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Modélisation des Relations Spatiales Entre Objets en Mouvement

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Dedication

To my grand father (Late Mian Sandhi) who has a lot of affection for me at my childhood and provide me shelter in lieu of my frivolous childhood and always pray for me —— and advised to be honest

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Declaration

This thesis contains no material which has been accepted for the award of any other degree or diploma in any University, and is less than 100,000 words in length. To the best of my knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference has been made.

Résumé:

les relations spatiales entre les différentes régions d' une image sont utiles pour la compréhension et l'interprétation d'une scène presentée. L'analyse Spatio-temporelle d'une scène implique l'intégration du temps dans des relations spatiales entre les objets en mouvement. Les relations spatio-temporelles sont définies dans un intervalle de temps utilisant la géométrie $3D$ ou l'extension de la géométrie $2D$ à la dimension temporelle. La modélisation des relations spatiales dynamiques prend en compte la position relative des objets et leur relations directionnelles, ceci implique les relations topologiques, directionnelles et de distance. Ces relations sont étendues aux domaine temporel.

Dans notre travail,on décrit une méthode de combinaison l' information topologique et directionnelle où les relations d'Allen floues $1D$ sont appliquées dans le domaine spatial. Cette méthode intègre le flou au niveau des relations. La méthode très gourmande initialement en temps de calcul en raison à l' approximation des objets ainsi qu'à l'algorithme de fuzzification des segments des sections longitudinales a été améliorée et la complexité initialement évaluée à $O(nM\sqrt{M})$ à été améliorée pour être de l'ordre de $O(nNlog(N))$, et ceci en utilisant une approximation polygonale adaptée pour les objets considérés, avec n, M et N représentent respectivement le nombre des directions,de pixels et des sommets des polygones. L'algorithme du fuzzification des segments d'une section longitudinale est quant à lui remplacé par les opérateurs d'agrégation floue. Dans la méthode proposée, Les relations topologiques $2D$ sont représentées par un histogramme. Cette méthode de représentation des relations spatiales a été améliorée. Les relations floues n'étant pas exhaustives et disjoints deux à deux (JEPD), un algorithme de défuzzification des relations spatiales a été proposé pour réaliser un ensemble JEPD de relations spatiales. Cet ensemble de relations spatiales est représenté par un graphe de voisinage où chaque nœud du graphe représente la relation topologique et directionnelle. Cette méthode définit des relations spatio-temporelles en utilisant le modèle de données Espace-Temps. Un ensemble de relations spatio-temporelles est également fourni à l'aide de la stabilité topologique.

Afin de valider le modèle, nous avons développé des applications fondées sur méthode de raisonnement spatio-temporelle proposée. Le raisonnement spatio-temporel a permit la création de tables de composition pour les relations spatiales. La table de composition pour les relations topologiques est structurées en sous-tables. Les entités de ces sous-tables sont liées les unes aux autres par des relations spatiales. Dans une seconde application, nous avons proposé une méthode de prédiction des évènements entre objets en mouvement fondée sur le même raisonnement spatio-temporel. Les objets en mouvement changeant de position à chaque instant, la prédiction de la nouvelle position d'un objet tient compte de l'historique de ces mouvements.

Mots-clés: Les relations spatiales floues, les relations spatio-temporelles, le raisonnement spatio-temporel, mouvement, prédiction.

Title: **Modeling Spatial Relations Between Moving Objects**

Abstract: Spatial relations between different image regions are helpful in image understanding, interpretation and computer vision applications. Spatio-temporal analysis involves the integration of spatial relations changing over time between moving objects of a dynamic scene. Spatio-temporal relations are defined for a selected time interval using $3D$ geometry or extension of $2D$ object geometry to the time dimension with sequence occurrence of primitive events for each snapshot. Modeling dynamic spatial relations takes into account the relative object position and their directional relations, this involves the topological, directional and distance relations and their logical extension to the temporal domain.

In this thesis, a method for combining topological and directional relations information is discussed where $1D$ temporal fuzzy Allen relations are applied in spatial domain. Initially, the method had a high computational cost. This computing cost is due to the object approximation and the algorithm for fuzzification of segments of longitudinal section. The computing time has been decreased from $O(nM\sqrt{M})$ to $O(nNlog(N))$ using polygonal object approximation and fuzzy aggregation operators for segments of a longitudinal section , where n, M and N respectively represents number of directions, pixels and vertices of polygons .

In this method, two dimensional topological relations are represented in a histogram. The representation method for two dimensional spatial relations has been changed. These fuzzy relations are not Jointly Exhaustive and Pairwise Disjoint (JEPD). An algorithm for defuzzification of spatial relations is proposed to realize JEPD set of spatial relations, these JEPD spatial relations are represented in a neighborhood graph. In this neighborhood graph, each node represents the topological and directional relation. This method is further extended for defining spatio-temporal relations using space and time data model, a set of spatio-temporal relations are also elaborated using the stability property in topology.

In an application, a method for spatio-temporal reasoning based on this new model is developed. Spatio-temporal reasoning consist of developing the composition tables for spatial relations. Composition table for topological relations are rearranged into sub-tables. Entities in these sub-tables are related to each other and mathematical rules are defined for composition of spatial relations which elaborate the relation between entities of sub-tables. In another application, we propose a method for motion event predictions between moving objects. It is a similar process to the spatio-temporal reasoning. Dynamic objects occupy different places at different time points, these objects have multiple choices for subsequent positions and a unique history. Prediction about motion events take into account the history of a moving object and predict about the semantics of a motion event.

Keywords: Fuzzy topological relations, spatio-temporal relations and motion events, spatio-temporal reasoning, motion events predictions

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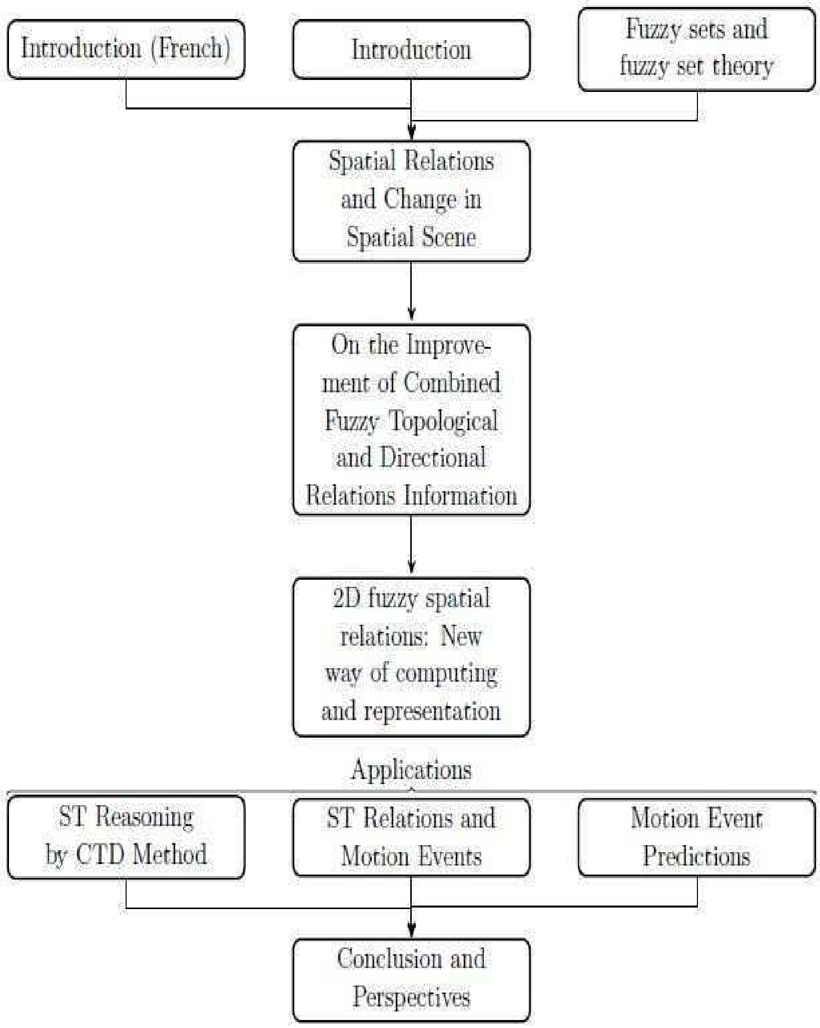
Dissertation Structure Flow Chart:

This dissertation is composed of independent and dependent chapters. The whole dissertation structure are shown in the dependance chart for chapters. This shows that how the chapters depend each other. A vector → shows that a chapter at the tail of vector should be read first and be followed by a chapter at the arrow head. Some chapters can be read parallel, they don't depend on the other chapter. First and second chapters are same and written in French and English language. A reader familiar with fuzzy set theory can skip third chapter, this chapter is related to fuzzy sets theory, operators and fuzzy relations.



Chapter 1

General Introduction

Numerous theories are developed to study the dynamic objects including the spatial relations theory, whereas the relative position of objects is elaborated through the binary spatial relations extended to temporal domain for modeling spatio-temporal relations. Digital data stored in snapshot format and single image analysis is extended to the analysis of sequence images, spatio-temporal phenomena is exploited to understand the dynamic visual world, moving objects and tracking moving objects.

Spatio-temporal relations play an important role in spatio-temporal reasoning, as in path planning task in robotics, cognition, trajectory annotation, automatic video interpretation and video understanding systems. Spatio-temporal relations are also used in modern devices like smart cell phones, hand held sets, wireless modems, Global Positioning Systems (GPS) devices, Temporal Geographic Information System (TGIS) and other new technologies. Effective representation of spatio-temporal data has been an important feature in many TGIS applications, general approach represents the video database in a sequence of scenes or static images. A group of frames, called key frames are selected. A spatio-temporal relation holds between objects if a spatial relation holds for a time duration consisting on durative span of key frames. Modeling these relations involves at least analysis and comparison of two snapshots or frames. Spatio-temporal relations are developed by describing the change in topological and directional relations in a sequential order.

In existing techniques, dynamic objects are represented by points and usually by their geometric center. As a result, some features of the spatio-temporal date is missed. Motion of an extended object is entirely dependant on the motion of its particles. Movement of an object can be modeled continuously when space and time are modeled as the adjacency spaces. In snapshot data models, motion is modeled as a sequence of change in positions, considering conceptual neighborhoods in spatial relations. Identity of objects as well as their topological and metric relations can be exploited to derive changes that occur between two consecutive snapshots. Automatic derivation of change from snapshots can add spatio-temporal reasoning power to GISs. This process will help to replace human intervention in the detection of change, which might consequently result in reduction of costs for parties with an interest in the evolution of spatial phenomena. knowledge about changes be used in methods to identify different types of change and sequential temporal order of spatial relations are used for developing spatio-temporal relations and motion events.

1.1 Moving Objects

An object is under motion if it occupies different positions in space at different times. Movement theory may be incorporate to theories of **time, space, objects and position**. Theory of time articulates the raw duration into time, it is an ordered relation and points are basic entities. In the space theory, standard mathematical tool for analysis is point set topology and length, area and volume are called sets of points. Something is considered as an object, if it has some sense for movement. Objects may be rigid or deformable and objects are modeled as sets. Position theory brings together the object and space theory by providing the means of specifying that where an object is located in space. Depending on how these theories are presented, we might end up with very different theories of movement. Motion is simply a mapping from time to space. Let us consider the theory of spatial relations. In this theory, spatial relations are grouped into three classes, topological, directional and distance relations. In study of moving objects, we study changes in spatial relations and these relations can change simultaneously or separately (see [Ibrahim 2007a, Frank 1992, Davis 2000, Gerevini 2002, Delafontaine 2008, Egenhofer 1992]). Spatial relations for two successive frames are compared and change in these relations is derived, this change is used for extending spatial relations in temporal domain. While describing the dynamic objects, we encounter the several problems like representation of moving objects in space and nature of movement.

1.1.1 Handling Moving Objects

Reasoning can be performed on quantitative as well as qualitative way. Motion seems to be orientation in space-time images, i.e., image sequences. It is a fundamental fact in all theories of motion analysis. The earliest work of modeling spatio-temporal relations appeared within other domains of Artificial Intelligence (AI) and computer science, with a variety of objectives. Research areas like robotics, physical reasoning, computer vision, natural language understanding, Geographic Information Systems(GIS) and Computer-Aided Design (CAD) have contributed to the study of representing and reasoning moving objects with spatial knowledge. A view widely held was that the ontology of space was unproblematic, topology and Euclidean geometry being the only mathematical models to be considered.

The representation of moving objects becomes much more compact as geometries are factored out and represented not in moving objects but once and for all with the network. For example, in the standard model a vehicle moving with constant speed along a road needs a piece of representation with every bend of the road. Some assumptions about moving objects are made in every theory of motion analysis, such as an assumption " moving objects don't change their brightness during movement" in optical flow theory. The theory of positions made following assumptions about moving objects.

- An object must be located at different parts of space at different times.

- A moving object cannot be at different places at the same time unless its spatial size is greater than the interval between places.
- Two objects are at rest relative to each other or absolutely if both objects are at rest.

These limitations don't prevent us from drawing information and conclusions from the data source. We use the change in spatial scene and spatial relations for reasoning the moving objects and modeling spatio-temporal relations.

1.1.2 Types of Movement

Objects may be rigid or deformable. A rigid object maintains its shape and size at all instants, whereas a deformable one can change its shape and size. Objects in nature, such as lakes, islands and forests are considered as non rigid objects. Artificial objects, such as land parcels, cars are considered as rigid objects. Both type of objects change position in space, moving objects can be studies through the spatial relations. Change in binary topological relations describe change in topological structure due to movement and change in directional and distance spatial relations describe only movement. In all the existing techniques, some limited cases for the change in spatial relations are studied due to moving objects. Such as, on road networks where topological relations are supposed to be disjoint and only directional changes are studied [Maeyera 2005, de Weghe 2006].

Movement is categorized into two types, objects moving along the networks and objects moving under certain constraints, where movement is limited in certain directions. This movement is one-dimensional along a path that is embedded into a two-dimensional space. This movement is called **Constraint Movements** and basic geometric entities are points, lines and two-dimensional objects. Objects can move without any constraint on the space. Under this type of movement, the objects move freely in two-dimensional space, this movement of spatial objects is called **Unconstraints movement**.

1.2 Spatial Relations

Spatial logic is more complex than temporal logic because of dimensionality of spatial objects, point of view of objects. Position of an object is specified in terms of topological and directional relations between the object pair for a spatial scene. In literature, many spatial relations like topological and directional relations are found and various theories are developed on this topic. Topology theory is used for modeling the classes of statements with an intuitionistic flavor and topological relations are characterized by the point set topology or Region Connection Calculus (RCC). Directional relations provide information about the spatial arrangement of these classes. These relations are determined by directional algebra, vector calculus or simple geometrical approaches. In this section, various sorts of binary topological and directional relations are discussed between two objects. These methods include the topological relations in $\mathbb{R}$ and $\mathbb{R}^2$ and can be divided into the following four classes between object pair.

Crisp Objects and Crisp Spatial Relations: Contents in an image can be understood with the help of certain properties of space. Topological properties of space provide us information about spatial organization of objects (sets) contained in space. Initially these methods were developed for crisp objects, simple point set topological notations or a pointless topology, Region Connection Calculus (RCC) is used to described these properties of space [Egenhofer 1991, Egenhofer 1993, Cohn 1996]. Binary spatial relations hold between object pair, when the objects are crisp or have determined contour. In such a case, objects are modeled as crisp sets and topological relations are crisp. These set of topological relations and modifications in these methods are discussed in [Du 2005a, Schockaert 2008c]. For modeling the directional relations, point based spaces are studied in [Frank 1996, Frank 1992, li 2008, Skiadopoulos 2007].

Crisp Objects and Fuzzy Spatial Relations: These spatial relations use the exact object models. This sort of methods associate fuzziness with relations and each relation in this class holds with a certain probability. Commonly, these relations represent the class of directional relations such approaches are developed in [Miyajima 1994, Wendling 1998, Bloch 1999, Polkowski 2003, Wang 2003] and some approaches in distance relations like method described in [Bloch 1995]. In topological domains, this class is ignored and much less developed. Main focus of our work is to develop such type of spatial relations.

These spatial relations can be used in many application. As an example, in road networks. Fuzzy topological relations can be used to maintain road side safety and security. A minimum distance is maintained between moving objects (vehicles) on road networks. In this case, a distance based fuzzy meet topological relation can be used to create alerts for vehicle driver to maintain minimum distance. Another example for possible application of a such relations is that the robot must stop at some distance from the artifacts on his path, fuzzy meet relation can be used to create the alert commands. Example for such methods is described in [Matsakis 2005]. Fuzzy spatial algebra is used to model these spatial fuzzy relations between object.

Fuzzy Objects and Crisp Spatial Relations: Fuzzy spatial objects exist in nature like cloud, fog and smoke etc, due to the spatial indeterminacy or fuzziness in the boundary of a spatial object. These objects are modeled as fuzzy object and treated in two ways, modeling fuzzy objects as crisp object using α-cut approach and modeling objects as fuzzy sets [Li 2004, Zhan 1998, Du 2005a, Du 2005b, Shi 2007, Tang 2004]. In this class of spatial relations, fuzzy set based and probabilistic models are used for object modeling. In fuzzy set techniques, object is determined by α-cut or fuzzy segmentation methods are applied. Once the object is determined, then exact object methods are applied to develop spatial relations between these objects. In this method fuzzy objects are due to spatial indeterminacy and three valued logic is used [Roy 2001].

Fuzzy Objects and Fuzzy Spatial Relations: Fuzzy spatial relations are also developed between fuzzy objects, for this, crisp topological methods are extended to deal fuzziness. In this class, objects are modeled using rough set and fuzzy set based approaches, such as concept of broad boundaries [Clementini 1996], egg-yolk model [Cohn 1996] for object representation. Then fuzzy set and fuzzy topological operators are applied to determine the topological relations between objects. In this area significant works are described in [Clementini 1997, Tang 2004, Schockaert 2008b, Liu 2009, Polkowski 2003, Roy 2001]. Different number of fuzzy topological relations are developed using these methods, for example, 44 fuzzy topological relations between simple $2D$ objects are expressed through point set topology and 46 relations are expressed using RCC theory.

1.3 Spatio-Temporal Relations

Dynamic spatio-relations are concerned with spatial relations of moving objects. Modeling the spatio-temporal relations are at the crossroads of AI and logic had been proposed for a long time. Starting from the basic need to deal with the structure of time in computational systems, theories and techniques have been developed that take into account efficiency and the need to integrate time in a more extended reasoning apparatus. Spatio-temporal reasoning will become a key factor in understanding references, retrieving relevant data (figures, video clip fragments, and so on), giving theoretical structure, and presenting the information. Many applications of spatio-temporal relations in computer vision and image and video understanding/interpretation are described in [Bremond 2007].

Modeling spatio-temporal relations is an extension of spatial relations modeling with time. For modeling spatio-temporal relations where topology governs changes, interval temporal logic or point temporal logic is used [Galton 2009, Allen 1983]. In interval temporal logic two-dimensional objects are extended into time and they behave like the volumetric objects and $3D$ topology is used [Muller 2002]. For point temporal logic, $2D$ topological is used and objects occupy different positions of space at each time instant and follows the theory of locations postulates [Gerevini 2002, Erwig 2003]. The method we proposed in chapters 5 and 6, we describe the spatial relationships among objects for a given key frames in a video scene description, where temporal relationships are handled through the logic and spatial relations hold in a sequence for the duration spanning over the key frames.

1.4 Thesis Objective

In applications such as Geographic Information Systems (GIS), Modern data is stored in digital form as snapshots. Initially computer vision researchers focus on single image analysis and exploitation. Research on sequence of images started with the study of motion and moving objects and sequence image analysis allows computers to understand the dynamic world and detect motion type and moving objects.

Modeling spatio-temporal relations is an interdisciplinary subject and spatial relations occur in description of a spatial scene. These spatial relations involve topological, directional and distance relations. These relations have following properties.

- Topological relations are invariant to the topological transformations on space such as translation, rotation, scale, shear.
- Directional relations vary with topological transformations, directional relations change with the rotation, scale and shear transformations.
- Distance relations also variant to the scale and shear transformations.

The objects in a scene are described by a set of spatial adjacent pixels corresponding to objects. Spatial relations are determined when the segmentation and labeling process terminates. Different steps in image analysis are expressed in figure 1.1 and problems at segmentation stage inherit by the spatial data to all the following process. To avoid such problems, we use the sketched images for verifying our method.

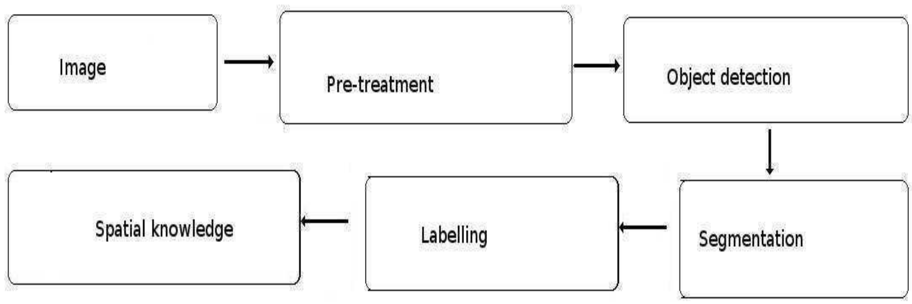


Figure 1.1: Process in image treatment

Modeling spatial relations between fuzzy objects is a slightly different topic from modeling fuzzy spatial relations between objects. For modeling fuzzy spatial relations between objects, we need only contour of objects. Thus a regular closed sets are considered as an object and such objects are taken only by their contours.

A sequence of snapshot, featuring identity states and topological relations of objects, allows an automatic derivation of changes that affect either identity, topology or a combination of identity and topology of objects. Similarly for directional relations. Automatic extraction of feature semantics form motion and provide a base for semantic-based motion detection. For modeling the spatio-temporal relations between moving objects, we need derivation of change in topological and directional relations.

Primary intention of this thesis is to study the moving objects through the spatio-temporal relations and extend the spatial relations theory to theory of spatio-temporal relations. The objective of our study is to develop a domain free method for modeling spatio-temporal relations between moving objects. This include modeling the fuzzy topological and directional relations at the same abstraction level for a snapshot and then extend this logic for modeling the spatio-temporal relations. For this purpose, we choose the method for combined extraction of topological and directional relations, introduced by Matsakis [Matsakis 2005]. This method can't be used for real time applications and modeling spatio-temporal relations due to high computational cost and histogram representation. We work for both the problems, i.e., reduce the computational cost and change the representation of visual results. We extend this method for modeling spatio-temporal relations and we also use this logic for modeling composite events which are the sequence of primitive events. In applications, we use this method for building composition tables of spatial relations used for spatio-temporal reasoning. We also propose a motion event predictions method using the combined topological and directional relations method.

1.5 Short Description of The Dissertation Chapters

This section summaries the whole thesis, in subsections, each following chapter is explained shortly.

1.5.1 Fuzzy Sets and Fuzzy Operators

In this chapter, we describe briefly the basic notation of fuzzy sets and fuzzy operators which are frequently used in the theory of fuzzy sets and fuzzy operators. Fuzzy sets are represented with the help of a fuzzy membership function. Different fuzzy member functions are described along with the fuzzy operators. Fuzzy relations are also expressed by the fuzzy membership functions, these relations are divided into two categories (i) fuzzy relations defined over the pair of crisp sets and (ii) fuzzy relations as a composition of two fuzzy membership functions, in this case sets over which the relations are defined are also fuzzy. The notation of the fuzzy sets and related terms, which are introduced in this chapter, are frequently used in the thesis hereafter. Fuzzy operators are frequently discussed in numerous books related to decision theory, fuzzy knowledge representation and many other related fields of Artificial Intelligence. Readers familiar with these notations can skip this chapter and readers who are new to fuzzy set, have to read this chapter to understand the basic terminology used in this thesis.

1.5.2 Spatial Relations and Change Detection in Spatial Scene

This chapter is concerned with the spatial changes in a spatial scene. These changes can be derived from spatial relations, which incorporate topological and directional relations. Methods for topological relations include the application of Allen relations in spatial domain, 9-intersections and Region Connection Calculus (RCC). Topological relations can be represented in neighborhood graphs and spatial change in topological aspects of space can be derived from graphs. Moving objects and change in their topological relations follow the common sense continuity and all these topological changes are derived by comparing the binary spatial relations at two consecutive states or snapshots. In next sections, methods for directional relations are discussed and possible changes in the directional relations are also described due to moving objects. In this chapter we also discuss different types of fuzziness involved into topological and directional relations and a method for combined fuzzy topological and directional relations.

1.5.3 On the Improvement of Combined Fuzzy Topological and Directional Relations Information

(Pattern Recognition (to be appear) and ICCS-09 [Salamat 2009])
In this part of the thesis, we propose our method for developing fuzzy spatial relations. It discusses in detail the method of combined topological and directional relations information, objects are considered as regular closed sets and these objects are decomposed into parallel segments in a direction. Allen relations are applied to each segment. There can exist multiple segments for a line in one direction called longitudinal section. This is due to two reasons, spatial indeterminacy, fuzziness or object has convex shape or disconnected boundary. This method has the time complexity of $O(nM\sqrt{M})$, where n and M respectively stands for number of directions and number of pixels. Time complexity is due to the algorithm for fuzzification of segments of a longitudinal section.

Longitudinal section is handled in two ways, existence of longitudinal section due to spatial indeterminacy or spatial fuzziness is eliminated by considering the polygonal object approximation and longitudinal section due to disconnected object boundary is coped with the application of fuzzy connectors. Application of fuzzy operators and polygonal object approximation decreases down the computation time for the method from $O(nM\sqrt{M})$ to $O(nNlog(N)$ where N represents the number of vertices of polygons. Affine transformations are also discussed, these properties are helpful for image retrieval in spatial database.

1.5.4 Two-Dimensional Fuzzy Topological Relations: A New Way of Computing and Representation

(Published in IPCV-10 [Salamat 2010b]and HAIS-10 [Salamat 2010a])
Chapter "Two-Dimensional Fuzzy Topological Relations: A New Way of Computing and Representation" concerns with new representation of fuzzy topological and directional relations, previously combined topological and directional relations information are represented in a histogram. One-dimensional topological relations are represented through histograms of fuzzy Allen relations, directional view of two-

dimensional objects for every direction from $[0,\ 2\pi]$. In complex situations it is hard to decide about $2D$ binary topological relation between objects. The histogram is replaced by the matrix representation where limited qualitative directions are represented. We also propose an algorithm for defuzzification of spatial relations, it is helpful to know the $2D$ topological relation between objects with some additional information that in which direction a particular topological relation holds. This provide us a Jointly Exclusive and Pairwise Disjoint (JEPD) topological and directional relation. JEPD topological and directional relations are represented in a neighborhood graph, this entails the common sense reasoning.

1.5.5 Spatio-Temporal Relations and Motion Events

Spatial logic provides us a formal view for any language interpreted over a class of structures featuring geometric entities and relations. Language of spatial logic considers the topological and directional relations as a fundamental entity and a sequence occurrence of these entities over time formulate the spatio-temporal relations. This chapter concerns with the spatio-temporal relations modeling and some visual representation of spatio-temporal events.

In modeling spatio-temporal relations, we use the point interval logic and spatio-temporal relations are considered as the continuous transitions from one snapshot to the next consecutive snapshot. These spatio-temporal relations are defined in the topological viewpoint, topology of space with time, relations are based on the stable and unstable categories. Topologically stable and unstable spatio-temporal relations play an important role in field of Artificial Intelligence (A.I) such as natural language processing and other areas of soft computing. In natural language processing, motion events describe the continuous transitions from one snapshot to other in a temporal ordering. We develop some motion events which are related to the topologically unstable spatio-temporal *Meet* and *Partially_ Overlap* relations.

1.5.6 Spatio-Temporal Reasoning Based on Combined Topological and Directional Relations

(HAL version **hal-00551282**)
Reasoning involves different aspects like common sense reasoning, mathematical reasoning etc. Common sense reasoning and reasoning about human thinking levels pledges to become the fundamental aspects of future knowledge representation techniques and systems, such a system must support a temporal projection. Separate methods for reasoning with topological and directional relations are developed and possible relations predictions are represented into composition tables. These tables consist of 64 entities (8×8 tables) for topological relations and for eight directional relations systems.

Spatial reasoning is more complex than temporal reasoning because more object dimensions, multiple perspective of space are involved. A suitable representation of entities and feasible perspective for automatic spatial reasoning is required. In this chapter, we propose a mechanism for spatio-temporal reasoning based on Combined Topological and Directional (CTD) relations method. Representation of neighborhood graph in our proposed method resembles with the method developed by Daniel Hernández in [Hernández 1991], our method involves some addition information by introducing the directional contents in TPP and $TPPI$ topological relations. In other words Daniel Hernández's approach becomes the refinement of our method. Mathematical formulas are also developed, this reduces the composition table and computational time for developing these composition tables and reasoning process.

1.5.7 Spatio-Temporal Motion Event Predictions

(Published in 4 th International Conference on Pattern Recognition and Machine Intelligence (PReMI-11) [Salamat 2011a])
Spatio-temporal reasoning is a process where subsequent positions or subsequent spatial relations are predicted between an object pair. Reasoning methods are largely developed for topological relations, point or interval temporal logic is used [Allen 1983, Ibrahim 2007a, Galton 2003]. In approaches [Gerevini 2002, Ibrahim 2007b, Cohn 1997, Wolter 2000], topological relations are computed through point set topology, Region Connection Calculus (RCC) and Allen relations [Guesgen 1989], reasoning process supports the common sense reasoning, i.e., method supports the space, time and cognitive representation of entities. A few methods are developed for spatio-temporal reasoning with directional relations [Frank 1992].

Objects in future and history have the similar behavior but they have multiple choices for future and a unique history. Moving objects change the topological, directional or both type of relations at same time. In this chapter we propose a method for spatio-temporal motion event predictions which uses the CTD method for computing static spatial relations. This method keeps into account topological and directional relations information simultaneously and history of spatial relations between moving objects. Motion event prediction is a similar process like the spatio-temporal reasoning, the difference is that this method holds the object history and can predict motion event considering the possible occurrence of a primitive event and its temporal ordering.

1.5.8 Conclusion and Perspectives

Spatio-temporal relations are largely used by research communities in fields of GIS, computer vision and AI. This chapter summarizes the findings of thesis and some outlines for the perspective. We point out possible extension of our work in future in the field of computer vision and image analysis.

Chapter 2

Fuzzy Sets and Fuzzy Operators

In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition. They can be used to represent black and white conceptual thinking. Oftentimes, when something is a member of a given crisp set it is then not a member of any other crisp set.

On the other hand, in 1965, concept of fuzzy sets was introduced by Zadeh [L.A 1965] as an extension of the classical notion of sets. Fuzzy set theory permits gradual assessment of the membership of elements in a set, this is described with the aid of a membership function. Commonly, object classes in physical world don't have precise membership criteria. They have ambiguities in their definitions. Let us consider examples, *class of tall men, class of good cricket players in the world, soup is hot, class of beautiful women, Elvira is blond.* In all these sets, fundamental property is not definitive. In this chapter we recall fundamental properties of fuzzy sets and fuzzy operators.

2.1 Fuzzy Sets

Fuzzy sets are used quite frequently in the real world. Most of the people in everyday thinking and in their linguistic phrases, use the concept of fuzzy sets. These classes don't constitute the exact sets in a usual way in mathematical sense. Such imprecise knowledge plays an important role in human thinking particularly in pattern recognition, artificial intelligence, cognitive sciences, decision theory etc. In this section we denote some basic definitions.

Fuzzy set: A fuzzy set is denoted by an ordered set of pairs. Fuzzy set A in a set X is a function, i.e., it is represented by its membership function $\mu_A : X \to [0, 1]$ such that $A = \{(x, \mu(x))|x \in X)\}$

Fuzzy point: A fuzzy set in X is called a fuzzy point if and only if it takes values 0 for all $x \in X$ except one point. Let $e \in X$ be a fuzzy point if its value at e is α $(0 < \alpha < 1)$. It is denoted by e_α.

Universal fuzzy set: It is a special fuzzy set where membership value is one everywhere in the set, i.e., $\mu(x) = 1 \ \forall\ x \in X$

Support of fuzzy set: The support of a fuzzy set A in X is a crisp set that contains all the elements of X that have nonzero membership values in A, i.e., $supp(A) = \{x \in X|\mu_A(x) > 0\}$. If the support of a fuzzy set is empty, it is called an **empty fuzzy** set.

Core of fuzzy set: Core of a fuzzy set A is the set of all points $x \in X$ such that $\mu_A(x) = 1$. i.e., $Core(A) = \{x|\mu_A(x) = 1\}$

Height of a fuzzy set: The height, $h(A)$, of a fuzzy set A is the largest membership grade obtained by any element in that set. i.e., $h(A) = Sup_{x \in X} A(x)$. A fuzzy set A is called **normal** when $h(A) = 1$, it is called **subnormal** when $h(A) < 1$. A fuzzy set can be normalized using $norm$ function. i.e. $B = norm(A) \Rightarrow \mu_B(x) = \frac{\mu_A(x)}{h(A)}$

Cross over point: The crossover point α of a fuzzy set A in X is the point in X whose membership value in A is 0.5, i.e., $\alpha = \{x|\mu_A(x) = 0.5\}$

Equality: Two fuzzy sets A and B are called equal if they have the equal membership values, i.e., $\mu_A(x) = \mu_B(x), \forall x \in X$

Containment or Subset: Fuzzy set A is contained in fuzzy set B if and only if $\mu_A(x) \leq \mu_B(x)$. $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \forall x \in X$

2.2 Fuzzy Membership Functions

Elements of a fuzzy set are represented by the membership functions. People working in this field developed a number of fuzzy membership functions according to their application. In this section different membership functions and their mathematical formulations are denoted.

S **Shape membership function:** This spline-based curve is a mapping on the vector x, and is named because of its S-shape. The parameters α and β locate the extremes of sloped portion of the curve.

$$\mu(x, \alpha, \beta) = \begin{cases} 0 & \text{if } x \leq \alpha \\ 2(\frac{x-\alpha}{\beta-\alpha})^2 & \text{if } \alpha < x < \frac{\alpha+\beta}{2} \\ 1 - 2(\frac{x-\beta}{\beta-\alpha} & \text{if } \frac{\alpha+\beta}{2} \leq x < \beta \\ 1 & \text{if } \beta \leq x \end{cases} \tag{2.1}$$

where $x, \alpha, \beta \in \mathbb{R}$ and $\alpha < \beta$. This fuzzy membership function is used to modal a step function in the forward sense. This function is explained in figure 2.1(a)

Z **Shape membership function:** This spline-based curve is a mapping on the vector x, and is named because of its z-shape. The parameters α and β locate extremes of the sloped portion of the curve, formally this function is written as

$$\mu(x, \alpha, \beta) = \begin{cases} 1 & \text{if } x \leq \alpha \\ 1 - 2(\frac{x-\beta}{\beta-\alpha} & \text{if } \alpha < x < \frac{\alpha+\beta}{2} \\ 2(\frac{x-\alpha}{\beta-\alpha})^2 & \text{if } \frac{\alpha+\beta}{2} \leq x < \beta \\ 0 & \text{if } \beta \leq \text{ x} \end{cases} \tag{2.2}$$

where $x, \alpha, \beta \in \mathbb{R}$ and $\alpha < \beta$. This fuzzy membership function is used to modal a step function in the backward sense. Graphical representation of this function is shown in figure 2.1(b).

Gaussian membership function: This membership function is mathematically defined as

$$\mu_A(x) = e^{\frac{-(x-\mu)^2}{2\delta^2}} \tag{2.3}$$

where μ and δ respectively represent the center and width of membership function. This fuzzy membership function is used to fuzzify a point. This function is shown in figure 2.1(c).

Sigmoid membership function: This type of membership function is defined as

$$\mu_A(x) = \frac{1}{1 - e^{-\alpha(x-\beta)}} \tag{2.4}$$

Sigmoid membership function can be determined by α which controls slop and β denotes the inflection point of curve. This function resembles with the S shaped fuzzy membership function for positive α and with z shaped membership function for negative value of α. An example is given with $\alpha = 1$ and $\beta = 6$ in figure 2.1(d).

Triangular membership function: Mathematically this function is written as:

$$\mu(x; \alpha, \beta, \gamma) = max(min(\frac{x - \alpha}{\beta - \alpha}, \frac{\gamma - x}{\gamma - \beta}), 0) \tag{2.5}$$

where $x,\ \alpha,\ \beta,\ \gamma\ \in \mathbb{R}$ and $\alpha < \beta < \gamma$ and this fuzzy membership function is used to fuzzify a point. This function is graphically shown in figure 2.1(e).

Trapezoidal membership function: This function can be defined using the max. and min. operators such as

$$\mu(x; \alpha, \beta, \gamma, \delta) = max(min(\frac{x - \alpha}{\beta - \alpha}, 1, \frac{\delta - x}{\delta - \gamma}), 0) \tag{2.6}$$

where $x,\ \alpha,\ \beta,\ \gamma,\ \delta\ \in \mathbb{R}$ and $\alpha < \beta \leq \gamma < \delta$ and this is used to fuzzify an interval. This function is shown in figure 2.1(f). Shape of this function can easily be changed like a triangular, S and Z shaped membership functions by changing parameters $\alpha,\ \beta,\ \gamma,\ \delta$.

Pi-shaped curve membership function: This membership function is defined as

$$\mu(x;\alpha,\beta,\gamma,\delta)=\begin{cases}0 & \text{if } x<\alpha\\ 2(\frac{x-\alpha}{\beta-\alpha})^2 & \text{if } \alpha\le x<\frac{\alpha+\beta}{2}\\ 1-2(\frac{x-\beta}{\beta-\alpha})^2 & \text{if } \frac{\alpha+\beta}{2}\le x<\beta\\ 1 & \text{if } \beta\le x<\gamma\\ 1-2(\frac{x-\delta}{\delta-\gamma})^2 & \text{if } \gamma\le x<\frac{\gamma+\delta}{2}\\ 2(\frac{x-\delta}{\delta-\gamma})^2 & \text{if } \frac{\gamma+\delta}{2}\le x<\delta\\ 0 & \text{if } \delta\le x\end{cases} \tag{2.7}$$

where $x,\alpha,\beta,\gamma\in\mathbb{R}$ and $\alpha<\beta\le\gamma$ and this function is used to fuzzify an interval. This function is represented graphically in figure 2.1(g). Its shape resembles with trapezoidal membership function.

Bell shaped membership function: Generalized bell curve membership function is defined as

$$\mu(x;\alpha,\ \beta,\ \delta)=\frac{1}{1+abs(\frac{x-\delta}{\alpha})^{(2\beta)}} \tag{2.8}$$

where β is a positive number, δ represents the curve center and α determine the curve shape. This membership function is an extension to the Cauchy probability distribution function. Graphically it is represented in figure 2.1(h).

2.3 Operations on Fuzzy Sets

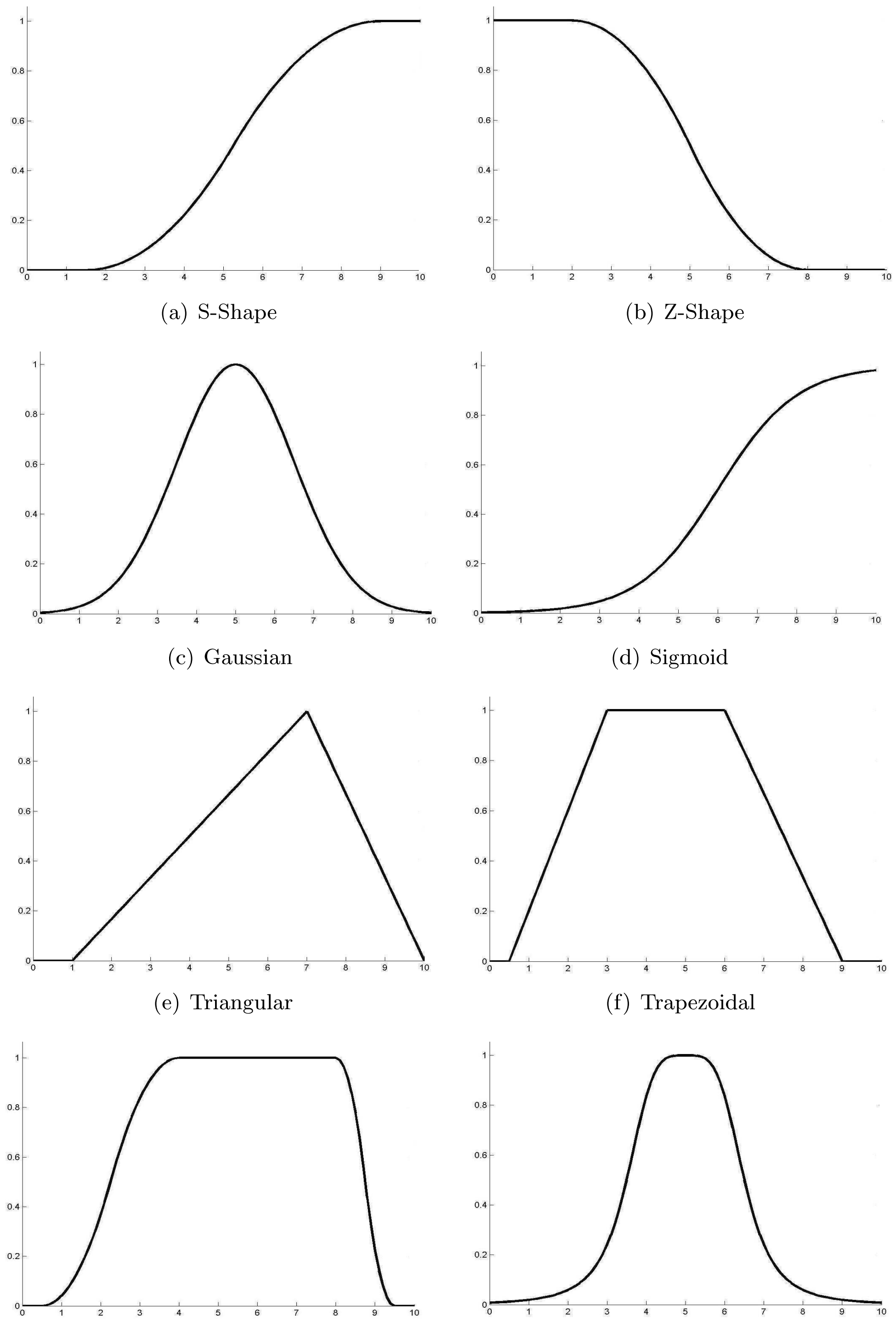
Operations on fuzzy sets are defined via their membership functions. In most of the cases, these operations are extended from the classical set theory. Many sorts of fuzzy set operations are introduced in the fuzzy set theory [L.A 1965], some largely used operators are discussed below.

Union: Union of two fuzzy sets A and B is defined as the maximum of two individual membership functions.
$\mu_{A\cup B}(x)=\{max(A(x),B(x))|\forall x\in X\}=max(\mu_A(x),\mu_B(x))$.

where $\mu_A(x)$ and $\mu_B(x)$ are fuzzy membership functions at x. The union operation in fuzzy set theory is equivalent to OR operation in boolean algebra. It is the smallest fuzzy set containing both of the fuzzy sets A,B.

Intersection: Intersection of two fuzzy sets A, B where $\mu_A(x)$ and $\mu_B(x)$ are fuzzy membership functions at x, is defined as the minimum of the two individual membership functions.
$\mu_{A\cap B}(x)=\{min(A(x),B(x))|\forall x\in X\}=min(\mu_A(x),\mu_B(x))$.

The intersection operation in fuzzy set theory is equivalent to AND operation in boolean algebra. It is the largest fuzzy set contained in fuzzy sets A and B.



(a) S-Shape

(b) Z-Shape

(c) Gaussian

(d) Sigmoid

(e) Triangular

(f) Trapezoidal

(g) Pi-Shape

(h) Bell shape

Figure 2.1: Fuzzy membership functions

Fuzzy Sum: Let $\mu_A(x)$ and $\mu_B(x)$ be two fuzzy membership functions in fuzzy sets A and B. The fuzzy sum in fuzzy sets is defined as

- Fuzzy algebraic sum operator : $\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x)$
- Fuzzy bounded sum: $\mu_{A \oplus B}(x) = min(1, \mu_A(x) + \mu_B(x))$
- Fuzzy symmetric sum: Let M_1, M_2 be called fuzzy symmetric sum, then these operators are defined as
  - $M_1(\mu_A(x), \mu_B(x)) = \frac{\mu_A(x)+\mu_B(x)-\mu_A(x)\mu_B(x)}{1+\mu_A(x)+\mu_B(x)-2\mu_A(x)\mu_B(x)}$
  - $M_2(\mu_A(x), \mu_B(x)) = \frac{\mu_A(x)\mu_B(x)}{1-\mu_A(x)-\mu_B(x)+2\mu_A(x)\mu_B(x)}$

Fuzzy Difference: Let A, B be two fuzzy sets and $\mu_A(x), \mu_B(x)$ are corresponding membership functions. Fuzzy difference is defined in a multiple fashion like

- Fuzzy difference operator: Set difference between crisp sets is defined as

  $$(A - B) = A - B = A \cap B^c$$

  This operator is extended in fuzzy sets as $\mu_{A \cap B}(x) = min(\mu_A(x), \mu_B(x))$. In fuzzy sets, it is represented by AND operator in boolean algebra.
- Absolute difference operator:

  $$\mu_{A \triangledown B}(x) = |\mu_A(x) - \mu_B(x)|$$

  where $\mu_A(x), \mu_B(x)$ are membership values in fuzzy sets A, B.
- Bounded difference operator: This operator is denoted by $(A| - |B)$ and it is defined as

  $$\mu_{A \ominus B}(x) = max(0, (A(x) - B(x))$$

  where $A(x), B(x) \in [0, 1]$
- Fuzzy symmetric difference: Let N_1, N_2 be called fuzzy symmetric differences, then these operators are defined as
  - $N_1(\mu_A(x), \mu_B(x)) = \frac{max(\mu_A(x), \mu_B(x))}{1+|\mu_A(x)-\mu_B(x)|}$
  - $N_2(\mu_A(x), \mu_B(x)) = \frac{min(\mu_A(x), \mu_B(x))}{1-|\mu_A(x)-\mu_B(x)|}$
    where A, B are fuzzy sets with memberships $\mu_A(x)$ and $\mu_B(x)$.

Complement: Complement of a fuzzy set$A$ with membership function $\mu_A(x)$ is defined as the negation of the specified membership function.
$\mu_{\overline{A}}(x) = 1 - \mu_A(x)$

2.4 Properties of Fuzzy Sets

Fuzzy sets have similar properties as the ordinary or crisp sets. Let A, B, C be fuzzy sets, X is a universal fuzzy set and ϕ is an empty fuzzy set, then following properties hold in fuzzy sets.

- Commutativity: $A \cup B = B \cup A$ and $A \cap B = B \cap A$
- Associativity: $A \cup (B \cup C) = (A \cup B) \cup C$ and $A \cap (B \cap C) = (A \cap B) \cap C$
- Idempotency: $A \cup A = A$ and $A \cap A = A$
- Distributivity: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap c)$
- Absorption: $A \cup \phi = A,\ A \cap X = A,\ A \cap \phi = \phi$ and $A \cup X = X$
- Transitivity: $A \subseteq B\ \wedge\ B \subseteq C \Rightarrow\ A \subseteq C$
- De Morgan's law: $\overline{A \cup B} = \overline{A} \cap \overline{B}$ and $\overline{A \cap B} = \overline{A} \cup \overline{B}$. In general De Morgan's laws can be extended to a finite number of sets.
- Involution: This property sometime called double complement. $\overline{\overline{A}} = A$
- Equivalence Formula: $(\overline{A} \cup B) \cap (A \cup \overline{B}) = (\overline{A} \cap \overline{B}) \cup (\overline{A} \cap \overline{B})$

2.5 Fuzzy Logic Connectors

In the decision or choice problem, when solution depends upon synthesizing of information supplied by diverse sources [Herrera 1997], it is more suitable to use fuzzy connectors. Aggregation refers to combine values into one aggregated value so that the final solution seems to be well addressed in a given fashion [Grabisch 1998]. Information obtained from different sources can be combined by using union, intersection and complements. These operators can be modeled by so called *t-norms*, *t-conorms*, fuzzy negation and fuzzy aggregation operators. It is a mapping

$$\tau : [0,1]^n \rightarrow [0,\ 1]$$

Fuzzy logic connectors are divided into three classes.

2.5.1 Conjunctive Operators

The conjunctive operators are generally include intersection of two fuzzy sets A and B, which is specified by a binary operation on the unit interval. It is a function of the form $T : [0,1] \times [0,1] \rightarrow [0,1]$. Let a, b, c, d be fuzzy membership values then fuzzy *t-norms* has the following properties

- Boundary condition: Following are the boundary conditions for operator T
  - $T(a,1) = T(1,a) = a$ and $T(1,1) = 1$
  - $T(0,1) = T(1,0) = 0$
  - $T(a,0) = T(0,a) = 0$
- Monotonicity: $T(a,b) \leq T(c,d)$ if $a \leq c$ and $b \leq d$.

- Commutativity: $T(a,b) = T(b,a)$
- Associativity:$T(a,T(b,c)) = T(T(a,b),c)$

Examples of $t-norms$ operators are AND, PRODUCT etc.

2.5.2 Disjunctive Operators

Commonly used disjunctive operators are triangular co-norms. Like fuzzy intersection, the general fuzzy union of two fuzzy sets A and B is specified by a function $S : [0,1] \times [0,1] \to [0,1]$. The argument to this function is the pair consisting of the membership grades of some element x in fuzzy sets A and B. Fuzzy *s-norms* has the following properties

- Boundary condition: Co-norm operators (*S-norms* denoted by S) have boundary conditions
  - $S(a,1) = S(1,a) = 1$ and $S(1,1) = 1$
  - $S(0,1) = S(1,0) = 1$
  - $S(a,0) = S(0,a) = a$
- Monotonicity: $S(a,b) \leq S(c,d)$ if $a \leq c$ and $b \leq d$.
- Commutativity: $S(a,b) = S(b,a)$
- Associativity:$S(a,S(b,c)) = S(S(a,b),c)$

t-norms and s-norms are dual to each other. Some examples are these operators are OR, Algebraic sum and many others.

2.5.3 Aggregation Operators

These operators are located between the maximum and minimum values, i.e., values of these operators are always between the *t-norms* and *t-conorms*(*s-norms*). Aggregation(averaging) operator have the compensative property. An example for the aggregation operator is γ operator. This function is defined as the combination of algebraic product operator and algebraic sum operator.

$$\mu_\gamma(x) = [\mu_{SUM}(x)]^\gamma \times [(\mu_{PROD}(x)]^{1-\gamma} \tag{2.9}$$

Where $0 \leq \gamma \leq 1$, out put of these operators is always between the maximum and minimum value. These operators are point wise injective, continuous, monotone and commutative. The operator using combination of min, max operators can be written as

$$\mu_1(\mu_A(x), \mu_B(x)) = \gamma \times min(\mu_A(x), \mu_B(x)) + (1-\gamma) \times max(\mu_A(x), \mu_B(x)) \tag{2.10}$$

The above defined operator in Eq. 2.9 can be rewritten as

$$\mu_\gamma(x) = \gamma \times [\mu_A(x).\mu_B(x)] + (1-\gamma) \times [\mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x)] \tag{2.11}$$

There are many aggregation operators defined in literature, the choice of a aggregation operator depends upon the certain criteria like numerical efficiency for computation, adoptability for the semantics interpretation, compensation of the participant operators etc.

2.6 Fuzzy Relations

A crisp relation represents the presence or absence of association, interconnection and interconnectedness between elements of two sets. This concept is generalized and it represent the degree of association or interconnectedness of two sets, called fuzzy relations. Fuzzy relations are fuzzy sets, they have the same set theoretical operations as fuzzy sets. These relations can be divided into two categories such as a fuzzy relation holds between two crisp sets or a fuzzy relation between two fuzzy sets. Let R and R_1 be two fuzzy relation from X to Y represented in example 1 and example 2.

2.6.1 Fuzzy Relations Between Crisp Sets

Let $X, Y \subseteq R$ be two crisp sets. A fuzzy relation $\tilde{R}$ is defined as

$$\mu_{\tilde{R}}: \; X \times Y \longrightarrow [0,\ 1]$$

This relation associates a membership degree to each pair $(x, y) \in X \times Y$. It is called a fuzzy relation between two crisp sets. A fuzzy relation between crisp sets X and Y is a fuzzy subset of $X \times Y$.

Example 1 *Let $X = \{x_1, x_2, x_3, x_4, x_5\}$ and R be a fuzzy relation defined by*

$R =$

	x_1	x_2	x_3	x_4	x_5
x_1	1	0.4	0.8	0.5	0.5
x_2	0.4	1	0.4	0.4	0.4
x_3	0.8	0.4	1	0.5	0.5
x_4	0.5	0.4	0.5	1	0.6
x_5	0.5	0.4	0.5	0.6	1

In this example, sets X, Y are crisp and degree is associated to relation, hence it is a fuzzy relation between crisp sets.

2.6.2 Fuzzy Relations Between Fuzzy Sets

Let X and Y be two non empty sets, $\tilde{A}$ be a fuzzy set on X and $\tilde{B}$ be a fuzzy set on Y, then a mapping $\mu_{\tilde{R}}: [0,\ 1]^2 \longrightarrow [0,\ 1]$ is called a fuzzy relation on fuzzy sets. Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy sets defined as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))|\ x \in X\}$ and $\tilde{B} = \{(y, \mu_{\tilde{B}}(y))|\ y \in Y\}$ be two fuzzy sets then a fuzzy relation $\tilde{R} = \{[(x, y), (\mu_{\tilde{R}}(x, y))]|\ (x, y) \in X \times Y\}$, is a fuzzy relation on $\tilde{A}$ and $\tilde{B}$ if

$$\mu_{\tilde{R}}(x,y) \leq \mu_{\tilde{A}}(x),\ \forall\ (x,y) \in X \times Y \tag{2.12}$$

and

$$\mu_{\tilde{R}}(x,y) \leq \mu_{\tilde{B}}(y),\ \forall\ (x,y) \in X \times Y \tag{2.13}$$

Fuzzy relations are fuzzy sets in product spaces and fuzzy relation between fuzzy sets is simply composition of two fuzzy sets.

Example 2 *Let R_1 be a fuzzy relation on $A \times B$ where $A = \{(3, 0.5), (5, 1), (7, 0.6)\}$ and $B = \{(3, 1), (5, 0.6)\}$ are fuzzy sets, then fuzzy relation R_1 defined as $R_1 = [(A \times B), min(\mu_A(x), \mu_B(x)]$, can be written*

$$R_1 = \left(\begin{array}{c|cc} & 3 & 5 \\ \hline 3 & 0.5 & 0.5 \\ 5 & 1 & 0.6 \\ 7 & 0.6 & 0.6 \end{array}\right)$$

2.6.3 Fuzzy Reflexive Relations

If $\{R(x,x) = 1\ \forall x \in X\}$, then R is called a reflexive (fuzzy) relation. If X is finite and $R = (r_{ij})_{n \times n}$, reflexivity implies that $r_{ii} = 1,\ (i = 1, 2, ..., n)$ and vice versa. As a result, we can observe the numbers on the principal diagonal of R to judge whether R is reflexive or not. In example 1, R is a reflexive because diagonal elements are 1.

2.6.4 Fuzzy Symmetric Relation

A fuzzy relation R is called a symmetric (fuzzy) relation if $\forall\ (x,y) \in X$ and $R(x,y) = R(y,x)$. Obviously, R is symmetric $\Leftrightarrow R = R^{-1}$. We know that $R^{-1} = R^T$ in the case of finite universe. Hence R *is a symmetric relation* if and only if R *as a matrix is symmetric*. In example 1, R is symmetric fuzzy relation($r_{ij} = r_{ji}$). In addition, it is easily checked that $R^2 = R$, and thus R is a fuzzy equivalence relation.

2.6.5 Fuzzy Transitive Relation

A fuzzy relation R is called a fuzzy transitive relation. If $R^2 \subseteq R$, $(x,y,z) \in R$ and $R(x,y) \wedge R(y,z) \Rightarrow R(x,z)$. i.e., R is transitive is and only if $(x,y,z) \in X$, $R(x,z) = R(x,y) \wedge R(y,z)$

2.7 Conclusion

In this chapter we recall some basic definitions of fuzzy sets and fuzzy operators. We also recall the definition of fuzzy relation. These relations are of two types, fuzzy relations between fuzzy sets and fuzzy relations between crisp sets. Through out this dissertation, we shall use fuzzy relations for relations between the crisp objects.

Chapter 3

Spatial Relations and Change Detection in Spatial Scene

Spatio-temporal relations are the spatial relation with time as third dimension, this involves the spatio-temporal object modeling or two-dimensional objects and a logical extension to temporal dimension. Cuboid object approximation or three dimensional geometry is used to model the former and for lateral two-dimensional objects occupy different spatial locations at different time points. Different approaches are used to represent the spatio-temporal data, for example, space-time or space-and-time data model. Changes in object's position often go along with changes to the topology and order information of objects in a spatial scenario. Various models have been proposed for topological and directional relations description of objects in space. The motivation for this chapter is a contribution to the collection of a general approaches that are frequently used for modeling change in a spatial scene. Here we describe the change in spatial relations and nature of object movement in detail.

3.1 Change in Spatial Scene and Its Spatial Relations

Mobile objects are mostly represented by disjoint topological relations. These approaches ignore some aspects of reasoning in the certain domains such as activity recognition in videos, automatic video surveillance, where objects can move within the limited study area. In such a case one object is virtually defined and movement of other object is studied with respect to the virtually defined object.

Events, changes and modeling of spatio-temporal relations has been made in the integration of time within Geographic Information Systems (GIS). GIS often deal with discrete data represented in snapshots so that changes can only be viewed as a sequence of changes in mutual relations between entities. These types of change either preserve topology of a spatial configuration (growing, shrinking and moving), or modify metric properties and topology. Temporal changes of spatial objects induce modifications of their mutual topological and metric spatial relations over time.

3.2 Change in Spatial Scene and Topological Viewpoint

This section explains how the object pair in a spatial scene as well as their binary topological relations can be exploited to derive changes that occur between two consecutive snapshots. Automatic derivation of changes from snapshots can add spatio-temporal reasoning power to GISs. The types of change that can be derived from information available in snapshots.

This section presented to derive changes from snapshots of qualitative spatial data uses spatial scenarios that include two or more spatial regions in one-dimensional or two-dimensional space. First step is to detect and identify specific types of change that affect topological relations of the involved objects in snapshots. Topological relations are derived for regions that undergo such a change. This analysis results in the set of topological relations that objects can have to a reference object. Change derivation from snapshots using qualitative measures promise to infer information that is not directly available. Topological change refers to the motion, shrinking, growing, shape transformation, splitting, merging, disappearing, or reappearing of a spatio-temporal objects. Different theories for describing the topological relations are presented in this section. In the next subsections, we describe theories for topological relations, continuous changes in topological relations with the help of neighborhood graphs to derive the nature of change in spatial scene.

3.2.1 Allen Relations

Allen [Allen 1983] introduced interval algebra, which is extensively used by the artificial intelligence and knowledge representation community. These relations are defined between two intervals of time such that A, and B be two intervals ($A = [x_1, x_2]$, $B = [y_1, y_2]$) where x_1, y_1 are initial and x_2, y_2 are terminal points of intervals. These relations are presented below in table 3.1 where interval B (one-dimensional object) as reference and interval A is considered as argument object.

Allen relation		Inverse	
Before	$x_2 < y_1$	After	$y_2 < x_1$
Meet	$x_2 = y_1$	Meet_by	$y_2 = x_1$
Overlap	$x_1 < y_1 < x_2 < y_2$	Overlapped_by	$y_1 < x_1 < y_2$, $y_2 < x_2$
Start	$x_1 = y_1 \wedge x_2 < y_2$	Started_by	$x_1 = y_1 \wedge y_2 < x_2$
During	$x_1 < y_1 \wedge y_2 < x_2$	During_by	$y_1 < x_1 \wedge x_2 < y_2$
Finishes	$x_1 < y_1 \wedge x_2 = y_2$	Finished_by	$x_1 < y_1$, $x_2 = y_2$
Equals	$x_1 = y_1 \wedge x_2 = y_2$	Equals	$x_1 = y_1 \wedge x_2 = y_2$

Table 3.1: Allen relations and their inverse

These are the 13 Jointly Exhaustive and Pairwise Disjoint (JEPD) interval relations. When directions are neglected, these temporal relations represent the eight topological relations in spatial domain in $\mathbb{R}$.

3.2.2 Conceptual Neighborhood Graph of One-Dimensional Allen Relations and Topological Change

Two relations between pairs of events are conceptual neighbors if they can be directly transformed into one another by continuous deformation (shortening, lengthening or translation) of the events [Freksa 1991]. These conceptual neighbor play an important role for spatio-temporal reasoning and modeling the spatio-temporal relations. These relations are used as the atomic relations in temporal reasoning. Allen relations represent eight topological relations in spatial domain [Allen 1983, Egenhofer 1993].

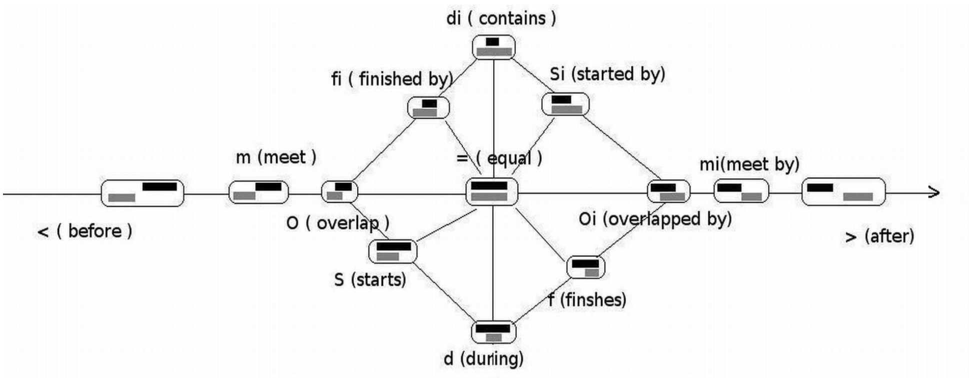


Figure 3.1: Allen relations where black object is reference and light grey object represents argument object.

Allen relations are represented in the neighborhood graph in figure 3.1. In spatial domain neighborhood graphs are used to predicate the topological relations and the change in the topological relation. These changes are categorized into different classes of motion which are summarized in table 3.2.

3.2.3 Two-Dimensional Topological Relations

Binary topological relations in 2D space between objects are richly developed and extensively used by the researchers in different fields of research. Two well-known theories are Regions Connection Calculus (RCC) and the 9-intersections method, are used to develop the topological relations. Both theories are discussed in detail here.

Region Connection Calculus (RCC): In *RCC* theory, a particular subset of topological relations, called *RCC*8 relations is considered and this theory defines the topological configuration between binary objects. These models are obtained from regular connected topological spaces and regions are considered as primitives. The relations defined in *RCC* are based on an atomic relation $C(X,Y)$, read as *"X connected to Y"* [Herring 1994, Randell 1992a, Randell 1992b]. This relation holds when two regions X and Y share a common point.

#	Topological change	Spatial effect
1	$< \leftrightarrow m$	movement
2	$m \leftrightarrow o$	movement
3	$o \leftrightarrow f_i$	movement
4	$f_i \leftrightarrow d_i$	movement
5	$d_i \leftrightarrow s_i$	movement
6	$s_i \leftrightarrow o_i$	movement
7	$o \leftrightarrow =$	movement
8	$= \leftrightarrow o_i$	movement
9	$o \leftrightarrow s$	movement
10	$s \leftrightarrow d$	movement
11	$d \leftrightarrow o_i$	movement
12	$o_i \leftrightarrow m_i$	movement
13	$m_i \leftrightarrow >$	movement
14	$f_i \leftrightarrow =$	Expansion of B or contraction of A
15	$= \leftrightarrow f$	Expansion of B or contraction of A
16	$d_i \leftrightarrow =$	Expansion of B or contraction of A
17	$= \leftrightarrow d$	Expansion of B or contraction of A
18	$s_i \leftrightarrow =$	Expansion of B or contraction of A
19	$= \leftrightarrow s$	Expansion of B or contraction of A

Table 3.2: Allen relations and motion type, A and B are intervals

RCC theory is based on axioms and proximity spaces form a useful intermediatory between topological and axiomatic theories based on contact relations. Eight topological relations, *Disconnected, Externally_connetced, Partially_Overlap, Tangent Proper Part, Non Tangent Proper Part, Tangent Proper Part Inverse, Non Tangent Proper Part Inverse* and *Equal*, are summarized in the table 3.3. There is no particular condition on object conceptualization, objects can be considered as open or closed sets, each time an interpretation is given, called closed and open object interpretation.

- Closed Interpretation: In this interpretation following fundamental concepts are considered
  1. A region is identified with a regular closed set of points
  2. Regions are connected if they share at least one point
  3. Regions overlap if their interiors share at least on point
- Open Interpretation: Postulates for open interpretation are
  1. A region is identified with a regular open set of points
  2. Regions are connected if their closures share at least one point
  3. Regions overlap if they share at least on point

All of the relations described in table 3.3 are defined on three atomic relations:

DC(X,Y)	$\equiv_{def}$	$\neg C(X,Y)$	X is not connected to Y
EQ(X,Y)	$\equiv_{def}$	$P(X,Y) \wedge P(Y,X)$	X coincides Y
PO(X,Y)	$\equiv_{def}$	$O(X,Y) \wedge \neg P(X,Y) \wedge \neg P(Y,X)$	X partially overlaps Y
EC(X,Y)	$\equiv_{def}$	$C(X,Y) \wedge \neg O(X,Y)$	X externally connects with Y
TPP(X,Y)	$\equiv_{def}$	$PP(X,Y) \wedge \exists Z[EC(Z,X) \wedge EC(Z,Y)]$	X is a tangential proper part of Y
NTPP(X,Y)	$\equiv_{def}$	$PP(X,Y) \wedge \neg\exists Z[EC(Z,X) \wedge EC(Z,Y)]$	X is a non tangential proper part of Y
TPPI(X,Y)	$\equiv_{def}$	TPP(Y,X)	X is a tangential proper part inverse of Y
NTPPI(X,Y)	$\equiv_{def}$	NTPP (Y,X)	X is a non tangential proper part inverse of Y

Table 3.3: Topological relation in *RCC*

- C(X,Y) a connection relation.
- $P(X,Y) \equiv_{def} \forall Z[C(Z,X) \rightarrow C(Z,Y)]$ read as X is a part of Y.
- $O(X,Y) \equiv_{def} \exists Z[P(Z,X) \wedge P(Z,Y)]$ read as X overlaps with Y.

The 9-Intersections Method: This theory was initiated by Max J. Egenhofer and R. D. Franzosa [Egenhofer 1991], point set topology is used to explore the topological relations. In this theory, topological parts of an object (objects are modeled as sets where interior, boundary and exterior of a set are called topological parts of sets hence topological parts of objects) participate. A function is defined on these topological parts of both objects and topological structure is studied by the empty ($\emptyset$) and non-empty ($\neg\emptyset$) intersections of topological parts, a topological relation based on these values is defined [Egenhofer 1994].

$$f(A,B) = \begin{cases} \phi \\ \neg\phi \end{cases}$$

Then the relation between two objects is defined as

$$R(A,B) = \begin{pmatrix} f(\dot{A},\dot{B}) & f(\dot{A},\partial B) & f(\dot{A},\overline{B}) \\ f(\partial A,\dot{B}) & f(\partial A,\partial B) & f(\partial A,\overline{B}) \\ f(\overline{A},\dot{B}) & f(\overline{A},\partial B) & f(\overline{A},\overline{B}) \end{pmatrix}$$

where $\dot{A}, \partial A, \overline{A}$ represents respectively interior, boundary and exterior of an object A, similarly for the case of object B. Eight basic topological configurations between $2D$ objects in $\mathbb{R}^2$ are observed out of 512 possibilities. These are similar configurations as in case of *RCC* theory with names *Disjoint, Meet, overlap, Covers, Contain, Inside, Covered_by* and *Equal*. Many extensions have been proposed in the 9-intersections method [Du 2005a, Zhan 1998, Egenhofer 1993, Egenhofer 1995, J. 2001].

3.2.4 Neighborhood Graph of $2D$ Topological relations and Topological Change

Topological relations in $\mathbb{R}^2$ has been explored extensively since a long time and these relations are represented in a neighborhood graph. These neighborhood graphs represent change in binary topological relations between the objects. These relations follow the common sense continuity, for example if two objects are disjoint at time t_1 and at time t_2 they overlap, then there must be a time point t between t_1 and t_2 when both objects have *meet* topological relation. Neighborhood graph of these relations is represented in figure 3.2.

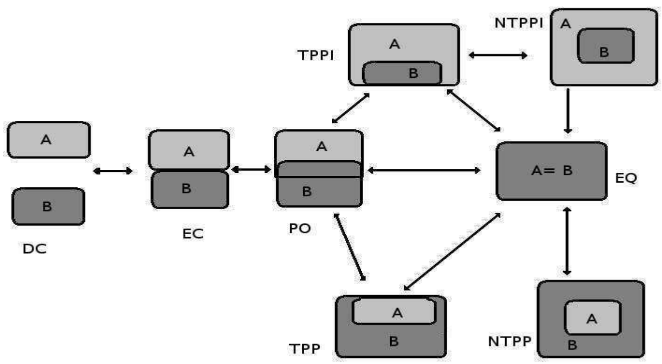


Figure 3.2: Neighborhood graph in 2D topological relations

When these topological relations change, they represent the certain motion types. The change in spatial relation and motion class is represented in table 3.4.

3.3 Fuzziness and Topological Relations

Modeling the fuzzy topological relations involve the extension principle of crisp or classical set theory to the fuzzy set theory. Fuzzy topological relations are developed between the spatial objects where inherit fuzziness holds in the data. This fuzziness is due to the fact that objects don't have the sharp boundary such as clouds, vegetation forest boundaries,etc. In this class of topological relations, problem is to model fuzzy spatial objects by fuzzy set theory. Fuzzy topological relations are also developed between the segmented images, here fuzzy topological relations are developed using extension principle from a crisp relation.

This section consist of two subsections, first we describe fuzziness and extension of crisp sets and crisp functions to fuzzy sets and fuzzy membership functions. Second section describe the existing theories for fuzzy topological relations and their extension to model fuzzy topological relations.

#	Topological change	Spatial effect
1	$D \leftrightarrow EC$	movement
2	$EC \leftrightarrow PO$	movement
3	$PO \leftrightarrow TPP$	movement
4	$TPP \leftrightarrow NTPP$	movement
5	$PO \leftrightarrow EQ$	movement
6	$PO \leftrightarrow TPPI$	movement
7	$TPPI \leftrightarrow NTPPI$	movement
8	$TPP \leftrightarrow EQ$	Expansion of A or contraction of B
9	$EQ \leftrightarrow TPPI$	Expansion of A or contraction of B
10	$NTPP \leftrightarrow EQ$	Expansion of A or contraction of B
11	$EQ \leftrightarrow NTPPI_i$	Expansion of A or contraction of B

Table 3.4: Change in 2D topological relations and motion type, A and B are 2D objects

3.3.1 Fuzziness

Almost all the information, we process in the real applications is incomplete, uncertain and imprecise. Fizziness is present at different levels in acquired data, there are many reasons for this fuzziness. It may be due to the observed phenomena like limited structure or objects or image acquisition process such as limited resolution and numerical reconstruction of images. Fuzziness may be due to the pre-processing steps in image analysis such as in object recognition steps, objects may have the weaker contours like clouds, storms, air pollution or due to the fact that rough segmentation is used. But in spite of all this, impression can also present in relationship semantics like approximately on *Left*, slightly *Right_ of* or it can be found in questions or knowledge representations [Bloch 2005].

In modeling fuzzy spatial relations, fuzziness may be present at object level or at relations level. In modeling fuzzy spatial relations where fuzziness is present in inherit data, fuzzy set theory is used to model the objects and membership functions are used at pixels level. It is a fuzzy membership function μ, which represents the impression at spatial extent of an object. It associate a membership degree to a pixel for which it belongs to object or not. On the other hand, extension principle for a relation is used for modeling fuzzy spatial relations between objects. A crisp and fuzzy relations are represented through a membership functions as

- A crisp binary relation R is a crisp subset of "$O \times O$" ($R \subseteq O \times O$). It is a crisp set and it is described by a membership function as

$$f : O \times O \rightarrow \{0, 1\}$$

$$f(O_i, O_j) = 1 \Leftrightarrow O_i R O_j$$

  (O_i is related to O_j through a relation R)

- A fuzzy membership is used to express the fuzzy relation between pair of objects. In this class of relations, an extension principle is used to extend the definition of relation. It is represented as

  $$f : O \times O \rightarrow [0, 1]$$

  in this definition, a relation holds with a certain probability and it is defined as

  $$0 < f(O_i, O_j) \leq 1 \Leftrightarrow O_i R O_j$$

  (O_i is related to O_j through a fuzzy relation R with a certain degree)

3.3.2 Fuzzy Topological Relations

Topological relations are derived from geometric description. An uncertain relation is a relation which exists with a certain probability. The 9-intersections (3×3) model depends upon the point set topology and topological structure is studied by the empty $(\emptyset)$ and non-empty $(\neg\emptyset)$ intersection of topological parts.

This method is extended to deal with fuzzy objects and 44 useful topological relations between two-dimensional simple fuzzy objects were developed using 9-intersections and objects with extended boundaries [Clementini 1996]. This method is also extended to 16 (4×4) intersections and a set of 152 useful configurations between simple fuzzy regions in $\mathbb{R}^2$ is realized [Tang 2004]. The 9-intersections method deals well with fuzzy objects, the objects where contour of the object is not strong due to the rough segmentation or any sort of noise. In this theory no distance based algebraic function is involved so this theory can't deal fuzziness at relation's level. It means this theory is used to develop the topological relations of type ***crisp objects- crisp topological relations, fuzzy objects- fuzzy topological relations*** and ***fuzzy objects- crisp topological relations***.

On the other hand RCC theory is largely developed to deal with fuzzy objects. In this theory the connection is an atomic relation. The connection relation can be defined by topological operators as well as algebraic operators. i. e.,
$C(X, Y) \equiv_{def} X \cap Y \neq 0$ or $C(X, Y) \equiv_{def} \{d(x, y) = 0, x \in X, y \in Y\}$ It is a distance based function and fuzziness can be introduced at relation's level. This deals with fuzziness at object level and according to the best of our knowledge, S. Schockaert et al. [Schockaert 2008b] have made efforts to extended theory to deal the fuzziness at relation's level. For this purpose he defined a pseudometric space and used the resemblance relation R as fuzzy relation. A fuzzy connection relation is defined as $C(A, B) = A \circ R \circ B$ where A, B are fuzzy regions, α β are real numbers and R is a resemblance relation written as

$$R_{(\alpha,\beta)}(x, y) = \begin{cases} 1 & \text{if } d(x, y) \leq \alpha \\ 0 & \text{if } d(x, y) > \alpha\beta \\ \frac{\alpha+\beta-d(x,y)}{\beta} & \text{otherwise} \end{cases}$$

It is a definition in terms of distance between points x and y, where parameter β defines the smooth transition from closeness to away and α defines that up to how much distance between points, they are considered close. This shows that this theory is used to develop ***crisp objects- crisp topological relations, fuzzy objects-crisp topological relations, fuzzy objects- fuzzy topological relations*** and ***crisp objects- fuzzy topological relations***. Having all this, one basic question remains unanswered, *RCC* theory did not reply that where a topological relation holds in the space.

3.4 Change in Spatial Scene and Order Relations Viewpoint

The moving objects change the spatial configuration of data represented in a snapshot, sometimes these objects change the topological configuration of a snapshot, resulting a change in the binary topological relations between them and sometimes they don't change the topological configuration and they only change order configuration of a data represented in a snapshot. In such a case the angular order relations or ordinal order relations are changed.

3.4.1 Theories to Represent the Angular Order

Order relations describe the position of an object in space with respect to another object. Many methods have been developed to study the binary directional relations between objects. The definition of directional relation concerns a reference object, target object and reference frame. For instance, the description *B north A* indicates that A is the reference object, B the target object, and north is a direction with the reference frame. As a reference frame could be relative or absolute, therefore directional relations can be described in a relative sense or an absolute sense. In a relative sense, such descriptors as front, back, left or right are often in use, while in an absolute sense the terms east, west, south or north are used.

Absolute Reference Frame: Absolute frame of reference is often based on cardinal directions with respect to a local meridian in large-scale spaces. In such a reference frame, compass directions are often used to partition space around a reference object and then to analyze position of the argument object.

Relative Reference Frame: Relative frame of reference use relative orientation in which positioning of a simple object is made with respect to an oriented line or an ordered set of points forming a vector or to some intrinsic properties of the reference object, e.g. front and back, left and right.

The methods for finding the directional relations can be divided into two classes, the crisp methods and fuzzy methods. Some methods from both categories are listed below.

- **Point Based Method:** In point based methods, $2D$ objects are approximated by a point, commonly it is the geometric center of the object. In point based methods, Sometimes one object is considered as point object and then projections are taken, examples are given in [Frank 1996, Moratz 2006]. These methods don't consider the neutral zone.

  These methods express the contextual orientation of a located point with respect to a reference point (both objects are represented by points), as seen from a perspective point. They apply to the reference point a local reference frame in which the frontal direction is fixed by the direction (reference point). In these methods, authors take the relation algebra approach and describe the inferential behavior of the primitive relations in transitivity tables. These methods don't work when the objects are very close to each other

- **Cone-shaped Method:** In this class of methods, two mutually perpendicular lines are drown passing through the geometric center of reference object for four directional systems. Four lines are drawn at slop of $\frac{\pi}{4}$ for eight directional systems. Relations are estimated through the intersection of cone and the argument object. This method is presented in [Peuquet 1987, Abdelmoty 1994].

- **Minimum Bounding Rectangle:** Methods using the minimum bounding rectangle (MBR) represent extended objects by rectangle and then directional relations are determined between them by using the Allen's interval algebra along projections of areas of both objects on the x-axis and y-axis. Example for such a method are presented in [Frank 1996, Sun 2008]. A method where both objects are considered by minimum bounding box, then Allen interval algebra is applied is described in [Papadias 1994]. A set of nine directional relations is released from (13×13) 169 possibilities.

- **Matrix Method:** This is the extension of the minimum bounding box proposed by Roop K. Goyal [Goyal 2001] where the reference object is approximated by its bounding box and then Max J. Egenhofer's [Egenhofer 1991] method of nine intersections is applied. The directional relations matrix will be

$$Dir(A,B) = \begin{pmatrix} NW_A \cap B & N_A \cap B & NE_A \cap B \\ W_A \cap B & O_A \cap B & E_A \cap B \\ SW_A \cap B) & S_A \cap B & SE_A \cap B \end{pmatrix}$$

  where the object$A$ is the reference object and $N_A, W_A, S_A, E_A, NE_A, NW_A$, SW_A and SE_A represents the directional tiles or partitions of space with respect to reference object. O_A represents the central tile, called neutral direction.

- **Internal Cardinal Directional Relations:** Geographical spaces are enough large but in small spaces, internal position of an object within the other object is also important. These objects may represent the case when an object is created through human imagination or convention, such as administrative units of a region. These objects can contain the other objects, to know the position of argument object inside the reference object. For reasoning in such a case Internal Cardinal Directional (ICD) relations method [Liu 2005] was introduced. In such a case, central tile of the matrix method for directional relations is further divided into four, nine or thirteen sub-tiles. Then intersection is taken between the sub-tiles and the argument object.

- **Fuzzy Directional Relations:** Extended objects are used for defining the binary directional relations in above cited methods and these methods ignore the geometric properties of objects. To overcome this drawback fuzzy methods were introduced. In these methods, a degree is associated to each relation, which represents that how much part of an object lies in a given direction. For this purpose different techniques are used, such as mathematical morphology [Bloch 1999], numerical methods such as angle histogram [Miyajima 1994] and force histograms [Matsakis 1999a]. These methods represent that what percentage of an object lies in a particular direction to the reference object.

3.4.2 Directional Relations and Angular Order Change

In MBR and the 9-intersections method for directional relations, region around the reference object is divided into tiles where central tile represents the extended object and then intersections are taken with the tiled region and the argument object. The 9-intersections method is used for deriving motion and trajectory representation of moving objects in [Li 1997]. There doesn't exist particular neighborhood graph.

#	Directional change	Spatial effect
1	$E \leftrightarrow NE$	movement
2	$E \leftrightarrow SE$	movement
3	$E \leftrightarrow O$	movement
4	$NE \leftrightarrow N$	movement
5	$NE \leftrightarrow O$	movement
6	$N \leftrightarrow NW$	movement
7	$N \leftrightarrow O$	movement
8	$NW \leftrightarrow W$	movement
9	$NW \leftrightarrow O$	movement
10	$W \leftrightarrow SW$	movement
11	$W \leftrightarrow O$	movement
12	$SW \leftrightarrow S$	movement
13	$SW \leftrightarrow O$	movement
14	$S \leftrightarrow SE$	movement
15	$S \leftrightarrow O$	movement

Table 3.5: Change in directional relations and motion

In methods, MBR and matrix based possible motion predictions are represented in table 3.5. This table represents the motion transition from one state to the next state, in these predictions O represents the neutral direction.

3.5 Combined Topological and Directional Relations Information

In the existing methods, topological and directional relations are studied separately. Topological relations are described by exact object geometry while in directional relations approximate object geometry is used. For the spatio temporal data, the objects in a snapshot changes their geometry or relative position, as a result their topological, order or both relations at the same time are changed. For example their relations for a snapshot at time t_1 are (α_1, β_1) where α is a topological and β is a directional relation. There are eight possibilities for change in relations for snapshot at time t_2, relations may change to (α_2, β_1), (α_1, β_2) and (α_2, β_2) and there are two possibilities for each α and β. Then we have to apply four different techniques to know the binary spatial relations at t_1 for a snapshot. As distance between the objects is inversely proportional to the angular distribution of objects thus the distance relations then can be discarded. P. Matsakis and Nikitenko [Matsakis 2005] introduced a method where $1D$ Allen relations are applied to a $2D$ object and the results are represented in a histogram. In this method object geometry is not approximated, moreover this method represent the topological and directional relations information at the same abstraction level. This method provide us information that in which direction a particular topological relation holds. Apparently this method has high computational cost and histogram representation.

3.6 Discussion and Conclusion

The 9-Intersections and $RCC8$ methods are basically developed for crisp objects, with the rise of fuzzy sciences and need to handle uncertainty, several extensions has been proposed in these methods. The 9-intersections method deals well with the fuzzy objects in two ways, the 9-intersections method for objects with extended boundaries and 44 binary fuzzy topological relations are released between simple two-dimensional objects. Another extension of the 9-intersections is the 16-intersections method and 152 binary fuzzy topological configurations are explored between simple two-dimensional objects. This theory has been extended through many ways, each time this deals with fuzzy objects and with inherit fuzziness in data due to weak spatial resolution, objects don't have sharp contours like clouds, storms, air pollution etc. Fuzziness in image acquisition process, digital image reconstruction and fuzziness introduced at image pre-processing steps or fuzziness due to rough segmentation. Fuzzy operators are applied to the topological parts of objects. Due to this reason different number of relations between fuzzy objects are released. In this theory, a certain degree cannot be associated to a relation.

On the other hand, *RCC* theory is largely developed and used for binary topological relations between regular objects (objects are modeled as sets and regular open ($A = int(\overline{A})$) and regular closed ($A = \overline{int(A)}$) sets are called regular objects, where $int(A)$ is called interior of set A and $\overline{A}$ is called closure of set A. The *RCC* theory is based on connection ($C(XY)$) relation. This theory deals fuzziness at two levels, fuzzy objects as well as fuzzy connection relation and 46 possible fuzzy topological relations are developed between fuzzy objects. Fuzzy connection relation is based on nearness, in such a case degree of fuzziness is associated to the relation [Schockaert 2008b] and possible number of relations between two-dimensional objects remains eight. This extension of topological relations represent the fuzziness in relations semantics and question's level. For example a mobile robot must stop at a certain distance from the other object for security, but as question we describe that mobile robot continue motion until meet topological relation holds between the mobile robot and the other object. Having all this, one basic question remains unanswered, *RCC* theory did not reply that where a topological relation holds in the space. The methods for the directional relations work autonomously (both topological and directional relations methods are independent from each other).

For the spatio-temporal relations we need topological, directional and distance relations at each snapshot, then for the next snapshot all these relations are calculated then compared for deriving spatial change. In qualitative methods (methods which provide yes or no answer. They don't provide answer about the quantity for example, consider the overlapping objects, these methods provide answer about the topological relation *overlap* but don't answer that what portion of two objects *overlap*), one have to apply the four methods simultaneously, one method for each binary topological, directional, internal cardinal directions and distance relations.

To extract all this information, a combined topological and directional relations information method introduced by Matsakis and Nikitenko [Matsakis 2005] can be used. This method represent both topological and directional relations information on the same abstraction level and provide us information that where in space a topological relation holds between object pair. Initially this method has high computational cost and histogram representation but for modeling the spatio-temporal relations between moving objects, we need definitive topological and directional relation between the binary objects. Due to this reason, histogram representation can't be used for extending this type of spatial relations to spatio-temporal relations. In this thesis, we work for the temporal complexity and change its representation. We also introduce an algorithm for defuzzification of spatial relations, this provide us JEPD set of topological and directional relations and these relations are used for modeling spatio-temporal relations.

Chapter 4

On the Improvement of Combined Fuzzy Topological and Directional Relations Information

Abstract

Concept of combined extraction of topological and directional relations information developed by E. Zahzah et al. [Zahzah 2002] by employing the Allen's temporal relations in $1D$ spatial domain was improved by Matsakis and Nikitenko [Matsakis 2005]. This latter algorithm has high computational complexity due to its limitations of object approximation and segment fuzzification.

In this paper, fuzzy Allen relations are used to define the fuzzy topological and directional relations information between different objects. Some extended results of N. Salamat and E. Zahzah [Salamat 2009] are discussed. Polygonal object approximation allows us to use fuzzy operators and this approach reduces computational complexity of the method for computing the combined topological and directional relations. To validate the method, some experiments are tested giving satisfactory and promising results. Affine transformation are depicted, these properties will be helpful for using the method in other areas of image analysis such as object tracking.

keyword: Fuzzy Allen relations, Topological and directional relations information, Computational complexity, Fuzzy operators.

4.1 Introduction

One of the fundamental tenet of modern sciences is that a phenomenon can't be claimed to be well understood, until it is characterized in quantitative manners. Advent of digital computers have determined an expansion in the use of quantitative methods where as vagueness and imprecise knowledge information give rise to fuzzy methods. Spatial relations belong to these categories. Spatial relations are used for content based image retrieval (CBIR), pattern recognition, database management, artificial intelligence (AI), cognitive science, perceptual psychology, robotics, linguistics expressions, medical imaging, image and video analysis [Millet 2005, Bloch 2003, Egenhofer 1992, Egenhofer 1993, Goyal 2001]. Reasoning on spatial objects needs support from representation of topological, directional, ordinal, distance relations, object size and shape.

Topological relations are derived from geometric descriptions, while an uncertain relation is a relation which exists with a certain probability. The well-known models for finding topological relations between spatial regions are the 9-intersections [Herring 1994] and Region Connection Calculus (RCC) model [Randell 1992b].

These theories are extended to deal with fuzzy objects [Liu 2009, Palshikar 2004, Shi 2007], in both theories, objects are modeled as fuzzy sets in $\mathbb{R}^2$. In 9-intersections method, 44 useful topological relations were developed between simple fuzzy regions. This method is also extended to 16 intersections and a set of 152 useful topological configurations between simple fuzzy regions in $\mathbb{R}^2$ are realized [Tang 2004]. This theory depends upon point set topology and simple set operators are used for modeling topological relations. This theory can't be extended to deal fuzziness at relation's level. In RCC theory number of fuzzy relations are 46. All of these relations represent the fuzziness at object's level. Fuzziness may be present in relation's semantic, in this theory this effort is made by S. Schockaert et al. [Schockaert 2008b] where connection relation based on closeness is defined. Different other approaches are adopted for representing the positional information as distance and orientation.

The methods for directional relations represent such information and fuzziness in relation's semantic are discussed in [Bloch 1999, Schockaert 2008d, Miyajima 1994, Wang 2003, Matsakis 1999b]. Most of these methods work only for a particularly *disjoint* topological relation and stop working as soon as objects *meet* or in some cases *overlap*. In the methods based on force histograms [Matsakis 1999b], $2D$ areal objects are represented by union of $1D$ segments.

The 9-intersections model for topological relations and the RCC theory have a rich support for the topological relations [Egenhofer 1991, Randell 1992a] and provide information about a topological relation without providing information that where a topological relation exists in the space? Consider for instance that two objects overlap, both of theories provide information about the topological relation overlap, but they don't give any information about where in space the two objects overlap. As object A overlaps object B from north or north_west or west, or other topological relation, object A disjoint from object B and lies in south of object B. To know the relative position of object A according to object B, we have to apply another method type, because qualitative methods provide information regarding the extended objects and they don't care about the topological relation.

Allen [Allen 1983] introduced 13 interval relations in temporal domain. These relations are commonly used to represent the knowledge in artificial intelligence. Different approaches for fuzzification of Allen temporal relations are proposed such as in [Schockaert 2005]. These temporal Allen relations are applied into the spatial domain due to homeomorphism between the temporal domain and $1D$ spatial domain. Allen relations represent the topological relations in $\mathbb{R}$. These relations are used to answer the question that where in space a certain topological relation holds?

Allen relations are applied in a $2D$ space by decomposition of a two-dimensional object into $1D$ parallel segments. This decomposition process is repeated in all directions and method is applied to get combined topological and directional relations data. Combined topological and directional relations were first introduced by J. Malki et al. [Zahzah 2002] by using the $1D$ Allen relations. Allen relations divide the whole space into 13 partitions and each partition corresponds to an Allen relation. This method was further improved by Matsakis and Nikitenko [Matsakis 2005] and fuzzification was introduced at relations level.

This latter method is costly regarding time constraint. The aim of this paper is to reduce time consuming for the computation of combined fuzzy Allen relations information developed in [Matsakis 2005]. Time is reduced in two ways, reducing and simplifying the number of computations and suggest an alternative way for the treatment of longitudinal section. As time constraint depends upon number of segments to be treated, to reduce the number of segments a polygonal object approximation is used. Algorithm for fuzzification of longitudinal section is replaced by fuzzy connectors. This approach decreases its time complexity from $O(nM\sqrt{M})$ to $O(nNlog(N))$, where n represents number of directions, N number of vertices of polygons and M number of pixels to be processed

This paper is arranged as follows, next section describes some preliminary definitions. Section 4.2 describes the terminology used, Allen relations in 1D space, fuzzification of Allen relations and finally the definition of a histogram of fuzzy Allen relations. Fuzzy logic connectors are discussed in section 4.3, these fuzzy operators are used for the treatment of a longitudinal section. Different affine transformations are discussed in section 4.4, experiments and comparison of results with other methods are described in section 4.5. Time complexity for both methods is compared in section 4.6, section 4.7 concludes the paper.

Preliminary Definitions

In this section we recall some basic definitions which are frequently used throughout the remainder of the paper.

Fuzzy membership function A membership function μ in a set X is a function $\mu : X \rightarrow [0, 1]$. Different fuzzy membership functions are proposed according to the requirements of the applications. For instance, Trapezoidal membership function is defined as

$$\mu(x; \alpha, \beta, \gamma, \delta) = max(min(\frac{x-\alpha}{\beta-\alpha}, 1, \frac{\delta-x}{\delta-\gamma}), 0) \tag{4.1}$$

it is written as $\mu_{(\alpha,\beta,\gamma,\delta)}(x)$ where $x, \alpha, \beta, \gamma, \delta \in \mathbb{R}$ and $\alpha < \beta \leq \gamma < \delta$.

Fuzzy set A fuzzy set A in a set X is a set of pairs $(X, \mu(x))$such that $A = \{(x, \mu(x)|x \in X)\}$ where μ represents a membership function.

Force histogram The force histogram attaches a weight to the argument object A that this lies *after* B in direction θ, it is defined as

$$\mathbf{F}^{AB}(\theta) = \int_{-\infty}^{+\infty} F(\theta, A_\theta(v), B_\theta(v))dv \tag{4.2}$$

The definition of Force histogram $\mathbf{F}^{AB}(\theta)$, directly depends upon the definition of real valued functions ϕ, f and F used for the treatment of points, segments and longitudinal sections respectively [Matsakis 1999b]. These functions are defined as

$$\left.\begin{aligned} \phi_r(y) &= \begin{cases} \frac{1}{y^r} & \text{if } y > 0 \\ 0 & \text{otherwise} \end{cases} \\ f(x_I, y^\theta_{IJ}, z_J) &= \int_{x_I+y^\theta_{IJ}}^{x_I+y^\theta_{IJ}+z_J} \int_0^{z_J} \phi(u-w)dw)du \\ F(\theta, A_\theta(v), B_\theta(v)) &= \sum_{i=1..n, j=1..m} f(x_{Ii}, y^\theta_{IiJj}, z_{Jj}) \end{aligned}\right\} \tag{4.3}$$

where n, m represents the number of segments of object A and object B respectively and variables (x, y, z) are explained in Fig. 4.1.

4.2 Histograms of Fuzzy Allen Relations

In this section, certain terms used for computations are explained. Drawing of oriented lines, segments and longitudinal sections are explained in subsection 8.3.1 and Allen relations in section 4.2.2. Fuzzification is explained in section 4.2.3 computation of histogram of fuzzy Allen relations are explained in section 4.2.4 and treatment of longitudinal section is explained in 4.2.5.

4.2.1 Oriented Lines, Segments and Longitudinal Sections

Let A and B be two objects and $(v, \theta) \in \mathbb{R}$, where v is any real number and $\theta \in [-\pi, \pi]$. Let $\Delta_\theta(v)$ be an oriented line at orientation angle θ and $A \cap \Delta_\theta(v)$ is the intersection of object A and oriented line $\Delta_\theta(v)$. It is denoted by $A_\theta(v)$, called segment of object A. If there exist more than one segment then it is called longitudinal section as in case of Fig.4.1 where $A \cap \Delta_\theta(v)$ has two segments, length of its projection points on x-axis is x. Similarly for object B where $B \cap \Delta_\theta(v) = B_\theta(v)$ is segment and z is length of its projection points. The variable y is the difference between the minimum of projection points of $A \cap \Delta_\theta(v)$ and maximum of projection points of $B \cap \Delta_\theta(v)$.

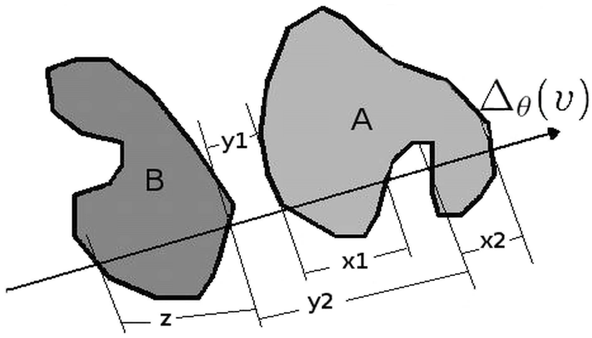


Figure 4.1: Oriented line, segment and longitudinal section

4.2.2 Allen Relations in Space

Allen [Allen 1983] introduced 13 mutually exclusive and exhaustive interval relations. These relations are arranged as $\mathcal{A} = \{<, m, o, s, f, d, eq, d_i, f_i, s_i, o_i, m_i, >\}$, where $\{<, m, o, s, f, d,\}$ *resp* $(\{d_i, f_i, s_i, o_i, m_i, >\})$ are the relations *before, meet, overlap, start, finish, during* (*resp* the inverse relations of the cited ones). The relation *eq* is the equality spatial relation. All the Allen relations in space are conceptually illustrated in Fig. 4.2. These relations have a rich support for the topological and directional relations.

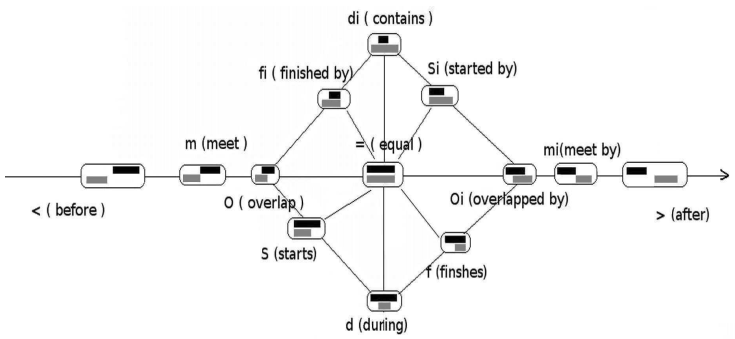


Figure 4.2: Black segments represent the reference objects and gray segments represent the argument objects

4.2.3 Fuzzification of Allen Relations

Different approaches are used for fuzzification of Allen temporal relations. Some of them use the fuzzification based on the human defined variables and fuzzification is described only for use in temporal domain, for the qualitative aspects of temporal knowledge and qualitative temporal reasoning processes. There is a homeomorphism between the Allen's temporal relations and $1D$ topological relations in spatial domain. Due to this homeomorphism, Allen relations are also used for extracting the combined topological and directional relations information

[Zahzah 2002, Matsakis 2005]. Fuzzy Allen relations are used to represent the fuzzy topological relations where fuzziness is represented at the relation's level.

Fuzzification of Allen relations doesn't depend upon particular choice of fuzzy membership function. Trapezoidal membership function (Eq.(8.2)) is used due to flexibility in shape. Let $r(I,J)$ be an Allen relation between segments I and J where $I \in A$(argument object) $J \in B$ (reference object), r' is the distance between $r(I,J)$ and it's conceptional neighborhood. We consider a fuzzy membership function $\mu : r' \longrightarrow [0\ 1]$, where μ is a trapezoidal membership function defined in Eq. 4.1. The fuzzy Allen relations defined by Matsakis and Nikitenko [Matsakis 2005] are

$$\left.\begin{aligned}
f_<(I,J) &= \mu_{(-\infty,\ -\infty,\ -b-3a/2,\ -b-a)}(y)\\
f_>(I,J) &= \mu_{(0,\ a/2,\ \infty,\ \infty)}(y)\\
f_m(I,J) &= \mu_{(-b-3a/2,\ -b-a,\ -b-a,\ -b-a/2)}(y)\\
f_{mi}(I,J) &= \mu_{(-a/2,0,0,a/2)}(y)\\
f_O(I,J) &= \mu_{(-b-a,-b-a/2,-b-a/2,-b)}(y)\\
f_{Oi}(I,J) &= \mu_{(-a,-a/2,-a/2,0)}(y)\\
f_f(I,J) &= min(\mu_{(-(b+a)/2,-a,-a,+\infty)}(y), \mu_{(-3a/2,-a,-a,-a/2)}(y), \mu_{(-\infty,-\infty,z/2,z)}(x))\\
f_{fi}(I,J) &= min(\mu_{-b-a/2,-b,-b,-b+a/2}(y), \mu_{(-\infty,-\infty,-b,-(b+a)/2)}(y), \mu_{(z,2z,+\infty,+\infty)}(x))\\
f_s(I,J) &= min(\mu_{-b-a/2,-b,-b,-b+a/2}(y), \mu_{(-\infty,-\infty,-b,-(b+a)/2)}(y), \mu_{(-\infty,-\infty,z/2,z)}(x))\\
f_{si}(I,J) &= min(\mu_{(-(b+a)/2,-a,-a,+\infty)}(y), \mu_{(-3a/2,-a,-a,-a/2)}(y), \mu_{(z,2z,+\infty,+\infty)}(x))\\
f_d(I,J) &= min(\mu_{(-b,-b+a/2,-3a/2,-a)}(y), \mu_{(-\infty,-\infty,z/2,z)}(x))\\
f_{di}(I,J) &= min(\mu_{(-b,-b+a/2,-3a/2,-a)}(y), \mu_{(z,2z,+\infty,+\infty)}(x))
\end{aligned}\right\} \tag{4.4}$$

where $a = min(x,z)$, $b = max(x,z)$, x is the length of segment (I), z is the length of segment (J) and (x,y,z) are computed as given in section 8.3.1.

Most of the relations are defined by a single membership function and some of them by minimum of multiple membership functions like $d(during)$, $d_i(during\_by)$, $f\ (finish)$, $f_i\ (finished\_by)$. Two relations are directly neighbors if the fuzzy Allen relations are shared between them. If $0 < r_1(I,J) < 1$ then $0 < r_2(I,J) < 1$ such that $r_1(I,J) + r_2(I,J) = 1$ and $r_1(I,J)$, $r_2(I,J)$ are neighbors expressed in the neighborhood graph (Fig. 4.2). This shows that sum of all the Allen relations is always one, from this $equal(f_=(I,J))$ relation is defined.

Histogram of fuzzy Allen relations represent the total area of subregions of A and B that are facing each other in a given direction θ [Matsakis 2005]. Formally, this definition is written as

$$\mathbf{F}_r^{AB}(\theta) = \int_{-\infty}^{+\infty} F_r(\theta, A_\theta(v), B_\theta(v))dv \tag{4.5}$$

$$F_r(\theta, A_\theta(v), B_\theta(v)) = \frac{x+z}{w} \sum_{k=1}^{c} \sum_{i=1}^{m_k} \sum_{j=1}^{n_k} [x_i^k z_j^k (a_k - a_{k-1})] r(I_i^k, j_j^k) \tag{4.6}$$

Where $\mathbf{F}_r^{AB}(\theta)$ represents the histogram of a fuzzy Allen relation in direction θ and$F_r(\theta, A_\theta(v), B_\theta(v))$ is the representation of histogram for a given v where $w = \sum_{k=1}^{c}\sum_{i=1}^{m_k}\sum_{j=1}^{n_k}[x_i^k z_j^k(a_k - a_{k-1})]$, $x = \sum_{i=1}^{m_k} x_i^k$ and $z = \sum_{j=1}^{n_k} z_j^k$.

In this case m_k and n_k represent the total number of segments of argument object A and reference object B and c represents number algorithm loop for fuzzification of a longitudinal section (algorithm is discussed in section 4.6).

4.2.4 Histogram of Fuzzy Allen Relations

Fuzzy Allen relation for each segment is a fuzzy set and fuzzy aggregation operators are used to combine different values of fuzzy grades. In polygonal object approximation, fuzzy Allen relations are calculated for a limited number of segments and within this region spatial relations don't change, so simply generalize the given relations. This technique minimizes the number of segments to be treated, decreasing the temporal complexity of algorithm. This changes mathematical equation (Eq. (4.6)) for histogram of fuzzy Allen relations. Formally,

$$F_r(\theta, A_\theta(v), B_\theta(v)) = r(I, J) \tag{4.7}$$

In discrete space, the integral can be written as a finite sum, thus above Eq. (4.5) for discrete space can be rewritten as

$$\mathbf{F}_r^{AB}(\theta) = (X + Z)\sum_{k=1}^{n} r(I_k, J_k) \tag{4.8}$$

Where Z is the area of reference object and X is area of argument object in direction θ, n is total number of segments to be treated and $r(I_k, J_k)$ is an Allen relation for segment pair (I_k, J_k) and $F_r(\theta, A_\theta(v), B_\theta(v))$ represents a histogram of a fuzzy Allen relation.

4.2.5 Treatment of Longitudinal Section

During the decomposition process of an object into segments, there can be multiple segments for a line $\Delta_\theta(v)$ depending on object shape and boundary which is called longitudinal section. Different segments of a longitudinal section are at a certain distance and these distances might affect final results. In this paper, we adopt the force histograms as in [Matsakis 1999b]. Here, we replace the sum operator in Eq. (7.5) by a fuzzy operator and use the logic connectors, i. e.

$$F_r(\theta, A_\theta(v), B_\theta(v)) = \odot(f_r(x_1, y_1^\theta, z), f_r(x_2, y_2^\theta, z), ..., f_r(x_n, y_n^\theta, z))$$

where $\odot$ is a fuzzy operator and $r \in \mathcal{A}$, $\mathcal{A}$ is denoted in section 4.2.2.

Example

We introduce an example to explain different steps of the method of combining topological and directional relations. This better explains difference between method developed by Matsakis and Nikitenko [Matsakis 2005] and the method we proposed in this paper.

Let us consider two simple objects for the computation of histogram of fuzzy Allen relations. Let these simple polygonal objects be A and$B$ where $A = \{(5,5),(15,5),(20,10),(20,15),(10,15)\}$ and
$B = \{(5,15),(15,12),(20,20),(15,25),(10,25),(5,20)\}$ are the vertices of polygons, it is shown in Fig. 4.3. Computing the histograms of Allen relation for objects A

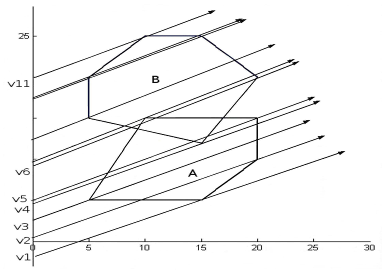


Figure 4.3: Showing the computation of relations by an example where each line passes through the vertex of a polygon and inclination angle is fixed at 30 degree

and B by our method involves the following steps.

1. Compute the boundary of both objects, for object A, line segments joining the vertices of polygon A, in a similar way for object$B$.

2. Fix the angle, let it be 30 degree and draw lines passing through the vertices of polygons, in our case, total number of vertices are 11 there will be 11 lines, each line passes through a vertex of a polygon.

3. Compute the intersection of line with boundary of an object, each line has one or two intersecting points with one object due to simple concave objects. In case of convex or objects with holes there may be more than two intersecting points. Consider only those lines which intersect both objects, for all the other lines relations are zero. As in Fig. 4.3, only $\Delta_{30}(v4), \Delta_{30}(v5)$, $\Delta_{30}(v6)$, $\Delta_{30}(v7)$ lines intersect both objects, for other lines fuzzy Allen relations are zero. These intersecting points are
$\Delta_{30}(v4) \cap A = \{(6,6),(20,14)\}, \Delta_{30}(v5) \cap A = \{(6,7),(20,15)\}$,
$\Delta_{30}(v6) \cap A = \{(9,13),(12,15)\}, \Delta_{30}(v7) \cap A = \{(10,15)\}$,
$\Delta_{30}(v4) \cap B = \{(15,12)\}, \Delta_{30}(v5) \cap B = \{(15,12),(16,13)\}$,
$\Delta_{30}(v6) \cap B = \{(9,13),(20,20)\}, \Delta_{30}(v7) \cap B = \{(8,14),(19,20)\}$

4. Take the projection (P) of these points on x-axis and calculate the values of (x, y, z). Here, for line $\Delta_{30}(v4)$ these values are $P(\Delta_{30}(v4) \cap A) = \{6, 20\}$ and $P(\Delta_{30}(v4) \cap B) = \{15\}$, from this we get (x, y, z)=(14,-9, 0).

5. Compute the fuzzy Allen relation for each segment by using Eq. (7.6). This system of equations will provide us the following results for the aforementioned (x, y, z) (segments obtained by taking intersection of objects with line

$\Delta_{30}(v4)$).
$f_>(I_4, J_4) = 0, f_<(I_4, J_4) = 0,\ f_m(I_4, J_4) = 0, f_{mi}(I_4, J_4) = 0,$
$f_O(I_4, J_4) = 0 f_{oi}(I_4, J_4) = 0,\ f_f(I_4, J_4) = 0 f_{fi}(I_4, J_4) = 0,$
$f_s(I_4, J_4) = 0 f_{si}(I_4, J_4) = 0,\ f_d(I_4, J_4) = 0,\ f_{di}(I_4, J_4) = 1,$
and $f_=(I_4, J_4) = 0,$

6. Repeat this process for lines $\Delta_{30}(v5), \Delta_{30}(v6)and\Delta_{30}(v7)$ (for other lines relations are already zero), sum all the relations and multiply the resultant relation by the sum of surface areas of two objects between line $\Delta_{30}(v4)$ and line $\Delta_{30}(v7)$. This will provide the number of pixels of both objects under a specific fuzzy Allen relation and for normalization, divide each fuzzy Allen by the sum of all the Allen relations for $\theta = 30$ degree, this will give us the percentage area of two objects under a fuzzy Allen relation.

7. Increase the angle by one degree and repeat the above steps(2-6).

We calculate the values (x, y, z) for Matsakis and Nikitenko [Matsakis 2005]. In this method objects are considered as regular closed sets. The difference between methods is the way to compute the triplet (x, y, z) and object approximation.

1. Fix the angle θ and draw the pencil of oriented lines $\Delta_\theta(v)$, in this case number of lines will be much greater than 11 due to pencil of lines.

2. Compute the intersection of a oriented line with both objects A and B. Let us consider the same line $\Delta_{30}(v4)$. Then
$\Delta_{30}(v4) \cap A = \{(6, 6), (7, 7), (8, 7), (9, 8), (10, 8), (10, 9), (11, 9),$
$(12, 10), (13, 10), (14, 11), (15, 11), (15, 12), (16, 12), (16, 13),$
$(17, 13), (18, 13), (18, 14), (19, 14), (20, 14)\}$.
Similarly for object B, $\Delta_{30}(v4) \cap B = \{(15, 12)\}$. Take projections on x-axis and compute the values of (x, y, z), here the values will be the same as in above case and produce the same results. i. e., $x = 14, y = -9, z = 0$, x and z are the diameter of point sets on real line and y is the difference between the minimum of argument point set and maximum of reference point set. Remaining process of computation is identical to our method.

3. Compute Allen relations for every line that intersect the objects. Pencil of lines is created then same calculations are made for every line.

This example elaborates the difference between two methods, and how the new proposed method reduces the time for computation, by simplifying and limiting the number of intersections in one line and an object. Secondly, it decreases the number of lines to be treated for one direction.

4.3 Fuzzy Logic Connectors

In decision theory, there arise some situations where solution depends upon combination of different information provided by different sources. In such a situation, fuzzy connectors are used. In this section, different fuzzy aggregation operators (fuzzy logic connectors) are studied. Aggregation refers of combining values into the one aggregated value so that the final solution seems to be well addressed in a given fashion [Grabisch 1998, Beliakov 2001]. It is a mapping $\tau : [0,1]^n \rightarrow [0,1]$, which combines different fuzzy grades. In literature of fuzzy set theory there exist a variety of operators such as fuzzy *T-norms, T-conorms* and so on, some commonly used operators are:

- Fuzzy OR: $\mu_{OR}(x) = max(\mu_A(x), \mu_B(x))$;
- Fuzzy AND: $\mu_{AND}(x) = min(\mu_A(x), \mu_B(x))$;
- Fuzzy Algebraic Product: $\mu_{PROD}(x) = \Pi_{i=1}^{2}(\mu_{(i)}(x))$;
- Fuzzy Algebraic Sum: $\mu_{SUM}(x) = 1 - \Pi_{i=1}^{2}(1 - \mu_i(x))$;
- Fuzzy γ operator: $\mu_\gamma(x) = [\mu_{SUM}(x)]^\gamma * [(\mu_{PROD}(x)]^{1-\gamma}$ where $\gamma \in [0,1]$

The *OR* operator is actually the *union* or *max* operator, while the *AND* is *intersection* or *min* operator. The contribution for resultant of *OR* (*AND*) fuzzy operator is a single input value, which is *maximum* (*minimum*). For other operators, both values contribute. The fuzzy algebraic *sum* (*product*) operator makes the set result larger than or equal to maximum (resp less than or equal to minimum) the contributing values while Fuzzy γ operator changes the result value from minimum to maximum values depending on the choice of γ. Application of these operators is explained in Annex-10.1. In this example, *t-norms* and *t-conorms* are explained.

4.4 Affine Properties of Histograms of Fuzzy Allen Relations

Affine properties are important in the pattern recognition especially object matching in a scene analysis. These properties of histograms of fuzzy Allen relations are depicted below which are independent from fuzzy membership functions.

- Object commutativity: Pair (A, B) be assessable, let r be a fuzzy Allen relations and for all relations, except the relation *during*(d) and *during_by*(d_i). $F_r^{A,B}(\Theta) = F_r^{B,A}(\Theta + \pi)$. For relation *during* and *during_by* we have $F_d^{A,B}(\Theta) = F_{di}^{B,A}(\Theta)$
- Orthogonal symmetry: Let orthogonal symmetry denoted by *sym* about the oriented line with slop α, then histogram of fuzzy Allen relations: $F_r^{sym(A),sym(B)}(\Theta) = F_r^{A,B}(2\alpha - \Theta)$
- Central dilation: Let central dilation(scale) denoted by *dil* and λ is dilation ratio, then $F_r^{dil(A),dil(B)}(\Theta) = \lambda^2 F_r^{A,B}(\Theta)$

- Stretch: Let stretch ($stre$) is orthogonal to x axis and k is stretch ratio, then histogram of fuzzy Allen relations satisfies.
$F_r^{stre(A),stre(B)}(\Theta) = kF_r^{A,B}(\Theta)$

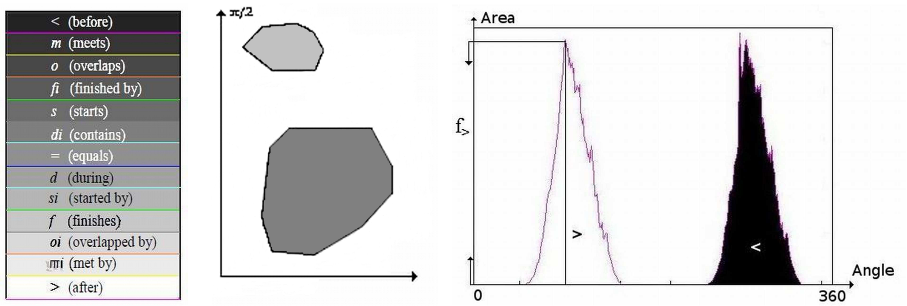
- Translation: Let translation denoted by $trans$ and (trans(A), trans(B)) is assessable then following relation holds for histogram of fuzzy Allen relations.
$F_r^{trans(A),trans(B)}(\Theta) = F_r^{A,B}(\Theta)$

- Rotation: Let rotation (rot) be a ρ-angle rotation and $(rot(A), rot(B))$ is assessable hence $F_r^{rot(A),rot(B)}(\Theta) = F_r^{A,B}(\Theta - \rho)$

4.5 Comparison With Others Works and Interpretation

In this section first we give the interpretation of results representation and then compare our approach with Matsakis and Nikitenko method [Matsakis 2005]. For the experiment purpose, 360 directions are considered. Instead of drawing pencil of lines in a direction, only those lines are considered which passes through vertices of polygons. Fuzzy Allen relations are computed for each segment, if there exit longitudinal section, then fuzzy aggregation operator is applied to obtain the resultant fuzzy Allen relation for an oriented line.

Each relation is associated with the grey scale value like $before$ with black, white color represents $after$, each relation has a different boundary color for better visualization of relations while opposite relations have the same boundary color such as $m(meet)$ and $mi(meet\_by)$ relations have the yellow boundary color. Interpretation of the grey level association to a relation and its boundary color is given in Fig. 4.4(a). Object A has the light grey color and object B is represented by dark grey. The thirteen histograms for directional and topological relations are represented by layers and each vertical layer represents the total area of objects in specific direction.



(a) Histograms representation (b) Object pair (c) Histograms

Figure 4.4: 4.4(a), Histogram representation for fuzzy Allen relations, 4.4(b), object pair representation and 4.4(c) represents histogram of fuzzy Allen relations for object pair in 4.4(b).

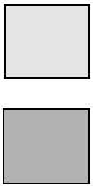
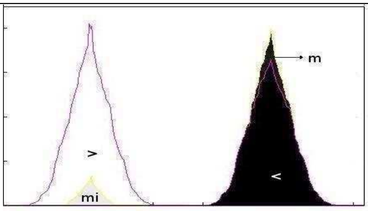
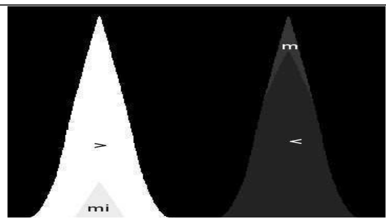
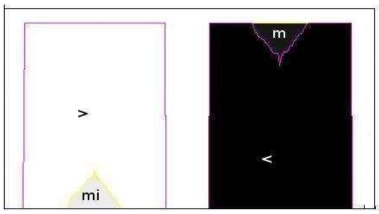
Obj. pairs	Our method	M & N method
		
		

Table 4.1: Comparison of normalized and un-normalized histograms of fuzzy Allen relations with the Matsakis and Nikitenko and our method(axis are same as in Fig. 4.4(b) and 4.4(c)).

These histograms are normalized as $A_j(\theta) = \frac{A_j(\theta)}{\sum_{i=1}^{13} A_i(\theta)}$ where $A_j \; \forall \; j = 1, ..., 13$ is an Allen relation. This technique of normalization provide us the percentage area of both objects under a histogram of fuzzy Allen relation. In this example (Figs. in table 4.1) first row represents the object pair and its histograms which are not normalized, computed by our method and method developed by Matsakis and Nikitenko (for Matsakis and Nikitenko method we copied the figure directly from [Matsakis 2005]). The second row represents normalized histogram of fuzzy Allen relations with the respective method. Both of the histograms seem similar, a small difference in the shape represents the small change in the total surface area of two objects under the specific fuzzy Allen relation, which is less important.

In this set of examples (Figs. in table 4.2), histograms for fuzzy Allen relations are compared with the method of Matsakis and Nikitenko [Matsakis 2005]. Where the first column represents object pairs at different distances, second column represents computation of histograms of fuzzy Allen relations by our method and in third column histograms are represented computed by Matsakis and Nikitenko method. These histograms of fuzzy Allen relations are distance dependant. As the objects become closer, the new histogram for fuzzy Allen relation *meet* and *meet_ by* emerges.

In both methods, similar histograms exist, there is a small difference in their shapes. Histogram shape represents the total area under the specific relation, as we consider object by the polygonal approximation and due to this approximation, there is a small change in object area under a particular histogram.

In this example (Figs. are shown in table 4.3), polygonal objects are considered. First, objects are at certain distances, only *after* and *before* relation exist. Histogram resembles with the histogram computed by the Matsakis and Nikitenko method. Sharp and sudden changes in the histogram shape computed by our method are due

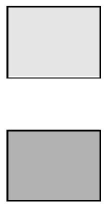
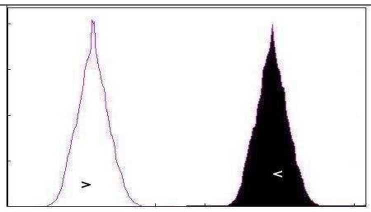
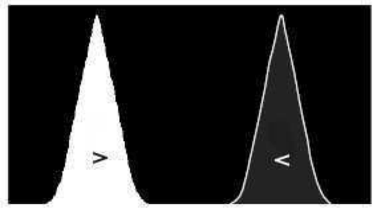
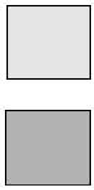
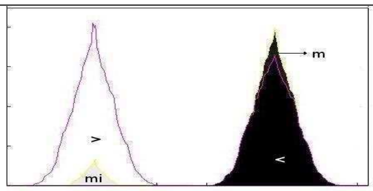
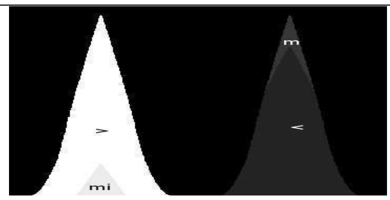
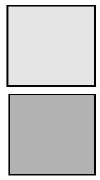
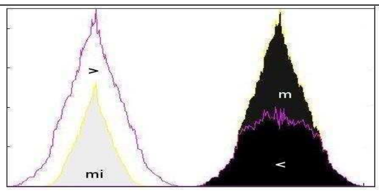
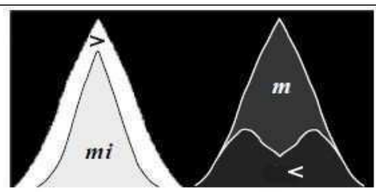
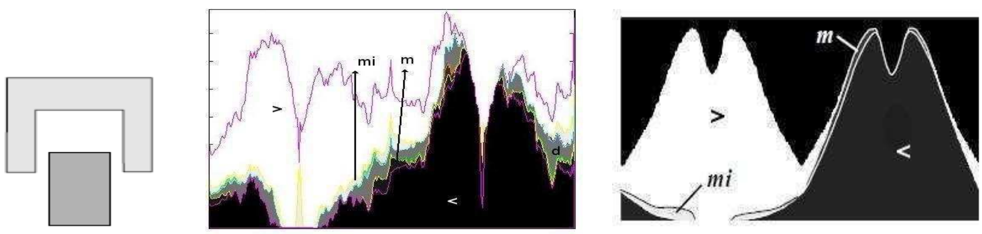
Obj. pairs	Our method	M & N method
		
		
		

Table 4.2: Comparison of histograms for rectangular objects (axis are same as in Fig. 4.4(b) and 4.4(c)).

to the problems of taking intersections in the 2D digital space. Empty intersection sometimes results a change in the object area (sharp ups and downs in histogram shape in first row) and sometimes it causes the existence of new spatial relation such as histogram in the *after* and *before* relations in third row.



(a) (b) (c)

Figure 4.5: 4.5(a) object pair, histograms of fuzzy Allen relations with our method in 4.5(b) and Matsakis and Nikitenko method in 4.5(c). (axis are same as in figure 4.4(b) and 4.4(c)).

For the example Fig. 4.5(a)-4.5(c), concave object is considered. Fig. 4.5(a) represents object pair, histogram in Fig. 4.5(b) is computed by our method and fuzzy operator OR is used. Histogram represented in Fig. 4.5(c) is computed by the method developed by Matsakis and Nikitenko. In certain direction object A is *before* as well as *after*, in the meanwhile there exist relation *meet* and *meet_ by* due

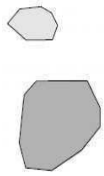
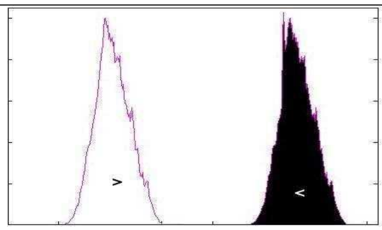
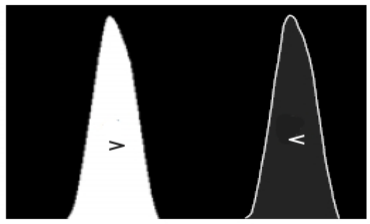
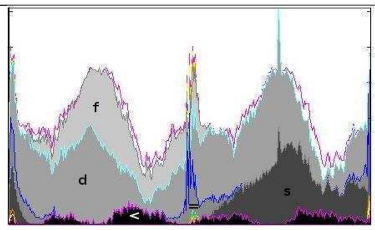
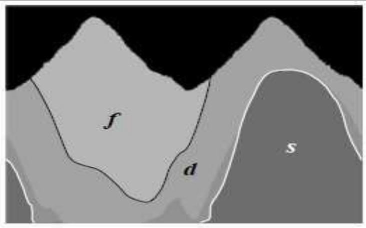
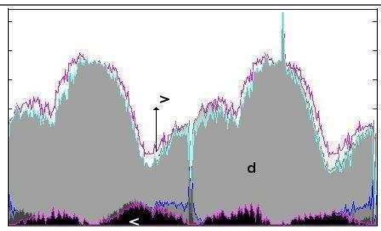
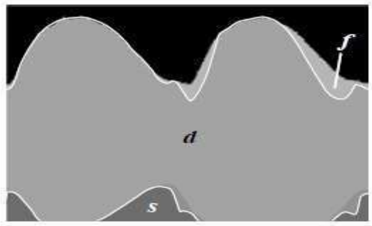
Obj. pairs	Our method	M & N method
		
		
		

Table 4.3: Comparison of histograms of fuzzy Allen relations with the Matsakis and Nikitenko method and our method for the polygonal objects (axis are same as in Fig. 4.4(b) and 4.4(c)).

to closeness of two segments. In our method, sharp change in histogram is due to the change in surface area of objects due to the digitization process. This example shows that two segments of an object have the opposite fuzzy Allen relations (a case considered in Fig. 10.1(b))

In this set of examples (from Fig. 4.6(a)- 4.6(g)), argument object is a concave object. As it changes its position, one segment changes its topological and directional relation with respect to argument segment. The other segment doesn't change its topological relation. As a result both segments have opposite Allen relations with the reference segment and resulting histogram represents both relations at the same time.

Examples given above show that histograms of fuzzy Allen relations are approximately the same while they are computed either with the help of Matsakis and Nikitenko [Matsakis 2005] method or polygonal object approximation. Our approach has a certain amount of decrease in the computation time. Computation time with our approach drops from $O(nM\sqrt{M})$ to $O(nNlog(N))$, where N represents the total number of vertices of polygons. For the treatment of longitudinal section, when there exist more than one segment, fuzzy Allen relations are computed between every pair of segments of reference and argument objects, then at the next stage fuzzy OR operator is used to integrate the available fuzzy Allen relations.

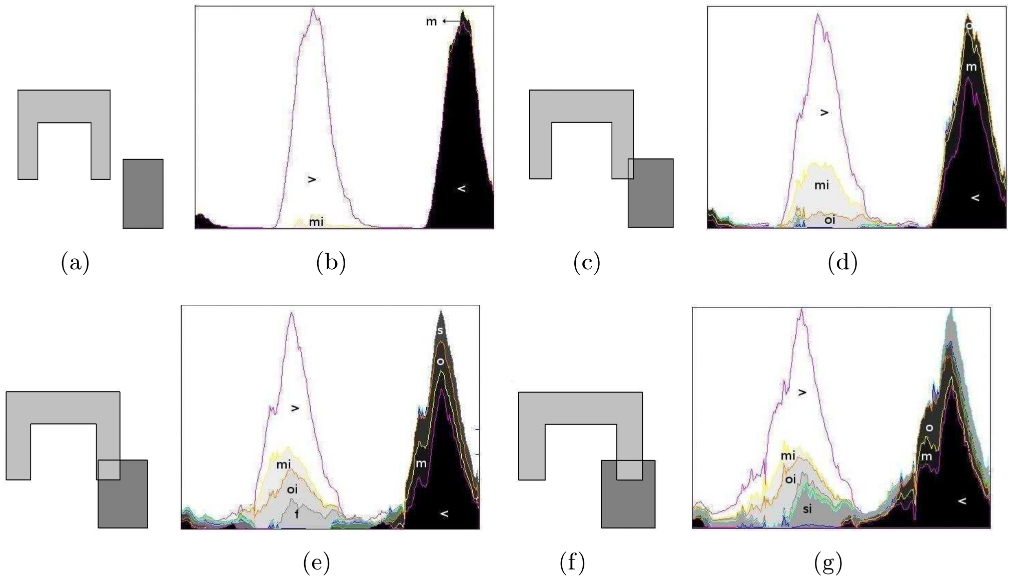


Figure 4.6: Concave & convex object pairs and their histogram of fuzzy Allen relations (axis are same as in figure 4.4(b) and 4.4(c)).

4.6 Complexity

Algorithm efficiency can be measured in terms of execution time. Method for finding the combined topological and directional relations information is to couple the force histograms with the fuzzy Allen relations. Time complexity for computing the combined topological and directional relations information depends upon the following three factors.

1. Object approximation
2. Algorithm for treatment of longitudinal section
3. Equation used for computation of histograms (Eqs. 4.6 and 4.7)

These different aspects of time complexity are discussed separately here below. We compare time complexity of both algorithms and at the end we note that there is a sufficient decrease in the execution time of the modified algorithm.

Time complexity due to object approximation: Objects are approximated as closed regular sets or by polygonal representation. Method for raster data starts directly with the scan of lines. For polygonal approximation, the methods first determines the polygonal representation of object and polygons vertices. Computation time for the polygon representation depends upon the algorithm used for polygon representation. For example, we use the method discussed in [Masood 2008], time complexity of this method is $O(MN_1^2)$ where M and N_1 represents respectively the number of dominant points and curve size. When we compute the force histogram, this time is also included in computation time for force histogram with polygonal object approximation.

In the method of computing the combined topological and directional relations information, force histograms are coupled with the fuzzy Allen relations. Time complexity of the method is directly related to the time complexity of force histograms. Time complexity of force histograms for polygonal object approximation is $O(nNlog(N))$ where N denotes the number of vertices of two polygons and n for number of directions to be computed. Computation time for raster data is $O(nM\sqrt{M})$ where M denotes the number of pixels of the processed image (see [Matsakis 2002]).

Time complexity of Algorithm for treatment of longitudinal section: Method for computing the combined topological and directional relations information directly depends upon the force histograms. In this method force histograms are coupled with the fuzzy Allen relations, where algorithm developed by Matsakis and Nikitenko [Matsakis 2005] (algorithm 1)for fuzzification of segments of a longitudinal section. This algorithm imposes restrictions on the assumption of an object. Its time complexity is added to the time complexity of the force histograms. For better elaboration, the algorithm used by Matsakis and Nikitenko [Matsakis 2005] is given below.

Algorithm 1 has the time complexity of order $O(n^3)$. Where n represents the number of segments for a longitudinal section of a line. With the proposed method, this algorithm is replaced by the fuzzy logic connectors, these logic connectors have less time complexity as compared to this algorithm, these fuzzy logic connectors are only used for dealing with the longitudinal section.

Time complexity of Eq. 4.6 and 4.7 Matsakis and Nikitenko used Eq. 4.6 for computing the combined topological and directional relation information. This equation has time complexity $O(n^3)$ where n represents the total number of segments exist in a longitudinal section. ($m + n$ represents the total number of segments for a longitudinal section and c represents the number of loops in algorithm 1, it maximum value equals to number of disjoint segments). Proposed Eq. 4.7 has time complexity N where N represents the total number of vertices of polygons. This shows that the Eq. 4.6 has higher time complexity as compared to the time complexity of Eq. 4.7.

Algorithm for fuzzification of longitudinal section used by Matsakis and Nikitenko (Algorithm 1) put limits for the object approximation. Due to this algorithm, time complexity of method for combined topological and directional relations information is $O(nM\sqrt{M})$, where n represents the number of directions M number of pixels to be processed, in this paper we replace the algorithm for fuzzification of segments of a longitudinal section (algorithm 1) with the fuzzy logic connectors. This substitution of algorithm for fuzzification of longitudinal section made it possible to consider objects by its polygon approximation, hence for the method we proposed,

Algorithm 1 Algorithm for the fuzzification of a longitudinal section I. The symbol $H(I_i^c \cup I_j^c)$ denotes the convex hull of $I_i^c \cup I_j^c$. Indexing is chosen in such a way that I_i, I_j are consecutive in I, c represents the number of iterations in while loop, its maximum value is n. The algorithm increases the degree of α_k associated with the open interval $J_k = H(I_i^c \cup I_j^c) - (I_i^c \cup I_j^c)$. Initially $\alpha_k = 0$ and α denotes the α-cut of a fuzzy membership function.

---

c 0;
α 1;
while $\alpha > 0$ **do**
..... There exits one set $\{I_i^c\}_{i \in 1 \ldots n_c}$ of mutually disjoint segments (and only one) such that: $\alpha I = \cup_{i \in 1 \ldots n_c}$. For any i and any j in 1....n_c, with $i \neq j$ the length of I_i^c is denoted by x_i^c and the distance between I_i^c and I_j^c is denoted by d_{ij}.
for any i in 1...$n_c - 1$ **do**
for any j in 1...n_c **do**
β $\alpha(1 - \frac{d_{ij}^c}{min(x_i^c, x_j^c)})$
for any k in 1...$n - 1$ **do**
if $J_k \subset H(I_i^c \cup I_j^c)$ **then**
α_k $max(\alpha_k, \beta)$
end if
end for
end for
end for
α $max\{\alpha_k\}_{k \in 1, \ldots n-1} \cap [0, \alpha]$
c $c + 1$
end while

---

time complexity for computing the combined topological and directional relations information drops from $O(nM\sqrt{M})$ to $O(nNlog(N))$, where N represents the total number of vertices of polygons. Obviously $N << M$ and n is number of directions to be computed, it ranges from 32 to 360 directions.

4.7 Conclusion

Fuzzy Allen relations can be used to detect and analyze object position in space and these relations have a rich support for defining the fuzzy topological relations. These relations represent fuzziness in relation's semantics, they can also answer the questions that where in space a certain topological relation exists.

Polygonal approximation of objects and application of fuzzy logic connectors simplifies the algorithm given by Matsakis and Nikitenko [Matsakis 2005]. This approach decreases its time complexity, it drops down from $O(nM\sqrt{M})$ to $O(nNlog(N))$, where n represents number of directions, N number of vertices of polygons and M number of pixels to be processed, due to using fuzzy logic connectors in lieu of the fuzzification of segments of a longitudinal section developed by

Matsakis.

This approach of using fuzzy operator will open new fields of applications for fuzzy logic connectors. This technique can further be developed for defining the dynamic spatial relations in a quantitative way. In this paper, computations for all the directions are calculated for experimental purpose and verification of affine properties will be helpful for affine invariant description of relative object positions in scene and image understanding applications by combined topological and directional relations information. The aim of this paper is to validate formally the method we propose, and for the future work we project to apply this method for real data obtained by video sequences of real applications after extracting objects and their polygonal approximation with the help of all the image processing techniques.

Chapter 5

Two-Dimensional Fuzzy Spatial Relations: A New Way of Computing and Representation

Abstract

In existing methods, fuzzy topological relations are based on computing topological relations between fuzzy objects. This imprecision is also found in relationships, hence, relations can also be fuzzy. In such a situation, fuzzy topological relations are needed between crisp objects. These relations are much less developed.

In this paper, we propose a method for combining fuzzy topological and directional relations which is called combined topological and directional relations (CTD) method. Moreover a single method is used to derive the fuzzy topological and directional relations and this method deals fuzziness in two ways, fuzziness in topological and directional. This is a quantitative method for finding the topological and directional relations and an algorithm for defuzzification of relations is proposed to determine the Jointly Exhaustive and Pairwise Disjoint (JEPD) topological and directional relations between a $2D$ object pair. These JEPD relations are represented in a neighborhood graph for spatial relations. For the validation and the assessment, a number of experiments have been performed on artificial data.

keywords:

Fuzzy reasoning, Combined topological and directional relations, Algorithm for defuzzification of spatial relations, neighborhood graph.

Introduction

The space can be studied through the objects and spatial relationship between them. Spatial relations between objects provide information regarding the image contents. These spatial relations provide information about topological structure, orientation and distance between objects, generally information about their relative locations.

Topological relations are derived from geometric descriptions. There are two well-known methods for finding the topological relations, the 9-intersections and Region Connection Calculus (RCC) method [Herring 1994, Randell 1992b]. The

9-intersections method depends upon point set topology where topological parts (interior, boundary and exterior) of an object participate. In this method, topological structure is studied by empty ($\emptyset$) and non-empty ($\neg\emptyset$) intersections of topological parts. Eight basic topological configurations between $2D$ objects in $\mathbb{R}^2$ are observed out of 512 possible configurations. Many extensions have been proposed in the 9-intersections model [Du 2005a, Egenhofer 1993, J. 2001, Zhan 1998, Shi 2007]. This method is extended to deal with fuzzy objects and 44 useful topological relations were developed, using the 9-intersections between objects with extended boundaries [Clementini 1996]. This method is also extended to 16 intersections and a set of 152 useful configurations between fuzzy regions in $\mathbb{R}^2$ is realized [Tang 2004]. This method doesn't represent fuzzy spatial relations between crisp objects.

On the other hand, RCC provides us information about the topological structure of an image which are corresponding to the eight basic topological relations. RCC models are applied to regular topological spaces. This calculus is based upon the well established axiomatic theory in which regions are primitives [CLARKE 1981]. In this theory spatial relations are based on a $C(x, y)$, called connection relation. This calculus is extended to fuzzy theory and fuzzy topological relations are developed between fuzzy objects [Liu 2009, Palshikar 2004, Schockaert 2008b, Schockaert 2008c], a set of 46 useful topological configurations are considered. This theory is also extended to deal fuzziness at relation's semantics and fuzzy connection relation based on nearness is defined [Schockaert 2008b]. In this case, the next question which comes into mind is that where a topological relation holds in space?

Topological relations ignore the directional contents between objects. The 9-intersections model for directional relations and a projection based cardinal directions relation method are developed in qualitative domains [Goyal 2001, Frank 1992]. In the 9-intersections method for directional relations, objects are considered by their Minimum Bounding Rectangle (MBR) and area around the reference object is divided into tiles. Intersections between argument object and these tiles are computed and relations are defined based on empty and non-empty intersections. In other methods, two-dimensional projections are taken on both axis then Allen relations are used. Both methods provide similar results and MBR of the reference object is called neutral or direction less region.

Reasoning about situations when argument object occupies space in the central tile of reference object, a method of Internal Cardinal Directional (ICD) relations was introduced [Liu 2005]. This method divides the central tile of the 9-intersections method for directional relations between extended objects into four, nine or 13 sub-tiles and internal directional relation between object pair is calculated by the intersection of sub-tiles and argument object. These methods describe the extended objects according to their relative position. Once a directional relation is determined then next question arises that what is the binary topological relation between object pair? Either two objects be *disjoint*, *meet*, *overlap* or one object is inside the other. These methods don't answer the question that what is the topological relation between the object pair?

In existing methods, One single method can't be used to adequately describe object's position in image understanding process. In qualitative domain commonly topological relations are studied between fuzzy objects such as in *RCC* and the 9-intersections methods. In languages, people use fuzziness in relation's semantics, for example *"Both objects are approximately equal"* or *"Both objects approximately meet"*. In this case both objects are crisp but relations are fuzzy. In second example, both objects don't meet, but relation depends upon nearness. A method which can describe fuzzy spatial relations between crisp objects is required. For this purpose, idea of combined topological and directional relations is not new, fuzzy methods [Malki 2000, Matsakis 2005, Salamat 2011c, Salamat 2010b] can be used to model the fuzzy directional and topological relations at the same time. This method represents fuzziness at relation's level along with the answer for question that where a topological relation holds in space.

Allen relations create 13 partitions around the reference interval or segment corresponding to each relation and these partitions represent eight topological relations. To represent fuzziness at relation's level along with the direction information, fuzziness is introduced at Allen's relations. This method deals with the positional fuzziness in topological relations. These fuzzy topological relations are used to model the positional uncertainty. This uncertainty present according to the directional viewpoint of relative position of object pair in a spatial domain. In this paper, the idea is to specify fuzzy topological relations between $2D$ objects along with qualitative directional information. For this purpose each relation is split into several components and $1D$ Allen relations are used.

This paper is structured as follows, Section 5.1 describes the related work. Some preliminary definitions are denoted in Section 5.2 and Section 5.3 discusses in detail the different terms and necessary computations for $1D$ Allen relations. In Section 5.4, our method for computing the topological relations along with directional aspects and their interpretation has been given. we also develop an algorithm for defuzzification of spatial relations. A neighborhood graph for defuzzified topological and directional relations is presented in Section 5.5. Results for different situation is given in Section 5.6 and Section 5.7 consist of conclusion and future work.

5.1 Related Work

One of the developed trends in geographic information science is a move from determinate geographic information science to the fuzzy geographic information science. Spatial relations is a major part of the Geographic Information Systems(GIS). Information on the spatial organization of objects in an image is useful for image understanding and reasoning process. Methods of extracting and representing this information greatly effect the obtained results.

In early years, fuzzy directional relations are studied separately and different approaches were adopted like mathematical morphology [Bloch 1995, Bloch 2003] and quantitative methods [Miyajima 1994], where fuzziness is associated to a relation. It is first numerical description of an object's relative position, called angle histogram. Force histograms [Matsakis 1999b] were the extension of angle histogram which deals only *disjoint* objects. In all these methods, fuzzy directional relations are studied and less attention has been paid to fuzzy topological relations while topological, directional and distance relations are considered essential to understand scene configuration, modeling common sense knowledge and spatial reasoning.

In most of the existing topological relations methods [Du 2005a, Du 2005b, Shi 2007, Tang 2004] uncertainty is represented at object level but uncertainty can also exist at relation's level. This type of uncertainty can be handled by assigning fuzzy membership value to a relation. According to our best knowledge, S. Schockaert et al. [Schockaert 2008b] are the pioneer in this field and they have defined the fuzzy "Connection" relation based on nearness and the topological fuzzy relations in RCC theory. Having all this, the basic question is that where a topological relation holds in space remains unanswered. For example, France touches Belgium from *north_ east* direction rather than this that France touches Belgium, then next question arises that France has common boundary with Spain, Monaco, Italy, Switzerland, Germany, Luxembourg, in which direction Belgium touches France?

To answer the question, that where a topological relation exists, the idea of combined topological and directional relations information was initiated by J. Malki et al. [Malki 2000] using the Allen [Allen 1983] relations of temporal domain in spatial domain. This work was revisited by Matsakis and Nikitenko [Matsakis 2005] and fuzziness in the Allen relations was introduced. The time complexity for this method is $O(nM\sqrt{M})$, this complexity is due to the algorithm for fuzzification of segments of a longitudinal section. This fuzzification algorithm is replaced by *t-conorms* along with polygonal object approximation [Salamat 2009]. The use of fuzzy connectors along with polygonal approximation decreases its computational complexity to $O(nNlog(N))$ where n number of directions and N represents the number of vertices of polygonal objects [Salamat 2010a]. This work is related to fuzzy spatial aspects where the topological and directional relations are evaluated according to fuzzy set theoretical viewpoint.

The method addressed fuzziness at two levels, in case of topological relations and fuzziness according to the directional viewpoint. Method of combined fuzzy topological and directional relations (CTD) has two fold impacts, Allen relations are combined in such a way that whole space can be analyzed by using the directions $[0, \pi]$ and this method answer well the question that where a topological relation exists in space. This method will be helpful to answer the query completely in managing database and detecting the small changes in scene descriptions.

5.2 Preliminary Definitions

In this section we denote some terms and definitions which are frequently used throughout the remainder of paper.

Fuzzy membership function: A membership function μ in a set X is a function

$\mu : X \rightarrow [0, 1]$. Different fuzzy membership functions are proposed according to the requirements of the applications. For instance, trapezoidal membership function is defined as

$$\mu(x; \alpha, \beta, \gamma, \delta) = max(min(\frac{x - \alpha}{\beta - \alpha}, 1, \frac{\delta - x}{\delta - \gamma}), 0) \tag{5.1}$$

it is written as $\mu_{(\alpha,\beta,\gamma,\delta)}(x)$ where $x, \alpha, \beta, \gamma, \delta \in \mathbb{R}$ and $\alpha < \beta \leq \gamma < \delta$.

Fuzzy set: A fuzzy set A in a set X is a set of pairs $(X, \mu(x))$such that $A = \{(x, \mu(x)|x \in X)\}$

where μ represents a fuzzy membership function.

Force histogram: The force histogram attaches a weight to the argument object A that object A lies *after* B in direction θ, it is defined as

$$\mathbf{F}^{AB}(\theta) = \int_{-\infty}^{+\infty} F(\theta, A_\theta(v), B_\theta(v))dv \tag{5.2}$$

The definition of Force histogram $\mathbf{F}^{AB}(\theta)$, directly depends upon the definition of real valued functions ϕ, f and F used for the treatment of points, segments and longitudinal sections respectively [Matsakis 1999b]. These functions are defined as

$$\left.\begin{aligned} \phi_r(y) &= \begin{cases} \frac{1}{y^r} & \text{if } y > 0 \\ 0 & \text{otherwise} \end{cases} \\ f(x_I, y^\theta_{IJ}, z_J) &= \int_{x_I + y^\theta_{IJ}}^{x_I + y^\theta_{IJ} + z_J} \int_0^{z_J} \phi(u - w)dw)du \\ F(\theta, A_\theta(v), B_\theta(v)) &= \sum_{i=1..n, j=1..m} f(x_{Ii}, y^\theta_{IiJj}, z_{Jj}) \end{aligned}\right\} \tag{5.3}$$

where n, m represents the number of segments of object A and object B respectively and variables (x, y, z) are explained in Fig. 5.1.

Conceptual Neighbor: Two relations between pairs of events are *conceptual* neighbors, if they can be directly transformed into one another by continuously deforming (by shortening, lengthening or moving) events in topological sense. A set of relations between pair of events form a *conceptual* neighborhood if its elements are path connected through *conceptual* neighbor relations [Freksa 1992].

5.3 Terminology Used for Computation of Fuzzy Allen Relations

This section describes the terminology used to decompose the space and computation of terms used in this paper.

5.3.1 Oriented lines, Segments and Longitudinal Sections

Let A and B be two spatial objects and $(v, \theta) \in \mathbb{R}$, where v is any real number and $\theta \in [0, 2\pi]$. Let $\Delta_\theta(v)$ be an oriented line at orientation angle θ and $A \cap \Delta_\theta(v)$ is the intersection of object A and oriented line $\Delta_\theta(v)$. It is denoted by $A_\theta(v)$, called segment of object A and length of its projection interval on x-axis is x. Similarly for object B where $B \cap \Delta_\theta(v) = B_\theta(v)$ is segment and length of its projection interval on x-axis is z, y is the difference between the minimum of projection points of $A \cap \Delta_\theta(v)$ and maximum value of projection points of $B \cap \Delta_\theta(v)$(for details [Matsakis 2005]).

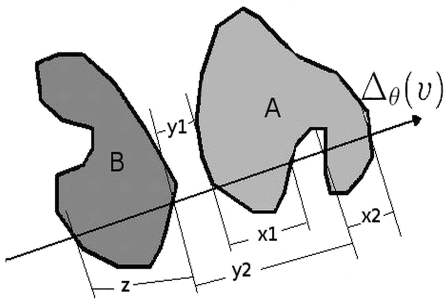


Figure 5.1: Oriented line $\Delta_\theta(v)$, segment as in B, longitudinal section as A.

In case of polygonal object approximation (x, y, z) can be calculated from intersecting points of line and object boundary. Only those oriented lines are considered which passes through at least one vertex of two polygons. If there exist more than one segment, then it is called longitudinal section as in case of $A_\theta(v)$ in Fig. 5.1.

5.3.2 $1D$ Allen Relations in Space

Allen [Allen 1983] introduced 13 JEPD interval relations based on temporal interval algebra. These relations are arranged as $\mathcal{A} = \{<, m, o, s, f, d, eq, d_i, f_i, s_i, o_i, m_i, >\}$ with meanings *before, meet, overlap, start, finish, during, equal, during_ by, finish_ by, start_ by, overlap_ by, meet_ by, and after.* All the Allen relations in space are conceptually illustrated in Fig. 5.2.

5.3.3 Fuzzification of Allen Relations

There is a homeomorphism between Allen's temporal relations and $1D$ topological relations, due to this homeomorphism, Allen relations are used for extracting the combined directional and topological relations information. Fuzzy Allen relations

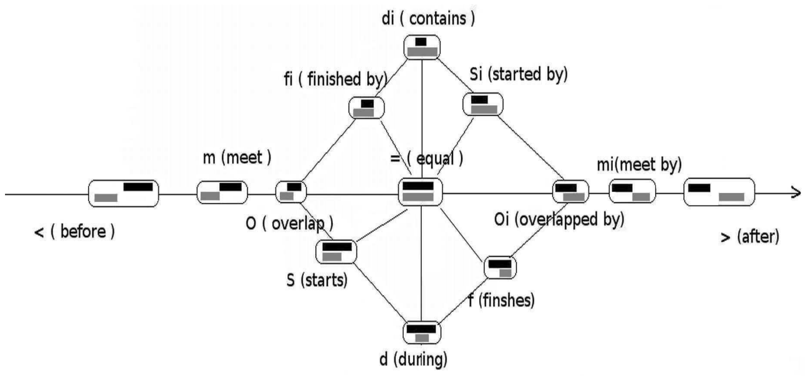


Figure 5.2: Black segment represents the reference object and gray segment represents argument object

are used to represent the fuzzy topological relations where fuzziness is represented at the relations level.

Fuzzification process of Allen relations doesn't depend upon particular choice of fuzzy membership function. Trapezoidal membership function (described in Eq. 5.1) is used due to flexibility in shape. Let $r(I,J)$ be an Allen relation between segments I and J where $I \in A$(argument object) and $J \in B$ (reference object), r' is the distance between $r(I,J)$ and it's conceptional neighborhood. We consider a fuzzy membership function such as defined in Eq. 5.1 and let $\mu : r' \longrightarrow [0,1]$. The fuzzy Allen relations are defined as:

- $f_{<}(I,J) = \mu_{(-\infty,-\infty,-b-3a/2,-b-a)}(y)$
- $f_{>}(I,J) = \mu_{(0,a/2,\infty,\infty)}(y)$
- $f_{m}(I,J) = \mu_{(-b-3a/2,-b-a,-b-a,-b-a/2)}(y)$
- $f_{mi}(I,J) = \mu_{(-a/2,0,0,a/2)}(y)$
- $f_{o}(I,J) = \mu_{(-b-a,-b-a/2,-b-a/2,b)}(y)$
- $f_{oi}(I,J) = \mu_{(-a,-a/2,-a/2,0)}(y)$
- $f_{f}(I,J) = min(\mu_{(-(b+a)/2,-a,-a,+\infty)}(y), \mu_{(-3a/2,-a,-a,-a/2)}(y), \mu_{(-\infty,-\infty,z/2,z)}(x))$
- $f_{fi}(I,J) = min(\mu_{-b-a/2,-b,-b,-b+a/2}(y), \mu_{(-\infty,-\infty,-b,-(b+a)/2)}(y), \mu_{(z,2z,+\infty,+\infty)}(x)$
- $f_{s}(I,J) = min(\mu_{-b-a/2,-b,-b,-b+a/2}(y), \mu_{(-\infty,-\infty,-b,-(b+a)/2)}(y), \mu_{(-\infty,-\infty,z/2,z)}(x)$
- $f_{si}(I,J) = min(\mu_{(-(b+a)/2,-a,-a,+\infty)}(y), \mu_{(-3a/2,-a,-a,-a/2)}(y), \mu_{(z,2z,+\infty,+\infty)}(x)$
- $f_{d}(I,J) = min(\mu_{(-b,-b+a/2,-3a/2,-a)}(y), \mu_{(-\infty,-\infty,z/2,z)}(x)$
- $f_{di}(I,J) = min(\mu_{(-b,-b+a/2,-3a/2,-a)}(y), \mu_{(z,2z,+\infty,+\infty)}(x)$

where $a = min(x, z), b = max(x, z)$ and x and z are the lengths of segment (I) and segment (J) respectively and y is the difference between the minimum value of projection points of $A_\theta(v)$ and maximum value of projection points of $B_\theta(v)$.

Most of relations are defined by one membership function like $f_<$, $f_>$, f_m, f_{mi} and others are represented as a conjunction of more than one fuzzy membership function, such as $f_f, f_{fi}, f_s, f_{si}, f_d, f_{di}$. In fuzzy set theory, sum of all the relations is one, this gives the definition for fuzzy *equal* relation. Fuzzy Allen relations are not JEPD because there exist at least two relations between two spatial objects. All these equations assign a numeric value to a spatial relation.

5.3.4 Treatment of Longitudinal Section

During the decomposition process of an object into segments, there can be multiple segments for a line depending on object shape and boundary that is called longitudinal section. Different segments of a longitudinal section are at a certain distance and these distances might affect end results. Fuzzy *T-norms*, *T-conorms* and fuzzy weighted operators are used for fuzzy integration of available information, here for simplicity only *T-conorm* is used.

$$\mu_{(OR)}(u) = max(\mu_{(A)}(u), \mu_{(B)}(u))$$

Where μ_A, μ_B represents the membership value for first and second segments of a longitudinal section. In this case each Allen relation has a fuzzy grade and objective is to accumulate the best available information. The choice for this operator is discussed in [Salamat 2011c]. When fuzzy operator *OR* is used, only one fuzzy value contributes for the resultant value that is *maximum*.

5.3.5 Normalization of Histogram of Fuzzy Allen Relations

Histograms of fuzzy Allen relations represent the total area of subregions of A and B that are facing each other in given direction θ. Mathematically it can be written as [Salamat 2009]

$$\mathbf{F}_r^{AB}(\theta) = \int_{-\infty}^{+\infty} F_r(\theta, A_\theta(v), B_\theta(v)dv \tag{5.4}$$

where $F_r(\theta, A_\theta(v), B_\theta(v))dv = r(I_k, J_k)$. In discrete space this integral can be written as sum of the surface areas.

$$\mathbf{F}_r^{AB}(\theta) = (X + Z)\sum_{k=1}^{n} r(I_k, J_k) \tag{5.5}$$

where Z is the area of reference object and X is area of augment object in direction θ, n is total number of segments to be treated and $r(I_k, J_k)$ is an Allen relation for segments (I_k, J_k) and $k = 1, 2, .., n$.

These histograms can easily be normalized by dividing an Allen relation by sum of them for every θ. It is represented by $\lceil F_r^{AB}(\theta) \rfloor$ where $r \in \mathcal{A}$. $\lceil F_r^{AB}(\theta) \rfloor =$

$\frac{F_r^{AB}(\theta)}{\sum_{\rho \in A} F_\rho^{AB}(\theta)}$. Each fuzzy Allen relation has its own weight in a specified direction θ. These normalized weights can be used to define the quantitative fuzzy directions.

5.3.6 Properties of Fuzzy Allen Relations

These fuzzy Allen relations verify some properties called reorientation of relations. Such as

- $f_>(\theta) = f_<(\theta + \pi), f_{mi}(\theta) = f_m(\theta + \pi)$
- $f_{oi}(\theta) = f_o(\theta + \pi),\ f_f(\theta) = f_s(\theta + \pi)$
- $f_{si}(\theta) = f_{fi}(\theta + \pi),\ f_=(\theta) = f_=(\theta + \pi)$
- $f_{di}(\theta) = f_{di}(\theta + \pi), f_d(\theta) = f_d(\theta + \pi)$

These relations can be written as $\mathcal{A}_1 = \{<, m, o, s, d, f_i, d_i, =\}$ and their inverses as $\mathcal{A}_2 = \{>, m_i, o_i, f, d, s_i, d_i, =\}$. Two-dimensional eight topological relations are possible combination of eight independent Allen relations. These relations and their reorientation show that the whole 2D space can be explored with the help of Allen relations using the oriented lines varying from $(0, \pi)$.

5.4 Topological and Directional Relations

This section consists of five subsections. In subsection 5.4.1, fuzzy membership functions for directional relations and possible combination of $1D$ Allen relations are described. Subsection 5.4.2 discusses relationship between $1D$ Allen and topological relations. An algorithm for defuzzification of fuzzy relations is proposed in subsection 5.4.3.

5.4.1 Fuzzy Membership Function for Directional Relations

The system of equations defined in section (5.3.3) assigns a numeric value to a spatial relation in a direction θ. To asses qualitative directions, directional fuzzy sets are used. A number of fuzzy membership functions has been proposed for assessing the directional relations, these functions include the trigonometric, triangular and trapezium membership function or by means of favorable and unfavorable forces in force histograms method.

In this paper, we prefer to use trigonometric functions which are easy to implement. The function $f(\theta)$ to model the direction *Right_ of* for four directional system, should have conditions, $f(+\frac{\pi}{2}) = 0 = f(-\frac{\pi}{2})$ and $f(0) = 1$. This function should be increasing in the interval $(-\frac{\pi}{2}, 0)$ and decreasing in the interval $(0, \frac{\pi}{2})$. At $\frac{\pi}{4}$ both relations *Above* or *North* and *Right_ of* have equal values and similarly at $-\frac{\pi}{4}$ both *Below* or *South* and *Right_ of* have equal values. $Cos^2(\theta) = Sin^2(\theta) = \frac{1}{2}$ at $\pm\frac{\pi}{4}$ are the best choices.

To formulate eight directions, straightway process is to narrow the interval, i.e., $f(+\frac{\pi}{4}) = 0 = f(-\frac{\pi}{4})$ and $f(0) = 1$ for relation *Right_ of*. For this purpose double angle trigonometric functions are used which satisfies all the above cited conditions. Directions are represented as $\{E, NE, N, NW, W, SW, S, SE\}$ with meanings *East, North_ East, North, North_ West, West, South_ West, South* and *South_ East*.

The angle distribution is taken to the half plane so opposite Allen relations are used to define the opposite directions except the direction East and West where union of both relations are used. This exception is used due to the reorientation of relations and domain of *East* and *West* relations lie in different Allen relations (domain of *East* directional relation is $[\frac{-\pi}{4}, \frac{\pi}{4}]$ and $f_<(\theta) = f_>(\theta + \pi)$, using this combination of relations results in the decrease of computational complexity due to angle distribution from $[0, \pi]$). Mathematically these relations can be written as

- $f_E = \sum_{\theta=0}^{\frac{\pi}{4}} \mathcal{A}_2 \times cos^2(2\theta) + \sum_{\theta=\frac{3\pi}{4}}^{\pi} \mathcal{A}_1 \times cos^2(2\theta)$
- $f_W = \sum_{\theta=0}^{\frac{\pi}{4}} \mathcal{A}_1 \times cos^2(2\theta) + \sum_{\theta=\frac{3\pi}{4}}^{\pi} \mathcal{A}_2 \times cos^2(2\theta)$
- $f_N = \sum_{\theta=\frac{\pi}{4}}^{\frac{3\pi}{4}} \mathcal{A}_{r_2} \times cos^2(2\theta)$
- $f_S = \sum_{\theta=\frac{\pi}{4}}^{\frac{3\pi}{4}} \mathcal{A}_{r_1} \times cos^2(2\theta)$
- $f_{NE} = \sum_{\theta=0}^{\frac{\pi}{2}} \mathcal{A}_2 \times sin^2(2\theta)$
- $f_{NW} = \sum_{\theta=\frac{\pi}{2}}^{\pi} \mathcal{A}_2 \times sin^2(2\theta)$
- $f_{SW} = \sum_{\theta=0}^{\frac{\pi}{2}} \mathcal{A}_1 \times sin^2(2\theta)$
- $f_{SE} = \sum_{\theta=\frac{\pi}{2}}^{\pi} \mathcal{A}_1 \times sin^2(2\theta)$

Where $\mathcal{A}_i \in \mathcal{A}_i, i = 1, 2$ given in section 5.3.6 are the fuzzy Allen relations with $\mathcal{A}_2$ is the reorientation of $\mathcal{A}_1$ and $f \in \{D, M, PO, TPP, NTPP, TPPI, NTPPI, EQ\}$ is a topological relation (*Disjoint, Meet, Partially overlap, Tangent proper part, Non Tangent proper part, Tangent proper part Inverse, Non Tangent proper part Inverse* and *Equal*). In this way the relations are manipulated as a (8×8) matrix (C(i,j)) where row hold the topological relations and columns have the qualitative directional aspects of 2D scene information. Relations are expressed in numeric values where each value (c(i,j)) represents the percentage area of two objects under a specific topological relation in that qualitative direction.

5.4.2 Driving Two-Dimensional Topological Relations from Allen relations

Simple $1D$ definitions can't be extended and applied directly to a $2D$ space. Some assumptions have to be adopt and these definitions are extended to two-dimensional case through logic.

In case of merging the topological and directional relations some of the topological relations depend upon a finite direction and limited set of points such as *EC, PO, TPP, TPPI.* Relations like *NTPP, NTPPI, EQ* hold if the relation holds in all directions. The topological relations which exist in a finite directions, they share with another topological relation existing in another direction. The relations which exist in all directions, they share directional relations. Temporal Allen relations represent the eight topological relations in spatial domain $\mathbb{R}$, these relations can be extended to spatial domain $\mathbb{R}^2$ through the logical implication. These relations are defined as:

Disjoint $D(A, B)$**:** In RCC theory, two objects are disjoint when there doesn't exist a connection relation (e.g., $Disjoint(A, B) \Rightarrow \neg C(A, B)$). A fuzzy connection relation based on nearness[1] is defined using the *resemblance relation.* This states that *if two objects are at a certain distance and resemblance relation is zero degree then objects will be disjoint.* In our method, functions f_m and f_{mi} plays the same role as the resemblance relation $R_{\alpha,\beta}(A, B)$ with variable $\alpha = 0$ for the $1D$ interval. In such a case both functions, $f_>$ and $f_<$, capture the semantics of $\neg R_{\alpha,\beta}(I, J)$, representing disjoint topological relation between two intervals on a real line, along with additional information that argument interval either *before* or *after* the reference interval. This definition of *Disjointness* of $1D$ intervals can be extended to two-dimensional objects by following relation

$$\text{(A,B) are disjoint} \Leftrightarrow \{\vee\theta \in [0, \pi] \text{ (I,J) are disjoint}\} \tag{5.6}$$

where A, B are $2D$ objects. Now follow Eq. 5.6, objects have fuzzy disjoint topological relation in a direction θ if all parallel segments are topologically disjoint in direction θ. A fuzzy disjoint topological relation exists in 2D domain if it exists in all directions, i. e., all the $f_<$ or $f_>$ behave like crisp functions and have a unit value along all oriented lines in $[0, \pi]$. All the other functions have zero values and objects are at certain distance. These relations are explained in table 5.1.

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[1] S. Schockaert et al. [Schockaert 2008b] defined a connection relation based on nearness as

$$R_{\alpha,\beta}(A, B) = \begin{cases} 1 & \text{if } d(a, b) \leq \alpha \\ 0 & \text{if } d(a, b) > \alpha + \beta \\ \frac{\alpha+\beta-d(a,b)}{\beta} & \text{otherwise } (\beta \neq 0) \end{cases}$$

Where $a \in A, b \in B$ and d is a Euclidian distance between objects A and B. They considered all of three, i. e., relation R, and both objects (A, B) are fuzzy, but the connection will remain fuzzy if we consider the resemblance relation R as fuzzy relation and A, B as crisp regions.

Meet $M(A,B)$**:** According to the topological view point, two objects have a $Meet$ topological relation when they share at least one boundary point and they don't share the interior regions of two objects. In our method of defining the topological relations, two functions are introduced f_m and f_{mi}. Both functions play similar role like the resemblance relation defined by S. Schockaert et al. in [Schockaert 2008b], where degree of closeness is one, when the intervals share a common point and smooth transition from *closeness to apart* depends upon the size of the smaller interval. Both functions capture some additional semantics regarding position of the interval, either the interval is after or before the reference interval. To make the sense in $2D$ scene, overall $Meet$ relation holds if at least one f_m or f_{mi} has some non zero value for any $\theta \in [0, \pi]$ and all the other directions have the *Disjoint* topological relation. These relations are explained in table 5.2.

Partially Overlap $PO(A,B)$**:** Partially overlap relation in topology (or overlap) exists when two objects share their interior region, in such a case their boundaries intersect at least from two points. In $\mathbb{R}$ the functions f_o and f_{oi} capture the overlap semantics on the interval along with the directional information. When $2D$ objects are decomposed into $1D$ segments, each pair of segments may have the different topological relations in different directions, e.g., the object's segments in direction θ_i may have overlap relation while in direction θ_j may have *meet* topological relation and in direction θ_k both the segments may be disjoint, where $i \neq j \neq k \wedge \theta_i, \theta_j, \theta_k \in [0, \pi]$. This shows that overlap in $2D$ objects have other topological relations between $1D$ segments of both objects. These relations are explained in table 5.3.

Tangent Proper Part $TPP(A,B)$ **and** $TPPI(A,B)$**:** $TPP(A,B)$ topological relation holds in $2D$ space when $A \subset B$ and they share a common point on the boundary. In $1D$ spaces the relation f_s or f_f shares the same semantics, if f is a crisp relation. In case of f is a fuzzy relation they represent the fuzzy semantics. When a $2D$ object is decomposed into $1D$ segments, in a limited number of directions they have the relation f_s or f_f while in other directions object is contained in the container, d (*during*) Allen relation exists. Similarly in case of $TPPI$ topological relation, in some directions the $1D$ segments share the common boundary point and f_{si} and f_{fi} fuzzy Allen relations exist while in other directions the d_i Allen relation exists. In our system of defining the topological and directional relations information, TPP ($TPPI$) relation exist in some direction while in other directions $NTPP$ (respectively $NTPPI$) relation exists. These relations are explained in tables 5.4 and table 5.5.

Non Tangent Proper Part $NTPP(A,B)$**:** This topological relation holds when argument object is contained in reference object and both objects don't share boundary. It means that object is contained in container object in all directions. When $1D$ Allen relations are applied to the spatial domain, relation d

captures same semantics. If each segment of an argument object is contained in the segment of a reference object in all directions, then argument object is topologically inside the reference object and they don't share boundary. Representing the fuzzy semantics, it is observed that this relation holds when there is a certain distance between boundaries of both objects. Situations having such relations are explained in first row in table 5.6 of section 5.6.6.

Non Tangent Proper Part Inverse $NTPPI(A, B)$**:** This topological relation holds when the container is an argument object and reference object is contained in the argument object, e. g., object B is a reference and A is an argument object and object B is contained in object A, both objects don't share boundary points. It means that the object is contained in the container object in all directions. When the $1D$ relations are applied to the spatial domain, the relation d_i captures the same semantics. If each segment of an argument object is contained in the segment of a reference object and this relation holds in all the directions, then there exist topological relation $NTPPI$ for a pair of $2D$ objects. As explained in second row in table 5.6 of section 5.6.6.

Equal $EQ(A, B)$**:** Two objects A and B are equal if they share the interior, exterior and boundary. Semantically both objects have the same interior, boundary and exterior. In RCC system equal topological relations are defined as $EQ(A, B) \equiv_{def} P(A, B) \wedge P(B, A)$. Geometrically two regions are called equal when both objects seem equal in all direction. Function $f_{=}$ in our system captures the semantics equal if two intervals are equal. When this relation is applied to $1D$ segments of $2D$ objects, segments of both objects must be equal in all directions. These results that degree of EQ topological relation distributed equally in all directions. As explained in third row in table 5.6 of section 5.6.6.

5.4.3 Defuzzification of Relations

Numerical values for a relation are stored in a matrix called fuzzy matrix of relations. The relations are manipulated in (8×8) matrix where topological relations are represented into rows and columns show directional distribution of each topological relation. The normalized histogram of fuzzy Allen relations represent a percentage surface area of two objects in a given direction θ. Multiplication with the directional fuzzy set normalises this histogram over the whole $2D$ space. Hence, each entity of this matrix represents percentage surface area of two objects, $C(i, j)$ represents the i^{th} topological relation in j^{th} direction. Rows and columns of the representation matrix are explained below[2].

[2] It is only representation and rows and columns explain how the relations are labeled

Explanation of rows and columns in representation matrix								
Row (i)	1	2	3	4	5	6	7	8
Topological relation	*D*	*M*	PO	*TPP*	*NTPP*	*TPPI*	*NTPPI*	*EQ*
Column (j)	1	2	3	4	5	6	7	8
Directional relation	E	NE	N	NW	W	SW	S	SE

These relations are not Jointly Exhaustive and Pairwise Disjoint (JEPD). To approximate $2D$ topological and directional relation, an algorithm for defuzzification of relations are proposed.

Defuzzification can be performed by a multiple ways, topological relations don't depend on numerical values. Two dimensional objects are decomposed into one-dimensional segments, two dimensional overlap topological relations give rise to all the one dimensional topological relations in different directions. Another reason is that our question at hand is that where a topological relation holds between the object pair, we proceed for extracting topological relation then proceed for directional relation. Fuzzy directional relations are distributed over multiple directions and reasoning about directional relations is performed by plausible occurrence of relations so we choose the maximum numerical value of directional relations.

Algorithm 2 Algorithm for defuzzification of 2D fuzzy topological and directional relations

for $i, j = 1$ to 8 **do**
  if $C(3, j) \neq 0$ **then**
    topological relation is overlap and directional relation is $max_j(C(2, j), C(3, j), C(4, j), C(6, j))$
  else if $C(4, j) \neq 0$ **then**
    topological relation is *TPP* and directional relation is $max_j(C(4, j))$
  else if $C(6, j) \neq 0$ **then**
    topological relation is *TPPI* and directional relation is $max_j(C(6, j))$
  else if all $C(5, :)$ are approximately equal for all j **then**
    topological relation will be NTPP
  else if all $C(7, :)$ are approximately equal for all j **then**
    topological relation is *NTPPI*
  else if all $C(8, :)$ are approximately equal for all j **then**
    topological relation will be *EQ*
  else if $C(2, j) \neq 0 \wedge C(i, j) = 0 \vee i \geq 3$ **then**
    topological relation is meet M and directional relation is $max_j(C(2, j))$
  else
    topological relation is Disjoint and directional relation is $max_j(C(1, j))$
  end if
end for

As discussed in section 5.4.2, that when a 2D object having a partially overlap relation is decomposed into 1D segments, then almost all the relations are possible between the segments in different directions. This distribution of $1D$ spatial relations depend upon the size of objects and overlapping surface area of objects. Due to this reason extraction method must be started from overlap relation. If partial overlap relation don't exist any where then we search for other possible topological relation. Proposed algorithm for 2D topological relation from 1D is described as algorithm 2. This algorithm releases a pair of JEPD topological and directional relations. These relations are possible to represent in a neighborhood graph for topological and directional relations.

5.5 Neighborhood Graph

Conceptual neighborhood graphs describe the possible continuous transitions. Graphs in topological relations describe the topological transitions between object pair when topological transformations are applied. They denote the movement, expansion and contraction of objects. *On* the other hand neighborhood graph in directional relations captures only movement.

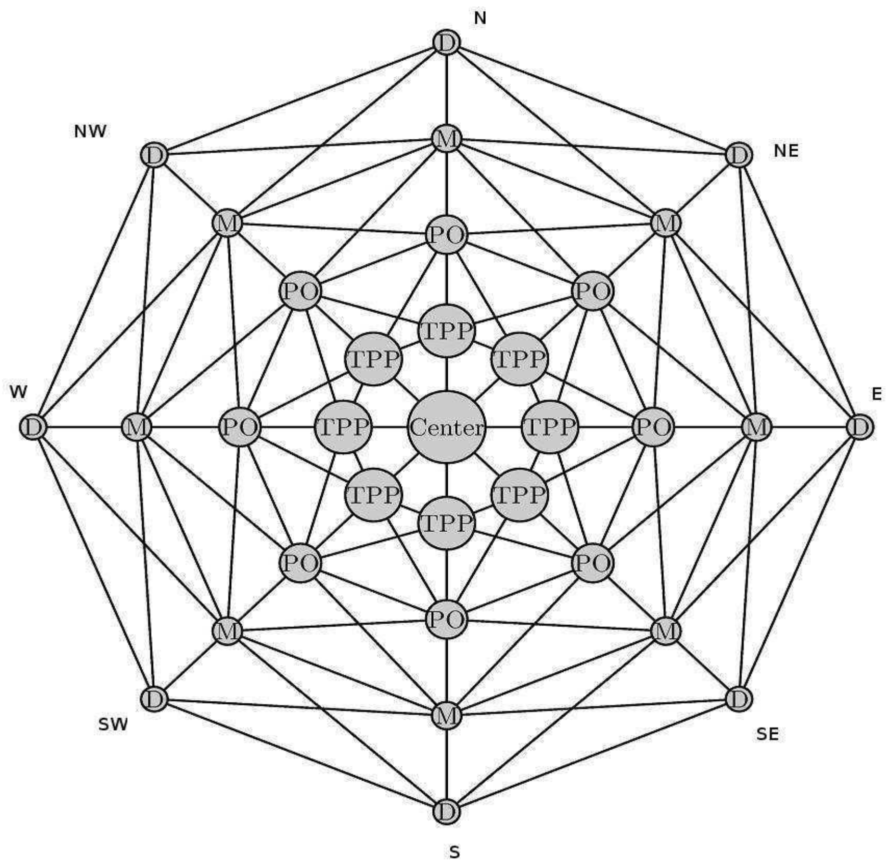


Figure 5.3: Neighborhood graph in the system of combined topological and directional relations (CTD) method.

These neighborhood graphs are frequently used for common sense reasoning and path planing in many A.I. fields. These graphs are developed between the crisp relations. Combined topological and directional relations, after the defuzzification process behave like the crisp relations. A graph can describe the three type of movements, here for simplicity we represent one part of the complete graph.

In Fig. 5.3, it is shown that every node of the neighborhood graph has eight edges except from the nodes representing the D and TPP relations. Each node represents a pair (α, β) of relations where α represents the topological and β represents the orientation relation. In this neighborhood graph each node has eight neighbors *circular, straight* and *diagonal* neighbors, respectively represents *directional, topological* and *directional and topological* neighbors.

5.6 Experiments

The fuzzy relations are manipulated in (8×8) matrix where topological relations are represented into rows and columns show the directional distribution of each topological relation. Values in each cell represents the strength of the relation between pair of the objects. Throughout this paper reference object B is represented by dark grey color and light grey object represents the argument object A. Different set of experiments are shown, some other examples are presented in annex (section 10.3). In these tables, object pairs are represented in first column, second column shows the graphical representation of results. In third column, results for the defuzzified topological and directional relations are represented, these results are produced by the algorithm 2.

5.6.1 Fuzzy Disjoint (D) Topological Relation

In this section the fuzzy disjoint topological relations in different directions are considered. As in table 5.1, first column represents the object spatial position and second column represents the overall 2D topological and directional relations and in third column the generated defuzzified topological and directional relation by algorithm 2 are depicted. As object A changes its position, its directional relation also changes, in first row of table 5.1, argument object lies in E to the reference object B. The Algorithm generates the topological relation *Disjoint* and directional relation *East*. In second row, argument object lies in direction *North_ West*, it histogram representation shows that in the *North_ West* direction, it has the maximum membership value, as a result, algorithm allocates it a directional relation *North_ west*.

5.6.2 Fuzzy Meet (M) Topological Relation

Fuzzy meet M relations in topology exist when the objects are exactly meeting or very close to each other and it seems that they are sharing the boundary. The table 5.2 represents the fuzzy *meet* topological relation (M).

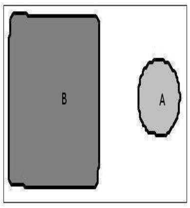
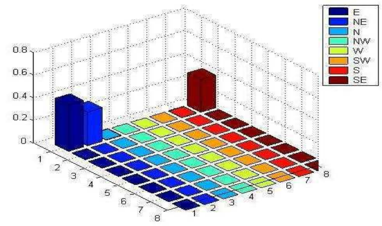
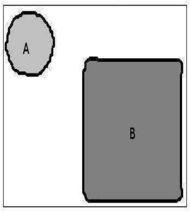
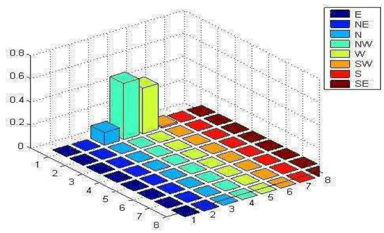
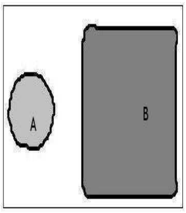
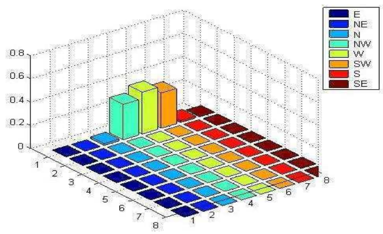
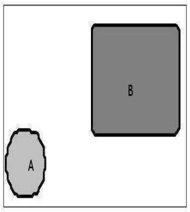
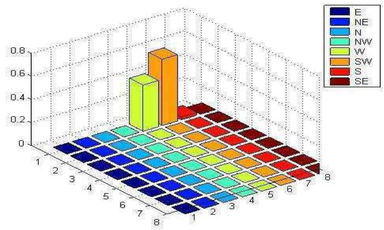
Object pairs	Matrix rep. of relations	Algo. Output
		Topological rel.= Disjoint Direction= East
		Topological rel.= Disjoint Direction= North_West
		Topological rel.= Disjoint Direction= West
		Topological rel.= Disjoint Direction= South_West

Table 5.1: Topological relation D

First column shows the object locations at different orientations of argument object A with respect to the reference object B. First rows shows that the argument object A touches the object B from the *East* direction (first column). Second column shows its histogram representation of its relations where the relations are shared between the *Disjoint* and *Meet* (second column) and their 2D topological relation generated by the algorithm. Similarly second row represents the argument object touches the reference object from *North* direction, for this object pair algorithm produces the result that M topological relation with *North* directional relation. In the third row argument object seems touching from the north_ west direction, hence the output of algorithm shows that M topological relation holds with directional relation *North_ West*. Similarly argument object in last row nearly touches from west as a result the algorithm for defuzzification of spatial relations produces the

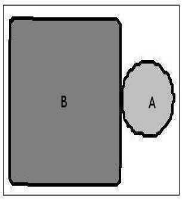
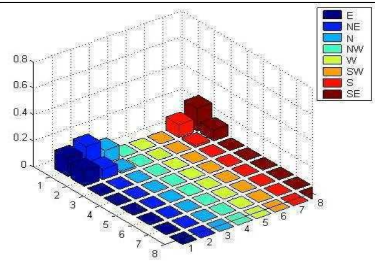
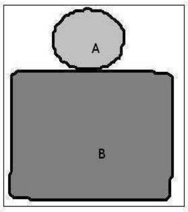
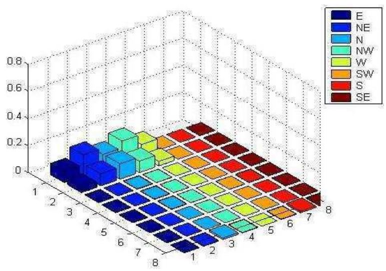
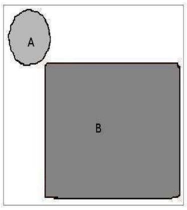
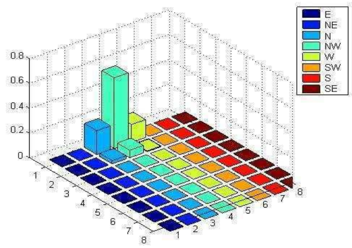
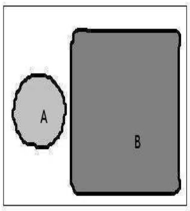
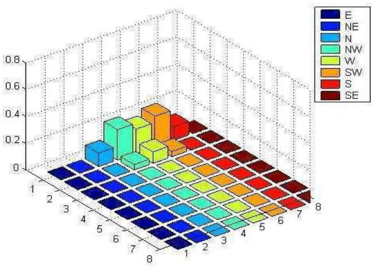
<table>
<tr><th>Object pairs</th><th>Matrix rep. of relations</th><th>Algo. Output</th></tr>
<tr><td>
</td><td>
</td><td>Topological rel.= M
Direction= East</td></tr>
<tr><td>
</td><td>
</td><td>Topological rel.= M
Direction= North</td></tr>
<tr><td>
</td><td>
</td><td>Topological rel.= M
Direction= North_West</td></tr>
<tr><td>
</td><td>
</td><td>Topological rel.= M
Direction= West</td></tr>
</table>

Table 5.2: Topological relation M

resultant direction as *West*.

5.6.3 Fuzzy Overlap (PO) Topological Relation

In this example we consider the overlapping objects in different directions. The object relative position, topological and directional relations and the topological and directional relations generated by the algorithm 2 are described in table 5.3.

In the first column, object pairs are represented having the topological relation *Partially_ Overlap* in different directions, second column represents topological and

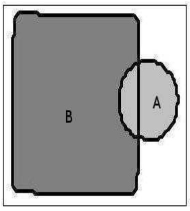
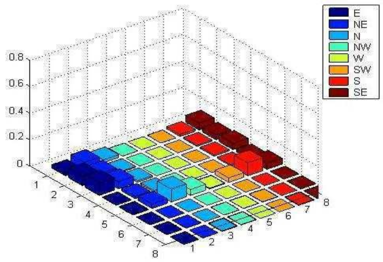
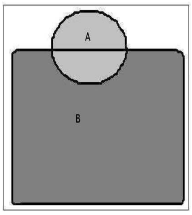
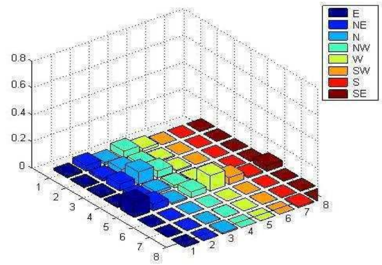
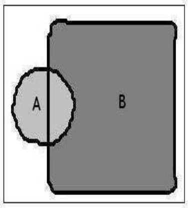
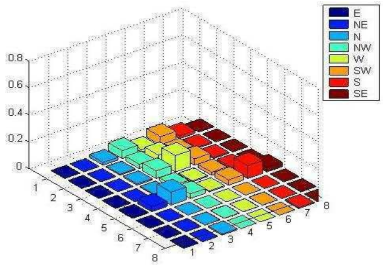
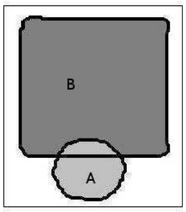
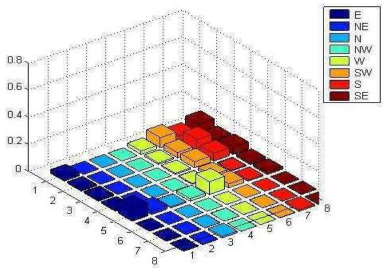
Object pairs	Matrix rep. of relations	Algo. Output
		Topological rel.= PO Direction= East
		Topological rel.= PO Direction= North
		Topological rel.= PO Direction= West
		Topological rel.= PO Direction= South

Table 5.3: Object pairs with PO topological relation

directional relations in a histogram representation. The results generated by algorithm are denoted in third column. As soon as object changes their position, topological and directional relations matrix also changes. The directional relation between overlapping objects depends upon the relative size, shape and overlapping surface area of objects. In all the above cited examples, object A is smaller relatively to object B.

5.6.4 Fuzzy *TPP* Topological Relation

In crisp topological relations, this relation holds when the argument object lies inside the reference object and share the boundary with the reference object. In fuzzy relations, this relation (*TPP*) holds when the argument object lies inside the reference object near the edge. In this method an entity in the matrix represents the degree of topological relation in a particular direction, as a result this relation holds in a particular direction along with the other fuzzy topological *NTPP* relation in other directions, for example in first row of the table 5.4 where object lies near the eastern edge of reference object, there relations show that highest value of topological relation *TPP* exists in direction East while in all other directions fuzzy topological relation *NTPP* holds.

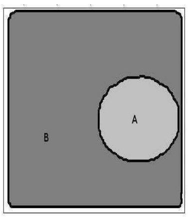
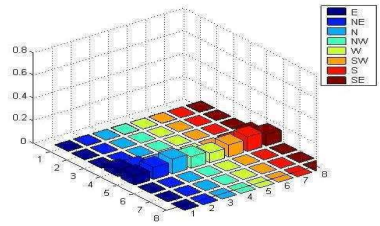
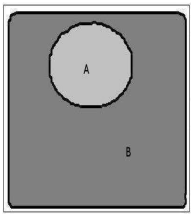
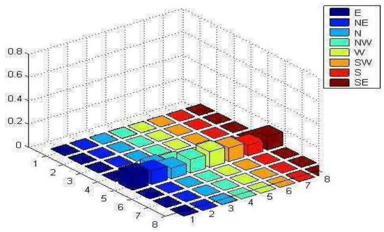
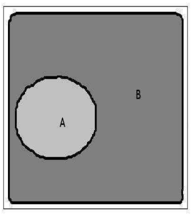
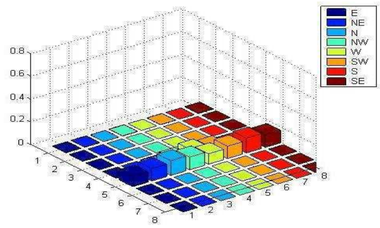
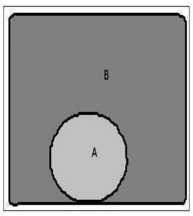
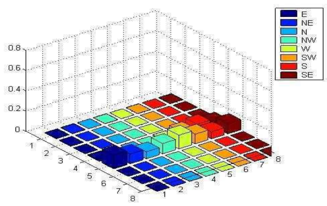
Object pairs	Matrix rep. of relations	Algo. Output
		Topological rel. = TPP Direction= East
		Topological rel.= TPP Direction= North
		Topological rel.= TPP Direction= West
		Topological rel.= TPP Direction= South

Table 5.4: Object pairs with *TPP* topological and their directional relations

5.6.5 Fuzzy *TPPI* Topological Relation

In this example reference object B is considered inside the argument object A. The object pairs are shown in first column of the table 5.5. Second column shows the histogram representation of relations and third column shows the results produced by the algorithm 2.

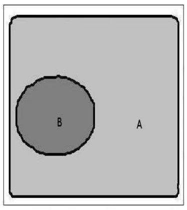
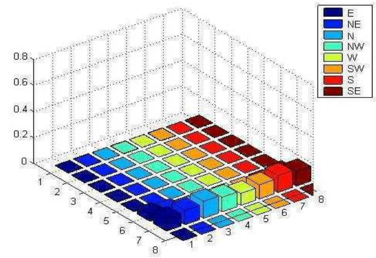
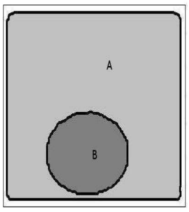
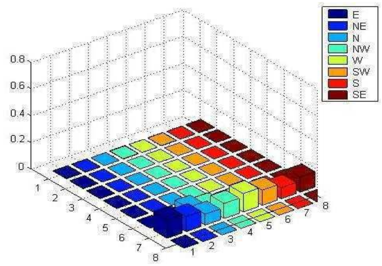
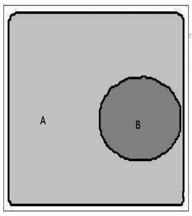
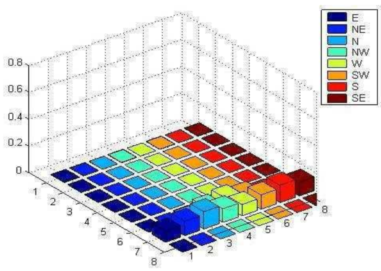
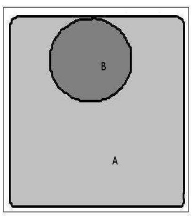
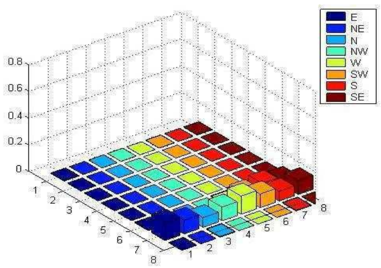
Object pairs	Matrix rep. of relations	Algo. Output
		Topological rel.= TPPI Direction= East
		Topological rel.= TPPI Direction= North
		Topological rel.= TPPI Direction= West
		Topological rel.= TPPI Direction= South

Table 5.5: Object pairs with *TPPI* topological and their directional relations

Obviously it is an inverse relation as a result visually reference object seems to be in opposite direction of the directional relation. In first row of the table, visually reference object lies near the *West* edge of the argument object, but its relation is

East, this is due to the inverse topological relation. When the objects commute, the topological and directional relations become inverse, for example consider the object pairs in third row of table 5.4 and first row of table 5.5, both represents the same object pair, when the objects commute, both the topological and directional relations become inverse to each other. Similarly for the other object pairs in table 5.4 and table 5.5, same object is used to represent the object pair when objects commute the topological and directional relations become inverse.

5.6.6 Fuzzy *NTPP, NTPPI* and *EQ* Topological Relations

In these examples we consider the all those topological relations which must exist in all the directions.

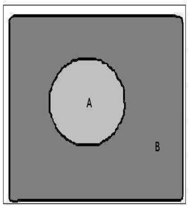
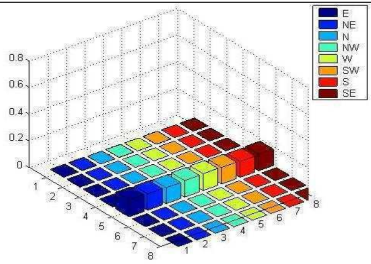
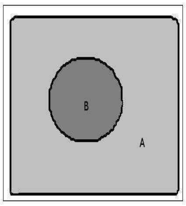
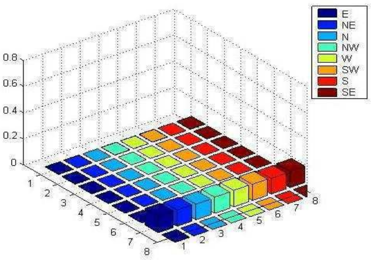
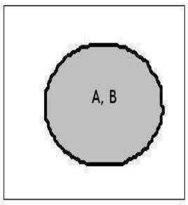
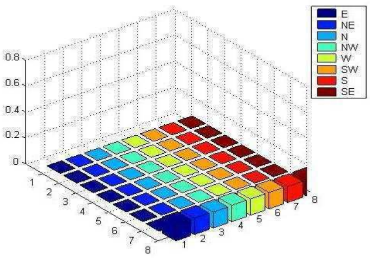
Object pairs	Matrix rep. of relations	Algo. Output
		Topological rel.= NTPP Direction= All
		Topological rel.= NTPPI Direction= All
		Topological rel.= EQ Direction= All

Table 5.6: Topological relations NTPP, NTPPI and EQ

Here, it is explained that a single topological relation must be held in all the directions for reference the object pairs represented in first column of the table 5.6. In first row, argument object A lies inside the reference object B as a result the *NTPP* relations holds equally in all directions, these results are represented in first row of the table, the algorithm generates the directional relation *All* which means that this topological relation holds in all directions. In second row reference object B lies inside the argument object, as a result inverse topological relations hold in all directions. Third rows shows the situation, when both objects are equal in size, thus fuzzy equal relation holds if both objects have equal size in all directions.

5.7 Conclusion and Future Work

In this paper a new method for finding fuzzy topological and directional relations was proposed and all the topological relations are generated using fuzzy Allen relations and directions are evaluated with the help of trigonometric functions. This method deals with fuzziness at two levels, fuzziness in the topological relations due to their geometrical description and fuzziness in directional relations. This method also deals objects with holes or convex objects. Longitudinal section holds when objects with holes or convex objects are decomposed into 1D segments. A method is described in section 5.3.4 to deal with a longitudinal section. It is a numerical description of relative position of objects and value in each cell represents the strength of the relation between object pair.

This method can be used in designing and managing spatial database, where one single model can answer adequately a query. This method can detect small changes in a spatial scene when implemented to the same pair of objects at two different time instant. In this way this method can replace the implementation of four methods (topological, directional, distance and internal cardinal directional (ICD) relations) of spatial relations which are used to compare a scene.

An algorithm for defuzzification of spatial relations is also given such that we can estimate the 2D fuzzy topological relation along with the directional components. These defuzzified spatial relations are represented by a neighborhood graph. Spatio-temporal relations are the emerging issue in GIS and other sciences and hopefully these results will be helpful in extending this work to a spatio-temporal aspects and fuzzy spatio-temporal reasoning and natural language processing. These results will be used in future to develop the spatio-temporal relations and motion verbs.

Chapter 6

Spatio-Temporal Relations and Modeling Motion Classes by Combined Topological and Directional Relations Method

Abstract

Defining the spatio-temporal relations and modeling motion events are the emerging issue of current research. Motion events are the subclasses of the spatio-temporal relations, where stable and unstable spatio-temporal topological relations and temporal order of occurrence of a primitive event play an important role.

In this paper we proposed a method of spatio-temporal relations based on topological and directional perspective. This method characterized the spatio-temporal relations into different classes according to the application domain and topological stability. This proposes a common sense reasoning and modeling motion events in diverse application with the motion classes as primitives, which describe the changes in topological and directional relations. Topological relations have a locative symmetry, to remove this symmetry we add the directional information in each motion event and these events are defined as a systematic way. This will help to improve the understanding of spatial scenario in spatio-temporal applications.

keywords: Spatio-temporal relations, motion events, event modeling, motion classes.

6.1 Introduction

Automatic event detection in spatio-temporal data is gaining more and more attention in computer vision and video researchers community. Visual scene description takes into account the ontological viewpoint of relative object positions with other objects and spatial relations between them. It is sufficient to emphasize the modeling of moving object's spatial relations such as modeling video events [Li 1997, Markus Schneider 2007]. Modeling spatio-temporal relations between moving objects involve the modeling of motion events such as durative events. Spatial relations for a snapshot is called a primitive event. Durative events are the union of primitive events holding in a sequence with a particular temporal order.

Defining spatio-temporal relations have the two main domains of research, spatio-temporal object and spatio-temporal relations modeling. Cuboid object approximation or three dimensional geometry is used to model the former and for lateral two-dimensional objects occupy different spatial locations at different time points [Worboys 2005]. Several types of logics for mechanizing the spatio-temporal relations and reasoning process are used like interval temporal logic [Allen 1983, Zaidi 2006], point temporal logic [Galton 2003] and propositional model logic [Halpern 1996]. The point temporal logic supports the instantaneous snapshots of the world. A snapshot represents the current situation and a spatio-temporal relation is defined if a particular spatial relation holds for every snapshot during that interval. It is considered that time and space are bounded to each other. Spatio-temporal relations are modeled between moving objects by taking transaction from one snapshot to the next snapshot and spatio-temporal relations are called the spatio-temporal features of a moving object. These features could be used for modeling the spatio-temporal events and reasoning moving objects [Fernyhough 1999].

D. Vakarelov et al. in [Nenov 2008, Vakarelov 2010] provided the strong mathematical and logical bases for defining the general spatio-temporal topological relations and divided them into two categories (i) stable spatio-temporal relations and (ii) unstable spatio-temporal relations. A stable spatio-temporal relation is a relation which holds for every frame or snapshot in the interval and unstable relations are those which exist at least one snapshot during the temporal interval. Some spatio-temporal relations are strictly stable such as *Disjoint*(D), *Non Tangent Proper Part*(NTPP), *Non Tangent Proper Part Inverse*(NTPPI) and *Equal*(EQ) and others may be stable or unstable like *Meet*(M), *Partially Overlap*(PO), *Tangent Proper Part*(TPP), *Tangent Proper Part Inverse*(TPPI). This provides a way to use the spatio-temporal relations in linguistics and modeling motion events.

Most of the existing theories of spatio-temporal relations are domain based. Domain knowledge imposes conditions to a spatio-temporal relation to be topologically stable or unstable. A domain where the spatio-temporal relation is topologically stable, directional or distance relations are unstable such as spatio-temporal relations on the road networks. In defining the motion events or verbs which represents the transitive movement, stability or un-stability of topological relations and sequence occurrence of primitive events play an important role where as directional relations along with the topological relations remove the certain symmetries and help the user to understand the real scene situation. Consider the following two examples.

1. Mr John (object, name of a person) *crosses* (relation) the football ground (object).

2. Mr John (object, name of a person) *crosses* (relation) the football ground (object) from north to south.

In proposition 1, there is no confusion about the topological relation that the object A (John) has a sequence of topological relations changing over time with object B (football ground). There is a symmetry about the directional relations and the user did not know the exact direction of object A to object B before and after the occurrence of spatio-temporal event *cross*. In proposition 2 when directional constraints are added, they remove the confusion about the directional relations and symmetry of topological relations that object A (john) crosses the object B (ground) from *North* to *South*. It justifies that how the topological relations in two objects change, what was the temporal order of occurrence the events?

In our approach, we used Combined Topological and Directional (CTD) relations method [Salamat 2010a, Salamat], which is more suitable for reasoning about moving objects and developing the motion events. A spatial situation is represented by the relationship between the considered objects. It is natural to represented the information using these spatial relations. Events can be expressed by interpreting collective behavior of physical objects over a certain period of time. The main focus of this work is to formalize the spatio-temporal relations and the spatio-temporal events in a systematic way.

In this paper we propose a method for defining the spatio-temporal relations. In this method, a point temporal logic is used for defining the general spatio-temporal relations and spatio-temporal motion events. A general description of topologically stable and unstable spatio-temporal relations are provided and these relations are used for modeling the motion events using the sequence occurrence of primitive events. A directional constraint is also introduce in modeling the motion events, this removes the symmetry of topological relations and provides a sound description of spatial situation.

This paper is arranged as follows, related work is discussed in Section 6.2 and Section 6.3 composed of preliminary definitions. Section 6.4 explains the combined topological and directional relations method, spatio-temporal relations are defined in Section 6.5 and Section 6.6 composes on geometric representation of some motion events and these motion events are defined in Section 6.7. Section 6.8 concludes the paper.

6.2 Related Work

A moving object occupies the different positions at different time points and relative motion means that the object changes its position with respect to another object. This relative motion can be studied through different aspects of space and spatial relations are one of them. A spatial relation for a snapshot is considered as a primitive event, then spatial relations between moving objects for an interval are characterized as spatio-temporal relations and sequence occurrence of primitive events over a period of time is known as spatio-temporal events.

Commonly adopted approaches are qualitative and domain based such as Qualitative Trajectory Calculus (QTC) [de Weghe 2006, Maeyera 2005]. This calculus describes the relation between moving point objects. Stewart Hornsby et al. [Hornsby 2008] modeled the different spatio-temporal relations between moving objects on road network. All these relations represent certain class of motion verbs and objects are approximated as point objects. When the objects are under motion, especially on road networks, the relations are purely directional relations where the objects change their position, but don't change the topological structure of scene.

A mereo-topological approach is extend to define the spatio-temporal relations and a notation of temporal slice is used. A temporal slice is called an episode of history for a given interval(see [Muller 2002, Ibrahim 2007a]). The primitive events are defined using Allen's temporal logic and defining the relation *holds(P,i)*(Property P holds during the time interval i) [Allen 1994]. In this method, interval temporal logic is used and a primitive temporal interval is defined as a smallest interval where the relation doesn't change. For composite events another property *"occurs"*, defined as *occurs(e,i)= event e occurs during the interval i* and different *hold* predicates are combined together through logical connectors in a sequential order.

Max. J. Egenhofer and Khaled K. Al-Taha [Egenhofer 1992] described a method for spatio-temporal reasoning based on continuous transection from one state to another state. In this method, topological relations are computed by the 9-intersections method [Egenhofer 1993]. A spatio-temporal reasoning method for reasoning topological changes is developed using the instantaneous point temporal logic and snapshot model is used for representing spatio-temporal data [Bittner 2009, Galton 2002, Adaikkalavan 2005]. This method of topological relations is used by Markus schinder et al. [Erwig 2003, Ma 2004] to model the motion events or motion classes. They model the motion events which involve the topological changes at each frame. Spatio-temporal motion events between moving objects are also effected by the environment regarding its application domain such as modeling the relation *cross, enter, leave* shows that one object is only on concept level. i.e., it is a region of interest which is defined by the observer itself. These regions are also defined for a network, visual tracking, image understanding and activity recognition or freely moving objects like defining the relations for road networks and modeling relations for ecological movement behavior analysis of animals in social sciences.

Spatio-temporal topological relations are divided into two categories, stable and unstable spatio-temporal relations [Vakarelov 2010]. Both stable and unstable spatio-temporal relations play an important role in modeling the spatio-temporal events. A snapshot represents the situation of the world at time instant t which is called a primitive event. Spatio-temporal events are embedded in time, they have temporal boundaries, they have their relationship to time and they don't occupy space but they are related to space. Spatio-temporal events are defined as composite events. Next question is, how their different parts(primitive events) are interrelated? A property *holds_ at(P,t)* is used along with the instantaneous temporal logic. The primitive events are defined for each snapshot during an interval T using the Allen's temporal logic and defining the relation *holds(P,T)*. If interval T has a zero duration, then it represents a snapshot and a relationship between *holds* and *holds_ at* can be represented as $holds(P,T) = \forall t \in T, holds\_at(P,t)$. This extension in definition provides us a relation between *holds* and *holds_ at* such that *a property P holds for an interval T if it holds_ at for every point during the interval.*

Motion events are the subclasses of spatio-temporal relations with a temporal ordering in a primitive events and they don't formulate the necessitate of a calculus, they are only logical representation with temporal ordering of primitive events. Modeling motion events, where the property (P) changes at each instant, it is suitable to use the sequential logic, $seq\_ev(t, e_1, e_2, s)$ (event e_1 occurs before e_2 in S during time t) where seq_ev represents the sequence event. Composite events are the initial conditions dependant, when an initial primitive event occurs at a certain time point t_0, it set up the superclass and name of the possible composite event to be happening.

Topological relations have a certain type of locative symmetries, they don't explain the symmetric location of path and motion direction of argument object. To remove this symmetry in topological relations about the locative perspective, relevant spatial direction is added in motion verbs. In language semantics, motion events are divided into three classes, an initial, median and terminal [Muller 1997] direction based events. Some motion classes are explained with the help of a single directional relation such as *enter, release, touch* and some needs two directions like *cross, graze* etc.

We used CTD method [Salamat] to develop such motion events where topological components plays role for defining the motion events, directional components are used to overcome the locative symmetries. For motion classes where directional components are important, topological components can be used for controlling variables such as moving objects on road networks. We hope this paper will create a bridge between the two approaches of modeling the spatio-temporal events, approach based on interval logic and point logic.

6.3 Preliminary Definitions

In this section we recall some basic definitions which are frequently used throughout the remainder of the paper.

Fuzzy membership function: A membership function μ in a set X is a function $\mu : X \rightarrow [0, 1]$. Different fuzzy membership functions are proposed according to the requirements of the applications. For instance, trapezoidal membership function is defined as

$$\mu(x; \alpha, \beta, \gamma, \delta) = max(min(\frac{x-\alpha}{\beta-\alpha}, 1, \frac{\delta-x}{\delta-\gamma}), 0) \tag{6.1}$$

it is written as $\mu_{(\alpha,\beta,\gamma,\delta)}(x)$ where $x, \alpha, \beta, \gamma, \delta \in \mathbb{R} \wedge \alpha < \beta \leq \gamma < \delta$.

Fuzzy set: A fuzzy set A in a set X is a set of pairs $(X, \mu(x))$such that $A = \{(x, \mu(x)|x \in X)\}$

where μ represents the fuzzy membership function.

Force histogram: The force histogram attaches a weight to the argument object A that this lies *after* B in direction θ, it is defined as

$$\mathbf{F}^{AB}(\theta) = \int_{-\infty}^{+\infty} F(\theta, A_\theta(v), B_\theta(v))dv \tag{6.2}$$

The definition of Force histogram $\mathbf{F}^{AB}(\theta)$, directly depends upon the definition of real valued functions ϕ, f and F used for the treatment of points, segments and longitudinal sections respectively [Matsakis 1999b]. These functions are defined as

$$\left.\begin{aligned} \phi_r(y) &= \begin{cases} \frac{1}{y^r} & \text{if } y > 0 \\ 0 & \text{otherwise} \end{cases} \\ f(x_I, y^\theta_{IJ}, z_J) &= \int_{x_I+y^\theta_{IJ}}^{x_I+y^\theta_{IJ}+z_J} \int_0^{z_J} \phi(u-w)dw)du \\ F(\theta, A_\theta(v), B_\theta(v)) &= \sum_{i=1..n, j=1..m} f(x_{Ii}, y^\theta_{IiJj}, z_{Jj}) \end{aligned}\right\} \tag{6.3}$$

where n, m represents the number of segments of object A and object B respectively and variables (x, y, z) are explained in figure 6.1. These are the definitions of Force histograms, directly depending upon the definition of function ϕ. Notice that $\mathbf{F}^{AB}(\theta)$ is actually a real valued function.

6.4 Combined Topological and Directional Relations Method

In this section we explain different steps of the CTD method. This explains different terms used in computation of combined topological and directional relations.

6.4.1 Oriented Lines, Segments and Longitudinal Sections

Let A and B be two spatial objects and $(v, \theta) \in \mathbb{R}$, where v is any real number and $\theta \in [0, 2\pi]$. Let $\Delta_\theta(v)$ be an oriented line at orientation angle θ and $A \cap \Delta_\theta(v)$ is the intersection of object A and oriented line $\Delta_\theta(v)$. It is denoted by $A_\theta(v)$, called segment of object A. Length of its projection interval on x-axis is denoted by x. Similarly for object B where $B \cap \Delta_\theta(v) = B_\theta(v)$ is segment and length of its projection interval on x-axis is denoted by z. Let y be the difference between the minimum of projection points of $A \cap \Delta_\theta(v)$ and maximum value of projection points of $B \cap \Delta_\theta(v)$(for details [Matsakis 2005]).

In case of polygonal object approximation (x, y, z) can be calculated from intersecting points of line and object's boundary. Only those oriented lines are considered

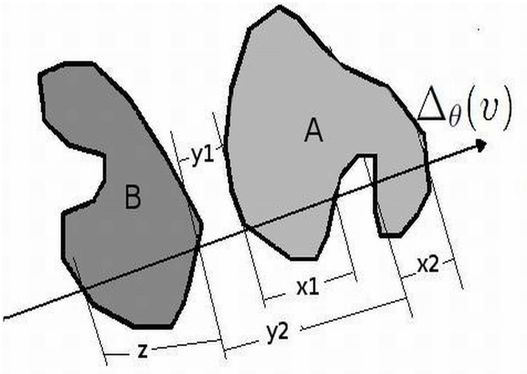


Figure 6.1: Oriented line $\Delta_\theta(v)$, segment as in case of object B, longitudinal section as in case of object A

which passes through at least one vertex of two polygons. If there exist more than one segment, then it is called longitudinal section as in case of $A_\theta(v)$ in figure 6.1.

6.4.2 Allen Temporal Relations in Spatial Domain and Fuzziness

Allen [Allen 1983] introduced the 13 Jointly Exhaustive and Pairwise Disjoint (JEPD) interval relations. These relations are $\mathcal{A} = \{<, m, o, s, f, d, eq, d_i, f_i, s_i, o_i, m_i, >\}$ with meanings *before, meet, overlap, start, finish, during, equal, during_ by, finish_ by, start_ by, overlap_ by, meet_ by, and after.* All the Allen relations in space are conceptually illustrated in figure 6.2. These relations have a rich support for the topological relations and represents the eight topological relations in one-dimensional spatial domain. Fuzzy Allen relations are used to represent nearness based fuzzy topological relations.

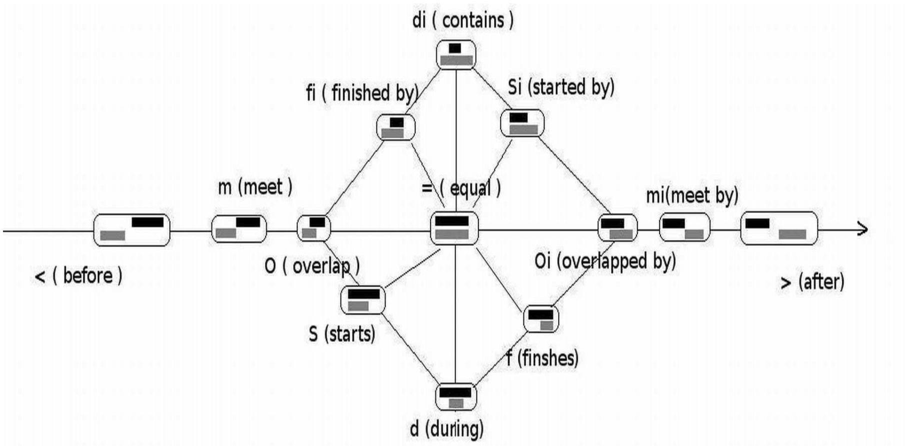


Figure 6.2: Black segment represents the reference object and gray segment represents argument object

Fuzzification process of Allen relations doesn't depend upon particular choice of fuzzy membership function. The trapezoidal membership function (defined in Eq. 6.1) is used due to flexibility in shape. Let $r(I, J)$ be an Allen relation between segments I (Segment of an argument object) and J (Segment of an reference object), r' is the distance between $r(I, J)$ and it's conceptional neighborhood. We consider a fuzzy membership function as $\mu : r' \longrightarrow [0\ 1]$. The fuzzy Allen relations defined in [Salamat 2010b, Salamat 2011c] are

$$\left.\begin{array}{r}
f_<(I,J) = \mu_{(-\infty,\ -\infty,\ -b-3a/2,\ -b-a)}(y) \\
f_>(I,J) = \mu_{(0,\ a/2,\ \infty,\ \infty)}(y) \\
f_m(I,J) = \mu_{(-b-3a/2,\ -b-a,\ -b-a,\ -b-a/2)}(y) \\
f_{mi}(I,J) = \mu_{(-a/2,0,0,a/2)}(y) \\
f_O(I,J) = \mu_{(-b-a,-b-a/2,-b-a/2,-b)}(y) \\
f_{Oi}(I,J) = \mu_{(-a,-a/2,-a/2,0)}(y) \\
f_f(I,J) = min(\mu_{(-(b+a)/2,-a,-a,+\infty)}(y), \mu_{(-3a/2,-a,-a,-a/2)}(y), \mu_{(-\infty,-\infty,z/2,z)}(x)) \\
f_{fi}(I,J) = min(\mu_{-b-a/2,-b,-b,-b+a/2}(y), \mu_{(-\infty,-\infty,-b,-(b+a)/2)}(y), \mu_{(z,2z,+\infty,+\infty)}(x)) \\
f_s(I,J) = min(\mu_{-b-a/2,-b,-b,-b+a/2}(y), \mu_{(-\infty,-\infty,-b,-(b+a)/2)}(y), \mu_{(-\infty,-\infty,z/2,z)}(x)) \\
f_{si}(I,J) = min(\mu_{(-(b+a)/2,-a,-a,+\infty)}(y), \mu_{(-3a/2,-a,-a,-a/2)}(y), \mu_{(z,2z,+\infty,+\infty)}(x)) \\
f_d(I,J) = min(\mu_{(-b,-b+a/2,-3a/2,-a)}(y), \mu_{(-\infty,-\infty,z/2,z)}(x)) \\
f_{di}(I,J) = min(\mu_{(-b,-b+a/2,-3a/2,-a)}(y), \mu_{(z,2z,+\infty,+\infty)}(x))
\end{array}\right\} \tag{6.4}$$

where $a = min(x, z), b = max(x, z)$, x and z represent respectively the length of segment (I) and (J). The triplet (x, y, z) are computed as described in section 6.4.1.

Most of relations are defined by a single membership function like $f_<$, $f_>$ and some relations are defined by conjunction of more than one membership functions like $d(during)$, $d_i(during\_by)$, f $(finish)$, f_i $(finished\_by)$. In fuzzy set theory. Sum of all the relations is one, this gives the definition for fuzzy *equal* relation. These are the topological relations which represent the fuzziness at relation's level, for example here *Meet* topological relation is represented based on nearness and length of the smaller interval defines the smooth transition between the *Meet (Meet_by)* and *before (after)* relation. In spatial domain, *before (after)* are called the *disjoint* topological relations. These relations has the following properties.
$f_<(\theta) = f_>(\theta + \pi),\ f_m(\theta) = f_{mi}(\theta + \pi),\ f_o(\theta) = f_{oi}(\theta + \pi),$
$f_f(\theta) = f_s(\theta + \pi),\ f_{fi}(\theta) = f_{si}(\theta + \pi),\ f_d(\theta) = f_d(\theta + \pi),$
$f_{di}(\theta) = f_{di}(\theta + \pi),\ f_=(\theta) = f_=(\theta + \pi)$

Eight topological relations are possible combination of eight independent Allen relations in one-dimensional spatial domain. These relations and their reorientation show that the whole 2D space can be explored with the help of 1D Allen relations using the oriented lines varying from $(0, \pi)$.

6.4.3 Combining Topological and Directional Relations

The topological relations between $2D$ objects are developed largely by the Max. J. Egenhofer [Egenhofer 1993] using the point set topological method and topological relations between two regions is represented by the 9-intersections method. It turns out eight meaningful matrices, which correspond to the eight topological relations. These topological relations are *Disjoint, Meet, Overlap, Covers, Contain, Covered_by, Contained, Equal.* It considers only one piece region without holes in two-dimensional space.

On the other hand, Randell et al. [Randell 1992a] developed RCC theory, it is an axiomatic theory. A reasonable set of topological relations in this theory is called $RCC8$ relations and same eight topological relations with different names are released between an object pair. Relations are $\{DC, EC, PO, TPP, NTPP, TPPI, NTPPI, EQ\}$, respectively *DisConnected, Externally Connected, Partially Overlap, Tangent Proper Part, Non Tangent Proper Part, Tangent Proper Part Inverse, Non Tangent Proper Part Inverse* and *Equal.* The difference of names is due to English language semantics[1] otherwise both theories represent the same topological relations.

Same eight topological relations are represented in one-dimensional space by the Allen's temporal relations in spatial domain. We extend these Allen relations for the $2D$ objects through the logical implication, where a $2D$ object is decomposed into parallel segments of a $1D$ lines in a given direction and the relation between each pair of line segments are computed.

The process of object decomposition is repeated for each direction varying from 0 to π. Two-dimensional topological relations are then defined as it provides us information that how the objects are relatively distributed. These relations are not JEPD [Salamat 2010a]. For obtaining JEPD set of topological and directional relations, an algorithm was advocated in [Salamat], it provides us the JEPD set of relations. Different steps of computing the combined topological and directional relations are explained as

- Fix an angle θ and draw lines passing through the vertices of polygons representing the objects.

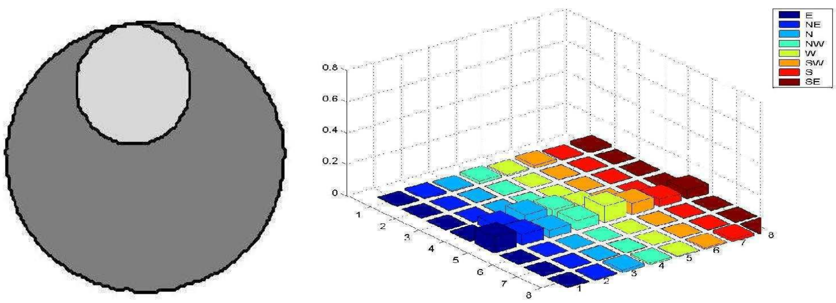
- For each line, compute the variables (x, y, z) as depicted in section 6.4.1 and compute Allen relation for each segment as given by equation (6.4). In case of longitudinal sections, use fuzzy operators to integrate the information, usually the disjunction operators are suitable. These relations are computed for each line in a direction, then obtained information are integrated into a single value. Normalize these relations for a direction θ by dividing sum of all Allen relations to each Allen relation.

[1] In RCC "DC= Disconnected" and the 9-intersections represent the "D=Disjoint" same semantics we use in our terminology "D" for disjoint topological relation, similarly for "M" for "meet".

- These normalized fuzzy Allen relation is then multiplied by fuzzy directional set to find the degree of an Allen relation in a direction.

- For qualitative directions, these information are summarized and different topological relations with directional contents are defined. For example topological relation for *East* direction defined as $f_E = \sum_{\theta=0}^{\frac{\pi}{4}} \mathcal{A}_2 \times cos^2(2\theta) + \sum_{\theta=\frac{3\pi}{4}}^{\pi} \mathcal{A}_1 \times cos^2(2\theta)$, where f represents a topological relation and E represents the *East* direction and $\mathcal{A}_2$ is the reorientation of $\mathcal{A}_1$. Similarly for other relations.

- This information is represented in a matrix, an algorithm proposed in [Salamat] is then used which enables us to have JEPD set of topological and directional relations. For an example object pair is represented in Fig. 6.3(a), the fuzzy relations are represented in the Fig. 6.3(b).



(a) (b)

Figure 6.3: Object pair and their combined topological and directional relation information qualitative topological and directional relations are *(TPP, N)*

This method describes well the possible topological relations between every sort of objects.

6.5 Spatio-Temporal Relations

Spatio-temporal relations can be defined as a spatial relation holds for an interval. These relation holds for a certain time interval and it doesn't change. In spatio-temporal object theory it is defined as *spatio-temporal relation (P) is a relation holding between all temporal slices of two entities during the relevant period of time.* All the eight spatio-temporal relations are defined below in terms of theorems.

6.5.1 Spatio-Temporal Relation

Theorem 1 *A spatio-temporal* disjoint *relation between object pair* (X, Y) *holds during the interval* T *if and only if a* disjoint *topological relation holds for every* *snapshot during the interval. In other words,* $D(XY,T) \Leftrightarrow \forall\, t \in T$*, the relation* $D(XY,t)$ *holds.*

Proof 1 ($\Rightarrow$) *A spatio-temporal* disjoint $D(XY,T)$ *relation is defined as object pair* (X,Y) *are* disjoint *during the interval* T*, it means* $X \equiv_t Y$ *(*X *is temporally equivalent to* Y*). Let* $t_a = t_1 < t_2 < t_3 ... < t_n = t_b$ *be the partition of temporal interval* $T = [t_a\ t_b]$*. Each* $t_i \in T,\ i = 1, 2, .., n$ *represents discrete point of the interval* T *and this representation is equivalent to a snapshot, typically a snapshot is a result of sampling process, which represents zero duration temporal slice of a spatio-temporal object. There are* n *snapshots in the interval, as a result a* disjoint *relation exists for each snapshot separately. Thus* $\forall\, t \in T,\ D(XY,t)$ *holds.*

($\Leftarrow$) *Let us consider the* n *snapshots where temporal ordering holds.* $t_1, t_2, ..., t_n$ *be points such that* $t_1 < t_2 < t_3 ... < t_n$ *and all these points form the partition of an interval* T*. If the* disjoint *topological relation holds at the discrete points of an interval, it is* disjoint *throughout interval. i.e.,* $D(XY,t_i) \wedge D(XY,t_{i+1}) \rightarrow D(XY,[t_i\ t_{i+1}])$. Disjoint *topological relation holds between object pair for each snapshot, it means both the objects are temporally equivalent(*$x \equiv_t y$*). Hence* $D(XY,\ T)$ *holds during the whole interval* T*.*

Theorem 2 *A spatio-temporal* Meet *relation holds for an interval* T *if* meet *relation holds at least for one snapshot and for all other snapshots it is a* disjoint *topological relation. i.e.,* $M(XY,T) \Leftrightarrow\ \exists\, t \in T$ *s. t.* $M(XY,t) \wedge \forall\, t_i \in T\ \wedge t_i \neq t\ \Rightarrow D(XY,t_i)$ *holds.*

Proof 2 ($\Rightarrow$) *A spatio-temporal relation* meet $M(XY,\ T)$ *holds between object pair* (X,Y) *over interval* T*, where* $X \equiv_t Y$*. Let* $t_a = t_1 < t_2 < t_3 < ... < t_n = t_b$ *be partition of interval* $T = [t_a\ t_b]$*, if* $\forall\, t \in T,\ M(XY,t)$ *holds then a stable* $M(XY,T)$ *holds. We consider on contrary, that* $\exists\, t_j$*, where the topological relation* $M(XY,t_j)$ *doesn't hold but it holds at* $M(XY,t_{j-1})$*, then according to the temporal logic and continuity of topological relations* $\bigcirc(M(XY,t_{j-1})) \Rightarrow (D(XY,t_j) \vee M(XY,t_j) \vee PO(XY,t_j)$. *This shows that any of the three relations are possible(*$\bigcirc$ *stands for future position). If* $PO(XY,\ t_j)$ *holds, then the whole spatio-temporal relation is changed and it becomes the* spatio-temporal partial overlap *relation. This possibility is ruled out. In other case, spatio-temporal relation remains* meet *and* j *is an arbitrary variable, this shows the minimum condition. Hence* $\exists\, t \in T, s.t.\ M(XY,t)$ *holds.*

($\Leftarrow$) *Let us consider that there are* n *snapshots in an order, which construct an interval* T*. Now consider that there exist at least one snapshot during the whole interval, where the spatial* meet *relation holds, and for all the other snapshots, the spatial relation is* disjoint*. This shows that during the temporal interval* T*, the unstable spatio-temporal* meet *relation holds. It satisfies the minimum conditions for a spatio-temporal* meet *relation, hence* $M(XY,T)$ *holds during the interval* T*.*

Theorem 3 *A spatio-temporal* Partial Overlap *(PO) relation holds over interval* T *such that* $PO(XY,T) \Leftrightarrow \exists\, t \in T, s.t.\ PO(XY,t)$.

Proof 3 *Spatio-temporal relations have the spatial and temporal boundaries, A stable spatio-temporal relation holds during the temporal slice, if it holds at every point of the interval. As temporal slice is the union of finite points of temporal domain, spatio-temporal* partial overlap *holds during the whole slice, if this relation holds at least one snapshot, at remaining points any of the spatial relation may exist. Hence* $\exists\, t \in T, s.\, t.,\ PO(XY, t)$ *and* $(\exists\, t_1 \neq t \Rightarrow CO(XY, t_1) \vee M(XY, t_1) \vee D(XY, t_1))$[2]. *If there doesn't exist such a* t_1*, then the relation holds for every* $t \in T$*, which shows that a stable* PO *relation holds.*

($\Leftarrow$) *We suppose on contrary that* $\nexists\ t \in T,\ s.\ t.\ PO(XY, t)$ *holds. It means that at all the points of temporal interval either the relations are complete overlap or disjoint and meet. if the relations are complete overlap.* $\forall\ t \in T, CO(XY, t)$ *holds, then the spatio-temporal relation will be a part of complete overlap. In case of other choice,* $\exists\ t \in T$ *such that* $M(XY, t)$ *or* $\forall\ t \in T, s.\ t.\ M(XY, t)$ *holds, then the spatio-temporal relation will be unstable or stable meet respectively. In case* $\forall\ t \in T\ s.\ t.\ D(XY, t)$*, the relation will be disjoint. The choice,* $\exists t_1 \in T$ *and* $M(XY, t)$ *and* $\exists\ t_2 \in T$ *such that* $CO(XY, t_2)$ *holds is impossible because in a such a case, common sense continuity of spatial relations provide us information that* $\exists\ t \in T$ *such that* $t_1 < t < t_2$ *and* $PO(XY, t)$ *holds, which is contradiction to the assumption that* $\nexists\ t \in T$ *for which* $PO(XY, t)$ *holds (continuity of spatial relations).*

Theorem 4 *A spatio-temporal* Tangent Proper Part *(TPP) relation holds over interval T, i. e.,* $TPP(XY, T) \Leftrightarrow \exists\ t_1 \in T, s.t.\ TPP(XY,\ t_1)\ \forall\ t_2$ *and* $t_2 \neq t_1,\ NTPP(XY, t_2)$ *holds.*

Proof 4 ($\Rightarrow$) *Let* O *be the space contains the objects* (X, Y) *during the interval* T*.* O_t *represents the space corresponding the time point t, called snapshot during the temporal interval* T*, then* $(X_t\ Y_t)$ *represents the objects in the snapshot* O_t*. Let us consider that a spatio-temporal* $TPP(XY)$ *relation holds during the interval* T*.*

Let $t_i\ i = 1, 2, 3\ .\ .\ n$ *be the partition of interval* T*, if this relation holds for every* t_i *then it is a stable* TPP *relation. If* TPP *relation doesn't hold for every* t_i*, then relations must be* $NTPP(XY)$*. In case of contrary, the relations becomes the* $PO(XY, T)$ *because* $\exists\ t \in T$ *and* $PO(XY, t)$ *holds.*

($\Leftarrow$) *Consider that* $\exists\ t_i$ *is* $TPP(XY, t_i)$ *holds for some* $t_i \in T$*. We consider on contrary that* $\exists t_{i+1} \vee t_{i-1}$ *such that* $NTPP(XY, t_{i-1})$ *or* $NTPP(XY, t_{i+1})$ *doesn't hold. Then possible topological relations at* t_{i-1} *are* $TPP(XY, t_{i-1}, PO(XY, t_{i-1},$ *similarly for* t_{i+1}*. Other possibilities are ruled out due to continuity of topological relations.* $EQ(XY, t_{i-1}$ *doesn't hold because objects are considered under motion and expansion or zooming of one object is not allowed.*

In case of the topological relation $PO(XY, t_{i-1})$ *holds then whole the spatio-temporal relation over the interval* T *becomes Partial overlap. Similarly for instant* t_{i+1} *which contradicts the fact. Since i is an arbitrary point, so this is impossible for whole the interval* T*. For the topological relation* $TPP(XY, t_{i-1})$*, the spatio-temporal relation becomes the stable spatio-temporal* TPP*.*

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[2] $CO(XY)$ stands for complete overlap of objects (XY), s. t., $CO(XY) = TPP(XY) \vee NTPP(XY) \vee TPPI(XY) \vee NTPPI(XY)$

Theorem 5 *A spatio-temporal* Non Tangent Proper Part*(NTPP) relation holds over interval T,i. e.,* $NTPP(XY,T) \Leftrightarrow NTPP(XY,t)$ *holds for all* $t \in T$.

Proof 5 $\Rightarrow$ *Let us suppose on contrary that* $\exists\ t_i \in T$ *s.t.* $NTPP(XY,t_i)$ *doesn't holds and at temporal points* t_{i-1}, t_{i+1} *the relation* $NTPP(XY,t_{i-1})$ *or* $NTPP(XY,t_{i+1})$ *holds. Then continuity of spatial relations force the existence of* $TPP(XY,t_i)$ *or* $EQ(XY,t_i)$ *spatial relations. This contradicts the existence of the spatio-temporal* $NTPP(XY,T)$ *relation. Hence* $NTPP(XY,t)$ *holds for all* $t \in T$.

$\Leftarrow$ *It is given that* $\forall t \in T$, $NTPP(XY,t)$ *holds. If a spatial relation between object pair holds at every point of the interval, it means it hold throughout the interval. i. e.,* $NTPP(XY,T)$ *holds.*

Theorem 6 *A spatio-temporal* Tangent Proper Part Inverse *(TPPI) relation holds over interval T,i. e.,* $TPPI(XY,T) \Leftrightarrow \exists\ t \in T, s.t.\ TPPI(XY,t)$ *and* $\forall\ t_1 \neq t,\ NTPPI(XY,t_1)$ *holds.*

Proof 6 *Proof is similar to the* $TPP(XY,T)$, *just replace* TPP *by* $TPPI$ *and* $NTPP$ *by* $NTPPI$.

Theorem 7 *A Spatio-temporal* Non Tangent Proper Part Inverse*(NTPPI) relation holds over temporal interval T,i. e.,* $NTPPI(XY,T) \Leftrightarrow NTPPI(XY,t)$ *holds for all* $t \in T$.

Proof 7 *Proof is similar to the* $NTPP(XY,T)$.

Theorem 8 *A spatio-temporal relation* Equal *(EQ) holds between the object pair* XY, $EQ(XY,T) \Leftrightarrow \forall\ t \in T, s.t.,\ EQ(XY,t)$ *holds.*

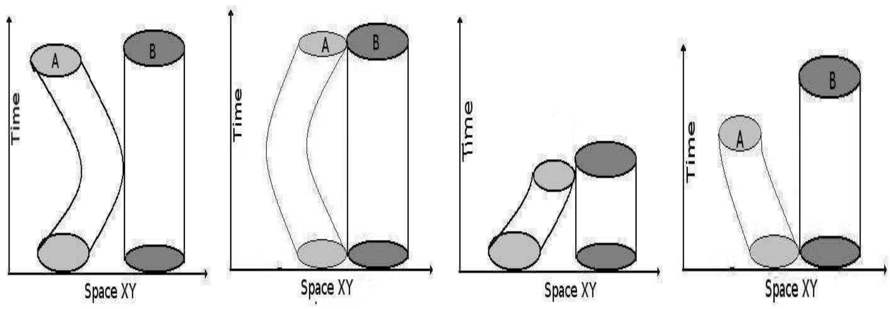
Proof 8 $(\Rightarrow)$ *We suppose on contrary that there exist a* $t \in T$ *where the* $EQ(XY,t)$ *relation doesn't hold. It shows that there are two possibilities that either the relation at* t *is a complete overlap or partial overlap. If the relation at* t *is complete overlap, then the spatio-temporal relation becomes* TPP *or* $TPPI$. *In the second case, the spatio-temporal relation becomes the* $PO(XY,T)$ *during the whole interval. Thus both cases prove the contrary conditions, hence* $\nexists\ t$ *such that* $EQ(XY,t)$ *doesn't hold, i. e.,* $\forall\ t \in T$, $EQ(XY,t)$ *holds.*

$(\Leftarrow)$ *Converse of this proof is very simple and straight forward. Let* T *be the interval for which we have to define the spatio-temporal relation, both the objects are temporally comparable* $(X \equiv_t Y)$. *Let* $t \in T$ *be an arbitrary point of the interval and relation* $EQ(XY,t)$ *holds for every* $t \in T$. *Since* t *is an arbitrary point so relation* $EQ(XY)$ *holds throughout interval* T. *i. e.,* $EQ(XY,T)$ *holds.*

6.6 Visual Interpretation: A Three Dimensional View

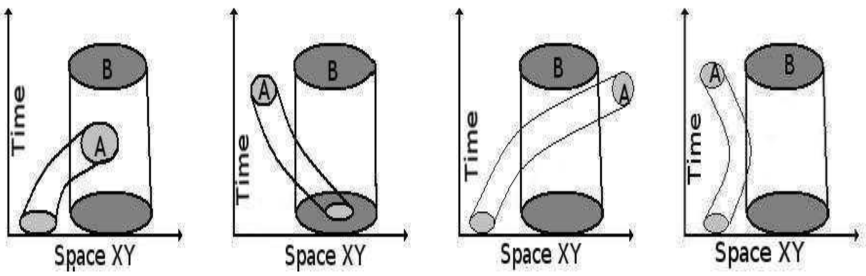
Geometrical figures can better elaborate the underlying concepts. A moving object changes its position at each instant t. These objects in a spatio-temporal domain can be represented by their envelops, a two dimensional object becomes the volume.

Here spatio-temporal *meet* and *partially overlap* relations are represented by their envelops in Figs. 6.4(a)-6.4(d) and 6.5(a)-6.5(h). These are possible representation of motion events. Spatial relations between moving objects are used in modeling the motion verbs or motion events in natural language processing. A set of motion relations is introduced that capture the semantic between pairs of moving objects. This information of spatial relations are useful about reasoning the moving objects.

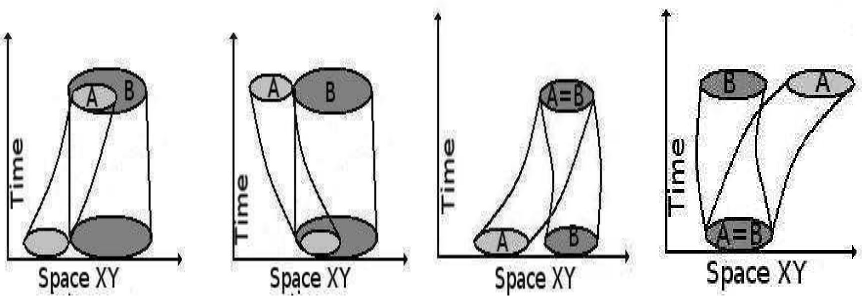


(a) Touch (b) Excurse (c) Snap (d) Release

Figure 6.4: Spatio-temporal *Meet* relation (Unstable Meet))



(a) Enter (b) Leave (c) Cross (d) Graze



(e) Into (f) Out of (g) Melt (h) Spring

Figure 6.5: Spatio-temporal *Partial_ Overlap* relation (Unstable Overlap))

6.7 Modeling Motion Classes

Visual images may illustrate cases of a definition, giving us a more visual grasp of its applications. They may help us understanding the description of a mathematical situation or steps in reasoning. These relations can be defined as the transection of relations at time t_i to t_{i+1}. This change may be in topological or metric relations and different classes of spatial relations between moving objects have been defined [de Weghe 2006, Maeyera 2005, Hornsby 2002]. Motion classes based on intuitive logics or motion verbs have been defined by Phillipe Muller [Muller 1998] and Ralf H. Güting and Markus Schneider in [Güting 2005]. In this paper, we define motion events where topological relations capture changes between situations. These motion events can be defined using predicates *holds-at, holds, occurs-at, occurs* and *sequence*.

6.7.1 Unstable Meet Spatio-Temporal Relation

Unstable spatio-temporal relation is a relation where objects changes their states at each time instant. A spatio-temporal *Meet* relation is characterized by different motion events depending upon the logical and temporal order of primitive events. These motion events are explained in detail by adding directional contents to these events.

Touch(XY,T): A spatio-temporal meet relation can be characterized as a motion event *Touch*, s. t. $\exists\ t_1, t_2, t_3 \in\ T$and $t_1 < t_2 < t_3$ where primitive events occur in an order and defined as

$$Touch(XY,T) = seq\_eve(holds(D(XY,t_1)) \wedge holds(M(XY,t_2)) \wedge holds(D(XY,t_3)))$$

where *seq_eve* stands for *sequence_ event*. An institutive view of this spatio-temporal relation is shown in Fig. 6.4(a). This relation can be expressed by a single direction, where a meet topological relation holds. It means,

$$Dir(Touch(XY,T)) = holds(Dir(XY,t_2))$$

Snap(XY,T): A spatio-temporal meet relation is called *Snap* if $\exists\ t_1, t_2 \in T$ and $t_1 < t_2$ such that

$$Snap(XY,T) = seq\_eve(holds(D(XY,t_1)) \wedge holds(M(XY,t_2)))$$

A geometric representation is shown in Fig. 6.4(c). This relation can be expressed by a single direction, where a meet topological relation holds. It means,

$$Dir(Snap(XY,T)) = holds(Dir(XY,t_2))$$

Release(XY,T): A spatio-temporal Meet is called *Release* ($Release(XY, T)$, read as X releases Y during interval T) if it has a certain temporal ordering, $\exists\, t_1, t_2 \in\ T$ and $t_1 < t_2$ such that

$$Release(XY, T) = seq\_eve(holds(D(XY, t_1)) \wedge holds(M(XY, t_1)))$$

A three dimensional geometric view of this relation is shown in Fig. 6.4(d). This relation can be expressed by a single direction, which is the destination direction. For example object X *releases* (motion event) object Y towards *east* (destination direction). Direction for such a relation is defined as

$$Dir(Release(XY, T)) = holds(Dir(XY, t_2))$$

Bypass(XY,T): A spatio-temporal Meet(XY,T) is called $Bypass(XY, T)$, read as X bypasses Y during interval T if it has a certain temporal ordering, i. e., $\exists\, t_1, t_2, t_3, t_4 \in\ T$ such that $t_1 < t_2 < t_3 < t_4$

$$Bypass(XY, T) = seq\_eve(holds(D(XY, t_1)) \wedge holds(M(XY, t_2)) \wedge \\ holds(M(XY, t_3)) \wedge holds(D(XY, t_4)))$$

This relation can be expressed by a single direction, where a meet topological relation holds. It means,

$$Dir(Touch(XY, T)) = holds(Dir(XY, t_2))$$

Excurse(XY,T): A spatio-temporal Meet(XY,T) is called $Excurse(XY, T)$, read as X excurse Y during interval T if it has a certain temporal ordering, an intuitive view of this relation is shown in Fig. 6.4(b). $\exists\, t_1, t_2, t_3 \in\ T, s.t.\ t_1 < t_2 < t_3$

$$Excurse(XY, T) = seq\_eve(holds(M(XY, t_1)) \wedge holds(D(XY, t_2)) \wedge holds(M(XY, t_3)))$$

This relation is expressed by an initial and destination directions, the direction for this relation can be defined as

$$Dir(Excurse(XY, T)) = seq\_eve(holds(Dir(XY, t_1)) \wedge holds(Dir(XY, t_3))$$

6.7.2 Unstable Overlap Spatio-Temporal Relation

Enter(XY,T): An unstable spatio-temporal overlap relation is called *Enter*, generally denoted by Enter(XY,T) and read as "X enters in Y during interval T". If $\exists\, t_1, t_2, t_3, t_4 \in\ T$ such that $t_1 < t_2 < t_3 < t_4$, then relation is define as

$$Enter(XY, T) = seq\_eve(holds(D(XY, t_1)) \wedge holds(M(XY, t_2)) \wedge \\ holds(PO(XY, t_3)) \wedge holds(TPP(XY, t_4))$$

An intuitive view of this relation is shown in Fig.6.5(a). This relation can be expressed by a single direction, because the destination point is inside and can be expressed without direction, a direction for the Enter spatio-temporal event is the direction where a meet topological relation holds. i. e.,

$$Dir(Enter(XY,T)) = holds(Dir(XY,t_2))$$

Leave(XY,T): A spatio-temporal partial overlap relation is called *Leave*, denoted as Leave(XY,T) "X leaves Y during interval T". If $\exists\ t_1, t_2, t_3, t_4 \in T$ such that $t_1 < t_2 < t_3 < t_4$, then relation is define as

$$Leave(XY,T) = seq\_eve(holds(NTPP(XY,t_1)) \wedge holds(TPP(XY,t_2)) \wedge \\ holds(PO(XY,t_3)) \wedge holds(M(XY,t_4) \wedge holds(D(XY,t_5)))$$

An intuitive view of this relation is shown in Fig. 6.5(b). This relation can be expressed by a single direction which is the destination point, i. e.,

$$Dir(Leave(XY,T)) = holds(Dir(XY,t_4))$$

Cross(XY,T): A spatio-temporal partial overlap relation is called *Cross(XY,T)* "X crosses Y during the interval T". Its geometric view is given in Fig. 6.5(c). If $\exists\ t_1, t_2, t_3, ..., t_9 \in\ T$ such that $t_1 < t_2 < ... < t_9$, then relation is define as

$$Cross(XY,T) = seq\_eve(holds(D(XY,t_1)) \wedge holds(M(XY,t_2)) \wedge holds(PO(XY,t_3)) \wedge \\ holds(TPP(XY,t_4)) \wedge holds(NTPP(XY,t_5)) \wedge holds(TPP(XY,t_6)) \wedge \\ holds(PO(XY,t_7)) \wedge holds(M(XY,t_8)) \wedge holds(D(XY,t_9)))$$

This spatio-temporal relation is expressed by a initial as well as destination direction such as object X crosses (motion event) object$Y$ from *north*(direction) towards *east*(direction) during the interval T.

$$Dir(Cross(XY,T)) = seq\_eve(holds(Dir(XY,t_1)) \wedge holds(Dir(XY,t_9)))$$

Into(XY,T): A spatio-temporal partial overlap relation is called *Into(XY,T)* read as "X get into Y during the interval T". If $\exists\ t_1, t_2, t_3 \in\ T$ such that $t_1 < t_2 < t_3$, then relation is define as

$$Into(XY,T) = seq\_eve(holds(M(XY,t_1)) \wedge holds(PO(XY,t_2)) \wedge holds(TPP(XY,t_3))$$

Its three-dimensional geometric view is given in Fig. 6.5(e). This relation can be expressed by a single direction in language semantics, where a meet topological relation holds. For example, object A get *into* (spatio-temporal event) object B from *north* (direction). It means,

$$Dir(Into(XY,T)) = holds(Dir(XY,t_1))$$

Out_of(XY,T): A spatio-temporal partial overlap relation *Outof(XY,T)* read as "X comes out of Y during the interval T", its intuitive view is considered in Fig. 6.5(f). If $\exists\, t_1, t_2, t_3, t_4 \in T$ such that $t_1 < t_2 < t_3 < t_4$, then relation is define as

$$Out\_of(XY,T) = seq\_eve(holds(TPP(XY,t_1)) \wedge holds(PO(XY,t_2)) \wedge holds(D(XY,t_3)))$$

This relation can be expressed by a single direction. Object X go *out_ of* (motion event) object Y towards *east* (direction). where a meet topological relation holds. It means,

$$Dir(out\_of(XY,T)) = holds(Dir(XY,t_3))$$

Melt(XY,T): A spatio-temporal partial overlap relation *Melt(XY,T)* read as "X, Y melts during the interval T". If $\exists\, t_1, t_2, t_3, t_4 \in T$ such that $t_1 < t_2 < t_3 < t_4$, then relation is define as

$$Melt(XY,T) = seq\_eve(holds(D(XY,t_1)) \wedge holds(M(XY,t_2)) \wedge holds(PO(XY,t_3)) \wedge holds(EQ(XY,t_4)))$$

An intuitive view of this relation is shown in Fig. 6.5(g). This relation can be expressed by a single direction because its destination point is dimensionless. This can be its direction where initial spatial relation holds.

$$Dir(Melt(XY,T)) = holds(Dir(XY,t_1))$$

Spring(XY,T): A spatio-temporal partial overlap relation *Spring(XY,T)* also called Separate(XY,T) read as "X separates Y during the interval T". If $\exists\, t_1, t_2, t_3, t_4 \in T$ such that $t_1 < t_2 < t_3 < t_4$, then relation is define as

$$Spring(XY,T) = seq\_eve(holds(EQ(XY,t_1)) \wedge holds(PO(XY,t_2)) \wedge holds(M(XY,t_3)) \wedge holds(D(XY,t_4)))$$

Its three-dimensional geometric view is given in Fig. 6.5(h). This relation can be expressed by a single direction because its destination point is dimensionless. This can be its direction where terminal spatial relation holds.

$$Dir(Spring(XY, T)) = holds(Dir(XY, t_4))$$

Graze(XY,T): A spatio-temporal partial overlap relation *Graze(XY,T)* read as "X grazes Y during the interval T". If $\exists\ t_1, t_2, t_3, t_4, t_5 \in\ T$ such that $t_1 < t_2 < t_3 < t_4 < t_5$, then relation is define as

$$Graze(XY, T) = seq\_eve(holds(D(XY, t_1)) \wedge holds(M(XY, t_2)) \wedge holds(PO(XY, t_3)) \wedge holds(M(XY, t_4)) \wedge holds(D(XY, t_5)))$$

This relation is represented in a three-dimensional perspective in Fig. 6.5(d). This spatio-temporal relation is expressed by a initial as well as destination direction such as object X grazes (motion event) object$Y$ from *north*(direction) toward *east*(direction).

$$Dir(Graze(XY, T)) = seq\_eve(holds(Dir(XY, t_1)) \wedge holds(Dir(XY, t_4)))$$

6.8 Conclusion and Future Work

In this paper we define spatio-temporal relations where the discrete time space is used and spatial relations are extended to the temporal domain using stability and un-stability of topological relations. Motion events represent the subclass of spatio-temporal relations, we denote the unstable *meet* and *overlap* spatio-temporal relation which represents the certain number of motion classes. In these spatio-temporal relations temporal order of a primitive event is more important and this order has a pivotal role in natural language semantics. Topological relations have a locative symmetries, to remove these symmetries we add a directional components, this enhance the expressivity of a motion event. In this paper CTD method [Salamat] is used to model the motion events, where topological and directional information are captured at same abstract level. Hopefully this work will bring a significant change in video understanding, modeling video events and other related areas of research.

Chapter 7

Spatio-Temporal Reasoning by Combined Topological and Directional Relations Information

Abstract

Spatio-temporal reasoning is extensively used in many areas of computer vision and Artificial Intelligence (AI). Different methods for spatio-temporal reasoning are proposed based on topological and directional relations separately in respective domains. Reasoning about moving objects in a spatial scene or description about the two-dimensional scene simultaneously needs both topological and directional reasoning. We introduced a reasoning system for two-dimensional spatial scene based on Combined Topological and Directional(CTD) relations method, where we obtain both topological and directional information. Main task in spatial reasoning is the construction of composition tables for topological and directional relations. Entities in these composition-tables follows the mathematical rule for composition of spatial relations, these rules are elaborated and composition table for topological relations is divided and re-arranged into sub-tables.

Key words: Spatio-temporal reasoning, combined topological and directional relations, composition rules.

7.1 Introduction

Common sense knowledge representation is qualitative and this type of knowledge represents the superset of quantitative knowledge [Guesgen 1989]. Spatial knowledge has a central point in many domains like AI including spatio-temporal reasoning, natural language processing, human machine interaction and automated reasoning. Spatio-temporal reasoning plays an important role in many computer vision applications, such as path planning in robotics, visual object recognition at higher level computing which includes the interpretation and integration of visual information and video scene interpretation.

The objects in an image constitute a situation which is described through relationship between these objects. Moving objects change their relative position, they change the relative information like distance between them, directional or topological information. Topological transformation represent the different types of changes in spatial scene such as translation, uniform expansion or contraction and tearing or shearing of an object. It is supposed that objects don't change the topological configuration, object doesn't split into multiple objects. Under these conditions, change in topological or in directional or in both type of relations are considered as the objects are moving. Reasoning methods with these conditions are called spatio-temporal reasoning for moving objects.

Spatial knowledge representation takes into account the topological, directional, distance relations, shape and size information. Methods for topological reasoning are introduced like Region Connection Calculus (RCC) based methods [Cohn 1997, Cohn 1994, Cohn 2007, Gotts 1996, Wolter 2000], fuzzy methods [Bao 2005, Homaifar 1997, Schockaert 2009, Li 2004] and the 9-intersections method [Egenhofer 1992]. In spatial knowledge representation techniques, knowledge about topological and directional relations is represented at different abstract levels. Two separate systems for reasoning with topological and directional knowledge are developed [Freksa 1991, Muller 2002, Museros 2003, Galton 2009, Wolter 2010, Schockaert 2008a] and a combined reasoning method is developed in [Hernández 1991, Li 2009]. It is observed that most of the existing spatio-temporal formalism is domain based and are developed for a particular application.

CTD-relations method [Salamat] considers the intrinsic frame of reference and both topological and directional relations are well-defined without generalizing object's geometry. The concept of directional and topological information are integrated into the single method. CTD-relations method represents the topological and directional relations information at same level of abstraction and combines the features of directional and topological relations calculi. Spatial reasoning with topological and directional relations are combined in this method. Topological and directional relations are computed with CTD-method and composition tables for these relations are developed separately.

In this paper, we proposed a method for the composition of spatial relations. we divide the composition tables for topological relations into sub-tables, this table is rearranged and it consist of nine sub-tables. Entities in these sub-tables are related to each other under a mathematical formula. This mathematical formula combines the inverse and commutative properties of a relation as a result computation for constructing composition tables become easy. we develop the mathematical relations between entities of these sub-tables as a result, less computation is required for building composition tables for topological relations. This composition of relations follows the common sense continuity of objects and all these entities in composition tables belong to the neighboring spatial relations of participant relations. To make the paper self contained, an introduction of the CTD-relations method is given.

This paper consists of following parts, section 7.2 explains the related work and section 7.3 explains the relevant terms which are frequently used in this paper. Combined topological and directional relations method is explained in section 7.4. Section 7.5 explains the neighborhood graph and change in spatial scene, section 7.6 explains the relation between the object pair and their converse relation. Composition tables are explained in section 7.7, in this section we explain the mathematical formula between entities of the composition tables. Section 7.9 concludes the paper.

7.2 Related Work

Spatial reasoning attempts to represent data in linguistic variables, if the observers or human controllers are involved, linguistic variables are related to numeric or geometric representation. Spatial reasoning, especially computing the composition tables for topological and directional relations can be done at a coarse level information or finer levels depending on the kinds of available information.

Many methods have been introduced for spatio-temporal reasoning during the recent years [Bao 2005, Ibrahim 2007b, Cohn 1997, Cohn 2001, Cohn 2007, Egenhofer 1992, Gotts 1996, O'smiaiowski 2010]. Fuzzy methods are among them and fuzzy topological relations based on nearness are developed in [Schockaert 2009] for fuzzy reasoning. In this method, there are 144 general transitive rules and reasoning is based on these rules. This method related to topological reasoning and the question about the directional relations still remains unanswered.

Allen [Allen 1983] introduced thirteen Jointly Exclusive and Pairwise Disjoint (JEPD) interval relations for temporal reasoning. There is a direct homeomorphism between the physical structure of time and one-dimensional spatial structure. Nowadays, these relations are used for modeling the directional relations [Matsakis 1999b] and combined topological and directional relations information [Matsakis 2005, Salamat 2011c]. Topological and qualitative directional relations are combined in [Salamat 2010b, Salamat 2010a]. We used the CTD-relations method, presented in [Salamat] for reasoning the topological and directional relations.

Our method for reasoning about the space is inspired by the method of representing the spatial knowledge adopted in [Hernández 1991, Li 2009], where the combined spatial knowledge of projection and orientation for a two-dimensional scene is presented. In our method relative directional information for the inclusion case is also introduced. Inclusion topological relation is further explored and directional contents are also introduced for the topological relations TPP and $TPPI$ (*Tangent Proper Part, Tangent Proper Part Inverse*). These cases are explored due to certain reasons like all objects contained in the other object can be manipulated by the parent objects. Describing the global scene, they can be relatively described with respect to parent object. For example, objects inside the room inherit the orientation of that room with respect to other rooms in the building. But how can be objects described lying inside the room and what will be the orientation of objects inside room relative to the room?

The method proposed in this paper also deals to the extension of knowledge representation through $RCC5$ to $RCC8$ in region connection calculus theory. In this method, inclusion spatial relation is further divided into four spatial relations, namely *TPP, NTPP, TPPI, NTPPI*. The relations *TPP, TPPI* also have the directional contents and reasoning for these relations is also introduced. Two separate systems of composition tables are developed for topological and directional relations. Entities in these composition tables are interrelated through the mathematical rules. These rule are explained in section 7.7. This reasoning process helps the user to get the finer spatial knowledge from coarse information.

7.3 Preliminary Definitions

In this section we recall some basic definitions which are frequently used in the rest of paper.

Fuzzy membership function: A membership function μ in a set X is a function $\mu : X \to [0,1]$. Many fuzzy membership functions are proposed according to the requirements of the application. For instance, trapezoidal membership function is defined as

$$\mu(x;\alpha,\beta,\gamma,\delta) = max(min(\frac{x-\alpha}{\beta-\alpha}, 1, \frac{\delta-x}{\delta-\gamma}), 0) \tag{7.1}$$

it is written as $\mu_{(\alpha,\beta,\gamma,\delta)}(x)$ where $x,\alpha,\beta,\gamma,\delta \in \mathbb{R} \wedge \alpha < \beta \leq \gamma < \delta$.

Fuzzy set: A fuzzy set A in a set X is a set of pairs $(X, \mu(x))$such that $A = \{(x, \mu(x)|x \in X)\}$

where μ represents a membership function.

Converse of a relation: Let R be a relation between the object pair (A, B), written as $R(A, B)$. Converse relation $\tilde{R}$ of R is a relation between the same object pair when objects commute. The converse relation is then defined as

$$\tilde{R}(A, B) = R(B, A) \tag{7.2}$$

Transitivity: The composition $R_1 \odot R_2$ of two general relations R_1, R_2 is a relation R defined as

$$R_1(A, B) \odot R_2(B, C) \Rightarrow R(A, C) \tag{7.3}$$

where (A, B, C) are three objects and R is called a transitive relation.

Conceptual neighbor: Two relations between pairs of events are conceptual neighbors if they can be directly transformed into one another by continuously deforming (shortening, lengthening or moving) events in topological sense. A set of relations between pair of events forms a conceptual neighborhood graph if its elements are path connected through conceptual neighbor relations [Freksa 1992].

Force histogram: The force histogram attaches a weight to the argument object A that object A lies *after* object B in direction θ. This is defined as

$$\mathbf{F}^{AB}(\theta) = \int_{-\infty}^{+\infty} F(\theta, A_\theta(v), B_\theta(v))dv \tag{7.4}$$

The definition of Force histogram $\mathbf{F}^{AB}(\theta)$, directly depends upon the definition of real valued functions ϕ, f and F. These functions are used for the treatment of points, segments and longitudinal sections respectively [Matsakis 1999b]. These functions are defined as

$$\left.\begin{aligned} \phi_r(y) &= \begin{cases} \frac{1}{y^r} & \text{if } y > 0 \\ 0 & \text{otherwise} \end{cases} \\ f(x_I, y^\theta_{IJ}, z_J) &= \int_{x_I+y^\theta_{IJ}}^{x_I+y^\theta_{IJ}+z_J} \int_0^{z_J} \phi(u-w)dw)du \\ F(\theta, A_\theta(v), B_\theta(v)) &= \sum_{i=1..n, j=1..m} f(x_{Ii}, y^\theta_{IiJj}, z_{Jj}) \end{aligned}\right\} \tag{7.5}$$

where n, m represent the number of segments of object A and object B respectively. Variables (x, y, z) are explained in section 7.4.1 and diagrammatically shown in Fig. 7.1. These definitions of Force histograms directly depend upon the definition of function ϕ. Notice that $\mathbf{F}^{AB}(\theta)$ is a real valued function.

7.4 Oriented Lines and Fuzzy Allen Relations

In this section terms used for computation of fuzzy Allen relations are explained. Drawing of oriented lines, segments and longitudinal sections are explained in subsection 7.4.1 and Allen relations in section 7.4.2. Combination of topological and directional relations are explained in section 7.4.3 and representation is elaborated in section 7.4.4.

7.4.1 Oriented Lines, Segments and Longitudinal Sections

Let A and B be two spatial objects and $(v, \theta) \in \mathbb{R}$, where v is any real number and $\theta \in [0, 2\pi]$. Let $\Delta_\theta(v)$ be an oriented line at orientation angle θ and $A \cap \Delta_\theta(v)$ is the intersection of object A and oriented line $\Delta_\theta(v)$. It is denoted by $A_\theta(v)$, called segment of object A and length of its projection interval on x-axis is denoted by x. Similarly for object B where $B \cap \Delta_\theta(v) = B_\theta(v)$ is segment, its length of its projection interval on x-axis is denoted by z and y is the difference between minimum value of projection points of $A \cap \Delta_\theta(v)$ and maximum value of projection points of $B \cap \Delta_\theta(v)$(for details [Matsakis 2005]). In case of polygonal object approximation (x, y, z) can be calculated from intersecting points of line and object boundary, oriented lines are considered which passes through at least one vertex of two polygons. If there exist more than one segment, then it is called longitudinal section as in case of $A_\theta(v)$ in Fig. 7.1.

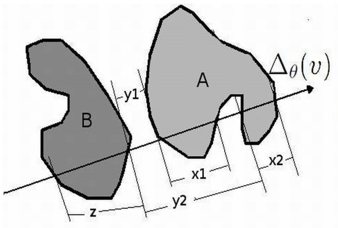


Figure 7.1: Oriented line $\Delta_\theta(v)$, segment as in case of object B, longitudinal section as in case of object A.

7.4.2 Allen Temporal Relations in Spatial Domain and Fuzziness

Allen [Allen 1983] introduced 13 Jointly Exhaustive Pairwise Disjoint (JEPD) interval relations. These relations are $\mathcal{A} = \{<, m, o, s, f, d, eq, d_i, f_i, s_i, o_i, m_i, >\}$ with meanings *before, meet, overlap, start, finish, during, equal, during_ by, finish_ by, start_ by, overlap_ by, meet_ by and after.* Allen relations in space are conceptually illustrated in Fig. 7.2.

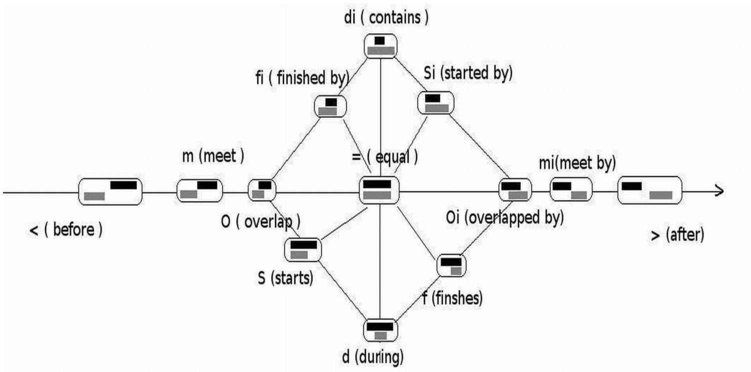


Figure 7.2: Black segment represents the reference object and gray segment represents argument object

These relations represent the eight topological relations in one-dimensional spatial domain. Eight relations are possible combination of eight independent Allen relations. Inverse of these relations and their reorientation are represented in table 7.1 and table 7.2. These tables show that whole two-dimensional space can be explored with the help of one-dimensional Allen relations using oriented lines varying from $[0, \pi]$.

Fuzzy Allen relations are used to represent the fuzzy topological relations where fuzziness is represented at the relation's level. Fuzzification process of Allen relations don't depend on a particular fuzzy membership function. Commonly a trapezoidal membership function (Eq. 7.1) is used due to flexibility in shape change.

Let $r(I,\ J)$ be an Allen relation between segments I (argument object) and J (reference object), r' is the distance between $r(I, J)$ and it's conceptional neighborhood. We consider a fuzzy membership function $\mu : r' \longrightarrow [0, 1]$. The fuzzy Allen

Relation	Inverse
$<$	$>$
m	mi
o	oi
s	si
f	fi
d	di
$=$	$=$

Table 7.1: Allen relations and their inverse

Relation	Re-orientation
$<$	$>$
m	mi
o	oi
s	f
fi	si
d	d
di	di
$=$	$=$

Table 7.2: Allen relations and their re-orientation

relations are defined in [Salamat 2010b] as

$$\left.\begin{aligned}
f_{<}(I,J) &= \mu_{(-\infty,\ -\infty,\ -b-3a/2,\ -b-a)}(y)\\
f_{>}(I,J) &= \mu_{(0,\ a/2,\ \infty,\ \infty)}(y)\\
f_{m}(I,J) &= \mu_{(-b-3a/2,\ -b-a,\ -b-a,\ -b-a/2)}(y)\\
f_{mi}(I,J) &= \mu_{(-a/2,0,0,a/2)}(y)\\
f_{O}(I,J) &= \mu_{(-b-a,-b-a/2,-b-a/2,-b)}(y)\\
f_{Oi}(I,J) &= \mu_{(-a,-a/2,-a/2,0)}(y)\\
f_{f}(I,J) &= min(\mu_{(-(b+a)/2,-a,-a,+\infty)}(y), \mu_{(-3a/2,-a,-a,-a/2)}(y), \mu_{(-\infty,-\infty,z/2,z)}(x))\\
f_{fi}(I,J) &= min(\mu_{-b-a/2,-b,-b,-b+a/2}(y), \mu_{(-\infty,-\infty,-b,-(b+a)/2)}(y), \mu_{(z,2z,+\infty,+\infty)}(x))\\
f_{s}(I,J) &= min(\mu_{-b-a/2,-b,-b,-b+a/2}(y), \mu_{(-\infty,-\infty,-b,-(b+a)/2)}(y), \mu_{(-\infty,-\infty,z/2,z)}(x))\\
f_{si}(I,J) &= min(\mu_{(-(b+a)/2,-a,-a,+\infty)}(y), \mu_{(-3a/2,-a,-a,-a/2)}(y), \mu_{(z,2z,+\infty,+\infty)}(x))\\
f_{d}(I,J) &= min(\mu_{(-b,-b+a/2,-3a/2,-a)}(y), \mu_{(-\infty,-\infty,z/2,z)}(x))\\
f_{di}(I,J) &= min(\mu_{(-b,-b+a/2,-3a/2,-a)}(y), \mu_{(z,2z,+\infty,+\infty)}(x))
\end{aligned}\right\} \quad (7.6)$$

where $a = min(x,z)$, $b = max(x,z)$ and x is the length of segment (I) and z is the length of segment (J) and (x,y,z) are computed as described in section 7.4.1.

Most of relations are defined by one membership function but some of them are defined by conjunction of more than one membership functions like $d(during)$, $d_i(during\_by)$, f $(finish)$, f_i $(finished\_by)$. In fuzzy set theory, sum of all the relations is one, this gives the definition for fuzzy *equal* relation.

These are the topological relations which represent fuzziness at relation's level. For example, *Meet* topological relation is represented based on nearness and length of smaller interval defines the smooth transition between the *Meet (Meet_ by)* and *before (after)* relation. In spatial domain, *before (after)* are called the *disjoint* topological relations.

7.4.3 Combining Topological and Directional Relations

The approaches for defining binary topological relations between object pair are largely developed by the Max. J. Egenhofer method [Egenhofer 1991] using the point set topology. This relationship between two regions is represented by a 3×3 matrix, called the 9-intersections method and relations are *Disjoint, Meet, Overlap, Covers, Contain, Covered_by, Contained, Equal.* The 9-intersections method considers only one piece region without holes in two-dimensional space.

On the other hand, the *RCC* method was developed by Randell et al. [Randell 1992a] which is an axiomatic theory for representing the topological relations. Relations in this theory are based on a single atomic relation called *connection*. This theory also represents the same eight topological relations, called $RCC8$ relations between object pair. These relations are represented with different names which are $\{DC, EC, PO, TPP, NTPP, TPPI, NTPPI, EQ\}$, called respectively *DisConnected, Externally Connected, Partially Overlap, Tangent Proper Part, Non Tangent Proper Part, Tangent Proper Part Inverse, Non Tangent Proper Part Inverse, Equal* . The difference of names is due to English language semantics[1] otherwise both theories represent the same topological relations. Allen's temporal relations in spatial domain represents the eight topological relations in one-dimensional space. We have extended these Allen relations for the $2D$ objects through the logical implication. A $2D$ object is decomposed into parallel segments of a $1D$ lines in a given direction. Allen relations between each pair of segments from both objects are computed. This process is repeated for directions $[0, \pi]$. This is called CTD-relations method [Salamat]. The steps involved for computing the two dimensional topological relations are explained below

7.4.4 CTD-relations Method

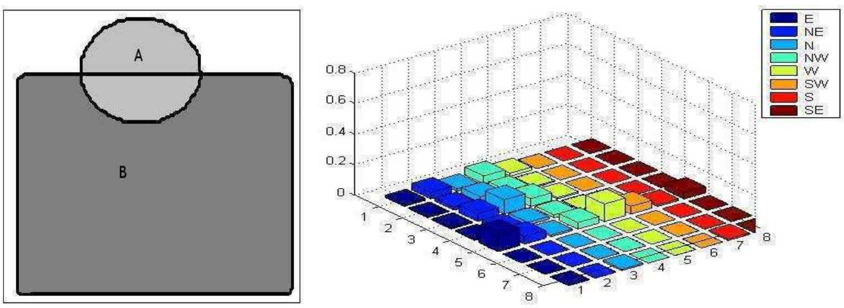
The process of object decomposition is repeated for each direction varying from 0 to π. The $2D$ topological and directional relations are represented as a matrix. This matrix provides us information that how the objects are relatively distributed ($\sum_{i,j=1}^{8} = 1$). In this representation, lines represent topological relation such as *{D, M, TPP, NTPP, TPPI, NTPPI, EQ}* with meanings *disjoint, meet, tangent proper part, non tangent proper part, tangent proper part inverse, non tangent Proper part inverse and equal.* Columns represent directional relations between object pair such as *{E, NE,N, NW, W,SW,S,SE}* with meanings *east, north east, north, north west, west, south west, south and south east.*

These relations are not JEPD relations. To obtain JEPD set of topological and directional relations, an algorithm was advocated in [Salamat]. This algorithm provides us the JEPD set of relations. These JEPD relations are represented as a

[1] In *RCC* "DC= Disconnected" and the 9-intersections represent the "D=Disjoint" same semantics we use in our terminology "D" for disjoint topological relation, similarly for "M" for "meet".

pair (R_T, R_{Dir}), where R_T, R_{Dir} represents topological and directional relations respectively. Objects are approximated through the polygon approximation. Different steps of computing the matrix of CTD relations are

- Fix an angle θ and draw lines passing through the vertices of polygons representing the objects. For this purpose, simple line drawing formula of slop intercept form are used.
- For each line compute variables (x, y, z) and use mathematical Eqs. given in section 8.3.2 to compute Allen relation. For longitudinal section, fuzzy aggregation operators are used to integrate the information, usually the disjunction operators are suitable. These relations are computed for each line in a direction. The obtained information are integrated into a single value. These relations are normalized for a given direction θ by dividing sum of all Allen relations to each relation.
- Normalized fuzzy Allen relation are multiplied by a fuzzy directional set to find the degree of the Allen relation in a given direction.
- Topological relations with qualitative directional contents are defined. For example topological relations with *East* direction are defined as $f_E = \sum_{\theta=0}^{\frac{\pi}{4}} \mathcal{A}_2 \times cos^2(2\theta) + \sum_{\theta=\frac{3\pi}{4}}^{\pi} \mathcal{A}_1 \times cos^2(2\theta)$, where f and E represents respectively the topological relation and the *East* direction and $\mathcal{A}_1$ is reorientation of an Allen relation $\mathcal{A}_1$.
- This information is represented in a matrix, an algorithm proposed in [Salamat] is then used which enables us to have JEPD set of topological and directional relations. For an example object pair are represented in Fig. 7.3(a), the fuzzy relations are represented in the Fig. 7.3(b).



(a) (b)

Figure 7.3: Object pair and their combined topological and directional relation information representation where qualitative topological and directional relations are *(PO, N)*

7.5 Conceptual Neighborhood Graph and Change in Spatial Relations

This section composed of three subsections. Section 7.5.1 explains changes in the topological and directional relations, conceptual neighborhood graph for the CTD method is elaborated in section 7.5.2 and section 7.5.3 explains continuity of spatial relations.

7.5.1 Conceptual Neighborhood Graph and Change in Spatial Relations

In real world situation, spatial and temporal relationships between objects change continuously. The objects change their positions to the next neighboring position, resulting the spatial relations also change to the neighboring spatial relations. The continuous change in topological relations can be depicted as:

$$D(A,B) \Rightarrow M(A,B) \Rightarrow PO(A,B) \Rightarrow TPP(A,B) \Rightarrow NTPP(A,B)$$

The movement can be represented as change in directional relations,

$$N(A,B) \Rightarrow NE(A,B) \Rightarrow E(A,B) \Rightarrow SE(A,B) \Rightarrow S(A,B) \Rightarrow ..$$

Similarly for other topological and directional relations sequence of change in spatial relations can be determined. These topological and directional relations are taken together, they might change topological or directional part of the relation or both topological and directional relations simultaneously between two objects. This change in spatial relation between two objects can be represented as a change in pair of relations. For better explanation let us consider the possible changes from initial position of $(M(A,B), E(A,B))$ (A, B *meets each other from East*)

$$(M(A,B),E(A,B)) \Rightarrow \begin{cases} (M(A,B),NE(A,B)) & \text{Change in directional relation} \\ (M(A,B),SE(A,B)) & \text{Change in directional relation} \\ (PO(A,B),NE(A,B)) & \text{Change in topological and directional relation} \\ (PO(A,B),E(A,B)) & \text{Change in topological relation} \\ (PO(A,B),SE(A,B)) & \text{Change in topological and directional relation} \\ (D(A,B),NE(A,B)) & \text{Change in topological and directional relation} \\ (D(A,B),E(A,B)) & \text{Change in topological relation} \\ (D(A,B),SE(A,B)) & \text{Change in topological and directional relation} \end{cases}$$

This shows that there are eight possible future positions from topological relation *Meet* and directional relation *East*. Similarly there will be 12 future positions from

PO topological relation. The other possible transitions from one spatial position to the other can easily be derived. The whole possible transitions are represented in a transition graph, where connected pairs of the spatial relations are called neighbors and the graph is called the neighborhood graph. This graph depicts all physical transitions between spatial relations that can occur through the deformation of intervals.

A neighborhood graph in topological relations describes three types of deformation, **A**-deformation, **B**-deformation and **C**-deformation [Freksa 1992]. **A**-deformation occurs when one interval expands or contracts with one point fixed (one interval is stretched or sheared). A deformation is called **B**-deformation when intervals don't change the size, only interval change their position with time. **C**-deformation is characterized as uniform expansion or contraction of interval take place. This graph also represent all these types of deformation. For simplicity, we represent here only one possible transition from *partially overlap* relation.

7.5.2 Conceptual Neighborhood and Neighborhood Graph

Neighborhood graph of topological relations represents three types of topological deformations. Here for simplicity of the graph, only one branch of **B**-deformation is considered and possible transitions are presented into the graph.

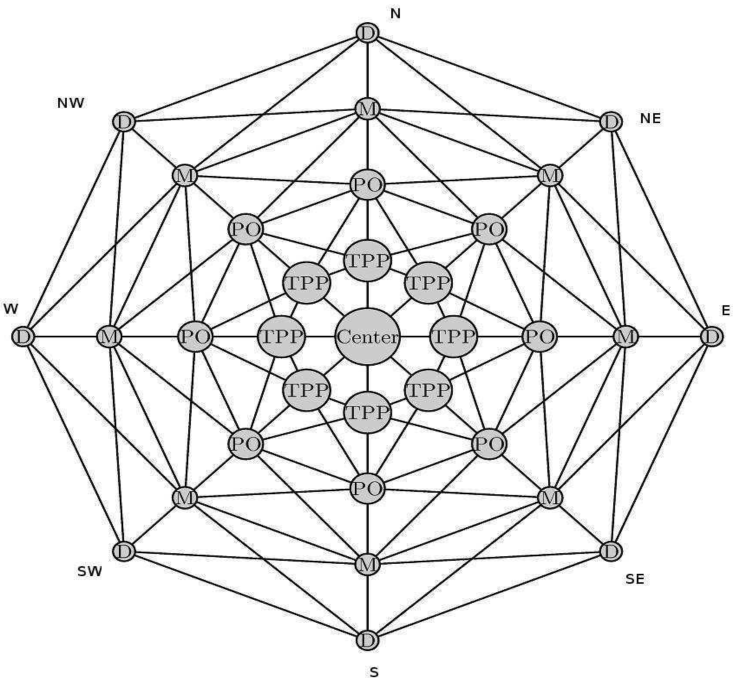


Figure 7.4: Neighborhood graph in the system of combined topological and directional relations

In the Fig. 7.4, object can move to a circular, straightened and diagonal path and neighborhoods are called *directional*, *topological* and *topological and directional* neighborhood respectively. Neighborhood graph shows allowable transitions from one relation to other relation. These transitions are possible when the objects move or spatial scene changes. In Fig. 7.4, a node represents the pair of spatial relations and an edge represents the transition or change from one spatial relation to other spatial relations. Spatial relations are represented by a pair (R_T, R_{Dir}) where R_T represents the topological and R_{Dir} represents directional relation between object pair.

In Fig. 7.4, it is shown that every node of a neighborhood graph has eight edges except from the node representing the topological relation D and TPP. This is due to the fact that when an object has a disjoint topological relation with the other object and this object moves away from its counter part, there is only change in directional relations and topological relations are stable in this case. Similarly when an object has TPP topological relation, the object can't change the topological relation towards $NTPP$ and directional relations simultaneously.

7.5.3 Conceptual Neighborhood Graph and Continuity

Continuity in motion is modeled as a continuous change in the spatial relations. In neighborhood graph two connective nodes represent two states of moving objects and the edge between them represent the time. Let us consider that $(0, 0)$ shows the position of a node and $+1$ shows the possible change in a relation and -1 shows change in relation in opposite direction. If topological relation is in outermost nodes, then topological change is not possible in outward direction, as a result, in outward direction, object change the distance relation or distance and directional relation at the same time.

For example if $(0, 0)$ position shows the relation (M, E), then -1 in topological neighborhood may represent the PO then 1 represents the D and vice versa. Similarly -1 in directional neighborhood may represent the directional relation NE (SE) then 1 will represent the relation SE (NE). Similar rule applies for the diagonal neighborhood. If the objects have the EQ spatial relations then shell representing the relation $TPP/TPPI$ disappears and $NTPP$, $NTPPI$ relation is replaced with EQ relation.

7.6 Topological and Directional Relations and Their Converse

The computation of spatial relations with CTD method provides the topological and directional relations at the same abstraction level. Let R defines relation between object pair (A, B) then R has two components, first component corresponds to topological and second component corresponding to directional relation. Topological and directional components provide us information about the topological and directional relation respectively. Formally this can be written as

$$R(A, B) = (R_T(A, B), R_{Dir}(A, B)) \tag{7.7}$$

Where R_T, R_{Dir} represents topological and directional parts respectively. Converse relation (recall Eq. 7.2) of $R(A,B)$ has two components (Eq. 7.7). Each component acts independently, it is written as

$$\tilde{R}(A,B) = (\tilde{R}_T(A,B), \tilde{R}_{Dir}(A,B)) \tag{7.8}$$

Topological part has the topological converse and directional converse of a directional part commutes with the symmetric properties of directional relations. Converse of a directional relation can be written as

$$\tilde{R}_{Dir}(A,B) = R_{Dir}(B,A) = R_{Dir}(A,B) + \pi \tag{7.9}$$

By combining the Eq. (7.7), (7.8) and (7.9), The converse relation (7.2) can be rewritten for this system of spatial relations as

$$\tilde{R}(A,B) = (\tilde{R}_T(B,A), R_{Dir}(A,B) + \pi) \tag{7.10}$$

The table 7.3 represents converse of all the topological relations. This table shows that most topological relations have the same converse relation. Topological relation doesn't depend on the order of objects. The table 7.4 shows the converse

Topological relation (R_T)	Symbol	Converse topological relation ($\tilde{R}_T$)	Symbol
Disjoint	D	Disjoint	D
Meet	M	Meet	M
Partially overlap	PO	Partially overlap	PO
Tangent proper part	TPP	Tangent proper part inverse	TPPI
Non tangent proper part	NTPP	Non tangent proper part inverse	NTPPI
Tangent proper part inverse	TPPI	Tangent proper part	TPP
Non tangent proper part inverse	NTPPI	Non tangent proper part	NTPP
Equal	Eq	Equal	EQ

Table 7.3: Topological relations and their converse

of directional relations. These relations have a difference π with their converses.

Directional relation (R_{Dir})	Symbol	Converse dir relation ($\tilde{R}_{Dir}$)	Symbol
East	E	West	W
North_East	NE	South_west	SW
North	N	South	S
North_West	NW	South_East	SE

Table 7.4: Directional relations and their converse

7.7 Composition Tables

Composition tables play important role in the spatio-temporal reasoning. These tables specify the relation obtained by composing the relation in the corresponding row to the relation in the corresponding column. These tables provide information that which spatial relation is possible between the two objects at next time point. CTD method consist of the 43 predicates. Composition table for CTD method is composed of 1849 (43×43) entries. It is a huge table, difficult to analyze and reasoning is intractable. Both topological and directional relations are not interconnected, we develop separate composition tables for topological and directional relations.

7.7.1 Composition Table For Topological Relation

We construct the composition tables for topological relations(table 7.5). In this composition table, first we fixe one type of a relation then find the possibilities for the other sort of spatial relations. Here, first we fix the directional relation for example N, then we build a composition table for topological relations.

	D	M	PO	TPP	NTPP	TPPI	NTPPI
D	**No info.**	D, M, PO, TPP, NTPP	D, M, PO, TPP, NTPP	D, M, PO, TPP, NTPP	D, M, PO, TPP, NTPP	D	D
M	D, M, PO, TPPI, NTPPI	D, M, PO, TPP, TPPI, EQ	D, M, PO, TPP, NTPP	M, PO, TPP, NTPP	PO, TPP, NTPP	D, M	D
PO	D, M, PO, TPPI, NTPPI	D, M, PO, TPPI, NTPPI	**No info.**	PO, TPP, NTPP	PO, TPP, NTPP	D, M, PO, TPPI, NTPPI	D, M, PO, TPPI, NTPPI
TPP	D	D, M	D, M, PO, TPP, NTPP	TPP, NTPP	NTPP	D, M, PO, TPP, TPPI, EQ	D, M, PO, TPPI, NTPPI
NTPP	D	D	D, M, PO, TPP, NTPP	NTPP	NTPP	D, M, PO, TPP, NTPP	**No info.**
TPPI	D, M, PO, TPPI, NTPPI	M, PO, TPPI, NTPPI	PO, TPPI, NTPPI	PO	PO, TPP, NTPP	TPPI, NTPPI	NTPPI
NTPPI	D, M, PO, TPPI, NTPPI	PO, TPPI, NTPPI	PO, TPPI, NTPPI	PO, TPPI, NTPPI	PO, TPP, NTPP, TPPI NTPPI, EQ	NTPPI	NTPPI

Table 7.5: Composition table for topological relations

In this composition table (table 7.5) EQ topological relation is not computed because it is an idempotent relation. We decompose the composition table for topological relations (table 7.5) into multiple sub-tables, this table is composed of nine sub-tables, separated by double lines. These tables represent the coarse knowledge usually it is a disjunction of relations. This derived knowledge can further be refined through the introduction of a directional contents between the object pair.

7.7.1.1 Properties of Composition Tables

We can define some composition rules based on these tables as follows

1. If the relation between object pairs are changed to their converse relations along with order, then resulting relation also becomes its converse. Let R be a topological relation between object pair (A, B) and $\tilde{R}$ is the relation between the object pair (B, A), topological inverse, then composition will be

$$R_1(A, B) \odot R_2(B, C) = R_3(A, C)$$

   and

$$\tilde{R}_2(A, B) \odot \tilde{R}_1(B, C) = \tilde{R}_3(A, C)$$

2. Composition table for topological relations(table 7.5 when equal relation is included. For details see composition tables for topological relations in [Cohn 1997, Gotts 1996, Li 2004]) has the 21 different entries, out of 64, these 21 entries have a certain relation and can be computed through the above cited relation. It is written as

$$R_1(A, B) \odot R_2(B, C) = R(A, C) \Rightarrow \tilde{R}_2(A, B) \odot \tilde{R}_1(B, C) = \tilde{R}(A, C)$$

   These relations can be expressed as shown below.

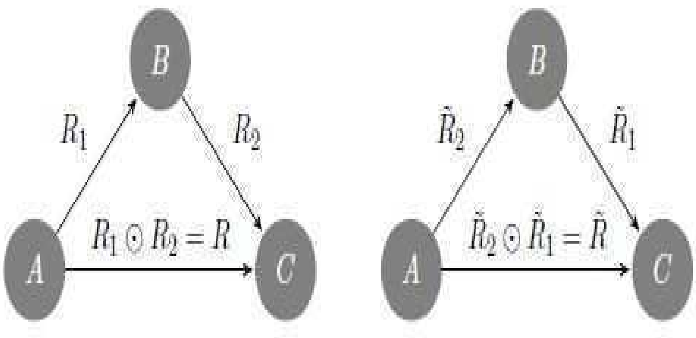


Figure 7.5: Commutative-inverse composition relation

3. Equal relation is an identity element, it does not effect the composition of a topological relation. If inverse topological relations are involved in the composition table, then all the topological relations are possible.

4. Composition relations are always in topological neighbors.
5. A transitivity relation shows constraints that are satisfied by the spatial reasoning algorithm.

7.7.2 Composition Table for Directional Relations

We compute the composition table for the composition of spatial relations. For this, we first fix the topological relations then proceed for the composition of directional relations. Table 7.6 represents the composition table for directional relations. This

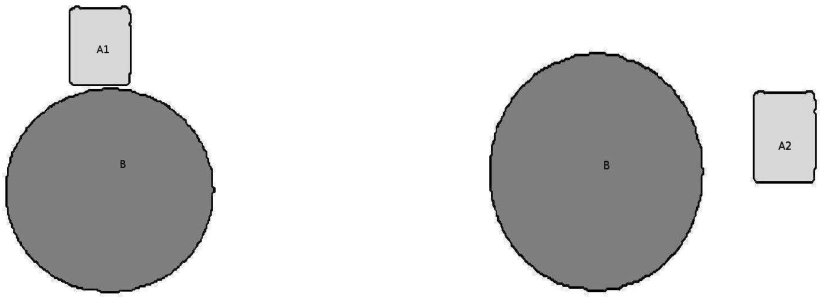
	E	NE	N	NW	W	SW	S	SE
E	E	E, NE	E, NE, N	E, NE, N, NW	**No info.**	SW, S, SE,E	E, SE, S	E, SE
NE	E, NE	NE	NE, N	NE, N, NW	NE, N, NW, W	**No info.**	E, NE, SE, S	E, NE, SE
N	E, NE, N	NE, N	N	N, NW	N, NW, W	N, NW W, SW	**No info.**	N, NE, E, SE
NW	E, NE, N, NW	NE, N, NW	N, NW	NW	W, NW	NW, W, SW	NW, W, SW, S	**No info.**
W	**No info.**	NE, N, NW, W	N, NW, W	NW, W	W	W, SW	W, SW, S	W, SW, S, SE
SW	E, SE, S, SW	**No info.**	N, NW, W, SW	NW, W, SW	W, SW	SW	SW,S	SW, S, SE
S	S, SE, E	NE, E, SE, S	**No info.**	NW, W, SW, S	W, SW, S	S,SW	S	S,SE
SE	E, SE	NE, E SE	N, NE, E, SE	**No info.**	W, SW, S, SE	SW, S, SE	S, SE	SE

Table 7.6: Composition table between directional relations

composition table can be refined by introducing the topological relations. A general rule can be followed that if the neighboring direction and same topological relation are involved in composition. In such a case, topological relation will be changed. If they have opposite or perpendicular direction, then topological relation between object pair are identical.

7.8 Spatio-Temporal Reasoning as Position Evaluation and Path Navigation in Video Understanding Systems

Spatio-temporal reasoning is a multi-task process, it is used for approximating the subsequent position, history as well as trajectory of a moving object, path navigation and as the problem solvers in many areas of artificial intelligence. Here, we use spatio-temporal reasoning for finding the relative position of a moving object with respect to it previous position and as path navigation. We consider here a set of information derived from Figs. 7.6(a) and 7.6(b), which represent the position of a moving object A, where $A1$ and $A2$ represent respectively position at t_1 and t_2.



(a) (b)

Figure 7.6: Object pairs at time t_1 and t_2

We use the CTD method to derive the information from these primary images. This method produces the results $R(A1, B) = (M, N)$ and $R(A2, B) = (D, E)$. The set of derived information from these object pairs are represented as

- $A1$ *Meets* B
- $A1$ at *North* of B
- $A2$ and B are *Disjoint*
- $A2$ lies on *Right* of B

In the next stage we used the proposed method to know the position of $A2$ with respect to $A1$ and the path adopted by object A from $A1$ to $A2$.

Spatio-temporal reasoning as position evaluation: In moving objects, motion direction is an important feature and this motion direction can be determined by the spatial relations between two consecutive positions of an object. Then a natural question arises that what is the relation between $A1$ and $A2$? First of all, we use proposed method for composition of spatial relations. This composition of spatial relation will provide us answer that what are the topological and directional relations between the two positions of a moving object? Summarized information are represented as

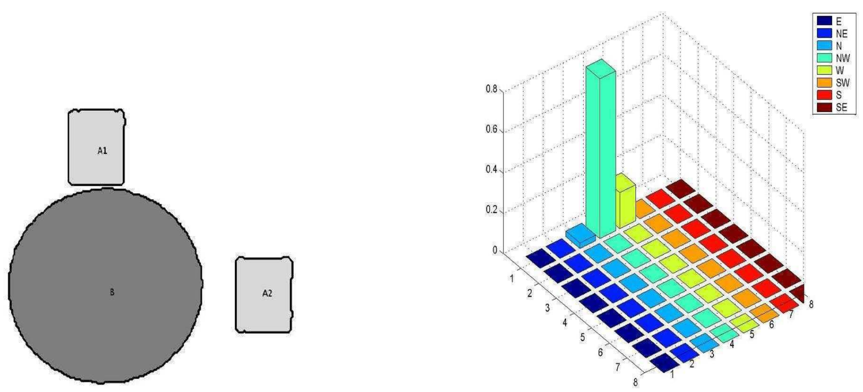
- $R_T(A_1, B) = M(A_1, B)$ and $R_{Dir}(A_1, B) = N(A_1, B)$
- $R_T(A_2, B) = D(A_2, B)$ and $R_{Dir}(A_2, B) = E(A_2, B)$

To know the position of A_1 with respect to A_2, first we need to know the inverse of spatial relation between object pair $(A2, B)$. Consider the inverse of topological and directional relation separately.
$\tilde{R}_T(A_2, B) = R_T(B, R_2)$ similarly $\tilde{R}_{Dir}(A_2, B) = R_{Dir}(B, A_2) = W(B, A_2)$

At this stage, we use the proposed method for spatial relations composition, this composition of relations provide us the spatial relations between objects $A2$(reference) and $A1$ (argument) objects.

$$\left.\begin{aligned} R(A_1, A_2) &= R(A_1, B) \odot R(B, A_2) \\ &= (R_T(A_1, B), R_{Dir}(A_1, B)) \odot (R_T(B, A_2), R_{Dir}(B, A_2)) \\ &= (M(A_1, B), N(A_1, B)) \odot (D(B, A_2), W(B, A_2)) \\ &= (\{M(A_1, B) \odot D(B, A_2)\}, \{N(A_1, B) \odot W(B, A_2)\}) \\ &= (\{D, M, PO, TPPI, NTPPI\}(A_1, A_2), \{N, NW, W\}(A_1, A_2)) \end{aligned}\right\}$$

This shows that topological relations between object $A1$ and $A2$ is one of the $\{D, M, PO, TPPI, NTPPI\}$ and directional relations are $\{N, NW, W\}$. Some of the topological relations like $TPPI, NTPPI$ are not possible because both object have same size and these topological relations depend upon size of objects. Hence, topological relations between two objects are $\{D, M, PO\}$. This represents the coarse knowledge of spatial relations between objects. This knowledge is improved by introducing some constraints. For example, if we know that topological relations are D then this knowledge improves the spatial knowledge about directional relations. Now we use the method of CTD directly to the combined image of three objects. These objects are represented in Fig. 7.7(a) and its relations are represented in 7.7(b). This method results out the spatial relations between objects $A2$ to $A1$ is *(D,NW)*.



(a) (b)

Figure 7.7: Objects and spatial relations where the spatial relations are (D, W)

Spatio-temporal reasoning as path navigation: $A1$ and $A2$ are the same object and represents the positions of A at t_1 and t_2. then what is the path of A from position $A1$ to position $A2$.

One can follow the forward and backward reasoning using the graph of spatial relations described in Fig. 7.5. This provides us the different navigation plans like

$P_1 = (M, N, t_1), (PO, N, t_2), (TPP, N, t_3), (NTPP, t_4), (TPP, E, t_5),$
$(PO, E, t_6), (M, E, t_7), (D, E, t_8)$
$P_2 = (M, N, t_1), (M, NE, t_2), (D, E, t_3)$
$P_3 = (M, N, t_1), (M, NE, t_2), (M, E, t_3), (D, E, t_4)$

Where P_1, P_2 and P_3 denote the paths. Similarly, some other paths can also be determined depending on the neighborhood graph of spatial relations. At this level, domain knowledge is used to remove some possibilities of holding spatial relation and possible path between object pairs.

7.9 Conclusion

Spatio-temporal reasoning is an important part of the Artificial Intelligence and many applications in related fields, existing approaches in this field are domain based. Commonly spatial reasoning consist of composition tables for spatial relations and these tables consist of disjunction of spatial relations. The disjunction of relations represents the coarse knowledge and this information could be refined by adding the topological information to the composition table for directional relations and vice versa. Many techniques have been developed for reasoning with topological and directional relations separately. Continuous moving objects can change the topological or directional or both relations simultaneously.

In this paper, we used the CTD method for reasoning which leads from the coarse knowledge to the finer knowledge of the spatial domain. This method represents the fuzziness at relation's level. We established that some entities of the composition tables are interrelated and follow a mathematical framework. This mathematical relation is generalized and the proposed composition table is rearranged and divided into sub-tables. These rules are used to construct the composition tables for topological relations and less calculation is required as compared to the existing approaches. This method can be used for modeling predictive topological and directional movements of object pair. We plan to implement this method in realistic applications such as motion event predictions and video scene analysis.

Chapter 8

Spatio-Temporal Motion Event Predictions

Abstract

Motion is an open ended process while motion events are bounded subsets of this process. Predictions about motion events is similar to spatio-temporal reasoning. This process predicts motion event if interval for existing motion event is extended to the next time point. Language semantics of a motion event also changes as soon as a new spatial relation is added due to change in temporal bound of an event. In this paper we proposed a method for the motion event predictions. This method takes into account the topological and directional relations predicate simultaneously along with the history of moving objects. These motion events predictions can be used to build spatio-temporal queries in spatio-temporal database.

Key words: Motion event predictions, Spatio-temporal reasoning, Topological predicates, Directional predicates.

8.1 Introduction

Different spatial languages are used to represent spatial objects and their relative position in space. Spatial objects in space are studied through the topological and metric aspects of space. For instance, in Geographic Information System(GIS), time and spatial changes are handled through the sequence of static snapshots at discrete time points. An individual snapshot at a discrete time t_i represents the objects relative position in terms of the topological and directional relations. Change in spatial scene or spatio-temporal event under way is captured through the difference in topological and directional relations between two consecutive snapshots.

The change in relative position may bring change in mutual topological, distance or directional relations between spatial objects. The change in any sort of relations needs analysis in topological, distance and directional view point between object pair at each step. The change detection is helpful for modeling spatio-temporal relations and motion events between moving objects. The spatio-temporal predictions are based on the predictions in the topological and directional relations [Erwig 1999, Cohn 2001]. These predictions are useful for implementing the machine learning based event recognition modules, decisions about the nature of events.

Topological relations are studied through the 9-intersections method or Region Connection Calculus (RCC) [Egenhofer 1991, Cohn 1997]. Eight topological configurations are considerd between objects. Many extensions have been proposed to deal with fuzzy objects. Neighborhood graph for topological relations is used for modeling the topological and spatio-temporal predictions. These predictions are based on topological changes in spatial scene. Directional relations are studied through qualitative, quantitative and fuzzy methods [Frank 1996, Goyal 2001, li 2008, Miyajima 1994, Matsakis 1999b, Bloch 1999]. In fuzzy methods, each directional relation holds with a certain degree. These relations are less used to model the spatio-temporal relations. Directional relations commonly work for disjoint objects. A method for reasoning when reference object is contained in the argument object is developed in [Liu 2005], called Internal Cardinal Directions (ICD) relations method.

A set of motion events are modeled in [Erwig 1999, Muller 2002, Salamat 2011a] where topology governs the changes at each snapshot. Composite events are the union of primitive events and an event is a bounded subsets of a process. These events can be derived from a process [Galton 2002] and events are bounded in temporal intervals. After the addition of a time point to temporal interval, the composite event also changes from a sub-event to super-event. Motion event prediction is a process closely related to the spatio-temporal reasoning. Spatio-temporal reasoning provide us information about the topological or directional relation for the next time point. Spatio-temporal motion event prediction stores the history of a moving object and provide us information about motion event for next time point. In other words, if analysis interval is extended to the next time instant, then how the motion event will change?

In this paper, we study the change in motion events with the extension of temporal interval and propose a method for spatio-temporal motion events predictions. Combined Topological and Directional relations (CTD) method [Salamat 2010a, Salamat] is used for finding topological and directional relations for a snapshot at instant t. These primitive events are stored into the history-tables for a moving object and then new prediction for the topological and directional relations are added to the history of a moving object. A spatio-temporal relation holds during an interval T, it represents a particular motion event. Motion is a process and its subsets are called the motion events, composite event are sequential existence of the primitive events. A new primitive events can exist with the change in bounds of temporal interval. This new primitive event is then added to the existent event and it changes the existing motion event. We propose a method where spatial reasoning is logically extended to motion event predictions.

The paper is arranged as follows, Section 8.2 describes related works. A short description of CTD method is described in Section 8.3. Spatial predictions are given in Section 8.4 and Section 8.5 compose of spatio-temporal motion event predictions. The theoretical explanations of method is explained in Section 8.6. An example is considered in Section 8.7 and Section 8.8 concludes the paper.

8.2 Related Work

Naive knowledge takes into account different external forms of representation, which are used by human being to encode language semantics, mathematical and formal logic. Spatio-temporal event predictions is a similar process to the spatio-temporal reasoning. Spatio-temporal relations hold the history of a moving object and spatial prediction is added in the history-table of a moving object. This addition of spatial prediction in the history-table of moving object can change spatio-temporal relation or motion event over the interval. Spatio-temporal events are syntactic summary of spatio-temporal relations of visual objects in an interval.

Predictions in spatial relations represent possible transition in topological and directional relations. Methods for defining the topological predicates are developed in [Egenhofer 1995, Gerevini 2002, Cohn 2001]. In these methods, topological relations are studied in context of point set topology or region connection calculus [Egenhofer 1991, Cohn 1997, Cohn 2001]. In these methods, point temporal logic is used to extend the spatial relations in temporal domain with neighborhood graph for topological relations [Freksa 1991, Museros 2003]. The change in topological relations follows the neighborhood graph.

Allen introduced interval temporal logic with two predicates *holds(P,i) (property P holds during interval i)* and *occurs(e,i)(event e occurs during interval i)* [Allen 1983, Allen 1994]. Philippe Muller [Muller 2002] used this logic and developed a class of motion events. This logic is further extended to the discrete domain and two new predicates are introduced[Galton 2003](*holds-at* and *occurs-at*(*holds-at(P,t) property P holds at time instant t*, *occurs-at(e,t) event e occurs at time instant t*). In [Erwig 1999] Markus Schinder used point interval logic and introduced different motion event predictions. These predicates are frequently used in linguistics.

Actions are decomposable into sub-actions with a simple temporal ordering. These actions are parts of a process. M. Worboys in [Worboys 2005] introduced a method where time can be handled as continuous process. It is known that one event proceed the other event like a real number line. Each point of this line is corresponding to an instant of time. In this method, time is considered as a continuous and sequenced *ticks*. Two consecutive *ticks* are connected through a channel. for example

$$tick_1 \xrightarrow{next_{12}} tick_2 \xrightarrow{next_{23}} tick_3 \xrightarrow{next_{34}} tick_4 \; .... \tag{8.1}$$

Where *tick* corresponds to a snapshot. Time between two consecutive snapshots is correspond to $next_{ij}$. In the motion event predictions, duration from snapshot $tick_i$ to snapshot $tick_j$ is matched as a prediction. Spatial relations for snapshot $tick_i$ is considered to the spatial relations at time t_i and an edge in neighborhood graph is correspond to the $next_{ij}$. As time changes from $tick_i$ to $tick_j$, a snapshot also changes from $snapshot_i$ to $snapshot_j$ and spatial relations between the objects of snapshot also changes continuously.

In this paper, we propose a method, which stores the current spatial relations in history-tables and spatial predictions are added into these histories. Topological and directional relations are determined with combined topological and directional relations method [Salamat 2010b, Salamat 2011b]. These fuzzy relations are defuzzified and represented in neighborhood graph. There are 43 topological and directional relations predicates [Salamat]. These spatial predictions are used for modeling the spatio-temporal event predictions. This model anticipate the motion events where the spatial predictions for topological and directional relations are taken from the CTD method.

8.3 Combined Topological and Directional (CTD) Relations Method

In this section we describe the terminology and different steps used for the computation of combined topological and directional relations method.

8.3.1 Oriented Lines, Segments and Longitudinal Sections

Let A and B be two spatial objects and $(v, \theta) \in \mathbb{R}$, where v is any real number and $\theta \in [0, 2\pi]$. Let $\Delta_\theta(v)$ be an oriented line at angle θ and $A \cap \Delta_\theta(v)$ is the intersection of object A with line $\Delta_\theta(v)$. It is denoted by $A_\theta(v)$, called segment of object A and length of its projection interval on x-axis is x. Similarly for object B where $B \cap \Delta_\theta(v) = B_\theta(v)$ is segment and length of its projection interval on x-axis is z. Whereas y is the difference between the minimum of $A \cap \Delta_\theta(v)$ and maximum of $B \cap \Delta_\theta(v)$(for details [Matsakis 2005]).

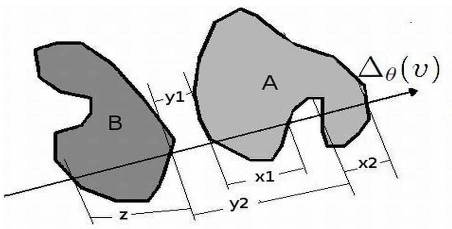


Figure 8.1: Oriented line $\Delta_\theta(v)$, segment as in case of object B, longitudinal section as in case of object A.

In case of polygonal object approximation (x, y, z) can be calculated from intersecting points of line and object boundary. Only those oriented lines are considered which passe through the vertices of a polygon. If there exist more than one segments, then it is called longitudinal section as in case of $A_\theta(v)$ in Fig. 8.1.

8.3.2 Allen Temporal Relations in Spatial Domain and Fuzziness

Allen [Allen 1983] introduced the 13 Jointly Exhaustive and Pairwise Disjoint (JEPD) interval relations. These relations are $\mathcal{A} = \{<$

$, m, o, s, f, d, eq, d_i, f_i, s_i, o_i, m_i, >\}$ where each relation means respectively *before, meet, overlap, start, finish, during, equal, during_by, finish_by, start_by, overlap_by, meet_by, and after.* All the Allen relations in space are conceptually illustrated in Fig. 8.2.

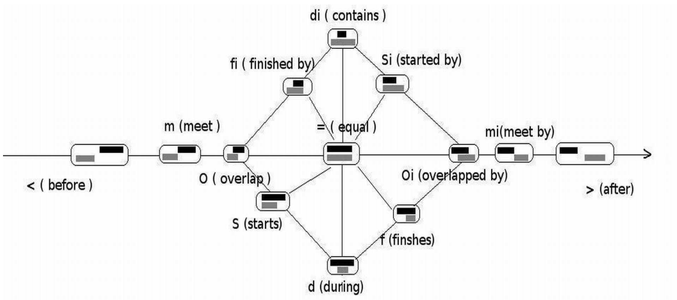


Figure 8.2: Black segment represents reference and gray segment represents argument object

Allen relations represent eight topological relations in one-dimensional spatial domain. Fuzzy Allen relations represent fuzziness at the relation's level. Fuzzification process of Allen relations don't depend on a particular choice of a fuzzy membership function. For example, trapezoidal membership function is defined as

$$\mu(x; \alpha, \beta, \gamma, \delta) = max(min(\frac{x-\alpha}{\beta-\alpha}, 1, \frac{\delta-x}{\delta-\gamma}), 0) \tag{8.2}$$

where $x, \alpha, \beta, \gamma, \delta \in \mathbb{R}$ and $\alpha < \beta \leq \gamma < \delta$. The fuzzy Allen relations using equation 8.2 are defined as

$$f_>(I, J) = \mu_{(0, a/2, \infty, \infty)}(y) \tag{8.3}$$

$$f_m(I, J) = \mu_{(-b-3a/2, -b-a, -b-a, -b-a/2)}(y) \tag{8.4}$$

where $a = min(x, z), b = max(x, z)$ and x is the length of segment (I), z is the length of segment (J) and (x, y, z) are computed as described in section 8.3.1.

Relations are fuzzified in such a way that a small movement in object effects the relation and its neighboring Allen relation. Such as in case (I_1, J_1), $f_{mi}(I_1, J_1) < 1$ and $f_>(I_1, J_1) < 1$. Similarly for (I_2, J_2), $f_{mi}(I_2, J_2) = 0$ and $f_>(I_2, J_2) = 1$. These relations are fuzzified in such a way that for one line, $\sum_{r \in A} r(I, J) = 1$. For the treatment of longitudinal section, as in case of (I_{3_1}, J_3) and (I_{3_2}, J_3) in Fig. 8.3, fuzzy Allen relations are computed for each segment separately then fuzzy aggregation operators are applied. Commonly fuzzy disjunction operators are easy to implement and better fit the situation. Histograms of Allen relations are normalized as $\sum_{i=1}^{13} A_i(\theta) = 1$, normalized histograms are represented by $\lceil A_i(\theta) \rfloor$. Fuzzy Allen histograms have reorientation property. These properties are
$f_<(\theta) = f_>(\theta + \pi),\ f_m(\theta) = f_{mi}(\theta + \pi),\ f_o(\theta) = f_{oi}(\theta + \pi),$

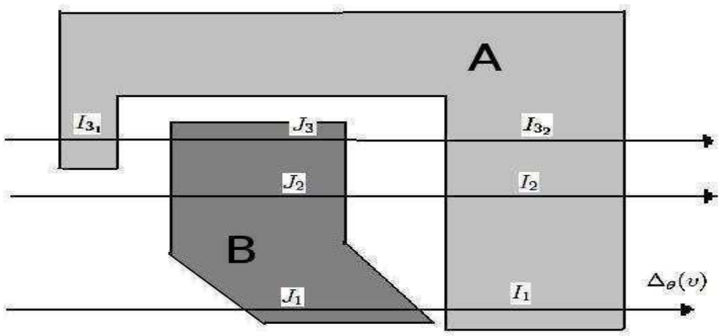


Figure 8.3: Fuzzification of Allen relations between reference and argument segments where $\Delta_\theta(v)$ is oriented line

$f_f(\theta) = f_s(\theta+\pi),\ f_{fi}(\theta) = f_{si}(\theta+\pi),\ f_d(\theta) = f_d(\theta+\pi),$
$f_{di}(\theta) = f_{di}(\theta+\pi)$ and $f_=(\theta) = f_=(\theta+\pi)$

Topological relations are named as $\{D, M, PO, TPP, NTPP, TPPI, NTPPI, EQ\}$ called respectively *Disjoint, Meet, Partially Overlap, Tangent Proper Part, Non Tangent Proper Part, Tangent Proper Part Inverse, Non Tangent Proper Part Inverse and Equal.* Eight topological relations are possible combination of eight independent Allen relations in one-dimensional spatial domain. These relations and their reorientation show that the whole $2D$ space can be explored with the help of $1D$ Allen relations using the oriented lines varying from $(0, \pi)$. Normalized fuzzy Allen relations and their reorientation is used to find the qualitative directions for a topological relation. For example

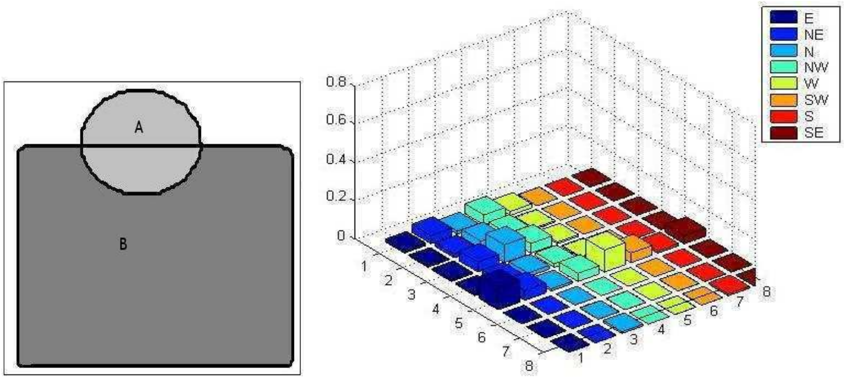
$$f_E = \sum_{\theta=0}^{\frac{\pi}{4}} \lceil A_2(\theta) \rfloor \times cos^2(2\theta) + \sum_{\theta=\frac{3\pi}{4}}^{\pi} \lceil A_1(\theta) \rfloor \times cos^2(2\theta) \tag{8.5}$$

Where $f, E, \lceil A_2(\theta) \rfloor$ represents respectively topological, directional and normalized Allen relation, A_2 is reorientation of A_1 and this information is represented in a matrix. An example in Fig. 8.4(a) and 8.4(b) for representing fuzzy spatial relations is illustrated.

This matrix represents the fuzzy spatial and directional relations for a two-dimensional case. An algorithm is proposed in [Salamat] for defuzzification of these spatial relations. This enables us to have a JEPD set of topological and directional relations and it is possible to represent these topological and directional relations in a neighborhood graph.

8.3.3 Conceptual Neighborhood Graph in CTD Method

A neighborhood graph represents continuous transition in the spatial relations. Neighborhood graph of spatial relations in CTD method describes all types of topological transformations, namely translation, uniform and non uniform expansion or contraction of an object. Here, for simplicity of graph only one branch of translation



(a) (b)

Figure 8.4: Object pair and its combined topological and directional relation information. Qualitative topological and directional relations are *(PO, N)* between this object pair

transformation is taken into account and possible transitions are presented into the neighborhood graph (Fig. 8.5). object can move in a *circular, straight* and *diagonal* path, called *directional*, *topological* and *topological and directional* neighborhood.

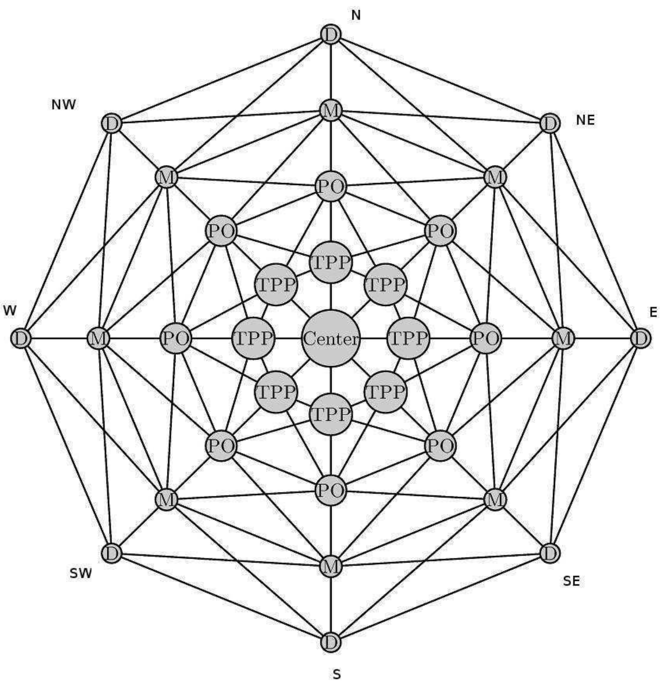


Figure 8.5: Neighborhood graph in the system of combined topological and directional relations

Neighborhood graph shows allowable transitions among the relations. These transitions are possible when the objects move or change occurs in a spatial scene. In

Fig. 8.5, it is shown that every node of a neighborhood graph has eight edges. Each node corresponds to (α, β), where α and β respectively represent the topological and directional relation. For better explanation let us consider the possible changes from the initial position of $(M(A,B), E(A,B))$ (called A, B *meets each other from East*)

$$(M(A,B), E(A,B)) \Rightarrow \begin{cases} (M(A,B), NE(A,B)) & \text{Change in directional relation} \\ (M(A,B), SE(A,B)) & \text{Change in directional relation} \\ (PO(A,B), NE(A,B)) & \text{Change in topological and directional relation} \\ (PO(A,B), E(A,B)) & \text{Change in topological relation} \\ (PO(A,B), SE(A,B)) & \text{Change in topological and directional relation} \\ (D(A,B), NE(A,B)) & \text{Change in topological and directional relation} \\ (D(A,B), E(A,B)) & \text{Change in topological relation} \\ (D(A,B), SE(A,B)) & \text{Change in topological and directional relation} \end{cases}$$

8.4 Spatial Predicates

Temporal changes of spatial objects induce modifications of their mutual topological and directional relations over time. Spatial predicates are divided into three categories namely topological, directional and distance predicates. Distance relations are inversely proportional to the directional relations. In the following subsections we discuss the predicates for topological and directional relations separately.

8.4.1 Topological Predicates

Topological predicates are the most investigated topic by researchers community. Method for topological predicates is discussed in [Egenhofer 1992] where a set of possible conceptual topological neighborhoods are defined. Topological predictions are based on the largely developed models for topological relations in this domain, most popular methods are the 9-intersections and Region Connection Calculus (RCC) [Egenhofer 1991, Randell 1992a]. In both methods eight topological relations are released and they are represented in neighborhood graph as in Fig. 8.6.

A topological change occurs if different topological relationship between two objects holds at time t_i and t_{i+1}. Neighborhood graph of topological relations depicts all physical transitions between spatial relations that can occur through the topological deformation. Topological predicates are represented in table 8.1.

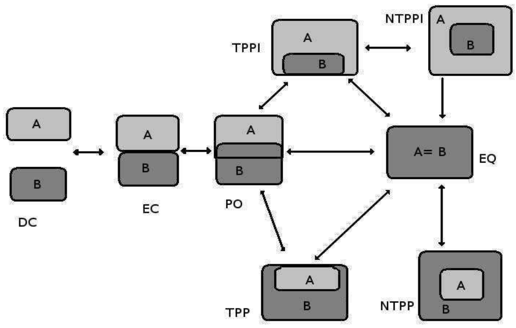


Figure 8.6: Neighborhood graph of spatial relations in 2D space

Topological Relation	Topological Prediction
D	D, EC
EC	D, EC PO
PO	EC, PO,TPP, TPPI, EQ
TPP	PO, TPP, NTPP
NTPP	NTPP, TPP
TPPI	PO,TPPI, NTPPI
NTPPI	NTPPI, TPPI
EQ	EQ, PO

Table 8.1: Topological Predictions

8.4.2 Directional Predicates

A method to describe the directional changes is developed in [Li 1997]. This method describes directional neighborhood of objects in matrix based system of directional relations and this method is used to annotate the trajectory of moving objects. Actually, this method discusses the directional predicates and has no concern with topological relations. Directional relations are not JEPD and no neighborhood graph is defined. Predictions in MBR based nine-directions systems are described in the following table 8.2. In this table, symbol O represents the neutral direction.

8.5 Spatio-Temporal Event Predicates

Motion events constitute the unbounded motion process and these events are bounded subsets in this process. These events change with the change into their temporal bounds. The spatio-temporal event predications help us to estimate the spatio-temporal event when the analysis interval is extended, these event predictions keep into account the history of spatio-temporal relation of moving object.

For defining the spatio-temporal relations and motion events, we need at least two snapshots or primitive intervals. A motion event for two snapshots is a subclass

Directional Relation	Directional Prediction
E	SE, E, NE,O
NE	E, NE, N, O
N	NE, N, NW , O
NW	N, NW, W,O
W	NW, W, SW, O
SW	W, SW, S, O
S	Sw, S, SE, O
SE	S, SE,E, O

Table 8.2: Directional predictions in eight directional relations system

of all those events which stands for more than two snapshots provided that these initial snapshots are included into the interval. For example, following spatio-temporal motion events.

- $Snap(XY,T) := seq\_ev(DC(XY,t_i), EC(XY,t_{i+1}))$
- $Touch(XY,T) := seq\_ev(DC(XY,t_i), EC(XY,t_{i+1}), DC(XY,t_{i+2}))$
- $Enter(XY,T) := seq\_ev(DC(XY,t_i), EC(XY,t_{i+1}), PO(XY,t_{i+2}), TPP(XY,t_{i+3}))$

where seq_ev represents a sequence event and $t_i,\ t_{i+1},\ t_{i+2},\ t_{i+3}\ \in\ T$ and $t_i <\ t_{i+1} <\ t_{i+2} <\ t_{i+3}$. All these primitive events hold in a sequence. This shows that $Touch \subseteq Enter$ and $Snap \subseteq Touch$. When the interval is extended, a new primitive event is added to the existing event, motion event may also change to its superclass.

8.5.1 Temporal Composition

Event types are considered as necessity and sufficient conditions for their occurrence. The composite events are detected when last primitive event occurs, it means composite event is detected at the end point of its occurrence interval. Commonly, there is a sequential composition of primitive events, concatenation of primitive events in time space. Motion event predicates are the process of concatenation of instantaneous events in time space with existing motion events. Time can be handled by two ways, logical combination and time as a process.

Logical Combination: In this method, logical combination of operators are used. The properties *holds, holds_ at, occurs, occurs_ at* are used along with the sequence operators *seq_ eve* and property *In.*

- *Holds(P,i)*: Property P holds during interval i.
- *Holds_ at(P,t)*: Property P holds at time point t.
- *Occurs(E,i)*: Event E occurs during interval i.

- *Occurs_ at(E,i)*: Event E occurs at time instant t.
- *Seq_eve*(E_1, E_2, i): Event E_1 occurs before event E_2 during interval i.
- *Occurs_in*$(E, i) \exists t(t_1 < t < t_2) \wedge$ *occurs*$(E, [t_1 t])$: Event E occurs in interval i.

when the temporal interval is extended to the next point, existing property turns into the *In* property.

Time as Process: M. Worboys in [Worboys 2005] introduced the process-oriented model for time. Ticking of a clock is a real world process linked to the continuous time. Time is considered as a collection of sequenced tick process, where two continuous ticks $(tick_i, tick_{(i+1)})$ are joined through the link $next_{i(i+1)}$ (Eq. 8.1). Moving objects are considered as the temporally referenced identity. The spatio-temporal relations between moving objects are corresponding to the ticks process and each primitive event represents a tick at a particular snapshot and the joint $next_{i(i+1)}$ stands for the change of spatial relations between two consecutive snapshots.

In the following examples, logic concatenation of primitive spatial relation is used. The properties *Holds, Holds_ at, occurs, occurs_ at* and their relationship are used, such as $holds(P, t_i \cup t_{i+1}) = \text{holds-at}(P, t_i) \wedge \text{holds-at}(P, t_{i+1})$ (A property P holds during the interval $[t_i, t_{i+1}]$ $\forall$ $i = 1, 2, ...$ when the same property holds at each point of an interval.

8.5.2 Spatio-Temporal Motion Event Predicates

Spatio-temporal predictions about the motion events take place when objects are continuously changing their positions and temporal component is added to spatial relations. One end of the time space is closed and other end is open, as time passes, a new point is added to the existing interval. The spatial relations at the new time points, which follows the static predictions described in tables 8.1 and 8.2 are also added to the existing spatio-temporal relation or motion event.

A single method could not be used to adequately predicate the spatio-temporal motion events. This prediction may hold in topological, directional or in combined properties of the space. In this paper, we use the CTD method, which enables us to know binary topological and directional relations simultaneously. This method represents fuzzy topological and directional relations. These relations are defuzzified and represented in a neighborhood graph as described in Section 8.3.3.

In our method, we combine the information provided from topological and directional relations. A single method works in each situation with some additional information. The method holds the object's spatio-temporal history and change the linguistic description of a motion event as soon as object's new position and spatial relations are added to the history of spatial relations. This logical addition in time domain can be represented formally by one of the subsequent relations.

$$STR_i = STR_{i-1} \cup \ SR_i, \forall\, i = 2, 3, ... \tag{8.6}$$

This equation can be rewritten as

$$STR_i = STR_{i-1} \cup SP_{i-1}, \forall\, i = 2, 3, ... \tag{8.7}$$

where STR, SR and SP respectively stands for spatio-temporal relations, spatial relation and spatial prediction. This represents direction, topological or both relations and i denotes the time instant at which the relations are evaluated. Spatio-temporal predictions can be formulated by

$$STP_i = STR_{i-1} \cup \ SP_i, \forall\, i = 2, 3, ... \tag{8.8}$$

where STP represents the spatio-temporal prediction. All these equations provide us mathematical formulas that how the temporal aggregation of different primitive events effects the initial event.

8.6 Method Explanation

Motion is a continuous phenomenon and it is analyzed at discrete time points, between these time points it can be directly interpolated. These time points can be chosen at a predefined time intervals or a specific snapshot of a video frame. In modeling the spatio-temporal relations and motion events, we need at least two snapshots. Spatio-temporal relations are defined based on stable and unstable topological relations. We start from two snapshots, at initial point, both spatio-temporal and spatial predictions represent the same semantics. All the possible predicates for motion events are given in table 8.3 where initially argument object lies in *North* with *disjoint* topological relation.

SR_1	SP_1	STR_2orSTP_1
(D,N)	(D,NE)	Changing direction from N to NE
(D,N)	(D,NE)	getting closer or going away from N to NE
(D,N)	(D,N)	getting closer or going away from N
(D,N)	(D,NW)	Changing direction from N to NW
(D,N)	(D,NW)	getting closer or going away from N to NW
(D,N)	(M,NE)	(Snap NE)
(D,N)	(M,N)	(snap, N)
(D,N)	(M,NW)	(Snap, NW)

Table 8.3: Spatio-temporal predictions for two frames

When the interval is extended to one snapshot, where the spatio-temporal relation is not changed. Every point has eight possible spatial predictions. We discuss here the possible spatial predictions from (M, NE). Sequence of existing spatial relations is (D, N, t_i), (M, NE, t_{i+1}) and existing spatio-temporal relation at this point is *Snap from NE*. Spatio-temporal predicates are depicted in table 8.4 when moving objects have the history of spatio-temporal relations as $SNAP$. These spatio-temporal predicates depend upon the current spatial predicates.

STR_{i-1}	SP_{i-1}	STR_iorSTP_{i-1}
Snap from NE	(D, E)	Touching form NE
Snap from NE	(M, E)	Bypass
Snap from NE	(PO, E)	Graze, Enter, Into from E
Snap from NE	(D, NE)	Touch from NE
Snap from NE	(PO, NE)	Enter, Into from NE
Snap from NE	(D, N)	Touching from NE
Snap from NE	(M, N)	Bypass towards N
Snap from NE	(PO, N)	Graze, Enter, Into from N

Table 8.4: Spatio-temporal predictions for 3^{rd} snapshot

Similarly if we extend current interval to the next time point. The history of spatio-temporal relation is $(D, N,\ t_i), (M, N,\ t_{i+1}), (PO, N,\ t_{i+2})$. The current spatial prediction is (TPP, N), then possible spatio-temporal motion event predictions are depicted in table 8.5.

STR_{i-1}	SP_{i-1}	STR_iorSTP_{i-1}
Entering from N	(PO, N)	Entering from N
Entering from N	(PO, NE)	Graze from N towards NE
Entering from N	(PO,NW)	Graze from N towards NW
Entering from N	(M, NW)	Graze from N towards NW
Entering from N	(M, N)	Move backward
Entering from N	(M,NE)	Graze from N towards NE
Entering from N	(TPP,N)	Entered from N
Entering from N	(TPP, NW)	Entered from NW
Entering from N	(TPP, NE)	Entered from NE

Table 8.5: Spatio-temporal predictions for 4^{th} frame

8.7 Example

For the experimental purpose, we analyze an image sequence of traffic scene on a road crossing point($http : //www.youtube.com/watch?v = 6prp1J1RcwU\&feature = related$). In this sequence we observe that vehicles have many sorts of spatio-temporal relations with each other. For modeling the spatio-temporal relations and video events point of view, we consider the junction point as a reference object and a moving vehicle is considered as argument object. Here two types of spatio-temporal relations (motion events) are possible, U-turn and junction crossing of a vehicle.

Both the relations have similar initial and final conditions. In such a case only directional relation at each instant plays an important role for a specific linguistic motion verb. Addition of a directional relation for modeling the motion verb may help to understand the actual situation and point out where a specific event happen in space. As a test case, we may consider the different frames of the video and observe the movement of a white car. In figure it is mentioned in a blue rectangle. Visually this car has a different motion direction, on can predict that this car is taking a U-turn at frame mentioned in Fig. 8.7(e). To avoid the segmentation problems we draw the sketches of each snapshot with two objects and consider their binary spatial relations theoretically for each snapshot in a sequence.

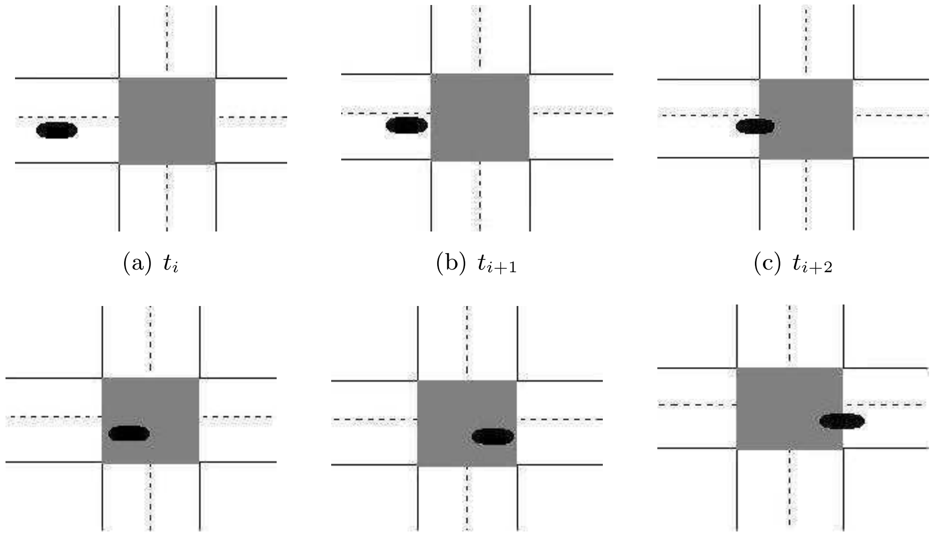
(a) Frame 0700 (b) Frame 0721 (c) Frame 0742

(d) Frame 770 (e) Frame 792 (f) Frame 0811

Figure 8.7: Description of different frames of a road junction scene image sequence

In this example, we consider two different scenarios. A car is approaching to a road junction. In first scenario, car crosses the road junction and in second case, it takes a U turn . In this example, the crossing point of roads (square) is considered as the reference object (object B) and a moving object (car is considered as an argument object A).

First consider junction crossing scenario, after the segmentation stage, object positions are demonstrated in figures from 8.8(a) to 8.8(f). In this case, images in the sequence are in such a way that time points are $t_i < t_{i+1} < t_{i+2} < t_{i+3} < t_{i+4} < t_{i+5}$. For image at point t_i, it spatial relations are written as (D, W, t_i). Prediction for spatial relation is (M, W, t_{i+1}) (no other possibility due to the domain knowledge, car can't in reverse direction on road network). Motion event prediction becomes *Snap* from *West*, with the addition of this image, *Snap* is completed and spatio-temporal prediction turn to all those events which have same starting point. Motion event prediction predicts the pos-



(a) t_i (b) t_{i+1} (c) t_{i+2}

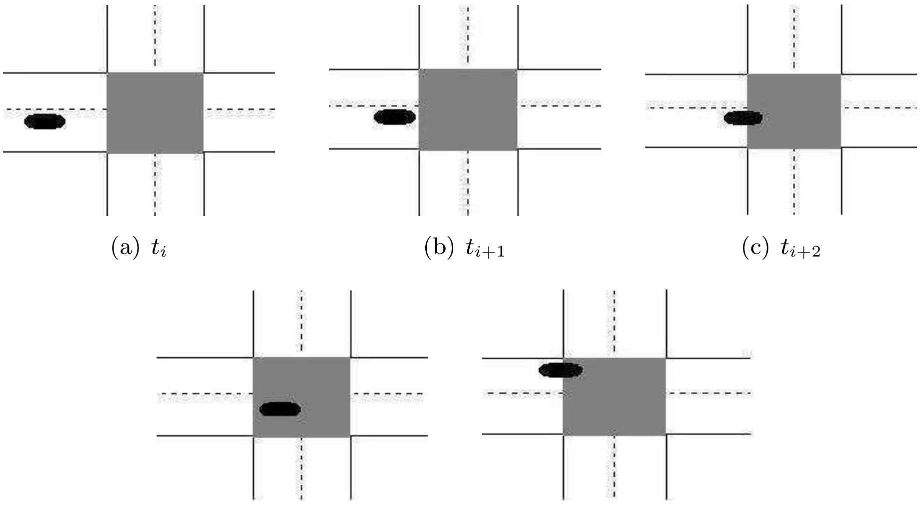
(d) t_{i+3} (e) t_{i+4} (f) t_{i+5}

Figure 8.8: Car, crossing a road junction

sible occurrence of spatio-temporal relation of *Touch, Entering* for the third snapshot. At this stage, spatial predictions for t_{i+2} are $(PO, W), (PO, NW), (PO, SW), (M, NW), (M, SW), (D, W), (D, NW), (D, SW)$. It is the domain knowledge and motion direction which adds the constraints. These constraints omit certain spatial and spatio-temporal prediction *Touch*.

This continues for the third frame and spatial relation for this snapshot is $(PO,\ W, t_{i+2})$. Spatio-temporal relation changes from *Snap* to *Entering* or *Graze* from *West*. This sequence of images continues and the next spatial relation is $(TPP,\ W, t_{i+3})$, spatio-temporal relation becomes *Entered* from *West*. There are different spatial predictions, for t_{i+4} spatial relation is $(TPP,\ E, t_{i+4})$. Spatial relation for t_{i+5} is $(PO,\ E, t_{i+5})$. This shows that the spatio-temporal relation is Crossing the road junction and directional relations add the knowledge that car continue its motion direction in the *East* direction (Car crosses the junction from *West* towards *East*). This shows that some data is missing here, the continuity of spatial relations also show that there is some data missing, such that, there exist a $t_{i+3} < t < t_{i+4}$, where $(NTPP, t)$.

Now we consider scenario of U turn, after the segmentation stage scenario looks like mentioned Figs. 8.9(a) to 8.9(e). In this case, first four images remains in the same sequence and spatio-temporal motion event prediction also same. After this, if the sequence of images is changed and the next image becomes as showed in Fig. 8.9(e)(some data is missing, object can't move directly to that point). Spatial relation for this image is $(PO,\ W, t_5)$, spatial predictions for this time point are $(PO, NW), (PO, SW), (M, SW), (M, W)$ and (PO, NW). Some predictions are discarded due to domain knowledge. When the prediction (M, W) is added to existing spatio-temporal spatial relation or motion event. It turns to the cross, but



(a) t_i (b) t_{i+1} (c) t_{i+2}

(d) t_{i+3} (e) t_{i+4}

Figure 8.9: Car, U turn at road junction

knowledge of direction relations describe this particular cross relation as a U turn of car. Directional relations at initial and final frames also specify the location of the initial and possibly final location of car in spatial scene. It differentiate the particular car taking U-turns from other vehicles taking U-turns from other directions.

8.8 Conclusion

Spatio-temporal reasoning and prediction are important fields in Artificial Intelligence. Existence of topological and directional predicates in a sequential order constitutes the motion events. Motion event prediction is a similar process to the spatial predictions. Events are bounded in time, they constitute the process. Language semantics of an event also change with change in temporal interval.

In this paper we develop a method for motion event predictions. This method holds the history of a moving object and predict the occurrence of a motion event. Time is a continue process and motion events are bounded into time process. A snapshot or primitive interval is added to the existing sequence of images, when the temporal interval is extended. The spatial relations for new snapshot are also added to the history, related spatio-temporal relation and motion event may also change. Hopefully this work will help in modeling spatio-temporal queries in moving object database, modeling the natural language processing and some related research areas like medical images for generating automatic medical reports, remote sensing images etc.

Chapter 9

Conclusion and Perspectives

This chapter concludes the thesis work and discuss some future aspects of our work. In spatio-temporal applications, snapshot data model is commonly used for representing spatial data by time-stamping layers. We used this technique and develop the method for spatio-temporal relations modeling and motion events. Some main features of our work are discussed in section 9.1 and some perspectives of the work is elaborated in section 9.2.

9.1 Conclusion

Spatio-temporal relations are extensively used in Geographic Information Sciences. For modeling spatio-temporal relations, spatial relations are extended to the temporal dimension. We have to deal the topological, directional and distance relations for each snapshot. Distance relations are inversely proportional to the directional relations, hence these relations are ignored. For modeling spatio-temporal relations we use combined topological and directional relations information method. This method provides us information that where a topological relation holds in space.

This method uses Allen temporal relations for modeling $2D$ topological relations. Initially this method has a high computation time which is due to object approximation and an algorithm for fuzzification of segments of a longitudinal section. we work for computation time of the method and decrease computational time from $O(nM\sqrt{M})$ to $O(nNlog(N))$ where M, N, n respectively represents the number of pixels to be treated, vertices of polygons and directions. Histogram representation of the method could not be used to infer and visualize the change in topological and directional relations. We change representation of fuzzy spatial relations and develop an algorithm for defuzzification of spatial relations (Chapters 4,5). This defuzzification provides us answer to the question that *where in space a topological relation holds?* This defuzzification also provide us a JEPD set of spatial relations and these relations are represented in a neighborhood graph. This method also decrease the computation time as information are obtained by using orientation angle $\theta \in [0,\ \pi]$.

Combined topological and directional relations (CTD) method is extended to time dimension for developing the spatio-temporal relations. These relations are based on stability property in topology and we define some spatio-temporal motion events for unstable spatio-temporal relations. Some motion events are developed and locative symmetry in motion events is removed by introducing the directional constraints in unstable spatio-temporal topological relations. A reasoning method with topological and directional relations is developed based on Combined topological and directional relations method. Commonly reasoning is performed by constructing the composition tables for spatial relations, a mathematical formula is developed which expresses the relation between entities of composition table for topological relations. This table is divided into sub-tables and entities of table are rearranged. These tables represent the coarse knowledge and this is improved by gradually introducing the directional relations constraints in composition tables for topological relations and vice versa. This newly introduced method is also used for prediction of motion events. This approach resembles to the topological or directional relations prediction or reasoning. This method stores the history of a moving object. In this approach, it is considered that if an analysis interval is extended to the next snapshot or primitive interval how the spatio-temporal motion event changes.

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9.2 Perspectives

Nowadays, Geographic Information Systems (GISs) are combined with expert system, decision support system or other methods in AI, called Enhanced Geographic Information Systems (EGISs) or Intelligent GISs. This is a hybrid information system which deals with complicated applications of GISs and widely used in many areas, such as in agriculture, forestry, ecosystem, traffic, transportation, environmental protection and public health. For problem solving methodologies, appropriate knowledge based intelligent systems (GISs) are designed for a particular problems. Solution to problem made by a human or a computer, the question is that what is the role of GIS-based system? Is it a management tool or intelligent decision maker? Our work is extendable in both dimensions, in future I would like to extend this work in the following directions.

1. **Fusion of CTD with Hidden Morkov Model (HMM) for Motion Event Predictions:** Qualitative spatio-temporal representation and reasoning models represent objects as abstract entities. Combined Topological and Directional (CTD) relations method after the defuzzification behaves like a qualitative model of spatial relations. These spatial relations are represented into a conceptual neighborhood graph. HMM represents the random process and used in applications concerned to prediction and recognition, qualitative HMM can be used for the discrete time along with the qualitative spatial relations to model the predictions for consequent spatial relations and motion events. We shall use CTD method as fuzzy qualitative relations method along with the qualitative probabilities and a qualitative HMM model for spatio-temporal reasoning in our future works.

2. **Image Interpretation and Computing With Words:** Commonly known approaches for image understanding and image interpretation algorithms start

after the segmentation processes. Spatial relations are used in different fields of image understanding, image analysis, knowledge representation techniques etc. Similarly spatio-temporal relations are used for trajectory annotation of moving objects, video understanding, video analysis etc. In modern ages, data is in digital format and natural language descriptions is created from numerical data for interaction between machine and user.

Computing With Words (CWW) is a reverse process of spatial reasoning where a language or locative expressions are developed to express a situation. This process consist of many modules, in encoder module, verbs are developed using the images. Fuzzy sets and fuzzy spatial relations are used for developing the adverbs like almost, probably, definitively etc. This conceptual frame work will represent, manipulate, measure and human machine interaction in natural language semantics of images about topological and directional information. In the design of CWW encoder based on spatial relations, these spatial relations provide information about binary topological and directional relations between image objects.

We will extend our method to develop such a language, where fuzzy predicates will be used for locative expressions. This can be used for automatic image and video interpretation, automatic medical image interpretations for example ultrasound image interpretations, based on spatial relations.

3. **Fusion With other Aspects:** Spatio-temporal reasoning supports the spatial and temporal attributes of moving objects, certain spatio-temporal functions are developed which deals with both attributes simultaneously. This involves many applications in social sciences such as behavior analysis, fraud detection etc. We will use our method for interpretation of short and long term behavior analysis between image and video objects in an environment.

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Chapter 10

Annexes

10.1 Annex-A

Now let's consider that the following situations arise for the segments of a longitudinal section when the objects are concave or objects have disconnected boundary.

(a) (b)

Figure 10.1: Different positions of reference object's segments in case of longitudinal sections

Figure 10.1(a) We discuss here the cases arising in figure 10.1(a) with all aggregation operators. For the above cited examples, we use terms $f(x_1, y_1, z)$ and $f(x_2, y_2, z)$ to express histograms of fuzzy Allen relations for first and second segments, here x_1 is the length of first segment of the argument object and x_2 is the length of second segment, similarly y_1 is the difference between the first segment of argument object and the reference object and y_2 is the difference between the second segment of argument object and the reference object, in this case consider both $y_1 > \frac{a_1}{2}$ and $y_2 > \frac{a_2}{2}$ where $a_1 = min(x_1, z)$ and $a_2 = min(x_2, z)$.

$$\begin{aligned} f(x_1, y_1, z) &= (\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ )^t \\ f(x_2, y_2, z) &= (\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ )^t \end{aligned}$$

It means $f_>(I_1, J) = 1$ and $f_>(I_2, J) = 1$ and all the other values of histogram are zero. The possible outcomes with the application of different fuzzy operators are

$$\begin{aligned} F_{OR}(\theta, A_\theta(v), B_\theta(v) &= max(f(x_1, y_1, z), f(x_2, y_2, z)) \\ &= (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t \end{aligned}$$

$$F_{AND}(\theta, A_\theta(v), B_\theta(v) = min(f(x_1, y_1, z), f(x_2, y_2, z)) \\ = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t$$

$$F_{PROD}(\theta, A_\theta(v), B_\theta(v) = f(x_1, y_1, z) \times f(x_2, y_2, z) \\ = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t$$

$$F_{SUM}(\theta, A_\theta(v), B_\theta(v) = 1 - ((1 - f(x_1, y_1, z)) \times (1 - f(x_2, y_2, z))) \\ = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t$$

In this case, the two segments of argument object have the same Allen relations with the reference segment. Both segments behave like the crisp Allen relation. In such a case, all fuzzy operator provide us similar information.

Figure 10.1(b) The cases arising in figure 10.1(b) with all aggregation operators. In this case consider $y_1 < -b - \frac{3a_1}{2}$ and $y_2 > \frac{a_2}{2}$ where $a_1 = min(x_1, z)$, $b_1 = max(x_1, z)$ and $a_2 = min(x_2, z)$ then $f(x_1, y_1, z)$ and $f(x_2, y_2, z)$ are used to express the histograms of fuzzy Allen relations for first and second segments.

$$\begin{array}{lcl} f(x_1, y_1, z) & = & (\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ )^t \\ f(x_2, y_2, z) & = & (\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ )^t \end{array}$$

The possible outcomes are

$$\begin{array}{lcl} F_{OR}(\theta, A_\theta(v), B_\theta(v) & = & max(f(x_1, y_1, z), f(x_2, y_2, z)) \\ & = & (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1)^t \end{array}$$

$$\begin{array}{lcl} F_{AND}(\theta, A_\theta(v), B_\theta(v)) & = & min(f(x_1, y_1, z), f(x_2, y_2, z)) \\ & = & (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t \end{array}$$

$$\begin{array}{lcl} F_{PROD}(\theta, A_\theta(v), B_\theta(v)) & = & f(x_1, y_1, z) \times f(x_2, y_2, z) \\ & = & (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t \end{array}$$

$$\begin{array}{lcl} F_{SUM}(\theta, A_\theta(v), B_\theta(v)) & = & 1 - ((1 - f(x_1, y_1, z)) \times (1 - f(x_2, y_2, z))) \\ & = & (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1)^t \end{array}$$

The above cited examples explain that, in a particular situation, *AND* and *PROD* operators cannot be used for the decision making process, when both segments of one longitudinal section of argument object have the same Allen relation with the segment of reference object, all fuzzy operators have the same results. In real situation different cases may arise and segments may have opposite relation as in figure 10.1(b). In case of conjunction operators, all the information may be lost.

These results show that fuzzy conjunction operators give results counter intuitive (both *AND, PROD* represents the conjunction operators) and the disjunction operators better fits the human intuition and provides here a better fusion of available fuzzy information. The third type of fuzzy operators such as Fuzzy γ operator can also be used to make possible contributions of two fuzzy values, but in this case finding compensation values of γ is a problem and for each case we have to adjust γ.

Concave objects or objects with holes have several segments, called longitudinal section. In such a case, fuzzy Allen relations are calculated for each segment separately then fuzzy connectors are applied for combining information from both segments.

10.2 Annex-B

Histogram of Allen relations is defined as *"Area of subregions of object A and B in a particular direction having a relation $r(I,J)$ where r is any Allen relation"*. Proof of affine transformations can be divided into three parts

1. Effect on change in direction of oriented line.
2. Change in fuzzy membership value
3. Effect of affine transformations on object area

10.2.1 Change in Direction

These are the histogram of forces coupled to the Allen relations and F_r^{AB} represents the histogram of fuzzy Allen relations in direction θ. Concerning to the change in direction of oriented line all the proofs are similar as in case of force histogram [Matsakis 1998].

10.2.2 Change on Object Area

It is considered that an object is a polygon, if A_k is a transformation matrix of affine transformation T then its effect on polygon Δ is represented by [Dionisio 2006]: $Area(T(\Delta)) = |A_k| Area(\Delta)$ Now we calculate effect on area of each object due to transformation. Translation is not a affine transformation hence proof for translation is omitted, it has no effect on object area.

- **Scale:** Determinant for a matrix of $2d$ affine transformation for scale is: $\begin{vmatrix} k & 0 \\ 0 & k \end{vmatrix} = k^2$ Object area is scaled by k^2 times where k is scale ratio.
- **Rotate:** Determinant for a matrix of $2d$ affine transformation for rotate is:

  $$\begin{vmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{vmatrix} = cos^2(\theta) + sin^2(\theta) = 1$$
  Determinant has value is 1. Rotation doesn't change the object area.
- **Shear:** Determinant for a matrix of $2d$ affine transformation for shear is:

  $$\begin{vmatrix} 1 & 0 \\ 0 & k \end{vmatrix} = k$$
  Object area is sheared by k times where k is shear ratio.
- **Reflection:** Determinant for a matrix of $2d$ affine transformation for reflection is:

  $$\begin{vmatrix} cos(2\theta) & sin(2\theta) \\ sin(2\theta) & -cos(2\theta) \end{vmatrix} = -(cos^2(2\theta) + sin^2(2\theta)) \Rightarrow -1$$
  Its absolute value is 1 so it will not change object area.

10.2.3 Effect on Fuzzy Membership Value

Grades of fuzzy membership value depends upon triplet (x, y, z). These are the projective distance between two points, so we calculate effect on each point. Let (x_1, x_2) be coordinates of a points before transformation and (x_1', x_2') are coordinates of a $2d$ point after transformation.

- **Scale:** Effect of scale transformation on a projection of a segment:

$$\begin{aligned} x_1' &= \left(\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right)^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} kx_1 \\ kx_2 \end{pmatrix}^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= kx_1 \end{aligned}$$

  A point after the scale transformation changes k times similarly a segment will also change k times. Now we calculate its effect on a fuzzy membership function. $x' = kx$ where x' is length of segment after scale transformation and x is length of segment before scale transformation.

$$\begin{aligned} \mu_{(\alpha',\beta',\gamma',\delta')}(y') &= max(min(\tfrac{y'-\alpha'}{\beta'-\alpha'}, 1, \tfrac{\delta'-y'}{\delta'-\gamma'}), 0) \\ &= max(min(\tfrac{ky-k\alpha}{k\beta-k\alpha}, 1, \tfrac{k\delta-ky}{k\delta-k\gamma}), 0) \\ &= max(min(\tfrac{y-\alpha}{\beta-\alpha}, 1, \tfrac{\delta-y}{\delta-\gamma}), 0) \\ &= \mu_{(\alpha,\beta,\gamma,\delta)}(y) \end{aligned}$$

  This result shows that scale transformations does not change the value of a fuzzy membership function. This is due to proportional change in value of triplet (x, y, z).

- **Rotate:** Effect of rotation on a projection of a segment :

$$\begin{aligned} x_1' &= \left(\begin{pmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right)^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} cos(\theta)x_1 - sin(\theta)x_2 \\ sin(\theta)x_1 + cos(\theta)x_2 \end{pmatrix}^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= cos(\theta)x_1 - sin(\theta)x_2 \end{aligned}$$

  A point after the rotate transformation changes $x' = cos(\theta)x_1 - sin(\theta)x_2$ which can be rewritten as $x' = k_1x_1 - k_2x_2$
  where (x_1, x_2) is segment length along x and y axis respectively and k_2x_2 is the projection of $x' - x$ on x axis. This formula (ratio formula in analytic geometry) shows that the length of a segment after the transformation divides the original segment in a particular ratio called k_1, k_2. we can write it as:

$$x' = (k_1 - k_2)x = kx \; .$$

  Where $k = k_1 - k_2$ and x' is segment length after transformation and x is segment length before transformation. Now we calculate its effect on a fuzzy membership function.

$$\begin{aligned} \mu_{(\alpha',\beta',\gamma',\delta')}(y') &= max(min(\tfrac{y'-\alpha'}{\beta'-\alpha'},1,\tfrac{\delta'-y'}{\delta'-\gamma'}),0) \\ &= max(min(\tfrac{ky-k\alpha}{k\beta-k\alpha},1,\tfrac{k\delta-ky}{k\delta-k\gamma}),0) \\ &= max(min(\tfrac{y-\alpha}{\beta-\alpha},1,\tfrac{\delta-y}{\delta-\gamma}),0) \\ &= \mu_{(\alpha,\beta,\gamma,\delta)}(y) \end{aligned} .$$

This proves that rotation does not change the value of a fuzzy membership function. This is due to proportional change in value of triplet (x, y, z).

- **Shear:** Effect of shear on a projection of a segment:

$$\begin{aligned} x_1' &= \left(\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right)^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} x_1 \\ kx_2 \end{pmatrix}^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= x_1 \end{aligned}$$

A point after the shear transformation does not change. Now we calculate its effect on a fuzzy membership function. $x' = (x_2^2 - x_1^1) \Rightarrow x$ where x' is length of segment after shear transformation and x is length of segment before shear transformation.

$$\begin{aligned} \mu_{(\alpha',\beta',\gamma',\delta')}(y') &= max(min(\tfrac{y'-\alpha'}{\beta'-\alpha'},1,\tfrac{\delta-y'}{\delta'-\gamma'}),0) \\ &= max(min(\tfrac{y-\alpha}{\beta-\alpha},1,\tfrac{\delta-y}{\delta-k\gamma}),0) \\ &= \mu_{(\alpha,\beta,\gamma,\delta)}(y) \end{aligned}$$

This completes the proof for shear transformation.

- **Reflection:** Effect of reflection on a projection of a segment :

$$\begin{aligned} x_1' &= \left(\begin{pmatrix} cos(2\theta) & sin(2\theta) \\ sin(2\theta) & -cos(2\theta) \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right)^t \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} cos(2\theta)x_1 + sin(2\theta)x_2 \\ sin(2\theta)x_1 - cos(2\theta)x_2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ &= cos(2\theta)x_1 + sin(2\theta)x_2 \end{aligned}$$

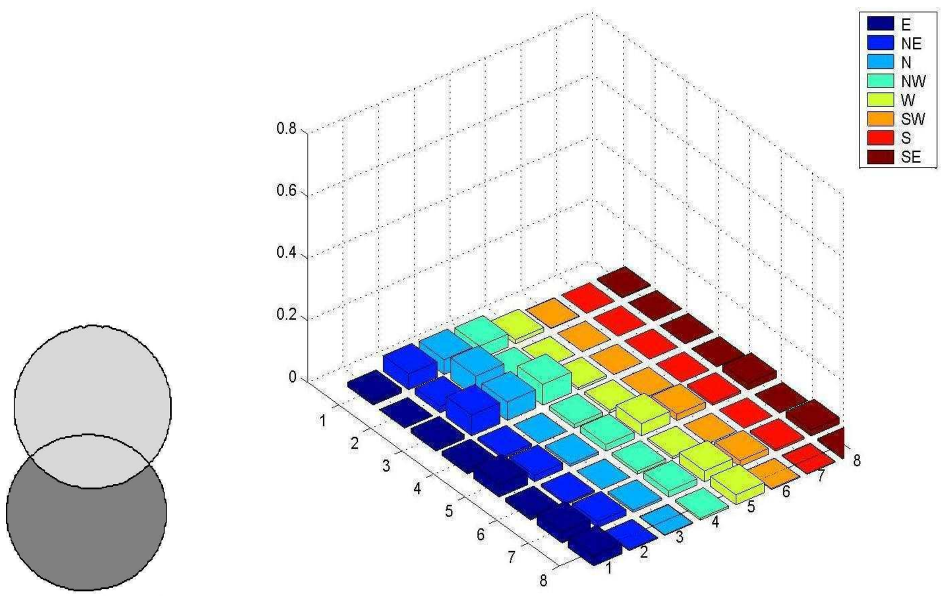
A point after the reflection transformation changes $x' = cos(2\theta)x_1 - sin(2\theta)x_2$ which can be rewritten as: $x' = k_1x_1 - k_2x_2$ where (x_1, x_2) is segment length along x and y axis respectively and k_2x_2 is the projection of $x' - x$ on x axis. This formula (ratio formula in analytic geometry) shows that the length of a segment after the transformation divides the original segment in a particular ratio called k_1, k_2. we can write it as: $x' = (k_1 - k_2)x = kx$
Where $k = k_1 - k_2$ and x is segment length before transformation and x' is segment length after transformations. Now we calculate its effect on a fuzzy membership function.

$$\begin{aligned} \mu_{(\alpha',\beta',\gamma',\delta')}(y') &= max(min(\tfrac{y'-\alpha'}{\beta'-\alpha'},1,\tfrac{\delta'-y'}{\delta'-\gamma'}),0) \\ &= max(min(\tfrac{ky-k\alpha}{k\beta-k\alpha},1,\tfrac{k\delta-ky}{k\delta-k\gamma}),0) \\ &= max(min(\tfrac{y-\alpha}{\beta-\alpha},1,\tfrac{\delta-y}{\delta-\gamma}),0) \\ &= \mu_{(\alpha,\beta,\gamma,\delta)}(y) \end{aligned}$$

This proves that reflection does not change the value of a fuzzy membership function. This is due to proportional change in value of triplet (x, y, z).

10.3 Annex-C

For the better visualization, we elaborate the spatial relations and color attribution in following example 10.2. We consider overlapping objects and their spatial relations. Object A (light gray) as argument object and object B (dark gray) as the reference object. Here colors show the directional relations and lines show the topological contents. These relations are arranged in order as $\{D, M, PO, TPP, NTPP, TPPI, NTPPI, EQ\}$ with meanings *Disjoint, Meet, Partially-Overlap, Tangent-Proper-Part, Non-Tangent-Proper-Part, Tangent-Proper-Part-Inverse, Non-Tangent-Proper-Part-Inverse* and *Equal.* value of an entity shows the percentage area of objects under a specific topological relation in a given direction. i.e., $\sum_{i,j=1}^{8} = 1$. The object pair is shown in figure 10.2(a) and its spatial relations are shown in figure 10.2(b).



(a) Object Pair (b) Spatial Relations

Figure 10.2: Overlaping objects and their combined topological and directional relations representation

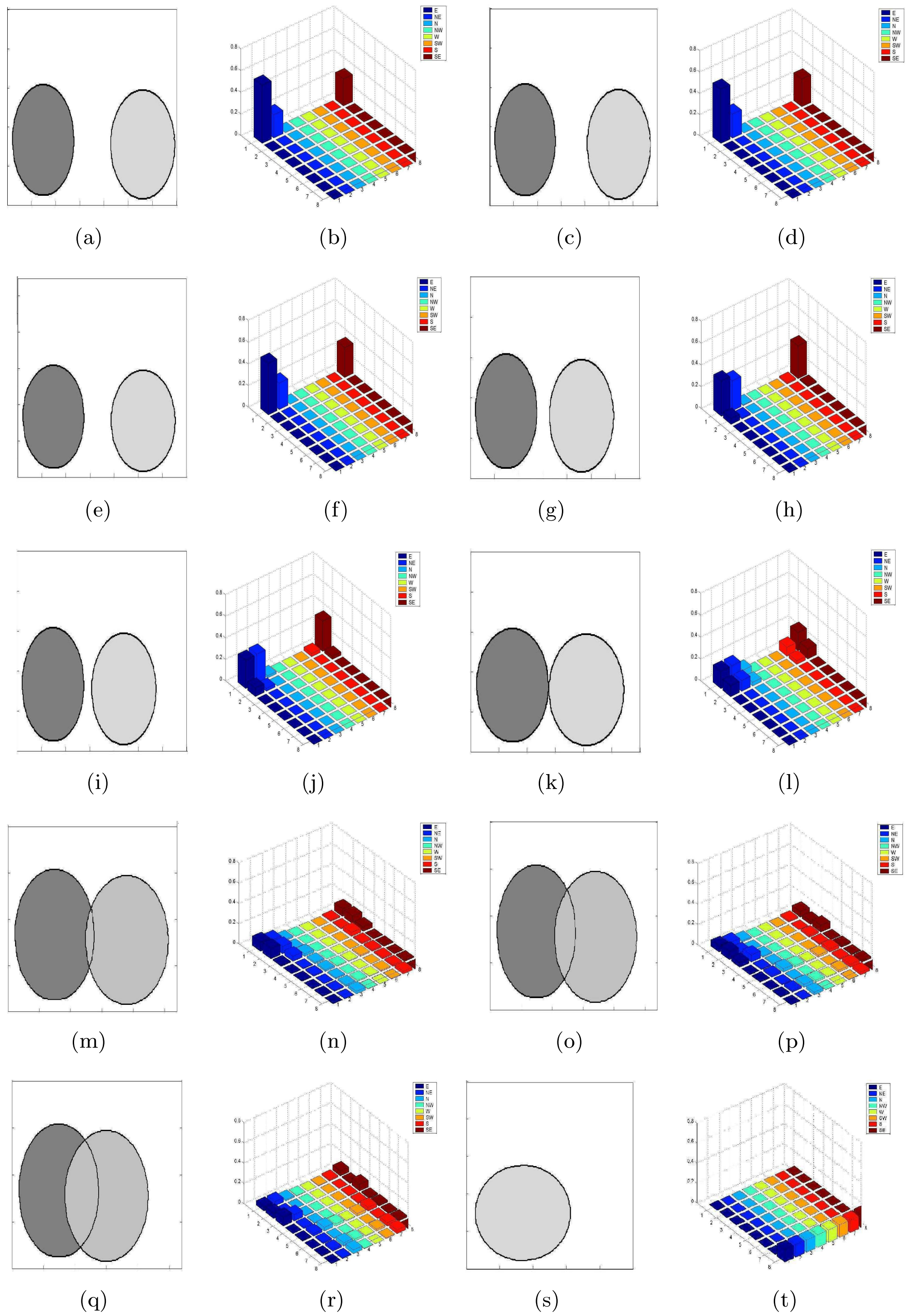


Figure 10.3: Objects at change positions from disjoint to equal topological relation, spatio-temporal *Merge* relation from east direction and their graphical representation

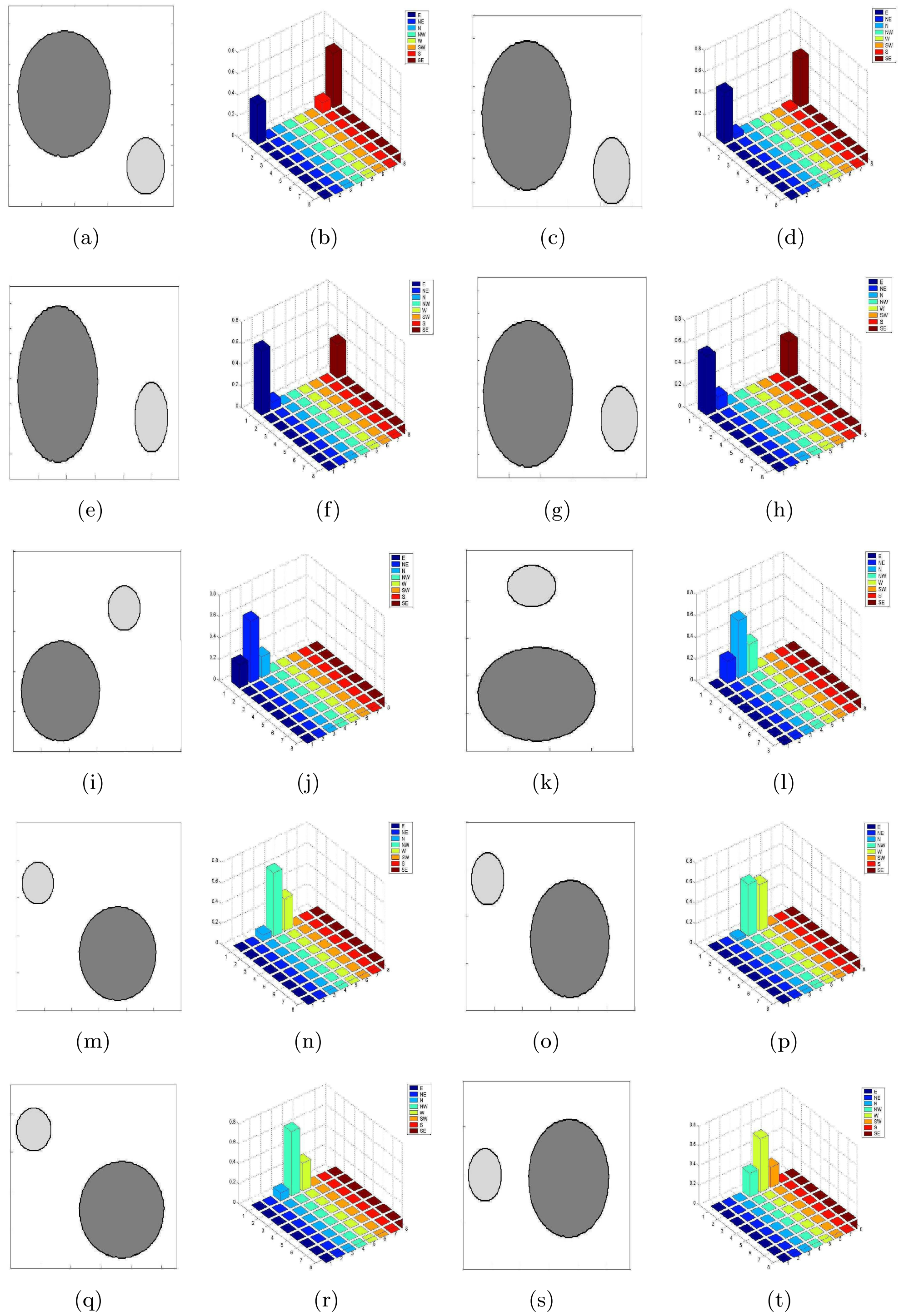


Figure 10.4: Description of object pair changing directional relations and their graphical representation

Hands are levers of influence on the world that made intelligence worth having. Precision hands and precision intelligence co-evolved in the human lineage and the fossil record shows that hands led the way.

Steven Pinker, in How the Mind Works