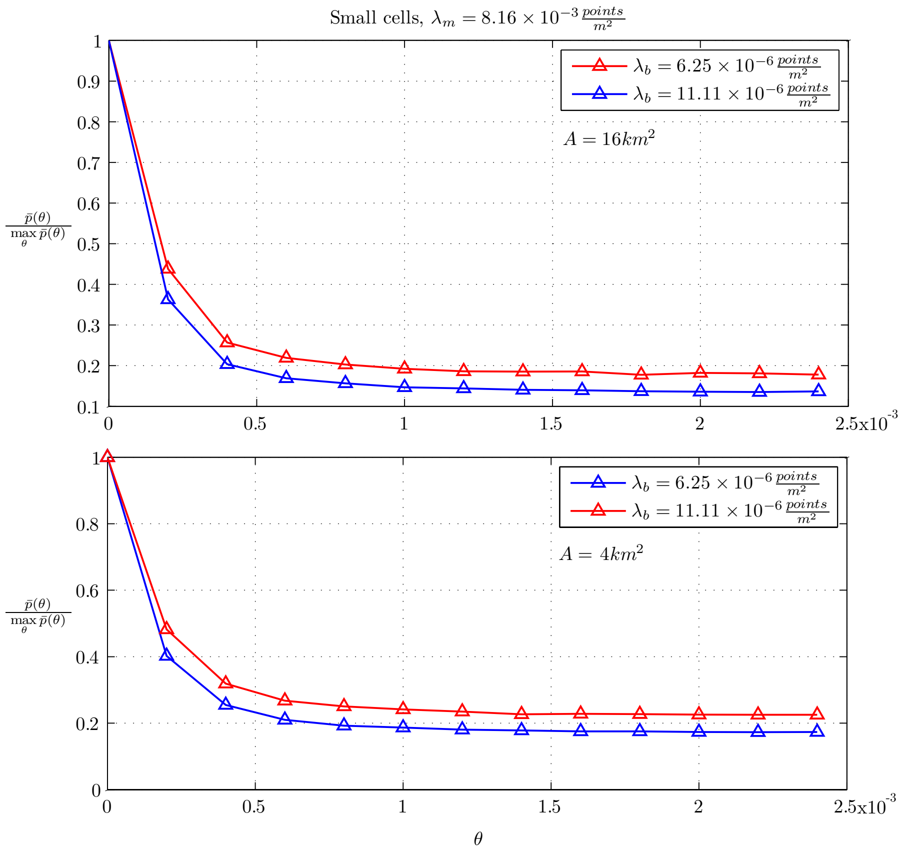


Figure 4.8: Small cells: Change of the normalized average total power with respect to θ.

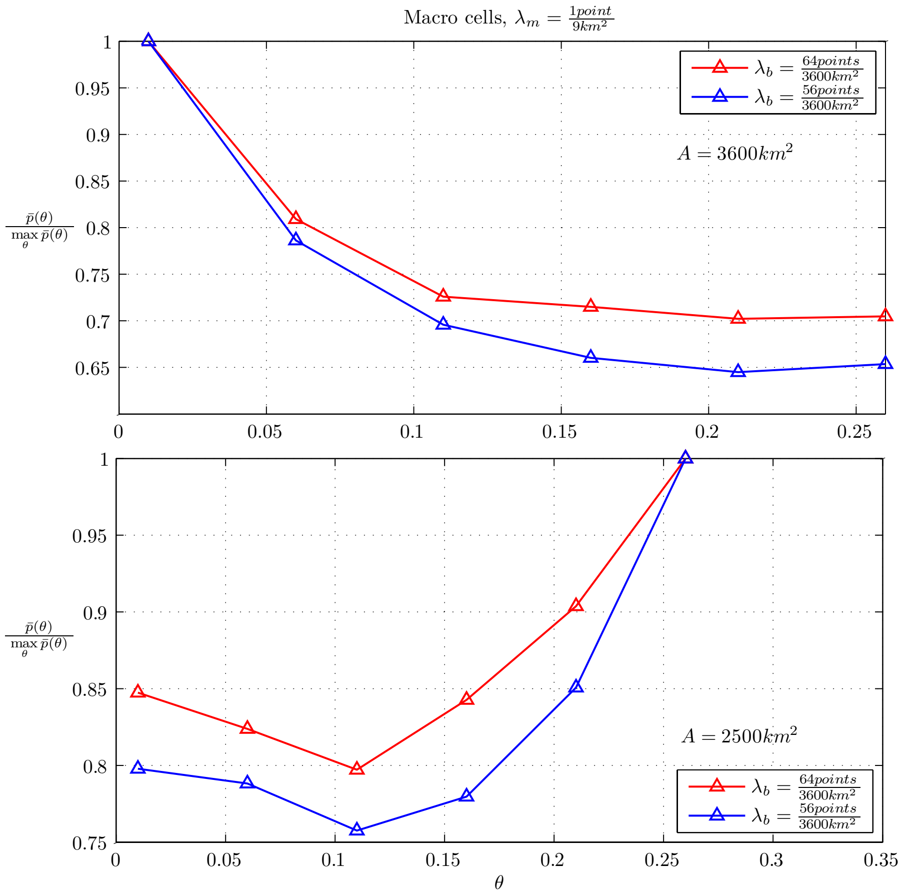


Figure 4.9: Macro cells: Change of the normalized average total power with respect to θ.

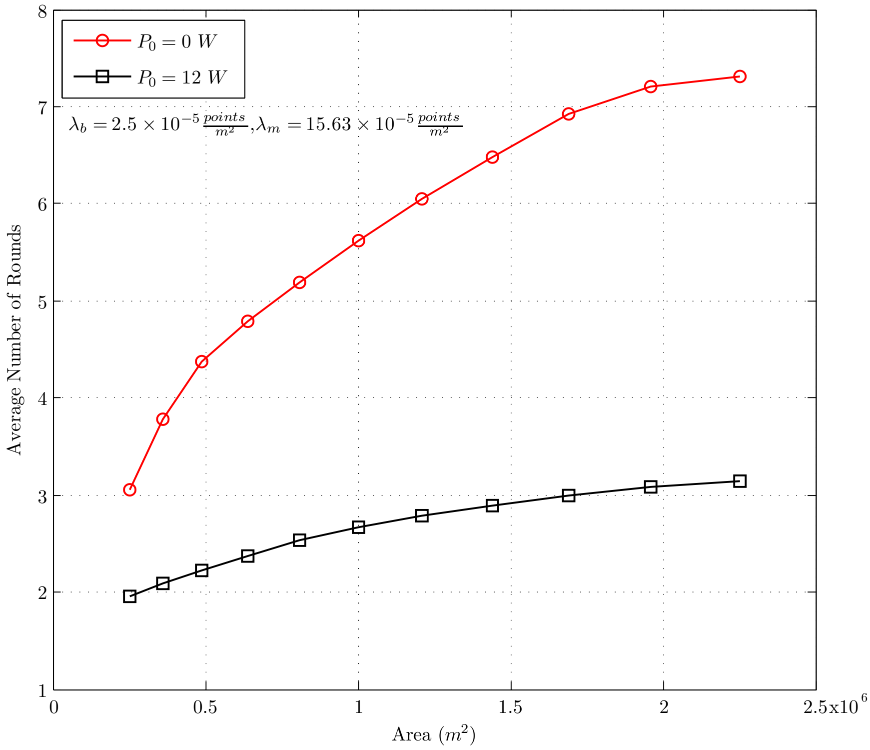


Figure 4.10: Change of the average number of rounds with respect to the area.

5

THE MOBILE ASSIGNMENT PROBLEM IN BROADCAST TRANSMISSION: COALITIONAL GAME ASPECTS

5.1 INTRODUCTION

We study the combined problem of (i) deciding what subset of the mobiles would be assigned to each BS, and then (ii) sharing the BSs' cost of multicast among the mobiles. The subset that we wish to assign to a given BS is said to be its target set of mobiles. A cost sharing rule consists of a pricing policy that determines the share that each mobile within the target set would pay. We are interested in the sharing policies that are stable in the sense that no subset of the M mobiles could pay strictly less than their cost share by forming a new separate multicast group.

This work builds on [45] who studied the case of a single BS. They studied (i) the cost sharing problem as well as (ii) the combined association and cost sharing problem. In the latter, each mobile was able to decide whether to join a dedicated unicast channel or to join the multicast session, in which case it was a part of the coalition game at the BS. The analysis strongly depended on the submodularity property which held in the case of a single BS. We here prove that submodularity doesn't hold in the case of multiple BSs.

5.1.1 *Our Contribution*

The starting point here has been our attempt to extend the submodularity property to the case of two BSs. Instead, however, we provide a counter example that shows that indeed already in the case of two BSs, submodularity does not hold.

We appreciate it as a coalitional game played by mobiles and prove that this game has an incentive to form grand coalition where all players join to the game. Furthermore, using Bondareva-Shapley theorem [62], we show that this coalition game has a non-empty core which means that grand coalition is stable. Then, we examine the cost allocation policy for different methods such as egalitarian allocation, proportional repartition of total cost, the Shapley value [29] and the nucleolus [65].

5.2 THE COALITIONAL GAME

The *players* are the mobiles, and they are subject to optimal assignments executed by the cellular network. Consider a subset $S \subseteq M$ representing a coalition. The total cost arising due to S is given by

$$C(S) = f\left(\underbrace{\min_{\mathbf{S}\in\mathscr{A}_S} \sum_{i\in S} \max_{j\in N} \mathbf{S} \otimes \mathbf{P}_S}_{\text{optimal power cost } p_S}\right), \tag{5.1}$$

where $\mathscr{A}_S$ is the all possible assignment matrices, $\mathbf{S}$ is the assignment matrix, and $\mathbf{P}_S$ is the power cost matrix due to the coalition S. The cost $f(p_S)$ can be some amount of money for subscription a multicast/broadcast service, which is a function of the optimal power cost p_S arising due to coalition S.

5.2.1 *Properties of the Coalitional Game*

In the sequel, we order the properties of the considered coalitional game.

Lemma 5.2.1 *Monotonically Increasing: Let $A \subseteq B \subseteq M$. Then $C(A) \leq C(B)$.*

Lemma 5.2.2 *Subadditivity: Let $A, B \subseteq M$, and $A \cap B = \emptyset$. This property means that*

$$C(A) + C(B) \geq C(A \cup B). \tag{5.2}$$

Grand coalition meaning that each player demands to join the coalition, is guaranteed due to subadditivity. This arises from the fact that all players have incentive to minimize the cost.

Lemma 5.2.3 *The problem is not submodular.*

Proof 5.2.1 *Let $A, B \subseteq M$. Submodularity is defined by the following property*

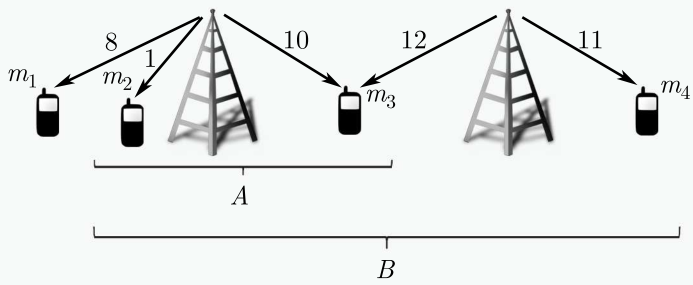
$$C(A) + C(B) \geq C(A \cup B) + C(A \cap B). \tag{5.3}$$

Submodularity is known to possess some important properties in coalitional game theory. If a coalitional game owns the submodularity property, it is called as convex game. Convex games have many nice properties such as the core of a submodular game is nonempty, it is a unique von Neumann-Morgenstern solution, and the Shapley value is the barycenter of the core [69].

The following equivalent characterization of submodularity can be used to prove Lemma 5.2.3 [66]

$$C(A \cup i) - C(A) \geq C(B \cup i) - C(B), \forall A \subseteq B \subseteq M \setminus i, \tag{5.4}$$

for all $i \in N$. In many cases it is easy to prove that submodularity holds. However, different counter examples can be obtained, especially when a new mobile arriving induces an association change for another mobile, as illustrated in the counter example in Figure 5.1 (m_3 is not associated with the same BS when considering subset A or B).



Counter example where the submodularity does not hold

m_3 is not associated with the same BS when considering subset A or B

Figure 5.1: A counter example scenario of submodularity.

$$C(A) = 10,$$
$$C(A \cup i) = 10,$$
$$C(B) = 12 + 1 = 13,$$
$$C(B \cup i) = 12 + 8 = 20$$

These values do not provide submodularity,

$$C(A \cup i) - C(A) < C(B \cup i) - C(B) \tag{5.5}$$

Lemma 5.2.4 *This game is totally balanced. Therefore, its core is* non-empty. *The core is defined in the sequel.*

Proof 5.2.2 *According to the Bondareva-Shapley theorem if a coalitional game is balanced, it has a non-empty core. Balancedness is satisfied if and only if the inequality [62]*

$$C(M) \geq \sum_{S \subseteq M} \lambda_S C(S) \tag{5.6}$$

holds for all $S \subseteq M$ where balanced weights $(\lambda_S)_{S \subseteq M} \geq 0$ and $\sum_{S \subseteq M: i \in S} \lambda_S = 1$, $\forall i \in M$. Let us consider the following linear program

$$\max \sum_{S \subseteq M} \lambda_S C(S) \quad \textit{subject to}$$
$$(\lambda_S)_{S \subseteq M} \geq 0 \textit{ and } \sum_{S \subseteq M: i \in M} \lambda_S = 1, \forall i \in M \tag{5.7}$$

which can be written in matrix form as following:

$$\max \boldsymbol{\lambda} \mathbf{C} \quad \textit{subject to}$$
$$\boldsymbol{\lambda} \mathbf{A} = 1, \quad \boldsymbol{\lambda} \geq 0. \tag{5.8}$$

Clearly, if the solution of this problem is $C(M)$, then we are able to conclude that there does not exist balanced weights $(\lambda_S)_{S\subseteq M}$ that does not satisfy the balancedness conditions.

For solution, we examine the dual program of this problem given by

$$\begin{aligned} &\min \boldsymbol{\nu} \quad \textit{subject to} \\ &\mathbf{A}\boldsymbol{\nu} \geq \mathbf{C}, \quad \boldsymbol{\nu} \textit{ unbounded}. \end{aligned} \tag{5.9}$$

where and $boldsymbol \nu$ represents the dual variables. Thanks to the subadditivity property of the cost function C, the result of the dual program is $C(M)$. All possible combinations in $\mathbf{A}\boldsymbol{\nu}$ will surpass $C(M)$.

Furthermore, the association game is totally balanced. By definition, if each subgame of a coalitional game is balanced, it is called as totally balanced game [66]. Since each subgame of considered problem is an association game, we can conclude that the association game of mobiles is totally balanced.

5.3 COST ALLOCATION METHODS

In this section, the solution concepts of this coalitional game are studied. Minimized total cost is distributed to the players using some methods as described in the following.

We represent as $\phi \in \mathbb{R}^M$ the cost allocation method where p_i is the cost of player i. The question here is: What is the most desirable allocation method for distribution of total cost? Cost allocation methods should own some properties in order to make convinced the players. We order them as following. However, there is no method which possesses all these properties [67]:

- ϕ is said to be an *efficient* cost allocation method if $\sum_{i\in M} \phi_i = C(M)$.
- ϕ is said to be an *individually rational* cost allocation method if $\phi_i \leq C(i)$.
- ϕ is said to be a *stable* cost allocation method if it lies in the core. Provided that the core is non-empty, *no coalition has incentive to leave the grand coalition* and receive a smaller cost.
- ϕ is said to have the *dummy player* property if $C(S \cup i) - C(S) = C(i)$ then $\phi_i = C(i)$ for all $i \in M$ and $S \subset M \setminus i$.
- ϕ is said to have the *symmetry* property if $C(S \cup i) = C(S \cup j)$ then $\phi_i = \phi_j$ for all $S \subset M \setminus i \cap j$.
- ϕ is said to have the *additivity* property if $\phi(C_1 + C_2) = \phi(C_1) + \phi(C_2)$ where C_1 and C_2 are cost functions.

5.3.1 *Egalitarian Allocation*

The simplest distribution of the total cost is egalitarian allocation. It divides equally the total cost to the players [67], i.e.

$$\phi_i = \frac{C(M)}{M}. \tag{5.10}$$

This method is not individually rational and does not possess the dummy, symmetry player, and additivity property. It does not lie in the core, as well [67].

5.3.2 *Proportional Repartition of Total Cost*

A fairer allocation is proportional repartition of total cost [68]:

$$\phi_i = \frac{C(i)}{\sum_{j\in M} C(j)} C(M). \tag{5.11}$$

However, it is not stable as well as does not satisfy the dummy, symmetry player, and additivity property.

5.3.3 *The Shapley Value*

Shapley [29] has proved that there exists one and only one allocation that satisfies all the properties except stability, which is given by

$$\phi_i = \sum_{S\subseteq M\setminus i} \frac{|S|!\,(M-|S|-1)!}{M!} \left(C(S\cup i) - C(S)\right). \tag{5.12}$$

This captures the "average marginal contribution" of player i, averaging over all the different sequences according to which the grand coalition could be built up from the empty coalition.

5.3.4 *The Nucleolus*

Define the excess

$$e(\phi, S) = C(S) - \sum_{i\in S} \phi_i \tag{5.13}$$

which measures the "happiness degree" of each coalition S. The nucleolus is the imputation that lexicographically maximizes the minimal excess. In other words, the nucleolus minimizes maximum unhappiness. The solution of the following linear program gives the nucleolus [68]

$$\max \delta \quad \text{subject to} \tag{5.14}$$

$$\sum_{i\in S} \phi_i + \delta \le C(S), \forall S \subset M \setminus \emptyset \tag{5.15}$$

$$\sum_{i\in M} p_i = C(M), \forall i, \phi_i \ge 0. \tag{5.16}$$

If the core is non-empty, the nucleolus is in the core.

Table 5.1: Allocation of costs for different methods

	EA	PR	SV	N
ϕ_1	5.00	5.33	6.25	8.00
ϕ_2	5.00	0.67	0.42	0.00
ϕ_3	5.00	6.67	4.08	2.00
ϕ_4	5.00	7.33	9.25	10.00

5.4 POSSIBLE SCENARIO

We calculate the allocations for the scenario considered in Figure 5.1 with respect to explained allocation methods. First, let us obtain the core of this scenario. It can be found by the following linear program

$$\begin{aligned}
&\max(\phi_1 + \phi_2 + \phi_3 + \phi_4) \quad \text{subject to} \\
&\phi_1 \leq 8, \phi_2 \leq 1, \phi_3 \leq 10, \phi_4 \leq 11, \\
&\phi_1 + \phi_2 \leq 8, \phi_1 + \phi_3 \leq 10, \phi_1 + \phi_4 \leq 19, \\
&\phi_2 + \phi_3 \leq 10, \phi_2 + \phi_4 \leq 12, \phi_3 + \phi_4 \leq 12, \\
&\phi_1 + \phi_2 + \phi_3 \leq 10, \phi_1 + \phi_2 + \phi_4 \leq 19, \\
&\phi_2 + \phi_3 + \phi_4 \leq 13, \phi_1 + \phi_3 + \phi_4 \leq 20, \\
&\phi_1 + \phi_2 + \phi_3 + \phi_4 \leq 20.
\end{aligned}$$

A solution of this linear program is the cost allocation vector given by $(\phi_1, \phi_2, \phi_3, \phi_4) = (8, 0, 2, 10)$ which lies in the core. The nucleolus (N) in Table 5.1 overlaps with this solution from the core. This shows the stability property of the nucleolus which means that *minimization of total cost for broadcast transmission* is provided. But it does not satisfy dummy player, symmetry, and additivity property. As for the Shapley value (SV), it fails in stability. For example, $\phi_1+\phi_3 = 6.25+4.08 = 10.33 > 10$. Since considered scenario is not submodular that we proved above, the Shapley value does not lie in the core. However, in submodular case, the game becomes convex and the Shapley value is the center of gravity of convex game's core [69]. Egalitarian allocation (EA) divides equally the total cost which is the least complex algorithm compared to the others. However, it is not fair and stable. Proportional repartition of total cost (PR) is fairer than egalitarian allocation, but it fails in stability, dummy player, symmetry, and additivity properties.

In summary, if the nucleolus is used as a cost allocation algorithm, the players maintain the grand coalition. Therefore, the nucleolus satisfies the objective of an optimal total cost. In the case of the other methods, players have incentive to break the grand coalition and look for other smaller coalitions which could result in non-optimal results.

5.5 CONCLUSION

We considered the association problem of mobiles using a coalitional game approach. It is an optimal assignment problem between BSs and mobiles in order to minimize the total cost which is determined by the transmission power. We proved that the players of the game form grand coalition and the core of this game is non-empty. Moreover, we also studied the game for some cost sharing methods and showed that in case of the nucleolus the grand coalition is stable, and it minimizes the total cost in broadcast transmission.

6

SWITCHING OFF BASE STATIONS: DOWNLINK CONSIDERATIONS

6.1 INTRODUCTION

Energy consumption can be reduced by dynamically switching off/on cells, base stations (BSs) and other radio resources (e.g. transmit antennas), according to observed traffic load, resource utilization, quality and coverage.

We consider downlink transmission in cellular networks where we target to reduce the energy consumption by switching off some BSs by such a way that the distribution of SINR remains unchanged. We assume full frequency reuse. Each mobile is associated with the BS being nearest to it. All BSs being out the nearest one cause interference to the mobile. The question that we ask is "How many BSs can be switched off in order that the distribution of the SINR remains unchanged?". We model the problem as a homogenous independently marked Poisson point process.

We analyze for line and plane cases, the gain in power consumption obtained after switching off BSs. It turns out from calculations that the more the operational cost the less the gain in power consumption, and similarly, the higher the dimension (distribution of BSs in line and plane means one and two dimensions, respectively) the less the gain in power consumption.

6.2 THE MODEL

We assume full frequency reuse. Each mobile is associated with the BS being nearest to it. All BSs being out the nearest one cause interference to the mobile.

We consider a homogenous independently marked Poisson p.p. of BSs represented by $\tilde{\Phi} = \sum_i \delta_{(X_i, M_i)}$ where X_i shows the location of BS i, and M_i denotes the mark corresponding to the BS i. Indeed, a mark shows the energy profile of related SP. Consider a tagged mobile at an arbitrary point on the Euclidean plane, say the origin (Figure 6.1).

Let p_0 denote the point in $\tilde{\Phi}$ which is the closest to it, and represents the BS to which it is connected. Let d_i be the distance of p_i to the origin. Moreover, suppose that average transmission power of a typical BS is P_t. This power should be understood as resulting from a long-term observation the SP performs. We consider an attenuation due to a path-loss with exponent α as well as the effect of fading denoted by the random variable h. The transmission power received at the tagged mobile from p_0 is thus given by $P_t h_0 d_0^{-\alpha}$. Thus,

Example deployment as Poisson point process

The deployment of base stations of a service provider is assumed to follow a homogenous Poisson point process

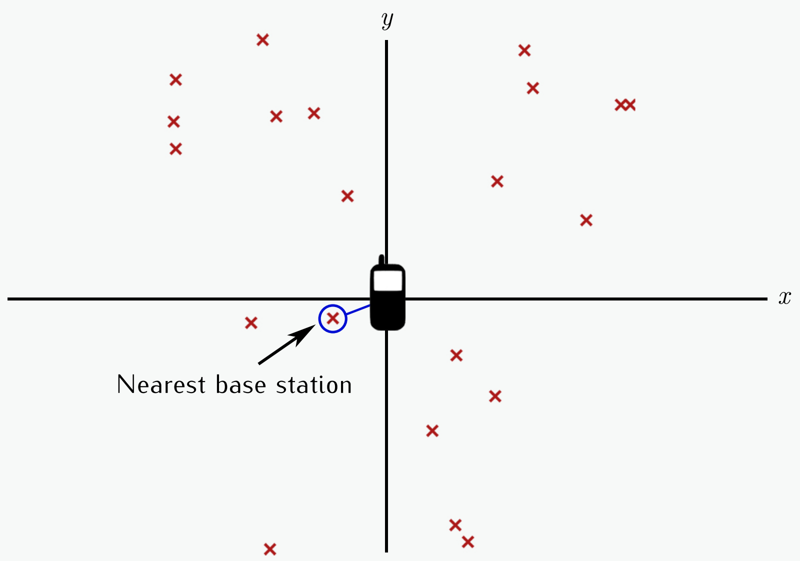


Figure 6.1: Example deployment.

the total average interference from other BSs is $P_t \sum_{i \in \Phi \setminus p_0} h_i d_i^{-\alpha}$. Hence, the SINR at the mobile is [12]

$$\text{SINR} = \frac{P_t h_0 d_0^{-\alpha}}{P_t \sum_{i \in \Phi \setminus p_0} h_i d_i^{-\alpha} + \sigma^2} = \frac{P_t h r^{-\alpha}}{I + \sigma^2}, \tag{6.1}$$

where σ^2 stands for additive noise variance.

6.2.1 *Base Station Energy Profile*

The energy consumption model is mandatory to predict the power consumption of a typical BS as a function of the traffic load. In this work, we adopt a linear energy profile model for a typical BS formulated as

$$P = P_0 + \beta P_{tot}, \tag{6.2}$$

where P_0 denotes the power consumption for operational tasks, β is the slope of the traffic-dependant part, and P_{tot} is the total transmitted power by the corresponding BS. Further, we assume that the average total transmission power is a function of the traffic intensity, given by

$$P_{tot}(T) = p + wf\left(\frac{T}{\lambda}\right), \tag{6.3}$$

where p is the minimum average total transmission power, e.g. signaling overhead in common pilot or control signal, w represents the power used per throughput (W/bps) as well as T is the traffic intensity (bps/m^2) and λ is the intensity of BSs ($points/m^2$) of the corresponding SP. In this work, we suppose that p and w are equal of each SP. Note that T/λ is the traffic per BS ($bps/point$). The function $f(T/\lambda)$ represents the power consumption as a function of the traffic.

Actually, different energy profiles could be proposed [71, 15, 16]. Here, since we focus to work on the coalitional game, we consider without loss of generality a simple linear model.

6.2.2 *SINR Distribution*

By considering only the SINR, it means that we do not take into account any power control at the transmission. Here, the transmission power is constant for all mobiles.

First, we give the definition of coverage probability [13]:

$$p_C = \mathbf{P}\{\mathsf{SINR} > \rho\}, \tag{6.4}$$

where ρ is the target SINR that ensures the coverage. The distribution of the SINR is thus the complementary probability of the coverage probability, i.e. $p_{\mathsf{SINR}} = 1 - p_C$.

The distance between the origin and the nearest BS has the following probability density function [13]:

$$f(r) = e^{-\lambda\pi r^2} 2\pi\lambda r. \tag{6.5}$$

Conditioning on the nearest BS being at a distance r from the mobile, the probability of coverage is

$$p_C(\lambda) = \int_0^\infty \mathbf{P}\left\{ h > \frac{\rho(\sigma^2 + I)}{P_t r^{-\alpha}} \middle| r \right\} \exp(-\pi\lambda r^2) 2\pi\lambda r dr. \tag{6.6}$$

If we consider the case of Rayleigh fading, the random variable $h \sim \exp(\mu)$ follows an exponential distribution with mean $1/\mu$, and therefore [13]

$$p_C(\lambda) = \int_0^\infty \exp\left(-\frac{\mu\rho\sigma^2}{P_t r^{-\alpha}}\right) \mathscr{L}_I\left(\frac{\mu\rho}{P_t r^{-\alpha}}\right) \exp(-\pi\lambda r^2) 2\pi\lambda r dr, \tag{6.7}$$

where $\mathscr{L}_I(s)$ is the Laplace transform of random variable I given as

$$\begin{aligned}
\mathscr{L}_I(s) &\triangleq \mathbf{E}\{\exp(-sI)\} = \mathbf{E}_{\Phi,h_i}\left\{ \exp\left(-sP_t \sum_{i\in\Phi\backslash p_0} h_i d_i^{-\alpha} \right) \right\} \\
&= \mathbf{E}_\Phi \left\{ \prod_{i\in\Phi\backslash p_0} \mathbf{E}_h \left\{ \exp\left(-sP_t h d_i^{-\alpha}\right) \right\} \right\} \\
&= \exp\left(-\int_r^\infty \left(1 - \mathbf{E}_h\left\{\exp\left(-sP_t h x^{-\alpha}\right)\right\}\right) 2\pi\lambda dx \right) \\
&= \exp\left(-2\pi\lambda \int_r^\infty \frac{x}{1 + \frac{\mu}{sP_t x^{-\alpha}}} dx \right)
\end{aligned} \tag{6.8}$$

where $s = \frac{\mu\rho}{P_t r^{-\alpha}}$ and the expectation over the fading and the p.p. are independent. Since the all h_i have the same distribution, we are able to calculate the expectation over only one variable denoted as h. Hence, the distribution of SINR can be calculated by

$$\begin{aligned} p_{\mathsf{SINR}}(\lambda) &= 1 - p_C(\lambda) \qquad (6.9)\\ &= 1 - 2\pi\lambda \int_0^\infty \exp\Bigg(-\pi r^2 \lambda \Bigg(1 + \\ &\frac{2\rho\, {}_2F_1\left(1, 1-\frac{2}{\alpha}; 2-\frac{2}{\alpha}; -\rho\right)}{\alpha - 2} \Bigg) - \frac{\mu\rho\sigma^2}{P_t r^{-\alpha}} \Bigg) r dr, \end{aligned}$$

in which hypergeometric function ${}_2F_1(a, b; c; z)$ is a special function represented by the hypergeometric series defined for $|z| < 1$ by the power series

$${}_2F_1(a, b; c; z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!} \qquad (6.10)$$

provided that $c \neq 0, -1, -2, \ldots$. Here $(f)_n$ is the Pochhammer symbol defined by

$$(f)_n = \begin{cases} 1, & \text{if } n = 0, \\ f(f+1)\cdots(f+n-1), & \text{if } n > 0. \end{cases} \qquad (6.11)$$

For $\alpha = 4$, the coverage probability can be found to be as following:

$$\begin{aligned} p_C(\lambda, \alpha = 4) &= \frac{\lambda}{2\sigma} \sqrt{\frac{\pi^3 P_t}{\mu\rho}} \operatorname{erfc}\left(\frac{\pi\lambda}{2\sigma} \sqrt{\frac{P_t}{\mu\rho}} \left(1 + \sqrt{\rho} \tan^{-1}\left(\sqrt{\rho}\right)\right) \right) \times \\ &\exp\left(\left(\frac{\pi\lambda}{2\sigma} \sqrt{\frac{P_t}{\mu\rho}} \left(1 + \sqrt{\rho} \tan^{-1}\left(\sqrt{\rho}\right)\right) \right)^2 \right). \qquad (6.12) \end{aligned}$$

6.3 SWITCHING OFF BASE STATIONS

In this section, we introduce the underlying approach to the problem in which the BSs are turned off. We assume that a SP has an observation that a typical BS is activated with some probability q. According to data traffic the SP turns on or off a BS. It gathers the information of this operation and set the activation probability q during that long enough period observations. However, we look for such an optimum value of q by which the SP maximizes its own energy saving introduced in Section 6.4.4.

6.3.1 *Scaling*

In this section, we adopt the technic of thinning a p.p which is performed through scaling. We put forward in [12] this approach. Also, here, we give a proof which does not exist in [12].

Consider Figure 6.2 in which we depict the scaling effect on a p.p. (if no otherwise stated we assume that p.p. is always homogenous Poisson one). We scale up the p.p. from the origin. The dark red points correspond to the p.p. which is obtained after scaling of the initial one (showed by shiny red points).

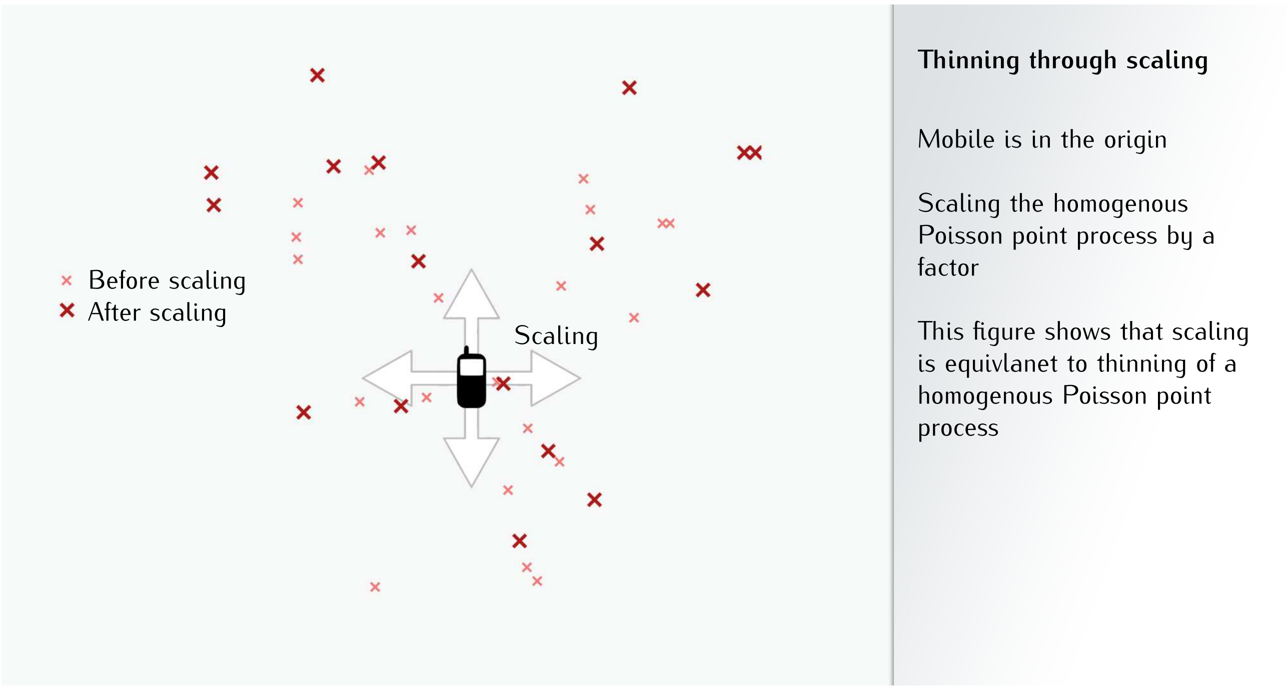


Figure 6.2: Scaling.

Recall that we model the network as an independently marked Poisson p.p. $\tilde{\Phi}$. In the following, we derive the intensity measure of points of $\tilde{\Phi}^q$ being a thinned version of the initial one.

Lemma 6.3.1 ***Thinning through scaling***: Choose some $0 \leq q \leq 1$. Scaling each coordinate by $\sqrt{q}$ in $\mathbb{R}^2$ results in a p.p. Φ^q of intensity measure $q\Lambda$ if the initial p.p. Φ has some intensity measure Λ.

Proof 6.3.1 *Let the coordinates of a typical point on some $E \subset \mathbb{R}^2$ be x and y, respectively. Scaling up each coordinate by q gives the new coordinates $x' = x/q$ and $y' = y/q$, respectively. We know that the distance from the origin before scaling is $d = \sqrt{x^2 + y^2}$. After scaling the distance becomes $d' = \frac{1}{q}\sqrt{x^2 + y^2}$ which means that each point moves away from the origin by*

$$d' = \frac{d}{q}. \tag{6.13}$$

It is straightforward to understand that when scaling up only one coordinate by q results in a new p.p. with intensity measure $q\Lambda$. Then, we are able to state that scaling up each coordinate by q brings out a new p.p. with intensity measure $q^2\Lambda$.

Eventually, a new p.p Φ^q is obtained by scaling each coordinate by $\sqrt{q}$ of the original one Φ which corresponds to the deleting independently points with probability $1 - q$. Deleted points should be imagined as the BSs that are switched off. Consequently, the new intensity measure is $\Lambda^q = q\Lambda$.

6.3.2 *SINR Distribution of the Scaled Network*

Let us now calculate the new SINR distribution while the initial p.p. is scaled by $\sqrt{q}$. If we replace all d_i in (6.1) by d_i' and replace P_t by $\bar{P}_t$ then we can interpret the SINR distribution of the original p.p. as the one corresponding to a network where BSs are located according to a new p.p. with intensity parameter $\lambda^q = q\lambda$ where $\bar{P} = P_0 + \beta \bar{P}_{tot}$.

Using the relation of transmission power given in eq. (6.3), the coverage probability given in eq. (6.9) can be expressed as

$$p_C(\lambda) = 2\pi\lambda \int_0^\infty \exp\left(-\pi r^2 \lambda(1 + 2f(\alpha, \rho)) - \frac{\mu\rho\sigma^2}{P_t r^{-\alpha}}\right) r dr. \tag{6.14}$$

The scaling of initial process requires $\lambda \to q\lambda$ then $r \to r/\sqrt{q}$, $dr \to dr/\sqrt{q}$ which gives the following coverage probability

$$p_C(q\lambda) = 2\pi q\lambda \int_0^\infty \exp\left(-\pi \left(\frac{r}{\sqrt{q}}\right)^2 q\lambda(1 + 2f(\alpha, \rho)) - \frac{\mu\rho\sigma^2}{P_t q^{\alpha/2} r^{-\alpha}}\right) \frac{r dr}{q}. \tag{6.15}$$

It is straightforward that if we choose $P_t \to P_t q^{-\alpha/2}$, then $p_C(q\lambda) = p_C(\lambda)$. Moreover, the average total transmission power of a BS of the thinned network is obtained by $P_{tot} \to P_{tot} q^{-\alpha/2}$. This result indicates that the SINR distribution remains unchanged while the average total transmission power is increased by $q^{-\alpha/2}$ of a typical BS.

6.4 MULTIPLE SERVICE PROVIDERS

In this section, we extend the analysis to the multiple SPs case. We derive the formulation for two SPs but the results can be immediately obtained for more than two SPs case.

6.4.1 *Non-cooperation of SPs*

Assume that there are two SPs. We denote by Φ_1 and Φ_2 the location of the BSs of the SP 1 and SP 2 with intensity parameters λ_1 and λ_2, respectively where the energy profiles of SP 1 and SP 2 are given as P_1 and P_2, respectively. Furthermore, let the scaling factors of two SPs be $\sqrt{q_1}$ and $\sqrt{q_2}$. Hence, the p.p. after scaling of SP 1 and SP 2 is represented as $\Phi_1^{q_1}$ and $\Phi_2^{q_2}$, respectively. The points of network which is a result of the sum of thinned version of SP 1 and SP 2 can thus be represented as $\Phi_1^{q_1} + \Phi_2^{q_2}$ with intensity measure $q_1\Lambda_1 + q_2\Lambda_2$.

Consider the Figure 6.3. We assume that the mobile is a customer of SP 1. Provided that SPs do not cooperate, we suppose that the thinning is performed independently by each SP. The SINR of the mobile before scaling can be calculated as following:

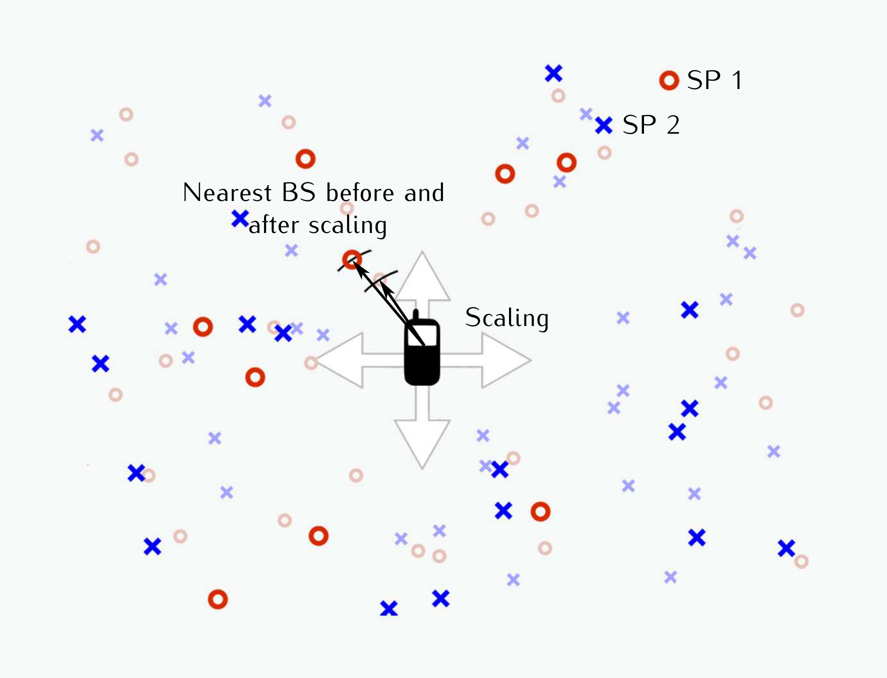
$$\mathsf{SINR} = \frac{P_{t,1} h_0 d_0^{-\alpha}}{P_{t,1} \sum_{i \in \Phi_1 \setminus p_0} h_i d_i^{-\alpha} + P_{t,2} \sum_{j \in \Phi_2} h_j d_j^{-\alpha} + \sigma^2}, \tag{6.16}$$

where d_0 denotes the distance of the mobile to the nearest BS of SP 1 as well as $P_{t,i}$ is the transmission power of SP i.

6.4.2 *SINR Distribution in case of Non-cooperation*

In order to derive the distribution of SINR when SPs do not cooperate, we first represent the SINR as following:

$$\mathsf{SINR} = \frac{P_{t,1} h r^{-\alpha}}{I_1 + I_2 + \sigma^2}. \tag{6.17}$$



Different scaling of two SPs

SP 1 switches off its own BSs with probability q_1 as well as SP 2 switches off with probability q_2. Mobile is a subscriber of SP 1.

Figure 6.3: Scaling in case of non-cooperation of two SPs.

The coverage probability conditioning to the nearest BS of SP 1 is formulated as

$$p_C(\lambda_1, \lambda_2) = \int_0^\infty \mathbf{P}\left\{ h > \frac{\rho(\sigma^2 + I_1 + I_2)}{P_{t,1} r^{-\alpha}} \middle| r \right\} \exp(-\pi\lambda_1 r^2) 2\pi\lambda_1 r dr, \quad (6.18)$$

where r is the distance between mobile and the nearest BS of SP 1. Then, the coverage probability can be expressed as

$$p_C(\lambda_1, \lambda_2) = \int_0^\infty \exp\left(-\frac{\mu\rho\sigma^2}{P_{t,1} r^{-\alpha}}\right) \mathscr{L}_{I_1}\left(\frac{\mu\rho}{P_{t,1} r^{-\alpha}}\right) \mathscr{L}_{I_2}\left(\frac{\mu\rho}{P_{t,1} r^{-\alpha}}\right) \times \exp(-\pi\lambda_1 r^2) 2\pi\lambda_1 r dr. \quad (6.19)$$

The Laplace transform of the interferences arising due to SP 1 and SP 2 are given as

$$\mathscr{L}_{I_1}(s) = \exp\left(-2\pi\lambda_1 \int_r^\infty \frac{x}{1 + \frac{\mu}{s P_{t,1} x^{-\alpha}}} dx\right), \quad (6.20)$$

$$\mathscr{L}_{I_2}(s) = \exp\left(-2\pi\lambda_2 \int_0^\infty \frac{x}{1 + \frac{\mu}{s P_{t,2} x^{-\alpha}}} dx\right), \quad (6.21)$$

respectively, where $s = \frac{\mu\rho}{P_{t,1} r^{-\alpha}}$. Note that the lower limit of the Laplace transform integral of I_2 is zero which takes into account the interference that occurs from the points of SP 2 being nearer than the nearest BS of SP 1. Thus, the following integral gives the coverage probability of non-cooperation case

$$p_C(\lambda_1,\lambda_2) = 2\pi\lambda_1 \int_0^\infty \exp\left(-\pi r^2\lambda_1\left(1+\frac{2\rho\,{}_2F_1\left(1,\frac{\alpha-2}{\alpha};2-\frac{2}{\alpha};-\rho\right)}{\alpha-2}+\frac{2\pi\frac{\lambda_2}{\lambda_1}\left(\frac{P_{t,1}}{\rho P_{t,2}}\right)^{-2/\alpha}}{\alpha\sin\left(\frac{2\pi}{\alpha}\right)}\right)-\frac{\mu\rho\sigma^2}{P_{t,1}r^{-\alpha}}\right) r\,dr. \quad (6.22)$$

For specific values of α, closed form coverage probability expressions can be found. For example, let $\alpha = 4$. The coverage probability can be given by

$$p_C\left(\lambda_1,\lambda_2,\alpha=4\right) = \frac{\lambda_1}{2\sigma}\sqrt{\frac{\pi^3 P_{t,1}}{\mu\rho}}\times \operatorname{erfc}\left(\frac{\pi^2\lambda_2\sqrt{\rho P_{t,2}}+2\pi\lambda_1\sqrt{P_{t,1}}\left(\sqrt{\rho}\tan^{-1}\left(\sqrt{\rho}\right)-1\right)}{4\sigma\sqrt{\mu\rho}}\right)\times \exp\left(\left(\frac{\pi^2\lambda_2\sqrt{\rho P_{t,2}}+2\pi\lambda_1\sqrt{P_{t,1}}\left(\sqrt{\rho}\tan^{-1}\left(\sqrt{\rho}\right)-1\right)}{4\sigma\sqrt{\mu\rho}}\right)^2\right). \quad (6.23)$$

The coverage probability of the networks of SP 1 and SP 2 scaled by $\sqrt{q_1}$ and $\sqrt{q_2}$, respectively can be calculated by the following integral,

$$p_C(q_1\lambda_1,q_2\lambda_2) = 2\pi q_1\lambda_1 \int_0^\infty \exp\left(-\pi\left(\frac{r}{\sqrt{q_1}}\right)^2 q_1\lambda_1\left(1+\frac{2\rho\,{}_2F_1\left(1,\frac{\alpha-2}{\alpha};2-\frac{2}{\alpha};-\rho\right)}{\alpha-2}+\frac{2\pi\frac{q_2\lambda_2}{q_1\lambda_1}\left(\frac{P_{t,1}}{\rho P_{t,2}}\right)^{-2/\alpha}}{\alpha\sin\left(\frac{2\pi}{\alpha}\right)}\right)-\frac{\mu\rho\sigma^2}{P_{t,1}\left(\frac{r}{\sqrt{q_1}}\right)^{-\alpha}}\right)\frac{r\,dr}{q_1}. \quad (6.24)$$

By increasing the transmission power of SP 1 and SP 2 as $P_{t,1} \to P_{t,1}q_1^{-\alpha/2}$, $P_{t,2} \to P_{t,2}q_2^{-\alpha/2}$, respectively, the SINR distributions of two SPs do not change in the non-cooperation case.

6.4.3 *Cooperation of SPs*

In the case of cooperation among the SPs, any mobile is associated to the nearest BS of any SP. Thus, SPs share their resources in order to obtain a better SINR level for each customer. By this way, the power consumption can be lowered providing a "green" approach to the BS deployment.

Let $S = (1,2)$ show the coalition of SP 1 and SP 2. We assume that in case of a cooperation the operators control jointly the network such that the activation probability q_S of a BS is determined in order to maximize the energy saving density. Formally, the traffic and the network intensity that reveals by cooperation are supposed to be additive, i.e. $\sum_{i\in S} T_i$ and $\sum_{i\in S}\lambda_i$, respectively.

It is also considered that the network formed by cooperation has the equal average total transmission power per BS given by

$$P_{tot}^{S} = p + wf\left(\frac{\sum_{i\in S} T_i}{\sum_{i\in S} \lambda_i}\right). \tag{6.25}$$

Thus, the SINR can be expressed as following:

$$\mathsf{SINR} = \frac{P_t^S h_0 d_0^{-\alpha}}{P_t^S \sum_{i\in\Phi_S} h_i d_i^{-\alpha} + \sigma^2}, \tag{6.26}$$

where $\Phi_S = \Phi_1 + \Phi_2$ denotes the p.p. which is a result of sum of the p.p. of SP 1 and SP 2 having the intensity $\lambda_1 + \lambda_2$ which is a direct result of superposition property stated in Definition 3.1.1, and P_t^S is the transmission power between mobile and the tagged BS in case of cooperation.

Scaling each coordinate by $\sqrt{q_S}$ results in a network which corresponds to a thinned one as a consequence of cooperation. The points of the network resulting from the scaling is denoted as Φ^{q_S} which has the intensity measure $q_S(\lambda_1 + \lambda_2)$. Thus, we can adopt the same result obtained for the single operator where the transmission power is adjusted as $P_t^S \to P_t^S q_S^{-\alpha/2}$. Moreover, the energy profile of a typical BS of SP i is given by

$$P_i^S = P_{0,i} + \beta_i P_{tot}^S. \tag{6.27}$$

We also denote by $\bar{P}_i^S = P_{0,i} + \beta_i P_{tot}^S q_S^{-\alpha/2}$ the energy profile of SP i corresponding to the thinned network.

6.4.4 *Energy Saving*

We are interested to see what is the energy saving by switching off BSs (independently) with probability $1 - q_S$, given that at the same time we increase the transmission energy to compensate for decreasing the resources in a way that the probability distribution of the SINR is unchanged.

Now, we introduce the energy saving density when SPs form a coalition S. The power consumption density of SP i can be calculated by $\lambda_i P_i^S$. We can also calculate the power consumption density by $q^S \lambda_i \bar{P}_i^S$ when considering the thinned network. So, in case of coalition S, the energy saving density is characterized as $\sum_{i\in S} \lambda_i P_i^S - q_S \sum_{i\in S} \lambda_i \bar{P}_i^S$ resulting in following function

$$G(S) = (1 - q_S)\sum_{i\in S} \lambda_i P_{0,i} + \left(1 - q_S^{1-\alpha/2}\right) P_{tot}^S \sum_{i\in S} \lambda_i \beta_i. \tag{6.28}$$

Let us represent by $U_S = \sum_{i\in S} \lambda_i P_{0,i}$ and $V_S = P_{tot}^S \sum_{i\in S} \lambda_i \beta_i$. The meaning of these variables can be interpreted as following: U_S corresponds to the energy saving density of the operational power costs arising due to the BSs of coalition S as well as V_S is the energy saving density due to transmission power in case of coalition S. Then, the energy saving density can be expressed as

$$G(S) = (1 - q_S)U_S + \left(1 - q_S^{1-\alpha/2}\right) V_S. \tag{6.29}$$

6.5 OPTIMAL ENERGY SAVING FOR UNIQUE SERVICE PROVIDER

6.5.1 *The line*

We are interested to see what is the gain in energy by switching off BSs (independently) with probability $(1 - q)$, given that at the same time we increase the transmission energy to compensate for decreasing the resources in a way that the probability distribution of the SINR are unchanged.

After switching off base stations, the power consumption density of the network is

$$\lambda q(P_0 + \beta P') = \lambda q(P_0 + \beta P q^{-\alpha}). \tag{6.30}$$

So that the gain in power consumption density is

$$\begin{aligned} G(q) &= \lambda(P_0 + \beta P) - \lambda q(P_0 + \beta P') \\ &= \lambda(P_0(1-q) + \beta P[1 - q^{1-\alpha}]). \end{aligned} \tag{6.31}$$

The switching probabilities that maximize this gain are obtained by solving

$$\frac{dG(q)}{dq} = -P_0 - (1-\alpha)\beta P q^{-\alpha} = 0 \tag{6.32}$$

which gives

$$1 - q^* = 1 - \left(\frac{\beta P(\alpha - 1)}{P_0}\right)^{\frac{1}{\alpha}}. \tag{6.33}$$

6.5.2 *The plane*

We calculate by the same way the power consumption density of the network after switching off BSs

$$\lambda q(P_0 + \beta P') = \lambda q(P_0 + \beta P q^{-\alpha/2}). \tag{6.34}$$

The gain in power consumption density is given by

$$\begin{aligned} G(q) &= \lambda(P_0 + \beta P) - \lambda q(P_0 + \beta P') \\ &= \lambda(P_0(1-q) + \beta P[1 - q^{1-\alpha/2}]). \end{aligned} \tag{6.35}$$

The switching probabilities that maximize this gain are obtained by solving

$$\frac{dG(q)}{dq} = -P_0 - \beta P(1-\alpha/2) q^{-(\alpha/2)} = 0 \tag{6.36}$$

which gives

$$1 - q^* = 1 - \left(\frac{\beta P(\alpha/2 - 1)}{P_0}\right)^{\frac{1}{\alpha/2}}. \tag{6.37}$$

6.5.3 *Simulation Results*

In this section, we compare the optimal switching off probabilities with respect to path loss α for different operational costs P_0, and we also match the optimal switching off probabilities in terms of β for some α. Moreover, gain in power consumption is compared with respect to α for different P_0.

In Figure 1 and 2, we depict the change of switching off probabilities in terms of path loss α. From figures, we observe that for higher path loss values, the number of switched off BSs is decreased. In other words, we need to keep more BSs switched on. Also, for the same path loss value optimum switching off probability is higher for higher P_0. That means, the switching off strategy tells us to remove base stations with a higher probability for higher P_0. On the other hand, if we compare the optimal switching off probabilities with respect to the dimension (line or plane), we remark that it is necessary to switch on more BSs.

We depict in Figure 3 and 4 the comparison of switching off probabilities in terms of β for $\alpha = (2.5, 4, 6)$. We interpret that for higher values of β the number of switched on BSs is increased. Furthermore, in case of plane the used switched on BSs is higher than that of line.

In Figure 5 and 6, the comparison of gain in power consumption $G(q^*)$ with respect to α is given. We calculate $G(q^*)$ in terms of optimal switching off probabilities. It is assumed to be unit intensity parameter λ. We observe that as long as P_0 increases, the obtained $G(q^*)$ increases. This means that for high operational costs the gain in power consumption by switching off BSs is also high.

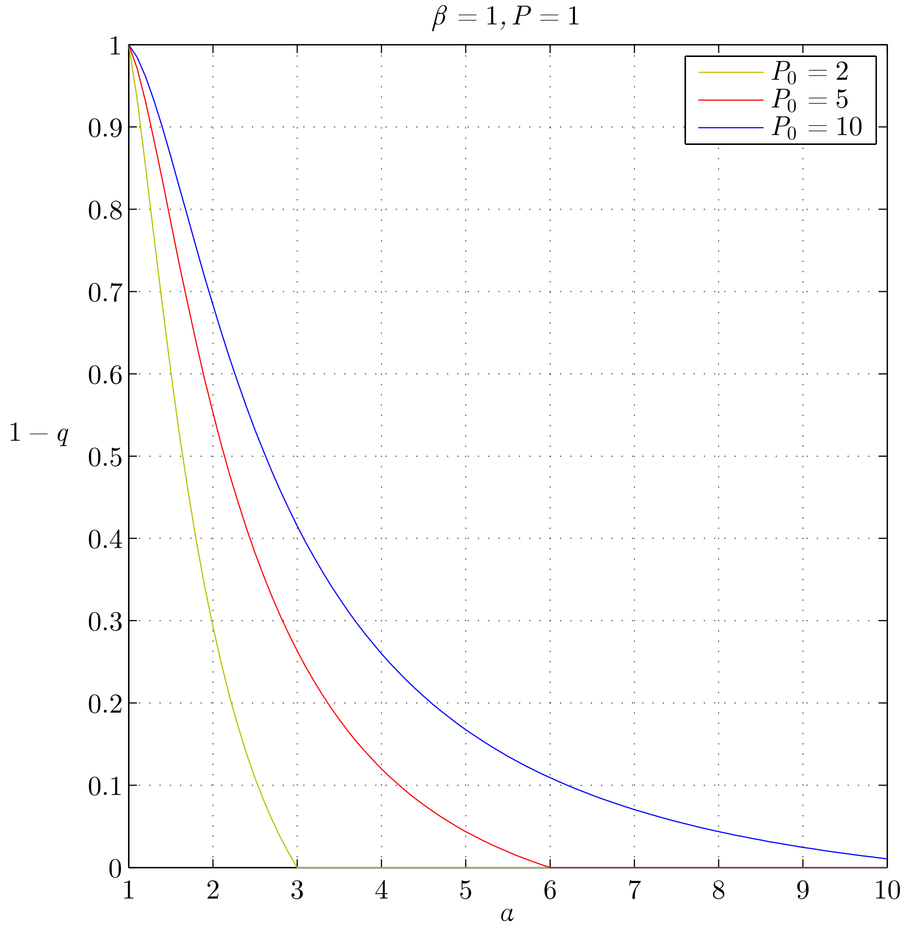


Figure 6.4: The change of optimal switching off probabilities with respect to path loss in case of line

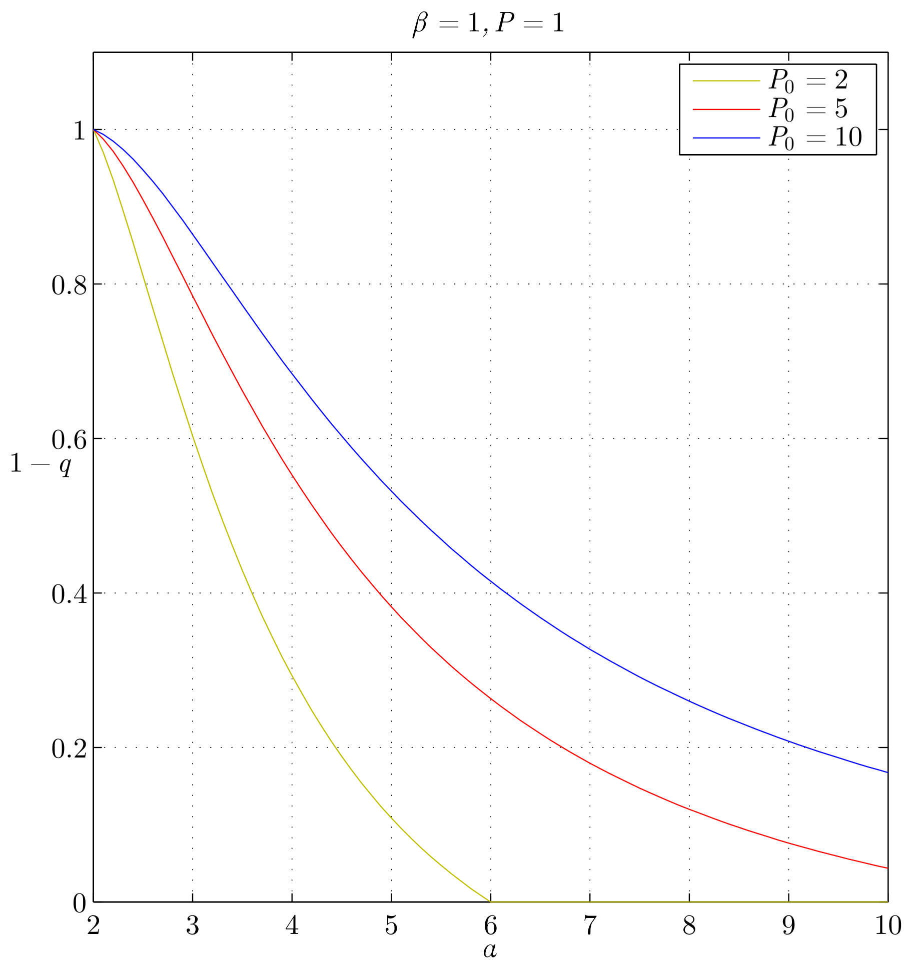


Figure 6.5: The change of optimal switching off probabilities with respect to path loss in case of plane

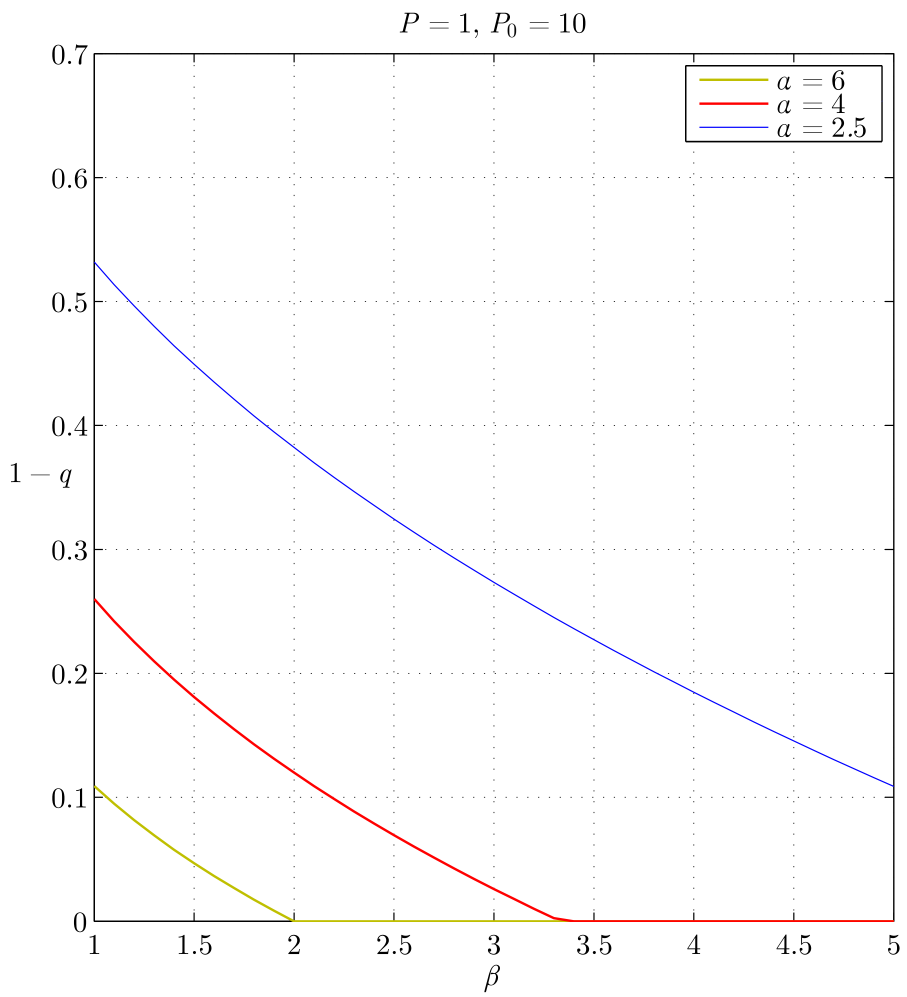


Figure 6.6: The change of optimal switching off probabilities with respect to β in case of line

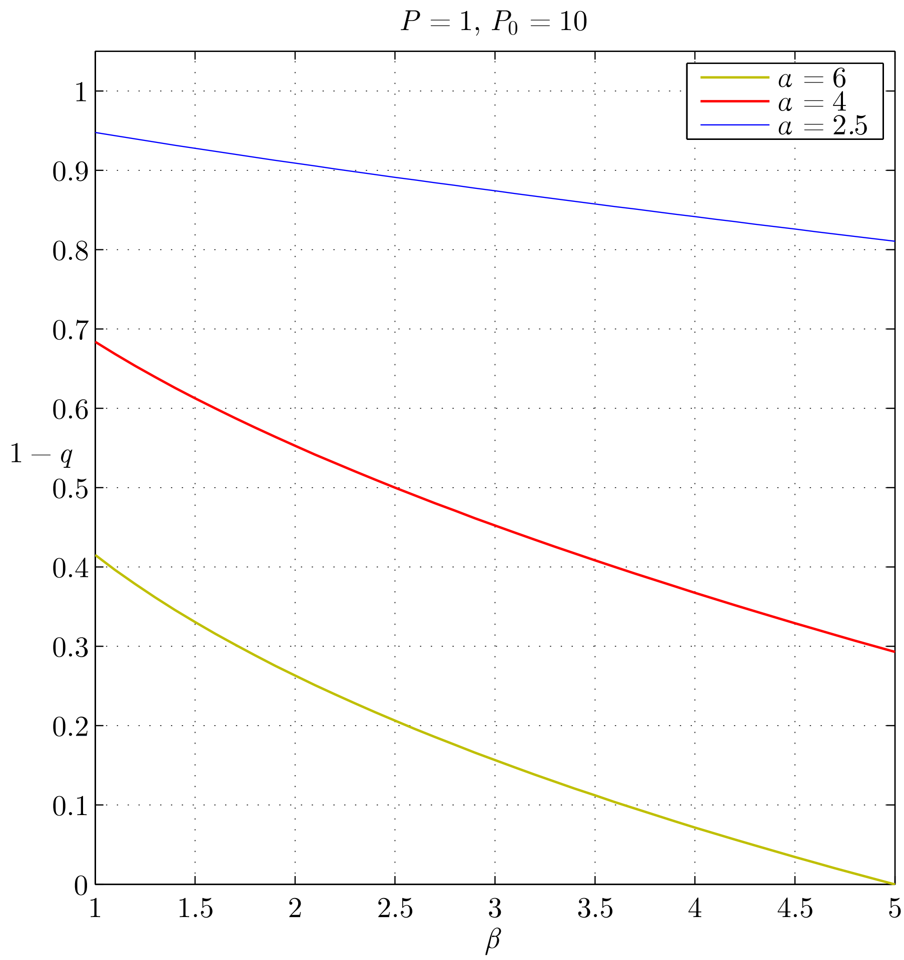


Figure 6.7: The change of optimal switching off probabilities with respect to β in case of plane

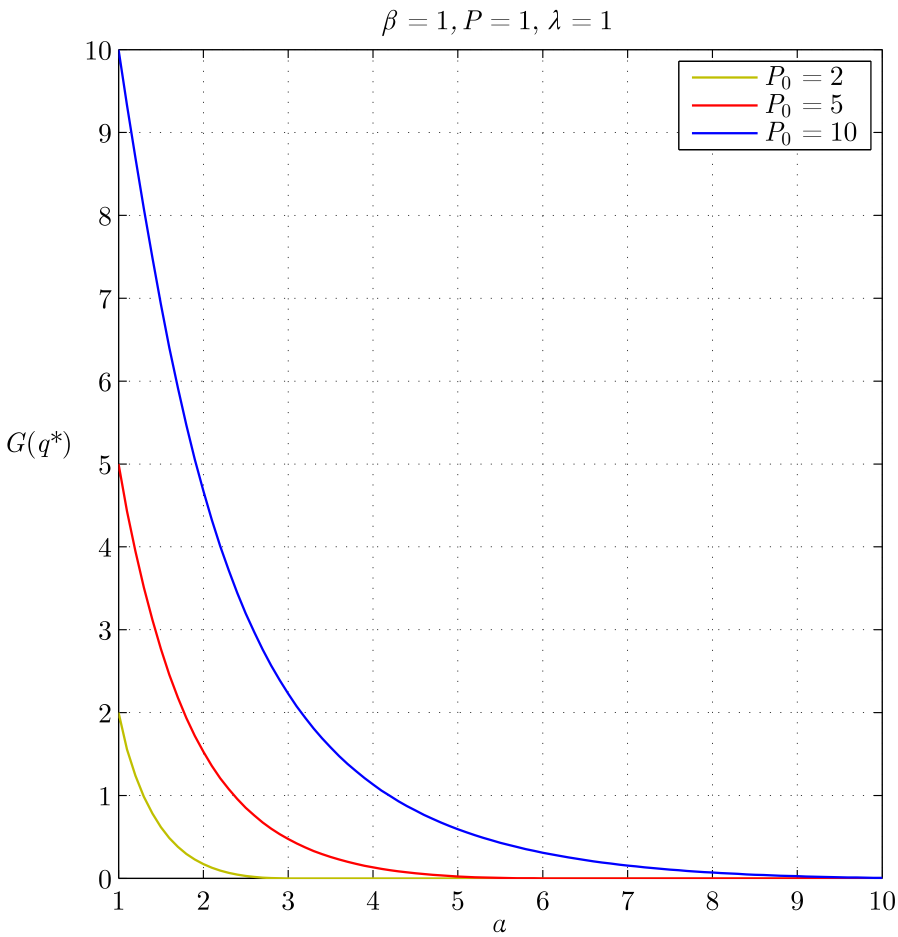


Figure 6.8: The gain in power consumption with respect to path loss in case of line

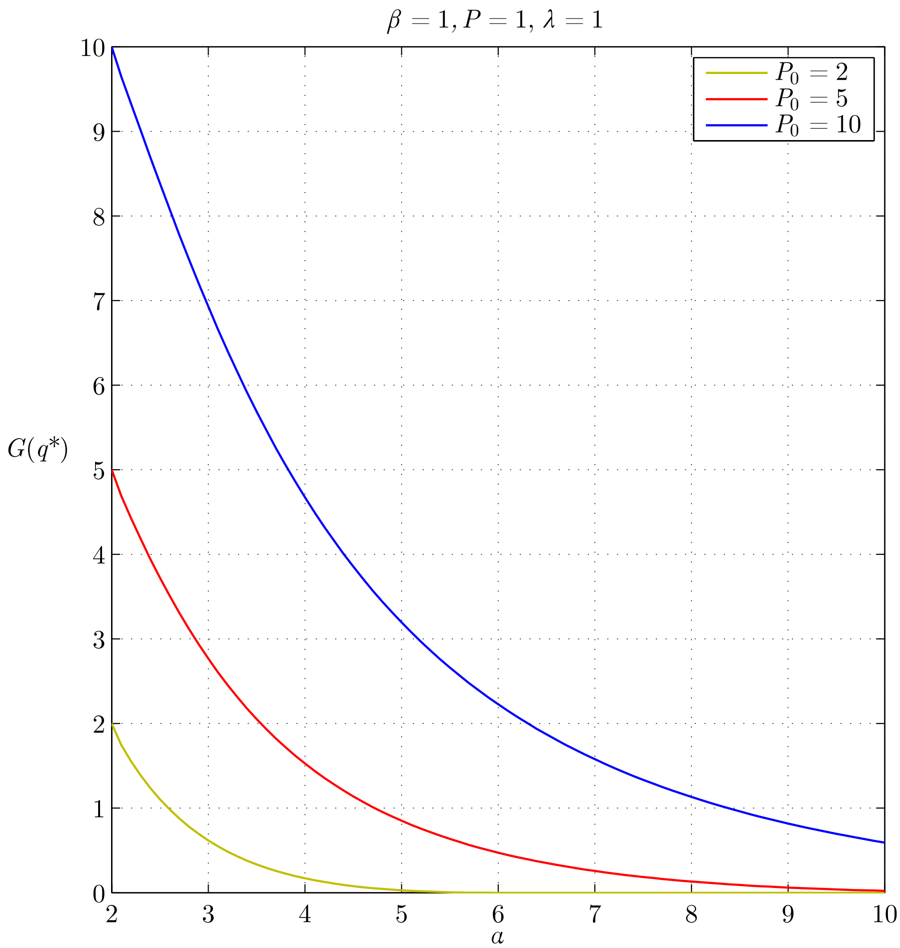


Figure 6.9: The gain in power consumption with respect to path loss in case of plane

7

THE COALITIONAL SWITCH OFF GAME

In this chapter, we examine the cooperation of SPs in terms of hedonic coalition formation games. We denote by $\langle N, \succ, u\rangle$ the coalitional switch off game in which $N = (1, 2, \ldots, n)$ is the set of SPs, $\succ$ is the preference profile of SPs, and u is the utility function. First, we determine the utility function of cooperation, then we study the properties of it.

7.1 THE UTILITY FUNCTION OF COOPERATION

Above, we mentioned that it is necessary to define a gain or cost function that characterizes the problem of cooperation. In our context, we need to analyze such a characteristic that should explain the total switch off gain, denoted by u, of a coalition. Precisely, we formalize this utility as in terms of maximization of energy saving density given in eq. (6.29), i.e.,

$$u(S) = f\left(\max_{q_S} G(S)\right) \quad \text{subject to} \quad 0 \leq q_S \leq 1. \tag{7.1}$$

The physical meaning of the utility function is to measure the total gain (it could some amount of money) when the switching off probability gives the global maximum of the energy saving density. For example, think of that two SPs However, in the sequel we assume that $f\left(\max_{q_S} G(S)\right) = \max_{q_S} G(S)$ Let us find the optimal value of q_S which can be calculated as

$$\frac{\partial G(S)}{\partial q_S} = -U_S + \left(\frac{\alpha}{2} - 1\right) q_S^{-\alpha/2} V_S = 0. \tag{7.2}$$

We know that the maximum of activation probability is $q_S \leq 1$, then the following gives the optimal value

$$q_S^* = \min\left\{\left(\frac{\left(\frac{\alpha}{2}-1\right)V_S}{U_S}\right)^{2/\alpha}, 1\right\} \tag{7.3}$$

by which the utility function can be expressed as

$$u(S) = \begin{cases} U_S + V_S - \frac{V_S^{2/\alpha}}{U_S^{2/\alpha-1}}\left(\frac{\alpha}{\alpha-2}\right)\left(\frac{\alpha}{2}-1\right)^{2/\alpha}, & \text{if } q_S^* < 1, \\ 0, & \text{if } q_S^* = 1. \end{cases} \tag{7.4}$$

Further, we could find the limit which guaranties that $u(S) > 0$,

$$\left(\frac{\left(\frac{\alpha}{2}-1\right)V_S}{U_S}\right)^{2/\alpha} < 1 \quad \Rightarrow \quad U_S > \left(\frac{\alpha}{2}-1\right)V_S. \tag{7.5}$$

What we infer from this result is that as long as energy saving density of operational power costs is higher than the total transmission energy saving multiplied by $\frac{\alpha}{2} - 1$, there exists a non-zero utility of a typical coalition S.

Furthermore, recall that the allocation of the utility $u(S)$ to player i being in coalition S is denoted as $\phi_i(S)$. This gain corresponds to the energy saving allocated to player i. Thus, we say that player i obtains $\phi_i(S)$ gain when the BSs are activated by q_S of the joint network formed by coalition S.

7.1.1 *Properties of the Utility Function*

In the following, we enumerate the properties of the utility function of the coalitional switch off game.

Lemma 7.1.1 Monotonicity: The coalitional switch off game is not always monotonic.

Proof 7.1.1 *Assume that there are two SPs, i.e.* $S = (1,2)$*, and each SP has equal* λ*,* P_0*, and* β *as well as different traffics* T_1*,* T_2*. Also, suppose that* $\alpha = 4$*,* $p = 0$ *and* $w = 1$*. Then, we are able to find the following:*

$$u(1) = \lambda P_0 + \beta T_1 - 2\sqrt{\lambda P_0 \beta T_1}, \tag{7.6}$$

$$u(1,2) = 2\lambda P_0 + \beta\left(T_1 + T_2\right) - 2\sqrt{2\lambda P_0 \beta\left(T_1 + T_2\right)}. \tag{7.7}$$

If we can prove that $u(1,2) < u(1)$*, then we could conclude that the monotonicity does not hold. To this end, let us denote the difference of these utilities as*

$$\Delta = u(1,2) - u(1) = \lambda P_0 + \beta T_2 - 2\sqrt{2\lambda P_0 \beta\left(T_1 + T_2\right)} + 2\sqrt{\lambda P_0 \beta T_1}, \tag{7.8}$$

which means that if $\Delta < 0$*, then the utility function has no monotonicity property. Assuming that* $\lambda P_0 = 1$ *and* $\beta = 1$*, let us look at the limit of this difference by converging the traffic of SP 1 to infinity,*

$$\lim_{T_1 \to \infty} \Delta = \lim_{T_1 \to \infty} \left(1 + T_2 - 2\sqrt{2(T_1 + T_2)} + 2\sqrt{T_1}\right) = -\infty \tag{7.9}$$

Consequently, we are able to state that when traffic of SP 1 increases to high levels then the monotonicity might not hold.

We can conclude that the SPs have an incentive to deviate from grand coalition if they play the coalitional switch off game. Because, non-monotonicity implies that sometimes the utility might not increase when a player joins the game. Therefore, we come up with the coalition formation problem. We consider the hedonic approach to the coalition formation of SPs in this work.

7.2 EXAMPLE SCENARIO

In this section, we study an example scenario in which the introduced concepts are explained practically. We compare the results for different allocation methods.

Assume there are three SPs, $N = (1,2,3)$. Let the gain function yield the following results for all possible coalitions

$$u(S_1) = 7.52 \times 10^{-1}, u(S_2) = 125.24 \times 10^{-1},$$
$$u(S_3) = 30.36 \times 10^{-1}, u(S_4) = 141.76 \times 10^{-1},$$
$$u(S_5) = 39.61 \times 10^{-1}, u(S_6) = 159.71 \times 10^{-1},$$
$$u(S_7) = 171.29 \times 10^{-1} \quad (7.10)$$

where $S_1 = (1)$, $S_2 = (2)$, $S_3 = (3)$, $S_4 = (1,2)$, $S_5 = (1,3)$, $S_6 = (2,3)$, $S_7 = (1,2,3)$. These utilities denote the maximal energy saving density that can be obtained with cooperation.

From these gains, we see that $u(S_7) < u(S_4) + u(S_3)$ meaning that the game is not superadditive. However, the utility function is monotonic. Because of non-superadditivity the grand coalition can not be set. Therefore, we state that the core of this game is empty. To be sure, the reason that the grand coalition is not possible is due to the fact that the players in S_4 deviates from the grand coalition because they can do better by leaving the grand coalition. On the other hand, in case of the aim is to find a Nash-stable solution, the grand coalition might occur. In the following, we have the results that players form the grand coalition.

7.2.1 *Finding the Nash-stable Core*

Here, we would like to obtain an efficient allocation method that will result in a Nash-stable partition. Clearly, we need a distribution method arranging the gains by such a way that the players will form a coalition formation which they will not deviate from.

Let us enumerate all possible partitions and check the conditions of Nash-stable partition:

1. $\Pi = \{(1),(2),(3)\}$:

$$\max\left(\phi_1^{12} + \phi_1^{13} + \phi_1^{123} + \phi_2^{12} + \phi_2^{23} + \phi_2^{123} + \phi_3^{13} + \phi_3^{23} + \phi_3^{123}\right) \text{ s.t.}$$
$$7.52 \times 10^{-1} \geq \phi_1^{12},$$
$$7.52 \times 10^{-1} \geq \phi_1^{13},$$
$$125.24 \times 10^{-1} \geq \phi_2^{12},$$
$$125.24 \times 10^{-1} \geq \phi_2^{23},$$
$$30.36 \times 10^{-1} \geq \phi_3^{13},$$
$$30.36 \times 10^{-1} \geq \phi_3^{23},$$
$$\phi_1^{12} + \phi_2^{12} = 141.76 \times 10^{-1},$$
$$\phi_1^{13} + \phi_3^{13} = 39.61 \times 10^{-1},$$
$$\phi_2^{23} + \phi_3^{23} = 159.71 \times 10^{-1},$$
$$\phi_1^{123} + \phi_2^{123} + \phi_3^{123} = 171.29 \times 10^{-1} \quad (7.11)$$

There is no a feasible solution for this linear program. Therefore $\{(1),(2),(3)\}$ is not a Nash-stable partition.

2. $\Pi = \{(1,2),(3)\}$:

$$\begin{aligned}
&\max\left(\phi_1^{12} + \phi_1^{13} + \phi_1^{123} + \phi_2^{12} + \phi_2^{23} + \phi_2^{123} + \phi_3^{13} + \phi_3^{23} + \phi_3^{123}\right) \text{ s.t.}\\
&\phi_1^{12} \geq 7.52 \times 10^{-1}, \phi_1^{12} \geq \phi_1^{13},\\
&\phi_2^{12} \geq 125.24 \times 10^{-1}, \phi_2^{12} \geq \phi_2^{23},\\
&30.36 \times 10^{-1} \geq \phi_3^{123},\\
&\phi_1^{12} + \phi_2^{12} = 141.76 \times 10^{-1},\\
&\phi_1^{13} + \phi_3^{13} = 39.61 \times 10^{-1},\\
&\phi_2^{23} + \phi_3^{23} = 159.71 \times 10^{-1},\\
&\phi_1^{123} + \phi_2^{123} + \phi_3^{123} = 171.29 \times 10^{-1}.
\end{aligned} \tag{7.12}$$

Solution of this linear program results in $\phi_1^{12} = 13.51 \times 10^{-1}$, $\phi_1^{13} = -45.92 \times 10^{-1}$, $\phi_1^{123} = 105.67 \times 10^{-1}$, $\phi_2^{12} = 128.25 \times 10^{-1}$, $\phi_2^{23} = 38.14 \times 10^{-1}$, $\phi_2^{123} = 105.67 \times 10^{-1}$, $\phi_3^{13} = 85.53 \times 10^{-1}$, $\phi_3^{23} = 121.57 \times 10^{-1}$, $\phi_3^{123} = -40.06 \times 10^{-1}$ which produces the following preference profile

$$\begin{aligned}
&(1,2,3) \succ_1 (1,2) \succ_1 (1) \succ_1 (1,3)\\
&(1,2) \succ_2 (2) \succ_2 (1,2,3) \succ_2 (2,3)\\
&(2,3) \succ_3 (1,3) \succ_3 (3) \succ_3 (1,2,3).
\end{aligned} \tag{7.13}$$

This means that $\{(1,2),(3)\}$ is a Nash-stable partition. Thus, this result indicates that the Nash-stable core is non-empty for that example scenario.

3. $\Pi = \{(1,3),(2)\}$:

$$\begin{aligned}
&\max\left(\phi_1^{12} + \phi_1^{13} + \phi_1^{123} + \phi_2^{12} + \phi_2^{23} + \phi_2^{123} + \phi_3^{13} + \phi_3^{23} + \phi_3^{123}\right) \text{ s.t.}\\
&\phi_1^{13} \geq 7.52 \times 10^{-1}, \phi_1^{13} \geq \phi_1^{12},\\
&125.24 \times 10^{-1} \geq \phi_2^{123},\\
&\phi_3^{13} \geq 30.36 \times 10^{-1}, \phi_3^{13} \geq \phi_3^{23},\\
&\phi_1^{12} + \phi_2^{12} = 141.76 \times 10^{-1},\\
&\phi_1^{13} + \phi_3^{13} = 39.61 \times 10^{-1},\\
&\phi_2^{23} + \phi_3^{23} = 159.71 \times 10^{-1},\\
&\phi_1^{123} + \phi_2^{123} + \phi_3^{123} = 171.29 \times 10^{-1}.
\end{aligned} \tag{7.14}$$

There is a solution of this linear program as well: $\phi_1^{12} = -52.88 \times 10^{-1}$, $\phi_1^{13} = 9.24 \times 10^{-1}$, $\phi_1^{123} = 70.72 \times 10^{-1}$, $\phi_2^{12} = 194.64 \times 10^{-1}$, $\phi_2^{23} = 197.20 \times 10^{-1}$, $\phi_2^{123} = 29.83 \times 10^{-1}$, $\phi_3^{13} = 30.36 \times 10^{-1}$, $\phi_3^{23} = -37.49 \times 10^{-1}$, $\phi_3^{123} = 70.72 \times 10^{-1}$. The resulted preference profile is as following:

$$\begin{aligned}
&(1,2,3) \succ_1 (1,3) \succ_1 (1) \succ_1 (1,2)\\
&(2,3) \succ_2 (1,2) \succ_2 (2) \succ_2 (1,2,3)\\
&(1,2,3) \succ_3 (1,3) \succ_3 (3) \succ_3 (2,3).
\end{aligned} \tag{7.15}$$

4. $\Pi = \{(2,3),(1)\}$:

$$\begin{aligned}
&\max\left(\phi_1^{12}+\phi_1^{13}+\phi_1^{123}+\phi_2^{12}+\phi_2^{23}+\phi_2^{123}+\phi_3^{13}+\phi_3^{23}+\phi_3^{123}\right) \text{ s.t.}\\
&7.52\times 10^{-1} \geq \phi_1^{123},\\
&\phi_2^{23} \geq 125.24\times 10^{-1}, \phi_2^{23} \geq \phi_2^{12},\\
&\phi_3^{23} \geq 30.36\times 10^{-1}, \phi_3^{23} \geq \phi_3^{13},\\
&\phi_1^{12}+\phi_2^{12} = 141.76\times 10^{-1},\\
&\phi_1^{13}+\phi_3^{13} = 39.61\times 10^{-1},\\
&\phi_2^{23}+\phi_3^{23} = 159.71\times 10^{-1},\\
&\phi_1^{123}+\phi_2^{123}+\phi_3^{123} = 171.29\times 10^{-1}.
\end{aligned} \tag{7.16}$$

This partition is also Nash-stable with the following allocated utilities: $\phi_1^{12} = 105.19\times 10^{-1}$, $\phi_1^{13} = 72.75\times 10^{-1}$, $\phi_1^{123} = -59.35\times 10^{-1}$, $\phi_2^{12} = 36.56\times 10^{-1}$, $\phi_2^{23} = 127.06\times 10^{-1}$, $\phi_2^{123} = 115.32\times 10^{-1}$, $\phi_3^{13} = -33.14\times 10^{-1}$, $\phi_3^{23} = 32.64\times 10^{-1}$, $\phi_3^{123} = 115.32\times 10^{-1}$. The following preference profile is obtained:

$$\begin{aligned}
&(1,2) \succ_1 (1,3) \succ_1 (1) \succ_1 (1,2,3)\\
&(2,3) \succ_2 (2) \succ_2 (1,2,3) \succ_2 (1,2)\\
&(1,2,3) \succ_3 (2,3) \succ_3 (3) \succ_3 (1,3).
\end{aligned} \tag{7.17}$$

5. $\Pi = \{1,2,3\}$:

$$\begin{aligned}
&\max\left(\phi_1^{12}+\phi_1^{13}+\phi_1^{123}+\phi_2^{12}+\phi_2^{23}+\phi_2^{123}+\phi_3^{13}+\phi_3^{23}+\phi_3^{123}\right) \text{ s.t.}\\
&\phi_1^{123} \geq 7.52\times 10^{-1},\\
&\phi_2^{123} \geq 125.24\times 10^{-1},\\
&\phi_3^{123} \geq 30.36\times 10^{-1},\\
&\phi_1^{12}+\phi_2^{12} = 141.76\times 10^{-1},\\
&\phi_1^{13}+\phi_3^{13} = 39.61\times 10^{-1},\\
&\phi_2^{23}+\phi_3^{23} = 159.71\times 10^{-1},\\
&\phi_1^{123}+\phi_2^{123}+\phi_3^{123} = 171.29\times 10^{-1}
\end{aligned} \tag{7.18}$$

Recall that the sufficient condition of grand coalition in a Nash-stable setting is $u(N) \geq \sum_{i\in N} u(i) \rightarrow u(1,2,3) \geq u(1)+u(2)+u(3)$. $u(1,2,3) = 171.29\times 10^{-1}$, $u(1)+u(2)+u(3) = 163.12\times 10^{-1}$. Therefore, there must exist a utility allocation method that result in the Nash-stable partition $\Pi = \{1,2,3\}$. The solution of tha last linear program results in the following utility allocations: $\phi_1^{12} = 70.88\times 10^{-1}$, $\phi_1^{13} = 19.80\times 10^{-1}$, $\phi_1^{123} = 7.53\times 10^{-1}$, $\phi_2^{12} = 70.88\times 10^{-1}$, $\phi_2^{23} = 79.85\times 10^{-1}$, $\phi_2^{123} = 132.94\times 10^{-1}$, $\phi_3^{13} = 19.80\times 10^{-1}$, $\phi_3^{23} = 79.85\times 10^{-1}$, $\phi_3^{123} = 30.81\times 10^{-1}$ which produce the following preference profile:

$$\begin{aligned}
&(1,2) \succ_1 (1,3) \succ_1 (1,2,3) \succ_1 (1)\\
&(1,2,3) \succ_2 (2) \succ_2 (2,3) \succ_2 (1,2)\\
&(2,3) \succ_3 (1,2,3) \succ_3 (3) \succ_3 (1,3).
\end{aligned} \tag{7.19}$$

7.2.2 *A Solution Based on Relaxed Efficiency*

Relaxed efficiency provides to calculate the symmetric gain of the players which can be given by

$$\begin{aligned}
&\max v(1,2)+v(1,3)+v(2,3) \text{ subject to}\\
&v(1,2) \leq 4.5\times 10^{-1},\\
&v(1,3) \leq 0.865\times 10^{-1},\\
&v(2,3) \leq 2.055\times 10^{-1},\\
&v(1,2)+v(1,3)+v(2,3) \leq 3.585\times 10^{-1}.
\end{aligned} \tag{7.20}$$

The solution of this linear program results in $v(1,2) = 1.0089\times 10^{-1}$, $v(1,3) = 0.5263\times 10^{-1}$, and 2.0497×10^{-1}. According to that solution the utilities of each player are $\phi_1^{12} = 8.5289\times 10^{-1}$, $\phi_1^{13} = 8.0463\times 10^{-1}$, $\phi_1^{123} = 9.0553\times 10^{-1}$, $\phi_2^{12} = 126.2489\times 10^{-1}$, $\phi_2^{23} = 127.2897\times 10^{-1}$, $\phi_2^{123} = 128.2987\times 10^{-1}$, $\phi_3^{13} = 30.8863\times 10^{-1}$, $\phi_3^{23} = 32.4097\times 10^{-1}$, $\phi_3^{123} = 32.9361\times 10^{-1}$, which produces the following preference profile:

$$\begin{aligned}
&(1,2,3) \succ_1 (1,2) \succ_1 (1,3) \succ_1 (1)\\
&(1,2,3) \succ_2 (2,3) \succ_2 (1,2) \succ_2 (2)\\
&(1,2,3) \succ_3 (2,3) \succ_3 (1,3) \succ_3 (3).
\end{aligned} \tag{7.21}$$

Thus, the Nash-stable partition is $(1,2,3)$.

Remark 7.2.1 *Let us compare all obtained utilities of each player. Note that in case of relaxed efficiency player 1 gains* $\phi_1^{123} = 9.0553\times 10^{-1}$*, but it receives at most* $\phi_1^{12} = 13.51\times 10^{-1}$ *when efficiency is considered. Player 2 gains* $\phi_2^{123} = 128.2987\times 10^{-1}$ *in case of relaxed efficiency. It can do better in case of efficiency; player 2 gains at most* $\phi_2^{123} = 132.94\times 10^{-1}$*. Player 3 has* $\phi_3^{123} = 32.9361\times 10^{-1}$ *utility in case of relaxed efficiency, while it gains* $\phi_3^{23} = 32.32.64\times 10^{-1}$*. The results of all players except player 3 show that players have better gains in case of efficiency compared to relaxed efficiency. But, it is not valid for player 3. Therefore, it is reasonable to implement relaxed efficiency when individual deviations occur in a problem.*

7.3 CONCLUSION

We analyzed the cooperation of SPs on switch off operation of BSs in the context of green networking. The homogeneous Poisson p.p. approach to the deployment of BSs has been used in order to study the SINR distribution of SPs. Furthermore, in the case of non-cooperating SPs, the SINR distribution is obtained of the original and thinned network of SPs, respectively. We also found the SINR distribution of cooperation case used in the context of coalition formation of SPs. The operations on the network formed by cooperation are assumed to be run jointly by SPs meaning that they share their resources such that any mobile is tagged to the nearest BS of any SP. The maximal energy saving density of a cooperation is supposed to be the utility of the coalition. We derive the closed form results of the utility. We compared the utilities of SPs in case of both the allocation based on efficiency and relaxed efficiency. We showed that in case of individual deviations the importance of efficiency is not significant in the side of SPs.

8

NON-COOPERATIVE ASSOCIATION OF MOBILES

8.1 INTRODUCTION

There has been a growing interest in last years in modelling access decisions to networks as non-competitive games. Indeed, it is quite frequent that the network leaves it to the user to decide to which access point to connect.The association problem is in fact related in nature to the channel selection problem. This motivates the use of games with incomplete information, also known as Bayesian games, where the partial information refers to the system load in [72] or to the channel quality in [73].

The access point may differ from one another by their technology and by the quality of radio channels between each of them and each mobile. Such state dependent competitive decision making in networking have been modelled in the past as stochastic games and structure of equilibrium policies has been derived for one or two dimensional problems. By one dimensional problem we mean problems in which each mobile has a choice between an access point in which resources are shared and between a dedicated channel. In such problem the information needed for taking the association decision is how many mobiles are connected to the shared resource (therefore the information is said to be one-dimensional). An example for a problem that falls into this category is [74]. The equilibrium policy there consists of a threshold policy with randomization at the threshold. In [75] the author study a two dimensional problem in which the choice is between accessing a 3G wireless cellular network or a wireless local area network. The information available is of two dimensions: the number of mobiles in each one of the networks. In [76] equilibrium policies were shown to have a switching curve form with possible randomizations at the boundary between regions corresponding to connecting to different access point. A problem of association to one of several access points of a wireless local area network was considered in [77]. In all the above problems, we assumed that once a connection decision is made, the mobile stays connected to the access point till the end of the call.

In contrast, in this work, we consider the problem where at any time period, mobiles can update their association decision. We consider the choice between two access points or more, where the access decisions may depend on the number of mobiles connected to each one of the access points. We obtain new results using elementary tools in congestion and in crowding games. We show in particular that at equilibrium, mixed (randomized) actions are not required. We moreover show the convergence of sequence of best response strategies.

Our results are based on congestion games [80] and on crowding games [79]. We further study (i) multihoming in which a user can connect simultaneously to more than one access point. (ii) the "elastic" case in which there is also an option not to connect at all.

8.2 THE GENERIC GAME PROBLEM

There is a set Σ containing r resources and a set M of m users (players). Player i has access to a subset $\Sigma_i \subset \Sigma$ of these and has to choose to which resource it associates. We assume that the cost C_{ji} for player i of associating with resource j only depends on the number n_j (including himself) of players that use this resource. Each one of the costs C_{ji} is assumed to be monotonically non-decreasing in n_j. We wish to know whether an equilibrium exists, i.e. whether each player can choose one resource such that no player can get a strictly cheaper resource by deviating unilaterally. We further are interested to know when do we have convergence to equilibrium. Before answering these questions, we first introduce applications to the association problem of mobiles to base station.

We study below problems where each one of m mobiles has to decide to which one of r base stations to associate. We assume that the association is determined by the downlink conditions.

8.2.1 *Association to a base station (BS): TDMA*

Mobiles are served cyclically by the BS they associate to. Thus, if $n_j > 0$ mobiles connect to BS j then the time dedicated to transmission to each mobile is one frame in every n_j consecutive frames.

The utility of a user is the difference between a payoff and some cost. Here is an example of utilities and of costs.

8.2.1.1 *The throughput as payoff*

We assume that each BS has its own frequency so that there is no interference. We further introduce the concept of effective bandwidth [84] which allows us to associate an effective bandwidth to each mobile depending on its class and location relative to a target cell. Assume that a maximum of L users are allowed to be served by a BS. The utility that player i obtains can be expressed as

$$u_i(j, n_j) = \begin{cases} \underbrace{\frac{W_{ji} \log\left(1 + \frac{|h_{ji}|^2 P_{ji} d_{ji}^{-\alpha}}{\rho}\right)}{n_j}}_{\text{throughput}} - \underbrace{\frac{\delta \tilde{P}_j}{n_j}}_{\text{cost}}, & \text{if } n_j \leq L; \\ -\infty, & \text{otherwise.} \end{cases}, \quad (8.1)$$

where W_{ji} is the bandwidth that BS j allocates to the mobile i, h_{ji} captures the effects of fading between mobile i and BS j, d_{ji} is the distance between BS j and mobile i, α is the path loss exponent, ρ is the variance of additive noise, P_{ji} is transmitted power from BS j to mobile i and δ is called the *price* of switching on a BS which bears $\tilde{P}_j$ power cost if the corresponding BS is the jth one.

Remark 8.2.1 *In the above formulation, L denotes the capacity constraints of a BS (maximum number of mobiles that can be associated with a BS). We included* *implicitly capacity constraints, by assigning an infinite cost to joining a BS j if the total number of mobiles that associate to this BS exceeds L. Instead of using* $-\infty$ *one can use any other number sufficiently small. In both cases any equilibrium solution will have the property that all capacity constraints are satisfied for all players. Note that crowding games with capacity constraints and a special cost structure have been studied already in [81]. By assigning sufficiently negative utilities to association to BSs for the case that the number of mobiles exceeds some threshold, we manage to include these constraints in the framework of [80].*

We notice that the throughput that a player gains decreases when some group of players are served by the same BS. However, the cost that the corresponding player has to pay decreases as well. Note that the utility function is player-specific.

Finally, we assume that a mobile has the option not to connect to any BS in which case its utility is zero.

8.2.1.2 *Monotonicity of utility*

In order the considered game to be a crowding game, the utility must be a monotonically decreasing function, i.e.

$$\frac{u_i(j,k+x)-u_i(j,k)}{x} \leq 0, \quad \forall k \geq 1 \text{ and } x \geq 1 \tag{8.2}$$

Therefore,

$$\frac{u_i(j,k+x)-u_i(j,k)}{x} = \frac{\frac{W_{ji}\log\left(1+\frac{|h_{ji}|^2 P_{ji} d_{ji}^{-\alpha}}{\rho}\right)-\delta\tilde{P}_j}{k+x} - \frac{W_{ji}\log\left(1+\frac{|h_{ji}|^2 P_{ji} d_{ji}^{-\alpha}}{\rho}\right)-\delta\tilde{P}_j}{k}}{x} \tag{8.3}$$

$$= \left[\delta\tilde{P}_j - W_{ji}\log\left(1+\frac{|h_{ji}|^2 P_{ji} d_{ji}^{-\alpha}}{\rho}\right)\right]\left(\frac{1}{k(k+x)}\right) \leq 0. \tag{8.4}$$

Note that $\frac{1}{k(k+x)} \geq 0 \ \forall k \geq 1$ and $\forall x \geq 1$. Therefore, the following must hold in order to guarantee the monotonicity of utility function

$$\delta \leq \frac{W_{ji}\log\left(1+\frac{|h_{ji}|^2 P_{ji} d_{ji}^{-\alpha}}{\rho}\right)}{\tilde{P}_j}, \quad \forall j,i. \tag{8.5}$$

Let us assume that the bandwidth W_{ji} allocated to a player be a component of a set $\mathscr{W}$ (the set of different bandwidth classes), i.e. $W_{ji} \in \mathscr{W}$, and the SNR takes a value from the set $\mathscr{G}$, i.e. $SNR_{ji} = \frac{|h_{ji}|^2 P_{ji} d_{ji}^{-\alpha}}{\rho} \in \mathscr{G}$. Also, the operational power cost $\tilde{P}_j \in \mathscr{P}$. In order to determine the upper bound of pricing δ, we need to calculate the following

$$\min_{j,i} \frac{W_{ji}\log\left(1+SNR_{ji}\right)}{\tilde{P}_j}. \tag{8.6}$$

8.2.2 *Association to a base station (BS): HSDPA*

We adopt the model of S. Borst [82] for opportunistic scheduling according to the proportional fairness criterion. Time is divided into slots, and each BS schedules at each slot transmission to one mobile among those connected to it. A weakly symmetric channel model is used in which the channel statistics from BS j to mobile i are such that the throughput available to that mobile, if the channel is assigned to it is a random variable of the form $R_{ji} = Q_{ji}Y_{ji}Z_j$ where for each given j, $\{Y_{ji}\}$ are independent and identically distributed random variables, Z_j is some random variable (that may be used to bring correlation) with a unit mean, and Q_{ji} is representing the time-average rate of user i [82]. Thus, the probability distribution of the normalized available throughput of all the mobiles connected to BS j are the same. The proportional fair allocation at BS j schedules transmission to the connected mobile for which the normalized rate (i.e. ratio R_{ji}/Q_{ji}) is the largest. The expected average throughput of mobile i when connecting to BS j is then given by $G(n_j)/n_j$ times its rate R_{ji}, where $G(k) := \max_{i=1,\dots,k} Y_{ji}$ is the opportunistic gain. Hence, the utility of player i is given by

$$u_i(j, n_j) = \begin{cases} R_{ji}\frac{G(n_j)}{n_j} - \frac{\delta \tilde{P}_j}{n_j}, & \text{if } n_j \le L; \\ -\infty, & \text{otherwise.} \end{cases} \tag{8.7}$$

By the law of iterated logarithm we know that $G(k)/k$ converges to 0.

In particular,

- for the Gilbert channel [83] in which Y_{ji} can take two values, say a and b with $b \ge a$ and with corresponding probabilities p and $1-p$, we have $G(k) = b(1-p^k) + ap^k$. Let us analyze in which condition the utility is always monotonically decreasing:

$$\frac{\partial u_i(j,k)}{\partial k} = \frac{-bR_{ji} + \tilde{P}_j\delta + (a-b)p^k R_{ji}(k \ln p - 1)}{k^2} \le 0 \tag{8.8}$$

  We would like to know the value of pricing δ in which the monotonically decreasing property maintains. Hence,

$$\delta \le \min_{j,i} \frac{R_{ji}\left(b + p^k(b-a)(k \ln p - 1)\right)}{\tilde{P}_j}, \tag{8.9}$$

- choose the distribution of the mean SNR as a bi-modal distribution either SNR_1 or SNR_2 with equal probability. If the instantaneous rate R is linear in the instantaneous SNR, i.e. $R = W \times SNR$, then the relative fluctuations $\{Y_{ji}\}$ have an exponential distribution, and the gain factor can be derived in closed form as $G(n_j) = \max_{i=1,\dots,n_j} Y_{ji} = \sum_{i=1}^{n_j} 1/i$ [82]. The harmonic numbers are given by $H^l(k) = \sum_{i=1}^{k} 1/i^l$ with $H(k) = H^1(k)$. It is suitable for both symbolic and numerical manipulation. The monotonically decreasing property requires the following

$$\delta \le \min_{j,i} \frac{R_{ji}(H(n_j) - n_j\psi(n_j+1))}{\tilde{P}_j} \tag{8.10}$$

  where $\psi(k)$ is the logarithmic derivative of the gamma function, given by $\psi(k) = \Gamma'(k)/\Gamma(k)$. Denote $\Delta(k) = H(k) - k\psi(k+1)$. Figure 8.1 plots how $\Delta(k)$ changes with respect to k. $k = 1$ minimizes $\Delta(k)$ which is $\Delta(1) = 2 - \frac{\pi^2}{6} = 0.355$.

8.3 THE ASSOCIATION PROBLEM AS A CROWDING GAME

Theorem 8.3.1 *Consider the association problem described in Section 8.2.1. Then the conclusions of Theorem 2.5.1 hold.*

Proof 8.3.1 *The game described in Section 8.2.1 satisfies the conditions of Theorem 2.5.1 except possibly two condition.*

1. *If for some mobile i and BS j, we have*

$$W_{ji} \log\left(1 + \frac{|h_{ji}|^2 P_{ji} d_{ji}^{-\alpha}}{\rho}\right) < \delta \tilde{P}_j$$

   then the utility of player i to associate with BS j increases with the number n_j that associate to that BS. Let H_1 be the set of pairs (i, j) that have this property.

2. *In the [80], if a resource is available to one player then it is available to all players. Let H_2 be the set of pairs (i, j) for which j is not available for i.*

Let $H = H_1 \cup H_2$. Consider a new game in which all BSs are accessible to all players. We set $u_i(j, n_j) = -1$ for all $(i, j) \in H$. This is a crowding game that satisfies the conditions of Theorem 2.5.1. Moreover, any best response sequence in the original game is also a best response in this game since any player i will never chooses a BS j with $(i, j) \in H$ as a best response since choosing not to connect at all gives a strictly better utility (of zero). This establishes the proof.

8.4 COMPUTATIONAL RESULTS

In this section, we show the computational results that are performed in the context of crowding games for non-cooperative association of mobiles to access points.

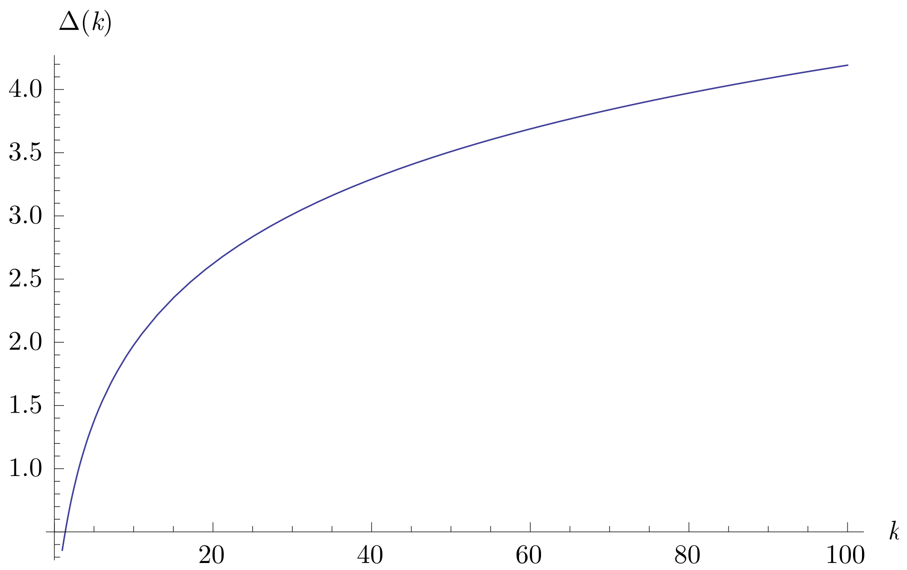


Figure 8.1: The change of Δ with respect to number of mobiles that share the same BS.

The mean of any variable x was calculated by Monte Carlo simulations by running the algorithm for different generated values x for some iteration number t and taking the mean of the result, which can be given by

$$\begin{aligned} x_s(i+1) &= x_s(i) + x, \quad i = 1, \dots, t \\ \bar{x} &= \frac{x_s}{t}. \end{aligned} \tag{8.11}$$

8.4.1 *Scenario 1: The Rayleigh Fading and Path Loss Model*

The deployment scenario in the Figure 8.3 and 8.4 is considered to be a small cell network context instead of macro or micro cells. Clearly, the general term "small cell networks" covers a range of radio network design concepts which are all based on the idea of deploying BSs much smaller than typical macro cell devices to offer public or open access to mobile terminals [10]. Therefore, we consider the deployment of BSs as random rather than a hexagon-type. The cellular network model consists of BSs arranged according to uniform distribution of r points over an area A in the Euclidean plane. Also, we consider an independent collection of mobile users, located according to uniform distribution of m points over the same area A. In MATLAB, we used the following code to produce the collection of BSs and mobiles:

```
pointsOfBSs = sqrt(A)*rand(r,2);
xOfBSs = pointsOfBSs(:,1); % x axis
yOfBSs = pointsOfBSs(:,2); % y axis
pointsOfMobiles = sqrt(area)*rand(m,2);
xOfMobiles = pointsOfMobiles(:,1); % x axis
yOfMobiles = pointsOfMobiles(:,2); % y axis
```

We also assume that within 200 meters a BS is deployed. The area over which the BSs and mobiles are distributed is supposed to increase as $A = (200r)^2\ m^2$. Furthermore, we enumerate the BSs and mobiles according to the distance between the corresponding node (BS, mobile) and the origin assumed to be $(0,0)$ (Figure 8.2).

The path loss model is supposed to be in the form $P_r = P_t(1+d)^{-\alpha}$ where P_r is the received power while the transmission power is P_t.

In Figure 8.3 and 8.4, we set $W_{ji} = 1Mhz$, $P_{ji} = 32mW, \forall j \in \Sigma, \forall i \in M$, and $\rho = 10^{-12}$, $\tilde{P}_j = 12W, \forall j \in \Sigma$. All channels were assumed to be subject to slow varying Rayleigh fading which is a result of a circularly symmetric complex Gaussian random variable with zero mean and unit variance. Moreover, we adjust the simulations such that the minimum SNR cannot be lower than $-4dB$, i.e. $SNR_{min}(dB) = -4dB$. Moreover, the multiple access model is assumed to be TDMA. Consequently, the upper bound of pricing δ is given by

$$\delta \le \frac{W\log(1+SNR_{min})}{\tilde{P}} = \frac{10^6 \log\left(1+10^{-4/10}\right)}{12}, \tag{8.12}$$

$$\delta \le 40289.6 \tag{8.13}$$

In Figure 8.3 and 8.4 the change of mean utility, throughput and number of mobiles that share the same BS of player 1[1] ($\bar{u}_1$, $\bar{\theta}_1$ and $\bar{n}$ respectively) with regards to pricing and number of mobiles, respectively are plotted. It is an

1 Without lose of generality, in all simulations, we plot the functions according to player 1. The same characteristics are valid for each player.

inevitable result that mean utility in equilibrium decreases while the pricing goes up. But mean throughput, conversely, increases.

In equilibrium, mean throughput depends on δ. Let us consider the following representation of the utility of player 1,

$$u_1(\delta, m, r) = \frac{c_1(m,r) - \delta \tilde{P}}{n(m,r)} \tag{8.14}$$

where n represents the mean number of mobiles that are served by the same BS with player 1 and c_1 is the capacity of player 1. Notice that the throughput of player 1 is $\theta_1(m,r) = c_1(m,r)/n(m,r)$ which depends on m and r but not on δ. However, in equilibrium the throughput is a function of δ. Mean throughput in equilibrium of player 1 is given by

$$\bar{\theta}_1(\delta, m, r) = \bar{u}_1(\delta, m, r) + \frac{\delta \tilde{P}}{\bar{n}(m,r)} \tag{8.15}$$

in which mean number of mobiles that share the same BS with player 1 $\bar{n}$ does not depend on δ (we observe this result from Figure 8.3).

Let us now answer the question why does mean throughput increase while mean utility decreases? (recall Figure 8.3). In fact, consider the issue reversely. The payoff (throughput) of the player can not compensate the cost $\delta\tilde{P}/\bar{n}$ while the pricing is augmented. Thus, the profit (utility) of the player diminishes.

From Figure 8.4 we conclude that for a specific value of pricing $\delta = 3 \times 10^4$ while m increases $\bar{u}_1$ and $\bar{\theta}_1$ diminishes as well as $\bar{n}$ increases. Since $r = 5$, $\bar{n}$ remains constant for $m \leq 5$. This means that there are more resources than players. Consequently, the players tend to be alone in one resource resulting in one player per resource: $\bar{n} = 1$. On the other hand, since the capacity depends on m mean throughput and consequently mean utility decreases while m is increased.

8.4.2 *Scenario 2: The Bi-modal Distribution of Mean SNR*

We suppose that mean SNR possesses a bi-modal distribution. Hence, the SNR takes a component from $\mathcal{G} = \{SNR_1, SNR_2\} = \{-4dB, 2dB\}$ which occurs with probability 0.5. Moreover, we set $W_{ji} = W, \forall j, i$ and $\tilde{P}_j = 12, \forall j$. Let us calculate the upper bound of pricing from (4) and (8) for TDMA and HSDPA cases, respectively

$$\delta_{TDMA} \leq \frac{W \log(1 + 10^{-4/10})}{12} = 0.0403W, \tag{8.16}$$

$$\delta_{HSDPA} \leq \frac{W \times 10^{-4/10} \Delta(1)}{12} = 0.0118W \tag{8.17}$$

8.4.2.1 *TDMA Case*

Figure 8.5 plots the curve of $\bar{u}_1$ and $\bar{\theta}_1$ with respect to δ for different values of $r = \{3, 4, 5\}$. Furthermore, we set $m = 7$, $W = 1$ and $L = 8$. The figure demonstrates the same characteristic of mean utility and throughput in equilibrium like in Figure 8.3. In addition, we observe that while the number of resources increases, mean utility and throughput in equilibrium also go up.

Figure 8.6 depicts the change of $\bar{u}_1$ and $\bar{n}$ with respect to L for $\delta = \{0, 2 \times 10^4, 4 \times 10^4\}$ where $r = 3$, $m = 12$ and $W = 10^6$. In the figure, the region $L < 4$ implies that some players can not join to the game. For example, let $L = 2$. If each BS serves to 3 mobiles, there will be 6 mobiles receiving transmission. Within the region $L \leq 4$, we observe from the figure that $\bar{n} = L$. For $L > 4$, $\bar{n}$ remains constant which is due to the fact that the ratio $\lceil m/r \rceil$ gives mean number of mobiles served by the same BS.

8.4.2.2 *HSDPA Case*

The interpretations of Figure 8.7 and 8.8 are the same like for Figure 8.5 and 8.6, respectively.

However, if compare $\bar{u}_1$ and $\bar{\theta}_1$ of TDMA and HSDPA, we conclude that in case of HSDPA, mean utility and throughput in equilibrium is always better than that of TDMA. For example, in case of TDMA (Figure 8.5) for $\delta = 0.005$ and $r = 3$, player 1 has $\bar{u}_1 = 0.4463$ and $\bar{\theta}_1 = 0.5512$ while in HSDPA (Figure 8.7) the same player gains $\bar{u}_1 = 0.9697$ and $\bar{\theta}_1 = 0.9960$.

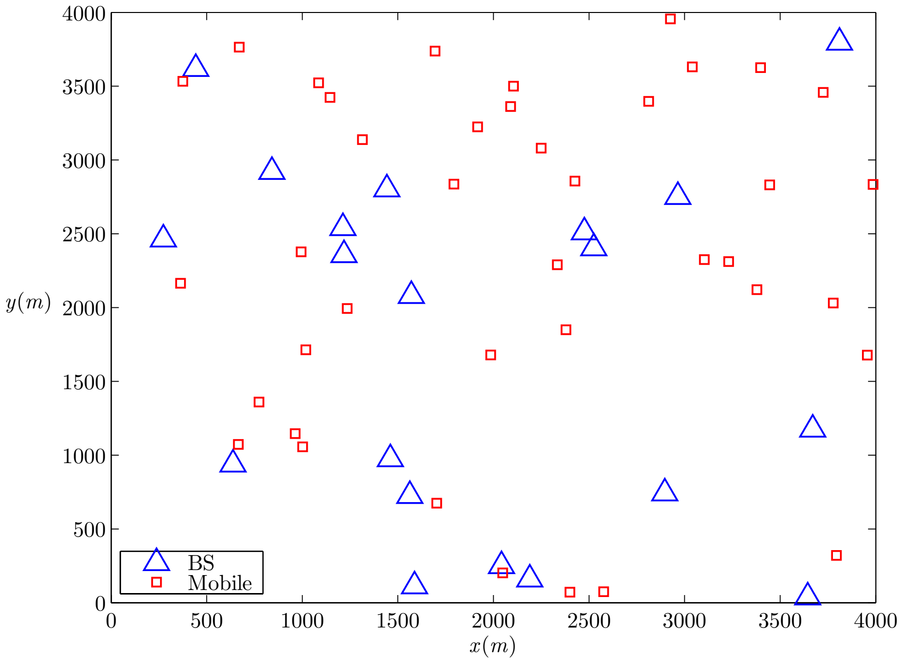


Figure 8.2: Distribution of BSs and mobiles in 2D plane. $r = 20$, $m = 40$

8.5 CONCLUSION

We studied the association problem of mobiles in wireless networks in the downlink transmission. We considered the problem as a crowding game in which the utility of a player is specific to player and a function of the number of the players that share the same resource. The utility considered to be a difference of a payoff and cost. Using the tools of crowding game we analyzed the problem for the TDMA and HSDPA cases. The throughput was taken as payoff. The cost has considered to be a function of operational power cost of a BS. From the computational results, we observed for several metrics that

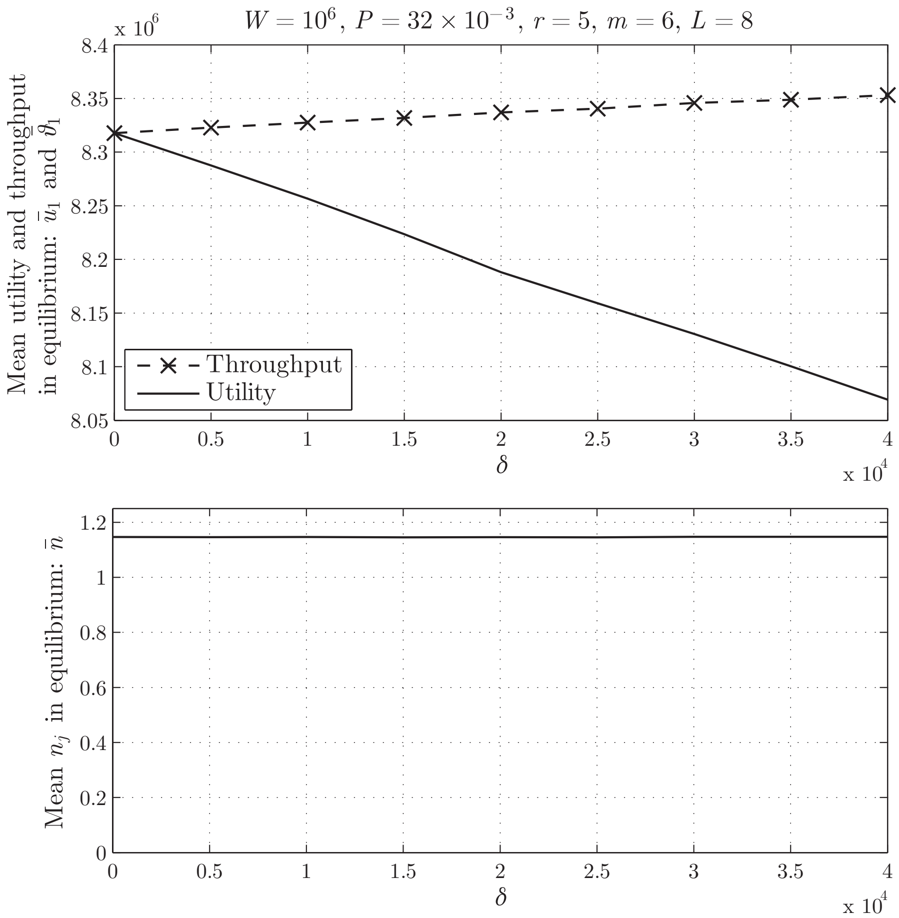


Figure 8.3: Mean utility, throughput and number of mobiles sharing the same BS with respect to pricing in case of Rayleigh and path loss model.

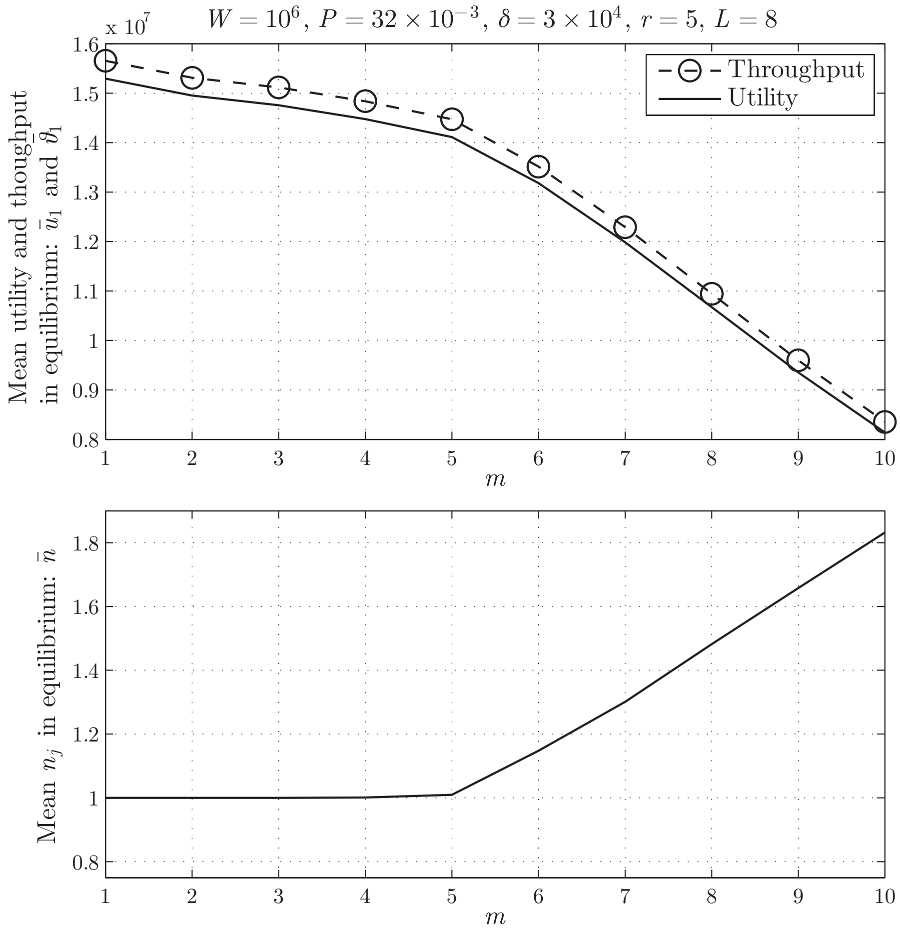


Figure 8.4: Mean utility, throughput and number of mobiles sharing the same BS with respect to m in case of Rayleigh and path loss model.

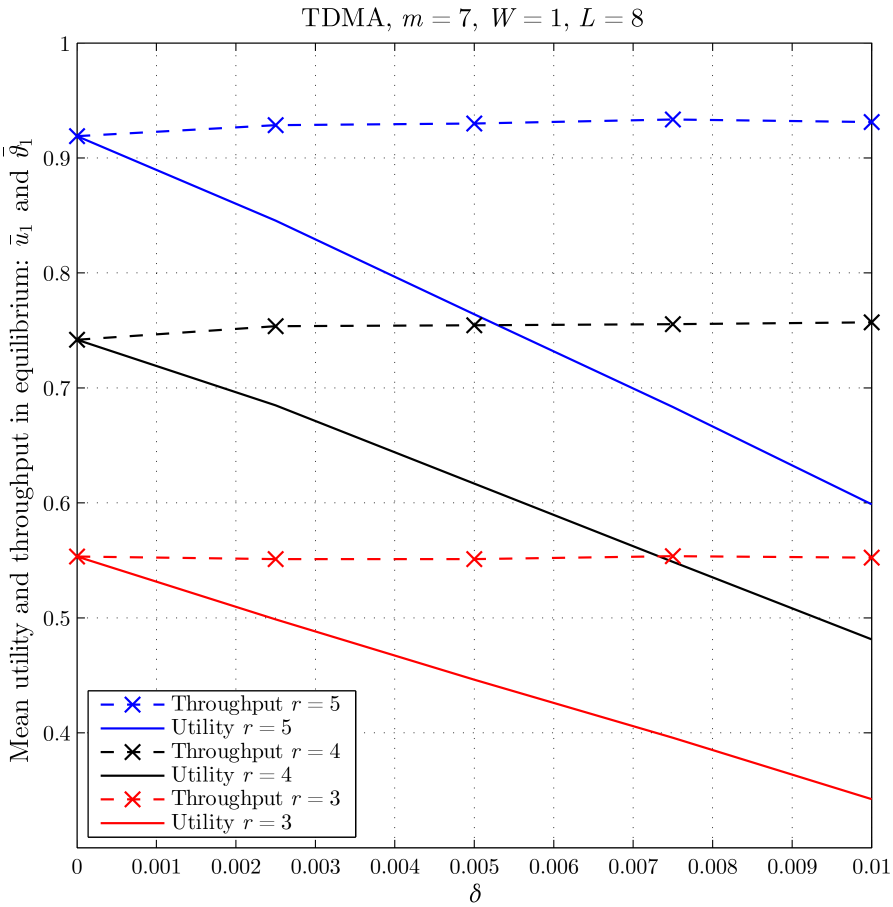


Figure 8.5: The effect of pricing in case of TDMA for Scenario 2: Mean utility and throughput for different values of r.

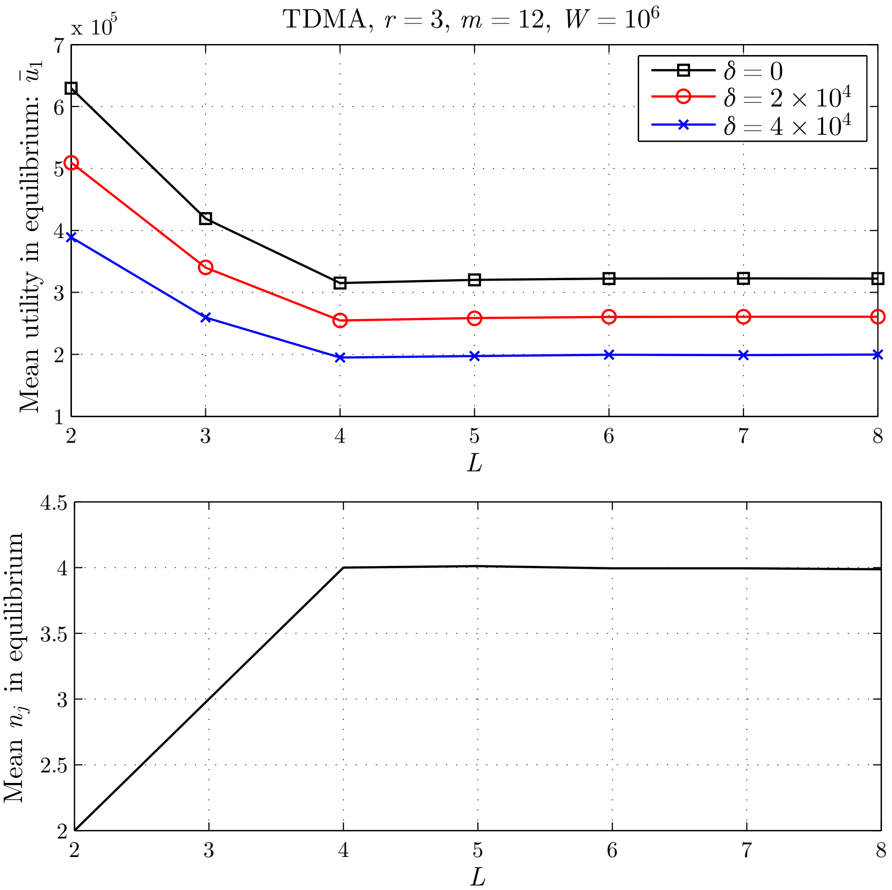


Figure 8.6: Mean utility, throughput and number of mobiles sharing the same BS with respect to L in case of TDMA for Scenario 2.

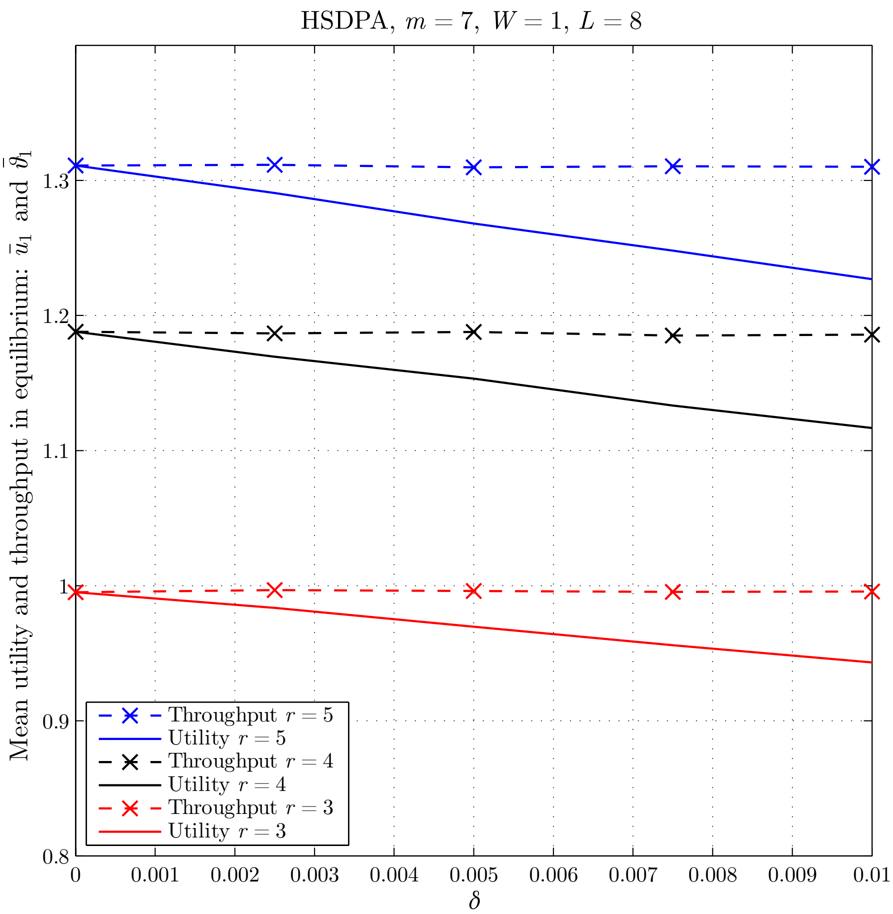


Figure 8.7: The effect of pricing in case of HSDPA for Scenario 2: Mean utility and throughput for different values of r.

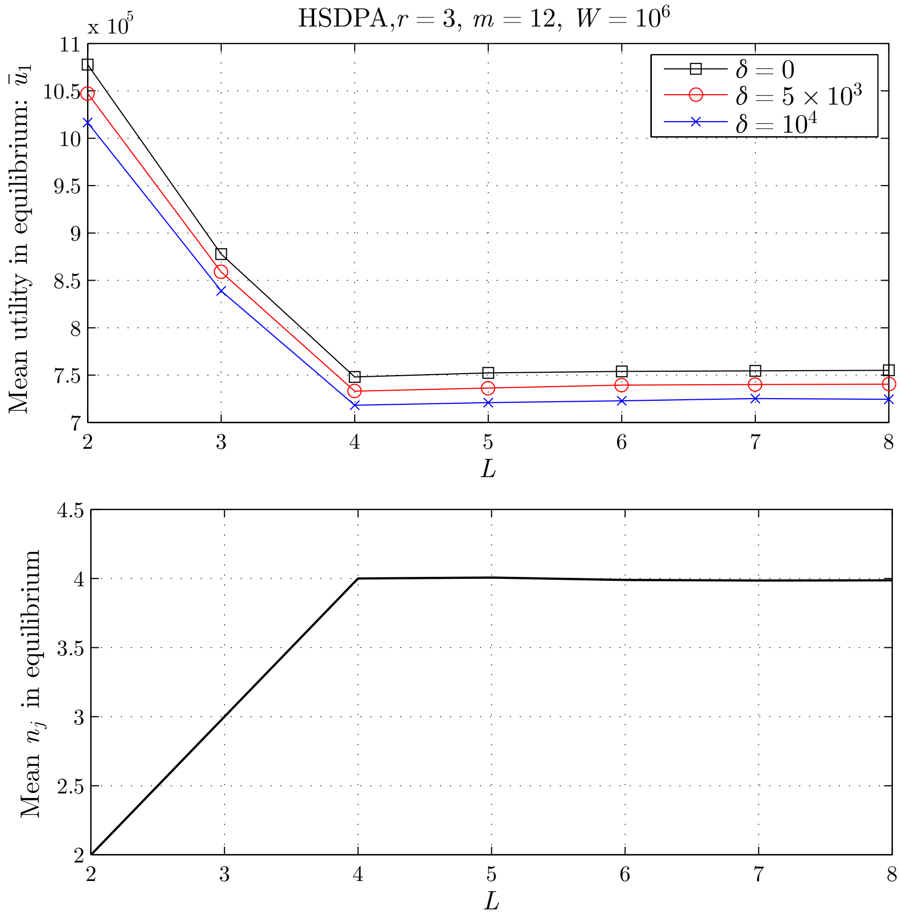


Figure 8.8: Mean utility, throughput and number of mobiles sharing the same BS with respect to L in case of HSDPA for Scenario 2.

mean utility and throughput in equilibrium that a player gains are always better when the used multiple access method is HSDPA.

9

PARTNER SELECTION FOR COOPERATIVE RELAYING

9.1 INTRODUCTION

Let be considered a scenario where a set of users, or agents, aim at transmitting their own data to a common destination. This scenario may correspond for instance either to mobiles in a single cell in uplink mode or to a wireless sensor network with a single sink. The reference protocol used to transmit the whole information to the common destination relies on a layer 2 MAC approach which divides the resources and schedule the allocation of each resource to each agent. Even if the agents are all in the range of the destination, cooperative transmissions can significantly improve the efficiency of the system. The efficiency can be measured by the total capacity, the total energy or by any criteria related to the QoS such as the packet error rate. Here, we focus on the outage probability. Transmission powers and capacity needs for each node are constant and cooperative transmission is used to reduce the outage probability. The formulation of the problem is based on the relay channel model as described in the early work of Laneman [89]. In the simplest approach, each source agent requires the help of another agent to improve its transmission by forming togeter an equivalent relay channel. We consider the special case where the agents associate by pairs such that each agent relays the data of the other. In the framework of network information theory, these nodes form a cooperative multiple access channel (CMAC).

Albeit the whole network can be considered as a large size CMAC, we rather propose to form several small coalitions. In this work, these small coalitions are even limited to pairs. In order to optimize the overall performance of the network, the *partner selection* process is therefore crucial. Each agent aims here at finding a "good" partner in order to exploit efficiently the spatial diversity achieved with cooperation. This process can be identified as a *matching problem*. In the game theoretic sense where the players are "strategic decision makers", the partner selection process appears to be an example of the *stable roommates problem*. *Stable matching theory* was established by Shapley and Gale by their seminal work [85]. Gale and Shapley analyzed matching at an abstract, general level. They used marriage as one of their illustrative examples. How should ten women and ten men be matched, while respecting their individual preferences? The main challenge involved designing a simple mechanism that would lead to a stable matching, where no couples would break up and form new matches which would make them better off. The solution–the Gale-Shapley "deferred acceptance" algorithm–was a set of simple rules that always led straight to a stable matching.

The stable marriage problem is an example of a so called *two-sided market* due to the gender issue. However, this kind of problem can be broaden to matching problems with no gender issue and are referred to as *one-sided* market. This is the case of the stable roommates problem. In this problem, each person targets matching with the best partner to share a room. We shall show that the partner selection problem in the context of cooperative relaying can be studied as a one-sided market. By determining the ranking rule of partners, we seek a stable matching. Although a stable matching is always possible in stable marriage problems, this is not the case in stable roommates problem. Further, even if a stable matching exists, there was no polynomial-time algorithm to find it until the recent work of Irving [86].

9.1.1 *Related Work*

The partner selection problem in cooperative communications has been already studied in the literature in [90] and [88]. More recently, Lee and Lee [91] extended the problem to relay assignment for multi-user DF-AF cooperative wireless networks while in [92] the authors proposed a new selection method which requires neither error detection methods at relay nodes nor feedback information at the source. The thesis in [93] includes many new approaches for matching the cooperating agents. In [94] the authors study the relay selection in heterogeneous relay networks, i.e. where relays with different protocols can co-exist. While varying algorithms are proposed in these papers, none of them uses the coalition formation principle. Cooperative game theoretic approaches exist in the literature for wireless problems, where "coalitions formation" problems are studied. A coalition can be of any size, from a single player to all players. For instance, [95] studies coalition formation of mobiles and destinations. Another coalitional game approach is formulated in [96] to examine how coalitions can form in a distributed manner, as well as possible resource allocation methods within groups. Moreover, in [97], a Markov chain model is proposed to investigate the stability of the coalitional structures.

9.1.2 *Our Contribution*

We propose the use of matching theory and more specifically the stable roommates problem, to solve the partner selection problem in cooperative transmissions. This work shows that this formalism is perfectly convenient and further a natural tool for this problem. The reason is that it provides a fair and stable sectioning process if we consider the source nodes (agents) as strategic decision makers. Here, we use this tool to analyse the partner selection problem.

9.2 PROBLEM FORMULATION AND SETTINGS

The system model is depicted in Fig.9.1. Let $N = (1, \ldots, n)$ be the set of *players* and d the *destination* node, or base station (BS). In this work, we rather target the case where the players are in a common area, sufficiently far from the BS such that the inter-mobile channels are statistically better than the mobile-destination channels. This is a favourable situation for mobile cooperation. This assumption is not always necessary and most of our results apply for any scenario. However, especially in the CMAC case, this assumption may drive our settings.

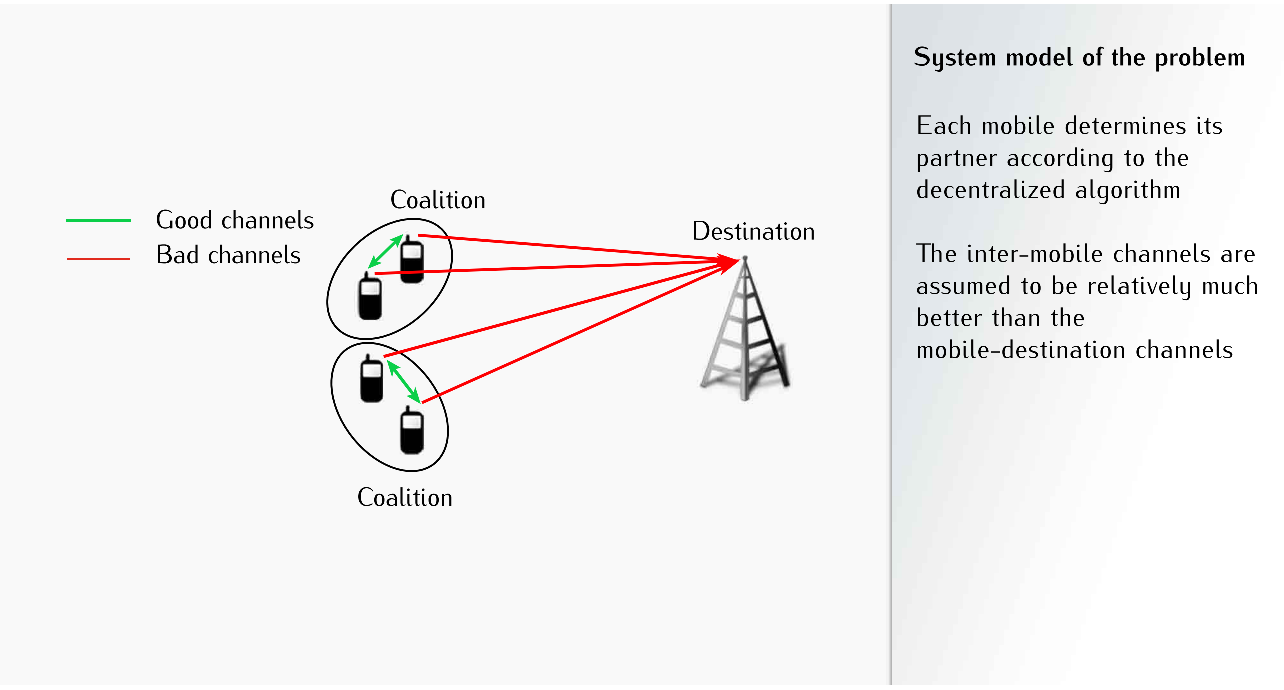


Figure 9.1: The problem.

In the default setting a resource unit (RU) is allocated to each player. These blocks may be time slots, frequency channels or time/frequency RU in LTE. We assume a perfect orthogonality between the blocks. Without loss of generality and for the sake of simplicity, we consider that each player receives a unique RU, and all have the same capacity.

9.2.1 *Cooperative Relaying*

Albeit it cannot achieve the upper bound capacity, the decode-and-forward strategy is nearly capacity achieving when the source-relay channel is much better than the others which is the case in our scenario here. We assume that each node is equipped only with one antenna. Interference-free uplink is considered where the transmissions of bipartite coalitions (in the sequel, we introduce these coalitions) do not interfere each other. Half-duplex transmission mode is applied in the communication between mobiles. There are adequate reasons for limiting the communication in half-duplex mode; because of insufficient electrical isolation between the transmit and receive circuitry, a terminal's transmitted signal drowns out the signals of other terminals at its receiver input [89]. All channels are assumed to be subject to slow varying block fading.

The physical channel between node i and j has the following instantaneous signal to noise ratio (SNR): $\gamma_{i,j} = \Gamma_{i,j}|h_{i,j}|^2$, where $|h_{i,j}|$ is the Rayleigh distributed fading coefficient with variance $\sigma_{i,j}^2$. Moreover, we assume that $\forall i, j \in N, \sigma_{i,j} = 1$. The term $\Gamma_{i,j}$ is the average SNR and is modelled as following:

$$\Gamma_{i,j} = \left(\frac{P}{N_0}\right) S_{i,j} d_{i,j}^{-\beta}, \tag{9.1}$$

where

- P is transmission power which is equal for all mobiles.
- $S_{i,j}$ is a zero-mean log-normal shadowing component with standard deviation σ_S.
- $d_{i,j}$ is the distance between nodes i and j as well as β is the path loss exponent.

The SNR in the transmitter part is P/N_0. Both the fading and shadowing components are i.i.d for each $\{i,j\}$ pair. The shadowing components are constant for a given network realization and are assumed to be reciprocal, $S_{i,j} = S_{j,i}$.

9.2.2 *The Protocol*

We consider the following decode-and-forward protocol:

S_1: *Source sends its data to relay and destination.*

S_2: *Relay tries to decode. If relay succeeds, then source and relay resend the packet. If relay fails, source resends alone.*

S_3: *Destination combines all copies of data.*

We assume here that the source and relay transmit simultaneously without phase synchronization during $\mathbf{S_2}$, and these transmissions do not interfere each other.

In case of repetition coding at the relay, the mutual information (bps/Hz) can be readily shown to be

$$\mathscr{I}_i = \begin{cases} \frac{1}{2}\log\left(1+\gamma'_{i,d}+\gamma''_{i,d}+\gamma_{j,d}\right) & \text{if } \frac{1}{2}\log\left(1+\gamma_{i,j}\right) > R_i \\ \frac{1}{2}\log\left(1+\gamma'_{i,d}+\gamma''_{i,d}\right) & \text{otherwise.} \end{cases} \tag{9.2}$$

Here, source is i, and relay is j. Relay can retransmit the data of source with rate R_i. $\gamma'_{i,d}$ and $\gamma''_{i,d}$ are the instantaneous SNRs of source-destination transmissions in $\mathbf{S_1}$ and $\mathbf{S_2}$ defined in the protocol as well as $\gamma_{j,d}$ is the instantaneous SNR of relay-destination transmission in $\mathbf{S_2}$. $\gamma'_{i,d}$ and $\gamma''_{i,d}$ are assumed independent.

Remark 9.2.1 *Here, we focus only on the "selective" decode-and-forward transmission where the relay station only decodes the data and retransmits it to the destination. One can improve the context of this work by applying the compress-and-forward transmission as well as MIMO attributes to the nodes.*

9.2.3 *Outage Probability Calculation*

9.2.3.1 *Direct Transmission–No Cooperation*

When a player i stays alone the outage probability is given by [89]: $p_{\mathscr{O}} = 1 - \exp\left(-\frac{2^{R_i}-1}{\sigma_d^2\Gamma_{i,d}}\right)$. This is a result when the player utilizes all degrees of freedom.

9.2.3.2 *Cooperation*

The outage probability can be calculated as following: $p_{\mathscr{O}} = p_{\mathscr{C}}p_{\mathscr{O}|\mathscr{C}} + (1 - p_{\mathscr{C}})p_{\mathscr{O}|\mathscr{N}\mathscr{C}}$, where

- $p_{\mathscr{C}}$ is the probability of successfully reception of source's data at the relay given by

$$p_{\mathscr{C}} = \Pr\left[\frac{1}{2}\log(1+\gamma_{i,j}) > R_i\right] = \Pr\left[|h_{i,j}|^2 > \frac{2^{2R_i}-1}{\Gamma_{i,j}}\right]$$
$$= \exp\left(-\frac{2^{2R_i}-1}{\sigma_m^2\Gamma_{i,j}}\right) \quad (9.3)$$

Note that $|h|^2$ follows an exponential distribution.

- $p_{\mathscr{O}|\mathscr{C}}$ is the conditional probability of outage in destination when the relay decodes correctly the source's data:

$$p_{\mathscr{O}|\mathscr{C}} = \Pr\left[\frac{1}{2}\log\left(1+\gamma'_{i,d}+\gamma''_{i,d}+\gamma_{j,d}\right) < R_i\right]$$
$$= \Pr\left[\gamma'_{i,d}+\gamma''_{i,d}+\gamma_{j,d} < 2^{2R_i}-1\right], \quad (9.4)$$

where $\gamma'_{i,d}$ and $\gamma''_{i,d}$ are independent. The sum of k exponential random variables $X = \sum_{i=1}^{k} X_i$ where each X_i has different mean λ_i, follows *hypo-exponential* distribution of which the probability density function is given by

$$f_X(x) = \sum_{i=1}^{k}\lambda_i\left(\prod_{j=1,j\neq i}^{k}\frac{\lambda_j}{\lambda_j-\lambda_i}\right)\exp(-\lambda_i x). \quad (9.5)$$

The sum $\gamma'_{i,d}+\gamma''_{i,d}+\gamma_{j,d}$ follows a 3rd order hypo-exponential distribution with means $\lambda_1 = \lambda_2 = (\Gamma_{i,d}\sigma_d^2)^{-1}$ and $\lambda_3 = (\Gamma_{j,d}\sigma_d^2)^{-1}$. Therefore,

$$p_{\mathscr{O}|\mathscr{C}} = 1 - \frac{e^{-\frac{2^{2R}-1}{\sigma_d^2\Gamma_{i,d}}}\Gamma_{i,d}\left(\sigma_d^2\Gamma_{i,d}+2^{2R}-1\right)}{\sigma_d^2\left(\Gamma_{i,d}-\Gamma_{j,d}\right)^2}$$
$$+ \frac{e^{-\frac{2^{2R}-1}{\sigma_d^2\Gamma_{i,d}}}\Gamma_{j,d}\left(2\sigma_d^2\Gamma_{i,d}+2^{2R}-1\right)}{\sigma_d^2\left(\Gamma_{i,d}-\Gamma_{j,d}\right)^2} - \frac{\Gamma_{j,d}^2 e^{-\frac{2^{2R}-1}{\sigma_d^2\Gamma_{j,d}}}}{\left(\Gamma_{i,d}-\Gamma_{j,d}\right)^2}. \quad (9.6)$$

- $p_{\mathscr{O}|\mathscr{N}\mathscr{C}}$ is the conditional outage probability when relay fails and source repeats its own data:

$$p_{\mathscr{O}|\mathscr{N}\mathscr{C}} = \Pr\left[\frac{1}{2}\log\left(1+\gamma'_{i,d}+\gamma''_{i,d}\right) < R_i\right]$$
$$= \Pr\left[\gamma'_{i,d}+\gamma''_{i,d} < 2^{2R_i}-1\right]$$
$$= 1 - \left(\frac{2^{2R}-1}{\sigma_d^2\Gamma_{i,d}}+1\right)e^{-\frac{2^{2R}-1}{\sigma_d^2\Gamma_{i,d}}} \quad (9.7)$$

9.2.4 *Preference Functions*

Here, we intend to show how a mobile designs its *preference list* which is the ranking of possible partners (including itself) from the most preferable to the least.

Long Term–Outage Probability Ranking: The motivation here is to determine a long-term partnership. The channel state information has statistical characterization, for example, the type of fading distribution. We can utilize this characterization in determining the partnership. By knowing the variance of the fading each mobile is able to calculate the outage probability. Thus, a mobile evaluates its partners by means of that metric. The preference list of each mobile is composed of ranking the possible partners according to the outage probability in a way that

> *the first ranked provides the lowest outage probability*

In that setting, each mobile also ranks itself in the preference list.

9.2.5 *Decentralized Approach to the alg-*IRVING

We consider that each mobile is able to communicate in a separated control channel to look for partners.

Algorithm 6 Decentralized *alg*-IRVING

Learn:

- Each mobile listens to the other partners continuously or randomly when each of them broadcast his averaged path loss and shadowing.
- Each mobile maintains a preference list from the messages sent by the other mobiles.

Phase 1:

- Randomly, each mobile does a bid until accepted by a partner in his preference list.
- Each mobile deletes some partners from his preference list according to **Reductions** procedure.

Phase 2:

- Each mobile broadcasts his second player of preference list to the other mobiles
- Each mobile performs *rotation* according to the received message. In case of an even party, then each mobile transmits alone without cooperation.
- Each mobile runs **Reductions** procedure according to the rotation, and continues Phase 2 until having only one partner in the preference list.

9.3 GLOBAL OPTIMUM

In this section, we analyse the problem in terms of global optimum and fairness criterion. The aim is to measure how much the stable matching is far from the global optimum.

9.3.1 *Minimum Total Outage Probability*

In terms of global outage minimization, the problem can be considered as a special case of the classical set-partitioning problem, which aims at finding the best partition of the N players which minimizes the total outage probability. In the considered problem, the partitioning is made only of singleton and bipartite coalitions:

$$\min_{\mathcal{M}} \sum_{S \in \mathcal{M}} \sum_{i \in S} p^i_{\mathcal{O}}. \tag{9.8}$$

In the computational results, we find the optimal solution with a *brute-force search* which enumerates all possible solutions and chooses the one which produces the lowest total outage probability.

9.4 COMPUTATIONAL RESULTS

This section includes the comparison of partner selection for different paradigms: stable matching, global optimum, and random selection. In case of random selection, the matching of mobiles is performed randomly. The locations of mobiles are denoted as $\Phi = (x_i, y_i)_{i \in N}$ and the destination (x_d, y_d) such that the distance between node i and j is given by $d_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$. Also, we assume that x_i and y_i follow uniform distribution on some area.

We generate randomly a rate R according to uniform distribution for each mobile; the shadowing component S follows log-normal distribution. In the calculations, the results are obtained per mobile. We utilize *the law of total probability* which is formulated as $\bar{p}_{\mathcal{O}}(\Theta) = \mathbf{E}_{R,S,\Phi}[\Pr[\mathcal{O}|R,S,\Phi]]$, where $\bar{p}_{\mathcal{O}}(\Theta)$ is called as “average outage probability” for any case

$$\Theta = \{\text{Matching, Global Optimum, Random Selection}\}$$

which can be calculated as the ergodic mean over (R, S, Φ), i.e.

$$\bar{p}_{\mathcal{O}}(\Theta) = \frac{1}{T} \sum_{t=1}^{T} \Pr[\mathcal{O}|R,S,\Phi],$$

where T is the number of iterations, R_t is the rate, S_t is the shadowing component, and Φ_t denotes the locations of mobiles in iteration t.

9.4.1 *Test-bed*

In all simulations, we consider the block fading channels with Rayleigh distribution. The variance of the fading is assumed to be 1. The shadowing variance $\sigma_S = 8$ dB for all links, the path loss exponent $\beta = 3$. The locations $\Phi = (x_i, y_i)_{i \in N}$ of mobiles follow uniform distribution within $x \in [85, 100]$, $y \in [85, 100]$ like a bagel where the location of destination is chosen as $(x_d, y_d) = (50, 50)$ which could be seen as the center of the bagel. Moreover, additive white Gaussian noise channel is considered in all simulations.

9.4.2 *Comments and Corollaries*

Figure 9.2 – Average outage probability with respect to average received SNR: The rate of the mobiles is assumed to be uniformly distributed within

$R \in [1,2]$. Also, the number of mobiles is fixed to $n = 6$. First, we observe that the cooperation is beneficial to the mobiles for the considered conditions. Note that the result obtained by stable-matching is near to global optimum, and it is better than random selection; for example, there is 3.25 dB gain when average outage probability is equal to 10^{-4}.

Figure 9.3 – Average outage probability with respect to the number of mobiles: Average received SNR is equal to 30 dB, and the distribution of the rate is chosen as $R = [1,3]$. This figure shows that the cooperation is always beneficial on average. Increasing number of mobiles has a positive effect since the probability of finding a good partner increases. Actually, increasing the number of mobiles in some area corresponds with the increasing the intensity of mobiles homogeneously. Observe that there exists a critical value of n that can be seen as a *saturation* after which the average outage probability becomes constant. For different scenarios, the saturation point changes. We observe here the fact that random selection is not useful compared to the stable-matching result. For example, when $n = 10$, the average outage probability is equal to 6.50×10^{-3} and 4.36×10^{-3} in case of random selection and stable-matching, respectively.

Figure 9.4 – Probability of cooperation with respect to the number of mobiles: Here, we depict the probability of cooperation of a mobile with another one. The transmitted SNR is $\frac{P}{N_0} = (70, 75, 80, 85, 90)$ dB. We set the rate to be distributed within $R = [1,3]$. Observe that with increasing transmitted SNR, the probability of cooperation is getting one. However, it is not so while the number of mobiles decreases. This is due to the fact that the probability of finding good partner is low when the intensity of mobiles decreases on some area.

Remark 9.4.1 *As a concluding remark about the usage of stable matching algorithm as a partner selection method, we can state that the results related to average outage probability and probability of cooperation show the advantage of Irving's algorithm. It is also fair in terms of the dynamics of matching games where there does not exist a pair that would deviate. Therefore, the decentralized version of this algorithm introduced in Section 9.2.5 is very practical for real implementations.*

9.5 CONCLUSION

We formalized the partner selection problem in decode-and-forward relaying favoured to stable roommates problem. The outage probability for a special protocol has been calculated and chosen as the ranking strategy in the preference lists of players. We proposed a decentralized version of Irving's algorithm for partner selection. Further, we compared the coupling of players with global optimum. In computational results, we showed that stable-matching gives near global optimum results. We also depicted the superior advantage of the stable-matching compared to random selection.

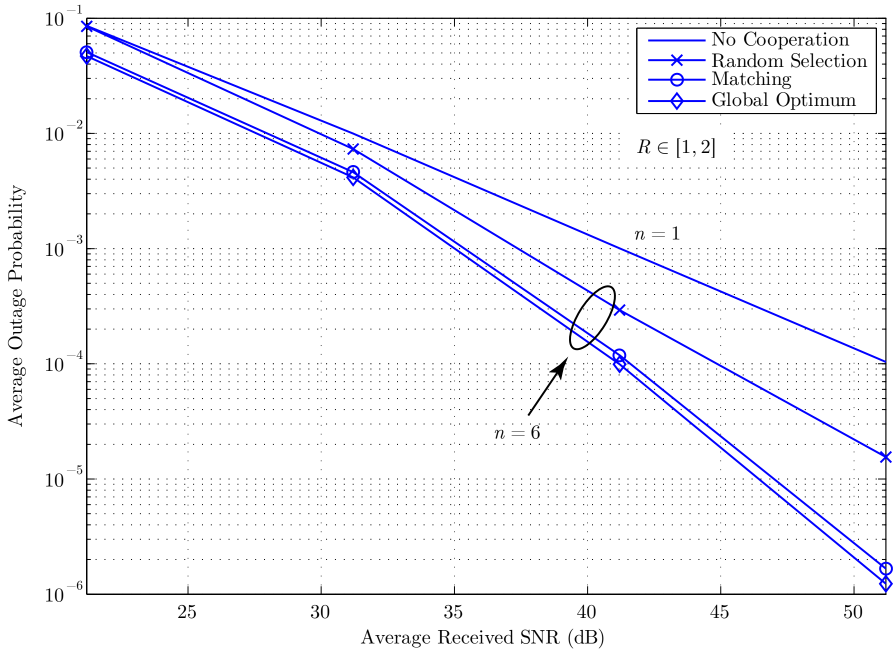


Figure 9.2: Average outage probability with respect to average received SNR.

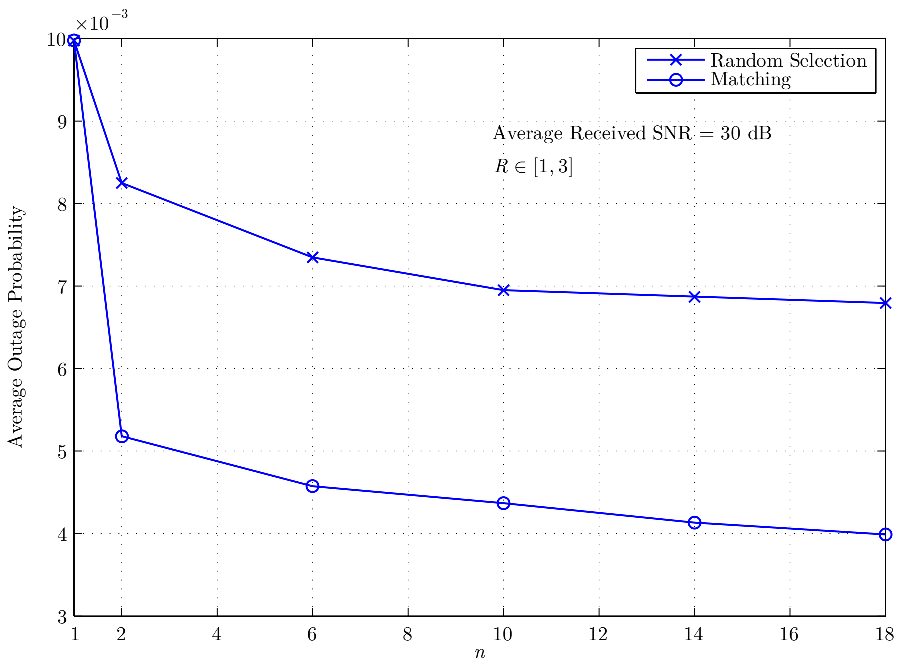


Figure 9.3: Average outage probability with respect to the number of mobiles.

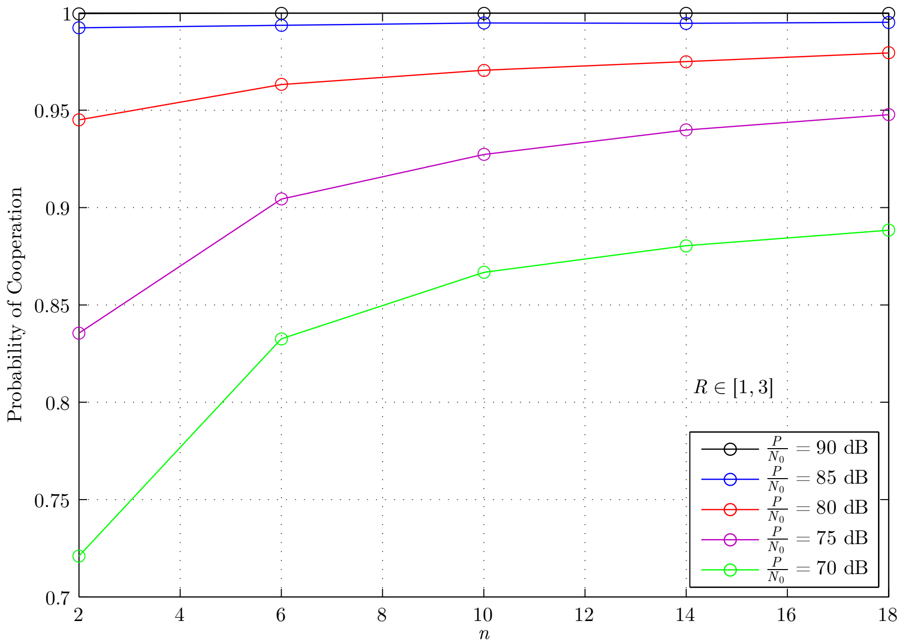


Figure 9.4: Probability of cooperation with respect to the number of mobiles.

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CONCLUSIONS

In Chapter 2, we suggested a decentralized algorithm for finding the Nash stability in a game whenever there exists always at least one. The problem of finding the Nash stability is considered as a non-cooperative game. We consider a *random round-robin* fashion where each player determines its strategy in its turn according to a *scheduler* which is randomly generated for each round. Under this condition, we proved that the algorithm converges to an equilibrium which is the indicator of the Nash stability. Moreover, we answer the following question: Is there any utility allocation method which could result in a Nash-stable partition? We proposed the definition of the Nash-stable core. We analyzed the cases in which the Nash-stable core is non-empty, and prove that in case of the relaxed efficiency condition there exists always a Nash-stable partition.

In Chapter 4, the mobile assignment problem in broadcast transmission in the "green" context is studied. We proposed a centralized optimal recursive algorithm (the HM) as well as a centralized polynomial-time heuristic algorithm (the CC). Further, we developed a distributed approach to the CC algorithm (the DCC), and another distributed one called the NBS algorithm. We also introduced a new algorithm based on group formation games, which we call as the hedonic decision (HD) algorithm. This formalism is constructive: a new class of group formation games is introduced where the utility of players within a group is additively separable and symmetric being a concept in hedonic coalition formation games. Simulation results were used to verify the performance of the algorithms. We realized that the HD algorithm produces near-optimal solutions. On the other hand, we analyzed the mobile assignment problem using a coalitional game approach. We proved that the players of the game form grand coalition and the core of this game is non-empty. Moreover, we also studied the game for some cost sharing methods and showed that in case of the nucleolus the grand coalition is stable, and it minimizes the total cost in broadcast transmission in Chapter 5.

In Chapter 6, we analyzed the cooperation of SPs on switch off operation of BSs in the context of green networking. The homogeneous Poisson p.p. approach to the deployment of BSs has been used in order to study the SINR distribution of SPs. It was proven that scaling the coordinates of $\mathfrak{R}^2$ by $\sqrt{q}$ from the origin of a homogeneous Poisson point process result in a thinned homogeneous Poisson point process with intensity modified by q. The SINR distribution of the original network was derived and by increasing the transmission power by some factor of q, it was proven that the SINR distribution of the thinned network (obtained by scaling the locations of BSs) remains unchanged. Furthermore, in the case of non-cooperating SPs, the SINR distribution is obtained of the original and thinned network of SPs, respectively.

We also found the SINR distribution of cooperation case used in the context of coalition formation of SPs. The operations on the network formed by cooperation are assumed to be run jointly by SPs meaning that they share their resources such that any mobile is tagged to the nearest BS of any SP. The maximal energy saving density of a cooperation is supposed to be the utility of the coalition. We derive the closed form results of the utility.

The cooperation of SPs on switch off operation of BSs in the context of green networking is studied in Chapter 7. The homogeneous Poisson p.p. approach to the deployment of BSs has been used in order to study the SINR distribution of SPs. Furthermore, in the case of non-cooperating SPs, the SINR distribution is obtained of the original and thinned network of SPs, respectively. We also found the SINR distribution of cooperation case used in the context of coalition formation of SPs. The operations on the network formed by cooperation are assumed to be run jointly by SPs meaning that they share their resources such that any mobile is tagged to the nearest BS of any SP. The maximal energy saving density of a cooperation is supposed to be the utility of the coalition. We derive the closed form results of the utility. We compared the utilities of SPs in case of both the allocation based on efficiency and relaxed efficiency. We showed that in case of individual deviations the importance of efficiency is not significant in the side of SPs.

In Chapter 8, we considered the association problem of mobiles as a crowding game in which the utility of a player is specific to player and a function of the number of the players that share the same resource. The utility considered to be a difference of a payoff and cost. Using the tools of crowding game we analyzed the problem for the TDMA and HSDPA cases. The throughput was taken as payoff. The cost has considered to be a function of operational power cost of a BS. From the computational results, we observed for several metrics that mean utility and throughput in equilibrium that a player gains are always better when the used multiple access method is HSDPA.

In Chapter 9, we formalized the partner selection problem in decode-and-forward relaying favoured to stable roommates problem. The outage probability for a special protocol has been calculated and chosen as the ranking strategy in the preference lists of players. We proposed a decentralized version of Irving's algorithm for partner selection. Further, we compared the coupling of players with global optimum. In computational results, we showed that stable-matching gives near global optimum results. We also depicted the superior advantage of the stable-matching compared to random selection.

Part III

OPTIMISATION D’ALLOCATION DES RESSOURCES AUX PETITS RÉSEAUX DE CELLULES: UNE APPROCHE EN RÉSEAU VERT

11

SOMMAIRE DES CONTRIBUTIONS

11.1 SUR LA STABILITÉ NASH DANS LES HÉDONISTES COALITION JEUX FORMATION

La coopération entre les agents (les joueurs) être en mesure de prendre des décisions stratégiques devient un *coalition jeu de formation* quand les joueurs peuvent, pour diverses raisons personnelles souhaitent appartenir un *petite coalition* relative plutôt "grande coalition". Partitionnement des joueurs est donc cruciale dans le contexte du jeu puisque la stabilité de la partition des joueurs résultats de l'équilibre. Dans [24], les auteurs proposent une approche abstraite de la formation de la coalition qui se concentre sur une simple fusion et des règles de stabilité fendus transformation partitions d'un groupe de joueurs. L'outil conceptuel principal est une notion spécifique d'une partition stable. Les résultats sont paramétrées par une relation de préférence entre les partitions d'un groupe de joueurs. D'autre part, un jeu de formation de coalition est appelée à être *hédonique*, si

- *le gain d'un joueur ne dépend que des membres de la coalition à laquelle le joueur appartient*, et
- *les coalitions se forment en raison des préférences des joueurs sur l'ensemble de leurs éventuelles coalitions*.

La définition de la stabilité Nash est assez simple :

> *une partition de joueurs est stable Nash quand il n'ya pas de joueur déviant de son / sa coalition à l'autre coalition dans la partition*.

Nous nous référons à [25] pour de plus amples discussions sur les concepts de stabilité dans le contexte des jeux hédoniques de formation de la coalition.

11.1.1 *Nos Contributions*

Tout d'abord, nous développons un algorithme décentralisé trouver la stabilité Nash dans un jeu quand il existe toujours au moins un. L'algorithme est basé sur *la meilleure stratégie de réponse* dans lequel chaque joueur décide série son/sa coalition. Ainsi, le problème est considéré comme un jeu non coopératif. Nous considérons un *random round-robin* mode où chaque joueur détermine sa stratégie à son tour selon un *programmateur* qui est généré de façon aléatoire pour chaque tour. Sous cette condition, nous montrons que l'algorithme converge vers un équilibre qui est l'indicateur de la stabilité Nash.

Deuxièmement, nous posons la question suivante: Y at-il affectation d'utilité méthode qui pourrait aboutir à une partition Nash-stable? Nous abordons cette question dans la suite. Nous proposons la définition de *le noyau Nash-stable* qui est l'ensemble de toutes les méthodes de répartition des services publics possibles résultant de partitions Nash-stables. Nous analysons les cas dans lesquels le noyau Nash-stable est non vide, et de prouver que sous *la détendue efficacité* condition (le gain attribué total de joueurs au sein d'une coalition pourrait être inférieure ou égale à l'utilité de cette coalition) il existe toujours une partition Nash-stable.

11.2 LE PROBLÈME DE L'ASSIGNATION MOBILE EN BROADCAST TRANSMISSION : ASPECTS D'OPTIMISATION

C'est une exigence de l'évolution à long terme (LTE) spécifications pour appuyer la prestation de données de diffusion/multidiffusion sous le nom de l'Multimedia Broadcast/Multicast service (MBMS) [30]. Il n'y a pas de différence entre la diffusion et de multidiffusion des transmissions de données en liaison descendante à la couche physique. Alors que les services de diffusion sont disponibles à tous les utilisateurs sans avoir besoin d'abonnements à des services particuliers, la multidiffusion peut donc être considérée comme "diffusion via abonnement", avec la possibilité de faire payer l'abonnement [30].

Broadcast est particulièrement bien adapté aux canaux sans fil, où l'on peut utiliser les ressources (en fréquence et/ou de temps) qui sont communs à toutes les destinations. Nous supposons que le coût d'une station de base (BS) de transmettre à une multidiffusion groupe est proportionnel à la puissance nécessaire pour atteindre le mobile le plus éloigné au sein du groupe, et que celui-ci est une fonction de la distance par rapport à ce que mobile et l'effet de l'ombrage. A BS peut diffuser la même information à plusieurs groupes de multidiffusion. Dans ce cas, chaque groupe multicast est chargé de la coût pour atteindre le mobile la plus éloignée de ce groupe. Dans notre contexte, le plus mobile à distance doit être comprise comme celui pour lequel on a besoin le plus haut la puissance de transmission, et non un éloignement grâce à une mesure géographique.

Nous considérons la situation où il ya une information commune que chacun de M mobiles est intéressé à recevoir, et qui peut être obtenu à partir de l'une des stations de base n. Les informations de diffusion pourrait être un peu de contenu, tels que la transmission en continu d'un événement culturel ou sportif, ou il pourrait y avoir une signalisation comme un phare pour la synchronisation horaire ou à des fins de contrôle de puissance. Nous recherchons ces affectations de mobiles au SRS pour que *la puissance totale est minimisée*. De plus, nous prenons en compte le *coûts de l'énergie opérationnelles* (par exemple, les amplificateurs de puissance, radiateur, etc) d'un BS typique. En effet, le problème d'affectation mobile (MAP) dans le contexte de la transmission de diffusion que nous étudions dans ce travail est construit sur *min-size k-clustering problem* proposée et examinée dans [31]. En min-taille k-clustering problème, l'objectif est d'attribuer des points au plus k grappes de sorte que la somme de toutes les distances entre les points dans le même cluster (k-clustering) est minimisé. Dans [31], le coût typique pour une paire BS-mobile est supposé être uniquement fonction de la distance entre la BS et le mobile, qui est formulée comme $\sum_{n_j \in \mathscr{C}} \max_{i \in n_j} d_{ij}^{\alpha} + P_0^j$. Ici, n_j est un *groupe* de mobiles affectées à BS j, $\mathscr{C}$ est l'ensemble des groupes appelés comme *regroupement*, d_{ij} est la distance entre le mobile i et BS j, α est l'exposant de perte de trajet, P_0^j est le coût de l'énergie opérationnel chargé de BS j. Cependant, ici, nous ajoutons l'effet d'ombre à un tel coût produisant le coût total suivante $\sum_{N_j \in \mathscr{C}} \max_{i \in N_j} d_{ij}^{\alpha}/\Psi_{ij} + P_0^j$ où Ψ_{ij} désigne l'effet d'ombrage entre mobile i et BS j.

11.2.1 *Nos Contributions*

Les documents visés concentrer sur l'aspect de la géométrie de la carte où fondamentalement, la zone de couverture d'une BS est supposé être un disque qui délivre du diagramme d'antenne omnidirectionnel. Cependant, *l'effet de observation*, spécialement conçu *diagrammes d'antenne* ainsi que *la coûts de l'énergie opérationnelles* pourraient modifier les affectations BS-mobiles. Ici, nous prenons en compte ces effets. À cette fin, nous représentons par *matrice de coût d'énergie* chacun de coût d'appariement alimentation BS-mobile. Ensuite, nous proposons Des algorithmes de programmation dynamique effectuant des opérations sur le coût d'alimentation matrice.

Par ailleurs, "approches sensibles au vert" [44] qui visent pour réduire la consommation d'énergie dans des environnements sans fil doit être pris en compte dans la conception des algorithmes pertinents. Dans ce contexte, la coupure une fraction de BSS est considéré comme un moyen de réduire considérablement le La consommation totale d'énergie. Réseaux hétérogènes comprennent des cellules macro et petite avec coordination. Dans ce travail, nous supposons que les petites cellules sont soumises à éteignant opération tandis que les cellules de macros sont toujours activées, qui servent déplacement mobiles afin de diminuer le nombre de la main-offs. Les petites cellules sont déployés de manière intensive, par conséquent, la puissance d'émission est inférieure à macro cellules.

En comparant la puissance d'émission étant en niveaux milliwatts avec l' coûts de l'énergie opérationnelle, près de dizaines de Watts, il est très efficace pour éteindre une certaine fraction de stations de base pour réduire la consommation totale d'énergie. Cette méthode pourrait être abondante lorsque les utilisateurs ne bougent pas. Cependant, l'évolution rapide les utilisateurs sont considérées être servi par macro stations de base qui ne sont pas éteinte généralement [11].

Par la suite, nous proposons un algorithme récursif appelé *the hold minimum algorithm* qui résout le problème considéré de manière optimale. L'attente minimum algorithme opère d'une manière centralisée, ce qui nécessite toute connaissance pour chaque coût de l'énergie appariement BS-mobile. Nous proposons également un autre centralisé algorithme heuristique en temps polynomial appelé *the column control algorithm* qui produit des affectations optimales en tenant compte de la puissance opérationnelle coûts. En outre, nous développons une approche distribuée à la commande de colonne, où chaque mobile recueille les informations locales de la stations de base qui peut transmettre à elle. Nous D'autre part, *the nearest base station algorithm*, un algorithme heuristique distribué qui fonctionne en temps polynomial est offert. Cette algorithme est efficace pour les utilisateurs qui se déplacent rapidement desservis par les stations de base macro. Nous introduisons également un nouvel algorithme basé sur les jeux de formation de groupe, que nous appelons aussi le *the hedonic decision algorithm*. Ce formalisme est constructif: une nouvelle classe de jeux de formation de groupe est introduit où l'utilité de joueurs au sein d'un groupe est séparable et symétrique est une version généralisée de la parité jeux affiliation. En outre, l'algorithme de décision hédonique peut être adapté à un ensemble couvrant problème.

11.3 LE PROBLÈME DE L'ASSIGNATION MOBILE EN BROADCAST TRANSMISSION : ASPECTS DE JEU DE COALITION

Nous étudions le problème combiné de (i) de décider ce sous-ensemble des mobiles serait attribué à chaque BS, puis (ii) le partage du coût de la multidiffusion BSS parmi les mobiles. Le sous-ensemble que nous souhaitons attribuer à un BS donnée est dit être son objectif fixé de mobiles. Une règle de partage des coûts se compose d'une politique de prix qui détermine la part que chaque mobile dans l'objectif fixé paierait. Nous sommes intéressés par les politiques de partage qui sont stables en ce sens qu'aucun sous-ensemble de M mobiles pourrait payer strictement inférieure à leur part des coûts de la formation d'un nouveau groupe de multidiffusion séparé.

Ce travail s'appuie sur [45] qui a étudié le cas d'une seule station de base. Ils ont étudié (i) le problème de partage des coûts ainsi que (ii) l'association combiné et problème de partage des coûts. Dans ce dernier, chaque mobile a été en mesure de décider de rejoindre un canal de diffusion unique dédié ou à se joindre à la session de multidiffusion, dans ce cas, c'était une partie de la partie de la coalition au BS. L'analyse fortement dépendu de la propriété sous-modularité qui a tenu dans le cas d'une seule station de base. Nous montrons ici que sous-modularité ne tient pas dans le cas de plusieurs stations de base.

11.3.1 *Nos Contributions*

Le point de départ a été notre tentative d'étendre la propriété de sous-modularité pour le cas de deux stations de base. Au lieu de cela, nous fournissons un contre-exemple qui montre qu'en effet, déjà dans le cas de deux stations de base, sous-modularité ne tient pas.

Nous apprécions cela comme un jeu coalitionnelle joué par les mobiles et prouver que ce jeu a tout intérêt à former la grande coalition où tous les joueurs se joignent à la partie. En outre, en utilisant Bondareva-Shapley théorème [62], nous montrons que ce jeu de coalition a un noyau non vide ce qui signifie que la grande coalition est stable. Ensuite, nous examinons la politique de répartition des coûts pour différentes méthodes telles que l'allocation égalitaire, la répartition proportionnelle du coût total, la valeur de Shapley [29] et le nucléole [65].

11.4 ETEINDRE LES STATIONS DE BASE: CONSIDÉRATIONS LIAISON DESCENDANTE

La consommation d'énergie peut être réduite en tournant et éteignant dynamiquement les cellules, les stations de base et les autres ressources radio (par exemple antennes d'émission), selon observé charge de trafic, l'utilisation des ressources, la qualité et la couverture.

Nous considérons transmission en liaison descendante dans les réseaux cellulaires où nous pour objectif de réduire la consommation d'énergie en éteignant certaines stations de base d'une telle manière que la distribution de SINR reste inchangé. Nous supposons que la réutilisation des fréquences. Chaque mobile est associée à la station de base étant la plus proche de lui. Toutes les stations de base étant la principale cause l'interférence la plus proche du mobile. La question que nous posons est "Combien de stations de base peut être désactivé afin que la distribution du SINR reste inchangé?". Nous modélisons le problème comme un processus ponctuel de Poisson marqué indépendamment homogène.

Nous analysons les affaires en ligne et le plan, le gain en consommation d'énergie obtenue après la coupure BS. Il s'avère de calculs que plus le coût d'exploitation moins le gain en consommation d'énergie, et même, plus la dimension (distribution des stations de base dans la ligne et le plan signifie une et deux dimensions, respectivement), moins le gain en consommation d'énergie.

Nous analysons également la coopération des fournisseurs de services sur désactiver le fonctionnement de stations de base dans le contexte de la mise en réseau vert. L'approche processus de Poisson homogène pour le déploiement de stations de base a été utilisée pour étudier la distribution SINR des prestataires de services. En outre, dans le cas des prestataires de services n'ayant pas coopéré, la distribution SINR est obtenue du réseau d'origine et amincie de prestataires de services, respectivement. Nous avons également constaté la distribution SINR de cas de coopération utilisé dans le cadre de la formation d'une coalition de fournisseurs de services. Les opérations sur le réseau formé par la coopération sont supposés être géré conjointement par les fournisseurs de services qui signifie qu'ils partagent leurs ressources de telle sorte que n'importe quel mobile est marqué à la station de base la plus proche de n'importe quel fournisseur de services. La densité d'économie d'énergie maximale d'une coopération est censé être l'utilité de la coalition. Nous obtenons les résultats du formulaire fermés de l'utilitaire. Nous comparons les services de prestataires de services dans les deux cas de l'allocation basée sur l'efficacité et l'efficience détendue. Nous avons montré que dans le cas d'écarts individuels de l'importance de l'efficacité n'est pas significative sur le côté de fournisseurs de services.

11.5 ASSOCIATION NON-COOPÉRATIVE DE MOBILES

Il ya eu un intérêt croissant pour les dernières années de modéliser les décisions d'accès aux réseaux que des jeux non compétitifs. En effet, il est assez fréquent que le réseau laisse à l'utilisateur de décider à quel point d'accès à connect.The problème d'association est en fait lié à la nature du problème de sélection de canal. Ce qui motive l'utilisation des jeux avec des informations incomplètes, également connu sous le nom des jeux bayésiens, où l'information partielle se réfère à la charge du système dans [72] ou à la qualité de canal dans [73].

Le point d'accès peut être différent de l'autre par leur technologie et par la qualité des canaux radio entre chacun d'entre eux et chaque mobile. État cette mise en réseau décision compétitif dépendants ont été modélisés dans le passé que des jeux stochastiques et la structure des politiques d'équilibre a été obtenue pour un ou deux problèmes dimensionnels. Par un problème tridimensionnel, nous entendons problèmes dans lesquels chaque mobile a le choix entre un point d'accès dont les ressources sont partagées et entre un canal dédié. Dans un tel problème les informations nécessaires pour prendre la décision d'association est combien de mobiles sont connectés à la ressource partagée (donc l'information est dite unidimensionnelle). Un exemple pour un problème qui tombe dans cette catégorie est [74]. La politique d'équilibre, il se compose d'une politique de seuil avec randomisation sur le seuil. Dans [75] l'étude d'auteur un problème à deux dimensions dans lequel le choix est entre l'accès à un réseau cellulaire sans fil 3G ou un réseau local sans fil. L'information disponible est de deux dimensions: le nombre de stations mobiles dans chacun des réseaux. Dans [76] politiques d'équilibre ont été montré pour avoir une forme de la courbe de commutation avec randomisations possibles à la frontière entre les régions correspondant à la connexion au point d'accès différent. Un problème d'association à l'un des nombreux points d'accès d'un réseau local sans fil a été examinée dans [77]. Dans tous les problèmes ci-dessus, nous avons supposé que, une fois une décision de la connexion est établie, le mobile reste connecté au point d'accès jusqu'à la fin de l'appel.

En revanche, dans ce travail, nous considérons le problème où à n'importe quelle période de temps, les portables peuvent mettre à jour leur décision d'association. Nous considérons le choix entre deux points d'accès ou plus, où les décisions d'accès peut dépendre du nombre de mobiles connectés à chacun des points d'accès. Nous obtenons de nouveaux résultats en utilisant des outils élémentaires de la congestion et des jeux éviction. Nous montrons en particulier que, à l'équilibre, les actions mixtes (aléatoire) ne sont pas nécessaires. Nous montrons par ailleurs la convergence de la séquence de meilleures stratégies d'intervention.

Nos résultats sont basés sur la congestion games [80] et sur l'éviction games [79]. Nous étudions en outre (i) Multihoming dans lequel un utilisateur peut se connecter simultanément à plus d'un point d'accès. (ii) le cas " élastique" dans laquelle il ya aussi une possibilité de ne pas se connecter à tous.

11.5.1 *Sélection des Partenaires pour Relayer Cooperative*

Soit être considéré comme un scénario où un ensemble d'utilisateurs ou agents, visent à transmettre leurs données vers une destination commune. Ce scénario peut correspondre, par exemple, soit vers les mobiles dans une seule cellule en mode de liaison montante ou à un réseau de capteurs sans fil avec un évier. Le protocole de référence utilisée pour transmettre l'ensemble des informations à la destination commune repose sur une approche MAC de couche 2 qui divise les ressources et programmer l'allocation de chaque ressource à chaque agent. Même si les agents sont tous dans la gamme de la destination, les transmissions coopératives peuvent améliorer considérablement l'efficacité du système. L'efficacité peut être mesurée par la capacité totale, l'énergie totale ou par des critères liés à la qualité de service tels que le taux d'erreur de paquet. Ici, nous nous concentrons sur la probabilité de panne. puissances d'émission et des besoins de capacité pour chaque nœud sont constants et transmission coopérative est utilisé pour réduire la probabilité de panne. La formulation du problème est basée sur le modèle de canal relais comme décrit dans les premiers travaux de Laneman [89]. Dans l'approche la plus simple, chaque agent source nécessite l'aide d'un autre agent pour améliorer sa transmission en formant togeter un canal de relais équivalent. Nous considérons le cas particulier où les agents associés par paires tels que les relais chaque agent les données de l'autre. Dans le cadre de la théorie de l'information du réseau, ces nœuds forment un canal d'accès multiple coopérative (CMAC).

Mais l'ensemble du réseau peut être considéré comme un grand CMAC de taille, nous proposons plutôt de former plusieurs petites coalitions. Dans ce travail, ces petites coalitions sont encore limités à deux. Afin d'optimiser la performance globale du réseau, le processus *sélection des partenaires* est donc cruciale. Chaque agent vise ici à trouver un bon partenaire pour exploiter efficacement la diversité spatiale réalisé avec la coopération. Ce processus peut être identifié comme un *problème d'appariement* . Dans le sens théorique de jeu où les joueurs sont "décideurs stratégiques", le processus de sélection du partenaire semble être un exemple de la *problème de colocataires stable*. *Théorie d'appariement stable* a été créé par Shapley et Gale par leur travail séminal [85]. Gale et Shapley analysés correspondant à, un niveau général abstrait. Ils ont utilisé le mariage comme l'un de leurs exemples illustratifs. Comment dix femmes et dix hommes doivent être adaptées, tout en respectant leurs préférences individuelles? Le principal défi consistait à concevoir un mécanisme simple qui mènerait à un appariement stable, où aucun couples se briser et former de nouveaux matchs qui les rendrait mieux. La solution – la Gale-Shapley "acceptation différée" algorithme – est un ensemble de règles simples qui toujours conduit tout droit à un appariement stable.

Le problème du mariage stable est un exemple de ce qu'on appelle *recto-verso marché* en raison de la question du genre. Toutefois, ce genre de problème peut être d'élargir à l'appariement des problèmes avec aucun problème du genre et sont considérés comme des *unilatérale marché*. C'est le cas du problème des colocataires stable. Dans ce problème, chaque personne cible correspondant à la meilleure partenaire pour partager une chambre. Nous allons montrer que le problème de la sélection des partenaires dans le cadre de relayer coopérative peut être étudié comme un marché à sens unique. En déterminant la règle classement des partenaires, nous recherchons un appariement stable. Même si un appariement stable est toujours possible dans les problèmes de mariage stable, ce n'est pas le cas dans le problème des colocataires stable. En outre, même si un appariement stable existe, il n'y avait pas d'algorithme polynomial pour trouver jusqu'à ce que les travaux récents de Irving [86].

11.5.2 *Nos Contributions*

Nous vous proposons l'utilisation de la théorie de l'appariement et plus spécifiquement le problème des colocataires stable, pour résoudre le problème de la sélection des partenaires dans les transmissions de coopération. Ce travail montre que ce formalisme est parfaitement pratique et en outre un outil naturel pour ce problème. La raison en est qu'il fournit une procédure de découpe stable et équitable si l'on considère les nœuds sources (agents) que les décideurs stratégiques. Ici, nous utilisons cet outil pour analyser le problème de la sélection des partenaires.

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