Robust energy shaping control of nonlinear systems

José Guadalupe Romero Velázquez

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José Guadalupe ROMERO VELAZQUEZ

Commande Robuste par façonnement d'énergie de systèmes non-linéaires

Directeur de thèse :	Romeo ORTEGA	Chercheur (L2S- Supelec)
Co-directeur de thèse :		

<u>Composition du jury</u> :

Président du jury :	Brigitte D'ANDREA-NOVEL	Chercheur (Mines PARISTECH)
Rapporteurs :	Alessandro ASTOLFI	Chercheur (Imperial College)
	Claude SAMSON	Chercheur (INRIA)
Examinateurs :	Brigitte D'ANDREA-NOVEL	Chercheur (Mines PARISTECH)
	William PASILLAS	Chercheur (L2S- Supelec)
	Antoine CHAILLET	Chercheur(Supelec)
Membres invités :		

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Publications

Journal papers

(i) R. Ortega and J.G. Romero. Robust integral control of port-Hamiltonian systems: The case of non-passive outputs with unmatched disturbances, Systems & Control Letters 61, 2012, pp.11-17

(ii) J.G. Romero, A. Donaire and R. Ortega. Robust energy shaping control of mechanical systems, Systems & Control Letters. (accepted)

(iii) J.G. Romero, I. Sarras and R. Ortega. A Globally exponentially stable tracking controller for mechanical systems using position feedback, IEEE Transactions on Automatic Control (TAC). (under review)

(iv) D.Navarro-Alarcon, Y. Liu, and J.G Romero. Visually servoed deformation control, IEEE Transaction on Robotic. (under review)

Conference papers

(v) R. Ortega and J.G. Romero. Robust integral control of port-Hamiltonian systems: The case of non-passive outputs with unmatched disturbances, 50st IEEE Conference on Decision and Control (CDC), 2011, pp 3222-3227.

(vi) N. Crasta, R. Ortega, H.K. Pillai, J.G. Romero V. The matching equations of energy shaping controllers mechanical systems are not simplified with generalized forces, 4th IFAC Workshop on Lagrangian and Hamiltonian Methods for Non Linear Control (LHMNLC), 2012, pp 48-53.

(vii) J.G Romero, A. Donaire and R. Ortega. Simplifying robust energy shaping controllers for mechanical systems via coordinate changes, 4th IFAC Workshop on Lagrangian and Hamiltonian Methods for Non Linear Control (LHMNLC), 2012, pp 60-65.

(viii) J.G. Romero, A. Donaire and R. Ortega. Robustifying energy shaping control of mechanical systems, 51st IEEE Conference on Decision and Control (CDC) 2012.

(ix) J.G. Romero, I. Sarras and R. Ortega. A globally exponentially stable tracking controller for mechanical systems using position feedback. American Control Conference (ACC), 2013. (accepted)

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(xii) J.G Romero, D. Navarro-Alarcon, R. Ortega and Y. Liu, Robust GES controllers for perturbed mechanical systems in free/constrained-motion tasks: theory and experiments. (Preparing to CDC'13)

(xiii) J.G. Romero and R. Ortega. A globally asymptotically convergent I&I speed adaptive–observer for mechanical systems insensitive to unknown constant disturbances. (Preparing to CDC'13)

Résumé

En 1975, Francis et Wonham [1] introduit le principe du modèle interne que c'était une percée dans l'étude des systèmes LTI compte tenu des perturbations (servo-systèmes), ce qui donne des conditions nécessaires et suffisantes sur le contrôleur pour assurer la stabilité asymptotique lorsque les signaux de référence et les perturbations sont générées par un de dimension finie exosystème. Le principe du modèle interne de systèmes LTI suggère qu'une copie de la exosystème doit être inclus dans le contrôleur. Par exemple, pour éliminer l'erreur en régime permanent pour les signaux de référence ou de perturbation étape, nous avons besoin d'intégrateurs dans la boucle. Cependant, dans le contexte de ports hamiltoniens (pH) des systèmes où sont considérées les perturbations appariés(matched), qui peuvent être considérés comme des effets du bruit de mesure, entrée inconnue et d'autres phénomènes "oubliées" par les hypothèses du modèle est limitée.

Dans [39] est exploitée action intégrale(IA) sur la sortie passive à résoudre ce problème ainsi que sa robuste régulation. La méthodologie donne comme résultat une boucle fermée étendu, préservant la form pH , de sorte que la robuste régulation et le rejet de perturbations appariés est satisfaite. La faiblesse cruciale (talon d'Achille) de cette méthode apparaît lorsque le signal de régulariser n'est pas la sortie passive et les perturbations ne sont pas appariés (unmatched) à l'entrée. Des exemples simples sont des systèmes mécaniques et de moteurs électriques, où les vitesses sont sorties passives et des courants, respectivement, mais la sortie de l'intérêt est souvent de position. Ce à dire que IA de [39] est insuffisante pour résoudre ce problème.

Dans [2] a été proposé une première discussion sur l'action intégrale et une redéfinition de la structure de pH afin d'offrir robustesse en présence de l'incertitude des paramètres. Toutefois, la première bosser où le pH se prolonge par IA sur la non-sortie passive viennent de [3]. Dans leur procédé, une transformation canonique généralisée est utilisé qui permet l'extension de l'état avec les intégrales des sorties d'intérêt et, en même temps, l'obtention d'un hamiltonien défini positif. Cette approche nécessite de résoudre un ensemble d'équations aux dérivées partielles (PDEs).

Plus récemment, dans [32] une technique ingénieuse sur la régulation des non-passifs sorties via action intégrale a été donnée, cette méthode nous permet en préservant la structure du pH et, par conséquent, la stabilité de boucle fermée. En outre, cette approche est doter de propriété de robustesse au présences des perturbations pas appariés.

Le grand apport de [32], c'est que le pH boucle fermée et la fonction d'énergie est conçu par changement de coordonnées, telles que la comparaison avec [3], la nécessité

de résoudre les PDEs est évitée. La formulation est illustrée par des simulations sur une PMSP(Permanent Magnet Synchronous Motors), où se considèrent les couples inconnus charge constante.

Basé sur [32] , une formulation au rejet de la déviation constant état stable à des systèmes mécaniques est présenté dans [49] . Cependant, le problème se limite au cas linéaire. Clairement, nous pouvons voir que s'il existe initialement une linéarisation par retour d'etats, alors le problème est descendu aux systèmes LTI. Si la question à traiter est le rejet de perturbations appariés alors un commande PI classique fera l'affaire.

Motivé par l'approche dans [32] et les nouveaux développements au sujet du changement de coordonnées [48] , [63] , dans le présent travail, nous proposons une conception constructive de commande intégrale robustes per à la régulation sur sorties non passives aux une large classe de systèmes physiques, aussi le rejet des perturbations non appariés est maintenu. De plus les conditions nécessaires et suffisantes pour la solvabilité du problème, en termes de certaines propriétés rang et la contrôlabilité du système linéarisé, sont fournis.

Lorsque le cas à considérer est non-linéaire des systèmes mécaniques, nous montrent deux méthodes de rejet de perturbations constantes (appariées) et pour variant dans le temps perturbations , des propriétés fortes sur l'IISS et ISS sont fournis.

Sur l'autre main, pendant une longue période une recherche incessante a été réalisé sur les commandes de suivi dans les systèmes mécaniques avec seulement position connue. Tout vient du fait que les systèmes mécaniques sont généralement équipés de capteurs de mesure de position seulement, ce qui a impliqué une recherche constante pour trouver de commandes robustes indépendantes de la vitesse.

Beaucoup semi-globales résultats au problème de retour de position suivi global ont dominé le scénario. Régimes intrinsèquement semi-globale s'appuyer sur haute - injection de gain à élargir le domaine d'attraction ou de la connaissance du modèle exact est exigé comme dans [27] et [28].

Parlant d'une solution globalement asymptotiquement stable, nous pouvons voir [54] et [58], où d'abord un, la solution est limitée a un degré de liberté, et une seconde, souffre malheureusement d'inconvénients graves, le plus important correspondait au fait que le requiert un changement de coordonnées en utilisant les fonctions de saturation où son inversibilité ne peut pas être garanti globalement [40].
De la discussion ci-dessus, dans ce travail, nous proposons un commande globalement exponentiellement suivi sans mesure de vitesse. Ceci est possible combinant un immersion et invariance ($I\&I$) observateur exponentiellement stable récemment publié et une conception appropriée de un retour d'état passivité commande avec l'aide de emph changement de coordonnées.

Ce travail de thèse est composé de quatre chapitres:

Le premier chapitre présente certains matériaux milieux, des concepts et des résultats. Nous commençons avec les notions de stabilité quand on considère les signaux d'entrée. Cadre de modélisation à port-Hamiltonien du système est présentée, montrant l'équivalence intrinsèque entre les équations d'Euler-Lagrange et du cadre hamiltonien. Après une brève introduction où les idées principales de l'immersion et de l'invariance sont illustrés, le principe de conception d'observateurs pour les systèmes non linéaires générales par I &I est donnée.

Dans les trois premières sections du chapitre 2, nous rappelons quelques résultats sur la robustesse aux perturbations appariées et de la régulation sur la sortie passive. Les dernières sections décrit les conditions assorties à des perturbations via commande intégrale , aussi la preuve au assure la régulation de la non-sortie passive est donnée.

Sous certaines hypothèses techniques sur les systèmes mécaniques, nous montrons le rejet de perturbations appariés et non appariés pour matrices d'inertie constante dans le chapitre 3.

En outre, plus forte propriété d'entrée à l'état de stabilité, cette fois par rapport aux perturbations appariées et non appariées, est assurée. Finalement, il est démontré que le commande peut être simplifiée, y compris un changement partiel de coordonnées sur les *momenta* si on considère les perturbations appariées uniquement .

Pour totalement actionnés systèmes mécaniques, il est montré dans le chapitre 4 que le suivi des références continues sans une information de vitesse peut être obtenue en combinant à observateur exponentiellement stable et une conception appropriée de une commande retour d'état à base de passivité, qui assigne à la boucle fermée à structure port-hamiltonien via changement de coordonnées tel qu'il est utilisé dans le chapitre 3.

Préliminaires

Nous commençons par quelques définitions basiques sur la stabilité des systèmes non linéaires d'entrée, où l'objet est d'exprimer les états d'information restent bornées pour l'entrée bornée. Port hamiltonien représentation des systèmes physiques est décrit, changement de coordonnées pour les systèmes mécaniques sont également résumés. Ceci est important car elle se trouve sur sur la plupart des résultats présentés.

Finalement une brève introduction sur l'immersion et l'invariance ($I\&I$) est illustré comme clé de la stabilisation et de la conception d'observateur dans les systèmes de systèmes non linéaires.

Notions de stabilité avec entrée externe

Dans la conception des commandes, l'un des principaux problèmes est d'étudier la sensibilité en boucle fermée à des perturbations, comme des erreurs de mesure, et qui sont délimitées, finalement petite ou convergentes. Dans cette section présente quelques définitions et théorèmes dans l'étude de cette question. Nous renvoyons le lecteur à des informations détaillées à [36],[44],[47].

Nous commençons la discussion aussi simple que possible, de sorte que nous considérons au cours de cette section que nous avons affaire à des systèmes avec des entrées de la forme:

$$\dot{x} = f(x, d) \tag{1}$$

avec l'état $x \in \mathbb{R}^n$, entrée étant inconnu et essentiellement délimitée. Le map $f : \mathbb{R}^n \times \mathbb{R}^m$ est supposée être localement lipschitzienne avec $f(0,0) = 0$. Les fonctions de comparaison à une définition formelle de la stabilité des systèmes présentant des perturbations sont utiles [47], tels que:

Definition 1. *Une classe $\mathcal{K}_\infty$ est une fonction $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ est continue, strictement croissante, non borné et vérifie $\alpha(0) = 0$.*

Definition 2. *Une classe $\mathcal{KL}$ est une fonction beta $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ tel que$\beta(\cdot, t)$ $\in \mathcal{K}_\infty$ pour chaque t et $\beta(r, t)$ strictement décroissante que $t \to \infty$.*

Stabilité entrée-état (ISS)

Le système (1) est dit ISS si et seulement si il existe un fonction $\beta(\mathcal{KL})$ et un fonction γ $(\mathcal{K}_\infty)$, de sorte que

$$|x(t)| \leq \beta(|x_0|, t) + \gamma(||d||_\infty)$$

est satisfaite pour tout $t \geq 0$

La définition de ISS exige que, pour grand t , l'etat doit être délimitée par une fonction $\gamma(||u||_\infty)$ l'correspondent à des entrées (parce que $\beta(|x_0|, t) \to 0$ ainsi $t \to 0$). En outre, le terme $\beta(|x_0|, 0)$ peuvent finir par prédominer pour t petit, ce qui permet de quantifier l'ampleur du comportement transitoire (dépassement) comme une fonction de la taille de l'état initial x_0. (voir pour plus de détails [47] section 2.9)

Une fonction du Lyapunov ISS pour (1.1) est par définition une fonction stockage lisse définie positive $V : \mathbb{R}^n \to \mathbb{R}$ qui est, $V(0) = 0$ et $V(x) > 0$ pour $x \neq 0$, et appropriée, qui est, $V(x) \to \infty$ comme $|x| \to \infty$. Pour V il existe des fonctions γ, $\alpha \in \mathcal{K}_\infty$ de sorte que:

$$\dot{V} \leq -\alpha(|x|) + \gamma(|d|) \quad \forall \ x, d \tag{2}$$

Finalement, nous pouvons conclure que un système est ISS si il ya toujours un bon ISS fonction du Lyapunov satisfaisant l'estimation (2) [47].

Intégrale Stabilité entrée-état (ISS)

Le système (1.1) est dit être $IISS$ prévus qu'il existe deux fonctions α et γ qui sont $\mathcal{K}_\infty$ et une fonction β à savoir $\mathcal{KL}$ de telle sorte que l'estimation

$$\alpha(|x(t)|) \leq \beta(|x_0|, t) + \int_0^t \gamma(|d(s)|)$$

est satisfaite le long de toutes les solutions.

De plus un système est $IISS$ si et seulement si il existe une fonction $\beta \in \mathcal{KL}$ et γ_1, $\gamma_2 \in K_\infty$ telle que

$$|x(t)| \leq \beta(|x_0|, t) + \gamma_1\Big(\int_0^t \gamma(|d(s)|)\Big)$$

pour tout $t \geq 0$, $x_0 \in \mathbb{R}$ et d Aussi, nous pouvons noter que si le système (1.1) est $IISS$, il est alors 0-GAS, qui est le système avec zero d'entrée.

$$\dot{x} = f(x, 0)$$

est Globalement Asymptotiquement Stable (GAS)

Dans le théorème 1 de papier élémentaire [44], il a été établi que l'existence d'un fonction de Lyapunov IISS lisse est nécessaire ainsi que suffisant pour le système (1) être IISS. Ceci est valable si:

(1) Il ya une certaine sortie qui rend le système dissipatif lisser et faiblement détectable zéro.

(2) Le système est 0-GAS et sortie-zéro dissipatif lisser.

Il est à noter que nous avons résumé le théorème, pour plus de détails et des preuves voir la proposition II.5 et la section III des exemples.

Le cadre port - hamiltonien

Fondamentalement, la représentation hamiltonien se pose de la mécanique analytique et commence à partir du principe de moindre action, et procède, en passant par les équations d'Euler-Lagrange et Legendre, la transformation vers les équations hamiltoniennes du mouvement [6]. Nous savons que, normalement, l'analyse des systèmes physiques a été réalisée dans le cadre Lagrangien et hamiltonien, le point de vue du réseau est en vigueur dans la modélisation et la simulation de (complexe) des systèmes d'ingénierie physiques.

Cependant, le cadre de porto-hamiltoniens (pH) des systèmes combinant les deux formulations, en associant à la structure d'interconnexion du modèle de réseau d'une structure géométrique donnée par une structure de Dirac (en général). Avec cette brève description, on peut dire que la dynamique hamiltonienne est définie par rapport à cette structure de Dirac et l'hamiltonien donné par l'énergie totale emmagasinée.

D'ailleurs les systèmes port-hamiltoniens sont des systèmes dynamiques ouverts, qui agissent l'un sur l'autre avec leur environnement par des ports tels qu'une grande classe des systèmes (non linéaires) comprenant les systèmes mécaniques passifs, les systèmes électriques, les systèmes électromécaniques, systèmes mécaniques avec les contraintes nonholonomic et des systèmes thermiques peuvent être décrits par le cadre hamiltonien.

Pour plus de détails au sujet de l'histoire du pH nous avons invité à lire [6], [7].

Comme mentionné la forme Port-hamiltonienne est déterminée par l'intermédiaire d'Euler-Lagrange, tel que des équations du mouvement d'Euler-Lagrange bien connues

$$\frac{d}{dt}\nabla_{\dot{q}}L(q,\dot{q}) - \nabla_{q}L(q,\dot{q}) = u \tag{3}$$

alors si le lagrangien L=K-V est
emph qui régulier c'est-à -dire l'hessien est différent de zéro, en définissant les nouvelles variables

$$\mathbf{p} = \nabla_{\dot{q}}L \tag{4}$$

qui s'appellent les *impulsions généralisés*, il est possible d'employer un changement des coordonnées[1] de $(q,\ \dot{q})$ à $(q, \mathbf{p})$. Ensuite, une fonctions scalaire est définie, dite l'Hamiltonien,

$$H(q, \mathbf{p}) = \mathbf{p}^\top \dot{q} - L(q, \dot{q}) \tag{5}$$

qui représente l'énergie totale du système. Cette procédure est appelée habituellement la *transformation de Legendre*. Par conséquent, les equations du mouvement d'*Euler-Lagrange* deviennent maintenant les équations d'Hamilton :

$$\begin{array}{rcl} \dot{q} & = & \nabla_{\mathbf{p}} H \\ \dot{\mathbf{p}} & = & -\nabla_q H + G(q)u \end{array} \tag{6}$$

Nous observons que l'application de la transformation de Legendre remplace le système de n équations du second ordre par un ensemble de $2n$ équations de premier ordre avec une structure simple et symétrique. Dans les systèmes mécaniques standards ou simples, l'énergie potentielle est habituellement une fonction des positions généralisées $V(q)$ tandis que l'énergie cinétique est une fonction quadratique des vitesses (impulsions), décrit comme $K = \frac{1}{2}\mathbf{p}^\top M(q)\mathbf{p}$, tels que le plein rendement hamiltonien de fonction rendements à être $H = V + K$.

Avec $G(q)$ comme la matrice de force d'entrée et $G(q)u$ décrivant les *forces généralisées* résultants de la commande $u \in \mathbb{R}^m$. Dans le cas où $m = n$ nous parlons de systèmes mécaniques *complètement actionnés* et dans le cas où $m \leq n$ des système mécaniques *sous–actionné*. La représentation dans l'espace d'état (6) avec états $(q,\ \mathbf{p})$ est habituellement appelé l'*espace de phase*. Une généralisation additionnelle de (6) aux systèmes hamiltoniens avec entrées et sorties, est donée par

$$\begin{array}{rcl} \dot{x} & = & \left[\mathbb{F}(x) - \mathcal{R}(x)\right]\nabla_x H(x) + \mathbb{G}(x)u \\ y & = & \mathbb{G}^\top \nabla_x H(x) \end{array} \tag{7}$$

avec la sortie $y \in \mathbb{R}^m$, $\mathbb{J} = -\mathbb{J}^\top$ et $\mathcal{R} = \mathcal{R}^\top \geq 0$. Le système (7) est appelé système *hamiltonien commandé par ports* (PCH) avec une matrice de structure $\mathbb{J}$, matrice de dissipation $\mathcal{R}$ et l'hamiltonien H.

Immersion et Invariance

Récemment surgi une nouvelle méthodologie pour concevoir les commandes adaptatifs pour les systèmes non linéaires (incertains), appelés Inmersion et l'invariance $(I\&I)$. La méthode repose sur les notions systèmes des Inmersión et invariante variété, qui sont des outils classiques de la théorie du régulateur non linéaire et géométrique du contrôle non linéaire [51].

[1] La dynamique d'Euler-Lagrange possède la propriété remarquable d'invariance par rapport à des changements quelconques de coordonnées [8]

Plus précisément, l'approche de $I\&I$ consiste donc à trouver une variété qui peut être rendue invariante et attractive, avec la dynamique interne une copie de la dynamique en boucle–fermée désirée, et à concevoir une loi de commande qui oriente l'état du système suffisamment proche de cette variété.

Une illustration graphique de l'approche de $I\&I$ est montrée dans la figure 1. Nous avons cela $\pi(\cdot)$ maps une trajectoire sur le space ξ à une trajectoire sur l'espace x, qui est limité à le variété $\mathcal{M}$ qui contenant l'origen. D'ailleurs, toutes les trajectoires commençant extérieur du $\mathcal{M}$ convergent à l'origine.

Stabilisation

Le résultat central par la stabilisation de $I\&I$, à savoir un ensemble de conditions suffisantes pour la construction de commande return d'etats globalement asymptotiquement stabilisants, commande affine, et sont données dans le théorème suivant.

Theorem 1. *Considérer le système*

$$\dot{x} = f(x) + g(x)u, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m \tag{8}$$

avec un point d'équilibre $x^* \in \mathbb{R}^n$ à stabiliser. Supposez que là existent les lisses mappage $\alpha : \mathbb{R}^p \to \mathbb{R}^p$, $\pi : \mathbb{R}^p \to \mathbb{R}^n$, $\phi : \mathbb{R}^n \to \mathbb{R}^{n-p}$, $c : \mathbb{R}^p \to \mathbb{R}^m$ et $v : \mathbb{R}^{n\times(n-p)} \to \mathbb{R}^m$, avec $p < n$, de telle sorte que la suivante est vérifiée.

- *(A1) le système cible*

$$\dot{\xi} = \alpha(\xi), \quad \xi \in \mathbb{R}^p \tag{9}$$

- *(A2) Pour tous ξ*

$$f(\pi(\xi)) + g(\pi(\xi))c(\pi(\xi)) = \nabla_\xi(\pi(\xi))\alpha(\xi) \tag{10}$$

- *(A3) L'ensemble identité*

$$\{x \in \mathbb{R}^n | \phi(x) = 0\} = \{x \in \mathbb{R}^n | x = \pi(\xi),\, \xi \in \mathbb{R}^p\} \tag{11}$$

- *(A4) Toutes les trajectoires du système*

$$\dot{z} = \nabla_x \phi(f(x) + g(x)v(x,z)), \tag{12}$$
$$\dot{x} = f(x) + g(x)v(x,z) \tag{13}$$

sont bornées et (1.12) *a un équilibre globalement asymptotiquement uniformément stable à $z = 0$.*

Alors x^* est un équilibre globalement asymptotiquement stable du système en boucle fermée

$$\dot{x} = f(x) + g(x)v(x, \phi(x)) \tag{14}$$

La preuve de ce théorème apparaît dans la section 2,1 de [51]

Contrairement à la commande optimale où l'objectif est d'optimiser un coût scalaire de performance, l'approche I&I ne requiert aucune opération de minimisation. En outre, en raison de son approche en deux temps (immersion et invariance), celle-ci est conceptuellement différente des méthodologies qui reposent sur l'utilisation de fonctions de Lyapunov. Des similitudes existent avec la commande par modes glissants à ceci près que la convergence ne se fait pas en temps fini mais est asymptotique, de plus les lois de commande obtenues ne reposent a priori sur aucun phénomène discontinu, caractéristique de la commande par modes glissants.

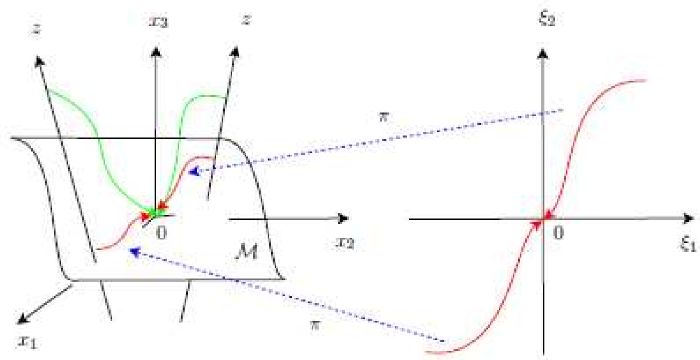


Figure 1: Représentation graphique de l'approche par immersion et invariance.

Les méthodes de commande basées sur des fonctions Lyapunov sont duales de celles présentées ci-dessus. En effet, il s'agit de déterminer une fonction V définie positive telle que $\dot{V} = \alpha(V)$ le long des trajectoires du système, ait un point d'équilibre (globalement) asymptotiquement stable à zéro. A noter que la fonction $V : \mathbb{R}^n \to I$, où I est un intervalle de l'axe réel, peut être considérée comme une submersion et la dynamique cible, puisque la dynamique de la fonction de Lyapunov est de dimension un, voir Figure 2. Une procédure similaire à l'I&I a été proposée dans [11], avec la différence fondamentale que l'application correspondante s'agit d'une transformation de coordonnées et pas une immersion.

Conception d'observateurs

Le problème de la reconstruction des vitesses des systèmes mécaniques, d'un grand intérêt pratique, a été intensivement étudié dans la littérature. Depuis la publication du premier résultat fondateur [12] dans 1990, de nombreuses solutions ont été proposées. Une approche efficace mais restrictive consiste à rendre linéaire la dynamique du système par rapport aux vitesses non mesurées via des changements de coordonnées partiels. Le problème de la construction d'observateurs et de lois de commande devient

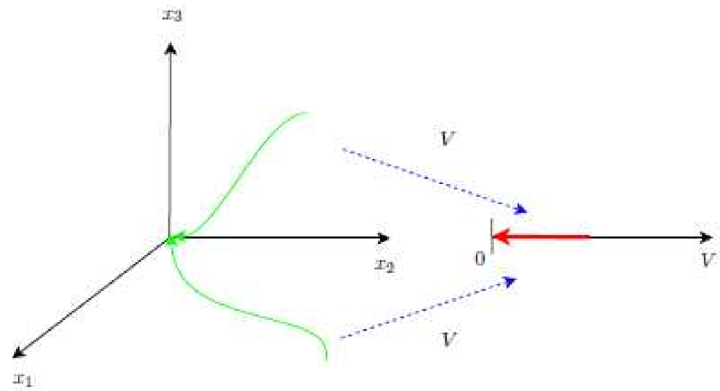


Figure 2: Interprétation par submersion des techniques basées sur l'approche de Lyapunov.

alors aisé [25, 14, 15, 16, 17].

Un observateur, qui exploite la structure Riemannienne du système, est présenté dans [18], [19], [20] tandis qu'une solution pour une classe de systèmes ayant deux degrés de liberté est exposée dans [21]. Pour une liste détaillée de références, le lecteur pourra se reporter aux ouvrages suivants [51, 22, 40].

La première utilisation des variétés invariantes et attractives pour la construction d'observateurs est initialement remonte aux travaux de Luenberger sur les systèmes linéaires, puis elle fut étendu récemment aux systèmes non linéaires [11], [23], [24]. Dans [11], un observateur est défini comme un système linéaire asymptotiquement stable, qui reçoit en entrée les mesures disponibles dont on définit une sortie à l'aide d'une application non linéaire. L'estimé de l'état est ensuite obtenu par inversion de cette application. Sous des conditions de non résonance, il peut être prouvé, à l'aide du théorème auxiliaire de Lyapunov, que le système étendu composé du système et de l'observateur possède une variété invariante et attractive (localement), qui garantit une erreur d'estimation nulle sur celle-ci. Une version globale de ces résultats est proposée dans [23].

Dans tous les travaux mentionnés ci-dessus l'observateur possède une dynamique linéaire. L'existence (locale ou globale) et l'invariance de la variété sont assurées sous des conditions de non résonance ou des hypothèses de complétude. L'attractivité est assurée par la stabilité de la dynamique de l'observateur.

Le problème de la conception d'observateurs via la perspective I&I, en opposition aux travaux précédents, considère que la variété est paramétrisée. La dynamique de l'observateur est choisie de telle sorte que cette variété soit invariante. Ainsi, par rapport au problème de stabilisation, la dynamique cible n'est pas donnée a priori mais est induite par l'observateur à construire. Le point clé revient à résoudre un ensemble d'équations différentielles partielles (EDPs) qui assurent l'attractivité de la variété. Dans l'article récent [25], un observateur d'ordre plein pour une classe des systèmes non linéaires qui obvie aux restrictions dérivant de la solubilité des EDPs en employant une extension dynamique se composant d'un filtre de sortie et d'un paramètre dynamique de graduation.

Nous rappelons la définition d'un observateur d'après [11]. Soit le système non linéaire décrit par les équations différentielles ordinaires suivantes :

$$\begin{aligned} \dot{y} &= f_1(\eta, y) \\ \dot{\eta} &= f_2(\eta, y), \end{aligned} \tag{15}$$

où $\eta \in \mathbb{R}^n$ est la partie de l'état nonmesurée et $y \in \mathbb{R}^k$ est la partie mesurée.

Définition 1. *Le système dynamique :*

$$\dot{\xi} = \alpha(\xi, y), \tag{16}$$

avec $\xi \in \mathbb{R}^s$, $s \geq n$, est appelé observateur I&I du système (15), s'il existe des applications $\beta : \mathbb{R}^s \times \mathbb{R}^k \to \mathbb{R}^s$ et $\phi : \mathbb{R}^n \times \mathbb{R}^k \to \mathbb{R}^s$ inversibles (à gauche par rapport à leur premier argument) et telles que la variété :

$$\mathcal{M} = \{(\eta, y, \xi) \in \mathbb{R}^n \times \mathbb{R}^k \times \mathbb{R}^s \ : \ \beta(\xi, y) = \phi(\eta, y)\} \tag{17}$$

vérifie les propriétés suivantes :

(i) toute trajectoire du système étendu (15,16) initialisée sur la variété $\mathcal{M}$ reste sur celle-ci pour tout temps futur, i.e., *$\mathcal{M}$ est positivement invariante par rapport au système étendu.*

(ii) toute trajectoire du système étendu (15,16) initialisée dans un voisinage de $\mathcal{M}$ converge asymptotiquement vers $\mathcal{M}$, i.e., *$\mathcal{M}$ est attractive par rapport au système étendu.*

Cette définition implique qu'un estimé asymptotique de l'état η est donné par :

$$\hat{\eta} = \phi^L(\beta(\xi, y), y), \tag{18}$$

où ϕ^L est l'inverse à gauche de ϕ. Ainsi, l'erreur d'estimation $\hat{\eta} - \eta$ est nulle sur $\mathcal{M}$.

Introduction

In 1975, Francis and Wonham [1] introduced the internal model principle than was a breakthrough in the study of LTI systems considering disturbances (servo systems), giving necessary and sufficient conditions on the controller to assure asymptotic stability when the reference and disturbance signals are generated by a finite-dimensional exosystem. The internal model principle for LTI systems suggests that a copy of the exosystem must be included in the controller. For example, to eliminate the steady-state error for step reference or disturbance signals, we need integrators in the loop.

However in the context of port Hamiltonian(pH) systems when are considered matched disturbances that can be viewed as effects of measurement noise, unknown input and other phenomena "forgotten" by the model assumptions is limited.

In [39] is exploited integral action(IA) on the passive output to solved this issue plus its robust regulation. The methodology give as result an extended closed loop preserving the pH structure such that robust regulation and the rejection of matched disturbances is hold.

The crucial weakness (achilles heel) for this method appears when the signal to be regulated is not the passive output and the disturbances are not matched (unmatched) with the input. Simple examples are mechanical systems and electrical motors, where the passive outputs are velocities and currents, respectively, but the output of interest is often position. This imply that IA from[39] is inadequate to solve this issue.

In [2] was proposed a first discussion about integral action and redefinition of the pH structure in order to provide robustness in the presence of parameter uncertainty. However the first work where pH is extended via IA on non-passive output come of [3]. In their method, a generalized canonical transformation is used which allows extending the state with the integrals of the outputs of interest and, simultaneously, obtaining a positive definite Hamiltonian. This approach requires solving a set of Partial Differential Equation's(PDEs).

More recently in [32] an ingenious technique about regulation of non-passive outputs via integral action was given. This method allow us preserving pH structure and,thus, closed loop stability. Furthermore this approach is endow with robustness property at presences of unmatched disturbances. The great contribution of [32] is that, the closed loop pH and energy function is designed via change of coordinates, such that comparing with [3], the need to solve PDEs is avoided. The formulation is illustrated via simulations on an PMSM where is consider the unknown piecewise constant load torques.

Based on [32] a formulation to rejection of constant steady state deviation at mechanical systems is presented in [49]. However the problem is limited to *linear case*. Clearly we can see that if there exist initially a feedback linearization, then the problem is come down to LTI systems. If the issue to deal is the rejection of matched disturbances then a classical PI controller will do the job.

Motivated by the approach in [32] and the further developments about change of coordinates [48],[63], in the present work we propose a constructive design of robust ICs to regulation on non-passive outputs to a large class of physical systems ,also rejection on unmatched disturbances (constant) is held. Moreover necessary and sufficient conditions for the solvability of the problem, in terms of some rank and controllability properties of the linearized system, are provided.

When the case to consider is non-linear mechanical systems, we show two methodology to rejection of constant matched disturbances and for time-varying disturbances, strong properties about IISS and ISS are provided.

On other hand, during a long time an incessant research has been realized about tracking controllers on mechanical systems with uniquely known position. Everything come from the fact that mechanical systems are generally equipped with only position measurement encoders, this has implied a constant search to find robust controllers independent of velocity.

Many semi-global results to the aforementioned position feedback global tracking problem have dominated the scenario. Semiglobal schemes intrinsically rely on high–gain injection to enlarge the domain of attraction or exact model knowledge is required as in [27] and [28]. Talking of globally asymptotically stable solution, we can see [54] and [58], where to first one, the solution is limited a one-degree of freedom, and second one, unfortunately suffers from serious drawbacks, once the most significant corresponded to fact that the controller requires a change of coordinates using saturation functions where its invertibility cannot be globally guaranteed [40].
From the above discussion, in this work we propose an globally exponentially tracking controller without velocity measurement. This is possible combining a recently reported exponentially stable immersion and invariance observer and a suitably designed state-feedback passivity-based controller via *change of coordinates*.

This thesis work is composed of four chapters:

The first chapter presents some backgrounds materials, concepts and results. We start with notions of stability when are considers input signals. Modeling framework to port-Hamiltonian system is presented, showing intrinsic equivalency between the Euler-Lagrange equations and Hamiltonian framework. After a brief introduction where the main ideas of immersion and invariance are illustrated, then the design principle of observers for general nonlinear systems by $I\&I$ is given.

In the first three sections of Chapter 2 we recall some results on the robustness to matched disturbances and regulation on passive output . The last sections describes the conditions to matched disturbances via integral controlled, also the proof to ensures regulation of non-passive output is given.

Under some technical assumptions on mechanical systems , we show the rejection of matched and unmatched disturbances to constant inertia matrix in Chapter 3. Moreover, stronger property of input-to-state stability, this time with respect to matched and unmatched disturbances, is ensured. Finally, it is shown that the controller can be simplified, respect to matched disturbances including a partial change of coordinates on *momenta*.

For fully actuated mechanical systems, it is shown in Chapter 4 that the tracking of a continuous references without velocity information can be achieved by combining a exponentially stable immersion and invariance observer and a suitably designed state-feedback passivity-based controller, which assigns to the closed-loop a port-Hamiltonian structure via change of coordinates as used in Chapter 3.

Chapter 1

Preliminaries

In this chapter theoretical background and concepts useful are presented.

We begin with some basics definitions on Stability of Nonlinear *Input* Systems, where the object is to express the fact states remain bounded for bounded input. Port Hamiltonian representation of physical systems is described, change of coordinates to mechanical systems also is summarized. This is important on since it lies on the most of the results presented.

Finally a brief introduction about Immersion and Invariance ($I\&I$) is illustrated as key to stabilization and observer design in nonlinear systems systems.

1.1 Notions of stability with respect to input

In control design, one of the main problems to study is the closed-loop sensitivity to disturbances, as measurement errors, and that are bounded, eventually small or convergent. In this section presents some definitions and theorems in the study of this issue. We refer the reader to extensive information to [36],[44],[47].

We begin the discussion as simple as possible, such that we consider during this section that we are dealing with systems with inputs of the form:

$$\dot{x} = f(x, d) \tag{1.1}$$

with state $x \in \mathbb{R}^n$, input being unknown and essentially bounded $d : [0, \infty) \to \mathbb{R}^m$. The map $f : \mathbb{R}^n \times \mathbb{R}^m$ is assumed to be locally Lipschitz with $f(0, 0) = 0$.

Comparison functions to one formal definition of stability of systems with disturbances are useful [47], such as:

Definition 3. *A class $\mathcal{K}_\infty$ is a function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ which is continuous, strictly increasing, unbounded and satisfies $\alpha(0) = 0$.*

Definition 4. *A class $\mathcal{KL}$ is a function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that $\beta(\cdot, t) \in \mathcal{K}_\infty$ for each t and $\beta(r, t)$ strictly decreasing as $t \to \infty$.*

1.1.1 Input to State Stability(ISS)

The system (1.1) is said ISS if and only if there exist a $\mathcal{KL}$ function β, and a $\mathcal{K}_\infty$ function γ so that

$$|x(t)| \leq \beta(|x_0|, t) + \gamma(||d||_\infty)$$

holds for all $t \geq 0$

The definition of ISS requires that, for t large, the state must be bounded by some function $\gamma(||u||_\infty)$ the correspond to inputs (because $\beta(|x_0|, t) \to 0$ as $t \to 0$). Furthermore the $\beta(|x_0|, 0)$ term may dominate for small t, and this serves to quantify the magnitude of the transient (overshoot) behavior as a function of the size of the initial state x_0. (see for more details [47] section 2.9)

An ISS-Lyapunov function for (1.1) is by definition a smooth storage function positive definite $V : \mathbb{R}^n \to \mathbb{R}$, that is, $V(0) = 0$ and $V(x) > 0$ for $x \neq 0$, and proper, that is, $V(x) \to \infty$ as $|x| \to \infty$. For V there exist functions γ, $\alpha \in \mathcal{K}_\infty$ so that

$$\dot{V} \leq -\alpha(|x|) + \gamma(|d|) \quad \forall \; x, d \tag{1.2}$$

Finally we can to conclude that a system is ISS if there is always a smooth ISS-Lyapunov function satisfying the estimate (1.2) [47].

1.1.2 Integral Input to State Stability($IISS$)

The system (1.1) is said to be $IISS$ provided that there exist two $\mathcal{K}_\infty$ functions α and γ, and a $\mathcal{KL}$ function β, such that the estimate

$$\alpha(|x(t)|) \leq \beta(|x_0|, t) + \int_0^t \gamma(|d(s)|)$$

holds along all solutions.

Moreover a system is $IISS$ if and only if there exist function $\beta \in \mathcal{KL}$ and γ_1, γ_2 $\in K_\infty$ such that

$$|x(t)| \leq \beta(|x_0|, t) + \gamma_1\Big(\int_0^t \gamma(|d(s)|)\Big)$$

for all $t \geq 0$, $x_0 \in \mathbb{R}$ and d

Also we can note that if system (1.1) is $IISS$, then it is 0-GAS, that is, the 0-input system

$$\dot{x} = f(x, 0)$$

is globally asymptotically stable (GAS).

In theorem 1 from elemental paper [44], it was established that the existence of a smooth IISS–Lyapunov function is necessary as well as sufficient for the system (1.1) to be IISS. This is hold if:

(1) There is some output that makes the system smoothly dissipative and weakly zero-detectable.

(2) The system is 0-GAS and zero–output smoothly dissipative.

It is noteworthy that we have summarized the theorem, for more details and proofs see proposition II.5 and section III to examples.

1.2 The port–Hamiltonian framework

Basically the Hamiltonian representation arises of analytical mechanics and starts from the principle of least action, and proceeds, via the Euler-Lagrange equations and the Legendre transform, towards the Hamiltonian equations of motion [6]. We know that normally analysis of physical systems has been performed within the Lagrangian and Hamiltonian framework, the network point of view is prevailing in modeling and simulation of (complex) physical engineering systems[40],[41]. However the framework of port-Hamiltonian(pH) systems combines both formulations, by associating with the interconnection structure of the network model a geometric structure given by a Dirac structure(generally). With this brief description we can say that the Hamiltonian dynamics is defined with respect to this Dirac structure and the Hamiltonian given by the total stored energy. Moreover the port-Hamiltonian systems are open dynamical systems, which interact with their environment through ports such that a large class of (nonlinear) systems including passive mechanical systems, electrical systems, electromechanical systems, mechanical systems with nonholonomic constraints and thermal systems can be described by hamiltonian framework. For more details about the history of pH we invited to read [6], [7].

As mentioned the Port-Hamiltonian form is determinate via Euler-Lagrange, such that from well known Euler-Lagrange equations of motion

$$\frac{d}{dt}\nabla_{\dot{q}}L(q,\dot{q}) - \nabla_q L(q,\dot{q}) = u \tag{1.3}$$

then whether the Lagrangian L=K-V is *regular* that is its Hessian is different from zero, by defining the new variables

$$\mathbf{p} = \nabla_{\dot{q}}L \tag{1.4}$$

that are called the generalized momenta, we can apply a change of coordinates[1] from $(q,\dot{q})$ to $(q,\mathbf{p})$. Then, we define a new scalar function, referred as the *Hamiltonian*

$$H(q,\mathbf{p}) = \mathbf{p}^\top \dot{q} - L(q,\dot{q}) \tag{1.5}$$

that represents the total energy of the system. This procedure is commonly called the *Legendre transformation.* Now, the Euler-Lagrange equations of motion become *Hamilton' s* equations

$$\begin{aligned} \dot{q} &= \nabla_{\mathbf{p}}H \\ \dot{\mathbf{p}} &= -\nabla_q H + G(q)u \end{aligned} \tag{1.6}$$

[1]Euler-Lagrange dynamics have the property of invariance with respect to arbitrary transformations of the coordinates[8]

Hence, we see that the application of Legendre's transformation replaces the system of n second-order differential equations with a set of 2n first-order differential equations with a simple and symmetric structure. In standard or simple mechanical systems, the potential energy is usually a function of the generalized positions $V(q)$ while the kinetic energy is a quadratic function of the velocities (momenta) described as $K = \frac{1}{2}\mathbf{p}^\top M(q)\mathbf{p}$, such that the full Hamiltonian function yields to be $H = V + K$.

With $G(q)$ as the input force matrix and $G(q)u$ denoting the generalized forces resulting from the control inputs $u \in \mathbb{R}^m$. In the case where $m = n$ we speak of fully actuated mechanical systems while when $m \leq n$ of *underactuated* mechanical systems. The state-space representation (1.6) with states (q, $\mathbf{p}$) is usually called a phase space. A further generalization of (1.6) to Hamiltonian systems with (collocated) inputs and outputs, is given in the form

$$\begin{aligned} \dot{x} &= \left[\mathbb{F}(x) - \mathcal{R}(x)\right]\nabla_x H(x) + \mathbb{G}(x)u \\ y &= \mathbb{G}^\top \nabla_x H(x) \end{aligned} \tag{1.7}$$

with the output $y \in \mathbb{R}^m$, $\mathbb{J} = -\mathbb{J}^\top$ and $\mathcal{R} = \mathcal{R}^\top \geq 0$. The system (1.7) is called a *Port–Controlled Hamiltonian* (PCH) system with structure matrix $\mathbb{J}$, dissipation matrix $\mathbb{R}$ and Hamiltonian H.

1.3 Immersion and Invariance

Recently arose a novel methodology to design adaptive controllers for (uncertain) nonlinear systems called Inmersion and Invariance ($I\&I$). The method relies upon the notions of systems inmersion and manifold invariance, which are classical tools from nonlinear regulator theory and geometric nonlinear control [51].

More precisely, the $I\&I$ approach relies on finding a manifold in state-space that can be rendered invariant and attractive, with internal dynamics a copy of the desired closed-loop dynamics, and on designing a control law that steers the state of the system sufficiently close to this manifold. A graphical illustration of the $I\&I$ approach is showed in Fig. 1.1. We have that $\pi(\cdot)$ maps a trajectory on the ξ-space to a trajectory on the x-space, which is restricted to the manifold $\mathcal{M}$ containing the origen. Moreover, all trajectories starting outside $\mathcal{M}$ converge to the origin.

1.3.1 Stabilization

The basic result for $I\&I$ stabilization, namely a set of sufficient conditions for the construction of globally asymptotically stabilising, static, state feedback control laws for general, control affine, nonlinear system, and are described in the next theorem.

Theorem 2. *Consider the system*

$$\dot{x} = f(x) + g(x)u, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m \tag{1.8}$$

with an equilibrium point $x^* \in \mathbb{R}^n$ *to be stabilized. Assume that there exist smooth mappings* $\alpha : \mathbb{R}^p \to \mathbb{R}^p$, $\pi : \mathbb{R}^p \to \mathbb{R}^n$, $\phi : \mathbb{R}^n \to \mathbb{R}^{n-p}$, $c : \mathbb{R}^p \to \mathbb{R}^m$ *and* $v : \mathbb{R}^{n\times(n-p)} \to \mathbb{R}^m$, *with* $p < n$, *such that the following holds.*

- *(A1) the target system*

$$\dot{\xi} = \alpha(\xi), \quad \xi \in \mathbb{R}^p \tag{1.9}$$

- *(A2) For all ξ*

$$f(\pi(\xi)) + g(\pi(\xi))c(\pi(\xi)) = \nabla_\xi(\pi(\xi))\alpha(\xi) \tag{1.10}$$

- *(A3) The set identity*

$$\{x \in \mathbb{R}^n | \phi(x) = 0\} = \{x \in \mathbb{R}^n | x = \pi(\xi), \, \xi \in \mathbb{R}^p\} \tag{1.11}$$

- *(A4) All trajectories of the system*

$$\dot{z} = \nabla_x \phi(f(x) + g(x)v(x, z)), \tag{1.12}$$
$$\dot{x} = f(x) + g(x)v(x, z) \tag{1.13}$$

are bounded and (1.12) *has a uniformly globally asymptotically stable equilibrium at* $z = 0$.

Then x^* is a globally asymptotically stable equilibrium of the closed-loop system

$$\dot{x} = f(x) + g(x)v(x, \phi(x)) \tag{1.14}$$

The proof of this theorem appears in section 2.1 of [51]

I&I should be contrasted with the optimal control approach where the objective is captured by a scalar performance index to optimize. In addition, because of its two-step approach, it is conceptually different from existing (robust) stabilization methodologies that rely on the use of control Lyapunov functions. However, it resembles the procedure used in sliding-mode control [10], where a given manifold–the sliding surface–is rendered attractive by a discontinuous control law. The key difference is that, while in sliding-mode control the manifold must be reached by the trajectories, in the proposed approach the manifold need not be reached.

Lyapunov-based design methods are somewhat dual to the approach (informally) described above. As a matter of fact, in Lyapunov design one seeks a function $V(x)$, which is positive-definite (and proper, if global stability is sought after) and such that the system $\dot{V} = \alpha(V)$, for some function $\alpha(\cdot)$, has a (globally) asymptotically stable equilibrium at zero. Note that the function $V : x \to I$, where I is an interval of the real axis, is a submersion and the "target dynamics", namely the dynamics of the Lyapunov function, are one-dimensional, see Figure 1.2. A procedure similar to I&I is proposed in [11], with the fundamental difference that corresponding mapping is not an immersion but a change of coordinates.

1.3.2 Observer design

The problem of velocity reconstruction of mechanical systems is of great practical interest and has been extensively studied in the literature. Since the publication of the first result in the fundamental paper [12] in 1990, many interesting partial solutions have been reported. Of particular attention has been the case in which the mechanical

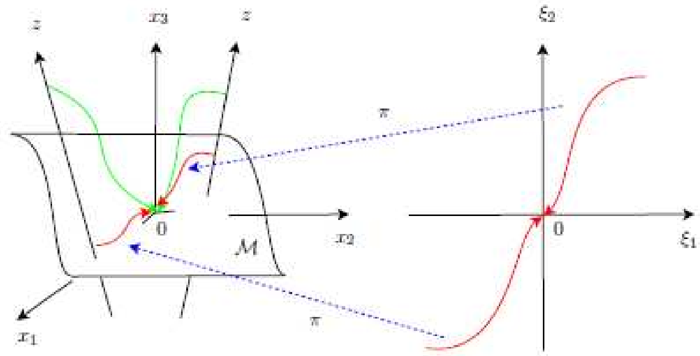


Figure 1.1: Graphical representation of the immersion and invariance approach.

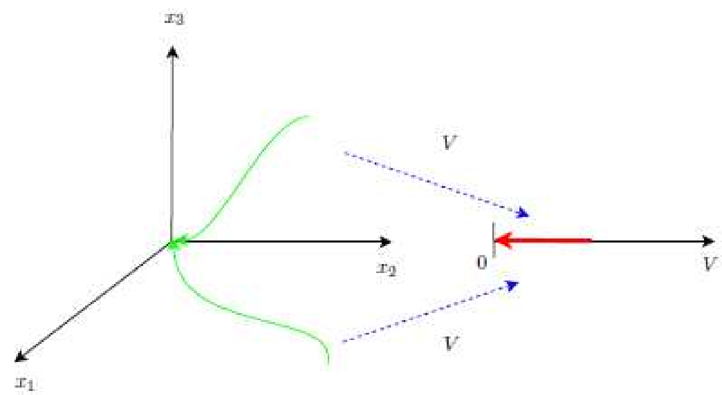


Figure 1.2: Submersion interpretation of Lyapunov based techniques.

system can be rendered linear in the unmeasured velocities via partial changes of coordinates since it simplifies considerably the observation as well as the control problem, see [25, 14, 15, 16, 17]. An intrinsic observer, exploiting the Riemannian structure of the system, has been reported in [18], [19], [20] while a solution for a class of two-degrees-of-freedom systems was reported in [21]. For an exhaustive list of references, the interested reader is referred to the recent books [51, 22, 40].

The use of invariant and attractive manifolds in observer design first appears in the work of Luenberger for linear systems while recently, it has been generalized to general nonlinear systems [11], [23], [24]. In [11], an observer is defined as a linear asymptotically stable system, driven by the available measurements and with a nonlinear output map, and the state estimate is obtained by inversion of such an output map. Under non-resonance conditions, by application of Lyapunov's auxiliary theorem, it is proved that the extended plant-observer system has a (locally) well defined invariant and attractive manifold, with the property that the estimation error is zero on the manifold. A global version of the results was reported in [23]. In all the aforementioned works the observer has linear dynamics, the (local or global) existence and invariance of the manifold is ensured by non-resonance conditions or completeness assumptions, and the attractivity is implied by the stability of the observer dynamics. In the recent paper [25], a full order I&I observer for a class of nonlinear systems has been proposed that obviates the restrictions deriving from the solvability of the PDEs by the use of a dynamic extension consisting of an output filter and a dynamic scaling parameter.

In the spirit of [11] we now give the definition of an I&I observer. To this end, consider the general nonlinear system described as

$$\begin{aligned} \dot{y} &= f_1(\eta, y) \\ \dot{\eta} &= f_2(\eta, y), \end{aligned} \tag{1.15}$$

where $\eta \in \mathbb{R}^n$ is the unmeasured part of the state and $y \in \mathbb{R}^k$ is the measured one.

Definition 5. *The dynamical system*

$$\dot{\xi} = \alpha(\xi, y), \tag{1.16}$$

with $\xi \in \mathbb{R}^s$, $s \geq n$, is called an I&I observer of the system (1.15), if there exist mappings $\beta : \mathbb{R}^s \times \mathbb{R}^k \to \mathbb{R}^s$ and $\phi : \mathbb{R}^n \times \mathbb{R}^k \to \mathbb{R}^s$ that are left-invertible with respect to their first argument and such that the manifold

$$\mathcal{M} = \{(\eta, y, \xi) \in \mathbb{R}^n \times \mathbb{R}^k \times \mathbb{R}^s \;:\; \beta(\xi, y) = \phi(\eta, y)\} \tag{1.17}$$

has the following properties.

(i) All trajectories of the extended system (1.15,1.16) that start on the manifold $\mathcal{M}$ remain there for all future times, i.e., *$\mathcal{M}$ is positively invariant.*

(ii) All trajectories of the extended system (1.15,1.16) that start in a neighborhood of $\mathcal{M}$ asymptotically converge to $\mathcal{M}$, i.e., *$\mathcal{M}$ is attractive.*

This definition implies that an asymptotically converging estimate of the state η is given by

$$\hat{\eta} = \phi^L(\beta(\xi, y), y), \tag{1.18}$$

where ϕ^L denotes a left inverse of ϕ. Thus, the estimation error $\hat{\eta} - \eta$ is zero on the manifold $\mathcal{M}$.

Chapter 2

Robust integral control of port Hamiltonian (pH) systems

Regulation of passive outputs of nonlinear systems can be easily achieved with an integral control (IC). In many applications, however, the signal of interest is not a passive output and ensuring its regulation remains an open problem. Also, IC of passive systems rejects constant input disturbances, but no similar property can be ensured if the disturbance is not matched. In this chapter we address the aforementioned problems and propose a procedure to design robust ICs for port–Hamiltonian models, that characterize the behavior of a large class of physical systems. Necessary and sufficient conditions for the solvability of the problem, in terms of some rank and controllability properties of the linearized system, are provided. For a class of fully actuated mechanical systems, a globally asymptotically stabilizing solution is given. Simulations of the classical pendulum system illustrate the good performance of the scheme.

2.1 Introduction

One of the central features of passivity–based control (PBC), where the first step is passivation of the system [42], is that the passive output can be easily regulated using integral control (IC)—with arbitrary positive gains. The regulation is, moreover, robust with respect to constant input disturbances. In many applications, however, the signal to be regulated is not a passive output and the disturbances are not matched with the input. Classical examples are mechanical systems and electrical motors, where the passive outputs are velocities and currents, respectively, but the output of interest is often position.

In this chapter we propose a procedure to design ICs to regulate non–passive outputs, which are robust to unmatched disturbances. We restrict our attention to port–Hamiltonian (pH) models that, as is widely known, characterize the behavior of a large class of physical systems [37, 7]. Another motivation to consider pH systems is that the popular interconnection and damping assignment PBC design technique [38, 39]—and the closely related canonical transformation PBC [33]—endow an arbitrary nonlinear system with a pH structure. The aim of the additional IC is then to ensure that output regulation is robust *vis–à–vis* external disturbances.

The controller design is formulated as a feedback equivalence problem, where a dynamic feedback controller and a change of coordinates such that the transformed closed–loop system takes a desired pH form are sought. To avoid the need to solve partial differential equations, the interconnection and damping matrices of the target system, as well as its energy function, are kept equal to the ones of the original system, and only add to it an integral action in the non–passive output. This construction is largely inspired by the one proposed in [32], but here we explicitly take into account the presence of the disturbances, which significantly complicates the task. An additional contribution is that necessary and sufficient conditions for feedback equivalence, in terms of some rank and controllability properties of the linearized system, are given. The method is applied to linear and mechanical systems for which robust globally asymptotically stabilizing solutions are obtained, under some reasonable assumptions.

2.2 Perturbed port–Hamiltonian systems and problem formulation

2.2.1 Class of systems and control objectives

The perturbed pH systems considered in the chapter are of the form

$$\begin{aligned} \dot{x} &= F(x)\nabla H(x) + g(x)u + d \\ y &= g^\top(x)\nabla H(x) \end{aligned} \tag{2.1}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m$, $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ is the full rank input matrix, $d \in \mathbb{R}^n$ is a constant disturbance, $H : \mathbb{R}^n \to \mathbb{R}$ is the energy function and

$$F(x) + F^\top(x) \leq 0.$$

As is well–known [37, 7], unperturbed pH systems define cyclo–passive operators $u \mapsto y$, with storage function $H(x)$. This property is strengthened to passivity if $H(x)$ is bounded from below.

We are interested in the scenario where the energy–shaping and damping injection stages of PBC, for the unperturbed system, have been accomplished. That is, it is assumed that an output feedback proportional term has already been added[1] and, consequently,

$$\nabla H^\top(x)[F(x) + F^\top(x))]\nabla H(x) \leq -\alpha|g^\top(x)\nabla H(x)|^2, \tag{2.2}$$

for some $\alpha > 0$, where $|\cdot|$ is the Euclidean norm. Furthermore, it is assumed that a suitable energy function $H(x)$ has been assigned. The choice of this function is

[1] This control action is also known in the literature as L_gV control [40, 7].

a delicate point that, as explained below, depends on whether the disturbances are matched or unmatched.

The control objectives are now, to preserve stability of a desired equilibrium and to drive a given output towards zero, in spite of the presence of disturbances. It will be shown below that, for matched disturbances, *i.e.*, those that enter in the image of $g(x)$, and the passive output y, an IC around y achieves the objectives. In this chapter we are interested in the cases where the disturbance is not matched and the signal to be regulated is not the passive output—but is also zero at the equilibrium.

2.2.2 Notational simplifications

In writing the chapter we have decided to sacrifice generality for clarity of presentation. Consequently, two assumptions that, without modifying the essence of our contribution, considerably simplify the notation are made. First, since we consider the case where disturbances enter in the $n-m$ non–actuated coordinates, the internal model principle indicates that it is necessary to add $(n-m)$ integrators. To ensure solvability of the problem it is reasonable to assume that the number of control actions is sufficiently large. This leads to the following assumption

$$m \geq n - m. \tag{2.3}$$

If less integrators are added this restriction can be relaxed—without modifying the essence of the calculations—but then the notation gets very cumbersome.

The second simplification that we introduce concerns the matrix $g(x)$. Dragging this matrix through the calculations significantly complicates the notation, therefore it will be assumed in the sequel that, after redefinition of the inputs and the states, the input matrix takes the form

$$g(x) = \begin{bmatrix} I_m \\ 0 \end{bmatrix}, \tag{2.4}$$

where I_m is the $m \times m$ identity matrix.

For notational convenience, we partition the state and disturbance vectors as

$$x = \mathrm{col}(x_1, x_2),\ d = \mathrm{col}(d_1, d_2),$$

where $d_1, x_1 \in \mathbb{R}^m$ and $d_2, x_2 \in \mathbb{R}^{n-m}$. Similarly, the matrix $F(x)$ is block partitioned as

$$F(x) = \begin{bmatrix} F_{11}(x) & F_{12}(x) \\ F_{21}(x) & F_{22}(x) \end{bmatrix},$$

with $F_{11}(x) \in \mathbb{R}^{m\times m}$ and $F_{22}(x) \in \mathbb{R}^{(n-m)\times(n-m)}$. With this notation the passive output is

$$y = \nabla_1 H(x).$$

For future reference we also define a second output to be regulated as the $(n-m)$–dimensional vector

$$r = \nabla_2 H(x). \tag{2.5}$$

2.2.3 Some remarks about equilibria

In the absence of disturbances the desired assignable equilibrium $x^\star \in \mathbb{R}^n$ is an isolated minimizer of $H(x)$, that is,

$$x^\star = \arg\min H(x),$$

ensuring that $H(x)$ is positive definite. In view of (2.2), when $u = 0$ and $d = 0$, we have that

$$\dot{H} \leq -\alpha|y|^2 \leq 0,$$

and $x^\star$ is a stable equilibrium of the unperturbed open–loop system with Lyapunov function $H(x)$. Furthermore, invoking standard LaSalle arguments it is possible to prove that $\lim_{t\to\infty} y(t) = 0$ and, if y is a detectable output, that $x^\star$ is asymptotically stable. See, for instance, [40, 7].

To simplify the presentation, in the sequel we identify the set of minimizers of $H(x)$ with

$$\mathcal{M} := \{x \in \mathbb{R}^n \,|\nabla H(x) = 0,\ \nabla^2 H(x) > 0\}. \tag{2.6}$$

Since the second order (Hessian positivity) condition is sufficient, but not necessary, for $x^\star$ to be a minimizer of $H(x)$, the set $\mathcal{M}$ is a subset of the minimizer set, hence the consideration is taken with a slight loss of generality.

In the perturbed case, the set of assignable equilibria of (2.1), (2.4) is given by

$$\mathcal{E} := \{x \in \mathbb{R}^n \mid F_{21}(x)\nabla_1 H(x) + F_{22}(x)\nabla_2 H(x) = -d_2\}. \tag{2.7}$$

It is clear that, if the disturbances are matched, *i.e.*, $d_2 = 0$,

$$\mathcal{M} \subseteq \mathcal{E}.$$

That is, all energy minimizers are assignable equilibria and it is desirable to preserve in closed–loop the open–loop equilibria. On the other hand, in the face of unmatched disturbances, that is, when $d_2 \neq 0$,

$$\mathcal{M} \cap \mathcal{E} = \emptyset. \tag{2.8}$$

In other words, it is not possible to assign as equilibrium a minimizer of the energy function. As will become clear below, this situation complicates the task of rejection of unmatched disturbances.

Remark 1. A problem with the equilibria, similar to the one described above, appears when the desired value for the output to be regulated is different from zero, which is discussed in point 3 of Subsection 4.3.2.

2.3 Robust IC of the passive output

In this section the output regulation and disturbance rejection properties of IC of the passive output of a pH system are revisited. Although both properties are widely referred in the literature, to highlight the differences with our main result, a detailed analysis and some comments and extensions are given below.

2.3.1 Robustness to matched disturbances

Proposition 1. *Consider the perturbed pH system*

$$\begin{aligned} \dot{x} &= F(x)\nabla H(x) + \begin{bmatrix} I_m \\ 0 \end{bmatrix}(u + d_1) \\ y &= \nabla_1 H(x) \end{aligned} \tag{2.9}$$

with an equilibrium $x^\star \in \mathcal{M}$, *and* $d_1 \in \mathbb{R}^m$ *a constant disturbance, in closed–loop with the IC*

$$\begin{aligned} \dot{\eta} &= K_i y \\ u &= -\eta, \end{aligned} \tag{2.10}$$

where $K_i \in \mathbb{R}^{m\times m}$ *is an arbitrary symmetric positive definite matrix.*

(i) (Stability of the equilibrium) *The equilibrium* $(x^\star, d_1)$ *is stable.*

(ii) (Output regulation) *There exists a (closed) ball, centered in* $(x^\star, d_1)$ *such that for all initial states* $(x(0), \eta(0)) \in \mathbb{R}^n \times \mathbb{R}^m$ *inside the ball the trajectories are bounded and*

$$\lim_{t\to\infty} y(t) = 0.$$

(iii) (Asymptotic stability) *If, moreover, y is a detectable output for the closed–loop system* (2.9), (2.10)*, the equilibrium is asymptotically stable.*

The properties (i)–(iii) are global if $H(x)$ *is globally positive definite and radially unbounded.*

Proof. Define the Lyapunov function candidate

$$W(x, \eta) := H(x) + \frac{1}{2}(\eta - d_1)^\top K_i^{-1}(\eta - d_1). \tag{2.11}$$

The closed–loop system (2.9), (2.10) *may be written in the pH form*

$$\begin{bmatrix} \dot{x} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} F(x) & \begin{bmatrix} -K_i \\ 0 \end{bmatrix} \\ \begin{bmatrix} K_i & 0 \end{bmatrix} & 0 \end{bmatrix} \nabla W(x, \eta). \tag{2.12}$$

Clearly, in view of (2.2) *and* (2.4),

$$\dot{W} \leq -\alpha |y|^2. \tag{2.13}$$

The proof is completed invoking standard Lyapunov and LaSalle arguments [36, 7].
□□□ □□□

Remark 2. It is clear from (2.9) that, to ensure $x^\star \in \mathcal{M}$ remains an equilibrium of the closed–loop system, the desired value for u, and consequently for $-\eta$, is $-d_1$. This aspect is also reflected in (2.11). The fact that in IC the disturbances fix the equilibrium value of their state, will also be exploited in the case of unmatched disturbances, allowing us to concentrate our attention on the x components of the equilibrium set.

2.3.2 Discussion and extensions

1. Proposition 1 is a global result that holds for arbitrary positive values of the damping injection and integral gains. The fact that PBC yields high–performance, easily tunable, simple designs (like PI control) explains its wide–spread popularity in applications.

2. Proposition 1 applies *verbatim* for a general input matrix $g(x)$. In this case, the closed–loop is the pH system

$$\begin{bmatrix} \dot{x} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} F(x) & \begin{bmatrix} -g(x)K_i \\ 0 \end{bmatrix} \\ \begin{bmatrix} K_i g^\top(x) & 0 \end{bmatrix} & 0 \end{bmatrix} \nabla W(x,\eta).$$

On the other hand, the presence of $g(x)$ in the subsequent material considerably complicates the notation. Hence, our assumption (2.4).

3. Looking at the linearization of the closed–loop system (2.9), (2.10), it is possible to show that, if $x^\star \in \mathcal{M}$ and the $(2,2)$ block of the matrix $F(x)$, evaluated at $x^\star$ is full rank, $x^\star$ is an exponentially stable equilibrium. Moreover, the rank condition holds if and only if the triple

$$\left(F^\star, \begin{bmatrix} -K_i \\ 0 \end{bmatrix}, \begin{bmatrix} K_i & 0 \end{bmatrix} \right)$$

has no transmission zeros at the origin. This assumption is standard for integral control of nonlinear systems. See, *e.g.*, Section 12.3 of [36].

4. If the desired value for the output y is different from zero, say $y_d \in \mathbb{R}^m$, it is common in practice to use a PI controller

$$\begin{aligned} \dot{\vartheta} &= \nabla_1 H(x) - y_d \\ u &= -K_p[\nabla_1 H(x) - y_d] - K_i \vartheta, \end{aligned}$$

where the proportional term, with $K_p \geq 0$, replaces the previous damping injection. Local stability of this scheme can be established looking at its linearization. It is not clear to the authors under which conditions is it possible to establish a global result—like the one obtained in Proposition 1. A particular case when this is so is when the matrix $F(x)$ is constant. Then, following the analysis of [35], it is possible to show that the shifted Hamiltonian qualifies as a global Lyapunov function.

5. Another difficulty that arises when $y_d \neq 0$ is that a necessary condition to achieve output regulation is the existence of $x^\star \in \mathbb{R}^n$ verifying

$$x^\star \in \mathcal{E} \cap \{x \in \mathbb{R}^n \mid \nabla_1 H(x) = y_d\}.$$

That is, an assignable equilibrium such that the output function, evaluated at this equilibrium, takes the desired value. If $y_d \neq 0$, it is clear that $x^\star \notin \mathcal{M}$. This, unfortunately, makes the expression of the linearized system rather complicated and it does not seem to be possible to easily complete the analysis with an assumption like the rank condition of point 3 above.

2.4 A feedback equivalence problem

As shown in the proof of Proposition 1 the key property to prove that IC of the passive output rejects matched disturbances is the preservation of the pH structure, moreover, with a separable energy function, see (2.11) and (2.12).[2] A key contribution of the chapter is the proof that, under some conditions, it is possible to retain these properties in the unmatched disturbance case. More precisely, it is proposed to add a new dynamic extension and a change of coordinates, without modifying the functional relations in the matrix $F(x)$ nor the energy function $H(x)$.[3] Preserving the energy function avoids the need to solve a partial differential equation, while keeping the same interconnection and damping matrix, simplifies the nonlinear algebraic equations. This motivates the following definition of feedback equivalence.

Definition 6. *The perturbed system*

$$\dot{x} = F(x)\nabla H(x) + \begin{bmatrix} I_m \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ d_2 \end{bmatrix} \tag{2.14}$$

is said to be feedback equivalent to a matched disturbance integral controlled system—for short, MDICS equivalent—if there exists two mappings

$$\hat{u}, \psi : \mathbb{R}^m \times \mathbb{R}^{n-m} \times \mathbb{R}^{n-m} \to \mathbb{R}^m,$$

with

$$\text{rank}\,\{\nabla_1 \psi(x_1, x_2, \zeta)\} = m, \tag{2.15}$$

such that the system in closed–loop with the "integral" control

$$\begin{aligned} \dot{\zeta} &= K_i[\nabla_2 H(\psi(x_1, x_2, \zeta), x_2)] \\ u &= \hat{u}(x_1, x_2, \zeta), \end{aligned} \tag{2.16}$$

expressed in the coordinates,

$$\begin{aligned} z_1 &= \psi(x_1, x_2, \zeta) \\ z_2 &= x_2 \\ z_3 &= \zeta, \end{aligned} \tag{2.17}$$

takes the pH–form

$$\dot{z} = \begin{bmatrix} F(z_1, z_2) & \begin{bmatrix} 0 \\ -K_i \end{bmatrix} \\ \begin{bmatrix} 0 & K_i \end{bmatrix} & 0 \end{bmatrix} \nabla U(z), \tag{2.18}$$

where

$$U(z) := H(z_1, z_2) + \frac{1}{2}(z_3 - d_2)^\top K_i^{-1}(z_3 - d_2). \tag{2.19}$$

It is said to be robustly MDICS equivalent if the mappings $\psi(x_1, x_2, \zeta)$ and $\hat{u}(x_1, x_2, \zeta)$ can be computed without knowledge of d_2.[4]

[2] This property is a consequence of the well–known fact that power–preserving interconnections of pH systems—through power–port variables—preserve the pH structure with energy the sum of the energies of the pH systems. See [37] for a detailed study of this property.

[3] See Remark 4 for a clarification of this point.

[4] See Remark 6 for a clarification of this point.

MDICS equivalence guarantees that the transformed closed–loop system takes the desired form (2.18). (Compare with (2.12).) The rank condition (2.15) ensures that (2.17) is a diffeomorphism that maps the set of equilibria of the (x, ζ)–system into the equilibria of the z–system. This is, of course, necessary to be able to infer stability of one system from stability of the other one. Robust MDICS equivalence guarantees that the control law (2.16) can be implemented without the knowledge of the disturbance d_2.

At this point we make the important observation that choosing the desired value for z_3 to be equal to d_2 is necessary to be able to solve the robust MDICS equivalence problem. Indeed, since in the change of coordinates (2.17) we fixed $z_2 = x_2$, and these are unactuated coordinates, it is necessary that d_2, which appears in $\dot{x}_2$, appears also in $\dot{z}_2$. This fact will become evident in the next section, when we give the solution to the MDICS equivalence problem. Remark that, since $z_3 = \zeta$, the equilibrium value for ζ is also d_2.

As explained in Subsection 2.2.3 the equilibrium sets of (2.14), (2.16) and (2.18) are not just different, but they are actually disjoint, see (2.8). Indeed, while the (x components of the) former are in the set

$$\mathcal{E}_{cl} := \mathcal{E} \cap \{x \in \mathbb{R}^n \mid [\nabla_2 H](\psi(x_1, x_2, d_2), x_2) = 0\}, \tag{2.20}$$

the (z_1, z_2) components of the latter are in $\mathcal{M}$. In spite of that, the fact that (2.17) is a diffeomorphism ensures that the implication

$$[(x_1, x_2) \in \mathcal{E}_{cl} \;\Rightarrow\; (\psi(x_1, x_2, d_2), x_2) \in \mathcal{M}], \tag{2.21}$$

is true, which will be essential for future developments.

Remark 3. The proposed control (2.16) is, in general, not an integral action because of the possible dependence of $\psi(x_1, x_2, \zeta)$ with respect to ζ. We have decided to keep the name because in the z coordinates it is, indeed, an integral action of the form

$$\dot{z}_3 = K_i \nabla_2 H(z_1, z_2). \tag{2.22}$$

Remark 4. It is important to underscore that in the feedback equivalence problem considered here the matrix $F(z_1, z_2)$ and energy function $H(z_1, z_2)$ are just the evaluations of the original functions of the x system in the z coordinates, without applying the (inverse) change of coordinates.[5] That is, $H(x_1, x_2) \neq H(z_1, z_2) \circ \psi(\chi)$, but simply $H(z_1, z_2) = H(x_1, x_2)|_{x_1 = z_1, x_2 = z_2}$. This, rather arbitrary, choice is done to be able to translate MDICS equivalence into an algebraic problem.

2.5 Conditions for MDICS equivalence

In this section we present two propositions that identify conditions for MDICS equivalence. The first one is global and identifies the matching conditions that the mapping $\psi(x_1, x_2, \zeta)$ has to satisfy. The second one gives a necessary and a sufficient condition

[5] To avoid cluttering the notation the same symbols, $H(\cdot)$ and $F(\cdot)$, have been used for both functions.

for existence of a local result in terms of controllability and a rank condition of the linearized systems, respectively. To simplify the notation we introduce the $2n-m$ state vector

$$\chi := \mathrm{col}(x_1, x_2, \zeta).$$

2.5.1 Global MDICS equivalence

Proposition 2. *The perturbed pH system* (2.14) *satisfying condition* (2.3) *is MDICS equivalent if the mapping $\psi(\chi)$ verifies* (2.15) *and the following algebraic equation:*

(DyM) (Dynamics matching)

$$\begin{array}{rcl} \zeta & = & -F_{21}(x)\nabla_1 H(x) - F_{22}(x)\nabla_2 H(x) + \\ & + & F_{21}(\psi(\chi), x_2)[\nabla_1 H(\psi(\chi), x_2)] + \\ & + & F_{22}(\psi(\chi), x_2)[\nabla_2 H(\psi(\chi), x_2)]. \end{array} \tag{2.23}$$

Moreover, the control signal $\hat{u}(\chi)$ is independent of d_2 if $\psi(\chi)$ verifies

(DiM) (Disturbance matching)

$$\nabla_2 \psi(\chi) d_2 = 0. \tag{2.24}$$

Proof. We will prove that, under the condition (2.23)*, there exists $\hat{u}(\chi)$ such that the closed–loop system* (2.14), (2.16) *takes, in the z–coordinates, the pH form* (2.18)*. Furthermore, if* (2.24) *holds, the mapping $\hat{u}(\chi)$ is independent of d_2. For, computing $\dot{\psi}$ and setting it equal to $\dot{z}_1$, as defined in* (2.18)*, yields*

$$\begin{array}{rcl} \dot{\psi} & = & \nabla\psi(\chi)\dot{\chi} \\ & = & \nabla_1\psi(\chi)[F_{11}(x)\nabla_1 H(x) + F_{12}(x)\nabla_2 H(x) + \hat{u}(\chi)] + \\ & & +\nabla_2\psi(\chi)[F_{21}(x)\nabla_1 H(x) + F_{22}(x)\nabla_2 H(x) + d_2] \\ & & +\nabla_3\psi(\chi)[\nabla_2 H(\psi(\chi), x_2)] \\ & \equiv & \dot{z}_1 = [F_{11}(z)\nabla_1 H(z) + F_{12}(z)\nabla_2 H(z)]|_{z_1=\psi(\chi), z_2=x_2} \end{array} \tag{2.25}$$

Since $\nabla_1\psi(\chi)$ is full rank, this equation has a unique solution that defines the mapping $\hat{u}(\chi)$. Notice that the disturbance enters through the term $\nabla_2\psi(\chi)d_2$, which cancels if $\psi(\chi)$ satisfies (2.24).

Proceeding now with $\dot{x}_2$, and setting it equal to $\dot{z}_2$, leads to

$$\begin{array}{rcl} \dot{x}_2 & = & F_{21}(x)\nabla_1 H(x) + F_{22}(x)\nabla_2 H(x) + d_2 \equiv \dot{z}_2 = \\ & = & [F_{21}(z)\nabla_1 H(z) + F_{22}(z)\nabla_2 H(z) - (z_3 - d_2)]|_{z_1=\psi(\chi), z_2=x_2, z_3=\zeta}, \end{array}$$

which is the matching equation (2.23)*. It is important to note that the disturbance d_2, that enters through $\dot{x}_2$, is canceled with the term $\dot{z}_2$, which also contains this signal.*

Finally, the third coordinate $\dot{z}_3$ is equal to $\dot{\zeta}$ by construction. □□□ □□□

2.5.2 Local MDICS equivalence

To streamline the presentation of the next result define the linearization of the pH system (2.14) at the points $x^\star \in \mathcal{E}_{cl}$ and $\bar{x} \in \mathcal{M}$ as

$$A := \nabla(F(x)\nabla H(x))|_{x=x^\star}, \quad E := (F(x)\nabla^2 H(x))|_{x=\bar{x}}. \tag{2.26}$$

These $n \times n$ matrices are block partitioned as

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$

with $A_{11} \in \mathbb{R}^{m \times m}$ and $A_{22} \in \mathbb{R}^{(n-m)\times(n-m)}$, with a similar partition for E. Notice that, since $\nabla H(\bar{x}) = 0$, the linearization at a point in the minimizer set takes a simpler form.

Proposition 3. *Consider the perturbed pH system* (2.14) *satisfying condition* (2.3) *and two points: $x^\star \in \mathcal{E}_{cl}$ and $\bar{x} \in \mathcal{M}$.*

(S1) *A necessary condition for MDICS equivalence is that the linearizations of the pH system at the points $x^\star$ and $\bar{x}$ are controllable. That is, the pairs*

$$\left(A, \begin{bmatrix} I_m \\ 0 \end{bmatrix}\right), \left(E, \begin{bmatrix} I_m \\ 0 \end{bmatrix}\right)$$

are controllable pairs.

(S2) *A sufficient condition for MDICS equivalence is that the* $(2,1)$ *blocks of the matrices A and E defined in* (2.26) *are full rank. That is,*

$$rank\ \{A_{21}\} = rank\ \{E_{21}\} = n - m. \tag{2.27}$$

Moreover, the system is robustly MDICS equivalent if

$$\begin{aligned} A_{22} &= E_{22} && (2.28) \\ A_{21}x_1^\star &= -d_2. && (2.29) \end{aligned}$$

Proof. *Since we are interested in local solutions we will solve the MDICS equivalence problem for the linearization of the systems* (2.14), (2.16) *and* (2.18)*—around their corresponding equilibrium points. In particular, we are interested in their unactuated dynamics, x_2 and z_2, for which we get*[6]

$$\dot{x}_2 = A_{21}(x_1 - x_1^\star) + A_{22}(x_2 - x_2^\star),$$

and

$$\dot{z}_2 = E_{21}(z_1 - \bar{x}_1) + E_{22}(z_2 - \bar{x}_2) - z_3 + d_2.$$

The linearization of the the mapping $\psi(\chi)$ at $(x^\star, d_2)$ yields

$$\psi(\chi) = \psi^\star + T_1(x_1 - x_1^\star) + T_2(x_2 - x_2^\star) + T_3(\zeta - d_2), \tag{2.30}$$

[6]With an obvious abuse of notation the same symbols for the original equations and their linearizations are used.

where the constant matrices

$$T_i := \nabla_i \psi^\star,\ i = 1, 2, 3,$$

have been defined. Setting $\dot{x}_2$ equal to $\dot{z}_2$—evaluated at (2.17)—yields the dynamics matching equation

$$\begin{aligned} A_{21}(x_1 - x_1^\star) + A_{22}(x_2 - x_2^\star) &\equiv \\ E_{21}[T_1(x_1 - x_1^\star) + T_2(x_2 - x_2^\star) + T_3(\zeta - d_2)] &+ \\ +E_{22}(x_2 - x_2^\star) - \zeta + d_2, \end{aligned}$$

where the identities $x_2^\star = \bar{x}_2$ and $\psi^\star = \bar{x}_1$, that stem from (2.21), are used. Equation (2.31) has a solution if and only if the matrices T_i satisfy

$$A_{21} = E_{21}T_1,\ A_{22} = E_{21}T_2 + E_{22},\ E_{21}T_3 = I_{n-m}. \tag{2.31}$$

Remark that, in view of (2.3), the matrices A_{21} and E_{21} are not tall, being either square or fat.

We now proceed to prove (S2). Assume rank $\{A_{21}\}$ = rank $\{E_{21}\} = n - m$. Then, $E_{21}E_{21}^\top$ is invertible and, defining the pseudo-inverse,

$$E_{21}^\dagger := E_{21}^\top (E_{21}E_{21}^\top)^{-1},$$

propose

$$T_1 = E_{21}^\dagger A_{21},\ T_2 = E_{21}^\dagger (A_{22} - E_{22}),\ T_3 = E_{21}^\dagger \tag{2.32}$$

as solutions of (2.31). Notice that T_1 is the product of full–rank matrices, hence is full–rank, and the condition (2.15) is satisfied.

To prove (S1) assume a solution of (2.31) exists. Then,

$$\text{rank}\ \{E_{21}T_3\} = \text{rank}\ \{I_{n-m}\} = n - m.$$

Since rank $\{AB\} \le \min\{\text{rank}\ \{A\}, \text{rank}\ \{B\}\}$, the identity above implies that rank $\{E_{21}\} = n - m$. Now, from Popov–Belevitch–Hautus test we have that the linearized system (E, g) is controllable if and only if, for all $v \in \mathbb{C}^{n-m}$, the following implication is true

$$\left(v^\top E_{21} = 0,\ E_{22}^\top v = \lambda v, \lambda \in \mathbb{C} \Rightarrow v = 0\right). \tag{2.33}$$

The rank condition ensures then that the system (E, g) is controllable. It only remains to prove that (A, g) is also controllable. Towards this end, note that $A_{21} = E_{21}T_1$. The rank condition on T_1, (2.15), imposes that A_{21} is full–rank that, once again, implies controllability of (A, g).

The claim of robust MDICS equivalence follows noting that, on one hand, (2.28) and (2.31) imply $T_2 = 0$, hence ensuring (2.24). On the other hand, replacing (2.29) and (2.32) in (2.30), yields the resulting mapping

$$\psi(\chi) = \bar{x}_1 + E_{21}^\dagger (A_{21}x_1 + \zeta), \tag{2.34}$$

which is, obviously, independent of d_2. □□□ □□□

Unfortunately, there is a gap between the necessary and the sufficient conditions of Proposition 3. Indeed, controllability of the linearized systems is necessary, but not sufficient, for MDICS equivalence. The gap stems from the fact that, without further qualifications on E_{22}, the implication (2.33) does not ensure that rank $\{E_{21}\} = n-m$. On the other hand, it is obvious that (2.27) implies controllability.

Proposition 3 establishes that, if (2.27), (2.28) and (2.29) hold, the system is locally robustly MDICS equivalent—in a neighborhood of $(x_1^\star, x_2^\star, d_2)$—with the linear mapping (2.34). Of course, there might be other, possibly nonlinear, admissible mappings valid in a large region of the state space. It is shown in Section 3.8, that this is the case for linear systems and nonlinear mechanical systems.

Remark 5. Condition (2.28) imposes restrictions on the dependence of $F(x)$ and $H(x)$ with respect to the unactuated coordinate x_2. Condition (2.29), on the other hand, is related with the form of the assignable equilibrium set $\mathcal{E}$. Recalling that the matrices A and E are linearizations of the same vector field at two different points, it is clear that both sets $\mathcal{E}_{cl}$ and $\mathcal{M}$ play a role in these assumptions. Interestingly, even though these assumptions are now technical, they are satisfied in the examples of Section 3.8, as well as in the motor example of [32].

Remark 6. In Definition 6 the feedback equivalence was said to be robust—for obvious reasons—if the mappings $\psi(\chi)$ and $\hat{u}(\chi)$ can be computed without knowledge of the disturbance d_2. As seen from the proof of Proposition 2, $\hat{u}(\chi)$ may, indeed, depend on d_2. However, from the dynamics matching equation (2.23) that defines $\psi(\chi)$, it is not clear why would it depend on d_2. The reason is that, as shown in Proposition 3, when looking for a local solution around the equilibria, these depend on d_2. See (2.30) and (2.31).

2.6 Robust integral control of a non–passive output

In this section the main result of the chapter is presented. Namely, the design of an IC, which is robust *vis–à–vis* unmatched disturbances. More precisely, the controller preserves stability of the equilibrium and ensures regulation (to zero) of the signal (2.5) that, being of relative degree larger than one, is not a passive output.

Proposition 4. *Consider the perturbed pH system* (2.14) *satisfying condition* (2.3)*. Assume there exist two points, $x^\star \in \mathcal{E}_{cl}$ and $\bar{x} \in \mathcal{M}$, that is, an assignable equilibrium and a minimizer of the energy $H(x)$, such that* (2.27)–(2.29) *hold, with A and E defined in* (2.26)*. Under these conditions, there exist two mappings*

$$\hat{u}, \psi : \mathbb{R}^m \times \mathbb{R}^{n-m} \times \mathbb{R}^{n-m} \to \mathbb{R}^m,$$

such that the "integral" control (2.16) *ensures the following properties.*

(i) (Stability of the equilibrium) *The equilibrium $(x_1^\star, x_2^\star, d_2)$ is stable.*

(ii) (Regulation of the passive output) *There exists a (closed) ball, centered at the equilibrium, such that for all initial states $(x(0), \zeta(0)) \in \mathbb{R}^n \times \mathbb{R}^{n-m}$ inside the* *ball the trajectories are bounded and*

$$\lim_{t\to\infty} y(t) = 0.$$

(iii) (Asymptotic stability) *If, moreover, y is a detectable output for the closed–loop system* (2.14), (2.16)*, the equilibrium is asymptotically stable.*

(iv) (Regulation of the non–passive output) *Under the condition of (iii), there exists a (closed) ball, centered at the equilibrium, such that for all initial states $(x(0), \zeta(0)) \in \mathbb{R}^n \times \mathbb{R}^{n-m}$ inside the ball the trajectories are bounded and the output* (2.5) *satisfies*

$$\lim_{t\to\infty} r(t) = 0.$$

The properties (i)–(iv) hold globally if the function $H(x)$ is globally positive definite and proper (with respect to $\bar{x}$) and the mapping $\psi(x_1, x_2, \zeta)$ satisfies (globally) the conditions (2.23) *and* (2.24) *of Proposition 15.*

Proof. The proof is an immediate corollary of Proposition 2 and Proposition 3. Indeed, under the conditions of the proposition, the perturbed pH system (2.14) *is robustly MDICS equivalent to* (2.18)*. That is,* (2.17) *is a diffeomorphism that transform the closed–loop system into* (2.18)*. Now, since $\bar{x} \in \mathcal{M}$, $U(z)$ is a positive definite function with respect to $(\bar{x}, d_2)$. Computing the derivative of $U(z)$ along the trajectories of* (2.18)*, and using* (2.2)*, yields*

$$\dot{U} \leq -\alpha|y|^2.$$

The proof of (i)–(iii) is completed, as the proof of Proposition 1, invoking standard Lyapunov and LaSalle arguments. Claim (iv) follows from asymptotic stability of the equilibrium and the fact that $\nabla_2 H(\bar{x}) = 0$. □□□ □□□

2.7 Examples

In this section we prove that the proposed IC ensures global asymptotic stability for linear systems and nonlinear mechanical systems.

2.7.1 Linear systems

Proposition 5. *Consider the linear perturbed pH system* (2.14) *satisfying condition* (2.3)*, with F constant verifying*

$$x^\top (F + F^\top) x \leq -\alpha |x|^2, \quad \alpha > 0,$$

for all $x \in \mathbb{R}^n$, and with[7]

$$H(x) = \frac{1}{2}|x|^2.$$

Assume

$$\text{rank}\ \{F_{21}\} = n - m.$$

[7] The choices of decoupled energy function and zero equilibrium are done for simplicity and without loss of generality.

The IC

$$\begin{aligned}\dot{\zeta} &= K_i x_2 \\ u &= -F_{21}^{\dagger} K_i x_2 + F_{11} F_{21}^{\dagger} \zeta,\end{aligned}$$

ensures the equilibrium $(-F_{21}^{\dagger} d_2, 0, d_2)$, *is globally asymptotically stable with Lyapunov function*

$$\begin{aligned}V(x,\zeta) &:= \frac{1}{2}\left(|x_1 + F_{21}^{\dagger}\zeta|^2 + |x_2|^2\right) + \\ &+ \frac{1}{2}(\zeta - d_2)^{\top} K_i^{-1} (\zeta - d_2).\end{aligned}$$

Proof. In this case

$$\begin{aligned}\mathcal{E} = \mathcal{E}_{cl} &= \{x \in \mathbb{R}^n \mid F_{21} x_1 = -d_2,\ x_2 = 0\} \\ \mathcal{M} &= \{x = 0\},\end{aligned}$$

and $F = A = E$. *Hence, the conditions for robust MDICS equivalence of Proposition 3 are satisfied. The mapping* (2.34) *takes the form*

$$\psi(\chi) = x_1 + F_{21}^{\dagger}\zeta.$$

The proof is completed computing the expression of u *from* (2.25), *which yields the expression above.* □□□ □□□

2.7.2 Mechanical systems

Proposition 6. *Consider an* m*–degrees of freedom, fully–actuated, fully–damped, perturbed mechanical system represented in pH form* (2.14), *with state*

$$x = col(\mathbf{p}, q),$$

where $q, \mathbf{p} \in \mathbb{R}^m$ *are the generalized positions and momenta respectively, and*

$$F = \begin{bmatrix} -K_p & -I_m \\ I_m & 0 \end{bmatrix}.$$

The energy function is given by

$$H(x) = \frac{1}{2} x_1^{\top} M^{-1} x_1 + P(x_2),$$

with $M \in \mathbb{R}^{m\times m}$ *the positive definite, constant inertia matrix, and* $P(x_2)$ *the potential energy function. Assume*

$$\bar{x}_2 = \arg\min P(x_2)$$

and it is isolated and global.

The IC

$$\begin{aligned}\dot{\zeta} &= K_i \nabla P(x_2) \\ u &= -K_p \zeta - M K_i \nabla P(x_2),\end{aligned}$$

ensures the equilibrium $(-Md_2, \bar{x}_2, d_2)$ *is globally asymptotically stable with Lyapunov function*

$$\begin{aligned} V(x,\zeta) &:= \frac{1}{2}(x_1+M\zeta)^\top M^{-1}(x_1+M\zeta)+P(x_2)+ \\ &+ \frac{1}{2}(\zeta-d_2)^\top K_i^{-1}(\zeta-d_2). \end{aligned}$$

Proof. A global solution to the dynamics matching equation (2.23) *is given by*

$$\psi(\chi) = x_1 + M\zeta,$$

which clearly satisfies (2.24)*. Hence, the conditions for global asymptotic stability of Proposition 11 are satisfied. The proof is completed computing the expression of* u *above from* (2.25)*.* □□□ □□□

The disturbance considered in the example represents a bias term in the measurement of velocity that propagates into the system through the damping injection. This fact is clear writing the dynamics of the open–loop system in Euler–Lagrange form

$$M\ddot{q} + K_p(\dot{q} - d_2) + \nabla P(q) = u.$$

It is interesting to note that, after differentiation, the closed–loop system is given by

$$M\dddot{q} + K_p\ddot{q} + (I_m + MK_i)\nabla^2 P(q)\dot{q} + K_pK_i\nabla P(q) = 0.$$

Hence, the stabilization mechanism is akin to the introduction of nonlinear gyroscopic forces plus a suitable waiting of the potential energy term.

The result can be extended—under some assumptions—to the case of nonconstant inertia matrix. Indeed, it is easy to verify that the mapping

$$\psi(\chi) = x_1 + M(x_2)\zeta,$$

is a global solution of the dynamics matching equation (2.23). However, additional constraints on $M(x_2)$ and–or d_2 are needed to satisfy the disturbance matching equation (2.24). Namely, that the i–th component of the disturbance vector is zero if $M(x_2)$ depends on the i–th element of x_2, that is,

$$e_i^\top d_2 \frac{\partial M(x_2)}{\partial x_{2_i}} = 0$$

where $x_{2_i} := e_i^\top x_2$, with $e_i \in \mathbb{R}^{n-m}$ the i–th vector of the Euclidean basis.

Remark 7. Note that

$$\begin{aligned} \mathcal{E}_{cl} &= \{x \in \mathbb{R}^n \mid x_1 = -Md_2,\ \nabla P(x_2) = 0\} \\ \mathcal{M} &= \{x \in \mathbb{R}^n \mid x_1 = 0, \nabla P(x_2) = 0\}, \end{aligned}$$

thus, as expected, $\psi(\chi)$ verifies the implication (2.21).

2.7.3 Simulation of the classical pendulum system

Simulations for the simple pendulum system of length l and mass m were carried out to illustrate the performance of the proposed IC. The equilibrium to be stabilized is the upward position, hence the gravity force is compensated with a linear spring of stiffness $K > mgl$, as proposed in [43]. This yields the (shaped) energy function

$$H(x) = \frac{1}{2ml^2}x_1^2 + mgl(1 - \cos(x_2)) + \frac{K}{2}x_2^2$$

Damping injection is also added with a gain K_p. Since the velocity measurement is perturbed by a constant disturbance d_2, the system takes the form (2.14). The IC (2.35) becomes

$$\begin{aligned} \dot{\zeta} &= K_i[mgl\sin(x_2) + Kx_2] \\ u &= -K_p\zeta - ml^2K_i[mgl\sin(x_2) + Kx_2] \end{aligned}$$

The simulations were done with the values $m = 0.57$, $l = 0.5$, $d_2 = 0.13$ and $K = 5$, yielding the equilibrium of the closed–loop system $(-0.0185, 0, 0.13)$. Figure 3.1 shows the transient behavior of the closed–loop system with initial condition $(0, 0.3, 0)$, $K_p = 3.4$ and different values of K_i. The three–dimensional plot of Fig. 2.2 depicts the state trajectories for initial conditions of x on a disk in the plane $\xi(0) = 0$.

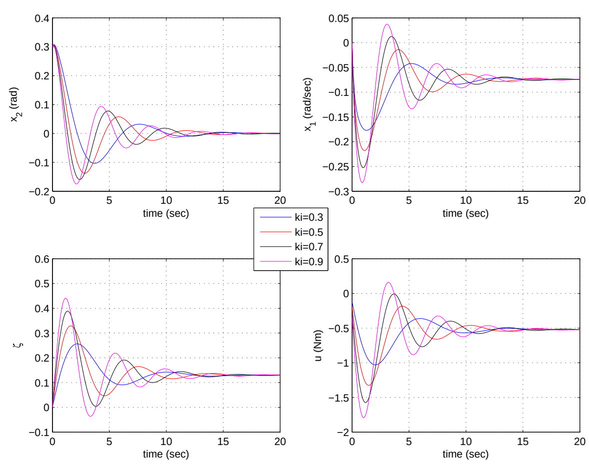


Figure 2.1: Trajectories of the state variables and control signal for different values of K_i.

2.8 Conclusions

Motivated by the developments of [32] a new IC that ensures regulation (to zero) of the passive output, as well as the non–passive output $\nabla_2 H(x)$, of the pH system (2.14)—in spite of the presence of disturbances in the non–actuated coordinates—has been

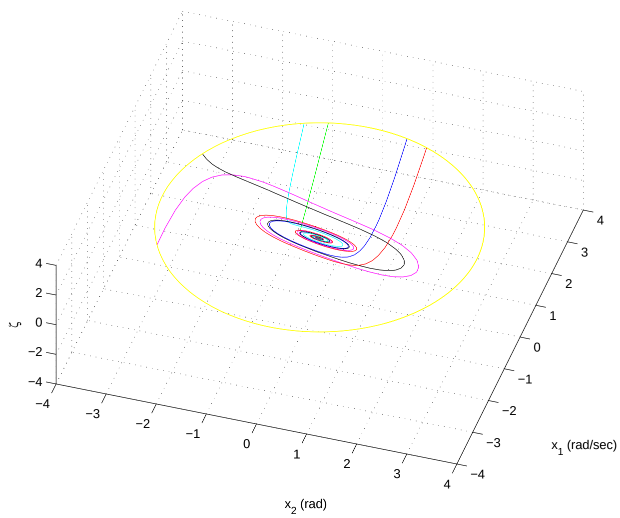


Figure 2.2: Trajectories in state space for initial conditions of x_1, x_2 on a disk in the plane $\xi(0) = 0$.

proposed. Because of its simplicity and widespread popularity, we have concentrated here on basic IC solutions. An alternative approach to reject the unmatched disturbance is to use the well–known output regulation techniques as done, for instance, in [29, 31, 34], which clearly lead to more complicated state–feedback designs. See also [30].

Robustness with respect to input disturbances of the proposed IC is unclear and is currently been investigated. If the system is fully damped, it can be shown that it is input–to–state stable and, consequently, for a constant input it has a steady state [47]. However, it would be interesting to analyze the effect of adding to the new IC a standard integral action in the passive output, as done in the simulation example of [32].

Finally, as pointed out in Remark 5, we have a poor understanding of the meaning of conditions (2.28) and (2.29) that, at this point, are just technically motivated.

Chapter 3

Unmatched and matched disturbances: mechanical systems

The problem of robustness improvement, *vis à vis* external disturbances, of energy shaping controllers for mechanical systems is addressed in this chapter. First, it is shown that, if the inertia matrix is constant, constant disturbances (both, matched and unmatched) can be rejected simply adding a suitable integral action—interestingly, not at the passive output. For systems with non–constant inertia matrix, additional damping and gyroscopic forces terms must be added to reject matched disturbances and, moreover, enforce the property of integral input–to–state stability with respect to matched disturbances. The stronger property of input–to–state stability, this time with respect to matched and unmatched disturbances, is ensured with further addition of nonlinear damping. Finally, it is shown that including a partial change of coordinates, the controller can be significantly simplified, preserving input–to–state stability with respect to matched disturbances.

3.1 Introduction

Passivity–based controllers (PBC), which achieve stabilization shaping the energy function of the system, are widely popular for mechanical systems. It is well–known that PBC is robust with respect to parametric uncertainty and passive unmodelled dynamics (like friction), in the sense that stability—with respect to a shifted equilibrium—is preserved. However, very little is known about their robustness in the face of *external disturbances*, due to measurement or system noise.

In this chapter is to address this practically important issue for fully actuated fully damped mechanical systems whose energy function has an isolated minimum at the desired equilibrium, but are subject to external, matched and unmatched, disturbances.

As witnessed by the ubiquity of PI controllers, one of the most popular and natural approaches to robustify a controller design is to add an integral action on the signal to be regulated. If this signal turns out to be a passive output, stability is preserved in spite of the addition of the integral action.

We shown that applying this procedure to mechanical systems, where the passive output is velocities, generates, even in the absence of disturbances, a *set* of equilibria and an *invariant foliation* in the extended state space, rendering asymptotic stability (practically) impossible.

Surprisingly enough, if the inertia matrix is constant the robustification problem has a very simple solution. Indeed, it is shown that adding a PI controller around the *potential energy forces* ensures the rejection of matched and unmatched constant disturbances using the methodology presented in chapter 2 . To quantify the robustness for time–varying disturbance we adopt the, by now standard, formalism of input–to–state stability (ISS), and the weaker property of integral ISS (IISS). More precisely, several controllers, with increasing complexity, that ensure these properties are proposed for mechanical systems. Finally, it is shown that including the *partial change of coordinates* proposed in [48], we obtain very simple controller that ensures ISS with respect to matched disturbances.

3.2 Problem formulation

Throughout the chapter we consider n-degrees of freedom, fully-actuated mechanical system described in port–Hamiltonian (pH) form by

$$\begin{bmatrix} \dot{q} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & -k_p \end{bmatrix} \nabla H(q,\mathbf{p}) + \begin{bmatrix} 0 \\ I_n \end{bmatrix} u + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \tag{3.1}$$

with Hamiltonian function

$$H(q,\mathbf{p}) = \frac{1}{2}\mathbf{p}^\top M^{-1}(q)\mathbf{p} + V(q). \tag{3.2}$$

$q, \mathbf{p} \in \mathbb{R}^n$ are generalized positions and momenta, respectively, and are assumed measurable, $u \in \mathbb{R}^n$ is the control input, d_1 and $d_2 \in \mathbb{R}^n$ are the matched and unmatched disturbances—possibly time–varying, but bounded and unmeasurable. The mass matrix $M(q) = M^\top(q) > 0$, and satisfies

$$m_1 I_n \leq M^{-1}(q) \leq m_2 I_n \tag{3.3}$$

$K_p = K_p^\top > 0$ is the dissipation matrix and I_n is the $n \times n$ identity matrix. We assume that the Hamiltonian (A.2) has a minimum at the desired equilibrium $(q^\star, 0)$, that is,

$$q^\star = \arg\min V(q),$$

and it is isolated. Note that $(q^\star, 0)$ is an asymptotically stable equilibrium of the mechanical system when $d_1 = 0$ and $d_2 = 0$—the stability is almost global if $V(q)$ is proper and has a unique minimum [7].

The control objective is to design a dynamic state feedback controller such that the closed–loop system ensures some stability properties in spite of the presence of the disturbances (d_1, d_2). In particular, we are interested in the following.

P1 Preserving asymptotic stability for constant, matched and–or unmatched, disturbances.

P2 Ensuring IISS and ISS, with respect to, matched and–or unmatched, disturbances.

To satisfy these objectives, besides a suitable integral action, additional gyroscopic and damping forces are added to the system. Instrumental for our developments is the introduction of coordinate changes, similar to the one used in [32] and chapter 2, that preserve the pH structure of the system with the *same* Hamiltonian function.

To motivate our developments consider first the standard addition of an integral action on the passive output, *e.g.*, the velocities $\dot{q} = M^{-1}(q)\mathbf{p}$. Thus, define

$$\begin{aligned} u &= -\eta \\ \dot{\eta} &= K_i M^{-1}(q)\mathbf{p} \end{aligned}$$

with $K_i = K_i^\top > 0$. If d_1 is a non–zero constant the system admits no constant equilibrium, and if $d_1 = 0$ and d_2 is constant there is an equilibrium set given by

$$\mathcal{E} = \Big\{(q, \mathbf{p}, \eta) \,|\, \mathbf{p} = 0,\, \nabla V(q) + \eta = d_2\Big\}.$$

Moreover, it is easy to see that, with or without disturbances, the foliation

$$\mathcal{M}_\kappa = \Big\{(q, \mathbf{p}, \eta) \,|\, K_i q - \eta = \kappa,\, \kappa \in \mathbb{R}\Big\},$$

is invariant with respect to the flow of the closed–loop system. Consequently, convergence to the desired equilibrium $(q^\star, 0, d_2)$ is attained only for a zero measure set of initial conditions. See Fig. 3.1 for a pictorial description of the state space.

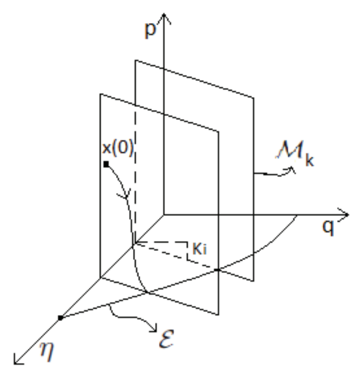


Figure 3.1: Graph of the state space showing two sheets of the invariant foliation $\mathcal{M}_\kappa$, the equilibrium set $\mathcal{E}$ and a trajectory $x(t) := ((q(t), \mathbf{p}(t), \eta(t))$.

Remark 8. *When M is constant the dynamics of the system (3.1) in Euler–Lagrange form takes the form*

$$M\ddot{q} + \nabla V(q) + K_p(\dot{q} - d_1) + K_p\dot{q} = u + d_2.$$

Hence, the disturbance d_2 represents either a constant force acting on the system or an input measurement noise, while d_1 is noise in the measurement of velocity that is propagated to the system by the injection of the damping K_p. For non–constant inertia matrix a term $\nabla_q(\dot{q}^\top M(q))d_1$, whose physical interpretation is less clear, appears in the dynamics.

3.3 Constant inertia matrix

In this section the particular case of constant inertia matrix is considered. For this case, the problem of rejection of constant disturbances has a surprisingly simple solution: adding a PI control around the potential energy forces. However, to enforce the important property of ISS, damping must be added to all the coordinates, which is achieved incorporating suitable gyroscopic forces.

3.3.1 Rejection of constant disturbances

Proposition 7. *Consider the system (3.1) with constant inertia matrix M and* constant *disturbances (d_1, d_2) in closed–loop with the PI control*

$$\begin{aligned} u &= -K_p z_3 - M K_i \nabla V \\ \dot{z}_3 &= K_i \nabla V, \end{aligned} \tag{3.4}$$

with $K_i = K_i^\top > 0$.

(i) The closed–loop dynamics expressed in the coordinates,

$$\begin{aligned} z_1 &= q \\ z_2 &= \mathbf{p} + M(z_3 - K_p^{-1} d_2) \end{aligned} \tag{3.5}$$

takes the pH form

$$\dot{z} = \begin{bmatrix} 0 & I_n & -K_i \\ -I_n & -K_p & 0 \\ K_i & 0 & 0 \end{bmatrix} \nabla H_z(z), \tag{3.6}$$

with energy function

$$H_z(z) := H(z) + \frac{1}{2}(z_3 - z_3^*)^\top K_i^{-1}(z_3 - z_3^*), \tag{3.7}$$

where $z_3^* := d_1 + K_p^{-1} d_2$.

(ii) The desired equilibrium point $z^\star := (q^\star, 0, z_3^*)$, is asymptotically stable*. The stability is almost global if $V(z_1)$ is proper and has a unique minimum.*

Proof.

(i) First, we take the time derivative of the first equation in (3.5), and we replace $\dot{q}$ and $\dot{z}_1$ by the corresponding state equations of the open- and closed–loop dynamics. This yields

$$\begin{aligned}\dot{q} &= \dot{z}_1 \\ &= M^{-1}z_2 - (z_3 - d_1 - k_p^{-1}d_2) \\ &\equiv M^{-1}\mathbf{p} + d_1,\end{aligned} \tag{3.8}$$

which is satisfied if and only if z_2 is as in the second row of (3.5).

Second, we take the time derivative of z_2 in (3.5), replace $\dot{\mathbf{p}}$, $\dot{z}_2$ and $\dot{z}_3$ by the corresponding state equations of the open- and closed–loop systems. From the resulting equation, we compute the control law (3.4)—that is independent of disturbances—and yields pH closed–loop dynamics (3.6). Finally, the last row of the closed–loop is given by the integral action of the non–passive output.

(ii) We consider the Hamiltonian (3.7) as a candidate Lyapunov function for (3.6). Its derivative along the trajectories of the system is

$$\dot{H}_z = -z_2^T M^{-1} K_p M^{-1} z_2 \leq 0,$$

which proves that the origin is stable. Moreover, the trajectories will converge to largest invariant set contained in $\mathcal{S} = \{z \,|\, z_2 = 0\}$. From (3.6), we can conclude that the largest invariant set in $\mathcal{S}$ is $z^* = (q^*, 0, z_3^*)$, then, the equilibrium point z^* is asymptotically stable. Using the change of coordinate (3.5), we conclude that the desired equilibrium in original coordinates $(q^*, -Md_1, d_1 + K_p^{-1}d_2)$ is asymptotically stable.

□□□

Remark 9. From (3.6) it is clear that, besides the addition of the integral action, the Poisson structure of the open–loop system (3.1) is preserved in closed–loop, in the new coordinates. Moreover, the Hamiltonian function (3.7) exactly coincides with the energy of the system (3.6). These are the two key steps first introduced for general pH systems in the chapter 2.

3.3.2 ISS for time–varying disturbances

Proposition 8. *Consider the system (3.1) with constant mass matrix M and* time–varying *disturbances $d(t) := col(d_1(t), d_2(t))$, in closed–loop with the control law*

$$\begin{aligned} u &= -\Big(k_1 \nabla^2 V M^{-1} + K_3 R_3\Big)\mathbf{p} - K_4 \nabla V - K_5 z_3 \\ \dot{z}_3 &= K_6 \nabla V + R_3 \mathbf{p}, \end{aligned} \tag{3.9}$$

where

$$\begin{aligned} K_4 &:= k_1 K_p M^{-1} + k_1 K_3 R_3 + K_3 M^{-1} \\ K_5 &:= \Big(K_p M^{-1} + M R_3\Big) K_3 \\ K_6 &:= M^{-1} + k_1 R_3, \end{aligned}$$

$k_1 > 0$, $K_3 = K_3^\top > 0$ and $R_3 = R_3^\top > 0$.

(i) The closed–loop dynamics expressed in the coordinates

$$\begin{aligned} z_1 &= q \\ z_2 &= \mathbf{p} + k_1 \nabla V(q) + K_3 z_3, \end{aligned} \tag{3.10}$$

takes the perturbed pH form

$$\dot{z} = \begin{bmatrix} -k_1 M^{-1} & I_n & -M^{-1} \\ -I_n & -K_p & -MR_3 \\ M^{-1} & R_3 M & -R_3 \end{bmatrix} \nabla \bar{H}_z + \begin{bmatrix} I_n & 0 \\ k_1 \nabla^2 V(z_1) & I_n \\ 0 & 0 \end{bmatrix} d(t) \tag{3.11}$$

with

$$\bar{H}_z(z) = H(z) + \frac{1}{2} z_3^\top K_3 z_3. \tag{3.12}$$

(ii) If $V(z_1)$ satisfies

$$0 < r_v I_n \leq \nabla^2 V(z_1) \leq r_w I_n,$$

then (3.11) is ISS *with respect to the time varying input disturbances $(d_1(t), d_2(t))$ with ISS Lyapunov function $\bar{H}_z(z)$.*

(iii) If $d_1 = 0$ and d_2 is constant, then the desired equilibrium

$$z^\star := (q^*, 0, K_3^{-1}(K_p M^{-1} + MR_3)^{-1} d_2)$$

is asymptotically stable.

Proof.

(i) The procedure to prove that the closed–loop dynamics can be written as the pH (3.11) is the same as in Proposition 7. That is, differentiate the first row of (3.10) with respect to time, and replace the derivative of the state by the state equations to obtain the second row of (3.10). Then, differentiate the later change of coordinate respect to time, replace the derivative of the states by the state equations and solve for u to find the control law.

(ii) We choose (3.12) as an ISS-Lyapunov function. We compute the derivative of $\bar{H}_z$ along the solutions of (3.11), which yields

$$\begin{aligned} \dot{\bar{H}}_z &\leq -k_1 \|\nabla_{z_1} H_z\|^2_{M^{-1}} - \|\nabla_{z_2} H_z\|^2_{K_p} - \|\nabla_{z_3} H_z\|^2_{R_3} + \\ &\quad + [\nabla_{z_2} H_z]^T \left[k_1 \nabla^2_{z_1} V(z_1) d_1 \right] + [\nabla_{z_2} H_z]^T d_2 + [\nabla_{z_1} H_z]^T d_1 \\ &\leq \frac{1}{2k_1 m_1} |d_1|^2 + \frac{1}{\lambda_{min}(K_p)} |d_2|^2 + \frac{k_1^2 r_w^2}{\lambda_{min}(K_p)} |d_1|^2 - \\ &\quad - \frac{k_1 m_1}{2} |\nabla V(z_1)|^2 - \frac{\lambda_{min}(K_p)}{2} |M^{-1} z_2|^2 - \lambda_{min}(R_3) |K_3 z_3|^2 \\ &\leq -\alpha(|z|) + \beta(|d|), \end{aligned} \tag{3.13}$$

with $\alpha, \beta \in \mathcal{K}_\infty$. From (3.13) and the fact that the Hamiltonian function $\bar{H}_z(z)$ is positive definite and radially unbounded we conclude that the closed–loop is ISS.

(iii) As proposed in [45], we use

$$H_0(z) = \bar{H}_z(z) - z^\top \nabla \bar{H}_z(z^*) - \Big[\bar{H}(z^*) - {z^*}^\top \nabla \bar{H}_z(z^*)\Big],$$

which has a minimum at z^*, as Lyapunov candidate function for the perturbed system. The time derivative of H_0 along the trajectories of the system (3.11) yields

$$\dot{H}_0 = -\|\nabla \bar{H}_z(z) - \nabla \bar{H}_z(z^*)\|_Q^2,$$

where

$$Q := block\ diag\{k_1 M^{-1}, K_p, R_3\} > 0,$$

which proves asymptotic stability of z^*.

□□□

Remark 10. The assumption $d_1 = 0$ in (iii) of Proposition 8 is needed to ensure that the equilibrium of the system is the desired equilibrium. Indeed, when this assumption holds, the momentum vector p is zero at steady state. This fact, together with the dynamics of the controller (3.9) at steady state, ensures that the position vector at equilibrium satisfies $\nabla V(q) = 0$, which happens at the desired position q^*. If $d_1 \neq 0$, the disturbance shifts the steady state from the desired equilibrium.

Remark 11. Comparing the closed–loop dynamics (3.6) and (3.11) we observe that, to enforce the ISS property, it was necessary to add damping in the $(1,1)$ and $(3,3)$ terms of the damping matrix of (3.11). This is achieved with the terms, added to the basic PI control (3.4), in (3.9).

Remark 12. The changes of coordinates used in Propositions 7 and 8 are different. The one in Proposition 7 incorporates the disturbances into the closed–loop pH system, and proves stability when disturbances are constant. The objective of the change of coordinates in Proposition 8 is to inject damping in all coordinates and ensure ISS with respect to time-varying disturbances.

Remark 13. As is well–known [47], ISS endows the mapping $d \mapsto z$ with the important properties of bounded–input bounded–states and converging–input converging–states. Since ISS is invariant under change of coordinates, also the mapping $d \mapsto (q, \mathbf{p}, z_3)$ enjoys these properties.

3.4 Non–constant inertia matrix

The derivation of the controller for non–constant inertia matrix M follows the same procedure used above. However, the expressions of the control laws become more complicated because of the need to differentiate M.

Throughout the section the following well–known identity is used

$$\nabla_q \Big[\mathbf{p}^\top M^{-1}(q)\mathbf{p}\Big] = \sum_{i=1}^{n} e_i \mathbf{p}^T \nabla_{q_i} M(q)^{-1}\mathbf{p} = -\nabla_q \Big[\dot{q}^\top M(q)\dot{q}\Big]$$

3.4.1 IISS for time–varying matched disturbances

Proposition 9. *Consider the system (3.1) with* constant *matched disturbance* d_2 *and* no unmatched disturbance, e.g., $d_1 = 0$, *in closed loop with the control law*

$$\begin{aligned} u &= -k_1 K_p M^{-1} \nabla V - k_1 \nabla^2 V M^{-1} \mathbf{p} - z_3 + v(q, \mathbf{p}) \\ \dot{z}_3 &= K_i M^{-1} \Big[\mathbf{p} + k_1 \nabla V \Big], \end{aligned} \tag{3.14}$$

where

$$\begin{aligned} v(q, \mathbf{p}) \quad := \quad & \frac{k_1}{2} \sum_{i=1}^{n} e_i \mathbf{p}^\top M^{-1} \nabla_{qi} M M^{-1} \nabla V - \Big(M J_{12} M^{-1} + J_{12}^\top \Big) \nabla V - \\ & - \frac{1}{k_1} J_{12}^\top M J_{12} M^{-1} \Big[\mathbf{p} + k_1 \nabla V \Big] \end{aligned} \tag{3.15}$$

with

$$J_{12}(q, \mathbf{p}) := -\frac{k_1}{2} M^{-1} \sum_{i=1}^{n} e_i \Big[\mathbf{p} + k_1 \nabla V \Big]^\top M^{-1} \nabla_{q_i} M,$$

$k_1 > 0$, $K_i = K_i^\top > 0$, $K_3 = K_3^\top > 0$ *and* $e_i \in \mathbb{R}^n$ *the* i*–th vector of the Euclidean basis.*

(i) The closed–loop dynamics expressed in the coordinates

$$\begin{aligned} z_1 &= q \\ z_2 &= \mathbf{p} + k_1 \nabla V(q), \end{aligned} \tag{3.16}$$

takes the pH form $\dot{z} = F(z) \nabla U_z$, *with*

$$U_z(z) := H(z) + \frac{1}{2} (z_3 - d_2)^\top K_i^{-1} (z_3 - d_2), \tag{3.17}$$

and

$$F(z) := \begin{bmatrix} -k_1 M^{-1}(z_1) & I_n + J_{12}(z) & 0 \\ -I_n - J_{12}^\top(z) & -K_p & -K_i^\top \\ 0 & K_i & 0 \end{bmatrix}$$

(ii) The desired equilibrium $z^\star := (q^\star, 0, d_2)$ *is asymptotically stable. The stability is almost global if* $V(z_1)$ *is proper and has a unique minimum.*

(iii) If the disturbance d_2 *is time–varying, the closed–loop system, which can be alternatively written as*

$$\dot{z} \quad = \quad F(z) \nabla \bar{U}_z + col(0, d_2(t), 0)$$

with

$$\bar{U}_z(z) := H(z) + \frac{1}{2} z_3^\top K_i^{-1} z_3. \tag{3.18}$$

is IISS with respect to the input $d_2(t)$ *with IISS Lyapunov function* $\bar{U}_z(z)$.

Proof.

(i) The proof follows the procedure in Proposition 7, using the time derivative of the change of coordinates (3.29). Note that the interconnection and damping matrix $F(z)$ of the closed–loop pH system was suitably chosen such that the control law does not depend on the unknown disturbance.

(ii) Considering (3.17) as a candidate Lyapunov function and taking its derivative along the system's trajectories, it follows

$$\dot{U}_z = -\|M^{-1}z_2\|^2_{K_p} - k_1\|\nabla_{z_1}H\|^2_{M^{-1}} \leq 0, \tag{3.19}$$

which proves that the equilibrium is stable. Asymptotic stability is concluded applying LaSalle's Invariance Principle [36].

(iii) Using $\bar{U}_z(z)$ as IISS Lyapunov function and computing its time derivative yields

$$\begin{aligned} \dot{\bar{U}}_z &= -\|M^{-1}z_2\|^2_{K_p} - k_1\|\nabla_{z_1}H\|^2_{M^{-1}} + z_2^\top M^{-T}d_2 \\ &\leq \frac{1}{k_p}|d_2|^2 - \frac{k_p}{2}|M^{-1}z_2|^2, \end{aligned}$$

where $k_p := \lambda_{\min}\{K_p\}$. The latter inequality proves that the system is smoothly dissipative. Now, from the fact that

$$\left(d_2(t) \equiv 0,\ M^{-1}z_2(t) \equiv 0 \ \Rightarrow \ z(t) \to 0\right),$$

we have that the system is weakly zero–state detectable form the output $M^{-1}z_2$. This two properties imply IISS [44].

□□□

Remark 14. Comparing Propositions 8 and 9, we observe that if the damping term R_3 is removed, only the weaker property of IISS with respect to matched disturbances, can be established.

3.4.2 ISS for time–varying matched and unmatched disturbances

It is well–known that IISS is not enough to ensure the important property of bounded–input-bounded–states. This motivates us to redesign the controller to endow the system with the stronger property of ISS, as indicated in Remark 13 which implies bounded–input-bounded–states .

Proposition 10. *Consider the system (3.1) under the action of unmatched and matched* *disturbances $d_1(t)$ and $d_2(t)$, in closed–loop with the control law*

$$\begin{aligned} u &= -k_1 K_p M^{-1}\nabla V - k_1 \nabla^2 V M^{-1}\mathbf{p} - K_3\Big[\Big[M^{-1}+k_1R_3\Big]\nabla V + R_3\mathbf{p}\Big] \\ &\quad -\Big[\frac{1}{2}\sum_{i=1}^{n} e_i \mathbf{p}^\top \nabla_{q_i} M^{-1} + K_p M^{-1} + J_{23}^\top\Big] K_3 z_3 - \\ &\quad -\frac{1}{k_1}\Big[I_n + J_{12}^\top\Big] M J_{12} M^{-1} K_3 z_3 + v(q,\mathbf{p}) \\ \dot z_3 &= \Big[M^{-1}+k_1R_3\Big]\nabla V + R_3\mathbf{p} \end{aligned} \tag{3.20}$$

where $v(q,\mathbf{p})$ is given in (3.15) with

$$\begin{aligned} J_{23} &:= -\frac{1}{k_1} J_{12} + R_3 M \\ J_{12} &:= -\frac{k_1}{2} M^{-1} \sum_{i=1}^{n} e_i \Big[\mathbf{p} + k_1\nabla V + K_3 z_3\Big]^\top M^{-1}\nabla_{q_i} M \end{aligned}$$

(i) The closed–loop dynamics expressed in the coordinates (3.10) takes the perturbed pH form

$$\dot z = \begin{bmatrix} -k_1M^{-1} & I_n + J_{12} & -M^{-1} \\ -J_{12}^\top - I_n & -K_p & -J_{23}^\top \\ M^{-\top} & J_{23} & -R_3 \end{bmatrix} \nabla \bar H_z + \begin{bmatrix} I_n & 0 \\ k_1\nabla^2 V(z_1) & I_n \\ 0 & 0 \end{bmatrix} \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} \tag{3.21}$$

with $\bar H_z(z)$ given in (3.12).

(ii) The closed–loop system is ISS with respect to the input disturbances $(d_1(t), d_2(t))$, provided that the Hessian of the potential energy satisfies condition (ii) in Proposition 8.

(iii) The unperturbed *system (3.21) has an asymptotically stable equilibrium at the desired state $z^\star = (q^*, 0, 0)$.*

Proof.

(i) The pH closed–loop (3.21) in z coordinates results differentiating (3.10) with respect to time and the control law (3.20).

(ii) We propose (3.12) as candidate ISS Lyapunov function and we compute its time derivative along the solutions of (3.21) as follows

$$\begin{aligned} \dot{\bar H}_z &= -k_1\|\nabla_{z_1}H_z\|^2_{M^{-1}} - \|\nabla_{z_2}H_z\|^2_{K_p} - \|\nabla_{z_3}H_z\|^2_{R_3} + \\ &\quad +[\nabla_{z_2}H_z]^T\left[k_1\nabla^2_{z_1}V(z_1)d_1\right] + [\nabla_{z_2}H_z]^T d_2 + [\nabla_{z_1}H_z]^T d_1 \qquad (3.22) \\ &\le -\frac{\lambda_1}{2}|\nabla_{z_1}H_z|^2 - \lambda_2|\nabla_{z_2}H_z|^2 - \lambda_3|\nabla_{z_2}H_z|^2 + \\ &\quad + \frac{1}{2\lambda_1}|d_1|^2 + \frac{1}{2\lambda_2}|d_2|^2 + \frac{\lambda_v^2}{2\lambda_2}|d_1|^2 \qquad (3.23) \\ &\le -\lambda_z|\nabla_z H_z|^2 + \lambda_{d_1}|d_1|^2 + \frac{1}{2\lambda_2}|d_2|^2 \qquad (3.24) \end{aligned}$$

Inequality (3.37) and the fact that the function $\bar{H}_z(z)$ is positive definite and radially unbounded, guarantees that for any bounded disturbances $d_1(t)$ and $d_2(t)$, the state $z(t)$ will be bounded and the dynamic system (3.21) is ISS.

The parameters λ in (3.23) and (3.37) are $\lambda_1 = k_1 m_1$, $\lambda_2 = \frac{1}{2}\lambda_{min}(K_p)$, $\lambda_3 = \lambda_{min}(R_3)$, $\lambda_v = k_1 r_w$, $\lambda_{d_1} = \frac{1}{2\lambda_1} + \frac{\lambda_v^2}{2\lambda_2}$ and $\lambda_z = \min\{\frac{\lambda_1}{2}, \lambda_2, \lambda_3\}$.

(ii) Taking $d_1(t) \equiv 0$, $d_2(t) \equiv 0$ in (3.22) asymptotic stability of the equilibrium z^* is proved, where it is considered (3.12) as Lyapunov function .

□□□

Remark 15. The difficulty introduced by the non–constant inertia matrix is clearly revealed comparing the interconnection matrices (3.11) and (3.21), which differ on the appearance of complex, state–dependent, expressions on the $(1,2)$ and $(2,3)$ sub–blocks.

3.5 A simplified controller for matched disturbances

As discussed in Remark 15 the controllers for non–constant inertia matrix are highly complex. To overcome this practical shortcoming we follow [48] and propose to change the generalized momentum coordinates to "remove" the inertia matrix from the energy function(see proof at Appendix A1).[1] Unfortunately, this modification achieves the desired objective only if there are no unmatched disturbances, *i.e.* if $d_1 = 0$, an assumption that is made throughout the remaining of the chapter. Also, for simplicity, we remove the damping injection term from (3.1) and add it in the new control law.

Fact 1. *Consider the system (3.1) without damping ($K_p = 0$) and no unmatched disturbances ($d_1 = 0$). Let $T \in \mathbb{R}^n \to \mathbb{R}^{n\times n}$ be the square root of the matrix $M^{-1}(q)$. That is,*

$$M^{-1}(q) = T^2(q).$$

The change of coordinates

$$(q,p) = (q, T(q)\mathbf{p}).$$

transforms the dynamics into

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & T(q) \\ -T(q) & S(q,p) \end{bmatrix} \nabla W + \begin{bmatrix} 0 \\ I_n \end{bmatrix} w + \begin{bmatrix} 0 \\ Td_2 \end{bmatrix}, \tag{3.25}$$

with $w := T(q)u$ the new control signal, new Hamiltonian function

$$W(q,p) = \frac{1}{2}|p|^2 + V(q), \tag{3.26}$$

[1] The motivation for this change of coordinates in [48] was speed observer design.

and the gyroscopic forces matrix[2]

$$\begin{aligned} S(q,p) &:= \nabla(T\mathbf{p})T - T\nabla^\top(T\mathbf{p})|_{\mathbf{p}=T^{-1}p}, \\ &= \sum_{i=1}^{n}\Big[\big[\nabla_{q_i}(T)T^{-1}p\big](Te_i)^\top - (Te_i)\big[\nabla_{q_i}(T)T^{-1}p\big]^\top\Big], \end{aligned} \tag{3.27}$$

with $e_i \in \mathbb{R}^n$ *the* i*–th basis vector of* $\mathbb{R}^n$

Remark 16. *From the definition of a square root of a positive definite matrix (Theorem 1 in Section 5.4 of [46]), it is clear that* $T(q)$ *is symmetric and satisfies*

$$0 < r_{t_1} I_n \leq T(q) \leq r_{t_2} I_n,$$

consequently, $T(q)$ *is positive definite and* $T(q)d_2(t)$ *is bounded.*

3.5.1 IISS and GAS for matched disturbances

Proposition 11. *Consider the system (3.25) in closed loop with the control law*

$$\begin{aligned} w &= -(\nabla^2 VT + R_2)p - (R_2 - S)\nabla V - Tz_3 \\ \dot{z}_3 &= K_i T\Big(p + \nabla V\Big), \end{aligned} \tag{3.28}$$

with $K_i = K_i^\top > 0$ *and* $R_2 = R_2^\top > 0$

(i) The closed–loop dynamics expressed in the coordinates

$$\begin{aligned} z_1 &= q \\ z_2 &= p + \nabla V(q), \end{aligned} \tag{3.29}$$

takes the perturbed pH form

$$\dot{z} = F(z)\nabla\mathcal{W}(z) + \begin{bmatrix} 0 \\ T(z)d_2 \\ 0 \end{bmatrix},$$

where

$$\mathcal{W}(z) \quad := \quad W(z_1, z_2) + \frac{1}{2} z_3^\top K_i^{-1} z_3, \tag{3.30}$$

with

$$W(z_1, z_2) := \frac{1}{2}|z_2|^2 + V(z_1)$$

and

$$F(z) := \begin{bmatrix} -T & T & 0 \\ -T & S - R_2 & -TK_i \\ 0 & K_i T & 0 \end{bmatrix}$$

[2]Clearly, $S(q,p) = -S^\top(q,p)$.

(ii) The closed loop system is IISS, with respect to the disturbance $d_2(t)$, with IISS Lyapunov function $\mathcal{W}(z)$.

(iii) If d_2 constant, the desired equilibrium $z^\star := (q^\star, 0, d_2)$ is asymptotically stable.

Proof.

(i) The proof follows the procedure in Proposition 7, computing the time derivative of the change of coordinates (3.29). Note that, as done before, the interconnection and damping matrix $F(z)$ of the closed–loop pH system was suitably chosen such that the control law does not depend on the unknown disturbance.

(ii) Using $\mathcal{W}(z)$ as IISS Lyapunov function and computing its time derivative yields

$$\begin{aligned} \dot{\mathcal{W}} &= -\|z_2\|_{R_2}^2 - \|\nabla V(z)\|_T^2 + z_2^\top T d_2 \\ &\leq -r_{t_2}|\nabla V(z)|^2 - \frac{r_2}{2}|z_2|^2 + \lambda_t |d_2|^2, \end{aligned} \tag{3.31}$$

where $r_2 := \lambda_{\min}\{R_2\}$ and $\lambda_t := \lambda_{\min}\{\frac{r_{t_2}}{r_2}\}$. The latter inequality proves that the system is smoothly dissipative. Now, from the fact that

$$(d_2(t) \equiv 0,\ z_2(t) \equiv 0\ \Rightarrow\ z(t) \to 0)\,,$$

we have that the system is weakly zero–state detectable form the output z_2. This two properties imply IISS.

(iii) If d_2 is constant the closed–loop system can be alternatively written as

$$\dot{z} = F(z)\nabla W_z$$

with

$$W_z(z) = W(z_1, z_2) + \frac{1}{2}(z_3 - d_2)^\top K_i^{-1}(z_3 - d_2). \tag{3.32}$$

Considering (3.32) as a candidate Lyapunov function and taking its derivative along the system's trajectories, it follows

$$\dot{W}_z = -\|z_2\|_{R_2}^2 - r_{t_2}|\nabla V(z)|^2 \leq 0, \tag{3.33}$$

which proves stability of the equilibrium. Asymptotic stability is concluded applying LaSalle's Invariance Principle [36].

□□□

3.5.2 ISS for time–varying matched disturbances

Proposition 12. *Consider the system (3.25) in closed–loop with the control law*

$$\begin{aligned} v &= -(\nabla^2 V T + R_2 + R_3)p - (R_2 + R_3 - S)z_3 - (T + R_2 + R_3 - S)\nabla V \\ \dot{z_3} &= (T + R_3)\nabla V + R_3 p. \end{aligned} \tag{3.34}$$

(i) The closed–loop dynamics expressed in the coordinates

$$\begin{aligned} z_1 &= q \\ z_2 &= p + \nabla V(q) + z_3 \end{aligned} \tag{3.35}$$

takes the perturbed pH form

$$\dot{z} = \begin{bmatrix} -T & T & -T \\ -T & S - R_2 & -R_3 \\ T & R_3 & -R_3 \end{bmatrix} \nabla U + \begin{bmatrix} 0 \\ T d_2(t) \\ 0 \end{bmatrix}$$

with

$$U(z) := \frac{1}{2}|z_2|^2 + V(z_1) + \frac{1}{2}|z_3|^2 \tag{3.36}$$

(ii) The closed–loop system (3.36) is ISS with respect to the disturbance $d_2(t)$.

(iii) The unperturbed *system has an asymptotically stable equilibrium at the desired state $z^\star = (q^*, 0, 0)$*

(iv) Let

$$R_2 = Tc_2, \quad R_3 = Tc_3 + S(q, \mathbf{p}),$$

with c_2, c_3 two positive scalars. Assume there exist $t_2 \geq 0$ such that

$$\lim_{t \to t_2} d_2(t) = \bar{d}_2,$$

with $\bar{d}_2$ constant. Then, all trajectories converge to $z^\star := (q^\star, 0, \alpha)$, where

$$\alpha := (c_2 + c_3)^{-1}\, \bar{d}_2$$

Proof.

(i) The pH closed–loop system (3.36) is obtained via direct computations.

(ii) Taking $U(z)$ as candidate ISS Lyapunov function and computing its time derivative along the solutions of (3.36) yields

$$\begin{aligned} \dot{U} &= -\|\nabla V\|_T^2 - \|z_2\|_{R_2}^2 - \|z_3\|_{R_3}^2 + z_2^\top T d_2 \\ &\leq -r_{t_2}|\nabla V|^2 - r_2|z_2|^2 - r_3|z_3|^2 + \frac{r_{t_2}}{2r_2}|d_2|^2 \\ &\leq -\lambda_z|\nabla U|^2 + \lambda_d|d_2|^2 \end{aligned} \tag{3.37}$$

where

$$r_2 := \lambda_{\min}(R_2),\ r_3 := \lambda_{\min}(R_3),\ \lambda_d := \{\frac{r_{t2}}{2r_2}\},\ \lambda_z := \min\{\frac{r_2}{2}, \frac{r_3}{2}, r_{t_2}\}.$$

Inequality (3.37) and the fact that the function $U(z)$ is positive definite and radially unbounded, guarantees that for any bounded disturbances $d_2(t)$, the state $z(t)$ will be bounded and the dynamic system (3.36) is ISS.

(iii) Taking $d_2(t) \equiv 0$ in (3.37) and invoking uniqueness of the minimum of V yields the proof.

(iv) Finally, with $d_2(t) = \bar{d}_2$, and the given selection of R_2, R_3, we can do a new change of coordinates to the closed–loop system (3.36):

$$\begin{aligned} \bar{z}_1 &= z_1 \\ \bar{z}_2 &= z_2 - \alpha \\ \bar{z}_3 &= z_3, \end{aligned} \tag{3.38}$$

that yields

$$\dot{\bar{z}} = \begin{bmatrix} -T & T & -T \\ -T & -Tc_2 + S & -Tc_3 - S \\ T & Tc_3 - S & -Tc_3 + S \end{bmatrix} \nabla \mathcal{U}$$

with the function

$$\mathcal{U}(\bar{z}) := \frac{1}{2}|\bar{z}_2|^2 + V(\bar{z}_1) + \frac{1}{2}|\bar{z}_3 - \alpha|^2 \tag{3.39}$$

Taking (3.39) as Lyapunov function yields

$$\dot{\mathcal{U}} = -\|\nabla_{\bar{z}_1} V\|_T^2 - \|\bar{z}_2\|_{c_2 T}^2 - \|\bar{z}_3 - \alpha\|_{c_3 T}^2, \tag{3.40}$$

from which the claim follows immediately.
Furthermore if the potential energy is defined as $V(q) = \frac{1}{2}(q - q^*)K(q - q^*)$, exponential convergence of the equilibrium point is proved. This follow from

$$\dot{\mathcal{U}} = -\|\bar{z}_1 - q^*\|_{KTK}^2 - \|\bar{z}_2\|_{c_2 T}^2 - \|\bar{z}_3 - \alpha\|_{c_3 T}^2, \tag{3.41}$$

and with basics bounds we can written as

$$\dot{\mathcal{U}} = -\delta \mathcal{U}(\bar{z}), \tag{3.42}$$

with

$$\delta := 2\min\{\frac{\lambda_{\min}(KTK)}{\lambda_{\max}(K)},\ c_2\lambda_{\min}(T),\ c_3\lambda_{\min}(T)\} > 0.$$

□□□

Remark 17. *To underscore the controller simplification gained using the change of coordinates, compare the control law (3.14), (3.15) of Proposition 9 with (3.28) (plus $w = Tu$) of Proposition 11, and note that both controllers enjoy the same robustness properties. It is evident that the achieved simplification can hardly be overestimated.*

3.6 Case study: prismatic robot

In this section, we use the two DoF prismatic robot[3] example of [44] to illustrate in simulations our results. Similarly to [44], the initial condition vector is $[q_{1_0},\, q_{2_0},\, p_{1_0},\, p_{2_0},\, z_{31_0},\, z_{32_0}] = [0,\, 0.1,\, 0.1,\, 0.1,\, 0.1,\, 0.2]$ and the desired equilibrium is the origin. The bounded disturbance vector is taken as $d_2 = \alpha \tanh(\dot{q})$, with $\alpha = 3, 10$. The parameters of the model are the same as in [44], and are repeated here for ease of reference. The mass matrix is

$$M = \begin{bmatrix} m_1 q_2^2 + \frac{m_2 L^2}{3} & 0 \\ 0 & m_1 \end{bmatrix}$$

where m_1 and L is the mass and length of the arm, and m_2 is the mass of the hand. The states $q = [q_1\ q_2]^\top$ and $p = [p_1\ p_2]^\top$ are the generalised position and momenta respectively. The subscript 1 and 2 indicates variables of the arm and the hand respectively. The system has no potential energy and no dissipation.

We present simulations with two controllers—denoted IISS and ISS Controllers in the sequel (3.14) and (3.20) corresponding to Propositions 3 and 4, respectively. The motivation for the names stems from the fact that the first controller ensures only IISS, while the second one strengthens this to ISS. As will be illustrated below, although both controllers yield bounded trajectories, the transient behavior of the ISS controller is far superior. As is well–known, and also indicated in Subsection 4.2, IISS does not ensure bounded–input–bounded–state behavior—for all inputs—but in this particular case they turn out to be bounded. It should be remarked that in [44] it is claimed that the trajectories are unbounded. The problem is that, to observe this fact, the simulation has to run in a longer horizon than the one used in [44].

The expression for the IISS controller is

$$\begin{aligned} u &= -(k_1 K_p M^{-1} + I_n) K_d \tilde{q} - (k_1 + 1) K_d M^{-1} p - z_3 + \\ &\quad + \begin{bmatrix} \frac{3 k_1 k_{d2} q_2 \tilde{q}_2}{3 m q_2^2 + M L^2} (p_1 + k_1 k_{d1} \tilde{q}_1) - k_1 m q_2^2 (p_1 + k_1 k_{d1} \tilde{q}_1)^3 \\ \frac{9 k_1 k_{d1} q_2 \tilde{q}_1}{(3 m q_2^2 + M L^2)^2} [(1 + m) p_1 + k_1 k_{d1} m \tilde{q}_1] \end{bmatrix} \\ \dot{z}_3 &= K_i M^{-1} \Big[p + k_1 K_d \tilde{q} \Big], \end{aligned} \tag{3.43}$$

with $\tilde{q} = q - q^*$. The expression for the ISS controller is

$$\begin{aligned} u &= -(k_1 K_p M^{-1} + I_n) K_d \tilde{q} - (k_1 + 1) K_d M^{-1} p - (K_p M^{-1} + M R_3) K_3 z_3 - \\ &\quad - \begin{bmatrix} -\frac{27 k_1 m q_2^2 (p_1 + k_1 k_{d1} \tilde{q}_1 + k_{31} z_{31})^2}{(3 m q_2^2 + M L^2)^3} & \frac{3 q_2 (p_1 + k_1 k_{d1} \tilde{q}_1 + k_{31} z_{31})}{(3 m q_2^2 + M L^2)} \\ \frac{9 m q_2 (k_1 k_{d1} \tilde{q}_1 + k_{31} z_{31})}{(3 m q_2^2 + M L^2)^2} & 0 \end{bmatrix} K_3 z_3 + \\ &\quad + \begin{bmatrix} \frac{3 k_1 k_{d2} q_2 \tilde{q}_2}{3 m q_2^2 + M L^2} (p_1 + k_1 k_{d1} \tilde{q}_1 + k_{31} z_{31}) - k_1 m q_2^2 (p_1 + k_1 k_{d1} \tilde{q}_1 + k_{31} z_{31})^3 \\ \frac{9 k_1 k_{d1} q_2 \tilde{q}_1}{(3 m q_2^2 + M L^2)^2} [(1 + m) p_1 + k_1 k_{d1} m \tilde{q}_1 + k_{31} z_{31}] \end{bmatrix} \\ \dot{z}_3 &= R_3 p + \Big[M^{-1} + k_1 R_3 \Big] K_d \tilde{q} \end{aligned} \tag{3.44}$$

[3]Notice that this robot does not satisfy condition (A.3)

The values of the model and controllers parameters are as follows: $m_1 = 1$, $ML^2 = 3$, $K_p = \text{diag}(2, 1)$, $K_d = \text{diag}(k_{d1}, k_{d2}) = \text{diag}(2, 1)$, $R_3 = \text{diag}(4, 4)$, $K_i = K_3 = \text{diag}(k_{31}, k_{32}) = \text{diag}(3, 3)$ and $k_1 = 2$.

Figs. 3.2–3.6 show the behavior of the system for the smaller disturbance, that is $\alpha = 3$, while the case of $\alpha = 10$ is depicted in Figs. 3.7–3.11. In all cases, the superior performance of the ISS controller is evident. It is interesting to note that the improved performance is not achieved injecting larger gains in the loop. Actually, as shown in Figs. 3.5 and 3.10, which show the control signals, the control action of the IISS controller is far more demanding than that of the ISS controller.

The bounded disturbances acting on the system are shown in Figs 3.6 and 3.11. As expected, the performance is deteriorated for bigger disturbances. However, the ISS controller still shows acceptable transients. On the other hand, the behavior of the IISS controller might not be practically acceptable—as the demanded forces might exceed the actuator limits.

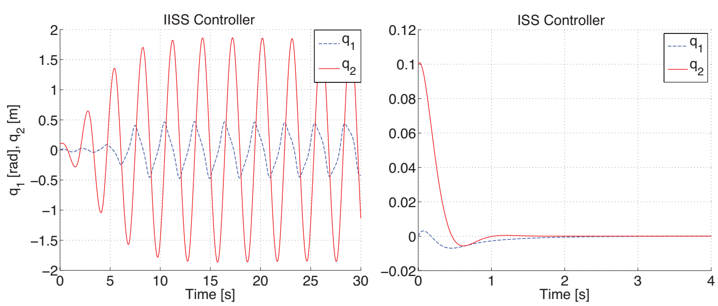


Figure 3.2: Angle of the arm q_1 and position of the hand q_2 for $\alpha = 3$.

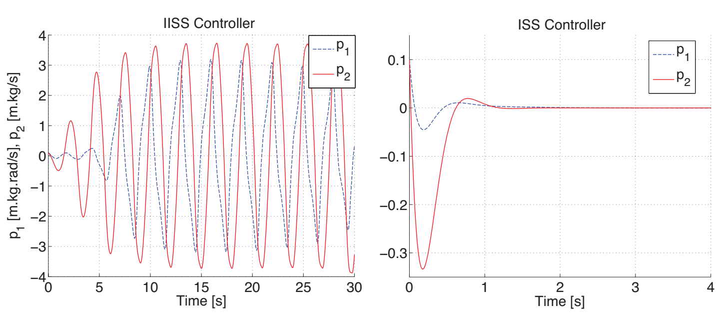


Figure 3.3: Momenta of the arm p_1 and the hand p_2 for $\alpha = 3$.

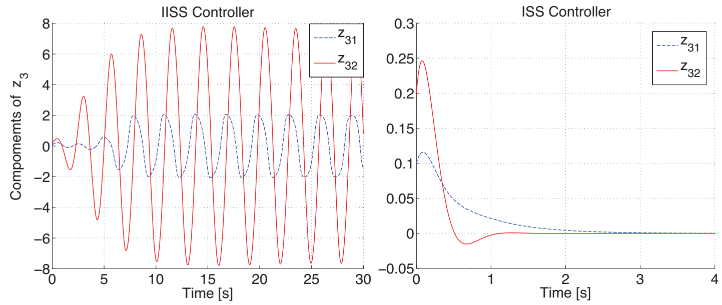


Figure 3.4: States of the controller for $\alpha = 3$.

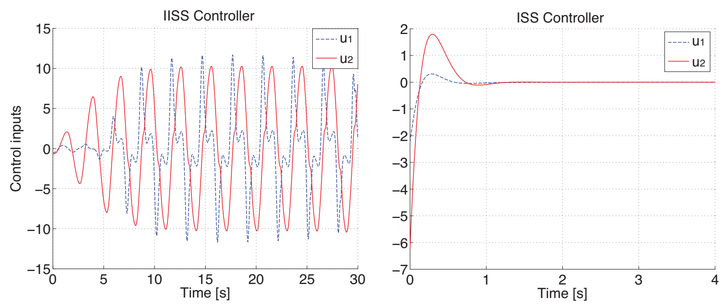


Figure 3.5: Control torque on the arm u_1 and force on the hand u_2 for $\alpha = 3$

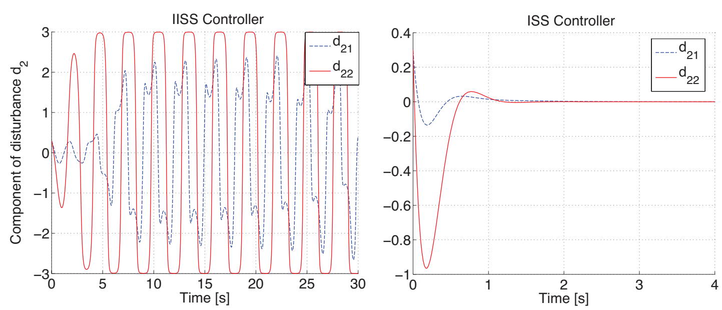


Figure 3.6: Disturbances $d_2 = 3\tanh(\dot{q})$.

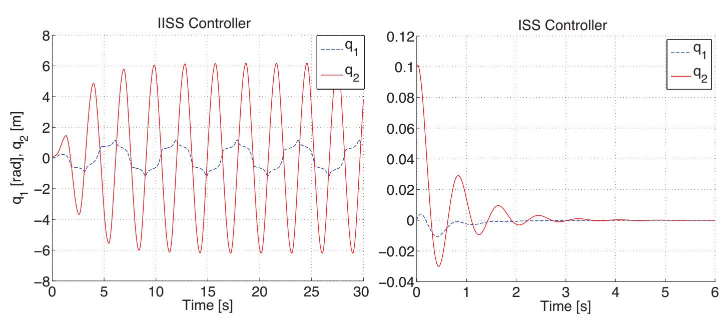


Figure 3.7: Angle of the arm q_1 and position of the hand q_2 for $\alpha = 10$.

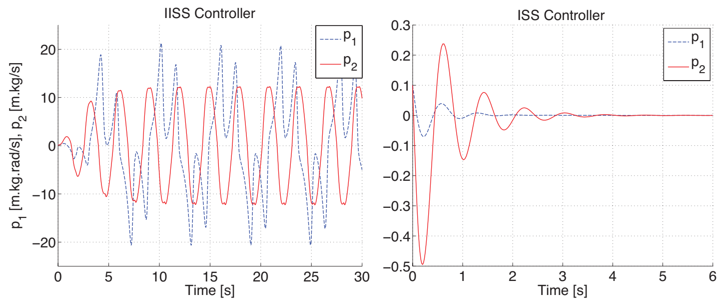


Figure 3.8: Momenta of the arm p_1 and the hand p_2 for $\alpha = 10$.

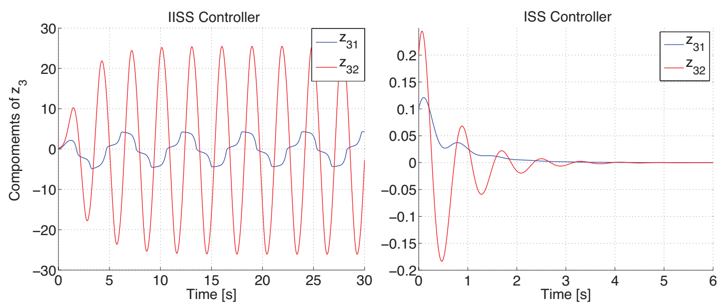


Figure 3.9: States of the controller for $\alpha = 10$.

3.7 Experiments

In this section we shown an experimental set-up to exponential convergence controllers at robots manipulators considering matched disturbances as the section 3.5.2 (change of coordinates). An extra contribution in this section is the simplicity of the controller structure, that is coming from the modified $S(q,p)$ presented in Appendix A.2 on the no-dependent of the complex $\nabla_q T(q)$ term.

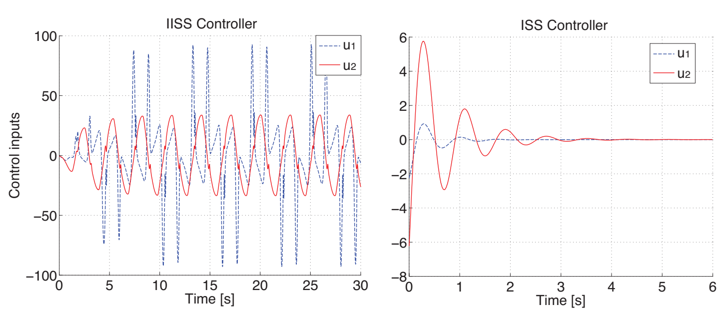


Figure 3.10: Control torque on the arm u_1 and force on the hand u_2 for $\alpha = 10$

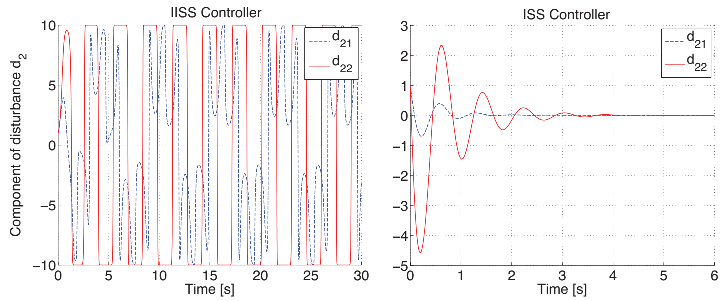


Figure 3.11: Disturbances $d_2 = 10\tanh(\dot{q})$.

3.7.1 Set-up

The robotic system used for our experimental study is a TX-60 Stäubli robot arm (see Fig. 3.12a). In this study, we only considered motion of the robot's second and third joints, that is, a planar manipulator as the one conceptually depicted in Fig. 3.12b.

To command the motion of the robot, the control system counts with the *Low-Level Interface* [60] that allows to explicitly set the torque on each of the joints. In order to provide a deterministic real-time behavior to the controller (a key feature to guarantee a *constant* sample time), a RT-Linux PC [61] processes all the feedback signals and computes the dynamic control law. The desired torque command is then transmitted via TCP/IP to the robot's low-level controller (see Fig. 3.13 for a conceptual representation of the control architecture). All the algorithms reported in this section were implemented at a real-time servo loop of 4 milli-seconds.

The implementation of the proposed control algorithms requires knowledge of the robot's dynamic model as well as the frictional forces, therefore, parametric identification of these physical parameters had to be performed. The analytic expressions of the kinetic co-energy and gravitational potential of the 2-DOF manipulator are given as follows [62]:

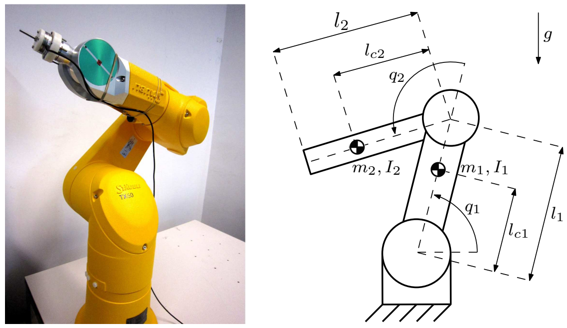
$$\begin{aligned} \tfrac{1}{2}\|\dot{q}\|_M^2 &= \tfrac{1}{2}(m_1 l_{c1}^2 + m_2 l_1^2 + I_1)\dot{q}_1^2 + \tfrac{1}{2}(m_2 l_{c2}^2 + I_2)(\dot{q}_1 + \dot{q}_2)^2 \\ &\quad + m_2 l_1 l_{c2}\cos(q_2)\left(\dot{q}_1^2 + \dot{q}_1\dot{q}_2\right), \\ V_g(q) &= g(m_1 l_{c1} + m_2 l_1)\sin(q_1) + g m_2 l_{c2}\cos(q_1 + q_2) \end{aligned}$$

To this end, the dynamic equations of the robot manipulator (expressed in Euler-Lagrange form) were linearly parameterized with respect to the constant vector of parameters θ, whose definition and identified *numerical* values are given as follows

$$\begin{aligned} \theta &= \left[\ \tfrac{1}{2}(m_1 l_{c1}^2 + m_2 l_1^2 + I_1) \quad m_2 l_1 l_{c2} \quad \tfrac{1}{2}(m_2 l_{c2}^2 + I_2) \right. \\ &\qquad \left. (m_1 l_{c1} + m_2 l_1)g \quad m_2 l_{c2} g \quad D_1 \quad D_2\ \right], \\ &= [\ 1.82, 0.29, 0.51, 48.18, 13.21, 16.27, 6.7\ ] \end{aligned}$$

Then, for this manipulator, the analytic expression of the mass matrix is

$$M(q) = \begin{bmatrix} \theta_1 + \theta_2\cos(q_2) & \theta_3 + \theta_2\cos(q_2) \\ \theta_3 + \theta_2\cos(q_2) & \theta_3 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{bmatrix},$$



(a) TX60 Stäubli robot. (b) Planar manipulator.

Figure 3.12: Experimental robotic system.

inverse of the mass matrix given by

$$M^{-1} = \frac{1}{Det}\begin{bmatrix} M_{22} & -M_{12} \\ -M_{12} & M_{11} \end{bmatrix} \tag{3.45}$$

with $Det = M_{11}M_{22} - M_{12}^2$. For this configuration, the square-root matrix takes the following simple form

$$T = \frac{1}{\varrho}\begin{bmatrix} M_{22} + \sqrt{Det} & -M_{12} \\ -M_{12} & M_{11} + \sqrt{Det} \end{bmatrix} \tag{3.46}$$

and $\varrho = \sqrt{Det}\sqrt{M_{22} + M_{11} + 2\sqrt{Det}}$. The joint controller $u = T^{-1}v$ was computed using the available joint velocity measurements (which correspond to $\dot{q} = M^{-1}\mathbf{p}$). Using Appendix A.2, we obtain the following simple control implementation

$$\begin{aligned} u \quad = \quad & skew(*)(\dot{q} + 2c_1 TK(q - q^*)) - \\ & -(c_2c_1 + c_3c_1 + 1)K(q - q^*) + (c_2 + c_3)z_3 - \\ & -(c_2 + c_3)T^{-1}\dot{q} - c_1 T^{-1}K\dot{q}, \end{aligned} \tag{3.47}$$

where the numerical integrator is computed as

$$z_3 = -(T skew(*) - c_3)\dot{q} - (c_1 T skew(*) - c_1 I + I)TK(q - q^*),$$

with $skew(*)$ as the skew-symmetric matrix

$$skew(*) = \frac{1}{2}\Big[e_2\big[\nabla_{q_2} M\dot{q}\big]^\top - \big[\nabla_{q_2} M\dot{q}\big]e_2^\top\Big].$$

To test the performance of the system, we defined a desired joint position of $q^* = q_0 + [0.3, 0.25]^\top$, where $q_0 = q(0)$ represents the starting joint configuration. Fig. 3.14a and 3.14b show how the joint position 1 and 2 converge to its desired reference. Fig. 3.15a and 3.15b respectively show the applied joint torque u and its integral action z_3. Note in this free-motion experiment, the integrator converges to a value different from zero. This simply means that the integral action is compensating unknown measurements or external force.

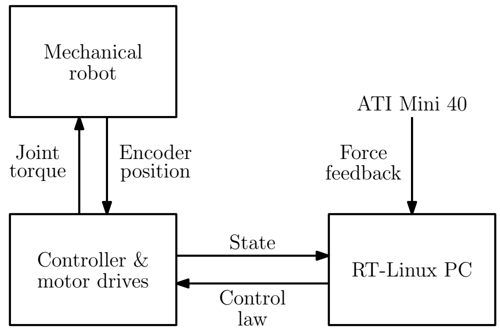
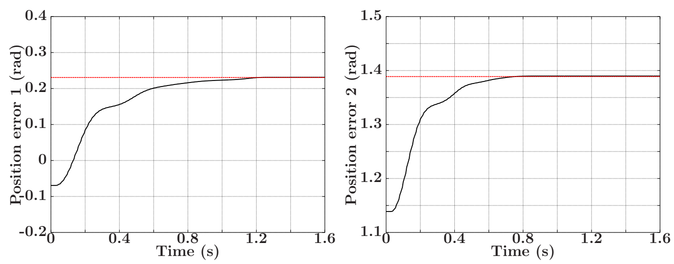


Figure 3.13: Conceptual representation of the real-time control architecture.



(a) Position q_1. (b) Position q_2.

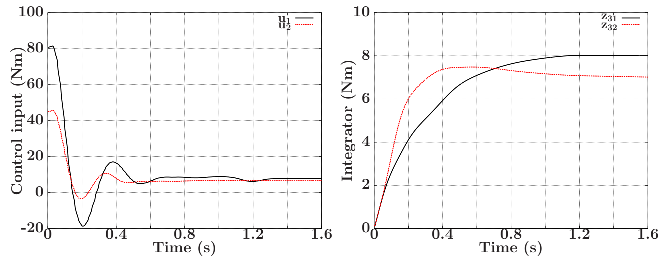
Figure 3.14: Response of the joint positions.

To test the robustness of the algorithm, we numerically perturbed the closed-loop system by adding a constant vector $d_2 = [2, 2]^\top$ to the control input u. For this numerically-perturbed case, Fig. 3.16a and 3.14b show the convergence of the joint position to its desired reference. Similarly, Fig. 3.17a and 3.17b respectively show the applied joint torque u and its integral action z_3. Compared to Fig. 3.15b, note that the integrator's signal converges to a higher value(shifted). As presented before, this value corresponds to $(c_2 + c_3)^{-1} d_2$

3.8 Conclusions

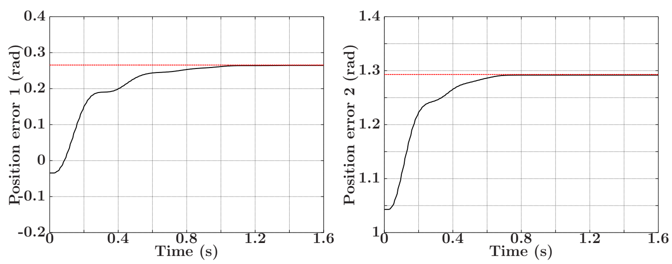
In this chapter, we have presented a control design that improves the robustness of energy shaping controllers for mechanical systems with external disturbances. Robustness is achieved with a dynamics state feedback that adds integral actions, as well as gyroscopic and damping forces. It should be underscore that none controllers carries out cancelation of nonlinearities, instead they inject the required forces to achieve the robustification objective. The solution for mechanical systems with constant mass matrix is simple, whilst the control laws are more involved when the mass matrix is non-constant. The proposed controllers ensure asymptotic stability for constant, matched and–or unmatched, disturbances. In the case of time–varying disturbances, the robustness properties have been quantified establishing IISS and ISS properties.

We show that with the simple incorporation of a change of coordinates the con-



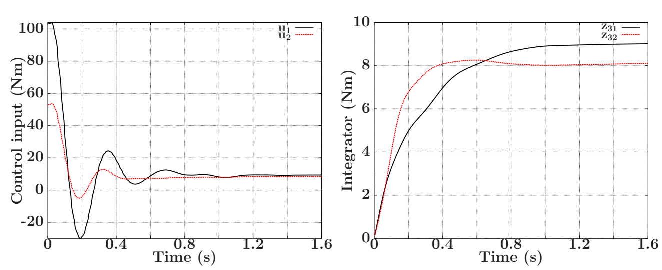
(a) Control input u. (b) Integral action z_3.

Figure 3.15: Resulting joint controls.



(a) Position q_1. (b) Position q_2.

Figure 3.16: Response of the joint positions under a constant disturbance d_2.



(a) Control input u. (b) Integral action z_3.

Figure 3.17: Resulting joint controls for the perturbed system.

trollers are significantly simplified preserving the same nice robustness properties. Furthermore, convergence to matched disturbances that converge to constant values was established, a result that is unavailable for the controllers of Section 4.4. Unfortunately, the new developments are limited to the case when there are no unmatched disturbances. This limitation stems from the fact that, in the presence of unmatched disturbances, a new term involving $\dot{T}$ appears in the disturbance vector of the transformed system (4.2).

Chapter 4

Tracking controller for mechanical systems

A solution to the problem of global exponential tracking of mechanical systems without velocity measurements is given in this chapter. The proposed controller is obtained combining a recently reported exponentially stable immersion and invariance observer and a suitably designed state–feedback passivity–based controller, which assigns to the closed–loop a port–Hamiltonian structure with a desired energy function. The result is applicable to a large class of mechanical systems and, in particular, no assumptions are made on the presence—and exact knowledge—of friction forces.

4.1 Introduction

A long standing open problem for mechanical systems is the construction of a (smooth) controller that ensures, without velocity measurements, global tracking of position and velocity for all desired reference trajectories. A major contribution towards the solution of this problem is due to [50], where invoking the Immersion and Invariance (I&I) techniques developed in [51], the first globally exponentially convergent speed observer is reported. In this chapter we prove that the certainty equivalent combination of (a slight variation of) this speed observer with a, suitably tailored, static state–feedback passivity–based controller (PBC) yields a solution to this problem with the following properties:

P1 The closed–loop is uniformly globally *exponentially* stable (UGES) that, via total stability arguments, ensures strong robustness properties.

P2 To achieve *asymptotic* stability only a lower bound on the inertia matrix is assumed—if it is also upper-bounded then the stronger exponential stability is ensured. Hence, the result is applicable to a large class of mechanical systems, including robots with prismatic joints.

P3 The fragile assumption of existence (and exact knowledge) of friction is conspicuous by its absence.

P4 The stabilization mechanism does not rely on the injection of high gain into the loop. Indeed, although the observer of [50] includes a dynamic scaling factor, it acts only during the transients and is shown to actually converge to one.

To the best of our knowledge, this is the strongest result available to date for this important problem. The reader is referred to [52, 40, 59], and references therein, for a review of the literature.

Many *semi-global* results to the aforementioned position feedback global tracking problem have been reported. Semi-global schemes intrinsically rely on high–gain injection to enlarge the domain of attraction, hence the interest in truly global controllers. In [58] a globally asymptotically stable solution is claimed to be found that, unfortunately, suffers from serious drawbacks. First, the design critically depends on the existence, and exact knowledge, of a positive definite friction matrix. As is well–known this fragile assumption considerably simplifies the controller design, see [55] for an example. Second, besides the requirement of an upper–bounded inertia matrix, some additional (technically motivated) assumptions on the inertia matrix and the potential energy function, which rule out many mechanical systems of practical interest, are imposed, *e.g.*, systems with linear springs. Third, and more importantly, as the controller requires a change of coordinates using saturation functions—first introduced in this context in [54] for the solution of the one–degree–of–freedom case—the invertibility of these functions cannot be globally guaranteed and, as clearly indicated in page 111 of [40], the claim in [58] is unfounded. Acknowledging (alas, obliquely) the problem, the same authors reported in [59] a variation of their previous controller that still suffers from the two first drawbacks indicated above. Notice that the *exact* knowledge of the friction coefficient is required in [59]. Indeed, the claim for the adaptive version of the scheme is unfortunately incorrect, since the argument used to prove the invariance of the estimated domain of attraction S_1 is not valid in this case.[1] Moreover, the unusual requirement of having the *controller initial conditions equal to zero*, see Remark 2 in [59], puts a serious question mark on the robustness of the scheme—see also Footnote 2 in Section 4.2.

More recently, a claim of a UGES scheme was reported in [52]. Unfortunately, it can easily be shown that this controller cannot be implemented without velocity feedback, see equation (32) in [52].

4.2 Main result

In the chapter we consider n–degrees of freedom, fully-actuated, friction–less, mechanical systems described in port–Hamiltonian (pH) form by

$$\begin{bmatrix} \dot{q} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \nabla H(q, \mathbf{p}) + \begin{bmatrix} 0 \\ I_n \end{bmatrix} u \tag{4.1}$$

[1] More precisely, the set where the derivative of the Lyapunov function is zero is not compact in the whole state space, that now contains the parameter errors, see equation (40) in [59].

with total energy function $H : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$

$$H(q, \mathbf{p}) = \frac{1}{2}\mathbf{p}^\top M^{-1}(q)\mathbf{p} + V(q),$$

where $q, \mathbf{p} \in \mathbb{R}^n$ are the generalized positions and momenta, respectively, $u \in \mathbb{R}^n$ is the control input, the inertia matrix $M : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ verifies the (uniform in q) bounds

$$m_{\max} I_n \geq M(q) = M^\top(q) \geq m_{\min} I_n,$$

for some constants $m_{\max} \geq m_{\min} > 0$, and $V : \mathbb{R}^n \to \mathbb{R}$ is the potential energy function.

Proposition 13. *Consider the mechanical system* (4.1)*. For all twice differentiable, bounded, reference trajectories* $(q_d(t), \mathbf{p}_d(t)) \in \mathbb{R}^n \times \mathbb{R}^n$*, there exists a dynamic position–feedback controller that ensures UGES of the closed–loop system. More precisely, there exist two (smooth) mappings*

$$\begin{aligned} \mathbf{F} &: \mathbb{R}^{3n+1} \times \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{3n+1} \\ \mathbf{H} &: \mathbb{R}^{3n+1} \times \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n \end{aligned}$$

such that, for all initial conditions

$$(q(t_0), \mathbf{p}(t_0), \varpi(t_0)) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^{3n} \times \mathbb{R}_{\geq 0}$$

the system (4.1) *in closed–loop with*

$$\begin{aligned} \dot{\varpi} &= \mathbf{F}(\varpi, q, t) \\ u &= \mathbf{H}(\varpi, q, t) \end{aligned}$$

verifies

$$\left|\left| \begin{bmatrix} q(t) - q_d(t) \\ \mathbf{p}(t) - \mathbf{p}_d(t) \\ \varpi(t) \end{bmatrix} \right|\right| \leq \kappa \exp^{-\alpha(t-t_0)} \left|\left| \begin{bmatrix} q(t_0) - q_d(t_0) \\ \mathbf{p}(t_0) - \mathbf{p}_d(t_0) \\ \varpi(t_0) \end{bmatrix} \right|\right|,$$

for some constants $\alpha, \kappa > 0$ *(independent of* t_0*) and all* $t \geq t_0 \geq 0$.

Moreover, the controller ensures uniform global asymptotic *stability (UGAS) even if the inertia matrix is* not bounded from above.

Remark 18. Our choice of a pH representation of the mechanical systems stems from the fact that the full–state feedback controller (described in the next section) is a PBC that shapes the energy function and assigns a suitable pH structure to the system.

Remark 19. As indicated in the introduction the proposed controller is a certainty equivalent version of this PBC where the unknown momenta is replaced by its estimate, generated with (a slight variation of) the observer of [50]. Hence, (4.2) is (essentially) the observer dynamics.

Remark 20. The initial conditions of the last component of the controller state, ϖ, is restricted to be positive. This coordinate corresponds to the (shifted) dynamic scaling factor of the I&I observer of [50], that is shown to remain bounded away from zero for all times. This restriction should be compared with the condition that the controller state should be initialized at zero imposed to the controller of [59].[2]

4.3 Full–state feedback PBC

The design of the full–state feedback PBC proceeds in two steps. First, the change of coordinates in momenta proposed in [48] for observer design is used to assign a constant inertia matrix in the energy function. Second, the change of coordinates used in the chapter 3 to add integral actions to mechanical systems is combined with a suitable state–feedback PBC to assign a pH structure with a desired energy function.[3]

4.3.1 A suitable pH representation

As shown in the section 3.5, the change of coordinates

$$(q,p) \mapsto (q, T(q)\mathbf{p}),$$

with $T : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ the positive definite, uniquely defined, square root of the inverse inertia matrix that is

$$M^{-1}(q) = T^2(q),$$

transforms (4.1) into

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & T(q) \\ -T(q) & S(q,p) \end{bmatrix} \nabla W + \begin{bmatrix} 0 \\ I_n \end{bmatrix} v, \tag{4.2}$$

with $v := T(q)u$ the new control signal, new Hamiltonian function $W : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$

$$W(q,p) = \frac{1}{2}|p|^2 + V(q),$$

and the gyroscopic forces matrix $S : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ given by (3.27).

4.3.2 The PBC and its pH error system

Proposition 14. *Consider the pH system* (4.2). *Define the mapping* $v^\star : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n$

$$\begin{aligned} v^\star(q,p,t) \quad = \quad & -T(q)\Big[K(q-q_d(t)) - \nabla V(q)\Big] - S(q,p)p_d(t) + \dot{p}_d(t) - \\ & -R(p-p_d(t)) - c_1 K T(q)(p-p_d(t)) - c_1\Big[S(q,p) - R\Big]K(q-q_d(t)). \end{aligned} \tag{4.3}$$

[2]Actually, as seen from Theorem 1 of [59], the initial condition can lie on an interval around zero, but this interval reduces to zero as the number of degrees of freedom increases.

[3]A similar coordinate transformation has been proposed in [57] to generate a sign-indefinite damping injection term for stabilization of mechanical systems without the standard detectability assumption.

where

$$p_d := T^{-1}(q)\dot{q}_d, \tag{4.4}$$

$c_1 \in \mathbb{R}_{>0}$, *and* $K, R \in \mathbb{R}^{n\times n}$ *are positive definite gain matrices.*

(i) The closed–loop dynamics obtained setting

$$v = v^\star(q,p,t)$$

expressed in the coordinates

$$\begin{aligned} w_1 &= \tilde{q} \\ w_2 &= c_1 K\tilde{q} + \tilde{p}, \end{aligned} \tag{4.5}$$

where

$$\tilde{q} := q - q_d,\ \tilde{p} := p - p_d,$$

takes the pH form

$$\dot{w} = \begin{bmatrix} -c_1 T(q) & T(q) \\ -T(q) & S(q,p) - R \end{bmatrix} \nabla H_w \tag{4.6}$$

with Hamiltonian function $H_w : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_{>0}$

$$H_w(w) = \frac{1}{2}|w_2|^2 + \frac{1}{2}\|w_1\|_K^2 \tag{4.7}$$

(ii) The zero equilibrium point of (4.6) *is UGES with Lyapunov function* $H_w(w)$*. Consequently,* $(\tilde{q}(t), \tilde{p}(t)) \to 0$ *exponentially fast.*

(iii) If the inertia matrix is not bounded from above, the zero equilibrium point of (4.6) *is UGAS with Lyapunov function* $H_w(w)$*.*

Proof. Taking the time derivative of the change of coordinates given in (4.5) and using the control law (4.3) yields the closed–loop (4.6), establishing the claim (i). Now, taking the time derivative of (4.7), along the system's trajectories, it follows

$$\dot{H}_w = -c_1\|w_1\|_{KTK}^2 - \|w_2\|_R^2 \leq -\delta H_w, \tag{4.8}$$

where

$$\delta := \min\{2c_1\frac{\lambda_{\min}(KTK)}{\lambda_{\max}(K)},\ 2\lambda_{\min}(R)\} > 0. \tag{4.9}$$

This proves, after some basic bounding, the claim (ii).

The difficulty in establishing UGES when $T(q)$ is not (uniformly) bounded from below—that would be the case if $M(q)$ is not (uniformly) upper–bounded—is due to the term $\|w_1\|_{KTK}^2$ in $\dot{H}_w$, which cannot be bounded from below by $|w_1|^2$—notice that δ in (4.9) zero. On the other hand, from the first inequality in (4.8) we can conclude uniform global Lyapunov stability. The attractivity part of the proof is established doing some standard signal chasing.

□□□

Remark 21. Of course, there are many full–state feedback controllers ensuring exponential tracking [40]. The interest of the PBC presented above relies on the preservation of the pH structure that is instrumental for the development of the position–feedback version.

Remark 22. The requirement of upper–bounded inertia matrix, needed for the exponential stability property, stems from the fact that the inverse of its square root, *i.e.*, $T(q)$, is the damping in the $\tilde{q}$ coordinates. See the $(1,1)$–block of the damping matrix in (4.6). As discussed in [50] and shown in the next section, this assumption is not needed for UGES of the observer.

4.4 An exponentially convergent momenta observer

In order to estimate directly the momenta p, in this section we slightly modify the exponentially convergent speed I&I observer reported in [50]. Also, motivated by the developments in [56], we consider an alternative Lyapunov function for the stability analysis and add some degrees of freedom to robustify the observer design. The latter feature is essential for the proof of our main result. Since the proof closely mimics the one given in [50].

Proposition 15. *Consider the system* (4.2)*, and assume v is such that trajectories exist for all $t \geq 0$. There exist smooth mappings*

$$
\begin{aligned}
\mathbf{A} &: \mathbb{R}^{3n+1} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{3n+1} \\
\mathbf{B} &: \mathbb{R}^{3n+1} \times \mathbb{R}^n \to \mathbb{R}^n
\end{aligned}
$$

such that the interconnection of (4.2) *with*

$$
\begin{aligned}
\dot{\mathrm{X}} &= \mathbf{A}(\mathrm{X}, q, v) \\
\hat{p} &= \mathbf{B}(\mathrm{X}, q),
\end{aligned}
$$

where $\mathrm{X} \in \mathbb{R}^{3n+1}$, $\hat{p} \in \mathbb{R}^n$, *ensures*

$$
\lim_{t \to \infty} e^{\alpha t}[p(t) - \hat{p}(t)] = 0,
$$

for some $\alpha > 0$, and for all initial conditions

$$
(q(0), p(0), \mathrm{X}(0)) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^{3n} \times \mathbb{R}_{\geq 0}.
$$

This implies that (4.10) *is an exponentially convergent momenta observer for the mechanical system* (4.2).

Proof. The basic idea of I&I observers is to find a *measurable* mapping $\beta : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ such that the (so–called) off–the–manifold coordinate

$$
z = \xi + \beta(q, \mathtt{q}, \mathtt{p}) - p, \tag{4.10}
$$

asymptotically converges to zero, where $\xi, \mathfrak{q}, \mathfrak{p} \in \mathbb{R}^n$ are (part of) the observer state. If this is the case

$$\hat{p} := \xi + \beta(q, \mathfrak{q}, \mathfrak{p}) \tag{4.11}$$

is a consistent estimate of p. We, therefore, study the dynamic behavior of z and compute

$$\dot{z} = \dot{\xi} + \nabla_q \beta \dot{q} + \nabla_{\mathfrak{q}} \beta \dot{\mathfrak{q}} + \nabla_{\mathfrak{p}} \beta \dot{\mathfrak{p}} - S(q,p)p + T(q)\nabla V - v.$$

We have that the mapping S defined in (3.27) verifies the following properties(see Appendix A.1):

(P.i) S is linear in the second argument, that is

$$S(q, \alpha_1 p + \alpha_2 \bar{p}) = \alpha_1 S(q,p) + \alpha_2 S(q, \bar{p})$$

for all q, p, $\bar{p} \in \mathbb{R}^n$, and α_1, $\alpha_2 \in \mathbb{R}$.

(P.ii) There exists a mapping $\bar{S} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n \times n}$ satisfying

$$S(q,p)\bar{p} = \bar{S}(q, \bar{p})p.$$

Hence, proposing

$$\begin{aligned} \dot{\xi} \quad := \quad & -\nabla_{\mathfrak{q}} \beta \dot{\mathfrak{q}} - \nabla_{\mathfrak{p}} \beta \dot{\mathfrak{p}} + S(q, \xi + \beta)(\xi + \beta) - \\ & -T(q)\nabla V + v - \nabla_q \beta T(q)(\xi + \beta), \end{aligned} \tag{4.12}$$

together with Properties (P.i) and (P.ii) yields

$$\dot{z} = [S(q,p) + \bar{S}(q, \xi + \beta) - \nabla_q \beta T] z. \tag{4.13}$$

It is clear that if the mapping β solves the partial differential equation (PDE)

$$\nabla_q \beta = [\psi I_n + \bar{S}(q, \xi + \beta)] T^{-1}(q),$$

the z–dynamics reduces to

$$\dot{z} = [S(q,p) - \psi I_n] z,$$

which is asymptotically stable provided ψ (that may be state–dependent) is positive. To avoid the solution of the PDE, which may not even exist, an approximate solution is proposed. Towards this end, define an *ideal* expression for $\nabla_q \beta$ as

$$[\psi I_n + \bar{S}(q, \xi + \beta)] T^{-1}(q) =: \mathcal{H}(q, \xi + \beta). \tag{4.14}$$

and, following [53], define β as[4]

$$\beta(q, \mathfrak{q}, \mathfrak{p}) := \mathcal{H}(\mathfrak{q}, \mathfrak{p}) q. \tag{4.15}$$

The above choice yields $\nabla_q \beta = \mathcal{H}(\mathfrak{q}, \mathfrak{p})$, which may be written as

$$\nabla_q \beta = \mathcal{H}(q, \xi + \beta) - [\mathcal{H}(q, \xi + \beta) - \mathcal{H}(\mathfrak{q}, \mathfrak{p})]. \tag{4.16}$$

[4]This construction avoids the cumbersome calculations proposed in [50], where the mapping β is defined computing several integrals.

Now, since the term in brackets in (4.16) is equal to zero if $\mathfrak{p} = \xi + \beta$ and $\mathfrak{q} = q$, there exist mappings

$$\Delta_q, \Delta_p : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$$

verifying

$$\Delta_q(q, \mathfrak{p}, 0) = 0, \quad \Delta_p(q, \mathfrak{p}, 0) = 0, \tag{4.17}$$

and such that

$$\mathcal{H}(q, \xi + \beta) - \mathcal{H}(\mathfrak{q}, \mathfrak{p}) = \Delta_q(q, \mathfrak{q}, e_q) + \Delta_p(q, \mathfrak{p}, e_p), \tag{4.18}$$

where

$$e_q := \mathfrak{q} - q, \quad e_p := \mathfrak{p} - (\xi + \beta). \tag{4.19}$$

Substituting (4.14), (4.16) and (4.18) in (4.13), yields

$$\dot{z} = [S(q,p) - \psi I_n]z + (\Delta_q + \Delta_p)T(q)z.$$

The mappings Δ_q, Δ_p play the role of disturbances that are dominated with a dynamic scaling and a proper choice of the observer dynamics. For, define the dynamically scaled off–the–manifold coordinate

$$\eta = \frac{1}{r}z, \tag{4.20}$$

where r is a scaling factor to be defined. The dynamic behavior of η is given by

$$\dot{\eta} = (S - \psi I)\eta + (\Delta_q + \Delta_p)T(q)\eta - \frac{\dot{r}}{r}\eta. \tag{4.21}$$

Mimicking [50] select the dynamics of $\mathfrak{q}$, $\mathfrak{p}$ as

$$\begin{aligned} \dot{\mathfrak{q}} &= T(q)(\xi + \beta) - \psi_1 e_q \\ \dot{\mathfrak{p}} &= -T(q)\nabla V + v + S(q, \xi + \beta)(\xi + \beta) - \psi_2 e_p \end{aligned} \tag{4.22}$$

where ψ_1, ψ_2 are some positive functions of the state defined later. Using (4.22), together with (4.19), we get

$$\begin{aligned} \dot{e}_q &= T(q)\eta r - \psi_1 e_q \\ \dot{e}_p &= (\nabla_q \beta)T(q)\eta r - \psi_2 e_p. \end{aligned} \tag{4.23}$$

Moreover, select the dynamics of r as

$$\dot{r} = -\frac{\psi}{4}(r-1) + \frac{r}{\psi}(\|\Delta_p T\|^2 + \|\Delta_q T\|^2),\ r(0) \geq 1, \tag{4.24}$$

with $\|\cdot\|$ the matrix induced 2–norm. Notice that the set $\{r \in \mathbb{R} : r \geq 1\}$ is invariant for the dynamics (4.24).

We show now that the (non–autonomous) *error* system (4.21), (4.23), (4.24)—with the shifted coordinate $r \mapsto (r-1)$—has a UGES equilibrium at zero. For, define the proper Lyapunov function candidate[5]

$$V(\eta, e_q, e_p, r) := \frac{1}{2}[|\eta|^2 + |e_q|^2 + |e_p|^2 + (r-1)^2]. \tag{4.25}$$

[5] The choice of this function as well as the use of additional degrees of freedom in the functions ψ_i was suggested in [56].

Following the calculations done in [50] we obtain

$$\begin{aligned}\dot{V} &\leq -\left(\tfrac{\psi}{4}-1\right)|\eta|^2 - \left(\psi_1 - \tfrac{1}{2}r^2\|T\|^2\right)|e_q|^2 - \\ &\quad - \left(\psi_2 - \tfrac{1}{2}r^2\|\nabla_q\beta\|^2\|T\|^2\right)|e_p|^2 + (r-1)\dot{r}. \end{aligned} \tag{4.26}$$

Clearly, if we set

$$\psi = 4(1+\psi_3),\ \psi_1 = \frac{1}{2}r^2\|T\|^2 + \psi_4 \tag{4.27}$$

and

$$\psi_2 = \frac{1}{2}r^2\|\nabla_q\beta\|^2\|T\|^2 + \psi_5,$$

where ψ_3, ψ_4, ψ_5 are positive functions of the state defined below, one gets

$$\dot{V} \leq -\psi_3|\eta|^2 - \psi_4|e_q|^2 - \psi_5|e_p|^2 + (r-1)\dot{r}.$$

Let us look now at the last right hand term above

$$(r-1)\dot{r} = -\frac{\psi}{4}(r-1)^2 + (r-1)\frac{r}{\psi}(\|\Delta_p T\|^2 + \|\Delta_q T\|^2).$$

Now, (4.17) ensures the existence of mappings $\bar{\Delta}_q,\ \bar{\Delta}_p : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ such that

$$\begin{aligned}\|\Delta_q(q,\mathfrak{p},e_q)\| &\leq \|\bar{\Delta}_q(q,\mathfrak{p},e_q)\|\ |e_q| \\ \|\Delta_p(q,\mathfrak{p},e_p)\| &\leq \|\bar{\Delta}_p(q,\mathfrak{p},e_p)\|\ |e_p|.\end{aligned}$$

Hence

$$\|\Delta_p T\|^2 + \|\Delta_q T\|^2 \leq \|T\|^2 |(\|\bar{\Delta}_p\|^2|e_p|^2 + |\bar{\Delta}_q\|^2|e_q|^2).$$

Finally, setting

$$\begin{aligned}\psi_3 &= \kappa \\ \psi_4 &= \frac{r(r-1)}{4(1+\psi_3)}\|T\|^2\|\bar{\Delta}_q\|^2 + \kappa \\ \psi_5 &= \frac{r(r-1)}{4(1+\psi_3)}\|T\|^2\|\bar{\Delta}_p\|^2 + \kappa,\end{aligned}$$

for some positive constant κ, yields

$$\dot{V} \leq -\kappa[|\eta|^2 + |e_q|^2 + |e_p|^2 + (r-1)^2] \leq -2\kappa V.$$

This completes the proof of UGES of the equilibrium of the error system.

From (4.10), (4.11) and (4.20), boundedness of r and the exponential convergence of η we get that z and the estimation error $\hat{p} - p$ also converge to zero exponentially fast.

The proof is completed selecting the observer state as

$$\mathrm{X} := (\xi, \mathfrak{q}, \mathfrak{p}, r-1),$$

defining $\mathbf{A}(\mathrm{X}, q, v)$ from (4.12), (4.22) and (4.24) and $\mathbf{B}(\mathrm{X}, q)$ via (4.11).
□□□

4.5 Proof of proposition 13

The certainty equivalent version of the full–state feedback controller (4.3) of Proposition 13 is obtained replacing p by its estimate $\hat{p}$ generated with the observer of Section 4.4. Notice that (4.3) contains a term $\dot{p}_d$ that, as seen from (4.4), depends on the unknown $\dot{q}$. To define the certainty equivalent version of (4.3) we must compute

$$\begin{aligned} \dot{p}_d &= \Big[\nabla_q (T^{-1}\dot{q}_d)\Big]\dot{q} + T^{-1}\ddot{q}_d \\ &= \Big[\nabla_q (T^{-1}\dot{q}_d)\Big]T^\top p + T^{-1}\ddot{q}_d \end{aligned} \tag{4.28}$$

Using (4.28) we get the implementable controller

$$\begin{aligned} v &= -T(K\tilde{q} - \nabla V) - S(q,\hat{p})p_d - R(\hat{p} - p_d) + \Big[\nabla_q (T^{-1}\dot{q}_d)\Big]T\hat{p} \\ &\quad + T^{-1}\ddot{q}_d - c_1 K T(q)(\hat{p} - p_d) - c_1\Big[S(q,\hat{p}) - R\Big]K\tilde{q}. \end{aligned} \tag{4.29}$$

We invoke now the key property (P.i) of Section 4.4, namely that $S(q,\hat{p})$ is *linear* in $\hat{p}$. Consequently, since all other $\hat{p}$–dependent terms in (4.29) are linear, there exists mappings

$$\Psi : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n, \quad \Theta : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{n\times n},$$

such that (4.29) can be written as

$$v = \Psi(q,t) + \Theta(q,t)\hat{p}.$$

Moreover, using (4.10) and (4.11) it can be expressed as

$$v = v^\star(q,p,t) + \Theta(q,t)z.$$

Replacing the latter in (4.2), and using (4.20), yields the perturbed pH system

$$\dot{w} = \begin{bmatrix} -c_1 T(q) & T(q) \\ -T(q) & S(q,p) - R \end{bmatrix} \nabla H_w + \begin{bmatrix} 0 \\ \Theta(q,t) \end{bmatrix} r\eta, \tag{4.30}$$

with the Hamiltonian function given by (4.7). The overall non–autonomous system (*e.g.*, closed–loop plant (4.30) plus observer (4.10)) is $5n+1$–dimensional and has a state $\big(w_1, w_2, e_q, e_p, \eta, r-1\big)$.

To establish the UGES claim consider the proper Lyapunov function

$$\mathcal{V}(w_1, \eta, e_q, e_p, r-1) = H_w(w) + V(\eta, e_q, e_p, r-1),$$

where the functions H_w and V have been defined in (4.7) and (4.25), respectively. From the derivations of the previous two sections it is clear that the only troublesome term is the sign–indefinite cross product $w_2^\top \Theta(q,t) r\eta$, that appears in $\dot{H}_w$.

To dominate this term, consider the bound

$$w_2^\top \Theta(q,t) r\eta \leq \frac{1}{2}|w_2|^2 + \frac{r^2}{2}\|\Theta(q,t)\|^2|\eta|^2. \tag{4.31}$$

From (4.8), (4.26) and (4.27) we see that there is the constant gain R and the free gain function ψ_3, that can be used to dominate the cross–term.[6] More precisely, setting

$$R = (\frac{1}{2} + \kappa)I_n$$

and $\psi_3 : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$

$$\psi_3(q, (r-1), t) = \frac{r^2}{2}\|\Theta(q,t)\|^2 + \kappa,$$

yields $\dot{\mathcal{V}} \leq -\alpha\mathcal{V}$, establishing the UGES claim.

The UGAS claim follows immediately from the derivations above and the arguments invoked in the proof of UGAS of Proposition 14.

4.6 Conclusions

We have given in this chapter a final, definite answer to the question of global exponential tracking of mechanical systems without velocity measurements. The result is applicable to a large class of mechanical systems, without assumptions on the friction forces, the inertia matrix or the potential energy function. In particular, the result does not rely on the existence—and exact knowledge—of pervasive friction, nor on boundedness of gravity forces. To achieve UGES it is required that the inertia matrix be bounded from above. For systems that do not satisfy this condition the weaker UGAS property is proven.

[6]For simplicity, in Proposition 15 ψ_3 is taken as constant.

Chapter 5

Conclusions and future work

5.1 Concluding remarks

The development of a methodology for a class of nonlinear system in pH form with unmatched disturbances exhibits two distinguishing features:

- Regulation of the non passive output and rejections of unmatched disturbances are satisfies adding simple integral action.

- The ingenious methodology formulated via change of coordinates avoid solve PDEs.

  For the nonlinear mechanical systems case:

- The rejection of constant matched disturbances is satisfied.

- To varying time disturbances(matched and unmatched), the system is endow of the IISS and ISS properties.

- Exponential performance was showed via experimental set-up to a manipulator of 2DOF.

- Exponential tracking of position and velocity for all desired reference trajectories without information in velocity is proved.

5.2 Future directions

Some future directions of research are the following: The simplification of the controllers via change of coordinates presented in the Chapter 3 unfortunately are limited to the case when there are no unmatched disturbances. This limitation stems from the fact that, in the presence of unmatched disturbances, a new term involving $\dot{T}$ appears in the disturbance vector of the transformed system (3.25). This discussion give a open question to deal.

In force feedback problem, normally appears the phenomena named "steady-state deviation" given by the interaction of a force transducer and the environment, such that a extension of robust controllers can be exploited to solve this problem.

From Chapter 4 arises interesting axes to research.

- An open question is the robustness of the design *vis–à–vis unmodeled, viscous* friction in the system. In this case we have

$$\begin{bmatrix} \dot{q} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & -D \end{bmatrix} \nabla H(q, \mathbf{p}) + \begin{bmatrix} 0 \\ I_n \end{bmatrix} u,$$

  where $D \in \mathbb{R}^{n \times n}$ is an unknown, positive semi-definite matrix. Some preliminary calculations show that it is possible to re-design the proposed scheme ensuring converge of the error signal to a bounded residual set.

- Another challenging problem is the extension of the result to the case of uncertain parameters. An adaptive version of Proposition 14 is easily obtained with standard techniques. However, it is far from clear how to implement an adaptive observer.

- The observer proposed in [50] is applicable for systems with non–holonomic constraints. How to formulate the position–feedback tracking problem in that case is still to be resolved.

Appendix A

Appendix

A.1 Partial change of coordinates in mechanical systems

Consider the nonlinear mechanical system

$$\begin{bmatrix} \dot{q} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \nabla H(q, \mathbf{p}) + \begin{bmatrix} 0 \\ I_n \end{bmatrix} u \tag{A.1}$$

with Hamiltonian function

$$H(q, \mathbf{p}) = \frac{1}{2}\mathbf{p}^\top M^{-1}(q)\mathbf{p} + V(q). \tag{A.2}$$

$q, \mathbf{p} \in \mathbb{R}^n$ are generalized positions and momenta, respectively, and are assumed measurable, $u \in \mathbb{R}^n$ is the control input. The mass matrix $M(q) = M^\top(q) > 0$, and satisfies

$$m_1 I_n \leq M^{-1}(q) \leq m_2 I_n \tag{A.3}$$

I_n is the $n \times n$ identity matrix. We assume that the Hamiltonian (A.2) has a minimum at the desired equilibrium $(q^\star, 0)$, that is,

$$q^\star = \arg\min V(q),$$

and it is isolated.

Lemma 1. *The system* (A.1) *admits a state space representation in the coordinates* $(q, \mathbf{p}) \mapsto (q, p)$ *of the form*

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & T(q)^\top \\ -T(q) & S(q, p) \end{bmatrix} \nabla W(q, p) + \begin{bmatrix} 0 \\ v \end{bmatrix} \tag{A.4}$$

with $v := T(q)u$ *the new control signal, new Hamiltonian function*

$$W(q, p) = \frac{1}{2}|p|^2 + V(q), \tag{A.5}$$

and the gyroscopic forces matrix as

$$\begin{aligned} S(q,p) &:= \nabla^\top(T\mathbf{p})T^\top - T^\top\nabla(T\mathbf{p})|_{\mathbf{p}=T^{-1}p}. \\ &= \sum_{i=1}^{n}\Big[\big[\nabla_{q_i}(T)T^{-1}p\big](Te_i)^\top - (Te_i)\big[\nabla_{q_i}(T)T^{-1}p\big]^\top\Big], \end{aligned} \quad (A.6)$$

with e_i the $i-th$ basis vector of $\mathbb{R}^n$. Furthermore, $S(q,p)$ verifies the following properties.

(i) S is skew symmetric, that is, $S+S^\top=0$.

(ii) S is linear in the second argument, that is, $S(q,\alpha_1 p+\alpha_2\bar{p}) = \alpha_1 S(q,p) + \alpha_2 S(q,\bar{p})$, for all $q\in\mathbb{R}^n$, $p\in\mathbb{R}^n$, $\bar{p}\in\mathbb{R}^n$ and $\alpha_1,\alpha_2\in\mathbb{R}^n$.

(iii) There exists a mapping $\bar{S}\in\mathbb{R}^n\times\mathbb{R}^n$ such that $S(q,p)\bar{p}=\bar{S}(q,\bar{p})p$, for all $q\in\mathbb{R}^n$, p and $\bar{p}\in\mathbb{R}^n$

Proof. From the change of coordinates $(q,\mathbf{p})\mapsto(q,p)$, we define $p=T\mathbf{p}$ and a factorization to the inertia as

$$M(q)^{-1} = T(q)^\top T(q) \quad (A.7)$$

such that differentiating $\bar{p}$ yields

$$\dot{p} = \dot{T}\mathbf{p} - T\nabla_q\Big(\frac{1}{2}\mathbf{p}^\top M^{-1}\mathbf{p}\Big) - T\nabla V(q) + Tu \quad (A.8)$$

Note now that

$$\begin{aligned} \dot{T}\mathbf{p} &= \sum_{i=1}^{n}(\nabla_{q_i}T)(e_i^\top\dot{q})\mathbf{p} = \sum_{i=1}^{n}(\nabla_{q_i}T)\mathbf{p}(e_i^\top M^{-1}\mathbf{p}) \\ &= \sum_{i=1}^{n}(\nabla_{q_i}T)T^{-1}p(e_i^\top T)p \end{aligned} \quad (A.9)$$

and that

$$\begin{aligned} \nabla_q\{\frac{1}{2}\mathbf{p}^\top M^{-1}\mathbf{p}\} &= \nabla_q\{\frac{1}{2}\mathbf{p}^\top T^\top T\mathbf{p}\} = \sum_{i=1}^{n}\{(\nabla_{q_i}T)\mathbf{p}\}^\top p \\ &= \sum_{i=1}^{n} e_i\{(\nabla_{q_i}T)T^{-1}p\}^\top p \end{aligned} \quad (A.10)$$

Replacing (A.9) and (A.10) in (A.8) yields (A.4) with $S(q,p)$ as in (A.6). Properties $(i)-(iii)$ follow immediately from skew-symmetry and linearity of $S(q,p)$ in (A.6). □□□

A.2 Avoiding the gradient on T matrix

The change of coordinates presented in the section A.1 mapped as skew-symmetric matrix the nonlinearity of the system into the interconnection matrix, given by (A.6). However the gradient at T can be obviated assuming that $M(q)$ is totally know. From the definition $M^{-1} = T^\top T$, we can to get its gradient in every coordinates i as

$$\begin{aligned} \nabla_{q_i} M^{-1}(q) &= \nabla_{q_i}(T^\top)T + T^\top \nabla_{q_i} T \\ &= 2T^\top \nabla_{q_i} T \end{aligned}$$

such that

$$\begin{aligned} -M^{-1}\nabla_{q_i} M(q) M^{-1} &= 2T^\top \nabla_{q_i} T \\ -\frac{1}{2} T \nabla_{q_i} M(q) T^\top T &= \nabla_{q_i} T \end{aligned} \tag{A.11}$$

Finally replacing (A.11) in (A.6) we acquire an expression to $S(q,p)$ independent of $\nabla_{qi} T(q)$ that yields to be:

$$S(q,p) = \frac{1}{2}\sum_{i=1}^{n} \Big[(Te_i)\big[\nabla_{q_i}(M)T^\top p\big]^\top T^\top - T\big[\nabla_{q_i}(M)T^\top p\big](Te_i)^\top \Big] \tag{A.12}$$

This equivalence will be useful to carry out the implementation of the controllers (experiments) presented in the section 3.8

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