The influence of the cross section shape on channel flow: modeling, simulation and experiment

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Outline

I. Introduction and objectives

II. Model

III. Data

IV. Validation

V. Conclusions and perspectives

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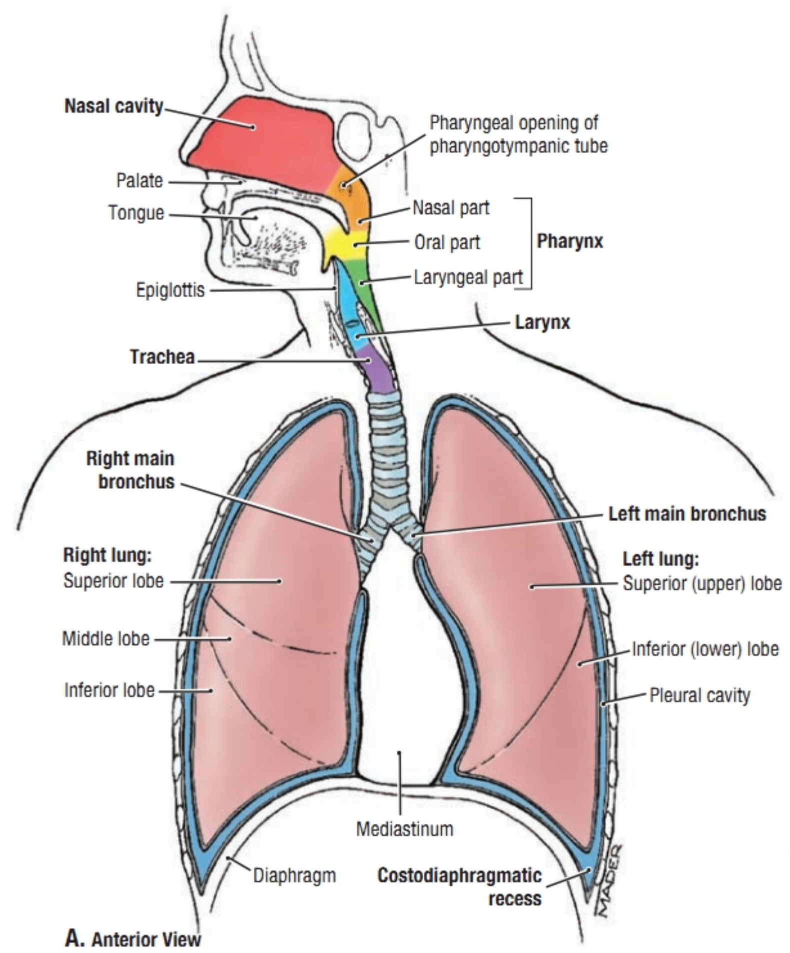
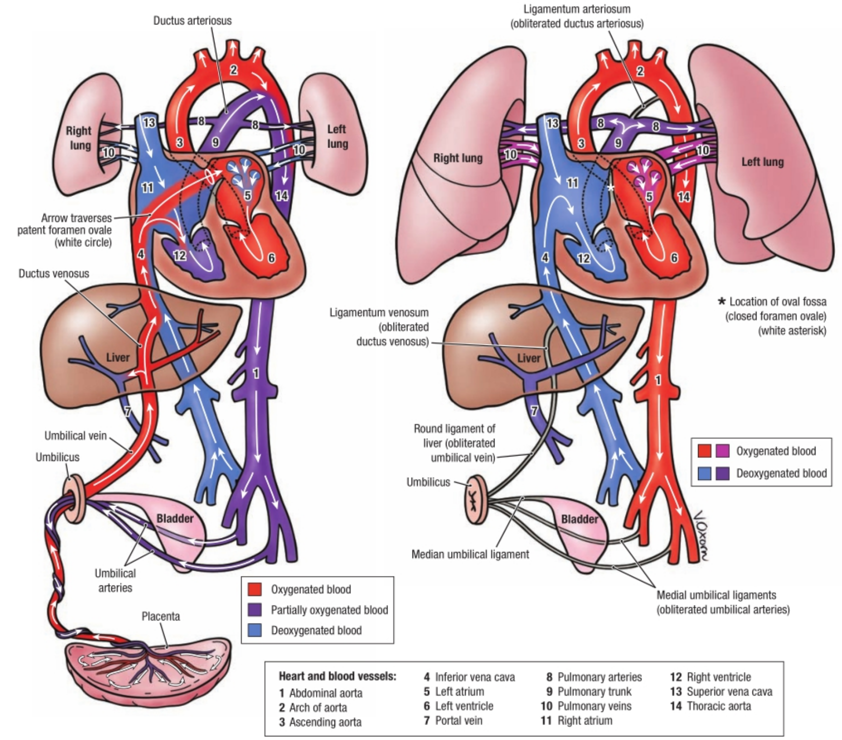
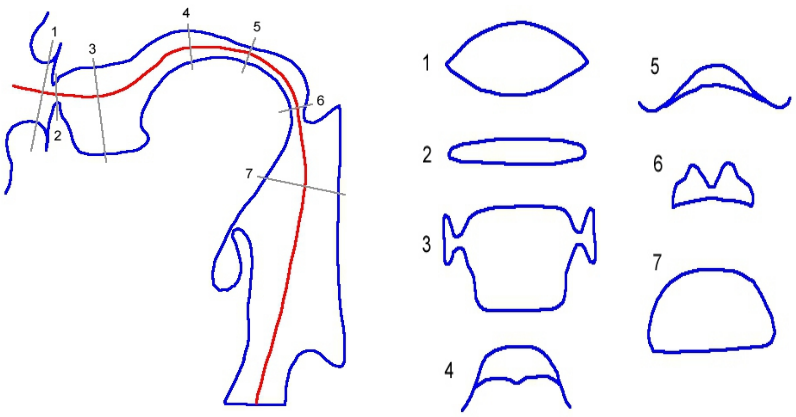


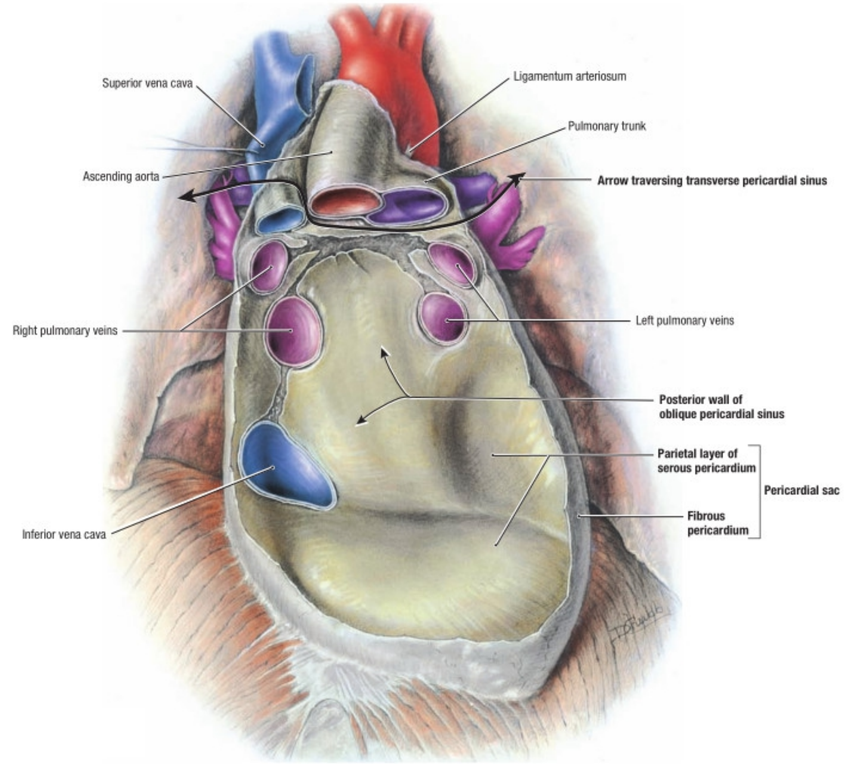
Human airway system



Blood circulation system



Upper airways



Heart

Problem: flow through highly varying cross section shape ??

3D flow model

Navier – Stokes equation

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) = -\nabla P + \nabla\cdot\mathbf{T} + \mathbf{f}$$

unsteady convective pressure-driven stress sensor body force

+ : realistic

- : high computational cost

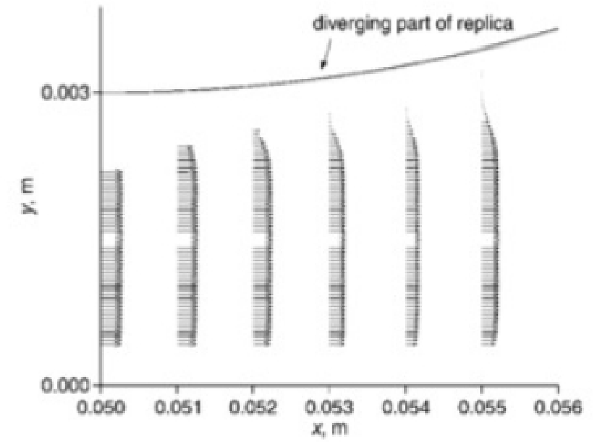
- : no parameterized general geometry

• 1D/2D models

1D model :

$$\bar{u}\frac{d\bar{u}}{dx} = -\frac{1}{\rho}\frac{dP}{dx}$$

convective pressure-driven

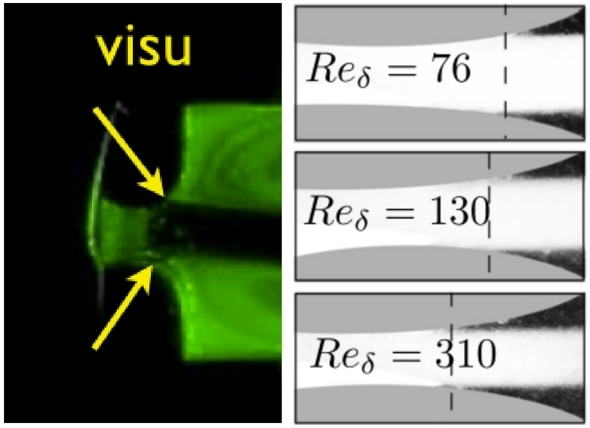


Anemometry

2D model :

$$\bar{u}\frac{d\bar{u}}{dx} + \frac{1}{\rho}\frac{dP}{dx} = \nu\frac{\partial^2 u}{\partial y^2}$$

convective viscosity



+ : area, flow separation, jet formation, viscosity

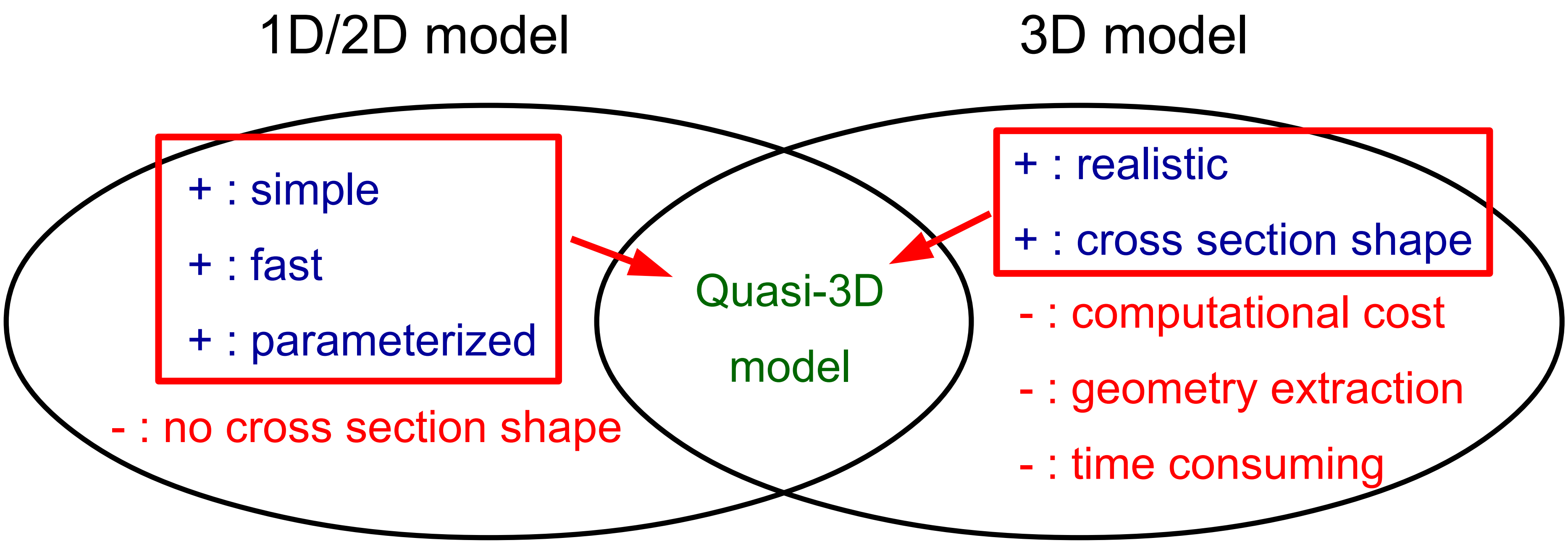
- : no cross section shape

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) = -\nabla P + \mu\Delta\mathbf{u} + \mathbf{f}$$

viscosity

- Influence of the cross section shape on the viscous contribution to the flow
  - qualitatively
  - quantitatively
  - propose a simple flow model

- Objectives



- Analyze: model, IB method, experiment

- Applications: phonation, biological circulation systems
  physical equations

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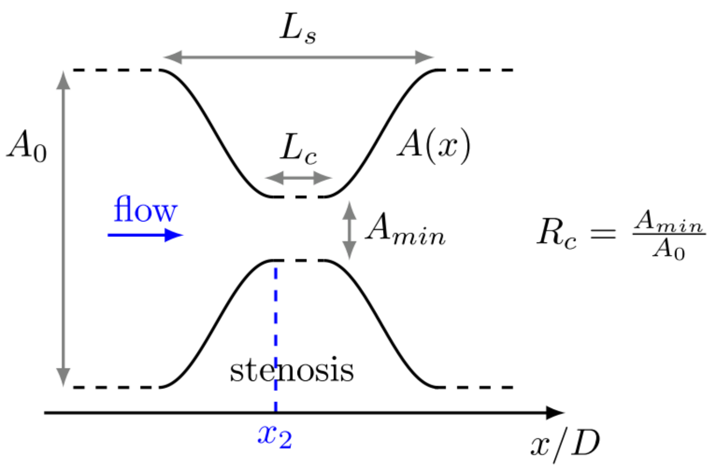
$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right) = -\nabla P + \mu\Delta\mathbf{u} + \mathbf{f}$$

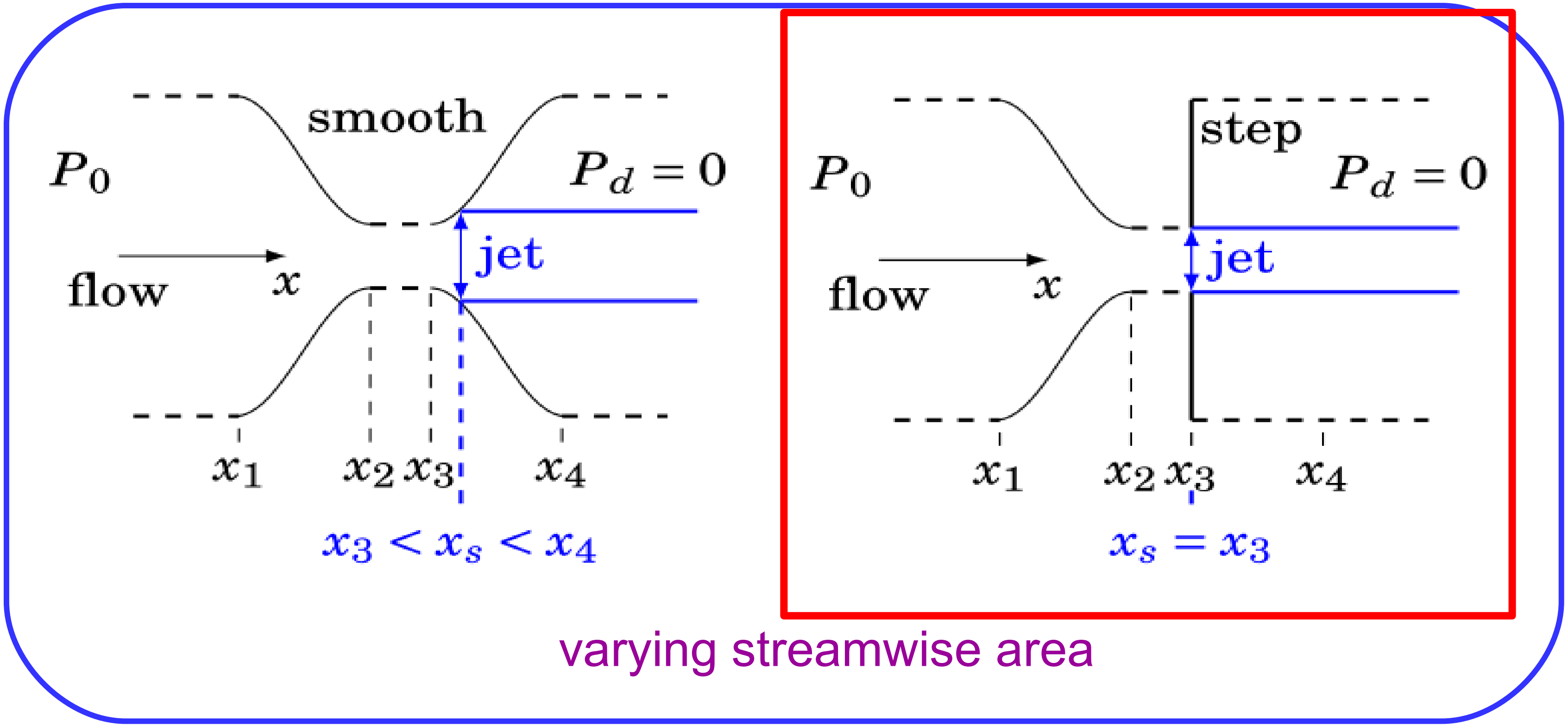
- Flow acceleration => changing area R_c
- Viscosity => cross section shape
- Viscosity => length of constriction L_c



+ cross section shape

$$dQ/dx = 0$$

$$-\frac{Q^2}{A^3}\frac{dA}{dx} + \frac{1}{\rho}\frac{dP}{dx} = \nu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$



- abrupt expansion: position of flow separation is fixed
- smooth expansion: position of flow separation depend on the ad–hoc criterion

$$P_0 - P_d = \Delta P_{visc} + \Delta P_{ber}$$

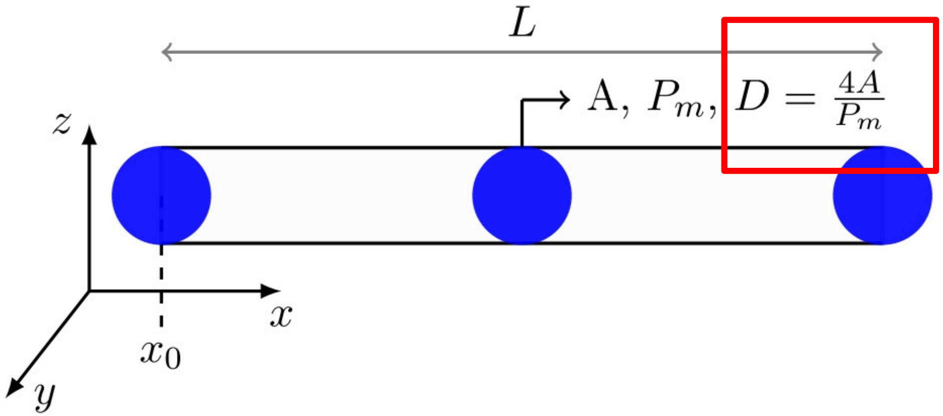
viscosity

1D ideal flow

$$\frac{1}{\mu}\frac{dP}{dx} = \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

$$\bar{u}\frac{d\bar{u}}{dx} = -\frac{1}{\rho}\frac{dP}{dx}$$

• Uniform channel



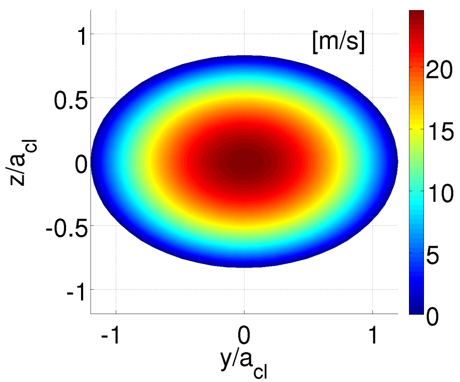
$$Re = \frac{\overline{U}D}{\nu}$$

- ➢ Academic cross section shape
  - quasi–analytical solution
- ➢ General / arbitrary cross section shape

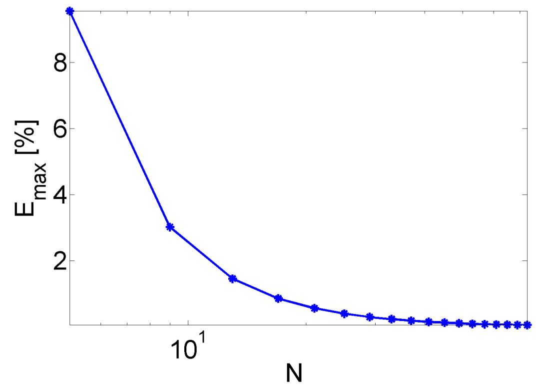
$$r(\theta) = g(\theta)\left[\left|\frac{\cos(\frac{m\theta}{4})}{a}\right|^{n_2} + \left|\frac{\sin(\frac{m\theta}{4})}{b}\right|^{n_3}\right]^{-1/n_1} = g(\theta)\cdot f(\theta), \quad m > 0$$

  - pseudo–spectral method
  - quasi–analytical solution

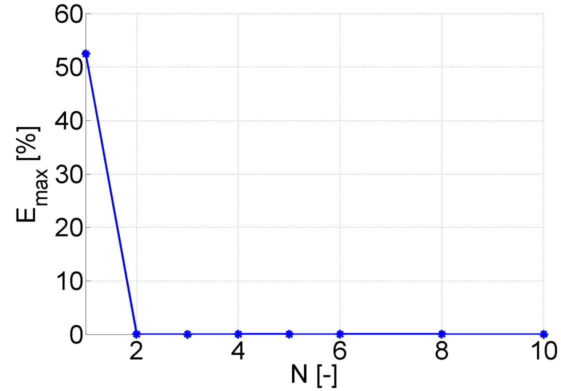
- # Uniform channel



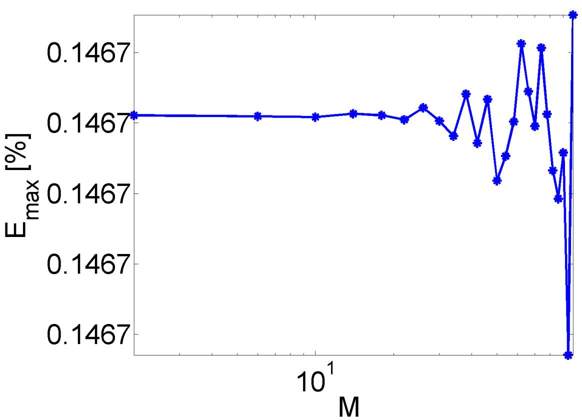
dP/dx=75Pa/m



pseudo-spectral (M=72)



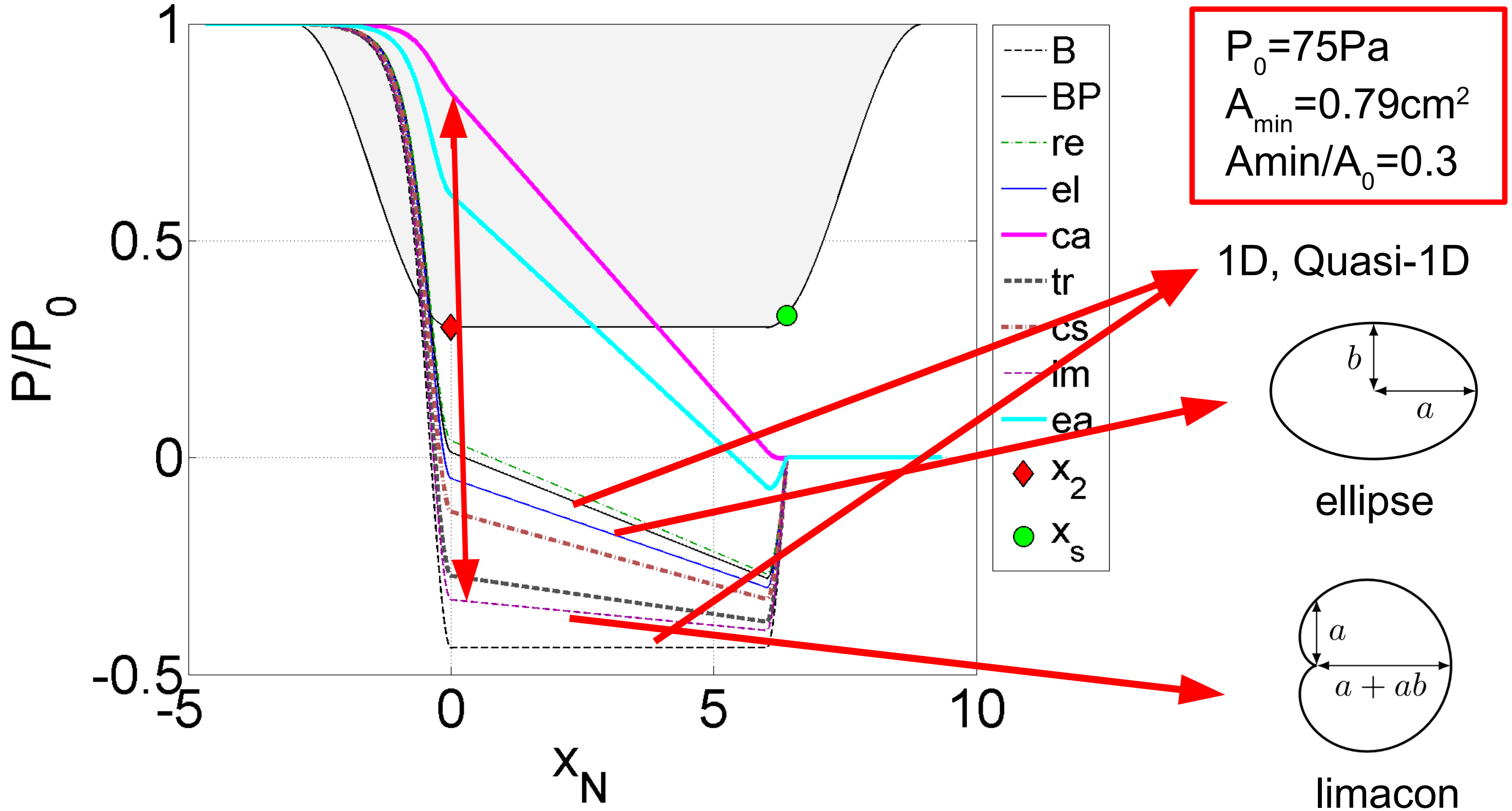
quasi-analytical



pseudo-spectral (N=41)

Approach	circle	rectangle	ellipse	equilateral triangle
quasi–analytical [%] $(N > 6)$	0	0.393	1.44e-14	0.479
numerical [%] $(N > 40)$	0.046	0.046	4.09	10.69

- Fixed width

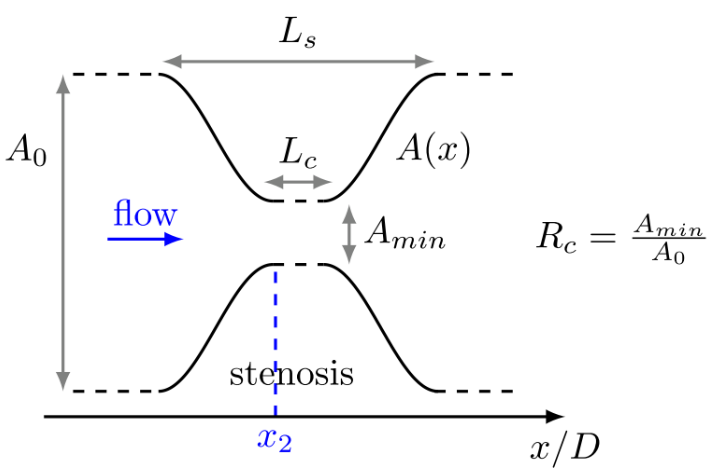


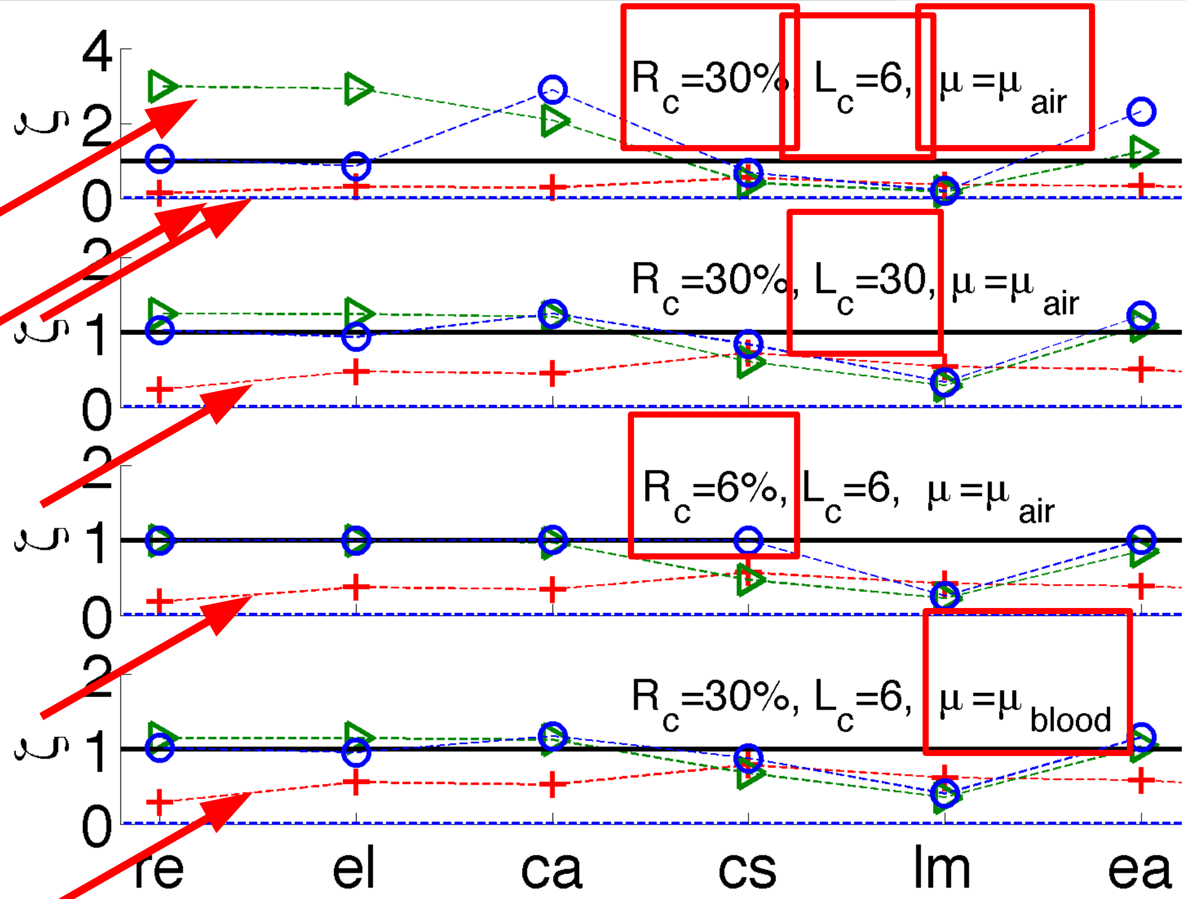
➢ 1D/BP overestimate, underestimate the results

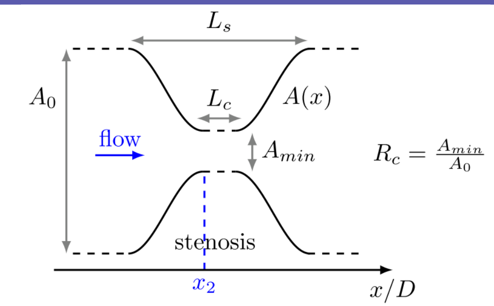
➢ 80% variation due to cross section shape

• Some model considerations

- developed viscous flow
- laminar incompressible flow
- cross section shape
- fluid
- constriction ratio R_c
- length of constriction L_c







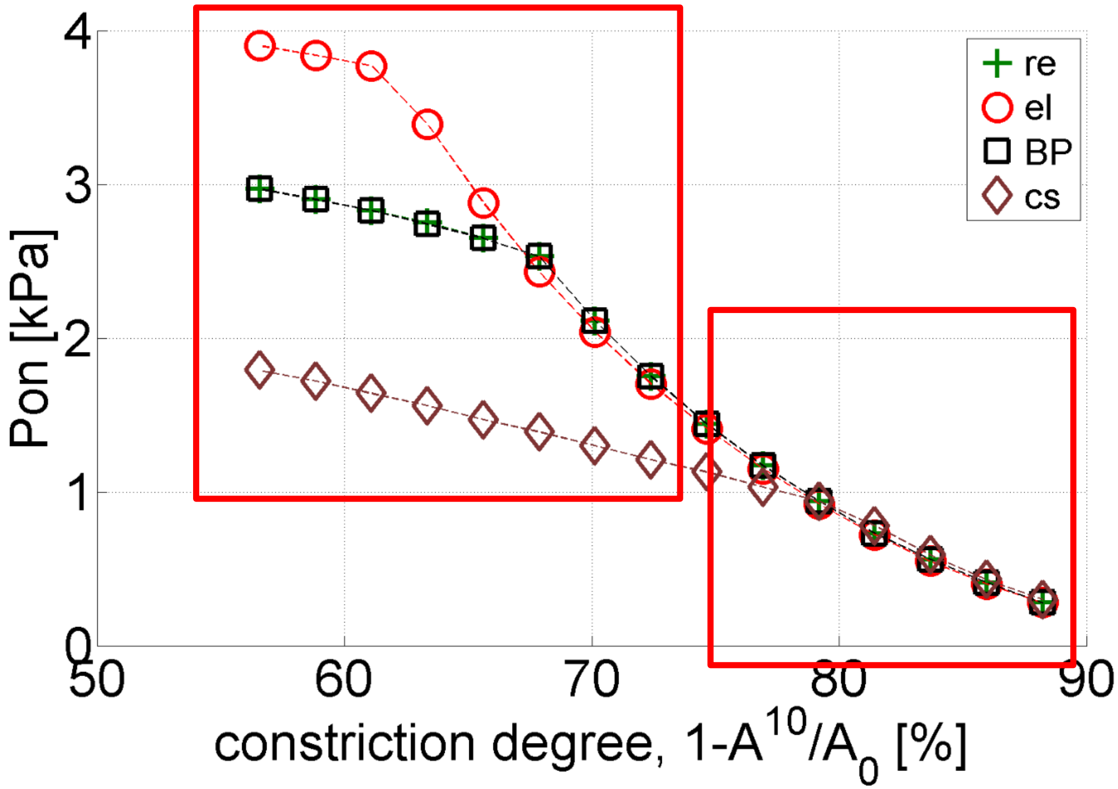
$$\zeta = \frac{|P_{min} - P(x_2)|}{|P_{min} - P(x_2)|_{BP}}$$

P_0=75Pa
A_{min}=0.79cm^2

- $+$: parameter set 1
- $\triangleright$: parameter set 2
- $\circ$: fixed width

- BP overestimates for **parameter set 1** → • depend on configurations
- BP underestimates for **parameter set 2**: re, el, ca and ea
- Underestimation of BP reduces when decreasing **constriction ratio R_c**
- Underestimation of BP reduces when increasing **length L_c**
- Underestimation of BP reduces when increasing **dynamics viscosity**
- Overestimation of BP is less sensitive to configurations for **parameter set 1**

1. Stability analysis of a physical phonation model



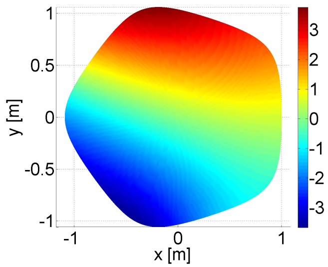
A^{10}: initial minimum area

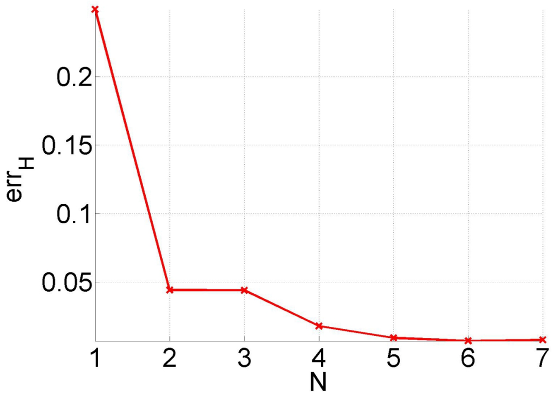
- neglected for **large** constriction degree
- important for **medium** constriction degree

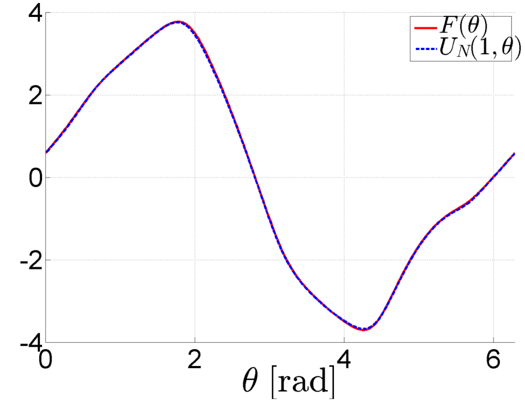
- Formulate physical equations for an arbitrary shape on 2D and 3D
  - Laplace equation
  - Helmholtz equation
  - Wave equation

  for different boundary or/and initial conditions

$$err_N = \frac{\| U_N - F(\theta) \|}{F(\theta)}$$







N = 7

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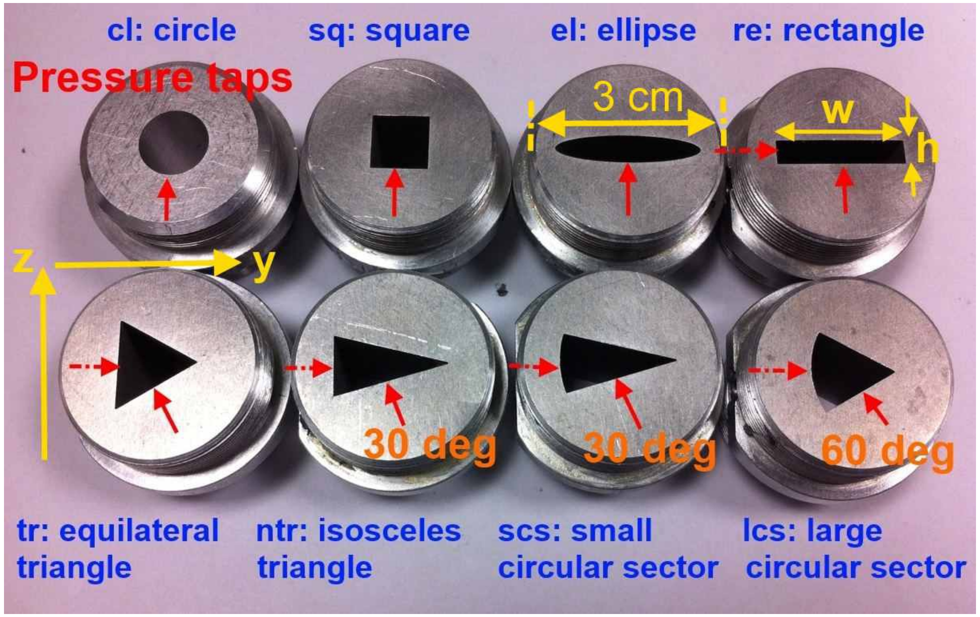
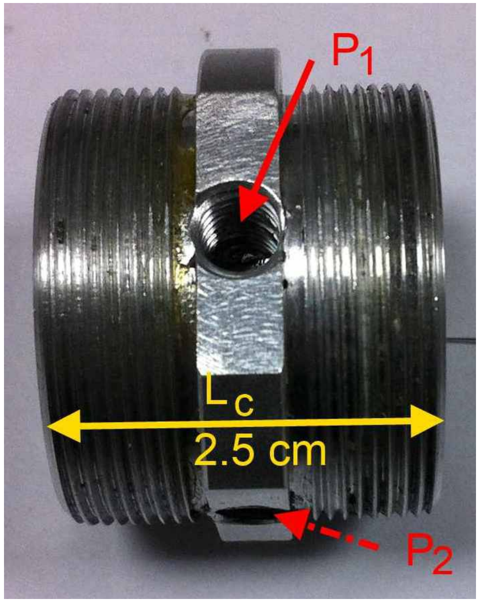


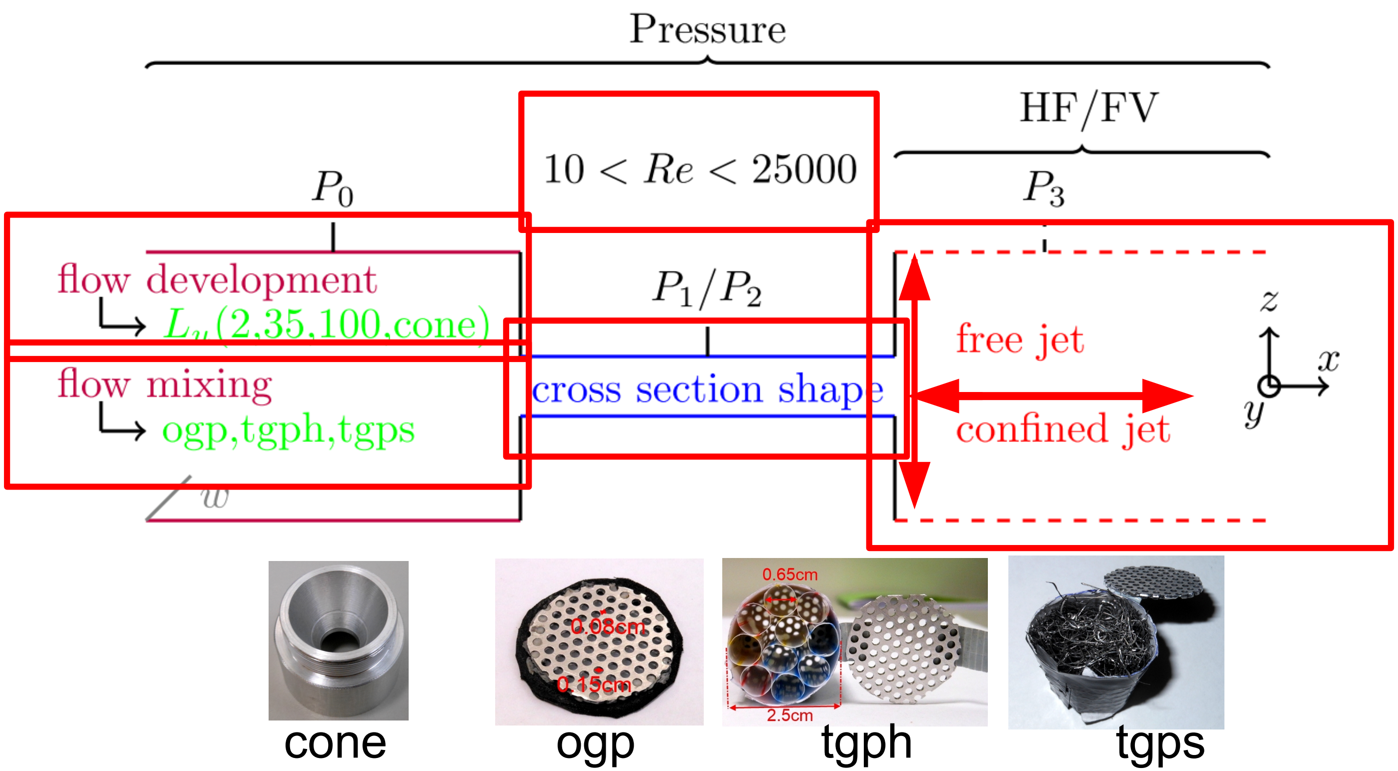


$A_c = 0.79 cm^2$

	cl	sq	lcs	tr	scs	ntr	el	re
D [mm]	10	8.9	8.4	7.8	7.2	7.0	6.7	6.6

- fixed area
- fixed length
- sharp inlet and sharp outlet

→ position of jet formation, flow separation



- Pressure
- Velocity profiles downstream the constriction
    - at the outlet (transverse)
    - along the centerline (longitudinal)
- Jet properties by flow visualization

- # Overview

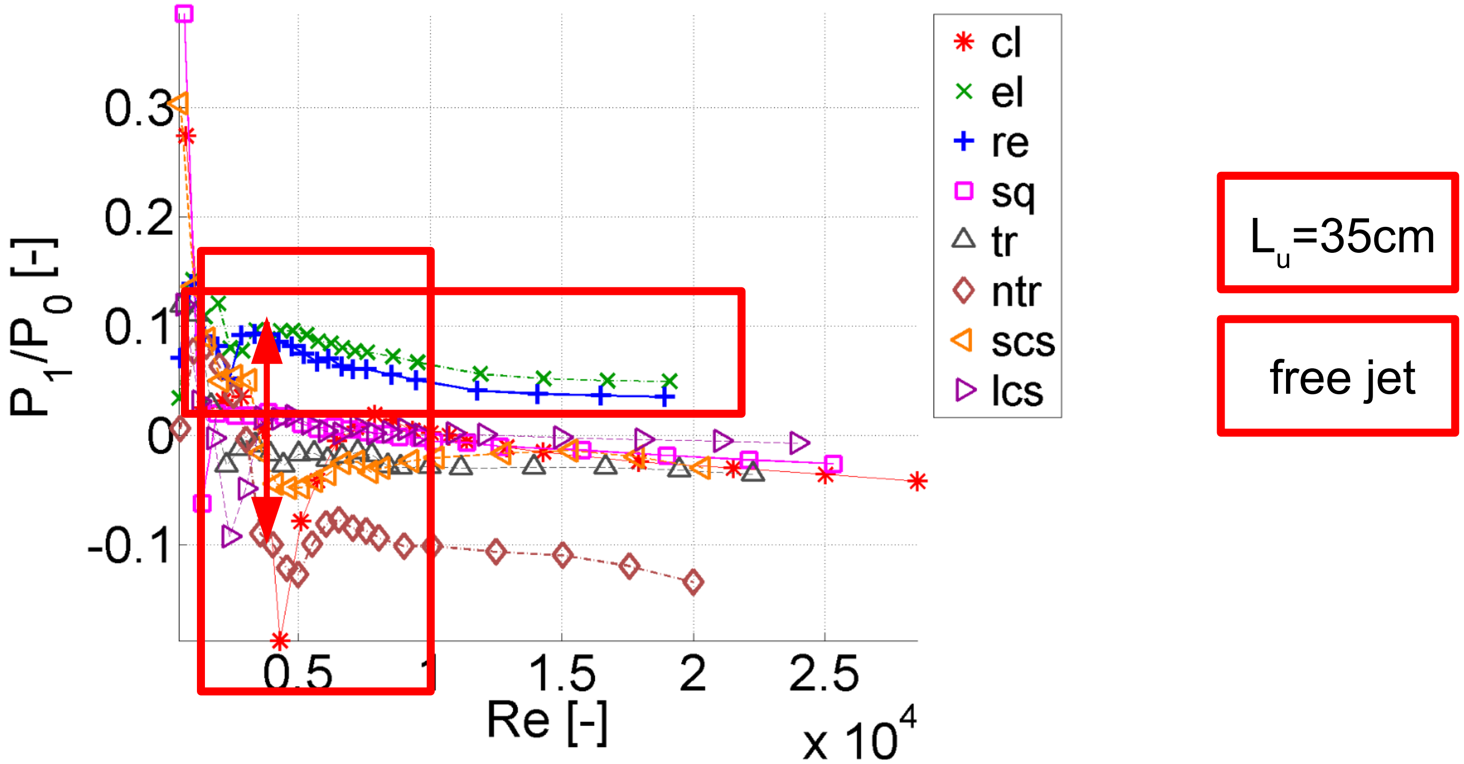
Label	L_u	L_d	Pressure sensors[1]	Flow field[2,3]	Comment	
	Inlet condition: sharp edges (Fig. 5.9) → flow development					
A	2cm	0cm	P_0, $P_1(P_2)$		free jet	all shapes
		15cm	P_0, $P_1(P_2)$, P_3		confined jet	
B	35cm	0cm	P_0, $P_1(P_2)$		free jet	
		15cm	P_0, $P_1(P_2)$, P_3		confined jet	
C	1m	0cm	P_0, $P_1(P_2)$	HF, FV	free jet	
		15cm	P_0, $P_1(P_2)$, P_3		confined jet	
	Inlet condition: use of mixing element (Fig. 5.10) → flow mixing					
D	35cm (ogp)	0cm	P_0, $P_1(P_2)$	HF	free jet	
		15cm	P_0, $P_1(P_2)$, P_3		confined jet	
E	35cm (tgph)	0cm	P_0, $P_1(P_2)$	HF	free jet	circular shape
		15cm	P_0, $P_1(P_2)$, P_3		confined jet	
F	35cm (tgps)	0cm	P_0, $P_1(P_2)$	HF	free jet	
		15cm	P_0, $P_1(P_2)$, P_3		confined jet	
	Inlet condition: no sharp edges (Fig. 5.8) → flow development					
G	35cm (cone)	0cm	P_0, $P_1(P_2)$	HF	free jet	
		15cm	P_0, $P_1(P_2)$, P_3		confined jet	
H	1m (d1cm)			HF	free jet	

(1) Steady flow for $0 < Q \leq 200$l/min or $Re \leq 25000$.
(2) Steady flow for $0 < Q \leq 100$l/min or $Re \leq 15000$.
(3) In hot-film anemometry the upstream channel length yields 1m.

- Experimental data: quantitatively

  - Influence of cross section shape
  - Influence of flow development
  - Influence of flow mixing
  - Influence of jet condition: free or confined

- Influence of cross section shape



- rectangle and ellipse have almost the same results
- complexity of flow dynamics
- 20% of variation between ellipse and isosceles triangle

- Influence of cross section shape

	$\Delta P_1/P_0$ [%]		$\Delta(P_1/P_0)$ [%]	
	$L_d = 0$cm	$L_d = 15$cm	$L_d = 0$cm	$L_d = 15$cm
Overall impact of cross section shapes: Eq. (5.1) and Eq. (5.2)				
A (L_u =2cm)	21%(1)	27%(1)	33%(3) 20%(4)	25%(3) 27%(4)
B (L_u =35cm)	18%(1)	23%(1)	18%(3) 17%(4)	27%(3) 21%(4)
C (L_u =1m)	16%(1)	19%(1)	13%(3) 15%(4)	18%(3) 19%(4)
D (ogp)	10%(2)	9%(2)	11%(3) 10%(4)	11%(3) 7%(4)

(1) $P_0 \approx 1500$Pa for $L_d = 0$cm and $P_0 \approx 1300$Pa for $L_d = 15$cm.

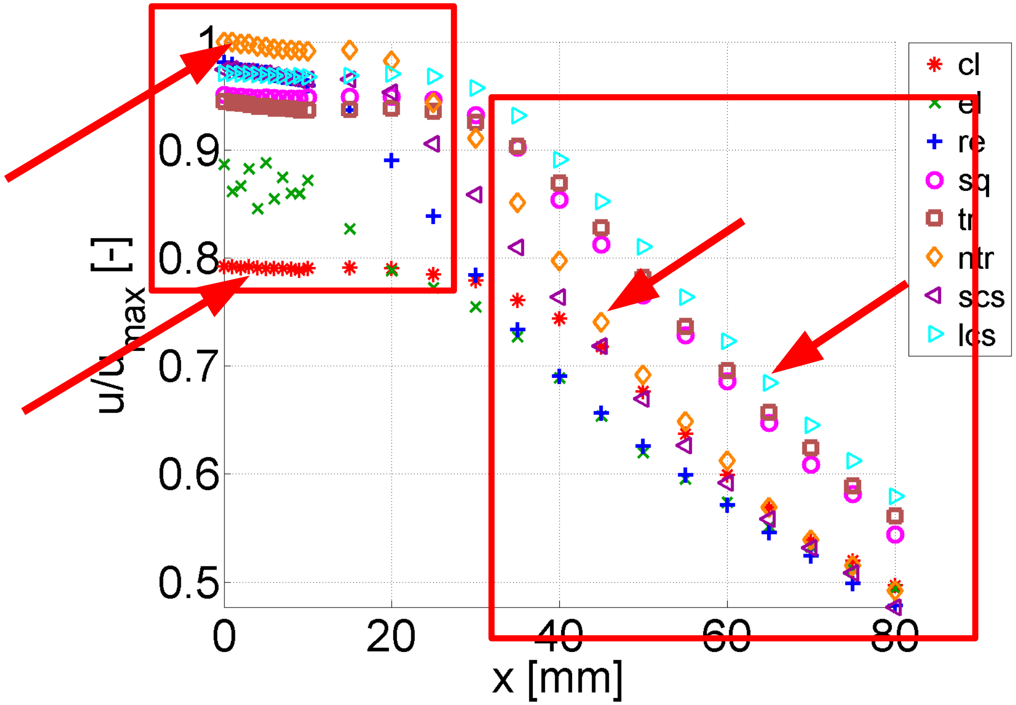
(2) $P_0 \approx 3400$Pa for $L_d = 0$cm and $P_0 \approx 3500$Pa for $L_d = 15$cm.

(3) $Re \approx 3600$ for $L_d = 0$cm and $L_d = 15$cm.

(4) $Re \approx 17500$ for $L_d = 0$cm and $L_d = 15$cm.

- 20% of variation for all flow conditioning

- Influence of cross section shape



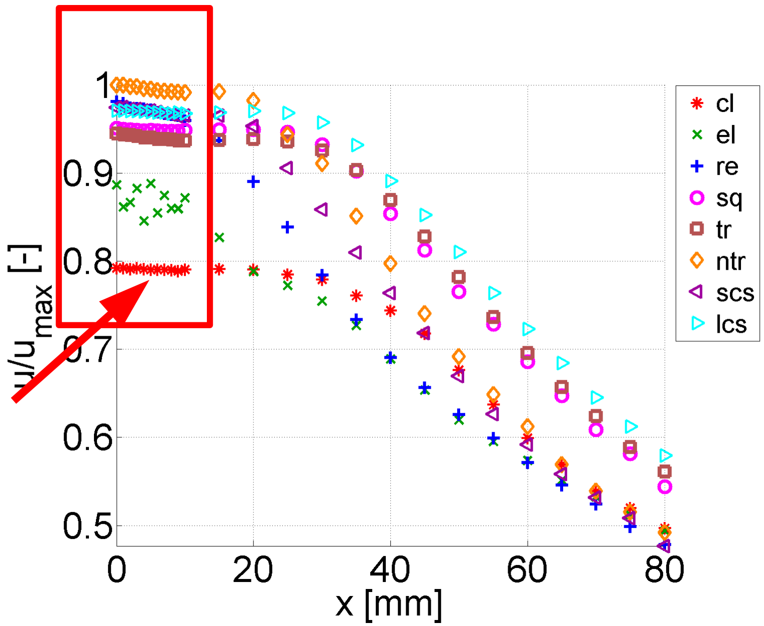
L_u=100cm

Q=35l/min

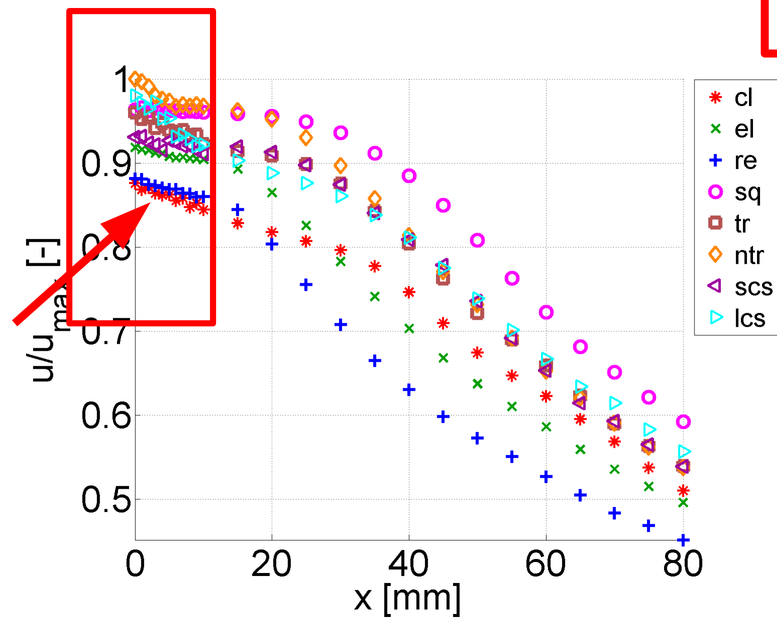
- Differences:
  - cross section shape amplitude → 20%
  - extent of potential cone → $1<X_{pc}/D<7$
  - cross section shape decay rate

Influence of flow mixing

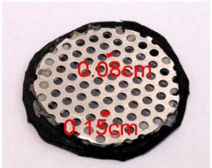
L_u=100cm



Q=35l/min

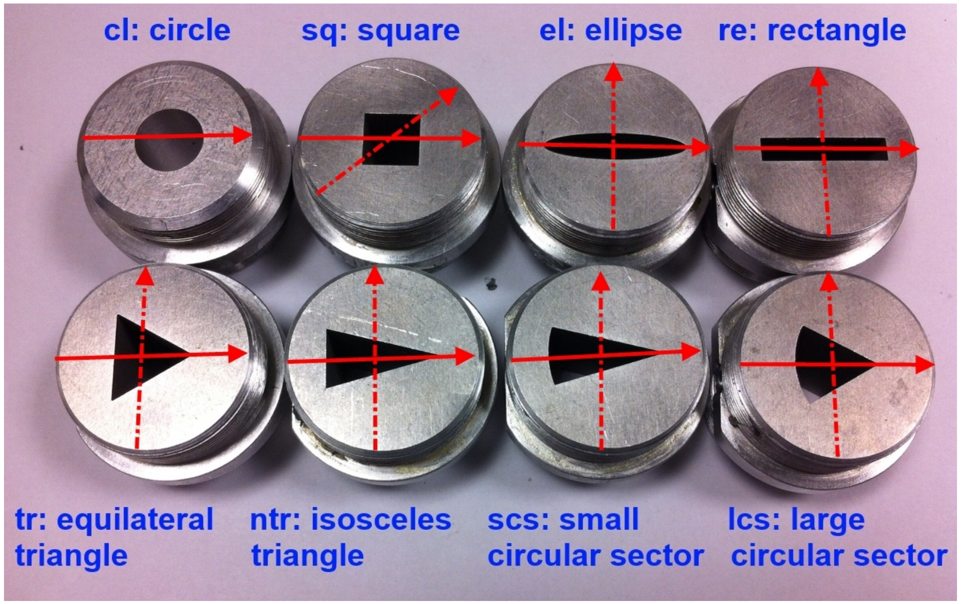


ogp + Q=35l/min

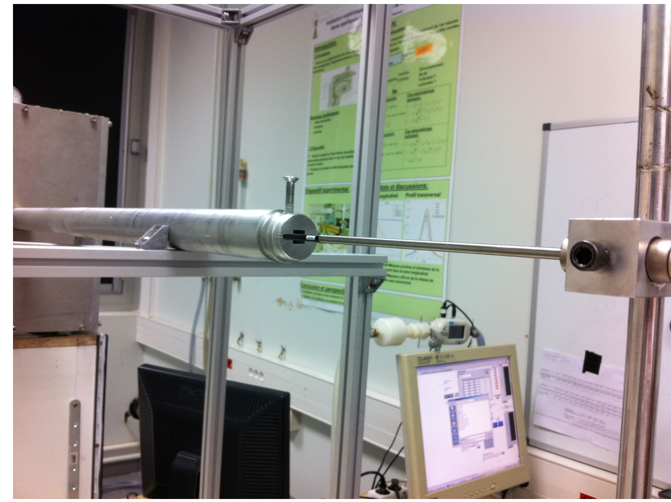


ogp

- Differences:
  - cross section shape decay rate
  - cross section shape amplitude → 10%
  - extent of potential cone → 5 times difference



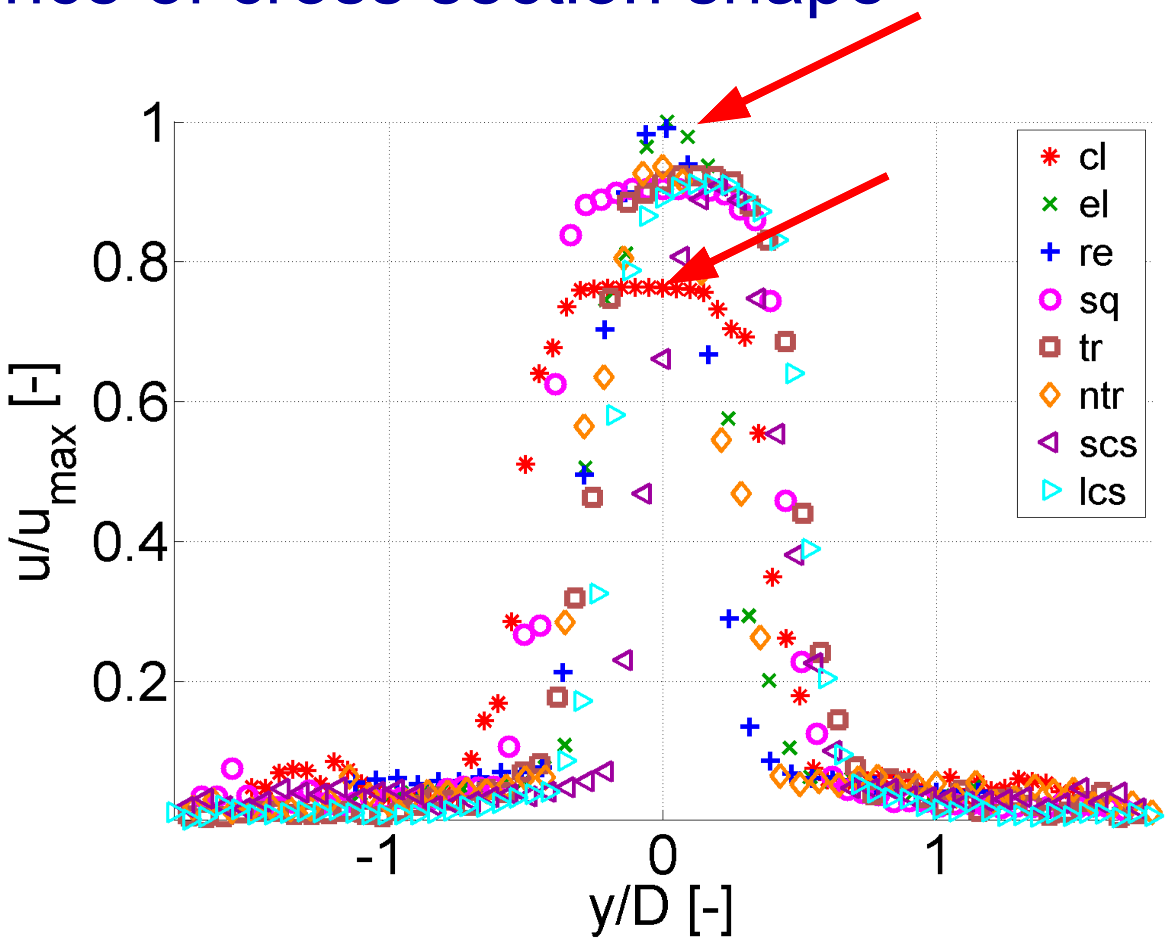
$A_c = 0.79 cm^2$



L_u=100cm

- major axis: →
- minor axis: - · - · →

- Influence of cross section shape



Q= 5l/min

'minor' axis

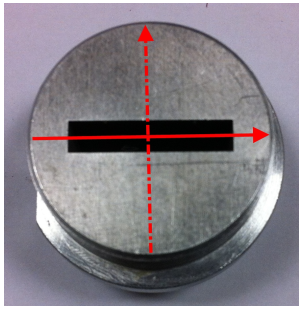
- cross section shape → boundary layer development

Experimental data: flow visualization

L_u=60cm

Q=5l/min

'major' axis



- Flow with free jet:
  - observation: flow structures (vortices)

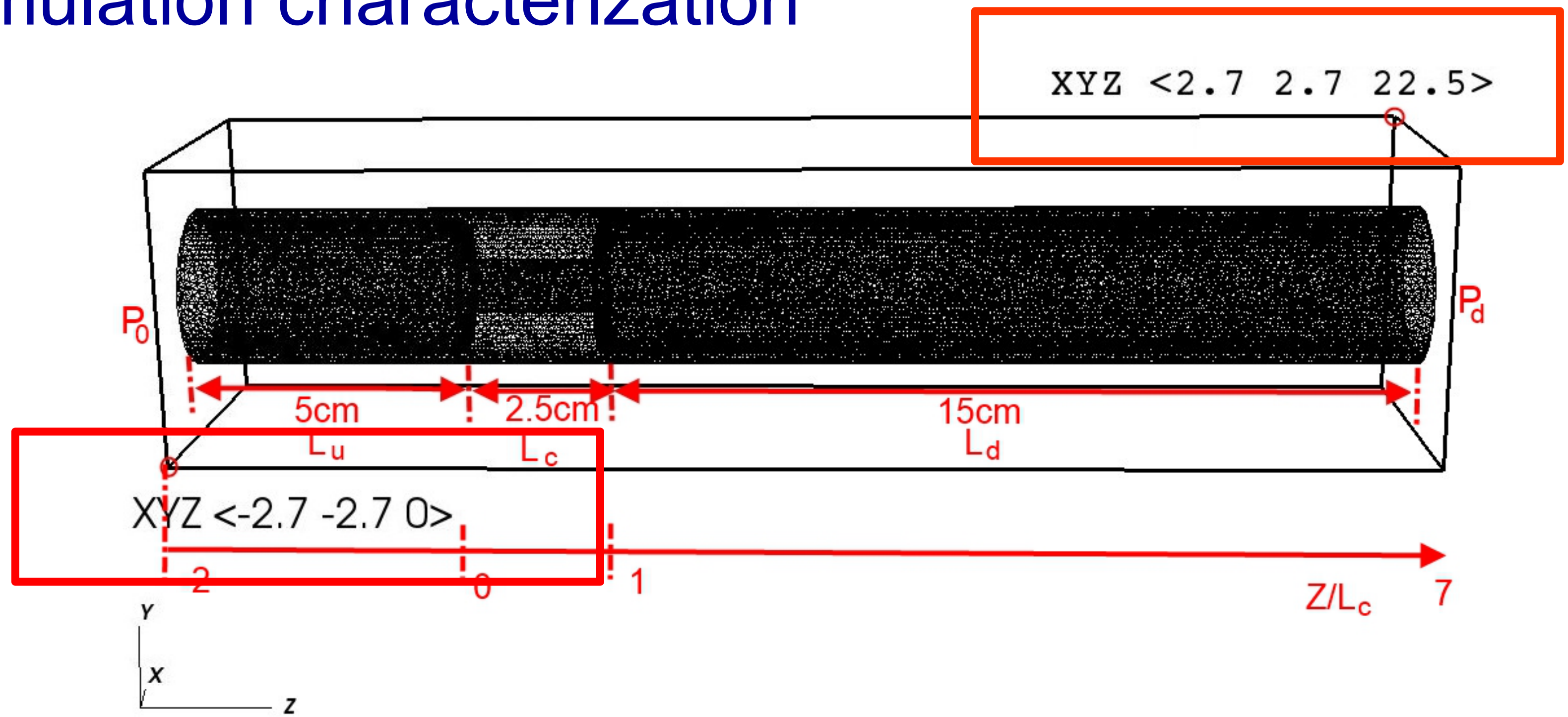
• Immersed boundary method

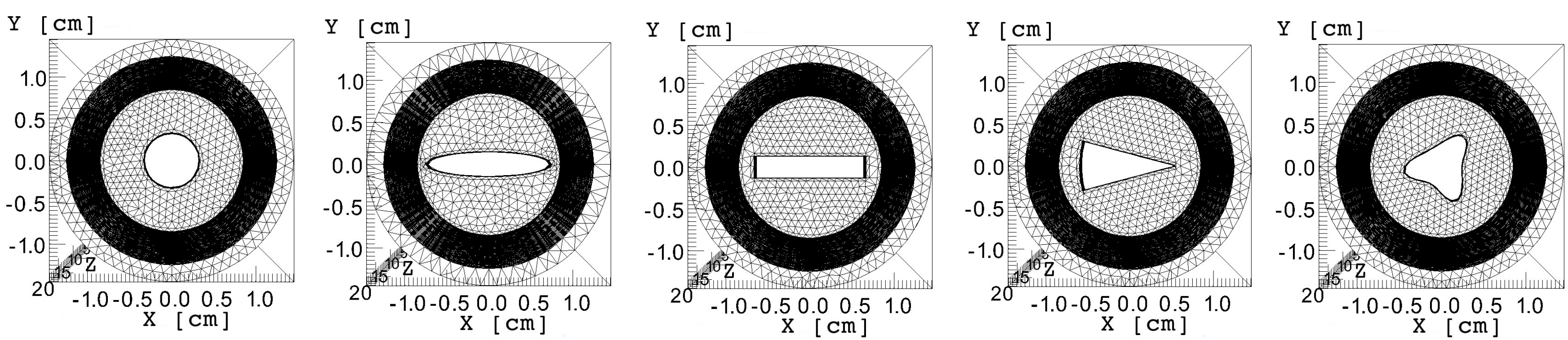
$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}(\mathbf{x},t)+\mathbf{u}(\mathbf{x},t)\cdot\nabla\mathbf{u}(\mathbf{x},t)\right)=-\nabla P(\mathbf{x},t)+\mu\nabla^2\mathbf{u}(\mathbf{x},t)+\mathbf{f}(\mathbf{x},t)$$

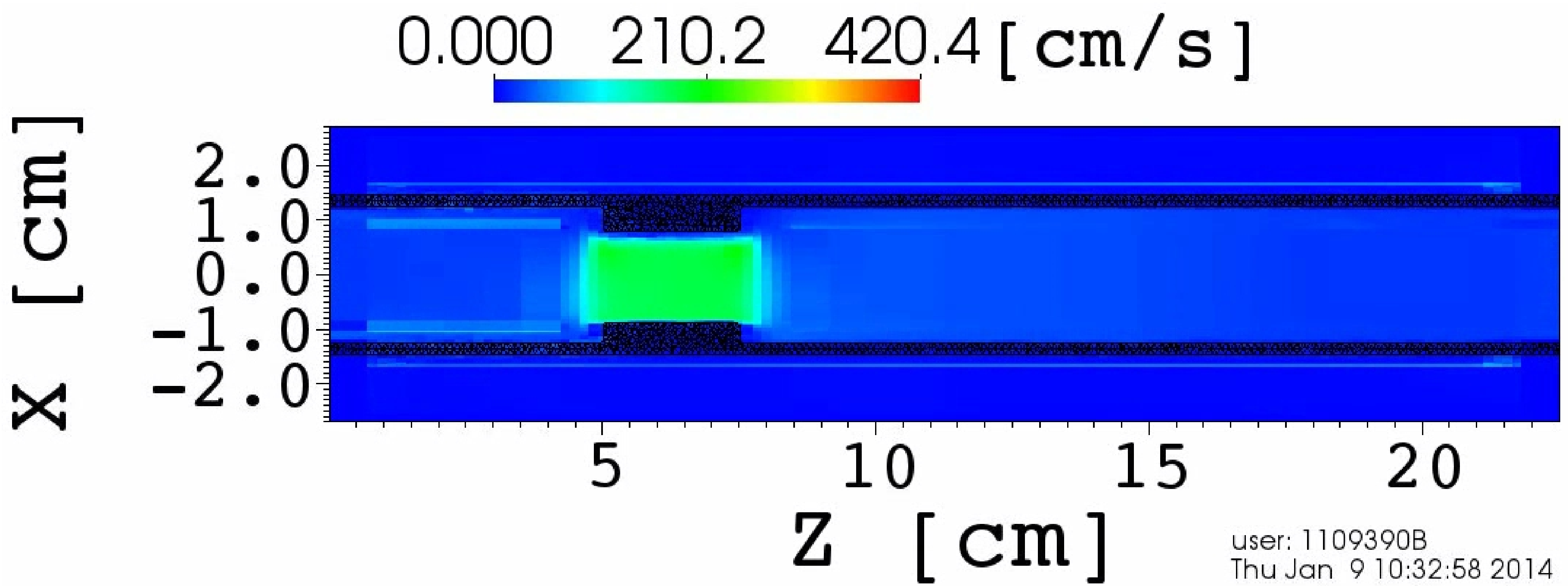
$$\mathbf{f}(\mathbf{x},t)=\int_U \mathbf{F}(\mathbf{s},t)\delta(\mathbf{x}-\chi(\mathbf{s},t))d\mathbf{s}$$

- Incompressible laminar flow
- Rigid, no-moving structure
- Impose pressure gradient: inlet & outlet
- Impose P=0Pa for remainder boundary
- t_{tot} >0.04s, quasi-steady: t>0.02s

- Simulation characterization







P_0=35Pa, P_d=0Pa + circular sector

Observations:

- flow development / vena contracta
- reattachment of jet

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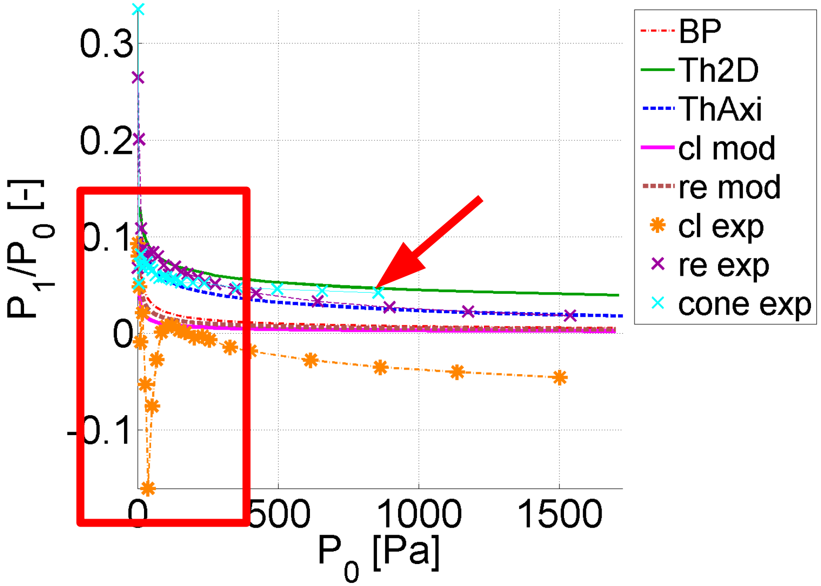
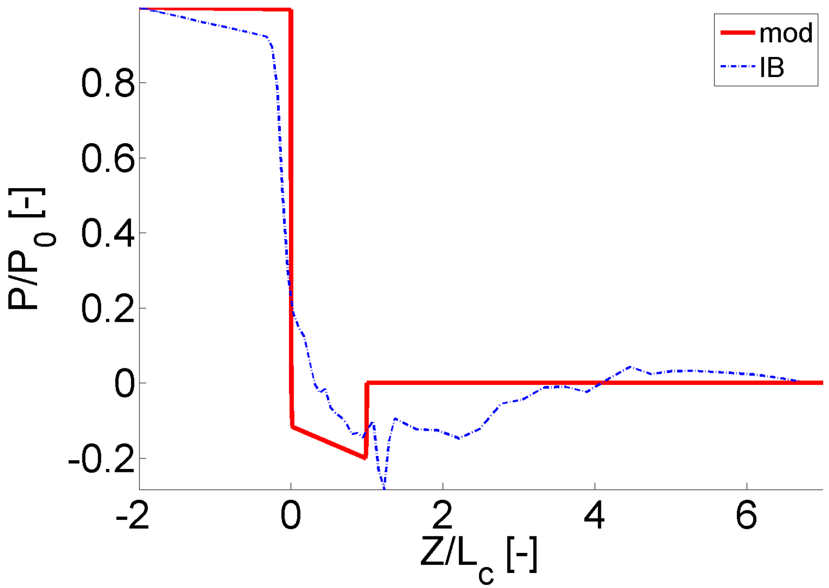




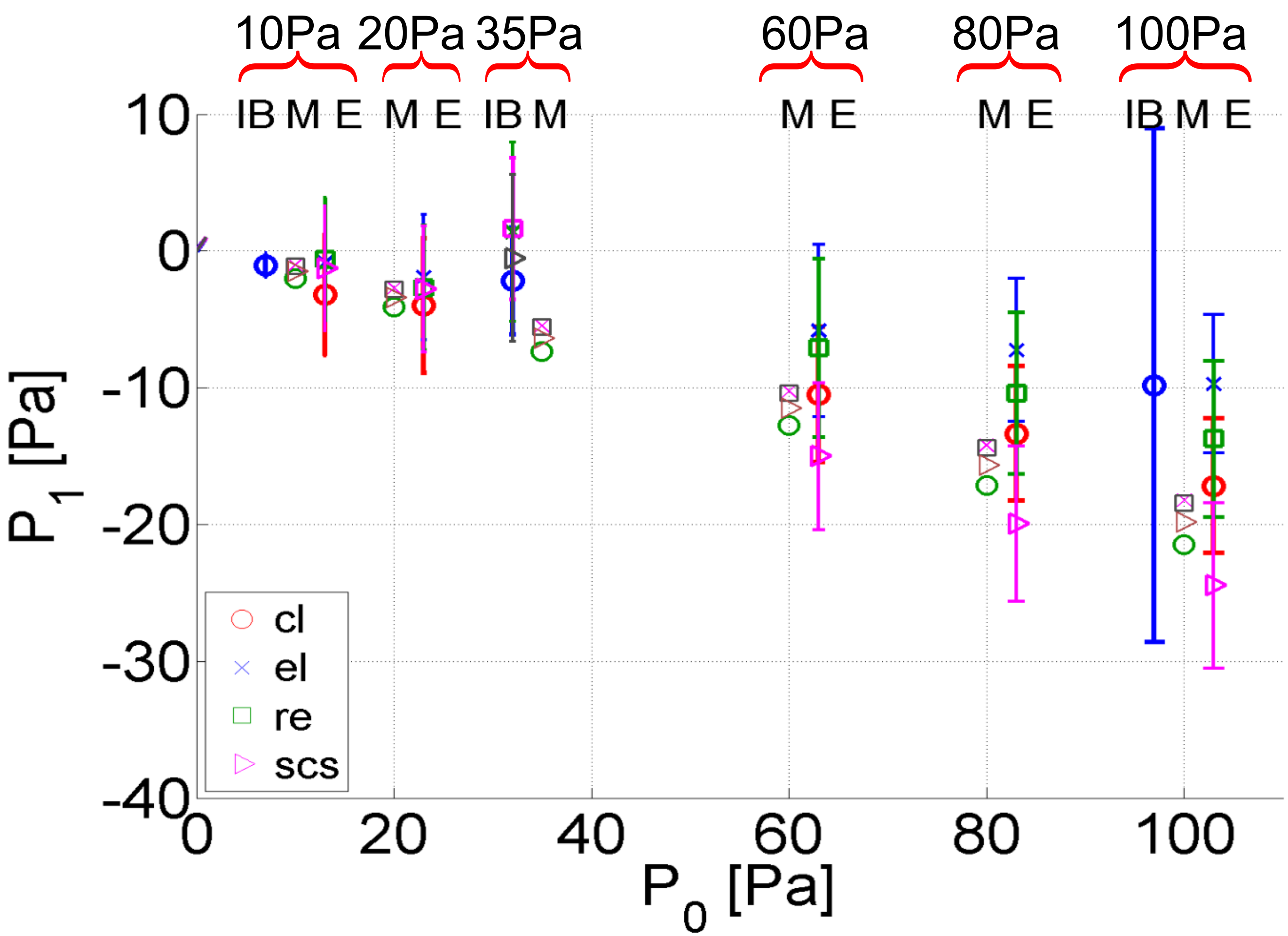
mod + exp

mod + IB

- can not capture flow dynamics
- agreement between modeled and measured when using **cone**
- < 5% for P_0 > 300Pa
  > 5% & <20% for P_0 < 300Pa

- useless at downstream
- 10% within constriction

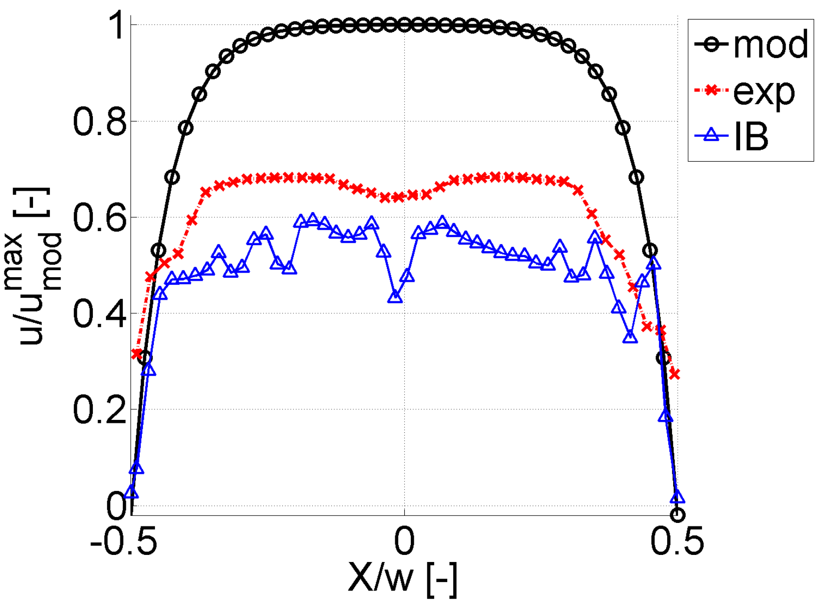
- Quantify the variation due to the cross section shape



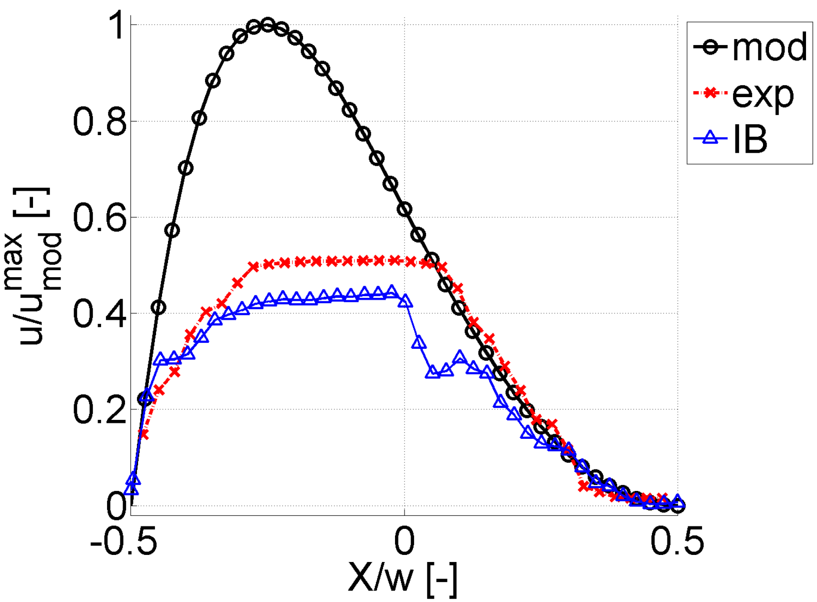
IB : IB method

M : model

E : experiment



rectangle



circular sector

mod + exp + IB

- 40% difference of maximum velocity
- match near the wall
- 20% better approximation: IB>mod

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Experiment

- influence of cross section shape
- influence of flow development
- influence of flow mixing
- complex flow dynamics

Quasi-3D model

validated influence of geometry
(cross section shape, L_c, A_r)

- within the constriction (pressure)
- at the outlet (velocity)

- not useful downstream the constriction
- can not capture complex flow dynamics
- no turbulence

1D / 2D models

- a small aperture
- fast
- simple

- negative points

3D model

- capture some of flow dynamics

- geometry
- high computational cost
- no variation of initial condition
- no turbulence

- Stability of the flow patterns
- Transition mechanism : laminar → turbulent
- Flow model → turbulent + unsteady + arbitrary/ varying CSS
- Interactive boundary layer method
- IB method → full fluid-structure interaction
- Refine applications, such as speech production, blood arteries, geo-physical flows
- Evolution of replicas, such as rounded corner, smooth inlet → all cross section shapes

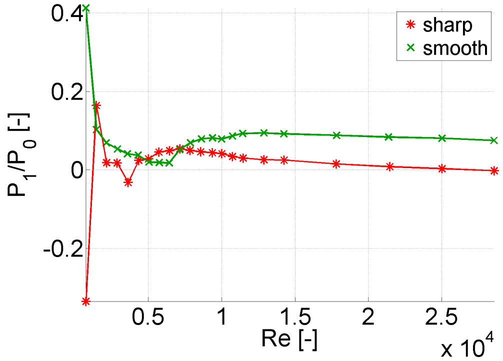
- Unsteady quasi-3D model

$$\frac{d\overline{u}}{dt} - \frac{Q^2}{A^3}\frac{dA}{dx} = -\frac{1}{\rho}\frac{dP}{dx} + \nu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

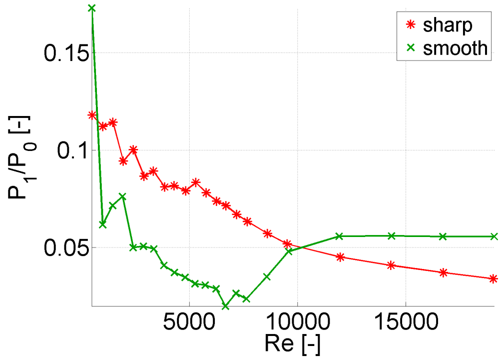
• Rounded corner

3D print

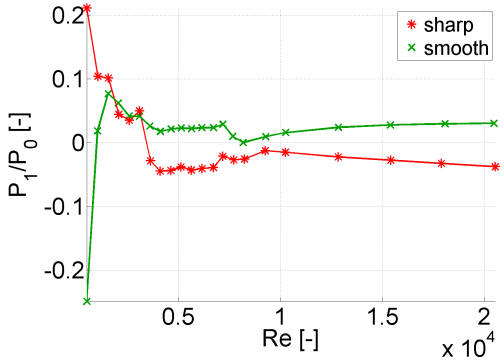
free jet



(a) circle

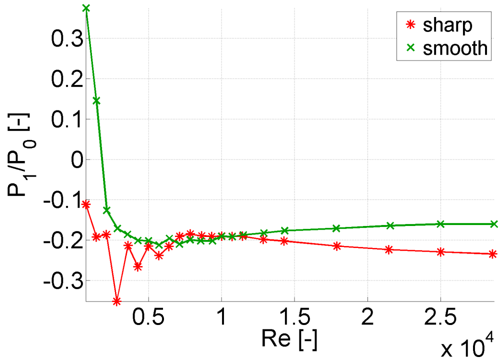


(b) ellipse

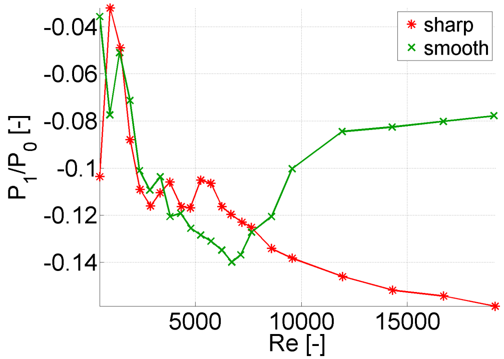


(c) small circular sector

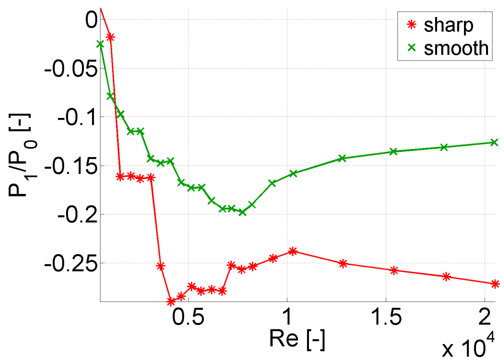
confined jet



(a) circle

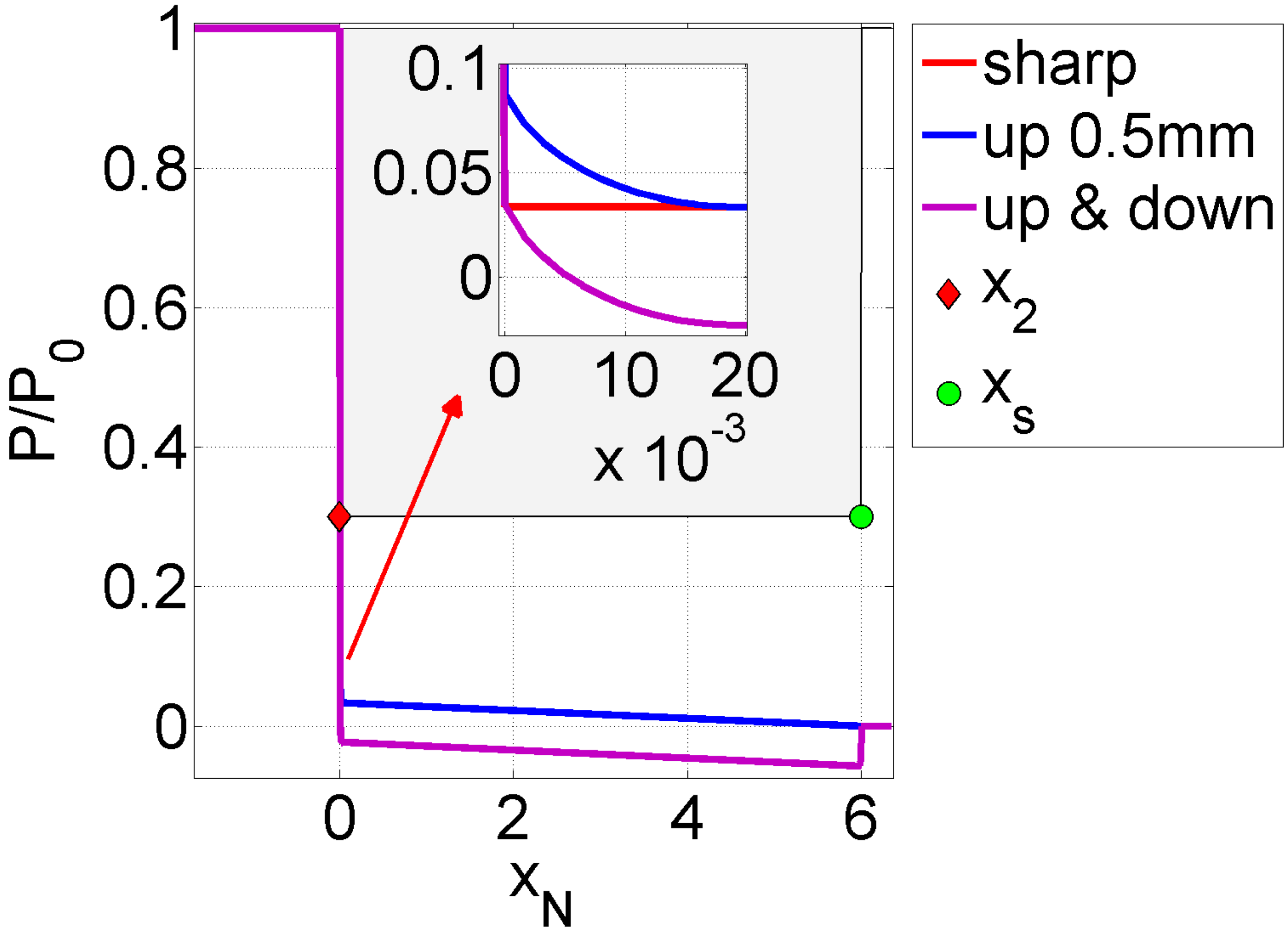


(b) ellipse

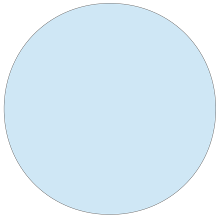


(c) small circular sector

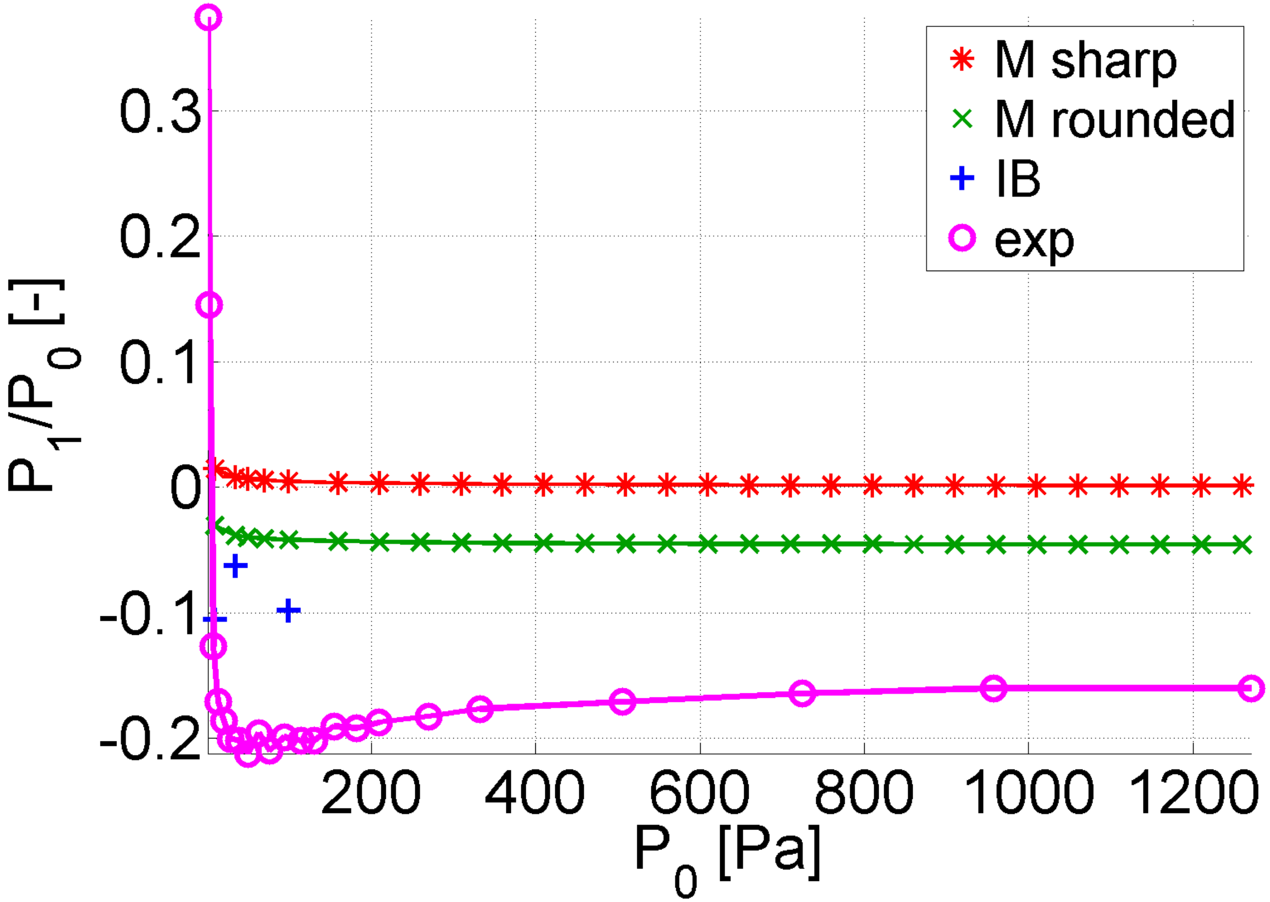
- Rounded corner



P_0=75Pa
A_{min}=0.79cm²
Amin/A_0=0.3

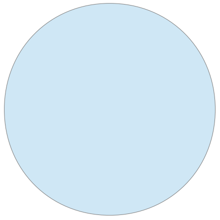


- Rounded corner



A_{min}=0.79cm²

confined jet



Publications

- ## Journal articles (3 published)

1. Wu B., Van Hirtum A., Luo X.Y., 2013. Pressure driven steady flow in constricted channels of different cross section shapes. International Journal of Applied Mechanics

2. Wu B., Van Hirtum A., Pelorson X., Luo X.Y., 2013. The influence of glottal cross-section shape on theoretical flow models. Journal of the Acoustical Society of America

3. Grandchamp X., Fujiso Y., Wu,B., Van Hirtum, A., 2012. Steady laminar axisymmetrical nozzle flow at moderate Reynolds numbers: modelling and experiment. Journal of Fluids Engineering

- ## International conference papers (4 published)

1. Wu B., Van Hirtum A., Luo X.Y., 2013. Influence of cross section shape on the outcome of a two-mass model. 21st International Congress on Acoustics

2. Pelorson X., Van Hirtum A., Wu B., Silva F., 2013. Theoretical and experimental study of glottal geometry in phonation. 21st International Congress on Acoustics

3. Wu B., Van Hirtum A., Luo X.Y., 2013. Influence of cross section shape on steady and unsteady flow through a constricted channel. 10th International Workshop on computational system biology

4. Wu B., Van Hirtum A., Luo X.Y., 2012. Analytical solution for pressure driven viscid flow in ducts of different shape: application to human upper airways. Acoustics 2012 Nantes

Thanks for your attention!