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multi-physique du comportement acoustique de milieux
poroélastiques : application aux mousses de faible
densité**

Minh Tan Hoang

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**Modélisation et Simulation Multi-Échelle et Multi-Physique du
Comportement Acoustique de Milieux PoroElastiques :
Application aux Mousses de Faible Densité**

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Modélisation et Simulation Multi-Échelle et Multi-Physique du Comportement Acoustique de Milieux PoroElastiques – Application aux Mousses de Faible Densité

Résumé

L'objectif de ce mémoire de recherche est de déterminer les propriétés acoustiques des milieux poroélastiques à partir d'une démarche multi-échelle et multi-physique. Il traite d'échantillons réels de mousses, à cellules ouvertes ou partiellement fermées, dont les propriétés microstructurales sont caractérisées par des techniques d'imagerie. Cette information est utilisée afin d'identifier une cellule périodique idéalisée tridimensionnelle, qui soit représentative du comportement acoustique du milieu poreux réel. Les paramètres gouvernant les propriétés acoustiques du milieu sont obtenus en appliquant la méthode d'homogénéisation des structures périodiques. Dans une première étape, la structure des mousses est supposée indéformable. Il a été montré que pour le cas d'une distribution étroite de tailles caractéristiques de la géométrie locale, le comportement macroscopique d'une mousse à cellule ouverte peut être calculé à partir des propriétés géométriques locales de manière directe. Dans le cas d'une distribution étendue, le comportement acoustique du milieu est gouverné par des tailles critiques qui sont déterminées à partir de la porosité et de la perméabilité statique pour une mousse à cellules ouvertes ; pour une mousse à cellules partiellement fermées il est nécessaire d'identifier en plus une dimension connue de la géométrie locale. Nos résultats sont comparés avec succès à des données expérimentales obtenues par des mesures au tube d'impédance. Dans une seconde étape, les propriétés élastiques effectives du milieu poreux sont déterminées. Une modélisation par éléments finis de la cellule représentative a été mise en œuvre. Les paramètres élastiques calculés sont finalement comparés avec les données de la littérature, ainsi qu'à des essais mécaniques.

Mots-Clés: microstructure – acoustique – mousse – membrane – périodique – méthode des éléments finis – transport – absorption – perméabilité – élastique

Multi-Scale and Multi-Physics Modeling and Simulation of Acoustic Behavior of Poroelastic Media - Application to Low Density Foams

Abstract

This work aims at determining the acoustical properties of poro-elastic media through a multi-scale method. Some imaging techniques (tomography and micrographs) allow to estimate some quantitative microstructure properties of foams containing open or partially closed cells. These properties are used in order to clarify the features of a representative three-dimensional unit cell of a periodic structure, which mimics the behaviour of the real foam. All parameters controlling the acoustical properties of the porous foam are obtained by using the homogenization of periodic structures. In a first step, the structure of the foam is assumed to be rigid. It was shown that, in the case of a narrow distribution of the characteristic size of the local geometry, a direct computation of the macroscopic behaviour from the local geometrical properties is consistent with the measured acoustical properties. For a wide distribution of pore size, the acoustical behaviour is controlled by critical sizes that are obtained from porosity and static permeability for an open-cell foam, while for partially closed cells, the identification of a complementary characteristic dimension within the pores becomes necessary (e.g. closure rate of membranes). Our results compare well with data obtained from an impedance tube set-up. In a second step, effective elastic properties are computed through a modelling of the foam structure by finite elements. The computed elastic parameters are finally compared with data coming from the literature and with results of mechanical tests.

Key-words: microstructure – acoustics – foams – membranes – periodic – finite element method – transport – absorption – permeability – elasticity

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Principales notations

\sim	Valeur complexe
$\langle \cdot \rangle$	Moyenne spatiale
V	Volume (m^3)
S	Surface (m^2)
L	Longueur des ligaments (m)
r	Rayon des ligaments (m)
R	Rayon des sphères aux nœuds (m)
D	Taille des pores (m)
t	Épaisseur des membranes (m)
δ/δ_{\max}	Taux de fermeture des membranes (-)
T_s	Etranglement (m)
\vec{u}	Vitesse locale du fluide (m.s^{-1})
p	Pression acoustique (Pa)
τ	Température ($^{\circ}\text{K}$)
ϕ	Porosité ouverte (-)
k_0	Perméabilité statique visqueuse (m^2)
σ	Résistivité statique à l'écoulement (N.s.m^{-4})
Λ'	Longueur caractéristique thermique (m)
Λ	Longueur caractéristique visqueuse (m)
k_0'	Perméabilité statique thermique (m^2)
α_{∞}	Tortuosité (limite haute fréquence) (-)
α_0	Tortuosité statique (-)
α_0'	Tortuosité statique thermique (-)
λ	Longueur d'onde (m)
η	Viscosité dynamique de l'air (Pa.s)
κ	Conductivité thermique de l'air ($\text{W.m}^{-1}.\text{K}^{-1}$)
ρ_0	Masse volumique de l'air (kg.m^{-3})
c_0	Célérité du son dans l'air (m.s^{-1})
Z_0	Impédance caractéristique de l'air (kg.m^{-3})
Pr	Nombre de Prandtl (-)
ν	Viscosité cinématique ($\text{m}^2.\text{s}^{-1}$)
γ	Rapport de chaleurs spécifiques (-)
C_p	Chaleur spécifique à pression constante ($\text{J.kg}^{-1}.\text{K}^{-1}$)
C_v	Chaleur spécifique à volume constante ($\text{J.kg}^{-1}.\text{K}^{-1}$)
K_a	Module d'incompressibilité adiabatique de l'air (Pa)

Principales notations

f	Fréquence (Hz)
ω	Pulsation (rad.s^{-1})
$\tilde{\alpha}(\omega)$	Tortuosité dynamique (–)
$\tilde{\beta}(\omega)$	Compressibilité dynamique adimensionnée (–)
$\tilde{k}(\omega)$	Perméabilité dynamique visqueuse (m^2)
$\tilde{k}'(\omega)$	Perméabilité dynamique thermique (m^2)
$\tilde{\rho}(\omega)$	Masse volumique dynamique effective (kg/m^3)
$\tilde{K}(\omega)$	Module d'incompressibilité dynamique effectif (Pa)
$\tilde{Z}(\omega)$	Impédance caractéristique (Pa.s.m^{-1})
$\tilde{Z}_s(\omega)$	Impédance de surface (Pa.s.m^{-1})
E_μ	Module d'Young de la matrice solide (Pa)
ν_μ	Coefficient de Poisson de la matrice solide (–)
σ, Σ	Champs de contrainte micro et macroscopique (Pa)
ε, \mathbf{E}	Champs de déformation micro et macroscopique (–)
c, C	Constante de rigidité micro et macroscopique (Pa)
s, S	Constante de souplesse micro et macroscopique (Pa^{-1})
E_L	Module élastique longitudinal homogénéisé (Pa)
E_I	Module d'Young homogénéisé (Pa)

Introduction : Contexte et objectifs de la thèse

1 Introduction : Contexte et objectifs de la thèse

1.1. Contexte

De manière concomitante avec la modernisation de la société, l'exigence de réduction des nuisances sonores est de plus en plus importante. Elle concerne plusieurs domaines de l'industrie tels que le bâtiment, les transports (automobile, ferroviaire, aéronautique...). Dans ce domaine, les matériaux poreux sont connus pour leurs qualités d'absorption sonore et sont donc couramment utilisés dans l'industrie.

En effet, lorsque ces matériaux font obstacle à l'onde acoustique, celle-ci induit dans le milieu poreux des mouvements relatifs de l'air dans les pores du matériau, et une partie de son énergie est dissipée sous forme de chaleur par des pertes liées aux effets visqueux, thermiques ou élastiques.

La performance acoustique des matériaux isolants a fait l'objet de nombreuses études depuis plusieurs années et il existe principalement deux méthodes pour approcher le problème. La première méthode plutôt empirique est d'ordre purement expérimental et repose sur les mesures acoustiques permettant de mesurer des paramètres acoustiques sur le plan macroscopique. Cette méthode a permis pendant de nombreuses années d'améliorer les performances acoustiques d'un matériau, mais il est relativement difficile de l'utiliser pour optimiser ces matériaux car la physique de la dissipation interne n'est prise en compte que de façon qualitative. La deuxième méthode utilise justement une étude fine de la physique de la dissipation locale dans le matériau en cherchant à lier microstructure et comportement acoustique du milieu poreux. Il s'agit donc d'une approche multi-échelle, qui a pu être mise en œuvre progressivement grâce à l'existence de codes de calcul numérique efficaces. L'objectif de cette thèse est de proposer une contribution visant à établir des liens entre la microstructure et les propriétés acoustiques des mousses isolantes à cellules ouvertes ou partiellement fermées par une approche multi-échelle.

1.2. Objectifs généraux de la thèse

Cette thèse porte sur la propagation d'ondes acoustiques dans le domaine des grandes longueurs d'ondes dans un milieu poreux dont la structure apparaît comme homogène à l'échelle macroscopique. Les propriétés du milieu « effectif » représentant à l'échelle macroscopique la propagation de l'onde dans le milieu poreux dépendent alors de ses paramètres physiques ainsi que de la géométrie du réseau poreux. L'objectif principal de ce

travail est d'établir des relations entre la microstructure et le macro-comportement acoustique de milieux poreux réels, avec applications aux mousses de faible densité. Noter au passage que le terme densité est utilisé abusivement, et qu'il désigne en réalité la masse volumique effective du matériau réel. Précisions aussi que par mousses de faible masse volumique effective, on entend essentiellement des mousses dont la porosité est très élevée (ce qui permet de disposer des modèles géométriques de la littérature issus de la physique des mousses). Pour autant, la démarche développée dans cette thèse peut être étendue sans limitation à des matériaux de faible porosité.

L'étude des milieux poreux a conduit depuis de nombreuses années à la construction de plusieurs modèles semi-phénoménologiques qui comportent un nombre de paramètres limité qui sont fonctions de la microstructure. L'utilisation de ces modèles nécessite donc à nouveau la mise en œuvre d'essais sur des matériaux réels ou de modélisations permettant d'accéder à ces paramètres, dits paramètres acoustiques.

L'étude de la structure complexe de ces milieux poreux par modélisation fine à l'échelle des pores reste donc incontournable. Cette structure est caractérisée par une porosité importante à l'intérieur d'un squelette fin généré par un assemblage désordonné de filaments et membranes interconnectés. La modélisation complète d'un volume élémentaire représentatif d'un tel matériau n'est pas envisageable sur le plan numérique. De plus, une telle modélisation « à l'aveugle » ne permettrait pas de pointer les paramètres importants de la structure vis à vis du comportement acoustique du milieu poreux. Par ailleurs, la méthode d'homogénéisation des milieux périodiques permet par une analyse bien fondée au plan mathématique de déterminer les propriétés effectives d'une structure périodique à partir de l'étude d'un problème aux limites portant sur une cellule de base.

La première idée fondamentale à la base de cette thèse est de construire une cellule périodique qui comporte le maximum de données géométriques et physiques obtenues à partir d'observations et de mesures sur la structure réelle, puis de déterminer les paramètres acoustiques nécessaires à la mise en œuvre des modèles semi-phénoménologiques.

Sur la base de cette idée, il est possible de mettre en œuvre une modélisation reposant sur la résolution des équations de la dynamique de l'écoulement de fluide à l'échelle des pores. Toutefois, ce type de modélisation nécessite la résolution d'un problème aux limites fréquence par fréquence sur une large gamme de fréquences, comportant de plus des difficultés numériques compte tenu de l'existence d'une couche limite à hautes fréquences. Mais par ailleurs, il a été montré que la détermination

des paramètres acoustiques peut s'effectuer également en résolvant des problèmes statiques ou quasi-statiques indépendants de la fréquence. La deuxième idée à la base de la thèse est justement d'utiliser uniquement la résolution de ces problèmes statiques ou quasi-statiques dans la détermination des paramètres acoustiques.

Dans ce travail, on adopte donc une **méthode hybride** combinant la modélisation par éléments finis avec l'apport des modèles phénoménologiques que prennent en compte la physique sous-jacente.

En résumé, la méthode mise en œuvre comporte les phases suivantes :

- observation de la microstructure du matériau par des outils d'imagerie adaptés en vue de la détermination de données qualitatives et quantitatives sur la microstructure qui sont pertinentes vis-à-vis du comportement acoustique ;
- construction d'une cellule périodique de propriétés géométriques déterminées à partir des résultats de l'observation de la microstructure ;
- résolution des problèmes aux limites statiques ou quasi-statiques sur la cellule périodique et obtention des paramètres acoustiques du matériau considéré ;
- comparaison de la réponse du milieu effectif finalement obtenu avec des résultats expérimentaux réalisés sur le matériau réel.

1.3. Plan du mémoire

Ce mémoire de recherche contient trois grandes parties dont les contenus principaux seront présentés.

Le **chapitre 2** expose la méthodologie pour atteindre les objectifs spécifiques énoncés. Il s'agit des outils importants concernant toutes les étapes principales de la caractérisation jusqu'au calcul des propriétés effectives.

Premièrement, les techniques d'imagerie permettant l'acquisition de données sur la géométrie locale sont présentées. Dans cette présentation, on distinguera l'identification bidimensionnelle avec microscopie optique ou microscopie électronique d'une part et l'identification tridimensionnelle par microtomographie à rayons-X. Pour chaque méthode d'imagerie, on se focalise sur la présentation générale du fonctionnement, sur les limites d'application et les performances (résolution, grossissement...). On mettra en lumière aussi les avantages et les inconvénients de chaque technique.

Dans un deuxième temps, quelques résultats obtenus par l'homogénéisation des structures périodiques, seront rappelés dans le cas de

l'écoulement en milieu poreux quasi-statique et dans le cas de la poroélasticité.

Ensuite, les modèles semi-phénoménologiques sont abordés. Il s'agit des théories classiques de fluide équivalent, développées depuis plusieurs années, et permettant de décrire le comportement dynamique de milieux poreux représenté par des fonctions complexes dépendant de la fréquence et définis par un nombre fini de paramètres acoustiques.

Enfin le chapitre 2 se termine par la présentation du modèle « hybride » qui combine les modélisations par éléments finis et les modèles semi-phénoménologiques.

Le **chapitre 3** qui porte sur la détermination des propriétés de transport des milieux poreux en squelette indéformable constitue la part la plus importante de ce mémoire. Il se compose de trois sous-chapitres.

Le premier sous-chapitre étudie le cas d'un milieu peu dispersé, où les éléments géométriques d'une cellule périodique sont déterminés à partir des mesures directes des tailles caractéristiques locales. Les propriétés acoustiques sont ensuite calculées par la méthode hybride combinant les éléments finis et les modèles semi-phénoménologiques. Les résultats de l'application de la méthode sur une mousse d'aluminium à cellules ouvertes sont alors comparés aux mesures acoustiques obtenues par la méthode du tube d'impédance.

Le deuxième sous-chapitre traite du comportement acoustique des échantillons réels de mousses pour lesquels la distribution de tailles de pores est étendue. Pour ce type de matériau, il est montré que les dimensions microstructurales issues des mesures directes ne sont pas représentatives du macro-comportement du matériau. Ce dernier est alors gouverné par au moins deux tailles critiques. La mousse est tout d'abord supposée ouverte et sans membrane. Les tailles caractéristiques de sa microstructure sont déterminées à partir de deux paramètres macroscopiques : la porosité et la perméabilité statique. L'application de cette méthode sur plusieurs échantillons réels de mousse donne des résultats qui sont satisfaisants dans certains cas, mais montrent qu'il est nécessaire pour d'autres cas d'introduire une deuxième taille critique. Les micrographies montrent que ces matériaux sont caractérisés par l'existence de membranes. On introduit alors la représentation de ces membranes dans la géométrie 3D, ainsi qu'une dimension caractéristique liée à ces membranes. Les membranes sont alors modélisées dans la cellule périodique. Dans ce travail, on se limite à une modélisation des membranes qui les fait apparaître de manière homogène, pour toutes les facettes de cellules, et donc pour toutes les cellules *à fortiori*, ce qui exclut de fait une

modélisation des cellules en cul-de-sac (la porosité est ouverte). La confrontation des calculs aux mesures à tube d'impédance montre une bonne adéquation de la méthodologie adoptée.

Le chapitre 3 se termine par des applications et des comparaisons aux données disponibles dans la littérature; avec mise en exergue de relations entre longueurs caractéristiques locales et longueurs caractéristiques macroscopiques.

Le **chapitre 4** concerne la détermination des propriétés élastiques des mousses acoustiques. La géométrie de la cellule périodique est identique à celle déterminée dans le chapitre 3. Cette cellule ayant une symétrie cubique les propriétés élastiques du matériau anisotrope cubique sont calculées en résolvant les équations de l'élasticité linéaire par éléments finis. En supposant que la mousse réelle correspond à la l'orientation aléatoire de telles cellules cubiques obtenue par rotation autour d'un axe quelconque dans l'espace ou autour d'un axe quelconque dans le plan, on détermine les propriétés d'un matériau isotrope ou isotrope transverse équivalent.

Les calculs sont effectués sur deux mousses cellulaires à faible densité ; les propriétés élastiques calculées sont comparés aux résultats des essais de compression uniaxiale. Finalement, les propriétés acoustiques de ces deux échantillons réels de mousse sont calculées en prenant en compte ces paramètres élastiques dans le modèle poroélastique. Une comparaison de ces propriétés acoustiques calculées aux résultats de mesures utilisant un tube d'impédance est ensuite effectuée.

1.4. Choix de présentation du mémoire

Ce mémoire a fait l'objet de plusieurs publications ou projets de publications dans des revues internationales à comité de lecture ou des chapitres de livres. Compte tenu de la diffusion internationale de ces documents, qui est d'ailleurs de plus en plus souhaitée pour les travaux de thèse, il a été jugé préférable d'intégrer ces documents dans le mémoire dans la forme sous laquelle ils ont été publiés ou soumis, mais en conservant une logique de présentation d'ensemble. De cette façon, le lecteur pourra évaluer non seulement le contenu du travail de thèse mais également le rayonnement international propre à ces travaux.

Dans une telle présentation, il est apparu que de nombreuses références sont communes à plusieurs articles et il a donc été choisi de regrouper les références correspondantes en fin de chapitre.

Caractérisation de la microstructure de matériaux poreux et méthodologie de prévision de leurs propriétés acoustiques

2 Caractérisation de la microstructure de matériaux poreux et méthodologie de prévision de leurs propriétés acoustiques

Ce chapitre porte sur la prévision des propriétés acoustiques des milieux poreux par une méthodologie semi-analytique. Cette méthode se situe à la frontière entre les méthodes phénoménologiques qui n'utilisent pas de modélisation de la microstructure et les méthodes d'homogénéisation des propriétés dynamiques, qui supposent connue la microstructure et qui prévoient les propriétés acoustiques par une résolution des équations de la dynamique à l'échelle locale. La méthodologie semi-analytique utilise les modèles phénoménologiques, mais alimente ces modèles par des informations obtenues par des calculs numériques portant sur une géométrie simplifiée de la microstructure. Celle-ci est elle-même définie en prenant en compte les propriétés de la micro-géométrie obtenues par une caractérisation spécifique sur un échantillon du matériau réel.

Ce chapitre comporte donc quatre parties. La première présente les méthodes adoptées pour caractériser la géométrie sur échantillons réels, la deuxième présente les modélisations par éléments finis utilisées dans le mémoire et la troisième partie présente les modèles phénoménologiques qui sont à la base de la méthode semi-analytique. Les méthodes présentées dans ces trois parties sont combinées dans un modèle hybride, montré dans la dernière partie du chapitre 2, visant à déterminer les fonctions dynamiques sur une gamme de fréquence complète.

2.1. Caractérisation de la micro-géométrie

Les propriétés macroscopiques d'un matériau dépendent fortement de sa microstructure ; c'est pour cette raison que l'étude de la géométrie locale du matériau occupe un rôle très important dans la science des matériaux. Pour les chercheurs, la connaissance de la microstructure d'un matériau permet d'interpréter et d'expliquer physiquement ses propriétés d'usage, et particulièrement c'est une information indispensable pour les méthodes d'homogénéisation des matériaux afin de déterminer leurs caractéristiques macroscopiques. La microstructure est caractérisée à différentes échelles par des techniques d'imagerie bidimensionnelle conventionnelle: microscopie optique et microscopie électronique, qui ont été complétées récemment par des techniques d'imagerie tridimensionnelle plus avancées comme les différents types de tomographie et en particulier la microtomographie à rayon-X. Ces méthodes produisent des images

numériques dont il faut ensuite extraire des paramètres quantitatifs de la microstructure.

Ce sous-chapitre présente les techniques de caractérisation morphologique du matériau utilisées dans le cadre de ce travail de thèse, ainsi que les avantages et les inconvénients de chaque méthode.

2.1.1. Identification bidimensionnelle

2.1.1.1. Microscopie optique

Afin de comprendre la relation entre la microstructure et les propriétés macroscopiques des matériaux, il est nécessaire d'observer des détails de la microstructure qui ne sont pas visibles à l'œil nu. La méthode de caractérisation la plus couramment utilisée reste encore la microscopie optique.

Il existe de nombreuses techniques de microscopie optique (Arnold, 1993) ^[1]: La microscopie en lumière directe, la microscopie de polarisation en lumière transmise ou réfléchie, la microscopie en contraste de phase... Les valeurs usuelles des grossissements des objectifs varient entre 1,25 et 125 (Roblin, 1999) ^[2]. La Fig. 2.1 représente un microscope binoculaire LEICA MZ6 avec des grossissements variant entre 6.3 – 40 ainsi qu'une micrographie d'une mousse polyuréthane de faible densité. Cet équipement, disponible au Laboratoire Géomatériaux et Géologie de l'Ingénieur (LG2I) de l'Université Paris-Est Marne-la-Vallée, a été utilisé dans le cadre de la thèse.

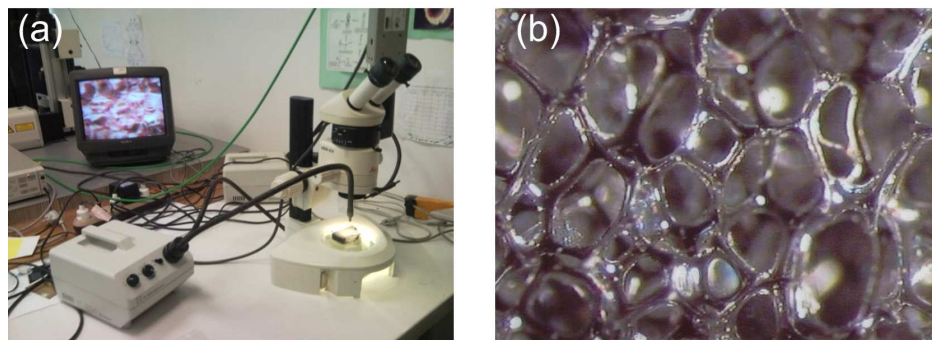


Figure 2.1. (a) Binoculaire LEICA MZ6; (b) Exemple d'une micrographie optique.

Les matériaux solides présentent des propriétés optiques très diverses. Chaque matériau se caractérise par différentes grandeurs physiques dues à sa réaction à la lumière ; ces dernières dépendent de la constitution structurale modulée par l'état de surface. On distingue donc

plusieurs catégories de matériaux (Degallaix, 2007) ^[3]: Un matériau « opaque » réfléchit une part de la lumière reçue et absorbe l'autre sur une épaisseur très faible, un matériau « transparent » ou « translucide » transmet presque toute la lumière incidente et la réfracte à chaque interface, certains matériaux sont « colorés » car ils absorbent le spectre complémentaire de leur couleur apparente...

La visualisation des images observées se fait maintenant généralement à l'écran. En ce qui concerne le stockage des images obtenues, soit les microscopes sont équipés des dispositifs de reproduction d'écran et l'image est imprimée sur papier par impression thermique, laser ou jet d'encre (suivant le degré de qualité et de coût) ; soit les images sont numérisées en reliant le microscope à un système de numérisation d'image. Pour la binoculaire LEICA MZ6 à l'LG2I, dans un premier temps les images peuvent être imprimées sur papier ou numérisées en vue d'un post-traitement.

Le microscope optique permet d'utiliser un grossissement pouvant aller jusqu'à 1500. L'observation produit usuellement des formes, des dimensions et une morphologie de la microstructure dans une représentation quasi-plane. En particulier, pour les mousses à faible densité qui font l'objet de cette recherche, le microscope optique permet d'estimer la forme (nombre d'arêtes/face et nombre de faces/cellule) et la taille (longueur et épaisseur de ligaments) des éléments d'une cellule.

La microscopie optique est un moyen peu coûteux et d'utilisation facile. Elle permet à l'ingénieur et au chercheur d'obtenir des informations importantes sur la microstructure du matériau, mais reste limitée par son pouvoir séparateur et sa faible profondeur de champ. Son grossissement utile est normalement inférieur à 1500. Dans de nombreux cas, elle est complétée par la microscopie électronique à balayage.

2.1.1.2. Microscopie électronique

La principale limitation de la microscopie optique est sa résolution, car la limite d'observation est d'environ $0.2\mu\text{m}$ (Paqueton et Ruste, 2006) ^[4] et les microstructures les plus fines ne peuvent être observées. Dans ce cas, on utilise alors la microscopie électronique avec une résolution beaucoup plus fine, de l'ordre du nanomètre.

On distingue classiquement deux catégories de microscopie électronique : La microscopie électronique à balayage (MEB ou « Scanning Electron Microscopy » SEM) et la microscopie électronique à transmission (MET, ou « Transmission Electron Microscopy » TEM). La microscopie électronique à balayage est une technique puissante d'observation de la topographie des surfaces avec très grand grossissement tandis que la

microscopie électronique à transmission est utilisée principalement pour visualiser les structures internes des échantillons avec une résolution supérieure à la microscopie à balayage. Au cours de cette thèse, seule a été utilisée la microscopie électronique à balayage.

La microscopie électronique à balayage est fondée principalement sur la détection des électrons secondaires émergeant de la surface sous l'impact d'un très fin pinceau d'électrons primaires qui balaye la surface observée et permet d'obtenir des images avec un pouvoir séparateur souvent inférieur à 5 nm et une grande profondeur de champ. La MEB donne une résolution latérale de l'ordre de 3 à 10 nm, un grossissement pouvant aller jusqu'à 300000 fois, et une profondeur de champ pouvant varier de plusieurs centimètres à faible grossissement à quelques micromètres au grossissement maximal. La Fig. 2.2 présente un microscope électronique à balayage S-3000N Hitachi disponible au centre de caractérisation des matériaux (CCM) de l'Université de Sherbrooke - Canada ainsi qu'une micrographie d'une mousse polyuréthane à faible densité prise par ce microscope et réalisée dans le cadre de ce travail de thèse.

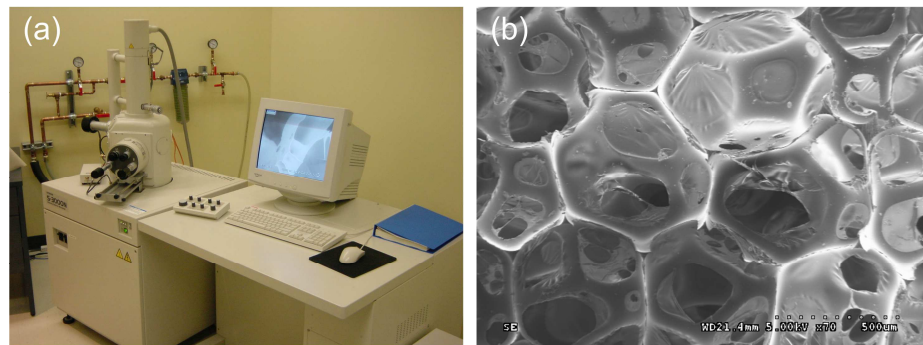


Figure 2.2. (a) Microscope électronique à balayage S-3000N Hitachi; (b) Exemple d'une micrographie électronique à balayage

L'utilisation du microscope électronique à balayage est simple et comporte cinq étapes principales :

- Démarrage de l'instrument (machine, PC, programme correspondant)
- Préparation de l'échantillon
- Introduction de l'échantillon
- Observation
- Acquisition et enregistrement des images.

Concernant la préparation de l'échantillon, elle vise à travailler sur une surface du matériau qui soit sèche et conductrice d'électricité. Par conséquent, il est nécessaire de faire une métallisation de l'échantillon si

celui-ci n'est pas conducteur (on dépose avant l'observation une fine couche métallique par évaporation sous vide).

Pour les mousses de faible densité qui ont fait l'objet de travail de thèse, le MEB a permis d'estimer les formes et les dimensions caractéristiques des constituants de la cellule (forme des sections des ligaments, épaisseur des ligaments). Il permet également d'observer les parties les plus fines de la microstructure telles que les membranes et les très petits pores situés sur les ligaments...

2.1.2. Identification tridimensionnelle par microtomographie à rayon-X

Les outils de microscopie optique ou à balayage précédemment décrits ne permettent pas normalement d'obtenir une représentation tridimensionnelle de la microstructure, sauf dans le cas de MEB munis de dispositifs permettant de réaliser une vision tridimensionnelle dans la zone située à proximité de la surface. La visualisation bidimensionnelle présente l'inconvénient de ne visualiser que des coupes du matériau et nécessite un traitement plus ou moins sophistiqué des données obtenues pour étendre les données obtenues à la caractérisation d'une géométrie tridimensionnelle et qui nécessiteraient idéalement de traiter statistiquement un grand nombre de coupes. L'utilisation de la microtomographie axiale à rayon-X (μ CT) permet de contourner cette difficulté.

La Fig. 2.3(a) représente un dispositif Skyscan 1172, qui constitue un équipement de laboratoire comportant une source de rayons-X pour μ CT axiale. Ce dispositif a été utilisé dans le cadre de cette thèse au centre de caractérisation des matériaux (CCM) de l'Université de Sherbrooke - Canada. La figure 2.3(b) comporte également la présentation d'une reconstruction 3D de la structure d'une mousse polyuréthane obtenue par ce dispositif.

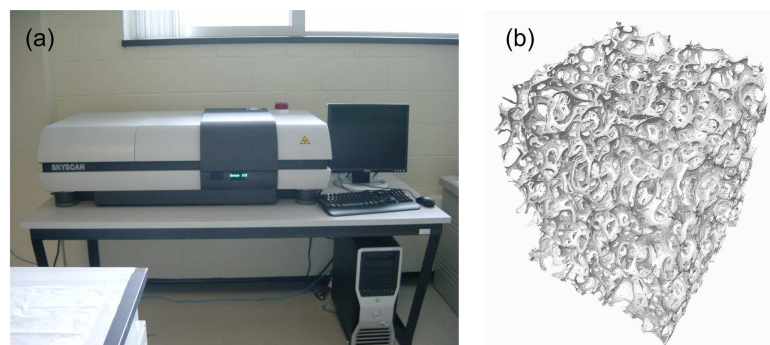


Figure 2.3. (a) Microtomographie à rayon-X Skyscan 1172; (b) Exemple d'une reconstruction 3D de la mousse polyuréthane.

Sommairement, une reconstruction tridimensionnelle par μ CT nécessite trois étapes principales. La première consiste à faire l'acquisition de radiographies de l'échantillon pour plusieurs positions angulaires du porte échantillon. Dans cette étape, quelques paramètres importants comme ceux de la source (voltage, courant), le temps d'exposition, et la résolution par réglage de la position relative source-échantillon-récepteur, sont soigneusement choisis afin d'obtenir un bon contraste entre les phases. La deuxième étape consiste à reconstruire les sections transversales de l'échantillon étudié, par rétroprojections filtrées des radiographies obtenues selon les différentes positions du porte échantillon. Des artefacts de reconstruction sont inhérents à la micro-tomographie utilisant une source de laboratoire (polychromatique). Le travail principal de cette étape est de limiter ces artefacts au moyen d'algorithmes inclus dans les logiciels commerciaux fournis par Skyscan. Une fois les sections transversales reconstruites, on peut procéder dans la dernière étape à la reconstruction 3D de l'échantillon par (i) la segmentation basée sur critères visuels, et (ii) l'assemblage des coupes (reconstruction de l'objet tridimensionnel). La mesure des paramètres morphologiques est finalement réalisée au moyen d'algorithmes inclus dans les logiciels commerciaux fournis par Skyscan.

Avec la microtomographie à rayon X, on dispose d'une méthode de caractérisation morphologique non-destructive et tridimensionnelle sur l'ensemble de l'échantillon. De plus, plusieurs outils informatiques ont été développés pour cet instrument afin de mesurer de nombreux paramètres morphologiques importants du matériau comme l'épaisseur, le diamètre des cellules, le degré d'anisotropie... Par contre, cet instrument reste d'utilisation moyennement répandue dans les laboratoires de recherche en raison de son coût élevé. De plus, même si la machine possède une résolution assez élevée (jusqu'à $0.9 \mu\text{m}$ pour le Skyscan 1172 au CCM), il est compliqué de reconstruire de très minces structures comme les membranes des mousses polyuréthanes à cause des artefacts et aussi en raison de la résolution (car les membranes sont normalement d'épaisseur inférieure au micromètre). Pour les mousses sans membranes, l'utilisation de microtomographie à rayon X a permis de reconstruire dans de bonnes conditions la microstructure du milieu réel (Perrot et *al.*, 2007) ^[5].

2.2. Homogénéisation des structures périodiques

D'une façon générale, les techniques d'homogénéisation sont destinées à prévoir les propriétés macroscopiques d'un matériau à partir de sa microstructure et des propriétés des constituants à l'échelle locale. Il existe de nombreuses techniques, mais dans le cadre de ce travail, seule la technique d'homogénéisation des structures périodiques sera utilisée.

Après une présentation générale de la problématique de l'homogénéisation, la technique d'homogénéisation de structures périodiques sera abordée puis appliquée à la dynamique des milieux poreux et enfin l'utilisation qui en est faite dans le cadre de la thèse est donnée.

2.2.1 Introduction aux méthodes d'homogénéisation

Les matériaux sont toujours hétérogènes à une échelle ou à une autre. Cette hétérogénéité réside soit dans la présence d'un réseau cristallin à l'échelle moléculaire, soit dans la présence de zones formées de constituants sensiblement homogènes se partageant l'espace. C'est ce dernier type de matériau qui se prête réellement aux techniques d'homogénéisation. Un tel matériau peut être décrit par deux modèles : l'un à l'échelle dite microscopique ou locale où le comportement est hétérogène ; et l'autre à l'échelle dite macroscopique où le comportement est considéré comme homogène. Le comportement macroscopique des matériaux peut, soit être identifié directement par des essais sur le matériau, soit faire l'objet d'une prédiction de comportement, lorsque le comportement des matériaux constitutifs et leur microstructure sont connus, à l'aide de la technique d'« homogénéisation ». Ces deux méthodes sont évidemment complémentaires. En pratique, les techniques d'homogénéisation servent surtout à comprendre le comportement de matériaux et à prévoir la constitution de nouveaux matériaux. Le comportement estimé doit évidemment être testé par des essais à l'échelle macroscopique.

Autrement dit, les méthodes d'homogénéisation cherchent à remplacer le matériau hétérogène par un milieu dit « homogène équivalent » caractérisé par des propriétés effectives (Fig. 2.4).

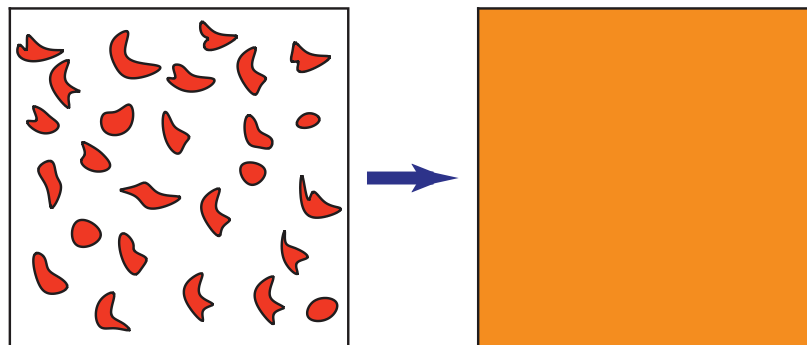


Figure 2.4. Un matériau hétérogène. (a) Modélisation à l'échelle des hétérogénéités, (b) Modélisation homogène « équivalente »

Concernant les méthodes d'homogénéisation les plus courantes, on peut citer ci-dessous :

- La méthode auto-cohérente, qui a été utilisée et développée par de nombreux auteurs (Brinkman, 1949 ; Hershey, 1954 ; Hill, 1965 ; Hashin, 1968 et Zaoui, 1987) ^{[6]-[10]}. On peut décrire l'esprit de cette approche de la façon suivante : considérons un matériau hétérogène dont on connaît les caractéristiques et les distributions des constituants. La première étape, dite localisation, consiste à analyser les interactions entre des inclusions contenant l'un des matériaux constitutifs et le reste du matériau qui est assimilé à une zone homogène possédant les propriétés du «matériau équivalent», de propriétés inconnues. La formulation de ces interactions permet ensuite d'exprimer les relations entre les moyennes volumiques des variables locales. Ces moyennes constituent les variables à l'échelle macroscopique et les relations obtenues constituent les propriétés du «milieu homogène équivalent». Cette méthode distingue bien les deux phases essentielles des techniques d'homogénéisation : une phase de localisation qui permet d'estimer les variables locales et une phase d'homogénéisation proprement dite permettant d'obtenir les propriétés à l'échelle macroscopique par moyenne sur les variables locales. Toutefois, cette méthode est bien adaptée pour les milieux totalement désordonnés, mais ne fournit qu'une approximation pour des distributions quelconques des matériaux constitutifs.
- La méthode de modélisation statistique développée par Kröner (1986) ^[11] pour une structure aléatoire et stationnaire de dimension infinie. Ce modèle nécessite la connaissance d'informations statistiques sur la répartition spatiale des constituants, qui restent toutefois difficiles à obtenir, même en utilisant les moyens modernes de caractérisation de la microgéométrie.
- Les méthodes variationnelles, développées dans les années 1960, permettent d'obtenir des bornes sur les coefficients effectifs à partir d'informations sur la distribution des constituants à l'échelle locale. Cette approche a été détaillée dans Torquato (2002) et Milton (2002) ^{[12],[13]}.
- La méthode de l'Homogénéisation des Structures Périodiques (HSP) développée dans les années 1970 par Sanchez-Palencia (1974) ^[14] et Bensoussan et *al.* (1978) ^[15]. La méthode utilise des développements asymptotiques à échelle multiples pour les matériaux de structure périodique. Cette dernière méthode sera présentée plus précisément dans le paragraphe suivant.

La condition fondamentale qui est commune à presque toutes les techniques d'homogénéisation même si elle est seulement implicite pour certaines

méthodes est la séparation des échelles, à savoir la séparation entre l'échelle microscopique (échelle d'hétérogénéité) et l'échelle macroscopique à laquelle le milieu équivalent est recherché. Si les échelles sont séparées, cela signifie que le milieu contient un grand nombre d'hétérogénéités dont la dimension est petite par rapport à l'échelle macroscopique. La séparation des échelles peut être exprimée comme suit :

$$\frac{l}{L} \ll 1 \quad (2.1)$$

où l est la longueur caractéristique microscopique (dimension caractéristique des hétérogénéités) et L est la longueur caractéristique macroscopique (dimension caractéristique du volume de matériau ou du phénomène étudié).

Une fois que la condition de séparation d'échelle est satisfaite, il est alors possible de définir un « volume élémentaire représentatif » (VER) du matériau hétérogène considéré, qui reste suffisamment petit par rapport à la dimension caractéristique de la structure macroscopique et qui doit être suffisamment grande à l'échelle microscopique pour décrire l'hétérogénéité. L'échelle de la structure est dite *macroscopique* et celle des hétérogénéités est dite *microscopique*.

2.2.2 Homogénéisation des structures périodiques

Parmi les différentes méthodes d'homogénéisation, la technique d'homogénéisation des structures périodiques est une méthode puissante et efficace. Dans le cas de matériaux périodiques, elle permet de déterminer de façon systématique le problème qu'il est nécessaire de résoudre sur la cellule élémentaire pour déterminer les propriétés macroscopiques, ainsi que la structure des équations de conservation et de comportement à l'échelle macroscopique. Cette méthode permet de définir avec précision la notion de comportement homogénéisé. Aussi, le domaine d'application de cette méthode s'étend graduellement. Dans le cadre de cette thèse, cette méthode sera appliquée à des matériaux poreux pour lesquels la distribution des hétérogénéités n'est pas strictement périodique, mais pour lesquels la microstructure est constituée de cellules de tailles sensiblement identiques et de morphologie comparables. Il est alors possible de constituer un modèle périodique à partir d'une cellule « moyenne » reproduite périodiquement, dont les propriétés géométriques sont bien adaptées pour simuler la morphologie réelle. Dans ce qui suit, on suppose donc que la structure est strictement périodique et l'on verra par la suite l'application pratique qui est faite de la méthode.

2.2.2.1. Cellule de base

Un milieu périodique est défini par une cellule de base reproduite périodiquement dans les différentes directions de l'espace. Le choix de la cellule de base n'est pas unique (Fig. 2.5). Par contre, les propriétés effectives du matériau sont déterminées de façon unique et ne dépendent pas du choix de la cellule. En pratique, on utilise des cellules de géométrie simple, rectangulaire pour les problèmes plans et parallélépipédique pour les problèmes tridimensionnels.

Concernant la détermination de la cellule de base, celle-ci est construite à partir de données physiques identifiées à partir de données morphologiques. La méthode de la cellule idéalisée (Perrot et *al.*, 2007) ^[5] définit une cellule moyenne « idéale » représentant l'ensemble de la microstructure à partir de données morphologiques simples et la méthode de la morphologie mathématique (Peyrega, 2010) ^[16], construit la cellule à partir de données plus élaborées synthétisées à partir de la microstructure réelle.

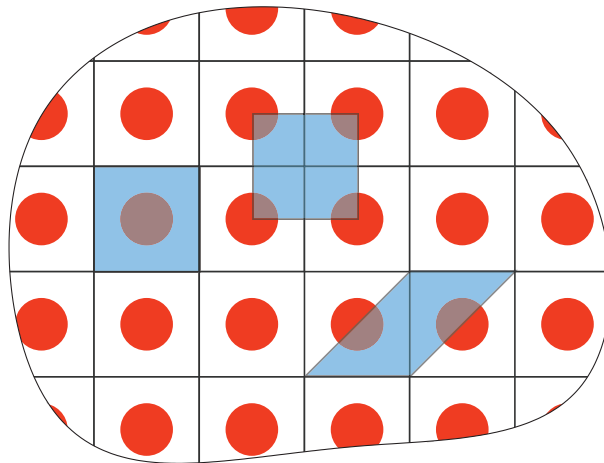


Figure 2.5. Plusieurs choix possibles de cellules de base pour une structure périodique.

2.2.2.2. Méthode de développements à échelles multiples

La méthode d'homogénéisation de structures périodiques repose sur l'utilisation d'échelles multiples. Elle a été initialement introduite par Sanchez-Palencia (1974) ^[14], Keller (1977) ^[18], Bensoussan et *al.* (1978) ^[15] et Sanchez-Palencia (1980) ^[17], puis complétée par une analyse dimensionnelle des équations aux dérivées partielles du problème par Auriault (1991). ^[19] Cette méthodologie est présentée dans les paragraphes ci-dessous. Le lecteur est invité à consulter les travaux de Auriault et *al.* ^[20] pour plus de détails.

2.2.2.2.1. Séparation d'échelle

Comme signalé précédemment, la séparation d'échelles est une condition indispensable pour les méthodes de passage micro-macro, et donc pour la méthode d'homogénéisation des structures périodiques. Dans le cas d'une structure périodique, la cellule périodique suffit à décrire la totalité de la structure. La période de cette cellule constitue la longueur caractéristique microscopique l . Soit L la longueur macroscopique (longueur caractéristique du volume de matériau considéré ou longueur caractéristique macroscopique de la sollicitation). La condition de séparation d'échelles s'écrit :

$$\varepsilon = \frac{l}{L} \ll 1 \quad (2.2)$$

La séparation d'échelle doit être aussi prise en compte pour les grandeurs physiques. Par exemple, dans le cas d'une propagation d'onde dans un milieu poreux, la méthode ne peut être utilisée que dans le cas où la taille des pores D est beaucoup plus petite que la longueur d'onde λ :

$$\frac{D}{\lambda} \ll 1 \quad (2.3)$$

2.2.2.2.2. Variable d'espace

On introduit deux variables d'espace (adimensionnelles) en fonction des longueurs caractéristiques et de la variable d'espace physique \mathbf{X} :

$$\mathbf{x} = \mathbf{X} / L \text{ et } \mathbf{y} = \mathbf{X} / l \quad (2.4)$$

La variable \mathbf{x} est la variable d'espace macroscopique (variable lente, mesure de la distance \mathbf{X} en utilisant la distance L comme longueur unité) et \mathbf{y} est la variable d'espace microscopique (variable rapide, mesure de la distance \mathbf{X} en utilisant la distance l comme longueur unité). En raison de la séparation d'échelle, chaque inconnue Φ dépend de ces deux paramètres adimensionnels. Il existe donc deux descriptions possibles :

- Point de vue macroscopique (la longueur caractéristique L est utilisée comme longueur de référence) :

$$\Phi = \Phi(\mathbf{x}, \mathbf{y}) = \Phi(\mathbf{x}, \varepsilon), \quad \mathbf{y} = \mathbf{x}/\varepsilon \quad (2.5)$$

- Point de vue microscopique (longueur caractéristique l est utilisée comme longueur de référence) :

$$\Phi = \Phi(\mathbf{x}, \mathbf{y}) = \Phi(\mathbf{y}, \varepsilon), \quad \mathbf{x} = \varepsilon \mathbf{y} \quad (2.6)$$

La séparation d'échelle repose physiquement sur le schéma de la Fig. 2.6 en observant que la variation de $\langle \Phi \rangle$ en fonction de \mathbf{x} sur une distance $O(l)$ est petite et tend vers zéro dans le cas limite où ε tend vers zéro. Cela signifie dans le cas d'une structure périodique que Φ vérifie une propriété de périodicité en \mathbf{y} . Notant que le symbole $O(\cdot)$ signifie :

$$A = O(B) \quad \text{si} \quad \varepsilon \ll (A/B) \ll \varepsilon^{-1}. \quad (2.7)$$

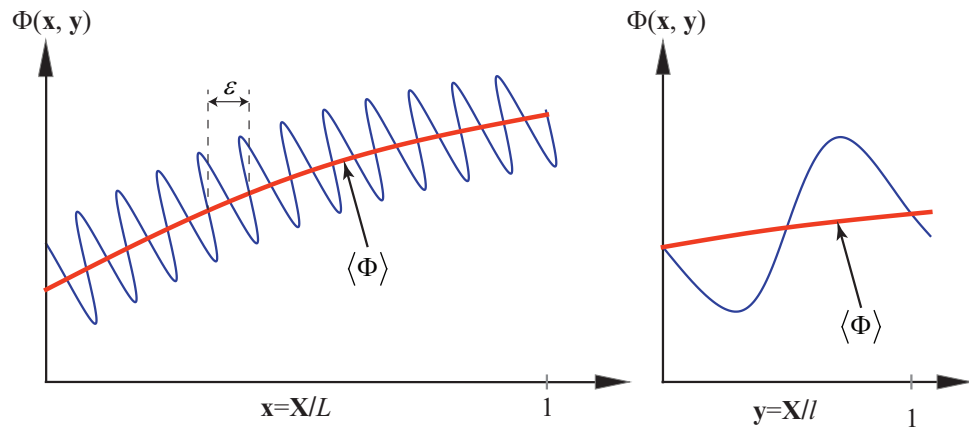


Figure 2.6. Séparation d'échelle : Description macroscopique (gauche) et locale (droite) de Φ .^[20]

2.2.2.2.3. Développement asymptotique à échelles multiples

Comme indiqué précédemment, toutes les grandeurs physiques peuvent s'exprimer en fonction des deux variables d'espace adimensionnelles \mathbf{x} et \mathbf{y} qui sont reliées l'une à l'autre par un petit paramètre ε . Cette observation a conduit à la recherche de ces grandeurs physiques inconnues sous la forme de développements asymptotiques en fonction de ε (Bensoussan et *al.*, 1978 ; Sanchez-Palencia, 1980).^{[15],[17]} Le développement s'écrit alors comme suit pour une grandeur adimensionnelle Φ :

$$\Phi(\mathbf{x}, \mathbf{y}) = \Phi^{(0)}(\mathbf{x}, \mathbf{y}) + \varepsilon \Phi^{(1)}(\mathbf{x}, \mathbf{y}) + \varepsilon^2 \Phi^{(2)}(\mathbf{x}, \mathbf{y}) + \dots \quad (2.8)$$

L'homogénéisation idéale repose sur une analyse asymptotique lorsque ε tend vers zéro. La propriété de périodicité en \mathbf{y} des fonctions $\Phi^{(i)}(\mathbf{x}, \mathbf{y})$ assure une bonne séparation d'échelle.

2.2.2.2.4. Méthodologie

On présente dans cette partie la méthodologie détaillée par Auriault (1991) ^[19]. Les étapes à suivre sont les suivantes :

- *Description à l'échelle microscopique* : C'est une étape indépendante de la méthode d'homogénéisation, qui porte sur la description à l'échelle microscopique des équations aux dérivées partielles et des conditions d'interface. Il s'agit dans cette étape d'identifier les paramètres à l'échelle locale de la microstructure comme la morphologie, les propriétés physiques (viscosité, densité...), les variables physiques (pression, vitesse, température...) et les équations qui les relient. Une fois que la description locale est donnée, on peut passer aux étapes suivantes pour chercher la description macroscopique équivalente.
- *Point de vue macroscopique ou microscopique ?* Les deux choix sont *a priori* équivalents et dépendent seulement de la commodité du problème étudié. Les développements sont pris sous la forme (2.8) en utilisant respectivement (point de vue macro- ou micro-scopique) $\mathbf{y} = \mathbf{x}/\varepsilon$ et $\mathbf{x} = \varepsilon\mathbf{y}$. Les développements dépendent donc d'une variable d'espace et du petit paramètre ε .
- *Normalisation de la description locale* : Cette étape, appelée normalisation, signifie que les descriptions locales sont adimensionnées et évaluées en fonction des puissances de ε . Une quantité adimensionnelle Φ est dite être $O(\varepsilon^p)$ si :

$$\varepsilon^{p+1/2} \leq |\Phi| \leq \varepsilon^{p-1/2} \quad (2.9)$$

- *Dérivation de la description macroscopique* : Tous les variables physiques sont exprimées sous la forme de développement asymptotique en puissance de ε :

$$\Phi(\mathbf{x}, \mathbf{y}) = \Phi^{(0)}(\mathbf{x}, \mathbf{y}) + \varepsilon\Phi^{(1)}(\mathbf{x}, \mathbf{y}) + \varepsilon^2\Phi^{(2)}(\mathbf{x}, \mathbf{y}) + \dots \quad (2.10)$$

où chaque terme $\Phi^{(i)}$ est \mathbf{y} -périodique. La méthode consiste alors à appliquer ces développements dans la description locale (microscopique). Cela conduit à des équations de conservation et des conditions aux limites aux ordres successifs de ε qui permettent de définir des problèmes aux limites sur une cellule périodique. La détermination de la description macroscopique consiste en une résolution de ces problèmes aux limites. Concrètement, les équations à résoudre sont sous la forme suivante :

$$\operatorname{div}_{\mathbf{y}} \Phi^{(i)} = -\operatorname{div}_{\mathbf{x}} \Phi^{(i-1)} + \mathbf{W} \quad (2.11)$$

où $-\operatorname{div}_{\mathbf{x}} \Phi^{(i-1)} + \mathbf{W}$ est un terme de source déjà déterminé en résolvant un problème aux limites à l'ordre précédent. Quand $\Phi^{(i)}$ est localement périodique (ou stationnaire, dans le cas du matériau aléatoire), la moyenne de terme de source doit s'annuler :

$$\langle -\operatorname{div}_{\mathbf{x}} \Phi^{(i-1)} + \mathbf{W} \rangle = 0 \quad (2.12)$$

Ce qui produit une équation de conservation à l'échelle locale. In fine, la description macroscopique dimensionnelle est obtenue en retournant aux quantités dimensionnelles.

2.2.2.3. Résultats sur l'écoulement en milieu poreux et sur la poroélasticité

2.2.2.3.1. Application à l'écoulement quasi statique en milieu poreux

On considère l'écoulement d'un fluide visqueux newtonien dans un matériau poreux périodique constitué d'une partie solide indéformable correspondant au volume Ω_s et d'une partie fluide correspondant au volume Ω_f

Les équations du problème à l'échelle locale sont :

- Équation de conservation de la quantité de mouvement (Équation de Stokes) :

$$\eta \Delta \mathbf{v} - \nabla p = 0 \quad \text{dans } \Omega_f \quad (2.13)$$

où Δ est un opérateur laplacien, ∇ désigne l'opérateur gradient, \mathbf{v} la vitesse et p la pression

- Équation de continuité :

$$\nabla \cdot \mathbf{v} = 0 \quad \text{dans } \Omega_f \quad (2.14)$$

- Condition d'adhérence :

$$\mathbf{v} = 0 \quad \text{sur } \partial\Omega \quad (2.15)$$

L'équation (2.13) introduit un nombre adimensionnel Q :

$$Q = \frac{\nabla p}{\eta \Delta \mathbf{v}} \quad (2.16)$$

On considère maintenant un écoulement forcé par un gradient macroscopique de pression :

$$\nabla p = O(p/L) \quad (2.17)$$

où L est la longueur macroscopique (la taille de l'échantillon examiné).
En notant l la taille caractéristique des pores, on a la relation suivante :

$$\eta \Delta \mathbf{v} = O(\eta v/l^2) \quad (2.18)$$

A partir de l'équation (2.13), on déduit :

$$\eta v/l^2 = O(p/L) \quad (2.19)$$

Le nombre sans dimension Q est donc :

$$Q = O\left(\frac{p}{l} \frac{l^2}{\eta v}\right) = O(\varepsilon^{-1}) \quad (2.20)$$

Dans ce qui suit, on choisit le point de vue microscopique, les inconnues étant développées sous la forme des développements asymptotiques sous la forme :

$$\begin{aligned} \mathbf{v}(\mathbf{x}, \mathbf{y}) &= \mathbf{v}^{(0)}(\mathbf{x}, \mathbf{y}) + \varepsilon \mathbf{v}^{(1)}(\mathbf{x}, \mathbf{y}) + \varepsilon^2 \mathbf{v}^{(2)}(\mathbf{x}, \mathbf{y}) + \dots \\ p(\mathbf{x}, \mathbf{y}) &= p^{(0)}(\mathbf{x}, \mathbf{y}) + \varepsilon p^{(1)}(\mathbf{x}, \mathbf{y}) + \varepsilon^2 p^{(2)}(\mathbf{x}, \mathbf{y}) + \dots \end{aligned} \quad (2.21)$$

Avec $\mathbf{x} = \varepsilon \mathbf{y}$, $\mathbf{v}^{(i)}$ et $\mathbf{p}^{(i)}$ sont Ω -périodique en \mathbf{y} .

Le rapprochement des différents termes du développement asymptotique conduit à $\mathbf{p}^{(0)} = \mathbf{p}^{(0)}(\mathbf{x})$, puis au système d'équations :

$$\begin{aligned} \nabla_{\mathbf{y}} \mathbf{p}^{(0)} &= 0, \quad (a) \\ \eta \Delta_{\mathbf{y}} \mathbf{v}^{(0)} - \nabla_{\mathbf{x}} \mathbf{p}^{(0)} - \nabla_{\mathbf{y}} \mathbf{p}^{(1)} &= 0, \quad (b) \end{aligned} \quad (2.22)$$

Avec des conditions de périodicité en \mathbf{y} sur \mathbf{v} , $\mathbf{p}^{(1)}$ qui constitue le problème de cellule dont la solution est linéairement dépendante de $\nabla_{\mathbf{x}} \mathbf{p}^{(0)}$, en particulier :

$$\mathbf{v}_i^{(0)} = -\frac{k_{0ij}^*}{\eta} \frac{d\mathbf{p}^{(0)}}{dx_j} \quad (2.23)$$

la considération de la compatibilité des équations conduit alors à

$$\nabla_{\mathbf{x}} \cdot \langle \mathbf{v}^{(0)} \rangle_{\Omega} = 0, \quad \langle \mathbf{v}^{(0)} \rangle_{\Omega} = -\frac{\mathbf{k}_0}{\eta} \nabla_{\mathbf{x}} \mathbf{p}^{(0)}, \quad \mathbf{k}_0 = \langle k_0^* \rangle_{\Omega} \quad (2.24)$$

où k_0^* est un tenseur fonction de \mathbf{y} et où la partie entre crochets correspond à la moyenne volumique sur la période.

$$\langle \bullet \rangle_{\Omega} = \frac{1}{|\Omega|} \int_{\Omega_f} \bullet \, d\Omega, \quad (2.25)$$

Ce qui permet de retrouver la loi de Darcy entre la vitesse moyenne et le gradient de pression macroscopique.

2.2.2.3.2. Application à la détermination du comportement dynamique de milieux poreux

Concernant l'étude de la dynamique de milieux poreux saturés déformables, la théorie classique est due à Biot (1956). [22]. [23] Contrairement à l'approche de Biot [22]. [23], qui décrit le comportement du milieu poreux directement à l'échelle macroscopique, de nombreux auteurs ont utilisé l'homogénéisation de structures périodiques pour décrire le comportements biphasique du matériau, à partir de l'échelle locale par les développements asymptotiques. On peut citer ici les travaux de Lévy (1979), Auriault (1980), Boutin (1983) [24]-[26], Auriault et al. (2009). [20]

Pour ce problème, les équations locales comportent les équations de la dynamique pour un fluide visqueux et les équations de l'élastodynamique pour le solide en petites déformations.

Moyennant une analyse dimensionnelle adaptée, le comportement du matériau à l'échelle macroscopique s'écrit (Auriault et al. ^[20]) :

$$\nabla \cdot \langle \boldsymbol{\sigma}^T \rangle = -\langle \rho_s \rangle \omega^2 \mathbf{u}_s + \rho_f i \omega \mathbf{v} + \bar{O}(\varepsilon), \quad (2.26)$$

$$\langle \boldsymbol{\sigma}^T \rangle = C : \mathbf{e}_s - \mathbf{a} \mathbf{p} + \bar{O}(\varepsilon), \quad (2.27)$$

$$\nabla \cdot (\langle \mathbf{v} \rangle - \phi i \omega \mathbf{u}_s) = -i \omega \mathbf{a} : \mathbf{e}_s - i \omega \mathbf{b} \mathbf{p} + \bar{O}(\varepsilon), \quad (2.28)$$

$$\langle \mathbf{v} \rangle - \phi i \omega \mathbf{u}_s = -\mathbb{L} (\nabla \mathbf{p} - \rho_f \omega^2 \mathbf{u}_s) + \bar{O}(\varepsilon), \quad (2.29)$$

d'où :

- $\langle \cdot \rangle$ est un opérateur de moyen sur la période Ω .
- $\bar{O}(\varepsilon)$ est un terme petit, d'ordre ε relativement aux autres termes de l'égalité.
- $\boldsymbol{\sigma}^T$ est la contrainte locale définie par, $\boldsymbol{\sigma}^T = \boldsymbol{\sigma}_s$ dans la phase solide et $\boldsymbol{\sigma}^T = \boldsymbol{\sigma}_f$ dans la phase fluide.
- ρ_s et ρ_f sont respectivement les masses volumiques de la phase solide et de la phase fluide.
- \mathbf{u}_s est le déplacement de la phase solide.
- \mathbf{v} est la vitesse de la phase fluide.
- C est le tenseur d'élasticité du squelette (macroscopique).
- \mathbf{e}_s est la déformation de la phase solide.
- \mathbb{L} est la conductivité hydraulique dynamique.
- \mathbf{a} est un tenseur symétrique d'ordre 2 et \mathbf{b} est un coefficient de compressibilité.

Le modèle dépend de : perméabilité, porosité, tenseur d'élasticité du squelette solide, \mathbf{a} et \mathbf{b} .

Dans le cas d'un milieu isotrope, les valeurs \mathbf{a} et \mathbf{b} sont données par $\mathbf{a} = a \mathbf{I}$ avec :

$$a = 1 - \frac{K_s}{K_m}, \quad (2.30)$$

K_s : Module d'incompressibilité du squelette (macroscopique)

K_m : Module d'incompressibilité du matériau constituant du squelette (microscopique)

et b devient

$$b = \frac{a - \phi}{K_m} + \frac{\phi}{K_f}, \quad (2.31)$$

où K_f est le module d'incompressibilité du fluide.

Dans le cadre de cette thèse, les matériaux considérés sont des mousses de faible densité (grande porosité). Sous l'hypothèse d'isotropie, les coefficients a et b suivent les formules (2.30) et (2.31). De plus, K_s est très petit devant K_m et la porosité ϕ est très proche de l'unité, les termes K_s/K_m et $(a - \phi)/K_m$ sont proches de zéro. On obtient donc :

$$a \cong 1 \text{ et } b \cong \frac{\phi}{K_f} \quad (2.32)$$

Les équations (2.26)-(2.29) sont équivalents à celles décrites dans le modèle de Biot. ^{[22],[23]} Il s'agit dans ce système d'équations de sept inconnues scalaires u_{si} , $\langle v_i \rangle$ et p , pour sept équations scalaires. Ces équations permettent donc de décrire complètement le comportement dynamique du milieu déformable.

2.2.3 Utilisation de l'homogénéisation des structures périodiques dans le cadre de la thèse

Dans le cadre de cette thèse, la méthode de l'homogénéisation des structures périodiques (HSP) présentée ci-dessus sera utilisée dans deux situations différentes :

Dans une première étape, pour des mousses à faible densité ayant des structures supposées rigides, trois problèmes différents sur la cellule périodique sont mis en œuvre afin de déterminer les paramètres de transport macroscopiques du milieu poreux considéré.

- Dans le premier problème (i.e. limites basses fréquences), le fluide se comporte comme un fluide visqueux régi par l'équation de Stokes, ce qui correspond au premier exemple décrit ci-dessus.

- Dans le deuxième problème (i.e. limites hautes fréquences), l'effet visqueux se concentre dans la couche limite à proximité de l'interface avec le solide, alors qu'à l'extérieur de la couche limite, le fluide se comporte comme un fluide parfait. Dans ce cas, les conditions aux limites pour une cellule périodique sont décrites par Avellaneda (1991) ^[27], qui montre que le problème est équivalent à celui de la conduction électrique en milieu hétérogène.

- Dans le troisième problème, les équations de la conduction thermique sont résolues en utilisant les conditions aux limites proposées par Rubinstein (1988) ^[28].

Dans une deuxième étape, le squelette est considéré comme déformable et le problème de cellule produit par l'homogénéisation de structures périodiques est résolu dans le cas de l'élastostatique. Une simplification est apportée en tenant compte des symétries de la cellule pour remplacer les conditions de périodicité sur la frontière de celle-ci par des conditions aux limites standard.

2.3 Modèles de comportement dynamique de milieux poreux

2.3.1 Introduction sur les modèles dynamiques de milieu poreux

Pour un milieu poreux indéformable et un fluide incompressible, le comportement du milieu poreux est régi complètement par la perméabilité dynamique qui est une quantité complexe dépendant de la fréquence. Pour un fluide compressible, il faut ajouter une autre fonction complexe dépendant de la fréquence. Par exemple, on peut utiliser le couple densité effective et module d'incompressibilité effectif $(\tilde{\rho}(\omega), \tilde{K}(\omega))$ que l'on peut relier à d'autres variables plus accessibles à l'expérience. De manière équivalente, deux autres fonctions sont souvent utilisées dans la littérature : ce sont les perméabilités dynamiques visqueuse et thermique $(\tilde{k}(\omega), \tilde{k}'(\omega))$. Pour la détermination de ces propriétés acoustiques, il existe dans la littérature de nombreux types de modèles.

Les modèles empiriques sont construits sur la base de nombreuses mesures afin de retrouver un comportement général. Le modèle le plus connu est celui de Delany et Bazley (1970). ^[29] Effectuant des mesures de l'impédance caractéristique et du nombre d'onde d'un grand nombre de matériaux fibreux, les auteurs ont trouvé des lois (de puissance) pour l'impédance caractéristique et le nombre d'onde (définis en annexe A de cette thèse) en fonction du rapport fréquence f sur résistivité au passage de l'air σ . Après Delany et Bazley, de nombreux autres modèles empiriques ont été proposés pour différents types des matériaux. ^{[30]-[34]}

Aujourd'hui, on peut considérer que ces modèles empiriques ne sont pas suffisants pour la modélisation acoustique de matériaux poreux qui n'ont pas été pré-étudiés.

Les approches phénoménologiques visent à chercher les modèles acoustiques pour des géométries complexes, à partir des solutions associées aux géométries plus simples pour lesquelles les solutions analytiques existent, en introduisant un ou plusieurs facteurs de forme dépendant de la géométrie locale. S'inspirant de la théorie de Kirchoff (1868) ^[35], l'un de premiers modèles phénoménologiques est celui de Zwikker et Kosten (1949) ^[36] avec l'hypothèse que le matériau poreux est constitué d'un réseau de pores cylindriques parallèles de section circulaire, en traitant séparément les effets visqueux et la conductivité thermique. Equivalent à ce modèle de Zwikker et Kosten, Biot (1956) ^{[22],[23]} a développé une théorie sur la propagation des ondes élastiques dans un milieu poreux saturé. Les travaux équivalents de Zwikker et Kosten et de Biot sont des modèles phénoménologiques de base. Par la suite, de nombreux autres modèles ont proposé de modifier ces théories de base afin d'étendre leurs champs d'application. ^{[37]-[40]}

Les deux approches abordées ci-dessus constituent une base importante ; par contre ils sont plus ou moins imprécis ou limités par les types de matériaux appliqués. De nos jours, les modèles semi-phénoménologiques - plus précis et prédictifs, sont les plus utilisés. Ces derniers se basent sur l'approche consistant à déduire analytiquement des fonctions de réponses approchées d'un fluide saturant l'espace poreux d'un milieu poreux soumis à une excitation acoustique. Ces fonctions dépendent de paramètres macroscopiques ayant une signification physique liée aux comportements asymptotiques basse ou haute fréquences du milieu poreux considéré. Bien adaptés aux différents types de matériaux poreux et à une large gamme de fréquences, ces modèles semi-phénoménologiques permettent aussi de comprendre physiquement le comportement du matériau poreux via les paramètres macroscopiques attachés à chaque modèle. Compte tenu de ces avantages, quatre de ces modèles semi-phénoménologiques seront présentés en détail dans la partie suivante.

2.3.2 Modèles semi-phénoménologiques

Apparu en 1987 dans les travaux de Johnson, Koplik et Dashen ^[41] avec un modèle comportant quatre paramètres macroscopiques, les modèles semi-phénoménologiques ont été par la suite étudiés et améliorés par nombreux auteurs en introduisant de nouveaux paramètres importants liés à une prise en compte plus fine de la physique.

2.3.2.1 Le modèle de Johnson et al.

En 1987, Johnson et ses collaborateurs ^[41] ont développé un premier modèle semi-phénoménologique pour définir la densité dynamique complexe ($\tilde{\rho}(\omega)$). Ils étudient les effets de dissipations visqueuses d'un fluide Newtonien saturant un milieu poreux rigide et isotrope soumis à un gradient de pression oscillant à travers l'échantillon. Ils supposent de plus que la longueur d'onde est beaucoup plus grande que la taille des pores et que la conductivité thermique du fluide est négligeable. Ils définissent alors la tortuosité dynamique $\tilde{\alpha}(\omega)$ et la perméabilité dynamique visqueuse $\tilde{k}(\omega)$ selon les deux formules équivalentes :

$$\tilde{\alpha}(\omega)\rho_0\frac{\partial\mathbf{v}}{\partial t}=-\nabla\mathbf{p}\text{ et } \phi\mathbf{v}=-\frac{\tilde{k}(\omega)}{\eta}\nabla\mathbf{p}\quad (2.33)$$

En étudiant les propriétés analytiques de la fonction de réponse linéaire aux limites basse et haute fréquences, ils en ont déduit des expressions pour déterminer la perméabilité dynamique visqueuse $\tilde{k}(\omega)$ et la tortuosité dynamique $\tilde{\alpha}(\omega)$.

Les formules obtenues par Johnson et al. sont les suivantes :

$$\tilde{\alpha}(\omega)=\alpha_\infty-\frac{i\eta\phi}{\omega k_0\rho_0}\left\{1+\frac{4i\alpha_\infty^2 k_0^2\rho_0\omega}{\eta\Lambda^2\phi^2}\right\}^{\frac{1}{2}}\quad (2.34)$$

$$\tilde{k}(\omega)=\frac{k_0}{\left\{1+\frac{4i\alpha_\infty^2 k_0^2\rho_0\omega}{\eta\Lambda^2\phi^2}\right\}^{\frac{1}{2}}+\frac{i\alpha_\infty k_0\rho_0\omega}{\eta\phi}}\quad (2.35)$$

où η et ρ_0 sont respectivement la viscosité dynamique et la densité du fluide saturant. Les quatre paramètres intrinsèques du matériau poreux sont les suivantes : la porosité ϕ , la tortuosité en fluide parfait α_∞ , la perméabilité statique visqueuse k_0 et la longueur caractéristique visqueuse Λ .

2.3.2.2 Le modèle de Champoux et Allard

Johnson et al. s'intéressent dans leur travail à des fluides ayant une faible conductivité thermique (dans le cadre de la prospection pétrolière). Dans le cas des matériaux saturés par l'air, les effets de dissipation thermique ne sont plus négligeables. Pour tenir compte de ces effets, Champoux et Allard (1991) ^[42] ont proposé un nouveau modèle en ajoutant un paramètre

caractérisant le comportement asymptotique hautes fréquences de la réponse thermique : la longueur caractéristique thermique Λ' . L'approche suivie par Champoux et Allard est similaire au cas de l'effet visqueux traité par Johnson et *al.* La longueur caractéristique thermique Λ' est un paramètre intrinsèque du matériau qui est purement géométrique, elle coïncide avec la longueur visqueuse Λ dans le cas d'un matériau composé de pores uniformes et non intersectés. Ce modèle comportant cinq paramètres macroscopique est souvent appelé modèle JCA (pour Johnson, Champoux, Allard)

2.3.2.3 *Le modèle de Lafarge*

En 1993, puis 1997 ^{[43],[44]}, Lafarge a modifié le model de Champoux et Allard (1991) concernant la dissipation thermique en introduisant un nouveau paramètre, la perméabilité thermique statique k'_0 qui permet de caractériser les phénomènes dissipatifs thermiques en basses fréquence. Il propose alors une nouvelle fonction dynamique d'interpolation, la perméabilité dynamique thermique $\tilde{k}'(\omega)$, qui est une fonction analogue à la perméabilité dynamique visqueuse $\tilde{k}(\omega)$:

$$\tilde{k}'(\omega) j\omega\rho = \phi\kappa\langle\tau\rangle, \quad (2.36)$$

où κ est la conductivité thermique du fluide et τ sa température. Le modèle de Lafarge comporte finalement 6 paramètres, et est dénommé par le sigle JCAL, du nom des différents auteurs précités.

2.3.2.4 *Le modèle visqueux de Pride et thermique de Lafarge*

En 1993, Pride, Morgan et Gangi ^[45] constatent que dans les modèles de Johnson et *al.* la partie imaginaire de la tortuosité dynamique $\tilde{\alpha}(\omega)$ (ou perméabilité dynamique visqueuse $\tilde{k}(\omega)$) est sous-estimée dans le cas d'un fluide soumis à des fortes variations de sections le long du parcours de l'onde acoustique. Ils proposent donc d'introduire un nouveau paramètre macroscopique, la tortuosité visqueuse basses fréquences α_0 pour améliorer le comportement asymptotique basses fréquences des dissipations visqueuses. De façon similaire, Lafarge (1997) raffine leur modèle en corrigeant le comportement basses fréquences des dissipations thermiques en ajoutant la tortuosité thermique basses fréquences α'_0 . ^[44] Avec ces deux nouveaux paramètres, le modèle comporte finalement 8 paramètres. Son sigle est JCAPL.

Les modèles semi-phénoménologiques dépendent alors du nombre de paramètres macroscopique qu'ils comportent. En introduisant les fonctions

de forme et les facteurs de forme, les modèles semi-phénoménologiques abordés ci-dessous sont présentés sous une forme plus synthétique dans l'annexe B à la fin de cette thèse.

2.4 Modèles hybrides

Le comportement acoustique des mousses dépend de deux paramètres : la perméabilité dynamique visqueuse $\tilde{k}(\omega)$ et thermique $\tilde{k}'(\omega)$. Ces deux paramètres peuvent être obtenus par la méthode des éléments finis (MEF) en régime dynamique comme dans le travail de Gasser et ses collaborateurs (2005).^[46] Dans ce travail, Gasser et ses collaborateurs ont déterminé les fonctions dynamiques (densité effective et compressibilité effective) du fluide saturant un squelette poreux de type FCC (face centered cubic). Les équations du problème sont celles obtenues par l'homogénéisation de structures périodiques en prenant en compte l'écoulement du fluide visqueux et la conduction thermique. Elles sont résolues sur la cellule périodique à l'aide de la méthode des éléments finis. La méthode permet de calculer directement les fonctions de réponse acoustique dynamique pour une fréquence quelconque. Cette méthode donne des résultats intéressants, mais nécessite un calcul fréquence par fréquence, ce qui nécessite de nombreux calculs. De plus, les calculs hautes fréquences doivent tenir compte d'une couche limite d'épaisseur de plus en plus faible, ce qui nécessite des maillages de plus en plus fins.

Dans le cadre de cette thèse, on utilise une méthode appelée « hybride » qui nous permet de déterminer les deux perméabilités dynamiques sur une gamme de fréquence complète, en couplant les résultats obtenus par éléments finis avec la structure des modèles semi-phénoménologiques. Déjà utilisé et validé dans le travail de Perrot et *al.* (2008)^[47] pour une structure périodique bidimensionnelle de type fibreux, l'idée de la méthode hybride est la suivante :

- La première étape consiste à étudier la microstructure des matériaux considérés. Il s'agit dans cette étape de l'étude de la morphologie locale en identifiant les caractéristiques saillantes de la géométrie locale. Cette étude permet de déduire une cellule périodique représentative du matériau poreux lorsqu'il existe une séparation d'échelle. L'acquisition de la géométrie locale est obtenue par les techniques d'imageries décrites précédemment : la microscopie optique, la microscopie électronique à balayage et la microtomographie axiale à rayons-X.
- Ensuite, les paramètres de transport macroscopiques du matériau considéré sont calculés en résolvant les problèmes asymptotiques basses fréquences (problème Stokes), hautes fréquences (conduction

électrique) et conduction thermique. Ces trois problèmes peuvent-être résolus par la méthode d'homogénéisation des structures périodiques (HSP, présenté précédemment) sur la cellule de base caractérisée, en utilisant un maillage unique et en résolvant des équations aux dérivées partielles standard. Cette étape sera présentée en détails dans les chapitres suivants. Les paramètres macroscopiques sont alors déterminés individuellement.

- Finalement, les paramètres macroscopiques calculés sont implémentés dans les modèles semi-phénoménologique décrits précédemment pour déterminer les propriétés effectives du milieu ($\tilde{\alpha}(\omega)$ et $\tilde{K}(\omega)$). Connaissant ce couple de fonctions dynamiques, il est possible de déterminer complètement le comportement acoustique du milieu poreux, et en particulier les perméabilités dynamiques visqueuse et thermique.

Les étapes citées ci-dessus sont appliquées pour un matériau de structure supposée rigide. Dans le cas d'un matériau de structure déformable, il faut ajouter une étape consistant à calculer les propriétés élastiques de la matrice poreuse. Les calculs seront effectués sur la partie solide, complémentaire de la partie fluide utilisée pour les propriétés de transport. Le calcul permet de déterminer les propriétés élastiques du squelette solide.

La méthode hybride oblige à fournir de nombreuses informations sur le matériau: l'étude de la géométrie locale donne des informations sur la microstructure des mousses, les paramètres de transport macroscopiques liés au comportement basses et hautes fréquences permettent de bien comprendre la physique des phénomènes de transport dans le milieu poreux étudié. Ces derniers paramètres permettent de calculer les fonctions de réponse dynamique du matériau poreux.

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Propriétés de transport en squelette indéformable

3 Propriétés de transport en squelette indéformable

Ce chapitre traite de la détermination des propriétés de transport en squelette indéformable. Une phase importante est de déterminer les tailles critiques de la géométrie locale gouvernant le comportement acoustique d'une mousse de structure indéformable ; ces tailles critiques dépendent de la structure interne du milieu poreux. Les matériaux de différents types de structure sont alors étudiés, en déroulant la méthodologie présentée dans le chapitre 2.

Dans un premier temps, il est montré que dans le cas d'une structure simple comme celle d'une mousse dont les tailles de pores sont peu dispersées, les dimensions trouvées par mesure directe de la géométrie locale sont représentatives du comportement acoustique du milieu. Dans ce cas, il a été appliqué pour ce type de matériau une méthode purement micro-macro avec utilisation directe de la microstructure caractérisée par l'observation afin d'obtenir le comportement acoustique macroscopique. Il a été observé que lorsque la structure de la mousse est plus complexe, les mesures directes de la géométrie locale ne sont plus suffisantes.

Dans un deuxième temps, l'idée est alors d'utiliser soit des paramètres d'entrée accessibles simplement au plan expérimental, telles que la porosité ou la perméabilité statique (cas d'une mousse dont la taille de pores est dispersée), soit une combinaison de ces paramètres avec une dimension microstructurale mesurée (cas d'une mousse avec membranes). Les méthodes proposées dans cette partie sont validées par des comparaisons aux résultats de mesure tant au niveau des paramètres de transport qu'au niveau des propriétés dynamiques.

Le chapitre se termine par l'application des méthodes proposées à des résultats de la littérature, lorsque la connaissance de la microstructure est suffisante.

3.1. Cas d'une distribution étroite de tailles caractéristiques de la géométrie locale

Lorsque le milieu réel est peu dispersé, comme dans le cas d'une mousse d'aluminium à cellules ouvertes de type Duocel[®], il est possible de définir un volume élémentaire représentatif en mesurant directement les caractéristiques (de forme et de taille) à l'échelle locale du matériau poreux sur quelques cellules (nombre de faces par cellule, nombre d'arêtes par face ; longueurs des ligaments, épaisseurs des ligaments). Un modèle tridimensionnel de cellule périodique, qui soit compatible avec les caractéristiques de forme mesurées, est alors dimensionné à partir des données collectées à l'échelle locale (moyenne ou médiane). Les coefficients de transports asymptotiques basses et hautes fréquences sont ensuite calculés par homogénéisation numérique, et les propriétés acoustiques estimées par le modèle de Pride-Lafarge (JCAPL). C'est l'objet de la Sec. 3.1 ayant été publiée sous forme de chapitre de livre dont la référence est :

C. Perrot, F. Chevillotte, L. Jaouen, and M. T. Hoang, Acoustics properties and applications, in Metallic Foams: Fundamentals and Applications, Edited by N. Dukhan, DEStech publications, Lancaster, PA, USA (2012). ^[1]

Contributions du candidat

Ce paragraphe permet de resituer précisément la contribution du candidat à ce chapitre de livre résultant d'un travail d'équipe. Pour ce faire, les contributions respectives du candidat ont été listées ci-dessous:

- Analyse des données de la morphologie cellulaire du milieu réel, et mise au point de cellules idéalisées.
- Modélisation de la géométrie tridimensionnelle.
- Contribution au développement du code d'éléments finis tridimensionnel, en particulier sur les aspects thermiques, et ce avec l'aide de F. Chevillotte ici remercié.
- Création de la figure et du tableau de résultats, et participation à l'analyse des résultats.

3.1.1. Introduction

A major issue in automobile, aeronautical, and building industries concerns the need to increase or adapt the sound absorption spectrum of metallic foams. However, semi-phenomenological models used to characterize and predict sound absorbing material performances are mainly based on interdependent macroscopic parameters, which do not account explicitly for the local geometry of these porous media (i.e., their microstructure). Thus, optimizing sound absorbing materials from the manufacturing step remains a difficult task mostly done by trial and error. A strict optimization method would firstly rely on our ability to predict the acoustic properties of real metallic foam samples from the description of their local geometry. Secondly, it would propose process-compatible modifications of their microstructure having predictable impacts on their absorption spectrum. Based on fundamental mechanisms governing audible sound waves propagation and dissipation through metallic foams, emphasis of this chapter is on linking scales in computational acoustics of porous media: how microstructure and macro-scale properties of real metal foam samples are related, with engineering guidelines for sound proofing.

What is the influence of the micro-structural morphology (e. g. aperture size, pore size, ligament diameter, ligament shape, etc.) of metallic foam on its acoustical performance? What can be done to the foam's structure to make it a better absorber? These are many questions that are dominating studies of relationships between microstructure and acoustic properties of metallic foams. Such questions may be addressed in different manners.

- (1) A common method consists in conducting a lot of laboratory measurements on samples of varying microstructural parameters ^{[5]-[19]}.
- (2) Alternatively, in a search for a theoretical understanding, one may try to better understand the mathematical and physical basis of the macroscopic equations governing acoustic dissipation phenomena ^{[10]-[19]}.
- (3) Numerical studies based on simulations can be considered ^{[20]-[28]}.
- (4) Semi-empirical approaches that combine numerical predictions of key physical parameters used as input data in empirical models can be employed ^[29].
- (5) Lastly, one can consider hybrid numerical approaches combining numerical predictions of key physical parameters used as input data in theoretical models ^{[30]-[33]}.

Each of these ways of considering these questions has advantages and drawbacks.

- (1) Laboratory measurements are of indisputable value; however, their interpretation may be limited to a specific group of morphologies (e.g. open-cell, cracked closed-cell, perforated closed-cell, etc.).
- (2) Theoretical studies at the macroscopic scale lead to robust semi-phenomenological models; but they also require measurements of macroscopic parameters, and this may involve great expense.
- (3) Numerical simulations usually attempt to bridge the gap between theory and experiments. They are nevertheless typically restrained by either the need to simplify the geometry, physics, or both.
- (4) Semi-empirical approaches suffer from the weakness of empirical models providing poor physical insight, and being unable to consider non-already existing microstructural configurations.
- (5) In recent years, a hybrid numerical approach to the study of long-wavelength acoustic waves propagation through rigid porous media has gained some popularity. The idea is to numerically solve elementary transport equations in a realistic local geometry model, and then to study how key physical parameters computed from volume-averaged fields, relate to frequency-dependent acoustic properties through approximate but robust semi-phenomenological models. Compared to direct numerical approaches, such studies offer the ability to identify the micro geometry features governing the macro transport and acoustic properties; they are however limited to micro geometries made of idealized periodic unit-cells.

The classical numerical homogenization approach study the long-wavelength acoustic properties of porous media by direct solutions of the linearized Navier-Stokes equation in harmonic regime with the local incompressibility condition ^[21] (dynamic viscous problem), and of the linearized heat equation in harmonic regime ^[17] (dynamic thermal problem) with appropriate boundary conditions.

For the case of the dynamic viscous problem, solutions mainly based on finite element methods (FEM) have been investigated. Craggs and Hildebrandt ^[20] solved the viscous problem for specific cross-sections of uniform pores. Zhou and Sheng ^[21] treated the case of a cylindrical tube with sinusoidal modulation of its cross section, three-dimensional (3D) fused-spherical-bed and fused-diamond lattices. Chapman and Higdon ^[22] considered the three cubic lattices [simple cubic (SC), body-centered cubic (BCC), and face-centered cubic (FCC)] from a very accurate semi-analytical method, with overlapping or non-overlapping spheres depending on the prescribed porosity. Firdaouss et al. ^[23] paid attention to a corrugated pore channel. Finite elements results obtained by Firdaouss et al. were subsequently confirmed by Cortis and Smeulders using Schwartz-Christoffel transformations ^[24]. The overall disseminated conclusions

related to two-dimensional porous media whose internal surface contains sharp-edged wedges were finally summarized in a clarifying paper [25]. Cortis et al. [26] also studied the case of bi-dimensional (2D) configurations made of a square arrangement of solid cylinders.

An attempt to grasp the viscous dynamic behavior of more complex microstructures, such as a real open-cell aluminum foam sample, has been carried out thanks to a basic 2D hexagonal model of solid fibers [28],[30],[34]. The 2D periodic foam model geometry provided a reliable estimate of the dynamic permeability, except in the low frequency range. In the 2D periodic foam model geometry, ligaments are always perpendicular to the flow direction, thus artificially decreasing the static permeability of the viscous flow.

For the case of the dynamic thermal problem, another approach has been the application of the random-walker simulation method, as proposed by Lafarge [35]. The principle of the method consists in simulating Brownian motion for a large number of the fluid-phase particles, and to link their mean square displacements to the thermal conduction properties of the confined fluid. An important point of the method is that, once the mean square displacements of a large number of particles has been estimated, the dynamic thermal response might be obtained for all frequencies. Contrary to finite element analysis, the solution does not need to be computed at each frequency.

The random-walker simulation method has been implemented in two and three dimensions for computing the trapping constant of a 2D arrangement of overlapping fibers of circular cross-sections [36], and 3D digitalized geometries [37]. The trapping constant provides the asymptotic low frequency behavior of the thermal problem. The first numerical simulations in harmonic regime have been achieved for the case of 2D regular and random arrangements of fibers with circular cross-sections [35], and extended to three-dimensional micro geometries [38] with application to an open cell aluminum foam sample [39].

Other contributions at the pore scale, of industrial interest, and which can be applied in order to properly determine both viscous and thermal dissipation phenomena in specific open-cell metallic structures are addressed in this section. Wang and Lu [40] determined the optimized acoustic properties of polygonal ducts through semi-analytical solutions. The optimized cell size found for best sound absorbers is on the order of ~ 0.1 mm for practical combinations of sample thickness, cavity depth, and porosity. Gasser et al. [27] treated the 3D case of face centered cubic nickel hollow spheres packings with a view of absorbing sound inside the turboengines of aircraft. A special care was given to properly model solder

joints. Alternatively, prescribed porosity and correlation length(s) have been used for the reconstruction process, or three-dimensional images of the real samples [41]. Lee et al. paid attention to the three-dimensional hexagonal-closed pack structure which has not appeared in the acoustic literature previously [32]; and showed that multi-periodic composite structures, defined as periodically-layered media wherein each layer is composed of periodic unit-cells, could lead to frequency stop-bands [33]. The sound absorption properties of metallic hollow sphere structures were also analyzed experimentally by Pannert et al. [9]. Very recently, membrane, anisotropy, and pore size dispersion effects of real foam samples, mostly open-cell, were investigated by the implementation of a 3D polyhedron unit-cell, a truncated octahedron with ligaments of circular cross-section shapes and spherical nodes at their intersections [2].

Contrary to open-cell foams, closed-cell foams are poor sound absorbers. However, they generally present a better structural rigidity and a lower production cost than open-cell foams. Two main methods can be used for enhancing the sound absorption of closed-cell foams. The first method consists of fracturing its cell walls via compression or rolling [42]. In an effort to model the acoustic properties of such a porous medium, it was shown that the fractured foam may be seen as a semi-open cell material, i.e., a two-dimensional foam model geometry consisting of infinitely long arrays of cracked cells [43]. The second method consists of hole drilling the closed-cell foam [44]. These two methods follow the same principle, which aims at increasing the viscous effects by enabling a relative motion between the two phases of the porous medium. The literature on both methods revealed good practical results; however it was limited in the sense that it did not systematically quantify the effects of microstructure modifications introduced by compression, rolling or hole drilling. Such an attempt was further performed by Chevillotte et al. [31].

Let us also mention that sound absorption characteristics of lotus-type porous magnesium and copper plates fabricated by unidirectional solidification were studied experimentally [6]. For the samples under study, it was found that the sound absorption coefficient increased with increasing porosity (43 % to 62 %), while it decreased with increasing pore diameter (from 460 μm to 660 μm).

This chapter is devoted to the hybrid numerical study of long-wavelengths acoustic waves propagation and dissipation through periodically-reconstructed images of rigid porous media. The simulations are performed by a finite element method. Since this chapter is also dedicated to non-specialists, we begin our discussion with a brief review of acoustic wave's dissipation mechanisms in addition to providing some

orders of magnitude for typical dissipative pore sizes. We then describe the hybrid numerical method through which we simulate the key physical parameters and acoustical properties of metallic foams. A study of both an open and a perforated closed-cell aluminum foam sample are provided, including a study of how acoustic properties vary as a function of common local geometry features. Our results compare qualitatively well to laboratory measurements. They thus serve to validate the application of the hybrid numerical method to periodically-reconstructed metallic foams, in addition to providing a detailed study of how acoustic properties vary with morphology.

3.1.2. Principles of acoustical energy dissipation through metallic foams

Metallic foams, composed of a metal frame and a connected pore network saturated with air, can be used as passive noise control packages to reduce both structure and air-borne sound and vibrations. The acoustical energy dissipation through such porous media involves 3 phenomena: visco-inertial dissipation, thermal dissipation and structural dissipation.

The visco-inertial dissipation results from the friction of air particles with the metal frame walls while the acoustic waves propagates inside the foam. The thermal dissipation results from the thermal exchanges between the air particles and the frame. These two dissipation phenomena thus depend mainly on the geometry of the pore network. Obviously the structural dissipation depends on the mechanical properties of the material.

Below are further details of these mechanisms of acoustical energy dissipation. A simple physical analysis of these mechanisms will lead to recommendations for producing acoustic absorber foams, to be used as standalone sound packages for acoustic correction purposes, or in association with heavier partition walls for sound insulation purposes.

A large amount of literature is available addressing the dissipation mechanisms of the acoustic energy in porous materials. Readers are invited to refer to the books by Zwikker and Kosten ^[45] or by Allard and Atalla ^[46] for example.

3.1.2.1. *Mechanisms of acoustical energy dissipation*

Porous materials presenting a single scale of porosity such as glass wool or polyurethane foams are constituted by two components called hereafter phases. One phase, the skeleton, is composed by the matter from which the porous medium is made of. The second phase is composed by the fluid saturating the skeleton: air in acoustics. When such diphasic material is submitted to an airborne or a structural vibration, the two phases can interact with each other and can dissipate energy. Three types of

interactions can occur: visco-inertial, thermal and structural (if the material's skeleton is deformable).

3.1.2.1.1. Visco-inertial dissipation mechanisms

Visco-inertial effects in an acoustical porous material are due to the fact that the pore saturating fluid does not move in phase with the skeleton.

In the low frequency range, the viscous forces dominate the inertial ones. The air flow inside the material is described by Darcy law. In the high frequency range, inertial forces dominate the viscous ones. A characteristic angular frequency ω_v has been introduced [10]-[11],[12] to separate the low and high frequency behaviors of visco-inertial effects. For pulsations much smaller than ω_v , the flow is purely viscous, for pulsation much larger than ω_v the flow is purely inertial. See Fig. 3.1 for a schematic view of these basic principles on visco-inertial dissipation mechanisms.

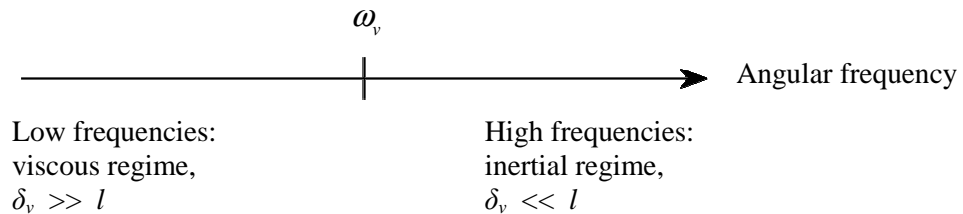


Figure 3.1. Frequency- dependent visco-inertial dissipation mechanisms

3.1.2.1.2. Thermal dissipation mechanisms

While a sound wave propagates through a porous medium, it experiences successive compressions and dilatations. During these successive state transformations, a thermal wave is created. Heat is thus exchanged between the air and the frame. The resulting thermal dissipative mechanism follows a similar two-asymptotic-state behavior as the visco-inertial one. Below the thermal characteristic angular frequency ω_t introduced by Lafarge et al. [17] the air compressions and dilatations are isothermal (heat exchanges exist between the whole fluid phase and the frame). Above ω_t transformations in the fluid phase can be considered as adiabatic due to the high frequency of the acoustic propagation phenomenon: The heat exchange phenomenon takes longer to establish than a cycle of the pressure wave. See Fig. 3.2 for a schematic view of these basic principles on thermal dissipation mechanisms.

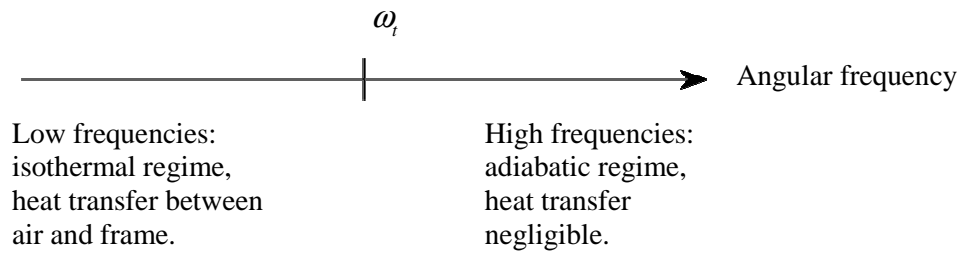


Figure 3.2. Frequency- dependent thermal dissipation mechanisms

3.1.2.1.3. Structural dissipation mechanisms

The Biot's theory [10]-[11] states that in a porous frame saturated by a fluid such as an air-saturated metal foam, three waves can propagate:

1. a longitudinal wave in the air,
2. a longitudinal wave in the frame and,
3. a shear wave in the frame.

Structural dissipation may thus occur in the frame of the metal foam.

In case of air-borne excitation, Zwikker and Kosten [45] introduced an angular frequency ω_{dec} for which the inertial effects in the frame are equal in magnitude to the viscous effects in the fluid phase, Fig. 3.3. For angular frequencies much larger than ω_{dec} the two phases can be considered as decoupled, assuming stiffness effects of the material sample frame are negligible compared to inertial ones. At such frequencies, the frame cannot be significantly set in motion by the fluid-borne wave having a high frequency, as the frame motion takes longer to establish than a cycle of the pressure wave in the pore network.

Zwikker and Kosten expression of ω_{dec} shows that this angular frequency depends on the pore morphology of the porous medium. ω_{dec} is also inversely proportional to the mass density of the material. For metal foams, the vibration of the frame can usually be neglected as the numerical value of the decoupling angular frequency is in the low part of the audible frequency range.

Obviously, for structure-borne sound no phase decoupling can be considered.

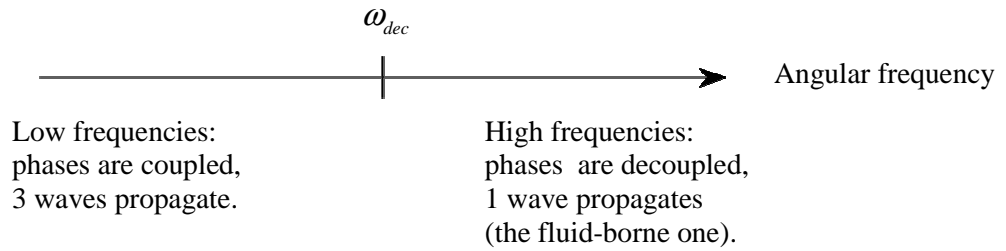


Figure 3.3. Frequency- dependent structural dissipation mechanisms

3.1.2.2. Conditions for a metal foam to be an acoustic absorber

To summarize the points above, two conditions on the pore network morphology can be stated for a metal foam to be an acoustic absorber:

1. An interconnected pore network for the dissipation mechanisms to occur in the largest possible volume.
2. Visco-inertial and thermal dissipation effects will significantly affect the sound wave propagation when the pores have a size of the same order of magnitude as that of the viscous and the thermal boundary layers.

From the work by Kirchhoff ^{[47]-[48]} it is known that the boundary layer thickness in a cylindrical tube for a plane wave incidence assuming a laminar flow of the air particle, a no-slip condition and no temperature jump at the air-frame interface, can be calculated as $\delta_v = \sqrt{\eta / \rho_0 2\pi f}$ where η is the dynamic viscosity of the air (1.8×10^{-5} N.s.m⁻² at a temperature of 20 °C and an atmospheric pressure of 101325 Pa), ρ_0 is the mass density of the air at rest (1.2 kg.m⁻³ for the same temperature and atmospheric pressure conditions) and f is the frequency of the incident sound wave.

A quick numerical application leads to a variation of δ_v between 3.5×10^{-4} m and 1.1×10^{-5} m in the audible frequency range: [20 – 20 000] Hz.

Kirchhoff has also reported the expression of the thermal boundary layer thickness, for the same conditions as above: $\delta_t = \sqrt{\kappa / \rho_0 C_p 2\pi f}$ where κ is the thermal conductivity of air and C_p is its specific heat at constant pressure. For the same temperature and atmospheric pressure conditions as above, the numerical values of these two quantities are respectively 2.6×10^{-2} W.m⁻¹.K⁻¹ and 1.0×10^3 J.kg⁻¹.K⁻¹. Again, a quick numerical application leads to a variation of δ_t between 4.1×10^{-4} m and 1.3×10^{-5} m in the audible frequency range. From the simple physical analysis above on visco-inertial and thermal effects, it appears that the second condition for a metal foam to be an acoustic absorber, is to present a pore size in the approximate range [10 – 1000] micrometers.

Let us finally mention that efficient transmission losses are generally obtained with multilayered systems, where the poroelastic layer acts as a decoupling element. This is obviously not the case for metallic foams, which are too rigid to be attractive for this application. Consequently, metallic foams for sound insulation should be closed-cell, or used for their sound absorption properties.

Possible effects of altering the microstructure of a specific foam and the resulting acoustical properties to achieve optimal structural acoustic performance in a given application were discussed by Lind-Nordgren and Göransson recently ^[49].

3.1.3. Micro-macro simulation method

3.1.3.1. *Metallic foams' cellular morphology*

Without being exhaustive, the intent of this section is to provide a brief overview of the diversity of microstructures which can be encountered when dealing with metallic foams and acoustic applications, together with possible corresponding periodic unit cell local geometry models; Fig. 3.4. Fig. 3.4(a) illustrates a 40 ppi Duocel[®] aluminum cylindrical foam sample mostly open cell. The foam sample diameter is 10 mm. The corresponding periodic unit cell foam model geometry is a regular truncated octahedra, also called tetrakaidecahedron, having ligaments of circular cross-section shapes. The reader is referred to Ref. [38] for a detailed study on this kind of foam sample microstructure and local geometry models. In Fig. 3.4(b), an initially closed cell aluminum foam sample is presented, showing cracks at the surface of the pores obtained by fracturing the cell walls via rolling. See Ref. [44] for more details about this technique used for improving sound absorbing properties of closed cell metallic foams. The related periodic unit cell is a simple body-centered cubic system with spheres allowed to interpenetrate in order to model interconnected openings. Fig. 3.4(c) presents a 22.6 mm thickness perforated metallic foam sample. Associated local geometry model is a simple cylinder with interconnected polydispersed spheres. See Ref. [31] for more information. It is also probably the place to underline the importance of collecting information on the fabrication process and related physics for modeling the typical cellular morphology of the metallic foam under interest. For an introduction of the different manufacturing routes for metallic foams, the reader is for instance referred to Ref. [50]. Despite the variability of metallic foam's cellular morphology, a common question to be addressed by engineers and researchers interested in metallic foams' acoustic properties, and more

specifically in bottom-up approaches for improving their sound absorbing properties, might be formulated as the following. What are the local geometry parameters to be introduced in the micro-macro modeling of the porous media? i.e., pore size and interconnection distributions, property gradients and (an)isotropy. The answer to this question is not unique. But among the advantages to consider an idealized periodic unit cell reconstruction approach instead of addressing explicitly the disordered nature of porous media, are its effective ability to (i) grasp the main local geometry features having a significant impact at the upper scale as well as (ii) suitability for optimization purposes by means of a hybrid numerical approach. The work considered in this chapter leads to the determination of a periodic unit cell, from which macro-properties are derived. In particular, this method is illustrated through the case of a 40 ppi Duocel[®] aluminum foam sample as depicted in Fig. 3.4(a), for which an extensive literature exists on both microstructure and physical macro-behavior. In this application, reconstruction is carried out from the standpoint of the ligament length L and thickness t distributions, which have been acquired by means of X-ray computed axial microtomography [38]. A simple isotropic three-dimensional model is considered for the representative idealized periodic unit cell, with ligaments of circular cross-section shapes, and spherical nodes at their intersections (of diameter $d = 1.5 \times t$ typical of lump modeling).

The open porosity ϕ of a porous solid is defined as the fraction of the interconnected pore fluid volume Ω_f to the total bulk volume of the porous aggregate Ω ,

$$\phi = \Omega_f / \Omega \quad (3.1)$$

The thermal characteristic length Λ' , which is a generalization of the hydraulic radius, is equal to twice the interconnected pore fluid volume Ω_f to pore wet surface $\partial\Omega$ ratio,

$$\Lambda' = 2 \Omega_f / \partial\Omega \quad (3.2)$$

The purely geometrical macroscopic properties – open porosity and thermal characteristic length of the idealized reconstructed PUC, might then be determined by spatial integration and compared with experimental measurements, with a view to validating the proposed idealized PUC prior to first principles computations of transport properties. It is worth mentioning that the studied thermal characteristic length – commonly used in acoustics of porous media – is a parameter closely related to the specific surface of the solid porous frame [51].

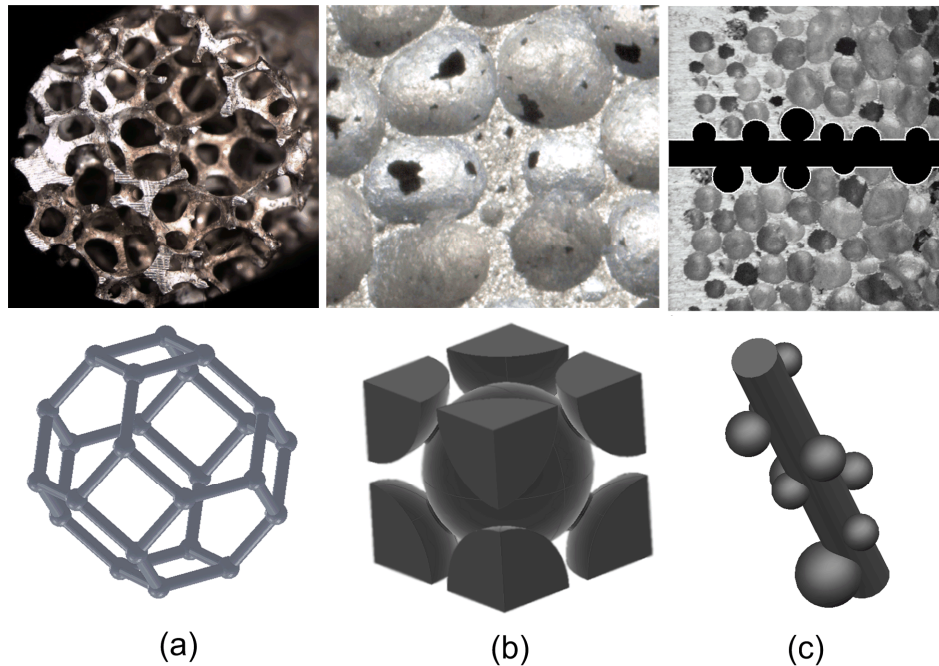


Figure 3.4. Illustration of the diversity of microstructures which can be encountered when dealing with metallic foams together with possible corresponding periodic unit cell local geometry models: (a) open cell, (b) fractured, and (c) perforated closed cell aluminum foam samples.

Here, we do not use the mean values as input parameters for L and t . Instead, values corresponding to the main peaks of the modal distributions are considered (“mean1” in the notations of Ref. [38]): $L = 1000 \mu\text{m}$ and $t = 330 \mu\text{m}$.

3.1.3.2. Hybrid numerical approach

3.1.3.2.1. First principles calculations of transport properties

Previous studies [18],[52] have shown how the long-wavelengths acoustic properties of rigid-frame porous media can be numerically determined by solving the local equations governing the asymptotic frequency-dependent visco-thermal dissipation phenomena in a periodic unit cell with adequate boundary conditions. In the following, it is assumed that $\lambda \gg D$, where λ is the wavelength of an incident acoustic plane wave. This means that for characteristic lengths on the order of $D \sim 0.5 \text{ mm}$, this assumption is valid for frequencies reaching up to a few tens of kHz. The asymptotic macroscopic properties of sound absorbing materials are computed from the numerical solutions of:

- (1) the low Reynolds number viscous flow equations (the static viscous permeability k_0 , and the static viscous tortuosity α_0);
- (2) the non-viscous flow or inertial equations (the high-frequency tortuosity α_∞ , and Johnson's velocity weighted length's parameter Λ);
- (3) the equations for thermal conduction (the static thermal permeability k_0' , and the static thermal tortuosity α_0').

(a) Viscous flow

At low frequencies or in a static regime, when $\omega \rightarrow 0$, viscous effects dominate, and the slow fluid motion in steady state regime created in the fluid phase Ω_f of a periodic porous medium having a unit cell Ω , is the solution of the following boundary value problem defined on Ω by ^[53]:

$$\eta \Delta \mathbf{v} - \nabla p = -\mathbf{G}, \quad \text{in } \Omega_f, \quad (3.3)$$

$$\nabla \cdot \mathbf{v} = 0, \quad \text{in } \Omega_f, \quad (3.4)$$

$$\mathbf{v} = \mathbf{0}, \quad \text{on } \partial\Omega, \quad (3.5)$$

$$\mathbf{v} \text{ and } p \text{ are } \Omega\text{-periodic}, \quad (3.6)$$

where $\mathbf{G} = \nabla p^m$ is a macroscopic pressure gradient acting as a source term, η is the viscosity of the fluid, and $\partial\Omega$ is the fluid-solid interface. This is a steady Stokes problem for periodic structures, where \mathbf{v} is the Ω -periodic velocity, p is the Ω -periodic part of the pressure fields in the pore verifying $\langle p \rangle = 0$, and the symbol $\langle \cdot \rangle$ indicates a fluid-phase average. It can be shown that the components v_i of the local velocity field are given by

$$v_i = -\frac{k_{0ij}^*}{\eta} G_j. \quad (3.7)$$

The components of the static viscous permeability tensor are then given by ^{[19],[54]}

$$k_{0ij} = \phi \langle k_{0ij}^* \rangle. \quad (3.8)$$

And the components of the tortuosity tensor are obtained from

$$\alpha_{0ij} = \frac{\langle k_{0\,pi}^* k_{0\,pj}^* \rangle}{\langle k_{0\,ii}^* \rangle \langle k_{0\,jj}^* \rangle}, \quad (3.9)$$

wherein the Einstein summation notation on p is implicit. In the present work, the symmetry properties of the microstructure under consideration imply that the second order tensors \mathbf{k}_0 and $\boldsymbol{\alpha}_0$ are isotropic. Thus $k_{0ij} = k_0 \delta_{ij}$ and $\alpha_{0ij} = \alpha_0 \delta_{ij}$, where δ_{ij} is the Kronecker symbol.

(b) Inertial flow

At the opposite frequency range, when ω is large enough, the viscous boundary layer becomes negligible and the fluid tends to behave as a perfect one, having no viscosity except in a boundary layer. In these conditions, the perfect incompressible fluid formally behaves according to the problem of electric conduction ^{[25],[55],[56]}, i.e. :

$$\mathbf{E} = -\nabla\varphi + \mathbf{e}, \quad \text{in } \Omega_f, \quad (3.10)$$

$$\nabla \cdot \mathbf{E} = 0, \quad \text{in } \Omega_f, \quad (3.11)$$

$$\mathbf{E} \cdot \mathbf{n} = 0, \quad \text{on } \partial\Omega, \quad (3.12)$$

$$\varphi \text{ is } \Omega\text{-periodic}, \quad (3.13)$$

where \mathbf{e} is a given macroscopic electric field, \mathbf{E} the solution of the boundary problem having $-\nabla\varphi$ as a fluctuating part, and \mathbf{n} is unit normal vector to the boundary of the pore region.

Then, the components $\alpha_{\infty ij}$ of the high frequency tortuosity tensor can be obtained from ^[18]

$$e_i = \alpha_{\infty ij} \langle E_j \rangle. \quad (3.14)$$

In the case of isotropy, the components of the tensor $\boldsymbol{\alpha}_\infty$ reduce to the diagonal form $\alpha_{\infty ij} = \alpha_\infty \delta_{ij}$. In this case, the tortuosity can also be obtained from the computation of the mean square value of the local velocity through:

$$\alpha_\infty = \frac{\langle \mathbf{E}^2 \rangle}{\langle \mathbf{E} \rangle^2}. \quad (3.15)$$

As for the low frequency tortuosity, an extended formula can be used for anisotropic porous media.

Having solved the cell conduction problem, the viscous characteristic length Λ can also be determined (for an isotropic medium) [12]

$$\Lambda = 2 \frac{\int_{\Omega} \mathbf{E}^2 dV}{\int_{\partial\Omega} \mathbf{E}^2 dS}. \quad (3.16)$$

(c) Thermal effect

When a vibration occurs, the pressure fluctuation induces a temperature fluctuation inside the fluid, due to the constitutive equation of a thermally conducting fluid. If one considers the solid frame as a thermostat, it can be shown that the mean excess temperature in the air $\langle \tau \rangle$ is proportional to the mean time derivative of the pressure $\partial \langle p \rangle / \partial t$. This thermal effect is described by $\langle \tau \rangle = (k_0' / \kappa) \partial \langle p \rangle / \partial t$, where $\langle \tau \rangle$ is the macroscopic excess temperature in air, κ is the coefficient of thermal conduction, and k_0' is a constant. The constant k_0' is often referred to as the “static thermal permeability”. As the usual permeability, it has the dimensions of area and was named by Lafarge *et al.* [17]. It is related to the “trapping constant” Γ of the frame by $k_0' = 1 / \Gamma$ [56]. In the context of diffusion-controlled reactions, it was demonstrated by Rubinstein and Torquato [57] that the trapping constant is related to the mean value of a “scaled concentration field” $u(\mathbf{r})$ by

$$\Gamma = 1 / \langle u \rangle, \quad (3.17)$$

where $u(\mathbf{r})$ solves

$$\Delta u = -1, \quad \text{in } \Omega_f, \quad (3.18)$$

$$u = 0, \quad \text{on } \partial\Omega. \quad (3.19)$$

It is worthwhile noticing that Δu is dimensionless. Therefore, u and k_0' have the dimension of area.

Similar to the tortuosity factors obtained from viscous and inertial boundary value problems, a “static thermal tortuosity” is given by :

$$\alpha_0' = \frac{\langle u^2 \rangle}{\langle u \rangle^2}. \quad (3.20)$$

3.1.3.2.2. Estimates of the frequency dependent visco-inertial and thermal responses

The acoustic response of foams depends on the dynamic viscous permeability and the “dynamic thermal permeability”. Both of these parameters could be obtained from dynamic FEM computations as in Ref. [27]. The approach presented here relies on the fact that the finite element computations presented previously are easy to implement, and provide the asymptotic behavior for both dynamic “permeabilities”. This asymptotic behavior constitutes the input data for the models which are used for predicting the full frequency range of the dynamic “permeabilities”. Therefore the hybrid approach employed in our study makes use of the asymptotic parameters of the porous medium obtained by finite elements. Then, it will be possible to provide the dynamic permeabilities and to compare these values to experimental ones. In a first step, the three different models which are used to build the dynamic permeabilities from asymptotic parameters are briefly recalled.

Johnson *et al.* [12] and, later, Pride *et al.* [15] considered the problem of the response of a simple fluid moving through a rigid porous medium and subjected to a time harmonic pressure variation across the sample. In such systems they constructed simple models of the relevant response functions, the effective dynamic viscous permeability $\tilde{k}(\omega)$ or effective dynamic tortuosity $\tilde{\alpha}(\omega)$. The main ingredient to build these models is to account for the causality principle, and therefore for the Kramers-Kronig relations between real and imaginary parts of the frequency-dependent permeability. The parameters in these models are those which correctly match the frequency dependence of the first one or two leading terms of the exact results for the high- and low- frequency viscous and inertial behaviors.

Champoux and Allard [14] and thereafter Lafarge *et al.* [17],[18],[52], in adopting these ideas to thermally conducting fluids in porous media, derived similar relations for the frequency dependence of the so-called effective “dynamic thermal permeability” $\tilde{k}'(\omega)$ or effective dynamic compressibility $\tilde{\beta}(\omega)$, which varies from the isothermal to the adiabatic value when frequency increases. The models for effective dynamic permeabilities were shown to agree with those calculated directly or independently measured. An important feature of this theory is that all of the parameters in the models can be calculated independently, most of them

being, in addition, directly measurable in non-acoustical experimental situations. In this regard, these models are very attractive because they avoid computing the solution of the full frequency range values of the effective permeabilities/susceptibilities. These models are recalled in Sec. 3.1.3.2.3. They are based on simple analytic expressions in terms of well-defined high- and low- frequency transport parameters which can be determined from first principles calculations [Sec. 3.1.3.2.1].

Such a hybrid approach was extensively used by Perrot, Chevillotte and Panneton in order to examine micro-/macro relations linking local geometry parameters to sound absorption properties for a two-dimensional hexagonal structure of solid fibers ^[28]. This method was recently completed by the use of easily obtained parameter (porosity ϕ and static viscous permeability k_0) of real foam samples, and by utilizing three-dimensional numerical computations ^[2].

As explained, spatial integration provides the purely geometrical macroscopic parameters – the open porosity ϕ and the thermal characteristic length Λ' ; and the five remaining input parameters for the models, α_0 , α_∞ , Λ , k_0' , and α_0' can be obtained by means of first-principles calculations by appropriate field-averaging in the PUC (Fig. 3.4(a)).

Finally, the predictions of the three models for the effective dynamic permeabilities described in Sec. 3.1.3.2.3 may be considered. In summary, the Johnson-Champoux-Allard” [JCA] model uses 5 parameters (ϕ , k_0 , α_∞ , Λ , Λ'), Johnson-Champoux-Allard-Lafarge” model [JCAL] uses in addition k_0' , and Johnson-Champoux-Allard-Prade-Lafarge” [JCAPL] model uses the full set of parameters (ϕ , k_0 , k_0' , α_∞ , Λ , Λ' , α_0 , and α_0').

3.1.3.2.3. Models for motionless skeleton materials

To describe the macro-scale acoustic properties of rigid-frame air-saturated porous media, also called “equivalent fluid” by some authors, the knowledge of two complex response factors are required. The dynamic tortuosity $\tilde{\alpha}_{ij}(\omega)$ is defined by analogy with the response of an ideal (non-viscous) fluid for which α_{ij} is real-valued and frequency independent,

$$\rho_0 \tilde{\alpha}_{ij}(\omega) \frac{\partial \langle \mathbf{v}_j \rangle}{\partial t} = -\mathbf{G}_j. \quad (3.21)$$

$\tilde{\alpha}_{ij}(\omega) = \tilde{\rho}_{ij}(\omega) / \rho_0$ is related to the dynamic viscous permeability by $\tilde{\alpha}_{ij}(\omega) = v\phi / i\omega \tilde{k}_{ij}(\omega)$. In these expressions, $\tilde{\rho}_{ij}(\omega)$ is the effective

density of air in the pores, ρ_0 is the density of air at rest, and $\nu = \eta / \rho_0$ is the air kinematic viscosity.

Similarly, a compressibility effect is also observed at macro-scale in the acoustic response of a thermo-conducting fluid filled porous media, where a second convenient response factor is the normalized dynamic compressibility $\tilde{\beta}(\omega)$ which varies from the isothermal to the adiabatic value when frequency increases,

$$\frac{\tilde{\beta}(\omega)}{K_a} \frac{\partial \langle p \rangle}{\partial t} = -\nabla \cdot \langle \mathbf{v} \rangle. \quad (3.22)$$

Here, $\tilde{\beta}(\omega) = K_a / \tilde{K}(\omega)$ is directly related to the dynamic (scalar) thermal permeability ^[17] by means of the relation $\tilde{\beta}(\omega) = \gamma - (\gamma - 1) i \omega \tilde{k}'(\omega) / \nu' \phi$. In these equations, $\tilde{K}(\omega)$ is the effective dynamic bulk modulus of air in the pores, $K_a = \gamma P_0$ is the air adiabatic bulk modulus, P_0 the atmospheric pressure, $\gamma = C_p / C_v$ is the specific heat ratio at constant temperature, $\nu' = \kappa / \rho_0 C_p$, and C_p and C_v are the specific heat capacity at constant pressure and volume.

With a locally plane interface, having no fractal character, the long-wavelength frequency dependence of the visco-thermal response factors $\tilde{\alpha}_{ij}(\omega)$ and $\tilde{\beta}(\omega)$ have to respect definite and relatively universal behaviors, namely causality through the Kramers-Kronig relation ^{[12],[20],[56]} similar to models used for relaxation phenomena in dielectric properties. The equivalent dynamic tortuosity of the material and the equivalent dynamic compressibility of the material are $\tilde{\alpha}_{eq\ ij}(\omega) = \tilde{\alpha}_{ij}(\omega) / \phi$ and $\tilde{\beta}_{eq}(\omega) = \phi \tilde{\beta}(\omega)$.

Simple analytic admissible functions for the fluid phase effective properties for *isotropic* porous media respecting the causality conditions are

$$\tilde{\alpha}(\omega) = \alpha_\infty \left[1 - \frac{1}{i\omega} f(\varpi) \right], \quad \tilde{\beta}(\omega) = \gamma - (\gamma - 1) \left[1 - \frac{1}{i\omega} f'(\varpi') \right]^{-1}, \quad (3.23)$$

where f and f' are form functions defined by

$$f(\varpi) = 1 - P + P \sqrt{1 + \frac{M}{2P^2} i\varpi}, \quad f'(\varpi') = 1 - P' + P' \sqrt{1 + \frac{M'}{2P'^2} i\varpi'}, \quad (3.24)$$

and ϖ and ϖ' are dimensionless viscous and thermal angular frequencies given by the following expressions,

$$\varpi = \frac{\omega k_0 \alpha_\infty}{\nu \phi}, \quad \varpi' = \frac{\omega k_0'}{\nu' \phi}. \quad (3.25)$$

The quantities M , M' , P and P' are dimensionless shape factors,

$$M = \frac{8k_0\alpha_\infty}{\Lambda^2\phi}, \quad M' = \frac{8k_0'}{\Lambda'^2\phi}, \quad P = \frac{M}{4\left(\frac{\alpha_0}{\alpha_\infty} - 1\right)}, \quad P' = \frac{M'}{4(\alpha_0' - 1)}. \quad (3.26)$$

- For $M' = P = P' = 1$, $k_0' = \phi\Lambda'^2/8$, the dynamic visco-inertial and thermal response functions reduce to 5 parameters (ϕ , k_0 , α_∞ , Λ , Λ') named throughout the paper as “Johnson-Champoux-Allard” [JCA] model.
- When the requirement $k_0' = \phi\Lambda'^2/8$ is not fulfilled, k_0' must be explicitly taken into account, this is the 6 parameters “Johnson-Champoux-Allard-Lafarge” [JCAL] model, where M' may differ from unity.
- A complete model relies on 8 parameters (ϕ , k_0 , k_0' , α_∞ , Λ , Λ' , α_0 , and α_0') and correctly matches the frequency dependence of the first two leading terms of the exact result for both high and low frequencies. This is the refined “Johnson-Champoux-Allard-Pride-Lafarge” [JCAPL] model.

Looking for plane waves solutions varying as $\exp[i(\omega t - \tilde{q}x)]$, Eqs. (3.21) and (3.22) yield the equivalent dynamic wave number $\tilde{q}_{eq}(\omega)$ of the material and equivalent characteristic impedance $\tilde{Z}_{eq}(\omega)$ of the material

$$\tilde{q}_{eq} = \omega \left(\tilde{\alpha}_{eq}(\omega) \tilde{\beta}_{eq}(\omega) \frac{\rho_0}{K_a} \right)^{\frac{1}{2}}, \quad \tilde{Z}_{eq} = \left(\frac{\tilde{\alpha}_{eq}(\omega)}{\tilde{\beta}_{eq}(\omega)} \rho_0 K_a \right)^{\frac{1}{2}}. \quad (3.27)$$

Thus, $\tilde{\alpha}_{eq}(\omega)$ and $\tilde{\beta}_{eq}(\omega)$ provide all pertinent information on the propagation and dissipation phenomena in the equivalent homogeneous material. Assuming an absorbing porous layer of thickness L_s that is backed by a rigid wall, the normal incidence sound absorption coefficient is

$$A_n = 1 - \left| \frac{\tilde{Z}_{sn} - 1}{\tilde{Z}_{sn} + 1} \right|^2, \quad (3.28)$$

with the normalized surface impedance of the porous medium defined as

$$\tilde{Z}_{sn} = \frac{\tilde{Z}_{eq}}{\rho_0 c_0} \coth(i\tilde{q}_{eq}L_s), \quad (3.29)$$

where c_0 is the sound speed in air.

3.1.4. Results and discussion

3.1.4.1. Experimental validations

Transport parameters and normal incidence sound absorbing behavior were derived on the basis of an idealized reconstructed PUC as described through Sec. 3.1.3.1. with the computational method presented in Sec. 3.1.3.2. See Tab. 3.1 and Fig. 3.5 for the corresponding numerical results, and their experimental counterparts, as obtained by the techniques and methods described below. These results validate our approach. Also shown is a typical perforated closed-cell metallic foam sound absorption spectrum, Fig. 3.6. See Ref. [31] for a detailed presentation of these results. Note that, contrary to open cell foam samples, perforated closed cell metallic foam samples as well as perforated plates present selective (as opposed to large frequency bands) sound absorption spectrums. A subsequent step consists in numerical experiments to provide insight about microstructure effects on acoustical macro-behavior.

The absorption performances of acoustical materials are usually measured in diffuse sound field according to ISO 354 (Acoustics – Measurement of sound absorption in a reverberation room). However, due to the small sample size of metallic foams usually available their sound absorption properties are usually measured for plane waves at normal incidence according to ISO 10534 (Determination of sound absorption coefficient and impedance in impedance tubes). Either method 1 or 2 of this latter standard test can be used.

Method	ϕ (-)	Λ '(mm)	k_0 (m ²)	α_0 (-)	Λ (mm)	α_∞ (-)	k_0 '(m ²)	α_0 '(-)
Computations	0.91	2.05	10.34×10^{-8}	1.42	1.17	1.07	19.92×10^{-8}	1.21
Measurements ^{a, b}	0.91 (± 0.01)		10.39×10^{-8} ($\pm 1.23 \times 10^{-8}$)					
Characterization ^{c, d}		2.01 (± 0.43)		NA	0.99 (± 0.06)	1.07 (± 0.01)	NA	NA

^aReferences [65]-[66].

^bReference [62].

^cReference [61].

^dReference [60].

Table 3.1. Comparison between computed, measured, and characterized macroscopic parameters for a Duocel® 40 ppi aluminum foam sample.

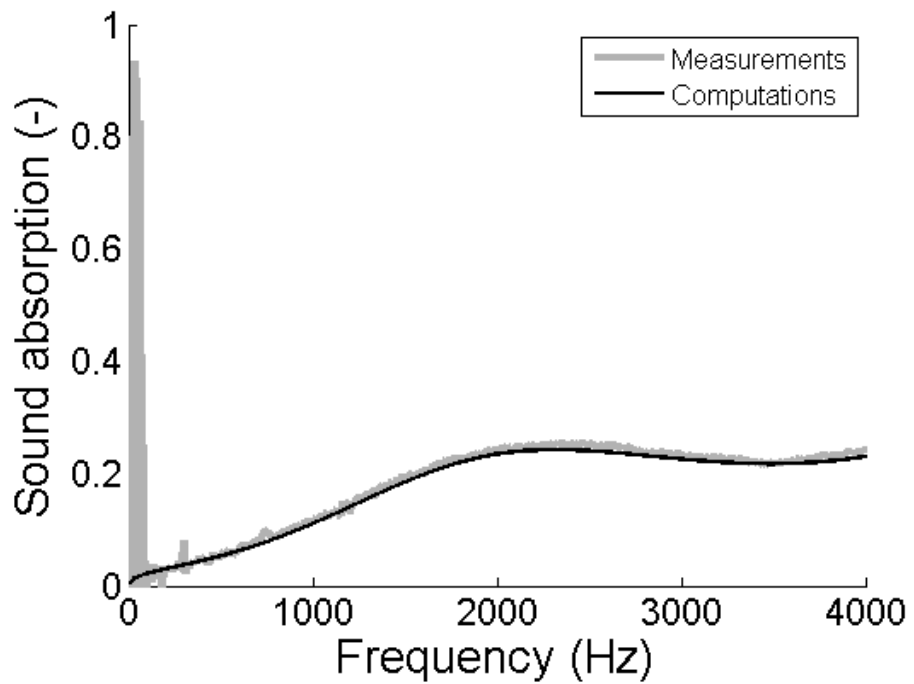


Figure 3.5. Comparison between computed and measured normal incidence, plane waves, sound absorbing behavior of a 40 ppi Duocel® aluminum foam sample. Thickness, 48 mm.

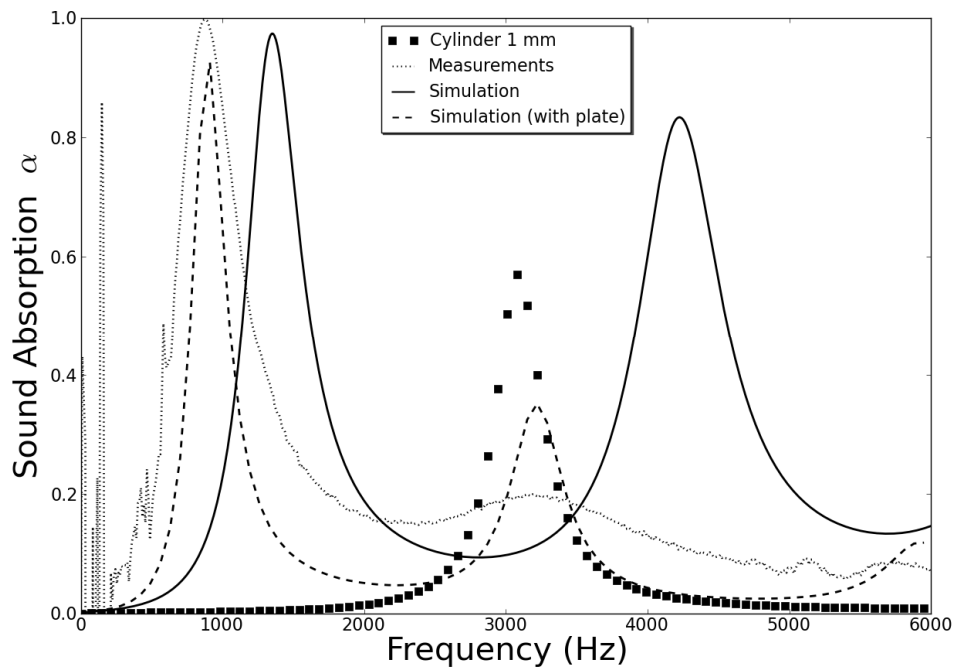


Figure 3.6. Normal incidence sound absorption coefficient of a real perforated closed-cell aluminum foam. Measurements (dotted line) compared to numerical computations with (dashed line) and without (solid line) a perforated facing plate.

Measurement of the sound absorption in normal incidence using an impedance tube ^[59] (cf. Fig. 3.7) can also be advantageous to carry out the estimations of 4 parameters introduced previously ^{[60],[61]}: the high-frequency limit of the tortuosity, the viscous and thermal characteristic lengths and the static thermal permeability.

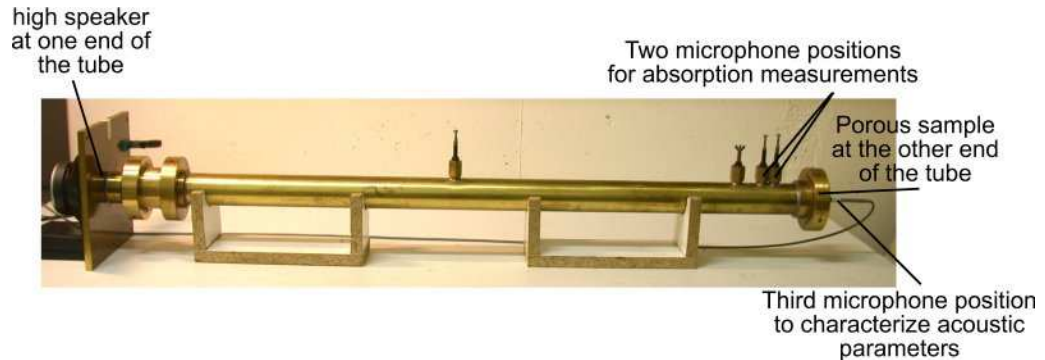


Figure 3.7. Picture of a 1 meter-long impedance tube which can be used (i) to measure the sound absorption properties of metal foams for plane waves at normal incidence and (ii) to estimate 4 parameters of the JCAL model.

The two remaining parameters of the JCAL model can be directly measured: (i) the static permeability was obtained by means of accurate measurements of differential pressures across serial mounted calibrated and unknown flow resistances, with a controlled steady and non-pulsating laminar volumetric air flow as described by Stinson and Daigle ^[62] and recommended in the corresponding standard ISO 9053 (method A) or ASTM C522 (cf. Fig. 3.8); (ii) and the open-porosity using methods such as those described by Champoux *et al.* ^[14] based on a previous work by Beranek ^[63] (cf. Fig. 3.9), Leclaire *et al.* ^[64], or Panneton *et al.* ^{[65],[66]}.

A direct measurement of the high frequency limit of the tortuosity α_∞ has been presented by Brown ^[55]. The method based on the measurement of the electrical conductivity of the porous material requires the material's frame to be saturated with a conducting fluid and can only be applied to materials for which the frame is composed with a dielectric material (i.e. it does not conduct electricity). As an alternative to direct measurement or estimation from impedance tube measurements, ultrasonic methods also exist. Allard *et al.* ^[67] have proposed a method to estimate α_∞ from the increase of flight time and the damping of an ultrasonic pulse, when a material sample is placed in between two ultrasonic transducers. From this work and in particular works by Johnson *et al.* ^[68] and Nagy ^[69], Leclaire *et al.* ^[70] have proposed a method to estimate α_∞ , Λ , Λ' using ultrasonic transmission measurements with the same porous material frame saturated successively with two different gases (usually air and helium). Recently, Groby *et al.* ^[71] have adapted the works by Panneton and Olny ^[60]

and Olny and Panneton ^[61] to estimate the four 4 last parameters of the JCAL model in the ultrasound domain from measurements of the transmitted and the reflected coefficients.

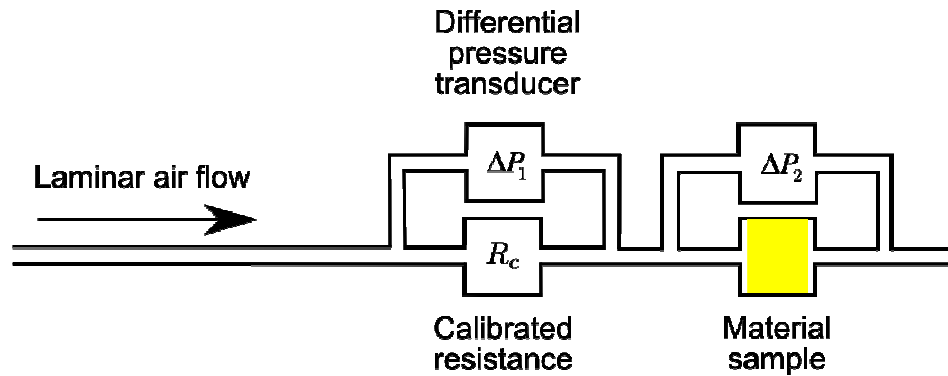


Figure 3.8. Scheme of the experimental setup used for the measurement of the static air flow resistivity.

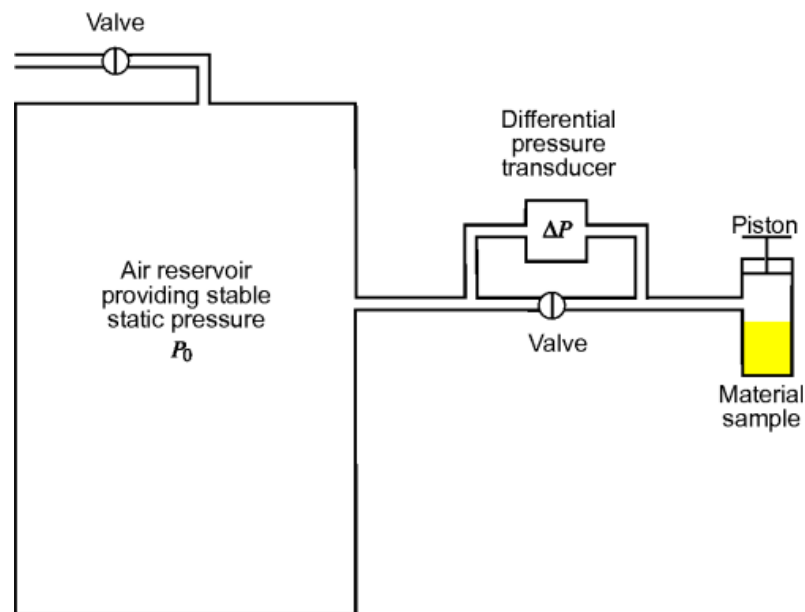


Figure 3.9. Scheme of the porosity measurement apparatus after Beranek ^[63] and Champoux et al. ^[14]. Porosity is measured from the pressure increase when reducing a reference volume containing the sample using Boyle-Mariotte law.

3.1.4.2. Numerical experiments: Microstructure effects on acoustical macro-behavior

Two main cellular morphology parameters were found to dominate the effects on acoustical macro-behavior of rigid porous media ^{[28],[30],[31],[34]}. It is worth mentioning that conclusions found by these authors on the basis of

a simple two-dimensional lattice of hexagonal solid fibers [28],[30],[34], were confirmed for other kinds of metallic foams apparently very different [31] such as the closed cell perforated metallic foam sample system as illustrated by Fig. 3.4(c). These conclusions might thus be considered as general acoustical micro-macro relationships. They report the existence of an emerging knowledge in which key local geometry features, having a significant impact on the *long-wavelength* acoustical macro-behavior of *motionless* porous media in general and metallic foams in particular, might be isolated from the standpoint of idealized periodic unit cells.

The throat size, which might be defined as the smallest aperture in a regular array of interconnected pores, appears as being the most important local geometry parameters in terms of acoustical macro-behavior. For instance, the throat size is the distance between two solid inclusions if one considers a regular array of solid fibers; it becomes the diameter of perforations when one deals with a perforated closed cell metallic foam sample. Numerical experiments have shown that the throat size directly controls the static viscous permeability k_0 (or the resistivity σ , since $k_0 = \eta / \sigma$) of the porous media and, as a consequence, the overall level of sound absorption (mainly by viscous dissipation mechanisms). In other words, if the aperture by which the compressional sound wave is allowed to penetrate is too small, reflection occurs and the sound wave can neither propagate, nor dissipate. By contrast, if the aperture is too large, the viscous boundary layer interacts only with a small fraction of the possible surface by which viscous interactions develop. In between there must exist an optimal opening zone, which was already known by Kirchhoff for pores of cylindrical cross-section shapes (Sec. 3.1.2.1.1.), and might be estimated by the proposed approach for real metallic foam samples.

The second microstructural parameter revealed from numerical experiments as having a significant and direct impact on the acoustical macro-behavior is the pore size. For instance, twice the inter-fiber distance in case of a hexagonal lattice of solid fibers ($2l$ in the notations of Fig. 1 in Ref. [30]); the characteristic bubble size when one considers perforated closed cell metallic foams (a in the notations of Fig. 2 in Ref. [31]). Pore size effect can be interpreted in terms of sound absorption modulation of the main peak: the overall sound absorption level is essentially unchanged by the pore size at constant throat size, whereas the frequency at which maximum absorption occurs might be advantageously modified according to the knowledge of a noise source spectrum. This is another fruitful property for sound absorbers design. The phenomenological reasons behind this micro-macro pore-size/modulation-spectrum linkage might be described in terms of tortuosity: pore size increases with the infinite

tortuosity factor α_∞ , which tends to lower the frequency of the sound absorption peak.

3.1.5. Further remarks on the evaluation of acoustic properties of metal foams

To summarize, we have presented a general approach for linking scales in acoustics of porous media in which the acoustic properties computation of various three-dimensional metallic foam microstructures can be considered in a unified framework. Comparison with experiments yields very good agreements. This paves the road for a systematic microstructure optimization of real sound absorbing materials. The transport and acoustic properties dependence of the local geometry model to membrane, anisotropy, and polydispersity effects will be published elsewhere.

3.2. Cas d'une distribution étendue de tailles caractéristiques de la géométrie locale

3.2.1. Identification d'une première taille critique de la géométrie locale

Lorsque le milieu réel est très dispersé et/ou hétérogène (distribution étendue de taille de pores, présence de membranes), l'identification d'une cellule élémentaire représentative n'est plus aussi directe. Pour s'en convaincre, il suffit de penser à un ensemble de tubes de sections constantes arrangés en parallèle ou en série, de tailles hétérogènes. La perméabilité du milieu est alors donnée par les tubes de plus grandes sections dans le cas parallèle, de plus petites sections dans le cas en série ; et non par la section moyenne. Il est alors utile, pour identifier une cellule représentative, de recourir au concept de chemin critique développé dans les années 70 pour étudier la conductivité de semi-conducteurs hétérogènes. L'image transposée qui s'en dégage distingue essentiellement trois types de perméabilités locales :

- (i) Les zones I de fortes perméabilités.
- (ii) Les zones II de perméabilités plus faibles qui interconnectent les précédentes et donnent la perméabilité d'ensemble.
- (iii) Les zones III de beaucoup plus faibles perméabilités court-circuitées, et qui ne participent donc pas à la perméabilité d'ensemble.

La perméabilité du milieu est donc donnée par la perméabilité de type II, identifiable par une mesure macroscopique. Un modèle tridimensionnel de cellule périodique sans membranes, qui soit compatible avec les caractéristiques de forme mesurées à l'échelle locale, est cette fois dimensionné à partir d'une mesure de porosité et de perméabilité à l'échelle macroscopique.

L'idée de la méthode de dimensionnement est la suivante. En principe, puisque l'on dispose d'un modèle de cellule tridimensionnelle principalement dépendant de deux paramètres locaux (longueur et épaisseur de ligaments), une inversion analytique requiert deux paramètres macroscopiques (porosité et longueur thermique ou surface spécifique). La longueur thermique et la surface spécifique sont difficilement accessibles par mesures directes non acoustiques. On remarque cependant que la longueur thermique est homogène à une longueur et la perméabilité à une longueur au carré. On peut d'autre part résoudre les équations de Stokes à l'échelle locale de la cellule adimensionnelle pour une porosité locale mesurée, et déduire la perméabilité de cette cellule par moyenne de champ. Puisqu'il doit exister une relation affine entre la longueur thermique et la

racine de la perméabilité, on peut donc identifier la longueur thermique correspondant à la perméabilité mesurée. La cellule tridimensionnelle sans membranes est donc dimensionnée au moyen de cette procédure pour le milieu dispersé hétérogène.

Les coefficients de transports asymptotiques complémentaires basses et hautes fréquences sont ensuite calculés par homogénéisation numérique, et les propriétés acoustiques à nouveau estimées par les modèles de Pride-Lafarge.

A l'échelle macroscopique, l'essentiel des déviations entre la prédiction et la mesure se situe au niveau de la longueur visqueuse, qui tend à être surestimée par le modèle (surestimation de la taille d'étranglement). De plus, on constate à l'échelle de la géométrie locale que les dimensions de la cellule modèle sans membrane sont systématiquement inférieures ou égales à celles des cellules réelles ; et ce d'autant plus que le milieu réel possède un taux important de membranes. Ces deux déviations, tant à l'échelle macroscopique que microscopique, concordent et concourent à promouvoir l'implémentation d'une caractéristique supplémentaire de la géométrie locale du milieu réel à l'échelle du modèle cellulaire, les membranes.

Ce travail fait l'objet de la section 3.2.1, ayant été publiée dans une revue internationale à comité de lecture dont la référence est :

C. Perrot, F. Chevillotte, M. T. Hoang, G. Bonnet, F.-X. Bécot, L. Gautron, A. Duval, Microstructure, transport, and acoustic properties of open-cell foam samples: Experiments and three-dimensional numerical simulations, Journal of Applied Physics, 111, 014911-16 (2012). [2]

Contributions du candidat

Ce paragraphe permet à nouveau de resituer précisément la contribution du candidat à cet article, publié dans une revue internationale à comité de lecture, et résultant d'un travail d'équipe. Pour ce faire, les principales contributions respectives du candidat ont été listées ci-dessous:

- Acquisition des paramètres de taille et de forme de la morphologie cellulaire d'échantillons réels de mousse, par microscopie optique.
- Production de l'ensemble des résultats et de leurs mises en forme.
- Identification des longueurs critiques calculées sur des microscopies électroniques à balayages.
- Participation de premier plan à la démarche de réponse aux referees : simulations supplémentaires visant à étayer les hypothèses avancées dans l'explication de certains résultats, analyses subséquentes, et production des figures supplémentaires.

Abstract

This article explores the applicability of numerical homogenization techniques for analyzing transport properties in real foam samples mostly open-cell, to understand long-wavelength acoustics of rigid-frame air-saturated porous media, on the basis of microstructural parameters. Experimental characterization of porosity and permeability of real foam samples are used to provide the scaling of a polyhedral unit-cell. The Stokes, Laplace, and diffusion-controlled reaction equations are numerically solved in such media by a finite element method in three-dimensions; an estimation of the materials' transport parameters is derived from these solution fields. The frequency-dependent visco-inertial and thermal response functions governing the long-wavelength acoustic wave propagation in rigid-frame porous materials are then determined from generic approximate but robust models and compared to standing wave tube measurements. With no adjustable constant, the predicted quantities were found to be in acceptable agreement with multi-scale experimental data, and further analyzed in light of scanning electron micrograph observations and critical path considerations.

3.2.1.1. Introduction

The determination from local scale geometry of the acoustical properties which characterize the macro-behavior of porous media is a long-standing problem of great interest ^{[45],[46],[73]}, for instance for the oil, automotive, and aeronautic industries. Recently, there has been a great interest in understanding the low Reynolds viscous flow, electrical, and diffusive properties of fluids in the pore structure of real porous media on the basis of microstructural parameters, as these transport phenomena control their long-wavelength frequency-dependent properties. ^{[12],[19],[21],[41],[54]} Each of these processes can be used to estimate the long-wavelength acoustic properties of a porous material. ^{[13],[75]-[78]} Our aim in this paper is to get insight into the microstructure of real porous media and to understand how it collectively dictates their macro-scale acoustic properties, from the implementation of first-principles calculations on a three-dimensional idealized periodic unit-cell.

In this purpose, one needs first to determine a unit cell which is suitable for representing the the local geometry of the porous medium, and second to solve the partial differential equations in such a cell to obtain the parameters governing the physics at the upper scale. The first problem is addressed through idealization of the real media. For instance open-cell foams can be modeled as regular arrays of polyhedrons. A presentation of various idealized shapes is given by Gibson and Ashby ^[79] for cellular

solids, and more specifically by Weaire and Hutzler ^[80] for foams. The second problem consists in the determination of the macroscopic and frequency-dependent transport properties, such as the dynamic viscous permeability ^[12]. The number of media which can be analytically addressed is deceptively small ^[81], and many techniques have been developed in the literature, such as estimates combining the homogenization of periodic media and the self-consistent scheme on the basis of a bicomposite spherical pattern (see, for instance, the recent work of Boutin and Geindreau, and references therein ^{[19],[54]}).

The purpose of this paper is to present a technique based on first-principles calculations of transport parameters ^[21] in reconstructed porous media ^[82] which can be applied to model the acoustic properties of real foam samples (predominantly open-cell) and to compare its predictions to multi-scale experimental data. The main difficulty in modeling the frequency-dependent viscous and thermal parameters characterizing the dissipation through open-cell foams lies in accurately determining microstructural characteristics, and in deducing from these features how they collectively dictate the acoustical macro-behavior. Since the variability in the foam microstructures makes it very difficult to establish and apply local geometry models to study the acoustics of these foams, the use of a representative periodic cell is proposed to quantitatively grasp the complex internal structure of predominantly open-cell foam samples. Such a periodic cell, named thereafter Periodic Unit Cell, has characteristic lengths which are directly deduced from routinely available porosity and static viscous permeability measurements - two parameters practically required to determine acoustical characteristics of porous absorbents in the classical phenomenological theory. ^[83]

The studies on the acoustic properties derivation from the local characteristics of a porous media can be split into two classes, which address the reconstruction problem differently. The first class uses prescribed porosity and correlation length(s) for the reconstruction process, or three-dimensional images of the real samples ^{[41],[72]}. In the second class, idealization of the microstructure, whether it is granular- ^{[27],[32]}, fibrous- ^[29], or foam- ^{[28],[31],[38],[39]} like types is performed. This provides with a PUC having parameterized local geometry characteristics depending on the fabrication process, helpful for understanding the microphysical basis behind transport phenomena as well as for optimization purposes ^[30].

The approach to be presented in this paper is a hybrid. From the first-principles calculations method ^[21], we take the idea to compute for three-dimensional periodic porous media models the asymptotic parameters of the dynamic viscous $\tilde{k}(\omega)$ and thermal $\tilde{k}'(\omega)$ permeabilities ^{[12],[17]}

from the steady Stokes, Laplace, and diffusion-controlled reaction equations. Then, instead of using this information for comparison with direct numerical simulations of $\tilde{k}(\omega)$ and $\tilde{k}'(\omega)$ (which would require the solutions of the harmonic Stokes and heat equations to be computed for each frequency), we use these results as inputs to the analytical formulas derived by Pride *et al.* [15] and Lafarge *et al.* [18],[58]. As we will show, the results obtained in this manner are satisfying for the various foam samples used in the experiments.

This paper is divided into six sections. Sec. 3.2.1.2 is devoted to the direct static characterization of foam samples. Sec. 3.2.1.3 describes the methodology which is used to determine the local characteristic lengths of a three-dimensional periodic unit-cell, from which all the transport parameters are computed. Sec. 3.2.1.4 details a hybrid numerical approach employed to produce estimates of the frequency dependent visco-inertial and thermal responses of the foams. An assessment of the methodology through experimental results is made in Sec. 3.2.1.5. In addition, keys for further improvements of the methodology are reported in light of scanning electron micrographs of the foam samples. Sec. 3.2.1.6 provides a supplementary justification and validation of the proposed method through conceptual and practical arguments as well as uncertainty analysis. Sec. 3.2.1.7 concludes this paper.

3.2.1.2. Direct static characterization of foam samples

3.2.1.2.1. Microstructure characterization

Three real and commercially available polymeric foam samples have been studied. They are denoted R_1 , R_2 , and R_3 . These samples have been chosen for the following reason: contrary to previously studied open-cell aluminum foam samples [28],[38],[39] their apparent characteristic pore size D is around a few tenths of a millimeter and small enough, so that the visco-thermal dissipation functions characterizing their acoustical macro-behavior are, *a priori*, accurately measurable on a representative frequency range with a standard impedance tube technique. [59]

Real foam samples are disordered [84],[85] and possess a complex internal structure which is difficult to grasp quantitatively. However, our objective is to be able to quantify the local geometry of such foams by an idealized packing of polyhedral Periodic Unit Cells (PUC). Apart from the intrinsic need for characterizing the cell morphology itself, insight into the morphology of an idealized PUC is helpful for understanding the microphysical basis behind transport phenomena.

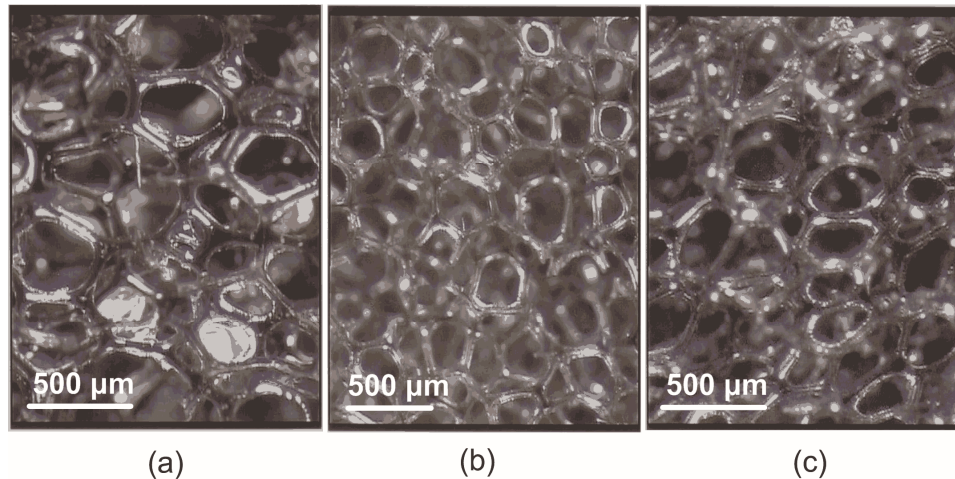


Figure 3.10. Typical micrographs of real foam samples: (a) R_1 , (b) R_2 , and (c) R_3 . The average numbers n of edges per face for each photomicrograph are as follows: (a) R_1 , $n_1 = 5.21 \pm 0.69$; (b) R_2 , $n_2 = 4.94 \pm 0.56$; (c) R_3 , $n_3 = 4.84 \pm 0.80$.

Fig. 3.10 shows typical micrographs of these real polyurethane foam samples (based on a polyester or polyether polyol), taken with the help of a binocular (Leica MZ6). Although X-ray microtomography analysis and Scanning Electron Micrography (SEM) provide a precise microstructure characterization, a stereomicroscope remains affordable for any laboratory, and enables reaching the primary objective related to the quantitative characterization of the foam cell shapes, or more simply stated, to verify that the local geometry model to be used will be compatible with the real disordered system under study. The maximum magnification is $\times 40$, with a visual field diameter of 5.3 mm. Foam samples were cut perpendicularly to the plane of the sheet. To get an idea of the cellular shape of these samples, the number of edges per face n was measured from 30 different locations for each material. Each location is associated with one photomicrograph. For each picture, the number of analyzed faces, having continuously connected edges, is ranging between 5 and 53 with an average value of 23. From these measurements follows an average number of edges per face for each foam sample: R_1 , $n_1 = 5.10 \pm 0.82$; R_2 , $n_2 = 5.04 \pm 0.68$; R_3 , $n_3 = 5.03 \pm 0.71$. Next, ligaments lengths were measured on optical micrographs of the foam samples. Since the surface contains exposed cells whose ligament lengths are to be measured on micrographs obtained by light microscopy, great care was taken during measurements to select only ligaments lying in the plane of observation. Ligament length measurements were performed on three perpendicular cross-sections of each sample. Assuming transverse isotropy of the foam samples cellularity, results of ligament length measurements were reported in Tab. 3.2 and their

distribution plotted in Fig. 3.11. Ligament thicknesses constitute also an important geometrical parameter. However, they were difficult to measure because lateral borders of the ligaments are not well defined on optical photomicrographs (due to reflections caused by thin residual membranes). Therefore, the ligament thicknesses were not primarily used.

Foams	Horizontal and vertical cross-sections	Horizontal cross-section	Vertical cross-sections
R ₁	$L_{m1HV} = 205.0 \pm 41.6$	$L_{m1H} = 192.9 \pm 43.3$	$L_{m1V} = 211.7 \pm 39.3$
R ₂	$L_{m2HV} = 229.5 \pm 57.3$	$L_{m2H} = 226.6 \pm 58.3$	$L_{m2H} = 236.5 \pm 54.5$
R ₃	$L_{m3HV} = 182.4 \pm 41.7$	$L_{m3H} = 167.5 \pm 32.1$	$L_{m3H} = 193.7 \pm 44.6$

Table 3.2. Averaged measured ligament lengths from optical photomicrographs, L_m

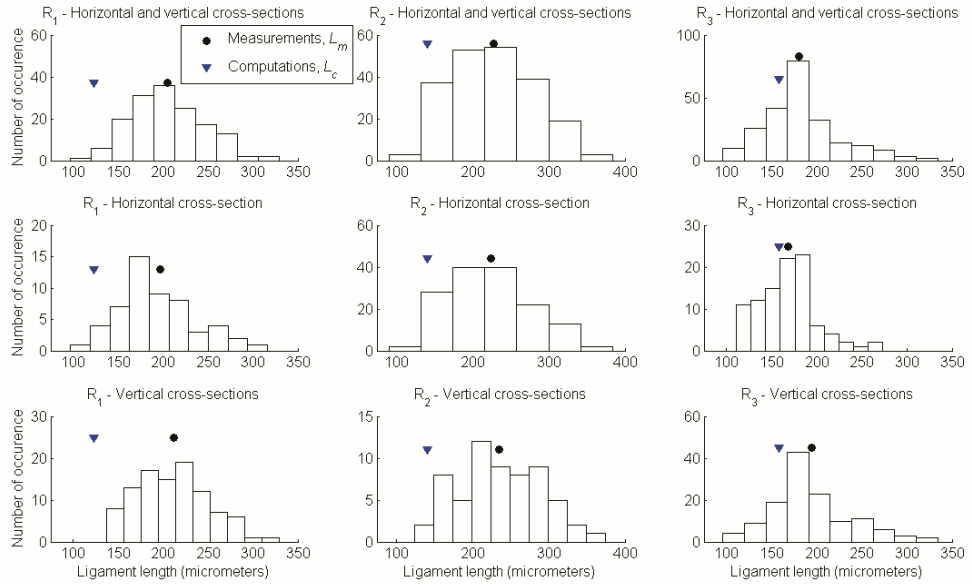


Figure 3.11. Ligament length distributions for real foam samples R₁ (left), R₂ (center), and R₃ (right). Labels (●) give the measured averaged ligament lengths L_m obtained from micrographs, whereas labels (▼) indicate the computed ligament length L_c of the truncated octahedron unit-cell used for numerical simulations.

3.2.1.2.2. Direct determination of porosity and static permeability

The porosity was non-destructively measured from the perfect gas law properties using the method described by Beranek ^[63]. It is found to range between 0.97 and 0.98: R_1 , $\phi_1 = 0.98 \pm 0.01$; R_2 , $\phi_2 = 0.97 \pm 0.01$; R_3 , $\phi_3 = 0.98 \pm 0.01$. The experimental value of the static permeability k_0 was obtained by means of accurate measurements of differential pressures across serial mounted calibrated and unknown flow resistances, with a controlled steady and non-pulsating laminar volumetric air flow as described by Stinson and Daigle ^[62] and further recommended in the corresponding standard ISO 9053 (method A). Results summarized in Tab. 3.3 are as follows: R_1 , $k_0 = 2.60 \pm 0.08 \times 10^{-9} \text{ m}^2$; R_2 , $k_0 = 2.98 \pm 0.14 \times 10^{-9} \text{ m}^2$; R_3 , $k_0 = 4.24 \pm 0.29 \times 10^{-9} \text{ m}^2$.

These measurements were performed by Matelys-AcV using equipments available at ENTPE (Lyon, France). To measure k_0 , the volumetric airflow rates passing through the test specimens have a value of $1.6 \text{ cm}^3/\text{s}$. A sample holder of circular cross-sectional area was used, with a diameter of 46 mm (which allows using the same samples for impedance tube measurements). This corresponds to a source such as there is essentially laminar unidirectional airflow entering and leaving the test specimen at values just below 1 mm/s, and for which quasi-static viscous permeability measurements are supposed to be independent of volumetric airflow velocity.

Foams	Method	ϕ (-)	Λ' (μm)	k_0 (m^2)	α_0 (-)	Λ (μm)	α_∞ (-)	k_0' (m^2)	α_0'
R_1	Computations		506		1.22	297	1.02	5.01×10^{-9}	1.13
	Measurements ^{a, b}	0.98		2.60×10^{-9}					
	Characterization ^{c, d}		440			129	1.12	8.30×10^{-9}	
R_2	Computations		477		1.26	279	1.02	5.85×10^{-9}	1.14
	Measurements ^{a, b}	0.97		2.98×10^{-9}					
	Characterization ^{c, d}		330			118	1.13	9.70×10^{-9}	
R_3	Computations		647		1.22	373	1.01	8.18×10^{-9}	1.13
	Measurements ^{a, b}	0.98		4.24×10^{-9}					
	Characterization ^{c, d}		594			226	1.06	13.10×10^{-9}	

^aReference [63].

^bReference [62].

^cReference [60].

^dReference [61].

Table 3.3. Comparison between computed and measured macroscopic parameters.

3.2.1.3. Prediction of transport properties from a three-dimensional periodic unit-cell

3.2.1.3.1. The local geometry

As observed from the micrographs, the network of ligaments appears to be similar to a lattice within which the ligaments delimit a set of polyhedra. In this work, it is therefore considered that a representation of the microstructure which can be deduced from this observation is a packing of identical polyhedra.

More precisely, truncated octahedra with ligaments of circular cross section shapes and a spherical node at their intersections were considered as in a similar work on thermal properties of foams ^[86]. It will be shown that the FEM results are not significantly affected by this approximation (see Secs. 3.2.1.3 and 3.2.1.4), even if the real cross-section of ligaments can be rather different ^[87].

A regular truncated octahedron is a 14-sided polyhedron (tetrakaidecahedron) having six squares faces and eight hexagonal faces, with ligament lengths L and thicknesses $2r$. The average number of edges per face, another polyhedron shape indicator, is equal to $(6 \times 4 + 8 \times 6) / 14 \approx 5.14$ and close to the experimental data presented in Sec. 3.2.1.2.1. The cells have a characteristic size D equal to $(2\sqrt{2})L$, between two parallel squared faces. An example of regular truncated octahedron for such packings is given in Fig. 3.12.

The simplest macroscopic parameter characterizing a porous solid is its open porosity, defined as the fraction of the interconnected pore fluid volume to the total bulk volume of the porous aggregate, ϕ . The porosity of such a packed polyhedron sample might be expressed as a function of the aspect ratio $L/2r$,

$$\phi = 1 - \left(\frac{3\sqrt{2}\pi}{16} \right) \left(\frac{2r}{L} \right)^2 - \left(\frac{\sqrt{2}\pi C_1}{16} \right) \left(\frac{2r}{L} \right)^3, \quad (3.30)$$

with $C_1 = -f^3 + 2(f^2 - 1)\sqrt{f^2 - 1}$, and f is a node size parameter related to the spherical radius R by $R = f \times r$, with $f \geq \sqrt{2}$. This last constraint on the node parameter ensures that the node volume is larger than the volume of the connecting ligaments at the node.

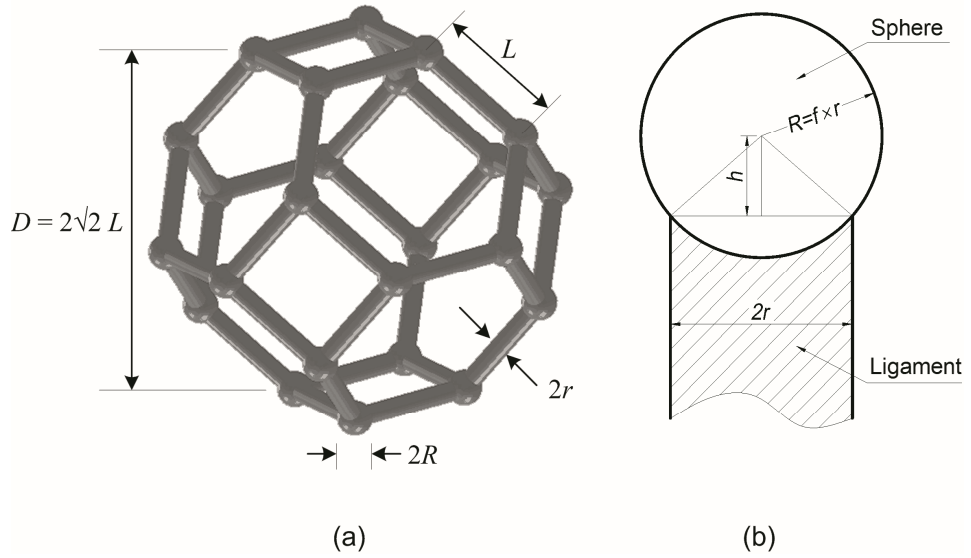


Figure 3.12. Basic 3D periodic foam model geometry: (a) a regular truncated octahedron with ligaments of circular cross-section shape (length L , radius r), and (b) spherical nodes (radius R) at their intersections. Note that f is a spherical node size parameter which is set to 1.5.

The second parameter which is widely used to characterize the macroscopic geometry of porous media, and thus polyhedron packing, is the specific surface area S_p , defined as the total solid surface area per unit volume. The hydraulic radius is defined as twice the ratio of the total pore volume to its surface area. This characteristic length may also be referred to as the “thermal characteristic length” Λ' in the context of sound absorbing materials ^[14], so that $\Lambda' = 2\phi/S_p$. As for the porosity, the “thermal characteristic length” might be expressed in terms of the microstructural parameters by

$$\Lambda' = \left[\frac{16\sqrt{2} / \left(\frac{2r}{L}\right)^3 - 6\pi / \left(\frac{2r}{L}\right) - 2\pi C_1}{3\pi \left(2 / \frac{2r}{L} + C_2\right)} \right] \times r, \quad (3.31)$$

with $C_2 = -f^2 + 2(f-1)\sqrt{f^2-1}$.

It might be useful to specify that, by definition, Eqs. (3.30) and (3.31) are valid in principle only for foams with non-elongated and fully reticulated cells.

3.2.1.3.2. Determination of the unit cell aspect ratio from porosity

When a laboratory measurement of porosity is available, the unit-cell aspect ratio $L/2r$ can be identified through Eq. (3.30). For a given value of the spherical node size parameter f , the unit-cell aspect ratio $L/2r$ is given by the solution of a cubic equation that has only one acceptable solution.

Once $2r/L$ is obtained, Eq. (3.31) gives r if a laboratory measurement of S_p is available. Then, the idealized geometry of the foam could be completely defined. The main problem in this method is that the specific surface area evaluation from non-acoustical measurements, such as the standard Brunauer, Emmett, and Teller method (BET) [51],[88] based on surface chemistry principles, is not routinely available. Moreover, the application of physical adsorption is usually recommended for determining the surface area of porous solids classified as microporous (pore size up to 2 nm) and mesoporous (pore size 2 to 50 nm). This tends to promote alternative techniques for macropore size analysis (i.e. above 50 nm width).^[89] In fact, the most widely measured parameter after the porosity to characterize the physical macro-behavior of real porous media is unarguably the static viscous (or hydraulic) permeability k_0 as defined in Sec. 3.2.1.3.3.a, a quantity having units of a surface (squared length).

Therefore, obtaining the local characteristic sizes of the PUC will be performed thereafter in four steps. Step 1 consists in acquiring the aspect ratio $L/2r$ from the porosity measurements as explained before. For a given spherical node size parameter, this produces all characteristic length ratios of the cell. At this stage, the ligament length of the cell is still unknown, but a non-dimensional PUC can be built. Step 2 is to characterize the permeability of the foam from routine measurements. Step 3 is to get the permeability of the set of non-dimensional periodic cells from first principle calculations. As explained before, the non-dimensional cell has a unit side of square faces. The Finite Element computation described thereafter implemented on the non-dimensional cell produces the non-dimensional permeability k_d . Let D_h be the side of square faces of homothetic periodic cells producing the static permeability k_0 . Then, a simple computation shows that $k_0 = D_h^2 \times k_d$. Finally, comparing the non-dimensional permeability to the true permeability produces in step 4 the size of the PUC. All other parameters are obtained from the non-dimensional results through a similar scaling.

3.2.1.3.3. First principles calculations of transport properties

Previous studies ^{[18],[58]} have shown how the long-wavelengths acoustic properties of rigid-frame porous media can be numerically determined by solving the local equations governing the asymptotic frequency-dependent visco-thermal dissipation phenomena in a periodic unit cell with the adequate boundary conditions. In the following, it is assumed that $\lambda \gg D$, where λ is the wavelength of an incident acoustic plane wave. This means that for characteristic lengths on the order of $D \sim 0.5$ mm, this assumption is valid for frequencies reaching up to a few tens of kHz. The asymptotic macroscopic properties of sound absorbing materials are computed from the numerical solutions of:

- (1) the low Reynolds number viscous flow equations (the static viscous permeability k_0 , and the static viscous tortuosity α_0);
- (2) the non-viscous flow or inertial equations (the high-frequency tortuosity α_∞ , and Johnson's velocity weighted length's parameter Λ);
- (3) the equations for thermal conduction (the static thermal permeability k_0' , and the static thermal tortuosity α_0').

(a) Viscous flow

At low frequencies or in a static regime, when $\omega \rightarrow 0$, viscous effects dominate, and the slow fluid motion in steady state regime created in the fluid phase Ω_f of a periodic porous medium having a unit cell Ω is solution of the following boundary value problem defined on Ω by ^[53]:

$$\eta \Delta \mathbf{v} - \nabla p = -\mathbf{G}, \quad \text{in } \Omega_f, \quad (3.32)$$

$$\nabla \cdot \mathbf{v} = 0, \quad \text{in } \Omega_f, \quad (3.33)$$

$$\mathbf{v} = 0, \quad \text{on } \partial\Omega, \quad (3.34)$$

$$\mathbf{v} \text{ and } p \text{ are } \Omega\text{-periodic}, \quad (3.35)$$

where $\mathbf{G} = \nabla p^m$ is a macroscopic pressure gradient acting as a source term, η is the viscosity of the fluid, and $\partial\Omega$ is the fluid-solid interface. This is a steady Stokes problem for periodic structures, where \mathbf{v} is the Ω -periodic velocity, p is the Ω -periodic part of the pressure fields in the pore verifying $\langle p \rangle = 0$, and the symbol $\langle \cdot \rangle$ indicates a fluid-phase average. It

can be shown that the components v_i of the local velocity field are given by

$$v_i = -\frac{k_{0ij}^*}{\eta} G_j. \quad (3.36)$$

The components of the static viscous permeability tensor are then specified by ^{[19],[54]}

$$k_{0ij} = \phi \langle k_{0ij}^* \rangle. \quad (3.37)$$

And the components of the tortuosity tensor are obtained from

$$\alpha_{0ij} = \langle k_{0pi}^* k_{0pj}^* \rangle / \langle k_{0ii}^* \rangle \langle k_{0jj}^* \rangle, \quad (3.38)$$

wherein the Einstein summation notation on p is implicit. In the present work, the symmetry properties of the microstructure under consideration imply that the second order tensors \mathbf{k}_0 and $\boldsymbol{\alpha}_0$ are isotropic. Thus $k_{0ij} = k_0 \delta_{ij}$ and $\alpha_{0ij} = \alpha_0 \delta_{ij}$, where δ_{ij} is the Kronecker symbol.

(b) Inertial flow

At the opposite frequency range, when ω is large enough, the viscous boundary layer becomes negligible and the fluid tends to behave as a perfect one, having no viscosity except in a boundary layer. In these conditions, the perfect incompressible fluid formally behaves according to the problem of electric conduction ^{[25],[55],[56]}, i.e. :

$$\mathbf{E} = -\nabla \varphi + \mathbf{e}, \quad \text{in } \Omega_f, \quad (3.39)$$

$$\nabla \cdot \mathbf{E} = 0, \quad \text{in } \Omega_f, \quad (3.40)$$

$$\mathbf{E} \cdot \mathbf{n} = 0, \quad \text{on } \partial\Omega, \quad (3.41)$$

$$\varphi \text{ is } \Omega\text{-periodic}, \quad (3.42)$$

where \mathbf{e} is a given macroscopic electric field, \mathbf{E} the solution of the boundary problem having $-\nabla \varphi$ as a fluctuating part, and \mathbf{n} is unit normal to the boundary of the pore region.

Then, the components $\alpha_{\infty ij}$ of the high frequency tortuosity tensor can be obtained from ^[18],

$$e_i = \alpha_{\infty ij} \langle E_j \rangle. \quad (3.43)$$

In the case of isotropy, the components of the tensor α_{∞} reduce to the diagonal form $\alpha_{\infty ij} = \alpha_{\infty} \delta_{ij}$. In this case, the tortuosity can also be obtained from the computation of the mean square value of the local velocity through:

$$\alpha_{\infty} = \frac{\langle \mathbf{E}^2 \rangle}{\langle \mathbf{E} \rangle^2}. \quad (3.44)$$

As for the low frequency tortuosity, an extended formula can be used for anisotropic porous media.

Having solved the cell conduction problem, the viscous characteristic length Λ can also be determined (for an isotropic medium) by ^[12]

$$\Lambda = 2 \frac{\int_{\Omega} \mathbf{E}^2 dV}{\int_{\partial\Omega} \mathbf{E}^2 dS}. \quad (3.45)$$

(c) Thermal effect

When the vibration occurs, the pressure fluctuation induces a temperature fluctuation inside the fluid, due to the constitutive equation of a thermally conducting fluid. If one considers the solid frame as a thermostat, it can be shown that the mean excess temperature in the air $\langle \tau \rangle$ is proportional to the mean time derivative of the pressure $\partial \langle p \rangle / \partial t$. This thermal effect is described by $\langle \tau \rangle = (k_0' / \kappa) \partial \langle p \rangle / \partial t$, where $\langle \tau \rangle$ is the macroscopic excess temperature in air, κ is the coefficient of thermal conduction, and k_0' is a constant. The constant k_0' is often referred to as the “static thermal permeability”. As the usual permeability, it has the dimensions of a surface and was thus named by Lafarge *et al.* ^[17]. It is related to the “trapping constant” Γ of the frame by $k_0' = 1/\Gamma$. ^[56] In the context of diffusion-controlled reactions, it was demonstrated by Rubinstein and Torquato ^[57] that the trapping constant is related to the mean value of a “scaled concentration field” $u(\mathbf{r})$ by

$$\Gamma = 1 / \langle u \rangle, \quad (3.46)$$

where $u(\mathbf{r})$ solves

$$\Delta u = -1, \quad \text{in } \Omega_f, \quad (3.47)$$

$$u = 0, \quad \text{on } \partial\Omega. \quad (3.48)$$

It is worthwhile noticing that Δu is dimensionless. Therefore, u and k_0' have the dimension of a surface.

Similarly to tortuosity factors obtained from viscous and inertial boundary value problems, a “static thermal tortuosity” is given by :

$$\alpha_0' = \frac{\langle u^2 \rangle}{\langle u \rangle^2}. \quad (3.49)$$

3.2.1.3.4. Dimensioning the unit cell from static permeability

The permeability k_0 obtained from a computational implementation of the low Reynolds number viscous flow equations as described in Sec. 3.2.1.3.3.a can be determined from the non-dimensional PUC. Then it is well known that for all homothetic porous structures, the permeability k_0 is proportional to the square of the hydraulic radius, which was previously renamed as “thermal characteristic length” Λ' . Thus, for an isotropic medium, a generic linear equation $k_0 = S \times \Lambda'^2 + 0$ must exist, where S is the non-dimensional slope to be numerically determined. At a fixed porosity, S depends only on the morphology of the unit cell and not on the size of the cell.

As a consequence, knowing k_0 from experimental measurements and S from computations on the non-dimensional structure produces the specific thermal length Λ' , and $D_h = \Lambda' \sqrt{S/k_d}$. Making use of Eqs. (3.30) and (3.31), local characteristic lengths L and r follow. Hence, there are *a priori* two routinely available independent measurements to be carried out in order to define the foam geometry: the porosity ϕ and the static viscous permeability k_0 . This method for periodic unit-cell reconstruction circumvents the necessary measure of the specific surface area. As previously mentioned, all this procedure assumes that the spherical node size parameter f is known. In our computations, f was set to 1.5. This value respects the constraint $f \geq \sqrt{2}$ and is in a rather good agreement with

microstructural observations, considering the absence of lump at the intersection between ligaments (see Fig. 3.10). Application of the above procedure yields the local characteristic sizes of a unit cell ligament for each foam sample: R_1 , $L_1 = 123 \pm 13 \mu\text{m}$ ($L_{m1} = 205 \pm 42 \mu\text{m}$), $2r_1 = 19 \pm 7 \mu\text{m}$ ($2r_{m1} = 31 \pm 7 \mu\text{m}$); R_2 , $L_2 = 141 \pm 12 \mu\text{m}$ ($L_{m2} = 229 \pm 57 \mu\text{m}$), $2r_2 = 27 \pm 7 \mu\text{m}$ ($2r_{m2} = 36 \pm 8 \mu\text{m}$); R_3 , $L_3 = 157 \pm 19 \mu\text{m}$ ($L_{m3} = 182 \pm 42 \mu\text{m}$), $2r_3 = 25 \pm 10 \mu\text{m}$ ($2r_{m3} = 30 \pm 6 \mu\text{m}$). Comparison between computed and measured characteristic sizes estimations are thoroughly discussed in Secs. 3.2.1.5 and 3.2.1.6; see also Appendix 3.2.1.C.

Uncertainties for the critical characteristic sizes of the PUC correspond to the standard deviations computed when considering input macroscopic parameters ϕ and k_0 associated with their experimental uncertainties. This enables evaluating the impact of porosity and permeability measurement uncertainties on the estimation of local characteristic lengths.

Note that, for anisotropic medium, k_0 varies with the direction of the airflow inside the foam (see for example the flow resistivity tensors presented in Ref. [90]) and the equation $k_0 = S \times \Lambda'^2$ is no more valid. Thus, the size of the PUC depends on the direction of the airflow used during the static permeability measurements. To be more complete, k_0 should be measured along three directions, leading to three pairs of critical lengths to estimate the possible anisotropy. This issue will be addressed in a forthcoming paper.

3.2.1.3.5. Results on asymptotic transport properties obtained from finite element modeling

An example of calculated viscous flow velocity, inertial flow velocity, and scaled concentration fields obtained through a Finite Element mesh is shown in Fig. 3.13 for foam sample R_1 . The number of elements and their distribution in the fluid phase regions of the PUC were varied, with attention paid especially to the throat and the near-wall areas, to examine the accuracy and convergence of the field solutions. The symmetry properties of the permeability/tortuosity tensors were also checked [34], as a supplementary test on convergence achievement. As previously noticed by several authors, such as Martys and Garboczi [91], due to the slip condition, the fluid flow paths are more homogeneous for the electric-current paths than for the viscous fluid flow.

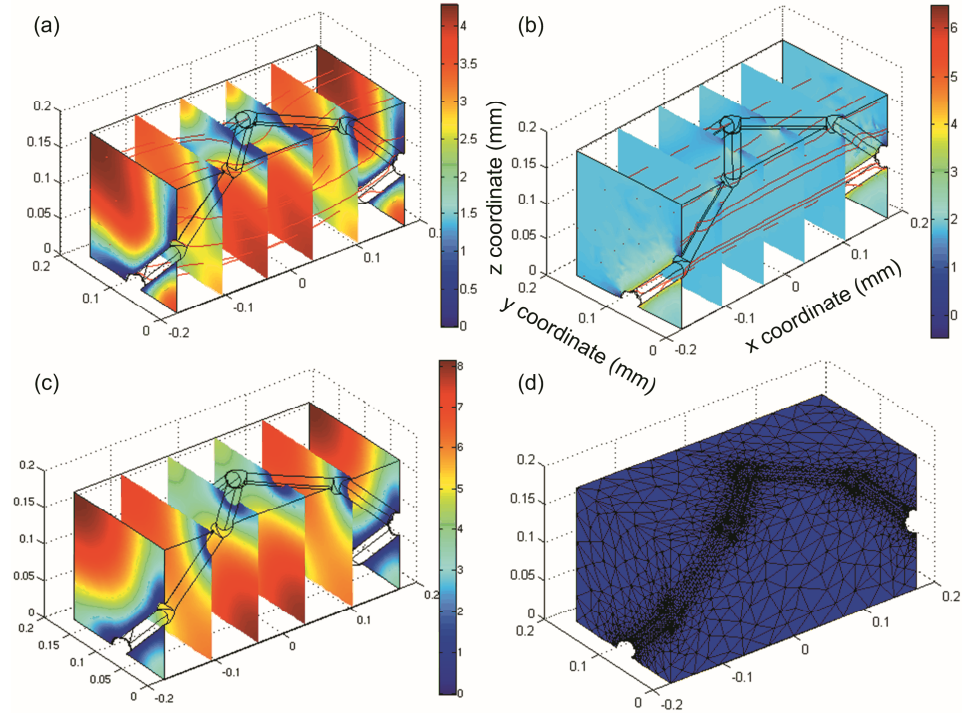


Figure 3.13. Asymptotic fields for 1/4th of the reconstructed foam sample period R_1 : (a) low-frequency scaled velocity field $k_{0,xx}^*$ [$\times 10^{-9} \text{ m}^2$], (b) high-frequency scaled velocity field $E_x / \nabla \phi$ [-] for an external unit field e_x , (c) low-frequency scaled temperature field k_0' [$\times 10^{-9} \text{ m}^2$], and (d) corresponding mesh domain with 41 372 lagrangian P_2P_1 tetrahedral elements.

Direct numerical computations of the complete set of macroscopic parameters were performed in reconstructed unit cells from adequate asymptotic field averaging as described in Secs. 3.2.1.3.3 a-c. Results are reported in Tab. 3.3. Some values are compared to estimations obtained from impedance tube measurements, see Sec. 3.2.1.5.1.

We also note that our results are consistent with the inequalities $\alpha_0 > \alpha_\infty$ and $\alpha_0 / \alpha_\infty \geq \alpha_0' > 1$, as introduced by Lafarge^[18] from physical reasons.

3.2.1.4. Estimates of the frequency dependent visco-inertial and thermal responses by a hybrid numerical approach

The acoustic response of foams depends on dynamic viscous permeability and “dynamic thermal permeability”. Both of these parameters could be obtained from dynamic FEM computations as in Ref. [27]. The approach presented here relies on the fact that the finite element computations presented previously are easy to implement and provide the asymptotic behavior for both dynamic “permeabilities”. This asymptotic behavior constitutes the input data for the models which are used for predicting the

full frequency range of the dynamic “permeabilities”. Therefore the hybrid approach employed in our study makes use of the asymptotic parameters of the porous medium obtained by finite elements. Then, it will be possible to provide the dynamic permeabilities and to compare these values to experimental ones. In a first step, the three different models which are used to build the dynamic permeabilities from asymptotic parameters are briefly recalled.

Johnson *et al.* [12] and, later, Pride *et al.* [15] considered the problem of the response of a simple fluid moving through a rigid porous medium and subjected to a time harmonic pressure variation across the sample. In such systems they constructed simple models of the relevant response functions, the effective dynamic viscous permeability $\tilde{k}(\omega)$ or effective dynamic tortuosity $\tilde{\alpha}(\omega)$. The main ingredient to build these models is to account for the causality principle and therefore for the Kramers-Kronig relations between real and imaginary parts of the frequency-dependent permeability. The parameters in these models are those which correctly match the frequency dependence of the first one or two leading terms on the exact results for the high- and low- frequency viscous and inertial behaviors.

Champoux and Allard [14],[46] and thereafter Lafarge *et al.* [17],[18],[58], in adopting these ideas to thermally conducting fluids in porous media, derived similar relations for the frequency dependence of the so-called effective “dynamic thermal permeability” $\tilde{k}'(\omega)$ or effective dynamic compressibility $\tilde{\beta}(\omega)$, which varies from the isothermal to the adiabatic value when frequency increases. The model for effective dynamic permeabilities were shown to agree with those calculated directly or independently measured. An important feature of this theory is that all of the parameters in the models can be calculated independently, most of them being in addition directly measurable in non acoustical experimental situations. In this regard, these models are very attractive because they avoid computing the solution of the full frequency range values of the effective permeabilities/susceptibilities. These models are recalled in Appendix 3.2.1.B. They are based on simple analytic expressions in terms of well defined high- and low- frequency transport parameters which can be determined from first principles calculations [Secs. 3.2.1.3.3 a-c].

Such a hybrid approach was used by Perrot, Chevillotte and Panneton in order to examine micro-/macro relations linking local geometry parameters to sound absorption properties for a two-dimensional hexagonal structure of solid fibers (Ref. [28]). Here, this method is completed by the use of easily obtained parameter (porosity ϕ and static

viscous permeability k_0) of real foam samples, as explained previously and by utilizing three-dimensional numerical computations.

As explicated, the comparison between non dimensional permeability obtained from finite element results and the measured permeability provides the thermal characteristic length Λ' ; and five remaining input parameters for the models, α_0 , α_∞ , Λ , k_0' , and α_0' can be obtained by means of first-principles calculations by appropriate field-averaging in the PUC.

Finally, we considered the predictions of the three models for the effective dynamic permeabilities, described in Appendix 3.2.1.B. In summary, the Johnson-Champoux-Allard” [JCA] model which uses the 5 parameters (ϕ , k_0 , α_∞ , Λ , Λ'), Johnson-Champoux-Allard-Lafarge” model [JCAL] which uses in addition k_0' , and Johnson-Champoux-Allard-Pride-Lafarge” [JCAPL] model which uses the full set of parameters (ϕ , k_0 , k_0' , α_∞ , Λ , Λ' , α_0 , and α_0').

3.2.1.5. Assessment of the methodology through experimental results

3.2.1.5.1. Experimental results and comparison with numerical results

Experimental values of the frequency dependent visco-inertial and thermal responses were provided using the impedance tube technique by Utsuno *et al.* [59], in which the equivalent complex and frequency-dependent characteristic impedance $\tilde{Z}_{eq}(\omega)$ and wave number $\tilde{q}_{eq}(\omega)$ of each material were measured, and the equivalent dynamic viscous permeability $\tilde{k}_{eq}(\omega) = \tilde{k}(\omega)/\phi$, the equivalent dynamic thermal permeability $k_{eq}'(\omega) = k'(\omega)/\phi$, and the sound absorption coefficient at normal incidence $A_n(\omega)$, derived from $\tilde{Z}_{eq}(\omega)$ and $\tilde{q}_{eq}(\omega)$.

One main objective of this section is to produce a comparison between hydraulic and thermal permeabilities coming from experimental results and from numerical computations. In this context, intermediate results were obtained for the acoustic parameters of the JCAL model through the characterization method described in Ref. [60] for viscous dissipation and Ref. [61] for thermal dissipation. This kind of characterization also provides the viscous (respectively thermal) transition frequencies between viscous and inertial regimes (respectively isothermal and adiabatic), $f_v = \nu\phi/2\pi k_0\alpha_\infty$ ($f_t = \nu'\phi/2\pi k_0'$). These results will be thereafter referenced to on the figures and tables as obtained from “characterization”.

Figures 3.14, 3.15 and 3.16 produce the sound absorption coefficient simultaneously with the estimation of hydraulic and thermal permeability obtained from experiment, from characterization and from numerical computations. Because all viscous and thermal shape factors recalled in Appendix 3.2.1.B significantly diverge from unity, large deviations are noticeable between JCA, JCAL, and JCAPL semi-phenomenological models. This tends to promote JCAL and JCAPL as the models to be numerically used for the real polymeric foam samples under study. Characterized values for M' thermal shape factors are on the order of 0.35, 0.73, and 0.30, respectively for foam samples R_1 , R_2 , and R_3 . Computed values are of similar magnitude: 0.16, 0.21, and 0.16. For real foam samples R_1 and R_3 , the ratio between characterized and computed thermal shape factors is around 2, whereas for foam sample R_2 it reaches approximately 3. Because we largely overestimated the thermal length for foam sample R_2 (that exhibits anisotropy, see next sections), the later overestimate is amplified through the square involved in M' computation (Appendix 3.2.1.B). We note that significant deviations from unity of the thermal shape factors characterized for real porous materials were already observed in the literature, for instance for the glass wool ($M' = 1.34$) and rock wool ($M' = 2.84$) samples studied in Ref. [61], Table 2. The large deviations from unity for the thermal shape factors reveal the striking importance of k'_0 parameter in the accurate description of the frequency-dependent thermal dissipation effects (see also Figs. 9 and 10 of the previously mentioned reference). Once again, at some computed shape factors P and P' well below one (0.29 - 0.4), the effect of α_0 and α'_0 is strong and large frequency range. One might therefore expect a frequency-dependant acoustical macro-behavior with the JCAPL model for the three real foam samples under study very distinct from the one described by the JCAL model. Instead, the computed values of $\tilde{k}(\omega)$ and $A_n(\omega)$ are of different magnitudes, especially around and after the viscous transition frequencies (since the low frequency behavior of $\tilde{k}(\omega)$ is essentially governed by k_0). Despite the simplicity of the local geometry model used to study the multi-scale acoustic properties of real foam samples predominantly open-cell, there is a relatively good agreement between computed (present microstructural method), measured (impedance tube measurements), and characterized dynamic quantities: $\tilde{k}_{eq}(\omega)$, $\tilde{k}'_{eq}(\omega)$, and $A_n(\omega)$. Furthermore, the general trend given in term of normal incidence sound absorption coefficient by our microstructural approach appears as being particularly relevant, if we notice that it requires only ϕ and k_0 as input parameters, and proceeds without any adjustable parameter.

Discrepancies between measured and computed sound absorption coefficient at normal incidence can be primarily explained from the comparison of a set of parameters obtained from numerical results and from the characterization method reported in Tab. 3.3; namely Λ' , Λ , α_∞ , and k_0' . Note, however, that this comparison is limited by the fact that the characterization method is JCAL model-dependent. From that comparison, it can be seen that:

- α_∞ is slightly underestimated by the numerical results;
- Λ' is slightly overestimated by the numerical results for R_1 and R_3 , but overestimated by around 44% for R_2 ;
- Λ is overestimated by a factor between 1.6 and 2.4;
- k_0' is underestimated (around 40% for all results).

Considering primarily visco-inertial dissipation phenomena, the most significant difference is the large overestimation provided for Λ . This means that, at high frequencies, the windows size of the local geometry model, which respectively plays the role of weighting the velocity field for Λ and rapid section changing for α_∞ by their small openings (the squares in the case of a truncated octahedron unit-cell), is presumably overestimated by a monodisperse, isotropic, and membrane-free local geometry model. Consequently, an improvement of the local geometry model would result in the introduction of a second set of characteristic sizes.

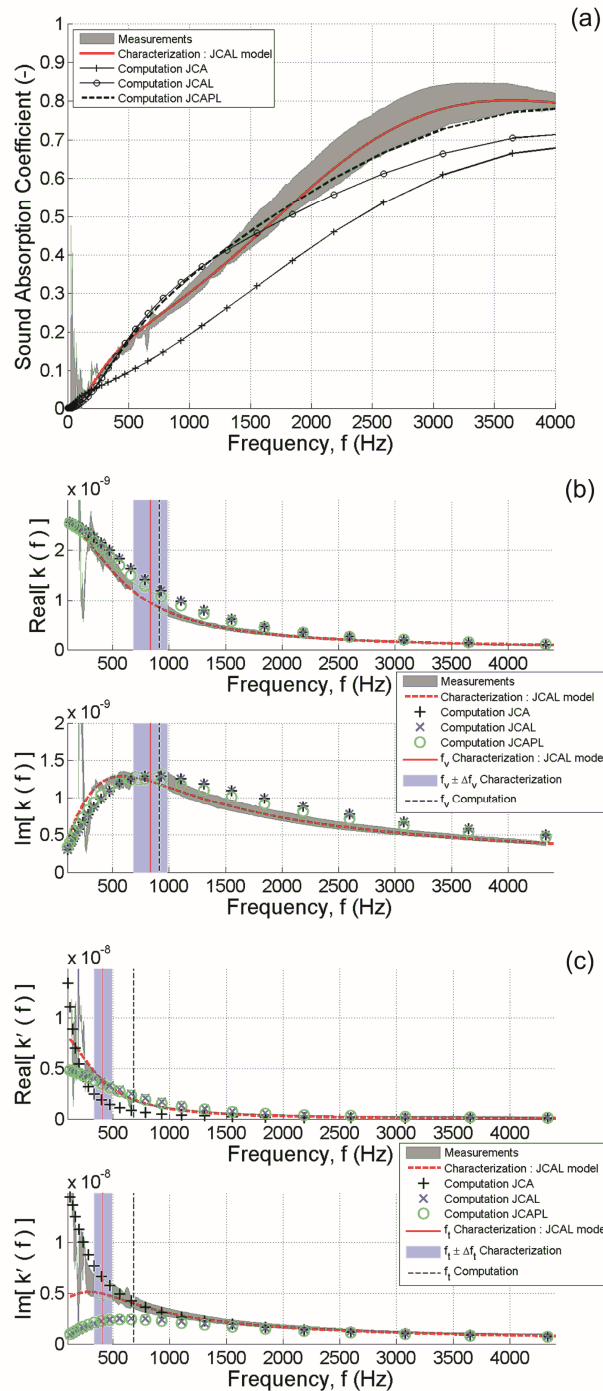


Figure 3.14. (a) Normal incidence sound absorption coefficient, (b) dynamic viscous permeability $k(f)$, and (c) dynamic thermal permeability $k'(f)$ for foam sample R_1 : Comparison between measurements (Ref. [59]), characterization (Ref. [60] and [61]) combined with JCAL model described in Appendix 3.2.1.B), and computations (this work). The errors of the characterizations of the transition frequencies Δf_v and Δf_t follow from the errors of the measurements of ρ_0 , ϕ , k_0 and from the errors of the characterizations of α_∞ and k_0' , through Gauss' law of error propagation. Sample thickness: 25 mm.

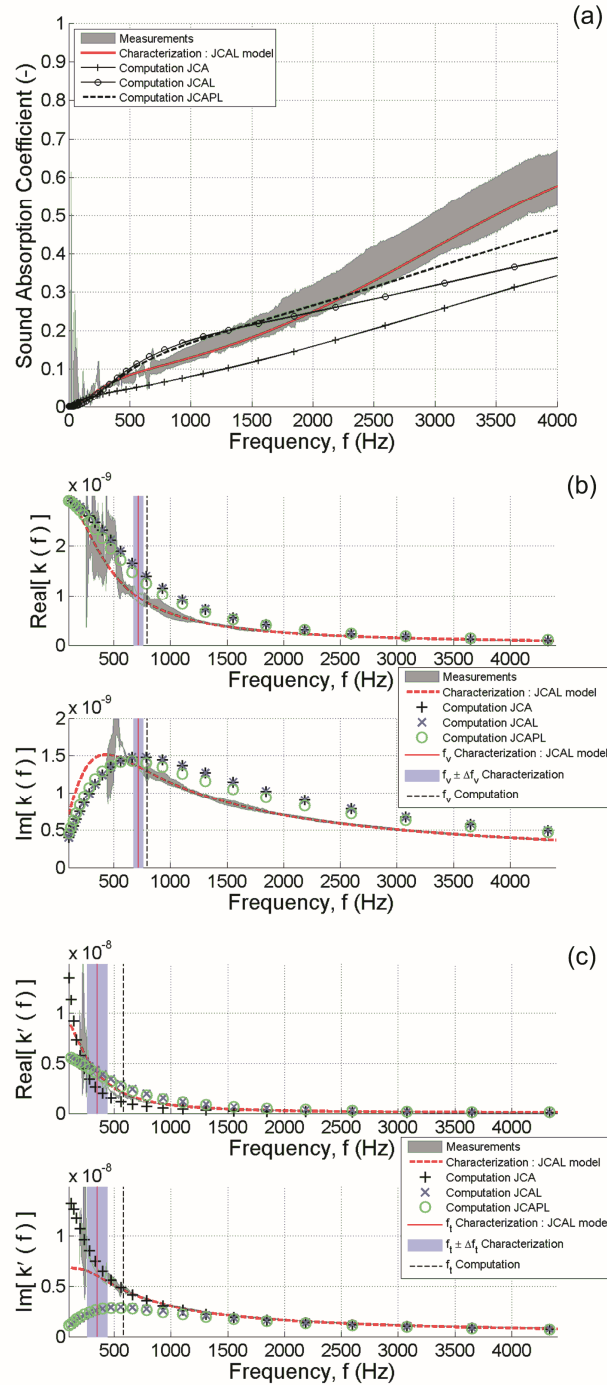


Figure 3.15. (a) Normal incidence sound absorption coefficient, (b) dynamic viscous permeability $k(f)$, and (c) dynamic thermal permeability $k'(f)$ for foam sample R_2 : Comparison between measurements (Ref. [59]), characterization (Ref. [60] and [61] combined with JCAL model described in Appendix 3.2.1.B), and computations (this work). The errors of the characterizations of the transition frequencies Δf_v and Δf_t follow from the errors of the measurements of ρ_0 , ϕ , k_0 and from the errors of the characterizations of α_∞ and k_0' , through Gauss' law of error propagation. Sample thickness: 15 mm.

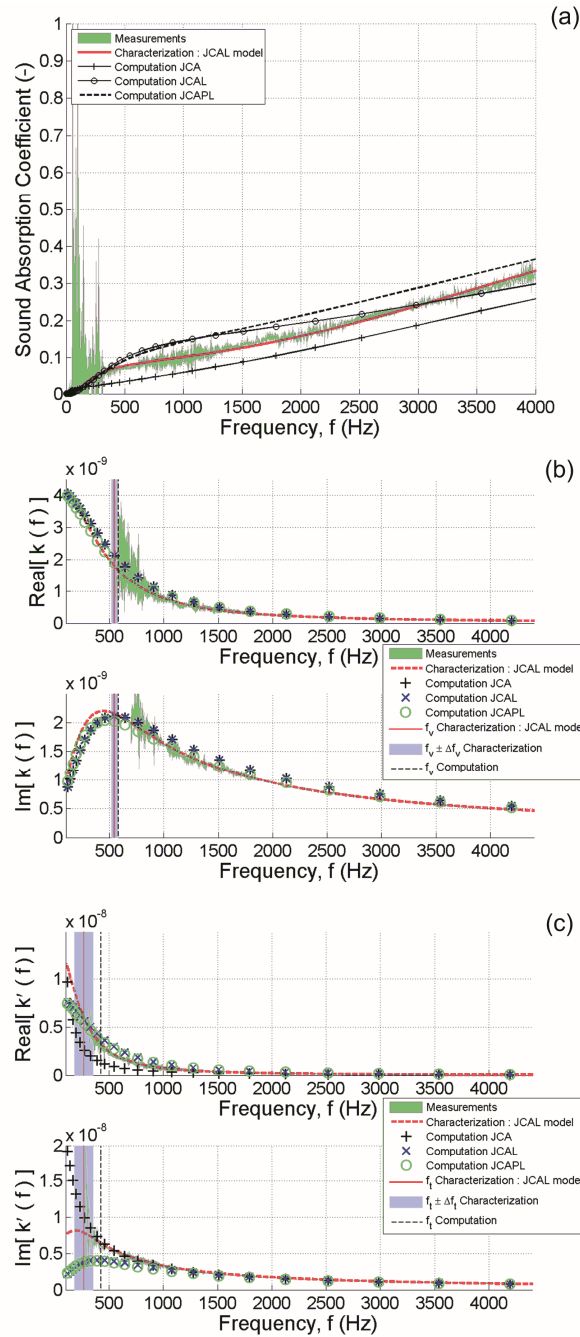


Figure 3.16. (a) Normal incidence sound absorption coefficient, (b) dynamic viscous permeability $k(f)$, and (c) dynamic thermal permeability $k'(f)$ for foam sample R_3 : Comparison between measurements (Ref. [59]), characterization (Ref. [60] and [61] combined with JCAL model described in Appendix 3.2.1.B), and computations (this work). The errors of the characterizations of the transition frequencies Δf_v and Δf_t follow from the errors of the measurements of ρ_0 , ϕ , k_0 and from the errors of the characterizations of α_∞ and k_0' , through Gauss' law of error propagation. Sample thickness: 15 mm.

A local geometry model having ligaments with concave triangular cross-section shapes and a fillet at the cusps was also implemented (not detailed here). For circular cross-section shapes, the deviations between computed and characterized thermal lengths are on the order of 15%, 44%, and 9%; for foam samples R_1 , R_2 , and R_3 , respectively (Tab. 3.3). It is also worth to mention that taking into account the inner concave triangular nature of the ligament cross-section shapes reduces discrepancies between computed and characterized thermal lengths, since the relative differences were found to decrease to 3%, 25%, and 8%, respectively. The erroneous underestimation of the $2r/L$ ratio introduced by the circular cross-section shape model does not exceed 10%.

Λ' large overestimation for R_2 might be due to the cell elongation of the real foam sample (see next section for cell elongation evidences). Indeed, from a purely geometrical point of view, it can be shown by using an elongated tetrakaidecahedron unit cell model ^[92], that a cell elongation of the tetrakaidecahedron may be obtained without modification of the ligaments lengths and thicknesses if there is an increase of the inclination angle θ (which defines the orientation of the hexagonal faces with respect to the rise direction as well as the obtuse angle of the vertical diamond faces, 2θ). By doing so, one can analytically derive a monotonic decreasing thermal length Λ' with increasing degree of anisotropy DA. For instance, $\Lambda' = 350 \mu\text{m}$ with $DA = 1.79$.

It is further fruitful for our purpose to think about the implications of a thermal reticulation process on the cellular morphology of real foam samples. During the thermal reticulation process, a high temperature, high speed flame front removes most of the cell membranes from the foam. This process melts the cell membranes, occurs as the membranes have a high surface area to mass ratio, and fuses them around the main cell ligaments. Consequently, membranes associated to large windows are predominantly depolymerised, and membranes attached to the smallest windows tend to be maintained. As a result, even apparently membrane-free foam samples conserve very small apertures around the smallest windows. This could explain why the open cell PUC generates an overestimation of the viscous length (by around 65%) for foam sample R_3 .

The purpose of the following is to examine more thoroughly the microstructure in order to provide some means aimed at improving the methodology.

3.2.1.5.2. Keys for further improvements of the methodology

A supplementary visual cell inspection is given by electron micrographs at very low magnification, as presented in Fig. 3.17. These pictures were obtained with a Environmental Scanning Electron Microscope (ESEM), S-3000N HITACHI, using an accelerating voltage of 5 or 15 kV, available at Université de Sherbrooke. The characteristic ligament length L_c obtained for the periodic cell is reported on the micrographs, which allows a first visual comparison between observed and computed cell size.

Another element of discussion is provided in Fig. 3.11 where the distribution of the measured ligament lengths is reported (together with its mean value L_m), simultaneously with the length L_c obtained from the numerical results and from the calibration coming from (k_0, ϕ) .

The characteristic ligament length L_c of the local geometry model provides a basis for understanding the influence of certain local geometry features, such as membrane effects and cell anisotropy, on the static viscous permeability of the real foam samples – in connection with ligaments length distribution.

More precisely, if the distribution of the ligament lengths is sharply peaked, one would expect the overall system behavior to be similar to that of the individual elements. This is a configuration close to the one observed for foam sample R_3 , where only isolated residual membranes (thermal reticulation process) and no specific cell elongation were observed, as illustrated on the electron micrograph in Fig. 3.17(c); and for which the distribution of the ligaments length combining horizontal and vertical surfaces is relatively sharp (see Fig. 3.11 top right). As a result, the ligaments length of the local geometry model for foam sample R_3 is (actually lower and) relatively close to the averaged value measured on the micrographs, especially for the horizontal surface through which permeability measurements were performed ($L_c = 158 \mu\text{m}$, $L_{m3H} = 167 \mu\text{m}$, and $L_{m3H} / L_c = 1.06$).

On the other hand, if the distribution is broader as shown for foam sample R_2 in Fig. 3.11 (top center), because of cell elongation as it can be seen on Fig. 3.17(b), the critical path - made by the small windows at the openings of the cells - is expected to dominate (in Fig. 3.11, for the horizontal surface $L_{m2H} = 227 \mu\text{m}$, whereas $L_c = 141 \mu\text{m}$, and L_{m2H} / L_c is now equal to 1.61).

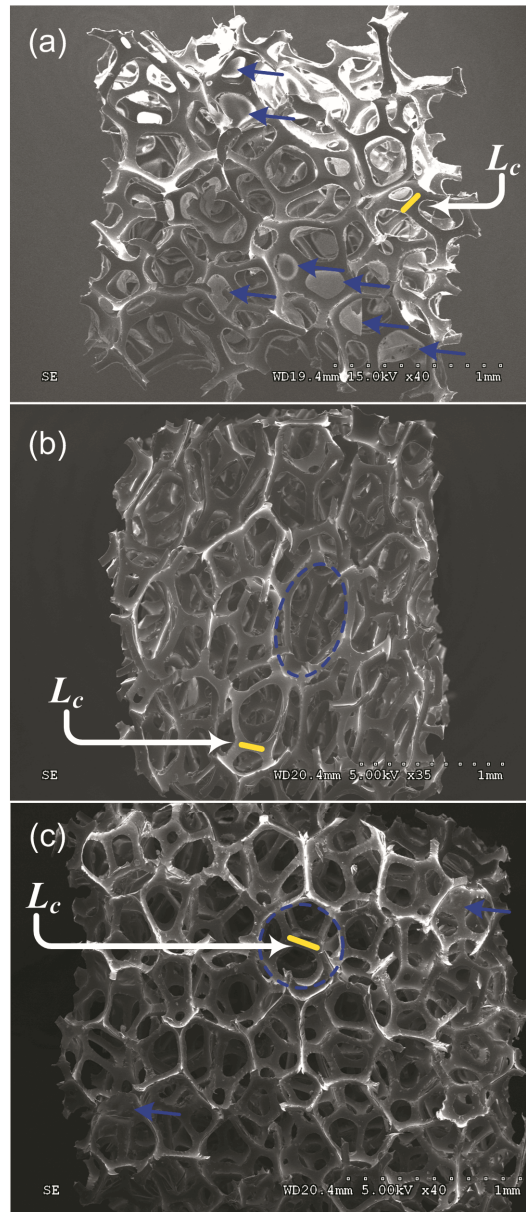


Figure 3.17. Typical scanning electron microscope images of real foam samples. (a) R_1 , showing a relatively great number of membranes (indicated by arrows) compared to R_2 and R_3 foams. (b) R_2 , having a degree of anisotropy equal to 1.75 as illustrated with a superimposed ellipse. (c) R_3 , exhibits only few isolated residual membranes (thermal reticulation process), with rather spherical pore shapes (schematically represented by a circle). For each real foam sample, a line corresponding to the specific length L_c clearly shows the typical size of an opening which could participate to a critical path.

Similarly, as observed for foam sample R_1 in Fig. 3.17(a), the presence of membranes occludes or significantly reduces the size of some windows which might belong to unit-cells in the class of local permeability

sites k_{ij} (in the sense of critical path considerations, see Appendix 3.2.1.A) much greater, or of the order of k_c . This has in addition the effect of disconnecting some critical subnetworks. In this later case, the unit-cells which were belonging to the permeability sites with $k_{ij} \leq k_c$, may now significantly contribute by participating in a new critical subnetwork, lowering drastically k_c (in Fig. 3.11, for the horizontal surface, $L_{m1H} = 193 \mu\text{m}$, whereas $L_c = 123 \mu\text{m}$, and L_{m1H} / L_c gives 1.57).

As explained before, reporting the value of L_c on the electron micrograph of Fig. 3.17 can illustrate what is the typical size of a critical path opening. It is also worth mentioning that L_c and $D_c = (2\sqrt{2})L_c$ provide a rather reliable rough estimate of the characterized values for Λ and Λ' respectively, see Tab. 3.4. This tends to confirm the customarily assumed idea that the small openings (windows) and the pore itself (cell) are respectively associated to viscous and thermal dissipation effects. What could be the consequences of isotropy and fully-reticulated cells assumptions related to Eqs. (3.30) and (3.31) in the determination of the PUC sizes ? (1) An elongation of a fully reticulated unit cell (obtained by an increase of the inclination angle θ) would presumably not significantly modify the critical sizes in the longitudinal direction, and accordingly nor the above mentioned characterized viscous and thermal length rough estimates (only a slight reduction in the thermal length is anticipated - see Sec. 3.2.1.5.1). But a permeability reduction, to be characterized (see Sec. 3.2.1.3.4), might be anticipated in the transverse direction. (2) Ignoring membranes results in a significant artificial reduction of both r_c and L_c compared to the PUC sizes that would be obtained for an isotropic unit cell with non fully reticulated membranes (R_1 case). In this last situation, it seems reasonable to infer the following rules of thumbs: $\Lambda \sim L_c - 2(r_c + \delta)$, where δ is taken as a typical membrane size; and $\Lambda' \leq 2(L_c\sqrt{2} - r_c)$, where the inequality would tends to a strict equality for $\delta \rightarrow 0$.

Characteristic lengths	Method	R ₁	R ₂	R ₃
L_c (μm)		123	141	158
$D_c = (2\sqrt{2})L_c$ (μm)		348	399	447
Λ (μm)	Computations	297 (-59%)	279 (-49%)	373 (-58%)
	Characterization ^a	129 (-5%)	118 (+19%)	226 (-30%)
Λ' (μm)	Computations	506 (-31%)	477 (-16%)	647 (-31%)
	Characterization ^b	440 (-21%)	330 (+21%)	594 (-25%)

^aReference [60].

^bReference [61].

Table 3.4. Local characteristic lengths L_c and D_c of the reconstructed idealized unit cells compared to macroscopic viscous and thermal characteristic lengths Λ and Λ' for the three polyurethane foam samples R₁, R₂, and R₃. Parentheses indicate the relative difference when L_c is compared to Λ , and D_c is compared to Λ' .

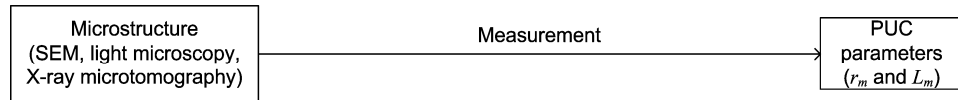
3.2.1.6. Additional justification and validation of the proposed method

What could be the microstructural characteristic lengths governing the long-wavelengths acoustic properties of real motionless foam samples? This is a question dominating the studies on the microphysical basis behind transport phenomena we addressed from critical path considerations in the present paper. In other words, why should we use the new method presented in Fig. 3.18(b) compared to the one presented in Fig. 3.18(a)? And can we really base our understanding of the foam acoustic behavior on the L_c parameter? To answer these questions and thus convince the reader to use the method presented here, a conceptual and practical justification is given, and an analysis of the uncertainties associated to L_c determination is then provided.

3.2.1.6.1. Conceptual and practical justification

The characteristic lengths governing transport and acoustic properties of real foam samples depend on the distributions of pore and window sizes. Although they might be determined from the average value of numerous cells captured with microtomography ^[38] (Fig. 3.18(a)), this would be justified only in the specific case of sharply peaked distributions ^[21] (when the averaged and critical lengths coincide, Fig. 3.11 R₃). Furthermore, even if the pore and window size distributions of the real porous system to be analyzed are sharply peaked, the approach presented in this paper for the analysis of transport and acoustic properties in real porous media allows circumventing microtomography techniques, which remain not commonly available and time consuming. Our work was inspired by critical-path ideas borrowed from statistical physics. ^[93] For instance, critical path considerations suggest that viscous fluid transport in a real system of polyhedral open cells with a broad distribution of ligament lengths is dominated by those polyhedral cells of permeabilities greater than some critical value k_c and thus by their corresponding critical ligament length L_c . The critical permeability k_c represents the largest permeability, such that the set of permeabilities $\{k|k>k_c\}$ still forms an infinite, connected cluster. Hence, viscous transport in such a system reduces to a critical path problem with threshold value k_c . We thus interpreted viscous transport within foam pore spaces in terms of these critical path ideas in order to identify what could be a basic ingredient to the microstructural key linkages governing the long-wavelengths acoustic properties of real motionless foam samples (necessary but not sufficient, see Sec. 3.2.1.5). Since the local viscous permeability is a function of the ligament length L , the threshold permeability k_c defines a critical length L_c , which is a length that was

(a) Method presented in Ref. 23



(b) Method presented in this paper

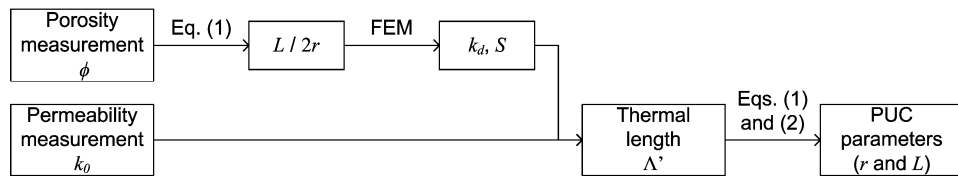


Figure 3.18. Schematic comparison between two different methods leading to the Periodic Unit Cell parameters (PUC) expressed as: (a) the average ligament lengths L_m and thicknesses $2r_m$; or, alternatively, (b) the ligament lengths L and thicknesses $2r$ governing the permeability k_0 of the real foam sample under study and interpreted in terms of critical characteristic lengths.

identified from measurements of the viscous permeability k_0 over a real foam sample. Moreover, the length that marks the permeability threshold in the critical viscous permeability problem also defines the threshold in the experimental viscous permeability case; see Appendix 3.2.1.A. This means that, in general, L_c for the viscous permeability is different from the averaged ligament lengths L_m . In other words, the very long ligaments have an excessive weight in the computation of the predicted permeability, except if the distribution of the ligament lengths is sharply peaked, Fig. 3.11 R₃. This property is quantitatively illustrated below, Sec. 3.2.1.6.2. We derived some general results concerning the relationship between experimental permeability k_0 and critical ligament length L_c by specifying the function $k_0 = f(L_c)$ for a given polyhedral shape. These relationships hold as long as the cellular shape of the local geometry model is compatible with real foam samples.

3.2.1.6.2. Quantitative validation through uncertainty analysis

To confirm further the correspondence between experiment and modeling, we tested the prediction that computation of the normal incidence sound absorbing behavior with the average ligament length L_m and thickness $2r_m$ as direct input parameters for the local geometry model should diminish the agreement (Figs. 3.19 and 3.20), except for a real foam sample exhibiting a rather sharply peaked ligament length distribution with isolated membranes and anisotropy (Fig. 3.21). Only in this last case, using directly measured

ligament lengths $L_m = 157 (\pm 19) \mu\text{m}$ and thicknesses $2r_m = 25 (\pm 10) \mu\text{m}$ as input parameters to the local geometry model increases the agreement without any adjustable parameter; see Appendix 3.2.1.C for the measurement procedure of the ligament thicknesses. Remark that in the computations, JCAPL model was used. Adding to the model experimental uncertainties for k_0 and ϕ helps improving the correspondence between experiments and modeling: $R_1 (\phi^-, k_0^-)$, $R_2 (\phi^-, k_0^-)$, $R_3 (\phi^+, k_0^+)$; where subscripts - and + are used to designate the lower and upper bounds of related quantities with respect to some experimental uncertainties (0.01 for porosity, 10 % of the mean measured value for permeability). Introducing concave triangular cross-section shapes with a fillet at the cusps instead of circular cross-section shapes in the model does not significantly modify the overall acoustical macro-behavior. This, and the results above justify and validate the proposed method and indicate that it captures the essential physics of the asymptotic low-frequency fluid-structure interactions in a real foam sample.

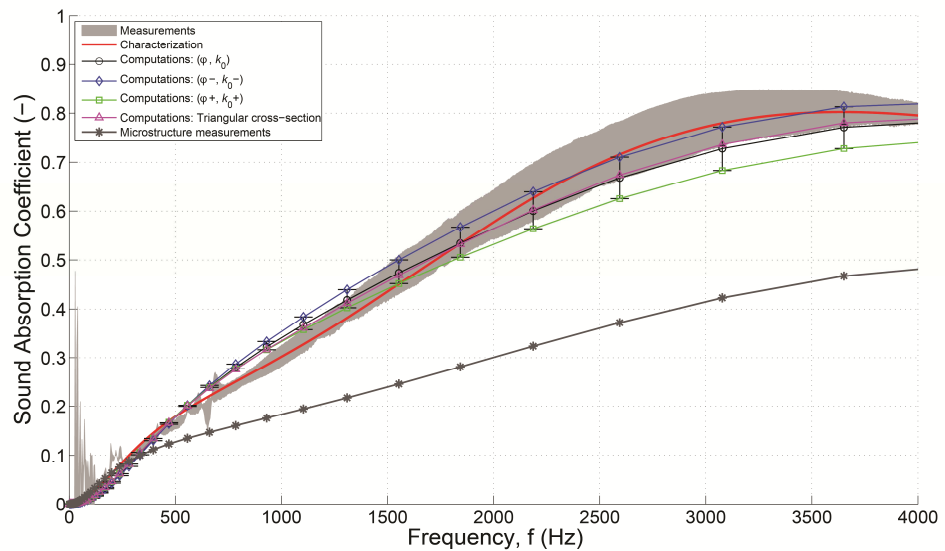


Figure 3.19. Normal incidence sound absorption coefficient for foam sample R_1 . Comparison between measurements (Ref. [59]), characterization (Ref. [60] and [61] combined with models described in Appendix 3.2.1.B), and computations (this work). Adding to the model experimental uncertainties for k_0 and ϕ helps improving the correspondence between experiments and modeling. Direct microstructure measurements are also used as input parameters of the three-dimensional local geometry model. The triangular cross-section shapes local model does not significantly modify the overall sound absorbing behavior. Sample thickness: 25 mm.

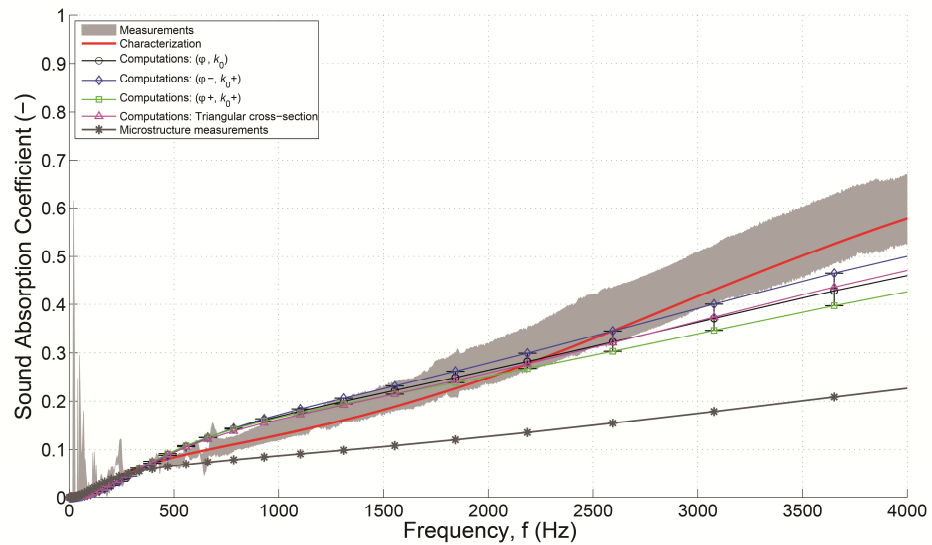


Figure 3.20. Normal incidence sound absorption coefficient for foam sample R_2 . Comparison between measurements (Ref. [59]), characterization (Ref. [60] and [61] combined with models described in Appendix 3.2.1.B), and computations (this work). Adding to the model experimental uncertainties for k_0 and ϕ helps improving the correspondence between experiments and modeling. Direct microstructure measurements are also used as input parameters of the three-dimensional local geometry model. The triangular cross-section shapes local model does not significantly modify the overall sound absorbing behavior. Sample thickness: 15 mm.

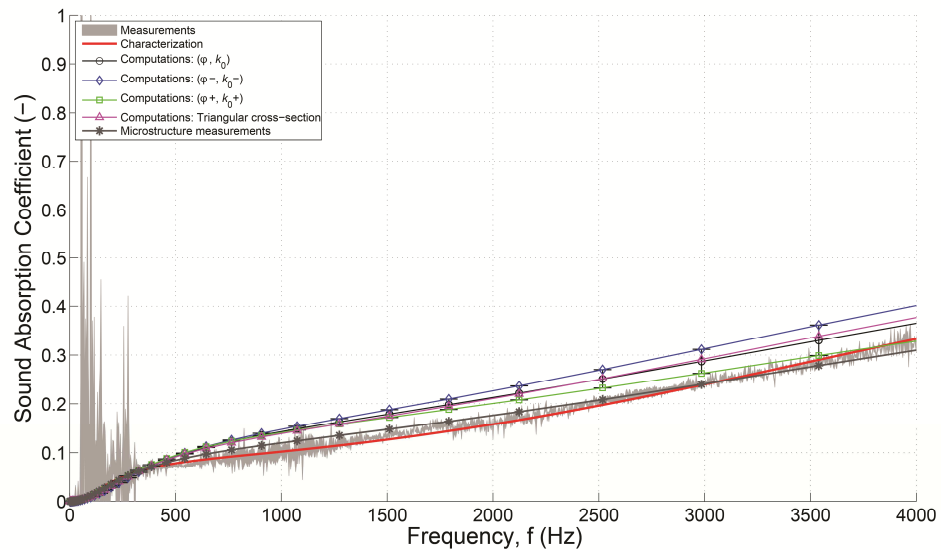


Figure 3.19. Normal incidence sound absorption coefficient for foam sample R_3 . Comparison between measurements (Ref. [59]), characterization (Ref. [60] and [61] combined with models described in Appendix 3.2.1.B), and computations (this work). Adding to the model experimental uncertainties for k_0 and ϕ helps improving the correspondence between experiments and modeling. Direct microstructure measurements are also used as input parameters of the three-dimensional local geometry model. The triangular cross-section shapes local model does not significantly modify the overall sound absorbing behavior. Sample thickness: 15 mm.

3.2.1.7. Conclusion

A three-dimensional idealized periodic unit-cell (PUC) based method to obtain the acoustic properties of three predominantly open-cell foam samples was described. The first step was to provide the local characteristic lengths of the representative unit cell. For isotropic open cell foams, two input parameters were required, the porosity and the static viscous (hydraulic) permeability. Long-wavelengths acoustic properties were derived from the three-dimensional reconstructed PUC by solving the boundary value problems governing the micro-scale propagation and visco-thermal dissipation phenomena with adequate periodic boundary conditions, and further field phase averaging. The computed acoustic properties of the foams were found to be in relatively good agreement with standing wave tube measurements. A close examination of the real foam sample ligament length distribution as observed from micrographs, and its comparison with the characteristic size of the local geometry model, showed evidences of membrane and cellular anisotropy effects discussed by means of critical path considerations. In summary, we have developed a microcellular approach in which the local characteristic length L_c governing the static viscous permeability of a real foam sample can be identified; and from which rough estimates of the viscous Λ and thermal lengths Λ' may follow (small openings and pore size itself). The overall picture that emerges from that work is that the acoustical response of these materials is governed by their three-dimensional micro-cellular morphology, for which an idealized unit-cell based method is a convenient framework of multi-scale analysis displaying the microgeometry features having a significant impact on the overall response function of the porous media.

The deviations between numerical and experimental results in the high frequency range were related to membrane and anisotropy effects, which were not taken into account by a simple three-dimensional non-elongated and open-cell geometry model. It was shown through preliminary simulations that the deviations were therefore significantly reduced when the three-dimensional unit-cell was allowed to present membranes at the peripheral of its windows, and forced to follow the elongation of the real foam sample as measured from microscopy. This confirms the validity of the proposed approach, and indicates that it captures the essential physics of the fluid-structure interactions in a real foam sample. In a forthcoming paper, it will be shown that the predictions of transport parameters and long-wavelength acoustical macro-behavior are in excellent agreement with measurements for three-dimensional membrane-based elongated local geometry models exhibiting the essential features of real foam sample microstructures having a significant impact at the upper scale.

Appendix 3.2.1.A. Critical path considerations

The purpose of this appendix is to present how a “critical path argument” can be used for helping to estimate the characteristic dimensions of a three-dimensional unit-cell which can represent at the best the physics occurring through a real foam sample having macro-scale static viscous permeability k_0 . Following Ambegaokar, Halperin, and Langer ^{[21],[93]} for the explanation of the hopping conductivity in disordered semiconductors, it is useful to think for our purpose, of a real foam sample as a network of randomly distributed unit-cells with a broad distribution of ligament lengths L and having polyhedral shapes linked between two sites i and j by local permeabilities k_{ij} . In general, any unit-cell in the network will be connected by an appreciably large permeability only to its close neighbors, and the discussion of the possible applicability of a simple three-dimensional unit-cell to model the over-all static viscous permeability k_0 of a real foam sample involves the relations between a set of such sites. Obviously, the geometry does not correspond to a percolation threshold, but some features of random media used in percolation studies can be of use here. In this picture, the correct choice for the characteristic unit-cell corresponds to the critical permeability k_c such that the subset of unit-cells with $k_{ij} > k_c$ still contains a connected network which spans the entire sample. Since the local viscous permeability is a function of the length L the threshold permeability k_c defines a characteristic length L_c .

The reasoning behind this statement is as follows. A real foam sample can be considered as composed of three parts:

- (i) A set of isolated regions of high permeability, each region consisting of a group of unit-cells with long ligament lengths and permeabilities $k_{ij} \gg k_c$.
- (ii) A relatively small number of resistive unit-cells with k_{ij} of order k_c and ligament lengths of order L_c , which connect together a subset of the high permeability clusters to form an infinite network which spans the system. The set of unit-cells in categories (i) and (ii) is the critical subnetwork.
- (iii) The remaining unit-cells with $k_{ij} \ll k_c$ and $L \ll L_c$.

The permeabilities in category (i) and their corresponding ligament lengths could all be set equal to infinity without greatly affecting the total permeability - the permeability would still be finite because the flow has to pass through unit-cells with permeabilities of order k_c and ligament lengths of order L_c to get from one end of the sample to the other. On the other end, the unit-cells with $k_{ij} \ll k_c$ and $L \ll L_c$ make a negligible contribution to the permeability because they are effectively shorted out by the critical subnetwork of unit-cells with $k_{ij} \geq k_c$ and $L \geq L_c$. It is now clear that the unit-cells with permeabilities of order k_c and ligament lengths of

order L_c determine the permeability of the real foam sample k_0 , i.e. $k_c = k_0$ and $L = L_c$.

In contrast, the choice of a length in the neighborhood of the averaged ligament lengths L_m would alter the value of the predicted permeability from the exaggerated contribution of the very large ligaments and would not be directly relevant to the representative unit-cell for the viscous flow.

Appendix 3.2.1.B. Different levels in modeling the acoustics of porous media

To describe the macro-scale acoustic properties of rigid-frame air-saturated porous media, the knowledge of two complex (\sim) response factors are required. The dynamic tortuosity $\tilde{\alpha}_{ij}(\omega)$ is defined by analogy with the response of an ideal (non-viscous) fluid for which α_{ij} is real-valued and frequency independent,

$$\rho_0 \tilde{\alpha}_{ij}(\omega) \frac{\partial \langle v_j \rangle}{\partial t} = -G_j. \quad (3.50)$$

$\tilde{\alpha}_{ij}(\omega) = \tilde{\rho}_{ij}(\omega) / \rho_0$ is related to the dynamic viscous permeability by $\tilde{\alpha}_{ij}(\omega) = \nu \phi / i \omega \tilde{k}_{ij}(\omega)$. In these expressions, $\tilde{\rho}_{ij}(\omega)$ is the effective density of air in the pores, ρ_0 is the density of air at rest, and $\nu = \eta / \rho_0$ is the air kinematic viscosity.

Similarly, a compressibility effect is also observed at macro-scale in the acoustic response of a thermo-conducting fluid filled porous media, where a second convenient response factor is the normalized dynamic compressibility $\tilde{\beta}(\omega)$ which varies from the isothermal to the adiabatic value when frequency increases,

$$\frac{\tilde{\beta}(\omega)}{K_a} \frac{\partial \langle p \rangle}{\partial t} = -\nabla \cdot \langle \mathbf{v} \rangle. \quad (3.51)$$

Here, $\tilde{\beta}(\omega) = K_a / \tilde{K}(\omega)$ is directly related to the dynamic (scalar) thermal permeability ^[17] by means of the relation $\tilde{\beta}(\omega) = \gamma - (\gamma - 1) i \omega \tilde{k}'(\omega) / \nu' \phi$. In these equations, $\tilde{K}(\omega)$ is the effective dynamic bulk modulus of air in the pores, $K_a = \gamma P_0$ is the air adiabatic bulk modulus, P_0 the atmospheric pressure, $\gamma = C_p / C_v$ is the specific heat ratio at constant temperature, $\nu' = \kappa / \rho_0 C_p$, and C_p and C_v are the specific heat capacity at constant pressure and volume.

With a locally plane interface, having no fractal character, the long-wavelength frequency dependence of the visco-thermal response factors $\tilde{\alpha}_{ij}(\omega)$ and $\tilde{\beta}(\omega)$ have to respect definite and relatively universal

behaviors ^{[12],[52],[56]}. (namely causality through the Kramers-Kronig relation), similarly to models used for relaxation phenomena in dielectric properties. The equivalent dynamic tortuosity of the material and the equivalent dynamic compressibility of the material are $\tilde{\alpha}_{eq\ ij}(\omega) = \tilde{\alpha}_{ij}(\omega)/\phi$ and $\tilde{\beta}_{eq}(\omega) = \phi\tilde{\beta}(\omega)$.

Simple analytic admissible functions for the fluid phase effective properties for *isotropic* porous media respecting the causality conditions are

$$\tilde{\alpha}(\omega) = \alpha_{\infty} \left[1 - \frac{1}{i\omega} f(\varpi) \right], \quad \tilde{\beta}(\omega) = \gamma - (\gamma - 1) \left[1 - \frac{1}{i\omega'} f'(\varpi') \right]^{-1}, \quad (3.52)$$

where f and f' are form functions defined by

$$f(\varpi) = 1 - P + P \sqrt{1 + \frac{M}{2P^2} i\varpi}, \quad f'(\varpi') = 1 - P' + P' \sqrt{1 + \frac{M'}{2P'^2} i\varpi'}, \quad (3.53)$$

and ϖ and ϖ' are dimensionless viscous and thermal angular frequencies given by the following expressions,

$$\varpi = \frac{\omega k_0 \alpha_{\infty}}{\nu \phi}, \quad \varpi' = \frac{\omega k_0'}{\nu' \phi}. \quad (3.54)$$

The quantities M , M' , P and P' are dimensionless shape factors,

$$M = \frac{8k_0 \alpha_{\infty}}{\Lambda^2 \phi}, \quad M' = \frac{8k_0'}{\Lambda'^2 \phi}, \quad P = \frac{M}{4 \left(\frac{\alpha_0}{\alpha_{\infty}} - 1 \right)}, \quad P' = \frac{M'}{4(\alpha_0' - 1)}. \quad (3.55)$$

- For $M' = P = P' = 1$ (with the requirement that $k_0' \approx \phi \Lambda'^2 / 8$), the dynamic visco-inertial and thermal response functions reduce to a 5 parameters (ϕ , k_0 , α_{∞} , Λ , Λ') named throughout the paper as “Johnson-Champoux-Allard” [JCA] model.
- When the requirement $k_0' \approx \phi \Lambda'^2 / 8$ is not fulfilled, k_0' must be explicitly taken into account, this is the 6 parameters “Johnson-

Champoux-Allard-Lafarge” [JCAL] model, where M' may differ from unity.

- A complete model relies on 8 parameters (ϕ , k_0 , k_0' , α_∞ , Λ , Λ' , α_0 , and α_0') and correctly matches the frequency dependence of the first two leading terms of the exact result for both high and low frequencies. This is the refined “Johnson-Champoux-Allard-Pride-Lafarge” [JCAPL] model.

Looking for plane waves solutions varying as $\exp[i(\omega t - \tilde{q}x)]$, Eqs. (3.50) and (3.51) yield the equivalent dynamic wave number $\tilde{q}_{eq}(\omega)$ of the material and equivalent characteristic impedance $\tilde{Z}_{eq}(\omega)$ of the material

$$\tilde{q}_{eq} = \omega \left(\tilde{\alpha}_{eq}(\omega) \tilde{\beta}_{eq}(\omega) \frac{\rho_0}{K_a} \right)^{\frac{1}{2}}, \quad \tilde{Z}_{eq} = \left(\frac{\tilde{\alpha}_{eq}(\omega)}{\tilde{\beta}_{eq}(\omega)} \rho_0 K_a \right)^{\frac{1}{2}}. \quad (3.56)$$

Thus, $\tilde{\alpha}_{eq}(\omega)$ and $\tilde{\beta}_{eq}(\omega)$ provide all pertinent information on the propagation and dissipation phenomena in the equivalent homogeneous material. Assuming an absorbing porous layer of thickness L_s that is backed by a rigid wall, the normal incidence sound absorption coefficient is

$$A_n = 1 - \left| \frac{\tilde{Z}_{sn} - 1}{\tilde{Z}_{sn} + 1} \right|^2, \quad (3.57)$$

with the normalized surface impedance of the porous medium defined as

$$\tilde{Z}_{sn} = \frac{\tilde{Z}_{eq}}{\rho_0 c_0} \coth(i \tilde{q}_{eq} L_s), \quad (3.58)$$

where c_0 is the sound speed in air.

Appendix 3.2.1.C. Ligament thicknesses measurement procedure

An estimation of the ligament thicknesses was provided through complementary measurements on SEM. Ligament thickness measurements

were performed on two perpendicular cross-sections of each foam sample, on the basis of 10 SEM for each perpendicular cross-section. These pictures were obtained with an Environmental Scanning Electron Microscope (ESEM), Electroscan E3 available at Laboratoire Géomatériaux et Environnement from Université Paris-Est, using an accelerating voltage of 20 or 30 kV, with a lanthanum hexaboride (LaB_6) cathodes electron source. Using a typical magnification, spanning from $\times 150$ to $\times 200$, measurements were carried out at the center of the ligaments (at the midpoint between two nodes). Note that the light zone situated at the peripheral of the ligaments, and attributed to membranes, was deliberately ignored during measurements. This is consistent with typical dimensions of observable transverse cross-sections. For each foam sample, the average ligament thickness estimates yield: R_1 , $2r_{m1} = 31 \pm 7 \mu\text{m}$ ($2r_1 = 19 \pm 7 \mu\text{m}$); R_2 , $2r_{m2} = 36 \pm 8 \mu\text{m}$ ($2r_2 = 27 \pm 7 \mu\text{m}$); R_3 , $2r_{m3} = 30 \pm 6 \mu\text{m}$ ($2r_3 = 25 \pm 10 \mu\text{m}$). Note that there is a reasonable agreement between computed and measured ligament thickness estimates providing standard deviations.

3.2.2. Identification d'une seconde taille critique de la géométrie locale

Pour modéliser les effets de membrane sur les propriétés de transport d'échantillons réels de mousse, une stratégie itérative a été retenue. Le point de départ est une cellule tridimensionnelle sans membrane, tel qu'explicité à la section précédente. Ensuite, on fait croître le taux de fermeture de membranes, d'un incrément égal à une fraction de taille de fenêtres. La porosité de la cellule tridimensionnelle ainsi modifiée est quasi-inchangée. En revanche, pour conserver la perméabilité de l'échantillon réel, la taille de la cellule modèle doit être augmentée. Le processus itératif est arrêté lorsque la longueur des ligaments de la cellule modèle est comparable à celle mesurée en moyenne sur l'échantillon réel de mousse. Pour faciliter la comparaison avec les paramètres de transport caractérisés ^{[60],[61]}, les propriétés acoustiques sont cette fois estimées par un modèle de Johnson-Lafarge ; avec un bon accord expérimental (tube de 44,5 mm de diamètre, échantillon placé en fond rigide, norme ISO 10534).

De plus, la perméabilité statique thermique s'avère être l'indicateur le plus sensible au contenu membranaire des échantillons réels de mousse parmi ceux testés, ce qui en fait un paramètre de choix pour la modélisation des films solide.

Ce travail clos la modélisation des paramètres de transports d'échantillons réels de mousses. Il est publié dans une revue internationale à comité de lecture dont la référence est :

M. T. Hoang and C. Perrot, Solid films and transports in cellular foams, Journal of Applied Physics, 112, 054911-6 (2012). ^[3]

Remarque visant à clarifier le dernier paragraphe de la page 108. Si on travaille sur une cellule isolée, on s'aperçoit en créant un réseau de cellules qu'il manque des membranes sur certaines faces. Les membranes susceptibles d'être oubliées sont au nombre de 12 pour une cellule complète. Et elles se situent sur les plans médians au parallélépipède cubique dans lequel la cellule est inscrite. Voir Fig. R1.

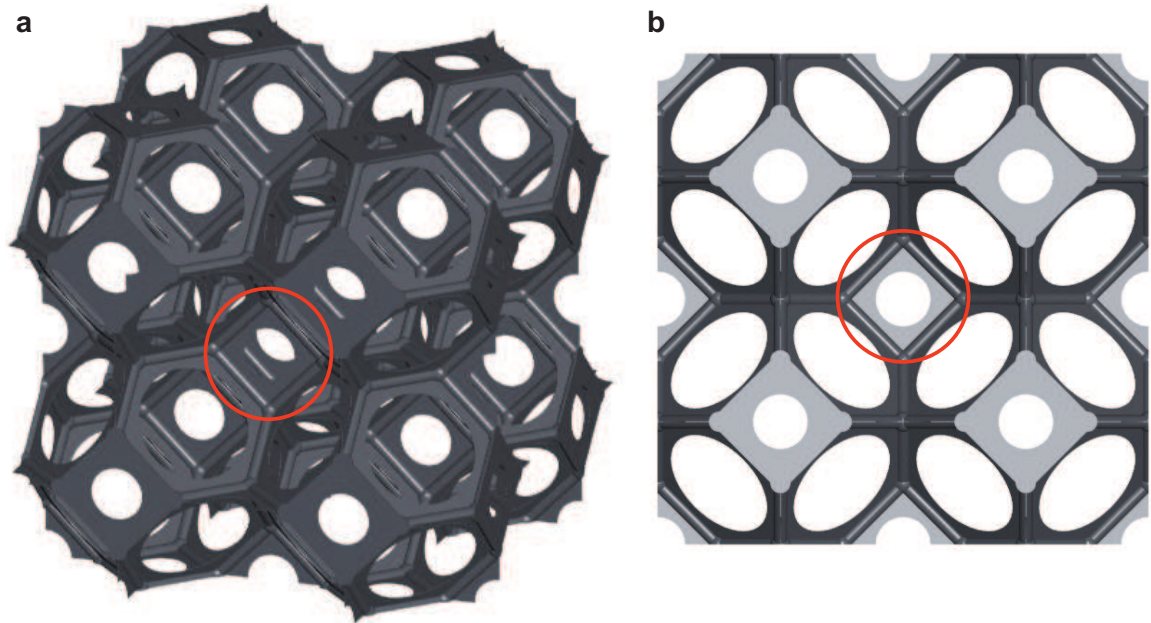


Fig. R1. Illustration d'un pavage de cellules périodiques. La membrane mentionnée est indiquée par un cercle rouge, (a) vue 3D et (b) vue 2D.

Abstract

We show that critical path ideas lead to the identification of two local characteristic sizes for the long wavelength acoustic properties in cellular solids, the pore and throat sizes. Application of the model to real foam samples which may contain solid films or membranes yields quantitative agreement between a finite-element numerical homogenization approach and experimental results. From three routinely available laboratory measurements: the open porosity ϕ , the static viscous permeability k_0 , and the average struts length L_m obtained from microscopy analysis; asymptotic transport parameters at high-frequencies and the normal incidence sound absorption coefficient are derived with no adjustable parameters.

3.2.2.1 Introduction

Solid films or membranes in real porous media such as polyurethane or metallic foams only account for a very small fraction of material in the overall mass of the porous media. Yet, their role might be of primary importance in the understanding of transport and acoustical properties of these foams. As a long wavelength wave propagates, the visco-inertial and thermal interactions between the disordered interconnected pores and the surrounding air pose a fundamental physical challenge in the microstructural identification of features which are characteristic of the overall transport phenomena. Part of the solution of this problem lies in the fact that the overall dissipation of the real porous media is expected to be dominated by a few key linkages^{[94],[95]} responsible for the main energy dissipation mechanisms; and in the successful identification of the critical-path responsible for viscous ones.^[2] Here, we demonstrate that a complementary part of the solution involves the fluid-structure interaction between the (thermally conducting) air inside the interconnected pores and the membranes that partially close them. Using finite element analysis on a periodic unit-cell local geometry model, experimental estimations of transport parameters, and high resolution imaging of real foam samples, we characterize the closure rate of membranes at the junction between interconnected pores. We find that the presence of non-closed membranes between pores effectively corresponds to the introduction of a second set of critical characteristic sizes, which governs the inertial effects and meanwhile enables a correct description of the thermal ones. For an increasing rate of semi-open membranes, because of the fact that the throat size reduces, then the length Λ which has been identified as a weighted volume-to-surface ratio for the porous medium diminishes, whereas the infinite tortuosity factor α_∞ increases as a result of strong cross section changes.^[12] An increasing membrane closure rate promotes the emergence

of a stronger contrast between two distinctive critical characteristic sizes inside one periodic unit-cell, the size of a pore itself and the size of the interconnections between pores, which provides a scaling behavior of real polyurethane foam samples for both viscous and inertial effects. The presence of membranes also favors surface effects known to have a strong influence in transport phenomena such as diffusion controlled reactions. As characterized from the low frequency asymptotic behavior of thermal exchanges between the solid frame and the surrounding air measured in a standing wave tube, the trapping constants $\Gamma = 1/k_0'$ of the real foam samples tend to agree well with the ones simulated from the previously identified visco-inertial scaling behavior of the unit-cells. A combination of advanced experiments and detailed numerical modeling of fluid-structure interactions at the pore scale reveals the basic microstructural features behind transport phenomena and shows quantitatively how these thin elements are crucial to the correct microstructural description of long-wavelengths acoustic waves' propagation and dissipation in real foam samples.

3.2.2.2 *Materials and method*

A three-dimensional cellular solid or foam is an interconnected network of solid struts or plates forming the edges or faces of polyhedra which pack to fill space. Foamed or cellular solids may be classified according to the type of cell structures: open-celled if the solid of which the foam is made is contained in the cell edges only, closed-cell if solid membranes close off the cell faces too. In many types of foams, this thin skin framed by thicker edges is partially open and partially closed. Carefully controlling the closure rate of membranes at the junction between interconnected pores allows for the fine tuning of foam transport and acoustical properties. When an acoustic wave propagates through an air saturated porous media having a motionless and isothermal skeleton, the frequency-dependent interplay between viscosity and inertia, and thermal conduction produce a spatial gradient of fluid velocity and temperature; and the shear combined with heat transfer between successive layers of fluid causes compression wave attenuation. ^{[45],[47],[48],[97]} The characteristic lengths of the compression waves created by an oscillating body in which the main visco-inertial and thermal dissipation processes occur are the viscous and thermal boundary layers. They respectively scale as $\sqrt{\nu/\omega}$ and $\sqrt{\nu'/\omega}$, in which $\nu = \eta/\rho_0$ is the fluid's cinematic viscosity, $\nu' = \kappa/\rho_0 C_p$ is its thermal diffusivity, ω is the angular frequency of motion; and where η is the fluid's dynamic viscosity, ρ_0 is its density at rest, κ is its thermal conductivity, C_p is the specific heat at constant pressure. For small perturbations, because

the wavelength λ of the incident compression wave greatly exceeds the typical pore size D of the motionless porous media at audible frequencies, visco-inertial motions and thermal conduction can be decoupled within a pore. On the other hand, the elastic behavior of the solid matrix may be subsequently incorporated as an additional pressure source term for the fluid flow. The viscous forces at low frequencies (Stokes flow), inertial forces at high frequencies (potential flow formally identical to electric conduction), and thermal conduction at low frequencies (similar to diffusion-controlled reactions) together determine all the transport parameters within a porous material.

Although ligament lengths distributions can be measured directly from microscopy analysis techniques,^[2] a quantitative appreciation of the critical size governing low-Reynolds number hydrodynamics in real foam samples can be obtained from the associated permeability (Fig. 3.20). When a viscous fluid moves through a three-dimensional periodic local geometry model of opened-cell solid foam structure, the ligament length of the unit cell with the same average permeability is always lower or equal to the average length measured from microscopy analysis. One might therefore expect a significant local geometry feature for the real solid foam samples omitted in the model. Indeed, a careful observation of the real foam samples microstructure from micrographs may reveal residual solid films (at the periphery of the foam ligaments). Because we computed the permeability for three-dimensional periodic unit cells that lacked solid films, the corresponding ligament's length value is about a half of the typically reported foam from micrographs on non-fully reticulated real foam samples. We note that these solid film contents are also significant on other transport parameters related to inertial flow and diffusion-controlled reactions. The large differences between the transport parameters for an open cell and for a partially closed-cell of three-dimensional local geometry models with identical viscous permeability reveal the striking role that solid films in real polymeric foam samples may play in the visco-inertial and thermal dissipation processes involved during acoustic wave propagation in polymeric foams at audible frequencies.

The first-principles calculation procedures of transport properties from a non-dimensional unit cell have been described.^[2] Although audible frequency sound waves propagating through air-saturated rigid porous media may be sensitive to a large range of local geometry characteristic sizes, their frequency-dependent macro-behavior is essentially determined by visco-inertial asymptotic transport parameters and their thermal counterparts. To estimate these transport parameters, we constructed a microstructure model of a solid foam unit-cell with air as a Newtonian

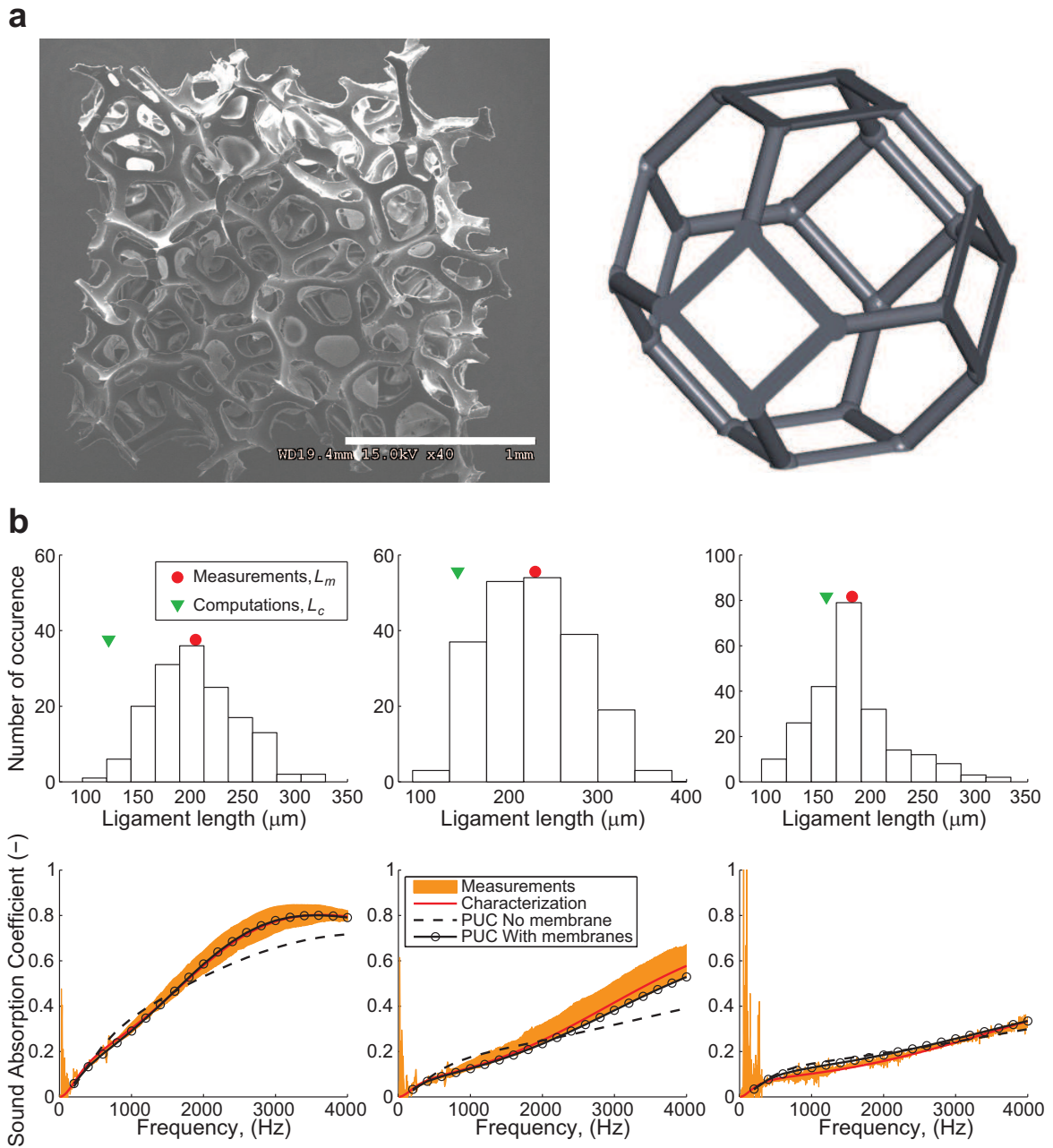


Fig. 3.20. Structure and sound-absorbing properties of three real polyurethane foam samples.

(a) A scanning electron micrograph of a polyurethane foam sample illustrates a number of semi-open or even closed membranes, at the interconnection between pores which are filled with air, a visco-thermal fluid. The calibration bar corresponds to 1 mm. Right: a schematic diagram depicts a single idealized periodic unit cell of polyurethane foam, without membranes at the fringe of its struts. (b) Ligament lengths distributions for three real foam samples (namely R₁, R₂, and R₃ from left to right) together with superimposed average values of the experimental

measurements L_m (red circle) and the ligament length of the opened-cell solid foam structure with the same average permeability (green triangle) convey the impression of a missing microstructural ingredient in the periodic unit cell. This is confirmed by the computed sound absorption with the unit cell containing membranes (bottom). Computations were performed using first principles calculations of transport parameters, and Johnson et al. [12] combined with Lafarge et al. [17] semi-phenomenological robust models. The thicknesses of the real polyurethane foam samples are 25 mm, 15 mm, and 15 mm, respectively from left to right.

viscous and thermally conducting filling liquid, preserving the scaling between the physical quantities of importance in the low-frequency asymptotic behavior (porosity and quasi-static viscous permeability). A simplified three-dimensional local geometry model of the opened-cell structure of polyurethane foam samples in which Stokes equations were solved was used to provide the initial characteristic ligament length estimate, L_c . A solid film was implemented at growing rates of $\Delta\delta = \delta_{\max}/20$ while the unit-cell was rescaled to account for the film frictional forces, until ligament length L is converged (Fig. 3.21). We estimated the transport parameters for the non-fully reticulated models of the polyurethane foam samples, which are in much better agreement than the ones computed without solid films compared to standing waves tube measurements (Table 3.5). This agreement demonstration confirmed that even a very small content of solid films plays an important role in the overall visco-inertial and thermal dissipation phenomena, indicating the importance of this particular local geometry feature in the identification of a representative unit-cell.

Viscous transport becomes dominant in the low frequency regime and inertial transport becomes dominant in the high-frequency regime of long wavelength acoustic wave propagation in rigid porous media. To quantify these transport properties as a function of cellular morphology, a finite-element model in which we could manipulate the solid-film content of the cellular structure while explicitly representing the velocity field in and around the cellular morphology was used. The models have about 400 000 degrees of freedom and rigorously resolve the liquid motion in the pore structure of non-fully reticulated polymeric foams as well as the temperature field outside the polymer films.

Due to the periodicity of the unit cell, $12 \times 1/4$ solid films are not lying along the faces of the polyhedron, contrary to the 14 remaining ones (Fig. 3.21). Here, the solid film thickness t over ligament thickness $2r$ was mainly set to a constant ratio, $t/2r \approx 1/20$. The porosities were therefore reduced to less than a percent when the solid films were implemented. The

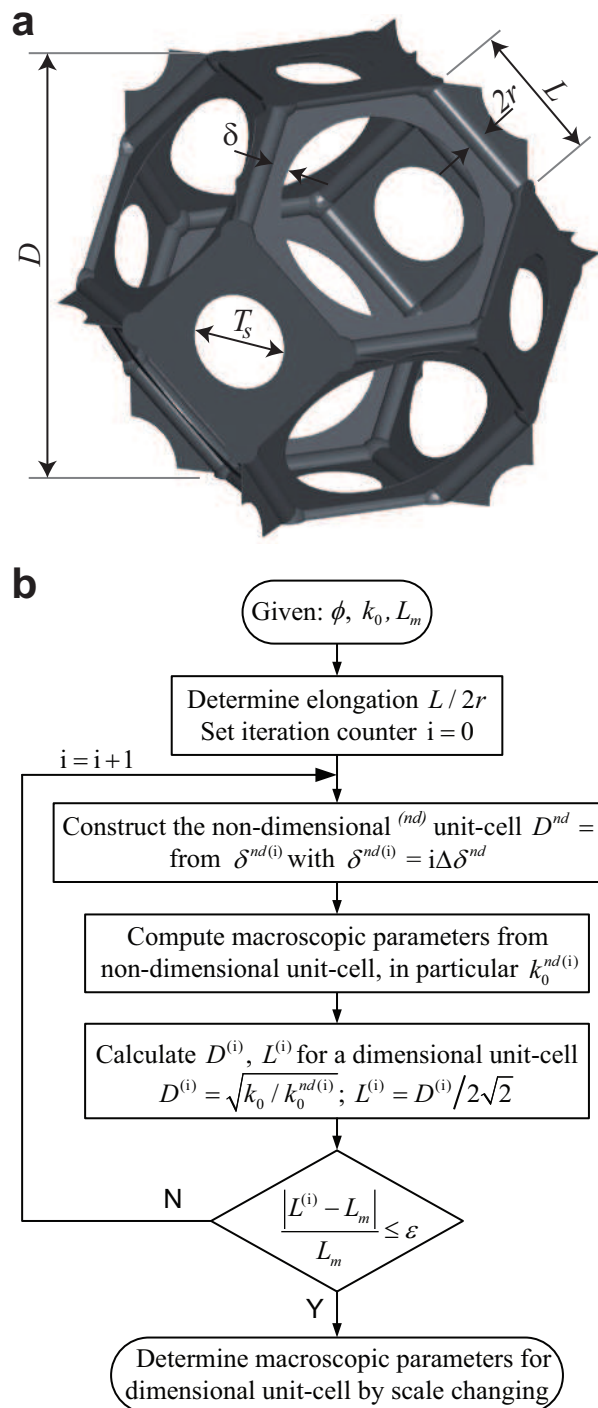


Fig. 3.21. Solid film implementation in a three-dimensional opened-cell model. (a) Illustration of a model with solid films implemented at the periphery of the struts, this feature of the microgeometry introduces a supplementary degree of freedom, from which a representative unit cell of the acoustical macro-behavior is found. The closure rate of the solid films δ/δ_{\max} may progress from an

opened-cell structure without any solid film ($\delta = 0$) to a relatively closed-cell structure when $\delta/\delta_{\max} \rightarrow 1$ ($\delta_{\max} = L/2 - r$), the throat size T_s being the smallest interconnecting size between opened cells. b, The first-principles calculation procedures of transport properties from a non-dimensional unit cell have been described.^[2] Here, this iterative procedure illustrates the importance of porosity ϕ , permeability k_0 , and average ligaments length L_m measurements from which all the macroscopic parameters of interest for a real cellular solid with solid film content are computed without any adjustable constant. The reported macroscopic parameters are computed with a closure increment $\Delta\delta = \delta_{\max}/20$, and a relative difference between modeled and measured ligament lengths $\epsilon = 2\%$.

finite element model with solid films replicated the thermal length (mesh quality), when calculated with a geometrical model (analytical solution). Unlike the elastic properties in the deformable solid frame model for which the solid film thickness effect is strong, the film thickness in this instance does not affect the transport coefficients. This is because the volume of the solid films is comparable to the volume of the solid ligaments (significant role on the elastic properties), whereas the volume of the solid films is negligible compared to the fluid phase volume (insignificant film thickness effect on the transport properties as long as they remain thin).

Tab 3.5. Comparison between measured, characterized and computed macroscopic parameters for non-elongated unit-cells (with and without membranes). The porosity ϕ and viscous permeability k_0 are experimental data of the real foam sample, taken as input parameters for scaling the local geometry model, by solving Stokes equations in the periodic three-dimensional microstructure. This provides initial ligament lengths L and thickness $2r$ of the isotropic unit-cell. Then, the generalized hydraulic radius also known as the thermal characteristic length Λ' is deducted from integration over the scaled unit-cell. All the other macroscopic parameters are derived from first-principles calculations.^{[2],[12],[95]} Furthermore, an iterative strategy is used to increase the closure rate of membranes. With no adjustable parameters, the iteration counter is stopped when the ligament length of the membrane-based three-dimensional local geometry model is comparable with the measurements obtained from standard micrographs.

Foams	Method	ϕ (-)	Λ' (μm)	k_0 ($\times 10^{-9} \text{ m}^2$)	Λ (μm)	α_∞ (-)	k_0' ($\times 10^{-9} \text{ m}^2$)	L_m (μm)	$2r$ (μm)	$\delta/\delta_{\text{max}}$ (-)
R ₁	Measurements	0.98 ± 0.01		2.60 ± 0.08				205 ± 42	31 ± 7	
	Computations without membranes		506 ± 114		297 ± 66	1.02 ± 0.01	5.01 ± 0.22	123 ± 13	19 ± 7	0
	Computations with membranes		288 ± 4		138 ± 7	1.17 ± 0.02	8.56 ± 0.10	207 ± 4	33 ± 10	0.40 ± 0.04
	Characterization		440 ± 202		129 ± 23	1.12 ± 0.2	8.30 ± 1.60			
R ₂	Measurements	0.97 ± 0.01		2.98 ± 0.14				230 ± 57	36 ± 8	
	Computations without membranes		477 ± 70		279 ± 41	1.02 ± 0.01	5.85 ± 0.34	141 ± 12	27 ± 7	0
	Computations with membranes		308 ± 7		147 ± 6	1.18 ± 0.02	10.04 ± 0.23	229 ± 1	44 ± 9	0.40 ± 0.04
	Characterization		330 ± 54		118 ± 9	1.13 ± 0.04	9.70 ± 2.40			
R ₃	Measurements	0.98 ± 0.01		4.24 ± 0.29				182 ± 42	30 ± 6	
	Computations without membranes		647 ± 147		373 ± 86	1.01 ± 0.01	8.18 ± 0.68	157 ± 19	25 ± 10	0
	Computations with membranes		438 ± 25		261 ± 19	1.04 ± 0.01	9.27 ± 0.67	178 ± 6	28 ± 9	0.05 ± 0.05
	Characterization		594 ± 80		226 ± 35	1.06 ± 0.02	13.10 ± 4.20			

3.2.2.3 Results and discussion

First, we examined the model including only visco-inertial dissipation effects for non-elongated unit-cells through their corresponding transport parameters, the quasi-static viscous permeability k_0 , the viscous characteristic length^[12] Λ and the tortuosity^[12] α_∞ . In Fig. 3.22 we plot the variation of the three asymptotic parameters Λ , α_∞ , and k_0' for partially closed-cell models as a function of the closure rate of membranes δ/δ_{\max} . Since at $\delta/\delta_{\max} \rightarrow 1$ the throat tends to pinch off, α_∞ tends to diverge and the values of Λ tend to T_S . Adding to the open cell model motionless solid films strongly increase α_∞ , and reduce Λ . It is noted that specific values for the closure rate of membranes δ/δ_{\max} may be chosen such that the ligament lengths of the non-fully reticulated unit-cells will match the average value of the experimental measurements performed on optical micrographs ($L = L_m$).

Using a transfer function method for measuring the characteristic impedance and propagation constant^[59] of the real foam samples from a standing wave tube (the corresponding dynamic viscous and thermal permeability $k(\omega)$ and $k'(\omega)$ were calculated) allowed us to calculate the dynamic density and dynamic bulk modulus of the materials, some quantities that could be directly compared with Johnson et al.^[12] and Lafarge et al.^[17] models. Assuming porosity ϕ and quasi-static viscous permeability k_0 known from direct non-acoustical measurements, an experimental estimation of Λ , α_∞ and k_0' was obtained from the analytic inversion of the models.^{[60],[61]} With no adjusted constant, the computed transport parameters corresponding to the aforementioned specific values of membrane closure rates are in good agreement with experimental characterization data. Adding to the three-dimensional local geometry model a closure rate of membranes, rather than no solid films, introduces an accurate description of the throat size area, which is critical to the determination of Λ and α_∞ . Consequently, this provides us with a cellular morphologically based method for the identification of the second critical linkage dominating, at high-frequencies, the dynamic viscous permeability (from the knowledge of the first critical linkage dominating, at lower frequencies, the dynamic viscous permeability). Let k_c and α_c be the two scaling parameters^[95] for the real and non-fully reticulated foam samples, $k(\omega)$ is thus expected to be entirely determined by simple measurements of ϕ , k_0 and L_m (since $k_c = k_0$ and α_c derives from our cellular morphologically based method).

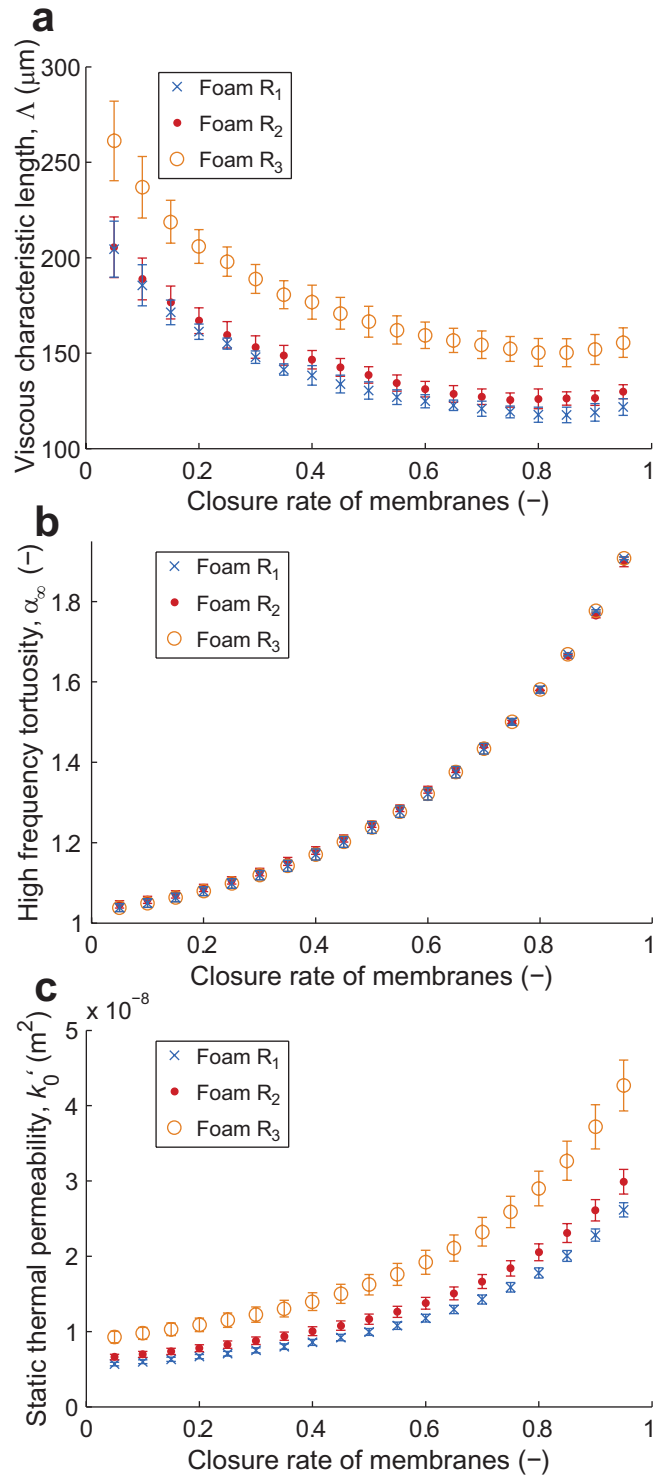


Fig. 3.22. Finite element analysis of transport properties in a cellular solid. The membrane closure rate dependence of the viscous characteristic length Δ (a), the high frequency limit of the viscous tortuosity α_∞ (b), and the static thermal

permeability k_0' (c) are obtained from the cellular solid model with three configurations of the measured porosity ϕ and permeability k_0 : $\phi = 0.98 \pm 0.01$ and $k_0 = 2.60 \pm 0.08 \times 10^{-9} \text{ m}^2$ (foam sample R₁, blue crosses), $\phi = 0.97 \pm 0.01$ and $k_0 = 2.98 \pm 0.14 \times 10^{-9} \text{ m}^2$ (foam sample R₂, red circles), $\phi = 0.98 \pm 0.01$ and $k_0 = 4.24 \pm 0.29 \times 10^{-9} \text{ m}^2$ (foam sample R₃, orange circles). The error bars were calculated by standard deviations when taking into account all possible cellular configurations associated with porosity and permeability experimental uncertainties.

Next, we considered more thoroughly the evolution of the thermal permeability $k_0' = 1/\Gamma$ as a function of the closure rate of solid membranes δ/δ_{\max} , a quantity related to the pore space diffusion.^[57] In this regard, it is not clear whether the above quantity provides an accurate description of the pore geometry. Indeed, contrary to k_0 , Λ and α_∞ ; k_0' is not directly related to connectivity of the pore space. If the links between the pores were gradually eliminated by solid films, the permeability would rapidly approach zero while the value of k_0' would not be greatly affected.^[96] Because pore sizes increase with membrane closure rates whereas throat sizes decrease for a given non-zero static viscous permeability k_0 , diffusion-controlled reactions progressively become very sensitive to the wetted surface area of solid films. As the pore links subside, the k_0' coefficient provides the most accurate probe to solid film content of the real porous structure (Fig. 3.22). It is helpful here to restate that the diffusing particles corresponding to a scalar field move randomly and explore the three-dimensional porous space without any privileged directions. This is to be contrasted with the visco-inertial transport parameters k_0 , α_∞ and Λ which relate to vector fields and favored directions. Therefore, for connected pores, the k_0' coefficient gives a better estimate of the membrane closure rate for a cellular solid containing solid films. This remark is corroborated by the periodic homogenization and consistent estimates of transport parameters through polyhedron packings modeling study (Sec. V D).^[19] It should be noted that Refs. [99] give an evaluation for k_0' based on acoustic experiments that, when combined with Fig. 3.22, would give an estimate of the membrane closure rate δ/δ_{\max} . Predictions of δ/δ_{\max} and k_0' using direct measurements on two-dimensional or three-dimensional images are not yet established, however, and is thus beyond the scope of this paper.

In conclusion, because all parts of the motionless disordered network of struts or plates forming a real cellular solid with the air as filling phase do not play the same role in the overall frequency-dependent response function, with a long wavelength acoustic signature governed

essentially from the low and high frequency asymptotic regimes, most microstructural details inside the real foam samples are inferred from simple measurements of open porosity, permeability, and ligament lengths. Although solid films might be thought to be non-significant details, the cellular solids microstructure actually involves membranes in visco-inertial and thermal energy dissipation, making it critical for the multi-scale modeling process to design solid foams with tailored transport properties. The enhanced fluid-structure interface due to solid films even suggests static thermal permeability as an asset by using it as a simple means of characterizing the closed pore content of real porous media.

3.3. Application des méthodes développées à des résultats de la littérature

A partir du travail de modélisation et simulation multi-échelle réalisé (sections 3.1, 3.2.1, et 3.2.2), la stratégie suivante d'utilisation des méthodes développées se dégage :

Si la mousse solide est pleinement réticulée d'une part (c'est-à-dire constituée de cellules parfaitement ouvertes) et qu'elle est constituée d'une distribution étroite de tailles de pores, d'autre part, il est possible alors de reconstruire une cellule représentative des propriétés de transport du milieu réel directement à partir des longueurs et épaisseurs médianes ou moyennes de ligaments.

Si la mousse solide est partiellement réticulée (c'est-à-dire faite de cellules semi-ouvertes) ou encore qu'elle repose sur une distribution étendue de tailles de pores, l'identification d'une cellule représentative est alors moins triviale. Cette fois, deux paramètres macroscopiques facilement mesurables sont utilisés pour dimensionner la cellule ouverte de base, la porosité et la perméabilité, auxquels s'ajoute une taille caractéristique de la micro-géométrie, telle que la longueur moyenne de ligaments.

Cette stratégie différenciée d'utilisation des méthodes développées a été appliquée à partir des données de la littérature disponibles. On constate cette fois que l'accord qualitatif (visuel) et quantitatif (à l'échelle) entre cellules modèles et réelles est satisfaisant. Ces données sont ensuite utilisées afin d'établir une correspondance directe entre tailles caractéristiques microstructurales et macroscopiques, les limites des correspondances établies étant précisées. En particulier, on trouve que le rayon de la plus petite interconnexion s'apparente à la longueur visqueuse. L'évolution du rapport des longueurs thermique et visqueuse avec le taux de fermeture de membranes est ensuite étudiée. Ce rapport est proche de deux pour des mousses à cellules essentiellement ouvertes, qui s'apparentent à des milieux fibreux ; et croit ensuite fortement avec le taux de fermeture de membranes, lorsque le contraste entre étranglement et taille de pores augmente. Cette étude et ces résultats font l'objet de la section 3.3, dont le manuscrit correspondant est publié dans un journal international à comité de lecture dont la référence est :

M. T. Hoang and C. Perrot, Identifying local characteristic lengths governing sound wave properties in solid foams, Journal of Applied Physics, 113, 084905-7 (2013). ^[4]

Abstract

Identifying microscopic geometric properties of and fluid flow through opened-cell and partially closed-cell solid structures is a challenge for material science, in particular for the design of porous media used as sound absorbers in building and transportation industries. We revisit recent literature data to identify the local characteristic lengths dominating the transport properties and sound absorbing behavior of polyurethane foam samples by performing numerical homogenization simulations. To determine the characteristic sizes of the model, we need porosity and permeability measurements in conjunction with ligament lengths estimates from available scanning electron microscope images. We demonstrate that this description of the porous material, consistent with the critical path picture following from the percolation arguments, is widely applicable. This is an important step towards tuning sound proofing properties of complex materials.

3.3.1. Introduction

Some important characteristics of a sound absorbing material are the transport properties, from which an approximate but robust description of its long wavelengths behavior derives. ^{[12],[14],[15],[17],[18]} Transport properties include the rate of either diffusive, electrical, or fluid movement through fluid-filled pore space. But a more detailed understanding comes from identifying other microstructural parameters. For disordered rigid-framed solid foams in which a sound wave is propagating and dissipated by visco-inertial and thermal interactions, transport properties are often very sensitive to the local variability of the geometry, their real microstructure. Thus, a systematic identification of local characteristic lengths has not yet been performed in polyurethane foams. In this paper, we present precise estimates of microscopic geometric quantities of a three-dimensional periodic unit-cell – with representations of pore and throat sizes that explicitly show the microphysical basis behind transport phenomena.

In classical semi-phenomenological acoustics, no clear relationships occur with the microgeometry itself. The situation is quite different in numerical homogenization. Indeed, J. L. Auriault ^[100], and others (see references therein) showed that even a disordered porous medium can undergo a homogenization to a quite simple form, a periodic structure, provided that the situation obeys scale separation (any feature of the local geometry is much smaller compared to the incident wavelength). The application of this theory to real rigid porous media in the long wavelengths' regime captures the essential of the physics at both the micro

and macro scales, and the way to compute explicitly the coefficients involved at the upper scale level. [76],[78],[101]

Porous media that obey scale separations, such as disordered solid foam samples, undergo a transition between viscous (isothermal) and inertial (adiabatic) regimes by forming the three-dimensional periodic unit cells that acts as the overall porous material. For this to happen, local characteristic lengths identification is necessary. In the simplest cases, the transport parameters are determined experimentally. [102] This characterization may be correlated to the closed pore content of polyurethane foam samples (the ratio of the number of closed windows to the total number of windows visible on micrographs). [103] The resulting semi-empirical relationships are typically applicable to a wide range of real foam samples. However, the micro-macro relations emerging from these correlations are only directly applicable to the actual fabrication processes. The physical insight in this numerical homogenization scheme is much larger than experimental characterization and semi-empirical correlations by allowing identification of local geometry features which have a significant impact at the upper scale.

The application of the numerical homogenization has proven to be widely applicable to synthesized periodic porous materials. [27],[30],[32],[33],[104] Beyond conventional characterization techniques [60],[61], it describes the complex interplay of the fluid flow within the porous media, in the frequency-dependent response, or in asymptotic behaviors at low and high frequencies. There is an important class of synthetic porous materials that do not follow the periodic structures paradigm. These materials seem too disordered to be described by a periodic unit cell that starts from a regular structure. However, a completely new way to interpret transports within disordered pore spaces has emerged – critical path ideas, whose conceptual strength can be used to identify characteristic lengths of the pore space. [21],[93],[94] To determine the characteristic lengths of real solid foam samples, one model is used here to cope with different experimental studies in an iterative loop of experimentation and computations without any adjustable constant. [2],[3] Hoang and Perrot describe this class of model, using the example of three real foam samples which may contain solid films. [3]

3.3.2. Materials and method

The general strategy to determine the three-dimensional periodic unit cell representative of a given real foam sample is to measure the porosity ϕ and permeability k_0 , in such a way that a Kelvin cell without solid films corresponding to the measured experimental data might be found.

Equivalently, by causing the closure rate of membranes δ/δ_{\max} to grow, a relative measure of the membranes' size when compared to the size of the smallest windows,^[3] one can determine the new size of the cell until the computed and measured ligament lengths become comparable. We measure the ligament length from published scanning electron micrographs, when this data is not reported in literature. The remaining transport parameters are then directly determined from the local geometry model and accurately computed by numerically solving the appropriate boundary value problems.

First, we test the model against a series of measurements on polyurethane foam samples recently reported by Kino *et al.*^[102] Data (non-acoustical) for porosity ϕ and permeability k_0 were provided for three polyurethane foam samples, namely 1, 2, and 5 (Tab. 6 of Ref. [102]); together with a scanning electron micrograph per foam sample taken at magnification $\times 50$ (Fig. 1 of Ref. [102]) – and from which rough estimates of the corresponding ligament lengths lying in the plane of observation might be obtained (to limit the 2D projection bias), yielding $L_m \pm \Delta L_m$. We next tested the quantitative accuracy of our model's predictions against those of semi-empirical models derived for fully and non-fully reticulated polyurethane foam samples.^[103] In this paper, Dutres, Atalla, and Dong (DAD) carried out extensive measurements. They provide expressions allowing correlations between macroscopic parameters and local geometry features of polyurethane foam samples from fifteen different materials, together with the normal incidence sound absorption coefficient for three of them (namely M7, M9, and M11). Finally, data on various polyurethane foam samples, namely R₁, R₂, and R₃, were taken from Tabs. I and II of Ref. [2]. We can therefore compare our model with three new complete sets of experimental data available in the literature (Ref. [2], Ref. [102], and Ref. [103]).

The details on the numerical homogenization method used in this study were described elsewhere.^[2] Briefly, quasi-static viscous permeability measurements on real porous materials should respect the essentially laminar airflow entering and leaving the test specimen. The finite-element method provides approximate solutions to partial differential equations reflecting the polymeric foam's local geometry, assumed to be motionless. The scale separation allows the elimination of the coupling terms between visco-inertial and thermal dissipations. Locally, the fluid flow is described by incompressibility, and the fluid pressure is constant throughout a pore cross-section transverse to the compression wave propagation direction. The asymptotic transport coefficients were computed numerically from field averaging by solving the Stokes (static viscous permeability k_0 , static viscous tortuosity α_0), potential-flow (tortuosity α_∞ ,

and viscous length Λ), and Laplace equations (static thermal permeability k_0' also known as the inverse of the trapping constant Γ , static thermal tortuosity α_0'). Open porosity ϕ and thermal characteristic length Λ' also known as the generalized hydraulic radius are purely geometrical macroscopic parameters respectively derived from the fluid volume to bulk volume ratio and from twice the fluid volume to wetted surface area ratio. The linear response functions of the visco-thermal fluid entailed in a rigid porous medium and subjected to an infinitesimal oscillatory pressure gradient across the sample were estimated analytically by using semi-phenomenological models, which are valid as long as the wavelength of sound (in the fluid) is much larger than the characteristic sizes of pores. In what follows, we test the results of the numerical model directly with literature data.

3.3.3. Results

Our method was tested against information available for a series of nine polyurethane foam samples that can be studied in two independent ways: by direct measurements of ligament lengths and thicknesses when the solid foam is free of solid film (Fig. 9(a) of Ref. [2]), and using the iterative approach (Ref. [3]) when the foam samples are not fully reticulated. These determinations yield the identification of the local characteristic lengths governing sound wave properties (Fig. 3.23). Fig. 3.23 also shows the localization of these characteristic lengths on the scanning electron micrographs. Our data show good agreement with the real foam sample micrographs. Fixing from measurements ϕ , k_0 , and L_m , or L_m and the ligaments thickness $2r$, our method ^{[2],[3]} yields a prediction for Λ' , α_0 , Λ , α_∞ , k_0' , α_0' , in acceptable agreement with Ref. [102] and Ref. [103] (Table I and II), but also in agreement with the microstructure. Note that there are only three samples chosen from DADs' fifteen because the measured values of acoustic properties are given by DAD for only three samples. In other words, the acoustical prediction's usefulness as a direct prediction of Λ' , α_0 , Λ , α_∞ , k_0' , α_0' , based on two or three experimental measurements ϕ , k_0 , and L_m ; or L_m and $2r$) might be fully established only for samples M7, M9, and M11 with Ref. [103].

From the macroscopic parameters, we obtain all the other acoustic quantities of the rigid porous material (Ref. [2], Appendix B). Fig. 3.24 shows the sound absorption coefficient at normal incidence (SAC_{NI}) as a function of frequency. At low frequencies, the SAC_{NI} is governed by the fixed permeability k_0 . At high frequencies, the SAC_{NI} is dominated by the asymptotic high frequency parameters (Λ , α_∞ , and Λ'). In between, the effect of the elastic properties of the real foam samples may be observed.

For instance, a dramatic rise of the SAC_{NI} is observed for Sample 1 around 1500 Hz (Fig. 3.24, top left). Such a feature in the experimental values of the SAC_{NI} is characteristic of local resonances of an elastic solid phase, which may be taken into account, for example, by using classical Biot theory. ^{[10],[11]}

We shall now examine some elements based on experimental characterizations of the porous materials that can serve as a basis to simply interpret discrepancies between experimental and numerical results, as shown in Fig. 3.24(a) (left and center). Such attempts can be found in the literature on the acoustics of porous materials where geometrical, transport, and elastic properties were related empirically to an elasto-acoustic coupling criterion, the Frame Acoustical Excitability (FAE).^[105] For instance, because the permeability of Sample 5 is very low (Tab. 3.6) and its skeleton deformable (relatively low Young's modulus, Tab. I. of Ref. [102]), the elasto-acoustic coupling with the acoustic wave is strong, Fig. 3.24(a) (left). This corresponds to a high FAE, $FAE > 2$ (W/kg). Although the Zwikker and Kosten decoupling frequency f_d ^[45] indicates, in the frequency range of interest, the existence of partial decoupling between the solid and fluid phases of the porous sample, the FAE criterion also formulates this acoustical energy transfer to the solid phase in terms of the resonance frequency of the frame-borne wave. These are combined with a geometrical shape factor of the foam sample, in order to deal with the effect of the edge constraint and mounting conditions in a standing wave tube. Interestingly, because of the very low permeability of Sample 5, a small penetrating depth of the wave is expected. This favors structural dissipation mechanisms by resonance of the frame corresponding to a high sound absorption peak, in addition to a surface absorption phenomenon. And it also explains why, contrary to less resistive foams for which a local sound absorption decrease is generally observed around the first quarter wavelength resonance frequency of the frame-borne wave, an absorption peak is observed in the same way as for closed-cell elastic foams (Fig. 7 of Ref. [106]).

In models of rigid-framed porous media, such jumps had been inferred by making empirical corrections to the Johnson-Champoux-Allard model to fit experimental data that implied a resonance. ^[102] Arguably this approach does better than the numerical method advocated in the current paper - at least for the samples 1 and 2. It should also be mentioned that Sample 5 was prepared especially to remove all membranes. Moreover Ref. [102] goes on to explain and elaborate on the presence of the 'frame resonance' in impedance tube data by reference to poroelastic theory. It is important to note, however, that Ref. [102] does not give a prediction for

the acoustic properties of a porous material because it uses semi-empirical fitted factors N_1 and N_2 . The main objection to them in respect of the authors' work might be that they are applicable only to the tested materials and therefore not as potentially general as the parameters in the proposed approach. A second important consideration is that these factors cannot be simply interpreted, since they are not directly related to the pore structure.

In this work, we do not expect to predict the elastic behavior very close to the structural resonance, because the skeleton is assumed to be undeformable (this will be the topic of a forthcoming paper). The results (Fig. 3.24) show that most of the sound absorbing behaviors are well captured, within the error bars. It might be worth mentioning that the good agreement for foam sample M11 between the DAD semi-empirical estimates, and the standing wave tube measurements, is partly due to the fact that the data used to generate the model from fits are also the ones taken to evaluate it. Another important remark is that DADs start from windows that are either opened or closed (Figs. 1 and 2 of Ref. [103]), whereas our geometrical model relies on membrane sizes δ that are identical for all the windows (Fig. 3.23). One may respectively speak about a 'binary' situation, as opposed to a rather 'continuous' case (both of these modeling choices being induced by cellular morphology observations, and fabrication processes). A comparison between Fig. 3.24(b) (right) and Fig. 3.23 (M11) suggests that an improvement of the sound absorption prediction is expected for - a local geometry model that would combine the 'binary' and 'continuous' features in a common periodic unit cell. One can find many other porous materials in literature on the subject where such a cellular morphology could be applicable. Dupont et al. [107] refer to air saturated materials containing dead-end porosities. The sound wave properties, prediction based on such a candidate needs more development and validation before its ultimate usefulness is made clear, and thus its discussion is beyond the scope of this paper.

Considering the small amount of input information $\{[\phi, k_0, L_m]$ (Ref. [3]), or $[L_m$ and $2r]$ (Ref. [2], Fig. 9(a)), the predictions are surprisingly close to the measured values of SAC_{NI} . This behavior is consistent with our macroscopic parameters, (see Tabs. 3.6 and 3.7, and Tab. I of Ref. [3]), but the elastic resonance is not resolved. Unlike the previous experimental or semi-empirical determinations of macroscopic parameters for real foam samples, [102],[103] we essentially determine high frequency transport parameters directly from the unit cells, finding a local geometry model in agreement with the key multi-scale experimental data ϕ , k_0 , and L_m .

After an analysis of the proposed approach at the microstructural level, transport coefficients and acoustic properties were shown to be predicted, with good agreement with experiment. The next question that must be addressed is: even if identifying local characteristic lengths is a good base of prediction for some transport coefficients and acoustic properties, does it work for analyzing transport coefficients in terms of microstructural parameters? In particular, does it work for polyurethane foam samples containing solid films or membranes which may have holes? From the computation of the viscous characteristic length Λ (see Ref. [12]), we can obtain a reduced geometrical representation Λ/T_{sm} as a function of the closure rate of membranes δ/δ_{max} (Fig. 3.25(a)); the closure rate of membranes δ/δ_{max} being defined as the ratio between the actual membrane size and the maximum membrane size of the squared windows. First, let δ be the membrane size; and δ_{max} the maximum membrane size such that the squared windows are entirely closed, $\delta_{max} = L/2 - r$ (Fig. 3.23). One can see that the closure rate of membranes δ/δ_{max} may progress from an opened-cell structure without any solid film ($\delta = 0$) to a fully closed-cell structure when $\delta/\delta_{max} \rightarrow 2.1$. More precisely, the maximum value of δ/δ_{max} depends on the porosity ϕ . In the interval of ϕ from 0.9 to 0.99, this maximum closure rate of membranes $\Omega = 20(\sqrt{3}(L/r) - 2) / ((L/r) - 2)$ varies from 2.1 to 1.8. Then, let T_{sm} be the mean throat size, $T_{sm} = (T_{s1} + T_{s2})/2$. In the interval of δ/δ_{max} from 0 to 1, two throat sizes exist, T_{s1} and T_{s2} , corresponding to the small (squared) and large (hexagonal) windows. Thus, for $\delta/\delta_{max} > 1$, $T_{s1} = 0$ and only T_{s2} differs from zero. In the interval of δ/δ_{max} from 0 to 1, the reduced representation of Λ attains a maximum of $\Lambda/T_{sm} = 1.4$ at Λ/T_{sm} as $\delta/\delta_{max} = 1/20$, and then decreases at larger closure rates, as expected for a perfect fluid flow in a porous medium whose viscous length is dominated by the smallest interconnections. Around $\delta/\delta_{max} = 1$, a singular behavior is observed, corresponding to the closing of squared windows. In the interval of δ/δ_{max} from 1 to around 1.75, the reduced representation of Λ is close to unity, $\Lambda/T_{sm} \approx 1$, and therefore the viscous characteristic length is close to the radius of the opening between interconnected pores, $\Lambda \approx T_{s2}/2$. This is a key result, relating explicitly the viscous characteristic lengths Λ to the throat size radius $T_{s2}/2$.

From the unit cell geometry and perfect fluid flow, we can obtain the thermal over viscous characteristic lengths ratio Λ'/Λ as a function of δ/δ_{max} (Fig. 3.25b). At low closure rates, Λ'/Λ is close to the typical macroscopic characteristic lengths ratio of a fibrous medium. Above $\delta/\delta_{max} = 1$, Λ'/Λ is always large compared with $\Lambda'/\Lambda = 2$. This shows that partially closed cell solid foams cannot be described in terms of a fibrous medium picture, although some sound absorbing properties agree surprisingly well

with the expectation of a fibrous medium. ^[102] Above $\delta/\delta_{\max} = 1.25$, Λ'/Λ begins to strongly increase over the value expected for a reticulated solid foam, which we interpret as an increasing contrast between the pore and interconnected throat sizes. Membranes have a strong overall contribution to this contrast. Although the closure rate of membranes and the closed pore content are not directly comparable, a similar trend is observable on DAD measurements who related the thermal over viscous lengths ratio to the closed pore content by a power law [see Fig. 6(f) and Eq. (7) of Ref. [103]]. Above this $\delta/\delta_{\max} = 1.25$, we also obtain a dependency of Λ'/Λ as a function of ϕ .

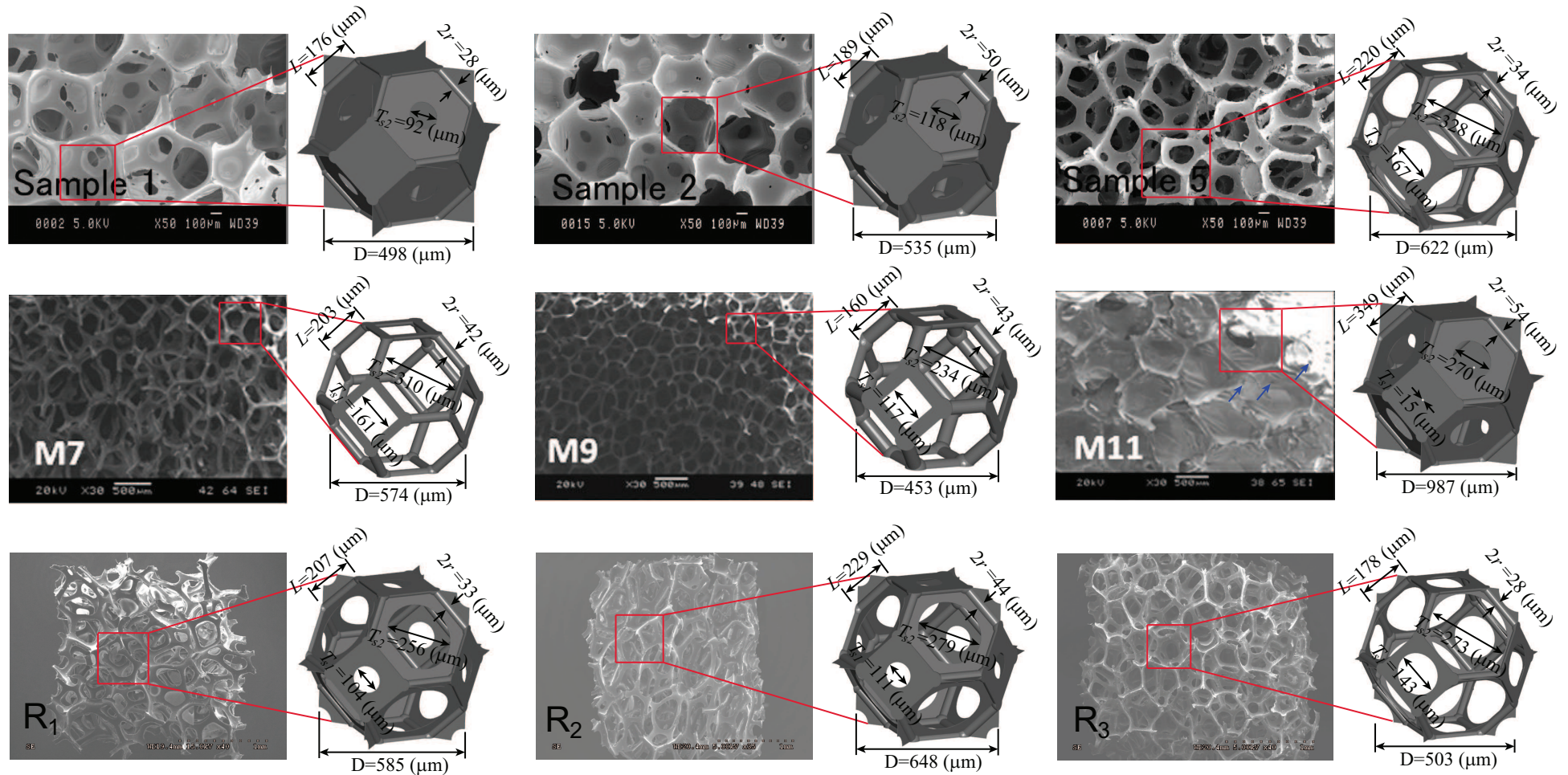


Fig. 3.23. SEM images and localization of specific unit cells to the critical paths (of the fluid flows) in real foam samples. (Top line) Micrographs and unit cells of Sample 1, Sample 2, and Sample 5 in Ref. [102]. (Middle line) Micrographs and unit cells of foam samples M7, M9, and M11 in Ref. [103]. Arrows represent some positions of T_{s1} . The difficulty to visualize T_{s1} in M11 is attributed to the low magnification of the corresponding SEM. (Bottom line) Micrographs and unit cells of foam samples R₁, R₂, and R₃ in Refs. [2] and [3].

Table 3.6. Comparison between experimental (Ref. [102]) and numerical estimates based on three-dimensional unit-cells (Ref. [3])

Sample	Method	ϕ (-)	Λ' (μm)	k_0 ($\times 10^{-9} \text{ m}^2$)	Λ (μm)	α_∞ (-)	k_0' ($\times 10^{-9} \text{ m}^2$)	L (μm)	$2r$ (μm)	$\delta/\delta_{\text{max}}$ (-)
1 ₍₃₎	Experimental estimates (Ref. [102])	0.978	47	0.124	23	1.111	-	178 ± 45	-	-
	Numerical computations	0.98	156 ± 42	0.124	39 ± 4	2.856 ± 0.551	3.74 ± 2.14	176 ± 52	28 ± 8	1.25 ± 0.13
2 ₍₃₎	Experimental estimates (Ref. [102])	0.946	70	0.242	35.1	1.1592	-	183 ± 33	-	-
	Numerical computations	0.945	168 ± 24	0.242	53 ± 4	2.333 ± 0.246	4.23 ± 1.18	189 ± 31	50 ± 8	1.15 ± 0.10
5 ₍₃₎	Experimental estimates (Ref. [102])	0.978	324	5.81	161	1.0584	-	219 ± 57	-	-
	Numerical computations	0.98	482 ± 175	5.81	277 ± 111	1.050 ± 0.053	13.38 ± 2.84	220 ± 46	34 ± 8	0.1 ± 0.15

Table 3.7. Comparison between semi-empirical correlations (Ref. [103]) and numerical estimates of multi-scale parameters derived from membrane-free [2] (M7 and M9, direct microstructural data) and membrane-based [3] (M11, iterative approach) three-dimensional unit-cells.

Sample	Method	ϕ (-)	Λ' (μm)	k_0 ($\times 10^{-9} \text{ m}^2$)	Λ (μm)	α_∞ (-)	k_0' ($\times 10^{-9} \text{ m}^2$)	L (μm)	$2r$ (μm)	$\delta/\delta_{\text{max}}$ (-)	
M7	Semi-empirical estimates (Ref. [103])	(a)	0.987	435	10.97	269	1.042	Na	203	42	0
		(b)	± 0.01	± 38	± 0.79	± 6	± 0.006	Na	± 43	± 3	± 0
	Numerical computations	(a)	0.990	499	10.09	322	1.05	Na	Na	Na	0
		(b)	± 0.004	± 216	± 8.67	± 139	± 0	Na	Na	Na	± 0
M9	Semi-empirical estimates (Ref. [103])	(a)	0.97	641	5.84	366	1.026	11.53	203	42	0
		(b)	± 0.02	± 314	± 3.99	± 181	± 0.015	± 7.51	± 43	± 3	± 0
	Numerical computations	(a)	0.968	268	4.94	183	1.059	Na	160	43	0
		(b)	± 0.01	± 16	± 0.16	± 1	± 0.001	Na	± 20	± 3	± 0
M11	Semi-empirical estimates (Ref. [103])	(a)	0.984	304	3.79	196	1.05	Na	Na	Na	0
		(b)	± 0.005	± 80	± 1.96	± 51	± 0	Na	Na	Na	± 0
	Numerical computations	(a)	0.94	398	2.94	228	1.043	6.02	160	43	0
		(b)	± 0.02	± 119	± 1.29	± 68	± 0.02	± 2.47	± 20	± 3	± 0
M11	Semi-empirical estimates (Ref. [103])	(a)	0.977	286	1.62	59	2.301	Na	358	85	Na
		(b)	± 0.01	± 30	± 0.16	± 12	± 0.092	Na	± 10	± 8	Na
	Numerical computations	(a)	0.987	321	2.14	48	2.396	Na	Na	Na	Na
		(b)	± 0.003	± 39	± 1.29	± 17	± 0.455	Na	Na	Na	Na
Numerical computations	(a)	0.98	336	1.62	96	1.907	16.30	349	54	0.95	
	(b)	± 0.02	± 8	± 1.29	± 7	± 0.118	± 0.92	± 12	± 16	± 0.04	

(a) direct measurements and characterization techniques;

(b) semi-empirical estimates.

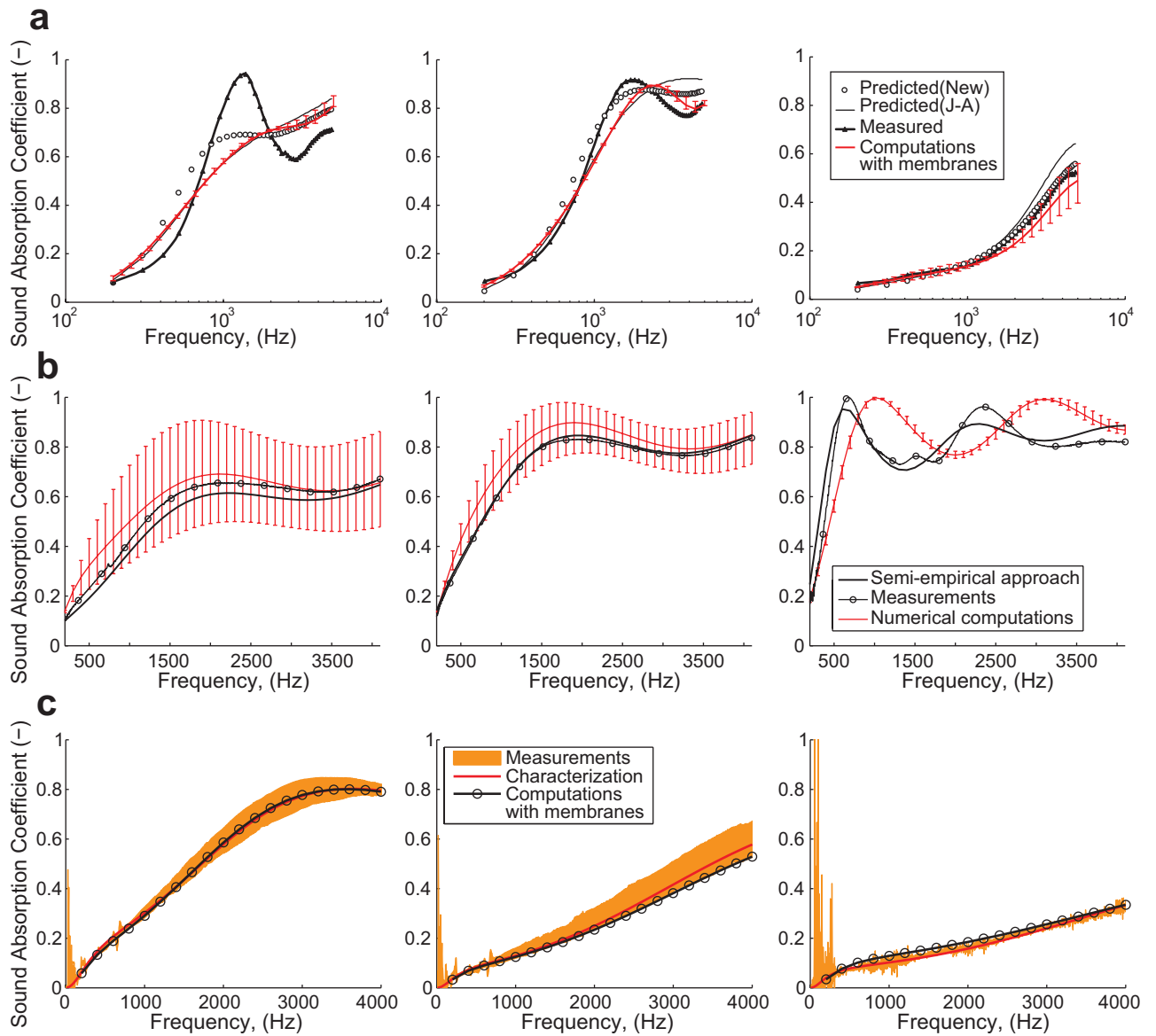


Fig. 3.24. Sound absorption coefficients at normal incidence (SAC_{NI}) of real foam samples. (a) SAC_{NI} vs frequency from Sample 1 (left), Sample 2 (middle), and Sample 5 (right) in Ref. [102]. The red thin lines in (a) correspond to our computations when using the unit cells shown in Fig. 3.23 to be compared with standing wave tube measurements (black thick lines with triangles). Also shown in black thin lines and circles are respectively the so-called Predicted(J-A) and Predicted(New) SAC_{NI} as obtained by Kino et al. in Ref. [102]. The error bars in red are dominated by the statistical errors in the ligament lengths measurements, which were determined from SD uncertainties using micrographs shown in Fig. 3.23 (top line). These uncertainties, in turn, determine the corresponding uncertainties in the SAC_{NI} . (b) SAC_{NI} vs frequency for foam samples M7, (left), M9, (middle), and M11 (right) in Ref. [103]. Error bars: the uncertainty region determined from all possible combinations of input parameters (doublets or triplets of input parameters, where an input parameter can take three possible values: mean, mean + SD, and mean - SD). (c) SAC_{NI} vs frequency for foam samples R_1 (left), R_2 (middle) and R_3 (right) in Ref. [3].

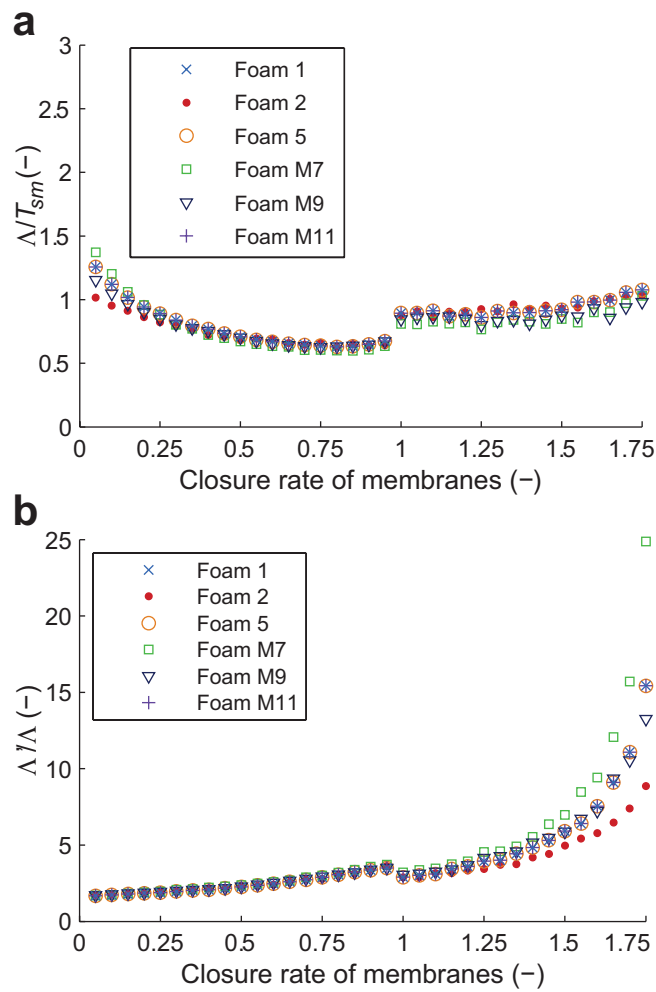


Fig. 3.25. Δ/T_{sm} (a) and Δ/Λ (b) vs closure rate of membranes for simulated foam samples of Refs. [102] and [103] shown in Fig. 3.23.

3.3.4. Discussion and conclusion

We performed numerical homogenization experiments of the compression wave propagation across an air-filled saturated rigid foam sample at the long wavelength regime, without any fits at micro- or macro- scales, enabling validation of the method for polyurethane foam samples. Similar unbiased methods can be applied to other porous materials, for example aluminum foams with spherical cells [108] or stratified foams made of recycled materials [109] and porous bones.

The results of this comparative investigation are a milestone in the quantitative analysis and understanding of this problem. The computation of transport parameters – how sound propagation and dissipation properties

relate to the cellular morphology – relies on determining the local velocity and temperature fields as a function of the low and high frequency asymptotic behaviors from idealized periodic unit cells. Assuming long wavelengths incident sound waves, we determined the local characteristic lengths, transport, and acoustic properties of solid foams with unprecedented multi-scale consistency. In particular, we determined the high frequency transport parameters of the foam by relating the permeability and closure rate of membranes up to the measured cell size. This new method for characterizing solid foams improved the identification of local characteristic lengths governing sound wave properties compared to previous experiments, ^{[102],[103]} and also allowed the observation of the idealized unit cell in a direct manner (see the Figs. 3.23 and 3.25). This work provides precise values of interdependent macroscopic parameters, such as ϕ , k_0 , k_0' , Λ , Λ' , α_0 , α_0' , and α_∞ ; which are essentially the physical signatures of the low Reynolds numbers hydrodynamics, electric conduction and diffusion-controlled reaction processes; and remain controversial from the experimental point of view. ^[110] Indeed, the numerical results are now accurate enough to benchmark novel experimental methods for the characterization of porous media properties problem.

An import step in our evaluation of the acoustic properties is the reduction of the disordered porous material to an equivalent three-dimensional periodic unit-cell (3D PUC). We remark that the critical path ideas that we used to obtain the scaling of the equivalent 3D PUC is essential for a correct understanding of the problem, and that correct local characteristic sizes can be obtained for the overall transport properties governing acoustic propagation at the upper scale if this feature is taken into account. An incorrect evaluation of the local characteristic sizes is obtained, for example, if one ignores the local heterogeneity in the pore sizes, and neglects the strong cross-section changes induced by the presence of membranes. In this picture, the 3D PUC linking the active sites of the real porous material is representative of the long-wavelengths acoustic properties; and, thus, the microstructure of the material can be optimized for specific purposes by adjusting the local geometry parameters of the model owing to the progress in manufacturing methods. ^[111] In addition, the computations shed new light on the interpretation of the viscous and thermal characteristic lengths in terms of either the throat size (Fig. 3.25(a)) or the cell size (Fig. 3.25(b)); the general features of the numerical results are substantiated by experimental data of the literature. In this sense, 3D PUC identification can serve as a kind of bridge that allows solving multi-scale problems that are intractable otherwise.

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Prise en compte des propriétés élastiques

4 Prise en compte des propriétés élastiques

Dans la partie 3, sous l'hypothèse d'un squelette poreux indéformable, il a été possible de déterminer la géométrie de cellules périodiques tridimensionnelles idéalisées, représentatives des propriétés de transport du milieu poreux réel. Toutefois, l'hypothèse d'indéformabilité du squelette n'est adaptée qu'à condition que celui-ci soit suffisamment rigide. Une fois la géométrie de la cellule périodique tridimensionnelle déterminée par les méthodes du chapitre 3, il est alors tentant d'utiliser à nouveau cette géométrie pour en déduire les propriétés linéaires élastique du squelette ainsi défini. Il est alors possible d'en déduire les propriétés poroélastiques de l'ensemble solide + fluide. On peut alors, soit vérifier si les hypothèses de solide indéformable sont admissibles pour le matériau considéré, soit utiliser les propriétés poroélastiques dans la prévision de l'isolement acoustique. Cette considération est importante pour évaluer les propriétés d'isolation phonique, car la souplesse du squelette donne généralement lieu à une augmentation des pertes par transmission dans la couche poreuse.

Ce chapitre est donc consacré à la détermination des propriétés élastiques du squelette solide par résolution d'un problème aux limites sur la cellule tridimensionnelle, en considérant cette fois-ci la partie solide de la cellule. Les conditions aux limites de la cellule sont celles issues de l'homogénéisation de structures périodiques. Toutefois, compte tenu des propriétés de la cellule, il est possible de remplacer les propriétés de périodicité des champs locaux par des conditions aux limites portant sur les composantes de déplacement ou de vecteur contrainte en bord de cellule.

La méthodologie adoptée n'est pas nouvelle, mais les travaux de la littérature portent généralement sur des cellules ouvertes ou complètement fermées. Une des originalités de ce travail est de traiter de cellules semi-ouvertes comportant des membranes.

Ce travail fait l'objet d'un article dont le manuscrit correspondant a été soumis pour publication dans un journal international à comité de lecture dont la référence est :

M. T. Hoang, G. Bonnet and C. Perrot, Linear elastic properties of sound absorbing foams, submitted to Journal of the Acoustical Society of America (2012).

Abstract

The present paper reports on the modeling of linear elastic properties of acoustically insulating foams with unit cells containing solid films or membranes at the junction between interconnected pores from a numerical homogenization technique. It combines fluid-flow induced microstructure identification with simulations of the effective Young's modulus and Poisson ratio from a mixture of routinely available laboratory measurements (porosity, permeability, cell size) and finite element calculations when the boundary conditions of the periodic unit cell take particular symmetric forms. This combination results in microstructural determination of the macroscopic coefficients entering into the Biot-Allard theory of wave propagation and dissipation through porous media. Precise control over pore morphology and mechanical properties of the base material renders this multi-scale approach particularly suitable for various advanced applications.

4.1 Introduction

Foams and cellular materials are very useful in the fields of building and transportation industries for sound proofing applications. ^[1] Those containing both an interconnected fluid phase and a deformable solid phase are highly desirable from the standpoint of pore accessibility (visco-thermal dissipations) and structural isolation (decoupling effect). Although semi-phenomenological approaches of modeling this type of materials have dominated the landscape, ^{[2]-[4]} multi-scale methods ^{[5]-[9]} have received attention due to the need to improve their acoustical properties using a synthetic control of their microstructure. Partially open-cell solid foams are an interesting class of cellular solids that are versatile precursors to advanced poroelastic materials with well-defined pore structures, sizes, and interface functionality. ^{[10]-[11]} However, there are three key issues that currently limit the utility of partially open-cell foams in sound proofing application. The straightforward calculation of elastic properties with a well-defined pore structure has only been achieved in a few cases. ^{[12]-[15]} Modeling the closure rate of membranes at the junction between interconnected pores has been challenging due to the non-homogeneity of the membrane closure rate itself. ^{[16]-[17]} In addition, fluid and solid mechanics at the cell size level that determine macroscopic transport and mechanical responses of foam are generally considered separately. Here, we present a new solid foams membrane-based strategy for the performance prediction of advanced poroelastic materials that simultaneously address all three of these issues.

Numerical simulations of the linear elastic properties of solid foams are mostly performed by finite element and FFT techniques. Gong *et al.* have compared the finite element approach and other models for predicting the response of open cell foams to uniaxial compression by using the space filling Kelvin cell characterized by the geometric characteristics found in polyester urethane foams.^[12] The cells are elongated in the rising direction; the ligaments are assumed to be straight, to have Plateau border cross-sections and non-uniform cross-sectional area distribution. The amount of material in the nodes is represented accurately. The linear elastic properties of the base material are measured directly from foam ligaments. Comparison between measurements and predictions is very favorable. This paper also contains a thorough review on elastic properties of solid foams to which the reader is referred. Jang *et al.* built on the work of Gong *et al.* by providing a more systematic and detailed study of the microstructure of the same foams and by using micro-computed X-ray tomography.^[13] The rendering are used in FE models including irregular ones to estimate the elastic properties of the foams. The elastic moduli predicted by the Kelvin cell models were found to be within engineering accuracy of predictions for random foams. Sullivan *et al.* have used finite element models of two elongated tetrakaidecahedron unit cells: one with cell faces and one without faces; they illustrated it by some analytical, numerical and experimental data.^[14] Thiyagasundaram *et al.* have derived a finite element based micromechanics procedure to calculate the elastic properties of equisized and elongated tetrakaidecahedral unit-cells without cell faces, which have a very high porosity and where the effect of varying strut cross sections plays an important role.^[15] Our main objective is to perform numerical homogenization simulations of linear elastic properties by the finite element technique in solid foams containing membranes with prescribed closure rates, in order to derive Young's modulus and Poisson ratio for real foam samples with various reticulation rates. Assuming scale separation, such a numerical estimate enables us to derive elastic coefficients for describing acoustic wave propagation in poroelastic materials on a microstructural basis.^[18]

We combined fluid-flow induced cellular morphology determination with linear elastic properties simulations by applying a version of periodic homogenization method ^[19] in the presence of polyhedral unit cells with cubic symmetry (namely non-elongated Kelvin cell models). Fluid-flow and heat transfer simulations in three-dimensional periodic unit cells of solid foams favor the formation of structured materials with essentially two local characteristic lengths, the pore and throat sizes.^{[20]-[21]} Importantly, the introduction of solid films or

membranes at the interconnection between pores that partially close them allows for capturing visco-inertial and thermal dissipation mechanisms in a consistent way at both micro and macro scales. Thus, the morphology of the three-dimensional unit cell with a specified closure rate of membranes reaches the point of becoming compatible with the *long-wavelength picture* ^[22] of acoustic waves propagating in poroelastic media, giving a non-elongated, acoustically robust, and microstructured geometry. The resultant bicontinuous structure is prevented from being deformable by virtue of the rigidity of the matrix. By employing an acoustically representative volume as the unit cell, microporous structures with a linearly elastic solid skeleton can be generated. Relying on a finite element based micromechanics procedure to calculate the elastic properties of unit cells obtained by simulated fluid-structure interactions, this strategy is distinguished from other micromechanical methods that result in the formation of (regular or random) microstructured cells arranged in a spatially periodic domain - but with fully opened or fully closed cells ^{[12]-[15]} ; and that generate nearly monodisperse foams with a low contrast between its local characteristic lengths (pore and throat sizes).

This paper is organized as follows. Materials and methods are presented in Sec. 4.2. First, local geometry features of the porous media which are going to be studied are described, together with the corresponding microstructure models. Then, purely geometrical macroscopic properties are derived: porosity ϕ and thermal length Λ' . They are completed by a measurement of the static permeability. The properties of transport parameters are briefly analyzed, such as the viscous length Λ , and the tortuosity α_∞ . Finally, effective mechanical properties of this class of microporoelastic materials are studied. Sec. 4.3 details the micromechanical model, and demonstrates how the main properties introduced at the end of Sec. 4.2 were obtained. Some general features of effective linear elastic properties were firstly recalled, and the version of periodic homogenization method developed in the framework of small perturbation hypothesis is presented when the boundary conditions of the periodic unit cell take particular symmetric forms. Polyhedral unit cells with cubic symmetry were analyzed. Results are presented and discussed in Sec. 4.4. This paper is ended by some concluding remarks.

4.2 Materials and methods

4.2.1. Regular solid foam structure with a specified closure rate of solid films or membranes

The spatial structure of the solid foam models proposed here is derived from routinely available laboratory measurements (permeability, porosity, ligament length) and transport processes simulation inside porous media. A general view on the representation of solid foams and on the generation processes can be found in the studies of Perrot *et al.* [20] and Hoang and Perrot [21]. The solid foam models which are going to be used are spatially periodic in all directions of space. Hence, the unit cell provides the basic structure from which the whole medium can be derived by translation along three coordinate axes. Two different samples of solid foams were studied; let us consider them more precisely in Fig. 4.1 which summarizes their more important geometrical characteristics. The ordered networks, analyzed in this article and shown in Fig. 4.1, are truncated octahedron networks with tetrahedral vertices; the ligaments of circular cross-section shape connect the spherical center of a regular tetrahedron, with plates partially opened or not forming the faces of this polyhedron. This regular foam structure of cubic symmetry follows this geometry from an initially open-cell structure, whose close rate of solid films or membranes is allowed to grow. This implies that the length of the ligaments of the model becomes compatible with measurements on scanning electron micrographs (see Ref. [21] for further details).

4.2.2. Purely geometrical macroscopic properties

The two samples, namely H_1 and H_2 were generated with the following measured porosity [23], $\phi = 0.93 \pm 0.01$ and $\phi = 0.97 \pm 0.01$ respectively. In the first sample, the ligament length L is smaller with larger ligament thickness $2r$ than in the second one (See Fig. 1. and Eq. (1) of Ref. [20]).

The pore solid surface S and pore volume V_p were systematically calculated for each solid foam sample with elementary spatial integration. These two quantities can be combined in order to define the length scale $\Lambda' = 2V_p / S$, a generalized hydraulic radius also named thermal characteristic length in the context of sound absorbing material. [4]

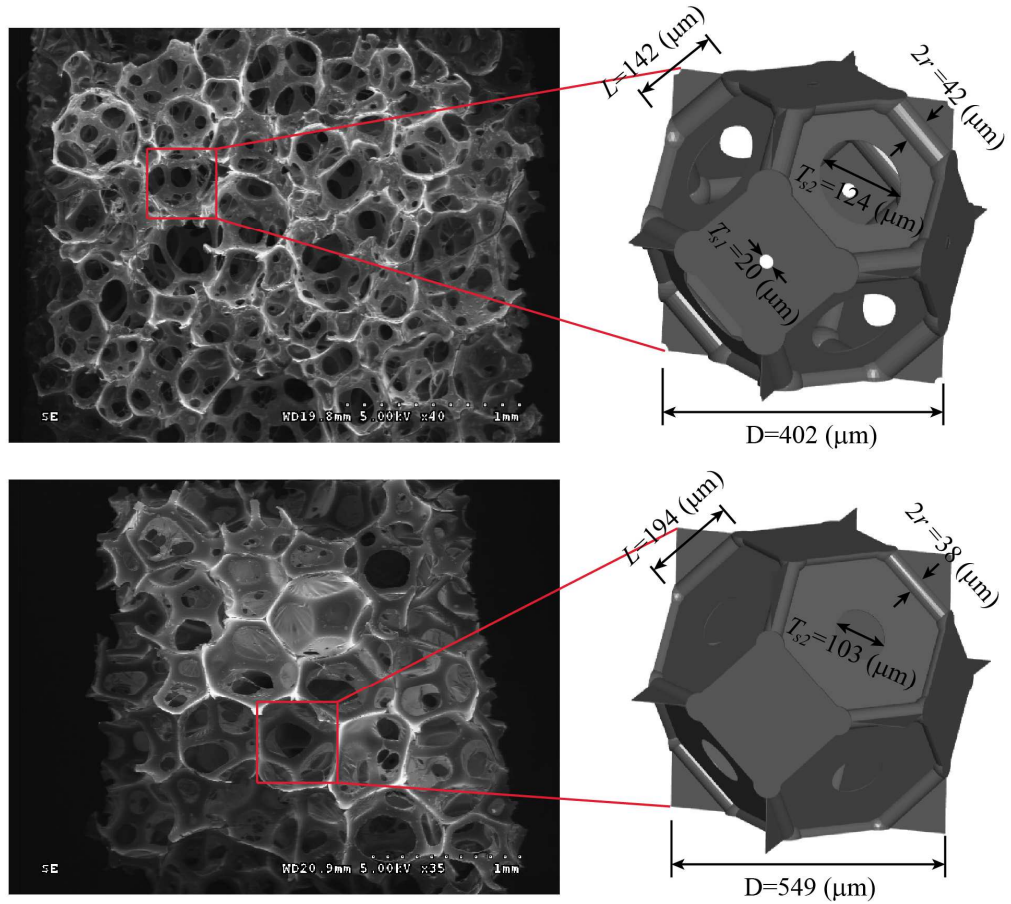


Figure 4.1. Geometrical characteristics of two different solid foam samples, H_1 (top) and H_2 (bottom).

4.2.3. Transport parameters

The macroscopic permeability k_0 of each solid foam sample was measured ^[24] and computed on a unit cell by solving the Stokes equations by a finite element method (Ref. [20]; Ref. [21]). The size of the cell is adapted in order to recover the measured permeability. The viscous characteristic length Λ can be used to characterize the throat size T_s of a porous media. Λ was introduced by Johnson *et al.* ^[3] It is essentially a volume-to-surface pore ratio with a measure weighted by the local value of the electric field $\mathbf{E}(X)$ in a conduction process,

$$\Lambda = \frac{2 \int |\mathbf{E}(X)|^2 dV}{\int_S |\mathbf{E}(X)|^2 dS}. \quad (4.1)$$

The tortuosity α_∞ can be calculated when the electric field is known,

$$\alpha_\infty = \frac{\langle \mathbf{E}(\mathbf{X})^2 \rangle}{\langle \mathbf{E}(\mathbf{X}) \rangle^2}, \quad (4.2)$$

where $\langle \bullet \rangle$ denotes a fluid phase average.

This quantity was also obtained from our computation on the unit cell.

4.2.4. Effective mechanical properties

The effective mechanical properties were obtained for the two different solid foam models either from both finite element calculations or from experiments on real samples, namely H_1 and H_2 . The effective linear-elastic properties of the solid networks are determined by a finite element scheme operating on discretized representations of the structure, see Sec. 4.3 for further details. The material is assumed to be locally isotropic and linear-elastic; the fluid phase is modeled as a vacuum.

The effective elastic longitudinal modulus E_L and Poisson ratio ν_L are obtained by applying two macroscopic external strains on the cube that bounds the solid foam model: a tensile strain and a shear strain related to the principal coordinate directions. Since the solid foam models have cubic symmetry, three independent elastic constants C_{11} , C_{22} and C_{44} (in contracted notations) exist in the elasticity matrix, whose identification requires two numerical experiments using periodic boundary conditions, to express E_L and ν_L . The effective elastic longitudinal modulus E_L and Poisson ratio ν_L are obtained by applying the relations $E_L = (C_{11}^2 + C_{11}C_{12} - 2C_{12}^2) / (C_{11} + C_{12})$ and $\nu_L = C_{12} / (C_{11} + C_{12})$ which are valid for the effective elastic properties along the principal coordinate directions, [100], [010] or [001] of materials with cubic symmetry.

The effective elastic properties of an effective transversely isotropic material and of an effective isotropic material correspond to an average of the properties obtained by random orientation of the unit cell obtained by arbitrary rotation around a given axis or for any arbitrary rotation in space. In particular, the detailed expression obtained by this angular averaging procedure yields for the isotropic case : $E_I = [5(C_{11} - C_{12})(C_{11} + 2C_{12}) C_{44}] / [C_{44} (3C_{11} + C_{12}) + 2(C_{11} - C_{12})(C_{11} + 2C_{12})]$, and $\nu_I = [- (C_{11} - 3C_{12}) C_{44} + (C_{11} - C_{12}) (C_{11} + 2C_{12})] / [C_{44} (3C_{11} + C_{12}) + 2(C_{11} - C_{12}) (C_{11} + 2C_{12})]$;

Fig. 4.2 shows the displacement and stress fields of the solid foam model corresponding to the real sample H₂ during the shear strain and the tensile strain numerical experiments.

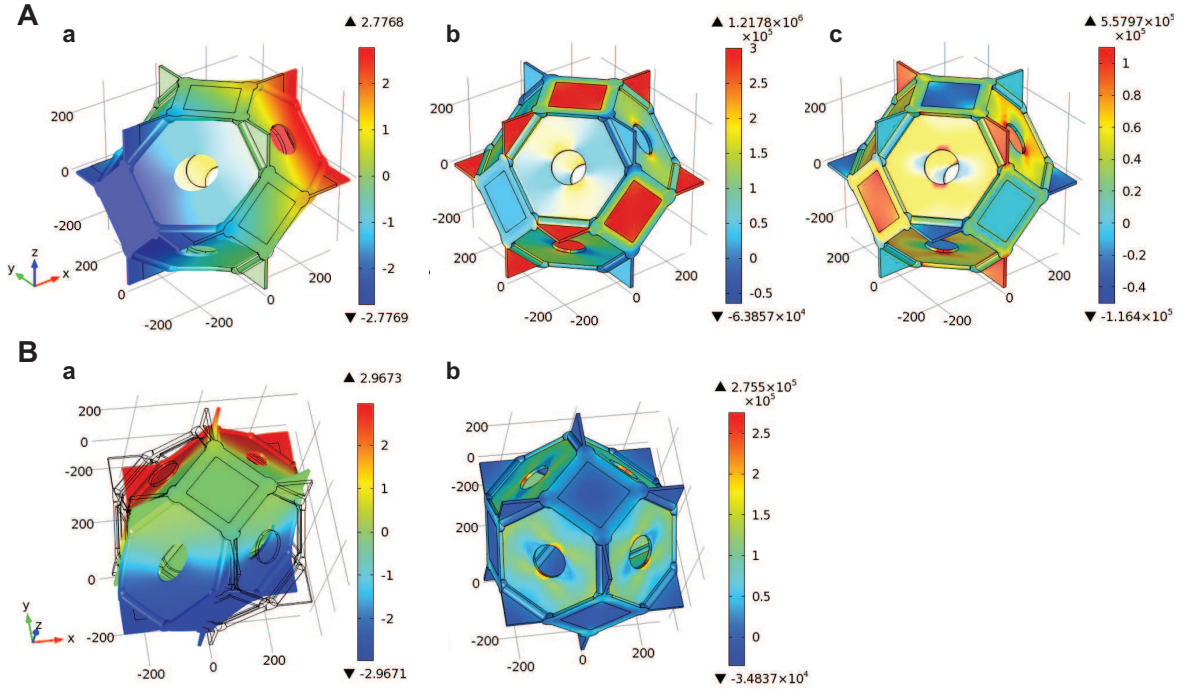


Figure 4.2. Numerical experiments allowing to identify the elastic constants C_{11} , C_{22} and C_{44} (in contracted notations) of the elasticity matrix for a solid foam model having cubic symmetry; illustrated with foam sample H₂. **(A)** Tensile strain numerical experiment. A uniform strain vector $E = E_{11} \mathbf{e}_1 \otimes \mathbf{e}_1$ is applied (along the principal coordinate direction [100]) in the equivalent homogeneous material with E_{11} showing (a) the displacement field (μm), (b) the stress field σ_{11} (N/m^2) and (c) the stress field σ_{22} (N/m^2). The data are then averaged over the periodic unit cell as follows: $\Sigma_{11} = \frac{1}{V} \int \sigma_{11} dV$, $\Sigma_{22} = \frac{1}{V} \int \sigma_{22} dV$, and $\Sigma_{33} = \frac{1}{V} \int \sigma_{33} dV$. Elastic constants C_{11} and C_{12} are computed using the relations $\Sigma_{11} = C_{11} E_{11}$ and $\Sigma_{22} = C_{12} E_{11}$ (or $\Sigma_{33} = C_{12} E_{11}$). With a transversely isotropic configuration, the effective Young's modulus was found to be unchanged; $E_{Tl} = E_L$ and $\nu_{Tl} = \nu_L$. Contrary to the transversely isotropic configuration, the effective Young's modulus and Poisson ratio for an isotropic material configuration were modified; $E_l \neq E_L$ and $\nu_{Tl} \neq \nu_L$. **(B)** Shear strain numerical experiment. A uniform strain vector $E = E_{12} (\mathbf{e}_1 \otimes \mathbf{e}_2 + \mathbf{e}_2 \otimes \mathbf{e}_1)$ is applied in the equivalent homogeneous material with $E_{12} = 0.01$, showing (a) the displacement field (μm) and (b) the stress field σ_{12} (N/m^2). The data are then averaged over the periodic unit cell as follows: $\Sigma_{12} = \frac{1}{V} \int \sigma_{12} dV$ (Eq. 4.17). The last elastic constant C_{44} is computed using the relation $\Sigma_{12} = C_{44} E_{12}$. See also Sec. 4.3.

4.3 Microporoelastic analysis

4.3.1. Elastic stiffnesses and compliances

Rendering to a fixed coordinate system $x_1 = x_2 = x_3$, let σ_{ij} and ε_{ij} be the stress and strains, respectively, in an anisotropic elastic material. The stress-strain law can be written as

$$\sigma_{ij} = c_{ijks} \varepsilon_{ks}, \quad (4.3)$$

in which c_{ijks} are the elastic stiffness coefficients which are components of a fourth rank tensor. They satisfy symmetry conditions. The inverse of this relation is written as

$$\varepsilon_{ij} = s_{ijks} \sigma_{ks}, \quad (4.4)$$

where s_{ijks} are the elastic compliances which are components of a fourth rank tensor. They possess the same symmetry conditions as the stiffness tensor.

4.3.2. Contracted notations

Introducing the contracted notation (Voigt, 1910) the stress-strain law (4.3) and (4.4) can be written as

$$\sigma_i = c_{ij} \varepsilon_j, \quad c_{ij} = c_{ji} \quad (4.5)$$

In other words, due to the symmetry $\sigma_{ij} = \sigma_{ji}$, only six independent components can appear in the stress tensor. These six independent components of stress and strain can be “contracted” to a single index notation by the following notation:

$$\sigma_{ii} = \sigma_i, \quad \varepsilon_{ii} = \varepsilon_i \quad (i=1, 2, 3). \quad (4.6)$$

This contraction of the stress and strain tensors to vector (with six components) according to the following indices exchange rule $(1, 1) \rightarrow 1$, $(2, 2) \rightarrow 2$, $(3, 3) \rightarrow 3$, $(2, 3) \rightarrow 4$, $(1, 3) \rightarrow 5$, $(1, 2) \rightarrow 6$, means that the fourth order elastic constant tensor may be contracted to a two-index notation by the application of the following conventions:

$$\underline{\underline{\sigma}} \rightarrow \underline{\underline{\sigma}} := \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sqrt{2}\sigma_{23} \\ \sqrt{2}\sigma_{31} \\ \sqrt{2}\sigma_{12} \end{pmatrix} \quad \text{and} \quad \underline{\underline{\varepsilon}} \rightarrow \underline{\underline{\varepsilon}} := \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \sqrt{2}\varepsilon_{23} \\ \sqrt{2}\varepsilon_{31} \\ \sqrt{2}\varepsilon_{12} \end{pmatrix}. \quad (4.7)$$

Note that there is a factor of $\sqrt{2}$ involved in the contraction of $\underline{\underline{\sigma}}$ and $\underline{\underline{\varepsilon}}$ in order to maintain a comprehensive description of $\underline{\underline{\sigma}}$ and $\underline{\underline{\varepsilon}}$, such as the equality $\underline{\underline{\sigma}}:\underline{\underline{\varepsilon}} = \underline{\underline{\sigma}}.\underline{\underline{\varepsilon}}$ holds. Following this convention, the generalized Hooke's law relationship between the elements of the stress and strain tensor (represented as six element column vectors) can be compactly written in matrix notations as

$$\underline{\underline{\sigma}} = \underline{\underline{c}}.\underline{\underline{\varepsilon}}, \quad (4.8)$$

where $\underline{\underline{c}}$ is a six-by-six symmetric matrix. An expanded form of the matrix notation is given below:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sqrt{2}\sigma_{23} \\ \sqrt{2}\sigma_{31} \\ \sqrt{2}\sigma_{12} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ & & c_{33} & c_{34} & c_{35} & c_{36} \\ \text{sym} & & & c_{44} & c_{45} & c_{46} \\ & & & & c_{55} & c_{56} \\ & & & & & c_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \sqrt{2}\varepsilon_{23} \\ \sqrt{2}\varepsilon_{31} \\ \sqrt{2}\varepsilon_{12} \end{pmatrix}. \quad (4.9)$$

As indicated previously, it is frequently useful to express the strain in terms of the stress,

$$\underline{\underline{\varepsilon}} = \underline{\underline{s}}.\underline{\underline{\sigma}}, \quad (4.10)$$

Where $\underline{\underline{s}}$ is the compliance tensor comprised of the 21 independent elements s_{ij} . The quantity $\underline{\underline{s}}$ is the inverse of $\underline{\underline{c}}$ in the matrix sense $\underline{\underline{s}} = \underline{\underline{c}}^{-1}$. The twenty-one coefficients, s_{ij} , are called the compliance constants.

4.3.3. Material symmetries

This paper addresses the linear elastic properties of partially closed cell solid foams with membrane-based tetrakaidecahedral cellular

morphologies. This geometry exhibits a cubic symmetric. In this case, the elasticity tensor is defined by three independent coefficients, the elastic stiffnesses:

$$\mathbf{C}^{cubic} = \begin{pmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ & C_{11} & C_{12} & 0 & 0 & 0 \\ & & C_{11} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ sym & & & & C_{44} & 0 \\ & & & & & C_{44} \end{pmatrix} . \quad (4.11)$$

Consequently, the elastic behavior can be described based on only three independent elastic parameters e.g. C_{11} , C_{12} , C_{44} . Alternatively, the elastic compliances S_{ij} might be expressed in terms of the elastic stiffnesses:

$$\begin{cases} S_{11} = S_{22} = S_{33} = \frac{C_{11} + C_{12}}{C_{11}^2 + C_{11}C_{12} - 2C_{12}^2} \\ S_{12} = S_{21} = S_{13} = S_{31} = S_{23} = S_{32} = \frac{-C_{12}}{C_{11}^2 + C_{11}C_{12} - 2C_{12}^2} \\ S_{44} = S_{55} = S_{66} = \frac{1}{C_{44}} \end{cases} \quad (4.12)$$

Attention will be directed in the next sections to (1) the specification of two kinds of numerical experiments required to completely characterize the elastic compliances and stiffnesses in terms of the base material's properties and to (2) the obtention of transversely average isotropic properties or isotropic properties obtained from the results of the numerical experiments.

4.3.4. Numerical experiments

A simple tensile numerical experiment is such that σ_{11} is different from zero and all other loads deleted such as $\sigma_{ij} = 0$ for i or $j = 2, 3, \dots, 6$. Under these loading conditions, the relationship $\boldsymbol{\varepsilon} = \underset{\sim}{\mathcal{S}}\boldsymbol{\sigma}$ greatly simplifies to yield

$$\begin{cases} \varepsilon_{11} = S_{11}\sigma_{11} \\ \varepsilon_{22} = S_{12}\sigma_{11} \\ \varepsilon_{33} = S_{13}\sigma_{11} \\ \varepsilon_{23} = \varepsilon_{31} = \varepsilon_{12} = 0 \end{cases} \quad (4.13)$$

The longitudinal elastic modulus and Poisson ratios were deduced by definition:

$$\begin{cases} E_L = \frac{\sigma_{11}}{\varepsilon_{11}} = \frac{1}{S_{11}} \\ \nu_{12} = \frac{\varepsilon_{22}}{\varepsilon_{11}} = -\frac{S_{12}}{S_{11}} \\ \nu_{13} = \frac{\varepsilon_{33}}{\varepsilon_{11}} = -\frac{S_{13}}{S_{11}} \end{cases} \quad (4.14)$$

For materials with cubic symmetry, substituting the expressions in (4.12) into equation (4.14) leads to

$$\begin{cases} E_L = \frac{C_{11}^2 + C_{11}C_{12} - 2C_{12}^2}{C_{11} + C_{12}} \\ \nu_{12} = \nu_{13} = \frac{C_{12}}{C_{11} + C_{12}} \end{cases} \quad (4.15)$$

which relates the longitudinal modulus and Poisson ratios to the elastic constants.

To calculate the macroscopic elastic constants of materials a macroscopic strain is applied to the unit cell (Fig. 4.2). The displacement field inside the cell is the solution of the cell problem obtained from the homogenization of periodic media.

It is given by:

$$\mathbf{u} = \mathbf{E} \cdot \mathbf{x} + \mathbf{u}_{\text{per}} \quad (4.16)$$

where \mathbf{u}_{per} complies with periodicity conditions on the cell boundary. It can be shown ^[19] that, accounting for the symmetries of the cell, these periodicity conditions can be changed into mixed boundary conditions enforcing that some components of \mathbf{u} are equal to the similar components of $\mathbf{E} \cdot \mathbf{x}$, while expressing that the other components of the traction vector are null. For further details the reader is referred to Sec. 4.2.1 of Ref. [25].

The components of the macroscopic effective stress tensor $\underline{\underline{\Sigma}}$ induced by the macroscopic strain $\underline{\underline{E}}$ are obtained by averaging the local stress tensor $\underline{\underline{\sigma}}$ obtained after solving the cell problem.

$$\underline{\underline{\Sigma}} = \langle \underline{\underline{\sigma}} \rangle_V = \frac{1}{V} \int_V \underline{\underline{\sigma}} dV . \quad (4.17)$$

But, from another point of view, the macroscopic stress tensor is related to the macroscopic strain tensor by:

$$\forall \underline{\underline{E}}, \quad \underline{\underline{\Sigma}} = \langle \underline{\underline{\sigma}} \rangle_D = \underline{\underline{C}} \cdot \underline{\underline{E}} \quad (4.18)$$

This computation produces therefore some components of the elasticity tensor.

For the materials with cubic symmetry which contain only three independent elastic coefficients, only two numerical experiments are required to find out completely the elasticity matrix, one by using a macroscopic tensile strain and another one by using a macroscopic shear strain.

In a first step we pay attention to a tensile strain numerical experiment for which we impose a uniform macroscopic strain, $\underline{\underline{E}} = E_{11} \mathbf{e}_1 \otimes \mathbf{e}_1$, from which two elastic constants are found from the macroscopic stress tensor:

$$\begin{cases} C_{11} = \Sigma_{11} / E_{11} \\ C_{12} = \Sigma_{22} / E_{11} \end{cases} \quad (4.19)$$

In a second step, we impose a uniform macroscopic strain $\underline{\underline{E}} = E_{12} (\mathbf{e}_1 \otimes \mathbf{e}_2 + \mathbf{e}_2 \otimes \mathbf{e}_1)$ to model a shear strain numerical experiment. This leads to

$$C_{44} = \Sigma_{12} / E_{12} \quad (4.20)$$

This completes the elasticity tensor.

4.3.5. Macroscopically transversely isotropic and isotropic material configurations

The cubic cell used previously has been built by using some properties of the local geometry. However, the real material seen in Fig. 4.1 presents cells which are similar with the cubic cell, but whose shape and orientation departs from regularly aligned cells. In practice, real foams display either

transversely isotropic properties (with a rotational symmetry axis along the growth direction of the fibers) or fully isotropic properties.

The properties of an equivalent isotropic material can be built on the basis of the cubic cell by considering that the real material is made of cubic cells which have an arbitrary orientation. It can be obtained by computing the elasticity tensor $S'_{ij}(\psi, \theta, \phi)$ for an arbitrary orientation of the axes of the cell, using the usual axes transformation of a tensor, and by averaging these properties over all orientations, i.e. on all possible Euler angles (ψ, θ, ϕ) :

$$S_{ij}^I = \frac{1}{8\pi^2} \int_0^{2\pi} \int_0^\pi \int_0^{2\pi} S'_{ij}(\psi, \theta, \phi) \sin \theta d\psi d\theta d\phi \quad (4.21)$$

This produces finally a relationship between the components of the equivalent isotropic tensor S_{ij}^I and those of the elastic properties of the cubic material.

$$\begin{cases} S_{11}^I = S_{22}^I = S_{33}^I = \frac{3S_{11} + 2S_{12} + 2S_{44}}{5} \\ S_{12}^I = S_{21}^I = S_{13}^I = S_{31}^I = S_{23}^I = S_{32}^I = \frac{S_{11} + 4S_{12} - S_{44}}{5} \\ S_{44}^I = S_{55}^I = S_{66}^I = \frac{2S_{11} - 2S_{12} + 3S_{44}}{5} \end{cases} \quad (4.22)$$

For a transversely isotropic material, the computation is the same, but by restricting the orientation of the axes to the ones perpendicular to the growth direction, i.e. by averaging over only all values of θ .

$$S_{ij}^{TI} = \frac{2}{\pi} \int_0^{\pi/2} S'_{ij}(\theta) d\theta \quad (4.23)$$

This produces finally the components of the compliance tensor, which possess the properties of a transversely isotropic tensor, including $S_{22} - S_{23} = S_{44}$:

$$\left\{ \begin{array}{l} S_{11}^{II} = S_{11} \\ S_{22}^{II} = S_{33}^{II} = \frac{3S_{11} + S_{12} + S_{44}}{4} \\ S_{44}^{II} = \frac{S_{11} - S_{12} + S_{44}}{2} \\ S_{55}^{II} = S_{66}^{II} = S_{44} \\ S_{12}^{II} = S_{21}^{II} = S_{13}^{II} = S_{31}^{II} = S_{12} \\ S_{23}^{II} = S_{32}^{II} = \frac{3S_{12} + S_{11} - S_{44}}{4} \\ S_{24}^{II} = S_{42}^{II} = 0 \\ S_{34}^{II} = S_{43}^{II} = 0 \end{array} \right. \quad (4.24)$$

4.4 Results and discussion

4.4.1. Geometrical and transport properties

Before looking at the mechanical results, it is of interest to consider the purely geometrical macroscopic properties (ϕ , Λ') and transport parameters (k_0 , Λ , α_∞) which were obtained on the cubic cell.

Note that in the regular solid foam structure, each unit cell can be generated by taking into account the experimental uncertainty in the input parameters $\phi \pm \Delta\phi$ ($\phi^+ = \phi + \Delta\phi$, $\phi^- = \phi - \Delta\phi$), $k_0 \pm \Delta k_0$ ($k_0^+ = k_0 + \Delta k_0$, $k_0^- = k_0 - \Delta k_0$) and $L \pm \Delta L$ ($L^+ = L + \Delta L$, $L^- = L - \Delta L$). Such a generation process can be useful to obtain more detailed results for a given foam sample. For instance, the successive values of Λ' , Λ and α_∞ were computed using 1 (mean) + 2^3 unit cells; and the standard deviation Δ were subsequently calculated from these $1+2^3$ numerical results.

The experimental data (ϕ , k_0 , Λ , Λ' , α_∞) were obtained at the Groupe d'Acoustique de l'Université de Sherbrooke (GAUS) laboratory (Québec, Canada). Three of these parameters were characterized using experimental techniques; ϕ [23], k_0 [24] and α_∞ [26]. Then, the determination of the missing parameters, Λ' and Λ , is based on an inverse procedure using a cost function minimization algorithm. The principle of the method consists in minimizing the differences between measured and modeled sound absorption curves, from a standing wave tube setup and Johnson-Champoux-Allard [3]-[4] model respectively (FoamX software).

The purely geometrical macroscopic properties (Λ') and transport parameters (Λ , α_∞) computed from the course of this multi-scale approach are in a good agreement with independently obtained experimental data, especially when

standard deviations are taken into account as seen in Table 4.1. This means that the identified microstructures presented in Fig. 4.1 are representative of the macroscopic properties of the real porous materials under study from both purely geometrical macroscopic quantities and transport processes point of views within experimental uncertainties. We shall now try to examine the representativity of these microstructures in terms of their effective linear poroelastic properties.

Foam	Method	ϕ (-)	Λ' (μm)	k_0 ($\times 10^{-10} \text{ m}^2$)	Λ (μm)	α_∞ (-)
H ₁	Computation		139 ± 26		44 ± 5	1.58 ± 0.33
	Measurements	0.93 ± 0.01		3.87 ± 0.32		2.44 ± 0.57
	Inverse characterization		136 ± 36		45 ± 16	
H ₂	Computation		171 ± 48		41 ± 6	2.79 ± 0.61
	Measurements	0.97 ± 0.01		1.59 ± 0.31		1.24 ± 0.05
	Inverse characterization		103 ± 108		48 ± 19	

Table 4.1. Macroscopic parameters: comparison between computational and experimental (measurements and inverse characterization) results

4.4.2. Linear elastic properties

The basic ingredients of flexible urethane foams of the type considered in this study are ester resin (or polyol), diisocyanate, water, catalysts and surfactants. [27] Foam H₁ was manufactured by Faurecia from the standpoint of these typical ingredients with a view of lowering significantly the Young's modulus of the resulting porous material when compared to standard plastic foams, and Foam H₂ is a commercial product. The samples represent cylindrical subsections of large panels of diameter equal to 44.5 mm. Their heights are equal to 25 mm for H₁; and 10 mm, 15 mm, and 20 mm for H₂.

Accurate literature values for the microscopic Young's modulus E_μ and Poisson ratio ν_μ are not available because these values are depending on processing strategy. The values obtained in the literature are scattered within a range of more than one order of magnitude, lying between 2 and 30 MPa [28]. As mentioned by Gong et al. [12], some foam chemists believe that the polymer flow resulting from the foaming process may cause preferential alignment of the long molecules of the material along the ligaments. Since these characteristics may not be easily achievable in bulk material, they recommend that the mechanical properties of the polymer be measured directly from foam ligaments. To do so, a modified microscopic testing stage was used, to test ligaments of a 3 ppi foam sample ($E_L \approx 4.5 \pm 0.3 \text{ MPa}$). However, the main conclusion which can be drawn is that specific measurements must be made on the material.

Without these measurements at the present time, the range 2-30 MPa will be kept in the following.

Due to the strong dependence of the foam properties, all results of the computations performed along the lines of the previous section were expressed in terms of non-dimensional values; for instance $E_L^{nd} = E_L/E_\mu$. These non-dimensional data were first computed with membranes. However, a parameter which has a strong effect on the results is the membrane thickness. A rough estimate of the membrane thickness $t \approx 10 \mu\text{m}$ was obtained by optical microscopy. Yet, it is worth to mention that the thickness is probably not constant, and that the estimation of thickness can also be overestimated through optical microscopy observation. For these reasons, two results for the model were given in Tab. 4.2: with and without membranes.

Foam	Method	Σ_{11}^{nd} ($\times 10^{-3}$)	Σ_{22}^{nd} ($\times 10^{-3}$)	Σ_{12}^{nd} ($\times 10^{-3}$)	C_{11}^{nd} ($\times 10^{-3}$)	C_{12}^{nd} ($\times 10^{-3}$)	C_{44}^{nd} ($\times 10^{-3}$)	E_{TI}^{nd} ($\times 10^{-3}$)	ν_{TI} (-)	E_I^{nd} ($\times 10^{-3}$)	ν_I (-)
H ₁	Without membranes	0.12 ± 0.02	0.09 ± 0.01	0.03 ± 0.01	12.44 ± 2.30	9.04 ± 1.25	2.91 ± 0.86	4.83 ± 1.44	0.42 ± 0.01	4.4 ± 1.29	0.43 ± 0.01
	With membranes	0.40 ± 0.03	0.15 ± 0.01	0.17 ± 0.02	39.51 ± 2.63	15.09 ± 1.04	17.12 ± 1.90	31.17 ± 2.14	0.28 ± 0.01	25.60 ± 2.19	0.32 ± 0.01
H ₂	Without membranes	0.044 ± 0.019	0.038 ± 0.015	0.005 ± 0.004	4.39 ± 1.89	3.84 ± 1.46	0.50 ± 0.38	0.81 ± 0.63	0.47 ± 0.01	0.76 ± 0.59	0.47 ± 0.01
	With membranes	0.30 ± 0.07	0.13 ± 0.02	0.17 ± 0.03	30.21 ± 6.81	12.64 ± 2.21	16.74 ± 3.00	22.75 ± 5.81	0.30 ± 0.01	22.18 ± 4.61	0.30 ± 0.00

Table 4.2. Computed non-dimensional elastic properties of foam samples H₁ and H₂

The experimental values of Young's modulus are given in Table 4.3. They are obtained by uniaxial compression experiments [29]-[30] on polyurethane foam samples, namely H₁ and H₂; see Fig. 4.1. For comparison with the computed results, the relative Young's moduli of both foams are given for both ultimate values of E_μ (2-30MPa). These values can be compared on the same table with relative Young's moduli coming from the computations.

Obviously, the range of the experimental relative moduli E_{exp}/E_μ follows the one of the polyurethane base material modulus E_μ . However, some conclusions can still be drawn. It can be seen for H₁ that the estimation of the relative modulus with membranes does not lie between the values given by experiments. By contrast, the computed relative modulus without membranes is compatible with this experimental range of values. It means that the membrane thickness used for this foam is probably

overestimated when constructing the periodic cell. On the contrary, the value of the relative Young's modulus obtained for H₂ without membranes is clearly outside the admissible range of relative moduli. It means that the membranes must be taken into account in estimating the Young's modulus of the foam. This last result confirms the need to take into account the membranes when estimating the elastic properties of such foams.

Foam	$1000E_{\text{exp}}$ (MPa)	$1000E_{\text{exp}}/E_{\mu(\text{min})}$	$1000E_{\text{exp}}/E_{\mu(\text{max})}$	$1000E_{\text{comp}}/E_{\mu}$ (with membranes)	$1000E_{\text{comp}}/E_{\mu}$ (without membranes)
H ₁	18	9	0.6	25.6	4.4
H ₂	111	55	3.7	22.2	0.76

Table 4.3. Comparison between measured E_{exp} and computed E_{comp} elastic Young's moduli. E_{comp} stands for the macroscopically isotropic computed value, E_I

4.4.3. Acoustical properties

Let us go further to one of the major purpose of this study, namely the determination of the acoustical properties of the poroelastic medium. The multi-scale determination of purely geometrical parameters, transport, and elastic properties was studied for the regular packing of partially open cell structures for two samples of solid foams. The numerical and experimental data relative to these macroscopic parameters are gathered in Tables 4.1 and 4.3. These results can be used as input parameters of approximate but robust semi-phenomenological models ^{[3]-[4]} ("universal curves") as summarized in Appendix B of Ref. [20]. We shall focus the presentation on the sound absorption $\alpha(\omega)$ and sound transmission STL(ω) properties relative to the reconstructed poroelastic media (and we shall point out the features specific to each of these solid foams, H₁ and H₂).

Some results relative to $\alpha(\omega)$ and STL(ω) are displayed in Fig. 4.3. In Fig. 4.3A(b), it is seen that a rigid model fails at describing the low frequency behavior of the sound transmission loss, whereas the acoustical behavior of foam sample H₁ of measured density equal to 68 kg/m³ is well described by a limp model ^[31]; Fig. 4.3A(a-b). It is possible to verify this property with the experimental behavior measured from a standing wave tube, since $\lim_{\omega \rightarrow 0} \text{STL}(\omega) \approx 0$ (rigid body motion of the frame); Fig. 4.3A(b).

Using the measured density of H₂ of 25 kg/m³, the sound absorbing behavior of the low-density foam was estimated using a limp model, in a good agreement with the measurements in the impedance tube; Fig. 4.3B(a). The additional sound absorption generated using a limp model for the low density foam was determined to be due to an added rigid body

motion of the overall sample (because of the apparent mass added in the equivalent fluid at the resonant frequency). The sound transmission loss that dropped in the low frequency range with the resonance frequency were more elastically sensitive compared with sound absorbing properties whose essential behavior can be captured with a limp model.

Using the elastic properties of the foam sample (H_1), which can be established using the method of micromechanics described in this paper, we obtain a prediction for the sound absorption and sound transmission loss in agreement with experimental results in a standing wave-tube, Fig. 4.3A. This is done without prior experimental knowledge of the sound transmission loss behavior in the asymptotic low frequency range. Fig. 4.3B also illustrates the utility of the elastic results derived in the preceding sections. We begin by considering the Frame Acoustical Excitability (FAE) of the foam sample.^[32] By computing the FAE, our interest is (1) to examine the edge constraint effect on the acoustic behavior of the foam and (2) to illustrate the effect of the elastic properties of the foam on the behavior of $\alpha(\omega)$ and $STL(\omega)$ in agreement with the results simulated in a standing wave tube numerical experiment. The values of permeability k_0 and bulk density ρ_1 are much lower for H_2 ($\rho_1 = 25 \text{ kg/m}^3$) than for H_1 ($\rho_1 = 68 \text{ kg/m}^3$). For some foam samples H_1 and H_2 having identical thicknesses and radii with a similar stiffness, we note that the value of the FAE for H_2 increases over more than one order of magnitude. This indicates that the boundary conditions in the tube will influence the overall sound absorption and sound transmission results, and that their effect must be simulated within an axisymmetric poroelastic FE formulation of the Biot-Allard equations for better agreement with experiments. As expected, our estimates using the elastic properties of the frame with a Biot-Allard model are close to laboratory measurements of sound absorption, Fig. 4.3B(a). By contrast, it was also necessary to use an axisymmetric poroelastic FE formulation to properly model the bounded edge boundary condition for the sound transmission loss, Fig. 4.3B(b), because the frame acoustical excitability was quite strong. The results of this analysis therefore show that the elastic properties derived from the micromechanical model are therefore directly relevant to the acoustical properties determination for real foam samples.

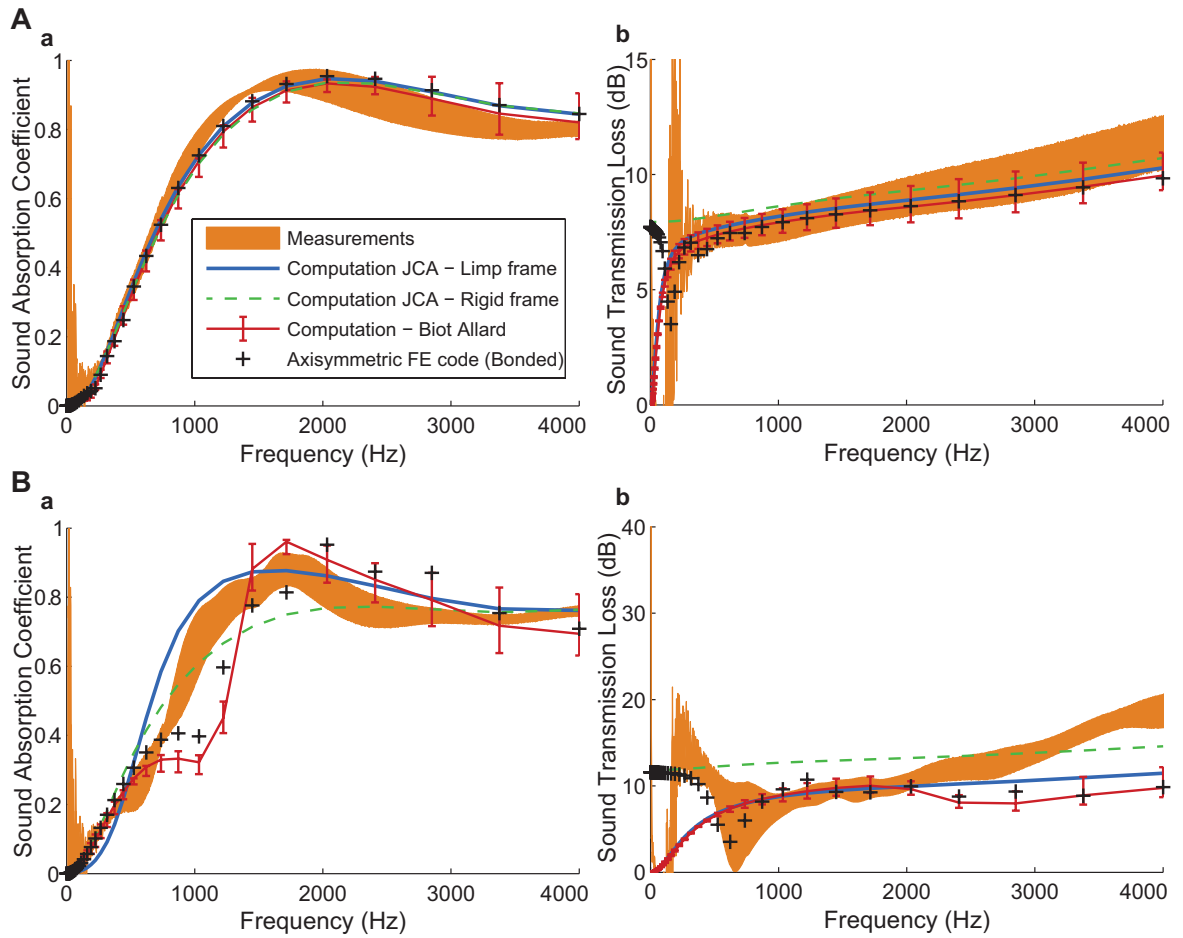


Figure 4.3. Acoustical properties of two different poroelastic foam samples, H_1 (A) and H_2 (B)

4.4.4. Conclusion

Let us have an overall view of the results that were derived from this multi-scale approach with the poroelastic foam samples used in this paper (Fig. 4.1). For the sake of clarity, let us recall the procedure. The porosity ϕ , permeability k_0 and ligament length L are assumed to be known from measurements. In the multi-scale approach, the extension of the solid film was implemented at growing rates, while the unit cell was rescaled to account for the film frictional forces, until ligament length L is converged. Macroscopic parameters are then computed from numerical homogenization and compared to the values that were measured at macro-scale in Sec. 4.4 and gathered in Table 4.1 and 4.3. These later values serve in a sense as bridges between microstructure and acoustical macro-behavior with microphysical and micromechanical foundations. The numerical simulations are generally in good agreement with the standing

wave tube measured values. The comparison for elastic properties is less impressive here since accurate values for the microscopic Young's modulus and Poisson ratio of the matrix were not yet available in the timeline of this study, while the estimation of the membrane thickness is rather rough at this stage. So, an important further step will be to refine the observation of membrane thickness from suitable observation means and to give a precise estimation of polyurethane Young's modulus by using means such as Mercury Intrusion Porosimetry (MIP) or nanoindentation techniques.

Results for the sound absorption and transmission look much better when the circumferential constraint effect of the tube is taken into account through an axisymmetric poroelastic FE formulation (Fig. 4.3). Therefore, the proposed multi-scale and multi-physics modeling approach of the acoustics of poroelastic foams provides a suitable strategy to characterize and optimize this class of materials. Note that this method provides new and complementary information when compared to inverse estimates of anisotropic open-cell foams.^[33] Considering the utility of cellular solids and foam structures in various applications, the versatility of periodic unit cell identification, and the chemical tunability of foam microstructures and mechanical functionality, this strategy can find ever broader application (biomedical and biomechanics, food industry, catalysis, etc.) because of added advantages of simple microstructure generation and parameterization, macroscopic properties estimation, and processability.

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Conclusion générale

5 Conclusion générale

Nous rappelons que l'objectif de cette thèse était de mettre au point une méthode reliant la géométrie locale d'une mousse cellulaire à faible densité à son macro-comportement acoustique.

La clef de voûte de cette thèse réside dans l'identification des caractéristiques de la géométrie locale d'une cellule périodique idéalisée tridimensionnelle afin que cette cellule soit représentative du comportement acoustique du milieu poreux à l'échelle macroscopique. Nous avons donc mis en œuvre des outils d'observation permettant de préciser la géométrie locale, puis mis en application une méthode hybride permettant de déterminer les paramètres acoustiques semi-phénoménologiques du milieu poreux.

La méthodologie a tout d'abord été mise en œuvre sur des structures simples (peu dispersées, cellules 100% ouvertes), puis nous avons augmenté de plus en plus la complexité de la microstructure (tailles de pore dispersées, membranes...). Finalement, une méthode itérative reposant sur un modèle géométrique contenant des membranes a été proposée et validée avec succès sur des échantillons réels de mousse. Cette méthode permet de déterminer complètement les tailles critiques gouvernant les propriétés acoustiques des mousses cellulaires. Cet outil ouvre la voie à l'optimisation au point de vue acoustique de telles mousses cellulaires.

Pour ce qui concerne la détermination des propriétés élastiques effectives, une méthode d'homogénéisation reposant sur des principes classiques a permis de calculer les propriétés élastiques macroscopiques des mousses. De tels calculs ont déjà été mentionnés dans la littérature, mais l'originalité a été ici d'inclure la présence de membranes dans la détermination des propriétés élastiques et de démontrer leur importance dans la prévision de ces propriétés. La comparaison des propriétés obtenues avec les résultats expérimentaux n'est pas encore complètement satisfaisante. La raison en est que les paramètres élastiques sont sensibles à l'épaisseur des membranes et que le module Young du composant solide des mousses identifié dans la littérature n'est connu qu'à l'intérieur d'une large plage d'extension supérieure à une décade. Néanmoins, au niveau de l'optimisation, notre méthode peut évidemment être appliquée en jouant sur les paramètres microstructuraux et sur les paramètres élastiques de la matrice.

Enfin, il est bien sûr nécessaire de citer les perspectives de cette thèse :

- Un point important serait de préciser l'épaisseur des membranes observées dans le cadre de cette thèse en utilisant les acquisitions par microtomographie à rayons-X. Cet objectif est d'autant plus accessible que l'acquisition des données sur des échantillons réels de mousse par μ CT a été réalisée lors de séjours de mobilité de l'auteur à l'Université de Sherbrooke. Par contre, les post-traitements restent à réaliser pour préciser la géométrie des membranes.
- Un des points bloquants pour la prévision des propriétés élastiques des mousses réside dans la méconnaissance avec une précision suffisante des propriétés élastiques du matériau constitutif des mousses. Les premiers contacts ont été pris pour la mesure de telles propriétés par une technique d'indentation ou par porosimétrie au mercure. L'utilisation de ces résultats permettra à court terme de valider l'approche retenue pour la prévision des propriétés élastiques de mousses à faible densité contenant des membranes.
- A moyen terme, le parachèvement de ce travail de thèse sera dans la mise au point d'une technique d'optimisation de mousses réelles à partir de la méthodologie présentée dans ce mémoire, afin de déterminer les caractéristiques de la géométrie locale pour lesquelles les propriétés acoustiques sont optimales vis-à-vis d'applications industrielles. Pour se convaincre de l'applicabilité des outils et méthodes développés dans une logique d'optimisation industrielle, le lecteur est encouragé à se reporter à un acte de conférence récent portant sur ce sujet, dont les références sont indiquées ci-dessous :
[A. Duval, M. T. Hoang, V. Marcel, and C. Perrot, Development of acoustically effective foams: A new micro-macro optimization method, VDI-Conference Polyurethan, 7-8 November, Stuttgart (2012)].

Annexes de la thèse

Annexe A : Propagation d'ondes dans les milieux poreux: fluide équivalent, matériau équivalent et mesure des paramètres acoustiques

A.1. Introduction

Pour la recherche du comportement acoustique des matériaux poreux, il est nécessaire de pouvoir décrire la propagation d'ondes dans de tels milieux afin de pouvoir prendre en compte et optimiser leurs propriétés. Cette annexe se compose de deux parties principales. La première aborde la propagation d'ondes dans les fluides, définit les paramètres importants de propagation, et représente la relation entre fluide équivalent et matériau homogène équivalent. Dans la deuxième partie, la caractérisation de l'impédance caractéristique ainsi que du nombre d'onde sont présentées en se basant sur la méthode à deux microphones et deux cavités de Utsuno. La mesure de l'impédance caractéristique est ensuite reliée aux propriétés acoustiques du milieu poreux.

Les deux références les plus importantes pour cette annexe sont le livre de Allard et Atalla ^[1] et l'article de Utsuno et al. ^[2].

A.2. Propagation d'ondes dans un fluide et un milieu poreux

A.2.1. Equation du mouvement

On considère une onde se propageant dans un fluide parfait d'incompressibilité K et de masse volumique ρ .

La pression p est reliée à la dilatation volumique θ par la relation

$$K = -p / \theta = \lambda \quad (\text{A.1})$$

Le vecteur de déplacement \mathbf{u} peut être cherché sous la forme :

$$\mathbf{u} = \nabla \varphi \quad (\text{A.2})$$

où φ est potentiel de déplacement, solution de :

$$\nabla \left[K \nabla^2 \varphi - \rho \frac{\partial^2 \varphi}{\partial t^2} \right] = 0 \quad (\text{A.3})$$

La pression p est alors déduite par :

$$p = -K\theta = -K \nabla^2 \varphi = -\rho \frac{\partial^2 \varphi}{\partial t^2} \quad (\text{A.4})$$

Une solution de l'équation qui décrit la propagation d'une onde dont la direction de propagation est parallèle à l'axe x_3 s'écrit:

$$\varphi = \frac{A}{\rho \omega^2} \exp[j(-kx_3 + \omega t) + \alpha] \quad (\text{A.5})$$

où A et α sont des constantes et k est le nombre d'onde :

$$k = \omega(\rho / K)^{1/2} \quad (\text{A.6})$$

la célérité (vitesse de propagation du son) est donnée par:

$$c = \omega / \text{Re} k \quad (\text{A.7})$$

ω est la pulsation.

La partie non nulle du déplacement u_3 et la pression p sont respectivement donnés par:

$$u_3 = \frac{\partial \varphi}{\partial x_3} = -\frac{jkA}{\rho\omega^2} \exp[j(-kx_3 + \omega t + \alpha)] \quad (\text{A.8})$$

$$p = -\rho \frac{\partial^2 \varphi}{\partial t^2} = A \exp[j(-kx_3 + \omega t + \alpha)] \quad (\text{A.9})$$

A.2.2. Impédance acoustique du fluide

A.2.2.1 Impédance caractéristique d'un fluide(en milieu infini)

On considère un fluide de module d'incompressibilité et de masse volumique respectivement K et ρ . Comme il a été dit dans le paragraphe précédent, une solution simple du potentiel de déplacement de l'équation (A.3) est la suivante, qui représente l'équation d'une onde progressive harmonique se propageant selon la direction x pour un fluide non visqueux:

$$\varphi(x,t) = \frac{A}{\rho\omega^2} \exp[j(-kx + \omega t)] \quad (\text{A.10})$$

La pression et les composantes du vecteur de déplacement valent respectivement:

$$p(x,t) = A \exp[j(-kx + \omega t)] \quad (\text{A.11})$$

et :

$$\begin{cases} u_x(x,t) = \frac{-jkA}{\rho\omega^2} \exp[j(-kx + \omega t)] \\ u_y = u_z = 0 \end{cases} \quad (\text{A.12})$$

A partir de l'équation (A.12), on peut calculer le vecteur vitesse $v = \partial \mathbf{u} / \partial t$:

$$\begin{cases} v_x(x,t) = \frac{kA}{\rho\omega} \exp[j(-kx + \omega t)] \\ v_y = v_z = 0 \end{cases} \quad (\text{A.13})$$

L'**impédance caractéristique** du fluide est définie comme le rapport entre la pression et la vitesse:

$$Z_c = \frac{p(x,t)}{v(x,t)} = \frac{\rho\omega}{k} \quad (\text{A.14})$$

En combinant avec l'équation (A.6), on a l'expression suivante de l'impédance caractéristique Z_c :

$$Z_c = (\rho K)^{1/2} \quad (\text{A.15})$$

A.2.2.2

Impédance correspondant à une onde réfléchiée par un monocouche

Si l'on considère une onde qui se propage dans le sens inverse, i.e. dans la direction négative de l'axe x , les expressions de sa pression et de sa vitesse sont respectivement:

$$p'(x,t) = A' \exp[j(kx + \omega t)] \quad (\text{A.16})$$

et

$$v'(x,t) = -\frac{A'}{Z_c} \exp[j(kx + \omega t)] \quad (\text{A.17})$$

On considère maintenant l'onde incidente sur le monocouche de la Fig. A.1 et son onde réfléchiée, ainsi que l'onde transmise dans le monocouche et son onde réfléchiée. Dans chaque cas, on a une superposition de deux ondes de directions de propagation inverses dont les pressions et les vitesses sont respectivement p , v et p' , v' [Voir équations (A.11), (A.13), (A.16), (A.17)] ; la pression totale et la vitesse totale s'expriment comme suit:

$$\begin{cases} p_T(x,t) = A \exp[j(-kx + \omega t)] + A' \exp[j(kx + \omega t)] \\ v_T(x,t) = \frac{A}{Z_c} \exp[j(-kx + \omega t)] - \frac{A'}{Z_c} \exp[j(kx + \omega t)] \end{cases} \quad (\text{A.18})$$

Le rapport entre la pression et la vitesse $p_T(x,t)/v_T(x,t)$ est l'impédance en x du fluide.

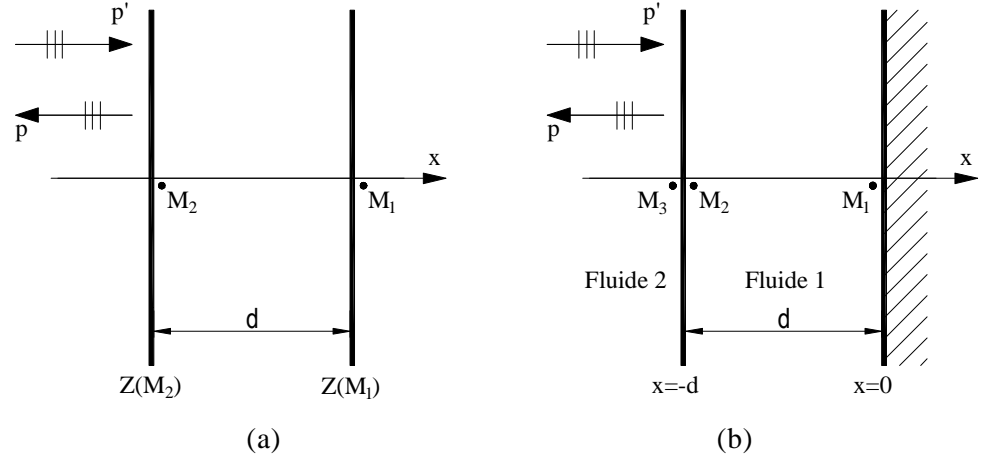


Figure A.1. Ondes planes se propageant dans la direction x et dans la direction opposée: (a) Fluide libre et (b) Une couche de fluide en contact avec une paroi rigide et imperméable.

La Fig. A.1(a) représente deux ondes se propageant dans deux directions opposées et parallèles à l'axe x , et on suppose que l'impédance $Z(M_1)$ au point M_1 est connue. En utilisant les équations de (A.18) pour la pression et la vitesse totale, on peut écrire impédance $Z(M_1)$ sous la forme suivante:

$$Z(M_1) = \frac{p_T(M_1)}{v_T(M_1)} = Z_c \frac{A \exp[-jkx(M_1)] + A' \exp[jkx(M_1)]}{A \exp[-jkx(M_1)] - A' \exp[jkx(M_1)]} \quad (\text{A.19})$$

et de même pour l'impédance au point M_2 :

$$Z(M_2) = \frac{p_T(M_2)}{v_T(M_2)} = Z_c \frac{A \exp[-jkx(M_2)] + A' \exp[jkx(M_2)]}{A \exp[-jkx(M_2)] - A' \exp[jkx(M_2)]} \quad (\text{A.20})$$

A partir de l'équation (A.19) on peut déduire:

$$\frac{A}{A'} = \frac{Z(M_1) - Z_c}{Z(M_1) + Z_c} \exp[-2jkx(M_1)] \quad (\text{A.21})$$

Finalement, en utilisant les équations (A.20) et (A.21) on obtient l'impédance au point M_2 du fluide considéré:

$$Z(M_2) = Z_c \frac{-jZ(M_1) \cotgkd + Z_c}{Z(M_1) - jZ_c \cotgkd} \quad (\text{A.22})$$

où d est la distance entre les deux points M_1 et M_2 : $d = x(M_1) - x(M_2)$.

Dans le cas de la Fig. A.1(b) où il y a deux milieux : Fluide 1 et Fluide 2, la couche de fluide 1 d'épaisseur d est en contact avec une paroi rigide et imperméable, l'impédance au point M_1 (qui est dans la phase de solide 1 et est en contact avec la paroi rigide) devient infinie: $Z(M_1) = \infty$. L'impédance en M_2 (à la surface de contact fluide 1/ fluide 2 et dans la phase de fluide 1) est obtenue en remplaçant la valeur de $Z(M_1)$ à l'équation(A.22).

$$Z(M_2) = -jZ_{c1} \cotg k_1 d \quad (\text{A.23})$$

avec Z_{c1} et k_1 sont respectivement l'impédance caractéristique et le nombre d'onde du fluide 1.

Si l'on considère un point M_3 qui se trouve à la surface de contact fluide 1/ fluide 2 mais dans la phase de fluide 2, comme la pression et la vitesse sont continues à la surface de contact, l'impédance en ce point doit être égale à celle au point M_2 .

$$Z(M_3) = Z(M_2) \quad (\text{A.24})$$

L'impédance mesurée au point M3 permet donc de déterminer l'impédance caractéristique du fluide 1. Cette relation va être utilisée dans la suite en remplaçant le milieu 1 par un milieu poreux

A.2.3. Fluide équivalent et Matériau homogène équivalent

A.2.3.1 Introduction

On s'intéresse maintenant à la propagation acoustique en grandes longueurs d'ondes dans un milieu poreux dont la structure, supposée indéformable dans un premier temps, apparaît homogène au niveau macroscopique. Pour des pores cylindriques, les caractéristiques de propagation (Z, k) peuvent être localement définies à l'échelle microscopique en fonction de la distance à l'axe des pores. Mais dès qu'il s'agit de microstructures complexes, il est très difficile de déterminer ces caractéristiques à l'échelle microscopique à cause de la géométrie compliquée de la structure. Dans ce dernier cas, on s'intéresse à la moyenne macroscopique qui est normalement calculée à partir d'un volume élémentaire représentatif. On suppose le squelette poreux rigide. L'air dans les pores de géométrie complexe est remplacé par un « fluide équivalent » qui a le même volume que le fluide interstitiel et qui a des valeurs complexes de l'impédance

caractéristique \tilde{Z}_c et du nombre d'onde \tilde{k} . Cette approche est généralement utilisée en acoustique des milieux poreux. Une deuxième approche est également possible en remplaçant le matériau poreux étudié par un matériau homogène équivalent dont l'impédance caractéristique \tilde{Z}_{eq} et le nombre d'onde \tilde{k}_{eq} sont aussi complexes. Dans le cadre de cette annexe, la première approche qui a fait l'objet de nombreuses études dans la littérature acoustique est abordée de façon simplifiée et on a supposé que les propriétés du fluide équivalent sont connues ; les parties suivantes visent à présenter le matériau homogène équivalent et la relation entre ses caractéristiques acoustiques avec le fluide équivalent correspondant.

A.2.3.2 *Matériau homogène équivalent*

A.2.3.2.1. Paramètres caractéristiques du matériau équivalent

On considère un matériau de volume total $\Omega = \Omega_f + \Omega_s$ où Ω_f et Ω_s sont respectivement le volume de la phase fluide connectée et le volume de la phase solide. L'idée de ce travail est de remplacer ce matériau par un matériau homogène équivalent qui a les caractéristiques d'un fluide, représenté par l'impédance caractéristique \tilde{Z}_{eq} et le nombre d'onde \tilde{k}_{eq} , et avec celui-ci les caractéristiques de surface comme l'impédance de surface, le coefficient de réflexion et le coefficient d'absorption, ces coefficients étant caractéristiques du matériau réel.

On définit l'impédance caractéristique du milieu poreux équivalent comme le rapport de la pression interstitielle sur la vitesse macroscopique du milieu homogène équivalent :

$$\tilde{Z}_{eq} = \frac{P}{\langle v \rangle_{\Omega}} \quad (\text{A.25})$$

elle s'exprime en fonction de $\tilde{\rho}_{eq}$ et \tilde{K}_{eq} par:

$$\tilde{Z}_{eq} = \sqrt{\tilde{\rho}_{eq} \tilde{K}_{eq}} \quad (\text{A.26})$$

Similairement à l'équation (A.6), le nombre d'onde caractérisant la propagation dans le milieu équivalent est aussi complexe et vaut:

$$\tilde{k}_{eq} = \omega \sqrt{\tilde{\rho}_{eq} / \tilde{K}_{eq}} \quad (\text{A.27})$$

La connaissance de $\tilde{\rho}_{eq}$ et \tilde{K}_{eq} , ou de \tilde{Z}_{eq} et \tilde{K}'_{eq} , permet de caractériser complètement la propagation dans le milieu homogène équivalent.

A.2.3.2.2. Caractéristiques de surface du matériau équivalent

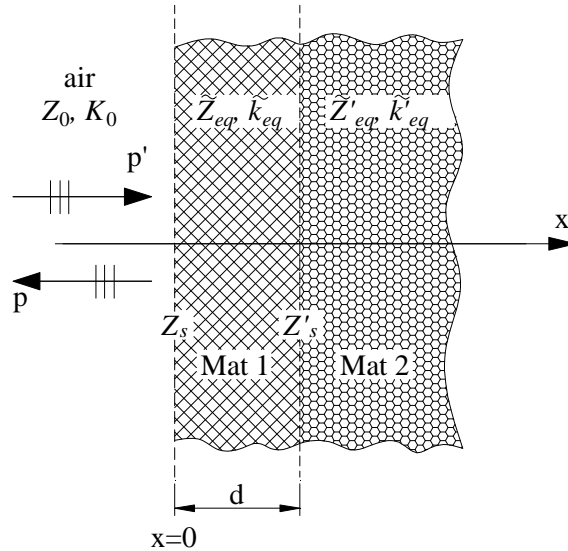


Figure A.2. Caractérisation de surface d'un matériau multicouche.

Les matériaux poreux acoustiques sont employés souvent sous la forme des systèmes multicouches fixés sur une surface rigide, ou séparés du support par une couche d'air. Pour la caractérisation acoustique de ces systèmes, on utilise différents paramètres qui sont les caractéristiques de surface du matériau.

Comme montré sur la Fig. A.2, on considère le cas d'une onde plane aérienne se réfléchissant en incidence normale sur un matériau multicouche, chaque couche pouvant être caractérisée par une impédance caractéristique. Trois grandeurs caractérisant le matériau au sein de la structure sont particulièrement utilisées pour quantifier ces effets de réflexion:

- **L'impédance de surface**, est définie comme le rapport entre la pression et la composante normale de la vitesse à la surface du matériau.

$$\tilde{Z}_s = \left. \frac{p}{\vec{v} \cdot \vec{x}} \right)_{x=0} \quad (\text{A.28})$$

Dans le cas d'une onde plane de direction x perpendiculaire à la surface d'un matériau d'épaisseur d , placé sur un autre matériau d'impédance de surface \tilde{Z}'_s , l'impédance de surface vaut, d'après équation (A.22) :

$$\tilde{Z}_s = \tilde{Z}_{eq} \frac{-j\tilde{Z}'_s \cot g\tilde{k}_{eq}d + \tilde{Z}_{eq}}{\tilde{Z}'_s - j\tilde{Z}_{eq} \cot g\tilde{k}_{eq}d} \quad (\text{A.29})$$

En particulier si le matériau est placé sur un support rigide et imperméable, l'impédance est similaire à l'équation (A.23):

$$\tilde{Z}_s = -j\tilde{Z}_{eq} \cot g\tilde{k}_{eq}d \quad (\text{A.30})$$

- **Le coefficient de réflexion:** Rapport des pressions réfléchie et incidente, une grandeur complexe dépendante de la fréquence, s'exprime dans ce cas en fonction de l'impédance de surface par:

$$R = \frac{p_r}{p_i} = \frac{\tilde{Z}_s - Z_0}{\tilde{Z}_s + Z_0} \quad (\text{A.31})$$

où Z_0 est l'impédance caractéristique de l'air, calculé par le produit de la masse volumique de l'air et de sa célérité: $Z_0 = \rho_0 c_0$

- **Le coefficient d'absorption:** il évalue l'énergie absorbée par rapport à l'énergie de l'onde incidente. Pour une onde plane, en incidence normale:

$$\alpha = 1 - |R|^2 \quad (\text{A.32})$$

Le coefficient d'absorption est réel, et varie par définition entre 0, pour un matériau parfaitement réfléchissant, et 1 pour un matériau totalement absorbant.

A.2.3.3 *Relation entre fluide équivalent et matériau homogène équivalent*

Il est courant de trouver dans la littérature acoustique la notion de « fluide équivalent », plutôt que de « matériau homogène équivalent ». Il est intéressant d'établir la correspondance entre ces deux notions. On considère un matériau dont le fluide équivalent a deux paramètres de propagation \tilde{Z}_c et \tilde{k} supposés connus, le matériau homogène équivalent a des

caractéristiques \tilde{Z}_{eq} et \tilde{k}_{eq} qui sont définies dans la partie A.2.3.2.1. On va chercher la relation entre ces paramètres à partir de leurs définitions. (Fig. A.3)

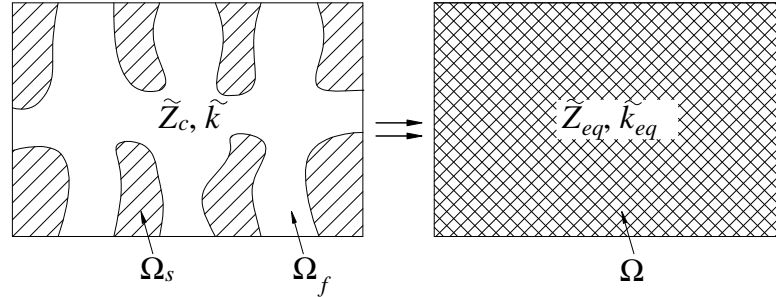


Figure A.3. Matériau poreux (à gauche) et son matériau homogène équivalent (à droite)

A.2.3.3.1. Impédances caractéristiques

Les impédances caractéristiques du matériau homogène équivalent et du fluide équivalent valent respectivement [voir l'équation (A.25)]:

$$\tilde{Z}_{eq} = \frac{P}{\langle v \rangle_{\Omega}} \quad (\text{A.33})$$

et

$$\tilde{Z}_c = \frac{P}{\langle v \rangle_{\Omega_f}} \quad (\text{A.34})$$

Le rapport entre les deux côtés de ces deux équations vaut:

$$\frac{\tilde{Z}_{eq}}{\tilde{Z}_c} = \frac{\langle v \rangle_{\Omega_f}}{\langle v \rangle_{\Omega}} \quad (\text{A.35})$$

De plus, les deux vitesses macroscopiques $\langle v \rangle_{\Omega}$ et $\langle v \rangle_{\Omega_f}$ sont les moyennes spatiales rapportée aux volumes correspondant (i.e.: Ω , Ω_f) des champs de vitesses microscopiques. Ces champs de vitesse sont les mêmes pour le fluide équivalent et le matériau homogène équivalent (la vitesse est nulle pour la phase solide) :

$$\langle v \rangle_{\Omega_f} = \frac{1}{|\Omega_f|} \int_{\Omega_f} v d\Omega \quad (\text{A.36})$$

$$\langle v \rangle_{\Omega} = \frac{1}{|\Omega|} \int_{\Omega} v d\Omega = \frac{1}{|\Omega|} \int_{\Omega_f} v d\Omega \quad (\text{A.37})$$

Donc, on obtient:

$$\frac{\langle v \rangle_{\Omega_f}}{\langle v \rangle_{\Omega}} = \frac{|\Omega|}{|\Omega_f|} = \frac{1}{\phi} \quad (\text{A.38})$$

où ϕ est la porosité ouverte du matériau calculée par le rapport entre le volume de la phase fluide sur le volume total. Les équations (A.35) et (A.38) donnent:

$$\tilde{Z}_{eq} = \frac{\tilde{Z}_c}{\phi} \quad (\text{A.39})$$

A.2.3.3.2. Nombre d'onde

Pour trouver la relation entre le nombre d'onde du matériau homogène équivalent et celui du fluide équivalent, on se base sur le fait que l'impédance de surface ne change pas de l'un à l'autre.

On considère un volume de matériau poreux qui est placé sur un support rigide imperméable (Fig. A.4(a)), la surface du matériau étant en contact avec l'air et étant soumise à une onde d'incidence normale. Deux points M_1 et M_2 sont choisis, M_2 dans l'air et M_1 dans le matériau poreux. On appelle $v(M_1)$ la vitesse moyenne de l'air dans un pore près de la surface et $v(M_2)$ la vitesse moyenne dans l'air libre. Les pressions $p(M_1)$ et $p(M_2)$ sont respectivement les pressions dans le pore et dans l'air. La continuité du flux d'air et de la pression à la surface du matériau poreux donne :

$$p(M_2) = p(M_1) \quad (\text{A.40})$$

$$v(M_2) = v(M_1)\phi_s \quad (\text{A.41})$$

où ϕ_s est la porosité surfacique à la surface du matériau. $\phi_s = \phi$ pour un matériau isotrope. Sachant que le rapport p/v nous donne l'impédance à la surface considérée, les rapports entre les deux côtés droit et gauche des deux équations (A.40) et (A.41) donnent:

$$\tilde{Z}(M_2) = \tilde{Z}(M_1)/\phi \quad (\text{A.42})$$

En regardant l'équation (A.23), on obtient finalement:

$$\tilde{Z}(M_2) = -j \frac{\tilde{Z}_c}{\phi} \cotg \tilde{k}d \quad (\text{A.43})$$

où \tilde{Z}_c et \tilde{k} sont respectivement l'impédance caractéristique et le nombre d'onde du fluide équivalent, d est l'épaisseur du matériau selon la direction de propagation.

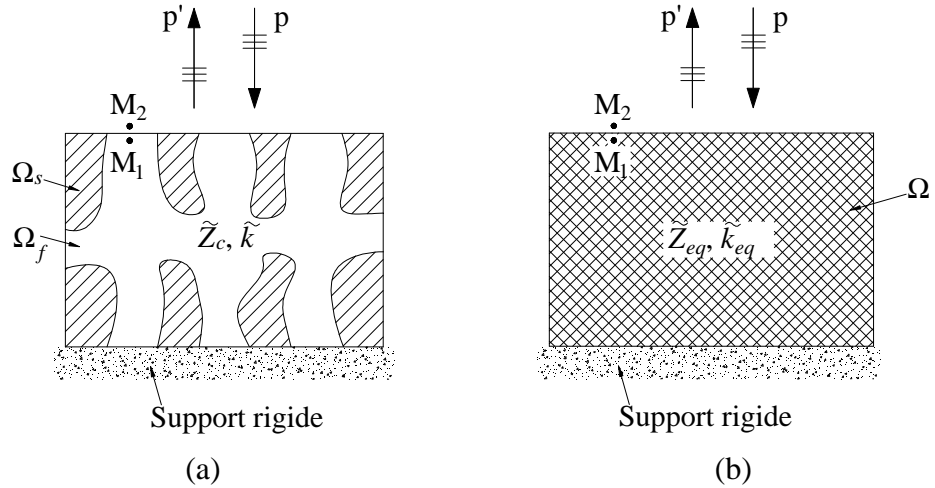


Figure A.4. Matériau poreux (a) et son matériau homogène équivalent (b) soumis aux ondes se propageant dans la direction normale à leurs surfaces.

Maintenant on considère la Fig. A.4(b) où l'on remplace le matériau poreux par un matériau homogène équivalent. En utilisant l'équation (A.24), on peut écrire:

$$\tilde{Z}(M_2) = -j \tilde{Z}_{eq} \cotg \tilde{k}_{eq}d \quad (\text{A.44})$$

Comme l'impédance de surface reste la même pour les cas (a) et (b), on peut déduire à partir des équations (A.43) et (A.44):

$$-j \frac{\tilde{Z}_c}{\phi} \cotg \tilde{k}d = -j \tilde{Z}_{eq} \cotg \tilde{k}_{eq}d \quad (\text{A.45})$$

et comme l'exprime l'équation (A.39): $\tilde{Z}_{eq} = \tilde{Z}_c/\phi$, on obtient:

$$\cotg \tilde{k}d = \cotg \tilde{k}_{eq}d \quad (\text{A.46})$$

Pour que cette équation soit vraie pour toutes les épaisseurs d , il faut que:

$$\tilde{k} = \tilde{k}_{eq} \quad (A.47)$$

ce qui indique que le nombre d'onde ne change pas quand on remplace le fluide équivalent par un matériau homogène équivalent.

A.2.3.3.3. Masse volumique et incompressibilité

Les équations (A.6) et (A.15) ont représenté le nombre d'onde et l'impédance caractéristique Z_c d'un milieu en fonction de sa masse volumique ρ et son incompressibilité K . On a donc:

$$\begin{cases} \tilde{k} = \omega(\tilde{\rho} / \tilde{K})^{1/2} & (a) \\ \tilde{Z}_c = (\tilde{\rho}\tilde{K})^{1/2} & (b) \end{cases} \quad (A.48)$$

pour le fluide équivalent et

$$\begin{cases} \tilde{k}_{eq} = \omega(\tilde{\rho}_{eq} / \tilde{K}_{eq})^{1/2} & (a) \\ \tilde{Z}_{eq} = (\tilde{\rho}_{eq}\tilde{K}_{eq})^{1/2} & (b) \end{cases} \quad (A.49)$$

pour le matériau homogène équivalent.

Les équations (A.47), (A.48)-(a) et (A.49)-(a) donnent:

$$\tilde{\rho} / \tilde{K} = \tilde{\rho}_{eq} / \tilde{K}_{eq} \quad (A.50)$$

et les équations (A.39), (A.48)-(b) et (A.49)-(b) donnent:

$$\tilde{\rho}\tilde{K} = \phi^2 \tilde{\rho}_{eq}\tilde{K}_{eq} \quad (A.51)$$

Finalement on obtient, à partir des relations (A.50) et (A.51):

$$\begin{cases} \tilde{\rho} = \phi\tilde{\rho}_{eq} & (a) \\ \tilde{K} = \phi\tilde{K}_{eq} & (b) \end{cases} \quad (A.52)$$

La formule (A.52)-(a) semble contre-intuitive car si l'on met un fluide équivalent de volume Ω_f dans un volume Ω qui est plus grand, la masse volumique de la deuxième est physiquement plus petite. Mais dans ce cas, comme $\tilde{\rho} = \phi \tilde{\rho}_{eq}$ et $\phi < 1$, la masse volumique du matériau homogène équivalent est plus grande que celle du fluide équivalent (masse volumique effective). Par contre, comme il a été montré dans la partie précédente, il est exact d'utiliser ces relations pour remplacer un matériau poreux réel par un matériau (fluide) homogène équivalent qui peut se présenter pour ce matériau au niveau acoustique. Avec ce dernier, les paramètres acoustiques du matériau comme l'impédance de surface, le coefficient d'absorption et le coefficient de réflexion restent inchangés. D'autre part, on peut montrer que la formule (A.52)-(a) est compatible avec la conservation de la quantité de mouvement :

$$\tilde{\rho} \langle v \rangle_{\Omega_f} = \tilde{\rho}_{eq} \langle v \rangle_{\Omega} \quad (A.53)$$

A partir des équations (A.38) et (A.53), on obtient donc :

$$\tilde{\rho} = \frac{\langle v \rangle_{\Omega}}{\langle v \rangle_{\Omega_f}} \tilde{\rho}_{eq} = \phi \tilde{\rho}_{eq} \quad (A.54)$$

On note toutefois que la formule donnant la quantité de mouvement obtenue ne correspond pas à l'approche classique de la théorie des mélanges. En effet ces deux quantités de mouvement correspondent à la quantité de mouvement par volume de fluide et non à la quantité de mouvement par volume solide+fluide.

A.3. Caractérisation acoustique des matériaux poreux par la méthode à deux microphones

Le comportement acoustique d'un matériau poreux peut se caractériser soit par l'impédance caractéristique et le nombre d'onde du matériau homogène équivalent $(\tilde{Z}_{eq}, \tilde{k}_{eq})$, soit par sa masse volumique et son incompressibilité $(\tilde{\rho}_{eq}, \tilde{K}_{eq})$. La connaissance d'un de ces deux couples de paramètres permet de déterminer les paramètres restants par des relations mathématiques abordées dans la partie précédente. Les mesures les plus souvent utilisées pour la caractérisation de ces paramètres sont celles en tube d'impédance utilisant la méthode de la fonction de transfert ^{[3]-[5]}. Il y a beaucoup de recherches basées sur cette méthode parmi lesquelles la méthode à deux microphones et deux cavités proposée par Utsuno et al. ^[2] qui permet la mesure des deux variables $(\tilde{Z}_{eq}, \tilde{k}_{eq})$ de façon fiable et rapide.

On représente dans les paragraphes suivants la méthode de Utsuno et *al.* [2]. Le lecteur intéressé par les méthodes les plus récentes pourra consulter les références [6], [7], [8] et [9].

A.3.1. Formulation de la méthode à deux cavités

La Fig. A.5 représente un schéma du tube à impédance proposé par Utsuno et *al.* [2]. Le matériau homogène d'épaisseur d est suivi d'une cavité d'épaisseur L et d'un piston rigide et mobile. \tilde{Z}_{S1} et \tilde{Z}_{S2} sont respectivement l'impédance de surface du matériau au contact matériau/cavité d'air et à la surface de référence. On considère $(\tilde{Z}_{eq}, \tilde{k}_{eq})$ qui sont les variables relatives au matériau homogène équivalent au matériau étudié. A partir de l'équation (A.22). En utilisant l'égalité $\cot g(\tilde{k}_{eq}d) = -j \coth(j\tilde{k}_{eq}d)$, on déduit:

$$\tilde{Z}_{S2} = \frac{\tilde{Z}_{S1} \cosh(j\tilde{k}_{eq}d) + \tilde{Z}_{eq} \sinh(j\tilde{k}_{eq}d)}{\tilde{Z}_{S1} \sinh(j\tilde{k}_{eq}d) + \tilde{Z}_{eq} \cosh(j\tilde{k}_{eq}d)} \quad (A.55)$$

L'équation (A.55) donne:

$$\frac{\tilde{Z}_{S2} + \tilde{Z}_{eq}}{\tilde{Z}_{S2} - \tilde{Z}_{eq}} \frac{\tilde{Z}_{S1} - \tilde{Z}_{eq}}{\tilde{Z}_{S1} + \tilde{Z}_{eq}} = \exp(2j\tilde{k}_{eq}d) \quad (A.56)$$

La côté à droite de l'équation (A.56) est une fonction de l'épaisseur d et du nombre d'onde \tilde{k}_{eq} qui sont constants. Donc si l'on change l'épaisseur de la couche d'air L par L' , \tilde{Z}_{S1} et \tilde{Z}_{S2} seront remplacées par \tilde{Z}'_{S1} et \tilde{Z}'_{S2} , ce qui mène à l'égalité suivante:

$$\frac{\tilde{Z}_{S2} + \tilde{Z}_{eq}}{\tilde{Z}_{S2} - \tilde{Z}_{eq}} \frac{\tilde{Z}_{S1} - \tilde{Z}_{eq}}{\tilde{Z}_{S1} + \tilde{Z}_{eq}} = \frac{\tilde{Z}'_{S2} + \tilde{Z}_{eq}}{\tilde{Z}'_{S2} - \tilde{Z}_{eq}} \frac{\tilde{Z}'_{S1} - \tilde{Z}_{eq}}{\tilde{Z}'_{S1} + \tilde{Z}_{eq}} \quad (A.57)$$

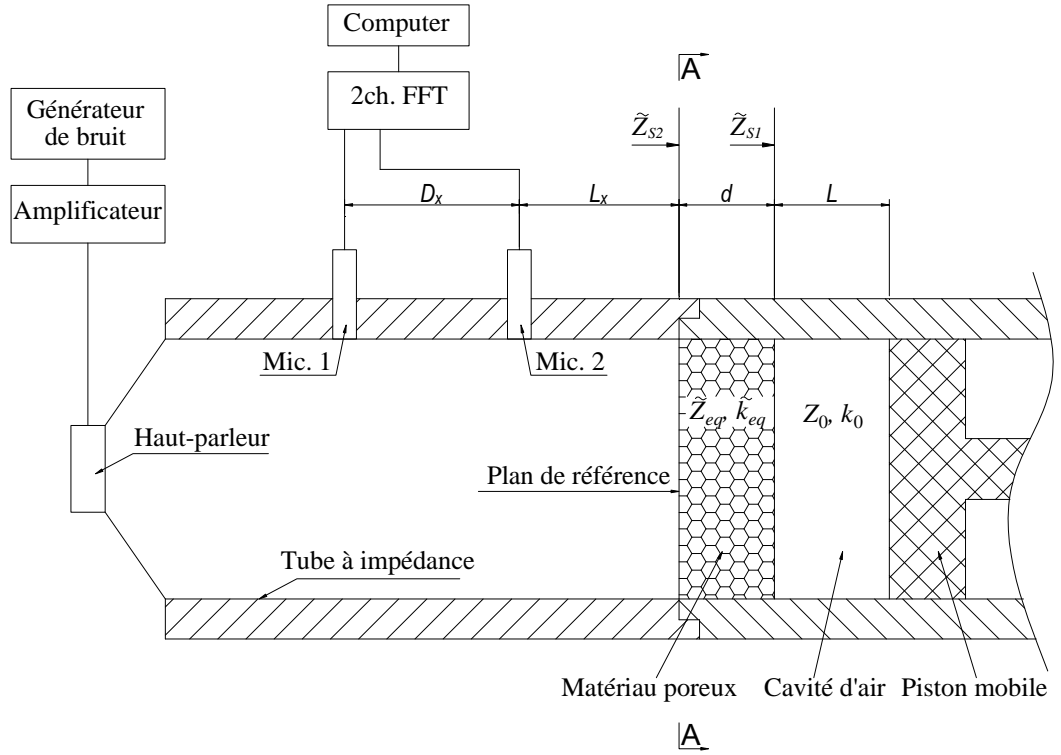


Figure A.5. Schéma du Tube à impédance à deux microphones et des accessoires

Par la résolution des équations (A.56) et (A.57), on obtient:

$$\tilde{Z}_{eq} = \pm \left(\frac{\tilde{Z}_{S2} \tilde{Z}'_{S2} (\tilde{Z}_{S1} - \tilde{Z}'_{S1}) - \tilde{Z}_{S1} \tilde{Z}'_{S1} (\tilde{Z}_{S2} - \tilde{Z}'_{S2})}{(\tilde{Z}_{S1} - \tilde{Z}'_{S1}) - (\tilde{Z}_{S2} - \tilde{Z}'_{S2})} \right)^{1/2} \quad (\text{A.58})$$

et

$$\tilde{k}_{eq} = \frac{1}{2jd} \ln \left(\frac{\tilde{Z}_{S2} + \tilde{Z}_{eq}}{\tilde{Z}_{S2} - \tilde{Z}_{eq}} \frac{\tilde{Z}_{S1} - \tilde{Z}_{eq}}{\tilde{Z}_{S1} + \tilde{Z}_{eq}} \right) \quad (\text{A.59})$$

le signe dans la formule (A.58) est choisi pour que la partie réelle de \tilde{Z}_{eq} soit positive. Pour calculer l'impédance caractéristique et le nombre d'onde du matériau selon les formules (A.58) et (A.59), il faut connaître les valeurs des impédances aux surfaces S1 et S2. On peut les calculer facilement selon la formule (A.23):

$$\tilde{Z}_{S1} = -jZ_0 \cot gk_0 L \quad (\text{A.60})$$

$$\tilde{Z}'_{S1} = -jZ_0 \cot gk_0 L' \quad (\text{A.61})$$

où (Z_0, k_0) sont l'impédance caractéristique et le nombre d'onde lié à la propagation de l'air dans le tube.

On utilise la méthode de la fonction de transfert (Iso 10534-2) pour déterminer $\tilde{Z}_{S2}, \tilde{Z}'_{S2}$. En ce qui concerne cette méthode, le champ de pression est mesuré à deux positions 1 et 2, séparées l'une de l'autre d'une distance D_x suivant l'axe du tube. La fonction de transfert H_{12} entre les pressions mesurées aux deux points 1 et 2 est par la suite calculée.

$$H_{12} = \frac{P_1}{P_2} \quad (\text{A.62})$$

Les impédances à la surface de référence correspondant aux distances L et L' sont respectivement calculées comme suit:

$$\tilde{Z}_{S2} = jZ_0 \frac{-H_{12} \sin(k_0 L_x) + \sin[k_0(L_x + D_x)]}{H_{12} \cos(k_0 L_x) - \cos[k_0(L_x + D_x)]} \quad (\text{A.63})$$

$$\tilde{Z}'_{S2} = jZ_0 \frac{-H'_{12} \sin(k_0 L_x) + \sin[k_0(L_x + D_x)]}{H'_{12} \cos(k_0 L_x) - \cos[k_0(L_x + D_x)]} \quad (\text{A.64})$$

où H_{12} et H'_{12} sont les fonctions de transferts correspondant aux distances L et L' . L_x est la distance du microphone 2 à la surface de référence (i.e. la surface de l'échantillon).

La connaissance des impédances de surface permet de calculer l'impédance caractéristique du matériau homogène équivalent \tilde{Z}_{eq} selon la formule (A.58), puis on remplace cette valeur dans la formule (A.59) pour trouver le nombre d'onde du matériau homogène équivalent.

A.3.2. Résultats et Discussions

Utsuno et ses collaborateurs ont réalisé des mesures au tube à deux microphones avec différentes couples de (L, L') pour mesurer l'impédance caractéristique et le nombre d'onde de deux types de matériaux: laine de verre et mousse aluminium. Ils constatent que les bons résultats sont obtenus si la conditions suivante est vérifiée:

$$f(L - L') \neq nc/2 \quad (\text{A.65})$$

où f et c sont respectivement la fréquence et la vitesse du son dans l'air et n est un nombre entier ($n=1, 2, 3, \dots$)

Les résultats d'impédance de surface calculés à partir de la formule (A.55) sont comparés avec ceux mesurés par la méthode de la fonction de transfert pour la vérification des mesures à deux microphones et conduit à de bons résultats.

A.3.3. Application de la méthode à deux microphones

La mesure à deux microphones fournit deux paramètres caractéristiques importants du matériau, qui permettent de déterminer de nombreux paramètres soit acoustiques soit non-acoustiques dont les formules seront présentées dans les deux parties suivantes:

A.3.3.1. Détermination des propriétés acoustiques

Selon la formule (A.52), la masse volumique effective et le module d'incompressibilité effectif du matériau peuvent se calculer par les formules suivantes:

$$\tilde{\rho} = \frac{\phi \tilde{Z}_{eq} \tilde{k}_{eq}}{\omega} \quad (\text{A.66})$$

$$\tilde{K} = \frac{\phi \omega \tilde{Z}_{eq}}{\tilde{k}_{eq}} \quad (\text{A.67})$$

Ensuite, le coefficient de réflexion et le coefficient d'absorption peuvent être déterminés respectivement par les formules:

$$R = \frac{\tilde{Z}_s - Z_0}{\tilde{Z}_s + Z_0} \quad (\text{A.68})$$

$$\alpha = 1 - |R|^2 \quad (\text{A.69})$$

où \tilde{Z}_s est l'impédance de surface du matériau, calculée par les formules (A.29) et (A.30).

A.3.3.2. Détermination des propriétés non-acoustiques

Les quatre paramètres non-acoustiques les plus importants pour un matériau poreux peuvent être calculés par la méthode indirecte proposée

par Panneton et Only ^{[10],[11]} en supposant que la porosité du milieu est connu (par la mesure directe):

- **Résistivité au passage de l'air:**

$$\sigma = -\frac{1}{\phi} \lim_{\omega \rightarrow 0} [\text{Im}(\omega \tilde{\rho})] \quad (\text{A.70})$$

- **Tortuosité:**

$$\alpha_{\infty} = \frac{1}{\rho_0} \left(\text{Re}(\tilde{\rho}) - \sqrt{\text{Im}(\tilde{\rho})^2 - \left(\frac{\sigma \phi}{\omega} \right)^2} \right) \quad (\text{A.71})$$

- **Longueur caractéristique visqueuse:**

$$\Lambda = \alpha_{\infty} \sqrt{\frac{2\rho_0\eta}{\omega \text{Im}(\tilde{\rho})(\rho_0\alpha_{\infty} - \text{Re}(\tilde{\rho}))}} \quad (\text{A.72})$$

- **Longueur caractéristique thermique**

$$\Lambda' = \delta_t \sqrt{2(-\text{Im}(\left(\frac{\phi - \tilde{K}/K_0}{\phi - \gamma\tilde{K}/K_0}\right)^2))^{-1/2}} \quad (\text{A.73})$$

où: K_0 est la module d'incompressibilité adiabatique de l'air, $K_0 = \gamma P_0$ où P_0 est la pression ambiante et γ est le rapport de chaleur spécifique. ρ_0 est la masse volumique de l'air au repos. η est la viscosité dynamique de l'air. $\delta_t = \sqrt{2\eta/\rho_0\omega\text{Pr}}$ où Pr est nombre de Prandtl, $\text{Pr} = C_p\eta/\kappa$ où C_p est la chaleur massique à pression constante et κ est la conductivité thermique de l'air.

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Annexe B : Modèles semi-phénoménologiques

Le comportement acoustique des milieux poreux à structure rigide saturés par l'air peut être déterminé à l'aide d'un couple de fonctions de réponse complexe, une caractérise l'effet visqueux et l'autre concerne l'effet thermique.

Pour l'effet visqueux, il existe plusieurs choix de fonctions possibles, suivant les auteurs et suivant le paramètre physique initial, choisi en hautes fréquences ou en basses fréquences, qui est étendu sur la bande de fréquences intermédiaires. On peut citer ici la tortuosité dynamique $\tilde{\alpha}(\omega)$, la perméabilité dynamique visqueux $\tilde{k}(\omega)$ et la masse volumique effective $\tilde{\rho}(\omega)$. Ces fonctions sont reliées entre elles par :

$$\tilde{\alpha}(\omega) = \tilde{\rho}(\omega) / \rho_0 \text{ et } \tilde{\alpha}(\omega) = \nu \phi / i \omega \tilde{k}(\omega) \quad (\text{B.1})$$

où ρ_0 et ν sont respectivement la masse volumique et la viscosité cinématique de l'air.

Similairement, les différentes fonctions de réponse dynamique alternatives caractérisant la dissipation thermique de l'air dans les pores du milieu sont la compressibilité dynamique normalisée $\tilde{\beta}(\omega)$, la perméabilité dynamique thermique $\tilde{k}'(\omega)$ et le module d'incompressibilité effectif $\tilde{K}(\omega)$. Une de ces trois fonctions peut se trouver à partir de la connaissance d'une autre par les relations suivantes :

$$\tilde{\beta}(\omega) = K_a / \tilde{K}(\omega) \text{ et } \tilde{\beta}(\omega) = \gamma - (\gamma - 1) i \omega \tilde{k}'(\omega) / \nu' \phi \quad (\text{B.2})$$

où K_a est le module d'incompressibilité de l'air, γ est le rapport des capacités thermiques massiques $\gamma = C_p / C_v$, C_p et C_v étant respectivement les capacités thermiques à pression constante et à volume constant, $\nu' = \kappa / \rho_0 C_p$ où κ est la conductivité thermique de l'air.

Les modèles phénoménologiques présentés dans la partie 2 visent alors à déterminer les fonctions de réponse dynamique ci-dessus. Ces modèles s'appuyant sur des paramètres macroscopiques peuvent se présenter sous une forme générale, exprimée en fonction de la masse volumique effective et du module de compressibilité effectif comme suit :

$$\tilde{\rho}(\omega) = \rho_0 \alpha_\infty \left[1 + \frac{1}{i\omega} f(\omega) \right] \quad (\text{B.3})$$

$$\frac{1}{\tilde{K}(\omega)} = \frac{1}{K_a} \left\{ \gamma - (\gamma - 1) \left[1 + \frac{1}{i\bar{\omega}'} f'(\bar{\omega}') \right]^{-1} \right\} \quad (\text{B.4})$$

où $\bar{\omega}$ et $\bar{\omega}'$ sont respectivement les pulsations adimensionnelles visqueuses et thermiques, données par les expressions :

$$\bar{\omega} = \frac{\omega k_0 \alpha_\infty}{\nu \phi}, \quad \bar{\omega}' = \frac{\omega k_0'}{\nu' \phi} \quad (\text{B.5})$$

et avec les fonctions de forme f et f' suivantes :

$$\tilde{f}(\bar{\omega}) = 1 - P + P \sqrt{1 + \frac{M}{2P^2} i\bar{\omega}} \quad (\text{B.6})$$

$$\tilde{f}'(\bar{\omega}') = 1 - P' + P' \sqrt{1 + \frac{M'}{2P'^2} i\bar{\omega}'} \quad (\text{B.7})$$

où M , M' , P et P' sont les facteurs de forme adimensionnels, donnés par :

$$M = \frac{8k_0 \alpha_\infty}{\Lambda^2 \phi}, \quad M' = \frac{8k_0'}{\Lambda'^2 \phi} \quad (\text{B.8})$$

$$P = \frac{M}{4 \left(\frac{\alpha_0}{\alpha_\infty} - 1 \right)}, \quad P' = \frac{M'}{4 (\alpha_0' - 1)} \quad (\text{B.9})$$

Les paramètres caractéristiques des modèles phénoménologiques sont alors présentés dans le tableau B.1.

Modèles	Paramètres macroscopiques							
JCA	ϕ	k_0	Λ'	Λ	α_∞	$k_0' = \frac{\phi \Lambda'^2}{8}$	$\alpha_0 = \alpha_\infty \left(1 + \frac{2k_0 \alpha_\infty}{\phi \Lambda^2} \right)$	$\alpha_0' = 1.25$
JCAL	ϕ	k_0	Λ'	Λ	α_∞	k_0'	$\alpha_0 = \alpha_\infty \left(1 + \frac{2k_0 \alpha_\infty}{\phi \Lambda^2} \right)$	$\alpha_0' = 1.25$
JCAPL	ϕ	k_0	Λ'	Λ	α_∞	k_0'	α_0	α_0'

Tableau B.1. Modèles phénoménologiques et paramètres caractéristiques correspondants.

Post Scriptum

1. The reviews in Chapters 2 and 3 (e.g. page 41) omit four relevant aspects:
 - (i) the availability of ‘exact’ solutions for specific idealised pore microstructures (these include cylinders, slits, triangles and rectangles - see for example Ch.3 of *Predicting Outdoor Sound*, K. Attenborough, K. M. Li and K. Horoshenkov, Taylor and Francis, 2006)
 - (ii) reference to the cell model approach in general and ‘exact’ results for stacked spheres in particular (see O. Umnova, K. Attenborough and K. M. Li, “Cell model calculations of dynamic drag parameters in packings of spheres”. *J. Acoust. Soc. Am.* **107** (3) 3113 – 3119 (2000), O. Umnova, K. Attenborough and K. M. Li, “A cell model for the Acoustical properties of Packings of Spheres”, *Acustica combined with Acta Acustica*, **87** 226 – 235 (2001)) [Note that these references offer a model for the acoustical properties of stacked spheres (this idealisation is mentioned specifically on page 42) requiring knowledge only of sphere radius and porosity without the need for any adjustable parameters]
 - (iii) reference to the relaxation time approach [D. K. Wilson “Relaxation-matched modeling of propagation through porous media, including fractal pore structure,” *J. Acoust. Soc. Am.* **94**, 1136–1145 (1993)] and
 - (iv) models based on pore size distributions (see for example K.V. Horoshenkov, Keith Attenborough and S.N. Chandler-Wilde, “Pade approximants for the acoustical properties of rigid frame porous media with pore size distribution”, *J. Acoust. Soc. Am.*, **104** (3) 1198-1209 (1998); see also K. Attenborough, “Models for the acoustical properties of air-saturated granular media”, *Acta Acustica* **1** 213-236 (1993)).

2. The statements ‘1.’ and ‘2.’ in section 3.1.2.1.3 on page 46 about the waves in Biot Theory for dynamics of poroelastic media are true only if the waves are decoupled (which is the case only under the conditions described in the following paragraph). Statement ‘3.’ is only true if the saturating fluid does not support shear.

3. There is a low frequency measurement problem which is evident in all of the data presented (starting with Fig. 3.6.) which is neither remarked nor explained.

It is due to the small inter-microphonic distance of the standard standing wave tube, which reduces significantly the low-frequency (long wavelength) accuracy measurements.

4. Only two of the ‘validating’ parameter values in Table 3.1 have been obtained by non-acoustic measurements. Although various ‘acoustical’ (including ultrasonic) methods of estimating the other six parameters are mentioned, it is not explained clearly how the estimated values in Table 3.1 have been obtained. Fig. 3.1 gives the impression that the acoustical data used for the estimations are confined to the normal incidence absorption coefficient spectrum (see also the unspecific reference to ‘standing wave tube measurements’ on page 108). The Thesis would be more self-contained if the methods of parameter estimation based on acoustical measurements (i.e. as in Refs. [60] and [61]) were spelled out. **Note that Ref. [60] uses dynamic density and Ref. [61] uses dynamic compressibility.**

5. If the same acoustical data used to estimate four of the parameters have been used also for validation, how can the validation of the ‘bottom-up’ parameter calculations be regarded as independent?

It is true that the transport parameters and the acoustical data which have been used for the validation of the ‘bottom-up’ approach cannot be regarded as independent (macroscopic data are interdependent). However, once the [micro (local geometry) or macro (porosity and permeability)] input parameters have been obtained by measurements, the output parameters are computed directly on the periodic unit cell from first principle calculations. Then, the resulting parameters are compared with independently obtained measurements [acoustical data and (model-dependent) macroscopic parameters estimates]. Furthermore, none of the input parameters are obtained from acoustical data. Therefore, the acoustical properties of porous media are obtained primarily from microstructure, and independently from any acoustical data.

6. The pore surface area is mentioned on page 49 as part of the definition of thermal characteristic length and specific (internal) surface area is mentioned again as important for this definition on page 73. The difficulty that the BET apparatus for measuring this quantity is not ‘routinely available’ is mentioned on page 74. Why hasn’t the water suction method [see H.L. Ritter and L. C. Drake Ind. Eng. Chem. **17** 782 – 786 (1945)] for obtaining the pore size distribution and hence total internal surface area been considered or used?

Authors of the present manuscripts were not aware of the water suction method. This will be however investigated in a forthcoming work.

7. What is particularly novel about the ‘key result’ stated on page 123 that the viscous characteristic length is related to the smallest openings (throat size)? Surely this stems from its definition and has been stated many times in the literature, for example by Allard.

To the knowledge of the authors’ manuscript, this statement appears in the literature as a general trend, a purely qualitative statement not substantiated with precise quantitative arguments. This explains why, this may be seen as a ‘key result’ delineating clearly the limits of applicability of this micro-macro link.

8. In Table 4.1 on page 153, reference is made to parameter values described as ‘inverse characterisation measurements’. However these are not measured values. They have been deduced from measurements using a commercially-available inversion procedure. Also it should be noted that Kino [“A comparison of two acoustical methods for estimating parameters of glass fibre and melamine foam materials” Applied Acoustics **73** 590 – 603 (2012)] has shown that parameters obtained using Foam-X software (see page 153) are not always accurate.

This is true that they are not directly measured. However, their characterization follows successive steps including (i) impedance tube measurements, and (ii) a specific iterative inversion process based on a constrained cost function minimization algorithm. In this sense, the successive combination of steps (i) and (ii) constitutes an inverse characterization method based on impedance tube measurements. According to the opinion of the authors’ of the present manuscript, the analytic inversion process following Refs. [60] and [61] is more rigorous than the iterative one, because it involves a unique theoretical solution when considering an input signal without noise. In practice, the low frequency signals are generally poorly accurate. This may justify the use of the iterative process, which has proven to be useful and robust anyway, following proper recommendations (as for any characterization methods) [Foam-X, User’s Manual]. This was apparently not the case in the aforementioned Kino’s work.

9. The conclusion that the presence of membranes has an important influence on bulk elastic properties in polymer foams (pages 155 and 156) was reached by Kino [Ref. [102] for Ch.3] already through measurements as well as modelling. But this is neither remarked nor acknowledged.

This point was corrected in the original manuscript. Note however that this conclusion was reached by Kino through measurements and macro modelling only, without any micro-macro (multi-scale) evidence.

10. To what extent is it true as stated on page 130 that ‘progress in manufacturing methods’ can make use of the results of this Thesis? Reference is made on page 154 to the fact that the frame elasticity of polymer foams can be controlled to some extent and page 159 refers to ‘chemical tunability’ (without explaining it). But what about controlling the microstructure? It would seem that, for example, the presence of holed membranes (in polymer foams), which are found to be important factors in acoustical performance, is an accidental consequence of deficiencies in a manufacturing process that aims at their complete removal.

A proper reference was added in the original manuscript to justify the new ‘chemical tunability’ evidences, and subsequent ‘morphological tunability’ possibilities. In this sense, membranes are not an accidental consequence of deficiencies in a manufacturing process that aims at their complete removal, but a new degree of freedom in the manufacturing process.

11. The Thesis is aimed at making models and numerical experiments that enable understanding of the influence of existing microstructures in metallic and polymer foams on their bulk acoustical properties. But what might be the merits of a different approach? This would specify a microstructure from first principles, aimed at bespoke acoustical characteristics and take advantage, for example, of new rapid prototyping methods enabling 3D fabrication of pre-programmed structures?

This is a possible route. However, is it really compatible with a large industrial process? At the present time, we believe that rapid prototyping might be a suitable option for validation purposes, but is still far from being realistic for a large scale industrial task.