Application of the TLM method to the sound propagation modelling in urban area

Gwenaël Guillaume

Laboratoire Central des Ponts et Chaussées (LCPC)

Thesis director:	Judicaël Picaut (LCPC-Nantes)
Thesis co-director:	Christophe Ayrault (LAUM-Le Mans)
Steering Committee:	Isabelle Schmich (CSTB-Grenoble)
	Guillaume Dutilleux (LRPC-Strasbourg)

October 13th, 2009

Global issue: noise annoyances prevention and abatement

- **health and societal impact of noise**
- **legislative and regulation framework**

LCPC research topic: predicting the noise level in urban environment

- sound propagation modelling in urban area

Global issue: noise annoyances prevention and abatement

- health and societal impact of noise
- legislative and regulation framework

LCPC research topic: predicting the noise level in urban environment

- **sound propagation modelling in urban area**

The urban noise: a complex issue

- street ≡ « opened » waveguide
  ⇒ steady-state phenomena, acoustic « leaks » by the open-tops
- frontages morphology
  ⇒ diffuse reflections, edge diffraction, absorption
- long distance propagation
  ⇒ atmospheric effects, ground effects, « unusual » micrometeorological conditions
- temporal variations
  ⇒ moving/time varying noise sources[1], micrometeorological conditions fluctutations[2]

Thesis objective: sound propagation modelling in urban area

- development of a **specific time-domain** numerical model

⇒ TLM method (*Transmission Line Modelling*)

[1] A. Can. *Représentation du trafic et caractérisation dynamique du bruit en milieu urbain*. *PhD Thesis*, Lyon, 2008.

[2] F. Junker *et al.*. *Meteorological classification for environmental acoustics - Practical implications due to experimental accuracy and uncertainty*. *ICA*, Madrid (Espagne), 2007.

The urban noise: a complex issue

- street ≡ « opened » waveguide
  - ⇒ steady-state phenomena, acoustic « leaks » by the open-tops
- frontages morphology
  - ⇒ diffuse reflections, edge diffraction, absorption
- long distance propagation
  - ⇒ atmospheric effects, ground effects, « unusual » micrometeorological conditions
- temporal variations
  - ⇒ moving/time varying noise sources[1], micrometeorological conditions flucturations[2]

Thesis objective: sound propagation modelling in urban area

- development of a **specific time-domain** numerical model

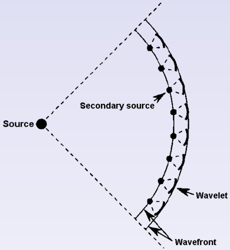
  ⇒ TLM method (*Transmission Line Modelling*)

[1] A. Can. *Représentation du trafic et caractérisation dynamique du bruit en milieu urbain*. *PhD Thesis*, Lyon, 2008.

[2] F. Junker *et al.*. *Meteorological classification for environmental acoustics - Practical implications due to experimental accuracy and uncertainty*. *ICA*, Madrid (Espagne), 2007.

Huygens principle (1690)

A **wavefront** can be broken down into a **set of secondary sources** that radiate spherical wavelets of identical frequency, amplitude and phase.

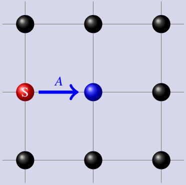


Numerical adaptation in electromagnetism[1]

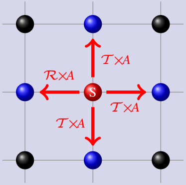
- The **secondary sources** are assimilated to **nodes**.
- The « **diffusion** » of the field between nodes is performed by means of **transmission lines** in term of pulses.

[1] P.B. Johns and R.L. Beurle. *Numerical solution of two dimensional scattering problems using a transmission line matrix.* Proc. IEE, **118**(9), 1971.

Simple case in 2D



Incident pulse



Scattered pulses from the node

- **Nodal reflection and transmission coefficients:**

$$\mathcal{R} = \frac{Z_T - Z_L}{Z_T + Z_L}, \quad \mathcal{R} < 0$$

$$\mathcal{T} = 1 + \mathcal{R} = \frac{2Z_T}{Z_T + Z_L}$$

Z_T : impedance of the termination
Z_L : impedance of the incident transmission line

$\left(\text{here, } Z_L = Z \text{ and } Z_T = Z/3, \text{ so } \mathcal{R} = -\frac{1}{2} \text{ and } \mathcal{T} = \frac{1}{2}\right)$

General case in 2D

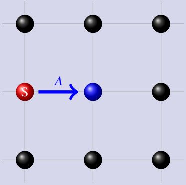
- **Matrix relation:** $_t\mathbf{S} = \mathbf{D} \times {}_t\mathbf{I}$

where $\quad {}_t\mathbf{I} = \left[{}_tI^1, {}_tI^2, {}_tI^3, {}_tI^4\right]^T,$

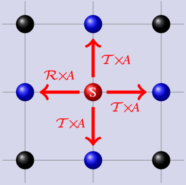
$${}_t\mathbf{S} = \left[{}_tS^1, {}_tS^2, {}_tS^3, {}_tS^4\right]^T,$$

and $\quad \mathbf{D} = \begin{bmatrix} \mathcal{R} & \mathcal{T} & \mathcal{T} & \mathcal{T} \\ \mathcal{T} & \mathcal{R} & \mathcal{T} & \mathcal{T} \\ \mathcal{T} & \mathcal{T} & \mathcal{R} & \mathcal{T} \\ \mathcal{T} & \mathcal{T} & \mathcal{T} & \mathcal{R} \end{bmatrix}.$

Simple case in 2D



Incident pulse



Scattered pulses from the node

- **Nodal reflection and transmission coefficients:**

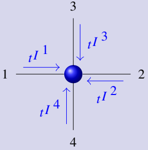
$$\mathcal{R} = \frac{Z_T - Z_L}{Z_T + Z_L}, \quad \mathcal{R} < 0$$

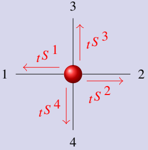
$$\mathcal{T} = 1 + \mathcal{R} = \frac{2Z_T}{Z_T + Z_L}$$

Z_T : impedance of the termination
Z_L : impedance of the incident transmission line

$\left(\text{here, } Z_L{=}Z \text{ and } Z_T{=}Z/3\text{, so } \mathcal{R}{=}-\frac{1}{2} \text{ and } \mathcal{T}{=}\frac{1}{2}\right)$

General case in 2D





- **Matrix relation: $_t\mathbf{S} = \mathbf{D} \times {_t\mathbf{I}}$**

$$\text{where} \quad {_t\mathbf{I}} = \left[{_tI^1}, {_tI^2}, {_tI^3}, {_tI^4}\right]^{\mathrm{T}},$$

$$_t\mathbf{S} = \left[{_tS^1}, {_tS^2}, {_tS^3}, {_tS^4}\right]^{\mathrm{T}},$$

$$\text{and} \quad \mathbf{D} = \begin{bmatrix} \mathcal{R} & \mathcal{T} & \mathcal{T} & \mathcal{T} \\ \mathcal{T} & \mathcal{R} & \mathcal{T} & \mathcal{T} \\ \mathcal{T} & \mathcal{T} & \mathcal{R} & \mathcal{T} \\ \mathcal{T} & \mathcal{T} & \mathcal{T} & \mathcal{R} \end{bmatrix}.$$

Diffusion in the transmission lines network ⇒ **connexion laws**

$$_{t+\Delta_t}I^{1}_{(i,j)} = {}_tS^{2}_{(i-1,j)}$$

$$_{t+\Delta_t}I^{2}_{(i,j)} = {}_tS^{1}_{(i+1,j)}$$

$$_{t+\Delta_t}I^{3}_{(i,j)} = {}_tS^{4}_{(i,j-1)}$$

$$_{t+\Delta_t}I^{4}_{(i,j)} = {}_tS^{3}_{(i,j+1)}$$

Nodal pressure definition

$$_tp_{(i,j)} = \frac{1}{2}\sum_{n=1}^{4} {}_tI^{n}_{(i,j)}$$

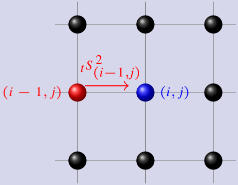
Diffusion in the transmission lines network ⇒ **connexion laws**

$$_{t+\Delta t}I^{1}_{(i,j)} = {}_{t}S^{2}_{(i-1,j)}$$

$$_{t+\Delta t}I^{2}_{(i,j)} = {}_{t}S^{1}_{(i+1,j)}$$

$$_{t+\Delta t}I^{3}_{(i,j)} = {}_{t}S^{4}_{(i,j-1)}$$

$$_{t+\Delta t}I^{4}_{(i,j)} = {}_{t}S^{3}_{(i,j+1)}$$



Nodal pressure definition

$$_{t}p_{(i,j)} = \frac{1}{2}\sum_{n=1}^{4} {}_{t}I^{n}_{(i,j)}$$

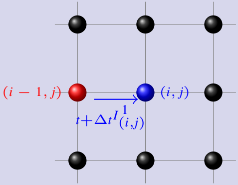
Diffusion in the transmission lines network ⇒ **connexion laws**

$${}_{t+\Delta t}I^{1}_{(i,j)} = {}_{t}S^{2}_{(i-1,j)}$$

$${}_{t+\Delta t}I^{2}_{(i,j)} = {}_{t}S^{1}_{(i+1,j)}$$

$${}_{t+\Delta t}I^{3}_{(i,j)} = {}_{t}S^{4}_{(i,j-1)}$$

$${}_{t+\Delta t}I^{4}_{(i,j)} = {}_{t}S^{3}_{(i,j+1)}$$



Nodal pressure definition

$${}_{t}p_{(i,j)} = \frac{1}{2}\sum_{n=1}^{4} {}_{t}I^{n}_{(i,j)}$$

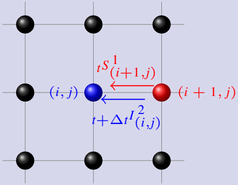
Diffusion in the transmission lines network ⇒ **connexion laws**

$$_{t+\Delta t}I^{1}_{(i,j)} = {}_{t}S^{2}_{(i-1,j)}$$

$$_{t+\Delta t}I^{2}_{(i,j)} = {}_{t}S^{1}_{(i+1,j)}$$

$$_{t+\Delta t}I^{3}_{(i,j)} = {}_{t}S^{4}_{(i,j-1)}$$

$$_{t+\Delta t}I^{4}_{(i,j)} = {}_{t}S^{3}_{(i,j+1)}$$



Nodal pressure definition

$$_tp_{(i,j)} = \frac{1}{2}\sum_{n=1}^{4} {}_tI^n_{(i,j)}$$

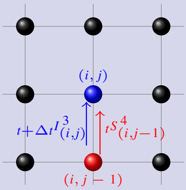
Diffusion in the transmission lines network ⇒ **connexion laws**

$$ {}_{t+\Delta t}I^{1}_{(i,j)} = {}_{t}S^{2}_{(i-1,j)} $$

$$ {}_{t+\Delta t}I^{2}_{(i,j)} = {}_{t}S^{1}_{(i+1,j)} $$

$$ {}_{t+\Delta t}I^{3}_{(i,j)} = {}_{t}S^{4}_{(i,j-1)} $$

$$ {}_{t+\Delta t}I^{4}_{(i,j)} = {}_{t}S^{3}_{(i,j+1)} $$



Nodal pressure definition

$$ {}_{t}p_{(i,j)} = \frac{1}{2}\sum_{n=1}^{4} {}_{t}I^{n}_{(i,j)} $$

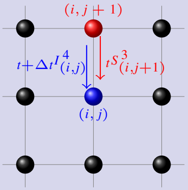
Diffusion in the transmission lines network ⇒ **connexion laws**

$$_{t+\Delta t}I^{1}_{(i,j)} = {}_{t}S^{2}_{(i-1,j)}$$

$$_{t+\Delta t}I^{2}_{(i,j)} = {}_{t}S^{1}_{(i+1,j)}$$

$$_{t+\Delta t}I^{3}_{(i,j)} = {}_{t}S^{4}_{(i,j-1)}$$

$$_{t+\Delta t}I^{4}_{(i,j)} = {}_{t}S^{3}_{(i,j+1)}$$



Nodal pressure definition

$$_{t}p_{(i,j)} = \frac{1}{2}\sum_{n=1}^{4} {}_{t}I^{n}_{(i,j)}$$

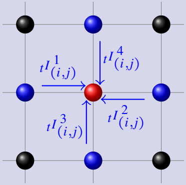
Diffusion in the transmission lines network ⇒ **connexion laws**

$$_{t+\Delta t}I^{1}_{(i,j)} = {}_{t}S^{2}_{(i-1,j)}$$

$$_{t+\Delta t}I^{2}_{(i,j)} = {}_{t}S^{1}_{(i+1,j)}$$

$$_{t+\Delta t}I^{3}_{(i,j)} = {}_{t}S^{4}_{(i,j-1)}$$

$$_{t+\Delta t}I^{4}_{(i,j)} = {}_{t}S^{3}_{(i,j+1)}$$



Nodal pressure definition

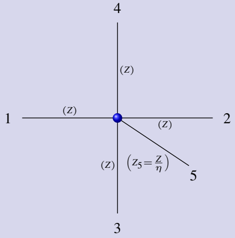
$$_{t}p_{(i,j)} = \frac{1}{2}\sum_{n=1}^{4} {}_{t}I^{n}_{(i,j)}$$

Heterogeneous propagation medium modelling (micrometeorological conditions)

addition of an open-circuited branch, of impedance Z/η, to the nodal original configuration to introduce **refraction** and **turbulence** where the parameter η is calculated by[1]:

$$ {}_t\eta_{(i,j)} = 4\left[\left(\frac{c_0}{{}_tc_{\text{eff}(i,j)}}\right)^2 - 1\right], $$

where $${}_tc_{\text{eff}(i,j)} = \sqrt{\gamma R\, {}_tT_{(i,j)}} + {}_t\mathbf{W}_{(i,j)} \cdot {}_t\mathbf{u}_{(i,j)}.$$

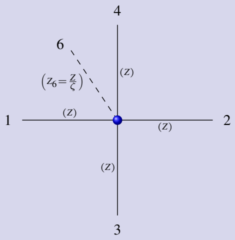


[1] G. Dutilleux. *Applicability of TLM to wind turbine noise prediction*, 2^{nd} *Int. Meeting on Wind Turbine Noise*, Lyon (France), 2007.

Dissipative propagation medium modelling (atmospheric attenuation)

addition of an anechoic terminated branch, of impedance Z/ζ, to the original nodal configuration where the attenuation factor ζ is defined by[1]:

$$ {}_t\zeta_{(i,j)} = -\alpha\sqrt{{}_t\eta_{(i,j)} + 4}\,\Delta l \frac{\ln(10)}{20}, $$



with $\alpha = f\,(\mathrm{T},\mathrm{P_0},\mathrm{H})$ the **atmospheric absorption coefficient** (expressed in $\mathrm{dB.m^{-1}}$) and Δl the spatial step (in m).

[1] J. Hofmann and K. Heutschi. *Numerical simulation of sound wave propagation with sound absorption in time domain*. Appl. Acoust., **68**(2), 2007.

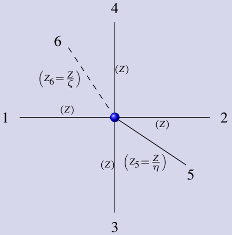
Heterogeneous and dissipative propagation medium modelling

- **Matrix relation:** ${}_t\mathbf{S}_{(i,j)} = {}_t\mathbf{D}_{(i,j)} \times {}_t\mathbf{I}_{(i,j)}$,

where $${}_t\mathbf{I}_{(i,j)} = \left[{}_tI^1 ; {}_tI^2 ; {}_tI^3 ; {}_tI^4 ; {}_tI^5\right]^{\mathrm{T}},$$

$${}_t\mathbf{S}_{(i,j)} = \left[{}_tS^1 ; {}_tS^2 ; {}_tS^3 ; {}_tS^4 ; {}_tS^5\right]^{\mathrm{T}},$$

and $${}_t\mathbf{D}_{(i,j)} = \frac{2}{{}_t\eta_{(i,j)} + {}_t\zeta_{(i,j)} + 4} \, {}_t\begin{bmatrix} a & 1 & 1 & 1 & \eta \\ 1 & a & 1 & 1 & \eta \\ 1 & 1 & a & 1 & \eta \\ 1 & 1 & 1 & a & \eta \\ 1 & 1 & 1 & 1 & b \end{bmatrix}_{(i,j)},$$



with $${}_ta_{(i,j)} = -\left(\frac{{}_t\eta_{(i,j)}}{2} + \frac{{}_t\zeta_{(i,j)}}{2} + 1\right) \quad \text{and} \quad {}_tb_{(i,j)} = \frac{{}_t\eta_{(i,j)}}{2} - \left(\frac{{}_t\zeta_{(i,j)}}{2} + 2\right).$$

- **Connexion laws:** ${}_{t+\Delta t}I^5_{(i,j)} = {}_tS^5_{(i,j)}$

- **Nodal pressure:** ${}_tp_{(i,j)} = \frac{2}{{}_t\eta_{(i,j)} + {}_t\zeta_{(i,j)} + 4}\left(\sum_{n=1}^{4} {}_tI^n_{(i,j)} + {}_t\eta_{(i,j)}\, {}_tI^5_{(i,j)}\right)$

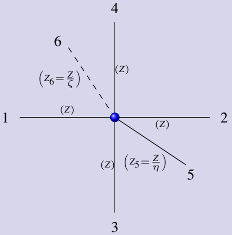
Heterogeneous and dissipative propagation medium modelling

- **Matrix relation:** ${}_t\mathbf{S}_{(i,j)} = {}_t\mathbf{D}_{(i,j)} \times {}_t\mathbf{I}_{(i,j)}$,

where $${}_t\mathbf{I}_{(i,j)} = \left[{}_tI^{\,1};\, {}_tI^{\,2};\, {}_tI^{\,3};\, {}_tI^{\,4};\, {}_tI^{\,5}\right]^{\mathrm{T}},$$

$${}_t\mathbf{S}_{(i,j)} = \left[{}_tS^{\,1};\, {}_tS^{\,2};\, {}_tS^{\,3};\, {}_tS^{\,4};\, {}_tS^{\,5}\right]^{\mathrm{T}},$$

and $${}_t\mathbf{D}_{(i,j)} = \frac{2}{{}_t\eta_{(i,j)} + {}_t\zeta_{(i,j)} + 4}\ {}_t\begin{bmatrix} a & 1 & 1 & 1 & \eta \\ 1 & a & 1 & 1 & \eta \\ 1 & 1 & a & 1 & \eta \\ 1 & 1 & 1 & a & \eta \\ 1 & 1 & 1 & 1 & b \end{bmatrix}_{(i,j)},$$



with $${}_ta_{(i,j)} = -\left(\frac{{}_t\eta_{(i,j)}}{2} + \frac{{}_t\zeta_{(i,j)}}{2} + 1\right) \quad \text{and} \quad {}_tb_{(i,j)} = \frac{{}_t\eta_{(i,j)}}{2} - \left(\frac{{}_t\zeta_{(i,j)}}{2} + 2\right).$$

- **Connexion laws:** ${}_{t+\Delta t}I^{\,5}_{(i,j)} = {}_tS^{\,5}_{(i,j)}$
- **Nodal pressure:** ${}_tp_{(i,j)} = \frac{2}{{}_t\eta_{(i,j)} + {}_t\zeta_{(i,j)} + 4}\left(\sum_{n=1}^{4} {}_tI^{\,n}_{(i,j)} + {}_t\eta_{(i,j)}\ {}_tI^{\,5}_{(i,j)}\right)$

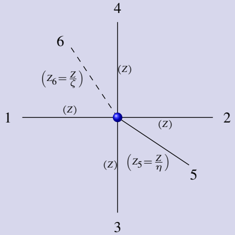
Heterogeneous and dissipative propagation medium modelling

- **Matrix relation:** ${}_t\mathbf{S}_{(i,j)} = {}_t\mathbf{D}_{(i,j)} \times {}_t\mathbf{I}_{(i,j)}$,

where $${}_t\mathbf{I}_{(i,j)} = \left[{}_tI^1\,;\,{}_tI^2\,;\,{}_tI^3\,;\,{}_tI^4\,;\,{}_tI^5\right]^{\mathrm{T}},$$

$${}_t\mathbf{S}_{(i,j)} = \left[{}_tS^1\,;\,{}_tS^2\,;\,{}_tS^3\,;\,{}_tS^4\,;\,{}_tS^5\right]^{\mathrm{T}},$$

and $${}_t\mathbf{D}_{(i,j)} = \frac{2}{{}_t\eta_{(i,j)} + {}_t\zeta_{(i,j)} + 4}\,{}_t\begin{bmatrix} a & 1 & 1 & 1 & \eta \\ 1 & a & 1 & 1 & \eta \\ 1 & 1 & a & 1 & \eta \\ 1 & 1 & 1 & a & \eta \\ 1 & 1 & 1 & 1 & b \end{bmatrix}_{(i,j)},$$



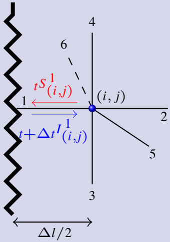
with $${}_ta_{(i,j)} = -\left(\frac{{}_t\eta_{(i,j)}}{2} + \frac{{}_t\zeta_{(i,j)}}{2} + 1\right) \quad \text{and} \quad {}_tb_{(i,j)} = \frac{{}_t\eta_{(i,j)}}{2} - \left(\frac{{}_t\zeta_{(i,j)}}{2} + 2\right).$$

- **Connexion laws:** ${}_{t+\Delta t}I^5_{(i,j)} = {}_tS^5_{(i,j)}$
- **Nodal pressure:** ${}_tp_{(i,j)} = \frac{2}{{}_t\eta_{(i,j)} + {}_t\zeta_{(i,j)} + 4}\left(\sum_{n=1}^{4} {}_tI^n_{(i,j)} + {}_t\eta_{(i,j)}\,{}_tI^5_{(i,j)}\right)$

Wall characterized by a pressure reflection coefficient

- Example: node (i,j) located at the vicinity of a west side wall defined by a pressure reflection coefficient R_1

$$
\begin{aligned}
{}_{t+\Delta t}I^{1}_{(i,j)} &= R_1 \times {}_{t}S^{1}_{(i,j)} \\
{}_{t+\Delta t}I^{2}_{(i,j)} &= {}_{t}S^{1}_{(i+1,j)} \\
{}_{t+\Delta t}I^{3}_{(i,j)} &= {}_{t}S^{4}_{(i,j-1)} \\
{}_{t+\Delta t}I^{4}_{(i,j)} &= {}_{t}S^{3}_{(i,j+1)}
\end{aligned}
$$



- Relation between the pressure reflection coefficient R_1 and the absorption coefficient in energy α_1:

$$\alpha_1 = 1 - |R_1|^2$$

Analogy with the wave equation

- **Combination** of the matrix relation, the connexion laws and the nodal pressure definition:

$$\frac{{}_t\eta_{(i,j)}+4}{2}\frac{\Delta t^2}{\Delta l^2}\overbrace{\frac{{}_{t+\Delta t}p_{(i,j)}-2\,{}_tp_{(i,j)}+{}_{t-\Delta t}p_{(i,j)}}{\Delta t^2}}^{\partial^2_{tt}p_{(i,j)}}+{}_t\zeta_{(i,j)}\frac{\Delta t}{\Delta l^2}\overbrace{\frac{{}_{t+\Delta t}p_{(i,j)}-{}_{t-\Delta t}p_{(i,j)}}{2\,\Delta t}}^{\partial_t p_{(i,j)}}=\underbrace{\frac{{}_tp_{(i+1,j)}-2\,{}_tp_{(i,j)}+{}_tp_{(i-1,j)}}{\Delta l^2}}_{\partial^2_{xx}p_{(i,j)}}+\underbrace{\frac{{}_tp_{(i,j+1)}-2\,{}_tp_{(i,j)}+{}_tp_{(i,j-1)}}{\Delta l^2}}_{\partial^2_{yy}p_{(i,j)}}$$

- **Helmholtz equation in a heterogeneous and dissipative medium**:

$$\left[\Delta+\left(\frac{\omega^2}{c^2_{\mathrm{TLM}}}-j\frac{\omega\,\zeta_{(i,j)}}{c\,\Delta l}\right)\right]P_{(i,j)}=0,\quad c=\frac{\Delta l}{\Delta t}$$

- **Celerity correction**:

$$c_{\mathrm{TLM}}=\sqrt{\frac{2}{{}_t\eta_{(i,j)}+4}}\,c\quad\Rightarrow\quad c=\sqrt{\frac{{}_t\eta_{(i,j)}+4}{2}}\,c_0$$

Analogy with the wave equation

- **Combination** of the matrix relation, the connexion laws and the nodal pressure definition:

$$\frac{{}_t\eta_{(i,j)}+4}{2}\,\frac{\Delta t^2}{\Delta l^2}\,\overbrace{\frac{{}_{t+\Delta t}p_{(i,j)} - 2\,{}_tp_{(i,j)} + {}_{t-\Delta t}p_{(i,j)}}{\Delta t^2}}^{\partial^2_{tt}p_{(i,j)}} + {}_t\zeta_{(i,j)}\,\frac{\Delta t}{\Delta l^2}\,\overbrace{\frac{{}_{t+\Delta t}p_{(i,j)} - {}_{t-\Delta t}p_{(i,j)}}{2\,\Delta t}}^{\partial_t p_{(i,j)}} =$$

$$\underbrace{\frac{{}_tp_{(i+1,j)} - 2\,{}_tp_{(i,j)} + {}_tp_{(i-1,j)}}{\Delta l^2}}_{\partial^2_{xx}p_{(i,j)}} + \underbrace{\frac{{}_tp_{(i,j+1)} - 2\,{}_tp_{(i,j)} + {}_tp_{(i,j-1)}}{\Delta l^2}}_{\partial^2_{yy}p_{(i,j)}}$$

- **Helmholtz equation in a heterogeneous and dissipative medium**:

$$\left[\Delta + \left(\frac{\omega^2}{c^2_{\mathrm{TLM}}} - j\,\frac{\omega\,\zeta_{(i,j)}}{c\,\Delta l}\right)\right] P_{(i,j)} = 0, \quad c = \frac{\Delta l}{\Delta t}$$

- **Celerity correction**:

$$c_{\mathrm{TLM}} = \sqrt{\frac{2}{{}_t\eta_{(i,j)}+4}}\,c \quad \Rightarrow \quad c = \sqrt{\frac{{}_t\eta_{(i,j)}+4}{2}}\,c_0$$

Analogy with the wave equation

- **Combination** of the matrix relation, the connexion laws and the nodal pressure definition:

$$\frac{{}_t\eta_{(i,j)}+4}{2}\frac{\Delta t^2}{\Delta l^2}\overbrace{\frac{{}_{t+\Delta t}p_{(i,j)}-2\,{}_tp_{(i,j)}+{}_{t-\Delta t}p_{(i,j)}}{\Delta t^2}}^{\partial^2_{tt}p_{(i,j)}}+{}_t\zeta_{(i,j)}\frac{\Delta t}{\Delta l^2}\overbrace{\frac{{}_{t+\Delta t}p_{(i,j)}-{}_{t-\Delta t}p_{(i,j)}}{2\,\Delta t}}^{\partial_t p_{(i,j)}}=$$
$$\underbrace{\frac{{}_tp_{(i+1,j)}-2\,{}_tp_{(i,j)}+{}_tp_{(i-1,j)}}{\Delta l^2}}_{\partial^2_{xx}p_{(i,j)}}+\underbrace{\frac{{}_tp_{(i,j+1)}-2\,{}_tp_{(i,j)}+{}_tp_{(i,j-1)}}{\Delta l^2}}_{\partial^2_{yy}p_{(i,j)}}$$

- **Helmholtz equation in a heterogeneous and dissipative medium**:

$$\left[\Delta+\left(\frac{\omega^2}{c^2_{\mathrm{TLM}}}-j\frac{\omega\,\zeta_{(i,j)}}{c\,\Delta l}\right)\right]P_{(i,j)}=0,\quad c=\frac{\Delta l}{\Delta t}$$

- **Celerity correction**:

$$c_{\mathrm{TLM}}=\sqrt{\frac{2}{{}_t\eta_{(i,j)}+4}}\,c\quad\Rightarrow\quad c=\sqrt{\frac{{}_t\eta_{(i,j)}+4}{2}}\,c_0$$

Analogy with the wave equation

- **Combination** of the matrix relation, the connexion laws and the nodal pressure definition:

$$\frac{{}_t\eta_{(i,j)}+4}{2}\frac{\Delta t^2}{\Delta l^2}\overbrace{\frac{{}_{t+\Delta t}p_{\,(i,j)}-2\,{}_tp_{\,(i,j)}+{}_{t-\Delta t}p_{\,(i,j)}}{\Delta t^2}}^{\partial^2_{tt}p_{(i,j)}}+{}_t\zeta_{(i,j)}\frac{\Delta t}{\Delta l^2}\overbrace{\frac{{}_{t+\Delta t}p_{\,(i,j)}-{}_{t-\Delta t}p_{\,(i,j)}}{2\,\Delta t}}^{\partial_t p_{(i,j)}}=$$

$$\underbrace{\frac{{}_tp_{\,(i+1,j)}-2\,{}_tp_{\,(i,j)}+{}_tp_{\,(i-1,j)}}{\Delta l^2}}_{\partial^2_{xx}p_{(i,j)}}+\underbrace{\frac{{}_tp_{\,(i,j+1)}-2\,{}_tp_{\,(i,j)}+{}_tp_{\,(i,j-1)}}{\Delta l^2}}_{\partial^2_{yy}p_{(i,j)}}$$

- **Helmholtz equation in a heterogeneous and dissipative medium**:

$$\left[\Delta+\left(\frac{\omega^2}{c^2_{\text{TLM}}}-j\frac{\omega\,\zeta_{(i,j)}}{c\,\Delta l}\right)\right]P_{(i,j)}=0,\quad c=\frac{\Delta l}{\Delta t}$$

- **Celerity correction**:

$$c_{\text{TLM}}=\sqrt{\frac{2}{{}_t\eta_{(i,j)}+4}}\,c\quad\Rightarrow\quad c=\sqrt{\frac{{}_t\eta_{(i,j)}+4}{2}}\,c_0$$

Room acoustics applications

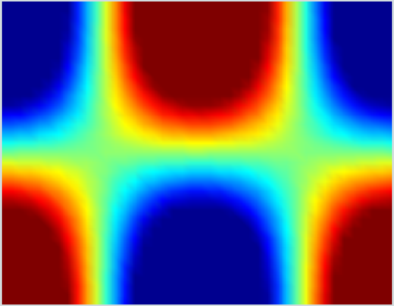


Figure: Eigenmode (2, 1) of a 2D room with perfectly reflecting walls

- dimensions: (6.75 m $\times$ 5.23 m)
- discretization: $\Delta l = 16$ cm and $\Delta t = 0.3$ ms
- sinusoidal source frequency: 60.5 Hz

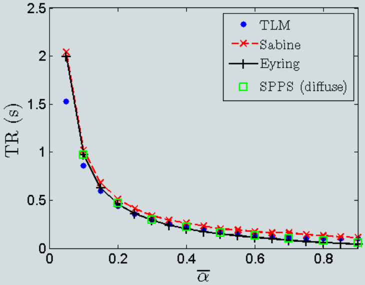
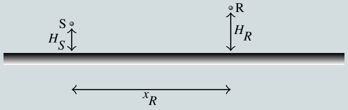


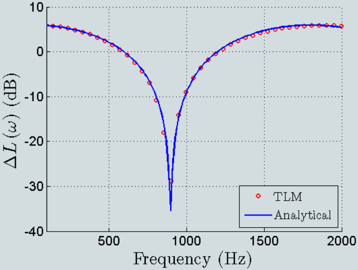
Figure: Reverberation time of a 3D room

- dimensions: (5 m $\times$ 4 m $\times$ 3 m)
- discretization: $\Delta l = 5$ cm and $\Delta t = 8 \times 10^{-5}$ s
- gaussian pulse source frequency: 500 Hz

« Open-space » application



- $H_S = 1$ m, $H_R = 2$ m and $x_R = 20$ m
- perfectly reflective ground
- discretization: $\Delta l = 2$ cm and $\Delta t = 4.1 \times 10^{-5}$ s
- gaussian pulse source frequency: 1500 Hz



Literature study

- few developments
- few validations sometimes limited or even arguable

Thesis contributions

- analytical formulation of a TLM model combining most of the propagative phenomena
- achievement of a generic 2D/3D formulation and numerical implementation
- rigorous validation of the model for academic cases

Main limitations of the model

- no relevant virtual boundary condition formulation in TLM for acoustic modelling
- no realistic boundaries conditions

Literature study

- few developments
- few validations sometimes limited or even arguable

Thesis contributions

- analytical formulation of a TLM model combining most of the propagative phenomena
- achievement of a generic 2D/3D formulation and numerical implementation
- rigorous validation of the model for academic cases

Main limitations of the model

- no relevant virtual boundary condition formulation in TLM for acoustic modelling
- no realistic boundaries conditions

Literature study

- few developments
- few validations sometimes limited or even arguable

Thesis contributions

- analytical formulation of a TLM model combining most of the propagative phenomena
- achievement of a generic 2D/3D formulation and numerical implementation
- rigorous validation of the model for academic cases

Main limitations of the model

- no relevant virtual boundary condition formulation in TLM for acoustic modelling
- no realistic boundaries conditions

Classical impedance boundary condition formulation:

- **in the frequency domain:**

$$P_{(b)}\left(\omega\right) = Z\left(\omega\right) \times V_{n_{(b)}}\left(\omega\right)$$

- **in the time domain:**

$$p_{(b)}\left(t\right) = z\left(t\right) * v_{n_{(b)}}\left(t\right) = \int_{-\infty}^{+\infty} z\left(t'\right) \times v_{n_{(b)}}\left(t - t'\right) dt'$$

where $z\left(t\right) = \mathfrak{F}^{-1}\left[Z\left(\omega\right)\right]$

Necessary conditions to transpose $Z\left(\omega\right)$ in the time domain:[1]

- causality
- passivity
- reality

[1] S.W. Rienstra. *Impedance models in time domain including the extended Helmholtz resonator model.* 12^{th} AIAA/CEAS Conf., Cambridge, Massachusetts (USA), 2006.

Classical impedance boundary condition formulation:

- **in the frequency domain:**

$$P_{(b)}(\omega) = Z(\omega) \times V_{n_{(b)}}(\omega)$$

- **in the time domain:**

$$p_{(b)}(t) = z(t) * v_{n_{(b)}}(t) = \int_{-\infty}^{+\infty} z(t') \times v_{n_{(b)}}(t - t')\, dt'$$

where $z(t) = \mathfrak{F}^{-1}[Z(\omega)]$

Necessary conditions to transpose $Z(\omega)$ in the time domain:[1]

- causality
- passivity
- reality

[1] S.W. Rienstra. *Impedance models in time domain including the extended Helmholtz resonator model.* *12th AIAA/CEAS Conf.*, Cambridge, Massachusetts (USA), 2006.

Classical impedance boundary condition formulation:

- **in the frequency domain:**

$$P_{(b)}(\omega) = Z(\omega) \times V_{n_{(b)}}(\omega)$$

- **in the time domain:**

$$p_{(b)}(t) = z(t) * v_{n_{(b)}}(t) = \int_{-\infty}^{+\infty} z(t') \times v_{n_{(b)}}(t - t')\, dt'$$

where $z(t) = \mathfrak{F}^{-1}[Z(\omega)]$

Necessary conditions to transpose $Z(\omega)$ in the time domain:[1]

- **causality**
- passivity
- reality

[1] S.W. Rienstra. *Impedance models in time domain including the extended Helmholtz resonator model.* *12^{th} AIAA/CEAS Conf.*, Cambridge, Massachusetts (USA), 2006.

Classical impedance boundary condition formulation:

- **in the frequency domain:**

$$P_{(b)}(\omega) = Z(\omega) \times V_{n_{(b)}}(\omega)$$

- **in the time domain:**

$$p_{(b)}(t) = z(t) * v_{n_{(b)}}(t) = \int_{-\infty}^{+\infty} z(t') \times v_{n_{(b)}}(t - t')\, dt'$$

where $z(t) = \mathfrak{F}^{-1}[Z(\omega)]$

Necessary conditions to transpose $Z(\omega)$ in the time domain:[1]

- causality
- **passivity**
- reality

[1] S.W. Rienstra. *Impedance models in time domain including the extended Helmholtz resonator model.* *12th AIAA/CEAS Conf.*, Cambridge, Massachusetts (USA), 2006.

Classical impedance boundary condition formulation:

- **in the frequency domain:**

$$P_{(b)}(\omega) = Z(\omega) \times V_{n_{(b)}}(\omega)$$

- **in the time domain:**

$$p_{(b)}(t) = z(t) * v_{n_{(b)}}(t) = \int_{-\infty}^{+\infty} z(t') \times v_{n_{(b)}}(t - t')\, dt'$$

where $z(t) = \mathfrak{F}^{-1}[Z(\omega)]$

Necessary conditions to transpose $Z(\omega)$ in the time domain:[1]

- causality
- passivity
- **reality**

[1] S.W. Rienstra. *Impedance models in time domain including the extended Helmholtz resonator model.* *12th AIAA/CEAS Conf.*, Cambridge, Massachusetts (USA), 2006.

Impedance representation by a sum of 1st order linear systems[1]

- **in the frequency domain** (response of K linear systems):

$$Z(\omega) = \sum_{k=1}^{K} \frac{A_k}{\lambda_k - j\omega}$$

where λ_k are real poles ($\lambda_k > 0$)

- **in the time domain** (sum of K impulse responses):

$$z(t) = \sum_{k=1}^{K} A_k e^{-\lambda_k t} H(t)$$

where $H(t)$ is the Heaviside function

[1] Y. Reymen *et al.*. *Time-domain impedance formulation based on recursive convolution.* *12th AIAA/CEAS Conf.*, Cambridge, Massachusetts (USA), 2006.

Zwikker and Kosten impedance model application[1]

- **Model expression:**

$$Z(\omega) = Z_\infty \sqrt{\frac{1 + j\omega\tau}{j\omega\tau}} \qquad \text{where} \quad \tau = \frac{\rho_0 q^2 \gamma}{R_S \Omega} \quad \text{a time constant}$$

$$\text{and} \quad Z_\infty = \frac{\rho_0 c_0 q}{\Omega} \quad \text{the impedance at the limit} \quad \omega\tau \rightarrow \infty$$

- **Transposition in the time domain[2]:**

$$z(t) = Z_\infty \left[\delta(t) + \frac{1}{\tau} f(\bar{t}) \right] \qquad \text{where} \quad \bar{t} = t/\tau$$

- **Impulse response approximation $f(\bar{t})$:**

$$f(\bar{t}) = \frac{e^{-\bar{t}/2}}{2} \left[I_1\left(\frac{\bar{t}}{2}\right) + I_0\left(\frac{\bar{t}}{2}\right) \right] H(\bar{t}) = \sum_{k=1}^{K} A_k e^{-\lambda_k \bar{t}} H(\bar{t})$$

[1] C. Zwikker and C. W. Kosten. *Sound absorbing materials*. *Elsevier Ed.*, New York, 1949.

[2] V. E. Ostashev *et al.* *Padé approximation in time-domain boundary conditions of porous surfaces*. *JASA*, **122**(1), 2007.

Time domain impedance boundary condition formulation[1]

$$p\left(m\Delta t\right) = Z'\left[v_n\left(m\Delta t\right) + \sum_{k=1}^{K} A'_k \psi_k\left(m\Delta t\right)\right]$$

where the accumulators ψ_k are given by ($\Rightarrow$ **recursive convolution method**)

$$\psi_k\left(m\Delta t\right) = v_n\left(m\Delta t\right)\frac{\left(1 - e^{-\lambda_k \Delta t'}\right)}{\lambda_k} + e^{-\lambda_k \Delta t'}\psi_k\left(\left(m-1\right)\Delta t\right)$$

- Zwikker and Kosten model: $Z' = Z_\infty$, $A'_k = A_k$ and $\Delta t' = \overline{\Delta t}$
- Miki model[2]: $Z' = Z_0$, $A'_k = \frac{A_k \mu}{\Gamma(-b_M)}$ and $\Delta t' = \Delta t$.

[1] Y. Reymen *et al.* *Time-domain impedance formulation based on recursive convolution*. *12th AIAA/CEAS Conf.*, Cambridge, Massachusetts (USA), 2006.

[2] B. Cotté *Propagation acoustique en milieu extérieur complexe: problèmes spécifiques au ferroviaire dans le contexte des trains à grande vitesse*. *PhD Thesis*, LMFA, École Centrale de Lyon (France), 2008.

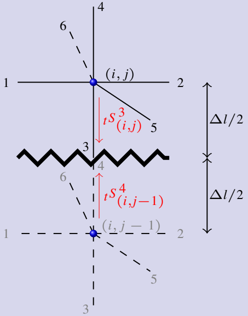
Boundaries modelling in a TLM model: case of the ground

- Introduction of a virtual node
- Boundary **pressure** definition:

$$p\left(t+\frac{\Delta t}{2}\right) = {}_tS^3_{(i,j)} + {}_tS^4_{(i,j-1)}$$

- **Normal particle velocity**:

$$v_n\left(t+\frac{\Delta t}{2}\right) = \frac{{}_tS^3_{(i,j)} - {}_tS^4_{(i,j-1)}}{\rho_0\, c}$$

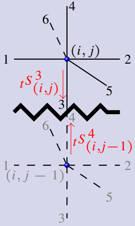


TLM model matching

- **Scattered pulse from the virtual node:**

$$ {}_{t_m}S^{\,4}_{(i,j-1)} = {}_{t_m}S^{\,3}_{(i,j)} \left[\frac{-1+\Lambda_k}{1+\Lambda_k} \right] + \frac{Z'}{1+\Lambda_k} \sum_{k=1}^{K} A'_k e^{-\lambda_k \Delta t'} {}_{t_m-\Delta t}\psi_k $$

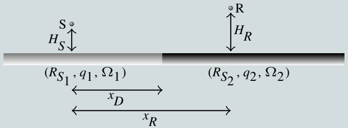
$$ \text{where} \quad \Lambda_k = \frac{Z'}{\rho_0\, c} \left(1 + \sum_{k=1}^{K} A'_k \frac{1-e^{-\lambda_k \Delta t'}}{\lambda_k} \right) $$



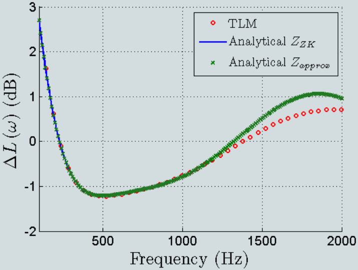
- **Accumulators:**

$$ {}_{t_m-\Delta t}\psi_k = \left(\frac{{}_{t_m-\Delta t}S^3\,(i,j) - {}_{t_m-\Delta t}S^4\,(i,j-1)}{\rho_0\, c} \right) \left(\frac{1-\mathrm{e}^{-\lambda_k \Delta t'}}{\lambda_k} \right) + \mathrm{e}^{-\lambda_k \Delta t'} {}_{t_m-2\Delta t}\psi_k $$

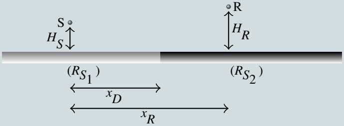
Heterogeneous plane ground: Zwikker and Kosten impedance model



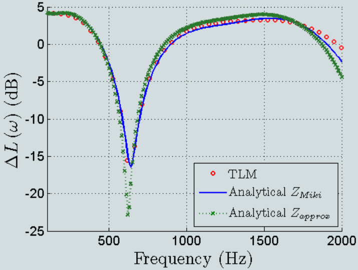
- $H_s = 1$ m, $H_R = 2$ m and $x_R = 20$ m
- discontinuity at $x_D = 10$ m from the source
- $R_{S_1} = 10$ kN.s.m^{-4}, $q_1 = \sqrt{3.5}$ and $\Omega_1 = 0.2$
- $R_{S_2} = 100$ kN.s.m^{-4}, $q_2 = \sqrt{10}$ and $\Omega_2 = 0.5$



Heterogeneous plane ground: Miki impedance model



- $H_S = 1$ m, $H_R = 2$ m and $x_R = 20$ m
- discontinuity at $x_D = 10$ m from the source
- $R_{S_1} = 10$ kN.s.m^{-4}
- $R_{S_2} = 1000$ kN.s.m^{-4}



Absorbing boundaries: application of a « non-reflective » termination

- definition of the pressure field on the limit by a **Taylor series expansion**[1]
- application of a **purely real impedance condition**[2]

Absorbing layers: introduction of an anisotropic absorbing region

- Perfectly Matched Layers (PML)?
- modification of the whole connexion laws for the nodes located inside the layer[3]

[1] S. El-Masri *et al.*. *Vocal tract acoustics using the transmission line matrix (TLM)*. *ICSLP*, Philadelphia (USA), 1996.

[2] J. Hofmann and K. Heutschi. *Numerical simulation of sound wave propagation with sound absorption in time domain*. *Appl. Acoust.*, 2007.

[3] D. De Cogan *et al.*. *Transmission Line Matrix in Computational Mechanics*. *CRC Press*, 2005.

Absorbing boundaries: application of a « non-reflective » termination

- definition of the pressure field on the limit by a Taylor series expansion[1]
- application of a purely real impedance condition[2]

Absorbing layers: introduction of an anisotropic absorbing region

- Perfectly Matched Layers (PML)?
- modification of the whole connexion laws for the nodes located inside the layer[3]

[1] S. El-Masri *et al.*. *Vocal tract acoustics using the transmission line matrix (TLM)*. *ICSLP*, Philadelphia (USA), 1996.

[2] J. Hofmann and K. Heutschi. *Numerical simulation of sound wave propagation with sound absorption in time domain*. *Appl. Acoust.*, 2007.

[3] D. De Cogan *et al.*. *Transmission Line Matrix in Computational Mechanics*. *CRC Press*, 2005.

Absorbing boundaries: application of a « non-reflective » termination

- definition of the pressure field on the limit by a Taylor series expansion[1]
- application of a purely real impedance condition[2]

Absorbing layers: introduction of an anisotropic absorbing region

- Perfectly Matched Layers (PML)?
- modification of the whole connexion laws for the nodes located inside the layer[3]

[1] S. El-Masri *et al.*. *Vocal tract acoustics using the transmission line matrix (TLM)*. *ICSLP*, Philadelphia (USA), 1996.

[2] J. Hofmann and K. Heutschi. *Numerical simulation of sound wave propagation with sound absorption in time domain*. *Appl. Acoust.*, 2007.

[3] D. De Cogan *et al.*. *Transmission Line Matrix in Computational Mechanics*. *CRC Press*, 2005.

Absorbing boundaries: application of a « non-reflective » termination

- definition of the pressure field on the limit by a Taylor series expansion[1]
- application of a purely real impedance condition[2]

Absorbing layers: introduction of an anisotropic absorbing region

- Perfectly Matched Layers (PML)?
- modification of the whole connexion laws for the nodes located inside the layer[3]

[1] S. El-Masri *et al.*. *Vocal tract acoustics using the transmission line matrix (TLM)*. *ICSLP*, Philadelphia (USA), 1996.

[2] J. Hofmann and K. Heutschi. *Numerical simulation of sound wave propagation with sound absorption in time domain*. *Appl. Acoust.*, 2007.

[3] D. De Cogan *et al.*. *Transmission Line Matrix in Computational Mechanics*. *CRC Press*, 2005.

Proposed absorbing layers formulation

- modification of the connexion law only for the incident pulse propagating in the direction of the computational domain limit

De Cogan *et al.* formulation

$$_{t+\Delta t}I^{1}_{(i,j)} = \mathbf{F}_{(\mathbf{i},\mathbf{j})} \times {}_{t}S^{2}_{(i-1,j)}$$
$$_{t+\Delta t}I^{2}_{(i,j)} = \mathbf{F}_{(\mathbf{i},\mathbf{j})} \times {}_{t}S^{1}_{(i+1,j)}$$
$$_{t+\Delta t}I^{3}_{(i,j)} = \mathbf{F}_{(\mathbf{i},\mathbf{j})} \times {}_{t}S^{4}_{(i,j-1)}$$
$$_{t+\Delta t}I^{4}_{(i,j)} = \mathbf{F}_{(\mathbf{i},\mathbf{j})} \times {}_{t}S^{3}_{(i,j+1)}$$

$\Longrightarrow$

Proposed formulation

$$_{t+\Delta t}I^{1}_{(i,j)} = \mathbf{F}_{(\mathbf{i},\mathbf{j})} \times {}_{t}S^{2}_{(i-1,j)}$$
$$_{t+\Delta t}I^{2}_{(i,j)} = {}_{t}S^{1}_{(i+1,j)}$$
$$_{t+\Delta t}I^{3}_{(i,j)} = {}_{t}S^{4}_{(i,j-1)}$$
$$_{t+\Delta t}I^{4}_{(i,j)} = {}_{t}S^{3}_{(i,j+1)}$$

Proposed absorbing layers formulation

- modification of the connexion law only for the incident pulse propagating in the direction of the computational domain limit

De Cogan *et al.* formulation

$$
\begin{aligned}
{}_{t+\Delta t}I^{1}_{(i,j)} &= \mathbf{F}_{(\mathbf{i,j})} \times {}_{t}S^{2}_{(i-1,j)}\\
{}_{t+\Delta t}I^{2}_{(i,j)} &= \mathbf{F}_{(\mathbf{i,j})} \times {}_{t}S^{1}_{(i+1,j)}\\
{}_{t+\Delta t}I^{3}_{(i,j)} &= \mathbf{F}_{(\mathbf{i,j})} \times {}_{t}S^{4}_{(i,j-1)}\\
{}_{t+\Delta t}I^{4}_{(i,j)} &= \mathbf{F}_{(\mathbf{i,j})} \times {}_{t}S^{3}_{(i,j+1)}
\end{aligned}
$$

$\Longrightarrow$

Proposed formulation

$$
\begin{aligned}
{}_{t+\Delta t}I^{1}_{(i,j)} &= \mathbf{F}_{(\mathbf{i,j})} \times {}_{t}S^{2}_{(i-1,j)}\\
{}_{t+\Delta t}I^{2}_{(i,j)} &= {}_{t}S^{1}_{(i+1,j)}\\
{}_{t+\Delta t}I^{3}_{(i,j)} &= {}_{t}S^{4}_{(i,j-1)}\\
{}_{t+\Delta t}I^{4}_{(i,j)} &= {}_{t}S^{3}_{(i,j+1)}
\end{aligned}
$$

Attenuation factor for an absorbing layer of thickness e_{AL}

Looking for a function such as: $\begin{cases} F\left(d_{(i_N,j_N)} = 0\right) = 1 & \text{at the interface} \\ F\left(d_{(i_1,j_1)} = e_{\mathrm{AL}}\right) = \epsilon & \text{on the limit, } \epsilon \in]0,1] \end{cases}$

$$F\left(d_{(i,j)}\right) = (1+\epsilon) - \exp\left[\frac{-\left(d_{(i,j)} - e_{\mathrm{AL}}\right)^2}{B}\right]$$

with $e_{\mathrm{AL}} = \frac{\lambda N_{\lambda_{\mathrm{AL}}}}{\Delta l}$ and $B = -\frac{e_{\mathrm{AL}}^2}{\ln \epsilon}$

Comparison of the virtual boundary conditions efficiency

$$\text{error}\,(x,y) = 10\log_{10} \frac{\sum_{t=0}^{T} \left|p_{\text{ff}}\,(x,y,t) - p\,(x,y,t)\right|^2}{\sum_{t=0}^{T} \left|p_{\text{ff}}\,(x,y,t)\right|^2}$$

Figure: Computational domain

Comparison of the virtual boundary conditions efficiency

$$\text{error}\,(x,y) = 10\log_{10}\frac{\sum_{t=0}^{T}\left|\mathbf{p}_{\text{ff}}\left(\mathbf{x},\mathbf{y},\mathbf{t}\right) - p\left(x,y,t\right)\right|^2}{\sum_{t=0}^{T}\left|\mathbf{p}_{\text{ff}}\left(\mathbf{x},\mathbf{y},\mathbf{t}\right)\right|^2}$$

Figure: Free-field computation

Comparison of the virtual boundary conditions efficiency

$$\text{error}(x,y) = 10\log_{10}\frac{\sum_{t=0}^{T}\left|p_{\text{ff}}(x,y,t) - \mathbf{p}(\mathbf{x},\mathbf{y},\mathbf{t})\right|^2}{\sum_{t=0}^{T}\left|p_{\text{ff}}(x,y,t)\right|^2}$$

Figure: Virtual free-field computation

Comparison of the virtual boundary conditions efficiency

$$\text{error}\left(x,y\right) = 10\log_{10} \frac{\sum_{t=0}^{T} \left| p_{\text{ff}}\left(x,y,t\right) - p\left(x,y,t\right)\right|^2}{\sum_{t=0}^{T} \left| p_{\text{ff}}\left(x,y,t\right)\right|^2}$$

Figure: Virtual boundary conditions efficiency (AL: $N_{\lambda_{\text{AL}}} = 5$ and $\epsilon = 10^{-5}$)

Figure: Virtual free-field computation

Urban application

Figure: Street section

Figure: Sound levels along the receivers axis

Work done

- analytical formulation and numerical implementation of a 2D/3D TLM model integrating most of the propagative phenomena
- improvement of the method
  - matched impedance boundary condition formulation
  - new formulation of absorbing layers
- validation of the model by comparison with analytical and numerical solutions in academic cases (room acoustics, outdoor sound propagation)

Parallel streets geometry (quiet street)

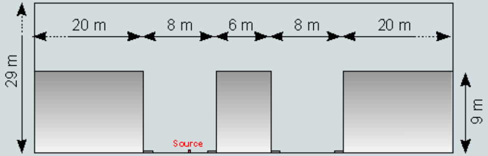


Figure: Gaussian pulse propagation

Urban noise barriers

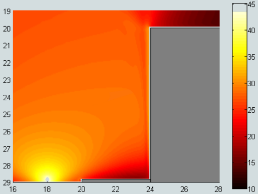


Figure: Without barrier

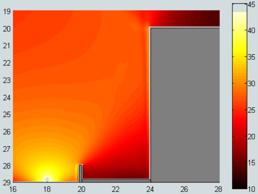


Figure: Green flat barrier

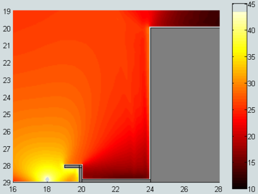


Figure: Perfectly reflective L-shaped barrier

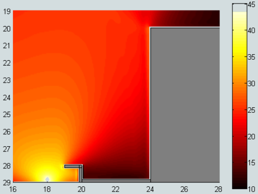


Figure: Green L-shaped barrier

Outlook concerning our contribution

- thickness consideration in the impedance boundary condition
  $\Rightarrow$ coefficients identification in the frequency domain
- rigorous PML formulation for TLM in acoustics

Outlook concerning the TLM model

- atmospheric attenuation frequency dependency
  $\Rightarrow$ digital filters[1]
- sound transmission
  $\Rightarrow$ transmission coefficient
  $\Rightarrow$ wall acoustic propagation modelling
- tetrahedral 3D mesh[2]
  $\Rightarrow$ 3D simulations with 2D cartesian simulations computational burden
- numerical scheme analysis

[1] T. Tsuchiya. *Numerical simulation of sound wave propagation with sound absorption in time domain.* *13th Int. Cong. Sound Vib.*, Vienne, 2006.

[2] S.J. Miklavcic and J. Ericsson *Practical implementation of the 3D tetrahedral TLM method and visualization of room acoustics.* *ITN Resarch Report ISSN*, 2004.

Outlook concerning our contribution

- thickness consideration in the impedance boundary condition

  ⇒ coefficients identification in the frequency domain
- rigorous PML formulation for TLM in acoustics

Outlook concerning the TLM model

- atmospheric attenuation frequency dependency

  ⇒ digital filters[1]
- sound transmission

  ⇒ transmission coefficient

  ⇒ wall acoustic propagation modelling
- tetrahedral 3D mesh[2]

  ⇒ 3D simulations with 2D cartesian simulations computational burden
- numerical scheme analysis

[1] T. Tsuchiya. *Numerical simulation of sound wave propagation with sound absorption in time domain.* *13^{th} Int. Cong. Sound Vib.*, Vienne, 2006.

[2] S.J. Miklavcic and J. Ericsson *Practical implementation of the 3D tetrahedral TLM method and visualization of room acoustics.* *ITN Resarch Report ISSN*, 2004.

Outlook in terms of validation

- micrometeorological conditions implementation
- comparison with experimental results

Outlook in terms of applications

- auralization (soundscape virtual modelling)
- coupling with road trafic models
- ...

Outlook in terms of validation

- micrometeorological conditions implementation
- comparison with experimental results

Outlook in terms of applications

- auralization (soundscape virtual modelling)
- coupling with road trafic models
- ...

Thank you for your attention

This thesis' work is supported by the following organizations' scientific drafts:

- LCPC - Opération 11M061: *« Prévoir le bruit en milieu urbain »* (*« Forecast the noise level in urban environment »*)
- IRSTV CNRS 2488, PRF *« Environnements sonores urbains »* (*« Sound urban environments »*)
- GdR CNRS 2493, thème 2: *« Propagation en espace urbain et en milieu ouvert »* (*« Propagation in urban area and in open-space »*)

Wave propagation equations in absorbing layers

- theoretical wave propagation equation in PML[1]:

$$\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = -\frac{1}{c_0^2} q_x \frac{\partial p}{\partial t} + \rho_0 q_x \frac{\partial u_x}{\partial x} + \rho_0 u_x \frac{\partial q_x}{\partial x}$$

- discrete wave propagation equation obtained with the proposed method:

$$\frac{\Delta t^2}{\Delta l^2}\frac{{}_{t+\Delta t}P_{(i)} - 2\,{}_tP_{(i)} + {}_{t-\Delta t}P_{(i)}}{\Delta t^2} - \frac{{}_tP_{(i+1)} - 2\,{}_tP_{(i)} + {}_tP_{(i-1)}}{\Delta l^2} =$$
$$-F_{(i)}\frac{\Delta t^2}{\Delta l^2}\frac{{}_tP_{(i)} - {}_{t-\Delta t}P_{(i)}}{\Delta t} + \rho_0\,F_{(i)}\frac{{}_tu_{(i+1)} - {}_tu_{(i)}}{\Delta l} + \rho_0\,{}_tu_{(i)}\frac{F_{(i+1)} - F_{(i)}}{\Delta l} + \frac{\Theta}{\Delta l^2}$$

$$\text{with}\quad \Theta = -F_{(i+1)}\left[{}_tS^2_{(i)} - {}_tS^1_{(i)}\right] + \left[F_{(i)} - 1\right]{}_{t-\Delta t}S^1_{(i)} - 2\,F_{(i)}\,{}_tS^2_{(i-1)}$$
$$- \left[F_{(i+1)} - 1\right]{}_{t-\Delta t}S^2_{(i)} + {}_tS^1_{(i)} + {}_tS^2_{(i)} - {}_{t-\Delta t}S^1_{(i)}.$$

[1] Q. Qi and T.L. Geers. *Evaluation of the perfectly matched layer for computational acoustics*. J. Comput. Phys., **139**(1), 1997.